Uncertainty Analysis of Control Inputs for Diesel Engines

Master’s Thesis

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Diesel engine emissions regulations and demands for better fuel economy are causing engine manufacturers to develop more advanced engine platforms. Multiple fuel injection events are now possible with high pressure common rail systems. The air path system can be controlled using combinations of actuators and advanced turbocharger systems. This increased flexibility dramatically increases the complexity of the control strategy and calibration efforts needed for new engine platforms. This study presents a new control structure for diesel engines using scheduling variables that are related to the cylinder conditions. This can reduce the number of calibrations needed compared to conventional techniques.

A list of candidate parameters to be used as the scheduling variables is developed. Accurately quantifying the scheduling variables during engine operation is a primary requirement for the control structure to function effectively. Equations for each of the parameters are developed based on production sensors and models. The quality in predicting the parameters is derived based on the uncertainties in the models and measurements resulting in analytical uncertainty equations. These equations can be applied to any engine platform providing support for control input selection paired with the most appropriate sensing system. The uncertainty equations are applied to a 2.8 L
diesel engine to show the application and benefit to quantifying the uncertainty in the control inputs.
Dedicated to my family and girlfriend

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CHAPTER 1
INTRODUCTION

1.1 Motivation

Diesel engine emissions regulations have become increasingly more stringent causing engine manufacturers to develop more advanced engine platforms. Manufacturers are expected to meet these emissions standards while maintaining fuel economy and customer satisfaction. Aftertreatment systems, like diesel particulate filters, lean NO\textsubscript{x} traps, oxidation catalysts and urea SCR systems, are being developed and coupled to diesel engines to help meet these emissions regulations. However, these aftertreatment systems are bulky and add a great deal of cost to the engine platform. To help combat these regulation standards, diesel engines must have more flexibility in controlling combustion to reduce engine out emissions. This flexibility is achieved with better control of the fuel and air path systems through advancements in diesel engine hardware and software.

The introduction of electronic diesel control has allowed for vast advancements in diesel engine systems. Current fuel injection systems are capable of multiple injection events (pilots, main, split main, posts) independently regulating the timing and quantity of each event. Actuators have been implemented into the air path system that allow for control of
EGR and boost pressure or fresh air flow rate. These advanced systems are used to influence combustion, however, as more degrees of freedom are added to the system hardware, the control strategy becomes more complex. A large effort is now placed on developing control strategies that meet emissions regulations while balancing the tradeoffs between emissions, fuel economy, performance, and drivability.

### 1.2 Current Diesel Practice

Diesel engine control strategies have specifics that are unique to the manufacturer, but the overall control structure is similar and can be found in open literature [42][43][44][45]. The injection strategy and air handling system set points are stored in ECU maps. The engine speed and load are the scheduling variables known as the combustion references. The engine speed is a characteristic of the engine operating condition and the engine load is determined through models that relate pedal position to fuel mass. Fuel mass is then directly related to torque production for diesel engines. This model takes into account the torque desired by the driver and external disturbances, like accessory loads, when determining the fuel injection quantity.

In order for the diesel engine to meet the requirements of emissions regulations, fuel economy, durability and drivability, ECU maps must be adapted for all operating conditions. The ECU maps currently in practice include the following,
1. Steady State Map - Used during steady state engine operation. Focuses on developing the best trade-offs between fuel economy and emissions.


3. Altitude Maps – Used as altitude increases. Multiple maps are developed at different elevations. Compensates for the reduction in air density.

4. Cold Start Map – Used at low ambient temperatures for initial cranking and engine warm up. Focuses on engine startability with less regards to emissions.

5. Thermal Maps – Used to regulate exhaust temperature for functionality of the emissions equipment. For example, increasing the exhaust temperature to regenerate the DPF.

Correction factors are also added to the ECU map set points to add compensation for effects like coolant temperature. The flow chart on the right of Figure 1 gives an example of the interactions between the ECU maps and correction factors to get the final main injection timing command. At each of the junctions, ECU map setpoints are blended together or correction factors are added to the setpoints. Controllers are then used to ensure that the final fuel and air handling commands are achieved. These ECU maps, correction functions, and controllers are essential to meet the current diesel engine emissions and consumer demands. This is currently done through extensive calibration efforts.
The calibration procedure is broken down into four stages, calibration of the basic engine ECU map (steady state), control function configuration (EGR, idle speed, fuel rail pressure, etc.), calibration of the compensation ECU maps (altitude, cold start, etc.), and calibration of the transient ECU map [42]. The steady state ECU map is developed first and then adaptations are made to find the remaining ECU maps. The control function configuration also includes the calibration of the controller gains once the controller structure is determined. The flow chart on the left in Figure 1 shows the procedure used for calibration. The calibration is done primarily on the engine test bench, where multiple iterations are needed to get a final control strategy that is ready for implementation. This procedure becomes very time consuming and costly due to the number of parameters that need calibrated.
1.3 Research Objective

With the increased complexity in diesel engines, efforts are being researched in order to try and curb the calibration time and cost for development of new engine platforms. The
majority of this research has been on model based calibration techniques [32][47][48].

This research focuses on building models that are used to find base engine calibrations. These calibrations are then fine tuned on the engine test bench. This process helps reduce the time needed on the test bench and costs incurred with experimental testing.

In parallel with model based calibration techniques, calibration efforts can be reduced by condensing the conventional diesel control strategy. This research focuses on restructuring the diesel engine control strategy by taking a root cause approach. Once the intake valve closes, all that matters for combustion are the conditions that are trapped within the cylinder. Therefore, there is no difference between steady state conditions, transient conditions or altitude conditions once the valves have closed. All that matters is what is physically inside the cylinder. Scheduling the injection parameters based on the cylinder conditions, instead of the operating and ambient conditions, can lead to a reduction in number of maps needed for the control algorithm.

Root cause parameters are developed that relate to the physics of diesel combustion and are used as the basis for a new control structure. The root cause parameters are implemented as additional scheduling variables, with the conventional speed and load parameters. Due to the nature of the root cause parameters having a direct correlation to diesel combustion, implementing them into the control strategy can reduce the number of ECU maps needed. For example, the altitude tables can be discarded if the control structure uses root cause parameters that command injection parameters based on oxygen
concentration. The combustion reference will compensate for the air density instead of using tables that are calibrated at different altitudes.

Quantifying the root cause parameters within the cylinder during all operating conditions and capturing their effects on diesel combustion are essential for the control structure to function as intended. The overall objective of the project is to find the root cause parameters that accomplish these two tasks most effectively. The following steps are needed to carry out this objective,

1. Develop a list of root cause parameters that influence diesel combustion
2. Determine the accuracy in which these parameters can be measured or predicted at steady state conditions using production sensors and models
3. Develop a transient model that will predict the root cause parameters during transient engine operation
4. Determine the additional uncertainties in measuring and predicting the root cause parameters due to delays in the transient model
5. Develop the trends between the desired injection parameters and the root cause parameters
6. Compare the results and determine the root cause parameters that will add the most benefit to the control strategy

This thesis specifically focuses on tasks 1 and 2. The first task is extremely important since the root cause parameters are the basis for the control structure. An extensive list of parameters is developed to cover all aspects of the diesel combustion process to ensure
the most effective parameter is chosen in the end. The second task is used to determine whether the root cause parameters developed can be quantified to within an acceptable uncertainty. This is a necessary requirement for a parameter to be used as a combustion reference. A scheduling variable that is unable to be quantified is useless in the diesel control structure. For example, the scheduling variable is EGR fraction with a nominal value of 10% but the measurement system has a 50% uncertainty and predicts 15% EGR fraction. The control strategy would command the injection parameters based on the calibration for 15% EGR fraction even though the cylinder actually contains 10% EGR fraction. The difference in the injection parameters for 10% EGR fraction and 15% EGR would result in unwanted emissions or reduced fuel economy.

1.4 Thesis Outline

- Chapter 2: Introduces a comprehensive literature review on compensations used during calibration for the different operating conditions. This is used during development of the root cause parameters and trends between the root cause parameters and injection strategies.
- Chapter 3: Describes the engine models and experimental setup used throughout this thesis. The background for the uncertainty analysis is also shown which is used in Chapter 7.
- Chapter 4: Introduces the root cause parameters developed and discusses the control structure using root cause parameters.
• Chapter 5: Presents the cylinder content analysis which determines the sources of the root cause parameters. The sources are used to quantify the root cause parameters.

• Chapter 6: Develops the equations for quantifying the root cause parameters based on the available engine sensors and models. The equations are based on the results from Chapter 5.

• Chapter 7: Develops the uncertainty equations for each of the root cause parameters based on the equations developed in Chapter 6. The uncertainty equations are used to determine the accuracy in which the parameters can be quantified.

• Chapter 8: Applies the uncertainty equations to the 2.8 L diesel engine to obtain numerical uncertainty results and draw conclusions about the ability to quantify the root cause variables.
CHAPTER 2
LITERATURE REVIEW

2.1 Introduction

The literature review covers topics in diesel engine calibration and models used to predict trapped residual mass. The focus of the calibration literature is towards the compensation that is used in current practice for the different engine operating condition. This part is divided into five sections covering thermal management, steady state calibration, cold starts, altitude compensation, and transient calibration. The goal is to use the literature to find parameters that affect combustion and identify any relationships that exist between the injection parameters and factors that affect combustion. The trapped residual mass literature is used as a reference when developing the trapped residual model.

2.2 Thermal management information

DPF regeneration has been studied extensively. The main thrust of the research is on safe regeneration procedures and determining the optimal regeneration frequency. Most of the papers only consider the exhaust side of DPF regeneration. The temperature of the exhaust is the control variable most often considered and not the fueling parameters.
SCR control is also widely studied but, the focus is urea dosing control. Thermal management using fuel control variables is not a well researched topic.

[1] studied the flow through a DPF by analyzing the flow distribution at a cross section near the outlet of the catalyst. It was discovered that the highest flow rates are near the periphery of the DPF when room temperature air was used (diesel exhaust would be too harsh). However, after real exhaust gas was used to the load the DPF, the flow distribution of ambient air was even more concentrated near the periphery (Figure 2). This result would suggest that the largest soot loads are located at the center which in turn suggests the flow rate for the clean catalyst was highest near the center. Although the results were not conclusive, they demonstrate that the flow through a DPF is very complex and has significant radial variations. Better results could be achieved if the sensor could withstand the harsh diesel exhaust environment.
Figure 2: Mass flow rate distribution at the outlet of a partially filled DPF [1].

The flow through a catalyst was also studied in [3]. 3-D X-rays of a DPF were taken and used to determine the exact microstructure. This data was then used to predict the flow through the filter walls. Some of the results are shown in Figure 3. This model was then used to predict how soot is oxidized within the catalyst.
[11] attempted to model the thermal response of a DPF to an oscillating inlet temperature. Because the soot oxidation rate is exponentially dependent on temperature, producing a sinusoidal exhaust temperature trajectory will oxidize more soot than if a constant temperature were used. Although the magnitude of the response agreed, the phasing between the measured and predicted outlet temperatures was significant.
A mean-value thermal model of an engine was developed in [8]. This model captured the gas exchange process, the heat transfer process and the combustion process. Due to the inherent feedback nature of the turbocharger, the model had to be iterated several times before converging to a fuel mass. The overall model shown in Figure 5 contains many table approximations calculated using the thermodynamic software EnginOS Tiger. Once calibrated this model was used to evaluate the fuel consumption during a warm up test.
[10] analyzed both passive and active methods of increasing and maintaining higher exhaust temperatures. One of the results showed that although a dual walled exhaust
manifold reduces the thermal mass of the pre-catalyst exhaust system, it does not materially shorten the light-off time. Using cylinder deactivation to avoid low load conditions was shown to provide a significant improvement in terms of the both average exhaust temperature and BSFC. This paper also provided some useful heat transfer breakdowns like the one shown in Figure 6.

![Figure 6: Breakdown of the heat transfer sources during a warm up FTP cycle [10].](image)

The only paper that discussed thermal management directly in terms of fuel injection strategies was [7]. In this paper, both early and late injection strategies were compared to direct fuel injection into the exhaust system, Figure 7. For the in-cylinder strategies, the
torque production was fixed but the fuel quantity was increased. Two sets of experiments were conducted. In the first set, all of the cylinders used the same injection strategy. For the second set, only one cylinder injected extra fuel (but the amount of extra fuel was significantly higher). The results highlighted in Figure 8 showed that only early injection can be used during cold start. During normal operation, all of the strategies perform similarly.

Figure 7: Illustration of exhaust temperature increasing injection strategies [7].
Regulating the exhaust temperature has also been explored from the engine coolant side. [4] tested an advanced coolant system shown in Figure 9. This system allows for flexible control of the coolant flow rate. When the conventional engine is cruising on the highway, too much energy is transferred from the engine to the coolant. With this system, the coolant flow can be regulated to a higher temperature and thus reduce the heat transfer between the coolant and the engine. As a result, the engine can reach higher peak temperatures and consume less fuel. The results showed a steady-state improvement of 2% in BSFC at the cost of a 10-15% increase in NOx.
Figure 9: Experimental setup for an advanced coolant control system [4].

A similar concept was also analyzed in [5]. With an auxiliary heater, variable speed water pump and adjustable thermostat valve it was proposed that an arbitrary coolant temperature profile could be tracked. This claim was validated on a simplified experimental apparatus where a heating coil was used to simulate the heat rejection by the engine.
2.3 Steady-state fuel parameter calibration information

All of the steady-state calibration papers relied on some form of statistical analysis to reduce the experimental burden. A neural network was used in [14]. DoE techniques were used in [15] and [16]. Stochastic methods were used in [17]. Figure 11 shows the regression coefficients between the smoke output and the inputs generated by the DoE from [15]. From these relationships the optimal combination of fueling parameters was determined. Almost all of the paper relied on calibrators to make these decisions. To facilitate the decision process, a series of trade-off graphs with constant BSFC were created. An example is shown in Figure 12.
Figure 11: Regression analysis for the smoke output [15].
[17] presents a stochastic controller that tunes the calibration online. Drivers tend to drive a vehicle in the same manor. This controller tries to model the driver’s decision process and use that information to actively adjust both the steady-state and transient fueling parameters. Instead of calibrating the fueling parameters directly, the fuel parameter update law must be calibrated. In simulation, the controller performed very well. However, the set points of the controller often change at a very high frequency as shown in Figure 13.
Figure 13: Comparison between the baseline and learned injection timings [17].

[18] describes some guidelines for calibration software development which include ease-of-use, functionality, copy protection and documentation requirements.

2.4 Cold Start Literature Information

The ignition delay associated with diesel combustion is directly related to the temperature of the charge that the fuel is injected into. The following equation shows the ignition delay dependency on temperature,

\[ \tau_{id} = A \rho^{-\eta} \exp\left(\frac{E_A}{RT}\right) \]  

(1)
where $p$ is the cylinder pressure, $E_a$ is the activation energy of the fuel, $\hat{R}$ is the universal gas constant, $T$ is the cylinder temperature and $A$ and $n$ are constants dependant on the fuel. As the cylinder temperature decreases the ignition delay increases, which causes combustion to occur later in the expansion stroke. If the delay is extended too far then combustion will not occur which results in a misfire.

The limits of combustion are also a function of the injection quantity. Figure 14 shows how the ignition limits are affected by the minimum amount of fuel injected and the chamber temperature [20]. As the temperature is decreased the ignition limits become more pronounced and more fuel in needed to ensure combustion. However as the cylinder contents warm, the surface flattens out and there is no longer any dependency on the injection amount. This figure is based off a theoretical formula relating reaction speed theory and heat transfer theory.
The focus in the literature was based on methods to reach stability of normal engine operation with little regards to the emissions produced during the cold start. The white smoke is related to misfires therefore a higher stability will reduce the white smoke content. Other emissions were probably neglected because to this date emissions standards are not required for cold starts, however the reliable to start your engine in severe ambient conditions is very important.

All of the cold start calibration papers, except for the one on noise control [21], used the fuel system as the means to compensate for engine instability. The studies used combinations of pilot injections along with different quantities and timings in order to
reach the desired stability during a cold start. The following discussion will be ordered in the number of pilot injections used.

2.4.1 Conventional Combustion (single injection)

In order to compensate for the longer ignition delays caused by the reduced ambient conditions, the optimal solution is to retard the injection timing so that the cylinder contents would be at a higher temperature and pressure. However the timing can only be retarded to the ignition limit curve and as the temperature decreases this curve moves towards more advanced timings. This can be seen in Figure 15, which is data based on an 8.5 L directed injection turbocharged diesel engine [19]. As the temperature is decreased the ignition limit curve moves towards a more advanced timing. The area above each curve is the misfiring zone and the area below each curve is the firing zone. At low temperatures the ignition limit curve is bounded by a maximum engine speed. This is shown the most pronounced at -10 °C where the limit becomes almost vertical at 600 rpm. If the engine speed were to climb above 600 rpm at this temperature then combustion would not occur by solely advancing the injection timing. Also the ignition limits cause the injection to occur earlier in the compression stroke where lower pressure and temperatures are present thus increasing the ignition delay. Wall wetting begins to play a role in combustion stability as the injection timing is advanced because the fuel droplets adhere to the cylinder wall before the combustion can occur [19]. This creates a lower bound on the injection timing.
The long ignition delays and wall wetting affects of single injection lead to combustion occurring intermittently at temperatures below -5 °C. This can be seen in Figure 16, which shows the cylinder pressure of cylinder #4 during a cold start at -5 °C [20]. This occurs because the fuel vapor from any previous misfires creates an activated atmosphere known as a “Cold Flame Reaction” [20]. The fuel vapors during compression go through initial reactions, which increase the pressure within the cylinder. This helps to reduce the ignition delay and moves the operating point away from the minimum fuel quantity limit. A study in [20] on a 6 cylinder turbocharged diesel engine with a common rail showed
the ignition delay to decrease by one third for a cold flame reaction combustion compared to a normal ignition cycle.

![Cylinder pressure for cylinder #4 during a cold start at -5 °C](image)

Figure 16: Cylinder pressure for cylinder #4 during a cold start at -5 °C [20].

2.4.2 Pilot Injection Strategies

Single Pilot Injection:

Pilot injections have proven to drastically reduce the ignition delay during diesel combustion. [23] showed that a single pilot injection was able to reduce the ignition delay in half. The pilot injection uses the concept of the cold flame reaction to promote an activated atmosphere which raises the cylinder pressure as shown in Figure 17 [24]. However, instead of using previous misfires the pilot injection is able to create this type
of reaction every cycle. This lessens the dependency on the main injection timing and on the minimum fuel quantity.

Figure 17: Difference between firing cycles with and without a pilot injection: (a) comparison between the pressure and gross heat release rate traces for the cases of single injection ($B_t=0$ and $B_q=40 \text{ mm}^3/\text{stroke}$) and single injection plus pilot injection ($A_t=10.5$, $A_q=5 \text{ mm}^3/\text{stroke}$, $B_t=0$, and $B_q=40 \text{ mm}^3/\text{stroke}$) [24].

An experimental study [22] on a single cylinder version of a Ford 2.0 L diesel engine proved a single pilot injection was able to have good stability up to 900 rpm for temperatures down to -20 °C. However above 900 rpm the covariance of the IMEP begins to rise at low temperatures, which means misfires. This can be seen in Figure 18. The pilot was kept at 10 degrees before main injection and the covariance was based off the first 25 fueled cycles. Figure 19 shows the covariance for the single injection based on the main injection timing, for 1000 rpm and -20 °C. This shows that the combustion stability is still strongly based on optimizing the main injection timing at low temperatures. However there is an improvement from no pilot injections where
intermittent combustion occurs because stability can be reached up to 900 rpm at low temperatures. In Figure 18 the covariance at the lower engine speed becomes higher due to the increased blow-by that occurs from a longer cycle. This results in reduced pressures and temperatures in the cylinder affecting the combustion.

Figure 18: Coefficient of variation of work output as a function of engine speed at four ambient temperatures, main injection timing 8° BTDC [22].
Figure 19: Coefficient of variation of work output as a function of main injection timing at -20 °C, engine speed is 1000 RPM [22].

An experimental study on a hydra single cylinder engine conducted by [24] showed that the pilot injection needs to be close coupled to the main injection in order to keep the IMEP from decreasing. The maximum separation between the pilot and main injection was shown to be 20 degrees before the IMEP started to decrease, Figure 20. The pilot injection quantity was shown to have no effects on the IMEP of the cycle as shown in Figure 21. This means that the proportion of the fuel should go to the main injection to increase fuel conversion efficiency. The objective of the pilot injection is to limit the misfires that occur by affecting the ignition delay of the main injection, not to add to the IMEP of the cycle. The reason the IMEP drops in Figure 20 as the pilot injection is advanced is because the number of misfires is increasing which reduces the IMEP.
Another advantage to using a small quantity in the pilot injection is that it limits the cooling affects as the fuel evaporates.

Figure 20: The influence of the pilot injection timing on IMEP for the first five fueled cycles for a pilot plus main injection. (main injection timing 0°) [24].
Multiple Pilot Injections:

As the number of pilot injections are added better combustion stability is reached for lower temperatures and higher engine speeds. This decreases the dependency on the main injection timing and quantity [22]. This can be seen in Figure 19 which compares the IMEP covariance for a single pilot injection strategy to a dual pilot injection strategy. The dual pilot injection strategy is more stable over a broader range of main injection timings. A plot of the best and worst heat release rates for a single and dual pilot injection strategy are shown in Figure 22 for the first 25 fueled cycles from study [22]. The covariance for IMEP is much better for the dual pilot strategy at 15.5 % compared to 31.1 % for single pilot injection. The figure also shows that the ignition for the twin pilot injection strategy occurs sooner than the single pilot injection strategy.

Figure 21: The influence of the pilot injection quantity on IMEP for the first five fueled cycles for a pilot plus main injection. (main injection timing 0°) [24].
In the same study [22] the triple pilot injection proved to produce even better stability than the dual pilot strategy. The heat release rate curve for dual and triple pilot injections is shown in Figure 23. The covariance in the IMEP drops from 15.5 % to 8.6 %. Again the triple pilot injection strategy produces earlier ignition than the dual pilot strategy. However when a fourth pilot was introduced the results for covariance decreased. It was thought that the system was optimized at three pilot injections and the fourth injection create wall wetting effects due to very early injection of the first pilot.
The advantages of adding more pilot injections are that the ignition delay is decreased and the mixture is more uniform. This means that using two to three pilot injections will allow for the ignition delay of the main injection to be relatively short, thus decoupling it from the main injection timing and creating a stable cold start combustion. Since the white smoke is also decoupled from injection timings with pilot strategies the timings can be optimized for other emissions. Also the proportion of fuel should be directed to the main injection in order to obtain the highest fuel conversion efficiency and the main ignition should occur just after TDC in order to have the minimum amount of fuel requirement low. Finally the pilot injections should be closed coupled to the main injection in order to ensure the wall wetting does not affect the combustion.
2.4.3 Cold Start Modeling

A cold start model was developed in [25] to investigate the interactions that are occurring during a cold start. The model incorporated a multicomponent fuel model, wall-film model, spray film model and an ignition model. The multicomponent model was used to associate centane numbers to the molecular weights since the centane number impacts the ignition delay. The wall film model was necessary to catch the interactions of fuel impingement to the wall and puddling effects. The model produced similar results to the above discussion by showing how the fuel evaporates and is combusted.

2.4.4 Noise Reduction Strategies

A study was conducted in [21] to reduce the noise level of a diesel engine when warming up during a cold start. The noise was determined by a combination of thermodynamics and structural evaluation functions. An optimization was conducted for noise and emissions from test bench data using a DoE. The varying parameters were the pre and main injection timing, pre injection quantity, EGR and rail pressure. The optimization was able to reduce the engine noise without any detrimental effects on the engine’s ability to start, combustion quality, or emissions degradation. Figure 44 shows the noise level reduction potential from the optimization.
As altitude is increased the density of air decreases due to the reduced oxygen concentration. This causes the performance of a diesel engine operating at altitude to deteriorate as elevation is increased. The reduced air density changes the initial conditions for the turbocharger and exhaust backpressure. In order to compensate for altitude changes, calibration is needed to help reduce the deteriorating performance and to ensure emissions regulations are met. The following section will describe causes of reduced performance and show compensation techniques currently being used.

2.5 Altitude Calibration Information

Figure 24: Noise level reduction potential from optimization [21].
2.5.1 Performance and Thermodynamics

An experimental study in [26] on a 2.5 L direct injected diesel engine was performed to compare altitude results for diesel and biodiesel fuel. Figure 25 shows the in-cylinder pressure at 500 m and 2400 m for B0 which is diesel fuel and B100 which is bio-diesel [26]. The pressure before the compressor decreases as the altitude increases, which causes the pressure at IVC to decrease. The reduced ambient pressure carries all the way through the combustion cycle. The lower pressure over the complete thermodynamic cycle causes the brake thermal efficiency to decrease. This would then cause the BSFC to increase as shown in study [27], which was a GT power model investigation of a Cummins direct injection diesel engine.
If the fueling is kept constant as altitude increases, the AFR will decrease due to the reduction in oxygen concentration. This has many affects on the combustion process, which causes it to stray from the base calibration. The decrease in AFR causes the smoke to increase out of the engine. This can be seen in Figure 26 for an experimental study on diesel performance at altitude [28]. Decreasing the AFR causes the mixing process during combustion to be longer, therefore increasing the burn duration further into the expansion stroke. This along with the lower heat capacity within the cylinder causes the exhaust enthalpy at the turbine inlet to increase. This can be seen in Figure 27, which shows the increased cylinder temperature at the end of the combustion for different altitudes [26].
Figure 26: Comparison of smoke opacity for different altitudes of a 9.4 L direct injection diesel engine [28].
The increased temperature to the turbine inlet causes the turbine speed to increase as seen in Figure 28 [28]. This creates a natural compensation effect for the reduced oxygen concentration because the compressor speed increases which increases the compressor pressure ratio. However this compensation is not enough to fully offset the oxygen reduction. A disadvantage to this affect is that the over-speed limit of the turbocharger decreases on the speed load map [29]. This can be seen in Figure 28 where the turbocharger speed reaches a maximum value at a lower fueling quantity. This means that turbocharger speed may have to be limited at lower fueling quantities depending on turbocharger design.
2.5.2 Altitude Compensation

In the study [27] a variable geometry turbine was shown to be able to compensate for some of the deteriorating performance caused as higher altitudes. Using a GT-power model of a 10.8 L Cummins engine the VGT position was optimized for BSFC and
maximum power restricting the turbocharger speed to maximum allowable and surge to within 10% at a constant pressure ratio. Figure 29 shows the optimized rack position for the VGT at full load across the engine speed range, with 0 corresponding to fully closed (smallest area) and 1 fully open (largest area). At the higher engine speeds the rack position decreases at increasing altitude in order to increase turbine speed to get higher pressure ratios across the turbine. At the lower engine speeds the altitude has a reverse affect on the rack position. This is needed in order to keep the turbocharger from operating across the surge line. This can be seen in Figure 30, which shows the engine operating line on the compressor map for different altitudes.
Figure 29: VGT control strategy for rack position at full load conditions [27].
Figure 30: Engine operating lines for FGT and VGT for different altitudes on the compressor map [27].

The optimized VGT rack positions are used to compare the power output and bsfc to a fixed geometry turbocharger system for different altitudes in Figure 31. The VGT outperforms the FGT at the 0 m and 3000 m altitudes for both power and bsfc. However at high altitudes and low engine speeds at 5000 m the VGT performs worse. This is due to the fact that it was optimized to avoid the surge line to run reliably. The VGT shows that it can help to compensate for some of the performance losses at altitude however a full recovery is not possible.
Figure 31: Power output and bsfc corresponding to full load conditions for optimized VGT rack position [27].

The study also optimized the VGT position at 40% load, as can be seen in Figure 32. The rack position is much different than the previous optimization. The rack position increases over the entire engine speed range because the surge limit is not an issue at the lower engine loads. The rack position decreases with increasing altitude for the jump between 3000 m to 5000 m, however the jump from 0 m to 3000 m does not. This was caused by the fact that the 0 m operating conditions were in the low efficiency range of the compressor map. The point however is shown in Figure 33, which is the power output and bsfc for the optimized VGT compared to the FGT at different altitudes. The VGT has the flexibility at the lower engine speeds to compensate almost fully for power and bsfc losses at the lower engine loads as altitude increases.
Figure 32: VGT control strategy for rack position at 40 % load conditions [27].

Figure 33: Power output and bsfc corresponding to 40 % load conditions for optimized VGT rack position [27].
An older paper published by GM [31] shows how their EGR and fuel system was updated in order to meet emissions legislation at increased altitude. The EGR system was updated to an electronic control system, which was able to compensate for changes in barometric pressure. The control function can be seen in Figure 34. This function is able to automatically adjust the diagram in order to maintain the correct EGR flow. The key here is that the EGR must be kept at constant proportions with the air as it decreases with increasing altitude to maintain the desired emissions.

Figure 34: EGR electronic control function [31].
The fuel system was updated in order to compensate for the ignition delay effects caused by the reduced pressure at IVC. This was needed in order to ensure that the emissions are kept within the desired constraints due to the dependency on ignition timing.

Further efforts have been done to try and predict the performance of diesel engines at increasing altitudes. In studies [28] and [30] iterative type ECUs were used for these predictions. The former used a relationship between the isfc at altitude and isfc at SAE conditions along with the turbocharger maps and part load engine data. The latter picks arbitrary values for compressor outlet, turbine inlet, and efficiencies, then iterates until the conditions match on the turbocharger maps. An empirical relationship is used for the thermal efficiency of the combustion.

2.6 Transient Calibration Information

2.6.1 Transient Emissions

During a transient operation, emissions spikes are produced due to the dynamics of the diesel engine. A study [33] was conducted on a 6.0 L diesel engine, for use in the Ford series pickup, to investigate the causes of these emissions spikes during transients. An instantaneous step from 1 to 9 bar at 2000 RPM was used as the transient test. The NO\textsubscript{x} spike can be seen in Figure 35 along with the injection pressure. The main causes of the NO\textsubscript{x} spike are due to the increase fuel rail pressure and the lack of EGR present, followed by the lag in the intake manifold pressure. The dynamics of the air systems can be seen
in Figure 36 and Figure 37. The EGR valve is momentarily closed in order to meet the driver demand, thus creating a tradeoff between torque and emissions.

Figure 35: NOx production and injection pressure during an instantaneous load step from 1 to 9 bar [33].
Figure 36: NOx production and intake manifold pressure during an instantaneous load step from 1 to 9 bar [33].

Figure 37: NOx production and EGR rate during an instantaneous load step from 1 to 9 bar [33].
The particulate spike for the same transient can be seen in Figure 38. The particulate spike has a magnitude over ten times that of steady state. The main cause is from the reduction in the AFR ratio due to the response of the turbocharger. Secondary effects are caused by the lower swirl ratio, higher fuel injection, which causes wall impingement, and EGR present in the intake manifold due to emptying effects.

![Figure 38: Particulate production during a load step from 1 to 9 bar for three different transient rates [33].](image)

2.6.2 Transient Calibration

In [32] model based calibration was used to reduce brute force calibration techniques in developing the transient maps. The process was comprised of five steps:

1. Design of transient experiments
2. Data collection needed for modeling
3. Dynamic engine modeling
4. Calibration optimization
5. Verification and validation

The study was conducted on a heavy duty Detroit diesel engine. The model that was developed was able to run at 200x real time and was comprised of physical and heuristic models, and neural networks. The transient maps were determined by optimizing a cost function using the virtual engine model, as seen in Figure 39. The procedure used was able to improve fuel consumption by 4.5 % with a 15-20 % reduction in NOx, HC, and PM compared to the base calibration which was performed through conventional techniques over a FTP cycle. The five step process took less time to complete than brute force calibration.
Figure 39: The Calibration Optimization Process (off-line). This represents engine simulation using the virtual engine [32].

An optimization was conducted in [35] for fuel quantity, EGR and VGT, however the scheduling variables were oxygen concentration before and after the combustion chamber instead of engine speed and load. These parameters were optimized to make the oxygen concentrations follow a desired trajectory during a transient operation. The oxygen concentrations were determined using a combination of models and an oxygen sensor which was placed up stream of the turbine. Figure 40 shows the oxygen concentration before and after the combustion chamber for a transient operation from 4 to 9 bar at 1650 rpm for the oxygen based calibration and the standard calibration. The desired trajectories are much smoother with less overshoot and oscillations then when the standard calibration is used. Figure 41 shows the actuator controls that were needed to
follow these desired trajectories and Figure 42 shows the emissions and torque results. There was a large reduction in emissions with little change in the torque produced.

Figure 40: $O_{2,BC}$ and $O_{2,AC}$ for a torque transient from 4 bar to 9 bar at 1650 rpm [35].
Figure 41: Comparison of injected fuel quantity and air path actuators for a transient from 4 bar to 9 bar at 1650 rpm [35].
Study [33] placed a limit on the AFR and EGR fraction to reduce the particulate and NO\textsubscript{x} spikes that occur during a transient operation. The AFR was limited to 28 and the EGR fraction was limited to 10 percent on the cylinder content. These strategies were both implemented on a Ford Puma 2.0 direct injected diesel engine. Figure 43 shows the experimental results for NO\textsubscript{x} and particulates for these control strategies when compared to the base calibration. These two control strategies are implemented with little affect on the torque production of the engine, which is shown in Figure 44. There is a small affect on torque using the low PM strategy since it limits the AFR, which means it limits the fuel quantity. However the NO\textsubscript{x} strategy can still be implemented if it is decided that the torque decrease is undesirable.

Figure 42: Comparison of NO\textsubscript{x}, PM and torque from 4 bar to 9 bar at 1650 rpm [35].
Figure 43: Comparison of standard ECU emissions during a load transient and emissions when the model based limit on both the cylinder burned gas fraction and air fuel ratio is applied [33].
Figure 44: Comparison of standard ECU emissions during a load transient and emissions when the model based limit on the cylinder burned gas fraction is applied [33].

2.7 Closed-loop conventional diesel combustion control information

The focus of this search was CLCC for conventional diesel combustion. Both of the identified papers demonstrated that the SOC could be regulated to a desired value. This optimal value must be calibrated ahead of time though. Figure 45 shows the disturbance rejection performance of the controller developed in [13]. In [13], the SOC was calculated by comparing the measured pressure to the equivalent motored pressure. They found that a pressure difference to 10 bar corresponded to 2 CAD after SOC. In [12], the
injection parameters were correlated with the location of the first heat release peak, the size of the first heat release peak and the location of the second heat release peak. The locations of the heat release peak were regulated well (standard deviations of 0.5 and 1.0 CAD) for EGR fractions less than 15%. The standard deviations doubled when the EGR fraction was increased to 30%. The size of the first heat release peak was not regulated very well.

Figure 45: Closed-loop start of combustion control demonstration [12].
2.8 Trapped Residual Mass Models

[36] uses the first law of thermodynamics to calculate the mass flow rate across the exhaust valve from intake valve opening to exhaust valve closing. The trapped mass is determined from the initial trapped mass at IVO plus the difference that flows across the exhaust valve until EVC. The model was based on the time averaged intake and exhaust pressure, and exhaust temperature.

The trapped mass at IVO is calculated with the following equation with the assumption that the pressure inside the cylinder is the same as the pressure in the exhaust manifold.

\[ m_{tr-0} = \frac{P_{exh}}{RT_{exh}} \cdot V_c(\varphi_{IVO}) \]  

(2)

The mass flow rate across the valves is determined using the first law with the assumptions that the change in the heat transfer and cylinder temperature during the valve overlap is zero.

\[-p_c \frac{dV}{d\varphi} + h_e \frac{dm_e}{d\varphi} + h_i \frac{dm_i}{d\varphi} = u_e \frac{dm_e}{d\varphi} \]  

(3)

Using the mass balance and the simplification that \( h_e = h_i = h_c \) the first law becomes

\[ \frac{p_c}{RT_c} \frac{dV}{d\varphi} = \rho_c \frac{dV}{d\varphi} = \frac{dm_e}{d\varphi} + \frac{dm_i}{d\varphi} \]  

(4)

The cylinder charge is considered incompressible so the exhaust and intake mass flow rates were determined using the Bernoulli equation.
\[
\frac{dm_{\text{exh}}}{d\phi} = SGN(p_{\text{exh}} - p_c)A_{\text{eff}_{-exh}} \sqrt{2 \cdot \rho \cdot ABS(p_{\text{exh}} - p_c)} \cdot \frac{dt}{d\phi}
\]
\[
\frac{dm_{\text{int}}}{d\phi} = SGN(p_{\text{int}} - p_c)A_{\text{eff}_{-exh}} \sqrt{2 \cdot \rho \cdot ABS(p_c - p_{\text{int}})} \cdot \frac{dt}{d\phi}
\]  

(5)

After some manipulations and integrating the exhaust valve mass flow rate, the change in trapped residual mass during the overlap period was,

\[
\Delta m_{\text{tr}} = \rho \sum_{\phi_{\text{ivo}}} \left[ \frac{dV}{d\phi} \frac{\alpha_{K_{-exh}}}{\alpha_{K_{-exh}}^2 + \alpha_{K_{-int}}^2} \Delta \phi \right] + SGN(p_{\text{exh}} - p_{\text{int}})A \cdot \int_{\phi_{\text{ivo}}}^{\phi_{\text{ivo}}+\Delta \phi} \left[ \frac{\alpha_{K_{-exh}}}{\alpha_{K_{-exh}}^2 + \alpha_{K_{-int}}^2} \right] \frac{dt}{d\phi} \cdot \sum_{\phi_{\text{ivo}}} \frac{\alpha_{K_{-int}}}{\alpha_{K_{-exh}}^2 + \alpha_{K_{-int}}^2} \Delta \phi
\]  

(6)

where \( A_{\text{eff}} = A_K a_K \) and \( A_K \) = piston area. The first term above is due to the piston motion which causes the cylinder volume to change and the second term results from the pressure differential between the intake and exhaust manifold. The summations for both of the terms remain constant as long as the system does not have any type of variable valve actuation. The only thing that is needed is precise flow data to determine the effective areas. Models were also developed for the exhaust pressure and temperature if production measurements are not available, however these are based on regressions for individual engines.

The model was used to estimate the trapped residual on a single cylinder version of a BMW Rotax F650 engine. The results are compared to a calibrated GT-power model, gas exchange model, and trapped residual models derived by Heywood and Bertling. The gas exchange model is based on determining the intake and exhaust flow rates and
cylinder pressure. This model has been validated experimentally using a fast gas sampling valve. The results for BMW engine for different models can be seen in Figure 46 and Figure 47 shows results for a large diesel engine, the engine is not specified.

Figure 46: The trapped residual fraction results for the BMW engine calculated using five different models.
Figure 47: The trapped residual fraction results for a large diesel engine calculated using three different models.

[37] and [38] uses a similar approach; however they model the complete cycle using the cylinder pressure as well. This model is good for offline calculation but could not be used for production due to the lack of cylinder pressure. The paper by Cavina is a grey box model that uses a similar zero dimensional approach. However the model also incorporates regressions for other parameters that they found influences the trapped residual mass. These variables include the engine speed, air to fuel ratio and the EGR fraction.
2.9 Conclusion

The majority of the literature focuses on model based calibration and design of experiment techniques to determine the corrections for the injection strategy at the different operating or ambient conditions. These processes reduce the time and cost needed for calibration of the conventional diesel engine control maps. Information is not available on the actual corrections that are made for the injection parameter calibrations. This information is proprietary to the engine manufactures and not published in literature. Attempts are made to force desired cylinder conditions to try and compensate for changes in operating or ambient conditions using advanced hardware components. However, there are no attempts made to schedule the injection parameters based directly on the cylinder conditions. The literature is based on advances in the conventional control techniques.

Scheduling the injection commands based on cylinder conditions decouples the commands from the operating and ambient conditions. The literature covers the affects that the ambient and operating conditions have on the cylinder conditions compared to normal steady state operation. These changes are used to influence the design of a new control structure that focuses on the cylinder conditions and captures the cylinder effects caused by the ambient and operating conditions.
CHAPTER 3
MODELS, EXPERIMENTAL SETUP AND METHODS

3.1 Introduction

The background tools and procedures used throughout this thesis are presented. Two engine models and an experimental apparatus are used for analysis and data collection. The models were developed using the commercial software GT-Power by Gamma Technologies. A brief overview of GT-Power is supplied, and details and validation of the models are given. The experimental apparatus is described, including the instrumentation implemented for data collection. The chapter concludes by presenting the propagation of variance method, which is used to find the uncertainty in the root cause parameters in Chapter 7.

3.2 GT-Power Models

GT-power is commonly used among engine manufacturers for engine design and development, including powertrain control. GT-Power is based on one dimensional gas dynamics and is capable of steady state and transient simulations. The gas dynamics are solved using the conservation of mass, energy, and momentum incorporating heat
transfers between elements and to the exterior of the control volumes. Multiple combustion models are available within GT-power ranging from empirical models to simplified CFD models. Engine components, like heat exchangers, crankshafts, turbochargers, and fuel injectors, can easily be modeled and integrated. These complete models are able to predict engine performance, emissions and fuel economy. This provides a very effective tool for engine development due to the ease in which components and controls can be interchanged.

The physical modeling used within GT-power also allows for many variables to be determined during simulations. An example of some of these variables includes:

- Pressures
- Temperatures
- Flow Rates
- Composition
- Burn Fractions
- Volumetric Efficiency
- IMEP

This is very helpful when analyzing the results of the models. The GT-power models are used in place of experimental data for analyses where experimental data is not feasible due to the physical ability to measure the required parameters.
3.2.1 Cummins 8.9 L Diesel Engine

The first model used in this study is for a Cummins 8.9 L direct injection engine. The engine was equipped with a high pressure common fuel rail and a variable geometry turbocharger. The specifications for the engine can be seen in Table 1.

Table 1: Engine specifications for the Cummins 8.9 L Engine

<table>
<thead>
<tr>
<th>Engine Type</th>
<th>6-Cylinder, 4 - valve DOHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>8.85 L</td>
</tr>
<tr>
<td>Bore X Stroke</td>
<td>114 mm X 114.5 mm</td>
</tr>
<tr>
<td>Compression Ratio</td>
<td>16.6</td>
</tr>
<tr>
<td>Combustion System</td>
<td>Direct Injection</td>
</tr>
<tr>
<td>Intake System</td>
<td>VGT with intercooler</td>
</tr>
<tr>
<td>Fuel Injection System</td>
<td>High Pressure Common Rail</td>
</tr>
</tbody>
</table>

The 8.9 L engine was equipped with a charge air cooler, high pressure EGR loop, and turbocharger. The configuration of these components can be seen in Figure 48. The bleed point and introduction of EGR into the intake manifold can be seen in Figure 49. It will be noted the EGR bleed point occurs only from cylinders four, five and six.
Figure 48: The 8.9 L engine configuration.
The direct-injection diesel jet model within GT-power is used to model the combustion process. This is a predictive model used exclusively for direct injection compression ignition engines based on work done in references [50][51]. This model is effective in predicting the burn rate and NOx emissions. At every time step of the model, an axial slice of fuel is injected into the cylinder. Each axial slice is broken into five radial slices, seen in Figure 50. The fuel mass injected is dependent on the injection pressure and fuel injector nozzle characteristics. The liquid fuel, unburned fuel vapor and entrained air, and burned gasses are determined for each time step for every zone. When cylinder pressure, zonal temperature, and fuel-to-air ratio are favorable ignition occurs.
Using experimental data, 149 operating conditions are developed in the model. These operating conditions provide sufficient coverage on the engine speed/load map, Figure 51. To validate the engine model, the 149 model points are compared to experimental data for ISFC, NOx, IMEP, and particulate matter. The validation plots are shown in Figure 52 to Figure 55 and were taken from Ming Fang Master’s Thesis [49]. The DI-jet combustion model provided acceptable results for IMEP, NOx and ISFC as expected.
Figure 51: 149 calibrated GT-power operating conditions.
Figure 52: ISFC validation for 149 points.

Figure 53: NOx validation for 149 points.
Figure 54: IMEP validation for 149 points.

Figure 55: PM validation for 149 points.
3.2.2 Cummins 2.8 L Diesel Engine

The second model used in this thesis is for a Cummins 2.8 L directed injection diesel engine. This engine is equipped with a charge air cooler, high pressure EGR loop, turbocharger and intake air throttle. The component configuration can be seen in Figure 56. The intake air throttle is used to increase the EGR flow rate by restricting the fresh air flow.

![Figure 56: The configuration of the 2.8 L diesel engine](image)

The lack of available experimental cylinder pressure and flow characteristics of the fuel injectors provided a barrier when developing the combustion model. However, this engine model is used for analysis of the air path system only, so a combustion model was developed that produced desired exhaust temperatures using a single Wiebe function.

75
This allows for the air path system to be modeled accurately since the turbocharger response is driven by the exhaust temperature. The torque production and engine out emissions are not modeled accurately due to the simplified combustion model.

Wiebe functions are used to model the burn rate during combustion and are based on empirical data. Single Wiebe functions are typically used for spark ignited engines. Diesel combustion requires multiple Wiebe functions to capture the burn rate profile due to the influences of multiple injections. The addition of Wiebe functions increases the flexibility of the model to fit more complex experimental burn rate profiles. Since experimental cylinder pressure is unavailable to tune the model, the complexity of the model does not mean better accuracy. Therefore, a simple single Wiebe function was chosen for the basis of the combustion model.

The aim of the combustion model is to produce the desired exhaust temperatures. The single Wiebe function has a variable that controls the timing of combustion, known as the anchor angle, Figure 57. Changing the anchor angle has a direct impact on the exhaust temperature. As the burn rate is shifted to a more advanced timing, the exhaust temperature increases due to fuel burning later in the expansion and exhaust stroke. As the burn rate is shifted to a more retarded timing, the exhaust temperature decreases due to the energy having more time to be consumed within the cylinder. However, as the combustion is retarded further into the compression stroke, there reaches a point where the exhaust temperature begins to increase again. Combustion during early stages of the compression stroke causes extremely high temperatures and pressures which affects the
exhaust temperature. This threshold occurs at an anchor angle of 10 degrees before the piston reaches top dead center.

Figure 57: Anchor angle control of single Wiebe combustion model.

Using experimental data, 90 operating conditions covering the speed/load map were developed in the model, Figure 58. These are the steady state calibration points for the 2.8 L engine.
Figure 58: The operating conditions used for the uncertainty analysis which are evenly distributed over the speed/load map.

To ensure the model matches the experimental data, controllers are in place on the anchor angle, fresh air throttle, and EGR throttle, to control exhaust temperature, fresh air flow rate, and EGR flow rate. Due to the coupling of these three variables, a feedforward controller is used placed on the fresh air throttle and anchor angle. The EGR throttle has a feedback controller since it is the most sensitive when tuning the air path model, Figure 59. The fresh air throttle is only used when the EGR throttle becomes saturated. The feed forward controllers were tuned by sweeping anchor angle and intake air throttle position.
Figure 59: Feed-back controller implemented on the EGR throttle.

Figure 60 to Figure 64 show the validation of the 2.8 L model with respect to the experimental data for the 90 operating conditions. These validation plots are for EGR flow rate, fresh air flow rate, exhaust temperature, intake manifold pressure, and turbocharger speed. It will be noted that the goal is to correctly model the fresh air flow rate and the EGR flow rate. The outliers in the results for the EGR flow rate and fresh air flow rate are due to limitations of the system and coupling of the EGR and fresh air system. For example, when the model EGR flow rate is lower than the experimental flow rate, the fresh air throttle could be closed further, but this would adversely affect the fresh air flow rate. The air path system is modeled accurately, however, characteristics of combustion, like torque, emissions, and fuel economy, are not corrected modeled due to the simplified combustion model implemented.
Figure 60: The 2.8 L model EGR flow rate validation.
Figure 61: The 2.8 L model fresh air flow rate validation.

Figure 62: The 2.8 L model exhaust temperature validation.
Figure 63: The 2.8 L model intake manifold pressure validation.

Figure 64: The 2.8 L model turbocharger speed validation.
3.3 Experimental Setup

A Cummins 2.8 L diesel engine is used for experimental data collection. This is the same engine that is modeled in the previous section. The engine is coupled to a 300 Hp AC dynamometer. The dynamometer is used to absorb the engine power and is able to operate in constant speed or torque mode using a dyne system controller. For all testing purposes, the dynamometer is operated in constant speed mode.

The engine is instrumented with sensors to record and monitor temperatures, pressures, flows, emissions, speed and torque. The location of these sensors are strategically placed to ensure the quality and accuracy of the measurements. Modifications to the engine and components are made as necessary to equip the engine with all the sensors, without compromising the functionality of the engine. The key measurements needed for this study included, the fresh air flow rate, the EGR flow rate, the charge flow rate, and the CO₂ fraction in the intake manifold. The engine and measurement specifics are not described due to the request of the manufacturer, so only an a brief overview is given.

The fresh air flow rate is measured using a laminar flow element upstream of the intake air box. The flow rate is determined by measuring the differential pressure across the flow element and making corrections for humidity and temperature. The EGR flow rate is measured using a venturi in line of the EGR flow path. The differential pressure is measured across the venturi and corrections are made for the fluid density. The charge flow rate is determined as the sum of the fresh air flow rate and EGR flow rate.
The CO₂ fraction is measured using a Horriba MEXA-7500DEGR exhaust gas analyzer. Three vertical sample ports are present in the intake manifold that can be sampled individually or in any combination of the three. There is also a horizontal tube that runs down the center of the intake manifold, with precisely designed holes, that can be sampled to get an average CO₂ fraction measurement. A measurement is not possible through the horizontal and vertical ports at the same time, only one or the other. Figure 65 shows the design and layout of the different sample ports.

![Figure 65: Intake manifold design to measure CO₂ fraction.](image)

The DAQ system is composed of three input modules to compensate for all the measurements. Modules 1 and 2 are National Instruments NI SCXI-1100 boards and module 3 is a National Instruments NI SCXI-1102 board, which is specifically designed for thermocouple measurements. These boards are used due to the programmable gain and filter settings for conditioning the signals and the ability to multiplex the inputs into a
The DAQ system is setup to sample at 100 Hz. The data acquired with this system is cycle averaged data, meaning that each acquisition is averaged over the complete sampling time. This results in a single steady state value for each engine variable.

3.4 Uncertainty Analysis

The quality in which a systems variables are measured or predicted is an important aspect in the design of a measurement system. The goal is to minimize the uncertainties in the measurement system to acquire the most accurate results. The effectiveness of a control strategy is directly related to the quality of the measured inputs. Control strategies are based on input/output relationships, so uncertainty in the input directly affects the output and performance of the controller. Systematic methods have been developed to quantify these uncertainties. The propagation of variance method allows for uncertainties in the independent variables to be projected onto the dependant variable [39]. This method is especially useful for dependant variables with multiple inputs. The propagation method is used to find the uncertainty in a nonlinear function $f$, composed of the mean value and the uncertainty value, equation (7).

$$f = \tilde{f} + \sigma_f$$  \hspace{1cm} (7)

Linearizing equation (7) using the Taylor series expansion,
\[ f' \approx f_0 + \sum \frac{\partial f}{\partial x_i} = k + \sum a_i x_i = k + a^T x \quad (8) \]

The covariance matrix is defined as shown in equation (9)

\[
M = \begin{pmatrix}
\sigma_{x_1}^2 & \sigma_{x_1 x_2} & \ldots & \sigma_{x_1 x_n} \\
\sigma_{x_2 x_1} & \sigma_{x_2}^2 & \ldots & \sigma_{x_2 x_n} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{x_n x_1} & \sigma_{x_n x_2} & \ldots & \sigma_{x_n}^2 \\
\end{pmatrix} 
\quad (9)
\]

Using the covariance matrix, the uncertainty in \( f \) is shown in equation (10)

\[
\sigma_f^2 = a^T M a
\quad (10)
\]

If the covariance terms, off diagonal terms, in \( M \) are zero, the uncertainty simplifies to equation (11)

\[
\sigma_f^2 = \sum \sigma_{x_i}^2 \left( \frac{\partial f}{\partial x_i} \right)^2 
\quad (11)
\]

This is the fundamental equation used for the uncertainty analysis in Chapter 7.

For independent variables with more than one uncertainty, the root-sum-squares method is used to find the total uncertainty in that variable [39],

\[
\sigma_{tot} = \pm \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots \sigma_k^2} 
\quad (12)
\]

As an example, the root sum squares method is needed to combine the uncertainties in a pressure sensor, which has errors due to linearity, calibration, hysteresis and other factors.

The total uncertainty in the pressure sensor measurement is then used as the input uncertainty in the propagation of variance method.
3.5 Conclusion

A GT-power model for a 2.8 L and 8.9 L diesel engine are validated and available for analysis throughout the thesis. The limitations of the models are clearly stated to ensure the boundaries are not crossed during analysis. A brief description of the experimental apparatus for the 2.8 L diesel engine is given. Experimental data is the primary source of data collection in this study. Simulation is only used where experimental data is not feasible. A method is presented to determine the uncertainty in the dependant variable based on uncertainties in the independent variables for nonlinear functions. The method is applied to possible diesel engine control inputs to determine the prediction quality during engine operation.
CHAPTER 4
ROOT CAUSE PARAMETERS

4.1 Introduction

Root cause parameters are variables that relate to the physics of combustion. Their influence changes the combustion characteristics, which is key to the control approach undertaken in this research. Currently, diesel engine control strategies schedule the air and fueling parameters based on the engine speed and load. To account for changes in operating conditions, multiple ECU maps are calibrated for the specific conditions. For example, maps are developed for steady state, transient, and altitude conditions. A new control strategy, based on root cause parameters, is developed to reduce the number of ECU maps needed. This will decrease the calibration efforts for development of new engine platforms.

After the intake valve closes, the only factors that affect combustion are the conditions that are trapped inside the cylinder. From the cylinder point of view, there are no distinction between steady state conditions, transient conditions, altitude conditions, and all other conditions. Scheduling the injection parameters based on cylinder conditions would thus reduce the need for multiple ECU maps. This statement holds if the cylinder conditions can be quantified and trends between the cylinder contents and desired
injection parameters are developed. For trends to exist, the scheduling variables must have a direct impact on the combustion process. This is the idea of using root cause parameters as the basis for the control structure.

Using open literature, factors affecting diesel combustion were researched to establish a comprehensive list of root cause parameters. A preliminary control structure is then presented to show how root cause parameters could enhance the diesel control strategy. This thesis focuses on the quality in which the control inputs can be quantified, so the presented structure is tentative and intended to provide the general direction of the research.

4.2 Root Cause Parameter Categories and Influence

Seven main categories were found to have an influence on the diesel engine combustion process, affecting emissions, fuel economy and noise [41][42][43][44]. These categories and their influence are shown in Table 2. The root cause parameters were developed by refining these seven categories into specific variables. For example, the unburned air is further refined into mass based variables and concentration based variables. Within each of these categories a list of root cause parameters is populated. An example of the mass based unburned air parameters is shown in Figure 66, which populates 6 root cause variables. This top down approach provides a rich list of root cause parameters as desired. A diagram showing all the root cause parameters is shown in Appendix A.
There are 40 total variables. Appendix A also includes a visual representation of the possible sources that determine each of the root cause parameters in subsequent pages. This information is used in the proceeding chapter for the cylinder content analysis.

Table 2: The root cause parameter categories and associated combustion influences.

<table>
<thead>
<tr>
<th>Root Cause Categories</th>
<th>Combustion Influence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Total Charge</td>
<td>Relates to the temperatures and pressures in the cylinder</td>
</tr>
<tr>
<td>2. Unburned Air</td>
<td>Mass Based: Mass of oxygen relates to mass of fuel to be burned</td>
</tr>
<tr>
<td></td>
<td>Concentration Based: Relative amount of oxygen relates to chemical kinetics of combustion</td>
</tr>
<tr>
<td>3. Residuals</td>
<td>Mass Based: Residual mass displaces air and acts as inert gas during combustion</td>
</tr>
<tr>
<td></td>
<td>Concentration Based: Relative amount of residuals relates to chemical kinetics of combustion</td>
</tr>
<tr>
<td>4. Engine Speed</td>
<td>Time Based: Time available for combustion</td>
</tr>
<tr>
<td></td>
<td>Charge Motion: Mixing dynamics</td>
</tr>
<tr>
<td>5. Temperature</td>
<td>Combustion kinetics, charge density and heat transfer</td>
</tr>
<tr>
<td>6. Load/Fuel Mass</td>
<td>Affects the overall air to fuel ratio and combustion duration</td>
</tr>
<tr>
<td>7. Fuel Properties</td>
<td>Ignition and combustion properties</td>
</tr>
</tbody>
</table>
4.3 Root Cause Variable Control

Combustion references are used to schedule the fuel injection strategy and air handling set points for diesel engines. Current practice stores the set points in maps based on engine speed and load (refer to Chapter 2 for a thorough description of current practice). Developing trends between commanded injection strategies or air set points and combustion references will lead to more sophisticated control techniques in place of extensive calibration efforts. To narrow the scope of this project, the focus is directed towards developing a more intelligent fuel injection strategy assuming the air handling set points are already optimal. A requirement when developing the new injection control strategy, is to be able to command the near optimal set points regardless of the engine
operating conditions. Therefore, the air handling control strategy can be developed in the future and the fuel control strategy will still be effective.

The basis of enhancing the injection control strategy relies on the relationship between root cause parameters and injection commands. The root cause parameters developed have properties that effect combustion, so the idea is to identify the trends and make appropriate corrections. Accurate prediction of the root cause parameters is necessary for the corrections to be applied accurately. This includes transient operation.

A preliminary control structure for the injection strategy is shown in Figure 67. The structure uses four combustion references, two being the conventional references. Out of the remaining two, one is a measure of fresh air and one is a measure of residual gas. A trapped residual model and transient model will be developed to dynamically determine the two additional combustion references. All four of the combustion references are used as inputs to the fuel recipe.
The following control structure has the capability of condensing the steady state, transient, thermal and altitude calibration maps into a single feed-forward engine map with correction factors. The effectiveness of the control structure is heavily dependent on developing trends between the second set of combustion references and desired injection commands. For example, if the trends due not hold at increased altitude then calibration maps will still be needed as elevation increases and correction factors will not be able to be used. Using the root cause parameters as the basis for additional combustion references will provide the desired trends.

4.4 Strategy to Determine Most Effective Combustion References

There are two key requirements for a set of root cause parameters to be effective in the control strategy presented. The parameters must be quantifiable to within a desired
uncertainty during engine operation and encompass a repeatable trend with the injection strategy. The following five steps will achieve the most effective combustion reference set based on these two criteria.

1. Define a subset list of root cause parameters that would be feasible for implementation into the diesel engine control strategy from the comprehensive list.

2. Determine the uncertainty in predicting the combustion references during steady state operation for all sensor set combinations.

3. Incorporate the uncertainty associated with the transient model.

4. Develop trends between the combustion references and the optimal injection strategy.

5. Compare the results to obtain the root cause parameters that exhibit repeatable trends are accurately quantified.

The first step reduces the comprehensive list of root cause parameters to a feasible set based on the ability to quantify the parameters, repetitive nature of the parameters, and acceptance to common practice. The second step ensures the root cause parameters can be quantified at steady state operating conditions using available measurements or models implemented on production engines. This step includes comparison between different sensing methods to determine the most accurate measurement system for a set of combustion references. The third step incorporates the uncertainty in predicting the transient delays during dynamic engine behavior with the steady state uncertainty. The
fourth step investigates the trends between the desired injection strategy and root cause parameters. The final step is a comparison of the results from the uncertainty analysis and analysis of the trends to find the most effective combustion references. After trade-offs between the uncertainty analysis and observed trends are resolved, parameter sets to be implemented in the control strategy would result. The last four processes incorporate multiple sub problems which are not straightforward and require thorough analysis. This thesis focuses on the first two steps in the process.

4.5 Subset List of Root Cause Parameters

The comprehensive list of root cause parameters covers a wide variety of engine variables. Not all of these parameters would be effective for implementation into the diesel engine control strategy. For example, the charge motion root cause parameters, which include swirl, tumble, squish and a turbulence metric, are known to affect the diesel combustion process, however, there is not a good way to quantify these variables using available sensors and cost-effective production calibration techniques. This would make it difficult to schedule fueling parameters based on these variables. The fuel properties root cause parameters are also dropped due to the lack of sensors available to quantify these variables.

Other variables from the comprehensive list are similar in nature. For example, the time based engine speed parameters, which includes engine speed in revolutions per minute,
mean piston speed, and time per cycle, are multiples of each other. These parameters are repetitive and have the same effectiveness as combustion references. Therefore, engine speed in revolutions per minute is included in the subset list of variables and the other variables are dropped. The same reasoning is applied to the load/fuel mass variables where the engine torque is included and all other variables are neglected. Engine speed and torque are chosen from these lists due to the acceptability of these variables since they are used in current practice.

The acceptability for engine manufacturers of new combustion references is an important consideration in choosing possible combustion references. Oxygen number density and diluents number density are abstract variables uncommon to the diesel environment and are dropped from the subset list of parameters. Parameters are only disregarded based on this reasoning if other parameters from the same category are chosen. For example, the oxygen number density is part of the unburned air concentration based variables. The oxygen mass concentration and molar concentration are considered in the subset list from this category.

After applying the three reduction processes above, the subset list of variables includes the root cause parameters listed in Table 3 and Table 4. Table 3 lists the burned and unburned root cause parameters. Table 4 includes the total charge variables and the conventional combustion references. The control structure introduced in the previous section is based on four combustion references, two of them being the conventional references. After refining the comprehensive list, the remaining root cause parameters
are measures of the cylinder composition. Therefore, the enhancement of the control structure is based on parameters that measure the cylinder composition.

Table 3: The burned and unburned root cause variables considered for combustion references.

<table>
<thead>
<tr>
<th>Burned Air Variables</th>
<th>Unburned Air Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass Based</strong></td>
<td><strong>Mass Based</strong></td>
</tr>
<tr>
<td>EGR Mass Rate</td>
<td>Fresh Mass Rate</td>
</tr>
<tr>
<td>EGR Mass Rate per Cylinder</td>
<td>Fresh Mass Rate per Cylinder</td>
</tr>
<tr>
<td>EGR Mass per Cylinder</td>
<td>Fresh Mass per Cylinder</td>
</tr>
<tr>
<td>Exhaust Mass Rate</td>
<td>Unburned Mass Rate</td>
</tr>
<tr>
<td>Exhaust Mass Rate per Cylinder</td>
<td>Unburned Mass Rate per Cylinder</td>
</tr>
<tr>
<td>Exhaust Mass per Cylinder</td>
<td>Unburned Mass per Cylinder</td>
</tr>
<tr>
<td>Burned Mass Rate</td>
<td></td>
</tr>
<tr>
<td>Burned Mass Rate per Cylinder</td>
<td></td>
</tr>
<tr>
<td>Burned Mass per Cylinder</td>
<td></td>
</tr>
<tr>
<td><strong>Concentration Based</strong></td>
<td><strong>Concentration Based</strong></td>
</tr>
<tr>
<td>Diluent Mass Concentration</td>
<td>Oxygen Mass Concentration</td>
</tr>
<tr>
<td>Diluent Molar Concentration</td>
<td>Oxygen Molar Concentration</td>
</tr>
</tbody>
</table>

Table 4: The charge root cause variables and conventional control variables to be considered as combustion references

<table>
<thead>
<tr>
<th>Charge Variables</th>
<th>Conventional Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mass Based</strong></td>
<td></td>
</tr>
<tr>
<td>Charge Mass Rate</td>
<td>Engine Speed</td>
</tr>
<tr>
<td>Charge Mass Rate per Cylinder</td>
<td>Engine Torque</td>
</tr>
<tr>
<td>Charge Mass per Cylinder</td>
<td></td>
</tr>
</tbody>
</table>
4.6 Conclusion

The effectiveness of the control structure developed is based on the ability of the root cause parameters to be related to the injection parameters and the quality in which the parameters can be quantified. Scheduling the injection parameters on a variable with high prediction uncertainty causes the control structure to lose its effectiveness. The remainder of this thesis will focus on quantifying the root cause parameters, using production sensors and models, and their associated uncertainty. This is step 2 in the overall process of determining the most effective combustion references. The subset list of root cause parameters are the most probable candidates for implementation as combustion references in the new control structure and will be the parameters of interest for the remainder of the thesis. The remaining root cause parameters are based on the cylinder composition which leads to the cylinder content analysis in the following Chapter.
CHAPTER 5
CYLINDER CONTENT ANALYSIS

5.1 Introduction

The objective of the cylinder content analysis is to determine the sources that have significant contribution to the final cylinder mass. This information is needed when developing the equations to predict the root cause parameters. Starting from all possible mass sources of the cylinder, this analysis provides results on the sources that could be considered negligible and sources that have significant contribution. This analysis is conducted using the 8.9 L GT-Power model and the results are validated using the 2.8 L GT-Power model. The GT-Power models are used because the analysis focused on mass quantities that could not be measured experimentally.

5.2 Possible Cylinder Content Sources

There are six possible flow sources that can contribute to the cylinder content,

1. Fresh Air
2. External EGR
3. Trapped Residual
4. Intake to Exhaust Transfer during Valve Overlap (Flow-through).

5. Exhaust to Intake Transfer during Overlap (Back-flow)

6. Blow-by Losses

The contribution of these sources can be seen visually in Figure 68.

Figure 68: Visual of the possible flow sources of trapped mass within the cylinder prior to combustion.

The definition for the first two sources are straightforward. The fresh air mass is the mass that flows through the compressor of the turbocharger and the EGR mass is the mass that flows through the exhaust gas recirculation loop that siphons mass from the exhaust manifold and circulates it back to the intake manifold. The remaining four
sources are not as evident and therefore will be defined in more detail as they are used in this thesis.

Trapped residual mass is any mass that remains inside the cylinder once the intake and exhaust valves close that originated from the previous burned cycle. Figure 69 shows the exhaust stroke of an engine just after combustion on the left and the intake stroke of the next cycle on the right. The yellow dots show the mass from the previous cycle and the blue and red dots are mass from the intake manifold. The mass that remains in the cylinder after the exhaust valve closes is considered trapped residual mass. As shown in the right figure, the single yellow mass inside the cylinder would be considered trapped residual mass.

![Figure 69: Representation of cylinder mass that would be considered trapped residual mass.](image)

Flow-through is mass that flows directly from the intake manifold to the exhaust manifold during the valve overlap period and is trapped in the exhaust manifold or ports.
once the exhaust valve closes. This mass does not participate in combustion unless any of this mass flows back into the cylinder when the exhaust valve opens during the next cycle. Figure 70 shows the intake stroke of an engine during the valve overlap on the left and just after the exhaust valve closes on the right, for the same intake stroke. The mass that remains trapped in the exhaust manifold once the exhaust valve closes would be considered flow-through.

![Cylinder Mass Diagram](image)

**Figure 70:** Representation of cylinder mass that would be considered flow-through mass.

Back-flow is any mass that is trapped in the intake manifold that originated from the previously burned cycle. This can occur if mass from the previous cycle flows into the intake manifold during any part of the intake process and remains there once the intake valve closes. This mass would then participate in the next combustion cycle once the intake valve opens. Figure 71 shows the intake stroke of an engine on the left and just after the intake valve closes during the compression stroke on the right. The yellow dots in the left figure are considered trapped residual mass at this point because they
originated from the previously burned cycle and remained in the cylinder once the exhaust valve closed. On the right figure, one yellow particle makes it into the intake manifold and remains there once the intake valve closes. This mass would now be considered back-flow and no longer trapped residual mass. The two yellow particles in the cylinder would still be considered trapped residual mass.

Figure 71: Representation of cylinder mass that would be considered back-flow mass.

Blow-by loss is mass that flows past the cylinder rings into the engine crankcase during the compression stroke. Figure 72 shows the mass that is considered blow-by. Regulations force manufacturers to recirculate this mass back to the intake system for emissions control.
Figure 72: Representation of cylinder mass that would be considered blow-by mass.

For this analysis it is known that fresh air and EGR have significant contributions to the cylinder mass. The focus is on determining whether the remaining four sources have any effect on the cylinder mass. There is currently no convenient way to measure the flow-through, back-flow and trapped residual mass in an experimental setting, therefore, the analysis on these three sources had to be completed using simulation tools. These tools are not capable of modeling the blow-by losses, so experimental data is used to quantify these losses. The analysis will be carried out on two different diesel engines to ensure the results obtained can be generalized for conventional diesel combustion.

5.3 Cylinder Content Analysis – 8.9 L Engine

The operating conditions chosen for the cylinder content analysis are based on the 13-mode test used for steady state emissions testing in the United States. These data points
are chosen because they are intended to represent common operating conditions of heavy-duty diesel engines. The 13-modes are based on the rated speed and load of the specific engine. Figure 73 shows the 13-mode points along with the calibrated GT-Power operating conditions for the 8.9 L engine. The closest calibration point to each mode is chosen to be used in the cylinder content analysis. The thirteen mode test included modes at idle and high idle that did not correspond to any of the calibrated GT-power points, so these two points are dropped from the analysis.

![Figure 73: Operating conditions chosen for the cylinder content analysis based on 13-mode test.](image)

The GT-power model does not directly differentiate between trapped residual mass and EGR mass. In order to determine these masses independently, the residual mass is
tracked across the intake valves to get the incoming EGR fraction. The remaining residual is considered trapped residual mass. This calculation is based on the assumption that the EGR and fresh air mixture is homogeneous. The assumption is not entirely accurate but is acceptable since the purpose of this analysis is to find flows that are negligible, not exact values. The resulting trapped residual fraction for each of the eleven modes is shown in Figure 74. The average of the trapped residual fraction is about 4 percent with some operating conditions reaching over 8 percent. This is a significant amount which leads to the conclusion that trapped residual mass may not be neglected in the cylinder content.
Figure 74: Quantifying the trapped residual mass for all eleven modes.

Figure 75 shows the mass flow across the valves for one of the 11-modes under study. During the valve overlap period, flow-through can occur because the intake and exhaust valves are open at the same time. Back-flow can also occur during the valve overlap period due to reverse flow through the intake valves as shown. Prior to intake valve closing, a small amount of mass from the cylinder flows back into the intake manifold. A portion of this mass is considered back-flow because it contains a fraction of trapped residual mass. These areas are at the beginning and end of the valve profiles where the piston does not have a strong influence on the flow direction. At exhaust valve opening, these effects are not present since the cylinder pressure is high enough to force flow out of the cylinder.
Figure 75: Intake and exhaust valve mass flow rates over an engine cycle for mode 10.

Figure 76 shows the mass that flows from the cylinder to the intake system during the valve overlap period as a percentage of the total cylinder mass. This represents the maximum possible back-flow percentage possible if all the mass remains in the intake manifold once the intake valve closes. The results show that for most of the operating conditions there is no flow from the cylinder to the intake manifold during the valve overlap period, therefore, no possibility of backflow. For mode three, there is a possible four percent contribution to total cylinder mass from backflow. However, the valve overlap period is during the beginning of the intake stroke, so the mass that enters the intake system is reconsumed by the cylinder prior to intake valve closing.
Figure 76: The maximum possible back-flow percentage with respect to total cylinder content if all mass remained in intake manifold and was not reconsumed.

Tracking the residual mass across the intake valves verifies the assumption that the mass that enters the intake manifold from the cylinder, during the valve overlap period, is reconsumed. This is done for mode three since it has the highest potential for back-flow to occur. Figure 77 shows the flow rate across the intake valves with the EGR valve closed. The figure breaks the flow rate into fresh air and residual mass. When the intake valve opens, the residual from the cylinder enters the intake manifold. As the intake process continues the residual is reconsumed by the cylinder. Integrating the residual mass flow across the intake valves results in zero total mass, which means that no residual is left within the intake manifold. This calculation is done for all eleven modes and the same result is obtained.
Figure 77: Flow rate through the intake valves for Mode 1 with the EGR valve closed, distinguishing between residual and fresh air flow.

At the end of the intake stroke, reverse flow through the intake valves occurs. This is shown in Figure 78. The reverse flow is trapped within the intake manifold and ports. If any of this mass is trapped residual mass, it becomes classified as backflow and will be consumed in the next cycle when the intake valves open. The amount of back-flow mass is calculated as,

\[ m_{\text{back-flow}} = m_{\text{rev}} \cdot X_{\text{res}} \]  \hspace{1cm} (13)

where \( m_{\text{rev}} \) is the total reverse flow across the intake valves and \( X_{\text{res}} \) is the trapped residual fraction inside the cylinder.
Figure 78: Flow through the intake valves for one cycle showing the backflow at the end of the intake process.

Figure 79 shows the back-flow that is trapped in the intake manifold due to the reverse flow at the end of the intake stroke. The back-flow is represented as a percentage of the total cylinder mass. The percentages for all 11 modes is less than .02%, which would not affect the combustion process. The back-flow contribution at the end of the intake stroke can be neglected.
Figure 79: Quantifying the back-flow percentage that occurs at the end of the intake stroke with respect to total cylinder mass.

The flow through is calculated within GT-power and the results for the 11 modes are shown in Figure 80. The flow-through mass never sees combustion because it passes directly from the intake to the exhaust during valve overlap. Once the exhaust valves open, this mass is pushed downstream by the exhaust mass from the cylinder. The flow-through mass is less than .5% of the total cylinder mass and can be neglected.
The flow-through and back-flow are insignificant to the overall cylinder content. The fresh air mass, EGR mass and trapped residual mass are the main factors that contribute to the overall cylinder contents prior to fuel injection. Figure 81 shows an example of the cylinder composition for one of the 11 modes. Table 5 shows the range of the fresh air, EGR, and trapped residual fraction for the 11 modes.
Table 5: The range of the fresh air, EGR, and trapped residual fraction for the 11 modes.

<table>
<thead>
<tr>
<th></th>
<th>Fresh Air Fraction</th>
<th>EGR Fraction</th>
<th>Trapped Residual Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>min</td>
<td>57.43</td>
<td>16.85</td>
<td>2.11</td>
</tr>
<tr>
<td>max</td>
<td>81.03</td>
<td>32.21</td>
<td>10.36</td>
</tr>
</tbody>
</table>

Experimental data from the 8.9 L Cummins engine is used to quantify the blow-by losses. This data was provided by the engine manufacturer. Figure 82 shows the blow-by fraction, with respect to the measured charge flow, at each operating condition on the speed/load map. The fraction of mass that is lost to blow-by is a little more than .5%.
This is a small percentage in relation to the total mass within the cylinder. The cylinder contents are assumed to completely mixed, so the percentage of the root cause parameter that is lost is the same as the percentage of blow-by mass that is lost. Thus the blow-by mass is not considered when quantifying the root cause variables and can be accounted for after the root cause parameters are determined.

Figure 82: Experimental data from the 8.9 L engine showing the blow-by percentage with respect to the fresh air flow.
5.4 Cylinder Content Analysis – 2.8 L Engine

To ensure the results obtained using the 8.9 L model are not specific to the engine, the analysis is repeated using the 2.8 L engine model. The same procedure is used for the 2.8 L engine as is shown for the 8.9 L engine analysis, so only the results are shown. Figure 83 shows the trapped residual fraction for each of the 11 modes. The trapped residual fraction remains relatively constant around 4 % of the total cylinder charge. This confirms the result that trapped residual fraction is a significant contributor to the total cylinder contents.
Figure 83: Trapped residual fraction at each of the 11 modes for the 2.8 L engine.

Figure 84 shows the mass that flows from the cylinder to the intake system during the valve overlap period as a percentage of the total cylinder mass. This mass would be considered back-flow if it remains in the intake once the intake valve closes. The percentage is less than 1% which is consistent with the 8.9 L engine results. This mass would be reconsumed by the cylinder during the intake stroke and would be considered trapped residual mass. This concludes that the back-flow mass during the valve overlap period is negligible.
Figure 84: The maximum possible backflow that could occur during the valve overlap period for the 2.8 L engine.

Figure 85 shows the back-flow that occurs at the end of the intake stroke with respect to the total cylinder contents. This mass is less than .06 percent of the trapped cylinder mass and would not have an effect on the combustion process. This confirms that the back-flow at the end of the intake stroke is negligible.
Figure 85: The backflow that occurs at the end of the intake for the 2.8 L engine.

Figure 86 shows the flow-through for each of the eleven modes with respect to the total cylinder contents. The intake mass lost to the exhaust system is less than .07 %. This confirms the result that flow-through is negligible.
5.5 Conclusions

The cylinder content analysis proved the back-flow and flow-through masses are negligible. The sources that need considered when quantifying the cylinder mass are the fresh air mass, EGR mass, and trapped residual mass. The blow-by mass is neglected for the remainder of the thesis. The lost mass can be accounted for in implementation by scaling the final root cause parameter equations by the percentage of cylinder mass lost to blow-by.
CHAPTER 6

ROOT CAUSE PARAMETER EQUATIONS

6.1 Introduction

The results from the cylinder content analysis are used to determine the equations for each of the root cause variables at steady state conditions. The derived equations are based on available measurements or models common to production diesel engines. This allows for the root cause variables to be implemented into a production control algorithm. Table 6 shows the production sensors and models available to diesel engines that would be useful in the prediction of the root cause parameters [36][41][42][44]. These sensors and models allow for the EGR and fresh air flow rate to be measured directly or determined indirectly as a difference between two other measurements. This causes the equations for the root cause parameters not to be unique to a specific sensor configuration. Therefore, equations are developed for all possible sensor combinations. Once the equations are developed for the root cause parameters, the uncertainty in the sensors and models can be used to find the total uncertainty in the root cause parameter. The uncertainty results for the parameters will aid in the decision of determining the most effective combustion references.
Table 6: The available production sensors and models used for calculating root cause variables.

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>Intake Manifold, EGR Loop, exhaust</td>
</tr>
<tr>
<td>Delta Pressure</td>
<td>EGR Loop</td>
</tr>
<tr>
<td>Temperature</td>
<td>Intake Manifold, EGR Loop, exhaust</td>
</tr>
<tr>
<td>Flow</td>
<td>Fresh Air</td>
</tr>
<tr>
<td>Speed</td>
<td>Engine</td>
</tr>
</tbody>
</table>

6.2 Determination of Cylinder Mass

From the cylinder content analysis fresh air flow, $m_f$, exhaust gas recirculation, $m_e$, and trapped residual mass, $m_r$, contribute to the cylinder mass prior to fuel injection, shown in the control volume in Figure 87. Since the diesel engine typically operates lean of stoichiometry, these sources have components of unburned air, burned air and burned fuel. The unburned air is defined as the fraction of air that does not react with fuel during combustion and is a measure of the oxygen in the composition. For example, the incoming fresh air only has an unburned component since it has yet to participate in combustion. The burned air component is the fraction of air that combusts stoichiometrically with the fuel mass injected. The burned fuel is the total fuel mass assuming 100% combustion. After combustion burned air and burned fuel do not physically exist, instead there are combustion products like $CO_2$, $H_2O$, $NO_x$, THC and...
CO, however, the mass components can be determined in this manner. The total burned mass is a measure of the CO₂ mass.

\[ m_{air, b} = AFR_b \cdot m_{f, b} \]  

(14)

\[ m_{air, b} + m_{air, ub} = AFR_m \cdot m_{f, b} \]  

(15)

The total mass flows, \( m_{fr} \), \( m_e \), \( m_t \), and \( m_f \), are determined using available sensors or models. To calculate the remaining burned and unburned mass components, relationships using the AFR are utilized. Equation (14) determines the burned air based on the stoichiometric air fuel ratio and the injected fuel mass. Equation (15) relates the measured, or actual air fuel ratio in the cylinder, and total injected fuel mass to the burned and unburned air components.
These two AFR equations are valid for EGR and trapped residual masses since the composition is the same at steady state conditions. To solve for the three trapped residual burned and unburned masses, the two AFR equations and total flow equation are used.

\[
m_{air,b}^t + m_{air,ub}^t + m_{f,b}^t = m^t
\]
\[
m_{air,b}^t - AFR_s \cdot m_{f,b}^t = 0
\]
\[
m_{air,b}^t + m_{air,ub}^t - AFR_m \cdot m_{f,b}^t = 0
\]

(16)

Arranging the three equations to be in matrix form,

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -AFR_s \\
1 & 1 & -AFR_m
\end{bmatrix}
\begin{bmatrix}
m_{air,ub}^t \\
m_{air,b}^t \\
m_{f,b}^t
\end{bmatrix}
= \begin{bmatrix}
m^t \\
0 \\
0
\end{bmatrix}
\]

(17)

Solving the equations to get the three individual components of the trapped residual mass,

\[
m_{air,ub}^t = \left(\frac{AFR_m - AFR_s}{AFR_m + 1}\right) \cdot m^t = F_{air,ub} \cdot m^t
\]

(18)

\[
m_{air,b}^t = \left(\frac{AFR_s}{AFR_m + 1}\right) \cdot m^t = F_{air,b} \cdot m^t
\]

(19)

\[
m_{f,b}^t = \left(\frac{1}{AFR_m + 1}\right) \cdot m^t = F_{f,b} \cdot m^t
\]

(20)

The EGR burned and unburned masses result in the same fractions as shown for the trapped residual mass since the composition of these masses are the same. Equations (21), (22), and (23) show the EGR components.
The cylinder composition, based on unburned and burned masses, is now fully determined and is used to develop the equations of the root cause parameters based on total incoming flows.

### 6.3 Models used for Control

**EGR Flow rate**

Direct EGR flow rate measurement in production engines is typically determined by placing an orifice inside the EGR loop and measuring the differential pressure. Equation (24) shows the orifice flow equation used to determine the EGR flow rate. The measured absolute pressure and temperature are also needed to compensate for fluid density.
\[
\dot{m}_{egr} = \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{\frac{2 P_{EGR}}{R T_{EGR}}} \Delta P_{EGR}
\]

(24)

Volumetric Efficiency

The volumetric efficiency is a parameter used to measure the effectiveness of an engine’s induction process. The induction process is a result of the intake system geometry and restrictions. Volumetric efficiency is used to determine the mass flow rate into the engine using the model shown in equation (25),

\[
\dot{m}_{chrg} = \eta_{vol} \frac{\omega_{Eng} V_{Eng} P_{chrg}}{60 \times 2 \ R T_{chrg}}
\]

(25)

which is based on the volumetric efficiency, engine speed, engine displacement and air density. Relationships are developed for the volumetric efficiency over the operating range of the engine. These relationships are typically functions of engine speed, and manifold pressures and temperatures. These functions are tuned for specific engines.

Air Fuel Ratio

The actual air fuel ratio inside the cylinder at steady state operating conditions can easily be determined as,

\[
AFR_m = \frac{m_{fr}}{m^f}
\]

(26)
Which is the mass of fresh air divided by the mass of injected fuel. This only holds for steady state operating conditions.

*Trapped Residual Mass*

Trapped residual models in the literature review were initially explored. These models are based upon the initial assumption that the cylinder density is equal to the exhaust density at IVO. This allows for the trapped residual mass inside the cylinder to be determined at IVO and used as the initial condition for the thermodynamic model. This assumption is investigated using the 90 calibrated GT-power operating conditions for the 2.8 L diesel engine. The trapped residual mass at IVO is calculated using the density of the exhaust manifold and volume at IVO and compared to the actual trapped residual mass calculated in the model. Figure 88 shows the error between the suggested model and the actual trapped residual mass. On average assuming the cylinder density to be equal to the exhaust density results in a 15-20% error of initial trapped residual mass. This would affect the final trapped residual mass prediction proportionally even if the rest of the thermodynamic model is accurate. Therefore, there is no benefit to using a physically based trapped residual model.
A model for the trapped residual mass is developed using the results from the GT-power model. The trapped residual mass is quantified over the calibrated GT-power simulation points and used to fit a regression model. The accuracy of this model is based on the accuracy of GT-powers to predict the trapped residual mass. Literature shows the prediction results to be within 10% [36], which is better than results using the physically based model. Also, a regression model is easier to develop and implement. The fit of the regression model is shown in Figure 89. The model is a function of the intake pressure, exhaust pressure and total cylinder mass.
Figure 89: Trapped residual regression model based on GT-Power points.

6.4 Deriving Root Cause Parameter Equations

The root cause parameter equations are not unique due to the ability to measure the EGR and fresh air flow rate directly or indirectly. The direct EGR measurement method measures the EGR flow rate directly by placing an orifice in the EGR flow path. The differential pressure is measured across the orifice and the flow rate is determined using the orifice flow equation. This method is referred to as the dP based method. The indirect EGR method determines the EGR flow rate as the difference between the charge model and the measured fresh air flow using the MAF sensor. The direct fresh air
measurement system using the MAF sensor to directly measure fresh air flow rate, the indirect method determines the fresh air flow rate as the difference between the direct EGR measurement system and the charge flow model. Root cause parameter equations for direct and indirect measurement systems are derived to compare the uncertainty between sensor sets.

The mass based root cause parameters include total mass rate, total mass rate per cylinder, and total mass per cylinder. The derivation between these three variables is very simplistic, so equations for total mass rate are derived in this chapter and the equations for total mass rate per cylinder and total mass per cylinder are listed in Appendix B. A simple example using EGR mass will be shown for the conversion between the three variables. The mass rate per cylinder is the total mass rate multiplied by the fraction of mass directed towards each cylinder, equation (27).

\[
\dot{m}_{\text{egr-cyl}} = \dot{m}_{\text{egr}} F_{\text{egr}}
\]  \hspace{1cm} (27)

The nominal value for the fraction term in the per cylinder equations is equal to one divided by the number of cylinders. The per cylinder variables are presented using a fraction term, instead of dividing by the number of cylinders, to incorporate the uncertainty in flow distribution that occurs between the cylinders due to engine geometry and mixing.

The mass per cylinder parameter is the mass rate per cylinder normalized by the engine speed, equation (28).
\[ m_{egr-cyl} = \frac{120 \cdot \dot{m}_{egr-cyl}}{\omega_{eng}} \]  

(28)

The constant is a property of converting between revolutions and cycles and appropriate time units. Since these parameters only differ by simple conversions, their effectiveness in the control strategy is the same. Combustion occurs on a per cylinder basis, but the per cylinder parameters are determined from the total flow rate per cylinder parameters, which are determined from the total flow rate parameters. These three parameters represent three different methods to measure the same metric. Even though the parameters are the same with respect to the control strategy, the uncertainty in the parameters differs due the mass that they represent. For example, the per cylinder flow rate includes the uncertainty in cylinder to cylinder variation where the total flow does not incorporate this uncertainty since it is defined as the overall flow into the cylinders. This is shown in the uncertainty analysis in the following chapter.

6.4.1 Residual Mass Based Equations

The residual mass based root cause parameters include the total EGR, the total residuals, and total burned residuals. The total EGR mass variables are determined using two different sensor/model combinations. The first method measured the EGR flow rate directly and the second method measured the EGR flow rate indirectly.

\textit{EGR Mass Rate – Method 1 (dP based)}
\[ \dot{m}^e = \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}} \]  

(29)

**EGR Mass Rate – Method 2 (MAF and VE based)**

\[ \dot{m}^e = \eta_\text{vol} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \dot{m}_{\text{MAF}} \]  

(30)

The total residual mass is defined as the total mass that has already been through a combustion cycle.

\[ \dot{m}_{\text{tot,res}} = \dot{m}^e + \dot{m}^r \]  

(31)

This results in the same equations developed for the total EGR flow with the addition of the trapped residual model.

**Total Residual Gas Mass Rate – Method 1 (dP based)**

\[ \dot{m}_{\text{tot,res}} = \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}} + \dot{m}' \]  

(32)

**Total Residual Gas Mass Rate – Method 2 (MAF and VE based)**

\[ \dot{m}_{\text{tot,res}} = \eta_\text{vol} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \dot{m}_{\text{MAF}} + \dot{m}' \]  

(33)
The total burned mass is the fraction of air and total fuel that combusts at stoichiometry, and is a measure of the CO₂ in the cylinder. The only two sources of burned air are from the EGR system and trapped residual mass, which have a burned air and burned fuel component.

\[
\dot{m}_{\text{tot},b} = \dot{m}_{\text{air},b}^e + \dot{m}_{\text{fuel},b} + \dot{m}_{\text{air},b}^i + \dot{m}_{\text{fuel},b}^i
\]  

(34)

Combining with equations (19), (20), (22), and (23) to get the total burned mass in terms of total flows,

\[
\dot{m}_{\text{tot},b} = \left( \frac{AFR_e + 1}{AFR_m + 1} \right) \left( \dot{m}_{\text{inj}}^e + \dot{m}_{\text{inj}}^i \right)
\]  

(35)

Similar to the other burned parameters, the burned flow rate is determined using the direct and indirect EGR measurement system.

**Total Burned Gas Mass Rate – Method 1 (dP based)**

\[
\dot{m}_{\text{tot},b} = \left( \frac{AFR_e + 1}{AFR_m + 1} \right) \left[ \frac{C_d A_2}{2 \sqrt{RT_{\text{EGR}}} \Delta P_{\text{EGR}}} \right] \left[ \frac{A_2}{A_1} \right] + \left( \frac{AFR_e + 1}{AFR_m + 1} \right) \dot{m}_{\text{inj}}^i
\]  

(36)

**Total Burned Gas Mass Rate – Method 2 (MAF and VE based)**
\[
\dot{m}_{\text{tot},b} = \left( \frac{A FR_s}{A FR_m + 1} \right) \cdot \left( \eta_{\text{vol}} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \dot{m}_{\text{MAF}} \right) + \left( \frac{A FR_s}{A FR_m + 1} \right) \cdot \ddot{m}
\]  

(37)

6.4.2 Residual Concentration Based Equations

The total residual concentration is the fraction of the total burned mass to the total charge. The mass rate, mass rate per cylinder, or mass per cylinder could be used to find the residual mass concentration,

\[
\left[ \dot{m}_{\text{tot},b} \right]_{\text{mass}} = \frac{\dot{m}_{\text{tot},b}}{m_{\text{tot,chg}}}
\]

(38)

where \( m_{\text{tot,chg}} \) is the total mass in the cylinder. The volume based concentration is determined by multiplying the mass based concentration by the ratio of the molecular weights of the two compositions,

\[
\left[ \dot{m}_{\text{tot},b} \right]_{\text{volume}} = \frac{\dot{m}_{\text{tot},b}}{m_{\text{tot,chg}}} \cdot \frac{M_{\text{tot,chg}}}{M_{\text{tot},b}}
\]

(39)

The mass based concentration is determined using three different sensing configurations. Configuration 1 uses the direct EGR measurement system, to determine the EGR flow rate and the volumetric efficiency model and trapped residual mass model to determine the total cylinder charge.
Equation (40) is left in total mass flow rates since the total mass flow rate equations are derived in the previous section.

Configuration 2 uses the MAF sensor and volumetric efficiency model to indirectly calculate the EGR flow rate and the volumetric efficiency model and trapped residual model to determine the total cylinder charge.

\[
\left[ \dot{m}_{\text{tot,b}} \right]_{\text{mass}} = \frac{\left( \dot{m}^e + \dot{m}' \right) \cdot \left( AFR_s + 1 \right)}{\dot{m}_{\text{chrg}} + \dot{m}'} \tag{40}
\]

\( \dot{m}_{\text{MAF}} \) is the fresh air flow measured with the MAF sensor. Configuration 3 is based on the direct EGR measurement system to determine the EGR flow rate and the total cylinder charge with the addition of the MAF sensor and trapped residual model.

\[
\left[ \dot{m}_{\text{tot,b}} \right]_{\text{mass}} = \frac{\left( \dot{m}_{\text{chrg}} - \dot{m}_{\text{MAF}} + \dot{m}' \right) \cdot \left( AFR_s + 1 \right)}{\dot{m}_{\text{chrg}} + \dot{m}'} \tag{41}
\]

\[
\left[ \dot{m}_{\text{tot,b}} \right]_{\text{mass}} = \frac{\left( \dot{m}^e + \dot{m}' \right) \cdot \left( AFR_s + 1 \right)}{\dot{m}_{\text{MAF}} + \dot{m}^e + \dot{m}'} \tag{42}
\]

The volume based concentration would be determined by multiplying any of these equations by the molecular weight ratio.
6.4.3 Unburned Air Mass Based Equations

The unburned air mass parameters consist of fresh air and total unburned air. Fresh air is defined as the air mass that originates through the compressor. The fresh air can be determined using the direct or indirect fresh air measurement system.

**Fresh Air Flow Rate – Method 1 (dP and VE based)**

\[
\dot{m}_{\text{fr, air, un}} = \dot{m}_{\text{chrg}} - \dot{m}^e
\]  

(43)

\[
\dot{m}_{\text{fr, air, un}} = n_{\text{ve}} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{R T_{\text{chrg}}} - \frac{C_d A_2}{2 \sqrt{RT_{\text{EGR}} \Delta P_{\text{EGR}}}} \left(1 - \frac{A_2}{A_1}\right)^2
\]

(44)

**Fresh Air Flow Rate – Method 2 (MAF based)**

\[
\dot{m}_{\text{fr, air, un}} = \dot{m}_{\text{MAF}}
\]

(45)

The total unburned air included the fresh air and the unburned air that enters in the EGR and trapped residual mass that does not participate in combustion. The unburned air is a measure of the O2 in the cylinder.

\[
\dot{m}_{\text{tot, ub}} = \dot{m}_{\text{fr, air, ub}} + \dot{m}_{\text{air, ub}} + \dot{m}_{\text{air, ub}}
\]

(46)

To get the function in terms of total mass flow rates it is combined with equations (18) and (21).
\[ \dot{m}_{\text{tot,abs}} = \dot{m}_{fr} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}^e + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}' \]  

(47)

The total unburned air is determined using three different sensor/model configurations. The first method used the direct EGR flow rate measurement and the charge model. The second method uses the direct measurement of the fresh air flow and the charge model. The third method uses the direct measurement of the EGR flow rate and fresh air flow rate.

Total Air Mass Rate – Method 1 (dP and VE based)

Defining the fresh air flow rate using the indirect fresh air measurement system,

\[ \dot{m}_{fr} = \dot{m}_{chrg} - \dot{m}^e \]  

(48)

Combining equations (47) and (48),

\[ \dot{m}_{\text{tot,abs}} = \dot{m}_{chrg} - \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \dot{m}^e + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}' \]  

(49)

Plugging in for the total flow rates, the final equation becomes,
\[
\dot{m}_{tot,ab} = \eta_{vol} \frac{\omega_{Eng} V_{Eng}}{60 \times 2} \frac{P_{chrg}}{RT_{chrg}} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \cdot \sqrt{\frac{2 P_{EGR}}{RT_{EGR}} \Delta P_{EGR} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}^t}
\] (50)

Total Air Mass Rate – Method 2 (MAF and VE based)

Defining the EGR flow rate using the indirect method since the MAF sensor and charge model are used in this configuration,

\[
\dot{m}^e = \dot{m}_{chrg} - \dot{m}_{MAF}
\] (51)

Combining equations (47) and (51),

\[
\dot{m}_{tot,ab} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \dot{m}_{MAF} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}_{chrg} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}^t
\] (52)

Plugging in for the total flow rates, the final equation becomes,

\[
\dot{m}_{tot,ab} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \dot{m}_{MAF} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \left( \eta_{vol} \frac{\omega_{Eng} V_{Eng}}{60 \times 2} \frac{P_{chrg}}{RT_{chrg}} \right) + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \cdot \dot{m}^t
\] (53)
Total Air Mass Rate – Method 3 (MAF and dP based)

The direct EGR and fresh air measurement systems are used in this configuration so equation (47) becomes,

\[
\dot{m}_{\text{tot,ub}} = \dot{m}_{\text{MAF}} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \left( \frac{C_d A_2}{2} \sqrt{\frac{P_{\text{EGR}}}{RT_{\text{EGR}}}} \right) + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \dot{m}' \tag{54}
\]

6.4.4 Unburned air Concentration Based Equations

The total unburned air concentration is the fraction of the total unburned air to the total charge. The mass rate, mass rate per cylinder, or mass per cylinder can be used to find the unburned concentration.

\[
\left[ \dot{m}_{\text{tot,ub}} \right]_{\text{mass}} = \frac{\dot{m}_{\text{tot,ub}}}{\dot{m}_{\text{tot,chg}}} \tag{55}
\]

The volume based concentration is determined by multiplying the mass based concentration by the ratio of the molecular weights of the two compositions.

\[
\left[ \dot{m}_{\text{tot,ub}} \right]_{\text{volume}} = \frac{\dot{m}_{\text{tot,ub}} \times M_{\text{tot,chg}}}{\dot{m}_{\text{tot,chg}} \times M_{\text{tot,ub}}} \tag{56}
\]
The same three configurations that are shown for the burned air concentrations are used to determine the unburned air concentrations. Using configuration 1, which is based on the direct EGR measurement and the volumetric efficiency model, the unburned concentration is,

\[
\left[ \dot{m}_{tot,ab} \right]_{mass} = \frac{\left( \dot{m}_{chrg} - \dot{m}^e \right) + \left( \dot{m}^e + \dot{m}' \right) \cdot \frac{(AFR_m - AFR_s)}{(AFR_m + 1)}}{\dot{m}_{chrg} + \dot{m}'}
\]  \hspace{1cm} (57)

The second configuration, based on the MAF sensor and volumetric efficiency model, results in equation (58),

\[
\left[ \dot{m}_{tot,ab} \right]_{mass} = \frac{\dot{m}_{MAF} + \left( \dot{m}_{chrg} - \dot{m}_{MAF} + \dot{m}' \right) \cdot \frac{(AFR_m - AFR_s)}{(AFR_m + 1)}}{\dot{m}_{chrg} + \dot{m}'}
\]  \hspace{1cm} (58)

and the third configuration, based on the MAF sensor and the direct EGR measurement system, is shown in equation (59).

\[
\left[ \dot{m}_{tot,ab} \right]_{mass} = \frac{\dot{m}_{MAF} + \left( \dot{m}^e + \dot{m}' \right) \cdot \frac{(AFR_m - AFR_s)}{(AFR_m + 1)}}{\dot{m}_{MAF} + \dot{m}^e + \dot{m}'}
\]  \hspace{1cm} (59)

These three equations can be converted to volume based concentration by multiplying by the molecular weight ratio.
6.4.5 Total Charge Equations

The total charge is defined as the total mass inside the cylinder prior to fuel injection.

\[ m_{tot-chrg} = m^r + m^e + m^{fr} \]  \hspace{1cm} (60)

This differs from the charge model described which only accounts for the mass entering through the intake valves.

\[ m_{chrg-model} = m^r + m^e \]  \hspace{1cm} (61)

The total charge is determined using two different methods. The first method uses the charge model and the trap residual mass model. The second method measures the EGR and fresh air flow directly in addition to the trapped residual mass model.

*Total Charge Mass Rate – Method 1 (VE based)*

\[ \dot{m}_{tot,chrg} = \dot{m}_{chrg} + \dot{m}' \]  \hspace{1cm} (62)

Plugging in for the total flow rates,

\[ \dot{m}_{tot,chrg} = \eta_{val} \omega_{Eng} V_{Eng} \frac{P_{chrg}}{RT_{chrg}} + \dot{m}' \]  \hspace{1cm} (63)
Total Charge Mass Rate – Method 2 (MAF and dP based)

\[ \dot{m}_{\text{tot,chg}} = \dot{m}^f + \dot{m}^e + \dot{m}' \]  \hspace{1cm} (64)

Plugging in for the total flow rates,

\[ \dot{m}_{\text{tot-chg}} = \dot{m}_{\text{MAF}} + \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{2 \frac{P_{\text{EGR}}}{R_{\text{EGR}}} \Delta P_{\text{EGR}}} + \dot{m}' \]  \hspace{1cm} (65)

6.5 Conclusion

The results from the cylinder content analysis are used to derive the equations for the root cause variables. The equations are based on typical sensors and models available to production diesel engines. Multiple sensor configurations are possible when quantifying the root cause parameters, so all combinations are explored. Using the equations developed, the uncertainty in the root cause parameters can be determined based on uncertainties in the equations inputs. For example, the uncertainty in the MAF sensor measurement will affect the prediction of the total unburned air flow rate. The uncertainty in quantifying the root cause parameters will play a key role when determining the most appropriate combustion references. The uncertainty in the prediction of the root cause parameters is derived in the following chapter.
CHAPTER 7

UNCERTAINTY ANALYSIS

7.1 Introduction

Accurately quantifying the combustion references leads to a more effective control strategy. Uncertainty in a combustion reference will lead to errors in the feed forward commands directly impacting the performance of the engine. Therefore, candidate variables for combustion references must have small uncertainties during engine operation. Using the equations from the previous section, which derived the root cause parameter based on available sensors and models, the uncertainty associated with each root cause parameter and sensor configuration is derived. The uncertainty in the root cause parameters is due to the uncertainty in the equations inputs. The uncertainty equations from this study are applied to the 2.8 L diesel engine to obtain numerical results. The input uncertainties are determined in this chapter and the results are shown in Chapter 8.
7.2 Deriving Uncertainty Equations

7.2.1 Mass Based Equations

The propagation of variance method is used to determine the uncertainty in each of the root cause variables. This method projects the independent variable uncertainties onto the output. The root cause parameter equations are functions of measurements, models, and constants, which are referred to as the equation inputs, Table 7. The equations are derived assuming there is an uncertainty associated with each equation input, except for the engine speed. Electronic controllers demand very precise engine speed measurement to accurately command the fuel injection strategy [44]. Crankshaft speed sensors are inductive devices triggered by a rotating ferromagnetic wheel coupled to the engine crankshaft [42]. The sensor outputs a frequency, based on the number of teeth on the wheel, which is easily translated into engine speed. This concept produces very accurate engine speed measurements.

Table 7: Inputs to the root cause parameters equations that introduce uncertainty to the equations.

<table>
<thead>
<tr>
<th>Category</th>
<th>Inputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurements</td>
<td>$P_{chrg}, T_{chrg}, MAF, dP_{egr}, P_{egr}, T_{egr}, \omega_{eng}$</td>
</tr>
<tr>
<td>Models</td>
<td>$\eta_{vol}, AFR_m, m_tr$</td>
</tr>
<tr>
<td>Constants</td>
<td>$V_{eng}, C_d, A_1, A_2, F_{chrg}, F_{trag}, F_{egr}, F_{faf}, R, AFR_s$</td>
</tr>
</tbody>
</table>

The derivation of the root cause parameter uncertainty equations is cumbersome, so an example derivation is shown for the EGR mass flow rate, EGR mass flow rate per
cylinder and EGR mass per cylinder. The example only includes the derivation for the configuration using the MAF sensor and volumetric efficiency model. The rest of the uncertainty equations are listed in Appendix C.

**EGR Mass Rate – Method 2 (MAF and VE based)**

Starting from the EGR mass rate equation,

\[
\dot{m}^e = \eta_{vol} \frac{\omega_{Eng} V_{Eng} P_{chrg}}{60 \times 2 \cdot RT_{chrg}} - \dot{m}_{MAF}
\]

(66)

the simplified propagation of variance method is applied to get the EGR mass rate uncertainty, equation (67). The uncertainty in the engine speed is assumed to be zero, so the engine speed term drops out.

\[
\sigma_{\dot{m}^e}^2 = \left( \frac{\partial \dot{m}^e}{\partial \eta_v} \right)^2 \sigma_{\eta_v}^2 + \left( \frac{\partial \dot{m}^e}{\partial \omega_{Eng}} \right)^2 \sigma_{\omega_{Eng}}^2 + \left( \frac{\partial \dot{m}^e}{\partial V_{Eng}} \right)^2 \sigma_{V_{Eng}}^2 + \left( \frac{\partial \dot{m}^e}{\partial P_{chrg}} \right)^2 \sigma_{P_{chrg}}^2 + \left( \frac{\partial \dot{m}^e}{\partial RT_{chrg}} \right)^2 \sigma_{RT_{chrg}}^2 + \left( \frac{\partial \dot{m}^e}{\partial m_{MAF}} \right)^2 \sigma_{m_{MAF}}^2
\]

(67)

Inserting the actual derivatives, equation (67) becomes,

\[
\sigma_{\dot{m}^e}^2 = \left( \frac{\omega_{Eng} V_{Eng} P_{chrg}}{120 \cdot RT_{chrg}} \right)^2 \sigma_{\eta_v}^2 + \left( \frac{\eta_v \omega_{Eng} P_{chrg}}{120 \cdot RT_{chrg}} \right)^2 \sigma_{\omega_{Eng}}^2 + \left( \frac{\eta_v \omega_{Eng} V_{Eng}}{120 \cdot RT_{chrg}} \right)^2 \sigma_{P_{chrg}}^2 + \left( \frac{\eta_v \omega_{Eng} V_{Eng}}{120 \cdot RT_{chrg}} \right)^2 \sigma_{RT_{chrg}}^2 + \left( \frac{\eta_v \omega_{Eng} V_{Eng} P_{chrg}}{120 \cdot RT_{chrg}^2} \right)^2 \sigma_{m_{MAF}}^2
\]

(68)
Simplifying equation (68) to be in terms of the total flow rates, the uncertainty equation for the EGR flow rate becomes,

\[
\sigma_{m_e}^2 = \left( \frac{\sigma_{\eta_v}}{\eta_v} \right)^2 + \left( \frac{\sigma_{V_{eng}}}{V_{eng}} \right)^2 + \left( \frac{\sigma_{P_{chrg}}}{P_{chrg}} \right)^2 + \left( \frac{\sigma_{R}}{R} \right)^2 + \left( \frac{\sigma_{m_{MAF}}}{m_{MAF}} \right)^2 + \left( \frac{\sigma_{\omega\eta}}{\omega\eta} \right)^2 \]

(69)

**EGR Mass Rate per Cylinder – Method 2 (MAF and VE based)**

Starting from the EGR mass rate per cylinder equation,

\[
\dot{m}_{cyl}^e = \left( \frac{\eta_{vol} \omega_{Eng} V_{Eng}}{60 \times 2} \right) \frac{P_{chrg}}{RT_{chrg}} \dot{m}_{MAF} F_{egr}
\]

(70)

the simplified propagation of variance method is applied to get the EGR mass rate per cylinder uncertainty, equation (71). The uncertainty in the engine speed is assumed to be zero, so the engine speed term drops out. This equation is very similar to the uncertainty equation for EGR mass rate. The only difference is the additional uncertainty that captures the mass distribution across the cylinders.

\[
\sigma_{\dot{m}_{cyl}}^2 = \left( \frac{\sigma_{\eta_v}}{\dot{m}} \right)^2 + \left( \frac{\sigma_{V_{eng}}}{\dot{m}} \right)^2 + \left( \frac{\sigma_{P_{chrg}}}{\dot{m}} \right)^2 + \left( \frac{\sigma_{R}}{\dot{m}} \right)^2 + \left( \frac{\sigma_{m_{MAF}}}{\dot{m}} \right)^2 + \left( \frac{\sigma_{\omega\eta}}{\dot{m}} \right)^2 \]

(71)
Applying the derivatives, equation (71) becomes,

\[
\sigma_{m_{cyl}}^2 = \left(\frac{\sigma_{v} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\frac{\sigma_{V} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\frac{\sigma_{P} \eta_{v} \omega_{eng} V_{eng} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \\
\left(\frac{\sigma_{\eta_{v}} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot R^2 T_{chrg}}\right)^2 + \left(\frac{\sigma_{V_{chrg}} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\sigma_{\eta_{v}} F_{egr}\right)^2 + \\
\left(\frac{\sigma_{F_{egr}} \left(\frac{\eta_{v} \omega_{eng} V_{eng} P_{chrg}}{120 \cdot RT_{chrg}} - \dot{m}_{MAF}\right)}{\dot{m}_{MAF}}\right)^2
\]

(72)

Simplifying equation (72) to be in terms of total flow rates, the EGR flow rate per cylinder uncertainty becomes,

\[
\sigma_{m_{cyl}}^2 = \left(\frac{\sigma_{v} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\frac{\sigma_{V} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\frac{\sigma_{P} \eta_{v} \omega_{eng} V_{eng} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \\
\left(\frac{\sigma_{\eta_{v}} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot R^2 T_{chrg}}\right)^2 + \left(\frac{\sigma_{V_{chrg}} \eta_{v} \omega_{eng} V_{eng} P_{chrg} F_{egr}}{120 \cdot RT_{chrg}}\right)^2 + \left(\sigma_{\eta_{v}} F_{egr}\right)^2 + \\
\left(\frac{\sigma_{F_{egr}} \left(\frac{\eta_{v} \omega_{eng} V_{eng} P_{chrg}}{120 \cdot RT_{chrg}} - \dot{m}_{MAF}\right)}{\dot{m}_{MAF}}\right)^2
\]

(73)

**EGR Mass per Cylinder – Method 2 (MAF and VE based)**

Starting from the EGR mass per cylinder equation,

\[
m_{cyl} = \left(\frac{\eta_{vol} V_{Eng} P_{chrg}}{RT_{chrg}} - \frac{120 \cdot \dot{m}_{MAF}}{\omega_{eng}}\right) F_{egr}
\]

(74)

the simplified propagation of variance method is applied to get the EGR mass per cylinder uncertainty, equation (75). The uncertainty in the engine speed is assumed to be zero, so the engine speed term drops out.
\[
\sigma_{m_{vir}}^2 = \left( \sigma_{\eta_v} \frac{\partial m_{egr}}{\partial \eta_v} \right)^2 + \left( \sigma_{\eta_v, P_{chrg}} \frac{\partial V_{eng}}{\partial \eta_v, P_{chrg}} \right)^2 + \left( \sigma_{P_{chrg}} \frac{\partial m_{egr}}{\partial P_{chrg}} \right)^2 + \left( \sigma_{P_{chrg}} \frac{\partial V_{eng}}{\partial P_{chrg}} \right)^2 + \left( \sigma_{F_{egr}} \frac{\partial m_{egr}}{\partial F_{egr}} \right)^2 + \left( \sigma_{F_{egr}} \frac{\partial V_{eng}}{\partial F_{egr}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{\partial m_{egr}}{\partial \omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{\partial V_{eng}}{\partial \omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{\partial F_{egr}}{\partial \omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{\partial m_{egr}}{\partial \omega_{eng}} \right)^2 \] (75)

Calculating the derivatives, equation (75) becomes,

\[
\sigma_{m_{vir}}^2 = \left( \sigma_{\eta_v} \frac{V_{eng} P_{chrg} F_{egr}}{RT_{chrg}} \right)^2 + \left( \sigma_{\eta_v, P_{chrg}} \frac{V_{eng} P_{chrg} F_{egr}}{RT_{chrg}} \right)^2 + \left( \sigma_{P_{chrg}} \frac{\eta_v V_{eng, P_{chrg}} F_{egr}}{RT_{chrg}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{\eta_v V_{eng, P_{chrg}} F_{egr}}{RT_{chrg}} \right)^2 + \left( \sigma_{F_{egr}} \frac{120 \cdot F_{egr}}{\omega_{eng}} \right)^2 \] (76)

Simplifying the equation to be in terms of the total mass rates, equation (76) becomes,

\[
\sigma_{m_{vir}}^2 = \left( \sigma_{\eta_v} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 + \left( \sigma_{\eta_v, P_{chrg}} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 + \left( \sigma_{P_{chrg}} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 + \left( \sigma_{\omega_{eng}} \frac{120 \cdot \dot{m}_{chrg} F_{egr}}{\omega_{eng}} \right)^2 \] (77)

Since the engine speed uncertainty term is negligible, the uncertainty in the EGR flow rate per cylinder and EGR mass per cylinder is the same, equation (78).
This result holds for all mass per cylinder root cause parameters. Therefore, the uncertainty equations for the per cylinder variables are not listed.

7.2.2 Concentration Based Equations

The concentration based uncertainty equations are derived with the intention of using results from the mass based uncertainties. The concentration based equations are functions of EGR flow rate, fresh air flow rate, charge flow rate, trapped residual flow rate and the measured and stoichiometric air fuel ratios. The EGR flow rate corresponds to the direct EGR measurement system, the fresh air flow rate corresponds to the MAF sensor, the charge flow rate corresponds to the volumetric efficiency model, and the trapped residual flow rate corresponds to the trapped residual model. The flow rate uncertainties, using these measurement methods, are determined in the mass based uncertainty analysis. Therefore, the uncertainty is in respect to the mass flow rates instead of the uncertainty for each input variable. This approach does not change the accuracy of the results, only reduces computations.

The derivation for the uncertainty in the burned concentration for configuration 2 will be used as an example. Starting from the burned concentration, which is based on the MAF sensor and volumetric efficiency model,
\[
\left[ \dot{m}_{\text{tot,b}} \right]_{\text{mass}} = \frac{(\dot{m}_{\text{chrg}} - \dot{m}_{\text{MAF}} + \dot{m}^t) \cdot (AFR_s + 1)}{\dot{m}_{\text{chrg}} + \dot{m}^t} (AFR_m + 1)
\]

(79)

the propagation of variance method is applied to the mass flow rates and air fuel ratio terms.

\[
\sigma^2[\dot{m}_{\text{tot,b}}]_{\text{mass}} = \left( \frac{\sigma_{\dot{m}_{\text{chrg}}} \dot{m}_{\text{MAF}} (AFR_s + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\dot{m}_{\text{MAF}}} (AFR_s + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\dot{m}_{\text{chrg}}} \dot{m}_{\text{MAF}} (AFR_s + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\dot{m}^t} (AFR_m + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{AFR_s} [CO_2]_{\text{mass}} (AFR_m + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{AFR_m} [CO_2]_{\text{mass}} (AFR_m + 1)}{\left( \dot{m}_{\text{chrg}} + \dot{m}^t \right) (AFR_m + 1)} \right)^2
\]

(80)

No simplification is needed since the equation is already in terms of total mass flow rates.

The assumption is made that the volume based concentration uncertainty is the same as the mass based concentration uncertainty. This assumption is based on the notion that the uncertainty with respect to the molecular weights is negligible. The assumption will be proven in section 7.3.3. The remaining uncertainty equations for burned and unburned air are shown in Appendix C.

### 7.3 Obtaining Input Uncertainties

The uncertainty associated with each individual input had to be determined to accurately obtain the root cause parameter uncertainty. The input uncertainties are all derived in
terms of $3\sigma$ values based on the 2.8 L diesel engine. The derivations are broken into measurement uncertainties, model uncertainties and parameter uncertainties.

7.3.1 Measurement Uncertainties

The uncertainties for the production sensors are obtained from the manufacturer specifications. The uncertainty values include effects due to temperature, hysteresis, linearity, repeatability, bias noise, and calibrations. The MAF and EGR delta pressure sensor also include uncertainties due to aging. Two categories that are not included in the sensor error, are the spatial and installation effects. Spatial effects account for the measurement not being uniform across the area the sensor represents. For example, the air flow through a dirty air cleaner causing flow eccentricity through the pipe which affects the MAF sensor reading. Installation effects include the performance of the sensor due to mounting to a specific engine.

An example of a sensor specification sheet is shown for the intake manifold pressure sensor in Figure 90. The uncertainty is a function of the fluid temperature. In the uncertainty analysis for the 2.8 L engine, the error is coded to incorporate the temperature effect, however, a constant value could be used instead. For example, the mean sensor error could be used for all operating conditions.
Figure 90: The error specification sheet from the manufacturer for the MAP sensor [53].

### 7.3.2 Model Uncertainties

Eighty experimental data points, covering the speed and load range of the engine, are used to find the uncertainty in the volumetric efficiency. The flow measurement systems installed on the engine are used to determine the charge flow rate into the engine. Using a disclosed VE model, the parameters are regressed to fit the experimental data. Assuming the experimental measurements have no uncertainties, the fit of the model to the experimental data is the only uncertainty in the volumetric efficiency. Figure 91 shows the error distribution of the VE model compared to the VE calculated using the experimental data. The distribution is bi-modal so a $3\sigma$ value is not obtained. Instead, the uncertainty is chosen to be 5 % since it incorporates all the points. This is essentially the same as using a $3\sigma$ value which includes 99.73 % of the results [39].
The uncertainty in the air fuel ratio model is determined by applying the propagation of variance method to the model,

\[
\left( \frac{\sigma_{\text{AFR}_m}}{\text{AFR}_m} \right)^2 = \left( \frac{\sigma_{m_{\text{AF}}}}{m_{\text{AF}}} \right)^2 + \left( \frac{\sigma_{m_{\text{fuel}}}}{m_{\text{fuel}}} \right)^2
\]  

(81)

The uncertainty in the fresh air flow was taken from the MAF sensor specification sheet and the fuel uncertainty was supplied by the engine manufacturer. The uncertainty in the measured AFR is,
\[ \frac{\sigma_{AFR_m}}{AFR_m} = 5.4\% \]  

(82)

The trapped residual model has two uncertainty components. The first component is due to the accuracy in which a GT-Power model can predict the trapped residual mass. The second component is due to the accuracy in which a model can be fit to the data. The uncertainty in the GT-Power model prediction is shown to be 10 \% [36]. The accuracy in which a model could be fit to the data is 7 \%, using the same approach as shown for the VE model fit uncertainty. Combining these two errors, using the root sum squared method, the total uncertainty in the trapped residual model is,

\[ \frac{\sigma_{\dot{m}_{\text{trap}}}}{\dot{m}_{\text{trap}}} = 12.2\% \]  

(83)

The uncertainty in the trapped residual model is higher than other variable uncertainties, however, the contribution of trapped residual mass to the cylinder contents is relatively low. This causes the trapped residual uncertainty to have a small effect on the overall root cause parameter uncertainty.
7.3.3 Parameter Uncertainties

The uncertainties in the EGR measurement system and engine displacement were supplied by the manufacturer based on the 2.8 L engine. This information is sensitive so is not discussed in this study. The rest of the parameter uncertainties are determined.

In theory, the fraction of flow that enters each cylinder should be the total flow rate divided by the number of cylinders. Due to the geometry of the engine and mixing dynamics of incoming EGR and fresh air, the mass flow distributions across the cylinders are not equal. This increases the uncertainty in the cylinder content estimation, which can affect the control strategy. The distribution across the cylinders is determined for charge flow, fresh air flow, EGR flow and trapped residual flow. The 2.8 L GT-Power model is used to classify the effects of geometry on the flow distributions and the experimental EGR measurement system is used to classify the mixing effects on the flow distributions. The GT-power model is used to classify the geometry effects of the engine because experimental techniques to quantify these variables are not available.

Figure 92 shows the EGR flow distribution across the cylinders using the GT-Power model. This distribution does not include any mixing effects because GT-Power solves one dimensional equations, and assumes complete mixing at flow junctions. Distributions are obtained for the other three flow rates using the same approach.
Figure 92: The flow distribution across the cylinders based on engine geometry from the GT-Power model.

The mixing uncertainty only affects the distribution in the EGR mass and fresh air mass. There are no mixing effects with regards to the trapped residual mass since all the mass in the exhaust manifold has the same composition. The charge mass is defined as the composition of fresh air and EGR mass, so mixing does not affect the overall charge distribution into the cylinder, only the composition changes. Twenty five experimental operating conditions are used to quantify the mixing dynamics of the EGR and fresh air flow rates. The CO₂ fraction was recorded through each vertical port in the intake manifold at every operating condition. The distribution of the CO₂ between the three ports can be seen in Figure 93.
The distribution in CO\textsubscript{2} is directly related to the distribution in the EGR flow rate since CO\textsubscript{2} is essentially zero in the atmosphere. It is assumed that this distribution is possible anywhere in the intake manifold, which would create this same distribution in EGR mass across the cylinders. Some outliers are present, which are highlighted in red. These are due to the repeatability of the CO\textsubscript{2} measurement system used. The repeatability of the measuring device is ±\%CO\textsubscript{2} fraction. This has a large impact on the at low EGR fraction measurements.

Figure 94 shows the uncertainty in CO\textsubscript{2} fraction as a function of the CO\textsubscript{2} fraction measured in the intake manifold for all the experimental data points collected. There is an obvious trend in the uncertainty, which is due to the measurement system errors since
the CO₂ fraction is so low in the intake manifold. As the CO₄ fraction increases, the envelope levels off at the actually CO₂ distribution in the intake manifold. The EGR mixing uncertainty is chosen to be 20% to remain conservative in the estimate.

![Figure 94: The CO2 measurement system error as a function of CO2 percent.](image)

The fresh air flow mixing distribution is base on the assumption that the EGR mass displaces the fresh air mass and vice versa. Figure 95 shows the distribution in fresh air inside the intake manifold calculated using the CO₂ distribution and the stated assumption. The distribution is smaller since the fresh air mass is larger than the EGR mass. The mixing uncertainty in the fresh air flow is 6%.
Figure 95: The distribution in fresh air fraction inside the intake manifold.

The gas constant value is a function of the gas composition [46]. The limits of the engine composition vary from fully burned air to fully fresh air. To get a worst case scenario uncertainty, the gas constant is determined for fully burned air and fully unburned air and compared. The gas constant is determined using,

\[ R = \frac{\overline{R}}{M} \]  

where \( M \) is the molecular weight of the composition and \( \overline{R} \) is the universal gas constant. The molecular weight for ambient air was found to be 28.97 kg/mol [46]. The molecular weight for burned composition is found using the following equation from [41],
\[ M_b = \frac{32 + 4\phi \left( 1 + \frac{8}{4+y} \right) + 28.16\psi}{\left( 1 - \frac{4}{4+y} \right) \phi + 1 + \psi} \]

(85)

where \( \phi \) is the equivalence ratio, \( y \) is HC ratio of the fuel, and \( \psi \) for air = 3.773. Since the objective is to find the molecular weight of fully burned air the equivalence ratio is set to 1. The HC ratio for diesel fuel is 1.8 [41]. Using equation (85), the molecular weight of totally burned air is 29.07. The uncertainty in the molecular weight is,

\[
\frac{\sigma_M}{M} = \frac{(M_{\text{max}} - M_{\text{mean}})}{M_{\text{mean}}} = .2 \%
\]

(86)

Since there is no uncertainty in the universal gas constant, the gas constant uncertainty is equal to the molecular weight uncertainty.

The uncertainty in the stoichiometric air fuel ratio for diesel combustion is determined by comparing the values listed from different sources. Table 8 shows the four sources used and the corresponding stoichiometric air fuel ratio value.

Table 8: The listed stoichiometric air fuel ratio for diesel combustion.

<table>
<thead>
<tr>
<th>Source</th>
<th>Author</th>
<th>AFR&lt;sub&gt;s&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal Combustion Engines</td>
<td>Basshuysen</td>
<td>14.8</td>
</tr>
<tr>
<td>Internal Combustion Engines</td>
<td>Ferguson</td>
<td>14.7</td>
</tr>
<tr>
<td>Automotive Handbook</td>
<td>Bosch</td>
<td>14.5</td>
</tr>
<tr>
<td>Internal Combustion Engines</td>
<td>Heywood</td>
<td>14.4</td>
</tr>
</tbody>
</table>
The uncertainty is calculated as the difference between the mean and the max value,

\[
\frac{\sigma_{AFR}}{AFR_s} = 1.4\%
\] (87)

where \(AFR_s\) is the mean value.

### 7.4 Uncertainty Analysis Inputs

The uncertainty values for the input variables are derived in terms of percentage of the variable. This becomes very useful in simplifying the uncertainty equations. Using the EGR flow rate uncertainty equation as an example,

\[
\sigma_m^2 = \left(\frac{\sigma_{\eta_v} \dot{m}_{chrg}}{\eta_v}\right)^2 + \left(\frac{\sigma_{\frac{V_{eng}}{\eta_v} \dot{m}_{chrg}}}{\eta_v}\right)^2 + \left(\frac{\sigma_{\frac{P_{chrg}}{V_{chrg}} \dot{m}_{chrg}}}{\frac{P_{chrg}}{V_{chrg}}\dot{m}_{chrg}}\right)^2 + \left(\frac{\sigma_{\frac{P_{chrg}}{R} \dot{m}_{chrg}}}{\frac{P_{chrg}}{R}\dot{m}_{chrg}}\right)^2 + \left(\frac{\sigma_{\frac{P_{chrg}}{T_{chrg}} \dot{m}_{chrg}}}{\frac{P_{chrg}}{T_{chrg}}\dot{m}_{chrg}}\right)^2 + \sigma_{m_MAF}^2
\] (88)

and inserting the uncertainty values obtained from the previous section, equation (88) becomes equation (89)

\[
\sigma_m^2 = \left(\sigma_{\%\eta_v} \dot{m}_{chrg}\right)^2 + \left(\sigma_{\%\frac{V_{eng}}{\eta_v} \dot{m}_{chrg}}\right)^2 + \left(\sigma_{\%\frac{P_{chrg}}{V_{chrg}} \dot{m}_{chrg}}\right)^2 + \left(\sigma_{\%\frac{P_{chrg}}{R} \dot{m}_{chrg}}\right)^2 + \left(\sigma_{\%\frac{P_{chrg}}{T_{chrg}} \dot{m}_{chrg}}\right)^2 + \left(\sigma_{\%\dot{m}_{MAF}} \dot{m}_{MAF}\right)^2
\] (89)
Where the $\sigma_{%}$ terms would be the uncertainty percentage of each variable. For example,

$$\sigma_{AFR} = 1.4\%AFR = \sigma_{%AFR} AFR$$

so $\sigma_{%AFR} = 1.4\%$. Equation (89) can then be normalized by the EGR flow rate to get equation (91).

$$\left(\frac{\sigma_{m^e}}{\bar{m}^e}\right)^2 = \left(\sigma_{%A_{\text{in}}}^2 + \sigma_{%\text{var}_{\text{in}}}^2 + \sigma_{%\text{var}_{\text{t}}}^2 + \sigma_{%\text{R}_{\text{ch}}}^2 + \sigma_{%\text{R}}^2\right) \left(\frac{\bar{m}_{\text{chrg}}}{\bar{m}^e}\right)^2 + \left(\sigma_{%\text{MAF}_{\text{in}}} \frac{\bar{m}_{\text{MAF}}}{\bar{m}^e}\right)^2$$

The uncertainty in the EGR mass rate is now in terms of a percentage of total EGR flow rate. The only inputs needed to find the EGR mass flow uncertainty, are the EGR mass rate, fresh air mass rate, charge mass rate and uncertainty in each of the input variables. The inputs to the EGR mass rate equation drop out of EGR mass rate uncertainty equation.

This makes the uncertainty equations developed for the root cause parameters very attractive in the process of determining the most appropriate combustion references and sensor set combinations for an individual engine. This result was valid for all the root cause variable uncertainty equations, which requires minimal information from the engine. The only inputs needed to quantify the root cause parameter uncertainties are the
mass flow rates and constants shown in Table 9 and the input uncertainties determined in section 7.3.3.

Table 9: The inputs needed for the uncertainty analysis.

<table>
<thead>
<tr>
<th>Mass Flows</th>
<th>Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\dot{m}_{fuel}$</td>
<td>Number Cylinders</td>
</tr>
<tr>
<td>$\dot{m}_{PAF}$</td>
<td>Upstream Orifice Area</td>
</tr>
<tr>
<td>$\dot{m}_{egr}$</td>
<td>Orifice Area</td>
</tr>
<tr>
<td>$\dot{m}_{trap}$</td>
<td></td>
</tr>
</tbody>
</table>

### 7.5 Uncertainty Results Method

The analytical equations derived are valid for all engine platforms. A method is now defined to show the application of these equations to a specific engine to acquire useful information and results. The process includes the following five steps:

1. Determine the uncertainty in each of the input variables with respect to the engine under study.
2. Define a set of operating conditions where the uncertainty values are of interest.
3. Determine the EGR, fresh air, trapped residual, and charge flow rate at each of the defined operating conditions.
a. A model needs to be developed for the trapped residual mass, this can be done experimentally or using GT-power.

b. In the case where a trapped residual model is unavailable, the comparison between sensor configurations for the same root cause parameter will still be valid.

4. Obtain the inside diameter of the EGR tubing and a rough idea of the orifice diameter that would be used for the specific engine.

5. Input the information obtained into the analytical equations to obtain numerical uncertainty values at each of the defined operating conditions.

7.6 Conclusions

The analytical uncertainty equations are derived for the root cause parameters. The equations are valid for any engine platform. The only inputs that are needed to obtain numerical results are the uncertainties in the input variables, with respect to the specific engine, the total mass flow rates, and the dimensions of the direct EGR measurement system. This is very useful in the application to all engine platforms because minimal information is needed to obtain uncertainty results. A procedure consisting of five steps is presented to apply the analytical equations to a specific engine platform to obtain numerical results. These steps are applied to the 2.8 L diesel engine. The specific input uncertainties are obtained towards the end of this chapter and the results are shown in Chapter 8.
CHAPTER 8
UNCERTAINTY ANALYSIS APPLICATION

8.1 Introduction

The objective of the uncertainty analysis is to help determine the root cause parameters that would be the most effective for implementation into the diesel engine control strategy. This includes determining the most appropriate sensors to install for the specific application. Low uncertainty in the prediction or measurement of a combustion reference is a necessary requirement. Scheduling the injection parameters becomes very ineffective when the scheduling variable has large uncertainties. For example, an engine is using total burned air as the combustion reference and is operating at 20% burned air fraction, but the measurement system has 50% uncertainty and predicts the unburned air to be 30%. The injection commands for 20% and 30% burned air fraction would not be the same, leading to higher emissions or lower fuel economy than expected. The injection commands would be different because the combustion references require that a trend exists between the injection strategy and the variable. If trends do not exist, the variable would not be effective as a combustion reference.

The numerical results of the uncertainty analysis applied to the 2.8 L diesel engine are shown. The 90 steady state calibration operating conditions are chosen for the analysis to
provide uncertainty values over the entire operating range of the engine. Comparisons are made between the different root cause parameters and between the sensor configurations for individual parameters. The results are explored thoroughly to determine the sources of the uncertainty differences.

Throughout the discussion, the direct and indirect measurement system phrases are used to define a particular measurement system for the burned and unburned parameters. The direct measurement system for any of the burned parameters involves the direct EGR measurement system. This refers to the direct measurement of the EGR flow rate using an orifice in the EGR flow path. The flow rate is determined by measuring the pressure differential across the orifice and correcting for the fluid density. The indirect measurement system for any of the burned parameters involves the indirect EGR measurement system. The indirect EGR measurement system determines the EGR flow rate by measuring the fresh air flow rate with a MAF sensor and predicting the charge flow rate using the volumetric efficiency model and taking the difference.

The opposite is true for the direct and indirect measurement of the unburned parameters. The direct measurement system for the unburned parameters includes the direct measurement of fresh air flow using the MAF sensor. The indirect measurement system for the unburned parameters includes the indirect measurement of fresh air flow. This is determined by predicting the charge flow rate with the volumetric efficiency model and subtracting the measured EGR flow rate using the direct EGR measurement system.
Results are also shown at the end of the chapter for an engine with different operating conditions and input uncertainties. The results are different than seen for the 2.8 L engine. This shows the uncertainty results dependence to a specific engine and the value in the process developed to find the numerical results. Analysis of the results are not shown for this engine, the purpose is to show other possible outcomes.

### 8.2 Applied Method to 2.8 L Diesel Engine

The five steps developed for the obtaining numerical uncertainty results from the analytical equations are applied to the 2.8 L engine. The steps are as follows:

1. Determine the uncertainty in each of the input variables with respect to the engine under study. (This is done for the 2.8 L diesel engine in the previous chapter)

2. Define a set of operating conditions where the uncertainty values are of interest. (The 90 steady state calibration points are chosen for the 2.8 L diesel engine to provide uncertainty values over the operating range of the engine)

3. Determine the EGR, fresh air, trapped residual, and charge flow rate at each of the defined operating conditions. (EGR flow rate and fresh air flow rate were experimentally measured for the 90 steady state operating conditions of the 2.8 L engine. Charge flow was determined by adding these two flow rates together and the trapped residual model was applied to the 90 operating conditions to find the trapped residual mass)
4. Obtain the inside diameter of the EGR tubing and a rough idea of the orifice diameter that would be used for the specific engine. (A beta ratio of .63 was used to find the orifice diameter of the 2.8 L diesel engine)

5. Input the information obtained into the analytical equations to obtain numerical uncertainty values at each of the defined operating conditions. (The results are shown in this chapter)

The advantage to using the steady state calibration points is that a map of uncertainty values is acquired. Using the map points, a surface plot can be used to show the uncertainty over the operating range of the engine. As an example, results from the 2.8 L diesel engine are shown in Figure 96. This example is for the indirect measurement of EGR flow rate using the volumetric efficiency model and MAF sensor. The accuracy in measuring the root cause variable over the operating range of the engine is easily interpreted and decisions about the control strategy and sensing configuration can be made.
Due to the number of root cause parameters and sensor configurations, statistics are used to compare the results from the 2.8 L analysis instead of showing individual surface plots. For each root cause parameter and sensor configuration the mean, min, and max uncertainty is determined from the surface. These statistics are plotted for all the burned variables, unburned variables, and charge variables.

8.3 Mass Based Uncertainty Results

Figure 97 shows the uncertainty results for the burned root cause parameters. The parameters are listed on the x-axis and grouped by the sensor set. The most obvious
result is that the indirect method of predicting burned parameters produces larger uncertainties. This is most evident in the EGR flow rate. The indirect measurement method is using two large numbers to predict a small number. A small error in both of the large numbers transfers to a large error in the small number. For example, a hypothetical operating condition with 15% EGR fraction would be predicted as 24% EGR fraction with a 5% error in charge flow and fresh air flow. This is a 60% error in the estimation of EGR fraction. This amount of uncertainty might be unacceptable in using the parameter as a combustion reference in the diesel control strategy.

Focusing on the indirect measurement system, the uncertainty for the residual flow rate and burned flow rate appeared to be better than the EGR flow rate. This is due to the operating conditions of the engine. About half of the 90 operating conditions are with the EGR valve closed. The uncertainty in points with the EGR valve closed is only a function of the trapped residual mass uncertainty, which is 12.2%. The large error in predicting the EGR flow rate is only present in points with the EGR valve open. This lowers the statistical average of the residual and burned flow rate uncertainties. The results shown for the two parameters in Figure 97 still hold but it is important to remember that there is a distinct difference in uncertainty between points with the EGR valve open and EGR valve closed. Figure 98 verifies this conclusion by showing the uncertainty for all 90 operating conditions on a histogram for the total residual flow rate. The bar at 12.2% represents all the operating conditions with the EGR valve closed.
Predicting the burned parameters using the direct measurement method produces uncertainties in an acceptable range between 5-15%. The tight tolerance in the statistical extremes also makes the direct measurement method attractive due to the consistent uncertainty over the entire engine operating region. Implementing the direct EGR measurement system would decrease the uncertainty in the burned parameters compared to the indirect EGR measurement system.

The residual flow rate and burned flow rate uncertainty increase compared to the EGR flow rate uncertainty for the direct EGR measurement system. This is opposite the case for the indirect measurement system. This occurs because the trapped residual uncertainty is higher than the EGR flow rate uncertainty of the direct EGR measurement system, which causes the residual and burned flow rate uncertainties to increase. The burned flow rate parameter had a slightly higher uncertainty than the residual flow rate uncertainty since the uncertainty in the predicted and stoichiometric air fuel ratio is included. This is true for both measurement systems.

The uncertainty increases for parameters on a per cylinder basis compared to total flow parameters. The total flow parameters do not account for the uncertainty in mass distribution across the cylinders, from the engine geometry and mixing dynamics. The per cylinder parameters capture these affects, which causes the uncertainty to increase. It will be noted that these two types of parameters would function the exact same as combustion references in the control structure. There currently is not a way to quantify the mass distribution across the cylinders during engine operation for production engine
operation. The per cylinder parameters would be determined by normalizing the total flow by the number of cylinders. Since combustion occurs on a per cylinder basis, the per cylinder parameter uncertainties should be used. However, for implementation in the control strategy, the combustion reference could be based on total flow or per cylinder flow.
Figure 97: The statistics for the burned root cause parameters.
Figure 98: The total residual flow rate uncertainty for the indirect method quantified for the 90 operating conditions.

Figure 99 shows the uncertainties of the burned parameters when normalized by the charge mass instead of the actual parameter. The large uncertainties seen for the indirect method drop to a more understandable uncertainty. The majority of the cylinder contents is fresh air, thus large uncertainties in the burned parameters do not affect the total cylinder charge by the same percentage. The burned flow rate and burned flow rate per cylinder parameters drop to less than 4% mean error since their mass contribution to the total cylinder contents is so small. However, the main result still holds that the indirect method has a much less favorable uncertainty.
Figure 99: The uncertainty in the burned root cause parameters normalized by the charge mass.

The uncertainty in the burned parameters is directly related to the EGR fraction when using the indirect method. This is shown in Figure 100 for the indirect measurement of EGR flow rate. The peak uncertainties occur when the EGR fraction is the smallest. As the EGR fraction decreases the uncertainty increases due to the small mass that is being predicted.
Figure 100: EGR flow rate and corresponding EGR fraction shown on the engine speed/load map (MAF and VE method).

Figure 101 shows the uncertainty results for the unburned root cause parameters. The parameters are grouped by the sensing system. The first group is based on the indirect fresh air flow measurement system. The second group is based on the direct fresh air flow measurement system. The indirect fresh air flow measurement system uses the direct EGR measurement system to calculate the fresh air flow rate. This allows for the EGR flow rate to be measured directly when determining the unburned fraction in the EGR mass. On the other hand, the direct fresh air measurement system must be paired with either the volumetric efficiency model or the direct EGR measurement system to determine the unburned portion of the EGR flow rate. The additional measurement used is listed on the figure above the error bar.

The unburned root cause parameters have less uncertainty when the direct fresh air measurement system is used. This is a result of the MAF sensors ability to measure the
fresh air flow rate more accurately than indirect fresh air method, 4% compared to 5.9%. The majority of the unburned air is fresh air, thus the uncertainty in the fresh air flow prediction is the driving factor for the uncertainty in the unburned flow parameters. Therefore, the uncertainty in comparing unburned flow rate to fresh air flow rate or unburned flow rate per cylinder to fresh air flow rate per cylinder is essentially the same when looking at an individual measurement system.
Figure 101: The statistics for the unburned parameters.

One result that appears in the unburned uncertainty analysis that is not intuitive is that the MAF plus volumetric efficiency model sensing system had a lower uncertainty on average than the MAF and direct EGR measurement sensing system. Both of these sensing systems use the MAF sensor to measure the fresh air flow rate, the difference appears in the method of measuring the EGR flow rate to determine the unburned air present in the EGR mass. The MAF plus VE method predicts EGR flow rate using the indirect EGR measurement system and will be referred to as method 1 for this discussion. The MAF plus direct EGR measurement system obviously uses the direct EGR...
measurement system and will be referred to as method 2 for this discussion. Since the unburned air from trapped residual mass affects these two methods equally, it is dropped from the discussion for simplicity. The uncertainty of determining the EGR flow rate using these direct and indirect measurement systems is shown in Figure 97. The mean uncertainty in the indirect measurement system is 43% compared to 7% for direct EGR measurement system. Since the only difference between these two methods is the determination of EGR flow rate, and method 1 uses the indirect EGR measurement system, it was expected that method 2 would have the better unburned air uncertainty on average. This is not the case as shown in Figure 101.

The equation for method 1 in terms of total mass flow rates is,

\[
\dot{m}_{\text{tot, unb}} = \dot{m}_{\text{MAF}} + \frac{A_{\text{FR}_m} - A_{\text{FR}_l}}{A_{\text{FR}_m} + 1} (\dot{m}_{\text{chrg}} - \dot{m}_{\text{MAF}})
\]

(92)

where the charge flow mass would be predicted with the volumetric efficiency model and the fresh air flow rate would be measured with the MAF sensor. The term on the left calculates the fresh air flow through the compressor and the term inside the parentheses calculates the EGR flow rate. The AFR terms in front of the EGR flow rate calculate the fraction of EGR flow that is unburned air. The unexpected result for the uncertainty in this equation is due to the redundant MAF measurement. This can be seen most easily with an example using a hypothetical operating condition. A operating condition is used where the charge flow rate is 64 kg/min, the fresh air flow rate is 54 kg/min, the EGR flow rate is 10 kg/min, and the measured AFR is 33. The example calculates the
uncertainty in the total unburned air assuming there is a ±5% error in the prediction of charge flow and measurement of fresh air flow.

Table 10 summarizes the results for all four error combinations. For case 3 and case 4, the uncertainty is solely due to the uncertainty in the fresh air flow measurement through the turbocharger. When the MAF and VE model have error in the same direction, the EGR flow rate prediction would be very accurate and very little uncertainty is produced from the EGR mass. The uncertainty would be due to the error in which the MAF sensor measured the fresh air flow rate. For cases 1 and 2, the uncertainty is very low. Using case 1, the high prediction in fresh air flow causes the unburned air through the turbocharger to be higher than actually present, however, the high prediction in fresh air flow and low prediction in charge flow causes a very low prediction in EGR flow rate, 61% error. This reduces the unburned air contribution from the EGR flow which compensates for the high prediction in fresh air flow. The opposite is true for case 2, the low fresh air flow measurement is compensated by the high EGR flow rate prediction.

This relationship is repeatable due to the redundancy of the fresh air flow measurement. This compensation effect is the reason for the lower mean uncertainty in the total unburned air parameter using the MAF sensor and VE model compared to the MAF sensor and direct measurement of the EGR flow rate. The amount of compensation is a function of the EGR flow rate and air fuel ratio, which are used to determine the amount of unburned air in the EGR flow. Directly measuring the EGR flow rate would not
incorporate the automatic compensation effect of the redundant measurement. The compensation would only occur due to probability.

Table 10: The unburned uncertainty calculated for all error combinations.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\dot{m}_{MAF}$</th>
<th>$\dot{m}_{chrg}$</th>
<th>Uncertainty [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.05</td>
<td>0.95</td>
<td>0.8</td>
</tr>
<tr>
<td>2</td>
<td>1.05</td>
<td>1.05</td>
<td>-0.8</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>0.95</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>1.05</td>
<td>-5</td>
</tr>
</tbody>
</table>

Figure 102 shows the statistics for the charge flow parameters. The x-axis shows the parameters names which are lumped by the sensing system. Using the direct EGR measurement system with the direct fresh air measurement system results in better charge flow prediction than using the volumetric efficiency model. However, both of methods produce low uncertainties and should be considered as possible prediction methods for charge flow. As seen for the burned and unburned parameters, the per cylinder parameters have higher uncertainty than the total flow parameters due to the uncertainty in mass distribution between the cylinders.
8.4 Concentration Based Uncertainty Results

The uncertainty in the burned concentration for the three configurations is shown in Figure 103. High uncertainties are seen in the second configuration which predicts the EGR flow rate using the indirect EGR measurement system. This type of EGR measurement is shown to be very poor from Figure 97. Since the majority of the burned
air entering the cylinder is from the EGR path, an inaccurate measurement of EGR will result in high burned air uncertainty as seen.

Configurations 1 and 3 had similar uncertainty results which are much better than configuration 2. This is due to the fact that the EGR flow rate is measured directly for both of the configurations. The direct EGR measurement is shown to have a mean uncertainty of 6% for EGR flow rate compared to the indirect method which has a mean uncertainty of 40%.

The uncertainty for the third configuration is slightly better than the first configuration. The first configuration determines the charge flow rate using the volumetric efficiency model and configuration 3 determined the charge flow rate using the sum of the EGR flow rate from the direct EGR measurement system and the fresh air flow from the MAF sensor. The charge flow prediction using the MAF sensor and direct EGR measurement had a lower uncertainty than the charge flow prediction using the volumetric efficiency model. A redundant measurement also occurs in the third configuration because EGR flow rate is used in determining the burned mass and the charge mass, equation (93).

\[
\left[ \dot{m}_{\text{tot,b}} \right]_{\text{mass}} = \left( \dot{m}^c + \dot{m}' \right) \cdot \frac{(AFR_s + 1)}{(AFR_m + 1)}
\]

The redundant EGR measurement reduces the uncertainty in the burned air concentration. The compensation is small since the contribution of the EGR flow rate to the burned mass is large compared to the charge mass, however, the compensation is present. The
compensation is also a function of the measured air fuel ratio. The fraction of burned air in the EGR flow rate changes the compensation effect of the redundant measurement.

![Burned Concentration - Measurement Comparison](image)

Figure 103: A comparison of the burned concentration uncertainties for different sensing configurations.

The unburned concentration uncertainty is shown in Figure 104 for all three configurations. Configuration 1 determines the unburned concentration using the volumetric efficiency model and direct EGR measurement, equation (94).
\[
\left[ \dot{m}_{\text{tot,ab}} \right]_{\text{max}} = \frac{\left( \dot{m}_{\text{chrg}} - \dot{m}^e \right) + \left( \dot{m}^r + \dot{m}^t \right) \cdot \frac{AFR_n - AFR_s}{AFR_n + 1}}{\dot{m}_{\text{chrg}} + \dot{m}^t}
\]  
(94)

The majority of the unburned air fraction is from fresh air flow. This method calculates the fresh air flow using the indirect fresh air measurement system. The charge flow prediction is redundant in the numerator, for prediction of the fresh air flow, and in the denominator, for total charge flow. This reduces the uncertainty in the fresh air concentration because the numerator and denominator uncertainty compensate for each other. An example is shown for clarity. Focusing on the fresh air concentration,

\[
\left[ \dot{m}_{\text{fr,ab}} \right]_{\text{max}} = \frac{\left( \dot{m}_{\text{chrg}} - \dot{m}^e \right)}{\dot{m}_{\text{chrg}} + \dot{m}^t}
\]  
(95)

A hypothetical operating condition is chosen with \( \dot{m}_{\text{chrg}} = 100 \, \text{kg/min}, \dot{m}_{\text{egr}} = 15 \, \text{kg/min}, \) and \( \dot{m}^t = 5 \, \text{kg/min}. \) The nominal fresh air concentration is,

\[
\left[ \dot{m}_{\text{fr,ab}} \right]_{\text{max}} = \frac{\left(100 - 15\right)}{100 + 5} = .818
\]  
(96)

A 5% error in charge flow prediction results in the fresh air concentration,

\[
\left[ \dot{m}_{\text{fr,ab}} \right]_{\text{max}} = \frac{\left(105 - 15\right)}{105 + 5} = .810
\]  
(97)
This is a very small difference in fresh air flow concentration where a large uncertainty in charge flow is present. The redundant measurement helps to cancel out the uncertainty in the charge flow rate. The direct EGR measurement system is also very accurate in EGR flow rate measurement, which causes the unburned air from the EGR to be predicted accurately. These two reasons combine to give a low uncertainty in the total unburned air concentration.

Configuration 2 determines the concentration using the MAF sensor to measure fresh air flow and the volumetric efficiency model to predict charge flow, equation (98).

\[
\left[ m_{\text{tot,unb}} \right]_{\text{mass}} = \frac{m_{\text{MAF}} + (m_{\text{chrg}} - m_{\text{MAF}} + \hat{m}) \cdot \frac{(AFR_{m} - AFR_{s})}{(AFR_{m} + 1)}}{m_{\text{chrg}} + \hat{m}}
\]  

(98)

The fresh air flow and charge flow are redundant measurements in this configuration. However, the uncertainty for this configuration is higher than for the first configuration. The redundant MAF sensor measurement decreases the uncertainty in the unburned air mass prediction as explained in the previous section. This decreases the uncertainty in the unburned air concentration. However, since the fresh air flow is measured using the MAF sensor, which is the majority of the unburned air, and the MAF sensor does not appear in the denominator for charge flow prediction, then there is no compensation effect in the fresh air concentration. Using the same example as for configuration 1, with a 5% uncertainty in charge flow and no uncertainty in fresh air flow measurement, the fresh air flow concentration is,
\[ \left[ \dot{m}_{fr,ub} \right]_{mass} = \frac{85}{105 + 5} = .772 \]  

(99)

This is a much bigger difference than shown for configuration 1 since the charge flow model is not used in the fresh air flow prediction. Compensation does exist in configuration 2, for the unburned air flow rate, but is dependent on the unburned air fraction in the EGR flow rate. This does not create the amount of compensation seen when using redundant measurements in the numerator and denominator.

Configuration 3 determines the unburned concentration by measuring the fresh air flow rate with the MAF sensor and the EGR flow rate directly, equation (100).

\[ \left[ \dot{m}_{tot,ub} \right]_{mass} = \frac{\dot{m}_{MAF} + \left( \dot{m}^e + \dot{m}' \right) \cdot \left( AFR_m - AFR_s \right)} {\dot{m}_{MAF} + \dot{m}^e + \dot{m}'} \]  

(100)

The redundant measurement in the numerator and denominator for the fresh air flow rate and EGR flow rate measurement decreases the unburned air concentration uncertainty the same as is shown for configuration 1. The better prediction for the charge flow using the MAF sensor and direct EGR measurement compared to the volumetric efficiency model, gives configuration 3 a slightly smaller uncertainty than configuration 1.
8.5 Further Uncertainty Results

The uncertainty results are engine specific due to the dependence on operating conditions and input uncertainties. Results are shown for an engine with different operating conditions and input uncertainties compared to the 2.8 L engine. The depth of analysis done for the 2.8 L engine is not shown, these results are used to demonstrate other
possible outcomes. Figure 105 shows the statistics for the uncertainty of the burned parameters. For this engine, the indirect EGR measurement system has a lower uncertainty than the direct EGR measurement system. This is opposite the results seen for the 2.8 L engine. However, the rest of the trends discussed previously for the 2.8 L engine burned parameters are valid. For example, the per cylinder parameters have a higher uncertainty than the total flow parameters due to the uneven distribution across the cylinders.
Figure 105: The statistics for the burned parameters.

Figure 106 shows the statistics for the uncertainty results of the unburned parameters. The direct fresh air flow measurement system has a lower uncertainty than the indirect measurement system. However, the difference between the two systems is dependent on the root cause parameter. There is a larger difference between the two measurement systems for the fresh air flow than the unburned air flow. This is not seen in the 2.8 L results. The direct and indirect fresh air measurement systems are similar for all the unburned parameters.
Figure 106: The statistics for the unburned parameters.

Figure 107 shows the statistics for the uncertainty results of the charge parameters. The volumetric efficiency model has a lower uncertainty in predicting the charge flow than the direct fresh air and EGR measurement systems by a small margin. The opposite is true for the 2.8 L engine. The difference is in the accuracy of the volumetric efficiency models for the two engines. However, for both engines the uncertainty difference is small, so either measurement system is acceptable.
Figure 107: The statistics for the charge parameters.

Figure 108 shows the statistics for the uncertainty results of the burned air concentration. The major difference in these results compared to the 2.8 L engine, is the major improvement in the second sensing configuration. The mean uncertainty for the 2.8 L engine is 22% compared to 6.8% for this engine. The improvement is due to the increased accuracy that indirect EGR measurement system is able to predict EGR flow rate for this engine.
Figure 108: A comparison of the burned concentration uncertainties for different sensing configurations.

Figure 109 shows the statistics for the uncertainty results of the unburned air concentration. The uncertainty in all the sensing configurations have acceptable results. However, there are differences compared to the 2.8 L engine which stem from the operating conditions of the engine.
8.6 Conclusion

The numerical uncertainty results are shown for the root cause parameters applied to the 2.8 L diesel engine. Some of the results obtained are not intuitive. For example, the uncertainty in the unburned concentration is better for the direct EGR measurement system paired with the volumetric efficiency model compared to the direct fresh air
measurement system paired with the volumetric efficiency model. The result is due to the way a redundant measurement is used when calculating the root cause parameter. Redundant measurements appear in multiple root cause parameters, giving results that are not initially obvious. This shows the value of the uncertainty analysis.

The second set of results shows the difference in uncertainty outcomes for specific engines. This shows the need for the process developed in determining the numerical uncertainty results. Since the uncertainty values are obtained over the entire operating range of the engine, then actual implementation of the root cause parameter and sensor set should perform within the calculated uncertainty values. The numerical results of the uncertainty analysis will be used when determining the most effective combustion references and sensors needed to quantify the parameters.
9.1 Conclusion

A structure for a new control strategy is presented for the enhancement of the diesel engine platform. The strategy introduces new control inputs that schedule the injection parameters based on cylinder conditions instead of operating and ambient conditions. This has the possibility of greatly reducing the number of calibrated ECU maps for development of new diesel engine platforms. The effectiveness of the control strategy is dependent on finding relationships between the control inputs and desired injection commands, to account for changes in cylinder conditions, and on the ability to accurately quantify these new control inputs during engine operation.

An extensive list of parameters is developed to be considered as the control inputs. These parameters are termed root cause parameters because they incorporate a direct relationship to the diesel combustion process. Parameters that relate to the combustion process provide the most probable chance that trends can be developed between the inputs and desired injection commands. Developing the trends is left to future work of the project, this research focused on the ability to accurately quantify the root cause parameters.
Equations are developed to quantify the root cause parameters during steady state engine operation based on production sensors and models. Multiple sensing combinations are possible for the different root cause parameters so all configurations are explored. Using the propagation of variance method, analytical equations are derived for the uncertainty in each of the root cause parameters with respect to the uncertainties in the sensors and models employed in the equations. The uncertainty equations developed are generic to the engine platform and require minimal inputs to obtain numerical results.

The numerical uncertainty results provide a measure on the ability to quantify a specific root cause parameter using a particular sensing technique. Comparisons of the numerical results, between different root cause parameter uncertainties and different sensing techniques for the same root cause parameter, is one essential piece in determining the control inputs for the new control strategy. The control inputs for diesel engines are not uniform across engine platforms, so the ease in which these equations can be applied to different platforms is very attractive for determining the most effective combustion references.

A simple process is presented to determine the numerical uncertainty results from the analytical equations. The process is applied to the 2.8 L diesel engine as a case study to show the effectiveness of the equations. Some of the results are not intuitive due to the nature of the equations. When redundant measurements are used in quantifying the root cause parameters, compensation effects occur which are initially not expected. This case
study shows the importance of applying the uncertainty analysis to obtain a numerical means for comparison.

9.2 Future Work

The uncertainty analysis is based on steady state equations that do not capture the transient effects. A transient model has been developed by a fellow PhD student that accounts for the delays in the air handling system during transient operation. The model uses the steady state flow equations as inputs to determine the cylinder composition. The additional uncertainties at transient conditions are due to the prediction in the air system delays caused by the steady state equation uncertainties. To get accurate uncertainty values for transient conditions, the delay uncertainties need to be added to the steady state uncertainty equations.

A thorough analysis needs to be conducted to find trends between the root cause parameters and the desired injection strategy. This will be done using extensive experimental testing and DOE analyses. To find the desired injection strategy, tradeoffs between fuel economy and NOx will have to be established. Once useful relationships are developed, the results will be combined with the parameter uncertainties to determine a set of combustion references and a sensing configuration that will be most effective for implementation into the new control strategy. The control strategy also requires some modeling and interpolation techniques to be developed, but the primary obstacle is to
determine the appropriate combustion references. These remaining tasks are more straightforward and can be dealt with during implementation.
BIBLIOGRAPHY


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APPENDIX A: ROOT CAUSE DIAGRAMS
Root Cause:
1. Mass of Oxygen relates to mass of fuel to be burned
Root Cause:
1. Relative amount of oxygen relates to chemical kinetics of combustion process.
Root Cause:
1. Relative amount of residuals relates to chemical kinetics of combustion process.

Diagram:
- Residual Concentration
  - Equivalent Oxidant Partial Pressure
  - Equivalent Oxidant Concentration
  - Equivalent Oxidant Number Density
  - Total Charge Molar per cylinder
    - Molar Breakdown
      - Mol Burnt gas
        - mol mixture
      - Mol Burnt gas/volume
      - Grant angle reference
Root Cause:
1. Time Available for combustion
2. Charge Motion
Root Cause:
1. Combustion kinetics
2. Charge density
3. Heat loss through cylinder walls
Fuel Properties

- Octane Number
- Heating Value
- Joule's constant
- Viscosity
- Density
- BioDiesel Content
APPENDIX B: ROOT CAUSE PARAMETER EQUATIONS

EGR

EGR Mass Rate per Cylinder – Method 1 (dP Based)

\[
\dot{m}_{cyl}^e = \frac{C_d A_z F_{egr}}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 \frac{P_{EGR}}{RT_{EGR}}} \Delta P_{EGR}
\]

EGR Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[
\dot{m}_{cyl}^e = \left(\eta_{val} \omega_{Eng} \frac{V_{Eng}}{60 \times 2} \frac{P_{cchg}}{RT_{cchg}} - \dot{m}_{MAF} \right) F_{egr}
\]

EGR Mass per Cylinder – Method 1 (dP Based)

\[
m_{cyl}^e = \frac{120 \cdot C_d A_z F_{egr}}{\omega_{eng} \sqrt{1 - \left(\frac{A_2}{A_1}\right)^2}} \sqrt{2 \frac{P_{EGR}}{RT_{EGR}}} \Delta P_{EGR}
\]

EGR Mass per Cylinder – Method 2 (VE and MAF based)

\[
m_{cyl}^e = \left(\eta_{val} V_{Eng} \frac{P_{cchg}}{RT_{cchg}} - \frac{120 \cdot \dot{m}_{MAF}}{\omega_{eng}} \right) F_{egr}
\]
Total Residual

Total Residual Gas Mass Rate per Cylinder – Method 1 (dP based)

\[
\dot{m}_{\text{tot, res-cyl}} = \frac{C_d A_2 F_{\text{egr}}}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2 P_{\text{EGR}}}{R T_{\text{EGR}}}} \Delta P_{\text{EGR}} + \dot{m}' F_{tr}
\]

Total Residual Gas Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[
\dot{m}_{\text{tot, res-cyl}} = \left( \eta_\text{vol} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{R T_{\text{chrg}}} - \dot{m}_{\text{MAF}} \right) F_{\text{egr}} + \dot{m}' F_{tr}
\]

Total Residual Gas Mass per Cylinder – Method 1 (dP based)

\[
m_{\text{tot, res-cyl}} = \frac{120 C_d A_2 F_{\text{egr}}}{\omega_{\text{eng}}} \sqrt{\frac{2 P_{\text{EGR}}}{R T_{\text{EGR}}}} \Delta P_{\text{EGR}} + \frac{120 \cdot \dot{m}' F_{tr}}{\omega_{\text{eng}}}
\]

Total Residual Gas Mass per Cylinder – Method 2 (VE and MAF based)

\[
m_{\text{tot, res-cyl}} = \left( \eta_\text{vol} V_{\text{Eng}} \frac{P_{\text{chrg}}}{R T_{\text{chrg}}} - \frac{120 \cdot \dot{m}_{\text{MAF}}}{\omega_{\text{eng}}} \right) F_{\text{egr}} + \frac{120 \cdot \dot{m}' F_{tr}}{\omega_{\text{eng}}}
\]
Total Burned Gas

Total Burned Gas Mass Rate per Cylinder – Method 1 (dP based)

\[
\dot{m}_{tot, \text{cyl}} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \left( \frac{C_d A_f F_{egr}}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right)^2 \sqrt{\frac{2}{P_{EGR} \Delta P_{EGR}}} + \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \dot{m}^{1} F_{tr}
\]

Total Burned Gas Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[
\dot{m}_{tot, \text{cyl}} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \eta_{vol} \frac{\omega_{Eng} V_{Eng}}{60 \times 2} - \dot{m}_{MAF} F_{egr} + \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \dot{m}^{1} F_{tr}
\]

Total Burned Gas Mass per Cylinder – Method 1 (dP based)

\[
m_{tot, \text{cyl}} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \left( \frac{120 C_d A_f F_{egr}}{\omega_{eng}} \sqrt{1 - \left( \frac{A_2}{A_1} \right)^2} \right)^2 \sqrt{\frac{2}{P_{EGR} \Delta P_{EGR}}} + \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \frac{120 \cdot \dot{m}^{1} F_{tr}}{\omega_{eng}}
\]

Total Burned Gas Mass per Cylinder – Method 2 (VE and MAF based)

\[
m_{tot, \text{cyl}} = \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \eta_{vol} \frac{V_{Eng}}{P_{chrg}} - \frac{120 \cdot \dot{m}_{MAF}}{\omega_{eng}} F_{egr} + \left( \frac{AFR_s + 1}{AFR_m + 1} \right) \cdot \frac{120 \cdot \dot{m}^{1} F_{tr}}{\omega_{eng}}
\]
Fresh Air

**Fresh Air Flow Rate per cylinder – Method 1 (dP and VE based)**

\[
\dot{m}_{\text{air,un-cyl}}^\text{fr} = \eta_{\text{vol}} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \frac{C_d A_2}{\sqrt{1 - \left(\frac{A_2}{A_1}\right)^2 \left[2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}\right]^2}} F_{\text{FAF}}
\]

**Fresh Air Flow Rate per cylinder – Method 2 (MAF based)**

\[
\dot{m}_{\text{air,un-cyl}}^\text{fr} = \dot{m}_{\text{MAF}} \cdot F_{\text{FAF}}
\]

**Fresh Air per cylinder – Method 1 (dP and VE based)**

\[
m_{\text{air,un-cyl}}^\text{fr} = \eta_{\text{vol}} V_{\text{eng}} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \frac{\omega_{\text{eng}} C_d A_2}{120 \cdot \sqrt{1 - \left(\frac{A_2}{A_1}\right)^2 \left[2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}\right]^2}} F_{\text{FAF}}
\]

**Fresh Air per cylinder – Method 2 (MAF based)**

\[
m_{\text{air,un-cyl}}^\text{fr} = \frac{120 \cdot \dot{m}_{\text{MAF}} \cdot F_{\text{FAF}}}{\omega_{\text{eng}}}
\]
Total Unburned Air

Unburned Air Mass Rate per Cylinder – Method 1 (dP and VE based)

$$\dot{m}_{\text{tot,ub-cyl}} = \left( \eta_{\text{vol}} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2}{RT_{\text{EGR}}}} \frac{P_{\text{EGR}}}{\Delta P_{\text{EGR}}} \right) F_{FAF} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \dot{m}' F_{tr} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \dot{m}' F_{tr}$$

Unburned Air Mass Rate per Cylinder – Method 2 (MAF and VE based)

$$\dot{m}_{\text{tot,ub-cyl}} = \dot{m}_{\text{MAF}} F_{FAF} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \left( \eta_{\text{vol}} \frac{\omega_{\text{Eng}} V_{\text{Eng}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} - \dot{m}_{\text{MAF}} \right) F_{egr} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \dot{m}' F_{tr}$$

Unburned Air Mass Rate per Cylinder – Method 3 (MAF and dP based)

$$\dot{m}_{\text{tot,ub-cyl}} = \dot{m}_{\text{MAF}} F_{FAF} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \left( \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2}{RT_{\text{EGR}}}} \frac{P_{\text{EGR}}}{\Delta P_{\text{EGR}}} \right) F_{egr} + \left( \frac{AFR_m - AFR_s}{AFR_m + 1} \right) \dot{m}' F_{tr}$$
Unburned Air Mass per Cylinder – Method 1 (dP and VE based)

\[
\begin{align*}
\dot{m}_{\text{tot,ub-cyl}} &= \left( \eta_{\text{vol}} \frac{V_{\text{Eng}}}{RT_{\text{chrg}}} - \frac{120 \cdot C_d A_2}{\omega_{\text{Eng}} \sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \left( 2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}} \right) F_{\text{EGR}} + \left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \\
120 \cdot \dot{m}_{\text{tr}} F_{\text{tr}} &+ \left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \left( \frac{120 \cdot C_d A_2 F_{\text{EGR}}}{\omega_{\text{Eng}} \sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \left( 2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}} \right) F_{\text{EGR}} + \left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right)
\end{align*}
\]

Unburned Air Mass per Cylinder – Method 2 (MAF and VE based)

\[
\begin{align*}
\dot{m}_{\text{tot,ub-cyl}} &= \left( \frac{120 \cdot \dot{m}_{\text{MAF}} F_{\text{EGR}}}{\omega_{\text{Eng}}} + \left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \left( \frac{120 \cdot \dot{m}_{\text{MAF}}}{\omega_{\text{Eng}}} \right) \right) \left( \eta_{\text{vol}} \frac{V_{\text{Eng}}}{RT_{\text{chrg}}} - \frac{120 \cdot \dot{m}_{\text{MAF}}}{\omega_{\text{Eng}}} \right) F_{\text{EGR}} + \\
&\left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \left( \frac{120 \cdot \dot{m}_{\text{tr}} F_{\text{tr}}}{\omega_{\text{Eng}}} \right)
\end{align*}
\]

Unburned Air Mass per Cylinder – Method 3 (MAF and dP based)

\[
\begin{align*}
\dot{m}_{\text{tot,ub-cyl}} &= \left( \frac{120 \cdot \dot{m}_{\text{MAF}} F_{\text{EGR}}}{\omega_{\text{Eng}}} + \left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \left( \frac{120 \cdot \dot{m}_{\text{MAF}}}{\omega_{\text{Eng}}} \right) \right) \left( \frac{120 \cdot C_d A_2}{\omega_{\text{Eng}} \sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \left( 2 \frac{P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}} \right) F_{\text{EGR}} + \\
&\left( \frac{\text{AFR}_m - \text{AFR}_s}{\text{AFR}_m + 1} \right) \left( \frac{120 \cdot \dot{m}_{\text{tr}} F_{\text{tr}}}{\omega_{\text{Eng}}} \right)
\end{align*}
\]
Total Charge

Total Charge Mass Rate per Cylinder – Method 1 (VE based)

\[
m_{\text{tot, chrg}} = \eta_{\text{vol}} \frac{\omega_{\text{Eng}} V_{\text{Eng}} F_{\text{chrg}}}{60 \times 2} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} + m' F_{\text{trap}}
\]

Total Charge Mass Rate per Cylinder – Method 2 (MAF and dP based)

\[
m_{\text{tot, chrg}} = \left( \frac{C_d A_2}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \right) \sqrt{\frac{2 P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}} \ F_{\text{chrg}} + m' F_{\text{trap}}
\]

Total Charge Mass per Cylinder – Method 1 (VE based)

\[
m_{\text{tot, chrg}} = \eta_{\text{vol}} V_{\text{Eng}} F_{\text{chrg}} \frac{P_{\text{chrg}}}{RT_{\text{chrg}}} + \frac{120 \cdot m' F_{\text{trap}}}{\omega_{\text{Eng}}}
\]

Total Charge Mass per Cylinder – Method 2 (MAF and dP based)

\[
m_{\text{tot, chrg}} = \frac{120}{\omega_{\text{Eng}}} \left( m_{\text{MAF}} F_{\text{MAF}} + \frac{C_d A_2 F_{\text{EGR}}}{\sqrt{1 - \left( \frac{A_2}{A_1} \right)^2}} \sqrt{\frac{2 P_{\text{EGR}}}{RT_{\text{EGR}}} \Delta P_{\text{EGR}}} \ F_{\text{chrg}} + \frac{120 \cdot m' F_{\text{trap}}}{\omega_{\text{Eng}}} \right)
\]
APPENDIX C: ANALYTICAL UNCERTAINTY EQUATIONS

EGR

**EGR Mass Rate – Method 1 (dP Based)**

\[
\sigma_{m}^2 = \left( \frac{\sigma_{s_p} \dot{m}^e}{2\Delta P_{\text{op}}} \right)^2 + \left( \frac{\sigma_{e} \dot{m}^e}{C_d} \right)^2 + \left( \frac{\sigma_{s_p} \dot{m}^e}{2R} \right)^2 + \left( \frac{\sigma_{e} \dot{m}^e}{2T_{\text{op}}} \right)^2 + \left( \frac{\sigma_{A_s} \dot{m}^e}{A(A_1^2 - A_2^2)} \right)^2 + \left( \frac{\sigma_{e} \dot{m}^e}{A(A_1^2 - A_2^2)} \right)^2
\]

**EGR Mass Rate – Method 2 (VE and MAF based)**

\[
\sigma_{m}^2 = \left( \sigma_{m_{\text{VE}}} \right)^2 + \left( \frac{\sigma_{\eta_{\text{VE}}} \dot{m}_{\text{VE}}}{\eta_{\text{vol}}} \right)^2 + \left( \frac{\sigma_{p_{\text{VE}}} \dot{m}_{\text{VE}}}{P_{\text{VE}}} \right)^2 + \left( \frac{\sigma_{\varphi_{\text{VE}}} \dot{m}_{\text{VE}}}{T_{\text{VE}}} \right)^2 + \left( \frac{\sigma_{V_{\text{VE}}} \dot{m}_{\text{VE}}}{\varphi_{\text{VE}}} \right)^2
\]

**EGR Mass Rate per Cylinder – Method 1 (dP Based)**

\[
\sigma_{m_{\text{cyl}}}^2 = \left( \sigma_{p_{\text{cyl}}} \dot{m}^e \right)^2 + \left( \frac{\sigma_{s_p} F_{\text{EGR}} \dot{m}^e}{2\Delta P_{\text{op}}} \right)^2 + \left( \frac{\sigma_{e} F_{\text{EGR}} \dot{m}^e}{C_d} \right)^2 + \left( \frac{\sigma_{s_p} F_{\text{EGR}} \dot{m}^e}{2R} \right)^2 + \left( \frac{\sigma_{e} F_{\text{EGR}} \dot{m}^e}{2T_{\text{op}}} \right)^2 + \left( \frac{\sigma_{p_{\text{cyl}}} F_{\text{EGR}} \dot{m}^e}{2P_{\text{op}}} \right)^2 +
\left( \frac{\sigma_{A_s} F_{\text{EGR}} \dot{m}^e}{A(A_1^2 - A_2^2)} \right)^2 + \left( \frac{\sigma_{e} F_{\text{EGR}} \dot{m}^e}{A(A_1^2 - A_2^2)} \right)^2
\]

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EGR Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[ \sigma_{\text{VE,MAF}}^2 = (\sigma_{\text{VE,MAF}} \cdot F_{\text{egr}})^2 + (\sigma_{\text{MAF}} \cdot \dot{m}^e)^2 + \left( \frac{\sigma_{\eta_{\text{vol}}} F_{\text{egr}} \dot{m}_{\text{charg}}}{\eta_{\text{eng}}} \right)^2 + \left( \frac{\sigma_{P_{\text{charg}}} F_{\text{egr}} \dot{m}_{\text{charg}}}{P_{\text{charg}}} \right)^2 + \left( \frac{\sigma_{R} F_{\text{egr}} \dot{m}_{\text{charg}}}{R} \right)^2 + \left( \frac{\sigma_{T_{\text{charg}}} F_{\text{egr}} \dot{m}_{\text{charg}}}{T_{\text{charg}}} \right)^2 + \left( \frac{\sigma_{\text{eng,vol}}} {V_{\text{eng}}} \right)^2 \]

Total Residual

Total Residual Gas Mass Rate – Method 1 (dP based)

\[ \sigma_{\text{res,1}}^2 = (\sigma_{\text{res}})^2 + \left( \frac{\sigma_{\dot{m}_{\text{egr}}}} {2\Delta P_{\text{egr}}} \right)^2 + \left( \frac{\sigma_{C_{d}}} {C_{d}} \right)^2 + \left( \frac{\sigma_{\dot{m}^e}} {2R} \right)^2 + \left( \frac{\sigma_{\dot{m}^e}} {2T_{\text{egr}}} \right)^2 + \left( \frac{\sigma_{\eta_{\text{eng}}} \dot{m}_{\text{charg}}}{\eta_{\text{eng}}} \right)^2 + \left( \frac{\sigma_{A_{\text{egr}}} \dot{m}^{e^2}} {A_{\text{egr}}(A^2 - A_{\text{egr}}^2)} \right)^2 + \left( \frac{\sigma_{A_{\text{egr}}} \dot{m}^{e^2}} {A_{\text{egr}}(A^2 - A_{\text{egr}}^2)} \right)^2 \]

Total Residual Gas Mass Rate – Method 2 (VE and MAF based)

\[ \sigma_{\text{res,2}}^2 = (\sigma_{\text{res}})^2 + \left( \frac{\sigma_{\dot{m}_{\text{egr}}}} {\eta_{\text{eng}}} \right)^2 + \left( \frac{\sigma_{\dot{m}_{\text{charg}}}} {P_{\text{charg}}} \right)^2 + \left( \frac{\sigma_{R} \dot{m}_{\text{charg}}}{R} \right)^2 + \left( \frac{\sigma_{T_{\text{charg}}} \dot{m}_{\text{charg}}}{T_{\text{charg}}} \right)^2 + \left( \frac{\sigma_{\text{eng,vol}}} {V_{\text{eng}}} \right)^2 \]
Total Residual Gas Mass Rate per Cylinder – Method 1 (dP based)

\[
\sigma_{\text{res, cyl}}^2 = (\sigma_{\text{v, exp}} F_{\text{exp}})^2 + (\sigma_{\text{m, exp}} m_{\text{exp}})^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{2\Delta P_{\text{exp}}}\right)^2 + \left(\frac{\sigma_{\text{v, exp}} m_{\text{exp}}}{C_d}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{2R}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{2T_{\text{exp}}}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{A(\hat{A} - \hat{A}_1)}\right)^2
\]

Total Residual Gas Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[
\sigma_{\text{res, cyl}}^2 = (\sigma_{\text{v, exp}} F_{\text{exp}})^2 + (\sigma_{\text{m, exp}} m_{\text{exp}})^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{\eta_{\text{vol}} F_{\text{exp}} m_{\text{chrg}} / P_{\text{chrg}}}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{\eta_{\text{vol}} F_{\text{exp}} m_{\text{chrg}} / V_{\text{eng}}}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{\eta_{\text{vol}} F_{\text{exp}} m_{\text{chrg}} / T_{\text{chrg}}}\right)^2 + \left(\frac{\sigma_{\text{m, exp}} m_{\text{exp}}}{\eta_{\text{vol}} F_{\text{exp}} m_{\text{chrg}} / R}\right)^2
\]

Total Burned Gas

Total Burned Gas Mass Rate – Method 1 (dP based)

\[
\sigma_{\text{burn, cyl}}^2 = \left(\sigma_{\text{v, exp}} F_{\text{exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{m, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{v, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{m, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{v, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{m, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{v, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2 + \left(\sigma_{\text{m, exp}} m_{\text{exp}} / (\text{AFR}_{\text{n}} + 1)\right)^2
\]
Total Burned Gas Mass Rate – Method 2 (VE and MAF based)

\[
\sigma^2_{\text{m, tot}} = \left( \sigma_{AFRm} \left( \frac{\dot{m}^e_{AFR} + \dot{m}^e_{MAF}}{AFR_n + 1} \right)^2 + \sigma_{AFRm} \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2
\]

Total Burned Gas Mass Rate per Cylinder – Method 1 (dP based)

\[
\sigma^2_{\text{m, cyl}} = \left( \sigma_{AFRm} \left( \frac{\dot{m}^e_{AFR} + \dot{m}^e_{MAF}}{AFR_n + 1} \right)^2 + \sigma_{AFRm} \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2
\]

Total Burned Gas Mass Rate per Cylinder – Method 2 (VE and MAF based)

\[
\sigma^2_{\text{m, cyl}} = \left( \sigma_{AFRm} \left( \frac{\dot{m}^e_{AFR} + \dot{m}^e_{MAF}}{AFR_n + 1} \right)^2 + \sigma_{AFRm} \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \frac{AFR + 1}{AFR_n + 1} \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2 + \left( \sigma_{\text{tmscylbtot}, F_{\text{AFRm}}} \left( \frac{AFR + 1}{AFR_n + 1} \right) \right)^2
\]
Residual Concentration

**Burned Air Concentration – Mass Based (dP and VE Based)**

\[
\sigma_{\text{res}}^2 = \left( \frac{\sigma_{\text{chrg}} (m_{\text{chrg}} + m') (AFR_s + 1) }{ (m_{\text{chrg}} + m')^2 (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\text{e}} (AFR_s + 1) }{ (m_{\text{chrg}} + m') (AFR_m + 1)} \right)^2 + \frac{\sigma_{\text{chrg}} (m_{\text{chrg}} - m') (AFR_s + 1)}{ (m_{\text{chrg}} + m')^2 (AFR_m + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_s + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_m + 1)}
\]

**Burned Air Concentration – Mass Based (MAF and VE Based)**

\[
\sigma_{\text{res}}^2 = \left( \frac{\sigma_{\text{chrg}} m_{\text{MAP}} (AFR_s + 1) }{ (m_{\text{chrg}} + m')^2 (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\text{e}} (AFR_s + 1) }{ (m_{\text{chrg}} + m') (AFR_m + 1)} \right)^2 + \frac{\sigma_{\text{chrg}} m_{\text{MAP}} (AFR_s + 1)}{ (m_{\text{chrg}} + m')^2 (AFR_m + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_s + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_m + 1)}
\]

**Burned Air Concentration – Mass Based (MAF and dP Based)**

\[
\sigma_{\text{res}}^2 = \left( \frac{\sigma_{\text{e}} m_{\text{MAP}} (AFR_s + 1) }{ (m_{\text{e}} + m_{\text{MAF}} + m')^2 (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\text{e}} m_{\text{MAP}} (AFR_s + 1) }{ (m_{\text{e}} + m_{\text{MAF}} + m')^2 (AFR_m + 1)} \right)^2 + \frac{\sigma_{\text{chrg}} m_{\text{MAP}} (AFR_s + 1)}{ (m_{\text{e}} + m_{\text{MAF}} + m')^2 (AFR_m + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_s + 1)} + \frac{\sigma_{\text{CO}_2} [CO_2]_{\text{mass}} }{ (AFR_m + 1)}
\]
Fresh Air

**Fresh Air Flow Rate – Method 1 (dP and VE based)**

\[
\sigma_{\bar{m}_{\text{unair}}}^2 = \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{\eta_{\text{vol}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{P_{\text{chrg}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{T_{\text{chrg}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{V_{\text{eng}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{2\Delta P_{\text{egr}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{C_d} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{\Delta} \right)^2
\]

**Fresh Air Flow Rate – Method 2 (MAF based)**

\[
\sigma_{\bar{m}_{\text{unair}}}^2 = \left( \bar{m}_v \right)^2
\]

**Fresh Air Flow Rate per cylinder – Method 1 (dP and VE based)**

\[
\sigma_{\bar{m}_{\text{unair-cyl}}}^2 = \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{\eta_{\text{vol}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{P_{\text{chrg}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{T_{\text{chrg}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{V_{\text{eng}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{2\Delta P_{\text{egr}}} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{C_d} \right)^2 + \left( \frac{\sigma_{\bar{m}} \cdot \bar{m}_{\text{deg}}}{\Delta} \right)^2
\]

**Fresh Air Flow Rate per cylinder – Method 2 (MAF based)**

\[
\sigma_{\bar{m}_{\text{unair-cyl}}}^2 = \left( \bar{m}_v \cdot F_{\text{FAF}} \right)^2 + \left( \bar{m}_{\text{deg}} \cdot \bar{m}_{\text{deg}} \right)^2
\]
Total Unburned Air

Unburned Air Mass Rate – Method 1 (dP and VE based)

\[
\sigma_{\text{unw}}^2 = \left( \frac{\dot{m}_{\text{veh}}}{R} + \frac{\dot{m} (AFR_n + 1)}{2R(AFR_n + 1)} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2
\]

Unburned Air Mass Rate – Method 2 (MAF and VE based)

\[
\sigma_{\text{unw}}^2 = \left( \frac{\dot{m}_{\text{veh}}}{\eta_{\text{veh}}} \right)^2 + \left( \frac{\dot{m} (AFR_n + 1)}{R} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2
\]

Unburned Air Mass Rate – Method 3 (MAF and dP based)

\[
\sigma_{\text{unw}}^2 = \left( \frac{\dot{m}'}{C_j} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2 + \left( \frac{\dot{m}'}{AFR_n + 1} \right)^2
\]
Unburned Air Mass Rate per Cylinder – Method 1 (dP and VE based)

\[
\sigma_{\text{error}}^2 = \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{R} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{2R(AFR_n + 1)} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2
\]

Unburned Air Mass Rate per Cylinder – Method 2 (MAF and VE based)

\[
\sigma_{\text{error}}^2 = \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{R} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{2R(AFR_n + 1)} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2
\]

\[
\sigma_{\text{error}}^2 = \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{R} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{2R(AFR_n + 1)} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2 + \left( \frac{m_{\text{inj}} F_{\text{AFR}}}{AFR_n + 1} \right)^2
\]

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Unburned Air Mass Rate per Cylinder – Method 3 (MAF and dP based)

$$\sigma_{\text{unburned}}^2 = \left( \frac{\sigma_m \bar{n'}_f \left( AFR_m - AFR_s \right)}{2R} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{C} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{A} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{2A \Delta} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{222} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{222} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{222} \right)^2 + \left( \frac{\sigma_{n'} \bar{n'}_e \left( AFR_m - AFR_s \right)}{222} \right)^2$$

Unburned Air Concentration

Unburned Air Concentration – Mass Based (dP and VE Based)

$$\sigma_{\text{unburned}}^2 = \left( \frac{\sigma_m \left( m' + m' \left( AFR_s + 1 \right) \right)}{(AFR + 1)(m' + m')^2} \right)^2 + \left( \frac{\sigma_{n'} \left( AFR_s + 1 \right)}{m' + m' (AFR + 1)} \right)^2 + \left( \frac{\sigma_{n'} \left( m' + m' \left( AFR_s + 1 \right) \right)}{(AFR + 1)(m' + m')^2} \right)^2 + \left( \frac{\sigma_{n'} \left( m' + m' \left( AFR_s + 1 \right) \right)}{(AFR + 1)(m' + m')^2} \right)^2 + \left( \frac{\sigma_{n'} \left( m' + m' \left( AFR_s + 1 \right) \right)}{(AFR + 1)(m' + m')^2} \right)^2 + \left( \frac{\sigma_{n'} \left( m' + m' \left( AFR_s + 1 \right) \right)}{(AFR + 1)(m' + m')^2} \right)^2$$

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Unburned Air Concentration – Mass Based (MAF and VE Based)

\[ \sigma^2_{\text{massubtot}} = \left( \frac{\sigma_{\text{m}_{\text{chrg}}} (\dot{m}_{\text{MAF}} (AFR_s + 1))}{(\dot{m}_{\text{chrg}} + \dot{m}^t)^2 (AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{\text{m}_{\text{MAF}}} (AFR_s + 1)}{(\dot{m}_{\text{chrg}} + \dot{m}^t)(AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{AFR_s} (\dot{m}_{\text{chrg}} - \dot{m}_{\text{MAF}} + \dot{m}^t)(AFR_s + 1)}{(\dot{m}_{\text{chrg}} + \dot{m}^t)(AFR_m + 1)} \right)^2 \]

Unburned Air Concentration – Mass Based (MAF and dP Based)

\[ \sigma^2_{\text{massubtot}} = \left( \frac{\sigma_{\text{m}_{\text{MAF}}} \cdot (AFR_s + 1)}{(AFR_m + 1)(\dot{m}^e + \dot{m}_{\text{MAF}} + \dot{m}^t)} \right)^2 + \left( \frac{\sigma_{\text{m}_{\text{MAF}}} (AFR_s + 1)}{(AFR_m + 1)(\dot{m}^e + \dot{m}_{\text{MAF}} + \dot{m}^t)(AFR_m + 1)} \right)^2 + \left( \frac{\sigma_{AFR_s} (\dot{m}^e + \dot{m}^t)}{(AFR_m + 1)(\dot{m}^e + \dot{m}_{\text{MAF}} + \dot{m}^t)} \right)^2 + \left( \frac{\sigma_{AFR_s} (\dot{m}^e + \dot{m}^t)(AFR_s + 1)}{(\dot{m}^e + \dot{m}_{\text{MAF}} + \dot{m}^t)^2 (AFR_m + 1)} \right)^2 \]
Total Charge

Total Charge Mass Rate – Method 1 (VE based)

\[
\sigma_{\text{w,chg}}^2 = \left( \frac{\sigma_{\text{m,chg}}}{\eta_{\text{vol}}} \right)^2 + \left( \frac{\sigma_{\text{P,chg}}}{P_{\text{chg}}} \right)^2 + \left( \frac{\sigma_{\text{T,chg}}}{T_{\text{chg}}} \right)^2 + \left( \frac{\sigma_{\text{V,chg}}}{V_{\text{eng}}} \right)^2 + \left( \sigma_{\text{w}} \right)^2
\]

Total Charge Mass Rate – Method 2 (MAF and dP based)

\[
\sigma_{\text{w,chg}}^2 = \left( \sigma_{\text{w,m}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2\Delta P_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{C_{\text{d}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2T_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2P_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{A}}A_{\text{t}} \Delta m'}{A_{\text{t}}(A_{\text{t}}^2 - A_{\text{t}}^2)} \right)^2 + \left( \sigma_{\text{w}} \right)^2
\]

Total Charge Mass Rate per Cylinder – Method 1 (VE based)

\[
\sigma_{\text{w,chg-cyl}}^2 = \left( \sigma_{\text{m,chg}} \right)^2 + \left( \sigma_{\text{m'}} \right)^2 + \left( \frac{\sigma_{\text{F,chg}}}{\eta_{\text{vol}}} \right)^2 + \left( \frac{\sigma_{\text{P,chg}}}{P_{\text{chg}}} \right)^2 + \left( \frac{\sigma_{\text{T,chg}}}{T_{\text{chg}}} \right)^2 + \left( \frac{\sigma_{\text{V,chg}}}{V_{\text{eng}}} \right)^2 + \left( \sigma_{\text{F}} \right)^2
\]

Total Charge Mass Rate per Cylinder – Method 2 (MAF and dP based)

\[
\sigma_{\text{w,chg-cyl}}^2 = \left( \sigma_{\text{m,chg-MAF}} \right)^2 + \left( \sigma_{\text{m'}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2\Delta P_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{C_{\text{d}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2T_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{m'}}}{2P_{\text{t}}} \right)^2 + \left( \frac{\sigma_{\text{A}}A_{\text{t}} \Delta m'}{A_{\text{t}}(A_{\text{t}}^2 - A_{\text{t}}^2)} \right)^2 + \left( \sigma_{\text{F}} \right)^2
\]