Performance Evaluation of RF Systems on Rotorcrafts

A Thesis

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By

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RF systems onboard rotorcrafts are susceptible to a periodic variation in both the magnitude and phase of the received signals due to the rotation of the rotor blades. This is often referred to as rotor blade modulation (RBM). As its name indicates, RBM causes a modulation of the incident signal, which is dependent on the frequency and direction of the incident signal. As one might imagine, RBM has the potential to degrade a given RF system. Therefore, RBM must be accounted for when characterizing the performance of any RF system onboard a rotorcraft. The first part of this thesis develops methods for incorporating RBM in computer or hardware-in-the-loop (HITL) simulations of RF systems onboard rotorcrafts. The methods are verified using the response of an antenna mounted on a simple rotorcraft that is analyzed through numerical electromagnetic computations as well as characterized by measurements. In the second part of this thesis the effects of RBM on digital communication systems are discussed. It is demonstrated that one can mitigate the effects of RBM within the receiver using spatial diversity and simple equalization techniques.
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CHAPTER 1

INTRODUCTION

For complete performance evaluation of a RF system (such as a navigation, communication, or radar system), one must include the \textit{in situ} response of the antenna in the performance evaluation. The \textit{in situ} antenna response accounts for the characteristics of the antenna and the platform it is mounted on. In order to characterize the antenna and platform effects for a particular system, one can carry out field tests or use simulations. Typically, field tests are very expensive, and therefore, it is beneficial to be able to carry out computer or hardware-in-the-loop (HITL) simulations.

If the platform of interest is a fixed platform, it is straight forward to include the platform and antenna effects. One can measure/analyze the \textit{in situ} response of the antenna in the direction of the incident signal over the frequency band of the incident signal. The incident signal then can be convolved with this frequency response to include the antenna and platform effects. To carry out the convolution, one can design a FIR filter whose frequency response matches the \textit{in situ} response of the antenna in the direction of the signal. The incident signal is then passed through this FIR filter to include the antenna and platform effects.

For antennas mounted on rotorcrafts, the \textit{in situ} antenna response (frequency dependence) in a given direction will vary with blade orientation due to the change in
platform geometry. Furthermore, as the rotor blades rotate continuously at a given speed, the change in platform geometry causes the response of the antenna to vary periodically as a function of time. This is often referred to as rotor blade modulation (RBM). Rotor blade modulation is a well known phenomenon that has been explored in the open literature. Most of the work has focused on methods for computing the electromagnetic scattering from a set of rotating blades as well as computing the associated doppler spectrum. Both analytic and numerical solutions have been explored and the results have been compared with measurements [1, 2, 3, 4]. It is important to note that the RBM is dependent on the look direction as well as the frequency of the incident signal. As one might imagine, RBM presents a challenging environment for RF systems to operate in, and therefore, RBM can degrade the performance of RF systems. Several efforts using field tests and computer simulations have been made to quantify the effects of RBM on specific RF systems [5, 6, 7]. The main contribution of this thesis is the development of an approach for including RBM in RF simulations that is independent of the incident signal structure, accurate, and efficient to implement in HITL simulations. Additionally, this thesis will analyze the effects of RBM on a digital communication receiver and propose several approaches for improving the receiver’s performance.

This thesis begins by developing three approaches for including the effects of RBM in simulations of RF systems. The approaches presented here are independent of the incident signal structure; therefore, they can be used to simulate any RF system. All three approaches assume that one has knowledge of the time domain response of the antenna at discrete frequencies covering the frequency band of interest.
In the first approach, the finite bandwidth signal incident on the antenna is represented as the sum of CW signals at discrete frequencies. Each of the CW signals is then multiplied by the time domain response of the antenna at that frequency. The signal received by the antenna is then the sum of the antenna-modulated signals at the discrete frequencies. Note that this approach does not involve any approximation and is quite intuitive. The approach can be easily implemented in computer simulations but is not practical in hardware simulations. It will take a lot of resources to decompose an incoming signal into discrete CW signals and to modulate the various CW signals. This approach will be referred to as the “Time Domain Approach (TDA).”

The second approach, which will be referred to as the “Fourier Series Approach (FSA),” exploits the periodic nature of the response of an antenna mounted on a rotorcraft. Since this repetition period is independent of the frequency of the incident signal, the antenna response at various frequencies can be represented using the same harmonics (exponentials). Note that the Fourier coefficients will change with the frequency of the incident signal. Thus, the Fourier coefficients (for a given harmonic) over the bandwidth of interest define the frequency response of a filter. One needs a filter for every harmonic used in the Fourier expansion. The signal received by the antenna is found by summing the output of the various filters after they are shifted in frequency by the harmonic associated with that filter. This is consistent with the analysis of periodically time-varying systems found in [10]. The FSA does not involve any approximation as long as enough harmonics are used in the Fourier series expansion. This approach is quite suitable for hardware simulation; however, for large
number of Fourier harmonics, the hardware cost could be prohibitively large, which makes FSA not very desirable.

In the third approach, the time domain response of the antenna (mounted on a rotorcraft) at a given frequency is approximated as a linear combination of the time domain response of the antenna at $J$ frequencies uniformly spaced over the frequency band of interest. The $J$ linear coefficients are selected to minimize the error between the true antenna response and the approximated antenna response at a given frequency. Like the Fourier coefficient, a given linear coefficient varies with the frequency of the incident signal. Therefore, the linear coefficients (for one of the $J$ frequencies) over the frequency band of interest define the frequency response of a filter. Again, there will be a separate filter for each of the $J$ frequencies used in the approximation. The signal received by the antenna is found by summing the output of the $J$ filters after they are multiplied by the antenna response at the selected frequencies. Note that this approach can be implemented using a few filters and a few modulators. Also, this approach combines features of the TDA and FSA. It will be shown that for most practical system (2 - 3% bandwidth), one needs to use two or three frequencies in the approximation. Thus, this approach can be implemented in hardware simulations without a very high cost. This approach will be called the “Approximate Simulator.”

After describing the TDA, FSA, and Approximate Simulator, several examples using each of the approaches are presented. These examples will show that all three approaches can provide very similar representations of the RBM. In these examples, the time domain antenna response is calculated with the Numerical Electromagnetic
Code - Basic Scattering Code (NEC-BSC) [9], which is a numerical electromagnetic software package.

This thesis also discusses how to measure the time domain antenna response of an antenna on a rotorcraft. Using the measured response of an antenna on a model rotorcraft, it is shown that one can measure the antenna response while the blades rotate at a fixed speed or while the blades are fixed at discrete blade locations. The measured antenna response is also used to evaluate the accuracy of the Approximate Simulator. It will be shown that a second order (two discrete frequencies are used in the approximation) Approximate Simulator is sufficient for most practical systems.

The TDA, FSA, or Approximate Simulator can be used to include RBM in simulations when the antenna is mounted on a dual rotor aircraft. However, one must know the time domain antenna response of the antenna over the frequency band of interest. For antennas mounted on dual rotor aircrafts, the time domain antenna response is periodic, however, the antenna response will depend on the starting location of the blades as well as the blade rotation speed of each rotor. Due to the number of measurements and the amount of time it would take, it is not feasible to obtain the antenna response for all combinations of blade speeds and starting locations. Therefore, this thesis also describes how to efficiently obtain the time domain antenna response at a given frequency in the presence of two rotors. The method assumes that the antenna response for a given set of blade orientations can be approximated as the sum of the antenna response in the absence of both rotors and the antenna response due to each of the rotors. This will be referred to as the “Composite Antenna Response (CAR)”. The major benefit of using the CAR is that the number of computations or
measurements needed to obtain the time domain antenna response for a given set of rotor speeds and starting locations is greatly reduced.

In the second part of this thesis, the effects of the rotorcraft platform on the performance of a digital communication receiver are discussed. First it is shown that when the blades are fixed to a particular location, the performance of the receiver can be severely degraded. The performance loss is caused by two factors. The first factor is a drop in the received signal power caused by the rotor blades blocking the line of sight (LOS) signal path. The second factor that causes a loss in performance is the phase distortion in the received signal, which is caused by the platform. It is shown that the phase distortion in the received signal can be removed using a linear equalizer (LE). The LE combines current and past samples of the received signal to provide the best estimate of the transmitted symbols. Although the LE can account for any phase distortion in the received signal, it cannot account for the drop in received signal power that is caused by the blockage of the LOS signal path. Subsequently, spatial diversity is introduced as an approach to increase the received signal power. Spatial diversity uses multiple antenna elements to receive the incident signal. Since it is unlikely that the rotor blades will simultaneously block the LOS path for all of the antenna elements, the signal can be received with high SNR. It is important to note that in this work, each antenna element has its own linear equalizer and the coefficients of the equalizers are found jointly for all antenna elements. When spatial diversity and equalization are used together, the received signal power is increased and the phase distortion in the received signal is removed. This leads to an overall improvement in the receiver’s performance. This will be referred to as the “Diversity Linear Equalizer (DLE).”
Although the LE and DLE can be implemented, it is important to note that they must be designed for each blade location. In order to reduce this requirement, this thesis proposes to use an average equalizer that is obtained by averaging each equalizer coefficient over one blade rotation period. This will be referred to as the “Average Linear Equalizer (ALE)” when a single antenna element is used and the “Average Diversity Linear Equalizer (ADLE)” when multiple antenna elements are used. It will be shown that the ALE and ADLE can be used with hardly any loss in performance as compared to the LE and DLE, respectively. Note that the LE and DLE provide the best performance since the equalizer coefficients are obtained for each blade location. Furthermore, the ALE and ADLE can be used in the presence of RBM with the blades rotating at any speed. As in the case of fixed blades, it will be shown that when diversity is used with equalization (ADLE) one can mitigate the effects of the RBM. However, it may not be practical to implement the ALE or ADLE because a large number of training symbols must be used to obtain the equalizer coefficients. This is undesirable because a large number of training symbols decreases channel capacity. Furthermore, the algorithm used to find the ALE and ADLE coefficients requires the inversion of a matrix that is of size $N_t \times N_t$, where $N_t$ is the number of training symbols. The matrix inversion is prohibitively expensive in terms of computational costs; therefore, a recursive equalizer that is computationally efficient is also proposed in this thesis. The recursive equalizer uses a recursive least-squares (RLS) algorithm to iteratively update the equalizer coefficients, which change as a function of time. This will be referred to as the “Recursive Least-Squares Equalizer (RLSE)” when one antenna element is used and the “Diversity Recursive Least-Squares Equalizer
(DRLSE)” when multiple antenna elements are used. As expected, the effects of RBM are mitigated when the DRLSE is used.

The rest of the thesis is organized as follows. Chapter 2 starts by describing how to include antenna and platform effects in simulations for antennas mounted on fixed platforms. Next, the TDA, FSA, and Approximate Simulator are discussed in Chapter 3. Chapter 4 discusses the measurement approach used to obtain the response of an antenna mounted on a model rotorcraft. The chapter also evaluates the accuracy of the Approximate Simulator using the measured antenna data. Chapter 5 goes on to discuss how to include RBM in simulations for antennas mounted on dual rotor aircrafts. In chapter 6, the effects of RBM on a digital communication receiver are discussed. Finally, the thesis ends with a summary and conclusions in Chapter 7.
2.1 Overview

This chapter lays the foundation for the remainder of the thesis by describing how to include the antenna and platform effects in simulations when the antenna is mounted on a fixed platform. It will be shown that the antenna and platform effects can be included in the simulations by filtering the incident signal with a filter whose frequency domain response is given by the in situ response of the antenna in the direction of the incident signal. To keep in context with the rest of the thesis, the platform used in the simulations is a rotorcraft with the rotor blades fixed to one position.

2.2 Development

Let $H(f_i, \theta, \phi)$ denote the response of an antenna mounted on a fixed platform at frequency $f_i$ in a given direction $(\theta, \phi)$. This antenna response represents the in situ response that can be obtained through measurement or numerical electromagnetic simulations. Let a signal at frequency $f_i$ (CW signal) be incident on the antenna
from a given direction \((\theta, \phi)\). The incident signal, \(x(t)\), can be written as

\[
x_i(t) = b_i e^{j2\pi f_i t}
\]  

(2.1)

where \(b_i\) is the complex amplitude of the incident signal with respect to the coordinate system origin. The signal received by the antenna is then given by

\[
y(t) = b_i H(f_i, \theta, \phi) e^{j2\pi f_i t}.
\]  

(2.2)

Thus, for a CW incident signal at a given frequency in a particular direction, the signal received by the antenna is found by multiplying the incident signal with the *in situ* response of the antenna in the corresponding direction at the given frequency.

Now let the incident signal, \(x(t)\), be a finite bandwidth signal. This incident signal can be written as a superposition of \(M\) CW signals; i.e.,

\[
x(t) = \sum_{i=1}^{M} b_i e^{j2\pi f_i t}.
\]  

(2.3)

In (2.3), \(b_i\) is the complex amplitude of the \(i^{th}\) frequency component \(f_i\). Combining (2.1) and (2.3), one can write

\[
x(t) = \sum_{i=1}^{M} x_i(t).
\]  

(2.4)

Then from (2.2),

\[
y(t) = \sum_{i=1}^{M} b_i H(f_i, \theta, \phi) e^{j2\pi f_i t}.
\]  

(2.5)

Note that the summation in (2.5) represents the convolution of the incident signal \(x(t)\) with a filter whose frequency response is given by \(H(f_i, \theta, \phi)\), \(i = 1, 2, \ldots M\). Thus the signal received by the antenna is found by filtering the incident signal with the *in situ* antenna response over the frequency band of interest. This is depicted in the block diagram in 2.1.
It should be noted that one has the option to carry out the filtering operation either in the time domain or in the frequency domain. Typically, for computer simulations it is equally feasible to carry out the filtering in the time domain or in the frequency domain, however, for HITL simulations it is typically easier to carry out the filtering in the time domain. In the following sections filtering in the time domain and in the frequency domain will be reviewed.

### 2.2.1 Frequency Domain Filtering

Given the frequency domain response of a filter $G(f)$ and an input signal $x(t)$, the filtered signal $y(t)$ is found by

$$y(t) = \mathcal{F}^{-1}\{\mathcal{F}\{x(t)\} \cdot G(f)\} \quad (2.6)$$

where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and inverse Fourier transform, respectively. To be formal, one must account for the fact that sampled versions of the input signal as well as the frequency response are used on a computer. Given $N$ samples of the input signal $x[n]$ for $n = 1 \cdots N$ and $M$ samples of the frequency response of the filter $G[f_i]$ for $i = 1 \cdots M$, the filtered signal $y[n]$ is found in the following way.

1. Interpolate the $M$ samples of $G[f_i]$ to $N$ samples equally spaced between $f_1$ and $f_M$. This sequence will be denoted as $\tilde{G}[f_q]$ for $q = 1 \cdots N$.

2. Compute the $N$ point discrete Fourier Transform (DFT) of $x[n]$, which will be denoted as $X[f_q]$ for $q = 1 \cdots N$. 

![Figure 2.1: Block diagram of simulator for antennas mounted on fixed platforms.](image-url)
3. Multiply $\tilde{G}[f_q]$ and $X[f_q]$ term by term to obtain $Y[f_q] = G[f_q] \cdot X[f_q]$.

4. Finally, $y[n] = IDFT\{Y[f_q]\}$, where IDFT denotes the inverse discrete Fourier transform.

This section has described how to carry out filtering in the frequency domain. In the following section, the equivalent time domain operation is described.

### 2.2.2 Time Domain Filtering

Given the frequency domain samples of a filter $G[f_i]$ for $i = 1 \cdots M$ one can obtain infinite impulse response (IIR) or finite impulse response (FIR) filter coefficients to obtain a time domain filter that has the same frequency response as $G[f_i]$. However, FIR filters are easier to work with since they are always stable and they are less sensitive to round off errors of the filter coefficients [8]. To this end, only FIR filters are considered here.

![Figure 2.2: Block diagram of time domain filter.](image)

Fig. 2.2 shows a block diagram of the time domain filtering operation. The output of the filter is given by

$$y[n] = \sum_{l=0}^{L} x[n - l] g[l] \quad (2.7)$$
where $L$ is the order of the filter. There are many ways to obtain the filter coefficients $g[l]$ that are described in [8]. A least-squares based approach is considered in this work. The basis for this approach is the relationship between the time domain filter coefficients and the frequency domain response. The two are related by the DFT; i.e.,

$$G[f_i] = \sum_{l=0}^{L} g[l]e^{j2\pi f_i(l-L/2)T_s}. \quad (2.8)$$

In (2.8), $T_s$ is the sampling period or delay between filter taps, which is assumed to be the inverse of bandwidth of interest. (2.8) can also be written using vector notation as

$$G[f_i] = f_i^T g \quad (2.9)$$

where

$$f_i = \left[e^{-j2\pi f_i \frac{1}{2} T_s} \ldots 1 \ldots e^{j2\pi f_i \frac{1}{2} T_s}\right]^T \quad (2.10)$$

and

$$g = [g[0] \ldots g[1] \ldots g[L]]^T. \quad (2.11)$$

Note that lowercase bold print denotes a vector, uppercase bold print denotes a matrix, superscript T denotes the transpose operator, superscript H denotes the Hermitian transpose operator, and * denotes the conjugate. Given the frequency domain response at $M > (L + 1)$ frequency points, one can write a system of equations to solve for $g[l]$.

$$\gamma = Fg \quad (2.12)$$

where

$$\gamma = [G[f_1] \ldots G[f_M]]^T. \quad (2.13)$$

and

$$F = [f_1 \ldots f_M]. \quad (2.14)$$
Thus, the least-squares solution for \( g \) is

\[
g = (F^H F)^{-1} (F^H \gamma)
\]  

(2.15)

This section has discussed the filtering of a signal in time and frequency domain. The following sections will show that the two methods produce the same results. Before presenting these results, the platform and antenna used in the simulations are discussed next.

2.3 Model Rotorcraft

Fig. 2.3 shows the model rotorcraft used in simulations throughout this thesis. The body of the rotor craft is modeled by a six corner flat plate of length 40 ft. The platform is 10 ft. wide at the nose and 2 ft. wide at the tail. The rotor blades are 25 ft. long and 1 ft. wide and they are tilted at an angle of 15° along their axis of length. In this configuration the antenna response repeats every 90°. In the figure, the blades are shown with a blade rotation angle of 0°. An in-house numerical EM code named NEC-BSC [9] was used to calculate the antenna response in various directions over the frequency band of interest. NEC-BSC is a high frequency EM code that uses ray tracing and uniform theory of diffraction to analyze antennas mounted on arbitrary platforms. For these simulations the antenna was assumed to be a crossed slot antenna fed with 90° phase difference to generate right-hand circular polarization (RHCP). The antenna is mounted in the center of the plate at a distance of 21.25’ from the nose. The antenna response was calculated over a 32 MHz bandwidth around the center frequency of 2.2 GHz. It should be noted that in all the calculations, the coordinate origin is assumed to be at the center of the antenna. Also, the x axis is along the length of the rotorcraft and the z axis is perpendicular to the rotorcraft.
body. The y axis is defined to be consistent with a right handed coordinate system. In this coordinate system, the azimuth angle ($\phi$) is measured counter-clockwise from the nose of rotorcraft and the elevation angle ($\theta$) is measured from the z axis to the x-y plane; i.e., $\phi = 0^\circ$ is along the nose of the rotorcraft and $\theta = 0^\circ$ is at zenith.

![Figure 2.3: Model Rotorcraft.](image)

2.4 Frequency Response Examples

In this section, several examples comparing the frequency response of the FIR filter coefficients to the *in situ* response of an antenna mounted on a fixed platform are presented. Throughout these examples an approximation error is used to evaluate how well the FIR filter represents the calculated antenna response. The error is defined as

$$E(f) = 20\log_{10} \left| \frac{H(f, \theta, \phi) - \hat{H}(f, \theta, \phi)}{|H(f, \theta, \phi)|} \right|$$

(2.16)

where $\hat{H}(f, \theta, \phi)$ is the approximated antenna response from the FIR filter.

Fig. 2.4 shows the magnitude and phase of the calculated antenna response versus frequency along the direction $\theta = 25^\circ, \phi = 30^\circ$. Also shown in the figure is the magnitude and phase of the frequency response of the FIR filter found using the LS approach described in this chapter. Additionally, the error between the calculated
antenna response and FIR filter response is shown in the figure. In these examples, the sampling period is 31.25 nsec. The top plots show the magnitude, phase, and error when the filter order is 10. One can see that there are oscillations in the frequency response and that there are very large errors especially near the edge of the frequency band. The bottom figures show the magnitude, phase, and error when the filter order is 50. It is clear from the figures that oscillations are less near the center of the frequency band when the filter order is increased; however, there are still large oscillations near the edge of the frequency bands. The figures also show an overall decrease in error as the filter order increases.

Figure 2.4: Simulated antenna response, FIR filter response, and error along $\theta = 25^\circ, \phi = 30^\circ$. 
Fig. 2.5 and Fig. 2.6 show the magnitude and phase of the calculated antenna response versus frequency along $\theta = 45^\circ, \phi = 30^\circ$ and $\theta = 65^\circ, \phi = 30^\circ$, respectively. Also shown in the figures is the magnitude and phase of the frequency response of the FIR filter used to represent the antenna response and the approximation error. Again, the figures show an overall decrease in error as the filter order is increased, however, it is evident that there is still a large amount of error due to the oscillations near the edge of the frequency band.

Figure 2.5: Simulated antenna response, FIR filter response, and error along $\theta = 45^\circ, \phi = 30^\circ$. 
The above results show that the FIR filter can accurately represent the antenna response near the center of the frequency band but not near the edge due to oscillations in the frequency response. These oscillations can be attributed to the finite length of the FIR filter. Ideally, an infinite number of coefficients are needed to represent the antenna response. Thus, by truncating the filter to a finite length impulse response, oscillations appear in the frequency response. This is the well known Gibb’s phenomenon [8]. It should be noted that if the incident signals are limited to a smaller bandwidth than that of which the antenna response is known, then the FIR filter may provide an acceptable representation of the antenna response. In the
examples presented above, the antenna response was defined over a 32 MHz band-
width. The results showed that with \( L = 50 \), the antenna response is represented
very well between 2182 and 2214 MHz (28 MHz bandwidth). If the incident signals
were limited to 28 MHz or less, one could expect to obtain a good representation of
the signal received by the antenna.

In order to reduce the oscillations in the frequency response, one can reformulate
the LS problem. In doing so, it is assumed that the measured or calculated \textit{in situ}
antenna response is known over a larger bandwidth than the frequency band of in-
terest; i.e., larger than the bandwidth of the incident signals. Thus, the goal is to
find a set of FIR filter coefficients that best match the measured/calculated antenna
response over the frequency band of interest. To achieve this, a weighted LS (WLS)
solution is used.

Let \( W \) be a \( M \times M \) diagonal weighting matrix. The entries of the diagonal
are used to indicate whether a particular frequency \( f_i \) is in the desired or undesired
portion of the frequency band. There is one entry along the diagonal of the matrix
for each frequency \( f_1 \ldots f_M \). If the frequency is in the desired band, the value of the
matrix is 1, otherwise it is 0. With the weighting matrix defined, one can write the
system of equations to solve as

\[
WF\gamma = W\gamma
\]  

(2.17)

Letting \( F' = WF \) and \( \gamma' = W\gamma \) one obtains

\[
F'g = \gamma'.
\]  

(2.18)
Note that the least-squares solution to (2.18) is given by (2.15). Substituting \( F = F' \) and \( \gamma = \gamma' \) in (2.15), one obtains that the WLS solution is

\[
g = (F'HF')^{-1} ((F')^H \gamma')
\]

\[
= ((WF)^H(WF))^{-1} ((WF)^HW \gamma).
\]

(2.19)

\[
= (F^HW^HWF)^{-1} (F^HW^HW \gamma).
\]

Fig. 2.7 shows the magnitude and phase of the calculated antenna response versus frequency along the directions \( \theta = 25^\circ, 60^\circ \) and \( 110^\circ, \phi = 30^\circ \). The antenna response was calculated over a 32 MHz bandwidth. Also shown in the figures is the magnitude and phase of the frequency response of the FIR filter used to represent the antenna response and the approximation error. The filter coefficients were found using the WLS solution. In these examples, the desired frequency band is between 2186 MHz to 2214 MHz (28 MHz), the filter order is 50, and the sampling period is 31.25 nsec. It is clear from the figures that the WLS solution provides an extremely accurate representation of the antenna response over the frequency band on interest. The error is around -130 dB or less over the desired frequency band. Since the FIR filter coefficients provide such an accurate representation of the antenna response, it is expected that the time domain and frequency domain filtering approaches are equivalent. For completeness, a few examples are provided in the following section to demonstrate their equivalence.

2.5 Filtering Examples

Fig. 2.8 shows the real and imaginary parts of received signal when the input signal (black) is a CW signal at 2202 MHz that arrives along \( \theta = 25^\circ, \phi = 30^\circ \). Note that a small portion of the total signal has been zoomed in on to show the fine details. Also
Figure 2.7: Simulated antenna response, FIR filter response, and error along $\theta = 25^\circ$, $45^\circ$, and $65^\circ$, $\phi = 30^\circ$. $L = 50$. Desired frequency band is 2186 MHz to 2214 MHz.

shown in the figures is the received signal when filtering is carried out in the frequency domain (blue) and in the time domain (red). The time domain FIR filter coefficients were obtained using the WLS solution with a desired frequency band of 2186 MHz to 2214 MHz, a filter order of 50, and a sampling period of 31.25 nsec. One can see
that the antenna attenuates the signal and changes the phase by approximately 180° for this particular frequency and direction. Moreover, it is also clear that the time domain and frequency domain filtering provide exactly the same results as expected.

Fig. 2.9 shows the real and imaginary parts of received signal when the input signal consists of two CW signals of equal power at 2196 MHz and 2202 MHz. All other parameters are the same as Fig. 2.8. Again, one can see the time domain and frequency domain filtering provide the same results.

![Figure 2.8](image)

Figure 2.8: Signal received by antenna when the input signal is a CW signal at 2202 MHz (black) arriving along $\theta = 25^\circ$, $\phi = 30^\circ$. Received signal is obtained using frequency domain filtering (blue) and time domain filtering (red).

### 2.6 Summary

This chapter described how to include the antenna and platform effects in simulations when the antenna is mounted on a fixed platform. It was shown that the signal received by the antenna can be obtained by convolving the incident signal with the in situ response of the antenna in the direction of the incident signal. It was demonstrated that this can be achieved by filtering the incident signal in the frequency
domain or in the time domain. To this end, unless otherwise stated, all filtering discussed in the remainder of this thesis is carried out in the frequency domain with the understanding that one can design an equivalent time domain filter using the approach presented in this chapter. The following chapter introduces antennas mounted on rotorcrafts and describes how to include the effects of RBM in simulations.
CHAPTER 3

INCLUSION OF ROTOR BLADE MODULATION IN SIMULATIONS

3.1 Overview

This chapter starts by discussing antennas mounted on rotorcrafts. A simple example is used to illustrate the effect of the rotor blades on the *in situ* antenna response. The chapter will also develop three approaches for including RBM in simulations. Namely, the Time Domain Approach (TDA), Fourier Series Approach (FSA), and the Approximate Simulator. Example results are provided to demonstrate the use of the three approaches. In the examples shown in this chapter, the antenna and rotorcraft are the same as in Chapter 2.

3.2 Antennas Mounted on Rotorcrafts

Fig. 3.1 shows the magnitude and phase of the antenna response at 2200 MHz. The trace in black shows the antenna response when the blades are fixed to a blade rotation angle of 0°. Also shown in the figures is the antenna response as a function of blade rotation angle (red). The antenna response was calculated using NEC-BSC as the blade rotation angle was changed from 0° to 90° in 0.5° steps. The antenna
response is shown along the directions $\theta = 25^\circ, 45^\circ$, and $65^\circ, \phi = 30^\circ$. One notices a significant variations in both the magnitude and phase of the antenna response that is caused by the change in orientation of the rotor blades. Moreover, the variations are very different for all three directions. Along $\theta = 25^\circ$ and $45^\circ$, one notices that the magnitude of the antenna response varies as much as 10 dB. This will certainly have an adverse effect on any RF system. Along, $\theta = 65^\circ$ there is less variation in the antenna response. This is expected as the signal arrives from an elevation angle that is below the plane of the blades.

It should be noted that if the blades are moving at a fixed speed, one can associate a repetition period with the response of the antenna. Thus knowing the antenna response at a set of blade rotation angles is equivalent to knowing the response of the antenna as the blades rotate at a fixed speed. This is true provided that the antenna response is calculated/measured at a sufficiently fine blade increment angle. This will depend on the size of the rotor blades as well as the frequency band of interest. A general rule of thumb is that the arc length traced out by the rotor blades should be less than half a wavelength as the blade rotation angle changes between calculations/measurements. Throughout the remainder of the thesis, the antenna response shown in Fig. 3.1 will be referred to as the time domain response of the antenna at a given frequency. It should be emphasized that the time domain antenna response will vary with both look direction and frequency. The following section develops a method to include RBM in simulations assuming that the time domain response of the antenna mounted on a rotorcraft is known over the frequency band of interest.
3.3 Time Domain Approach

Let $h(t, f_i, \theta, \phi)$ be the time domain response of an antenna mounted on a rotorcraft along a given direction $(\theta, \phi)$ at frequency $f_i$. Assuming the blades rotate at a constant speed and the shape of the blades does not change, $h(t, f_i, \theta, \phi)$ will be
periodic with a period of $T_b$ seconds. Note that $T_b$ will depend on the blade rotation speed and the number of blades. If there are $B$ blades and $S$ is the rotation speed (revolution per second), then $\frac{1}{BS} \leq T_b \leq \frac{1}{S}$. If all blades are similar, $T_b = \frac{1}{BS}$. On the other hand, if all blades are dissimilar, $T_b = \frac{1}{S}$. It should be noted that unless otherwise stated, the response of the antenna is being considered along a single direction. Therefore, the directional dependence will be dropped from the notation from this point forward.

Let a signal at frequency $f_i$ (CW signal) be incident on the antenna from a given direction ($\theta, \phi$). The incident signal, $x(t)$, can be written as

$$x_i(t) = b_i e^{j2\pi f_i t}$$  \hspace{1cm} (3.1)

where $b_i$ is the complex amplitude of the incident signal with respect to the coordinate system origin. The signal received by the antenna is then given by

$$y(t) = b_i h(t, f_i) e^{j2\pi f_i t}.$$  \hspace{1cm} (3.2)

Note that $h(t, f_i)$ is periodic, therefore, one only needs to measure/analyze $h(t, f_i)$ over one period ($T_b$). Thus, knowing the time domain response of the antenna at frequency $f_i$, one can simulate the signal received by the antenna for all time instants using (3.2).

Now let the incident signal, $x(t)$, be a finite bandwidth signal. This incident signal can be written as a superposition of $M$ CW signals; i.e.,

$$x(t) = \sum_{i=1}^{M} b_i e^{j2\pi f_i t}.$$  \hspace{1cm} (3.3)

In (3.3), $b_i$ is the complex amplitude of the $i^{th}$ frequency component ($f_i$). Combining (3.1) and (3.3), one can write

$$x(t) = \sum_{i=1}^{M} x_i(t).$$  \hspace{1cm} (3.4)
Then from (3.2),
\[ y(t) = \sum_{i=1}^{M} b_i h(t, f_i) e^{j2\pi f_i t}. \] (3.5)

Thus, knowing the time domain response of the antenna over the frequency band of interest, the signal received by the antenna can be simulated using (3.5). Fig. 3.2 illustrates the process described by (3.5), which will be referred to as the “Time Domain Approach” (TDA) from this point forward. The incident signal is first decomposed into \( M \) CW signals and then each of these CW signals is modulated by the time domain response of the antenna at the corresponding frequency. Finally, the signal received by the antenna is found by summing each of the modulated signals together.

The TDA is quite intuitive and can be easily implemented in computer simulations. However, it is not suitable for implementation in HITL testing as it would require a significant amount of resources to decompose the incoming signal into many narrowband signals. The next section develops the Fourier Series Approach (FSA) for including RBM in simulations. It is shown that the FSA is more suitable for implementation in HITL testing.
3.4 Fourier Series Approach

As mentioned in the previous section, the response of an antenna, \( h(t, f_i) \), mounted on a rotorcraft at a given frequency \( f_i \) is periodic with time period equal to \( T_b \). Since any periodic function can be represented as a Fourier series, one can write \( h(t, f_i) \) as a Fourier series; i.e.,

\[
h(t, f_i) = \sum_{k=-K}^{K} A_k(f_i) e^{j\omega_0 kt} \tag{3.6}
\]

where \( A_k(f_i) \) is the \( k \)th Fourier coefficient at frequency \( f_i \) and \( \omega_0 = \frac{2\pi}{T_b} \). Knowing \( h(t, f_i) \) over one period, \( A_k(f_i) \) can be calculated as

\[
A_k(f_i) = \frac{1}{T_b} \int_{-T_b/2}^{T_b/2} h(t, f_i) e^{-j\omega_0 kt} dt. \tag{3.7}
\]

Recalling that one can write the signal received by the antenna as

\[
y(t) = \sum_{i=1}^{M} b_i h(t, f_i) e^{j2\pi f_i t}, \tag{3.8}
\]

and substituting (3.6) in (3.8), one obtains

\[
y(t) = \sum_{k=-K}^{K} \left( \sum_{i=1}^{M} b_i A_k(f_i) e^{j2\pi f_i t} \right) e^{j\omega_0 kt}. \tag{3.9}
\]

Note that the inner summation in (3.9) represents the convolution of the incident signal \( x(t) \) with a filter whose frequency response is given by \( A_k(f_i), i = 1, 2, \ldots M \). Let \( z_k(t) \) be the output of the \( k \)th filter after convolution. Then \( z_k(t) \) can be expressed as

\[
z_k(t) = \sum_{i=1}^{M} b_i A_k(f_i) e^{j2\pi f_i t} \tag{3.10}
\]

and \( y(t) \) can be expressed as

\[
y(t) = \sum_{k=-K}^{K} z_k(t) e^{j\omega_0 kt}. \tag{3.11}
\]
The exponential term in (3.11) represents a frequency shift of $k\omega_0 = \frac{k \pi}{T_b}$. Thus, one can frequency shift the output of the various filters and sum these frequency shifted outputs to simulate the signal received by the antenna. This is consistent with the analysis of periodically time-varying systems found in [10]. The “Fourier Series Approach” (FSA) is illustrated in Fig. 3.3. Note that the output of each filters goes through a different amount of frequency shift. Also, the frequency shift depends on $T_b$. The smaller $T_b$ is (blades are moving faster), the larger the frequency shift will be, as expected.

Although the FSA is not very intuitive, it can be easily be implemented in HITL testing. However, as one can see from Fig. 3.3, this approach requires $2K + 1$ filters and $2K$ frequency shifters for each incident signal. For large $K$, this approach leads to a very expensive signal simulator. In the next section, a simple approximation of the time domain antenna response is proposed. It will be shown that by using this approximation, one needs a small number of filters and modulators to incorporate the effects of rotor blade modulation in simulations. Thus it is a suitable for implementation in HITL simulations.

\[
\begin{align*}
    x(t) & \rightarrow A_0(f) \times e^{jK\omega_0 t} \rightarrow y(t) \\
    & \vdots \\
    & \rightarrow A_K(f) \times e^{jK\omega_0 t} \\
    & \rightarrow \sum e^{-jK\omega_0 t} \\
    & \rightarrow e^{-jK\omega_0 t} \\
    & \rightarrow \sum e^{jK\omega_0 t} \\
\end{align*}
\]

**Figure 3.3:** Block diagram of FSA.
3.5 Approximate Simulator

Let the time domain response of an antenna mounted on a rotorcraft be approximated as

\[ \hat{h}(t, f_i) = \sum_{j=1}^{J} \alpha_{ji} \cdot h(t, \bar{f}_j) \]  

(3.12)

where \( \alpha_{ji} \) is a complex scalar and \( h(t, \bar{f}_j) \) is the time domain response of the antenna at frequency \( \bar{f}_j \). The frequencies \( \bar{f}_j, j = 1 \ldots J \) are a subset of the frequencies \( f_i, i = 1 \ldots M \). Note that at a given frequency \( f_i \), the response of the antenna is approximated as a linear combination of the true response of the antenna at \( J \) different frequencies. The frequencies \( \bar{f}_1, \bar{f}_2, \ldots, \bar{f}_J \) should be spread evenly throughout the frequency band of interest. Recalling that one can write the signal received by the antenna as

\[ y(t) = \sum_{i=1}^{M} b_i h(t, f_i) e^{j2\pi f_i t} \]  

(3.13)

and substituting the approximated antenna response given by (3.12) into (3.13), one finds that the approximated received signal is given by

\[ \hat{y}(t) = \sum_{j=1}^{J} h(t, \bar{f}_j) \cdot \sum_{i=1}^{M} b_i \alpha_{ji} e^{j2\pi f_i t}. \]  

(3.14)

The approximated received signal can be simulated as follows. First, the incident signal is passed through a bank of \( J \) FIR filters \( (\alpha_1(f), \alpha_2(f), \ldots, \alpha_J(f)) \). Next, the output of each of these filters is modulated with the time domain response of the antenna at frequency \( \bar{f}_1, \bar{f}_2, \ldots, \bar{f}_J \), respectively. Finally, the output of all of the modulators are summed together to obtain the received signal \( \hat{y}(t) \). This is depicted in the block diagram in Fig. 3.4. This will be referred to as the “\( J^{th} \) Order Approximate Simulator”, where \( J \) denotes the number of filters used in the implementation of the Approximate Simulator. Note that the zeroth order simulator \((J = 0)\) is special case
in which there are no FIR filters used in the simulator. In this case, the signal received by the antenna is simulated by modulating the incident signal with the response of the antenna at the center frequency.

\[ x(t) = \sum_{j=1}^{J} \alpha_j(f) h(t, \bar{f}_j) + \sum_{j=1}^{J} y(t) \]

Figure 3.4: Block diagram of Approximate Simulator.

The Approximate Simulator is very similar to that of the FSA, which is shown in Fig. 3.3. The main difference between the two simulators is that the frequency shifters in the FSA have been replaced with modulators whose time domain response is given by the response of the antenna at the frequencies \( \bar{f}_1, \bar{f}_2, \ldots \bar{f}_J \). Note that the implementation cost of this approach depends on the order \( (J) \) of the simulator.

Up until this point, it has not been specified how one should obtain the response of the FIR filters \( \alpha_j(f), j = 1 \ldots J \). For \( J > 0 \) the filters are designed so that the error between the true antenna response and the approximated antenna response over one blade rotation period \( (T_b) \) is minimized in the least-squares (L.S.) sense at each frequency \( f_i \). The filter responses are found as follows. For a given frequency \( f_i \), assume that the antenna response is known at \( N \) discrete time points over one blade rotation period. Note that \( N \) must be large enough so that the antenna response is properly sampled as was in discussed in Section 3.3. The antenna response at
frequency $f_i$ at $N$ time points can be written in vector notation as

$$h_i = \begin{bmatrix} h(t_1, f_i) & h(t_2, f_i) & \ldots & h(t_N, f_i) \end{bmatrix}^T \quad (3.15)$$

One can now rewrite (3.12) as

$$\hat{h}_i = \sum_{j=1}^{J} \alpha_{ji} \bar{h}_j, \quad (3.16)$$

where

$$\bar{h}_j = \begin{bmatrix} h(t_1, \bar{f}_j) & h(t_2, \bar{f}_j) & \ldots & h(t_N, \bar{f}_j) \end{bmatrix}^T. \quad (3.17)$$

(3.16) can also be expressed as a matrix-vector product; i.e.,

$$\hat{h}_i = \mathbf{H} \cdot \mathbf{a}_i \quad (3.18)$$

where

$$\mathbf{H} = \begin{bmatrix} \bar{h}_1 & \bar{h}_2 & \ldots & \bar{h}_J \end{bmatrix} \quad (3.19)$$

and

$$\mathbf{a}_i = \begin{bmatrix} \alpha_{1i} & \alpha_{2i} & \ldots & \alpha_{ji} \end{bmatrix}^T. \quad (3.20)$$

The objective here is to find the vector $\mathbf{a}_i$ that minimizes

$$||h_i - \mathbf{H} \cdot \mathbf{a}_i||^2 \quad (3.21)$$

where $|| \cdot ||$ denotes the Euclidean norm. The $\mathbf{a}_i$ that minimizes (3.21) in the L.S. sense is

$$\mathbf{a}_i = (\mathbf{H}^T \mathbf{H})^{-1} \cdot (\mathbf{H}^T \mathbf{h}_i). \quad (3.22)$$

Now that the filters $\alpha_j(f), j = 1, 2, \ldots, J$ have been specified, one can simulate the signal received by the antenna using (3.14). One can evaluate the accuracy of the Approximate Simulator by comparing the true antenna response at a given frequency $f_i$ to the approximated antenna response found using (3.12). Note that at
the frequencies $\bar{f}_1, \bar{f}_2, \ldots, \bar{f}_J$, the approximation is exact and that as $J$ becomes very large, $\alpha_j(f) \approx \delta(f - \bar{f}_j)$, where $\delta(f)$ denotes the Dirac delta function. In other words, the approximate simulator becomes more and more like the TDA as $J$ increases.

3.6 Example Results

The previous sections presented three method for including RBM in computer or HITL simulations. In this section, some examples are provided to demonstrate the use of the TDA, FSA, and Approximate Simulator. Again, the antenna and rotorcraft are the same as in Chapter 2.

Fig. 3.5 shows the magnitude and phase of the received signal in the presence of RBM when the incident signal is a CW signal at 2214 MHz that arrives along $\theta = 25^\circ, \phi = 30^\circ$. The traces in black show the received signal when the RBM is included using the TDA and the traces in red show the received signal when the RBM is included using the FSA with various number of coefficients. One notices that the FSA provides very similar results with $K = 10, 35, \text{ or } 50$ in this case. The antenna response in this direction changes very smoothly thus it can be represented using a small number of harmonics.

Fig. 3.6 shows the magnitude and phase of the received signal in the presence of RBM when the incident signal is a CW signal at 2214 MHz that arrives along $\theta = 65^\circ, \phi = 30^\circ$. One notices that in this direction there is a very abrupt changes in the antenna response around 20 msec. It should be noted that these abrupt changes are caused by the limitations of the numerical tool used to simulate the antenna response. Although the antenna response should be smooth, these abrupt changes are useful in evaluating the performance of the FSA. The traces in black show the
Figure 3.5: Received signal in the presence of RBM when the input signal is a CW signal at 2214 MHz along the directions $\theta = 25^\circ$, $\phi = 30^\circ$. The TDA (black) and the FSA (red) is used to simulate the received signal in the presence of RBM.

received signal when the RBM is included using the TDA and the traces in red show the received signal when the RBM is included using the FSA with various number of coefficients. One notices that when $K = 10$ the FSA approach provides results that are very similar to the TDA. However, around 20 ms it is clear that the abrupt
Figure 3.6: Received signal in the presence of RBM when the input signal is a CW signal at 2214 MHz along the directions $\theta = 65^\circ$, $\phi = 30^\circ$. The TDA (black) and the FSA (red) is used to simulate the received signal in the presence of RBM.

Changes in the received signal are not represented very well. In order to represent these fast changes, one must use more harmonics in the Fourier Series. One can see that as $K$ is increased to 35 and 50, the received signal provided by the FSA is more
similar to the received signal provided by the TDA. One also notices that there is little benefit in using 50 harmonics as opposed to 35.

Fig. 3.7 shows the magnitude and phase of the received signal in the presence of RBM when the incident signal is a CW signal at 2214 MHz that arrives along \( \theta = 25^\circ, \phi = 30^\circ \). The traces in black show the received signal when the RBM is included using the TDA and the traces in red show the received signal when the RBM is included using the Approximate Simulator with \( J = 0, 1, 2, \) and 3. The frequencies that were used in the Approximate Simulator are provided in Table 3.1. When \( J = 0 \) or 1, one observes that there is some noticeable differences between the received signal produced by the Approximate Simulator and the TDA. However, when the order of the Approximate Simulator is increased to 2 or 3 both the TDA and Approximate Simulator provide very similar results.

Table 3.1: Filter order and frequencies used for approximate simulator.

<table>
<thead>
<tr>
<th>Simulator Order</th>
<th>Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2200</td>
</tr>
<tr>
<td>1</td>
<td>2200</td>
</tr>
<tr>
<td>2</td>
<td>2294, 2206</td>
</tr>
<tr>
<td>3</td>
<td>2190, 2200, 2210</td>
</tr>
</tbody>
</table>
Figure 3.7: Received signal in the presence of RBM when the input signal is a CW signal at 2210 MHz along the directions $\theta = 25^\circ$, $\phi = 30^\circ$. The TDA (black) and the Approximate Simulator (red) is used to simulate the received signal in the presence of RBM.
Fig. 3.8 shows the magnitude and phase of the received signal in the presence of RBM when the incident signal arrives along $\theta = 65^\circ, \phi = 30^\circ$. All other parameters are the same as Fig. 3.7. Again, there are some slight differences between the received signal produced by the Approximate Simulator and the TDA when $J = 0$ or 1. As expected, very similar results are produced when the order of the Approximate Simulator is increased to 2 or 3.

Fig. 3.9 shows the magnitude of the received signal in the presence of RBM when the incident signal consists of two CW signals at 2196 and 2214 MHz. The signal arrives along $\theta = 45^\circ, \phi = 30^\circ$. Fig. 3.9a shows the received signal when the RBM is included using the TDA and Fig. 3.9b-d shows the received signal when the RBM is included using the FSA with $K = 10, 35, \text{ and } 50$, respectively. Again, one can see that with $K = 10$ the FSA does not provide all of the fine details in the received signal that the TDA provides. However, with $K = 35$ or 50 the received signal is very similar to the received signal produced by the TDA.

Fig. 3.10 shows the magnitude of the received signal in the presence of RBM when the incident signal consists of two CW signals at 2196 and 2214 MHz. The signal arrives along $\theta = 45^\circ, \phi = 30^\circ$. Fig. 3.10a shows the received signal when the RBM is included using the TDA and Fig. 3.10b-e shows the received signal when the RBM is included using the Approximate Simulator with $J = 0, 1, 2, \text{ and } 3$, respectively. The frequencies used in the Approximate Simulator are defined in Table 3.1. Again, one can see that with $J = 0$ or 1 the Approximate Simulator does not provide all of the fine details in the received signal that the TDA provides. However, with $J = 2$ or 3 the received signal is very similar to the received signal produced by the TDA.
Figure 3.8: Received signal in the presence of RBM when the input signal is a CW signal at 2210 MHz along the directions $\theta = 65^\circ$, $\phi = 30^\circ$. The TDA (black) and the Approximate Simulator (red) is used to simulate the received signal in the presence of RBM.
Figure 3.9: Received signal in the presence of RBM when the input signal consists of two CW signal at 2196 and 2214 MHz. The signal arrives along $\theta = 45^\circ, \phi = 30^\circ$. The TDA (a) and FSA (b-d) is used to simulate the received signal in the presence of RBM.
Figure 3.10: Received signal in the presence of RBM when the input signal consists of two CW signal at 2196 and 2214 MHz. The signal arrives along $\theta = 45^\circ$, $\phi = 30^\circ$. The TDA (a) and Approximate Simulator (b-e) is used to simulate the received signal in the presence of RBM.
3.7 Summary

In this chapter, three approaches for including the effects of RBM in simulations were developed. Initially, the TDA and FSA approach were presented. Although these two methods provide an exact model of the RBM, it was clear that they were not suitable for implementation in HITL testing. Subsequently, the Approximate Simulator was developed and it was shown that RBM can be included in simulations using $J$ filters and modulators. This approach is well suited for implementation in HITL simulation since a good representation of the RBM can be obtained with a relatively small value of $J$. In the following chapter, the accuracy of the Approximate Simulator is evaluated using the measured response of an antenna mounted on a model rotorcraft. The next chapter describes the model rotorcraft, the antenna used in the measurements, and the measurement procedure used to obtain the antenna response. Additionally, the antenna response obtained using the Approximate Simulator is compared to the measured antenna response in order to evaluate the accuracy of the Approximate Simulator.
CHAPTER 4

MEASUREMENTS

4.1 Overview

Chapter 3 described how to include RBM in simulations assuming that the time domain antenna response over the frequency band of interest is known. The time domain antenna response can be obtained through numerical EM calculations or through measurements. Often times, it may not be feasible to measure the antenna response as the blades rotate at their operational speed. This may be due to safety concerns or because the measurement equipment is not capable of measuring the data fast enough. This chapter provides guidelines for measuring the time domain response of an antenna mounted on a rotorcraft. In order to facilitate this discussion, the response of an antenna on a model rotorcraft was measured. The measured antenna data will be used for three purposes. First, it will be demonstrated that once the antenna response is known for one blade rotation speed it is known for all blade rotation speeds. Furthermore, it will be shown that one can also measure the antenna response at discrete blade locations without any loss of information. This provides an approach that is more feasible than measuring the time domain antenna response as the blade rotate at their operational speed. Next, the measured antenna data will
be compared to the antenna response calculated in NEC-BSC. Since the calculated antenna data is used widely throughout this thesis, it is important to verify that the calculations provide a reasonable prediction of the antenna response. Finally, the measured antenna data will be used to verify the accuracy of the Approximate Simulator when it is used to approximate the time domain antenna response of a real antenna on a model rotorcraft.

4.2 Rotorcraft Model and Antenna

A picture of the model rotorcraft as well as a diagram with its dimensions are provided in Fig. 4.1 and Fig. 4.2, respectively. The body of the rotorcraft was constructed from two thin sheets of metal that were bent into the shape shown in Fig. 4.1. The overall length of the model rotorcraft is 5.5’. It consists of a top plate that is 2’ wide at the top and 5” wide at the bottom as well as a set of side plates that extend 2’ behind the top plate. The rotor blade is 49” long and 2.5” wide. It consists of a thin strip of metal that is backed by a strip of rigid foam. The foam was used to keep the shape of the rotor blade from changing throughout the measurements. The rotor blade was mounted 5” above the body of the rotorcraft using a metallic shaft that is 3/8” in diameter.

An open-ended waveguide antenna that operates from 6 GHz to 8 GHz was used as the antenna on the model rotorcraft. The antenna was mounted 4” in front of the rotor blade shaft. Two perpendicular slots were cut out of the top plate of the rotorcraft so that the open-ended waveguide could be mounted to receive either a horizontally polarized (H-Pol) or vertically polarized (V-Pol) wave. Note that the dimensions of the rotorcraft were scaled down from typical rotorcraft dimensions. Also, the
frequency range of the antenna was chosen according to this scaling assuming that the RF systems on board the rotorcraft operates in the L-Band. In the following section, the approach used to measure the response of the antenna is described.
4.3 Measurement of Antenna Response

The response of the antenna was measured using two different measurement approaches. In the first approach, the response of the antenna was measured as a function of time as the rotor blade rotated at a fixed speed. In the second approach, the response of the antenna was measured as the rotor blade was fixed at discrete blade rotation angles. The measured data obtained using the two approaches will be discussed separately and then the two data sets will be compared.

4.3.1 Rotating Blade

For the rotating blade measurements, the response of the antenna was measured at 7000 MHz in a particular look direction as the rotor blade rotated at a fixed speed. For each of the look directions, the antenna response was recorded over a 10 second interval. The antenna response was obtained at five different look directions in the $\phi = 90^\circ$ plane (see Fig. 4.2). They were $\theta = 0^\circ, 30^\circ, 75^\circ, 100^\circ$, and $115^\circ$. For each of these look directions, the response of the antenna was collected at two different blade rotation speeds. The second speed being approximately twice that of the first speed. Both the H-Pol and V-Pol response of the antenna was measured.

Fig. 4.3 shows the measured response of the antenna at 7000 MHz along $\theta = 0^\circ$. Fig. 4.3a and Fig. 4.3b show the H-Pol response of the antenna at speed 1 and speed 2, respectively. It is evident that the response of the antenna is periodic at both speed 1 and speed 2. At speed 1, which is 24.5 revolutions per minute (RPM), the response of the antenna repeats every 2.45 seconds. One can also see from the figures that the response of the antenna at speed 2 is the same as the response at speed 1. As expected, the only difference in the response of the antenna at speed 1 and
speed 2 is that the repetition period at speed 2 is about half the repetition period at speed 1. Fig. 4.3c and Fig. 4.3d show the V-Pol response of the antenna at speed 1 and speed 2, respectively. Again, it is evident that the response of the antenna is periodic at both speeds. Note that speed 1 is now 16 RPM and that the response of the antenna repeats every 3.75 seconds. Again, the response of the antenna at speed 2 is the same as the response at speed 1 except for the repetition period, which is smaller by a factor of two.

Figure 4.3: Measured antenna response along $\theta = 0^\circ$ at 7000 MHz. The antenna response was obtained at two different blade rotation speeds.
Fig. 4.4 shows the measured response of the antenna at 7000 MHz along $\theta = 30^\circ$. Fig. 4.4a and Fig. 4.4b show the H-Pol response of the antenna at speed 1 and speed 2, respectively. Again, at speed 1, the response of the antenna is periodic with a repetition of 2.45 seconds. The response of the antenna at speed 2 is the same as the response at speed 1 except for the repetition period. The repetition period of the antenna response at speed 2 is about half the repetition period of the antenna response at speed 1. Fig. 4.4c and Fig. 4.4d show the V-Pol response of the antenna at speed 1 and speed 2, respectively. Note that speed 1 is 19.7 RPM and that the response of the antenna repeats every 3 seconds. Again, the response of the antenna at speed 2 is the same as the response at speed 1. The only difference in the response at speed 2 is the repetition period, which is smaller by a factor of two.

Fig. 4.5 shows the measured response of the antenna at 7000 MHz along $\theta = 115^\circ$. Fig. 4.5a and Fig. 4.5b show the H-Pol response of the antenna at speed 1 and speed 2, respectively. Again, the response of the antenna at speed 1 is periodic with a repetition of 2.45 seconds. The response of the antenna at speed 2 is the same as the response at speed 1 except for the repetition period. The repetition period of the antenna response at speed 2 is about half the repetition period of the antenna response at speed 1. Fig. 4.5a and Fig. 4.5b show the V-Pol response of the antenna at speed 1 and speed 2, respectively. Note that speed 1 is 19.7 RPM and that the response of the antenna repeats every 3 seconds. Again, the response of the antenna at speed 2 is the same as the response at speed 1. The only difference in the response at speed 2 is the repetition period, which is smaller by a factor of two.

From these results, one can conclude that antenna response is periodic and that the response is independent of the blade rotation speed. This is true assuming that
the shape of the blades is the same for all speeds. Therefore, once the response of
the antenna is known at a given blade rotation speed, the response of the antenna
is known for all blade rotation speeds. Next, the antenna response measured when
the rotor blade was rotating at a fixed speed is compared to the measured antenna
response when the rotor blade was fixed at a set of discrete blade positions.

4.3.2 Discrete Blade Angle

For the discrete blade angle measurements, the response of the antenna was mea-
sured at a 7000 MHz along $\phi = 90^\circ$, $\theta = 0^\circ$ as the rotor blade was fixed at a particular
Figure 4.5: Measured antenna response along $\theta = 115^\circ$ at 7000 MHz. The antenna response was obtained at two different blade rotation speeds.

blade rotation angle. The response of the antenna was collected as the blade rotation angle was changed from $0^\circ$ to $360^\circ$ in $0.5^\circ$ increments. Note that a blade rotation angle of $0^\circ$ corresponds to the rotor blade being directly aligned with the x-axis in Fig. 4.2. Again, both the H-Pol and V-Pol response of the antenna was measured.

Fig. 4.6 shows the measured response of the antenna at 7000 MHz along $\theta = 0^\circ$. Fig. 4.6a and Fig. 4.6b show the H-Pol response of the antenna. Fig. 4.6a shows the response of the antenna measured at discrete blade rotation angles. The response of the antenna repeats every $180^\circ$ as expected. Fig. 4.6b shows the response of the antenna measured as the blade rotated at 24.5 RPM. Again, the response of the
antenna repeats every 2.45 seconds. One can see from the Fig. 4.6a and Fig. 4.6b that the response of the antenna is the same whether it is collected at discrete blade rotation angles or as the blade rotates at a fixed speed. Fig. 4.6c and Fig. 4.6d show the V-Pol response of the antenna. Fig. 4.6c shows the response of the antenna measured at discrete blade rotation angles. Again, the response of the antenna repeats every 180° as expected. Fig. 4.6d shows the response of the antenna measured as the blade rotated at 16 RPM. The response of the antenna repeats every 3.75 seconds. Again, one can conclude from Fig. 4.6c and Fig. 4.6d that the response of the antenna is the same whether it is collected at discrete blade rotation angles or as the blade rotates at a fixed speed.

The results in this section have shown that the response of the antenna is the same whether it is measured at discrete blade rotation angles or as the blade rotates at a fixed speed. Thus, knowing the response of the antenna at discrete blade angles is synonymous with knowing the response of the antenna as the blade rotates at any fixed speed. There are two underlying assumptions that should be pointed out here. The first assumption is that the shape of the blade remains the same for all blade rotation speeds. The second assumption is that the response of the antenna is measured at a sufficiently fine blade increment angle. The discrete blade rotation angle measurements can be particularly helpful because one may not be able to obtain the response of the antenna as the rotor blades are rotating at a particular speed of interest due to safety concerns. Additionally, one may not have the proper equipment to collect the required data fast enough. In this case, one can collect the data at discrete blade rotation angles as was shown in this section. In the following section, the measured antenna response is compared to the antenna response obtained using
Figure 4.6: Comparison of the measured antenna response at fixed discrete blade rotation angles with the antenna response measured as the blades are rotating at a fixed speed. The response of the antenna is measured at 7000 MHz along $\theta = 0^\circ$.

NEC-BSC. It will be shown that the measured antenna response agrees with the numerical antenna response.

### 4.4 Comparison with Calculated Antenna Response

This section starts by describing the model rotorcraft that was simulated in NEC-BSC. After this, the antenna response obtained using NEC-BSC is compared to the measured antenna response. The antenna response will be compared in both the absence of the rotor blade as well as in the presence of the rotor blade.
4.4.1 NEC-BSC Simulation

The platform of interest is modeled in NEC-BSC using a set of flat plates that are attached to each other in order to make the platform. Fig. 4.7 shows a picture of the rotorcraft model used in NEC-BSC. The main body consists of five plates. Note that this model has the same dimensions as the rotorcraft that was built and these dimensions can be found in Fig. 4.2. The rotor blade is modeled as a single flat plate that is 49" long and 2.5" wide. The rotor blade is located 5" above the body of the rotorcraft. Note that the rotor blade is tilted at an angle of 15° along its axis of length in the simulation. In this configuration, the response of the antenna repeats every 360°. The tilt of the rotor blade was also included in the measurements that will be discussed below.

Figure 4.7: Model rotorcraft geometry used in NEC-BSC.

4.4.2 No Blades

For the no blade simulation and measurements, the response of the antenna was measured/simulated as a function of elevation angle (θ) in the φ = 90° plane from 6900 MHz to 7100 MHz in 10 MHz increments. Fig. 4.8-Fig. 4.10 shows the measured and
simulated response of the antenna in the absence of the rotor blade versus elevation angle at 6950, 7000, and 7060 MHz, respectively. The H-Pol response is shown on the left and the V-Pol response is shown on the right. One can see from the figures that the measured response and simulated response are very similar. Although there are slight differences between the simulated and measured responses, the overall trends are the same. These differences can be attributed to mismatch between the antenna and model rotorcraft defined in the NEC-BSC simulation and the actual antenna and model rotorcraft that was used in the measurements. Additionally, the measured data was calibrated using a standard gain horn. Any errors in the horn’s calibration data will also cause an error in measured antenna response. Finally, for elevation angles below the horizon \((\theta > 90^\circ)\), the antenna response is very weak. Therefore, the measured antenna response approaches the noise floor of the measurement system. As a result, some error is introduced into the measured antenna response.

![Figure 4.8: Simulated and measured antenna response in the absence of blades at 6950 MHz.](image)

(a) H-Pol  
(b) V-Pol
4.4.3 Fixed Blade

For the fixed blade simulation and measurements, the response of the antenna was measured/simulated as the rotor blade was fixed to a particular blade rotation angle. Again, a blade rotation angle of 0° corresponds to the rotor blade being directly aligned with the x-axis in Fig. 4.2. The antenna response was measured/simulated for blade rotation angles ranging from 0° to 180° in 20° increments. For each of these blade rotation angles, the antenna response was measured/simulated as a function
of elevation angle ($\theta$) in the $\phi = 90^\circ$ plane from 6900 MHz to 7100 MHz in 10 MHz increments.

Fig. 4.11 - Fig. 4.13 shows the measured and simulated response of the antenna versus elevation angle at 7000 MHz when the rotor blade is fixed at a blade rotation angle of 20°, 100°, and 160°, respectively. The H-Pol response is shown on the left and the V-Pol response is shown on the right. One can see from the figures that there are slight discrepancies between the measured response and simulated response, however, the overall trends are very similar between the measured and simulated antenna response. Again, these discrepancies are caused by mismatch between the simulated and experimental antenna and model rotorcraft, uncertainties in the calibration data, and noisy measurements below the horizon.

Figure 4.11: Simulated and measured antenna response at 7000 MHz when the rotor blade angle is fixed to 20°.
Figure 4.12: Simulated and measured antenna response at 7000 MHz when the rotor blade angle is fixed to 100°.

![Simulated and measured antenna response at 7000 MHz](image)

(a) H-Pol  
(b) V-Pol

Figure 4.13: Simulated and measured antenna response at 7000 MHz when the rotor blade angle is fixed to 160°.

![Simulated and measured antenna response at 7000 MHz](image)

(a) H-Pol  
(b) V-Pol

4.4.4 Fixed Look Direction

For the fixed look direction simulation and measurements, the response of the antenna was measured/simulated as the look direction was fixed to a particular direction. The antenna response was measured/simulated at $\theta = 0^\circ, 30^\circ, 75^\circ, 100^\circ, \text{ and } 115^\circ$ in the $\phi = 90^\circ$ plane. For each of these directions, the antenna response was
measured/simulated as a function of blade rotation angle in discrete steps from 0° to 360° in 1° increments. For each of the blade rotation angles, a frequency scan was carried out from 6900 MHz to 7100 MHz in 10 MHz increments. Fig. 4.14 - Fig. 4.18 shows the measured and simulated response of the antenna versus blade rotation angle at 7000 MHz when the the look direction is fixed at $\theta = 0°, 30°, 75°, 100°$, and $115°$, respectively. The H-Pol response is shown on the left and the V-Pol response is shown on the right. Along $\theta = 0°, 30°$, and $75°$, it is apparent that there is some discrepancy in the depth of the nulls in the patterns. However, aside from the null depths, there is a very good agreement in the trends of the measured and simulated antenna response. Along $\theta = 100°$ and $115°$, there is more discrepancy between the trends of the simulated and measured antenna response. It should be noted that for these aspect angles, the response of the antenna is significantly weaker. Therefore, the measured antenna response approaches the noise floor of the measurement system for some blade rotation angles. As a result, this leads to some of the discrepancies between the measured and simulated antenna response.

Figure 4.14: Simulated and measured antenna response at 7000 MHz along $\theta = 0°$. 

(a) H-Pol.  
(b) V-Pol.
4.5 Accuracy of Approximate Simulator

In this section, the measured antenna response is used to evaluate the accuracy of the Approximate Simulator. In order to quantify the accuracy, a prediction error
Figure 4.17: Simulated and measured antenna response at 7000 MHz along $\theta = 100^\circ$.

Figure 4.18: Simulated and measured antenna response at 7000 MHz when the aspect angle is $\theta = 115^\circ$.

is defined as

$$\Gamma_i = \sqrt{\frac{\sum_T |\hat{h}(t, f_i) - h(t, f_i)|^2}{\sum_T |h(t, f_i)|^2}} \times 100$$  \hspace{1cm} (4.1)$$

where $\Gamma_i$ is the normalized root mean-squared error (RMSE) in percentage at frequency $f_i$ and $\sum_T$ denotes the sum over one blade rotation period. Fig. 4.19 shows the prediction error versus the frequency offset of the incident signal for the zeroth
order simulator. The prediction error is shown for $\theta = 0^\circ, 30^\circ, 100^\circ, \text{ and } 115^\circ$. As expected, at the center frequency, the prediction error is zero. The figure also shows that as the amount of frequency offset increases, the prediction error increases and that the amount of error varies with the look direction. The normalized RMSE is as much as 22% when $\theta = 0^\circ$, 25% when $\theta = 30^\circ$, 31% when $\theta = 100^\circ$, and 41% when $\theta = 115^\circ$. This error is too large for a good simulator. Typically, the error should be 10% or smaller. Note that 10% corresponds to -20 dB, which is quite desirable.

Figure 4.19: Prediction error versus frequency offset of the incident signal for the zeroth order simulator. The prediction error is shown for $\theta = 0^\circ, 30^\circ, 100^\circ, \text{ and } 115^\circ$.

Fig. 4.20 shows the prediction error versus the frequency offset of the incident signal for the first order simulator. The prediction error is shown for $\theta = 0^\circ, 30^\circ, 100^\circ,$
and 115°. Again, the prediction error is zero at the center frequency as expected. It is also apparent that as the amount of frequency offset increases, the prediction error increases and the amount of error varies with the look direction. The RMSE is as much as 15% when \( \theta = 0^\circ \), 19% when \( \theta = 30^\circ \), 15% when \( \theta = 100^\circ \), and 31% when \( \theta = 115^\circ \). Although there is some improvement in performance when the first order signal simulator is used instead of the zeroth order simulator, the error is still too large. One, therefore, needs to increase the simulator order.

![Figure 4.20](image)

**Figure 4.20:** Prediction error versus frequency offset of the incident signal for the first order simulator. The prediction error is shown for \( \theta = 0^\circ, 30^\circ, 100^\circ, \) and \( 115^\circ \).

Fig. 4.21 shows the prediction error versus the frequency offset of the incident signal for the second order simulator. The second order simulator was implemented
using the true antenna response at the frequencies 6960 and 7040 MHz. These frequencies are denoted as a frequency offset of ±40 MHz in the figure. The prediction error is shown for θ = 0°, 30°, 100°, and 115°. It is clear from the figure that there is a large improvement in performance when the second order simulator is used instead of the first order simulator (compare Fig. 4.21 with Fig. 4.20). As expected, the prediction error is zero when the frequency offset is ±40 MHz. The RMSE is limited to 5% when θ = 0°, 30°, or 100°. In the case when θ = 115°, the error is as much as 20%. However, over most of the frequency band the normalized RMSE is less than 10%. Thus, a second order Approximate Simulator may be sufficient for most RF systems.

Figure 4.21: Prediction error versus frequency offset of the incident signal for the second order simulator. The prediction error is shown for θ = 0°, 30°, 100°, and 115°.
Fig. 4.22 shows the prediction error versus the frequency offset of the incident signal when the antenna response is approximated using the third order simulator. The third order simulator was implemented using the true antenna response at the frequencies 6960, 7000, and 7040 MHz. These frequencies are denoted in the figure as a frequency offset of −40, 0, and 40 MHz, respectively. The prediction error is shown for $\theta = 0^\circ, 30^\circ, 100^\circ$, and $115^\circ$. The figure shows that there is a further improvement in the simulator performance if the third order simulator is used instead of the second order simulator (compare Fig. 4.22 with Fig. 4.21). The prediction error is zero when the frequency offset is −40, 0, and 40 MHz, as expected. In particular, the RMSE is less than 5% when $\theta = 0^\circ, 30^\circ$, or $100^\circ$. Even in the case when $\theta = 115^\circ$, the error is less than 10%, which is very good.

From Fig. 4.19 - Fig. 4.22, it can be concluded that one needs to use at least a second order simulator to obtain a good model of the rotor blade modulation. It should be noted that the prediction error shown in these figures is the average error over one blade rotation period. In the remainder of this section, the measured antenna response at a given frequency is directly compared to the approximated antenna response at that frequency for the second and third order simulator. For illustration, the antenna response at offset frequencies of ±70 MHz (6930 and 7070 MHz) are compared. Neither of these frequencies are used in the design of the second order or third order Approximate Simulator. It should be noted that the data at other frequencies were compared as well and the simulator performance was similar or better at the other frequencies.

Fig. 4.23 (left plots) shows the magnitude of the measured and approximate antenna response versus rotor blade angle at 6930 MHz. The approximate antenna
Figure 4.22: Prediction error versus frequency offset of the incident signal for the third order simulator. The prediction error is shown for $\theta = 0^\circ$, $30^\circ$, $100^\circ$, and $115^\circ$.

Response was found using the second order simulator which was implemented using the true antenna response at the frequencies 6960 and 7040 MHz. These frequencies correspond to a frequency offset of $\pm 40$ MHz. Also plotted (right plots) in the figure is the normalized error (right) which is defined as

$$E(t, f_i) = \frac{|h(t, f_i) - \hat{h}(t, f_i)|}{\sum_T |h(t, f_i)|}.$$  \hspace{1cm} (4.2)

Note that this error is normalized by the average magnitude response of the antenna at frequency $f_i$. Again, an error less than -20 dB corresponds to an error that is less than 10%. The antenna response and approximation error is shown along $\theta = 0^\circ$, $30^\circ$, $100^\circ$, and $115^\circ$ when the frequency is offset by -70 MHz from the center frequency. Along $\theta = 0^\circ$, one can see that the predicted antenna response is very similar to the measured.
antenna response. Fig. 4.23b shows that the error is below -20 dB for almost all blade angles. However, for blade angles around 200° and 340° the error is significantly larger. If one compares Fig. 4.23a and Fig. 4.23b, it can be seen that the spikes in the error plot occur when there are deep nulls in the antenna pattern. These deep nulls are difficult to predict accurately and thus the approximation error is larger in these regions. Along θ = 30°, and 100°, the approximate antenna response is almost an exact replica of the measured antenna response. As expected, the approximation error is below -20 dB for all blade angles. Along θ = 115° there are some discrepancies between the measured and predicted antenna response. One can see that there is an overall increase in the approximation error for θ = 115°. Additionally, for blade angles between 200° and 300° the error is frequently above -20 dB.

Fig. 4.24 shows the magnitude of the measured antenna response and approximate antenna response versus rotor blade angle at 7070 MHz. Again, the approximate antenna response was found using the second order simulator. Also plotted in the figure is the normalized approximation error. The antenna response and approximation error is shown along θ = 0°, 30°, 100°, and 115° when the frequency is offset by 70 MHz from the center frequency. Along θ = 0°, one can see that the predicted antenna response is very similar to the measured antenna response. Additionally, Fig. 4.24b shows that the prediction error is below -20 dB for almost all blade angles. However, there is a significant increase in error for blade angles around 200° and 340°. Again, the spikes in the error plot occur when there are deep nulls in the antenna response. Along 30°, and θ = 100° the approximated antenna response is very similar to the measured antenna response and accordingly the approximation error is below -20 dB for all rotor blade angles. Along θ = 115°, it is evident that there is some discrepancy.
between the predicted and measured antenna response. Again, one can see an overall increase in the approximation error along $\theta = 115^\circ$ and the error is frequently above -20 dB for blade angles between 200° and 300°. Overall, the second order simulator does a good job of approximating the true response of the antenna.

Figures 4.25 compares the magnitude of the approximated antenna response to the true antenna response at 6930 MHz when a third order signal simulator is used. The third order simulator was implemented using the true antenna response at 6960, 7000, and 7040 MHz. These frequencies correspond to a frequency offset of $-40, 0,$ and $40$ MHz, respectively. Also shown in these figures is the approximation error. All other parameters are the same as in Figures 4.23. Note that with the third order signal simulator the approximated antenna response for the four directions looks quite similar to the true antenna response. Along $\theta = 0^\circ$, one can see that the approximation error is again well below -20 dB for almost all blade angles. Furthermore, the spikes in the error plots have been significantly reduced when the third order simulator is used instead of the second order simulator (compare Fig. 4.25b and Fig. 4.23b). Along $30^\circ$, and $\theta = 100^\circ$ the approximation error is well below -20 dB for all blade angles and there is a slight decrease in error when the third order simulator is used instead of the second order simulator. Along $\theta = 115^\circ$, the approximation error for the third order simulator is slightly less than that of the second order simulator for most of the blade angles. Furthermore, for blade angles between 200° and 300°, there is significant reduction in error when the third order simulator is used instead of the second order simulator.

Figures 4.26 compares the magnitude of the approximated antenna response to the true antenna response at 7070 MHz when a third order signal simulator is used.
Also shown in these figures is the approximation error. All other parameters are the same as in Figures 4.24. Again, when the third order signal simulator is used, the approximated antenna response is almost an exact replica of the true antenna response in the four directions. Along $\theta = 0^\circ$, $30^\circ$, and $100^\circ$, one can see that the approximation error is now below -20 dB for all blade angles and there is a slight decrease in error when the third order simulator is used instead of the second order simulator (compare with Fig. 4.24). Note that there are still spikes in the error plots along $\theta = 0^\circ$, however, they do not exceed -25 dB in this case. Along $\theta = 115^\circ$, the approximation error for the third order simulator is slightly less than that of the second order simulator for almost all of the blade angles. Moreover, for blade angles between $200^\circ$ and $300^\circ$, there is significant reduction in error for the third order simulator.

4.6 Summary

In this chapter, it was first demonstrated that the response of an antenna mounted on a rotorcraft is independent of the blade rotation speed. Next, the antenna response obtained using NEC-BSC was compared to the measured antenna response. Overall, the comparisons showed that the antenna response obtained using NEC-BSC is very similar to the measured antenna response. Finally, it was shown that there is a tradeoff between the accuracy of the Approximate Simulator and the implementation costs. As $J$ increases, the number of filters and modulators required in the implementation of the approximate simulator increases. However, the results in this chapter have shown that the approximate simulator can provide a very accurate model of the rotor blade modulation with $J$ as small as two or three.
For antennas mounted on a rotorcraft, a large amount of data must be measured or calculated since the antenna response must be measured/calculated for each blade angle and look direction over the frequency band of interest. However, with the speed of today’s measurement equipment and computers it is feasible to obtain the antenna response for antennas with a single rotor. The next chapter considers how to obtain the response of antenna mounted on a dual rotor aircraft. In this case, the antenna response is also periodic, however, the antenna response will depend on the starting location of the rotors. Additionally, the antenna response will depend on the speed at which each rotor is rotating. One can see that the antenna response must be collected for each combination of rotor starting locations and rotor speeds. Clearly, it is not feasible to obtain the antenna response for all combinations of rotor speeds and starting locations because the amount of time that is required is too large. The following chapter describes how to efficiently collect the time domain antenna response when the antenna is mounted on dual rotor aircrafts.
Figure 4.23: Magnitude of measured and approximated antenna response (left) and normalized error (right) for the second order simulator. The frequency is offset -70 MHz from the center frequency. The antenna response and error is shown along $\theta = 0^\circ, 30^\circ, 100^\circ,$ and $115^\circ$. 

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Figure 4.24: Magnitude of measured and approximated antenna response (left) and normalized error (right) for the second order simulator. The frequency is offset 70 MHz from the center frequency. The antenna response and error is shown along $\theta = 0^\circ, 30^\circ, 100^\circ, \text{ and } 115^\circ$. 
Figure 4.25: Magnitude of measured and approximated antenna response (left) and normalized error (right) for the third order simulator. The frequency is offset -70 MHz from the center frequency. The antenna response and error is shown along $\theta = 0^\circ, 30^\circ, 100^\circ, \text{and } 115^\circ$. 

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Figure 4.26: Magnitude of measured and approximated antenna response (left) and normalized error (right) for the third order simulator. The frequency is offset 70 MHz from the center frequency. The antenna response and error is shown along $\theta = 0^\circ, 30^\circ, 100^\circ, \text{ and } 115^\circ$. 

(a) $\theta = 0^\circ$. Magnitude.  
(b) $\theta = 0^\circ$. Error.  
(c) $\theta = 30^\circ$. Magnitude.  
(d) $\theta = 30^\circ$. Error.  
(e) $\theta = 100^\circ$. Magnitude.  
(f) $\theta = 100^\circ$. Error.  
(g) $\theta = 115^\circ$. Magnitude.  
(h) $\theta = 115^\circ$. Error.
5.1 Overview

This chapter considers how to obtain the time domain response of an antenna mounted on a dual rotor aircraft. In order to facilitate this task, the time domain antenna response is approximated using the Composite Antenna Response, which is described below. Example results are provided to demonstrate the usefulness of the Composite Antenna Response.

5.2 Composite Antenna Response

Let \( h(\theta, \phi, f_i, \psi_1(t), \psi_2(t)) \) denote the time domain response of an antenna mounted on a dual rotor aircraft, where \( \theta, \phi, \) and \( f_i \) are the elevation angle, azimuth angle, and frequency of the antenna response, respectively. \( \psi_1(t) \) and \( \psi_2(t) \) are the blade rotation angles at time \( t \) that is associated with rotor 1 and rotor 2, respectively. It is important to note that there will be a common repetition period associated with the two rotors assuming that each rotor rotates at a fixed speed. The common repetition period, \( T_b \), is given by the least common multiple (LCM) of the two individual rotor repetition periods (\( T_{b1} \) and \( T_{b2} \)). Thus to find the time domain response of the antenna
at a given frequency, one must simulate/measure the antenna response as a function of the two rotor angles over the common repetition period. Once the time domain antenna response is known over the common repetition period, one can use the TDA, FSA, or Approximate Simulator to incorporate the RBM in simulations. However, it should be noted that any change in the starting location or repetition period of the rotors would require recollection of this data. One can see that using this brute force approach can be very time consuming and expensive.

As an alternative, one can approximate the time domain response of an antenna mounted on a dual rotor aircraft as

\[ h_c(\theta, \phi, f_i, \psi_1(t), \psi_2(t)) = h_{NB}(\theta, \phi, f_i) + h_{B1}(\theta, \phi, f_i, \psi_1(t)) + h_{B2}(\theta, \phi, f_i, \psi_2(t)) \]  \hspace{1cm} (5.1)

where \( h_c(\theta, \phi, f_i, t) \) is the “Composite Antenna Response” (CAR) that approximates the true antenna response. \( h_{NB}(\theta, \phi, f_i) \) is the response of the antenna in the absence of the rotor blades and \( h_{Bn}(\theta, \phi, f_i, \psi_n(t)) \) is the portion of the antenna response at time \( t \) due to the blade associated with rotor \( n \), which is given as

\[ h_{Bn}(\theta, \phi, f_i, \psi_n(t)) = h_{TBn}(\theta, \phi, f_i, \psi_n(t)) - h_{NB}(\theta, \phi, f_i). \]  \hspace{1cm} (5.2)

In (5.2), \( h_{TBn}(\theta, \phi, f_i, \psi_n) \) is the total antenna response in the presence of the blade associated with rotor \( n \) only. Substituting (5.2) into (5.1), the composite antenna response is given as

\[ h_c(\theta, \phi, f_i, \psi_1(t), \psi_2(t)) = h_{TB1}(\theta, \phi, f_i, \psi_1(t)) + h_{TB2}(\theta, \phi, f_i, \psi_2(t)) - h_{NB}(\theta, \phi, f_i). \]  \hspace{1cm} (5.3)

From (5.3), one can see that the measurement/simulation requirements are significantly decreased by using this approximation. In order to obtain the time domain
response of the antenna, one first needs to simulate/measure the response of the antenna in the absence of all rotor blades. Next, the time domain response of the antenna for a given rotor blade is measured/simulated. Then, for any combination of blade angles, the CAR can be found using (5.3). The major benefit of the CAR is that once this data is collected, the antenna response is known for all rotor starting position and repetition period combinations. Thus, the amount of simulations/measurements needed to obtain the time domain response of the antenna is greatly reduced. The following section provides some example results using the CAR.

5.3 Small Rotorcraft Example

As an initial example, a small model rotorcraft, which is shown in Fig. 5.1 is considered. The body of the rotorcraft is made from a six sided flat plate that is 10’ long. The plate is 5’ wide at the front and 1’ wide at the back. The rotor blades are modeled as flat plates that are in the shape of a cross. The blades are 5’ long on each side and they are 1’ wide. The front blade (blade 1) is mounted 1’ away from the nose and 2’ above the body plate. The rear blade (blade 2) is mounted 1’ away from the tail and 4’ above the body plate. The antenna used in the simulations is a \( \frac{\lambda}{4} \) monopole that is located at 5’ away from the nose. The antenna response was simulated using NEC-BSC and FEKO [11], a full-wave electromagnetic simulator that uses the Method of Moments to analyze antennas. FEKO was used to demonstrate that the validity of the CAR is independent of the simulation tool used. As stated earlier, NEC-BSC is based on ray tracing and UTD. Thus, the code uses the principle of superposition by adding all the rays together to obtain the total antenna response. As a result, one would expect the CAR to reasonably approximate
the antenna response that is obtained using NEC-BSC since the CAR is based on superposition as well. The examples below show that the CAR is able to represent the antenna very well no matter which simulation tool is used to find the antenna response.

![Image of rotorcraft structure](image)

(a) NEC Model.  
(b) FEKO Model.

Figure 5.1: Simulation model of rotorcraft structure.

Fig. 5.2 shows the magnitude and phase of the true and approximate antenna response at 1227 MHz over the angular region $\theta = 0^\circ$ to $120^\circ$, $\phi = 0$ to $360^\circ$. The figures on the left show the results from the simulation carried out in NEC-BSC and the figures on the right show the results from the simulations carried out in FEKO. The blade location angle for both blades is $0^\circ$ ($\psi_1 = \psi_2 = 0^\circ$). Note that the blade angle is measured from the x-axis to the y-axis (see Fig. 5.1) and a blade rotation angle of $0^\circ$ corresponds to one of the arms of the blades being aligned with the x-axis. Both the magnitude and phase of the antenna response is shown in the figure. It is clear from the plots in the figure that the approximate antenna response is very similar to the true antenna response for both the NEC and FEKO simulations. It is also clear that the antenna response computed in NEC and FEKO are very similar.
Figure 5.2: True and approximate antenna response at 1227 MHz. $\theta = 0^\circ$ to $120^\circ$. Blade 1 = $0^\circ$. Blade 2 = $0^\circ$.

Fig. 5.3 and Fig. 5.4, respectively show the magnitude and phase of the true and approximate antenna response when $(\psi_1, \psi_2) = (5^\circ, 15^\circ)$ and $(\psi_1, \psi_2) = (45^\circ, 45^\circ)$. 

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All other parameters are the same as in Fig. 5.2. One can see from both figures that
the approximate antenna response is very similar to the true antenna response for
both the NEC and FEKO simulations and the antenna response computed in NEC
and FEKO are very similar.

To further quantify the accuracy of the CAR, an approximation error is defined
as
\[ E(\theta, \phi, f_i, \psi_1, \psi_2) = \left| \frac{h(\theta, \phi, f_i, \psi_1, \psi_2) - h_c(\theta, \phi, f_i, \psi_1, \psi_2)}{h(\theta, \phi, f_i, \psi_1, \psi_2)} \right| \times 100. \] (5.4)

As stated in Chapter 4, it is desirable to have the approximation error less than 10%.

Fig. 5.5 shows the approximation error from the NEC simulation for various blade
locations over the angular region $\theta = 0^\circ$ to $120^\circ$, $\phi = 0$ to $360^\circ$. From the figures it
is clear that the approximation is very good over most of the angular region as it is
less than 10% for most look directions. However, near zenith it is apparent that there
is slightly more error and at some angles below the horizon. The antenna response
in these directions is very weak, therefore, the normalized error can be very large.
Overall, the composite antenna response is a good approximation of the true antenna
response.

Fig. 5.6 shows the approximation error from the FEKO simulations. All other
parameters are the same as in Fig. 5.5. Again, the approximation is very good over
most of the angular region except near zenith and at some angles below the horizon
where the antenna response is very weak. Overall, the composite antenna response is
a good approximation of the true antenna response.
5.4 Large Rotorcraft Examples

In this section, two additional rotorcraft models that were simulated using NEC-BSC are considered. The body of the rotorcraft is made from a six sided flat plate.
Figure 5.4: True and approximate antenna response at 1227 MHz. \( \theta = 0^\circ \) to 120\(^\circ\). Blade 1 = 45\(^\circ\). Blade 2 = 45\(^\circ\).

that is 40’ long. The plate is 10 ft wide at the front and 2’ wide at the back. There is also a rear stabilizer that extends 5 feet above the main plate. The top plate of the
stabilizer is 2’ wide and 3’ long. As mentioned above, two versions of this rotorcraft will be considered. The first version, “Rotorcraft3”, is shown in Fig. 5.7. This model contains a set of blades that are modeled using four plates that are all tilted at angle of 20°. The blades are 20’ long and are 1’ wide. The front blades (blade 1) is mounted 15’ away from the nose of the rotorcraft and 3’ above the body plate. The rear blade (blade 2) is also modeled as four tilted plates, however, they are 4’ long and 0.5’ wide. The tilt angle is 10° for these blades. Additionally, the blades are mounted vertically
Figure 5.6: Error in FEKO approximate antenna response for various blade orientations. $\theta = 0^\circ$ to $120^\circ$.

in the x-z plane. They are mounted 1’ away from the rear stabilizer and 4.25’ above the body plate. The antenna used in the simulations is a crossed slot antenna that is located at one of two locations. Location A is located 25’ back of the nose of the rotorcraft on the surface of the main plate and Location B is in the middle of the top plate of the stabilizer. The second version of the rotorcraft, “Rotorcraft4”, is shown in Fig. 5.8. Blade 1 and blade 2 are both 20’ long and 1’ wide. Blade 2 is located 10’ back from the nose of the rotorcraft and is 3’ above the main plate. Blade 2 is
located 1 ft above the center of the top plate of the stabilizer. Again, the antenna is a crossed slot and the location is either at location A or location B described above.

Figure 5.7: Simulation model of Rotorcraft3.

In the following examples, (5.4) is used to evaluate the approximation error for the various rotorcrafts and antenna locations. Fig. 5.9 shows the error when the antenna is mounted at location A on Rotorcraft 3. Error plots for various blade locations are shown in the figure. The error is plotted over the angular region $\theta = 0^\circ$ to $120^\circ$, $\phi = 0$ to $360^\circ$. Note that the antenna is a crossed slot antenna which provides an omnidirectional pattern over the upper hemisphere. From the plots it is clear that
the approximation is very good over most of the angular region. The only problem
spot is below the horizon around $\phi = 180^\circ$, which corresponds to the rear of the
rotorcraft. Note that this direction is looking through the stabilizer and therefore the
antenna response is hard to predict. Overall, the composite antenna response is a
good approximation of the true antenna response.

Fig. 5.10 shows the error when the antenna is mounted at location B on Rotorcraft
3 for various blade rotation angles. The error is shown over the angular region $\theta = 0^\circ$
to $120^\circ$, $\phi = 0$ to $360^\circ$. Again, it is clear that the approximation is very good over
most of the angular region for all blade orientations. The only problem spot is below
the horizon around $\phi = 0^\circ$, which corresponds to the nose of the rotorcraft. Note that
the antenna is mounted on top of the stabilizer and these look directions are through
the main plate. Therefore, the antenna response is harder to predict.

Fig. 5.11 shows the error when the antenna is mounted at location A on Rotorcraft
4 for various blade rotation angles. The error is shown over the angular region $\theta = 0^\circ$
to $120^\circ$, $\phi = 0$ to $360^\circ$. When both blade angles are $0^\circ$ (Fig. 5.11a), it is apparent
that there is slightly more error than the other blade angles. In this case, both blades
are directly aligned over the antenna and therefore, it is harder to predict the antenna
response. From the other figures it is clear that the approximation is very good over
most of the angular region. Again, the only problem spot is below the horizon around
the rear of the rotorcraft as expected.

Fig. 5.12 shows the approximation error when the antenna is located at location B
on Rotorcraft 4 for various blade rotation angles. The error is shown over the angular
region $\theta = 0^\circ$ to $120^\circ$, $\phi = 0$ to $360^\circ$. One can see that when both blade orientations
are $0^\circ$ (Fig. 5.12a), the error is about the same as the other blade angles. For antenna
location B, the antenna is located under the center of Blade 2, and therefore, it is not as difficult to predict the antenna response. From the other figures it is clear that the approximation is very good over most of the angular region. Again, the only problem spot is below the horizon and around the nose of the rotorcraft as expected. The blade angles shown in the figures above cover 0° to 45°, which is a small sample of all the possible blade angles. It should be noted that the error has been evaluated for blade angles from 0° to 90° in 1° steps and similar conclusions were drawn.

\begin{align*}
(a) & \ B_1 = 0^\circ, \ B_2 = 0^\circ. \\
(b) & \ B_1 = 0^\circ, \ B_2 = 15^\circ. \\
(c) & \ B_1 = 0^\circ, \ B_2 = 45^\circ. \\
(d) & \ B_1 = 15^\circ, \ B_2 = 0^\circ. \\
(e) & \ B_1 = 15^\circ, \ B_2 = 15^\circ. \\
(f) & \ B_1 = 15^\circ, \ B_2 = 45^\circ. \\
(g) & \ B_1 = 45^\circ, \ B_2 = 0^\circ. \\
(h) & \ B_1 = 45^\circ, \ B_2 = 15^\circ. \\
(i) & \ B_1 = 45^\circ, \ B_2 = 45^\circ.
\end{align*}

Figure 5.9: Error in approximate antenna response for various blade orientations for Rotorcraft 3 Antenna Location A. $\theta = 0^\circ$ to $120^\circ$. 

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Figure 5.10: Error in approximate antenna response for various blade orientations for Rotorcraft 3 Antenna Location B. $\theta = 0^\circ$ to $120^\circ$. 

(a) $B_1 = 0^\circ$. $B_2 = 0^\circ$. 
(b) $B_1 = 0^\circ$. $B_2 = 15^\circ$. 
(c) $B_1 = 0^\circ$. $B_2 = 45^\circ$. 
(d) $B_1 = 15^\circ$. $B_2 = 0^\circ$. 
(e) $B_1 = 15^\circ$. $B_2 = 15^\circ$. 
(f) $B_1 = 15^\circ$. $B_2 = 45^\circ$. 
(g) $B_1 = 45^\circ$. $B_2 = 0^\circ$. 
(h) $B_1 = 45^\circ$. $B_2 = 15^\circ$. 
(i) $B_1 = 45^\circ$. $B_2 = 45^\circ$. 

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Figure 5.11: Error in approximate antenna response for various blade orientations for Rotorcraft 4 Antenna Location A. $\theta = 0^\circ$ to $120^\circ$. 
Figure 5.12: Error in approximate antenna response for various blade orientations for Rotorcraft 4 Antenna Location B. $\theta = 0^\circ$ to $120^\circ$. 
5.5 Summary

In this chapter, the Composite Antenna Response was developed to approximate the time domain response of an antenna mounted on a dual rotor aircraft. The major benefit of the CAR is that it significantly reduces the amount of measurements/calculations that are needed to obtain the time domain response of the antenna. Several examples of antennas mounted on dual rotor platforms were used to demonstrate that the CAR provides an accurate approximation of the true antenna response.

In the following chapter, the focus of this thesis turns to the effects of RBM on digital communication systems. The FSA is used to derive the receiver’s correlator outputs in the presence of RBM. These correlator outputs are distorted by the RBM; therefore, the symbol detection capability of the receiver is compromised. The following chapter demonstrates the level of performance loss that the receiver encounters in the presence of RBM and proposes several techniques to mitigate the effects of RBM within the receiver.
CHAPTER 6

DIGITAL COMMUNICATIONS IN THE PRESENCE OF RBM

6.1 Overview

Communication over wireless channels can be significantly impaired if the effects of the channel are not accounted for in the receiver [12]. Typically, this means one must account for the effects of multipath, which is generated from scattering off of objects nearby the receiver. When the transmitter and receiver are both stationary the channel is said to be frequency selective as the multipath has a different effect at different frequencies. When there is relative motion between the transmitter and receiver, the multipath characteristics also will change as a function of time. In this case, the channel is said to be doubly-selective since the frequency characteristics of the channel change as function of time. As shown in [13], doubly-selective channels can be modeled as a linear time-varying (LTV) system. Many recent research efforts, which are summarized in [14], have focused on how to mitigate the effects of doubly-selective channels. In several of these papers, [15, 16, 17, 18], the LTV channel is modeled using a basis expansion model (BEM). The BEM allows the rapidly time-varying channel to be modeled using a linear combination of time-invariant (or
slowly changing) coefficients and a basis that accounts for the fast variations in time. Moreover, once the coefficients are estimated, they can be used to improve the performance of a communication receiver in the presence of a doubly-selective channel. In this chapter, it is shown that the BEM is not practical for representing a communication channel in the presence of RBM even though the channel is a doubly-selective channel. Thus, alternative methods are developed for improving the receiver’s performance in the presence of RBM.

The chapter is organized as follows. First, a general model for a communication receiver is presented. After this, the receiver outputs are derived in the presence of several different types of channels. First, the noise only channel and fixed platform channel are described. Numerical simulations are provided to show the receiver performance over these types of channels and how to mitigate the effects of the fixed platform. Finally, the RBM channel is discussed and it is shown how one can mitigate the RBM effects within the receiver. Again, numerical examples are provided.

6.2 Digital Communication Basics

6.2.1 Transmitted Signal

The first component of the digital communication system considered here is the digital base-band representation of the transmitted signal. For linearly modulated digital communication systems, the system transmits symbols out of the set \( i_z \) for \( z = 1..Z \). The transmitted signal can be expressed as

\[
s[n] = \sum_{m=0}^{\infty} I_m g[t - mR]
\]  

(6.1)
where $I_m$ is the $m^{th}$ symbol transmitted out of the set $i_z$. In (6.1), $R$ is the symbol duration in samples which is defined as

$$R = \left\lfloor \frac{T}{T_s} \right\rfloor \quad (6.2)$$

where $T$ is the symbol duration in seconds and $T_s$ is the sampling period in seconds. $g[n]$ is the transmission pulse shape, which is generally defined as

$$g[n] = \begin{cases} f[n] & \text{for } 0 \leq n < R \\ 0 & \text{otherwise} \end{cases} \quad (6.3)$$

Note that $f[n]$ is an arbitrary function of $n$ and that this base-band model is very general. It can be used to represent many digital communication schemes such as pulse amplitude modulation (PAM), phase-shift keying (PSK), and quadrature amplitude modulation (QAM).

### 6.2.2 Receiver

Fig. 6.1 shows a block diagram of the receiver that is considered here. The transmitted signal ($s[n]$) is transmitted over a channel to be specified later. At the input of the receiver is the signal $c[n]$ that is subsequently passed through a correlator and sampled at the symbol rate. The correlator outputs are sufficient for estimating the transmitted symbols [12]. Note that this model assumes that the receiver has locked onto the carrier and that the symbol timing has been recovered.

![Figure 6.1: Block diagram of digital communication receiver.](image)

Figure 6.1: Block diagram of digital communication receiver.
6.2.3 AWGN Channel

The simplest channel to consider is the additive white Gaussian noise (AWGN) channel. The communication receiver in the presence of AWGN is illustrated in Fig. 6.2. The signal at the input of the receiver is

\[ c[n] = s[n] + v[n] \]  \hspace{1cm} (6.4)

\[ r_p = \sum_{n=(p-1)R}^{pR-1} c[n]g[n - (p - 1)R] \]

\[ = \sum_{n=(p-1)R}^{pR-1} (s[n] + v[n])g[n - (p - 1)R] \]

\[ = \tilde{s}_p + \tilde{v}_p \]  \hspace{1cm} (6.5)

where \( \tilde{v}_p \) is a zero-mean white gaussian noise with variance \( \sigma_n \). Using (6.4), the correlator output \( (\tilde{r}_p) \) can be expressed as

\[ \tilde{v}_p = \sum_{n=(p-1)R}^{pR-1} v[n]g[n - (p - 1)R] \]  \hspace{1cm} (6.6)
Using (6.1), one can write

\[
\tilde{s}_p = \sum_{n=(p-1)R}^{pR-1} s[n]g[n - (p-1)R]
\]

\[
= \sum_{n=(p-1)R}^{pR-1} \sum_{m=0}^{\infty} I_m g[n - mR]g[n - (p-1)R]
\]

\[
= \sum_{m=0}^{\infty} I_m \sum_{n=(p-1)R}^{pR-1} g[n - mR]g[n - (p-1)R]
\]

\[
= \sum_{m=0}^{\infty} I_m \Phi[(p-m)R]
\]

where \( \Phi[(p-m)R] \) is given by

\[
\Phi[(p-m)R] = \sum_{n=(p-1)R}^{pR-1} g[n - mR]g[n - (p-1)R].
\]

Letting \( z = n - (p-1)R \) and \( \alpha = (p-m)R \), one obtains

\[
\Phi[\alpha] = \begin{cases} 
\sum_{z=0}^{R-1} g[z + \alpha - R]g[z] & \text{for } 0 < \alpha < 2R \\
0 & \text{otherwise}
\end{cases}
\]

(6.9)

It is important to note that only a finite number of symbols will contribute to the correlator output at sample \( p \). In other words there is a limited range on the values of \( m \) given by

\[
0 < (p-m)R < 2R \quad (6.10)
\]

\[
0 < (p-m) < 2 \quad (6.11)
\]

\[
p - 2 < m < p \quad (6.12)
\]

\[
p - 1 \leq m \leq p - 1 \quad (6.13)
\]

From (6.13) it is clear that only one symbol contributes at each sample of \( \tilde{s}_p \), thus there is no inter-symbol interference (ISI). Taking this into account, the correlator output is given as

\[
\tilde{r}_p = I_{p-1} \Phi[R] + \tilde{v}[pR].
\]

(6.14)
(6.14) shows that the symbols at the output of the correlator are delayed by one symbol. The delay occurs because the correlator operates over the previous symbol interval. In the following section, the correlator outputs are derived for antennas mounted on fixed platforms.

6.3 Antennas on Fixed Platforms

6.3.1 Channel Model

![Diagram of Fixed Platform Channel Model]

Figure 6.3: Fixed platform channel model.

Fig. 6.3 shows the received signal model for a receiver mounted on a fixed platform. In this case, the platform is a rotorcraft with the rotor blades fixed to one position. As shown in Chapter 2, the channel can be modeled as a filter that is dependent on the look direction of the incident signal and the orientation of the blade. The signal at the input of the receiver is

\[ c[n] = y[n] + v[n] \]  \hspace{1cm} (6.15)

where

\[ y[n] = \sum_{l=0}^{L} h[l] s[n - l]. \]  \hspace{1cm} (6.16)
In (6.16), \( h[l] \) is the order \( L \) impulse response of the filter \( H(f, \theta, \phi, \psi) \). The impulse response is generally defined as

\[
h[l] = \begin{cases} f[l] & \text{for } 0 \leq l \leq L, \\ 0 & \text{otherwise} \end{cases}
\]  

(6.17)

The correlator output, \( \tilde{r}_p \), can be expressed as

\[
\tilde{r}_p = \sum_{n=(p-1)R}^{pR-1} c[n]g[n - (p - 1)R] \\
= \sum_{n=(p-1)R}^{pR-1} (y[n] + v[n])g[n - (p - 1)R] \\
= \tilde{s}_p + \tilde{\nu}_p
\]  

(6.18)

Using (6.1), one can write

\[
\tilde{s}_p = \sum_{n=(p-1)R}^{pR-1} y[n]g[n - (p - 1)R] \\
= \sum_{n=(p-1)R}^{pR-1} \sum_{l=0}^{L} h[l]s[n - l]g[n - (p - 1)R] \\
= \sum_{m=0}^{\infty} I_m \sum_{l=0}^{L} h[l] \sum_{n=(p-1)R}^{pR-1} g[n - mR - l]g[n - (p - 1)R] \\
= \sum_{m=0}^{\infty} I_m \sum_{l=0}^{L} h[l]\Phi[(p - m)R - l] \\
= \sum_{m=0}^{\infty} I_m \tilde{\Phi}[(p - m)R]
\]  

(6.19)

where

\[
\tilde{\Phi}[\alpha] = \begin{cases} \sum_{l=0}^{L} h[l]\Phi[\alpha - l] & \text{for } 0 < \alpha < L + 2R, \\ 0 & \text{otherwise} \end{cases}
\]  

(6.20)
Again, only a finite number of symbols will contribute at sample \( p \) and these values of \( m \) are given by

\[
0 < (p - m)R < 2R + L \quad (6.21)
\]

\[
0 < (p - m) < 2 + \frac{L}{R} \quad (6.22)
\]

\[
p - 2 - \frac{L}{R} < m < p \quad (6.23)
\]

\[
p - 1 - \frac{L}{R} \leq m \leq p - 1 \quad (6.24)
\]

\[
p - 1 - Q \leq m \leq p - 1 \quad (6.25)
\]

where \( Q = \lfloor \frac{L}{R} \rfloor \). Thus, (6.19) can be written as

\[
\tilde{s}_p = \sum_{m=p-1-Q}^{p-1} I_{m} \bar{\Phi}[(p - m)R]. \quad (6.26)
\]

Substituting \( b = p - m \) in (6.26) one obtains

\[
\tilde{r}_p = \sum_{b=1}^{Q+1} I_{p-b} \bar{\Phi}[bR] + \tilde{v}_p. \quad (6.27)
\]

From (6.27) it is clear to see that the correlator output contains inter-symbol-interference (ISI) and noise. It is expected that the performance of the receiver will be degraded as compared to the performance in the presence of noise only because of the ISI. It is important to note that the same samples of \( \bar{\Phi}[\alpha] \) are in every correlator output sample. Thus, if one can estimate these coefficients, the receiver’s performance can be improved. This is described in the following section.

### 6.3.2 Linear Equalization

In this section, linear equalization is introduced. A linear equalizer (LE) estimates the transmitted symbols by taking a linear combination of past correlator outputs.
(\tilde{r}_p). The output of the LE (\tilde{I}_{p-1}) can be written as

\[
\tilde{I}_{p-1} = \sum_{m=1}^{M} \tilde{r}_{p-m+1} w_m
\]  

(6.28)

where \(w_m, m = 1 \ldots M\) are the coefficients of the linear equalizer. In order to find these coefficients, it is assumed that one has knowledge of \(N_t\) training symbols that have been transmitted to the receiver. Thus, the only unknowns in (6.28) are the coefficients \(w_m\) for \(m = 1 \ldots M\). Using vector notation, one can write

\[
\tilde{I}_{p-1} = \mathbf{f}_p \mathbf{w}.
\]

(6.29)

Note that lowercase bold print denotes a vector, uppercase bold print denotes a matrix, superscript T denotes the transpose operator, and superscript H denotes the Hermitian transpose operator. In (6.29),

\[
\mathbf{f}_p = [\tilde{r}_p \cdots \tilde{r}_{p-M}]
\]

(6.30)

and

\[
\mathbf{w} = [w_1 \cdots w_M]^T.
\]

(6.31)

The \(N_t\) received samples can be expressed in vector form as

\[
\mathbf{i} = \mathbf{Fw}
\]

(6.32)

where

\[
\mathbf{i} = [I_{p-1} \quad I_{p-2} \cdots I_{p-N_t}]^T,
\]

(6.33)

and

\[
\mathbf{F} = 
\begin{bmatrix}
\mathbf{f}_1 \\
\mathbf{f}_2 \\
\vdots \\
\mathbf{f}_{N_t}
\end{bmatrix}
\]

(6.34)
Thus if $N_t$ is larger than the number of coefficients ($M$), an over-determined system of equations is obtained and the coefficients can be solved for in the least-squares sense. The solution of the coefficients is

$$\hat{w} = (F^H F)^{-1} F^H i. \quad (6.35)$$

Once the estimated channel coefficients are computed, they can be used to filter the correlator outputs. The symbols can then be detected from the LE output by selecting the symbol out of the set of all possible symbols that is closest to the estimated symbol. The detected symbols will be denoted as $\hat{I}_p$.

### 6.3.3 Results

This section presents some example results using the LE when the digital communication system is mounted on a rotorcraft with blades fixed to a particular blade position. In the simulations, a digital phase-shift keying (PSK) communication system is used. In digital PSK, $Z$ possible symbols are represented as

$$i_z = e^{j2\pi(z-1)/Z}, z = 1, 2, ..., Z. \quad (6.36)$$

In the presence of noise only, the transmitted signal is best detected by finding the symbol $i_z$ whose phase is closest to that of the received signal. To quantify the performance of a given receiver, the probability of error (PE) versus signal-to-noise ratio (SNR) is used. For $Z = 2$ (BPSK), one finds the probability of error (PE) in the presence of noise to be [12]

$$P_2 = Q(\sqrt{2SNR}) \quad (6.37)$$

where $Q(x)$ is related to the complimentary error function by

$$Q(x) = 0.5\text{erfc} \left( \frac{x}{\sqrt{2}} \right). \quad (6.38)$$
In order to evaluate the PE in the presence of the platform, Monte Carlo simulations were carried out. In the simulations, the pulse duration is 1 µsec. and the sampling frequency in the simulation was 32 MHz. Also, \( R = 32, \; \; L = 50, \; \; \text{and} \; \; Q = 1 \). Twenty-five trials were used to calculate the average PE for various SNR values. In each trial there were 2500 transmitted symbols. The platform used in the simulations is shown in Fig. 6.4. This is the same platform that was described in Chapter 2. It should be noted that for the results presented in this chapter, the antenna response in a given direction is normalized by the average response of the antenna over one blade rotation period in that direction. In doing this, any performance loss due to a drop in gain in the antenna pattern has been removed. The goal here is to obtain an understanding of how the platform effects the receiver and it is understood that the drop in gain due to the antenna pattern can always be accounted for by boosting the signal power.

Fig. 6.5 shows the probability of error versus SNR when the signal arrives along \( \theta = 25^\circ, \phi = 30^\circ \). The results are shown for three different blade locations, \( \psi = 0^\circ, 30^\circ, \; \text{and} \; 60^\circ \). The black trace shows the performance when the symbols are detected from the correlator output that is obtained in the presence of noise only. The blue trace shows the performance when the symbols are detected from the correlator output
that is obtained in the presence of the platform. In this case, there is no equalization used and this is denoted by “NE” in the figure. The red trace shows the performance when the symbols are detected from the output of the linear equalizer. The results were generated using 100 training symbols and with $M = 2$ and 6. It is clear from the figures that the platform can have a profound effect on the receiver’s performance (blue). However, it is also clear that the LE can mitigate most of this performance loss. It is interesting to note that the performance is still worse than the performance in the presence of noise only even when the LE is used. This can be explained by looking at the magnitude as well as the IQ diagram of the signal that is used to detect the transmitted symbols.

Fig. 6.6 shows the magnitude and IQ diagram of the correlator output for the noise only channel. In this figure the SNR is 8 dB. One notices that the average magnitude is 0 dB. The IQ diagram shows that the samples are spread around the two possible symbols -1 and 1. Note that the black dots represent symbols that should be -1 and the red dots represent symbols that should be 1. If there was no noise in the system, the IQ diagram would show two dots, one at -1 and the other at 1. However, as the figure shows, the noise causes the received signals to be spread around the two possible symbols. Thus, the IQ diagram provides a way to judge the amount of noise in the signal used for symbol detection. An increase in the spread indicates an increase in noise.

Fig. 6.7 shows the magnitude and IQ diagram of the correlator output obtained in the presence of the rotorcraft. Again, the signal arrives along $\theta = 25^\circ$ and the results are shown for blade angles of $\psi = 0^\circ$, $30^\circ$, and $60^\circ$. For blade angles of $0^\circ$ and $30^\circ$ one can see that the average magnitude of the correlator output is below 0 dB.
Figure 6.5: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions. Signal arrives along ($\theta = 25^\circ, \phi = 30^\circ$). The PE is shown for the AWGN channel (black) as well as the the fixed blade channel when symbols are detected from the output of the correlator with no equalization (blue) and when the symbols are detected from the output of the linear equalizer (red).

This loss in signal strength is due to the blockage of the direct signal path by the rotor blades. The IQ plot also shows that the symbols are rotated in the IQ plane. One can see that the symbols that are supposed to be -1 (shown in black) are now close to 1 and vice versa. This phase distortion causes the symbols to be detected incorrectly, which leads to a loss in performance. For a blade angle of $60^\circ$, one can see that the average magnitude of the correlator output is about 0 dB and there is significant phase distortion.
Figure 6.6: Correlator output magnitude and IQ diagram for AWGN channel. SNR = 8 dB.

(a) Mag. \( \psi = 0^\circ \). (b) IQ Plot. \( \psi = 0^\circ \).

(c) Mag. \( \psi = 30^\circ \). (d) IQ Plot. \( \psi = 0^\circ \).

(e) Mag. \( \psi = 60^\circ \). (f) IQ Plot. \( \psi = 60^\circ \).

Figure 6.7: Correlator output magnitude and IQ diagram for the fixed blade channel when the signal arrives along \((\theta = 25^\circ, \phi = 30^\circ)\) and the SNR = 8 dB.

(a) Mag. \( \psi = 0^\circ \). (b) Mag. \( \psi = 30^\circ \). (c) Mag. \( \psi = 60^\circ \).

(d) IQ Plot. \( \psi = 0^\circ \). (e) IQ Plot. \( \psi = 30^\circ \). (f) IQ Plot. \( \psi = 60^\circ \).

Figure 6.7: Correlator output magnitude and IQ diagram for the fixed blade channel when the signal arrives along \((\theta = 25^\circ, \phi = 30^\circ)\) and the SNR = 8 dB.

It is clear from the IQ diagrams that the phase distortion can be removed using the LE. The plot of the magnitude of the equalizer output also shows that the average
magnitude is restored to 0 dB when the LE is used. It is important to note that both the noise and signal components (of the correlator output) are multiplied by the equalizer coefficients; therefore, the SNR does not change at the output of the equalizer. One can see that the spread of the equalizer output has increased for $\psi = 0^\circ$ and $\psi = 30^\circ$. This shows that noise increases as the equalizer restores the average signal level to 0 dB. Thus, the PE performance still has room for improvement since this loss in SNR has not been accounted for. For $\psi = 60^\circ$, the PE performance is very similar to the performance in the presence of noise only when the LE is used since the average value of the correlator output was about 0 dB. It is important to note that there is only a slight penalty in performance when six coefficients are used instead of two. This is important because the actual number of coefficients may not be known to the receiver and one may have to estimate the number of coefficients to use on the fly.

Fig. 6.9 shows the probability of error vs. SNR and Fig. 6.10 and Fig. 6.11 show the correlator and LE outputs along $\theta = 45^\circ, \phi = 30^\circ$. All other parameters are the same as Fig. 6.5, Fig. 6.7, and Fig. 6.8, respectively. Again, one can see from the figures that the platform greatly degrades the receiver’s performance (blue curve). For this look direction, one can see that the received signal magnitude is above 0 dB for all three blade locations. Therefore, it is not a drop in SNR that causes this degradation. One can see from the IQ plots that there is a large rotation of the received signal in the IQ plane, which causes the loss in performance. As expected, the LE is able to remove the phase distortion for all three blade locations. In fact, the performance is slightly better than the noise only case for $\psi = 60^\circ$. This is expected since the average value of the correlator output is slightly more than 0 dB.
(a) Mag. $\psi = 0^\circ$.
(b) Mag. $\psi = 30^\circ$.
(c) Mag. $\psi = 60^\circ$.

(d) IQ Plot. $\psi = 0^\circ$.
(e) IQ Plot. $\psi = 30^\circ$.
(f) IQ Plot. $\psi = 60^\circ$.

Figure 6.8: Linear equalizer output magnitude and IQ diagram for the fixed blade channel when the signal arrives along ($\theta = 25^\circ$, $\phi = 30^\circ$) and the SNR = 8 dB.
Figure 6.9: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions. Signal arrives along ($\theta = 45^\circ$, $\phi = 30^\circ$). The PE is shown for the AWGN channel (black) as well as the fixed blade channel when symbols are detected from the output of the correlator with no equalization (blue) and when the symbols are detected from the output of the linear equalizer (red).
Figure 6.10: Correlator output magnitude and IQ diagram for the fixed blade channel when the signal arrives along ($\theta = 45^\circ$, $\phi = 30^\circ$) and the SNR = 8 dB.
(a) Mag. $\psi = 0^\circ$.  (b) Mag. $\psi = 30^\circ$.  (c) Mag. $\psi = 60^\circ$.

(d) IQ Plot. $\psi = 0^\circ$.  (e) IQ Plot. $\psi = 30^\circ$.  (f) IQ Plot. $\psi = 60^\circ$.

Figure 6.11: Linear equalizer output magnitude and IQ diagram for the fixed blade channel when the signal arrives along $(\theta = 45^\circ, \phi = 30^\circ)$ and the SNR = 8 dB.

Fig. 6.12 shows the probability of error vs. SNR and Fig. 6.13 and Fig. 6.14 show the correlator and equalizer output along $\theta = 65^\circ, \phi = 30^\circ$. All other parameters are the same as Fig. 6.5, and Fig. 6.7, and Fig. 6.8, respectively. Very similar conclusions can be drawn from these figures. Although, the LE is able to remove the phase distortion, there is still some loss in performance that is caused by a drop in SNR that is observed at the output of the correlator. In the next section, spatial diversity will be introduced as a means to account for the drop in SNR.

6.3.4 Diversity

Spatial diversity can be introduced into the system by using multiple antenna elements to receive the incoming signal. Fig. 6.15 shows the configuration of the antennas on the rotorcraft. Element 1 is located at the center of the rotorcraft 6.25’
Figure 6.12: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions. Signal arrives along $(\theta = 65^\circ, \phi = 30^\circ)$. The PE is shown for the AWGN channel (black) as well as the fixed blade channel when symbols are detected from the output of the correlator with no equalization (blue) and when the symbols are detected from the output of the linear equalizer (red).

behind the rotor blades. This is the element that was used to generate the results presented in the previous section. Elements 2 and 3 are located 3’ away from element 1. There is 1’ between these edge elements and the platform edge.

The basic idea behind diversity is to use all the antenna elements to increase the received signal strength. As an illustration, Fig. 6.16 shows the magnitude and phase of element 1 versus blade angle along $\theta = 25^\circ, 45^\circ, 65^\circ$. Also shown in these figures is the magnitude and phase of the diversity response. The diversity response is found by simply selecting the element at each blade angle that has the
Figure 6.13: Correlator output magnitude and IQ diagram for the fixed blade channel when the signal arrives along \((\theta = 65^\circ, \phi = 30^\circ)\) and the SNR = 8 dB.

largest SNR. One can see that there is an overall increase in the magnitude response by using diversity. The large jumps in phase in the diversity response is caused by the switching between elements. Although this simple approach can be used, another approach is to combine the output of all three antenna elements to obtain the best estimate of the transmitted symbols. In order to do this, the linear equalizer must be modified. When diversity is used, the linear equalizer estimates the transmitted signal as

\[
\hat{I}_{p-1} = \sum_{e=1}^{E} \sum_{m=1}^{M} r_{p-m+1}^{e} w_{em}
\]  

(6.39)

where \(E\) is the number of antenna elements, \(r_{p-m+1}^{e}\) is the correlator output of the \(e^{th}\) antenna element, and \(w_{em}\) is the \(m^{th}\) coefficient of element \(e\)’s equalizer. (6.39) shows that a linear equalizer is used behind each antenna element, however, it is important
Figure 6.14: Linear equalizer output magnitude and IQ diagram for the fixed blade channel when the signal arrives along ($\theta = 65^\circ, \phi = 30^\circ$) and the SNR = 8 dB.

to note that the coefficients $w_{em}$ are found jointly to provide the best estimate of the transmitted symbols. Again, if $N_t$ training symbols are known, one can write a system of equations to solve for the equalizer coefficients as

$$\hat{w} = (F^H F)^{-1} F^H i.$$  \hspace{1cm} (6.40)

In (6.40),

$$w = [w_{1,1} \cdots w_{1,M} \cdots w_{E,1} \cdots w_{E,M}]^T.$$  \hspace{1cm} (6.41)

and

$$F = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_{N_t} \end{bmatrix}.$$  \hspace{1cm} (6.42)

where

$$f_p = \begin{bmatrix} \tilde{r}^{(1)}_p & \tilde{r}^{(1)}_{p-M} & \cdots & \tilde{r}^{(E)}_p & \tilde{r}^{(E)}_{p-M} \end{bmatrix}.$$  \hspace{1cm} (6.43)
Once the coefficients are known, (6.39) can be used to estimate the transmitted symbols. This will be referred to as the Diversity Linear Equalizer (DLE). The following section shows the PE performance when the DLE is used.

### 6.3.5 Diversity Results

In order to evaluate the PE performance of the DLE, Monte Carlo simulations were carried out. Again, the pulse duration was 1 µsec. and the sampling frequency in the simulation was 32 MHz. In the simulations, $R = 32, L = 50$, and $Q = 1$. Three antenna elements were used as described in the previous section. Each element had an linear equalizer with $M = 2$.

Fig. 6.17 shows the PE versus SNR along $\theta = 25^\circ, \phi = 30^\circ$ when the rotor blades are fixed at $\psi = 0^\circ, 30^\circ$, and $60^\circ$. The performance in the presence of noise only is shown in black. The trace in blue shows the performance when the symbols are detected from the output of the linear equalizer when element 1 is used by itself to receive the signal. In this case, no diversity is used and this is denoted by “LE” in the figures. The trace in red shows the performance when symbols are detected from the output of the DLE. One can see that the performance improves well past the
Figure 6.16: Magnitude and Phase of element 1 antenna response and diversity response vs. blade angle.

noise only performance. Again, the magnitude of the equalizer output as well as the IQ diagram (shown in Fig. 6.18) can be used to understand this. As expected, the figure shows that the phase distortion is removed at the output of the equalizer and the average magnitude is 0 dB. The important thing to notice in the figure is that the spread around the possible symbols has been reduced (compare Fig. 6.18 with
Fig. 6.8). This indicates that the noise is reduced at the output of the DLE and thus the SNR has increased. This leads to large improvement in performance that is observed in Fig. 6.17.

Figure 6.17: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions when the signal arrives along \((\theta = 25^\circ, \phi = 30^\circ)\). The symbols are detected from the output of the DLE.

Fig. 6.19 and Fig. 6.20 show the PE versus SNR along \(\theta = 45^\circ, \phi = 30^\circ\) and \(\theta = 65^\circ, \phi = 30^\circ\). All other parameters are the same as in Fig. 6.17. One observes that when the DLE is used the performance exceeds the performance in the presence of noise only. Again, the performance improves because the DLE is able to remove the phase distortion and increase the SNR at the output of the equalizer.

The results above have shown that the equalizer coefficients must be known for each blade angle in order to improve the performance of the receiver in the presence of the platform. Obviously, it would be beneficial to be able to reduce this requirement. One approach is to average the equalizer coefficients over one blade rotation period for a given direction and use the average coefficients for all blade locations. This will
Figure 6.18: Diversity linear equalizer output magnitude and IQ diagram for the fixed blade channel when the signal arrives along ($\theta = 25^\circ, \phi = 30^\circ$) and the SNR = 20 dB.

(a) Mag. $\psi = 0^\circ$.  
(b) Mag. $\psi = 30^\circ$.  
(c) Mag. $\psi = 60^\circ$.

(d) IQ Plot. $\psi = 0^\circ$.  
(e) IQ Plot. $\psi = 30^\circ$.  
(f) IQ Plot. $\psi = 60^\circ$.

Figure 6.19: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions when the signal arrives along ($\theta = 45^\circ, \phi = 30^\circ$). The symbols are detected from the output of the DLE.

(a) $M = 2, \psi = 0^\circ$.  
(b) $M = 2, \psi = 30^\circ$.  
(c) $M = 2, \psi = 60^\circ$.  

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Figure 6.20: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades fixed at various positions when the signal arrives along $(\theta = 65^\circ, \phi = 30^\circ)$. The symbols are detected from the output of the DLE.

be referred to as the Average Linear Equalizer (ALE) when only one element is used and the Average Diversity Linear Equalizer (ADLE) when diversity is used.

Fig. 6.21 shows the PE versus rotor blade angle along $\theta = 25^\circ$ for SNR values of 0 and 8 dB. Note that in these figures the PE is set to $10^{-5}$ whenever it is smaller than $10^{-5}$. The black curves represent the noise only case. The figures on the left show the performance when no equalization is used. The blue curve shows the PE when the symbols are detected from element 1’s correlator output and no diversity or equalization is used (NDNE). The red curve shows the PE when the symbols are detected from the antenna with the strongest correlator output at each blade location. In this case diversity is used, however, no equalization is used. This is denoted as “DNE” in the figure. It is interesting to note that for some blade locations there is a large improvement in performance just by using the different elements. This is a result of the improvement in SNR. The center figures show the PE when the LE is used with $M = 2$ (blue) and when the DLE is used with $M = 2$ and $E = 3$ (red). This represents the best case performance since the equalizer coefficients are obtained
Figure 6.21: Probability of error vs. blade angle for a BPSK receiver mounted on a rotorcraft when the signal arrives along $(\theta = 25^\circ, \phi = 30^\circ)$ and the SNR is 0 and 8 dB.

for each blade location. Again, one can see that by using diversity the performance is improved. As expected, this improvement is more pronounced around the blade locations where the magnitude of element 1 drops off (compare with Fig. 6.16a). The figures on the right show the PE when the ALE is used (blue) and when the ADLE is used (red). One can see that the performance of the ALE and ADLE are very similar to that of the LE and DLE, respectively. This is a promising result as it implies that one set of equalization coefficients can be used over all blade locations.

Fig. 6.22 shows the PE versus rotor blade angle along $\theta = 45^\circ$. All other parameters are the same as in Fig. 6.21. Again, very similar conclusions can be drawn from
Figure 6.22: Probability of error vs. blade angle for a BPSK receiver mounted on a rotorcraft when the signal arrives along $(\theta = 45^\circ, \phi = 30^\circ)$ and the SNR is 0 and 8 dB.

these figures. When no equalization is used, diversity can lead to an improvement in performance at some blade locations because of the increase in SNR. When LE is used, there is still some performance loss that cannot be accounted for if element 1 is used alone. However, when the DLE is used the performance is very similar to the performance in the presence of noise only. These conclusions are also true when the ALE and the ADLE are used.

Fig. 6.23 shows the PE versus rotor blade angle along $\theta = 65^\circ$. All other parameters are the same as in Fig. 6.21. It is interesting to note that in the case of no equalization (figures on left), there is no improvement in performance even when
(a) SNR = 0 dB. No Eq.  
(b) SNR = 0 dB. LE.  
(c) SNR = 0 dB. ALE.  
(d) SNR = 8 dB. No Eq.  
(e) SNR = 8 dB. LE.  
(f) SNR = 8 dB. ALE.

Figure 6.23: Probability of error vs. blade angle for a BPSK receiver mounted on a rotorcraft when the signal arrives along \((\theta = 65^\circ, \phi = 30^\circ)\) and the SNR is 0 and 8 dB.

diversity is used. This is caused by the rotation of the signal in the IQ plane. In this case, equalization is needed in addition to diversity to improve the performance. This is illustrated in the figures. Again, one can see that the performance of the ALE and ADLE are very similar to that of the LE and DLE, respectively.

In this section, it was demonstrated that one set of equalizer coefficients can be used to improve the receiver’s performance for all blade locations. This is a promising results as it implies that the average equalizer coefficients can be used when the blades are rotating at any speed. In the following section, the correlator outputs are derived for a communication system operating in the presence of RBM.
6.4 Communication Channel in the Presence of RBM

6.4.1 Channel Model

In this section, the correlator outputs are derived for a digital communication system operating in the presence of RBM. Fig. 6.24 shows the received signal model for a receiver mounted on a rotorcraft when the rotor blades are rotating at a fixed speed. In this case, the RBM is modeled using the FSA that was described in Chapter 3. The correlator output at sample \( p \) is given by

\[
\tilde{r}_p = \sum_{n=(p-1)R}^{pR-1} c[n]g[n - (p - 1)R] \\
= \sum_{n=(p-1)R}^{pR-1} (y[n] + v[n])g[n - (p - 1)R] \\
= \tilde{s}_p + \tilde{v}_p
\]

In (6.44),

\[
y[n] = \sum_{k=-K}^{K} y_k[n]
\]

Figure 6.24: RBM channel model.
and

\[ y_k[n] = \sum_{l=0}^{L} a_k[l] s[n - l] e^{jk\omega_0 n T_s} \]  

(6.46)

where \( a_k[l] \) is the impulse response of the \( k^{th} \) Fourier series filter of length \( L + 1 \). Note that \( \omega_0 \) is the fundamental harmonic which is related to the blade repetition period by \( \omega_0 = \frac{2\pi}{T_b} \), where \( T_b \) is the blade repetition period. Thus one can write

\[ \tilde{s}_p = \sum_{k=-K}^{K} \tilde{s}_{pk}. \]  

(6.47)

Using (6.1) and (6.46), one can write

\[
\tilde{s}_{pk} = \sum_{n=(p-1)R}^{pR-1} \tilde{y}_k[n] g[n - (p - 1)R]
\]

\[ = \sum_{n=(p-1)R}^{pR-1} \sum_{l=0}^{L} a_k[l] s[n - l] e^{jk\omega_0 n T_s} g[n - (p - 1)R] \]

\[ = \sum_{m=0}^{\infty} \sum_{l=0}^{L} a_k[l] \sum_{n=(p-1)R}^{pR-1} g[n - mR - l] g[n - (p - 1)R] e^{jk\omega_0 n T_s} \]

\[ = \sum_{m=0}^{\infty} I_m \sum_{l=0}^{L} a_k[l] \Phi_{pk}[(p - m)R - l]. \]  

(6.48)

where

\[ \Phi_{pk}[(p - m)R - l] = \sum_{n=(p-1)R}^{pR-1} g[n - mR - l] g[n - (p - 1)R] e^{jk\omega_0 n T_s}. \]  

(6.49)

Letting \( z = n - pR + R \) one obtains

\[ \Phi_{pk} = \sum_{z=0}^{R-1} g[z + (p - m) R - R - l] g[z] e^{jk\omega_0 (z + pR - R) T_s} \]

(6.50)

Finally, letting \( \alpha = (p - m) R - l \) and

\[ \dot{\Phi}_k[\alpha] = \sum_{z=0}^{R-1} g[z + \alpha - R] g[z] e^{jk\omega_0 z T_s} \]  

(6.51)
one obtains

\[ \Phi_{pk}[\alpha] = \begin{cases} e^{j k \omega_0 (p-1) RT} \Phi_k[\alpha] & \text{for } 0 < \alpha < 2R \\ 0 & \text{otherwise} \end{cases} , \quad (6.52) \]

Using (6.52), (6.48) becomes

\[ \tilde{s}_{pk} = \sum_{m=0}^{\infty} I_m \sum_{l=0}^{L} a_k[l] \Phi_{pk}[(p-m)R - l] . \quad (6.53) \]

Again, (6.53) can be simplified by defining

\[ \tilde{\Phi}_{pk}[\alpha] = \begin{cases} \sum_{l=0}^{L} a_k[l] \Phi_{pk}[\alpha - l] & \text{for } 0 < \alpha < 2R + L \\ 0 & \text{otherwise} \end{cases} \quad (6.54) \]

Using (6.54), one obtains

\[ \tilde{s}_{pk} = \sum_{m=0}^{\infty} I_m \tilde{\Phi}_{pk}[(p-m)R] . \quad (6.55) \]

Again, only a finite number of symbols will contribute to each correlator output. As shown before, (6.55) can be written as

\[ \tilde{s}_{pk} = \sum_{b=1}^{Q+1} I_{p-b} \tilde{\Phi}_{pk}[bR] . \quad (6.56) \]

Using (6.56) in (6.47), one obtains

\[ \tilde{s}_p = \sum_{k=-K}^{K} \sum_{b=1}^{Q+1} I_{p-b} \tilde{\Phi}_{pk}[bR] \]
\[ = \sum_{b=1}^{Q+1} I_{p-b} \sum_{k=-K}^{K} \tilde{\Phi}_{pk}[bR] \]
\[ = \sum_{b=1}^{Q+1} I_{p-b} d_{bp} \quad (6.57) \]

where

\[ d_{bp} = \sum_{k=-K}^{K} \tilde{\Phi}_{pk}[bR] . \quad (6.58) \]

Finally, the correlator output can be expressed as

\[ \tilde{r}_p = \sum_{b=1}^{Q+1} I_{p-b} d_{bp} + \tilde{v}_p . \quad (6.59) \]
(6.59) shows that in the presence of RBM, the correlator output contains ISI and noise. However, the linear coefficients vary with time, thus the ISI is changing with time. In the following section, the Basis Expansion Model (BEM) is described and compared to the results developed in this section.

### 6.4.2 Basis Expansion Model

In this section, the Basis Expansion Model (BEM) described in [15] is presented. It should be noted that the notation used in this thesis is adopted as much as possible.

As mentioned earlier, the BEM was developed to model fast varying multipath, which causes multiple copies of the transmitted signal to arrive at the receiver. Each copy of the signal will arrive with a different delay and amplitude. Thus if the transmitted signal at the carrier frequency ($f_c$) is given by

$$s_c(t) = Re\left\{ \sum_{m=0}^{\infty} I_m g(t - mT) e^{j2\pi f_c t} \right\}$$  \hspace{1cm} (6.60)

the received signal is given by

$$y_c(t) = \sum_{l=1}^{L} A_l(t) s_c(t - \tau_l(t)) + v_c(t).$$  \hspace{1cm} (6.61)

In (6.61), $L$ is the number of paths, $A_l(t)$ is the time varying amplitude along path $l$, $\tau_l(t)$ is the time varying delay along path $l$, and $v_c(t)$ is the noise in the received signal. Using (6.60) in (6.61) and assuming perfect carrier removal the base band received signal is given as

$$y(t) = \sum_{l=1}^{L} A_l(t) \sum_{m=0}^{\infty} I_m g(t - mT - \tau_l(t)) e^{j2\pi f_c \tau_l(t)} + v(t).$$  \hspace{1cm} (6.62)
Assuming that the symbol timing is known and the path delay and amplitude are constant over each symbol, the output of the correlator can be written as

\[ r_p = \sum_{l=1}^{L} A_{lp} e^{j2\pi f_c \tau_{lp}} \sum_{b=0}^{Q} I_{p-b} \Phi[bR - \tau_{lp}] + \tilde{v}_p. \]  

(6.63)

If one assumes that the path delay is a linear function of time then one can write

\[ \tau_{lp} = \lambda_{lp} + \lambda_{lp}^0. \]  

(6.64)

Substituting (6.64) in (6.63), one obtains

\[ r_p = \sum_{l=1}^{L} A_{lp} e^{j2\pi f_c (\lambda_{lp} + \lambda_{lp}^0)} \sum_{b=0}^{Q} I_{p-b} \Phi[bR - \tau_{lp}] + \tilde{v}_p \]  

\[ = \sum_{b=0}^{Q} I_{p-b} \sum_{l=1}^{L} \tilde{A}_{lp} \Phi[bR - \tau_{lp}] e^{j\alpha_{lp}} + \tilde{v}_p \]  

\[ = \sum_{b=0}^{Q} I_{p-b} \sum_{l=1}^{L} \theta_{bl} e^{j\alpha_{lp}} + \tilde{v}_p \]  

(6.65)

where \( \tilde{A}_{lp} = A_{lp} e^{j2\pi f_c \lambda_{lp}^0} \), \( \alpha_l = 2\pi f_c \lambda_l \), and \( \theta_{bl} = \tilde{A}_{lp} \Phi[bR - \tau_{lp}] \). At this point it is important to note that the time varying channel is modeled using time-invariant coefficients \( \theta_{bl} \) and complex exponentials with frequencies \( \alpha_l \). Additionally, it should be reiterated that this model holds only when the path delays change linearly with time and there are a small number of paths [15]. When these conditions hold, one can estimate the coefficients \( \theta_{bl} \) and use them to equalize the channel. This of course assumes that the frequency of the exponentials \( (\alpha_l) \) are known. In practice, they must be estimated on the fly.

As stated earlier, RBM manifests itself as a doubly-selective channel. Therefore, one would expect that the BEM could be used to model RBM. In order to represent RBM using the BEM the derivation in the previous section must be slightly altered.
Using the explicit form of $\Phi_{pk}[\alpha]$ given in (6.52) one can write
\[\tilde{s}_{pk} = \sum_{m=0}^{\infty} I_m e^{jk\omega_0(p-1)RT_s} \sum_{l=0}^{L} a_k[l] \Phi_k[(p - m)R - l]. \quad (6.66)\]

Letting
\[\Phi_k[\alpha] = \sum_{l=0}^{L} a_k[l] \Phi_k[\alpha - l]. \quad (6.67)\]

one obtains
\[\tilde{s}_{pk} = \sum_{m=0}^{\infty} I_m e^{jk\omega_0(p-1)RT_s} \tilde{\Phi}_k[(p - m)R]. \quad (6.68)\]

Using (6.68) and switching the notation from $k$ to $l$, one can write the correlator output in the presence of RBM as
\[\tilde{r}_p = \sum_{b=1}^{Q+1} I_{p-b} \sum_{l=0}^{L} \tilde{\Phi}_l[bR] e^{jl\omega_0(p-1)RT_s}. \quad (6.69)\]

Comparing (6.69) with (6.65), one can see that $\theta_{bl} = \tilde{\Phi}_l[bR]$ and $\alpha_l = l\omega_0RT_s = 2\pi lT_s$.

At this point, a few comments are in order. First, it would seem that the estimation of the channel in the case of RBM would be easier since all the the frequencies ($\alpha_l$) are integer multiples of each other. In other words, it is reasonable to assume that the frequencies are known since they are determined by the rotation rate of the blades. However, as shown earlier in this thesis, $L$ should be at least 35 to obtain a good representation of the RBM. This means there are a large number of coefficients to estimate. As stated earlier the BEM model assumes a small value of $L$. As a result, the BEM is not appropriate for modeling RBM. The main reason is the model in (6.69) is an over parameterized model. Furthermore, the exponentials change relatively slowly in the case of RBM and it makes it difficult to estimate the coefficients. As shown in the previous section, the RBM model can be simplified down to several
time varying coefficients. This is a better way to model the RBM since the time-varying coefficients can be estimated directly. In the next section, two approaches for equalizing the channel in the presence of RBM are presented.

6.4.3 Equalization

It was previously shown in this chapter that one can use the ALE or ADLE to improve the receiver’s performance at all blade locations. Thus it is expected that the ALE or ADLE can be used as the blades rotate at any speed. In order to obtain the equalizer coefficients for the ALE or ADLE, one must observe the channel as the blades rotate over one repetition period. It is important to note that one must have enough training symbols to span the blade repetition period. One can see that the number of required training symbols can be large depending on the repetition period of the blades. An alternative approach for equalizing the channel in the presence of RBM is to account for the time variations of the equalizer coefficients. This can be achieved by recursively updating the estimate of the equalizer coefficients [12, 15]. In order to do this, a recursive least-squares algorithm is employed. This algorithm is summarized below.

1. Use $N_t$ training samples to obtain an estimate of the channel coefficients $w_0$.

2. Initialize $P$ as a $M \times M$ identity matrix.

3. For the successive samples,

   (a) Use $w_0$ and (6.28) to estimate the transmitted symbol $\hat{I}_{p-1}$.

   (b) Use the most recent detected symbol and past detected symbols to obtain the newest estimate of $w_p$. 

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In the algorithm above, step 3b is done using the RLS algorithm, which is summarized below.

1. Define \( \hat{x}_p = [\tilde{r}_p \quad \tilde{r}_{p-1} \quad \cdots \quad \tilde{r}_{p-M}]^T \).

2. Compute \( \epsilon_p = \hat{I}_{p-1} - w_{p-1}^T x_p \).

3. Compute \( g_p = P[p-1]x_p^* (\lambda + x_p^T P[p-1]x_p^*)^{-1} \).

4. Compute \( P[n] = \lambda^{-1} (P[p-1] - g_p x_p^T P[p-1]) \).

5. Compute \( w_p = w_{p-1} + \epsilon_p g_p \).

This will be referred to as the Recursive Least-Squares Equalizer (RLSE). Note that when diversity is used \( w_p \) becomes a \( EM \times 1 \) vector, \( P \) becomes a \( EM \times EM \) matrix, and \( x_p \) is given by

\[
 x_p = [\tilde{r}_1^p \quad \cdots \quad \tilde{r}_{p-M}^1 \quad \cdots \quad \tilde{r}_p^E \quad \cdots \quad \tilde{r}_{p-M}^E]^T \tag{6.70}
\]

This will be referred to as the Diversity Recursive Least-Squares Equalizer (DRLSE).

In the RLS algorithm \( \lambda \) is the forgetting factor that determines how much past samples effect the current estimation of the coefficients. \( \lambda \) is chosen between 0 and 1. As \( \lambda \) increases past samples contribute more and more to the current estimate of the coefficients. In the simulations presented below, \( \lambda \) was chosen to be 0.95. In the following section, simulation results are presented for the ALE, ADLE, RLSE, and DRLSE.

6.4.4 Results

In order to evaluate the receiver performance in the presence of RBM, Monte Carlo Simulations were carried out. In the simulations the RBM was included with \( K = 35 \).
The blade repetition period \( T_b \) was 60 ms. Again, the pulse width was 1 \( \mu s \) and the sampling rate was 32 MHz. Additionally, \( R = 32, \) \( L = 50, \) and \( Q = 1. \) 15 trials were used to calculate the average PE. In each trial, the symbols were transmitted for a time duration corresponding to 1.5 blade rotation periods.

![Figure 6.25](image)

(a) \( \theta = 25^\circ. \)  
(b) \( \theta = 45^\circ. \)  
(c) \( \theta = 65^\circ. \)

Figure 6.25: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades rotating at a fixed speed. The symbols are detected from the output of the ALE and ADLE.

Fig. 6.25 shows the PE versus SNR along \( \theta = 25^\circ, 45^\circ, \) and \( 65^\circ. \) In the figures, the black curves show the PE when the symbols are detected from the correlator output in the presence of noise only. The blue curves show the PE in the presence of RBM when the symbols are detected from element 1’s correlator output. In this case, no diversity and no equalization is used and this is denoted by “NDNE” in the figure. Clearly, the RBM causes a large degradation in performance for all three look directions. This is due to the drop in SNR for certain blade locations as well as the rotation of the received signal in the IQ plane. The green curves show the PE when the ALE is used. In the simulation, 60,000 training symbols were used. It is clear that the performance improves, however, there is still room for improvement. The red
curve shows the PE when the ADLE is used. One can see that the PE performance is better than the noise only case for all three directions. Again, this can be explained by looking at the magnitude as well as the IQ diagram of the signal that is used to detect the transmitted symbols.

![Magnitude of correlator output in the presence of RBM. SNR = 8 dB.](image)

**Figure 6.26:** Magnitude of correlator output in the presence of RBM. SNR = 8 dB.

Fig. 6.26 shows the magnitude of the correlator output in the presence of noise and RBM along $\theta = 25^\circ$, $45^\circ$, and $65^\circ$ when the input SNR is 8 dB. In the presence of noise only, one can see that envelope of the correlator output is constant. However, in the presence of RBM there are some variations in envelope of the correlator output that are caused by the change in the rotor blade position. Fig. 6.27 shows the IQ diagram of the correlator output in the presence of noise and RBM along $\theta = 25^\circ$, $45^\circ$, and $65^\circ$ when the SNR is 8 dB. Again, the black dots indicate symbols that should be
Figure 6.27: IQ diagram of correlator output in the presence of RBM. SNR = 8 dB.

-1 and the red dots indicate symbols that should be 1. In the presence of noise only, one can see that there are some samples that will be detected in error since they cross over the imaginary axis. In the presence of RBM, one can see that if the correlator output is used to detect the transmitted symbols, almost all of the samples will be detected incorrectly due to the rotation of the samples in the IQ plane.

Fig. 6.28 shows the magnitude of the correlator output in the presence of noise and the magnitude of the ALE output in the presence of RBM along $\theta = 25^\circ$, $45^\circ$, and $65^\circ$ when the input SNR is 8 dB. One can see that there are still significant variations in the output of the ALE. Fig. 6.29 shows the IQ diagram of the correlator output in the presence of noise and the IQ diagram of the ALE output in the presence of RBM when the SNR is 8 dB. Clearly, the ALE is able to remove the rotation of the samples in the IQ plane that was observed in Fig. 6.27. This leads to the improved
performance that is observed in Fig. 6.25, however, the performance would improve if the variations in the magnitude of the signal were accounted for.

Fig. 6.30 shows the magnitude of the correlator output in the presence of noise and the magnitude of the ADLE output in the presence of RBM along $\theta = 25^\circ$, $45^\circ$, and $65^\circ$ when the input SNR is 8 dB. One can see that there is less variation in the envelope of the ADLE output as compared to the ALE. Fig. 6.31 shows the IQ diagram of the correlator output in the presence of noise and the IQ diagram of the ADLE output in the presence of RBM when the SNR is 8 dB. As expected, the ADLE is able to remove the rotation of the samples in the IQ plane that was observed in Fig. 6.27. Furthermore, one can see that the spread around the possible symbols of -1 and 1 has decreased. In fact, the spread is less than the noise only case. This is
the reason why the PE performance of the ADLE (see Fig. 6.25) is better than the PE performance in the presence of noise only.

Fig. 6.32 shows the PE versus SNR along $\theta = 25^\circ, 45^\circ$, and $65^\circ$. This figure is the same as Fig. 6.25, except the green curves shows the PE when the RLSE is used and the red curve shows the PE when the DRSLE is used. In the simulation, 100 training symbols were used to find the initial estimate of the equalization coefficients. As expected, when the RLSE is used, there is still room for improvement. When the DRLSE is used, the performance surpasses the noise only performance for SNR values above 0 dB.

Fig. 6.33 shows the magnitude of the correlator output in the presence of noise and the magnitude of the RLSE output in the presence of RBM along $\theta = 25^\circ, 45^\circ$, and

![IQ diagram of correlator output in the presence of noise and ALE output in the presence of RBM. SNR = 8 dB.](image)

Figure 6.29: IQ diagram of correlator output in the presence of noise and ALE output in the presence of RBM. SNR = 8 dB.
Figure 6.30: Magnitude of correlator output in the presence of noise and ADLE output in the presence of RBM. SNR = 8 dB.

65° when the input SNR is 8 dB. One observes that there are significant variations in the envelope of the RLSE output as compared to the correlator output in the presence of noise. Again, these variations are caused by the change in position of the rotor blades. Fig. 6.34 shows the IQ diagram of the correlator output in the presence of noise and the IQ diagram of the RLSE output in the presence of RBM when the SNR is 8 dB. As expected, the RLSE is able to remove the rotation of the samples in the IQ plane. However, there is still room for improvement in the performance since the RLSE cannot account for the fluctuations in the received signal power.

Fig. 6.35 shows the magnitude of the correlator output in the presence of noise and the magnitude of the DRLSE output in the presence of RBM along θ = 25°, 45°, and 65° when the input SNR is 8 dB. One observes that there is far less variation in
Figure 6.31: IQ diagram of correlator output in the presence of noise and ADLE output in the presence of RBM. SNR = 8 dB.

Figure 6.32: Probability of error vs. SNR for a BPSK receiver mounted on a rotorcraft with the blades rotating at a fixed speed. The symbols are detected from the output of the RLSE and DRLSE.

the envelope of the DRLSE output as compared to the RLSE output. The variations can be removed because diversity is used and the equalizer coefficients are constantly
Figure 6.33: Magnitude of correlator output in the presence of noise and RLSE output in the presence of RBM. SNR = 8 dB.

updated to track the changes in the channel. Fig. 6.36 shows the IQ diagram of the correlator output in the presence of noise and the IQ diagram of the DRLSE output in the presence of RBM when the SNR is 8 dB. The figure shows that the spread at the output of the DRLSE is less than that in the noise only case and at the output of the ADLE (see Fig. 6.31). From this figure, one would expect that the DRLSE would provide the best performance. However, if one compares the performance of the ADLE and DRLSE, it is clear that for lower SNR values, the ADLE is better. At lower SNR values, the noise causes errors in the estimates of the DRLSE coefficients, therefore, the performance degrades. However, when the SNR is above 0 dB, the ADLE and DRLSE provide similar performance.
It is important to note that the DRLSE approach is more reasonable for improving the performance in the presence of RBM. Although the ADLE is simple, the requirement of a large number of training symbols may not be acceptable for some systems since it is a waste of channel capacity. In the examples shown here, the blade repetition period was 60 ms, which means 60,000 training symbols were used. Furthermore, the averaging approach requires the inversion of a matrix that has a size of $N_t \times N_t$. This is a very expensive operation that is $O(N^2)$. The DRLSE requires much less computation cost as there is no inversion of a matrix in the algorithm. Furthermore, a smaller number of training symbols are needed for initialization. In these examples, 100 training symbols were used.
Figure 6.35: Magnitude of correlator output in the presence of noise and ADLE output in the presence of RBM. SNR = 8 dB.

6.5 Summary

In this chapter, the effects of RBM on a digital communication receiver were discussed. First, the case of fixed blades was considered to demonstrate how the platform degrades the receiver performance. The performance degradation was attributed to the drop in received signal power caused by the rotor blades blocking the line of sight signal path. Additionally, it was shown that the performance degrades because the platform causes a distortion in the phase of the received signal. Subsequently, the LE was introduced and it was shown to provide some improvement in PE performance by compensating for phase distortion in the received signal. However, the LE could not improve the SNR at its output and thus the PE performance was worse than the noise only case. Next, spatial diversity was introduced and it was shown that the
DLE could remove the phase distortion in the received signal and improve the SNR at its output. As expected, the PE performance was shown to be better than that of the noise only case when the DLE was used. Next, it was shown that one equalizer can be used over all blade locations by averaging the LE or DLE coefficients over one blade rotation period. This idea was introduced as a possible solution for a receiver operating in the presence of RBM.

In order to understand how RBM effects the received signal, the correlator output was derived using the FSA to model the RBM. It was shown that in the presence of RBM, the correlator output contains a linear combination of past symbols. However, in the case of RBM the coefficients are time varying. Monte Carlo simulations were used to demonstrate the degradation in performance that is incurred in the presence of RBM.
of RBM. The ALE was shown to improve the performance, however, as expected, the performance could still be improved since the ALE only removed the phase distortion in the received signal. As in the case of fixed blades, when diversity was introduced the performance exceeded the noise only performance. The RLSE was also introduced as another approach to improving the receiver performance. The RLSE attempts to track the variations in the equalizer coefficients. Again, the best performance was obtained when diversity was used. The DRLSE was shown to be the most practical approach to improving the receiver performance in the presence of RBM.
CHAPTER 7

CONCLUSIONS

7.1 Summary

For complete performance evaluation of RF systems, antenna and platform effects must be included in the performance evaluation. Performance evaluations can be carried out using computer or hardware-in-the-loop simulations. These options are beneficial because they can be significantly cheaper and easier to coordinate than field tests. This thesis considered how to include antenna and platform effects in simulations for RF systems onboard rotorcrafts. Additionally, the performance of a digital communication receiver onboard a rotorcraft was studied. A summary of each chapter is provided below.

Chapter 2 described how to include the antenna and platform effects in simulations when the antenna is mounted on a fixed platform. It was shown that the signal received by the antenna can be obtained by convolving the incident signal with the \textit{in situ} response of the antenna in the direction of the incident signal. It was also demonstrated that the filtering can be carried out in the time domain or frequency domain.
Chapter 3 described three approaches for including the effects of RBM in simulations. The first two methods, the TDA and FSA approach, provide an exact model of the RBM; however, they are not suitable for implementation in HITL testing. Subsequently, the Approximate Simulator was developed and it was shown that RBM can be included in simulations using $J$ filters and modulators. This approach is well suited for implementation in HITL simulation as it was shown that the RBM can be accurately modeled with $J$ as small as two or three.

Chapter 4 discussed the results from the measurement of a antenna mounted on a model rotorcraft. It was first demonstrated that the response of an antenna mounted on a rotorcraft is independent of the blade rotation speed. Next, the antenna response obtained using NEC-BSC was compared to the measured antenna response. Overall, the comparisons showed that the antenna response obtained using NEC-BSC is very similar to the measured antenna response. Finally, the accuracy of the Approximate Simulator was evaluated using the measured antenna data. Again, it was demonstrated that the approximate simulator can provide a very accurate model of the rotor blade modulation with $J$ as small as two or three.

Chapter 5 considered how to obtain the time domain antenna response for antennas mounted on dual rotor aircrafts. It was shown that the time domain antenna response depends on on the starting position and speed of each rotor. As a result, it is not practical to obtain the time domain antenna response for all combinations of starting locations and rotor speeds. Thus, the Composite Antenna Response was developed to approximate the time domain response of an antenna mounted on a dual rotor aircraft. The CAR significantly reduces the amount of measurements/calculations that are needed to obtain the time domain response of the antenna for different starting
positions and rotor speeds. Several different examples of antennas mounted on dual rotor platforms were used to demonstrate that accuracy of the CAR.

Chapter 6 discussed how the antenna and platform affect the detection capabilities of a digital communication receiver onboard a rotorcraft. First, it was demonstrated that when the rotor blades are fixed, the receiver’s performance can be severely degraded because the rotor blades can block the line of sight signal path and the platform can cause a distortion in the phase of the received signal. The LE was then proposed as a way to improve the receiver performance. However, the LE was only able to account for the phase distortion in the received signal and thus the performance could still be improved. Next, spatial diversity was introduced and it was shown that the DLE could remove the phase distortion in the received signal and improve the SNR at its output. As a result, the PE performance was shown to be better than that of the noise only case when the DLE was used.

The main drawback of the DLE is that the equalizer coefficients must be known for each blade angle in order to improve the performance of the receiver. In order to reduce this requirement, an averaging approach was introduced. It was shown that one can average the DLE equalizer coefficients over one blade rotation period to obtain one set of equalizer coefficients. This was referred to as the ADLE. It was demonstrated that the ADLE can be used to improve the receiver’s performance over all blade angles. Since the ADLE coefficients are independent of the blade location, it was expected that the ADLE would improve the receiver’s performance in the presence of RBM regardless of the blade speed. Although this was shown to be true, the ADLE was deemed to be impractical to implement since it requires a large number
of training symbols and the inversion of a matrix whose size is proportional to the number of training symbols.

Subsequently, a recursive approach was introduced. In this approach, the transmitted symbols are detected using a DLE, however, the coefficients of equalizer are updated using the RLS algorithm after each symbol is detected. This was referred to as the DRLSE. Since the equalizer coefficients are updated recursively, it was expected that the DRLSE could improve the receiver’s in the presence of RBM regardless of the blade speed. It was shown that the DRLSE provides very similar performance to the ADLE provided that the SNR is above 0 dB. Moreover, the DRLSE requires far fewer training symbols and there is no matrix inversion in the algorithm. Thus, it was concluded that the DRLSE is most reasonable approach to improve the receiver’s performance in the presence of RBM.

7.2 Future Work

Throughout this thesis, the time domain antenna response was obtained from numerical electromagnetic calculations that were carried out using NEC-BSC. These calculations are very time consuming since the code must be executed for each blade location. One possible area for future work would be to develop a numerical tool that is designed to efficiently calculate the response of an antenna on a rotorcraft. In doing so, the simulation time could be reduced and more complex models could be analyzed. Another possible area for future work would be to carry out measurements of an antenna mounted on a dual rotor rotorcraft in order to verify the accuracy of the Composite Antenna Response. The existing model rotorcraft that was used in the measurements can be modified to accommodate two rotor blades.
As stated earlier, the TDA, FSA, and Approximate Simulator are very general and thus they can be used to the study the performance of other RF systems on-board rotorcrafts. Additionally, the work on digital communication receivers can be extended by looking at different modulation schemes such as PAM, QAM or even non-linear modulation schemes. Another important topic to look at would be carrier synchronization and symbol timing recovery. In this thesis, it was assumed that these were known. In a real world receiver, it would be important to understand how RBM effects the algorithms for obtaining the carrier frequency and phase as well as the symbol timing as this is a fundamental part of the receiver.
BIBLIOGRAPHY


