STRATEGIC COMPETITION IN THE BANKING INDUSTRY

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By

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** * * * *

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1989
To my wife
Carol Ayogu
and to the loving memory of my aunt
Ada Mgbekw Ugoezue (19xx – 89)
ACKNOWLEDGMENTS

I express honest gratitude to Edward J. Kane, my mentor, to whom I am perpetually indebted intellectually. His "1985 Economics 821" was influential in framing this dissertation. I owe a tremendous debt to Paul D. Evans for assiduous guidance in the development of the structure of my model and during many technical difficulties. My sincere appreciation to Stephen J. Turnbull for his continuous encouragement and for letting me benefit from his mastery of strategy and conflict. I thank the entire faculty and staff of the Economics Department at the Ohio State University for contributing in various ways to this research. Gratitude is expressed to Andrew Paine, the Department's systems analyst for his invaluable assistance and for making his services available at odd times. To my family and friends for their understanding. To I.N.C. Aniebo, Ugorji Ebizie, my mother-in-law, Sarah Ogbalu, Cecilia Romero, Mary Adiego, Henry Hendler, John Small, James Gotcher, Dan Okoli, the entire "1981 staff" of the International Student Center at the University of California at Los Angeles; especially, Mia Valert, and Colette Boehm. I salute my wife, Carol for her uncommon valor. To my children, Chioma and Nworah for enduring my frequent absences, and to my dear parents for their faith in me. All errors remain my sole responsibility.
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SYNOPSIS OF THE RESEARCH

Model Inputs

1. **Objective Function:** maximizing of expected profit
2. **Decision Horizon:** one period
3. **Endogenous Variables:** Deposit and loan rates, \( R_{D_i}, R_{L_i} \) (i=1,2)
4. **Nonrandom Exogenous Variables and Parameters:**
   \( R_{D_j}, R_{L_j} \) = deposit and loan rates of a rival financial intermediary \( j \) (=1,2), \( \beta, \delta; \theta, \lambda \) = market (deposit and loan) demand parameters, \( \rho, \sigma_i \) = money-market borrowing rate, and firm-specific parameter that determines the marginal cost of borrowing, \( c_d, c_l \) = average (and marginal) real-resource cost of servicing loans and deposits
5. **Random Exogenous Variables and Parameters:**
   \( \alpha, \phi \) = sum of the intercept term and random disturbances in the demand function for deposits and loans respectively; \( \text{Ex post net liquidity position, } Z \) is known
6. **Operative Constraint:**
   Balance-sheet constraint, \( L = D + Z \) Ex post liquidity-adjustment cost that increases with the size of borrowing
Model Outputs

7. **Decisions:** $R_{D_{ij}}, R_{L_{ij}}$ (conditioned on expectations about $R_{D_{j}}, R_{L_{j}}$)

8. **Novel Features:**
   a. constrained liquidity-adjustment opportunities
   b. market structures - extension of monopoly models to duopoly model

9. **Story told: Applications**
   a. real-resource cost conditions deposit and loan rates
   b. the Nash equilibrium entails an "externality" which causes $R_{D}$ to be higher, and $R_{L}$ to be lower than otherwise
   c. illustrates the underlying structure of the problems that confront financial intermediaries in an evolving market environment of interstate banking
   d. reverses the analytic results - that optimal loan and optimal deposit policies are independent - of Sealey (for the risk-neutral case) and Klein; explains the source of the difference
   e. constrained liquidity-adjustment opportunities causes risk-neutral intermediaries to behave like risk-averse maximizers
   f. develops implications for antitrust analysis
   g. develops an analytic foundation for the empirical analysis of the "prime-rate controversy"

10. **Criticalisms or qualifications, and suggestions for further research:**
    a. need for explicitly dynamic models to study reactions, price wars, retaliations, and collusion; and possibly to formalize the concept of regulatory dialectic
    b. need for a model that incorporates explicitly the multidimensionality
of financial intermediary competition

c. extensions to test empirically the existence of conscious parallelism in the banking industry; and (2) to study the strategic incentives that influence optimal capital decisions of financial intermediaries, exploiting the analytical methods exposted in the seminal papers of Fudenberg and Tirole (1984), and Bulow, Geanakoplos, and Klemperer (1985)
CHAPTER I

Introduction

Strategic interdependence exists when, in a conflict, Player 1's best course of action depends on what he expects Player 2 to do, which 1 knows depends, in turn, on 2's expectation of 1's move. This dissertation takes the view that recent developments in the financial-services industry, particularly progress in interstate banking, has made strategic interdependence a critical factor in micro models of financial intermediaries. Accordingly, it develops a model of depository financial intermediaries that incorporates strategic interaction between these intermediaries, conditional on their marginal cost of borrowing in money markets. Our analyses focus on two points: (1) identifying strategic incentives that influence the behavior of financial intermediaries and (2) characterizing the resulting competition.

Recent advances in interstate banking have been driven mainly by technological progress, competitive financial deregulation, and deposit insurance subsidies to innovative forms of risk-bearing. Reregulation occurs whenever an inherited system of regulatory
instruments is adjusted in any way. Deregulation may be defined as the *ceteris paribus* relaxation of one or more regulatory restrictions. The concept of reregulation recognizes the fact that existing regulations never quite disappear; rather they are readjusted (by politicians and regulators) in response to political and economic pressures. Acting together, these forces have lowered the exclusionary cost of regulatory avoidance and thus promoted the fading away of lines of market cleavage that have long delimited inherited patterns of competition. Common examples of past cleavage are requirements for product-line heterogeneity such as the separation of banking and commerce (Glass-Steagall issues) and geographic market segmentation (McFadden Act and Douglas Amendment issues). This breakdown in traditional barriers and the resulting fusion of financial-services confirms the applicability to the financial services industry of the contestable market theory which makes market structure endogenous. Contestable market theory maintains that far from being an exogenous determinant of industry performance, market structure adapts through entry and exit to permit customer demand to be served at minimum cost. The number and size distribution of firms adapt to let demand be served by the most efficient producers (Baumol, Panzar, and Willig, 1982).

Tax and regulatory benefits available from forming bank holding companies help to explain the breakdown of entry barriers and the fusion of financial-services product lines. Table 1 on page 3 shows the trend in bank-holding-company formation as well as the ratio of
<table>
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<th>PERCENT OF INDUSTRY ASSETS HELD BY BHCs</th>
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<tr>
<td>1981</td>
<td>61.8</td>
<td>81.1</td>
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<tr>
<td>1984</td>
<td>78.7</td>
<td>89.7</td>
<td>88.6</td>
</tr>
<tr>
<td>1986</td>
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total assets and total deposit liabilities in the industry held by this form of banking organization.

An industry structure that is characterized by competition among a few firms is called oligopoly. The hallmark of oligopoly is the mutual recognition of strategic interdependence. Accordingly, models of financial-intermediary competition need to recognize a role for strategic interactions among incumbents. With proper modeling, this approach promises to improve our understanding of the spread and possible effects of interstate banking. Interstate banking, although considered unstoppable, evokes mixed feelings in legislative halls. These mixed feelings were expressed by the House Banking Committee in its H.R. 2707: the House was "deeply concerned that without appropriate safeguards, interstate banking could lead to an undue concentration of financial resources in the hands of a few giant banking companies." On the same issue, before a House subcommittee in 1985, the former Chairman of the Federal Reserve Board, Paul Volcker testified that:

. . . a counterargument to interstate banking has been a strong antipathy to concentration of economic power, particularly in the banking system, and a desire to maintain banking resources in significant measure under control of local banks, knowledgeable about the needs and circumstances of smaller businesses and individuals. Other traditional arguments are: that bigness will, through collusion, lead to prices in the industry that are higher than would prevail if the environment had been atomistic; and that large size ipso facto confers a 100% de facto deposit insurance guarantee because the
Federal Deposit Insurance Corporation is reluctant to liquidate (through deposit pay-offs) large insolvent institutions. With respect to the latter contention it should be explained that the "extensive de facto guarantee" argument identifies a structural distortion (imposed by structural inequities in the deposit insurance scheme) which causes departure from an efficient industry configuration.

This dissertation models interinstitutional competition in terms of a one-period duopolistic game with rivals producing similar but not necessarily identical products. In this model, rivals face uncertainties in what are assumed to be two relevant product markets: deposits and loans. Each incumbent posts both a deposit and a loan rate, observes the realization, and makes up for the possible differences between the quantity of loans demanded and deposits supplied by incurring a liquidity-adjustment cost through borrowing or investment (if deposits exceed loans) in money markets. Both players possess identical real-resource-cost functions. Although the industry is imperfectly contestable, incumbents nevertheless are assumed to have equal access to technology. They also face symmetric demand functions but are subject to differences in their liquidity-adjustment cost parameter. This cost parameter specifies the marginal cost of money-market borrowing by the firms.

In a static one-shot duopoly game, the appropriate solution concept is that of simple Nash equilibrium. The Nash equilibrium uses a no-regret criterion. It is selected because of its appealing self-enforcing property. When a Nash equilibrium strategy combination is played, no player could, in retrospect, have done better by selecting
a different strategy. Extending such a game to more than two periods produces supergames. Supergames which "was initially motivated by the theme that repetition enables cooperation," provide a formal way of focusing attention on long run average pay-off. However, the underlying economic environment is time-invariant, so that rivals are unable to make lasting commitments. "To study reactions, retaliations, price wars, and tacit collusion we require explicitly dynamic models of oligopoly." Unfortunately, progress in this area has been slow, presumably due to (among other reasons) tractability as well as lack "of a compelling method of choosing between equilibria." Despite their limitations, one-period models make it possible to explore concepts such as interdependence. Of interest is how in particular environments this interdependence can generate behavior that is alternatively relatively competitive or relatively collusive.

The rest of the dissertation is organized as follows: Chapter 2 presents a review of the literature on micro models of the banking firm. Through this review, the research objective stated in chapter 1 is fitted into the banking literature. Chapter 3 contains the formal structure of the model and links game-theory research to the banking literature. In Chapter 4 the optimization program developed in Chapter 3 is solved, and the resulting equilibrium fully characterized. Implications for antitrust and merger analysis are also developed. In order to compare the existing analytic findings in the literature to that derived here, a monopoly model within the basic framework presented in Chapter 3 is also developed in Chapter 4.
Chapter 5 presents a summary of the results from Chapter 4, qualifications to the results, and suggestions for extensions and empirical research.
CHAPTER II

Literature Survey

2.1 Introduction

Models of banking firms have developed from three distinguishable perspectives. These are the mean-variance portfolio-management approach, the firm-theoretic approach, and the simultaneity-of-function approach. Portfolio-theoretic models emphasize the role of intermediaries as institutional investors (their portfolio-management function), while firm-theoretic models stress the function of financial intermediaries as purveyors of financial services. Firm-theoretic models are rooted in the traditional theory of the firm, while portfolio models use the Markowitz-Tobin portfolio theory. The simultaneity-of-function approach endeavors to include both of the first two approaches. This class of models focuses on the simultaneity of a bank's asset and liability decisions.

Structurally, the portfolio approach explicitly incorporates risk and uncertainty but invariably ignores real-resource inputs. Such a modeling strategy would be untenable in a (firm-theoretic) framework...
that emphasizes the transactional aspects of an intermediary's functions. However, research by Sealey and Lindley (1977), and Stillson (1974) shows that results from these models are robust to the exclusion of real-resource costs. In contrast to the portfolio approach, firm-theoretic models can analyze the optimal behavior of banks as they seek to exploit market imperfections. These models incorporate real-resource costs and assume linear risk preferences.

In all three classes of models, intermediaries have been cast in one of two ways: as pure monopolists or, as authors who write in the finance tradition prefer to assume - as participants in perfect markets. For researchers who have modeled banks as subject to regulatory interference and having monopoly power in at least the loan market, this monopoly power (in a multi-period context) may be rooted in regulatory restrictions on entry or in informational advantages they possess in servicing an inherited base of loan customers. When monopoly power is assumed to exist in the deposit market, it is usually rooted in switching costs a customer faces either in developing a new banking relationship or in operating with banks that are less conveniently located. However, it bears repeating that contemporary industry reconfigurations make a new approach to bank modeling imperative. The few models incorporating liquidity-adjustment considerations have proceeded by assuming infinitely elastic (i.e., constant-cost) liquidity-adjustment opportunities, Sealey (1980), for example. But assuming constant-cost liquidity-adjustment opportunities is illogical from the capital-structure point of view. According to this view, financial distress is costly. Because
creditors know that a levered firm may fall into financial distress, they worry about the degree of leverage in a firm. Instead, this dissertation develops a simultaneity model in which incumbent financial intermediaries confront limited adjustment opportunities (i.e. liquidity-adjustment costs that rise with leverage). Both historical evidence [Simonson (1987), Wall Street Journal (1988)], and recent empirical findings by Hannan and Hanweck (1988) support the constrained liquidity adjustment-opportunities approach developed in this dissertation.

2.2 The Literature Review

This review begins with models that emphasize the simultaneity of bank decision-making, making competition for deposits and market imperfections the central features in models of the banking firm. These models include Kane and Malkiel (1965), Klein (1971), Pringle (1974), Mason (1979), Sealey (1980), Startz (1983), and Keeley and Zimmerman (1985).

Kane and Malkiel (1965) develop a simultaneous model that focuses on micro and macro analysis of credit rationing. Their model extends traditional (Markovitz-Tobin) analysis of portfolio optimization under risk-aversion by introducing risks of deposit variability. This allows them to portray the central dilemma that bankers sometimes confront in overall balance-sheet management. Part of this overall balance-sheet management involves controlling liquidity risks. The
existence of liquidity risks forces banks to recognize conceptually a class of core customers whose requests for credit accommodation lead financial intermediaries to modify their optimal asset decisions. Flannery (1982) defines core depositors as retail customers whose deposit balances are likely to remain with a bank despite market rate fluctuations. In Kane and Malkiel's model, management maximizes utility in risk-return space subject to a balance-sheet constraint and nonnegativity restrictions on loans and government securities. The model does not consider real resource costs. Both expected deposits and the quality of future relationship are conditioned on loans granted. The analysis assumes infinite leverage by banks, and purely competitive markets in securities but not in loans.

In the spirit of Kane and Malkiel (who emphasized the banker-customer relationship), Flannery (1982) explains how the notion of core deposits will lead banks to share set-up costs with depositors. The need to amortize set-up costs condition a bank's optimal decisions while on the part of the depositor, set-up costs provide strong incentives, among other reasons, to continue the relationship. Keeley and Zimmerman (1985) empirically investigate this notion of deposits as quasi-fixed factors of production in the context of money market deposit accounts.

Sealey (1980) "integrates the risk considerations of the portfolio-theoretic approach with the market conditions, cost considerations, and deposit rate-setting behavioral mode of the firm-theoretic approach." His model portrays bank decision-making as jointly determined by cost, liquidity, and risk considerations. He
demonstrates how the effect of risk aversion on optimal behavior of financial intermediaries critically depends upon liquidity and cost conditions facing the firm. He also explains why many of the conclusions of portfolio-theoretic models, which ignore deposit rate-setting behavior and resource costs, are not comparable to models that incorporate these factors. In Sealey’s model, intermediaries maximize expected utility of profits by optimally choosing a deposit rate and a quantity of loans subject to a balance-sheet constraint,

\[ L = D + Z, \]  

(1)

where \( L \) = loans, \( D \) = deposit liabilities assumed, and \( Z \) = net money market borrowing/lending. He assumes a perfectly competitive loan market, a random return on loans, a random deposit supply, and infinitely elastic liquidity-adjustment opportunities. Profit is assumed strictly concave and stated as

\[ \Pi = R_L L - R_D D - f(Z) - C_D(D) - C_L(L). \]  

(2)

Equation (2) states profit as gross revenue from loans less interest cost of deposit funds, liquidity-adjustment cost and the real-resource cost of servicing loans and deposits. There is no deposit-loan linkage in his model and some of his propositions rely on the assumed independence between the random return on loans (\( R_L \)) and the disturbance in deposit supply (\( \mu \)).
In some banking models developed before deposit-rate deregulation [Barro and Santomero (1972), Santomero (1979), Mitchell (1979), and Startz (1979, 1983)], the focus falls on how regulation creates the need for banks to pay implicit interest on deposits to circumvent ceilings on explicit rates and on ways in which this interest rate is rendered – concessionary loan rates and funds commitments in uncertainty models or concessionary provision of deposit services in models that build on the pioneering work of Michael Klein. Formally, Klein (1971) develops a monopoly model of a rate-setting intermediary. The model does not explicitly incorporate real resource costs except in relation to the payment of implicit interest on deposits. He applies his analysis to the problem of interest rate regulation by determining whether on a priori grounds, higher costs of funds are expected to induce banks into holding riskier (higher-yield) portfolio. According to Klein, this hypothesis was the original justification for deposit-rate regulation and as such warrants serious consideration. He finds no analytic support for this hypothesis in his model because "neither the cost of deposits nor the parameters of the deposit supply functions appear in the optimization condition and therefore, cannot affect asset selection." On the other hand, he finds that portfolio yield affects deposit rates. For this, he relies on (his) equations (34 & 35), reproduced below, to establish this relation:

\[ R_i = E - \frac{D_i(R_i)}{D_i'(R_i)} \quad , \quad i = 1,2, \]  

(3)
where $E_a = \text{return on assets}$, $D_i = \frac{dD_i}{dR_i}$, $D_1$, $D_2 = \text{demand}$, and time deposits respectively, and $R_1$, $R_2 = \text{deposit rates on } D_1$, and $D_2$ respectively. The model also demonstrates a longstanding intuitive proposition that banks may induce depositors to hold demand deposits by providing a positive implicit return on such deposits. Therefore, differences in effective average and marginal costs of demand and time deposits are less obvious than they appear. In Klein's model optimal portfolio structure is determined by equating the marginal return on loans and the implicit return on cash both to the expected (marginal) return on government securities. Structurally, it is a single-period model in which financial intermediaries maximize expected return on equity subject to a balance-sheet constraint. The bank distinguishes between two types of deposits (time and demand) and faces perfectly elastic (asset) market in government securities. Deposit outflow is assumed to follow a rectangular distribution.

Building on Klein (1971), but within the firm-theoretic framework, Mitchell (1979) develops a model that focuses on how regulation creates the need to pay implicit interest on deposits to circumvent ceilings on explicit rates and on the way in which this interest is rendered. By incorporating real resource costs into his model, he shows that the bank is able to subsidize intermediational products such as check clearing, funds transfers, and account maintenance cost, to encourage deposit balances. His principal analytic result is that the effect of an increase in explicit deposit rates on implicit payments is indeterminate. He also finds that when explicit
rate rises, the implicit rate does not go down equivalently due to divergence between the way consumers and banks value any given amount of implicit interest. This second finding is the well-known analytic result in welfare economics that deadweight loss will result when services offered are valued at less than cost by customers, mitigated by tax implications.⁸ Structurally, Mitchell (1979) is a model of a bank that maximizes profits by choosing an optimal amount of implicit interest (through the appropriate choice of service charges) when explicit deposit-rate ceilings are binding; or implicit and explicit rates on time deposits when ceilings are nonbinding. Profit is stated as

\[ \pi = rA - \sum_{i} (r_{d_i}D_i + [c-s]D_i), \]  

(4)

which says that profit is gross return from assets, less explicit cost of funds employed, and implicit interest on deposit services. \( c \) is the assumed-to-be-constant marginal cost of deposit services and \( s \) is the unit service charge; \( s \) is specified to be less than \( c \). Intermediaries are assumed to be price-takers in their assets market, but to confront imperfectly elastic demand for deposits. In Mitchell's model, the implicit interest paid is based on the number of transactions. A variation of this approach (adopted by Startz [1983]) is to base the subsidy on the volume of deposits.

Startz (1983) uses a microeconomic model of monopolistic competition [one that is structurally similar to Mitchell (1979)] to examine
the effect of changes in explicit deposit-rate ceilings on the level and interest sensitivity of money demand. His "central conclusion is that removal of explicit interest prohibition will force the banking industry closer to fully competitive behavior." He also obtains results similar to Mitchell that movements in explicit rates do not cause equivalent changes in implicit rates due to the way consumers and banks value any given amount of implicit interest. By using empirical estimates from his prior work, Startz (1979), he was able to constrain the theoretical parameters of his model and thus obtain qualitative predictions on the relationship between explicit and implicit rates under both regimes (binding and nonbinding ceilings). However, one must question the validity of applying the 1979 parameter estimates to 1983 banking environment.

In contrast to the firm-theoretic and the simultaneity models reviewed above, the third approach employs portfolio management models that apply EV (mean variance) analysis but treat a bank as possessing an exogenously fixed amount of loanable funds. Parkin (1970), Pyle (1971), and Hart and Jaffee (1974) are of this genre. Parkin (1970) presents an empirical investigation of the behavior of U.K. Discount Houses. The principal goal of the paper was to develop a simple but rigorously formulated model to analyze optimal debt and portfolio decisions of Discount Houses, and empirically test the predictions of the model. A novel feature of his model is that by treating liabilities as negative assets, it was able to deal simultaneously with the choice of assets and liabilities. He empirically tests the predictions derived. Although, applied to Discount Houses rather than
depository financial intermediaries, this work captures the basic idea behind this class of models, namely the optimal allocation of an exogenously determined amount of funds.

Pyle (1971) deals with the portfolio-management problem of intermediaries while abstracting from the important problems of liquidity, and real resource costs that confront these intermediaries. The principal focus of his model is to discover circumstances under which intermediation would take place. Pyle's main analytic finding is that the properties of the joint distribution of loan and deposit yields are conducive to intermediation. The covariance between the deposit and loan rates "fosters intermediation by encouraging the risk-averse maximizer to transform deposits into loans." Pyle's analysis leads him to conclude that asset and liability decisions of financial intermediaries are interdependent; a conclusion that contrasts with Klein (1971).

Hart and Jaffee (1974) develop a model that "is a direct application of portfolio theory to financial intermediaries." Like Pyle (1971), they abstract from liquidity considerations. Their two main results are: (1) that a separation theorem may be developed for a financial intermediary as is the case with individual portfolio-choice; and (2) the derivation of comparative-static properties that are stronger than the results of Parkin (1970). That separation theorem holds for a bank implies that the selected scale of the bank's operation can be segmented from its risk-return choice. However, these results are not robust to the restrictive assumptions on which they are based.
Structurally, these portfolio models have a standard format. Management maximizes a utility function concave in profits by restricting its asset choice to the efficient frontier of feasible sets of assets and liabilities of the firm. Optimality implies a point on the frontier where the marginal rate of substitution between risk and return along the objective function equals the marginal rate of transformation of risk into return ($MRT_{rE}$) along the efficient locus. This first-order equilibrium condition for utility-maximization (established at any point where the EV-efficient locus is tangent to an indifference curve) is stable since by assumption the second-order condition is also met in all these models (the utility functions demonstrate risk aversion and profit is concave in expected return).

2.3 Summary and Transition

In concluding this review, attention is drawn again to the major characteristics of these models: portfolio-theoretic models which cast intermediaries as participants in perfect markets emphasize the portfolio-management function but ignore the risk of being unable to service an unexpected surge in clearings - the kernel of an intermediary's problems - while those simultaneity models that incorporate this liquidity risk assume infinitely elastic liquidity-adjustment opportunities and cast intermediaries as either monopolists or as perfect competitors. This research promises to improve the literature
by developing a simultaneous model of mutually interdependent financial intermediaries operating in imperfectly contestable markets and confronting firm-specific constrained liquidity-adjustment opportunities. It may, thus, be viewed as integrating within an oligopolistic market structure, the risk considerations emphasized in portfolio-theoretic approach with the real-resource considerations of the firm-theoretic approach. The next chapter draws on this literature survey in presenting the formal structure of the model.
CHAPTER III

The Structure of the Models

3.1A The Behavioral Model with Uncertainty in Deposit and Loan Demands

Individual intermediaries are assumed to face stochastic demands in what are assumed to be two relevant product markets: deposits and loans. Uncertain demand is modeled by subjecting the relevant demand to an additive error term each time an output or pricing decision is carried out. Although under certainty the choice of strategy variable by a price-searching firm is unimportant, Leland (1972) has shown that it critically conditions performance under uncertainty. It is also true that within the class of static models, the precise way in which rivalry is modeled has profound impact on equilibrium behavior. When demand is random, at least three modes of behavior may be considered depending on the relative flexibility of output and/or price adjustment. These modes are quantity-setting, price-setting, and price/quantity-setting. Hart and Jaffee (1974) give an instance of price/quantity-setting to be "... [a] complicated situation of credit rationing in which the intermediary attempts to determine both
the price (yield) and quantity."

Authors writing in the finance tradition accept quantity-setting as the relevant behavioral mode. Parkin (1970), Pyle (1971), Hart and Jaffee (1974) are of this genre. These authors adopt the Markowitz-Tobin portfolio theory as their analytical tool. Although this approach allows for the explicit treatment of uncertainty, the choice of market structure is essentially one way—perfectly competitive markets. This invariably leads to quantity-setting as the relevant behavior. However, quantity-setting is not applicable, at least in the deposit market where banks in practice stand ready to issue deposit liabilities to all those that demand them at the set rates. Besides, the assumption of quantity-setting leads to the neglect of liquidity risk—the kernel of an intermediary's problems. For example, Sealey (1980) notes liquidity risk as having featured prominently in discussions of intermediary behavior. Even in the loan market, rate-setting is the prevalent behavior. Following Mills (1959), Klein (1971), Mason (1979), Sealey (1980), and consistent with the contestable-markets paradigm, rate-setting is framed in this dissertation as the relevant strategy in both the deposit and the loan markets. It should, however, be noted that Klein and Sealey adopt rate-setting strategy only in the deposit market. This dissertation assumes the absence of quantity constraints in both product markets. In the next two sections, the nature of the demands faced by these intermediaries in their two product markets is specified. The demand system derives from the utility maximization of a representative economic agent. The representative consumer subject to its budget
constraint (defined by the agent's wealth and the prevailing vector of prices) maximizes a utility function that is assumed to be quadratic and strictly concave in the numeraire good. This type of consumer maximization problem which is thoroughly analyzed in the literature [Dixit(1979), Vives (1984, 1985), Singh and Vives (1984)] yields as its solution demand systems of the type specified below.

3.1B The Deposit Market

Following Klein (1971) and Sealey (1980), financial intermediaries are assumed to face an imperfectly elastic deposit market. When market power is assumed to exist in the deposit market, it is usually rooted in switching costs a customer faces in either developing a new banking relationship or in operating with banks that are less conveniently located [Klein (1973), Benston and Smith (1976), and Flannery (1982)].

An intermediary's demand function for deposits denotes for each vector of deposit rates the expected value of the deposit liabilities assumed. The demand structure is assumed linear and symmetric. Linearity is imposed to ensure the existence and uniqueness of the Nash equilibrium. In fact, any convex demand function suffices. Symmetry is not required; but it improves the exposition by making it easier to focus attention on one of the main themes - financial intermediaries do not confront the same marginal cost of borrowing in the money markets. Uncertainty exists only about the respective quantity
intercepts of the demand functions over which the intermediaries have no control. This means that each random term is distributed independently of the deposit rates and thus a firm is unable to affect the amount of uncertainty by altering its deposit rate.

\[ D_1 = \alpha_1 + \beta R_{D_1} - \delta R_{D_2}, \quad (5) \]

\[ D_2 = \alpha_2 + \beta R_{D_2} - \delta R_{D_1}, \quad (6) \]

where \( D_1, D_2 \) = quantity of deposit services demanded of intermediaries 1 and 2;
\( R_{D_1}, R_{D_2} \) = deposit rates set by intermediaries 1 and 2;
\( \alpha_1, \alpha_2 \) = the sum of the intercept term and random disturbance for intermediaries 1 and 2 respectively.

As specified, \( \alpha_i(i=1,2) \) shifts the demand function parallel to itself in the quantity direction and each intermediary has a subjective probability distribution over \( \alpha_1 \). These expectations need not be the same across rivals but this chapter abstracts from such asymmetric information issues by assuming that rivals hold identical expectations over \( \alpha_i(i=1,2) \). Denote these expectations as \( \alpha_i^* = \alpha^*(i=1,2) \). In addition the following restrictions apply to the demand system:

\[ 0 \leq \alpha \leq \alpha_1, \alpha_2 \leq \bar{\alpha}; \quad \delta > 0, (\beta^2 - \delta^2) > 0. \]
The first part says that the random term assumes values between zero and a positive upper bound $\bar{\alpha}$, while $(\beta^2 - \delta^2) > 0$, $\delta > 0$, implies that the demand liabilities issued by the incumbents are regarded as imperfect substitutes by consumers. The first set assures us that deposits are forthcoming at positive deposit rates, while the second set states that with product differentiation, sales (deposit liabilities assumed) and profits are not discontinuous function of prices (deposit rates). The amount of deposits demanded of an intermediary is a continuous function of all deposit rates prevailing in the geographic market. There is no special significance to having all incumbents set a uniform rate. A higher rate by a rival does not result in a total loss of deposits from the other intermediaries.

1.C. The Loan Market

In this market intermediaries are assumed to offer slightly differentiated products. Some simultaneity models, Sealey (1980), for example, and all portfolio-theoretic models assume perfect asset markets. However, the assumption of perfectly competitive asset market is inconsistent with a key reason for the existence of financial intermediaries. Financial intermediaries are institutions that issue so-called direct debt of their own to finance their holdings of primary securities. The maturities, denominations, and risk characteristics of indirect debt are designed to be more attractive to most surplus-spending units than primary securities issued directly by
deficit-spending units. In effect, they repackage the primary securities they hold as a series of claims against their earnings from this debt.

In markets that are costless to enter and exit, and in which information is costless, competition among financial intermediaries for primary securities and surplus-unit financing would narrow the spread between primary securities and direct debt until it embodied the minimum cost of administering the accounts and (abstracting from deposit insurance) covering undiversifiable risks. As Pringle (1974) aptly summarized, "... in perfect markets, financial intermediaries would have no reason to exist." This research thus follows the lead of Kane and Malkiel (1965), Leland and Pyle (1977), and Diamond (1984) in suggesting that perfect information does not characterize the market for loans. In this model, intermediaries are assumed to face an imperfectly elastic demand for loans owing to taste (by consumers) for individual banks that may be rooted in relationship-specific benefits. Within an institution borrowers are viewed as a homogeneous group, and all non-interest loan terms are fixed and identical to all borrowers. The loan demand system is assumed to be symmetric:

\[ L_1 = \phi_1 - \Theta R_{L_1} + \lambda R_{L_2}, \]  

(7)

\[ L_2 = \phi_2 - \Theta R_{L_2} + \lambda R_{L_1}, \]  

(8)

where, \( L_1, L_2 \) = quantity of loan demanded of intermediaries 1 and 2;
\( R_{L1}, R_{L2} \) = lending rates of intermediaries 1 and 2;
\( \phi_1, \phi_2 \) = a measure of the intercept and random disturbances in the demand functions of intermediaries 1 and 2. Both intermediaries maintain identical expectations over these terms, i.e., \( E_1(\phi_i) = \phi^*_i \), \( i=1,2 \); and \( E_2(\phi_i) = \phi^*_i \), \( i=1,2 \); \( E \equiv \) expectations operator. The following restrictions imply bounded stochastic variability in demand, and that the products are differentiated. As those of the deposit demand system, these conditions assures that demand exists at positive loan rates and that lending (sales) and profits are not discontinuous function of loan rates.

\[ 0 \leq \phi \leq \phi_1, \phi_2 \leq \bar{\phi}, \lambda > 0, (\theta^2 - \lambda^2) > 0. \]

3.2 Ex post Liquidity-Adjustment Cost Functions

Liquidity-adjustment costs arise as a result of uncertainties that intermediaries confront in their daily operations. In this model, each incumbent posts both a deposit and a loan rate, observes the realization, and makes up for possible differences between the quantity of loans demanded and deposits supplied by incurring a liquidity-adjustment cost through borrowing or investment (if deposits exceed loans) in money markets. The firm-specific borrowing rate takes the form:
\[
\begin{align*}
    r_1(Z_1) &= \begin{cases} 
    \rho + \frac{1}{2} \sigma_1 Z_1, & Z_1 > 0, \\
    \rho, & Z_1 \leq 0,
    \end{cases} \\
    r_2(Z_2) &= \begin{cases} 
    \rho + \frac{1}{2} \sigma_2 Z_2, & Z_2 > 0, \\
    \rho, & Z_2 \leq 0,
    \end{cases}
\end{align*}
\]

(9) (10)

\(Z_1, Z_2\) are net money-market borrowings (investments if negative) of intermediaries 1 and 2. \(\rho\) is the money-market lending rate, and \(\sigma_1, \sigma_2\) (positive) are firm-specific parameters. The cost of borrowing increases with quantity borrowed. The higher the value of \(\sigma\), the more rapidly the marginal cost of borrowing rises.

3.3 Specification of the Real Resource Cost Functions

Both intermediaries have the same real resource cost of servicing loans and deposits:

\[
C_i = c_{iL} L_i + c_{iD} D_i, \quad i = 1, 2
\]

(11)

\(c_{iL}, c_{iD}\) are assumed to be strictly positive.
3.4 The Intermediaries' Objective Functions

Each intermediary is assumed to face the following simplified balance-sheet constraint:

\[ L_i = D_i + Z_i + K_i, \quad i = 1, 2, \]  

(12)

where \( L_i \) = loans of intermediary \( i \), 
\( D_i \) = deposits of intermediary \( i \). To simplify the analysis, deposit liabilities within an institution are assumed to be homogenous but differentiated from that of other institutions.

\( Z_i \) = net money-market funds borrowed/invested by intermediary \( i \).

\( K_i \) = equity capital of intermediary \( i \).

For analytic convenience, we follow Kane and Malkiel (1965) and Hart and Jaffee (1974) in fixing equity capital at zero. Accordingly, management (in this model) is constrained to distribute all profits to stockholders. Intermediaries are assumed to maximize expected profits. Risk neutrality is appropriate since the marginal rate of substitution between risk and return is not the focus of attention. The relevant maximization problem is:

\[
\begin{align*}
\max_{R_{D_i}, R_{L_i}} & \quad [\max E(\pi_i)] \\
\text{subject to} & \quad L_i = D_i + Z_i + K_i, \quad i = 1, 2.
\end{align*}
\]  

(13)
subject to:

\[ L_i = D_i + Z_i, \quad i = 1,2 \]  \hspace{1cm} (14)

and the demand systems facing the separate firms; where,

\[ \pi_i = \text{profit function of intermediary } i, \text{ and} \]

\[ \pi_i = \frac{R_L}{L_i} L_i - \frac{R_D}{D_i} D_i - Z_i r_i(Z_i) - C_i(L_i, D_i), \quad i = 1,2 \]  \hspace{1cm} (15)

with

\[ \frac{R_L}{L_i} L_i = \text{ gross revenue on loans}, \]

\[ \frac{R_D}{D_i} D_i = \text{ interest cost of deposit liabilities assumed}, \]

\[ Z_i r_i(Z_i) = \text{ cost of/revenue from money-market funds borrowed/invested, and} \]

\[ C_i(L_i, D_i) = \text{ real resource cost of servicing loans and deposits}. \]

Equation (13) says that by choosing the optimal vector of rates, a financial intermediary thus selects the optimal quantity of deposits and of loans that maximizes its total profits. Substituting (9 & 10) for \( r_i(Z_i) \), and using (14) to eliminate \( Z_i \) from (15) yields:

\[
\text{Max } E \left\{ \frac{R_L}{L_i} L_i - \frac{R_D}{D_i} D_i - (L_i - D_i) \left[ \rho + \frac{1}{2} \sigma_i(L_i - D_i) \right] - C_i(L_i, D_i) \right\}, \quad i = 1,2
\]  \hspace{1cm} (16)
To express (16) in terms of \textit{ex ante} decision variables only, equations (5) through (8) and (11) are used to eliminate $L_i, D_i$ and $C_i$ from (16) thus yielding:

$$\begin{align*}
\max E \left\{ R_{L_i} (\phi_i - \theta R_{L_i} + \lambda R_{L_i}) - R_{D_i} (\alpha_i + \delta R_{D_i} - \delta R_{D_j}) \\
- \rho(\phi_i - \theta R_{L_i} + \lambda R_{L_j} - \alpha_i - \delta R_{D_i} + \delta R_{D_j}) - \frac{1}{2}\sigma_1 (\phi_i - \theta R_{L_i} + \lambda R_{L_j} - \alpha_i - \delta R_{D_i} + \delta R_{D_j})^2 \\
- c_1(\phi_i - \theta R_{L_i} + \lambda R_{L_j}) - c_d(\alpha_i + \delta R_{D_i} - \delta R_{D_j}) \right\}. \quad i, j = 1, 2 \quad i \neq j. \quad (17)
\end{align*}$$

Equation (17) states that each financial intermediary maximizes its expected profit by making an \textit{ex ante} optimal choice of deposit and loan rates; where the expected profit is defined as the expected gross revenue from loans less the sum of expected interest cost of deposit liabilities assumed, expected liquidity-adjustment costs, and expected total real-resource costs of servicing loans and deposits.

Before employing this model to analyze the rivalry between mutually interdependent incumbents, two monopoly models that are consistent with this model's framework are presented. Doing this allows for comparison with existing models in the literature as well as enriching our understanding of the variety of behavioral strategies available to a financial intermediary.
Chapter IV

The Models

4.1 The Perfect-Foresight Monopoly Model

The models analyzed in this and the next section are derived by modifying the structure of the duopoly model developed in Chapter III. This modification is achieved by dropping the subscripts indexing the financial intermediaries, and by eliminating the cross-price effect in the demand functions for both products. Analysis of the perfect-foresight model involves examining the optimal behavior of a (monopolist) financial intermediary in the absence of demand uncertainties. Absence of demand uncertainties implies that liquidity problems are nonexistent. Therefore, the \textit{ex post} liquidity-adjustment cost function is considered only as specifying the costs of \textit{ex ante} alternative funding source. In the following maximization program we have conveniently excluded the money markets as an alternative funding source. This can easily be accomplished through appropriate quantitative restrictions on the parameters of the model. The effect of relaxing the restriction on the funding strategy is analyzed later.
in this section. We note that since this is a certainty model, the \textit{ex ante} constraint; namely that deposit liabilities assumed equal private securities purchased (i.e., loans granted), is binding. Furthermore, the \textit{ex ante} constraint coincides with the balance-sheet constraint - assets equal total liabilities and equity - given that the \textit{ex post} net liquidity position, $Z$, is always zero. That this correspondence does not always hold is exemplified by the uncertainty model developed in the next section. It is pertinent to add that excluding government securities in the investment portfolio of the financial intermediary is not unduly restrictive (investing in government securities is not the essence of banking). The quintessential bank "is an institution that both accepts transaction accounts and makes commercial loans."\textsuperscript{11}

At the beginning of the planning period, the financial intermediary by (posting) its choice of deposit and loan rates commits to a specific amount of loans which equals the amount of deposit liabilities assumed. Consequently, the intermediary's maximization program may be expressed thus:

$$
\text{Max } \Pi = R_L L - R_D D - c_d D - c_1 L, \quad \text{subject to the balance-sheet constraint}
$$

$$
R_L, R_D \quad \text{L} = D, \quad \text{and the demand system,}
$$

$$
D = \alpha + \bar{\alpha} R_D, \quad \text{subject to the balance-sheet constraint}
$$

$$
R_L, R_D \quad \text{L} = D, \quad \text{and the demand system,}
$$

$$
D = \alpha + \bar{\alpha} R_D, \quad \text{subject to the balance-sheet constraint}
$$

$$
R_L, R_D \quad \text{L} = D, \quad \text{and the demand system,}
$$

$$
D = \alpha + \bar{\alpha} R_D, \quad \text{subject to the balance-sheet constraint}
$$

$$
R_L, R_D \quad \text{L} = D, \quad \text{and the demand system,}
$$

$$
D = \alpha + \bar{\alpha} R_D, \quad \text{subject to the balance-sheet constraint}
$$

$$
R_L, R_D \quad \text{L} = D, \quad \text{and the demand system,}
$$

$$
D = \alpha + \bar{\alpha} R_D, \quad \text{subject to the balance-sheet constraint}
$$
\[ L = \phi - \Theta R_L. \]  

Equation (18) states that profit equals the gross revenue from loans, less the sum of the interest costs of deposit liabilities, and the total real-resource costs of servicing loans and deposits. Under demand certainty \( \alpha \) and \( \phi \) simply represent the intercept terms, the stochastic components being in this case identically zero. Substituting (20 & 21) into (18 & 19) for \( D \) and \( L \) respectively, and using the balance-sheet constraint to eliminate \( R_L \) from (18) yields an objective function in one choice variable:

\[
\max_{R_D} \{ \frac{\phi}{\theta} (\alpha + \beta R_D) - (1/\theta) (\alpha + \beta R_D)^2 - R_D (\alpha + \beta R_D) - \beta c_d (\alpha + \beta R_D) - \theta c_1 (\alpha + \beta R_D) \}. \quad (22)
\]

The first-order condition necessary for an optimum is

\[
\frac{\partial \Pi}{\partial R_D} = (\beta \phi / \theta) - (2 \beta / \theta) (\alpha + \beta R_D) - (\alpha + 2 \beta R_D) - \beta c_d - \beta c_1 = 0. \quad (23)
\]

The second-order sufficient condition is:

\[
\frac{\partial^2 \Pi}{\partial R_D^2} = -2 \beta (\beta + \theta) / \theta < 0. \quad (24)
\]

Optimal deposit and loan rates are respectively:

\[
R_D = \frac{1}{2(\beta + \theta)} \{ \phi - ([2 \beta + \theta] / \beta) \alpha - \theta (c_d + c_1) \}. \quad (25)
\]
\[ R_L = \frac{1}{2(\beta + \theta)} \left\{ \frac{(\beta + \theta)}{\alpha} + \theta(c_d + c_1) - \alpha \right\}. \tag{26} \]

To simplify the exposition, we employ the symbol \( c \) to represent the sum of the marginal costs of servicing loans and deposits, \( (c_d + c_1) \).

For the results to be economically interesting, the equilibrium loan rate must be positive. A sufficient condition is \( \phi > \alpha \). On the other hand, the condition \( \phi \geq \frac{1}{\beta}([2\beta + \theta]\alpha + \theta c) \) guarantees that optimal deposit rate is strictly positive for all parameter values. However this requirement is unnecessary as there is an economic interpretation for negative deposit rates: If investment opportunities are severely limited (\( \phi \) such that \( R_D < 0 \)) profit-maximizing financial intermediaries are not likely to refuse to accept deposits because in providing depository services they stand in the same position as common carriers. But results from this model indicate that profit-maximization requires that they exploit their market power by imposing charges in excess of the marginal real resource cost of servicing the deposits. In practice, however, charges would be rendered implicitly (longer teller lines, e.t.c). This behavior is of course a manifestation of the overall balance-sheet management (jointness of the asset and liability decisions).

We now examine the effect of relaxing the restriction on the funding strategy. Relaxing the constraint necessitates additional quantitative restrictions on the parameters of the model to ensure that the solutions are indeed optimal. For the perfect-foresight monopolist the existence of the money market as an alternative ex ante
funding source implies that under specific circumstances, it is optimal for it to invest (lend) part of its funds in this market in preference to making loans. One such possibility is illustrated by figure 1 on page 36. Under this scenario, the money-market lending rate is higher than the net marginal revenue from loans that equates the net marginal cost of issuing deposit liabilities, that is \( \rho > (MR_1 + c_1) = (MC_d - c_d) \). When this is the case, the effective marginal return from banking operations is specified by the curve \( \phi_{my} \). If the monopolist desires to fund entirely from the money market, the relevant marginal cost of borrowing is given by the line \( \phi_k \). A mixed funding tactics (part money-market borrowing, part deposit liabilities assumed) are also possible – specified by the marginal cost curve \( xyt \). However, as drawn, it never pays this intermediary to fund its operations from the money market. Thus the effective marginal cost of issuing deposit liabilities (net of the real-resource cost of servicing deposits) is represented by the line \( xny \). Given this configuration, profit maximization involves issuing deposit liabilities, \( \hat{D} \) at the rate \( R_D \), and lending the amount \( \hat{L} \) at the rate \( R_L \). (\( \hat{D} - \hat{L} \)) which is the difference between deposit liabilities issued and the quantity of loans extended is invested in the money market at the rate of return \( \rho \). Therefore all marginal conditions converge to \( \rho \) and, as will be discussed shortly, create the illusion of the separability of the optimal loan and deposit decisions of a financial intermediary. To rule out the nonlending strategy requires that: (1) the marginal cost of money-market borrowing be higher than the marginal cost (net of the real-resource cost) of issuing deposit
Figure 1

Nonlinear Pricing Schedule for the Perfect-Foresight Financial Intermediary
liabilities at all feasible levels of funding - i.e., levels of funding that guarantee interior solutions to the financial intermediary's optimization problem, and (2) that the money-market rate be less than the marginal return from loans (inclusive of the real-resource cost of servicing loans) that equates the marginal cost of issuing deposit liabilities (net of the real-resource cost of servicing deposits). By defining

\[ \text{mm} = \text{the financial intermediary's marginal cost of borrowing in the money markets; } \]

\[ \text{mr}_1 = \text{the marginal return from loans; and } \]

\[ \text{mc}_d = \text{the marginal cost of issuing deposit liabilities,} \]

this restriction can be summarized as

\[ \text{mm} \Big|_{Z>0} > (\text{mr}_1 + c_1) = (\text{mc}_d - c_d) > \rho = r(Z) \Big|_{Z \leq 0}, \quad (27) \]

It is interesting to note that this "non lending" phenomenon can be conceived as the *crowding out effect* in a different context than its traditional context (the fiscal-policy-efficacy controversy). The connection is rooted in the impact of Treasury-financing operations on \( \rho \). A higher \( \rho \) increases the probability (since it becomes more difficult to satisfy the condition, \( \text{mm} \Big|_{Z>0} > (\text{mr}_1 + c_1) = (\text{mc}_d - c_d) > \rho = r(Z) \Big|_{Z \leq 0} \)) of observing the "non lending" strategy. In effect, public sector borrowing requirements (PSBR) displace some of the private sector demand for loanable funds.
4.1.2 Properties of the Equilibrium

An appealing feature of the equilibrium is that the optimal deposit rate is paid net of real-resource cost of servicing loans and deposits, while the rate charged on loans is inclusive of these real-resource costs. Therefore, we have the expected result that no benefits should implicitly be transferred from banks to customers in a one-dimensional model of optimal bank behavior in the absence of binding rate ceilings.

In this equilibrium, the optimal deposit rate is positively related to \( \phi \). This is interesting because in testing empirically whether a competitive deposit market leads banks to invest in a riskier portfolio, Benston concluded that "... the interest rate on deposits offered by a banker is a function of the investment possibilities ... available to the banker, rather than the reverse."\(^{12}\) In this model, the greater the investment possibilities, \textit{ceteris paribus}, the greater will be the value of the loan demand parameter \( \phi \) and thus the value of \( R_D \), as stated in Benston's paper.

In comparison with Benston, it is noted that the characteristics of this perfect-foresight equilibrium is consistent with his findings that the "rate[s] of interest paid on demand deposits and yields on assets were functions of the nature of the market in which banks operated; rather than determinants of one another." We may also compare our equilibrium with Klein's results. Klein's result is
compared to the perfect-foresight model because the behavior of the financial intermediary in his model corresponds to that of an intermediary that sets deposit rates \textit{ex ante} with perfect-foresight. In his model, the financial intermediary sets deposit rates \textit{ex ante} such that a specified quantity of deposit liabilities, \( D \) is assumed. These funds are then allocated to a menu of assets. No \textit{ex ante} stochastic variability in deposits exists, so that there is no need to allow for \textit{ex post} liquidity-adjustment. This scenario corresponds to the perfect foresight equilibrium presented in this section; hence the comparison.

A comparison of the two equilibria reveals that both Benston's findings and the properties of this perfect-foresight equilibrium contrast with Klein's results. In Klein (1971), "neither the cost of deposits nor the parameters of the deposit supply functions appear in the optimization condition and therefore cannot affect asset selection," but he finds that the portfolio yield affects deposit rates. Klein's result follows directly from a key feature in the structure of his model - the existence (in the asset menu) of government securities in perfectly elastic supply. According to his model, all available funds are allocated among a menu of assets - government securities, private securities, and cash. Cash is held against the risk of being unable to service an unexpected surge in clearings (liquidity risk) but otherwise bears no specific return. Private securities are in imperfectly elastic supply. Given this framework, portfolio equilibrium involves setting the marginal return on all assets equal to the exogenously determined rate on government securities. The
deposit-loan linkage is, in effect, maximized out of the program, producing the illusion of independence between the optimal loan policy and the structure of the deposit market.

4.1.3 A Numerical Example

Numerical examples provided here and in the other sections serve identical purpose — to elucidate the models developed in this dissertation and to demonstrate that as a practical matter, the equilibrium values can be computed. Unfortunately the examples do not provide a means of comparing performance across different assumed market structures since these models are nonnested.

Although oligopoly fits conceptually between the extremes of monopoly and perfect competition, . . . it does not smoothly fill in the cases between pure monopoly and perfect competition. Schumpeter (1954, p. 981) notes, however, that "as we leave the case of pure monopoly, factors assert themselves that are absent in this case and vanish again as we approach pure competition," so that "the unbroken line from monopoly to competition is a treacherous guide." 14

It is also necessary to warn that the monopoly model is not a true benchmark for the duopoly model. A joint-profit maximizing model is appropriate, unless the duopolists are considered to be multi-plant monopolists. It must also be emphasized that the various assumed market structures do not a priori imply anything about conduct-performance. What this means is that because there exist circumstances in which a monopoly equilibrium pareto dominates (from a
welfare point of view) a duopoly outcome, it is tenuous to infer conduct from market structure. This idea is espoused by the contestability paradigm. As summarized in Brock (1986):

... under certain structural conditions on technology and certain technical requirements on demand, frictionless entry and exit together with equal access to technology lead to a type of competitive equilibrium with desirable welfare consequences, even though there may be only one active firm in equilibrium. A market with the foregoing characteristics of frictionless reversible entry and equal access to technology is called a "perfectly contestable market" by Baumol et al. ... .

Estimates of the real-resource costs employed in the examples were obtained from the figures provided in Benston, Hanweck, and Humphrey (1982); Benston, Berger, Hanweck, and Humphrey (1983); Startz (1979); and Rose, Kolari, and Riener (1985). The existing empirical literature on bank structure and competition does not contain any estimates of the rest of the parameters and thus is unable to provide any guidance. Table 2 on page 43 pertains to the perfect-foresight monopoly model. The parameter set contained therein is only illustrative and in no way represents an exhaustive list.

Φ and α, the intercepts of the loan and of the deposit demand curves respectively, have been scaled down and like the rest of the parameters are unit-free. The equilibrium values - optimal loan rate, optimal deposit rate, and the spread (R_L - R_D) - are expressed as rates of return per period and therefore are also unit-free. Certain values of Φ that violate the condition for strictly positive deposit rates are included in table 2 for illustrative purpose. It has already been explained that circumstances exist in which it is optimal for a
profit-maximizing financial intermediary to pay negative deposit rates. For ease of reference, we reiterate our argument as to why negative values for deposit rates are admissible for an incumbent financial intermediary. The reason traces to the fact that there is in addition to the demand for depository services by economic agents also a derived demand for deposits that comes from the lending role of depository financial intermediaries. Consequently, a circumstance in which lending opportunities are severely limited can force profit-maximizing intermediaries to pay negative deposit rates. This tactical manoeuvre can easily be accomplished if (as is in practice) competition is multi-dimensional. Table 2 as well as tables 3, 5 and 6 are separated into two sections. Each row in the first section contains a set of parameter values which generate corresponding equilibrium values given in the second section labelled appropriately. Each block of rows displays the resulting equilibrium values when the value of one parameter only is allowed to vary in the set. A caveat is in order. The difference between the optimal loan rate and the optimal deposit rate (called the spread) is only an indicator of the rate of return per dollar at the margin. It implies nothing about total profits, and in the case of the duopolists does not signify which of the incumbents earns higher total profits.
# Table 2

<table>
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<th>$\alpha$</th>
<th>$\theta$</th>
<th>$\beta$</th>
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<th>$c_l$</th>
<th>$R_L$</th>
<th>$R_D$</th>
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<td>0.0005</td>
<td>1.0000</td>
<td>1.0000</td>
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**Key to symbol used:** -do- means "ditto".
4.2.1 The Monopoly Model With Stochastic Variability in Deposit and Loan Demands

This section develops a monopoly model of a financial intermediary facing stochastic variability in loan and deposit demands. Following the same procedure as in section 4.1, this monopoly model is derived by modifying the structure of the duopoly model developed in chapter III. Doing this allows for comparison with existing models in the literature as well as enabling us to obtain some interesting comparative-static results. At the beginning of the planning period, the intermediary posts both a deposit and a loan rate; observes the realization (i.e., the quantity of deposit liabilities assumed and the quantity of loan demanded); and makes up for the possible difference by borrowing (if loans exceed deposits) or lending (if deposits exceed loans) in the money markets. This implies that the ex ante constraint — quantity of loans must equal total liabilities assumed — is non binding. We also note that ex ante funding from the money markets is not allowed. This assumption can be justified on the basis of prudent banking policy — short-funding is a risky business for financial intermediaries. Given this configuration, the expected profit can be stated as

\[ E(\Pi) = E[R_L - R_D - Z(\rho + \frac{1}{2}\sigma Z) - c_D - c_L], \quad Z > 0, \quad (28) \]

and
\[ E(\hat{\Pi}) = E\{R_L - R_D - Z\rho - c_d - c_1 L\}, \quad Z < 0. \quad (29) \]

Maximization of the objective function in the present form involves piecewise integration as well as the requirement to specify the laws of probability governing the evolution of the demand for loans and the demand for depository services. The outcome is considerable increase in analytical complexity and possibly intractability. Therefore, to expedite the analysis, we confine the optimization program to the case of \( Z \geq 0 \). In the circumstance, the relevant maximization may be expressed as:

\[ \max E(\Pi) = E\{R_L - R_D - Z(\rho + \frac{1}{2}\sigma Z) - c_d - c_1 L\}, \quad (30) \]

subject to the balance-sheet constraint

\[ L = D + Z, \quad (31) \]

and the demand system,

\[ D = \alpha + \beta R_D, \quad (32) \]

\[ L = \phi - \Theta R_L. \quad (33) \]
Equation (30) states that profit equals the gross revenue from loans, less the sum of the interest costs of deposit liabilities, the liquidity-adjustment costs, and the real-resource costs of servicing loans and deposits. Using (31) to eliminate $Z$ from (30) and replacing $D$ and $L$ with their respective demand functions, equations (32) and (33), yields:

$$\max \mathbb{E} (R_L(\phi + \rho R_D) - R_D(\alpha + \beta R_D) - \rho (\phi + \rho R_L - \alpha - \beta R_D))$$

$$= \frac{1}{2}(\sigma(\phi + \rho R_L - \alpha - \beta R_D)^2 - c_1(\phi + \rho R_L) - c_d(\alpha + \beta R_D)).$$

Equation (34) which expresses (30) in terms of ex ante decision variables only, states that expected profit is maximized by choosing optimal ex ante deposit and loan rates; where the expected profit equals the expected gross revenue from loans, less the sum of the expected interest cost of deposit liabilities assumed, the expected liquidity-adjustment costs, and the expected total real-resource cost of servicing loans and deposits.

Necessary Conditions for profit maximization are given by the following pair of equations:

$$\frac{\partial \pi}{\partial R_L} = \phi - 2 \theta R_L + \theta \sigma(\phi + \rho R_L - \alpha - \beta R_D) + \theta \rho + \theta c_1 = 0,$$

$$\frac{\partial \pi}{\partial R_D} = -\alpha - 2 \beta R_D + \beta \sigma(\phi + \rho R_L - \alpha - \beta R_D) + \beta \rho - \beta c_d = 0.$$

Second-Order Conditions are given by:
\[ \frac{\partial^2 \pi}{\partial R_L^2} = -\theta(2+\theta\sigma) < 0, \]  
\[ (37) \]

\[ \frac{\partial^2 \pi}{\partial R_d^2} = -\beta(2+\beta\sigma) < 0, \]  
\[ (38) \]

\[ \frac{\partial^2 \pi}{\partial R_L \partial R_d} = -\beta\theta\sigma < 0, \]  
\[ (39) \]

\[ \beta\theta(2+\theta\sigma)(2+\beta\sigma) - (\beta\theta\sigma)^2 > 0. \]  
\[ (40) \]

Conditions (37) through (40) constitute a sufficient condition for an optimum. By defining \( \Delta = 2\beta\theta(2+\beta\theta\sigma), \ t = (1+\beta\sigma), \ s = (1+\theta\sigma), \ a = (1+s), \) and \( b = (1+t), \) optimal deposit and loan rates may be expressed as:

\[ R_D = \frac{\theta}{\Delta} \{ \beta\sigma\phi^* - (a+2\beta\sigma)\alpha^* - \beta(ac_d + \sigma c_L) + 2\beta\rho \}, \]  
\[ (41) \]

\[ R_L = \frac{\theta}{\Delta} \{ (b+2\theta\sigma)\phi^* - \theta\sigma\alpha^* + \theta(bc_d + \sigma c_L) + 2\theta\rho \}. \]  
\[ (42) \]

For the results to be economically interesting, the equilibrium loan rate must be positive. A sufficient condition is \( \phi^* > \alpha^* . \)
4.2.2 Properties of the Equilibrium

These results indicate that optimal deposit and loan policies of financial intermediaries are strongly influenced by market conditions, real-resource cost considerations, and liquidity cost factors. They also show that optimal loan policy and optimal deposit policy are interdependent. This interdependence must be treated in the overall balance-sheet management of the financial intermediary. This model can be viewed as improving the existing literature by integrating the risk considerations emphasized in the portfolio approach with the market conditions and real-resource cost considerations of the firm-theoretic approach. Sealey (1980) develops another simultaneity model that integrates the risk considerations emphasized in the portfolio-theoretic approach with the market conditions, cost considerations, and deposit-rate-setting behavior emphasized in the firm-theoretic approach. Although the optimal deposit and loan decisions of a risk-averse intermediary is the focus of Sealey's model, his result for the risk-neutral intermediary may easily be compared with our equilibrium. For the risk-neutral intermediary in Sealey's model, optimal loan and deposit policies are independent. This independence differs sharply from the dependence featured in our equilibrium and from the analytic results of Pyle (1971). Sealey's result traces directly to his assuming infinitely elastic liquidity-adjustment opportunities.
In his model, the equilibrium condition for deposit liabilities is satisfied when the marginal cost of deposits equals the (marginal) liquidity-adjustment cost less marginal real-resource cost of servicing deposits; otherwise an intermediary can avoid the real-resource cost of depositor service by borrowing solely from the money markets. In the model, the possible difference between the deposits realized and the optimal quantity of loans chosen ex ante (at the beginning of the planning-period) is borrowed from the money markets. The equilibrium condition for the assets is met if loans are extended until the marginal revenue of loans equals the sum of the marginal real-resource cost of servicing loans and the constant-marginal cost of liquidity-adjustment.

It is therefore clear that envisaging an infinitely elastic source of liquidity adjustment creates an illusion of separability because all marginal conditions converge to it. Figure 1 on page 35 illustrates this phenomenon. Yet another implication of the assumption of infinitely elastic liquidity-adjustment opportunities in Sealey’s model concerns the robustness of the model to relaxing the assumption of a constant marginal cost of servicing loans. Once this assumption is adopted, the scale of the intermediary increases without bound. The intermediary gains a capacity to lend an infinite amount of money in the perfectly competitive asset market assumed in the model. Sealey’s model rules out this phenomenon by assuming that the marginal cost of servicing loans is increasing in the amount of loans extended. This assumption bounds the scale of his intermediary’s
operations by forcing marginal net returns from loans to become negative at a finite value for loans.

4.2.3 Numerical Examples

Table 3 contains the numerical examples for the monopoly model with demand uncertainties. A simple way of introducing table 3 is to emphasize its similarity to and differences with table 2. In terms of parameter set, table 3 differs from 2 by containing one additional parameter, $\sigma$. Sigma specifies how rapidly the marginal cost of liquidity adjustment rises. Such considerations of course, are nonexistent under perfect foresight. In choosing the values for the parameter set in table 3, additional restrictions not explicit in the model were imposed on a subset of the parameters $\phi$, $\theta$, $\beta$, $\sigma$. The values chosen for $\phi$ and $\theta$ within a certain range of values for $\theta$ are such as to generate a loan demand function that is finite. The values for $\beta$ and $\sigma$ are chosen such that the inequality $\beta \sigma > 2$ holds. This guarantees that, for all feasible levels of funding the intermediary ex ante prefers (in a cost-minimizing sense) the deposit market to the money market. This is overly restrictive but serves to rule out completely any nonlinearities in the outcome of the optimization program.
| Row | 1.0000 | 0.0005 | 1.0000 | 1.0000 | 2.1500 | 0.0005 | 0.0010 | 0.4209 | 0.1699 | 0.0251 |
| 2.   | 0.5000 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.2106 | 0.0846 | 0.1260 |
| 3.   | 0.2500 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.1054 | 0.0419 | 0.0635 |
| 4.   | 0.1500 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0633 | 0.0248 | 0.0385 |
| 5.   | 0.1000 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0423 | 0.0163 | 0.0260 |
| 6.   | 0.0050 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0024 | 0.0001 | 0.0023 |
| 7.   | 0.0006 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0005 | -0.0006 | 0.0011 |
| 8.   | 0.0005 | -do-   | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0005 | -0.0006 | 0.0011 |
| 9.   | 0.1000 | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0841 | 0.0190 | 0.0477 | 0.0109 | 0.0368 |
| 10.  | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0173 | 0.0200 | 0.0513 | 0.0074 | 0.0439 |
| 11.  | 0.0050 | -do-   | -do-   | -do-   | 3.7500 | 0.0005 | 0.0010 | 0.0025 | 0.0003 | 0.0022 |
| 12.  | -do-   | -do-   | -do-   | -do-   | 5.6700 | -do-   | -do-   | 0.0025 | 0.0003 | 0.0022 |
| 13.  | -do-   | -do-   | -do-   | -do-   | 6.0E01 | -do-   | -do-   | 0.0027 | 0.0004 | 0.0022 |
| 14.  | 0.1000 | 0.0005 | -do-   | -do-   | 2.1500 | -do-   | -do-   | 0.0423 | 0.0163 | 0.0260 |
| 15.  | -do-   | 0.0000 | -do-   | -do-   | -do-   | -do-   | -do-   | 0.0424 | 0.0167 | 0.0257 |
| 16.  | 0.0200 | 0.0005 | 5.0000 | -do-   | -do-   | -do-   | -do-   | 0.0031 | 0.0005 | 0.0026 |
| 17.  | -do-   | -do-   | 4.0000 | -do-   | -do-   | -do-   | -do-   | 0.0037 | 0.0008 | 0.0029 |
| 18.  | -do-   | -do-   | 2.0000 | -do-   | -do-   | -do-   | -do-   | 0.0058 | 0.0017 | 0.0041 |
| 19.  | -do-   | -do-   | 1.5000 | -do-   | -do-   | -do-   | -do-   | 0.0069 | 0.0021 | 0.0048 |
| 20.  | -do-   | -do-   | 1.0000 | -do-   | -do-   | -do-   | -do-   | 0.0087 | 0.0027 | 0.0060 |
| 21.  | 0.0500 | -do-   | 0.5773 | -do-   | -do-   | -do-   | -do-   | 0.0283 | 0.0093 | 0.0190 |
| 22.  | -do-   | -do-   | 0.1250 | -do-   | -do-   | -do-   | -do-   | 0.0700 | 0.0116 | 0.0584 |
| 23.  | -do-   | -do-   | 1.0000 | 5.0000 | 1.0000 | -do-   | -do-   | 0.0224 | 0.0028 | 0.0196 |
| 24.  | -do-   | -do-   | -do-   | 4.0000 | -do-   | -do-   | -do-   | 0.0219 | 0.0032 | 0.0187 |
| 25.  | -do-   | -do-   | -do-   | 2.0000 | 1.5000 | -do-   | -do-   | 0.0196 | 0.0051 | 0.0143 |
| 26.  | -do-   | -do-   | -do-   | 1.5000 | -do-   | -do-   | -do-   | 0.0199 | 0.0059 | 0.0140 |
| 27.  | -do-   | -do-   | -do-   | 0.5773 | 5.0000 | -do-   | -do-   | 0.0269 | 0.0116 | 0.0153 |

Key to the symbols used: * denotes "expected value"; -do- means "ditto".
4.2.4 The Comparative-Static Properties of the (Stochastic) Monopoly Equilibrium

Proposition 1:
Ceteris paribus, a rise in an intermediary's marginal cost of money-market borrowing (liquidity-adjustment cost) leads to a simultaneous rise in deposit and loan rates.

This result is easily understood by noting that an intermediary's marginal cost of borrowing in the money market can increase if its credit-rating is downgraded. Downgrading may trace to a variety of possible causes: increase in the variability of earnings, adverse information (such as a qualified audit report), loan losses, and anticipated or unanticipated (adverse) regulatory developments. An increase in the marginal cost of money-market borrowing, other things equal, means that the existing equilibrium has been disturbed. If such borrowing was part of the old position, that position is no longer optimal in that it seeks to raise too much funds from the money market. Re-establishing equilibrium requires that the amount of money-market borrowing be reduced. Given the balance-sheet constraint, and an unchanged loan portfolio and market-demand parameters, the financial intermediary must necessarily raise its deposit rate. In effect, imposing a tighter liquidity-adjustment constraint, ceteris paribus, causes an intermediary to become pessimistic in selecting an
expected value for its deposit liabilities, and the expected value of the amount of loan demand it can accommodate subject to its balance-sheet constraint. This pessimistic stance translates to higher deposit and loan rates. Therefore, a seemingly risk-averse behavior is observed from an assumed risk-neutral intermediary owing to the existence of constrained liquidity-adjustment opportunities.

4.3 The Perfect-Foresight Duopoly Model

This section contains an analysis of the strategic interaction between financial intermediaries in a very simple setting. A perfect-foresight model is useful in establishing perspective on the more complex stochastic model. In a perfect-foresight setting, liquidity considerations do not influence intermediary decisions, because incumbents are assumed to face nonstochastic demands for their loans and deposits. All structural parameters are assumed identical across the two rivals. The relevant maximization is:

$$\max_{R_{Di}, R_{Li}, L_i, D_i} \left[ \max E(\pi_i) \right] \quad (13)$$

subject to:

$$L_i = D_i + Z_i, \quad i = 1, 2 \quad (14)$$
and the demand system facing the separate firms given by equations (5) through (8). Under perfect foresight, and subject to the same restrictions on the pricing schedule as in the perfect-foresight monopoly model, the amount of deposit liabilities assumed by the financial intermediaries always equals the amount of loans granted. Therefore, by replacing \( L_i \) with \( D_i \), and setting \( Z_i \) equal to zero, equation (12) is transformed into:

\[
\text{Max} \quad \pi_i = (R_{L_i} - R_{D_i})D_i - cD_i, \quad i, j = 1, 2 \quad i \neq j. \quad \tag{43}
\]

Equation (43) is equation (15) modified in two ways: (i) by setting the disturbance terms in the demand systems equal to zero and (ii) by eliminating the liquidity-adjustment cost functions whose values are identically zero. Equation (43) states that total profit equals the gross margin earned on total funds employed, less the real-resource costs of servicing these funds. To obtain an expression for the gross margin, \((R_{L_i} - R_{D_i})\), the system of demand equations (5) through (8), and the balance-sheet constraint is restated as:

\[
\phi - \Theta R_{L_i} + \lambda R_{L_j} = \alpha + \beta R_{D_i} - \delta R_{D_j}, \quad i, j = 1, 2 \quad i \neq j \quad \tag{44}
\]

which leads to

\[
(R_{L_i} - R_{D_i}) = \frac{1}{\Theta}(\lambda R_{L_j} + \phi - D_i) - R_{D_i}. \quad \tag{45}
\]
The cross-price term, $R_{Lj}$, must first be eliminated. This allows the loan rate, $R_{Li}$, to be expressed in terms of the deposit rates, $R_{Dj}$ and $R_{Di}$. Using straightforward but tedious algebraic manipulations, and defining $\gamma = \lambda^2 - \delta^2 < 0$; we obtain:

$$n_i = \frac{1}{\gamma} \left( [\beta \theta - \delta \lambda - \gamma] R_{Di} + [\beta \lambda - \delta \theta] R_{Dj} \right. $$

$$+ [\theta + \lambda] \alpha - [\theta + \lambda] \phi (\alpha + \beta R_{Di} - \delta R_{Dj}) - c (\alpha + \beta R_{Di} - \delta R_{Dj}); \quad i,j = 1,2; \ i \neq j. \quad (46)$$

Equation (46) is the objective function expressed in terms of one choice set - deposit rates $R = \{R_{Di}, R_{Dj}\}$. The incumbent financial intermediaries simultaneously and noncooperatively post their deposit and loan rates. The solution to their interdependent maximization problem is a Nash equilibrium in prices. $p^* = \{R_{Di}, R_{Li}\}; \ i = 1,2 \}$ is a Nash equilibrium strategy combination when no financial intermediary has an incentive to reconsider its strategy choice if it believes that the opponent (j \neq i) will choose $p_j^*$, and it sees no reason to suppose that intermediary j will alter its choice. The Nash equilibrium is given by the solution to the following pair of first-order conditions defined by equation (47):

$$\partial n_i / \partial R_{Di} = \frac{1}{\gamma} \left( 2 \beta (\theta \lambda - \theta \delta) R_{Di} + \left( [\beta^2 \lambda - \delta \theta^2] - 2 \delta \gamma \delta \lambda \right) R_{Dj} + (2 \beta \theta + \lambda [\beta - \delta] - \gamma) \alpha \right.$$ 

$$- \beta (\theta + \lambda) \phi \right) - \beta c = 0. \quad i,j = 1,2; \ i \neq j. \quad (47)$$

The Second-Order Condition may be seen to be satisfied:
\[ \frac{\partial^2 \pi_i}{\partial R_{D_i}^2} = \frac{2\beta}{\gamma}(\beta\theta - \delta\lambda - \gamma) < 0. \quad i=1,2. \] (48)

To obtain the equilibrium deposit rates, the pair of equations defined by (47) is solved using Cramer’s rule:

\[
\begin{bmatrix}
m/\gamma & n/\gamma \\
n/\gamma & m/\gamma
\end{bmatrix}
\begin{bmatrix}
R_{D_1} \\
R_{D_2}
\end{bmatrix}
= \begin{bmatrix}
x/\gamma \\
x/\gamma
\end{bmatrix}
\]

The determinant of the coefficient matrix, \( \Delta = m \cdot n^2 > 0 \), where,

\[ m \equiv 2\beta(\beta\theta - \delta\lambda - \gamma), \]
\[ n \equiv (\beta^2\lambda - \theta^2\delta - 2\delta(\beta\theta - \delta\lambda)), \]
\[ x \equiv \beta(\theta + \lambda)\phi + \beta\gamma c - (2\beta\theta + \lambda[\beta - \delta] - \gamma)\alpha. \]

\[ R_{D_1} = R_{D_2} = \frac{x}{m+n} \]
\[ = \frac{\beta(\theta + \lambda)\phi + \gamma\beta c - (2\beta\theta + \lambda[\beta - \delta] - \gamma)\alpha}{2(\beta - \delta)(\beta\theta - \delta\lambda) - 2\beta\gamma + \beta(\beta\lambda - \theta(\delta\theta))}. \] (49)

The equilibrium loan rates for the financial intermediaries is obtained by replacing the deposit rates in equation (45) by their optimal values and then solving for the optimal loan rates. This yields:

\[ R_{L_1} = R_{L_2} = \frac{(1 + \theta - \gamma)}{(m+n)\theta\gamma} \left\{ \left( \delta\lambda[\lambda - \delta] - \beta\gamma \right)\alpha + \beta\gamma[\beta - \delta]c \right. \]
\[ - \left( \left( \delta^2 + \theta^2 - \delta\lambda - \beta\gamma + \lambda[\delta^2 - \beta\lambda] \right)\phi \right\}. \] (50)

Equilibrium loan rates are positive if and only if
\[(\delta \lambda \wedge \lambda - \delta - \delta \gamma) \alpha + (2[\beta - \delta] \gamma) c - ([\beta - \delta] [\delta + \delta^2 - \delta \lambda] - \delta \gamma + \lambda \delta^2 - \delta \lambda) \Phi) < 0.\]

A sufficient condition for this is \(\delta^2 > \beta \lambda\). This condition implies that a sure way to survival for a financial intermediary is through, among other ways, strong product differentiation, at least in the loan market. To explain this inference, we recall that
\[\delta < \beta \Rightarrow \delta^2 < \beta \delta.\]
Therefore, if \(\delta^2 > \beta \lambda\) holds, then
\[\delta^2 < \beta \delta,\]
and
\[-\delta^2 < -\beta \lambda\]
together implies
\[\delta > \lambda.\]

\[\lambda^2 \in [0,1]\] expresses the degree of product differentiation; from independent when \(\lambda^2 = 0\), to perfect substitutes when \(\lambda^2 = \theta^2\). Thus we see that a small value of \(\lambda\) implies a relatively high degree of product differentiation. It also raises the probability of observing positive equilibrium loan rates. A positive equilibrium loan rate is a necessary condition for financial viability particularly when financial intermediaries pay nonnegative deposit rates.

Economic sense requires that the equilibrium quantities of loans extended and of deposit liabilities assumed by a financial intermediary be nonnegative. This obtains if and only if the following additional condition holds:
\begin{align}
0 & \leq D_i = \alpha + \beta R_{D_i} - \delta R_{D_j}, \\
\Phi R_{D_i} & \geq \frac{\delta R_{D_j}}{\bar{\beta}} - \frac{\alpha}{\bar{\beta}}; \\
\text{and} \\
0 & \leq L_i = \phi - \Omega R_{L_i} + \lambda R_{L_j}, \\
\Phi R_{L_i} & \leq \frac{\lambda R_{L_j}}{\bar{\theta}} + \frac{\phi}{\bar{\theta}}.
\end{align}

\(R_{L_i} = \frac{\lambda R_{L_j}}{\bar{\theta}} + \frac{\phi}{\bar{\theta}}\) is the "choke price" of loans. \(\frac{\lambda^2}{\bar{\theta}^2}\) and \(\frac{\delta^2}{\bar{\beta}^2}\) \(\in [0, 1]\) denote the degree of product differentiation in the loan and deposit markets respectively; from independent when \(\lambda^2(\delta^2) = 0\) to perfect substitutes when \(\theta^2(\beta^2) = \lambda^2(\delta^2)\). In this model, \(\frac{\lambda^2}{\bar{\theta}^2}\) and \(\frac{\delta^2}{\bar{\beta}^2}\) are constrained to lie in the open interval, \((0, 1)\). This means that the products are considered as imperfect substitutes by consumers. The same condition is needed in the deposit and the loan demand uncertainties model analyzed in the section 4.4.
4.3.2 Numerical Examples

Table 4 on page 60 pertains to the perfect-foresight duopolist. As with the perfect-foresight monopoly model, ex post liquidity adjustment plays no role here. Accordingly, $\sigma$ is labelled "not applicable" in the table. A subset of the parameters, namely $\beta$, $\delta$, $\lambda$ have their values chosen so as to satisfy the sufficient condition ($\delta^2 > \beta \lambda$) for strictly positive equilibrium loan rates. The value for $\phi$, the intercept of the loan demand curve is also such that optimal deposit rates are positive. However, for illustrative purpose, two cases are shown (rows 7 and 8) in which this condition is violated. Table 4 shows only one value each for optimal deposit rate and optimal loan rate because both intermediaries possess identical equilibrium rates.
Table 4

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Key to the symbols used:
- do- means "ditto".
- n.a. means "not applicable".
4.4.1 The Duopoly Model With Stochastic Variability in Deposit and Loan Demands

This section contains analysis of the strategic interaction between incumbent financial intermediaries conditional on their marginal cost of liquidity adjustment. The rival financial intermediaries face stochastic variability in deposit and loan demands, and simultaneously and noncooperatively set their deposit and loan rates. The solution to their mutually interdependent optimization problem is a Nash equilibrium in prices. \( p^* = \{ R_{D_i}^*, R_{L_i}^* ; i=1,2 \} \) is a Nash equilibrium strategy combination when no financial intermediary has an incentive to reconsider its strategy choice if it believes that the opponent \((j\neq i)\) will choose \( p_j^* \), and it sees no reason to suppose that intermediary \( j \) will alter its choice. As stated in Chapter III, the objective of the financial intermediaries is assumed to be the maximization of expected profits:

\[
\text{Max E} \left( R_{L_i} \left( \phi_i - \Theta R_{L_i} + \lambda R_{L_j} \right) - R_{D_i} \left( \alpha_i + \beta R_{D_i} - \delta R_{D_j} \right) \right) \\
- \rho(\phi_i - \Theta R_{L_i} + \lambda R_{L_j} - \alpha_i - \beta R_{D_i} + \delta R_{D_j}) - \frac{1}{2} \sigma_i^2 (\phi_i - \Theta R_{L_i} + \lambda R_{L_j} - \alpha_i - \beta R_{D_i} + \delta R_{D_j})^2 \\
- \frac{1}{2} \sigma_j^2 (\phi_j - \Theta R_{L_i} + \lambda R_{L_j}) - \frac{1}{2} \sigma_j^2 (\alpha_i + \beta R_{D_i} - \delta R_{D_j}). \quad i,j = 1,2 \quad i \neq j
\]  

(17)
Equation (17) states that each financial intermediary maximizes its expected profit by choosing \textit{ex ante} optimal deposit and loan rates; where the expected profit is defined as the expected gross revenue from loans less the sum of expected interest cost of deposit liabilities assumed, expected liquidity-adjustment costs, and expected total real-resource costs of servicing loans and deposits. The solution to the above optimization program is the Nash equilibrium in prices given by the solution to the following pairs of first-order conditions:

\[
\frac{\partial E(\Pi_1)}{\partial R_{L_1}} = 0, \quad (51)
\]

\[
\frac{\partial E(\Pi_1)}{\partial R_{D_1}} = 0. \quad i,j = 1,2; \ i \neq j \quad (52)
\]

Equations (51) and (52) generate the following simultaneous system of four equations, which by defining

\[
s_i = (1+\theta \sigma_i), \quad t_i = (1+\beta \sigma_i), \quad a_i = (1+s_i), \quad b_i = (1+t_i), \quad v_i = \theta \sigma_i \alpha_i^* - s_i \phi_i^* - \theta(\rho + c_i), \quad \omega_i = t_i \alpha_i^* - \beta \sigma_i \phi_i^* - \beta(\rho - c_d), \quad i = 1,2;
\]

can be expressed as:

\[
-\theta \alpha_1 R_{L_1} - \beta \theta \sigma_1 R_{D_1} + \lambda s_1 R_{L_2} + \delta \theta \sigma_1 R_{D_2} = v_1, \quad (53)
\]
\[-\beta \theta \sigma_1 R_{L_1} - \beta b_1 R_{D_1} + \beta \lambda \sigma_1 R_{L_2} + \delta t_1 R_{D_2} = v_1, \quad (54)\]

\[\lambda \sigma_2 R_{L_1} + \theta \sigma_2 R_{D_1} - \theta a_2 R_{L_2} - \beta \theta \sigma_2 R_{D_2} = v_2, \quad (55)\]

\[\beta \lambda \sigma_2 R_{L_1} + \delta t_2 R_{D_1} - \beta \theta \sigma_2 R_{L_2} - \beta b_2 R_{D_2} = v_2, \quad (56)\]

Sufficient conditions for the maximization of profits require that

\[
\begin{vmatrix}
\frac{\partial^2 \pi_i}{\partial R_{L_i}^2} & \frac{\partial^2 \pi_i}{\partial R_{L_i} \partial R_{D_i}} \\
\frac{\partial^2 \pi_i}{\partial R_{D_i}^2} & \frac{\partial^2 \pi_i}{\partial R_{L_i} \partial R_{D_i}}
\end{vmatrix} \equiv \Omega_i > 0, \quad i = 1, 2
\]

\[
\frac{\partial^2 \pi_i}{\partial R_{L_i}^2} = -\theta a_i < 0, \quad (57)
\]

\[
\frac{\partial^2 \pi_i}{\partial R_{D_i}^2} = -\beta b_i < 0, \quad (58)
\]

\[\Omega_i \equiv 2 \beta \theta (2 + [\beta + \theta] \sigma_i) > 0, \quad i = 1, 2
\]

Using Cramer's rule to solve the system of equations (53) to (56) yields the equilibrium deposit and loan rates for the two financial
intermediaries. By defining groups of expressions in terms of new variables:

\[ \Delta = \frac{1}{2} \frac{1}{2} + R_{12} C_{21} + E_{21} C_{12} + B_{12} B_{21} + D_{12} D_{21} + F_{1} F_{2}, \]

\[ Q_1 = (\theta a_1 b_1 - [\theta a_1]^2), \]

\[ Q_2 = (\theta a_2 b_2 - [\theta a_2]^2), \]

\[ B_{12} = (\theta b_1 s_2 - \theta \theta^2 \delta \sigma_1 \sigma_2), \]

\[ B_{21} = (\theta^2 \delta \sigma_1 \sigma_2 - \theta \lambda b_2 s_1), \]

\[ C_{12} = (\theta \delta t_2 \sigma_1 - \theta^2 \lambda b_1 \sigma_2), \]

\[ C_{21} = (\theta \delta t_1 \sigma_2 - \theta^2 \lambda b_2 \sigma_1), \]

\[ D_{12} = (\theta^2 \theta \lambda \sigma_2 \sigma_1 - \theta \delta a_1 t_2), \]

\[ D_{21} = (\theta \delta a_2 t_1 - \theta^2 \theta \lambda \sigma_1 \sigma_2), \]

\[ E_{12} = (\theta^2 \delta a_1 \sigma_2 - \theta \theta \lambda s_2 \sigma_1), \]

\[ E_{21} = (\theta^2 \delta a_2 \sigma_1 - \theta \theta \lambda s_1 \sigma_2), \]

\[ F_1 = (\delta \lambda s_1 t_1 - \theta \delta \lambda \sigma_2), \]

\[ F_2 = (\delta \lambda s_2 t_2 - \theta \delta \lambda \sigma_2), \]

we have that:

\[ R_L = \{(Q_2 \theta \theta a_1 + B_{21} \delta \theta a_2 - E_{21} \delta t_2)w_1 + \sum_{D_{21} \delta \theta \sigma_1 - F_{21} \delta \theta \sigma_2 - C_{21} \theta \theta a_1 + F_{21} \delta t_2)}v_1 + \sum_{B_{12} \theta a_2 - C_{21} \theta \theta a_2 - Q_2 \theta \theta a_1}w_2 \}/\Delta, \quad (59) \]

\[ R_D = \{(E_{21} \theta \lambda \sigma_2 - B_{21} \lambda s_2 - Q_2 \theta a_1)w_1 + \sum_{E_{21} \theta \theta a_1 + F_{21} \lambda s_2 - D_{21} \theta a_1}w_2 \}/\Delta, \quad (60) \]
\[ R_{L2} = \{(Q_1 \beta \sigma_2 - B_{12} \delta \sigma_1 - E_{12} \delta t_1)w_2 - (D_{12} \delta \theta \sigma_2 + F_2 \delta \theta \sigma_1 + E_{12} \beta b_2)w_1
\]
\[ - (D_{12} \delta t_1 + C_{12} \delta \theta \sigma_1 + Q_1 \delta \theta) v_2 - (B_{12} \beta b_2 + C_{12} \beta \theta \sigma_2 - F_2 \delta t_1) v_1 \}/\Delta, \quad (61) \]

\[ R_{D2} = \{(E_{12} \delta \lambda \sigma_1 + B_{12} \lambda s_1 - Q_1 \delta \theta a_2)w_2 + (E_{12} \delta \theta \sigma_2 + F_2 \lambda s_1 + D_{12} \delta \theta a_2)w_1
\]
\[ + (Q_1 \delta \theta \sigma_2 + C_{12} \lambda s_1 + D_{12} \delta \lambda \sigma_1) v_2 + (C_{12} \delta \theta a_2 + B_{12} \delta \theta \sigma_2 - F_2 \delta \lambda \sigma_1) v_1 \}/\Delta. \quad (62) \]

**4.4.2 Properties of the (Stochastic) Duopoly Equilibrium**

The equilibrium is characterized by a price dispersion - the firm with the lower liquidity-adjustment cost charges a lower lending rate and pays a lower deposit rate than the high-cost rival. This price dispersion in equilibrium is due to the different liquidity-adjustment costs that confront these intermediaries. Symmetric choices cannot be an equilibrium. This is because the values differ that satisfies the marginal conditions for increasing and/or decreasing loans and deposits across rivals. Rivals consequently cannot be satisfied at the same deposit and loan rates.

Another way to characterize the equilibrium is by asking what does the equilibrium maximize? The answer to this question is neither industry profits (since this Nash equilibrium does not replicate collusion) nor Bergsonian social welfare (since prices are not equated to marginal costs). The Nash equilibrium does not replicate collusion because it attains neither the joint profit-maximizing outcome (cartel
profits) nor the monopoly equilibrium. When each financial intermediary maximizes its own profits, given its rival's prices \( \max \Pi_i(p_i | p_j, \sigma_i) \), \( p = \{ R_{L_i}^i, R_{D_i}^i \}, i, j = 1, 2, i \neq j \), the outcome cannot be maximal overall profits. Decreases (increases) in a single bank's lending (deposit) rate have a negative effect on its rival's profits - production externality. This "externality" causes the equilibrium to entail lower lending rates and higher deposit rates than does the purely collusive outcome.

4.4.3 The Prime Rate

The inability of the financial intermediaries to achieve the collusive outcome as a noncooperative equilibrium reflects the underlying Prisoners' Dilemma structure of the problem; each firm has an incentive to defect from collusion by chiselling and all incumbents end up with lower profits due to these defections. However, the existence of this "externality" provides incentives for incumbent financial intermediaries to improve upon what the firms consider to be an unsatisfactory outcome - bad equilibrium. One of the ways in which they seek to avoid this outcome is by the use of focal-point pricing. A focal point is an obvious benchmark by which prices or output could be tacitly coordinated such as the banking industry's prime rate and euromarkets' LIBOR (London inter-bank offered rate). Through the prime rate, the pricing tactics of financial intermediaries can be
tacitly coordinated within broad ranges. Incumbent financial intermediaries use their understanding of the industry to make decisions and anticipate the behavior of rivals.

What is this "prime rate" and what makes it appealing as a focal point? The answer to the first part of this question is furnished by providing a sampling of the definitions gathered from a survey of the ten largest U.S. banks in 1981.\textsuperscript{18} An updated ranking of these institutions as published in the American Banker of 9/27/88 is in Table 5 on page 71. At Bank of America National Trust and Savings Association of San Francisco, California "the prime rate means Bank of America's 'prime rate for 90 day loans for substantial commercial borrowers as from time to time in effect'". Security Pacific Corporation of Los Angeles, California provides the following bewildering definition:

In this context then, the "prime rate" would better be described as the rate offered to the broadest range of the most credit worthy business customers for the widest range of business needs and for which the source of money is the most widely available and least expensive pool of funds at the time.

Since most commentators dislike complexities and where at all possible will choose more simple descriptions, I don't think this definition will catch on. But there are no truly simple answers to complex questions.

The fact, as Kane plainly stated, is that:

It does not take an economics degree to see that, as the lowest interest rate at which banks admit to making loans, the prime rate represents an implicit agreement among banks to limit their competition for the biggest and best (i.e., the 'prime') customers.
Since October 1979, Morgan Guaranty Trust Co. of New York has adopted the following definition of the prime rate: "The 'Bank's Prime Rate' shall mean the rate of interest publicly announced by the Bank in New York City from time to time as its Prime Rate." Officially, both Citibank of New York and First National Bank of Chicago have no Prime Rate. Manufacturers Hanover Trust Company of New York although admitting to the use of Prime Rate has no definition of it.

Fernand J. St Germain, the former Chairman of House Committee on Banking, Finance and Urban Affairs is of the opinion that the response from these institutions:

... clearly establishes that the "prime rate" which at one time stood for the absolute lowest available business lending rate has gone the way of the bankers' green eyeshades. The prime rate has been so often misused, abused, and tortured in recent years that the phrase now seems beyond repair. ... Thanks to the actions of the banking industry, the prime has little meaning today and it should be consigned to the junkyard of abused banking terms.

The lack of consensus in the industry over the definition of prime rate does not seem to persist when it comes to quoting numbers for the prime rate. Consider the fact that Citibank has no prime rate. Its response to the House request for their definition of prime rate is that it uses a base rate 'which definition is not the definition of "prime rate" . . . quoted in your [House] letter'. Yet the same Citibank posted a base rate of 14% at the end of May, 1981; the same figure as the prime rate announced by Bank of America, Chase Manhattan, Morgan Guaranty, Chemical Bank, and First National Bank of Chicago. For First National, the 14% was their corporate base rate
(since they do not have prime rate). First National defines their base rate as "that rate which we announce from time to time." That disparity in words does not prevent consensus in numbers comes as no surprise. Semantics is by no means the essence of focal point pricing. What incumbents seek is to generate clues for coordinating behavior, creating a focal point for "each player's expectation of what the other expects him to expect to be expected to do." Schelling aptly characterizes the essence of focal point: "Any key that is mutually recognized as the key becomes the key." According to the Wall Street Journal:

The prime we use is based on the rates charged by the top 20 commercial banks in the U.S., ranked by assets according to American Banker, a trade publication. We show one rate only when all 20 banks agree. If just one bank posts a different rate, we will show a split rate--9\(\frac{3}{4}\%)--10\%--for example.

The following excerpt taken from the Staff Report to the House Committee on Banking, Finance and Urban Affairs underscores the power of information and communication in tacit bargaining, and may also explain why the prime rate is appealing as a focal point:

The phrase "prime rate" took on a formal meaning in 1933 when the industry adopted a 1 1/2 percent rate for its most preferred customers, a move prompted, in part, by efforts to stabilize the industry and to dampen what some felt could be excessive efforts to compete for the limited number of truly "prime" customers remaining in the economy. Prior to 1933, many banks did post prime rates but these were not publicized and tended to vary across the nation [emphasis added].

The prime rate obviously is entrenched in tradition. As originally perceived by the public, the prime as the rate charged to the most
trustworthy corporate customers is appealing because data on corporate credit ratings is relatively readily available. The criteria seemed identifiable and objective — desirable characteristics of a focal point. Schelling (1980) is an excellent treatment (with vivid illustrations) of the characteristics of a focal point.
### Table 5

15 Largest U.S. Bank Holding Companies Ranked by Assets as at 6/30/88

<table>
<thead>
<tr>
<th>Rank</th>
<th>Company Name</th>
<th>Location</th>
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<tbody>
<tr>
<td>1.</td>
<td>Citicorp, New York</td>
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<tr>
<td>2.</td>
<td>Chase Manhattan Corp., New York</td>
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<tr>
<td>3.</td>
<td>BankAmerica Corp., San Francisco</td>
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<td>5.</td>
<td>Chemical Banking Corp., New York</td>
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<td>7.</td>
<td>Manufacturers Hanover Corp., New York</td>
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<tr>
<td>8.</td>
<td>First Interstate Bancorp, Los Angeles</td>
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<td>9.</td>
<td>Bankers Trust New York Corp</td>
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<tr>
<td>10.</td>
<td>Wells Fargo &amp; Co., San Francisco</td>
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<td>11.</td>
<td>First Chicago Corp.</td>
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<td>12.</td>
<td>PNC Financial Corp., Pittsburgh</td>
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<tr>
<td>15.</td>
<td>Mellon Bank Corp., Pittsburgh</td>
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</table>

On the question of the power of information and communication in tacit bargaining, it bears repeating that albeit not a prisoners' dilemma game, the duopoly-pricing equilibrium of these financial intermediaries shares the same basic structure as the classic Prisoners' Dilemma game (and as in the classic game, each financial intermediary confesses). This confession takes the form of setting deposit and loan rates conditional on the firm-specific marginal cost of liquidity adjustment, an equilibrium strategy that is certain to be modified if some form of communication is allowed. "Although the prisoners' dilemma is not a faithful representation of a price game, it contains some elements that illustrate price competition."\(^{20}\) The essence of the Prisoners' Dilemma is that self-interest leads to a pareto-inefficient outcome (in that the gain to the suspects from both of them not revealing is unambiguously better than the payoff when both of them reveal). The classic game is recalled here for ease of reference.

Two suspects to a crime (with private information as to the details of the crime) are segregated without the ability to communicate with each other or to acquire information on the actions of the other. The following proposition from the district attorney is understood by both suspects: (i) if both reveal, they get a light sentence; (ii) if suspect A informs, he is released and recompensed, while B gets a long jail term, and vice versa; and (iv) if none reveals, the two suspects are simply released.
It is also the case that in some (real-world) conflicts - games of economic survival - participants avoid mutually unfavorable outcomes while in others they do not. Which outcome is likely to emerge depends, among other things, on the amount of information available to participants and opportunities for communication. It is, therefore, remarkable that in the financial-services industry, not only do the media herald changes in "the prime rate;" technological advances in communication and information systems simplify the task of mutual monitoring of actions by rivals. Presently, all that is necessary to monitor instantaneously (i.e., in real time) a competitor's rate is to subscribe to one of the many specialist wire services (TELERATE, for example). Part of the appeal of the prime rate as an obvious benchmark must, therefore, be rooted in the attention it commands, which facilitates some form of limited communication - an essential ingredient in resolving conflict.

A. Testing Hypotheses about the Prime

The analytic properties of the (noncooperative) duopoly equilibrium derived in this dissertation together with the preceding analysis of the prime rate enable us to develop a testable theory of the prime rate as an instrument of conscious parallelism in the banking industry. We may hypothesize that a focal-point pricing mechanism is operative in the banking industry. In this dissertation, this hypothesis is called conscious parallelism in the banking industry.
Note that the model does not predict conscious parallelism in the banking industry. Characterizations of the resulting competition follow from the properties of the Nash equilibrium and institutional analyses of the industry. Yet such characterizations generate refutable theories about the nature of the rivalry (conduct) of firms in the industry.

The null hypothesis that the prime rate is a focal-point price can be tested against the alternative that it is not, using either industry (macro) or individual-firm (micro) data. The null hypotheses for the test using industry aggregate data is based on the analysis that compete on their marginal cost curves (i.e., they all confess) unless there is some kind of (limited) communication. If the hypothesized role of information is correct, then a hypotheses based on the predicted differences in outcome between a state in which the players communicate (a sort of coordination) and that in which they remain incommunicado is valid. The proposed null hypothesis is that "ceteris paribus the dispersion of interest rates on loans based on the marginal cost of money-market borrowing (liquidity-adjustment) is higher than the dispersion of interest rates on loans indexed to the prime rate". For a test using micro data, the hypothesis may be based on the written response to a Congressional Committee inquiry of the ten largest U.S. banks in 1981. A photocopy of the letter is on page 74. A section of the questionnaire sent to the banks by Fernand St Germain specifically requested each respondent to explain how its prime rate is determined. Based on the response from the few banks that chose to address this question, the general impression is that
IDENTICAL LETTER TO CEO'S OF NATION'S TEN LARGEST BANKS

Mr. C. J. Medberry, Chairman
Bank of America
P. O. Box 37600
San Francisco, California 94137

Dear Mr. Medberry:

I am concerned about the widening credibility gap between the public announcement of changes in commercial banks' prime lending rates and actual day-to-day lending practices. I believe that it is a fundamental requirement for efficiency, equity, and free competition in a market that all participants have accurate and complete information about market prices.

Federal Reserve surveys indicate that upwards of two-thirds of the business loans made by large commercial banks in New York City were, at times last year, at interest rates below the publicly announced prime rate. I am informed that in May 1980 after the prime rate hit 20 percent in April the average interest rate charged on these loans was, in fact, more than four full percentage points below that advertised as the prime rate.

As you know, the phrase "prime rate" has gained wide acceptance in our vocabulary and, in fact, Webster's Dictionary defines the phrase thusly: "An interest rate at which preferred customers can borrow from banks and which is the lowest commercial interest rate available at a particular time". (Emphasis added)

The Federal Reserve survey clearly established that the prime rate, as announced by the commercial banks, is not the "lowest commercial rate available" as Mr. Webster and the American public have been led to believe.

In these inflationary times and in a period of crushing interest rates, I think it is highly important that banking corporations be precise, accurate and extremely careful in conveying information to the public about their corporate policies and activities.

It is a matter of record that news commentators and financial writers seize upon every prime rate announcement as a major indicator, often suggesting that the prime eventually affects every rate in the land from the finance company to the department store credit sales. These widely heralded announcements of a prime that is not a prime can only help add to the inflationary high interest psychology of the nation, particularly when we are talking about double digit rates and then some.
The misleading nature of these prime rate announcements is highly unfair to the consumer and the small businessman. They cannot afford to assign personnel to shop for the best discount from the advertised prime rates that different banks are willing to negotiate.

What is the small store owner, seeking a loan to remodel, to think when he is told by Walter Cronkite that the very best rate to the blue ribbon, Triple A commercial borrower is a prime of 20 percent? Isn't he at a distinct disadvantage when he sits down to negotiate with his local lender? Shouldn't he have the knowledge that the prime is not 20 percent, but in reality 16 percent? Perhaps your more sophisticated borrowers are well aware that the prime rate is not the prime rate, but the small businessman and the consumer are none the wiser and most are in full belief that the commercial banking industry's prime rate announcement is the real thing.

Even more important is the fact that many loan contracts across the nation are tied to the prime rate, with the rates moving up and down with the announcements of the money center banks. What is the status of these contracts when the de facto prime rate, as established by the Federal Reserve, is some four percent less than the publicly announced prime?

In addition to these specific contractual ties to the prime rate, many lenders informally adjust their rates in line with the prime rate announcements. It is difficult to estimate the total impact that these highly visible rates have on the economy as a whole, but I am convinced that it is substantial.

In this time of deregulation, I hesitate to suggest new statutory and administrative remedies. Frankly, I would like to think the banking industry, itself, would be concerned and would make a voluntary effort to make certain that its announcements are accurate and that the public can depend on what Mr. Webster suggests is the correct definition of a prime rate.

You are a leader in your industry. I need your help in remedying the problem caused by the present use of the prime rate. Your views and suggestions would be very helpful. Also, as an important guide in clarifying the present use of the prime I am asking your bank, along with some other large banks, to answer the enclosed questions. This kind of information will go a long way in informing the American public about the nature of the prime rate.

Sincerely,

[Signature]

Fernando J. St Germain
Chairman

Enclosure
SIX QUESTIONS ON THE PRIME RATE

1. Does your bank use a bank lending rate which you call your "prime rate" or an equivalent thereof? If so, exactly how is that rate defined? Is the rate stated publicly?

2. How does your bank set that rate? What officer or group of officers has responsibility for determining the rate?

3. Does your bank give loan customers discounts from the prime rate? If so, on what basis are these discounts given? Who has authority in the bank for granting discounts from the prime? Is any class of borrowers — with respect to size of borrowers or the type of business involved — more frequently given the discounts?

4. Please supply a statistically valid sample (using a sample of 100 or less) of all domestic commercial and industrial loans as reported under the Uniform Report of Condition provided to the Federal Financial Institutions Examination Council (without disclosing the identity of the borrower) made during May 1980 and January 1981. Please state the size of the loan, final maturity, and interest rates charged. Also please state what your bank's prime rate was during these two months.

5. Are your commercial and industrial loan customers informed of the range of interest rates charged different customers?

6. Does your bank have domestic commercial and industrial loans on which the interest rate floats with the prime rate, or with some other rate which is agreed upon in advance? Please describe the nature and extent of these loans as a percentage of your domestic commercial and industrial loans.

Please return your answers within four weeks.

individual banks set their prime rate (also known as, corporate base rate, and as reference rate) conditional on, inter alia, their "[marginal] cost of funds." Accordingly, the proposed hypotheses involves comparing (across institutions in the same geographic market) the marginal effect of changes in an individual bank's liquidity-adjustment cost of funds on its prime rate. The null hypothesis may be constructed to test that "for banks within the same geographic (product) market, ceteris paribus, changes across time in each bank's prime rate are caused by changes across time in each bank's liquidity-adjustment cost." These proposed tests (involving either or both micro and macro data) constitute a departure from existing empirical test-procedure of the prime rate in the literature. Previous tests invariably concentrated on testing (using aggregate data) whether or not the prime rate is competitively determined. According to these researchers, to reject the hypothesis that the prime rate is competitively determined implies that the prime rate is an "oligopolistic price," a term used in the context to refer to the presence of conscious parallelism. Unfortunately, rejecting the hypothesis that a particular price is competitively determined does not render it an "oligopolistic price." A null hypothesis of a competitively determined price when rejected could imply a monopolistically determined price unless the structure of the test is careful to exclude this alternative. The point we make here is simple. In economic science, markets are either perfect or imperfect. But imperfect markets can be either monopolistic or oligopolistic. Therefore, tests to determine
whether a particular conduct is within the realm of perfect or imperfect market is more useful as a policy guide if it can distinguish between what truly are three different market structures. It is also true that these tests—Mukherjee and Wade (1978), Goldberg (1982, 1984)—are ad hoc (as they are not based on any optimizing model of financial-intermediary behavior).

Mukherjee, and Wade address the following question, "Did a less than competitive situation exist in the process of prime rate determination before the floating prime was adopted?" They provide an answer to this question by comparing "the responsiveness of the prime rate to free market forces in two successive periods—January 1968 through October 1971, and November 1971 through December 1974." The proxy variable for "free market forces" is the commercial paper rate. Mukherjee and Wade conclude that if the commercial paper rate is considered a reasonable reflection of "the free money market forces," the second period characterized by the floating prime rate is considered to be more competitive than the (first) period when the prime rate was less flexible. Their study is perplexing in that it purports to determine the competitiveness of the prime rate through comparison with some market-determined interest rate. Instead, it tests the behavior of the prime across two regimes.

Goldberg (1982) empirically analyzes "recent prime movements in order to determine if the prime reacts immediately to changes in money market conditions, as would be the case of a competitively determined price, or whether the prime demonstrates properties of an oligopolistic price" — the stated hypotheses. The test hypotheses is that the
prime rate is determined by the marginal cost of funds against the alternative that it is based on the average cost of managed liabilities. What is doubtful is that this hypothesis test as constructed actually tests the stated hypothesis. Besides, average-cost pricing is not limited to oligopolists. It is used by monopolists. It can also exist outside imperfect markets. In perfectly contestable markets, for instance, a long run competitive equilibrium (sustainable industry configuration) may involve one active firm charging a price that is equal to average cost.

Goldberg (1984) basically deals with the same issue as Goldberg (1982):

Commercial banking's institutional setting can make one bank's profits dependent on the pricing strategies of its rivals. . . . widely disseminated prime rate quotes, . . . rule-of-thumb pricing techniques can result in prime rate outcomes that jointly maximize banks' market values. . . . the relationship between the prime and money market rates is examined over the last decade to determine if the prime rate behaves more like a competitive money market rate than an oligopolistic price.

The period covered in the study was January 1972 through December 1981, while Goldberg (1981) covered January 1975 through October 1980.

Further insight from the properties of the Nash equilibrium pertains to the use of aggregate data to test for whether or not the prime rate is competitively determined. Such tests implicitly assume a symmetric duopoly-pricing equilibrium in the sense of \( \sigma_1 = \sigma_2 = \sigma \) (i.e., all financial intermediaries are subject to an identical firm-specific liquidity-adjustment cost parameter). The effect of such an assumption is to bias the test by assuming away the real issue - is the degree of
observed price dispersion in the particular market justified by the observed underlying cost differentials?

Besides pricing strategies, financial intermediaries can also improve upon the "bad" equilibrium by adopting whenever possible strategies that mitigate some of the forces that can sabotage collusive arrangements. Take for instance, fringe suppliers that have always been a problem to producer-cartels. In our context, the recent wave of mergers in the banking industry can be interpreted partly in terms of a desire to achieve overall operating efficiency — namely, cost efficiency and market-structure efficiency. Cost efficiency in terms of scale economies, scope economies, and deposit insurance subsidies. Deposit insurance subsidies grow with size of the institution because of differential forbearance which lowers deposit rates for the large institutions. This differential forbearance takes the form of an extensive de facto deposit insurance guarantee versus de jure limited guarantee of $100,000 per individual account in each insured institution. The forbearance traces to the asymmetric insolvency resolution procedure of the federal deposit insurance corporations — FDIC and FSLIC. In other words, these agencies are more likely to use the pay-off method (which involves shutting down an insolvent institution, and paying off its depositors up to the legal limit of $100,000) when the insolvent institution is not large. The record has been to employ the deposit-assumption method (which involves only a change in the ownership of an existing firm) for insolvent insured large institutions.22 When insolvent institutions
are not shut down, depositors of funds in excess of the legal limit of $100,000 are implicitly covered.

Figure 2 on page 83 illustrates the strategic incentive to achieve market-structure efficiency. What are measured on the axes are the profit levels of mutually interdependent financial intermediaries. The direction of the arrows indicate increasing profit levels for each of the intermediaries. The entire space inside the profit possibility frontier represent various levels (depending on the respective values of the rivals' structural parameters) of feasible Nash equilibrium profits. The segment of the line $\Pi_1 = \Pi_2$ within the profit possibility frontier is the locus of equal levels of Nash equilibrium profits. The profits possibility frontier represent limits of profits achievable by mutually interdependent financial intermediaries even when they engage in a joint profit-maximization program, provided no side transfers are allowed. Levels of profits outside the profit-possibility frontier are only possible with joint profit maximization subject to side payments. $\Pi^*$, the maximal industry profit level is also a Nash bargaining solution.

That financial intermediaries are able to improve upon the "bad" equilibrium is easily understood as the operation of the "turpsy-turvy" principle of tacit collusion - "anything that makes more competitive behavior feasible or credible actually promotes collusion". If overt communication were allowed, rivals would be embroiled in bargaining over market shares and profits which would
Figure 2

Profit Levels of Financial Intermediary 1

Profit Possibility Frontier

Cartel Profits (Egalitarian)

Multiplant Monopolist (Joint-Profit Maximization with side payments legal)

Cartel Profits (Non-Egalitarian)

Max Equilibrium Profits

Profit Levels of Financial Intermediary 2

STRATEGIC INCENTIVES
precipitate a breakdown and thus lead to price wars (one of the reasons for the instability of collusive arrangements). In the absence of overt communication, their overriding interest is to achieve a consensus. Therefore, solutions reached in this case are more stable (i.e., likely to persist). Even those with cost advantage do not flex their muscle. Note that in situations such as this, "win" and "lose" may not be quite accurate, since both may lose by comparison with what they could have achieved through tacit communication. This suggests that in a local market in which a large bank competes with a small one, the outcome may not be different from a symmetric rivalry if the local market is contestable - a result that addresses an issue of key concern to interest groups and policy makers in the new environment of interstate banking. Although entry into the banking industry is neither ultra free nor costlessly reversible, entry from allied industry via product substitutes serves to increase the degree of ex post contestability in the industry. For example, depository institutions face competition from nonbank financial intermediaries that provide limited checking-account services; from finance companies, and from insurance companies that provide loans; from mutual funds that provide denomination intermediation services; and from corporate treasurers that increasingly resort to direct financing in the credit market. Recent ranking between Citicorp and Western Union over money transmission (liquidity intermediation) underscores this threat from allied industry. 

Ironically the concern of the small depository institutions and their interest groups (including regulators) is possibly misplaced.
Aggressive pricing (or cut-throat competition) from large competitors may not undo local banks as feared. The real source of small or local banks' demise in the future may lie in hostile or friendly mergers or acquisitions. For although the number and size distribution of firms serves to minimize the cost of serving the industry demand, once in equilibrium, the nature of the strategic incentives that confront these intermediaries (in markets that are imperfectly contestable) make mergers and acquisitions welfare improving for firms active in equilibrium.

### 4.4.3 Numerical Examples

Table 6 (on page 87) pertains to the duopoly model with uncertainties in the deposit and loan demands. Several remarks are in order. First, it should be reiterated that the spread merely indicates the difference between the optimal loan and deposit rates at the margin without providing any information on the relative level of total profits for the incumbent rivals. Second, note that a relatively lower value for sigma implies an *ex post* liquidity-adjustment cost advantage, i.e. $\sigma_1 < \sigma_2$ implies that financial intermediary 1 is the low-cost rival. Third, observe that the loan rates for the two incumbents are displayed next to each other to facilitate comparison. Similarly for the deposit rates and the respective spreads. This contrasts with the previous habit of stating the strategy variables of
each rival \( p_i = \{R_{Li}, R_{Di}, i=1,2\} \) together as a set. We have also constrained \( \sigma_i > 2 \) (i=1,2) so as to ensure that deposits are the ex ante preferred funding mode for all feasible funding levels.

Noteworthy is the derived-demand property of deposit funds, previously noted in connection with the monopoly model, which has carried over to the duopoly model; namely that if investment opportunities are limited, profit maximization dictates that financial intermediaries charge for deposit liabilities assumed. It must be emphasized that this scenario is theoretically in the nature of a corner solution, and empirically highly improbable. It is pointless to argue that Treasury securities exist in infinitely elastic supply. If financial intermediaries must perforce pay negative deposit rates, it is very unlikely that in the prevailing circumstance the Treasury can absorb the entire supply of deposits. For while an individual financial intermediary confronts an infinitely elastic supply of government securities, such is not the case for the entire banking system. Lastly, we note that table 6 shows the low-cost intermediary charging lower loan rates and paying lower deposit rates for all values of the parameter set therein. This observation is consistent with the equilibrium properties of the model.
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<th>Row</th>
<th>J</th>
<th>a</th>
<th>θ</th>
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<th>Expected Values of the Equilibrium rates</th>
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<td>do</td>
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<td>do</td>
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Key to the symbols used: * denotes "expected value"; do means "ditto"; si = (E_i - R_i), i = 1, 2.
4.4.4 Comparative-Static Properties of the (Stochastic) Duopoly Equilibrium

Introduction

The following analyses are designed to examine how strategic considerations influence the optimal response of mutually interdependent financial intermediaries to firm-specific shocks. We thereby demonstrate that some behavior which may superficially, under conventional microanalysis or ad hoc empirical investigation, appear anticompetitive (and thus may seem to furnish support for political efforts at deregulation) are compatible with competitive interaction.

Proposition 2(a):
Ceteris paribus, a rise in an intermediary's marginal cost of money-market borrowing (liquidity-adjustment cost) leads to a simultaneous increase in its loan and deposit rates.

Proposition 2(b):
These simultaneous adjustments (increase in loan rate and increase in deposit rate) in response to a liquidity-adjustment shock are invariant to any (liquidity-adjustment) cost differentials between the incumbent financial intermediaries.
Proposition 2(a) indicates that previous analytic finding that liquidity-adjustment constraint causes an otherwise risk-neutral monopolist financial intermediary to behave like a risk-averse maximizer extends to a duopoly market. Unfortunately, interpreting this result in the duopoly case is not straightforward because of the presence of strategic considerations.

Proposition (2b) supports the previous suggestion that in a local market in which a large bank competes with a small one, the outcome may not be different from a symmetric rivalry if there is some ease of entry into the local market. In other words, proposition (2b) tells us that a bank which uses the "puppy-dog ploy" (i.e., signals mutual cooperation) does not transform into a "top dog" (an aggressive rival) once the conflict involves a less threatening rival; a time-consistent behavior is necessary when there is credible threat of discipline (if local market is contestable).

Proposition 3:
Ceteris paribus, a rise in an intermediary’s marginal cost of money-market borrowing (liquidity-adjustment cost) leads to a sympathetic movement in the rival’s price vector (i.e. a simultaneous increase in the competitor’s loan and deposit rates).

Definition:
A strategic move is a move that induces the other player to choose in one’s favor. It constrains the other player’s choice by affecting its expectations.
Proposition 3 can be understood by asking how does a financial intermediary respond to a shock such that the opponent is manipulated into acting in its favor? In this model in which \( \sigma_i \) (i=1,2) does not directly enter into a rival's profit function, it affects \( \Pi_j \) (i\neq j; j=1,2) through its (the shock) effect on \( p_1 \) which affects \( p_j \), \( p = \{R_L, R_D\} \). In other words, by altering its behavior, it is able to manipulate its opponent's response. The sign of the strategic effect,

\[
\left. \frac{s \text{gn}(\partial R_L / \partial \sigma_1)}{\partial \sigma_1} \right|_{\sigma_1 \neq \sigma_2} = (\partial^2 \pi_1 / \partial R_L \partial \sigma_1) \left\{ (\partial^2 \pi_1 / \partial R_L \partial R_L) + (\partial^2 \pi_1 / \partial R_L \partial R_D) + (\partial^2 \pi_1 / \partial R_D \partial R_L) \right\},
\]

(63)

and

\[
\left. \frac{s \text{gn}(\partial R_D / \partial \sigma_1)}{\partial \sigma_1} \right|_{\sigma_1 \neq \sigma_2} = (\partial^2 \pi_1 / \partial R_D \partial \sigma_1) \left\{ (\partial^2 \pi_1 / \partial R_D \partial R_D) + (\partial^2 \pi_1 / \partial R_D \partial R_L) + (\partial^2 \pi_1 / \partial R_L \partial R_D) \right\},
\]

(64)

can be decomposed into the product of the effect of the shock on the pricing scheme of 1 with the sum of (a) through (e).

(a) the effect of 1's loan rate on 2's loan rate,

(b) the effect of 1's deposit rate on 2's loan rate,

(c) the effect of 1's loan rate on 2's deposit rate,

(d) the effect of 1's deposit rate on 2's deposit rate,
(e) the effect of the deposit-loan-linkage on 2's marginal profits (the cross effect of the deposit and loan rates of financial intermediary 2).

Bulow, Geanakoplos, and Klemperer have coined the term "strategic complements" to capture similar cases where player 2's optimal response to more aggressive play by 1 is to be more aggressive. The emphasis in their context as well as here falls on the (conditioning) effect of these tactical plays on marginal profitabilitys.

A key result in our analysis is that when a financial intermediary responds strategically to a (liquidity-adjustment) shock to its opponent, the resulting duopoly equilibrium could be favorable (i.e., entails higher profits) to both incumbents. The indirect effects (source of strategic interaction) thus raise the possibility that a prima facie unfavorable change can benefit one or more of the competitors and as noted by Dixit (1986) may cause one or more of the firms to raise costs. The intuition is that the cost increase leads to a new equilibrium in which incumbents produce less and charge higher prices - a more collusive outcome. The benefit from this can outweigh the direct effect of the higher cost and could therefore constitute a strategic incentive. As noted previously, a financial intermediary's marginal cost of borrowing in the money market can increase if its credit-rating is downgraded. Downgrading may trace to a variety of possible causes (some of which are under the direct control of the firm and thus can be manipulated by the management of
the firm): increase in the variability of earnings, adverse information (such as a qualified audit report), loan losses, inadequate (accounting or regulatory) capital position, and anticipated or unanticipated (adverse) regulatory developments.

What is the implication of proposition 3 for the financial-services industry studies? The answer is that showing that one cannot distinguish between sympathetic movements in prime rates as a strategic response or as a tacit coordinating mechanism, confounds empirical tests of the prime rate as an instrument of conscious parallelism in the banking industry. Therefore, one must exercise caution in constructing and interpreting such tests.
CHAPTER V

CONCLUSIONS

5.1 Summary of Results

This chapter presents a summary of the analytic results derived in Chapter 4, qualifications of these results, and suggestions for extensions and further analytic and empirical research. To recapitulate, the main purpose of this research is to develop a micro model of financial intermediaries that is appropriate to the contemporary structure of the financial-services industry. With proper modeling, this approach promises to improve our understanding of the spread and possible effects of interstate banking. In 1987, for example, 49% of all financial institution agreements and offers for mergers and acquisition involved interstate deals. 42% of these agreements and offers completed in 1987 were interstate mergers deals. As at February 4, 1988, 44 states and the District of Columbia have interstate banking legislation in place. Only six states – Arkansas, Colorado, Hawaii, Kansas, Montana, and North Dakota – do not yet have
laws that permit some kind of interstate banking activity. The model also improves the existing literature by integrating within an oligopolistic market structure, the risk considerations emphasized in portfolio-theoretic approach with the real-resource considerations emphasized in the firm-theoretic approach. This dissertation achieves this by developing a simultaneous model of mutually interdependent financial intermediaries that operate in imperfectly contestable markets; and confront firm-specific constrained liquidity-adjustment opportunities. The analysis of the Nash equilibrium focuses on: (1) identifying strategic incentives that influence the behavior of financial intermediaries and (2) characterizing the resulting competition.

Results indicate that the noncooperative equilibrium involves price dispersion with the low-cost firms charging a lower lending rate and paying a lower deposit rate than the high-cost rival. In the symmetric cost case, it pays competitors to mitigate the pressure of competition through increased product differentiation in, at least, the loan market. The Nash equilibrium does not maximize industry profits. When each financial intermediary maximizes its own profits, given its rival's rates, the outcome cannot be maximal overall profits. Decreases (increases) in a single bank's lending (deposit) rate have a negative effect on its rival's profits. The existence of this "externality" provides incentives for incumbent financial intermediaries to improve upon this "bad" equilibrium. One of the ways in which they seek to avoid this outcome is by the use of focal-point pricing.
Through focal-point pricing such as the banking industry’s prime rate, the pricing tactics of financial intermediaries can be tacitly coordinated within certain ranges. The inability of the financial intermediaries to achieve the collusive outcome as a noncooperative equilibrium reflects the underlying Prisoners’ Dilemma structure of the problem; each rival has an incentive to defect from collusion by chiselling and all incumbents end up with lower profits due to defections. A testable theory of the prime as an instrument of conscious parallelism in the banking industry is proposed. Alternative methods of testing the prime rate based on our analysis are contrasted with existing test procedure in the literature. Some of the more important theoretical weakness in these ad hoc tests are discussed. Proceeding dialectically as it were, we show: (1) how conventional analysis of the prime rate lead to a hypothesis of the prime rate as an instrument of conscious parallelism, and (2) how the same behavior can be justified in the anti-competitive sense; i.e. held to be a behavior that does not invite anti-competitive concerns. The implication of this dialectical analysis for empirical analyses of the prime rate controversy is noted.

Also, by identifying the strategic incentives that influence the behavior of financial intermediaries in the new environment of inter-state banking, the likely competitive effects and implications for regulatory policy are derived.

Other analytic findings are: (1) that (in the monopoly case) constrained liquidity-adjustment opportunities causes an otherwise risk-neutral intermediary to behave as a risk-averse maximizer and (2)
that optimal deposit and loan policies of financial intermediaries are strongly influenced by market conditions, real-resource cost considerations, and liquidity cost factors. (3) The results also show that optimal loan policy and optimal deposit policy are interdependent. This interdependence must be treated in in the overall balance-sheet management of the financial intermediary. This conclusion differs sharply from previous analytic findings for simultaneous models in the literature. Graddy and Kyle's (1979) regression results offer qualified support for the theory that optimal decisions of financial intermediaries are interdependent. This finding is important because of the plethora of applied work on structure-conduct-performance hypothesis in the banking industry - Edwards (1964, 1965), Flechsig (1965), Kaufman (1966), Meyer (1967), Bell, and Murphy (1969), Aspinwall (1970), Klein and Murphy (1971), Fraser, and Rose (1972), Edwards (1973), Heggestad, and Mingo (1976, 1977), Whitehead (1978), Rhodes (1979, 1980, 1981, 1982a), Smirlock (1983). 27 These tests are conducted using single-equation methods. Several authors such as Graddy and Kyle (1979), and Clark (1979) have argued that these tests may be inappropriate if bank-decision-making is interdependent. Single-equation estimation is appropriate if the regressors are strictly exogenous, otherwise the equation needs to be estimated as a simultaneous system.
5.2 Qualifications and Suggestions for Further Research

Immediate extensions of this research are along two dimensions: (1) An empirical test of the hypothesis on the fundamental property of the liquidity-adjustment cost-constrained Nash equilibrium — namely that the presence of price dispersion among incumbents and the existence of an "externality" provide incentives for rivals to improve upon what (from their perspective) is a "bad" equilibrium. Integrating the properties of the Nash equilibrium with our knowledge of the industry led to the development of a (focal-point) pricing theory based on the prime. The empirical investigation thus focuses on the prime rate as an instrument of conscious parallelism. We have presently identified two promising data sources. One source of industry (macro) data is from a quarterly publication of the Federal Reserve Board — Survey of Terms of Bank Lending (STBL). The other source is direct solicitation of (micro) data from banks. If test results support the theory of the prime as an instrument of conscious parallelism, it does not provide definite answer for public policy. An exception is in a circumstance in which the test is constructed to discriminate between conscious parallelism and strategic response.

The second avenue for extension of this research is to examine analytically the strategic incentives that influence the optimal capital decisions of financial intermediaries. The focus on bank capital is timely because of the continuing discussion on risk-based
capital requirements, and the growing concern over the potential impact of off-balance-sheet activities on the capital adequacy of financial intermediaries. The literature on bank capital is oriented largely toward the role of banks in the monetary system and the role of capital in bearing risk and protecting depositors against loss. There exists a small theoretical literature that explains bank capital position as optimizing behavior on the part of the individual bank. Sharpe (1978) and Buser, Chen, and Kane (1981) develop such a theory. These models deal explicitly with the effect of federal deposit insurance on an individual bank's optimal capital decision. Santomero (1984) comments that the overall literature on bank capital needs further development towards determining private optimal capital.

This extension, thus, complements the existing literature by emphasizing that strategic considerations are also important in explaining the capital structure of mutually interdependent financial intermediaries. An obvious implication of this is that the effect of the federal deposit insurance scheme on bank capital structure is not limited to its impact on insured banks; rather it has industry-wide ramifications if these intermediaries (insured or not) are linked strategically. The simplest framework within which this idea can be explored is in terms of a dynamic, state-space game (two-period duopolistic game) exploiting the analytic tools exposited in the seminal papers of Fudenberg and Tirole (1984), and Bulow, Geanakoplos, and Klemperer (1985).

A final question is to identify some of the shortcomings of this research. A major regret must be that the analysis of the prime rate
controversy is cast in a static model instead of the appropriate framework for studying reactions, retaliations, and tacit collusion—dynamic models of oligopoly: Green and Porter (1984); Brock and Scheinkman (1985)—on supergames (repeated games in which the underlying economic environment is time-invariant); and the truly dynamic models (that combine the repeated rivalry aspects of supergames with the commitment side of two-period games) such as the work of Kreps, Milgrom, Roberts, and Wilson (1982), and the trilogy of Maskin and Tirole (1986). As pointed out in the introductory chapter, progress in this area has been slow presumably due to the technical complexity involved; especially in the area germane to the banking industry studies—dynamic games of incomplete information.

The possibility of strategic entry deterrence in the banking industry is a timely issue, as depository financial intermediaries face increasing competition from thrifts, foreign banks, and nonbank financial intermediaries. We discuss, without developing this topic in the dissertation, two concepts fundamental to strategic entry deterrence: asymmetric information and dynamic rivalry. Gaskins (1971) notes that "the dependence of future market share on the current price level means that the dominant firm's pricing strategy can only be determined in a dynamic framework." If a potential entrant and the established firm are equally well-informed about the determinants of post-entry profitability as in a full information model, a potential entrant's preoccupation is with the post-entry game. After entry has occurred, the incumbent and the entrant are in a duopolistic game whose structure (and the resulting equilibrium) are independent of the
pre-entry price. Limit pricing results in a loss of first-period profits but no second-period gain. Therefore limit pricing would not emerge. Contemporary discussion of the contribution of asymmetric information to strategic entry deterrence is influenced by Milgrom and Roberts (1982). In asymmetric-information models, markets may differ in their post-entry profitability in a way that is not observable to potential entrants. Accordingly, entrants attempt to infer private information from prices and would avoid entry if prices were low. Thus an incumbent has incentives to invest in disinformation to deter entry. Consequently, in equilibrium, (under asymmetric information) limit pricing could emerge.

The other aspect of strategic entry deterrence is predation to induce exit. In full-information regimes, predation is not a credible threat because the prey understands that the price-cutting is temporary. Proponents of predation in the presence of symmetric information seek recourse in the "deep pocket" story according to which the resulting war of attrition is won by the firm that is able to withstand the losses that is concomitant with the price war. As Roberts (1985) observes, "the problem is to explain why the prey has limited access to capital. . . . Thus if the deep pocket story is to survive, we must justify limited access to capital markets in the face of positive expected profits." This pushes the concept of informational asymmetries once more onto the forefront.

Another major shortcoming of our research is the analytic complexity of the model, which reflects the nature of the institution being modeled as well as weaknesses in the art of modeling.
Restrictions derived from the optimization program developed in this dissertation indicate that a financial intermediary's optimal pricing problem may involve significant nonlinear programming. Throughout this dissertation convenient restrictions have ruled out nonlinear pricing tactics. We thereby avoid additional complexity while focusing on the strategic interaction that is the main focus of the analyses. It is clear that such restrictions constrain the opportunity set of the financial intermediaries and hence compresses the profit possibility frontier.

Finally, we note that although interinstitutional competition is multidimensional, this research casts financial intermediary competition in terms of explicit prices only. Ideally, the demand for a financial intermediary's loan commitment and depository services depends on confidence, user-convenience, and explicit price. A satisfactory theory of deposit-rate pricing and loan pricing must deal with these multidimensional aspects and the substitutions at the margin between the explicit and implicit prices. These extensions constitute a rich area for further research.
APPENDICES

A. Mathematical Derivation of the Monopoly Equilibrium with Demand Uncertainties (section 4.2.1)

For the monopoly model with stochastic variability in demands (presented in section 4.2.1), the first-order condition consists of the following pair of equations:

\[
\begin{align*}
-\theta(2+\theta\sigma)R_L - \beta\theta\sigma R_D &= -(1+\theta\sigma)\phi^* + \theta\sigma^* - \theta p - \theta c_1, \\
-\beta\theta\sigma R_L - \beta(2+\beta\sigma)R_D &= -\beta\sigma^* + (1+\beta\sigma)\alpha^* - \beta p + \beta c_d.
\end{align*}
\]

(65) (66)

Solving for optimal deposit and loan rates using Cramer's rule, we have:

\[
\begin{bmatrix}
-\theta a & -\beta b \\
-\beta a & -\beta b
\end{bmatrix}
\begin{bmatrix}
R_L \\
R_D
\end{bmatrix}
= \begin{bmatrix}
-\phi^* + \theta\sigma^* - \theta p - \theta c_1 \\
-\beta\sigma^* + t\alpha^* - \beta p + \beta c_d
\end{bmatrix},
\]

\[
\Delta = \beta c a b - (\beta\theta\sigma)^2 > 0, \ s = (1+\theta\sigma), \ t = (1+\beta\sigma), \ a = (1+s), \ b = (1+t).
\]

\[
R_D = \frac{-a(\beta\sigma^* + t\alpha^* - \beta p + \beta c_d) + \beta\theta\sigma(-\phi^* + \theta\sigma^* - \theta p - \theta c_1)}{\Delta}
\]

(67)
\[ R_D = \frac{\theta}{\Delta} [\beta \phi^* - (a + 2 \beta \sigma) \alpha^* - \beta (ac_d + \sigma c_1) + 2 \beta \rho]. \] (68)

\[ R_L = \{-\beta b (\theta \sigma \alpha^* - \phi^* - \theta \rho - \theta c_1) + \beta \theta \sigma (\alpha^* - \beta \rho + \beta c_d - \beta \sigma \phi^*)\}/\Delta \] (69)

\[ = \frac{\beta}{\Delta} [(b + 2 \theta \sigma) \phi^* - \theta \sigma \alpha^* + \theta (bc_d + \sigma c_d) + 2 \theta \rho]. \] (70)
B. Mathematical Derivation of the Comparative-Static Properties of the (Stochastic) Monopoly Equilibrium (section 4.2.4)

To derive the comparative-static properties of this equilibrium, optimal profit is restated compactly as:

$$\Pi = \pi(R_L(\beta, \Theta, \sigma), R_D(\beta, \Theta, \sigma), \sigma).$$  \hspace{1cm} (71)

We may conveniently write $\partial^2 \pi / \partial R_L^2$ as $\pi_{11}$, $\partial^2 \pi / \partial R_L \partial \sigma$ as $\pi_{1\sigma}$, and $\partial \pi / \partial \sigma$ as $\pi_{\sigma}$, etc.

Totally differentiating $\Pi$, we have:

$$\pi_{11} \partial R_L + \pi_{1d} \partial R_D + \pi_{1\sigma} \partial \sigma + \pi_{1\Theta} \partial \Theta + \pi_{1\beta} \partial \beta = 0, \hspace{1cm} (72)$$

$$\pi_{d1} \partial R_L + \pi_{dd} \partial R_D + \pi_{d\sigma} \partial \sigma + \pi_{d\Theta} \partial \Theta + \pi_{d\beta} \partial \beta = 0. \hspace{1cm} (73)$$

Performing the operations described by equations (72 & 73) on the pair of first-order conditions, equations (64 & 65); and defining $x = (\alpha - \phi) < 0$,

yields the following:

$$\begin{bmatrix} -\Theta a & -\beta \Theta \sigma \\ -\beta \Theta \sigma & -\beta b \end{bmatrix} \begin{bmatrix} \partial R_L / \partial \sigma \\ \partial R_D / \partial \sigma \end{bmatrix} = \begin{bmatrix} \Theta x \\ \beta x \end{bmatrix}. \hspace{1cm} \text{Using Cramer's rule, we solve for,}$$

$$\partial R_L / \partial \sigma = \partial R_D / \partial \sigma = -2 \beta \Theta x / \Delta. \hspace{1cm} (74)$$
C. Mathematical Derivation of the Perfect-Foresight Duopoly

Equilibrium (section 4.3.1)

\[
\max \pi_i = (R_{L_i} - R_{D_i})D_i - cD_i. \tag{75}
\]

Using equations (5) through (8), the balance-sheet constraint can be restated as:

\[
\phi - \Theta R_{L_i} + \lambda R_{L_j} = \alpha + \Theta R_{D_i} - \delta R_{D_j}, \quad i, j = 1, 2 \quad i \neq j \tag{76}
\]

which leads to \( R_{L_i} - R_{D_i} = \frac{1}{\Theta}(\lambda R_{L_j} + \phi - D_i) - R_{D_i} \). \( \tag{77} \)

To obtain an expression for \( R_{L_i} - R_{D_i} \) we first eliminate the cross-price term \( R_{L_j} \) and then solve for \( R_{L_i} \) in terms of \( R_{D_i} \) and \( R_{D_j} \). From (77),

\[
\lambda R_{L_2} - \Theta R_{L_2} = D_1 - \phi_1, \tag{78}
\]

\[
\lambda R_{L_1} - \Theta R_{L_1} = D_2 - \phi_2. \tag{79}
\]

To obtain an expression for the cross-price term \( R_{L_j} \), Cramer's rule is used to solve equations (78) and (79):

\[
\begin{bmatrix}
\lambda & -\Theta \\
-\Theta & \lambda
\end{bmatrix}
\begin{bmatrix}
R_{L_2} \\
R_{L_1}
\end{bmatrix} = \begin{bmatrix}
D_1 - \phi_1 \\
D_2 - \phi_2
\end{bmatrix},
\]

The determinant \( \gamma = \lambda^2 - \Theta^2 < 0. \)
\[ R_{L_2} = R_{L_1} = R_{L_j} = (\lambda D_i + \theta D_j - [1 + \theta] \phi)/\gamma. \quad i, j = 1, 2; \quad i \neq j \]  

(80)

Substituting (80) into (77) yields:

\[ (R_{L_i} - R_{D_i}) = \frac{1}{\gamma} \{ [(\beta \theta - \delta \lambda - \gamma) R_{D_i} + (\beta \lambda - \delta \theta) R_{D_j} + (\theta + \lambda) \alpha - (\theta + \lambda) \phi]. \]  

(81)

Therefore (75) can be restated as:

\[ \pi_i = \frac{1}{\gamma} \{ [(\beta \theta - \delta \lambda - \gamma) R_{D_i} + (\beta \lambda - \delta \theta) R_{D_j} + (\theta + \lambda) \alpha - (\theta + \lambda) \phi] \} \cdot (\alpha + \delta R_{D_i} - \delta R_{D_j}) - c(\alpha + \delta R_{D_i} - \delta R_{D_j}). \quad i, j = 1, 2; \quad i \neq j \]  

(82)

First-order condition is:

\[ \partial \pi_i /\partial R_{D_i} = \frac{1}{\gamma} \{ 2 \beta (\beta \theta - \delta \lambda - \gamma) R_{D_i} + [(\beta \lambda - \delta \theta)^2 - 2 \delta (\beta \theta - \delta \lambda)] R_{D_j} + (2 \beta \theta + \lambda [\beta - \delta - \gamma]) \alpha - \beta (\theta + \lambda) \phi \} - \beta c = 0. \quad i, j = 1, 2; \quad i \neq j \]  

(83)

A sufficient condition for an optimum is:

\[ \partial^2 \pi_i /\partial R_{D_i}^2 = \frac{2 \beta}{\gamma} (\beta \theta - \delta \lambda - \gamma) < 0. \quad i = 1, 2 \]  

(84)

The equilibrium deposit rates are the solution to the pair of simultaneous equations given by equation (83).

Using Cramer's rule again:

\[
\begin{bmatrix}
m/\gamma & n/\gamma \\
n/\gamma & m/\gamma
\end{bmatrix}
\begin{bmatrix}
R_{D_1} \\
R_{D_2}
\end{bmatrix}
=
\begin{bmatrix}
x/\gamma \\
x/\gamma
\end{bmatrix}.
\]
The determinant of the coefficient matrix is \( m^2 - n^2 = \Delta \).

\[
R_{D_1} = R_{D_2} = \frac{x}{(m+n)}
\]

\[
= \frac{\beta(\theta+\lambda) \phi + \gamma \beta c - (2 \beta \theta + \lambda [\beta - \delta] - \gamma) \alpha}{2(\beta - \delta)(\beta \theta - \delta \lambda) - 2 \beta \gamma + \beta(\delta \lambda) - \theta(\delta \theta)}
\]

Here,

\[
m = 2 \beta(\beta \theta - \delta \lambda - \gamma) > 0
\]

\[
n = (\beta^2 \lambda - \delta \theta^2) - 2 \delta (\beta \theta - \delta \lambda)
\]

\[
x = \beta(\theta+\lambda) \phi + 2 \beta \gamma c - (2 \beta \theta + \lambda [\beta - \delta] - \gamma) \alpha.
\]

To establish that \( m^2 - n^2 = \Delta > 0 \), first note that,

\[
(m+n) = 2 \beta(\beta \theta - \delta \lambda - \gamma) + (\beta^2 \lambda - \delta \theta^2) - 2 \delta (\beta \theta - \delta \lambda)
\]

\[
= 2 \beta(\beta \theta - \delta \lambda) - 2 \beta \gamma + (\beta^2 \lambda - \delta \theta^2) - 2 \delta (\beta \theta - \delta \lambda)
\]

\[
= 2(\beta - \delta)(\beta \theta - \delta \lambda) + (\beta^2 \lambda - \delta \theta^2) - 2 \beta \gamma
\]

\[
= (\beta - \delta)(2[\beta \theta - \delta \lambda] + \theta^2) + \beta \lambda(\beta - \lambda) - \beta \gamma > 0.
\]

Now,

\( m > 0 \), \((m+n) > 0 \Rightarrow m > -n \Rightarrow (m^2 - n^2) > 0 \) since whether \( n \leq 0 \),

\((m-n) > 0 \) and \( 0 < (m+n)(m-n) = (m^2 - n^2) > 0 \). A more direct way of establishing \((m-n) > 0 \) is by fully expanding

\[
(m-n) = \beta^2 \theta + \beta^2 \theta - 2 \beta \delta \lambda - 2 \beta \gamma + 2 \beta \delta \theta - \delta^2 \lambda - \beta^2 \lambda + \delta \theta^2
\]

\[
= (\theta - \lambda) (2 \beta \delta + \beta^2 + 2 \beta [\theta + \lambda]) + \beta^2 \theta + \delta \theta^2 - \delta^2 \lambda - \delta^2 \lambda > 0,
\]

since \( \beta^2 \theta > \delta^2 \lambda \), and \( \delta \theta^2 > \delta^2 \lambda \). \text{ Q.E.D.}

Substituting equation (86) for \( R_{D_1} \) in equation (81) yields:
\[ R_{L_1} = R_{L_2} = \frac{(1-\Theta-\gamma)}{(m+n)-(\theta+\gamma)[(\delta[\lambda-\delta]-\beta\gamma)\alpha + \beta\gamma[\beta-\delta]\phi]

- ([(\beta-\delta)(\beta^2+\beta\theta-\delta\lambda)-\beta\gamma+\lambda(\delta^2-\beta\lambda)]\phi}). \] (93)

Equilibrium loan rates are positive if and only if

\[ [(\delta[\lambda-\delta]-\beta\gamma)\alpha + (\beta-\delta)\beta\gamma - ([(\beta-\delta)(\beta^2+\beta\theta-\delta\lambda)-\beta\gamma+\lambda(\delta^2-\beta\lambda)]\phi) < 0. \]

A sufficient condition for this is \( \delta^2 > \beta\lambda \). If \( \delta^2 > \beta\lambda \), \((\beta-\delta)(\beta^2+\beta\theta-\delta\lambda)-\beta\gamma+\lambda(\delta^2-\beta\lambda))\) is positive and greater than \((\delta[\lambda-\delta]-\beta\gamma)\).
D. Mathematical Derivation of the Nash Equilibrium for the Duopolists with Demand Uncertainties (section 4.4.2)

Each financial intermediary is assumed to maximize:

\[
E(\pi_i) = R_{L_1} (\phi_1^* - \Theta R_{L_1} + \lambda \sigma_{L_1}) - R_{D_1} (\alpha_1^* + \beta \sigma_{D_1} - \delta R_{D_1}) - \rho (\phi_1^* - \Theta R_{L_1} + \lambda \sigma_{L_1}) - \alpha_1^* - \beta R_{D_1} + \delta R_{D_1}) - \frac{1}{2} \sigma_1 (\phi_1^* - \Theta R_{L_1} + \lambda \sigma_{L_1} - \alpha_1^* - \beta R_{D_1} + \delta R_{D_1})^2 - c_1 (\phi_1^* - \Theta R_{L_1} + \lambda \sigma_{L_1}) - c_1 (\alpha_1^* + \beta R_{D_1} - \delta R_{D_1}), \quad i,j=1,2; \quad i \neq j. \tag{94}
\]

The first-order conditions for the two intermediaries in this rate-setting game consist of the following two pairs of equations:

\[
\frac{\partial E(\pi_1)}{\partial R_{L_1}} = \phi_1^* - 2 \Theta R_{L_1} + \lambda \sigma_{L_2} + \rho \Theta + \Theta c_1 (\phi_2^* - \Theta R_{L_2} + \lambda \sigma_{L_2} - \alpha_1^* - \beta R_{D_1} + \delta R_{D_2}) + \Theta c_1 = 0, \tag{95}
\]

\[
\frac{\partial E(\pi_1)}{\partial R_{D_1}} = -\alpha_1^* - 2 \beta R_{D_1} + \delta R_{D_2} + \rho \beta + \beta \sigma_1 (\phi_2^* - \Theta R_{L_2} + \lambda \sigma_{L_2} - \alpha_1^* - \beta R_{D_1} + \delta R_{D_2}) - \beta c_1 = 0, \tag{96}
\]

\[
\frac{\partial E(\pi_2)}{\partial R_{L_2}} = \phi_2^* - 2 \Theta R_{L_2} + \lambda \sigma_{L_1} + \rho \Theta + \Theta c_1 (\phi_2^* - \Theta R_{L_2} + \lambda \sigma_{L_2} - \alpha_2^* - \beta R_{D_2} + \delta R_{D_1}) + \Theta c_1 = 0, \tag{97}
\]

\[
\frac{\partial E(\pi_2)}{\partial R_{D_2}} = -\alpha_2^* - 2 \beta R_{D_2} + \delta R_{D_1} + \rho \beta + \beta \sigma_2 (\phi_2^* - \Theta R_{L_2} + \lambda \sigma_{L_2} - \alpha_2^* - \beta R_{D_2} + \delta R_{D_1}) - \beta c_1 = 0. \tag{98}
\]

To satisfy the second-order condition requires that

\[
\frac{\partial^2 E(\pi_i)}{\partial R_{L_1}^2} < 0, \quad \frac{\partial^2 E(\pi_i)}{\partial R_{D_1}^2} < 0, \quad \text{and} \quad \frac{\partial^2 E(\pi_i)}{\partial R_{L_1} \partial R_{D_1}} = \Omega_i > 0, \quad i=1,2;
\]

\[
\begin{array}{|c|c|}
\hline
\frac{\partial^2 E(\pi_1)}{\partial R_{L_1}^2} & \frac{\partial^2 E(\pi_1)}{\partial R_{L_1} \partial R_{D_1}} \\hline
\frac{\partial^2 E(\pi_1)}{\partial R_{D_1}^2} & \frac{\partial^2 E(\pi_1)}{\partial R_{L_1} \partial R_{D_1}} \\hline
\end{array}
\]

\[
\frac{d^2 \pi_1}{dR_1^2} = -\theta(2 + \theta \sigma_1) < 0
\]
(99)

\[
\frac{d^2 \pi_1}{dR_{D_1}^2} = -\beta(2 + \beta \sigma_1) < 0
\]
(100)

\[
\Omega_i = \beta \theta(2 + \theta \sigma_1)(2 + \beta \sigma_1) - (\beta \theta \sigma_1)^2 > 0.
\]

Defining \( s_i \equiv (1 + \theta \sigma_i), \ t_i \equiv (1 + \beta \sigma_i), \ a_i \equiv (1 + s_i), \ b_i \equiv (1 + t_i) \), it is convenient to express these sufficient conditions as:

\[-\theta a_i < 0, \ -\beta b_i < 0, \text{ and } \beta \theta a_i b_i - (\beta \theta \sigma_1)^2 > 0. \ i = 1, 2\]

Algebraic manipulation of the first-order condition, equations (95) through (98) yields equations (100) to (103) whose solution determine the equilibrium set of deposit and loan rates:

\[
-\theta a_1 R_{L_1} - \beta \theta \sigma_1 R_{D_1} + \lambda s_1 R_{L_2} + \delta \theta \sigma_1 R_{D_2} = \theta \sigma_1 a_1^* - s_1 \phi_1^* - \theta(\rho + c_1),
\]
(101)

\[
-\beta \theta \sigma_1 R_{L_1} - \beta b_1 R_{D_1} + \beta \lambda s_1 R_{L_2} + \delta t_1 R_{D_2} = t_1 \alpha_1^* - \beta \sigma_1 \phi_1^* - \beta(\rho - c),
\]
(102)

\[
\lambda s_2 R_{L_1} + \theta \delta \sigma_2 R_{D_1} - \theta a_2 R_{L_2} - \beta \theta \sigma_2 R_{D_2} = \theta \sigma_2 a_2^* - s_2 \phi_2^* - \theta(\rho + c_1),
\]
(103)

\[
\beta \lambda s_2 R_{L_1} + \delta t_2 R_{D_1} - \beta \theta \sigma_2 R_{L_2} - \beta b_2 R_{D_2} = t_2 \alpha_2^* - \beta \sigma_2 \phi_2^* - \beta(\rho - c). \]
(104)

Equations (101) through (104) can be conveniently expressed in matrix form. We may define:

\[
v_i \equiv \theta \sigma_1 a_i^* - (1 + \theta \sigma_i) \phi_i^* - \theta(\rho + c_1),
\]

\[
v_i \equiv (1 + \beta \sigma_1) a_i^* - \beta \sigma_1 \phi_i^* - \beta(\rho - c), \ i = 1, 2.
\]

This lets us rewrite (101) to (104) as:
\[
\begin{bmatrix}
-\theta a_1 & -\beta \theta_1 & \lambda s_1 & \delta \theta_1 \\
-\beta \theta_1 & -\beta b_1 & \beta \lambda s_1 & \delta t_1 \\
\lambda s_2 & \delta \theta_2 & -\theta a_2 & -\beta \theta_2 \\
\beta \lambda s_2 & \delta t_2 & -\beta \theta_2 & -\beta b_2
\end{bmatrix}
\begin{bmatrix}
R_{L_1} \\
R_{D_1} \\
R_{L_2} \\
R_{D_2}
\end{bmatrix}
= \begin{bmatrix}
v_1 \\
w_1 \\
v_2 \\
w_2
\end{bmatrix}.
\]

The determinant of the system, \( \Delta \), is:

\[
\begin{vmatrix}
-\theta a_1 & -\beta \theta_1 & \lambda s_1 & \delta \theta_1 \\
-\beta \theta_1 & -\beta b_1 & \beta \lambda s_1 & \delta t_1 \\
\lambda s_2 & \delta \theta_2 & -\theta a_2 & -\beta \theta_2 \\
\beta \lambda s_2 & \delta t_2 & -\beta \theta_2 & -\beta b_2
\end{vmatrix}
= (\beta \theta a_2 b_2 - [\beta \theta]_2)^2 (\beta \theta a_1 b_1 - [\beta \theta]_1^2) + (\theta^2 \delta a_1 \sigma_2 - \beta \theta \lambda s_2 \sigma_1) (\beta \theta \delta t_1 \sigma_2 - \beta^2 \lambda b_2 \sigma_1) + (\theta^2 \delta a_1 \sigma_2 - \beta \lambda s_1 \sigma_2) (\beta \lambda b_1 \sigma_2 - \theta^2 \delta \sigma_1 \sigma_2) + (\beta^2 \theta \lambda \sigma_1 \sigma_2 - \theta \delta a_1 t_2) (\theta a_2 \sigma_1 - \beta^2 \theta \lambda \sigma_1 \sigma_2) + (\delta \lambda s_1 t_2 - \beta \theta \delta \lambda \sigma_2) (\delta \lambda s_1 t_1 - \beta \theta \delta \lambda \sigma_1).$

The determinant is positive \( (\Delta > 0) \). This is established in two ways:

1. Using a numerical search procedure that involved successive iterations over the domain of the parameters that comprise the determinant. 29

2. Implementing a straightforward heuristic argument:

\( \Delta = 0 \) if and only if one of the following conditions hold:

(i) \( \beta = 0 \),

(ii) \( \theta = 0 \),

(iii) \( \beta \) and \( \theta \) are equal to zero.

Since the domain of \( \beta \) and \( \theta \) are restricted to,

\[ 0^\circ < \arctan \beta, \arctan \theta < 90^\circ, \]

\( \Delta \neq 0 \).
Therefore, either $\Delta > 0$ or $\Delta < 0$.

It is sufficient to show that there is at least a set of admissible parameter values for which $\Delta > 0$.

To expedite the analysis, let us pick $\sigma_1 = \sigma_2 = \sigma$; and define $\Delta(\sigma)$ as follows:

$$\Delta = Q^2 + 2EC - (B^2 + D^2) + F^2,$$

$$Q = (\beta \Phi \alpha - [\beta \Theta r]^2),$$

$$B = (\beta \lambda \alpha - \beta^2 \delta \sigma^2),$$

$$C = (\beta \delta \lambda \rho - \beta^2 \lambda \rho \sigma),$$

$$D = (\Theta \delta t \sigma - \beta^2 \Theta \lambda \sigma^2),$$

$$E = (\Theta^2 \delta \alpha \sigma - \beta \Theta \lambda \rho \sigma),$$

$$F = (\delta \lambda \alpha t - \beta \Theta \delta \lambda \sigma^2),$$

Furthermore, we set $\beta = \Theta$, $\delta = \lambda$; which implies the following values for $Q$, $B$, $C$, $D$, $E$, $F$:

$$Q = (4 + 4 \beta \sigma) \beta^2,$$

$$B = (2 + 3 \beta \sigma) \beta \delta,$$

$$C = -\delta \sigma \beta^2,$$

$$D = (2 + 3 \beta \sigma) \beta \delta,$$

$$E = \delta \sigma \beta^2,$$

$$F = (1 + 2 \beta \sigma) \delta^2.$$

Since $E = -C$, and $B = D$, we may rewrite the determinant, $\Delta(\sigma)$, as

$$Q^2 + F^2 - 2(B^2 + C^2);$$

which when evaluated at the arbitrarily chosen numbers,

$\beta = 1$, $\delta = 1/2$, $\sigma = 3$, 

within the domain of the admissible parameter values yields

\[
([16]^2 + [49/16]) - 2([121/4] + [9/4]) \gg 0.
\]

Hence \( \Delta > 0 \).

Solving for the equilibrium deposit and loan rates using Cramer's rule yields:

\[
R_{L_1} = \frac{N_1}{\Delta} \quad (105)
\]

\[
N_1 = (\beta \theta a_2 b_2 - (\beta \theta \sigma_2)^2)(v_1 \beta \theta \sigma_1 - v_1 \beta b_1) + (\theta^2 \delta \sigma_1 \sigma_2 - \beta \lambda b_2 \sigma_1)(v_1 \delta \sigma_2 + v_2 \beta b_1) + (\theta \delta t_1 \sigma_2 - \beta^2 \lambda b_2 \sigma_1)(-v_1 \theta \delta \sigma_2 - v_2 \beta \theta \sigma_1) + (\theta \delta a_2 t_1 - \beta^2 \theta \lambda \sigma_1 \sigma_2)(v_1 \delta t_2 + v_2 \beta \theta \sigma_1) + (\theta^2 \delta a_2 \sigma_1 - \beta \theta \lambda s_1 \sigma_2)(-v_2 \beta b_1 - v_1 \delta t_2) + (\delta \lambda s_1 t_1 - \beta \theta \delta \lambda \sigma_1^2)(v_2 \delta t_2 - v_2 \theta \delta \sigma_2).
\]

\[
R_{D_1} = \frac{T_1}{\Delta} \quad (106)
\]

\[
T_1 = (\beta \theta a_2 b_2 - [\beta \theta \sigma_2]^2)(v_1 \beta \theta \sigma_1 - v_1 \theta a_1) + (\theta \delta t_1 \sigma_2 - \beta^2 \lambda b_2 \sigma_1)(v_2 \theta a_1 + v_1 \lambda s_2 - v_1 \lambda b_2) + (\theta \delta t_1 a_2 - \beta^2 \theta \lambda \sigma_1 \sigma_2)(-v_2 \theta a_1 - v_1 \beta \lambda s_2) + (\beta^2 \delta \sigma_1 \sigma_2 - \beta \lambda s_1 b_2)(-v_2 \beta \theta \sigma_1 - v_1 \lambda s_2) + (\theta^2 \delta a_2 \sigma_1 - \beta \theta \lambda s_1 \sigma_2)(v_2 \beta \theta \sigma_1 + v_1 \beta \lambda s_2) + (\delta \lambda s_1 t_1 - \beta \theta \delta \lambda \sigma_1^2)(v_2 \lambda s_2 - v_2 \beta \lambda s_2).\]
\[ R_{L_2} = \begin{vmatrix} -\theta a_1 & -\theta b_1 & v_1 & \sigma_1 \theta & \delta \\ -\theta b_1 & -\theta b_2 & \delta t_1 \\ \lambda s_2 & \delta \theta _2 & v_2 & -\sigma_2 \theta \beta \\ \beta \lambda s_2 & \delta t_2 & v_2 & -\beta b_2 \end{vmatrix} \equiv N_2 \quad \Delta \] 

\[ N_2 \equiv (\theta a_1 b_1 - [\theta b_1]^2) (v_2 \theta b_2 - \nu_1 \beta b_2) + (\theta \theta^2 \delta \sigma_2 \sigma_1 - \beta \lambda b_1 s_2) (v_2 \theta \sigma_1 + \nu_1 \beta b_2) 
+ (\theta \theta \delta t_2 \sigma_1 - \beta^2 \lambda b_1 s_2) (-\nu_1 \theta \sigma_1 - v_1 \beta \sigma_2) 
+ (\theta \delta a_1 t_2 - \beta^2 \theta \lambda \sigma_2 \sigma_1) (v_2 \delta t_1 + \nu_1 \beta \sigma_2) 
+ (\theta^2 \delta a_1 \sigma_2 - \theta \delta \nu_1 \beta \lambda s_2 \sigma_1) (-\nu_1 \beta b_2 - v_2 \delta t_1) 
+ (\delta \lambda s_2 t_2 - \theta \delta \lambda s_2^2) (v_1 \delta t_1 - \nu_1 \theta \delta \sigma_1). \]

\[ R_{D_2} = \begin{vmatrix} -\theta a_1 & -\theta b_1 & \lambda s_1 & v \\ -\theta b_1 & -\theta b_2 & \beta \lambda \sigma_1 & w \\ \lambda s_2 & \delta \theta_2 & -\theta a_2 & x \\ \beta \lambda \sigma_2 & \delta t_2 & -\theta \sigma_2 & y \end{vmatrix} \equiv T_2 \quad \Delta \] 

\[ T_2 \equiv (\theta a_1 b_1 - [\theta b_1]^2) (v_2 \theta \sigma_2 - w_2 \theta a_2) + (\theta \delta t_2 \sigma_1 - \beta^2 \lambda b_1 s_2) (v_1 \theta a_2 + v_2 \lambda s_1) 
+ (\theta \delta t_2 a_1 - \beta^2 \theta \lambda \sigma_2 \sigma_1) (-v_1 \theta a_2 - v_2 \beta \lambda \sigma_1) + (\theta \delta \sigma_2 \sigma_1 - \beta \lambda s_2 b_1) (-v_1 \theta \sigma_2 - v_2 \lambda s_1) 
+ (\theta^2 \delta a_1 \sigma_2 - \theta \lambda s_2 \sigma_1) (v_1 \theta \sigma_2 + v_2 \beta \lambda \sigma_1) 
+ (\delta \lambda s_2 t_2 - \theta \delta \lambda s_2^2) (v_1 \lambda s_1 - v_1 \beta \lambda \sigma_1). \]

By defining groups of expressions in terms of new variables:

\[ \Delta = q_1 q_2 + E_{12} C_{21} + E_{21} C_{12} + B_{12} B_{21} + D_{12} D_{21} + F_1 F_2, \]

\[ q_1 = (\beta a_1 b_1 - [\beta b_1]^2), \]

\[ q_2 = (\beta a_2 b_2 - [\beta b_2]^2), \]

\[ B_{12} = (\beta \lambda b_1 s_2 - \beta \theta^2 \delta \sigma_1 \sigma_2), \]

\[ B_{21} = (\beta \theta^2 \delta \sigma_1 \sigma_2 - \beta \lambda b_2 s_1), \]
\[ C_{12} = (\beta \theta t_2 \sigma_1 - \beta^2 \lambda b_1 \sigma_2), \]
\[ C_{21} = (\beta \theta t_1 \sigma_2 - \beta^2 \lambda b_2 \sigma_1), \]
\[ D_{12} = (\beta^2 \theta \lambda \sigma_2 \sigma_1 - \theta \delta a_1 t_2), \]
\[ D_{21} = (\theta \delta a_2 t_1 \sigma_1 - \beta \theta \lambda \sigma_1 \sigma_2), \]
\[ E_{12} = (\theta^2 \delta a_1 \sigma_2 - \beta \theta \lambda s_2 \sigma_1), \]
\[ E_{21} = (\theta^2 \delta a_2 \sigma_1 - \beta \theta \lambda s_1 \sigma_2), \]
\[ F_1 = (\delta \lambda s_1 t_1 - \beta \theta \delta \lambda \sigma_1^2), \]
\[ F_2 = (\delta \lambda s_2 t_2 - \beta \theta \delta \lambda \sigma_2^2), \]

the optimal deposit and loan rates for the financial intermediaries [defined by equations (105) through (108)] may be expressed in a more compact form:

\[
R_{L_1} = \{(Q_2 \beta \theta \sigma_1 + B_{21} \delta \theta \sigma_2 - E_{21} \delta t_2)w_1 + (D_{21} \beta \theta \sigma_1 - F_1 \delta \theta \sigma_2 - E_{21} \beta \lambda b_1)w_2 \]
\[ + (D_{21} \delta t_2 - C_{21} \delta \theta \sigma_2 - Q_2 \beta \lambda b_1)w_1 + (B_{21} \beta \lambda b_1 - C_{21} \beta \theta \sigma_1 + F_1 \delta t_2)w_2 \}/\Delta, \quad (109) \]

\[
R_{D_1} = \{(E_{21} \beta \lambda \sigma_2 - B_{21} \lambda s_2 - Q_2 \theta a_1)w_1 + (E_{21} \beta \theta \sigma_1 + F_1 \lambda s_2 - D_{21} \theta a_1)w_2 \]
\[ + (Q_2 \beta \theta \sigma_1 + C_{21} \lambda s_2 - D_{21} \beta \lambda \sigma_2)w_1 + (C_{21} \theta a_1 - B_{21} \beta \theta \sigma_1 - F_1 \beta \lambda \sigma_2)w_2 \}/\Delta, \quad (110) \]

\[
R_{L_2} = \{(Q_1 \beta \theta \sigma_2 - B_{12} \delta \theta \sigma_1 - E_{12} \delta t_1)w_2 - (D_{12} \beta \theta \sigma_2 + F_2 \delta \theta \sigma_1 + E_{12} \beta \lambda b_2)w_1 \]
\[ - (D_{12} \delta t_1 + C_{12} \delta \theta \sigma_1 + Q_1 \beta b_2)w_2 - (B_{12} \beta b_2 + C_{12} \beta \theta \sigma_2 - F_2 \delta t_1)w_1 \}/\Delta, \quad (111) \]

\[
R_{D_2} = \{(E_{12} \beta \lambda \sigma_1 + B_{12} \lambda s_1 - Q_1 \theta a_2)w_2 + (E_{12} \beta \theta \sigma_2 + F_2 \lambda s_1 + D_{12} \theta a_2)w_1 \]
\[ + (Q_1 \beta \theta \sigma_2 + C_{12} \lambda s_1 + D_{12} \beta \lambda \sigma_1)w_2 + (C_{12} \theta a_2 + B_{12} \beta \theta \sigma_2 - F_2 \beta \lambda \sigma_1)w_1 \}/\Delta, \quad (112) \]
E. Mathematical Derivation of the Comparative-Static Properties of the (Stochastic) Duopoly Equilibrium (section 4.4.4)

For this exercise, the equilibrium profits may be summarized as:

\[ E(\pi_1) = \pi_1(R_{L_1}(\sigma_1, \sigma_2), R_{D_1}(\sigma_1, \sigma_2), \sigma_1) \] (112)

\[ E(\pi_2) = \pi_2(R_{L_1}(\sigma_1, \sigma_2), R_{D_1}(\sigma_1, \sigma_2), \sigma_2), \ i=1,2. \] (113)

Comparative-static properties may be derived by totally differentiating equations (101) through (104). These set of equations determine explicitly the equilibrium deposit and loan rates for the financial intermediaries. The total differentiation yields a system of four equations which by defining,

\[ f_i = (\alpha_i - \phi_i) < 0, \ i=1,2 \]

may be conveniently represented thus:

\[
\begin{bmatrix}
-\theta_1 & -\beta_1 & \lambda_1 & \delta_1 \\
-\beta_1 & -b_1 & \beta_1 & \delta_1 \\
\lambda & \delta & -\theta_2 & -\beta_2 \\
\beta & \delta & -\beta_2 & -b_2 \\
\end{bmatrix}
\begin{bmatrix}
\partial R_{L_1} \\
\partial R_{D_1} \\
\partial R_{L_2} \\
\partial R_{D_2} \\
\end{bmatrix} =
\begin{bmatrix}
\partial \pi_1 \partial \sigma_1 \\
\partial \pi_1 \partial \sigma_1 \\
\partial \pi_2 \partial \sigma_2 \\
\partial \pi_2 \partial \sigma_2 \\
\end{bmatrix}
\]
and

\[
\Delta = \begin{vmatrix}
-\theta a_1 & -\beta \theta \sigma_1 & \lambda s_1 & \delta \theta \sigma_1 \\
-\beta \theta \sigma_1 & -\beta b_1 & \beta \lambda \sigma_1 & \delta t_1 \\
\lambda s_2 & \delta \theta \sigma_2 & -\theta a_2 & -\beta \theta \sigma_2 \\
\beta \lambda \sigma_2 & \delta t_2 & -\beta \theta \sigma_2 & -\beta b_2
\end{vmatrix} > 0.
\]

The comparative-static results are:

\[
\left( \frac{\partial R_L}{\partial \sigma_1} \right)_{\sigma_1 \neq \sigma_2} = -\frac{\Theta f_1}{\Delta} \left( \Theta a_2 [4 \beta^2 - \delta^2] + 2 \beta \theta \sigma_2 [2 \beta^2 - \delta^2] + \delta \lambda \sigma^2 \sigma_2 \right) > 0. \tag{114}
\]

\[
\left( \frac{\partial R_D}{\partial \sigma_1} \right)_{\sigma_1 \neq \sigma_2} = \frac{\beta f_1}{\Delta} \left( \beta b_2 [4 \theta^2 - \lambda^2] + 2 \beta \theta \sigma_2 [2 \theta^2 - \lambda^2] + \delta \theta \sigma^2 \sigma_2 \right) > 0. \tag{115}
\]

\[
\left( \frac{\partial R_L}{\partial \sigma_1} \right)_{\sigma_1 \neq \sigma_2} = -\frac{\Theta f_1}{\Delta} \left( \lambda s_2 [4 \beta^2 - \delta^2] + \beta \lambda \sigma_2 [2 \beta^2 - \delta^2] + \beta \theta \sigma_2 [\delta \beta] + \delta \theta \sigma_2 [\beta^2] \right) > 0. \tag{116}
\]

\[
\left( \frac{\partial R_D}{\partial \sigma_1} \right)_{\sigma_1 \neq \sigma_2} = -\frac{\beta f_1}{\Delta} \left( \delta t_2 [4 \theta^2 - \lambda^2] + \delta \theta \sigma_2 [2 \theta^2 - \lambda^2] + \beta \theta \sigma_2 [\theta \lambda] + \beta \lambda \sigma_2 [\theta^2] \right) > 0. \tag{117}
\]
Proof of proposition 2(b): that the qualitative results are unchanged when $\sigma_1 = \sigma_2 = \sigma$.

Remarks: (i) $\sigma_1$ does not appear anywhere in the numerators of equations (114) through (117).

(ii) The denominator, $\Delta$ is the same in equations (114) through (117).

(iii) $\sigma_1$ is featured only in the denominator, $\Delta$.

(iii) $\sigma_1 = \sigma_2 = \sigma$ does not change the algebraic sign of the denominator, $\Delta$.

Q.E.D.
References


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ENDNOTES

1. As most basic ideas of strategy and tactics found in the growing literature on strategic competition, this definition has roots in Schelling (1960).

2. The concept of exclusionary cost of regulatory avoidance was introduced into the literature in Kane (1981) to explain the role of technological innovation in decreasing the effectiveness of banking regulation.


6. At least one bank in Columbus Ohio now advertizes an innovative scheme for reducing consumer switching costs thus making it easier for rivals' customers to switch. In a telephone communication 15 August 1988, with Laura Thrime, Marketing Executive for Fifth Third Bank Columbus, she explained that the conclusion reached from a survey of potential customers is that a significant amount of inertia existed because "customers thought that changing banks was a hassle." In response to this, Fifth Third Bank now stands ready, at the potential deposit customer's option, to obtain a relatively simple checking account mandate plus an authorization that transfers to Fifth Third the responsibility of tracking the customer's outstanding checks at the previous bank, and obtaining the transfer of the account balance after all checks have cleared. This behavior, characteristic of an industry consisting of mutually interdependent firms reflects well the underlying prisoners' dilemma structure that confronts players in such situation - each financial intermediary has an incentive to lower switching costs for customers of competitors, but if all
incumbents do this, the outcome is a reduction in overall industry profits (in that all firms lose as a result of such cost reduction).


8. One can still recall various tax-free gifts from banks such as umbrellas, toasters, and tote bags.

9. See also Simonson (1986).

10. In contrast to the approach taken here, Kane and Malkiel (1965) distinguish between L and L* (L star) applicants. According to them, L applicants are customers whose past behavior is characterized by their tendency to maintain stable or improving relationships.


13. It is not clear how this liquidity risk that is absent ex ante arises ex post.


15. Details of the mathematical derivations of the monopoly equilibrium are displayed in Appendix A.

16. Mathematical derivation of the Perfect-Foresight Duopoly equilibrium is in Appendix B.

17. Mathematical derivation of the Nash equilibrium is in Appendix C.


21. Telerate of Telerate Systems Inc. (founded:1968) operates an international electronic network for the real-time distribution of money market and other financial data. The network also supplies Dow Jones news headlines as well as TELERATE's own news service. Additionally, TELERATE compiles historical financial data and
makes them available through commercial time-sharing companies. One noteworthy customer is the Board of Governors of the Federal Reserve System. Telephone conversation between this author and Debbie McMillan (Banking and Monetary Market Statistics) revealed that the Board relies on this service to compute its prime rate figures each business day.


22. For a record of FDIC transactions, see "Handling Bank Failures," Federal Deposit Insurance Corporation: The First Fifty Years (Washington D.C: Federal Deposit Insurance Corporation, 1984).


24. Mayberry, Nash, and Shubik (1953) provide a lucid example of this phenomenon.


29. This was easily incorporated into the computer program for the numerical examples.