The Semantics of Plurals: A Defense of Singularism

Dissertation

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In this dissertation, I defend *semantic singularism*, which is the view that syntactically plural terms, such as ‘they’ or ‘Russell and Whitehead’, are semantically singular. A semantically singular term is a term that denotes a single entity. Semantic singularism is to be distinguished from *syntactic singularism*, according to which syntactically plural terms are not required in the regimentation of natural language into a formal language; rather, syntactically singular terms suffice for the task.

The traditional semantic conception of plurals embraces syntactic singularism. In recent years, however, a number of theorists have argued against the traditional conception and in favor of both *syntactic pluralism* and *semantic pluralism*. According to syntactic pluralism, syntactically plural terms are required in the regimentation of natural language. According to semantic pluralism, a syntactically plural term is semantically plural in that it denotes many entities at once. In light of the arguments against the traditional conception, I reject syntactic singularism but I argue that semantic singularism is a viable alternative to semantic pluralism.

According to *object singularism*, which is the standard formulation of semantic singularism, plural terms denote objects. As I argue in Chapter 1, the object-singularist can sidestep many objections in the literature but she faces a serious challenge, since she cannot accommodate the possibility of absolutely unrestricted quantification. In response to this potential difficulty, I propose a novel construal of semantic singularism, *property singularism*, according to which a plural term denotes a property rather than an object. In Chapter 2, I argue that property singularism fares at least as well as the version of semantic pluralism that takes plural predicates, such as ‘being
two’ or ‘cooperate’, to denote plural properties. In Chapter 3 and 4, I argue against two more versions of semantic pluralism, one that takes plural predicates to denote properties as objects and one that takes plural predicates to denote superpluralities. I conclude that, whether or not the possibility of absolutely unrestricted quantification is admitted, semantic singularism remains a satisfactory approach to plurals.
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Introduction

Issues concerning the logic and meaning of plurals have been at the center of a lively debate for almost three decades. Both philosophers and linguists have been interested in providing an account of the semantic features of plural expressions. It is well-known that traditional logical analysis eliminates plurals in favor of singular constructions. First-order logic encompasses only one kind of terms, predicates, and quantifiers. Consider the following sentence:

(1) Socrates and Plato are philosophers.

One may attempt to symbolize (1) within first-order logic by first analyzing it as (2).

(2) Socrates is a philosopher and Plato is a philosopher.

In this spirit, Frege claimed that, in (1),

\[ \text{[W]e have two thoughts: Socrates is a philosopher and Plato is a philosopher, which are only strung together linguistically for the sake of convenience. Logically, Socrates and Plato is not to be conceived as the subject of which being a philosopher is predicated. (Frege (1980), p. 140)} \]

Since plurals have been eliminated in (2), it is now straightforward to symbolize the sentence in first-order logic.
However, it takes a moment to realize that this strategy cannot be generalized with respect to all sentences containing plurals, or at least it cannot be generalized in any obvious way. As Frege himself noticed, the following sentences are not amenable to the same treatment.

(3) Bunsen and Kirchhoff laid the foundations of spectral analysis.

(4) The Romans conquered Gaul.

Frege writes:

Here we must regard *Bunsen and Kirchhoff* as a whole. ‘The Romans conquered Gaul’ must be conceived in the same way. The Romans here are the Roman people, held together by custom, institutions, and laws.

(Ibidem)

The plural terms ‘Bunsen and Kirchhoff’ and ‘The Romans’ occurring in (3) and (4) cannot be separated in the way ‘Socrates and Plato’ could be separated in (1). So the question arises as to how plural predication should be understood from a logical point of view. Is there any satisfactory way to account for the kind of plural predication exhibited in (3) and (4) within first-order logic or within some variant of it? Or does plural predication require a more radical departure from first-order logic? The main goals of this dissertation are to explore these questions and to provide an answer to them. In pursuing these goals, I will also engage with important contributions to the literature on the logic and meaning of plurals.
Two kinds of singularism and pluralism

Customary logical analysis of natural language proceeds in two stages. First, natural language sentences are regimented into a formal language — the regimenting language. Second, the logical and semantic features of the regimenting language are characterized model-theoretically or proof-theoretically, thereby shedding light on the regimented language. For reasons that will become clear in due course, the focus of this dissertation is on model theory.¹ The model theory associated with the regimenting language models logical notions, such as logical consequence and logical truth, and semantic notions, such as denotation.

This two-tiered process of analysis is reflected in a standard approach in linguistic semantics, such as Montague semantics.² In Montague semantics, a fragment of natural language is mechanically translated via lambda calculus into a formal language whose logical and semantic properties are given model-theoretically.

Therefore, it is important to distinguish between two questions:

(i) what type of symbolic language should be used to regiment the relevant fragment of natural language?

(ii) once a regimenting language has been chosen, how should one construct a model theory for it?

In the present context, I will refer to the issue raised by the first question as the syntactic issue. Moreover, I will refer to the issue raised by the second question as

¹The focus on model theory is recommended by the same considerations that recommend focusing on model theory in the case of higher-order logic.

²For a presentation of Montague semantics, see Dowty et al. (1981).
the *semantic* issue. The contrastive labels ‘syntactic’ and ‘semantic’ are not meant to be descriptive and should be taken as merely suggestive. Regimentation is not a purely syntactic procedure, as it requires the preservation of some relevant semantic features of the regimented sentences. Thus, the syntactic issue does have a semantic component.

The traditional approach to plurals embraces **syntactic singularism**, namely, the view that syntactically plural terms are not required in the regimentation of natural language plurals; rather, syntactically singular terms suffice for the task. According to the syntactic singularist, no new type of terms needs to be introduced in the regimenting language: the basic machinery of first-order logic is adequate to capture the logic and meaning of plurals.³

In recent years, however, a number of theorists have argued against the traditional approach and in favor of both **syntactic pluralism** and **semantic pluralism**. In contrast with syntactic singularism, the syntactic pluralist holds that syntactically plural terms are required in the regimentation of natural language, thereby extending first-order logic with the introduction of a new type of terms — plural terms. On the other hand, semantic pluralism is a view about the semantic issue. It holds that, once syntactically plural terms have been introduced in the regimenting language, they should be regarded as *semantically plural*. A semantically plural term is a term that

³By ‘singular language’ I will mean a language that contains only singular expressions. A language containing plural expressions will be called a ‘plural language’.
denotes *many entities at once* or, as I will often say, a *plurality*.\(^4\,5\)

In this dissertation, I argue that, in light of the arguments against the traditional approach, there is reason to reject syntactic singularism. However, I also argue that **semantic singularism** is a viable alternative to semantic pluralism — thus the title "The Semantics of Plurals: A Defense of Singularism". According to **semantic singularism**, syntactically plural terms are *semantically singular*. A semantically singular term is a term that denotes *a single entity*.

**Varieties of semantic singularism and pluralism**

As we saw, semantic singularism and pluralism differ over how many entities each view takes a plural term to denote. According to semantic singularism, a syntactically plural term is semantically singular, denoting a single entity. Semantic pluralism, on the other hand, holds that a syntactically plural term is semantically plural, denoting many entities at once.\(^6\) Depending on what *types* of entities one takes a syntactically plural term to denote, semantic singularism and pluralism can have different construals.

\(^4\)Throughout this work talk of pluralities should be understood as a shorthand for genuine plural talk. When the use of the plural idiom requires expository and stylistic complications, talk of pluralities will be preferred. This means that, when I assert the existence of a plurality satisfying a certain condition (‘there is a plurality such that...’), I do not mean to assert the existence a particular kind of object. I just mean to assert that *there are some things* satisfying a certain condition.

\(^5\)I will assume that a plurality may contain only one thing. We may call such pluralities ‘degenerate’. This means that I am allowing for the possibility that there be some things such that there is a unique thing that is among them. Nothing important will turn on this assumption.

\(^6\)Henceforth, ‘plural term’ will typically mean *syntactically plural term*. 
The basic kinds of entities are objects and properties. This gives rise to four views: object singularism, property singularism, object pluralism, and property pluralism. The last view appears to be the least attractive, since it combines conceptual resources and controversial aspects of the other three views without offering any advantage over them. Thus, object pluralism may be considered the only pluralist option.

So we have three main views:

Object singularism: a plural term denotes an object;

Property singularism: a plural term denotes a property;

Object pluralism (‘pluralism’ for short): a plural term denotes many objects.

Within the pluralist camp, there is an additional dimension of classification concerning what one takes a plural predicate, such as ‘cooperate’ or ‘being two’, to denote. Here we have three main candidates. Before presenting them, let me introduce the notion of a plural property.

The traditional conception of reality is singular: there are objects and there are singular properties, that is, properties that are instantiated by objects separately. For example, the property denoted by the predicate ‘being a philosopher’ is instantiated separately by Socrates, Plato, Aristotle, and others. Plural properties differ from singular properties in that they are properties that are instantiated by many objects taken together. For example, one might take ‘co-wrote Principia Mathematica’ to denote a property that is instantiated by Russell and Whitehead together, not

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7I use ‘entity’ as an umbrella term covering both objects and properties.
According to **untyped pluralism**, a plural predicate denotes a plural property, where one does not postulate a type distinction between objects and properties.\(^8\) **Standard pluralism** is a similar view: a plural predicate is taken to denote a plural property but a type distinction between objects and properties is postulated.\(^9\) Finally, one might take a plural predicate to denote a *superplurality*, that is, a plurality of pluralities.\(^10\) We may call this view **superpluralism**.

As I argue in Chapter 1, the object-singularist can sidestep many objections in the literature. However, she might face a serious challenge. There is a controversy over whether it is possible to achieve *absolutely unrestricted quantification*, that is, quantification over absolutely everything there is. It has been argued that the object-singularist is in no position to accommodate the possibility of absolutely unrestricted quantification. In response to this potential difficulty, there is reason to prefer the second construal of singularism, property singularism, which can accommodate the possibility of absolutely unrestricted quantification. In Chapter 2, I argue that property singularism fares at least as well as standard pluralism. In Chapters 3 and 4, I argue against untyped pluralism and superpluralism respectively. I conclude that, whether or not the possibility of absolutely unrestricted quantification is admitted, semantic singularism remains a satisfactory approach to plurals.

As I mentioned above (and as it will be argued below), there is reason to reject syntactic singularism in favor of syntactic pluralism. That means that one needs to extend first-order logic by introducing plural terms, predicates, and quantifiers. I

\(^8\)This view is put forward by McKay (2006).
\(^9\)This approach has been defended by Yi (1999, 2005, 2006) and Oliver and Smiley (2001).
will now introduce the basic language that will be used in this work to regiment a relevant fragment of natural language containing plural expressions. Given that we are mostly interested in foundational semantic issues, a number of questions concerning the details of how one may best capture the logical form of natural language plurals will have to be put aside. For simplicity, I will adopt a regimenting language that contains only the basic elements of plural predication. This language will be called $\mathcal{L}_{PL}$.

Regimenting plurals: $\mathcal{L}_{PL}$

In $\mathcal{L}_{PL}$, every plural predicate takes a fixed number of arguments, each of which can be exclusively singular or exclusively plural. An argument place is singular if it can be occupied only by singular terms, that is, by singular constants and singular variables. It is plural if it can be occupied only by plural variables. A predicate is said to be plural if at least one of its argument places is plural. Special status is given to the binary plural predicate ‘being one of’ or ‘is among’, which will be treated as logical.

A note on plural constants: even though they might be useful for proof-theoretic purposes, plural constants will not be official components of the regimenting language. This will simplify the exposition of the various model theories.

The vocabulary of $\mathcal{L}_{PL}$ is composed of the usual vocabulary of first-order logic plus the following items.

A. Plural variables: $vv, xx, yy, zz, \ldots$. They correspond to the natural language pronoun ‘they’. Unofficially, we may think of the language as containing plural

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11This language is closely related to the one known in the literature as PFO+, plural first-order logic with plural predicates. See Rayo (2002) for details.
proper names as well: *aa, bb, cc, dd, ... .*

B. The plural existential quantifier (*∃*) binding plural variables: *∃vv, ∃xx ... .*

(‘There are some things such that...’)

C. Symbols for collective plural predicates of any finite arity. For any arity *n*, it is convenient to allow only predicate symbols whose first *m* argument places (1 ≤ *m* ≤ *n*) are plural and whose remaining *n* − *m* argument places are singular. A superscript enclosed in square brackets will indicate the number of plural arguments. The arity of the predicate, when marked, will precede the square brackets. For example, if the two-place predicate ‘lift together’ is represented by *L*₂[₁], the sentence ‘they lifted John together’ might be represented by *L*₂[₁](vv, j). For the sake of readability, I will often depart from this convention, leaving the superscripts implicit and allowing singular argument places to precede plural ones.

D. A distinguished binary predicate representing ‘is one of’ or ‘is among’, ≺, taking a singular term in the first argument place and a plural variable in the second.

The recursive clauses defining well-formed formulae and free or bound occurrences of variables are the obvious ones.

An important semantic phenomenon concerning plurals that has to be accounted for is the distinction between distributive and collective plural predicates. Distributivity is an analytic relation that holds between a plural predicate and its corresponding singular form — if this exists. Let Φ be a plural predicate and let φ be its corresponding singular form. Then Φ is said to be *distributive* if

(5) Analytically, for any things *xx, xx* are Φ if and only if anything that is one of *xx* is φ.
A plural predicate is said to be *collective* if it is not distributive.

In $\mathcal{L}_{PL}$, we can obtain the force of distributive predication without invoking distributive predicates. Consider:

(1) Socrates and Plato are philosophers.

The use of the distributive ‘are philosophers’ can be paraphrased away via the singular ‘is a philosopher’ as shown in (6).

(6) Everything that is one of Socrates and Plato is a philosopher.

This is why we have assumed that all plural predicates of $\mathcal{L}_{PL}$ are collective.

The existence of a good method of paraphrase also makes the introduction of a symbol for phrasal conjunction unnecessary in $\mathcal{L}_{PL}$. This is shown by the following pair of sentences.

(7) Russell and Whitehead cooperate.

(8) There are some things such that Russell is one of them, Whitehead is one of them, no other thing is one of them, and they cooperate.

Some additional concepts do not require a separate treatment as they can be introduced as abbreviations. The plural ‘are among’, symbolized by $\preceq$, can be defined thus:

(9) $yy \preceq xx \iff \forall y (y \preceq yy \rightarrow y \prec xx)$.

Plural identity, symbolized by $\approx$, is also definable:

(10) $yy \approx xx \iff \forall x (x \prec yy \leftrightarrow x \prec xx)$. 

This completes the presentation of our regimenting language.

Providing an adequate semantics of plural definite descriptions presents distinctive difficulties that, however, go beyond the scope of this dissertation. The main points I will make do not depend on any particular view about plural definite descriptions. So, despite its intrinsic semantic interest, I will have to put the issue aside. The following remarks will have to suffice.

To deal with plural definite descriptions, one might think of proceeding in the usual Russellian way. And the next pair of sentences might give some comfort to this idea.

(11) The authors of *Principia Mathematica* gathered.

(12) There are some things such that they are authors of *Principia Mathematica* and no other things are authors of *Principia Mathematica*, and they gathered.

This way of paraphrasing plural definite descriptions, however, does not always succeed. Consider:

(13) The men gathered.

(14) There are some things such they are men and no other things are men, and they gathered.

If there is more than one man, there is more than one plurality of men (we allow ‘degenerate’ pluralities). So if a plurality of men contains more than one man, every subplurality of it is a plurality of men. Thus, it is not true in that situation that ‘no other things are men’. Therefore, if there is more than one man, (14) cannot be true but (13) surely can. A better paraphrase of (11) is provided by (15).
There are some (maximal) things such that each of them is a man and they gathered.

As these cases show, paraphrasing plural definite descriptions has additional complications with respect to paraphrasing singular ones. I am not suggesting that one can avoid the introduction of a definite description operator and that $L_{PL}$ has the resources for a fully adequate treatment of plural definite descriptions. However, for our purposes, we may assume that $L_{PL}$ can at least capture some uses of plural descriptions, in particular those occurring in our examples.

A final remark on some proof-theoretic aspects of the regimenting language just introduced. Plural quantifiers are governed by rules of inferences analogous to those governing the first-order quantifiers. In addition, we have an axiom of plural comprehension analogous to the axiom of comprehension for second-order logic. Since, unlike second-order entities, pluralities cannot be empty, plural comprehension will require that, if something is $\phi$, then the things consisting of everything that is $\phi$ exist. Formally,

$$(CP) \quad \exists x \phi(x) \rightarrow \exists xx \forall x(x \prec xx \leftrightarrow \phi(x)),$$

where $\phi(x)$ is any formula, possibly with parameters. Depending on whether or not plural quantification occurs in $\phi(x)$, we have an impredicative or a predicative instance of the principle. Instances of comprehension will be constantly invoked both in the object language and in the metatheories that employ plural resources. An adequacy condition on any model theory for $L_{PL}$ is that it validate instances of plural comprehension. Of course, if any restriction is imposed on the acceptable instances of comprehension, the adequacy condition will have to be modified accordingly. Let us now examine the approaches to plurals introduced in the previous section.
Chapter 1

Plurals and Singularism

In this chapter, I present and evaluate two versions of syntactic singularism, a set-theoretic and a mereological one. After discussing a number of objections found in the literature, I conclude that both versions of syntactic singularism are problematic. However, I argue that object singularism, a particular construal of semantic singularism, provides a better way to characterize the role that sets or mereological structures might play in connection with plurals. I show that object singularism is immune to most objections raised against syntactic singularism. Moreover, I show that object singularism might face a serious challenge, since it cannot accommodate the possibility of absolutely unrestricted quantification, i.e., quantification over absolutely everything there is. Since it is controversial whether or not absolutely unrestricted quantification can be achieved, this need not be a knock-down objection. But if such a possibility is admitted, I argue that accommodating it is especially important in the case of a plural language. The conclusion is that the main threat to object singularism comes from admitting the possibility of quantifying over absolutely everything.
1.1   Syntactic singularism

1.1.1   Set-theoretic version

There are two rather natural ideas as to how one might regiment plurals into a formal language containing only singular variables. The first idea is to paraphrase a plural term like ‘Russell and Whitehead’ as ‘the set of Russell and Whitehead’ or, avoiding the repetition of the plural term, ‘the set such that Russell is a member of it, Whitehead is a member of it, and such that any member of it is either Russell or Whitehead’. The second idea is to paraphrase plural terms by appealing to a mereological sum, rather than a set. So ‘Russell and Whitehead’ is paraphrased as ‘the sum of Russell and Whitehead’, that is, ‘the object such that anything overlaps it if and only if it overlaps Russell or Whitehead’. In this section, I focus on the first idea.

Despite having been the target of a number of criticisms, the set-theoretic version of syntactic singularism, as a systematically developed view, has never been propounded in print. Perhaps this is because it has appeared as a trivial exercise in regimentation based on the idea that plural terms should be understood as terms for sets, plural quantifiers should be understood as singular quantifiers restricted to sets, and ‘being one of’ should be represented as membership. However, rendering such a view as an alternative worth considering requires some care. For example, how should plural predicates be understood? Should they be understood as predicates applying to sets? If so, a sentence such as

(1.1) Some students arrived

would be represented as

(1.2) \( \exists x (Set(x) \land S_p(x) \land A_p(x)) \),
where the subscript $p$ marks the predicates ($S$ for ‘are students’ and $A$ for ‘arrived’) as plural in order to distinguish them from their corresponding singular forms. It is clear, though, that without further assumptions this simple translation method will not give the right results. Consider the logically true sentence:

(1.3) If some students arrived, then any student among them arrived.

On a straightforward set-theoretic translation, (1.3) becomes

\[(1.4) \forall x (\text{Set}(x) \land S_p(x) \land A_p(x)) \rightarrow \forall y (y \in x \rightarrow A(x)).\]

Unlike the sentence it represents, (1.4) is not a logical truth of first-order logic. Similarly, a simple appeal to sets will not validate plural contraction and expansion. These phenomena are illustrated by the following sentences, respectively.\(^1\)

(1.5) Tom, Dick, and Harry are linguists

\[\text{Tom and Harry are linguists}\]

Dick and Harry are lawyers

(1.6) Tom is a lawyer

\[\text{Dick, Harry, and Tom are lawyers}\]

Consider, for example, (1.5). A straightforward regimentation using sets would deliver the following inference, which is not valid.

\[(1.7) \frac{L_p(\{t, d, h\})}{L_p(\{t, h\})}\]

\(^1\)See Mussey (1976).
In order to account for the validity of these inferences, one would need to introduce additional principles linking singular predicates and plural distributive ones. No matter whether, or how, this can be done, there seems to be a strong argument against this strategy for the regimentation of plurals. Let us look at this argument.

1.1.2 The paradox of plurality

Elaborating on an idea of Boolos, a number of authors have put forward variants of a general argument reminiscent of Russell’s paradox against the set-theoretic version of syntactic singularism. Because of this, the argument has been called the paradox of plurality. I will adopt this label. The argument attempts to show that a broad range of versions of syntactic singularism, including the set-theoretic one, are bound to regiment sentences that are true, or might be true, as sentences that express logical falsehoods. Let us start with a particular case of the paradox targeted against set-theoretic singularism. I will then present two fully general variants of the paradox and discuss some possible ways out for the syntactic singularist. The paradox — as it will be clear — threatens only syntactic versions of singularism. It does not threaten semantic construals of singularism.

Here is the particular version of the paradox. This sentence seems to express a set-theoretic truth:

(1.8) There are some sets such that any set is one of them if and only if it is not self-membered.

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According to the set-theoretic version of syntactic singularism, plural quantification is rendered as quantification restricted to sets. Moreover, the predicate ‘being one of’ is taken to stand for the set-theoretic relation of membership. So, if the quantifiers are understood as unrestricted, (1.8) would appear to be equivalent to (1.9).

(1.9) There is a set $x$ such that for every set $y$, $y$ is in $x$ if and only if $y$ is not self-membered.

By Russell’s famous argument, however, (1.9) is false. Thus, a set-theoretic truth such as (1.8) is rendered equivalent to a set-theoretic statement that is false.\(^4\) Let us now see how this argument can be generalized.

**First formulation**

The scheme of plural comprehension in English is as follows. Let $\Phi$ be any predicate.

(1.10) If something is $\Phi$, then there are some things such that anything is among them if and only if it is $\Phi$.

Its formal rendering in $\mathcal{L}_{LP}$ would be (1.11).

(1.11) $\exists x \Phi x \rightarrow \exists x \forall y (y \prec xx \leftrightarrow \Phi y)$.

As remarked in the introduction, instances of plural comprehension are taken to be logically true.

Consider now a syntactic singularist and call $\mathcal{L}_{\text{Sing}}$ her regimenting language. Such a singularist will pick a relation in $\mathcal{L}_{\text{Sing}}$, say $R(x, y)$, to regiment the object language.

\[^4\text{For discussions of this argument see, for example, Boolos (1984b) p. 447 and Lewis (1991) pp. 62-71.}\]
relation of ‘being among’, which occurs in (1.10). As the notation indicates, $R(x, y)$ will be a first-order binary relation. In the set-theoretic case, set-theoretic membership ($x \in y$) will be used for $R(x, y)$. Now suppose that the object language one is trying to regiment contains a binary singular relation $R$ that is also regimented as $R(x, y)$. For the set-theoretic singularist, this happens when the fragment of natural language under consideration contains the basic set-theoretic vocabulary. Finally, suppose that the following stands for a sentence of natural language that is, or might be, true.

(1.12) Something does not $R$ itself.

Apply plural comprehension to the object language predicate ‘does not $R$ itself’ and obtain:

(1.13) If something does not $R$ itself, then there are some things such that anything is among them if and only if it does not $R$ itself.

From (1.12) and (1.13), it follows that

(1.14) There are some things such that anything is among them if and only if it does not $R$ itself.

Since plural quantifiers are read as first-order quantifiers, on the assumption that the singularist regimentation will respect the basic logical structure of the sentences to be regimented, (1.14) will be regimented as (1.15).

(1.15) $\exists x(\psi(x) \land \forall y(R(y, x) \leftrightarrow \neg R(x, x)))$.

The formula $\psi(x)$ is introduced to allow the singularist to restrict the quantifiers to the kinds of objects with which plurals are associated. For example, if plural quantification is quantification over sets, (1.15) becomes (1.16).
Chapter 1. Plurals and Singularism

(1.16) \( \exists x (\text{Set}(x) \land \forall y (R(y, x) \leftrightarrow \neg R(x, x))) \).

If the range of \( \forall y \) includes, or is the same as, the range of \( \exists x \), then (1.15) entails the contradictory Russell sentence:

(1.17) \( \exists x (R(x, x) \leftrightarrow \neg R(x, x)) \).

Thus, singularism has the problematic consequence of turning a sentence like (1.13), which follows from plural comprehension and (1.12), into a logically false sentence.

Second formulation

The paradox can also be formulated using plural definite descriptions. As in the previous version, let \( R(x, y) \) be the relation chosen by the singularist to regiment ‘being among’. Suppose, as before, that the relevant fragment of natural language contains a singular relation \( R \) that is also regimented by \( R(x, y) \). Finally, assume that \( R \) is not reflexive. That is,

(1.12) Something does not \( R \) itself.

Then, the plural definite description ‘the things that do not \( R \) themselves’ is non-empty and, therefore, it can be used to express true sentences. This follows from a version of plural comprehension involving definite descriptions. For any predicate \( \Phi \),

(1.18) If something is \( \Phi \), then the things that are \( \Phi \) exist.

Let \( F \) be any predicate that makes the following sentence true.

(1.19) The things that do not \( R \) themselves are \( F \).

A natural way for the singularist to regiment (1.19) is along the lines of (1.20).
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(1.20) \( \exists x (\psi(x) \land \forall y (R(y,x) \leftrightarrow \neg R(y,y)) \land F(x)).^5 \)

This entails a contradiction if the range of \( \forall y \) includes that of \( \exists x \). But we chose \( F \) so that (1.19), hence (1.20), would be true. So any form of singularism committed to representing (1.19) as (1.20) faces the paradox.

Discussion

In this section, I discuss the paradox in connection with both syntactic and semantic singularism. I will argue that, while the paradox affects some versions of the former, it is easily avoided by the latter. I will also show that the syntactic version of mereological singularism survives the paradox.

A close look at both versions of the paradox reveals three major assumptions:

(i) plural comprehension;

(ii) the existence in the object language of a relation \( R \) that is properly regimented as \( R(x,y) \), where \( R(x,y) \) is the relation that also symbolizes ‘being among’;

(iii) the relation \( R \), hence \( R(x,y) \), is non-reflexive.

Let us examine whether denying any of them is a viable option.

It must be admitted that instances of plural comprehension enjoy remarkable plausibility. So denying them appears too costly. Of course, the form of comprehension involving plural definite descriptions requires that the relevant predicate \( \Phi \) have a distributive reading in the consequent. Otherwise, one has counterexamples like the following, which uses the predicate ‘lifted the piano’.

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^5The restriction of the first quantifiers to \( \psi(x) \) has the same motivation as before.
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(1.21) If someone lifted the piano [alone], then the people that lifted the piano [together] exist.

To avoid misgivings, one could restate the principle forcing the distributive reading in the consequent.

(1.22) If something is Φ, then the things such that each of them is Φ exist.

This is a plausible principle and it is enough to generate the paradox.

The second major assumption of the paradox has to do with a potential feature of the language one wants to regiment. It might just be that such a language does not contain any relation \( R \) that would have to represented as \( R(x, y) \), i.e., the relation already chosen to represent ‘being among’. This would be the case, for example, if the set-theoretic singularist was dealing with a language that did not include a relation of set-theoretic membership.\(^6\)

However, there seems to be an independent reason to be uncomfortable with the second assumption. It is commonly assumed that ‘being among’ is a logical relation.\(^7\) One does justice to this fact by treating \( R(x, y) \) as logical in the regimenting language. Now, the existence of another relation \( R \) also represented by \( R(x, y) \) implies that \( R \) will have to count as logical too. But on some construals of syntactic singularism this is an unwanted consequence. A set-theoretic singularist would be forced to treat set-theoretic membership as a logical relation. A speaker of a plural language whose singular fragment includes set-theoretic predicates should not be committed to the logicality of membership, hence, to the logicality of set-theoretic principles.

\(^6\)We will see later that the limitation brought up by the paradox is deeper. On pain of paradox, the language cannot include any relation that is coextensive with that of set-theoretic membership.

\(^7\)See Yi (2005), pp. 487-488, for an argument in support of this assumption.
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Here is another example of the problematic consequences of assumption (ii) of the paradox. Something is self-identical. The following claim is a consequence of plural comprehension applied to the predicate ‘being self-identical’:

(1.23) Some things are such that everything that is among them is self-identical and everything that is self-identical is among them.

This will be regimented by an instance of (1.24).

(1.24) \( \exists x (\psi(x) \land \forall y (R(y, x) \leftrightarrow y = y)) \).

Now, (1.24) entails (1.25).

(1.25) \( \exists x R(x, x) \).

But (1.25) translates the sentence ‘something \( R \)s itself’. The problem lies in the fact that (1.25) follows from two logical principles: plural comprehension plus the claim that something is self-identical. Therefore, (1.25) and, as a consequence, the sentence ‘something \( R \)s itself’ would be logical truths as well. This means that the set-theoretic singularist would be committed, as a matter of logic, to accepting the sentence ‘something is a member of itself’. Notice that this is compatible with (and independent of) assumption (iii) of the paradox, namely, the assumption that \( R(x, y) \) is not a reflexive relation. Given the standard conception of sets, ‘something is a member of itself’ is problematic. If one needs to avoid the logicality of ‘something \( R \)s itself’ and wants to treat ‘being among’ as a logical relation, one has to reject assumption (ii) above.

How does a singularist proceed without the second assumption? Let us consider the case in which we are more interested, namely, that of the set-theoretic version
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of syntactic singularism. Abandoning assumption (ii) means that the relation of set-theoretic membership should not be part of the object language and, therefore, it does not appear in the comprehension formula. The relation appears only in the regimenting language. This imposes an expressive limitation that one might want to overcome. How can it be done? An obvious alternative is to enrich the object language with a new relation, membership* (∈*), and use it to express set theory by characterizing its behavior axiomatically. In the presence of plural comprehension, however, the relationship between ∈ and ∈* is constrained. In particular, one can prove that

(1.26) \( \exists x(x \notin^* x) \rightarrow \neg \forall x \forall y(x \in y \leftrightarrow x \in^* y) \).

Suppose that \( \exists x(x \notin^* x) \) and suppose that ∈ and ∈* are coextensive. By comprehension applied to the formula ‘\( x \notin^* x \)’, it follows that \( \exists x \forall y (y \in x \leftrightarrow y \notin^* y) \). If ∈ and ∈* are coextensive, then one can deduce \( \exists x \forall y (y \in x \leftrightarrow y \notin y) \), which is inconsistent. To sum up: if, as is plausible, ∈* is not reflexive, then ∈ and ∈* cannot be coextensive.

Is the option of introducing a distinct membership relation satisfactory? There are two potential concerns that are worth mentioning. First, since ∈* is not a relation in the original object language but an artificial surrogate of ∈, one might object that ∈* is not understood and conclude that its introduction in the object language is illegitimate. But the regimenting language comes — we are assuming — with a rigorous definition of the consequence relation. Therefore, the behavior of ∈* is, in a sense, fully characterized. Moreover, the new relation will be governed by axioms. This gives support to the idea that, just like in the case of ∈, we do have an understanding of ∈*.

However, there is a more serious concern that may not be dismissed so easily. In proving (1.26), we have seen that, from the fact that ∈* is not reflexive and from the
principle of comprehension applied to the formula ‘\( x \notin^* x \)’, it follows that

\[(1.27) \; \exists x \forall y (y \in x \leftrightarrow y \notin^* y).\]

A close look at (1.27) reveals a problem. One would like to use \( \in^* \) to formulate (a well-founded) set theory. So one might want to assume (1.28).

\[(1.28) \; \forall x (x \notin^* x).\]

From (1.27) and (1.28) one may deduce (1.29).

\[(1.29) \; \exists x \forall y (y \in x).\]

A particular consequence of (1.29) is (1.30).

\[(1.30) \; \exists x (x \in x).\]

But \( \in \) was supposed to express a standard membership relation. Forcing \( \in^* \) to be well-founded, has turned \( \in \) — via comprehension — into a non-well-founded relation. This consequence makes the introduction of a surrogate membership relation a very unappealing solution to the paradox for the set-theoretic singularist. Is there any other way for such a singularist to get around the paradox?

As we have seen above, the paradox requires that \( R \) be non-reflexive. This suggests an alternative response to the paradox. Instead of using the non-reflexive \( \in \) to interpret ‘being among’, the singularist might try to use \( \in \) instead. This is the relation being a member of or being identical to. Since this relation is reflexive, the paradox is avoided. However, given its consequences, there is reason to resist this move. Suppose that ‘being among’ is interpreted as \( \in \). Then, comprehension applied to ‘\( x \in x \)’ gives us (1.31).
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(1.31) \( \exists x \ (x \subseteq x) \rightarrow \exists y \ \forall x \ (x \subseteq y \leftrightarrow x \subseteq x) \).

Since \( x \subseteq x \) holds of everything in the domain of discourse, (1.31) entails that there is some \( y \) such that everything other than \( x \) is in \( x \).

(1.32) \( \exists y \ \forall x \ (x \neq y \rightarrow x \in y) \).

The problem is that this is inconsistent with basic set-theoretic principles that may characterize the behavior of \( \in \). For example, the combination of pairing, union, and foundation is inconsistent with (1.32). By pairing, \( \{ a \} \) exists, where \( a \) is the witness of the existential quantifier in (1.32). Then, by union, we have that \( a \cup \{ a \} \) exists. But, by (1.32), if \( a \cup \{ a \} \neq a \), \( a \cup \{ a \} \in a \), which entails that \( a \in a \cup \{ a \} \in a \) and, therefore, violates foundation. So it must be that \( a \cup \{ a \} = a \), in which case, again, \( a \in a \).

Of course, the syntactic singularist who wanted to adopt this solution could resort to an alternative set theory allowing for a universal set. But being forced to abandon more familiar set theories should be regarded as too costly. Could \( a \), the witness of the existential quantifier in (1.32), be a proper class? Since by (1.32) everything other than \( a \) is in \( a \), there could be no proper class other than \( a \). This would prevent us from resorting to familiar class theories as well. Therefore, this solution to the paradox is unsatisfactory. We must conclude that the paradox of plurality casts serious doubts on the prospects of the set-theoretic version of syntactic singularism. How do other approaches, such as mereologic singularism or semantic singularism, fare with respect to the paradox?

I will present the mereological version of syntactic singularism in more detail below but a simple remark shows why this approach is not affected by the paradox. As we have seen, the paradox of plurality does not apply to syntactic versions of singularism.
that regiment ‘being among’ as a non-reflexive relation. The mereological singularist will regiment ‘being among’ as ‘being part of’, which is a reflexive relation. Later, I will discuss an attempt to produce a paradox of plurality for mereological singularism and I will conclude that is appears unsuccessful.

Finally, it should not be difficult to convince oneself that the paradox of plurality poses no problem for semantic versions of singularism. Semantic singularism, as I have defined it, relies on $L_{PL}$ for regimentation. So ‘being among’ is represented as $\prec$, which is a relation between a singular and a plural argument. There is no purely singular relation $R$ that is appropriately represented as $\prec$. Moreover, the distinction between singular and plural quantifiers is respected in $L_{PL}$. This blocks the paradox. Indeed, the paradox requires a collapse of plural quantifiers into singular ones. As will become clearer below, sentences (1.8) and (1.9) are not equivalent for the semantic singularist.

(1.8) There are some sets such that any set is one of them if and only if it is not self-membered.

(1.9) There is a set $x$ such that for every set $y$, $y$ is in $x$ if and only if $y$ is not self-membered.

Semantic singularism validates a very intuitive form of comprehension without danger of paradox. This is one more point in its favor.
1.1.3 Mereological version

In this section, I present a mereological version of syntactic singularism. The system presented here is due to Godehard Link.\textsuperscript{8} I call the system $\mathcal{L}_M$. For the purposes of our discussion some of its complications are immaterial, so Link’s original account will be simplified. Also, for the sake of clarity and uniformity, I will modify his original notation. Link aims to provide a unified account of plurals and mass terms (e.g., ‘gold’ or ‘water’); I will focus on plurals, neglecting mass terms.

In addition to the standard language of first-order logic with identity, $\mathcal{L}_M$ has a distinguished existence predicate $E$, a distinguished individual part relation $\leq$, a term-forming operator $\sigma$ representing the operation of individual fusion, a predicate prefix operator $\ast$ marking plural distributivity, and a definite description operator $\iota$. So, if $\phi(x)$ is a formula with only $x$ occurring free, then $\sigma x \phi(x)$ and $\iota x \phi(x)$ are terms.

If $P$ is an atomic predicate, such as ‘is a student’, $\ast P$ regiments the distributive plural reading of $P$, i.e., ‘are students’. The relation of ‘being one of’ or ‘being among’ is represented by the mereological relation $\leq$. We can also have a term-forming operator $\oplus$ corresponding to binary fusion, which will regiment phrasal conjunction.

A first question that arises is the following. What is the relation between the newly introduced notions of fusion ($\sigma$) and parthood ($\leq$), and the ordinary notions of mereological fusion and mereological parthood? For reasons that will become clear in due course, the two sets of notions have to be distinguished. Link suggests a distinction between individual fusion and individual parthood on the one hand, and material fusion and material parthood on the other hand. What seems to motivate

\textsuperscript{8}This approach to plurals was first proposed in Link (1983) and then developed in successive publications collected in Link (1998).
the distinction are primarily semantic needs, i.e., the needs of providing a singularist treatment of plurals. Take two material objects, say a car (named “Car”) and a bike (named “Bike”). The material fusion of Car and Bike is the material object, call it “Car-plus-Bike”, such that anything overlaps Car-plus-Bike if and only if it overlaps Car or Bike. However, this mereological relation cannot model the relation of ‘being one of’. Detailed arguments will be formulated below. What is needed in connection with plurals is a mereological relation that allows Car and Bike to form an object that has them as its only parts. Such a relation treats Car and Bike as basic mereological individuals. Clearly, given their complex material structure, Car and Bike are not basic with respect to material parthood. A rigorous characterization of notions of individual fusion and of individual parthood will be provided by the model theory that accompanies $\mathcal{L}_M$. Before outlining the model theory for this language, let us give some examples of regimentation in $\mathcal{L}_M$ to get a feeling for how the language works.

Phrasal conjunction is paraphrased in terms of fusion. Thus, ‘Russell and Whithead’ becomes ‘Russell $\oplus$ Whitehead’. The operation can be iterated:

(1.33) (a) Tom, Dick, and Harry are lawyers.
       (b) $^*L((t \oplus d) \oplus h)$

Plural definite descriptions are paraphrased in terms of fusions too. In this case, the fusion is built from an open formula. So, ‘$z$ is a boy’ gives rise to ‘the fusion of every $z$ such that $z$ is a boy’, as shown by the next example.

(1.34) (a) The boys surrounded the tent.
       (b) $S(\sigma zBz, tyTy)$
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The relation of ‘being one of’ is paraphrased by the singular relation of individual parthood (≤). The definite description ‘the tent’ is captured by the the ι-operator. This is illustrated by the following example.

(1.35) (a) Tom is one of the winners.
(b) \( t \leq \sigma x W x \)

The basic idea behind the model theory for \( \mathcal{L}_M \) is to associate the relevant mereological notions with a suitably structured domain. It turns out that a familiar algebraic structure works well in this context. Let us take a closer look.

A model \( M \) of \( \mathcal{L}_M \) is a quadruple \( (D, \leq, A, I) \) where \( D \), functioning as domain, is a complete atomic Boolean algebra with respect to \( \leq \), \( A \subseteq D \) is the set of atoms of \( D \), and \( I \) is an interpretation function.\(^9\) The interpretation function extends, or

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\(^9\) Let us define the notion of a complete atomic Boolean algebra. We need several auxiliary notions. Let \( S \) be a non-empty set and \( \leq \) a partial order on \( S \), i.e. a reflexive, antisymmetric, and transitive relation on \( S \). The following notions are defined with respect to \( \leq \). For any set \( X \subseteq S \), \( a \in S \) is a supremum (infimum) of \( X \) in \( S \) iff \( a \leq y \ (y \leq a) \) for every \( y \in S \) such that, for every \( x \in X \), \( x \leq y \ (y \leq x) \). I will often suppress the explicit reference to the set (usually \( S \)) in which the supremum (infimum) is taken. It follows that, if the supremum (infimum) of \( X \) exists, it is unique. The pair \( (S, \leq) \) is a lattice iff for any \( x, y \in S \), both the supremum and the infimum of \( \{x, y\} \) exist. If for any \( x, y \in S \) at least the supremum (infimum) of \( \{x, y\} \) exists, \( (S, \leq) \) is called a join (meet) semilattice. By \( 0 \) and \( 1 \) we denote, respectively, the infimum and the supremum of \( S \), if they exist. In a lattice the relation \( \leq \) induces two operations \( \sqcup, \sqcap : S \times S \to S \) such that, for every \( x, y \in S \), \( x \sqcup y \) is the supremum of \( \{x, y\} \) and \( x \sqcap y \) is the infimum of \( \{x, y\} \). A lattice \( (S, \leq) \) is called complete iff any non-empty \( X \subseteq S \) has both a supremum and an infimum. Also, it is called distributive iff, for every \( x, y, z \in S \), \( x \sqcup (y \sqcap z) = (x \sqcup y) \sqcap (x \sqcup z) \). If \( 0, 1 \in S \), then for any \( x, y \in S \), we say that \( y \) is the complement of \( x \) (denoted by \( \bar{x} \)) iff \( x \sqcap y = 0 \) and \( x \sqcup y = 1 \). In a distributive lattice with \( 0 \) and \( 1 \), the complement of any element is unique if it exists. A lattice \( (S, \leq) \) is called complemented iff,
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constrains, the usual first-order interpretation function in the following ways. For the sake of simplicity, I will be concerned only with the conditions on \( I \) with respect to atomic unary predicates. For any arbitrary predicate \( P \),

(a) the interpretation of \( P \) is a subset of the domain not containing the null element:

\[
I(P) \subseteq (D - \{0\});
\]

(b) if \( P \) is not a collective predicate, the interpretation of \( P \) is a subset of the atoms of the domain and the interpretation of \( *P \), the plural distributive form of \( P \), is the complete join subsemilattice\(^\text{10} \) generated by \( I(P) \) in \( D \), that is,

\[
\begin{align*}
(b1) & \quad I(P) \subseteq A, \\
(b2) & \quad I(*P) = \{x \in D : \text{there is a non-empty } X \subseteq I(P) \text{ such that } x \\
& \quad \text{is the supremum of } X \text{ in } S\};
\end{align*}
\]

for any \( x \in S, \bar{x} \) exists. Thus, in a complemented lattice \( \neg : S \to S \) is an operation. Finally, we define a Boolean algebra to be a lattice \( (S, \leq) \) with \( \mathbf{0} \) and \( \mathbf{1}, 0 \neq 1 \), that is both distributive and complemented. Any element \( a \) of a Boolean algebra \( (S, \leq) \) is an atom iff, for every \( x \in S - \{0\}, x \leq a \) implies \( x = a \). A Boolean algebra is atomic iff, for every \( x \in S - \{0\} \), there is an atom \( a \) such that \( a \leq x \). To summarize, let \( (S, \leq) \) be a lattice with \( \mathbf{0} \) and \( \mathbf{1}, 0 \neq 1 \). Then \( (S, \leq) \) is a complete atomic Boolean algebra if it is distributive and complemented (plus, of course, complete and atomic).

\(^{10}\)Let \( (S, \leq) \) be a join (meet) semilattice. Then \( (S, \leq) \) is said to be a complete join (meet) semilattice if for any non-empty \( X \subseteq S \), the supremum (infimum) of \( X \) in \( S \) exists. A non-empty \( X \subseteq S \) is a join (meet) subsemilattice of \( S \) with respect to \( \leq \) iff for any \( x, y \in X \), the supremum (infimum) of \( \{x, y\} \) in \( S \) is also in \( X \). Moreover, a join (meet) subsemilattice of \( S, X \), is called complete iff for every non-empty \( Y \subseteq X \), the supremum (infimum) of \( Y \) in \( S \) is also in \( X \). Finally, for \( X \subseteq S \) we define the complete join (meet) subsemilattice generated by \( X \) in \( S \) to be the intersection of all the complete join (meet) subsemilattices \( Y \) of \( S \) such that \( X \subseteq Y \) (i.e. \( \bigcap \{Y : Y \text{ is a complete join (meet) subsemilattice of } S \land X \subseteq Y\} \)). This can be shown to be identical to the type of set specified in condition (b2).
(c) if the interpretation of $P$ is a singleton and has a non-null element in its extension, say $\{d\}$, then the interpretation of the definite description $\iota xPx$ is $d$; otherwise the interpretation of the description denotes is the null element, i.e., $I(\iota xPx) = \emptyset$;

(d) the interpretation of a fusion with respect to $P$ is the supremum of the elements of $I(P)$: $I(\sigma xPx)$ is the supremum of $I(P)$, hence, it is $0$ if $I(P)$ is empty;

(e) for $\alpha$ and $\beta$ terms, $I(\alpha \oplus \beta)$ is the supremum of $\{I(\alpha), I(\beta)\}$ in $D$.

As usual, satisfaction ($\models$) is defined as a relation between a model, a formula, and a variable assignment from the domain of the model. The recursive clauses defining satisfaction are the familiar ones from first-order logic. In addition, we have clauses characterizing satisfaction for the existence predicate and for the individual parthood relation. Let $M$ be a model and let $s$ be a variable assignment from $D$, the domain of $M$. Then,

(S$E$) For any term $\alpha$, $M \models E\alpha [s]$ iff $I(\alpha) \neq 0$, the infimum of $D$.

(S$\leq$) For any terms $\alpha$ and $\beta$, $M \models \alpha \leq \beta [s]$ iff $I(\alpha) \leq I(\beta)$.

For simplicity, I discussed only the case of unary predicates but, once the unary case has been dealt with, extending the conditions on the interpretation function to $n$-ary predicates is relatively straightforward. In the $n$-ary case, however, the distributivity operator $\ast$ has to be relativized to specific argument places. For example, when trying to model the relation of loving in ‘Annie and Bonnie love Tom’, the distributivity operator must be associated only with the first argument place. So one could index the operator to indicate the argument places to which it applies. Thus, ‘Annie and Bonnie love Tom’ could be represented as $L^{\ast_1}(a \oplus b, t)$. In the discussion below, I will
leave the relativization unmarked. The context will make clear how the distributivity operator has to be relativized.

Now that the model-theoretic apparatus is in place, one can show how some valid inferences involving plurals are accounted for in this semantics. Here are some examples from Massey (1976) and Schein (1993).

Contraction, expansion, and symmetry come out valid in $\mathcal{L}_M$. As we saw before, contraction and expansion are inferences of the following kind.\footnote{See p. 15.} To the right of the natural language inference, a formalization in $\mathcal{L}_M$ is provided.

\begin{align}
(1.36) & \quad \text{Tom, Dick, and Harry are linguists} & *L((t \oplus d) \oplus h) \\
& \quad \text{Tom and Harry are linguists} & *L(t \oplus h) \\
& \quad \text{Dick and Harry are lawyers} & *L(d \oplus h) \\
(1.37) & \quad \text{Tom is a lawyer} & L(t) \\
& \quad \text{Dick, Harry, and Tom are lawyers} & *L((d \oplus h) \oplus t)
\end{align}

Symmetry is illustrated by the next example.

\begin{align}
(1.38) & \quad \text{Tom and Dick love Mary} & *L(t \oplus d, m) \\
& \quad \text{Dick and Tom love Mary} & *L(d \oplus t, m)
\end{align}

This inference from Schein (1993) comes out valid too.

\begin{align}
(1.39) & \quad \text{Every one of the elms is tall} & \forall x (x \leq \sigma xEx \rightarrow Tx) \\
& \quad \text{Every elm is tall} & \forall x (Ex \rightarrow Tx)
\end{align}

Moreover, if atomicity is expressed by the formula $\neg \exists y(y \neq x \land y \leq x)$, abbreviated by $\text{At}(x)$, the following versions of Plural Comprehension are validated in $\mathcal{L}_M$:
(1.40) $\exists x\phi(x) \rightarrow \textbf{E}x\phi(x)$

(1.41) $(\exists x\phi(x) \land \forall x(\phi(x) \rightarrow \textbf{At}(x))) \rightarrow \exists x\forall y((y \leq x \land \textbf{At}(y)) \leftrightarrow \phi(y))$

Let me briefly comment on these examples. There is a lattice-theoretic result that underwrites the validity of most of these inferences. It can be proved that, if $D$ is a complete atomic Boolean algebra and $I(*P)$ is the complete join subsemilattice generated in $D$ by a subset $I(P)$ of $A$ (recall that $A$ is the set of atoms of $D$), then, for every $x \in D - \{0\}$, if $x \leq y$ and $y \in I(*P)$, $x \in I(*P)$.\footnote{We prove the result by first proving two lemmas.}

**Lemma 1.** Let $D$ be an atomic Boolean algebra and let $A$ be the sets of its atoms. For every $x \in D$ define $A(x) := \{d \in D : d \in A$ and $d \leq x\}$. Then $x$ is the supremum of $A(x)$.

**Proof.** Since $D$ is atomic, $A(x)$ is non-empty. By construction, for every $y \in A(x)$, $y \leq x$. Assume that $z \in D$ and $y \leq z$ for every $y \in A(x)$. We need to show that $x \leq z$. Suppose not, i.e. $x \nless z$. If $x \cap \overline{z} = 0$, then using distributivity $z = z \cup 0 = z \cup (x \cap \overline{z}) = (z \cup x) \cap (z \cup \overline{z}) = (z \cup x) \cap 1 = z \cup x$ hence $x \leq z$. So it must be that $x \cap \overline{z} \neq 0$. Since $D$ is atomic, there is an atom $a \leq x \cap \overline{z}$. It follows that $a \leq x$, so $a \in A(x)$ and also $a \leq z$. Now, in general, for every $\alpha, \beta, \gamma \in D$, if $\alpha \leq \beta$ and $\alpha \leq \gamma$, $\alpha \leq \beta \cap \gamma$. Therefore, given that $a \leq z$ and $a \leq x \cap \overline{z}$, we have that $a \leq z \cap (x \cap \overline{z}) = x \cap (z \cap \overline{z}) = z \cap 0 = 0$. Thus $a = 0$, contradiction since $a$ is an atom. So, $x \leq z$ and we can conclude that $x$ is the supremum of $A(x)$. QED

**Lemma 2.** Let $D$ be a complete Boolean algebra with $X \subseteq D$. For any $d \in D$, define $d \cap X = \{d \cap x : x \in X\}$ and let $\sigma$ be the supremum of $X$. Then $d \cap \sigma$ is the supremum of $d \cap X$.

**Proof.** The completeness of $D$ is needed to guarantee the existence of $\sigma$. Now, in general, for every $\alpha, \beta, \gamma \in D$, if $\beta \leq \gamma$, then $\alpha \cap \beta \leq \alpha \cap \gamma$ and $\alpha \cup \beta \leq \alpha \cup \gamma$. Hence, in particular, if $\beta \leq \gamma$, then $\alpha \cap \beta \leq \gamma$ and $\beta \leq \alpha \cup \gamma$. Fix $d$. If $y \in d \cap X$, there is $x \in X$ such that $y = d \cap x$. Since $\sigma$ is the supremum of $X$, $x \leq \sigma$. Therefore, $d \cap x \leq d \cap \sigma$. So $y \leq d \cap \sigma$ for every $y \in d \cap X$. Suppose that $y \leq z$ for every $y \in d \cap X$. We need to show that $d \cap \sigma \leq z$. Let $y = d \cap x$ be in $d \cap X$. Then $d \cap x \leq z$. Also, the following hold:
Consider symmetry. Its validity results from the fact that \( I(t \oplus d) \) and \( I(d \oplus t) \) are both defined as the supremum of \( \{I(t), I(d)\} \); thus, if one is in the extension of \( I(L(x, m)) \), then so is the other.

In the case of contraction, the truth of the premise gives us that \( I((t \oplus d) \oplus h) \in I(\star L) \). We are inheriting the assumption of standard first-order logic that individual constants (e.g., \( t, d, \) and \( h \)) are denoting expressions. This is captured by the requirement that individual constants denote an atom of the domain. So \( I(t), I(d), \) and \( I(h) \) are all non-zero elements of the domain. Therefore, \( I(d \oplus h) \) is non-zero. Moreover, \( I(d \oplus h) \leq I(d \oplus t \oplus h) \), since \( I(t \oplus d \oplus h) \) is the supremum of \( \{I(t), I(d), I(h)\} \) and \( I(t \oplus h) \) is the supremum of \( \{I(t), I(h)\} \). It follows from the lattice-theoretic result

\[
\begin{align*}
1) & \quad x \leq \bar{d} \sqcup x = 1 \cap (\bar{d} \sqcup x) = (\bar{d} \sqcup d) \cap (\bar{d} \sqcup x) = \bar{d} \sqcup (d \cap x) \\
2) & \quad d \cap x \leq z \quad \Rightarrow \quad \bar{d} \sqcup (d \cap x) \leq \bar{d} \sqcup z
\end{align*}
\]

From \( d \cap x \leq z \), (1), and (2), it follows that \( x \leq \bar{d} \sqcup z \), for every \( x \in X \). Since \( \sigma \) is the supremum of \( X \), we get \( \sigma \leq \bar{d} \sqcup z \). So \( d \cap \sigma \leq d \cap (\bar{d} \sqcup z) = (d \cap \bar{d}) \sqcup (d \cap z) = \emptyset \sqcup (d \cap z) = (d \cap z) \leq z \). Thus, \( d \cap \sigma \leq z \), hence \( d \cap \sigma \) is the supremum of \( d \cap X \). QED

**Theorem.** Let \( D \) be a complete atomic Boolean algebra with \( A \) the set of its atoms. Let \( G \subseteq A \) and let \( \phi(G) \) be the complete join semilattice generated by \( G \) in \( D \). For any \( x, y \in D \), if \( y \in \phi(G) \) and \( 0 \neq x \leq y \), then \( x \in \phi(G) \).

**Proof.** Using the notation introduced in the statement of the theorem, we notice that because \( D \) is atomic, \( A(x) \) is non-empty. Let \( a \in A(x) \). Since \( y \in \phi(G) \), \( y \) is the supremum of some non-empty set \( X \subseteq G \). So, \( a \leq x \leq y \). Therefore, \( a = a \cap y \). By Lemma 2 and the fact that \( y \) is the supremum of \( X \), it then follows that \( a \) is the supremum of \( a \cap X \). The elements of \( X \) are atoms, so if \( a \notin X \), \( a \cap x = 0 \) for every \( x \in X \), hence \( a \cap X = \{0\} \). Since \( a \) is the supremum of \( a \cap X \), this entails that \( a = 0 \), contradiction given that \( a \) is an atom. Therefore, \( a \in X \) for every \( a \in A(x) \). So \( A(x) \subseteq X \subseteq G \), with \( A(x) \) non-empty. By Lemma 1, \( x \) is the supremum of \( A(x) \), thus \( x \) is the supremum of a non-empty subset of \( G \). This means that \( x \in \phi(G) \). QED
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mentioned above that \( I(t \oplus h) \in I(^*L) \), given that \( I(t \oplus d \oplus h) \in I(^*L) \).

As for expansion, the truth of the premises give us that \( I(d \oplus h) \in I(^*L) \) and \( I(t) \in I(L) \). From the fact that \( I(d \oplus h) \in I(^*L) \), it follows, in particular, that \( I(d), I(h) \in I(L) \). Since \( I(d \oplus h \oplus t) \) is the supremum of \( \{ I(d), I(h), I(t) \} \subseteq I(L) \), the clause (b2) about interpretations (p. 30) implies that \( I((d \oplus h) \oplus t) \in I(^*L) \). Thus, the conclusion is true as well.

Let us now turn to example (1.39).

\[
\text{(1.39) Every one of the elms is tall} \quad \forall x (x \leq \sigma x E x \rightarrow T x)
\]

\[
\text{Every elm is tall} \quad \forall x (E x \rightarrow T x)
\]

In any model \( M \) in which the premise is true, one must have that

\[
\{ d \in D : d \leq \text{the supremum of } I(E) \} \subseteq I(T).
\]

Suppose that \( x \in I(E) \). Then, by definition of supremum,

\[
x \leq \text{the supremum of } I(E).
\]

From the truth of the premise, it follows that \( x \in I(T) \). So, if \( x \in I(E) \), \( x \in I(T) \). This means that the conclusion is true in \( M \). The inference is therefore valid.

Recall the two versions of Plural Comprehension mentioned above:

\[
\text{(1.40) } \exists x \phi(x) \rightarrow E \sigma x \phi(x)
\]

\[
\text{(1.41) } (\exists x \phi(x) \land \forall x (\phi(x) \rightarrow A t(x))) \rightarrow \exists x \forall y ((y \leq x \land A t(y)) \leftrightarrow \phi(y))
\]

The first version asserts, model-theoretically, that in any interpretation, for any formula \( \phi(x) \), if \( I(\phi(x)) \) is non-empty, then there is a non-zero supremum of \( I(\phi(x)) \) in the domain. This is guaranteed by the fact that the domain is a complete lattice,
so every subset of the domain has a supremum. The second version asserts that, if $\emptyset \neq I(\phi(x)) \subseteq A$, then there is an object $x$ in the domain — the supremum of $I(\phi(x))$ — such that, for any $a, a \in I(\phi(x))$ just in case $a$ is an atom and $a \leq x$.\footnote{To prove this, one can appeal to the Theorem in the previous footnote. Suppose that $x$ is the supremum of $I(\phi(x)) \subseteq A$. Then, clearly, for any $y \in I(\phi(x))$, $y \leq x$. On the other hand, suppose that $y$ is an atom and $y \leq x$. Let $\lambda(I(\phi(x)))$ be the complete join semilattice generated by $I(\phi(x))$ in the domain of the model. Then, it follows from the Theorem that $y \in \lambda(I(\phi(x)))$. Since $y$ is also an atom, $y \in I(\phi(x))$.}

Later, I will discuss alternative formulations of Plural Comprehension in the $\mathcal{L}_M$ framework, as they play a role in connection with some objections to mereological singularism. The requirement is that there be adequate ways of capturing the informal principle Plural Comprehension in the system under discussion. Two plausible candidates have just been discussed.

It is worth pausing for a moment to reflect about an important feature of $\mathcal{L}_M$. This will give us a useful indication when, later, we will attempt to diagnose what goes wrong with this mereological system. It is an essential feature of $\mathcal{L}_M$ that natural language plural quantifiers are rendered as singular quantifiers. Thus, the distinction found in natural language between two distinct types of quantification collapses. The singular quantifier of $\mathcal{L}_M$ is taken to range, in each model, over the non-zero elements of the domain $D$ and not just the atoms.\footnote{Link (1998), p. 26.} As a consequence, the system appears to validate inferences like the following.\footnote{This inference is discussed in McKay (2006), p. 23.}

\begin{align*}
\text{(1.42) Some students are surrounding a building} \\
\text{Something is surrounding a building}
\end{align*}

Formally:

\footnotetext[13]{To prove this, one can appeal to the Theorem in the previous footnote. Suppose that $x$ is the supremum of $I(\phi(x)) \subseteq A$. Then, clearly, for any $y \in I(\phi(x))$, $y \leq x$. On the other hand, suppose that $y$ is an atom and $y \leq x$. Let $\lambda(I(\phi(x)))$ be the complete join semilattice generated by $I(\phi(x))$ in the domain of the model. Then, it follows from the Theorem that $y \in \lambda(I(\phi(x)))$. Since $y$ is also an atom, $y \in I(\phi(x))$.}

\footnotetext[14]{Link (1998), p. 26.}

\footnotetext[15]{This inference is discussed in McKay (2006), p. 23.}
∃x∃y (∗S(x) ∧ B(y) ∧ Sr(x,y))

∃x∃y (B(y) ∧ Sr(x,y))

But this is problematic, since (1.42) does not appear to be logically (or even analyti-
cally) valid. One would have to find a way to avoid the commitment to the validity
of inferences of this kind. For example, one could use the definable predicate At(x)
to signal that the quantification is genuinely singular and require that the inference
be regimented as in (1.43).

(1.43)  

∃x∃y (∗S(x) ∧ B(y) ∧ Sr(x,y))

∃x∃y (At(x) ∧ B(y) ∧ Sr(x,y))

So the conclusion would be interpreted as ‘there is an atom surrounding the building’.
Since (1.43) is invalid in $L_M$, the difficulty is avoided. However, this move is not en-
tirely satisfactory unless it is imposed systematically. This would, in effect, introduce
a way of distinguishing two types of quantification, corresponding to those found in
natural language. But, then, it is not clear why one should not simply introduce a
second type of quantifier serving to represent natural language plural quantification.
In light of further problems for mereological singularism to be discussed below, this
option will become even more appealing. It will ultimately lead us to reject syntactic
singularism, expand the regimenting language to include plural quantifiers and plural
variables, and adopt syntactic pluralism. Once the distinction between singular and
plural quantifiers is explicitly marked, and once the semantics has been appropriately
formulated, there is no risk of validating inferences like (1.42).

Since the paradox of plurality does not apply to versions of syntactic singularism
that regiment ‘being among’ as a reflexive relation, mereological singularism survives
it. Before considering the main objections raised against the mereological version of
syntactic singularism, I would like to comment very briefly on Schein’s attempt, in
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Schein (1993), to formulate a version of the paradox of plurality against mereological singularism.

As noticed in section 1.1.2, the scheme of Plural Comprehension in English is as follows.

(1.10) If something is \( \Phi \), then there are some things such that anything is among them if and only if it is \( \Phi \).

A straightforward mereological rendering of this principle, obtained by simply replacing the plural quantifiers with singular ones and ‘being among’ with parthood, is not validated in \( L_M \). Indeed, the schema:

(1.44) \( \exists x \phi(x) \rightarrow \exists x \forall y (y \leq x \leftrightarrow \phi(y)) \),

is false in some model of \( L_M \).\(^{16}\)

Schein attempts to formulate a paradox using a principle of plural comprehension that is more complex than (1.44). He appeals to (1.45).

(1.45) \( \exists x \phi(x) \rightarrow \exists x \forall y ((\text{At}(y) \land y \leq x) \leftrightarrow \phi(y)) \).

He instantiates this scheme with the formula \( \neg((\text{At}(x) \land x \leq x) \rightarrow \phi(y)) \) to get the paradox. However, as pointed out by Link, schema (1.45) fails in \( L_M \).\(^{17}\) Another possible version of the plural comprehension schema that is equally unacceptable in \( L_M \) is:

\(^{16}\)Consider the model \( M \) with a domain of four distinct element \( D = \{0, a, b, 1\} \), where the set of atoms \( A \) is \( \{a, b\} \). Let \( \phi(x) \) be \( x = \sigma y(y = y) \). Then, the extension of \( \phi(x) \) is \( \{1\} \) and the schema fails since no element witnesses the existential quantifiers in the consequent. For example, \( 1 \) does not because \( a \leq 1 \) but \( a \notin I(\phi(x)) \).

\(^{17}\)See Link (1998), p. 323. The model constructed in the previous footnote shows this as well.
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(1.46) \((\exists x \phi(x) \land \forall x (\phi(x) \rightarrow \text{At}(x))) \rightarrow \exists x \forall y (y \leq x \leftrightarrow \phi(y))\).^{18}

Earlier, we discussed at least two ways in which plural comprehension can be formulated in \(L_M\).\(^19\) So, to avoid the threat of paradox, the mereological singularist does not need to reject plural comprehension altogether. She just has to exercise some care in selecting which principles to invoke. Of course, an opponent of mereological singularism might try to argue that the formulations of plural comprehension validated in \(L_M\) are inadequate. However, crucial evidence in favor of or against a given principle of plural comprehension comes from its role in accounting for entailment relations among plural sentences. Schein (1993) offers two examples of inferences that, he argues, require suitable principle of plural comprehension.\(^20\)

(1.47) Elm \(a\) exists and elm \(b\) exists  
The elms, \(a\) and \(b\), exist

(1.48) There is an elm  
The elms exist

The principle of plural comprehension available in \(L_M\), (1.40) and (1.41), seem to capture adequately these inferences. So, pending a more thorough examination of the role of plural comprehension in accounting for natural language patterns of valid inferences, it is not clear that the mereological singularist is forced to adopt a paradoxical principle of plural comprehension.

\(^{18}\)That this schema fails can be shown by considering a model with three atoms \((A = \{a, b, c\})\), taking \(\phi(x)\) to be \(\text{At}(x)\).

\(^{19}\)Section 1.1.3, schemas (1.40) and (1.41).

\(^{20}\)Schein (1993), chapter 2, section 3.2.
1.1.4 Problems for the mereological version

As we have seen, the mereological version of syntactic singularism escapes the paradox of plurality. However, a number of objections have been raised against it. In this section, I will present and evaluate the main ones.\(^{21}\)

Oliver and Smiley (2001) argue as follows. Suppose that ‘Russell and Whitehead’ is rendered by ‘the mereological sum of Russell and Whitehead’. Moreover, it seems plausible to suppose that the mereological sum of Russell and Whitehead is the same as the mereological sum of molecules of Russell and Whitehead. Then, on their view, the plural phrase ‘the molecules of Russell and Whitehead’ should also be rendered by ‘the mereological sum of the molecules of Russell and Whitehead’. It seems that the mereological singularist is then committed to accepting the validity of the following inference.\(^{22}\)

\[
(1.49) \quad \text{Russell and Whitehead were logicians} \quad \quad \quad \text{The molecules of Russell and Whitehead were logicians}
\]

In \(L_M\), the fact that the mereological sum of Russell and Whitehead is the same as the mereological sum of molecules of Russell and Whitehead might be expressed by identity (1.50).

\[
(1.50) \quad (r \oplus w) = \sigma x(M(r, x) \lor M(w, x)).
\]

Adding (1.50) to the premise of (1.49) results in a valid inference in virtue of properties of the identity relation:

\(^{21}\)For an extensive discussion of the prospects of the mereological approach see Nicolas (unpublished). A large-scale defense of mereological singularism is found in Link (1998).

\(^{22}\)Oliver and Smiley (2001), p. 293.
The mereological singularist might reply that the argument is based on the conflation of two distinct senses of ‘mereological sum’. There is an ordinary sense that supports the claim that the mereological sum of Russell and Whitehead is the same as the mereological sum of molecules of Russell and Whitehead. However, there is a technical sense of ‘mereological sum’, namely, the sense in which this notion is used within the system $\mathcal{L}_M$. The distinction between these two senses is made apparent by the fact that, in $\mathcal{L}_M$, (1.50) entails that there are only two distinct molecules of Russell and Whitehead (i.e., Russell and Whitehead!). This shows that neither the $\sigma$ operator nor the $\oplus$ operator capture the ordinary sense of ‘mereological sum’. It is true ‘Russell and Whitehead’ is rendered in the system by ‘the mereological sum of Russell and Whitehead’, but this expression is understood in the technical sense. It is also true that ‘the molecules of Russell and Whitehead’ would be rendered by ‘the mereological sum of the molecules of Russell and Whitehead’ in the technical sense of ‘sum’. On the other hand, when we assume the metaphysical fact that the mereological sum of Russell and Whitehead is the same as the mereological sum of the molecules of Russell and Whitehead, we are using the ordinary notion of sum. Therefore, we cannot conclude that the mereological singularist is committed to (1.50) on metaphysical grounds, since identity (1.50) does not capture the ordinary metaphysical notion of sum. Ordinary metaphysical facts about sums have to be expressed differently, by appealing to some relation of parthood other than $\leq$.\footnote{This response to the argument is in line with Link’s distinction between a relation of individual parthood and one of material parthood. See Link (1998), pp. 16-19, 22-27.}
A similar argument against mereological singularism has been suggested by Rayo (2002). Imagine that we have some sand and that the grains of sand are grouped into piles. There is a possible scenario in which both of the following sentences are true.

(1.52) The piles of sand are scattered.

(1.53) The grains of sand are not scattered.

However, the grains of sand may be taken to form the same mereological sum as the piles of sand. On this assumption, it is argued that mereological singularism has to assign the same denotation to ‘the piles of sand’ and ‘the grains of sand’, as they will be rendered by ‘the mereological sum of the piles of sand’ and ‘the mereological sum of the grains of sand’. Therefore, (1.52) and (1.53) would be inconsistent, contrary to our intuition that there is a possible scenario in which they are both true.\(^{24}\)

A response similar to the one employed above is also available here. The mereological singularist may argue that the regimentation in \(L_M\) of the claim that the grains of sand form the same mereological sum as the piles of sand cannot use the technical notions expressed by \(\leq\), \(\oplus\), and \(\sigma\). Suppose that one formalizes the claim that the grains of sand form the same mereological sum as the piles of sand as in (1.54).

\[
(1.54) \quad \sigma x(Gx \land Sx) = \sigma x(Px \land Sx).
\]

Since the predicates involved are not collective, their extensions in every model are subsets of the set of atoms.\(^{25}\) Therefore, (1.54) entails that every grain of sand is a pile of sand and vice versa. This means there would have to be as many grains as there are


\(^{25}\)See condition (b) on interpretation in section 1.1.3.
piles of sand. Thus, the formal rendering (1.54) does not capture the metaphysical thought that the grains of sand form the same mereological sum as the piles of sand. The diagnosis is the same as in the previous objection. The mereological relation $\leq$ and the sum operation $\sigma$ found in the system do not express the same mereological relations expressed in a sentence like ‘the grains of sand form the same mereological sum as the piles of sand’.

There is another argument, inspired by Schein (1993) and discussed in Nicolas (unpublished), that is worth considering here. A version of the argument goes as follows. In the mereological framework, whatever the domain, plural definite descriptions such as ‘the atoms’ would get the same denotation as the description ‘the non-atoms’ or ‘the mereological sums of atoms’. That is because, in every model, all these terms denote the supremum of the domain. Then, it would seem that another inference would — implausibly — come out valid in $\mathcal{L}_M$.

\[ (1.55) \begin{align*} 
\text{The atoms are exactly three} \\
\text{The non-atoms are exactly three}
\end{align*} \]

We know that, if there are $n$ atoms, the number of mereological sums over them is $2^n - 1$. The number of non-atoms is the number of mereological sums minus the number of atoms, that is, $2^n - 1 - n$. Therefore, if the number of atoms is greater than two, there are more non-atoms than atoms, contrary to what is inferred in (1.55). How could the mereological singularist reply?

The intuitively false identity claim concerning the atoms and the non-atoms is rendered in $\mathcal{L}_M$ by (1.56), which is true in every model.

\[ (1.56) \sigma x \text{At}(x) = \sigma x \neg \text{At}(x). \]

\[ ^{26}\text{See Nicolas (unpublished) section 2.6.} \]
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Notice that, here, there seems to be only one kind of mereological notion involved, the technical one. This is why the response to the previous two objections is no longer available.

The singularist may reply that inference (1.55) is properly regimented in $\mathcal{L}_M$ by (1.57):

\[
\exists x_1 \exists x_2 \exists x_3 (x_1 \neq x_2 \land x_2 \neq x_3 \land x_1 \neq x_3 \land \\
\text{At}(x_1) \land \text{At}(x_2) \land \text{At}(x_3) \land \\
\forall y (\text{At}(y) \rightarrow y = x_1 \lor y = x_2 \lor y = x_3))
\]

Since (1.57) is invalid in $\mathcal{L}_M$, the problem is blocked. However, one might complain that this formalization is not a correct regimentation of the natural language inference (1.55), given that the regimentation does not contain the definite descriptions that give rise to the problem. This dispute is not easy to adjudicate. Regimentation might have different goals and it seems that, for a variety of purposes, the proposed regimentation would be satisfactory. The main complaint is that the form of the regimented sentences departs significantly from that of the natural language sentences.

Suppose, however, that one wants to reduce the distance between the natural language and the result of regimentation in $\mathcal{L}_M$. In particular, suppose that one wants to have a term corresponding to the natural language definite description. Then, instead of eliminating it through quantification, one might treat the numeral as a predicate applying to plural terms. So, let us introduce the one-place predicate THREE into the regimenting language and let us define it implicitly through to the following satisfaction clause:
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(1.58) For any model $M$ and interpretation $I$ and for any object language predicate $\phi$, $M \models \text{THREE}(\sigma x(\phi(x)))$ if and only if the cardinality of the set $I(\phi(x))$ is three.$^{27}$

With this strategy in place, consider the regimentation of the problematic inference, where the extra premise ‘the atoms are the non-atoms’ has now been made explicit.

\[
\sigma x \text{At}(x) = \sigma x \neg \text{At}(x)
\]
\[
\text{THREE}(\sigma x \text{At}(x)) \quad \text{THREE}(\sigma x \neg \text{At}(x))
\]

(1.59) Given rule (1.58) on the interpretation of numerical predicates, the inference comes out invalid. At the syntactic (proof-theoretic) level, however, this constitutes a violation of the usual inferential rule sanctioning the indiscernibility of identicals. The new predicate THREE violates extensionality. The problem for the mereological singularist is that the inference pattern appears generally valid. For example, consider (1.60).

\[
\text{The members of the team are John’s best friends}
\]
\[
\text{The members of the team are eight (in number)}
\]
\[
\text{John’s best friends are eight (in number)}
\]

It is interesting to notice that, according to the semantics of numerals just introduced, inference (1.60) comes out semantically valid, as desired. Indeed, the fact that the members of the team and John’s best friends are all mereological atoms (in the $L_M$ sense of ‘atom’), renders the inference semantically valid. Even if there seems to be no pressing problem at the semantic level, the mereological singularist has to pay a

27The case of numerical predication with respect to terms formed via phrasal conjunction is easily handled by recalling that, in $L_M$, $s \oplus t = \sigma x(x = s \lor x = t)$. 

45
high price at the proof-theoretic level: she has to give up the usual inference rules
codifying unrestricted indiscernibility of identicals.

Finally, I would like to mention another objection that exploits the same anomaly
that gave rise to the previous objection. Here is one way it can be formulated.28

Suppose that there is more than one thing. Then, there is something that
is not an atom. Thus, the atoms are not the same as the non-atoms. But,
according to $L_M$, the sentence ‘the atoms are the non-atoms’ is true in
every model.

Is this a new problem for the mereological singularist? It seems not. The argument
implicitly relies on a formulation of plural comprehension that is not validated in $L_M$.
It assumes that, if there is a non-atom $x$, then $x$ is not one of the atoms, since the
atoms are the things such that anything is one of them if and only if it is an atom.
Formally,

\[(1.61) \exists x \neg \text{At}(x) \rightarrow \exists x \forall y (y \leq x \leftrightarrow \neg \text{At}(y)).\]

But, as we saw earlier, (1.61) is not an instance of a valid form of plural comprehension
in $L_M$.29 This objection does not seem to raise a new problem; it highlights the
previously discussed issue of whether or not $L_M$ does justice to plural comprehension.

In conclusion, the syntactic version of mereological singularism can sidestep the
objections discussed in this section. In at least one case, however, it has to pay a high
price, as it has to give up some standard rules of inference concerning identity. As I will

28For another instance of this problem, involving the notion of sum, rather than atom, see Nicolas
(unpublished), section 2.8.

29See section 1.1.3 and 1.1.2.
argue in section 1.4.1 below, the main threat to mereological singularism comes from the fact that, on this view, plural sentences have ontological commitments that one would think they should not have. In light of the problem with the rules of inference for identity and in anticipation of the problem with ontological commitments, we should look for an alternative framework. I will now introduce *semantic singularism* and try to show that it is immune to the sorts of problems that trouble syntactic singularism.

### 1.2 Semantic singularism

The difficulties with the syntactic versions of singularism seem to have a common source, that is, the reduction of plural quantifiers to singular ones. It is therefore natural to adopt a regimenting language that distinguishes between two types of quantifiers. The language $L_{PL}$, presented in the Introduction, offers an example of such a regimenting language. Our formulations of semantic singularism will be based on this language.

Once $L_{PL}$ is adopted, sets do not need to be invoked in order to regiment the basic plural constructions. Do they still have a role to play in the semantics of plurals? According to the main formulation of object singularism, they do. They play a model-theoretic role by providing the semantic values of plural terms. This means, in particular, that they play a role in the metalanguage rather than the regimenting language. I will now present the main formulation of object singularism and illustrate some of its virtues.

The basic idea is to interpret plural terms as appropriate non-empty subsets of the domain and to interpret plural quantifiers as ranging over the power set of the domain minus the empty set. Let the set $D$ be a domain. Let $\mathcal{P}(D)$ be its power
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set and let us define $\mathcal{P}^*(D)$ to be $\mathcal{P}(D) - \{\emptyset\}$. Next, we define the following items: variable assignments from $D$, interpretations from $D$, and a relation of satisfaction holding among a formula, an interpretation, and a variable assignment. A variable assignment from $D$ is any function $s$ from the variables of the language into $D$, satisfying the conditions:

(1.62) (i) for every singular variable $v$ of the object language, $s(v) \in D$;

(ii) for any plural variable $vv$ of the object language, $s(vv) \in \mathcal{P}^*(D)$.

For any singular variable $v$, a variant $s(v/x)$ of a variable assignment $s$ from $D$ is an assignment such that $s(v/x)(v) = x \in D$ and $s(v/x)(u) = s(u)$ for any singular variable $u$ distinct from $v$. Analogously, for any plural variable $vv$, a variant $s(vv/y)$ of a variable assignment $s$ from $D$ is an assignment such that $s(vv/y)(vv) = y \in \mathcal{P}^*(D)$ and $s(vv/y)(uu) = s(uu)$ for any plural variable $uu$ distinct from $vv$.

An interpretation $i$ from $D$ is a function from the non-logical terminology of the language such that

(1.63) (i) for every constant $c$, $i(c) \in D$;

(ii) for every $n$-ary predicate $P^{n[m]}$, $i(P) \subseteq D_1 \times \ldots \times D_n$, where for each $j \in \{1, \ldots, m\}$, $D_j = \mathcal{P}^*(D)$ and for each $k \in \{m+1, \ldots, n\}$, $D_k = D$.

Satisfaction is defined as as a relation among an interpretation $i$ from some domain $D$, a variable assignment from $D$, and a formula of the object language: $\text{Sat}(\phi, i, s)$, which will be written as $i \models \phi [s]$. Let $t$ be a singular term, that is, a singular constant or a singular variable. We denote by $i/s(t)$ the unique $x$ such that $i(t) = x$ if $t$ is a constant, and $s(t) = x$ if $t$ is a variable. Satisfaction is characterized by the following clauses:
(1.64) (i) for any singular terms \( t \) and \( r \), \( i \models t = r [s] \) if and only if \( i/s(t) = i/s(r) \); 
(ii) for any singular term \( t \) and plural variable \( vv \), \( i \models t < vv [s] \) if and only if \( i/s(t) \in s(vv) \); 
(iii) for any \( n \)-ary predicate \( P^{n[m]} \), plural terms \( \Theta_1, ..., \Theta_m \), and singular terms \( \theta_{m+1}, ..., \theta_n \), 
\[
    i \models P(\Theta_1, ..., \Theta_m, \theta_{m+1}, ..., \theta_n) [s] \text{ if and only if } \\
    (s(\Theta_1), ..., s(\Theta_m), i/s(\theta_{m+1}), ..., i/s(\theta_n)) \in i(P); 
\]
(iv) for every formula \( \phi \) and singular variable \( v \), \( i \models \exists v\phi [s] \) if and only if there is \( x \in D \) such that \( i \models \phi [s(v/x)] \); 
(v) for every formula \( \phi \) and plural variable \( vv \), \( i \models \exists vv\phi [s] \) if and only if there is \( y \in P^*(D) \) such that \( i \models \phi [s(vv/y)] \); 
(vi) for every formula \( \phi \), \( i \models \neg \phi [s] \) if and only if it is not the case that \( i \models \phi [s] \); 
(vii) for every formula \( \phi \) and \( \psi \), \( i \models \phi \land \psi [s] \) if and only if \( i \models \phi [s] \) and \( i \models \psi [s] \).\(^{30,31}\)

\(^{30}\)As usual, disjunction, implication, biconditional, and the universal quantifiers are regarded as abbreviations.

\(^{31}\)Despite his rejection of singularism, McKay (2006), chapter 5, offers a friendly proposal for the semantic singularist akin to the one just presented. On pp. 104-105, McKay discusses two options for the set-theoretic singularist and argues that distributive predicates should be assigned the set of singletons of the objects that satisfy the corresponding singular predicate. He then proposes to interpret distributive predication as the subset relation. For example, if the singular ‘is a student’ is assigned the set \( \{a, b\} \) as its extension, the distributive plural ‘are students’ would have to be assigned the set \( \{\{a\}, \{b\}\} \). This requires that a plural terms such as ‘Annie and Bonnie’ would be interpreted as the set of singletons of the individual denotations of ‘Annie’ and ‘Bonnie’, say
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It follows that, in the case of a sentence, variable assignments are irrelevant, in the sense that, if an interpretation satisfies a sentence with respect to an assignment from its domain, then it satisfies the same sentence with respect to any other assignment from its domain. This means that, when we are only concerned with the satisfaction of sentences, we may drop any reference to assignments. The relations of logical consequence and logical truth are defined as usual.

(1.65) A sentence $\sigma$ is a logical consequence of the set of sentences $\Gamma$ ($\Gamma \models \sigma$) if and only if for every interpretation $i$, if $i \models \gamma$ for every $\gamma \in \Gamma$, then $i \models \sigma$. A sentence $\sigma$ is a logical truth if and only if $\emptyset \models \sigma$.

The model theory just illustrated resembles very closely the standard semantics for second-order logic. Plural quantifiers differ from second-order ones in that the former are interpreted as ranging over non-empty subsets of the domain of the first-order quantifier. Of course, in this semantics, there is no obvious analogue of quantification

$\{\{a\}, \{b\}\}$. Thus, ‘Annie and Bonnie are students’ would be true in the given interpretation if and only if $\{\{a\}, \{b\}\} \subseteq \{\{a\}, \{b\}\}$. However, this solution is problematic and may be avoided if the argument places of plural predicates are marked as distributive or collective, or if the reference to distributive predicate is eliminated via paraphrase as done $L_{PL}$. The solution is problematic because it forces an artificial ambiguity: for example, ‘Annie and Bonnie’ would denote the set of singletons of individual denotations, $\{\{a\}, \{b\}\}$, in cases of distributive predication, but the same term would denote the set of the individual denotations $\{\{a\}, \{b\}\}$ in cases of collective predication. Moreover, the solution may be avoided. For instance, one can derive a distributive predicate from the corresponding singular predicate via some predicate modifier similar to the $^*$ operator found in the system $L_M$. Once the distinction between collective and distributive arguments is appropriately marked, a distributive predicate can be assigned the power set (minus the empty set) of the objects that satisfy the corresponding singular predicate. In our example, the predicate ‘are students’ would be assigned the set $\{\{a\}, \{b\}, \{a, b\}\}$. 

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over \(n\)-ary relations. Boolos has famously shown how to interpret monadic second-order logic in a fragment of a language with plural quantifiers containing just the \(\prec\) relation, and vice versa.\(^{32}\) From the perspective of the semantic singularist under discussion, the result should not appear surprising.

It is relatively straightforward to verify that the singularist account just illustrated captures a variety of logical relations among plural expressions. For example, plural comprehension is valid in the simple form

\[
\exists x \phi(x) \rightarrow \exists x \forall x (x \prec xx \leftrightarrow \phi(x)).
\]

Moreover, if phrasal conjunction is interpreted as the set of the objects that interpret the component expressions (e.g., \(i(a \oplus b) = \{i(a), i(b)\}\)), then symmetry, contraction, and expansion are validated in their \(\mathcal{L}_{PL}\) regimentation:

(1.66) Tom and Dick love Mary

\[
\forall x (x \prec (t \oplus d) \rightarrow L(x, m))
\]

Dick and Tom love Mary

\[
\forall x (x \prec (d \oplus t) \rightarrow L(x, m))
\]

(1.67) Tom, Dick, and Harry are linguists

\[
\forall x (x \prec (t \oplus d \oplus h) \rightarrow L(x))
\]

Tom and Harry are linguists

\[
\forall x (x \prec (t \oplus h) \rightarrow L(x))
\]

Dick and Harry are lawyers

\[
\forall x (x \prec (d \oplus h) \rightarrow L(x))
\]

(1.68) Tom is a lawyer

\[
L(t)
\]

Dick, Harry, and Tom are lawyers

\[
\forall x (x \prec (d \oplus h \oplus t) \rightarrow L(x))
\]

Even if definite descriptions are not officially part of \(\mathcal{L}_{PL}\), their natural interpretations in this semantics would be a non-empty subsets of the domain: ‘the \(\phi\)s’ would be interpreted as the set of members of the domain that satisfy the formula \(\phi\).

\(^{32}\)See Boolos (1984b).
Finally, it should not be difficult to convince oneself that the paradox of plurality poses no problem for this construal of semantic singularism. Indeed, the relation of ‘being among’ is represented in $\mathcal{L}_{PL}$ as $\prec$, which is a relation between a singular and a plural argument. Therefore, there cannot be a purely singular relation $R$ in the object language that is appropriately represented as $\prec$. Moreover, since the paradox requires the collapse of plural quantifiers into singular ones, by distinguishing two types of quantifiers, the paradox is blocked. In the object-singularist semantics, the paradox generating sentences

(1.8) There are some sets such that any set is one of them if and only if it is not self-membered.

(1.9) There is a set $x$ such that for every set $y$, $y$ is in $x$ if and only if $y$ is not self-membered.

are not equivalent, as it would be easy to show model-theoretically.

In the next section, I present a well-known technical result showing that there is a very close connection between the object-singularist semantics and the mereological version of syntactic singularism. Specifically, the result shows that, while differing on the choice of the regimenting language, there is a common model-theoretic insight behind both approaches. This insight will lead to the view that I will call algebraic singularism.

### 1.2.1 Algebraic singularism

As we have seen in section 1.1.3, the mereological singularist appeals to structured domains in the formulation of her model theory. The domain of a model of $\mathcal{L}_M$ is a complete atomic Boolean algebra. Likewise, the construal of semantic singularism
presented in the previous section requires that the domain of the plural quantifiers $\mathcal{L}_{PL}$ have the structure of the power set of the domain of the first-order quantifiers. There is a well-known technical result that sheds light on the connection between these two model-theoretic structures. It is a particular version of Stone’s representation theorem for Boolean algebras.

**Theorem 1**: A complete atomic Boolean algebra is *isomorphic* to the power set algebra over the set of its atoms.

**Proof.** Let $(D, \leq)$ a complete atomic Boolean algebra and let $A$ be the set of atoms. Define the power set algebra over $A$ to be $(\mathcal{P}(A), \subseteq)$. It is not hard to verify that $(\mathcal{P}(A), \subseteq)$ is a complete atomic Boolean algebra and that $\{\{x\} : x \in A\}$ is the set of its atoms. It can also be verified that its join $(\sqcup)$ and meet $(\sqcap)$ operations correspond to set-theoretic union and intersection in $(\mathcal{P}(A), \subseteq)$. Let $(B, \leq)$ and $(B', \sqsubseteq)$ be two Boolean algebras with $\sqcup, \sqcap$ and $\sqcup', \sqcap'$ their respective induced join and meet operation. Then a function $f : B \rightarrow B'$ is Boolean homomorphism just in case for every $x, y \in B$, (i) $f(x \sqcup y) = f(x) \sqcup' f(y)$, (ii) $f(x \sqcap y) = f(x) \sqcap' f(y)$, and (iii) $f(\overline{x}) = \overline{f(x)}$, where the complement sign in the right-hand-side of the identity is taken in $B'$. As usual, an isomorphism is a bijective homomorphism.

Define a function $f : D \rightarrow \mathcal{P}(A)$ by

$$f(d) = A \cap \{x \in D : x \leq d\} \quad \text{for every } d \in D.$$  

We adopt the notation $A(d)$ for the set $A \cap \{x \in D : x \leq d\}$. So, $f(d) = A(d)$

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33See footnote 9 for definitions.
for every \( d \in D \). We want to verify that \( f \) is Boolean isomorphism. First, we verify that \( f \) is injective and surjective.

Suppose that \( f(x) = f(y) \), i.e. \( A(x) = A(y) \). Now, \( x \) is the supremum of \( A(x) \) and \( y \) is the supremum of \( A(y) \).\(^{34}\) Since suprema are unique, it follows from \( A(x) = A(y) \) that \( x = y \). Thus, \( f \) is injective.

Suppose that \( X \in \mathcal{P}(A) \), i.e. \( X \subseteq A \). If \( X = \emptyset \), then \( f(0) = X \). We may assume that \( X \) is non-empty. Since \((D, \leq)\) is complete, \( X \) has a supremum in \( D \). Call it \( x \). Since \( x \) is the supremum of \( X \), \( X \subseteq f(x) \). Let \( y \in f(x) \). We want to show that \( y \in X \). Since \( y \in f(x) \), \( y \leq x \). Thus, \( y = y \cap x \). Let \( y \cap X \) be the set \( \{y \cap z : z \in X\} \). Then, the fact that \( x \) is the supremum of \( X \) entails that \( y \) is the supremum of \( y \cap X \).\(^{35}\) But \( y \in A \) and \( X \subseteq A \). So, if \( y \notin X \), then \( y \cap X = \{0\} \) whose supremum is \( 0 \). Hence, \( y = 0 \), contradiction. Therefore, \( y \in X \), which means that \( f(x) \subseteq X \). We conclude that \( f(x) = X \) and that \( f \) is surjective.

Next, we prove the claim that for every \( x, y \in D \), \( x \leq y \) if and only if \( f(x) \subseteq f(y) \). Suppose that \( x \leq y \). Then, clearly, \( A(x) \subseteq A(y) \), i.e., \( f(x) \subseteq f(y) \). Now suppose that \( A(x) \subseteq A(y) \). Since \( y \) is the supremum of \( A(y) \), then for every \( z \in A(x) \), \( z \leq y \). But \( x \) is the supremum of \( A(x) \), so it must be that \( x \leq y \).\(^{36}\)

We are now ready to verify that the three conditions for Boolean homomorphism are satisfied with respect to \( f \). We need to show that for every \( x, y \in D \), \( f(x \sqcup y) = f(x) \cup f(y) \) (condition (i)). By definition of \( f \), we have

\(^{34}\)See Lemma 1 in footnote 12.

\(^{35}\)This follows from Lemma 2 in footnote 12.

\(^{36}\)For the last two claims, see again Lemma 1 in footnote 12.
that \( f(x \sqcup y) = A(x \sqcup y) \). So we want to show that

\[
A(x \sqcup y) = A(x) \cup A(y).
\]

The right-to-left inclusion (\( \supseteq \)) is trivial. Let us verify the other inclusion. Let \( K \) be an arbitrary set in \( \mathcal{P}(A) \) such that \( A(x) \subseteq K \) and \( A(y) \subseteq K \). Since \( f \) is surjective, there is \( k \in D \) such that \( f(k) = K \). So, \( f(x) \subseteq f(k) \) and \( f(y) \subseteq f(k) \). Thus, by the claim above, \( x \leq k \) and \( y \leq k \). It follows that \( x \sqcup y \leq k \), hence \( f(x \sqcup y) \subseteq f(k) = K \). But \( K \) was arbitrary. Taking \( K = A(x) \cup A(y) \), we have that \( f(x \sqcup y) \subseteq A(x) \cup A(y) \).

Now we verify that \( f(x \cap y) = f(x) \cap f(y) \) (condition (ii)). That is, we verify that \( A(x \cap y) = A(x) \cap A(y) \).

Suppose that \( z \in A(x \cap y) \). This means that \( z \in A \) and \( z \leq x \cap y \). Thus, \( z \leq x \) and \( z \leq y \). So \( z \in A(x) \cap A(y) \), hence \( A(x \cap y) \subseteq A(x) \cap A(y) \). For the other direction, suppose that \( z \in A(x) \cap A(y) \). Then, \( z \in A \), \( z \leq x \), and \( z \leq y \). It follows that \( z \leq x \cap y \). Thus, \( x \in A(x \cap y) \), hence \( A(x \cap y) \supseteq A(x) \cap A(y) \).

Finally, we verify that for every \( x \in D \), \( f(\overline{x}) = \overline{f(x)} \), namely \( f(\overline{x}) = A - f(x) \) (condition (iii)). It follows from the characterization of \( f \) that \( f(0) = \emptyset \) and \( f(1) = A \). By definition of complement, \( x \cap \overline{x} = 0 \) and \( x \cup \overline{x} = 1 \). So it follows from conditions (i) and (ii) that

\[
f(x) \cap f(\overline{x}) = f(x \cap \overline{x}) = f(0) = \emptyset, \quad \text{and} \quad f(x) \cup f(\overline{x}) = f(x \cup \overline{x}) = f(1) = A.
\]

Thus, \( f(\overline{x}) = A - f(x) = A - A(x) = \overline{f(x)} \). Therefore, \( f \) satisfies conditions (i), (ii), and (iii). Since we have already shown that \( f \) is a bijection, we conclude
that $f$ is a Boolean isomorphism between $(D, \leq)$ and $(P(A), \subseteq)$. □

As this result shows, both mereological singularism and semantic singularism impose the same model-theoretic structure on plurals, namely, the structure of a complete atomic Boolean algebra or, equivalently, the structure of the powerset of its atoms. The difference between the two approaches lies in the choice of the regimenting language to which the common model-theoretic structure is applied. However, in section 1.1.3 I have discussed a problem with mereological singularism resulting from the reduction, in $\mathcal{L}_M$, of plural quantifiers to singular quantifiers. Below, I will discuss another problem resulting from the same source. If, as I believe, these problems are enough to motivate the distinction between two two types of quantifiers, a natural move for the mereological singularist would be to modify the regimenting language along the lines of $\mathcal{L}_{PL}$ and apply the old model theory to the new language. This means, presumably, that plural quantifiers would range over the non-atomic elements of the complete atomic Boolean algebra whose atoms are the members of the domain of the singular quantifiers. Now, the above representation theorem tells us that this modification of mereological singularism would be model-theoretically indistinguishable from the version of semantic singularism outlined in the previous section.

I propose to characterize as *algebraic singularism* the semantic approach based on $\mathcal{L}_{PL}$ that assigns to the plural domain the structure of a complete atomic Boolean algebra whose set of atoms is the singular domain. These structures include those invoked in the above characterization of semantic singularism, that is, structures in which the plural domain is a powerset algebra. As the theorem confirms, for model-theoretic purposes we could confine ourselves to structures in which the plural is a powerset algebra. But there is no particular reason to impose this restriction. In
fact, adopting a more general formulation of semantic singularism, such as algebraic
singularism, will help to avoid some of the metaphysical questions that might arise
in connection with a formulation of semantic singularism in which set-theoretic mem-
bbership, or some parthood-like relation, plays a key role in interpreting ‘being one
of’. According to algebraic singularism, neither set-theoretic membership nor any re-
lation of parthood has a privileged status. Any relation that meets certain algebraic
conditions can function as the interpretation of $\prec$ in some model.

We now have a preferred version of object singularism, that is, algebraic singular-
ism. In the next two sections, I will discuss some potential difficulties for this view.
I will also go back to the problem of regimentation and argue that $L_{PL}$ is preferable
over $L_M$ as a regimenting language. In the end, I hope to show that the only serious
threat for algebraic singularism comes from admitting the possibility of quantifying
over absolutely everything.

1.3 Nonfirstorderizability: the Geach-Kaplan sen-
tence

A widely invoked argument against singularism is based on the particular properties of
the so-called Geach-Kaplan sentence.$^{37}$ As I will try to show in this section, arguments
of this kind threaten only the syntactic version of singularism. They do not pose a
problem for semantic singularism. In fact, they seem to presuppose it.

The argument based on the Geach-Kaplan sentence is meant to show that, given
some model-theoretic requirements on regimentation, there are English sentences that

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$^{37}$See, for example, Boolos (1984b), Resnik (1988), Rayo (2002), and Linnebo (2003).
cannot be regimented by any first-order sentence. It exploits a technical result applying to some second-order sentences. Let us start with the technical result. I will later discuss its connection with plurals.

Let $\mathcal{L}_1$ be a first-order language with the usual model-theoretic semantics and let $\mathcal{L}_2$ be a second-order language with standard semantics. There is a sense in which $\mathcal{L}_2$ has the same class of models as $\mathcal{L}_1$. A model for $\mathcal{L}_1$ can be thought of as a pair $(D, f)$, where $D$ is a set specifying the domain of quantification and $f$ is a function specifying the interpretation of the non-logical terminology of the language. Now, $\mathcal{L}_2$ extends $\mathcal{L}_1$ by introducing quantification over properties and relations, but this does not require an extension of the interpretation function, as the domain of the second-order quantifiers is fixed by the domain of first-order ones. Of course, the extension of the language will be reflected somewhere at the semantic level. In particular, the relation of satisfaction (or truth in a model) for $\mathcal{L}_1$ differs from that for $\mathcal{L}_2$. The satisfaction relation for sentences of $\mathcal{L}_2$ will have to include a satisfaction clause for the second-order quantifiers. So, when we say that the same pair $(D, f)$ is a model for sentences of both $\mathcal{L}_1$ and $\mathcal{L}_2$, we mean to say that $(D, f)$ stands in distinct satisfaction relations to sentences of $\mathcal{L}_1$ and $\mathcal{L}_2$. Analogously, when we say that two sentences, $\sigma_1$ of $\mathcal{L}_1$ and $\sigma_2$ of $\mathcal{L}_2$, have the same class of models, we mean to say that $\sigma_1$ stands in a certain relation of satisfaction to each model in the class and $\sigma_2$ stands in a distinct relation of satisfaction to each model in the class.

With this clarification in mind, we can proceed to report the technical result. We assume that $\mathcal{L}_1$, and $\mathcal{L}_2$, have enough symbols to formulate arithmetic. For convenience, let us assume that $\mathcal{L}_1$ and hence $\mathcal{L}_2$ include the usual arithmetical symbols

\footnote{For the definitions of these and related notions, including that of a standard semantics, Henkin semantics, and first-order semantics for a second-order language, see Shapiro (1991), chapter 3.}
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‘0’, ‘s’ (for successor), ‘+’, and ‘·’.

**Theorem 2:** There is a sentence $\Phi$ of $L_2$ such that, for no sentence $\phi$ of $L_1$, $\Phi$ has the same class of models as $\phi$.

*Proof.* Let PA1 be the set of first-order axioms of Peano arithmetic and let PA2 the set of second-order axioms of Peano arithmetic. Since PA2 is finite, let $\Phi$ be the conjunction of its members. By categoricity of second-order arithmetic, the class of models of $\Phi$ contains all and only models isomorphic to the intended model of arithmetic. In particular, it contains no uncountable model. If there were a sentence $\phi$ of $L_1$ with the same class of models as $\Phi$, there would be a first-order sentence with no uncountable model. But this would contradict the Löwenheim-Skolem theorem. Thus, there is no sentence of $L_1$ with the same class of models as $\Phi$. QED

Let us call *nonfirstorderizable* every second-order sentence to which the theorem applies. The proof of the theorem provides an example of a nonfirstorderizable sentence. It is not hard to construct other examples. We now want to see how this result has been invoked in discussions about singularism.

The way in which facts about nonfirstorderizability have seemed to be connected to singularism is through arguments of the following kind. Suppose that one can find an English sentence $\sigma$ and a sentence $\Phi$ of $L_2$ that meet the following conditions:

(i) $\Phi$ is a correct regimentation of $\sigma$;

(ii) $\Phi$ is nonfirstorderizable.

Moreover, assume this principle about correctness of regimentation:
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(iii) for any sentence \( \phi \) and \( \psi \), if \( \phi \) correctly regiments \( \sigma \), then \( \psi \) correctly regiments \( \sigma \) only if \( \psi \) has the same class of models as \( \phi \).

These assumptions yield the conclusion that no sentence of a singular language can correctly regiment \( \sigma \), which runs against the set-theoretic version of syntactic singularism. The Geach-Kaplan sentence, the classic example of a plural nonfirstorderizable sentence, is (1.69).

(1.69) Some critics admire only one another.

Supporters of nonfirstorderizability arguments typically claim that (1.69) is correctly regimented by the second-order sentence:

\[
(1.70) \exists X(\exists xXx \land \forall x(Xx \to Cx) \land \forall y\forall z(Xy \land Ayz \to Xz \land y \neq z)).
\]

David Kaplan has first shown that (1.70) is nonfirstorderizable.\(^{39}\) By the above argument, it might then be concluded that there is no correct first-order paraphrase of (1.69). Since (1.69) is a perfectly good sentence of English, if the argument is sound, the set-theoretic version of syntactic singularism would be undermined. Other examples of interesting nonfirstorderizable sentences have been offered by Boolos (1981, 1984a). Some remarks are in order.

\(^{39}\) Proof sketch. Let \( \text{PA}^+ \) be the theory

\[
\text{PA} \cup \{\forall x\forall y(A(x, y) \iff (x = 0 \lor x = y + s(0))) \land \forall x(Cx \iff x = x)\}.
\]

For any model \( M \) of \( \text{PA}^+ \), \( M \) satisfies (1.70) if and only if \( M \) is non-standard (see Boolos (1984b), pp. 432-433). Suppose that there is a first-order sentence \( \theta \) with the same class of models as (1.70). Then, the theory \( \text{PA}^+ \cup \{\neg \theta\} \) would have only standard models, hence in particular countable ones. This would contradict the Löwenheim-Skolem theorem, since \( \text{PA}^+ \cup \{\neg \theta\} \) is a first-order theory. So there is no first-order sentence with the same class of models as (1.70). QED
There are reasons to doubt that arguments of this kind are effective. First, claims about correctness of regimentation crucially depend on one’s goals. So it is doubtful, at least *prima facie*, that the requirement expressed by condition (iii) would apply across the board. This is evidenced by the fact that there seem to be reasonable first-order candidates for the regimentation of (1.69). For example, consider the following sentence.

(1.71) There is a collection $x$ such that $x$ is non-empty, $x$ includes only critics, and for all $y$ and $z$, if $y$ is in $x$ and $y$ admires $z$, then $y \neq z$ and $z$ is in $x$.

Consider also variants of (1.71) in which ‘collection’ is replaced by ‘set’, ‘class’, ‘property’, ‘group’, ‘aggregate’, or ‘mereological sum’ and the relevant relation of inclusion is modified accordingly. It seems that, especially in isolation, some of these sentences do not fail, at least obviously, to provide satisfactory regimentations of (1.69). But the argument based on nonfirstorderizability rules out that any of these sentences, even in isolation, can correctly regiment (1.70).

There might be other reasons to be unsatisfied with all these candidates. For example, one might be inclined to reject them because they have ontological commitments (to collections, sets, etc.) that the original sentence does not have. I will focus on this argument in the next section. For the time being, let me just note that it requires that correct paraphrases respect the ontological commitments of the original sentence. Moreover, let me point out that the rejection of the proposed paraphrases does not follow from the nonfirstorderizability argument presented above. Sentences without substantive ontological commitments might have different classes of models. There is perhaps some hope to find pairs of sentences, for instance conditionals with-

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out substantive existential consequences, with the same ontological commitments but different classes of models. On the other hand, it seems harder to produce uncontroversial examples of sentences with the same classes of models but different ontological commitments. This would show that one can deny the requirement on paraphrase found in the nonfirstorderizability argument without having to deny the requirement on paraphrase concerning ontological commitments.

The argument from nonfirstorderizability crucially relies on the claim that (1.69) is correctly regimented by (1.70). However, the argument requires that (1.70) be understood as a sentence of $L_2$, i.e., as a sentence of second-order logic with standard semantics, as opposed to Henkin or first-order semantics. This assumption is necessary to obtain premise (ii) of the argument to the effect that (1.69) is nonfirstorderizable. A defender of syntactic singularism may challenge this assumption. One would then need a strategy to rule out that (1.70), when thought of as a sentence of second-order logic with Henkin or first-order semantics, would be a correct regimentation of (1.69). Clearly, such a strategy could not appeal to deductive properties of (1.70), since the natural axiom system for second-order languages is sound with respect to all three types of semantics in question.\footnote{For a formulation of a deductive system for second-order languages and related semantic properties, see Shapiro (1991), chapter 3, section 2.} In the absence of a principled reason to accept (1.70) as a correct regimentation of (1.69) only when thought of as a sentence of second-order logic with standard semantics, the nonfirstorderizability argument might lose some of its force.

Notice that the semantic singularist can do justice to the intuition that there is a connection between (1.69) and (1.70). They have similar truth-conditions. Semantic singularism, however, has the advantage of not having to venture into controversial

\footnote{For a formulation of a deductive system for second-order languages and related semantic properties, see Shapiro (1991), chapter 3, section 2.}
claims about regimentation. In $L_{PL}$, the regimentation of (1.69) would be (1.72).

$$(1.72) \exists x (\forall x (x \prec xx \rightarrow Cx) \land \forall y \forall z ((y \prec xx \land Ayz) \rightarrow (z \prec xx \land y \neq z))).$$

Since the regimentation takes place in an extension of first-order logic, facts about nonfirstorderizability become irrelevant. Given the result discussed in the previous section and an appropriate identification of models, one is allowed to claim that (1.72) and (1.70) have the ‘same’ class of models. Thus, even if nonfirstorderizability arguments threaten syntactic singularism — although, as we have seen, this is not obvious — they do not threaten semantic singularism. Semantic singularism, on the contrary, gives some insight into nonfirstorderizability arguments and is able to make sense of the temptation to invoke second-order representations of plural sentences such as the Geach-Kaplan one. Indeed, according to the semantic singularist, the second-order and the plural regimentations of the Geach-Kaplan sentence are semantically indistinguishable, that is, indistinguishable with respect to models.

### 1.4 Further problems

In this section, I would like to discuss a number of further problems raised against singularism. Since the set-theoretic version of syntactic singularism, as we have seen, is undermined by the paradox of plurality, we may focus our attention on the system $L_M$ and on semantic singularism.

#### 1.4.1 Ontological consequences

A common complaint against syntactic singularism concerns its ontological consequences in the following sense. Consider this sentence.
(1.73) Russell and Whitehead wrote *Principia Mathematica*.

According to the set-theoretic and mereological version of syntactic singularism, (1.73) should represented as (1.74) and (1.75), respectively.

(1.74) \( \exists x \ (\text{Set}(x) \land \forall y (y \in x \leftrightarrow (y = r \lor y = w)) \land W(x, PM)) \).  

(1.75) \( W(r \oplus w, PM) \).

Now, (1.74) entails (1.76).

(1.76) \( \exists x \ \text{Set}(x) \).

On the other hand, (1.75) entails (1.77).

(1.77) \( \exists x \ (\forall y ((\text{At}(y) \land y \leq x) \leftrightarrow (y = r \lor y = w)) \land W(x, PM)) \).

On the assumption that Russell and Whitehead are distinct, it follows that

(1.78) \( \exists x \ \neg \text{At}(x) \).

This expresses the sentence ‘there is a mereological sum (distinct from each atom)’.

Therefore, a sentence such as ‘Russell and Whitehead wrote *Principia Mathematica*’ would entail ‘there is a set’ or ‘there is a mereological sum (distinct from each atom)’. However, these two sentences, which are about sets and sums, do not appear to be logical consequences of ‘Russell and Whitehead wrote *Principia Mathematica*’ in any intuitive way.\(^{42}\) We have already seen a version of this argument above when discussing the paradox of plurality.\(^{43}\) There, sentences like ‘something is a member of

\(^{43}\) See p. 22.
itself’ and ‘something is a part of itself’ were derived from plural comprehension and a purely logical principle (‘something is self-identical’). In both cases, it is essential for the derivation of the unwanted consequence that the plural quantifiers be regimented as singular ones. It seems that syntactic singularism, built around the collapse of plural quantifiers into singular ones, cannot escape this kind of objection. Substantive ontological consequences about sets or sums do follow from apparently innocent plural sentences. There might be ways to avoid this problem but they do not look entirely satisfactory. Let us briefly review them.

Can this problem be avoided by restricting the expressive resources of the target language? Suppose that the fragment of natural language we are studying did not contain set-theoretic or mereological vocabulary. Then, the problematic inferences would not be expressible. This limitation, however, appears too severe. By analogy with the move explored in response to the paradox of plurality, the singularist might try to rectify the situation by introducing surrogate set-theoretic or mereological concepts. Let us indicate these concepts with a superscripted star. Because of the expressive restrictions concerning the concepts of set, membership, etc., mereological sum, parthood, etc., the following inferences would not be expressible.

\[(1.79)\] Russell and Whitehead wrote *Principia Mathematica*
There is a set

\[(1.80)\] Russell and Whitehead wrote *Principia Mathematica*
There is a mereological sum

They would have counterparts but they would not be valid.

\[(1.81)\] Russell and Whitehead wrote *Principia Mathematica*
There is a set*
Russell and Whitehead wrote *Principia Mathematica*

There is a sum

Do semantic construals of singularism face a similar difficulty? I think not. As it can easily be verified, both (1.79) and (1.80) are invalid by the lights of the semantic singularist. According to semantic singularism, plural quantifiers do not collapse into singular ones. Therefore, plural sentences, such as ‘Russell and Whitehead wrote *Principia Mathematica*’, do not have substantial singular existential consequences, except for those associated with the proper names that occur in them. Sets are used in the model theory: they do not make their illegitimate way into the regimenting language.

Despite the fact that sets play a role only in the model theory, it has been argued that this is enough to generate unwanted ontological commitments. Here is how McKay makes this point. Consider these two inferences.

(A1) ‘Some students are surrounding the building’ is true
Some individual is such that it is surrounding the building

(M1) ‘I could have had a martini’ is true (although I did not have one)
Possible worlds exist

The first inference is similar to one we have discussed above. McKay’s view is as follows.

[T]he semanticist who uses possible worlds in giving the meaning of modal claims is committed to the semantic validity of M1. [...] [T]he assertion of the premise and affirmation of the semantics in terms of possibilities commits one to the existence of those possibilities. Similarly, if the correct interpretation of ‘These are more numerous than those’ require the
existence of relations, then, as a matter of meaning, if the sentence is true then relations exist; and if the correct interpretation of ‘John is walking slowly’ requires the existence of events, then, as a matter of meaning, if the sentence is true, then events exist. [...] If the sets (or other singular interpretants) are not just elements of an artificial model constructed for a formal consistency proof or some similar purpose, but rather are presented as what is needed to give the meaning of the plural, then the singularist must say that arguments like $A_1$ are semantically valid. If the premise is true, the conclusion must be true as well, and that is a consequence of what the words mean. (McKay (2006), pp. 25-26)

Several remarks are in order. First, issues of this kind are notoriously controversial and cannot be settled without a long detour into difficult questions concerning meaning and semantic theorizing. Nevertheless, there are some relatively uncontroversial things that should be mentioned — or repeated.

As far as I can see, there are two ways of understanding the claim that $A_1$ (or $M_1$) are semantically valid. On the first reading, the claim is that the inference comes out valid in the object language. As we have remarked several times already, semantic versions of singularism are immune to this problem. The inference is not valid. On the second reading, the claim is that some appropriate interpretation of the premise entails that, in the metalanguage, there are sets (or “other singular interpretants”). This appears true, but only trivially so. Sets or suprema in a Boolean algebra are assumed in the metalanguage and are part of the resources needed to formulate the semantics. The sentence ‘there are sets’ is true in the metalanguage but it is a

\[44\] McKay acknowledges this. See p. 25, footnote 9.
stipulation necessary to get the semantic theory off the ground. It is true even if the premise is false.

We do not usually take this kind of worry too seriously when dealing with a singular fragment of natural language. There, sets are regarded as tools we use to model the semantically relevant features of natural language. It is unclear that plural languages differ from singular ones in this respect. If there is anything problematic with the adoption of a model-theoretic framework involving sets, the problem is not limited to plurals and does not arise because of them. The problem arises for any model-theoretic semantics whatsoever. Of course, one might have qualms about model-theoretic approaches in general, but I presume that this is not the worry that is intended here.45

McKay’s remarks about the existence of relations or events are probably best understood as targeting analogues of syntactic singularism in other contexts. Given that we have not found any pressing way of making good the claim that A1 is semantically valid, it appears that semantic singularism avoids the problem discussed in this section. It might have to face other problems but, as the discussion of McKay’s view suggested, such problems would depend on controversial issues in the foundations of semantics. We now want to turn to the main objection against object singularism.

45McKay draws a distinction between sets as elements of an artificial model and sets as elements of part of a way to give the meaning of certain expressions. In light of model-theoretic approaches to meaning, the distinction might look questionable. See, for example, Dowty et al. (1981), chapter 1, for a brief discussion.
1.4.2 Absolute generality and singularism

I have tried to show that semantic singularism fares better than syntactic singularism. In this section, I will present what I take to be the main problem that semantic singularism has to face.

There is a controversy over whether or not absolutely unrestricted quantification, that is, quantification over absolutely everything there is, can be achieved. A number of authors have expressed skepticism about the possibility of absolutely general quantification and have variously argued against it. If the possibility of absolutely unrestricted quantification is admitted, though, it appears that semantic singularism cannot do justice to it.

According to semantic singularism, plural quantifiers range over the non-empty subsets of the domain of the singular quantifiers. From plural comprehension applied to the formula ‘x is not self-membered’, one can deduce this intuitively true sentence of the object language:

(1.8) There are some sets such that any set is one of them if and only if it is not self-membered.

If strong enough set-theoretic assumptions are made in the metalanguage to guarantee the existence of a model of the axioms of a suitable set theory (say ZFC), then (1.8) has a model in conjunction with those axioms. But the domain of that model will still be a set.

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46 See the Introduction to Rayo and Uzquiano (2006) for an overview of the main problem associated with absolutely unrestricted quantification.

There is something *prima facie* unsatisfactory about this situation. The sentence is intended to talk about *every* set there is, not just those contained in some appropriately large set. Object singularism is thus not able to capture models where the domain encompasses every set. Surely, the singularist could appeal to classes to provide such models. But, then, the same problem would reappear — not surprisingly — for another sentence.

(1.83) There are some classes such that any class is one of them if and only if it is not self-membered.

An appeal to super-classes or some other type of collections would only delay the problem. Here is how David Lewis gives voice to this worry.

> Whatever class-like things there may be altogether, holding none in reserve, it seems we can truly say that there are those of them that are non-self-members. Maybe the singularist replies that some mystical censor stops us from quantifying over absolutely everything without restriction.\(^{48}\)

The argument requires, as Lewis remarks, that one accept the possibility of absolutely unrestricted quantification. How could the semantic singularist cope with this possibility?

She might respond by pointing out that the problem appears to have little to do with plurals. It arises even for singular languages with the usual model-theoretic semantics in which domains are sets. In the framework of a standard set theory, since there is no universal set, no model can interpret the universal quantifier as ranging over *every* set. So sentence (1.84) seems on a par with (1.8).

\(^{48}\)To which he adds: “Lo, he violates his own stricture in the very act of proclaiming it!” (Lewis (1991), p. 68.)
(1.84) Every set is not self-membered.

(1.8) There are some sets such that any set is one of them if and only if it is not self-membered.

The burden is on the opponent of singularism to find something that sets the two cases apart and to show that the difficulty is tied to plurals. A promising way to accomplish this task is by reflecting on an important logical difference between standard first-order semantics and the proposed singularist semantics for plurals.

It can be shown that the relation of logical consequence sanctioned by object singularism is not compact.\(^{49}\) It follows that there is no effective deductive system that is sound and complete for the object-singularist semantics.\(^{50}\) One might then argue that a set-theoretic semantics of the kind proposed by the object-singularist can be satisfactory only with respect to a singular language. A famous argument by

\(^{49}\)There is an infinite set of sentences \(\Delta\) and a sentence \(\gamma\) such that \(\Delta \vDash \gamma\), but such that for no finite \(\Delta_0 \subseteq \Delta\), \(\Delta_0 \vDash \gamma\). Following Yi (2006), p. 262, here is a proof sketch. Suppose that there are infinitely many singular constants in the language \(c_0, ..., c_n, ...\). Let \(\Gamma^*\) be the set of sentences \(\{A(c_n, c_{n+1}) : n \in \mathbb{N}\}\). Then

\[
\Gamma^* \vDash \exists xx \forall x (x < xx \rightarrow \exists y (y < xx \land A(x, y))).
\]

However, there is no finite subset \(\Gamma^*_0\) of \(\Gamma^*\) such that

\[
\Gamma^*_0 \vDash \exists xx \forall x (x < xx \rightarrow \exists y (y < xx \land A(x, y))).
\]

\(^{50}\)Proof sketch. Suppose that \(S\) is such a system. Let \(\vdash_S\) denote its provability relation. If \(\Delta \vDash \gamma\) and \(S\) is complete, then \(\Delta \vdash_S \gamma\). Since \(S\) is effective, there is finite \(\Delta_0 \subseteq \Delta\) such that \(\Delta_0 \vdash_S \gamma\). But \(S\) is supposed to be sound, hence \(\Delta_0 \vDash \gamma\). Therefore, the consequence relation is compact. Contradiction.
Kreisel can be employed here. The argument appeals to the fact that there is a sound and complete deductive system for first-order logic with the usual set-theoretic semantics. Let $D_1$ be this system and let $\vdash_1$ be its provability relation. Let $\models_1$ be its semantic relation of consequence and, finally, let $\equiv$ be the intuitive relation of logical consequence for first-order sentences. This relation is meant to capture the idea that, for every possible model of the language, if all premises are true in it, then the conclusion is true in it. Kreisel’s argument runs as follows.

$D_1$ is intuitively adequate in the sense that, if $\Gamma \vdash_1 \theta$, then $\Gamma \models \theta$. That is, provable arguments are intuitively valid. Now, if $\Gamma \models \theta$, then, in particular, in every model with a set-sized domain, if all the members of $\Gamma$ are true, then $\theta$ is true. Thus, if $\Gamma \models \theta$, then $\Gamma \vdash_1 \theta$. Since $D_1$ is complete, if $\Gamma \models \theta$, then $\Gamma \vdash_1 \theta$. This establishes the following equivalences.

$$\Gamma \vdash_1 \theta \iff \Gamma \models \theta \iff \Gamma \vdash_1 \theta$$

Kraisel’s argument seems to show that the semantic relation of consequence for first-order logic corresponds to the intuitive relation of logical consequence for a singular language despite the fact that the semantics only appeals to models with set-sized domains. This — it might be argued — puts to rest worries about absolutely unrestricted quantification in the first-order case. At least for the purposes of characterizing logical consequence, set-sized domains are enough for a singular language.

Given that, as we saw, no sound and complete deductive system for the object-singularist semantics for plurals is available, Kreisel’s argument cannot be invoked by the singularist to put to rest worries about absolutely unrestricted quantification.

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51 Kreisel (1967).
in the plural case. We must conclude that there is a relevant difference between the singular and the plural case. Supporters of absolutely unrestricted quantification have an argument against semantic singularism.\textsuperscript{52}

In chapter 2, I will propose a novel version of semantic singularism, \textit{property singularism}, that accommodates absolutely unrestricted quantification. No matter whether absolutely unrestricted quantification turns out to be possible, singularism remains a viable semantic approach to plurals and plural quantification.

1.5 Conclusion

In this chapter, I have examined various construals of syntactic and semantic singularism. I have distinguished between syntactic and semantic singularism. The set-theoretic version of syntactic singularism falls prey to the paradox of plurality. The mereological version of syntactic singularism, as codified by the system $\mathcal{L}_M$, also faces some difficulties, though less severe than its set-theoretic counterpart. In $\mathcal{L}_M$, plural existential sentences, such as ‘some students surrounded the building’, incorrectly entail singular sentences, such as ‘something surrounded the building’. Moreover, the treatment of numerical predication in $\mathcal{L}_M$ requires that one abandon intuitive rules of inference about identity. Semantic singularism, on the other hand, offers a way out of these difficulties. The main threat to this view comes from admitting the possibility of quantifying over absolutely everything.

\textsuperscript{52}For an attempt to respond to the challenge of absolute generality, see the system of properties developed in Linnebo (2006).
Chapter 2

Is Two a Plural Property?

According to the singular conception of reality, there are objects and there are singular properties, that is, properties that are instantiated by objects separately. On the other hand, the plural conception of reality holds that there are plural properties — properties that are instantiated by many objects taken together — alongside singular ones.\(^1\) Pluralists have argued that semantic considerations about plurals force us to embrace a plural conception of reality. In this chapter, I present and compare two semantic accounts of plurals: property singularism and standard pluralism.\(^2\) I argue that property singularism is a satisfactory alternative to standard pluralism, even by the lights of the standard pluralist. First, property singularism can accommodate absolutely unrestricted quantification. Second, it is logically adequate in the sense that, on modest assumptions about the relation between properties and pluralities, it can been proved (see Appendix B) that property singularism delivers the same relation of logical consequence for \(\mathcal{L}_{PL}\) as standard pluralism. The viability of property

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\(^2\)By ‘pluralism’, in this chapter, I will often mean ‘standard pluralism’.
singularism thus undermines the semantic argument in favor of the plural conception of reality, that is, the argument that plural properties must be introduced to provide a proper semantic account of plurals.

I begin by discussing some terminology and some issues concerning plurals and properties (section 2.1). I then provide an informal presentation of pluralism and property singularism (section 2.2), and I compare the two accounts. Finally, I discuss some potential concerns about property singularism (section 2.3). In the appendix, I offer a more formal presentation of the two semantics and I prove that they deliver the same relation of logical consequence.

2.1 Pluralities and properties

In chapter 1, I argued that the main threat to object singularism comes from the possibility of absolutely unrestricted quantification. This difficulty might lead one to embrace pluralism, for the pluralist can do justice to the possibility of absolutely unrestricted quantification. Since plural resources are available in the metalanguage, the pluralist can take the plurality of everything to serve as the all-inclusive domain of quantification. That is, on some interpretations, the domain of quantification can be the things such that everything is among them.

However, it is important to appreciate the property-singularist too can do justice to the possibility of absolutely unrestricted quantification. Once properties are available, the property-singularist can take a universal property (e.g., the property of being self-identical) to serve as the all-inclusive domain of quantification. This means that, on some interpretations, the domain of quantification can be the property that is had by
Chapter 2. Is Two a Plural Property?

everything. So worries about absolutely unrestricted quantification do not settle the matter in favor of pluralism. If wanting to accommodate the possibility of absolutely unrestricted quantification might give us reason to reject object singularism, it does not give us reason to reject singularism altogether. Property singularism remains an option.

It is also important to construe properties as predicable entities rather than objects. This means that talk of properties will be achieved through higher-order quantification. In chapter 3, I discuss a Russell-style argument put forward by Williamson (2003) that shows that if one accepts an intuitive requirement on semantic interpretations and admits the possibility of absolutely unrestricted quantification, one cannot construe semantic interpretations as objects. To avoid this problem, the property-singularist has to work in a higher-order framework. As I will assume in this chapter and argue in the next, the pluralist must also work in a higher-order framework, where a type distinction is made between objects and properties. If plural properties, which provide the semantic values of plural predicates, are construed as objects, intuitive requirements about semantic interpretations run against a plural version of Cantor’s Theorem and lead to paradox.

Once a type distinction between objects and properties is introduced, it is natural to ascend to higher types and speak of higher-order properties, that is, properties that are instantiated by properties of lower types. There is no apparent need to postulate genuinely plural properties of higher order, i.e., properties instantiated by several properties of lower types taken together. Thus, we will only deal with higher-order properties that are singular in the sense of being instantiated by properties of

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3This is, indeed, the response to the problem of absolute generality endorsed by Williamson (2003).
lower types *separately*. For simplicity, empty properties of every type will also be postulated. Here are some terminological clarifications.

A *singular first-order relation* is any relation such that each of its arguments is an object. A *plural first-order relation* is any relation such that at least one of its arguments is a plurality and such that each of the remaining arguments is either a plurality or an object. Quantification over singular first-order relations will be called *singular second-order quantification*, while quantification over plural first-order relations will be called *plural second-order quantification*. It is important to keep in mind that, as we just defined it, plural second-order quantification is not quantification over many properties. It is singular quantification over plural properties.

We can then ascend to higher types. A *plural second-order relation* is any relation such that at least one of its arguments is a plural first-order relation and such that it has no argument that is a relation (plural or singular) of order higher than the first. Similarly, a *singular second-order relation* is one that has at least one singular first-order relation as argument and has no argument that is a relation (plural or singular) of order higher than the first. Quantification over plural second-order relations will be called *plural third-order quantification* and quantification over singular second-order relations will be called *singular third-order quantification*. We can subsequently define plural and singular third-order relations, and correlative plural and singular fourth-order quantification. And so on.

Two hierarchies result. The singular one corresponds to the hierarchy of standard higher-order logic generated from first-order logic. The plural hierarchy corresponds to the hierarchy that is generated in a similar fashion from $L_{PL}$. When needed, superscripts will be used to mark the type of relation — in the metalanguage the arity of a relation will always be unmarked. A plural $n$-th order relation may be accompanied
by the superscript \( np \), while a singular \( n \)-th order relation may be accompanied by the bare superscript \( n \). Each property type is accompanied by a distinctive identity relation, which is taken as primitive.

Notice that we are permitting singular relations to be arguments of plural ones. A pluralist might avoid resorting to singular properties altogether by significantly complicating the semantics. However, there seems to be no reason for doing so. If one admits plural properties, it is hard to see why one should not admit singular properties as well. The price of avoiding singular properties is that one must abandon the idea that the reference of a singular term is an object. It is a price that few would be willing to pay.

Notice also that the above characterization of the singular and plural hierarchies of relations need not be complete. One may want to admit more exotic relations, such as one that takes as arguments plural relations of order \( n \) and singular relations of order higher than \( n \). Since these types of relations will not play any role here, they may be left out.

Semantic needs will force the introduction of higher-order relations, and quantification over them. The most basic semantic concepts, such as interpretation, satisfaction, and logical consequence, will be defined in terms of relations of order higher than the first. The following table illustrates what resources are employed at each stage if concepts are defined in the straightforward way without using coding mechanisms that might help lowering the types. Details are provided in the Appendix.
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The presentation of the two model theories for $\mathcal{L}_{PL}$ — standard pluralism and property singularism — could be simplified if we allowed, as Yi (2005, 2006) and Oliver and Smiley (2006) seem to do, relations that, on a given argument place, admit different kinds of arguments. Let us call such relations *multifaceted*. For example, suppose that we want to define a variable assignment, $S$, as a relation between a linguistic object — a variable — and a semantic value for that object. If we want $S$ to assign a certain plurality to a given plural variable, we would have to regard $S$ as a plural first-order relation taking, respectively, an object and a plurality as arguments. However, we would also want to assign a single object to a given singular variable. This means that we would also have to regard $S$ as a first-order singular relation.
Admitting multifaceted relations would allow $S$ to do double duty, thereby avoiding the multiplication of concepts, that is, the introduction of two sorts of variable assignments, a variable assignment for plural variables and a variable assignment for singular variables. Since it is not clear that we have independent reasons to admit multifaceted relations, I will proceed without them at the cost of complicating the model theory.

### 2.2 Two semantic approaches

According to the pluralist approach advocated by Yi (2005, 2006) and Oliver and Smiley (2006), plural terms denote pluralities and plural predicates denote plural properties. Since we are disallowing multifaceted relations, we must construe pluralism as assigning different kinds of properties to the predicates of the object language. Singular predicates will be interpreted as denoting singular properties, while plural predicates will be interpreted as denoting plural properties. Similarly, we must introduce two distinct notions of denotation, one for singular terms and one for plural terms, and two distinct notions of variable assignment, one for singular variables and one for plural variables.

The property-singularist semantics coincides with the standard pluralist semantics with respect to the first-order fragment of the language. However, according to property singularism, plural terms denote *non-empty* singular properties and plural predicates denote singular second-order *extensional* properties. A singular second-order property is extensional just in case if it holds of a property, it holds of any property coextensive with it. I will discuss the extensionality requirement in the next section.

To illustrate the difference between the two semantics, consider a case of basic
plural predication: ‘Russell and Whitehead cooperate’. Let us use subscripts to indicate the relativization of a notion to a given interpretation. On the pluralist approach ‘Russell and Whitehead cooperate’ is true in a given interpretation \( I \) if and only if there are some things \( xx \) in the domain \( I \) such that anything is one of them just in case it is the denotation \( I \) of ‘Russell’ or the denotation \( I \) of ‘Whitehead’, and \( xx \) have collectively the plural property denoted \( I \) by ‘cooperate’.

On the property-singularist approach, ‘Russell and Whitehead cooperate’ is true in a given interpretation \( J \) if and only if there is a (non-empty singular first-order) property \( X \) holding of the denotation \( J \) of ‘Russell’, of the denotation \( J \) of ‘Whitehead’, and of nothing else, and the singular second-order property denoted \( J \) by ‘cooperate’ holds of \( X \). It is of course required that the denotation \( J \) of ‘Russell’ and of ‘Whitehead’ be in the domain \( J \).

The relations specifying the semantic values of plural expressions differ across the two semantics. For the pluralist, interpretations of plural terms are given by plural first-order relations holding between a plural term and some things in the domain. For the property singularist, they are given by singular second-order relations holding between a plural term and a non-empty first-order property whose extension is in the domain. As for plural predicates, the pluralist specifies their semantic values via a plural second-order relation holding between the plural predicate and a plural property. The property singularist, on the other hand, specifies the semantic value of plural predicates via a singular third-order relation holding between the plural predicate and an extensional second-order relation.

In both cases, models can be construed as relations whose arguments codify the interpretations of terms and predicates of the language. Satisfaction is then defined as a relation holding among a model, a formula, and a variable assignment. Finally,
logical consequence is defined as preservation of satisfaction from the premises to the conclusion. Details of the two semantics are given in Appendix A1 and Appendix A2. In the next section, I address some concerns about property singularism.

### 2.3 Plurals without pluralities?

The idea of using properties in connection with plural terms is not new. It has a rather long history: it may already be found in Russell and it has been suggested repeatedly since then.\(^4\) However, what has usually been proposed, which we may call the traditional approach, differs from property singularism in a critical way. The traditional approach employs property talk, thus the language of higher-order logic, for regimentation. Then it presumably relies on a set-theoretic semantics to characterize the relation of logical consequence for the higher-order language used for regimentation. By contrast, property singularism employs properties only in the semantics, where it uses higher-order logic taken at face value. Property talk does not appear at the level of regimentation. This is an essential feature of the view.

Yi (1999, 2005) has offered some arguments against the traditional approach.\(^5\) It is worth examining his arguments to see if any of the problems he raises for the traditional approach carries over to property singularism. First, consider this inference:

---


Russell and Whitehead cooperate

(2.1) Russell and Whitehead are the authors of *Principia Mathematica*

Yi points out that, on the traditional approach, the inference does not come out valid unless one is willing to assume in the object language that, as a matter of logic, the second-order predicate rendering ‘cooperate’ (or any other plural predicate) satisfies extensionality. Fortunately, on our construal of property singularism, (2.1) can be shown to be valid without any special assumption about the extensionality of properties in the metalanguage. Let us check this claim. The following appears to be a correct regimentation of the inference in $\mathcal{L}_{PL}$.

\[
\begin{align*}
\exists xx \ (\forall y (y \prec xx \leftrightarrow y = r \lor y = w) \land C(xx)) \\
\forall x (A(x, PM) \leftrightarrow x = r \lor x = w) \\
\exists xx \ (\forall y (y \prec xx \leftrightarrow A(y, PM) \land C(xx))
\end{align*}
\]

In any model of the language, the first premise is true just in case there is a singular first-order property $X^1$ that applies to the denotation of ‘Russell’, the denotation of ‘Whitehead’, and to nothing else; and such a property has the singular second-order property $Y^2$ denoted by ‘cooperate’. The second premise is true just in case the singular first-order relation $R^1$ signified by ‘is an author of’ only holds between the denotation of ‘Russell’ and the denotation of ‘Principia Mathematica’, and between the denotation of ‘Whitehead’ and the denotation of ‘Principia Mathematica’. So there is a singular property, namely $X$, such that it applies to all and only the things that $R$ relates to the denotation of ‘Principia Mathematica’ and it has $Y$. Therefore, the conclusion is true as well. The inference is thus valid according to property singularism.
Here is an example of a sentence that should be regarded as a logical truth, at least if one regards (2.1) as logically valid. One might say: ‘take some things and take some other things, if the first cooperate and the second are the same as the first, then the second cooperate too’. This can be regimented as (2.3).

(2.3) \( \forall xx \forall yy \ ((C(xx) \land yy \approx xx) \rightarrow C(yy)) \).

According to property singularism, this sentence is interpreted as saying that, relative to a given domain, if the second-order property \( X^2 \) interpreting ‘cooperate’ holds of any property \( Y \), then it holds of any property \( Z \) coextensive with \( Y \). We have called the properties that satisfy this condition \textit{extensional}. The worry is that there are non-extensional second-order properties, hence (2.3) would fail to be a logical truth. Some remarks are in order.

In reaction to this worry, one might be tempted to regard the object language relation \( \approx \) as primitive, rather than as an abbreviation, and take it to denote property identity. This would render (2.3) a logical truth without requiring extensionality. This move, however, would not work in general. The following sentence should have the same logical status of (2.3).

(2.4) \( \forall xx \forall yy \ ((C(xx) \land \forall x (x \prec xx \leftrightarrow x \prec yy)) \rightarrow C(yy)) \).

The appeal to plural identity would not help to avoid extensionality in the case of (2.4). It seems, then, that the property singularist must require that the higher-order property signified by ‘cooperate’ be extensional after all. This is, indeed, what we have done above when specifying the semantic values of plural predicates. No analogous requirement had to be imposed by the pluralist, as plural properties are usually assumed to be extensional. The extensionality requirement salvages the validity of (2.4) in the framework of property singularism, but is it acceptable?
Yi writes:

[It]t is one thing to hold the extensional conception, quite another to hold, more implausibly, that the truth of the conception rests on logic alone. [...] One cannot meet the objections [...] under the assumption that the property indicated by “COOPERATE” is one that Russell calls extensional (that is, a second-order property instantiated by any first-order property coextensive with one that instantiates it). This does not help unless the assumption holds by logic [...].

This applies to a view about the regimentation of plural terms, such as the traditional approach, but it does not apply to a semantic proposal like property singularism. The property-singularist does not have to embrace an extensional conception of relations — let alone embrace it as logically true. On semantic grounds, she just has to confine the semantic values of collective plural predicates to extensional second-order properties. Since properties are invoked in the semantics and not in the object language, there is no worry about having to assume the logicality of the theory of properties. Semantic theories do usually rely on axioms that are not logically true, for example those of set theory. So there is nothing special about property singularism in this regard.

Of course, there is no shortage of extensional relations. It follows from the principle of comprehension for second-order properties that for any second-order property $P^2$, there is an extensionalization of $P$:

\[
(2.5) \exists X^2 \forall Y^1 (X(Y) \leftrightarrow \exists Z^1 (P(Z) \land \forall x (Yx \leftrightarrow Zx))).
\]

Thus, the problem is not whether extensional relations are available. The problem, rather, is whether one may impose a restriction on the kind of the semantic values

---

assigned to plural predicates and require that they be extensional. As far as I can see, there is nothing that advises against the restriction.

Interestingly enough, there are arguments intended to show that we have reason to doubt that the extensionality requirement should be imposed in the first place. Let us introduce plural proper names such as ‘the Beatles’ or ‘the Joneses’. Now suppose that, unbeknown to most, the members of the Beatles also formed a rowing team, called ‘the Rowers’. Then, we have:

(2.6) If the Beatles were popular and the Beatles were the same as the Rowers, then the Rowers were popular.

Formally,

\[(2.7) \quad (P(bb) \land \forall x (x \preceq bb \leftrightarrow x \preceq rr)) \to P(rr),\]

where \(bb\) and \(rr\) are plural constants standing, respectively, for ‘the Beatles’ and ‘the Rowers’. This sentence is hardly a logical truth — it is indeed false in the scenario envisaged. So, if ‘being popular’ denotes a property, it is not an extensional one. The property-singularist can easily avoid the claim that (2.6) is a logical truth by dropping the extensionality requirement. The pluralist, on the other hand, seems to be committed to treat the sentence as logically true — at least according to the usual way of thinking about plural properties. Of course, the pluralist might just say ‘mea culpa’ and deny that plural properties are extensional. Admittedly, examples of this sort are hardly conclusive without further considerations. However, pending a more thorough discussion, extensionality should be regarded as a double-edged sword.

Another potential source of concern about property singularism is that it might illegitimately import logical principles that are only correct with respect to higher-order
logic. For example, one might worry that it would validate principles of comprehension for higher-order logic rather than plural comprehension. As we noted earlier, plural comprehension is generally taken to be valid.

(2.8) If something is \( \Phi \), then there are some things such that anything is one of them if and only if it is \( \Phi \).

It is relatively easy to check that (2.8) comes out valid in the semantics outlined in Appendix A2. On the contrary, the plural version of second-order comprehension does not come out valid.

(2.9) \( \exists x x \forall x (x \prec xx \leftrightarrow \phi(x)) \).

Since property singularism interprets plural quantification as quantification over non-empty properties, (2.9) is not unrestrictedly valid. Property singularism uses the resources of higher-order logic in the metalanguage, without imposing its structure on \( \mathcal{L}_{PL} \).

Finally, one might worry that property-pluralism generates a mismatch of types among predicates.\(^7\) Consider these two sentences.

(2.10) (a) Russell and Whitehead wrote a book.

(b) Wittgenstein wrote a book.

From these two sentences one can infer:

(2.11) Russell and Whitehead wrote a book and Wittgenstein did too.

\(^7\)See (Yi 1999), p.185.
The possibility of ellipsis displayed in (2.11) might be taken to support the thesis that the predicate ‘wrote a book’ occurs univocally in (2.10a) and (2.10b) and, thus, the thesis that it should be assigned the same semantic value. However, according to property-singularist semantics, the predicate denotes a second-order property in (2.10a), but a first-order property in (2.10b). Notice that the same difficulty is faced by the pluralist who disallows multifaceted relations. Such a pluralist has to assign a plural property to the predicate in (2.10a) and a singular property to the predicate in (2.10b). So this is a prima facie problem for both views.\(^8\)

A number of other tests could be invoked in support of the thesis that the predicate ‘wrote a book’ occurs univocally in (2.10a) and (2.10b).\(^9\) For example, it seems that one can quantify over both predicates.

(2.12) Wittgenstein did something that Russell and Whitehead did.

Moreover, the predicate can be used with disjunctive noun phrases with a plural and singular component such as ‘two famous English logicians or Wittgenstein’, as in the sentence:

(2.13) Two famous English logicians or Wittgenstein wrote a book.

Finally, both ‘Russell and Whitehead (did)’ and ‘Wittgenstein (did)’ would be appropriate answers to the question: ‘Who wrote a book?’.

I am skeptical that these tests provide conclusive evidence for the univocity claim, but arguing for skepticism would take us too far afield. What matters the most here

\(^8\) Notice that assigning different semantic values to the two occurrences of ‘wrote a book’ is inevitable if one wants to think of the language of plurals as an extension of first-order logic.

\(^9\) Thanks to Ben Caplan and Alex Oliver here. For a discussion, see Oliver’s “Against Predicative Theories of Plurality” (unpublished).
is that, even if there is reason to accept the univocity claim, both the pluralist and the property-singularist could cope with the consequences. As mentioned above, both could resort to multifaceted properties. The pluralist would have to introduce properties that take objects as well as pluralities as arguments. The property-singularist would have to introduce second-order properties that take objects as well as first-order properties as arguments. An alternative (but much less attractive) response could be based on ‘lifting the type’ of singular terms to match those of plural ones. For example, the pluralist might take ‘Wittgenstein’ to denote a plurality composed of one individual, while the property-singularist might take it to denote a property that has only one individual in its extension. In either case, both semantics have the resources to account for the data.

2.4 Conclusion

Semantic considerations about plurals have led pluralists to embrace a plural conception of reality. The plural conception of reality accepts the existence of plural properties in addition to singular ones. As I have argued, property singularism provides a viable alternative to pluralism and thus undermines the semantic motivation for the acceptance of plural properties. As the result in Appendix B shows, the two semantic approaches deliver the same relation of logical consequence. We must conclude that the singular conception of reality is rich enough to provide a satisfactory semantics for plurals. Semantic considerations do not give us reasons to abandon it.
Appendix A1: Pluralism

In this appendix, I present the pluralist approach put forward by Yi (2005, 2006) and Oliver and Smiley (2006). My presentation differs from theirs in that, as discussed in section 2.1, we do not allow multifaceted relations. This requires us to introduce three distinct types of variable assignments and two distinct types of interpretations. A model is then defined as a relation among a domain and a representative of each type of interpretation. Satisfaction becomes a relation among a formula, a model, and a representative of each type of variable assignment from the domain of the model. We define an \( n \)-ary relation to be \textit{exclusive at its} \( i \)-\textit{th argument place} \( (1 \leq i \leq n) \) just in case if it holds of entities or pluralities \( E_1, \ldots, E_n \) and \( E_1^*, \ldots, E_n^* \), then \( E_i = E_i^* \).\(^{10}\) If the relation holds of \( E_1, \ldots, E_n \) and is exclusive at its \( i \)-\textit{th argument place}, then \( E_i \) is said to be the exclusive \( i \)-\textit{th argument} of the relation. Derivatively, we call a relation \textit{exclusive} if it is exclusive at each of its argument places.

Domains are taken to be pluralities. Let us first define two types of variable assignments from a given domain. An assignment for a singular (plural) variable is a plural first-order relation holding among a domain, a singular (plural) variable, and an object in the domain (some things in the domain). For any things \( \alpha \) functioning as domain, we say that \( S_s \) is a \textit{singular variable assignment} from \( \alpha \) and we say that \( S_p \) is a \textit{plural variable assignment} from \( \alpha \) if and only if they are both exclusive at their first argument place and

\[
\text{(2.14) } \begin{align*}
\text{(i) for every singular variable } v \text{ of the object language, there is a unique} \\
x \prec \alpha & \text{ such that } S_s(\alpha, v, x);
\end{align*}
\]

\[
\text{(ii) for any plural variable } vv \text{ of the object language, there are some things}
\]

\(^{10}\)Identity with respect to pluralities should be understood as \( \approx \), defined on p. 10.
Chapter 2. Is Two a Plural Property?

Let us define interpretations of the object language. Let $\alpha\alpha$ be a domain. A plural first-order relation $I^1_n$ is a name interpretation with domain $\alpha\alpha$, a plural second-order relation $I^2_p$ is a plural interpretation with domain $\alpha\alpha$, and a plural second-order relation $I^2_s$ is a singular interpretation with domain $\alpha\alpha$, if and only if they are all exclusive at their first argument place and

(2.15) (i) for every constant $c$, there is a unique $x < \alpha\alpha$ such that $I_n(\alpha\alpha, c, x)$;

(ii) for every $n$-ary plural predicate $P$, there is a unique corresponding $n$-ary plural first-order relation $X$ such that (a) for every relatum $x$ or relata $xx$ of $X$, $x < \alpha\alpha$ and $xx \leq \alpha\alpha$, and (b) $I_p(\alpha\alpha, P, X)$;

(iii) for every $n$-ary singular predicate $P$, there is a unique $n$-ary singular first-order relation $X$ such that (a) for every relatum $x$ of $X$, $x < \alpha\alpha$, and

(b) $I_s(\alpha\alpha, P, X)$.

A model with domain $\alpha\alpha$ is an exclusive plural third-order relation $M_I$ such that $M_I(I_n, I_p, I_s)$ for some name, plural, and singular interpretations $I_n, I_p, I_s$, all with domain $\alpha\alpha$. Satisfaction can now be defined as a four-place relation $\mathbf{Sat}(\phi, M_I, S_s, S_p)$ holding among a formula of the object language, a model $M_I$ with domain $\alpha\alpha$, a singular variable assignment $S_s$, and a plural one $S_p$ (both assignments from $\alpha\alpha$). We
use the notation $M_I \models \phi [S_s, S_p]$ for $\text{Sat}(\phi, M_I, S_s, S_p)$. Let $t$ be a singular term — a singular constant or a singular variable. By $I_n/S_s(t, x)$ we denote the name interpretation $I_n$ such that $I_n(\alpha \alpha, t, x)$ if $t$ is constant, or the singular assignment $S_s$ such that $S_s(\alpha \alpha, t, x)$ if $t$ is a variable. The inductive clauses characterizing satisfaction are as follows. For every domain $\alpha \alpha$ and for every interpretation $M_I(I_n, I_p, I_s)$ with domain $\alpha \alpha$, and variable assignments $S_s$ and $S_p$ from $\alpha \alpha$:

\begin{enumerate}
  \item[(i)] for any singular terms $t$ and $r$, $M_I \models t = r [S_s, S_p]$ if and only if, for every $x$ and $y$, if $I_n/S_s(t, x)$ and $I_n/S_s(r, y)$, then $x = y$;
  \item[(ii)] for any singular term $t$ and plural variable $vv$, $M_I \models t \prec vv [S_s, S_p]$ if and only if, for any $x$ and any $xx$, if $I_r/S_s(t, x)$ and $S_p(\alpha \alpha, vv, xx)$, then $x$ is one of $xx$;
  \item[(iii)] for any $n$-ary plural predicate $P^{n[m]}$, plural terms $\Theta_1, \ldots, \Theta_m$, and singular terms $\theta_{m+1}, \ldots, \theta_n$, $M_I \models P(\Theta_1, \ldots, \Theta_m, \theta_{m+1}, \ldots, \theta_n) [S_s, S_p]$ if and only if $X(xx_1, \ldots, xx_m, x_{m+1}, \ldots, x_n)$, where $X$ is the first-order relation such that $I_p(\alpha \alpha, P, X)$ and for every for $i$ ($1 \leq i \leq m$) and $j$ ($m + 1 \leq j \leq n$), $S_p(\alpha \alpha, \Theta_i, xx_i)$ and $I_n/S_s(\alpha \alpha, \theta_j, x_j)$;
  \item[(iv)] for any $n$-ary singular predicate $P$ and singular terms $t_i, 1 \leq i \leq n$, $M_I \models P(t_1, \ldots, t_n) [S_s, S_p]$ if and only if $X(x_1, \ldots, x_n)$, where $X$ is the singular first-order relation such that $I_s(\alpha \alpha, P, X)$, and for each $i$, $x_i$ is the unique object such that $I_n/S_s(t_i, x_i)$;
  \item[(v)] for every formula $\phi$ and singular variable $v$, $M_I \models \exists v \phi [S_s, S_p]$ if and only if there is $x \prec \alpha \alpha$ such that $M_I \models \phi [S_s(v/x), S_p]$;
  \item[(vi)] for every formula $\phi$ and plural variable $vv$, $M_I \models \exists vv \phi [S_s, S_p]$ if and only if there are some $xx \lhd \alpha \alpha$ such that $M_I \models \phi [S_s, S_p(vv/xx)]$;
\end{enumerate}

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(vi) for every formula \( \phi \), \( M_I \models \neg \phi [S_s, S_p] \) if and only if \( M_I \not\models \phi [S_s, S_p] \);

(vii) for every formula \( \phi \) and \( \psi \), \( M_I \models \phi \land \psi [S_s, S_p] \) if and only if \( M_I \models \phi [S_s, S_p] \) and \( M_I \models \psi [S_s, S_p] \).

The final step is the definition of logical consequence and logical truth. In order to define them, one needs to be able to quantify over models. This is where we ascend to plural third-order quantification. We have:

\[(2.17) \]

(i) for any set of sentences \( \Gamma \) and any sentence \( \phi \), \( \phi \) is a logical consequence of \( \Gamma \) (\( \Gamma \models \phi \)) if and only if for any domain \( \alpha \alpha \), any model \( M_I \) from \( \alpha \alpha \), and any singular variable assignment \( S_s \) and plural variable assignment \( S_p \), both from \( \alpha \alpha \): if \( M_I \models \gamma [S_s, S_p] \) for every \( \gamma \) in \( \Gamma \), then \( M_I \models \phi [S_s, S_p] \);\(^{11}\)

(ii) a sentence \( \phi \) is a logical truth if \( \emptyset \models \phi \).

This completes a more formal presentation of the pluralist semantics for \( L_{PL} \).

**Appendix A2: Property singularism**

I now formulate the property-singularist semantics for \( L_{PL} \). The basic idea is simple: we replace any use of pluralities with corresponding singular first-order properties and any use of plural properties with corresponding singular second-order properties. We then redefine interpretations and satisfaction to take that into account. The definition of exclusive relation carries over from Appendix A1.

\(^{11}\) As in standard model theory, it can be proved that, when dealing with satisfaction of sentences, variable assignments are irrelevant in the sense that, if \( M_I \models \sigma [S_s, S_p] \) (for sentence \( \sigma \)), then \( M_I \models \sigma [S_s^*, S_p^*] \) for any other assignments \( S_s^* \) and \( S_p^* \) from the relevant domain.
A domain is a non-empty unary property $D$ and an object $x$ is in the domain if $Dx$. All-inclusive domains are available through universal properties such as being self-identical. The principle of comprehension delivers such properties. \(^{12}\) Let $D$ be a domain. For any property $X$, we define $\subseteq$ thus:

$$X \subseteq D \text{ if and only if for every } x, \ Xx \text{ only if } Dx.$$ 

As before, we define two types of variable assignments from $D$. An assignment $R_s$ for singular variables is a singular second-order relation holding among a domain, a singular variable, and an object. An assignment for plural variables is a singular second-order relation holding among a domain, a plural variable, and a non-empty singular first-order property. We say that $R^s_s$ is a singular variable assignment from $D$ and $R^2_p$ is plural variable assignment from $D$ if and only if they are both exclusive in the first-argument place and

\[\begin{align*}
\text{(2.18) (i) for every singular variable } v \text{ of the object language, there is a unique } x \\
&\text{ such that } Dx \text{ and } R_s(D, v, x); \\
\text{(ii) for any plural variable } v v \text{ of the object language, there is a unique } \\
&\text{non-empty singular first-order property } X^1 \text{ such that } X \subseteq D \text{ and} \\
&R^2_p(D, v v, X).
\end{align*}\]

\(^{12}\) The principle of comprehension for singular first-order properties states that:

\[\text{(CP1) } \exists X^1 \forall x_1, \ldots, x_n (X^1(x_1, \ldots, x_n) \leftrightarrow \phi_1(x_1, \ldots, x_n)),\]

where $\phi_1(x_1, \ldots, x_n)$ is a formula in which, besides logical vocabulary, only singular first-order expressions can occur and in which, obviously, $X^1$ does not occur free. An all-inclusive domain is given by the following instance:

\[\text{(CPA) } \exists X \forall x (Xx \leftrightarrow x = x).\]
Variants of variable assignments are defined as before. For example, if $R_p$ is a plural variable assignment from $D$ and $Y$ is any non-empty property such that $Y \subseteq D$, $R_p(vv/Y)$ denotes the variable assignment that is just like $R_p$ except, possibly, for the fact that it holds among $D$, $vv$, and $Y$, that is, $R_p(vv/Y)(D,vv,Y)$.

Next we define interpretations. A singular second-order relation $H_n^1$ is a name interpretation with domain $D$, a singular third-order relation $H_p^3$ is a plural interpretation with domain $D$, and a singular second-order relation $H_s^2$ is a singular interpretation with domain $D$ provided that they are all exclusive in their first argument place and

$$\text{(2.19)} \quad \begin{align*}
&\text{(i) for every constant } c, \text{ there is a unique } x \text{ such that } Dx \text{ and } H_n(D,c,x); \\
&(\text{ii) for every } n\text{-ary plural predicate } P, \text{ there is a unique corresponding } n\text{-ary singular second-order extensional relation } X^2 \text{ such that} \\
&(\text{a) for every relatum } x \text{ or } Y^1 \text{ of } X^2, \text{ } Dx \text{ and } Y \subseteq D, \text{ and} \\
&(\text{b) } H_p^3(D,P,X);^{13} \\
&(\text{iii) for every } n\text{-ary singular predicate } P, \text{ there is a unique corresponding } \\
&n\text{-ary singular first-order relation } X^1 \text{ such that } H_s(D,P,X).
\end{align*}$$

A model $M_H$ from a domain $D$ is an exclusive forth-order singular relation such that $M_H(H_n,H_p,H_s)$ for some name, plural, and singular interpretations $H_n$, $H_p$, $H_s$.

\textsuperscript{13}If the $i$-th argument place of $P$ is singular, a corresponding relation takes an object at its $i$-th place. If the $i$-th argument place of $P$ is plural, a corresponding relation takes a first-order property at its $i$-th place. This clarifies the intended meaning of corresponding relation. We imposed the condition that the interpretation of a plural predicate be an extensional relation. A second-order relation $X^2$ is extensional just in case if it holds of $\Omega_1,...,\Omega_i,...,\Omega_n$ and $\Omega_i$ ($1 \leq i \leq n$) is a first-order property, then for every first-order property $\Sigma$ coextensive with $\Omega_i$, $X^2$ also holds of $\Omega_1,...,\Sigma,...,\Omega_n$. Issues related to the extensionality requirement are discussed in section 2.3.
and $H_s$ with common domain $D$. For every model $M_H$ with domain $D$ and variable assignments $R_s$ and $R_p$ from $D$, satisfaction is characterized by the following clauses:

(2.20) (i) for any singular terms $t$ and $r$, $M_H \models t = r \ [R_s, R_p]$ if and only if, for every $x$ and $y$, if $H_n/R_s(t, x)$ and $H_nR_s(r, y)$, then $x = y$;

(ii) for any singular term $t$ and plural variable $vv$, $M_H \models t < vv \ [R_s, R_p]$ if and only if, for any $x$ and any singular first-order property $X$, if $H_n/R_s(t, x)$, and $R_p(D, vv, X)$, then $Xx$;

(iii) for any $n$-ary plural predicate $P_n[m]$, plural terms $\Theta_1, ..., \Theta_m$, and singular terms $\theta_{m+1}, ..., \theta_n$, $M_H \models P(\Theta_1, ..., \Theta_m, \theta_{m+1}, ..., \theta_n) \ [R_s, R_p]$ if and only if $Z^2(X_1, ..., X_m, x_{m+1}, ..., x_n)$, where $Z$ is the singular second-order relation such that $H_p(D, P, X)$ and, for every $i (1 \leq n \leq n)$ and $j (m + 1 \leq j \leq n)$, each $X_i$ and $x_j$ is such that

$$R_p(D, \Theta_i, X_i) \text{ and } H_n/R_s(\theta_j, x_j);$$

(iiib) for any $n$-ary singular predicate $P$ and singular terms $t_i, 1 \leq i \leq n$, $M_H \models P(t_1, ..., t_n) \ [R_s, R_p]$ if and only if $X(x_1, ..., x_n)$, where $X$ is the singular first-order property such that $H_s(D, P, Z)$ and, for each $i$, $x_i$ is the unique object such that $H_n/R_s(t_i, x_i);$

(iv) for every formula $\phi$ and singular variable $v$, $M_H \models \exists v \phi \ [R_s, R_p]$ if and only if there is $x$ such that $Dx$ and $M_H \models \phi \ [R_s(v/x), R_p];$

(v) for every formula $\phi$ and plural variable $vv$, $M_H \models \exists vv \phi \ [R_s, R_p]$ if and only if there is a non-empty singular first-order property $X^1$ such that, for every $x$, if $Xx$, then $Dx$, and $M_H \models \phi \ [R_s, R_p(vv/X)];$

(vi)-(vii) the clauses for negation and conjunction are the obvious ones.
Finally, we can define logical consequence and logical truth. Quantification over models requires us to ascend to fourth-order quantification. We have:

(2.21) (i) for any set of sentences \( \Gamma \) and any sentence \( \phi \), \( \phi \) is a logical consequence of \( \Gamma \) (\( \Gamma \models \phi \)) if and only if, for every domain \( D \), for any model with domain \( D \), for every singular variable assignment \( R_s \) and for every plural variable assignment \( R_p \) both from \( D \), if \( M_H \models \gamma [R_s, R_p] \) for every \( \gamma \) in \( \Gamma \), then \( M_H \models \phi [R_s, R_p] \);

(ii) a sentence \( \phi \) is a logical truth if \( \emptyset \models \phi \).

This concludes the exposition of the property-singularist semantics for \( L_{PL} \).

**Appendix B: The equivalence**

We start from the assumptions that for every plurality there is a singular first-order property that is coextensive with it and that, conversely, for any non-empty singular property there is a plurality that is coextensive with it. The assumption is certainly problematic if properties are thought of as objects. In that case, the plural version of Cantor’s theorem would bar the assumption. But once properties are thought of as predicatable entities, the difficulty is removed and we may assume:

(2.22) \( \forall xx \exists X \forall x (Xx \leftrightarrow x \prec xx) \).

(2.23) \( \forall X (\exists x Xx \rightarrow \exists xx \forall x (Xx \leftrightarrow x \prec xx)) \).

We also assume that plural relations and second-order relations match:

(2.24) \( \forall Y^2 \exists Y^1p \forall xx (Y^1p_xx \leftrightarrow \exists X^1(Y^2(X^1) \& X^1 \equiv x)) \)
We define the notion of extended model, which is a model combined with variable assignments. Let $M_I$ be an $I$-model, i.e., a model in the pluralist sense, and let $S_s$ and $S_p$ be pluralist variable assignments from the domain of $M_I$. Then $M_I$ is an extended $I$-model of $M_I$ with respect to $S_s$ and $S_p$ just in case $M_I$ is an exclusive relation such that $M_I(M_I, S_s, S_p)$.

Analogously, for an $M_H$ models, i.e., a model in the sense of property singularism, we say that $M_H$ is an extended $H$-model of $M_H$ with respect to $R_s$ and $R_p$ just in case $M_H$ is an exclusive relation such that $M_H(M_H, R_s, R_p)$, where $R_s$ and $R_p$ are property-singularist variable assignments from the domain of $M_H$.

We say that $M_I$ satisfies a formula $\phi$ ($M_I \models \phi$) if and only if $M_I \models [S_s, S_p]$. A similar definition holds for $M_H$. We use $\equiv$ to stand for coextensionality between singular first-order properties and pluralities, so

$$X \equiv xx \text{ if and only if for every } x, Xx \text{ just in case } x \prec xx.$$

Now we characterize an appropriate relation $\Phi$ holding between extended $I$-models and extended $H$-models. Let $M_I$ and $M_H$ be models with $M_I(dd, I_n, I_p, I_s)$ and $M_H(D, H_n, H_p, H_s)$. The extended models $M_I$ and $M_H$ are then such that $M_I(M_I, S_s, S_p)$ and $M_H(M_H, R_s, R_p)$. We say that $\Phi(M_I, M_H)$ just in case these conditions are met:

(2.26) (i) for every $x$, $x \prec dd$ if and only if $Dx$;

(ii) for every plural variable $vv$, for every plurality $xx$, and for every first-order singular property $X$, if $S_p(dd, vv, xx)$ and $R_p(D, vv, X)$, then

$$xx \equiv X;$$

\(^{14}\)For the definition of exclusive relation see Appendix A1.
(iii) for every singular variable $v$, and every object $x$ and $y$, if $S_s(dd, v, x)$ and $R_s(D, v, y)$, then $x = y$;

(iv) for every constant $c$, and every objects $x$ and $y$ such that $I_n(dd, c, x)$ and $H_n(D, c, y)$, $x = y$;

(v) for every $n$-ary plural predicate $P^{n[m]}$, for every plural first-order relation $Y^{1p}$, for every singular second-order relation $Z^2$ such that $I_p(dd, P, Y)$ and $H_p(D, P, Z)$, for every singular first-order properties $X_1, \ldots, X_m$, for every objects $x_{m+1}, \ldots, x_n$, and for every pluralities $xx_1, \ldots, xx_m$ such that, for each $i$ $(1 \leq i \leq m)$, $X_i \equiv xx_i$:

$$Z(X_1, \ldots, X_m, x_{m+1}, \ldots, x_n) \text{ if and only if } Y(xx_1, \ldots, xx_m, x_{m+1}, \ldots, x_n);$$

(vi) for every $n$-ary singular predicate $P$ and for every singular first-order relation $X$ and $Y$ such that $I_s(dd, P, X)$ and $H_p(D, P, Y)$, $X = Y$.

It is important to observe that it follows from (2.22) and (2.23) that for every $M_I$ there is $M_H$ such that $\Phi(M_I, M_H)$ — and vice versa.\textsuperscript{15} The next step is to prove by induction the following claim.

**Claim.** For every formula $\phi$, and for any $M_I$ and $M_H$, if $\Phi(M_I, M_H)$, then $M_I \models \phi$ if and only $M_H \models \phi$.

**Proof.** Let $M_I$ and $M_H$ be extended models such that $\Phi(M_I, M_H)$. To fix the notation assume that the models $M_I$ and $M_H$ are such that $M_I(dd, I_n, I_p, I_s)$ and

\textsuperscript{15}Strictly speaking, no relation denoted by $\Phi$ is part of the official hierarchy described in section 2.1. However, reference to $\Phi$ could always be eliminated in favor of the explicit characterization in (2.26).
$M_H(D, H_n, H_p, H_s)$, and assume that the extended models are such that $M_I(M_I, S_s, S_p)$ and $M_H(M_H, R_s, R_p)$. We work through the inductive steps. When left out, the proof of a certain direction of the biconditional can be recovered in a straightforward way from the proof of the opposite direction.

1) Suppose that $\phi$ is $P(t_1, \ldots, t_n)$ for singular terms $t_1, \ldots, t_n$. Then $M_I \models \phi$ if and only if $X(x_1, \ldots, x_n)$, with $X$ such that $I_s(dd, P, X)$ and $x_i$ ($1 \leq i \leq n$) such that $I_s/S_s(dd, t_i, x_i)$. By (2.26vi) and (2.26ii,iv), $H_s(D, P, X)$ and $H_s/R_s(D, t_i, x_i)$ for each $i$. Thus, $M_H \models \phi$ if and only if $X(x_1, \ldots, x_n)$. So $M_I \models \phi$ if and only if $M_H \models \phi$.

2) Suppose that $\phi$ is $P(\Theta_1, \ldots, \Theta_m, \theta_{m+1}, \ldots, \theta_n)$ for plural terms $\Theta_1, \ldots, \Theta_m$ and singular terms $\theta_{m+1}, \ldots, \theta_n$. If $M_I \models \phi$, then $Y(x_1, \ldots, x_m, x_{m+1}, \ldots, x_n)$ where $Y$ is such that $I_p(dd, P, Y)$ and for each $i$ ($1 \leq i \leq m$) and $j$ ($m + 1 \leq j \leq n$), $S_p(dd, \Theta_i, xx_i)$ and $I_n/S_s(dd, \theta_j, x_j)$. Now (2.26v) and (2.26ii,iii,iv) entail that, for $Z$ such that $H_p(D, P, Z)$ and for $X_1, \ldots, X_m$ such that, for each $i$ ($1 \leq i \leq m$), $R_p(D, \Theta_i, X_i)$, $Z(X_1, \ldots, X_m, x_{m+1}, \ldots, x_n)$. Since for each $j$ ($m + 1 \leq j \leq n$), $R_s(D, \theta_j, x_j)$, it follows that $M_H \models \phi$.

3) Now suppose that $\phi$ is $t < vv$ for a singular term $t$ and a plural variable $vv$. Suppose that $M_I \models \phi$. Then, by definition, $x < xx$ where $I_s/S_s(dd, t, x)$ and $S_p(dd, vv, xx)$. Clause (2.26iii,iv) and (2.26ii) entail that for $x$ and $X$ such that $R_s(D, vv, X)$ and $H_s/R_s(D, t, x)$, we have that $Xx$. So, by (2.20ii), $M_H \models \phi$.

4) For $\phi$ equal to $t = s$, $M_I \models s = t$ if and only if $x = y$, for $x$ and $y$ such that $I_s/S_s(dd, t, x)$ and $I_s/S_s(dd, s, y)$. By clause (2.26ii,iii), that is the case if and only if
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\[ H_s/R_s(D, t, x) \text{ and } H_s/R_s(D, s, y). \] This means that \( M_H \models s = t. \)

By inductive hypothesis, assume that the claim is true for formulae \( \phi \) and \( \psi \). The inductive cases for \( \neg \phi \) and \( \phi \land \psi \) are straightforward. For the existential quantifiers, we proceed as follows.

5a) Suppose that \( M_I \models \exists v \phi \). Then there is \( x \prec dd \) (hence \( Dx \)) such that \( M_I \models \phi [S_s(v/x), S_p] \). This means that \( M_I^* \models \phi \), where \( M_I^*(M_I, S_s(v/x), S_p) \). Let \( M_H^* \) be an extended \( H \)-model such that \( M_H^*(M_H, R_s(v/x), R_p) \). Since \( \Phi(M_I^*, M_H^*) \), it follows from the inductive hypothesis that \( M_H^* \models \phi \). That is, \( M_I \models \phi [R_s(v/x), R_p] \). Since \( Dx \), \( M_I \models \exists v \phi [R_s, R_p] \). Therefore \( M_H \models \exists v \phi \).

5b) Suppose that \( M_I \models \exists vv \phi \). This means that there are some \( xx \prec dd \) such that \( M_I \models \phi [S_s(vv/xx)] \). Let \( M_I^* \) be such that \( M_I^*(M_I, S_s, S_p(vv/xx)) \). Then \( M_I^* \models \phi \). Let \( X \) be a singular first-order property coextensive with \( xx \) and let \( M_H^* \) be such that \( M_H^*(M_H, R_s, R_p(vv/X)) \). It follows that \( \Phi(M_I^*, M_H^*) \). Thus, by inductive hypothesis, \( M_H^* \models \phi \). That is, \( M_H \models \phi [R_s, R_p(vv/X)] \), hence \( M_H \models \exists vv \phi [R_s, R_p] \). Therefore, \( M_H \models \exists vv \phi \).

As remarked earlier, when dealing with sentences, variable assignments are irrelevant. So, when we say that a model satisfies a sentence, we mean that it satisfies that sentence with any variable assignments. This justifies that notation \( M_I \models \sigma \) or \( M_H \models \sigma \) (for any sentence \( \sigma \)). It is a corollary of the claim just proved that for any sentence \( \sigma \), and for any extended models \( M_I \) and \( M_H \), if \( \Phi(M_I, M_H) \), then \( M_I \models \sigma \) if and only if \( M_H \models \sigma \).
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If $\Delta$ is a set of formulas, $M_I \models \Delta$ ($M_H \models \Delta$) will abbreviate: $M_I \models \delta$ for every $\delta \in \Delta$ ($M_H \models \delta$ for every $\delta \in \Delta$). Finally, $\Delta \models \sigma$ ($\Delta \models \sigma$) stands for the following: for every $I$-model $M_I$, if $M_I \models \Delta$, $M_I \models \sigma$ (for every $H$-model $M_H$, if $M_H \models \Delta$, $M_H \models \sigma$). We are now ready to prove the equivalence thesis.

**Equivalence Thesis.** For any sentence $\sigma$ and set of sentences $\Gamma$,

$$\Gamma \models I \sigma \text{ if and only if } \Gamma \models H \sigma.$$  

**Proof.** Suppose that $\Gamma \models I \sigma$. Assume for *reductio* that there is an $H$-model, call it $M_H$, such that $M_H \models \Gamma$ but $M_H \not\models \sigma$. Let $M_H$ be any extended model of $M_H$. We have that $M_H \models \Gamma$ and $M_H \not\models \sigma$. Now, there is an extended $I$-model $M_I$ such that $\Phi(M_I, M_H)$. If follows from the above claim that $M_I \models \Gamma$ and $M_I \not\models \sigma$. Therefore, there is an $I$-model $M_I$ such that $M_I \models \Gamma$ but $M_I \not\models \sigma$. This contradicts the assumption that $\Gamma \models \phi$. Thus, there is no $H$-model $M_H$ such that $M_H \models \Gamma$ but $M_H \not\models \sigma$. This means that $\Gamma \models H \phi$. The other direction is proved similarly.  

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Chapter 3

Untyped Pluralism

This chapter is a critical discussion of a pluralist approach to plurals that has been put forward by McKay (2006). According to this approach, the semantic values of plural predicates are relations conceived as objects. No type distinction between objects and properties is introduced. For this reason, I will call this view *untyped pluralism*. I will argue that untyped pluralism is problematic on two counts. First, since allowing unrestricted domains is crucial to pluralism, the proposal is subject to a variant of a Russell-style argument proposed by Timothy Williamson. Second, invoking relations as objects runs against a plural version of Cantor’s theorem. Both counts, I suggest, provide strong reasons to be dissatisfied with this approach.

### 3.1 Plural satisfaction and untyped relations

In this section, I present and discuss a simplified version of the model theory associated with untyped pluralism. The object language around which the model theory will be developed is $\mathcal{L}_{PL}$. The overall account of plurals offered by McKay is more complex...
than the one discussed here. However, it is more complex in ways that do not affect the main points I want to make. Given that our main focus is on foundational issues, some of the details of his work will have to be put aside. For instance, McKay provides an account of a variety of natural language quantifiers related to plurals; my discussion will be confined to existential ones. For the sake of exposition, I will also have to modify McKay’s notation and introduce some auxiliary notions.

3.1.1 Model theory: the basic idea

As remarked above, the metalanguage in which untyped pluralism is formulated is of the same type as the object language. That is, it encompasses singular and plural quantifiers, and no higher-order quantifiers. A pairing operation on objects is postulated: for any things \(a\) and \(b\), the ordered pair \((a, b)\) exists and it is distinct from \(a\) and \(b\).\(^1\) I will also assume the existence of at least one object that is not a pair.\(^2\) Untyped pluralism has two main features. First, it takes plural predicates to signify plural relations (properties are just unary relations). Second, relations fall within the range of the first-order metalanguage quantifiers. I will call them *untyped relations* to distinguish them from relations conceived as higher-order entities.\(^3\)

\(^1\)By itself pairing is not problematic in this context. Although \(n\)-tuples are usually construed as sets, they can be taken to be *sui generis* objects. For a neutral account of \(n\)-tuples and their logic, see Tennant (2007).

\(^2\)Clearly enough, the pairing operations will have to be constrained by further postulates. *Extensionality*: two pairs are identical if and only if their corresponding components are identical. *Well-foundedness*: no pair appears in the decomposition of any of its components, the components of its components, and so on.

\(^3\)I will often use ‘relation’ and ‘property’ interchangeably. A property may be regarded as a unary relation and a relation may be regarded as an \(n\)-ary property \((n > 1)\).
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The basic structure of the model theory is the following. Interpretations assign objects to singular terms, pluralities to plural terms, singular relations to singular predicates, and plural relations to plural predicates. An atomic plural predication, such as ‘\(P(xx)\)’, is true in an interpretation just in case the \textit{things} assigned to the variable \(xx\) collectively have the plural property assigned to the predicate \(P\). The next subsection gives a semi-formal treatment of untyped pluralism.

3.1.2 Interpretations and satisfaction

Let \(\alpha\) be some things — an arbitrary domain of quantification. We want to define an interpretation of the object language \(L_{PL}\) with domain \(\alpha\). In McKay’s framework, an interpretation is neither an object nor a higher-order relation between the items of the object language and their semantic values. An interpretation is a plurality, namely, \textit{some things} appropriately related to \(\alpha\). The things among \(ii\) are ordered pairs coding the semantic values of the non-logical terminology of the object language.

We start with the notion of variable assignment from \(\alpha\). Some things \(ss\) are a variable assignment from \(\alpha\) if and only if

\begin{equation}
\begin{aligned}
&(i) \text{ for every singular variable } v \text{ of the object language, there is a unique } x \prec \alpha \text{ such that } (v, x) \prec ss; \\
&(ii) \text{ for any plural variable } vv \text{ of the object language, there are some things } xx \text{ such that } xx \preceq \alpha \text{ and for every } x, x \prec xx \text{ if and only if } (vv, x) \prec ss.
\end{aligned}
\end{equation}

If \(ss\) are a variable assignment from \(\alpha\) and \(x \prec \alpha\), by \(ss(v/x)\) we denote an assignment just like \(ss\) except, possibly, for the the fact that \((v, x) \prec ss(v/x)\). Similarly, if \(xx \preceq \alpha\), by \(ss(vv/xx)\) we denote an assignment just like \(ss\) except, possibly, for the fact that for every \(x, (vv, x) \prec ss(vv/xx)\) if and only if \(x \prec xx\).
Next we define the notion of an interpretation. Some things \(ii\) are an interpretation with domain \(\alpha\) if and only if

\[(3.2) \begin{align*}
(i) & \text{ for every constant } c, \text{ there is a unique } x \prec \alpha \text{ such that } (c, x) \prec ii; \\
(ii) & \text{ for every } n\text{-ary plural predicate } P, \text{ there is a unique corresponding } n\text{-ary plural property } \pi \text{ such that } (P, \pi) \prec ii; \\
(iii) & \text{ for every } n\text{-ary singular predicate } P, \text{ there is a unique } n\text{-ary singular property } \rho \text{ such that } (P, \rho) \prec ii.
\end{align*}\]

Satisfaction is defined as a relation among an interpretation \(ii\) from some domain, some things \(ss\) forming a variable assignment from that domain, and formulas of the object language: \(\text{Sat}(\phi, ii, ss)\), which will be written as \(ii \models \phi [ss]\). For an interpretation \(ii\) with domain \(\alpha\) and a variable assignment \(ss\) from \(\alpha\), the following clauses provide an inductive characterization of the satisfaction relation:

\[(3.3) \begin{align*}
(i) & \text{ for any singular term } t \text{ and } s, ii \models t = s [ss] \text{ if and only if, for every } x \text{ and } y, \text{ if } (t, x) \prec ii \text{ or } (t, x) \prec ss, \text{ and } (s, y) \prec ii \text{ or } (s, y) \prec ss, \text{ then } x = y; \\
(ii) & \text{ for any singular term } t \text{ and for any plural variable } vv, ii \models t < vv [ss] \text{ if and only if, for any } x \text{ and for any } yy, \text{ if } (t, x) \prec ii \text{ or } (t, x) \prec ss, \text{ and if for every } y, y < yy \text{ just in case } (vv, y) \prec ss, \text{ then } x < yy; \\
(iii) & \text{ for any } n\text{-ary plural predicate } P^{n[m]}, \text{ plural terms } \Theta_1, ..., \Theta_m, \text{ and singular terms } \theta_{m+1}, ..., \theta_n, ii \models P(\Theta_1, ..., \Theta_m, \theta_{m+1}, ..., \theta_n) [ss] \text{ if and only if } \\
& \quad \quad \quad \quad xx_1, ..., xx_m, x_{m+1}, ..., x_n \text{ are related by } \pi \text{ in that order,} \\
& \quad \quad \quad \quad \text{where } (P, \pi) \prec ii \text{ and} \\
& \quad \quad \quad \quad (1) \text{ for every } i \ (1 \leq i \leq m) \text{ and for every } x, x \prec xx_i \text{ if and only if } (\Theta_1, x); 
\end{align*}\]
(2) for every $j$ ($m + 1 \leq j \leq n$), $(\theta_j, x_j) \prec ss$ if $\theta_j$ is a variable, and $(\theta_j, x_j) \prec ii$ if $\theta_j$ is a constant;

(iib) for any $n$-ary singular predicate $P$ and singular terms $t_1, \ldots, t_n$,

\[ ii \models P(t_1, \ldots, t_n) [ss] \text{ if and only if } \]

\[ x_1, \ldots, x_n \text{ are related by } \rho \text{ in that order,} \]

where $(P, \rho) \prec ii$ and, for every $i$ ($1 \leq i \leq n$), $(\theta_i, x_i) \prec ss$ if $\theta_i$ is a variable, and $(\theta_i, x_i) \prec ii$ if $\theta_i$ is a constant;

(iv) for every formula $\phi$ and singular variable $v$, \[ ii \models \exists v \phi [ss] \text{ if and only if } \]

there is $x \prec \alpha \alpha$ such that \[ i \models \phi [ss(v/x)] ; \]

(v) for every formula $\phi$ and plural variable $vv$, \[ ii \models \exists vv \phi [ss] \text{ if and only if } \]

there are some $xx \prec \alpha \alpha$ such that \[ ii \models \phi [ss(vv/xx)] ; \]

(vi) for every formula $\phi$, \[ ii \models \neg \phi [ss] \text{ if and only if } ii \not\models \phi [ss] ; \]

(vii) for every formula $\phi$ and $\psi$, \[ ii \models \phi \land \psi [ss] \text{ if and only if } ii \models \phi [ss] \text{ and } \]

\[ ii \models \psi [ss]. \]

Untyped relations play a crucial role. It is assumed that there is a special predicative relation of *instantiation* involving untyped relations and a corresponding number of (ordered) arguments, singular or plural. In clause (3.3iiia) and (3.3iiib), I have indicated this predicative relation by saying that some things are related by the untyped relation denoted by the predicated. We could have said, instead, that some things instantiate the relation.

Let me briefly illustrate how the appeal to untyped relations is meant to provide us with the truth-conditions of an atomic formula in a given interpretation. For example, we would like to have a model that captures an interpretation whose domain
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of quantification is certain students, $\alpha\alpha$, and some of them co-wrote a paper. Using $P(v)$ for ‘$v$ is a paper’ and $C(vv,v)$ for ‘$vv$ co-wrote $v$’, the sentence

\[(3.4) \exists vv \exists v (P(v) \land C(vv,v))\]

would have to come out true in that model. Unraveling the definition of satisfaction, we have that (3.4) is true in an interpretation $ii$ with domain $\alpha\alpha$ if and only if there are some things $xx$, there is a thing $x$ such that $x$ has the property $\rho$ denoted by $P$, and $xx$ stand to $x$ in the relation $\kappa$ denoted by $C$. That is, there is a singular property $\rho$, a plural relation $\kappa$, some things $xx$, and a thing $x$, such that

\[(3.5) \begin{align*}
(i) & \quad (C, \kappa) \prec ii, \\
(ii) & \quad (P, \rho) \prec ii, \\
(iii) & \quad xx \preceq \alpha\alpha, \\
(iv) & \quad x \prec \alpha\alpha, \\
(v) & \quad x \text{ has } \rho, \text{ and} \\
(vi) & \quad xx \text{ are collectively related to } x \text{ by } \kappa.
\end{align*}\]

The desired interpretation exists provided that the relevant $\rho$ and $\kappa$ can be found. It is of critical importance, then, that there exist enough untyped relations, holding of the right things, to be able to capture all the intuitive interpretations of the language. As shown in section 3.3 below, this is a problematic aspect of untyped pluralism.

3.1.3 Remarks

Two features of this semantic approach to $\mathcal{L}_{PL}$ are worth emphasizing. First, it employs expressive resources of the same type as the object language. The model
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theory is entirely formulated within $\mathcal{L}_{PL}$. Untyped pluralism does not produce, as it were, any ideological expansion. It is accessible to someone who understands the object language, with its singular and plural quantifiers. The versions of pluralism discussed in Chapter 2 and 4 invoke expressive resources that go beyond those of the object language. In particular, they introduce new types of higher-order quantification. Evaluating McKay’s proposal will allow us to draw some preliminary conclusions about the required expressive strength of the metalanguage for an adequate semantic theory for $\mathcal{L}_{PL}$.

Second, untyped pluralism is able to capture nicely the idea that the object language quantifiers can be absolutely unrestricted. Since the domain of quantification of every interpretation of the language are some things and not a thing, a set or a class, there is no obstacle in defining a domain of quantification that encompasses everything there it. The plurality of everything is obtained from the following trivial instance of plural comprehension:

$$\exists x (x = x) \rightarrow \exists xx \forall x (x < xx \leftrightarrow x = x).$$

Thus, there will be interpretations that interpret the object language quantifiers as ranging over those things, i.e., over the plurality of everything. Since that the main argument against object singularism is that it cannot accommodate the possibility of absolutely unrestricted quantification, this is a very important point in favor of pluralism.

However, as I will try to show, there is a tension between appealing to untyped relations as the semantic values of plural predicates and admitting the possibility of quantifying over absolutely everything, including untyped relations. The details of the background theory of untyped relations are obviously crucial for untyped pluralism. Worries about the consistency of the background theory of untyped relations have been
expressed by Burgess (2008) and Nicolas (2008). In the next section, I discuss this issue and try to substantiate Burgess’s and Nicolas’s worries by showing that, under some plausible semantic assumptions, the background theory of untyped relations is rendered inconsistent. I will rely on a variant of a Russell-style paradox put forward by Timothy Williamson.

A requirement on the background theory is that it be able to provide enough relations to capture all the intuitive interpretations of the language. In section 3.3, I will offer an argument based on a plural version of Cantor’s Theorem to the effect that an untyped theory of relations will fail to meet the requirement.

3.2 Interpretations of languages with unrestricted quantifiers

As I have just remarked, an important advantage of pluralism over object singularism lies in the fact that pluralism seems better suited to capture the idea that first-order quantifiers can range over absolutely everything. A Russell-style argument put forward by Timothy Williamson shows there is a tension between the view that the first-order quantifiers of the object language can be interpreted as absolutely unrestricted and a natural principle about what interpretations there are. According to this principle, an atomic predicate of the object language should be interpretable by any formula of the metalanguage. As Williamson shows, if the principle is adopted, one cannot think of semantic interpretations as objects. Therefore, one might hope that a model-theoretic proposal that construes semantic interpretations as pluralities, such as untyped pluralism, escapes the problem. However, I will argue that, if untyped relations are invoked in the account of plural predication, as it is done in McKay’s
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proposal, Williamson’s problem cannot be avoided — despite the fact that, on that view, semantic interpretations are not taken to be objects anymore.

3.2.1 Semantic interpretations

Standard model-theoretic semantics for first-order languages treats semantic interpretations as objects, specifically as maps from (a representation of) the expressions of the object languages to certain set-theoretic structures — the models. The quantifiers of the object language are systematically interpreted as having a restricted range. So, if quantification over absolutely everything is a coherent notion, standard model-theoretic semantics does not do justice to it and the question arises as to how to formulate a semantic theory that does justice to it. Williamson’s argument, which will be presented below, casts doubt on the prospects of understanding unrestricted quantification through an extension of the familiar model-theoretic framework in which interpretations are objects.\(^4\)

The notion of interpretation plays a crucial role in semantic theorizing. The fundamental logical notions, such as logical consequence and logical truth, are characterized from the notion of truth in an interpretation. Let us focus on a standard first-order object language \(\mathcal{L}\). It is customary to think of interpretations as determining, for each item of the non-logical vocabulary, a corresponding semantic value. The semantic values of terms are typically objects. There are more candidates to play the role of semantic value for the predicates: sets, classes, properties, or pluralities. In each case, however, a basic predication, such as \(P(v)\), is true in an interpretation just in case

\(^4\)This argument was first proposed by Williamson (2003). It has then been discussed by Glanzberg (2004), Glanzberg (2006), Linnebo (2006), McKay (2006), and Parsons (2006).
the semantic value of $v$, say $x$, satisfies a certain formula $\Phi(\xi)$ of the metalanguage. This formula tells us which objects satisfy the predicate. For example, if the semantic value of $P$ is the set $a$, then $P(v)$ is true in the relevant interpretation just in case $x$ satisfies the formula ‘$\xi \in a$’. That is, $P(v)$ is true in the relevant interpretation just in case $x \in a$.

Since we want to be able to discuss some general aspects of the notion of interpretation, abstracting from the specific frameworks in which the notion might be articulated, it is convenient to speak of the metalinguistic formulas associated with the predicates, rather than to speak of the particular semantic values assigned to the predicate. Thus, for any interpretation $I$, I will refer to a metalinguistic formula $\Phi(\xi)$ such that

\[ (*) \quad \text{for every } x \text{ in the domain of quantification, } I \text{ interprets } P \text{ as applying to } x \text{ if and only if } \Phi(x). \]

The expression ‘$I$ interprets $P$ as applying to $x$’ has to be understood as

$I$ satisfies the formula $P(v)$ when $x$ is assigned to the object language variable $v$.

In the context of standard model theory, ‘$I$ interprets $P$ as applying to $x$’ is understood as

\[ i \models P_v [s(v/x)]. \]

In the context of untyped pluralism, the same expression is understood as

\[ ii \models P_v [ss(v/x)]. \]

According to untyped pluralism, an interpretation $ii$ assigns a property $\rho$ to a singular predicate $P$. Then, relative to the domain $\alpha\alpha$ of $ii$, one has that
(3.6) for every $x \prec \alpha$, ii interpret $P$ as applying to $x$ if and only if $x$ has $\rho$.

Since logical notions are characterized with respect to possible interpretations, it is crucial that one’s model theory recognize all the legitimate interpretations. Disagreement on what interpretations there are might have an impact on what logic is sanctioned for the object language. A natural reaction, intended to avoid the risk of missing interpretations, is one according to which, for any admissible domain $\alpha$ and for any formula of the metalanguage $\Phi(\xi)$, there should be an interpretation from $\alpha$ assigning a semantic value associated with $\Phi(\xi)$ to a given predicate $P$. We may call this view the liberal conception of interpretations. Its core idea, which we may call in turn the liberal principle of interpretations (LPI for short), can be stated as follows.

Let $P$ be a predicate letter in the object language, let $D$ be an admissible domain, and let $\Phi(\xi)$ be any formula in the metalanguage. Then

(LPI) there is an interpretation $I$ such that, for every $x$ in the domain $D$, $I$ interprets $P$ as applying to $x$ if and only if $\Phi(x)$.

Depending on the details of one’s model theory, particularly what domains are admitted and how interpretations are defined, different versions of LPI might result. Let us note that parameters should be allowed to occur in $\Phi(\xi)$ but, on pain of inconsistency, no free occurrence of $I$ should be permitted. I now turn to discuss the idea that the quantifiers of $\mathcal{L}$ might be unrestricted.

### 3.2.2 Unrestricted quantifiers

How one thinks of a domain of quantification depends on the particular semantic theory in which one is working. In standard model-theoretic semantics for $\mathcal{L}$ domains
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are sets but — as previously remarked — classes, properties, or pluralities might provide, in different semantic frameworks, alternative ways of characterizing the notion of a domain. Domains of quantifications specify the range of the quantifiers of the object language in the sense that they contribute to determine the truth-conditions of quantified formulas.

At least for the sake of argument, I will assume that the notion of an all-inclusive domain of quantification is coherent and that the quantifiers of the object language can range over such a domain. As stressed in Chapter 1, this is an assumption that the pluralist should be happy to make, since it provides the most serious argument against object singularism. If $\Phi(\xi)$ is the formula associated with the semantic value of $P$ by the interpretation $I$, our satisfaction clause for universal formulas of $L$ becomes:

\begin{equation}
I \text{ satisfies the formula } \forall zP(z) \text{ with } S \text{ if and only if for every } x \text{ in the domain, } \Phi(x),
\end{equation}

where $S$ is any variable assignment from the domain of $I$. When the quantifiers of the object language are understood as absolutely unrestricted, we can drop the reference to the domain of quantification. The quantifiers of the object language and those of the metalanguage, in some sense, coincide. So, if we are dealing with an all-inclusive domain, the liberal principle of interpretations entails that for every $\Phi(x)$,

$LPI'$ there is an interpretation $I$ such that for every $x$, $I$ interprets $P$ as applying to $x$ if and only if $\Phi(x)$.

The metalinguistic quantifier in $(LPI')$ is meant to be free of contextual restrictions and range over all objects whose existence is accepted in the metalanguage. The argument in the next section shows that $(LPI')$ is inconsistent with the idea that interpretations are objects in the range of the metalinguistic quantifiers. Let us look at the details of the argument.
3.2.3 The main argument

Williamson (2003) has formulated a Russell-style argument to the effect that if we accept (LPI'), i.e., if we accept (LPI) and accept that the quantifiers of $\mathcal{L}$ can be interpreted as ranging over an all-inclusive domain, then we must reject the view that interpretations are objects. His argument runs as follows.

The main argument. Assume that semantic interpretations are objects, as opposed to pluralities or higher-order entities, and let $\Psi(\xi)$ be the following formula:

$$\Psi(\xi) := \xi \text{ is not an interpretation that interprets } P \text{ as applying to } \xi.$$ 

We can apply (LPI') to $\Psi(\xi)$ and infer that there is an interpretation $i$ such that

(i) for all $x$, $i$ interprets $P$ as applying to $x$ if and only if $x$ is not an interpretation that interprets $P$ as applying to $x$.

Since interpretations are objects and fall in the range of the metalinguistic quantifier in (i), we can instantiate the initial quantifier with $i$ itself and obtain that there is an interpretation $i$ such that:

(ii) $i$ interprets $P$ as applying to $i$ if and only if $i$ is not an interpretation which interprets $P$ as applying to $i$.

But (ii) is a contradiction.

Williamson’s own suggestion is that, when dealing with unrestricted languages, semantic interpretations are best construed as second-order relations. Then the liberal principle of interpretations would be reformulated as follows. For every admissible domain $D$ and any metalinguistic formula $\Phi(\xi)$,
(LPI*) there is a second-order relation $I$ such that for every $x$, $I$ interprets $P$ as applying to $x$ if and only if $\Phi(x)$.

One might, instead, reject the assumption that the quantifiers of $\mathcal{L}$ can be unrestricted. But this option, as we know, should not be too attractive for the pluralist: it would remove the main obstacle in the way of object singularism. Does the Russell-style argument threaten the untyped pluralist? One might think that such a pluralist would be safe. After all, an untyped pluralist construes semantic interpretations as pluralities, not objects. However, I will argue in the next section that the main argument puts pressure on the untyped pluralist. On the assumption that quantification over an all-inclusive domain can be achieved, appealing to untyped relations as semantic values of plural predicates is incompatible with (LPI).

### 3.2.4 A pluralist way out?

According to the version of untyped pluralism presented in section 3.1.2, singular and plural predication are treated uniformly. Consider, for example, the case of an atomic monadic predication. In both the plural and the singular case, the predication is true in a model just in case the object or objects denoted by the term stand in the relation denoted by the predicate. Since untyped relations are objects in the range of the metalinguistic quantifiers, admitting the possibility of quantifying over absolutely everything entails that untyped relations can be in the range of the object language quantifiers as well. What I want to show now is that the liberal principle of interpretations functions as a naïve comprehension principle for untyped relations,

---

5See (McKay 2006), pp. 147-154, for a discussion of Williamson’s argument from this pluralist perspective and a defense of the pluralist response to the argument.
leading to contradiction.

Within a plural metalanguage where interpretations are pluralities, the liberal principle of interpretations would take the following form. For any admissible domain \( \alpha \alpha \) and for any formula \( \Phi(\xi) \) in the metalanguage,

\((\text{LPI}^{**})\) there is an interpretation \( ii \) such that for every \( x \prec \alpha \alpha \), \( ii \) interpret \( P \) as applying to \( x \) if and only if \( \Phi(x) \).

Recall that saying that the \( ii \) interpret \( P \) as applying to an object \( x \) means that \( ii \models Pv\{ss(v/x)\} \) for some assignment \( ss \) from the domain of \( ii \). By clause (3.3iiib) of satisfaction, \((\text{LPI}^{**})\) can be rephrased (switching the order of the biconditional for readability) as

\((3.8)\) there is an interpretation \( ii \) such that, for every \( x \prec \alpha \alpha \), \( \Phi(x) \) if and only if for every property \( \rho \), if \((P, \rho) \prec ii\), then \( x \) has \( \rho \).

A plurality is an interpretation only if there is a unique property it associates with \( P \). Since we are admitting an all-inclusive domain, let \( \alpha \alpha \) be the plurality of everything. Also, let \( ii \) be the interpretation in (3.8) when the domain is \( \alpha \alpha \) and let \( \rho_{ii} \) be the unique property such that \((P, \rho_{ii}) \prec ii\). We take \( \Phi(\xi) \) to be the formula ‘\( \xi \) does not have \( \xi \)’ so that (3.8) yields that

\((3.9)\) for every \( x \), \( x \) does not have \( x \) if and only if for every property \( \rho \), if \((P, \rho) \prec ii\), then \( x \) has \( \rho \).

A contradiction can now be derived, since \( \rho_{ii} \) is in the domain of quantification:

Suppose that \( \rho_{ii} \) does not have \( \rho_{ii} \). It follows from (3.9) that, for every property \( \rho \), if \((P, \rho) \prec ii\), then \( \rho_{ii} \) has \( \rho \). But \( \rho_{ii} \) is such that \((P, \rho_{ii}) \prec ii\). So \( \rho_{ii} \) has \( \rho_{ii} \). Contradiction.
Suppose that \( \rho_{ii} \) has \( \rho_{ii} \). Since \( \rho_{ii} \) is the only property such that \((P, \rho_{ii}) \prec ii\), it follows that for every property \( \rho \), if \((P, \rho) \prec ii\), \( \rho_{ii} \) has \( \rho \). Thus, (3.9) entails that \( \rho_{ii} \) does not have \( \rho_{ii} \). Contradiction again.

This shows that the liberal principle of interpretations and the admission of an all-inclusive domain conflict with an account of predication based on untyped relations.

A couple of remarks are in order. First, even though the possibility of capturing interpretations with unrestricted domains is crucial to pluralism, the problem just raised cannot be avoided by merely denying that possibility. Giving up an all-inclusive domain is not enough to escape the problem. The assumption that there is a domain of all untyped properties, or one in which all of them are included, suffices to run Williamson’s argument in the pluralist framework. Second, the problem just stated does not invoke plurals in the object language or in the instantiating formula \( \Phi(\xi) \). Indeed, the object language predicate mentioned in the principle is singular and \( \Phi(\xi) \) was taken to be a singular sentence of the metalanguage, namely, ‘\( \xi \) does not have \( \xi \)’. The argument shows that an account of predication based on untyped entities cannot avoid the problem raised by Williamson simply by construing interpretations as pluralities. This might suggest a pluralist solution based on a revised account of predication. Of course, there are reasons to keep a uniform account of predication but, in light of this difficulty, the pluralist might be tempted to revise her account by characterizing singular predication without reference to untyped relations. However, it can be shown that this alternative will not succeed. Williamson’s argument can be reformulated to apply to plural predication.

Let \( R(vu, v) \) be a collective two-place relation of the object language taking a plural and a singular argument. The considerations that motivate (LPI**) seem to motivate the following principle as well. For any domain \( \alpha \alpha \) and any formula \( \Phi(\xi, \xi) \)
of the metalanguage

(LPIP) there are is an interpretation \( ii \) such that, for all \( xx \preceq \alpha\alpha \) and for every \( x \prec \alpha\alpha \), \( ii \) interpret \( R \) as applying to \( xx \) and \( x \) if and only if \( \Phi(xx, x) \),

where saying that the \( ii \) interpret \( R \) as applying to \( xx \) and \( x \) means that

\[
 ii \models R(vv, v) \left[ ss(vv/xx)(v/x) \right]
\]

for some assignment \( ss \) from the domain of \( ii \). Given clause (3.3iiia) of satisfaction, (LPIP) becomes (inverting again the order of the biconditional for readability)

(3.10) there is an interpretation \( ii \) such that, for all \( xx \preceq \alpha\alpha \) and for every \( x \prec \alpha\alpha \),

\[ \Phi(xx, x) \text{ if and only if, for every relation } \pi, \text{ if } (R, \pi) \prec ii, \text{ then } \pi \text{ relates } xx \] (collectively) to \( x \).

Even in this case there is an appropriate substitution for \( \Phi(\xi\xi, \xi) \) that yields a contradiction. Take \( \Phi(\xi\xi, \xi) \) to be ‘\( \xi \xi \) and \( \xi \) do not have \( \xi \)’. Then let the \( \alpha\alpha \) be the plurality of everything (or at least the plurality of all untyped relations) and let \( \pi_{ii} \) be the unique untyped relation such that \( (R, \pi_{ii}) \prec ii \). It follows from (3.10) that

(3.11) for all \( xx \) and for every \( x \), \( xx \) and \( x \) do not have \( x \) if and only if \( xx \) and \( x \) have \( \pi_{ii} \).

Let \( dd \) be any plurality. Instantiating the singular quantifier in (3.11) with \( \pi_{ii} \), one obtains a contradiction:

(3.12) \( dd \) and \( \pi_{ii} \) do not have \( \pi_{ii} \) if and only if \( dd \) and \( \pi_{ii} \) have \( \pi_{ii} \).
Giving up a uniform account of predication does not provide the untyped pluralist with a solution to Williamson’s problem. In the presence of unrestricted domains, an account of plural predication based on untyped relations violates the liberal principle of interpretations. As long as the liberal principle of interpretations has some strong intuitive motivation, the version of pluralism under discussion faces a serious difficulty. Since accommodating the possibility of an all-inclusive domains marks, by the lights of the pluralist, an important advantage over the object singularist, the problem raised in this section is especially pressing for the untyped pluralist.

3.3 Cantor and untyped relations

As I have already remarked, a model theory for \( \mathcal{L}_{PL} \) has to be able to represent all the intuitive interpretations of the language. For example, let \( P(vv) \) be an atomic one-place plural predicate. Here are some interpretations we can conceive. For every \( xx \) whatsoever, we can conceive an interpretation in which \( vv \) denotes \( xx \) and \( P \) applies to \( xx \) and to no other plurality. More formally:

(a) For any things \( dd \), we have an interpretation \( ii \) with an unrestricted domain such that

\[
(3.13) \quad ii \models P(xx) \land \forall yy (yy \not\approx xx \rightarrow \neg P(yy)) \left[ ss(xx/dd) \right]
\]

for some assignment \( ss \) from the domain.

(b) If plural constants are available, the previous point could be made even more forcefully. Add the plural constant \( cc \) to the language and extend the characterization of an interpretation \( ii \) with domain \( \alpha \alpha \) in the obvious way, namely, by requiring that there be some things \( dd \not\subsetneq \alpha \alpha \), such that for every \( d \), \( (cc, d) \prec ii \)
if and only if \( d \prec dd \). Then, for any things \( dd \), we have an interpretation \( ii \) with an unrestricted domain such that, for every object \( d \), \((cc, d) \prec ii \) if and only if \( d \prec dd \), and

\[
(3.14) \; ii \models P(cc) \land \forall xx (xx \not\approx cc \rightarrow \neg P(xx)) \, [ss]
\]

is satisfied by \( ii \) and any assignment \( ss \).

For the untyped pluralist, admitting the interpretations specified in (a) and (b) entails the following principle about untyped relations:

\[
(3.15) \forall yy \exists \pi \forall xx (xx \text{ have } \pi \leftrightarrow xx \approx yy).
\]

If, for all \( xx \) in the unrestricted domain, there is an interpretation in which \( P \) applies to \( xx \) and to no other plurality, it means that, for all \( xx \), there is an untyped property that applies to \( xx \) and to no other plurality.

However, (3.15) is incompatible with a plural version of Cantor’s Theorem. The intuitive idea is that, if at least two distinct things exist, there are more pluralities than objects. Therefore, there are more pluralities than untyped relations. But (3.15) implies that the instantiation relation between a plurality and a property \((xx \text{ having } \pi)\) can be used to obtain a one-to-one function from the pluralities into the properties. Cantor’s Theorem says that this cannot be the case.

Of course, what I have just said is strictly speaking incoherent. I talked of pluralities as if there were things and I used a cardinality comparison between pluralities and objects. I need to show that there is a way of making sense of this intuitive idea that is acceptable to the pluralist. It is slightly laborious but it can be done. The crucial step is to prove a version of Cantor’s Theorem within the framework of untyped pluralism. The proof of the Theorem is carried out in the metalanguage of
the untyped pluralist. Following the untyped pluralist, a pairing operation will be postulated and the existence of at least one object that is not a pair will be assumed.

Let \( xx \) and \( yy \) be some things, possibly different. Then we say that an object \( x \) codes \( yy \) in \( xx \) if and only if, for every \( y \), \( (x, y) \prec xx \) just in case \( y \prec yy \). Also, we say that \( yy \) are coded in \( xx \) if and only if there is an object \( x \) that codes \( yy \) in \( xx \). The result below shows that there is no plurality \( xx \) in which every plurality \( yy \) is coded.\(^6\)

**Plural Cantor’s Theorem:** there are no things \( xx \) in which every plurality \( yy \) is coded. That is, there are no things \( xx \) such that for all \( yy \) there is an \( x \) such that for all \( y \), \( (x, y) \prec xx \) if and only if \( y \prec yy \). Formally,

\[
\neg \exists xx \ \forall yy \ \exists x \ \forall y \ ((x, y) \prec xx \leftrightarrow y \prec yy).
\]

**Proof:** Suppose for reductio that

\[
(*) \ \exists xx \ \forall yy \ \exists x \ \forall y \ ((x, y) \prec xx \leftrightarrow y \prec yy).
\]

Call these things \( \alpha \alpha \). We first show that there is \( z \) such that \( (z, z) \not\prec \alpha \alpha \). Suppose not, i.e., suppose that for every \( z \), \( (z, z) \prec \alpha \alpha \). For every \( x \), using \( x \) as a parameter in plural comprehension, we get that

\[
\exists xx \ \forall y \ (y \prec xx \leftrightarrow y = x).
\]

Let us use \( p(x) \) to denote the plurality comprising \( x \) and only \( x \). Let \( x \) be arbitrary and let \( c_x \) code \( p(x) \) in \( \alpha \alpha \), i.e.,

\[
(**) \ \forall y \ ((c_x, y) \prec \alpha \alpha \leftrightarrow y \prec p(x)).
\]

\(^6\)For the analogues of this result in second-order logic see Bernays (1942) and Shapiro (1991). For a discussion of it, see Rayo (2002).
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Then \((c_x, x) \prec \alpha \alpha\). However, we are assuming that for every \(z\), \((z, z) \prec \alpha \alpha\). So, in particular, \((c_x, c_x) \prec \alpha \alpha\). From \((**\)) we obtain that \(c_x = x\). Thus, for every \(x\), if \(y\) codes \(p(x)\) in \(\alpha \alpha\), then \(y = x\). From \((\ast\)) which asserts the existence of a code for every plurality in \(\alpha \alpha\), it then follows that \(x\) codes \(p(x)\) in \(\alpha \alpha\). Now at least one object exists that is not a pair, call it \(b\). By the rules associated with pairing, \(b \neq (b, b)\), with \(b\) coding \(p(b)\) and \((b, b)\) coding \(p(b, b)\) in \(\alpha \alpha\). An instance of plural comprehension gives us that

\[
\exists x \forall y (y \prec xx \iff (y = b \lor y = (b, b))).
\]

Call these things \(\beta \beta\). From \((\ast)\) we have that there is an object \(c_{\beta \beta}\) that codes the \(\beta \beta\) in \(\alpha \alpha\). Thus, \((c_{\beta \beta}, b) \prec \alpha \alpha\) and \((c_{\beta \beta}, (b, b)) \prec \alpha \alpha\). But, from the assumption that, for every \(z\), \((z, z) \prec \alpha \alpha\), it also follows that \((c_{\beta \beta}, c_{\beta \beta}) \prec \alpha \alpha\). Since \(c_{\beta \beta}\) codes the \(\beta \beta\), then \(c_{\beta \beta} = b\) or \(c_{\beta \beta} = (b, b)\). If \(c_{\beta \beta} = b\), since \(b\) codes \(p(b)\), \((c_{\beta \beta}, (b, b)) \prec \alpha \alpha\) entails \(b = (b, b)\): contradiction. So \(c_{\beta \beta} \neq b\). The same reasoning shows that \(c_{\beta \beta} \neq (b, b)\). Therefore, there is \(z\) such that \((z, z) \not\in \alpha \alpha\). This, via plural comprehension using \(\alpha \alpha\) as parameters, allows us to reproduce the usual diagonal argument used to prove the traditional Cantor’s Theorem. That is, since there is \(z\) such that \((z, z) \not\in \alpha \alpha\), by comprehension we also have:

\[
\exists x \forall y (y \prec xx \iff (y, y) \not\in \alpha \alpha).
\]

Call these things \(\gamma \gamma\). Now, \((\ast)\) entails that

\[
\exists x \forall y ((x, y) \prec \alpha \alpha \iff y \prec \gamma \gamma),
\]

which, by the characterization of \(\gamma \gamma\), entails in turn that

\[
\exists x \forall y ((x, y) \prec \alpha \alpha \iff (y, y) \not\in \alpha \alpha).
\]

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Thus, for some $r$,

$$(r, r) \prec \alpha \leftrightarrow (r, r) \not\prec \alpha,$$

contradiction. Thus, we conclude that $(\ast)$ is false. □

Against the background of untyped pluralism, the plural Cantor’s Theorem prevents the existence of some intuitive interpretations of $\mathcal{L}_{PL}$. Even though the introduction of untyped relations as semantic values of plural predicates avoids an ideological expansion, there are simply not enough untyped relations to build a model theory that is intuitively adequate. Indeed, suppose that we have

(3.15) $\forall y y \exists \pi \forall x (x x \text{ have } \pi \leftrightarrow x x \approx yy).$

as a result of admitting the interpretations in (a) and (b) on p. 120. Then we can use plural comprehension to obtain a plurality in which every plurality is coded, against the Theorem. Consider the metalanguage formula

$$\phi(x) := \exists y \exists z (x = (y, z) \land \exists z z (z < z z \land z z \text{ have } y))$$

which says that $x$ is a pair such that the second coordinate $z$ is among some things $zz$ that collectively have $y$, the first coordinate. Clearly, there is some $x$ such that $\phi(x)$. For example, for some objects $\beta$ and $\gamma$, we might have the property $\pi_{\beta, \gamma}$ of being the things such that $\beta$ and $\gamma$ are among them and nothing else is among them. Then, $\phi(x)$ is true of $(\pi_{\beta, \gamma}, \beta)$. By plural comprehension we have that

$$\exists xx \forall x (x < xx \leftrightarrow \phi(x)).$$

Call these things $\gamma \gamma$. It follows from (3.15) that, for every $\alpha \alpha$, there is a property $\pi$ such that, for every $x$, $(\pi, x) < \gamma \gamma$ if and only if $x < \alpha \alpha$. This means that $\gamma \gamma$ code
every plurality, which is impossible. It should be concluded that the existence of the interpretations in (a) and (b) cannot be admitted by the untyped pluralist. Resorting to untyped relations and allowing the possibility of unrestricted domains comes at the cost of sacrificing the adequacy of the model theory. The intuitive notion of logical consequence as truth-preservation under every possible interpretation of the language is not captured.

3.4 Conclusion

In this chapter, I have raised two problems for untyped pluralism. First, I have shown that a version of Russell-style argument put forward by Timothy Williamson can be used against untyped pluralism. The argument shows that, even in the framework of untyped pluralism, one cannot accept both the possibility of quantifying over absolutely everything and the liberal principle of interpretations. Second, I have argued that, in light of a plural version of Cantor’s Theorem, untyped pluralism cannot capture all the intuitive interpretations of the object language. I submit that these are good reasons to be dissatisfied with this approach to the semantics of plurals.
Chapter 4

Plurals and Superpluralism

In this chapter, I offer a critical discussion of a third version of pluralism. Pluralism, as we have seen, can be construed in different ways depending on what semantic values are assigned to plural predicates. According to standard pluralism, plural predicates denote plural properties conceived as higher-order entities. Untyped pluralism holds that plural predicates denote plural properties conceived as objects. Unlike these two versions of pluralism, superpluralism does not appeal to plural properties at all. According to superpluralism, plural predicates denote superpluralities, that is, pluralities of pluralities. A superpluralist semantics for plurals has been laid out in Rayo (2006).

I will begin with some preliminary considerations concerning superplurals. First, I will address the issue of whether superplurals occur in English and what implications, if any, that would have for the semantic debate about plurals. Then I will present the basic superpluralist semantics for $\mathcal{L}_{PL}$. Later, I will look at the motivations behind superpluralism and compare it to alternative accounts of plurals. I will conclude that it is doubtful that superpluralism enjoys any advantage over its competitors.
4.1 Superplurals: what are they?

It would be easy to clarify what superplurals are if we could find such expressions in natural language. However, it is controversial whether they do occur in natural language. At least as far as English is concerned, the thesis that superplurals occur in it has been denied by some authors but has been endorsed by others.¹ In the next section, we will take a closer look at this issue. For the moment, we want to get some grip on what types of expressions superplurals are. One way of introducing superplurals would be via a formal system, and this is the way we will ultimately pursue below, following Rayo (2006). But, first, we would like to see if there is an informal way to do that as well. Before continuing I should remark that, when referring to superplural expressions, one refers to a class of expressions including terms (constants and variables), quantifiers, and predicates.

Different authors have used different analogies to introduce superplurals. For example, Uzquiano (2004) writes that superplural quantification “would be a variety of quantification related to plural quantification as plural quantification is related to singular quantification.”² Rayo, on the other hand, warns us against taking this analogy too far:

[T]hinking of super-plural quantification as an iterated form of plural quan-

¹ Rayo (2006), for example, claims that “there are, of course, no super-plural terms and quantifiers in English” (p. 227). Contrary to this claim, Linnebo and Nicolas (2008) argue that English does contain superplural terms — although, perhaps, no superplural quantifiers. They claim, for example, that ‘the square things, the blue things, and the wooden things’ as occurring in ‘The square things, the blue things, and the wooden things overlap’ is a superplural term. See section 4.2 for a discussion.
tification — plural quantification over pluralities — would be a serious mistake. Plural quantification over pluralities can only make sense if pluralities are to be taken to be ‘items’ of some kind of other. And a plurality is not an ‘item’.³

Instead, he suggests that we regard the relation between plural and superplural quantification by analogy with the relation holding between second-order and third-order quantification. There is a relevant difference though. Higher-order quantification is quantification over predicable entities, i.e., entities that can be predicated of other entities. In the case of plurals and superplurals, however, the quantification would bind variables that do not take predicative position. Plural or superplural variables take argument position.

These analogies concern superplural quantification. What about superplural terms and predicates? The analogies may be reproduced. Superplural terms (superplural predicates) may be thought of as a class of terms (predicates) related to plural terms (plural predicates) as plural terms (plural predicates) are related to singular terms (singular predicates). Alternatively, one might appeal to the second analogy and say that the relation between a superplural predicate and a plural predicate is similar to the relation holding between a second-order predicate and a third-order predicate. If the analogy involving predicates is found more illuminating than the analogy involving terms, one may give a derivative (rough) characterization of superplural terms as the terms to which superplural predicates apply.

If one wants to go beyond the analogies, one might try to provide a semantic characterization of superplurals. For example, Linnebo and Nicolas (2008) write:

A natural question that arises is whether the step from the singular to the plural can be iterated. Are there terms that stand to ordinary plural terms the way ordinary plural terms stand to singular terms? Let’s call such terms superplurals. A superplural term would thus, loosely speaking, refer to several ‘pluralities’ at once, much as an ordinary plural term refers to several objects at once.\(^4\)

After having introduced superplural terms via analogy, the authors suggest a semantic characterization of them. They suggest that superplural terms are those which refer to several pluralities at once. Of course, such a characterization cannot help someone who does not already have a grasp of superplural expressions. Indeed, it invokes superplural quantification and superplural relations: it says that a superplural term is one standing in the reference relation to some superplurality.

It seems, then, that unless English provides examples of superplural expressions, “the best way of attaining a genuine grasp of superplural quantification is by mastering the use of superplural quantifiers” (Rayo (2006), p. 227). A good way of accomplishing this is via a formal system. But, before we do that, it is worth exploring whether there is compelling evidence that superplural expressions occur in English.

### 4.2 Superplurals in English

As I mentioned above, it is controversial whether English contains superplural expressions. In discussing this issue, however, it is important to clarify how the controversy

should be understood in the present context. It seems clear that it cannot be sim-
ply understood as a semantic dispute as to whether one can find English expressions
whose semantic values are superplurals. This is because what we are tying to establish
here is exactly whether certain English expressions call for a semantic analysis based
on superplurals.

We must distinguish two aspects of the search for superplurals in English. First,
one might ask whether English contains expressions whose regimentation requires the
introduction a new type of terms? Suppose that the answer to this question is positive.
Just like in the case of plurals, admitting a new category of terms in the regimenting
language is compatible with a number of different ways of specifying the semantic
value of the terms in the new category. For example, a semantic singularist might
accept what we may call syntactic superpluralism, while denying that the semantic
value of a superplural term is a superplurality. Perhaps one might regard superplural
terms as denoting sets of sets or second-order properties.

It has been thought that finding superplural expressions in English would help
avoid the intelligibility objection to the use of superplurals, i.e., the objection that
superpluralism is not a legitimate semantic view because superplurals are unintelli-
gible. We will not attribute much force to this objection and, later, we will assume
that the formal characterization of superplurals given in section 4.3 suffices for intel-
ligibility.

The best case for superplurals in English has been made by Linnebo and Nicolas
(2008). Here is their main example.

(4.1) The square things, the blue things, and the wooden things overlap.\(^5\)

(4.2) These people, those people and these other people play against each other.\textsuperscript{6}

The contention is that the noun phrases in (4.1) cannot be understood as simply referring to the plurality composed of the square things, the blue things and the wooden things. Analogously, they contend that the noun phrase in (4.2) cannot be understood as referring to the plurality composed of these people, those people, and these other people.

Indeed, suppose that for any \( x \), \( x \) is one of things owned by Jones if and only if \( x \) is one of the square things or one of the blue things or one of the wooden things. Then consider the following sentence.

(4.3) The things owned by Jones overlap.

It is clear that (4.1) and (4.3) might differ in truth value.\textsuperscript{7}

One might attempt to eliminate the contentious expressions by analysis. For example, it might be suggested that (4.1) be analyzed as (4.4):

(4.4) There is something that is among the square things, among the blue things, and among the wooden things.

Linnebo and Nicolas respond that (4.4) is unacceptable as an analysis of (4.1) since ‘overlap’ is not a defined term — indeed, it could be learned directly — and thus it should be possible to introduce it in the language as primitive.

However, a challenge to the view that (4.1) and (4.2) exhibit superplural terms comes from considering examples of multigrade predicates. As it has been forcefully

\textsuperscript{6}Ibidem.

\textsuperscript{7}For a defense of the thesis that ‘the square things, the blue things, and the wooden things’ is semantically plural, see Ben-Yami (unpublished).
argued by Oliver and Smiley (2004), one should take seriously the idea that there are genuinely multigrade predicates. In particular, some lists of terms are best regarded as strings rather than terms. Consider, for example, the following sentences uttered in a context in which both single players and couples can take part in a competition:

(4.5) Annie, Bonnie and Connie, Danny, and Freddie are the past winners.

(4.6) Annie, Bonnie, Connie, Danny, and Freddie are the past winners.

For a pluralist, it is hard to distinguish the two sentences if one assumes that ‘Annie, Bonnie and Connie, Danny, and Freddie’ and ‘Annie, Bonnie, Connie, Danny, and Freddie’ are (plural) terms. Similarly, (4.7) and (4.8) should have the same truth value if ‘1 and 7’ and ‘1, 1, and 7’ are considered plural terms.

(4.7) The mean of 1 and 7 is 4.

(4.8) The mean of 1, 1, and 7 is 3.

A natural response to these example is to maintain that ‘1, 1, and 7’ and ‘Annie, Bonnie and Connie, Danny, and Freddie’ should not be regarded as terms, but as strings of terms followed by a genuinely multigrade predicate.

The same problem for the view that ‘1, 1, and 7’ and ‘Annie, Bonnie and Connie, Danny, and Freddie’ are plural terms seems to arise for the view that ‘the square things, the blue things and the wooden things’ and ‘these people, those people and these other people’ are superplural terms. Let us see why.

If a list of the form ‘aa, bb, and cc’ is taken to be a term denoting a superplurality, then it denotes the same superplurality as ‘aa and bb’ if the plural terms aa and cc are coreferential. This means that, on the view that ‘the square things, the blue things and the wooden things’ is a superplural term, it is unclear why the following sentences can differ in truth value.
(4.9) The square things and the square things overlap.

(4.10) The square things overlap.

Moreover, for the same reason, a superplural analysis commits one to the equivalence of (4.11) and (4.12).

(4.11) These people, those people, and these people [pointing to them again] play against each other.

(4.12) These people and those people play against each other.

However, we can think of games or competitions in which some people may play against themselves, as well as against other people, in which case the two sentences would not be equivalent. As in the case of plural lists, a natural response to these examples involves treating the lists as strings of terms followed by multigrade predicates, rather than treating them as superplural terms followed by a superplural predicate with a fixed adicity. We must conclude that lists present a challenge for the superplural analysis.

Let us assume, at least for the sake of argument, that there is reason to extend the regimenting language to encompass superplural terms. Does that mean that such terms should be taken to denote superpluralities? I think not. Suppose that, in spite of the examples discussed in the previous paragraph, one wants to regard ‘the square things, the blue things, and the wooden things’ as a term. The property-singularist, for example, would seem to have no problem avoiding superplurals in the semantic analysis of (4.1). She might take ‘the square things, the blue things, and the wooden things’ to denote the second-order property that holds of the properties denoted, respectively, by ‘the square things’, ‘the blue things’, and ‘the wooden things’. The
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predicate ‘overlap’ as used in (4.1) would then have to denote a third-order property. A parallel response is available to the object-singularist. If ‘the square things’ and ‘the blue things’ denotes a set, then ‘the square things and the blue things’ would denote the set of sets denoted by ‘the square things’ and the ‘the blue things’. This shows that the mere introduction of a new type of superplural expression in the regimenting language does not pose any special problem to singularism and, therefore, does not provide strong evidence for superpluralism. Does it at least provide some reason to prefer superpluralism over standard pluralism?

The difference between standard pluralism and superpluralism concerns the semantic values of plural predicates. The two views agree on the semantic values of plural terms: on both views a plural term denotes a plurality. It seems that (4.1) might provide support for the thesis that ‘the square things, the blue things, and the wooden things’ denote a superplurality but, importantly, does not provide any evidence that settles the dispute between the standard pluralist and the superpluralist, which is a semantic dispute about predicates in the context of $\mathcal{L}_{PL}$. A standard pluralist could acknowledge that ‘the square things, the blue things, and the wooden things’ requires an extension of $\mathcal{L}_{PL}$ and appeal to superplurals as the semantic values of terms of this sort. However, she might continue to hold that plural predicates signify plural properties rather than superpluralities. She will then have to maintain that superplural predicates, such as ‘overlap’ in (4.1), denote a new kind of property: a superplural property. The standard pluralist might insist that there are benefits to treating predicates as referring to properties rather than superpluralities. In particular, properties will give a better way to deal with empty predicates and with quantification into predicate positions. On the other hand, the superpluralist might stress the importance of ontological innocence in rejecting the standard pluralist view
in favor of superpluralism. The discussion below will help shed some light on this dispute.

For the moment, we may conclude that the examples offered by Linnebo and Nicolas give us interesting semantic data but they are not decisive in establishing syntactic or semantic superpluralism. A pluralist might have to weigh the pros and cons of standard pluralism vs. superpluralism, but the evidence from natural language does not put any pressure on semantic singularism.

4.3 The language of plurals and superplurals

In this section, superplurals are introduced via a formal system. Plurals were introduced by extending the standard language of first-order logic with plural terms, plural quantifiers, and plural predicates. Every plural predicate takes a fixed number of arguments, each of which can be exclusively singular or exclusively plural. Argument places can be occupied by singular terms — constants and variables — or plural terms. Special status is given to the predicate ‘being one of’ (or ‘being among’), which, as we know, is treated as logical. We called the resulting language $L_{PL}$, which is composed by the usual vocabulary of the language of first-order logic plus the following items.

A1. plural variables $vv, xx, yy, zz, ...$

A2. plural constants (omitted for simplicity);

B. plural existential quantifiers binding plural variables $\exists vv, ...$

C. symbols for plural predicates, i.e., predicates taking at least one plural argument;

D. a distinguished binary predicate representing ‘is one of’, $\prec$, taking a singular term in the first argument place and a plural variable in the second.
Superplurals are now added to $\mathcal{L}_{PL}$ in a way that, unsurprisingly, mirrors that in which plurals were added to the language of first-order logic. So we extend $\mathcal{L}_{PL}$ by introducing superplural terms, superplural quantifiers, and superplural (collective) predicates. Every superplural predicate takes a fixed number of arguments, each of which can be exclusively singular, exclusively plural, or exclusively superplural. Argument places can be occupied by singular terms, plural terms, or superplural terms. A new logical relation is also introduced. It holds between a plural and a superplural term and, intuitively, it expresses the condition that the relevant plurality ‘is one of’ the pluralities that make up the superplurality denoted by the superplural term. Since this relation and other logical relations might not have a counterpart in English, I will use boldface (e.g., are some of) to indicate them. Let us call the new language $\mathcal{L}_{SPL}$.

So $\mathcal{L}_{SPL}$ includes $\mathcal{L}_{PL}$ plus the following items.

E1. superplural variables $vvv, xxx, yyy, zzz, ...$;

E2. superplural terms (omitted for simplicity);

F. superplural existential quantifiers binding plural variables $\exists vvv, ...$;

G. symbols for superplural predicates, i.e., predicates taking at least one superplural argument. As in the case of plural predicates, for any arity $n$, it is convenient to allow only predicate symbols whose first $m_1$ argument places ($1 \leq m \leq n$) are superplural, the subsequent $m_2$ argument places ($0 \leq m_2 \leq n - m_1$) are plural, and whose remaining $n - (m_1 + m_2)$ argument places are singular. Two superscripta enclosed in square brackets will indicate the number of superplural and plural arguments respectively. The arity of the predicate, when marked, will precede the square brackets. For example, $L^{4[2,1]}$ is a superplural predicate with
two superplural arguments, one plural argument, and one singular argument. For the sake of practicality, superscripts will be often left implicit and we will be lenient about the order of arguments, allowing singular or plural argument places to precede plural or superplural ones.

H. a distinguished binary predicate \( \preceq^* \) (are some of, are among), taking a plural variable in the first argument place and a superplural variable in the second (e.g., ‘\( xx \preceq^* xxx \)’).

The recursive clauses defining well-formed formulae of \( \mathcal{L}_{SPL} \) are the obvious ones.

Additional notions can be introduced by definition as in the case of plurals. So the superplural ‘are some of’ (or ‘are among’), denoted by \( \preceq^* \), and superplural identity, denoted by \( \approx^* \). They are characterized by the following schemas.

\[
(4.13) \quad yyy \preceq^* xxx \leftrightarrow_{def} \forall yy (yy \preceq^* yyy \rightarrow yy \preceq^* xxx).
\]

\[
(4.14) \quad yyy \approx^* xxx \leftrightarrow_{def} \forall xx (xx \preceq^* yyy \leftrightarrow xx \preceq^* xxx).
\]

In order to gain mastery of superplural expressions, one needs to be introduced not only to the language of superplurals but to the logical principles that govern such expressions. As the analogies between plurals and superplurals suggest, superplurals are governed by logical principles that mirror the logical principles governing plurals. Proof-theoretically, we have the natural introduction and elimination rules for the existential (plural and superplural) quantifiers. Moreover, we have a principle of superplural comprehension. Below is the principle of plural comprehension followed by the superplural one.

\[
(4.15) \quad \exists x \, \phi(x) \rightarrow \exists xx \forall x (x \preceq xx \leftrightarrow \phi(x)).
\]
We may suppose that this gives us enough mastery of superplural expressions to avoid the charge of unintelligibility. Then we can proceed to formulate a superpluralist semantics for $L_{LP}$, using superplural resources in the metalanguage. That is, the superpluralist semantics will be formulated within $L_{SPL}$. Let us now look at the details of the semantics.

### 4.4 The model theory

#### 4.4.1 Preliminary notions

Let us begin with an informal presentation of the main idea, which will also help motivate some of the concepts we need to introduce. To avoid further complications, it will be assumed that an operation to form ordered pairs (hence ordered $n$-tuples), subject to the usual constraints, is available in the metatheory. As discussed above, the characteristic feature of superpluralism is the use of superpluralities as semantic values for plural predicates. In the semantics, we would like to use ordered pairs to pair, within each interpretation, a linguistic item with its semantic value. However, since pluralities and superpluralities are not objects, they cannot figure as the second coordinate of an ordered pair. This is why we have to develop a way of coding the relevant semantic information using the resources available in the metalanguage, i.e., in $L_{SPL}$. It turns out that one can simply construe an interpretation as a superplurality whose pluralities carry the relevant information. At the same time, since it would be

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8Here I present a simplified version of the model theory put forward in Rayo (2006). See in particular his Appendix.
self-defeating for superpluralism to use relations or any other higher-order resource to specify the notion of interpretation, we must be careful in avoiding any reference to relations of higher-order, plural or superplural.

As in the case of standard and untyped pluralism, the domain of an interpretation may be naturally taken to be a plurality of things, the things we intend the quantifiers to range over in the given interpretation. To be able to refer to a plurality as the domain of a given interpretation, such plurality will be codified as the plurality of the second coordinates of a plurality of ordered pairs whose first coordinate is $\forall$. For example, if we want our domain of quantification to be the plurality including $a$, $b$ and nothing else, we let our interpretation contain the plurality including $(\forall, a)$, $(\forall, b)$, and no other pair whose first coordinate is $\forall$. This will tell us that the domain is $a$ and $b$. More generally, if we want $xx$ to be our domain of quantification, we are going to include in the interpretation the plurality $yy$ such that

$$\forall x ((\forall, x) < yy \iff x < xx) \land \forall x (x < yy \rightarrow \exists z (x = (\forall, z))).$$

Given its relation to $xx$ and $\forall$, we may denote the plurality of $yy$ thus defined as $[\forall; xx]$, indicating that $yy$ result from prefixing $\forall$ to each object that is one of $xx$.

We can generalize this concept and the correlative notation to an arbitrary object combined with an arbitrary plurality. For any object $\alpha$ and plurality $xx$, we denote by $[\alpha; xx]$ the plurality of all and only the ordered pairs whose first component is $\alpha$ and whose second component is an object that is one of $xx$. In other words, the square brackets denote an operation that takes an object and a plurality and yields the ordered pairs in which the given object is always the first coordinate and a thing...
in the plurality figures as a second coordinates.\(^9\)

The interpretation of a singular term \(t\) of \(\mathcal{L}_{PL}\) is an object, say \(x\). This fact will be codified by the pair \((t, x)\). The semantic value of both a singular predicate and a plural variable is a plurality. Let \(P\) be an \(n\)-ary predicate and let \(vv\) be a plural variable. We codify the fact that the semantic value of \(P\) is, say, the plurality of \(xx\), or that \(vv\) is interpreted as the plurality of \(yy\), using \([P; xx]\) and \([vv; yy]\) respectively. Notice that when the arity of \(P\) is \(n\) strictly greater than one, then \(xx\) must be a plurality of \(n\)-tuples. Of course, we must require, for each interpretation, that the pluralities interpreting the predicates and the plural variables, \(xx\) and \(yy\) in our example, be subpluralities of the domain of quantification.

As we know, superpluralities are employed in the semantics to provide the semantic values of plural predicates — and to constitute the interpretations of the language. Let \(C\) be one such predicate. Suppose that we want \(xxx\) to be the semantic value of \(C\) on a given interpretation. Obviously, since superpluralities are not objects, we cannot codify this fact using the ordered pair \((C, xxx)\). But we may codify this fact using the superplurality \(yyy\) including all and only the pluralities \([C; xx]\), where \(xx\) are among \(xxx\). That is, \(yyy\) is such that

\(^9\)To give another example, consider objects \(a, b, c\) and the plurality \(xx\) such that

\[
\forall x (x \prec xx \leftrightarrow (x = b \lor x = c)).
\]

Then the operation \([a; xx]\) yields the plurality containing all and only the following objects:

\((a, b), (a, c)\).
∀xx ([C; xx] ∼* yyy ↔ xx ∼* xxx).

It is convenient to extend the bracket notation introduced above to superpluralities. So, if xxx is a plurality of the pluralities xx, yy, zz, ..., then [C; xxx] denotes the superplurality of [C; xx], [C; yy], [C; zz], ... .

We need an additional notion to be able to deal with n-ary plural predicates. To this end, we modified the notation just introduced. For any xx₁, ..., xxₘ, xxₘ₊₁, ..., xxₙ, we use [α; xx₁, ..., xxₘ, xxₘ₊₁, ..., xxₙ] to denote the plurality defined by the following schema:

\[\forall x (x \prec [\alpha; xx₁, ..., xxₘ, xxₘ₊₁, ..., xxₙ] \iff
\exists y₁, ..., \exists yₙ((y₁ \prec xx₁ \land ... \land yₘ \prec xxₘ \land yₘ₊₁ = xxₘ₊₁ \land ... \land yₙ = xxₙ) \land
x = (\alpha, (y₁, ..., yₙ)))).\]

This is gives us the plurality of ordered pairs such that the the first coordinate is α and the second coordinate is an n-tuple whose i-th coordinate (1 ≤ i ≤ m) is one of xxᵢ and whose j-th coordinate (m + 1 ≤ j ≤ n) is identical to xxₖ.

What should a model (an interpretation) of the language be? As I mentioned above, it is natural to think of a model here as a special kind of superplurality, that is, a superplurality that includes all the pluralities that codify the semantic values of all the relevant items in the language. There is only one wrinkle. The semantic values of some items of the language are not pluralities. For example, the semantic value of a singular term is an object. Now the fact that a singular term, say t, denotes an object, say x, is codified by an object, the ordered pair (t, x). But there is a way to ‘bloat’ objects into pluralities — and pluralities into superpluralities — so that they are of the appropriate kind to be included in a model. For example, let α be
an object. Then we have a plurality $xx$ consisting of $\alpha$ and only $\alpha$. This is possible, since we allow pluralities of one object. Analogously, we might have a superplurality $xxx$ such that $xx$ and no other plurality is such that $xx \prec^* xxx$. It is convenient to adopt a special notation to refer to ‘bloated’ pluralities or superpluralities. Let $\alpha$ be an object and let $\alpha \alpha$ be a plurality. Then we denote the ‘bloated’ plurality of $\alpha$ and the ‘bloated’ superplurality of $\alpha \alpha$ by

$$(\alpha)xx \quad \text{and} \quad (\alpha \alpha)xxx,$$

respectively.\textsuperscript{10}

One aspect of this approach to plurals requires a special attention. How should empty predicates be treated? If we want to interpret predicates as superpluralities, there is no room for empty predicates given that superpluralities are, by definition, non-empty. One way to proceed is to think of empty predicates as not having a denotation.\textsuperscript{11} This is how the semantics will be set up here. Some related philosophical issues will be discussed in section 4.5.1. An alternative, but more artificial way of proceeding would be to use a conventional object to codify that fact that a given predicate is empty on a given interpretation.

We are now ready to characterize the notion of an interpretation of $L_{PL}$.

\textsuperscript{10}The operation of bloating, of course, can be applied in succession. For example, we can start from $\alpha$ to obtain $(\alpha)xx$ first, and then obtain $((\alpha)xx)xxx$, namely, the superplurality consisting only of the plurality $(\alpha)xx$.

\textsuperscript{11}See Rayo (2006), Appendix.
4.4.2 The formal account

First, let us characterize the notion of variable assignment (for singular and plural variables) needed to formulate the satisfaction clauses for the quantifiers. Let the plurality $\alpha$ be a domain. A variable assignment from $\alpha$ is a plurality $ss$ such that

\begin{align}
(4.17) & \quad (i) \text{ for every singular variable } v \text{ of the object language, there is a unique } x \prec \alpha \text{ such that } (v,x) \prec ss; \\
& \quad (ii) \text{ for any plural variable } vv \text{ of the object language, there are some things } xx \preceq \alpha \text{ such that } [vv,xx] \preceq ss \text{ and for any } y, \text{ if } (vv,y) \prec ss, \text{ then } y \prec xx.
\end{align}

Variants of variable assignments are defined as follows. If $ss$ is a variable assignment with domain $\alpha$, then for every $x \prec \alpha$, the assignment $ss(v/x)$ is a plurality from $\alpha$ just like $ss$ except, possibly, for the fact that $(v,x) \prec ss(v/x)$ and no other $y$ is such that $(v,y) \prec ss(v/x)$. For plural variables, we have that for any $xx \prec \alpha$, $ss(vv/xx)$ is an assignment just like $ss$ except, possibly, for the fact that $[vv,xx] \preceq ss(vv/xx)$ and for every $y$, if $(vv,y) \prec ss$, then $y \prec xx$.\(^\text{12}\)

Next, let us characterize the notion of interpretation. We take an interpretation with domain $\alpha$ to be a superplurality $iii$ such that

\begin{align}
(4.18) & \quad (i) [\forall,\alpha] \prec^* iii \text{ and for all } xx, \text{ if } [\forall,xx] \prec^* iii, xx \approx \alpha. \\
& \quad (ii) \text{ for any constant } c \text{ of the object-language, there are is a unique } x \prec \alpha \text{ such that } ((c,x))xx \prec^* iii, \text{ and, for any } y \text{ and } yy, \text{ if } (c,y) \prec yy \prec^* iii, \\
& \quad \quad yy \approx ((c,x))xx.
\end{align}

12This very last condition ensures that $xx$, in $ss(vv/xx)$, is the maximal plurality associated with $vv$ in the assignment.
(iii) for any singular predicate \( P \) of the object-language, for all \( xx \) and all \( yy \),
if \([P, xx] \prec^* iii\) and \([P, yy] \prec^* iii\), then \( xx \preceq \alpha\alpha\), and \( xx \approx yy\).

(iv) no extra clause is required for plural predicates.

Some clarifications are in order. Clause (i) specifies that an interpretation has a
unique domain. Clause (ii) specifies that every constant is assigned an object as
its semantic value and, in every interpretation, there is a unique plurality in the
interpretation codifying this fact. Further, clause (iii) implies that there might be
no plurality interpreting the predicate, in which case the predicate is empty, but, if
there is a plurality interpreting the predicate, then there is ‘exactly one plurality’
that interprets it. For a singular predicate \( P \), it is important to specify that an
interpretation should contain only one plurality of the form \([P; \beta\beta]\) that interprets
the predicate. Otherwise, it would be unclear which plurality should be taken as the
extension of the predicate. This restriction is not required in the case of a plural
predicate. Indeed, if the interpretation of a plural predicate \( C \) contains multiple
pluralities of the form \([C; \beta\beta]\), say \([C; xx]\) and \([C; yy]\), this just means that both \( xx \)
and \( yy \) are are in the extension of the predicate — \( C \) applies to \( xx \) and to \( yy \). If there
is no such plurality, the plural predicate is empty.

For example, let \( C \) be a plural predicate and let \( iii \) be an interpretation. When
we define the relation of satisfaction, it will be made clear that for any plurality \( xx \),
if \([C, xx] \prec^* iii\), this is taken to mean that the predicate \( C \) applies to (holds of)
\( xx \). Since it is OK that there be \( xx \) and \( yy \), \( xx \not\approx yy \), such that both \([C, xx]\) and
\([C, yy]\) are among \( iii \), there is no need for an extra clause for plural predicates in the
characterization of the notion of interpretation. In the case of a singular predicate \( P \),
there cannot be \( xx \) and \( yy \) such that \([P, xx] \prec^* iii\) and \([P, yy] \prec^* iii\), but \( xx \not\approx yy \).
So, for the unique \( xx \) such that \([P, xx] \prec^* iii\), the predicate \( P \) applies to (holds of)
any $x$ among $xx$.

We can now define the notion of satisfaction holding among an interpretation, a formula of the language, and variable assignment: $\text{Sat}(\phi, iii, ss)$, which will be written as $iii \models \phi [ss]$. Let $iii$ be an interpretation with domain $\alpha\alpha$ and let $ss$ be a variable assignment from $\alpha\alpha$, the following clauses provide an inductive characterization of satisfaction:

\begin{align*}
\text{(4.19) } & \text{(i) for any singular term } t \text{ and } s, \ iii \models t = s [ss] \text{ if and only if, for every } x \\
& \text{and } y, \text{ if } (\langle t, x \rangle) xx \prec^* iii \text{ or } (t, x) \prec ss, \text{ and if } (\langle s, y \rangle) yy \prec^* iii \text{ or } \\
& (s, y) \prec ss, \text{ then } x = y; \\
& \text{(ii) for any singular term } t \text{ and plural variable } vv, \ iii \models t \prec vv [ss] \text{ if and only if, for any } x \text{ and any } yy, \text{ if } (\langle t, x \rangle) xx \prec^* iii \text{ or } (t, x) \prec ss, \text{ and if } \\
& [vv, yy] \prec ss, \text{ then } x \prec yy; \\
& \text{(iia) for any } n\text{-ary singular predicate } P \text{ and singular terms } t_1, ..., t_n, \\
& \text{iii } \models P(t_1, ..., t_n) [ss] \text{ iff, for some } xx, \ [P; xx] \prec^* iii \text{ and } (x_1, ..., x_n) \prec xx, \\
& \text{where, for each } i, \ (\langle t_i, x_i \rangle) xx \prec^* iii \text{ or } (t_i, x_i) \prec ss; \\
& \text{(iib) for any } n\text{-ary plural predicate } C^{\llbracket m \rrbracket}, \text{ plural terms } \Theta_1, ..., \Theta_m, \text{ and singular } \\
& \text{terms } \theta_{m+1}, ..., \theta_n, \\
& \text{iii } \models C(\Theta_1, ..., \Theta_m, \theta_{m+1}, ..., \theta_n) [ss] \text{ iff } [C; xx_1, ..., xx_m, x_{m+1}, ..., x_n] \prec^* iii, \\
& \text{where, for each } i, \\
& \text{if } 1 \leq i \leq m, \ [\Theta_i; xx_i] \preccurlyeq ss, \text{ and } \\
& \text{if } m + 1 \leq i \leq n, \ x_i \text{ is the unique object such that } (\langle \theta_i, x_i \rangle) xx < iii \\
& \text{or } (\theta_i, x_i) \prec ss; \\
\end{align*}
(iv) for every formula $\phi$ and singular variable $v$, $iii \models \exists v \phi \ [ss]$ if and only if there is an $x \prec \alpha \alpha$ such that $iii \models \phi \ [ss(v/x)]$;

(v) for every formula $\phi$ and plural variable $vv$, $iii \models \exists vv \phi \ [ss]$ if and only if there are some $xx \preccurlyeq \alpha \alpha$ such that $iii \models \phi \ [ss(vv/xx)]$;

(vi) - (vii) the clauses for $\neg$ and $\land$ are the obvious ones.

Finally, we can proceed to define the superpluralist notion of logical consequence and logical truth for $L_{PL}$.

(4.20) (i) for any set of sentences $\Gamma$ and any sentence $\phi$, $\phi$ is a logical consequence of $\Gamma$ ($\Gamma \models \phi$) if and only if for any domain $\alpha \alpha$, any interpretation $iii$ with domain $\alpha \alpha$, and variable assignment $ss$ from $\alpha \alpha$: if $iii \models \gamma \ [ss]$ for every $\gamma$ in $\Gamma$, then $iii \models \phi \ [ss]$;

(ii) a sentence $\phi$ is a logical truth if $\emptyset \models \phi$.

This completes our presentation of the superpluralist semantics. The conceptual resources needed in the metatheory lie within $L_{SPL}$. Superplural quantification, with the aid of ordered $n$-tuples, is enough to carry out the desired semantics and capture the idea that the semantic values of plural predicates are superpluralities. As we have seen, one can define semantic interpretations can be construed as special kinds of superpluralities. Let us now move to consider some philosophical aspects of superpluralism.

\[13\] Even in this case, it can be proved that, when dealing with satisfaction of sentences, variable assignments are irrelevant in the sense that, if $iii \models \sigma \ [ss]$ (for any sentence $\sigma$), then $iii \models \sigma \ [ss^\ast]$ for any other assignments $ss^\ast$ from the relevant domain.
4.5 Discussion of superpluralism

4.5.1 Empty predicates

Since superpluralities are not empty, the superpluralist cannot maintain, in general, that plural predicates denote superpluralities.\textsuperscript{14} That would imply that, for any plural predicate $C$, the sentence $\exists vv \ C(vv)$ is a logical truth. The way we made room for empty predicates in section 4.4.2 was by allowing predicates to fail to signify. An atomic formula, such as $C(vv)$, is true in an interpretation just in case the things denoted by $vv$ in that interpretation are some of ($\prec^*$) the superplurality that interprets the predicate $C$. If there is no superplurality that interprets the predicate, the predicate is understood to be empty and the formula is false. This breaks with the model-theoretic tradition in which any predicate denotes. Even in the various semantics for free logics, predicates do not fail to denote. In model-theoretic frameworks based on set theory, the empty set provides the signification for empty predicates. With respect to empty predicates, the superpluralist approach also differs notably from the other pluralist approaches, which appeal to properties as semantic values of predicates. If properties are available, empty predicates are just those that denote empty properties, i.e., properties whose extension is empty.

Notice that this problem would be exacerbated if one wanted to impose the semantic requirement that pluralities contain at least two objects. At the beginning of this work, I have assumed, without a detailed analysis, that pluralities can be composed of single object (degenerate pluralities). This means that a sentence such as ‘Superman and Clark Kent love Lois Lane’ can be true (in the fiction) even if ‘Superman is Clark

\textsuperscript{14}Similarly, she cannot maintain, in general, that singular predicates denote pluralities.
Kent’ is true. This assumption has not played any significant role in our previous discussion. However, this is a place where calling the assumption into question would have some non-trivial repercussion for the superpluralist. Indeed, if the requirement of non-degenerate pluralities were in force, by analogy with pluralities, superpluralities would have to contain at least two pluralities. As a consequence, the superpluralist would have to find some *ad hoc* way of making room for predicates that, in some interpretation, are true of one plurality only. That is, she would have to find some *ad hoc* way of avoiding that any sentence of the form

\[
\exists xx (Cxx \land \forall yy (Cyy \to yy \approx xx)),
\]

be a logical falsehood.

Although it does not appear that thinking of empty predicates as failing to denote produces any technical difficulty, this approach requires a commitment to an unpopular view about reference. It is hard to downplay the importance of this consequence, especially if one adopts a certain view about the role of model theory, which pluralists have usually endorsed. As Rayo puts it,

> [T]here is more to model-theory than a characterization of logical consequence. Conspicuously, model-theory might be thought to deliver a generalized notion of reference [...].\(^{15}\)

Providing an account of logical consequence is not thought to exhaust the role of a model theory. A model theory has to be adequate, at least in the sense of accommodating one’s intuitive conception of the semantic values it assigns to the expressions of the object language. This view about the role of model theory sits well with

\(^{15}\text{Rayo (2006), p. 244.}\)
pluralism, since it encourages one to take seriously intuitive interpretations of the language in which the domain of quantification encompasses absolutely everything. So, even if superpluralism can capture the correct relation of logical consequence, it might be accused of assigning intuitively incorrect or *ad hoc* semantic values to plural predicates.

Natural language seems to allows for quantification into predicate position. Here are some examples from Higginbotham (1998):

(4.21) John is everything we wanted him to be.\(^{16}\)

(4.22) John is mostly what we expected him to be

\[\text{The only things we expected John to be are: honest, polite, and scholarly}\]

\[\text{John is either honest or polite}\]\(^{17}\)

A treatment of examples like these requires variable binding of predicate positions. However, it is semantically controversial how this form of quantification should be understood. In particular, it is controversial whether a substitutional analysis is enough to do it justice, or whether this type of quantification calls for the introduction of a domain of non-linguistic entities.

Quantification into predicate position is not limited to singular predicates. It extends to plural predicates. Consider the following examples.

(4.23) John and Mary are what we wanted them to be.


\(^{17}\)Ibidem.
(4.24) John and Mary are what we expected them to be

We expected them to be two people who care for each other

John and Mary are two people who care for each other

If the substitutional approach turns out to be inadequate, it is natural to interpret quantification into predicate position as quantification over the entities that are taken to be denoted by the predicates. In other words, it is natural to try to specify the interpretation of a sentence like (4.23) by means of a quantified sentence in the metalanguage that reflects the quantificational structure of (4.23). For the superpluralist, that would be a sentence quantifying over superpluralities.

However, this would not work in general, or at least it would not work without introducing some semantic complication that would set the logical structure of (4.23) apart from the logical structure of the sentence that specifies its truth-conditions in the metalanguage. Consider the following consistent passage.

We expected John and Mary to be two people who care for each other.

But they are not what we expected them to be. That’s not surprising.

There are no people who care for each other!

In the last sentence, the plural predicate ‘care for each other’ is asserted to be empty. So, on the superpluralist account, it does not denote. But, then, the quantification into predicate position in the first two sentences cannot be simply interpreted as meaning that

(a) there is a superplurality of those who care for each other and that we expected John and Mary to be some of that superplurality, but

(b) John and Mary are not some of the superplurality of which we expected them to be some.
In light of the last sentence of the passage, (a) and (b) cannot provide the semantic interpretation of the first two sentences. Indeed, since there cannot be an empty superplurality, the last sentence entails that there is no superplurality of those who care for each other. Whatever solution is adopted by the superpluralist, it comes at the cost of complicating the semantics.

If there are reasons to avoid the thesis that empty predicates fail to denote, the superpluralist could use a dummy object to serve (conventionally) as the denotation of empty predicates and would have to revise the model theory accordingly. Although this might have some notable historical precedents (e.g., Frege and Carnap), it is unsatisfactory for someone like the pluralist who aims at assigning intuitively correct semantic values to the expressions of the object language. Regarding empty predicates as failing to denote seems then unavoidable on the superpluralist approach. This clashes with a popular view about reference and prevents semantic interpretations from mirroring the logical structure of the sentences they interpret. Moreover, it would not avoid all artificial moves in the semantics in case one required that pluralities, hence superpluralities, be composed of more than one entity of the appropriate kind.

4.5.2 Plural properties and superplurals

Within the pluralist camp, the direct competitor of superpluralism is standard pluralism, as defended by Yi (1999, 2005, 2006) and Oliver and Smiley (2006). According to this approach, plural predicates denote plural properties. Standard pluralism has a straightforward way to deal with empty predicates. It is interesting, then, to determine whether superpluralism has any advantage over standard pluralism.

Rayo writes:

In this paper I give no reason for favoring a hierarchy of higher and higher
level predicates [plurals, superplurals, etc.] over a hierarchy of higher and higher order predicates [higher-order logic]. I have chosen to focus on the former because it seems to me that second-level predicates deliver a more natural regimentation of English predicates with collective readings than their second-order counterparts. But either hierarchy will do, as far as the purposes of this paper are concerned.¹⁸

Unlike Rayo, we understand the dispute about plurals not as a dispute about regimentation, but as a dispute about semantics.¹⁹ In this passage, Rayo is concerned about how to provide a natural regimentation of English. For us, all semantic views about plurals adopt the same regimenting language. They differ over how they construct the model theory. What, if anything, recommends superpluralism then?

Properties have a venerable pedigree: they play an important role in various semantic analyses. By contrast, superplurals are a relatively recent innovation. However, there may be enough structural similarity between singular properties and pluralities, and plural properties and superpluralities, for the superpluralist to be able to claim that superpluralities (or pluralities) can replace properties in the contexts in which properties have been traditionally used. Although a limited version of this claim might be true, it is hard to see how it could apply across the board. Properties have been thought to be crucial in the analysis of intensional contexts for which plurals and superplurals, given their extensional nature, would seem inadequate.

The main reason why plurals and plural quantification have appeared semantically attractive is their alleged ontological innocence. The thought is that, by avoiding the use of properties, invoking superplurals would enable us to provide a semantic theory

²¹See the Introduction and Chapter 1 for a discussion of the distinction.
that is ontologically innocent and, therefore, preferable over standard pluralism or property singularism.

It has been claimed that plural quantification over some things commits us to nothing over and above what singular quantification over each of those things would commit us to. So, unlike second-order quantification, which would commit us to properties, plural quantification would be ontologically innocent. If one adopts this view, it might be tempting to regard superplural quantification, in analogy with plural quantification, as ontologically innocent. But do we have reason to think that superplural quantification is ontologically innocent? Even if plural quantification is innocent, does the analogy with plurals support the claim that superplural quantification is innocent? Let us turn to these questions. In the next section, I will try to give a tentative answer to them.

4.5.3 Is superplural quantification ontologically innocent?

Issues of ontological commitment and ontological innocence are notoriously hard to adjudicate. Without going too far into intricate questions about ontological commitments, let us consider whether, even \textit{prima facie}, superplural quantification could be regarded as ontologically innocent and, hence, whether considerations of ontological innocence could provide an argument in favor of superpluralism.

As discussed in section 4.1, two types of analogies have been used to help the novice understand what superplurals are. According to the first analogy, superplural quantification is related to plural quantification in the same way in which plural quantification is related to singular quantification. However, we saw that Rayo warns us against taking this analogy too seriously. What seems to motivate his warning is the worry that the analogy requires one to think of pluralities as entities of some
sort. If plural quantification commits us to the things it plurally quantifies over, i.e., objects, then superplural quantification should commit us to the entities it plurally quantifies over, i.e., pluralities. So superplural quantification would be ontologically committing, since it would commit us to pluralities as entities.

If one steers away from the first analogy and follows Rayo in embracing the second analogy, one would regard the relation between plural and superplural quantification by analogy with relation holding between second-order and third-order quantification. This analogy, unfortunately, is not obviously helpful to determine the ontological commitments of superplural quantification. Second- and third-order logic, as understood in this context, are both ontologically committing. In particular, they commit us to the existence of first- and second-order properties respectively. This means that third-order quantification introduces a new type of ontological commitment with respect to second-order quantification, namely, a commitment to second-order properties. In light of the analogy, should we conclude that superplural quantification introduces a new type of ontological commitment as well?

It looks as though none of the analogies offers strong support to the claim that superplural quantification is ontologically innocent. So, even if plural quantification is innocent, it is unclear whether one could legitimately take superplural quantification to be innocent. However, a dialectical issue arises with respect to the dispute between the superpluralist and the property singularist.

If one wants to pursue the first analogy and argue for the innocence of superplural quantification, one must assume that plural quantification is innocent: superplural quantification has no ontological commitment to entities other than objects only if plural quantification does not either. However, the dialectical situation between the superpluralist and the property-singularist is such that the superpluralist is in no
position to assume, without begging the question, the ontological innocence of plural quantification. Indeed, it might be thought that, if property singularism is correct, plural quantification commits us to the existence of properties. Thus, assuming the innocence of plural quantification amounts to ruling out property singularism.

In other words, the ontological innocence of plurals is needed by the superpluralist to argue against a view, property singularism, that seems to entail that plurals are not innocent; but, in the absence of independent motivations to think that plurals are innocent, the argument from ontological innocence has no dialectical force against the property-singularist. It is only in light of a proper semantic account of plurals — the property-singularist will argue — that we can determine whether plurals are ontologically innocent or not.

It appears that, no matter which analogy is taken to illuminate the relationship between plurals and superplurals, there is little support for the claim that superplural quantification is ontologically innocent. If so, superpluralism cannot claim to have a clear advantage over standard pluralism. Moreover, assuming the ontological innocence of plurals is necessary if one is to claim that superplural quantification is ontologically innocent. Unfortunately, that assumption begs the question against the property singularist. We have to conclude that considerations of ontological innocence do not provide any clear reason to prefer superpluralism over competing views.

4.6 Conclusion

In this chapter, I have discussed superpluralism, a version of pluralism that takes plural predicates to denote superplurals. As we have seen, there is no uncontroversial evidence that English contains a category of superplural terms. Moreover, even if natural language contained such terms, the claim that they should be understood as
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referring to superpluralities seems problematic. Therefore, one might question the legitimacy of a metalanguage that contains superplural expressions.

But, for theoretical purposes, one could introduce the language of superplurals in a purely formal way. However, it does not appear that the resulting semantic view, superpluralism, would have any advantage over the competing views. First, superpluralism has unattractive consequences for the treatment of empty plural predicates. Second, there is no clear reason to think that superplural quantification is ontologically innocent. Therefore, the view loses what has be taken to be its main theoretical advantage, namely, that of providing a semantic account of plurals without incurring any ontological commitment in addition to those incurred by the first-order quantifiers.
Conclusion

In the current philosophical debate about the semantics of plurals, pluralism seems to have the upper hand. I have attempted to clarify the nature of this debate and to rehabilitate singularism.

As I have emphasized throughout this work, it is important to distinguish two dimensions of the debate. The first dimension is occupied by the dispute between syntactic singularists and syntactic pluralists. The second dimension is occupied by the dispute between semantic singularists and semantic pluralists.

In Chapter 1, I have argued that there is reason to abandon syntactic singularism in favor of syntactic pluralisms. That is, one should not attempt to regiment natural language plurals within a purely singular language. On the contrary, one should adopt a regimenting language in which singular expressions are distinguished from plural ones.

Adopting a plural regimenting language, however, is compatible with a number of ways of constructing a model theory for the regimenting language. The semantic dispute between singularists and pluralists is a dispute about the shape that a model-theoretic semantics for plural expressions should take. Against a popular opinion among philosophers of language and philosophical logicians, I have argued that, if we look carefully at the semantic dispute, singularism is not an implausible view at
all. In fact, it fares at least as well as the most promising version of pluralism, i.e., standard pluralism. The technical result proved in the Appendix of Chapter 2 shows that, for the purpose of defining a notion of logical consequence for $\mathcal{L}_{PL}$, the pluralist’s appeal to pluralities and their properties is redundant. The same notion of logical consequence can be formulated using only singular resources. We must conclude that plural resources are not semantically indispensable.

Two main versions of semantic singularism have been outlined: object singularism and property singularism. Unlike the first, the second version employs higher-order resources. This view is primarily motivated by the desire to capture interpretations of the object language with an all-inclusive domain, remedying the object-singularist’s apparent inability to accommodate absolutely general quantification. I have tried to show that, once semantic singularism is distinguished from syntactic singularism, there is no obstacle in the way of object singularism for an opponent of absolute generality.

As the arguments in Chapter 3 and 4 have shown, some versions of pluralism are less attractive than it is usually thought. In Chapter 3, we have seen that untyped pluralism leads to paradox and runs against a plural version of Cantor’s theorem. In Chapter 4, I have argued that superpluralism seems to offer no significant theoretical advantage over standard pluralism and, indirectly, over singularism. We have found little evidence that supports the idea that superpluralism, if legitimate, provides an ontologically innocent framework to conduct semantic investigations.

Providing a comprehensive and fully adequate semantics of natural language plurals is a vast task, as attested also by the work done in linguistic semantics. In this dissertation, I have only addressed the foundational issues, philosophical and logical, concerning the construction of a model-theoretic semantics for a basic regimenting
language, such as $\mathcal{L}_{PL}$. In order to account for all the complexities of natural language plurals, much more is required. However, it is important to build one’s semantic edifice on a sound foundation. In light of the arguments presented in this dissertation, there is reason to think that singularism offers a viable theoretical option.
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