A THEORETICAL AND EXPERIMENTAL INVESTIGATION
OF ROLLER AND GEAR SCUFFING

A DISSERTATION

Presented in Partial Fulfillment of the Requirements for
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in the Graduate School of The Ohio State University

By

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Scuffing is a prominent surface failure mode of loaded, lubricated contacts of gears and rolling element bearings experiencing excessive relative sliding and high speeds. This temperature-induced failure occurs suddenly when the contact temperatures reach a critical level due to the frictional heat generated at the contact interface. Material properties and geometry of contacting surfaces, operating conditions (normal load, relative sliding and speed), surface texture (roughness amplitude and direction) as well as physical and chemical properties of the lubricant all influence the scuffing behavior of such components.

In this study, a physics-based methodology is proposed for predicting thermal conditions of lubricated contacts under combined sliding and rolling, and for relating these thermal conditions to the likelihood of scuffing. The methodology combines (i) a mixed thermal elastohydrodynamic lubrication (EHL) model to predict temperatures of the contacting surfaces and in the lubricant film in between, (ii) a convective heat transfer model to predict the time-varying temperature distributions of the contacting bodies, and (iii) a scuffing criterion to predict the onset of scuffing.
Considering a generic two-dimensional point (elliptical) lubricated contact problem formed by two rough roller surfaces that are in relative sliding, a new thermal EHL model is proposed first to predict the instantaneous pressure, film thickness, and temperature distributions under considerable metal-to-metal contact conditions in a robust and computationally efficient way. A novel iterative process is devised to combine a convective heat transfer model of the contacting bodies with the thermal EHL model. The bulk temperatures required by the thermal EHL model are predicted in this process by the heat transfer model while the heat flux into the contacting bodies and the heat partitioning coefficients required by the heat transfer model are provided by the thermal EHL model. An extensive set of two-disk experiments are performed to (i) establishing a link between the bulk temperatures and relative sliding during traction tests, and (ii) the limits of scuffing under various speed, load and sliding conditions. Both types of experiments are simulated by using the proposed model to demonstrate its accuracy.

The proposed general methodology is applied to a spur gear problem by considering variations of contact parameters along the tooth surfaces and incorporating a gear load distribution to predict contact loads. This spur gear scuffing model uses a one-dimensional (line contact) thermal EHL model and a convective heat transfer model of a gear pair in an iterative manner to predict the maximum instantaneous
contact temperatures, which are used with the scuffing temperature limits established by the experiments to determine the likelihood of scuffing to occur. At the end, the proposed methodology is compared to the conventional gear scuffing criteria to highlight its capabilities to overcome the major shortcoming these criteria.
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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Scuffing is one of the most common surface failure modes observed at lubricated, loaded contacts experiencing excessive relative sliding. The other two most common surface failure modes are wear and contact fatigue (micro-pitting and spalling) \[1.1, 1.2\]. Scuffing is a failure that occurs suddenly, resulting in complete destruction of contacting surfaces such as gear teeth. Scuffing is often characterized as a lubrication failure \[1.3, 1.4\], frequently accompanied by a sudden increase in friction and the instantaneous temperature at the contact zone. In case of contacts operating at high speeds, any breakdown of full elastohydrodynamic lubrication (EHL) film causes metal-to-metal contacts. The metal-to-metal contacts together with significant sliding motion generate considerable heat at the contact interface. The temperature at the interface may increase to levels that cause surface asperities weld together and then tear apart as the motion continues. The breakdown of full EHL is a necessary but not a
sufficient condition for scuffing to occur [1.5, 1.6]. Available evidence suggests that the film breakdown depends not only on the operating conditions but also on the physical and chemical nature of the lubricant as well as the material properties of the contacting surfaces. It is safe to consider that scuffing failure is induced by temperature, and the frictional heating due to interacting asperities impacts it drastically [1.7, 1.8].

From the point of view of tooth engagement, scuffing typically occurs either at the tips or at the roots on gear teeth especially if the gear teeth have little or no profile modification [1.9]. The sliding velocities have their maximum value either at the start of active profile (SAP) or at the tip of the gear tooth such that the built-up heat becomes more critical in these areas of the tooth surface. From the point of view of operation, scuffing is more likely to occur at high-load and high-speed conditions as in typical rotorcraft and aircraft gearing [1.10]. Increased contact loads tend to enhance the asperity contacts such that lubrication conditions are in a mixed EHL type. Under such conditions, both fluid film and asperity contacts share the normal load applied. As the frequency of loading at each tooth surface increases with increasing operating speed, the rate of heat generation increases as well. Under such severe conditions, the lubricant temperature might reach its flash point and the lubricant film starts to vaporize. The breakdown of lubricant film in the contact causes welding and tearing on the metal surfaces, which is often referred to as hot scuffing. However, a similar damage also occurs on the surfaces of gears operating at low speeds without the
instantaneous welding phenomenon. This damage typically occurs at high-load and low-speed conditions and is termed cold scuffing [1.1, 1.9].

While other modes of lubrication related tooth failures such as contact fatigue (pitting and micro-pitting) and wear generally take a long period of operation to reach the state of catastrophic failure, scuffing occurs early in the life span of a gear pair. In fact, gears are more vulnerable to scuffing when they are new and their tooth surfaces have not yet been smoothed by run-in. Therefore, scuffing is the most immediate failure mode that must be dealt with. The other failure modes become simply irrelevant once a gear scuffs. In other words, any well designed and manufactured gear set has the hurdle of avoiding scuffing before other considerations come to play.

Scuffing is not limited to gears, and may occur in many other mechanical components such as the piston-pin/bore bearings, piston skirt/liners, and cam/follower contacts. Since the lubrication conditions characterizing the rolling and sliding motion in the tooth contact differ from those in bearings or other components, and gear applications are the primary focus of this study, only gear scuffing phenomenon will be considered in this study. The main goal here is to develop validated physics-based models for prediction of the onset of scuffing for general two dimensional contacts in combined sliding and rolling, and to apply them to study scuffing of gears.
1.2 Literature Review

As various definitions of scuffing can be found in the literature [1.2, 1.11-1.13], all agree that scuffing is a gross damage of the contacting surfaces characterized by the occurrence of solid-phase welds between the sliding surfaces. Although several other terms such as scoring, burnishing, welding, smearing, and galling have also been used in many occasions and added more confusion to the definition of scuffing, they in fact practically refer to the same failure mode. While the critical role of scuffing performance of a lubricated contact has been studied extensively through experiments, contributions to the fundamental understanding of scuffing including its basic mechanisms have been quite limited. Complex mechanical, thermal and chemical interactions between the sliding surfaces make scuffing rather complicated. So far, various criteria have been proposed to explain the mechanisms of scuffing for a generic lubricated contact using different models or theories including the Frictional Power Model [1.14], Frictional Power Intensity Model [1.15], True Frictional Power Intensity Model [1.16], Critical Contact Temperature Theory [1.17], Collapse of Elastohydrodynamic Lubrication Film [1.18], Thermoelastic Instability [1.19], Desorption Failure Model [1.7, 1.20], Oil Oxidation [1.7], Lubricant Decomposition Model [1.21], Plasticity Index [1.22], Subsurface Plastic Deformation and Propagation [1.23], Adiabatic Shear Instability [1.13] and so on. While each of the above models, theories, or hypotheses provides plausible explanations for scuffing, none of them has
been widely accepted as the mechanism of scuffing for all types of components and applications.

The studies that focus on gear scuffing can be reviewed in two groups. The first group is formed by experimental studies using FZG type standard gear tests or two-disk tests designed to simulate the gear scuffing. One sub-group of experimental scuffing studies focused on the impact of the lubricant and how to modify the thermophysical, rheological, or chemical properties of lubricant by adding the suitable extreme pressure (EP) or antiwear (AW) additives [1.24-1.27]. Effects of aging of oils and more environmentally-friendly oils on the scuffing resistance have also been studied [1.28, 1.29]. Another sub-group of gear scuffing experiments focused on the impact of gear geometry, material properties, and surface finish on gear scuffing. Ideas such as optimization of tooth modifications to prevent edge loading in the areas of maximum sliding, heat treatment processes to enhance the surface hardness, coating of tooth surfaces with physical vapor deposition (PVD) coatings to reduce friction [1.30], and super-finishing (polishing) of tooth surfaces to reduce the possibility of asperity contact [1.31] were all tested in terms of their scuffing consequences. The scuffing resistance of structural ceramic materials (zirconia and polycrystalline) and powder metals was also a subject matter for several studies [1.32, 1.33]. The third sub-group of experiments focused on the impact of operating conditions on gear scuffing. The evaluation methods of scuffing load capacity for different types gears were established in the form of standardized tests of ASTM (American Society for Testing and
Materials) and ISO (International Organization for Standardization) [1.34-1.39]. Nevertheless, not all of the factors in the operation were taken into account. For example, under the dip lubrication condition, the influence of the sump oil level on the scuffing load capacity was not evaluated [1.40]. Alternative methods were also proposed to improve the shortcomings of standard FZG tests in terms of their resolution and sensitivity [1.41].

Several theoretical investigations were conducted to predict the temperature rise along a contact interface due to friction. A flash temperature model to obtain a closed-form steady-state solution for temperature rise within the contact zone was proposed by Blok [1.17]. This model leads to an equal temperature rise on both contact surfaces since it assumes an equal partitioning of heat flux to both surfaces. Blok’s model is valid only for ideally smooth surfaces and it requires a sufficient time of operation for the temperature distribution to reach its steady state. The validity of Blok’s model was examined in several studies in terms of its ability to predict scuffing, with some of these studies showing discrepancies in Blok’s criterion [1.25] and others showing agreement [1.42].

The effect from multi-spotted heat sources in the contact zone was the focus of several other studies. By assuming that the finite multi-spotted heat sources are randomly distributed (in position and in time) in the contact zone, Ling [1.43] used the stochastic approach to calculate the transient temperature rise in the contact zone for
two sliding surfaces and found that even small numbers of asperity contact can greatly influence the surface temperature in the contact. In addition, Cioc et al. [1.44] and Wang et al. [1.45] used the deterministic approach to estimate the temperature distributions under the mixed EHL condition for the surfaces with measured surface roughnesses.

1.3 Scope, Objectives and Overall Modeling Methodology

Despite the progress made by the investigations listed above, a widely accepted, validated gear scuffing model still does not exist. The most commonly used model, Blok’s model, has major problems in evaluating friction coefficient, which is the case for several other models as well [1.45-1.48]. These models also fail in terms of their ability to incorporate asperity contacts in the EHL model as well as predict the bulk temperatures. Accordingly, this study focuses on developing a physics-based model to predict thermal conditions of lubricated contacts under combined sliding and rolling and relate these thermal conditions to the likelihood of scuffing.

First, a generic two-dimensional (2D) lubricated contact formed by two rough disk surfaces that are in relative sliding will be considered. Focusing on the lubricant, surfaces and operating conditions resulting in the mixed EHL behavior with excessive asperity interactions, the modeling strategy illustrated in Figure 1.1(a) will be used to
Figure 1.1 The scuffing modeling methodology for (a) a two-disk contact and (b) a gear contact.
predict the thermal conditions of the contact surfaces in time. This strategy includes development of

(i) a non-Newtonian, transient, mixed, thermal EHL (TEHL) model that simulates the point (elliptical) contact problem under the heavily loaded conditions to predict the time-averaged friction force $F_{ave}$ and the time-averaged heat partitioning coefficient $f_{ave}$ at given surface roughness and lubricant bulk temperature conditions, and

(ii) a heat transfer model of the contacting bodies and their supporting structures to predict the change in bulk temperatures, $T_{b1}$ and $T_{b2}$, due to the heating by $F_{ave}$ and the cooling by the oil/air mist.

These two models will be exercised iteratively to predict the critical (maximum) instantaneous surface temperatures to be used as a measure for scuffing to occur.

As the main intended application is gear scuffing, a specialized version of the above 2D model will be developed for spur gears as well. The modeling methodology for gear scuffing, as illustrated in Figure 1.1(b), will differ from the earlier 2D model. First of all, in order to define the relationship between the loading and the contact pressures, a gear load distribution model must be incorporated in this methodology to predict the load distributions at a given input torque as gears roll in mesh. While such a load distribution model [1.49-1.51] is available, its implementation within this
methodology is required. Secondly, the contact shapes are representative of line contacts rather than being elliptical point contacts. Finally, gear contact conditions (load, rolling and sliding velocities, radii of curvature, etc.) vary with the rotational position of gears. Therefore, the thermal analysis of a gear contact requires (i) discretization of the mesh period into discrete rotational positions, and (ii) one-dimensional (line contact) thermal mixed EHL analysis at each simultaneous contact position. Finally, since there are multiple tooth pairs in contact at any given instant, a new heat transfer model is required to capture the heat flux generated by each contact to predict the bulk temperatures. According to Figure 1.1(b), the second set of objectives towards the study of gear scuffing is formed by:

(i) development of a 1D version of TEHL model proposed for generic point contacts,

(ii) implementation of a gear load distribution model for providing normal loads and geometric parameters required by the 1D TEHL model at each contact position, and

(iii) development of a new gear pair heat transfer model to be implemented the average heat fluxes, $\bar{q}_1$ and $\bar{q}_2$, iteratively from the 1D TEHL to predict the bulk temperatures.
With these two sets of models in place, the last objective of this study is the direct comparisons to measurements from the experiment for assessing their accuracy. For the validation of the generic contact model, an experimental study using a two-disk test methodology will be conducted and the measured temperatures and scuffing behavior will be compared to those predicted by the model. Likewise, the gear scuffing model will be evaluated by using the spur gear scuffing experiments performed in a companion study [1.52].

1.4 Dissertation Outline

As illustrated in the modeling methodologies of Figure 1.1, a number of models must be developed so that they can be used iteratively to predict the thermal conditions of the contacting surfaces in time. Chapter 2 details the methodology of a scuffing model for point (elliptical) contact problems. First, a non-Newtonian, transient, mixed, thermal EHL (TEHL) model is proposed to simulate point contact problems under the heavily loaded conditions to predict the distributions of pressure, film thickness, surface and oil temperatures, friction force as well as the heat partitioning coefficient at the given surface roughness and lubricant bulk temperature. Secondly, a transient heat transfer model of the two contacting disks and their supporting structures is proposed to predict the time-varying bulk temperatures due to the heating by the friction force on the contacting interface in the presence of the cooling by the oil/air mist surrounding contacting bodies. With simultaneous use of these two models, the critical (maximum)
instantaneous surface temperatures will be predicted and used as a measure for scuffing to occur.

Chapter 3 is devoted to two-disk experiments and validation of the scuffing model for elliptical contact problems. A two-disk test set-up designed and built for performing two-disk experiments will be described. Tests conducted on the two-disk machine include traction tests and scuffing tests. Comparisons between the experiments and the model predictions will be provided to assess the accuracy of the model.

In Chapter 4, a specialized version of scuffing model is developed for spur gears. Details of this model including the implementation of the gear load distribution model and development of a one-dimensional gear TEHL model are given in Chapter 4. A new gear pair heat transfer model is proposed and coupled in Chapter 4 with the gear TEHL model to iteratively predict the time-varying bulk temperatures and the critical surface temperatures of gear tooth surfaces. Comparisons are also provided at the end of the chapter to the measurements of Klein [1.52]

Finally, Chapter 5 provides a summary of the dissertation and lists its major conclusions and contributions. Several recommendations are also made for future work on this topic.
References for Chapter 1


CHAPTER 2

A SCUFFING MODEL FOR POINT CONTACT PROBLEMS

2.1 Introduction

Components such as gears and rolling element bearings transmit a certain amount of force through a thin elastohydrodynamic film of lubricant formed at the contact interfaces. Under most ideal conditions, two contact surfaces in combined rolling and sliding should be fully separated by the fluid film, avoiding any damages from direct metal-to-metal contacts of the asperities. This is often not the case when asperities of surfaces whose amplitude heights are comparable to the film thickness interact with each other. As a result, significant increases in instantaneous contact pressures are observed at the locations where asperity contacts take place. The contact surfaces also experience significant increases in surface shear stresses. In this less desirable regime of mixed EHL, the lubricant film and asperity contacts share the applied normal load simultaneously. The corresponding increases in surface temperatures due to increased rate of heat generation form the basis of scuffing failures.
of such components. This chapter focuses on the development of a scuffing model for the lubricated point (elliptical) contacts of two convex surfaces in the combined rolling and sliding action. A detailed review of the literature on this problem will be provided first, followed by the formulations governing the scuffing problem. At the end of the chapter, a set of results will be presented to demonstrate the capabilities of the model.

2.1.1 Literature Review

The modeling of lubricated contacts operating under mixed EHL conditions with significant asperity contacts has been done two drastically different ways. One group of studies treated surface roughness profiles stochastically by using certain statistical parameters and attempted to predict statistical behavior of the lubricated contact [2.1-2.5]. The greatest shortcoming of these stochastic models was that they failed to predict instantaneous changes due to asperity interactions including local pressure peaks, film thickness fluctuations and asperity deformation. Such instantaneous events are indeed critical to the surface friction and surface failures such as scuffing and fatigue.

The second group of EHL models can be characterized as deterministic models since they consider actual (measurements) roughness profiles of the contact surfaces to predict micro-scale normal pressure and film thickness distributions together with asperity interactions in the mixed lubrication regime. Digitized surface roughness
profiles were the main input to these deterministic models. Jiang et al. [2.6] developed a transient deterministic model to predict the pressure and film thickness distributions for point contacts lubricated with a Newtonian fluid, in which the lubricated regions and asperity interaction regions were handled separately. The Fast Fourier Transform (FFT) technique was used to accelerate the computation of surface deformations, recognizing that the surface elastic deformation is a convolution between the global pressure distribution and the corresponding influence function. Hu and Zhu [2.7] proposed a full-scale mixed EHL solution wherein the reduced Reynolds equations was utilized to calculate pressure in regions where the film thickness approaches to zero. Li and Kahraman [2.8] presented a mixed lubrication model where the computational domain was discretized by using a asymmetric integrated control volume (AICV) scheme. This model with its asymmetric discretization scheme was shown to be less sensitive to the grid size and discretization errors.

When a component (say a gear pair) is operated under full film EHL conditions, incremental increases in the lubricant temperature help reduce the load independent (spin) power losses associated with oil churning due to the reduced viscous shearing [2.9, 2.10]. However, in the mixed EHL regime, the instantaneous increase in the lubricant temperature within the contact reduces the viscosity and the film thickness, and as a result, increases the occurrence of asperity interactions [2.11]. Such thermal effects were not addressed in the models cited above [2.6-2.8] as they all considered isothermal contact conditions. In an earlier study, Blok [2.12] proposed a surface flash
temperature model to arrive at a closed-form steady-state solution for the temperature rise within the contact zone. This closed-form solution leads to an equal temperature rise on both rubbing surfaces since it assumes equal partition of heat flux to both surfaces. Blok’s model is valid only for ideally smooth surfaces and it requires sufficient time for the temperature distribution to reach a steady state.

The effect from multi-spotted heat sources in the contact zone has been the focus of several studies. By assuming that the finite multi-spotted heat sources are randomly, in position and in time, distributed in the contact zone, Ling [2.11] used the stochastic model to calculate the transient temperature rise in the contact zone for two sliding surfaces and found that even a small number of asperity contacts can greatly influence the surface temperature in the contact zone. Qiu and Cheng [2.13] simulated temperature rise across the contact using a deterministic model of elliptical contacts in the mixed EHL regime. They included computer-generated longitudinal and transverse profiles of surface roughness as well as measured profile of surface roughness. Qiu and Cheng’s thermal solutions were kept uncoupled from the pressure distribution solutions to obtain fully converged thermal EHL solutions. Cioc et al. [2.14] used the deterministic model to predict the temperature distributions under the mixed EHL line-contact condition for the measured surface roughnesses. Wang et al. [2.15] calculated the temperature rise at the contact interface by using the deterministic mixed EHL model of point contacts formed by surfaces having sinusoidal roughness profiles. Deolalikar et al. [2.16] also developed a deterministic thermal mixed lubrication model
to predict the temperature rise of elliptical contacts for a surface having a single asperity and for a computer-generated rough surface with specified statistical parameters.

All thermal EHL models mentioned [2.13-2.16] used two common assumptions: (i) the coefficient of friction in the lubricated region is known beforehand, and (ii) the bulk temperatures of the contacting surfaces are equal to the oil inlet temperature. The value of coefficient of friction in the lubricated region used in above models [2.13-2.16] ranged from 0.02 to 0.1, which is in a very wide range. Moreover, the bulk temperature has been shown experimentally to be much higher than the oil inlet temperature. Factors that influence the bulk temperature include the oil inlet temperature, torque, gear rotational speed, contact position, and the oil flow rate [2.17, 2.18]. In summary, a physics-based mixed thermal elastohydrodynamic lubrication (TEHL) model of point contacts formed by surfaces having realistic roughness profiles is still needed to overcome such shortcomings before any attempt can be made to predict scuffing.

2.1.2 Modeling Methodology

In this chapter, a generic, two-dimensional (2D) lubricated contact formed by a pair of rollers with rough surfaces that are in relative sliding will be considered. Focusing on the lubricant, surfaces and operating conditions resulting in the mixed
TEHL behavior with significant asperity interactions, the modeling methodology shown in Figure 1.1 will be used to predict the thermal conditions of the contact surfaces in time. This methodology consists of two different models that must be exercised iteratively. First model is a non-Newtonian, transient, mixed, TEHL model that simulates the point (elliptical) contact problem under heavily loaded conditions to predict the average friction force $F_{ave}$ and the average heat partition coefficient $f_{ave}$ for the measured surface roughness and lubricant bulk temperature conditions. The second model is a heat transfer model of the contacting bodies and their supporting structures to predict the change in bulk temperatures due to heat generated by $F_{ave}$ in presence of convective cooling by the oil or oil-air mixture around the contacting bodies. As these two models are exercised iteratively, the critical (maximum) instantaneous surface temperatures can be predicted and used as a measure for scuffing to occur.

2.2 Non-Newtonian Transient Mixed TEHL Model for Point Contacts

In a recent study, Li and Kahraman [2.8] proposed a transient, isothermal, mixed EHL model that is capable of handling excessive asperity contacts, which are commonly observed in mechanical components such as gears. This two-dimensional contact model used a unified approach first proposed by Hu and Zhu [2.7] to handle the asperity contact regions simultaneously with the fluid film contact regions. Li and
Kahraman [2.8] used an integrated asymmetric control volume scheme in discretizing the governing equations to enhance the robustness of the numerical solution while reducing the errors associated with the grid size. The new model proposed in this section combines surface temperature rise equations with the isothermal EHL model of Li and Kahraman [2.8] to include thermal effects in the lubrication analysis.

2.2.1 Governing Equations

In the mixed EHL regime, the fluid flow in the lubricated (wet) regions of the contact is governed by the two-dimensional transient Reynolds equation

\[
\frac{\partial}{\partial x} \left( \frac{\rho_f h}{12 \eta_{fx}^*} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho_f h}{12 \eta_{fy}^*} \frac{\partial p}{\partial y} \right) = \frac{\partial (\rho_e \rho_f h)}{\partial x} + \frac{\partial (\rho_f h)}{\partial t} \tag{2.1a}
\]

where \( x \) is the coordinate in the direction of sliding, \( y \) is the coordinate normal to the direction of sliding, \( t \) is the time, \( p \) is the pressure, and \( \rho_f \) is the density of the fluid film, \( h \) is the film thickness. \( u_e = \frac{1}{2} (u_1 + u_2) \) is the mean surface entraining velocity (rolling velocity) in the \( x \) direction where \( u_1 \) and \( u_2 \) are the surface velocities of the contacting bodies 1 and 2 in the direction of sliding. In Eq. (2.1a), \( \rho \), \( \rho_f \) and \( h \) are all functions of \( x \), \( y \), and \( t \). Terms \( \eta_{fx}^* \) and \( \eta_{fy}^* \) are the effective viscosities in the \( x \) and \( y \) directions, respectively. For line-contact problems and various types of non-
Newtonian fluids, \( \eta_{fx}^{*} \) can be expressed analytically [2.19, 2.20]. For point-contact problems, however, such analytical expressions are not available while approximate expressions for \( \eta_{fx}^{*} \) and \( \eta_{fy}^{*} \) are possible using a perturbation approach [2.21], that is based on the assumption that the effective shear stress is only coupled with the shear stress in the sliding direction (the \( x \) direction). Details of this perturbation analysis are available in References [2.22, 2.23]. For an Eyring type of non-Newtonian fluid, \( \eta_{fx}^{*} \) and \( \eta_{fy}^{*} \) can be approximated as [2.21]

\[
\eta_{fx}^{*} \approx \frac{\eta_{f}}{\cosh\left(\frac{\tau_{m}}{\tau_{0}}\right)}, \tag{2.1b}
\]

\[
\eta_{fy}^{*} \approx \frac{\eta_{f}}{\sinh\left(\frac{\tau_{m}}{\tau_{0}}\right)}, \tag{2.1c}
\]

where \( \eta_{f} \) is the dynamic viscosity of the lubricant, and \( \tau_{m} \) and \( \tau_{0} \) are the mean shear stress and Eyring shear stress acting on the lubricant, respectively. The term of \( \tau_{m}/\tau_{0} \) in Eqs. (2.1b,c) can be calculated as \( \tau_{m}/\tau_{0} = \sinh^{-1}[\eta_{f}(u_{2} - u_{1})/\tau_{0}h] \).
Equation (2.1) is solved by assuming that the pressure on the boundaries of the space domain is zero and the pressure $p$ within the computational space domain is not allowed to fall below zero (no cavitation conditions). Thus, the boundary and cavitation conditions for Eq. (2.1) can be respectively written as

$$ p(x_{in}, y; t) = p(x_{out}, y; t) = p(x, y_{in}; t) = p(x, y_{out}; t) = 0, $$  \hspace{1cm} (2.2a)  

$$ p(x, y; t) \geq 0 \text{ for } x_{in} < x < x_{out} \text{ and } y_{in} < y < y_{out} $$ \hspace{1cm} (2.2b)

where $x_{in}$ and $x_{out}$ are the boundaries in the $x$ direction of the space domain, and $y_{in}$ and $y_{out}$ are the boundaries in the $y$ direction of the space domain.

Equation (2.1) is only valid for hydrodynamic lubricated regions in the contact zone where two surfaces are actually separated by sufficient lubricant film ($h > 0$). For regions where asperities of mating surfaces are in direct interactions ($h = 0$), the residual errors from the Poiseuille flow terms, first two terms in Eq. (2.1a), often cause numerical instabilities when the condition of asperities interactions is severe. A reduced form of Reynolds equation first proposed by Hu and Zhu [2.7] is used here as

$$ \frac{\partial(u_{ij} h)}{\partial x} + \frac{\partial h}{\partial t} = 0 $$  \hspace{1cm} (2.3)

to resolve the instability issues.
Thermal effects are included through their influence on the viscosity and density of lubricant. Conventionally, the energy equation is used to calculate the temperature distribution for the problems of full film lubrication. In the mixed film lubrication, however, the fluid film becomes discontinuous due to the presence of asperity interactions. Such discontinuities obstruct the numerical solution of the energy equation. Although some studies [2.24-2.26] have attempted to solve the energy equation for the mixed film lubrication problems, their numerical solutions were not widely accepted.

Alternatively, the increments of surface temperatures of rubbing surfaces due to sliding heat are considered in the present study. In the mixed lubrication condition, the heat flux is generated both by (i) the viscous dissipation from the shearing lubricant film and (ii) the frictional heat from the sliding asperity contacts. According to the equation proposed by Carslaw and Jaeger [2.27], the surface temperature rise at any contact point \((x, y)\) on a semi-infinite solid at time \(t\) due to an instantaneous heat flux of \(q\) generated at another contact point \((x', y')\) at time \(t'\) can be calculated as

\[
dT(x, y; t) = \frac{q(x', y'; t') dx'dy'dt'}{4\rho_s c_s \left[\pi\alpha_s (t-t')\right]^{3/2}} \exp\left\{-\frac{[(x-x')^2 + (y-y')^2]}{4\alpha_s(t-t')}\right\}
\]

(2.4)

where \(T\) is the surface temperature, \(\rho_s\) and \(c_s\) are the density and the specific heat of the solid, respectively, and \(\alpha_s = k_s/\rho_s c_s\) is the thermal diffusivity of the solid defined
by the ratio of thermal conductivity $k_s$ to volumetric heat capacity $\rho_s c_s$. Equation (2.4) makes the assumption that no heat is lost to the environment and all the heat is transferred into one surface. The heat flux $q$ in Eq. (2.4) is calculated by

$$q = \mu p |u_s|$$  \hspace{1cm} (2.5)

where $\mu$ is the coefficient of friction and $p$ is the pressure at point $(x, y)$ at time $t'$, and $u_s = u_1 - u_2$ is the surface sliding velocity.

The heat dissipated in the mixed EHL contact is partitioned between the two contacting surfaces. This partition depends on the thermophysical properties of the surfaces and the operation conditions. By introducing a heat partition coefficient, equations governing the temperature rises on the contacting surfaces 1 and 2 can be written, respectively, as

$$dT_1 = [1 - f(x', y'; t')] \frac{q(x', y'; t') dx'dy'dt'}{4\rho_s c_s \left[ \pi \alpha_s (t-t') \right]^{3/2}} \exp \left\{- \frac{[(x-x')^2 + (y-y')^2]}{4\alpha_s (t-t')} \right\}, \hspace{1cm} (2.6a)$$

$$dT_2 = f(x', y'; t') \frac{q(x', y'; t') dx'dy'dt'}{4\rho_s c_s \left[ \pi \alpha_s (t-t') \right]^{3/2}} \exp \left\{- \frac{[(x-x')^2 + (y-y')^2]}{4\alpha_s (t-t')} \right\} \hspace{1cm} (2.6b)$$

where $f(x', y'; t')$ is the heat partition coefficient, which represents the fraction of heat going into surface 2 while the remaining $[1 - f(x', y'; t')]$ of the heat is transferred into surface 1. The instantaneous temperature rise distribution on surface $i \ (i = 1, 2)$ at time
$t$ caused by this heat flux is obtained by integrating Eq. (2.6) over the space domain $\Omega$ from time $t'=0$ to $t$ as

$$\Delta T_i(x, y; t) = \int_0^t \int_\Omega dT_i(x', y'; t').$$ (2.7)

In order to solve the Eq. (2.7) for temperature rises on surfaces, the heat partition coefficient $f(x', y'; t')$ must be known first. The temperature variation relationship across the lubricant film proposed by Lai and Cheng [2.28] is used to determine $f(x', y'; t')$. As described previously, the contact zone consists of lubricated contact regions and asperity contact regions. In lubricated contact regions, all the viscous shearing heat in the lubricant film is assumed to be concentrated in the mid-layer of lubricant. From the mid-layer, the film temperature varies linearly across the oil film with different gradients proportional to the heat flux. The expressions of temperature variation with this assumption can be written as

$$k_f \frac{T_f(x, y, \frac{h}{2}; t) - T_1(x, y; t)}{\frac{1}{2} h(x, y; t)} = q[1 - f(x, y; t)], \quad (2.8a)$$

$$k_f \frac{T_f(x, y, \frac{h}{2}; t) - T_2(x, y; t)}{\frac{1}{2} h(x, y; t)} = q f(x, y; t) \quad (2.8b)$$
where \( T_f(x, y; \frac{h}{2}; t) \) is the temperature on the mid-layer of the oil film, and \( T_1(x, y; t) \) and \( T_2(x, y; t) \) are the temperatures on surfaces 1 and 2, respectively. Subtracting Eq. (2.8b) from Eq. (2.8a), one obtains

\[
[T_{2b}(x, y; t) + \Delta T_2(x, y; t)] - [T_{1b}(x, y; t) + \Delta T_1(x, y; t)]
= \frac{q h(x, y; t)}{2k_f} [1 - 2f(x, y; t)]
\]

(2.9)

where \( T_{1b} \) and \( T_{2b} \) are the bulk temperatures of surfaces 1 and 2, respectively. Equation (2.9) establishes the relationship amongst \( T_1(x, y; t), \ T_2(x, y; t), \) and \( f(x, y; t) \) for lubricated contact regions. In the asperity contact regions, meanwhile, it is assumed that surface temperatures are equal. At points of asperity contact this assumption yields as

\[
T_2(x, y; t) - T_1(x, y; t) = 0
\]

(2.10a)

or

\[
[T_{2b}(x, y; t) + \Delta T_2(x, y; t)] - [T_{1b}(x, y; t) + \Delta T_1(x, y; t)] = 0.
\]

(2.10b)

Equation (2.9) can also be viewed as a general form of Eq. (2.10) since Eq. (2.9) is reduced to Eq. (2.10) when \( h(x, y; t) = 0 \). In summary, Equations (2.7) and (2.9) are
used to solve \( f(x', y'; t') \) for the lubricated contact regions while Equations (2.7) and (2.10) are used to solve \( f(x', y'; t') \) for the asperity contact regions.

The temperature of the lubricant is a critical parameter influencing its viscosity and density values within the contact zone. Some of the previously developed TEHL models (e.g. References [2.15, 2.16]) did not elaborate on how the lubricant temperature was computed, while there is convincing experimental evidence that neglecting lubricant temperature variations is not a valid assumption for mixed EHL problems, as the rheological and thermophysical properties of the lubricant in the contact zone has a strongly influence on the state of EHL conditions. In the present study, the viscous dissipation is assumed to be uniform across the film. In addition, the convection, compression, side flow terms are neglected such that the predicted lubricant temperatures provide an upper bound value. With these assumptions, a simplified heat transfer equation in the fluid film is given as

\[
k_f \frac{\partial^2 T_f(x, y, z; t)}{\partial z^2} = -\eta_f^* \left( \frac{\partial u_f}{\partial z} \right)^2
\]

(2.11)

where \( z \) is the coordinate across the lubricant film from surface 1 to surface 2. With the boundary conditions \( T = T_1 \) at \( z = 0 \) and \( T = T_2 \) at \( z = h \), the temperature distribution across the film at a point \( (x, y, z) \) is found to be
\begin{equation}
T_f(x, y, z; t) = \frac{\eta_f^*}{2k_f} \left( \frac{\partial u_f}{\partial z} \right)^2 \left[ zh(x, y; t) - z^2 \right] + \frac{[T_2(x, y; t) - T_1(x, y; t)]}{h(x, y; t)} z + T_1(x, y; t).
\end{equation}

According to Fourier’s law of thermal conduction, the heat flux on surface 1 is

\begin{equation}
q[1 - f(x, y; t)] = k_f \left. \frac{T_f(x, y, z; t)}{\partial z} \right|_{z=0} = \frac{1}{2} \eta_f^* \left( \frac{\partial u_f}{\partial z} \right)^2 h(x, y; t) + \frac{T_2(x, y; t) - T_1(x, y; t)}{h(x, y; t)}.
\end{equation}

Similarly, the heat flux on surface 2 is written as

\begin{equation}
q f(x, y; t) = -k_f \left. \frac{T_f(x, y, z; t)}{\partial z} \right|_{z=h} = \frac{1}{2} \eta_f^* \left( \frac{\partial u_f}{\partial z} \right)^2 h(x, y; t) - \frac{T_2(x, y; t) - T_1(x, y; t)}{h(x, y; t)}.
\end{equation}

Subtracting Eq. (2.13b) from Eq. (2.13a), one obtains

\begin{equation}
T_2(x, y; t) - T_1(x, y; t) = \frac{q h(x, y; t)}{2k_f} \left[ 1 - 2 f(x, y; t) \right],
\end{equation}

which is the same result as Eq. (2.9).
The temperature on the mid-layer of lubricant film is of special interest here since it will be used later to determine the viscosity and the density of the lubricant. The temperature on the mid-layer of lubricant film is given as

\[ T_f(x, y, \frac{h}{2}; t) = \frac{\eta_f h^2}{8k_f} \left[ \frac{\partial u_f}{\partial z} \right]_{z=\frac{h}{2}}^2 + \frac{1}{2} [T_1(x, y; t) + T_2(x, y; t)]. \] (2.15)

In the asperity contact regions, the corresponding surface temperature will be used as the mid-layer fluid temperature for numerical purposes.

### 2.2.2 Companion Equations

The film thickness \( h(x, y; t) \) between two rubbing surfaces is composed of the relative approach of two surfaces, the undeformed geometry of the two bodies, the surface roughness profiles of the two bodies, and the elastic deformations. An expression for \( h(x, y; t) \) can be written as [2.8]

\[ h(x, y; t) = h_0(t) + h_s(x, y) - \delta_1(x, y; t) - \delta_2(x, y; t) + V(x, y; t) \] (2.16a)

where \( h_0(t) \) is the reference film thickness, \( h_s(x, y) \) is the separation function for the original geometries of two bodies that include the radii of curvature of the surfaces and given as
\[ h_s(x, y) = \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \]  

(2.16b)

where \( R_x \) and \( R_y \) are the equivalent radii of curvature in the \( x \) and \( y \) directions, respectively. In Eq. (2.16a), \( \delta_1(x, y; t) \) and \( \delta_2(x, y; t) \) are the surface roughness heights of surfaces 1 and 2, respectively. \( V(x, y; t) \) is the elastic deformation experienced by the contact due to the pressure distribution \( p(x, y; t) \) that is given as

\[ V(x, y; t) = \frac{2}{\pi E'} \int_{\Omega} \frac{p(\xi, \zeta; t)}{\sqrt{(x - \xi)^2 + (y - \zeta)^2}} \, d\xi d\zeta. \]  

(2.17a)

Here \( \Omega \) is the space domain and \( E' \) is the equivalent Young’s modulus defined as

\[ E' = 2 \left[ \frac{1 - \nu^2}{E_1} + \frac{1 - \nu^2}{E_2} \right]^{-1} \]  

(2.17b)

where \( \nu \) and \( E_i \) are the Poisson’s ratio and the Young’s modulus of contacting body \( i \) \( (i = 1, 2) \), respectively.

The normal load \( W(t) \) applied to the contact at any arbitrary time \( t \) must be balanced by the normal force due to the predicted pressure distribution within the contact zone, i.e.
\[ W(t) = \int_{\Omega} p(x, y; t) dxdy. \] (2.18)

If the resultant normal load computed from the pressure distribution at a given instant is not equal to applied normal load, the reference film thickness \( h_0(t) \) in Eq. (2.16a) is adjusted within an iterative loop until Eq. (2.18) is satisfied.

In the mixed EHL regime, the surface traction in the contact zone consists of viscous shear exerted by the lubricant shearing in lubricated contact regions and the asperity shear exerted by the metal-to-metal shearing in asperity contact regions. The viscous shear stress \( \tau_f \) applied on the lubricant by the surface of body 1 for the non-Newtonian fluid can be derived from the reduced Navier-Stokes equations as

\[
\frac{\partial p_f}{\partial x} = \frac{\partial}{\partial z} \left( \eta^* \frac{\partial u_f}{\partial z} \right), \tag{2.19a}
\]

\[
\frac{\partial p_f}{\partial y} = \frac{\partial}{\partial z} \left( \eta^* \frac{\partial v_f}{\partial z} \right) \tag{2.19b}
\]

where \( z \) is the coordinate across the fluid film, \( \eta^*_f \) is the effective viscosity of the lubricant, and \( u_f \) and \( v_f \) are the velocity components of the fluid film in the \( x \) and \( y \) directions, respectively. Since the film thickness in the contact is much less than the dimension of contact area, the variation of pressure across the fluid film (z direction) is
insignificant. Therefore, the pressure can be assumed to be a sole function of \( x \) and \( y \) only. The gradients of the velocity components \( u_f \) and \( v_f \) can be found by integrating Eqs. (2.19a) and (2.19b) as

\[
\frac{\partial u_f}{\partial z} = \frac{z}{\eta_f} \frac{\partial p}{\partial x} + \frac{A_1(x, y)}{\eta_f},
\]

(2.20a)

\[
\frac{\partial v_f}{\partial z} = \frac{z}{\eta_f} \frac{\partial p}{\partial y} + \frac{B_1(x, y)}{\eta_f},
\]

(2.20b)

where \( A_1 \) and \( B_1 \) are the integration constants. Integrating Eqs. (2.20a) and (2.20b) once more and introducing the following boundary conditions with the assumption of no slip or velocity discontinuity between the fluid and solid at the boundaries

\[
z = 0, \quad u_f = u_1, \quad v_f = 0, \quad (2.21a)
\]

\[
z = h, \quad u_f = u_2, \quad v_f = 0, \quad (2.21b)
\]

the velocity components in the fluid film can be written as

\[
u_f = \left[ \int_0^z \frac{z'}{\eta_f} \, dz' - \int_0^z \frac{dz'}{\eta_f} \right] \frac{\partial p}{\partial x} + \left( \frac{u_2 - u_1}{F_0} \right) \int_0^z \frac{dz'}{\eta_f} + u_1,
\]

(2.22a)
\[ v_f = \left[ \int_0^z \frac{z'}{\eta_f} \, dz' - \frac{F_1}{F_0} \int_0^z \frac{dz'}{\eta_f} \right] \frac{\partial p}{\partial y}. \]  

(2.22b)

Here \( z' \) is a dummy variable corresponding to \( z \), and \( F_0 \) and \( F_1 \) are constants defined as

\[ F_0 = \int_0^h \frac{1}{\eta_f} \, dz', \]  

(2.23a)

\[ F_1 = \int_0^h \frac{z'}{\eta_f} \, dz'. \]  

(2.23b)

Generally, the relationship between \( \eta_f^* \) and \( z \) must be known before the integrals in Eqs. (2.23a) and (2.23b) can be evaluated analytically. If not, they must be evaluated numerically. Additional integrations performed across the film thickness to obtain \( \eta_f^* \) and further evaluate \( u_f \) and \( v_f \) increases the computational time significantly. For this practical reason, \( \eta_f^* \) is assumed to remain constant across the film thickness such that the velocity components in Eqs. (2.23a) and (2.23b) can be reduced to

\[ u_f = \left( \frac{z^2 - zh}{2\eta_f^*} \right) \frac{\partial p}{\partial x} + \left( u_2 - u_1 \right) \frac{z}{h} + u_1, \]  

(2.24a)
\[ v_f = \left( \frac{z^2 - zh}{2\eta_f^*} \right) \frac{\partial p}{\partial y}. \]  \hspace{1cm} (2.24b)

With this, the viscous shear stress along the \( x \) and \( y \) directions on the \( z \) plane for the non-Newtonian fluid can be expressed as

\[ \tau_{zx} = \eta_f^* \left( \frac{\partial w_f}{\partial x} + \frac{\partial u_f}{\partial z} \right), \]  \hspace{1cm} (2.25a)

\[ \tau_{zy} = \eta_f^* \left( \frac{\partial w_f}{\partial y} + \frac{\partial v_f}{\partial z} \right) \]  \hspace{1cm} (2.25b)

where \( w_f \) is the velocity component of the fluid in the \( z \) direction. Terms \( \frac{\partial w_f}{\partial x} \), \( \frac{\partial w_f}{\partial y} \) are at least an order of magnitude smaller than \( \frac{\partial u_f}{\partial z} \) and \( \frac{\partial v_f}{\partial z} \), respectively. Hence, Eqs. (2.25a) and (2.25b) can be reduced further to

\[ \tau_{zx} = \eta_f^* \frac{\partial u_f}{\partial z}, \]  \hspace{1cm} (2.26a)

\[ \tau_{zy} = \eta_f^* \frac{\partial v_f}{\partial z}. \]  \hspace{1cm} (2.26b)

Substituting for \( u_f \) and \( v_f \), Eqs. (2.26a) and (2.26b) are written as

\[ \tau_{zx} = \eta_f^* \left( \frac{u_2 - u_1}{h} \right) + \frac{1}{2} \left( 2z - h \right) \frac{\partial p}{\partial x}, \]  \hspace{1cm} (2.27a)
\[ \tau_{xy} = \frac{1}{2} (2z - h) \frac{\partial p}{\partial y}. \quad (2.27b) \]

Assuming that each of contacting bodies and surface roughness profiles are both symmetric about the \( xz \) plane, which is true for gear contacts, Eq. (2.27b) vanishes since shear stresses about the symmetry plane cancel out each other. The first term present on the right hand side of Eq. (2.27a) represents the traction term due to viscous shear stress and the second term is due to the pressure gradient causing rolling resistance. It is noted that the first term disappears when \( u_2 = u_1 \) (pure rolling). The rolling resistance has been claimed to be a result of energy dissipation in squeezing the lubricant [2.30] or a result of elastic hysteresis in the deformation of the contacting bodies [2.31]. The traction term in Eq. (2.27a) is by far the larger of the two terms. However, both terms in Eq. (2.27a) contribute to the friction when any sliding in the contact takes place. It is also noted that shear stress on the fluid film varies along the film thickness. According to the perturbation approach, the effective shear stress is only coupled to the shear stress in the sliding direction. Therefore, the viscous shear stress acting on the surface 1 \((z = 0)\) can be written as

\[ \tau_f = \tau_{zx} \bigg|_{z=0} \approx \eta \frac{u_2 - u_1}{h} - \frac{h}{2} \frac{\partial p}{\partial x}. \quad (2.28) \]

So far, there is no expression available to accurately calculate the shear stress in the regions of asperity interaction (dry contact regions). An approximation that is based on
the formula proposed by Rabinowicz [2.32] for the boundary film shearing is used in this study to determine the shear stress in the regions of asperity interaction as

\[
\tau_a = \sqrt{\tau_{a0}^2 + (\gamma_a P)^2}
\]

(2.29)

where \(\tau_{a0}\) is the initial shear strength of the boundary film and \(\gamma_a\) is the coefficient corresponding to the friction coefficient in the boundary lubrication. According to the experimental results of Hoglund [2.33], \(\gamma_a\) is in the same order as the coefficient corresponding to maximum friction coefficient in the hydrodynamic lubrication. In the present study, the numerical values of \(\tau_{a0} = 5.5\) MPa and \(\gamma_a = 0.15\) are adopted.

In the mixed EHL regime, the friction force is defined as the sum of the traction in lubricated contact regions and friction in asperity contact regions. According to the Coulomb/Amonton friction law, the coefficient of friction \(\mu\), a ratio of the friction force between two contacting bodies to the load pressing them together, can be then calculated as

\[
\mu = \frac{\Omega_f}{\Omega_a} \frac{\iint \tau_f \, dx \, dy + \iint \tau_a \, dx \, dy}{\iint \Omega \, p \, dx \, dy}
\]

(2.30)

where \(F\) is the friction force, \(\Omega_f\) is the space domain of lubricated contact regions, and \(\Omega_a\) is the space domain of asperity contact regions.
Both viscosity and density of the lubricant vary with pressure and temperature. Experiments have shown that the viscosity increases with increasing pressure. Any lubricant as a liquid can be significantly more viscous or may become an amorphous solid under sufficiently high pressure. Such high pressure condition forms the elastohydrodynamic lubrication, thus providing necessary film thickness to separate two rubbing surfaces. In 1893, Barus [2.34] proposed an empirical isothermal pressure-viscosity formula in which viscosity increases exponentially with pressure. Several other isothermal pressure-viscosity formulae have been proposed since then to improve Barus’ formula at higher pressure conditions. Meanwhile, viscosity of the lubricant decreases with an increase in temperature. One widely used empirical relationship of pressure-temperature-viscosity is the modified Roelands equation [2.35], which was shown recently to be accurate only within the low-temperature and low-pressure ranges [2.36].

Under the mixed EHL conditions, the changes in contact pressure with location are often sudden and significant. As a result, the instantaneous lubricant viscosity also changes drastically within the contact zone, in some cases, several orders of magnitude. Therefore, the accuracy of the viscosity-pressure relationship becomes critical as it dictates the accuracy of the resultant EHL model. In the present study, the two-slope viscosity-pressure model of Allen et al. [2.37] is used in the form modified later by Goglia et al. [2.38] with thermal effects [2.14] as
\( \eta_f = \begin{cases} 
\eta_{f0} e^{[\alpha_1 p - \gamma(T-T_0)]}, & p \leq p_a, \\
\eta_{f0} e^{[c_0 + c_1 p + c_2 p^2 + c_3 p^3 - \gamma(T-T_0)]}, & p_a < p < p_b, \\
\eta_{f0} e^{[\alpha_1 p + \alpha_2 (p-p_t) - \gamma(T-T_0)]}, & p_b \leq p, 
\end{cases} \)  
\tag{2.31}

where \( \eta_{f0} \) is the dynamic viscosity of inlet lubricant, \( T_0 \) is the oil inlet temperature, \( \gamma \) is the viscosity-temperature coefficient, and \( p_t \) is the transitional pressure, a value beyond which the increase in viscosity changes slope from \( \alpha_1 \) to \( \alpha_2 \). Here \( p_a = 0.7 p_t \) and \( p_b = 1.4 p_t \) where \( p_t = 0.38 \) GPa are proposed by Allen et al. [2.37] for mineral oils. The coefficients \( c_i \ (i \in [0,3]) \) are determined such that the transition between the two slopes is a smooth one.

In terms of dependency of lubricant density on pressure and temperature, the most widely used pressure-density relation in the EHL simulations was proposed by Dowson and Higginson [2.30]. Here, a modified form of it that considers the variation of pressure and temperature can be written for mineral oils as [2.14]

\[
\rho_f = \rho_{f0} \left[ 1 + \frac{0.6(10)^{-9} p}{1+1.7(10)^{-9} p} \right] \left[ 1 - \beta(T-T_0) \right] 
\tag{2.32}
\]

where \( \rho_{f0} \) is the density of the inlet lubricant, \( \beta \) is the thermal expansion coefficient of the lubricant, and \( p \) is in Pa.
The percentage of load carried by asperity contacts and the percentage of area occupied by asperity contacts can be used to measure the severity of asperity interaction in the mixed lubrication regime [2.8]. A load ratio of asperity contact $W_c$ is defined here as the ratio of load supported by asperity interactions to the applied normal load,

$$W_c = \frac{1}{W} \iint_{\Omega_a} p \, dx \, dy. \quad (2.33)$$

Similarly, an area ratio of asperity contact $A_c$ is defined as the ratio of area with asperity interactions to the nominal Hertzian contact area

$$A_c = \frac{\iint_{\Omega_a} dx \, dy}{\pi a_h b_h} \quad (2.34)$$

where $a_h$ and $b_h$ are semi-axis lengths of the Hertzian contact in the $y$ and $x$ directions, respectively.

### 2.2.3 Numerical Solution

The relationships among the pressure, viscosity, and density, together with the non-Newtonian behavior of the lubricant and thermal effects make the Reynolds equation highly nonlinear. Appropriate discretization and linearization schemes are
required to achieve a robust and accurate numerical solution to the mixed TEHL problem in hand. The asymmetric integrated control volume (AICV) method developed by Li and Kahraman [2.8] to solve the non-Newtonian transient mixed isothermal EHL problem is expanded in the present study to include the thermal effects as formulated in the previous section.

The overall numerical procedure used to implement the non-Newtonian transient mixed TEHL model is shown in Figure 2.1. With the assumed initial film thickness, pressure and temperature distributions along the contact area together with the initial bulk temperatures and the measured roughness profiles, the first step in the numerical procedure is to go through the pressure iterations under isothermal conditions. The lubricant viscosity and density are updated according to the pressure and temperature distributions. A flexibility matrix of influence coefficients is constructed. Each measured surface profile is approximated by the cubic spline interpolant of the nodal surface height. The elastic deformation defined by Eq. (2.17a) is written as a convolution problem and solved by using the Fast Fourier transform (FFT) based method [2.39-2.41]. Once the elastic deformation is computed, the film thickness between the two surfaces is calculated through Eq. (2.16a). Substituting the film thickness into the Reynolds Equations, Eqs. (2.1) and (2.3), another system of equations is formed and solved by using the Tri-Diagonal Matrix Algorithm (TDMA) for the pressure distribution. The calculated pressure distribution is checked against the applied normal load according to Eq. (2.18). If Eq. (2.18) is not satisfied, the
Figure 2.1 Computational flow chart used for the TEHL solution.
reference film thickness is adjusted by the under-relaxation method and the pressure iterations are repeated until a converged pressure is obtained.

As shown in Figure 2.1, after the converged isothermal pressure and film thickness distributions, $p^{(1)}(x,y;t)$ and $h^{(1)}(x,y;t)$, are obtained, the numerical procedure proceeds with the thermal model. First, viscosity and density distributions of the lubricant are updated to obtained $\eta_f^{(1)}(x,y;t)$ and $\rho_f^{(1)}(x,y;t)$ according to Eqs. (2.31) and (2.32) with the converged pressure distribution and the oil inlet temperature. The surface temperatures $T_1^{(1)}(x,y,t)$ and $T_2^{(1)}(x,y,t)$ are calculated from Eq. (2.7) by using the Moving Grid Method (MGM) [2.42-2.44]. The new temperature distribution on the mid-layer of oil film $T_f^{(1)}(x,y,\frac{h}{2},t)$ calculated from Eq. (2.15) is then compared with the oil inlet temperature. Here the superscript is the index for thermal iterations. If the difference of temperature on the mid-layer of oil between $T_f^{(1)}(x,y,\frac{h}{2},t)$ and the oil inlet temperature is not within a user defined threshold value $\varepsilon_T$, the new lubricant temperature distribution is also adjusted by using the under-relaxation method. The corresponding lubricant viscosity and density distributions are updated according to the adjusted lubricant temperature distribution and then fed into the isothermal EHL model to determine new converged pressure and film thickness distributions $p^{(2)}(x,y;t)$ and $h^{(2)}(x,y;t)$. Another thermal iteration is initiated to obtain $\eta_f^{(2)}(x,y;t)$, $\rho_f^{(2)}(x,y;t)$,
The temperature distribution from this iteration is compared to that from the previous iteration to check for its convergence. If the difference of temperature on the mid-layer of oil film between the two consecutive thermal iterations is larger than \( \varepsilon_T \), then another thermal iteration is initiated. In order to ensure that the pressure and temperature solutions are fully coupled, at least two thermal iterations are imposed. Once the differences in the temperature distribution and in the pressure distribution between two consecutive iterations are reasonably small and the thermal model has been run at least two times, the most recent temperature, pressure and film thickness distributions are recorded as thermal EHL results at this time instant \( t \).

After solutions at time instant \( t \) are obtained, the time is set to \( t + \Delta t \) and the surface roughness profiles are updated according to the time and their moving velocities. The same iterative procedure described above is repeated to perform the thermal EHL analysis at the next time increment.

### 2.3 Example Mixed TEHL Model Predictions

An example contact of a pair of disks will be introduced here to demonstrate the capabilities of the mixed TEHL model. This example two-disk contact problem will also be used in the experimental study presented in Chapter 3.
Table 2.1 lists the dimensions and material properties of the disks of this example system. The smaller of the two disks (disk 1) has a diameter of 31.75 mm and disk 2 has a diameter of 57.15 mm. Both disks are made out of carbon steel (4620M gear steel) with $E_1 = E_2 = 207$ GPa. Disk 2 has a circular crown in the axial direction at 76.2 mm radius. The contact of this slightly barrel shaped disk 2 with disk 1 avoids any edge loading conditions. As a result, the contact pair forms an elliptical contact shape.

A typical automatic transmission fluid (ATF) is used in this simulation as the lubricant. Table 2.2 lists parameters of this lubricant at 90°C. The density value listed in Table 2.2 is predicted by best linear fit from the measured density-temperature data. The dynamic viscosity value is converted from the kinematic viscosity obtained by the viscosity-temperature formula according to ASTM D341.

The contact problem is simulated with surface roughness profiles with root-mean-square (RMS) roughness values of $R_{q1} = 0.45$ μm and $R_{q2} = 0.35$ μm. These roughness values were measured using a surface profiler tracing along the rolling (circumferential) direction. The corresponding composite RMS surface roughness $R_q$ is 0.57 μm. Figure 2.2 shows these measured roughness profiles. The instantaneous roughness profiles $\delta_1(x,y;t)$ and $\delta_2(x,y;t)$ for each time step are approximated by using a cubic spline interpolation, and extended in the $y$ direction. As both contact
Table 2.1 Dimensions and material properties of disks of the example contact problem.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Disk 1</th>
<th>Disk 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter [mm]</td>
<td>31.75</td>
<td>57.15</td>
</tr>
<tr>
<td>Width [mm]</td>
<td>12.2</td>
<td>6.4</td>
</tr>
<tr>
<td>Radius of crown in axial direction [mm]</td>
<td>N/A</td>
<td>76.2</td>
</tr>
<tr>
<td>Young’s modulus [GPa]</td>
<td></td>
<td>207</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td></td>
<td>0.3</td>
</tr>
<tr>
<td>RMS surface roughness [μm]</td>
<td>0.45</td>
<td>0.35</td>
</tr>
<tr>
<td>Density [kg/m³]</td>
<td></td>
<td>7850</td>
</tr>
<tr>
<td>Specific heat [J/kg/K]</td>
<td></td>
<td>470</td>
</tr>
<tr>
<td>Thermal conductivity [W/m/K]</td>
<td></td>
<td>44.5</td>
</tr>
</tbody>
</table>
Table 2.2 Parameters of the automatic transmission fluid used in this study.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic viscosity at 90 °C [Pa s]</td>
<td>6.03 (10)^{-3}</td>
</tr>
<tr>
<td>Density at 90 °C [kg/m^3]</td>
<td>824</td>
</tr>
<tr>
<td>First viscosity-pressure coefficient [Pa^{-1}]</td>
<td>1.339 (10)^{-8}</td>
</tr>
<tr>
<td>Second viscosity-pressure coefficient [Pa^{-1}]</td>
<td>1.9 (10)^{-9}</td>
</tr>
<tr>
<td>Viscosity-temperature coefficient [K^{-1}]</td>
<td>0.029</td>
</tr>
<tr>
<td>Thermal expansion coefficient [K^{-1}]</td>
<td>8.4 (10)^{-4}</td>
</tr>
<tr>
<td>Thermal conductivity [W/m/K]</td>
<td>0.128</td>
</tr>
<tr>
<td>Specific heat [J/kg/K]</td>
<td>2132</td>
</tr>
</tbody>
</table>
Figure 2.2  Measured surface roughness profiles along the circumferential direction: (a) disk 1 profile having $R_q = 0.45 \, \mu m$ and (b) disk 2 profile having $R_q = 0.35 \, \mu m$. 
surfaces are textured axially along the $y$ direction, the roughness profiles are assumed to be maintained axially in this simulation while any three-dimensional roughness profile can be handled by this model.

In order to demonstrate the ability of the proposed mixed TEHL model to handle excessive asperity contacts, a severe set of operating conditions are considered, represented by a rolling velocity of $u_e = 4$ m/s, a slide-to-roll ratio of $SR = -0.25$, and normal load of $W = 2,500$ N. The slide-to-roll ratio is defined as $SR = u_s/u_e = 2(u_1 - u_2)/(u_1 + u_2)$. The maximum Hertzian contact pressure corresponding to the normal load is 2.2 GPa. Based on the disk dimensions, material properties, oil parameters, and operating conditions, the minimum smooth-surface film thickness calculated from the Hamrock and Dowson formula [2.45] and the corresponding lambda ratio (the ratio of minimum film thickness to the composite RMS surface roughness) are 0.1 $\mu$m and 0.2, respectively. This implies, prior to any TEHL analysis, that contact conditions are of mixed type with significant asperity interactions.

The major and minor semi-axis dimensions of the elliptical Hertzian contact zone at the specified normal load value are 1.36 and 0.38 mm, respectively. This forms an ellipticity parameter of 3.7 with the minor axis being along the $x$ direction. A relatively small computational domain of $-0.71 \leq x \leq 0.42$ mm and $-2.1 \leq y \leq 2.1$ mm is considered here in order to maximize the usage of measured surface roughness.
profiles. The spatial grid sizes used in the simulations are \( \Delta x = 7.9 \ \mu m \) and \( \Delta y = 67 \ \mu m \). The grid size in the \( x \) direction is somewhat smaller than the values used by Xu and Sadeghi [2.4], Ai and Cheng [2.46], and Zhu and Ai [2.47] for point contact EHL problems but larger than the valued used by Li and Kahraman [2.8] while the mesh size in the \( y \) direction is large since the roughness in the axial direction is assumed to be constant in the present simulation.

It is not feasible to present the results at every simulation time step, results at two example instances are presented in this section. Figures 2.3 depicts results of mixed TEHL for rough surface contacts at a time step of \( n_t = 36 \) with \( \Delta t = 2 \Delta x/u_1 \) and \( t = n_t \Delta t \). In Figure 2.3(a), grid nodes having asperity contacts are shown across the contact zone to illustrate the level of the metal-to-metal contact activity. The corresponding load \( W_c \) and area \( A_c \) contact ratios at this instant are 10.3\% and 26.3\%, respectively. As a result of such severe asperity interactions, the pressure distribution shown in Figure 2.3(b) fluctuates drastically in the contact zone along the \( x \) direction. In this simulation, the local pressure at an asperity contact location was not allowed to exceed 4 GPa. The instantaneous film thickness distribution shown in Figure 2.3(c) has zero values at points of asperity contact, illustrating that the lubricant film is broken down frequently due to asperity interactions.

Instantaneous temperature distributions on surface 1, surface 2, and the mid-layer of the oil film at the same time instant are shown in Figures 2.3(d), (e), and (f),
Figure 2.3 Mixed TEHL results of example two-disk contact at $n_r = 36$: (a) points of asperity contact, (b) the pressure distribution, (c) the film thickness distribution, (d) surface 1 temperature distribution, (e) surface 2 temperature distribution, and (f) mid-layer lubricant temperature distribution.
Figure 2.3 Continued.
respectively. In contrast to drastic pressure distribution fluctuations shown in Figure 2.3(b), temperature fluctuations of both surfaces across the contact zone are much more gradual, except in the locations where asperity contacts take place. Temperature values along the mid-layer of oil film in the lubricated regions, as shown in Figure 2.3(f), are much higher than those of each surface, and they exhibit significant fluctuations in the direction of the rolling and sliding. The maximum temperature on the mid-layer oil exceeds 120°C. The faster moving surface (surface 2) has a slightly higher temperature than surface 1, indicating that the faster moving surface receives slightly more dissipated heat. However, the temperature gradient between the two surfaces is not that pronounced due to proximity of the surfaces in such mixed lubrication conditions.

The mixed TEHL results of the same system at another time step of $n_t = 60$ are also shown in Figure 2.4. The corresponding $W_c$ and $A_c$ values of the contact shown in Figure 2.4(a) are 12% and 20%, respectively. In comparison to the Figure 2.3(a), most of asperity interactions in Figure 2.4(a) are shifted to the downstream, illustrating the highly transient nature of the problem originated mainly by the instantaneous matching of the surface roughness profiles. The pressure, film thickness and temperature distributions shown in Figure 2.4 are also somewhat different from those in Figure 2.3. In Figure 2.4(f), the maximum temperature on the mid-layer of oil film is close to 130°C at this instant.
Figure 2.4 Mixed TEHL results of example two-disk contact at $n_r = 60$: (a) points of asperity contact, (b) the pressure distribution, (c) the film thickness distribution, (d) surface 1 temperature distribution, (e) surface 2 temperature distribution, and (f) mid-layer lubricant temperature distribution.
Figure 2.4 Continued.
In order to further demonstrate the transient nature of this lubricated contact, the load and area ratios of asperity contact, $W_c$ and $A_c$, are plotted in Figure 2.5 against the time index $n_t$ for the first 100 time steps. Such severe fluctuations of $W_c$ and $A_c$ with time were also observed in the previous predictions for rough isothermal EHL contacts [2.8]. The corresponding time variation of the surface traction (friction) force $F$ and the corresponding average friction coefficient $\mu$ (both defined in Eq. (2.30)) are shown in Figures 2.6(a) and (b), respectively. The friction force values are very low in certain instances when almost a full-film lubrication condition exists, while it is as high as 150 N in some other instance when asperity contacts are dominant. As a result, the coefficient of friction varies from a fraction of a percentage point to nearly 0.06.

The spatial average value of the heat partition coefficient $f$ (from Eq. (2.8) within the nominal Hertzian contact area) is also transient and bounded by the values of 0 and 1. As shown in Figure 2.7, the spatial average values of $f$ remain relatively constant within this time segment. It is about 0.55 in this case, indicating that about 55% of the heat generated of the contact is transferred to surface 2 (the faster moving surface).

### 2.4 Transient Heat Transfer Model of a Pair of Rollers

The non-Newtonian transient thermal EHL model proposed in the previous
Figure 2.5 Variation of the (a) load and (b) area contact ratio values with time.
Figure 2.6 (a) Friction force and (b) coefficient of friction as a function of time.
Figure 2.7 The heat partition coefficient as a function of time.
section provides the required contact parameters in an efficient manner. However, it is not sufficient by itself to predict thermal conditions since it can only predict temperature rises above a user-defined set of bulk temperatures. As stated in Chapter 1 and illustrated in Figure 1.1, this mixed TEHL model must be coupled with a heat transfer model that can predict the bulk temperature required by it. While bulk temperature was often assumed to be equal to the oil inlet temperature \([2.14]\), a number of numerical and experimental studies showed clearly that this assumption is not valid \([2.17, 2.48-2.50]\). Especially in lubrication of components such as gears, the knowledge of bulk temperature of the contacting surfaces is a must. Therefore, a transient heat transfer model for a two-disk contact problem will be used to predict the bulk temperatures required by the mixed TEHL model. This heat transfer model will rely on the heat flux and heat partition coefficient values provided by the TEHL model.

### 2.4.1 Governing Equation

Figure 2.8 shows the schematic plot of a two-disk contact arrangement that consists of two cylindrical disks (with or without lead corrections) of radii \(R_i\) and width \(w_i\) \((i = 1, 2)\) that are in contact with each other. Their shafts are formed by cylindrical segments of radius \(r_j\) and lengths \(L_j\) \((j \in [1, 4])\). Since all shaft segments and two disks are axisymmetric, only half of each segment needs to be modeled.

The heat generated within the contact zone is assumed to be distributed evenly
Figure 2.8 Dimensions of disks in contact with their shafts.
on the running track of each disk. The governing equation for the heat transfer within each segment is a transient diffusion-convection equation that can be written in cylindrical coordinates as

\[
\frac{\partial T}{\partial t} = \alpha_s \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
\]

(2.35)

where \( t \) is the time, \( y \) and \( r \) are the axial and radial coordinates, respectively and \( \alpha_s \) is the thermal diffusivity of the solid.

Focusing on any disk or shaft segment in Figure 2.8, different types of boundaries are identified as (i) the centerline of axisymmetry (line AB), (ii) the perpendicular surfaces and (iii) the cylindrical surfaces. Assuming that the radiation heat transfer is insignificant, only the convective conditions are considered in these boundaries. The boundary condition along the symmetry centerline (line AB in Figure 2.8) is given as

\[
\frac{\partial T}{\partial r} = 0.
\]

(2.36a)

The boundary conditions for the perpendicular (e.g. surface AC in Figure 2.8) and cylindrical surfaces (e.g. surface CD in Figure 2.8) are given as

\[
k_s \frac{\partial T}{\partial y} = -h_p(T - T_{amb}),
\]

(2.36b)
respectively where $k_s$ is the thermal conductivity of the solid, $h_{ci}$ and $h_{pi}$ are the convective heat transfer coefficients on the cylindrical and perpendicular surfaces, respectively, subscript $i$ indicates the segment considered (a shaft segment of the disk), and $T_{amb}$ is the ambient temperature. The disk surface where contact takes place receives the heat generated at the contact while convective cooling occurs for the remainder of the circumference on both sides of the contact track. For this special condition, Eq. (2.36c) is modified for disks 1 and 2 respectively as

\begin{equation}
 k_s \frac{\partial T}{\partial r} = (1 - f_{ave}) \frac{F_{ave} \mu_s}{4 \pi R_1 a_h} - h_{ci}(T - T_{amb}), \tag{2.37a}
\end{equation}

\begin{equation}
 k_s \frac{\partial T}{\partial r} = f_{ave} \frac{F_{ave} \mu_s}{4 \pi R_2 a_h} - h_{ci}(T - T_{amb}), \tag{2.37b}
\end{equation}

where $f_{ave}$ is the time-averaged heat partition coefficient that is the fraction of heat dissipating to the disk 2, $F_{ave}$ is the time-averaged friction force in the contact zone, and $a_h$ is the semi-axis length of the Hertzian contact in the $y$ direction. In Eq. (2.37), both $f_{ave}$ and $F_{ave}$ are provided by the mixed TEHL model proposed in the Section 2.2.
The steady-state heat conduction equation is utilized to provide the initial condition for Eq. (2.35) in the section that disk is fitted as

\[-\frac{1}{r} \frac{d}{dr} \left( k_s r \frac{dT}{dr} \right) = 0 \]  \hspace{1cm} (2.38)

with the boundary conditions \( \partial T/\partial r = 0 \) at \( r = 0 \) and \( T = T_b \) at \( r = R_i \) \( (i = 1, 2) \), where \( T_b \) is the initial bulk temperature at the surface. The temperature distribution of any shaft segment \( j \) \( (j \in [1, 4]) \) is then given as [2.51]

\[ T_j(y) = \left[ \frac{\cosh m_j(L_j - y) + \frac{h_{pj}}{m_j k_s} \sinh m_j(L_j - y)}{\cosh m_j L_j + \frac{h_{pj}}{m_j k_s} \sinh m_j L_j} \right] (T_{rj} - T_{amb}) + T_{amb} \]  \hspace{1cm} (2.39)

where \( m_j = (2h_{cj}/k_s r_j)^{1/2} \), and \( T_{rj} \) is the temperature at the base of shaft segment \( j \) where it interfaces with the disk.

One challenge in using Eqs. (2.36b), (2.36c), and (2.37) is determining the values of the heat transfer coefficients on these convective boundaries. Previous research relied on accurate measurements to calculate Nusselt numbers, which can be converted to convective heat transfer coefficients. The convective heat transfer conditions depend greatly on the method of lubrication (dip lubrication, mist lubrication, or jet lubrication). The lubrication method simulated in the present study is
jet lubrication, in which a minimum amount of lubricant is applied directly to the contact interface. The characteristics of heat transfer on the cylindrical surface of disk are assumed to be different from those on the perpendicular surfaces.

A number of experimental studies were conducted to determine the convective heat transfer on the cylindrical surfaces. In one such study, Tachibana et al. [2.52] measured the convective heat transfer in the annular gap between the rotating inner cylinder and the stationary outer cylinder. Becker [2.53] developed the empirical formula for the Nusselt number from a rotating cylinder immersed in water with the limitation of the rotating Reynolds number ranging from 1,000 to 46,000. Smith and Greif [2.54] presented that measured results that were in agreement with the Becker’s formula for high Prandtl numbers and Reynolds numbers up to 200,000. Lebeck [2.55] validated the Becker’s formula for a class of oils and stated “In the absence of better information, the Becker formula can be used to estimate convection coefficients”. Shimada et al. [2.56] measured the local and average coefficient of convective heat transfer from a rotating cylinder using an interferometer while the same was done by Ozerdem [2.57] using a radiation pyrometer. Even in the view of more recent studies [2.58-2.60], the Becker’s formula remains as the most widely accepted and commonly used formula. Thus, the Nusselt number that fitted the Becker’s data and was validated by Lebeck [2.55] for a cylinder rotating in oil is employed here. It reads

\[
Nu_D = 0.133(Pr)^{1/3} (Re_D)^{2/3}, \quad 10^3 < Re_D < 2 \times 10^5
\]  

(2.40)
where $Nu_D$ is the Nusselt number, $Re_D$ is the rotating Reynolds number with the cylinder diameter as the characteristic length, and $Pr$ is the Prandtl number. These three dimensionless numbers are defined as [2.61]

\[ Nu_D = \frac{h_c d}{k}, \quad (2.41a) \]

\[ Pr = \frac{c \eta}{k}, \quad (2.41b) \]

\[ Re_D = \frac{\rho d^2 \omega}{2 \eta} \quad (2.41c) \]

where $h_c$ is the convective heat transfer coefficient on the cylindrical surface, $d$ is the cylinder diameter, $k$ and $c$ are respectively the thermal conductivity and the specific heat of the surrounding fluid, $\omega$ the angular velocity of the cylinder, $c$ is the specific heat of the surrounding fluid, and $\rho$ and $\eta$ are respectively the density and the dynamical viscosity of the surrounding fluid. From Eqs. (2.40) and (2.41), the convective heat transfer coefficient on the cylindrical surface is found as

\[ h_c = 0.133 \frac{k}{d} \left( \frac{\eta c}{k} \right)^{1/3} \left( \frac{\omega \rho d^2}{2 \eta} \right)^{2/3} \quad (2.42) \]

By using the radius of the cylinder as the characteristic length (instead of its diameter) and employing subscript $j$ to denote a particular $j$-th shaft segment, Eq. (2.42) is
modified to

\[
    h_{ej} = 0.0665 \frac{k}{r_j} \left( \frac{\eta c}{k} \right)^{1/3} \left( \frac{2 \omega r_j^2}{\eta} \right)^{2/3}.
\]  

(2.43)

The convective heat transfer coefficient on a perpendicular surface of the cylinder is estimated by using the results on rotating circular disks. A large number of studies on rotating circular disks are available in the literature. Only the ones relevant to this study will be mentioned here. Cobb and Saunders [2.62] investigated the average heat transfer of a rotating disk in the still air, taking place under laminar to mixed laminar-turbulent flow conditions. Owen et al. [2.63] presented a combined theoretical and experimental investigation of the heat transfer from a rotating disk with impinging air flow. Cardone et al. [2.64] measured the local heat transfer coefficient on a disk rotating in still air by using the infrared (IR) thermography and heated-thin-foil thermal sensors while Astarita and Cardone [2.65] relied on same measurement technique for determining the heat transfer of a rotating disk with impinging air jet. In this study, the formula proposed by Cardone et al. [2.64] is adopted to calculate the heat transfer on the rotating circular surface as

\[
    h_p = 0.2 \frac{k}{r} (Pr)^{0.5} \left( \frac{\eta c}{k} \right)^{0.5} \left( \frac{\omega r_j^2}{\eta} \right)^{0.5}.
\]  

(2.44)

Substituting \(Pr\) expression from Eq. (2.41b) the convective heat transfer coefficient on
a perpendicular surface of j-th shaft segment can be written as

\[
h_{pq} = 0.2k\left(\frac{c\eta}{k}\right)^{0.5}\left(\frac{\omega\rho}{\eta}\right)^{0.5}.
\] (2.45)

The method of determining \( h_c \) and \( h_p \) can also be applied to the disk segments of the system shown in Figure 2.8, with the consideration that the surrounding fluid may not be characterized by a single-phase flow as the surrounding medium is likely to be an air air/oil mixture (mist). A simple approach is adapted here to estimate the heat transfer in an air/oil mist environment. The convective heat transfer coefficient on the cylindrical and perpendicular surfaces of the disk are defined approximately as

\[
h_c = (1 - \phi)h_{ca} + \phi h_{cf}, \quad (2.46a)
\]

\[
h_p = (1 - \phi)h_{pa} + \phi h_{pf} \quad (2.46b)
\]

where subscripts \( c \) and \( p \) denote cylindrical and perpendicular surfaces of disk, \( \phi \) is the weighting factor that accounts for the volumetric fraction of the mixture occupied by the oil on the surface of disk, and \( h_a \) and \( h_f \) are the convective heat transfer coefficients on the surface surrounded by a single phase of air and oil, respectively.
2.4.2 Numerical Solution

The finite difference method is used to solve the governing equation, Eq. (2.35). A rectangular finite-difference grid is placed over the computation domain that covers only one half of the space domain of the simplified shaft geometry. The diffusion terms (first two terms on the right hand side) of Eq. (2.35) are discretized by a second-order central difference approximation while the convection term (the last term) is represented by a backward difference approximation. The time derivative term is represented by a forward difference approximation. The diffusion and convection terms at the boundaries are also treated separately. The time-averaged heat partition coefficient $f_{ave}$ and the time-averaged friction force $F_{ave}$ in Eqs. (2.37a) and (2.37b) are obtained from the mixed TEHL model by simply averaging values from a user-defined number of time steps.

The scheme proposed by Peaceman and Rachford [2.66] is used here to solve resulting finite difference equations in a computationally efficient way. This scheme is unconditionally stable and only requires the Tri-Diagonal Matrix Algorithm (TDMA) without any need for iterative procedures to solve the system of algebraic equation. The initial condition Eq. (2.38), however, is not solved by the finite difference method, but rather by a finite volume method. In this method, the steady-state heat conduction equation is integrated over an elemental control volume. When carrying out the integration, no interpolation between the adjacent grid points is needed since the only
variable at the finite volume boundaries is the thermal conductivity of solid, which is a constant. The system of algebraic equations formed by Eq. (2.38) produces a three-banded matrix that is diagonally dominant. The solution of these algebraic equations is also obtained by the TDMA. Once the solution of Eq. (2.38) is obtained, Eq. (2.39) is applied to obtain the temperature at each shaft segment.

2.5 Results of Transient Heat Transfer Model for a Two-Disk Contact Problem

The bulk temperatures of each disk in contact depend on the heat flux and the convective heat transfer. In order to illustrate the capabilities of the heat transfer model, the same example two-disk arrangement from Section 2.3 is used here as well. The only difference here is that the width of the second disk is much larger (44.2 mm) such that the contact track is only a small portion of it. Geometrical dimensions for the supporting shafts are given in Table 2.3. The length of the third shaft segment shown in Figure 2.8 was very narrow such that the simulation can be performed without it. Shaft segment 4, meanwhile, has the largest diameter and the longest length to warrant relatively low temperatures.

In the present simulation, disks and shafts are assumed to have the same thermophysical properties. Both disks are jet-lubricated with ATF at 90°C oil inlet temperature. Parameters of this lubricant at this temperature were listed earlier in Table 2.2. The disks are assumed to operate in a 25°C environment surrounded by air,
Table 2.3 Dimensions of shafts of the example two disk problem.

<table>
<thead>
<tr>
<th>Shaft</th>
<th>Length</th>
<th>Radius</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment 1 [mm]</td>
<td>49.4</td>
<td>6.4</td>
</tr>
<tr>
<td>Segment 2 [mm]</td>
<td>25.6</td>
<td>4.8</td>
</tr>
<tr>
<td>Segment 3 [mm]</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Segment 4 [mm]</td>
<td>246.3</td>
<td>14.4</td>
</tr>
</tbody>
</table>
whose properties are given in Table 2.4. The initial bulk temperatures of the cylindrical surfaces of disks are taken to be 51.5°C. The surface velocities of the disks are kept at $u_1 = 7.5$ m/s and $u_2 = 22.5$ m/s corresponding to a mean entraining velocity of $u_e = 15$ m/s and a slide-to-roll ratio of $SR = -1$, respectively. They are kept constant during this simulation. The normal load levels applied to the disks were defined such that discrete maximum Hertzian pressure levels ranging from 0.8 to 1.3 GPa (with the increments of 0.1 GPa) can be achieved. Operation time at each load value was taken to be 5 minutes starting from the lowest load to the highest. This loading sequence indeed represents a typical scuffing experiment. The initial surface roughness profiles shown in Figure 2.2 are considered here with the RMS roughness parameters of $R_{q1} = 0.45$ μm and $R_{q2} = 0.35$ μm.

A relatively small computational domain is employed in the mixed TEHL simulation at each load stage to maximize the usage of measured surface roughness. Each mixed TEHL simulation is performed for 140 time steps including the first 40 time steps of transition period that is from the lubricated smooth EHL analysis to the lubricated rough surface isothermal EHL analysis, and then to the lubricated rough TEHL analysis [2.8]. Table 2.5 lists Hertzian semi-axis lengths, computational domain and grid sizes at each 5-minute long load stage. As seen from Table 2.5, the grid size in the $x$ direction is kept relatively small to capture the changes of surface roughness in the $x$ direction. Likewise, a reasonably fine grid size is used for the transient two-
Table 2.4 Properties of air.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic viscosity at 25 °C [Pa-s]</td>
<td>$1.846 \times 10^{-5}$</td>
</tr>
<tr>
<td>Density at 25 °C [kg/m$^3$]</td>
<td>1.1855</td>
</tr>
<tr>
<td>Specific heat [J/kg-K]</td>
<td>1005</td>
</tr>
<tr>
<td>Thermal conductivity [W/m-K]</td>
<td>0.02605</td>
</tr>
</tbody>
</table>
Table 2.5 Parameters of the contact and the simulation of each load stage considered.

<table>
<thead>
<tr>
<th>$p_h$ [GPa]</th>
<th>$t$ [min]</th>
<th>$b_h$ [mm]</th>
<th>$a_h$ [mm]</th>
<th>$x$ [mm]</th>
<th>$y$ [mm]</th>
<th>$\Delta x$ [$\mu$m]</th>
<th>$\Delta y$ [$\mu$m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>0</td>
<td>0.14</td>
<td>0.50</td>
<td>[-0.26, 0.15]</td>
<td>[-0.76, 0.76]</td>
<td>2.9</td>
<td>24.2</td>
</tr>
<tr>
<td>0.9</td>
<td>5</td>
<td>0.15</td>
<td>0.58</td>
<td>[-0.29, 0.17]</td>
<td>[-0.85, 0.85]</td>
<td>3.2</td>
<td>27.1</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>0.17</td>
<td>0.65</td>
<td>[-0.32, 0.19]</td>
<td>[-0.96, 0.96]</td>
<td>3.6</td>
<td>30.4</td>
</tr>
<tr>
<td>1.1</td>
<td>15</td>
<td>0.19</td>
<td>0.70</td>
<td>[-0.35, 0.21]</td>
<td>[-1.03, 1.03]</td>
<td>3.9</td>
<td>32.8</td>
</tr>
<tr>
<td>1.2</td>
<td>20</td>
<td>0.21</td>
<td>0.77</td>
<td>[-0.38, 0.23]</td>
<td>[-1.14, 1.14]</td>
<td>4.3</td>
<td>36.1</td>
</tr>
<tr>
<td>1.3</td>
<td>25</td>
<td>0.22</td>
<td>0.83</td>
<td>[-0.42, 0.25]</td>
<td>[-1.23, 1.23]</td>
<td>4.7</td>
<td>39.1</td>
</tr>
</tbody>
</table>
disk heat transfer model. A uniform grid size of $\Delta x = \Delta y = 0.375$ mm and a time step of 1 s are used for heat transfer simulation of both disks. These grid density and time increment values were chosen such that discretization and simulation errors are minimal. Finally, the weighting factors defined in Eq. (2.46) was assumed to be $\phi = 0.07$.

As transient details of this simulation at each time step are not feasible to present, a set of simulations at a given time step of $n_t = 60$ is presented here after each time the load is incremented up. Figure 2.9 shows the asperity contact points at each load level at this time instance. It is noted here that there is no asperity contacts (i.e. full film conditions are present) for the first three load stages up to below 1.1 GPa, while asperity contacts increase with increased contact pressures for $p_h \geq 1.1$ GPa. The size of the computational domain is also shown in these figures, indicating that it was made larger at higher $p_h$ values to accommodate increasing sizes of contact zone. The increase in load is not the sole reason for increased asperity activity at higher $p_h$ values as temperature effects are also critical as it will be shown below.

Figure 2.10 shows the instantaneous pressure distributions for the same conditions as those of Figure 2.9. As a direct result of the increased asperity activity, sizable pressure spikes are observed at higher load conditions as shown in Figures 2.10(e) and (f). Likewise, the film thickness distributions shown in Figure 2.11 for the
Figure 2.9 Asperity contact regions at $n_f = 60$ for the $p_h$ values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
Figure 2.10  Pressure distributions at $n_t = 60$ for the $p_h$ values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
Figure 2.11 Film thickness distributions at \( n_t = 60 \) for the \( p_h \) values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
same simulation reveals reduction in film thickness due to the effect of increased temperature, and to a certain extent, increased load.

The corresponding temperature distributions along the surfaces 1 and 2 and the along the mid-layer of the fluid film are shown in Figures 2.12, 2.13 and 2.14, respectively. It is noted in these figures that the bulk temperatures of the surfaces climb with normal load impacting the contact conditions adversely. In addition to increase in bulk temperatures, the instantaneous temperature rises across the contact zone increase with the load level as well. As a result, instantaneous maximum surface temperatures and mid-layer oil temperature at $n_t = 60$ and at the last load stage reach 169, 169, and 226°C in Figures 2.12(f), 2.13(f) and 2.14(f), respectively. These values indicate that the maximum oil temperature on its mid-layer is nearly 60°C higher than those at the contacting surfaces.

Predicted bulk temperature distributions of the disks and their supporting shafts at the end of each load stage are shown in Figure 2.15. These temperature contours indicate that the contact track is the hottest spot on the disk surfaces, with temperatures reduce gradually as away from the contact. They also show the signs of heat built-up as each increased load level represents higher temperatures. For instance, in Figure 2.15(f), the maximum bulk temperatures for disks 1 and 2 at the end time of last load stage $p_h = 1.3$ GPa are nearly 185 and 193°C, respectively. The predicted variation of maximum disk bulk temperatures with time is shown in Figure 2.16. Here, each
Figure 2.12 Temperature distributions on surface 1 at $n_f = 60$ for the $p_h$ values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
Figure 2.13  Temperature distributions on surface 2 at $n_t = 60$ for the $p_h$ values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
Figure 2.14  Temperature distributions on the mid-layer of oil film at $n_t = 60$ for the $p_h$ values of (a) 0.8 GPa, (b) 0.9 GPa, (c) 1 GPa, (d) 1.1 GPa, (e) 1.2 GPa, and (f) 1.3 GPa.
Figure 2.15  Predicted bulk temperature distributions at the end of each load stage: (a) $t = 5$ min, (b) $t = 10$ min, (c) $t = 15$ min, (d) $t = 20$ min, (e) $t = 25$ min, and (f) $t = 30$ min.
Figure 2.15 Continued.

Disk 1

Disk 2

$r$ [mm]

$\theta$ [mm]

$y$ [mm]
Figure 2.15  Continued.

(e) Disk 1

(f) Disk 2

$r$ [mm]

$y$ [mm]
Figure 2.16 Predicted bulk temperatures as a function of time.
stepwise increase in load causes a nearly stepwise response with steady state temperatures at each load level increasing more with load. For instance, at the end of the second load stage the bulk temperature of disk 2 climbs from 58 to 67°C (only 9 degrees increase) while the same disk experiences 56 degrees climb (from 137 to 193°C) during the last load stage of the simulation. It is also noted that the larger of the two disks (disk 2) gets consistently hotter and the difference reaches 8°C at the end of the last load stage.

2.6 Summary

In this chapter, a two-dimensional, non-Newtonian, transient, mixed thermal elastohydrodynamic lubrication (TEHL) model is proposed for heavily loaded elliptical contacts formed by non-conformal rough surfaces. This model expands the isothermal EHL model developed by Li and Kahraman [2.8] to include thermal effects in the lubricated contact analysis. The mixed TEHL model couples together the thermal analysis and pressure calculation to reach a fully convergent solution. The thermal analysis employs the Moving Grid Method (MGM) to calculate the temperature rises on rubbing surfaces, and utilizes the simplified heat transfer equation to determine the temperature on the mid-layer of oil film. The pressure computation employs FFT based method to solve the elastic deformation distribution, the unified numerical approach to handle severe asperity interactions, and an asymmetric integrated control
volume discretization method to reach the equivalent accuracy and robustness at a lower grid density.

A two-dimensional transient heat transfer model is proposed next to predict the time-varying bulk temperatures due to heat generated in the contact. The heat transfer model provides bulk temperatures to the mixed TEHL model, which in turn predicts the instantaneous temperature rises on the rubbing surfaces. This heat transfer model uses the finite volume method to solve the initial temperature distribution in the disks, a closed-form solution to determine the initial temperature distribution in the shafts, and the Peaceman and Rachford scheme to solve the transient diffusion-convection equation. It also employs Becker empirical formulas to estimate the convective heat transfer coefficient on the cylindrical surfaces, a rotating disk model to calculate the convective heat transfer coefficient on the perpendicular surfaces, and a weighting factor to approximate the benefit on the heat transfer due to the air/oil mist environment.

In order to demonstrate the iterative solution obtained from integrating the mixed TEHL model and the transient two-disk heat transfer model, a disk scuffing experiment was simulated at the end to show the dependence of the models to each other. The example two-disk system and the scuffing experiment defined as part of this example analysis will be implemented in the next chapter to collect experimental data for validation of the modeling methodology proposed in this chapter.
References for Chapter 2


CHAPTER 3

TWO-DISK EXPERIMENTS AND MODEL VALIDATION

3.1 Introduction

As reviewed in Chapter 1, experimental scuffing studies were done in numerous test methods. Actual gearboxes operating under typical conditions were used by some (e.g. [3.1]), which is perhaps the most expensive and time-consuming way of evaluating gear scuffing. Other system-level influences are present in this case due to deflections of the supporting components as well as complexities associated with lubrication of the contact of interest. Other studies used standardized gears in power-circulation machines [3.2, 3.3]. In these studies, while they do not exactly represent the actual production gears, test gears provide a means of evaluating the impact of operating conditions, lubricant parameters and surface conditions of scuffing failures. These studies often used aggressive test conditions to ensure and accelerate the occurrence of scuffing.

Another group of studies available in the literature used different forms of tribometers, where more idealized contacts are tested under tightly controlled
conditions. The commonly used tribometers for simulating scuffing failures include

two-disk machines [3.4-3.27], ball-on-disk machines [3.28-3.30], pin-on-disk machines
[3.31-3.34], block-on-ring machines [35, 36], four-ball machines [37-47] and pin-on-
vee block machines [3.48]. Such devices have been proven to effectively test contact
performance, wear, traction, scuffing or contact fatigue under tightly-controlled
laboratory conditions. They are relatively inexpensive set-ups with basic speed and
lubricant temperature control schemes, and their specimens have simple, easy-to-
machine shapes. In addition, contact surfaces in these tribometers can be accessed
easily (in comparison to, say, contacts of gears) such that the surface characteristics can
be monitored throughout the test, allowing better means of defining the mechanisms
leading to failures.

Figure 3.1 shows a view of two spur gears in mesh. In this particular contact
instant with a contact point C, the shape of the tooth surfaces are nearly circular such
that two imaginary disks of radii \( R_1 = CA_1 \) and \( R_2 = CA_2 \) define all relevant contact
parameters. Tooth surfaces at the contact point C move at \( u_1 = R_1 \omega_1 \) and \( u_2 = R_2 \omega_2 \)
where usually \( u_1 \neq u_2 \) unless the contact point C is at the pitch point \( P \). Thus, a
relative sliding is generated at tooth contacts and is commonly quantified by the slide-
to-roll ratio \( SR = 2(u_1 - u_2)/(u_1 + u_2) \). This indicates that the contact of a gear tooth
at certain location with rolling and sliding velocities \( u_e = \frac{1}{2}(u_1 + u_2) \) and \( u_s = u_1 - u_2 \),
and normal load \( W \) can be simulated in a bench test by using a two-disk setup,
Figure 3.1 Representation of an instantaneous gear tooth contact by two imaginary cylindrical disks.
provided that the disks have radii $R_1$ and $R_2$, rotational speeds $\omega_1$ and $\omega_2$, and loaded normally at $W$. For this particular reason, a two-disk set-up will be used in this chapter (i) to provide an experimental investigation of the scuffing and traction behaviors of contacts in combined rolling and sliding and (ii) to generate data that can be used for validation of the disk scuffing model proposed in Chapter 2.

The rest of this chapter is organized in the following way. The test machine, specimens, procedures, instrumentation and other relevant hardware details are described in Section 3.2. Since the friction at the contact is the main mechanism for generating heat to cause scuffing, a set of traction experiments will be performed and documented in Section 3.3, followed by two-disk scuffing test results described in Section 3.4. At the end, model predictions are compared to the measurements in Section 3.5.

3.2 Test Methodology

3.2.1 General Description of Two-Disk Machine

The two-disk machine used in this study is shown in Figure 3.2. Its layout is provided in Figure 3.3 to specify its components. The test set-up consisted of two independent 3-phase AC motors that drive input and output shafts at desired speeds. Input shaft holds the smaller of the two disks, which will be called the roller, while the output shaft holds the larger of the two disks, called the disk. Each drive motor was rated for 10 HP with a maximum speed of 5,000 rpm. Two identical motor controllers
Figure 3.2 General view of the two-disk machine used in this study.
Figure 3.3 Schematic layout of the two-disk machine showing its main components.
with built-in PID were used to precisely control the speed of each motor. Each motor was connected to its shaft via a belt drive to increase the shaft speeds beyond the maximum motor speed as it will be described later. Independent speed controls allowed both shafts to spin at different speeds to achieve desired $u_e$ and $SR$ values at the two-disk contact. Input and output shafts were supported by relatively rigid bearings within the bearing housings. A loading arm mechanism was designed to push the roller towards the disk in the normal direction. In order to allow a small amount of play of roller in this direction, a flexible coupling was implemented between the input shaft and the roller.

The contact load $W$ between the disk and roller was generated by a pivoted loading arm that has a mechanical advantage of 4:1, as shown in Figure 3.4. The input load on the loading arm was applied by a pneumatic cylinder, which was adjusted by a pressure regulator up to the maximum pressure level of 550 kPa (80 psi). Figure 3.5 shows the roller-disk pair in their loaded configuration. The maximum value of the resultant contact load was 4,450 N (1,000 lbf), which was monitored by a miniature load cell of 1,112 N (250 lb) capacity attached between the loading arm and the pneumatic cylinder head. The load cell was powered by DC voltage supplied by the signal conditioning amplifier while the output of the load cell was fed into a 12 bit A/D converter through the same signal conditioning amplifier (Intertechology 2300). In Figure 3.5, the disk and roller were shrink-fitted on their shafts. One side of the disk rested against the shoulder of the shaft and fastened axially on the other side by using a
Figure 3.4 Test chamber of the two-disk test machine.
Figure 3.5 Top view of the roller-disk pair in their loaded configuration.
retaining lock nut. The roller also had a similar mounting arrangement with the exception that a pair of high-precision ball bearings were placed between the roller and the lock nut. This arrangement ensured the test-to-test repeatability of axial position of the disk and roller.

The test machine was equipped with a low-volume lubrication system. The oil was heated in a reservoir and pumped directly into the inlet wedge of the contact. This lubrication system, while simple, allowed the pressure and flow rate of the supply oil to be controlled, as well as its temperature. The lubrication system consisted of a series of stainless steel tubing, an anodized aluminum reservoir, and a 0.5 HP DC motor. The reservoir was capable of holding and heating a maximum of 3 liters of lubricant to temperatures as high as 150°C. The speed of the pump motor was adjustable to achieve any desired flow rate within the range from 0.4 to 3.3 lpm. An in-line, fine hydraulic filter was used to remove any particulates, preventing any wear debris from returning to the inlet wedge of the contact, as well as minimizing the contamination of the oil.

A J-type (positive lead: Fe, negative lead: Cu-Ni) thermocouple was used to monitor the lubricant temperature in the reservoir. The thermocouple extension was connected to the PID temperature controller and the measured lubricant temperature can be directly read from the controller display. A desired lubricant temperature was set up in the controller display as long as the heater switch is toggled on so that the
heater will automatically turn on when the measured lubricant temperature in the reservoir is lower than the desired temperature.

### 3.2.2 Instrumentation, Calibration and the Data Acquisition System

The data acquisition system used to collect the data of applied normal force, traction torque, and temperature consisted of sensors, signal conditioning amplifiers, thermocouple-to-analog converter, a shielded multi-channel connector block (BNC-2110, National Instruments Inc.), and an A/D board (PCI-M10-16E, National Instruments Inc.).

Three different types of sensors were used in the two-disk test machine: load cell, torque sensor and thermocouples. Signals from each type of sensor were fed into commercial data analysis software (DASYLab, DASYTec) through the connector block so that all incoming signals can be analyzed simultaneously.

Two sheath-enclosed K-type (positive lead: Ni-Cr, negative lead: Ni-Al) thermocouples were mounted on the test chamber cover. The thermocouples were spring-loaded to slightly touch the top edges of disk and roller, measuring the bulk temperatures of disk and roller. Two additional unsheathed thermocouples were used, one placed in the reservoir to record the reservoir temperature and another attached to the bearing housing to measure the bearing housing temperature.
A compression-only subminiature load cell was attached between the loading arm and the top of the pneumatic cylinder. The load cell had a capacity of 1,112 N (250 lb) and rated output of $2 \pm 0.5 \text{ mV/V}$ with an excitation of 5 VDC, such that the load cell outputs $10 \pm 5 \text{ mV}$ at full load. An amplifier gain of 1,000 was used to improve the measurement resolution to about 0.1 lb. The load cell was calibrated using the following procedure. After zeroing the load cell from the signal conditioning amplifier, a series of load levels were applied at an increment of 139.75 N (corresponding to 10 psi) first in ascending and then descending order covering an entire load range. A curve-fit of these readings provided the normal load value as a function of voltage output of the sensor.

As shown in Figure 3.3, an in-line digital rotary torque sensor (Sensor Developments, Model 01424) was placed on the output shaft between the disk and the drive motor to measure the torque applied to the disk to overcome the torque loss caused by the friction at the roller-disk contact. The torque sensor was rated for a maximum torque of 56 Nm with an uncertainty of $\pm 0.06\%$ of the full scale (0.034 Nm). This provided sufficient resolution and accuracy for the intended measurements at a sampling rate of 20,000 Hz. The maximum rotational speed of the torque sensor was 10,000 rpm.

A general-purpose signal conditioning amplifier (2310, Measurements Group, Inc.) was used for conditioning and amplifying low-level signal from the load cell for
display and recording. In order to make accurate temperature measurements, the low (mV) level output signals from the thermocouples must be compensated by using cold junction compensation (CJC). Four thermocouple-to-analog converters (SMCJ-K, Omega Inc.) were used to convert the thermocouple input signals to a cold junction compensated and linear analog output for use with the data logger. Each thermocouple was calibrated using an accurate temperature calibrator, thermocouple-to-analog converter and voltage multi-meter. It is observed that the connection of thermocouple and thermocouple-to-analog converter had an uncertainty of ±2°C.

3.2.3 Test Specimens and the Lubricant Properties

As shown in Figure 3.5 in their loaded position on the machine, two-disk set-up consists of a disk of 57.15 mm (2.25 in) diameter and a roller of 31.75 mm (1.25 in) diameter. Figure 3.6 shows examples of roller and disk specimens. The test disks and rollers were all made of AISI 4620 low carbon gear steel, a typical gear steel used in automotive transmission applications. Each specimen was case carburized to achieve a surface hardness of 60±2 HRC. In addition, disks were finished to have a (axial) crown of radius of 76.2 mm. No crowning was applied to roller specimens (i.e. they were cylindrical in shape). With these radii and axial crown values, an elliptical contact was achieved with $a/b = 3.75$ where $b$ and $a$ are the semi-axis dimensions of the contact ellipse in the direction of rolling and normal to the rolling, respectively.
Figure 3.6 Roller and disk test specimen.
As the exact shape of the roller and disk specimen was critical to achieving the intended pressure levels at the contact, the axial profiles of each roller and disk specimen were qualified before any test through measurements on a gear coordinate measurement machine (CMM). Figure 3.7 illustrates a set of such measurements, indicating that roller surfaces are rather flat while the disk surface is crowned at the specified radius. The other critical parameter in terms of the EHL behavior of the contact was the surface finish of the specimens. Two types of surface finishes applied to the test specimens. The first type of surface finish was designed to simulate surfaces of gears cut by the gear shaving process, in terms of both the direction of roughness pattern relative to the rolling direction and amplitude of the roughness profiles in the rolling direction. Figure 3.8 illustrates close-up views of roller and disk specimens from this batch to demonstrate that machining grooves are nearly perpendicular to the rolling (and sliding) direction. The targeted root-mean-square (RMS) amplitudes of the roughness were kept within the range of 0.35 – 0.45 μm. The other type of surface texture was selected to be chemically polished (super-finished) profiles with RMS surface roughness amplitudes within 0.07 – 0.08 μm. Two specimens with super-finished texture are shown in Figure 3.9. Typical measured roughness profiles in the rolling direction for rough and super-finished surfaces are shown in Figure 3.10 as the inspected on a surface profilometer with cut-off value of 0.25 mm.

The lubricant used in all tests reported in this chapter is a typical automatic transmission fluid (ATF). This lubricant has a kinematic viscosity of 6.0 cSt at 100°C.
Figure 3.7  Example measured axial profiles of (a) a roller and (b) a disk specimen.
Figure 3.8  Magnified surface images of brand new (a) roller and (b) disk specimens having simulated shaving marks.
Figure 3.9  Super-finished (a) roller and (b) disk specimens.
Figure 3.10  Measured surface roughness profiles of example specimens: (a) simulated shaved and (b) super-finished specimens (measured along the circumferential direction).
and 28.7 cSt at 40°C. The main properties of ATF are summarized in Table 3.1.

### 3.3 Traction Measurements

Two types of tests were conducted in this study: (i) traction tests to measure the friction coefficient as a function of \( SR \) at given temperature, speed and load level, and (ii) scuffing tests that provide a scuffing limit at given temperature, speed, sliding and load condition. Detailed description of the traction test procedures, the test matrix as well as the test results are presented in this section.

#### 3.3.1 Removal of Bearing Losses

Traction torque due to sliding at the contact interface was measured by the in-line torque sensor on the drive shaft of the disk. The measured torque includes contributions of the torque losses caused by the support bearings (between the sensor and the disk) that must be removed to determine the traction torque caused by the contact. According to Harris [3.49], load-dependent \( \Gamma_l \) (friction induced) and load-independent \( \Gamma_v \) (viscous induced) components of rolling element bearing torque losses constitute the total bearing loss \( \Gamma_b \) as

\[
\Gamma_b = \Gamma_l + \Gamma_v. \tag{3.1}
\]
Table 3.1 Properties of automatic transmission fluid (ATF).

<table>
<thead>
<tr>
<th>Property</th>
<th>ATF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific gravity at 15°C</td>
<td>844 kg/m³</td>
</tr>
<tr>
<td>Flash point</td>
<td>206 °C</td>
</tr>
<tr>
<td>Pour point</td>
<td>-54 °C</td>
</tr>
<tr>
<td>Kinematic viscosity at 40°C</td>
<td>29.8 cSt</td>
</tr>
<tr>
<td>Kinematic viscosity at 100°C</td>
<td>6.0 cSt</td>
</tr>
</tbody>
</table>
The torque losses of angular-contact and radial ball bearings due to a radial load are given, respectively, as

\[
\Gamma_l = 10^{-6} \left( \frac{0.5F_r}{C_s} \right)^{0.33} F_r d_m, \quad (3.2a)
\]

\[
\Gamma_l = 4(10)^{-6} \left( \frac{0.6F_r}{C_s} \right)^{0.55} F_r d_m \quad (3.2b)
\]

where \( \Gamma_l \) is the torque loss due to radial load in Nm, \( F_r \) is the radial load on each bearing in N, \( C_s \) is the static load rating in N, and \( d_m \) is the pitch diameter of bearing in mm. The torque loss due to viscous traction of radial ball bearings is given as [3.49]

\[
\Gamma_v = \begin{cases} 
2(10)^{-10} (\nu_0 n)^{2/3} d_m^3, & \nu_0 n \geq 2000, \\
3.2(10)^{-8} d_m^3, & \nu_0 n < 2000 
\end{cases} \quad (3.3)
\]

where \( \Gamma_v \) is the torque loss due to viscous traction (in Nm), \( \nu_0 \) is kinematic viscosity of lubricant (in cSt), and \( n \) is the rotational speed of shaft (in rpm). In the present setup, one paired angular-contact ball bearing set served as the fixed node and one paired light series radial ball bearing set as the redundant node in the bearing housing to support the applied normal load and shaft weight. Since the bearings are not dip lubricated, only the half of \( \Gamma_v \) value predicted by Eq. (3.3) is used this study. For the
lubricant in hand, a temperature-viscosity relationship, given as $v_0 = 8459.170^{-1.555}$ ($T_0$ is the oil temperature in °C), was used to determine the $v_0$ value.

### 3.3.2 Test Matrix, Test Procedure and Test Condition

The test matrix given in Table 3.2 specifies the conditions considered in the traction tests. It includes surface roughness effects (rough versus super-finished), speed and load effects (nine combinations of rolling velocity $u_e$ and maximum Hertzian contact pressure $p_h$) as well as oil temperature effect (40 and 90°C for the rough surfaces) through implementation of a total of 27 tests. The lubricant is jetted into the mesh inlet at the flow rate of 1 lpm for all traction tests including the run-in process for rough specimens.

For rough specimens, each new pair of specimens was first run-in for 10 minutes at $p_h = 0.5$ GPa, $u_e = 6$ m/s, $SR = 0.1$ and $T_0 = 40^\circ$C. The purpose of the run-in process was to avoid premature transitions to scuffing and let the rubbing surfaces conform to each other. The run-in process was not needed for the super-finished specimens since no tangible changes were observed between the roughness profiles before and after the run-in tests. After the run-in test was completed, the disk and the roller were disengaged and the torque sensor was zeroed again. The test specimens were removed together with their assembled shafts and fixtures from the machine to
Table 3.2 Traction test matrix.

<table>
<thead>
<tr>
<th>Surface type</th>
<th>Temperature [°C]</th>
<th>Rolling Velocity [m/s]</th>
<th>Max. Hertzian Pressure [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rough</td>
<td>40</td>
<td>2, 6, 10</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>Rough</td>
<td>90</td>
<td>2, 6, 10</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
<tr>
<td>Super-finished</td>
<td>90</td>
<td>2, 6, 10</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>2.0</td>
</tr>
</tbody>
</table>
measure the surface roughness profiles after the run-in process. The fixtures were then put back on the machine and an actual traction test was initiated.

The positive SR portion of each test was done first. Eighteen discrete SR values (from 0 to 0.2 at an increment of 0.02, from 0.2 to 0.4 at an increment of 0.05 and from 0.4 to 0.7 at an increment of 0.1) were considered and shown graphically in Figure 3.11. In the beginning, the lubricant was circulated while the specimens rotate at a very low speed (15 rpm) until the bulk temperatures were stabilized. At this point, the first sliding stage of \( SR = 0 \) was set up, the load cell was zeroed and the normal load was applied. After collecting data for 2.5 minutes, the next SR value was set and the process repeated until the maximum SR value of 0.7 was reached.

After the positive sliding portion of a test was completed, the specimens were removed from the machine and their roughness profiles were measured to quantify the changes to the surfaces. This was followed by another 18 tests at negative SR values with the same specimens. Surface roughness measurements were repeated at the end of the test as well.

### 3.3.3 Traction Test Results

The calculated \( \Gamma_b \) value and the measured total torque \( \Gamma_t \) were used to calculate the traction torque of the disk \( \Gamma_d = \Gamma_t - \Gamma_b \) such that traction coefficient can
Figure 3.11 Variation of $SR$ with time in the positive sliding portion of the traction test.
be determined as

$$\mu = \frac{\Gamma_d}{WR_2}$$  \hspace{1cm} (3.4) \\

where $W$ is the applied normal load and $R_2$ is the radius of disk 2.

Figures 3.12 and 3.13 show measured traction curves for rough surfaces at oil inlet temperatures of 40 and 90°C, respectively. Likewise, Figure 3.14 shows the traction curves for the super-finished specimens at 90°C. The kinematics of the problem suggests that a skew-symmetry should exist in the traction curves, i.e. the absolute value of $\mu$ should be the same for a given absolute value of $SR$. This was, however, not the case since the surface roughness changed significantly in the positive $SR$ portion of the tests, as it will be demonstrated later. Focusing mostly on the positive $SR$ regions of the data, the following observations can be made from these figures:

- Regardless of $SR$, the magnitude of $\mu$ increases with reduced $u_e$, as evident from Figures 3.12 to 3.14. This is primarily due to the reduction in the film thickness and increase in asperity interactions caused by reducing. For instance, in Figure 3.12(a) with $T_0 = 40^\circ$C, $SR = 0.4$ and $p_h = 2$ GPa, $\mu = 0.06$, 0.05 and 0.04 for $u_e = 2$, 6 and 10 m/s, respectively, which is a rather significant difference. While this indicates that more friction exists at lower speeds, it does
Figure 3.12 Measured $\mu$ values of specimens with rough surfaces at $T_0 = 40^\circ\text{C}$ and various $u_e$: (a) $p_h = 1 \text{ GPa}$, (b) $p_h = 1.5 \text{ GPa}$, and (c) $p_h = 2 \text{ GPa}$.
Figure 3.13  Measured $\mu$ values of specimens with rough surfaces at $T_0 = 90^\circ C$ and various $u_e$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.14 Measured $\mu$ values of specimens with super-finished surfaces at $T_0 = 90^\circ$C and various $u_c$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
not necessarily mean that the specimens will scuff at lower speeds easier. As it will be shown later, in spite of the reduced $\mu$ values, rate of heat generation is greater at higher $u_e$ values, creating the conditions for scuffing failures.

- While it can be extracted from Figures 3.12 to 3.14, Figure 3.15 is generated here from the same data shown in Figure 3.12 to show influence of $p_h$ quantitatively. Here, it is shown that the $\mu$ increases with $p_h$ for values $|SR|<0.1$. The qualitative shapes of the traction curves are also influenced by the value of $p_h$. At lower $p_h$ values (e.g. 1 GPa), a gradual increase in $\mu$ is observed with increased $SR$ and the maximum $\mu$ value occurs typically at maximum $SR$. In contrast, at higher load values, a very steep increase in $\mu$ is obtained with application of small amounts of $SR$ with $p_h$. In such cases, $\mu$ reaches its maximum value about $SR = 0.1$ and experiences slight decline after that point. Influence of $u_e$ on the shape of the traction curves is less pronounced. These trends agree well with previous experimental and theoretical studies under similar conditions [3.50].

- A comparison of the traction curves from Figures 3.12 and 3.13 yields the influence of oil temperature through its influence on the oil viscosity. While at lower $p_h$ values, the impact of $T_0$ on $\mu$ is secondary, a slight increase in $\mu$ is
Figure 3.15  Measured $\mu$ values of specimens with rough surfaces at $T_0 = 40^\circ$C and various $p_h$: (a) $u_c = 2$ m/s, (b) $u_c = 6$ m/s, and (c) $u_c = 10$ m/s.
evident at higher $p_h$ values. This agrees with Yoshizaki et al. [3.51] which demonstrated that lubricant viscosity while beyond 10 cSt had limited effects on gear tooth friction coefficient. Ikejo and Nagamura [3.52] also concluded from their measurements with a gear test setup that gear mesh frictional loss is weakly dependent on oil viscosity.

- Selected traction curves from Figures 3.13 and 3.14 are compared directly in Figure 3.16 to show the difference between the rough (shaved) and smooth (super-finished) surfaces. It is clear from this figure that the increased surface roughness amplitudes cause higher $\mu$ values regardless of the values of $u_e$, $p_h$, and $SR$. For instance at $u_e = 6$ m/s and $p_h = 1.5$ GPa, the $\mu$ values are respectively 0.06 and 0.03 for the rough and super-finished surfaces at $SR = 0.4$.

Two auxiliary side effects were found to impact the traction data presented in Figures 3.12 to 3.16. Measured average of the bulk roller and disk surface temperatures shown in Figures 3.17 to 3.19 for the same tests shown in Figure 3.12 to 3.14 to one of these side effects. Here, the average bulk temperature is defined as $T_{b,ave} = \frac{1}{2} (T_{b1} + T_{b2})$ where $T_{b1}$ and $T_{b2}$ are the bulk temperatures of the roller and the disk, respectively. These measurements indicate that the bulk temperatures increase significantly with $SR$ especially for the conditions when $p_h$ and $u_e$ are high. Temperatures reach levels of 200°C in Figure 3.18(c) at $SR = 0.7$, $p_h = 2$ GPa and
Figure 3.16 Effect of surface finish on measured $\mu$ values at $T_0 = 90^\circ$C and $u_e = 6$ m/s: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.17  Measured $T_{b,ave}$ values of specimens with rough surfaces at $T_0 = 40^\circ$C and various $u_e$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.18 Measured $T_{b,\text{ave}}$ values of specimens with rough surfaces at $T_0 = 90^\circ\text{C}$ and various $u_e$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.19  Measured $T_{b,ave}$ values of specimens with super-finished surfaces at $T_0 = 90°C$ and various $u_e$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
$u_e = 10$ m/s. Such changes in the bulk temperatures of the contact influence the lubrication characteristics as described by the TEHL formulation of the previous section. Therefore, the measured traction data must be viewed with the demonstrated bulk temperature changes in mind.

One other unintended change that took place during the traction tests was found to be the evolution of the surface roughness. While each traction data was identified by the surface roughness amplitudes of the brand new specimens at the beginning of the test, significant changes to roughness profiles were measured during interim inspections done at different stages of each test. Figures 3.20 to 3.22 show the RMS roughness amplitudes ($R_q$) at different test stages. Here each stage is defined as follows. Stage 1: before run-in test; Stage 2: after run-in test just before positive sliding test; Stage 3: after positive sliding test; and Stage 4: after negative sliding test. Figure 3.23 shows measured roughness profiles for one of these tests. A slight initial smoothing is evident between Stages 1 and 2 in Figures 3.20 and 3.21 for the rough specimens as a direct result of the run-in test. More significantly, significant reductions in the $R_q$ values are observed after the positive SR tests (from Stage 2 to 3). In some cases, the $R_q$ values are reduced from values around 0.5 µm to about 0.3 µm during the course of the positive SR tests. This brings up two issues with the data of Figures 3.12 to 3.14. The first one has to do with when the surface roughness changes the most during a positive SR test. Regardless of when the changes took place, the
Figure 3.20 Measured $R_q$ values of specimens with rough surfaces at $T_0 = 40^\circ$C and various $u_c$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.21 Measured $R_q$ values of specimens with rough surfaces at $T_0 = 90\degree C$ and various $u_c$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.22  Measured $q_R$ values of specimens with super-finished surfaces at $T_0 = 90^\circ$C and various $u_e$: (a) $p_h = 1$ GPa, (b) $p_h = 1.5$ GPa, and (c) $p_h = 2$ GPa.
Figure 3.23  Measured surface roughness profiles of (a) a roller and (b) a disk at different stages of a traction test with $p_h = 1.5$ GPa and $u_e = 6$ m/s.
beginning and end portions of the positive $SR$ tests represent contact of surface at different surface roughness, indicating that one of the test conditions changed unintentionally during these tests. The second issue is the compatibility of the positive $SR$ and negative $SR$ portions of the traction curves of Figures 3.12 and 3.13. According to measured roughness amplitudes of Figures 3.20 and 3.21, these segments of a given traction curve are no longer represented by the same roughness values. Due to the smoothening that takes place in the positive sliding test, the negative $SR$ portions represent surfaces with reduced roughness. This is the main reason for the differences in the absolute value of $\mu$ corresponding to $SR = a$ and $SR = -a$, the latter being less due to reduced roughness amplitudes.

3.4 Scuffing Measurements

The second type of tests performed using the same test machine and test specimens described were scuffing tests. Scuffing tests were fundamentally different from traction tests as their intent was to establish conditions when the surface will fail due to scuffing rather than measuring certain steady-state contact behavior under ideal safe sliding conditions. This section provides the details of the scuffing test procedure developed, specifies the test condition and presents the results of a family of scuffing tests performed.
3.4.1 Test Procedure and Test Condition

The scuffing procedure used in this study is based on incrementally increasing the normal load at specified constant values of $u_e$ and $SR$ until the scuffing takes place. This method was been used by others effectively to evaluate scuffing limits of other lubricants and materials [3.5, 3.53, 3.54]. The load schedule shown in Figure 3.24 was applied in the scuffing tests. Starting at $p_h = 0.6$ GPa, the load was increased every 5 minutes at a $p_h$ increment of 0.1 GPa until the specimens scuffed or the maximum load stage at $p_h = 2.3$ GPa was completed.

Before each test, the specimens were cleaned by 75% isopropyl alcohol to eliminate debris, dust and oil from fabrication and delivery processes. Prior to each test run, the oil in the reservoir is heated to 100°C and circulated through the test head at a flow rate of 1 lpm with specimens at unloaded and low speed conditions for a period of one hour. This allowed bulk temperatures of specimens stabilized. It was found there is a 10°C drop in oil temperature as it travels to the jet nozzle from the reservoir. For this reason, the oil temperature in the reservoir was maintained at 100 ± 3°C during each test run to maintain an oil inlet temperature of 90°C. Once the initial steady-state temperature conditions were achieved, the oil flow rate was reduced to 0.5 lpm. The speeds of the motors were set such that the desired $u_e$ and $SR$ values were achieved. The load cell was zeroed and the first load stage of 0.6 GPa was applied to
Figure 3.24 Normal pressure schedule for the scuffing test.
start the test.

### 3.4.2 Scuffing Test Results

A total of 23 scuffing tests with the simulated shaved surface were performed in this study. Five of these 23 tests resulted in no scuffing at the maximum load capacity of the test machine while the rest produced scuffing failures. This section presents the results for these scuffed tests as a function of operating conditions and various widely used scuffing parameters.

Typical results from a test that resulted in scuffing are presented here to better explain the nature of the failure as well as the changes that lead to the suspension of a test. In this particular test, $u_e = 15$ m/s and $SR = -1.0$ such that the corresponding absolute value of sliding speed is 15 m/s. In this test, scuffing took place approximately 30 seconds after the 9th load stage ($p_h = 1.4$ GPa or $W = 635$ N). The recorded $p_h$ and $W$ histories (as calculated from the load cell reading) up to the scuffing failure are shown in Figure 3.25. The variation of the measured traction torque $\Gamma_d$ and the corresponding friction coefficient $\mu$ recorded during this test is shown in Figure 3.26(a) and (b), respectively. The corresponding bulk temperatures are presented in Figure 3.27. It can be seen from these figures that, after the initial loading phases that act as run-in phases are passed, both traction torque and surface temperatures increase with load. Sudden increases were observed in the roller bulk
Figure 3.25 A typical recording of the applied normal load and the corresponding maximum contact pressure at $u_e = 15$ m/s and $SR = -1$. 

\[ W [N] \]

\[ P_h [GPa] \]

\[ t [\text{min}] \]
Figure 3.26  Typical (a) traction torque and (b) coefficient of friction measurements at $u_e = 15$ m/s and $SR = -1$. 
Figure 3.27  Typical recordings of measured bulk temperature during a scuffing test at \( u_e = 15 \) m/s and \( SR = -1 \).
temperature $T_{bl}$ as well as the traction torque $\Gamma_d$, marking the start of the scuffing when the test was suspended. At the same instant, heavy fuming of the lubricant also provided additional information that scuffing was indeed taking place. Views of the roller and disk surfaces after scuffing are shown in Figure 3.28. Discoloration due to excessive heat generation on surfaces is evident in this figure. Also evident is the transfer of material from one surface to the other.

The lead traces of the roller and disk profiles measured before and after this scuffing test are compared in Figure 3.29. Sizable material removal (about 15 μm in depth) from the roller surface is evident here as well as some added material to the disk surface indicating that some of the lost material from the roller surface (slower moving surface) was transferred to the disk (faster moving surface) confirming the observations tests carried out by Martin et al. [3.54].

As stated earlier, 5 of the 23 tests resulted in no scuffing. All of these tests were at sliding speed values of 10 m/s or less. The test conditions of the 18 tests that resulted in scuffing are listed in Table 3.3. These conditions are defined by the $u_e$ and $SR$ values from which $|u_s|$ values listed in this table were calculated. Here, the $u_e$ values varied from 9 to 21.2 m/s and the $SR$ values were kept at three levels of -0.8, -1.0 and -1.2 corresponding to a sliding velocity range of 12 to 21.6 m/s. Also listed in Table 3.3 are the maximum apparent Hertzian contact pressure values at the load stages when the scuffing takes place and the corresponding bulk temperatures of the roller and
Figure 3.28  Pictures of an example pair of scuffed specimens.
Figure 3.29 Comparison of measured lead profiles before and after the scuffing test: (a) roller and (b) disk.
Table 3.3 Summary of the scuffing test results.

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<td>1.9</td>
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Table 3.3  Continued.

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<td>$T_c$ [$^\circ$C]</td>
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<td>$h_{\text{min}}$ [$\mu$m]</td>
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<td>0.35</td>
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<td>FP [W]</td>
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<td>395.9</td>
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<td>413.1</td>
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<td>FPI [MW/m$^2$]</td>
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<td>352.2</td>
<td>864.8</td>
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disk surfaces. These parameters are denoted by $p_{h,s}$, $T_{b1,s}$ and $T_{b2,s}$. Also listed in Table 3.3 is the average bulk temperature at the moment of scuffing $T_{b,ave} = \frac{1}{2}(T_{b1,s} + T_{b2,s})$.

There are various ways to present the data listed in Table 3.3. One way is analogous to the PV index method (Almen scoring factor) [3.55-3.57]. This is one of the earliest methods devised to predict scuffing failures of automotive and small aircraft gears. If the PV index that is defined as the product of the maximum Hertzian contact pressure (P or in this study $p_{h,s}$) and the sliding velocity (V or in this study $u_s$) at the scuffing exceeds an empirically established critical value, then two rubbing surfaces are likely to scuff. Figure 3.30 plots the $p_{h,s}$ values against $|u_s|$. Here the data points at different $SR$ values are denoted by different symbols. This figure establishes a reasonably well-defined boundary between the safe sliding and scuffing zones on the PV plane. The trend line for this population is $p_{h,s} = -0.0819|u_s| + 3.0366$ with a standard deviation of 0.3274 ($p_{h,s}$ is in GPa and $u_s$ is in m/s), indicating that the contact can endure larger normal pressure values at lower sliding velocity conditions. Figure 3.30 also suggests that the value of $SR$ might be the main reason for the scatter of the data around this trend line. The product $p_{h,s}|u_s|$ (to correspond PV index) is plotted as a function of $|u_s|$ in Figure 3.31.
Figure 3.30  Scuffing pressure as a function of absolute sliding velocity at various $SR$ values.
Figure 3.31 \( p_{h,s} |u_s| \) as a function of absolute sliding velocity at various \( SR \) values.
value for scuffing to occur is established in this figure as 26.7 GN/ms with a standard deviation of 3.92 GN/ms.

The values of coefficient of friction $\mu_s$ at the onset of scuffing are also listed in Table 3.3. They are calculated by using Eq. (3.4) with the $\Gamma_f$ value measured right before the scuffing failure occurs. The $\mu_s$ values at scuffing vary from 1.7 to 4.5% suggesting that it would not be a reliable measure of scuffing.

The value of the lambda ratio $\lambda = h_{\text{min}}/R_q$ ($h_{\text{min}}$ is the minimum smooth surface film thickness as defined by the film thickness formula of Hamrock and Dowson [3.58]) has been proposed as a criterion of scuffing, with the assertion that contact conditions with $\lambda \leq 1$ would likely to scuff [3.59, 3.60]. Figure 3.32 plots the $\lambda_s$ values ($\lambda$ at the scuffing stage) as a function of $|\mu_s|$. The trend line for this dataset is $\lambda_s = 0.0129|\mu_s| + 0.2971$ and a standard deviation of 0.082, in which $u_s$ is in m/s. Not only all of these $\lambda_s$ values shown in Figure 3.32 are all less one, the $\lambda$ values for the five unscuffed specimens at the maximum load stage were also less than one. Therefore, the present scuffing tests do not support the scuffing criterion based on the lambda ratio. Same conclusions were made by Ku [3.61] and Patching et al. [3.5].

Several temperature-based criteria have also been proposed for establishing a scuffing limit. Wydler [3.62] proposes the critical flash temperature as a criterion for
Figure 3.32  Calculated specific film thickness at scuffing as a function of absolute sliding velocity for various slide-to-roll ratios.
scuffing. In the present work, the major axis is almost four times larger than the minor axis in the contact ellipse, the steady-state maximum flash temperature $T_{fla}$ values listed in Table 3.3 are calculated by using the formula proposed by Blok [3.63] for the contact area of a narrow rectangle and of mating surfaces with the same material:

\[ T_{fla} = \frac{1.11\mu_s W_s \sqrt{u_1 - u_2}}{2a_h \sqrt{2b_h k_s \rho_s c_s}} \]  

(3.5)

where $\mu_s$ is the coefficient of friction just prior to scuffing failure, $W_s$ is the scuffing load, $a_h$ and $b_h$ are respectively semi-axis lengths of the Hertzian contact in the $y$ direction and $x$ (sliding) directions as well as $k_s$, $\rho_s$ and $c_s$ are respectively the thermal conductivity, density and specific heat of the solid. Specimen dimensions and material properties used in this formula shown in Table 3.4. The $T_{fla}$ values corresponding to each scuffing test are listed in Table 3.3 and plotted in Figure 3.33 as a function of sliding velocity. Here the $T_{fla}$ values range from 50 to 149°C, failing to reveal any trends with $|\mu_s|$.

The maximum total contact temperature is adopted by AGMA as a scuffing criterion [3.64]. The maximum total contact temperatures $T_c$ in Table 3.3 are defined as $T_c = T_{b,ave} + T_{fla}$ where $T_{b,ave}$ is the measured average bulk temperature just prior to scuffing. Figure 3.34 shows the variation of $T_c$ with $|\mu_s|$. Here, while $T_{b,ave}$ and
Table 3.4  Disk dimensions and material properties used in the calculation of maximum flash temperature.

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<td>Radius of crown [mm]</td>
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<td>Poisson’s ratio</td>
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<td>Specific heat [J/kg/K]</td>
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<td>470</td>
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<tr>
<td>Thermal conductivity [W/m/K]</td>
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<td>44.5</td>
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Figure 3.33  Calculated maximum flash temperature at scuffing as a function of absolute sliding velocity for various slide-to-roll ratios.
Figure 3.34  Maximum contact temperature at scuffing as a function of absolute sliding velocity for various slide-to-roll ratios.
fail to define a limit for scuffing, their sum (total contact temperature just prior to scuffing) remains relatively constant for all the tests with wide ranges of $p_h$, $|\mu_s|$ and $SR$. Based on the data points in Figure 3.34, the $T_c$ value at scuffing limit for this particular contact is 271°C and standard deviation of 27°C. This shows that the tested ATF performs, in terms of its scuffing resistance, much better than the mineral oils tested by Blok [3.63].

Several other composite parameters including the friction power $FP = \mu_s W_s |\mu_s|$ [3.65] and the friction power intensity $FPI = \mu_s W_s |\mu_s|/(\pi a_h b_h)$ [3.66] were also proposed in the past. The $FP$ and $FPI$ values listed in Table 3.3 for these experiments do not support the claim that these parameters can be used as a measure of scuffing.

In summary, the data presented in this section indicates that the PV values at scuffing were rather uniform suggesting that the product of contact pressure and the sliding velocity is a good indicator for scuffing. Other immediate parameters such as $\lambda$, $T_{b,ave}$ and $T_{fla}$ as well as computed quantities $FP$ and $FPI$ were found to be ineffective measures of scuffing. The total contact temperature criterion of AGMA provides the most reliable indicator for scuffing, provided the bulk temperatures are known. As it will be discussed in Chapter 4, bulk temperature formulae of AGMA are not realistic, hampering the fidelity of this criterion. Therefore, instantaneous
temperature increases at the contact (in place of \( T_{fla} \)) and the bulk temperatures predicted by the model proposed in Chapter 2 can be used with the critical temperature criterion to predict scuffing.

3.5 Validation of the Disk Scuffing Model

For assessing the accuracy of the model proposed in Chapter 2, traction and scuffing tests presented above will be simulated and the predicted variation of bulk temperatures will be compared to the measured ones. For this purpose, one traction test and one scuffing test were chosen for simulations. The first case is a traction test using the rough surface specimens. The operating conditions for test were \( p_h = 1.5 \) GPa, \( u_e = 10 \) m/s, \( SR \geq 0 \), and \( T_0 = 90^\circ \mathrm{C} \). The second case was from the scuffing tests, at \( u_e = 15 \) m/s, \( SR = -1 \), and \( T_0 = 90^\circ \mathrm{C} \).

Figure 3.35(a) compared the predicted and measured bulk temperatures, \( T_{b1} \) and \( T_{b2} \), during the traction test. Here the measured bulk temperatures represent the thermocouple readings at the end time of each \( SR \) stage. The predicted bulk temperatures are the temperatures at the middle points of face widths of disks 1 and 2 on each disk circumferential surface. This figure shows that model predictions match rather well with the measurements. Deviations between the predicted and measured bulk temperatures for \( 10 \leq t \leq 30 \) min are potentially caused by the simplified shaft
Figure 3.35 Comparisons of measured and predicted bulk temperatures during (a) a traction test, and (b) a scuffing test.
dimensions, unchanged surface roughness as well as the critical assumption of constant ambient temperature and no structural vibration in the modeling.

The predicted and measured $T_b$ values for the selected scuffing test are compared in Figure 3.35(b). It should be noted that the first two load stages, $p_h = 0.6$ and 0.7 GPa are not considered in predictions since the run-in process might not be appropriately predicted by the disk scuffing model. The agreement between the measurements and predictions is again reasonably good for most of the $p_h$ stages from $t = 10$ to 30 min, with predictions being well within 5% of measurements. Deviations after $t > 30$ min are potentially caused by the simplified shaft dimensions, unchanged surface roughness as well as the critical assumption of constant ambient temperature and no structural vibration in the modeling. The comparison provided in Figures 3.35(a) and (b) for the two different test procedures demonstrates the of the disk scuffing model of Chapter 2 is reasonably accurate, in spite of the simplification of actual geometries of shafts in the heat transfer model.
References for Chapter 3


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CHAPTER 4

A SCUFFING MODEL FOR SPUR GEAR PAIRS

4.1 Introduction

Contacts at a gear mesh are highly transient since the contact parameters vary as the contact lines move along the tooth surfaces. These contact parameters include the normal load, surface velocities, and radii of curvature. While spur gears having involute tooth geometries can be considered as the simplest form of gearing, their lubrication behavior is still complex. Unavoidable shaft misalignments and gear manufacturing errors, lubricant effects such as non-Newtonian behavior, intricate surface finish topographies as well as the dynamic loading conditions all add to the complexity of gear lubrication problem. In addition, the thermal effects must be included if the scuffing behavior is a concern, as it is the case in this chapter.

4.1.1 Conventional Gear Scuffing Criteria

The PV index method, also known as Almen scoring factor [4.1], is one of the first empirical methods developed to consider scuffing in gear design calculations.
This method was modified in 1940s to the PVT index method, which was found to be effective in evaluating the scuffing risk of small aircraft gears made by the automobile gear manufacturers. In PVT parameter, P stands for the maximum Hertzian contact pressure, V represents the sliding velocity at the point where P acts, and T is the distance along the line of action from the pitch point to the point where P acts. The PVT index (in N/s) for gear $i$ of the gear pair $(i=1,2)$ as [4.1]

$$PVT_i = \frac{\pi n_1}{360} \left( 1 + \frac{N_1}{N_2} \right) \left( R_i - r_{pi} \sin \psi_t \right)^2 P_i$$ (4.1)

where $n_1$ is the rotational speed of gear 1 (in rpm), $N_1$ and $N_2$ are respectively the number of teeth on gears 1 and 2, $R$ is the radius of curvature at the mesh point of interest (in m), $r_{pi}$ is the pitch radius (in m), $\psi_t$ is transverse pressure angle, and $P_i$ is the maximum Hertzian contact pressure (in Pa).

The value of PVT is zero at the pitch point of spur gears and increases steadily as the contact moves away from the pitch point. For gears without tooth profile modifications, the maximum value of PVT index occurs at the tip and root of the tooth. Therefore, the value of PVT is conventionally calculated only at the tip and SAP of the pinion tooth, while Eq. (4.1) applies to all mesh points along the line of action. For gears with tooth profile modifications, the location of the maximum PVT index is not necessary at the tip or root, and evaluation of PVT index along the entire line of action
is preferable.

The main drawback of the PVT index is that its critical value at scuffing must be known for each lubricant-material combination. This requires extensive gear scuffing tests for each combined lubricant and gear material of interest. Very little information on the critical value of PVT is available in the literature. A design PVT limit of $8(10)^7$ N/s was considered to represent a scuffing-free gear operation for case-hardened gears having precise profiles, lubricated with medium-weight ordinary mineral oils, and in the high-speed operation (at least 10 m/s of pitch line velocity) [4.2]. Other PVT limits of $1.1(10)^8$ N/s for gear oils and $1.97(10)^8$ N/s for hypoid gear oils were also reported [4.3]. Besides absence of PVT limits for today’s gear materials and synthetic oils, use of this method is also problematic since it does not take into account parameters such as the surface roughness, lubricant viscosity, and temperature.

*Flash Temperature Criterion:*

The flash temperature approach was introduced by Blok [4.4] to estimate the instantaneous surface temperature assuming (i) the contacts are one-dimensional, (ii) surfaces are perfectly smooth, (iii) Hertzian pressure distribution can be used in place of the actual fluid firm pressure, and (iv) temperatures of both contact surfaces are equal to each other. In other words, most of the critical parameters dictating the
thermal EHL model of Chapter 2 (surface roughness, etc.) are ignored in the flash temperature approach.

The flash temperature concept was interpreted in different ways. One group of studies considered the flash temperature to be the instantaneous temperature increment over the bulk temperature [4.5-4.8]. Others interpreted as the sum of instantaneous temperature increment and bulk temperature [4.9, 4.10]. In this chapter, the former interpretation is adapted in line with the AGMA (American Gear Manufacturers Association) standards [4.6]. The flash temperature of a pair of spur or helical gears of the same material is defined as

\[ T_{\text{flash}} = \frac{0.8 \mu X_{\Gamma} w_n}{B_s \sqrt{b_h}} \left| \sqrt{u_1} - \sqrt{u_2} \right|. \]  

(4.2)

Here \( T_{\text{flash}} \) is the flash temperature (in °C) representing the instantaneous temperature rise at a given gear mesh, \( b_h \) is the semi-width of the Hertzian contact zone (in meters), \( \mu \) is the user-defined coefficient of friction (intended to include the effects of surface roughness indirectly), \( X_{\Gamma} \) is a dimensionless tooth-to-tooth load sharing factor, \( w_n \) is the normal unit load (in N/m), and \( u_1 \) and \( u_2 \) are the instantaneous tooth surface velocities of gears 1 and 2 at the mesh point (in m/s). \( B_s = \sqrt{k_s \rho_s c_s} \) is the thermal contact coefficient where \( k_s \) is the thermal conductivity of gear (in W/m/K), \( \rho_s \) is the density of gear (in kg/m³), and \( c_s \) is the specific heat of gear (in J/kg/K).
One important parameter on Eq. (4.2) is the coefficient of friction $\mu$. In the absence of an EHL analysis, AGMA standard [4.6] provides two approximate ways to define $\mu$. The first one is a constant friction coefficient assumed by Kelley [4.11] and AGMA Standard 217.01 [4.12] as

$$\mu = 0.06 \left( \frac{1.13}{1.13 - \bar{R}_q} \right)$$

(4.3)

where $\bar{R}_q = \frac{1}{2}(R_{q1} + R_{q2})$ is the average surface roughness in $\mu$m. Here $R_{q1}$ and $R_{q2}$ are the root-mean-square surface roughness values for gears 1 and 2 in $\mu$m, respectively. This equation was stated to be valid only if $1.13/(1.13 - \bar{R}_q) \leq 3$ (i.e. $R_{q1}$ and $R_{q2}$ values are less than 0.75 $\mu$m). The other one to estimate $\mu$ is Benedict and Kelley’s empirical formula [4.13] with a surface roughness correction as

$$\mu = 0.0127 \left( \frac{1.13}{1.13 - \bar{R}_d} \right) \log_{10} \left( \frac{7.42(10)^{-3} X \Gamma w_n}{\eta_0 |u_s| u_e^2} \right)$$

(4.4)

where $\eta_0$ is the dynamic viscosity of oil at inlet temperature (in Pa-s), $u_s = u_1 - u_2$ and $u_e = \frac{1}{2}(u_1 + u_2)$ (both in m/s). Equation (4.4) is not valid at or near the operating pitch point where $u_s$ goes to zero.
The flash temperature as predicted by Eq. (4.2) is only one of the two components of the surface temperature. The other component is the bulk temperature. For the lack of this information (and a heat balance model like the one proposed in Chapter 4), the oil inlet temperature was often used in place of the bulk temperature. However, Akazewa et al. [4.14] and Deng et al. [4.9] experimentally showed that the bulk temperature on the gear tooth flank could be significantly higher than the oil inlet temperature. For instance, in high-speed gear experiments of Akazewa et al. [4.14], the pinion bulk temperature was observed to be about 95°C higher than the oil inlet temperature. Deng et al [4.9] concluded that the bulk temperature rise rapidly with increased speed, and it is much higher than the oil inlet temperature. Experiments of Hurley [4.15] on hypoid gear pairs confirmed the same observations as well. A rough approximation of the bulk temperature was proposed as a function of the inlet oil temperature $T_0$ in °C and the maximum value of the flash temperature as [4.6]

$$T_b = X_l(T_0 + 0.47T_{\text{flash,max}})$$

(4.5)

where $T_b$ is the bulk temperature in °C, $X_l$ is respectively equal to 1.0 and 1.2 for the dip lubrication and the jet lubrication methods, $T_{\text{flash,max}}$ is the maximum flash temperature in °C along the line of action.

The maximum contact temperature $T_{c,\text{max}}$ is defined as the sum of the bulk temperature and the maximum flash temperature as
AGMA standard defines the scuffing temperature as the contact temperature at which scuffing is likely to occur for a given combination of lubricant and gear materials. The scuffing temperature is assumed to be a characteristic value for the material-lubricant pair. It also assumes the lubricant is a mineral oil with no or very low concentrations of anti-scuffing additives.

**Scuffing Criterion of Dudley:**

A scuffing (scoring) number $Z_c$ was proposed by Dudley [4.16] as a measure of the risk of scuffing. $Z_c$ (in °C) is defined as

$$Z_c = n_1 m_t^{0.25} \left( \frac{W_{te}}{F_e} \right)^{0.75}$$

where $m_t$ is the transverse gear module (in mm), $F_e$ is the effective gear face width (in mm), and $W_{te} = C_m W_t$ is the effective tangential tooth load (in N). Here $C_m$ is a correction factor that is introduced to consider the spacing error and $W_t$ is the tangential tooth load (in N). Dudley [4.16] specifies critical $Z_c$ values for aerospace spur and helical gears with AGMA rated, military, and synthetic oils at different bulk temperatures.
Considering $Z_c$ as a flash temperature component, Dudley defines the contact temperature at any point along the line of action as [4.16]

$$T_c = T_b + Z_t Z_s Z_c$$  \hspace{1cm} (4.8)

where $Z_t$ is a dimensionless scuffing geometry factor defined as

$$Z_t = \frac{0.0175 \left[ R_1^{0.5} - \left( \frac{R_2}{N_1 N_2} \right)^{0.5} \right]}{m_t^{0.25} \left( \cos \psi_t \right)^{0.75} \left( \frac{R_1 R_2}{R_1 + R_2} \right)^{0.25}}$$  \hspace{1cm} (4.9a)

and $Z_s$ is the surface roughness factor that is intended to represent rough surface effects in the form of

$$Z_s = \frac{1.27}{1.27 - R_q}.$$ \hspace{1cm} (4.10b)

In these expressions, $m_t$ is transverse module (in mm) and $R_q = \sqrt{R_{q1}^2 + R_{q2}^2}$ is the composite RMS surface roughness (in μm).

In Eq. (4.8), Dudley assumes that the bulk temperature is 25°C higher than the oil inlet temperature for high-speed aircraft gears and 15°C higher for turbine gears. Similar to the PVT index, the scuffing criterion of Dudley is conventionally calculated.
only at the pinion tips and gear tips for gears without tip relief. For gears with tooth profile modifications, the location of the maximum value of $Z_t$ usually occurs at the start of the tip relief. However, finding $Z_t$ values along the entire line of action, its maximum value can be determined regardless of the modifications applied to tooth profiles.

4.1.2 Recent Research in Gear Scuffing Modeling

Due to the complexities listed at the beginning of this chapter, only a very few models have been developed for gear lubrication problems, especially a gear lubrication model in the mixed lubrication regime with measured surface roughness profiles. These models vary based on their ability to include transient effects, non-Newtonian behavior or thermal effects. The model by Hua and Khonsari [4.17] used a Newtonian, transient, isothermal lubrication model of involute spur gear with assumption of rigid gear teeth and smooth surfaces. Using some of the same assumptions (rigid gear teeth, smooth surfaces, and simplified idealistic load variation), Larsson [4.18] developed a non-Newtonian, transient isothermal, lubrication model to predict the pressure, film thickness, and coefficient of friction at several contact points along the line of action, while the model by Wang et al. [4.19] can be considered as a Newtonian thermal version of this model. Kumar et al. [4.20] presented a transient isothermal lubrication solution for involute spur gears using compressible couple-stress
fluids to account the effects of fluid film thickeners and VI improvers from polymer additives. Wang and Cheng [4.21-4.23] developed a Grubin-type analytical approach to predict the dynamic tooth loads, lubricant film thickness, bulk and flash temperatures of spur gear contacts. As a part of this approach, finite element method was employed in determining the bulk temperature on the tooth surface by assuming a uniform convective heat transfer coefficient over the entire surfaces. This approach was extended by Patir and Cheng [4.24] to include the effects of non-uniform convective heat transfer coefficients over the surfaces of gear teeth. Most recently, a non-Newtonian, transient, isothermal mixed lubrication model for involute spur gear was proposed by Li and Kahraman [4.25]. This model stands out for its capabilities in including rough surfaces and actual asperity contacts.

Another group of models focused on heat transfer characteristics of tooth surfaces with special interest in the fling-off cooling of tooth surfaces under jet lubricated conditions. Model by DeWinter and Blok [4.26] dealt with a simplified analytical model to describe intermittent fling-off cooling on the gear teeth in terms of hydrodynamic and thermal effects. This model was later extended by van Heijningen and Blok [4.27] to include the continuous fling-off cooling effects due to a continuous supply of the oil jet to the tooth contact surfaces. By using the finite element method, El-Bayoumy et al. [4.28] extended the model of DeWinter and Block [4.26] by adding the influence of Coriolis forces on oil cooling of gears. Handschuh and Kicher [4.29] also conducted the heat transfer analysis on the spiral bevel gears using nonlinear finite
element method. The models proposed in Ref. [4.28, 4.29] required significant computational time. Koshigoe et al. [4.30] improved the model developed by El-Bayoumy et al. [4.28] computationally by using the Green’s function method to predict the instantaneous gear bulk temperature.

Assuming unity load sharing factor and constant friction coefficient along the contact path, Anifantis and Dimarogonas [4.31] developed a model to calculate the flank temperatures of mating gear teeth using finite element method. They found that the mean and maximum flank temperature values are both dependent on the Biot number and the number of teeth, while the heat partition is dependent mainly on the thermal conductivity, heat transfer coefficient and Biot number ratios of the two mating gears.

Long et al. [4.32] investigated the effects of gear geometry, rotational speed and applied load, as well as lubrication conditions on the bulk temperature of high-speed gear teeth by a finite element thermal model. They also conducted the temperature measurement on the out-of-mesh surface by an infrared imager, and in the gear tooth and gear body by thermocouples. The oil was jetted into the mesh inlet of the driving gear flank. They claimed that their FE thermal model compares reasonably well with their measurements except at higher rotational speeds and higher loads.

Based on above review of the literature on thermal behavior of gears, it can be stated that most of these thermal models did not include surface roughness related
effects. Instead of determining the friction coefficient distributions at the gear contact
interfaces using rough surface EHL models, they often opted to use a user-defined
friction coefficient value of formulae. They are also indifferent to the method of
lubrication applied, dip lubrication (gears are partially or fully immersed in oil) or jet
lubrication (no oil accumulation within the gearbox is allowed while oil is circulated
and applied to gear mesh interfaces via high-speed jets). Each lubrication method
cannot be expected to be dictated by different convective heat transfer coefficient that
must be taken into account in the gear thermal analysis. Most surprisingly, the
previous models provided no link between the thermal EHL analyses of the gear
contacts and the computation of gear bulk temperatures.

In this chapter, a new gear scuffing model will be proposed. This new model
will be capable of predicting the pressure, film thickness, surface and bulk
temperatures distributions as well as shear stresses acting on the surface and the
resultant coefficient of friction, and heat flux. For this purpose, the kinematic and
geometric conditions of the gear contacts will be described by using the involute gear
formulations. The instantaneous tooth load that act as the normal contact loads will be
determined by a quasi-static load distribution model [4.33-4.35]. A one-dimensional
thermal EHL model of gear contacts at given steady state rotational positions
(neglecting the transient effects mentioned earlier) will be developed based on the two-
dimensional thermal EHL formulation proposed in Chapter 2. This model will be used
to predict the heat generated at the gear mesh interfaces at a given set of bulk
temperatures. Generated heat will be incorporated in a convective heat transfer model of gears to predict the changes in the bulk temperatures. Implementing the heat transfer and EHL models iteratively, the bulk temperatures and the critical tooth surface temperatures will be predicted.

The rest of the chapter is structured to provide the details of (i) the implementation of a gear load distribution model for providing normal loads and geometric parameters required by the line-contact thermal EHL model for each contact segment, (ii) development of a line-contact thermal EHL model for generic gear contacts, and (iii) development of a gear pair heat transfer model that is exercised iteratively with the thermal EHL model.

4.2 Determining Gear Contact Conditions

Before the line-contact TEHL model can be exercised for a generic gear contact problem, the kinematic and operating conditions of the gear contact must be determined. Figure 3.1 defines the transverse geometric parameters of an involute spur gear pair, where $O_1O_2$ is the center distance, two inner dashed circles denoted by the radii $R_{b1}$ and $R_{b2}$ are respectively the base circles of driving gear 1 and driven gear 2, two larger dashed circles denoted by the radii $R_{p1}$ and $R_{p2}$ are the pitch circles, $\omega_1$ and $\omega_2$ are the angular velocities, $A_1A_2$ is the line of action, $\psi$ is the pressure angle,
$C$ is the instantaneous contact point, and $P$ is the pitch point. As indicated in Chapter 3, the contact between gears 1 and 2 can be approximated by two disks with different radii that are equal to the instantaneous radii of surface curvature on the tooth flanks:

$$R_1(\xi) = CA_1 = R_{b1}\theta_1 = -\xi + R_{b1}\tan\psi,$$  \hspace{0.5cm} (4.11a)

$$R_2(\xi) = CA_2 = R_{b2}\theta_2 = \xi + R_{b2}\tan\psi$$  \hspace{0.5cm} (4.11b)

where $\theta_1 = -\xi/R_{b1} + \tan\psi$ and $\theta_2 = \xi/R_{b2} + \tan\psi$ are the roll angle of gears 1 and 2, respectively. Here, $\xi$ is the coordinate along the line of action with its origin at the pitch point $P$. A tooth on gear 1 comes into contact with a tooth of gear 2 at the start of active profile (SAP) of gear 1 that is the lowest dedendum point with contact. As the gears roll, point $C$ moves upward towards the tip of tooth of gear 1, passing through the pitch point $P$ and leaving the contact at the end of active profile (EAP) that is typically at the tip of the tooth profile. Both $R_1$ and $R_2$ have their minimum values at their corresponding SAPs and maximum values at their corresponding EAPs.

The mating tooth surfaces roll and slide against each other along the line of action except at the pitch point, where there is a pure rolling condition. The tangential surface velocities at the gear contact point $C$ are

$$u_1(\xi) = \omega_1R_1(\xi),$$  \hspace{0.5cm} (4.12a)
\[ u_2(\xi) = \omega_2 R_2(\xi). \] (4.12b)

The instantaneous rolling and sliding velocities are given respectively as 
\[ u_e(\xi) = \frac{1}{2}[u_1(\xi) + u_2(\xi)] \] and \[ u_s(\xi) = u_1(\xi) - u_2(\xi), \] from which the slide-to-roll ratio can be defined as \[ SR(\xi) = u_s(\xi)/u_e(\xi). \] Here \[ u_1(\xi) > u_2(\xi) \] or \[ SR(\xi) > 0 \] when point \( C \) is between the SAP and point \( P \), while \[ u_1(\xi) < u_2(\xi) \] or \[ SR(\xi) < 0 \] for the contacts along the addendum of the gear 1. Under these conditions, a gear contact can experience a wide range of sliding.

In addition to kinematic and geometric parameters, the normal force (along the line of action) carried by a tooth also varies from the instant that the tooth comes to contact to the instant that it leaves the gear mesh. Figure 4.1 shows a schematic plot of variation of the load \( W(\xi) \) carried by a spur gear tooth with no deviations (manufacturing errors or intentional tooth profile modifications) in the gear rotation. Here the tooth loads depend on where the contact occurs along the tooth surface and how many teeth carry the gear mesh force. Sudden stepwise changes take place when number of teeth in contact changes from one to two and then from two to one. These transition points are defined in Figure 4.2 as the lowest point of single tooth contact (LPSTC) and the highest point of single tooth contact (HPSTC), respectively. The variation of tooth load might be altered significantly when the gear tooth profiles are modified or not machined accurately. For these reasons, a gear load distribution model
Figure 4.1 A schematic plot of variation of tooth load of a spur gear with no profile modifications.
Figure 4.2 A schematic plot of a spur gear tooth showing the locations of the start of active profile (SAP), the lowest and highest points of single tooth contact (LPSTC and HPSTC) as well as pitch and tip (EAP) points.
[4.33-4.35] will be used in this chapter to predict the gear tooth forces.

4.3 Non-Newtonian Mixed Thermal EHL and Heat Generation Models for Spur Gears

Since the contact shapes at gear mesh may not be always characterized by a simple ellipse with a reasonable aspect ratio, the mixed TEHL model developed in Chapter 2 for point (elliptical) contacts is reduced to a line-contact, mixed TEHL model to present the generic gear contacts. The model presented below is a thermal version of a previous one dimensional EHL model [4.36].

4.3.1 Governing Equations

With the characterization of line contacts for spur gears, the fluid flow in a contact zone at a given line of action location $\xi$ with no asperity interactions is governed by the transient Reynolds equation as

$$\frac{\partial}{\partial x} \left[ \phi_x \frac{\partial p(x,t)}{\partial x} \right] = \frac{\partial}{\partial x} \left[ u_e \rho_f (x,t) h(x,t) \right] + \frac{\partial}{\partial t} \left[ \rho_f (x,t) h(x,t) \right]$$

(4.13a)

where $x$ is a local coordinate in the direction of sliding (tangential to the gear contacting surface) at the mesh point, $t$ is the time as well as $p$, $h$, and $\rho_f$ are
pressure, thickness, and density of the fluid film, respectively. \( u_e \) represents the value of rolling velocity at position \( \xi \) and \( \phi_x \) is the non-Newtonian flow factor in the \( x \) direction which can be approximated as [4.37]

\[
\phi_x = \frac{\rho_f(x,t)[h(x,t)]^3}{12\eta_f(x,t)} \cosh \left[ \frac{\tau_m(x,t)}{\tau_0} \right]
\]  

(4.13b)

where \( \eta_f \) is the dynamic viscosity of the lubricant, and \( \tau_m \) and \( \tau_0 \) are the mean shear stress and the Eyring shear stress, respectively. Here the mean shear stress can be written as

\[
\frac{\tau_m(x,t)}{\tau_0} = \sinh^{-1}\left\{ \frac{\eta_f(x,t)u_s}{\tau_0 h(x,t)} \right\}
\]  

(4.13c)

where \( u_s \) is the sliding velocity at position \( \xi \) as defined in Sect. 4.2.

Equation (4.13) is only valid for fluid flow in the wet contact regions where the local surfaces are separated by the lubricant film \((h > 0)\). In the dry contact regions where asperity interactions occur \((h = 0)\), residual errors from the Poiseuille term often cause numerical instabilities when asperity interactions are severe. In order to avoid such instabilities, a reduced form of the Reynolds equation is applied [4.38],

\[
\frac{\partial}{\partial x} \left[ u_e \rho_f(x,t) h(x,t) \right] + \frac{\partial}{\partial t} \left[ \rho_f(x,t) h(x,t) \right] = 0.
\]  

(4.14)
Assuming a smooth transition between the wet and dry contact regions, Eqs. (4.13) and (4.14) can be solved simultaneously to predict the mixed EHL lubrication conditions. Here any film thickness less that the molecular size of a typical lubricant (say 1 nm) was assumed to represent an asperity contact.

The governing equations for temperatures on the moving surfaces, $T_1$ and $T_2$, with same material can be obtained as [4.39]

$$T_1(x,t) = T_{1b}(t) + \frac{1}{\sqrt{\pi \rho_s c_s k_s u_1}} \int_{x_{in}}^{x} \frac{q_1(x',t)}{\sqrt{x-x'}} dx'$$

(4.15a)

$$T_2(x,t) = T_{2b}(t) + \frac{1}{\sqrt{\pi \rho_s c_s k_s u_2}} \int_{x_{in}}^{x} \frac{q_2(x',t)}{\sqrt{x-x'}} dx'$$

(4.15b)

where $T_{1b}(t)$ and $T_{2b}(t)$ are respectively the bulk temperatures of surfaces 1 and 2 at time $t$, $x_{in}$ is inlet boundary of EHL contact, and $q_1(x,t)$ and $q_2(x,t)$ are the heat fluxes directed towards surfaces 1 and 2, respectively. In Eq. (4.15), $\rho_s$, $c_s$, and $k_s$ are the density, specific heat, and thermal conductivity of solid, respectively.

Assuming all of the heat generated goes into gear solids (i.e. none of the heat generated is removed by the oil), heat fluxes into surfaces 1 and 2 must equal the total heat generated $q(x,t) = \mu(t)p(x,t)|u_s|$ at location $\xi$ and time $t$, i.e.
\[ q_1(x,t) + q_2(x,t) = q(x,t). \] (4.16)

A heat partition coefficient \( f(x,t) \) is defined from Eq. (4.16) to determine how much of the heat generated goes to surface 2:

\[
f(x,t) = \frac{q_2(x,t)}{q(x,t)} \quad \text{and} \quad 1 - f(x,t) = \frac{q_1(x,t)}{q(x,t)}
\] (4.17)

Solutions for \( f \) can be achieved by matching the surface temperature at asperity interaction interfaces by the iterative method as illustrated in Chapter 2.

A thermal model for the temperature variation across the lubricant film proposed by Lai and Cheng [4.40] is used to calculate the temperature in the lubricant film as

\[
k_f(t) \frac{\partial^2 T_f(x,z,t)}{\partial z^2} = -\eta^*_f(x,t) \left[ \frac{\partial u_f(x,z)}{\partial z} \right]^2.
\] (4.18)

The temperature on the mid-layer of lubricant film is of special interest since it will be used later to determine the viscosity and density of the lubricant. It is given as

\[
T_f(x, \frac{h}{2}, t) = \frac{\eta^*_f h^2}{8 k_f} \left( \frac{\partial u_f}{\partial z} \bigg|_{z=\frac{h}{2}} \right)^2 + \frac{T_1(x,t) + T_2(x,t)}{2}.
\] (4.19)
4.3.2 Companion Equations

The film thickness \( h(x,t) \) between two contacting surfaces at \( \xi \) and \( t \) consists of the relative approach of two surfaces, the un-deformed geometry of the two surfaces, the surface roughness heights on the surfaces, and the elastic deformation. The expression for \( h(x,t) \) can be written as [4.41]

\[
h(x,t) = h_0(t) + h_s(x) - \delta_1(x) - \delta_2(x) + V(x,t)
\]  

(4.20)

where \( h_0(t) \) is the reference film thickness and \( \delta_i(x) \), \( i = 1, 2 \), is the surface roughness height at the coordinate of \( x \) and at the mesh position of \( \xi \). The term \( h_s(x) \) is the separation function that includes the radii of curvature of the surfaces before the elastic deformation occurs, defined at position \( \xi \) as

\[
h_s(x) = \frac{x^2}{2R_{eq}}
\]  

(4.21)

where \( R_{eq} \) is the \( \xi \)-dependent equivalent radius of curvature in the \( x \) direction that can be written as \( R_{eq} = \frac{R_1(\xi)R_2(\xi)}{R_1(\xi) + R_2(\xi)} \) with \( R_1(\xi) \) and \( R_2(\xi) \) as defined in Sect. 4.2. The term \( V(x,t) \) in Eq. (4.20) is the surface elastic deformation due to the pressure distribution \( p(x,t) \) acting on the two contacting elastic bodies, and is given as [4.42]
\[ V(x,t) = \int_{\Omega} K_i(x-x') p(x',t) dx' \]  \hspace{1cm} (4.22a)

where \( \Omega \) is the space domain and \( K_i(x) \) is the influence coefficient, given as

\[ K_i(x) = -\frac{4 \ln|x|}{\pi E'}. \]  \hspace{1cm} (4.22b)

Here, \( E' \) is the equivalent Young’s modulus.

The normal load \( W(\xi) \) applied to the contact at \( \xi \) must be balanced by the normal force due to the predicted pressure distribution within the contact zone such that

\[ W(\xi) = \int_{\Omega} p(x,t) dx. \]  \hspace{1cm} (4.23)

Here \( W(\xi) \) is obtained from the gear load distribution analysis. If the load imposed by the predicted pressure distribution at a given instant is not equal to \( W(\xi) \), i.e. Eq. (4.23) is not satisfied, then \( h_0(t) \) in Eq. (4.20) is adjusted within an iterative loop until Eq. (4.23) holds.

The surface traction in the contact zone consists of the viscous shear exerted by the lubricant shearing in the lubricated contact regions and the asperity shear caused by the metal-to-metal contact of asperities. The viscous shear stress acting on the lubricant near the first tooth surface is given as
\[
\tau_f(x,t) = \frac{\eta_f(x,t)u_s(\xi)}{h(x,t) \cosh \left( \frac{\tau_m(x,t)}{\tau_0} \right)} - \frac{h(x,t)}{2} \left[ \frac{\partial p(x,t)}{\partial x} \right].
\]

(4.24a)

By using an approximation based on the formula of Rabinowicz [4.43] for the boundary film shearing, the shear stress in the regions of asperity interaction is written as

\[
\tau_a(x,t) = \begin{cases} 
\sqrt{\tau_{a0}^2 + \left[ \gamma_a p(x,t) \right]^2}, & u_1(\xi) \neq u_2(\xi), \\
0, & u_1(\xi) = u_2(\xi),
\end{cases}
\]

(4.24b)

where \( \tau_{a0} \) is the initial shear strength of the boundary film and \( \gamma_a \) is the coefficient corresponding to the friction coefficient in the boundary lubrication. According to the experimental results given by Hoglund [4.44], \( \gamma_a \) is in the same order as the coefficient corresponding to maximum friction coefficient in the hydrodynamic lubrication. In this chapter, values of \( \tau_{a0} = 5.5 \text{ MPa} \) and \( \gamma_a = 0.15 \) are adopted.

In the mixed lubrication regime, the total friction is the sum of the traction in the lubricated contact regions and friction in the asperity contact regions. The coefficient of friction \( \mu \) at position \( \xi \) is defined as

\[
\mu(t) = \frac{F(t)}{W} = \frac{\int_{\Omega_f} \tau_f(x,t)dx + \int_{\Omega_a} \tau_a(x,t)dx}{\int_{\Omega_f} u_s(\xi)dx + \int_{\Omega_a} \tau_{a0} dx}
\]

(4.25)
where $F$ is the friction force, $\Omega_f$ is the space domain of lubricated contact regions, and $\Omega_a$ is the space domain of asperity contact regions.

In order to avoid overestimation of lubricant viscosity under very high pressure, a refined two-slope viscosity-pressure model expressed in Eq. (2.31) is used here. Finally, the lubricant density is calculated by density-pressure relationship of Dowson and Higginson [4.45] with the thermal effect [4.46], given in Eq. (2.32), is employed.

### 4.4 Transient Heat Transfer Model for Spur Gears

One major uncertainty of the thermal EHL model proposed above is that the bulk temperatures of the contacting surfaces must be known. Several previous studies used the value of the oil inlet temperature as the bulk temperatures as well [4.6, 4.46]. While this might be acceptable in cases the interest is limited to the lubrication analysis, it is not so for scuffing modeling where the bulk temperatures expected to rise far beyond the oil inlet temperature for scuffing to take place. A number of experimental studies [4.9, 4.10, 4.47] as well as the roller contact scuffing data presented in Chapter 3 have shown that the bulk temperature is quite different from the oil inlet temperature. Therefore, a more accurate method for the bulk temperature is essential on evaluating the total tooth surface temperatures in a gear mesh.

In this section, a heat transfer analysis is carried out a spur gear pair to predict the surface temperature distributions. This is a transient heat transfer problem where
the bulk temperatures change with time. As indicated in the Ref. [4.48], a numerical temperature simulations of an exact gear configuration would require great significant computational demand. Therefore, the heat transfer model considered here in is a simplification of an actual gear configuration. The main simplification is that the gears are represented by cylindrical bodies as shown in Figure 4.3. Outside diameter and width of each disk are equal to the pitch diameter and the face width of the corresponding gear. These simplified gears have the same thermal mass as the actual design. In addition to the geometrical simplification, the temperature within each representative disk is assumed constant in the angular (circumferential) direction. With this simplifying assumption and recognizing the symmetry with respect to the centerline along the face width, only a quarter of each disk shown in Figure 4.3 has to be modeled.

4.4.1 Heat Transfer Model Formulation

The formulation for the quarter of simplified gear body includes the heat flux from the contacting gear tooth surfaces, heat conduction in each gear body as well as heat convection to the surrounding environment. The instantaneous heat flux on the contact areas of gears 1 and 2 is obtained from the mixed TEHL model as

\[ q_{\text{ins},i}(\xi; t) = \frac{1}{x_{\text{out}}(\xi; t) - x_{\text{in}}(\xi; t)} \int_{x_{\text{in}}(\xi; t)}^{x_{\text{out}}(\xi; t)} q_i(x, t) dx, \quad i = 1, 2 \]  

(4.26)
Figure 4.3  Simplified gear blank geometries of a spur gear pair, showing the quarter segments considered as the computational domain.
where $x_{in}(\xi; t)$ and $x_{out}(\xi; t)$ are the inlet and outlet boundaries of EHL contact at mesh position $\xi$ and time $t$. Every tooth on a given gear receives the heat input once over one revolution during gear rotations. Therefore, once the instantaneous heat flux is computed, the respective heat flux must be averaged over one revolution of the gear [4.24, 4.32, 4.48]. The time required for the contact at any mesh point to travel over its contact width is $[x_{out}(\xi; t) - x_{in}(\xi; t)]/u_i(\xi)$ where $i = 1, 2$. Thus the heat flux that passes that area in one revolution is $q_{ins}(\xi; t)[x_{out}(\xi; t) - x_{in}(\xi; t)]/u_i(\xi)$. Then, the average heat flux over one revolution is obtained by dividing this expression by the period $2\pi/\omega_i$. As a result, the time-averaged heat flux generated at mesh points for gears 1 and 2 is given by

$$q_{ave,i}(\xi) = \frac{x_{out}(\xi) - x_{in}(\xi)}{u_i(\xi)} \frac{\omega_i}{2\pi} q_{ins,i}(\xi; t), \quad i = 1, 2.$$  

Thus, the average heat flux should further be averaged over one mesh cycle defined by one base pitch. In Figure 4.1, the segment from SAP to HPSTC (or from LSTC to EAP) is one entire mesh cycle. This averaging must account for the heat generated by two tooth pairs in contact between SAP and HPSTC. Since the heat generated by the third first tooth between SPA and LPSTC should be nearly equal to the heat generated by the second tooth from HPSTC to EAP (in Figure 4.1), an average of the heat generated by the second tooth from SPA to EAP is equivalent to an average over one mesh cycle with two tooth pair for a portion of the mesh cycle. With this, an average
over the loading period of a tooth, as shown in Figure 4.1 SAP to EAP represents the averaged heat flux

\[
\bar{q}_i = \frac{1}{M} \sum_{m=1}^{M} X_e q_{ave,i}(\xi_m), \quad i = 1, 2
\]  

(4.28)

where \( \bar{q}_i \) is an average value of \( X_e q_{ave,i}(\xi_m) \), \( \xi_m \) is the discrete coordinate along the line of action, and \( M \) is the total number of incremental rotational gear positions considered between SAP to EAP. In order to exclude the portion of the heat removed by the lubricant, a heat retention factor \( X_e \) is introduced in Eq. (4.28). A value of \( X_e = 1.0 \) indicates that no heat is removed by the lubricant such that all generated heat is transferred to the gear blanks while \( X_e = 0 \) means all of the heat generated is removed by the lubricant. Here \( X_e \) is assumed to be a constant at all points on the line of action.

The heat generated in the contact mesh is assumed to be distributed evenly over the periphery of the simplified gear body. Thus, the governing equation for the heat transfer in the simplified gear body is a transient diffusion-convection equation that can be written in cylindrical coordinates as

\[
\frac{\partial T}{\partial t} = \alpha_s \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)
\]  

(4.29)
where $T$ is the temperature, $t$ is the time, $y$ and $r$ are the axial and radial coordinates, and $\alpha_s$ is the thermal diffusivity of the gear material.

The initial condition of temperature distribution in the gear body is assumed to be equal to the oil temperature in the gearbox if the dip lubrication method is employed or a user-defined temperature if the jet lubrication method is employed. The cross-sectional computational domain ABCD shown in Figure 4.3(a) includes a symmetry plane AD, symmetry centerline CD, perpendicular (side) surface BC, and cylindrical peripheral) surface AB. Along AD, an adiabatic boundary condition is imposed as

$$\frac{\partial T}{\partial y} = 0. \quad (4.30a)$$

The adiabatic boundary condition is also specified on the symmetry centerline CD as

$$\frac{\partial T}{\partial r} = 0. \quad (4.30b)$$

These boundary conditions ensure that little or no heat flux across the plane or line of symmetry. The perpendicular surface BC is exposed to the surroundings, warranting a convective boundary condition

$$k_s \frac{\partial T}{\partial y} = -h_{pl}(T - T_{amb}) \quad (4.30c)$$
where $h_{pl}$ is the convective heat transfer coefficient on the perpendicular surface of gear 1 and $T_{amb}$ is the ambient temperature. Here, $T_{amb}$ is assumed to be a user-defined parameter, which can be constant or variable based on the gearbox application and the lubrication method employed. The cylindrical surface AB corresponds to the contact area where heat is input to the gear body due to the shearing lubricant and frictional asperity in addition to the cooling from the oil in the dip lubrication condition or from the mixture of oil and air in the jet lubrication condition. A convective boundary condition with heat source is specified on such surface as

$$k_s \frac{\partial T}{\partial r} = \bar{q}_1 - h_{c1}(T - T_{amb})$$

(4.30d)

where $\bar{q}_1$ is the average value of heat flux for gear 1, $h_{c1}$ is the convective heat transfer coefficient on the cylindrical surface of gear 1. The computation of $\bar{q}_1$ using the TEHL model must be done periodically with the updated bulk temperatures predicted by the heat transfer model. This is in fact the mutual two-way relationship between the heat transfer model and the TEHL model. The same boundary conditions also apply to the gear 2 as well with the gear switched to 2 from 1.

4.4.2 Heat Transfer Coefficients

Among the parameters required for the gear bulk temperature analysis, the
heat transfer coefficients are subject to more uncertainty [4.32, 4.49]. This is partly because they depend on how the gear pair is cooled (dip lubrication, jet lubrication, or mist lubrication). Most of the heat transfer coefficients in the literature are based on a single-phase flow, which may not be valid for the major cooling methods in the gear operations where medium can be a mixture of air and oil (two-phase flow).

In Eq. (4.30c) and (4.30d), the heat transfer coefficient on a cylindrical surface \( h_{c1} \) is defined to be different from \( h_{pl} \) on a perpendicular surface. If the gear is surrounded by a single-phase flow of oil, the convective heat transfer coefficient on a cylindrical surface is estimated by using Becker’s empirical formula [4.50] as

\[
h_{cf} = 0.0665 \frac{k_f}{r} \left( \frac{\eta_f c_f}{k_f} \right)^{1/3} \left( \frac{2 \omega \rho_f r^2}{\eta_f} \right)^{2/3}
\]

where \( h_{cf} \) is the convective heat transfer coefficient on the cylindrical surface surrounded by a single-phase flow of oil and \( \omega \) is the angular velocity. Here, \( k_f, \eta_f, c_f, \) and \( \rho_f \) are the thermal conductivity, dynamic viscosity, specific heat, and density of the lubricant, respectively. For a single-phase flow of air, this equation still applies, now with \( k_a, \eta_a, c_a, \) and \( \rho_a \) of air replacing \( k_f, \eta_f, c_f, \) and \( \rho_f \).

The convective heat transfer coefficient on the perpendicular surface surrounded by a single-phase flow of oil (or air with its proper parameters) was defined
based on heat transfer measurements of rotating circular disks [4.51] as

\[ h_{pf} = 0.2k_f \left( \frac{c_f \eta_f}{k_f} \right)^{1/2} \left( \frac{\omega_p f}{\eta_f} \right)^{1/2}. \] (4.32)

Here, the influence of two-phase flow is counted in a simplified way through modifications to the heat transfer coefficient

\[ h_c = \phi h_{cf} + (1-\phi)h_{ca}, \] (4.33a)

\[ h_p = \phi h_{pf} + (1-\phi)h_{pa} \] (4.33b)

where subscripts \( a \) and \( f \) represent single-flow air and oil conditions, respectively. Here, \( h_c \) and \( h_p \) are the equivalent convective heat transfer coefficients on the cylindrical and perpendicular surfaces with \( \phi \) is oil-to-air weighting factor that must be determined based on actual applications analyzed. This factor is estimated by the volume ratio of oil to air in the surrounding medium.

### 4.5 Iterative Computational Procedure

An iterative numerical procedure is devised here to implement the TEHL model of Sect. 4.3 and the heat transfer model of Sect. 4.4 simultaneously. Figure 4.4 shows the flow chart of this numerical procedure. It starts with computation of the gear load.
Figure 4.4 Flow chart of the computational procedure employed.
distribution from the given the geometric parameters of gear pair and torque transmitted. Starting at the first rotational position \( m = 1 \) \(( m \in [1, M])\) representing the position when the contact of a given driving gear tooth is initiated at its SAP, \( W(\xi_m) \), \( R_1(\xi_m), R_2(\xi_m), u_e(\xi_m), \) and \( SR(\xi_m) \) at every discrete position \( \xi = \xi_m \) are determined by the load distribution model [4.33-4.35].

These parameters are fed into the TEHL model next together with the measured tooth surface roughness profiles and lubricant properties to determine the heat flux \( q_{\text{ins},i} \) ( \( i=1,2 \) ) at an initial bulk temperature. A fixed computational domain of \(-2.5b_{h,\text{max}}(\xi_m) \leq x \leq 1.5b_{h,\text{max}}(\xi_m)\) is considered here for all discrete positions so that the EHL pressure distribution in the upstream at very discrete position can be captured. Here, \( b_{h,\text{max}}(\xi_m) \) is the maximum value of the half-Hertzian contact width during the gear mesh from SAP to EAP. A relatively refined grid size is used to achieve higher accuracy of solutions. In addition, in order to have faster convergence rate in solutions, the asymmetric integrated control volume discretization (AICV) method developed by Li and Kahraman [4.41] is employed in the present study to discretize the governing equations in the mixed TEHL model. The TEHL analyses at all of the rotational positions are combined using Eqs. (4.26) to (4.28) to determine the heat flux \( \overline{q}_i, i = 1,2. \)

With \( \overline{q}_i \), the numerical solution of the heat transfer equation, Eq. (4.29), is
obtained next by using the finite difference method together with the initial and boundary conditions specified. A rectangular uniform grid is placed over the computation domain that covers only quarter of the space domain of the simplified gear geometry. The details of discretization and the computational scheme in solving Eq. (4.29) can be found in Sect. 4.2.

The heat transfer model is used to predict the increases in bulk temperatures with time. The simulation is continued until these increases reach a user-defined threshold value to warrant a new TEHL analysis with the new (updated) bulk temperatures. At that point, the new bulk temperature values are fed into the TEHL model to predict the $\bar{q}_i$ value corresponding to the new bulk temperature values. Another heat transfer simulation is performed with the new $\bar{q}_i$ value until the same threshold is reached to seek another TEHL update. This iterative process is continued until the surface temperatures reach (i) a steady state value below a critical limit or (ii) they climb beyond the critical scuffing limit.

### 4.6 Example Simulations

In order demonstrate the capabilities of the gear scuffing model, an example spur gear pair having 16 and 24 teeth is considered. The design parameters of this gear pair are listed in Table 4.1 [4.52]. This is a standard test gear pair with a center distance of 91.5 mm that was designed to evaluate scuffing performance of gears. The
Table 4.1 Design parameters of example gear pair.

<table>
<thead>
<tr>
<th></th>
<th>Gear 1</th>
<th>Gear 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>Module [mm]</td>
<td>4.5</td>
<td></td>
</tr>
<tr>
<td>Center distance [mm]</td>
<td></td>
<td>91.5</td>
</tr>
<tr>
<td>Face width [mm]</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pressure angle [degrees]</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Pitch diameter [mm]</td>
<td>73.2</td>
<td>109.8</td>
</tr>
<tr>
<td>Addendum diameter [mm]</td>
<td>88.77</td>
<td>112.50</td>
</tr>
<tr>
<td>Profile shift factor</td>
<td>0.8532</td>
<td>-0.5</td>
</tr>
<tr>
<td>Length of recess path [mm]</td>
<td></td>
<td>14.7</td>
</tr>
<tr>
<td>Length of approach path [mm]</td>
<td></td>
<td>3.3</td>
</tr>
</tbody>
</table>
gear pair does not have any modification in the face width direction such that line contact conditions exist. A large tip relief which increases the portion of tooth flank exposed to excessive adhesive wear and the sensitivity to scuffing failure is designed in such gear pair as shown in Figure 4.5. Gear blanks are full bodies such that the general cylindrical shaped assumed in the heat transfer model is conformed.

The typical mechanical and thermal properties of such a gear pair (carburized ANSI 8620) are $E = 205 \text{ GPa}$, $\nu = 0.3$, $\rho_s = 7850 \text{ kg/m}^3$, $c_s = 475 \text{ J/kg/K}$, $k_s = 46.6 \text{ W/m/K}$. The applied torque and speed values on gear 1 of 450 Nm and 2,180 rpm are used in this simulation. This torque value corresponds to a maximum Hertzian contact pressure of 1.7 GPa at HPSTC on gear 1. The properties of automatic transmission fluid (ATF) used in this simulation at 90°C (initial oil temperature) are provided in Table 2.2. The lubrication method is chosen as the dip lubrication method with the oil level at the centerline of the gear axes as shown in Figure 4.6. An empirical relationship between the oil temperature and the run time for the operating conditions was given as $T_{oil} = 90 + 2.33t$ ($0 \leq t \leq 15 \text{ min}$) where $T_{oil}$ is the oil temperature in °C [4.53]. The ambient (mixture of air and oil) temperature is assumed to stay at 90°C at all times in the absence of better information. Measured surface roughness profiles (along the tooth involute with 0.25 mm of cut-off) of $R_{q1} = 0.22$ and $R_{q2} = 0.25 \mu m$ are used in the mixed TEHL model and shown in Figure 4.7. The surface roughness profiles were measured from a pair of run-in test gears from a companion gear scuffing
Figure 4.5 Tooth shapes of the gears used in the example simulation: (a) gear 1 and (b) gear 2.
Figure 4.6  Example gear pair shown in mesh, half immersed in the bath of oil.
Figure 4.7 Measured surface roughness profiles of an example spur gear pair: (a) roughness of the tooth surface of gear 1 with $R_{q1} = 0.22 \, \mu m$ and (b) roughness of the tooth surface of gear 2 with $R_{q2} = 0.25 \, \mu m$ (both measured after running-in and along the tooth profile direction).
The load distribution analysis was performed at $M = 29$ gear rotational positions to cover a complete cycle from SAP to EAP. A TEHL analysis was performed after every 3 minutes of operation at these conditions with the total test time kept at 15 minutes. This required 5 sets of TEHL analyses covering all 29 rotational positions (totaling 145 TEHL simulations). Meanwhile, within each 3 min segment of heat transfer simulation, the transient diffusion-convection Eq. (4.29) was solved at a time increment of 1 s. All heat transfer simulations were carried out using a spatial grid consisting of $41 \times 73$ and $41 \times 109$ nodes in the axial and radial directions, respectively, for the quarters of simplified configurations of gears 1 and 2, which corresponds an equal grid size of 0.25 mm in the both directions. The oil-to-air weighting factor $\phi$ is chosen as 0.5. The heat retention factor $X_e$ was also chosen as 0.5 in these simulations suggesting that half of the heat generated at the gear mesh interface is removed by the lubricant.

Figure 4.8 shows the variations of normal tooth force $W$, maximum Hertzian contact pressure $p_h$, the Hertzian contact width $2b_h$, the equivalent radius of curvature $R_{eq}$, the rolling velocity $u_e$, and slide-to-roll ratio $SR$ along the line of action. Five rotational positions of interest have been labeled as A, B, C, D, and E to represent SAP, Pitch, LPSTC, HPSTC, and EAP, respectively in Figure 4.8(a). The corresponding roll angles of gear 1 for these five significant rotational positions are $18^\circ$, $...$
Figure 4.8  Variation of (a) the tooth force, (b) the maximum Hertzian contact pressure, (c) Hertzian contact length, (d) the equivalent radius of curvature, (e) the rolling velocity, and (f) the slide-to-roll ratio with gear 1 roll angle.
23.7°, 26.2°, 40.4°, and 48.7° respectively. It should be noted that the pitch point is not between LPSTC and HPSTC as evident from Figure 4.8(f) that the position with $SR = 0$ is between SAP and LPSTC. While this is not typical for real life gear sets, it was done intentionally for this gear pair to increase the $SR$ value up to 1.5. In Figure 4.8(a), $W$ suddenly increases as the contact reaches point C (LPSTC) and suddenly drops at point D (HPSTC). The Hertzian contact width in Figure 4.8(c) increases as the contact moves up along the tooth of gear 1, reaching its maximum value of about 5 mm at point C (LPSTC). $R_{eq}$ shown in Figure 4.8(d) also increases with roll angle of gear 1, reaching its maximum value of 8.8 mm at a position between LPSTC and HPSTC. Figure 4.8(e) shows that the value of $u_e$ increases linearly with the roll angle of gear 1.

Figure 4.9 presents the results of the TEHL simulations at each of the 3-min updates (TEHL simulation results at $t = 0$ min are not shown here). Considering each set consisted of 29 individual TEHL analyses at various contact positions, it is not feasible to present all the simulation results. Therefore, in Figure 4.9, a number of key quantities are chosen to summarize the results. In Figure 4.9(a) the variations of the critical pressure value $p_{cr}$ (the maximum value of instantaneous pressure distribution) is plotted as a function of the roll angle of gear 1. The minimum film thickness value is not a meaningful parameter for contacts operating under the mixed EHL conditions since it becomes zero if any asperity contacts exist. Therefore, an alternative way to
Figure 4.9 Variation of (a) $p_{cr}$, (b) $h_{ave}$, (c) $T_{1,cr}$, (d) $T_{2,cr}$, (e) $q_{ave,1}$, (f) $q_{ave,2}$, (g) $W_c$, (h) $A_c$, (i) $\mu$, and (j) $\lambda'$ at $t = 3, 6, 9, \text{ and } 12 \text{ min.}$
Figure 4.9 Continued.

(e) $q_{ave,1}$ [MW/m²]

(f) $q_{ave,2}$ [MW/m²]

(g) $W_c$ [%]

(h) $A_c$ [%]

$t$ increased

Continued
Figure 4.9 Continued.
evaluate the film thickness under rough contacts is the average film thickness, which is given as

\[
h_{\text{ave}}(\xi) = \frac{x_{\text{in}}(t)}{x_{\text{out}}(t)} \left( \int_{x_{\text{in}}(t)}^{x_{\text{out}}(t)} h(x, t) dx \right)
\]

where \(x_{\text{in}}(t)\) and \(x_{\text{out}}(t)\) are respectively the inlet and outlet boundaries of elastohydrodynamic (EHL) contact at the rotational position \(\xi\) and time \(t\). In Figure 4.9(b), \(h_{\text{ave}}\) are plotted as a function of roll angle of gear 1.

The critical surface temperatures \(T_{1, cr}\) and \(T_{2, cr}\) (maximum of the instantaneous surface temperature distributions) of surfaces 1 and 2 are given in Figures 4.9(c) and (d), respectively. Likewise, the time-averaged heat flux values \(q_{\text{ave},1}\) and \(q_{\text{ave},2}\) are plotted as functions of roll angles \(\theta_1\) and \(\theta_2\) in Figures 4.9(e) and (f) for gears 1 and 2, respectively.

In order to quantify the degree of asperity interactions, two contact ratio parameters are defined here. One parameter is the load contact ratio \(W_c\), which is defined as the ratio of load supported by asperity interaction to the tooth-tooth normal load at the contact point \(\xi\).
The other parameter is the area contact ratio $A_c$ that is the ratio of area with the asperity interactions to the total EHL contact area at the contact point $\xi$:

$$W_c(\xi) = \frac{\int p(x,t)dx}{\Omega_a W(\xi)}. \quad (4.34a)$$

$$A_c(\xi) = \frac{\int dx}{\int_{x_{in}(t)}^{x_{out}(t)} dx}. \quad (4.34b)$$

Variations of $W_c$ and $A_c$ with the roll angle are shown as functions of roll angle $\theta_1$ and $t$ in Figures 4.9(g) and (h), respectively. Finally, the friction coefficient $\mu$ (Eq. (4.25)) and the lambda ratio (ratio of the minimum smooth surface tooth thickness to the composite surface roughness are plotted in Figures 4.9(i) and (j), respectively.

The following main observations can be made from Figure 4.9:

- While the value of $p_h$ remain relatively unchanged throughout the five TEHL analyses (3 min apart from each other within the heat transfer computation loop), $h_{ave}$ values are reduced in Figure 4.9(b) after each iteration, mainly because of the increases in the bulk temperatures coming from the heat transfer model.
The increase in the bulk temperatures is rather obvious in Figures 4.9(c) and (d) for $T_{1,cr}$ and $T_{2,cr}$ where distributions are shifted up with each iteration while the instantaneous increases across the contact remain relatively the same.

It is observed that the heat flux values $q_{ave,1}$ and $q_{ave,2}$ are increased with each TEHL update, suggesting that the contacts generate more heat as they get hotter, in line with the reduced $h_{ave}$ values.

The lubrication conditions appear to approach the boundary lubrication conditions as both $W_c$ and $A_c$ in Figures 4.9(g) and (h) are as high as 60% (i.e. more than half of the contact zone in asperity contact conditions with about half of the load carried directly by asperities in contact). While this might appear to be excessive, it is common occurrence in most of the automotive gearing applications. Figures 4.9(i) and (j) further emphasize these mixed EHL conditions as $\mu$ values reach 0.09 and the lambda ratio values are as low as 0.3.

It is also noted that the asperity interactions are increases with each TEHL iteration, with sizable increases in $W_c$, $A_c$ and $\mu$ values and some reduction in the lambda values.

The computed transient bulk temperature distributions for the gear blank simplified geometries at the beginning of 3-minute long simulation segment are shown
in the form of contour plots in Figure 4.10. As defined in Figure 4.3, these contours represent a quarter of each gear blank. The flow of heat from the contact takes place perpendicular to the isotherms illustrated in these contour plots. Near the meshing contact surface (cylindrical surface) the heat flux has an axial component along the face width direction indicating that heat is flowing to the sides of gears where heat transfers to the surroundings. Away from the contact surface the temperature gradient is reduced in the radial direction. While the meshing contact surfaces experience a significant amount of heating, the gear bodies away from the circumference remain relatively cool, demonstrating their heat-sink capability, drawing heat from the meshing surface, and dissipating it to the surrounding atmosphere.

Along the transverse symmetry plane (along line AD in Figure 4.3), the variation of bulk temperature at three different radii are plotted in Figures 4.11(a) and (b) for gears 1 and 2, respectively. Bulk temperatures at the pitch circles are shown to be about 15 and 25°C higher than those at the rotational axes of gears 1 and 2 respectively.

Surface roughness profiles $\delta_1(x)$ and $\delta_2(x)$ (required in Eq. (4.20)) were measured from actual scuffing test gears. In order to further investigate the impact of tooth to tooth variations from the same surface texture with different profiles, another nine simulations were carried out. The measured surface roughness profile of pinion $\delta_1(x)$ was divided into smaller segments to construct ten different variations of $\delta_1(x)$
Figure 4.10 Predicted bulk temperature distributions at different time instances.
Figure 4.11 Predicted variations of bulk temperature at various gear radii: (a) gear 1 and (b) gear 2.
by simply patching these roughness segments in different sequences. These ten different surface roughness profiles were intended to capture typical tooth-to-tooth surface roughness variations observed in on the surfaces of a real gear. Figure 4.12 shows the variations of critical pressure $p_{cr}$, average film thickness $h_{ave}$, critical surface temperatures $T_{1,cr}$ and $T_{2,cr}$ on gears 1 and 2, time-averaged heat fluxes $q_{ave,1}$ and $q_{ave,2}$ on gears 1 and 2 as well as load and area ratios $W_c$ and $A_c$ of asperity interactions. Here, at each contact position $\xi$ along the line of action, these values are represented by their mean values as well as standard deviations around the mean. These simulation results are at $t = 12$ min. Figure 4.12(a) shows that the mean critical pressure (average maximum instantaneous pressure magnitude) along the line of action is higher than its corresponding maximum Hertzian contact pressure on the ideal smooth surface contact. This is due to the transient surface roughness effect at each individual gear mesh point. If it is provided that the surface roughness profiles do not vary with time, the mean $h_{ave}$ shown in Figure 4.12(b) indicates that approximate 70% mesh points experience severe asperity contact conditions along the line of action. In Figures 4.12(c) and (d), the mean critical surface temperatures (average maximum instantaneous surface temperature magnitudes) reach values up to 400°C, the highest values being at the HPSTC on gear 1.

In Chapter 3, a set of two-disk scuffing experiments and a point contact thermal EHL model were used to establish the critical temperature values at which scuffing is
Figure 4.12 Mean variations of (a) $p_{cr}$, (b) $h_{ave}$, (c) $T_{1,cr}$, (d) $T_{2,cr}$, (e) $q_{ave,1}$, (f) $q_{ave,2}$, (g) $W_c$, (h) $A_c$ due to tooth-to-tooth roughness variations at $t = 12$ min.
Figure 4.12  Continued.

(e) $q_{1,ave}$ [MW/m$^2$] vs. $\theta_1$ [deg.]

(f) $q_{2,ave}$ [MW/m$^2$] vs. $\theta_2$ [deg.]

(g) $W_c$ [%] vs. $\theta_1$ [deg.]

(h) $A_c$ [%] vs. $\theta_1$ [deg.]
initiated. As shown in Figure 3.34 and a Table 3.3, the typical maximum temperature values ranged from 260 to 300°C, depending on the sliding ratio values. The surface roughness values of the test gears were similar to those of the roller specimens. In addition, same lubricant (automatic transmission fluid) at similar temperature was used in both test programs. The gear materials were also very similar to those used for the rollers. With this, the critical temperature values for this gear pair should be around 300°C as well.

Klein [4.53] reported that the load stage 11 (450.1 Nm) of the standard test resulted in the defined scuffing failure load [4.54]. Examining Figure 4.13, it can be seen that the model predicts $t_{i,cr}$ values reach 300°C at the HPSTC after $t = 3$ min, suggesting that the model predictions are in agreement with the experiments of Klein [4.53].

### 4.7 Summary

In this chapter, formulations to link the TEHL behavior of a spur gear pair to its heat transfer behavior were provided. The TEHL model was reduced from the 2D TEHL model of Chapter 2 while the heat transfer model was also a simplified version of the corresponding roller heat transfer model from the same chapter. The mutual interaction between the TEHL model and the heat transfer model were demonstrated
Figure 4.13 Variations of $T_{1,cr}$ along the line of action at $t = 0, 3, 6, 9$ and $12$ min.
through an example simulation. The heat flux, friction coefficient and the degree of asperity interactions were all shown to be increased with elevated bulk temperatures predicted by the heat transfer model. At the same time heat generated at the contact interfaces of gears that is the main input for the heat transfer model were shown to increase in the process.
References for Chapter 4


CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

5.1 Dissertation Summary

In this dissertation, a generalized modeling methodology was proposed to study the temperature-induced failures of lubricated load carrying contacts. This methodology consisted of two main models. One model focused on the thermal characteristics of a lubricated contact. Instantaneous surface and fluid film temperature rises at a contact interface were predicted by devising a thermal elastohydrodynamic lubrication (EHL) formulation that represents the asperity contacts within the mixed or boundary lubrication regimes. This model required the actual machined surface roughness profiles for a deterministic prediction of surface temperature distributions as well as the heat flux generated at the contact and the heat partitioning parameter. The second main model employed in the proposed methodology was a convective heat transfer model of contacting bodies, which predicts the variation of the bulk surface temperatures for given heat fluxes in and out of the control volume. The proposed methodology devised an iterative process where the heat generated at the contacts and
the bulk temperatures were periodically updated and fed into the respective models. At the end the maximum instantaneous temperature of the contact was proposed as a metric for temperature induced failures to occur.

This methodology was applied in two different levels. In one level, a generic, point contact problem was considered. Such contacts are common to many machine components where the contact zone has often an elliptical shape. A two-dimensional (2D) mixed, thermal EHL model and a 2D convective heat transfer model were developed for this purpose to be implemented iteratively according to the proposed methodology. Other level of application was focused specifically towards lubricated gear contacts. Incorporating the unique variations of contact parameters along the line of action, a one-dimensional version of the mixed thermal EHL model was developed together with a 2D heat transfer model to predict the maximum surface temperature distributions along the tooth profiles of spur gears.

Both levels of models, model for point contact problems and the model for line contact problems of spur gears both predicted the maximum contact temperatures that should be expected at the load, speed, surface and lubricant conditions dictating the contact, while falling short in determining whether these maximum temperatures should cause any temperature induced failures such as scuffing. In order to establish scuffing limits as well as validating the models in terms of time-varying bulk temperature predictions, an extensive experimental study was conducted using a two-
disk arrangement. These experiments designed to measure traction and actual scuffing limits generated surface bulk temperature data as a function of time and test parameters. Comparisons of these measurements to predictions of the model were used to validate the proposed methodology. These tests also established scuffing temperate limits for the range of surface roughnesses, the material and the lubricant of interest under typical operating conditions. Such limits were used with the models to assess the likelihood of scuffing to occur.

The conventional PV index and scuffing criteria were compared to the proposed model in terms of their accuracy in predicting the experimental behavior observed in this study. Main shortcomings of these conventional criteria were identified and conditions under which they can be used were defined.

5.2 Conclusions and Contributions

This dissertation provided a detailed treatment of the thermal conditions of point contact and gear contact problems in relation to temperature induced failure of scuffing. These modeling efforts were accompanied by extensive lab experiments for the validation of the model predictions. The following conclusions can be listed based on the results of the models and tests:
(i) Predicted bulk temperatures of the proposed scuffing model for a point contact problem show a good agreement with the corresponding measurements from both the traction and scuffing measurements. This can be interpreted such that both the mixed thermal EHL model and transient heat transfer model for the contacting disks and their supporting structures are accurate in their ability to capture the physics of the contacts.

(ii) The results of point and line contact scuffing analyses indicate that asperity interactions are significant at typical speed, load and temperature conditions with typical surface roughness profiles. The heat generated at the contact under these conditions is much larger to enhance the likelihood of scuffing significantly. With this, it can be stated that any scuffing model must have the capability to capture mixed EHL conditions accurately. This is especially true for most automotive, wind turbine and industrial applications.

(iii) The model predictions also point to the importance of the bulk temperature values. Assuming that the bulk temperatures are equal to (or higher by a certain amount) than the oil inlet temperature, as done by the conventional scuffing criteria, is flawed. Both the proposed model and the experiments show clearly that the bulk temperatures vary in time depending on the level of heat generation and removal. The model also indicates that the EHL conditions of the contact are strongly dependent on the actual bulk temperature values.
(iv) The models and the experiments point to the critical impact of the surface roughness amplitudes on the surface traction levels, heat generation as well as the occurrence of scuffing. The results in these studies show that the deterministic surface roughness profiles dictate the instantaneous friction coefficients as well as instantaneous surface temperatures. This effect not captured by the conventional scuffing criteria, which assume a constant friction coefficient formula determined empirically.

In view of above conclusions, the following can be listed as the main contributions of this study to the current stage.

(i) Extension of a previous isothermal mixed EHL model to include the thermal effects can be considered as one of the contributions of this study. The literature lacked a mixed thermal EHL model that can handle non-Newtonian fluids as well as contact conditions resulting in excessive asperity interactions. This new two-dimensional mixed thermal EHL model is unique in terms of its ability to handle (a) lambda ratios lower than 0.1, (b) transient effects brought about by the instantaneous matching of two rough surfaces moving at different speeds relative to each other, (c) non-Newtonian lubricant behavior caused by high sliding motions, and (d) thermal effects including temperature increases in the film and contacting surfaces caused by high sliding, and potentially localized instantaneous surface temperatures at the site of asperity collisions.
The formulations of the thermal EHL model to predict the heat flux and the heat partitioning parameter can also be viewed to be new and completely physics-based.

(ii) While the convective heat transfer model is not new, the overall methodology that requires its iterative use with the thermal EHL model is novel. Mutual dependence of the thermal behavior of the contact zone captured by the thermal EHL model and the bulk temperatures focused by the overall heat transfer model is fully comprehended in this methodology to predict the maximum contact temperatures to the onset of scuffing as the bulk temperature increase in time.

(iii) The experimental study performed in this study has its own contributions to the state-of-the-art. First of all, the two-disk test methodology proposed provides to detailed procedures to measure traction and scuffing limits of lubricated contacts in an accurate way. The experimental data base collected as part of this study is also significant in terms of friction and scuffing performance of automatic transmission fluids used with the automotive quality surfaces operated under speeds, loads and temperatures representative of automotive transmissions.

(iv) The extension of the scuffing methodology to spur gears is also new. In view of the literature, this is the first gear scuffing model that can include all gear
specific intricacies associated with load distribution, profile modifications as well as the variation of sliding and rolling speeds, radii of curvature and normal contact forces. Equally vital is the implementation of a mixed thermal EHL model and a convective heat transfer model to the spur gear pair with these intricacies included to allow a prediction of maximum temperatures, and hence, the occurrence of scuffing.

5.3 Recommendations for Future Work

Although the theoretical and experimental investigations conducted in this study demonstrate the comprehensive approach to disk and gear scuffing, they are by no means capable of addressing all aspects that possibly have impacts on the disk and gear scuffing. As the next-generation gear systems must have higher power densities and efficiency, it is essential to expand some of the stated objectives to enhance the accuracy of model predictions. The following recommendations for future work identify some of these areas of improvement.

(i) The proposed models assumed that the surface roughness profiles defined at the beginning of the simulations remain unchanged, while the measured surfaces at different stages of tests indicate clearly that the surfaces evolve with time depending on the load and sliding conditions. One way to capture such variations is to provide surface roughness profiles measured at different stages
of the tests to the proposed iterative methodology. While this enables the model to account for such changes, it is not desirable as the model becomes more empirical. Another way to discard the assumption used by the previously developed isothermal EHL model is to incorporate the form of surface wear or plastic deformation on asperity tips.

(ii) In the disk and gear heat transfer models, the ambient temperature is assumed to be given constants or known functions of time. With the gear blanks forming the control for the heart transfer model, the ambient temperature within the gearbox must be known for the proposed model to work properly. The entire gearbox can be chosen such that the ambient temperature inside the gearbox becomes another variable of the heat transfer model that now includes gears, the lubricant and air inside the gearbox as well as gearbox housing. This level of sophistication is required to capture the influence of the lubrication method (jet lubrication or dip lubrication) as well. It is also noted here that the model becomes more computational and application specific with a heat transfer model including gearbox housing.

(iii) Provided that the control volume of the heat transfer model is expanded to the entire gearbox to predict the ambient temperature within the gearbox as well, gear scuffing experiments according to standard procedures with dip lubricated gears should be carried out for further validation of this form of the model.

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(iv) The gear scuffing model proposed in this study can be extended to include other sources of heat generation in the form of drag and pocketing of the medium (churning of oil and windage for air).

(v) In scuffing measurements, the failure stage is currently detected by the sudden increased friction and bulk temperatures. Physical observations are such that the vibration and noise levels also increase at the onset of scuffing failure as well as amount of fume produced by the oil. A better diagnostic system can be developed and calibrated using all these indicators to better pin-point the onset of scuffing.

(vi) The scuffing model proposed here for spur gears can be modified to handle scuffing of other types of gears such as helical, spiral bevel and hypoid gears. This would require proper load distribution models for such gearing as well as a discretization of the contact lines in the face width direction.
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