Experimental Investigation of Judder in a
Floating Disc-Caliper Braking System with Focus on Pad Geometry

THESIS

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By
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Abstract

Brake judder in mid-size vehicles is characterized by 10 to 20 Hz brake torque variations on the order of 100 N-m when braking from speeds of approximately 120 kph. These vibrations can be perceived by the vehicle’s occupants through the seat track, chassis, steering wheel and/or brake pedal. The source of these vibrations is often attributed to thickness variations on the rotor surface which generate brake torque variations and brake pressure variations. This thesis suggests considerations for experimental studies as well as examines the effects of brake pad geometry on brake torque and brake pressure variations measured under laboratory conditions. A dynamic friction laboratory experiment has been constructed and instrumented for the purpose of investigating these specific circumstances at the source. This dynamic friction experiment consists of a motor-driven flywheel which stores 1/5th the kinetic energy of a mid-sized vehicle traveling at 115 kph. The flywheel is connected to a shaft terminating into a rotor. A floating caliper braking system affixed to the rotor dissipates the kinetic energy by means of friction at the disc-caliper interface. During the controlled braking event, dedicated instrumentation allows for quantification of brake torque variations and brake pressure variations under cold judder conditions.
A hypothesis related to changes in the pad-caliper contact stiffness due to changes in pad geometry is formulated from experimental results. Supporting evidence for this hypothesis is provided by means of pad bending strain measurement, computational stiffness calculations, and torque calculations using a four degree-of-freedom linear model of the judder source (floating caliper braking system). Analytical and experimental results suggest a strong correlation between computational caliper-pad contact stiffness values and levels of torque and pressure variation, when normalized with respect to the baseline pad. A recommendation for future work involves verification of the calculated contact stiffness values.
Dedication

I dedicate this thesis to my family and friends. It is through their endless support, encouragement, and love that the completion of this work has been made possible.
Acknowledgments

I would like to take this opportunity to acknowledge those who directly assisted my research efforts. Many thanks to my advisor Dr. Rajendra Singh for the opportunity, guidance, and advice he has provided. I would also like to thank Dr. Jason Dreyer and his dedicated assistance to The Ohio State University’s Acoustics and Dynamics Laboratory. Finally, I would like to thank my colleagues and all sponsors who in any way supported my research efforts.
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Field of Study

Major Field: Mechanical Engineering
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<td>$A$</td>
<td>area</td>
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<tr>
<td>$c$</td>
<td>viscous damping, kg/s</td>
</tr>
<tr>
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<td>damping matrix</td>
</tr>
<tr>
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### Subscripts

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</tr>
<tr>
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</tr>
<tr>
<td>d</td>
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</tr>
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<td>m</td>
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### Abbreviations

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<tr>
<td>BTV</td>
<td>brake torque variation</td>
</tr>
<tr>
<td>DTV</td>
<td>disc thickness variation</td>
</tr>
<tr>
<td>FRF</td>
<td>frequency response function</td>
</tr>
<tr>
<td>SD</td>
<td>standard deviation</td>
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Chapter I – Introduction

1.1 Motivation and Literature Review

Brake judder is defined as the vibration/shake experienced by a driver during a high-speed braking event, most often attributed to warped rotor surfaces. Research in this area is driven by the high costs incurred by the auto-industry which some estimate to total over one-billion dollars in customer warranties annually [1]. The driver can sense these variations that comprise the judder phenomena through the seat track, steering wheel, chassis, and brake pedal. Possible contributors to these variations include disc thickness variation, disc runout, uneven wear, corrosion, elastic deformations, changes in frictional contacts, uneven heating characterized by hotspots and coning, misalignments, as well as the phasing of torque variations between left and right side braking systems. Causes, observed effects, and analysis methods are summarized by Jacobsson [2, 3]; an extensive review of literature (as of 2003) has also been carried out by Jacobsson [2]. It has been generally accepted that disc thickness variation $u(t)$ is the main contributor to BTV. Although $u(t)$ is typically required produce judder, the parameters of a disc-caliper system can significantly contribute to overall BTV levels.

Most mathematical or computational models attempting to predict judder related BTV assume a uniform brake pad-rotor contact surface [4]. However, rotors which possess a
wavy braking surface will create a non-uniform contact area $A_b$ between it and the brake pad. This non-uniformity on the interfacial braking surface can affect heat distribution patterns on the rotor surface as well as the time-varying effective center of pressure [5]. Changes in the center of pressure can affect several parameters which can ultimately affect $T(t)$ and $p(t)$ levels. A simplified floating disc-caliper torque model similar to that described by Kang and Choi [4] can be used to illustrate the system’s sensitivity to parameters other than $u(t)$.

$$T(t) = 2\mu \cdot R_E \left( F_{n,m} + k_E \cdot u(t) \right)$$  \hspace{1cm} (1)

In this model, the coefficient of friction $\mu$, effective contact radius $R_E$, and mean normal force $F_{n,m}$ and effective stiffness $k_E$ are assumed to be constant while only $u(t)$ is allowed to vary with time, say as a sinusoidal function. The term $k_E$ can be represented as

$$k_E = \frac{1}{\frac{1}{k_h} + \frac{1}{k_{pist}} + \frac{1}{2k_a} + \frac{1}{2k_{pad}} + \frac{1}{k_{cal}}}$$  \hspace{1cm} (2)

which is the series combination of stiffness values for the hydraulic fluid $k_h$, pistons $k_{pist}$, pad abutments $2k_a$, brake pads $2k_{pad}$, and caliper body $k_{cal}$. These simplifications rely on the assumption that the system has identical properties on both sides of the rotor. In order to successfully predict $T(t)$, the model must be modified such that it represents system parameters more accurately. The goal of this research is to enhance the understanding of braking judder physics by investigating sensitivity of a disc-caliper braking system especially to the brake pad geometry.
1.2 Problem Formulation

The scope of this thesis is intended to focus on specific aspects of judder research which have not been adequately addressed by the existing literature [2-10]. Chapter I outlines considerations for the construction of a laboratory experiment specifically designed for the purpose of conducting brake judder research. The purpose of this dynamic friction test stand is to provide a means to measure the brake system’s response to a controlled simulated braking event with known inputs and parameters. Preliminary multi-physics and lumped parameter models of a disc-caliper braking system have been developed [4, 6]. Currently, torque variation within the braking system has been computationally investigated around nominal values of the system parameters. Although these models are useful for understanding judder sensitivity, they often rely heavily on many assumptions that may or may not represent the vehicle system. To assist modeling efforts, an experiment focused on investigating if brake pad geometry has an effect on brake torque and hydraulic pressure will be conducted. If an effect is observed, further studies will be conducted to investigate the possible mechanisms responsible.

1.3 Design Considerations

The experimental test stand constructed specifically for the proposed study is designed to quantify cold judder. The inertial scaling of the experiment is similar to that described and used by Beer [11] who investigated brake squeal. For instance, in a typical vehicle, the front brakes will account for absorbing approximately 3/4\textsuperscript{th} of the total kinetic energy. Using a single front disc-caliper system in the experiment, the total energy in the
The experiment is expressed here as follows where \( m_v \) is the total mass of the vehicle and \( v_v \) is a typical critical vehicle speed at which judder is known to develop.

\[
K_v = \frac{3}{8} [m_v v_v^2]
\]  

(3)

The rotational kinetic energy of the laboratory flywheel \( (K_f) \) can be expressed as follows where \( J_f \) is the mass moment of inertia and \( \omega_f \) is the angular velocity.

\[
K_f = \frac{1}{2} J_f \omega_f^2
\]  

(4)

To excite the braking system at similar frequencies, the initial angular velocity \( \omega_f \) of the flywheel should be equal that of the vehicle wheel \( \omega_v \). By equating \( K_f \) and \( K_v \), the flywheel inertia \( J_f \) is determined as

\[
J_f = \frac{3}{8} \left[ \frac{m_v v_v^2}{\omega_f^2} \right]
\]  

(5)

Using a \( v_v \) of 115kph and assuming a typical tire radius of 0.3m, the angular velocity was calculated to be \( \omega_f = \omega_v = 103.7 \text{ rad/s} \) or \( \Omega_f = \Omega_v = 990 \text{ RPM} \). Given a typical vehicle weight of \( m_v = 2,100 \text{ kg} \), the flywheel inertia \( J_f \) is 74.7 kg-m\(^2\). As Beer [11] outlines in his thesis, construction of a flywheel of this inertia would be expensive and unsafe to operate. For a more manageable experiment in terms of safety, cost, and fabrication, the flywheel inertia is scaled by a factor of \( 1/5 \), as defined below.

\[
\left[ J_f \right] = \frac{J_f}{5} = 14.9 \text{kg-m}^2
\]  

(6)

From here on it will simply be regarded as \( J_f \). The flywheel is to be constructed from A-36 steel of density \( \rho_f = 7,800 \text{ kg/m}^3 \) and yield strength of \( \sigma_y = 250 \text{ MPa} \). The
dimensions of a thin disc type flywheel are chosen after considering available stock dimensions, fabrication ease, cost, and safety. The selection of steel plate of stock thickness \( t_f = 4.45 \text{ cm} \) results in a flywheel of radius \( R_f = 40.64 \text{ cm} \) in order to achieve the calculated \( J_f \). To ensure the flywheel does not deform or shatter, the maximum principal stress when spinning at 1000 RPM \((\omega_f = 105 \text{ rad/s})\) will not exceed \( \sigma_i = \rho_f R_f^2 \omega_f^2 = 14.2 \text{ MPa} \) resulting in a factor of safety of 17.6 [12]. To ensure stability during rotation of the flywheel, the design, as shown in Figure 1 incorporates two automotive wheel hub bearings mounted into welded steel supports. A 5 hp AC vector motor is used to achieve shaft speed \( \Omega_s \) above 1000 RPM. Once this shaft speed is achieved, a disengagement coupler disconnects the motor from the flywheel to eliminate any motor dynamics. Due to the 1/5\(^{th}\) inertial scale of the experiment, generating and quickly transferring large amounts of thermal energy to the rotor surface is not possible. Therefore, the focus will be mainly on the topic of “cold” judder. Cold judder is classified as braking vibrations generated under “cold” disc thickness variation conditions or DTV which is already present on the rotor surface near ambient room temperature. This will provide an experimental platform to understand the effects of DTV, runout, pad shape, and hydro-mechanical coupling void of significant thermal effects. The experiment must also include a disc-caliper braking system and hydraulic brake actuation to supply the braking force. Quantification of dynamic torque, pressure, and rotational speed need consideration during the design of the test stand.
1.4 Torsional Modes of the Experiment

The torsional natural frequencies of the experiment must be determined to ensure they do not lie in the frequency range of interest (10-20 Hz) in order to get an accurate reading of the dynamic torque within this operating range. Once the motor has been decoupled, the system of Figure 1 can be simplified into a two degree-of-freedom torsional model as shown in Figure 2. This figure shows the flywheel inertia $J_f$ and rotor inertia $J_r$ connected by a shaft of torsional stiffness $k_s$. Automotive grade wheel bearings are located on both sides of the flywheel and a third bearing is housed within the knuckle on which the rotor is mounted as shown in Figure 1. These bearings restrict lateral movement of both the flywheel and rotor while allowing rotation.

![Figure 1. Schematic of the dynamic friction experiment](image-url)
The mass moment of inertia of the flywheel $J_f$ has been previously calculated while the rotor mass moment of inertia is obtained through CAD modeling as $J_r = 0.14 \text{ kg-m}^2$. A half shaft with corresponding splines to those found on the bearings will be used to couple the flywheel to the rotor. The torsional stiffness for shaft length $L_s$ and radius $R_s$ is

$$k_s = \frac{G_s I_s}{L_s}$$

where polar moment of inertia of the shaft is given as $I_s = \pi R_s^4/2 = 5.2 \times 10^{-8} \text{ m}^4$. Using a shear modulus of the steel shaft $G_s = 77 \times 10^9 \text{ Pa}$, shaft length $L_s = 0.7 \text{ m}$, and the shaft radius $R_s = 0.0135 \text{ m}$, $k_s$ is found to be $k_s = 5,650 \text{ N-m/ rad}$. Assuming all bearings provide ideal boundary conditions allowing only torsional motion, a lumped parameter torsional eigenvalue problem can be constructed. The boundary conditions at the rotor are controlled to simulate off-brake and applied brake scenarios. The torsional system, with
little clamping pressure applied to the rotor surface, is represented by setting the effective torsional stiffness of the braking system to $k_b = 0.1 k_s$. To represent the brake applied condition, a relatively high torsional stiffness of $k_b = 10 k_s$ reduces rotor movement with respect to ground. The effects on the natural frequencies ($f_1$ and $f_2$) and eigenvectors for these cases listed in Table 1 are based on $[M]\{\dot{\theta}\} + [K]\{\theta\} = \{0\}$ where $\{\theta\} = \{\theta_1, \theta_2\}^T$.

For clamped boundary conditions the natural frequencies are 2.9 Hz and 99.2 Hz. These predicted values are well outside the frequency range of interest (10-20 Hz). However, the slipping motion of the rotor relative to the brake pads can alter this boundary condition during a stop. Because of this, an experiment is required check if there is excitation of torsional modes or the support structure. Figure 3 is a waterfall FFT taken of torque data generated due to a constant 0.34 MPa brake apply. The diagonal lines represent the shaft speed ($\Omega_s$) and its harmonics ($2\Omega_s, 3\Omega_s$) as the system decelerates from 16.5 Hz (1000 RPM) to 0 Hz. The vertical band shown on the right of Figure 3 is a clear indication that the 31 Hz resonance predicted for the un-applied brake condition persists regardless of the clamped end condition. The excitation occurs when the 2nd and 3rd harmonics of shaft speed pass through the resonance frequency.
Table 1. Torsional natural frequencies ($f_1$ and $f_2$) and eigenvectors ($\{\theta_1 \theta_2\}^T$) of the experiment with two values of torsional caliper stiffness given $L_s = 0.7$ m

<table>
<thead>
<tr>
<th>$k_b$ case</th>
<th>$f_1$ (Hz)</th>
<th>${\theta_1}^T$</th>
<th>$f_2$ (Hz)</th>
<th>${\theta_2}^T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_b = 0.1k_s$</td>
<td>1.0</td>
<td>[1.0 0.9]</td>
<td>31.5</td>
<td>[-0.01 1.0]</td>
</tr>
<tr>
<td>$k_b = 10k_s$</td>
<td>2.9</td>
<td>[1.0 0.1]</td>
<td>99.2</td>
<td>[-0.001 1.0]</td>
</tr>
</tbody>
</table>

Figure 3. Time-frequency diagram of the torque with a shaft of $L_s = 0.7$ m

The system can be tuned by modifying the shaft stiffness in an attempt to reduce the unwanted effect of the system resonance. To shorten the shaft would have involved special tooling, a long lead time, and additional costs. Another option is to lengthen to $L_s = 1.4$ m which required a simple in-house fabrication of a coupler made from steel. Lengthening of the shaft reduces its stiffness to $k_s = 2,869$ N-m/rad. The eigenvalue solution when $L_s = 1.4$ is shown in Table 2.
Table 2. Effect Torsional natural frequencies ($f_1$ and $f_2$) and eigenvectors ($\{\theta_1 \theta_2\}^T$) of the experiment with two values of torsional caliper stiffness given $L_s = 1.4$ m

<table>
<thead>
<tr>
<th>$k_b$ case</th>
<th>$f_1$ (Hz)</th>
<th>${\theta_1}^T$</th>
<th>$f_2$ (Hz)</th>
<th>${\theta_2}^T$</th>
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<td>$k_b = 0.1k_s$</td>
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<td>${1.0 \ 0.9}$</td>
<td>24.9</td>
<td>${-0.01 \ 1.0}$</td>
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<tr>
<td>$k_b = 10k_s$</td>
<td>2.1</td>
<td>${1.0 \ 0.1}$</td>
<td>104</td>
<td>${-0.001 \ 1.0}$</td>
</tr>
</tbody>
</table>

Figure 4. Time-frequency diagram of torque with a shaft $L_s = 1.4$ m

The lengthened shaft lowers the system resonance to 25 Hz. This lower frequency allows for more than 2 seconds of data with minimal resonant participation once the brake is applied at 0.34 MPa. Figure 5 shows the effect of length on the torque time history before and after the shaft modification. The torque signal with $L_s = 1.4$ m is now clear of any resonances in the system up to 2 seconds. This length of data is adequate for experimental studies and analysis of pad geometry effects.
Figure 5. Effect of shaft length ($L_s$) on measured torque time history $T(t)$

The data acquisition and measurement transducers used in experiments are as follows: To capture torque variation, a full bridge strain-based torque transducer is used as shown in Figure 6. As torque is applied to the shaft, the voltage difference induced across the bridge is sensed and sent wirelessly via radio waves to a receiver which filters and amplifies the signal. The signal is then sent to the NI-CompactDAQ data acquisition system before it is sent to a PC for recording and post processing. Similar to the torque measurement, the use of strain gauges can be utilized to measure the deflection of components under load. This will be utilized in the pad geometry investigation. An
inductance probe sensing the lug nuts as they pass by is used as a tachometer for angular speed measurement. Initial temperature is logged using an infrared thermometer to record thermal variation between data sets. The CompactDAQ system provides simultaneous data sampling to ensure synchronized data. A full schematic of the power supplies, measurement devices, actuation devices and data acquisition system is shown in Figure 10. Table 3 gives more detailed description of the components depicted in Figure 10.

Figure 6. Full bridge strain based torque transducer, strain gauges (1-4) applied at 45° from shaft centerline for cancellation of bending effects.

The hydraulic system consists of a thermally compensated metal diaphragm type pressure transducer with a pressure sensing range of 0 to 6.8 MPa. The pressure transducer is strain based and therefore will measure both mean and alternating components of the hydraulic line pressure. The sensor is placed in line with the hydraulic system just after the master cylinder as shown in Figure 7.
Figure 7. Pressure transducer in line with hydraulic line and master cylinder

Spectrograms of time varying pressure data $p(t)$ are shown in Figure 8 ($L_s = 0.7$ m) and Figure 9 ($L_s = 1.4$ m). The shaft speed and harmonics are clearly visible but no hydraulic resonances are found.
Figure 8. Time-frequency plot of pressure with a shaft of $L_s = 0.7$ m

Figure 9. Time-frequency plot of pressure with a shaft of $L_s = 1.4$ m
Figure 10. Schematic of power supplies, transducers, actuators, power supplies, and data acquisition components

Table 3. Details of Figure 10 components

<table>
<thead>
<tr>
<th>Sensor or component</th>
<th>Specifications</th>
<th>Corresponding NI-DAQ C series module</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tachometer (inductance)</td>
<td>Omega: LD701-5/10 Noncontact linear displacement sensor</td>
<td>NI 9485</td>
</tr>
<tr>
<td>Torque transducer</td>
<td>Binsfeld: Torquetrak 10K torque telemetry system</td>
<td>NI 9239</td>
</tr>
<tr>
<td>Pressure transducer</td>
<td>Honeywell:MLH01KPSB06A</td>
<td>NI 9239</td>
</tr>
<tr>
<td>Pad strain gauges</td>
<td>Omega: 350 ohm, uniaxial, Gage Factor ~ 2.1</td>
<td>NI 9236</td>
</tr>
<tr>
<td>Solenoid (pneumatic brake actuator)</td>
<td>Mead MB12-3CSC, 24V, 3 way, normally closed</td>
<td>NI 9485</td>
</tr>
<tr>
<td>CompactDAQ chassis</td>
<td>National Instruments: cDAQ-9172</td>
<td>All C series modules</td>
</tr>
</tbody>
</table>
Chapter II – Dynamic Measurements under Judder Conditions

2.1 Objectives

Using the test stand described in Chapter I, a correlation between $T(t)$ and $p(t)$ and modifications of brake pad geometry is experimentally investigated to better understand the pad-rotor source regime. The pad geometry will be physically altered to change the area in contact at the rotor and pad friction surface interface. The objectives of the investigation are as follows: 1. Determine if pad geometry affects peak-to-peak values of the $T(t)$ and $p(t)$; 2. If a significant effect is observed, identify trends in data and gather supporting evidence which can be utilized in the development of a hypothesis explaining the effect.

2.2 Preliminary Experiments

To investigate the effects of pad geometry on judder, seven unique pad shapes (designated 0-6) will be used as shown in Figure 11. The unmodified full pad is used as a baseline (designated as set 0) for comparison. In addition to pad sets 0-6, an additional unmodified full pad set is used to maintain the rotor temperature at a steady state temperature of 70°C ±3°C between all experimental runs. A quasi-steady state is achieved after 5-6 stopping events due to the low amount of energy transferred and convective cooling during the speed up cycle. The pads have beveled portions removed
to observe the effects of contact pressure when the pad contact geometry is limited to the annular direction (1, 2), radial direction (3-5), and both annular and radial directions (6).

Since the variation between pad sets is of importance when comparing data sets, a series of baseline (set 0) test runs is needed to ensure that each pad set yields similar dynamic characteristics. Four of the pad sets are randomly selected in their unmodified state for comparison of peak-to-peak $T(t)$ and $p(t)$ levels. A constant deceleration stop is performed by applying a constant actuation pressure. The normalized $T(t)$ and $p(t)$ data for 4 sets are shown in Figure 12 and Figure 13 respectively. Both $T(t)$ and $p(t)$ are divided by the mean pressure of the stop as a means to remove any effects due to a drift in mean pressure between runs. For peak-to-peak comparison purposes, the mean is removed from each $T(t)$ and $p(t)$ data set and the peak-to-peak values are normalized from -1 to 1 by a common data set. Although the same rotor and caliper is used for all pad sets, there is a clear difference in the shape of the time-domain signals. This can be attributed to minute variations in the pad surface roughness, surface profile, and perhaps friction material composition. Despite this variation in the time-history curves of $T(t)$ and $p(t)$, the peak-to-peak amplitudes are very consistent between four pad sets. Thus the peak-to-peak metrics denoted $T_a$ (N-m/Pa) and $p_a$ (Pa/Pa) will be used as the basis of comparison for pad geometry studies.
Figure 11. Illustration of pad geometries studied; here the red area indicates where abrasive material remains. They are referred to set 1 to 6. In addition set 0 is the baseline (unmodified pad).
Figure 12. Normalized $p(t)$ variations of four random baseline pads (set 0)

Figure 13. Normalized $T(t)$ variations of four random baseline pads (set 0)
The next step in verifying the consistency of data is ensuring that frictional properties do not change significantly after several consecutive braking events and among the various pad sets used. This is done by calculating the effective coefficient of friction ($\mu_E$) for several stops from 1000 RPM at a mean brake line pressure of $p_{P,m} = 0.34$ MPa. The effective coefficient of friction $\mu_E$ is defined below as:

$$
\mu_E = \frac{(J_f + J_r)\Delta\omega_f}{2R_EA_p p_{P,m}\Delta t}
$$

(8)

The flywheel and rotor inertia’s $J_f$ and $J_r$ have been previously determined. Normal force is calculated as the product of piston area $A_p$ and mean pressure $p_{P,m}$ acting at an assumed effective braking radius $R_E$. The assumed location of the effective radius is at the midpoint of the pad material in contact with the rotor in the radial direction of the rotor. The change in angular velocity ($\Delta\omega$) is defined as over a $\Delta t$ time interval. Figure 14 shows the calculated value of $\mu_E$ approaching its limit over several runs. It is important to note that this $\mu_E$ value approaches an asymptotic limit without any significant variation in frictional properties after 3 to 4 braking events. This is a desired quality given that a consistent $\mu_E$ reduces the amount of variability between runs and thus reduces uncertainty in results. Figure 15 shows the reproducibility of $\mu_E$ for seven different pad sets which will be used in the pad geometry study.
Figure 14. Effective coefficient of friction $\mu_E$ approaching an asymptotic limit after several braking events

Figure 15. Effective coefficient of friction $\mu_E$ reproducible for the 7 pad sets
2.3 Effect of Pad Modification on Torque and Pressure Variations

To further the understanding of brake judder physics, a series of experiments to compare the effect of brake pad geometry on $T_a$ and $p_a$ levels is carried out. Through utilization of the dynamic friction test stand the experiment is conducted by employing the following steps: 1. A unique brake pad set (say set i) from those geometries described in section 2.2 is inserted into the test stand caliper; 2. The motor is engaged and the flywheel-rotor system is brought up to an angular speed of $\Omega_f = 1100$ RPM; 3. Once this angular speed is achieved the motor is disengaged and the data acquisition system is set to trigger once the brake pressure is applied; 4. Before $\Omega_f < 1000$ RPM, a solenoid is actuated which causes the pneumatic cylinder to apply a constant force on the hydraulic master cylinder connected to the caliper. The hydraulic pressure is preset to have a steady state pressure of $p_h = 0.34$ MPa which dissipates the kinetic energy via friction at the rotor-pad interface. The floating caliper braking system is shown in with brake pads. Time domain signals for $T(t)$ and $p(t)$ are recorded for each unique pad set. The same rotor is used for all test runs which possesses a second order sinusoidal thickness variation with peak-to-peak amplitude of $20 \mu$m.

Comparisons of $T(t)$ and $p(t)$ for set i to the unmodified baseline pad (set 0) are show in Figure 17 and Figure 18. As in the preliminary test stand experiments, scaling effects due to shifts in mean pressure are removed from all data sets of $T(t)$ and $p(t)$ by dividing by the mean pressure. Displayed results have been normalized with respect to the baseline pad set 0. Each modified set (set 1,…6) is indicated by red and the unmodified
set (set 0) is shown in blue. Pad shapes 1 and 2 are augmented by beveling the leading and trailing edges which result in a significant reduction in overall $T_a$ and $p_a$ levels. Pad set 6 combines removal of pad material in both the radial and circumferential directions and also demonstrates a similar decrease in both torque and pressure variation of over 40 percent. A decrease of up to 25 percent in $p_a$ with minimal effect on $T_a$ is observed for pad geometries 3 and 4 which focus on pad geometry in the radial direction. The least effective of these geometries in reducing $T_a$ and $p_a$ levels is pad set 4 which retains most of its material at the outer most rotor radius towards the base of the caliper fingers. Bar graph summaries of the peak-to-peak torque and pressure levels are shown in Figure 19 and Figure 20. Standard deviation (SD) of each pad set is also indicated using a sample size of 5 runs per pad geometry.

Figure 16. Schematic of floating caliper braking system with brake pads
Figure 17. Normalized pressure variation plots for unique pad geometries compared to full pad variation.
Figure 18. Normalized torque variation plots for unique pad geometries compared to full pad variation
Figure 19. Summary of normalized peak-to-peak pressure values for 6 pad geometries (set 1,..6) and the baseline (set 0) with error bars representing standard deviation

Figure 20. Summary of normalized peak-to-peak torque values for 6 pad geometries (set 1,..6) and the baseline (set 0) with error bars representing standard deviation
Table 4. Summary of peat-to-peak torques and pressure for modified pads corresponding to Figure 19 and Figure 20

<table>
<thead>
<tr>
<th>Set</th>
<th>$T_a$ (SD)*</th>
<th>$p_a$ (SD)*</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline (0)</td>
<td>1.00 (0.07)</td>
<td>1.00 (0.03)</td>
<td>Used to normalize results</td>
</tr>
<tr>
<td>Modified Sets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.70 (0.05)</td>
<td>0.59 (0.03)</td>
<td>Largest $p_a$ reduction</td>
</tr>
<tr>
<td>2</td>
<td>0.62 (0.07)</td>
<td>0.54 (0.03)</td>
<td>$p_a$ reduction without $T_a$ reduction</td>
</tr>
<tr>
<td>3</td>
<td>0.96 (0.08)</td>
<td>0.72 (0.05)</td>
<td>$p_a$ reduction without $T_a$ reduction</td>
</tr>
<tr>
<td>4</td>
<td>0.97 (0.04)</td>
<td>0.79 (0.03)</td>
<td>$p_a$ reduction without $T_a$ reduction</td>
</tr>
<tr>
<td>5</td>
<td>0.84 (0.06)</td>
<td>0.75 (0.02)</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.57 (0.07)</td>
<td>0.57 (0.02)</td>
<td>Largest $T_a$ reduction</td>
</tr>
</tbody>
</table>

* SD = standard deviation

Although sets 1, 2, and 6 have the largest effects in terms of torque and pressure variation, it is unclear whether this is due to the piston side pads, finger side pad interactions, or a combination of both. To better understand the effects of piston and finger side pad geometry interactions an additional experiment is required.

In this series of experiments, set 2 will be examined due to its significant influence. This geometry is also chosen to study because it the modifications are limited to the annular direction which should be easier to analyze later. This experiment will combine the modified pad with its unmodified counterpart. Again, $T_a$ and $p_a$ levels will be of main interest. The two configurations are shown in Figure 21 and Figure 22. For configuration A, the orientation of the pads is such that the modified pad (set 2) is located on the piston side and the unmodified pad (set 0) is on the finger side of the caliper. For configuration B the locations of the pads are reversed. The data collected from configuration A is shown in Figure 23. Under the conditions set for configuration A, the
pad set 2 is not effective in reducing $T_a$ and $p_a$. However, Figure 22 shows a significant reduction in variations for configuration B. In summary, experimental results show that pad shape does affect $T_a$ and $p_a$ values. In addition to applying a modification to the leading and trailing edges of the pad geometry, location within the caliper (finger or piston) is of significant importance in determining peak-to-peak values of $T(t)$ and $p(t)$. 
Figure 21. Configuration A, modified pad (set 2) on piston side

Figure 22. Configuration B, modified pad (set 2) on finger side

Figure 23. Normalized $T(t)$ and $p(t)$ variations of configuration A

Figure 24. Normalized $T(t)$ and $p(t)$ variations of configuration B
2.4 Hypothesis

The objective is now directed towards gaining an understanding of the results of section 2.3. In other words, the goal is to understand why the modified pad on the finger creates significantly more reduction in $T_a$ and $p_a$ values in comparison to its placement on the piston side. A hypothesis as to why the finger side of the caliper is more sensitive to changes in pad geometry than the piston side, specifically in the annular direction, is formulated below; it will be tested in section 2.5.

*Hypothesis:* When the caliper is pressurized, the pistons apply a normal load which is transmitted through the rotor and carried by the three caliper fingers. By varying the finger side pad geometry, the normal load distribution to the three fingers is altered. An illustration of this concept is shown in Figure 25 for pad set 0 and pad set 2. For pad set 0, the loading is directly opposite each finger compressing the pad material between the rotor surface and the three caliper fingers. This type of load can result in a high stiffness due to the high compressive stiffness of the brake material as well as the uniform load distribution to all three fingers. When the load is not directly opposite a finger, the load is transmitted indirectly to the finger by a shear force resulting in a bending moment within the pad material. This indirect force transmission can lead to non-uniform loading of the caliper fingers. If the three caliper fingers are not loaded uniformly, the effectiveness of each finger in terms of a spring action can decrease as a result. In the case of pad set 2, modifications applied to the leading and trailing edges should cause the pad to bend at the leading and trailing locations. An increase in a centralized load
leading to greater central deflection (center finger location) due to an increase in bending is illustrated in the static deflection study of a beam in Figure 26.

Figure 25. Finger side loading for full pad (left) and modified pad (right)

Figure 26. Representation of beam with fixed end conditions deflecting as a result of a uniform load (right) and a centralized load (left)
2.5 Strain Study

To test this hypothesis of uneven pad loading, strain gauges are applied to the backing plate of a full pad and the modified pad (set 2). Strain induced by the bend of the backing plate will be compared between the full pad and modified pads. Based on static calculations using a finite element model shown in Figure 27, the strain gauges are located close to the regions of maximum strain when compressed against a fixed boundary condition approximating the caliper finger contact geometry. The locations of the strain gauges are show in Figure 29. Data is collected using the same experimental procedure as described in Section 2.3 with the exception of the use of instrumented pads in configurations A and B of Figure 21 and Figure 22. Figure 28 is window of time-domain strain $e(t)$ with superimposed torque $T(t)$ and pressure $p(t)$ signals to show how the finger side strain data compares to these signals. The periods of strain, torque and pressure are related by shaft speed ($\Omega_s$). Pressure and strain signals exhibit a second order of $\Omega_s$ which matches the second order profile of $u(t)$. Torque is a much smoother sinusoidal curve close to resembling a first order signal with respect to $\Omega_s$.

![Figure 27](image.png)

Figure 27. Static finite element analysis of a compressed pad with finger and piston side boundary conditions for determination of maximum strain locations
Figure 28. Zoomed view of strain data for comparison of strain $e(t)$ to synchronized torque $T(t)$ and pressure $p(t)$ signals for configuration B

The results of the strain experiments using configurations A and B are shown in Table 5 and Figure 29. An increase in both mean and dynamic amplitudes of the finger side strain $e_F(t)$ measurement strongly suggests a significant increase in pad bending when configuration B is used. This verifies the hypothesis that the center finger is loaded only when the finger side pad is modified (set 2).

As the next step, the effects observed experimentally via $T_a$ and $p_a$ values due to pad geometry will be analytically modeled. Using finite the element analysis, effective caliper stiffness will be calculated for different loading conditions as affected by changes in pad geometry. The stiffness values will then be used to compare relative $T_a$ levels calculated using a lumped parameter model in Chapter III.
Table 5. Pad microstrain measurements $\mu e(t)$ during a braking event for configuration A (piston side: pad set 2, finger side: pad set 0)

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Mean Microstrain, $\mu e_a$</th>
<th>Peak-to-Peak Microstrain, $\mu e_{\text{pm}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Piston Side</td>
<td>Finger Side</td>
</tr>
<tr>
<td>A (Figure 21)</td>
<td>Lead 3</td>
<td>Trailing -8</td>
</tr>
<tr>
<td>B (Figure 22)</td>
<td>0</td>
<td>-6</td>
</tr>
</tbody>
</table>

Figure 29. Comparison of measured microstrain levels for configurations A and B
Chapter III: Analytical Experiments and Comparison

3.1 Overview of Modeling Approach

Using analytical and computational approaches, effects of changes in the contact stiffness introduced by pad modifications and caliper-finger interaction will be investigated using a static finite element model [13]. A linearized lumped parameter model of a disc-caliper braking similar to one by Leslie [6] will be utilized though it has been modified. Torque variations due to a displacement input at the rotor surface $u(t)$ will be examined while varying the contact stiffness. A comparison of experimental and mathematical studies will be used to better understand the concepts developed in Chapter II. These studies will focus on pad geometry effects on caliper-pad contact stiffness and how changes in contact stiffness can affect torque variation levels. Results from experiment and model are then compared in order to gain physical understanding while simultaneously evaluating the usefulness of the model to represent system behavior related to pad geometry effects.
3.2 Stiffness Calculation

As mentioned in section 1.1, the effective braking stiffness $k_E$ is the combination of several stiffness values representing all components which contribute to applying the braking force to the rotor surface. Therefore, a portion of $k_E$ involves the contact stiffness resulting from the caliper-pad interface. By means of a finite element code (COSMOS within SolidWorks) [13], the effective caliper-pad contact stiffness $k_{con}$ is calculated for each pad and caliper finger contact case (set 0-6). For each case, the caliper and pad are combined into a solid body model (assuming gray iron as the representative material) and then meshed. A static normal load of $F_N = 100$ N is distributed at the back of the caliper piston wells, and a fixed boundary condition is applied to the pad surface as indicated in Figure 30. Using a static deflection study, the effective caliper-pad contact stiffness is calculated below using

$$k_{con} = \frac{F_N}{\Delta z}$$

(9)

where $\Delta z$ is the deflection in $z$-direction (normal) which is then computationally averaged over the surface as displayed in Figure 31. The stiffness ($k_{con}$) values found using the static analysis are listed in Table 6. To determine if the static stiffness is dependent on the static load magnitude, static loads of $F_N = 500$ N and $F_N = 1,000$ N are also applied and the static stiffness is calculated. This study concludes that the computational static stiffness using the finite element code has no dependence on load in this range of loading.
Using the same finite element method used to find \( k_{\text{con}} \), the normalized caliper body stiffness without a pad \( k_{\text{cal}} \) is calculated to be \( k_{\text{cal}} = 2.22 \, k_{\text{con}} \). Assuming the caliper body and pad act as individual springs in series, their total stiffness \( k_{\text{con}} \) is expressed as

\[
\frac{1}{k_{\text{con}}} = \frac{1}{k_{\text{cal}}} + \frac{1}{k_{\text{pad}}}
\]

(10)

where \( k_{\text{cal}} \) and \( k_{\text{pad}} \) are the nominal the stiffness values of the caliper and pad respectively.

Solving for \( k_{\text{pad}} \) we obtain

\[
k_{\text{pad}} = \frac{k_{\text{cal}} \, k_{\text{con}}}{k_{\text{cal}} - k_{\text{con}}}
\]

(11)

By inserting \( k_{\text{cal}} = 2.22 \, k_{\text{con}} \) into equation (11) the nominal value for the pad stiffness is found to be \( k_{\text{pad}} = 1.85 \, k_{\text{con}} \). These nominal values found for \( k_{\text{pad}} \) and \( k_{\text{cal}} \) will be used in the lumped parameter dynamic model and scaled to match the \( k_{\text{con},i} \) value as reported in Table 6.

Table 6. Caliper contact stiffness values based on static finite element analysis

<table>
<thead>
<tr>
<th>Set</th>
<th>Normalized static contact stiffness ( k_{\text{con},i} )</th>
<th>Normalized caliper contact stiffness ( k_{\text{cal},i} )</th>
<th>Normalized pad contact stiffness ( k_{\text{pad},i} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Baseline Pad Set 0</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 0 )</td>
<td>( k_{\text{con},0} = k_{\text{con}} ) \quad (nominal value)</td>
<td>( k_{\text{cal},0} = k_{\text{cal}} = 2.22 , k_{\text{con}} ) \quad (nominal value)</td>
<td>( k_{\text{pad},0} = k_{\text{pad}} = 1.85 , k_{\text{con}} ) \quad (nominal value)</td>
</tr>
<tr>
<td><strong>Modified Pad Set</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( i = 1 )</td>
<td>( k_{\text{con},1} = 0.66 , k_{\text{con}} )</td>
<td>( k_{\text{cal},1} = 1.47 , k_{\text{con}} )</td>
<td>( k_{\text{pad},1} = 1.22 , k_{\text{con}} )</td>
</tr>
<tr>
<td>( i = 2 )</td>
<td>( k_{\text{con},2} = 0.52 , k_{\text{con}} )</td>
<td>( k_{\text{cal},2} = 1.15 , k_{\text{con}} )</td>
<td>( k_{\text{pad},2} = 0.96 , k_{\text{con}} )</td>
</tr>
<tr>
<td>( i = 3 )</td>
<td>( k_{\text{con},3} = 0.70 , k_{\text{con}} )</td>
<td>( k_{\text{cal},3} = 1.55 , k_{\text{con}} )</td>
<td>( k_{\text{pad},3} = 1.30 , k_{\text{con}} )</td>
</tr>
<tr>
<td>( i = 4 )</td>
<td>( k_{\text{con},4} = 0.77 , k_{\text{con}} )</td>
<td>( k_{\text{cal},4} = 1.71 , k_{\text{con}} )</td>
<td>( k_{\text{pad},4} = 1.42 , k_{\text{con}} )</td>
</tr>
<tr>
<td>( i = 5 )</td>
<td>( k_{\text{con},5} = 0.74 , k_{\text{con}} )</td>
<td>( k_{\text{cal},5} = 1.64 , k_{\text{con}} )</td>
<td>( k_{\text{pad},5} = 1.37 , k_{\text{con}} )</td>
</tr>
<tr>
<td>( i = 6 )</td>
<td>( k_{\text{con},6} = 0.44 , k_{\text{con}} )</td>
<td>( k_{\text{cal},6} = 0.98 , k_{\text{con}} )</td>
<td>( k_{\text{pad},6} = 0.81 , k_{\text{con}} )</td>
</tr>
</tbody>
</table>
Figure 30. Boundary conditions for the static finite element analysis using baseline pad set 0 (left) and modified pad set 2 (right) located on the finger side of the caliper.

Figure 31. Typical caliper deflections ($\Delta z$) using baseline pad (set 0) and modified pad (set 2) on the finger side of caliper.
3.3 Development of a Lumped Parameter Dynamic Model

The static stiffness contact as calculated in section 3.2 can now be embedded in a lumped parameter dynamic model. An illustration of the disc-caliper braking system and hydraulics is shown in Figure 32. Assuming the first natural frequency of the hydraulic actuation system is well above the operating frequency of 15 Hz, the entire hydraulic system is reduced to one element of bulk stiffness $k_h$. This bulk stiffness represents the combined compliance of the fluid within the piston chamber, hydraulic line, main brake chamber, and piston seal. This assumption should be valid as long as the inertial effects of the hydraulic system do not dominate. With this simplification, a linear 4-degree dynamic model is used to describe the system. Figure 33 shows a schematic of this system where the lumped masses include the piston mass $m_{pist}$, the piston side and finger side brake pad masses $m_{pad,P}$ and $m_{pad,F}$, and the caliper body mass $m_{cal}$. Spatial coordinates consist of the piston motion $x_{pist}$, the piston side and finger side pad motions $x_{pad,P}$ and $x_{pad,F}$, and the caliper body motion $x_{cal}$. The piston and finger side rotor surface motions $x_{rot,P}$ and $x_{rot,F}$ as displayed in Figure 33 are provided as inputs. Along with the combined hydraulic system stiffness $k_h$, the lumped stiffness values include the piston stiffness $k_{pist}$, the pad stiffness $k_{pad,P}$ and $k_{pad,F}$, the guide pin stiffness $k_{pin}$, the pad abutment stiffness $k_a$, and the caliper bridge stiffness $k_{cal}$. Lastly, the viscous damping coefficients include the hydraulic system damping $c_h$, the piston damping $c_{pist}$, the pad damping $c_{pad}$, the guide pin damping $c_{pin}$, the pad abutment damping $c_a$, and the caliper bridge damping $c_{cal}$. The most significant sources of damping are associated with $c_h$ and $c_{pad}$. All other damping coefficients are assumed to be negligible for our model.
Figure 32. Representation of disc-caliper braking system and hydraulic system components

Figure 33. Simplified 4 degree-of-freedom dynamic model of the system of Figure 32
The governing equations of motion are derived below using the 4 degree-of-freedom dynamic model of Figure 33.

\( m_{cal} \ddot{x}_{cal} = -k_{cal} (x_{cal} - x_{pad,P}) - k_h (x_{cal} - x_{pist}) - k_{pin} x_{cal} \)  \hspace{1cm} (12)

\( m_{pad,P} \ddot{x}_{pad,P} = -k_{pad,P} (x_{pad,P} - x_{rot,P}) - k_{pist} (x_{pad,P} - x_{pist}) - k_a x_{pad,P} \)  \hspace{1cm} (13)

\( m_{pist} \ddot{x}_{pist} = -k_h (x_{pist} - x_{cal}) - k_{pist} (x_{pist} - x_{pad,P}) \)  \hspace{1cm} (14)

\( m_{pad,F} \ddot{x}_{pad,F} = -k_{pad} (x_{pad,F} - x_{rot,F}) - k_{cal} (x_{pad,F} - x_{cal}) - k_a x_{pad,F} \)  \hspace{1cm} (15)

These equations can then be written in matrix form with mass matrix \( M \), stiffness matrix \( K \), damping coefficient matrix \( C \), displacement vector \( X \) and force vector \( F \).

\[
\left[ -\omega^2 M - i\omega C + K \right] \ddot{X}(\omega) = \ddot{F}(\omega)
\]  \hspace{1cm} (16)

\[
X = \begin{bmatrix} x_{cal} \\ x_{pad,P} \\ x_{pist} \\ x_{pad,F} \end{bmatrix}
\]  \hspace{1cm} (17)

\[
K = \begin{bmatrix} (k_{cal} + k_h + k_{pin}) & 0 & -k_h & -k_{cal} \\ 0 & (k_{pist} + k_{pad,P} + k_a) & -k_{pist} & 0 \\ -k_h & -k_{pist} & k_h + k_{pist} & 0 \\ -k_{cal} & 0 & 0 & k_{pad,F} + k_{cal} + k_a \end{bmatrix}
\]  \hspace{1cm} (18)

\[
C = \begin{bmatrix} (c_{cal} + c_h + c_{pin}) & 0 & -c_h & -c_{cal} \\ 0 & (c_{pist} + c_{pad} + c_a) & -c_{pist} & 0 \\ -c_h & -c_{pist} & c_h + c_{pist} & 0 \\ -c_{cal} & 0 & 0 & c_{pad} + c_{cal} + c_a \end{bmatrix}
\]  \hspace{1cm} (19)

\[
F = \begin{bmatrix} 0 \\ k_{pad,P} x_{rot,P} \\ 0 \\ k_{pad,F} x_{rot,F} \end{bmatrix}
\]  \hspace{1cm} (20)
The eigenvalue problem utilized in section 1.4 to predict torsional modes will now be applied to predict natural frequencies and mode shapes of the 4 degree-of-freedom braking system model. The mode shapes and natural frequencies are shown in Table 7.

Table 7. Natural frequencies and mode shapes of the 4 degree-of-freedom dynamic model

<table>
<thead>
<tr>
<th></th>
<th>1&lt;sup&gt;st&lt;/sup&gt; mode</th>
<th>2&lt;sup&gt;nd&lt;/sup&gt; mode</th>
<th>3&lt;sup&gt;rd&lt;/sup&gt; mode</th>
<th>4&lt;sup&gt;th&lt;/sup&gt; mode</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(r = 1)$</td>
<td>$(r = 2)$</td>
<td>$(r = 3)$</td>
<td>$(r = 4)$</td>
</tr>
<tr>
<td>$f_{n,r}$ (Hz)</td>
<td>445</td>
<td>1,227</td>
<td>2,548</td>
<td>13,187</td>
</tr>
<tr>
<td>${\theta}<em>r = \begin{bmatrix} x</em>{cal} \ x_{pad,P} \ x_{pist} \ x_{pad,F} \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0.00 &amp; -0.05 &amp; 0.00 \ 0.03 &amp; -0.98 &amp; 0.00 &amp; 1.00 \ 0.03 &amp; -1.0 &amp; 0.00 &amp; 0.75 \ 0.47 &amp; 0.01 &amp; 1.00 &amp; 0.00 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0.00 &amp; -0.05 &amp; 0.00 \ 0.03 &amp; -0.98 &amp; 0.00 &amp; 1.00 \ 0.03 &amp; -1.0 &amp; 0.00 &amp; 0.75 \ 0.47 &amp; 0.01 &amp; 1.00 &amp; 0.00 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0.00 &amp; -0.05 &amp; 0.00 \ 0.03 &amp; -0.98 &amp; 0.00 &amp; 1.00 \ 0.03 &amp; -1.0 &amp; 0.00 &amp; 0.75 \ 0.47 &amp; 0.01 &amp; 1.00 &amp; 0.00 \end{bmatrix}$</td>
<td>$\begin{bmatrix} 1 &amp; 0.00 &amp; -0.05 &amp; 0.00 \ 0.03 &amp; -0.98 &amp; 0.00 &amp; 1.00 \ 0.03 &amp; -1.0 &amp; 0.00 &amp; 0.75 \ 0.47 &amp; 0.01 &amp; 1.00 &amp; 0.00 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

The proportional damping using a damping ratio $\zeta$ of 1% is used to represent a lightly damped system to calculate the $C$ matrix using

$$C = [\theta^T]^{-1} [2\zeta \omega_n][\theta]^{-1}$$

where $r$ is the mass index. In addition to these proportional damping coefficients an approximated damping coefficient associated with the hydraulic fluid and piston seal $c_{pist}$ is calculated and the damping associated with the brake pads $c_{pad}$ is experimentally obtained through the use of a compression test machine. These values are appropriately inserting into $C$ according to equation (19). The frequency response functions $X_{pad,P}/X_{rot,P}$, $X_{pad,F}/X_{rot,P}$, $X_{pad,P}/X_{rot,F}$ and $X_{pad,F}/X_{rot,F}$ are shown before and after the $C$ matrix is augmented. It can be seen that the lightly damped system does little to affect the resonance peaks. However the augmented system which uses approximated damping values reduces all modes and increases the systems frequency response magnitudes in the...
low frequency range (0-50 Hz) which can be seen in Figure 35. This model also shows a weak coupling between the piston and finger sides of the system as the frequency response functions show that $x_{pad,F}/x_{rot,P} \ll x_{pad,F}/x_{rot,F}$ and $x_{pad,F}/x_{rot,F} \ll x_{pad,P}/x_{rot,P}$. As seen in Table 7 and illustrated in the frequency response functions in Figure 34, the caliper finger side dynamics dominate the lowest frequency mode, suggesting that the low frequency response of the system will be most sensitive to modifications on the caliper finger side.
Figure 34. Effects of augmented damping matrix on frequency response functions on the 4 degree-of-freedom dynamic model.
Figure 35. Effects of augmented damping matrix on frequency response functions on the 4 degree-of-freedom dynamic model on first resonance \( (f_1 = 445 \text{ Hz}) \)

Using the 4 degree-of-freedom dynamic model to calculate \( T_a \), we observe that the \( T_a \) levels associated with the augmented damping matrix \( C \) are much closer to values found in experiment compared to those found using only the proportional damping matrix where \( \zeta = 0.01 \). For this reason the augmented \( C \) matrix will be used in the calculation of \( T_a \) levels for the remainder of this section. An excitation frequency of \( f_{exe} = 30 \text{ Hz} \) is chosen to approximate a second order \( u(t) \) at the onset of braking from \( \Omega_f = 1000 \text{ RPM} \)
which represents the conditions found in experiment. Rotor surfaces $x_{rot,P}$ and $x_{rot,F}$ are the locations at which a sinusoidal displacement due to $u(t)$ excites the system. The displacement amplitudes are of equal and opposite magnitude representing DTV. A total displacement amplitude of 20 $\mu$m will be achieved through simultaneous displacements of $x_{rot,P} = -10$ $\mu$m and $x_{rot,F} = 10$ $\mu$m. Assuming harmonic excitation $F(t) = \tilde{F} e^{i\omega t}$ where $\omega$ is the excitation frequency, the complex valued displacement response $\tilde{X}(t) = \tilde{X} e^{i\omega t}$ is solved for using matrix inversion.

$$\tilde{X}(\omega) = \left[-\omega^2 M - i\omega C + K\right]^{-1} \tilde{F}(\omega) \quad (22)$$

Dynamic response is then calculated. With $|\tilde{X}(\omega)|$, the braking torque variation $T_a(\omega)$ is calculated using a simple torque equation written here as

$$T_a(\omega) = R_E \mu_E (F_{k_{pad,P}}(\omega) + F_{k_{pad,F}}(\omega)) \quad (23)$$

where the force acting on $k_{pad,P}$ is

$$F_{k_{pad,P}}(\omega) = k_{pad,P}(x_{rot,P} - x_{pad,P}) \quad (24)$$

and the force acting on $k_{pad,F}$ is

$$F_{k_{pad,F}}(\omega) = k_{pad,F}(x_{rot,F} - x_{pad,F}) \quad (25)$$

The effective radius is assumed to be $R_E = 0.14$ m and the effective coefficient of friction $\mu_E = 0.4$. Results for peak-to-peak $T_{a,i}$ levels corresponding to pad modification sets $i = 1, \ldots, 6$ are shown in section 3.4. The $T_{a,i}$ values are normalized with respect to a nominal baseline $T_a$ and compared in experimental results of section 2.3 Figure 36.
3.4 Comparison of Analytical Calculations and Measurements

For purposes of comparing results from the experimental and analytical studies, $T_a$ levels are normalized relative to a nominal torque for the baseline pad. It is important to note that the dynamic model is used here only for purposes of investigating the relative effects of contacts stiffness on torque variation. Exact torque levels can be affected by changes in other parameters due to pad modifications which are beyond the scope of this model. Affected parameters may include effective radius and coefficient of friction which are assumed to be constant in this study.

The normalized torque values $T_{a,i}$ where $i = 1, \ldots, 6$ are shown in Table 8. The results indicate that both calculated and experimental $T_a$ levels are sensitive to changes in pad geometry, particularly to pad modifications in the annular direction. This supports the hypothesis of Chapter II that a decrease in the contact stiffness value will result in a decrease in the torque variations.

Table 8. Experimental and modeled $T_{a,i}$ levels with finger side pad modifications

<table>
<thead>
<tr>
<th>Set</th>
<th>Measurement</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$T_{a,i}$ Level</td>
<td>$T_{a,i}$ Level</td>
</tr>
<tr>
<td>Baseline Pad Set 0</td>
<td>Experiment (nominal value)</td>
<td>Dynamic Model (nominal value)</td>
</tr>
<tr>
<td>$i = 0$</td>
<td>$T_{a,0} = 1.0 \ T_a$</td>
<td>$T_{a,0} = 1.0 \ T_a$</td>
</tr>
<tr>
<td>Modified Pad Set $i$</td>
<td>$T_{a,i} = 0.70 \ T_a$</td>
<td>$T_{a,i} = 0.79 \ T_a$</td>
</tr>
<tr>
<td>$i = 1$</td>
<td>$T_{a,1} = 0.62 \ T_a$</td>
<td>$T_{a,2} = 0.68 \ T_a$</td>
</tr>
<tr>
<td>$i = 2$</td>
<td>$T_{a,2} = 0.96 \ T_a$</td>
<td>$T_{a,3} = 0.81 \ T_a$</td>
</tr>
<tr>
<td>$i = 3$</td>
<td>$T_{a,3} = 0.97 \ T_a$</td>
<td>$T_{a,4} = 0.86 \ T_a$</td>
</tr>
<tr>
<td>$i = 4$</td>
<td>$T_{a,4} = 0.84 \ T_a$</td>
<td>$T_{a,5} = 0.84 \ T_a$</td>
</tr>
<tr>
<td>$i = 5$</td>
<td>$T_{a,5} = 0.57 \ T_a$</td>
<td>$T_{a,6} = 0.63 \ T_a$</td>
</tr>
</tbody>
</table>

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Figure 36. Comparison of torque variation levels between experiment and dynamic model for 6 pad geometries (set 1,…6) and the baseline (set 0)

A plot of normalized torque variation found experimentally and analytically against the computationally found caliper-pad contact stiffness for each pad case is shown in Figure 37. Because the two cases for modified pad sets 3 and 4 have pad material between the rotor and all three caliper fingers, a high compressive stiffness results. These pad sets also have material removed toward the inner most rotor radius which may cause the effective radius at which the normal force is applied to increase and therefore increase the normalized torque variation levels. Adjusting the radius in the model for these two cases may improve the predictive capacity of the model with respect to the experimental data. The measurement of a dynamic effective radius will be addressed in future experimental studies using the dynamic friction test stand. Both a linear and polynomial regression
models have been used to curve fit experimental data. From the results it is unclear which provides a better match because they both have a \( R^2 \) value 0.89.

![Normalized Torque Variation Graph](image1)

Figure 37. Comparison of normalized torque variation between experiment and 4 degree-of-freedom dynamic model

![Normalized Torque Variation Graph](image2)

Figure 38. Comparison of linear and polynomial curve fit of experimental data
Chapter IV: Conclusion

4.1 Summary and Contributions

This thesis documents the construction of the dynamic friction experiment which was used to investigate the response of a disc-caliper system with modifications to brake pad contact area. Torque and pressure measurements were made under laboratory conditions to provide a basis for comparing relative disc-caliper system sensitivity resulting from various pad geometry modifications. From the pad geometry experiments, a hypothesis was formed to explain the mechanisms related to changes in $T_a$ and $p_a$ levels. Supporting evidence of this hypothesis was gathered using pad strain measurements of the brake pad backing during a braking event and static finite element analyses of the caliper and modified pads. Furthermore, an analytical dynamic model was formulated to predict torque variation on a normalized basis.

A goal of this study was to investigate if pad geometry affects $T_a$ and $p_a$ levels. Measured $T_a$ and $p_a$ levels prove that the braking system is considerably sensitive to finger side pad geometry modifications. A change in the caliper-pad contact stiffness has been identified as largest the contributing factor for changes in system response.

The likely mechanism responsible for the significant reduction in $T_a$ and $p_a$ levels is the reduction in contact stiffness which occurs due to a non-uniform loading between the pad
and caliper fingers. When friction material is removed from leading and trailing edges of the finger side brake pad a compressive load is carried by the center finger while a load is applied to the outer fingers due to the bending of pad. Strain measurements and computational calculations using a finite element model demonstrate that these conditions can lead to lower caliper-pad contact stiffness. This reduction in contact stiffness generates lower normal forces due to DTV which can reduce $T_a$ and $p_a$ levels.

The purpose of the analytical model was to validate concepts derived experimental results. Predictions from the model reasonably match normalized levels in torque reduction found in experimental studies. As seen in both experiment and model, a change in pad geometry can dramatically change the caliper-pad contact stiffness and reduce $T_a$ levels. Although reducing the caliper stiffness has been shown here to reduce $T_a$ and $p_a$ levels, it is important to consider all aspects of the braking system including pedal feel and pad wear.

Further work is needed due to several sources of error which can affect the accuracy of predicted $T_a$ levels. When calculating $T_a$ levels, a possible source of error comes from a possible shift in the effective braking radius $R_E$ due to pad contact area modifications. Several sources of error involve in the contact stiffness calculation using the finite element model. These error sources are introduced during meshing of the model, use of a uniform-body caliper-pad model, as well as assuming an isotropic material. Error can also be caused by damping and stiffness matrices which are largely based on calculated and estimated values. For these reasons, the dynamic model is not used to predict exact
torque variation levels. Instead, it is utilized to predict relative system behavior due to changes in caliper-pad contact stiffness. The significant influence of damping on $T_a$ levels found when developing dynamic model suggests that further investigation is needed to better quantify $R_E$, stiffness and damping values.

4.2 Future Work

The dynamic friction test stand should be modified to continue efforts in expanding our understanding of brake judder physics. A modular tool head for caliper mounting can be replaced to accommodate a variety of caliper braking systems. Investigations related to the attenuation of torsional resonances can be accomplished by adjusting the shaft length. The inclusion of a vehicle suspension system consisting of the strut, lower ball joint, and steering rod may be useful for investigations of the coupling between the caliper and suspension. Additional dedicated instrumentation can also be introduced to help better capture judder phenomena. Runout and DTV can be captured through the use of inductance probes placed near the rotor surface. A modified caliper instrumented with load cells behind the brake pads and at the pad abutments can be used to measure normal and tangential loads for time-varying center of contact and coefficient of friction estimations. These types of time-varying estimations will aid in future investigations attempting to better understand brake judder phenomena. An improved understanding of judder physics along with the direct measurement of damping and contact stiffness values found by means of compression testing will undoubtedly prove useful as research efforts move towards a true predictive model.
References


