TRANSIENT VIBRO-ACOUSTIC CALCULATIONS AND MEASUREMENTS WITH APPLICATION TO RECTANGULAR PLATES AND ANNULAR RODS

A THESIS

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ABSTRACT

Since commercial noise prediction software codes usually do not provide transient radiation calculation capability, there is a need to develop a time domain based computational tool for the prediction of sound pressures radiated by vibrating structures subjected to impulsive or transient loads. Analytically, sound pressure radiated from structures can be predicted in the time domain using either Kirchhoff integral or Rayleigh integral provided the surface acceleration time history is given. Solving Kirchhoff integral is more difficult compared to Rayleigh integral but the Rayleigh integral is valid for only flat surfaces in baffle. A MATLAB program to predict the sound pressure radiated from flat surfaces in a baffle is developed by numerically solving the Rayleigh integral using boundary element techniques. The MATLAB program is first quantitatively validated for a rigidly vibrating plate radiator using a frequency domain formulation which assumes sinusoidal acceleration distribution across the plate.

A laboratory experiment is then performed on elastic structures. This consists of an impact hammer, an accelerometer and a microphone and the structure is suspended to simulate the free-free condition. Four elastic structures include a rectangular plate and three annular rods of different dimension and material. The data is acquired in the time domain and then analyzed in the frequency domain using transfer functions. The structural transfer function relates the excited force and surface acceleration in the
frequency domain while the acoustic transfer function relates excited force and measured pressure. The structural transfer function is dependent on the mechanical properties like mass, stiffness and damping constant of the system. The acoustic transfer function depends on the mechanical properties, distance of observation point from the elastic structure, surface area of structure and also, on the properties of the medium. Time domain analysis is then performed using structural and acoustic impulse responses. The impulse responses are obtained using inverse Fourier transform of the transfer functions. The structural and acoustic impulse responses are responses for acceleration and pressure when the excitation force is an impulse function. For determination of computational acoustic impulse response, measured structural impulse response is redistributed on the plate. Phasing between vibrating elements is obtained using the structural wavelength for a particular mode shape in the beam. This modal phasing is determined using Euler’s beam theory. Computational acoustic impulse response using measured structural impulse response (with and without modal phasing) is compared with measured acoustic impulse response for the elastic structures. When observed with phasing, the difference between peak-to-peak computational and measured acoustic impulse response is less for the annular rods. The experimental comparison for the time domain program suggests that this program offers a reasonable prediction of peak to peak sound pressures. Detailed explanations are provided for the experimental comparison with computational results, and the sources of error are discussed.
This is dedicated to my parents, my adorable sis and all of my wonderful friends …
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LIST OF SYMBOLS

Nomenclature

A, B, C, D    type of test sample

a    radius of sphere

B    coefficient dependent on value of current time step

c    speed of sound

d    diameter

E    Young’s modulus of material

e    exponential function

F    force

f    frequency (Hz)

G    Green’s function

g, h    acoustic and structural impulse responses

H    transfer function in frequency domain

J    Jacobi-determinants of the co-ordinate transformation

k    wave number

l    length

m    mode number

N1, N2, N3, N4    shape functions

N    number of sample points
n total number of time steps
b width
P sound pressure in frequency domain
p sound pressure in time domain
PI participation index
q volume velocity
r distance
S surface area in a acoustic field
s cross sectional area
t current time
T time period
u (t) unit step function
W weight for numerical integration
w thickness
x, y, z Cartesian coordinates
\dot{Z}, \ddot{Z} surface velocity and acceleration in frequency domain
\dot{z}, \ddot{z} surface velocity and acceleration in time domain
\Delta t time step (increment)
\beta coefficient relating space and time
\phi peak value at the resonant mode
\( \chi \) constant used in mode shape

\( \delta \) unit impulse function

\( \lambda \) structural wavelength for mode shape

\( \theta \) phasing from mode shape

\( \omega \) frequency (rad/s)

\( \gamma \) greatest distance between nodes of an element

\( \xi, \eta \) coordinates of master element

\( \rho \) density

\( \sigma \) constant used in mode shape

\( \tau \) duration of pulse

\( \psi \) orientation angle between the dipole axis and observation point

**Subscripts**

A acceleration

a air

anl analytical

avg average

com computational

cpt computational peak-to-peak using time domain code

cpf computational peak-to-peak using frequency domain code
dip dipole
<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>exp</td>
<td>experimental</td>
</tr>
<tr>
<td>e</td>
<td>element</td>
</tr>
<tr>
<td>F</td>
<td>force</td>
</tr>
<tr>
<td>eff</td>
<td>effective</td>
</tr>
<tr>
<td>gre</td>
<td>Green’s function</td>
</tr>
<tr>
<td>fft</td>
<td>obtained from FFT</td>
</tr>
<tr>
<td>i</td>
<td>location for impact hammer</td>
</tr>
<tr>
<td>iff</td>
<td>obtained from IFFT</td>
</tr>
<tr>
<td>IP</td>
<td>integration point</td>
</tr>
<tr>
<td>j</td>
<td>location for accelerometer</td>
</tr>
<tr>
<td>m</td>
<td>mode number</td>
</tr>
<tr>
<td>mex</td>
<td>using modal expansion</td>
</tr>
<tr>
<td>mtl</td>
<td>material (structure)</td>
</tr>
<tr>
<td>mon</td>
<td>monopole</td>
</tr>
<tr>
<td>o</td>
<td>observation point</td>
</tr>
<tr>
<td>p-p</td>
<td>peak-to-peak</td>
</tr>
<tr>
<td>s</td>
<td>source</td>
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**Superscripts**

- . . : time derivatives
- - : dimensionless
- ~ : complex number
<table>
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<tr>
<th>Abbreviation</th>
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<tbody>
<tr>
<td>a</td>
<td>acceleration</td>
</tr>
<tr>
<td>act</td>
<td>actual contribution from plate</td>
</tr>
<tr>
<td>bgd</td>
<td>background noise</td>
</tr>
<tr>
<td>com</td>
<td>computational</td>
</tr>
<tr>
<td>exp</td>
<td>experimental</td>
</tr>
<tr>
<td>f</td>
<td>frequency domain code</td>
</tr>
<tr>
<td>gre</td>
<td>Green’s function</td>
</tr>
<tr>
<td>i</td>
<td>inner</td>
</tr>
<tr>
<td>m</td>
<td>mode number</td>
</tr>
<tr>
<td>o</td>
<td>outer</td>
</tr>
<tr>
<td>p</td>
<td>pressure</td>
</tr>
<tr>
<td>p-p</td>
<td>peak to peak</td>
</tr>
<tr>
<td>rig</td>
<td>rigid piston</td>
</tr>
<tr>
<td>s</td>
<td>sampling frequency</td>
</tr>
<tr>
<td>sur</td>
<td>due to surroundings</td>
</tr>
<tr>
<td>t</td>
<td>time domain code</td>
</tr>
</tbody>
</table>

**Operators**

\[ \int \] \text{integral operator}

\[ < >_s \] \text{spatial averaging}

\[ \square^2 \] \text{D’Alembert operator}
\( \mu \) Fourier transform

\( \mu^{-1} \) inverse Fourier transform

Im imaginary

Re real

**Abbreviations**

BEM boundary element method

DAQ data acquisition system

FEM finite element method

FFT fast Fourier transform

FDTD finite difference time domain

FRF frequency response function

IFFT inverse fast Fourier transform

SPL sound pressure level
CHAPTER 1 INTRODUCTION

1.1 Motivation

Transient sounds radiated by vibrating structures are often undesirable. Examples for general acoustic transients are impact noises such as the one created by the opening and closing of injectors during fuel injection [1], a baseball striking a bat and other acoustic transients induced from machinery [2, 3]. Due to the introduction of legislation with regards to noise limits in factories, Richards et al. [2, 3] have worked in the prediction of impact noise. The impact noise has been classified into two parts, namely, acceleration noise [2] and ringing noise [3]. Acceleration noise [2] concentrates on the noise generated by impacting bodies due to high surface accelerations during contact period with the absence of flexural motion. Ringing noise [3] is the noise associated from the subsequent free vibration. The above discussed [1, 2, 3] are specific cases and they are not applicable to vibrating structure of different shapes. Thus there is a need to develop a time domain based computational tool for prediction of sound pressures radiated by vibrating elastic structures subjected to impulsive or transient loads.

1.2 Research Issues

Formulation of the analytical or computational (including boundary element) methods for transient acoustic problems involves making use of Green’s function [4]; it is the impulse response of the scalar wave equation [5]. One relatively straightforward (but a brute force) method to calculate the transient sound pressure would
be to utilize the existing frequency domain analysis in conjunction with the Fourier transform theory. The transient excitation could be resolved into its individual frequency components via the Fast Fourier Transform (FFT) method, analysis could then be performed in the frequency domain at each frequency and finally the time domain response could be reconstructed by using the Inverse Fast Fourier Transform (IFFT) method. Although this is a good first step, this approach has several limitations. The frequency content of a finite duration time signal (such as a rectangular pulse) would be inversely related to the signal duration. For a very short duration pulse, analysis must be performed at a large number of frequencies including the phase calculation. In addition, several signal processing issues such as the selection of appropriate sampling rate, windowing function, block size, etc. must be addressed. Windowing is not required when the pressure is calculated directly in the time domain and hence, the waveform is not distorted and better fidelity is maintained.

1.3 Literature Survey

A detailed literature survey was conducted to assess the current state of transient sound pressure level prediction. Very few analytical solutions for complex radiators are available in the literature and available solutions for ideal radiators include dipole, monopole and piston radiator in a baffle [5]. Analytically, sound pressure radiated from structures can be predicted in the time domain using either Kirchhoff integral or Rayleigh integral provided the surface acceleration time history is given. The piston radiator, as discussed by Pierce [5], is a simple case where an impulsively accelerated circular piston in a baffle radiates sound in a hemi - spherical space. The pressure is analytically predicted using the Rayleigh integral assuming surface acceleration as an
impulse function and results for transient pressure are given in the near and far field regions. Adrian et al. [6] have predicted acoustic pressure fields from irregular multi-sided polygons in an infinite rigid baffle with uniform velocity distribution across the surface by using the Rayleigh integral method. They have performed convolution theorem to impulse response (time-stepped function) and the time derivative of velocity to predict acoustic pressure fields. Schedina et al. [7] compared numerical and experimental results for transient sound fields from impacted plates. A finite difference scheme is used for the plate vibration model in time domain and the Rayleigh integral equation is numerically solved using the boundary element method for prediction of radiated acoustic pressure. A pulsed two-reference-beam holographic interferometry technique is employed by Schedina et al. [7] for transient sound pressure measurements. From the above studies, it is clear that Rayleigh integral equation can be used to calculate the transient acoustic fields radiated by flat radiators.

1.4 Problem Formulation

The Kirchhoff integral assumes the surface of the structure to be constructed with monopoles and dipoles [5]. If the vibrating surface is flat and placed in an infinite baffle, it consists of only monopoles since the net force produced due to dipoles is zero. Based on this assumption, the Rayleigh integral is deduced from Kirchhoff integral [5]. Even though the Kirchhoff integral is applicable for any shape, the Rayleigh integral is valid for only flat surfaces in baffle though it is easier to solve.

The transient sound pressure $p(\vec{r}_{so}, t_{eff})$ at the observation point ‘O’, as radiated by a vibrating elastic structure in a baffle, can be predicted using the Rayleigh Integral [4, 5]:

3
\[ p(\vec{r}_{so}, t_{eff}) = \int \frac{\rho_a}{2\pi r_{so}^2} z(\vec{r}_s, t) dS \]  
(1.1)

where S is the surface area of the vibrating source, \( \rho_a \) is density of air, \( c_a \) is speed of sound in air, \( t_{eff} = t - r_{so} / c_a \); \( r_{so} / c_a \) is the retarded time, \( \ddot{z}(t) \) is the normal surface acceleration for impulse excitation and \( r_{so} \) is the distance between the source and observation point ‘O’; refer to Figure 1.1. The distance \( r_{so} \) is obtained by integrating numerically the distance of observation point from all the elements on the surface and Equation 1.1 is numerically solved to calculate \( p(t) \) using \( \ddot{z}(t) \).

Figure 1.1 Transient radiation from a baffled plate with surface acceleration \( \ddot{z}(r_s,t) \) where \( \vec{r}_s \) is the position vector for source and \( O(\vec{r}_{so}) \) is the observation point. Key: (a) Side view, (b) front view.

Chief goal is the development of a Rayleigh integral time domain program (in MATLAB) for prediction of \( p(t) \) from a flat shaped vibrating elastic structure using...
the boundary element technique as discussed by Araujo et al. [8]. An elastic structure has many modes of vibration but only the first few will be analyzed. The elastic structures considered for experimental studies include a flat rectangular plate and annular rods of different dimension and material. Experimental results will be analyzed in both frequency and time domains. Eigenvectors and mode shapes will be used from the modal analysis of beams for determining the phasing between the elements of vibration. Then, computational results will be compared with measured results and the sources of error will be discussed.

The list of assumptions is: 1. The distance $r_{so}$ from observation point to either of any two surface points on opposite sides of the plate has the same value since the plate is assumed to be infinitesimally thin. 2. For analytical purpose, the flat plate will be assumed to be in a baffle during the experiment. 3. During experiment, the effect of surroundings on the measurements will be assumed to be negligible. 4. For the computational model, measured structural impulse response is assumed on the plate with reasonable logic. 5. Phasing obtained from the Euler’s beam theory will be used for rectangular plate assuming it behaves like a beam when observed as a one dimensional structure. 6. For experimental comparison, we assume the surface to be flat. In the case of annular rods an equivalent flat rectangular strip of same dimensions is assumed instead of a curved surface.

Specific objectives are: 1. Develop a MATLAB program for prediction of transient sound pressure radiated from vibrating elastic flat surfaces in a baffle by solving the Rayleigh integral numerically. 2. Validate the program for an elastic plate radiator using both frequency and time domain approaches for sinusoidal acceleration distribution.
across the plate. 3. Conduct experiment on a rectangular plate and three annular rods using impact hammer, accelerometer and microphone and analyze the data in time and frequency domains. 4. Determine modal phasing between vibrating elements in a beam using the structural wavelength for a particular mode shape. 5. Compare computational acoustic impulse response using measured structural impulse response (with and without modal phasing) with measured acoustic impulse response for the elastic structures.
CHAPTER 2 ANALYTICAL AND COMPUTATIONAL METHODS

2.1 Ideal Acoustic Sources

Text books such as Pierce [5] and Reynolds [9] provide analytical solutions for transient sound pressure radiated by monopole, dipole and a rigid piston. These analytical solutions are derived from the wave equation where $\Box^2$ is the D’Alembert operator.

$$\Box^2 p(r_{so}, t) = 0.$$  \hspace{1cm} (2.1)

Reynolds [9] has expressed the transient sound pressure $p_{\text{mon}}(r_{\text{mon}}, t_{\text{eff}})$ at a radial distance $r_{\text{mon}}$ from the monopole (a spherically diverging symmetric source of sound); it is given by

$$p_{\text{mon}}(r_{\text{mon}}, t_{\text{eff}}) = \frac{\rho_a q(t)}{4\pi r_{\text{mon}}},$$ \hspace{1cm} (2.2)

where $q(t)$ is the volume velocity of the monopole, $r_{\text{mon}}/c_a$ is the retarded time, $t_{\text{eff}} = t - r_{\text{mon}}/c_a$ is the effective retarded time, $c_a$ is the speed of sound in air and $\rho_a$ is the density of air. A dipole is a combination of two equal and opposite monopoles when the separation distance between them is small; its transient sound pressure $p_{\text{dip}}(r_{\text{dip}}, t_{\text{eff}})$ radiated in a spherical space is given by
\[ p_{\text{dip}}(r_{\text{dip}}, t_{\text{eff}}) = \frac{1}{4\pi} \left[ \frac{F_{\text{dip}}(t)}{r_{\text{dip}}^2} + \frac{1}{c_a} \frac{F_{\text{dip}}'(t)}{r_{\text{dip}}} \right] \cos \psi. \] (2.3)

where \( F_{\text{dip}} \) is the axial force acting along the dipole axis, \( r_{\text{dip}} / c_a \) is the retarded time, \( t_{\text{eff}} = t - r_{\text{dip}} / c_a \) is the effective retarded time, \( \psi \) is the orientation angle between the dipole axis and observation point and \( r_{\text{dip}} \) is the radial distance between the dipole and the observation point. A monopole is usually referred as a pulsating sphere whereas a dipole is referred as a transversely oscillating sphere. Since we are interested in transient sources, we will describe \( q(t) \) and \( F_{\text{dip}}'(t) \) via impulse function and then calculate \( p(t) \) for both monopole and dipole in time and frequency domains.

### 2.2 FFT/IFFT Method

For a pulsating sphere (monopole), assume that

\[ q(t) = 4\pi a^2 \left( -u(t) + u(t + \tau_{\text{mon}}) \right) \]

where \( u(t) \) is the unit step function, \( \tau_{\text{mon}} \) is the duration of the rectangular pulse for monopole and \( a \) is the radius of sphere. Equation 2.2 becomes

\[ p_{\text{mon}}(r_{\text{mon}}, t_{\text{eff}}) = \rho_a a^2 \left( \frac{d(u(t) - u(t + \tau_{\text{mon}}))}{dt} \right). \] (2.4)

The Fourier Transform \([\mu]\) of Equation 2.4 yields

\[ P_{\text{mon}}(r_{\text{mon}}, \omega) = \mu \left( p_{\text{mon}}(t - r_{\text{mon}} / c_a) \right) = \frac{\rho_a a^2 \left( e^{-j\omega r_{\text{mon}}/c_a} - e^{-j\omega(r_{\text{mon}}/c_a + \tau_{\text{mon}})} \right)}{r_{\text{mon}}}. \] (2.5)
Equation 2.4 is plotted in the frequency domain after performing FFT using MATLAB which yields $p_{\text{mon}}^{\text{fft}}(\omega)$. It is then compared with Equation 2.5. Equation 2.5 is then plotted in the time domain after performing IFFT using MATLAB which gives $p_{\text{mon}}^{\text{iff}}(t)$ and compared with Equation 2.4. In the case of monopole, $a = 1 \text{ m}$, $\rho_a = 1.3 \text{ kg m}^{-3}$, $c_a = 350 \text{ m s}^{-1}$, $r_{\text{mon}} = 3.5 \text{ m}$, $\tau_{\text{mon}} = 0.04 \text{ s}$, sampling frequency $f_s = 1 \text{ kHz}$ and number of sample points $N = 2^{14}$ for both FFT and IFFT.

For a dipole, assume that $F_{\text{dip}}(t) = u(t) - u(t + \tau_{\text{dip}})$ and $\psi = 0$, where $\tau_{\text{dip}}$ is the duration of the rectangular pulse for dipole. Equation 2.3 becomes

$$p_{\text{dip}}(r_{\text{dip}}, t_{\text{eff}}) = \frac{1}{4\pi} \left[ \frac{u(t) - u(t + \tau_{\text{dip}})}{r_{\text{dip}}^2} + \frac{1}{c_a} \left( \delta(t) - \frac{\delta(t - \tau_{\text{dip}})}{r_{\text{dip}}} \right) \right]. \quad (2.6)$$
The Fourier Transform \([\mu]\) of Equation 2.6 yields

\[
P_{\text{dip}}(r_{\text{dip}}, \omega) = \mu \left( p_{\text{dip}} \left( t - \frac{r_{\text{dip}}}{c_a} \right) \right)
= \frac{1}{4\pi} \left[ \frac{e^{-j\omega r_{\text{dip}}/c_a} - e^{-j\omega (r_{\text{dip}}/c_a + \tau_{\text{dip}})}}{j\omega r_{\text{dip}}} + \frac{1}{c_a} \left( e^{-j\omega r_{\text{dip}}/c_a} - e^{-j\omega (r_{\text{dip}}/c_a + \tau_{\text{dip}})} \right) \right].
\] (2.7)

Equation 2.6 is plotted in the frequency domain after performing FFT using MATLAB which yields \(P_{\text{dip}}^{\text{fft}}(\omega)\) and compared with Equation 2.7. Equation 2.7 is then plotted in time domain after performing IFFT using MATLAB which gives \(P_{\text{dip}}^{\text{ift}}(t)\); it is compared with Equation 2.6. In the case of dipole, \(c_a = 350 \text{ m/s}\), \(r_{\text{dip}} = 3.5 \text{ m}\), \(\tau_{\text{dip}} = 0.02 \text{ s}\) and number of sample points \(N = 2^{14}\) for both FFT and IFFT.

Figure 2.2 Transient sound pressure for a dipole in time and frequency domains with sampling frequency \(f_s = 1 \text{ kHz}\). (a) Time domain (b) frequency domain. Key: (a) \(\), \(P_{\text{dip}}^{\text{fft}}, \text{Pa}\); (b) \(\), \(P_{\text{dip}}^{\text{ift}}, \text{Pa}\).
Figure 2.3 Transient sound pressure for a dipole in time and frequency domains with sampling frequency $f_s = 64$ kHz. (a) Time domain (b) frequency domain. Key: (a) , $p_{\text{dip}}^{\text{t}}, Pa$; , $p_{\text{dip}}^{\text{iff}}, Pa$; (b) , $P_{\text{dip}}^{\text{fft}}, Pa$; , $P_{\text{dip}}^{\text{iff}}, Pa$.

The FFT and IFFT transforms for monopole and dipole are performed to compare sound pressure for ideal radiators in time and frequency domains. Figure 2.1 suggests that the transient sound pressures for a monopole match well in time and frequency domains. The FFT/IFFT method works well for a monopole given impulse volume acceleration. Figure 2.2 suggests that for a dipole with an impulse excitation in term of $F_{\text{dip}}^{\text{t}}$, there is a slight difference in the sound pressure in time domain between $p_{\text{dip}}^{\text{t}}(t)$ and $p_{\text{dip}}^{\text{iff}}(t)$. In the frequency domain however, there is a considerable difference at lower frequencies though the difference decreases with an increase in frequency. Overall, the FFT/IFFT method does not work well for a dipole at lower frequencies for $f_s = 1$ kHz. The discrepancy in Figure 2.2 can be decreased if we were to use a higher sampling frequency as shown in Figure 2.3 which exhibits the Gibbs phenomenon. It states that a more accurate representation of the function can be achieved in the frequency
domain by using more number of terms in the Fourier series expansion which is equivalent to the sampling frequency in this case. Even though the results match well in the frequency domain, there is a considerable difference in the time domain. We can observe that the difference between sound pressures in time domain have increased for $f_s = 64$ kHz when compared to $f_s = 1$ kHz. Thus the FFT/IFFT method poses a computational disadvantage when the two transforms are performed using MATLAB. If the FFT/IFFT method does not work well for a simple dipole whose transient sound pressure expression is known, it may not be feasible for a complex source. Thus one must calculate the transient pressure directly in the time domain since much of the waveform could be lost if we were to perform two computational transforms.

### 2.3 Computational Methods

Alternate numerical techniques such as Finite-Difference Time Domain (FDTD), Finite Element Method (FEM) and Boundary Element Method (BEM) have been employed to solve acoustic field equations and related disciplines [8, 11, 12]. The FDTD [11] is a popular computational electrodynamics technique used primarily to solve the Maxwell’s equations which consist of a set of four partial differential equations that relate electro-magnetic fields to their sources, charge density and current density. The FEM [12] numerically finds an approximate solution of the partial differential equation as well as of integral equations but it is mostly used for steady state structural dynamic problems.

The BEM solves a set of linear partial differential equations by reformulating them as integral equations [8]. In our case, the governing partial differential equation is the linear wave equation given by Equation 2.1. Using boundary conditions,
the integral equation obtained is the Rayleigh integral with boundary specified in terms of the surface acceleration [8]. Other reasons for using BEM are [8]: 1. It is a surface discretization method and a time varying acceleration distribution across the surface can be easily implemented. 2. Non-uniform acceleration distribution can be described. 3. It is applicable to any field problems as long as Green’s function is available. The Green’s function criterion places considerable restrictions on the range of problems to which the BEM can be successfully applied [8].

2.4 Numerical Form of Rayleigh Integral

With reference to Equation 1.1, from Araujo et al. [8], the Rayleigh integral for sound pressure in the numerical form is written as follows

\[
p(r_{so}, t_{eff}) = \sum_{m=1}^{n} \sum_{e=1}^{n_e} B^{(n-m+1)} \sum_{q=1}^{4} N_q(\xi, \eta) dS_e z^{(m)}. \tag{2.8}
\]

where \(t_{eff}\) is the effective retarded time and the current time \(t = n \Delta t\); \(n\), the total number of time step increments \(\Delta t\), \(n_e\) is the total number of elements on the surface, \(m\) and \(e\) are arbitrary constants, \(N_1, N_2, N_3\) and \(N_4\) are the shape functions, \(S_e\) is the surface area of the current element, \(\xi, \eta\) are coordinates of the master element and

\[
B^{(n)} = \frac{P_u}{4\pi r_{so}^3} \begin{cases} 
(n-1) c_u \Delta t < r_{so} < nc_u \Delta t \\
0, \text{ otherwise.} 
\end{cases}
\tag{2.9}
\]

Now Equation 2.8 can be numerically integrated using the Gauss quadrature method [13]. The final expression, from Araujo et al. [8], is of the form

\[
p(r_{so}, t) = \sum_{e=1}^{n_e} \sum_{IP=1}^{4} W_{IP} \Phi_{IPe}. \tag{2.10}
\]
where $IP$ is the integration point, $W_{IP}$ is the weight for numerical integral using Gauss quadrature and

$$\varphi_{IPe} =$$

$$\frac{\rho_a}{4\pi} \left[ -\frac{n^{(a)} J(\xi, \eta)}{r_{so(a)}} (if \ 0 < r_{so} < c_a\Delta t) + ... \right]$$

$$+ z \frac{n^{(1)} J(\xi, \eta)}{r_{so(1)}} (if \ (n-1)c_a\Delta t < r_{so} < nc_a\Delta t)$$

and $J(\xi, \eta)$ are the Jacobi-determinants of the coordinates of the surface. Equation 2.10 is a simplified numerical form of the Rayleigh integral. It could be used to predict the time domain pressure $p(\overline{r_{so}}, t)$ provided the surface acceleration of the vibrating structure $\ddot{z}(t)$ and observation point $(r_{so})$ are known. As explained by Araujo et al. [8], a constant $\beta = c_a\Delta t / \gamma$ is used to relate time and spatial domains where $\gamma$ is the greatest distance between nodes of an element.

As given by Araujo et al. [8], Equation 2.8 is obtained using a finite difference scheme of first order in time and first order in space i.e. constant interpolation. Equation 2.8 describes the working of the numerical technique i.e. contribution from each element is observed in time domain for each time step $\Delta t$ while implementing the condition given by Equation 2.9 for that element. Even though Equations 2.8 – 2.11 are provided by Araujo et al. [8], we have implemented them using MATLAB and the program structure (Table 2.1 – 2.2) and working (Figure 2.4) is given below.
Table 2.1 MATLAB files with corresponding function.

<table>
<thead>
<tr>
<th>Filename</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>“meshgen.m” (from Chandrupatla et al. [13])</td>
<td>Generation of 2-D mesh of any shape.</td>
</tr>
<tr>
<td>“plot2d.m” (from Chandrupatla et al. [13])</td>
<td>Plotting the created mesh including element and node numbers.</td>
</tr>
<tr>
<td>“rayleigh_integ.m”</td>
<td>Predict $p(t)$ for the flat plate of any shape.</td>
</tr>
</tbody>
</table>

Figure 2.4 Flow-chart describing the working of the MATLAB program. I/P = input file, O/P = output file.
Table 2.2 MATLAB files with necessary input and output.

| Input Data/Files:                           | • Input file for “meshgen.m” i.e. “IP_MESH.txt”.  
|                                            | • Output file of “meshgen.m” as input to “plot2d.m” i.e. “IP_RAY.txt”  
|                                            | • Output file of “meshgen.m” as input to “rayleigh_integ.m” i.e. “IP_RAY.txt”  
|                                            | Input Data for “rayleigh_integ.m”  
|                                            | • \( z(t) \) at every node.  
|                                            | • \( O(r_{so}) \) and \( \beta \).  
| Output Data                               | • \( p(t) \) for pre-defined \( O(r_{so}) \).  

Table 2.1 – 2.2 and Figure 2.4 gives us a descriptive working of the MATLAB files. Using the procedure described in Figure 2.4, \( p(t) \) is predicted, provided \( O(r_{so}) \), \( z(t) \) and \( \beta \) are given. Here, “IP_MESH.txt” and “IP_RAY.txt” are text files useful in creating mesh and prediction of \( p(t) \). MATLAB files “meshgen.m” and “plot2d.m” are used from Chandrupatla et al. [13] for its compatibility with MATLAB.

2.5 Analytical Validation

According to Hanish [10], the Green’s function for the acoustic wave equation can be expressed for both steady and transient states. In the steady state for a sinusoidal wave \( e^{i\omega t} \) at frequency \( \omega \), it is

\[
G' \left( r_{so}; \omega \right) = \frac{e^{-ikr_{so}}}{4\pi r_{so}}. \tag{2.12}
\]
where the superscript $^f$ implies the frequency domain and $k = \omega / c_a$ is the wave number. In the transient state, the Green’s function $G^t(t; r_{so})$ is as follows where $\delta(t - r_{so} / c_a)$ is an impulse function,

$$G^t(r_{so}; t) = \frac{\delta(t - r_{so} / c_a)}{4\pi r_{so}}. \tag{2.13}$$

where the superscript $^t$ implies the time domain. Two separate programs are developed in our work to numerically solve the Rayleigh integral; the time domain program is for transient state only and the frequency domain program is written for the steady state. Consider the numerical form of the time domain program given by Equations 2.10 and 2.11 with surface acceleration $\bar{z}(t) = i\omega v_o e^{i\omega t}$ where $v_o$ is the velocity amplitude. For plotting the time domain pressure, we consider only the real part Re for surface acceleration, i.e.

$$\bar{z}(t) = \text{Re}\left(i\omega v_o e^{i\omega t}\right) = -\omega v_o \sin(\omega t). \tag{2.14}$$

Accordingly, the Rayleigh integral for steady state free field pressure (for a spherical radiator) is given as follows from Reynolds [9]:

$$p^f(r_{so}; \omega) = \int \frac{e^{-ikr_{so}}}{4\pi r_{so}}(i\rho_o \omega v_o) dS. \tag{2.15}$$

The pressure using the frequency domain program has to be a sinusoidal function with time (since the acceleration $\bar{z}(t) = i\omega v_o e^{i\omega t}$ is sinusoidal) and thus it is expressed in the time domain as

$$p^f(r_{so}; t) = \text{Re}\left[\tilde{P} e^{i\omega t}\right]. \tag{2.16a}$$
\[ \tilde{P} = \text{Re}\left( \int \frac{i\rho_o \omega_o e^{-ikr_o}}{4\pi r_o} dS \right) + i \text{Im}\left( \int \frac{i\rho_o \omega_o e^{-ikr_o}}{4\pi r_o} dS \right). \]  

(2.16b)

In order to plot the time domain pressure using the frequency domain program, only real part is considered for \( p^f(r_o, t) \). Equation 2.16 is numerically solved for \( p^f(t; r_o) \) at \( r_o \) and it is useful in plotting \( p(t) \) for both frequency domain and time domain programs. As described in Equations 2.10 and 2.11, the time domain program is numerically solved taking into consideration both spatial and time domains whereas the frequency domain program, given by Equation 2.16, assumes only the steady state and hence it has only spatial domain at a given \( \omega_o \). The peak-to-peak pressure values are compared in section 2.6 for the two programs under the same sinusoidal excitation.

2.6 Example Case

A rigidly vibrating flat plate with length \( l = 240 \) mm and width \( b = 100 \) mm is discretized with ‘\( N_x \)’ elements along the x-axis and ‘\( N_y \)’ elements along the y-axis. Sinusoidal surface acceleration \( z(t) = -\omega_o v_o \sin(\omega_o t) \) is uniformly distributed across the entire plate at frequency \( f_o = \omega_o / 2\pi \) where \( v_o = 1 \) mm/s. The peak-to-peak acceleration is \( \ddot{z}_{p-p} = 2\omega_o |v_o| \) with the observation point O (120 mm, 50 mm, 1000 mm). The dimensionless computational pressures for time and frequency domain programs are given by

\[ \tilde{p}^t_{\text{com}}(t) = \frac{p^t_{\text{com}}(t)}{p_{p-p}^{\text{rig}}}. \]  

(2.17a)

\[ \tilde{p}^f_{\text{com}}(t) = \frac{p^f_{\text{com}}(t)}{p_{p-p}^{\text{rig}}}. \]  

(2.17b)
where $p_{p-p}^{\text{rig}}$ is the peak-to-peak reference pressure generated by an oscillating rigid piston in a duct, $p'_{\text{com}}(t)$ and $p^f_{\text{com}}(t)$ are computational pressures obtained using time domain and frequency domain programs. The peak-to-peak rigid piston pressure is given by

$$p_{p-p}^{\text{rig}} = \rho_a c_a A^{\text{rig}} \tau^{\text{rig}} z_{p-p}.$$  \hspace{1cm} (2.18)

where $\tau_A$ is the duration of acceleration pulse and $z_{p-p}$ is the peak-to-peak piston acceleration. Assume $\rho_a = 1.3 \text{ kg/m}^3$, $c_a = 350 \text{ m/s}$, $\tau^{\text{rig}} = 1 \text{ ms}$ and $z_{p-p} = 30 \text{ m/s}^2$ which results in $p_{p-p}^{\text{rig}} = 13.65 \text{ Pa}$. The peak-to-peak rigid piston pressure $p_{p-p}^{\text{rig}}$ is used to normalize the experimental and computational pressures.

Table 2.3 compares the peak-to-peak dimensionless pressures for the two programs given $\beta = 1$ and surface acceleration in the form of a sine wave with frequency range 100-2000 Hz.
Table 2.3 Peak-to-peak pressures for time domain $p_{cpt}$ and frequency domain $p_{cpf}$ programs at selected frequencies $\omega_o$ given sinusoidal surface acceleration $\ddot{z}(t) = -\omega_o v_o \sin(\omega_ot)$ for a rigidly vibrating plate.

<table>
<thead>
<tr>
<th>$f_o = \frac{\omega_o}{2\pi}$ (Hz)</th>
<th>Sinusoidal acceleration at $f_o$</th>
<th>Time domain program</th>
<th>Frequency domain program</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$p_{cpt}$, mPa</td>
<td>$\overline{p}_{cpt}$</td>
</tr>
<tr>
<td>100</td>
<td>$-0.63 \sin(2\pi100t)$</td>
<td>3.1</td>
<td>$2.27 \times 10^{-4}$</td>
</tr>
<tr>
<td>1000</td>
<td>$-6.28 \sin(2\pi1000t)$</td>
<td>6.2</td>
<td>$4.54 \times 10^{-4}$</td>
</tr>
<tr>
<td>2000</td>
<td>$-12.57 \sin(2\pi2000t)$</td>
<td>124</td>
<td>$0.91 \times 10^{-2}$</td>
</tr>
</tbody>
</table>
Figure 2.5 Computed sound pressure time history at observation point O (120 mm, 50 mm, 1000 mm) from the rigidly vibrating plate given sinusoidal acceleration 
\[ z = -12.57 \sin(2\pi 2000t) \] at \( f_o = 2000 \) Hz. (a) \( p_{com}^f, \text{mPa} \) (b) \( p_{com}^i, \text{mPa} \). Key: --, pressure using frequency domain program; ---, pressure using time domain program.
From Table 2.1, it is clear that the $p(t)$ results for the time domain and frequency domain programs match well at lower frequencies. The error appears to increase with an increase in $f$. Figure 2.5 shows sample sinusoidal pressure histories for both programs. For the time domain program, the pressure reaches a value other than zero only after a time which is actually the retarded time $r_{ao}/c_a$. This is not observed for the frequency domain program since $p(t)$ is in steady state. A good match between peak-to-peak pressures at lower frequencies between both the programs and the existence of $t_{eff}$ in the time domain program suggests that the time domain program provides a reasonable prediction.
CHAPTER 3 TRANSIENT VIBRO-ACOUSTIC EXPERIMENTS

3.1 Experimental Studies

The experiment is performed on four elastic structures as listed in Table 3.1 providing dimensions such as the length ‘l’ and width ‘b’ and thickness ‘w’ for the rectangular plate or inner diameter ‘d’

, outer diameter ‘d’

and thickness ‘w’ for the annular rods. These are suspended using fish wire to simulate the free-free boundary condition. Each structure is impulsively excited and the force \( F_i(t) \) is measured where i is the location for impact hammer. Structural acceleration \( \ddot{z}_j(t) \) and sound pressure \( p_o(t) \) are simultaneously acquired where j is the location for accelerometer and O is the observation point for \( p_o(t) \).

<table>
<thead>
<tr>
<th>Sample code</th>
<th>Elastic structure</th>
<th>Material</th>
<th>Dimensions in mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Rectangular plate</td>
<td>Steel</td>
<td>( l = 305, b = 152, w = 1.5 )</td>
</tr>
<tr>
<td>B</td>
<td>Annular rod</td>
<td>Steel</td>
<td>( l = 564, d^o = 19, d^i = 14.53, w = 2.24 )</td>
</tr>
<tr>
<td>C</td>
<td>Annular rod</td>
<td>Steel</td>
<td>( l = 376, d^o = 19, d^i = 14.53, w = 2.24 )</td>
</tr>
<tr>
<td>D</td>
<td>Annular rod</td>
<td>Aluminum</td>
<td>( l = 564, d^o = 19, d^i = 14.68, w = 2.16 )</td>
</tr>
</tbody>
</table>
As described in Figure 3.1, the position of microphone (O) for the plate is 152 mm, 76 mm, 305 mm from the bottom left side corner of the plate in the cartesian coordinate system and for the rod, O is 100 mm, 100 mm, 100 mm from the leftmost tangential point of contact to the rod in the cartesian coordinate system. Each experiment is carried out by using a roving impact hammer on the side of the plate which does not face the microphone. For the case of the rod, the structure is excited on the same side which is facing the microphone. The baffled model assumes that there is not interaction of sound waves between the left- and right-hand sides of the plate [7] and the pressure is supposedly zero outside the edges of the plate in the direction of the surface. During experiment, the baffle is achieved by not making comparisons outside the edges of the plate. And also the fact that the wavelength of the sound being measured is small compared to the dimensions of the plate ensures that the plate is in a baffle.

For plate A, the accelerometer is moved on its surface from \( j = 1 \) to 9 and \( \ddot{z}_j(t) \), \( p_o(t) \) and \( F_i(t) \) are obtained as given in Figure 3.2 (a). For annular rods B, C and D, the impact hammer is moved on its surface from \( i = 1 \) to 9 and \( \ddot{z}_j(t) \), \( p_o(t) \) and \( F_i(t) \) are obtained as given in Figure 3.2 (b). Assuming the system is linear, reciprocity is applied for both cases and \( \ddot{z}(t) \) on the entire surface of the test sample can be estimated by exchanging the position of accelerometer (j) and impact hammer (i). The numbered positions on the elastic structure for obtaining signals from the 3-channel data acquisition system (DAQ) are shown in Figure 3.2.
Figure 3.1 Vibro-acoustic experiment used to measure transient pressure radiation by applying an impulse excitation on the sample. The observation point is at O and the origin (0, 0, 0) is at one corner of the structure. The vibro-acoustic experiment was carried out in a realistic acoustic environment with carpeted floors, walls made of dry wall and ceiling made of acoustic ceiling tiles. (a) Rectangular plate (b) annular rod.
3.2 Typical Measurements and Analysis

Table 3.2 lists the sensors and DAQ. Continuous sampling is employed to acquire data with a sampling frequency \( f_s = 25 \text{ kHz} \) and number of sample points \( N = 25624 \).

### Table 3.2 Sensors and data acquisition system

<table>
<thead>
<tr>
<th>Sensor / System</th>
<th>Model No.</th>
<th>Sensitivity or Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impact Hammer</td>
<td>PCB 086C40</td>
<td>2.25 mV/N</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>PCB 352C65</td>
<td>106 mV/g</td>
</tr>
<tr>
<td>Microphone</td>
<td>PCB 130C21</td>
<td>40 mV/Pa</td>
</tr>
<tr>
<td>DAQ</td>
<td>NI cDAQ 9172</td>
<td>8 C series I/O modules, 11-30 VDC</td>
</tr>
</tbody>
</table>

Figure 3.2 The numbered positions on the test sample. (a) Rectangular plate (b) annular rod. Key: *, i; *, j.
An experiment was initially performed by placing five accelerometers on the rectangular plate, but this resulted in a poor signal-to-noise ratio. This is mainly due to the mass loading from the accelerometers and the effective reduction of the plate response amplitude. Also, the Rayleigh integral assumes the surface to be flat which is not the case when we place too many accelerometers on the surface of the elastic structure. Figure 3.3 compares the measured pressures in time and frequency domains for two cases namely, case I: one accelerometer and case II: five accelerometers on the plate.

Figure 3.3 Measured pressure in time and frequency domains for two rectangular plate cases. (a) t domain (b) f domain. Key: , case I; , case II.

Figure 3.3 (a) shows that the signal-to-noise ratio is poor for case II compared to case I; in case II, it is difficult to identify the $p_{exp}$ signal in time domain in Figure 3.3 (a). Due to poor signal-to-noise ratio, the peaks are not visible at the resonant
modes in the frequency domain for case II; conversely, clear peaks are observed for case I in Figure 3.3 (b). The measured data for the case I is then analyzed in time and frequency domains.

Figure 3.4 Block diagram describing input, outputs and domains of measurement and analysis for the experiment. Key: IP – Input, OP – Output.

Figure 3.4 suggests that the input is the excited force and the outputs are measured surface acceleration and pressure and all the data are acquired in the time domain. Continuous sampling is employed with a low time resolution 39 µs so that precise response waveforms can be obtained in the experiment. The measured pressure is affected by the background noise as well as the reflections from the surroundings.
Fourier $[\mu]$ and inverse Fourier transforms $[\mu^{-1}]$ are used to analyze the data in the frequency and time domains. During analysis, the same signal processing parameters such as $N = 25624$ and $f^* = 25$ kHz are used for both Fourier and inverse Fourier transforms. Explanations for the transfer functions and the impulse responses are provided in the sections 3.3 and 3.4.

3.3 Structural and Acoustic Transfer Functions

Now, the measurements are analyzed in the frequency domain. Two sinusoidal transfer functions are defined using the measured surface acceleration (at j), pressure (at O) and the excitation force (at i) and are shown below with reference to Figure 3.2

$$H^\beta_{z}(\omega) = \frac{\ddot{Z}(\omega)F^*(\omega)}{F(\omega)F^*(\omega)}, \quad (3.1a)$$

$$H^{oi}_{p}(\omega) = \frac{P(\omega)F^*(\omega)}{F(\omega)F^*(\omega)}, \quad (3.1b)$$

$$H^{pj}_{p}(\omega) = \frac{P(\omega)}{\ddot{Z}(\omega)} = \frac{H^{oi}_{p}(\omega)}{H^\beta_{z}(\omega)}, \quad (3.1c)$$

where $\ddot{Z}(\omega), P(\omega), F(\omega)$ are the Fourier transforms of measured acceleration, pressure and force signals, subscripts $z$ and $p$ correspond to acceleration and pressure and $*$ is the conjugate of $F(\omega)$. Equation 3.1a describes the structural transfer function which relates the excitation force and surface acceleration in the frequency domain. $H^\beta_{z}(\omega)$ is dependent on the mechanical properties like mass, stiffness and damping.
constant of the system. Equation 3.1b describes the acoustic transfer function which relates the excitation force and measured pressure. \( H_{p}^{oi}(\omega) \) depends on the mechanical properties, distance of observation point from the elastic structure, surface area of structure and also, on the properties of the medium. Equation 3.1c describes the vibro-acoustic transfer function which is the ratio of acoustic and structural transfer functions. It is mainly dependent on distance of observation point from the elastic structure, surface area of structure and also, on the properties of the medium. This is because these are the parameters relating pressure and surface acceleration as given by Rayleigh integral.

In Equation 3.1, \( j = 1 – 9 \), \( i = 1 \) and \( o = 1 \) for plate A, i.e. the accelerometer is moved on the surface of the rectangular plate whereas the impact hammer location and observation point are kept fixed. For the annular rods B, C and D, \( i = 1 – 9 \), \( j = 1 \) and \( o = 1 \), i.e. the impact hammer is moved on the surface of the rod whereas the accelerometer position and observation point are kept fixed. When we measure \( H_z(\omega) \), we can consider \( i = 1 – 9 \) using the reciprocity. Now for rods (B, C and D), when we measure \( H_p(\omega) \), we should consider only \( i = 1 \) and \( i = 9 \). This is because when we apply the reciprocity (exchange \( i \) and \( j \)), a fixed position for the impact hammer must be considered when the pressure is measured. Using the physical symmetry of the rod, it could be at either ends of the rod, i.e. \( i = 1 \) and \( 9 \). Now the spatially averaged transfer functions for acceleration and pressure in the frequency domain are defined as follows where \( \langle \cdots \rangle_s \) implies spatial averaging, i.e. estimating the average across the surface of the elastic structure.

\[
\langle H_z^i(\omega) \rangle_s = \frac{1}{9} \sum_{j=1}^{9} H_z^j(\omega).
\]  

(3.2a)
\[ \langle H_p^{oi}(\omega) \rangle_s = \frac{1}{9} \sum_{i=1}^{9} H_p^{oi}(\omega). \] (3.2b)

\[ \langle H_p^{mi}(\omega) \rangle_s = \frac{\langle H_p^{oi}(\omega) \rangle_s}{\langle H_z^{ji}(\omega) \rangle_s}. \] (3.2c)

The transfer function magnitude spectra \( \langle H_z^{ji}(\omega) \rangle_s \) and \( \langle H_p^{oi}(\omega) \rangle_s \) are plotted for all samples in Figure 3.5. \( \langle H_p^{oi}(\omega) \rangle_s \) for all samples is plotted in Figure 3.6.
Figure 3.5 Spatially averaged transfer function spectra $\langle H(\omega) \rangle$ for all structures. (a) Plate A (b) Rod B (c) Rod C (d) Rod D. Key: $\langle H_p^{oi}(\omega) \rangle$, (mPa / N); $\langle H_z^{ji}(\omega) \rangle$, (m / s^2 N).

The elastic structures considered have many modes of vibration but only the first few will be analyzed. For plate A, we can observe around 13 modes in the range...
0 – 1000 Hz for both transfer functions with peaks in the range 80 – 4500 m/s²N for \( \left\langle H_{z}^{ii} (\omega) \right\rangle_s \) and 90 – 1050 mPa/N for \( \left\langle H_{p}^{oi} (\omega) \right\rangle_s \). Rod B has 5 modes in the range 0 – 4300 Hz with peaks in the range 100 – 150 m/s²N for \( \left\langle H_{z}^{ii} (\omega) \right\rangle_s \) and 70 – 1200 mPa/N for \( \left\langle H_{p}^{oi} (\omega) \right\rangle_s \) whereas Rod C has only four modes in the range 0 – 6200 Hz with peaks in the range 80 – 800 m/s²N for \( \left\langle H_{z}^{ii} (\omega) \right\rangle_s \) and 90 – 1080 mPa/N for \( \left\langle H_{p}^{oi} (\omega) \right\rangle_s \). Rod D has 5 modes in the range 0 – 4300 Hz with peaks in the range 120 – 1000 m/s²N and 100 – 1200 mPa/N for \( \left\langle H_{p}^{oi} (\omega) \right\rangle_s \). For all the structures, a cluster of peaks is visible in the low frequency region (0 – 100 Hz) which is due to the background noise. In order to locate peaks at the resonant modes, structural transfer function provides better result (because of absence of background noise effects) when compared to acoustic transfer function. These transfer functions are useful in obtaining measured impulse responses for the system in time domain.
Figure 3.6 Spatially averaged vibro-acoustic transfer function spectra $\langle H_{\rho}^{ij}(\omega) \rangle_s$ for all structures. (a) Plate A (b) Rod B (c) Rod C (d) Rod D.

Even though $\langle H_{\rho}^{ij}(\omega) \rangle_s$ and $\langle H_{\rho}^{oi}(\omega) \rangle_s$ were found to be mode dependent, $\langle H_{\rho}^{oi}(\omega) \rangle_s$ is not mode dependent for the elastic structures as given by Figure
3.6. For \( \{H_{p}^{ai}(\omega)\}_{i} \), the peaks are not clearly visible at the resonant modes. It is dependent on parameters such as surface area of structure and distance of observation point from source.

3.4 Structural and Acoustic Impulse Responses

For each structure, there are nine measurement sets for acceleration, pressure and force in time domain. The inverse Fourier transform \( (\mu^{-1}) \) of the transfer function (in frequency domain) is the impulse response (in time domain). Thus the structural \( h_{zi}^{ji}(t) \) and acoustic \( g_{p}^{oi}(t) \) impulse responses for acceleration and pressure is given by

\[
\begin{align*}
    h_{zi}^{ji}(t) &= \mu^{-1}\left( H_{zi}^{ji}(\omega) \right). \\
    g_{p}^{oi}(t) &= \mu^{-1}\left( H_{p}^{oi}(\omega) \right). \\
    g_{p}^{oi}(t) &= \mu^{-1}\left( H_{p}^{oi}(\omega) \right).
\end{align*}
\]

Equation 3.3a describes the structural impulse response which relates the positional surface acceleration and excited force in the time domain. Equation 3.3b describes the acoustic impulse response which relates the measured pressure and positional excitation force. The structural \( h_{zi}^{ji}(t) \) and acoustic impulse responses \( g_{p}^{oi}(t) \) are responses for acceleration and pressure when the excited force is an impulse function. Similar to the transfer functions, \( h_{zi}^{ji}(t) \) is dependent on the mechanical properties while \( g_{p}^{oi}(t) \) depends on the mechanical properties, distance from the elastic structure, surface area of structure and also, on the properties of the medium. These impulse responses can
be used to obtain the forced responses such as pressure and acceleration in the time domain using convolution. Equation 3.3c describes vibro-acoustic response which is estimated by the inverse Fourier transform of the vibro-acoustic transfer function. As discussed for the vibro-acoustic transfer function, it is mainly dependent on distance of observation point from the elastic structure, surface area of structure and also, on the properties of the medium. Again, this is because these are the properties relating pressure and surface acceleration as given by Rayleigh integral.

Figures 3.7 – 3.12 describes the impulse responses using Equation 3.3 for an arbitrarily selected position (from Figure 3.2) for the elastic structure.

![Graphs showing impulse responses](image)

**Figure 3.7 Measured impulse responses for a numbered position in the plate A.** (a) \( g_{pi}^{oi} \) (b) \( h_i^{ii} \). Key: ———, 1.
Figure 3.8 Measured impulse responses for a numbered position in the rod B. (a) $g_{p}^{oi}$ (b) $h_{z}^{ii}$. Key: , 1.

Figure 3.9 Measured impulse responses for a numbered position in the rod C. (a) $g_{p}^{oi}$ (b) $h_{z}^{ii}$. Key: , 1.
Figure 3.10 Measured impulse responses for a numbered position in the rod D. (a) $g_p^{oi}$ (b) $h_z^{ii}$. Key: $-$, 2.

Figure 3.11 Measured impulse responses for a numbered position in the rod D. (a) $g_p^{oi}$ (b) $h_z^{ii}$. Key: $-$, 1.
From Figures 3.7, 3.8 and 3.9, the peak-to-peak acoustic impulse response $p-p g_p^{ai}$ is 0.029 mPa/N, 0.016 mPa/N and 0.030 mPa/N for rods B, C and D. The peak-to-peak structural acoustic impulse response $p-p h_s^{ii}$ is 16 m/s$^2$/N, 7 m/s$^2$/N and 28 m/s$^2$/N for rods B, C and D for the same numbered position “1”. Figures 3.10 – 3.11 suggests that the peak-to-peak impulse responses (0.075 mPa/N and 90 m/s$^2$/N for position “2” and 0.030 mPa/N and 28 m/s$^2$/N for position “1”) is dependent on the position as the peak-to-peak varies from position to position for the same structure. Figure 3.12 suggests that the magnitude of the vibro-acoustic response fluctuates about zero but the peak-to-peak is observed around $t = 0$. Since, it is difficult to analyze the vibro-acoustic transfer function and response, the structural and acoustic impulse responses is used for experimental comparison. Measured structural impulse response is provided as an input to the

**Figure 3.12 Vibro-acoustic response for a numbered position in the plate A. (a) 1 (b) 2.**
MATLAB program for prediction of computational acoustic impulse response and then compared with the measured acoustic impulse response.

3.5 Modal Synthesis

For the flat plate, plots for the spatially averaged transfer functions are compared with those obtained using modal synthesis in Figure 3.13. Modal synthesis is performed for the structural and acoustic transfer functions to analyze them in the frequency domain in the absence of background effects. The transfer functions are reconstructed using the peak values at the resonant modes and the modal frequencies. The expression for structural and acoustic impulse responses using modal expansion are given by

\[
\left\{ h_z^m(t) \right\}_s = \sum_{m=1}^{11} \varphi_z^m \sin(\omega_m t). \tag{3.4a}
\]

\[
\left\{ g_p^m(t) \right\}_s = \sum_{m=1}^{11} \varphi_p^m \sin(\omega_m t). \tag{3.4b}
\]

where \( \varphi_z^m \) and \( \varphi_p^m \) are the peaks at the resonant modes with mode number \( m \) and subscript \( \text{mex} \) corresponds to modal expansion. Now we obtain the corresponding transfer functions using

\[
\left\{ H_z^m(\omega) \right\}_s = \mu \left( \left\{ h_z^m(t) \right\}_s \right). \tag{3.5a}
\]

\[
\left\{ H_p^m(\omega) \right\}_s = \mu \left( \left\{ g_p^m(t) \right\}_s \right). \tag{3.5b}
\]
Figure 3.13 Spatially averaged transfer function spectra for the flat plate A. (a) $\langle H_z^{ii} (\omega) \rangle_s$, (m / s$^2$N) (b) $\langle H_p^{ii} (\omega) \rangle_s$, (Pa / N). Key: Using modal expansion: ****, from measurement.

Figure 3.14 Spatially averaged structural and acoustic impulse responses for the flat plate A. (a) $\langle h_z^{ii} \rangle_s$, (m / s$^2$N) (b) $\langle g_p^{ii} \rangle_s$, (Pa / N). Key: From measurement: **** using modal expansion.
Similar plots can be constructed for other structures using the first few modes and the modal frequencies. During modal synthesis, the effect of damping is not considered which is observed in Figure 3.14. For the plot using modal synthesis, the amplitude does not decrease with time whereas it decreases for measured plot. Since the peak-to-peak is not clearly visible for the \( \langle h_z^\mu \rangle_s \) and \( \langle g^\omega_r \rangle_s \) using modal synthesis, measured results is used during experimental comparison in Chapter 4.
CHAPTER 4 TRANSIENT PRESSURE PREDICTION AND COMPARISON WITH MEASUREMENTS

4.1 Comparison of Natural Frequencies

The measured natural frequencies \( f_{\text{exp}} \) for the elastic structures are first compared with calculated natural frequencies \( f_{\text{com}} \). They are given in Tables 4.1 – 4.4 where \( m \) is the mode number. For rectangular plate A, \( f_{\text{com}} \) is obtained using a finite element code (ANSYS) assuming ‘solid’ element type for the computational model. The computational model is a rectangular plate (A) of dimensions listed in Table 3.1 and is meshed using “sweep” command in ANSYS. “Modal analysis” is performed for the model to obtain \( f_{\text{com}} \).

Table 4.1 Comparison of computational and experimental natural frequencies for the rectangular plate A.

<table>
<thead>
<tr>
<th>Modal Index ( m )</th>
<th>Calculated ( f_{\text{com}} ), Hz using Finite Element</th>
<th>Measured ( f_{\text{exp}} ), Hz from ( \langle H_z^m \rangle_s )</th>
<th>Measured ( f_{\text{exp}} ), Hz from ( \langle H_p^m \rangle_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>84</td>
<td>81</td>
<td>81</td>
</tr>
<tr>
<td>2</td>
<td>104</td>
<td>104</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>229</td>
<td>224</td>
<td>224</td>
</tr>
</tbody>
</table>
For the case of annular rods, the natural frequencies are obtained using the analytical expression from Blevins [14]. According to the Euler’s beam theory, the natural frequencies in flexure are given by

\[ f_{mn}^2 = \frac{\chi_m^2}{\pi^2} \left( \frac{EI}{\rho_{mtl}l^4 s} \right)^{1/2}, \quad m = 1, 2, 3.. \]  

(4.1)

where \( E \) is the Young’s modulus, \( l \) is the length of beam, \( I \) is the area moment of inertia, \( \rho_{mtl} \) is the density of material, \( \chi^m = 0.5(2m+1)\pi \) is a constant and \( s \) is the cross sectional area. For rods B, C and D, \( f_{com} \) is obtained using ANSYS assuming element type ‘solid-shell’ for the computational model. The computational models are rods of dimensions listed in Table 3.1. They are meshed using “sweep” command and “Modal analysis” is performed to obtain \( f_{com} \). Tables 4.2 – 4.4 compare the natural frequencies for rods B, C and D.
### Table 4.2 Comparison of computational, experimental and analytical natural frequencies for rod B.

<table>
<thead>
<tr>
<th>Modal Index m</th>
<th>Calculated $f_{\text{com}}, \text{Hz}$ using Finite Element</th>
<th>Measured $f_{\exp}, \text{Hz}$ from $\langle H_{zz}^{\mu} \rangle_s$</th>
<th>Measured $f_{\exp}, \text{Hz}$ from $\langle H_{pp}^{\mu} \rangle_s$</th>
<th>Calculated $f_{\text{anl}}, \text{Hz}$ using Equation 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>338</td>
<td>344</td>
<td>344</td>
<td>346</td>
</tr>
<tr>
<td>2</td>
<td>924</td>
<td>948</td>
<td>948</td>
<td>953</td>
</tr>
<tr>
<td>3</td>
<td>1781</td>
<td>1830</td>
<td>1830</td>
<td>1870</td>
</tr>
<tr>
<td>4</td>
<td>2899</td>
<td>2969</td>
<td>2969</td>
<td>3091</td>
</tr>
<tr>
<td>5</td>
<td>4254</td>
<td>4324</td>
<td>4324</td>
<td>4617</td>
</tr>
</tbody>
</table>

### Table 4.3 Comparison of computational, experimental and analytical natural frequencies for rod C.

<table>
<thead>
<tr>
<th>Modal Index m</th>
<th>Calculated $f_{\text{com}}, \text{Hz}$ using Finite Element</th>
<th>Measured $f_{\exp}, \text{Hz}$ from $\langle H_{zz}^{\mu} \rangle_s$</th>
<th>Measured $f_{\exp}, \text{Hz}$ from $\langle H_{pp}^{\mu} \rangle_s$</th>
<th>Calculated $f_{\text{anl}}, \text{Hz}$ using Equation 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>752</td>
<td>763</td>
<td>763</td>
<td>779</td>
</tr>
<tr>
<td>2</td>
<td>2024</td>
<td>2078</td>
<td>2078</td>
<td>2145</td>
</tr>
<tr>
<td>3</td>
<td>3845</td>
<td>3936</td>
<td>3936</td>
<td>4209</td>
</tr>
<tr>
<td>4</td>
<td>6181</td>
<td>6239</td>
<td>6239</td>
<td>6959</td>
</tr>
</tbody>
</table>
Table 4.4 Comparison of computational, experimental and analytical natural frequencies for rod D.

<table>
<thead>
<tr>
<th>Modal Index</th>
<th>Calculated (f_{\text{com}}), Hz using Finite Element</th>
<th>Measured (f_{\text{exp}}, Hz) from (H_{\beta}^{z})</th>
<th>Calculated (f_{\text{anl}}), Hz using Equation 4.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>329</td>
<td>338</td>
<td>338</td>
</tr>
<tr>
<td>2</td>
<td>904</td>
<td>942</td>
<td>942</td>
</tr>
<tr>
<td>3</td>
<td>1760</td>
<td>1818</td>
<td>1818</td>
</tr>
<tr>
<td>4</td>
<td>2866</td>
<td>2941</td>
<td>2941</td>
</tr>
<tr>
<td>5</td>
<td>4179</td>
<td>4268</td>
<td>4268</td>
</tr>
</tbody>
</table>

We can infer from tables 4.2 – 4.4, that they behave like beams in flexure and thus the Euler’s beam theory is applicable for rods B, C and D. Also, the rods do not have any shell modes for measured \(f_{\text{exp}}\) when compared to calculated \(f_{\text{com}}\) within the frequency range of interest. Shell modes are usually visible when the diameter of the rod is large. In our case, the annular rods have a small diameter when compared to its length which is why the bending modes are more dominant. The natural frequencies for rods B and D are approximately the same because of same dimensions and same \(E / \rho_{\text{mol}}\). Comparing natural frequencies for rods B and C, rod C has a higher natural frequency because of the longer length even though the material properties and diameters are the same.
4.2 Phasing from Mode Shape in a Beam in Flexure

According to Blevins [14], the eigenfunction $z^m(x)$ for a thin beam of length $l$ under free-free boundary condition is given by

$$z^m(x) = \cosh\left(\frac{\chi^m x}{l}\right) + \cos\left(\frac{\chi^m x}{l}\right) - \sigma^m \left(\sinh\left(\frac{\chi^m x}{l}\right) + \sin\left(\frac{\chi^m x}{l}\right)\right). \quad (4.2)$$

where $m = 1, 2, 3, 4 \ldots$ is the modal index, $\sigma$ is a constant and $\chi^m = 0.5(2m+1)\pi$ is a constant for the corresponding ‘$m$’. Here, $\sigma^1 = 0.98251$, $\sigma^2 = 1.00077$, $\sigma^3 = 0.99996$, $\sigma^4 = 1.000001$, $\sigma^5 = 0.99999$ and the values used for $\sigma$ are useful in maintaining exactly same boundary conditions at the two ends of the beam. The eigenfunction is extremely sensitive to $\sigma$ and a slight change in $\sigma$ will produce different boundary values at $x = 0$ and $x = l$ even though we have same boundary conditions. Figure 4.1 plots the analytical mode shape for a beam using Equation 4.2.
Figure 4.1 Analytical mode shapes $z(x)$ for a free-free beam. (a) $m=1$ (b) $m=2$ (c) $m=3$ (d) $m=4$.

The modal phasing $\theta^m(x/l)$ between the plate elements can be obtained using the structural wavelength $\lambda^m = 2l/(m+1)$ for a particular mode shape. The phasing $\theta^m(x)$ in radian is given by
\[
\theta^m(x/l) = \frac{2\pi x}{\lambda^m}.
\] (4.3)

Phasing is obtained by assuming that the mode shape extends beyond \(x/l=1\) in the form of a wave with \(\lambda^m\) being its wavelength. \(\lambda^m\) is obtained by calculating the distance along the length of beam for one cycle i.e. the distance over which the wave’s shape repeats. Now, the beam is discretized into eight elements as described in Figure 3.2 (b) and using Equations 4.2 - 4.3, the phasing is obtained at nine nodes as given in Table 4.5.

**Table 4.5 Modal phasing \(\theta^m(x)\) between nine nodes in a beam.**

<table>
<thead>
<tr>
<th>Modal Index (m)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\chi^m), rad</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.71</td>
<td>7.85</td>
<td>11.00</td>
<td>14.14</td>
<td>17.28</td>
<td></td>
</tr>
<tr>
<td>(\sigma^m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.98251</td>
<td>1.00077</td>
<td>0.99996</td>
<td>1.000001</td>
<td>0.99999</td>
<td></td>
</tr>
<tr>
<td>(\lambda^m), m</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(l)</td>
<td>(2l/3)</td>
<td>(l/2)</td>
<td>(2l/5)</td>
<td>(l/3)</td>
<td></td>
</tr>
<tr>
<td>Node number</td>
<td>(\theta), rad</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.250 (\pi)</td>
<td>0.375 (\pi)</td>
<td>0.500 (\pi)</td>
<td>0.625 (\pi)</td>
<td>0.750 (\pi)</td>
</tr>
<tr>
<td>3</td>
<td>0.500 (\pi)</td>
<td>0.750 (\pi)</td>
<td>1.000 (\pi)</td>
<td>1.250 (\pi)</td>
<td>1.500 (\pi)</td>
</tr>
<tr>
<td>4</td>
<td>0.750 (\pi)</td>
<td>1.125 (\pi)</td>
<td>1.500 (\pi)</td>
<td>1.875 (\pi)</td>
<td>2.250 (\pi)</td>
</tr>
<tr>
<td>5</td>
<td>1.000 (\pi)</td>
<td>1.500 (\pi)</td>
<td>2.000 (\pi)</td>
<td>2.500 (\pi)</td>
<td>3.000 (\pi)</td>
</tr>
<tr>
<td>6</td>
<td>1.250 (\pi)</td>
<td>1.875 (\pi)</td>
<td>2.500 (\pi)</td>
<td>3.125 (\pi)</td>
<td>3.750 (\pi)</td>
</tr>
<tr>
<td>7</td>
<td>1.500 (\pi)</td>
<td>2.250 (\pi)</td>
<td>3.000 (\pi)</td>
<td>3.750 (\pi)</td>
<td>4.500 (\pi)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>8</td>
<td>1.750$\pi$</td>
<td>2.625$\pi$</td>
<td>3.500$\pi$</td>
<td>4.375$\pi$</td>
<td>5.250$\pi$</td>
</tr>
<tr>
<td>9</td>
<td>2.000$\pi$</td>
<td>3.000$\pi$</td>
<td>4.000$\pi$</td>
<td>5.000$\pi$</td>
<td>6.000$\pi$</td>
</tr>
</tbody>
</table>

For $m = 1$, $\theta = 0$ for node numbers 1 and 9 which is observed in Figure 4.1(a) as the mode shape represents a sine wave with same boundary values at $x = 0$ and $x = l$. For other modes, $\theta = 0$ for node numbers 1 and 9 since they represent the start and ends of a sine wave of different $\lambda$.

4.3 Computational Model

Measured structural impulse responses from the nine positions will be redistributed on a rectangular plate though more elements will be added for calculating the pressure. The distribution of $h_i^n(t)$ at nine positions in Figure 3.2 is shown in Figures 4.2 – 4.3. For the case of annular rods, we assume the rod to be a narrow flat rectangular strip of length and breadth equal to that of rod’s length and outer diameter, i.e. $l_{com} = l_{rod}$, $b_{com} = d'$, where $l_{rod}$ is the length of rod, $l_{com}$ and $b_{com}$ are length and breadth of computational rectangular plate.
Figure 4.2 Location of measured structural impulse response $h_z(t)$ at the numbered nine positions in Figure 3.2 (a) distributed on a 8 x 5 element discretization for plate A.
Figure 4.3 Location of measured structural impulse response $h_{ij}(t)$ at the numbered nine positions in Figure 3.2 (b) distributed on a 8 x 2 element discretization for rods B, C and D.

Measured structural impulse response $h_{ij}(t)$ is provided as an input to the nodes for prediction of computational acoustic impulse response. For the case of rectangular plate, it is carried out by redistributing on nodes which are closer to the numbered positions described in Figure 3.2 (a). For the case of annular rods, they are redistributed exactly on the numbered positions as given in Figure 4.3.
4.4 Comparison of Predicted and Measured Pressures

In the case of experiment on the rectangular plate, the data for each position is collected in the time domain without any usage of triggering during impact. This resulted in a loss of phase when the structural impulse responses for the nine positions was collected separately. In order to avoid this, phasing needs to be applied to the structural impulse response and it is performed using the structural transfer function as given in Equation 4.4. For application of phasing to the rectangular plate, it is assumed to behave like a beam when observed as a one dimensional structure. Assuming beam behavior for plate, phasing $\theta$ will be used from Table 4.5 and applied along $l$ and $b$ of the plate resulting in two cases for each mode.

In the case of experiment on annular rods, the triggering problem is resolved and the data is collected at the same instant for the nine different positions. Even though the synchronized triggering is performed, it addresses only the phase delay due to instrumentation. In an elastic structure, different modes get excited based on the excitation position and there is an inherent phase delay due to natural modes. Hence it is useful to observe the dominance of particular modes using phasing.

During the comparison of computational acoustic impulse response with the measured result, $\theta$ obtained from section 4.2 is applied in Equation 3.2a for observing the dominance of modes. This phasing is applied to $\left< H_\alpha^{ij} \right>$ for the numbered positions discussed in Figure 3.2. The steps involved are,

I. $H_\alpha^{ij} = H_\alpha^{ij} e^{i\theta}$. \hspace{1cm} (4.4)

where $\theta$ is used from Table 4.5

II. $h_\alpha^{ij} (t) = f^{-1}\left( H_\alpha^{ij} (\omega) \right)$. \hspace{1cm} (From Equation 3.3a)
III. Redistribution of $h_{iz}^j(t)$ at the nodes in the computational model described in section 4.3 for prediction of computational acoustic impulse response $g_p^{\text{com}}(t)$.

IV. Comparison between predicted $g_p^{\text{com}}(t)$ and measured $g_p^{\text{exp}}(t)$.

Another case is considered for the elastic structures where acceleration in the form of a unit impulse function is uniformly distributed across the surface. The impulse function is achieved by assuming a rectangular pulse of very short duration with the area of its profile equal to unity. Its peak value $p_{p-p}^{*} z_{\text{gre}} = 100 \text{ ms}^{-2}$ with duration of pulse $\tau_{A}^{\text{gre}} = 0.01 \text{ s}$. Now the computational pressure $p_{p}^{\text{gre}}(t)$ is the impulse response which is also referred as the Green’s function. The dimensionless computational pressure for the Green’s function is given by

$$
\frac{p_{p}^{\text{gre}}(t)}{p_{p-p}^{\text{rig}}} = \frac{p_{p}^{\text{com}}(t)}{p_{p-p}^{\text{rig}}}.
$$

(4.5)

where $p_{p-p}^{\text{rig}}$ is the peak-to-peak reference pressure generated by an oscillating rigid piston in a duct. The peak-to-peak rigid piston pressure is given by

$$
p_{p-p}^{\text{rig}} = \rho_{a} c_{a} \tau_{A}^{\text{rig}} z_{p-p}^{\text{rig}}.
$$

(4.6)

where $z_{p-p}^{\text{rig}}$ is the peak-to-peak piston acceleration. Assume $\rho_{a} = 1.3 \text{ kg/m}^{3}$, $c_{a} = 350 \text{ m/s}$, $\tau_{A}^{\text{rig}} = 0.90 \text{ ms}$ and $z_{p-p}^{\text{rig}} = 34.04 \text{ ms}^{-2}$ which results in $p_{p-p}^{\text{rig}} = 13.65 \text{ Pa}$. The peak-to-peak rigid piston pressure $p_{p-p}^{\text{rig}}$ is used to normalize the computational pressure.

The peak-to-peak impulse response for a rigid piston pressure $g_{p-p}^{\text{rig}}$ is used to normalize the acoustic impulse responses $g_{p}^{\text{exp}}(t)$ and $g_{p}^{\text{com}}(t)$ and it is given by
the following where $\tau_A$ is the duration of acceleration pulse and $h_z^{p-p}$ is the measured average of the peak-to-peak structural impulse response.

$$g_{p-p}^{rig} = \rho_c c_i \tau_A^{rig} h_z^{p-p}. \quad (4.7)$$

In experimental studies, $\tau_A^{rig}$ varies since the duration of pulse for force $\tau_F$ is between 0.86 and 0.90 ms. Assuming $\tau_A^{rig}$ to be dependent on $\tau_F$, select $\tau_A^{rig} = 0.90 \, ms$, the peak-to-peak structural impulse response (for measured acceleration) is given by

$$h_z^{p-p} = \frac{1}{9} \sum_{j=1}^{9} h_z^{p-pj} = 34.04 \, m / s^2 N. \quad \text{Hence} \quad g_{p-p}^{rig} = 13.66 \, Pa / N. \quad \text{Tables 4.7 – 4.10 compare the acoustic impulse responses for computations and measurements for elastic structures. In each case, results are given with and without phasing using Equation 4.4. Now the dimensionless peak-to-peak acoustic impulse responses are given as follows for computations and experiments.}

$$\frac{g_{p-p}^{com}}{g_{p-p}^{rig}} = \frac{g_{p-p}^{com}}{g_{p-p}^{rig}}. \quad (4.8a)$$

$$\frac{g_{p-p}^{exp}}{g_{p-p}^{rig}} = \frac{g_{p-p}^{exp}}{g_{p-p}^{rig}}. \quad (4.8b)$$

As discussed previously for rectangular plate, alternate models have been considered when phasing is applied. Table 4.6 lists the phasing and the node number (Figure 3.2 (a)) to which it has been applied along with the computational result. For model 1, $\theta = 0$, models 2 – 5 uses $\theta$ from Table 4.5 and model 6 uses arbitrary phasing. The arbitrary phasing selected for model 6 is based on the assumption that two edges of the plate have same phasing ($\theta=0$) with the central part being out of phase ($\theta=\pi/2$). Models 2 – 5 uses phasing $\theta$ along $l$ and $b$ of the plate.
Table 4.6 Alternate models with phasing and applied location and corresponding computational peak-to-peak acoustic impulse response for rectangular plate A.

<table>
<thead>
<tr>
<th>Model</th>
<th>1</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
<th>4a</th>
<th>4b</th>
<th>5a</th>
<th>5b</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node number using Figure 3.2 (a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0</td>
<td>$\pi$</td>
<td>0</td>
<td>1.5$\pi$</td>
<td>0</td>
<td>2.0$\pi$</td>
<td>0</td>
<td>2.5$\pi$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0</td>
<td>2$\pi$</td>
<td>0</td>
<td>3.0$\pi$</td>
<td>0</td>
<td>4.0$\pi$</td>
<td>0</td>
<td>5.0$\pi$</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$\pi$</td>
<td>0</td>
<td>1.5$\pi$</td>
<td>0</td>
<td>2.0$\pi$</td>
<td>0</td>
<td>2.5$\pi$</td>
<td>0</td>
<td>$\pi$/2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$\pi$</td>
<td>$\pi$</td>
<td>1.5$\pi$</td>
<td>1.5$\pi$</td>
<td>2.0$\pi$</td>
<td>2.0$\pi$</td>
<td>2.5$\pi$</td>
<td>2.5$\pi$</td>
<td>$\pi$/2</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>$\pi$</td>
<td>2$\pi$</td>
<td>1.5$\pi$</td>
<td>3.0$\pi$</td>
<td>2.0$\pi$</td>
<td>4.0$\pi$</td>
<td>2.5$\pi$</td>
<td>5.0$\pi$</td>
<td>$\pi$/2</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>2$\pi$</td>
<td>0</td>
<td>3.0$\pi$</td>
<td>0</td>
<td>4.0$\pi$</td>
<td>0</td>
<td>5.0$\pi$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>2$\pi$</td>
<td>$\pi$</td>
<td>3.0$\pi$</td>
<td>1.5$\pi$</td>
<td>4.0$\pi$</td>
<td>2.0$\pi$</td>
<td>5.0$\pi$</td>
<td>2.5$\pi$</td>
<td>0</td>
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<td>9</td>
<td>0</td>
<td>2$\pi$</td>
<td>2$\pi$</td>
<td>3.0$\pi$</td>
<td>3.0$\pi$</td>
<td>4.0$\pi$</td>
<td>4.0$\pi$</td>
<td>5.0$\pi$</td>
<td>5.0$\pi$</td>
<td>0</td>
</tr>
</tbody>
</table>

$\frac{g_{pp}^{com}}{g_{pp}} \times 10^{-3}$

| 34.4 | 23.7 | 22.7 | 24.2 | 27.1 | 30.7 | 27.1 | 38.8 | 42.5 | 11.6 |

$g_{pp}^{com}$, dB re 20 $\mu$Pa / N

| 87  | 84  | 84  | 84  | 85  | 86  | 85  | 88  | 89  | 78  |
Table 4.7 Comparison of peak-to-peak acoustic impulse response for computations and measurements for plate A with reference to Table 4.6.

<table>
<thead>
<tr>
<th>Model</th>
<th>Computational $p-p g_p^{\text{com}}$, dB re 20 $\mu$Pa / N</th>
<th>Computational $g_p^{\text{com}} \times 10^{-3}$</th>
<th>Experimental $p-p g_p^{\text{exp}}$, dB re 20 $\mu$Pa / N</th>
<th>Experimental $g_p^{\text{exp}} \times 10^{-3}$</th>
<th>Green’s function (Computational) $p-p P_p^{\text{com}}$, dB re 20 $\mu$Pa</th>
<th>Green’s function (Computational) $P_p^{\text{com}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>87</td>
<td>34.4</td>
<td>67 - 72</td>
<td>3.28 – 5.83</td>
<td>108</td>
<td>0.34</td>
</tr>
<tr>
<td>2a</td>
<td>84</td>
<td>23.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2b</td>
<td>84</td>
<td>22.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3a</td>
<td>84</td>
<td>24.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3b</td>
<td>85</td>
<td>27.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4a</td>
<td>86</td>
<td>30.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4b</td>
<td>85</td>
<td>27.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5a</td>
<td>88</td>
<td>38.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5b</td>
<td>89</td>
<td>42.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>78</td>
<td>11.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the case of annular rods, the modal phasing in Table 4.5 is directly applied to the nine numbered positions described in Figure 3.2 (b). At m = 0, the phasing is assumed to be zero at all the nine marked positions.
Table 4.8 Comparison of peak-to-peak acoustic impulse response for computations and measurements for rod B.

<table>
<thead>
<tr>
<th>m</th>
<th>Computational</th>
<th>Experimental</th>
<th>Green’s function (Computational)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p-p , g_p^{\text{com}}$</td>
<td>$g_p^{\text{exp}}$</td>
<td>$P_{P_{\text{com}}}$</td>
</tr>
<tr>
<td></td>
<td>$\text{dB re } 20 , \mu Pa / N$</td>
<td>$\text{dB re } 20 , \mu Pa / N$</td>
<td>$\text{dB re } 20 , \mu Pa$</td>
</tr>
<tr>
<td>0 ($\theta = 0$)</td>
<td>71</td>
<td>5.19</td>
<td>0.359</td>
</tr>
<tr>
<td>1</td>
<td>70</td>
<td>4.63</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>4.63</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>68</td>
<td>3.68</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>70</td>
<td>4.63</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>66</td>
<td>2.92</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9 Comparison of peak-to-peak acoustic impulse response for computations and measurements for rod C.

<table>
<thead>
<tr>
<th>m</th>
<th>Computational</th>
<th>Experimental</th>
<th>Green’s function (Computational)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p-p , g_p^{\text{com}}$</td>
<td>$g_p^{\text{exp}}$</td>
<td>$P_{P_{\text{com}}}$</td>
</tr>
<tr>
<td></td>
<td>$\text{dB re } 20 , \mu Pa / N$</td>
<td>$\text{dB re } 20 , \mu Pa / N$</td>
<td>$\text{dB re } 20 , \mu Pa$</td>
</tr>
<tr>
<td>0 ($\theta = 0$)</td>
<td>70</td>
<td>4.63</td>
<td>0.328</td>
</tr>
<tr>
<td>1</td>
<td>66</td>
<td>2.92</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>61</td>
<td>1.64</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>63</td>
<td>2.07</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>65</td>
<td>2.60</td>
<td></td>
</tr>
</tbody>
</table>
Table 4.10 Comparison of peak-to-peak acoustic impulse response for computations and measurements for rod D.

<table>
<thead>
<tr>
<th>m</th>
<th>Computational $g_{p-p}^{com}$, dB re 20 $\mu$Pa / N</th>
<th>Experimental $g_{p-p}^{exp}$, dB re 20 $\mu$Pa / N</th>
<th>Green’s function (Computational) $p-p P_{com}$</th>
<th>Green’s function (Computational) $p-p P_{com}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ($\theta = 0$)</td>
<td>81</td>
<td>16.4</td>
<td>64 - 69</td>
<td>1.64 – 2.07</td>
</tr>
<tr>
<td>1</td>
<td>74</td>
<td>7.34</td>
<td></td>
<td>108</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>8.23</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>72</td>
<td>5.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>71</td>
<td>5.19</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>70</td>
<td>4.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the flat plate, the difference between computational and measured results lie in between 6 – 22 dB re 20 $\mu$Pa / N. For an arbitrary phasing used in model 6 in Table 4.6, the results match well with difference of only 6 dB re 20 $\mu$Pa / N. During the application of phasing, some of the models (model 5a and 5b) have computational results which exceed the computational result without phasing (model 1). This suggests that proper phasing between the numbered positions is required for obtaining better results.

Tables 4.8 – 4.10 suggests a difference in the range 3 - 10 dB re 20 $\mu$Pa / N between computational and measured results for rod B while the difference is 0 - 11 dB re 20 $\mu$Pa / N and 1 - 17 dB re 20 $\mu$Pa / N for rods C and D. Even though rods B and D have same dimension, they have different measured peak-to-
peak acoustic impulse responses for the same observation point. With phasing, rods B and C have a smaller range for the difference when compared to rod D. This suggests that material property may be important during experimental comparison since rods B and C are made of steel while rod D is made of Aluminum. When the results are compared for particular modes, for rod B, it is as low as $3 \text{ } dB\text{ re } 20 \mu Pa / N$ while it is in the range $0 - 4 \text{ } dB\text{ re } 20 \mu Pa / N$ and $1 - 5 \text{ } dB\text{ re } 20 \mu Pa / N$ for rods C and D. For all the rods, the computational result is higher for the rigid body mode ($m = 0$) when compared to other modes ($m = 1 - 4$). This emphasizes on the fact that there is a phasing problem due to instrumentation or natural mode. The difference between computational and measured acoustic impulse response is less when observed with phasing.

The peak-to-peak Green’s function is found to be $108 \text{ } dB\text{ re } 20 \mu Pa$ and $107 \text{ } dB\text{ re } 20 \mu Pa$ for plate A and rod B. For rods B and C, it is 108 and 107 $dB\text{ re } 20 \mu Pa$. This is a rigid body case since the acceleration is uniformly distributed across the surface. The Green’s function is useful in a rough prediction of pressure for impulse acceleration. For all the elastic structures, the peak-to-peak Green’s function lies outside the measured range of the acoustic impulse response because they are based on the rigid body assumption, i.e. the acceleration is uniformly distributed across the surface. In the case of a rigid body, the entire plate is vibrating with same phase and the absence of cancellation of pressures between elements results in a larger peak-to-peak pressure. Since the plate A has a longer distance but greater surface area whereas rods B, C and D have smaller areas but shorter distances from observation point, computational peak-to-peak value remains the same for all the structures.
For all the elastic structures, the peak-to-peak dimensionless computational and experimental acoustic impulse responses are less than one. This suggests that the behavior of the pressure wave for the elastic structure approaches the rigid piston pressure. The rigid piston pressure is larger since it is independent of the distance from the observation point. The above experimental comparison for the time domain program suggests that the time domain program offers a reasonable estimation of sound pressure in time domain.

4.5 Sources of Error

Even when the results are compared with phasing for measured structural acoustic impulse response, there is a slight difference between measured and simulated results. The reasons could be:

- During experiment: The limited number of positions used for obtaining the measured structural impulse response will contribute to the overall error during experimental comparison. In our case, we have considered only nine positions to estimate measured structural impulse response across the surface. The larger the number of positions, the more accurate will be the distribution of structural impulse response on the surface of the structure. The Rayleigh integral assumes the flat plate to be in a baffle i.e. its boundary exists at an infinite extent which is not followed during the experiment. The effect of the surroundings which has been acoustically treated has not been included. The actual acoustic impulse response can be found from the expression below

\[
g_p^{\text{exp}}(t) = g_p^{\text{act}}(t) + g_p^{\text{bgd}}(t) + g_p^{\text{sur}}(t). \tag{4.7}
\]
where, $g_{p}^{\text{exp}}(t)$ is the measured acoustic impulse response at microphone, $g_{p}^{\text{act}}(t)$ is the actual acoustic impulse response contribution from the plate, $g_{p}^{\text{bgd}}(t)$ is the acoustic impulse response from background noise and $g_{p}^{\text{sur}}(t)$ is the acoustic impulse response due to reflection from surroundings. The effect of surroundings $g_{p}^{\text{sur}}(t)$ exists when the pressure wave coming from the plate has enough time to reflect from the surroundings and return to the microphone thereby contributing to the overall acoustic impulse response.

- During computation: Due to the number of elements used. The computational result depends on the number of elements and better results can be obtained with greater number of elements. The measured structural impulse response is spatially assumed on the plate for the computational model.

- Due to the assumptions made when comparing theory vs. experiment: For the rectangular plate, the plan of applying the phasing along $l$ and $b$ assumes the same phasing for at least 3 positions. For example, in Figure 3.2 (a), the numbered positions 1, 2 and 3 will have same phasing when applied along its width and the phasing may itself be different for these three positions. In the case of annular rods, we have made an approximation wherein the rod is replaced with a narrow rectangular strip of equivalent dimension.
CHAPTER 5 CONCLUSION

5.1 Summary

A time domain based computational tool for prediction of sound pressures radiated by vibrating elastic flat surfaces (in a baffle) subjected to impulsive or transient load is developed. Initially, the sound pressure radiation from ideal radiators with impulse excitation is evaluated in the time and frequency domains using analytical and computational Fourier transforms. The computational transient sound pressure is not consistent with the analytical results for an ideal dipole radiator using FFT/IFFT method. Hence the analysis must be performed directly in the time domain. Literature is surveyed to identify the numerical methods available for solving Rayleigh integral computationally. Araujo et al. [8] has discussed the boundary element technique used to solve Rayleigh integral numerically. Using Araujo et al. [8], a MATLAB program is developed to predict the transient sound pressure radiated from the flat surface in a baffle by solving the Rayleigh integral numerically using boundary element techniques provided surface acceleration is given on the plate. The proposed program is first computationally validated for a rigidly vibrating plate using a frequency domain formulation which assumes sinusoidal acceleration distribution across the plate. Comparing the peak-to-peak pressures for the two programs under same conditions, the time domain program is validated.
Experimental study is performed on a rectangular plate and three annular rods using an impulse hammer, accelerometer and microphone. Experimental results are analyzed initially in the frequency domain using structural and acoustic transfer functions. Inverse Fourier transform is performed for the transfer functions to obtain measured structural and acoustic impulse responses. Eigenvectors and mode shapes are used for the determination of phasing between the elements of vibration in a beam. Phasing between elements of vibration is determined using the Euler’s beam theory for distribution of measured structural impulse response across the elastic structure. Then, computational acoustic impulse response using measured structural impulse response (with and without phasing) is compared with measured acoustic impulse response for the elastic structures. For the flat plate, differences between measured and computed results are as low as $6 \text{ dB} r 20 \mu \text{Pa} / N$ for individual modes. For rod B, it is $3 \text{ dB} r 20 \mu \text{Pa} / N$ while it is in the range $0 - 4 \text{ dB} r 20 \mu \text{Pa} / N$ and $1 - 5 \text{ dB} r 20 \mu \text{Pa} / N$ for rods C and D for particular modes. The experimental comparison suggests that the time domain program provides a reasonable estimation of sound pressure in time domain. The sources of error are discussed and these include experimental errors, computational errors and errors due to assumptions made for comparison between theory and experiment.

To conclude, a time domain based MATLAB program for prediction of transient sound pressures radiated by vibrating elastic structures in a baffle is developed by numerically solving the Rayleigh integral. The proposed program is validated for a rigid plate radiator using a frequency domain formulation which assumes sinusoidal acceleration distribution across the plate. Modal phasing between vibrating elements in a
beam is determined using the structural wavelength for a particular mode shape. Applying impact hammer testing on a flat plate and rods, measured and computational acoustic impulse responses (with and without phasing) are compared and the sources of error are discussed.

5.2 Future Work

Based on this research, further development of the algorithm and methodology would include the following:

- Proper phasing between elements of vibration must be determined for the case of the rectangular plate.
- Rectangular plate experiment should be conducted with accelerometers placed across its surface in an anechoic chamber where the signal to noise ratio is high. Data should be obtained at the same instant of time instead of using phasing and the measured pressure compared with computational results.
- Mass loading due to the accelerometers must be incorporated when the Rayleigh integral is numerically solved or a non-contact displacement sensor or laser vibrometer should be used.
- MATLAB program for solving the Kirchhoff integral [8] numerically should be developed which is applicable for transient sound pressure prediction from an elastic structure of any complicated shaped. Measured sound pressure from the sound radiation experiment conducted for the annular rod should be compared with the computational result using the program for Kirchhoff integral.
REFERENCES


