Essays on Industrial Organization

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

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2010

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ABSTRACT

The three essays in this dissertation deal with three separate problems in modern Industrial Organization. Two of the essays are theoretical, dealing with the combination of price discrimination and exclusion, and the excessive risk taking behavior that is thought to have contributed to the recent difficulties of the economy. The third essay provides an empirical analysis of the effects of the expansion of the Internet and the associated reduction in consumer search cost.

The first essay, “Price Discrimination and Exclusion in Intermediate Input Markets,” assesses the role of a ban on price discrimination in intermediate input markets as mandated by the Robinson-Patman Act. Based on parameter values representing intermediate firms’ technology and efficiency levels, we are able to identify the welfare effects of the ban on price discrimination. The basic result depends on the degree of asymmetry between intermediate firms. If the technologies and efficiency levels of intermediate firms are moderately asymmetric, a ban on price discrimination in the input market will improve consumer welfare. However, if intermediate firms are very asymmetric, such a ban on price discrimination will reduce consumer welfare. This result can assist the antitrust authorities in separating different cases which will facilitate the enforcement of antitrust law. There is an argument in the extant literature that if price discrimination in the input market improves consumer welfare, then it must reduce the total social welfare. In contrast, by endogenizing the input supplier’s
decision to exclude one of the intermediate firms under uniform pricing, we show that
input price discrimination can retain more firms and competition in the final product
market, therefore, improve both consumer and total social welfare simultaneously.

The second essay, “How Online Purchasing Has Affected US Airline Ticket Pricing,” studies how airline ticket pricing changed after the introduction of the online travel agency. We find evidence supporting that increased search intensity associated with online ticket sales may lead to lower ticket prices and a decrease in price dispersion. The paper also discusses the welfare impact on two types of consumers. Simulation results show that too much competition online could potentially harm a group of consumers who do not search intensively for lower ticket price. For already competitive routes, adding more competition in terms of distribution channels can result in increased price dispersion and harm a non-searcher group of consumers. We find this prediction is supported by the data. These results have policy implications for regulation of online distribution channels and also point out that price dispersion can be a useful indicator for consumer welfare.

In the essay, “Why Do CEOs Take Risk,” I provide a new explanation for the corporate CEO gambling behavior that is widely identified as proximate cause of the recent economic crisis. In particular, the essay explains a manager’s risk taking as arising from inefficient incentive schedules in the presence of utility that follows the tenets of Prospect Theory. I show how the managerial effort and risk taking respond to the change in pay-for-performance sensitivity. I give a necessary condition for a higher pay-for-performance sensitivity to be able to reduce excessive risk taking and induce manager’s effort at the same time.
ACKNOWLEDGMENTS

I give thanks to my parents, who have always been supportive and understanding.

I am indebted to my advisor, Professor Marvel for his continuous encouragement, help and invaluable suggestions. Many thanks are also due to Professor Ye, for his concern, constant support, and help in my difficult times. I thank Professor Yang for his guidance. Through him I became better in thinking and writing. I also own thanks to Professor Lewis for teaching me and developing my interest in the subjects of IO. I thank Professor Blau for helpful suggestions. I thank Professor Miranda for teaching useful computation methods. Last but not least I thank the Department for providing a very good environment of learning.
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CHAPTER 1

PRICE DISCRIMINATION AND EXCLUSION IN INTERMEDIATE INPUT MARKETS

The paper assesses the role of a ban on price discrimination in input markets as mandated by Robinson-Patman Act. Based on parameter values representing intermediate firms' technology and efficiency levels, I am able to identify whether or not a ban on price discrimination is welfare enhancing. The basic result depends on the degree of asymmetry between the intermediate firms. If the technologies and efficiency levels of intermediate firms are moderately asymmetric, a ban on price discrimination in input market will improve consumer welfare. However, if the intermediate firms are very asymmetric, such a ban on price discrimination will reduce consumer welfare. This result can assist antitrust authorities in separating different cases, which will facilitate the enforcement of antitrust law. There is an argument in the extant literature that if price discrimination in the input market can improve consumer welfare, then it will reduce the total social welfare. In contrast, by endogenizing input supplier’s decision to exclude one of the intermediate firms under uniform pricing, we can show that input price discrimination can retain more firms and competition in the final product market, therefore, improve both consumer and total social welfare simultaneously.

1.1 Introduction

Upstream firms whose products reach their ultimate consumers only after passing through intermediate firms often face an array of choices that vary in efficiency, efficacy and competitiveness. Thus a manufacturer of a computer processor chip can deal with a variety of OEMs who differ in the efficiency of their operations and the
visibility of their brands, while the seller of a branded product must structure its
distribution by choosing whether to sell through big box stores, specialty retailers,
Internet shops, and more. The intermediate stage can be structured with an ar-
ray of contractual provisions, including vertical restraints and integration, and can
also give rise to price discrimination by the upstream producer, who may favor some
downstream partners without wishing to shut off sales through alternatives.

The problem of price discrimination in input markets has long attracted the
attentions of both economists and policymakers. The ability of a powerful buyer,
A&PG—the first major U.S. supermarket chain—to achieve substantial discounts from
suppliers caused considerable pain for “Ma & Pa” independent grocers whose out-
cry contributed substantially to the passage of the Robinson-Patman Act1. The
Robinson-Patman Act bans price discrimination unless the discriminating firm pro-
vides substantial evidence either that the price differences across retailers were cost
justified or a showing that lower prices to particular intermediate firms were in re-
response to competitive pressures. More recently, the rise and fall of a succession of
retail distribution formats—big-box stores, then mass merchants such as Walmart and
Target, high turnover, low inventory ”club stores” such as Sam’s Club and Costco,
and more recently Internet merchandisers such as Amazon, have brought attention
to the ability of some distributors to negotiate input purchases at favorable prices
compared to rivals, with the expectation that the disfavored rivals are thereby less
able to compete effectively.

American public policy and Robinson-Patman Act\(^2\) in particular prohibit price discrimination in input markets.\(^3\) This generates debate in the literature as to whether such a ban on price discrimination has beneficial or adverse effects. Due to the complexity of production processes, a model that can explicitly address intermediate firms’ differences in both input utilization efficiency and processing efficiency is needed in order to assess the role of input price discrimination in a wider range of industries.\(^4\)

Yoshida (2000) provides a model that is flexible enough to cover intermediate firms’ asymmetries in both input utilization and processing efficiencies. His model yields a striking welfare result that if third-degree input price discrimination increases total output of the final products (thereby increasing consumer welfare), that price discrimination must \textit{reduce} total social welfare. In other words, when input price discrimination can increase consumer welfare, it is accompanied by a distortion in production allocation that always dominates the gain in consumer welfare. This result contrasts with that obtained from models of price discrimination in a final product market\(^5\), where an increase in output (consumer welfare) is a necessary condition for total social welfare improvement. Yoshida’s surprising welfare implication is based


\(^{3}\) More precisely, the Robinson-Patman Act amended section 2 of the Clayton act by declaring unlawful sellers who attempt “to discriminate in price between different purchasers of commodities of like grade and quality” where the effect of the discrimination is to lessen competition substantially.” In practice, courts have interpreted the competition requirement as satisfied by a showing of injury to a competitor. See O’Brien and Shaffer (1994).

\(^{4}\) Most of the extant literatures (Cf. Katz (1987), Degraba (1990)) use a vertical relation between a monopoly manufacturer and downstream retailers as background where each unit of input is transformed into one unit of final product. However, the production process is usually more complicated than the simple manufacturer-retailer relation. For example, the upstream supplier could be a monopoly manufacturer who provides a patented input chemical and the intermediate firms use this input to produce a final product drug. Since the technologies downstream firms use are different, they may need different amount of input to produce the same final product and their processing cost could vary.

\(^{5}\) To achieve the welfare improvement, a gain in the form of output increase is needed to offset the distortion associated with price discrimination. See, for example, Schmalensee (1981), Varian
on the assumption that every intermediate firms always operate. When one of the intermediate firms is too inefficient, the upstream monopoly has the option to exclude the intermediate firm by simply withholding a favorable price for that firm. This essay incorporates the possibility of exclusion in this context.

In particular, this essay provides a simple vertical relationship between an upstream input producer and two intermediate firms where the upstream firm has the option to force one of the intermediate firms out of the market, if it finds profitable to do so. Because of this exclusion option, the number of intermediate firms may vary across the discriminatory and nondiscriminatory pricing regimes. We can show that the surprising welfare result from Yoshida (2000) does not always hold. When both intermediate firms are indeed in operation, we can confirm that if price discrimination leads to output increase then the total social welfare is diminished. On the other hand, we show that a rule against price discrimination is more likely to cause exclusion. As a result, by retaining intermediate firms, input price discrimination can increase output and total social welfare simultaneously.

Under this setting, we reevaluate the impact of a ban on third-degree input price discrimination on consumer welfare. Our evaluation can assist antitrust authorities in separating different cases which will facilitate the enforcement of antitrust law. The main result depends on the degree of asymmetry between the intermediate firms. Generally speaking, if input utilization and processing efficiency levels of intermediate firms are moderately asymmetric, a ban on price discrimination in input market will improve consumer welfare by lowering intermediate industry’s input cost. However, if (1985) and Schwartz (1990). For other papers challenging this welfare implication in final product market see Adachi (2005) and Galera and Zaratiegui (2006)
intermediate firms are very asymmetric, such a ban on price discrimination will cause
exclusion of intermediate firms and as a result reduce consumer welfare.

We also compare the upstream supplier’s incentive to vertically integrate and
invest across discriminatory and nondiscriminatory regimes. A ban on price discrimi-
nination gives the upstream supplier more incentive to vertically integrate into one of
the intermediate firms, and such integration can improve consumer welfare. As to
the upstream firm’s investment decision, we discuss two types of investments. For a
demand-increasing investment, the incentive to invest depends on the intermediate
industry’s input cost. For a cost-reducing investment, the incentive to invest depends
on intermediate good output in each pricing regime.

Based on the assumption of the upstream firm’s market power, the literature
on intermediate input market can be categorized into two classes. The first class,
represented by Katz (1987) and Inderst and Valletti (2009), discusses the behavior
of a constrained monopoly supplier and emphasizes the effect of downstream outside
option of input supply. Their models can generate input price differentials that are
consistent with those commonly observed. The second class, represented by Degraba
(1990) and Yoshida (2000), assumes an unconstrained monopoly supplier. This paper
belongs to the second class. Because of the extreme market power an unconstrained
monopoly supplier can exercise, the exclusion option needs especially to be considered
in this environment. Recently the possibility of exclusion in a vertical relationship
has been discussed by Marx and Shaffer (2007). They emphasize the retailers’ buyer
power and the role of an upfront payment in preventing small manufactures from
obtaining adequate distribution. Here we focus on a ban on price discrimination and
the relationship between output and welfare. Following Katz (1987) and Yoshida
(2000), we assume piecewise linear pricing for input and do not model more complex contracts in the vertical relationship. Recently Inderst and Shaffer (2009) and O’Brien (2008) have began to study the input price discrimination allowing both nonlinear pricing and bargaining between parties.

In the next section, the model and assumptions are provided. We derive the condition under which the upstream supplier wants to use the option of exclusion. Then we evaluate the consumer welfare across the discriminatory and uniform pricing regimes. A comparison with the Yoshida (2000) result is provided. In the extension section, we discuss the upstream supplier’s vertical integration and investment decisions in the presence of a ban on input price discrimination. All proofs can be found in the appendix.

1.2 The Model

We assume a monopoly producer of a product, which it sells to intermediaries who then sell the product to final consumers. For simplicity, we restrict the intermediate stage to two firms, indexed by \( i \in \{1, 2\} \). The producer’s product is homogeneous—the intermediaries do not transform or otherwise differentiate their products. We assume that the activities of the intermediaries do not include demand-increasing promotional activities or other services that could be subject to free riding, so that we can take final consumer demand for the product as given. Intermediate firms choose quantities to order independently, and place their entire stocks on the market, so that their competition is Cournot-Nash. They do, however, differ in their technology/efficiency, a difference that can be reflected both in the number of units of the producers’ product necessary to deliver a unit to consumers, and in the marginal
cost of each intermediary’s operation. We normalize the amount of input required by firm 1 to produce a unit of output to one, and denote the corresponding input required by firm 2 for a unit of output as $r > 0$. We follow the existing literature in assuming that the monopoly supplier can make take-it-or-leave-it offers to intermediate firms, but that these offers must consist of a fixed price per unit, with each intermediate firm permitted to choose the number of units it will acquire. If price discrimination is not permitted, the monopoly supplier offers a price $K$ to each firm. If discrimination is allowed, the monopolist sets prices of $k_i$, $i \in \{1, 2\}$.

While either firm may be more efficient in its ability to convert the product input into consumer output ($r \gtrless 1$), firm 2 is the less efficient in terms of marginal processing cost. Thus, we normalize firm 1’s constant marginal processing cost to 1, and denote firm 2’s constant marginal processing cost by $c > 1$. Inverse consumer demand is given by $p = a - Q$, where $Q = Q_1 + Q_2$ is the sum of the quantities ($Q_i$) brought to market by the two intermediaries. The relation between the intercept $a$ and the processing cost of the less efficient processor is constrained as $1 < c < \frac{a+1}{2}$. We show below that this restriction ensures that firm 2 will operate when price discrimination is permitted. That is, the restriction guarantees that firm 2’s inefficiency is not so large as to render it meaningless. Similarly, we adopt the restriction $r > 0.5$. Without this restriction, firm 2 could be so efficient relative to firm 1 in transforming input into output that it would operate in the absence of price discrimination no matter what its processing cost inefficiency.
To summarize, the total marginal cost of production under input price discrimination for each intermediate firm will be:

\[
C_1 = 1 + k_1, \text{ and } \quad C_2 = c + rk_2
\]

(1.1)

where \( r > 0.5, 1 < c < \frac{a + 1}{2} \).

If the producer is not permitted to discriminate, and instead sets a common price \( K \) for input products, the marginal costs in (1.1) become

\[
C_1 = 1 + K, \text{ and } \quad C_2 = c + rK.
\]

(1.2)

where \( r > 0.5, 1 < c < \frac{a + 1}{2} \).

We take the supplier’s marginal cost of production to be zero to simplify expressions.

This model is very similar to that considered by Yoshida (2000), who solved a general version with \( n \) intermediate firms. Yoshida showed that a monopoly producer can benefit by favoring its less efficient intermediaries, thereby inducing increased downstream competition. Given the homogeneous product, consumers benefit as well, purchasing more at a lower price than would otherwise have prevailed. Yoshida’s surprising result is that while this discrimination benefits both the producer and consumers, social welfare declines, since the inefficiency generated by shifting sales to less efficient intermediaries exceeds the benefits consisting of the sum of higher supplier profits and increased consumer surplus.

We show here that Yoshida’s result holds only if the helping hand that the producer chooses to lend to its less efficient intermediary is comparatively small. That
is, Yoshida assumes that without discrimination, the producer sets $K$ low enough so that the number of intermediaries that carry the producer’s product is unchanged. However, the upstream supplier will always have the option of raising its input price sufficiently high so as to exclude the less efficient intermediary, and it may wish to do so if it cannot selectively benefit the less efficient firm to keep it in operation. By considering possibility that the less efficient firm can disappear in the absence of discriminatory assistance, we in essence endogenize the number of intermediate firms. In contrast, Yoshida (2000) assumes that the output of each intermediate firm is positive under both pricing regimes.

We solve for the producer’s profit-maximizing choice of price or prices by first solving the two intermediate firms’ profit maximization problems. Each intermediate firm’s problem under Cournot competition is

$$\max_{Q_i} [P(Q_i + Q_{-i}) - C_i] \cdot Q_i$$

which yields

$$Q_i = \max \left\{ \frac{a - 2C_i + C_{-i}}{3}, 0 \right\}. \quad (1.3)$$

We then substitute this/these solutions into the producer’s problem, obtaining the solutions when price discrimination is and is not permitted.

### 1.2.1 Price Discrimination

When the input producer is permitted to price discriminate, its problem is that of choosing input prices $k_i$ that solve\(^6\)

\(^6\)Superscript describes the pricing scenario. ‘dis’ means price discrimination allowed, ‘uni’ means uniform pricing with both firm operating and ‘one’ means only one intermediate firm in operation. Subscript gives the name of the firm, for example ‘1’ means intermediate firm 1.
\[
\max_{(k_1, k_2)} \Pi^{\text{dis}} = k_1 \cdot Q^{\text{dis}}_1 + k_2 \cdot rQ^{\text{dis}}_2
\]

Substituting the results of the intermediate firms’ problem in (1.3) into this expression and optimizing, we obtain the producer’s price for each intermediate firm,

\[
k_{1\text{dis}} = \frac{a - 1}{2}
\]

\[
k_{2\text{dis}} = \frac{a - c}{2r}
\]

These prices yield manufacturer profit given by

\[
\Pi^{\text{dis}} = \frac{(a - c)^2 + (a - 1)^2 + (c - 1)^2}{12}
\]

Note that for the less efficient intermediary, firm 2, to produce a positive quantity, even when benefitting from favorable price discrimination, it must be the case that \(c < \frac{a+1}{2}\), as assumed above.

1.2.2 Uniform Pricing

If price discrimination is banned for sales to the intermediate market, the producer must choose a single price \(K\) that solves

\[
\max_K \Pi^{\text{uni}} = K \cdot Q^{\text{uni}}_1 + K \cdot rQ^{\text{uni}}_2
\]

As long as both intermediate firms order positive quantities. \(K\) will be chosen so that

\[
K^* = \frac{(a - 2 + c) + (a + 1 - 2c)r}{4(r^2 - r + 1)}
\]

But if \(K\) is set high enough so that it excludes intermediate firm 2, the supplier will set its price as follows: \(K^{\text{one}} = \frac{a - 1}{2}\) and realize a profit of \(\Pi^{\text{one}} = \frac{(a - 1)^2}{8}\)
The role of a second intermediate firm is simply to discipline its more efficient rival. Note, however, that if more general price discrimination were feasible, there would be no need for favoritism for an inefficient supplier. A simple two-part tariff charged to the more efficient intermediate firm would be sufficient to eliminate the downstream distortion while allowing the producer to capture all profits from the intermediate firm. But, of course, if such tariff schedules were feasible, there would be no reason for a policy banning price discrimination. Yoshida’s result is interesting because it suggests a reason for banning price discrimination that benefits inefficient firms. The model here explores the limitations of his conclusions without simply assuming them away.

Notice that when price discrimination is allowed, the upstream supplier will not want to serve only firm 1, because doing so reduces competition and generates less profit.\(^7\)

\[
\Pi_{\text{one}} = \frac{(a - 1)^2}{8} < \frac{(a - c)^2 + (a - 1)^2 + (c - 1)^2}{12} = \Pi_{\text{dis}}
\]

The upstream supplier is possible to exclude the less efficient firm, however, when there is a rule against input price discrimination. The following statement gives the threshold condition that triggers the exclusion.

*Lemma:* In the uniform pricing regime,

if

\[
c > \frac{(a - 2) + (a + 1)r - \sqrt{3}(a - 1)\sqrt{r^2 - r + 1}}{2r - 1} \equiv \hat{c}(r)
\]

then

\[
\Pi^{\text{uni}} < \Pi^{\text{one}}
\]

\(^7\)The inequality comes from the fact that \(2(a - c)(c - 1) < (a - c)^2 + (c - 1)^2\)
and if

\[ K = K^{one} \]

then

\[ Q_2^{uni} = 0 \]

If the firm 2 is too inefficient, under uniform pricing, the upstream supplier may find it not profitable to keep firm 2 in the market. It remains to be shown that the upstream supplier will charge a price that is too high for firm 2 to remain viable. The second part of the lemma insures the supplier to be able to do so. If the upstream supplier finds it profitable to only sell to firm 1 in the uniform pricing regime, it prices optimally for firm 1. This price will automatically make firm 2 unprofitable to operate.

**Proposition 1:** Uniform pricing is more likely to drive the less efficient firm 2 out of market.

Under a uniform pricing regime, the benefit of retaining the less efficient intermediate firm is to enhance the competition in the final product market, but the cost of doing so is that the upstream supplier also has to lower the input price charged to the efficient firm. When firm 2 is very inefficient, the cost can become too high so that the upstream supplier would rather exclude firm 2 from the market. However, this cost associated with retaining the less efficient firm does not occur, if input price discrimination is allowed, since the upstream supplier is free to charge different prices to different intermediate firms. As a result, exclusion is more likely to happen under a ban on input price discrimination. The threshold condition for the upstream supplier to force firm 2 out under uniform pricing is given by the lemma.
We can summarize the upstream monopolist’s action as the following. In the price discrimination regime the upstream supplier will set the input price pair \( \{ k_{dis1}, k_{dis2} \} \) specified above. In the uniform pricing regime, the upstream supplier will use Condition (1) to check whether firm 2 is efficient enough and set the uniform input price at \( K^* \) or \( K^{\text{one}} \) respectively. Then we can find the resulting final product quantities in these two pricing regimes and compare the welfare consequences between these two regimes.

### 1.2.3 Comparison

To investigate the difference between the price discrimination and uniform pricing regimes, let \( \Delta Q = Q^{\text{dis}} - Q^{\text{uni}} \) denote the difference in final product quantity between the two pricing regimes and let \( \Delta W = W^{\text{dis}} - W^{\text{uni}} \) denote the difference in total social surplus between the two pricing regimes. Let \( P^{\text{dis}} \) denote the final product price when input price discrimination is allowed and \( P^{\text{uni}} \) denote the final product price when there is a ban on price discrimination. We investigate these variables in parameter space \((c, r)\).

**Proposition 2:**

i) *(Less asymmetric intermediate firms)*

If \( r \in \left( \frac{a-c}{a-1}, 1 \right) \) and \( 1 < c < \hat{c}(r) \), then \( \Delta Q > 0 \) \((P^{\text{dis}} < P^{\text{uni}})\).\(^8\)

ii) *(Very asymmetric intermediate firms)*

If \( r > 0.5 \) and \( \hat{c}(r) < c < \frac{a+1}{2} \), then \( \Delta Q > 0 \) \((P^{\text{dis}} < P^{\text{uni}})\).\(^9\)

iii) *(Asymmetric intermediate firms)*

\( ^8\)Two firms in operation under uniform pricing in case i)

\( ^9\)Only one firm in operation under uniform pricing in case ii)
If \( r \in (0, \frac{a-c}{a-1}) \cup (1, 2) \) and \( 1 < c < \hat{c}(r) \), then \( \Delta Q < 0 \) \( (P_{\text{dis}} > P_{\text{uni}})\).\(^{10}\)

The above proposition separates the parameters space \((c, r)\) with the sign of \( \Delta Q \). The result is illustrated in Figure 1.1. Corresponding to Case i), in the triangle formed by line (1),(2) and (4), price discrimination gives consumers a lower price and both intermediate firms are in operation. Corresponding to Case ii), in the area to the right of line (4), \( \Delta Q > 0 \) and price discrimination is better for consumers, because uniform pricing will force out one of the intermediate firms resulting in a high price. Corresponding to Case iii), in the triangle formed by line (2),(3) and (4) and the area below line (1), \( \Delta Q < 0 \) and \( P_{\text{dis}} > P_{\text{uni}} \). The uniform requirement here plays a role to reduce the intermediate industry’s input cost and benefit final consumers.

Notice that the triangle formed by line (1),(2) and (4) corresponds to the proposition 3 of Yoshida (2000) where a higher consumer surplus resulting from input price discrimination implies a lower total surplus of consumers and manufactures combined. In Yoshida (2000)’s notation, the result is \( \Delta Q > 0 \Rightarrow \Delta W < 0 \). We can show that we have a consistent result in this triangle.

**Corollary 1:** (Corresponding to case i)

If \( r \in (\frac{a-c}{a-1}, 1) \) and \( 1 < c < \hat{c}(r) \), then \( \Delta Q > 0 \) and \( \Delta W < 0 \).

This confirms that in this simple setting as long as both intermediate firms are operating, we have the similar result that \( \Delta Q > 0 \Rightarrow \Delta W < 0 \). However, since here it is possible that one of the intermediate firm is forced out of market because of uniform pricing, we can also get the reverse result that \( \Delta Q > 0 \) and \( \Delta W > 0 \). This reverse result can happen in the area to the right of line (4) in Figure 1.1 and is stated in the following statement.

\(^{10}\)Two firms in operation under uniform pricing in case iii)
Corollary 2: (Corresponding to part of case ii)\textsuperscript{11}

If $\hat{c}(r) < c < \frac{17a+29}{46}$, then $\Delta Q > 0$ and $\Delta W > 0$.

The major result of Yoshida (2000) is surprising that if price discrimination in the input market makes consumers better off, it will harm the total social welfare. It seems to provide a dilemma to social planner who must choose between the consumer welfare and total social welfare. But Yoshida (2000) may omit the upstream supplier’s force-out option which can lead to a reverse result. In contrast, we are able to identify the condition under which both consumer and total social welfare can improve because of price discrimination. In Yoshida (2000), the deterioration of total social welfare under

\textsuperscript{11}With the assumption of $a > 1$, we have $1 < \frac{17a+29}{46} < \frac{a+1}{2}$, $\hat{c}(r)$ is decreasing in $r$ and $\hat{c}(2) = 1$, so $\exists r$, if $r > \hat{r}$ then $\hat{c}(r) < \frac{17a+29}{46}$. The set is nonempty. if $r > 0.5$, $\frac{d\hat{c}(r)}{dr} = 3(a-1)(\frac{\sqrt{2}}{2}/\sqrt{r^2} - r + 1 - 1) < 0$
price discrimination comes from the fact that price discrimination leads to distortion in the production allocation between the intermediate firms. Moreover, the cost from this distortion can outweigh the gain in consumer welfare. However, when uniform pricing is possible to exclude one of the intermediate firms, the consumers’ gain from one more source of final good production is much larger than the cost of distortion in production. As a result, under price discrimination, both the consumer and social welfare can be higher than uniform pricing.

We also give some policy suggestions in this section. When intermediate firms are very asymmetric, the price discrimination has the potential to retain more intermediate firms in the market, thus reduces final product price favoring consumers. However, when the intermediate firms’ asymmetry is acceptable to the input supplier, a uniform pricing requirement can produce better final product price for consumers, because it can reduce the intermediate industry’s input cost. The result will depend on the degree of asymmetry between the intermediate firms measured by a combination of parameter values \((c, r)\). Generally speaking, the policy suggestion is that when one of the intermediate firms (possibly a start-up with no economy of scale yet) may be out of business under uniform input pricing, allowing price discrimination in the input market can benefit final consumers. Otherwise, enforcing uniform pricing is better for consumers because it can reduce the intermediate industry’s input cost. There is a related example about Microsoft contracting its protocol license agreement. Microsoft develops product to make software developers have a better connection with Windows. In return, software developers pay royalty that is a fixed proportion of its software’s revenue. As a result, some of the start-up software developers will pay a lower fee to Microsoft and they are in better position for competition.
Figure 1.2: Difference in Total Social Surplus
Note: Social surplus under price discrimination minus that under uniform pricing

Figure 1.2 is a comparison of the total social surplus under the price discrimination scenario and the uniform pricing scenario. The surface shows the difference (Price discrimination - Uniform pricing) in the total social surplus between these two pricing scenarios. Notice that there is a jump in the difference of total welfare. If two intermediate firms are very asymmetric, it is optimal for the upstream supplier to serve only one intermediate firm under uniform pricing. Because of this reduced competition in the final product market, there will be a jump in the final product price, which will harm the consumer welfare a lot. Beyond the force-out threshold, because of this reduction in the number of intermediate firms (if uniform pricing is required), the price discrimination regime has an additional welfare benefit. Therefore, the total welfare difference of the two pricing scenarios is not a continuous function. But this does not imply that the upstream profit difference is not continuous. The upstream’s
profit is continuous under both uniform pricing and price discrimination regimes. In particular, under uniform pricing, at the threshold point, the upstream supplier is indifferent between raising the price to force one of the intermediate firms out and keeping the optimal price for both intermediate firms to operate. Both options give the upstream supplier the same profit by definition of the threshold condition. Beyond the threshold, the upstream profit is governed by the continuous solution to a single intermediate firm problem. Within the threshold, the profit is governed by the solution to an oligopoly problem. It is interesting that the decision to exclude one of the intermediate firms has a small effect on the upstream’s profit but has a big impact on the consumer welfare. The impact is magnified by an increase in $r$. As $r$ increases, the two increasingly different intermediate technologies make the upstream firm less willing to charge a uniform price that allows both intermediate firms to operate.

### 1.3 Extensions

#### Integration

In this section, we will briefly discuss the upstream supplier’s vertical integration decision in the presence of a ban on input price discrimination. Under uniform pricing and $c > \hat{c}(r)$, if vertical integration is allowed, the upstream supplier is willing to acquire the otherwise out-of-business firm 2. The reason is that by acquiring firm 2 and entering the final product market, the upstream supplier is able to prevent the efficient firm 1 from charging too large margin. Ideally, the upstream supplier wants to purchase the more efficient firm 1, but the upstream may find difficulty in taking over a profitable and strong firm.\textsuperscript{12} Even if the upstream supplier is able to integrate

\textsuperscript{12}It is more likely for the upstream supplier to acquire the less efficient intermediate firm, because the efficient one’s technology may give it some advantage in negotiation. Moreover, the managers
into the efficient intermediate firm 1, this problem will degenerate into the problem of a sole monopoly in final product market. We will instead focus on the situation in which the upstream supplier integrates vertically into the less efficient intermediate firm 2.

Denote the upstream supplier’s profit after integrating into the less efficient firm 2 as \( \Pi^{int} \), and the corresponding final product price as \( P^{int} \). The upstream firm sells the input at price \( K \) to the intermediate firm 1, and provides the subsidiary firm 2 for free. The marginal cost to two intermediate firms after integration will be \( C_1 = 1 + K \), in the efficient firm may have a higher ambition and better reason to keep the firm. Though the upstream supplier can threaten to raise the input price, it is not credible. If the upstream supplier is able to integrate into the more efficient firm 1, the setup will degenerate to the problem of a sole monopoly, because assuming firm 2 in operation, we can get the \( Q_2 = \frac{10c^2}{c^2} < 0 \) which is a contradiction. Therefore, the firm 2 will be forced out by the vertically integrated firm leaving a monopoly in both markets.
and $C_2 = c$ respectively. The optimal quantity for intermediate firm 1 will be:

$$Q_1^* = \frac{a - Q_2 - (1 + K)}{2}$$

The problem for the upstream firm is to choose both input price and the intermediate firm 2’s production:

$$\Pi_{int} = \max_{Q_2, K} [a - \frac{a - Q_2 - (1 + K)}{2} - Q_2 - c] \cdot Q_2 + K \cdot \frac{a - Q_2 - (1 + K)}{2}$$

So

$$Q_{2\text{int}} = \frac{a + 1}{2} - c$$

$$K_{int} = \frac{a - 1}{2}$$

$$Q_{1\text{int}} + Q_{2\text{int}} = \frac{a + Q_{2\text{int}} - (1 + K_{int})}{2} = \frac{a - c}{2}$$

$$P_{int} = \frac{a + c}{2}$$

Without integration under uniform pricing, the cost of keeping the less efficient firm 2 in the market can be too high. Therefore, firm 2 can be forced out under uniform pricing. But the existence of firm 2 in the intermediate market can discipline the efficient firm 1, and this can benefit both the upstream supplier’s profit and the final consumer welfare. The option of integration makes it possible to keep firm 2 in operation at a lower cost. We can further show a stronger statement that the integration into firm 2 can improve the upstream profit and the consumer welfare regardless of firm 2 being forced out or not under uniform pricing\(^\text{13}\).

**Proposition 3:** If $r > 0.5$ and $c < \frac{a+1}{2}$

then $\Pi_{int} > \Pi_{dis} > \Pi_{uni}$, $P_{int} < P_{dis}$, and $P_{int} < P_{uni}$.

\(^{13}\)Generally speaking, the integration will give a lower final product price than price discrimination regime, because the subsidiary firm 2 is effectively purchase the input for free, thus, the intermediate industry’s input cost is reduced.
Assuming firms only operate in one period and the fixed cost associated with integration is \( F \), whether the integration will happen depends on the difference between the integration benefit and cost to the upstream firm. We identify the requirement on this fixed cost for the integration to happen under different pricing regimes as follows:

**Corollary 3.1:**

1) If \( c < \hat{c}(r) \),

1.a) When \( F \in (0, \frac{(a+1-2c)^2}{12}) \) then integration will happen in both price discrimination and uniform pricing regimes.

1.b) When \( F \in \left( \frac{(a+1-2c)^2}{12}, \frac{(a+1-2c)^2}{8} + \frac{[r(a-1)-(a-c)]^2}{8(r^2-r+1)} \right) \), integration will only happen in the uniform pricing regime.

2) If \( c > \hat{c}(r) \),

2.a) When \( F \in (0, \frac{(a+1-2c)^2}{12}) \) then integration will happen in both price discrimination and uniform pricing regimes.

2.b) When \( F \in \left( \frac{(a+1-2c)^2}{12}, \frac{(a+1-2c)^2}{8} \right) \) then integration will happen only in the uniform pricing regime.

In sum, if the cost associated with the integration is relatively high, the integration is less likely to happen under the price discrimination allowed regime.

Taking into account the potential impact of integration, setting a ban on price discrimination in the input market will be a good policy choice. Since the profit increase from uniform pricing to integration will be more significant to the upstream supplier, a ban on price discrimination will give the upstream supplier more incentive to make the integration that improves the consumer welfare.\(^{14}\)

\(^{14}\)The higher profit in the integration scenario may also explain why some chain stores want to carry their own brand grocery product. They are more likely to do so, if there are few brand existing. Consumer welfare can improve similar to the case where firm 2 is brought back to operation because
In terms of total social surplus, even though the integration moves more of the production to the less efficient intermediate firm, the consumer welfare gain resulting from the integration can dominate the loss in production distortion. This can happen, for example, when firm 2 would have to exit the market under uniform pricing without integration.

**Corollary 3.2:** If \( \hat{c}(r) < c < \frac{7}{20}a + \frac{13}{20} \), then \( W^{\text{int}} > W^{\text{disc}} > W^{\text{uni}} \) and \( Q^{\text{int}} > Q^{\text{uni}} \).

The above statement identifies a region where integration gives both higher consumer welfare and total social surplus. If the less efficient firm cannot operate under the uniform pricing scenario, similar to a shift to price discrimination, the consumer welfare improvement resulting from integration generally dominates the production distortion, as long as intermediate firm 2 is not too inefficient\(^{15}\). As a result, this will give a higher total social surplus.

Without considering integration, uniform pricing can improve consumer welfare depending on whether the less efficient firm 2 can operate in the uniform pricing regime. With the option of integration, uniform pricing will not be an obstacle for the upstream supplier to keep firm 2 in operation, because the upstream supplier can bypass the uniform requirement by using integration. Because of the low upstream profit generated from a uniform pricing regime without integration, the uniform requirement can further induce the upstream to make the integration decision that can improve the consumer welfare. The policies on price discrimination and integration need to be coordinated (with a ban on price discrimination and allowing integration), of a vertical integration. Microsoft and Abbott Lab can also give examples where the upstream monopoly is eager to set up operation in the final product (applied software or final drug) market.

\(^{15}c < \frac{7}{20} a + \frac{13}{20} \)
so as to give the upstream supplier the most incentive to make the welfare enhancing integration.\textsuperscript{16}

**Incentive to Invest**

We now discuss the upstream supplier’s marginal incentive to make investments across discriminatory and nondiscriminatory regimes. There are two types of investments. One is the demand-increasing investment. This investment can increase the intercept of demand $a$.\textsuperscript{17} We denote this kind of investment as an $a$-type investment. The other kind of investment can reduce the upstream firm’s own marginal cost of producing the input. We denote this investment as an $m$-type investment.

**Demand Increasing Investment (Change in $a$)**

The effect of an increase in $a$ on the upstream profits in both pricing regimes are

$$\frac{\partial \Pi^{\text{dis}}}{\partial a} = \frac{k_1^{\text{dis}} + r k_2^{\text{dis}}}{3}$$

$$\frac{\partial \Pi^{\text{uni}}}{\partial a} = \frac{(1 + r)K^*}{3}$$

An increase in consumer demand will raise the number of final products sold. This, in turn, will increase the intermediate input quantity and the upstream supplier’s revenue. Therefore, the effect of an increase in consumer demand on the upstream supplier’s profit will be proportional to the intermediate industry’s input cost. If the upstream supplier finds it optimal to keep both intermediate firms in operation,

\textsuperscript{16}The policy suggestion is derived from this specific setting and the fact that integration can improve consumer welfare here. But in many cases, integration does not improve welfare. See for example Hart and Tirole (1990)

\textsuperscript{17}The upstream supplier can increase the final product demand by advertisement. For example, consumers may prefer Intel’s microprocessors in the personal computers they buy.
comparing the marginal incentive to make an $a$-type investment is equivalent to comparing the intermediate industry’s average input cost.\textsuperscript{18} Otherwise, if the upstream firm decides to force the less efficient intermediate firm out under a uniform pricing requirement, the derivative of the upstream supplier’s exclusion profit with respects to demand will be used to evaluate which scenario gives the upstream supplier more incentive to invest. It turns out that the reduction in the downstream competition makes the upstream profit less sensitive to consumer demand increase. The results are summarized below.

\textit{Proposition 4:}

(Less asymmetric intermediate firms)

If $r \in (\frac{a-c}{a-1}, 1)$ and $1 < c < \hat{c}(r)$, then uniform pricing gives more incentive to make $a$-type investments ($P_{dis} < P_{uni}$).

(Asymmetric intermediate firms)

If $r \in (0.5, \frac{a-c}{a-1}) \cup (1, 2)$ and $1 < c < \hat{c}(r)$, then price discrimination gives the upstream supplier more incentive to make $a$-type investments.

(Very asymmetric intermediate firms)

If $r > 0.5$ and $\hat{c}(r) < c < \frac{a+1}{2}$, then price discrimination gives the upstream supplier more incentive to make $a$-type investments.

\textbf{Cost Reducing Investment (Change in $m$)}

If the upstream supplier incurs a marginal cost $m$ to produce the input, an investment can be made to reduce the marginal cost, $m$. The corresponding threshold

\textsuperscript{18}If the upstream incurs a marginal cost $m$ to produce the input, we can adjust $a$ by $a - m$, and $k_s$ by $k - m$ and get the similar problem, so without loss of generality as in all the previous argument we assume $m = 0$ here.

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condition for the less efficient firm to remain in business will be:

\[ c < \dot{c}(r, m) = \frac{(a - 2) + (a + 1)r - A}{2r - 1} \]

where

\[ A = \sqrt{3(a - 1)^2 + 4m[(a - 2 + c) + (a + 1 - 2c)r] - 4m^2(r^2 - r + 1) \cdot \sqrt{r^2 - r + 1}} \]

The incentive to make a cost-reducing investment turns out to depend on the optimal quantity of input goods sold in the two pricing regimes. Intuitively, if the upstream supplier needs to sell a larger volume, it has more incentive to reduce the unit cost. This argument is confirmed by taking the derivative of the upstream profit with respect to the marginal cost of the input.

\[ \frac{\partial \Pi_{\text{dis}}}{\partial m} = Q_{\text{dis}}^1 + rQ_{\text{dis}}^2 \]
\[ \frac{\partial \Pi_{\text{uni}}}{\partial m} = Q_{\text{uni}}^1 + rQ_{\text{uni}}^2 \]

From a proposition\textsuperscript{19} in Yoshida (2000),

\[ Q_{\text{dis}}^1 + rQ_{\text{dis}}^2 = Q_{\text{uni}}^1 + rQ_{\text{uni}}^2 \]

Thus, both pricing regimes give the same volume of input products in transaction. Given the previous argument, this means that if both intermediate firms are in operation, price discrimination and uniform pricing give the upstream supplier the same marginal incentive to make \( m \)-type investments.\textsuperscript{20}

\textsuperscript{19} Proposition 1: Third-degree price discrimination has no effect on the aggregate quantity of the input supplied.

\textsuperscript{20} When \( m=0 \), it is easy to confirm this result. \( Q_{\text{dis}}^1 + rQ_{\text{dis}}^2 = \frac{(a+c-\frac{1}{3}) + (a+1-\frac{c}{3})r}{(a-2+c)+(r-2r)^{1/2}} \) and \( Q_{\text{uni}}^1 + rQ_{\text{uni}}^2 = \frac{(a+c-\frac{1}{6}) + (a+1-\frac{c}{6})r}{(a+c-2)+(a+1-2c)r} \).
If it is optimal for the upstream supplier to exclude the less efficient firm 2 under uniform pricing regime, then we need to compare the resulting input quantity with that in price discrimination regime so as to decide the incentive to invest. Results are summarized in the following statement.

*Proposition 5:*

1) Both intermediate firms operate

If $c \leq \hat{c}(r,m)$, then both intermediate firms operate. As a result, input price discrimination and uniform pricing give the upstream supplier the same marginal incentive to make $m$-type investments.

2) Exclusion

If $c > \hat{c}(r,m)$, then firm 2 will be excluded and price discrimination regime will give the upstream supplier more incentive to make $m$-type investments. (As long as $c < \frac{a-m+1}{2}$ and $r > 0.5$)

Under a ban on price discrimination, if the upstream supplier finds it optimal to retain the less efficient firm 2, the quantity of the input good sold will not change after imposing such a ban. As a result, the upstream supplier’s incentive to make a cost-reducing investment is the same across the two pricing regimes. If the upstream supplier finds it profitable to exclude the firm 2 under uniform pricing, the input quantity will be reduced. So in a uniform pricing regime with exclusion, the upstream supplier will be less likely to make cost reducing investments. The threshold condition for the less efficient firm 2 to remain in business is stricter when the upstream supplier has a non-zero marginal cost to produce input. The cost-reducing investment can relax the threshold condition and make firm 2 more likely to operate.
after the investment. So this cost-reducing investment can also benefit final consumers by enhancing the competition in the final product market.

1.4 Conclusion

This paper has analyzed the vertical relationship between a monopoly supplier and two intermediate firms. In particular, we assess the role of a ban on price discrimination in input markets on improving consumer welfare. We find that if one of the intermediate firms has an immediate threat of being driven out of the market, a ban on price discrimination will harm consumer welfare. Otherwise, a ban on price discrimination is welfare improving by reducing the intermediate industry’s input cost. The condition that determines whether the ban on price discrimination improves welfare or not is summarized in two parameters describing the intermediate firms’ efficiency levels and technologies of production. In the context of this model, we point out that the surprising welfare result of Yoshida (2000) is a special case that applies only for particular combinations of parameter values. In our more general setting, a reverse result can also happen. The reason is that the possibility of exclusion under uniform pricing can make the price discrimination regime more valuable to consumer welfare, so that the welfare improvement can outweigh the distortion in the production process. However, in Yoshida’s setting, the loss of production efficiency, because of input price discrimination, always dominates the consumer benefit. Notice that an upstream monopolist can use other ways to extract rents from the system more effectively. For example, if a two-part tariff is possible, the upstream monopoly may only keep the efficient intermediate firm and extract most of the surplus by an upfront fee. We do not consider this case. Our goal is to address the limitation
of Yoshida with similar assumptions about the contracts between the upstream and intermediate firms.
CHAPTER 2

SEARCHING THE WEB: HOW ONLINE PURCHASING HAS AFFECTED US AIRLINE TICKET PRICING

This paper investigates the response of air ticket pricing to the introduction of online travel agencies. Airline ticket prices are taken to be determined by price discrimination on the part of the airlines. Online travel agencies reduced consumer search costs, a reduction that had predictable consequences from a comparative statics analysis of airline pricing behavior. We find evidence that increased search intensity associated with online ticket sales results in decreased ticket prices and reduced price dispersion.

These results would appear to suggest that the introduction of online search has been an unalloyed good thing for consumers. However, it is also possible that as some consumers avail themselves of better price information, customers who do not will pay higher prices than they otherwise would. This paper investigates this possibility by modeling the welfare impact of cheaper search on two types of consumers. Simulation results show that too much competition online could indeed potentially harm a group of consumers who do not search intensively for lower ticket prices, and that the effect depends on the intensity of competition prior to the introduction of lower search costs. Empirical results provided indicate that price dispersion tends to increase after adding one more distribution channel on those routes previously subject to intense competition. These results have policy implications for regulation of online ticket distribution channels, implications that will permit policy makers to find a balance between airline profit and consumer welfare.

2.1 Introduction

This paper analyzes the effects of the introduction of online ticket sales on the prices of airline tickets. The article has two objectives. First, we wish to establish the
relationship between Internet penetration and price behavior as measured by both average prices and dispersion in those prices. It appears that the introduction of lower search costs has heightened the already intense competition between airlines. Second, we analyze the welfare consequences of such increasing competition online and demonstrate that price dispersion is a useful indicator of welfare, especially for the non-searcher group of consumers.

Owing to intense competition in the airline industry arising from nearly homogeneous products and low incremental costs, airlines achieve relatively low profit margins.\textsuperscript{1} The move from traditional travel agencies to their online counterpart is regarded to be a way to reduce distribution costs and boost airlines’ profit.\textsuperscript{2} Holding prices constant, the ability to squeeze costs out of distribution would appear to benefit the airlines. But, of course, a reduction in distribution costs that is linked to consumers’ ability to compare fares easily for themselves has the potential to increase competition and lower fares. That is, because the air tickets on different carriers serving the same route as close substitutes in the view of many consumers, Internet price comparisons may increase competition among carriers. Indeed, it is possible that the reduction in markups stemming from the increase in competition can pass more than the cost savings to consumers in the form of lower prices, so that squeezing costs out of distribution in this way may actually reduce airline profitability. When customers shop around traditional agencies, they incur a search cost. This may help

\textsuperscript{1}For example, Cheong Choong Kong, Former CEO, Singapore Airlines, has remarked that "The airline business is unlike any other business on earth. Over its entire history from day one till now, airlines as a whole have lost more money than they have ever earned." ‘Staying profitable in a lunatic industry’ \url{www.airliners.net/aviation-forums/general Aviation/read.main/1132732}

\textsuperscript{2}Alamdari and Mason (2006) estimate that distribution costs account for about 13% of the total ticket prices for tickets purchased through traditional travel agencies. Distribution costs rank third among airline operating cost components, trailing only fuel and labor. By converting to online travel agencies, airlines can reduce the distribution costs by 40%.
to maintain a profit margin. In addition, traditional agencies collect more information of customers. The move to the online travel agencies has deprived of the possibility to price discrimination based on consumers’ different search abilities.³

After investigating the data, we find Internet penetration is associated with intense competition leading to lower ticket prices and the reduced price dispersion. In addition, a higher number of traditional agencies is associated with higher ticket prices. On the other hand, we also find that in highly competitive routes, the additional competition from an extra online distribution channel could lead to an increase in the price dispersion.

The economics of airlines has generated a substantial empirical literature. Borenstein’s empirical work on ticket pricing has spawned a particular fruitful line of research. Borenstein (1989) provides estimates to show that an airline’s share of passengers on a route and at the endpoint airports significantly influence its ability to mark up price above cost, but that the higher prices of a dominant carrier on a route do not provide an “umbrella” under which prices of smaller carriers on dominated routes also rise. Borenstein and Rose (1994) study the dispersion in prices airlines charge to different passengers and argue that the data support models of price discrimination in monopolistically competitive markets. Both Borenstein (1989) and Borenstein and Rose (1994) focus on a short period of ticket data.⁴ We use data in two separate years to compare ticket pricing patterns before and after the introduction of online ticket agencies. Recently, the effect of Internet on pricing has spurred

³This is definitely good news to consumers. However, as Borenstein (1989, p. 655) have argued, given the cost structure of airlines, they may need some price discrimination in order merely to break even.

interest in the literature. Brynjolfsson and Smith (2000) find that compact disk and book prices are lower in online markets and that price dispersion is slightly smaller. Puller et al. (2008) have detailed data on airline tickets purchased through a single computer reservation system allowing them to ask what portion of fare differences is resulted from characteristics (restrictions) and what portion represents pure price dispersion. Clemons et al. (2002) find that prices available from online travel agents are just as dispersed as those available from traditional agents. Dana and Orlov (2008) identify the role of Internet in improving industry’s capacity utilization. Using a similar approach, Verlinda and Lane (2004) find that higher Internet usage increases the spread between unrestricted and restricted fares. Besides documenting the effect of Internet purchasing on average price and price dispersion, we also focus on a natural experiment provided by the launch of Orbitz to see the effect of an additional seller on market pricing behavior. Our simple example predicts that an additional seller can harm the non-searcher group of consumers and increase the price dispersion especially in highly competitive markets. The predicted increase in the price dispersion is supported by the data.

In terms of theory, when consumers differ in their costs of searching, firms may be able to segment the market by offering a distribution of prices. Salop (1977) discussed a model where a noisy monopolist uses price dispersion as a sorting device to separate consumers into sub markets to permit price discrimination. Until now, competitive analogs of Salop’s monopoly price discrimination have not been well developed. (Stole, 2003). Rosenthal (1980) and Carlson and McAfee (1983) discussed each firm offers a single price in equilibrium. Burdett and Judd (1983) developed the effect of sequential
and nonsequential search of identical consumers on price dispersion.\textsuperscript{5} Unlike Salop’s paper, they do not have multiple prices for each firm to choose. Here we want to extend Salop’s price distribution into an oligopoly (or monopoly competition) setting with each firm offering a distribution of prices for consumers. Then we can see whether the result is consistent with the data. Stahl (1989) and Janssen et al. (2005) are the few that discussed oligopoly firms offering price distributions derived from consumer searcher costs. Our model is related to their models. We have simpler assumptions on consumers’ search ability. Our simple model can generate more realistic predictions about pricing behavior under some scenarios.\textsuperscript{6}

In the following parts of the paper, we will describe the data and the basic observed pattern of price changes in section 2.2. We will do a regression to identify the factors that lead to the price changes in section 2.3. Then we use a simple model to illustrate how the intense competition online (from an increase in the number of online agents) can harm uninformed consumers in section 2.4 and we find some support from the data showing highly competitive routes actually tend to increase price dispersion after the introduction of an extra online distribution channel.

\section*{2.2 Data}

We analyze the Origin and Destination survey from the U.S. Department of Transportation (DB1B data set), which is a 10 percent random sample of all tickets that originate in the US operated by US carriers. We will compare the data in the fourth

\textsuperscript{5}Markets in the presence of sequential consumer search is also examined in Anderson and Renault (1999)

\textsuperscript{6}in particular, we assume that consumers differ in the total number of sites they compare before making decisions. We take this number as exogenous. In the case of increasing one more firm in the market, we predict that the average market price will drop slightly with searchers better off while non-searchers are harmed. In Stahl (1989) model, average price will even increase in this case.
quarter of 1995 and 2004. Because we think there were very few online tickets sold in 1995, while in 2004 it had already become very common to buy tickets online. Prices are measured as one-way fares. For round trips, the price is taken to be the half of the total ticket price. This will normalize the prices to one-way fares for the convenience of comparison, but for a few complicated itineraries, this normalization will cause problems. Therefore, we further restrict our sample to trips with no more than two stops round trip. We focus on restricted coach class. Tickets of Other classes are few in the observations, so this elimination does not affect the volume too much. This restriction will make the comparison of price distributions easier. Because the business and first class tickets are viewed as different products, including them will complicate the analysis of price distribution change. We only look at the pricing behavior of the vast majority of restricted economy class tickets. We also eliminate tickets that are less than 10, as these are most probably frequent flyer reward or key punch errors. A sample of 425 routes is chosen to do the analysis.

The number of traditional air travel agencies is collected from the census bureau. The land size for each city is obtained from Wikipedia. The Internet use is derived from current population survey 2003, which could give a relatively close estimate for the Internet use in 2004.

The average price drop for our 425 sample routes is 5 percent between 1995 and 2004.\textsuperscript{7} This price decline does not appear to have been based on labor or fuel costs. In particular, fuel cost (price per gallon) rose more than 30 percent over the period, while labor costs per available seat mile increased by 10 percent. Total operating

\textsuperscript{7}The price index from Bureau of Transportation Statistics shows that the general ticket price increases slightly by 2 percent from 1995 to 2004.
expenses per available seat mile increased by 25 percent from 1995 to 2004.\footnote{Source: MIT airline data program, http://web.mit.edu/airlinedata/www/default.html} It is therefore remarkable that prices fell as costs rose.

A comparison of price distributions in 1995 and 2004 could give us more details about how the ticket pricing has changed.\footnote{The typical route provided in figure 2.1 is DTW-LAX. The figure gives the general pattern of price distribution changes that we observe in the data. More illustrative route pairs can be found in the appendix.}

The main difference between these two distributions was that in 1995 there were two spikes located on a lower price and a higher price, however, in 2004 the higher price spike disappeared. It was as if some force had pushed the higher price spike to merge with the lower price one, and the whole distribution had moved to the left a little. As a result, the average price dropped. The US airline industry has a long history, most of the operations have already been established in the 90s.\footnote{Dana and Orlov (2008) argue that the sophisticated revenue management systems which help airlines to forecast demand and more efficiently utilize their aircraft and personnel resources were widely adopted in the 1980’s.} When we think about what has changed in these years, the only big thing that happened in
the industry is the online purchasing. This finding in price distribution change may indicate that firms are less able to price discriminate after the introduction of online ticket sales. It is less likely to sell a ticket at a premium price to some people who do not search carefully. After the introduction of online ticket sales, the search ability of most consumers has improved.

There is a gap in the 1995 histogram, while there is no gap in 2004. To quantitatively discern these two types of histograms, we calculate the distance between Q3 (75%) and the Median for each of our sample routes in respective years. For the 1995 data, if there is a gap, the Median is close to the lower price spike, and the Q3 is close to the bottom of the gap. There is 25% of the mass between Q3 and the Median. Thus if there is a gap in the 1995 distribution, we would expect a longer distance in 1995 to accumulate the 25% mass than in 2004 where no gap is observed. If we can show that the distance between Q3 and the Median shrinks from 1995 to 2004 in most of the routes, we can use it as evidence supporting the pattern of price distribution change described above.

As expected, the distance between Q3 (75 percent) and the Median (50 percent) is shortened from 1995 to 2004 in 75% of observations (routes). This is the result of filling up the gap between two peaks in the 1995 price distribution.

The distance between the Mean and the Median also shrinks from 1995 to 2004 in about 84% of observations (routes). The distance between the 90 percentile and the Median is also reduced due to a similar reason.
2.3 Regression

We run regressions to identify factors that affect the magnitude of price change. Our argument is that the traditional agencies provide the possibility of price discrimination according to consumers’ search ability. On the other hand, online agencies put competition too close and make searches easier for most of the buyers. As a result, the online travel sites lose this option of price discrimination and fall into fierce competition on prices. To test this argument, we need to identify the strength of traditional travel agencies in each route and the change of search intensity for each route due to the introduction of online ticket sales.

First, we introduce a variable of Agency Rate (AR). AR is defined as the number of traditional agencies per population in a city. If the people in the city relatively still tend to use traditional agencies, the number of agencies serving the population would be bigger. We expect AR to have a positive effect on the relative price change. A higher AR for a route means that the prices in 2004 tend to increase or at least not to drop more than average routes.

Because there are two end-point cities for each route, we first define the route agency rate from city i to city j as the average of the agency rates in each end-point city.

\[ AR_{ij} = \frac{ln(AR_i) + ln(AR_j)}{2} \]

Secondly, the distance between two agencies has a unique effect on the traditional search behaviors because people incur a travel cost that is proportional to the distance in the traditional search behavior. The distance between two agencies will affect the search intensity and agencies’ competition, so we introduce land rate (LR) to address
<table>
<thead>
<tr>
<th></th>
<th>MeanRate</th>
<th>StdRate</th>
</tr>
</thead>
<tbody>
<tr>
<td>IR</td>
<td>-0.51*</td>
<td>-1.42**</td>
</tr>
<tr>
<td></td>
<td>(-2.12)</td>
<td>(-3.70)</td>
</tr>
<tr>
<td>AR</td>
<td>0.10</td>
<td>0.56**</td>
</tr>
<tr>
<td></td>
<td>(1.93)</td>
<td>(3.05)</td>
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<tr>
<td>LR</td>
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<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(1.29)</td>
<td>(1.53)</td>
</tr>
</tbody>
</table>

\[ R^2 \] .08 .15 .27 .25 .17

Note: t-value in parenthesis

Table 2.1: Impact of the Internet on Ticket Prices

this effect. ‘LR’ is defined as the average distance between traditional agencies. LR is expected to have a positive effect on relative price change.

Further, we introduce the effect of Internet use intensity on the airline ticket price. We use the proportion of people using Internet as a measure of the online purchasing activity. The data describing the Internet use is Current Population Survey in 2003. We denote this Internet use rate by ‘IR’. We do regressions to see whether the Internet use rate is correlated with price change statistics. The regression results are summarized in the following table (2.1).

Internet use has a negative effect on average ticket prices. It is also negatively correlated with the price dispersion (standard deviation) change. The reason is that online ticket sales increase consumers’ search ability. As a result the price distribution would be more concentrated. AR is positively correlated with the price standard deviation change. The reason is that if more people still rely on traditional travel
agencies, the search ability is lower, as a result, contrary to a higher rate of Internet use, the price distribution will be more flat.

By comparing the same route before and after the introduction of online ticket sales, there are already some benefits of controlling variables that could potentially effect price. However, possibly, the demand for a route has changed over the years. The market structure and market dominance can also change. The number of tickets sold is used to indicate the demand shift. In addition, Herfindahl index is used to evaluate the market structure change. However, no significant effect on the regression is found after adding these control variables.

To illustrate the price discrimination based on consumer search argument and for further discussion of the impact of the increasing competition online as a result of the launch of Orbitz, we construct a simple model that can be used to predict the price distribution change after adding one more seller in the market.

2.4 A “Bare Bones” Model of Online Competition

Travel agencies did more than simply sell tickets. They could reveal to airlines information about consumer characteristics. Online ticket sales, in contrast, are much less revealing. Apart from exceptions due to membership in frequent flyer plans, competition is simply over posted prices, and customer information is limited to responses to these price postings.

Consider competition between two independent ticket sellers, each of which can obtain tickets from airlines at the same cost, $c$. The websites then set their ticket prices by setting markups over $c$. Denote markups by $m^{11}$. For a particular route,

\[ m^{11} \]

In the analysis, we treat prices as equivalent to markups.
one site selects its ticket price by drawing from a markup distribution denoted by $F(m)$ with density $f(m)$. The other site provides the ticket price markup from the distribution $G(m)$. The different notations for two distributions are adopted for convenience in solving optimality conditions. We will later focus on symmetric ticket sellers, that is, $F(m) = G(m)$. Customers are of two types. Loyal customers search only one site and buy without search across sites, acting as if unaware of the existence of the competing site. The remaining customers are searchers who compare prices between the two sites and choose the lower one. We assume, for each site, that the proportion of loyal customers is $(1 - x)$.

We can calculate the expected profit for the online sale sites:

$$\Pi(f) = x \int_0^M m(1 - G(m))f(m)dm + (1 - x) \int_0^M mf(m)dm$$

$$\Pi(f) = \int_0^M [mf(m) - xmg(m)f(m)]dm$$

$$\Pi(f) = \int_0^M \pi(m, f(m))dm$$

The maximization problem is to find a density function that can maximize the above profit function. According to optimization theorem, the first order condition is: $\frac{d\pi}{dm} = 0$ or

$$1 - xG(m) - xmg'(m) = 0$$

or

$$m - xmg(m) = constant$$

**Proof** by perturbation method:

Taking an arbitrary $\lambda(m)$ such that $\int_0^M \lambda(m)dm = 0$
replace the original optimal density function \( f(m) \) with the perturbed version 
\[
f(m) + \epsilon \lambda(m)
\]
integrating from 0 to \( M \)
\[
\int_0^M m[f(m) + \epsilon \lambda(m)] - xmG(m)[f(m) + \epsilon \lambda(m)] \, dm
\]
taking derivative with respect to \( \epsilon \)
\[
\int_0^M [m \lambda(m) - xmG(m)] \lambda(m) \, dm = 0
\]
\[
\int_0^M [m - xmG(m)] \lambda(m) \, dm = 0
\]
itegrate by part, we get
\[
\int_0^M \Lambda(m)d[m - xmG(m)] = 0
\]
because \( \lambda(m) \) is arbitrarily chosen, so 
\[
1 - xG(m) - xmG'(m) = 0
\]
\[
m - xmG(m) = \text{constant}
\]
Intuitively, we can think of the FOC as the condition under which there is no incentive to move any part of the density function to the left or right a little bit \( \epsilon \).
Without loss of generality, consider moving the density on any point \( m \) to \( \epsilon \) left. The benefit of doing so is attracting those customers who search two sites and find equal prices previously (earn a markup \( m \)). That benefit is \( x\epsilon G'(m)m \). On the other hand, the cost of doing so is the loss of profit when facing loyal customers who only go to one site and the searchers who have already found the price lower. Thus the cost is \( (1 - x)\epsilon + x(1 - G(m))\epsilon \). Setting the marginal benefit equal the marginal cost, we get
\[
1 - x + x - xG(m) = xG'm
\]
\[1 - xG(m) - xmG'(m) = 0\]

which is just the first order condition derived from the optimal control problem.

Focusing on symmetric equilibrium, we set \(G(m) = F(m)\).

\[F(m) = \frac{m - \text{Const}}{xm} = \frac{1}{x} - \frac{\text{Const}}{xm}\]

By the terminal condition that \(F(M) = 1\) and \(F(m) = 0\) we can get \(\text{Const} = M - xM\), \(m = M - xM\) so

\[F(m) = \frac{1}{x} - \frac{M - xM}{xm}\]

or

\[F(m) = \frac{1}{x} - \frac{m}{xm}\]

\[f(m) = \frac{M - xM}{x} \frac{1}{m^2}\]

\[\text{Mean}(m) = \int_{M-xM}^{M} mdF(m) = M - \int_{M-xM}^{M} F(m)dm\]

\[= M\left[-\frac{1}{x}ln(1 - x)\right]\]

Therefore,

\[x \to 0 \Rightarrow \text{Mean}(m) \to M, x \to 1 \Rightarrow \text{Mean}(m) \to 0\]

\[\text{Var}(m) = \int m^2dF(m) - (\text{Mean})^2 = \frac{M - xM}{x}xM - (\text{Mean})^2\]

\[= M^2[(1 - x) - (\frac{1}{x} - 1)^2(ln(1 - x))^2]\]

As shown in Figure 2.2, the mean of the offered price markup goes to zero as the proportion of two-site searchers increases. The variance of markups is not straightforward. It attains the highest level roughly when 60 percent of the people are two-site searchers. It can be seen from the facts that when all the consumers are
searchers, firms will set the same competitive price, and when all the consumers are non-searchers, firms will set the same monopoly price. Since there is no variance in these two extreme cases, a higher price variance will happen when the population is composed of a more diverse group of consumers.

The realized price for two-site searchers is the lower of the posted prices. We denote the distribution of this order statistic as $F_{(1)}(m)$, and the density as $f_{(1)}(m)$. Here ‘(1)’ means the lower price drawn from the two offer price distributions.

$$F_{(1)}(m) = 1 - (1 - \frac{1}{x} + \frac{M - xM}{xm})^2$$

$$f_{(1)}(m) = 2(1 - F(m))f(m)$$

$$f_{(1)}(m) = 2(1 - \frac{1}{x} + \frac{M - xM}{xm}) \frac{M - xM}{xm^2}$$

The realized price charged to a loyal customer who only goes to one seller is simply the price drawn from the offer price distribution. On the other hand, the realized price paid by a searcher is the lower price drawn from both sellers (order statistics). The overall realized price distribution is the weighted average of these two kinds of
price draws. By assumption, each site has a mass of \((1-x)\) loyal customers and shares a mass, \(x\), of searchers. The density function of realized prices will be:
\[
\frac{x}{2-x} f(1)(m) + \frac{2(1-x)}{2-x} f(m)
\]
The mean of the overall realized price will be
\[
\int_{M}^{M-xM} m \left[ \frac{x}{2-x} f(1)(m) + \frac{2(1-x)}{2-x} f(m) \right] dm
\]
which is
\[
\frac{2x}{2-x} \left[ \frac{(1-x)^2 M}{x^2} \ln(1-x) + \frac{(1-x)M}{x} \right] + \frac{2(1-x)}{2-x} M \left[ -\frac{1-x}{x} \ln(1-x) \right] = \frac{2(1-x)}{2-x} M
\]

Therefore, the overall realized price mean will be:
\[
Mean_r = \frac{2(1-x)}{2-x} M
\]

The variance of the realized price will be
\[
\int_{M-xM}^{M} m^2 \left[ \frac{x}{2-x} f(1)(m) + \frac{2(1-x)}{2-x} f(m) \right] dm - Mean_r^2,
\]
which is
\[
\frac{2(1-x)M}{2-x} [(x-1)M - \frac{(1-x)M}{x} \ln(1-x)] + \frac{2(1-x)}{2-x} M^2 (1-x) - \frac{4(1-x)^2}{(2-x)^2} M^2 =
\]
\[
\frac{-2(1-x)^2 M^2}{(2-x)^2} ln(1-x) - \frac{4(1-x)^2}{(2-x)^2} M^2
\]

Therefore, the overall realized price variance will be:
\[
Var_r = \left[ \frac{-2(1-x)^2 ln(1-x)}{(2-x)x} - \frac{4(1-x)^2}{(2-x)^2} \right] M^2
\]

Both the mean and the standard deviation of the realized price are lower than that of their offer-price counterparts, due to the existence of two-site searchers. The price interval representing the majority of ticket prices is also drawn. A tight band indicates that, with the competition from only two online sites, the price dispersion and extreme price observations are both reduced.

One of the problems of this model is that it simplifies the competition to only the online competition without addressing the role of different airlines. However, if
Figure 2.3: Mean Price and Standard Deviation

Realized price $mean \pm \sigma$

Realized price 90 percent interval

Figure 2.4: Realized Price Interval, N=2

Note: Realized price $mean \pm \sigma$ and Realized price 90 percent confidence interval
airlines act as whole sellers who do not interfere with the retail market and travelers view all airlines as identical, this setting can be an approximate to reality.

Another criticism may relate to the assumption of the mixed-strategy price distribution. How realistic is that assumption? For the same route, the ticket price is changing over time. Therefore, we can think that each day the price is drawn from an underlying distribution, which we derived in the model. By picking a day of departure, a customer effectively makes a draw from the underlying distribution. Two separate draws from different sites could be correlated, but for simplicity, we exclude this possibility by assuming independent draws. There is another form of price discrimination that arises from making the price as a function of the date of departure. However, we do not include this possibility and only concentrate on the price discrimination based on the different search abilities of consumers.

Some possible adjustments to the base line example include introducing asymmetric price distributions for different firms, increasing the number of firms in the market, and making the proportion of searchers vary with the price markup. In the next section, we will address the question of an increase in the number of sellers in the market as this question is related to our empirical test.

2.5 Increase to More Sites

It is straightforward to extend the example to three sites. We assume the same amount of loyal customers is divided equally among all three sites.\textsuperscript{12}

To see how the increase in the number of sellers in the market can make the offer price distribution less competitive, suppose there are three symmetric sites now. All

\textsuperscript{12}Even though the proportion of non-searchers does not change, the intensity of competition may decrease, because agency are less likely to be the lowest price draw.
the derivations above can go through without adjustment, except that $G(m)$ should be interpreted as the distribution function of the lowest price offered by all of the opponent sites. For three sites, $G(m) = 1 - (1 - F(m))^2$

$$\frac{m - \text{const}}{xm} = 1 - (1 - F(m))^2$$

by the terminal condition $F(M) = 1$, we can still get $\text{Constant} = M - xM$ so

$$F(m) = 1 - \sqrt{1 - \frac{m - (M - xM)}{xm}}$$

Use the subscript to denote the number of sites in the market.

$$F_3(m) = 1 - \sqrt{1 - \frac{m - (M - xM)}{xm}}$$

since $F_2(m) = \frac{m-(M-xM)}{xm}$

$$F_3(m) = 1 - \sqrt{1 - F_2(m)}$$

$F_3(m) - F_2(m) = [1 - F_2(m)] - \sqrt{1 - F_2(m)} < 0$

(since $0 \leq F_2(m) \leq 1$)

This means that $F_3(m)$ has more mass located at higher prices. Therefore, it is closer to a monopoly price offering.

More generally,

$$F_n(m) = 1 - \sqrt[n]{1 - F_2(m)}$$

The counter-intuitive result is that as the seller number increases, the price offering from a seller does not become more competitive, instead, consumers get an even worse price distribution of tickets.
Stahl (1989) presents an example with consistent results where, as the firm number increases, the price distribution becomes less competitive. This is because, as the firm number increases, the chance of being the lowest-priced seller decreases exponentially, thus undermining the incentive for lowering price.

Even though Stahl (1989) constructs an elegant model that endogenizes both the reservation price and the maximum of the price distribution support, the search cost is still exogenous and does not incorporate the possibility of an increasing marginal cost. Besides, in equilibrium, consumers will stop searching when they find a price no more than their reservation price. However, only when consumers visit all the sellers, could they know the whole price distribution. The assumption that consumers can understand the price distribution beforehand is not very reliable, although an elegant rational expectation equilibrium can be derived from this assumption. We assume that the proportion of consumers who make a specific number of searches
is exogenous. This assumption will simplify the problem to solve, but is still able to generate the predictions we need in terms of the effect of an additional seller on pricing.

Taking into account that the loyal customers are fewer when there are more sellers, we recalculate the model and analyze the prices that both groups of consumers are facing.

When there are 3 sites (N=3), the original total of \(2(1-x)\) loyal customers (non-searchers) are assumed to be equally divided among 3 sites. As a result, each site (seller) gets \(2(1-x)/3\) non-searchers. Each seller’s maximization problem is changed to

\[
x \int m(1 - G(m))f(m)dm + \frac{2(1-x)}{3} \int mf(m)dm, \text{ which is } (\frac{x}{3} + \frac{2}{3}) \int mf(m)dm - \int xmG(m)f(m)dm
\]

so the first order condition is

\[
\frac{\left(\frac{2}{3} + \frac{x}{3}\right)m - \text{constant}}{xm} = G(m)
\]

when N=3, \(G(m) = 1 - (1 - F_{\{3\}}(m))^2\) as a result

\[
F_{\{n\}} = F_{\{3\}}(m) = 1 - \left(1 - \frac{\frac{2}{3} + \frac{x}{3}}{x} + \frac{\frac{2}{3} - \frac{2x}{3}}{xm}\right)^{0.5}
\]

When N=3, normalize M=1. The realized price CDF (distribution function) for a searcher is

\[
F_{s\{3\}}(m) = 1 - \left(1 - \frac{\frac{2}{3} + \frac{x}{3}}{x} + \frac{\frac{2}{3} - \frac{2x}{3}}{xm}\right)^{1.5}
\]

The pdf (density function) is

\[
f_{s\{3\}}(m) = 1.5\left(1 - \frac{\frac{2}{3} + \frac{x}{3}}{x} + \frac{\frac{2}{3} - \frac{2x}{3}}{xm}\right)^{0.5}\left(\frac{\frac{2}{3} - \frac{2x}{3}}{x}\right)\frac{1}{m^2}
\]

13This assumption is not more surprising than Stahl’s assumption of a flat search cost. In particular, Stahl (1989) makes the assumption that a proportion of consumers have a common positive search cost. However, this model can not be easily adjusted to serve our purpose, as we think there are differences in searcher population across markets (routes). Therefore, we simply assume the proportion of searchers in the population is exogenous to avoid the complication of specifying search costs. Interestingly, we still get the same counter-intuitive result as Stahl (1989) that increasing the number of sellers makes the market prices less competitive.
When \( N = 3 \), the realized price CDF for a non-searcher is
\[
F_{n3}(m) = 1 - (1 - \frac{2}{3} + \frac{x}{3} + \frac{2 - 2x}{3})^{0.5}
\]
The pdf is
\[
f_{n3}(m) = 0.5(1 - \frac{2}{3} + \frac{x}{3} + \frac{2 - 2x}{3})^{-0.5}(\frac{2}{3} - \frac{2x}{3}) \frac{1}{m^2}
\]
As a comparison, when \( N = 2 \), the realized price CDF for a searcher is
\[
F_{s2}(m) = 1 - (1 - \frac{1}{x} + \frac{1 - x}{xm})^2
\]
The pdf is
\[
f_{s2}(m) = 2(1 - \frac{1}{x} + \frac{1 - x}{xm}) \frac{1 - x}{xm^2}
\]
When \( N = 2 \), the realized price CDF for a non-searcher is
\[
F_{n2}(m) = \frac{1}{x} - \frac{1 - x}{xm}
\]
The pdf is
\[
f_{n2}(m) = \frac{1 - x}{xm^2}
\]
When \( N = 3 \), we can also obtain the realized average price and variance:
\[
Mean_{3r} = \frac{x}{2 - x} \int_{\frac{2}{3} + \frac{x}{3}}^{\frac{2}{3} + \frac{2}{3} - x} f_{s3}(m) m dm + \frac{2 - 2x}{2 - x} \int_{\frac{2}{3} + \frac{x}{3}}^{\frac{2}{3} + \frac{2}{3} - x} f_{n3}(m) m dm
\]
\[
= 1.5 \frac{x}{2 - x} (\frac{2 - 2x}{3x})^{1.5} \cdot \{2(\frac{3x}{2 - 2x})^{0.5} - 2 \arctan[(\frac{3x}{2 - 2x})^{0.5}]\}
+ 0.5 \frac{2 - 2x}{2 - x} (\frac{2 - 2x}{3x})^{0.5} \cdot \{2 \arctan[(\frac{3x}{2 - 2x})^{0.5}]\}
\]

\(^{14}\)The subscript “n” means non-searcher, “s” means searcher. The number followed the letter denotes the total number of firms competing in the market. Together, for example, \( F_{(n3)} \) means the realized price CDF for non-searchers with 3 firms in the market.
Figure 2.6: Increase in Price Dispersion (Adding One More Seller)

Note: The ratio of $\text{Std}_{3\text{sites}}/\text{Std}_{2\text{sites}}$ is Increasing in the proportion of searchers. This implies that more competitive routes will incur a more significant increase in price dispersion.

$$Var_{3r} = \frac{x}{2-x} \int_{\frac{3}{2} - \frac{x}{3}}^{\frac{x}{x+2}} f_{s3}(m)m^2 dm + \frac{2 - 2x}{2-x} \int_{\frac{3}{2} - \frac{x}{3}}^{\frac{x}{x+2}} f_{n3}(m)m^2 dm - (Mean_{3r})^2$$

$$= 1.5 \frac{x}{2-x} (\frac{2 - 2x}{3x})^{1.5} \cdot \{ \arctan[(\frac{3x}{2 - 2x})^{0.5}] - (\frac{3x}{2 - 2x})^{0.5}/[1 + (\frac{3x}{2 - 2x})] \} + 0.5 \frac{2 - 2x}{2-x} (\frac{2 - 2x}{3x})^{0.5} \cdot \{ \arctan[(\frac{3x}{2 - 2x})^{0.5}] + (\frac{3x}{2 - 2x})^{0.5}/[1 + (\frac{3x}{2 - 2x})] \} - (Mean_{3r})^2$$

In Figure (2.6), the ratio between 3-site and 2-site price dispersion as a function of the searcher proportion in the population is drawn. The result shows that if the market is already very competitive (searcher majority), the increase of price dispersion will be more significant after introducing an additional seller into the market. We will test this prediction using a natural experiment given by the launch of Orbitz.
2.5.1 Comparing Searchers and Non-Searchers

The tables 2.2 below show the welfare effect of adding one more seller into the market on two different types of consumers. There are two cases. If non-searchers are in the majority of the population, increasing the number of sellers from 2 to 3 will reduce the average prices for both the searcher and non-searcher groups. On the other hand, in the case of searcher majority, adding one more seller will only reduce the searcher price, while increase the non-searcher price. As a result, non-searchers are harmed. The reason is that over competition will make sellers overreact by offering an increased probability of higher prices that may harm the non-searcher. Because both increasing the proportion of searchers and increasing the number of sellers can enhance the competition in the market, adding the number of sellers in already searcher dominated markets can cause over competition. Sellers are making so low a profit that they would rather offer a high price hoping to capture a non-searcher. This will harm non-searchers a lot. When non-searchers are in the majority, however, competition is on a low level originally. Adding one more seller in this case introduces the good competition that will drive prices down for both searchers and non-searchers.

The figure 2.7 below shows how the searcher and non-searcher price pdfs change from two sellers to three sellers. Both searcher and non-searcher price pdfs move to the left. The non-searcher group also incurs a heavy tail at the high price end. In the case of searcher majority, the effect of the heavy tail at the high price end outweighs the effect of horizontal move, therefore causes the average price to increase for non-searchers.

The idea that non-searchers are harmed after adding one more seller can be captured by the Stahl (1989) model. However, he does not separate the cases of searcher
Table 2.2: Comparison of Searcher and Non-searcher Prices

<table>
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<th>N=2</th>
<th>N=3</th>
</tr>
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<tbody>
<tr>
<td>Searcher mean:</td>
<td>0.903</td>
<td>0.870</td>
</tr>
<tr>
<td>Non-searcher mean:</td>
<td>0.927</td>
<td>0.924</td>
</tr>
<tr>
<td>Overall mean:</td>
<td>0.923</td>
<td>0.916</td>
</tr>
</tbody>
</table>

\[ x=0.15, \text{Non-searcher majority} \]

\[ x=0.85, \text{Searcher majority} \]

Figure 2.7: Effect of Number of Sellers on Price Distribution

Note: searcher and non-searcher pdf comparison varying the number of sellers and the proportion of searchers, \((x)\).
and non-searcher majority.\textsuperscript{15} In short, the above exercise shows that when facing the increased competition from a new entry into the market, sellers generally will increase the probability of higher prices. As a result, the price dispersion will increase. This effect of the increased dispersion will be so significant when the searcher type of consumers are also plentiful, that the non-searcher group will be harmed by the price dispersion. We want to test whether the more competitive routes will incur a more significant increase of the price dispersion after introducing one additional seller in the market.

Because Travelocity.com was created in 1996, as a subsidiary of Sabre Holdings, and Microsoft launched Expedia.com in 1996 as part of MSN and Orbitz.com was officially launched in 2001, as a result, the late comer Orbitz.com actually provided a natural experiment of the impact of adding one more seller in the market.

The following regression in Table 2.3 shows that after Orbitz, there are more price dispersion increases in previously competitive routes.(in terms of a lower price level and a narrower distribution before the launch of Orbitz)

In Table 2.3, the dependant variable is the price dispersion change rate after the introduction of Orbitz, and the independent variables include the relative average price and price dispersion before the launch of Orbitz to measure the implied competition level on a route. Regression suggests that the price dispersion tends to increase after Orbitz on routes that already have intense competition.\textsuperscript{16}

\textsuperscript{15}His conclusion is that non-searchers are always harmed and the average market price goes up after adding one more seller. In contrast, our example can predict that in some cases the non-searchers are harmed and the average market price goes down after adding one more seller.

\textsuperscript{16}Models about competitive type of price discrimination can predict the price dispersion increase after adding more firms in the market, but they generally do not make the welfare claim. In addition, we argue that price dispersion increases are not uniform across routes. More competitive routes will incur more price dispersion increases.
<table>
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<th>StdRate(_{04/98})</th>
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<tr>
<td>IR</td>
<td>0.75(*) (3.86)</td>
<td>0.81(*) (4.32)</td>
<td>0.79(*) (4.42)</td>
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<tr>
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<td>-1.02(*) (2.98)</td>
<td>-1.22(*) (3.71)</td>
<td>-1.26(*) (3.98)</td>
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<tr>
<td>StdRate(_{98/95})</td>
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<tr>
<td>MeanRate(_{98/95})</td>
<td>-</td>
<td>-0.64(*) (5.96)</td>
<td>-</td>
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</table>

Table 2.3: Impact of Competition Levels on Post-Orbitz Price Dispersion

### 2.5.2 Simulate the Data

Another interesting test is to see whether our simple model can simulate the price distribution change observed in the data. The 1995 distribution corresponds to a lower proportion of searchers in the population. The 2004 distribution corresponds to a higher proportion of searchers in the population due to the introduction of online ticket sales. Generally speaking, in figure 2.8, we can see two spikes when there is a lower proportion of searchers in the population. Moreover, when the proportion of searchers increases in figure 2.9, the higher price spike in the density function disappears and the whole curve moves to the left indicating a lower average price. This result is obtained by varying the proportion of searchers in the population from .65 to .85.
Figure 2.8: Simulated Market Price Density (Low searcher proportion)

Figure 2.9: Simulated Market Price Density (High searcher proportion)
2.5.3 Discussion

The international routes have capacity constraints due to the bilateral air service agreements that determine the cities served and the frequency of flights. These constraints will make international routes different from domestic ones. Due to these regulations, it is believed that the international operation can give airlines a higher profit margin. Therefore, we restrict the data analysis to only US domestic tickets. To extend this study to international ticket prices, we need to account for the effect of capacity constraints and different regulations imposed by different countries, which are relatively hard to measure.

Even though it is difficult to control the effects of regulations, a basic survey can give us some primary results. The Asia and European airlines are slow in adopting the technology of online ticket sales. However, most of the profitable airlines are in Europe and Asia. If there are enough data and information about regulations, we can run a regression of the airline profitability on the adoption of online ticket sales.

There is an alternative explanation of higher prices at traditional agencies, which is that they provide services that are not easily done by oneself online. Such services may include complicated connections. However, the routes we use to analyze price distributions are all simple routes, which do not include complicated connections. Further, we believe that the so-called personal services are just the process of assessing customer information in order to charge a specific price to maximize profit. Sometimes, this price discrimination may happen in a subtle way. It is very often that agents ask for some add-on services after a customer accepts the ticket price they quote.
Another criticism may be that the industry is cyclical and we happen to select two years that have opposite positions in the cycle. This may be able to explain the price drop, but this argument can not explain the regression result that the price change is correlated with the geographic variation in Internet penetration and the traditional agency rate. However, we can do a robustness check by using more years’ data in future work.

2.6 Conclusion

Competition among airlines, always intense due to the ease with which consumers can substitute across carriers, has been increased even further by the ability of consumers to compare prices easily using the Internet. Then it is natural to predict that the price would drop and price dispersion narrow. We found that the average price dropped after the introduction of online ticket sales in our sample routes, while airlines incurred an increase in operating costs during the period. Regression showed that over the years the rate of price dispersion change was associated with Internet penetration in endpoint cities of the route. The rate of average price change was also correlated to the Internet penetration, though to less degree of significance.\textsuperscript{17} The regression results also suggested that the more consumers relied on traditional travel agencies when purchasing tickets, the higher ticket prices tended to be. We related these findings to airline price discrimination that derived from differences in consumers’ search ability. We argued that access to online ticket sales, including

\textsuperscript{17}The R square of the regression is not big, suggesting there are other factors that will also affect the rate of change in price dispersion and average fare. The effect on average fare is not so significant may because the increased operating cost is becoming a binding point of collusion. It is possible that airlines know each other’s costs, since some research websites publish cost data. Therefore, the effect on average fare may be mitigated by airlines’ reactions to the intense competition.
posted prices from multiple airlines, decreased the cost of consumer search. In re-
response, prices fell and the distribution of prices changed in a predictable fashion. We
further argued that when the proportion of searchers was already large, the welfare
effect of adding one more online ticket agency could be mixed, with searchers better
off even while non-searchers were harmed. The predicted increase in price dispersion,
in highly competitive routes, after introducing one more seller, was supported by the
data.
CHAPTER 3

WHY DO CEOS TAKE RISK

This essay provides a new explanation for the corporate CEO gambling behavior that is widely identified as proximate cause of the recent economic crisis. In particular, the essay explains a manager’s risk taking as arising from inefficient incentive schedules in the presence of utility that follows the tenets of prospect theory. I show how the managerial effort and risk taking respond to the change in pay-for-performance sensitivity. I give a necessary condition for a higher pay-for-performance sensitivity to be able to reduce excessive risk taking and induce manager’s effort at the same time.

3.1 Introduction

Since the fall of Lehman Brothers and AIG, great public attention has focused on why CEOs in those companies want to engage in such huge systematic risk that brings about catastrophic results to shareholders and even to the whole economy. The theoretical and empirical literature on principal-agent problem, such as Holmstrom and Milgrom (1987) and Garen (1994) assumes that agents maximize expected utility. Recent developments in experiment and empirical tests \(^1\) have suggested that a reference-point-dependent utility as in prospect theory would have a greater explanatory power and flexibility in illustrating decision and risk attitudes. Post et al. (2008) have applied prospect theory to explain risk-seeking behavior evident in the “Deal or

\(^1\text{e.g. Tversky and Kahneman (1992). Post et al. (2008)}\)
No Deal" game. This is a simple setting compared to that in which managers must operate. Here I add a simple incentive scheme for managers to a setting in which their behavior is governed by prospect theory. Perhaps the interaction between the utility of CEO in terms of prospect theory and incentive scheme used by shareholders has not been well explored.\(^2\) This paper attributes risk-seeking to a combination of loss aversion and decreasing sensitivity (as pointed out by prospect theory) in the presence of a linear incentive scheme. We also try to suggest ways to mitigate risk-seeking from CEOs.

### 3.2 A Simple Discrete Model

The regular business of the firm is assumed to be a random variable \(N\) with distribution:

\[
N = \begin{cases} 
  n, & 0.5 \\
-n, & 0.5
\end{cases}
\]

The CEO can also engage in a zero sum gamble with firm’s resources. The return from the gamble is assumed to be \(mZ\). Here \(m\) measures how deeply the CEO is involved in the gamble.

\[
Z = \begin{cases} 
  1, & 0.5 \\
-1, & 0.5
\end{cases}
\quad (3.1)
\]

Shareholders cannot discern whether the return is from the normal business or the gamble. The total return is \(N + mZ\). The compensation for the CEO is the sum of base salary plus performance-based bonus

\[
w + a(N + mZ) 
\quad (3.2)
\]

\(^2\)Recently, Hart and Moore (2008) discuss the effect of reference point on the form of contract. In particular, they use entitlement and shading to explain the optimality of simple long-term contracts.
where $a$ measures the sensitivity of compensation to the firm’s return. We normalize the base salary to zero to focus on the incentive bonus.

The utility function of the manager around a reference point is

$$U(x) = \begin{cases} 
  x^\alpha, & x > 0 \\
  -\lambda(-x)^\alpha, & x < 0 
\end{cases}$$  \hspace{1cm} (3.3)

where $0 < \alpha < 1$ and $\lambda > 1$

Since the $\lambda > 1$, the utility captures the feature of loss aversion.

The executive’s problem is to choose the amount of gamble to maximize utility. We show that this combination of a simple compensation schedule and a reference-dependant utility function gives the executive an incentive to take excessive risk.

$$\max_m E[U(a(N + mZ))] = \frac{1}{4} a^\alpha (1 - \lambda)[(n + m)^\alpha + (n - m)^\alpha]$$

$$0 < m < n$$

**Proposition 1.** As long as $0 < \alpha < 1$ and $\lambda > 1$, the executive will take the risky gamble. This result is obtained especially when the individual is risk averse in the normal sense ($\alpha < 1$, risk averse in positive region).

Shareholders can, however, adjust the pay schedule in order to limit the executive’s risk taking behavior. This can work, for example, when the performance-based bonus is adjusted to treat gain and loss with different sensitivities.

$$B = \begin{cases} 
  a(N + mZ), & N + mZ > 0 \\
  b(N + mZ), & N + mZ < 0 
\end{cases}$$

$$\max_m E[U(B)] = \frac{1}{4} a^\alpha (1 - \lambda b^\alpha)[(n + m)^\alpha + (n - m)^\alpha]$$

$$0 < m < n$$
Proposition 2. As long as $\frac{b}{a} < \left(\frac{1}{\lambda}\right)^{\frac{1}{\alpha}}$, the executive will not take the risky gamble. This schedule means that shareholders treat the executive easily, by limiting downside risk and setting the target of the performance at average normal business return.

There is another way to prevent executives from excessive risk taking, that is adjusting the performance target. Denote the performance target relative to average business return as $t$. The bonus will then be

If $t > 0$

$$B = a(N + mZ - t)$$

The executive’s problem becomes

$$\max_m E[U(B)] = \frac{1}{4}a^{\alpha}[(n + m - t)^{\alpha} + (n - m - t)^{\alpha} - \lambda(n - m + t)^{\alpha} - \lambda(n + m + t)^{\alpha}]$$

s.t.

$$0 < m < n$$

Taylor expansion with respect to $m$ around zero gives

$$\frac{1}{4}a^{\alpha}[2(n - t)^{\alpha} - 2\lambda(n + t)^{\alpha} + 2\alpha(\alpha - 1)(n - t)^{\alpha - 2}\Delta m^2 - 2\lambda\alpha(\alpha - 1)(n + t)^{\alpha - 2}\Delta m^2]$$

As long as $\alpha(\alpha - 1)[(n - t)^{\alpha - 2} - \lambda(n + t)^{\alpha - 2}] < 0$, the executive will not take the gamble.

Proposition 3. if the performance target satisfies

$$1 > \frac{t}{n} > \frac{1 - \lambda^{\frac{1}{\alpha - 2}}}{1 + \lambda^{\frac{1}{\alpha - 2}}}$$

then the executive will not take the risky gamble.
For example, using the usual assumptions\(^3\) about the parameter values: \( \lambda = 2\), and \( \alpha = 0.8 \), we can get \( 1 > \frac{\mu}{n} > 0.28 \). Therefore, setting the yardstick of the firm’s return to \( \mu + 0.5n \) will be effective in deterring the executive from taking excessive risk, given the above simple assumption about the distribution of the underlying business.

In this simple setting, there are two ways to prevent executives from excessive risk. The first way may seem counterintuitive. An incentive scheme that permits the executive a share of profit in good periods but limits his exposure to losses\(^4\) would seem to encourage risk taking. However, in this setting, the reason for the executive to take the additional risk is the asymmetry of perceived loss and gain around the reference point. By reducing the downside punishment, the perceived effect of gain and loss can be balanced. On the other hand, the other way is harder for executives. Shareholders will set the yardstick high enough and not reduce the downside risk of the executive’s compensation. Setting a higher target of performance is able to move the executive towards the loss region even in a good period of normal business return. As a result, the asymmetry of perceived loss and gain could be mitigated. The executive will always have some sense of loss if taking the gamble. Shareholders should be consistent in their choices. Simply setting an easy target of average return and a constant sensitivity of compensation with respect to both gain and loss will produce risk-seeking behavior from CEOs.

It may need to be mentioned that in this simple setting, the decision to gamble is made before the realization of normal business return. In later sections, we will discuss the decision to gamble made after the normal business return is realized, and introduce

\(^3\)See for example Abdellaoui et al. (2008)

\(^4\)The yardstick of performance is set to the mean return and downside risk of executive compensation is reduced by setting \( b \) low enough and \( b < a \).
the manager’s effort that can affect the firm’s performance (return). Hart and Moore (2008) discuss the shading behavior as retaliation if a reference level of entitlement is not achieved between contracted parties. We do not focus on this shading possibility. We only consider the situation where the manager has the power to place a risky bet that has no added value to the firm except a chance to boost profit. Besides loss aversion and decreasing sensitivity, prospect theory also assigns subjective weights to probabilities of events with features of overweighing small probability and discounting large probability. The effect of these subjective weights is not discussed here.

This line of logic, that a combination of unfit compensation schedule and loss aversion results in a manager’s risk-seeking behavior, can be extended to other finite domain symmetric discrete distributions and some continuous distributions.

We generalize the result to a group of symmetric discrete distributions. If the return for normal business is of the following distribution,

\[ N = \begin{cases} 
\mu + n_i, & f_i \\
\mu - n_i, & f_i 
\end{cases} \]  

(3.4)

s.t.

\[ \sum_{i=1}^{s} 2f_i = 1 \]

then we have a similar argument.

**Proposition 4.** Given the discrete distribution of regular business return of (3.4) and the zero sum gamble distribution (3.1), loss averse CEO with utility function (3.3) and compensation schedule (3.2) will engage in the risky gamble.

**Proof.** We only need to show that taking a \( \Delta \) amount of the gamble can give the executive higher utility.

\[
E[U(N + \Delta Z)] = \sum_{j \in \{1,-1\}} \sum_{i=-s}^{s} U(n_i + j \cdot \Delta) \cdot f_i
\]

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\begin{equation*}
= (1 - \lambda) \sum_{i=1}^{S} u(n_i \pm \Delta) \cdot f_i
\end{equation*}

\begin{equation*}
= E[U(N)] + 2(1 - \lambda) \sum_{i=1}^{S} u''(n_i) \Delta^2 \cdot f_i
\end{equation*}

As long as the positive part of the utility function is concave and the executive is loss averse \((\lambda > 1)\), the executive will take the risky gamble and impose excessive risk on shareholders.

We also generalize the result to a group of symmetric continuous distributions. We have similar results, when the underlying distribution is as follows:

\begin{equation}
\begin{cases}
  f(-x), & x < -\sigma \\
  f(x), & x > \sigma \\
  0, & |x| < \sigma
\end{cases}
\end{equation}

(3.5)

This specific form (3.5) makes sure that for a small amount of gamble \(m < \sigma\), the firm’s performance will not be moved from loss region to positive region causing complication.

If the underlying return of business is of the form (3.5), \(\lambda > 1\) and \(\alpha < 1\), then the loss averse manager wants to take the risky gamble\(^5\) hoping to boost profit, because we can show that the manager’s utility with the gamble is higher than without the gamble.

\begin{equation*}
\int_{-\infty}^{0} -\lambda \frac{(-x - m)^\alpha + (-x + m)^\alpha}{2} f(-x) dx + \int_{0}^{\infty} \frac{(x - m)^\alpha + (x + m)^\alpha}{2} f(x) dx
\end{equation*}

\begin{equation*}
= (1 - \lambda) \int_{0}^{\infty} \frac{(x - m)^\alpha + (x + m)^\alpha}{2} f(x) dx > (1 - \lambda) \int_{0}^{\infty} x^\alpha f(x) dx
\end{equation*}

\begin{equation*}
= \int_{-\infty}^{0} -\lambda (-x)^\alpha f(-x) dx + \int_{0}^{\infty} x^\alpha f(x) dx
\end{equation*}

\(^5\)Assume the gamble amount satisfies \(m < \sigma\)

66
However, this logic cannot easily be taken to the limit. This result of risk-seeking does not necessarily follow, when the underlying distribution of the performance measure is a normal distribution. A counter example of normal distribution is given.

The measure of regular business performance, $B$, satisfies a normal distribution, $B \sim N(\mu, \sigma_b^2)$. The gamble, $G$, is also normally distributed, $G \sim N(0, \sigma^2)$. Therefore, the total measure of performance, $A$, depending on both the regular business and the gamble, is also normally distributed\(^6\), $A = B + mG \sim N(\mu, \sigma_b^2 + m\sigma^2)$. If the performance target is set to $\mu$, the manager’s problem is equivalent to choosing the variance of $A$ to maximize utility:

$$\max_{\sigma_A^2} (1 - \lambda) a^\alpha \int_0^\infty A^\alpha \frac{1}{\sqrt{2\pi} \sigma_A} \exp\left(-\frac{A^2}{2\sigma_A^2}\right) dA$$

$$\Leftrightarrow \max_{\sigma_A^2} (1 - \lambda) a^\alpha \frac{1}{2\sqrt{2\pi} \sigma_A} \int_0^\infty (A^2)^{\frac{\alpha+1}{2}-1} \exp\left(-\frac{A^2}{2\sigma_A^2}\right) dA^2$$

$$\Leftrightarrow \max_{\sigma_A^2} (1 - \lambda) a^\alpha \frac{1}{2\sqrt{2\pi} \sigma_A} \cdot \Gamma\left(\frac{\alpha+1}{2}\right) \cdot (2\sigma_A^2)^{\frac{\alpha+1}{2}}$$

In this situation, the manager actually wants to minimize the variance of the firm’s performance by eliminating the gamble.

In the normal distribution case, gambling can arise from the combination of setting a performance target of at least average return and limited liability on the part of the manager. This is a sufficient condition and consistent with our intuition, because

\(^6\)Alternatively, if we assume that the gamble distribution is discrete as in equation 3.1, we can show the ex-ante utility $EU = \frac{1}{2}\left[-\lambda \int_{-\infty}^0 (-x)^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+m)^2}{2}\right) dx + \int_0^\infty x^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2}\right) dx\right] + \frac{1}{2}\left[-\lambda \int_{-\infty}^0 (-x)^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2}\right) dx + \int_0^\infty x^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2}\right) dx\right]$. Therefore, $EU'(m) = (1 - \lambda) \int_0^\infty x^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x-m)^2}{2}\right) dx - (1 - \lambda) \int_0^\infty x^\alpha \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x+m)^2}{2}\right) dx$, which is less than zero when $\lambda > 1, \alpha = 0.8$. Therefore, this can give the same no-gamble result.
if there is not much to lose on the downside, the manager tends to take the gamble in order to increase the compensation.

More precisely, suppose that the measure of performance is the firm’s ranking among a peer group. This ranking is supposed to be normally distributed, and the compensation is a linear transformation of a certain year’s ranking. For example, firm A is in a group of five firms, and normally firm A ranks 3rd (the mean of the normal distribution of firm A’s ranking). The firm will get a score of zero if ranks 3rd. If the firm ends up in the 2nd place, it will get a score of 1 and so on. If the target score is set to $t$, and the firm’s realized score is $A$, then the bonus $B(A - t)$ will be proportional to $A - t$. For simplicity, we just denote the manager’s utility as $U(A - t)$. By assumption, the limited liability comes into play when the firm misses the target score by $2t$ or more. In that case, the manager will not receive a utility lower than $U(-2t) = -\lambda u(2t) = -\lambda(2t)^\alpha$. Suppose the target is designated at 2nd place, if the firm ends up in the 1st place, the manager will get a score of 1 earning 1 million dollar bonus. If the firm ends up in the 3rd place, the manager scores -1 incurring a million deduction in wage. But the deduction will not be larger than -2 million, no matter how badly the firm performs. We can calculate the manager’s ex ante utility under these assumptions, and show that the manager wants to take the risky gamble to increase the utility in a large range of performance targets values.\(^7\)

\[
EU = -\lambda \int_{t}^{\infty} u(2t) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{A^2}{2\sigma_A^2}\right)dA - \lambda \int_{-t}^{t} u(t - A) \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{A^2}{2\sigma_A^2}\right)dA
\]

\(^7\)Notice that this result is the opposite of our argument in the first discrete example. In that case a less sensitive schedule in the negative region can help reduce risk taking. But here when the performance measure is relatively precise and stable (normally distributed), providing limited liability to the manager will be a mistake as it actually encourages excessive risk taking.
+ \int_{t}^{\infty} u(A - t) \frac{1}{\sqrt{2\Pi} \sigma} \exp(-\frac{A^2}{2\sigma_A^2}) dA

Taking derivative with respect to $\sigma_A$

$$\frac{\partial EU}{\partial \sigma_A} = \lambda \int_{t}^{\infty} u(2t) \frac{1}{\sqrt{2\Pi}} \exp(-\frac{A^2}{2\sigma_A^2}) \frac{1}{\sigma^2} [1 - \frac{A^2}{\sigma_A^2}] dA$$

$$+ \lambda \int_{-t}^{t} u(t - A) \frac{1}{\sqrt{2\Pi}} \exp(-\frac{A^2}{2\sigma_A^2}) \frac{1}{\sigma^2} [1 - \frac{A^2}{\sigma_A^2}] dA$$

$$- \int_{t}^{\infty} u(A - t) \frac{1}{\sqrt{2\Pi}} \exp(-\frac{A^2}{2\sigma_A^2}) \frac{1}{\sigma^2} [1 - \frac{A^2}{\sigma_A^2}] dA$$

$\frac{\partial EU}{\partial \sigma_A}$ measures the manager’s marginal incentive to gamble. The figure 3.1 shows that the manager has an incentive to gamble in a large range of target values ($t > 0$), due to the limited liability.

Given the loss averse utility function, the executive is first-order risk averse at the reference point. This explains why when the underlying measure of performance is a normal distribution, the executive wants to reduce the risk. However, when the distribution is discrete two states choice, the bad-state positions the executive in the loss region of the utility function. Because the function is convex in the loss region, the risk taking behavior will follow. These are two polar cases, the example below illustrates how the executive is moved from risk averse to risk taking as the underlying distribution changes.

If the underlying distribution is

$$f(x) = \frac{1}{2} \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x + \mu)^2}{2}] + \frac{1}{2} \frac{1}{\sqrt{2\pi}} \exp[-\frac{(x - \mu)^2}{2}]$$

This is the average of two normal distributions with means at $\pm \mu$. Therefore, it has the features of two extreme cases mentioned above. When $\mu$ is zero, it indeed
Figure 3.1: Incentive to Gamble as A Function of Performance Target

Note: The manager has an incentive to gamble in a large range of target values with normally distributed performance measure and limited liability.

degenerates to the example of normal distribution. We will see how a change in \( \mu \) will affect the executive’s risk taking behavior. Intuitively, when \( \mu \) is small, the performance distribution is closer to the normal distribution example, so we will expect that the executive wants to reduce the risk. On the other hand, when \( \mu \) is large, it is closer to the two-state discrete example. Given the previous argument, we expect that the executive wants to engage in the gamble.

Derive the ex-ante utility for the executive facing the above distribution of performance measure.

\[
EU(f, \mu) = \int_{-\infty}^{0} -\lambda(-x)^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x + \mu)^2}{2}\right] dx + \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x + \mu)^2}{2}\right] dx
\]
\[ + \int_{-\infty}^{0} -\lambda(-x)^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2}\right] dx \\
+ \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2}\right] dx \\
= (1 - \lambda) \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x+\mu)^2}{2}\right] dx \\
+ (1 - \lambda) \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2}\right] dx \]

To simplify notation, denote,

\[ F(\mu) = \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x+\mu)^2}{2}\right] dx \\
+ \int_{0}^{\infty} x^{\alpha} \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{(x-\mu)^2}{2}\right] dx \]

We can show that whether or not the executive wants to take the risky gamble depends on the sign of \( F''(\mu) \). If \( F''(\mu) < 0 \) then the executive wants to increase risk taking. Otherwise, the executive wants to reduce the risk. The figure 3.2 below gives an illustration of \( F''(\mu) \) when \( \alpha = 0.8 \). Notice that when \( \mu > 2 \), then we have \( F''(\mu) < 0 \). This is consistent with our expectation that when \( \mu \) is large, the distribution is closer to the two-state discrete case where the manager wants to take the gamble to increase his/her utility.

Although assuming continuous distribution will give a more complex problem, it is not definite that this problem is more realistic. Given that the decision process invariably involves simplification of a complicated problem, a simple discrete illustration still has its relevance in terms of prediction and policy implication.

Based on the above arguments, our predictions are that firms with relatively stable measure of performance will see their managers less likely to take the gamble, while firms whose performance measure is sensitive to the business cycle tend to have problems of excessive risk taking.
There are two types of performance benchmarks. One is based on a firm’s own return-on-equity\(^8\), the other is based on relative performance ranking among selected peer competitor companies. Since a firm’s return-on-equity is subject to the business cycle and associated up-and-downs, the distribution of this measure is closer to that in the simple discrete example with good draws and bad draws. On the other hand, the relative ranking among peers is reasonably stable. Therefore, this measure is closer to a normal distribution.

In the real world, incentives usually come from two channels. One is how the executive’s decision will affect his/her current year compensation number. The other is how the decision will affect the accumulated equity wealth consisting of stocks and options. The main concern of the current year compensation is the number instead of the structure, because the structure of a firm’s compensation in terms of

\(^8\)AIG sets each executive’s yearly maximum performance compensation at 0.3 percent of AIG’s adjusted net income for the year, 46 million in 2006
cash, stocks, and options does not change too much over the years. The executive may only compare the number of current compensation to the previous year or some personally acceptable reference point.\(^9\) However, the structure of the compensation matters in the accumulated wealth channel, given that people value stocks and options differently (e.g., the option award does not have the downside risk).

Here is a brief survey of selected firms’ executive compensation plans. Bear Stearns used a benchmark of return-on-equity, and the incentive plan consisted mainly of restricted stock awards. Since the compensation number was exposed to the downside risk of business return, it was similar to our simple discrete example. Facing the looming dis-utility in down markets, the executive whose utility function was governed by prospect theory was inclined to take the gamble in the hope of breaking-even. For the other channel of incentives, the accumulated wealth, since the compensation consisted mainly of stock awards, the downside risk of stocks gave the executive an incentive to take excessive risk in handling the firm. In 2007, A minority group had proposed an executive compensation plan based on ranking among peer firms, but unfortunately it was not adopted by the board\(^{10}\).

In the case of Washington Mutual, the benchmark was set to the peer ranking. The compensation schedule only rewarded high ranking, but Washington Mutual had been in the lowest quartile among peers for years. The target and reward had been set too high, while the executive had nothing to lose. Their incentive compensation

\(^9\)One of the drawbacks of prospect theory is that it is usually hard to select a reference point. We use an exogenous given compensation level for average business performance as the reference point in the analysis.

\(^{10}\)Proxy 2007 P46
was low anyway. This compensation schedule put the executive directly in the loss region, encouraging the risky gamble.  

On the other hand, Wells Fargo is among the few good national banks in terms of a balance sheet and has the highest credit rating by Standard & Poor’s. Interestingly the incentive compensation consists of 100% option awards, and the benchmark of performance is set to the above average ranking among peers. Consider the first channel of incentive derived from the current year compensation number. The benchmark of ranking is a stable measure. In addition, Wells Fargo is among the top quartile for years in their selected comparison group. The bar is generally easy to achieve. Therefore, there is no tendency to take the risk. For the second channel about the accumulated wealth, since the downside risk for options is perceived to be smaller than stocks, the position may avoid putting the executive in a loss region. Therefore, this will not induce excessive risk taking.

However, the risk taking result and the effectiveness of a compensation structure will depend on the distribution of business returns and the timeline of decision processes in the real world. For example, if the regular business earning follows a normal distribution and the stock price tracks the earning of the business, then option awards

But for Washington Mutual, the structure of the compensation consists mostly of options. There could be two ways to explain incentives from the accumulated wealth channel. For simplicity, some firms report the option value at one fourth of the stock price. Therefore, contrary to intuition, those out of money options may have paper value. If the executive regards the value of options in this way, then the value of options is not much different from stocks in providing incentives. In this sense, choosing option awards may not eliminate the downside risk all together. Therefore, the accumulated equity including stock and option can move in the same direction as stock price. If the return-on-equity of the firm is the driving force of stock price and the normal business return is subject to the good and bad draws of the business cycle, the executive with loss aversion and decreasing sensitivity is likely to take the risky gamble. This is the incentive coming from the change of accumulated equity wealth. On the other hand, if the executive perceives the value of options in a different way from stock awards, then the prediction will be different. Therefore, overall, whether the executive at Washington Mutual takes the risky bet depends on the relative weights of incentives derived from the current year compensation number and the accumulated equity wealth. If the current year compensation has a higher weight, the executive will take the gamble.
can induce risk taking. This is because taking excessive risks can move the positive gain further away from the reference point, while there is no downside risk associated with option awards. Stock awards can however prevent this problem if the underlying earning follows a normal distribution, because reducing risk can increase the manager’s utility if the distribution has most of the weight located near the reference point. We will also discuss a different specification of the decision process in the next section.

3.3 Effort or Gamble

In the previous discussion the risky gamble decision is made before the realization of a firm’s performance. In this section, we look at the manager’s decision after a negative shock to the firm performance. If the manager’s decision only involves taking a gamble or not, it is intuitive that the risky gamble will be taken, because the utility function is convex after entering the loss region. To make the setting more realistic, we also include effort as an instrument to the manager. Now the manager’s problem is whether to use effort or gamble to affect the firm’s performance. If we believe that the manager makes his/her decision after observing some private information about the firm’s operation and that the manager’s efforts do indeed matter to the firm, then this setting will be relevant.

Under this setting when the decision to gamble is made after the state is realized, we find that if the pay-for-performance is sensitive enough, the manager will not take the risky gamble. In addition, there is a unique threshold value of pay-for-performance sensitivity, above which the excessive risk taking can be prevented. This threshold value is lower when the manager is more risk neutral in the positive region of his/her
utility function indicating that a lower pay-for-performance sensitivity is needed to prevent the risky gamble for this type of manager. Under a different setting, Goldman and Slezak (2006) show that pay-for-performance is a double-edged sword, inducing managers to exert productive effort but also to divert valuable firm resources to misrepresent performance.\footnote{Goldman and Slezak (2006) is motivated by the last episode of corporate scandals (Enron and WorldCom in 2002) where financial and accounting disclosures could not be trusted. It is usually easier for the manager to mislead the public with misrepresented accounting information than with a reckless gamble, because at least the result of the gamble has to be true. Therefore, when the pay-for-performance increases, the manager’s willingness to fabricate accounting information may grow but not the ambition to gamble.} But here we show that the higher pay-for-performance sensitivity can not only induce efforts but also prevent excessive risk taking, because if the manager notices that his/her real efforts will be rewarded, there will be less incentive to make a gamble. In fact, in this setting, the resources allocated to the gamble will reduce the positive multiplier effect of real efforts.

The firm has limited total resources $y$. The total resources are allocated to real business operation and risky gamble, so that $y_b + g = y$. The manager’s effort denoted by $e$ has a multiplier effect on the resources devoted to real business operation. Because of the business cycle, there could be a negative shock $\mu$ to the firm’s return. Therefore, the firm’s return will be $-\mu + e \cdot (y - g) \pm g$ depending on whether or not the gamble is successful. The gamble is assumed to be a coin flip with equal chance of success and failure. The gamble and effort decisions are made after the manager observes a negative shock $\mu$ to the firm. We want to learn how the pay-for-performance sensitivity $w$ will affect the manager’s gamble decision. The net utility of the manager is the utility from compensation minus the cost of effort. The reference point is normalized to zero. $u(.)$ is a utility function with decreasing sensitivity in
the loss region as described in prospect theory.

\[ U = \frac{1}{2} u\{w[e(y - g) + g - \mu]\} + \frac{1}{2} u\{w[e(y - g) - g - \mu]\} - c(e) \]

Consider the case that the negative shock is big \((\mu > y)\). As a result, managerial effort that is less than 1 or any gamble amount will not be able to bring the performance back to the reference point. We want to narrow down the amount of gamble that the manager wants to take. It turns out that, if \(\mu > y\), the possible optimal \((e, g)\) pair will be between \(g = 0\) and \((e = 0, g = y)\). This suggests that we only need to look at these two cases. One is that the manager takes no gamble at all. The other is that the manager allocates all the resources to the gamble. The reason is given by taking the derivative of the utility function with respect to the gamble amount.

\[ \frac{\partial U}{\partial g} = \frac{1}{2} u'\{w[e(y - g) + g - \mu]\}(1 - e)w + \frac{1}{2} u'\{w[e(y - g) - g - \mu]\}(-1 - e)w \]

if \(g = 0\), \(u' > 0\), and \(u'' > 0\) (for the negative part) then

\[ \frac{\partial U}{\partial g} < 0 \]
\[ \frac{\partial^2 U}{\partial g^2} > 0 \]

Therefore when effort is less than 1, a marginal increase of the gamble from zero has a negative effect on utility initially. However, when the gamble amount further increases, the marginal utility from a successful gamble increases, because we assume that the utility function is convex in the negative region (decreasing sensitivity to loss), and therefore the marginal effect of the gamble on utility can become positive eventually.

As a result, if the optimal action of the manager involves a gamble, it will be a maximum gamble using all the firm's resources. The optimal action depends on
whether no gamble or a full gamble can give a higher utility to the manager. We can compare these two possibilities. Denote the utility, derived from optimal effort, without the gamble, as \( V(w) \) and the corresponding derivative as \( V'(w) \).

\[
V(w) = \max_e U[w(ey - \mu)] - c(e)
\]

\[
V'(w) = U'[w(e^*y - \mu)](e^*y - \mu)
\]

Denote the utility derived from an all-in gamble as \( V_g(w) \) and the derivative as \( V'_g(w) \).

\[
V_g(w) = \frac{1}{2} U[w(y - \mu)] + \frac{1}{2} U[w(-y - \mu)]
\]

\[
V'_g(w) = \frac{1}{2} U'[w(y - \mu)](y - \mu) + \frac{1}{2} U'[w(-y - \mu)](-y - \mu)
\]

We still focus on the utility function specification below that is usually used to explain prospect theory.

\[
U(x) = -\lambda(-x)^\alpha, \quad x < 0 \tag{3.6}
\]

We can derive a property resulting from this class of utility functions.

\textit{Lemma: Under utility function (3.6)}

\[
V'(w) \lesssim V'_g(w) \iff U[w(e^*y - \mu)] \lesssim V_g(w)
\]

\textit{Proof.}

\[
V'(w) \lesssim V'_g(w) \iff
\lambda[-w(e^*y - \mu)]^{(\alpha-1)}(e^*y - \mu) \lesssim \frac{1}{2}\lambda\{[-w(y - \mu)]^{(\alpha-1)}(y - \mu) + [-w(-y - \mu)]^{(\alpha-1)}(-y - \mu)\}
\]

\[
\iff U[w(e^*y - \mu)] \lesssim V_g(w)
\]

\[ \square \]
With the lemma, we can show that whether the manager takes the risky gamble depends on whether the pay-for-performance is sensitive enough. There is a unique threshold value of pay-for-performance sensitivity, which changes the behavior of the manager.

**Proposition 5.** Under utility function (3.6), if there exists \( \hat{w} \) such that \( V(\hat{w}) = Vg(\hat{w}) \), \((\hat{w} > 0)\), then \( \hat{w} \) is unique, \( V(w) > Vg(w) \) for \( w > \hat{w} \), and \( V(w) < Vg(w) \) for \( w < \hat{w} \).

**Proof.** If \( w > 0 \), then \( e^* > 0 \) and \( c(e^*) > 0 \).

Therefore, if \( U[w(e^*y - \mu)] - c(e^*) = V(w) \geq Vg(w) \), then \( U[w(e^*y - \mu)] > Vg(w) \). From the previous lemma, we have \( V'(w) > Vg'(w) \). Therefore, for \( w > \hat{w} \), \( V(w) > Vg(w) \).

If there exists \( w < \hat{w} \), such that \( V(w) \geq Vg(w) \), then \( V(\hat{w}) > Vg(\hat{w}) \), a contradiction. Therefore, \( V(w) < Vg(w) \) for \( w < \hat{w} \). And if exist, \( \hat{w} \) is unique.

This result is different from that in the previous setting where the decision to gamble is made prior to a negative (or positive) shock to the firm’s performance. A lower pay for performance sensitivity can reduce the sense of loss, so that it will prevent the ex ante gamble in the previous setting. On the other hand, if a loss has already occurred, a higher pay for performance sensitivity can make the manager more responsible, thus, concentrated on exerting a real effort\(^{13}\). This can prevent an ex post risky gamble. Under this ex post decision setting, a simple proportional wage contract, if not sensitive enough to the performance, can encourage a gamble.

\(^{13}\)In this case, a high-powered compensation schedule can increase the reward of effort. By moving the executive deeper into the loss region, it can also reduce the benefit of the gamble.
Then we are interested in how the manager’s attitude towards risk measured by the parameter $\alpha$ can affect the value of this threshold pay-for-performance sensitivity. This may have some implications for selecting managers.

**Proposition 6.** The threshold $\hat{w}$ is decreasing in $\alpha$, if $\ln[\hat{w}(\mu - e^*y)] > -\frac{1}{\alpha}$ and $\ln[\hat{w}(\mu + y)] < \frac{2\alpha - 1}{(1-\alpha)^\alpha}$

**Proof.** Fix $\hat{w}$ and effort $e^*$, first we can show that under the above assumptions, $U[\hat{w}(e^*y - \mu)] - \frac{1}{2}\{U[\hat{w}(y - \mu)] + U[\hat{w}(-y - \mu)]\}$ is increasing in $\alpha$

$$\frac{\partial U[\hat{w}(e^*y - \mu)] - \frac{1}{2}\{U[\hat{w}(y - \mu)] + U[\hat{w}(-y - \mu)]\}}{\partial \alpha} =$$

$$-\lambda[\hat{w}(\mu - e^*y)]^\alpha \ln[\hat{w}(\mu - e^*y)] - \frac{1}{2}[\hat{w}(\mu - y)]^\alpha \ln[\hat{w}(\mu - y)] - \frac{1}{2}[\hat{w}(\mu + y)]^\alpha \ln[\hat{w}(\mu + y)]$$

if $\ln[\hat{w}(\mu - e^*y)] > -\frac{1}{\alpha}$, then $(x^\alpha \ln x)' > 0$ and $[\hat{w}(\mu - e^*y)]^\alpha \ln[\hat{w}(\mu - e^*y)] < (\hat{w}\mu)^\alpha \ln(\hat{w}\mu)$

if $\ln[\hat{w}(\mu + y)] < \frac{2\alpha - 1}{(1-\alpha)^\alpha}$, then $(x^\alpha \ln x)'' > 0$ and $(\hat{w}\mu)^\alpha \ln(\hat{w}\mu) < \frac{1}{2}[\hat{w}(\mu - y)]^\alpha \ln[\hat{w}(\mu - y)] + \frac{1}{2}[\hat{w}(\mu + y)]^\alpha \ln[\hat{w}(\mu + y)]$

Therefore,

$$\frac{\partial U[\hat{w}(e^*y - \mu)] - \frac{1}{2}\{U[\hat{w}(y - \mu)] + U[\hat{w}(-y - \mu)]\}}{\partial \alpha} > 0$$

Increasing $\alpha$ will make $V(\hat{w}) > Vg(\hat{w})$. To restore the equality, the new threshold $\hat{w}$ has to be lower than before.

The intuition is simply that as the manager becomes more risk neutral in the loss region, the option of gamble becomes less attractive. As a result, a lower pay-for-performance sensitivity is needed to induce effort and compensate the cost of effort.

An example is given in figure 3.3 showing how the threshold $\hat{w}$, s.t. $V(\hat{w}) = Vg(\hat{w})$ corresponds to changes in $\alpha$, the parameter on utility.
Figure 3.3: Effect of Risk Averse on Threshold PFP Sensitivity
Note: The threshold PFP $\hat{w}$ is decreasing in the manager’s risk coefficient, $\alpha$.

3.4 Loss Aversion and Adverse Selection

How does the loss aversion affect an adverse selection problem? When the outcome of effort is deterministic, but the state of the underlying business can incur some variation that is outside the control of the agent and unobserved by the principal, this is a standard adverse selection problem. It turns out that the loss aversion property in the utility function of the agent can cure some distortion in the second best solution to this adverse selection problem. We denote the bad and good state by $\mu_1$ and $\mu_2$. The principal’s problem will be maximizing the net profit subject to PC and IC constraints:

$$\max (1 - v)(\mu_1 + e_1 - t_1) + v(\mu_2 + e_2 - t_2)$$
\[
\begin{align*}
\text{s.t.} \\
v[u(t_1) - c(e_1)] + (1 - v)[u(t_2) - c(e_2)] &> u^o \\
u(t_2) - c(e_2) &\geq u(t_1) - l \cdot [u(t_2) - u(t_1)] - c(e_1 - \Delta \mu) \\
u(t_1) - c(e_1) &\geq u(t_1) - c(e_2 + \Delta \mu)
\end{align*}
\]

where \( u^o \) is the outside option utility, and \( l \) is the parameter for loss aversion. The feeling of loss occurs, when the agent reports a bad-state while the true state is good, because the agent will end up with a lower transfer payment for exerting a lower effort. We assume the reference level of payment is the one resulting from a truthful report. We only focus on the effect of loss aversion on the transfer payment but not on the effort level. This is because we know that the incentive compatibility condition will be binding only in the good-state, and that a misreporting in the good-state does not involve loss feeling from effort. Only a misreporting in the bad-state can result in a loss in terms of exerting more effort, but this condition is not binding. We can form the Lagrangian for this problem.

\[
L = (1 - v)(\mu_1 + e_1 - t_1) + v(\mu_2 + e_2 - t_2) + \lambda[(1 + l)(u(t_2) - u(t_1)) - c(e_2) + c(e_1 - \Delta \mu)] + \eta[(1 - v)[u(t_1) - c(e_1)] + v[u(t_2) - c(e_2)]]
\]

First order conditions will be,\(^{14}\)

\[
\begin{align*}
\lambda &= \frac{v(1 - v)(u_1' - u_2')}{(1 + l)u_1'u_2'} \\
\eta &= \frac{vu_1' + (1 - v)u_2'}{u_1'u_2'} \\
1 - \frac{(1 - v)(u_1' - u_2')}{(1 + l)u_1'u_2'}c'(e_2) - \frac{vu_1' + (1 - v)u_2'}{u_1'u_2'}c'(e_2) &= 0
\end{align*}
\]

\(^{14}\)\( u_1' = u(t_1)' \)
\[ 1 + \frac{v(u'_1 - u'_2)}{(1 + l)u'_1 u'_2} c'(e_1 - \Delta \mu) - \frac{vu'_1 + (1 - v)u'_2}{u'_1 u'_2} c'(e_1) = 0 \]

and the binding PC and IC,

\[ v[u(t_1) - c(e_1)] + (1 - v)[u(t_2) - c(e_2)] = 0 \]

\[ (1 + l) \cdot [u(t_2) - u(t_1)] = c(e_2) - c(e_1 - \Delta \mu) \]

When \( u(t_2) - u(t_1) \) is reduced as a result of loss aversion, the amount of payment \( vt_2 + (1 - v)t_1 \) is reduced because of the concavity assumption of \( u \). Without the adverse selection problem, the principal wants to pay the agent the same amount in each state, so as to reduce the agency cost. If the loss aversion concern of the agent can reduce the wedge between the good and bad states, this will reduce the agency cost.

When \( c^{(3)}(\cdot) \geq 0 \), then \( e_1 \) is decreasing in \( l \). This means that the loss aversion can create a countervailing incentive and cure part of the upward distortion in effort level in the bad state. This upward distortion in effort in the bad-state can induce the truth telling in the good-state, but this is not efficient. If the agent has the loss aversion concern, then he/she is not willing to accept the lower payment of a bad-state when the true state is good, so the distortion in effort can also be reduced. Denote

\[ A = \frac{vu'_1 + (1 - v)u'_2}{u'_1 u'_2} c'(e_1) - \frac{v(u'_1 - u'_2)}{(1 + l)u'_1 u'_2} c'(e_1 - \Delta \mu) = 1 \]

We have \( \frac{dA}{dl} > 0 \), and if \( c''(\cdot) > 0 \) \( c^{(3)}(\cdot) \geq 0 \) then

\[ \frac{dA}{de_1} = \frac{vu'_1 + (1 - v)u'_2}{u'_1 u'_2} c''(e_1) - \frac{v(u'_1 - u'_2)}{(1 + l)u'_1 u'_2} c''(e_1 - \Delta \mu) > 0 \]

As a result, the required effort in the bad-state is decreasing in the degree of loss aversion.

\[ \frac{de_1}{dl} < 0 \]
Table 3.1: Impact of Loss Aversion on Adverse Selection Problems

<table>
<thead>
<tr>
<th></th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$t_1$</th>
<th>$t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l = 0$</td>
<td>1.02</td>
<td>0.91</td>
<td>0.25</td>
<td>0.53</td>
</tr>
<tr>
<td>$l = 1$</td>
<td>0.98</td>
<td>0.95</td>
<td>0.30</td>
<td>0.47</td>
</tr>
<tr>
<td>$FB$</td>
<td>0.967</td>
<td>0.967</td>
<td>0.387</td>
<td>0.387</td>
</tr>
</tbody>
</table>

Note: Assume $u(t) = t^{0.8}$, $v = 0.5$, $\Delta \mu = 0.5$, $c(e) = 0.5 \cdot e^2$. In the first best situation, the principal can observe the true state $\mu$ and specify the effort level and payment. The problem for the principal in both good and bad state will be the same: $\max(e - t)$ subject to $u(t) - c(e) \geq 0$. This gives full insurance to the agent.

For example, Table (3.1) gives the effect of loss aversion increasing from zero to one.

3.5 Loss Aversion, Moral hazard and Gamble

In contrast to the previous section, this section focuses on the possibility that the effect of the manager’s real effort on firm’s performance is not deterministic. In a normal moral hazard problem when the outcome from an effort is random, the incentive condition is to induce the agent’s high effort level. It can be shown that when the agent’s utility function exhibits loss aversion and a low performance is not very informative, then the compensation schedule does not enter the punishment region (The compensation is usually set above the reference point). When there is a possibility of a gamble from the manager, further restrictions on the compensation schedule are needed to prevent the gamble. Therefore, the agency cost usually increases. We illustrate these points with the simple setup below. The effort is assumed to be discrete and $e \in \{0, 1\}$. The cost to the agent of a high effort is $\psi$. There are three possible
outcomes for the firm’s performance \{L, M, H\}. If a high effort is exerted, the possibility of different outcomes is respectively \(\Pi(L)\), \(\Pi(M)\) and \(\Pi(H)\). In the case of a low effort, the probability will be \(\Pi(L)\), \(\Pi(M)\) and \(\Pi(H)\). The decision to gamble is taken after the outcome is realized, and it is only considered by the manager when the outcome is \(M\). The gamble may boost the outcome to \(H\) or reduce it to \(L\) with equal chance. If the manager chooses to gamble after the other two performance outcomes, the result from the gamble can be outside the normal domain of performance outcomes. Therefore, their gamble behavior can be detected. Thus, we only focus on the possible gamble after a medium performance outcome.

First, we consider a pure moral hazard problem without the possibility of a gamble. The only difference is that the agent’s utility function exhibits loss aversion below the reference point. The principal’s problem is standard:

\[
\min_{X_L, X_M, X_H} X_L \cdot \Pi_L + X_M \cdot \Pi_M + X_H \cdot \Pi_H
\]

subject to

**MH:**

\[
\Delta \Pi(L) \cdot U(L) + \Delta \Pi(M) \cdot U(M) + \Delta \Pi(H) \cdot U(H) > \psi
\]

**PC:**

\[
\Pi(L) \cdot U(L) + \Pi(M) \cdot U(M) + \Pi(H) \cdot U(H) > U^R + \psi
\]

where \(\psi\) is the cost of a high effort and \(U^R\) is the outside option utility the agent can get.

As the statement below shows, the optimal compensation schedule usually does not use the punishment that gives utility below the reference income, simply because it is too costly to punish a loss aversion agent.
Proposition 7. In a pure moral hazard problem with agent’s utility function of the form
\[ U(x) = \begin{cases} 
 x^\alpha, & x > 0 \\
 -\lambda(-x)^\alpha, & x < 0
\end{cases} \] (3.7)
when low performance is not a very informative indicator of low effort \((\frac{\Delta\Pi(L)}{\Pi(H)} > \frac{\Delta\Pi(M)}{\Pi(L)})\) and outside option utility is high enough \((U^R > (\frac{\Pi(H)}{\Delta\Pi(H)} - 1) \cdot \psi)\). It is not optimal to use the punishment. In other words, the compensation under low performance is above the reference point and \(U(L) > 0\).

Proof. From the MH and PC condition, we can get \(U(H)\) and \(U(M)\) as functions of \(U(L)\).
\[
U(H) = -\frac{\Delta\Pi(L) - \Pi(L) \frac{\Delta\Pi(M)}{\Pi(M)}}{\Delta\Pi(H) - \Pi(H) \frac{\Delta\Pi(M)}{\Pi(H)}} U(L) + \frac{\psi - \frac{\Delta\Pi(M)}{\Pi(M)} (\psi + U^R)}{\Delta\Pi(H) - \Pi(H) \frac{\Delta\Pi(M)}{\Pi(H)}}\]
\[
U(M) = -\frac{\Delta\Pi(L) - \Pi(L) \frac{\Delta\Pi(H)}{\Pi(H)}}{\Delta\Pi(M) - \Pi(M) \frac{\Delta\Pi(H)}{\Pi(H)}} U(L) + \frac{\psi - \frac{\Delta\Pi(H)}{\Pi(H)} (\psi + U^R)}{\Delta\Pi(M) - \Pi(M) \frac{\Delta\Pi(H)}{\Pi(H)}}
\]
the first order conditions also give
\[
\frac{1}{U'(i)} = \mu + \eta \frac{\Delta\Pi(i)}{\Pi(i)}
\]
where \(\mu > 0\) and \(\eta > 0\) are Lagrangian multipliers.

Suppose not and \(U(L) < 0\). The owner can increase this low performance utility by \(\epsilon\). The direct effect on the agency cost is that the owner has to pay more for low performance. This increase in agency costs is \(-\frac{\epsilon}{U'(L)} \cdot \Pi(L)\). On the other hand, since the utility and associated payment for medium and high performance is reduced, the indirect effect is a reduction in agency costs of the amount
\[
-\frac{\epsilon}{U'(M)} \cdot \Pi(M) \cdot \frac{\Delta\Pi(L) - \Pi(L) \frac{\Delta\Pi(H)}{\Pi(H)}}{\Delta\Pi(M) - \Pi(M) \frac{\Delta\Pi(H)}{\Pi(H)}} - \frac{\epsilon}{U'(H)} \cdot \Pi(H) \cdot \frac{\Delta\Pi(L) - \Pi(L) \frac{\Delta\Pi(M)}{\Pi(M)}}{\Delta\Pi(H) - \Pi(H) \frac{\Delta\Pi(M)}{\Pi(M)}}
\]
\[
\Delta\text{Cost} = \frac{\epsilon}{U'(L)} \cdot \Pi(L) - \frac{\epsilon}{U'(M)} \cdot \Pi(M) \cdot \frac{\Delta\Pi(L) - \Pi(L) \frac{\Delta\Pi(H)}{\Pi(H)}}{\Delta\Pi(M) - \Pi(M) \frac{\Delta\Pi(H)}{\Pi(H)}}
\]
\[ -\frac{\epsilon}{U'(H)} \cdot \Pi(H) \cdot \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(M)}{\Pi(M)}}{\Delta \Pi(H) - \Pi(H) \frac{\Delta \Pi(M)}{\Pi(M)}} \]

when

\[ \frac{\Pi(L)}{\Pi(M)} > \frac{\frac{\Delta \Pi(H)}{\Pi(H)} - \frac{\Delta \Pi(M)}{\Pi(M)}}{\frac{\Delta \Pi(M)}{\Pi(M)} - \frac{\Delta \Pi(L)}{\Pi(L)}} \]

The probability of low performance is high enough and is not informative of low effort.

so

\[ \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(H)}{\Pi(H)}}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}} > 1 \]

when the outside option utility is high enough and

\[ U^R > \left( \frac{\Pi(H)}{\Delta \Pi(H)} - 1 \right) \cdot \psi \]

then

\[ \frac{\psi - (\psi + U^R) \frac{\Delta \Pi(H)}{\Pi(H)}}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}} > 0 \]

Therefore,

\[ U(M) = -\frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(H)}{\Pi(H)}}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}} U(L) + \frac{\psi - (\psi + U^R) \frac{\Delta \Pi(H)}{\Pi(H)}}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}} > -U(L) \]

From the specification of the utility function we have

\[ U'(L) = \left[ -U(L) \right]^{\frac{\alpha - 1}{\alpha}} \cdot \alpha \cdot \lambda^{\frac{1}{2}} \]

\[ U'(M) = \left[ U(M) \right]^{\frac{\alpha - 1}{\alpha}} \cdot \alpha \]

where \( \alpha < 1, \lambda > 1 \) As a result

\[ U'(L) > U'(M) > U'(H) \]

and

\[ U'(L) > U'(M) > U'(H) \]

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The change in agency costs, due to the increased utility in low performance, will be:

$$\Delta \text{Cost} < \frac{\epsilon}{U'(M)} \Pi(L) - \frac{\epsilon}{U'(M)} \cdot \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(H)}{\Pi(H)}}{\frac{\Delta \Pi(H)}{\Pi(H)} - \frac{\Delta \Pi(M)}{\Pi(M)}}$$

$$= \frac{\epsilon}{U'(M)} \cdot \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(M)}{\Pi(M)}}{\frac{\Delta \Pi(H)}{\Pi(H)} - \frac{\Delta \Pi(M)}{\Pi(M)}} < 0$$

Therefore, setting $U(L) < 0$ is not optimal. \[\square\]

We will then deviate from a pure moral hazard problem and allow the manager to make a gamble after the firm’s performance is realized. Since the gamble adds no real value to the firm, the owner of the firm wants to prevent this kind of gamble. The owner’s problem becomes:

$$\min_{X_L, X_M, X_H} X_L \cdot \Pi_L + X_M \cdot \Pi_M + X_H \cdot \Pi_H$$

subject to

MH:

$$\Delta \Pi(L) \cdot U(L) + \Delta \Pi(M) \cdot U(M) + \Delta \Pi(H) \cdot U(H) > \psi$$

No-gamble requires:

$$U(L) + U(H) < 2U(M)$$

PC:

$$\Pi(L) \cdot U(L) + \Pi(M) \cdot U(M) + \Pi(H) \cdot U(H) > U^R + \psi$$

We can see the condition, under which the additional no-gamble requirement affects the moral hazard problem and the agency cost.
Proposition 8. 1) If $\alpha \geq 0.5$ and $\frac{\Delta H(H)}{\Pi(H)} + \frac{\Delta L(L)}{\Pi(L)} > 2 \frac{\Delta M(M)}{\Pi(M)}$, then no-gamble restriction is stronger than Moral hazard. Adding this restriction will further increase the agency cost.

2) If $\alpha < 0.5$ and $\frac{\Delta H(H)}{\Pi(H)} + \frac{\Delta L(L)}{\Pi(L)} \leq 2 \frac{\Delta M(M)}{\Pi(M)}$, then no-gamble restriction is weaker than Moral hazard. Adding this restriction will not increase the agency cost.

Proof. Under the situation where the compensation is above the reference income, the first order conditions for the moral hazard problem are, $\frac{1}{U'(x_i)} = \mu + \eta \cdot \frac{\Delta H}{\Pi}$. Therefore, $U(x_i) = [\alpha \cdot (\mu + \eta \frac{\Delta H}{\Pi})]^{\frac{1}{1-\alpha}}$.

When $\alpha > 0.5$, $[\alpha \cdot (\mu + \eta \frac{\Delta H}{\Pi})]^{\frac{1}{1-\alpha}}$ is increasing and convex in $(\mu + \eta \frac{\Delta H}{\Pi})$. And if $\frac{\Delta H(H)}{\Pi(H)} + \frac{\Delta L(L)}{\Pi(L)} > 2 \frac{\Delta M(M)}{\Pi(M)}$, then

$$\frac{1}{2} \left[ \alpha \cdot (\mu + \eta \frac{\Delta H}{\Pi_H}) \right]^{\frac{1}{1-\alpha}} + \frac{1}{2} \left[ \alpha \cdot (\mu + \eta \frac{\Delta L}{\Pi_L}) \right]^{\frac{1}{1-\alpha}} > \left[ \alpha \cdot \left( \frac{\mu + \eta \frac{\Delta H}{\Pi_H} + (\mu + \eta \frac{\Delta L}{\Pi_L})}{2} \right) \right]^{\frac{1}{1-\alpha}}$$

Therefore,

$$\frac{1}{2} U(H) + \frac{1}{2} U(L) > U(M)$$

The manager is willing to take the risky gamble, given the optimal compensation schedule of a pure moral hazard problem.

Similarly, when $\alpha < 0.5$ $[\alpha \cdot (\mu + \eta \frac{\Delta H}{\Pi})]^{\frac{1}{1-\alpha}}$ is increasing and concave in $(\mu + \eta \frac{\Delta H}{\Pi})$. Therefore, the no-gamble condition is satisfied when the Moral hazard condition holds.

In the condition, the component $\frac{\Delta M}{\Pi}$ indicates the informativeness of the medium performance. When it is larger, this usually means that the medium performance is assigned a higher utility. Therefore, there is no need to gamble.
On the other hand, for example, when both medium and low performance levels have the same likelihood ratio, an option like compensation schedule, with the same payment for medium and low performance, can emerge as a solution to the moral hazard problem. But this compensation schedule will encourage the gamble after a medium performance. Therefore, the restriction to prevent excessive risk taking requires that there is a difference between medium and low performance. This will increase the agency cost as it distorts the optimal solution.

### 3.6 Generalization of Utility Functions and Specific Example

When the manager is loss averse and there is only real effort involved, it is not optimal to punish the manager for low performance. The reason is that the big loss in utility under low performance requires a higher compensation under better performance in order to satisfy the participation condition. This turns out to increase the agency cost. As a result, it is possible to get an option-like compensation schedule as optimal with both medium and low performance paying reference level income (without entering the loss region). This argument may shed some light on the widely used option grant in managers’ compensation packages.\(^{15}\) But we want to show that even though this argument is true, the option like compensation schedule will not be optimal if the owner wants to prevent the manager from taking the risky gamble.

In contrast to the first section where the gamble taking arose from the convexity part of the utility function, here it comes from the convexity of the the compensation schedule that is supposed to be optimal to motivate a loss averse agent.

\(^{15}\)de Meza and Webb (2007) use loss aversion to explain why compensation schemes often reward success but do not penalize failure and show that there will be intervals over which pay is insensitive to performance.
Proposition 9. Without no-gamble restriction, if an option like schedule without punishment \((U(M) = U(L) = U^R)\) is optimal (because of loss aversion), then after imposing no-gamble restriction, punishing the low performance will re emerge as optimal with \(U(L) < U^R\).

Notice that this argument is not trivial, because we need to show why it is not the case that the new solution simply moves utility levels in medium and high performance upwards without incurring punishment.

The intuition is simple. The need to prevent gamble adds one more reason to punish the agent when the performance is low, even though the cost of punishment is high to a loss averse agent\(^{16}\). In order to prevent the manager from taking risky gamble, the possibility of punishing low performance is higher. The argument does not need to assume a certain form of utility function, but for the simplicity of computation, a specific form of utility function is used in the example to give a finite derivative at the reference point. The program can handle this form of example and find a numerical solution easier.

Proof. First, we show that the optimal compensation schedule that minimize the agency cost and satisfying all PC, MH and No-gamble conditions have both participation and moral hazard incentive conditions binding.

\(^{16}\)If there is no need to have a difference between compensation levels, \(X_M\) and \(X_L\), setting them at the same level can save agency costs of contracting with a loss averse agent. Otherwise, if \(X_M\) is raised above the reference level and \(X_L\) is reduced below the reference level, to maintain the participation condition, \(X_M\) has to be raised more than \(X_L\) is reduced, due to the loss aversion. As a result, this will cost the principal more. On the other hand, if no-gamble requires that there has to be a difference between \(X_M\) and \(X_L\), raising \(X_M\) and reducing \(X_L\) together will be better than raising \(X_M\) alone. In fact, raising \(X_M\) alone by the same amount is not enough to deter the gamble. Moreover, it costs more to the principal.
If Participation condition is slack and the Moral Hazard Incentive condition is binding in the optimal solution with additional restraint on gamble, then one can always reduce $U(L)$ to reduce agency costs, without violating incentive condition ($\Delta \Pi_L < 0$) and no-gamble condition. This contradicts the fact that the solution is optimal. Similarly, if both participation and incentive compatibility conditions are slack, we can use the above argument to show that cannot be optimal.

If Participation condition is binding and Moral Hazard condition is slack, the lagrangian multiplier of Moral Hazard condition $\eta$ in the first order condition $\frac{1}{U'(i)} = (\mu + \eta \frac{\Delta \Pi}{\Pi})$ is zero. There are two cases. One is the no-gamble condition $2U(M) \geq U(H) + U(L)$ is slack, so the lagrangian multiplier associated with the no-gamble condition is also zero. Therefore, the first order conditions reduce to $\frac{1}{U'(i)} = \mu$. This suggests that $U(H) = U(M)$. If $U(L) < U(M)$ this cannot be optimal in terms of the agency cost. If $U(L) = U(M) = U(H)$ then this cannot be Moral Hazard incentive compatible. The other case is the no-gamble condition binding. Therefore, the first order conditions reduce to $\frac{1}{U'(H)} = \mu + \zeta \frac{1}{\Pi_H}$, $\frac{1}{U'(M)} = \mu - \zeta \frac{2}{\Pi_M}$, $\frac{1}{U'(L)} = \mu + \zeta \frac{1}{\Pi_L}$. Therefore, $U(M) < U(H)$ and $U(M) < U(L)$. This contradict the no-gamble condition.

The above arguments suggest that both the participation condition and moral hazard incentive condition are binding at the optimal solution to a maximization problem with an additional restriction of no-gamble.

If both conditions are binding, we know that:

$$U(H) = - \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(M)}{\Pi(M)}}{\Delta \Pi(H) - \Pi(H) \frac{\Delta \Pi(M)}{\Pi(M)}} U(L) + \frac{\psi - \frac{\Delta \Pi(M)}{\Pi(M)} (\psi + U^R)}{\Delta \Pi(H) - \Pi(H) \frac{\Delta \Pi(M)}{\Pi(M)}}$$

$$U(M) = - \frac{\Delta \Pi(L) - \Pi(L) \frac{\Delta \Pi(M)}{\Pi(M)}}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}} U(L) + \frac{\psi - \frac{\Delta \Pi(H)}{\Pi(H)} (\psi + U^R)}{\Delta \Pi(M) - \Pi(M) \frac{\Delta \Pi(H)}{\Pi(H)}}$$
the $U(M)$ moves in the opposite direction to $U(L)$, and $U(H)$ moves in the same
direction as $U(L)$. When $U(M) = U(L) = U^R$, to make the no-gamble condition
hold, either $U(H)$ has to be lowered or $U(M)$ increased. In either way, $U(L)$ will be
reduced below $U^R$ incurring punishment.

An example is given in table 3.2 to show how the possibility of a gamble makes
the punishment emerge in the compensation schedule. Here we use another form of
the utility specification with loss aversion for ease of computation.

\[ U(x) = \begin{cases} 
  x^\alpha, & x \geq x^R \\
  x - \left[l_0(U^R - x^\alpha) + l_1(U^R - x^\alpha)^2\right], & x < x^R 
\end{cases} \quad (3.8) \]

when $x < x^R$

\[ U' = (1 + l_0) \cdot \alpha x^{\alpha-1} + l_1 \cdot 2(U^R - x^\alpha)\alpha x^{\alpha-1} \]

\[ U'' = (1 + l_0) \cdot \alpha(\alpha - 1)x^{\alpha-2} + l_1 \cdot 2U^R\alpha(\alpha - 1)x^{\alpha-2} - 2l_1(2\alpha - 1)\alpha \cdot x^{2\alpha-2} \]

If $l_1 < 0$, $2\alpha - 1 > 0$ and $1 + l_0 + 2l_1 \cdot U^R = 0$, then $U' > 0$ and $U'' > 0$ at loss
region.

More specifically, we set $\Pi(H) = 0.4$, $\Pi(M) = 0.3$, $\Pi(L) = 0.3$, $\Delta\Pi(H) = 0.2$,
$\Delta\Pi(M) = -0.1$, $\Delta\Pi(L) = -0.1$, $\alpha = 0.8$, $l_0 = 1$, $l_1 = -0.2$, $\psi = 0.1$, $U^R = 5$ and the
corresponding reservation income $x^R = 7.48$ ($U^R = 5$).

When a moral hazard problem is combined with loss aversion, the payment to the
manager is never set below the reference income. However, when there is a possible
gamble decision for the manager, the payment for the low performance is set below
the reference income to deter this possibility. The optimal compensation schedule
resulting from a moral hazard problem with loss aversion is able to explain the once
widespread use of option awards that eliminate downside risks in compensation pack-
ages. But after the recent economic crisis, firms begin to improve the alignment of
the compensation and the firm’s return on the downside, especially in the financial industry where it is easy to place a gamble. That fact is consistent with this exercise. When the owners of the firm realize the possibility of using a gamble to boost performance, the option like compensation schedule is no longer efficient and punishment can re-emerge to prevent the risky gamble.

3.7 Conclusion

This essay explained the managers’ excessive risk taking decision by prospect theory utility function and unfit incentive schedules. In a simple ex ante risk-taking-decision setting, the essay provided ways to prevent excessive risk taking by adjusting the performance target and the pay-for-performance sensitivity. It also considered the manager’s risky decision when the decision was made after the state of the underlying business had been revealed. In this setting, the essay analyzed how the manager’s effort and gamble behaviors responded to the pay-for-performance sensitivity. It provided a necessary condition for a higher pay-for-performance sensitivity being able to reduce the excessive risk taking and induce the manager’s effort at the same time.
In the extensions, it explained how the manager’s loss aversion could affect adverse selection and moral hazard problems. In this specific setting of adverse selection, loss aversion can act as a countervailing incentive. As a result, it can reduce the agency cost and cure part of the distortion in the second best solution to a pure adverse selection problem. In terms of a moral hazard problem, loss aversion can make an option like compensation schedule without punishment optimal. However, given the possibility of the risky gamble, punishment for the low performance can re-emerge in the optimal compensation scheme even under loss aversion.
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APPENDIX A

PROOFS OF CHAPTER 1

Proof of Lemma: If the intermediate firm 2 is driven out of the market under uniform pricing, the upstream profit by selling through only firm 1 will be: \( \frac{a-1}{2} \cdot \frac{a-1}{4} \).

Under uniform pricing, the upstream is willing to keep both intermediaries in the market only when by doing so it can generate profit higher than the above one firm profit. Also the solution of firm 2’s quantity should be positive. It turns out that the profit condition is stronger than the quantity condition. The profit condition to keep both firms in the market under uniform pricing is:

\[
\frac{(a-1)^2}{8} < K^* \cdot (Q_1 + Q_2r)
\]

\( \Leftrightarrow \)

\[
\frac{(a-1)^2}{8} < \frac{1}{3}[(a-2+c) + (a+1-2c)r]K^* + (2r - 2 - 2r^2)K^{*2}]
\]

\( \Leftrightarrow \)

\[
3(a-1)^2 < 8\left[\frac{(a-2+c) + (a+1-2c)r^2}{4(r^2 - r + 1)} - \frac{[a-2+c + (a+1-2c)r]^2}{8(r^2 - r + 1)}\right]
\]

\( \Leftrightarrow \)

\[
\hat{c}(r) \equiv \frac{(a-2) + (a+1)r - \sqrt{3}(a-1)\sqrt{r^2 - r + 1}}{2r - 1} > c
\]
For the second part of the lemma that the upstream supplier is able to force firm 2 out, whenever it is profitable to do so.

If there is only one intermediate firm 1 in the market, the upstream will charge an input price at \( \frac{a-1}{2} \). If firm 2 want to operate, at this input price it will produce \( Q_2 = \frac{(a+1-2c)+(1-2r)a-1}{3} \). So the upstream is not able to force the firm 2 out of the market at input price \( \frac{a-1}{2} \) is equivalent to \( c < \frac{3a+1}{4} - \frac{(a-1)r}{2} \).

The profit condition for the upstream not optimal to drive firm 2 out of the market is \( \Pi^{one} \leq \Pi^{uni} \), \( \iff \)

\[
3(a-1)^2 \leq \frac{(a - 2 + c + (a + 1 - 2c)r)^2}{r^2 - r + 1}
\]

Notice that the right hand side is decreasing in \( c \) (if \( r > 0.5 \)). So if \( c_0 \) satisfies the inequality then any \( c < c_0 \) satisfy the inequality too. To show any \( c < \frac{3a+1}{4} - \frac{(a-1)r}{2} \) satisfy this inequality, we only need to show \( c = \frac{3a+1}{4} - \frac{(a-1)r}{2} \) satisfy the condition.

Substitute in we get \( \iff \)

\[
3(a-1)^2 \leq [(a - 2) + (a + 1)r + (1 - 2r)(\frac{3a + 1}{4} - \frac{(a-1)r}{2})]^2
\]

When \( r = 0.5 \) the ’=’ holds. Take derivatives on both sides with respect to \( r \), we only need to show that

\[
3(a-1)^2(2r - 1) < 2[(a - 2) + (a + 1)r + (1 - 2r)(\frac{3a + 1}{4} - \frac{(a-1)r}{2})] \times
\]

\[
[(a + 1) + (1 - 2r)(-\frac{a-1}{2}) - 2(\frac{3a + 1}{4} - \frac{(a-1)r}{2})]
\]

\[
= 2[(a - 2) + (a + 1)r + (1 - 2r)(\frac{3a + 1}{4} - \frac{(a-1)r}{2})] \cdot (2r - 1)(a-1)
\]

\( \iff \)

\[
3(a-1) < 2(a-1)(r^2 - r + \frac{7}{4})
\]

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\[ r > 0.5 \]

So if \( c < \frac{3a+1}{4} - \frac{(a-1)r}{2} \) then \( 3(a-1)^2 \leq \frac{|a-2+c+(a+1-2c)r|^2}{r^2-r+1} \). This means

\[
\{(c, r) \mid \text{it is not possible to drive firm 2 out by setting input price } \frac{a-1}{2}\} \subset \{(c, r) \mid \text{it is not optimal to drive firm 2 out for the upstream}\}.\]

Equivalently,

\[
\{(c, r) \mid \text{it is optimal to drive firm 2 out for the upstream}\} \subset \{(c, r) \mid \text{it is possible to drive firm 2 out by setting input price } \frac{a-1}{2}\}.\]

If the upstream supplier finds it profitable to only serve firm 1 in the uniform pricing regime, it can act as if there is only firm 1 in the market and set input price optimally. This price will automatically make firm 2 unprofitable to operate. QED.

Proof of Proposition 2:

Given both firms are operating in both scenarios, we know \( 1 < c < \hat{c}(r) \) from the lemma.

The intermediate’s quantity decision is given by:

\[
Q_i = \text{Max}\{(a - 2C_i + C_j)/3, 0\}
\]

In the discrimination allowed scenario,

\[
\Pi^{\text{dis}} = \frac{a - 2(1 + k_1) + (c + rk_2)}{3} k_1 + \frac{a - 2(c + rk_2) + (1 + k_1)}{3} r k_2
\]

\[\Rightarrow \]

\[
a - 2 + c + 2rk_2 = 4k_1
\]

\[
a - 2c + 1 + 2k_1 = 4rk_2
\]

\[\Rightarrow \]

\[
k_1^{\text{dis}} = \frac{a - 1}{2}, r k_2^{\text{dis}} = \frac{a - c}{2}
\]
\[ k_{1}^{\text{dis}} + r k_{2}^{\text{dis}} = a - \frac{1 + c}{2} \]

\[ P^{\text{dis}} = a - (Q_1 + Q_2) = a - \frac{2}{3} + \frac{1}{3}(1 + c) + \frac{1}{3}a - \frac{1 + c}{6} \]

\[ = \frac{2}{3}a + \frac{1 + c}{6} \]

Since in order for firm 2 to operate

\[ Q_2 = \frac{a - 2(c + r k_2) + (1 + k_1)}{3} > 0 \]

\[ \Rightarrow \]

\[ c < \frac{a + 1}{2} \]

When price discrimination is banned,

\[ Q_1^{\text{uni}} = \frac{(a - 2 + c) + (r - 2)K}{3} \]

\[ Q_2^{\text{uni}} = \frac{(a + 1 - 2c) + (1 - 2r)K}{3} \]

\[ \Pi^{\text{up}} = (a - 2 + c) + (a + 1 - 2c)r \]

\[ K^* = \frac{(a - 2 + c) + (a + 1 - 2c)r}{4(r^2 - r + 1)} \]

\[ P^{\text{uni}} = a - (Q_1 + Q_2) = a - \frac{(2a - 1 - c) + (-1 - r)K^*}{3} \]

\[ = \frac{a}{3} + \frac{1 + c}{3} + \frac{1 + r}{3}K^* \]

\[ ^1 \text{In the two intermediate firm price discrimination scenario, it is interesting that the upstream charges the amount as if there is only one intermediate firm. There are two forces. The increased competition makes firm produce less than single firm case, making the upstream tend to charge less. However, charging more on one firm will increase the production from another intermediate firm. These two forces happen to cancel each other, making the first order condition equivalent to the single intermediate firm case.} \]
\[ P_{\text{uni}} > P_{\text{dis}} \]
\[
\Leftrightarrow \quad \frac{1 + c}{6} + \frac{1 + r}{3} \cdot \frac{(a - 2 + c) + (a + 1 - 2c)r}{4(r^2 - r + 1)} > \frac{1}{3^a}
\]
\[
\Leftrightarrow \quad (2 + 2c - 4a)(r^2 - r + 1) + (a - 2 + c) + (a - 2 + c)r + (a + 1 - 2c)r + (a + 1 - 2c)r^2 > 0
\]
\[
\Leftrightarrow \quad (-4a + 2 + 2c) + (a - 2 + c) + (4a - 2 - 2c)r + (a - 2 + c)r + (a + 1 - 2c)r
\]
\[
+(-4a + 2 + 2c)r^2 + (a + 1 - 2c)r^2 > 0
\]
\[
\Leftrightarrow \quad (a - 1)r^2 + (c + 1 - 2a)r + (a - c) < 0
\]
\[
\Delta = (c - 1)^2
\]
\[
\Leftrightarrow \quad r \in \left(\frac{a - c}{a - 1}, 1\right)
\]
where
\[
1 < c < \frac{a + 1}{2}
\]

So If \( r \in \left(\frac{a - c}{a - 1}, 1\right) \) and \( 1 < c < \hat{c}(r) \), then \( \Delta Q > 0 \) (\( P_{\text{dis}} < P_{\text{uni}} \)). This is case i).

And If \( r \in (0.5, \frac{a - c}{a - 1}) \cup (1, 2)^2 \) and \( 1 < c < \hat{c}(r) \), then \( \Delta Q < 0 \) (\( P_{\text{dis}} > P_{\text{uni}} \)). This is case iii).

Notice that since \( c < \frac{a + 1}{2} \), \( a - c > \frac{a - 1}{2} \)

\(^2\)When \( r = 2, \hat{c}(r) = 1 \)

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\[ a - c \frac{a - c}{a - 1} > \frac{1}{2} \]

And if \( r = 1 \), two firms use the same proportion of intermediate to produce the final product, it would not be possible to get this result. Assuming both intermediate firms are in operation and use the same amount of input, the price from a ban on price discrimination will not be higher than the final price when discrimination is allowed.

If firm 2 is forced out by the upstream, from the lemma we know \( \hat{c}(r) < c < \frac{a + 1}{2} \). Because of the force-out, the price will be \( P_{\text{one}} = a - \frac{a - 1}{4} = \frac{3a}{4} + \frac{1}{4} \). The input discrimination final product price will be \( P_{\text{dis}} = \frac{2}{3}a + \frac{1+c}{6} \). \( P_{\text{one}} > P_{\text{dis}} \), as long as \( c < \frac{a + 1}{2} \). This is the case ii). QED.

Proof of Corollary 1: We only need to show \( \Delta W < 0 \).

\[
W^{\text{Dis}} = \frac{[2a - (Q^{\text{Dis}}_1 + Q^{\text{Dis}}_2)](Q^{\text{Dis}}_1 + Q^{\text{Dis}}_2)}{2} - (Q^{\text{Dis}}_1 + c \cdot Q^{\text{Dis}}_2)
\]

\[
W^{\text{Uni}} = \frac{[2a - (Q^{\text{Uni}}_1 + Q^{\text{Uni}}_2)](Q^{\text{Uni}}_1 + Q^{\text{Uni}}_2)}{2} - (Q^{\text{Uni}}_1 + c \cdot Q^{\text{Uni}}_2)
\]

\[
\Delta W = \frac{2a(Q^{\text{Dis}} - Q^{\text{Uni}})}{2} + (Q^{\text{Uni}}^2 - Q^{\text{Dis}}^2) + [Q^{\text{Uni}} - Q^{\text{Dis}} + c(Q^{\text{Dis}} - Q^{\text{Uni}})]
\]

Since \( Q^{\text{Dis}} - Q^{\text{Uni}} > 0 \), \( (Q^{\text{Uni}}^2 - Q^{\text{Dis}}^2) < 0 \), so it suffices to show that \( a(Q^{\text{Dis}} - Q^{\text{Uni}}) + [Q^{\text{Uni}} - Q^{\text{Dis}} + c(Q^{\text{Dis}} - Q^{\text{Uni}})] < 0 \), expand the left hand side:

\[
a[\frac{a - 2(1 + k_1) + (c + rk_2)}{3} + \frac{a - 2(c + rk_2) + (1 + k_1)}{3} - (a - 2 + c) + (r - 2)k] + \frac{3}{3} - (a + 1 - 2c) + (1 - 2r)k \]

\[
+ [\frac{a - 2(1 + k_1) + (c + rk_2)}{3} - \frac{a - 2(c + rk_2) + (1 + k_1)}{3} - a - 2 + c + (r - 2)k] \]

\[
+ c[\frac{a + 1 - 2c + (1 - 2r)k}{3} - \frac{a - 2(c + rk_2) + (1 + k_1)}{3} + a - 2(c + rk_2) + (1 + k_1)]
\]

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which is:

\[
(1 - a) \frac{(r - 2)k - (rk_2 - 2k_1)}{3} + (c - a) \frac{(1 - 2r)k - (k_1 - 2rk_2)}{3}
\]

Since in the triangle area, \( r \in \left(\frac{a-c}{a-1}, 1\right) \), so \((a - c) < r(a - 1), (c - a) > r(1 - a)\).

And since \( \frac{(1 - 2r)k - (k_1 - 2rk_2)}{3} = \frac{(k - k_1) + (2rk_2 - 2rk)}{3} < 0 \), \((k_2 < k < k_1)\). As a result \((c - a) \cdot \frac{(1 - 2r)k - (k_1 - 2rk_2)}{3} < r(1 - a) \cdot \frac{(1 - 2r)k - (k_1 - 2rk_2)}{3}\). Using this inequality, we can get

\[
(1 - a) \frac{(r - 2)k - (rk_2 - 2k_1)}{3} + (c - a) \frac{(1 - 2r)k - (k_1 - 2rk_2)}{3}
\]

\[
< (1 - a) \frac{(r - 2)k - (rk_2 - 2k_1)}{3} + (1 - a) \frac{r(1 - 2r)k - r(k_1 - 2rk_2)}{3}
\]

\[
= \frac{1 - a}{3} \left[ (2r - 2 - 2r^2)k + (2 - r)k_1 + (2r - 1)rk_2 \right]
\]

\[
= \frac{1 - a}{3} \left[ (2r - 2 - 2r^2) \cdot \frac{(a - 2 + c) + (a + 1 - 2c)r}{4(r^2 - r + 1)} + (2 - r) \cdot \frac{a - 1}{2} + (2r - 1) \cdot \frac{a - c}{2} \right]
\]

\[= 0\]

QED.

Proof of Corollary 2: The first inequality makes sure that firm 2 will be forced out under uniform pricing. Since the firm 2 will be excluded under uniform pricing regime, the total quantity and social surplus under uniform pricing \(Q^{uni}\) and \(W^{uni}\) equal to that in the one intermediate firm situation. we need to compare \(Q^{one}\) and \(W^{one}\) with \(Q^{dis}\) and \(W^{dis}\).

\[
Q_1^{one} = \frac{a-1}{4}, Q_1^{dis} = \frac{a}{3} - \frac{1+c}{6}. \text{ So } \Delta Q = \frac{a + 1 - 2c}{12} > 0 \text{ as long as } c < \frac{a+1}{2}, \text{ which is the condition for firm 2 to operate under price discrimination regime. We assume firm 2 is efficient enough to operate under input price discrimination regime, otherwise we are effectively discussing the situation with only one intermediate firm.}
\]

\[
W = (2a - Q) \cdot Q/2 - (Q_1 + cQ_2)
\]

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\[ Q^{one} = \frac{a-1}{4}, \text{ and } W^{one} = \frac{7}{32}(a-1)^2 \]

\[ Q^{dis}_1 = \frac{a+c}{6} - \frac{1}{3}, \text{ and } Q^{dis}_2 = \frac{a+1}{6} - \frac{c}{3}, \]

\[ W^{dis} = \frac{23}{72}c^2 - \left( \frac{5}{18}a + \frac{13}{36}c \right) + \frac{5}{18}a^2 - \frac{5}{18}a + \frac{23}{72} \]

\[ W^{dis} - W^{one} = \frac{23}{72}c^2 - \left( \frac{5}{18}a + \frac{13}{36}c \right) + \frac{17}{288}a^2 + \frac{46}{288}a + \frac{29}{288} \]

The roots

\[ c_{1,2} = \frac{(\frac{13}{36} - \frac{a}{18}) \pm \sqrt{\Delta}}{\frac{23}{36}} \]

where

\[ \Delta = \frac{9(a-1)^2}{16 \cdot 18^2} \]

\[ c_1 = \frac{17a+29}{46}, \text{ and } c_2 = \frac{a+1}{2} \]

So \( c < \frac{17a+29}{46} \) in order for \( \Delta W > 0 \) QED.

Proof of Proposition 3 and 3.1: \( \Pi^{int} = \frac{(a-c)^2 + (c-1)^2}{4} \), and \( \Pi^{dis} = \frac{(a-c)^2 + (c-1)^2 + (a-1)^2}{12} \).

From the fact that \((a-c)^2 + (c-1)^2 > 2(a-c)(c-1)\) we have

\[ \Pi^{int} > \Pi^{dis} \]

Since uniform pricing set more constraint on the maximization problem, we always have

\[ \Pi^{dis} > \Pi^{uni} \]

So we get the order of the upstream profit in these three scenarios.

\[ P^{int} = \frac{a+c}{2}, \text{ Compare the integration price with discrimination price } \]

\[ P^{dis} > P^{int} \]

\[ a + 1 > 2c \]

This happen to be the condition for firm 2 to stay in the market in the discrimination scenario. So generally, if the above condition holds, consumer welfare will be higher in the integration case than in price discrimination.
If \( c < \hat{c}(r) \) (or \( \frac{3(a-1)^2}{4} < [(a - 2 + c) + (a + 1 - 2c)r] \cdot K^* \)), \( P^{uni} = \frac{a}{3} + \frac{1+c}{3} + \frac{1+r}{3} K^* \).

Since \( P^{int} < P^{uni} \iff K^* > \frac{a+c-2}{2(1+r)} \), We only need to show \( \frac{3(a-1)^2}{4[(a-2+c) + (a+1-2c)r]} > \frac{a+c-2}{2(1+r)} \iff 3(a-1)^2(1+r) > 2[(a - 2 + c) + (a + 1 - 2c)r](a + c - 2) \iff (a - 1)^2 r + 2(a - 1)(c - 1) r + 4(c - 1)^2 r + (a - 1)^2 > 2(c - 1)^2 + 4(a - 1)(c - 1) \) (since \( r > 0.5 \))
\[ \iff \frac{3}{2}(a - 1)^2 > 3(a - 1)(c - 1) \iff \frac{a+1}{2} > c > 1. \]

So if two firm are in operation in the uniform pricing scenario, then \( P^{int} < P^{uni} \).

If \( c > \hat{c}(r) \), the less efficient firm 2 will be forced out of the market under uniform pricing, then \( P^{dis} < P^{uni} \). We know that \( P^{int} < P^{dis} \). So \( P^{int} < P^{uni} \).

For the fix cost part.

1) If \( c < \hat{c}(r) \), \( \Pi^{uni} = \frac{[(a-2+c) + (a+1-c)r]^2}{24(r^2 - r + 1)} \). \( \Pi^{dis} = \frac{(a-c)^2 + (c-1)^2 + (a-1)^2}{12} \).

So
\[
\Pi^{dis} - \Pi^{uni} = \frac{r^2 \cdot 3(a - 1)^2 + 3(a - c)^2 - 6r(a - 1)(a - c)}{24(r^2 - r + 1)} = \frac{[r(a - 1) - (a - c)]^2}{8(r^2 - r + 1)}
\]

\[
\Pi^{int} - \Pi^{dis} = \frac{(a + 1 - 2c)^2}{12}
\]

If \( F \in (0, \frac{(a+1-2c)^2}{12}) \) then integration will happen in both price discrimination and uniform pricing regime.

If \( F \in (\frac{(a+1-2c)^2}{12}, \frac{(a+1-2c)^2}{12} + \frac{[r(a - 1) - (a - c)]^2}{8(r^2 - r + 1)}) \), integration will only happen in the uniform pricing regime.

2) If \( c > \hat{c}(r) \), \( \Pi^{uni} = \frac{(a-1)^2}{8} \).

So
\[
\Pi^{dis} - \Pi^{uni} = \frac{(a + 1 - 2c)^2}{24}
\]

If \( F \in (0, \frac{(a+1-2c)^2}{12}) \) then integration will happen in both price discrimination and uniform pricing regime.
If \( F \in \left( \frac{(a+1-2c)^2}{12}, \frac{(a+1-2c)^2}{8} \right) \) then integration will happen only in the uniform pricing regime. QED.

Proof of Corollary 3.2: 

\[
W^{\text{int}} = \frac{1}{2}(2a - \frac{a-c}{2}) \cdot \frac{a-c}{2} - \left[ \frac{c}{2} + c(\frac{a+1}{2} - c) \right] = \frac{3}{8}a^2 - \frac{3}{4}ac + \frac{7}{8}c^2 - c + \frac{1}{2} \\
W^{\text{dis}} = \frac{1}{2}(2a - (\frac{a}{3} - \frac{1+c}{6}))(\frac{a}{3} - \frac{1+c}{6}) - \left[ \frac{a+c}{6} - \frac{1}{3} + c(\frac{a+1}{6} - \frac{c}{3}) \right] = \\
\frac{5}{18}a^2 - \frac{5}{18}ac + \frac{23}{72}c^2 - \frac{13}{36}c + \frac{23}{72} \\
W^{\text{int}} - W^{\text{dis}} = \frac{7}{72}a^2 - \frac{34}{72}ac + \frac{5}{18}a + \frac{40}{72}c^2 - \frac{23}{36}c + \frac{13}{72} = \\
\frac{5}{9}c^2 - (\frac{23}{36} + \frac{17}{36}a)c + \frac{7}{72}a^2 + \frac{5}{18}a + \frac{13}{72}c^1,2 = \left( \frac{23}{36} + \frac{17}{36}a \right) \pm \left( \frac{3}{36}a - \frac{3}{36} \right)
\]

So \( W^{\text{int}} > W^{\text{dis}} \iff c < \frac{7}{20}a + \frac{13}{20} \). Since if \( \hat{c}(r) < c < \frac{17a+29}{46} \), then \( W^{\text{dis}} > W^{\text{uni}} \), \( \frac{17a+29}{46} > \frac{7}{20}a + \frac{13}{20} \) \((a > 1)\).

So if \( \hat{c}(r) < c < \frac{7}{20}a + \frac{13}{20} \), then \( W^{\text{int}} > W^{\text{uni}} \). QED.

Proof of Proposition 4: In the price discrimination allowed regime, the upstream supplier’s profit is:

\[
\Pi^{\text{dis}} = \max_{\{k_1,k_2\}} k_1 \cdot Q_1^{\text{dis}} + k_2 \cdot rQ_2^{\text{dis}} \\
= \max_{\{k_1,k_2\}} k_1 \cdot \frac{a - 2(k_1 + 1) + (rk_2 + c)}{3} + k_2 \cdot r \cdot \frac{a - 2(rk_2 + c) + (k_1 + 1)}{3}
\]

By envelope theory, we have:

\[
\frac{\partial \Pi^{\text{dis}}}{\partial a} = \frac{k_1^{\text{dis}} + rk_2^{\text{dis}}}{3}
\]

Similarly for the uniform pricing regime, we have:

\[
\frac{\partial \Pi^{\text{uni}}}{\partial a} = \frac{(1 + r)K^*}{3}
\]

By plugging in the optimal input prices, we have,

\[
\frac{\partial \Pi^{\text{dis}}}{\partial a} = \frac{(a - c) + (a - 1)}{6}
\]

When both intermediate firms operate in the uniform pricing

\[
\frac{\partial \Pi^{\text{uni}}}{\partial a} = \frac{(1 + r)K^*}{3}
\]
When the less efficient firm 2 is driven out in the uniform pricing regime

\[ \frac{\partial \Pi^{one}}{\partial a} = \frac{a - 1}{4} \]

Notice that

\[ P^{dis} = \frac{a}{3} + \frac{1 + c}{3} + \frac{2a - 1 - c}{6} \]
\[ P^{uni} = \frac{a}{3} + \frac{1 + c}{3} + \frac{(1 + r)K^*}{3} \]

So comparing \( \frac{\partial \Pi^{dis}}{\partial a} \) and \( \frac{\partial \Pi^{uni}}{\partial a} \) is equivalent to comparing \( P^{dis} \) and \( P^{uni} \) which is solved in Proposition 2. If \( r \in (\frac{a-c}{a-1}, 1) \) and \( 1 < c < \hat{c}(r) \), then \( P^{dis} < P^{uni} \), and \( \frac{\partial \Pi^{dis}}{\partial a} < \frac{\partial \Pi^{uni}}{\partial a} \).

If \( r \in (0.5, \frac{a-c}{a-1}) \cup (1, 2) \) and \( 1 < c < \hat{c}(r) \), then \( P^{dis} > P^{uni} \) and \( \frac{\partial \Pi^{dis}}{\partial a} > \frac{\partial \Pi^{uni}}{\partial a} \). QED.

Proof of Proposition 5: If intermediate firm 2 is excluded under uniform pricing, the final product(input) quantity will be \( \frac{(a-m)-1}{4} \). Under pricing discrimination the quantity of input sold is \( \frac{(a-m)+c}{6} - \frac{1}{3} + r(\frac{(a-m)+1}{6} - \frac{5}{3}) \). So \( \frac{(a-m)+c}{6} - \frac{1}{3} + r(\frac{(a-m)+1}{6} - \frac{5}{3}) - \frac{(a-m)-1}{4} = \frac{1}{12}((a - m) + 1 - 2c)(2r - 1) > 0 \)

The corresponding threshold condition for firm 2 to operate when the upstream firm has a positive marginal cost of production:

When the upstream firm has a marginal cost of \( m \), the problem for upstream firm under uniform pricing will be:

\[ \max_{K(m)} \Pi^{uni} = \]
\[ \frac{1}{3} \left\{ [(a + c - 2) + (a + 1 - 2c)r]K + (2r - 2 - 2r^2)K^2 \right\} \]
\[ -m \left\{ \frac{(a + c - 2) + (a + 1 - 2c)r}{3} + \frac{(2r - 2 - 2r^2)K}{3} \right\} \]

The optimal input price will be

\[ K^*(m) = K^* + \frac{m}{2} \]
The condition for firm 2 to be forced out under uniform pricing when the upstream firm has a marginal cost of $m$ will be

$$\frac{(a - 1)^2}{8} > \frac{1}{3} \left\{ \frac{[(a + c - 2) + (a + 1 - 2c)r]^2}{8(r^2 - r + 1)} \right\}$$

$$-[(a + c - 2) + (a + 1 - 2c)r] \frac{m}{2} + (2r^2 - 2r + 2) \frac{m^2}{4}$$

$$\Leftrightarrow$$

$$c > \frac{(a - 2) + (a + 1)r - A}{2r - 1}$$

where

$$A = \sqrt{3(a - 1)^2 + 4m[(a - 2 + c) + (a + 1 - 2c)r] - 4m^2(r^2 - r + 1)} \cdot \sqrt{r^2 - r + 1}$$

QED.
APPENDIX B

SUPPLEMENTAL HISTOGRAMS FOR CHAPTER 2

Here we provide more examples to illustrate the price change pattern we observe in the data. One exception to this pattern is CMH-DCA. It may be due to a smaller sample in this market, and already lower and concentrated fares in 95.
Figure B.1: Histograms for BOS-LAX

Figure B.2: Histograms for SEA-ORD

Figure B.3: Histograms for DFW-DTW
Figure B.4: Histograms for DEN-ATL

Figure B.5: Histograms for CMH-DCA