Modeling Accelerating Trends of Displacement
in Geodetic Time Series

Thesis

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ABSTRACT

Geodesists and geophysicists engaged in crustal motion geodesy monitor the position (or displacement) time series associated with thousands of GPS stations worldwide. These time series are useful for studying a wide range of geodynamic phenomena including plate motion, mountain building, the earthquake deformation cycle, postglacial rebound, and environmental loading.

Station coordinate time series are expressed in a spatial reference frame which is typically a global, earth-centered, earth-fixed (ECEF) reference frame. The motion of a station in a given reference frame can be referred to as the trajectory of that station. The great majority of station trajectory models in use within the geodetic community are linear models, which consist of three component or sub-models characterizing: (i) the trend of displacement over time, (ii) jumps or discontinuities in the time series, and (iii) annual oscillations.

In this thesis, we use a constant velocity model for most trends, but a polynomial function for trends with time-varying velocity, Heaviside function to implement jumps if and when jumps are required, and a truncated Fourier series, typically composed of just annual and semi-annual terms to compose our standard linear trajectory model. We first illustrate the use of a polynomial trend model with reference to a GPS station of our GNET project which is well known to have a time-varying velocity, particularly in the
vertical component. We then consider an original problem: quantifying time-changes in
the velocity of a station COYQ near Coyhaique in southern Chile from CAP project
which manifests postseismic transient deformation in the aftermath of the great 1960
Chile earthquake.

We have shown that station trajectory models in which the secular trend of
displacement can be represented as a polynomial function of time can be very useful for
modeling GPS time series obtained in areas undergoing accelerating ice loss, and in areas
undergoing postseismic transient deformation a decade or more after a great earthquake.
Of course the great majority of GPS stations can be characterized perfectly adequately
using a constant velocity trend model. But the areas in which this is not true are areas of
considerable geodynamic interest. This thesis presents a new tool for studying those areas.
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CHAPTER 1

INTRODUCTION

1.1 The objectives of this thesis

Geodesists and geophysicists engaged in crustal motion geodesy monitor the position (or displacement) time series associated with thousands of GPS stations worldwide. These time series are useful for studying a wide range of geodynamic phenomena including plate motion, mountain building, the earthquake deformation cycle, postglacial rebound, and environmental loading. Station coordinate time series are expressed in a spatial reference frame which is typically a global, earth-centered, earth-fixed (ECEF) reference frame. The motion of a station in a given reference frame can be referred to as the trajectory of that station. The great majority of station trajectory models in use within the geodetic community are linear models in the sense that the parameters of these models can be isolated as the model vector \( \mathbf{m} \) of a linear matrix system \( \mathbf{A} \mathbf{m} = \mathbf{d} \).

In general, linear trajectory models consist of three component or sub-models characterizing: (i) the trend of displacement over time, (ii) jumps or discontinuities in the time series, and (iii) annual oscillations. In practice the trend component is nearly always a time-linear model, with the form

\[
\mathbf{p} = \mathbf{p}_0 + \mathbf{v}t
\]  

(1-1)
or more frequently

\[
p(t) = p_R + v(t - t_R)
\]  

(1-2)

where \(p\) is the position vector at time \(t\), \(p_R\) is the reference position at the reference time \(t_R\), and \(v\) is the velocity of the station. We refer to this model as the constant velocity model. Note that the trend model (1.2) is a first order polynomial in \(t\). In this thesis we extend linear trajectory models so that they can also be applied to stations whose motions are not steady by generalizing (1.2) to a quadratic or higher order polynomial in \(t\).

We first illustrate the use of a polynomial trend model with reference to a GPS station in Thule, Greenland which is well known to have a time-varying velocity, particularly in the vertical component (Khan et al, 2008, 2010; Jiang et al., 2010). We then consider an original problem: quantifying time-changes in the velocity of a station COYQ near Coyhaique in southern Chile which manifests postseismic transient deformation in the aftermath of the great (\(M_w 9.5\)) 1960 Chile earthquake (Wang et al, 2007). We use a quadratic or ‘constant acceleration’ model to determine the decay of the postseismic transient evident in the east component of displacement of COYQ since 1998.

**1.2 Historical background: the evolution of geodetic station trajectory models**

Despite the work of such visionaries as Alfred Wegener (1915), Arthur Holmes (1928), Alexander du Toit (1937), and S. Warren Carey (1958), nearly all professional geologists and geophysicists circa 1960 viewed the lithosphere (i.e. the solid earth's stiff
and strong outer layer) as stable and very nearly static - as argued, for example, by Jeffreys (1924) and Simpson (1943). Minor thermal (Dana, 1873) or gravitational (Chamberlin and Salisbury, 1906) contraction of the entire planet had been suggested to explain the shortening observed in mountain belts, but major horizontal displacements coherent over large portions of the globe, as suggested by Wegener's theory of continental drift, had been long rejected by all but a tiny minority of scientists. Of course, geologists, geophysicists and geodesists were aware that earthquakes often produced measurable displacements of the ground (Reid, 1910), and they were also aware of postglacial rebound (Jamieson, 1865; Haskell, 1935), but these movements were viewed as local deformations.

Geodesists had a similar view as geologists and geophysicists around 1960: they believed that (solid earth tides aside) it was possible to establish an earth-centered, earth-fixed reference frame that allowed almost any point on the surface of the earth to be described using three spatial coordinates that would not vary with time (earthquakes, landslides and volcanic deformations aside), or would vary at a rate too small to measure. Indeed, Wegener had attempted to measure continental drift using astro-geodetic techniques and had failed. (He had badly overestimated contemporary rates of continental drift, due to lack of adequate stratigraphic and radiometric age controls, and the actual rates of crustal motion are far too slow to be measured using the technology available in his time).

This essentially 'fixist' conception of the earth was overturned during the 1960's by the development of the kinematic theory of plate tectonics (Dietz, 1961; Vine and Matthews, 1963; Wilson 1965, McKenzie and Parker 1967; Morgan, 1968; Sykes, 1967;
Isacks, Oliver and Sykes, 1968). By the end of that decade it was almost universally accepted within the earth science community that the earth's surface was divided into a set of nearly rigid plates which were in relative motion, and that typical rates of relative motion across the boundaries between these plates fell in the range 1 - 18 cm/yr. It was widely realized that as soon as it became possible to measure positions with decimeter accuracy, it would no longer be viable to assign stations fixed (unchanging) spatial coordinates except within the stable core of a single plate, an even then only when using a reference frame attached to that plate.

Plate motions were first measured in the 1980s using Very Long Baseline Interferometry (VLBI) (Herring et al., 1986) and Satellite Laser Ranging (SLR) (Christodoulidis et al., 1986). By the time that the Global Positioning System (GPS) had emerged as a practical means for measuring spatial positions at very large numbers of points distributed all over the globe, geodesists were fully prepared for the idea that the trajectory of any given survey station relative to a global reference frame would, at its very simplest, involve a constant rate of motion (velocity) as well as a reference position at a reference time. In the widely-used global reference frame ITRF2005 (Altamimi et al., 2007), for example, the geocentric cartesian coordinate system \{X, Y, Z\} is defined by assigning reference positions and velocities to 608 geodetic stations located at 338 sites.

Most scientists and engineers are used to the concept that it is the existence of a set of axes (e.g. the X, Y, Z axes, where Z corresponds to the average position of the earth's spin axis, and X passes through the prime meridian as well as the equator) that allows us to assign coordinates \{x, y, z\} to a given point. But these axes are mental constructs and not physical entities, and, in effect, the reality is the reverse. We define the axes by
assigning coordinates to a set of points. Because the earth's surface is in constant motion, to define a frame consistently over time, the coordinates of the defining set S of geodetic stations must be provided at all times, and therefore the concept of station velocity has to be introduced. The realization of a reference frame in this way allows us to assign coordinates to other sets of points. One might argue that what is really happening is that new stations are being positioned relative to the established set S of stations whose coordinates are already known (as a function of time), and therefore the axes are not truly necessary. Nevertheless, the notion of the earth-centered, earth-fixed (ECEF) axis system \( \{X, Y, Z\} \) is a very useful one. For the space geodesist, reference frame realization and the positioning of stations within that frame are two sides of the same coin.

While SLR and, especially, VLBI continue to play an important part in the realization of global reference frames, by the mid 1990s the vast majority of geodetic reference stations were GPS stations. Geodetic GPS technology is relatively inexpensive, lightweight and robust, and as a result there are now many thousands of continuous GPS (CGPS) stations worldwide. GPS stations were being deployed in many plate boundary zones throughout the 1990’s, and a significant number of them eventually recorded coseismic jumps produced earthquakes. Other jumps or coordinate discontinuities were observed too, and many of these were non-tectonic in nature – sometimes the GPS antenna had actually be moved to another monument for engineering purposes, but more frequently these nearly instantaneous changes in station coordinates were artifacts associated with the change of a GPS receiver, the change of a GPS antenna, or the change of an antenna ray dome. Both tectonic and non-tectonic discontinuities in the coordinates of a geodetic reference station must be incorporated into the model for a station’s
coordinate time series or trajectory. By the mid 1990s it was standard procedure to model a station trajectory as a combination of a steady or constant velocity motion with no, one or more jumps or discontinuities superimposed on this trend. As discussed in the next Chapter, position discontinuities are easily modeled using Heavyside functions.

By the late 1990’s most geodesists had noticed the presence of annual oscillations in station positions, especially in the vertical component. At first it was not obvious if these oscillations were real or artificial, but in 2001 and 2002 it was recognized that these oscillations were actual ground motions manifesting earth’s elastic response to seasonal changes in the loads imposed on the solid earth by the atmosphere, the hydrosphere and the cryosphere (Heki, 2001; van Dam et al., 2001; Blewitt et al., 2001; Dong et al., 2002). This mainly vertical elastic response to environmental loading cycles occurs at global (Blewitt et al., 2001), regional (Heki, 2001) and local (Bevis et al., 2004; 2005) scales. Perhaps the most striking example of seasonal oscillation was provided by the GPS station in Manaus, located in the Central Amazon Basin. This station (MANA), which no longer exists, revealed an annual vertical displacement cycle with a peak-to-peak amplitude of 50 – 70 mm. This cycle was very strongly anti-correlated with the stage height of the nearby Amazon river (Figure 1.1). Bevis et al. (2005) showed that this annual displacement cycle was a purely elastic response to changes in the weight of the Amazon river system.
Figure 1.1 (a) Stage height time series $H(t)$ observed in Manaus, Brazil (b) daily solutions for the upwards component of displacement $U(t)$ at GPS station MANA (dots), and the model prediction (solid curve), (c) and (d) geodetic measurements (dots) and model predictions (solid curves) for the north and east components of displacement. (Reprinted from Bevis et al. 2005)
In response to these discoveries, geodesists modified their station trajectory models by incorporating a truncated Fourier series, typically composed of just annual and semi-annual terms, in addition to a constant velocity trend and Heavyside jumps if and when jumps were required. As we show in the next section, trend+cycle+jump trajectory models can be formulated as purely linear systems.

Geodetic observation of postseismic transient deformation in the near and medium field of large earthquakes has prompted geophysicists to add nonlinear components to the trajectory models for limited numbers of GPS stations. The existence of postseismic transient deformation has been known for many decades (Okada and Nagata, 1953; Kanamori, 1973; Thatcher and Rundle, 1984), but only since ground motion can be recorded continuously using CGPS stations has there been a lot of observational evidence for the precise functional form of its time dependence (e.g Shen et al., 1994; Heki et al., 1997; Perfettini et al., 2010). Synthetic Aperture Radar Interferometry, or InSAR, has also played an important role in recent studies of postseismic deformation by providing a spatially continuous image of the transient motion field (Peltzer et al., 1996).

Postseismic deformation is believed to result from at least two distinct processes: afterslip that is confined to the fault surface (Marone, 1991; 1998; Savage and Svarc, 1997; Bock et al., 1997) and bulk relaxation processes (Thatcher and Rundle, 1984; Ivins, 1996; Segall, 2009) affecting the volume surrounding the fault. A variety of bulk relaxation mechanisms have been suggested including viscoelastic relaxation of the asthenosphere and/or ductile portions of the crust (Elsasser, 1969; Thatcher and Rundle, 1984; Ivins, 1996) and poro-elastic deformation of a fluid saturated crust (Rice and Cleary, 1976; Peltzer et al., 1998; Fialko, 2004). Only afterslip models have generated a
simple analytical representation for the time-dependence of postseismic deformation which is widely supported by geodetic observations. This dependence has the form

\[ P(t) = A \log(1 + \delta t / \tau) \]  

where \( p \) is a component of the station position or displacement vector, \( A \) is the amplitude of the transient, \( \delta t \) is the time since the earthquake occurred, and \( \tau \) is the relaxation time. This expression is valid only for \( \delta t > 0 \). This equation is non-linear because the transient behavior is parameterized in terms of \( \tau \) and not just \( A \).

There is an emerging consensus that afterslip often dominates postseismic deformation in the early history of postseismic deformation (Perfetinni et al., 2010). Many authors believe that the postseismic deformation occurring many decades after truly great earthquakes such as the 1960 Chile and the 1964 Alaska megathrusts are dominated by viscoelastic relaxation of the asthenosphere (e.g. Wang et al., 2007).

This thesis focuses on linear and not nonlinear models for geodetic station trajectories. Such models are not suitable for modeling short-term transient deformation associated with afterslip). But they are quite capable of modeling the steady and non-steady motions observed at thousands of GPS stations worldwide, including, as we shall show, postseismic deformation occurring several decades after a great earthquake occurred.

1.3 Thesis outline

We explain our use of polynomial time trends, and how they fit into a general linear trajectory model, in Chapter 2. In using this approach to model geodetic time series we encounter a traditional problem of least squares fits (or inversions): they are very
sensitive to outliers. In the past members of the Geodesy and Geodynamics group at OSU went through an preliminary stage of data analysis that indentified and eliminated highly discrepant data or outliers. More recently we perform the data cleaning concurrently with the inversion of the trajectory models using an iterative reweighting scheme that has been quite widely used to achieve robust inversions (i.e. to implement inversions that essentially ignore outliers). This approach is described in Chapter 3. We then illustrate the use of a polynomial time trend to analyze the uplift of the GPS station THU3 (Thule, Greenland) in Chapter 4. This station is now routinely analyzed at OSU along with the stations of the Greenland GPS Project (GNET). The accelerating uplift evident at THU3 has previously been analyzed using piecewise linear functions (Khan et al., 2008; 2010; Jiang et al., 2010). We show that it is possible to use a quadratic trend instead. Finally in Chapter 5 we present an entirely new result, and one that we believe to have considerable significance. We detect and characterize time variations in the east component of station COYQ, in southern Chile, which is the only long-lived CGPS station established in the rupture zone of the great 1960 Chile earthquake - the most powerful earthquake ever recorded. We show that this station which initially moved seawards (Wang et al., 2008) has steadily slowed down and recently has reversed its motion so that it is now moving towards Argentina. This is the first time, to our knowledge, that anyone has resolved the rate of decay of a postseismic transient more than forty years after the main event occurred.
CHAPTER 2

USING LEAST SQUARES TO ESTIMATE LINEAR TRAJECTORY MODELS

2.1 Background

Station position time series are most commonly specified in a global or local cartesian coordinate system. The most commonly used global reference system is the well known earth-centered, earth-fixed (ECEF) cartesian axis system {X, Y, Z} whose Z axis roughly coincides with the earth’s spin axis. The most common local cartesian coordinate system has an origin located in the immediate vicinity of the station whose movements are being described, and {E, N, U} axes that are oriented east, north, and up, respectively, where the ‘up’ direction is parallel to the local normal of the reference ellipsoid (e.g. Hoffman-Wellenhof et al., 2001). The global axis system {X, Y, Z} is useful in that it allows us to define the position of an entire network of geodetic stations in a single axis system. In contrast when we describe crustal motion using local (or topocentric) coordinates each GPS station has its own local axis system. The advantage, of course, is that there is often a special significance to the horizontal and vertical components of motion, and these can only be described in a local coordinate system.

Suppose some station has a position vector \( \mathbf{p}(t) \) with components \( x(t) \), \( y(t) \) and \( z(t) \)
and there is a fixed point $p_R$ located very near to that station which can serve as a local point of reference. We can define the motion of this station by examining the temporal evolution of the vector $d(t) = p(t) - p_R$. This deviation or displacement vector $d$ is trivial to compute in the geocentric coordinate system. But it is also a simple matter to express the components of this vector in the local {E, N, U} system, thus:

$$d_e = T d_x$$

where $d_e = [e \ n \ u]'$, $d_x = [\delta x \ \delta y \ \delta z]'$, and $T$ is the appropriate transformation matrix (Hoffman-Wellenhof et al, 2001). Note that $d_e$ and $d_x$ are two representations of the same vector.

Crustal motion geodesists frequently move between $p(t)$ and $d_e(t)$ as essentially equivalent descriptions of crustal motion at a given material point on the surface of the solid earth. Most of the formulas developed below can be applied to either description, but we will refrain from pointing this out because of the repetition involved.

### 2.2 Linear Trajectory Models

The standard linear model for the trajectory of a GPS station (within a given reference frame) consists of three sub-models which we can refer to as the time-trend, the jumps and the oscillatory components of motion. We now describe each of these model components in turn.

(i) The trend. (Figure 2.1(a))

Traditionally the temporal trend of station position or displacement has been described under the assumption of constant station velocity, using a formula similar to
\[ \mathbf{p}(t) = \mathbf{p}_R + \mathbf{v}(t - t_R) \]  

which is equivalent to a set of three equations (one for each vector component):

\[
\begin{align*}
x(t) &= x_R + v_x(t - t_R) \\
y(t) &= y_R + v_y(t - t_R) \\
z(t) &= z_R + v_z(t - t_R)
\end{align*}
\]  

The parameters of this model are the reference position \( \mathbf{p}_R = [x_R \ y_R \ z_R]' \) and the station velocity \( \mathbf{v} = [v_x \ v_y \ v_z]' \). Note that the reference time \( t_R \) is not a model parameter but is adopted by convention. We normally choose the reference time \( t_R \) to be the mean of all times \( \{t_i\} \) at which \( \mathbf{p} \) was measured or observed.

We generalize the typical approach by assuming that the displacement trend can be described as a polynomial function of time, thus

\[ x(t) = p_0 + p_1 \Delta t + \cdots + p_n \Delta t^n \]  

where \( x \) is just one component of position or displacement, \( \Delta t = (t - t_R) \) and \( n \) is the maximum order of the polynomial. If \( n=1 \), then this reduces to the constant velocity model. In practice, \( n \) will rarely be assigned a value greater than 4 or 5. After the standard case \( n=1 \), we shall most often consider the quadratic case, \( n = 2 \):

\[ x(t) = p_0 + p_1 \Delta t + p_2 \Delta t^2 \]  

Note that the second derivative \( \frac{d^2 x}{dt^2} = 2p_2 \) which leads us to call this the ‘constant acceleration’ trend model. This contrasts with the cases \( n = 0 \) and \( n = 1 \) which are the ‘constant position’ and ‘constant velocity’ models.

(ii) Heavyside jumps (Figure 2.1(b))

Jumps in station position, ie. discontinuities in the position coordinate at a given time
are modeled using Heavyside function $H(t)$, where $H(t)=0$ for $t<0$, $H(t)=1/2$ for $t=0$, and $H(t)=1$ for $t>0$. The trajectory model for this jump component is

$$x(t) = \sum_{j=1}^{n} h_j H(t - T_j)$$  \hspace{1cm} (2-6)$$

where $T_j$ is the time of the $j$th jump and $h_j$ is the magnitude or amplitude of the $j$th jump. I will just use one time jump at $T$ as the example to illustrate our model, so the formula (2-6) can be reduced to

$$x(t) = h H(t - T)$$  \hspace{1cm} (2-7)$$

(iii) Oscillations or cycles (Figure 2.1(c))

Seasonal cycles of displacement are modeled using a truncated Fourier series

$$x(t) = \sum_{k=1}^{n} s_k \sin\left(\frac{2\pi t}{\tau_k}\right) + c_k \cos\left(\frac{2\pi t}{\tau_k}\right)$$  \hspace{1cm} (2-8)$$

Theoretically, the function (2-8) can model any shape of oscillation curve in the time series, but in practice, we found two terms of function (2-8), $k=1$, $\tau_1=1$ year and $k=2$, $\tau_2=0.5$ year is good enough for our model, which stands for the annual and semi-annual oscillations. So function (2-8) can be written as

$$x(t) = s_1 \sin(2\pi t) + c_1 \cos(2\pi t) + s_2 \sin(2 * 2\pi t) + c_2 \cos(2 * 2\pi t)$$  \hspace{1cm} (2-9)$$

Therefore an example of a linear trajectory model is (Figure 2.1(d))

$$x(t) = p_0 + p_1 \Delta t + p_2 \Delta t^2 + h H(t - T) + s_1 \sin(2\pi t) + c_1 \cos(2\pi t) + s_2 \sin(2 * 2\pi t) + c_2 \cos(2 * 2\pi t)$$  \hspace{1cm} (2-10)$$

which, in this case, invokes a quadratic trend with $\Delta t = (t - t_R)$, and there is a single jump at time $T$, and both annual and semi-annual oscillatory terms. In this example the model has 7 coefficients, and the model vector for this specific component, $x$, would be
\( \mathbf{m} = \begin{bmatrix} p_0 & p_1 & p_2 & h & s_1 & s_2 \end{bmatrix} \). Note that the models for the x, y and z components of a position vector in global Cartesian coordinates are essentially independent. Just as one can convert the position times series between the x,y,z and e,n,u representations, one can readily convert the total model vector \( \mathbf{m} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix} \) into topocentric coordinates.

Typically the model parameters are computed in global cartesian coordinates, and these are subsequently converted to local cartesian coordinates, if desired.
Figure 2.1 The simulation plot of our model, to illustrate the function of model would match the GPS observations, without considering any real conditions, parameters, and scales. (a) polynomial part (b) Heavyside jump part (c) oscillation part (d) composite final model.
Assume we have a set of \(n\) measurements \(\{t_i, p_i\}_{i=1}^{n}\), where \(t\) is the predictor variable (independent), the time of time series, and \(p_i = [x_i \ y_i \ z_i]'\) is the response variable (dependent), the position of stations.

Still keep discussing the following problems in one component \(x\),

So the model in matrix notation is

\[
x = \begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
\vdots \\
x_n
\end{bmatrix} = \begin{bmatrix}
1 & \Delta t_1 & \Delta t_1^2 & H(t_1 - T) \\
1 & \Delta t_2 & \Delta t_2^2 & H(t_2 - T) \\
1 & \Delta t_3 & \Delta t_3^2 & H(t_3 - T) \\
\vdots & \vdots & \vdots & \vdots \\
1 & \Delta t_n & \Delta t_n^2 & H(t_n - T)
\end{bmatrix}
\begin{bmatrix}
p_0 \\
p_1 \\
p_2 \\
h \\
s_1 \\
c_1 \\
s_2 \\
c_2
\end{bmatrix}
\]

which is the same as other two components, \(y, z\).

### 2.3 Model estimate

Once we got the model, the next step of our problem is to solve the parameters of the model. In order to the better understand and convenient description, following keep discussing in one component \(x\).

The method of least squares will be used from here:

formula (2-11) can be formulated like

\[
x \approx Am_x
\]

where \(A\) is the design model matrix, \(x=[x_1 \ x_2 \ x_3 \ldots x_n]'\) is the data vector,
and \( \mathbf{m_x} = [p_0 \ p_1 \ p_2 \ h \ s_1 \ c_1 \ s_2 \ c_2]' \) is the parameter vector, which needs to be solved.

Given some estimates for \( \mathbf{m_x} \), say \( \tilde{\mathbf{m_x}} \), can get the predicted value of \( \mathbf{x} \), say \( \hat{x} \), where

\[
\hat{x} \approx A\tilde{\mathbf{m_x}}
\]

and so the residual vector, referred as the observed value minus computed value.

\[
r = x - \hat{x}
\]

Traditional way is to find a model vector which minimizes the sum of the squares of the residuals,

\[
\text{minimize } r'r = \sum_{i=1}^{n} r_i^2 = \sum_{i=1}^{n} (x_i - \hat{x}_i)^2
\]

with the problem that \( x \) is known to have covariance matrix \( C_x \) then it is instead to

\[
\text{minimize } r'C_x^{-1}r
\]

The classical solution is

\[
\mathbf{m_x} = X^{-1}A'\mathbf{C_x}x
\]

where

\[
X = A'\mathbf{C_x}^{-1}A
\]

and the covariance matrix of \( \mathbf{m_x} \) is

\[
C_{\mathbf{m_x}} = X^{-1} = (A'C_x^{-1}A)
\]

But in our problem, nothing is reliably known about the errors in \( x \), and the matrix \( C_x \) is unknown. So the solution is

\[
\mathbf{m_x} = \text{inv}(A' * A) * A' * X
\]

and

\[
C_{\mathbf{m_x}} = (A'A)^{-1}
\]
The error analysis of our problem which required a large enough data set, is to ensure that the post-fit residuals are consistent with the covariance matrix $C_{m_x}$. The $\sigma^2$ is estimated, which is known as the variance of unit weight,

$$\sigma^2 = \frac{r^\prime r}{(n-j)}$$

(2-22)

Where $n$ is the dimension of vector $x$, also the number of observations, and $j$ is the dimension of vector $m_x$, also the number of parameters.

All the methods discussed above can be applied on the other two components, $y, z$ to estimate $m_y$ and $m_z$. 
CHAPTER 3

USING IRWLS TO EDIT THE DATA

3.1 Background

Once we can estimate the model through $\sigma^2$, formula (2-22), we have the ability to adjust our model through getting rid of the outliers. The method of Iteratively Re-Weighted Least squares (IRWLS) has been used, which is a robust inversion technique which massively downweights, and effectively ignores gross outliers in the data.

As we know general least squares is a very common method of optimal fitting. At present, the least squares method is widely used in the field of science and technology, many practical problems, such as curve fitting, data smoothing, state estimation, function approximation, system identification, and time series modeling. Although some practical problems with the least squares method can achieve excellent results, there are some limitations of the method. For example, when less data collected and data mixed with abnormal points, the results obtained with least square method become very unreliable, in which case the applications of regression equations to predict or model fitting would draw the accuracy of forecasts or fitting very low, even not acceptable.

In fact, when the data points include the abnormal points, these outliers would have
larger deviation, which leads to the relatively greater square value. In order to suppress this square value, which is the key principle of least square, we have to yield to these outliers, increasing the effect of these outliers to the regression line, which would pull the regression line close to the outliers, resulting in distortion of linear fitting.

The usual method is getting rid of outliers based on intuition and experience, this deal has two main disadvantages: First, after removing outliers, the obtained regression model would be badly affected because of reduced sample size, especially when there are fewer data available. On the other hand, outliers in some ways are just a true reflection of some special conditions, should not be arbitrarily removed. This is what happened in our most cases of trajectory modeling. Therefore, we use robust regression to deal with outliers, which leads to the method of Iteratively Re-Weighted Least squares.

In order to better understand how the method of IRWLS works, I have used MATLAB to simulate a simple example to illustrate that. (Figure 3.1) Assume we have 50 data points, and I included 10%, 5 points as outliers. Plot (a) is the raw sample with 50 data points without outliers. Plot (b) shows the distortion of regression line by the effect of 5 outliers. Plot (c) indicates that the less distortion of regression line after downweighting 50% of the effect of outliers. Plot (d) the regression line is similar with the raw sample after only weighting 5% of the effect from the outliers, can be also considered as completely downweighted.
Figure 3.1 The simulation plot to illustrate how the method IRWLS works. The blue dots are the data points, the red dots are the outliers, and the green lines are the regression lines. \( w \) indicates the percentage of weighted.
3.2 The weighting routine of model adjustment

General least squares estimate which is given for each sample data with equal weight (all 1), in the case of absence of unusual value, is the optimal estimation with smallest residual sum of squares. However, within the outliers, the same non-discriminatory weight applied, would greatly reduced the model accuracy. Thus, most of our trajectory modeling cases, which the outliers exist in the data set and cannot be simply removed, have required to use the robust regression, whose principle is based on the weighted least squares estimation form.

All in all, Iteratively Re-Weighted Least squares is an iterative algorithm for fitting a linear model in the case where the data may contain outliers that would distort the parameter estimates. The procedure uses weighted least squares, the influence of an outlier being reduced by giving that observation a small weight. The weight chosen in one iteration is related to the magnitudes of the residuals in the previous iteration — with a large residual earning a small weight.

The following is the basic weighting routine applied in our projects:

Assume we have a linear regression model as \( y = Ax \), same as the Chapter 2

1. Using least squares with initial standard linear trajectory model, to solve the parameter vector \( x \), resulted the initial estimate \( \hat{x} \).

2. Check the station data with high percentage of outliers, make sure the initial model is reasonable fit the measurements, if not, adjust the initial model and rerun the step (1).

3. From formula \( \hat{x} = \mathbf{A} \mathbf{m} \) (2-13) and \( \mathbf{r} = \mathbf{x} - \hat{x} \) (2-14), solved the magnitudes of the residuals.
(4) Using formula $\sigma^2 = \frac{r'r}{(n-j)}$ (2-22), in which sigma decides the weight of that outlier.

(5) Back to the step (1), solve another estimate, robust estimate, which based on the weight of the outliers.

(6) Repeat the above steps, until generated the optimal linear regression model with acceptable percentage of outliers.

With this routine the vector sigma represents the prior estimates of the standard deviations in station trajectory once the outliers are excluded. This is the starting point for weighting. But the weights will be adjusted according to the residuals on an iterative basis. In this way measurements identified as outliers will be completely downweighted.

### 3.3 Study example

To testify and better understand our modeling work on the geodetic time series, this research will draw some data examples from our group’s research project region, Greenland and Chile. In Greenland, the observations will exactly match the Earth’s behavior which we discussed before, uplift of vertical direction dues to the accelerating ice loss in the glacial region. However, the measurements in Chile are a slight different, most of the uplifts behaved strong in the horizontal direction, whose reason will be explained detailedly in Chapter 5, but still can be applied using the same model.

The background, field work and data processing of each project will be introduced in their own chapter, and all the example data are acquired, processed and analyzed by our
own group. The sample data used in this study had been processed by the method that is well described by Kendrick et al. (1999), using GAMIT (King and Bock, 1998) and GLOBK (Herring, 1998) software.
CHAPTER 4

STUDY CASE ONE – GREENLAND

4.1 Introduction of GPS network – GNET

OSU is leading the construction and analysis of the Greenland GPS Network (GNET), which is composed of about 50 CGPS stations ringing the entire sub–continent. This project is a collaboration between OSU, the DTU Space Institute in Denmark and the University of Luxembourg. The project is funded by the US National Science Foundation and Supported by the NSF – sponsored facilities UNAVCO and CPS. We plan to use this network to provide GRACE with an accurate PGR correction. We also intend to use GNET to measure seasonal and long-term ice mass changes by exploiting Earth’s elastic response to changes in surface loading. The geometry of GNET is shown in Figure 4.1
Figure 4.1 The Greenland GPS Network (GNET). Stations symbols indicate the year in which the stations were installed. The older stations whose four letter codes are underlined (such as THU3) are part of the global tracking network of the IGS.
**4.2 Previous geodetic study in Greenland**

Greenland has the second largest ice cap on Earth, after Antarctica. It contains about 1/20 of the ice on this planet, and if the Greenland ice cap was completely melted it would cause global sea level to rise by about 7 meters.

Many of the older GPS stations in Greenland have time-varying rate of uplift. In most cases tendency is toward accelerating uplift (Khan et al., 2007; 2008; Jiang et al., 2010).

So far the time variable uplift rate have been characterized using piecewise – linear trajectories, such as that produced by Khan et al. (2010) and reproduced here as Figure 4.2.

We contend that all of these uplift histories could be characterized using polynomial trend model, and we shall illustrate this claim by focusing on the uplift recorded by the station THU3 in Thule, NW Greenland.
Figure 4.2 The plot of acceleration of uplift in the time series in vertical component of station THU3, by using the linear piecewise function model (Khan et al., 2010).

### 4.3 An analysis of the uplift history at THU3

The GNET data have been processed following the same methods and reference used in our group’s previous work on West Antarctic GPS Network (WAGN) (Bevis et al., 2009).

The vertical time series observed as THU3 is shown in Figure 4.3(A). We have modeled this trajectory using a quadratic trend, and annual and semi–annual oscillation. This is a constant acceleration model, and it implies the velocity history shown in Figure
4.3 (B). We found that the uplift rate in Thule has increased by about a factor of five between early 2003 and late 2009. Accelerating rates of bedrock uplift at Thule implies regional ice loss has been accelerating during this same time period.

The mean annual oscillation is shown in greater detail in Figure 4.3(C). The oscillation involves two peaks a year. This displacement cycle is caused by seasonal changes in atmospheric pressure, as well as seasonal changes in ice mass.
Figure 4.3 The time series outcome of station THU3. (A) the uplift of vertical component, blue dots indicate the data observation, red line shows the trajectory model. (B) the velocity trend of vertical component. (C) the mean annual cycle of time series.
CHAPTER 5

STUDY CASE TWO - CHILE

5.1 Introduction of GPS network - CAP

Our group has been using GPS to measure crustal motion and deformation in South America since 1993 (Figure 5.1). The CAP team is Ohio State University, the University of Memphis, the University of Hawai’i, the Instituto Geografico Militar (IGM) de Chile, the Universidad de Concepcion (Ch.), the Centro de Estudios Cientificos (CECS, Ch.), the IGM de Argentina, the Universidad Nacional de Cuyo (Arg.) and the Universidad Nacional de Buenos Aires (Arg.).

The CAP network is being used to study the interseismic, the coseismic and the postseismic stages of the earthquake deformation cycle (Bevis et al., 1999; Bevis et al., 2001; Brooks et al., 2003; Kendrick et al., 1997; Kendrick et al., 2001; Kendrick et al., 2006; Wang et al., 2007).

Southern Chile was the site of the largest earthquake ever recorded – the (Mw 9.5) 1960 Chile event, and the fifth largest event ever recorded – the (Mw 8.8) Maule event of 2010. In this study we will focus on postseismic deformation associated with the 1960 event.
Figure 5.1 The map of continuous GPS stations of CAP in South America
5.2 Previous study of postseismic deformation following the 1960 megathrust

The 1960 great Chilean earthquake was the most powerful earthquake ever recorded, with moment magnitude of 9.5.

Kendrick et al. (1997) identified an inland CGPS station (COYQ) near the rupture zone of the 1960 earthquake, which had an anomalous seaward movement. Klotz et al. (2001) later presented the results from a survey GPS network and showed all coastal stations move landward, in contrast with the seaward movement of inland stations. This observation has been explained as postseismic deformation due to viscoelastic stress relaxation following the 1960 earthquake. The earthquake drives the upper plate to move seaward towards the trench, until the elastic stresses induced in the upper mantle terminate this motion. But afterwards, stress relaxation within the asthenosphere allows additional seaward motion of the upper plate to develop. This postseismic deformation persists for many decades (Hu et al., 2004; Wang et al., 2007).

Our focus in this thesis is to see if we can detect the slow temporal decay of the postseismic transient. There is only one CGPS station in the general area of the 1960 event which has a long enough time series to allow us to detect such velocity change – the station COYQ near Coyhaique.

5.3 GPS data results and estimate

Station COYQ is located a little north of the Nazca-South America-Antarctica triple junction, which was the southern terminus of the 1960 event, and not far from the Chilean border with Argentina. The GPS data from COYQ have been processed
following the method of Kendrick et al. (2007), in a South America-fixed reference frame.

We model the time series at COYQ using a quadratic trend and an oscillation with annual and semi-annual components (Figure 5.3). We clearly resolve an acceleration in the east component of motion. The quadratic trend model for the east component of displacement at COYQ is:

\[ x(t) = p_1(t - t_R) + p_2(t - t_R)^2 \]

where \( x \) is in mm, and \( t \) in years, and \( t_R \) equals to 2003.9852. We find that \( p_1 = -3.4947 \) and \( p_2 = 0.3390 \), which implies an acceleration of 0.678 mm/yr\(^2\).

The implied variation of eastward velocity is shown in Figure 5.4. This figure shows that COYQ recently changed from seaward (west) motion to landward (east) motion! This is the first time, to our knowledge, that anyone has resolved the rate of decay of a postseismic transient more than forty years after the main event occurred. According to our trajectory model, this reversal occurred in 2009.14. That is > 48 years after the 1960 Chile earthquake which occurred on 22 May 1960.
Figure 5.2 COYQ displacement time series in a SouthAmerica-fixed frame
Figure 5.3 The acceleration plot in east component of COYQ
CHAPTER 6

CONCLUSIONS

We have shown that station trajectory models in which the secular trend of displacement can be represented as a polynomial function of time can be very useful for modeling GPS time series obtained in areas undergoing accelerating ice loss, and in areas undergoing postseismic transient deformation a decade or more after a great earthquake. The Geodesy and Geodynamics Group is now modifying all of its software for reference frame realization and trajectory analysis so that the analyst has the option of assigning a quadratic, cubic or quadratic trend model instead of the more conventional constant velocity model (corresponding to a degree one polynomial). Of course the great majority of GPS stations can be characterized perfectly adequately using a constant velocity trend model. But the areas in which this is not true are areas of considerable geodynamic interest. This thesis presents a new tool for studying those areas.
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