ESSAYS ON MARKET FRICTIONS, ECONOMIC SHOCKS, AND BUSINESS FLUCTUATIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the
Graduate School of The Ohio State University

By
Seungho Nah, M.A.
Graduate Program in Economics

* * * * *

The Ohio State University

2010

Dissertation Committee:
Bill Dupor, Adviser
Paul Evans
Pok-sang Lam
ABSTRACT

My dissertation investigates what types of shocks cause business fluctuations and what roles the frictions existing in an economy are playing in propagating the shocks. Specifically, I focus on the propagation mechanism of the shocks to (i) agents’ expectation and (ii) frictions or uncertainty in financial or goods markets, all of which have gradually been the center of attention in the recent business cycle literature.

In the first essay, “Financial Frictions, Intersectoral Adjustment Costs, and News-Driven Business Cycles”, I show that an RBC model with financial frictions and intersectoral adjustment costs can generate sizable boom-bust cycles and plausible responses of stock prices in response to a news shock. Booms in the labor market, which make it possible for both consumption and investment to increase in response to positive news, are caused through two channels: the increases in value of marginal product of labor and the increases in value of collateral. Both of these channels enable firms to hire more workers. Intersectoral adjustment costs contribute to both channels by increasing the relative price of output and capital during expansions. Financial frictions enter in the forms of collateral constraints on firms, which influence the latter channel, and the financial accelerator mechanism driven by agency costs, which amplifies all the key variables. My model differs from previous studies in its ability to generate boom-bust cycles without restricting the functional form of
consumption in household preferences and without requiring investment adjustment costs, variable capital utilization, or any nominal rigidities.

In the second essay, “Financial and Real Frictions as Sources of Business Fluctuations”, I show that a negative shock to a financial or real friction in an economy can generate quantitatively significant and persistent recessions, even without a decrease in exogenous aggregate total factor productivity in a heterogeneous agents DSGE model. The increase in uncertainty that a firm is facing when it makes capital adjustment, however, is found to have a limited or dubious influence on economic activities. The roles of collateral constraints as a financial friction and nonconvex capital adjustment costs as a real friction in aggregate fluctuations are examined in this propagation mechanism. When these frictions become strengthened, the degree of capital misallocation is intensified, which leads to a drop of endogenous aggregate total factor productivity. As agents expect that the return to investment and endogenous TFP decrease, they reduce aggregate investment sharply, which also leads to a drop in employment. Interruption of efficient resource allocation coming from these two frictions is found out to be enough to generate a large and persistent aggregate fluctuations even without introducing heterogeneity in firm-level productivity.
Dedicated to my parents and family
ACKNOWLEDGMENTS

First of all, I am indebted to my advisor, Bill Dupor. This work could not have been completed without his comments, advices, suggestions and, most of all, encouragement. At the most frustrating and stressful time, his trust in me made it possible for me not to give up and to focus on my work. I am also indebted to Dr. Paul Evans and Dr. Pok-sang Lam, who offered extremely valuable comments and suggestions on my work and also gave lots of inspiration. Their advice improved the quality of this work. Dr. Aubhik Khan and Dr. Julia Thomas shared their deep insight and vast knowledge without reservation.

I would like to thank my wife, Soyoung Park, and my two sons, Sangwon and Sangwook, for their love and support during my study. My special gratitude goes to my parents and parents-in-law for their endless love. I also have an enormous debt to many former and current graduate students in the Department of Economics at the Ohio State University who shared good times and bad times together, although I cannot list all of them here.

I also thank the Bank of Korea for providing financial support and allowing me to stay at school until I finish the dissertation.
The second chapter was supported in part by an allocation of computing time from the Ohio Supercomputer Center.(Grant#: PAS0477-1)
October 6, 1970 ......... Born - Seoul, Republic of Korea

1995 .......................... B.B.A. Business Administration, Seoul National University, Republic of Korea


2007 ............................ M.A. Economics, The Ohio State University

2007–present ................... Graduate Teaching Associate, The Ohio State University

PUBLICATIONS

FIELDS OF STUDY

Major Field: Economics

Studies in:
- Macroeconomics
- Econometrics
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgments</td>
<td>v</td>
</tr>
<tr>
<td>Vita</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xi</td>
</tr>
<tr>
<td>Chapters:</td>
<td></td>
</tr>
<tr>
<td>1. Financial Frictions, Intersectoral Adjustment Costs</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 The Model</td>
<td>8</td>
</tr>
<tr>
<td>1.2.1 Firms</td>
<td>8</td>
</tr>
<tr>
<td>1.2.2 Households</td>
<td>11</td>
</tr>
<tr>
<td>1.2.3 Entrepreneurs</td>
<td>13</td>
</tr>
<tr>
<td>1.2.4 Competitive Equilibrium</td>
<td>19</td>
</tr>
<tr>
<td>1.2.5 Asset Prices</td>
<td>20</td>
</tr>
<tr>
<td>1.2.6 Parameterization</td>
<td>20</td>
</tr>
<tr>
<td>1.3 Results</td>
<td>22</td>
</tr>
<tr>
<td>1.3.1 Baseline Model</td>
<td>22</td>
</tr>
<tr>
<td>1.3.2 The Roles of the 3 Elements</td>
<td>24</td>
</tr>
<tr>
<td>1.3.3 How Important is Each Element?</td>
<td>34</td>
</tr>
<tr>
<td>1.3.4 Financial Accelerator Mechanism vs. Intertemporal Investment</td>
<td>36</td>
</tr>
<tr>
<td>Adjustment Costs</td>
<td></td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>----------</td>
<td>------------------------------------------------------------------------</td>
</tr>
<tr>
<td>1.4</td>
<td>Conclusion</td>
</tr>
<tr>
<td>2.</td>
<td>Financial and Real Frictions as Sources of Business Fluctuations</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>2.2</td>
<td>The Model</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Firms</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Households</td>
</tr>
<tr>
<td>2.2.3</td>
<td>Recursive Competitive Equilibrium</td>
</tr>
<tr>
<td>2.2.4</td>
<td>Decision Rules</td>
</tr>
<tr>
<td>2.2.5</td>
<td>Calibration</td>
</tr>
<tr>
<td>2.3</td>
<td>Results</td>
</tr>
<tr>
<td>2.3.1</td>
<td>The Steady State</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Decision Rules</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Responses to a Technology Shock</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Responses to a Shock to a Financial or Real Friction</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Responses to a Shock to the Uncertainty of a Real Friction</td>
</tr>
<tr>
<td>2.4</td>
<td>Conclusion</td>
</tr>
</tbody>
</table>

Appendices:

A. Appendix for Chapter 1 ................................. 114
   A.1 Aggregation for entrepreneurs ...................... 114
   A.2 Competitive Equilibrium ........................... 117
   A.3 A model with investment adjustment costs .......... 121
       A.3.1 Firms ........................................ 121
       A.3.2 Households .................................... 122
       A.3.3 Competitive Equilibrium ....................... 123

B. Appendix for Chapter 2 ................................. 124
   B.1 Footnote 16: The Definition of Maxium Debt Policy 124

Bibliography .................................................. 125
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 The Sequence of Events: For simplicity, an entrepreneur is assumed to be an average representative in some part.</td>
<td>41</td>
</tr>
<tr>
<td>1.2 Parameterization</td>
<td>42</td>
</tr>
<tr>
<td>2.1 Calibration</td>
<td>96</td>
</tr>
<tr>
<td>2.2 Business Cycle Moments</td>
<td>97</td>
</tr>
</tbody>
</table>


LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Circular-Flow Diagram</td>
<td>43</td>
</tr>
<tr>
<td>1.2</td>
<td>Responses to a News Shock : Baseline Model (Unrealized News)</td>
<td>44</td>
</tr>
<tr>
<td>1.3</td>
<td>Responses to a News Shock : Baseline Model (Realized News)</td>
<td>45</td>
</tr>
<tr>
<td>1.4</td>
<td>The Role of Each Element : ① Existence of collateral constraints gives an incentive to increase current investment. ② RCC channel ③ DMC channel ④ Investment-initiated boom leads to an increase in the price of capital. ⑤ Higher leverage exposes an economy to greater agency costs, which leads to an increase in the price of capital ⑥ Financial accelerator mechanism amplifies all the responses.</td>
<td>46</td>
</tr>
<tr>
<td>1.5</td>
<td>Responses to a News Shock : An RBC Model with Intersectoral Adjustment Costs only</td>
<td>47</td>
</tr>
<tr>
<td>1.6</td>
<td>Responses to a News Shock : An RBC Model with Intersectoral Adjustment Costs and Collateral Constraints on Firms</td>
<td>48</td>
</tr>
<tr>
<td>1.7</td>
<td>Baseline Model and Perturbed Model (No Financial Frictions on Entrepreneurs)</td>
<td>49</td>
</tr>
<tr>
<td>1.8</td>
<td>Responses of Financial Variables : Baseline Model</td>
<td>50</td>
</tr>
<tr>
<td>1.9</td>
<td>Financial Accelerator Mechanism in the Baseline Model: When a news shock hits the economy.</td>
<td>51</td>
</tr>
<tr>
<td>1.10</td>
<td>Financial Accelerator Mechanism in the Baseline Model: One period after a news shock hits the economy</td>
<td>52</td>
</tr>
<tr>
<td>Section</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>1.11 Baseline Model and Perturbed Model (No Collateral Constraint on</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>Firms)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.12 Baseline Model and Perturbed Model (No Intersectoral Adjustment</td>
<td>54</td>
<td></td>
</tr>
<tr>
<td>Costs)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.13 A RBC Model with Intersectoral Adjustment Costs, Collateral</td>
<td>55</td>
<td></td>
</tr>
<tr>
<td>Constraints and Investment Adjustment Costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.14 Baseline model and a Model with Investment Adjustment Costs</td>
<td>56</td>
<td></td>
</tr>
<tr>
<td>2.1 Maximum Debt Policy ($B^W$) and Threshold ($\hat{B}$), given $\xi$</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>and $z$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.2 Stationary Distribution of the Full Economy</td>
<td>99</td>
<td></td>
</tr>
<tr>
<td>2.3 Stationary Distribution of the Economy w/o financial frictions</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>2.4 The Budget Set of a Hypothetical Constrained Firm</td>
<td>101</td>
<td></td>
</tr>
<tr>
<td>2.5 The Objective Function of a Hypothetical Constrained Firm: when $\xi$</td>
<td>102</td>
<td></td>
</tr>
<tr>
<td>varies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.6 The Objective Function of a Hypothetical Constrained Firm: when $b$</td>
<td>103</td>
<td></td>
</tr>
<tr>
<td>varies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.7 Decision Rules: The Full Economy Case ($B = .02, \theta_b = .80$)</td>
<td>104</td>
<td></td>
</tr>
<tr>
<td>2.8 Decision Rules: The Economy with Stronger Financial Frictions Case</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>($B = .02, \theta_b = .56$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.9 Responses to a Technology Shock</td>
<td>106</td>
<td></td>
</tr>
<tr>
<td>2.10 Responses to a Financial Shock</td>
<td>107</td>
<td></td>
</tr>
<tr>
<td>2.11 Responses to a Real Shock</td>
<td>108</td>
<td></td>
</tr>
<tr>
<td>2.12 Responses to both Financial and Real Shock</td>
<td>109</td>
<td></td>
</tr>
<tr>
<td>2.13 Responses to a Financial Shock and Recovery</td>
<td>110</td>
<td></td>
</tr>
<tr>
<td>2.14 Responses to a Shock to the Second Moment of a Real Friction</td>
<td>111</td>
<td></td>
</tr>
</tbody>
</table>
2.15 PDF, Adjustment Hazard Rate, and Distribution of Capital for Low Uncertainty Economy  

2.16 PDF, Adjustment Hazard Rate, and Distribution of Capital for High Uncertainty Economy 

2.17 PDF, Adjustment Hazard Rate, and Distribution of Capital for Low Uncertainty Economy 

2.18 PDF, Adjustment Hazard Rate, and Distribution of Capital for High Uncertainty Economy
1.1 Introduction

Technology shocks, monetary shocks, and sunspot shocks have been considered as the main potential driving forces for business cycles in macroeconomics. Recently, some economists started to pay attention to another type of shock. They thought that a change in expectations about future exogenous variables — defined as a news shock — could be another source for economic fluctuations.¹ The 1999-2001 boom-bust cycle observed in the US and Asia may fit this idea. Many economists believe that people's overly optimistic view about future technological progress in the late 1990s indeed contributed to the high economic growth rates of 1999 and 2000 and that the subsequent disappointment caused the bust that followed. Another episode which may be explained by a news shock scenario is the U.S recession of 1990-91. Though various explanations were suggested by economists, no theory could explain the 1990-91 recession by giving a single main reason. Noting the limitations of the established

¹This idea has an old tradition. Pigou (1926, [40]) thought that a change in businessmen’s expectation about future profit could make business fluctuations. Keynes (1936, [30]) also noted that a spontaneous urge to action – animal spirits – might drive aggregate economic activities.
macroeconomic theories, Blanchard (1993, [8]) and Hall (1993, [27]) argue that a drop in consumer confidence caused by some noneconomic (hence nonfundamental) events might play an important role in the 1990-91 recession.

Inspired by the episodes, some economists argue that a news shock could be the main driving force for boom-bust business cycles, which means that a news shock alone can generate the main properties of business cycle data. They say that, as the main driving force, positive (negative) news can (i) generate positive (negative) responses of major economic aggregates (e.g. consumption, investment, hours worked, output and stock prices) and (ii) explain a sizable fraction of volatility of the variables. Indeed, if the main force that created the boom-bust in 1999-2001 was just news about future technological progress, the comovement between the aggregates observed during that period and sizable boom-bust should be generated by any reasonable economic model which includes only a news shock as the main driving force. One could argue that the episodes mentioned above were triggered not only by a news shock but also by other fundamental shocks and, hence, it is not known what properties of responses of the variables any reasonable news shock model should generate. So she could argue, for example, that the comovement between consumption and investment does not necessarily need to be generated by any reasonable news shock model. However, whether the episodes were caused only by a news shock alone or by some combination of a news shock and other fundamental shocks is an empirical question which has not been answered yet satisfactorily. So, at this point in time, what could be a worthy experiment would be to see if a news shock alone can generate the comovement phenomenon and the sizable volatility of the variables in a DSGE
setting and, if it can, to conclude that a news shock could be a good candidate for a main driving force causing business fluctuations.

Existing research finds that it is difficult to generate the comovement pattern in response to a news shock in a standard RBC model. In the standard RBC model, news about a positive technology shock give rise to a immediate boom in consumption due to consumption smoothing motives. Hours worked, however, decrease because of a positive wealth effect of the news shock which induces households to demand more leisure. Since hours worked are reduced and the news shock is not realized yet, an increase in consumption leads to a decrease in current investment. For this not to happen, a sufficient increase in hours worked is needed, so that expansion in output can support the increases in both consumption and investment.

To make this mechanism work, Christiano et al. (2007, [16]) and Jaimovich and Rebelo (2008, [29]) rely on a combination of specific forms of utility functions, investment adjustment costs, or varying capital utilization. These variations could successfully generate business fluctuations and comovement with news shocks.

Beaudry and Portier (2004, [5]), (2007, [6]) impose some friction directly on the process of transformation between consumption and investment to solve the negative comovement problem between the two sectors. This friction makes it costly for agents to substitute investment goods for consumption goods immediately in response to a shock and make it optimal to comove the two goods at the same time. This type of friction is often referred to as intersectoral adjustment costs between consumption and investment goods, the cost complementarity between the production of different goods, or the convex production possibility frontier. One advantage of using this type of friction is that, since the convex production possibility technology implies that the
marginal cost of production of investment goods increases as long as investment goods increases relatively more than consumption goods in a expansion period, the price of capital, which can be interpretable as the stock prices under some conditions, has a procyclical property.

The third type of research investigates the role of frictions in the financial market to generate the comovement. Kobayashi et al. (2007, [37]) show that when firms use land as collateral, a news shock can generate business cycles and comovement under the preferences discussed in King et al. (1998, [34]) and investment adjustment costs. Kobayashi and Nuthara (2007, [36]) consider a case in which capital goods are used as collateral. In both papers, news about positive technology pushes up the current price of asset (land or capital). This relaxes the current financial constraints of producers so that producers can use more labor and materials, which leads to the current boom.

While existing research has been successful in generating comovement between the major variables, they have some unsatisfactory factors in their model. For example, Christiano et al. (2007, [16]) could not generate stock market boom-bust cycles with news shocks in their RBC type model. In the model, the current price of capital drops when there is a positive news shock. Only after introducing monetary factors which include sticky wages and an inflation-targeting monetary policy could they generate plausible patterns of the price of capital under a news shock.\(^2\) Jaimovich and Rebelo (2008, [29]) also has marginal $q$ which is decreasing with a positive news shock in their

\(^2\)So they conclude that the nominal rigidity and inflation targeting regime are the main factors that led the stock market boom-bust episodes observed in the US history. However, the Fed has not always engaged in inflation targeting. For example, the pre-1914 period evidenced little stickiness of price and certainly no inflation targeting. So the phenomenon should have been especially absent then according to their model.
model. In order to cause stock prices to jump with positive news, they show that average $q$ is different from marginal $q$ in their model and the value of the firms can increase even when marginal $q$ decreases if the value of investment increases enough. But to make this mechanism work, they need another assumption that the production function has decreasing returns to scale.

The other limitations which are found in some literature are that unrealistically high intertemporal elasticity of substitution (IES) is required to generate the comovement phenomenon and that the size of responses of the aggregates are too small to explain the volatilities observed in data. For example, the model used in Beaudry and Portier (2007, [6]) needs the IES to be greater than or equal to 2 to generate the comovement in response to a news shock to neutral technology.\(^3\) This is because it does not have any mechanism in their model that dampens the strong positive response of current consumption and induces agents to increase investment immediately in response to a positive news about future technology.\(^4\) The high IES was also used in Kobayashi et al. (2007, [37]) or Kobayashi and Nuthara (2007, [36]).\(^5\) In addition to that, Kobayashi et al. (2007, [37]) and Kobayashi and Nuthara (2007, [36]) assume that the intermediate input is used as a production factor and is one component of final goods, which helps magnify the responses of the aggregates.

In this paper, I show that an RBC model with financial frictions and intersectoral adjustment costs can generate sizable boom-bust cycles and plausible response of the

\(^3\)This fact comes from my own replication of their paper. In their paper, Beaudry and Portier succeed in generating the comovement under the *Hansen-Rogerson* utility function with a news shock to the tax rate, but do not with a new shock to neutral technology.

\(^4\)Other models usually use intertemporal convex investment adjustment costs in lieu of a high IES.

\(^5\)They use 2 and 1.33 as IES, respectively.
stock prices in response to a news shock to neutral technology, with the Hansen-Rogerson utility function and without the assumptions of investment adjustment costs, variable capital utilization or any nominal rigidities. My model differs from Christiano et al. (2007, [16]) and Jaimovich and Rebelo (2008, [29]) as it does not impose any restriction on the functional form of consumption in household’s preference to generate the comovement phenomenon.\textsuperscript{6} It also does not require any nominal rigidities or a specific type of production function to generate the positive response of the price of capital and the stock prices.\textsuperscript{7} In my model, the response of the price of capital is driven not by investment adjustment costs but by intersectoral adjustment costs and agency costs. Most of the existing research about the news shock experiment use the assumption of convex investment adjustment cost to make the price of capital deviate from one. Some models that focus on financial frictions also rely on this assumption. In my model, however, intersectoral adjustment costs and agency costs in the production of capital goods make the price of capital change along with business cycles. One advantage of this setup is that we can be free from the issue what functional form of investment adjustment cost is relevant.\textsuperscript{8} Secondly, we can focus on the various types of effects of financial frictions that exist in the economy and sort out the effects of them more easily. In particular, I am interested in the

\textsuperscript{6}For example, Christiano et al. (2007, [16]) assume habit formation type utility function and Jaimovich and Rebelo (2008, [29]) use Greenwood et al. (1988, [26]) type preference.

\textsuperscript{7}Regarding this property of my model, another contribution is that the model can generate a stock market ‘crash’ in the sense that stock prices fall beneath the steady state level when news turns out to be false. This is a property that most of the previous studies could not generate.

\textsuperscript{8}The assumption of convex investment adjustment costs, which is used in most of the news shock literature is recently questioned by some economists because it is found out to be inconsistent with micro-level data. See Khan and Thomas (2008, [31]).
role that the financial accelerator mechanism plays in amplifying the effect of a news shock.

My model also differs from Beaudry and Portier (2007, [6]), Kobayashi et al. (2007, [37]), Kobayashi and Nuthara (2007, [36]) in that it does not need the assumption of high IES. In my model, the role of high IES is played by the collateral constraints on firms. This is because accumulating capital gives additional benefit to firms by relaxing the constraints in the future.

The basic mechanism underlying my model is as follows. Good news about the future induces the forward-looking agents to increase the current consumption due to the consumption smoothing motive. The existence of collateral constraints, however, induces the agents to increase the current investment as well because higher level of capital in the future can make the agents hire the level of employment closer to the optimal one by relaxing the future collateral constraints. With technology level fixed, increases in both consumption and investment can be supported only by an increase in labor. A boom in the labor market is made possible through an increase in the value of marginal product of labor and the loosened collateral constraints, both of which induce the firms to hire more workers. Intersectoral adjustment costs, the first friction in my model, contribute to these two channels because, during expansion, the costs increase the relative price of output and, at the same time, the price of capital which determines the tightness of collateral constraints on firms (the second friction). The third friction—the financial accelerator mechanism driven by the agency costs—amplifies all the responses.
1.2 The Model

The model economy consists of a continuum of agents of unit mass. The agents are of two types: households (fraction $1 - \eta$) and entrepreneurs (fraction $\eta$). Firms are owned by the two agents. There are also financial intermediaries which issue real bank notes and pass household savings (in terms of investment goods) onto entrepreneurs. Firms, which produce output and transform it into consumption and investment goods with convex technology, face the collateral constraints as in Kiyotaki and Moore (1995, [35]). Entrepreneurs, which produce capital goods, face the financial constraints as in the Carlstrom and Fuerst (1997,[14]) model, which includes an optimal financial contract under Costly State Verification (CSV). The circular flow diagram in Figure 1.1 provides a brief description of the model economy.

1.2.1 Firms

Firms hire households, $H_t$, and entrepreneurial labor, $H^e_t$. They also purchase capital, $K_t$, to produce output, $Y_t$.\footnote{In my model, $Y_t$ is used as an input to produce consumption goods and investment goods. In this sense, $Y_t$ can be referred to as intermediate goods rather than output. I call it, however, output, following a usual practice.} They transform the output into consumption goods, $C_t$, and investment goods, $I_t$, and supply them at the price of 1 and $p_t$, respectively.\footnote{The numeraire in the model is consumption goods.} When firms transform the output, they face some cost complementarity between consumption and investment goods. This makes the price of investment goods in consumption units deviate from 1. When they produce consumption and investment goods, they face some financial constraints. I assume that financial intermediaries issue real bank notes, $B_t$, that can be used in the economy as a payment
method. Firms needs to borrow bank notes in order to pay for wages in advance of production.\footnote{I interpret firms borrowing in my model, $B_t$, as bank lines of credit for firms rather than the whole debt that firms are usually liable for. This is because credit lines are heavily used by firms as a method of short-term liquidity management, especially for financing working capital which are usually used for paying wages or material. Even though the lines of credit is unsecured debt, banks usually evaluate firms’ net worth or cash flow before they set the credit limit and if firms fails to repay the debt, banks can seize some asset. In this sense, it can be said that borrowing from credit line is actually constrained by the firms’ net worth.} So a firm’s choice of $H_t$ and $H^e_t$ is constrained by its current assets, that is, working capital.

$$w_tH_t + w^e_tH^e_t \leq B_t$$

(1.1)

As in Kiyotaki and Moore (1995, [35]), firms cannot commit themselves to repay the debt. So, borrowing is constrained by some portion of the value of the capital which is owned by firms.

$$B_t \leq \Gamma q_t K_t$$

(1.2)

where $\Gamma$ is the ratio of the capital holding that can be used as collateral and $q_t$ is the price of capital applied during period $t$.

So the representative firm has the following objective function:

$$\Pi_t = C_t + p_t I_t - R_t K_t - w_t H_t - w^e_t H^e_t + q_t (1 - \delta) K_t$$

(1.3)

subject to the two production functions which can be seen as the production possibility frontier together:

$$Y_t = \theta_t K_t^{\alpha_h} H_t^{\alpha_h} (H_t^{e})^{\alpha_e}, \quad (C_t^\chi + I_t^\chi)^\frac{1}{\chi} = Y_t$$

(1.4)
and the collateral constraint\textsuperscript{12}:

$$w_t H_t + w_t^e H_t^e \leq \Gamma q_t K_t \quad (1.5)$$

where $w_t$ and $w_t^e$ is the wage for household and entrepreneurial labor, respectively, and $R_t$ is the purchasing price for capital at the beginning of period $t$.

In this setup, I assume that firms buy capital at the beginning of each period and resell it to households and entrepreneurs right after the production of output. This makes it possible for us to assume that firms supply their capital as collateral during the production. As is commonly known, this assumption does not make any difference from the case in which households own the capital and rent it to the firms. See Table 1.1 for the detailed description of the sequence of events.

The Lagrangian multipliers for the production possibility constraint and the collateral constraint are denoted as $\frac{1}{\Omega_t}$ and $\Lambda_t$, respectively. If each constraint is binding, then the FOCs are:

$$R_t = \frac{1}{\Omega_t} M P_{K,t} + q_t (1 - \delta) + \Lambda_t \Gamma q_t \quad (1.6)$$

$$w_t = \frac{1}{(1 + \Lambda_t) \Omega_t} M P_{H,t} \quad (1.7)$$

$$w_t^e = \frac{1}{(1 + \Lambda_t) \Omega_t} M P_{He,t} \quad (1.8)$$

\textsuperscript{12}Since financial intermediaries face no default risk in this financial contract and the contract is intra-period, competition among banks ensures that the return on the loan be zero in equilibrium and, as a result, the intermediaries supply whatever amount of the notes would be demanded.
\[ p_t = \left( \frac{I_t}{C_t} \right)^{\chi-1} \]  

where

\[ \Omega_t = \frac{\partial Y_t}{\partial C_t} = (C_t^\chi + I_t^\chi) \frac{1}{\chi} C_t^{\chi-1} \]  

and the three constraints.

### 1.2.2 Households

The household problem is standard. The representative household has the following objective function:

\[ E_0 \sum_{t=0}^{\infty} \beta^t (C_t^h)^{1-\sigma} - \frac{1}{1-\sigma} + \nu_l (1 - L_t) \]  

subject to the budget constraint,

\[ C_t^h + p_t I_t^h + q_t (1 - \delta) A_t = R_t A_t + w_t L_t + \Pi_t \]  

and the law of motion for capital,

\[ A_{t+1} = (1 - \delta) A_t + I_t^h / \bar{q}_t \]

where \( A_{t+1} \) denotes the demand for capital at the end of period \( t \), \( L_t \) is household labor supply, \( I_t^h \) is the demand for investment goods and \( \bar{q}_t \) is the price of capital in investment units. Using labor and capital income, households buy consumption goods at the price of unity and investment goods at the price of \( p_t \). They also buy back the undepreciated capital at the price of \( q_t \) as assumed in the previous section. They lend the investment goods to financial intermediaries who will also lend the goods to entrepreneurs under the optimal financial contract in a CSV setting. Since
the loan contract between households and financial intermediaries is intraperiod and the risk that financial intermediaries face when they lend can be perfectly diversified by virtue of the law of large numbers, the households get repaid the same value of investment goods which was already converted into capital goods by entrepreneurs’ capital producing technology.\footnote{If a household lends $I_h^t$ units of investment goods to financial intermediaries, (after he gets repaid), he can purchase $I_h^t/\bar{q}_t$ units of capital goods, which has the same value as $I_h^t$ units of investment good when it is measured in terms of investment goods.} When the borrowed investment goods is transformed into capital goods, the economy loses some part of them because of agency costs. This is the reason why $\bar{q}_t$ is in the equation for capital accumulation. The Lagrangian multipliers for the budget constraint and the capital accumulation equation are $\lambda_{C,t}$ and $\lambda_{A',t}$, respectively. Since the consumption good value of a newly installed unit of capital to be used in production at the beginning of period $t + 1$ (the price of capital, $q_t$, in the model) is defined as $\lambda_{A',t}/\lambda_{C,t}$,\footnote{This is the traditional definition of marginal $q$. On the other hand, Christiano and Fisher (2003, [15]) use a different definition of marginal $q$, which is (the consumption goods value of newly installed capital/the consumption goods value of investment goods), to make it a sufficient statistic for explaining investment under the model in which the price of capital can be decomposed into the price of investment and the price of capital in investment goods. According to this definition, marginal $q$ in my model is $\bar{q}_t$. Under the level specification of investment adjustment which is named by Christiano et al. (2007, [16]), marginal $q$ is equal to average $q$, so that it can be interpreted directly as (the stock market value of the firm/the replacement cost of the capital). However, under the flow specification, it is different from average $q$, as Jaimovich and Rebelo (2008, [29]) showed. In my model, the traditional definition of marginal $q$, $q_t$, is also different from average $q$, because the cost function for installing capital is not guaranteed to be homogeneous of degree one. I will show how average $q$ can be constructed in the later section.} we can see from the FOCs that it can be decomposed into:

$$q_t \equiv \frac{\lambda_{A',t}}{\lambda_{C,t}} = p_t \bar{q}_t \quad (1.14)$$

The capital-demand curve and the labor supply curve are
\[ U_{C,t} = \beta E_t \left[ U_{C,t+1} \frac{R_{t+1}}{q_t} \right] \]  \hspace{1cm} (1.15)

\[ \frac{U_{L,t}}{U_{C,t}} = w_t \]  \hspace{1cm} (1.16)

### 1.2.3 Entrepreneurs

The average\(^{15}\) entrepreneur has the following objective and constraints.

\[ E_0 \sum_{t=0}^{\infty} (\beta \gamma)^t [C_t^e + \nu_2(1 - L_t^e)] \]  \hspace{1cm} (1.17)

where \(C_t^e, L_t^e\) represent average entrepreneurial consumption and labor supply, respectively. Since there is heterogeneity in the size of entrepreneurs’ net worth, I consider first each entrepreneur’s budget constraint before I get the aggregate one. Entrepreneurs get income from supplying labor and capital. So their net worth is, in terms of investment goods, given by

\[ n_t(j) = \frac{1}{p_t} (w_t^e l_t^e(j) + R_t z_t(j) + \pi_t(j)) \]  \hspace{1cm} (1.18)

where \(n_t(j)\) and \(z_t(j)\) denote net worth and capital holdings of the entrepreneur \(j\), respectively. After entrepreneurs receive his income in terms of consumption goods and firms finish their production, entrepreneurs repurchase his undepreciated capital \((1 - \delta)z_t(j)\) and, with the capital and the remnant of consumption goods, they purchase \(n_t(j)\) units of investment goods, including \(i_t^e(j)\).\(^{16}\)

\(^{15}\) In this model, due to the linear nature of the capital goods and monitoring technolgies, the aggregate value of each variable is equivalent to its mean. See appendix A.1 for details.

\(^{16}\) \(i_t^e(j)\) is defined as the amount of investment goods purchased by an entrepreneur in the investment goods market. Since I assume that there are two processes for the transformation from output, \(Y_t\), to
Another source of income comes from investing $\bar{t}(j)$ units of investment goods into a technology that produces $\omega_t(j)i_t(j)$ units of capital goods, where a random variable, $\omega$, is i.i.d, both across agents and over time and lognormally distributed with log-mean $-\frac{1}{2}\sigma^2_\omega$ and log-variance $\sigma^2_\omega$. The CDF and PDF are denoted by $\Phi(\omega)$ and $\phi(\omega)$, respectively. An entrepreneur who borrows $\bar{t}_t - n_t$ units of investment goods has to repay $(1 + r_l)(\bar{t}_t - n_t)$ in capital goods. So there exists a default threshold technology level, $\bar{\omega}_t$, such that if

$$\omega_t \leq (1 + r_l)(\bar{t}_t - n_t)/\bar{t}_t = \bar{\omega}_t$$

then, the entrepreneur defaults and all the returns are confiscated. In that case, a lender has to pay a monitoring cost of $\mu\bar{t}_t$ units of capital goods, since $\omega_t$ is observed only by the entrepreneur.

Before we proceed to construct a budget constraint for entrepreneurs, we need to specify the characteristics of financial contracts and deal with an aggregation problem for entrepreneurs.

**Financial Contracts and Financial Intermediary**

capital goods, $K_{t+1}$, there are actually two different markets for investments goods. First, there are markets for the investment goods which is transformed from output, $Y_t$. In this market, firms supply $I_t$ and, household and entrepreneurs demand $I^h_t$ and $I^e_t$, respectively ($i_t^e(j)$ represents the individual entrepreneur’s demand, while $I^e_t$ represents the aggregate or average). Financial Intermediaries collect households’ investment goods, $I^h_t$, and lend it to entrepreneurs. After entrepreneurs finish capital transactions, they now have their net worth, some part of which is the investment goods, $I^e_t$. Adding their net worth, $N_t$ to the borrowing from households, $I^h_t$, the entrepreneurs get the investment goods, $I^e_t$, which will be put into production of capital goods, $I^S_t$, which can be directly transformed to $K_{t+1}$. Here, we have the second investment goods market, where the demand for investment goods as an input factor of capital goods is $I^d_t$ and the supply is $I^h_t + N_t$. See Table 1.1 for details.

---

17This gives us $E[\omega] = 1$, $Var[\omega] = e^{\sigma^2} - 1$.

18For expositional convenience, $j$ is omitted in some part of the paper.
Financial intermediaries pass households' investment goods onto entrepreneurs. The expected income of an intermediary that finances a project with a loan in the amount of \(i_t - n_t\) is:

\[ q_t i_t(j) g(\bar{\omega}_t) \] (1.20)

where

\[ g(\bar{\omega}_t) = \int_0^{\infty} \min\{\omega_t(j), \bar{\omega}_t\} d\Phi[\omega_t(j)] - \Phi(\bar{\omega}_t)\mu \] (1.21)

is the fraction of the expected net output of capital goods collected by the lender.

The fraction of the expected output of capital goods received by the entrepreneur is:

\[ f(\bar{\omega}_t) = \int_0^{\infty} \max\{\omega_t(j) - \bar{\omega}_t, 0\} d\Phi[\omega_t(j)] \] (1.22)

Under this circumstance, an entrepreneur solves:

\[
\max_{i_t(j), \bar{\omega}_t} q_t i_t(j) f(\bar{\omega}_t) \\
\text{s.t} \quad q_t i_t(j) g(\bar{\omega}_t) \geq p_t [i_t(j) - n_t(j)]
\] (1.23)

Solving this problem, along with the restriction given by equation (1.14), leads us to the two FOCs

\[ i_t(j) = \frac{1}{1 - \bar{q}_t g(\bar{\omega}_t)} n_t(j) \] (1.25)

\[ \frac{1}{\bar{q}_t} = 1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \] (1.26)

Solutions for these are the optimal default threshold and project size:
\[ \bar{\omega}_t = \bar{\omega}(\bar{q}_t) \]  

(1.27)

\[ i_t(j) = i(\bar{q}_t, n_t(j)) \]  

(1.28)

This implies that the optimal lending rate is

\[ r^l(j) = r^l(\bar{q}_t) \]  

(1.29)

So the equilibrium threshold and lending rate is the same across agents, but the equilibrium project size differs across entrepreneurs. In fact, it is proportional to the given net worth. From comparative statics, it can also be proven that \( \bar{\omega}_t = \bar{\omega}(\bar{q}_t) \) is increasing in \( \bar{q}_t \). The intuition is that an entrepreneur is more willing to pay a high lending rate and to increase leverage as the price of capital increases. Note that the threshold is a function of lending rate and leverage ratio. It can also be proven that

\[ \frac{\partial i}{\partial q}, \frac{\partial i}{\partial n} \geq 0. \]  

Also note that, in equilibrium,

\begin{equation}
\text{Expected return to external funds} = \frac{\bar{q}_t i_t g(\bar{\omega}_t)}{(i_t - n_t)} = 1
\end{equation}

(1.30)

\begin{equation}
\text{Expected return to internal funds} = \frac{\bar{q}_t i_t f(\bar{\omega}_t)}{n_t} > 1
\end{equation}

(1.31)

From this, it can be shown that \( \frac{\bar{q}_t i_t f(\bar{\omega}_t)}{n_t} \) is increasing in \( \bar{q}_t \).

After \( \omega_t(j) \) is realized, each \( j \) entrepreneur who is still solvent makes their consumption and capital accumulation decision. So the budget constraint for each entrepreneur is given by
\[ c_t^e(j) + q_t z_{t+1}(j) = q_t \hat{i}(\hat{q}_t, n_t(j)) \max\{\omega_t(j) - \bar{\omega}_t, 0\} \] (1.32)

That is, for successful entrepreneurs and unsuccessful ones, respectively

\[ c_t^e(j) + q_t z_{t+1}(j) = q_t \hat{i}(\hat{q}_t, n_t(j))(\omega_t(j) - \bar{\omega}_t) \] (1.33)

\[ c_t^e(j) + q_t z_{t+1}(j) = q_t \hat{i}(\hat{q}_t, n_t(j)) \cdot 0 \] (1.34)

This implies that optimal decision will satisfy the Euler equation:

\[ 1 = E_t^{\beta \gamma} \left[ \frac{R_{t+1}}{q_t} R_{t+1}^d \right] \] (1.35)

where \( R_t^d = (1 + r_t^d)\bar{q}_t \) can be defined as gross external finance premium (or default premium) paid by an entrepreneur. Since the loan contracts are intra-period ones, you cannot lend resources intertemporally. So the risk free rate is one under this financial contract. If you lend it, you get a risky \((1 + r_t^d)\bar{q}_t\) consumption goods. This is the reason why \((1 + r_t^d)\bar{q}_t\) is the gross external finance premium. As for the Euler equation, it means, intuitively, that if you consume one unit today, your opportunity costs are the return on capital amplified by risk premium.

**Aggregation for Entrepreneurs**

In the setup assumed above, the aggregate value of each variable is the first moment of the variable times the mass of each type of agents. The linearity of each individual’s budget constraint, (1.32), net worth equation, (1.18), and the equation for optimal investment size, (1.25), allows us to get the equations for the aggregate economy by simply adding up all the equations of each entrepreneur. See appendix A.1
for details. Let’s suppose capital letters represent the first moment of the variables.

For example, \( I_t^d = E[\iota_t(j)] \), \( N_t = E[n_t(j)] \). Then, the aggregate budget constraint is

\[
\eta C_t^e + \eta q_t Z_{t+1} = \eta q_t I_t^d \int_0^\infty \max\{\omega_t(j) - \omega_1, 0\} d\Phi[\omega_t(j)] \tag{1.36}
\]

\[
= \eta q_t I_t^d f(\omega_t) \tag{1.37}
\]

(1.18), (1.25) turn out to be:

\[
\eta N_t = \frac{\eta}{p_t} (w_t^e N_t^e + R_t Z_t + \Pi_t) \tag{1.38}
\]

\[
\eta I_t^d = \eta \frac{1}{1 - \tilde{q}_t g(\omega_t)} N_t \tag{1.39}
\]

Also, the aggregate, expected, production of capital goods can be constructed.

This function can be seen as the supply curve for capital goods.

\[
I^S(\tilde{q}_t, N_t) = \eta I_t^d [1 - \mu \Phi(\omega_t)] \tag{1.40}
\]

Using (1.37), (1.38), (1.39), I obtain the aggregate budget constraint:

\[
C_t^e + q_t Z_{t+1} = \frac{p_t \tilde{q}_t f(\omega_t)}{1 - \tilde{q}_t g(\omega_t)} \left\{ \frac{1}{p_t} (w_t^e N_t^e + R_t Z_t + \Pi_t) \right\} \tag{1.41}
\]

\[
= q_t I_t^d f(\omega_t) \tag{1.42}
\]

\[
= (1 + \rho_t) p_t N_t \tag{1.43}
\]

where \( 1 + \rho_t = \frac{q_t f(\omega_t)}{1 - \tilde{q}_t g(\omega_t)} \) (the gross expected return to internal funds).
Note that the net capital supply function is:

$$I_t^{NS} = \eta I_t^d [1 - \mu \Phi(\bar{\omega}_t)] - \eta (1 - \delta) Z_t - \frac{\eta C_t^e}{q_t}$$  \hfill (1.44)

The extra two terms are subtracted because, after the average entrepreneur puts his whole wealth (labor income, \(w_t^e L_t^e\), return on capital, \([R_t - q_t (1 - \delta)] Z_t\), and undepreciated capital, \((1 - \delta) Z_t\)) into the capital production in terms of investment goods, she gets the capital goods, \(\eta I_t^d f(\bar{\omega}_t)\) and, then, use the proceeds to purchase \(C_t^e\) and repurchase \((1 - \delta) Z_t\).\(^{19}\)

Using (1.43), I get the aggregate FOCs for the average entrepreneur.

\[ FOCs: \]

1. \[1 = \beta \gamma E_t \left[ \frac{R_{t+1}}{q_t} \right] (1 + \rho_{t+1}) \] \hfill (1.45)

2. \[ \frac{U_{L^e, t}}{U_{C^e, t} (1 + \rho_t)} = w_t^e \] \hfill (1.46)

1.2.4 Competitive Equilibrium

A competitive equilibrium is a set of sequences of quantities, \(\{K_{t+1}, Z_t, C_t, C^h_t, C^e_t, \]
\(H_t, H^e_t, L_t, L^e_t, I_t, I_t^d, I_t^h, I_t^e, I_t^{NS}, N_t, Y_t\}\)\(^{\infty}\), prices, \(\{R_t, w_t, w_t^e, q_t, q_t^e, p_t, \bar{\omega}_t, \Omega_t, \Lambda_t, \rho_t\}\)\(^{\infty}\), given \(K_0, Z_0\), satisfying (i) firms optimization, that is, FOCs (6-10); (ii) households optimization, FOCs (12-16); (iii) the optimal financial contract, FOCs (25-26); (iv) entrepreneurs optimization, (43), (45, (46); and (v) the market clearing conditions. Refer to appendix A.2.

\(^{19}\)In this model, I assume that when the average entrepreneur goes through this process, she purchase only \(I_t^e\) units of investment goods in the market and, as for the transaction for the other goods, she can transform them into investment goods or vice versa as a social planner does. This assumption enable us to get the condition for clearing the market for an input factor (investment goods) used in the production of capital goods: \(\eta I_t^d = (1 - \eta) I_t^h + \eta N_t\).
1.2.5 Asset Prices

To investigate the implication for the stock markets, I change some of the given assumptions. I assume that firms own and accumulate the capital, and households and entrepreneurs own stocks on these firms. As is well known, the model under this new setup is equivalent to the previous one. In this setup, value of the firms can be thought of the stock prices. Let \( V_t \) denote the value of the firm measured in terms of period-\( t \) consumption goods. Then,

\[
V_t = E_t \left[ \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{C,t+j}}{\lambda_{C,t}} \left( \frac{Y_{t+j}}{\Omega_{t+j}} - w_{t+j} H_{t+j} - w_{t+j}^e H_{t+j}^e - p_{t+j} I_{t+j} \right) \right] \tag{1.47}
\]

One can write \( V_t \) recursively as:

\[
V_t = \frac{Y_t}{\Omega_t} - w_t H_t - w_t^e H_t^e - p_t I_t + \beta E_t \frac{\lambda_{C,t+1}}{\lambda_{C,t}} V_{t+1} \tag{1.48}
\]

I use (1.7), (1.8), and the property of Cobb-Douglas production function to obtain\(^{20}\)

\[
V_t = \frac{\alpha + \Lambda_t}{(1 + \Lambda_t) \Omega_t} Y_t - p_t I_t + \beta E_t \frac{\lambda_{C,t+1}}{\lambda_{C,t}} V_{t+1} \tag{1.50}
\]

1.2.6 Parameterization

Table 1.2 shows the parameter values, most of which follow standard ones found in the literature. The entrepreneurial labor share, \( \alpha_e \), is set arbitrarily small so that the entrepreneurial labor and the entrepreneurial labor income play very minor roles in the

\(^{20}\)Dividing value of the firms by \( K_t \), I also can get:

\[
q_{a,t} = \frac{\alpha + \Lambda_t}{(1 + \Lambda_t) \Omega_t} Y_t - p_t I_t + \beta E_t \frac{\lambda_{C,t+1}}{\lambda_{C,t}} \frac{K_{t+1}}{K_t} q_{a,t+1} \tag{1.49}
\]

where \( q_{a,t} \) denotes average \( q \).
production of output and net worth dynamics. This makes my baseline model comparable with the standard RBC model. I use the steady state quarterly bankruptcy rate (0.974%), reported by Fisher (1994, [24]) and the annual risk premium (1.87%), which is the average spread between the prime rate and the 3-month CP rate for April 1971 to June 1996, to obtain \( \sigma_\omega = 0.207 \) and \( \gamma = 0.947 \). In particular, \( \gamma \) is selected such that it exactly offsets the steady-state internal return and hence, self-financing does not occur at steady states. The empirical estimates of the parameter for agency costs, \( \mu \), which can also be interpreted as bankruptcy costs, range from 0.04 to 0.36, depending on how bankruptcy costs are defined. In this paper, I set \( \mu \) equal to 0.25.

Following Altman (1984, [2]), Carlstrom and Fuerst (1997, [14]) argue that this is a reasonable value for the total (direct and indirect) costs of bankruptcy. \( \sigma \) is set to 1 (log utility) for the baseline model and \( \nu_1 \) is set such that, at steady states, \( L \) is equal to 0.3. \( \Gamma \) is set to 0.05, so that the collateral constraint is binding. This comes from Sufi (2007, [44]) who estimates that the total line of credit in non-financial U.S firms between 1996 and 2003 represents 16% of book assets and the used portion ranges from 4.7% to 6%, depending on the samples.\(^{21}\) The coefficient for the intersectoral adjustment costs, \( \chi \), is set to 1.8. This is obtained from Vallés (1997, [48]), where he chooses the value to match the simulated responses of investment with the actual ones to various shocks. My results are independent of the fraction of entrepreneurs, \( \eta \).

\(^{21}\)So credit limit is not usually binding. The reason why, nevertheless, I set \( \Gamma \) to 5% rather than 16%, and argue that the constraints are binding is that since the interest rate applied to credit line is usually higher than the one applied to usual debts, we can consider it as almost always binding even though firms usually do not take the loan up to the credit limit. In this sense, the ratio of the asset that can be used as collateral is the ratio of outstanding line of credit to asset.
1.3 Results

Under this news shock experiment, I assume that the neutral technology is exoge-
nously evolving according to,

$$\theta_t = (1 - \rho) + \rho \theta_{t-1} + \xi_{t-p} + \nu_t$$  \hspace{1cm} (1.51)

where $\xi_t$ and $\nu_t$ are uncorrelated both across time and with each other. $\xi_{t-p}$ is a news
shock which occurred $p$ periods ago from current time $t$. For example, suppose $p = 4$.
If $\xi_1 > 1$ and $\nu_5 = 0$, this means that in period 1, there was a news shock which
said that there would be a positive technology shock after 4 periods and it is realized
in period 5. This news shock also shifts up all the values of $E_t \theta_{t|t\geq 5}$. If $\xi_1 > 1$ and
$\nu_5 = -\xi_1$, this means that the news shock is not realized in period 5.

1.3.1 Baseline Model

Figure 1.2 shows how economic variables respond when a news shock hits the
economy and turns out to be false after four quarters.\(^{22}\) Output, household consump-
tion, investment, hours worked, price of capital, and value of firms all rise upon the
shock in period 5.\(^{23}\) When agents find out that the news is not realized in period 9, all
variables fall. On the other hand, Figure 1.3 shows the case in which the news shock
is realized. The variables demonstrate the positive hump-shaped patterns which can
be observed in many empirical studies.

\(^{22}\)I denote the first vertical dotted line as the period when a news shock hits the economy, and
the second vertical dotted line as the period when the news turns out to be false or true.

\(^{23}\)The increase of the total consumption is less than that of household consumption.(It increases
by 0.07% upon the shock.) This is because I assume that entrepreneurs are risk neutral, which makes
their IES = $\infty$. This assumption, even though unrealistic, makes the model tractable and, hence,
is technically essential in this type of model. If I made an assumption of risk averse entrepreneurs,
which is more realistic, the decrease in the entrepreneurial consumption would be more moderate.
Figure 1.4 provides a schematic description of how this model works. The intuition is as follows. When good news about the future productivity hits the economy, agents with consumption smoothing motives have a strong incentive to increase the current consumption. Firms are, on the other hand, financially constrained. Therefore, capital accumulation is more attractive than it otherwise would be. More capital enables firms to hire the number of workers closer to the optimal level of workers in the future, since collateral constraints are loosened. This makes the future return on capital higher, causing the consumption smoothing motive to be dampened. The strong incentive to increase current investment, however, is also dampened, because higher levels of investment can be financed by a larger amount of borrowing which exposes the economy to greater agency costs, and hence greater loss in resources. As a result, entrepreneurs, who internalize the expected agency cost, delay their investment. The gradual increases in both consumption and investment allow for both to increase simultaneously, as opposed to consumption crowding out investment or vice versa.

How does, then, labor increase when consumption increases? This is necessary so that expansion in output supported by higher employment can pay both increases in consumption and investment.24 There are two channels. One is the decreasing marginal cost of production of consumption channel (hereby referred to as the DMC channel) and the other is the relaxed collateral constraint channel (or RCC channel). The DMC channel is related to intersectoral adjustment costs. Since expansion causes

24In a standard RBC model, this is impossible because an increase in consumption implies the decrease in leisure, so that leisure gets more valuable which drives up the real wage, which, in turn, decrease firms demand for labor.
investment to increase relatively more than consumption, convex production possibilities make output \( (Y_t) \) more expensive relative to consumption goods. This makes the value of marginal product of labor increase, so that firms want to hire more labor at any given real wage. The RCC channel comes from the collateral constraints on firms. The boom causes an increase in the price of capital and the stock prices, which relaxes firms’ collateral constraints. This enables the financially constrained firms to hire more workers. In all, these two channels can make it possible for equilibrium labor to comove with the consumption.

Why, then, are the price of capital and stock prices procyclical? Intersectoral adjustment costs and the financial frictions on entrepreneurs (the existence of agency costs) cause this phenomena. Because the ratio of investment to consumption increases in expansion, the price of investment goods, one component of the price of capital, goes up. In addition to that, the larger loss incurred from producing capital goods caused by higher leverage ratio upon the shock makes the marginal cost of producing capital goods increase. Together with increased demand for capital goods, the rise in the marginal cost increases the price of capital. These two channels make the price of capital procyclical.

### 1.3.2 The Roles of the 3 Elements

In this section, I discuss the role played by each element in the baseline model in detail.

**Intersectoral Adjustment Costs**
Intersectoral adjustment costs make it possible for both consumption and hours to respond positively to a positive news shock, so that output and investment also respond positively to it. However, this mechanism works only when the IES is relatively high. Figure 1.5 shows how each variable reacts to a positive news shock for different levels of the IES when I introduce only intersectoral adjustment costs into the standard RBC model. In the standard RBC model, consumption smoothing motives induce agents with log utility (IES=1) to increase a little bit of current consumption and decrease the substantial size of investment. However, in an RBC model with only intersectoral adjustment costs, agents with log utility, who think it is optimal for the consumption and investment to comove, decrease their current consumption as they experience a large reduction of investment. But when IES=2, the direction gets reversed because the high IES makes agents start to increase their investment. The mechanism that helps consumption, investment and hours worked comove under high IES can be explained in more detail as follows. In the model, the linearity of leisure in the utility function means that the IES for leisure (labor supply) and wage elasticity of leisure (labor supply) are infinite. So if current leisure gets expensive intertemporally or intratemporally, agents decrease the current leisure infinitely (increase the current labor infinitely). This means that the labor supply curve is flat and the equilibrium labor is determined only by demand side. This can be seen in the following labor market equilibrium condition in the RBC model with only intersectoral adjustment costs.

\[
\frac{\nu}{C_t^{-\sigma}} = w_t = \frac{1}{\Omega_t} A_t \left( \frac{K_t}{L_t} \right)^\alpha
\]

(1.52)
If $C_t$ increases, MRS of $L_t$ for $C_t$ increases (leisure gets more valuable), which means that higher wages are needed at equilibrium. Since the real wage has increased, firms decrease $L_t$ to get higher marginal product of labor, $MP_{L,t}$. This causes consumption and labor to negatively comove in the standard RBC model. But in the perturbed model, if $\Omega_t$ decreases enough, $L_t$ can increase. $\Omega_t$ is the marginal cost of production of $C_t$ in $Y_t$ units (or the price of $C_t$ in $Y_t$ units)

$$\Omega_t = \frac{\partial Y_t}{\partial C_t} = (C_t^\chi + I_t^\chi) \frac{1-\chi}{\chi} C_t^{\chi-1} \quad (1.53)$$

Since $w_t$ is the price of leisure in $C_t$ units and $MP_{L,t}$ is measured in $Y_t$ units, we should multiply $MP_{L,t}$ by $\frac{1}{\Omega_t}$ before equating it to the wage. Intuitively speaking, when a firm hires one more unit of labor, it gets $MP_{L,t}$ units of output, which it converts into consumption goods at the rate of $\frac{1}{\Omega_t}$. Finally it gets $\frac{1}{\Omega_t} MP_{L,t}$ which should be equal to $w_t$ units of consumption goods at equilibrium. In the perturbed model, an increase in the expectation of future technology and high IES make the current investment increase relatively more than consumption. Under a convex production possibility curve, this means that the marginal cost of production of $C_t$ decreases ($\Omega_t$ decreases). This makes the value of the marginal product of labor measured in consumption units increase at any given real wage. This means that labor demand curve shifts to the right. So, if the labor demand curve moves to the right sufficiently, then equilibrium labor can be increased even when the wage goes up.

The reason why we have the positive response of the price of capital also comes from higher growth in investment than consumption induced by high IES.\(^{25}\) This

\(^{25}\)In this perturbed model, the price of capital in consumption units is the same as the price of investment goods in consumption units, $p_t$ because there is no friction in transformation from investment goods to capital goods.
relationship can be easily found in equation (1.9). The problem is that the above mechanism only works under the high IES and the size is not large enough.

**The Collateral Constraints Imposed on Firms**

The main role of the collateral constraints imposed on firms is to dampen the strong positive response of consumption, thus inducing the investment. The high IES plays this role in the previous perturbed model. So, intuitively, we can expect that the value of IES can be lowered by introducing this type of constraint. The reason why the consumption smoothing motive is dampened is because when firms are confronted with the collateral constraints, accumulating capital gives additional benefits to firms by enabling them to hire the amount of labor that is closer to the optimal level under relaxed collateral constraints in the future. Without it, an increase in investment happens only when IES is relatively high. The other role for collateral constraints on firms is to shift the labor demand curve outward just as the intersectoral adjustment costs do. Figure 1.6 shows how the variables react to different degrees of intersectoral frictions when we introduce the collateral constraints on firms as well as the intersectoral adjustment costs into a RBC model. When the coefficient for intersectoral adjustment cost \((\chi)\) is 1, agents increase current investment and decrease consumption because of higher future expected returns on capital caused by the collateral constraints even though they have a strong consumption smoothing motive \((\text{IES} = 1)\). Since the transformation rate from consumption to investment goods is one regardless of the ratio of the two goods, the price of the investment goods (and hence the price of capital) does not change. When the value of \(\chi\) increases, the response of
the price of capital gets stronger. The increase of the price of capital relaxes the collateral constraint. This, along with the effect from the DMC channel, shifts the labor demand curve to the right. Firms can hire more workers, which makes it possible for output to expand enough so that the consumption can also increase. This fact can be shown from the equilibrium condition for factor markets. There are two wedges to prevent the market from reaching at the level of employment in the perfect financial market case with no intersectoral adjustment costs.

\[ R_t = \frac{1}{\Omega_t} M P_{K,t} + q_t(1 - \delta) + \Lambda_t \Gamma q_t \]  

\[ w_t = \frac{1}{(1 + \Lambda_t) \Omega_t} M P_{L,t} \]  

\( \frac{1}{1 + \Lambda_t} \) represents the wedge created by the collateral constraints on firms and \( \frac{1}{\Omega_t} \) is the wedge created by the intersectoral adjustment costs. \( \Lambda_t \) is a Lagrangian multiplier for the collateral constraints imposed on firms. Thus, if the collateral constraint gets relaxed by an increase of the price of capital, the value decreases so that the labor demand curve moves to the right. In Figure 1.6, you can see that the marginal cost of consumption (\( \Omega_t \)) and the labor market inefficiency (\( \Lambda_t \)) are indeed decreasing upon the shock under high value of \( \chi \).

**Financial Frictions Imposed on Entrepreneurs**

From Figure 1.6, we could see that the two frictions, that is, intersectoral adjustment costs and collateral constraints on firms, are sufficient to generate the positive co-movement between the major aggregates and the plausible response of the price of capital qualitatively. However, the size of the responses is too small to explain the
volatilities that can be observed in U.S. data. Figure 1.7 compares the baseline model with the RBC model with intersectoral adjustment costs and collateral constraints on firms explained in the previous section. The sizes of responses in the baseline model are substantial, compared to those in the previous model. So, the financial frictions imposed on entrepreneurs amplify the responses of the variables in the baseline model. As mentioned in the introduction, the magnitude of the response must be sizable for the news shock to be the main driving force for business cycles. Introducing financial frictions on entrepreneurs is essential for this purpose.

It seems to be inappropriate to compare the baseline model directly with the RBC model with intersectoral adjustment costs and collateral constraints on firms to quantify the effect of the introduction of another financial friction–agency costs– on the other variables. This is because, in the baseline model, the capital accumulation is affected by not only households but also entrepreneurs who are assumed to be risk neutral, so that their capital accumulation is very sensitive to the return on capital. For example, the non-trivial fraction of the amplified responses in major variables at the first period when the news shock hits the economy in Figure 1.7 may come from this fact. But we can see that there is a substantial and persistent increase of the major variables even after the first period. This amplified response comes from the financial accelerator mechanism as will be explained below.

Before analyzing how agency costs amplify the responses of major variables in response to a news shock through net worth dynamics, it is worthwhile to note that $q_t$ in the baseline model can be decomposed into $p_t$ and $\bar{q}_t$, and that $p_t$ is driven by intersectoral adjustment costs and $\bar{q}_t$ is driven by agency costs. Therefore, to know how the agency costs affect the price of capital, $q_t$, it is enough to analyze the behavior
of \( \bar{q}_t \) and the effect of \( q_t \) on other variables. Also note that, from FOCs derived from the financial contract (equation (1.25) and (1.26)), the decision rules that govern the optimal financial contract are independent of \( p_t \) and, in particular, the capital goods supply function, equation (1.40), is just a function of \( \bar{q}_t \) and \( N_t \).

Note that the capital goods supply curve, equation (1.40), is upward sloping. In other words, the supply of capital goods is increasing in the price of capital in investment goods, \( \bar{q}_t \). The intuition is as follows. If investment for producing capital goods increases with net worth fixed, entrepreneurs’ borrowing should increase, so that leverage ratio goes up.\(^{27} \) Hence, agency costs, which are internalized by entrepreneurs,\(^{28} \) increase. This leads to an increase in the marginal cost of production of capital goods, which means the supply of capital goods is increasing in the price of capital, \( \bar{q}_t \). To be specific, when investment for capital goods increases, entrepreneurs increase the leverage ratio, setting a higher default threshold, \( \bar{\omega}_t \), which, in turn, raises the bankruptcy rate, \( \Phi(\bar{\omega}_t) \). This exposes the economy to larger agency costs and drives up the external finance premium. Thus, when the economy increases investment, it loses resources at an increasing rate, so that the total cost of production of capital goods increases at an increasing rate and hence, the marginal cost of production of capital goods rises. Equation (1.40) embodies this fact. An outline of the

\(^{26}\)See Carlstrom and Fuerst (1997, [14]) for the proof. It can also be proven that the claims explained here are applied to the price of capital in consumption goods, \( q_t \).

\(^{27}\)Note that the leverage ratio defined in the equation (1.25), \( \frac{1}{i_n} = \frac{1}{1-\bar{q}_t g(\bar{\omega}_t)} \) can be proven to be increasing in \( \bar{q}_t \) from comparative statics.

\(^{28}\)This fact can be observed by looking at the entrepreneurs’ maximization problem in the financial contract. The participation constraint (equation (1.24)) should always bind, so that the opportunity cost of the intra-period loan is one in equilibrium. This means that entrepreneurs guarantee financial intermediaries (or equivalently households) only the principal and, hence, internalize the expected agency cost.
proof is as follows. When \( q_t \) increases, \( I^d_t \) increases.\(^{29}\) But the term \( [1 - \mu \Phi(\bar{\omega}_t)] \) is decreasing in \( \bar{q}_t \) because \( \bar{\omega}_t \) is increasing in \( \bar{q}_t \). As a result, the capital supply function, \( I^S_t \) is concave in \( I^d_t \).\(^{30}\) Thus, the marginal cost of production of capital goods is increasing in \( I^S_t \) and, hence, \( I^S_t \) is increasing in \( \bar{q}_t \).

Another thing to note is the capital supply function is also increasing in net worth, \( N_t \).\(^{31}\) If net worth increases–ceteris paribus–entrepreneurs’ borrowing and leverage ratio are reduced. This exposes entrepreneurs to smaller agency costs, so that the marginal cost of production of capital goods decreases. This means that the capital supply curve move to the right. In other words, \( N_t \) can be considered as a supply shifter.

What happens if a news shock hit the economy at time \( t \)? Figure 1.8 shows how the financial variables respond under the baseline model. We can see a small increase in the price of capital in investment goods, \( \bar{q}_t \), and a substantial increase of the price of investment, \( p_t \), which leads to a large increase in the price of capital in consumption goods, \( q_t \). When the news shock occurs and the current investment increases at time \( t \), net worth is almost unchanged because the news shock is not realized yet, so little change in entrepreneurial income (which is equivalent to the change of \( MP_{H^e,t} \), \( MP_{K,t} \)) occurs at time \( t \) and the entrepreneurial capital, the change of which is the main source for the change for net worth, is initially predetermined. Thus, the upward sloping capital goods supply curve does not shift much at time \( t \). On the other hand,

\(^{29}\)This fact also comes from the comparative statics.

\(^{30}\)Concavity of capital production function implies the convexity of the cost function, which means that the marginal cost of production of capital goods is increasing.

\(^{31}\)This can be seen easily by taking derivative of (1.25) with respect to \( N_t \).
a news shock makes the households’ demand curve for capital goods shift outward so that it drives up $\bar{q}_t$. To see this, if we iterate (1.15) forward with (1.6), we get:

$$
\bar{q}_t = \frac{1}{p_t} E_t \sum_{i=1}^{\infty} \frac{1}{[(1 - \delta) + \Gamma \Lambda_t]} \left\{ \prod_{j=1}^{i} \lambda_{t+j} \lambda_{t+j-1}^{-1} \left[ (1 - \delta) + \Gamma \Lambda_{t+j-1} \right] \right\} \frac{MP_{K,t+i}}{\Omega_{t+i}} (1.56)
$$

Thus, the rise in the future value of $MP_{K,t+i}$s makes the demand curve for $K_{t+1}$ move to the right.

The same logic can be applied to the entrepreneurs’ demand curve for capital goods. If we iterate (1.45) forward with (1.6), we get:

$$
\bar{q}_t = \frac{1}{p_t} E_t \sum_{i=1}^{\infty} \frac{1}{[(1 - \delta) + \Gamma \Lambda_t]} \left\{ \prod_{j=1}^{i} \beta \gamma \lambda_{t+j}^{-1} \lambda_{t+j-1}^{-1} \left[ (1 - \delta) + \Gamma \Lambda_{t+j-1} \right][1 + \rho_{t+j-1}] \right\} \frac{MP_{K,t+i}}{\Omega_{t+i}} (1.57)
$$

In all, the dynamics of the demand and supply curve for capital goods drives up the price of capital in investment goods, $\bar{q}_t$, at time $t$. Figure 1.9 depicts how the equilibrium at the capital goods market is reached at $(\bar{q}_t, I_t)$ planes in which the capital demand function and the capital supply function are drawn. To see what happens to the price of capital in consumption goods, $q_t$, the same result is drawn at $(q_t, I_t)$ planes in Figure 1.9. We can see that while the increase in $\bar{q}_t$ is relatively small, $q_t$ increases substantially, largely driven by $p_t$.

Financial accelerator mechanism driven by agency costs starts to work from time $t + 1$. At the end of time $t$, entrepreneurs start to accumulate their capital, induced by a higher return on capital which is amplified by the expected return to internal funds (see equation (1.45)). The chosen capital now becomes the major part of net worth at time $t + 1$ and, hence, the net worth increases substantially. This shifts the capital supply function to the right sufficiently, so that $\bar{q}_{t+1}$ decreases. However, the
ratio of investment to consumption still rises, so that $p_{t+1}$ goes up further and, hence, $q_{t+1}$ still increases (refer to Figure 1.10). External finance premium drops below the steady state level. This enlarges the swing in the size of borrowing, investment, and output, even though the decrease in the price of capital in investment goods, $\bar{q}_t$, dampens the response of the price of capital in consumption goods, $q_t$. We can see this fact in Figure 1.10. Under the environment in which there is no agency costs, capital supply is just a function of the price of capital. However, in a Costly State Verification setting, the size of net worth, which serves as proxy for the balance sheet condition of entrepreneurs, affects the marginal cost of production of capital goods because entrepreneurs internalize expected default costs when they supply capital. So when entrepreneurs’ financial condition is bad (when his net worth is low so that leverage ratio is high), the marginal cost of production of capital increases, which means that the capital supply function moves to the left. If net worth grows rapidly, leverage ratio goes down, so that default costs that they have to pay goes down. Thus, the marginal cost of production of capital falls, which means the supply curve move to the right. Since the capital supply function moves to the right, the response of investment gets larger than it otherwise would have been (denoted as blue arrow lines in Figure 1.10). This process keeps going before the news reveals its truth. If the news shock turns out to be false, investment decreases but net worth is at its highest level. Thus, leverage ratio drops drastically, so that $\bar{q}_t$, which is the part of the price of capital driven by the agency cost, falls substantially. Hence, the price of capital in consumption goods, $q_t$, goes down, along with the decrease in $p_t$ which reflects the drop of the ratio of investment to consumption goods.
Another role of agency costs is that it dampens the strong positive response of the investment, so that it does not crowd out consumption. Actually, the contribution of Carlstrom and Fuerst (1997, [14]) model lies in the ability to generate hump-shaped responses of the major aggregates, including investment, rather than amplified responses. The main reason why the responses are delayed is because it takes time for entrepreneurs to build their net worth. That is, the delayed response in net worth delays the movements of the other variables. This result, of course, comes from the agents' maximization behavior. Carlstrom and Fuerst (1997, [14]) assert:

This hump-shaped [investment] behavior arises because [entrepreneurs] delay their investment decision until agency costs are at their lowest – a point in time several periods after the initial shock.

As discussed previously, the expected agency costs are internalized by entrepreneurs in the model. As a result, the entrepreneurs tend to find out which period will have the lowest agency cost and delay his investment until they can enjoy it. In summary, the existence of agency costs dampens the immediate and strong response of investment and makes it increase gradually. This gradual expansion in investment makes room for consumption to increase.

1.3.3 How Important is Each Element?

In the previous section, I compared the baseline model with the perturbed one (the baseline model without financial frictions on entrepreneurs) and found out that the intersectoral adjustment costs and the collateral constraint on firms are essential to solve the negative comovement problem, while the financial frictions on entrepreneurs

\[^{32}\text{See Carlstrom and Fuerst (1997, [14]). The magnitude of the responses of major variables in response to a neutral technology shock is smaller or similar under their model, compared to the one under the standard RBC model.}\]
help us obtain the sizable responses of the aggregates. In this section, I consider the comparison between the baseline model and the baseline model from which intersectoral adjustment costs or the collateral constraints on firms is eliminated, respectively.

Figure 1.11 shows the case in which only the collateral constraint on firms are eliminated from the baseline model. As expected, the absence of the additional benefit to accumulate capital induces agents to increase consumption and decrease the investment. More consumption and less investment under convex production possibility curves means that the marginal cost of production of consumption increases, so that there is no way that firms can hire more workers because the value of the marginal product of labor decreases. So labor decreases and, hence, output is reduced.

Figure 1.12 shows the case in which only the intersectoral adjustment costs are eliminated from the baseline model. Since we have an additional benefit to accumulate capital, consumption is dampened and investment increases. However, the strong positive response of investment in the first a few periods is limited by the agency cost as explained above. Compare this one (agency cost + collateral constraint on firms) with the one with $\chi = 1$ in Figure 1.6 (collateral constraint on firms only). Since the initial response of investment is very small and the intersectoral adjustment costs, which are main forces driving $q_t$, are absent, the response of the stock prices is also small. Because the response of the stock price is not sufficient and the decreasing marginal cost effect caused by intersectoral adjustment costs is absent, the loosening collateral effect itself is not enough to generate the sizable response of labor, which, in turn, limits the positive response of output and consumption.
1.3.4 Financial Accelerator Mechanism vs. Intertemporal Investment Adjustment Costs

After it is shown that the additional financial friction – agency costs – amplifies the responses of the variables through the financial accelerator mechanism, a natural question that can arise is whether any other friction can replace the role of agency costs in magnifying the responses. We can expect that any friction that enlarges the response of the price of capital can play this role. To show this, I consider a economy where there is intertemporal frictions in adjusting capital stock with the two basic frictions. I show that the intertemporal investment adjustment costs can replace the financial accelerator mechanism in the baseline model in magnifying the responses of the major variables but the amplified magnitude is very limited. Even though there could be a debate about using this type of adjustment cost in capital installing technology, it is worth investigating how the amplified response of the price of capital – no matter where it comes from – leads to the amplification of other variables.

Now, convex capital adjustment costs are introduced in an RBC model with intersectoral adjustment costs and collateral constraints on firms. The capital accumulation equation that household faces is:

$$K_{t+1} = (1 - \delta)K_t + I_t - \Psi \left( \frac{I_t}{K_t} \right) K_t$$

(1.58)

where

$$\Psi \left( \frac{I_t}{K_t} \right) = \frac{1}{2\delta\sigma_{\Psi}} \left( \frac{I_t}{K_t} - \delta \right)^2$$

(1.59)

Here, $\sigma_{\Psi}$ is the elasticity of the investment-capital ratio with respect to the price of capital. For the details for the model, refer to appendix C. Figure 1.13 shows how this new model reacts to the different value of $\sigma_{\Psi}$. When $\sigma_{\Psi}=1$, which is consistent with the empirical evidence, the economy has the larger responses in response to a news
shock than when $\sigma_\Psi = \infty$, which is equivalent to the case in which the economy has only intersectoral adjustment costs and collateral constraints on firms as frictions. This result also arises through the DMC and the RCC channel mentioned before. When capital producing technology is concave because of intertemporal adjustment costs, capital goods get more and more expensive when the production of it expands. This makes the collateral constraints relaxed more and more, in which case firms can hire more workers. DMC channel also works. As is well known, convex investment adjustment costs induce agents to increase the current investment in response to a positive news shock because the agent does not want to increase the investment drastically when the news is realized. So this force leads to an increase the ratio of investment to consumption, which makes the price of investment rise more and the marginal cost of production of consumption falls more. So the relative price of output to consumption goods rises, so that the marginal product of labor increases. Again, this leads to higher employment.

Figure 1.14 compare the baseline model with the new models which have different values of $\sigma_\Psi$. Even when this economy has implausibly high investment adjustment coefficient ($\frac{1}{\sigma_\Psi} = 10$), The responses of major variables is trivial compared to those in the baseline.

From the experiment given in this section, we can see that investment adjustment costs can amplifies the response of major variables via the amplified response of the price of capital, but the effect is very limited.

1.4 Conclusion

Why is a news shock important? Emphasizing the role of animal spirits in business cycles, Farmer (2007, [23]) says:
Why should we care if shocks arise in the productivity of the technology or in the minds of entrepreneurs? The answer is connected to the efficiency question. If business cycles arise as the consequence of the optimal allocation of resources in the face of unavoidable fluctuations in the technology, then there is not much that government can or should do about them. But if they arise as the consequence of avoidable fluctuations in the animal spirits of investors then the fluctuations that result are avoidable and the allocations are Pareto suboptimal. Animal spirit driven business cycles provide a reason for countercyclical stabilization policy and the cause of cycles is therefore an important question.

Farmer argues that if the substantial part of fluctuations comes from any change in expectations of agents, it should and can be avoided. Should the fluctuation caused by a news shock, indeed, be avoided? It depends on the form of the social welfare function that a society has. If the social welfare function is just the same as the lifetime utility function of a representative agent, a news-driven fluctuation does not need to be avoided as long as it raises the lifetime utility. If the social welfare function has a volatility of the economic aggregates as an argument that decreases the social welfare severely, then it should be avoided. In this sense, whether it should be avoided or not is not directly relevant to the mechanism by which news shocks make boom-bust cycles. However, whether we can avoid the fluctuations caused by news shocks or not depends on the mechanism. If the main reason why news-driven cycles occur lies in the intrinsic features of the economy like a specific characteristic of people’s preference or production technology as the existing research argues, then there are few policies that can be used. Maybe managing people’s expectations is the best that policymakers can do. However, if the news-driven boom-busts come from the imperfection in the financial markets and goods markets such as information asymmetry, limited enforcement or imperfect substitutability between sectors as I suggested in the paper, then elimination of such imperfection would be the main task.
for the policymaker who are faced with a wave of expectation shocks. Thus, the model suggested in this paper shows that the latter may be the case.

Another thing to note is that the hump-shaped patterns of the economic variables, which are observed in many empirical studies, may not be the result of any nominal rigidities, but rather people’s maximization procedures related to expectation shocks. People usually get some information about a future technological innovation and take some action because of it. After that, the technology is realized and commercialized. In contrast, the standard technology shock experiment could be considered somewhat unrealistic because it assumes that technological innovation occurs totally unexpectedly and is materialized and commercialized immediately. As shown in Figure 1.3, the time in which the major variables reach their highest points depends on the time length between when a news shock occurs and when it is realized. In this sense, the delayed of response does not come from some sort of frictions, but rather from the process of agents’ adaptation to new information in this model.

There is a lot of room for future research. First, it would be interesting to estimate how much news shocks can contribute to the volatility of macroeconomic variables by using simulation methods. Using my model, one can get some moments not only of real variables but also financial variables such as stock returns or default premia when news affect an economy. In this case, this model could be used to explain the equity premium puzzle.

Secondly, it may be possible to develop a model which assumes that people update their beliefs about the distribution of an idiosyncratic stochastic capital-producing technology in a Bayesian fashion, since, in reality, people update their beliefs about the moments of the idiosyncratic technology.

Thirdly, we can consider how a news shock to the mean of the stochastic capital producing technology, $\omega$, affects the economy. This shock can interpreted as a news
shock to investment-specific technology. The similar experiment can be done when we assume that there is a news shock to the variance of the stochastic capital producing technology, $\sigma_{\omega,t}$. Then, we can evaluate how people’s expectations about uncertainty affect the economy.

Lastly, the identification of news shocks could be a promising topic. Compared to the existing news shock models, my model has many “information variables” which have information about future neutral technology – stock price, lending rate, bankruptcy rate, default premium. As the inclusion of a broader set of information variables is known to help the identification of news shocks, my model can be used as a benchmark for identifying news shocks by using a richer set of variables and, then, estimating the parameters by matching the impulse responses from the data with the ones from my model.

\[33\text{See Sims (2009, [43]) or Barsky and Sims (2008, [4]).}\]
1. At the beginning of period $t$, household capital ($A_t$) and entrepreneurial capital ($Z_t$) are sold to firms, making up of total capital ($K_t$).

2. Aggregate productivity ($\theta_t$) is realized.

3. The capital is used as collateral by firms. Firms produce output ($Y_t$), using capital ($K_t$), household labor, ($H_t$), and entrepreneurial labor, ($H^e_t$).

4. Firms transforms the output into consumption goods ($C_t$) and investment goods ($I_t$).

5. After production of output ($Y_t$), the capital ($K_t$) is resold to households and entrepreneurs. Households now get $(1 - \delta)A_t$ and entrepreneurs get $(1 - \delta)Z_t$.

6. Households buy investment goods ($I_t^h$) and lend it to entrepreneurs via Capital Mutual Fund (CMF)

7. Entrepreneurs buy investment goods ($I_t^e$). The remnants of $(1 - \delta)Z_t$, $w^e_t L^e_t$ and the part of the proceeds having accrued from the previous capital transactions after the purchase of $I_t^e$, are also transformed into investment goods at the respective transformation rates.

8. Entrepreneurs put the investment goods ($I_t^d$) which consists of net worth ($N_t$) and borrowing from household ($I_t^d - N_t$) into the production of capital goods.

9. Each $\omega_t(j)$ is realized. If $\omega_t(j)$ is less than $\bar{\omega}_t$, then the entrepreneurs go bankrupt and the whole returns are confiscated. On average, entrepreneurs get $f(\bar{\omega}_t)I_t^d$. Total amount of capital goods produced is $I_t^d[1 - \mu \Phi(\bar{\omega}_t)]$.

10. At the end of period $t$, the aggregate capital goods production is finished. Now household and entrepreneurs choose $A_{t+1}$ and $Z_{t+1}$ by adding $I_t^d[1 - \mu \Phi(\bar{\omega}_t)]$ to their given capital, $(1 - \delta)A_t$ and $(1 - \delta)Z_t$.

Table 1.1: The Sequence of Events: For simplicity, an entrepreneur is assumed to be an average representative in some part.
<table>
<thead>
<tr>
<th>Parameter/Name</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_k$</td>
<td>0.36</td>
<td>capital share</td>
</tr>
<tr>
<td>$\alpha_h$</td>
<td>0.6399</td>
<td>labor share</td>
</tr>
<tr>
<td>$\alpha_e$</td>
<td>0.0001$^a$</td>
<td>entrepreneurial labor share</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.947</td>
<td>additional discount factor for entrepreneurs</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.02</td>
<td>depreciation rate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.25$^c$</td>
<td>agency cost</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1/EIS</td>
<td>1</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.95</td>
<td>autocorr. coefficient for technology</td>
</tr>
<tr>
<td>$\sigma_\omega$</td>
<td>0.207$^b$</td>
<td>log-standard deviation of $\omega$</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.05</td>
<td>ratio of capital put up as collateral</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.33$^d$</td>
<td>fraction of entrepreneurs</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1.8</td>
<td>coefficient for intersectoral adjustment costs</td>
</tr>
</tbody>
</table>

$^a$ The positive share of entrepreneurial labor guarantees positive net worth with probability one. It is also deliberately chosen so that the entrepreneurial labor and labor income play a very minor role in the production of output and net worth dynamics. This makes the baseline model comparable with the standard RBC model.

$^b$ This is derived from matching the quarterly bankruptcy rate of 0.974 (Fisher (1994)) and the annual risk premium of 187 basis points (for the period April 1971 to June 1996) with the model’s bankruptcy rate and risk premium. In particular, $\gamma$ is selected such that it exactly offset the steady-state internal return.

$^c$ Following Altman (1984), Carlstrom and Fuerst (1997) argue that this is a reasonable estimate of the total (direct and indirect) costs of bankruptcy.

$^d$ Our results are independent of the value of the share of entrepreneurs.

Table 1.2: Parameterization
Households

Firms

Entrepreneurs

Financial Int.

\[ C_t^h \quad I_t^h \quad L_t \quad A_t \]

\[ C_t^e \quad I_t^e \quad Z_t \]

\[ I_t^d = I_t^h + N_t \]

\[ I_t^d f(\bar{\omega}_t), \quad I_t^d g(\bar{\omega}_t) \]

KM Collateral Constraint

CF Financial Friction

leakage

\[ \mu I_t^d \phi(\bar{\omega}_t) \]

Figure 1.1: Circular-Flow Diagram
Figure 1.2: Responses to a News Shock: Baseline Model (Unrealized News)
Figure 1.3: Responses to a News Shock: Baseline Model (Realized News)
Figure 1.4: The Role of Each Element:

1. Existence of collateral constraints gives an incentive to increase current investment.
2. RCC channel
3. DMC channel
4. Investment-initiated boom leads to an increase in the price of capital.
5. Higher leverage exposes an economy to greater agency costs, which leads to an increase in the price of capital.
6. Financial accelerator mechanism amplifies all the responses.
Figure 1.5: Responses to a News Shock: An RBC Model with Intersectoral Adjustment Costs only
Figure 1.6: Responses to a News Shock: An RBC Model with Intersectoral Adjustment Costs and Collateral Constraints on Firms
Figure 1.7: Baseline Model and Perturbed Model (No Financial Frictions on Entrepreneurs)
Figure 1.8: Responses of Financial Variables: Baseline Model
Demand: $q_t = D[I^P_t, E_t \theta_{t+1}, E_t \Omega_t, ...]$  
Supply: $q_t = S[I^S_t, N_t]$  

Figure 1.9: Financial Accelerator Mechanism in the Baseline Model: When a news shock hits the economy.
Demand: $q_t = \frac{1}{\bar{p}_t} D[I_d^P, E_t^{\theta_{t+1}}, E_\theta^{t+1}, \ldots]$  
Supply: $q_t = p_t S[I_s^N, N_t]$  

Figure 1.10: Financial Accelerator Mechanism in the Baseline Model: One period after a news shock hits the economy
Figure 1.11: Baseline Model and Perturbed Model (No Collateral Constraint on Firms)
Figure 1.12: Baseline Model and Perturbed Model (No Intersectoral Adjustment Costs)
Figure 1.13: A RBC Model with Intersectoral Adjustment Costs, Collateral Constraints and Investment Adjustment Costs
Figure 1.14: Baseline model and a Model with Investment Adjustment Costs
2.1 Introduction

A recession is a period during which the conditions in financial markets and goods markets deteriorate and economic uncertainty increases.\footnote{1}{Bloom (2009, [9]) and Bloom et al. (2009, [10]) provide empirical evidence that economic uncertainty is countercyclical.} For example, the supply of credit from financial institutions is reduced with the bad financial environment during a recession. In goods market, firms face higher adjustment costs when they make capital adjustment than in good times.\footnote{2}{The anecdotal examples that the previous studies take for explaining fixed capital adjustment costs are the costs of the planning, budgeting, and committee work that accompany most investment. The re-tooling of a factory, or the temporary closure of a retail store to redesign it are another example. (Gourio and Kashyap (2007, [25])) There is no direct evidence that this types of costs increase in recessions. However, we can infer this from the fact that the number of investment spikes, which are found out to be decreasing in the average size of fixed cost in the simple lumpy investment model, is highly procyclical in data.} In addition, firms sometimes face more disperse adjustment costs in bad times. For example, in the periods of ‘flight to quality,’ once a firm turns out to be financially healthy in the financial market, they can finance their investment more easily than in normal times. A firm that has bad financial rumors, however, should make higher level of effort to convince the
lenders that it is financially sound.\textsuperscript{3} This paper asks how important each of these three factors – (i) financial market conditions, (ii) goods market conditions, and (iii) uncertainty that an individual firm is facing when it makes capital adjustment – is in explaining the cause of a recession and the magnitude of it.

The question is rephrased as follows: Can a negative shock to a financial or real friction in an economy or a more dispersion in the magnitude of the frictions that firms are facing generate a large and persistent recession that we observe in reality? To answer this, I introduce two frictions into a heterogeneous agents DSGE model: nonconvex capital adjustment costs as a real friction and collateral constraints as a financial friction, investigate how the economy responds to a shock to each friction or its distribution, and evaluate the quantitative significance of the responses. I show that the model economy, when it suffers from the aggravated state of the frictions, falls into a quantitatively significant and persistent recession even though there is no drop of the exogenous aggregate total factor productivity. The main mechanism that works here is that the deteriorated frictions prevent the economy more strongly from allocating its resources efficiently, which leads to a fall in the endogenous aggregate total factor productivity. The agents in the economy regard this event as the one where there is a negative shock to the exogenous aggregate total factor productivity and hence, reduce investment and employment. The increase in uncertainty that a firm is facing when it makes capital adjustment, however, is found out to have a limited or dubious influence on economic activities.

The traditional role of financial frictions in the business cycle literature has been mainly the amplification of the responses of an economy when a small shock – mainly a technology or monetary shock – hits the economy. The amplification mechanism has

\textsuperscript{3}In this sense, the fixed capital adjustment costs introduced in this paper is related with the costs that the firms have to pay in financial markets to make capital adjustment. However, since the firms have to pay these costs in terms of labor or other goods, I consider these costs as the costs related to goods market conditions.
been emphasized particularly to deal with the problem that the size of the exogenous shocks that is required to justify such a large aggregate fluctuations as we observe in reality is hardly found in the data (Cochrane (1994, [17]), Summers (1986, [45])). The role of financial frictions in the development literature, on the other hand, has focused more on the perspective of resource allocation. For example, the recent study by Buera and Shin (2008, [11]) highlights how the different level of the financial frictions across the nine countries lead to the different level of misallocation of capital and, hence, level of endogenous total factor productivity, which in turn, causes differences in the per-capita GDP across the the countries. In this paper, I evaluate the importance of the latter role of the financial frictions in the business fluctuations. Thus, I show that the negative shock to financial friction, by itself, can deteriorate the endogenous TFP of the economy by interrupting the efficient allocation of resources when the economy is populated with a large number of agents who are heterogeneous in capital or debt. This aspect would provide the financial frictions with a new role, which has rarely been investigated in the business fluctuation literature.

The existence of real frictions, on the other hand, has been justified in the business cycle literature because it can give the model-generated data some externally consistent properties, that is, the properties which are consistent and compatible with microeconomic (or macroeconomic) evidences found in the real data. The role of real frictions in the development literature, on the other hand, has rarely been investigated. However, if some real frictions also hinder an economy from allocating its resources efficiently, we can expect that the real frictions could have the same

---

4There are, however, some disagreement about the magnitude of the amplification. See Kiyotaki and Moore (1997, [35]), Bernanke et al.(1998, [7]), Cooley et al. (2004, [19]) for advocating the significance of the amplification. See Kocherlakota (2000, [38]), Cordoba and Ripoll (2004, [21]) for the contradictory results.

5For the role of nonconvex capital adjustment costs, see Caballero and Engel (1999, [12]), Cooper and Haltiwanger (2006, [20])
role as mentioned above and, hence, it would be a worthwhile experiment to evaluate quantitatively the magnitude of the aggregate fluctuations coming from the change of the real frictions.

Thus, the goal of my paper is to shed light on a new role of these frictions and evaluate the role, as sources of business fluctuations, quantitatively in a calibrated DSGE setting.\textsuperscript{6} The rest of the paper is organized as follows: in Section 2, I set up and solve my model and outline my calibration strategy; in Section 3, I present the main simulation results. Finally, Section 4 offers some concluding remarks.

2.2 The Model

The model extends the basic real business cycle model to introduce nonconvex costs in undertaking capital adjustment and collateral constraints in firms’ financing as in Kiyotaki and Moore (1997, \cite{35}). The model economy consists of two types of agents: households and firms, each of which has a unit mass. Firms are heterogeneous in their capital and debt, but households are assumed to be identical. The introduction of heterogeneity in the firm-level state vector leads me to use the non-linear solution algorithm suggested by Krusell and Smith (1997, \cite{39}).

\textsuperscript{6}There are few studies that consider a real and financial friction at the same time as main factors in the models to address business cycle issues. The exceptions are Caggese (2007, \cite{13}) and Khan and Thomas (2010, \cite{33}) to which my model is similar. The differences from the latter are that (i) they use investment irreversibility, rather than fixed capital adjustment costs, as a real friction, (ii) include heterogeneity in firm-level productivity to generate endogenously determined aggregate TFP, which is abstracted from in my model. In their model, it is argued that the persistent aggregate fluctuations are attributed largely to the persistent and idiosyncratic property of firm-level productivity. In my model, however, I show that the plausible magnitude of persistency of the aggregates can be generated even though there is no heterogeneity in the firm-level productivity.
2.2.1 Firms

Firms produce a homogeneous output using an increasing and concave production technology, \( y = zF(k, n) \), with capital stock \( k \) and labor \( n \). \( z \) represents exogenous stochastic total factor productivity. I assumed that productivity follows a Markov Chain, \( z = z_1, ..., z_{N_z} \), where \( \Pr(z' = z_m | z = z_l) \equiv \pi_{lm} \geq 0 \), and \( \sum_{m=1}^{N_z} \pi_{lm} = 1 \) for each \( l = 1, ..., N_z \).

At the beginning of each period, a firm is defined by its predetermined stock of capital, \( k \in K \subseteq \mathbb{R}_+ \), and the level of one-period debt it incurred in the previous period, \( b \in B \subseteq \mathbb{R} \). Let the state space \( S \) be the Cartesian product \( S = K \times B \) and \( B_S \) its Borel sigma-algebra. For a measurable space, \((S, B_S)\), and any set, \( S = K \times B \in B_S \), let \( \mu(S) \) be the measure of firms in the set \( S \). Then, the distribution of firms over \((k, b)\) can be denoted by the probability measure \( \mu \) defined on the Borel sigma algebra, \( B_S \).

The aggregate state of economy is then described by \((z, \mu)\). I also assume the the distribution of firms evolves over time according to the mapping, \( \Gamma \), from the current aggregate states; \( \mu' = \Gamma(z, \mu) \).

At the beginning of each period, a firm with \((k, b)\) observes the current total productivity and then makes a series of decisions to maximize the expected discounted value of the current and future dividends returned to its shareholders, the households in the economy. First, it chooses employment and then produces output. And then it pays wages and repays the debt. Just before it makes a decision about investment, the firm learns whether it will survive to continue its production activity in the next period.\(^7\) Conditional on survival, the firm chooses its investment, \( i \), current dividends, etc.\(^7\)

\(^7\)The exogenous exit and entry in the model are introduced to prevent all the firms from growing enough that none will experience the binding collateral constraints. In particular, I assume that \( \pi_d \in (0, 1) \) is the fixed probability that the firms will be forced to exit after production. The equal number of firms are assumed to replace the exiting ones. Each new firm that enters economy is assumed to start its business with zero debt and a fixed \( \chi \) fraction of the typical capital stock held.
$D$, and the level of debt with which it will enter into the next period, $b'$. When it makes investment and borrowing decisions, it must consider the following two frictions in the economy.

**Nonconvex Capital Adjustment Costs**

After learning that it can exist in the next period, the firm must decide whether it adjusts its capital to the desired level by paying some fixed cost, $\xi \in [0, B]$, or simply allows capital depreciate without any positive or negative investment, hence avoiding the fixed cost. The fixed cost is assumed to be drawn from a time-invariant distribution, $G(\xi)$, common across firms and denominated in hours of labor. The law of motion of capital and the adjustment cost, which is measured in units of output using the wage rate $\omega$, for each case can be summarized as follows:

<table>
<thead>
<tr>
<th>Fixed Cost paid</th>
<th>Law of Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i \neq 0$:</td>
<td>$w\xi\gamma k' = (1 - \delta)k + i$</td>
</tr>
<tr>
<td>$i = 0$:</td>
<td>$0\gamma k' = (1 - \delta)k$</td>
</tr>
</tbody>
</table>

Throughout the paper, primes indicate one-period-ahead values, and all variables measured in units of output are deflated by the trend level of labor augmenting technological progress.\(^8\)

**Financial Frictions**

Along with the real frictions mentioned above, firms also face financial constraints when they make investment and borrowing decisions. This collateralized borrowing across all firms in the long-run of the full economy; that is, $k_0 = \chi \int k\bar{\mu}(d[k \times b])$, where $\bar{\mu}$ represents the steady state distribution.

\(^8\)I assume efficiency units of labor grow at the exogenous rate $\gamma^{1-\theta} - 1$, where $\theta$ is capital share of output. In this case the trend growth rate for output is $\gamma - 1$. 

62
constraint takes the following form

\[ b' \leq \theta_b k \]  

(2.1)

where \( \theta_b \in \mathbb{R}_+ \). As a result, only some fraction of capital can be used as collateral and it is the upper bound of the firm’s debt.

**Firm’s Problem**

Let \( v_0(k, b; z, \mu) \) represent the expected discounted value of a firm that enters the period with \((k, b)\) when the aggregate state of economy is \((z, \mu)\) but does not know yet whether it will survive into next period and what value of fixed adjustment cost it should have to pay for undertaking investment. Then the firm’s dynamic optimization problem can be defined as the following functional equations, (2.2)-(2.6) below.

\[
v_0(k, b; z, \mu) = \max_n \left[ zF(k, n) - w(z, \mu)n + (1 - \delta)k - b \right] 
+ (1 - \pi_d)v(k, b; z, \mu)
\]  

(2.2)

Each line of the equation (2.2) corresponds to the case in which the firm exits or survives, respectively. If it is forced to exit, the firm simply chooses labor, given predetermined capital stock, to maximize its current dividend payment, which will be the residual from output and undepreciated capital, less wage payment and debt repayment. If the firm knows that it will survive into the next period, it should choose labor, dividend, next-period capital and debt before the fixed adjustment cost of capital is drawn. That is, \( v(k, b; z, \mu) \) is the expected discounted value of a firm after it recognizes it will survive, but before it knows how much the fixed adjustment cost of capital will be. Let \( v^1(k, b, \xi; z, \mu) \) be the expected discounted value of a firm
after the fixed adjustment cost for the firm, $\xi$ is drawn, then,

$$v(k, b; z, \mu) \equiv \int_0^B v^1(k, b, \xi; z, \mu) G(d\xi)$$ (2.3)

After $\xi$ is drawn, the firm’s value is determined as the result of the following binary choice problem.

$$v^1(k, b, \xi; z, \mu) = \max \left\{ v^A(k, b, \xi; z, \mu), v^{NA}(k, b, \xi; z, \mu) \right\}$$ (2.4)

where $v^A$ is the expected discounted value of a firm which decides to adjust its capital to the desired level with the payment of the fixed cost whereas $v^{NA}$ is the value of a firm which decides not to adjust. We can define each of these value functions as follows. Assume that $d_m(z, \mu)$ is the discount factor applied by firms to their next-period expected value if the aggregate productivity at that time is $z_m$ and the current aggregate state is $(z, \mu)$. In each case, a firm chooses employment, the debt with which it will enter into the next period, along with the binary decision on investment, which, finally, determine the level of current dividends to maximize its expected discounted dividends. We use $q$ as a discount rate applied to the bond that the firm issues. That is, for each unit of debt it incurs for next period, a firm receives $q$ units of output. Note that the price of bond, $q(z, \mu)$ is also a function of aggregate state, as is the wage rate. Then, the dynamic optimization problem for each case is as follows

$$v^A(k, b, \xi; z, \mu) = \max_{n, k', b', D} \left[ D + \sum_{m=1}^{N_z} \pi_{lm}^z d_m(z, \mu) v^0(k', b'; z_m, \mu') \right]$$ (2.5)

To be consistent with the discrete Markov process defined above, the current total factor productivity should be denoted by $z_l$, rather than $z$. For notational convenience, however, I denote it by $z$, hereafter.
subject to  \( 0 \leq \Delta \leq zF(k, n) - w(z, \mu)n + q(z, \mu)b' - b - [\gamma k' - (1 - \delta)k] - w \xi \)

\[ b' \leq \theta b k \]

\[ v^N_A(k, b, \xi, z, \mu) = \max_{n, b, D} \left[ D + \sum_{m=1}^{N} \pi_{lm} d_m(z, \mu) \psi^0 \left( \frac{1 - \delta}{\gamma} k, b'; z_m, \mu' \right) \right] \]  \hspace{1cm} (2.6)

subject to \( 0 \leq \Delta \leq zF(k, n) - w(z, \mu)n + q(z, \mu)b' - b \)

\[ b' \leq \theta b k \]

Note that the problem for each case has the two more additional constraints, along with the budget constraint: (i) dividends should be non-negative\(^{10}\) and (ii) the size of debt issued is bounded above the level of collateral. Also notice that, during the whole process of the firm’s maximization, the firm’s optimal employment decision is independent of not only the current level of debt but also its continuation value because the adjustment cost the firm draws, and the decision about the next period capital and debt do not affect the current production. As a result, the optimal choice for employment can be denoted by \( N(k; z, \mu) \) and the associated production by \( y(k; z, \mu) \), which means that the firms which are at the same level of capital, \( k \), hire the same level of employment and produce the same level of output. The choice for the capital and the debt for the next period is, however, affected by the current level of debt and the adjustment cost and hence are denoted by \( K(k, b, \xi; z, \mu) \) and \( B(k, b, \xi; z, \mu) \), respectively.

\(^{10}\)In Aiyagari (1994, [1]), the assumption of the non-negative consumption, along with one of the the Inada conditions, \((\lim_{c \to +0} u'(c) = \infty)\), leads to the existence of the natural debt limit in incomplete market. In other words, when agents keeps this limit, the possibility of negative consumption is ruled out. Therefore, the assumption is essential for the existence of optimal solution of the maximization problem. The non-negative constraint for dividend here, with an exogenous debt limit (that is, the collateral constraint), leads to the existence of the maximum debt threshold and the maximum debt policy, each of which will be explained in the following sections. In this sense, it has nothing to do with the existence of the optimal solution.
2.2.2 Households

Households are identical. Households own firms by holding one-period shares, which will be denoted by the measure $\lambda$.\(^{11}\) Let $\rho_0(k, b; z, \mu)$ be the price of shares of the firms with $(k, b)$ at the beginning of each period, given the current aggregate state is $(z, \mu)$ and $\rho_1(k', b'; z, \mu)$ the price of new shares of the firms with $(k', b')$. Then, the representative household who maximizes her discounted lifetime expected utility in the economy solves the following Bellman equation:

$$V^h(\lambda; z, \mu) = \max_{c, n^h, \lambda'} \left[ U(c, 1 - n^h) + \beta \sum_{m=1}^{N_z} \pi^z_{lm} V^h(\lambda'; z_m, \mu') \right]$$  \hspace{1cm} (2.7)

subject to

$$c + \int_S \rho_1(k', b'; z, \mu) \lambda' (d[k' \times b']) \leq w(z, \mu) n^h + \int_S \rho_0(k, b; z, \mu) \lambda (d[k \times b])$$  \hspace{1cm} (2.8)

Let $C^h(\lambda; z, \mu)$ be the household choice of current consumption, and let $N^h(\lambda; z, \mu)$ be its labor supply. Finally, let $\Lambda^h(k', b', \lambda; z, \mu)$ be the quantity of shares the household purchase in firms that begin the next period with $k'$ units of capital and $b'$ units of debt.

2.2.3 Recursive Competitive Equilibrium

A recursive competitive equilibrium in this economy is a set of functions

$$\left\{ w, q, (d_m)_{m=1}^{N_z}, \rho_0, \rho_1, v^0, N, K, B, V^h, C^h, N^h, \Lambda^h \right\}$$  \hspace{1cm} (2.9)

such that:

\(^{11}\)So $\lambda$ measures how many shares are held for each firm with type $(k, b)$. 

(i) $v^0$ solves (2.2)-(2.6) and $N$ is the associated policy function for exiting firms, and 
$(N, K, B)$ are the ones for continuing firms.

(ii) $V^h$ satisfies (2.7) and $(C^h, N^h, \Lambda^h)$ are the associated policy functions for households.

(iii) $\Lambda^h(k', b', \lambda; z, \mu) = \mu'(k', b')$, for each $(k', b') \in B_s$.

(iv) $N^h(\lambda; z, \mu) = \int S \left( N(k; z, \mu) + \int_0^B \xi J \left( \frac{1-\delta}{\gamma} k - K(k, b, \xi; z, \mu) \right) G(d\xi) \right) \\
\mu(d[k \times b])$, where $J = 0$ if $x = 0$; $J = 1$ if $x \neq 0$.

(v) $C^h(\lambda; z, \mu) = \int_S \int_0^B \left[ zF(k; N(k; z, \mu)) - (1 - \pi_d)J(K(k, b, \xi; z, \mu) - \frac{(1-\delta)}{\gamma} k) \right. \\
(\gamma K(k, b, \xi; z, \mu) - (1 - \delta)k + \pi_d[(1-\delta)k - k_0]) G(d\xi) \mu(d[k \times b])$, where $J = 0$ if $x = 0$; $J = 1$ if $x \neq 0$.

(vi) $\mu'(D) = (1 - \pi_d) \int_{\{(k,b)|(K(k,b,\xi;z,\mu),B(k,b,\xi;z,\mu))\in D\}} G(d\xi) \mu(d[k \times b]) + \pi_d H(k_0)$, for all $D \in B_s$, defines $\Gamma$, where $H(k_0) = \{1$ if $(k_0, 0) \in D; 0$ otherwise $\}$

Following Khan and Thomas (2008, [32]), I use the above market clearing conditions and the values of household consumption and hours worked, to obtain a single Bellman equation which describes the same allocation results as the recursive competitive equilibrium as follows. Defining $p(z, \mu)$ as an output price at which firms value current dividends and using the result of household maximization problems, I obtain:

$$
p(z, \mu) = D_1 U(C, 1 - N)$$

$$w(z, \mu) = \frac{D_2 U(C, 1 - N)}{p(z, \mu)}$$

$$q(z, \mu) = \beta \sum_{m=1}^{N_z} \pi_{im} \frac{p(z_m, \mu')}{p(z, \mu)}$$

67
Then, we can reformulate the firm’s problem, (2.2)-(2.6) by multiplying each side of the equations by \( p(z, \mu) \), which yields an equivalent set-up of a firm’s dynamic optimization problem where each firms’s value is now measured in units of marginal utility, rather than output. Suppressing the argument of the price functions for simplicity and using indicator function \( J(\cdot) \), the problem can be defined as:

\[
V^0(k, b; z, \mu) = \pi_d \max_n p[zF(k, n) - wn + (1 - \delta)k - b] + (1 - \pi_d)V(k, b; z, \mu) \tag{2.13}
\]

where

\[
V(k, b; z, \mu) \equiv \int_0^B V^1(k, b, \xi; z, \mu)G(d\xi) \tag{2.14}
\]

and

\[
V^1(k, b, \xi; z, \mu) = \max_{n,k',b',D} \left[ pD + \sum_{m=1}^{N_z} \pi^{z_m}_{lm} V^0(k', b'; z_m, \mu') \right] \tag{2.15}
\]

subject to

\[
0 \leq D \leq zF(k, n) - w(z, \mu)n + q(z, \mu)b' - b - J(k' - (1 - \delta)k' + \gamma k') + w\xi
\]

\[b' \leq \theta_b k\]

### 2.2.4 Decision Rules

From previous discussion, it is obvious that the firms having the same \((k, b)\) enjoy the common current profit \( \pi(k, b) \) since they choose the same employment and hence produce the same output with the repayment of the same debt. I define the profit as:

\[
\pi(k, b)^{12} \equiv \pi(k, b; z, \mu) = zF(k, N(k; z, \mu)) - w(z, \mu)N(k; z, \mu) - b \tag{2.16}
\]

\(^{12}\)For notational convenience, I will suppress the \( z, \mu \) arguments of the firm-level state vector and in price in some instances below.
To get decision rules for the firms in the economy, we categorize the firms into the two groups: the firms which are financially constrained because they do not have enough wealth yet and the ones which are wealthy enough to be financially unconstrained now and forever.

**Constrained Firms’ Problem**

It can be shown that, in the presence of this type of financial constraints, as long as the sum of the present and the expected value of future multipliers on the collateral constraints is not zero, the multiplier on the present non-negative dividend constraint (or no-equity issuance constraint) is greater than zero. This means that if a firm thinks that its collateral constraint is binding currently or will be binding for some possible future states, the non-negative dividend constraint binds so that the firm never pays dividend now \((D = 0)\).\(^{13}\) This fact simplifies the above problem, since in this case, the firm’s choice of \(k'\) directly implies its next-period debt, \(b'\). We call these type of firms *constrained* firms. For this type of firms, the equation (2.15) and its constraints turn out to be:

\[
V^C(k, b, \xi; z, \mu) = \max_{k' \geq 0} \beta \sum_{m=1}^{N_z} \pi^z_{lm} V^0(k', b'; z_m, \mu') \\
\text{subject to } \quad b' = \frac{1}{q} \left[ -\pi(k, b) + J(k' - \frac{(1-\delta)k}{\gamma}) \left( [\gamma k' - (1-\delta)k] + w\xi \right) \right] \\
\quad b' \leq \theta_b k
\]

and the equations (2.13),(2.14) are defined correspondingly.

**Unconstrained Firms’ Problem**

\(^{13}\)See, for example, Caggese (2007, [13]).
If the sum of the present and the expected value of future multipliers on the collateral constraints is zero, the multiplier on the present non-negative dividend constraint is also zero. That is, if a firm is wealthy enough by accumulating its capital or savings (increasing $k$ or decreasing $b$) so that its collateral constraint is not binding currently and will never be binding for all possible future states, the non-negative dividend constraint does not bind. This means that there could be the case where $D > 0$, which implies that the firm is indifferent between dividend payment and saving. Without loss of generality, I assume that the firm does not incur debt in the current and all the future date ($b' = 0$) and give all the dividends to the shareholders.\footnote{This, of course, is assumed to make the unconstrained firms' problem simple so that we can define the value function of unconstrained firms more easily. This does not imply that the unconstrained firms never incur debt in my model economy. For example, in some bad states, there could be a case where a firm's optimal capital decision forces it to choose negative dividends if the firm does not incur the next-period debt. In that case, in order not to violate non-negative constraint for dividends, the unconstrained firms need to finance their investment from issuing debt. But, since these firms are unconstrained and, hence Miller and Modigliani theorem is applied to them, we can think of this situation equivalent to the case where the amount of money they get by incurring debt is financed by the same amount of negative dividends, that is, equity. Therefore, the firm's value does not depend on the fact that we assume that $b' = 0$ and firms give all the dividends (regardless of whether it is positive or negative) to shareholders. To sum up, even though we assume that $b' = 0$ when we calculate the value function because the assumption is justified by Miller-Modigliani theorem and gives us computational convenience, we do not need to maintain this assumption when we choose the plausible set of policy that the firms select in the model. In the later section, I derive the firms' policy about the decision for the next-period debt which makes the firms stay unconstrained without violating the non-negative constraint for dividends. Specifically, the unconstrained firms in my model are assumed to follow the maximum debt policy (or the minimum savings policy equivalently), in order to stay unconstrained in the simulation implemented in this paper. The policy, by definition, makes the firms unconstrained. So in the model, unconstrained firms have some amount of debt or saving. But again, as long as the firms keep this policy so that they are unconstrained, the value of these firms can be derived from the alternatively equivalent policy where the next-period debt is transformed into negative dividends.} We refer to this type of firms as unconstrained firm. The optimization problem for this type of firms is:
where

\[ W^0(k, b; z, \mu) = \pi_d p \left[ \pi(k, b) + (1 - \delta)k \right] + (1 - \pi_d) W(k, b; z, \mu) \] (2.19)

and

\[ W(k, b; z, \mu) \equiv \int_{0}^{B} W^1(k, b, \xi; z, \mu) G(d\xi) \] (2.20)

This problem can be simplified more by using the fact that the value of a firm, which has just been unconstrained with some debt (or savings) \( b \neq 0 \), is actually linearly reduced (or raised) by the associated reduction (rise) in current dividends. Thus, we can alternatively express the value of any unconstrained firm of type \((k, b, \xi)\) given \((z, \mu)\) as follows.

\[ W^1(k, b, \xi; z, \mu) = W^1(k, 0, \xi; z, \mu) - pb \equiv w^1(k, \xi; z, \mu) - pb \]

This can be done to the firm’s beginning-of-period expected value: \( W^0(k, b; z, \mu) = w^0(k; z, \mu) - pb \), where \( w^0(k; z, \mu) \equiv W^0(k, 0; z, \mu) \). Defining the unconstrained firms’ value functions as above helps us solve the original problem without considering firms’ debt.
Unconstrained Firms’ Decision Rules

To get the decision rules for the *unconstrained* firm, I represent the equation (2.18) as follows:

\[
W^1(k, b, \xi; z, \mu) = p\pi(k, b) + p(1 - \delta)k \\
+ \max \left\{ -\xi wp + \max_{k'} \left( -\gamma k'p + \beta \sum_{m=1}^{N_z} \pi_{lm}^z w^0(k'; z_m, \mu') \right) \right\} \\
- \left( p(1 - \delta)k + \beta \sum_{m=1}^{N_z} \pi_{lm}^z w^0 \left( \frac{(1 - \delta)}{\gamma} k; z_m, \mu' \right) \right)
\]

where

\[
w^0(k; z, \mu) = \pi_d p \left[ \pi(k, b) + b + (1 - \delta)k \right] + (1 - \pi_d) w(k; z, \mu)
\]

and

\[
w(k; z, \mu) \equiv \int_0^B w^1(k, \xi; z, \mu) G(d\xi) \equiv \int_0^B W^1(k, 0, \xi; z, \mu) G(d\xi)
\]

I define the gross value of undertaking active capital adjustment as

\[
E(z, \mu) = \max_{k'} \left( -\gamma k'p + \beta \sum_{m=1}^{N_z} \pi_{lm}^z w^0(k'; z_m, \mu') \right)
\]

Here, we can see that the target capital solving this maximization problem is independent of \(k\) and \(\xi\). As a result, all *unconstrained* firms that actively adjust their capital choose a common level of capital next period, \(k' = k^*(z, \mu)\). Returning to the binary decision problem in the second and third line of the equation (2.22), we
can see that, given $(z, \mu)$, there is some $k$-specific threshold value $\hat{\xi}_k = \hat{\xi}(k; z, \mu)$ that equates these two lines.

$$-\hat{\xi}_k w(z, \mu) p + E(z, \mu) = -p(1 - \delta)k + \beta \sum_{m=1}^{N_z} \pi_{lm} w_0 \left( \frac{(1-\delta)}{\gamma} k ; z_m, \mu^l \right)$$

Next, define $\bar{\xi}(k; z, \mu) \equiv \min\{B, \max\{0, \hat{\xi}_k\}\}$ so that $0 \leq \bar{\xi}(k; z, \mu) \leq B$. Then, firms with adjustment costs at or below $\bar{\xi}_k$ will adjust their capital and the ones with adjustment costs above this will simply allow its capital stock to depreciate. Thus, the decision rules for the unconstrained firms with firm-level state vector $(k, b, \xi)$ given the aggregate state vector, $(z, \mu)$ are:

$$k' = K^W(k, \xi; z, \mu) = \begin{cases} 
  k^*(z, \mu) & \text{if } \xi \leq \bar{\xi}(k; z, \mu) \\
  \frac{(1-\delta)}{\gamma} k & \text{if } \xi \geq \bar{\xi}(k; z, \mu)
\end{cases} \quad (2.26)$$

As mentioned in the previous section, in the model, the unconstrained firms should adopt the maximum debt (or minimum savings) policy to stay unconstrained and not to violate the non-negative dividend constraint, even though I assume that $b' = 0$ when I derive the value function. How can we get the conditions that satisfy these two constraints? Intuitively, if a firm is wealthy enough that his current debt, $b \leq \hat{B}$ and adopts some prudent debt financing policy small enough that $b' \leq B^W$, the firm’s collateral constraint is not binding currently and will never be binding for all future possible states, even without equity financing. Then what are $\hat{B}$ and $B^W$? How are they related?
To answer these questions, let’s define profits after debt repayment and the optimal investment expenditure as

\[ P^W(k, b, \xi; z, \mu) \equiv \pi(k, b) - J(K^W(k, \xi; z, \mu) - \left(1 - \delta\right) k (\left[\gamma K^W(k, \xi; z, \mu) - (1 - \delta) k\right] + w \xi) \tag{2.27} \]

Then, define \( \hat{B}(k, \xi; S) \) as the beginning of period maximum (threshold) debt level that enables a firm to be unconstrained, given the firm-level vector, \((k, \xi)\) and the aggregate state, \( S \equiv (z, \mu) \), when the firm choose the maximum debt policy, \( b' = B^W(k, \xi; S) \). That is, if a firm’s debt is less than \( \hat{B} \), its collateral constraint is not binding currently and will not be for all possible future states. Then:

\[ \hat{B}(k, \xi; S) = P^W(k, 0, \xi; S) + q(S) \min \{ B^W(k, \xi; S), \theta_b k \} \tag{2.28} \]

where \( B^W(k, \xi; S) \) is defined as the maximum debt (or minimum savings) policy, which ensures that, given \( S \), a firm of type \((k, \xi)\) will be unconstrained across all possible future states, \((\xi', S')\).\(^{15}\) That is,

\[ B^W(k, \xi; S) \equiv \min_{\{\xi' | G(d\xi) > 0 \text{ and } z_m | \pi_{lm} > 0\}} \hat{B}(K^W(k, \xi; S), \xi' ; z_m, \mu') \tag{2.29} \]

Notice that the equation (2.28) and (2.29) define the beginning of period maximum debt level and the maximum debt policy recursively. Also notice that the firm’s dividend policy, \( D^W \) can be derived from the budget constraint, \( D^W = P^W + qB^W \).

\(^{15}\) An unconstrained firm is, by definition, a firm whose collateral constraints is not binding currently and for all possible future states. In this sense, if a firm gets unconstrained, using the maximum debt policy, it stays unconstrained forever. That is, the unconstrained state is the absorbing state. The constrained state, however, is not absorbing. That is, a constrained firm can become unconstrained, depending on the aggregate states and its individual state. Also note that a firm whose collateral constraint is not binding currently, could get constrained in the next period, if it has sufficiently small saving (or sufficiently large current debt), which forces it to incur the debt greater than the maximum debt policy. This type of firm, however, is not an unconstrained firm, by definition of an unconstrained firm.
Equation (2.28) is simply saying that the maximum level of current debt that permits a firm to be unconstrained is increasing in the current profit under optimal capital decision before debt payment and the amount of the debt financing from the maximum debt policy which makes the firm unconstrained for all future states.

To understand the implication of the equation, let’s first think about the case where \( B^W \leq \theta_b k \) so that it is financially feasible to adopt the policy, \( B^W \) if the current level of debt is equal to or less than a certain threshold level which we are trying to derive now. In order to get the maximum threshold level of current debt, given already the optimal capital level and some debt policy for the next period, we should consider the case where we give the shareholders as less dividend as possible, which is zero in this case so that we can pay for the current debt as much as possible. Thus, if \( \hat{B} \) is the threshold, \( D = P^W(k, \hat{B}, \xi; S) + qB^W(k, \xi; S) = 0 \). This condition gives us \( \hat{B}(k, \xi; S) = P^W(k, 0, \xi; S) + qB^W(k, \xi; S) \), which we can get from the equation (2.28). We also see that since \( D = P^W(k, \hat{B}, \xi; S) + qB^W(k, \xi; S) = 0 \), the non-negative dividend constraint is satisfied. And since the firm adopts \( b' = B^W(\leq \theta_b k) \), the collateral constraint is not binding currently and will not be for all possible future states.

Consider the case where \( B^W > \theta_b k \). This is the case where a firm does not have sufficient collateral to borrow \( B^W \). Thus, at the threshold level of current debt we are trying to derive now, the firm borrows \( \theta_b k \). Then, as above, at the maximum threshold level of current debt, it should be the case that \( D = P^W(k, \hat{B}, \xi; S) + q\theta_b k = 0 \). This gives \( \hat{B}(k, \xi; S) = P^W(k, 0, \xi; S) + q\theta_b k \), which we can get from the equation (2.28). We see that since \( D = P^W(k, \hat{B}, \xi; S) + q\theta_b k = 0 \), the non-negative dividend constraint

\[ 16 \text{Refer Appendix B.1 to see a different approach.} \]

\[ 17 \text{As a trivial case, if the firm’s current debt is at the threshold and } b' = B^W = \theta_b k, \text{ the collateral constraint is currently binding (and satisfied) and, since the firm adopts } B^W, \text{ it gets unconstrained in the next period.} \]
is satisfied. When the firm’s current debt is less than this threshold, the firm adopts some level of debt, $b'$ which is less than $\theta_b k(< B^W)$ in this case so that its collateral constraint is not binding currently (and satisfied) and for all possible future states.\footnote{As a trivial case again, if the firm’s current debt is at this threshold, the firm chooses $b' = \theta_b k$. Thus the collateral constraint is currently binding (and satisfied), but since $\theta_b k < B^W$, the firm gets unconstrained in the next period.}

Figure 2.1 is one example of the maximum debt policy of an unconstrained firm, given a specific value of $\xi$ and $z$ around the middle of grid points. First, notice that, at any given level of $k$, the maximum debt, $B^W$ is increasing along $K$ axis, reflecting the fact that an individual firm’s profit is increasing in aggregate capital. That is, the larger the aggregate capital is in an economy, the more profitable a firm gets and, hence, the more debt it can hold. Second, along $k$ axis at any given $K$, $B^W$ is largely constant, except some points. This is because the target capital, $K^W$, is the same for any level of individual capital, if the level of individual capital deviates from $(S, s)$ region.

**Constrained Firms’ Decision Rules**

To derive the decision rules for a constrained firm, we need to determine first whether a firm is unconstrained or not. This can be done by checking whether a firm’s current net wealth, $b$, has exceeded a certain threshold level at which the constrained firms get unconstrained. If a firm’s wealth is under the threshold, the collateral constraint will continue to affect its investment decisions. If there is, for example, a sufficiently favorable aggregate TFP shock at the beginning of the next period, this new state could make a currently constrained firm become unconstrained in the next period for some choices of the next period firm-level vector. Thus, we have to use, for that possible case, the unconstrained firms’ value function when we derive the continuation value of a constrained firm. Then, how can we determine whether, given the next period aggregate state and firm-level states, a firm actually
will be unconstrained? By the recursive structure of the problem, this question is 
the same as the one whether a firm is unconstrained or not. From the discussion in 
the previous section, we can simply see that a firm is unconstrained when its current 
debt is less than \( \hat{B}(k, \xi; S) = P^W(k, 0, \xi; S) + q \min \{ B^W(k, \xi; S), \theta_b k \} \). One thing to 
note is that the constrained firm we are considering now is the firm whose collateral 
constraint is currently binding and \( \theta_b k < B^W \). \(^{19}\)

Let’s denote as \( b^{TW} \) the threshold level of the current debt that divide *unconstrained* and *constrained* firms. Then \( b^{TW} \) is simply \( \hat{B}(k, \xi; S) \) with the restriction, 
\( \theta_b k < B^W \). That is,

\[
b^{TW}(k, \xi; z, \mu) \equiv -J\left(K^W(k, \xi; z, \mu) - \frac{(1 - \delta)k}{\gamma} \right) - (1 - \delta)k + w\xi \\
+ q\theta_b k + y(k, z, \mu) - wN(k, z, \mu) 
\]  

(2.30)

Thus, these *unconstrained* firms, which have \( b < b^{TW} \), adopts the decision rules (2.26) 
- (2.27) and achieve value \( W^1(k, b, \xi; z, \mu) \) from (2.18).

To get the decision rules for the constrained firm, I represent the equation (2.17) 
as follows:

\[
V^C(k, b, \xi; z, \mu) = \max \left\{ \max_{k' \geq 0} \beta \sum_{m=1}^{N_s} \pi^{z}_{lm} V^0(k', b'_A(k'); z_m, \mu'), \beta \sum_{m=1}^{N_s} \pi^{z}_{lm} V^0((1 - \delta)k, b'_NA(k'); z_m, \mu') \right\} 
\]

(2.31)

with

\(^{19}\)If the firm were classified as a constrained firm not because its collateral constraint is currently 
binding but because the constraint will be binding for some possible states, we cannot expect that 
the firm will be unconstrained for all the possible future so that we can not use an unconstrained 
firm’s value function. This implies \( b' = \theta_b k < B^W \).
\[ b_A'(k') = \frac{1}{q} \left[ -\pi(k, b) + \gamma k' - (1 - \delta)k + w\xi \right] \]

\[ b_{NA}'(k') = \frac{1}{q} \left[ -\pi(k, b) \right] \]

subject to

\[ b_A'(k') \leq \theta_b k \]

\[ b_{NA}'(k') \leq \theta_b k \]

Here, there exist, among constrained firms, some very poor firms which have so much debt that they should sell their undepreciated capital to make their current debt and wage payment. That is, there also exists a threshold level of current debt which makes even zero investment not feasible, given \( k \). We can see that this is the case if \([- y(k) + wN(k) + b] > q\theta_b k\). Thus among any group of firms sharing \( k \), only those with debt less than \( b^{TC}(k, \xi; z, \mu) \) can consider the above maximization problem, where the threshold debt level is

\[ b^{TC}(k, \xi; z, \mu) \equiv q\theta_b k + y(k; z, \mu) - wN(k; z, \mu) \]  

(2.32)

Firms with \( b > b^{TC} \) do not solve the problem (2.31) because they cannot choose even zero investment.\(^{20}\) They solve only the first line of the equation (2.31). That is, they solve the first line conditional on the negative investment as follows

\[ V^C_d(k, b, \xi; z, \mu) = \max_{k' \in \Lambda^d(k, b, \xi)} \beta \sum_{m=1}^{N_z} \pi_{lm}^z V^0(k', \left\lfloor -\pi(k, b) + \gamma k' - (1 - \delta)k + w\xi \right\rfloor) / q \]

where

\[ \Lambda^d(k, b, \xi) = \left[ 0, \min\left\{ \frac{(1 - \delta)k}{\gamma}, \frac{(1 - \delta)k - w\xi + q\theta_b k + \pi(k, b)}{\gamma} \right\} \right] \]  

(2.34)

\(^{20}\)There could be a case where zero investment is feasible but only negative investment is possible. This type of firm, however, just solve the problem (2.31)
Notice that the constraint set of $k'$ is bounded above the maximum capital permitted by the borrowing constraint under capital adjustment.

Firms with $b^{TW} \leq b \leq b^{TC}$ solve the problem (2.31), we also identify the maximum capital permitted by the borrowing constraint under capital adjustment, and impose the relevant restrictions on the constraint set of $k'$. The set is:

$$
\Lambda^u(k, b, \xi) = \left[ 0, \frac{(1-\delta)k - w\xi + q\theta_b k + \pi(k, b)}{\gamma} \right]
$$

Substituting in the debt implied by each capital choice and restricting the choice of $k'$ to the set derived above, we can express the value of the constrained firms as follows;

$$
V^C_u(k, b, \xi; z, \mu) = \max \left\{ \max_{k' \in \Lambda^u(k, b, \xi)} \beta \sum_{m=1}^{N_s} \pi^z \pi^m V^0(k', \frac{-\pi(k, b) + \gamma k' - (1-\delta)k + w\xi}{q}; z_m, \mu'), \beta \sum_{m=1}^{N_s} \pi^z \pi^m V^0(\frac{(1-\delta)k - w\xi + q\theta_b k + \pi(k, b)}{\gamma}; z_m, \mu') \right\}
$$

and $V^C_d(k, b, \xi; z, \mu)$. (Either of which will be applied, depending on the level of the current debt.)

where

$$
\begin{align*}
V^0(k, b; z, \mu) &= \pi d p \left[ \pi(k, b) + (1-\delta)k \right] + (1-\pi_d)V(k, b; z, \mu) \\
V(k, b; z, \mu) &= \int_0^B V^1(k, b, \xi; z, \mu) G(d\xi) \\
V^1(k, b, \xi; z, \mu) &= \begin{cases} 
W^1(k, b, \xi; z, \mu) & \text{if } b < b^{TW} \\
V^C_u(k, b, \xi; z, \mu) & \text{if } b^{TW} \leq b \leq b^{TC} \\
V^C_d(k, b, \xi; z, \mu) & \text{if } b > b^{TC} 
\end{cases}
\end{align*}
$$
Here, I define the value of undertaking active capital adjustment as

\[
V^A(k, b, \xi; z, \mu) \equiv \max_{k' \in \Lambda^u(k, b, \xi)} \left( \beta \sum_{m=1}^{N_z} \pi_{im}^z V^0(k', \left[ -\pi(k, b) + \gamma k' - (1 - \delta)k + w \xi \right] q; z_m, \mu') \right)
\]

(2.37)

Here, we can see that the target capital solving this maximization problem is not independent of \(k\), \(b\), and \(\xi\). As a result, all \textit{constrained} firms of type \((k, b, \xi)\) that actively adjust their capital choose their own target capital next period, \(k' = k^*(k, b, \xi; z, \mu)\).

Note that the level of target capital is not uniform across firms, in contrast with the case for \textit{unconstrained} firms.

I also define the value of not adjusting as

\[
V^{NA}(k, b; z, \mu) \equiv \beta \sum_{m=1}^{N_z} \pi_{im}^z V^0((1 - \delta) k, -\pi(k, b) q; z_m, \mu')
\]

(2.38)

Returning to the binary decision problem in the first and second line of the equation (2.36), we can see that a firm of type \((k, b)\) will undertake capital adjustment if its fixed adjustment cost, \(\xi\) falls under a specific value, \(\bar{\xi}(k, b)\). Let \(\hat{\xi}(k, b; z, \mu)\) denote the level of \(\xi\), given current \((k, b)\), that leaves a firm indifferent between

\[
V^A(k, b, \hat{\xi}(k, b); z, \mu) = V^{NA}(k, b; z, \mu)
\]

(2.39)

Next, define \(\bar{\xi}(k, b; z, \mu) \equiv \min\{B, \max\{0, \hat{\xi}(k, b)\}\}\) so that \(0 \leq \bar{\xi}(k, b; z, \mu) \leq B\). Then, firms with adjustment costs at or below \(\bar{\xi}(k, b)\) will adjust their capital and the ones with adjustment costs above this will simply allow its capital stock to depreciate. Thus, the decision rules for the \textit{constrained} firms with firm-level state vector \((k, b, \xi)\) given the aggregate state vector, \((z, \mu)\) are:

80
\[ k' = K^C(k, b, \xi; z, \mu) = \begin{cases} 
 k^*(k, b, \xi; z, \mu) & \text{if } \xi \leq \bar{\xi}(k, b : z, \mu) \\
 (1-\delta) k & \text{otherwise} 
\end{cases} \]  

(2.40)

Given the capital rule, and, using the fact, \( D^C(k, b, \xi) = 0 \), we can derive the firm’s optimal debt rule.

\[ B^C(k, b, \xi; z, \mu) = \frac{1}{q} \left[ J \left( K^C(k, \xi; z, \mu) - \frac{(1-\delta)}{\gamma} k \right) \left[ \gamma K^C(k, \xi; z, \mu) - (1-\delta) k + w \xi \right] \right] - \pi(k, b) \]

(2.41)

### 2.2.5 Calibration

I evaluate the qualitative and quantitative implication of collateral constraints, nonconvex capital adjustment costs and their interaction with uncertainty with some numerical experiments. I fix the length of a period to correspond to one year. This enables me to target plant-level data suggested by Cooper and Haltiwanger (2006, [20]) when calibrating some parameters in my model. I calibrate the benchmark economy, which has both the financial and real friction in it and is called ‘the full economy’ in the following sections, and then use the same parameter values across the reference model economies except the extent of financial frictions, \( \theta_b \), and the upper bound of fixed adjustment costs, \( B \). Most experiments are based on 1,000-period model simulation, and the same random draw of aggregate TFP is used in each.

The representative household’s period utility is assumed to take the form, \( u(c, L) = \log c + \varphi L \), following Hansen (1985, [28]) and Rogerson (1988, [42])). The firm’s production function takes the Cobb-Douglas form, \( zF(k, n) = zk^\alpha n^\nu \). Aggregate productivity is assumed to follow AR(1) process: \( \log z' = \rho_z \log z + \eta'_z \) with \( \eta'_z \sim N(0, \sigma^2_{\eta_z}) \). The value of \( \rho_z \) and \( \sigma_{\eta_z} \) is estimated from Solow residuals measured using National Income and Product Accounts (NIPA) data on U.S. real GDP and private capital, and
the total employment hours series constructed by Prescott, Ueberfeldt, and Cociuba (2005, [41]). This productivity process is discretized with 3 grids points ($N_z = 3$) to obtain ($z_t$) and ($\pi_{lm}$), using Tauchen (1986, [46]) method.

The mean growth rate of technological progress, $\gamma$, is set to imply a 1.6% average annual growth rate of real per capita output, and $\beta$ is chosen to imply an average real interest rate of 4%. The depreciation rate is selected to match an average investment-to-capital ratio of 10 percent. The production parameter, $\nu$, is chosen to imply an average labor share of income, 0.60 (Cooley and Prescott (1995, [18])). Given these values, capital share of income, $\alpha$, and the parameter for the preference for leisure, $\varphi$, is determined by targeting an average capital-to-output ratio and average available time spent on market work. The exit rate, $\pi_d$, is set to be 0.10. The fraction of the steady state aggregate capital stock held by each entering firm, $\chi$, equals 0.10, following the data suggested by Davis and Haltiwanger (1992, [22]). The upper bound of the fixed capital adjustment costs, $B$, is chosen so that the firm level-data generated by the full economy can match the two aspects of establishment-level investment data documented by Cooper and Haltiwanger (2006, [20]). These targets are (i) the average standard deviation of investment rates ($i/k$) across establishment: $\sigma_{i/k} = 0.337$. (ii) the fraction of observations of a positive investment spike ($i/k > 0.20$) in the average year: 0.186. The degree of financial frictions, $\theta_b$, is chosen to target an average debt-to-capital ratio of nonfarm nonfinancial businesses over 1952-2005 in the Flow of Funds, 0.366. The resulting parameter values are presented in Table 2.1.
2.3 Results

2.3.1 The Steady State

Figure 2.2 gives some insight about the life-cycle of the firms in my model economy. A firm start with a low level of capital (about 0.10) and zero debt. As time goes by, it accumulates capital while incurring debt. After accumulating sufficient capital, it goes to the region for the high level of capital and the maximum debt policy. Figure 2.3 shows the stationary distribution of the economy which has only the real frictions. Absent financial frictions, a starting firm can go directly to the region of the target capital and the maximum debt policy, even though the real frictions make some of the firms remain a little far from the target capital.

2.3.2 Decision Rules

To get a better understanding of how these decision rules work, I give an example of the budget set and the objective function of a hypothetical constrained firm with a firm level vector, \((k, b, \xi)\), given an aggregate state vector, \((z, \mu)\). Figure 2.4 represents the budget set of such a firm. Given the aggregate and firm-level state vectors, the firm has a budget set of \(ABC\) and the point \(D\). Note that the line \(AB\) can be represented by the equation, \(k' = \frac{qb' + (1-\delta)k + \pi(k,b) - w\xi}{\gamma}\), or equivalently, \(b' = \frac{\gamma k' - (1-\delta)k - \pi(k,b) + w\xi}{q}\). Also notice that, since this firm has a collateral constraint, the size of the debt it can incur is bounded above \(\theta_b k\) (point \(C\)). If the firm does not adjust its capital, its next period capital should be \(\frac{(1-\delta)}{\gamma} k\) (point \(F\)) and it incurs debt of \(-\frac{\pi(k,b)}{q}\) (point \(G\)). Actually, in this case, it is a loan. Note that this is located outside of \(ABC\) because the firm does not pay a fixed cost for this choice.

The hypothetical constrained firm’s objective function, which is the firm’s expected value function as shown in the constrained firms problem, the functional
equation (2.36), is drawn in Figure 2.5. The function is represented as a sliced sphere in the figure. X-axis, Y-axis, and Z-axis represent the choice of $b'$ and $k'$, and the corresponding expected value of a firm.\textsuperscript{21}

Thus, when the constrained firm solves its maximization problem, the equation (2.36), it compares the value of not adjusting (point $E$) with the maximum value it can get when it adjusts (point $P^*$). Thus, in this case, the firm makes positive investment ($k \rightarrow k^*$) and has a position of net worth, $b^*$. If a firm has a slightly more adjustment cost than in the above case, then the firm would choose inaction (point $E$) since the budget line shifts down so that the point $P^*$ will be located under the point $E$. Thus, depending on the size of the adjustment cost, a firm choose action or inaction.

The change in the current level of debt also shift the budget line. Let’s suppose a firm with a relatively large amount of current debt, $\hat{b}$, holding other states constant. In this case, the budget line shifts down so that the budget constraint becomes $A'B'C$ and the point $D'$ as in Figure 2.6. Note that the wage and debt payment net of output, $-\frac{\pi(k,b)}{q}$, exceeds the borrowing limit of the firm, $\theta_b k$ (point $C$). Thus, even the zero investment is not feasible for this firm and, hence, the firm should sell their capital in this case so that $\hat{P}^*$ would be the optimal choice for the firm.

Figure 2.7 represents the example for the actual decision rules of the two firms which have a low and a high $\xi$,\textsuperscript{22} respectively at the steady state.\textsuperscript{23} The values of the vertical axis of the graph indicate firms decisions. Unconstrained firms are located at the part where the level of capital is relatively high, which is the top left part of the

\textsuperscript{21}Even though a value function should be defined over state vector, I set the arguments here as choice vector in order to make it easy to explain firm’s decisions. Notice that, here, the Z-axis represents expected value function, $\sum_{m=1}^{N} \pi^z_{lm} V^0(k', b'; z_m, \mu')$.

\textsuperscript{22}16.7 percentile and 66.7 percentile, respectively.

\textsuperscript{23}To be exact, these are the decision rules that firms evenly distributed at each mass point adopt when the aggregate TFP is at its unconditional mean.
They choose 5 or 4, which means active capital adjustments and inaction, respectively. Constrained firms are located at the middle (or bottom) right part of the graph, which represents relatively low level of capital. They chooses 2, 1 or -3, which means active capital adjustments, inaction, or negative investment. If we look at the unconstrained firms, we can see that their decision actually does not depend on the level of current debt and that they make active capital adjustments only when the level of current capital is far enough away from the target capital which are located at the middle of the inaction region. If we look at the constrained firms, we can see that their decisions are affected by the level of current debt. For example, if they have a relatively high level of debt given a medium level of capital, the maximum value that the firms can achieve by actively adjusting the level of capital, would be lower than the value of not adjusting because the budget set, the size of which is decreasing in the level of current debt, is very small. When the level of current capital is extremely small, the same thing can happen even if the level of current debt is relatively low, because the budget set is increasing in the level of current capital. The inaction regions which correspond to these two cases are represented by the green part (The vertical axis value is 1). In the region where firms have a very low level of capital and relatively high debt, which is the dark blue part, firms just make negative investment (The vertical axis value is -3). They cannot even choose inaction because they have to pay back a large amount of debt with wage payment by selling their capital. One more thing to note is that the the inaction region becomes larger when a firm draw a higher capital adjustment cost. We can see from Figure 2.7 that the size of the light red (axis value: 4) and green part (axis value: 1) becomes larger for a high $\xi$ firm.

24‘Active’ capital adjustments in this section means positive or negative investment.
2.3.3 Responses to a Technology Shock

Figure 2.9 shows how the economic aggregates respond to the 2 standard deviation of technology shock in the full economy ($B = 0.02, \theta_b = 0.80$) and the reference economy, which has stronger financial friction ($B = 0.02, \theta_b = 0.56$). Across the economies, output, consumption, investment and employment all increase after the shock. As is well known in the literature, a positive technology shock moves the uniform level of desired stock of capital upward and adjustment hazard to the right. So, along with intensive margin, extensive margin of capital adjustment increases, which make aggregate investment increase. Under the collateral constraints introduced, another type of extensive margin should be considered, the margin between unconstrained and constrained firms. That is, positive shock loosens the collateral constraints among the constrained firms (usually small firms), so the number of unconstrained firms in the economy increases. To be exact, a positive technology shock makes the constrained firms at the margin get unconstrained by letting them produce more output than before. This effect amplifies the responses of economy to the positive technology shock. As seen in Figure 2.9, the more restricted an economy gets (lower $\theta_b$) financially, the larger the size of the responses of economy gets.

Table 2.2 presents the business cycle moments of the full and various reference economies. A couple of points should be noted. First, the means of the major economic variables decrease when each frictions are added and strengthened. For example, when we go from the economy without real and financial frictions to my full economy, the mean of output decreases by 0.03 and that of capital falls by 0.16. If the financial frictions are strengthened, they fall further by 0.009 and 0.03, respectively.

When I solve the model, the outerloop simulates the economy 1000-periods during innerloop-outerloop iteration. All the calibration and the business cycle moments that will be given below, are based on this 1000-periods simulation. Given the solution, I simulate the economy 20 times for 30-periods to get the average impulse responses.
Second, we can see that the relative volatility of investment is amplified when we consider financial frictions. We can see this fact by comparing the second and third row, the fourth and fifth row in the column for the relative standard deviation of investment. Thus, the low level of enforcement in financial contracts leads more volatile investment activities.

2.3.4 Responses to a Shock to a Financial or Real Friction

To see the qualitative aspects of my model more, I implement two experiments: What happens to an economy if the economy suddenly changes from the situation where a financial or a real friction is favorable to the one where the frictions become strengthened?\textsuperscript{26} For this, I define the first experiment as follows. In period zero, an economy is at its steady states and hence, the joint distribution of capital and debt is at its stationary equilibrium. In period one, the collateral value of firms’ capital stock suddenly drops so that we can regard this as the case where there is a shock to the financial markets. Agents in the economy now use their value function and the forecasting functions corresponding to the bad financial state and the economy converges to the new steady state for the bad financial state. I consider the case where the value of $\theta_b$ falls by 30\% in the baseline simulation, so $\theta^\text{original}_b = 0.80$ drops down to $\theta^{\text{bad}}_b = 0.56$.

The second experiment is defined as follows. An economy starts at the same original steady states in period zero. In period one, agents face stronger real frictions than before, which means that the maximum possible adjustment cost the firms would

\textsuperscript{26}This experiments can be seen as an analysis of transition dynamics often made in the development literature. So these are the experiment for analyzing transition path from the regime in which each friction is favorable to the regime in which each friction is unfavorable.
have to pay if they want to adjust their capital stock, increases. We can think of this as a shock to the goods markets. Now $B^{\text{original}} = 0.02$ rises to $B^{\text{bad}} = 0.04$.

Figure 2.10 shows the responses of the economy to a financial shock only. Note that there is no change in the exogenous aggregate TFP from its steady state during this experiment. When a financial shock occurs in period one, the expected ratio of capital that can be put up as collateral drops so drastically that the future return to investment decreases, even though the shadow value of future capital as collateral increases due to the tightened financial condition. From the representative household’s viewpoint, the tightened financial condition means more misallocation in capital across firms than in the original condition, so the household expects the return to saving to be low, which discourages the supply of capital.

Through what mechanism does the stronger financial friction cause the drop of endogenous TFP? My model does not have heterogeneity in firm-level productivity. Thus, there is no endogenous movement of aggregate TFP that comes from the disproportionate distribution of capital, for example, the case where too much capital is owned by the relatively low productive firms. The endogenous movement of aggregate TFP in my model during these experiments comes from the fact that the stronger the financial (or real) friction gets, the more firms, who otherwise would have adjusted their capital and operate at the optimal level under a frictionless neoclassical economy, become inactive. Another factor that gives rise to the decrease in the endogenous TFP is that the fraction of the constrained firms in an economy increases under the stronger financial frictions. To see both of these facts, refer to the Figure 2.8, which is an example of the capital choice for the firms in the economy with stronger financial frictions. First, we can see that the size of unconstrained firms’

27 We can also think of this real shock in connection with financial shock. For example, for the firms which have to sell their capital because of the debt payment or the reduction of its credit limits during a financial crisis, the increased fixed costs can be considered as capital loss coming from dumping their equipment.
regions in Figure 2.8 is smaller than that of the region in Figure 2.7 for the same size of $\xi$. We can also see that the inaction region for the constrained and unconstrained firms under the stronger financial friction is much larger than in the full economy case. Note that not only the inaction region of the constrained firms but also that of the unconstrained firms are larger under the stronger financial friction. For the constrained firms’ case, this is evident because the stronger financial friction means the smaller budget set, which leads the constrained firms, who otherwise could have achieved the higher value by adjusting their capital in the larger budget set, to remain inactive under the shrunken budget set. The reason why the unconstrained firms’s inaction region shrinks is because the return to investment under stronger financial friction is lower than that under the weak financial friction. This causes the threshold fixed cost, $\xi(k; z, \mu)$, which is the maximum cost a firm is willing to pay, to fall at any given state $(k, z, \mu)$. Thus, other things being equal, more firms, which get now disappointed about the decreased return to investment, remain inactive. In other word, the value of adjusting gets lower when the financial friction gets stronger, so even when an unconstrained firm draws the same size of fixed costs as before, the firm which otherwise would have adjusted capital, does not adjust it.

To sum up, the expansion of the inaction regions and the increase in the fraction of the constrained firms under the stronger financial friction means that more firms are now giving up the opportunity to operate their businesses at the optimal level of capital in the frictionless economy. As a result, the endogenous TFP and hence the return to saving drop.

Returning to the dynamics, the reduced return to investment and saving leads to a sharp drop in investment in equilibrium, which causes a small increase in consumption. The reduction in the current return to capital also discourages the firms from hiring. (With the capital predetermined, the reduction in employment is the only
way to decrease the marginal return to capital.) This increases real wages, which is consistent with the increase in consumption coming from both the substitution and income effect. With the capital predetermined, the reduction in employment decreases output. Output, investment, and employment drop further by period three and then rebound to its new steady state level. Consumption and capital show significantly persistent negative responses. The rebound shown in investment and labor seems to come from firms’ effort to outgrow financial constraint.

Figure 2.11 represents the case where there is a real shock only. The responses are similar to the case for a financial shock, even though the size of the response is an order of magnitude smaller. The higher level of fixed capital adjustment costs means that more firms become inactive, which will lead to a fall in the endogenous aggregate TFP. According to the logic similar to the above case, the aggregate investment decreases, the level of capital gradually drops and hence, the economy experience a persistent recession.

Figure 2.12 is the case where there is a shock to both the financial and real frictions. When combined, these two types of shock generate a quantitatively significant and persistent recession.

Figure 2.13 represents the case where an economy is fully recovered from the financial crisis after period 5. So for this experiment, I replace the value of $\theta_b$ and agent’s value function by the original ones at period 5. Even though absent the heterogeneity in firm-level productivity across the firms, the persistence of recession looks strong, because it takes much time for the aggregate capital to reach its original steady state level. Note that it takes almost 10 years for consumption and output to recover their original steady state level. This is, of course, because the financial constraints in the economy, along with the real frictions, prevent the firms from adjusting their capital quickly.
2.3.5 Responses to a Shock to the Uncertainty of a Real Friction

The next experiment asks the following question: When firms are confronted with higher uncertainty regarding the magnitude of the fixed capital adjustment costs, can this generate a quantitatively significant recession? In other words, if an economy faces a more disperse distribution of the fixed capital adjustment costs, can this alone generate a quantitatively significant recession? A question similar to this is answered recently by Bloom (2009, [9]), Bloom et al. (2009, [10]) and Bachman and Bayer (2009, [3]) empirically and theoretically. They try to evaluate the effect of uncertainty in an economy. Bloom (2009, [9]) and Bloom et al. (2009, [10]) suggest various empirical evidence that shows that economic uncertainty, which is measured, for example, as the second moment of the aggregate TFP or firm-level Solow residuals, has a countercyclical pattern and, hence, negative correlation with the main economic aggregates. To explain this phenomenon, Bloom (2009, [9]) builds a decision theoretic model of firms with various types of labor and capital adjustment costs (both convex and nonconvex), analyzes the effect of uncertainty shock on firms’ investment decision and finds that uncertainty shocks generate short sharp recessions and recoveries. Bloom et al. (2009, [10]) extends this model into a general equilibrium environment. Including nonconvex labor and capital adjustment costs in an otherwise RBC model, they find that the transition from low uncertainty state to high uncertainty state gives rise to about 2% drop of output and employment without a decrease in the aggregate TFP. In both papers, the main mechanism by which uncertainty affects an economy is ‘the wait-and-see (or real option) effect.’ That is, when uncertainty increases, the value of waiting without action for investment and employment increases and, hence,
inaction region of current state of capital and labor expands, which results in the negative investment and employment by their depreciation.

Bachman and Bayer (2009, [3]) propose some empirical evidence that is a little bit different from the above. Their empirical VAR results show that consumption decreases, but investment and employment increase upon the shock and then decrease, whereas in Bloom (2009, [9]), all the aggregate variables fall upon the shock. They also use a model a la Khan and Thomas (2008, [32]) to generate these empirical results. The main difference in their model from the previous studies is that they consider only the volatility of firm-level Solow residuals as a source for the uncertainty shock because they find that only the firm-level uncertainty is empirically important in German data (USTAN). And then they use an additional restriction on the relationship between the process of the aggregate TFP shock and the firm-level volatility.

The main difference in my experiment from the previous studies mentioned above is that I do not consider the uncertainty shock as the shock to the second moments of the aggregate or the firm level total factor productivity. Rather than considering the shock to the second moment of productivity, I focus on the second moment of the fixed capital adjustment cost. By excluding the volatility of productivity as a source of uncertainty, my model can be free from the implausible argument that technological regress or the possibility thereof causes recessions.28 Second, even though the previous studies show empirically that the size of these seconds moments is procyclical, it is not clear whether the procyclicality is the cause of recessions or the result of recessions. By introducing the more explicitly exogenous shocks as defined above in my model, my model can also be free from this kind of criticism.

28Even though the average total factor productivity remains the same in the experiments of Bloom et al. (2009, [10]) and Bachman and Bayer (2009, [3]), agents in those economies predict more disperse aggregate or firm-level productivity. This means that the agents predict technological regress in aggregate or firm level with some probability.
So now, the fixed cost, \( \xi \), is assumed to follow the uniform distribution defined on \([A, B]\).^29 For the low uncertainty case, I use \( A = 0.03, B = 0.05 \) and for the high uncertainty case, \( A = 0, B = 0.08 \). This parameterization implies that the variance of the high uncertainty economy is 16 times as big as that of the low uncertainty economy with the mean held constant. All other parameters are assumed to be identical to those for the full economy case in the previous chapters.

The experiment is implemented similarly as the first and the second ones. An economy starts at the original low uncertainty steady state in period zero. In period one, agents face a more disperse distribution of the fixed costs. Agents in the economy now use their value function and the forecasting functions corresponding to the high uncertainty state and the economy converges to its new high uncertainty steady state.

Figure 2.14 shows the responses of the economy. The major aggregates falls upon the shock. The magnitudes of response, however, are extremely limited. The reason why an increase in uncertainty in the fixed capital adjustment costs potentially leads to a recession is because it could generate more disperse (or unequal) distribution of capital, which could lead to more inefficient allocation of capital. To see why this happens, refer to Figure 2.15 and 2.16 which represent the probability density function for \( \xi \), the adjustment hazard rate, \( G(\xi) \), and the distribution over capital, \( \mu(k) \) for the two economies that has two different degrees of uncertainty.^30 We can see that, in the high uncertainty economy, the adjustment hazard rate is flatter than in the low uncertainty economy. This is because the probability density for the high uncertainty economy is more disperse. As a result, given a threshold adjustment cost greater than the mean of it, for example, less firms adjust their capital at relatively

---

29In this case, the first and second moment of \( \xi \) are simply, \( \frac{B+A}{2}, \frac{(B-A)^2}{12} \).

30For expositional convenience, I present the case where there is no financial frictions. In the presence of financial frictions, the adjustment hazard rate for the constrained firms should also be taken into account, which will make it harder to understand the underlying mechanism intuitively.
low levels of current capital and hence more firms remain at those levels. But due to the symmetry of the distribution of the adjustment cost, this effect is partially offset.\footnote{In the example, the steady state level of the aggregate capital for the low uncertainty economy is 1.1148 and the one for the high uncertainty economy is 1.1120, which is only 0.25\%p low.} That is, given a threshold adjustment cost less than the mean, more firms do not adjust their capital at the relatively high level of current capital in the high uncertainty economy. This is the reason why the magnitude of the response is not significant.\footnote{In the experiment under which an order of magnitude smaller mean of $\xi$ is applied ($A = 0.003, B = 0.005$ for the low uncertainty economy and $A = 0, B = 0.008$ for the high uncertainty economy), the result is actually reversed. So the steady state level of the aggregate capital for the low uncertainty economy is 1.1209 and the one for the high uncertainty economy is 1.1211, which is only 0.018\%p high. Refer to Figure 2.17 and 2.18 for this case. We can see the fact that the left tail in the high uncertainty economy is relatively short compared to that with the high mean, which seems to reduce the degree of inequity of the redistribution of capital.} To sum up, in the high uncertainty economy, more firms remain at the relatively low level of capital and do not go to the optimal level of capital under the frictionless economy than in the low uncertainty economy. This could lead to more inefficient allocation of capital which causes the aggregate TFP to fall. But the effect is dampened, since at the relatively high level of capital, less firms remain there and go to the optimal level of capital.

2.4 Conclusion

Real business cycle models are often criticized for not taking into account the financial markets, the malfunctioning of which is one of the longstanding suspects that cause recessions. The skeptics about RBC-based models also argue that the models have the implausible implication that recessions are times of technological regress. In this paper, I attempt to fill these gaps between the RBC models and reality by explicitly introducing firms’ borrowing constraints and making experiments under which the shocks apply to financial and real frictions. The deterioration of
the frictions in financial and goods markets prevents agents from allocating capital efficiently, which leads to a fall in the aggregate total factor productivity. Firms and households consider these as a negative productivity shock and reduce investment and employment. An increase in uncertainty with respect to the magnitude of real frictions can also generate the fluctuations in the aggregate total factor productivity and the economic activities through a change in the distribution of capital. Among these, a negative shock to the financial markets is a leading candidate that can explain a sizable and persistent recession, while a negative shock to the real friction and its distribution is found to have a limited influence on economic activities.
<table>
<thead>
<tr>
<th>Parameter/Name</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>Mean growth rate of technological progress</td>
<td>1.016 Average annual growth rate of real per capita output: 1.6%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.96 Average real interest rate: 4.0%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.06 Average investment-to-capital ratio: 10 %</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share of income</td>
<td>0.25</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Labor share of income</td>
<td>0.60$^a$</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Preference for leisure</td>
<td>2.15</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Autocorr. coefficient for technology</td>
<td>0.852</td>
</tr>
<tr>
<td>$\sigma_{n_t}$</td>
<td>Standard deviation of technological innovation</td>
<td>0.014</td>
</tr>
<tr>
<td>$\pi_d$</td>
<td>Firms exit or entry rate</td>
<td>0.10</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Fraction of capital stock held by entering firms</td>
<td>0.10$^b$</td>
</tr>
<tr>
<td>$B$</td>
<td>Upper bound of fixed capital adj. cost</td>
<td>0.02$^c$ Positive investment spike ($i/k &gt; 0.20$): 0.186</td>
</tr>
<tr>
<td>$\theta_b$</td>
<td>Ratio of capital put up as collateral</td>
<td>0.80 Average debt-to-capital ration: 0.366</td>
</tr>
</tbody>
</table>

$^a$Cooley and Prescott (1995)

$^b$Davis and Haltiwanger (1992)

$^c$Cooper and Haltiwanger (2006)
<table>
<thead>
<tr>
<th>Model</th>
<th>$B$</th>
<th>$\theta_b$</th>
<th>$\text{mean}(x)$</th>
<th>$\sigma_x/\sigma_Y$</th>
<th>$\text{corr}(x,Y)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economy w/o Real and Financial Frictions</td>
<td>$B = .00, \theta_b = 1000$</td>
<td>0.5436</td>
<td>0.4486</td>
<td>0.0952</td>
<td>0.3379</td>
</tr>
<tr>
<td>Full economy</td>
<td>$B = .02, \theta_b = .80$</td>
<td>0.5131</td>
<td>0.4303</td>
<td>0.0829</td>
<td>0.3352</td>
</tr>
<tr>
<td>Full economy w/ Stronger Financial Frictions</td>
<td>$B = .02, \theta_b = .56$</td>
<td>0.5046</td>
<td>0.4240</td>
<td>0.0808</td>
<td>0.3383</td>
</tr>
<tr>
<td>Full economy w/ Stronger Real Frictions</td>
<td>$B = .04, \theta_b = .80$</td>
<td>0.5113</td>
<td>0.4293</td>
<td>0.0822</td>
<td>0.3376</td>
</tr>
<tr>
<td>Full economy w/ Stronger Real, Financial Frictions</td>
<td>$B = .04, \theta_b = .56$</td>
<td>0.5061</td>
<td>0.4247</td>
<td>0.0812</td>
<td>0.3446</td>
</tr>
</tbody>
</table>

Table 2.2: Business Cycle Moments
Figure 2.1: Maximum Debt Policy ($B^w$) and Threshold ($\hat{B}$), given $\xi$ and $z$. 

Debt Policy ($B_w$) and Threshold ($\hat{B}$)
Figure 2.2: Stationary Distribution of the Full Economy

SS distribution

Capital(k)

mu(k,b)
Figure 2.3: Stationary Distribution of the Economy w/o financial frictions
Figure 2.4: The Budget Set of a Hypothetical Constrained Firm

\[ k' = \frac{qb' + (1 - \delta)k + \pi(k, b) - w\xi}{\gamma} \]
\[ \text{or} \]
\[ b' = \frac{\gamma k' - (1 - \delta)k - \pi(k, b) + w\xi}{q} \]
Figure 2.5: The Objective Function of a Hypothetical Constrained Firm: when $\xi$ varies
Figure 2.6: The Objective Function of a Hypothetical Constrained Firm: when $b$ varies
Figure 2.7: Decision Rules: The Full Economy Case ($B = 0.2, \theta_b = 0.80$)
Figure 2.8: Decision Rules: The Economy with Stronger Financial Frictions Case ($B = .02, \theta_b = .56$)
Figure 2.9: Responses to a Technology Shock

\[ \theta_b = 0.8 \quad \theta_b = 0.56 \]
Figure 2.10: Responses to a Financial Shock
Figure 2.11: Responses to a Real Shock
Figure 2.12: Responses to both Financial and Real Shock
Figure 2.13: Responses to a Financial Shock and Recovery
Figure 2.14: Responses to a Shock to the Second Moment of a Real Friction
Figure 2.15: PDF, Adjustment Hazard Rate, and Distribution of Capital for Low Uncertainty Economy

Figure 2.16: PDF, Adjustment Hazard Rate, and Distribution of Capital for High Uncertainty Economy
Figure 2.17: PDF, Adjustment Hazard Rate, and Distribution of Capital for Low Uncertainty Economy

Figure 2.18: PDF, Adjustment Hazard Rate, and Distribution of Capital for High Uncertainty Economy
APPENDIX A

APPENDIX FOR CHAPTER 1

A.1 Aggregation for entrepreneurs

The aggregate budget constraint for entrepreneurs can be derived as follows. Since each \( \omega_t(j) \) has a different value, each entrepreneur now faces different budget constraint. So to get the aggregate budget constraint, I use the law of large numbers. Suppose a variable \( y_t(j) \) is a variable chosen by an agent \( j \) after each idiosyncratic shock is realized and there is a continuum of agents with mass \( \eta \). To aggregate \( y_t(j) \), I integrate it over \( j \in [0, \eta] \). But this Riemann sum is actually equivalent to the value of \( \eta \) times the limit of the sample mean. Uhlig (1996, [47]) shows that the sample mean converge to its population mean as follows

\[
\int_0^\eta y_t(j)dj \equiv \lim_{n \to \infty} \frac{\eta}{n} \sum_{j=1}^n y \left( \frac{j\eta}{n} \right) \to \eta E[y_t(j)] \equiv \eta \int_0^\infty y_t(j)f(y_t(j))dy \equiv \eta Y_t \quad (A-1)
\]

So, the aggregated value of each variable is the first moment of the variable, \( Y_t \) times the fraction of the subinterval of agents. To get aggregated equation for (1.32), I integrate them over \( j \in [0, \eta] \). Capital letters represent the first moment of the variables. For example, \( I_t = E[i_t(j)], N_t = E[n_t(j)] \).
\[
\int_0^\eta c_t^e(j) + q_t z_{t+1}(j) \, dj = \int_0^\eta q_t i(\tilde{q}_t, n_t(j)) \max\{\omega_t(j) - \tilde{\omega}_t, 0\} \, dj \quad (A-2)
\]

\[
\eta \int_0^\infty c_t^e(j) + q_t z_{t+1}(j) \, d\Phi[\omega_t(j)] = \\
\eta \int_0^\infty \int_0^\infty q_t i(\tilde{q}_t, n_t(j)) \max\{\omega_t(j) - \tilde{\omega}_t, 0\} \, d\Phi[\omega_{t-1}(j)] \, d\Phi[\omega_t(j)] \quad (A-3)
\]

\[
\eta C_t^e + \eta q_t Z_{t+1} = \eta q_t \int_0^\infty \max\{\omega_t(j) - \tilde{\omega}_t, 0\} \, d\Phi[\omega_t(j)] \quad (A-4)
\]

\[
\eta I_t^d = \eta \frac{1}{1 - \tilde{\omega}_t g(\tilde{\omega}_t)} \int_0^\eta \omega_t(j) i(\tilde{q}_t, n_t(j)) - 1_{\{\omega_t(j) \leq \tilde{\omega}_t\}} \mu i(\tilde{q}_t, n_t(j)) \, dj \quad (A-5)
\]

(1.18), (1.25) turn out to be:

\[
\eta N_t = \frac{\eta}{p_t} (u_t^e N_t^e + R_t Z_t + \Pi_t) \quad (A-7)
\]

\[
\eta I_t^d = \eta \frac{1}{1 - \tilde{\omega}_t g(\tilde{\omega}_t)} \int_0^\eta \omega_t(j) i(\tilde{q}_t, n_t(j)) - 1_{\{\omega_t(j) \leq \tilde{\omega}_t\}} \mu i(\tilde{q}_t, n_t(j)) \, dj \quad (A-8)
\]

Also, the aggregate, expected, production of capital goods can be constructed by simply adding all the optimal investment policies of each entrepreneur. This function can be seen as the supply curve for capital goods.

\[
I_t^S(\tilde{q}_t, N_t) = \int_0^\eta \omega_t(j) i(\tilde{q}_t, n_t(j)) - 1_{\{\omega_t(j) \leq \tilde{\omega}_t\}} \mu i(\tilde{q}_t, n_t(j)) \, dj \quad (A-9)
\]

\[
= \eta \left( \int_0^\infty \omega_t(j) i(\tilde{q}_t, n_t(j)) \, d\Phi[\omega_{t-1}(j)] \, d\Phi[\omega_t(j)] - \int_0^{\tilde{\omega}_t} \int_0^\infty \mu i(\tilde{q}_t, n_t(j)) \, d\Phi[\omega_{t-1}(j)] \, d\Phi[\omega_t(j)] \right) \quad (A-10)
\]

\[
= \eta I_t^d [1 - \mu \Phi(\tilde{\omega}_t)] \quad (A-11)
\]
Using (A-6), (A-7), (A-8), I also get the aggregate budget constraint:

\[
C_t^e + q_t Z_{t+1} = \frac{p_t \bar{q}_t f(\bar{\omega}_t)}{1 - \bar{q}_t g(\bar{\omega}_t)} \left\{ \frac{1}{p_t} \left( w_t^e N_t^e + R_t Z_t + \Pi_t \right) \right\} \quad (A-12)
\]

\[
= q_t I_t^d f(\bar{\omega}_t) \quad (A-13)
\]

\[
= (1 + \rho_t) p_t N_t \quad (A-14)
\]

where \(1 + \rho_t = \frac{\bar{q}_t f(\bar{\omega}_t)}{1 - \bar{q}_t g(\bar{\omega}_t)}\) (the gross return to internal funds).
A.2 Competitive Equilibrium

A competitive equilibrium is a set of sequences of quantities, \( \{K_t, Z_t, C_t, C^h_t, C^e_t, H_t, H^c_t, L_t, L^c_t, I_t, I^d_t, I^h_t, I^e_t, I^{NS}_t, N_t, Y_t\}_{t=0}^{\infty} \), prices, \( \{R_t, w_t, w^e_t, q_t, \bar{q}_t, p_t, \bar{\omega}_t, \Omega_t, \Lambda_t, \rho_t\}_{t=0}^{\infty} \), given \( K_0, Z_0 \) and satisfying the following:

\[
U_{C,t+1} = \beta E_t \left[ U_{C,t+1} \frac{R_{t+1}}{q_t} \right] \quad (A-15)
\]

\[
q_t = p_t \bar{q}_t \quad (A-16)
\]

\[
\frac{U_{L,t}}{U_{C,t}} = w_t \quad (A-17)
\]

\[
1 = \beta \gamma E_t \left[ \frac{R_{t+1}}{q_t} \right] (1 + \rho_{t+1}) \quad (A-18)
\]

\[
\frac{U_{L^e,t}}{U_{C^e,t}} (1 + \rho_t) = w^e_t \quad (A-19)
\]

\[
\rho_t = \frac{\bar{q}_t f(\bar{\omega}_t)}{1 - \bar{q}_t g(\bar{\omega}_t)} - 1 \quad (A-20)
\]

\[
C^c_t + q_t Z_{t+1} = q_t I^d_t f(\bar{\omega}_t) \quad (A-21)
\]

\[
= (1 + \rho_t) p_t N_t \quad (A-22)
\]

\[
Z_{t+1} = (1 - \delta) Z_t + I^c_t / \bar{q}_t \quad (A-23)
\]
\[ p_t = \left( \frac{I_t}{C_t} \right)^{\chi-1} \] (A-24)

\[ R_t = \frac{1}{\Omega_t} MP_{K,t} + q_t(1 - \delta) + \Lambda_t \Gamma q_t \] (A-25)

\[ w_t = \frac{1}{(1 + \Lambda_t) \Omega_t} MP_{H,t} \] (A-26)

\[ w_t^e = \frac{1}{(1 + \Lambda_t) \Omega_t} MP_{H^e,t} \] (A-27)

\[ w_t H_t + w_t^e H_t^e = \Gamma q_t K_t \] (A-28)

\[ \Omega_t = (C_t^\chi + I_t^\chi)^{\frac{1}{\chi}} C_t^{\chi-1} \] (A-29)

\[(C_t^\chi + I_t^\chi)^{\frac{1}{\chi}} = Y_t \] (A-30)

\[ Y_t = \theta_t K_t^{\alpha_k} H_t^{\alpha_h} (H_t^e)^{\alpha_e} \] (A-31)

\[ \frac{1}{\bar{q}_t} = 1 - \Phi(\bar{\omega}_t)\mu + \phi(\bar{\omega}_t)\mu \frac{f(\bar{\omega}_t)}{f'(\bar{\omega}_t)} \] (A-32)

\[ I_t^d = \frac{1}{1 - \bar{q}_t g(\bar{\omega}_t)} N_t \] (A-33)
\[ N_t = \frac{1}{p_t} (w^e_t L^e_t + R_t Z_t + \Pi_t) \] 

(A-34)

Labor market:

\[ H_t = (1 - \eta) L_t \] 

(A-35)

\[ H^e_t = \eta L^e_t \] 

(A-36)

Output market:

\[ \frac{Y_t}{\Omega_t} = C_t + p_t I_t \] 

(A-37)

Consumption goods market:

\[ C_t = (1 - \eta) C^h_t + \eta C^e_t \] 

(A-38)

Investment goods market:

\[ I_t = (1 - \eta) I^h_t + \eta I^e_t \] 

(A-39)

Investment goods (as an input factor) market:

\[ \eta I^d_t = (1 - \eta) I^h_t + \eta N_t \] 

(A-40)

Capital goods market:
\[ K_{t+1} = (1 - \delta)K_t + I_t^{NS} \] (A-41)
A.3 A model with investment adjustment costs

A.3.1 Firms

The Lagrangian for firm’s maximization problem is:

\[
\mathcal{L} = C_t + p_t I_t - R_t K_t - w_t L_t + q_t (1 - \delta) K_t + \frac{1}{\Omega_t} \left[ A_t K_t^\alpha L_t^{1-\alpha} - (C_t^\chi + I_t^\chi)^{1/\chi} \right] + \Lambda_t \left[ \Gamma q_t K_t - w_t L_t \right]
\]

(A-42)

FOCs are:

\[
[C_t] \quad \Omega_t = (C_t^\chi + I_t^\chi)^{1-\chi} C_t^{\chi\chi-1}
\]

(A-43)

\[
[I_t] \quad \frac{1}{\Omega_t} (C_t^\chi + I_t^\chi)^{\frac{1-\chi}{\chi}} I_t^{\chi-1}
\]

(A-44)

\[
[K_t] \quad R_t = \frac{1}{\Omega_t} MP_{K,t} + q_t (1 - \delta) + \Lambda_t \Gamma q_t
\]

(A-45)

\[
[L_t] \quad w_t = \frac{1}{(1 + \Lambda_t)\Omega_t} MP_{L,t}
\]

(A-46)

and the two constraints.
A.3.2 Households

The Lagrangian for household’s maximization problem is:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - L_t) + \lambda_{C,t} [w_t L_t + R_t K_t - C_t - q_t (1 - \delta) K_t - p_t I_t] \\
+ \lambda_{K',t} [(1 - \delta) K_t + I_t - \Psi \left( \frac{I_t}{K_t} \right) K_t - K_{t+1}] 
\]

(A-47)

FOCs are:

\[
[C_t] \quad U_{C,t} = \lambda_{C,t} 
\]

(A-48)

\[
[L_t] \quad U_{L,t} = \lambda_{C,t} w_t 
\]

(A-49)

\[
[I_t] \quad \lambda_{C,t} p_t = \lambda_{K',t} \left[ 1 - \Psi' \left( \frac{I_t}{K_t} \right) \right] 
\]

(A-50)

\[
[K_{t+1}] \quad q_t = \beta E_t \left[ \frac{\lambda_{C,t+1}}{\lambda_{C,t}} \left\{ R_{t+1} + q_{t+1} \left( \Psi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \frac{1}{K_{t+1}} - \Psi \left( \frac{I_{t+1}}{K_{t+1}} \right) \right) \right\} \right] 
\]

(A-51)

and the two constraints.
A.3.3 Competitive Equilibrium

A competitive equilibrium is a set of sequences of quantities, \( \{K_{t+1}, C_t, L_t, I_t\}_{t=0}^{\infty} \), prices, \( \{R_t, w_t, q_t, p_t, \Omega_t, \Lambda_t\}_{t=0}^{\infty} \), given \( K_0 \), satisfying (i) firms optimization (ii) households optimization (iii) the market clearing conditions:
APPENDIX B

APPENDIX FOR CHAPTER 2

B.1 Footnote 16: The Definition of Maximum Debt Policy

To see why the equation (2.28) holds, let’s consider two cases. First, suppose $B^W(k, \xi; S) > \theta_bk$. Then, the threshold debt will be $\bar{B}(k, \xi; S) = P^W(k, 0, \xi; S) + q\theta_bk$. Suppose a firm’s current debt, $\tilde{b}$, is slightly greater than this. That is, $\tilde{b} = \bar{B}(k, \xi; S) + \epsilon$. If we substitute $\tilde{b}$ for $b$ in the budget constraint, then the budget constraint will be:

\[ qb' - D = -y + wN + \tilde{b} + \mathcal{J}(K^W(k, \xi; z, \mu) - \frac{(1 - \delta)}{\gamma}k)([\gamma K^W(k, \xi; z, \mu) - (1 - \delta)k] + w\xi) - q\theta_bk + \epsilon \]

Thus, the firm with $\tilde{b}$ has to incur more than $q\theta_bk$ even when it does not give any dividend (in other words, since we are deriving the maximum threshold, we should set $D = 0.$), which is infeasible. Only when $b < \bar{B}(k, \xi; S)$, the firm’s collateral constraint is not binding currently and since the next period debt, $b' < \theta_bk < B^W$, the constraint will also not be binding for all possible future states. Second, suppose $B^W(k, \xi; S) \leq \theta_bk$. Then, the threshold debt will be $\bar{B}(k, \xi; S) = P^W(k, 0, \xi; S) + \epsilon$.
$q B^W(k, \xi; S)$. Suppose a firm’s current debt, $\tilde{b}$, is slightly greater than this. That is, 

$$\tilde{b} = \hat{B}(k, \xi; S) + \epsilon.$$ 

If we substitute $\tilde{b}$ for $b$ in the budget constraint, then the budget constraint will be:

\[
qb' - D = -y + wN + \tilde{b} + J(\gamma K^W(k, \xi; z, \mu) - \frac{(1 - \delta)}{\gamma} k) \\
(1 - \delta)k + \hat{N}
\]

\[
= q B^W(k, \xi; S) + \epsilon
\]  

(B-2)

Thus, the firm with $\tilde{b}$ has to incur more than $q B^W(k, \xi; S)$ even when it does not give any dividend. $B^W$ is, by definition, the maximum permissible debt that enables a firm to be unconstrained for all possible future states. If the firm incurs more debt than that, it could get constrained in some state of the next period. Thus only when $b < \hat{B}(k, \xi; S)$, the firm’s collateral constraint will not be binding for all possible states and since $b' = B^W < \theta_b k$, the constraint is not binding currently.
BIBLIOGRAPHY


