THE PRESENT VALUE MODEL OF THE
EXCHANGE RATE DETERMINATION

DISSERTATION

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By

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* * * * *

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ABSTRACT

In my dissertation, I examine issues of the present value model applied to the exchange rate determination. The first part deals with a shortcoming of the popular Campbell and Shiller test of the present value models. In present value models, expectations of future variables are used to determine the current value of the variables of interest. Campbell and Shiller's technique uses vector autoregressive (VAR) framework and assumes a particular forward looking solution, but rational expectations actually gives rise to an infinite number of solutions. My research reviews this problem and shows that, in the absence of additional assumptions about the agents' information set, it is impossible to discriminate between forward and backward looking solutions and their linear combinations within the VAR framework. In that essay, I also offer possible explanations as to why the restrictions are often rejected.

Given the frequent rejection of rationality, the second part revisits the debate between rational and adaptive expectations for pricing the nominal exchange rate. A monetary approach to the exchange rate in a dynamic error-correction framework is used to develop a test for adaptive expectations. Unlike previous work on this problem, I allow for nonstationarity in the data. The restrictions imposed on the data by the rational expectations versus adaptive expectations are tested for several currencies. The model, combined with rational expectations, is rejected for plausible values of the model parameters. However, when combined with adaptive expectations, the model is not rejected in most cases, indicating that adaptive expectations help the model fit the data better than rational expectations.
The third part investigates whether structural breaks in exchange rates can reconcile the present value model with rational or adaptive expectations. Unlike previous work, the model tests separately for different sources of breaks: changes in money demand, policy, or expectations. Empirically, my results indicate that breaks in the exchange rate are consistent with adaptive expectations and shifts in the money demand function.
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CHAPTER 1

INTRODUCTION

The main purpose of this research is to investigate the applications of the present value model to exchange rate determination. The present value model is the most widely used model for exchange rate determination which arises when the monetary model is applied to the exchange rate. The monetary approach considers the exchange rate to be an asset price in which expectations about the future development of economic fundamentals influence the current exchange rate (see, for example, Mussa (1976)). Among numerous uses of the model are, for example, exchange rate forecasting (MacDonald and Taylor, (1993) and others), target zones analysis (Krugman (1992) and others), speculative attack models (Flood (1994) and others), and peso problems (Lewis (1989) and others). The explanation of why the model is so widely used can be twofold: theoretically, it makes the most sense, and nobody has offered a better model yet. However, present value rational expectations models have performed poorly over the past 20 years, both in empirical econometric work and in experimental asset markets (e.g. Camerer and Weigelt (1991), Sunder (1992)).

The current research tries to investigate why empirical support for these models is so poor. The overview of the research is as follows. First, it reviews the popular present value
model tests which use vector autoregressive (VAR) framework and assume a particular forward looking solution. This research shows that, in the absence of additional assumptions about the agents' information set, it is impossible to discriminate between forward and backward looking solutions and their linear combinations within the VAR framework. Given the frequent rejection of rationality, it further revisits the debate between rational and adaptive expectations for pricing the nominal exchange rate. A monetary approach to the exchange rate in a dynamic error-correction framework is used to develop a test for adaptive expectations, which, unlike previous work, allows for nonstationarity in the data. Further, the restrictions imposed on the data by the rational versus adaptive expectations are tested for several currencies. The next step of the research is to use a time variant specification for present value model tests and see if taking structural breaks into account can reconcile the exchange rate movements with a particular type of expectations formation. Unlike previous work, the model tests separately for different sources of breaks: changes in money demand, policy, or expectations.

In short, the research mainly concentrates on the following aspects of the present value model: expectations formation, testing methodology and time-invariant specification. The next section considers each of these aspects one at a time and explains the particular importance of each.

A. Expectations Formation.

The present value model requires that an expectations mechanism of the agents must be specified. The default type of expectations formation for the present value model is rational expectations (RE). When RE was first introduced into macro models during the early 1970's, it was a radical idea. Many papers debated rational expectations versus other types of expectations
(see, for example, Swamy et.al (1982), Rappoport (1982) and Bruno (1989)), but RE eventually won out because people recognized its internal consistency.

Most recent macroeconomic theories are based on the assumption that expectations are rational, taking for granted that rational expectations are optimal and any other type of behavior is sub-optimal and can, thus, be ignored. A rational expectation is an informed forecast of the future, normally coinciding with the predictions of the model itself. Assuming a model with imperfect information, rational expectations involve solving a general equilibrium problem in order to make an informed prediction of the future values of important variables. Thus, rational expectations require more information and computational abilities than a simple backward looking forecasting rule, based on past values of key variables. But all the information is costly. For example, if some people are not behaving rationally, then rational people must determine how these people react before they can determine their behavior. Therefore, a forecasting rule based on rational expectations is optimal if and only if the extra benefit outweighs the costs, compared with alternative types of pricing behavior.

Cochrane (1989) found that if a consumer sets consumption equal to income each period, rather than following the rational expectation based optimal permanent income decision rule, he or she loses between $0.10 and $1.00 per quarter in environments specified by popular tests on aggregate data. Cochrane concluded that a test can statistically reject the optimal decision rule in favor of alternatives with small utility costs, or, equivalently, that the likelihood function could be more curved than the utility function. Therefore, a statistical rejection of a non-rational expectations driven model may be caused by modeling simplifications rather than by the failure of the basic theory. Cochrane’s conclusions are important because they support the idea that rational expectations may not be optimal, given the cost of formulating these expectations.
An adaptive expectations mechanism, on the other hand, relies on market participants looking back in order to forecast the future. That is, people form their expectations as a “Rule-of-Thumb” based on previous experiences. In an uncertain world where the future is highly unpredictable such an expectations formation mechanism should not necessarily be ruled as “irrational.” An important feature of the adaptive expectations model is that it provides a psychologically attractive view of expectation formation. People hold an on-going expectation that is adjusted on the basis of currently available information (Winer (1985)). Adaptive expectations are, thus, consistent with the anchoring and adjustment-type processes often found in information-processing models (Tversky and Kahneman (1974)).

Smith, Suchanek and Williams (1988) examined the stock market’s rational expectations model in a laboratory environment in which they could control the dividend distribution. Their finding is that expectations and price adjustment are both adaptive, but that the adaptation over time and across experiments with increasing trader experience tended to a risk adjusted, rational expectations.

Some recent studies have shown that there is little difference in the predictive qualities for the various exchange rate models (Pilbeam (1995)). The author used a nonparametric test of popular modern exchange rate models under static, regressive, extrapolative, adaptive, rational and heterogeneous expectations mechanism. He wrote, that “at best, it seems economic fundamentals provide only a marginal amount of forecasting information” (p. 1013) and showed that the expectations mechanism is far more important than the economic model in terms of forecasting. Rational expectations performed poorly relative to the extrapolative and adaptive hypotheses and “the flexible monetary model under adaptive expectation is the best overall performer” compared with other modern exchange rate models under alternative expectations specifications (Pilbeam (1995), p. 1014).
In my research, I revise the debate between rational and adaptive expectations for pricing the nominal exchange rate. A monetary approach to the exchange rate in a dynamic error-correction framework is used to develop a test for adaptive expectations. To answer the question of whether any expectation formation theory is consistent with reality, rational and adaptive expectations must be tested against a more general hypothesis. This research offers a procedure which allows for testing of a main expectations formation mechanism for the monetary approach to the exchange rate against a more general null. Unlike the Chow approach, the proposed approach will incorporate the possibility of nonstationarity in the data. In addition, to test rational expectations, this paper estimates the parameters of the model (which are the semielasticity of the interest rate $\alpha$ for money demand function for RE; and the semielasticity $\alpha$ and adaptive expectation parameter $\eta$ for adaptive expectations), as part of the model and then tests for plausibility as opposed to MacDonald and Taylor (1993), who estimate it separately and then impose that value upon the model.

B. Testing Methodology

The second question which arises about testing present value model is the methodology used. The Campbell and Shiller approach is among one of the most popular tests in current literature for testing forward looking rational expectations. Below there are just few examples of when this approach has been applied. Engsted (1995) uses the technique to test the implications of the Fisher hypothesis, that an increase (decrease) in the spread between the long-term, or multiperiod, interest rate and the one-period inflation rate signals an increase (decrease) in future one-period inflation. Further, Engsted applies Campbell and Shiller's test to test Philip Cagan's (1956) model of hyperinflation (1993) and to the linear quadratic adjustment cost models under

It can be seen from the above that Campbell and Shiller technique is very popular in testing and forecasting in the models which use forward looking rational expectations assumptions. However, rational expectations actually gives rise to an infinite number of solutions. My research reviews this problem and shows that, in the absence of additional assumptions about the agents' information set, it is impossible to discriminate between forward and backward looking solutions and their linear combinations within the VAR framework. Therefore, it is impossible to make predictions or do forecasting based on the particular forward looking rational expectations solution wherever Campbell and Shiller restrictions cannot be rejected and the model in question is found to be consistent with rational expectations.
C. Time Invariant Specification

In the last 30 years, there have been many institutional changes in the international financial environments. They include: financial innovations; abandonment of Bretton Woods fixed exchange rate system; elimination of capital controls in Great Britain and Japan; creation of an European monetary union; economic convergence; inflation targeting; NAFTA agreement; and financial liberalization in developing countries and emerging markets. There is empirical evidence in the literature that all these changes have affected many important economic variables and resulted in their structural breaks. However, testing of present value model assumes a time invariant specification. Therefore, the next step of the current study is to try to determine how structural breaks would affect testing of present value models.

The research shows that in present value models, if the expectation mechanism does not change, breaks in the exchange rates can be explained by shifts in the preferences for one currency in terms of another and by the shifts in the process that governs the exchange rate fundamentals and expectations. It is shown that, within the monetary models framework, a possible shift in the cointegrating relationship between the exchange rate and fundamentals can be explained only by the shift in preferences for one currency in terms of another, and not by a change in the fundamentals process, as it has been done in the previous literature.

The outline of the research is the following. Chapter 2 shows that the Campbell and Shiller forward-looking restrictions actually test as a null hypothesis not only forward-looking, but any rational expectations restrictions. Therefore, by rejecting these restrictions, one rejects the null hypothesis of rationality in a very broad sense. Chapter 3 first reviews and discusses the
application of present value model tests to the exchange rate models with and without structural break in the underlying variables. The sources of possible structural breaks are investigated. The importance of the invoked parameters values is shown. Next, the restrictions for adaptive expectations are derived when the variables in question are integrated of order 1 (are \(I(1)\) variables). Chapter 4 reviews econometric methodology used for the research. Chapter 5 discusses the data and compares the results of present value model tests applied to the exchange rate for rational and adaptive expectations. Chapter 6 discusses how the present value model tests will change if the structural breaks are taken into account. The results of these tests applied to the exchange rate for rational and adaptive expectations when the structural break are taken into account are discussed. Chapter 7 concludes.
CHAPTER 2

PRESENT VALUE MODELS AND THEIR TESTING

2.1 Introduction

Present value models are among the simplest dynamic stochastic models of economics. A present value model for two variables $y_t$ and $x_t$ states that $y_t$ is a linear function of the present discounted value of expected future $x_t$. Therefore, $y_t$ is determined by what the agents think $x_t$ will be in the future. Therefore, the dynamic of $y_t$ depends to a high extent on agents’ expectations. The hypothesis of rational expectations (RE) has stimulated much interesting research in economics, and the validity of the RE hypothesis is tested by applying it to present value models. A number of studies have attempted to estimate and test present value models under the RE assumption (Campbell and Shiller (1987), Fama and French (1988), Poterba and Summers (1987), West (1988) etc.). Many of these studies have found the restrictions imposed by the RE hypothesis on present value models to be inconsistent with the data, but the authors usually prefer to reject the models tested while maintaining the RE hypothesis.

Chow (1989) tested implications of present value models using data on stock prices and dividends. He concluded that the hypothesis that current value is a linear function of the conditional expectations of the next-period value and current determining variable combined
with rational expectations is strongly rejected, but combined with adaptive expectations is accepted.

Campbell and Shiller (1987) developed a present value model test for rational expectations for I(1) variables. They apply their restrictions to test present value models combined with forward-looking rational expectations for stocks and bonds. They find that both models are rejected statistically at conventional significance levels. They also pointed out that the strength of the evidence depends sensitively on the discount factor assumed in the test.

The current chapter shows that in models where expectations of future variables affect the current value of the variables in question, rational expectations gives an infinite number of solutions. Any solution can be written as a weighted sum of two kinds of solutions: a backward looking and a forward looking solution. Recent macro-economic models use the assumption of a forward-looking solution. The Campbell and Shiller technique has been used to determine if reality is consistent with the assumption that our models hold and expectations are forward looking.

The purpose of this chapter is to revise RE present value models restrictions, develop the AE restrictions applied to the variables integrated of order (1), and show how the present value models can be applied to the exchange rate. The chapter proceeds as follows. In 2.2 Chow’s technique for rational and adaptive expectations is summarized. 2.3 revises RE restrictions for the case where the variables are integrated of order (1). It is shown that, in general, when no assumptions are made about the agents’ information set, it is impossible to discriminate between backward, forward or a weighted sum of the two solutions. I then show why it may be important. This section also discusses the solutions to the present value model equation and shows that the Campbell and Shiller restrictions do not discriminate between backward and forward-looking
types of solutions to the equation and that when our information set is the same as the agents', the restrictions actually test the null of a backward looking solution. Section 2.4 develops the proper restrictions for adaptive expectations.

2.2 Present Value Models

Present value models start with the relationship:

$$y_t = \varphi \sum_{i=0}^{\infty} E_t \delta^i x_{t+i}^e.$$  \hspace{1cm} (2.1)

where $x_{t+i}^e$ is the subjective expectations of $x_{t+i}$ in the minds of the economic agents at time $t$; $\delta$ is a time-invariant discount factor, $0 < \delta < 1$ and $\varphi$ is a parameter of the model which is positive. In the stock price present value models, $y_t$ denotes the price of a stock (or the total value of a number of stocks) at the beginning of period $t$ while $x_t$ denotes the dividend derived from the above stock(s) during period $t$, and $\varphi = \delta$. In the interest rate present value model, $y_t$ is a long-term rate at time $t$ while $x_t$ is a short-term rate at time $t$, and $\varphi = \delta - 1$. For the exchange rate present value model, as it will be shown later, $y_t$ is an exchange rate at time $t$ and $x_t$ is the fundamentals of exchange rate, $\delta = \alpha/(1+\alpha)$, where $\alpha$ is the semi-elasticity of the interest rate in money demand function and $\varphi = 1 - \delta$.

Most present value model testing concentrates on the following implication of (2.1). Using (2.1) to evaluate $x_{t+1}^e$ and subtracting $\delta y_{t+1}^e$ from (2.1) gives:

$$y_t = \delta y_{t+1}^e + \varphi x_t.$$  \hspace{1cm} (2.2)
(2.2) can be tested after the expectation mechanism of the agents is specified.

2.3 Testing Rational versus Adaptive Expectations for Present value models: Chow’s Approach

To estimate (2.2) under rational expectations (RE) as defined by Muth (1961), the subjective expectations in the minds of the economic agents are equated with the mathematical expectations generated by the econometric model used by the econometrician. $H_t$ denotes the information available to the econometrician at time $t$, which includes, at minimum, the values of the variables of the selected model up to time $t$. $H_t$ is usually assumed to be a subset of $I_t$, the information available to the economic agents at time $t$. (2.2) can be rewritten as:

$$y_t = \delta E_t y_{t+1} + \varphi x_t. \quad (2.3)$$

If $E_t$ in (2.3) originally refers to $E(\cdot | I_t)$, equation (2.3) is also valid for $E_t$ interpreted as $E(\cdot | H_t)$, since $E(E(\cdot | I_t) | H_t) = E(\cdot | H_t)$.

Assume that under RE $y_{t+1}$ can be written as:

$$y_{t+1} - E_t y_{t+1} = u_{t+1} \quad (2.4)$$

where $u$ is serially independent. Using (2.4) to replace $E_t y_{t+1}$ in (2.3) the following may be obtained:

$$y_t = \delta y_{t+1} + \varphi x_t - \delta u_{t+1}. \quad (2.5)$$

Chow (1983) reduces the time subscripts of the resulting equation by one to give:
\[ y_t = \delta^{-1} y_{t-1} - \phi \delta^{-1} x_{t-1} + u_t \]  

(2.6)

Restrictions implied by equation (2.4) are RE restrictions.

To estimate (2.1) under adaptive expectations (AE), specify the AE hypothesis as

\[ y_{t+1}^e - y_t^e = \eta (y_t - y_t^e) + \epsilon_t \]  

(2.7)

where \( \epsilon_t \) summarizes all factors other than \( (y_t - y_t^e) \) which may affect the change in expectations. \( \epsilon_t \) must be orthogonal to variables in \( H_t \) since otherwise these variables would need to be included in (2.7). The residual may include factors that affect short-term expectations.

If (2.7) is correct, the following equation may be obtained:

\[ y_t = (1-\eta)(1-\delta \eta)^{-1} y_{t-1} + \phi (1-\delta \eta)^{-1} x_t - \phi (1-\eta)(1-\delta \eta)^{-1} x_{t-1} + \delta(1-\delta \eta)^{-1} \epsilon_t \]  

(2.8)

Restrictions implied by equation (2.8) are AE restrictions. Equations (2.6) and (2.8) imply different coefficients for \( x_t \) and \( y_{t-1} \). Under RE, the coefficient on \( x_t \) is zero, because \( x_t \) is not among the regressors in (2.6), and the coefficient on \( y_{t-1} \) is greater than one (\( \delta \), the discount factor is usually greater than zero and less than one). Under AE, the coefficient on \( x_t \) is greater than zero (\( \phi \) is greater than zero and \( \delta \) and \( \eta \) are both less than one, guaranteeing that \( (1-\delta \eta)^{-1} \) is positive) and the coefficient on \( y_{t-1} \) is less than one (which is implied by the condition that \( \delta \) is less than one). However, any testing based on these restrictions provides reliable parameter estimates only if \( x_t \) and \( y_t \) are stationary (are I(0) variables), which is usually not the case.

Chow (1989) uses data on stock prices and dividends, and long-term interest rates to test (2.6) and (2.8). His finds that the implications of the present value models under RE are strongly rejected by the data. However, under adaptive expectations, the data can be explained very well. Adaptive expectations, if accepted, implies a model which is inconsistent with the RE formulation. The data support the former and reject the latter. The AE model is also capable of
explaining the observed negative relationship between the rate of return and stock price. However, in this paper adaptive expectations are accepted based only upon the rejection of a rational expectations null. Furthermore, the possibility of nonstationarity in the data is not taken into account.

2.4 Restrictions under Rational Expectations: Campbell and Shiller Technique

2.4.1 Solutions for Stochastic Expectational Linear Difference Equation

If \( y_t \) and \( x_t \) are I(1) variables, the present value model equation can be rewritten in terms of stationary (I(0)) variables. Adding and subtracting \( E_t y_{t+1} \) to the right-hand side of equation (2.4) and subtracting \( (\varphi + \delta)x_t \) from both sides of the equation will not change equation (2.3) but will allow it to be expressed in terms of stationary variables:

\[
y_t - (\varphi + \delta) x_t = \delta E_t y_{t+1} - \delta E_t x_{t+1} + \delta E_t x_{t+1} - \delta x_t
\]  

(2.9)

or:

\[
L_t = \delta E_t L_{t+1} + \delta E_t \Delta x_{t+1},
\]  

(2.10)

where \( L_t = y_t - (\varphi + \delta) x_t \)  

(2.11)

and \( \Delta \) is the first difference of the corresponding variables.

It has been shown (Campbell and Shiller (1988)) that a variable \( L_t \) is stationary. The proof of stationarity proceeds as follows: (2.10) implies:
\[ L_t = \delta \sum_{i=0}^{\infty} \delta^i E_t \Delta x_{t+i+1} \]  

(2.12)

and \( \Delta x_{t+1} \) is stationary by definition. If the expectations are rational, then \( E_t \Delta x_{t+1} \) is stationary as well. Therefore, \( L_t \) is also stationary, as it was originally stated.

There are two types of solutions to equation (2.10): forward and backward looking (Blanchard (1979), Gourieroux, Laffont and Monfort (1982), Sargent (1987)). They can be defined as follows. A forward-looking solution for \( L_t \) is sought as a linear function of the variables that \( E_t x_{t+1}, E_t x_{t+2}, \ldots, x_{t+n} (n \to \infty) \) depend on. A backward-looking solution is such that the control variable depends on the past directly, as compared with the forward-looking case where the control variable depends on the past only through its effect on the future expected values of state variable. A weighted combination of a forward and backward looking solution is a solution as well.

Originally, \( E_t \) refers to the expectation conditional of the agents information set \( I_t \). If \( H_t \), the information set consisting of \( L_t, x_t, \) and their lags, is a subset of \( I_t \) then projecting equation (2.10) onto set \( H_t \) would give:

\[ L_t = \delta E L_{t+1} \bigg| H_t + \delta E \Delta x_{t+1} \bigg| H_t \]  

(2.13)

which can be rewritten as:

\[ E L_{t+1} \bigg| H_t = 1/\delta L_t - E \Delta x_{t+1} \bigg| H_t. \]  

(2.14)

Define \( z_t \) as:

\[ z_t = [ \Delta x_t, \ldots, \Delta x_{t+p+1}, L_t, \ldots, L_{t+p+1} ]' \]  

(2.15)

In general, \( L_t \) and \( \Delta x_t \) can be written as a function of the variables from \( z_t \) and an error term

\[ L_t = k_0 \Delta x_t + k_{11} \Delta x_{t+1} + k_{12} L_{t+1} + k_2 \Delta z_{t+2} + u_t \]  

(2.16)

\[ \Delta x_t = c_{11} \Delta x_{t+1} + c_{12} L_{t+1} + c_2 \Delta z_{t+2} + \varepsilon_t \]  

(2.17)
where $E_t (u_t \mid H_{t-1}) = 0$ and $E_t (e_t \mid H_{t-1}) = 0$. Equations (2.10) and (2.17) imply certain restrictions on the coefficients in equation (2.16). Substituting (2.16) and (2.17) into (2.10) gives:

$$L_t = \delta \left( k_0 \left( c_{11} \Delta x_t + c_{12} \Delta I_t + c_2 z_{t-1} \right) + k_{11} \Delta x_t + k_{12} I_t + k_2 z_{t-1} \right)$$

$$+ \delta \left( c_{11} \Delta x_t + c_{12} I_t + c_2 z_{t-1} \right)$$

(2.18)

There are two possible scenarios for which restrictions (2.18) will hold. First, when $L_t$ is an exact function of other variables in $z_t$, and second, when the sum of the coefficients for each variable in (2.18) is equal to zero. $L_t$ would be an exact linear function of the other variables in $H_t$ when $H_t$ is equal $I_t$, and the expectations are forward-looking. Rearranging (2.18) will give:

$$L_t = \{(\delta k_0 c_{11} + \delta k_{11} + \delta c_{11}) \Delta x_t + (\delta k_0 c_2 + \delta k_2 + \delta c_2) z_{t-1} \}/(1 - \delta k_0 c_{12} - \delta k_{12} - \delta c_{12})$$

(2.19)

In the case where $H_t \neq I_t$, $L_t$ will no longer be an exact linear function of the variables in $H_t$. In this case, the only way for equation (2.18) to hold is for the following to be true: there should be $2n$ (where $n$ is the number of lags in equation (2.15)), cross equation restrictions on the equations (2.15) and (2.16):

$$(k_0 + 1) c_{11} = - k_{11}; \quad 1/\delta + (k_0 + 1) c_{12} = - k_{12} ; \quad (k_0 + c_2) = - k_2 ;$$

(2.20)

There is no way to know if these restrictions are the results of the forward-looking rational expectations restrictions based on some unobservable variable, or the results of a pure backward looking solution, or some mixture of the two. Therefore, testing restrictions (2.20) is testing the hypothesis that expectations are rational and the agents’ information set is different from the researcher’s but not testing a specific hypothesis of forward-looking expectations.
From (2.18) it can be seen that for any specific process $\Delta x_t$, there could be an infinite number of different processes for $H_t$, depending upon which particular solution we want to consider and what the agents' information set is.

The restrictions implied by (2.20) can be written differently. Projecting the right hand side of equation (2.20) onto $H_{t-1}$, yields:

$$L_4 = k_0(c_{11} \Delta x_{t-1} + c_{12} L_{t-1} + c_2 z_{t-2}) + k_{11} \Delta x_{t-1} + k_{12} L_{t-1} + k_2 z_{t-2} + (e_t + k_0 u_t)$$

or:

$$L_4 = (k_0 c_{11} + k_{11}) \Delta x_{t-1} + (k_0 c_{12} + k_{12}) L_{t-1} + (k_0 c_2 + k_2 z_{t-2} + (e_t + k_0 u_t)$$

Denote $(k_0 c_{11} + k_{11})$ as $a_{11}$, $(k_0 c_{12} + k_{12})$ as $a_{12}$, etc. Equation (2.22) can be rewritten as:

$$L_4 = a_{11} \Delta x_{t-1} + a_{12} L_{t-1} + a_2 z_{t-2} + w_t$$

From (2.23):

$$E_{t-1} L_4 = a_{11} \Delta x_{t-1} + a_{12} L_{t-1} + a_2 z_{t-2}$$

Thus, in the case where $H_t$ is strictly less then $I_t$, equation (2.24) implies the following restrictions:

$$c_{11} = -a_{11}; \quad c_{12} + 1/\delta = -a_{11}; \quad c_2 = -a_2$$

Restrictions (2.25) will hold for any rational expectations solution where $I_t \neq H_t$.

### 2.4.2 Campbell and Shiller Approach

Campbell and Shiller start with the present value model equation (2.10), define $L_t$, $\Delta x_t$, and $z_t$ as in (2.11) and (2.15). They then show that by the multivariate form of Wold's decomposition (Hannan (1970)), there exists a Wold representation that may be approximated by a vector autoregression (VAR) of lag length $p$. A qth-order error-correction model gives a pth-
order VAR, which defines a new vector \( z_t = [\Delta x_t, \ldots, \Delta x_{t-p+1}, L_t, \ldots, L_{t+p-1}]' \) which follows a vector autoregression (VAR) process. Further, assume that \( z_t \) follows a VAR (p) process \( z_t = E(z_t | H_{t-1}) + \nu_t \):

\[
z_t = Az_{t-1} + \nu_t
\]

where \( A \) is a companion matrix of the VAR and has the form:

\[
A = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1p} \\
1 & 0 & \cdots & 0 \\
a_{21} & a_{22} & \cdots & a_{2p} \\
0 & 1 & \cdots & 0 \\
\end{bmatrix}
\]

(2.27)

The multi-period forecasting formula for \( z_{t+k} \) is:

\[
E(z_{t+k} | H_t) = A^k z_t.
\]

(2.28)

As a next step, Campbell and Shiller show that forward looking solution implies restrictions on matrix \( A \). To show this, first define \( g' \) to be a row vector which picks out \( L_t \) from the vector \( z_t \):

\[
L_t = g'z_t
\]

(2.29)

(The vector \( g' \) has all elements zero except the (p+1)st element, which is unity.) A row vector \( h' \) is defined similarly, as a row vector that picks out \( \Delta y_t \):

\[
x_t = h'z_t.
\]

(2.30)

Substituting (2.29-2.30) into the present value model equation (2.10) would yield:

\[
g'z_t = \delta \sum_{i=0}^{\infty} \delta^i E_t h'z_{t+1+i}
\]

(2.31)

Further, substituting (2.28) into (2.31) gives:

\[
g'z_t = \delta \sum_{i=0}^{\infty} \delta^i h' A^i z_{t+1+i}
\]

(2.32)

which is equivalent to:
\[ g'z_t = \delta' A (I-\delta A)^{-1} z_t \]  

which can be rewritten as:

\[ c_{11} = -a_{11}; \quad -c_{12} + 1/\delta = -a_{11}; \quad -c_2' = -a_2' \]  

yielding restrictions identical to (2.25) from above. Restrictions (2.34) are obtained by imposing forward-looking restrictions on the present value model equation. However, they are identical to the restrictions for any rational expectation solution. Thus, the restrictions not only test the finite number of forward-looking solutions, but also the continuum of combinations of forward and backward looking solutions to the present model equation (2.10).

It is straightforward to show that, if the information set \( I_t \) is equal to the information set \( H_t \), it is impossible to apply Campbell and Shiller restrictions to the forward-looking expectations solution. The reason is that, in this case, the variable \( L_t \) is an exact linear function of the variables in \( z_t \). Therefore, the test cannot be applied to forward-looking solution due to multicollinearity.

### 2.4.3 Why the Type of the Solution is Important

It is important to discriminate between backward and forward-looking solutions when one wants to draw conclusions about the implications of economics models. The usual assumption of economics models is that agents' behavior depends on their views of future movements of the relevant variables. In some models, it is impossible to predict the short- or long-run effects of changes in macroeconomic variables without assumptions about how expectations are formed.

Forward looking behavior is consistent with the case where the agents evaluate \( L_t \) based on their expectations of the future movements of \( \Delta x_t \). Backward looking evaluation of \( L_t \)
depends on all the past movements in $\Delta x_t$. A proper-weighted combination of forward and backward looking solutions makes agents evaluate $L_t$ based on all the future and past movements in $\Delta x_t$. Although the paths for $L_t$, and the way $L_t$ reacts to the future and past shocks in $\Delta x$ are different, the VAR restrictions are nevertheless the same.

As is shown above, when the researcher does not know the information set of the agents, a VAR representation cannot discriminate between different types of rational expectations. They could be forward looking, backward looking, or sun-spot types of solution. In order to rule out, for example, backward looking solutions, some assumptions about the variables in $I_t$ have to be formed. (For example, if all variables in $I_t$ do not depend on the lagged values of $L_t$, then the backward looking solution is unstable.) However, without these assumptions, none of the solutions can be excluded as impossible.

Blanchard (1979) shows that for the model where $x_t$ is the log of nominal money supply, $L_t$ is the log of price level, and expectations are forward-looking, the model implies that the real money stock stays unchanged. The backward looking solution, however, leaves the price level unchanged at the time of the shock. In the current analysis it can be seen from equation (2.16): $L_t = k_0 \Delta x_t + k_{11} \Delta x_{t-1} + k_{12} L_{t-1} + k_2 z_{t-2} + u_t$. For a pure backward looking solution $k_0$ is equal to zero, therefore a shock in $\Delta x$ at time $t$ will not affect the values of $L$ at time $t$. Therefore, there will be a delayed reaction in the price level and implications of the monetary policy will be different. However for a forward looking solution, $k_0$ will be different from zero, and therefore the shock in $\Delta x_t$ will affect $L_t$.

There have been a variety of papers that have shown that short-run dynamics for backward and forward-looking solutions differ substantially. For example, Yavas (1993)
analyzes the money demand function and shows different implications for the conduct of monetary policy for backward and forward-looking approaches.

Model (2.10) is often applied to the permanent income hypothesis. If restrictions (2.15) are not rejected, it is concluded that a positive unexpected shocks in income increase consumption. However, as can be seen in Figures 1 and 2, this is not necessarily the case. Figure 1 and 2 show how two different solutions to equation (2.10) respond to an unexpected increase in fundamentals. Suppose at time 1 the fundamentals and the control variable are in equilibrium (denoted as zero on the graphs). At time 2 there is an unexpected shock to fundamentals. Figure 1, 2 and 3 show the response of the fundamentals and the control variable to the shock in fundamentals. As can be seen from the graphs, a positive shock to fundamentals can either increase (Figure 1) or decrease (Figure 2) the equilibrium value of the fundamentals and the control variable. Figure 3 shows the case of a sun-spot solution.

Restrictions (2.25) are used in the literature to test forward-looking restrictions, and usually, in case of non rejection, it is concluded that reality is consistent with the model and forward-looking expectations. For example, Sheffrin and Woo use the Campbell and Shiller restrictions to show, that in the case where the restrictions are not rejected, it means that the reality is consistent with the implication of the Ricardian Equivalence Principle. The Principle states that, holding the path of government spending constant, changes on the timing of taxes will not change consumption or the current account. However, as can be seen from above, it is not necessarily true if expectations are rational but not forward looking.

In another example of a VAR approach to testing a macroeconomic model, Ghosh (1995) applies the Campbell and Shiller restrictions to the current account model, where the current account deficit is the negative present discounted value of future net output changes. The author concludes that when restrictions (3.29) cannot be rejected, it is consistent with an expected
positive shock to the future output improvements in the current account. As can be seen above, a
positive expected shock to the future output might actually cause the current account deficit rise,
and it will also be consistent with restrictions (3.29).

Usually some other requirements are imposed to ensure that only a forward-looking
solution can exist. Among those requirements usually are: expectational stability, stationarity,
minimum variance and consistency with the economic behavior. However, if the agents’
information set is not known, it is impossible to check if a solution satisfies the first three of the
above criteria. The argument about consistency with economic behavior can work for both
solutions as well. Except for the pure backward looking solution, both the current and the
expected value of the control variable move in response to an expected change in a state variable.
Therefore, the existence of both forward and backward looking solutions can be justified by
theory.

2.5 Restrictions Under AE if y_t and x_t are I(1) Variables

By applying AE hypothesis (2.7) to present value formula in form of (2.13), the
following equation can be gotten:

\[ y_t - x_t = (1-\eta)(1-\delta\eta)^{-1}(y_{t-1} - x_{t-1}) \cdot \delta(1-\eta)(1-\delta\eta)^{-1}(x_t - x_{t-1}) + \delta(1-\delta\eta)^{-1}e_t \]  (2.35)

or, replacing \((y_t - x_t)\) with \(L_t\):

\[ L_t = -\delta(1-\eta)(1-\delta\eta)^{-1} \Delta x_t + (1-\eta)(1-\delta\eta)^{-1} L_{t-1} + \delta(1-\delta\eta)^{-1} e_t \]  (2.36)

\(L_t\) and \(\Delta x_t\) may be written as:

\[ \begin{align*}
L_t & = \sum_{j=0}^{\infty} \delta^j \Delta x_t + \sum_{j=0}^{\infty} \delta^j L_{t-1} + e_t \\
\Delta x_t & = \sum_{j=0}^{\infty} (1-\delta^j) \Delta x_{t+j} \\
\end{align*} \]
\[ L_t = (\text{Proj } L_t \mid H_{t-1}) + u_t \text{ and } \Delta x_t = (\text{Proj } \Delta x_t \mid H_{t-1}) + \varepsilon_t. \]

As it has been shown in the previous section, it will imply that there exist a vector autoregression (VAR) process (2.26). Thus, adaptive expectations restrictions on VAR process (2.26) can be derived.

Substituting (2.36) into (2.26) and taking (2.29) into account will yield:

\[ g'z_t = -\delta(1-\eta)(1-\delta\eta)^{-1}h^*z_t + (1-\eta)(1-\delta\eta)^{-1}g'z_{t-1} + \delta(1-\delta\eta)^{-1}\varepsilon_t \] (2.37)

or:

\[ (g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)z_t = (1-\eta)(1-\delta\eta)^{-1}g'z_{t-1} + \delta(1-\delta\eta)^{-1}\varepsilon_t \] (2.38)

Multiplying (2.38) by \((g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)\) will give:

\[ (g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)z_t = (g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)A z_{t-1} + (g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)\varepsilon_t. \] (2.39)

Equation (2.39) implies the following equality:

\[ (1-\eta)(1-\delta\eta)^{-1}g' = (g' + \delta(1-\eta)(1-\delta\eta)^{-1}h^*)A. \] (2.40)

Then the restrictions on the variance-covariance matrix would mean that \(\text{Cov}(u_t, \varepsilon_t)\) is equal to \(-\delta(1-\eta)(1-\delta\eta)^{-1}\). Notice, that for naïve expectations \((\eta = 1)\), the restrictions will reduce to

\[ g'A = 0, \] (2.41)

with no restrictions on the covariance matrix.
CHAPTER 3

PRESENT VALUE MODELS AND EXCHANGE RATE DYNAMICS

3.1 Introduction

The purpose of the chapter is to show how present value models can be applied to the nominal exchange rate. First, Section 3.2 describes how the application of the monetary model to the exchange rate results in the present value model equation and describes the exchange rate fundamentals. The monetary approach to the exchange rate implies that the purchasing power parity holds. Section 3.3 shows how the conclusions of the model will change if this assumption is violated. The case of no structural breaks in the underlying variables (Section 3.4) and the case with structural breaks (Section 3.5) are considered. The results of the previous empirical work and its shortcomings are given in Section 3.6.

3.2 Application of Monetary Approach to the Exchange Rate

For the current research, the flexible monetary model has been chosen to describe the exchange rate dynamics. It seems that the assumptions of the flexible monetary model describe
reality better. Moreover, the exchange rate forecasting based on the flexible model outperform the forecasting based on the sticky monetary model and portfolio balance model.

The monetary approach to the exchange rate is based on the assumption that the money market consists of the money demand and money supply relationship in the home and foreign countries. The money demand function is given by:

\[ m_t^d - p_t = a_0 - a_1 y_t - \alpha i_t, \quad \alpha > 0, \ a_1 < 0 \]

\[ m_t^{d*} - p_t^* = a_0^* - a_1 y_t^* - \alpha i_t^*, \quad (3.1) \]

where \( m_t \) and \( m_t^{d*} \), \( y_t \) and \( y_t^* \) are the natural logarithms of money supply and the level of real national income in the home and foreign countries at time \( t \) respectively, \( i_t \) and \( i_t^* \) are home and foreign interest rates at time \( t \), \( \alpha \) is the semielasticity of the interest rate, and \( a_1 \) is the income elasticity of the demand for money. It has been assumed, for simplicity, that the income elasticity and interest rate semielasticity are equal across countries. The money markets are assumed to be continuously equilibrated; thus:

\[ m_t^d = m_t^* = m_t \]

\[ m_t^{d*} = m_t^{d*} = m_t^* \]

(3.2)

By substituting (3.2) into (3.1), subtracting the resulting foreign money market relationship from the domestic expression and solving for the relative price level we obtain:

\[ p_t - p_t^* = -(a_0 - a_0^*) + (m_t - m_t^*) + a_1 (y_t - y_t^*) + \alpha (i_t - i_t^*) \]

(3.3)

or, assuming that the purchasing power parity (PPP) holds:

\[ s_t = p_t - p_t^*, \quad (3.4) \]

and therefore:

\[ s_t = -(a_0 - a_0^*) + (m_t - m_t^*) + a_1 (y_t - y_t^*) + \alpha (i_t - i_t^*). \]

(3.5)
As it has been shown in previous chapters, the conventional assumption that \( s_t, m_t, m_t^*, y_t \) and \( y_t^* \) are all integrated of order (1) and thus are \( I(1) \) variables, and \((i_t - i_t^*)\) is a \( I(0) \) variable, would imply that there exists a cointegrating relationship between \( s_t, m_t, m_t^*, y_t \) and \( y_t^* \). Denote \( x_t \) as:

\[
x_t = - (a_0 - a_0^*) + (m_t - m_t^*) + a_1 (y_t - y_t^*)
\]

(3.6) implies that \( s_t \) and \( x_t \) are cointegrated with the cointegrating coefficient vector equal to \( \{1; -1\} \).

If the monies are non-substitutable, the bonds are perfect substitutes, and the asset-holders can adjust their portfolios instantly after a disturbance, then capital is perfectly mobile. Therefore, uncovered interest rate parity holds:

\[
\Delta s_{t+1}^e = (i_t - i_t^*)
\]

(3.7) where \( \Delta s_{t+1}^e \) denotes the (logarithmic) expected change in the exchange rate, one period ahead.

Therefore (3.6) can be rewritten as:

\[
s_t = x_t + \alpha \Delta s_{t+1}^e
\]

(3.8)

Collecting all the terms containing \( s_t \) on the left side and taking into account that:

\[
\Delta s_{t+1}^e = (s_{t+1}^e - s_t),
\]

(3.9)

(3.8) can be rewritten as:

\[
s_t = x_t/(1+\alpha) + \alpha/(1+\alpha) \ s_{t+1}^e.
\]

(3.10)

(3.10) is analogous to a present model equation (2.2) described in section (2.2). Therefore, equation (3.10) can be tested using the present value model technique described in sections (2.2) and (2.3).
3.3 Purchasing Power Parity and Exchange Rate Dynamics

One of the assumptions of the current model is that purchasing power parity (PPP) holds in the form of equation (3.4): \( s_t = p_t - p_t^* \). However, there is some evidence in the literature (Hamilton, (1994)), that PPP holds rather in the form:

\[
s_t = d^* p_t - g^* p_t^*, \tag{3.11}
\]

where \( d \) and \( g \) are some positive coefficients. The formulation of PPP in (3.11) stems from the fact that, while the exchange rate is still the ratio of prices in home and foreign countries, \( S_t = \frac{P_t}{P_t^*} \), these prices \( P_t \) and \( P_t^* \) cannot be measured. What can be measured are \( P_t \) and \( P_t^* \), where:

\[
P_t = (P_t^*)^{l/d} \quad \text{and} \quad P_t^* = (P_t^*)^{l/g} \tag{3.12}
\]

Then the relationship between \( S_t \), \( P_t \), and \( P_t^* \) is:

\[
S_t = \frac{(P_t)^d}{(P_t^*)^g} \tag{3.13}
\]

and, after taking logs of (3.13), equation (3.11) can be obtained.

If assumption (3.11) is true and the money demand functions for home and foreign countries can be described by equation (3.1), then the exchange rate between these two countries can be written as:

\[
s_t = - \left( d a_0 - g a_0^* \right) + d m_t - g m_t^* + d a_1 y_t - g a_1 y_t^* + \alpha (d-g) i_t^* + \alpha d (i_t - i_t^*), \tag{3.14}
\]

or, assuming that uncovered interest rate parity (3.7) holds and replacing \((i_t - i_t^*)\) with \(\Delta S_{t+1}^c\), (3.14) can be rewritten as:

\[
s_t = - \left( d a_0 - g a_0^* \right) + d m_t - g m_t^* + d a_1 y_t - g a_1 y_t^* + \alpha (d-g) i_t^* + \alpha d \Delta S_{t+1}^c. \tag{3.15}
\]
There has been evidence in the literature that $m_t$, $m_t^*$, $y_t$, $y_t^*$ and $i_t^*$ are I(1) variables. $\Delta s_{t+1}$ is assumed to be stationary. Then for the case where $d \neq g$, the model implies a cointegrating relationship between the variables $s_t$, $m_t$, $m_t^*$, $y_t$, $y_t^*$ and $i_t^*$ with a cointegrating vector: $\{1, -d, -g, da_t, ga_t, \alpha(d-g)\}$. The variables $s_t$, $m_t$, $m_t^*$, $y_t$, $y_t^*$ and $i_t^*$ are the fundamentals $x_t$ of the exchange rate as well. (3.15) can be rewritten as:

$$s_t = x_t + \alpha d \Delta s_{t+1}.$$

(3.16)

where

$$x_t = d m_t - g m_t^* + d a_t y_t - g a_t y_t^* + \alpha (d-g) i_t^*.$$

(3.17)

(3.16) is analogous to (3.8), and therefore rational and adaptive expectations can be tested against more general VAR representation in the same way as described in the previous section for the case when PPP holds in the form of (3.4). A superconsistent estimate for $d$ can be found from the cointegrating relationship between $s_t$, $m_t$, $m_t^*$, $y_t$, $y_t^*$ and $i_t^*$.

If coefficients $d$ and $g$ are the same, $d = g$, then equation (4.56) can be written as:

$$s_t = -d(a_0 - a_0^*) + d (m_t - m_t^*) + d a_t (y_t - y_t^*) + \alpha d \Delta s_{t+1}.$$

(3.18)

and will imply that there exists a cointegrating relationship between $s_t$, $(m_t - m_t^*)$ and $(y_t - y_t^*)$ with a cointegrating vector: $\{1, -d, da_t\}$. The fundamentals of the exchange rate are the same as the case where PPP holds in the form (3.4). The only difference is that the coefficient on $(m_t - m_t^*)$ is not equal to one.

Pilbeam (1995) shows that including $i_t$ or $i_t^*$ (or both) as fundamentals for the exchange rate does not help the exchange rate forecasting, but, on the contrary, makes it worse. Therefore, for the rest of the paper the fundamentals of the exchange rate are assumed to be $(m_t - m_t^*)$ and $(y_t - y_t^*)$. 

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3.4 Case with no Structural Breaks

Equation (3.9) can be rewritten in a form:

\[ s_t = (\alpha / (1+\alpha)) E_t s_{t-1} + (1/(1+\alpha)) x_t, \]  
(3.19)

where \( E_t \) is a conditional expectation given the information \( I_t \) available to the economic agents at time \( t \), \( \alpha \) is the interest rate semi-elasticity of the money demand function, \( s_t \) and \( x_t \) are logarithmic values of the level terms of the exchange-rate and the fundamentals and \( x_t \) is defined as in (3.6). Equation (3.19) is analogous to any present value models equation in the form of (2.1). Therefore, the present value models expectations restrictions can be applied to the exchange rate model, where \( L_t \) is now defined as \( L_t = s_t - x_t \), and \( x_t \) are the fundamentals. Notice, that RE present value model restrictions also imply that \( \Delta s_t \) follows the process:

\[ \Delta s_t = (1/\alpha)(s_{t-1} - x_{t-1}) + u_t, \]  
(3.20)

or, replacing \( s_{t-1} - x_{t-1} \) with \( \alpha \Delta s_t^e \),

\[ \Delta s_t = \Delta s_t^e + u_t, \]  
(3.21)

as predicted by RE.

The restrictions implied by AE may also be applied for testing AE against more general VAR specification. These restrictions would imply the following process for \( \Delta s_t \):

\[ \Delta s_t = \{1/(1+\alpha-\alpha \eta)\} \Delta x_t - \{\eta/(1+\alpha-\alpha \eta)\}(s_{t-1} - x_{t-1}) + \alpha/(1+\alpha-\alpha \eta)e_t, \]  
(3.22)

or, replacing \( s_{t-1} - x_{t-1} \) with \( \alpha \Delta s_t^e \),

\[ \Delta s_t = \{1/(1+\alpha-\alpha \eta)\} \Delta x_t - \{\eta \alpha(1+\alpha-\alpha \eta)\} \Delta s_t^e + \alpha/(1+\alpha-\alpha \eta)e_t. \]  
(3.23)
From equation (3.23), it can be seen that if the expectations are indeed adaptive, then it helps to explains the negative relationship between \( \Delta s_t \) and \( \Delta s_t^c \), the usual result of standard “Fama” regression.

3.5 Case with Structural Breaks

The present value model tests assume that there are no structural breaks in the relationship between the exchange rate and fundamentals. However it does not necessarily have to be true. According to the monetary theory there are two possible sources for a structural break in the exchange rate. One possible source of structural changes in the exchange rate could be the change in people’s demand for one currency in terms of another. It will result in the structural shift in the money demand function equation, and, therefore, in the cointegrating relationship between the exchange rate and fundamentals, no matter what the mechanism for expectations formation is. As a result of the shift in the cointegrating relationship, the coefficients in (3.6) will change. Another possible source of structural breaks is changes in expectations and/or process that governs the fundamentals. Changes in expectations and/or fundamentals would affect not the cointegrating relationship, but the error term in the cointegrating relationship.

First source of structural breaks between the exchange rate and fundamentals is a shift in the cointegration relationship between the exchange rate and fundamentals (3.6), which results in a different set of coefficients for (3.6). The question of whether it is possible for a cointegrating vector to undergo structural changes is still an open question in the literature. Some researchers argue that cointegrating relationship describes a long run equilibrium, and is, therefore, stable.
They deny the possibility of structural breaks and instead argue that we just do not have enough data to estimate this long term relationship.

Although the question of whether a long term equilibrium exists is rather philosophical, it is still possible to investigate what defines the long run equilibrium and what can change it. According to monetary theory, the cointegrating relationship between exchange rates and fundamentals arises because there exists a stable (at least for some period) money demand function. Therefore, if there exists a stable money demand which is a function of some measurable variables, then there exists a long run cointegrating relationship between the exchange rate and countries differences in nominal money plus these variables, which are the fundamentals of the exchange rate. Any change in this money demand function will automatically result in a shift of the cointegrating relationship between the exchange rate and fundamentals.

The long run equilibrium is very common for natural sciences such as chemistry and physics. For example, for a chemical reaction \( A + B \rightleftharpoons C \) under specified conditions, it is possible to predict the long run equilibrium amounts of A, B and C. These equilibrium amounts stay the same because the optimization principle for any given molecule does not change: each molecule "prefers" to be at the lowest energy level. Compared to molecules people can change their preferences because of some political events, fashions, etc. Therefore, changes in money demand function may be due to a change in peoples’ preferences.

Another reason that the demand for money might undergo changes is that some variables explaining the demand for money are difficult to measure, and are therefore not included in the money demand equation. For example, the flexible monetary model assumes that the demand for real money is a function of real output and the nominal interest rate. The well-known identity states that money divided by price is equal to real output divided by income velocity. Therefore,
the flexible monetary model implicitly assumes that velocity is a function of only real output and the nominal interest rate. However velocity may also be a function of transaction costs of getting cash, and the transaction costs in turn can be a function of real output (for example, the higher developed society can have better technology which lowers transaction costs of receiving cash from the banking accounts etc). Thus, if the transaction costs are not included in the money demand equation, it is possible that the flexible monetary demand function will undergo shifts when there are velocity shocks.

A second source of changes in the cointegrating relationship can be the change in the way the data are collected. Suppose that the money demand function includes all the relevant variables, but some of the variables are measured with an error. Write \( M/P = k Y^n \). If instead of \( Y \) we have data for \( Y + a \), then the money demand function for \( Y + a \) is: \( M/P = k Y^m \), where \( m = n \ln(Y)/\ln(Y + a) \). If \( a \) is large enough, then the elasticity of measured output with respect to \( M/P \) will change when the measured output changes from \( Y + a \) to \( Y \). Therefore, changing the way the data are collected will result in a structural shift in money demand function, in which the variables are measured in logs.

The complexity of the question of whether the long run relationship are stable in general makes it interesting to at least look at the possibility of structural break in the cointegrating relationship between the exchange rate and fundamentals. Another reason is that the rational expectations monetary approach cannot reconcile the exchange rate dynamics with the data. This approach is one of the most popular approaches to describe the exchange rates movements, and the ability to show that it is consistent with the data is very important.

The second source of structural breaks in the exchange rates comes from the structural breaks in policies and expectations. However, as it was shown earlier in this chapter, it will not
result in the change of the cointegrating relationship and only change the short run dynamics of
the exchange rate. Therefore, the second type of breaks will result in structural shifts in the VAR
(2.26), formed with 2 variables: changes in the fundamentals $\Delta x_t$ and the cointegrating error $L_t$.

If the expectations are rational, then, as has been shown in Chapter 2.3, the following
restrictions on $L_t$ that relate the path of $L_t$ to the process that governs fundamentals are implied:

$$L_t = -\bar{E}_{t-1} \Delta x_t + (1/\alpha + 1) L_{t-1} + u_t,$$

(3.24)

where $L_t$ is equal to $(s_t - x_t + a)$ and $E_t$ in (3.24) refers to $E(\bullet | H_t)$. $H_t$ denotes the information
available to the econometrician at time $t$, which includes, at minimum, the values of the variables
of the selected model up to time $t$. Thus, the following restrictions are implied:

$$\Delta x_t = \delta_1 \Delta x_{t-1} + \kappa_1 L_{t-1} + \ldots + \delta_2 \Delta x_{t-k} + \kappa_2 L_{t-k} + \epsilon_{1t},$$

$$L_t = -(\delta_1 \Delta x_{t-1} + \kappa_1 L_{t-1} + \ldots + \delta_2 \Delta x_{t-k} + \kappa_2 L_{t-k}) + (1/\alpha + 1) L_{t-1} + \epsilon_{2t},$$

(3.25)

If the expectations are adaptive in the form of:

$$E_t (s_{t+1} - E_{t+1} s_t) = \eta (s_t - E_{t-1} s_t) + \epsilon_t,$$

denoting $(s_t - x_t + a)$ as $L_t$, the restrictions for adaptive expectations developed in Chapter 2.4 can
be rewritten as:

$$\Delta x_t = \delta_1 \Delta x_{t-1} + \kappa_1 L_{t-1} + \ldots + \delta_2 \Delta x_{t-k} + \kappa_2 L_{t-k} + \epsilon_{1t},$$

$$L_t = -\alpha(1-\eta)/(1+\alpha-\alpha\eta)(\delta_1 \Delta x_{t-1} + \kappa_1 L_{t-1} + \ldots + \delta_2 \Delta x_{t-k} + \kappa_2 L_{t-k}) + (1+\alpha)(1-\eta)/(1+\alpha-\alpha\eta)L_{t-1} + \epsilon_{2t},$$

(3.26)

From (3.26) it can be seen, that in the case of adaptive expectations, provided that $\alpha$ and $\eta$ do not
change, the structural break in $L_t$ will correspond to a shift in the process that governs
fundamentals.

To summarize, the main implications of present value models to the exchange rate,
structural breaks in the cointegrating relationship will correspond to the shifts of the money
demand function. The structural breaks in the VAR (2.26) will correspond to changes in how people form their expectations about the future exchange rates. The expected exchange rate is a discounted expected sum of the future changes in the fundamentals. By matching the changes in the expected value of the exchange rate to the changes in fundamentals, I am going to learn how different expectations formations can explain the data.

3.6 MacDonald’s and Taylor’s Approach: Importance of Specifying the Semielasticity of Interest Rate in Money Demand Function $\alpha$

The restrictions for rational expectations derived above are identical to the restrictions obtained by MacDonald and Taylor (1993) (labeled (16) in their paper). They tested these restrictions against more general VAR representation, using Wald statistics to test for rational restrictions. In order to be able to use the Wald test for either rational or adaptive expectations in this framework, it is necessary to “know” (or assume to know) $\alpha$, the semielasticity of the interest rate in the money demand function. MacDonald and Taylor try to determine $\alpha$ “from the cointegration parameter in the basic flexible price equation $s_t = (m_t - m^*) - a_1 (y_t - y^*) + \alpha(i_t - i^*)$” (p.92). However, the cointegration equation in this case cannot include $i_t$ because the model requires $i_t$ to be I(0). It is still possible to find an estimator for $\alpha$, but it will no longer be super-consistent and, therefore, cannot be used in non-stochastic restrictions, as in MacDonald and Taylor’s work. Substituting the estimator for $\alpha$ into the present value model restrictions will result in stochastic restrictions.
MacDonald and Taylor also propose to use extraneous estimates of $\alpha$ ($\lambda$ in their notation) from the literature. The estimator for $\alpha$ is found for interest rate data written in the form $i_t \cdot 100$, where $i_t$ is a net interest rate, as opposed to its percentage form. The net interest rate is defined as an interest rate written in its decimal and not in its percentage form. An example of a net interest rate would be, for example, 0.07 if the interest rate is 7%. In order to test a monetary theory, it is crucial to have a semielasticity only for net interest rate. It is important because in this case uncovered interest parity (UIP) is used to substitute for $i_t$:

$$1 + i_t = (1 + i_t^*) \cdot \frac{S^e_{t+1}}{S_t}$$  \hspace{1cm} (3.27)

where $S_t$ is the current spot exchange rate and $S^e_{t+1}$ is the spot exchange rate expected to prevail in period $t + k$. Equation (2.40) can be rearranged as

$$i_t = i_t^* + \frac{(S^e_{t+1} - S_t)}{S_t}$$  \hspace{1cm} (3.28)

or as

$$i_t - i_t^* = \Delta s^e_{t+1}$$  \hspace{1cm} (3.29)

where $\Delta s^e_{t+1}$ is $\ln S^e_{t+1} - \ln S_t$.

According to monetary theory:

$$\ln M_t - \ln M_t^* = \alpha_t (\ln Y_t - \ln Y_t^*) + \alpha (i_t - i_t^*)$$  \hspace{1cm} (3.30)

where $M_t$ and $M_t^*$, $Y_t$ and $Y_t^*$, and $i_t$ and $i_t^*$ are the money supply, the level of real national income and nominal interest rate in the home and foreign countries at time $t$ respectively. $\alpha_t$ is the income elasticity of the demand for money and $\alpha$ is the interest rate semielasticity of the money demand function. It can be seen that the home and foreign money supply and (or) the real
income can be multiplied by the same factors, for example d, and it would not change the relationship (3.30):

\[
\ln (d \cdot M_t) - \ln(d \cdot M_t^*) = \ln M_t - \ln M_t^* + \ln(d) - \ln(d) = \ln M_t - \ln M_t^* \tag{3.31}
\]

However, because the interest rates are not expressed in logs, multiplying the interest by some factor d will change equation (3.30):

\[
\alpha (d \cdot i_t - d \cdot i_t^*) = \alpha \cdot d \cdot (i_t - i_t^*) \tag{3.32}
\]

Therefore, if trying to estimate \( \alpha \) using net interest multiplied by 100, the resulting estimator will not be the estimate for \( \alpha \), but rather for \( (\alpha/100) \). If this estimator is used as superconsistent for testing rational restrictions, it will not be surprising if the null hypothesis is incorrectly rejected.

In order to avoid this problem of imposing artificial semielasticity of the interest rates, these parameters are instead estimated as part of the model. For both types of expectations, the restrictions will be overidentified for a lag equal to or greater than one. The number of overidentifying restrictions will be equal to the number of lags (p) minus one. Thus, under the null hypothesis, the test asymptotically has a \( \chi^2 \) distribution with degrees of freedom equal to (p-1).

Thus, it is possible to decide on the expectations mechanism using the results of the LR tests and the plausibility of the estimated values of \( \alpha \) and \( \eta \). This approach helps deal with nonstationarities of \( x_t \) and \( s_t \) without presupposing one kind of expectations formation to be the right one.
CHAPTER 4

ECONOMETRIC METHODOLOGY

4.1 Introduction

The purpose of this chapter is to describe the econometric methodology that has been used for the current research and explain why one technique is chosen in favor of another. The chapter proceeds as follows. First, the tests for units roots are described (Section 4.2). The present value models techniques developed and applied in this research are designed for I(1) variables, therefore before applying the present value models tests, the unit root tests are performed. Present value models applied to the exchange rate imply that a cointegrating relationship exists between the exchange rate and the fundamentals. The vector error correction model (VECM) and estimation of a cointegrating relationship are discussed in Section 4.3. In Section 4.4 I discuss testing the unitary income elasticity hypothesis using the cointegrating estimation results. The particular present value hypothesis is tested against a more general VAR representation, therefore a technique used for estimating an unrestricted VAR based on the VECM variables is discussed in Section 4.5. Present value model restrictions are discussed in 4.6. Section 4.7 deals with maximum likelihood estimation of the restricted and unrestricted VAR’s. The technique used for maximum likelihood estimation is discussed in 4.8. Section 4.9 describes the likelihood ratio test in the case of both rational and adaptive expectations in the

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context of present value models. Section 4.10 gives the overview of the tests for structural breaks and explains why CUSUM and likelihood ratio tests have been chosen for testing the current problem. A brief description of CUSUM and likelihood-ratio-like tests is given in Sections 4.11 and 4.12 correspondingly.

4.2 Tests for Unit Root

4.2.1 Dickey-Fuller t Test

Before applying the VAR restrictions to the data, it is necessary to test if the exchange rate and fundamentals are I(1) variables. A Dickey-Fuller test and the t-test modification suggested by Phillips (1987) has been used for this purpose.

Suppose a variable, $Y_t$, can be described by the following equation:

$$Y_t = \alpha + \beta t + \rho Y_{t-1} + \lambda_1 \Delta Y_{t-1} + \lambda_2 \Delta Y_{t-2} + \ldots + \lambda_k \Delta Y_{t-k} + u_t$$  \hspace{1cm} (4.1)

Dickey and Fuller derived the distribution for the estimator of $\rho$ that holds when $\rho = 1$, and tabulated the critical values for the estimate of $(\rho^* - 1) / \sigma_p^*$, where a star indicates a maximum likelihood estimate of the relevant parameter.

The Dickey-Fuller test is valid in the presence of some heterogeneity in the error sequence provided the errors are martingale differences. In general, it is not appropriate when the errors are serially correlated. However, it has good small sample properties even in the case when the error terms are serially correlated.
Although the Dickey-Fuller test is widely used, its power is limited. It only allows one to reject (or fail to reject) the hypothesis that a variable is not a random walk. A failure to reject is only weak evidence in favor of the random walk hypothesis.

4.2.2 Phillips and Perron Tests

Phillips and Perron (1988) generalized the results of OLS estimation of $\alpha$ and $\rho$ in (4.1). They derived their statistics assuming that $u_i$ in (4.1) is a zero-mean, but otherwise heterogeneously distributed, process satisfying certain restrictions on serial dependence and higher moments. The test is considered to be nonparametric, because no parametric specification of the error process is involved. Phillips and Perron postulated the critical values for the adjusted $t$ statistic, which is calculated as:

$$\left( \gamma_0 / \lambda^2 \right)^{1/2} \times \frac{T}{\sqrt{\frac{1}{T} \sum_{t=1}^{T} E(u_i^2)}}$$

(4.2)

where

$$\gamma_0 = \lim_{T \to \infty} T^{-1} \sum_{t=1}^{T} E(u_i^2)$$

(4.3)

and:

$$\lambda^2 = \lim_{T \to \infty} T^{-1} E(u_1 + u_2 + \ldots + u_T)^2$$

(4.4)

$s_T$ is the estimated variance of the error term, $u_i$, in equation (4.1). A consistent estimate of $\gamma_0$ is

$$\gamma_0 = T^{-1} \sum_{t=1}^{T} u_i^2$$

(4.3a)
where $u_t^*$ is the OLS sample residual. However Phillips and Perron (1988) suggested using the standard OLS estimate:

$$
\gamma_0 = (T-2)^{-1} \sum_t u_t^*, 
$$

(4.3b)

One of the possible estimates for $\lambda^2$ is considered in the following section.

One of the possible drawbacks of the Phillips and Perron tests is that when the correlation coefficient between $u_t$ and $u_{t-1}$ in (4.1) is close to minus one, the test is likely to reject the null hypothesis of a unit root in finite samples even if it is true. For the purposes of the current paper this does not seem to be a problem: the error term in a cointegrating relationship between the exchange rate and fundamentals, as shown in the literature, is positively correlated. However because of some indications that Phillips and Perron nonparametric tests may be less reliable than Dickey-Fuller tests, both tests are used in the current research.

### 4.2.3 Newey and West Approach to Estimating $\lambda^2$

A Newey-West estimator can be applied in cases when $u_t$ is MA($\infty$), and is particularly convenient when the process for $u_t$ is not known. Newey and West (1987) suggested the estimate of $\lambda^2$, $S^2$:

$$
S^2 = \Gamma_0 + \sum_v [1 - v/(q + 1)] (\Gamma_v + \Gamma_v^T) 
$$

(4.5)

where a superscript $T$ denotes a transpose of a corresponding matrix and $\Gamma_v$ is:

$$
\Gamma_v = (1/T) \sum_t (y_{1t} - E(y_{1t})) (y_{vt} - E(y_{1t}))^T 
$$

(4.6)
They showed that their estimate, $S^2$, is positive semidefinite by construction and can provide a consistent estimate of $\lambda^2$ for an MA ($\infty$) process, provided that $q$, the lag truncation parameter, goes to infinity as the sample size grows, and provided that $q$ grows sufficiently slowly relative to $T$. Phillips (1987) established such consistency assuming that $q_T \to \infty$ and $q_T / T^{1/4} \to 0$

4.3 Model-Building with Cointegrated Variables

4.3.1 Error-Correction Model

One way of dealing with non-stationary variables is to difference them as many times as needed to make them stationary. Once all series have been transformed to stationarity, dynamic regression models may be specified in the usual way and standard asymptotic results apply. However the differencing eliminates the opportunity to estimate any relationships between the levels of the dependent and independent variables, thus ignoring the information that economic theory could offer concerning the role of long-run equilibria.

A second approach is to estimate an error-correction model, or ECM. First, the ECM suggests explanatory variables for inclusion in this equation; and second, it identifies long-run equilibrium relationships among economic variables, which if not exactly satisfied will set in motion economic forces affecting the variable being explained. The equation is usually developed in two stages. First, a traditional econometric equation is specified, with a sufficient lag structure (which is later reduced by testing procedures) on all the explanatory variables, including lagged values of the dependent variable. Second, this equation is manipulated to
reformulate it in terms that are more easily interpreted, producing a term representing the extent to which the long-run equilibrium is not met. The last term is called an error-correction term since it reflects the current “error” in achieving long-run equilibrium. This type of model has consequently come to be known as an error-correction model, or ECM.

If variables are cointegrated, it means that although individually they are I(1), a particular linear combination of them is I(0). The cointegrating combination is interpreted as an equilibrium relationship, since it can be shown that variables in the error-correction term in an ECM must be cointegrated, and, vice versa, that cointegrated variables much have an ECM representations.

An error-correction model, includes, among regressors, not only lags of differenced variables, but also lagged cointegration errors which are stationary. Suppose that the variables $y_t$ and $x_t$ are I(1) variables which are cointegrated with a cointegrating vector $\{1, -a\}$. Then the VAR for this model will look like:

$$\Delta x_t = \alpha_{11} \Delta x_{t-1} + \ldots + \alpha_{1p} \Delta x_{t-p} + \alpha_{21} (y_{t-1} - a x_{t-1}) + \ldots + \alpha_{2p} (y_{t-p} - a x_{t-p}) + \epsilon_t$$

$$y_t - a x_t = \beta_{11} \Delta x_{t-1} + \ldots + \beta_{1p} \Delta x_{t-p} + \beta_{21} (y_{t-1} - a x_{t-1}) + \ldots + \beta_{2p} (y_{t-p} - a x_{t-p}) + u_t$$

One of the methods of estimating the VAR in error-correction representation is to first estimate the cointegrating relationship, and second estimate the coefficients in the VAR. This approach is particularly convenient when the economic theory implies different cross-equation restrictions on the system above.
4.3.2 Estimation of a Cointegrating Relationship with OLS with Leads and Lags

An \((n\times1)\) vector \(y_t\) is said to be cointegrated if each of its elements individually is \(I(1)\) and if there exists a nonzero \((n\times1)\) vector \(a\) such that \(z_t = a'y_t\) is stationary. If this is the case, \(a\) is called a cointegrating vector. If \(a\) is not known and \(z_t\) is stationary and ergodic for second moments, the estimates of \(a\) can be obtained by minimizing \(E(z_t^2)\) subject to some normalization condition on \(a\). Such an estimator is superconsistent, converging at rate \(T\) rather than \(T^{1/2}\).

If the cointegrating vector has a nonzero coefficient for the first element of \(y_t\) \((a_1 \neq 0)\), then \(a_1\) can be set \(a_1 = 1\). In this case the objective is to choose \((a_2, a_3, \ldots, a_n)\) to minimize \(T^{-1}\Sigma_t(z_t^2)\), which is achieved by an OLS regression of the first element of \(y_t\) on all of the others:

\[
y_{1t} = -a_2 y_{2t} - a_3 y_{3t} - \ldots - a_n y_{nt} + u_t
\]

(4.7)

Consistent estimates of the negatives of \(a_k\), \(\gamma_k = -a_k\), are also obtained when a constant term is included in (4.7), as in:

\[
y_{1t} = \alpha + \gamma_2 y_{2t} + \gamma_3 y_{3t} + \ldots + \gamma_n y_{nt} + u_t
\]

(4.8)

or:

\[
y_{1t} = \alpha + \gamma' y_{2t} + u_t
\]

(4.9)

where \(\gamma' = (\gamma_2, \gamma_3, \ldots, \gamma_n)\) and \(y_{2t} = (y_{2t}, y_{3t}, \ldots y_{nt})'\) and:

\[
y_{2t} = y_{2t-1} + u_{2t}
\]

(4.10)

If \(u_t\) and \(u_{2t}\) are correlated, it will imply that the error term, \(u_n\), in (4.9) becomes correlated with the regressors. This correlation will result in a bias of the estimate of the \(t\) and \(\chi^2\) statistics for testing the hypothesis about the cointegrating relationship. To correct for this
asymptotically, possible correlation between \( u_t \) and \( u_{2t} \) can be corrected by augmenting (4.9) with leads and lags of \( \Delta y_{2t} \) (Phillips and Loretan (1991), Stock and Watson (1993), Wooldridge (1991)). It will remove the effects of short-run dynamics in the equilibrium error on the estimates of the statistics based on the cointegrating relationship. Then equation (4.9) can be written as:

\[
y_{1t} = \alpha + \gamma' y_{2t} + \sum s \beta_s' \Delta y_{2,t+s} + z_t
\]  \hspace{1cm} (4.11)

and can be estimated by OLS.

4.4 Testing the Coefficients of a Cointegrating Relationship

To test a hypothesis that a cointegrating coefficient is equal to a particular value, a modified \( t \)-statistic needs to be used. A modified \( t \)-statistic is obtained by multiplying the regular \( t \)-statistic by \((\lambda_{11}^2/s_n^2)^{1/2}\). This correction is necessary in order to take care of the possible correlation between \( u_t \) and \( u_{2t} \) in (4.8) and (4.10). \( \lambda_{11} \) can be estimated using Newey-West technique using (4.5).

Another way to calculate \( \lambda_{11} \) is the following, \( \lambda_{11} \) is equal to:

\[
\lambda_{11} = \sigma_1 \ast \psi_{11}.
\]  \hspace{1cm} (4.12)

where \( \sigma_1 \) is the square root of the variance of the error term in the equation:

\[
u_t = \phi_1 u_{t-1} + \phi_2 u_{t-2} + \ldots + \phi_p u_{t-p} + \epsilon_t
\]  \hspace{1cm} (4.13)

and

\[
\psi_{11} (L) \epsilon_t = u_t.
\]  \hspace{1cm} (4.14)
Therefore, another way to calculate $\lambda_{i2}^2$ is run the regression of $u_t$ on $p$ of its lagged values and then an estimate of $\lambda_{i2}^2$ is:

$$\lambda_{i2}^2 = \sigma_1^2 / (1 - f_1 - f_2 - \ldots - f_p).$$

(4.15)

where

$$\sigma_1^2 = (T - p)^{-1} \Sigma_t e_t^2$$

(4.16)

and $f_p$ is the OLS estimate of $\phi_p$.

4.5 Estimation of the Unrestricted VAR

4.5.1. Unrestricted VAR

The unrestricted VAR for present value models estimation has the following form:

$$\Delta x_t = \theta_1 \Delta x_{t-1} + \theta_2 \Delta x_{t-2} + \ldots + \theta_p \Delta x_{t-p} + \kappa_1 L_{t-1} + \kappa_2 L_{t-2} + \ldots + \kappa_p L_{t-p} + \epsilon_t$$

and

$$L_t = \gamma_1 \Delta x_{t-1} + \gamma_2 \Delta x_{t-2} + \ldots + \gamma_p \Delta x_{t-p} + \mu_1 L_{t-1} + \mu_2 L_{t-2} + \ldots + \mu_p L_{t-p} + u_t$$

(4.17)

where the $\Delta x_t$'s are the first differences of the fundamentals, and $L_t$ is the cointegrating error in the relationship between the exchange rate and fundamentals.

Equation (4.17) can be rewritten as:

$$y_t = \Pi x_t + v_t,$$

(4.18)
and $\Pi$ is defined as:

$$
\Pi = \begin{vmatrix}
\theta_1 & \theta_2 & \cdots & \theta_p \\
\kappa_1 & \kappa_2 & \cdots & \kappa_p \\
\gamma_1 & \gamma_2 & \cdots & \gamma_p \\
\mu_1 & \mu_2 & \cdots & \mu_p
\end{vmatrix},
$$

(4.19)

$$
y_t = [\Delta x_t \ L_{t-1}]'; x_t = [\Delta x_{t-1} \ \Delta x_{t-2} \ \cdots \ \Delta x_{t-p} \ L_{t-1} \ L_{t-2} \ \cdots \ L_{t-p}]' \text{ and } v_t = [u_t \ v_t]' \sim \text{i.i.d. } N(0, \Omega).
$$

(4.20)

Each element of $\Omega$ can be denoted as $\Omega_{ij}$ where i is the corresponding row and j is the corresponding column of the matrix $\Omega$.

4.5.2. Determining the Lag Length

Akaike’s (AIC) criterion and Schwarz’s (SC) criterion have been used to determine the optimal lag length of the VAR. They are defined as follows:

$$
\text{AIC}(n) = \ln \det (\Sigma^*) + 2M^2 n / T
$$

(4.21)

$$
\text{SC}(n) = \ln \det (\Sigma^*) + M^2 n \ln T / T
$$

(4.22)

where $M$ is the number of variables in the system, $T$ is the sample size, and $\Sigma^*$ is an estimate of the residual covariance matrix $\Sigma$, obtained with a VAR (n) model. The elements of $\Sigma^*$ are computed as:

$$
\sigma_{ij} = (y^i - X \theta_i)' (y^j - X \theta_i) / T
$$

(4.23)
The order \( p \) is chosen so that the AIC or SC criterion is minimized. That is, models with order \( n = 0, 1, \ldots, P \) are estimated with \( P \) being a prespecified upper bound for the VAR order. Then the matrices \( \Sigma_n \) for \( n = 0, 1, \ldots, P \) and the corresponding values AIC \((n)\) or SC\((n)\) are computed. The value \( p \) (AIC) is the order that minimizes AIC \((n)\) over \( n = 0, 1, \ldots, P \) and \( p \) (SC) is the order minimizing SC\((n)\). In this procedure the sample size, \( T \), is held fixed: in other words, in each estimation \( P \) observations for each variable are treated as presample values.

### 4.5.3 Testing for Serial Correlation and Autoregressive Conditional Heteroskedasticity

\textbf{(ARCH) of Order m}

The Lagrange multiplier (LM) test (Engle (1982)) has been used to determine if the residuals in a VAR are serially correlated and/or conditionally heteroskedastic. These tests are relatively simple because you do not need to calculate serial correlation or ARCH parameters. The tests proceed as follows. First the regression equation:

\[ y_t = x_t' \beta + u_t. \]  \hspace{1cm} (4.24)

is estimated by OLS for observations \( t = -m + 1, -m + 2, \ldots, T \) and the OLS estimates of the residuals are saved. Here \( x_t \) denotes a vector of predetermined explanatory variables, which could include lagged values of \( y \).

For the ARCH \((m)\) test, the residuals from (4.24) are then regressed on a constant and \( m \) of their own lagged values. The sample size, \( T \), times the uncentered \( R^2 \) from this regression,
converges in distribution to a $\chi^2$ variable with $m$ degrees of freedom under the null hypothesis that $u_t$ is actually i.i.d. $N(0, \sigma^2)$.

The LM statistic for serial correlation is calculated in a similar fashion. The residuals from (4.24) are regressed on the regressors, $x$, and on the lagged residuals of order that corresponds to the order of serial correlation. The LM statistic from this regression is $TR^2$ and is distributed as a $\chi^2$ variable with $m$ degrees of freedom under the null hypothesis that $u_t$ is actually i.i.d. $N(0, \sigma^2)$, with $m$ being the order of serial correlation.

4.6 Restricted VAR estimation

4.6.1 Restricted VAR. Rational Expectations Restrictions

A present value models approach to the exchange rate combined with rational expectations implies that there are the following cross-equation restrictions on the VAR (4.17):

\[
\Delta x_t = \theta_1 \Delta x_{t-1} + \theta_2 \Delta x_{t-2} + \ldots + \theta_p \Delta x_{t-p} + \kappa_1 L_{t-1} + \kappa_2 L_{t-2} + \ldots + \kappa_p L_{t-p} + \epsilon_t^R
\]

\[
L_t = -\theta_1 \Delta x_{t-1} - \theta_2 \Delta x_{t-2} - \ldots - \theta_p \Delta x_{t-p} - (\kappa_1 - (1 - 1/\alpha)) L_{t-1} - \kappa_2 L_{t-2} - \ldots - \kappa_p L_{t-p} + u_t^R.
\]

(4.25)

These are Campbell and Shiller restrictions (2.16) rewritten in a VAR form, and $\alpha$ is the semielasticity of the interest rate.

Analogous to (4.21), (4.25) can be rewritten as:

48
\[ y_i = \Pi^R x_i + v_i^R, \]  
\hspace{1cm} (4.26)

and \( \Pi^R \) and \( v_i^R \) are defined as:

\[
\Pi^R = \begin{bmatrix}
\theta_1 & \theta_2 & \cdots & \theta_p & \kappa_1 & \kappa_2 & \cdots & \kappa_p \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
-\theta_1 & -\theta_2 & \cdots & -\theta_p & (-\kappa_1 + 1-1/\alpha) & -\kappa_2 & \cdots & -\kappa_p
\end{bmatrix}
\hspace{1cm} (4.27)
\]

\[ v_i^R = [u_i^R \varepsilon_i^R]’ \sim \text{i.i.d. } N(0, \Omega^R) \hspace{1cm} (4.28) \]

and \( y_i \) and \( x_i \) defined as in (4.20).

### 4.6.2 Restricted VAR. Adaptive Expectations Restrictions

Adaptive expectations combined with the present value models imply the following non-linear restrictions on the VAR (4.21):

\[
\Delta x_i = \theta_1 \Delta x_{i-1} + \theta_2 \Delta x_{i-2} + \cdots + \theta_p \Delta x_{i-p} + \kappa_1 L_{i-1} + \kappa_2 L_{i-2} + \cdots + \kappa_p L_{i-p} + \varepsilon_i
\]

\[
L_i = -f \theta_1 \Delta x_{i-1} - f \theta_2 \Delta x_{i-2} - \cdots - f \theta_p \Delta x_{i-p} - (f \kappa_1 - g) L_{i-1} - f \kappa_2 L_{i-2} - \cdots - f \kappa_p L_{i-p} + u_i
\]

\hspace{1cm} (4.29)

where:

\[ f = \alpha(1-\alpha)/(1+\alpha-\alpha) \]  
\[ g = (1-\alpha)/(1+\alpha-\alpha). \hspace{1cm} (4.30) \]

Again, rewrite (4.29) in the form:
\[ y_t = \Pi^A x_t + \nu_t^A, \] (4.31)

then \( \Pi^A \) and \( \nu_t^A \) are defined as:

\[
\Pi^A = \begin{bmatrix}
\theta_1 & \theta_2 & \cdots & \theta_p \\
\kappa_1 & \kappa_2 & \cdots & \kappa_p \\
-f\theta_1 & -f\theta_2 & \cdots & -f\theta_p \\
-(-f\kappa_1 + g) & -\kappa_2 & \cdots & -\kappa_p
\end{bmatrix}
\] (4.32)

\[ \nu_t^A = [u_t^A \ \varepsilon_t^A]^\prime \sim \text{i.i.d.} \ N(0, \Omega^A) \] (4.33)

and \( y_t \) and \( x_t \) defined as in (4.20).

Adaptive expectations also implies the restrictions on the variance-covariance matrix of the VAR:

\[ \Omega_{12}^A = -\Gamma \Omega_{11}^A. \] (4.34)

4.7 Maximum Likelihood Estimation of the Unrestricted and Restricted VAR

4.7.1 Unrestricted VAR

The purpose of maximum likelihood estimation is to find the maximum likelihood estimates (MLE) of \( A \); the coefficients of a vector autoregression; and \( \Omega \), the variance-covariance matrix of the VAR. As has been shown in Hamilton (1994), OLS regressions provide the maximum likelihood estimates of \( A \) under the following assumptions (Assumptions (4.7.1)): 

50
(i) \( \{v_t\} \) is a homoskedastic innovation process \( v_t = [u_t, \varepsilon_t]' \sim \text{i.i.d. } N(0, \Omega); \)

(ii) \( z_{t+1} \) is weakly exogenous for \((A, \Omega)\);

(iii) \((A, \Omega)\) are constant \(\forall t\).

All these assumptions hold for the present case.

The MLE estimate of \(\Omega, \Omega^*\) can be found as:

\[
\Omega^* = \frac{1}{T} \sum_t e_t e_t'
\]

(4.35)

where \(e_t\) is the vector residual \((m*1)\) from the OLS estimation. In the present value models case, \(m\) is equal 2.

4.7.2. Restricted VAR: Rational Expectations

A VAR can be viewed as the reduced form of a general dynamic structural model, when a sufficient number of lags are included so that the error term is vector white noise. The restricted VAR equation describing present value models under rational expectations is defined in (4.26): \(y_t = \Pi^R x_t + v_t^R\). Under Assumptions (4.7.1), the log-likelihood function \(l(\Pi^R, \Omega | y_t, x_t)\) depends on the multivariate normal distribution:

\[
l(\Pi^R, \Omega | y_t, x_t) = T^* m^* \ln (2^* \pi)/2 - T^* \ln |\Omega|/2 - \frac{1}{2} \sum_t v_t' \Omega^{-1} v_t
\]

(4.36)

which, using the following relations:

\[v_t', \Omega^{-1} v_t = \text{tr} (\Omega^{-1} v_t v_t') \text{ and } |\Omega| = |\Omega^{-1}|\]
and further simplifying can be rewritten as:

$$I (\Pi^R, \Omega | y_t, x_t) = K + T^* \ln |\Omega^i|/2 - \frac{1}{2} \text{tr} [\Omega^i \Sigma_t v_t v_t']$$

(4.37)

where \(\text{tr}[..]\) is the trace and \(K\) is the constant, which does not depend on the parameters of the model.

Equation (4.37) can be differentiating with respect to \(\Omega^i\) using the following relations:

$$\frac{\partial \ln |\Omega^i|}{\partial \Omega^i} = \Omega^i$$ and $$\frac{\partial \text{tr} [\Omega^i \Sigma_t v_t v_t']}{\partial \Omega^i} = [\Sigma_t v_t v_t']^\top$$ and then equating the score to zero to give:

$$T^* \Omega/2 - \Sigma_t v_t v_t'/T = 0.$$  

(4.38)

Solving equation (4.38) for \(\Omega\) gives, an expression for the conditional maximum of \(\Omega, \Omega_c:\)

$$\Omega_c = \frac{\Sigma_t v_t v_t'/T}{T}$$

(4.39)

The resulting likelihood function, which is conditional on \(\Omega\), is obtained by substituting (4.39) into (4.37) and using the following equalities: \(|\Omega^i| = |\Omega|\) and \(|\Sigma_t v_t v_t'/T| = (1/T)^m |\Sigma_t v_t v_t'|:\)

$$I (\Pi^R | y_t, x_t, \Omega) = \left|K - T^* \ln |\Sigma_t v_t v_t'| - \frac{1}{2} T m^* \ln T - \frac{1}{2} T m = \right.$$

$$K^* - T/2 \ln |\Sigma_t (y_t - \Pi^R x_t) (y_t - \Pi^R x_t)^\top|$$

(4.40)

The value of \(\Pi^R\) which provides the maximum of (4.40) will be the maximum likelihood estimate of \(\Pi^R\).
4.7.3 Restricted VAR: Adaptive Expectations

The expression for the maximum likelihood function for present value models and adaptive expectations is analogous to (4.40):

\[ l (\Pi^A \mid y_t, x_t, \Omega) = K^* - \frac{T}{2} \star \ln \left| \Sigma_t (y_t - \Pi^A x_t) (y_t - \Pi^A x_t)' \right| \]  

(4.41)

with additional variance-covariance restriction \( \Omega_{12}^A = -\Gamma \Omega_{11}^A \), and \( \Pi^A \) defined as in (4.32).

4.8 Non-Linear Optimization Technique

4.8.1 Downhill Simplex Method

The advantage of the downhill simplex method is the following: the method requires only function evaluations, not derivatives. The storage requirement is of order N. The optimization technique is completely different from the simplex method of linear programming. Here a simplex is the geometrical figure consisting, in N dimensions, of N+1 points and all their interconnecting line segments, polygonal faces, etc. The optimization technique must be started with N+1 points, defining an initial simplex, where N is the number of parameters to be optimized. The next N points as a result of optimization would be:
\[ P_i = P_0 + \lambda e_i \]  

where the \( e_i \)'s are \( N \) unit vectors, and where \( \lambda \) is a constant which is a guess of the problem's characteristic length scale.

Termination criteria can be different. It is possible to terminate when the vector distance moved in that step is fractionally smaller in magnitude than some tolerance. Alternatively, it could be required that the decrease in the function value in the terminating step be fractionally smaller than some tolerance. The problem with these criteria is that all of them might be fooled by a single anomalous step that, for some reason failed to get anywhere. Therefore, sometimes it is worthwhile to restart a multidimensional routine at a point where it claims to have found a maximum.

4.8.2 Powell's Method

Powell's method is a direction-set method and does not require computation of first derivatives. This technique, however, requires a one-dimensional maximization sub-algorithm. The optimization proceeds as follows. First, a start point, \( P \), in \( N \)-dimensional space is defined and a set of directions is set, for simplicity, by unit vectors \( e_1, e_2, \ldots, e_N \). Then, using a one-dimensional maximization algorithm, the routine moves along the first direction to its maximum, and so on, cycling through the whole set of directions as many times as necessary, until the function stops increasing in its value.

The problem with this method can be the following. If the function's second derivatives are much larger in magnitude in some directions than in others, then many cycles through all \( N \)
basis vectors will be required in order to get anywhere. This problem, however, can be overcome by choosing a better set of directions than the \( e_i \)'s. There are few methods which update the set of directions as the method proceeds. The most popular are conjugate directions and Powell's method discarding the direction of largest increase.

The conjugate method works as follows. The vectors \( u \) and \( v \) are said to be conjugate if \( u^t A v = 0 \), where \( A \) is the Hessian matrix of the function \( f \). The conjugate direction method does successive line maximization along a conjugate set of directions. The advantage of the conjugate method is that moving along a direction \( v \) after some direction \( u \) has reached its maximum will not "harm" the maximization along \( u \) because the gradient will stay perpendicular to \( u \) if \( u \) and \( v \) are conjugate vectors. Powell offered a method which can calculate mutually conjugate vectors without actually calculating the Hessian. Repeated cycles of \( N \) line maximizations will converge quadratically to the maximum.

Unfortunately the quadratically convergent algorithm has its problems. The procedure tends to produce sets of directions that become linearly dependent. Once this happens, the procedure finds the maximum of the function \( f \) only over a subspace of the \( N \)-dimensional case.

To avoid this problem, one needs to give up the property of quadratic convergence. It can be achieved by the following. The initial set of directions is the set of basis vectors. After that, \( (P_N - P_0) \) is taken as a new direction, where \( P_0 \) is the starting position, and \( P_N \) is the next one. This direction is the average direction moved after trying all \( N \) possible directions. Then proceed with discarding the old direction along which the function \( f \) made its largest increase. This paradoxical decision of eliminating the best direction allows one to avoid a buildup of linear dependence.
Powell's method requires choosing a one-line optimization technique. Two different techniques have been used for this purpose: golden section search and Brent's technique. These techniques are used because they are among the most popular methods which do not include first derivatives. The idea of the golden section search method is the following. Suppose there are three points, such as \( a < b < c \). It is known, that if \( f(b) \) is larger than both \( f(a) \) and \( f(c) \), then a nonsingular function \( f \) has a maximum in the interval \((a, c)\). Then the function is evaluated at a triplet of points: \( a, b \) and \( c \), after which a new point \( x \) in the largest of the intervals \([a, b]\) or \([b, c]\) is chosen. If the interval \([a, b]\) is larger than interval \([b, c]\), then \( x \) will belong to \([a, b]\) and the next interval will be either \([a, b]\) with the intermediate point \( x \), or \([x, c]\) with the intermediate point \( b \). The golden section search finds a point \( x \) such that the distances of these two possible new intervals \([a, b]\) and \([x, c]\) are equal. The process continues until the bracketing interval is acceptably small. The convergence of the process is linear. The initial interval \([a, c]\) has its intermediate point \( b \) a fractional distance 0.3820 from one end and 0.6180 from the other end. It implies that the new point \( x \) will divide the interval \([a, b]\) in the same fashion as \( b \) divides the interval \([a, c]\).

Another method of a one-line maximization is parabolic interpolation. The method is similar to the golden section search, but now a parabola is drawn through the three original points on the function. The function is evaluated at the parabola's maximum which replaces one of the original points. New parabolas are evaluated through the two sets of new triplets. The parabola with the maximum which is closer to the maximum of the function is selected for further maximization.
4.8.3 Davidon-Fletcher-Powell (DFP) Algorithm

The Davidon-Fletcher-Powell method is used to receive the final optimization results. The Davidon-Fletcher-Powell method is a gradient method which belongs to a class of variable metric or quasi-Newton methods. The method requires the ability to compute the function’s gradient at any point and uses a linear Taylor series approximation to achieve the maximum. This is a very effective class of algorithms which eliminates second derivatives (the change in the first derivatives is used as an estimate of second derivatives) and has excellent convergence properties.

The quasi-Newton methods store and update information which is accumulated, including the estimate of the negative inverse of the second derivatives matrix. Different methods differ in the way the information is updated. The current research uses the Broyden-Fletcher-Goldfarb-Shanno (BFGS) variant of the DFP optimization technique. The BFGS technique has proved to be superior to the regular DFP algorithm. This method is very similar to DFP and the major difference is that the BFGS method uses a rank one update, whereas the regular DFP algorithm uses a rank two update (the rank here refers to the rank of a matrix which is used for updating).

4.8.4 Non-Linear Optimization for the Present Value Models

Present value model restrictions result in non-linear restrictions on the VAR. The maximum likelihood function for rational expectations (4.34) (see Sections 4.6.1 and 4.7.2) and
adaptive expectations (4.35) (Sections 4.6.2 and 4.7.3) is maximized using the following multidimensional optimization technique. First, the downhill simplex method of Nelder and Mead is implemented (briefly described in Section 4.8.1). Then the results of the maximization serve as initial conditions for Powell’s method (Section 4.8.2). Neither method requires computation of first derivatives, and both have proved to work well for the optimization of the maximum likelihood function. Powell’s method is especially good for functions whose maximums are located in long, twisty valleys. Along the long directions quadratically convergent methods do not typically work. The advantage of Powell’s method is that it follows the “twists of the valley” and finds the maximums more easily. The results of Powell’s method are used as initial values for maximization using the Broyden-Fletcher-Goldfarb-Shanno method.

The downhill simplex routine requires, as its inputs, an \((n+1)\times n\) starting simplex matrix, where the first row is the maximizing vector and the other \(n\) rows are the vectors that set the direction (the vertices of the starting simplex). The maximizing vector is the unrestricted VAR coefficients for the equation describing \(\Delta x_t\). The values for the parameters \(\alpha\) and \(\eta\) are selected from the intervals \((\alpha \in [0, 100] \text{ and } \eta \in [-1, 1])\). The results of the optimization are robust to the different values of the initial maximizing vector. The routine is required to stop if one of the following is true: the optimization time exceeded 10 hours, the number of iterations exceeded 500 or the decrease in the function value is smaller than \(10^{-5}\).

To avoid the termination problem described in 4.8.1, after the first results have been obtained, the downhill simplex method routine is repeated again, using the results as a starting value. The downhill simplex method converges rather quickly, and the cost of the second optimization is not high. After the second results of the downhill simplex method have been obtained, they are used as a starting value for Powell’s method. The direction vectors chosen for
Powell’s method are the unit vectors. The conditions to terminate Powell’s routine are analogous to the conditions described for the downhill simplex method.

Powell’s method requires choosing a one-line optimization technique. Two different techniques have been used for this purpose: golden section search and Brent’s technique. Both methods gave comparable results for optimization. However the golden section search technique has proved to be more robust. Therefore, all the results of optimization are given for the golden section search routine.

4.9 Likelihood Ratio Test for Rational and Adaptive Expectations Restrictions

The maximum likelihood function for an unrestricted VAR is equal to:

$$ I(\Pi, \Omega) = -\frac{1}{2} T m \ln(2\pi) + T/2 \ln|\Omega| - \frac{1}{2} T m. \quad (4.43) $$

For rational expectations and present value models:

$$ \hat{I}^R(\Pi^R, \Omega^R) = -\frac{1}{2} T m \ln(2\pi) + T/2 \ln|\Omega^R| - \frac{1}{2} T m. \quad (4.44) $$

For adaptive expectations and present value models:

$$ \hat{I}^A(\Pi^A, \Omega^A) = -\frac{1}{2} T m \ln(2\pi) + T/2 \ln|\Omega^A| - \frac{1}{2} T m. \quad (4.45) $$

Likelihood ratio test statistic is calculated according to:

$$ 2(I(\Pi, \Omega) - \hat{I}^{Res} = \Pi^{Res}, \Omega^{Res}) = \chi^2 \{ \ln|\Omega| - \ln|\Omega^{Res}| \}, \quad (4.46) $$

where $$\Omega^{Res}$$ is either $$\Omega^R$$ or $$\Omega^A$$ depending on which hypothesis is tested as a null: rational or adaptive expectations. Under the null, the likelihood ratio test asymptotically has a $$\chi^2$$
distribution with degrees of freedom equal to the number of restrictions imposed under the null. For rational expectations, there are \(4p\) coefficients to be estimated for the unrestricted VAR, and only \((2p + 1)\) coefficients for the restricted (the coefficients for the autoregressive process and \(\alpha\), the semielasticity of the interest rate). Therefore the likelihood ratio statistic for rational expectations present value models restrictions will be distributed as \(\chi^2\) with \((2p - 1)\) degrees of freedom. For adaptive expectations, \((2p + 2)\) coefficients need to be estimated (the coefficients for the autoregressive process, \(\alpha\) and the adaptive expectations coefficients \(\eta\)). In addition there are restrictions on the VAR. Thus the likelihood ratio statistic for adaptive expectations present value models restrictions will be distributed as \(\chi^2\) with \((2p - 1)\) degrees of freedom as well.

4.10 Testing for a structural break

In the time series literature a lot of attention has recently been given to the possibility of detecting structural change. Chow (1960) proposed a test that assumes one structural change at a known point in time. Procedures which do not require such knowledge include the Quandt likelihood ratio test (Quandt (1958), (1960)), the CUSUM and CUSUM of squares tests (Brown et al. (1975) Ploberger and Kramer (1992)), and fluctuation test (Ploberger et al. (1989)). Kramer et al. (1988) developed a CUSUM test for lagged dependent variables. Kao and Ross (1995) proposed a CUSUM test for serially correlated disturbances. Kao and Ross (1995) and James et al. (1987) found a distribution applicable to the likelihood-ratio-like test for independent observations. Andrews (1993) and Vogelsang (1997) developed Wald, Lagrange multiplier (LM), and likelihood ratio tests (Quandt ratio) in a general framework that allows dependent and
heterogeneously distributed data. Diebold and Chen (1996) investigated the small sample properties of these tests. Different tests for data with a unit root have been proposed by Banerjee et al. (1992), Perron (1989, 1990), Hansen (1992), Perron and Vogelsang (1992) and Zivot and Andrews (1992). Tests for multiple structural changes with a known number of break points are developed by Bai and Perron (1997). Issues about the distributional properties of the parameter estimates, in particular those of the break dates, have also been considered (Bai (1995), (1996)).

4.11 CUSUM Test for Structural Breaks

The regression model considered for the CUSUM test is of the form:

$$ y_t = x_t' \beta_t + \varepsilon_t $$  \hfill (4.47)

Recursive residuals $w_r$ can be calculated for (4.47) as follows. First, let $b_r$ be the least-squares estimate of $\beta$ based on the first $r$ observations, i.e. $b_r = (X_r'X_r)^{-1}X_r'Y_r$. Then $w_r$ can be defined as:

$$ w_r = (y_r - x_r'b_{r-1})/(1+x_r'(X_{r-1}'X_{r-1})^{-1}x_r)^{1/2}, \quad r = k + 1, \ldots, T $$  \hfill (4.48)

where $X_{r-1}' = [x_1, \ldots, x_{r-1}]$ and $Y_r' = [y_1, \ldots, y_r]$.

If $\beta$ is constant up to time $t = t_0$ and differs from this constant value after $t_0$, the $w_r$'s will have zero means for $r$ up to $t_0$ but in general will have non-zero means subsequently. The main idea of the CUSUM test is to examine a plot designed to reveal departures of the means of the $w_r$'s from zero as one travels along the series through time, using a pair of lines lying symmetrically above and below the line such that the probability of crossing one or both lines is $\alpha$, the required significance level.
The plot considered is the plot of the CUSUM quantity:

\[ W_r = \frac{1}{\sigma_T} \sum w_j \] (4.49)

given \( r = k + 1, \ldots, T \), where \( \sigma_T \) denotes the estimated standard deviation determined by \( \sigma_T^2 = S_T / (T - k) \). The two straight lines surrounding the CUSUM plot are drawn through the points \( \{ k, \pm a (T - k)^{1/2} \} \), \( \{ T, \pm 3a (T - k)^{1/2} \} \), where \( a \) is a parameter which varies with the significance level. For \( \alpha = 0.01 \), \( a = 1.143 \); for \( \alpha = 0.05 \), \( a = 0.948 \) and for \( \alpha = 0.1 \), \( a = 0.850 \).

Ploberger and Kramer (1992) show that the CUSUM test can be performed with OLS residuals as well. They also show that the performance of these two tests (CUSUM with recursive residuals and CUSUM with OLS residuals) is very comparable.

4.12 Likelihood Ratio - Like Test for Structural Change with Unknown Change Point

Another statistic used in the current research for testing structural breaks is the "supremum" test of Andrews (1993):

\[ \text{SupLR} = \max_{\pi} T \left[ \frac{e'e}{(e_1'e_1 + e_2'e_2)} \right] \] (4.50)

Where \( e \) is the \( T \times 1 \) vector of OLS residuals from the unrestricted model (i.e. with no structural break); \( e_1 \) is the \( T' \times 1 \) vector of OLS residuals from the subsample with \( t \) changing from 1 till \( T' \); \( e_2 \) is the \( (T - T') \times 1 \) vector of OLS residuals from the subsample 2, with \( t \) changing from \( (T' + 1) \) to \( T \); and \( \pi \) is equal to \( T'/T \). The standard procedure is to impose \( \pi \in [0.15, 0.85] \). This statistic is sequential, which means that it is computed over subsamples \( t = 1, \ldots, m \) for \( m = m_0 \).
..., $T$ where $m_0$ is a startup value and $T$ is the size of the full sample. The statistic is computed using the full sample, sequentially incrementing the date of the hypothetical break.
CHAPTER 5

RESULTS FOR THE CASE WITHOUT STRUCTURAL BREAKS

5.1 Introduction

The main purpose of this chapter is to demonstrate the results of the model for a case without a structural break. The chapter proceeds as follows. 5.2 describes the data used for testing the model. 5.3 shows the results of testing for the unit root in the raw data. 5.4 describes how the cointegrating relationship is found. 5.5 gives the results of estimating of VAR. 5.6 discusses the results of testing the present value hypothesis under adaptive and rational expectations formation mechanism. 5.7 describes the results of forecasting based on the present value model with different types of expectation formations.

5.2 Data

The data for this study relate to the monthly DM, UK pound and Japanese yen versus U.S. dollar exchange rates from March 1973 through November 1993. The exchange rates are all
expressed as home currency per unit of foreign currency. All monetary aggregates are monthly M1 and the income measure is monthly industrial production.

5.3 Tests for a Unit Root in the Data

Table 1 displays estimation results for the raw data. The reported numbers are studentized Dickey-Fuller statistics and the t-test modification suggested by Phillips (1987) and Perron (the tests are described in Sections 4.2.1. and 4.2.2. correspondingly). In constructing these statistics, a Bartlett lag window is used to ensure positive definiteness (Newey and West (1987), the technique is described in Section 4.2.3).

Dickey-Fuller and Phillips and Perron tests are performed with the number of lags equal to 2, 4 and 6. The results of the tests with different numbers of lags are comparable. \( \gamma_0 \) from the Phillips and Perron test has been calculated using the approaches suggested by both (4.3a) and (4.3b), and the results are comparable as well. Table 1 gives the results for the lags equal 6 and \( \gamma_0 \) calculated as in (4.3b). For both sets of statistic the null hypothesis is that the series in question is I(1). The critical value at the 5% level is -2.88, with rejection region (\( \phi: \phi < -2.88 \)). The tests for a unit root indicate that all series are I(1) processes. For some of the series, there is some evidence that the series has a trend. However, other series indicate no trend. Because these tests have relatively low power and the literature supports an I(1) processes without a trend, this assumption has also been made here.
5.4 Cointegrating relationship

Theory implies that $s_t$, $m_t$ and $y_t$ are cointegrated with a cointegrating vector \{1, -1, -a_t\}, where $a_t$ is less than zero, and, therefore, $-a_t$ is positive (in some papers the hypothesis that $a_t$ is equal to one for Germany cannot be rejected). Under the assumption that the variables are cointegrated, the ordinary least squares estimator is superconsistent. Therefore $a_t$ is estimated by OLS. Lags and leads were added to the OLS regression in order to obtain the correct $t$ and $\chi^2$ statistics as suggested by Stock and Watson (1993) (this approach is described in Section 4.3). The regression is estimated with lags and leads equal to 2, 4 and 6. The estimate of $a_t$ for the three currencies with lags and leads equal to 4 is given in Table 2. Inference based on the $t$-ratio of this estimator is then entirely standard if the error term is serially uncorrelated, homoskedastic, and uncorrelated with all leads and lags (Hamilton, (1994), pp. 602-608).

Although that the coefficient $a_t$ (estimate of income elasticity) for Germany has a wrong sign, the null that $a_{t1}$ is equal to -1 cannot be rejected for any of the countries (the specifics of testing for a cointegrating relationship are in Section 4.4). Income elasticity for Germany has been estimated when a number of years were omitted. If the years 1981-1986 are omitted, the estimate of $a_{t1}$ is equal to 1.82, and the estimate for $a_{t1}$ for the years 1981-1986 is equal to -4.27.

To test the hypothesis that coefficient $a_{t1}$ is equal to one, the modified $t$-statistics described in Section 4.4 have been used. $\lambda_{t1}$ is calculated by both using the Newey-West technique and applying formula (4.15). The conclusions of the tests are very similar. The results of the tests can be explained by the fact that monetary factors are relatively more important than real income in predicting exchange rate movements. Throughout the paper all tests are performed using the original cointegrating relationship and one where $a_t$ is equal to -1.
Resulting $\Delta x_t$ and $L_t$ (as defined in equations (2) and (7)) are subjected to ADF and Phillips-Perron tests of unit root. The results are given in Table 3. The hypothesis that there is a unit root cannot be rejected, since the 10-percent critical value is -2.57. However, it has been shown that the power of generic unit-root tests is bounded above by the size of the test (Blough (1992)), so $L_t$ is assumed to be stationary, although quite persistent.

5.5 Estimation of the Unrestricted VAR

The first step in VAR estimation is to find the optimal number of lags, in this case by minimizing Schwarz (SC) and Akaike (AIC) information criterion (both described in Section 4.5.2). The results of both procedures are very similar and given in Table 4. The prespecified upper bound used for both tests is equal to 4, 7 and 13. The results when the prespecified bound is equal to 13 are given. The lag of two is chosen for all future tests.

The VAR was estimated by OLS (see Section 4.7.1) and the results for the unrestricted VAR are given in Table 5. The Lagrange multiplier (LM) tests for first- and third-order residual serial correlation and for third-order residual ARCH are implemented (see Section 4.5.3 for a brief description of the tests). The null hypothesis is that there is no serial correlation. P-values refer to the marginal significance levels for a one tail test. It is further assumed that the error term is white noise.

The common feature for the three countries is that $L_t$, the deviation of the exchange rate from fundamentals, is quite persistent. According to the monetary model, if $L_t$ is persistent, $\Delta s_{t+1}^e$ is persistent as well ($L_t = \alpha \Delta s_{t+1}^e$), meaning that on average, the relatively long periods when the
agents expect the exchange rate to go up, alternate with the periods when they expect the exchange rate to go down.

5.6 Testing the expectations restrictions

Table 6 shows the results of the LR test for the rational expectations restrictions (the test is described in Section 4.9). Maximum likelihood estimation yields an estimate for $\alpha$ which is negative when no restrictions are placed upon it. When the sign of $\alpha$ is restricted to positive values, the null that expectations are rational and the monetary model is true is rejected at the 1% percent significance level for all countries. These results are presented in Table 7. If rational expectations are rejected even with the maximum likelihood estimate for $\alpha$, then they would be rejected for any value of the parameter. Therefore it can be seen that the rejection of RE by MacDonald and Taylor (1993) is not due to a wrong value of $\alpha$.

The negative estimate for an unrestricted $\alpha$ means the following. According to RE, if at time $t$, the exchange rate is above its equilibrium value $x_t$, it means that the agents expect $\Delta s_{t+1}$ to be positive ($s_t - x_t = \alpha \Delta s_{t+1}$). The explanation is that the agents know that at time $t+1$ $x$ will go up, and incorporate this information at time $t$ by allowing the exchange rate to overshoot its equilibrium value at time $t$. A negative $\alpha$ means that, on the contrary, when the exchange rate is above its equilibrium value $x_t$, (the agents expect the exchange rate in the next period to go up), the exchange rate of the next period most likely will go down. Therefore, a negative estimate of $\alpha$ is consistent with the fact, that on average the agents are wrong about the prediction of the sign of the next exchange rate movement.
The results of the testing for the AE null are given in Table 8. The estimator for $\eta$ has plausible values between zero and one and the estimator for $\alpha$ is positive, as it predicted by theory. However it is smaller than expected. The standard error of the coefficients is rather high which can be explained by the presence of multicollinearity. The hypothesis of AE cannot be rejected at the 5% significance level for Great Britain in both cases (when using the cointegrating relationship and when the income elasticity is restricted to be equal to minus one), at the 1% significance level for Germany in both cases and once for Japan when income elasticity is restricted.

If expectations are indeed adaptive, then the value of $\eta$ shows how fast people correct their mistakes in forecasting. If the parameter $\eta$ in the range found in this paper, then reality is consistent with the fact that, within some time period, when the relative demand for the currency is stable, people adjust their expectations of the exchange rate relative to what the current value of the exchange rate is very slowly and do not update the information very often. When $\eta \rightarrow 0$, people’s expectation of $s_{t+1}$ does not change with time to a great extent. If it is equal to zero, then the people do not change their expectations based on the recent movements in the exchange rates at all, and $y_{t+1}^{e} = y_{t}^{e} + e_{t}$. It would mean, for example, when the exchange rate goes up in one period, people expect it to go down in the next, and when it goes down in one period, the expectations would be that it will go up in the next. In other words, “current appreciation generates the expectation of future depreciation” (Frankel and Froot, 1987, p.133). If the exchange rate is an I(1) process, as assumed in this and in the majority of other studies, such behavior would be difficult to interpret if agents are considered fully rational. However, it is fully consistent with the findings of Gourinchas and Tornell: (1996) “while the data fail to exhibit significant transitory components, market participant implicitly assume that a sizable
portion of the shocks is transitory” (p. 38) and of Cecchetti, Lam and Mark (1996), who showed that the fact that the economic agents expect the currency to excessively depreciate over booms and appreciate over recessions can explain several empirical puzzles involving equity and short-term bond returns.

5.7 In-sample and Out-of-Sample Forecasting using Rational and Adaptive Expectations.

Discussion

Figures 4 through 6 graph the actual and theoretical $\Delta s_t$ for in-sample forecasting where using the restrictions implied by RE and AE. The graphs reveal important differences between actual and theoretical $\Delta s_t$ for RE. Overall, the AE can reconcile the monetary approach to the exchange rate and the data quite well, unlike the RE hypothesis.

According to RE, the predicted exchange rate is too volatile. The explanation this is as follows. According to RE, the equation describing the exchange rate dynamic is: $\Delta s_t = (1/\alpha)l_{t-1} + u_t$. Therefore, volatility of the predicted rational expectations exchange rate can be explained by the high volatility of $l_t$.

Restrictions on RE and AE enable us to forecast the future movements of the exchange rate using out-of sample estimates. The procedure employed is as follows. The first 40 observations are used to make the first out of sample predictions, then the first 41 observations are used to predict the second out of sample observations, et cetera. The fundamentals are calculated for every additional sample point as well. The results are shown in Figures 7-9. It can be seen that out of sample forecast works well in case of AE, and performs poorly in case of RE.
The estimation is also done for a case where the fundamentals are estimated for the entire sample. The results, which are very similar to a case in which the fundamentals are re-estimated for every sample point, are not reported but are available upon request.
CHAPTER 6

EXCHANGE RATE DYNAMICS, EXPECTATIONS AND STRUCTURAL BREAKS

6.1 Introduction

This chapter employs exchange rate data to investigate whether or not recurrent shifts can reconcile the present value hypothesis with rational expectations and improve the predictions of present value models combined with adaptive expectations. When no breaks are assumed, the hypothesis for rational expectations has been strongly rejected in the literature (MacDonald and Young (1986), MacDonald and Taylor (1993), the present research Chapter 5). As has been shown in the previous chapter, adaptive expectations seem to explain the exchange rate dynamics fairly well, however, the estimates of the semielasticity of the interest rate in money demand function are somewhat lower then one would have expected.

The evidence of structural breaks in macroeconomics data is overwhelming. Below is a far from complete list of papers on instability of some important macroeconomic variables. Garbade (1977) shows that the money demand function is, most likely, unstable. Existence of structural changes in GNP has been shown, among others, by Perron (1989), Christiano (1992), Perron and Vogelsang (1992), Raj (1992) and Andrews (1992). Stock and Watson (1996) show
that the majority of macroeconomic data in general exhibit some degree of structural instability. Boughton (1987) shows that reduced-form exchange rate models are not stable. Papell (1997) finds that exchange rate data are consistent with the possibility of a structural break. When a composite monetary model is applied to the exchange rate Goldberg and Frydman (1996) find that there are shifts in the cointegrating vector.

The fact that the exchange rate and its fundamentals show strong signs of structural instability casts serious doubts as to the appropriateness of testing the present value hypothesis without taking into account the possibility of structural breaks. The main purpose of the current chapter is to try to fill this gap. The rest of the chapter is as follows. 6.2 will describe the data used for the model testing. 6.3 describes the results of structural breaks in the cointegrating relationship between the exchange rate and fundamentals. 6.4 shows the results of estimation of unrestricted VAR. 6.5 gives the results of testing the restrictions for rational and adaptive expectations for present value models. 6.6 discusses the results and compares the case with structural breaks to the results of the testing when no structural breaks are assumed.

6.2 Data

The data for this study are the same as the data used in Chapter 5. They relate to the monthly DM, UK pound and Japanese yen versus U.S. dollar exchange rates from March 1973 through November 1993. The exchange rates are all expressed as home currency per unit of foreign currency. All monetary aggregates are monthly M1 and the income measure is monthly industrial production.
6.3 Testing for Structural Breaks in the Cointegrating Relationship Between Exchange Rate and Fundamentals

It can be seen from the above that there is no single test in the literature that would be optimal when there are an unknown number of break at unknown times. An approach used in the literature for testing an unknown number of periodic shifts in the cointegrating vector at the unknown dates is to first use the CUSUM test (see Section 4.10) to determine whether a break point has occurred, and then the likelihood ratio-like test (Section 4.11) to determine its approximate location (Frydman (1983), Boughton (1987), Goldberg and Frydman (1996)). As has been shown in (Goldberg and Frydman (1996)) these procedures search the data recursively for the possibility of one or more break points, rather than relying on tests that require the choice of break points a priori. Another feature of the CUSUM test that it is valid for a model involving I(1) variables. The CUSUM test requires only that the OLS residuals of the equation in question be estimated consistently, which is the case with I(1) variables (Stock, 1987). The error in the cointegrating relationship between the exchange rate and its fundamentals is known to be serially correlated, which can distort the size of the test. Therefore, the CUSUM test for serially correlated disturbances (Kao and Ross (1995)) is used.

To test for the possibility of a number of break points in the cointegrating vector, the following strategy is employed. Starting at the beginning of the sample, March 1973, the first possible point of parameter instability is searched recursively using the CUSUM test and the likelihood ratio test. Once the first break point is found, the CUSUM and likelihood ratio tests are rerun with a subsample that starts immediately after the location of the first break point. Both the CUSUM test for recursive and the CUSUM test for OLS residuals were employed, and the
results of the tests were very similar. The results for the CUSUM test with recursive residuals are
reported. \( \pi \) in the range \([0.15 \ 0.85]\) is used for the likelihood-ratio like test.

The order for testing the structural breaks is as follows. First, the structural breaks in the
cointegrating relationship that correspond to the shifts in the monetary demand function
according to the monetary theory are found and proper fundamentals \( x_t \) are constructed. Second,
the structural breaks in \( \Delta x_t \) are considered. Third, the structural breaks in \( L_t \) are investigated.
After all the structural breaks are taken into account, the testing for rational and adaptive
expectations is performed.

From Table 9 and Figures 10-12 it can be seen that the tests for structural breaks
indicate that within the period from March 1973 through November 1993, Japan and Great
Britain underwent two and Germany underwent three structural breaks. Overall, if the structural
breaks are taken into account, the fundamentals describe the exchange rate’s movement much
better (Figures 10-12). The first series of breaks occurred between March 1976 and March 1977
for each country and coincides with the end of so called early floating period (1973-1977). The
major characteristic of the period is that there was no full confidence that the floating system
would survive, some people believed that the exchange rate regime would become fixed again.
Plus, the volume of government intervention for U.S., Japan, Great Britain and Germany during
that period was relatively small. However, by the end of 1976 and the beginning of 1977, it
became clear that the exchange rate would continue to float. Therefore, the breaks which
occurred in 1976-1977 could be indicative of the fact that the people reevaluated the demand for
currencies under the new floating regime.

The next structural break for the Dollar-Pound cointegrating relationship happened in
October-November 1982. The break occurred after the pound underwent significant appreciation
in 1979 -1981 and further depreciation in 1982; in the period between 1979 and 1981 the
attempts by the Thatcher administration to reduce the UK money supply at times led to very high real interest rates in the UK relative to the USA and this in turn led to a sharp appreciation of the nominal sterling exchange rate. This structural break, indicating the change in the relative demand for pounds versus dollars could also be linked to the Falklands/Malvinas conflict between Great Britain and Argentina (March - June 1982).

The period between 1977 and 1987 can in general be described as a period when the governments began participating more intensively in financing interventions. During this period, the governments conducted foreign exchange operations primarily to counter disorderly markets. Disorderly markets are characterized by sharp exchange rate movements, thin trading, and wide spreads between the rates at which market participants are willing to buy and sell. However, in general, the period is characterized by wide deviations of exchange rates from their fundamentals, and toward the end of this period, the governments began to intervene not only to calm the markets, but also to push the currencies towards their fundamental values. The major episode of U.S. intervention in 1985 occurred after the meeting of the five industrialized G-5 nations at the Plaza Hotel in September (the G-5 includes the United States, Japan, Germany, France and United Kingdom). As a result of this policy by early 1987 the dollar had fallen to its lowest level in seven years.

At a meeting on February 1987 at the Louvre in Paris, G-7 officials (the G-7 includes the G-5 countries plus Canada and Italy) reevaluated the exchange rate fundamentals and decided that exchange rates reflected economic fundamentals relatively well. As a result of it, the officials decided to cooperate closely to foster stability of exchange rates around current levels. It is not surprising that the tests show that the second break in Dollar-Yen cointegrating relationship happened in February - March 1987 and the break in the Mark-Dollar exchange rate in October-November 1987. The break in Mark-Dollar exchange rate might also reflect the
October stock market crash and the associated easing of monetary policy by the Federal Reserve, which put downward pressure on the dollar.

The Louvre Accord has guided international cooperation in exchange rate policy to the end of 1993 (the end of the sample). It is not surprising, that only Germany has a break in 1990 which coincides in time with the reunification of the country (October 1990).

The results are consistent with some of the previous work on the subject. Boughton (1987), using different models and monthly exchange rate data for 1973-1984, shows that Deutsche mark and Japanese yen exchange rates have shifts around 1975-1979 (time varies for different models). Papell (1997) applies Dornbusch’s (1976) model to the DM exchange rate and tests for a single shift in the resulting cointegrating relationship using a similar technique. Using the data of the same length as in this paper, he finds that the most plausible time for a single shift is January 1985. Goldberg and Frydman (1996) test a cointegrating relationship between the exchange rate and fundamentals for multiple breaks for DM-Dollar within a framework of a Dornbusch model for a sample from March 1973 to March 1988. They also find two breaks, however their tests indicate that the breaks occurred in November 1979 and September 1984.

A structural break at time t changes the cointegrating relationship between the fundamentals and exchange rate and therefore changes the fundamentals $x_t$ from $x_t^{\text{old}}$ to $x_t^{\text{new}}$. Therefore the value $(s_t - x_t)$ and $\Delta s_{t+1}^e$ will change at the time of structural breaks as well ($(s_t - x_t) = \alpha \Delta s_{t+1}^e$). Denote $\Delta s_{t+1}^{\text{old}}$ as $\Delta s_{t+1}^{\text{old}} = 1/\alpha (s_t - x_t)_{\text{old}}$ and $\Delta s_{t+1}^{\text{new}}$ as $\Delta s_{t+1}^{\text{new}} = 1/\alpha (s_t - x_t)_{\text{new}}$. If $(\Delta s_{t+1}^{\text{new}} - \Delta s_{t+1}^{\text{old}})$ is greater than zero, it means that the structural break in the money demand function is favorable for the foreign country currency, and if $(\Delta s_{t+1}^{\text{new}} - \Delta s_{t+1}^{\text{old}})$ is less than zero, then it is favorable for the home country currency at time t. All the breaks but one (for the British pound in November 1982) were favorable for the foreign currencies. According to
(2.7), if \( \eta \) is small, \( s_{t+1} \) does not change much with the changes in \( s_t \). As \( \eta \) is getting close to zero, \( s_{t+1} \) approaches \((s_t + \epsilon_t)\). Thus, the time of structural breaks can be also considered as the time at which \( s_t \) undergoes significant changes as well, whereas between the breaks \( s_{t+1} \) on average is not significantly different from \( s_t \). Therefore, the time of a structural break will also be the time when the shock \( \epsilon_t \) in (2.7) is relatively large.

### 6.4 Estimation of the Unrestricted VAR

The first step in VAR estimation is to find the optimal number of lags, in this case by minimizing Schwarz (SC) and Akaike (AIC) information criteria (see section 4.5.2 for a brief description of these procedures). The results of the tests are given in Table 10. A lag of two is chosen for all future tests.

For the rest of the tests, it is assumed that \( \Delta x_t \) and \( L_t \) follow second order VAR:

\[
\Delta x_t = b_0 + \delta_1 \Delta x_{t-1} + \kappa_1 L_{t-1} + \delta_2 \Delta x_{t-2} + \kappa_2 L_{t-2} + \epsilon_{1t}
\]

\[
L_t = a_0 + \gamma_1 \Delta x_{t-1} + \mu_1 L_{t-1} + \gamma_2 \Delta x_{t-2} + \mu_2 L_{t-2} + \epsilon_{2t}
\]

(6.1)

First, the test for structural breaks in \( \Delta x_t \) and \( L_t \) have been performed (Sections 4.10 and 4.11). The results show no breaks in \( \Delta x_t \) and \( L_t \) for Japan and Germany. However, for Great Britain, the test indicates that there is a structural break in \( \Delta x_t \) in January 1976, and a structural break in \( L_t \) in May 1976. Both breaks in \( \Delta x_t \) and \( L_t \) are very close in time to the shift in the cointegrating relationship (March 1976). Therefore, for the purpose of the rest of the tests, the data sample for Great Britain is divided into two parts: the first from March 1973 till January
1976, and the second from June 1976 till the end of the sample. The first part will be referred to as "early floating", and the latter as "mature floating".

The results for the unrestricted VAR are given in Table 11. The Lagrange multiplier (LM) tests for first- and third-order residual serial correlation and for third-order residual ARCH are implemented (Section 4.5.3). The null hypothesis is that there is no serial correlation. P-values refer to the marginal significance levels for a one tail test. It is further assumed that the error term in the unrestricted VAR equation is white noise.

The results in Table 11 show that the process for fundamentals $\Delta x_t$ are similar for all the countries except the early floating period for Great Britain. The majority of countries have the following similarities: the fundamentals at time $t$ negatively depend on the fundamentals at time $t-1$, and their dependence on $L_{t-1}$ is weak. However, for the period of early floating in Great Britain, the fundamentals depend positively on the fundamentals a period before, and their dependence on $L_{t-1}$ is much stronger. The process for $L_t$, is also similar for all the countries except for the early floating period in Great Britain. $L_t$, for the majority of the countries, are highly persistent, and depend positively on the last period fundamentals. The process for $L_t$ for the early floating period in Great Britain is much less persistent.

### 6.5 Testing the expectations restrictions

Table 12 shows the results of the LR test (Section 4.9) for the null hypothesis that expectations are rational. Maximum likelihood estimation yields an estimate for $\alpha$ which is negative when no restrictions are placed upon it. Therefore, it can be concluded that the
structural breaks cannot salvage rational expectations. When the sign of $\alpha$ is restricted to positive values, the optimization for BFGS does not converge: the parameter $\alpha$ estimated within the model goes to infinity. Therefore, it can be seen that the rejection of RE in the previous chapter cannot be explained by the fact that the structural breaks have not been taken into account.

The results of the testing for AE null are given in Table 13. The estimator for $\beta$ has plausible values between zero and one and the estimator for $\alpha$ is positive, as it predicted by theory. For the case with no structural breaks (Chapter 5), the interest rate semielasticity for adaptive expectations is found to be around 0.01, which is smaller than expected. Previous work on estimating money demand function has shown that monthly interest rate semielasticity should be expected to be around 30-40. As it can be seen from Table 13, the interest rate semielasticity estimated when the structural breaks are taken into account falls pretty much within the expected region. The hypothesis that the data are consistent with the present value models, adaptive expectations and the structural breaks in the money demand function cannot be rejected at the 1% significance level for all the countries.

6.6 Discussion: Structural Breaks Case versus the Case with no Structural Breaks

As it can be seen from the results of the tests described in the previous section, exchange rate data in general do not agree with the present value hypothesis and rational expectations, even if the possible structural breaks in preferences, expectations and policy are taken into account. The interest rate semielasticity of the money demand function $\alpha$, estimated with RE is negative, and no convergence is achieved if $\alpha$ was assumed greater than zero.
However, when combined with AE, the present value model seems to describe the exchange rate dynamics fairly well. The null of AE is not rejected for all the countries at the conventional levels of significance. Plus, the estimates of the model parameters are within the expected range.

The structural breaks in the cointegrating relationship between the exchange rate and fundamentals seem to indicate the shifts in preferences for one currency relative to another. However, the reason why these shifts has occurred is not clear in the majority of cases. When shifts in preferences occur, the cointegrating relationship between the exchange rate and the fundamentals undergoes a shift, which has to be taken into account if testing for the present value models.

Adaptive expectations imply the following behavior for the exchange rates. Adaptive expectations are expectations which can be described by equation (2.7): $y_{t-1}^e - y_t^e = \eta (y_t - y_t^e) + e_t$. $\eta$ is the adjustment factor which indicates the relative emphasis that the agent places on the current realization and the past expectation. As $\eta \to 1$, more weight is given to the current realization. An adjustment factor of one is often referred to as naïve expectations. As $\eta \to 0$, the agent increasingly disregards the current realization. An adjustment factor of zero implies that expectations are never updated. The parameters $\eta$'s found in the current research is equal 0.065 for Japan, 0.043 for the later period for Britain and 0.051 for Germany. If, indeed, expectations are adaptive, and the parameters $\eta$'s are similar to those found in this research, it means that the agents have some value of the exchange rate in mind to which they expect it to converge, and update this value very slowly.
If the expectations are not rational, it implies that there is a possibility of arbitrage in the exchange rate market. To see what kind of arbitrage opportunities exist, rewrite the equation (2.7) applied to the exchange rate as:

\[ s_{t+1}^e = (1 - \eta) s_t^e + \eta s_t + e_t. \]  

(2.7a)

Each time the exchange rate exceeds the expected value at time t (the home currency depreciates too much compared with the agents' expectations), the agents expect the exchange rate to go back to this value (expect the home currency to appreciate) in the period t+1 (assuming that on average the expected value for \( e_t \) is equal to zero). However the actual exchange rate at time t+1 will most likely be higher than expected. Therefore, at time t the currency is overvalued (\( s_t \) is lower than its equilibrium value \( x_t \)) and you can make money by selling home currency and buying foreign currency. If at time t the exchange rate goes down (home currency appreciates) by more than the agents expected, then the agents expect the exchange rate to go up in the next period (expect the currency to depreciate). Therefore, at time t the currency at time t+1 is undervalued (\( s_t \) is above \( x_t \)), and on average it is possible to make money by buying the home currency and selling the foreign currency.

Shortly, the trading strategy above can be described as: buy “winners” (currency which recently appreciated) and sell “losers” (currency which depreciated). Receiving significantly high returns based on a particular strategy in not new, for example, for the stock market. It has been shown (Jegadeesh, and Titman, 1993), who actually use the terminology “winners” and “losers” applied to the stock market) that in the stock market such a strategy realized significant abnormal returns over the period 1965-1989 which cannot be explained by systematic risk. The authors attribute their results to delayed stock price reactions. As shown in Section 2.2, present value
models can be applied to stock market price movements. Therefore, adaptive expectations may explain some peculiarities of the stock market behavior as well.

According to adaptive expectations, during exchange rate appreciation, the people would most likely expect the exchange rate to depreciate and visa versa. From the monetary approach to the exchange rate it follows that \( s_t - x_t = \alpha \Delta s_{t+1}, \) where \( \alpha \) is positive. Therefore, during periods of appreciation or depreciation the values of the exchange rate will deviate from its equilibrium value \( x. \) As has been shown in the previous paragraph, the smaller \( \eta, \) the longer it takes for the exchange rate to converge to its equilibrium value \( x. \) Empirically, it has been shown in the literature, that the time period during which the exchange rates deviate from their equilibrium values can be quite significant. The small values of \( \eta \) found in this research may explain these quite persistent deviations.

The presence of structural breaks may help to explain why the tests show that agents’ expectations are consistent with AE. The current model assumes that, after the break has occurred, agents have perfect information about the break, including when it occurred and what the new parameters of the model are. In reality, it could be that at the time of the break agents do not have full information about the changes in the fundamentals. Therefore, they try to extract this information from the data and learn about the structural change adaptively. The time between breaks may not be enough for the system to converge to the rational expectations equilibrium. Hence, throughout the sample, agents exhibit adaptive learning behavior which, when being tested in present value model framework, is more consistent with adaptive than rational expectations.

An argument can be made that, if a researcher can determine a time of a break from the data, the agents should have this information as well. However, the tests for structural breaks are

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designed in a such way that the possibility that a break has occurred is estimated when the information about the whole sample is known. If agents do not have full information about the break, they might not immediately realize that the break has occurred. Therefore, it seems interesting to see if the tests for structural breaks show the existence of break points near the time of the break when only the information about the past is used. The time when a break can first be detected could be thought of as the time when the agents first realize that the break has occurred. If a break can be detected relatively soon after it has occurred, then it is plausible to assume that the agents know about the break as well.

The tests are rerun as follows. A subsample, which starts immediately after the start of a break and ends when the next break occurs (time t), is tested for the presence of structural breaks (the first subsample starts with the first observation) for all three countries. If the data do not indicate the presence of a structural break, then the tests are rerun with one more observation (at time (t+1)) added to the subsample. This procedure is repeated until the data show the presence of a structural break. The last observation in such subsamples can be thought of as the time when the agents realize that the structural break has taken place.

The results are as follows. The first break for Japan happened in July 1976, however the earliest it can be detected from the data is 10 months later, in May 1977. The first break for Great Britain occurred in March 1976, but can be detected only 9 months later, in December 1976. The first break for Germany occurred in March 1977 and can be detected 4 months later, in July 1977. However, the rest of the breaks can be detected immediately after they occur. Therefore, it might be plausible to assume that, after the agents know about the break, they still do not know the new parameters of the model. By using adaptive learning they are trying to learn about the parameters of the structural break. As a result of this learning, the adaptive
expectations hypothesis combined with the present value model can describe the empirical data better than the rational expectations hypothesis.
CHAPTER 7

CONCLUSION

The primary purpose of the current research has been to study the present value models for exchange rate determination. Briefly, the main findings of the research are:

1. The Campbell and Shiller restrictions which are known in the literature as forward-looking restrictions do not test just forward-looking restrictions, but any forward or backward looking solution and the continuum of their linear combinations.

2. In general, it is impossible to test a particular rational expectations solution within a VAR framework.

3. Rational expectations present value models restrictions are strongly rejected for three countries by exchange rate data for plausible values of the model parameters.

4. The importance of specifying the right interest rate semielasticity in money demand function for testing implications of present value models to the exchange rate is shown.

5. Adaptive expectations present value models restrictions for I(1) variables are developed.
6. Adaptive expectations restrictions cannot be rejected by the exchange rate data for three countries. However, the estimate for interest rate semielasticity, obtained as a result of such restrictions, is somewhat smaller that one would expect.

7. Different sources of structural breaks in the exchange rate implied by present value models are considered.

8. It is shown that the data are consistent with the hypothesis that the cointegrating relationship between the exchange rate and fundamentals undergoes structural breaks. From the present value model point of view, such shifts would indicate shifts in preferences for one currency relative to another.

9. It is impossible to reconcile the rational expectation and present value approach to the exchange rate even taking structural breaks into account.

10. The combined hypothesis of adaptive expectations, structural breaks and present value models to the exchange rate cannot be rejected for any of the three countries at the conventional significance levels. The estimate for interest rate semielasticity, obtained as a result of such restrictions, is within the expected range. It is shown that exchange rate behavior is consistent with present value models, shifts in preferences for one currency relative to another and adaptive expectations.

The paper starts with an analysis of the popular Campbell and Shiller test for forward looking rational expectations models in the form:

\[ L_t = \delta L_{t+1} \varepsilon + \delta \Delta x_{t+1} \varepsilon, \]  

(7.1)

It has been shown that the test does not discriminate between any kind of rational expectations solution to an equation (7.1) which can be either backward or forward looking, or a linear combination of both.
Because it does not test a forward looking type of solutions only, the Campbell and Shiller test does not have any obvious advantages over other present value model test. A rational expectations solution to the present value model equation can, for example, be tested by regressing \((L_{t+1} + \Delta x_{t+1})\) on \(L_t\), and restricting the coefficient on \(L_t\) to \(1/\delta\):

\[
(L_{t+1} + \Delta x_{t+1}) = 1/\delta \ L_t + \varepsilon_t
\]  

(7.2)

Testing the small sample size properties of the Campbell and Shiller test was outside the scope of this research. However, it seems that their test will be inferior to the test suggested by (7.2): in (7.2) there is no auxiliary hypothesis for \(\Delta x_{t+1}\) process, which exists in Campbell and Shiller test. Therefore, for future research it would be interesting to look at the small sample properties of different present value model tests and compare their performance. To date there has been no research done on this topic yet.

Another important topic which was discussed in this study is the presence of structural breaks in the exchange rate. The current paper is not the first paper which has paid attention to this problem. There has been substantial research done where the authors looked at either the breaks in the exchange rates or the variables which form the exchange rate’s fundamentals, and papers which dealt with the breaks in the cointegrating relationship between the exchange rates and fundamentals. The current research is the first attempt to look at the different sources of breaks in the exchange rates which are implied by the monetary model. The monetary model treats the exchange rate as the expected present value of future fundamentals. Therefore, the exchange rate dynamic depends on what exchange rate fundamentals are, the dynamic of these fundamentals and how people form expectations about their future values. The exchange rate fundamentals are determined from the money demand function equations for both countries. If the demand for money in one country changes relative to the demand for money in another
country, then the cointegrating relationship between the exchange rate and fundamentals undergoes a structural break and the fundamentals of the exchange rate change. The research shows that reality seems to be consistent with such reoccurring shifts in relative preferences for different currencies.

The question of rational versus adaptive expectations has been considered. The rejection of RE both when structural breaks are and are not taken into account casts serious doubt on the appropriateness of using RE as the default type of expectations for economic models without further justifications, especially when a simple quadratic utility function is used. Rejection of the Campbell and Shiller restrictions means rejection of any rational expectation solution for people with a quadratic utility function. It further means that people behave irrationally in the sense that they reconcile with permanent arbitrage. However, there could be some other utility functions for which the presence of such arbitrage could be rational. For example, in the recent Wall Street Journal survey, traders from around the country were asked if they considered themselves rational and the answer was “Yes.” However, when asked if their behavior is affected by what other traders do or did, they answered that they much prefer to be wrong with other traders, than by themselves. Thus, sometimes they are making decisions based not on their assessment of the situation, but on what they think the other traders may do, even if they expect it to be wrong. If this feature of traders’ psychology could be incorporated into traders’ utility function, then combined with rational expectations it may describe reality fairly well. Therefore, the conclusion of the current research is not to show that the world consists of the people with simple utility functions who form their expectations adaptively, but rather that the world is populated with complex people who behave to maximize their utility function which may be very difficult to model. The current research suggests that a simple utility function and adaptive expectations may be just a better approximation of reality. The research also confirms the fact
that imposing the assumption of rational expectations on a correct present value model may lead to quite implausible estimates of the parameters of the model, and therefore, reject the model itself. At the same time, combined with adaptive expectations, the model can explain the data reasonably well.

Overall, the fact that the exchange rate data seem to be consistent with the present value model and AE supports some other research which shows that people’s behavior is consistent with a case in which people mistake permanent shocks in the exchange rate for transitory shocks, thus expecting the currency go up in value when the currency depreciates and expecting the currency to depreciate when the currency appreciates. Adaptive expectations can also explain the negative relationship between $\Delta s_i$ and $\Delta s_i^e$, the usual result of a standard “Fama” regression.

A good example of people mistaking permanent shocks for transitory shocks is the economic crisis in South America in 1980’s, where the people in charge of economic policies mistakenly thought that the increase in oil prices was transitory and behaved correspondingly. Another example could be the recent appreciation of the dollar (the end of 1996-1997); according to the Wall Street Journal, a lot of international companies have suffered losses because of a hedging strategy aimed at obtaining protection from depreciation of the dollar.

The findings of the current research may be applied to other models of exchange rate dynamics as well. For example, for target zone models, it may be applied by making assumptions about people’s response to Central Bank intervention. They may overestimate the degree of the Central Bank’s reaction and, as a result, the exchange rate is not as mean-reversed as they think it is. When the band’s “realignment” occurs, it may coincide with a structural breaks in fundamentals (if the government now targets different value for the exchange rate, or people
believe it does, the people’s demand for one currency in terms of the other may change, changing the relationship between the exchange rate and the fundamentals as well).

As another direction for future research, it would be interesting to see if adaptive learning which takes the structural breaks into account can describe the data better than a pure adaptive or rational mechanism. If the agents know that the break has occurred, but do not know the new parameters of the model, they will be using adaptive learning in order to learn about these parameters. A Kalman filter-based variable parameter learning framework that takes structural breaks into account could be used for this study. However, analysis of these models remains open as future research that would have to proceed along these lines.
<table>
<thead>
<tr>
<th></th>
<th>ADF t-stat (no trend)*</th>
<th>ADF t-stat (trend)**</th>
<th>PP t-stat (no trend) *</th>
<th>PP t-stat (trend) **</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>raw money data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>0.578</td>
<td>-2.345</td>
<td>1.098</td>
<td>-1.843</td>
</tr>
<tr>
<td>Japan</td>
<td>-2.382</td>
<td>-2.302</td>
<td>-2.825</td>
<td>-3.012</td>
</tr>
<tr>
<td>Great Britain</td>
<td>-2.334</td>
<td>-2.320</td>
<td>-7.760</td>
<td>-2.237</td>
</tr>
<tr>
<td>Germany</td>
<td>0.118</td>
<td>-1.540</td>
<td>0.855</td>
<td>-1.40</td>
</tr>
<tr>
<td><strong>raw output data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td>-0.218</td>
<td>-3.344</td>
<td>-0.512</td>
<td>-3.105</td>
</tr>
<tr>
<td>Japan</td>
<td>-0.982</td>
<td>-3.081</td>
<td>-0.745</td>
<td>-1.836</td>
</tr>
<tr>
<td>Great Britain</td>
<td>-0.664</td>
<td>-2.703</td>
<td>-0.855</td>
<td>-3.132</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.875</td>
<td>-2.480</td>
<td>-0.994</td>
<td>-2.979</td>
</tr>
<tr>
<td><strong>raw exchange rate data</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-2.364</td>
<td>-2.364</td>
<td>-0.193</td>
<td>-2.325</td>
</tr>
<tr>
<td>Great Britain</td>
<td>-1.915</td>
<td>-2.030</td>
<td>-1.979</td>
<td>-2.148</td>
</tr>
<tr>
<td>Germany</td>
<td>-1.202</td>
<td>-1.647</td>
<td>-1.677</td>
<td>-1.949</td>
</tr>
</tbody>
</table>

* probability that the statistics for a sample size of 250 is less than -2.57 is 10%
** probability that the statistics for a sample size of 250 is less than -3.13 is 10%

Table 1: Studentized coefficient for ADF and Phillips-Perron test of unit root test for fundamentals ΔX and cointegrating error L using Newey-West Estimator
<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>-0.265</td>
<td>-1.932</td>
<td>-1.293</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.012</td>
<td>0.019</td>
<td>0.032</td>
</tr>
<tr>
<td>$a_1$</td>
<td>-1.921</td>
<td>-1.565</td>
<td>0.473</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.146</td>
<td>0.166</td>
<td>0.315</td>
</tr>
<tr>
<td>$N(0,1)$</td>
<td>-0.554</td>
<td>-0.337</td>
<td>0.394</td>
</tr>
</tbody>
</table>

Table 2: Coefficients for the cointegrating relationship between exchange rate and fundamentals found by OLS and corrected for correlation $N(0,1)$ statistics of the null hypothesis that income elasticity is equal to -1.
<table>
<thead>
<tr>
<th></th>
<th>ADF t-stat (no trend)*</th>
<th>ADF t-stat (trend)**</th>
<th>PP t-stat (no trend)</th>
<th>PP t-stat (trend)</th>
<th># lags</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta x_t$ ($\Delta x_t = \Delta((m_t - m_t^<em>) - a_t (y_t - y_t^</em>))$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-15.75</td>
<td>-15.74</td>
<td>-20.73</td>
<td>-20.74</td>
<td>4</td>
</tr>
<tr>
<td>Germany</td>
<td>-11.35</td>
<td>-11.33</td>
<td>-16.04</td>
<td>-16.04</td>
<td>4</td>
</tr>
<tr>
<td>$L_t$    ($L_t = (s_t - x_t)$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>-1.32</td>
<td>-1.55</td>
<td>-1.67</td>
<td>-1.91</td>
<td>4</td>
</tr>
<tr>
<td>Great Britain</td>
<td>-1.58</td>
<td>-1.74</td>
<td>-1.62</td>
<td>-1.82</td>
<td>4</td>
</tr>
<tr>
<td>Germany</td>
<td>-1.36</td>
<td>-1.46</td>
<td>-1.52</td>
<td>-1.57</td>
<td>4</td>
</tr>
</tbody>
</table>

• probability that the statistics for a sample size of 250 is less than -2.57 is 10%
  ** probability that the statistics for a sample size of 250 is less than -3.13 is 10%

Table 3: Studentized coefficient for ADF and Phillips-Perron test of unit root test for fundamentals $\Delta X$ and cointegrating error $L$ using Newey-West Estimator
<table>
<thead>
<tr>
<th>Lag(I)</th>
<th>Cointegrating relationship</th>
<th>When income elasticity is 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SC</td>
<td>AIC</td>
</tr>
<tr>
<td>3</td>
<td>-13.11</td>
<td>-13.29</td>
</tr>
<tr>
<td>4</td>
<td>-13.04</td>
<td>-13.28</td>
</tr>
<tr>
<td>5</td>
<td>-12.96</td>
<td>-13.25</td>
</tr>
<tr>
<td>6</td>
<td>-12.87</td>
<td>-13.22</td>
</tr>
<tr>
<td>7</td>
<td>-12.79</td>
<td>-13.20</td>
</tr>
<tr>
<td>8</td>
<td>-12.71</td>
<td>-13.18</td>
</tr>
<tr>
<td>9</td>
<td>-12.66</td>
<td>-13.19</td>
</tr>
<tr>
<td>10</td>
<td>-12.63</td>
<td>-13.22</td>
</tr>
<tr>
<td>12</td>
<td>-12.64</td>
<td>-13.35</td>
</tr>
</tbody>
</table>

(Cont.)

Table 4: Choosing the optimal lag length for the VAR formed with fundamentals $\Delta X_t$ and cointegrating error $L$ (data for Japanese yen)
Table 4 (Con’t.)

<table>
<thead>
<tr>
<th>Lag(l)</th>
<th>Cointegrating relationship</th>
<th>When income elasticity is 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SC</td>
<td>AIC</td>
</tr>
<tr>
<td>0</td>
<td>-10.10</td>
<td>-10.10</td>
</tr>
<tr>
<td>8</td>
<td>-13.01</td>
<td>-13.49</td>
</tr>
<tr>
<td>9</td>
<td>-12.93</td>
<td>-13.46</td>
</tr>
<tr>
<td>10</td>
<td>-12.84</td>
<td>-13.43</td>
</tr>
<tr>
<td>11</td>
<td>-12.76</td>
<td>-13.41</td>
</tr>
<tr>
<td>12</td>
<td>-12.75</td>
<td>-13.45</td>
</tr>
</tbody>
</table>

(data for British pound)
Table 4 (Con’t.)

<table>
<thead>
<tr>
<th>Lag(l)</th>
<th>Cointegrating relationship</th>
<th>When income elasticity is 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SC</td>
<td>AIC</td>
</tr>
<tr>
<td>0</td>
<td>-10.82</td>
<td>-10.82</td>
</tr>
<tr>
<td>1</td>
<td>-14.57</td>
<td>-14.63</td>
</tr>
<tr>
<td>2</td>
<td>-14.51</td>
<td>-14.63</td>
</tr>
<tr>
<td>5</td>
<td>-14.27</td>
<td>-14.57</td>
</tr>
<tr>
<td>7</td>
<td>-14.11</td>
<td>-14.52</td>
</tr>
<tr>
<td>12</td>
<td>-13.79</td>
<td>-14.49</td>
</tr>
</tbody>
</table>

(data for German mark)
\[ \Delta X_t = b_0 + g_1 \Delta X_{t-1} + \kappa_1 L_{t-1} + g_2 \Delta X_{t-2} + \kappa_2 L_{t-2} + \varepsilon_{1t} \quad (\Delta X_t = \Delta ((m_t - m_t^*) - a_1 (y_t - y_t^})) \]

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_0 )</td>
<td>0.003</td>
<td>-0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.027</td>
<td>0.027</td>
<td>0.001</td>
</tr>
<tr>
<td>( g_1 )</td>
<td>-0.149</td>
<td>-0.084</td>
<td>-0.127</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.628</td>
<td>0.729</td>
<td>0.824</td>
</tr>
<tr>
<td>( \kappa_1 )</td>
<td>-0.616</td>
<td>-0.521</td>
<td>-0.365</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.483</td>
<td>0.540</td>
<td>0.497</td>
</tr>
<tr>
<td>( g_2 )</td>
<td>-0.050</td>
<td>-0.059</td>
<td>-0.014</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.630</td>
<td>0.738</td>
<td>0.808</td>
</tr>
<tr>
<td>( \kappa_2 )</td>
<td>0.613</td>
<td>0.508</td>
<td>0.358</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.491</td>
<td>0.547</td>
<td>0.499</td>
</tr>
</tbody>
</table>

LM test for first- and third-order residual serial correlation:

<table>
<thead>
<tr>
<th>( \chi_{[1]}^2 ) (t-1)</th>
<th>0.452</th>
<th>0.103</th>
<th>2.934</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.501</td>
<td>0.748</td>
<td>0.087</td>
</tr>
<tr>
<td>( \chi_{[1]}^2 ) (t-3)</td>
<td>0.081</td>
<td>0.076</td>
<td>1.644</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.776</td>
<td>0.782</td>
<td>0.200</td>
</tr>
<tr>
<td>( \chi_{[1]}^2 ) (t-6)</td>
<td>0.147</td>
<td>0.797</td>
<td>2.323</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.702</td>
<td>0.180</td>
<td>0.127</td>
</tr>
<tr>
<td>( \chi_{[3]}^2 ) (t-1, t-3, t-6)</td>
<td>1.272</td>
<td>3.780</td>
<td>5.564</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.736</td>
<td>0.286</td>
<td>0.135</td>
</tr>
</tbody>
</table>

LM test for third-order residual ARCH

| \( \chi_{[3]}^2 \) (t-1, t-3, t-6) | 1.151 | 3.703 | 5.489 |
| marginal level of sign.   | 0.765 | 0.295 | 0.139 |
| \( R^2 \)                    | 0.665 | 0.499 | 0.388 |

(Con’t.)

Table 5: Estimates for the VAR (using a cointegrating relationship)
Table 5 (Con’t.)

\[ L_t = a_0 + \gamma_1 \Delta X_{t-1} + \mu_1 L_{t-1} + \gamma_2 \Delta X_{t-2} + \mu_2 L_{t-2} + \varepsilon_{2t} \quad (L_t = (s_t - x_t)) \]

<table>
<thead>
<tr>
<th></th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>0.004</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>(SD)</td>
<td>0.057</td>
<td>0.056</td>
<td>0.043</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.249</td>
<td>0.106</td>
<td>0.163</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.316</td>
<td>1.355</td>
<td>1.623</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.826</td>
<td>0.968</td>
<td>0.858</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.012</td>
<td>1.804</td>
<td>0.978</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>-0.009</td>
<td>0.034</td>
<td>0.236</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.320</td>
<td>1.137</td>
<td>1.591</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>0.150</td>
<td>0.011</td>
<td>0.136</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.029</td>
<td>1.017</td>
<td>0.983</td>
</tr>
</tbody>
</table>

LM test for first- and third - order residual serial correlation:

<table>
<thead>
<tr>
<th>( \chi^2_{(t-1)} )</th>
<th>0.089</th>
<th>0.038</th>
<th>0.342</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.765</td>
<td>0.846</td>
<td>0.558</td>
</tr>
<tr>
<td>( \chi^2_{(t-3)} )</td>
<td>0.762</td>
<td>0.903</td>
<td>0.552</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.383</td>
<td>0.342</td>
<td>0.458</td>
</tr>
<tr>
<td>( \chi^2_{(t-6)} )</td>
<td>0.483</td>
<td>2.409</td>
<td>3.462</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.487</td>
<td>0.121</td>
<td>0.063</td>
</tr>
<tr>
<td>( \chi^2_{(t-1, t-3, t-6)} )</td>
<td>2.952</td>
<td>3.663</td>
<td>3.912</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.399</td>
<td>0.300</td>
<td>0.271</td>
</tr>
</tbody>
</table>

LM test for third - order residual ARCH

<table>
<thead>
<tr>
<th>( \chi^2_{(t-1, t-3, t-6)} )</th>
<th>2.039</th>
<th>3.383</th>
<th>3.406</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.564</td>
<td>0.336</td>
<td>0.333</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.926</td>
<td>0.943</td>
<td>0.972</td>
</tr>
</tbody>
</table>

(Con’t.)

(using a cointegrating relationship)
Table 5 (Con’t.)

\[
\Delta X_t = b_0 + 9 \Delta X_{t-1} + \kappa_1 L_{t-1} + 9 \Delta X_{t-2} + \kappa_2 L_{t-2} + \varepsilon_{1t} \quad (\Delta X_t = \Delta((m_t - m_{t-1}^*) - a_t (y_t - y_t^*)))
\]

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_0)</td>
<td>0.002</td>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.02</td>
<td>0.023</td>
<td>0.021</td>
</tr>
<tr>
<td>(\delta_1)</td>
<td>-0.226</td>
<td>-0.098</td>
<td>-0.126</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.776</td>
<td>0.866</td>
<td>0.839</td>
</tr>
<tr>
<td>(\kappa_1)</td>
<td>0.391</td>
<td>-0.291</td>
<td>-0.335</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.490</td>
<td>0.484</td>
<td>0.487</td>
</tr>
<tr>
<td>(\delta_2)</td>
<td>-0.103</td>
<td>-0.064</td>
<td>-0.009</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.788</td>
<td>0.878</td>
<td>0.825</td>
</tr>
<tr>
<td>(\kappa_2)</td>
<td>0.392</td>
<td>0.277</td>
<td>0.328</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.453</td>
<td>0.489</td>
<td>0.489</td>
</tr>
</tbody>
</table>

LM test for first and third - order residual serial correlation:

<table>
<thead>
<tr>
<th>(\chi^2_{(1)}(t-1))</th>
<th>0.969</th>
<th>0.137</th>
<th>2.398</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.325</td>
<td>0.711</td>
<td>0.121</td>
</tr>
<tr>
<td>(\chi^2_{(1)}(t-3))</td>
<td>0.247</td>
<td>0.585</td>
<td>0.908</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.619</td>
<td>0.444</td>
<td>0.341</td>
</tr>
<tr>
<td>(\chi^2_{(1)}(t-6))</td>
<td>0.075</td>
<td>0.802</td>
<td>1.893</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.785</td>
<td>0.370</td>
<td>0.168</td>
</tr>
<tr>
<td>(\chi^2_{(3)}(t-1, t-3, t-6))</td>
<td>1.076</td>
<td>5.107</td>
<td>4.880</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.783</td>
<td>0.164</td>
<td>0.181</td>
</tr>
</tbody>
</table>

LM test for third - order residual ARCH

<table>
<thead>
<tr>
<th>(\chi^2_{(3)}(t-1, t-3, t-6))</th>
<th>0.981</th>
<th>4.990</th>
<th>4.817</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.806</td>
<td>0.173</td>
<td>0.186</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.465</td>
<td>0.282</td>
<td>0.358</td>
</tr>
</tbody>
</table>

(when income elasticity is equal to -1)
Table 5 (Con't.)

\[ L_t = a_0 + \gamma_1 \Delta X_{t-1} + \mu_1 L_{t-1} + \gamma_2 \Delta X_{t-2} + \mu_2 L_{t-2} + \varepsilon_{2t} \quad (L_t = (s_t - x_t)) \]

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_0)</td>
<td>0.002</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.042</td>
<td>0.047</td>
<td>0.042</td>
</tr>
<tr>
<td>(\gamma_1)</td>
<td>0.308</td>
<td>0.056</td>
<td>0.169</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.601</td>
<td>0.786</td>
<td>1.687</td>
</tr>
<tr>
<td>(\mu_1)</td>
<td>0.916</td>
<td>1.046</td>
<td>0.871</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.012</td>
<td>0.998</td>
<td>0.978</td>
</tr>
<tr>
<td>(\gamma_2)</td>
<td>0.035</td>
<td>-0.014</td>
<td>0.246</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.627</td>
<td>1.810</td>
<td>1.658</td>
</tr>
<tr>
<td>(\mu_2)</td>
<td>0.068</td>
<td>-0.062</td>
<td>0.123</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.017</td>
<td>1.009</td>
<td>0.982</td>
</tr>
</tbody>
</table>

LM test for first-and third - order residual serial correlation:

<table>
<thead>
<tr>
<th>(\chi_{[1]}^2) (t-1)</th>
<th>0.718</th>
<th>0.005</th>
<th>5.542</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.397</td>
<td>0.945</td>
<td>0.019</td>
</tr>
<tr>
<td>(\chi_{[1]}^2) (t-3)</td>
<td>0.657</td>
<td>0.712</td>
<td>0.152</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.417</td>
<td>0.399</td>
<td>0.697</td>
</tr>
<tr>
<td>(\chi_{[1]}^2) (t-6)</td>
<td>0.909</td>
<td>1.923</td>
<td>3.111</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.340</td>
<td>0.165</td>
<td>0.078</td>
</tr>
<tr>
<td>(\chi_{[3]}^2) (t-1, t-3, t-6)</td>
<td>2.148</td>
<td>3.097</td>
<td>3.453</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.542</td>
<td>0.377</td>
<td>0.327</td>
</tr>
</tbody>
</table>

LM test for third - order residual ARCH

<table>
<thead>
<tr>
<th>(\chi_{[3]}^2) (t-1, t-3, t-6)</th>
<th>1.699</th>
<th>2.929</th>
<th>2.966</th>
</tr>
</thead>
<tbody>
<tr>
<td>marginal level of sign.</td>
<td>0.637</td>
<td>0.403</td>
<td>0.397</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.959</td>
<td>0.961</td>
<td>0.972</td>
</tr>
</tbody>
</table>

(income elasticity is equal to -1)
### Using a Cointegrating Relationship

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{[M]}$)</td>
<td>119.7</td>
<td>59.63</td>
<td>60.61</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.00</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>-19.06</td>
<td>-19.81</td>
<td>-35.31</td>
</tr>
<tr>
<td>(SE)</td>
<td>$1 \times 10^4$</td>
<td>$5 \times 10^4$</td>
<td>$10^5$</td>
</tr>
</tbody>
</table>

### When Income Elasticity is Equal to -1

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{[M]}$)</td>
<td>51.34</td>
<td>12.27</td>
<td>51.99</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.000</td>
<td>0.007</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>-34.79</td>
<td>-35.21</td>
<td>-36.89</td>
</tr>
<tr>
<td>(SE)</td>
<td>$1 \times 10^4$</td>
<td>$1 \times 10^4$</td>
<td>$1 \times 10^4$</td>
</tr>
</tbody>
</table>

$^a\alpha$ is the semi-elasticity of the interest rate (equation (1)):

$$s_t - x_t = \alpha E_t \Delta s_{t+1},$$

(1)

Table 6: Testing of unconstrained model against rational expectations null
using a cointegrating relationship

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{[3]}$)</td>
<td>597.2</td>
<td>622.7</td>
<td>853</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>1.355</td>
<td>1.272</td>
<td>1.101</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.326</td>
<td>0.249</td>
<td>0.094</td>
</tr>
</tbody>
</table>

when income elasticity is equal to -1

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{[3]}$)</td>
<td>743.7</td>
<td>826</td>
<td>831</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>1.203</td>
<td>1.155</td>
<td>1.167</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.189</td>
<td>0.134</td>
<td>0.120</td>
</tr>
</tbody>
</table>

\(\alpha^*\) is the semielasticity of the interest rate (equation (1)):

\[ s_t - x_t = \alpha E_t \Delta s_{t+1}, \tag{1} \]

Table 7: Testing of unconstrained model against rational expectations null when $\alpha$ is bounded to be positive
Using a Cointegrating Relationship

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{[3]}$)</td>
<td><strong>15.04</strong></td>
<td>1.546</td>
<td>10.53</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.002</td>
<td>0.672</td>
<td>0.015</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>0.010</td>
<td>0.022</td>
<td>0.0015</td>
</tr>
<tr>
<td>(SE)</td>
<td>2.293</td>
<td>3.707</td>
<td>5.586</td>
</tr>
<tr>
<td>$\eta^{**}$</td>
<td>0.028</td>
<td>0.024</td>
<td>0.017</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.105</td>
<td>0.108</td>
<td>0.068</td>
</tr>
</tbody>
</table>

When Income Elasticity is Equal to -1

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Japanese Yen</th>
<th>British Pound</th>
<th>German Mark</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR, ($\chi^2_{[3]}$)</td>
<td>10.86</td>
<td>0.613</td>
<td>9.393</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.013</td>
<td>0.894</td>
<td>0.025</td>
</tr>
<tr>
<td>$\alpha^a$</td>
<td>0.015</td>
<td>0.015</td>
<td>0.018</td>
</tr>
<tr>
<td>(SE)</td>
<td>6.665</td>
<td>14.56</td>
<td>6.6</td>
</tr>
<tr>
<td>$\eta^{**}$</td>
<td>0.075</td>
<td>0.230</td>
<td>0.0038</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.138</td>
<td>0.611</td>
<td>0.057</td>
</tr>
</tbody>
</table>

$^a$ $\alpha$ is the semielasticity of the interest rate (equation (1)): $s_t - x_t = \alpha E_t \Delta s_{t+1},$ \hspace{1cm} (1)

$^{**}$ $\eta$ is the adaptive expectation coefficient (equation (2)): $E_t s_{t+1} - E_{t+1} s_t = \eta \eta (s_t - E_{t+1} s_t) + e_t$ \hspace{1cm} (2)

Table 8: Testing of unconstrained model against the adaptive expectations null using a cointegrating relationship
<table>
<thead>
<tr>
<th>Time of the Regimes</th>
<th>$a_0$ (Standard Error)</th>
<th>$a_1$ (Standard Error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03.73 - 07.76</td>
<td>4.011 (0.200)</td>
<td>-0.387 (1.503)</td>
</tr>
<tr>
<td>08.76 - 02.87</td>
<td>4.209 (0.153)</td>
<td>0.599 (2.101)</td>
</tr>
<tr>
<td>03.87 - 11.93</td>
<td>4.604 (0.109)</td>
<td>-0.008 (1.311)</td>
</tr>
<tr>
<td>Great Britain</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03.73 - 03.76</td>
<td>2.833 (0.148)</td>
<td>-0.610 (0.705)</td>
</tr>
<tr>
<td>04.76 - 10.82</td>
<td>3.328 (0.325)</td>
<td>1.764 (2.581)</td>
</tr>
<tr>
<td>11.82 - 11.93</td>
<td>2.631 (0.149)</td>
<td>-0.067 (2.761)</td>
</tr>
<tr>
<td>Germany</td>
<td></td>
<td></td>
</tr>
<tr>
<td>03.73 - 03.77</td>
<td>2.780 (0.503)</td>
<td>-1.937 (3.112)</td>
</tr>
<tr>
<td>04.77 - 10.87</td>
<td>2.958 (0.471)</td>
<td>-3.012 (5.195)</td>
</tr>
<tr>
<td>11.87 - 07.90</td>
<td>3.331 (0.071)</td>
<td>-1.573 (1.992)</td>
</tr>
<tr>
<td>08.90 - 11.93</td>
<td>3.505 (0.123)</td>
<td>-0.909 (1.077)</td>
</tr>
</tbody>
</table>

Table 9: Estimated coefficients for the cointegrating relationship between exchange rate and the fundamentals: $s_t = a_0 + m_t + a_1 y_t$ for different regimes.
<table>
<thead>
<tr>
<th>Lag</th>
<th>Japan SIC</th>
<th>AIC</th>
<th>Great Britain 1 SIC</th>
<th>AIC</th>
<th>Great Britain 2 SIC</th>
<th>AIC</th>
<th>Germany SIC</th>
<th>AIC</th>
</tr>
</thead>
</table>

Table 10: Choosing the optimal lag length for the VAR formed with fundamentals Δx and cointegrating error L for a case with structural breaks
### Table 11: Estimates for the VAR formed with fundamentals ΔX for a case with structural breaks

<table>
<thead>
<tr>
<th>Equation</th>
<th>Japan</th>
<th>Gr. Brit. I</th>
<th>Gr. Brit. II</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔXₜ = b₀ + δ₁ΔXₜ⁻¹ + κₚₚ + δ₂ΔXₜ⁻² + κ₂ + ϵₜₜ</td>
<td>0.002</td>
<td>-0.007</td>
<td>0.000</td>
<td>-0.002</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.026</td>
<td>0.030</td>
<td>0.019</td>
<td>0.025</td>
</tr>
<tr>
<td>δ₁</td>
<td>-0.288</td>
<td>0.155</td>
<td>-0.166</td>
<td>-0.197</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.199</td>
<td>1.479</td>
<td>1.100</td>
<td>1.156</td>
</tr>
<tr>
<td>κ₁</td>
<td>0.043</td>
<td>0.696</td>
<td>-0.003</td>
<td>0.035</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.709</td>
<td>1.529</td>
<td>0.484</td>
<td>0.635</td>
</tr>
<tr>
<td>δ₂</td>
<td>-0.237</td>
<td>0.048</td>
<td>0.037</td>
<td>-0.059</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.992</td>
<td>0.961</td>
<td>1.010</td>
<td>1.004</td>
</tr>
<tr>
<td>κ₂</td>
<td>-0.046</td>
<td>0.220</td>
<td>-0.004</td>
<td>-0.035</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.702</td>
<td>1.650</td>
<td>0.478</td>
<td>0.630</td>
</tr>
</tbody>
</table>

#### LM test for first and third - order residual serial correlation:

<table>
<thead>
<tr>
<th>Test</th>
<th>Japan</th>
<th>Gr. Brit. I</th>
<th>Gr. Brit. II</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>χₚₚ² (t-1)</td>
<td>0.344</td>
<td>1.353</td>
<td>0.453</td>
<td>0.632</td>
</tr>
<tr>
<td>Marginal level of sign.</td>
<td>0.558</td>
<td>0.245</td>
<td>0.501</td>
<td>0.426</td>
</tr>
<tr>
<td>χₚₚ² (t-3)</td>
<td>0.162</td>
<td>0.469</td>
<td>10.88</td>
<td>0.325</td>
</tr>
<tr>
<td>Marginal level of sign.</td>
<td>0.687</td>
<td>0.494</td>
<td>0.001</td>
<td>0.568</td>
</tr>
<tr>
<td>χₚₚ² (t-6)</td>
<td>0.029</td>
<td>4.372</td>
<td>0.416</td>
<td>1.836</td>
</tr>
<tr>
<td>Marginal level of sign.</td>
<td>0.865</td>
<td>0.037</td>
<td>0.519</td>
<td>0.175</td>
</tr>
</tbody>
</table>

#### LM test for third - order residual ARCH

<table>
<thead>
<tr>
<th>Test</th>
<th>Japan</th>
<th>Gr. Brit. I</th>
<th>Gr. Brit. II</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>χₚₚ² (t-1, t-3, t-6)</td>
<td>0.934</td>
<td>2.007</td>
<td>3.990</td>
<td>3.450</td>
</tr>
<tr>
<td>Marginal level of sign.</td>
<td>0.817</td>
<td>0.571</td>
<td>0.263</td>
<td>0.327</td>
</tr>
<tr>
<td>R²</td>
<td>0.128</td>
<td>0.402</td>
<td>0.033</td>
<td>0.053</td>
</tr>
</tbody>
</table>
\[ L_t = a_0 + \gamma_1 \Delta X_{t-1} + \mu_1 L_{t-1} + \gamma_2 \Delta X_{t-2} + \mu_2 L_{t-2} + \varepsilon_t \quad (L_t = (s_t - x_t)) \]

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_0 )</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.001</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.044</td>
<td>0.029</td>
<td>0.044</td>
<td>0.046</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>0.213</td>
<td>0.053</td>
<td>0.210</td>
<td>0.252</td>
</tr>
<tr>
<td>(SE)</td>
<td>2.049</td>
<td>1.448</td>
<td>2.487</td>
<td>2.104</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>0.958</td>
<td>0.418</td>
<td>1.030</td>
<td>0.896</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.210</td>
<td>1.497</td>
<td>1.094</td>
<td>1.156</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>0.357</td>
<td>-0.076</td>
<td>-0.043</td>
<td>0.157</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.695</td>
<td>0.940</td>
<td>2.284</td>
<td>1.827</td>
</tr>
<tr>
<td>( \mu_2 )</td>
<td>-0.005</td>
<td>-0.521</td>
<td>-0.073</td>
<td>0.065</td>
</tr>
<tr>
<td>(SE)</td>
<td>1.200</td>
<td>1.615</td>
<td>1.083</td>
<td>1.148</td>
</tr>
</tbody>
</table>

**LM test for first and third-order residual serial correlation:**

- \( \chi^2_{(1)} \) (t-1)  
  - marginal level of sign.: 0.091  
  - marginal level of sign.: 0.763
- \( \chi^2_{(1)} \) (t-3)  
  - marginal level of sign.: 1.900  
  - marginal level of sign.: 0.168
- \( \chi^2_{(1)} \) (t-6)  
  - marginal level of sign.: 0.202  
  - marginal level of sign.: 0.653

**LM test for third-order residual ARCH**

- \( \chi^2_{(1)} \) (t-1, t-3, t-6)  
  - marginal level of sign.: 2.007  
  - marginal level of sign.: 0.571
- \( R^2 \)  
  - 0.883  
  - 0.256

Table 12: Estimates for the VAR formed with cointegrating error L for a case with structural breaks
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{31}$)</td>
<td>3.231</td>
<td>0.026</td>
<td>0.017</td>
<td>0.416</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.357</td>
<td>0.999</td>
<td>0.999</td>
<td>0.937</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>-21.29</td>
<td>-10.29</td>
<td>-22.44</td>
<td>-23.70</td>
</tr>
<tr>
<td>(SE)</td>
<td>3.189</td>
<td>5.771</td>
<td>3.125</td>
<td>3.260</td>
</tr>
</tbody>
</table>

$^a$ $\alpha$ is the semielasticity of the interest rate (equation (1)):

$$ s_t - x_t = \alpha E_t \Delta s_{t+1}, \quad (1) $$

Table 13: Testing of unconstrained model against rational expectations null for a case with structural breaks
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LR ($\chi^2_{12}$)</td>
<td>0.911</td>
<td>3.653</td>
<td>0.831</td>
<td>5.502</td>
</tr>
<tr>
<td>marginal level of sign.</td>
<td>0.823</td>
<td>0.301</td>
<td>0.842</td>
<td>0.139</td>
</tr>
<tr>
<td>$\alpha^*$</td>
<td>35.43</td>
<td>35.92</td>
<td>43.35</td>
<td>43.96</td>
</tr>
<tr>
<td>(SE)</td>
<td>10.72</td>
<td>13.56</td>
<td>16.82</td>
<td>14.18</td>
</tr>
<tr>
<td>$\eta^{**}$</td>
<td>0.065</td>
<td>0.725</td>
<td>0.043</td>
<td>0.051</td>
</tr>
<tr>
<td>(SE)</td>
<td>0.804</td>
<td>1.146</td>
<td>0.899</td>
<td>0.855</td>
</tr>
</tbody>
</table>

$^a\alpha$ is the semi-elasticity of the interest rate (equation (1)):

$$s_t - x_t = \alpha \Delta s_{t+1},$$

(1)

$^{**}\eta$ is the adaptive expectation coefficient (equation (2)):

$$E_t s_{t+1} - E_t s_t = \eta (s_t - E_t s_t) + e_t$$

(2)

Table 14: Testing of unconstrained model against the adaptive expectations null for a case with structural breaks
Figure 1: A rational expectation solution where a positive shock to fundamentals leads to the increase in the fundamentals and control variable
Figure 2: A rational expectation solution where a positive shock to fundamentals leads to the decrease in the fundamentals and control variable.
Figure 3: A rational expectation solution where a positive shock to fundamentals leads to the erratic behavior of the fundamentals and control variable
Figure 4: Actual versus predicted (in-sample) exchange rate for Japanese yen
Figure 5: Actual versus predicted (in-sample) exchange rate for Great Britain pound
Figure 6: Actual versus predicted (in-sample) exchange rate for German mark
Figure 7: Actual versus predicted (out-of sample) exchange rate for Japanese yen
Figure 8: Actual versus predicted (out-of-sample) exchange rate for Great Britain pound
Figure 9: Actual versus predicted (out-of-sample) exchange rate for German mark
Figure 10: Fundamentals (with and without breaks) versus exchange rate for Japanese yen
Figure 11: Fundamentals (with and without breaks) versus exchange rate for British pound
Figure 12: Fundamentals (with and without breaks) versus exchange rate for German mark
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