The Demand for Owner-occupied Housing: A Study
of the Simultaneity among Housing Demand,
the Choice of Loan-Value Ratio and the Length of Stay.

DISSERTATION

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* * * * *

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To my parents
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Chapter I
INTRODUCTION

The purpose of this dissertation is to develop a structural model of the demand for owner-occupied housing. Unique aspects include the incorporation of interactions among a household's decision variables, and the treatment of the price of housing services as an endogenous variable in the model. The result of these assumptions is that the price of housing varies among households depending on their loan-value ratio and the planned length of stay; this result representing a substantial departure from current analyses.

Our main goal is to test whether the decision variables are simultaneously chosen. The simultaneous decision mechanism is compared with a sequential choice mechanism that is consistent with prior studies. In order to test the hypothesis about the simultaneity among the housing decision variables, a structural equation system is estimated using the Panel Study of Income Dynamics data from 1979.

Traditionally, the demand for owner-occupied housing is specified as a function of a household's income and the
price of housing services. The usual specification of the price of housing services for owner-occupied housing is a market based price index modified by a household's income tax rate (Buth(1969), Laidler(1969), Aaron(1972), Struyk(1975)). More recently, a measure of the user cost of housing has replaced the traditional definition of the price of housing services when analyzing the demand for housing services. (Fenderson(1979), Kearl(1979), Rosen(1980), Pollain(1982)). The user cost of housing is developed by applying neoclassical investment theory to owner-occupied housing and it introduces the expected capital gain from housing as a major determinant of the price of owner-occupied housing. However, all previous housing demand studies have ignored the point that a utility-maximizing household simultaneously selects the value of the house purchased, the loan-value ratio, and the length of stay. In other studies, the price of housing services is independent of the loan-value ratio and the length of stay. This independence is justified by unrealistically assuming that the owner has an infinite length of stay and that the borrowing rate always equals the lending rate.

The importance of these results is that the range of housing policies to provide adequate housing to all households can be expanded. Our model and empirical results indicate that if you can affect the choice of loan-value
ratic or length of stay, housing demand will also be modified. The range of contending policies is thereby expanded by including policies concerning the mortgage rate structure and various types of government guaranteed mortgages. A larger income elasticity in the structural form than in the reduced form indicates that policies in the mortgage market are complementary to housing subsidies in promoting the consumption of housing services. The effect of the length of stay on the housing demand indicates that housing subsidies to younger household are more effective than those to older households because a younger household expects a rising income profile compared to an older household's flat or falling income profile. Another important aspect of these results is that the comparison between the structural and reduced forms of the housing demand equation can be used to explain both the observed temporal variations in housing demand and the variations in the estimate of housing's income elasticity with regard to income or wealth. These results also indicate that every household's tax benefit differs with its combinations of the loan-value ratio, the length of stay, and the quantity of housing demanded; and that low-wealth households can enter the owner-occupied sector of the housing market by adjusting the loan-value ratio, the length of stay, and the quantity of housing simultaneously.
Estimated parameters of the loan-value ratio and the length of stay in the housing demand equation are positive and significant. The estimated parameter of housing services in both the loan-value ratio equation and the length of stay equation are also positive and significant. These two results confirm the proposed hypothesis. The income elasticity of housing in the structural form is greater than that in the reduced form. The estimation results show a number of sign reversals of the estimated parameters between reduced form and structural form thereby demonstrating the danger of ignoring the simultaneity in the housing decision. The level of initial wealth, the expected opportunity cost of capital, and the marginal income tax rate in both the housing demand equation and the length of stay equation have different signs in the structural and the reduced forms.

This paper consists of the following chapters. Chapter 2 reviews the previous studies and presents their perspectives on housing demand. A simultaneous decision model is constructed in Chapter 3 and the testable hypotheses are also presented. Chapter 4 deals with the econometric issues encountered when estimating the structural model. Chapter 5 describes the specification of the variables and the data. Empirical results are presented in Chapter 6. In Chapter 7, we summarize the findings and discuss their
policy implications. The limitations of the analysis are also discussed in Chapter 7.
Chapter II

LITERATURE REVIEW

The estimates of the price and income elasticities of the demand for owner-occupied housing differ greatly across housing demand studies. Micro data studies conducted during the 1970's generally found income elasticities to be between 0.25 to 0.87 and price elasticities from -0.57 to -1.02 (Mayo (1981), Macrae and Turner (1981), Goodman and Kawai (1982)).

2.1 PRICE OF HOUSING SERVICES

Muth (1969) argues that there exists a unique before-tax price for housing services in a market, with the only difference in the price of housing services being between owner-occupied housing and rental housing, due to the favorable tax treatment of owner-occupied housing. Laidler (1969), Aaron (1972), and Struyk (1974, 1975) followed Muth's line of thought and specified the price of owner-occupied housing as a function of price of the housing services and the income tax rate.
The price and quantity of housing services are unobserved. Only housing expenditure or value of a house are observed. Although the housing characteristics approach has developed recently, the approach that continues to predominate is to measure the quantity of a dwelling's housing services by the market value of the house. The quantity of housing services is assumed to be proportional to the housing expenditure or the market value of the house. The price of housing services is housing expenditure or house value divided by the quantity of housing services. Difficulties in measuring the quantity of housing services due to the multi-dimensional nature of housing services are bound to create problems in measuring price. Different methods have been used to measure unobserved housing prices. They are based on: (1) the Bureau of Labor Statistics 'family workers budgets' which contain relative price information (deLeeuw (1971), Maisel, Furnham and Austin (1971), Carliner (1973), and Lee and Kong (1977)); (2) the parameters of a housing production function which are used to transform factor price information on housing output prices (Fclinsky and Ellwood (1979)); and (3) a hedonic index of housing services, which is used to define the price of a standard bundle of services which may be used to establish relative prices (Straszheim (1973) and Goodman and Kawai (1982)).
The FLS method specifies a particular set of housing characteristics and computes for each city the annual cost of living in a house with those characteristics. This method rules out the possibility of factor substitution in the production of housing and does not allow for unit price variation within a city.

The production function method estimates a constant returns to scale production function for housing using the prices of land and housing capital. Then, the corresponding unit cost function is calculated, which is used for constructing the price of housing under the assumption of perfect competition among housing producers. The housing price index constructed in this approach varies over all observations. While this method need not specify the standard housing unit, perfect competition in all markets must be assumed for the procedure to be valid.

The hedonic price index approach, first, estimates the hedonic price functions for housing submarkets. The hedonic price function is estimated by regressing housing expenditure on housing characteristics. From the estimated hedonic price function, implicit prices of housing characteristics are calculated by taking partial derivatives with respect to corresponding characteristics. Then, a hedonic price of housing services is constructed for each submarket by applying these estimated implicit characteris-
tic prices to the specified standard market bundle of housing characteristics. The hedonic price index is more attractive than the other two methods because it accounts for the multi-dimensional nature of a dwelling unit. However, the crucial problem of this approach is the arbitrariness of the standard bundle. It does not explain how housing characteristics are converted into a composite housing services. Another problem is the requirement that the estimated hedonic equations have to be significantly different from one another among submarkets in order to have a valid hedonic index of housing prices.

Different specifications of the price of housing services for owner-occupied housing result in different estimates of the income and price elasticities of the demand for owner-occupied housing. Housing demand analyses based on BLS price data indicated price elasticities from -0.57 (Lee and Kong (1977)) to -0.89 (Maisel, Burnham and Austin (1977)). Income elasticities from the analyses based on BLS price data are within the range from 0.05 (Maisel, Burnham and Austin (1977)) to 0.87 (Lee and Kong (1977)). Polinsky and Ellwood (1979) derived the price of housing based on a production function for housing and the prices of the inputs. Use of these micro-based housing prices resulted in lower estimates of the elasticities for income and price compared to the estimates found when using a
price index based on aggregated data, such as the BLS price data (Polinsky and Fillmore, 1979). Macrae and Turner (1981) found that adjusting for the tax benefits of home ownership resulted in an even lower income elasticity estimate of 0.25. Their price elasticity estimate was -0.88. Housing demand analyses based on the hedonic price index approach indicated a price elasticity ranging from -0.53 (Straszheim (1973)) to -1.0 (Goodman and Kawai (1982)). The estimates of income elasticities were 0.42 (Straszheim (1973)) and 0.98 (Goodman and Kawai (1982)).

These measures of housing prices imply that owner-occupied housing is essentially the same as rental housing in that it provides the same housing services as rental housing does. However, owner-occupied housing is not the same as rental housing since owner-occupied housing is not only a consumption good but also an investment good. The investment aspect of owner-occupied housing was made obvious because it yielded a higher rate of return than alternative financial assets during the 1970s (Hendershott and Hu (1979)). Therefore, another measure of the price of housing services which incorporates the investment aspect of owner-occupied housing, its user cost, will be discussed in the next section.
2.2 User Cost of Housing

Litzenberger and Sosin (1978), Downs (1979), and Hendershott (1979) incorporate the investment aspects of owner-occupied housing into the price of housing services by introducing expected capital gains. The user cost of housing capital is now widely used as the relevant price of housing services for owner-occupied housing (Hendershott and Huf (1979), Follain (1979), Pearl (1979), and Rosen (1979)).

The user cost of housing begins with the notion that, in equilibrium, one expects that the present value of the net cash flows from a house will equal the initial equity investment. In deriving the user cost of housing, it is assumed that: (1) the expected appreciation rate of housing equals the expected rate of increase in the rental price; (2) no transaction costs accrue in selling the owner-occupied housing; (3) the expected holding period of housing is infinite; and (4) the required after-tax rate of return on equity invested in owner-occupied housing equals the expected after-tax mortgage interest rate. Assumption (4) implies that a utility-maximizing household equalizes the after-tax rate of return on available investment opportunities.

The user cost of owner-occupied housing is described by:

\[ UC = \left( (1-T_v) \hat{i} - \pi_M d - (1-T_v) T_p \right) P_M / P_r. \] (2.1)
$\tau_\gamma$ is the household's marginal income tax rate; $i$ is the mortgage interest rate; $\tau_p$ is the property tax rate; and $d$ is the depreciation rate. The expected appreciation rate of housing is described by $\Pi_H$. $P_H$ is the price of housing capital and $P_r$ is the rental price of housing capital. The crucial difference between user cost in equation (2.1) and other housing price index is the presence of the expected appreciation rate of housing. When homeowners expect capital gains, it lowers their effective cost of housing. This implies that, when a high appreciation rate of housing is expected, a larger house should be purchased for investment purposes even if some rooms provide low imputed rental values.

The user cost of owner-occupied housing has been widely used to explain the reasons why households showed a strong preference toward owner-occupied housing during the 70's (Bendershott and Hu(1979), Rosen and Rosen(1980)). The user cost of housing also enables us to specify the household's tenure choice as a function of the income and the ratio of the price of housing services between owner-occupied housing and rental housing. Previously, the household's tenure choice function was specified as a function of its income and other demographic variables (Struyk(1974, 1975)). The price term did not appear in the tenure choice function because a difference in income was
thought to be sufficient to explain the difference in the housing price between owner-occupied housing and rental housing. However, when the user cost of housing is used, the difference in the housing price between owner-occupied housing and rental housing can also be explained by the expected appreciation rate of housing. The expected appreciation rate of housing is independent of the household's income. Therefore, the price ratio between owner-occupied housing and rental housing is included in the household's tenure choice function in addition to its income (Mendeshott (1979), Rosen (1979)). Moreover, empirical work has generated significant parameter estimates of the price ratio with the expected sign. Although the user cost of housing capital is widely used as the relevant price of housing services, it omits some important aspects of cost.

Prior analysts have assumed the user cost of housing capital is independent of the loan-value ratio. This relationship is based on the following three assumptions. First, the after-tax mortgage interest rate is equal to the marginal after-tax rate of return on equity invested in owner-occupied housing. Second, the expected opportunity cost of capital is unique and the same for all households. Third, the mortgage interest rate is independent of the loan-value ratio. Hanney (1979), Kane (1979) and Van-
dell (1980) have questioned the validity of these three assumptions.

Kane holds that the expected opportunity cost of capital varies among households based on differences in age and wealth due to the following reasons: deposit-rate ceilings, restricted opportunities for small-denomination marketable instruments, and the high cost of acquiring sufficient sophistication to track suitable investment opportunities. Vandell (1980) and Plaut (1984) argue that the mortgage interest rate becomes higher as the loan-value ratio becomes larger since lending institutions charge a higher mortgage interest rate for a household with a larger loan-value ratio. This is to guard against the possibility of a larger loss if the mortgage loan is defaulted.

Ranney (1979) developed an intertemporal utility maximization model in which a household's loan-value ratio is a choice variable, in addition to the quantity of owner-occupied housing. The model introduces an externally imposed upper limit on the loan-value ratio, and the mortgage interest rate is assumed to be independent of the loan-value ratio. It is also assumed that the mortgage rate under the standard fixed-rate mortgage system is always greater than the expected opportunity cost of capital. The price of housing services for owner-occupied housing differs across households whether the loan-value ratio equals the exter-

nally imposed limit or not. If the loan-value ratio attains its upper limit, then the price of housing services for owner-occupied housing depends upon the loan-value ratio. If the loan-value ratio is smaller than the upper limit of the loan-value ratio, the price of housing services for owner-occupied housing is dependent upon the ratio of the mortgage repayment rate to the expected opportunity cost of capital, which is independent of the loan-value ratio. However, Ranney's model fails to include the effect of an unconstrained loan-value ratio on the price of housing services because it ignores the positive relationship between the mortgage interest rate and the loan-value ratio.

Kearl (1979) and Kane (1979) also recognize the effect of the loan-value ratio on the quantity of owner-occupied housing demanded. However, by ignoring the effect of the loan-value ratio on the price of owner-occupied housing services, these studies confined the role of the loan-value ratio to supplementing income and wealth in acquiring the desired quantity of owner-occupied housing.

By assuming either an infinite holding period (Mender-shott (1979)) or an exogenously determined length of stay (Ranney (1979)), authors have eliminated the effect of the length of stay on the user cost of housing. However, these
assumptions are too rigid and unrealistic when both the variation in the quantity of housing services demanded at each stage of life cycle and the uncertainty of future capital gains from a housing asset are considered.

Shelton (1968) shows that high transaction costs for selling a house limit an owner-occupier's ability to move into another house. However, he points out that the length of stay affects the price of housing services since the annual burden of the transaction costs for a house sale is negatively related to the length of stay. Muth (1974) also shows that a household's expenditure on owner-occupied housing depends upon the length of stay in addition to income and the price of housing services. For the households headed by a person over 60 years of age, the elasticity of housing expenditure with respect to the length of stay is estimated as 0.71. The estimated elasticity is 0.8 for the households headed by a person over 30 years of age.

The studies by Shelton (1968) and Muth (1974) recognize that the length of stay affects the demand for owner-occupied housing. However, their studies miss two important points. The first question is: How does a household determine its length of stay? The second question is: How does the quantity of owner-occupied housing affect the length of stay?
2.3 **Perspectives**

Previous studies on the demand for owner-occupied housing ignore important determinants of the price of housing services. The model developed here analyzes the demand for owner-occupied housing in the context of a simultaneous equation system; including as choice variables the quantity of owner-occupied housing, the loan-value ratio, and the length of stay.

Use of a simultaneous equation system raises econometric issues because of the inclusion of limited dependent variables and the possibility of sample selection bias. Using a two-stage procedure based on the probit function, Lee and Trost (1979) and Rosen (1979) solve the problem of sample selection bias created by the effect of tenure choice on the demand for owner-occupied housing. However, our proposed model is expected to have simultaneous relationships among the quantitative choice variables in addition to the effects of qualitative choices on the quantitative variables. Therefore, an estimation method dealing with a simultaneous equation system having limited dependent variables will be applied to the proposed model to obtain consistent estimates of the parameters.

---

1. If the value of the dependent variable should be positive in a theoretical reason or observations on the dependent variable below some values are not reported and treated as value of zero, the dependent variable is defined as a limited dependent variable.
Chapter III

MODEL

3.1 BASIC MODEL

3.1.1 The Household Utility Function

This chapter develops a two period model of the housing decisions of a family. The household buys a house in the beginning of the first period and plans to live there for a certain periods. It then plans to move into another house, where it remains until the terminal period T. The length of the stay in a house is a variable which differs across households. The household realizes utility each period from the consumption of housing services and a composite good.

The quantity of housing services consumed within a period is constant. Let \( h_A \) and \( h_B \) be the quantity of housing services consumed each year during the first period (A) and second period (B) and let \( q_A \) and \( q_B \) be the quantity of the composite good consumed yearly (also assumed constant within each period) during the first period and the second period respectively. The household's intertemporal utility function is:
\[ U = \int_{0}^{s} U(h_A, q_A) e^{-\delta t} dt + \int_{s}^{T} U(h_B, q_B) e^{-\delta t} dt \]

\[ = k_0 U(h_A, q_A) + k_0 e^{-\delta s} U(h_B, q_B), \quad (3.1) \]

where \( \delta \) : time preference rate,

\( T \) : the length of the life cycle,

\( k_0^s = \int_{0}^{s} e^{-\delta t} dt = (1 - e^{-\delta s}) / \delta \), and \( (3.2) \)

\( k_0^t = \int_{s}^{T} e^{-\delta (t - s)} dt = (1 - e^{-\delta (T - s)}) / \delta. \) \( (3.3) \)

The utility function is assumed to be increasing, twice differentiable and concave in its arguments.

We also assume that preferences are intertemporally separable. It is assumed that the flow of housing services from housing capital is constant, and housing does not depreciate. A household selects the planning horizon, in addition to the quantities of housing and other goods. It is important to note that \( h_A \) and \( h_B \) represent the flow of housing services per year and the number of years within a period is a choice variable because of the endogeneity of \( s \).

3.1.2 Constraints

An owner faces three different constraints when maximizing intertemporal utility: the intertemporal budget constraint and each period's borrowing constraint for housing. The following assumptions are used in the specification of the constraints. The price of other goods equals one and remains constant over the entire period.
housing capital, $P_H$, increases at a rate of $\Pi_H$; a household can borrow only for a house purchase; and a fixed-rate, fixed-payment mortgage system is assumed. The mortgage interest rate is positively related to the loan-value ratio and it is also assumed that the mortgage interest rate is always greater than a household's opportunity cost of capital.

The appreciation rate of housing is assumed to be smaller than the mortgage interest rate. Otherwise, householders would face a negative (marginal) price of housing services for owner-occupied housing. Muellbauer (1981) points out that the price of housing services for owner-occupied housing can, in principle, fall to zero or below, which should lead consumers to demand infinite quantities attempting to profit from the capital gains. A negative (marginal) price of housing services prohibits the derivation of a demand function for housing services because it destroys the convexity of the budget constraint.

The intertemporal budget constraint requires the household's total resources to be equal to its total expenditures, which implies that a household has no bequest. The present value of the owner's total resources consists of

---

2 It is impossible for a household to expect a negative (marginal) price of housing services even though the actual price of housing services can be negative during a period with a high appreciation rate. The reason is that a household would treat the high appreciation rate as a temporary and extraordinary phenomenon.
initial wealth, \( w_0 \), and the present value of the expected income during the two periods, \( Y_A \) and \( Y_B \). \( Y_A \) and \( Y_B \) are defined as level streams of income over periods A and B respectively by using the average of the expected income for each period. A household's total expenditure consists of the expenditure on housing and other goods.

Owners pay a downpayment and make periodic mortgage payments during the length of stay.\(^3\) The present value of mortgage payments for period \( j(j=A,B) \) is specified as follows. Let \( r_m^j \) be the mortgage payment rate during period \( j \) assuming a fixed-rate, fixed-payment mortgage system. The mortgage payment rate, \( r_m^j \), depends on the mortgage interest rate, \( i_j \), and the term-to-maturity of the mortgage loan, \( N_j \).

\[
r_m^j = i_j (1+i_j)^{N_j}/((1+i_j)^{N_j}-1) \tag{3.4}
\]

Let the ratio of the mortgage interest payment to the mortgage payment be \( \alpha_j(t) \). Under the fixed-rate, fixed-payment mortgage, the variable \( \alpha_j(t) \) decreases over time because of the amortization of the mortgage principal. This implies that a household pays relatively more interest in the early years of the loan. Let \( h_j \) be the loan-value ratio of the mortgage loan for period \( j \) and \( V_j \) be the present value of

\(^3\) The cost of the mortgage loan consists of the after-tax mortgage interest payment and the repayment of the mortgage principal. The difference between the gross mortgage interest payment and the after-tax mortgage interest payment is due to the deductibility of the mortgage interest payment in the federal income tax.
the house purchased in period $j$. Then, the after-tax mortgage payment is:

$$(1-\tau \alpha_j(t))l_j r m_j v_j,$$  \hfill (3.5)

where $\tau$ is the marginal income tax rate. The term, $\tau \alpha_j(t) l_j r m_j v_j$, captures the income tax deductibility of the mortgage interest payment for year $t$ during the period $j$. Using the discount factor $K_j$, the present value of the income tax deductibility of the mortgage interest payment for period $j$ is approximated by:

$$\tau \alpha_j(z/2) b_j r m_j K_j v_j \text{ with } z=s \text{ if } j=A \text{ and } \quad z=T-s \text{ if } j=B. \hfill (3.6)$$

The present value of the mortgage payment for period $j$ is now specified as:

$$(1-\tau \alpha_j(z/2)) b_j r m_j K_j v_j \text{ with } z=s \text{ if } j=A \text{ and } \quad z=T-s \text{ if } j=B. \hfill (3.7)$$

The present value of the downpayment for period $j$ is $(1-b_j) v_j$.

The present value of all housing expenditures made during period $j$ is the sum of the present value of the downpayment and the present value of the mortgage payment. The present value of total housing expenditures for period $j$, symbolized as $X_{1j} v_j$, is:

$$X_{1j} v_j = ((1-b_j) + (1-\tau \alpha_j(z/2)) b_j r m_j K_j v_j \text{ with } z=s \text{ if } j=A \quad z=T-s \text{ if } j=B. \hfill (3.8)$$
Owner-occupied housing is different from rental housing in that it not only provides capital gains during the length of stay but also is associated with considerable transaction costs upon sale at the end of the stay. The present value of the capital gain from housing for period and $E$ is defined by:

$$(e^{(\Pi_H - r)z} - 1)V_j,$$

where $Z$ is defined as before, $\Pi_H$ is the expected appreciation rate of housing, and $r$ is the discount rate. Capital gains taxes are avoided because a household can move into another house thereby deferring the tax and eventually the one-time exemption for owners aged 55 or more effectively eliminates the tax. The ratio of the transaction cost to the house value, $\beta$, is assumed to be a constant. The present value of the household's receipts from the house sale net of the transaction costs, $V_{2j}$, is defined as:

$$V_{2j} = e^{(\Pi_H - r)z}(1 - \beta)V_j.$$  \hfill (3.9)

Owners also accumulate wealth by building up equity in housing during the period of stay. Wealth accumulated from the equity build-up is measured by the changes in the remaining mortgage principal. The present value of the remaining mortgage principal at the end of the period $j$, $V_{3j}v_j$, is:

$$V_{3j}v_j = \left(\frac{b_j}{\alpha_j(0)}\right)e^{-rz^j}h_j(1 - 1/\alpha_j(0))e^{-(i_j - r)z}.$$  \hfill (3.10)
The variable $\alpha_j(0)$ refers to the ratio of the interest portion of the mortgage payment to the total mortgage payment at the time of the first mortgage payment.

In summary, the present value of the owner's net expenditure on housing for each period is described by:

\[(I_{1j} + I_{3j} - I_{2j}) \cdot V_j \quad \text{for} \quad j = A, B \tag{3.11}\]

where

\[I_{1j} = (1-b_j) + (1 - \tau_j(z/2)) \cdot b_j \cdot r_j \cdot m_j \cdot Y_j,\]
\[I_{2j} = \left( \frac{\Pi_H - x}{1 - \beta} \right) \cdot z,
\[I_{3j} = \left( \frac{b_j}{\alpha_j(C)} \right) \cdot e^{-xz} + b_j \cdot (1 - 1/\alpha_j(0)) \cdot e^{-(i_j - x)z},\]

\[z = \begin{cases} s & \text{if } j = A \\ T - s & \text{if } j = F, \text{ and} \end{cases}\]

\[V_j = \begin{cases} \left( \frac{P_H}{\gamma} \right) h_A & \text{if } j = A \\ \left( \frac{P_H}{\gamma} \right) e^{(\Pi_H - x)s} h_B & \text{if } j = F. \end{cases}\]

Here $\gamma$ is the factor that converts housing capital into the flow of housing services and $r$ is the discount rate, which is equal to the after-tax opportunity cost of capital.

The term $(I_{2j} - I_{3j}) \cdot V_j$ represents the present value of wealth accumulated through investment in owner-occupied housing during period $j$. A proportional income tax rate is assumed thereby avoiding the problem of distinguishing the average income tax rate from the marginal tax rate. It is also assumed that the opportunity cost of capital, the appreciation rate of housing asset, and the income tax rate remain constant throughout the period.
Using equation (3.11), the intertemporal budget constraint can be derived:

\[ w_0 + (1 - \tau) \sum_j Y_j - \sum_j (X_{1j} + X_{2j})V_j - \sum_j Q_j = 0, \quad \text{for } j = A, B, \]  

(3.12)

where \( Q_j \) is the present value of the cost of the composite good in period \( j \),

\[ C_A = \int_0^S q_A e^{-rt} \, dt = c_A^S, \quad \text{and} \]
\[ C_B = \int_T^S q_B e^{-rt} \, dt = q_B^e - rS, \int_S^{T-e} r(t-s) \, dt = c_B^e - rS, \]

(3.13)

The assumed relationship among the mortgage payment rate, the expected appreciation rate of housing, and the expected opportunity cost of capital is: \( r_m \geq \eta_H \) and \( r_m > r \). These assumptions assure the convexity of the intertemporal budget constraint with respect to housing services.

The owner also faces borrowing constraints. An owner must have enough wealth to meet the downpayment constraint at the time of house purchase. Households face two borrowing constraints in the model:

\[ w_0 - (1 - l_A) v_A > 0, \]  

(3.14)
\[ w_A - (1 - l_B) v_B > 0, \]  

(3.15)

where \( w_A \) is the present value of the wealth at the end of the first period.

\[ w_A = w_0 + (1 - \tau) v_A - (X_{1A} + X_{3A} - X_{2A}) v_A - Q_A. \]

(3.16)

Constraint (3.15) requires that the present value of wealth at the end of the first period be positive because the loan-value ratio must be less than one. A positive
value for $W_A$ implies that a household cannot borrow against period B income to make expenditures during period A.

3.1.3 A Housing Decision Model

A household maximizes their intertemporal utility function in (3.1) subject to the borrowing constraints in (3.14) and (3.15) and the intertemporal budget constraint in (3.12). Choice variables include the quantities of housing services, $h_A$ and $h_B$, the quantities of the composite goods, $q_A$ and $q_B$, the loan-value ratios, $h_A$ and $h_B$, and the length of stay, $s$.

$$\text{Max } U = \int_0^S \Phi(h_A, q_A) + \int_0^T e^{-\delta s} U(h_B, q_B)$$  \hspace{1cm} (3.17)

subject to

$$W_0 + (1-t)(y_A + y_B) - (x_{1A} + x_{3A} - x_{2A})v_A - (x_{1B} + x_{3B} - x_{2B})v_B - O_A - C_B = 0,$$

$$W_0 - (1-t_A)v_A \geq 0,$$

$$W_A - (1-t_B)v_B \geq 0.$$

Note that the intertemporal budget constraint assumes no bequest. The assumption of no bequest implies that a household must spend wealth on other goods during the second period, requiring the use of a financing scheme such as a reverse annuity mortgage. This result is not a serious problem because this study does not focus on the analysis of consumption behavior in period B.
The problem is modified into the following Lagrangian function.

\[ L = R^S_U(h_A, q_A) + R^T_O e^{-\delta U}(h_B, q_B) + \lambda_1(\mathbb{V}_0^A + (1 - \gamma)(Y_A + Y_B) - (Y_{1A} + Y_{3A} - Y_{2A})v_A - (Y_{1B} + Y_{3B} - Y_{2B})v_B - c_A - c_B) + \lambda_2(\mathbb{V}_0^A - (1 - b_A)v_A) + \lambda_3(\mathbb{V}_A^A - (1 - b_B)v_B) \]  

(3.18)

A household selects the loan-value ratio when deciding on the value of the house. Increasing the loan-value ratio allows a household to purchase a larger house given its initial wealth. On the other hand, if the mortgage interest rate is greater than the expected opportunity cost of capital and/or the mortgage interest rate varies with the loan-value ratio, an increase in the loan-value ratio may lead a household to bear higher costs. The household will choose its loan-value ratio so that these two effects are reconciled in a way that maximizes its utility.

Vandell (1980) argues that the mortgage interest rate becomes larger as the loan-value ratio becomes larger. A reason for this is that lending institutions charge a higher mortgage interest rate to guard against the possibility of a larger loss if the household defaults on the mortgage loan. Therefore, the mortgage interest rate is assumed to increase with increase in the loan-value ratio. Mortgage data compiled by the Federal Home Loan Bank Board confirms the existence of a positive relationship between the house-
hold's mortgage interest rate and the loan-value ratio. In 1979, mortgage interest rates were 11.20%, 11.27% and 11.54% for loan-value ratios of 0.5, 0.75 and 0.9, respectively. The marginal interest rates are 11.41% for an increase in the loan-value ratio from 0.5 to 0.75, and 12.9% for an increase in the loan-value ratio from 0.75 to 0.9.

The choice variables for a household include the length of stay. To provide some insight into the determinants of a household's choice of length of stay, the simpler problem of optimal refinancing is investigated. The household's problem is to select the timing of the change in its financing, but assuming they remain in the same house. Suppose that a household buys a house valued \( V \) with the loan-value ratio of \( t \) and keeps it over the entire period, \( T \). Each year the household accumulates wealth; the accumulation of wealth from homeownership comes from both capital gains due to the appreciation of housing and the equity build-up through the amortization of the mortgage-loan. A household's income profile also influences the profile of wealth accumulation over time. In the case where \( r_m > \Pi_H \) and \( r > r \), it is advantageous for a household to lower its loan-value ratio using the accumulated wealth since a decrease in the loan-value ratio lowers the expenditure on housing. On the other hand, the decision to refinance with a lower loan-value ratio increases the down-
payment at the time of refinancing. Therefore, a household is expected to refinance with a lower loan-value ratio as long as its accumulated wealth is large enough to pay the increased downpayment. This implies that a household's optimal refinancing time is determined by its income profile, the appreciation rate of housing, the loan-value ratios for period A and B, the term-to-maturity of the mortgage loan, and the household's marginal income tax rate. Suppose that a household's income grows steadily over time and the appreciation rate of housing is greater than the opportunity cost of capital; then an increase in the length of stay allows a household to reduce its newly selected loan-value ratio further. However, in the case where \( r_m > \Pi_m \) and \( r_m > r \), a longer length of stay implies that a household bears a higher expenditure on housing for a longer period. The point is that even when holding the value of the house constant, a household generally has an incentive to refinance at time \( s < T \).

The length of stay is also closely related to the value of the house purchased, since the length of stay affects the intertemporal price of housing services. I will show that the length of stay affects the net expenditure on owner-occupied housing as long as the length of stay is smaller than the term-to-maturity of the mortgage loan. This effect depends upon the relationships among the expected
appreciation rate of housing, \( \Pi_H \), the mortgage payment rate, \( r_m \), and the expected after-tax opportunity cost of capital, \( r \). For example, if \( r_m > \Pi_H > r \) for the entire period, the annual net expenditure on housing for period B becomes larger than that for period A as the length of stay increases. Additional wealth accumulated due to the longer length of stay may relax the downpayment burden in the second period. However, this effect is weakened by the increase in the downpayment required because of the appreciation of housing. Suppose that \( h_A = h_B \) and \( b_A = b_B \). An increase in the length of stay results in a lower price of housing services in the first period relative to the second period because the increase in the second period's net expenditure, due to the appreciation of housing, dominates the effect due to the additional wealth accumulated as the length of stay increases. Therefore, an increase in the length of stay results in a lower price of housing services for the first period relative to the second period. In this case, a household will purchase a larger house for the extended first period. If \( r > \Pi_H > r_m \), nominal house value rises with increases in the length of stay, but the present value does not increase. In this case, an increase in the length of stay results in a higher price of housing services for the first period relative to the second period, thus purchases of a smaller house with a shorter length of stay.
The first-order conditions of the intertemporal utility maximization problem yield the equilibrium values of $h_A$, $h_B$, $q_A$, $q_B$, $b_A$, $b_B$, and $s$

\[
\frac{\partial L}{\partial h_A} = u_{h_A} K^S - (\lambda_1 + \lambda_3) (X_{1A} + X_{3A} - X_{2A}) + \lambda_2 (1 - b_A) \text{ (P)} \frac{\gamma}{\gamma} = 0, \tag{3.19}
\]

\[
\frac{\partial L}{\partial q_A} = u_{q_A} K^S - (\lambda_1 + \lambda_3) K^S = 0, \tag{3.20}
\]

\[
\frac{\partial L}{\partial b_A} = \gamma \quad \frac{\partial (X_{1A} + X_{3A} - X_{2A})}{\partial b_A} = 0, \tag{3.21}
\]

\[
\frac{\partial L}{\partial s} = u(h_A, q_A) - u(h_B, q_B) e^{-\delta s} - \lambda_1 (q_A e^{-\gamma s} - q_B e^{-\gamma s}) + \gamma \frac{\partial (X_{1A} + X_{3A} - X_{2A})}{\partial s} + \gamma \frac{\partial (\Pi_H - \gamma)}{\partial s} (X_{1B} + X_{3B} - X_{2B}) = 0, \tag{3.22}
\]

\[
\frac{\partial L}{\partial h_B} = u_{h_B} K^{I} e^{-\delta s} - (\lambda_1 + \lambda_3) (1 - b_B) \text{ (P)} \frac{\gamma}{\gamma} = 0, \tag{3.23}
\]

\[
\frac{\partial L}{\partial q_B} = u_{q_B} K^{I} e^{-\delta s} - \lambda_1 e^{-\gamma s} K^{I} = 0, \tag{3.24}
\]

\[
\frac{\partial L}{\partial b_B} = \gamma \quad \frac{\partial (X_{1B} + X_{3B} - X_{2B})}{\partial b_B} = 0, \tag{3.25}
\]

\[
\frac{\partial L}{\partial \lambda_1} = \gamma (\Pi_H - \gamma) (Y_A + Y_B - (X_{1A} + X_{3A} - X_{2A}) V_A = 0, \tag{3.26}
\]

\[
\frac{\partial L}{\partial \lambda_2} = \gamma (1 - \gamma) (Y_A + Y_B - (X_{1A} + X_{3A} - X_{2A}) V_A = 0, \tag{3.27}
\]

\[
\frac{\partial L}{\partial \lambda_3} = \gamma (1 - \gamma) (Y_A + Y_B - (X_{1A} + X_{3A} - X_{2A}) V_A = 0, \tag{3.28}
\]

In the equation (3.22), $y(s)$ is the expected income at the time of the move into another house.

Corner solutions of the choice variables are not considered in this study. We assume that there are no corner solutions for $h_A$, $h_B$, $q_A$, and $q_B$ because households should maintain at least subsistence levels of $h_A$, $h_B$, $q_A$, and $q_B$. 

A zero length of stay is also ruled out. The corner solutions of \( b_A = 0 \), \( b_B = 0 \), and \( s = T \) are possible but are not examined in this study.

Our model yields three different prices of housing services: the price of housing services for period A, the price for period B, and the intertemporal price of housing services. The price of housing services for owner-occupied housing, \( P_A \), is derived by combining the first-order conditions for the quantity of housing services and the quantity of other goods. The price of housing services in the period A, \( P_A \), is:

\[
P_A = \frac{U_{HA}}{U_{QA}} = \left( (X_{1A} + X_{2A} - X_{3A}) + \left( \lambda_2 / (\lambda_1 + \lambda_3) \right) (1-b_A) \right) P_H / (K^T \gamma)
= P_A \left( b_A, s, \gamma, P_H, \Pi_H, \beta, \tau, \lambda_1, \lambda_2, \lambda_3, \tau, N \right). \tag{3.29}
\]

The price of housing services in the period B, \( P_B \), is:

\[
P_B = \frac{U_{HB}}{U_{QB}} = \left( (X_{1B} + X_{2B} - X_{3B}) + \left( \lambda_3 / \lambda_1 \right) (1-b_B) \right) (e^{\Pi_H}) P_H / (K^T \gamma)
= P_B \left( b_B, s, \gamma, P_H, \Pi_H, \beta, \tau, \lambda_1, \lambda_3, \tau, N \right). \tag{3.30}
\]

The determinants of the price of housing services for owner-occupied housing can be classified into two groups. One group contains the variables that vary across households such as the loan-value ratio, the length of stay, the opportunity cost of capital, the income tax rate, the term-to-maturity of the mortgage loan, and the shadow prices of the constraints. The other group contains the variables that are invariant across households such as \( \gamma, \Pi_H, P_H, \) and \( \beta \). The price of housing services for rental housing
depends upon the price of housing capital, $P_H$, the rate of return on rental housing, $CR$, and the factor that converts the stock of housing capital into the flow of housing services, $\gamma$. All of these determinants are exogenous to households. Therefore, there is a unique market price for rental housing and every household within the market faces the same price of housing services for rental housing.*

Equation (3.29) and (3.30) show that the price of housing services for owner-occupied housing varies across households according to their choices of the loan-value ratio and the length of stay. The length of stay also affects the price of housing services for each period through its effects on the intertemporal price of housing services. The intertemporal price of housing services, $P_{AB}$, mainly depends upon the length of stay and the difference in the loan-value ratio between the two periods.

$$P_{AB} = \frac{P_A}{P_B} = \frac{\left( (X_{1A} - X_{2A}) - \left( \lambda_2 / (\lambda_1 + \lambda_3) \right) (1-b_A) \right) e^{-\gamma S} K^I_y}{\left( (X_{1B} - X_{2B}) - \left( \lambda_2 / \lambda_1 \right) (1-b_B) \right) e^{(\Pi_H - \gamma) S} K^S_y}$$

$$= \frac{P_{AB} (s, b_A, b_B, \Pi_H, r, \tau, M, \beta, \lambda_1, \lambda_2, \lambda_3)}{\ldots} \quad (3.31)$$

This model implies that households select both the quantity and the price of housing services when maximizing intertemporal utility.

* This theoretical analysis clearly abstracts from housing price variations derived from spatially varying input prices typical of metropolitan areas.
The observation that some determinants of the price of housing services of owner-occupied housing are endogenous distinguishes this model from others in the literature. Summarizing the differences: first, the price of housing services for owner-occupied housing varies across households. Secondly, the variation in the price of housing services implies that the housing's budget share varies across owners depending on the loan-value ratio and the length of stay. Third, the loan-value ratio, the length of stay, and the quantity of housing are simultaneously chosen.

3.2 **Demand Functions**

The demand equation for owner-occupied housing is derived by using the first-order conditions of the model. Both the structural equations and the reduced form demand functions will be analyzed. Note that the use of the reduced form demand function avoids the problems associated with the simultaneous decision mechanism.

3.2.1 **The System of Structural Equations**

The system of structural equations is formulated using the concept of the $\lambda$-constant demand function.$^5$

---

$^5$ A $\lambda$-constant demand function represents the marginal utility of wealth (or income) constant demand function for a particular form of the lifetime preference function.
Philips (1983) and Macurdy (1981) use the concept of $\lambda$-constant functions to derive the structural-intertemporal model of a household's simultaneous choice of its consumption and labor supply. The $\lambda$-constant function treats the Lagrangian multipliers as variables in formulating the consumption function and the labor supply function. The structural equations of the model are nothing but the first-order conditions since the constraints are imposed through the values of the $\lambda$'s. The Lagrangian multiplier is a sufficient statistic for the future information that is relevant to the current choice of consumption and labor supply (Macurdy, 1981, p1031). Consumption and labor supply decisions at a point in time are related to variables out-

$\lambda$ is the marginal utility of wealth (or the shadow price associated with the lifetime budget constraint). The $\lambda$-constant demand functions are simply the first-order conditions derived from maximizing lifetime utility subject to the wealth constraint. By substituting the $\lambda$-constant functions into the wealth constraint, optimal values of $\lambda$ can be derived as implicit functions of all variables. Concavity of preference implies $\partial \lambda / \partial w(0) < 0$ and $\partial \lambda / \partial y(t) < 0$, $t = 1, \ldots, T$. Conventional demand functions are derived by substituting the optimal values of $\lambda$ into the $\lambda$-constant demand functions. In intertemporal analysis, $\lambda$-constant demand functions are widely used rather than conventional demand functions since they are more convenient. Convenience includes; first, $\lambda$-constant demand functions reveal that the decisions at a point in time are related to variables outside the sample period only through $\lambda$, affording a considerable simplification of the analysis. Second, $\lambda$-constant demand functions are not indexed on time $t$, nor is the quantity $\lambda$. This implies that at any age, any path of income over a consumer's lifetime that keeps $\lambda$ and the current variables constant implies the same optimal current consumption be-
side the decision period only through \( \lambda \). A \( \lambda \)-constant function assumes that the time preference rate remains constant over time. The Lagrangian multipliers in the static, intertemporal optimization problem are not indexed on time. This implies that the static, intertemporal optimization problem does not deal with the changes in the Lagrangian multipliers over time. Note that the costate variables, which are the dynamic equivalents of the Lagrangian multipliers of the static optimization problem, are indexed on time since the costate variables in the optimal control problem can vary over time. It is also noted that, on the optimal path, the relative rate of change of the shadow price of wealth must equal the difference between the rate of time preference and the rate of return from the portfolio. In the model, the rate of return from the household's portfolio depends upon the opportunity cost of capital, loan-value ratio, and the appreciation rate of housing asset.

The first-order conditions and the constraints in our model comprise a complete representation of the demand for owner-occupied housing. This system of equations reveals the simultaneous relationships among the choice variables.

In an empirical context, MacCurdy (1981) interprets \( \lambda \) as a 'fixed effect' unique to each individual and approximates it as a function of initial wealth, income at each age, and the family background variable such as family size. He also assumed that the parameters are constant across consumers.
Also it shows that the equilibrium values, at a point in
time, of the quantity of housing services, the composite
good, and the loan-value ratio, are related to variables
outside the decision period only through the lagrangian
multipliers, $\lambda_1$ and $\lambda_3$. The equilibrium values of the
choice variables are non-linear function of the other
choice variables, exogenous variables, and the lagrangian
multipliers.

The intertemporal utility function given in equation
(3.31) is respecified, simplifying the structural equation
system.

$$
U = k_0^s (\gamma_1 \ln h_A + \gamma_1 \ln q_A) + k_0^1 e^{-\delta s} (\gamma_2 \ln h_B + \gamma_2 \ln q_B),
$$

(3.32)

where $\gamma_1 > 0$, and $\gamma_2 > 0$.

Preferences for $h$ and $q$ are strongly separable in equation
(3.32) and this assumption restricts the range of consumer
behavior. However, it is generally believed that, for suffi-
ciently broad groups of goods, the restrictions derived
from strong separability are not obviously inappropriate
(Barnett(1974), Muellbauer and Deaton(1980), Theil and Lai-
tinen (1981)).

---

6 An example of the strong separability is the Stone-Geary
utility function. This form of utility function has been
suggested as suitable for studying the demand for housing
services (Mayo, 1981). Apart from a very peculiar
case (see Silberberg, p.363), strong separability requires
negative $U_{ii}$ in the Hessian matrix in order to meet the
second-order condition of the utility maximization. Strong separability rules out both the inferiority and the complementarity of goods. All goods are 'general
substitutes'.
Our main interest is the analysis of the household's choices of $h_A$, $q_A$, and $h_A^*: the reason being that equilibrium values of $h_A^*$, $q_A^*$, and $h_A^*$ are executed values. On the other hand, the equilibrium values of $h_B$, $q_B$, and $h_B^*$ are planned values which are subject to replanning when new information is available.

The first-order conditions of the model yield explicit solutions for $h_A$ and $q_A$. The quantity of housing services demanded for the period $A$ is a function of the loan-value ratio for period $A$, the length of stay, the Lagrangian multipliers, and the exogeneous variables in period $A$.

\[
\ln h_A = \ln \gamma_1 + \ln K_0^5 - \ln \left( (\lambda_1+\lambda_3) (x_{1A} \lambda_3 x_{2A} + \lambda_2 (1-h_A)) \right) \quad \Leftrightarrow \quad h_A = H(\gamma_1, h_A^*, s, \lambda_1, \lambda_2, \lambda_3, \delta, r, \tau, \Pi_H, \pi, \beta). \quad (3.33)
\]

The quantity of other goods demanded in period $A$ is also a function of the length of stay, the Lagrangian multipliers, and the exogeneous variables in period $A$.

\[
\ln q_A = \ln \gamma_2 + \ln K_0^5 - \ln (\lambda_1+\lambda_3) - \ln K_r^5 \quad \Rightarrow \quad q_A = C(\gamma_2, s, \lambda_1, \lambda_3, \delta, r). \quad (3.34)
\]

The first-order conditions yield only general functions for the loan-value ratio and the length of stay.

\[
b_A = F(s, \lambda_2, \lambda_1, \lambda_3, r, \tau, \Pi), \quad \text{and} \quad (3.35)
\]

\[
s = S(h_A^*, q_A^*, h_B^*, c_B^*, b_B^*, \lambda_1, \lambda_3, \delta, r, \Pi_H, \gamma_1, \gamma_2), \quad i = 0, \ldots, T. \quad (3.36)
\]

Equations (3.33)-(3.35) do not directly involve the intertemporal aspect of the decision. The length of stay
equation, (3.36), is different in that all of the intertemporal decisions are directly involved. This equation also shows that the income profile affects the household's length of stay.

Comparative statics are performed by totally differentiating the first-order conditions of the model. Given the assumption of strong separability in preferences, the second-order condition requires diminishing marginal utility in housing services and in other goods. It also requires that the mortgage interest rate be greater than the after-tax opportunity cost of capital. Derivation of the expected signs of the partial derivatives is confined to the case where all constraints are binding\(^7\) and the appreciation rate of housing is greater than the opportunity cost of capital.\(^8\) A binding borrowing constraint implies that a household uses all of its accumulated wealth for the downpayment. The expected directions of the increase in the variables in the equations of (3.33)-(3.36) are summarized in Table 1.

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\(^7\) The intertemporal budget constraint is always binding due to the no-guest assumption. A non-binding borrowing constraint would result in a zero value for the Lagrangian multipliers of \(\lambda_2\) and \(\lambda_3\).

\(^8\) We confine the analysis to this case to reflect the attractiveness of the investment in owner-occupied housing during the last decade, and the model is tested in a time period when this assumption was fulfilled.
<table>
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<tr>
<th>Dependent Var.</th>
<th>$b_A$</th>
<th>$q_A$</th>
<th>$b_A$</th>
<th>$s$</th>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$c_A$</td>
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<td></td>
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<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$s$</td>
<td>+     ?# - $\neq$</td>
<td></td>
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<td></td>
<td>?#</td>
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<td></td>
<td>?#</td>
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<td></td>
<td></td>
<td>+</td>
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<td></td>
<td></td>
<td>?</td>
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<td></td>
<td>0</td>
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<td></td>
<td>?</td>
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</table>

**Note:** The relationship shows a zero value if $\delta=r$.  

**Table 1** shows the structural relationships among the three housing related choice variables that are in the model. Zero values for $c_A$ in the $b_A$ equation and for $b_A$ in the $q_A$ equation are due to the assumption of separability in preferences. The length of stay affects the household's choice of the loan-value ratio and the quantity of housing services, given the values of $\lambda_1$, $\lambda_2$, and $\lambda_3$. The inter-
temporal budget constraint does not affect the household's length of stay since a household spends all of its resources during its lifetime regardless of its choice of the length of stay. Table 1 also reveals the intertemporal aspect involved in the demand for owner-occupied housing since the signs of variables $s$, $\lambda_1$, and $\lambda_3$ are non-zero. The quantity of owner-occupied housing demanded is found to be independent of the equilibrium loan-value ratio, given the values of the length of stay and the Lagrangian multipliers for the binding constraints.

3.2.2 Estimatable Structural Equations

The structural equation system of the demand for owner-occupied housing shows clearly how the choice variables are interrelated in the housing decision process. However, these equations cannot be estimated directly since the variables $\lambda_1$, $\lambda_2$, and $\lambda_3$ are not observed. Moreover, the constraints in our model are non-linear with respect to each choice variable. The optimal values of the Lagrangian multipliers are derived by substituting the first-order conditions of the model into the constraints and solving

---

9 If the borrowing constraints for the period $A$ is not binding, a household can choose any loan-value ratio. However, a household will choose a corner solution of $h_A = 0$ since the mortgage interest rate is greater than the opportunity cost of capital. This implies that a household use all of its initial wealth as a downpayment for a house purchase unless its initial wealth is greater than the value of the house purchased.
the resulting system of three equations. However, explicit solutions for the optimal values of the Lagrangian multipliers can not be derived. The implicit forms imply highly non-linear regression equations.

The non-linear constraints are linearly approximated to derive estimatable structural equations for the demand for owner-occupied housing. In approximating the Lagrangian multipliers in each structural equation, the relationships among the value of the house purchased, the loan-value ratio and the length of stay should be considered. The quantity of housing demanded depends upon the loan-value ratio and the length of stay due to their effects on the price of housing services. The loan-value ratio and the length of stay, in turn, depend upon the quantity of housing demanded. In a somewhat similar problem, Deaton and Muellbauer (1980, pp248-249) show how a nonlinear constraint can be linearly approximated as a function of the exogenously given input prices and the income at the optimum quantity of output. In this problem, independence between price and output is established by assuming constant returns to scale in producing output. Then, the demand equation is derived by treating the prices of output as if they were exogenous. In fact, this demand equation disregards the linkage of the endogenous variables to both exogenous and other endogenous variables. This problem is solved by treating the demand equation as a structural equation of the model.
Our model cannot follow Deaton and Muellbauer's method because input prices are not constant here. The price of the loan-value ratio is not constant since the mortgage interest rate varies with the loan-value ratio. The price of the length of stay can not be measured. In this case, an alternative is to approximate the input prices at the equilibrium values of the inputs. The equilibrium values of the loan-value ratio and the length of stay are used as proxies of the variable input prices and they are treated as if they were exogenous. Given the equilibrium values of the loan-value ratio and the length of stay, the price of housing services is independent of the quantity of housing demanded. Then, the nonlinear constraints are linearly approximated as a function of the equilibrium values of the loan-value ratio and the length of stay, other exogenous variables and income. This method is applied to the linear approximation of the Lagrangian multipliers in each structural equation of the system. In an estimation context, the equilibrium values of the value of the house purchased, the loan-value ratio and the length of stay are replaced by the observed value of the corresponding variables.

When approximating the variables, the choice variables in the model are classified into two groups. One group consists of the quantity variables $b_A$, $b_B$, $g_A$, and $g_B$ and the other group is composed of $b_A$, $b_B$, and $s$. In the
structural equations for \( h_A \) and \( q_A \), the equilibrium values of \( b_A \), \( b_B \), and \( s \) are treated as exogenous variables when approximating the \( \lambda \) variables. For the structural equations of \( b_A \) and \( s \), the equilibrium values of \( h_A \), \( h_B \), \( q_A \), and \( q_B \) are treated as exogenous variables.

Our model also has three constraints. Each of the \( \lambda \) variables in the system of the structural equations of the demand for owner-occupied housing shows a partial effect since the parameter of each \( \lambda \) variable is derived under the assumption that the remaining \( \lambda \) variables remain constant. The \( \lambda \) variables in the system of the structural equations can be approximated as linear functions of initial wealth, income, and the equilibrium values of the choice variables in the different group. Individual background variables such as family size are also included in approximating the Lagrangian multiplier of the lifetime budget constraint in the housing service and composite good expenditure equations. The measures of initial wealth, income, and family size reflect the 'fixed effect' specific to each household in its housing decision.

In the structural equations for \( h_A \) and \( q_A \), the \( \lambda \) variables are approximated as:

\[
\lambda_1 = \lambda_1 (w_o, y, \tau, b_A, q_B, P_S), \quad (3.37)
\]

\[
\lambda_2 = \lambda_2 (w_o, b_A), \quad \text{and} \quad (3.38)
\]
\[ \lambda_3 = \lambda_3 ( \mathbb{w}_O, \tau, b_A, \varepsilon, b_B ). \]

In the structural equations for \( b_A \) and \( s \), the \( \lambda \) variables are approximated as:

\[ \lambda_1 = \lambda_1 ( \mathbb{w}_O, \mathbb{y}, \tau, b_A, b_B, q_A, q_B ). \]

\[ \lambda_2 = \lambda_2 ( \mathbb{w}_O, b_A ), \text{ and} \]

\[ \lambda_3 = \lambda_3 ( \mathbb{w}_O, \tau, b_A, q_A, b_B ). \]

\( \mathbb{w}_O \)
The estimable system of the structural equations for is derived by substituting equations (3.37)-(3.42) for the variables in equations (3.33)-(3.36). The system for period A is:

\[ h_A = H(\gamma, s, \lambda_1, \lambda_2, \lambda_3, \delta, \gamma, \Pi, W, \beta, \tau) \]

\[ = H(\gamma, b_A, s, b_b, \Pi, \delta, \gamma, \Pi, W, \beta, W, \gamma, \Pi, FS), (3.43) \]

\[ + ? + ? + - + ? - - + + + \]

\[ q_A = Q(\gamma, \lambda_1, \lambda_3, \delta, \gamma, s) \]

\[ + - - + - ? \]

\[ = Q(\gamma, W_o, \gamma, \tau, b_A, b_b, s, \delta, \gamma, \Pi, FS), (3.44) \]

\[ + + + - - ? ? + - + \]

\[ b_A = B(\gamma, s, h_A, b_b, q_A, b_b, \delta, \gamma, \Pi, W_o, \gamma, \Pi, FS). (3.45) \]

\[ + - - + + + \]

\[ s = S(h_A, b_A, q_A, h_b, b_b, q_b, \gamma_1, \gamma_2, \lambda_3, \delta, \gamma, \Pi, \delta, \gamma, \Pi, W, \gamma(1)) \]


\[ = S(b_A, q_A, b_A, b_b, q_b, \gamma_1, \gamma_2, W_o, \Pi, \delta, \gamma, \Pi, W, \gamma(1)). (3.46) \]


Note that the mortgage interest rate variable is not included in the system of equations (3.43)-(3.46) since the mortgage interest rate is directly related to the loan-value ratio. The relationship between the mortgage interest rate and the loan-value ratio is determined in the mortgage
market and exogenous to a household. Each household's borrowing is so small that it can not influence this relationship.

The analysis is focused on the structural equation for the quantity of housing services since this equation is instrumental in testing the simultaneity of the choice variables in the housing decision. The structural equation of housing services, (3.43), shows that the loan-value ratios have ambiguous effects on the demand for owner-occupied housing when all of the constraints are binding, since the loan-value ratios affect the shadow prices of the constraints in different directions. The loan-value ratio for period A, \( b_A \), has a negative effect on the shadow price of the borrowing constraints for the period A, a positive effect on the shadow price of the intertemporal budget constraint, and a positive effect on the shadow price of the borrowing constraint for the period B. The loan-value ratio for the period B, \( b_B \), has a negative effect on the shadow price of the borrowing constraint for the period B and a positive effect on the shadow price of the intertemporal budget constraint. The length of stay has a positive effect on the demand for owner-occupied housing due to the negative effect of the length of stay on the intertemporal price of housing services. The positive sign for initial wealth, \( w_0 \), and income, \( Y \), and the appreciation rate of
housing, \( \Pi_H \), are plausible. The negative sign for the share of transaction costs for a house sale, \( \beta \), is also plausible. An increase in the opportunity cost of capital, \( r \), is expected to reduce the demand for owner-occupied housing. The negative sign for the opportunity cost of capital is plausible since the attractiveness of the investment in owner-occupied housing decreases if it increases.

The sign of the income tax rate is ambiguous. An increase in the income tax rate decreases disposable income in spite of the tax savings from the tax deductibility of mortgage interest payments. This causes a decrease in the demand for owner-occupied housing. However, an increase in the income tax rate lowers the price of housing services for owner-occupied housing and increases the demand for owner-occupied housing at the expense of the consumption of other goods. Finding a negative sign implies the income effect dominates the substitution effect. For a given length of stay, the remaining mortgage principal is an increasing function of the term-to-maturity. Nevertheless, the mortgage payment is a decreasing function of the term-to-maturity. Thus, a sign of the coefficient of the term-to-maturity in the demand for owner-occupied housing equation is ambiguous.
3.2.3 The Reduced Form Demand Function

The overall effect of the exogenous variables on the demand for housing, considering the simultaneity among endogenous variables, is shown by deriving the standard (reduced form) demand function for housing. The standard demand function for owner-occupied housing is:

\[ h = H(\gamma_1, \gamma_2, \delta, \epsilon, \tau, \kappa, \pi_H, \beta, \omega_0, Y, y(i), F_5), \]  \hspace{1cm} (3.47)

for \( i = 0, \ldots, T. \)

The appearance of the variable \( y(i) \) implies that the income profile has an effect on the demand for owner-occupied housing. Table 1 shows that an increasing income profile has a positive effect on the length of stay. Equation (3.43) shows that the length of stay has a positive effect on the demand for owner-occupied housing. This implies that a household which expects an increase in its future income will buy a larger house in period \( t \) with a longer length of stay compared to a household (holding wealth constant) with a flat or falling income profile.

The expected directions of the effects of the variables on the demand for owner-occupied housing can be investigated by using comparative statics analysis. However, the comparative statics from our model are too complicated to assign definite directions to the effects of the variables on the demand for owner-occupied housing. The endogeneity of the length of stay aggravates the complexity of the de-
derived comparative statics since we can not disregard the effect of the preference structure in period B while analyzing the choices made in period A.

3.3 HYPOTHESES

Our model hypothesizes that a household simultaneously selects the value of the house purchased, the loan-value ratio, and the length of stay. This simultaneity is directly related to the endogeneity of the price of housing services for owner-occupied housing. The price of housing services for owner-occupied housing is determined by the household's choice of its loan-value ratio and its length of stay. The existence of a simultaneous relationship among the choice variables will be tested in the empirical specification of the demand function for owner-occupied housing.

One choice mechanism different from the simultaneous one suggested in our model is a sequential choice mechanism. A sequential choice mechanism means that housing decisions are made in a related series of actions: first, a household chooses the size of the house and then decides the loan-value ratio based on its decision of the size of house. A household's moving decision in response to the changes in family size determines the (actual) length of stay. The sequential choice mechanism rules out the possibility that
the loan-value ratio and the length of stay affect the demand for owner-occupied housing. It also results in the prediction that the loan-value ratio and the length of stay will have no effect on the current demand for owner-occupied housing. Previous studies of housing demand have shown that the price of housing services for owner-occupied housing is independent of the loan-value ratio if the capital market is perfect; that is, every household can freely borrow and lend at the same interest rate. The assumption that all households have an infinite length of stay results in the price of housing services being independent of the length of stay. This assumption ignores the effect of the length of stay on the accumulation of wealth from the investment in owner-occupied housing, an important element of the price of housing services. If the price of housing services for owner-occupied housing is independent of the loan-value ratio and the length of stay, a household follows a sequential choice procedure. The value of the house is determined in the first stage based on the price of housing services, income, and wealth. The loan-value ratio is determined in the second stage in a manner so that the household does not violate the borrowing constraint.

The simultaneous choice mechanism in our model shows that the demand for owner-occupied housing is interrelated with the loan-value ratio and the length of stay. This si-
multaneous choice mechanism is required due to the addition of new choice variables to the utility-maximization scheme. In this sense, the choice mechanism in our model can be regarded as a more general model which contains the sequential choice mechanism as a special case. As shown in the equation (3.43), our model predicts an ambiguous sign for the loan-value ratio and a positive sign for the length of stay in the demand equation for owner-occupied housing. The ambiguity of the expected sign of the loan-value ratio in the demand equation does not allow us to derive a testable hypothesis for our maintained hypothesis since finding a zero coefficients is consistent with both the sequential mechanism and the simultaneous mechanism. If the estimated coefficients of the loan-value ratio are insignificant, we can not tell which mechanism is supported by the empirical evidence. However, if the estimated coefficients of the loan-value ratio and the length of stay are positive and significant, the sequential choice mechanism is ruled out in describing the household's decision process since it predicts coefficients of zero for the loan-value ratio and the length of stay.
3.4 IMPLICATIONS

The test of the simultaneity among the housing decision variables is important since this simultaneity in housing decisions suggests new directions for housing demand studies. First, the estimation of the demand for owner-occupied housing using a single equation in the simultaneous equation system would result in biases in the coefficients of the variables. Second, the recognition of the simultaneity in the housing decisions, especially in the case of owner-occupied housing, also enables us to distinguish the effects of the variables in the demand equation between direct and indirect effects by comparing the coefficients of the structural form with those of the reduced form; the variables initial wealth and expected income will be good examples. Third, the simultaneous equation system also rigorously suggests new determinants of the demand for owner-occupied housing, such as the income profile. Fourth, the simultaneous relationship implies that the rate structure of mortgage loans is very important to the household's demand for owner-occupied housing and its tenure choice, since the rate structure of mortgage loans determines the slope of the functional relationship between the price of housing services and the loan-value ratio.

The simultaneous choice of key housing variables also requires that previous studies dealing with the impact of
tax benefits from owner-occupied housing on the household's tenure choice and the quantity of owner-occupied housing demanded, be reevaluated. The simultaneous relationships imply that every household's tax benefit differs with its combinations of the loan-value ratio, the length of stay, and the quantity of owner-occupied housing demanded. The simultaneous relationship also implies that low-wealth households can enter the owner-occupied sector of the housing market by increasing their loan-value ratio and reducing the quantity of owner-occupied housing simultaneously.
Chapter IV

ESTIMATION PROCEDURE

The system of structural equations for owner-occupied housing is specified and estimation procedures are discussed in this chapter.

4.1 SPECIFICATION OF ESTIMATING EQUATIONS

The system of equations (3.43)-(3.46) cannot be estimated directly due to several problems in our data base. The value of the house and the loan-value ratio are reported only for owners. Our data base does not report variables such as the planned length of stay, expenditures on other goods, or the income profile. Planned variables such as the value of the house purchased for period $k$ and its loan-value ratio cannot be observed in our data base. This limited data base and the endogeneity of the tax rate and the term-to-maturity of the mortgage loan raise several econometric issues regarding the consistent estimation of parameters.
4.1.1 Incorporation of Tenure Choice

Households obtain housing services either by owning or by renting a house.

Prior studies show that tenure choice and the quantity of owner-occupied housing consumed are simultaneously determined, and that the neglect of the tenure choice in the demand equation results in biased and inconsistent estimates unless the error terms of the demand equation and the tenure choice equation are uncorrelated\(^\text{10}\) (Lee and Trost (1978), Gillingham and Hagemann (1983)). This implies that empirical estimation using data only from owners yields biased and inconsistent parameter estimates, but the problem can be overcome by taking into account the tenure

\(^{10}\) Using the data from the Panel Study of Income Dynamics of 1972, Lee and Trost (1978) rejected the hypothesis that error terms of the demand equation and the tenure choice equation are uncorrelated. Gillingham and Hagemann (1983) also showed that the error terms of the demand equation and the tenure choice equation are correlated by finding a positive and significant parameter estimate of "\(\lambda\)" in the demand equation. The "\(\lambda\)" variable reflects the effect of the probit tenure choice function on the demand equation, where:

\[
\lambda_{\text{obs}}(\lambda) = -\sigma_1 \epsilon f(1)/F(I),
\]

with \(\sigma_1\) the covariance between the error term of the probit tenure choice function and the error term of the housing demand equation,

\(I\) : a household's tenure choice index

\(1\) : own a house

\(0\) : rent a house,

\(f(\cdot)\) : ordinate of the standard normal distribution, and

\(F(\cdot)\) : distribution function of the standard normal variable.

choice decision.

In incorporating tenure choice into the system of structural equations for owner-occupied housing, the selectivity criterion can be specified in two ways: a probit-type index or the tobit-type index. Lee and Trost (1978), Rosen (1979) and Gillingham and Hagemann (1983) use the probit model to describe the household’s tenure choice. In the probit model, each household has an index $I_j$ which measures the likelihood that it chooses owning over renting a house. The index for the $j$th household, $I_j$, can not be measured directly, but it is a function of the arguments of the indirect utility function i.e. the price of housing services for owner-occupied housing, $P_A$, the price of housing services from rental housing, $P_r$, and permanent income, $Y$.

$$I_j = I(P_A, P_r, Y).$$ (4.1)

For each household, it is assumed that there exists some critical value of the index, $I^*$, such that if $I > I^*$, the family will own a house, otherwise it will rent. It is also assumed that the critical value, $I^*$, varies randomly among households. On the basis of the observed dummy variable, $I$, the parameters of the tenure choice function can be estimated by the probit method only up to a proportionality factor. Therefore, the variance of the probit-type criterion function is restricted to be unity. However, the model with a probit-type criterion function is a more gen-
eral model in that it allows the variables in the selection function to be different from the variables in the regression equations of the model. This difference is a necessary condition for identification of the parameters in the selection function equation which is different from the regression equation.

The Tobit analysis assumes that the criterion function index can be described by one of the observed limited dependent variables in the system of equations (Lee, Maddala and Trost (1980), pp. 493-494). Therefore, the Tobit-criterion function can be used when the variables in the criterion function are identical with or comprise a subset of the variables in the regression equation. In the Tobit-type criterion function, the index I is observed only if it is positive. The variance of the Tobit-type criterion function need not be restricted to be one since the index can be observed for some of the observations.

Using the price of housing services for owner-occupied housing in equation (3.29) and the linear approximations of the λ's in equations (3.37)-(3.39), the tenure choice index can be written as:

\[ I = 1( P \cdot P_{A} (b_{A}, s, \gamma, \Pi_{H}, \tau, \beta, \lambda_{1}, \lambda_{2}, \lambda_{3}, \Gamma, W_{0}, Y) ) \]

\[ = 1( P \cdot P_{A} (b_{A}, b_{B}, s, \tau, \Pi_{H}, \beta, \gamma, \delta, \Gamma, W_{0}, Y, FS) ) \]  \( (4.2) \)

Since \( P \cdot \gamma \) and \( \beta \) are assumed to be same for all of the households, the index can be rewritten:

\[ I = 1( b_{A}, b_{B}, s, \tau, \Pi_{H}, \Gamma, W_{0}, Y, FS ) \]  \( (4.3) \)
Note that the structural equation of housing services in equation (3.43) includes all of the determinants of the tenure choice index in equation (4.3).

A household's tenure choice also affects its loan-value ratio. A mortgage loan is available only if a household decides to own a house. Households would have a zero loan-value ratio in two possible cases. One is that they decide to rent a house. The other is that they decide not to use a mortgage loan in purchasing their houses. The truncated normal distribution which is assumed for the loan-value ratio is directly related to the tenure choice.

Lee, Madala, and Trest (1980) show that it is reasonable to specify a simultaneous equations model with the selectivity criterion function of the tobit-type rather than the model with the selectivity criterion function of the probit-type, when the variables in the selection criterion function comprise a subset of the variables in the regression equations. However, the tobit-type selection function implicitly imposes the restriction that the effect of the regressors on the probability of owning a house (normalized coefficients) are proportional to the effects of the regressors on the quantity of the house pur-

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11 Tobit type selection function is reasonable in the sense that it avoids a specification of a separate choice index which is unnecessary when the variables in the selection criterion function comprise a subset of the variables in the regression equation.
chased (regression coefficients). The tenure choice index in equation (4.3) shows that the variables in the index equation comprise a subset of the variables in the system of structural equations for owner-occupied housing. Moreover, the estimation of the model with a tobit-type criterion function is easier than with a probit-type criterion function. Therefore, the tobit-type index is chosen to incorporate the tenure choice index into the system of structural equations for the demand for owner-occupied housing.

The system of equations with the selectivity criterion of the tobit type is:

\[ h_A = H(\gamma_1, b_A, b_B, s, \Pi, \delta, \tau, \rho, \theta_0, \theta_1, \theta_2, \theta_3) \quad \text{if } h_A > 0 \]  

\[ b_A = B(\epsilon, h_A, b_B, s, \Pi, \delta, \tau, \rho, \theta_0, \theta_1, \theta_2, \theta_3) \quad \text{if } h_A > 0 \]

---

17 The tobit model refers to the regression model in which the range of the dependent variable is constrained. A typical tobit model is defined as:

\[ y_i^* = x_i' \beta + u_i \quad i = 1, 2, \ldots, n \]  

\[ y_i = y_i^* \quad \text{if } y_i^* > 0 \]  

\[ 0 \quad \text{if } y_i^* \leq 0 \]

The \( u_i \) are assumed to be independent, identically distributed drawings from \( N(0, \sigma^2) \). It is assumed that \( y_i \) and \( x_i \) are observed for \( i = 1, 2, \ldots, n \), but the \( y_i^* \) are unobserved if \( y_i^* < 0 \). Using Tobin's terminology, \( y_i^* \) is a limited dependent variable. The likelihood function of the typical tobit model is given by:

\[ L = \prod_{i \in S} \Phi(y_i^*/\sigma) \prod_{i \in O} \phi((y_i^* - x_i' \beta)/\sigma) \]

where \( \Phi \) and \( \phi \) are the distribution and density function of the standard normal variable respectively. The tobit maximum likelihood estimator (Tobit MLF) is derived by finding the partial derivatives of the log-likelihood function and setting them equal to zero.

\[ \log L = \sum_{i \in S} \log(1 - \Phi(x_i' \beta/\sigma)) - (n_1/2) \log \sigma^2 \]  

\[ - (1/2\sigma^2) \sum_{i \in O} (y_i - x_i' \beta)^2 \]

\[ \frac{\partial \log L}{\partial \beta} = -\frac{1}{\sigma^2} \sum_{i \in S} \left( \phi(x_i' \beta/\sigma) x_i \right) / (1 - \Phi(x_i' \beta/\sigma)) \]

\[ + \frac{1}{\sigma^2} \sum_{i \in O} (y_i - x_i' \beta) x_i = 0 \]

\[ \frac{\partial \log L}{\partial \sigma^2} = \frac{1}{2\sigma^3} \sum_{i \in S} \left( x_i' \Phi(x_i' \beta/\sigma) \right) / (1 - \Phi(x_i' \beta/\sigma)) - (n_1/2 \sigma) \]
\[ h_A = 0 \quad \text{otherwise} \]
\[ b_A = 0 \quad \text{otherwise} \]
\[ s = S(h_A q_A r_A h_B q_B b_B \gamma_1 \gamma_2 \omega_0 \Pi_H \delta r \tau \nu y(i)) \quad \text{and (4.6)} \]
\[ q_A = 0 (\gamma_2 b_A b_B s \omega_0 y \delta r, FS) \quad \text{(4.7)} \]

4.1.2 **Unobserved Endogenous Variables**

The system of estimable equations contains several unobserved endogenous variables. Unobserved endogenous variables are classified into two groups. One group consists of the length of stay and expenditures on goods for period A and the other group consists of the quantity of housing purchased, the loan-value ratio, and the expenditure on non-housing goods in period B. These outside-the-

\[ \left(1/2\sigma^2\right) \sum_{i=1}^{n} (y_i - x_i'\beta)^2 = 0. \quad \text{(6)} \]

The equations are nonlinear in the parameters and hence must be solved iteratively. Tobit MLE is consistent and asymptotically normal.

A typical tobit model has been extended to contain the general simultaneous-equations tobit model (Nelson and Olson (1978)).

\[ \Gamma'y_i = \Gamma'x_i + u_i, \quad i=1,2, \ldots, n. \quad \text{(7)} \]

where the elements of vector \( y \) contain the types of variables

1. always completely observable,
2. completely observed only if \( y_i > 0 \), and
3. completely observed only if \( y_i > 0 \) and \( j \neq i \).

The general simultaneous-equations tobit model describes our system of equations of the demand for owner-occupied housing. The general simultaneous-equations tobit model is estimated using a two-stage procedure, which is described in detail in section 4.3. "Estimation". The estimated parameters of structural-form equations are consistent and asymptotically normal.

Finally, the consistency of the Tobit MLE is de-
sample-period variables are important because of the intertemporal framework.

4.1.2.1 Unobserved Variables for Period A

A household's planned length of stay is unobservable. Omission of the length of stay may cause a serious bias of the parameter estimates in our system of equations since the length of stay is closely related with the other housing decision variables. A statistical procedure that can deal with the unobserved variable is to use an instrumental variable technique (Hanushek and Jackson, Chapter 10). The instrument for the unobserved length of stay is developed using a two-stage procedure which is based on the observed length of stay from the earlier years of the Panel data and also some other information. This instrument replaces the unobserved length of stay in our system of equations and this method yields consistent parameter estimates in of equations of (4.4)-(4.7).

The length of stay for a household is estimated based on Panel data from 1969 to 1979. First, households who moved into new houses in 1969 are identified. They are checked each year to see whether they moved into another house. For example, if a household moved into another house in 1974, its length of stay is 5 years. The observed length stroyed under non-standard assumptions such as heteroscedasticity and non-normality.
of stay cannot be greater than 10 years due to data limitations. The observed length of stay is always greater than zero by definition of the variable. To create the instrument: first, the natural log of the observed length of stay of a household is regressed upon other variables which are thought to vary systematically with the length of stay. The natural log of the length of stay is used in the regression equation since the length of stay is always greater than zero. Let \( Z_0 \) be the set of the observed variables varying systematically with the length of stay and \( S_0 \) be the observed length of stay. Then,

\[
\ln S_0 = Z_0 \alpha_0 + \epsilon_0, \tag{4.8}
\]

where \( Z_0 \) represents the regressors observed in 1969.

In specifying equation (4.8), the environment is assumed to be time homogeneous so that \( Z_0 \) remains constant over time for each household. The parameter \( \alpha_0 \) is estimated using the tobit method since the observed length of stay has an upper limit of 10 years. Next, the instrument for the length of stay for a household (in 1979) is calculated by:

\[
\ln s = Z_{0,s} \hat{\alpha}_0, \tag{4.9}
\]

where \( Z_{0,s} \); regressors observed in 1969 for the households in our sample of 1979 panel data, and

\( \hat{\alpha}_0 \); tobit estimates of \( \alpha_0 \) in equation (4.8).
In constructing the instrument for the length of stay, regressors observed in 1969 are used rather than the regressors observed in 1979 because with duration analysis in a time homogeneous environment the distribution of the length of stay is determined by the values of the regressors at the beginning of the stay. This implies that the estimated value of $\alpha_0$ is closely tied to the values of the regressors observed in 1969. This also implies that, in constructing the instrument for the length of stay, the regressors observed in 1969 should be used rather than the regressors observed in 1979 if the observed values of the regressors are different.

The length of stay varies across households depending upon factors which are consumption-related, job-related, or demographic. The variables representing the consumption-related factors are shown in equation (4.6), some of which are not available for the Panel data of 1969. The loan-value ratio for period $A$ is measured with error using the ratio of the annual mortgage payment to the value of house, $PAY/V_A$. The income profile $y(i)$ is approximated by the projected income growth rate, $g$. The income growth rate is calculated for each household using an estimated lifetime wage profile equation. Following the method of Macurdy (1981), the lifetime wage equation is specified as a varying parameter function of age. The parameter is assumed
to vary with the age-invariant characteristics such as the household-head's education and the on-the-job training. The income growth rate $g$ is:

$$g = \partial \ln y(t)/\partial t = \alpha_3 + \alpha_4 t + \alpha_5 T + 2\alpha_6 t^2 + 2\alpha_7 t F + 2\alpha_8 t T, \quad (4.10)$$

where $E$ : education,

$T$ : on-the-job training, and

$t$ : household-head's age,

with $\ln y(t) = \alpha_0 + \alpha_1 E + \alpha_2 T + (\alpha_3 + \alpha_4 T + \alpha_5) t + (\alpha_6 + \alpha_7 T + \alpha_8) t^2 + u$.

The length of stay of the household-head in his or her current job is assumed to represent the degree of the household-head's on-the-job training. The reason is that on-the-job training is more firm specific compared with the general training such as education. The previous tenure, $PT$, is used to represent the effect of a previous homeowner's wealth from its house. The variable for the expected opportunity cost of capital, $r$, the income tax rate, $\tau$, and the term-to-maturity of the mortgage loan, $M$, are not available in the Panel data of 1969.

The length of stay of a household is, in part, determined by nonconsumption factors such as demographic factors and job related factors. The number of children, $NC$, and the household-head's age, $AGE$, are included to control for the effect of the demographic factors on the length of stay. The household-head's job security, employment status, and expected date of retirement also affects the
household's length of stay. The job factor is important because the expectation of a continuous inflow of a certain level of income for a long period of time is required for owner-occupiers since they have to repay the mortgage for a long period of time. The household-head's job security is measured whether he is a union member, UM, or not.\footnote{The union is characterized as maximizing an objective function that contains the wage rate and employment as arguments, subject to a trade-off between these two variables given by the employer's labor demand function (Pencavel(1984), Blair and Crawford(1984)). The elasticity of substitution between wages and employment in the union's objective function is estimated to be low(Pencavel(1984)). This means that it would take a considerable increase in wages to compensate for a reduction in employment. This also implies that the union is concerned with the members' job security, therefore, the household-head's job security is measured, in part, by his or her union membership.} The square of the household-head's age, $\text{AGE}^2$, is used with the household-head's age, $\text{AGE}$, to reflect the effect of the household-head's possible retirement on its length of stay. The household-head's employment status, UP, is also included in the length of stay equation. The set of the observed variables varying systematically with the length of stay, $Z$, is specified by:

$$Z = (v_{A}, \text{PAY} / v_{A}, \text{Y}, \Pi_{H}, \text{NC}, \text{AGE}, \text{AGE}^2, \text{UE}, \text{UM}, \text{PT}, g), \quad (4.11)$$

where $\text{PAY}$: amount of mortgage repayment for each year,

$\text{AGE}$: household-head's age,

$\text{AGE}^2$: $\text{AGE} \times \text{AGE}$,

$\text{NC}$: number of children.
UI = 1: unemployed
0: otherwise,

UF = 1: union member
0: otherwise,

PI = 1: owned a house previously
0: otherwise.

g: income growth rate.

The expenditure on other goods for period i, q_A, is not directly observed in the Panel data. However, the expenditure for other goods in period i can be measured by subtracting housing expenditures from disposable income. Disposable income is calculated using data on the total income of head and wife and estimated federal income taxes of head and wife. For homeowners, housing expenditures are composed of the mortgage payment and property taxes, while for renters, the annual rental payment is the measure of housing expenditure. This approximation of the expenditure for other goods does not allow a household to invest its savings in nonhousing assets.\(^{14}\)

\(^{14}\) Our measurement of the expenditure on other goods is based on the argument that if rm > \(\Pi_H\) and rm > r, the prior model implies that a household will accumulate its wealth only through investments in owner-occupied housing. However, this method of measurement causes a problem in that it rules out the possibility of a renter accumulating wealth. To partially investigate the effect of selecting this measure of consumption of the composite good, the system of structural equations is estimated using a sample which contains only homeowners and compared with the estimation results based on the whole sample. A comparison of the estimation results suggests
4.1.2.2 Unobserved Future Variables

Including the decision variables for period B reflects the fact that a household accounts for the future effects of its present housing decisions. These variables cannot be observed and are subject to replanning when new information is available.

Optimal control theory is widely used to avoid the variables outside the sample period when specifying a demand equation derived in an intertemporal framework (Hotz (1980), Philips (1983), pp. 282-287). The intertemporal utility function is defined as a discounted sum of instantaneous utility functions. This definition does not accommodate planning in periods of different length in the household's life cycle consumption plan. Our model shows that the length of stay is closely related to the household's choice of the quantity of owner-occupied housing. It would be difficult to apply optimal control methods to a model which allows for households to choose different lengths of stay. Such a restriction on the model would be costly because the measurement of the effects of including an endogenous length of stay variable is one of the primary goals of the model.

that the selectivity bias due to the neglect of tenure choice is more serious than the bias from our mismeasurement of the expenditure on other goods.
Another method to deal with the unobserved variables outside the sample period is to predict their values by using additional information. Macurdy (1981) predicts the wage outside the sample period by specifying and estimating the lifetime profile wage equation as a varying parameter function of age. The parameter is assumed to vary with age-invariant characteristics of the consumer, such as education. However, this method cannot be applied to the prediction of unobserved endogenous variables such as \( b_B \), \( b_{\tau} \), and \( q_B \). The reason is that these variables are determined within the system of equations rather than by outside information.

Unobservability of future planned variables, coupled with the unavailability of a suitable method to deal with them, forces us to delete the future planned variables \( b_B \), \( b_{\tau} \), and \( q_B \) from the system of estimating equations in spite of the danger of biases in parameter estimates of the structural equations.

The danger of bias in the reduced form equations depends upon the validity of the assumption that the exogenous variables remain the same between period A and B. If the exogenous variables are different between periods A and B, omitted exogenous variables for period B cause a bias in the estimated parameters in the reduced form equations. Differences in the marginal income tax rate and the term-
to-maturity of the mortgage loan between period A and B are good examples. However, endogeneity of the length of stay in our model makes it difficult to measure the different values of the exogenous variables in period A and B. The only way to avoid this problem is to introduce a lifetime profile of the exogenous variables, but this method is not feasible. Therefore, we assumed that the exogenous variables remain the same between periods A and B.

4.1.3 Exogenous Variables

Among the exogenous variables, the most controversial one is the income tax rate. Rosen (1979) points out the possibility of a spurious positive correlation between the price of housing and the quantity of housing demanded due to the variation in the marginal income tax rate with house value. The ideal solution is to parameterize the entire tax structure to completely represent the choices open to the household. Instead of the ideal solution, Rosen (1979) tried to correct this relationship by evaluating net price and income at standardized levels of house value but failed since the estimates had nonsensical values. Most studies recognize this problem but treat the marginal income tax rate as an exogenous variable in their empirical work since a suitable method to correct this problem has not yet been found (Rosen (1979), Gillingham and Hagemann (1983)).
The term-to-maturity of a mortgage loan is treated as an exogenous variable. Borrowing costs for the mortgage loan depend on the term-to-maturity of the mortgage loan and the loan-value ratio because the mortgage interest rate varies with the loan-value ratio. However, I assume that the loan-value ratio is the household's prime concern among the terms of mortgage credit. The reason is that the loan-value ratio affects the cost of housing capital through its effect on the mortgage interest rate. The term-to-maturity does not affect the cost of housing capital. The term-to-maturity affects the credit risk of mortgage loan calculated by the ratio of mortgage payment to income. The credit risk of the mortgage loan is primarily a concern of the mortgage lending institutions. Therefore, the lending institutions would decide the term-to-maturity based on the household's income and the size of the mortgage loan accounting for the credit risk of the loan. This implies that given the loan-value ratio, the term-to-maturity is an exogenous variable to the household given by the mortgage lending institution.

The household's income profile is approximated by its estimated income growth rate which is based on MaCurdy's lifetime wage equation. Finally some unobserved, exogenous variables such as taste parameters and the time preference rate are simply omitted from the system of estimating equations.
4.2 IDENTIFICATION

The system of estimating equations is rewritten as follows after dealing with the unobserved endogenous and exogenous variables.

\[ h_A = H( b_A, (\ln s), \pi_H, r, \tau, \tau', \omega_0, v, FS) \text{ if } h_A > 0, \]  
\[ b_A = B( h_A, q_A, (\ln s), r, \tau, \omega_0, v) \text{ if } h_A > 0, \]  
\[ h_A = 0 \text{ otherwise} \]  
\[ b_A = 0 \text{ otherwise} \]  
\[ q_A = Q( h_A, (\ln s), \omega_0, v, r, FS) \]  
\[ (\ln s) = S( h_A, q_A, b_A, \omega_0, \pi_H, r, \tau, \tau', \omega) \]  

where \((\ln s) = Z_{QA}\) is the instrument for the length of stay, and

\[ Z = (v_A, \text{PAY}/v_A, y, \pi_H, NC, AGF, AGF^2, UF, UN, PT, g). \]

Equation system, \((4.12)-(4.15)\), can not be identified without additional restrictions. First, the location of the household, specifically, the distance from the center of the city, is introduced into the equation system. The traditional housing model by Ruth (1969) shows that the location of the household affects the quantity of housing consumed. The reason is that land prices decline with increases in the distance from the center of the city, resulting in a variation in the price of housing services with changes in the location of the house. This argument implies that the quantity of housing services consumed by a household is affected by its location. Moreover, this locational factor is related only to housing consumption, and
not expenditures on other goods, the loan-value ratio, and
or length of stay.

Second, the parameter of the term-to-maturity variable
in equation (4.12) is restricted to be zero. The rationale
for this restriction is that, when the loan-value ratio
appears in equation (4.12), the term-to-maturity variable
in that equation affords no new information concerning the
choice of the quantity of owner-occupied housing. In our
model, a household's choice of loan-value ratio determines
the cost of the mortgage loan thereby affecting the price
of housing services. The term-to-maturity of the mortgage
loan affects the choice of both the loan-value ratio and
the length of stay, but it is assumed to not directly af-
fact the choice of the quantity of housing. Indirect ef-
facts through variations in the price of housing are cap-
tured by the endogenous variables present in the housing
demand equation. The estimating equations can now be writ-
ten as:

\[ b_A = h_A (\ln s)_{\Pi H, r, \tau, W_0, Y, FS, LOC} \] if \( b_A > 0 \) . \hspace{1cm} (4.16)

\[ b_A = b_A (\ln s, r, \tau, N, W_0, Y) \] if \( b_A > 0 \) . \hspace{1cm} (4.17)

\[ b_A = 0 \] otherwise

\[ b_A = 0 \] otherwise

\[ q_A = q_A (\ln s, W_0, Y, r, FS) \] and

\[ (\ln s) = s (\ln s, q_A, b_A, \Pi H, r, \tau, N, W_0, FS) \]. \hspace{1cm} (4.19)

where \( FS \) : the family size of a household, and

\( LOC \) : the household's distance from the center of
the city.

The system of estimation equations is overidentified due to the large number of restrictions in specifying the loan-value ratio and the expenditure on other goods. The zero restriction on the parameter of $h_A$ in the $q_A$ equation comes from the separability assumption made when specifying the utility function. All other zero restrictions, except for the zero restrictions related to LOC and $W$ come from the specification of the theoretical model in the previous chapter.

4.3 ESTIMATION

The system of estimating equations (4.16)–(4.19) is a simultaneous equation system with the selection criterion being of the tobit type. In estimating a simultaneous equations model with tobit-type selection function, Amemiya (1974) uses only the observations where the dependent variables have positive values, and suggests an indirect least square procedure. Suppose that the dependent variable $y$ has a normal distribution. Also suppose that the dependent variable $y$ is observed only if it has a positive value: the dependent variable is truncated from below. By truncating the distribution of $y$ at the point $y=0$, no observations are drawn for $y<0$. The truncation of the dependent variable results in a truncated normal distribution.
of the error term in the reduced form equation. Considering the truncated normal distribution of the error terms, he respecifies the reduced form equations and estimates the coefficients based on the initial consistent estimates of the density functions and the distribution functions. An instrumental variable method is used to get the initial consistent estimates of the density functions and the distribution functions. Error terms in the reduced form equations are assumed to have a bivariate normal distribution. The estimated coefficients of the reduced form equations are transformed into the coefficients of the structural form following the logic of indirect least squares.

Amemiya's method can be criticized on the following grounds. First, if the model is overidentified, his method yields multiple solutions for the structural coefficients. A second criticism is that he uses only a part of the observations available. A theoretical proof that efficiency is increased by using the other observations is very difficult. However, it is important, in a practical point, not to throw away available observations.

Nelson and Olson (1978) specify a simultaneous equation model by replacing the observed, truncated normal endogenous variable with the latent (not directly observable) and unconstrained form of the endogenous variables. They argue that the respecification is better in that the latent
unconstrained form of the endogenous variable is more relevant when the truncation is simply an artifact of data measurement or reporting. Even if the truncation of a variable is real in the sense that a nonnegativity constraint causes the truncation, the latent and unconstrained form is intuitively appealing. The latent form of the right hand side endogenous variables in the tobit model allow for a different outcome depending on whether the latent variable is well below the threshold value or just marginally below it. The value of the latent endogenous variables equal the observed value only if they are greater than zero.

In estimating the system of equations of (4.16)-(4.19), we follow the procedure developed by Nelson and Olson. In our system, variables $h_A$ and $b_A$ are regarded as latent and unconstrained endogenous variables. Nelson and Olson (1978) ruled out the full information maximum likelihood method because the presence of multiple integrals in the likelihood function makes computational costs prohibitive. Instead, they suggested a two-stage procedure for the estimation of the simultaneous equation system. The first stage of this procedure is to estimate the coefficients of the reduced form equations for the latent and unconstrained dependent variables using a tobit maximum likelihood estimation and estimate the coefficients of the completely ob-
served dependent variable by ordinary least squares. Next form the instruments of the right hand side endogenous variables of the simultaneous-equations tobit model using the reduced form estimation results. The second stage is to replace the endogenous variables on the right hand sides of the equations by the corresponding instruments and treat these instruments as fixed regressors and the resulting equations as single equation models. Next estimate the structural-form equations for the latent and unconstrained dependent variables using a tobit maximum likelihood estimation method and estimate the equation for the completely observed dependent variables by ordinary least squares. Nelson and Olson argue that the estimates of the structural parameters are consistent based on both the consistency of the reduced form estimates and the straightforward extension of proofs for two-stage least squares. They also argue that these estimates are asymptotically normal.
Chapter V

DATA

This study uses data from 'A Panel Study of Income Dynamics', 1979. A panel data set is chosen since only the panel data provides the time series data for each household which are needed to calculate variables such as the length of stay and the income growth rate. Our sample consists of families who moved during the period from the spring of 1978 to the spring of 1979. The moving households are selected in spite of the danger of selectivity bias since moving households are thought to better reveal their preferences for housing compared to nonmoving households. The transaction costs for a house sale prevents owners from instantaneously adjusting the quantity of housing services demanded. Therefore, the nonmoving owner's housing value is expected to deviate more from the equilibrium value than that of a moving owner. The sample consists of 1677 households.

The rest of this chapter describes the methodology and data used to construct the variables.

Quantity of owner-occupied housing \( h_A \)
TABLE 2

The Composition of The Sample

<table>
<thead>
<tr>
<th>Tenure Choice Decision on Mortgage</th>
<th>Owner $V_A &gt; 0$</th>
<th>Renter $V_A = 0$</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Mortgage Loans ($b_A &gt; 0$)</td>
<td>330</td>
<td>-</td>
<td>330</td>
</tr>
<tr>
<td>Without Mortgage Loans ($b_A = 0$)</td>
<td>64</td>
<td>1263</td>
<td>1347</td>
</tr>
<tr>
<td>Total</td>
<td>394</td>
<td>1263</td>
<td>1677</td>
</tr>
</tbody>
</table>

The quantity of owner-occupied housing, $b_A$, is defined as $V_A/P_H$ under the assumption that $γ$ equals 1. The value of $γ$ does not have any real impact on the measurement of housing services since $γ$ is same for all households. The data on the value of owner-occupied housing, $V_A$, is available from the panel data. The regional data on the price index of a constant-quality house, $P_H$, are obtained from the Construction Reports C-25. The price index of the constant-quality house indicates the changes over time in the sales price of a new one-family house which is the same with respect to ten important characteristics. The ten characteristics used are: floor area, number of stories, number of bathrooms, geographic division, metropolitan location, air-conditioning, type of parking facilities, type of foundation, presence of fireplace, and size of lot.

Mortgage payment rate (IR)
The mortgage payment rate, $r_m$, is calculated from the household's annual mortgage payment, $PAY$, its remaining mortgage principal, $M_t$, and the remaining years to pay the mortgage loans, $N$, using the following relationship.

$$\frac{PAY}{M_t} = \frac{r_m}{(1-(1+r_m)^{-N})}. \quad (5.1)$$

The relationship in equation (5.1) assumes a standard fixed-rate, fixed-payment mortgage.

**Term-to-maturity ($N$)**

Since our sample consists of the households who moved during the period from the spring of 1978 to the spring of 1979, the maturity period of mortgage loans is derived as $N+1$ under the assumption that new owners can not assume the previous owner's mortgage loan.

**Loan-value ratio ($b_A$)**

The derivation of the mortgage payment rate and the maturity period of the mortgage loan enables us to estimate the loan-value ratio at the point of the house purchase.

$$b_A = \frac{M_A}{V_A}$$

$$= \frac{PAY \times (1-(1+r_m)^{-N+1})/r_m \times (V_A/(1+DPH))}{r_m V_A}, \quad (5.2)$$

where $M_A$: the value of mortgage principal at the time of house purchase, and

$DPH$: the observed appreciation rate of housing during 1979, measured by region.
A regional price deflator is the best available measure of individual house appreciation. The use of the regional price variable for the loan-value ratio results in measurement error, which causes a downward bias in the estimated coefficients of the loan-value ratio.

**Length of Stay (ln s)**

The instrumental variable for the length of stay, ln s, is calculated using the previous results from the regression based on the observed length of stay between 1969-1979. Equations (4.9) and (4.11) indicate that this regression requires additional variables such as the number of children, the age of household-head, the household-head's union membership status, the amount of expenditures on repair of the house, the employment status of the household-head, the previous tenure status, and the projected income growth rate. All of the variables except for the projected income growth rate are available in the Panel data.

The projected income growth rate is calculated using an estimation method based on Macurdy's income profile equation as in equation (4.10). The income profile equation is estimated as:

---

15 Stepwise regression is used to estimate the equation. A stepwise regression eliminates the explanatory variables which have a smaller contribution to the explanation of the dependent variable than a prespecified tolerance limit. In the stepwise regression, the tolerance limit was set to either 0.05 or 0.01. This change results in
\[ \ln Y = 5.97520 + 0.00467 \times (E \times AGE) + 0.01815 \times T - 0.00112 \times (AGE)^2 \]

\((37.54) \quad (32.09) \quad (8.58) \quad (12.07)\)

\(+ 0.07355 \times AGE - 0.11733 \times 10^{-3} \times (T \times AGE), \quad (5.3)\)

\((10.16) \quad (12.46)\)

\[ R^2 = 0.32 \]

where \(Y\) : household's income in 1968,

\(AGE\) : household-head's age as of 1968,

\(E\) : household-head's education as of 1968,

\(T\) : number of months the household-head has spent in the current job as of 1968.

The figures in the parentheses show the t-statistics of each parameter. This estimation result allows us to calculate the projected income growth rate, \(g\).

\[ g = 0.07855 - 0.00224 \times AGE + 0.00467 \times E - 0.11733 \times 10^{-3} \times T. \quad (5.4)\]

The actual length of stay of households who relocated between the spring of 1968 and the spring of 1969 is calculated using the Panel data of 1969-1979; with results summarized in the Table 3.

The original tenure status is listed first and the post-1969 status is listed below, implying that the length of stay on 314 owners and 819 renters is analyzed. Table 3 reveals that a considerable portion of the owners have an expected length of stay greater than ten years. Since the

a different set of the explanatory variables but does not affect the overall explanatory power, \(R^2\). It is also noted that the projected income growth rates are almost same whichever set of the explanatory variables is used.
TABLE 3

Actual Length of Stay of Moving Households

<table>
<thead>
<tr>
<th>Length of Stay</th>
<th>Changes in Tenure</th>
<th>Renter</th>
<th>Owner</th>
<th>Renter</th>
<th>Owner</th>
<th>Renter</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>years</td>
<td></td>
<td>Owner</td>
<td>Owner</td>
<td>Renter</td>
<td>Owner</td>
<td>Renter</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>24</td>
<td>36</td>
<td>326</td>
<td>78</td>
<td>464</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>18</td>
<td>20</td>
<td>139</td>
<td>14</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>22</td>
<td>16</td>
<td>63</td>
<td>14</td>
<td>115</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>11</td>
<td>15</td>
<td>26</td>
<td>6</td>
<td>58</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>10</td>
<td>9</td>
<td>37</td>
<td>1</td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>6</td>
<td>10</td>
<td>16</td>
<td>5</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>7</td>
<td>9</td>
<td>13</td>
<td>3</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>8</td>
<td>3</td>
<td>12</td>
<td>5</td>
<td>28</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>8</td>
<td>0</td>
<td>8</td>
<td>0</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>0</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>&gt;10</td>
<td></td>
<td>46</td>
<td>27</td>
<td>40</td>
<td>5</td>
<td>118</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>164</td>
<td>150</td>
<td>688</td>
<td>131</td>
<td>1133</td>
<td></td>
</tr>
</tbody>
</table>

The observed length of stay is artificially truncated to an upper limit of 10 years, so the upper limit tobit method is used in the regression. The tobit-regression results are:

\[
\ln s = -0.271e2 + 0.000035588y_A + 3.0641z (PAY/y_A) - 0.000012028zY \\
(0.90) (8.91) (5.38) (1.67)
- 0.014816zNC + 0.16214zP_i + 0.017581zAGE - 0.000083109z(AGE)^2 \\
(1.19) (2.79) (1.91) (0.85)
- 0.026149zN_E - 0.34899zPT + 0.27276zUM - 0.033726zG, \\
(0.23) (4.75) (3.80) (2.74)

Log-likelihood = -1498.82

The estimation results are believable based on the specification of the length of stay equation (3.46) in Chapter 3.
This estimation result is used to calculate the instrumental variable of the household's length of stay as specified in equation (4.9).

**Expenditures on other goods** ($g_A$)

Expenditures on other goods are measured following the method described in section 4.1.2.1.

**Income** ($I$)

Income, $I$, is defined as the average of the household's future expected income. This definition of income is consistent with the standard argument that permanent income rather than current income should be used in housing demand studies. However, it is not obvious how to compute permanent income and this problem has been dealt with in various ways. Carliner (1973) and Rosen (1979) take an average of several years' worth of previous annual income. Struyk (1976) and Goodman and Kawai (1982) use the fitted value of a regression of income on a set of personal characteristics for the permanent income measure. The widely-used set of the regressors in the income equation consists of the household-head's age, education, and the degree of the on-the-job training; these being closely related to the household-head's earning power. However, neither of these two methods is a perfect measure of permanent income. The first method causes both an overestimation for older households and an underestimation for younger households. The
second method requires a full set of the regressors which affect the household-head's earning power. Goodman and Kawai's study showed that the instrumental variable method for permanent income improves the predictive power of housing demand equation. However, Gillingham and Magee (1983) used measured income as a proxy for permanent income because the pay off for instrumental variable estimation for permanent income was so low.

The weighted average of past incomes is used as a proxy for permanent income in our study. As for the weighting scheme, Calimer (1973) and Rosen (1979) use a simple average, their argument being that the parameter estimates in the housing-demand equation are not very sensitive to variations in the weighting schemes used to generate permanent income. However, they anticipated the estimated income elasticity to be biased downward when using the average of past incomes as the proxy for permanent income. Following their method, permanent income is measured by the simple three-year average of a household's past income. The measure used for permanent income is defined as after-tax income, subtracting the simple three-year average of federal income taxes.

**Expected opportunity cost of capital (E)**

In the previous studies, the expected opportunity cost of financial capital is assumed to be constant for all
households. Hendershott and Hu (1979) assumed that the after-tax opportunity cost of capital equals the after-tax mortgage interest rate. Ranney (1979) defines the expected opportunity cost of capital as the interest rate paid on savings. In her paper, savings are defined as financial assets and assumed to earn an exogenous rate of return that is constant through time. However, Kane (1979) points out that the opportunity cost of financial capital differs across households depending on the level of income because of deposit-rate ceilings and the restricted opportunities for small-denomination marketable instruments result in low-income households having a limited opportunities for financial investment. Using the Survey of Consumer Finance for 1962 and 1970, he shows that the expected opportunity cost of financial capital for low-income households is very low. During a period of accelerating inflation, the effects of deposit-rate ceilings and other constraints on a low-income household's expected opportunity cost of capital increases. Therefore, this effect was more important in the 1970's than now because the inflation rate accelerated during the 1970's. Hendershott (1979) also suggests that a household's marginal income tax rate affects the expected opportunity cost of financial capital since it is the after-tax opportunity cost of capital that matters when evaluating the value of investment in financial assets rather than the before-tax opportunity cost of capital.
Based on these arguments, a household's expected opportunity cost of financial capital is assumed to be a linear function of its income and it is specified as:
\[ r/(1-\tau) = a_0 + a_1 Y. \]  \hfill (5.6)

The assumption of the linear relationship in the above equation is consistent with the Kane's estimates of the anticipated 1970 portfolio rate of return for each income class in that his results show a roughly linear relationship between income and the rate of return. The return from financial assets for the lowest-income households is "a0" and following the arguments by Kane and Penney, "a0" is the rate of return on a regulated asset such as the return on savings deposits. The average rate of return and average income by income class are derived from the latest Consumer Expenditure Survey which was conducted in 1972-1973 by the Bureau of Labor Statistics. Marginal income tax rates are from the 1972 tax rate schedule in the Statistics of Income:1972, Individual Income Tax Returns by Internal Revenue Service. Equation (5.6) is specified as:
\[ r/(1-\tau) = 0.5107 + 0.0001972Y, \]  \hfill (5.7)
\[ (7.13) \quad (0.79) \quad R^2 = 0.8357. \]

It is necessary to analyze the changes in the estimated value of "a0" and "a1" over time. Based on the specification of "a0", the changes in "a0" over time are mainly described by the changes in the maximum interest rate on
passbook savings deposits set by the Federal Home Loan Bank Board. The changes in \(a_1\) largely depend upon innovations in the marketable investment instruments; however, the estimation of changes in \(a_1\) is beyond the scope of this study. Therefore, we simply assume that the estimated value of \(a_1\) remains the same between 1972-1979 even though the time period was long enough to suspect the validity of a fixed value of \(a_1\). The expected opportunity cost of financial capital, \(r\), is calculated by:

\[
r = (a_{0A} + \hat{a}_1 \tau) (1 - \tau),
\]

where \(a_{0A}\) is adjusted for the changes in the maximum rate of interest on passbook savings deposits (\(a_{0A} = 5.0\) in 1979), \(\hat{a}_1\) is estimated from equation (5.7).

**Marginal Income Tax Rate (\(\tau\))**

The marginal income tax rate is available in the panel data.

**Initial Wealth (\(W_0\))**

Initial wealth is derived by dividing the three-year average of a household's capital income by the estimated opportunity cost of financial capital, equation (5.8). The household's capital income is available in the panel data. In the case of a homeowner, the owner's equity in the previous house is added in calculating initial wealth. An owner's equity in the previous house is calculated by sub-
tracting the remaining mortgage principal from the previously house's value. However, this method results in nonsensical estimated parameters for income and wealth in the housing demand equation. This indicates that the above measure of initial wealth may contradict the assumptions of our model. Therefore, in calculating initial wealth, we impose the assumptions that \( r_m > r \) and that homeowners accumulate their wealth only through investments in owner-occupied housing. The household's initial wealth is now determined by its previous and present tenure choices.

\[
W_0 = \begin{cases} 
\frac{YC}{(\min (I, r_m))} & \text{for previous renter and present owner,} \\
\frac{YC}{r} & \text{for previous renter and present renter, and} \\
EV_{-1} & \text{for previous homeowner,}
\end{cases} \tag{5.9}
\]

where \( YC \): a three-year average of capital income, and \( EV_{-1} \): equity value of the previously owned house.

**Expected appreciation rate of housing \( (\Pi_H) \)**

The expected capital gains from housing are proxied for by the expected appreciation rate of housing, \( \Pi_H \). This proxy assumes a zero tax rate on capital gains from housing. This assumption is plausible since a household can delay the capital gains tax by purchasing another house and eventually claim a substantial exemption after age 55. The expected appreciation rate of housing has been estimated various ways, however an adaptive-expectations model is generally used in the estimation (Mendershott and
Hu (1979), Diamond (1980), Dougherty and Van Order (1982). On the other hand, Pollain (1982) imposes competitive and efficient housing market conditions and derives the expected appreciation rate of housing as the difference in the price of housing services between owner-occupied housing and rental housing. The price of housing services from owner-occupied housing varies across households with their income and wealth. Pollain's method results in different expected appreciation rates across households for the same housing asset based on differences in income. However, when information on past movements in housing prices is available, it is hard to expect different appreciation rates for housing assets across households based on differences in income.

Hamilton and Schwab (1982) showed that 60% of the variation in capital gains from housing are explained by a combination of the variables; past capital gains, income growth, and population growth. However, they found that their equation has no power to predict subsequent actual capital gains on housing. This implies that all of the contemporaneously available information is not efficiently used in formulating the expectation of the appreciation rate of housing, therefore, they rejected the applicability of the rational expectations model in deriving the expected capital gains on housing. They argue that the inefficient
utilization of the available market information in formulating measures of the capital gains on housing assets is due to the high cost of getting and coordinating all information for households and individual landlords. Therefore, an adaptive-expectations model is used to estimate the expected appreciation rate of housing.

In applying the adaptive-expectations model, Hendershott and Hu (1979), and Dougherty and Van Order (1982) estimate the expected appreciation rate of a housing asset as a lagged distribution of previous values. On the other hand, Diamond (1980) prefers the simple average of the past actual appreciation rates of housing, because this measure is conservative during a period of accelerated appreciation of the price of housing. Consistent with Diamond's argument, Hamilton and Schwab argue that households discounted the rapid appreciation of housing during the mid-70's when forming their expectations since the households believed the rapid appreciation was an abnormal case. This view implies that households have regressive expectations during a period of rapid appreciation of housing; however, a lag-distribution method of estimating expected appreciation cannot capture the household's regressive attitude. Therefore, using a simple average of past appreciation rates seems to be a reasonable assumption in the estimation. This paper calculates the expected appreciation rate of
housing as the simple average of the prior six year actual appreciation rate by region. The actual appreciation rate of housing is available in the Construction Report, C-25. The expected appreciation rates of housing are: 7.5% for Northeast, 8.7% for Northcentral, 8.3% for South, and 12.0% for West.

Projected income growth rate ($g$)

The projected income growth rate is measured as of 1979 using Macurdy's income profile equation. The estimated equation is:

$$\ln Y = 5.2441 + 0.10046 + 0.00426 + 10^{-4} - 0.00166 (AGE)^2 \quad (42.38) \quad (23.59) \quad (6.44) \quad (22.73)$$

$$+ 0.13631 (AGE) - 0.31064 \times 10^{-4} (T \times AGE), \quad (5.10)$$

$$R^2 = 0.28$$

Therefore, the projected income growth rate is calculated as:

$$g = 0.13631 - 0.00332 \times AGE - 0.31064 \times 10^{-4} \times T, \quad (5.11)$$

where $AGE$ is the household-head's age as of 1979, and $T$ is the number of months the household-head has spent in the current job.

Other variables

The family size of a household, $FS$, is reported in the Panel data. The location of a household, $LOC$, is measured by the distance from the center of the city. The house-
hold's distance from the center of city is reported in the Panel data in the form of a bracket variable. The locational variable, LOC, is defined as follows.

\[
LOC = \begin{cases} 
1 & \text{if the distance is less than 5 miles}, \\
2 & \text{if the distance is } 5 - 14.9 \text{ miles}, \\
3 & \text{if the distance is } 15 - 29.9 \text{ miles}, \\
4 & \text{if the distance is } 30 - 49.9 \text{ miles}, \\
5 & \text{if the distance is longer than } 50 \text{ miles}, \text{ and} \\
6 & \text{if the household is outside the city limit}. 
\end{cases}
\]
Chapter VI

EMPIRICAL RESULTS

This chapter reports the parameter estimates of the system of equations for the demand for owner-occupied housing. The estimates are reported in two different forms: parameters of structural equations and parameters of the equations in the reduced form. The parameter estimates in the structural equations reveal the existence of a simultaneous relationship among the quantity of owner-occupied housing demanded, the loan-value ratio, and the length of stay. The parameter estimates in the reduced form show the overall effects of the variables and can be used as the basis for comparison with other studies.

Table 4 contains the parameter estimates for the reduced form of the structural model.

In the housing demand equation, all the variables, except for the expected appreciation rate of housing and the opportunity cost of capital, are significant at 95% level and they all have positive signs, except the income growth rate. The elasticity of permanent income, evaluated at means, is 0.61. This estimate is a bit lower than the
TABLE 4

Tobit Estimates of the Reduced Form

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Housing Services</th>
<th>Loan-Value Ratio</th>
<th>Other Goods</th>
<th>Length of Stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-4.8939</td>
<td>-1.3845</td>
<td>117.976</td>
<td>0.62445</td>
</tr>
<tr>
<td></td>
<td>(4.13)</td>
<td>(5.49)</td>
<td>(55.42)</td>
<td>(3.53)</td>
</tr>
<tr>
<td>Expected Income</td>
<td>0.650 x 10^-2</td>
<td>-0.796 x 10^-5</td>
<td>4.119</td>
<td>-0.122 x 10^4</td>
</tr>
<tr>
<td></td>
<td>(3.00)</td>
<td>(1.74)</td>
<td>(86.83)</td>
<td>(3.08)</td>
</tr>
<tr>
<td>Marginal Income</td>
<td>296.91</td>
<td>1.2076</td>
<td>-116.810</td>
<td>1.1001</td>
</tr>
<tr>
<td>Tax Rate</td>
<td>(2.13)</td>
<td>(4.14)</td>
<td>(47.28)</td>
<td>(5.35)</td>
</tr>
<tr>
<td>Initial Wealth</td>
<td>0.213 x 10^-2</td>
<td>-0.174 x 10^-5</td>
<td>-0.036</td>
<td>0.373 x 10^5</td>
</tr>
<tr>
<td></td>
<td>(5.73)</td>
<td>(1.74)</td>
<td>(4.65)</td>
<td>(5.81)</td>
</tr>
<tr>
<td>Opportunity Cost of Capital</td>
<td>171.44</td>
<td>11.874</td>
<td>-228.500</td>
<td>14.473</td>
</tr>
<tr>
<td>Appreciation Rate of Housing</td>
<td>87.543</td>
<td>-1.8664</td>
<td>-848.63</td>
<td>-1.4454</td>
</tr>
<tr>
<td>Term-to-Maturity of Mortgage</td>
<td>13.635</td>
<td>0.0401</td>
<td>-122.47</td>
<td>0.592 x 10^3</td>
</tr>
<tr>
<td>Income Growth Rate</td>
<td>-1264.2</td>
<td>-1.7439</td>
<td>-338.84</td>
<td>-0.6056</td>
</tr>
<tr>
<td></td>
<td>(7.04)</td>
<td>(4.46)</td>
<td>(1.16)</td>
<td>(2.46)</td>
</tr>
<tr>
<td>Family Size</td>
<td>21.779</td>
<td>0.0503</td>
<td>662.15</td>
<td>-0.0502</td>
</tr>
<tr>
<td></td>
<td>(4.07)</td>
<td>(4.45)</td>
<td>(7.27)</td>
<td>(6.62)</td>
</tr>
<tr>
<td>Household's Location</td>
<td>17.292</td>
<td>0.0413</td>
<td>111.10</td>
<td>-0.0006</td>
</tr>
<tr>
<td></td>
<td>(5.18)</td>
<td>(5.64)</td>
<td>(1.82)</td>
<td>(1.12)</td>
</tr>
<tr>
<td>Log-Likelihood</td>
<td>-2862.4</td>
<td>-486.9</td>
<td>0.89</td>
<td>0.10</td>
</tr>
<tr>
<td>R-square</td>
<td>0.52</td>
<td>0.60</td>
<td>0.89</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: Numbers in Parentheses are t-statistics for each parameter.

A widely accepted value of 0.75, but it lies within a range of 0.3-1.0 which has been reported in earlier cross section studies (Folinsky(1977), Rosen(1979), Macrae and Turner(1981), Faye(1981), and Goodman and Kawai(1982)).

The results of the earlier cross section studies are based on a single housing demand equation, and include as expla-
natory variables: household income, the expected appreciation rate of housing, the expected opportunity cost of capital, the marginal income tax rate, and family size. This specification is quite similar to our reduced form of the housing demand equation. Therefore, the between study comparison of the estimated income elasticity may not be completely correct, but it is tolerable. The estimated parameter of wealth is positive and significant. The estimates reveal the overall effects of the variables including the simultaneity. If the reduced form was interpreted as a structural equation, the interpretation commits the serious error of ignoring the simultaneity bias, yielding a biased interpretation of the estimates. Good examples are the opportunity cost of capital and the income growth rate. The positive sign of the opportunity cost of capital reflects both the direct effect and the indirect effect. The direct effect is to increase the opportunity cost of a downpayment thereby decreasing the demand for owner-occupied housing. The indirect effect is mainly due to the positive effect of the opportunity cost of capital on the choice of the loan-value ratio. If the simultaneity between loan-value ratio and the housing demand was properly treated, the estimated parameter of the opportunity cost of capital would reflect only the direct effect, and the result should have a negative sign in the housing demand equation. The income
growth rate is solely related with the choice of the length of stay. If the simultaneity among the choice variables in the housing decision is properly treated, the negative sign of the income growth rate would be unexpected in the housing demand equation.

As for the loan-value ratio equation, all the variables are significant at 95% level except income, the level of initial wealth, and the appreciation rate of housing. A household’s income and initial wealth, the expected appreciation rate of housing, and the income growth rate have negative signs. The elasticity of wealth evaluated at the mean is -0.05. The marginal income tax rate, the opportunity cost of capital, and the term-to-maturity show positive signs.

In the length of stay equation, all the variables, except for the term-to-maturity and the household’s location, are significant at 95% level. The household’s income, the expected appreciation rate of housing asset, and the income growth rate show negative signs. The opportunity cost of capital, the marginal income tax rate, and the household’s initial wealth show positive signs. As for the equation of the expenditure on other goods, all the variables, except for the expected appreciation rate of housing, the income growth rate, and location are significant at 95% level. The coefficients of the household’s income and the family
size variables are positive. The coefficients of the opportunity cost of capital, the marginal income tax rate, and the term-to-maturity are negative. Initial wealth has a negative sign, contrary to intuition.

To test whether there exist simultaneous relationships among the demand for housing, the demand for mortgage loans, and the length of stay, the structural equations of the simultaneous equation system should be estimated. The reason is that the parameter estimates in the structural form enable us to separate the overall effects of the variables into a direct effect and an indirect effect from the simultaneous relationships among the endogenous variables.

The parameter estimates of the structural equations in the simultaneous equation system are contained in Table 5. All of the coefficients in the housing demand equation, except for initial wealth, are significant at 95% level. The loan-value ratio, the length of stay, a household's income, the expected appreciation rate of housing, family size, and a household's location have a positive effect on the demand for owner-occupied housing. The income elasticity evaluated at the mean is 2.19, far greater than the estimated income elasticity from the reduced form. The income elasticity calculated only for homeowners is 1.5. This is smaller than the estimated income elasticity which considers the renter's probability of owning a house and the expected
TABLE 5

Tobit Estimates of the Structural Form

<table>
<thead>
<tr>
<th>Dependent Variables</th>
<th>Housing Services</th>
<th>Loan-Value Ratio</th>
<th>Other Goods</th>
<th>Length of Stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing Services</td>
<td>* 0.1320 ± 10^{-2}</td>
<td>-</td>
<td>0.08006</td>
<td>(4.15)</td>
</tr>
<tr>
<td>Loan-Value Ratio</td>
<td>322.95</td>
<td>* -2226.3</td>
<td>-33.146</td>
<td>(4.21)</td>
</tr>
<tr>
<td>Other Goods</td>
<td>- 0.728 ± 10^{-4}</td>
<td>* -0.134 ± 10^{-3}</td>
<td>(1.74)</td>
<td></td>
</tr>
<tr>
<td>Length of Stay</td>
<td>1157.6</td>
<td>0.4289</td>
<td>-77803</td>
<td>* (4.24)</td>
</tr>
<tr>
<td>Constant</td>
<td>-765.12</td>
<td>-9.7292</td>
<td>131950</td>
<td>16.757</td>
</tr>
<tr>
<td>Initial Wealth</td>
<td>-0.163 ± 10^{-2}</td>
<td>-0.376 ± 10^{-5}</td>
<td>0.25847</td>
<td>-0.231 ± 10^{-3}</td>
</tr>
<tr>
<td>Expected Income</td>
<td>0.232 ± 10^{-1}</td>
<td>-0.313 ± 10^{-3}</td>
<td>2.642</td>
<td>-</td>
</tr>
<tr>
<td>Marginal Income</td>
<td>-1366.5</td>
<td>8.9861</td>
<td>-5.2576</td>
<td>(3.33)</td>
</tr>
<tr>
<td>Opportunity Cost</td>
<td>20417</td>
<td>173.51</td>
<td>-815390</td>
<td>-48.009</td>
</tr>
<tr>
<td>Appreciation Rate</td>
<td>2363.5</td>
<td>-</td>
<td>-71.963</td>
<td>(4.29)</td>
</tr>
<tr>
<td>Term-to-Maturity</td>
<td>-0.0307</td>
<td>-</td>
<td>0.2142</td>
<td>(4.51)</td>
</tr>
<tr>
<td>Income Growth Rate</td>
<td>-</td>
<td>-</td>
<td>0.1743</td>
<td>(4.01)</td>
</tr>
<tr>
<td>Family Size</td>
<td>63.647</td>
<td>-3001.1</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Household Location</td>
<td>10.524</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Log-likelihood</td>
<td>-2882.4</td>
<td>-489.5</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>R-square</td>
<td>0.52</td>
<td>0.60</td>
<td>0.83</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Note: Numbers in Parentheses are t-statistics for each parameter.

The difference between these two income elasticities supports the argument that the value of an owner-occupied house...
lower income households have a higher income elasticity of housing because renters, in general, have a lower income compared with homeowners. An increase in the marginal income tax rate or the expected opportunity cost of capital have negative effects on the demand for owner-occupied housing. An increase in the marginal income tax rate has a dual effect on the demand for owner-occupied housing. It has a negative effect on the demand for owner-occupied housing because of the reduction in after tax income. On the other hand, an increase in the income tax rate decreases the after-tax mortgage interest rate, thus increasing the demand for owner-occupied housing. The negative sign of the marginal income tax rate in the housing demand equation shows that the income effect of the marginal income tax rate dominates the effect on the after-tax mortgage interest rate. A positive sign of the location of a household implies that a household demands a larger house as it moves farther from the center of the city. This reflects the traditional theory that the land price declines with the distance from the center of the city. A lower land price results in a lower price of housing thereby resulting in an increase in the housing demand as a household moves away from the center of the city. The loan-value ratio also shows a positive sign. An increase in the loan-value ratio relaxes the burden of downpayment thereby allowing a
household to buy a larger house with given initial wealth. On the other hand, an increase in the loan-value ratio increases the mortgage interest rate, thereby resulting in a higher price of owner-occupied housing. The positive sign of the loan-value ratio in the housing demand equation implies that, in purchasing a house, the downpayment burden is generally a more important factor than the increase in the price of housing due to a higher loan-value ratio. The signs of the estimated parameters are also compared with the expected signs based on the comparative statistics of the theoretical model. The signs of the estimated parameters of the length of stay, income, the appreciation rate of housing, and family size in the housing demand equation are consistent with expectations. The level of initial wealth has a negative sign, contrary to our expectation, however, the estimated parameter of initial wealth is insignificant.

The variables in the loan-value ratio equation are significant at 95% level except for the expenditure on other goods, the length of stay, a household's income and the expected opportunity cost of capital. However, the estimated parameters of the expenditure on other goods, a household's income, and the opportunity cost of capital are significant at the 90% level. The coefficient of initial wealth and income have negative signs. The elasticity of the loan-
value ratio with respect to the initial wealth is -0.12. The income elasticity of the loan-value ratio is -12.7. The quantity of housing services, expenditures on other goods, the length of stay, the opportunity cost of capital, the marginal income tax rate, and the term-to-maturity have positive signs. The coefficients of the term-to-maturity and the opportunity cost of capital are as expected. The positive sign of the marginal income tax rate reflects the effect of the income tax deductibility of mortgage interest payments. The income tax deductibility of mortgage interest payments reduces the after-tax mortgage interest rate as the marginal income tax rate increases thereby allowing a household to have a higher loan-value ratio as its marginal income tax rate increases. An increase in the expected opportunity cost of capital increases the attractiveness of mortgage loans and allows a household to bear a higher mortgage interest rate. Therefore, a household uses a larger mortgage loan in purchasing a house if the expected opportunity cost of capital increases. A longer term-to-maturity stretches out the household's burden of mortgage payment thereby allowing a household to use a larger mortgage loan for the house purchase. The positive sign of housing services demanded shows that households increase their equilibrium loan-value ratio to purchase a larger house.
Expenditures on other goods are negatively related to the loan-value ratio and the length of stay. The coefficients of initial wealth and income are positive as expected. All the estimated parameters are significant at 95% level, and all of the variables, except for the length of stay and family size, have the expected signs.

As for the length of stay equation, all the variables are significant at 95% level. The quantity of housing services, the term-to-maturity, and the income growth rate show positive signs. The loan-value ratio, expenditures on other goods, initial wealth, the opportunity cost of capital, the expected appreciation rate of housing, and the marginal income tax rate show negative signs. The parameters of housing services, the loan-value ratio, initial wealth, and the income growth rate are as expected; in other cases, the theoretical model was ambiguous in its prediction of signs. Table 6 summarizes the comparison of the estimation results with the theoretically expected signs of the variables.

The most important result concerns the question of whether the housing demand equation is part of a simultaneous system of equations. The estimated parameters of the loan-value ratio and the length of stay, in the housing demand equation, the quantity of owner-occupied housing, in the loan-value ratio equation, and housing in the length of
### Table 6
Correspondence of Results and Theory

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Housing Services</th>
<th>Loan-Value Ratio</th>
<th>Other Goods</th>
<th>Length of Stay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Housing Services</td>
<td>*</td>
<td>+ (?)</td>
<td>*</td>
<td>+ (+)</td>
</tr>
<tr>
<td>Loan-Value Ratio</td>
<td>+ (?)</td>
<td>*</td>
<td>- (-)</td>
<td>- (-)</td>
</tr>
<tr>
<td>Other Goods</td>
<td>±</td>
<td>0 (-)</td>
<td>*</td>
<td>- (?)</td>
</tr>
<tr>
<td>Length of Stay</td>
<td>+ (+)</td>
<td>0 (-)</td>
<td>- (?)</td>
<td>*</td>
</tr>
<tr>
<td>Initial Wealth</td>
<td>0 (+)</td>
<td>- (?)</td>
<td>+ (+)</td>
<td>- (-)</td>
</tr>
<tr>
<td>Expected Income</td>
<td>+ (+)</td>
<td>0 (+)</td>
<td>+ (+)</td>
<td>*</td>
</tr>
<tr>
<td>Marginal Income</td>
<td>- (?)</td>
<td>+ (?)</td>
<td>*</td>
<td>- (?)</td>
</tr>
<tr>
<td>Tax Rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Opportunity Cost</td>
<td>- (-)</td>
<td>0 (+)</td>
<td>- (-)</td>
<td>- (?)</td>
</tr>
<tr>
<td>of Capital</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Appreciation Rate</td>
<td>+ (+)</td>
<td>±</td>
<td>*</td>
<td>- (?)</td>
</tr>
<tr>
<td>of Housing</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Term-to-Maturity</td>
<td>±</td>
<td>+ (+)</td>
<td>*</td>
<td>+ (?)</td>
</tr>
<tr>
<td>of Mortgage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income Growth Rate</td>
<td>±</td>
<td>±</td>
<td>*</td>
<td>+ (+)</td>
</tr>
<tr>
<td>Family Size</td>
<td>+ (+)</td>
<td>±</td>
<td>- (+)</td>
<td>*</td>
</tr>
<tr>
<td>Household's Location</td>
<td>+ (?)</td>
<td>±</td>
<td>±</td>
<td>*</td>
</tr>
</tbody>
</table>

**Note:** The first entry is the actual regression results where
* : not entered in the equation,
+ : positive and significant at 95% level,
- : negative and significant at 95% level, and
0 : not significant at 95% level.
The second entry in the parentheses is the theoretically derived predicted sign with + : positive, - : negative, and ? : uncertain.

Stay equation, are significant and positive. This result supports the proposed hypothesis that there exist a simul-
taneous relationship among the quantity of owner-occupied housing demanded, the loan-value ratio, and the length of stay.

The existence of the simultaneity among the choice variables of owner-occupied housing implies that the demand for owner-occupied housing can be correctly estimated using the simultaneous equation system. The direct estimation of only one equation in the structural system would result in a simultaneity bias in the estimated coefficients of the variables. The danger of ignoring the simultaneity in the housing decision is shown by a number of sign reversals of the estimated parameters between the reduced form and the structural form. The level of initial wealth, the marginal income tax rate, and the opportunity cost of capital in the housing demand equation have different signs in the structural form and the reduced form. In the length of stay equation, the expected opportunity cost of capital, initial wealth, the marginal income tax rate, and the income growth rate have different signs. We can also compare the estimated parameters from the structural form with those from the reduced form. Our estimates of income and wealth elasticities indicate large differences between the structural form estimates and those of the reduced form (see Table 7).

The estimated elasticities from the structural form show the impacts of income and wealth on the dependent variable
TABLE 7
Income and Wealth Elasticities

<table>
<thead>
<tr>
<th>Effect</th>
<th>Dependent Va.</th>
<th>Owner-occupied Housing Income</th>
<th>Loan-Value Ratio Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent Va.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Structural Form</td>
<td>2.19</td>
<td></td>
<td>~0.12</td>
</tr>
<tr>
<td>Reduced Form</td>
<td>0.61</td>
<td></td>
<td>~0.05</td>
</tr>
</tbody>
</table>

when other endogenous variables are controlled for. Therefore, the large difference in the estimates of elasticities between the structural form and reduced form implies that the simultaneity in housing decisions should be considered when estimating the key coefficients for housing policies such as the income elasticity of demand for owner-occupied housing.
Chapter VII

CONCLUSION

The goal of this thesis was to examine the simultaneity among the choices of the quantity of owner-occupied housing, the loan-value ratio, and the length of stay. An important point of this approach to modeling the demand for housing is the justification of the introduction of new variables, the loan-value ratio and the length of stay, into the housing demand equation. Differing from the previous studies, our model also shows that, beside a household's permanent income, its initial wealth and income profile also affect the demand for owner-occupied housing.

Estimated parameters of the loan-value ratio and the length of stay in the housing demand equation are significant and the estimated parameter of housing services in both the loan-value ratio equation and the length-of-stay equation are significant. These results confirm the proposed hypothesis that there is a simultaneous relationship among the demand for owner-occupied housing, the loan-value ratio, and the length of stay. Another implication is that a simultaneous equation model is required for estimating

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the structural housing demand equation. Our empirical results also indicate that the income elasticity estimate found in the structural housing demand equation is substantially greater than the income elasticity estimate in the reduced form housing demand equation.

Prior studies of housing demand have omitted the loan-value ratio and the length of stay from the housing demand equation. Our interpretation of that body of empirical research is that it estimates the reduced form demand for housing. Both the structural equation and the reduced form are of interest, and furthermore it is of interest to determine the difference between structural and reduced form income elasticities. Our estimated income elasticity is 2.19 in structural form and 0.61 in reduced form. The fact that our reduced form estimate is so typical of that derived in other studies suggests that the substantial difference in estimate is not due to a peculiar data set or inappropriate proxy variables.

Reduced form coefficients may change over time due to variations in any of the structural relationships. For example, a change in the exogenous relationship between the mortgage interest rate and the loan-value ratio will affect the choice of the loan-value ratio, then affect and coefficients in the housing demand equation even though the structural housing demand equation is unchanged. Early es-
imates of the income elasticity of demand for housing were substantially greater than current estimates. Reid (1962) estimated the income elasticity to be about 2.0. However, current estimates of the income elasticity center about 0.75 (Rosen (1979), Polinsky and Ellwood (1979)). A number of explanations for the decline in the estimate are possible. Because these estimates were derived in reduced form housing demand equation, it is possible that changes in the mortgage market resulted in the observed changes in the parameter estimates while the structural equation did not change. Whether the distinction between structural and reduced form relations helps to explain observed temporal variations in housing demand is an open question that can be answered by applying the structural model to data sets from different time periods.

Another example of the potential value of separate analyses of structural and reduced form housing demand concerns the variations in the estimate of household's income elasticity with the level of income or wealth. Tests of difference in income elasticities using a reduced form confound the differences (if any) in the structural housing equations with differences derived from the loan-value ratio or the length of stay equations. Wealth and income affect the choice of the loan-value ratio, and for relatively high wealth households, the downpayment constraint may no
longer binding. As long as the choice of loan-value ratio affects housing demand, differences in the sizes of effects of income and wealth on the loan-value ratio will be reflected in income or wealth elasticity estimates in the reduced form housing demand equation. If the reduced form is interpreted as a structural equation, variations in the elasticity estimates among income groups could be interpreted as reflecting some type of differences in tastes. However, the true structural estimates might not vary among groups.

Another possible use of the structural housing demand equation is in policy analysis. Assume that one goal of government is to provide some minimal level of 'adequate' housing to all households. Various types of income subsidies or reductions in the price of housing could be employed. Our model and empirical results indicates that if you can affect the choice of loan-value ratio (or length of stay), the demand for housing will also be modified. The range of contending policies is thereby expanded by including housing programs that flatten the relationship between mortgage interest rates and loan-value ratio and various types of government guaranteed mortgages. The higher income elasticity estimate in the structural equation, compared to that in the reduced form equation, indicates that these mortgage policies are complementary with income subsidies in promoting the consumption of housing services.
The simultaneous relationships among the demand for owner-occupied housing, the loan-value ratio, and the length of stay requires previous studies on the impact of tax benefit for owner-occupied housing to be reevaluated. Most of the studies on the impact of tax benefit concentrates on the effect on both the quantity of housing services demanded and the tenure choice. They calculated the effect of tax benefit on the price of housing services and applied this result to both the estimated demand equation and the tenure choice equation. Previous studies argued that a decrease in the price of owner-occupied housing due to tax benefit is roughly proportional to the marginal income tax rate. They ignored the effect of the loan-value ratio and the length of stay on the price of housing services. This study suggests that the price of owner-occupied housing depends on both the loan-value ratio and the length of stay. It also shows that the marginal income tax rate affects the choices of the loan-value ratio and the length of stay. This implies that previous studies on the impact of tax benefit ignores the effect of the marginal income tax rate on the choices of the loan-value ratio and the length of stay. This also implies that every household's tax benefit differs with its combinations of the loan-value ratio, the length of stay, and the quantity of owner-occupied housing demanded. As for the impact of tax benefit on
tenure choice, the simultaneous relationship implies that low-wealth households can enter the owner-occupied sector of the housing market by increasing the loan-value ratio and simultaneously reducing the quantity of owner-occupied housing. Given the tax benefit to owner-occupied housing, a low-wealth household takes a higher loan-value ratio to relax the downpayment burden in the house purchase. Meanwhile, a higher loan-value ratio causes a higher price of owner-occupied housing due to a higher mortgage interest rate. Under the simultaneous decision mechanism, the household can choose the loan-value ratio and the quantity of housing simultaneously resulting in a lower price of owner-occupied housing than the price of rental housing. If so, the household is expected to enter the owner-occupied sector of the housing market. Previous studies on the impact of tax benefit rule out this possibility thereby underestimating the impact of tax benefit on tenure choice.

The results of this study are derived using some restrictive assumptions and a limited database. First, households are assumed to be risk neutral and preferences are assumed to be strongly separable, simplifying the analytical model. Second, assuming that the price of housing capital and its rental rate are unique is restrictive if the housing market is not spatially integrated. Third, the assumption of a constant conversion factor of housing capi-
tal into the flow of housing services rules out any effects of the vintage of housing capital on the housing decision. Finally, dynamic aspects of housing decisions, such as the timing of a house purchase, are not considered but could be investigated if maintenance decisions and the life cycle aspect of housing demand are incorporated into the model.
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