EXCHANGE RATE TARGET ZONE ANALYSIS: A THEORETICAL AND EMPIRICAL APPROACH COMBINING IMPERFECT CREDIBILITY AND FOREIGN EXCHANGE RESERVE DEFICIENCIES

DISSERTATION

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the Degree Doctor of Philosophy in the
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By

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* * * * *

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ABSTRACT

In this dissertation, I identify and test a model of exchange rate target zones. This model modifies the standard model to incorporate issues related to the credibility and ability of monetary authorities to maintain a target zone regime. I am responding to recent difficulties in reconciling theoretical target zone models with the data. These modifications are an economical and effective response to the failure of the empirical literature to find strong evidence of the predicted S-curve effect on exchange rates. My solution modifies the model and improves the method of testing the model.

My results indicate that previous empirical studies of target zone models appear to place inadequate restrictions on the possible shape of the S-curve. In particular, I show that the so-called inverted S-curve (where exchange rates become more volatile than their underlying fundamentals) is theoretically implausible, as it implies a target zone regime must collapse if the monetary authority is ever asked to defend it.

In addition, this work identifies separates target zone arrangements by their ability to withstand speculative attack. I show a solution that identifies which target zone arrangements are likely to succumb to speculative attack and which are not. Thus, it may be possible to predict possible speculative attack, even under conditions where forward and/or options markets are not open and therefore those prices cannot be used to calculate implied market forecasts of probabilities of realignment.
My model also allows for policy analysis. I allow the researcher to adjust policy parameters so that they can make conclusions about how particular parameter settings affect the stability and durability of the regime being studied. This framework may be used to analyze many different fixed or nearly fixed exchange rate arrangements, including the European Monetary System (EMS), metallic standards like the gold standard (where the "gold points" are reinterpreted as target zone bands), pegged exchange rates (where only one monetary authority actively maintains the arrangement, such as is often practiced in LDCs and the newly industrialized countries of East Asia), and possible future transnational currency arrangements that may arise from the current wave of free-trade pacts.
To my parents, Walter and Jean Cornell, and my sister Kathleen.
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On a technical note, this document is composed entirely using the \LaTeX open-source scientific typesetting package, although it has been modified on several different types of computers (both PCs and Macintoshes) over the course of this project, and easily could have been converted to Linux or other Unix platforms if so desired. The flexibility of \LaTeX made this otherwise daunting feat fairly simple. Conversion to the Ohio State dissertation format was made simple by using \LaTeX style sheets developed by various members of the Department of Electrical Engineering at Ohio State. General diagrams of the S-curve in early chapters were rendered in Macromedia FreeHand (and later in Adobe FreeHand) in EPS format on a Macintosh, which converts nicely for use on a PC, Linux, or other platform. Diagrams in Chapter 7 were rendered

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in Microsoft Excel and pasted into the document at a later point. Statistical analysis of the model was done in the GAUSS econometrics package, while derivations of equations used were double-checked by machine using the Waterloo MAPLE package.

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CHAPTER 1

INTRODUCTION

Theoretical and empirical analysis of exchange rate target zones, such as the Exchange Rate Mechanism (ERM) of the European Monetary System (EMS), has exploded tremendously in the past ten years. Strangely, the recent burst of research is motivated less by concern over the viability of the EMS and more by the recent development of techniques that have rendered a deep analysis of target zones possible.

While the development of stochastic calculus techniques have been most useful for asset price research in general and with exchange rate research in particular, the specific innovation was a theoretically simple and intuitively appealing target zone model. This model, first developed by Paul Krugman in the late 1980s, has led to an explosion of research on the subject. Following the literature, I refer to the model in this dissertation as the “S-curve” model, so named due to the elongated S-shape of a relationship between the spot exchange rate and its underlying fundamentals implied by the model. The model describes a systematic market reaction to exchange rates under target zones that biases exchange rate movement in a predictable, systematic fashion.

Unfortunately, theory has not been accompanied by confirming empirical analysis. In fact the literature has been characterized throughout the 1990s by conflict between
theory and empirics. While the theory appears clean, consistent, and intuitive, empirical studies have not confirmed the reduced volatility of exchange rate in target zones that the S-curve theory predicts. The obvious agenda of researchers in the field has been one of attempting to reconcile theory and econometric evidence, either by experimenting with different variations of the theory or by developing alternative, superior techniques to those first employed.

My dissertation is mostly concerned with the job of moving theory closer to empirics rather than the alternative. Specifically, I construct a theoretical target zone model that allows for closed-form solutions of the S-curve where the exogenous policy parameters include such variables as the central bank’s foreign exchange reserve level, degree of credibility of the central bank, and the degrees of band shifting and widening.

My model specifically addresses the issue of whether an exchange rate target zone has a stabilizing or destabilizing effect on exchange rates. The original S-curve model, reviewed in Chapter 3, presents an argument that exchange rate target zones must, by their nature, stabilize exchange rates. The S-curve model predicts that foreign exchange arbitrage must be stabilizing, and depends on the action of arbitrageurs to drive the result. This result is sometimes called the “inverted S-curve”, describing this alternative relationship between the spot exchange rate and monetary fundamentals. Several researchers have countered that target zones may actually destabilize exchange rates. That is, the actions of arbitrageurs in foreign exchange markets may, under a proper set of circumstances, push exchange rates toward the edges of the bands, precipitate a series of crises, and thus destabilize exchange rates, thus causing
the opposite of the intended effect desired by monetary authorities. These arguments, along with a summary of the rest of the target zone literature, can be found in Chapter 2.

In my model, presented in Chapter 4, I demonstrate that the destabilizing effect described above only occurs with a particular set of choices about how one manages a target zone. In other words, I identify a set of reasonable monetary policies which guarantee that exchange rates in target zones exhibit lower volatility than freely floating exchange rates do. I go further in Chapter 5 to prove that if an inverted S-curve should arise, then the monetary authority finds it impossible to defend the target zone arrangement, thus pushing exchange rates into a de facto freely floating regime. Therefore, an inverted S-curve cannot depict a well-functioning target zone arrangement.

I also perform an econometric assessment of the model from Chapter 4. Chapter 6 outlines the econometric theory needed to do the simulated method of moments (SMM) technique used in the analysis. Chapter 7 provides both the results of that analysis and additional tests based on the simulated data generated by the test. I find reasonable point estimates for the parameters of interest, but very high standard errors. I derive a reasonable explanation why those standard errors should be high. In short, they are high because the S-curve effect, even if true, must be extremely small. Based on that idea, I explore the remaining implications of the S-curve theory, such as examining the number and timing of realignments, examining the distribution of deviations from central parity (which is an issue in the present literature), and an assessment of the size of S-curve effects.
In short, the dissertation is organized as follows. In Chapter 2, I briefly review the exchange rate band literature. In Chapter 3, I review the basic Krugman exchange rate target zone model. In Chapter 4, I generalize this target zone model to allow for imperfect credibility, along the lines of the generalization offered by Bertola and Caballero. In Chapter 5, I further generalize the target zone model to account for imperfect credibility and foreign exchange reserve deficiencies, along the lines of the generalization offered by Krugman and Rotemberg. In Chapter 6, I propose the manner in which this generalized model may be tested against actual data to verify or reject its predictive power. I employ this test in Chapter 7. I conclude in Chapter 8.
CHAPTER 2

A REVIEW OF THE LITERATURE

2.1 The Standard Model and Empirics

The exchange rate target-zone literature arose from Krugman’s [19] innovative study of the stabilizing role of target-zones in a simple monetary environment. Krugman’s original study focused on how the actions of rational, forward-looking agents can cause exchange rates to stabilize in response to exchange rate bands, although the monetary authority only intervenes at the margins. The original reason cited by monetary authorities for using target zones is the implied ability to engage in limited discretionary monetary policy when the exchange rate lies within the target zone. That idea is highlighted in Svensson’s early target zone literature review, as well as a later article on monetary policy and target zones. [34, 35]

Krugman [19] used the continuous-time monetary model in which uncovered interest parity (UIP) and purchasing-power parity (PPP) hold at each instant. He then applied stochastic calculus techniques to analyze exchange rates when rational agents know that perfectly credible exchange rate bands exist.\(^1\) He found that the existence of the bands dampened the volatility of the exchange rate in the interior of the band.

\(^1\)The exact derivation is given in Chapter 3
He called this phenomenon the “S-curve” after the elongated S shape in the mapping from the fundamental to the spot exchange rate as shown in Figure 2.1 below. Note that the slope of the S-curve is one at central parity, zero at the margins (due to the smooth pasting conditions), and between zero and one otherwise (also known as the “honeymoon effect”).

A co-implication of this theory is the so-called “U-curve”, first identified by Bertola and Caballero [3]. The U-curve describes the idea that the distribution of derivations of the spot exchange rate from central parity ought to exhibit a U-shaped distribution (what I term the “absolute U-curve”), or at least ought to exhibit more mass near the edge and less mass at central parity than a freely floating exchange rate (what I term the “relative U-curve”).

Empirical studies typically reject Krugman’s [19] model. The standard (and first) empirical study of the S-curve model was done by Flood, Rose, and Mathieson [16]. They identified distributional implications of the Krugman model and compare these
stylized facts with EMS data using the generalized method of moments (GMM) methodology. Among the predictions of the Krugman model, as identified by Flood, Rose, and Mathieson, are the nonlinear relationship (i.e., the S-curve) between the natural logarithm of the monetary fundamentals and the natural logarithm of the spot exchange rate (including a divergence on the proper estimation of the fundamental as Krugman defines it), and the behavior of interest-rate differentials and the U-shaped distribution of deviations of the log spot exchange rate from central parity (the absolute U-curve mentioned above). These predictions emerge given Krugman’s simplifying but unrealistic assumptions of continuously-holding UIP and PPP. The authors find no empirical support for these implications of the Krugman model. Svensson’s [34] literature review provides further insight into the early responses to Krugman’s initial target zone model.

2.2 Alternative Theoretical Models

In response to criticism of the Krugman [19] model, Bertola and Caballero [2, 3] developed an extension of the Krugman model using an imperfectly credible exchange rate target zone, upon which I base this work. Bertola and Caballero modified Krugman’s model to allow for the possibility that the monetary authority does not always maintain the announced target-zone. Instead, when the exchange rate reaches the edge of the band, the monetary authority either defends the band (as in Krugman’s model) with some fixed, exogenous probability $p$ or realigns the entire band up or down by the amount of the original bandwidth, recentering the band around the fundamental with probability $(1-p)$. If the probability of realignment is more than 50%, an "inverted S-curve" relationship arises in equilibrium. An example of an inverted
Figure 2.2: The Standard and Inverted S-curves

S-curve can be seen in Figure 2.2 below. Exchange rate bands are then destabilizing, producing an exchange rate that is more volatile than the underlying fundamental, and the distribution of the log exchange rate is hump-shaped. Bertola and Caballero conclude that their inverted S-curve is more consistent with empirical evidence.

Flood, Rose, and Mathieson [16] also commented on the Bertola-Caballero [2, 3] model, then at the working paper stage. They note that there is little empirical support for the assumptions or the implications of the Bertola-Caballero model (although they note that it matches the empirical evidence more closely than Krugman's [19] model). In their conclusion, Flood, Rose, and Mathieson express pessimism on the current state of S-curve models of banded exchange rates and conclude that this class of models may be too restrictive to match the empirical phenomena. Most notably, they argue that S-curve models incorporate only a single state variable and therefore ignore sticky prices and certain types of devaluation risk.
In response to the first point of criticism, a growing body of sticky-price target zone literature has been developed. It mainly arises out of the Miller and Weller studies, mostly published in the *Economic Journal*. The model is explicitly derived in Miller and Weller [26], which uses a Dornbusch [10] sticky-price model of exchange rate determination as the baseline model rather than the monetary model used by Krugman [19]. In such a model, a target zone not only provides decreased exchange rate volatility relative to a measure of fundamentals but also eliminates the possibility of the exchange rate moving along its possible divergent trajectory. One may recall from Dornbusch [10] that an exchange rate may take either a convergent path or a divergent path. He assumes that an exchange rate will attain its convergent path, but leaves open the theoretical possibility of the divergent path. Because I do not use the techniques or results of the sticky-price target zone literature in this work, I shall end my discussion of it at this point.

An alternative model to Bertola and Caballero [2, 3] that offers the possibility of “devaluation risk” was proposed by Bertola and Svensson [4]. This model has proven to be the workhorse of a fruitful and solid body of both theoretical and empirical analysis of target zones. Its primary contribution was to offer the theoretical possibility of intramarginal interventions, assumed to occur with some fixed probability. It was followed up with several empirical studies (discussed in the appropriate subsection below) and theoretical variants. The major empirical studies of this model include Rose and Svensson [30] (which estimated EMS realignment expectations and failed to detect a drop in credibility before the September 1992 crisis) and Rose and Svensson [31] (which tested the Bertola-Svensson model against DEM/FRF data).
Working along similar lines, Lindberg and Söderlind [23] focused on Swedish exchange rates rather than the EMS. In the Swedish case, the *Sveriges Riksbank* (Central Bank of Sweden) left the EMS in the summer of 1977 and set up an alternative system to the EMS. It chose to unilaterally peg the krona to a basket of currencies and explicitly set bands around the central parity, where it pledged to maintain the value of the krona. However, most of the intervention took place intramarginally. To model this behavior, Lindberg and Söderlind [23, 24] constructed a variant of the Bertola-Svensson [4] model where the fundamental follows an Ornstein-Uhlenbeck process, which may be thought of as a continuous-time analog to an AR(1) process (in much the same way a Wiener process is a continuous-time analog to a random walk). Empirical analyses of this model, conducted by Lindberg, Söderlind, and Svensson [25] and by Lindberg and Söderlind [23, 24] produced some positive results for the Swedish case. Most notably, the papers concluded that the mean-reverting fundamental provided a mean-reverting exchange rate, and the mean-reverting exchange rate offered a better fit with the target zone data than an exchange rate following a random walk.

Tristani [36] offers an intriguing critique and theoretical alternative to the Bertola-Svensson [4] framework. Tristani argues that Bertola and Svensson’s assumption of a fixed probability of intramarginal realignment (that is, realignments that take place while the exchange rate still lies between the target zone bands) implied a degree of irrationality on the part of traders, and that a better-constructed alternative would include a variable probability of realignment that was positively related to the degree to which the exchange rate deviates from its central parity. Such assumptions

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2The interested reader will find a solid treatment of stochastic calculus techniques in Dixit [7]. Further background technique, including an introduction to Wiener and Ornstein-Uhlenbeck processes, may be found in Chapter 3 of Dixit and Pindyck [8].
lead to a “steeper S-curve” model where the “honeymoon effect” vanishes while the U-shaped distribution of exchange rates is retained. Tristani offers the possibilities of a fundamental following a Wiener process (random walk) or an Ornstein-Uhlenbeck process (mean-reverting), and (like Lindberg and Söderlind) concludes that the exchange rate is (is not) mean-reverting if its underlying fundamentals process is (is not, respectively) also mean-reverting.

There is a developing literature on speculative attacks on fixed exchange rates and target zones which strives to integrate the possibility of foreign exchange reserve deficiencies into S-curve models. Mathematical models of speculative attack on fixed exchange rate regimes date back at least as far as Flood and Garber’s [14] now-classic treatment using a standard monetary model of exchange rate determination. These same authors published a number of papers outlining the mathematical techniques for demonstrating the potential for incorporating speculative attack into a monetary model, first under movement from a free float to a fixed exchange rate in Flood and Garber [13], then with target zones in Flood and Garber [15].

However, it was Krugman and Rotemberg [20] who identified that the key variable for speculative attack is the level of foreign exchange reserves in the central bank of the country whose currency is under attack. The solution offered by Krugman and Rotemberg in the perfect credibility case is a “critical mass” solution. If the central bank has sufficient reserves, speculative attack may be prevented, but if they have fewer reserves, collapse of the target zone into a freely floating exchange rate is inevitable. In this light, Krugman and Rotemberg were able to re-interpret the smooth-pasting conditions as guarantees of sufficient reserves, since a lack of sufficient reserves coincided exactly with the inapplicability of the smooth pasting conditions.
2.3 Alternative Empirical Studies

As discussed earlier, the Flood, Rose, and Mathieson [16] paper set the stage for the exchange rate target zone literature in the 1990s. This literature has been characterized by conflict between theory and empirics. This paper showed that the canonical theoretical target zone model was in conflict with the canonical empirical analysis of it, and one or the other would have to change if the current generation of target zone models had any hope of predicting exchange rate behavior under target zones. The above subsection provides several ways the theory has been altered to satisfy the empirics. This subsection reviews the ways in which the empirical analysis has been brought into question.

Smith and Spencer [32] generalize Flood, Rose, and Mathieson’s [16] test into an environment where the theoretical moments of a desired target zone model cannot be readily generated. In this environment, the authors outline a procedure for implementing a simulated method of moments (SMM) technique for empirical assessment of target zone models, based heavily on Hansen’s [18] original work on the generalized method of moments (GMM) technique. This SMM technique was later readily adapted in the literature.

Bodnar and Leahy [5] test Krugman’s [19] initial target zone model using the generalized method of moments (GMM) technique. This is remarkable, as the exact theoretical moments are remarkably difficult to identify in target zone models. However, they fail to find significant evidence of target zone effects on exchange rates, and therefore their study is seen as inadequate at the current levels of sophistication.
DeJong [6] uses both SMM and maximum likelihood (ML) estimation of the Krugman [19] model using EMS data from 1987 to 1990. His ML estimators detect significant nonlinearities in three of the six exchange rates studies, while his SMM estimators fail to detect any nonlinearities. These results are clarified by autoregressive conditional heteroskedastic (ARCH) and higher-moment tests which detect misspecification error, significant ARCH effects, and leptokurtosis in the data for all exchange rates except one. In other words, there is some support for S-curve effects under ML tests, but no such support under SMM tests.

Bekaert and Gray [1] propose an atheoretical approach to S-curve testing. To approach the possibility of S-curve phenomena without testing the Krugman model, they use a purely econometric model to investigate the stability of target zones. They conduct ML estimation of this model, which accounts for both marginal and intramarginal jumps in the exchange rate. These jumps are designed to account for realignments as well as foreign exchange reserve crises in the weak country of an exchange rate band system. By comparing the results of such estimation on the banded DEM/FRF exchange rate with the unbanded DEM/USD exchange rate, Bekaert and Gray find both significant nonlinearities in the banded DEM/FRF exchange rate and support for effects on the time series behavior of exchange rates similar to the predictions of the S-curve. The results of the Bekaert and Gray study hints that S-curve effects may indeed exist in the data, but current theoretical models inadequately capture their true nature. An implication is that a new class of theoretical S-curve models is desired, but models that either approach the problem from a new theoretical perspective or attack several assumptions in Krugman’s [19] model at once rather than individually.
A strand of the empirical target zone literature has focused on extracting expected probabilities of realignment and therefore credibility from exchange rate data. Notable papers in this line include Rose and Svensson [30, 31], both of which were discussed earlier. An innovative approach to this problem was recently discussed in Mizrahi [28], which manipulates option prices through the standard Black-Scholes model. While attempting to resolve the empirical irregularity known as the “volatility smile,” Mizrahi was able to manipulate option price data to extract market-based expectations of realignment probabilities. His option-based prediction method enjoys a higher degree of success than the S-curve-based analysis of Rose and Svensson [31] for DEM/FRF data or Rose and Svensson [30] for EMS currencies before the September 1992 crisis.
CHAPTER 3

FULLY CREDIBLE TARGET ZONES

This chapter is a technical review of the standard S-curve, first described by Krugman [19]. It does not consist of original material, but is given so as to provide background and insight on the class of exchange rate target zone models in a manner not possible in a standard review of the literature.

3.1 A Simple Model of Exchange Rates

Consider the canonical first-order differential equation describing the evolution of the log spot exchange rate.

\[ s_t = m_t + v_t + \alpha \frac{E_t[ds_t]}{dt} \]  

(3.1)

where \( s_t \) is the natural logarithm of the spot exchange rate, \( m_t = \ln \left( \frac{M_t^H}{M_t^F} \right) \) describes the natural logarithm of the ratio of the domestic \( (M_t^H) \) to foreign \( (M_t^F) \) money supplies, \( v_t \) is a money demand shock term or “velocity” shock term, \( E_t \) is the mathematical expectation operator conditioned on the set of information available at some date \( t \), and \( \alpha \) is a positive constant corresponding to the interest semielasticity of money demand in the context of a monetary model of exchange rate determination. This equation may be derived from the standard monetary model of exchange rates in the special case where one assumes that the fundamentals follow a random walk.
Note that \( m_t + v_t \) is usually referred to collectively as the “fundamentals” term, since it describes the monetary fundamentals that determine the value of the spot exchange rate in a monetary model of exchange rate determination. If one assumes that the monetary fundamentals follow in discrete time a random walk, or in continuous time a Brownian motion, then the natural logarithm of the spot exchange rate at any given rate is equal to the natural logarithm of the monetary fundamentals. If one adds a drift to the fundamentals, then the drift term is simply added onto the solution for the log spot exchange rate.

The primary advantage of separating the monetary fundamentals into two separate terms is that we can consider what happens to the two separate components. In particular, we can consider one component, \( v_t \), to be exogenously determined by a Brownian motion, thus representing the idea of a money demand shock, seen here as a shock to velocity. On the other hand, we can consider the second component, \( m_t \), to be endogenously determined within the model, constant except for an occasional monetary intervention made to support a target zone regime. In that spirit, and employing a continuous-time framework, allow \( v_t \) to follow a Brownian motion described by

\[
dv_t = \theta dt + \sigma dW_t
\]  

where the standard deviation of \( dW_t \) is equal to \( \sqrt{dt} \), \( \theta \) is a constant, and \( \sigma \) is a positive constant. Equations (3.1) and (3.2) may be combined to solve for the current value of the spot exchange rate, which gives us

\[
s_t = \frac{1}{\alpha} \int_t^\infty E_t[m_{\tau} + v_{\tau}] \exp \left[ \frac{(\tau - t)}{\alpha} \right] d\tau
\]  

(3.3)
Combining equations (3.2) and (3.3), we find the following convenient closed-form solution for the log spot exchange rate:

\[ s_t = m_t + v_t + \alpha \theta \]  

(3.4)

since \( E_t[(m_\tau + v_\tau)|(m_t + v_t)] = m_t + v_t + \theta(\tau - t) \) for all \( t \) and for all \( \tau \geq t \).

### 3.2 A Simple Model of Target Zones

Now consider a fully credible target zone with unlimited available reserves so that the exchange rate is to be kept within the band \([\underline{s}, \bar{s}]\). The theory should imply the existence of a one-to-one function, described in an abstract fashion as \( s_t = g(v_t; m_t, \underline{s}, \bar{s}) \).

The one-to-one relationship between \( v_t \) and the exchange rate implies the target zone can be rewritten as a target zone on the fundamentals described by \([\underline{v}, \bar{v}]\). Therefore, what will be shown to be the S-curve equation may be rewritten in an abstract fashion as \( s_t = g(v_t; m_t, \underline{v}, \bar{v}) \).

In this context, the target zone is perfectly credible. If the exchange rate reaches a margin of the target zone, the monetary authorities "tap" the fundamental back so that the exchange rate remains in the center of the band. In mathematical language, the target zone acts as a classical reflecting barrier. Please note that in imperfect credibility models of target zones, this is not the case, and the process of defense or realignment follows a different mathematical pattern. This idea is further discussed in Chapter 4.

In this model, we note that \( \frac{E[d\ln s_t]}{dt} \) is typically nonzero. We appeal to Itô’s Lemma for its exact value. In this case, Itô’s Lemma argues that the following equation must hold

\[ E_t ds_t = \left( \frac{E_t \partial g}{\partial t} \right)(dt) + \left( \frac{E_t \partial g}{\partial v_t} \right)(dv_t) + \frac{1}{2} \left( \frac{E_t \partial^2 g}{\partial v_t^2} \right)(dv_t^2) \]
\[ = 0dt + \left( \frac{\partial g}{\partial v} \right)(\theta dt + \sigma du_t) + \frac{1}{2} \alpha \left( \frac{\partial^2 g}{\partial v^2} \right)(\theta^2 dt^2 + \sigma^2 dt) \]

Solving for \( \frac{E_t[ds_t]}{dt} \), we find that the equation above simplifies to

\[ \frac{E_t[ds_t]}{dt} = \left( \frac{\partial g}{\partial v} \right)(\theta + \sigma \sqrt{dt}) + \frac{1}{2} \alpha \left( \frac{\partial^2 g}{\partial v^2} \right)(\theta^2 dt + \sigma^2) \]

Of course, the two terms on the right-hand side that are functions of \( dt \) vanish as \( dt \) converges to zero, and so are dropped. As a result, the final form of \( \frac{E_t[ds_t]}{dt} \) is

\[ \frac{E_t[ds_t]}{dt} = \theta \left( \frac{\partial g}{\partial v} \right) + \frac{1}{2} \alpha \sigma^2 \left( \frac{\partial^2 g}{\partial v^2} \right) \quad (3.5) \]

Equation (3.5) is then substituted into equation (3.1) to form a second-order stochastic differential equation once one notes that \( s_t = g(v_t; m_t, \nu, \nu) \). For convenience, we use the notation \( g' \equiv \frac{\partial g}{\partial v} \) and \( g'' \equiv \frac{\partial^2 g}{\partial v^2} \).

\[ g = m_t + \nu_t + \alpha \theta + \alpha \left[ \theta g' + \frac{1}{2} \sigma^2 g'' \right] \]

Equation (3.5) then simplifies to

\[ \frac{1}{2} \alpha \sigma^2 g'' + \alpha \theta g' - g = -m_t - \nu_t - \alpha \theta \quad (3.6) \]

The characteristic equation for equation (3.6) is \( \frac{1}{2} \alpha \sigma^2 \lambda^2 + \alpha \theta \lambda - 1 = 0 \). Solving for \( \lambda \), we find that

\[ \lambda = \frac{-\theta}{\sigma^2} \pm \frac{\sqrt{\alpha^2 \theta^2 + 2 \alpha \sigma^2}}{\alpha \sigma^2} \quad (3.7) \]

Given \( \alpha > 0 \) and \( \theta, \sigma \in \mathbb{R} \), both values of \( \lambda \) are guaranteed to exist in real space. For convenience, call the value of \( \lambda \) where the right hand term of equation (3.7) is positive \( \lambda_1 \) and the other \( \lambda \) where the right hand term of (3.7) is negative \( \lambda_2 \). It may be readily proven that first, \( \lambda_1 \) and \( \lambda_2 \) must differ in sign and second, \( |\lambda_1| \gtrless |\lambda_2| \) according as \( \theta \leq 0 \).
Now we wish to find the solution to equation (3.6). The only remaining step is to impose the smooth-pasting conditions. The interested reader may refer to Dixit’s [7, 8] and Dixit and Pindyck’s [7, 8] detailed discussions of smooth-pasting conditions. In this context, the smooth-pasting condition is the condition that the solution function \( g(\bullet) \) must have a slope of zero at the point of tangency with the edges of the target zone band. That point must also reflect the maximum tolerable drift of the velocity shock from its central parity, thus implying that \( g'(\bar{y}) = g'(\bar{v}) = 0 \). Given our assumption of a perfectly credible defense of the target zone, the smooth-pasting conditions are given in Krugman [19] and are equal to

\[
g'(\bar{y}) = 1 + \lambda_1 \tilde{A} \exp(\lambda_1 \bar{y}) + \lambda_2 \tilde{B} \exp(\lambda_2 \bar{y}) = 0
\]

\[
g'(\bar{v}) = 1 + \lambda_1 \tilde{A} \exp(\lambda_1 \bar{v}) + \lambda_2 \tilde{B} \exp(\lambda_2 \bar{v}) = 0
\]

The solution to this system of two equations is

\[
s_t = m_t + v_t + \alpha \theta + \tilde{A} \exp(\lambda_1 v_t) + \tilde{B} \exp(\lambda_2 v_t) \tag{3.8}
\]

where

\[
\tilde{A} = \frac{\exp(\lambda_2 \bar{y}) - \exp(\lambda_2 \bar{v})}{\lambda_1 [\exp(\lambda_1 \bar{y} + \lambda_2 \bar{v}) - \exp(\lambda_1 \bar{y} + \lambda_2 \bar{v})]} < 0
\]

and

\[
\tilde{B} = \frac{\exp(\lambda_1 \bar{v}) - \exp(\lambda_1 \bar{y})}{\lambda_1 [\exp(\lambda_1 \bar{y} + \lambda_2 \bar{v}) - \exp(\lambda_1 \bar{y} + \lambda_2 \bar{v})]} > 0
\]

For convenience, let \( A \equiv \exp(-\lambda_1 \bar{v}) \tilde{A} < 0 \) and \( B \equiv \exp(-\lambda_2 \bar{v}) \tilde{B} > 0 \). Then equation (3.8) simplifies to Krugman’s [19] S-curve, which is

\[
s_t = m_t + v_t + \alpha \theta + A \exp[\lambda_1 (v_t - \bar{y})] + B \exp[\lambda_2 (v_t - \bar{v})] \tag{3.9}
\]
CHAPTER 4

IMPERFECTLY CREDIBLE TARGET ZONES

Now consider the model of Chapter 3 in a setting with target zones that are not always defended by the monetary authorities that create them. The idea that monetary authorities do not always defend their target zones implies that such target zones do not enjoy the sort of credibility that they would otherwise, and therefore arbitrageurs behave differently than they would had they had considered the target zones fully credible. For that reason, models such as the present one are referred to as “imperfectly credible” target zone models. Theoretical results of such models are often not as sharp or distinct as they are in the case of full or perfect credibility. Further, as cited in Chapter 2, there exists the theoretical possibility that imperfect credibility might actually make exchange rates less stable than they would be in a free float. I will disprove that concept in Chapters 4 and 5.

Following the work of Bertola and Caballero [2, 3], I allow the monetary authority to make only marginal interventions; that is, to intervene only when the exchange rate is at one of the edges of the band, and not at any point inside the band. Although unrealistic on the surface, this modeling strategy permits the researcher to consider a constant probability of realignment rather than a series of probabilities,
which one presumes increases the further the exchange rate is from its central parity. The primary difference between the present model and the work of Bertola and Caballero [2, 3] is that I add parameters describing the realignment policy of the monetary authorities. One such variable, labeled “a”, describes the amount by which the monetary authorities shift the target zone upon realignment. One presumes that the more the new band is shifted away from the old band, the more unstable the regime becomes. I prove that below. The other such variable, labeled “b”, describes the amount by which the monetary authorities widen the target zone upon realignment. One also presumes that the wider the target zone, the more sustainable it is and the less stabilizing it is. I prove that below as well.

In short, this model permits estimation and policy analysis of a simple imperfect credibility model of exchange rate target zones. This model also permits an econometric assessment of target zones, which appear below as Chapters 6 and 7.

4.1 Deriving the Model

As noted above, I use the symbol \( s_t \) to denote the natural logarithm of the spot exchange rate at some point in time \( t \) and \([s, \bar{s}]\) as the lower and upper edges of the band, respectively. Because exchange rate bands are imposed symmetrically, I can alternatively refer to central parity and the band’s width rather than the two edges of the band. However, I will want to focus my attention on the fundamentals, with the knowledge provided above that there must exist a one-to-one relationship between the monetary fundamentals and the spot exchange rate. Use the symbol \( c_t \) to denote central parity (measured in fundamentals) at some time \( t \) and \( v^* \) as the bandwidth of the target zone (again measured in fundamentals space), so that the edges of the band
may be restated in terms of fundamentals as \([c_t - v^*, c_t + v^*]\). Naturally, if the S-curve effect described above is true, the effective bandwidth placed on the fundamentals by a target zone should be wider than the bandwidth on the exchange rate. In other words, if exchange rates are permitted to deviate by plus or minus 3% of central parity in a target zone regime, then \(v^*\) should be greater than \(\ln 1.03\).

Let the only possible interventions occur at the band’s edges, as stated above. Such interventions are called “marginal” interventions for that reason, not because the interventions are said to be marginal in size. I retain use of the word “marginal” for the purpose of retaining consistency with the rest of the literature. An alternative assumption, which I do not pursue, is that of intramarginal intervention, so that interventions may happen within the bands as well as at the band’s edges.

Following the idea of imperfect credibility, I must now address how one handles the consequences of whether monetary authorities choose to defend a particular target zone or not. Following the literature, I describe the choice to defend a target zone as “defense”. The act of defense as a discrete, instantaneous monetary intervention of sufficient size to immediately move the exchange rate back to its central parity value. The alternative case, where the monetary authority chooses not to defend the target zone, is described by me and in the literature as “realignment”. The act of realignment is simultaneously a monetary intervention of sufficient size to immediately move the exchange rate to its new central parity value, and a redefinition of the edges of the target zone. The choice of whether to defend or realign is modeled as probabilistic and distributed binomially, where I assign the probability \(p\), \(0 \leq p \leq 1\), to the act of defense and the probability \(1 - p\) to the act of realignment.
To finish the description of realignment, return to the description of the bands in what one might term "fundamentals space". Let the parameter $a$, a nonnegative real number, represent the degree to which the band shifts upon realignment, so that a shift to a new band where the far edge of the old target zone becomes the central parity of the new target zone (as the EMS aspired to do but sometimes failed to do) is reflected by $a = v^*$ and realignment to an unchanged band is reflected by $a = 0$. Further, let the parameter $b$, a nonnegative real number, represent the degree to which the band widens upon realignment, as it did after the September 1992 crisis in the EMS for the remaining countries in the arrangement. Let $b = 0$ reflect the idea that the band does not widen upon realignment, and $b > 0$ reflect the opposite. Given this, one may consider a devaluation of the home currency as a realignment from the band $[c_t - v^*, c_t + v^*]$ to the band $[c_t - v^* + a - b, c_t + v^* + a + b]$, while revaluation of the home currency would be realignment to the band $[c_t - v^* - a - b, c_t + v^* - a + b]$.

Figure 4.1 below shows a typical realignment, defined in fundamental space, under such a policy. Note that the figure allows for both a shift in the location of the new band and a widening of the band. The upper thick line defines the limits of the old band, while the lower thick line defines the limits of the new band.
Although I do not model it, I know that monetary authorities also face costs of maintaining a given target zone, primarily consisting of spent foreign exchange reserves when the exchange rate depreciates to the point of reaching the upper edge of the target zone. If the monetary authority chooses to defend the target zone, it spends some amount of reserves determined by the policy variables, primarily those that determine the size of the target zone and the probability of realignment. I assume that the home monetary authority has entered into a cooperative agreement with its foreign counterpart to maintain a target zone, so that the home monetary authority is also committed to purchase of foreign exchange when the home currency appreciates to the lower edge of the target zone. (Of course, this assumption allows me to abstract from the question of which central bank actually intervenes. This issue is further explored in Chapter 5 of this dissertation.) For that reason, when one considers normative aspects of this research, one should weigh these costs with the benefits, however defined, of imposing a target zone arrangement on a particular exchange rate.

In terms of notation, let the variable $m_t$ denote the natural logarithm of the ratio of the money stock held by the home monetary authority to the money stock held by the foreign monetary authority. In light of the cooperative arrangement assumption above, one may either assume that this money stock ratio $m_t$ is either jointly controlled by the two monetary authorities, or that one of the two monetary authorities controls it while the other promises not to adjust monetary policy for purposes of exchange rate stability.
To make realignment meaningful, I impose the trivial condition that the realignment cannot be to the original band, implying \( a + b > 0 \). As in Bertola and Caballero [2, 3], the fundamental is still recentered after realignment. The two possible interventions are realignment and defense, with constant and exogenously given probabilities as described earlier. Defense is the same as before. I then define the monetary authority’s policy by the set \( \{a, b, p\} \), where \( a, b, \) and \( p \) are chosen at the discretion of the monetary authority and assumed to be known by market participants. I now evaluate of the shape of the S-curve by finding a closed-form solution for \( A \) and \( B \) as a function of \( a, b, \) and \( p \).

Bertola and Caballero [2, 3] also identify a method for deriving the solution for the S-curve under these assumptions. Since interventions are only at the margin, one may nail down the solution by considering, in place of the smooth-pasting condition, a pair of zero expected arbitrage profit conditions (also known as no unexpected jump conditions or zero profit conditions). For this model, consider first the zero profit condition for the situation where the exchange hits the upper edge of the target zone, then the zero profit condition for the situation where the exchange rate hits the lower edge of the target zone.

The zero profit condition is described in general by:

\[
s(t_0) = ps(t_0 + dt|R) + (1 - p)s(t_0 + dt{|D})
\]  

(4.1)

where \( R \) describes the state of the world where the monetary authority chooses to realign the target zone and \( D \) describes the opposite state of the world where the monetary authority chooses to defend the target zone.

The zero profit condition, or more correctly the zero expected arbitrage log profit condition, is founded on a variant of the arbitrage pricing theory of assets from the
finance literature. The constraint is an efficiency condition: the foreign exchange market is considered to be efficient if and only if an arbitrageur cannot expect to achieve arbitrage profits by holding one currency long and the other short. This condition has a linear form in logarithms because it should hold in percentage form in levels. That is, this condition is an assumption of the “normal market return” on foreign exchange market holdings, corresponding to an interest rate concept, not unlike the interest arbitrage conditions of international finance.

To solve this model, proceed through the exercise of considering the moment the band edge is hit, the moment after given realignment, and the moment after given defense. Put together the solutions in these three cases into a zero profit condition. Do this procedure first for hitting the upper edge of the band, then for the lower edge. The two zero profit conditions will have two primary variables of interest we wish to solve for (A and B). Under general conditions, closed-form solutions are attainable for A and B, respectively.

First, calculate the exchange rate at some moment $t_0$ when the exchange rate hits its upper band and immediately before the monetary authority faces its choice. That exchange rate must be

$$s(t_0) = m_t + c_t + \nu^* + \alpha \theta + A + B \exp(2\lambda_2 \nu^*)$$

Second, calculate the exchange rate at the moment $t_0 + dt$ in the state of the world where the monetary authority realigns to a new target zone. That exchange rate must be

$$s(t_0 + dt|R) = m_t + c_t + a + \alpha \theta + A \exp[-\lambda_1 (\nu^* + b)] + B \exp[\lambda_2 (\nu^* + b)]$$
Third, calculate the exchange rate at the moment $t_0 + dt$ in the state of the world where the monetary authority defends the current target zone. That exchange rate must be

$$s(t_0 + dt|\textbf{D}) = (m_t - v^*) + (c_t + v^*) + \alpha \theta + A \exp(-\lambda_1 v^*) + B \exp(\lambda_2 v^*)$$

Combining those into the first zero-profit condition, I find

$$[v^* - pa] = A[p \exp[-\lambda_1(v^* + b)] + (1 - p) \exp(-\lambda_1 v^*) - 1]$$

$$+ B[p \exp[\lambda_2(v^* + b)] + (1 - p) \exp(\lambda_2 v^*) - \exp(2\lambda_2 v^*)] \quad (4.2)$$

Repeating the procedure for the lower edge of the band, I find that at $t_0$,

$$s(t_0) = m_0 + c_t - v^* + \alpha \theta + A \exp(2\lambda_1 v^*) + B$$

If the monetary authority realigns, I know

$$s(t_0 + dt|\textbf{R}) = m_0 + c_t - a + \alpha \theta + A \exp[-\lambda_1(v^* + b)] + B \exp[\lambda_2(v^* + b)]$$

And if the monetary authority defends (assuming that they have agreed to cooperate with the foreign monetary authority), I know

$$s(t_0 + dt|\textbf{D}) = (m_0 + v^*) + (c_t - v^*) + \alpha \theta + A \exp(-\lambda_1 v^*) + B \exp(\lambda_2 v^*)$$

Combining those into the second zero-profit condition, I find

$$-[v^* - pa] = A[p \exp[-\lambda_1(v^* + b)] + (1 - p) \exp(-\lambda_1 v^*) - \exp(-2\lambda_1 v^*)]$$

$$+ B[p \exp[\lambda_2(v^* + b)] + (1 - p) \exp(\lambda_2 v^*) - 1] \quad (4.3)$$
Equations (4.2) and (4.3) can be combined in order to derive closed-form solutions for $A$ and $B$. Those solutions are

$$A = \frac{[1 + 2pk_2(1 - k_4)][v^* - pa]}{k_1[1 - p(1 - k_3)] + k_2[1 - p(1 - k_4)] - 2k_1k_2[pk_3 - 1] + (pk_4 - 1) - 1}$$

(4.4)

$$B = \frac{-[1 + 2pk_1(1 - k_3)][v^* - pa]}{k_1[1 - p(1 - k_3)] + k_2[1 - p(1 - k_4)] - 2k_1k_2[pk_3 - 1] + (pk_4 - 1) - 1}$$

(4.5)

where $k_1 = \exp(-\lambda_1 v^*)$, $k_2 = \exp(\lambda_2 v^*)$, $k_3 = \exp(-\lambda_1 b)$, $k_4 = \exp(\lambda_2 b)$, $0 < k_1, k_2 < 1$, and $0 < k_3, k_4 \leq 1$. The implications of this conclusion may be best seen as a series of propositions.

4.2 Propositions

**Proposition 1** If there is no trend drift in the exchange rate (i.e., $\theta = 0$), then the closed-form solutions for $A$ and $B$ are

$$A = -B = \frac{v^* - pa}{\exp(-2\lambda v^*) - 1}$$

The proof is straightforward. $\theta = 0$ implies $\lambda \equiv \lambda_1 = -\lambda_2$, which in turn implies $k_1 = k_2$ and $k_3 = k_4$. Given that information, $A$ and $B$ reduce to

$$A = -B = \frac{[2pk_1(k_3 - 1) - (k_1^2 + 1)][v^* - pa]}{(k_1^2 - 1)[2pk_1(k_3 - 1) - (k_1^2 + 1)]}$$

$$= \frac{v^* - pa}{k_1^2 - 1} = \frac{v^* - pa}{\exp(-2\lambda v^*) - 1}$$

This proposition describes one way in which the target zone model might be transformed into a short closed-form equation and therefore tested. However, observed exchange rates in the EMS appear to have significant trend drifts that should be estimated. Given that idea, I calculate an alternative simplification of the generalized S-curve, described below.
Proposition 2  If the monetary authority chooses no band-wideining policy (i.e., \( b = 0 \)), then the closed-form solutions for \( A \) and \( B \) are

\[
A = -B = \frac{v^* - pa}{\exp(-\lambda_1 v^* + \lambda_2 v^*) - 1}
\]

The proof is straightforward. \( b = 0 \) implies \( k_3 = k_4 = 1 \). Using this information, equations (4.4) and (4.5) are easily reduced to the closed-form stated above.

This form of the S-curve equation allows me to study a period of the EMS before the September 1992 crisis, as bandwidths were not expanded until then. Since the EMS started in March 1979, a total of 13 1/2 years worth of data are available for analysis under this assumption. I pursue this line of investigation in Chapter 7.

Proposition 3  If there is no trend drift in the exchange rate, then a standard S-curve results (i.e., the spot exchange rate must be less volatile than its underlying fundamental) if and only if \( p < \frac{v^*}{\alpha} \).

Consider the closed-form in Proposition 1. Given \( v^* > 0 \) and \( \lambda > 0 \), the denominator of \( A \) is negative. Therefore \( A < 0 \) if its numerator is positive; that is, a standard S-curve results if the numerator is positive. It is when \( p < \frac{v^*}{\alpha} \). Intuitively, this proposition argues that the further out a monetary authority shifts a target zone upon realignment the higher the level of credibility the monetary authority must have.

The significance of this proposition is to outline the circumstances under which a realignment might lead to a target zone becoming destabilizing rather than stabilizing. It describes two regions of interest: one region, where small shifts in the band (where the new central parity can be anywhere from the former central parity to the edge of the former band) cannot destabilize exchange rates, and another region (beyond
the first) where exchange rates can only be destabilized with high probabilities of realignment.

**Proposition 4** *If there is no trend drift in the exchange rate, then the stabilizing effect of the S-curve is weakened by a greater band shift size.*

Consider the closed-form in Proposition 1. The partial derivative of $A$ with respect to the band shifting parameter $a$ is $\frac{\partial A}{\partial a} = \frac{-p}{\exp(-2\lambda_0^*) - 1} > 0$. That implies that $A$ moves closer to zero as $a$ rises.

In other words, if a monetary authority chooses to shift a band by more than the width of the former band, as the Bank of France did on three occasions in the early 1980s under the EMS, the more the monetary authority risks destabilizing the exchange rate. The more credible the monetary authority is, the less likely the destabilizing effect is to occur. One casually notes that the Bank of France must have been credible enough in the early 1980s, as the EMS did not collapse then.

**Proposition 5** *If there is no trend drift in the exchange rate, then the stabilizing effect of the S-curve is invariant to the degree of band widening.*

Consider the closed-form in Proposition 1. $\frac{\partial A}{\partial b} = 0$ proves the point. One might note that it appears to contradict the intuitive idea that a “realignment” to freely floating exchange rates should imply $A = 0$. However, realignment to a system of freely floating exchange rates requires $b \rightarrow \infty$, $p = 1$, and $e = v^*$. That is, a realignment to freely floating exchange rates occurs when the realignment is certain, has an infinite bandwidth, and does not alter the position of the fundamental. Plugging in these terms, one quickly deduces that they imply $A = 0$.
In practical terms, this proposition is the reassurance monetary authorities in the EMS relied upon when they chose to widen the bands on the remaining banded currencies after the September 1992 crisis. But this proposition does not speak to the size of the S-curve effects yielded. With wider bands, the EMS became more viable; i.e., more likely to survive future crises.

**Proposition 6** If there is no trend drift in the exchange rate, then the stabilizing effect of the S-curve is weakened by a decrease in credibility.

The result is simple and intuitive. One need only calculate \( \frac{\partial A}{\partial p} = \frac{-a}{\exp(-2\lambda v^*) - 1} > 0 \). In other words, credibility is the key to strong S-curve effects. If the monetary authority does not maintain its target zone, if it realigns too often, or is perceived as being very likely to realign, arbitrageurs will assign a smaller weight to the possibility of defense of the target zone and therefore the price assigned to the exchange rate through exchange rate markets will be closer to what it would have been in the absence of target zones. Of course, this model does not describe how credibility is formed, so I cannot whether the reality of a highly credible monetary or the mere perception of credibility is the most important factor. A more complicated, and far harder to empirically test, model would be required to describe the process of how credibility expectations are formed over time.

**Proposition 7** If the target zone is not permitted to widen upon realignment, then a standard S-curve results if and only if \( p < \frac{\nu^*}{a} \).

Recall Proposition 2. Using the same assumptions, I note that \( A < 0 \) and \( B > 0 \) if and only if \( p < \frac{\nu^*}{a} \) and \( \exp(-\lambda_1 v^* + \lambda_2 v^*) - 1 < 0 \). The first assumption need not
hold in general, and so I make it a condition of the proposition. The second condition is certain to hold once I plug in the values of \( \lambda_1 \) and \( \lambda_2 \). Doing so, I find

\[
\exp \left[ -\left( -\frac{\theta}{\sigma^2} + \frac{\sqrt{\alpha^2 \theta^2 + 2 \alpha \sigma^2}}{\alpha \sigma^2} \right) v^* + \left( -\frac{\theta}{\sigma^2} - \frac{\sqrt{\alpha^2 \theta^2 + 2 \alpha \sigma^2}}{\alpha \sigma^2} \right) v^* \right] - 1
\]

\[
= \exp \left[ -2v^* \frac{\sqrt{\alpha^2 \theta^2 + 2 \alpha \sigma^2}}{\alpha \sigma^2} \right] - 1 < 0
\]

because \( \alpha > 0, v^* > 0, \sigma^2 > 0 \), and \( \theta^2 \geq 0 \).

Again, this result breaks out two different sets of results for the target zone models. In the first group are sustainable target zones, where the standard S-curve results. In the second group are unsustainable target zones, where the inverted S-curve results. Trivially, there is a third group where no S-curve effects result, and the model is operationally equivalent to a pure free floating regime. What distinguishes the first group from the second is how far the new target zone is shifted upon realignment and how credible the monetary authority is. If the target zone is shifted by no more than half the original bandwidth of the target zone, than it automatically falls into the first class. In the target zone is shifted by more than half the original bandwidth of the target zone, than it only falls in the first group if the monetary authority is sufficiently credible (i.e., has a sufficiently low probability of realignment).

### 4.3 Implications of Imperfect Credibility

In short, the significance of these findings is this: for a broad class of realignment policies, including policies that closely resemble those carried out by the EMS for most of its duration, exchange rate target zones must yield more stabilizing exchange rates than would exist under a pure free float. Band widening or narrowing does not
cause crises. Instead, it merely strengthens or weakens S-curve effects, and perhaps makes crises easier or harder to avert given the other parameters.

There are necessary and sufficient conditions for a crisis. The necessary condition is an policy of large shifts in the band upon realignment. The sufficient condition is low degree of credibility of the monetary authorities managing the target zone. Credibility is of course always important. But if the necessary condition above is not satisfied, a low degree of credibility will only result in weaker S-curve effects than would otherwise prevail. If the necessary condition above is satisfied, a low degree of credibility can yield exchange rates that are actually more unstable than they would be under a pure free float. I show in the next chapter that this will also inevitably lead to a collapse in the exchange rate regime. The key to a successful target zone regime, therefore, is sustainable policies and high credibility, and in that order.
CHAPTER 5

CREDIBILITY AND RESERVE CRISES

In chapter 4, I argue that the stabilizing effects of exchange rate target zones is a function of the policy parameters. Specifically, it is primarily a function of the degree to which the monetary authorities shift a target zone band upon realignment, and secondarily it is a function of the credibility of the monetary authorities. I hinted, but did not prove, that a destabilizing target zone, also called an “inverted S-curve” by Bertola and Caballero [2, 3], is also not sustainable, while a standard S-curve is. I prove that proposition in this section. Thus, I derive a model that shows the conditions under which an exchange rate target zone is expected to collapse or not.

5.1 Deriving the Model

In the model given in Section 4, I did not have to explicitly state which monetary authority or authorities realigned or defended the target zone. The variables of interest did not include the home and foreign money stocks except for the manner in which they affected the exchange rate. Here I am interested in the money stocks because of the possibility that a monetary authority may not be able to defend a target zone due to a lack of either foreign exchange reserves or willingness to expand the money supply as needed.
Therefore, a general model of foreign exchange reserve crises must include not only the individual monetary authorities’ money supplies but also an equation or series of equations describing the institutional arrangement whereby the target zone is defended. These ideas are briefly outlined in Dumas’s [12] discussion of the Krugman and Rotemberg [20] paper. Similar ideas are explored in the literature on speculative attack, including papers by Flood and Garber [13, 14, 15] and Froot and Obstfeld [17]. In the model given below, I consider a specific institutional arrangement (similar to the one used implicitly above) and derive the implications of the model given that arrangement. This specification allows for future work describing this model under alternative institutional frameworks.

Consider a simple model of foreign exchange reserve crises under imperfect credibility, adapted from my work in Chapter 4 and from Krugman and Rotemberg [20]. This model examines an target zone arrangement between two countries, generically labeled “home” and “foreign.” In general, both the home and foreign monetary authority will have an interest in maintaining the target zone. The existence of a target zone presumes the existence of an arrangement for maintaining that target zone, which can be cooperative or noncooperative in nature. Under a cooperative arrangement, the home and foreign monetary authorities act in concert under the terms of an agreement outlining how they should behave in the event the target zone is threatened. On the other hand, a noncooperative arrangement implies no such agreement exists, so that the monetary authorities work independently.

Consider again the S-curve given in equation (3.9), where I explicitly account for the monetary policy instruments available to the home and foreign monetary
\[ s_t = m_t^H - m_t^F + v_t + \alpha\theta + A\exp[\lambda_1 (v_t - \overline{v})] + B\exp[\lambda_2 (v_t - \overline{v})] \]

Finally, consider a target zone arrangement where the home monetary authority takes on the entire burden of maintaining the target zone. The foreign monetary authority, freed from the burden of exchange rate maintenance, pursues other monetary policy goals and ignores their implications for exchange rate policy. For that reason, I model the foreign money stock \( M_t^F \) (and therefore \( m_t^F \)) as an exogenous variable whose value does not change in response to foreign exchange reserve crises.

On the other hand, I shall consider the home money stock in different scenarios, and recognize that it alone is the policy variable of interest in foreign exchange reserve crises. Consider a monetary authority with either limited means or limited will to maintain a target zone under all circumstances. In particular, let \( \overline{M}^H \) denote the maximum money stock and \( M^H \) denote the minimum money stock the home monetary authority is willing to tolerate to facilitate defense of the target zone. (Recall that the units on \( m_t^H \) are in natural logarithms, while the units on \( \overline{M}^H \) and \( M^H \) are in levels.)

In the event of speculative attack, the maximum or minimum value described above will be achieved in the event of a significant appreciation or depreciation, respectively. For that reason, the exchange rate after speculative attack must be equal to the value achieved the moment before the speculative attack (by the “no expected jumps” condition), and must be freely floating after speculative attack (as the monetary authorities are no longer able to regulate exchange rate movements).

One can think of this modeling strategy as incorporating two implicit assumptions. The first, which I shall whimsically label the “no George Soros assumption”, argues
that no individual market participant considers or behaves as though their actions can possibly affect the stability of a given target zone regime. This assumption is similar to the assumptions made about individual consumer and producer behavior under perfect competition, and are consistent with the restrictions placed on exchange rate dynamics by various assumptions listed earlier. That is, speculative attack occurs because (1) the target zone is vulnerable and (2) the monetary authority cannot successfully defend it.

The second assumption is the “Alamo assumption”. It argues that a monetary authority will use all available resources to defend a target zone if it so chooses that policy until either defense is successful or until monetary reserves are depleted and the target zone regime is indefensible. Like the defenders of the Alamo in American history, I assume that the monetary authority continues to defend a target zone regime until it runs out of ammunition – or monetary reserves.\(^3\) That is, upon speculative attack, we know that the money supply must be equal to its minimum or maximum tolerable level, depending on whether the home currency had depreciated or appreciated, respectively.

For the sake of consistent notation, let \(\xi\) be an indicator function which can take on values of 0 or 1, where the value of 1 denotes the state of the world where speculative attack occurs and 0 denotes the opposite state of the world where speculative attack

---

\(^3\)Strictly speaking, these events occurred in the history of the independent Republic of Texas prior to her annexation to the Union. As Texas joined the Union over 150 years ago, one may include these events of her history into American history, much as the French settlement of Louisiana or pre-Columbian Native American history are part of American history, even though those events took place outside the domain of the country we call the United States of America. I ignore this historical detail in this discussion. Further, I ignore recent historical studies of the diaries of Santa Ana’s soldiers, which indicate that Davy Crockett and other Texans at the Alamo did not die in the siege of the Alamo but rather were captured by the Mexican Army and then executed. Such is history. In the oft-quoted words of Henry Ford, “History is more or less bunk.” Or, as Warren Harding put it, “I love Paul Revere, whether he rode or not.”
does not occur. Thus, an exchange rate at the upper band of a target zone in a state of the world $\xi$ is denoted as $\bar{s}(\xi)$.

Using the S-curve given earlier, I know that the following must be true

\begin{align}
\bar{s}(1) &= \ln(M^H) - m_t^F + c_t + v^* + \alpha \theta \\
\underline{s}(1) &= \ln(M^H) - m_t^F + c_t - v^* + \alpha \theta
\end{align}

(5.1)

(5.2)

At first, equations (5.1) and (5.2) appear to be counterintuitive. Why would $\bar{s}$ correspond with $M^H$? The answer is that if $M = M^H$, and the monetary authority wishes to defend, it must cut the money stock so as to revalue $s$ from the upper band of $\bar{s}$ to its central parity value. But if the money stock is at its minimum tolerable value, this is impossible and speculative attack is imminent. A similar argument explains why $\underline{s}$ goes with $M^H$ in equation 5.2.

On the other hand, if speculative attack is averted, the stock of reserves is not drawn down because they did not have to be used — no realignment took place and the target zone was successfully defended. Therefore, one may use equation (3.9) to determine the value of the exchange rate the moment before the speculative attack was averted

\begin{align}
\bar{s}(0) &= \ln(M_t^H) - m_t^F + c_t + v^* + \alpha \theta + A + B \exp(2\lambda_2 v^*) \\
\underline{s}(0) &= \ln(M_t^H) - m_t^F + c_t - v^* + \alpha \theta + A \exp(-2\lambda_1 v^*) + B
\end{align}

(5.3)

(5.4)

The “no expected jumps” condition argues that the $\bar{s}$ given in equations (5.1) and (5.3) must be equal and that the $\underline{s}$ given in equations (5.2) and (5.4) must be equal. That is, the following equations must hold
\[
\ln(M^H_t) - m^F_t + c_t + v^* + \alpha \theta = \ln(M^H_t) - m^F_t + c_t + v^* + \alpha \theta + A + B \exp(2\lambda_2 v^*) 
\]
(5.5)

\[
\ln(M^H_t) - m^F_t + c_t + v^* + \alpha \theta = \ln(M^H_t) - m^F_t + c_t - v^* + \alpha \theta + A \exp(-2\lambda_1 v^*) + B 
\]
(5.6)

Let me focus on equation (5.5) first. Solving for the ratio of the minimum tolerable money stock to the pre-attack money stock, I find

\[
\frac{M^H_t}{M^H_t} = \exp[-A - B \exp(2\lambda_2 v^*)] 
\]
(5.7)

Thus, in the case where the home currency has weakened, a target zone will resist speculative attack if and only if

\[
\frac{M^H_t}{M^H_t} < \exp[-A - B \exp(2\lambda_2 v^*)] 
\]

That is, if the minimum tolerable money stock \(M^H_t\) is sufficiently low, then the ratio \(\frac{M^H_t}{M^H_t}\) must be sufficiently low to fend off speculative attack.

Now I shall focus on equation (5.6). Solving for the ratio of the pre-attack money supply to the maximum tolerable money stock, I find

\[
\frac{M^H_t}{M^H_t} = \exp[-A \exp(-2\lambda_1 v^*) - B] 
\]
(5.8)

Thus, in the case where the home currency has strengthened, a target zone will resist speculative attack if and only if

\[
\frac{M^H_t}{M^H_t} < \exp[-A \exp(-2\lambda_1 v^*) - B] 
\]

That is, if the maximum tolerable money stock \(M^H_t\) is sufficiently high, then the ratio \(\frac{M^H_t}{M^H_t}\) must be sufficiently low to fend off speculative attack.
Note that the solution contains the two target zone coefficients $A$ and $B$. These are the same $A$ and $B$ that I derived above in the section on imperfect credibility. We may substitute those same solutions in and find closed-form solutions to the two cutoff points described above. Once those substitutions have been made, equations (5.7) and (5.8) should be cumbersome in general. As in the imperfect credibility section, I will show that the closed-form solutions for some special cases are remarkably simple to solve for and yield simple, intuitive results.

5.2 Propositions

Proposition 8 If there is no band widening (i.e., $b = 0$), then the implied cutoffs simplify to

$$\frac{M_H^H}{M_t^H} = \exp\left[\frac{[v^* - pa][1 - \exp(2\lambda_2 v^*)]}{\exp(-\lambda_1 v^* + \lambda_2 v^*) - 1}\right]$$

$$\frac{M_t^H}{M_H^H} = \exp\left[\frac{[v^* - pa][1 - \exp(-2\lambda_1 v^*)]}{\exp(-\lambda_1 v^* + \lambda_2 v^*) - 1}\right]$$

Note that if $b = 0$,

$$A = -B = \frac{v^* - pa}{\exp(-\lambda_1 v^* + \lambda_2 v^*) - 1}$$

Therefore, equation (5.7) simplifies to $\exp((1 - \exp(2\lambda_2 v^*))A$ and equation (5.8) simplifies to $\exp((1 - \exp(-2\lambda_1 v^*))A$. These can be easily manipulated into the result shown above.

Proposition 9 If there is no trend drift in the exchange rate (i.e., $\theta = 0$), then $\lambda = \lambda_1 = -\lambda_2$ and the implied cutoffs simplify to

$$\frac{M_H^H}{M_t^H} = \exp(-[v^* - pa])$$
\[
\frac{M_i^H}{M^H} = \exp(-[v^* - pa])
\]

This is a simple extension of Proposition 8. Substitute \( \lambda \equiv \lambda_1 = -\lambda_2 \) into the closed-form equations in Proposition 8. Note that I have not assumed \( b = 0 \) in this proposition. As in Proposition 1 in Chapter 4, the band widening \( (b) \) term disappears in the closed-form solution for \( A, B \), and by implication here as well.

Propositions 8 and 9 describe exact solutions for the cutoff points in closed form. Although they appear to be empirically testable, they are only testable if we are willing to identify values for the cutoff minimum and maximum tolerable money stocks. Without that knowledge, they are only interesting insofar as they describe circumstances where the solution identified is in fact trapped between zero and one or not. Why is that important? If the right-hand sides of the terms are in fact between zero and one, then so are the left-hand sides, and that implies that the present conditions identified imply no collapse of the target zone regime. However, if circumstances were to arise so that at least one of the terms is below zero or above one, then the current money stock would be outside the acceptable range, and collapse of the target zone regime would be imminent at the next time the exchange rate reached an edge of the target zone, if not before.

**Proposition 10** If either there is no band widening \( (b = 0) \) or there is no trend drift in the exchange rate \( (\theta = 0) \), then if the S-curve moves into an inverted shape (as in Bertola and Caballero, where \( A > 0 \) and \( B < 0 \)), then the target zone will collapse the next time the exchange rate hits its upper or lower bound.
This is a surprising and intriguing result, although its proof is relatively simple. In the inverted S-curve case, \( A > 0 \) and \( B < 0 \). But if either \( b = 0 \) or \( \theta = 0 \), \( A = -B \). In this case the appropriate cutoff points will be given by the following equations

\[
\frac{M_h^H}{M_t^H} = \exp[(1 - \exp(2\lambda_2v^*))A] > 1
\]

\[
\frac{M_t^H}{M_h^H} = \exp[(1 - \exp(-2\lambda_1v^*))A] > 1
\]

However, \( M_h^H/M_t^H \) and \( M_t^H/M_h^H \) are both trapped between zero and one. Thus, it must be true that no change in the money supply can be enough to prevent speculative attack. Because speculative attack cannot be averted in this case, it must be inevitable by the time the exchange rate hits either the upper or lower bound of the target zone.

Therefore, Proposition 10 describes why the inverted S-curve first described by Bertola and Caballero \([2, 3]\) is theoretically implausible. If conditions were such that the S-curve took on an inverted shape, then collapse would be imminent upon the next crisis point reached. Therefore, one may or may not expect to see inverted S-curve phenomena for short periods of time, perhaps immediately prior to a total or partial collapse of a target zone regime. And perhaps this describes what happened in September 1992, but I have no proof of that.

What it definitely says is that an inverted S-curve cannot be a persistent feature of a target zone regime, and certainly cannot characterize a system of target zones that has gone through several periods of realignment and/or defense. If a target zone had successfully been defended, then it must have withstood the possibility of speculative attack, and if that reflects reality, then the S-curve must have its conventional, stabilizing properties.
If that is true, then a persistent target zone must, virtually by definition, be stabilizing. Exchange rates under a target zone must exhibit lower volatility than they would have under a free float. This analysis does not address the costs of maintaining a target zone, or whether a target zone is desirable in a normative sense. But the stabilizing effects of a target zone should be beyond dispute given the logic of this analysis.
CHAPTER 6

ECONOMETRIC THEORY AND THE MODEL

If it provides an adequate explanation of reality, the simple theoretical model outlined above has some fascinating implications. First, it describes the movement of targeted exchange rates facing problems of imperfect credibility in a remarkably simple setting. Second, it rejects the Bertola-Caballero [2, 3] possibility of heightened volatility relative to a free-float regime. Third, it describes the degree to which the standard S-curve effects are lessened by common beliefs in less than completely credible central bankers. Fourth, it provides a method of determining the amount of reduced volatility that can be "bought" by central bankers, relative to a free-float regime. Finally, it provides an empirical method for predicting the approximate timing of realignments, or at least the mean time until the next realignment. All of these provide an incentive to empirically test the model above.

Empirical analysis of this model requires a degree of econometric sophistication. The data is time-series, and according to conventional unit-root tests is difference-stationary, or $I(1)$. This model has stranger implications. If $p = 1$, monetary authorities always realign, the variance of the sequence of exchange rates is infinite in infinite time, and the exchange rate can be shown to be $I(1)$. If $p = 0$, monetary authorities always defend, the variance of the sequence of exchange rates is finite in infinite time,
and the exchange rate must be $I(0)$. Cases of imperfect credibility ($0 < p < 1$) lie on a strange border between being $I(0)$ and $I(1)$. In other words, the natural logarithm of the spot exchange rate, measured in discrete time, is expected to follow a random walk, and differences of this variable are expected to be covariance-stationary. The expected value of the log spot exchange rate is determined by monetary growth and expectations of that growth, all shocks are permanent, and the variance of the spot exchange rate may become infinite in infinite time, depending on the level of credibility, as discussed above.

The parameters of the model do not enter in a linear fashion, so no simple correction to least squares can be used. The model does yield testable implications on different moments of the data. Further, the probability of realignment parameter yields special difficulties in estimating, as it only affects the data a handful of times. In theory, the ideal technique would be maximum likelihood. However, calculation of the theoretical log likelihood function is impractical in practice given the highly complex model in question. A method of moments-based test, in spite of its typically low power, provides a practical method of estimating parameter values with a minimum of complication and a useful method for comparing these estimated parameters with typical values found in the literature.

However, my closed-form solution for the exchange rate, shown above, does not easily yield equations describing higher order moments, let alone derivatives of those moments with respect to the various parameters. I attempted to calculate these moments in theoretical fashion using the Maple symbolic manipulation program. The results, fuzzy and dozens of pages long, speak for themselves. But the model does easily lend itself to simulation, since the assumed distribution of the shocks can easily
be transformed into a simulated data series. For that reason, the implications of the model can be tested using the simulated method of moments (SMM) technique, as described by Lee and Ingram [21], Duffie and Singleton [11], and Stern [33], among others.

6.1 The Data

I test my model in Chapter 4 against three sets of exchange rates: the Deutschmark - Italian lira (DEM/ITL), Deutschmark - French franc (DEM/FRF), and Deutschmark - U.S. dollar (DEM/USD) exchange rates. All three are nominal spot exchange rates, observed in daily intervals (excluding market holidays) throughout an exchange rate regime. The first two exchange rates were part of the ERM, the third was a free float throughout the period in question and is used as a test of the power of the estimators. The DEM/ITL exchange rate was a part of the ERM from its inception on March 13, 1979 until Italy dropped out on “Black Wednesday,” 16 September 1992. The data analyzed covers the period where both the DEM/ITL and DEM/FRF exchange rates were under the ERM; that is, from 13 March 1979 until 15 September 1992. Because markets are closed on Saturdays, Sundays, and occasional holidays, 3,390 data points are available for analysis. Data used are daily quotes taken from the PACIFIC Exchange Rate Service at the University of British Columbia, Vancouver, B.C., Canada. The service passes along the data, which are noon spot quotes at the Bank of Canada and are denominated in Canadian dollars. The numbers I cite are actually cross rates, but converge within seconds to the actual rates due to triangular

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4The data are available online at http://pacific.commerce.ubc.ca/ and can be accessed at no charge, provided the source and their copyright (1998, 1999) is noted. I thank Professor Werner Antweiler of U.B.C. and the PACIFIC Exchange Rate Service for making this online resource available to the academic community.
arbitrage. Actual dates of realignments in the data are cited by Mizrach [27] for the DEM/ITL and DEM/FRF exchange rates.

For the sake of analysis, I assume that the entire ERM experience can be characterized as a single regime, with a single value of all five of the parameters, notably including the probability of realignment. It should be noted that many authors, including Li [22] and Dominguez and Kenen [9] describe a reduction in exchange rate volatility in the ERM after the Basle-Nyborg agreement was signed in 1987. Li [22] specifies that this agreement provided “limited use of EMS credit facilities for intra-marginal interventions and, fuller use of the exchange rate band.” Such a move is difficult to directly assess in my model, as I assume all interventions take place at the margin in order to clarify the intuition of the model, but suffice to say the volatility-reducing effects of target zones should be more prominent as exchange rates spend more time near the edges of the bands.

A more complete analysis of the model, made untestable by the small number of realignments in the actual data, might divide the ERM experience into pre- and post-Basle-Nyborg periods, where we would predict the volatility of the exchange rate (denoted by $\sigma$ in the model) takes on a lower value after the agreement is signed. The probability of realignment may or may not change its value in response to this, but we cannot tell with the information we have available. The model does not describe the optimal behavior of a monetary authority; rather, it is a policy analysis tool to describe to a monetary authority what they might expect to observe when managing an exchange rate target zone.

Because a simulated method of moments uses simulated shocks and compares the observed moments of the data with simulated moments conditioned on chosen values
of the parameters, it is not necessary to find values of the fundamentals described in
the model. The shocks to the fundamentals are described by the logic of the model,
and the discretionary monetary interventions are described by the model itself. With
a description of the fundamentals fully given this way, I do not need to use data
describing the relevant fundamentals terms, like money stocks or real GDP levels. In
this way, I can take advantage of the thick data source provided by daily exchange rate
values and avoid the thin data sources provided by monthly or quarterly descriptions
of monetary stocks and real GDP levels. Exchange rates are prices, and are quite
easy to observe and with little observational error. Money stocks and real GDP
levels, on the other hand, are quantities and therefore are hard to observe and are
often observed with a great deal of error.

6.2 The SMM Technique

The simulated method of moments (SMM) technique is a method for extracting
estimated parameter values of a specified model by comparing observed moments of
the data with the moments generated by a model of interest. As I stated above,
it differs from the GMM technique by using moments of the data generated by a
simulation rather than by direct calculation of the theoretical moments implied by
a model. SMM is preferred to GMM when the moments cannot or cannot easily be
calculated from a theoretical model, or when the independent variables of the model
do not appear in sufficiently high frequency. Both of these conditions, as noted above,
are present in this study.

The technique, as roughly outlined in Lee and Ingram [21], Duffie and Single-
ton [11], and Stern [33], and to a lesser extent in Smith and Spencer [32], and modified
for the present requirements of this study, uses the following methodology. Although
Smith and Spencer [32] is one of the pioneering studies on empirical analysis of target
zone models, it has a number of undesirable econometric suggestions that make its
adoption for my purposes highly problematic. Unfortunately, I only found that out
after a large amount of what in the end was wasted effort.

First, write out the theoretical model and its underlying assumptions and method
of derivation. In part, proper analysis and testing of model will go back to these
details. Preferably, this model will lead to a closed-form solution of how the time path
of a dependent variable is generated through vectors of shocks, vectors of independent
variables, and a vector of parameters. In the context of the present model, that
equation is \( s_t = g(m_t, v_t, c_t; \alpha, p, \sigma, v^*, \theta) \).

Second, make a provisional choice of values for the parameters. Each run of the
simulation requires provisional choices of parameter values in order to help calculate
values of the dependent variables and therefore the simulated moments. For different
runs of the simulation, different values of the parameters can be chosen, and so the
impact of different parameter choices on the simulated moments may be determined
numerically. In the case of the model at hand, each run of the simulation is done
with a choice of five parameters, identified as \( \hat{\alpha}, \hat{v}^*, \hat{p}, \hat{\sigma}, \) and \( \hat{\theta} \).

Third, determine sample values for the stochastic variables in the model. The
variables and their theoretical distributions are given by the model. In the case of
this model, there are two stochastic variables, \( \mu_t \sim N(0, 1) \) and \( \pi_t \sim U[0,1] \). Of
course, one saves these draws so as to prevent resampling errors.

Fourth, calculate simulated data using the assumed values for the parameter val-
ues. Using an equation like \( s_t = g(m_t, v_t, c_t; \alpha, p, \sigma, v^*, \theta) \), and using the assumed
values $\alpha = \hat{\alpha}$, $\nu^* = \hat{\nu}^*$, $p = \hat{p}$, $\sigma = \hat{\sigma}$, and $\theta = \hat{\theta}$, one can form a series of simulated values indexed by $j$, where $j = 1, \ldots, M$, of the simulated exchange rates in the sequence $\{s_j\}_{j=1}^M$. In cases like the present example, where the value of the simulated exchange rate is a function of the current central parity ($c_j$), these simulated exchange rates must be calculated in an iterative fashion, as the value of this period's simulated exchange rate ($s_j$) is an implicit function of the value of the prior period's simulated exchange rate ($s_{j-1}$).

Fifth, moments of the simulated data can be calculated from the simulated data. But which moments to use? While the exact choice of moments is arbitrary and econometric theory makes no specific recommendations, a prudent choice of moments would include the first few moments of the first differences (not the levels, as those are $I(1)$ and therefore possess undesirable econometric properties), and might well include moments of the autocovariances. This suggestion differs with the choices made by Smith and Spencer [32], but I find that their choices are misleading. The first two moments should certainly be chosen, and the often-identified leptokurtosis and ARCH effects present in exchange rate data imply that one should compare the third and fourth moments. The first few autocovariances should be chosen so as to insure the number of moments exceeds the number of parameters to be estimated. Finally, the number of moments chosen must exceed the number of parameter values being estimated, so as to create “overidentifying restrictions” and thus permit a nontrivial estimation of parameter values in the method of moments framework. One might also consider adding day of the week effects (the fifth autocovariance), but the number of holidays in the data set makes this approach impractical. Following that logic, I
select the first four moments of the first difference of the data and the first three autocovariances of the first difference of the data. That choice permits analysis of seven moments, which easily exceeds the five parameter values being analyzed ($\hat{\alpha}, \hat{\sigma}^*, \hat{\rho}, \hat{\sigma}$, and $\hat{\theta}$).

Sixth, generate point estimates of the parameters to be estimated. This is done at first through an iterative process, examining one parameter at a time while holding the others constant. The relative validity of the parameter values are considered by finding the values that minimize a quadratic loss function. Once one finds preliminary point estimates using this grid search technique, various methods of numerical optimization may be employed to find more precise point estimates.

Let $\beta$ denote the five element vector of parameters to be estimated, where $\beta = [\nu^*, \alpha, \theta, \sigma, p]'$. The observed time series is designated $\{s_t^{OBS}\}$. Let the superscript OBS denote observed data and the superscript SIM denote simulated data, when needed for purposes of emphasis. Let $T$ denote the number of observations in the observed data series, indexed by $t = 1, \ldots, T$, and $M$ denote the number of simulated data points generated in the computer simulation, indexed by $j = 1, \ldots, M$, using the parameter vector $\beta$. Let $h(s_t)$ be a vector moment function, which has seven elements since I consider seven moments. Since I have chosen to consider the first four moments of the first difference of the log spot exchange rate along with the first three autocovariances of the same, my vector moment function is summarized as

$$h(s_t) = [\Delta s_t, \Delta s_t^2, \Delta s_t^3, \Delta s_t^4, \Delta s_t \Delta s_{t-1}, \Delta s_t \Delta s_{t-2}, \Delta s_t \Delta s_{t-3}]'$$

Let the vector $H(s_t)$ denote the arithmetic mean of the moment vectors $h(s_t)$ identified above, so that $H_T(s_t)$ and $H_M(s_j(\beta))$ denote observed and simulated arithmetic means of the moment vectors, respectively. Let $g_{T,M}(\beta) = H_T(s_t) - H_M(s_j(\beta))$
denote the difference between the observed and simulated arithmetic means of the moment vectors. Let the vector \( u_t = h_t(s_t^{OBS}) - H_T(s_t^{OBS}) \) denote deviations of the moment vector from its mean at some given observation \( t \). Finally, let the weighting matrix \( W_T \) be estimated using the Newey-West\([29]\) estimate of the long-run variance-covariance (VCV) matrix of \( u_t \), which is defined as

\[
\hat{W}_T = \hat{\Omega}_0 + \sum_{k=1}^{K} (1 - \frac{k + 1}{K})(\hat{\Omega}_k + \hat{\Omega}'_k)
\]

where \( \hat{\Omega}_0 = \frac{1}{T} \sum_{t=1}^{T} u_t u_t' \) and \( \hat{\Omega}_k = \frac{1}{T} \sum_{t=k}^{T} u_t u_{t-k}' \) for a sufficiently large number \( K \) so that \( \hat{W}_T \) is a positive definite matrix. For my purposes, \( K = 20 \) suffices.

The SMM estimator is \( \hat{\beta}^{SMM} \) that minimizes the quadratic form

\[
g_{T,M}(\beta)'[W_{T,M}^{-1}]g_{T,M}(\beta)
\]

where \( W_{T,M} = (1 + \frac{T}{M})\hat{W}_T \) and \( \hat{W}_T \) is the Newey-West estimate of the weighting matrix defined above.

Examine each parameter one at a time to find the parameter value that, holding all other parameter values constant, minimizes the loss function. Do so for each other parameter value, again holding the others constant, and continue to do in repetition until convergence is reached for a set of parameter values. While doing this process, be sure to hold constant (using fixed seed values) the set of draws of the random variables \( \mu_t \) and \( \pi_t \) defined above. And, as mentioned above, use numerical optimization algorithms to identify more precise point estimates after finding preliminary estimates using this grid search technique.

Seventh, find standard error estimates for the point estimates of the parameters. The standard errors are derived from examining the diagonal elements of the estimated asymptotic VCV matrix, described in the asymptotic distribution of the
vector of parameters to be estimated. Lee and Ingram [21] derive the distribution of the SMM estimator $\hat{\beta}_{SMM}$, relative to its true value $\beta_0$, to be

$$\sqrt{T}(\hat{\beta}_{SMM} - \beta_0) \sim N(0, V)$$

where $V = [B'WB]^{-1}$ and $W_{T,M}$ is used to calculate $W$ in the estimation of standard errors, and the matrix of partial derivatives of the moment vector functions with respect to the parameters, $B$, is given by $B = E[\frac{\partial{h_i(s_1(\beta))}}{\partial{\beta}}]$. I estimate the value of the matrix $B$ using the numerical derivative subroutine in GAUSS, with each of element of $B$ estimated to be equal to the arithmetic mean of the respective vector of numerical derivatives generated by the GAUSS subroutine.

Let $V_i$ be the $i^{th}$ diagonal element of VCV matrix $V$ identified above, which corresponds to the $i^{th}$ element of the optimal parameter values vector $\hat{\beta}_{SMM}$. Then the standard error for $\hat{\beta}_i^{SMM}$ is given by $se(\hat{\beta}_i^{SMM}) = (T^{-\frac{1}{2}})\sqrt{V_i}$.

A final test of note is a test for the overall goodness of fit of the model. Lee and Ingram [21], following the GMM theory of Hansen [18], describe a chi-square test to estimate the goodness of fit of a model estimated using a method of moments technique. The test statistic is simply calculated to be the number of observations of the data multiplied to the minimized value of the quadratic loss function, assessed using the point estimates $\hat{\beta}_{SMM}$ found above. Lee and Ingram [21] prove that this statistic must have a $\chi^2$ distribution, with the number of degrees of freedom equal to the difference of the number of moment conditions used and the number of parameter estimates identified. In my case, using seven moment conditions and estimating five parameter values, my test statistic must be distributed $\chi^2$ with $7 - 5 = 2$ degrees of freedom. If the value of this test statistic is greater than the critical value for the chi-square distribution with that number of degrees of freedom at the 5% level.
(for two degrees of freedom at the 95% level, this cutoff value is about 5.99), then I must reject the proposed model as not fitting the data well; otherwise, I cannot reject it as a good fit of the data. (On a side note, if I reverse the test and fail to give the null hypothesis the benefit of the doubt, so that 95% of the distribution is set as the rejection region instead of the reverse, then the cutoff point reverts to about 0.1026, where I would reject the DEM/USD model but fail to reject the DEM/ITL and DEM/FRF models.) This test shall provide a good test for whether the present target zone model describes EMS data, even if individual parameters prove difficult to identify.

6.3 Manipulating the Model for Testability

Recall from Chapter 3, equation (3.9) that

\[ s_t = m_t + \alpha \theta + A \exp \lambda_1 (v_t - \bar{v}) + B \exp \lambda_2 (v_t - \bar{v}) \]

Chapter 4, equations (4.4) and (4.5) provide solutions for \( A \) and \( B \), and the propositions listed provide simplified solutions for \( A \) and \( B \) given particular assumptions. For empirical research, I accept for purposes of analyzing ERM data the assumption that the new exchange rate band is centered at the relevant edge of the former exchange rate band; that is, \( a = v^* \), \( b = 0 \), and the assumptions of Proposition 2 in Chapter 4 are satisfied. Thus,

\[ A = -B = \frac{(1 - p)v^*}{\exp(-\lambda_1 v^* + \lambda_2 v^*) - 1} \]

Chapter 5 extends this theoretical model to the possibility of foreign exchange reserve crises. However, the derivation of this extension implies no substantive change in exchange rate behavior. Instead, it merely derives cutoff values for a viable target
zone regime, and shows that an inverted S-curve cannot describe a viable, working target zone regime. Because the "half the distance" assumption \((a = v^*)\) implies an inverted S-curve cannot prevail (as I proved in Chapter 5), the theoretical restriction is supported as economically correct. I am now ready to derive the testable assumptions of the model.

Begin by substituting the solutions for \(A\) and \(B\) into equation (3.9). Thus,

\[
s_t = m_t + v_t + \alpha \theta + \left[ \frac{(1 - p)v^*}{e^{-\lambda_1 v^* + \lambda_2 v^*}} - 1 \right] \left[ e^{\lambda_1 (v_t - v)} - e^{\lambda_2 (v_t - v)} \right]
\]

But \(\bar{v} = c_t + v^*\) and \(\underline{v} = c_t - v^*\) by earlier assumptions. Further, \(\lambda_1\) and \(\lambda_2\) are known from equation (3.7). Thus, a fully expanded closed-form solution for the spot exchange rate under a target zone is given by

\[
s_t = m_t + v_t + \alpha \theta + \left[ \frac{(1 - p)v^*}{e^{-\lambda_1 v^* + \lambda_2 v^*}} - 1 \right] \left[ e^{\lambda_1 (v_t - c_t - v^*)} - e^{\lambda_2 (v_t - c_t + v^*)} \right] \tag{6.2}
\]

where \(\lambda_1\) and \(\lambda_2\) are constants given by

\[
\lambda_1 = \frac{-\theta}{\sigma^2} + \frac{\sqrt{\theta^2 + 2\alpha\sigma^2}}{\alpha\sigma^2}
\]

\[
\lambda_2 = \frac{-\theta}{\sigma^2} - \frac{\sqrt{\theta^2 + 2\alpha\sigma^2}}{\alpha\sigma^2}
\]

This closed-form solution is cumbersome to convert into the appropriate theoretical moments needed to test the model in a GMM setting. This further justifies use of the simulation-based variant of GMM, namely SMM, used as the empirical technique to analyze the model.

The closed-form solution may thus be summarized by

\[
s_t = g(v_t, m_t, c_t; \alpha, \sigma, \theta, v^*, p)
\]

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Because the model is estimated by simulation, we know that each simulation must be run with choices for values of the five parameters listed, where the econometric derivation of the parameter estimates used is detailed above.
CHAPTER 7

ECONOMETRIC PRACTICE AND THE MODEL

The most surprising aspect of this research has been the numerous practical difficulties encountered in econometric analysis of target zones. In the end, the problems primarily concerned search algorithms, debugging GAUSS source code, “hilly” loss functions, poor standard errors, long numerical optimization searches, and the underlying reasons behind these unexpected difficulties. In short, econometric analysis using the techniques outlined in Chapter 6 can be compared to an Alfred Hitchcock movie, where the plot seems to lead down a straightforward path until an unexpected plot twist at the end alters the ending in a way no one in the theater expected. The insights and conclusions that one draws are significant, important, and yet not those originally anticipated. Yet as one ponders the final conclusion, there is a sense of wondering, ”Why didn’t I think of that a long time ago?”

7.1 Difficulties Encountered

One of my first problems was the process of identifying a search algorithm. SMM methodology, described above, does not include a proper statement of the search process. Instead, it presumes that an optimal search process will emerge from a careful analysis of the problem being considered. Smith and Spencer [32] suggest that
a proper econometric analysis start with ten or so reasonably good guesses of the true parameter values, describes roughly how one should search over those parameters, then chooses the parameter values that yield the lowest loss function values, then repeating until consistent estimates emerge. This approach, which one might term "one at a time", or more sarcastically, "plug in, pray, repeat 'till Doomsday", is essentially a manual method, requiring countless hours of active manipulation and running of GAUSS code until one is reasonably assured of having achieved a local (and hopefully global) minimum of the loss function, so that the parameter values calculated are, in fact, proper SMM estimates.

Alternatives to this approach exist. For example, one could recode GAUSS so that a vast array of numerical minimization algorithms, such as the famous Newton-Rhapson technique, can be utilized to find the optimal parameter values, much as is done in work using maximum likelihood functions. On the other hand, a simple grid search technique could be employed over an economically reasonable set of parameter estimates in order to find a good set of parameters. A modified version of this technique would start with a grid search, then refine the point estimates with an iterative process as is outlined above.

The numerical optimization approach proved to be the most difficult alternative. First, numerical optimization algorithms in GAUSS are long, costly in time, and required a dramatic rewriting of my source code to a form amenable to the algorithms. Second, identifying and programming the necessary modifications is a large and time-intensive task. Finally, the quadratic loss function has hilly properties, thus making it quite possible that a long numerical optimization program run may push estimates away from economically meaningful values. Since this danger, and the other problems
outlined above are difficult to overcome, I abandoned this option as a first stage. As discussed above, this technique becomes viable and useful if one has already identified the approximate range of the loss function minimizing parameter values, presumably from some grid search process.

The grid search technique seems the most useful approach, as far as identifying particular regions of parameter values to focus on. The primary drawbacks to this technique are time-intensity and small search areas. The algorithm for calculating the simulated exchange rates is slow, primarily due to a do-loop needed at every observation to determine if a crisis point is reached, and if so, whether the monetary authority should defend or realign at that point. To calculate ten times the number of simulations as there are observed values (in this case 33,900 simulated data points) takes about two minutes on a Pentium-II 450 MHz computer in my office. I must estimate five parameter values. Assume I narrow my grid to ten possible values for each parameter to be estimated, as Smith and Spencer [32] recommend. I would then be estimating a $10^5 = 10,000$ point grid. Since calculating each point of the grid takes about two minutes, such a run should take 20,000 minutes, or 13 days, 21 hours, and 20 minutes. This is prohibitively long, and yields poorly refined point estimates which may be subject to large standard errors. Narrowing the grid to six possible values for each parameter estimate yields a run of $4^6 \times 2 = 2,048$ minutes, or 1 day, 10 hours, and 8 minutes – essentially a Friday night until a Sunday morning on a dedicated computer. While debugging and otherwise attempting various methods of analysis, many weekends were wasted running through one set of parameter estimates, only to find the underlying program that generated them was flawed, so the results were worthless and had to be rerun. However, reducing search time to a reasonable level
(i.e., one weekend of dedicated processor time) grossly shrinks the viability of the grid search technique. Therefore, I must abandon it, or at least incorporate it into another technique as a hybrid method.

Therefore, I select the iterative method as a primary search technique, occasionally allowing a search of two or occasionally three parameter values simultaneously, thus incorporating in a small way the advantages of the grid search method. Because some of the variables, primarily the probability of realignment, appear to be closely related to other variables being estimated, I found it necessary to search over probabilities of realignment as well as some other variable in order to properly find estimates of the other variable in question. If I felt uncertain of the validity of the point estimates derived, I could derive them again using a different set of seed values for the shock terms, then examine how close the parameter estimates are in each case. Once those parameter estimates were calculated, I reverted to numerical optimization to yield refined point estimates.

A question remains: which parameter values are “economically meaningful” starting points? Some parameter values have natural choices. The probability of realignment, \( p \), is theoretically restricted to between 0% and 100%, thus I impose the restriction that \( 0 \leq p \leq 1 \). The size of the bandwidth in units of the monetary fundamentals, \( v^* \), should be slightly larger than the bandwidth in units of the exchange rate if the S-curve theory is true, and exactly equal if it is false. For an exchange rate with a 2.25% bandwidth (such as the DEM/FRF exchange rate), in logs that bandwidth is equal to \( \ln(1.0225) = 0.022250609 \). Thus, a reasonable restriction on \( v^* \) is \( v^* \geq \ln(1.0225) \), while an analogous restriction is placed on the DEM/ITL exchange rate, with its 6% bandwidth. The interest semielasticity of money demand, \( \alpha \), has

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been studied for decades in the money demand literature, and reasonable estimates of it range between 0.2 and 1. Thus, I restrict \( \alpha \) to \( 0.2 \leq \alpha \leq 1 \), but am open to any positive value of \( \alpha \), as this restriction is more one of common sense than deeper issues. But the drift and variance terms for the velocity shock term, namely \( \theta \) and \( \sigma \), respectively, have no seemingly natural value. How to find reasonable restrictions on those, other than the fact that \( \sigma \) must be strictly positive?

My final answer is to adopt a simple process for deriving reasonable good guesses using the data and a simple simulation of a random walk. I start with data \( \{s_t\} \), the values of the log spot exchange rate for a pair of countries I am examining. In the event of no target zone effects, the exchange rate is said to be determined by the equation

\[
s_t = \rho s_{t-1} + \theta + \epsilon_t
\]

However, that relationship is said to be characterized by cointegration given the very realistic idea that \( \rho \approx 1 \). Instead, begin determining the "reasonable first guesses" by fitting the first difference of that equation with OLS, where that equation is

\[
\Delta s_t = \beta_0 + \beta_1 \Delta s_{t-1} + \xi_t
\]

where I assume that \( \xi_t \sim N(0, \sigma^2) \). That yields parameter estimates \( \hat{\beta}_1 \), which is meaningless for the purpose at hand, and \( \hat{\beta}_0 \), which should yield a reasonable first guess at the value of the drift term \( \theta \). To estimate \( \sigma \), the standard deviation of the error term, I calculated the prediction error by considering the difference between fitted and true values of \( \Delta s_t \) as follows:

\[
\Delta s_t - \hat{\Delta} s_t = (\Delta s_t - \Delta s_{t-1}) - \hat{\beta}_0 = \hat{\xi}_t
\]

where s.d.(\( \hat{\xi}_t \)) is my first guess for the estimator of \( \sigma \).
\[ \chi^2 \text{ goodness of fit test statistic } = 0.0043894908 \]

Table 7.1: Initial Grid Search Results for DEM/ITL

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\nu}^* )</td>
<td>ln(1.0604)</td>
<td>7611.8515</td>
</tr>
<tr>
<td>( \hat{\alpha} )</td>
<td>1.0</td>
<td>5.2587789 \times 10^{10}</td>
</tr>
<tr>
<td>( \hat{\theta} )</td>
<td>-0.00012</td>
<td>888815.99</td>
</tr>
<tr>
<td>( \hat{\sigma} )</td>
<td>0.001800</td>
<td>1197007.7</td>
</tr>
<tr>
<td>( \hat{p} )</td>
<td>0.8</td>
<td>8470006.1</td>
</tr>
</tbody>
</table>

\[ \chi^2 \text{ goodness of fit test statistic } = 0.0032395940 \]

Table 7.2: Initial Grid Search Results for DEM/FRF

To find reasonable point estimates of those, I find point estimates of \( \nu^*, \alpha, \) and \( p \) assuming the values mentioned above are true. Then I estimate \( \theta \) and \( \sigma \) assuming the values found above for \( \nu^* \) and \( \alpha \) are true, re-estimating \( p \) in the process. Then I iterate that process until consistent answers appear. These are the point estimates I report below, along with the appropriate standard errors.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\hat{\nu}^*)</td>
<td>ln(1.0234)</td>
<td>0.046214077</td>
</tr>
<tr>
<td>(\hat{\alpha})</td>
<td>1.0</td>
<td>(4.7398521 \times 10^{10})</td>
</tr>
<tr>
<td>(\hat{\theta})</td>
<td>0.000009</td>
<td>5.2167485</td>
</tr>
<tr>
<td>(\hat{\sigma})</td>
<td>0.002000</td>
<td>5.3655016</td>
</tr>
<tr>
<td>(\hat{p})</td>
<td>0.0</td>
<td>416.37422</td>
</tr>
</tbody>
</table>

\(\chi^2\) goodness of fit test statistic = 0.13305076

Table 7.3: Initial Grid Search Results for DEM/USD

7.2 Estimation Results

Using the grid search procedure and noting the difficulties outlined above, I find the parameter estimates and standard error estimates. I list these estimates in Tables 7.1 – 7.3. The point estimates are listed first, followed by their respective standard error estimates. Simulations using these parameter values are graphed out below in Figures 7.1, 7.2, and 7.3.

The most disappointing result of this dissertation is the absurdly high standard errors generated above. No variable estimated above can pass a simple test of whether it is statistically significant or not. Some variables, \(\alpha\) in particular, have remarkably high standard errors, in a range of \(10^{10}\)-fold higher than their respective point estimates. To make matters worse, the chi-square goodness of fit tests fail to reject the target zone model, including for the untargeted DEM/USD case. How can I know whether one model is to be preferred to another in an environment such as this? How do I explain such high standard errors, especially in the case of the interest
Figure 7.1: 3,390 Observed (S-obs), Simulated (S-sim), and "Counterfactual" (cfact) DEM/ITL Exchange Rates
Figure 7.2: 3,390 Observed (S-obs), Simulated (S-sim), and “Counterfactual” (cfact) DEM/FRF Exchange Rates
Figure 7.3: 3,390 Observed (S-obs), Simulated (S-sim), and “Counterfactual” (cfact) DEM/USD Exchange Rates
semielasticity of money demand (\(\alpha\))? Is there any ray of hope in these results, or is all statistical analysis of models of this class doomed to failure?

The obvious place to turn is numerical optimization. After all, I may have made poor choices as to which region to search and therefore not found minimizing parameter values. Further, I may have made poor choices of moments to search over, and therefore may not have moments that have a “hilly” surface or may not yield the desired loss function minimizing point. So I proceed to experiment with both.

I employ a series of numerical optimization algorithms, originally derived from Professor Bo Honore’s GAUSS translation of the numerical optimization algorithms in Numerical Recipes, modified slightly by my adviser, Professor Nelson Mark\(^5\). The primary search algorithms include the Davidon-Fletcher-Powell (DFP), Fletcher-Reeves-Polak-Ribiere (FRPR), Amoeba (AMOeba), and Powell (POWELL) algorithms, which can be used with either the Golden Search (LINMIN) or Brent’s (LINMIB) line minimization algorithm\(^6\).

In practice, the LINMIN line minimization algorithm appears incompatible with the present model, while Brent’s alternative algorithm (LINMIB) yields reasonable results. The FRPR and Amoeba algorithms appear to crash with a high degree of frequency, while the DFP and Powell algorithms exhibit a sufficient amount of stability. The catch is that they yield substantially different results, leaving me to compare multiple possible minima. In particular, the minima generated from the DFP algorithm is generally close to results from the grid search, while the Powell

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\(^5\)I thank both Prof. Honore and Prof. Mark for use of these algorithms.

\(^6\)The interested reader can find these and other numerical optimization algorithms available at the Gaussians web site at American University, http://www.american.edu/academic.depts/cas/econ/gaussres/GAUSSIDX.HTM
algorithm yields results quite far from the grid search results. I will argue that the DFP results are superior to the Powell results on several important criteria.

I begin the refined analysis by finding refined parameter estimates with the DFP algorithm, then find and examine new parameter estimates based on a reasonably specified alternative set of moment conditions. The choice of alternative moments comes from using information about the U-curve, described earlier. Because the deviation of the exchange rate from central parity has natural limits to its distribution (if it exceeded a certain value, a realignment or defense question would be triggered). Thus, \( s - c \) has a finite mean and variance and must be \( I(0) \), giving it similarly desirable econometric properties to other moments used. I employ both \( s - c \) and \( (s - c)^2 \) as moment conditions, replacing the third autocovariance, and thus providing a total of eight moment conditions in place of the former seven conditions. Finally, I assess this new set of moment conditions using the Powell algorithm and analyze the results.

The results of the refined analysis of the original moments used is described below and are identified as Tables 7.4, 7.5, and 7.6.

These results appear to be remarkable for only two reasons. First, the point estimates and chi-square statistics are quite close to those found above in the standard grid search above. Graphs of simulations using these parameters and using the grid search parameters are virtually indistinguishable, and so the qualitative results described below are essentially unchanged. (For convenience, I have used simulated data from the grid search for the qualitative analysis below, but it looks essentially the same if I had used simulated data using these parameters.) Second, the statistics indicate an even poorer fit than the grid search yielded! This result is perhaps more
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^*$</td>
<td>ln(1.9604)</td>
<td>4309.69</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>$1.09965 \times 10^{10}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.000120127</td>
<td>58752.1</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.0018</td>
<td>511700</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.8</td>
<td>5736320</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 0.577535

Table 7.4: DFP Algorithm Results for DEM/ITL Using Initial Point Estimates From Table 7.1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^*$</td>
<td>ln(1.0230)</td>
<td>1347.50</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>1.0</td>
<td>$2.71213 \times 10^{11}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>-0.0000977190</td>
<td>102493</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.0012</td>
<td>135603</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.5</td>
<td>1347340</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 0.00306223

Table 7.5: DFP Algorithm Results for DEM/ITL Using Initial Point Estimates From Table 7.2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^*$</td>
<td>ln(1.02854)</td>
<td>1401.45</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.999972</td>
<td>$3.88521 \times 10^{13}$</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.00113319</td>
<td>3154.16</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.00675813</td>
<td>834.577</td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>0.0100471</td>
<td>381808</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 0.0293778

Table 7.6: DFP Algorithm Results for DEM/USD Using Initial Point Estimates From Table 7.3
intriguing, as it points broadly to the power and usefulness of the simple grid search technique, especially in highly nonlinear models such as the one I study. Use of the nonlinear optimization algorithms has done nothing more than confirm the correctness of my previously analysis using a simple grid search. Of course this is comforting, as it asserts that the remainder of my conclusions are undamaged.

However, one asks if these results are essentially linked to my choice of moment conditions. As an alternative, consider moment conditions related to the position of the exchange rate relative to central parity. Including moments based on this idea would tend to enhance an analysis of the parameters, as it focuses on the U-curve prediction of the S-curve model. However, if the U-curve that emerges from empirical analysis is relative rather than absolute, and I shall show below that the point estimates generated above imply that the U-curve in EMS data is a relative rather than an absolute U-curve, then there is little reason to believe these moment conditions will contribute substantially to the analysis.

Consider for the sake of analysis the use of eight moment conditions: the first four moments of $\Delta s$, its first two autocovariances, and the first and second moment of the deviation of the exchange rate from central parity, described mathematically as $(s - c)$ and $(s - c)^2$. Tables 7.7, 7.8, and 7.9 summarize these results, again using the DFP algorithm for the nonlinear optimization.

I am, of course, disheartened by the results generated. When comparing these results to those identified above, two things stand out. First, the point estimates generated are close to those found earlier, except that the point estimates of $\theta$ appear to be lower than those found previously by a factor of 10. Since this parameter determines the degree of drift exhibited, and capturing the drift is an important part
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\phi}^*$</td>
<td>ln(1.06040)</td>
<td>15670.6</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>1.0</td>
<td>$3.72862 \times 10^9$</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>-0.0000176995</td>
<td>67896.4</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.00180457</td>
<td>119032</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.8</td>
<td>25708900</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 13.6888

Table 7.7: DFP Algorithm Results for DEM/ITL Using Alternative Moment Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\phi}^*$</td>
<td>ln(1.02305)</td>
<td>1034.54</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>1.0</td>
<td>$2.46332 \times 10^9$</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>-0.00000609877</td>
<td>17071.5</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.00120856</td>
<td>17394.6</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.5</td>
<td>12401200</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 18.0834

Table 7.8: DFP Algorithm Results for DEM/FRF Using Alternative Moment Conditions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Point Estimate</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\phi}^*$</td>
<td>ln(1.002270)</td>
<td>29548.488</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>1.0</td>
<td>$6.2608243 \times 10^{10}$</td>
</tr>
<tr>
<td>$\hat{\theta}$</td>
<td>-0.0000141992</td>
<td>918238.88</td>
</tr>
<tr>
<td>$\hat{\sigma}$</td>
<td>0.00150192</td>
<td>858382.16</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>0.9</td>
<td>813134730</td>
</tr>
</tbody>
</table>

$\chi^2$ goodness of fit test statistic = 2.1047417

Table 7.9: DFP Algorithm Results for DEM/USD Using Alternative Moment Conditions
of bringing the observed and simulated series close together, missing the drift by that wide a margin should lead to a poor fit between observed and simulated exchange rates. Naturally, that leads to the second difference, that of much higher chi-square test statistic values. The natural explanation is that since this technique consistently underestimated the drift parameter, the example least dominated by a drift (the DEM/USD case) yields the best results, while those examples strongly dominated by a drift (the DEM/ITL and DEM/FRF cases) yield chi-square values that reject the goodness-of-fit test.

As an alternative, I searched for parameter estimates using Powell’s algorithm. For each of three exchange rates studied, the parameter estimates under this algorithm exhibited a similar pattern: point estimates for \( \sigma \) dropped slightly and for \( \theta \) dropped dramatically as point estimates for \( \alpha \) shot up to the neighborhood of 1000-4000. Point estimates for \( u_{star} \) and \( p \) remained fairly steady, although their estimates did move around somewhat in the optimization process. Although 4000 does not appear to be an economically reasonable estimate of \( \alpha \), one does ask if it does at least yield reasonable standard error estimates. I examined that question for this case and discovered disturbing results. For two of the three cases (DEM/ITL and DEM/FRF), complex-valued standard error estimates emerged, implying negative values along the diagonal of the variance-covariance matrix. For the remaining case (DEM/USD), the standard errors were every bit as high as in other cases. Chi-square values were also similarly high. Thus, I reject the Powell estimates, and conclude that the algorithms merely identified a economically meaningless set of hills in the quadratic loss function.

Consequently, I am left with perhaps a more puzzling result. The high standard errors generated in the grid search are neither the result of a poor grid search nor
improperly identified. In fact, high standard errors appear to be a normal part of this model. The question is why this is so. I will argue that there is a two-part answer to this question. First, the parameters enter the S-curve function in a highly nonlinear way, and so are difficult to identify, so that the rank-order of standard error estimates gives a clue about how easy or hard it is to identify each of these. Second, even if we assume the estimated parameters to be correct (ignoring the standard errors for the moment), the implied S-curve effect must be incredibly small – in fact, so small that it is unobservable for the vast majority of the observed period. Therefore, even if the model is correctly specified, I will argue that the S-curve is essentially an untestable proposition, as I cannot discern its existence even if I assume it to be true and merely seek to correctly parameterize the model.

7.3 Implications of the Analysis

As I stated above, I am left with a puzzle: why such bad standard errors? The answer to this will be first to rely on additional statistical tests, then to examine the model more carefully to uncover the deeper reasons behind the result.

The first piece of good news comes in noting results of the chi-square test. In the results from the grid search, the test statistic does yield better results in the cases of targeted exchange rates than in the cases of freely floating exchange rates. The chi-square test statistic for the DEM/USD case is 41 times higher than that for the DEM/FRF case, and the same test statistic for the DEM/USD case is 30 times higher than that for the DEM/ITL case.

The second piece of good news comes in understanding possible sources of upward bias in the standard error estimates. The great puzzle of the standard errors comes
from examining the simulated and observed time series paths of the same exchange rate. Figures 7.1, 7.2, and 7.3 show the dilemma in greater detail, using point estimates from the grid search. In the first two cases (the targeted exchange rates), the fit between the two series appears quite strong. In the third case (a free float), the fit between the two series is terrible. Numerically, I calculate correlation coefficients between the observed and simulated series for each of the three exchange rates. For the DEM/ITL case, it is 0.9426. For the DEM/FRF case, it is 0.9520. But for the DEM/USD case, it is -0.03143. While the first two appear to be good fits of the data, the third appears to be a terrible fit. Yet in each case, the parameter values leading to these results is the set that minimized the quadratic loss function in SMM simulations, and each has similarly terrible standard errors attached to it. Why the gap between poor econometric evidence of target zone and good anecdotal evidence of target zones?

The primary reason for the poor DEM/USD fit appears to be the selection of $\hat{p} = 0$; that is, that the “target zone” between the U.S. dollar and the Deutschmark is defended with complete certainty, and neither the U.S. Federal Reserve nor the Bundesbank ever realigns the target zone. Not only is this result out of touch with reality (in fact, no target zone was placed on the USD/DEM exchange rate), but it is also out of touch with the data (the period 1979-1992 included the Great Appreciation of the early 1980s and the Great Depreciation of the late 1980s). So this point estimate is especially puzzling.

Further, one must explain the remarkably high standard error estimates for $\alpha$, the interest semielasticity of money demand. I discovered through trial and error that $\alpha$ only affects the value of the quadratic loss function if $p$, the probability of
realignment, is less than one, and even in that case only has small effects on the value of the quadratic loss function. At \( p = 1 \), the S-curve effects are negated, the theoretical model reverts to \( s_t = m_t + v_t + \alpha \theta \), and \( \alpha \) and \( \theta \) are estimated jointly. However, the estimated value of \( \theta \) does not appear to vary much as the values of other variables change, while the estimated value of \( \alpha \) is quite sensitive to the estimated value of other variables. This makes identifying the value of \( \alpha \) particularly difficult and prone to error. I posit that this is also related to the high standard errors.

Perhaps there are other explanations for the high standard errors beyond this. One might prove (I make no such attempt) that an SMM-based test simply yields low power. Below, I argue that the remarkably small nature of the S-curve effect is at least partially to blame. But I doubt incorrect choices of parameter values are at fault, at least for most of the parameter values in question. Naturally, I dispute the DEM/USD estimate of \( \hat{p} = 0 \), but I view this as an exception rather than part of a rule. Furthermore, I performed a sensitivity test on the standard error calculation algorithm by plugging in bogus point estimates. Standard errors derived from those wrong point estimates proved to be even higher than the ones cited above. For those reasons, I do not doubt at a basic level that the point estimates identified above are correct, or at least nearly correct.

Further, one can quantify this problem. Using Maple’s symbolic manipulation software, I can program my S-curve function into a computer, find the partial derivatives of the function with respect to each parameter value, then plug in values for the point estimates and therefore find the marginal effect of changes in each variable on the overall function. To do this in a general setting, I select point estimates that are broadly reflective of the point estimates identified above. Further, I choose values for
the variables in the model that reflect points where the S-curve effect is strongest, thus permitting the parameter estimates to have their greatest possible impact. I note that if the theory is true, the S-curve should have zero impact at central parity and therefore none of the point estimates should have any impact, save the drift term. Therefore the only truly usable data points are those where the exchange rate is near the edge of the bands. This argument shall take on central importance as I proceed in this section.

Using Maple, I enter the S-curve in functional form, then assign parameter values that make the S-curve effect most pronounced. Specifically, I assign the values $c_1 = m_t = 0$ (a normalization), $c^* = \ln(1.06)$, $v^* = \ln(1.0602)$, $\sigma = 0.0015$, and $p = 0.5$. I then have the computer calculate $\frac{\partial s}{\partial \alpha}$. A long and complicated-looking function results. I then plug in the value $\alpha = 1$ and assign different values to $\theta$. In every case, $\frac{\partial s}{\partial \alpha}$ comes out to be equal to the value I have plugged in for $\theta$, just as one would expect if the S-curve effect were absent and the equation being estimated were merely $s_t = m_t + v_t + \alpha \theta$. Given that analysis, I have no reason to expect any estimation technique to produce an accurate assessment of the value of $\alpha$ with any degree of accuracy whatsoever, as it is impossible to detect it in theory, even with assuming the theoretical model is true and parameters take on reasonable values. Other parameters yield very small effects on $s$, but none as small as $\alpha$ yields.

Given the drawbacks of the standard statistical tests, how does one then distinguish targeted exchange rates from nontargeted? Ex ante, I thought that targeted exchange rates could reject a test where the null hypothesis was that the probability of realignment was equal to one, while untargeted exchange rates would fail to reject such a null hypothesis. This is obviously not so, due to absurdly high standard errors
on all estimated parameters. Of course, even the DEM/USD exchange rate should show some, although small, effects of targeting, due to the Plaza accord.

The DEM/USD exchange rate should show some evidence of targeting. There were some efforts to control the rise of the dollar during the Great Appreciation of the early '80s. The Plaza Accord of 1985 reflects a desire of the G-7 countries to intervene in the foreign exchange markets so as to prevent the further appreciation of the dollar. The dollar subsequently sank in value during the Great Depreciation of the late '80s, so that the dollar in the early '90s was worth about as much on foreign exchange markets as it was a decade earlier. Obviously it would be difficult for a target zone model of the nature of the one I cited to pick up the Great Appreciation and Great Depreciation, but the parameter values, even if statistically insignificant in all ten cases for \( \theta \) and \( \sigma \), did show sufficient variation to account for movements like the actual movements of the dollar in the 1980s, although as special bizarre realizations of the innovations rather than in expected value.

An alternative test would be a simple comparison of simulated realignments to the actual realignments observed in the data. Mizrach [27] lists both dates and sizes of realignments for the DEM/ITL and DEM/FRF exchange rates. Obviously the DEM/USD did not experience any realignments in the sense of target zone realignments, as it was not targeted. But I display those as a point of comparison only. This test should be intriguing in that realignment and defense as practiced in the ERM is different than that assumed in theory. First, intervention often takes place between the bands ("intramarginally") rather than at the edge of the band ("marginally"). Second, new central parities are set differently at different realignments, so that a 6% shift in one realignment and a 3% shift in the next realignment is not viewed a
<table>
<thead>
<tr>
<th>Action</th>
<th>DEM/ITL</th>
<th>DEM/FRF</th>
<th>DEM/USD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Defend, DEM revalued</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Defend, DEM devalued</td>
<td>1</td>
<td>10</td>
<td>12</td>
</tr>
<tr>
<td>Realign, DEM not revalued</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Realign, DEM not devalued</td>
<td>10</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 7.10: Simulated Realignments and Defenses

fundamental violation of the target zone regime rules, as it is considered in my model. So, can a tightly (perhaps too tightly) modeled abstraction of the ERM come close to mimicking observed behavior? Tables 7.10 through 7.14, again calculated using the estimates from the grid search, make the case that it can.

Table 7.10 summarizes the number of simulated realignments and defenses for all three simulated exchange rates. I note that the simulated DEM/ITL and DEM/FRF mimic observed realignments in one important way: there are no devaluations of the DEM/ITL or DEM/FRF exchange rate, only revaluations. Details on how those realignments mimic observed realignments appear below. Defenses are more difficult to compare to observed data, as defense in the real world is often a continual process and not a once-and-for-all event, as realignments are. So ultimately, the test of how good a fit the parameter $p$ is not that it be equal to some set value between zero and one but rather that the number of implied realignments be close to the number of observed realignments.
In the case of the DEM/ITL exchange rate, this number (10) is predicted exactly. Both the simulated and observed DEM/ITL exchange rate contain ten realignments over 3,390 data points. In the case of the DEM/FRF exchange rate, this number (6) is predicted to be 11, almost double the number observed. But upon closer inspection, it would be hard for my model as specified to guess correctly. Unlike the DEM/ITL case, realignments in the DEM/FRF case were often quite big. In fact, for three realignments in the early 1980s, the realignment was a shifting of central parity of over 8%, while the bandwidth remained at $2\frac{1}{4}$%. Since my model would force all realignments to be shifts of central parity by exactly the size of the bandwidth, my model would have to produce more realignments in order to put the exchange rate on a path close to the observed path. That process is observed quite well in Figure 7.2. So 11 realignments is not unreasonable in such an environment. In fact, as I mentioned in Chapter 4, shifting central parity by more than the bandwidth carries a heightened risk of collapse of the target zone regime, depending on how credible the relevant monetary authorities are.

At this point, I present detailed information on both the observed and simulated realignments, so as to more directly compare them and describe how well and how poorly the simulations match the observed data. For the DEM/ITL case, Table 7.11 identifies the dates on which observed and simulated realignments occurred, where the simulated dates are the data points where simulated realignments took place, then mapped back into a list describing the calendar dates that match up to each observed data point. Table 7.13 gives summary statistics comparing the observed and simulated realignments. Note that two major events happened in September 1992 for the DEM/ITL exchange rate. First, a realignment on the 14th designed as
<table>
<thead>
<tr>
<th>DEM/ITL simulated</th>
<th>Business days until next realignment</th>
<th>DEM/ITL observed</th>
<th>Business days until next realignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/13/1979</td>
<td>284</td>
<td>3/13/1979</td>
<td>136</td>
</tr>
<tr>
<td>10/20/1982</td>
<td>506</td>
<td>10/5/1981</td>
<td>171</td>
</tr>
<tr>
<td>10/24/1984</td>
<td>270</td>
<td>6/14/1982</td>
<td>192</td>
</tr>
<tr>
<td>1/13/1989</td>
<td>245</td>
<td>4/7/1986</td>
<td>192</td>
</tr>
<tr>
<td>1/4/1990</td>
<td>330</td>
<td>1/12/1987</td>
<td>752</td>
</tr>
</tbody>
</table>

Table 7.11: Dates of Observed and Simulated Realignments, DEM/ITL

<table>
<thead>
<tr>
<th>DEM/FRF simulated</th>
<th>Business days until next realignment</th>
<th>DEM/FRF observed</th>
<th>Business days until next realignment</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/13/1979</td>
<td>139</td>
<td>3/13/1979</td>
<td>136</td>
</tr>
<tr>
<td>9/27/1979</td>
<td>523</td>
<td>9/24/1979</td>
<td>508</td>
</tr>
<tr>
<td>10/30/1981</td>
<td>148</td>
<td>10/5/1981</td>
<td>171</td>
</tr>
<tr>
<td>6/7/1982</td>
<td>171</td>
<td>6/14/1982</td>
<td>192</td>
</tr>
<tr>
<td>2/10/1983</td>
<td>71</td>
<td>3/21/1983</td>
<td>761</td>
</tr>
<tr>
<td>1/30/1984</td>
<td>574</td>
<td>1/12/1987</td>
<td>1430</td>
</tr>
<tr>
<td>5/16/1986</td>
<td>110</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/23/1986</td>
<td>787</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12/11/1989</td>
<td>224</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10/31/1990</td>
<td>170</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7/8/1991</td>
<td>302</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.12: Dates of Observed and Simulated Realignments, DEM/FRF
<table>
<thead>
<tr>
<th>Statistic</th>
<th>Simulations</th>
<th>Observations</th>
<th>Observations(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of realignments</td>
<td>10</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Median of time between realignments</td>
<td>283</td>
<td>192</td>
<td>192</td>
</tr>
<tr>
<td>Standard deviation of time between realignments</td>
<td>115.6</td>
<td>250.7</td>
<td>241.9</td>
</tr>
<tr>
<td>Maximum time between realignments</td>
<td>506</td>
<td>752</td>
<td>752</td>
</tr>
<tr>
<td>Minimum time between realignments</td>
<td>78</td>
<td>2</td>
<td>156</td>
</tr>
</tbody>
</table>

Table 7.13: Summary Statistics for Realignment Timing, DEM/ITL

\(^a\)Calculates summary statistics as if the final realignment of 14 September 1992, which was an unsuccessful attempt to save the dying DEM/ITL target zone, had never occurred.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Simulations</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of realignments</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>Median of time between realignments</td>
<td>171</td>
<td>192</td>
</tr>
<tr>
<td>Standard deviation of time between realignments</td>
<td>224.0</td>
<td>476.4</td>
</tr>
<tr>
<td>Maximum time between realignments</td>
<td>787</td>
<td>1430</td>
</tr>
<tr>
<td>Minimum time between realignments</td>
<td>71</td>
<td>136</td>
</tr>
</tbody>
</table>

Table 7.14: Summary Statistics for Realignment Timing, DEM/FRF

81
a last-minute attempt to keep Italy in the ERM. Second, final Italian abandonment of the ERM on the 16th, the day after I cut off the observed exchange rates, as that exchange rate truly belongs to a different, freely-floating, regime. So I also calculate summary statistics that assume no additional realignment took place on the 14th, so that the anomalous time between the final realignment and collapse of 2 days is pushed out of the way. For the DEM/FRF case, comparable tables describing the dates of observed and simulated realignments is found in Table 7.12, while summary statistics are provided in Table 7.14.

An examination of Tables 7.11 and 7.12 makes clear one strong difference between observed and simulated realignment durations: they are more similar in the simulations and more dispersed in the observed data. An examination of the path of observed exchange rates in both cases reveals a strong appreciation of the Deutschmark up through the late 1980s, and no clear trend after that point.

Several explanations have been cited. Li [22] and Domínguez and Kenen [9], among others, cite the signing of the 1987 Basle-Nyborg accords which allowed for closer cooperation between the various central banks of the ERM, which I discussed in detail above. Others recognize an important shift in German (then West German) fundamentals around late 1988 and early 1989: the influx into West Germany of refugees from the collapsing East German state. Through a long series of events, refugees from East Germany slowly, then quickly moved through several Warsaw Pact states willing to admit them, then eventually forward them to West Germany, where West Germans and the West German state (through constitutional provision) offered care and assistance. This process came to a head when the Berlin Wall was opened on 9 November 1989, the West German Deutschmark and the East German
Östmark were united into a currency union on 1 July 1990, and finally the German Democratic Republic (East Germany) was assimilated into the Federal Republic of Germany (West Germany) on 3 October 1990. German policy goals switched rapidly toward the care of 16 million former East German citizens and the rebuilding of eastern Germany from the ruination of almost 45 years of Communist rule.

I made no *ad hoc* attempt to model such accords or events. Rather, I considered the target zone regime as a single, continuous regime from March 1979 until September 1992, subject to occasional realignment, and did not attempt to break the data in an *ad hoc* fashion into subperiods simply to provide better goodness-of-fit measures or lower standard errors, although that is precisely what arises from such an effort.

### 7.4 How Small Is the S-curve? Does That Explain Why Parameters Are Hard to Identify?

As I stated above, one of the remaining questions is to ask why the standard errors found above are so large. One reason cited, but not fully explored, is that perhaps S-curve effects are too small to be properly identified. That is, S-curve effects may be statistically insignificant precisely because they are economically insignificant.

To explore this idea, begin with the presumption that the point estimates identified above are correct, ignoring the high standard errors. Then use the simulation to explore the size of the nonlinear (S-curve) terms, thus estimating the size of the S-curve. With this information, I can then determine if this problem is significant.

How to isolate the S-curve effect? Begin by recalling equation 6.2. For simplicity, restate this equation as

\[ s_t = m_t + v_t + \alpha \theta + \kappa_t \]
where $\kappa_t$ denotes the nonlinear S-curve effect first identified by Krugman [19]. Moving terms, I find $\kappa_t = s_t - (m_t + v_t + \alpha \theta)$. Substituting my point estimates for the parameter values, I can estimate this S-curve effect in the simulation as
\[
\hat{\kappa}_t = \hat{s}_t^{SIM} - (\hat{m}_t^{SIM} + \hat{v}_t^{SIM} + \hat{\alpha} \hat{\theta})
\].

Figures 7.4 - 7.6 graph out $\hat{\kappa}_t$ for the DEM/ITL, DEM/FRF, and DEM/USD cases. All three show the nonlinear or "S-curve" effect to be both very small and not relevant for the vast majority of observations. The figures in all three cases appear to be flat lines at zero, except for occasional spikes. These spikes correspond to periods where the simulated exchange rate approaches one or the other edge of the target zone band, in line with the S-curve idea that arbitrageurs would shy away from the edges of a target zone band as its edge was approached. If I alternatively graphed the exchange rate, its monetary fundamentals, and their difference ($\hat{\kappa}_t$) together, $\hat{\kappa}_t$ would look like a fine powder lying on the horizontal axis while the other two would be virtually indistinguishable.

But these terms are in logarithms, and are difficult to relate to the foreign exchange quotes in the newspaper we are all familiar with. The question remains: how to relate the S-curve effect to the real world? To answer that, I convert the log spot exchange rate and its monetary fundamentals (which would be exactly equal in the present model without any target zone being in place) and consider the difference between those two to be the S-curve effect in levels. Formally, let $S_t^{SIM} = \exp(s_t^{SIM})$ and $F_t^{SIM} = \exp(m_t^{SIM} + v_t^{SIM} + \hat{\alpha} \hat{\theta})$, so that $\hat{K}_t = S_t^{SIM} - F_t^{SIM}$. $\hat{K}_t$ therefore must represent the difference between the observed exchange rate quote (in levels) in the newspaper and the quote one would observe if there were no S-curve effects.
Figure 7.4: S-curve Effect ($\kappa_t$), DEM/ITL
Figure 7.5: S-curve Effect ($\kappa_t$), DEM/FRF
Figure 7.6: S-curve Effect ($\tilde{\kappa}_t$), DEM/USD
Figure 7.7: Levels S-curve Effect ($|\hat{K}_{ij}|$), DEM/ITL
Figure 7.8: Levels S-curve Effect (|$\hat{K}_d$|), DEM/FRF
Figure 7.9: Levels S-curve Effect (|$\hat{K}_i$|), DEM/USD
<table>
<thead>
<tr>
<th>Cutoff Points (fraction of $S_1$)</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to $10^{-6}$</td>
<td>3097</td>
<td>91.4%</td>
</tr>
<tr>
<td>$10^{-6}$ to $10^{-5}$</td>
<td>117</td>
<td>3.5%</td>
</tr>
<tr>
<td>$10^{-5}$ to $10^{-4}$</td>
<td>115</td>
<td>3.4%</td>
</tr>
<tr>
<td>$10^{-4}$ to $10^{-3}$</td>
<td>61</td>
<td>1.8%</td>
</tr>
<tr>
<td>$10^{-3}$ to $10^{-2}$</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>$10^{-2}$ to $10^{-1}$</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>$10^{-1}$ and higher</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 7.15: Distribution of Levels S-curve Effect ($|\tilde{K}_t|$), DEM/ITL

<table>
<thead>
<tr>
<th>Cutoff Points (fraction of $S_1$)</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to $10^{-6}$</td>
<td>2988</td>
<td>88.1%</td>
</tr>
<tr>
<td>$10^{-6}$ to $10^{-5}$</td>
<td>215</td>
<td>6.3%</td>
</tr>
<tr>
<td>$10^{-5}$ to $10^{-4}$</td>
<td>136</td>
<td>4.0%</td>
</tr>
<tr>
<td>$10^{-4}$ to $10^{-3}$</td>
<td>51</td>
<td>1.5%</td>
</tr>
<tr>
<td>$10^{-3}$ to $10^{-2}$</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>$10^{-2}$ to $10^{-1}$</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>$10^{-1}$ and higher</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 7.16: Distribution of Levels S-curve Effect ($|\tilde{K}_t|$), DEM/FRF

<table>
<thead>
<tr>
<th>Cutoff Points (fraction of $S_1$)</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 to $10^{-6}$</td>
<td>2809</td>
<td>82.9%</td>
</tr>
<tr>
<td>$10^{-6}$ to $10^{-5}$</td>
<td>280</td>
<td>8.3%</td>
</tr>
<tr>
<td>$10^{-5}$ to $10^{-4}$</td>
<td>192</td>
<td>5.7%</td>
</tr>
<tr>
<td>$10^{-4}$ to $10^{-3}$</td>
<td>92</td>
<td>2.7%</td>
</tr>
<tr>
<td>$10^{-3}$ to $10^{-2}$</td>
<td>17</td>
<td>0.5%</td>
</tr>
<tr>
<td>$10^{-2}$ to $10^{-1}$</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>$10^{-1}$ and higher</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 7.17: Distribution of Levels S-curve Effect ($|\tilde{K}_t|$), DEM/USD
Figures 7.7 – 7.9 graph out a histogram of the levels S-curve effects ($\tilde{K}_t$), while Tables 7.15 – 7.17 lists the numbers used in the figures to make the points easy to see numerically. The bins chosen in the histogram are grouped so as to identify how significant (as a fraction of the exchange rate) the S-curve effect is. In other words, the bin “0 to $10^{-6}$” would identify S-curve effects that are significant to less than six significant digits, the bin “$10^{-6}$ to $10^{-5}$” would do the same for effects significant to six significant digits, the next would be for five significant digits, and so forth. Thus, in the case of the targeted exchange rates (the DEM/ITL and DEM/FRF exchange rates), S-curve effects are not significant at six significant digits for over 90% of observations, and are never significant to one, two, or three significant digits.

Those are certainly subtle effects. But how significant are they? After all, one presumes an arbitrageur could leverage any difference in price of any arbitrarily small size into significant monetary gains. But foreign exchange quotes are only commonly quoted to four or five significant digits, and on occasion to six. The data I have done my analysis with are only significant to five significant digits. That means that in the DEM/ITL case, even if the S-curve theory is true, Table 7.15 tells me that I should only notice an S-curve effect 1.8% of the time, and fail to notice it 98.2% of the time. In the DEM/FRF case, that percentage falls to 1.5%, and therefore says that I should miss the S-curve effect 98.5% f the time. And that small percentage represents points where the exchange rate approaches one band or another, and everyone involved knows that a realignment or defense choice is rapidly facing the appropriate monetary authorities.

This is hardly an economically significant and large force for smoothing exchange rate instability. Worse, even if the effect is true, that means the data a researcher uses
to look for the S-curve will offer no useful information for 98.2% of the observations in the DEM/ITL case, 98.5% in the DEM/FRF case. So much for the advantage of a thick data set of 3,390 observations. Once one considers the relevant 1.5%-1.8% of that, the researcher is left with around 50–60 observations. This number is hardly enough to do OLS, let alone a more sophisticated and subtle econometric method like SMM.

That said, what (if anything) is important about exchange rate target zones? The only apparent difference can be seen in Figures 7.1 – 7.3. Consider a separate calculation, where the monetary authority never intervenes and the shock term \( u_t \) is the only stochastic variable to affect the exchange rate. In the figures, this corresponds to the graph labeled “cfact”. This counterfactual exchange rate slowly diverges from the simulated exchange rate, and eventually (after 3,390 simulated iterations) takes on a very different value than the corresponding simulated targeted exchange rate. Why? With target zones, an exchange rate is subject to a series of monetary policies designed to promote lower exchange rate volatility. But that effect of controlling drift through a commitment to a sequence of monetary policy interventions appears to have a much stronger effect on the direction of exchange rates than the S-curve effect ever did. In other words, if a monetary authority commits to the goal of exchange rate stability, they may achieve that goal, but only through continual monetary intervention, and not because of a special nonlinear S-curve effect.

Figures 7.10 – 7.12 address this point clearly. Monetary shocks, represented by the “staircase” shaped line and measured along the right axis, is seen to have a profound effect on the direction of the exchange rate, represented by the “jiggly” line and measured on the left axis. These monetary interventions occur upon defense of
Figure 7.10: Simulated Monetary Shocks (right axis) on the Simulated DEM/ITL Exchange Rate (left axis)
Figure 7.11: Simulated Monetary Shocks (right axis) on the Simulated DEM/FRF Exchange Rate (left axis)
Figure 7.12: Simulated Monetary Shocks (right axis) on the Simulated DEM/USD Exchange Rate (left axis)
the target zone, and therefore shock an exchange rate away from one of the edges of the band and toward central parity instantaneously.

A final issue to address is the distribution of deviations from central parity. That is, I wish to address the question of whether the U-curve is absolute, is merely relative, or simply cannot be found. One of the arguments of Bertola and Caballero [2, 3] is that exchange rates under target zones ought to yield deviations from central parity \((s_t - c_t)\) in the terminology of this paper) that are U-shaped. That is, the distribution ought to have large amounts of mass near the edges of the target zone band and small amounts near central parity. However, observed data have the opposite distribution; that is, a hump-shaped distribution, with most of the mass near central parity and a lesser amount of mass near the bands of the target zone.

An examination of the simulated data generated in my model shows that the model also yields hump-shaped distributions. Figures 7.13–7.15 above display how that proposition is contradicted. Like the other figures above, this analysis is generated by examining simulated runs of the model, using the same seed values for the shocks as in the econometric tests, and assuming the estimated parameters are the true values. In each of the three cases studied, the simulated deviations from central parity exhibit a hump-shaped distribution, although in some cases the hump is more pronounced than in others.

At first this seems surprising, as the S-curve effect is so small. If these deviations from central parity are hump-shaped, could freely floating exchange rates generate a hump-shaped distribution as well? With a small amount of introspection, the answer is, in fact, yes. Consider a nonstochastic exchange rate with zero drift, summarized
Figure 7.13: Histogram of Deviations of the Simulated DEM/ITL Exchange Rate from Central Parity
Figure 7.14: Histogram of Deviations of the Simulated DEM/FRF Exchange Rate from Central Parity
Figure 7.15: Histogram of Deviations of the Simulated DEM/USD Exchange Rate from Central Parity
as the equation $s_t = c$. The time path of this exchange rate would be constant, exactly at its central parity. This case can be thought of as the ultimate hump-shaped distribution, with all mass at central parity and no mass at the bands. Consider instead a nonstochastic exchange rate with a nonzero drift, summarized by the equation $s_t = \theta t + c$. The distribution of deviations from central parity would be uniform between central parity and the edge of the band that the exchange rates drifts to, which depends on the sign of $\theta$. Finally, consider a stochastic exchange rate with a Gaussian innovation term, following the standard definition of a random walk. The distribution of a random walk is well-known to be bell-curve shaped (or hump-shaped), and adding mass to the center of this distribution through occasional (although trivial) realignment will add to the hump-shaped character of that distribution. The only force that may push mass toward the edges would be something like the “honeymoon effect” associated with the S-curve, which I have described above as the S-curve effect.

So I weaken Bertola and Caballero’s [2, 3] argument to the proposition that target zone regimes tend to make the distribution of exchange rates from their central parity more U-shaped than they would have been otherwise. In other words, I ask whether the honeymoon effect yields a U-shaped distribution of deviations from central parity.

To answer the question, I ran two dummy simulations: one where the target zone is perfectly credible, and the other where there is no target zone arrangement, so the monetary authority always realigns. In each, I entered dummy parameter values that mimic the experience of the DEM/ITL and DEM/FRF exchange rates. Specifically, I assume that the initial value of the exchange rate (in logs) is zero, that the bandwidth is 2.25%, and that the parameters of the model would be set at $\nu^* = \ln(1.0227)$, $\alpha = 1$, $\theta = -0.0001$, and $\sigma = 0.001$. In the first dummy run, I set
\( p = 0 \), and in the other I set \( p = 1 \). A comparison of the distributions of the deviation from central parity for each will help to address the question in common terms.

Figures 7.16 and 7.17 above summarize the results. Although both appear to be hump-shaped, Figure 7.17 is more hump-shaped than Figure 7.16. That is, if I further weaken Bertola and Caballero's [2, 3] assertion to say that exchange rate target zones tend to move mass away from central parity and toward the edges of the bands, then I have a correct assertion. In other words, targeted exchange rates should spend more time near the margins than freely floating exchange rates, but both freely floating and targeted exchange rates spend a tremendous amount of time near their "central parities" (in the latter case just after realignments and defenses).

What, then, do exchange rate target zones do? They do not yield substantial decreases in the volatility of exchange rates; S-curve effects, if true, must be effectively nil for over 98% of the business days simulated. Whether they exist or not, I fail to detect them using a simulated method of moments analysis. The model I develop is useful insofar as it makes reasonable guesses on the number of realignments likely to occur. In some circumstances, such as the DEM/FRF exchange rate in the late 1970s and early 1980s, I predict realignment dates with remarkable precision (the first three realignments are predicted within 17 business days, while the fourth is predicted to occur about a month before it actually did), while at other times the predictions are wildly far from the mark. These predictions are all the more remarkable insofar as I do not update these predictions to account for the innovations observed. Innovations are only included to the extent that they aid my prediction of the drift and variance parameters in the estimation. The only important effect of target zone regimes is to serve as a way of committing a monetary authority or authorities to a series
Figure 7.16: Histogram of Deviations of a Simulated Exchange Rate from Central Parity Assuming Perfect Credibility
Figure 7.17: Histogram of Deviations of a Simulated Exchange Rate from Central Parity Assuming a Free Float
of monetary interventions designed to promote the goal of exchange rate stability, perhaps at the expense of other goals such as price stability, promoting investment, or reducing unemployment.

In the end, I am left to question the desirability of target zones on normative grounds. After all, the act of a monetary authority maintaining a target zone, or conducting monetary policy in general, can be thought of as a sort of "buy high, sell low" strategy, in that they systematically buy up underperforming currencies rejected by the market and sell off overperforming currencies accepted by the market in an attempt to regulate their price. If this were truly a profit-making strategy, one wonders why relatively large (to the size of the market) arbitrageurs do not conduct similar strategies so as to shore up the market and personally profit. Put simply, if one argues that central banks profit from their various interventions, why doesn't George Soros conduct monetary policy in competition with the Federal Reserve? The answer is that monetary policy is essentially costly, although costly in terms of interest and dividend payments rather than in terms of the overall prices of the assets held. If target zones really do yield social benefits not achievable by individuals in the market, and if central banking is essentially costly, one could imagine a sort of cost-benefit analysis characterizing the question of whether a policymaker ought to target exchange rates or not.

Of course, such analysis is impossible in the present context. Not only would one have to carefully measure the costs of conducting monetary policy (which is tough enough), one would have to determine the function describing how socially beneficial a given reduction in exchange rate volatility is to a given country, a given region, or perhaps the world at large. But the above analysis of the small economic significance
of target zones yields an intuitive answer. The costs of conducting monetary policy ought to be fairly large, and moreover, permanent. Monetary policy, like all economic activity, involves opportunity costs, and as the old saying goes, there is no such thing as a free lunch. However, the benefits of target zones, which one presumes increase with the implied reduction in exchange rate volatility, are shown in this present study to be rather small, and of course, temporary. What is left? A sketchy outline of the normative case against target zones: they may yield benefits, but the small and temporary benefits are likely to be outweighed by the large and permanent costs.
CHAPTER 8

CONCLUSION

In this work, I propose an alternative to both the simple first-generation models of imperfectly credible target zones and their more complicated descendants. This framework allows for simple policy analysis of target zones, allowing for analysis of policy alternatives which feed back on the market-generated S-curve effect. This framework is extended to account for the possibility of foreign reserve crises, and therefore offers a generalization of the problem of a monetary authority pursuing the goal of exchange rate stability.

I show theoretically that target zones cannot destabilize exchange rates, but rather may have either a large or small effect on them. This idea stands in contrast to the received wisdom in the literature, and changes the question that the researcher must ask when studying target zones from “Is this target zone stabilizing or destabilizing?” to “Does this target zone yield large or small effects on the exchange rate I am studying?”

Empirically, I fail to find evidence to prove or disprove the existence of the S-curve effect in both the two EMS exchange rates that I study (DEM/ITL and DEM/FRF) and in the freely floating exchange rate that I study (DEM/USD). I offer as an
explanation the idea that even if true, the S-curve effects must be so small as to make their detection and estimation nearly impossible.

Using the point estimates generated from my econometric analysis, I address several puzzles in the target zone literature, including the inability of the literature to find evidence of target zone effects and the hump-shaped distribution of deviations of targeted exchange rates from their central parities. In short, this work offers both a simplification of target zone models and, in its simplicity, an attempt to resolve many of the puzzles facing researchers in this area of international finance.
BIBLIOGRAPHY


