MODELS FOR UNDERSTANDING THE DYNAMICS
OF HUMAN WALKING

DISSERTATION

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By

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*****

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To my parents,

Corrine and Gordon
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NOMENCLATURE

t

\text{time}

\text{gravitational constant (9.81 m/s}^2\text{)}

i

\text{link number}

j

\text{muscle force number}

N

\text{total number of degrees of freedom}

R

\text{number of redundant degrees of freedom}

m

\text{maximum number of muscles acting}

n

\text{maximum number of links present}

i, i, k

\text{unit vectors along three mutually perpendicular coordinate axes}

S_1, S_{12}

\sin\theta_1, \sin(\theta_1+\theta_2), \text{etc;}

C_1, C_{12}

\cos\theta_1, \cos(\theta_1+\theta_2), \text{etc;}

\hat{i}, \hat{z}_i

\text{unit vector indicating that the joint velocity}

k

\text{spring constant}

C_D

\text{damping constant}

A

\text{spring free length}

a_i

\text{link length}

\alpha_i

\text{link twist}

d_i

\text{joint offset}
\[ x, y \] displacement of hip coordinates

\[ \dot{x}, \dot{y} \] velocity of hip coordinates

\[ \ddot{x}, \ddot{y} \] acceleration of hip coordinates

\[ z \] spring displacement

\[ r_k \] moment arm of spring force about the knee joint

\[ \theta_k, \theta_k' \] angle which spring makes with vertical
angle between stance shank and spring shown in Figure E-2

\[ F_x, F_y \] horizontal and vertical ground reactions

\( (\cdot)^{-1} \) indicates the inverse of a square matrix

\( (\cdot)_r \) indicates quantity composed of redundant rows of equations of motion

\( (\cdot)_{nr} \) indicates quantity composed of nonredundant rows of equations of motion

\( (\cdot)^T \) indicates transpose of a matrix

\[ F_{ji}, \quad \mathbf{I}_{ji} \] force of \( j^{th} \) muscle exerted upon the \( i^{th} \) link
moment arm of \( j^{th} \) muscle force from distal end of link \( i \)

\[ \mathbf{C}(\theta) \] inertia matrix of the open chain (NxN)

\[ \mathbf{g}(\theta) \] vector of gravitational terms (Nx1)

\[ \mathbf{C}(\theta, \dot{\theta}) \] vector of coriolis terms (Nx1)

\[ \theta, \dot{\theta}, \ddot{\theta} \] vector of joint displacements, velocities and accelerations (all are Nx1)
\( F_m(\theta) \)

Vector of muscle forces (Nx1)

\( F_{ab} \)

Force generated in hip abductors

\( F_{ap} \)

Force generated in ankle plantarflexors

\( F_{hf} \)

Force generated in hip flexors

\( \dot{p}_i \)

Position vector of a point \( i \) relative to the base

\( i^{-1}T_i \)

4 X 4 homogeneous transformation matrix relating frames \( i \) and \( i^{-1} \)

\( \theta_i, \dot{\theta}_i, \ddot{\theta}_i \)

Displacement, velocity and acceleration of a revolute joint expressed in the \( i^{th} \) frame

\( i \omega_i, i \dot{\omega}_i \)

Angular velocity and acceleration of link \( i \) expressed in the \( i^{th} \) reference frame

\( i \gamma_i, i \dot{\gamma}_i \)

Linear velocity and acceleration of the origin expressed in the \( i^{th} \) frame

\( i \nu_i, i \dot{\nu}_i \)

Linear velocity and acceleration of the center of mass of link \( i+1 \) expressed in frame \( i+1 \) and acceleration are directed along the \( z \) axis

\( i p_{i+1}, i p_{ci} \)

The position of joint \( i+1 \) relative to joint \( i \)

The position of the center of mass of link \( i \) relative to the reference frame attached to link \( i+1 \)

\( i+1 R_i \)

A rotation matrix which relates all quantities expressed in reference frame \( i \) to that
expressed if frame i+1

the transpose of the matrix \( i_{+1}^R \)
inertial force acting at the center of mass of
link i+1, expressed in frame i+1

inertial torque acting about the center of
mass of link i+1, expressed in frame i+1

moment of inertia of link i+1 about its center
of mass, expressed in frame i+1

force acting at the joint i expressed in the
reference frame attached to link i

moment acting at joint i expressed in the
reference frame of link i

joint moment (force) acting at joint i for a
revolute (prismatic) joint

linear (sliding) velocity and acceleration of
a prismatic joint i+1

linear velocity and acceleration of link n
expressed in frame n

linear velocity and acceleration of the
constrained foot expressed in frame n

angular velocity and acceleration of link n
expressed in frame n

position of a point on the constrained foot
expressed in frame n

linear velocity and acceleration of the constrained foot expressed relative to the base link frame

\( \mathbf{K}(\theta) \) Jacobian which transforms joint rates into the linear foot velocity vector (RxN)

\( \mathbf{k} \) vector of ground reaction forces acting under the constrained foot during double support (Rx1)

\( \mathbf{I} \) vector of joint torques (forces) for revolute (prismatic) joints (Nx1)

\( \mathbf{b} \) vector of bias torques (equation 2.3) (Nx1)

\( \dot{\theta}_r, \ddot{\theta}_r, \dddot{\theta}_r \) displacement, velocity and acceleration vector of redundant coordinates (all are Rx1)

\( \dot{\theta}_{nr}, \ddot{\theta}_{nr}, \dddot{\theta}_{nr} \) displacement, velocity and acceleration vector of nonredundant coordinates (all are (N-R)x1)

\( \mathbf{K}_a(\theta), \mathbf{K}_b(\theta) \) matrices containing the redundant and nonredundant columns of the Jacobian \( \mathbf{K}(\theta) \), respectively. \( \mathbf{K}_a(\theta) \) is RxR and \( \mathbf{K}_b(\theta) \) is Rx(N-R)

\( \mathbf{K}_c(\theta,\dot{\theta}) \) vector containing all position and velocity terms of the acceleration constraint equations (equation 2.13) (Rx1)

\( \mathbf{C}_1(\theta), \mathbf{C}_2(\theta) \) matrices containing the redundant and
nonredundant columns of the inertia matrix $[H(\theta)]$, respectively. $[H_1(\theta)]$ is $N \times R$ and $[H_2(\theta)]$ is $N \times (N-R)$

overall inertia matrix defined by equation 2.17 ($N \times (N-R)$)

overall bias vector defined by equation 2.17 ($N \times 1$)

the inertia matrices containing the redundant and nonredundant rows of the equations of motion. $[H_r(\theta)]$ is $R \times (N-R)$ and $[H_{nr}(\theta)]$ is $(N-R) \times (N-R)$

the bias vectors containing the redundant and nonredundant rows of the equations of motion. $b_r(\theta)$ is $R \times 1$ and $b_{nr}(\theta)$ is $(N-R) \times 1$

matrices of the Jacobian, containing the redundant and nonredundant rows of the equations of motion. $[K_r(\theta)]$ is $R \times R$ and $[K_{nr}(\theta)]$ is $(N-R) \times R$

redundant and nonredundant input vectors. $\tau_r$ is $R \times 1$ and $\tau_{nr}$ is $(N-R) \times 1$

inertia matrix for the closed chain

bias vector for the closed chain

vector of muscle forces for closed chain

vector of joint moments for closed chain

length of link $i$
$r_i$ distance of center of mass of link $i$ from distal end of link
Chapter I
INTRODUCTION

Locomotion is the process by which animals transport themselves from one location to another. Among the most common of all gaits is walking. Human walking, in particular, is a process of locomotion involving alternate single bases of support for an upright, moving body. These single support phases are further interrupted by brief periods of double support, where load is uniformly transferred from the trailing to the leading extremity. Walking, therefore, is characterized by a speed which insists that either one leg or the other remains in contact with the ground.

Most succinctly put, locomotion entails the ability to maintain balance during gait execution. While walking, more generally, includes such transitory activities as starting and stopping, these are superimposed on a basic pattern comprising the rhythmic displacements of various segments during forward progression. It is this basic pattern that molds a framework for the study of normal human gait.

Concurrent with the motion of bodily parts is the appearance of a ground reaction force vector. Defining support of the erect body, the ground reaction produces restraint when directed downwards and
forwards, and propulsion when in the downward and backward directions. Since this vector reflects the inertia forces generated during gait, it is an essential constituent of the aforementioned framework.

Any adequate description of human walking must also include such factors as energy and power requirements, and phasic muscular activity. Integration of these elements then lays a foundation for understanding the mechanics and control of this subtle, but most complex, phenomenon.

1.1 DETERMINANTS OF GAIT

The essential elements of normal walking are those mechanisms contributing to a smooth, undulating pathway of the body's center of gravity. Saunders et al. (1953) have referred to these as the principal determinants of human locomotion.

In normal level walking, the center of mass describes a sinusoidal curve when projected onto the sagittal plane. Within the restrictions of a normal pattern, the maximum vertical displacement is approximately 5 cm. The peaks of this curve are located at midstance during single support; the troughs coinciding with double stance. In addition to translating in the direction of progression, the body is also displaced in a direction perpendicular to the sagittal plane. In fact, typically, the total amount of transverse displacement is comparable to that observed vertically.
The inadequacies of the compass gait argue strongly for the importance of jointed limbs. The objection to this form of stiff-legged walking is twofold: firstly, it produces a vertical displacement twice that observed normally; and secondly, it demands abrupt changes in the body’s forward acceleration. This form of locomotion is characterized by a series of intersecting arcs and, therefore, excessive energy expenditure.

Fortunately, the human skeletal system has evolved into a structure capable of flattening and smoothing out the compass gait trajectory. This mobility is visible in the form of six major determinants: pelvic list, transverse pelvic rotation, stance knee flexion-extension, foot and knee interaction and lateral pelvic displacement. The former three determinants serve primarily to reduce the net vertical displacement of the body’s center of mass in the sagittal plane.

Transverse pelvic rotation acts to raise the points of intersection between two compass-gait arcs, while pelvic list further flattens the trajectory by lowering the body’s center of mass as the pelvis tilts from the weight bearing limb to the limb approaching toe-off. Finally, stance knee flexion-extension brings an obvious reduction in the vertical displacement predicted by stiff-legged walking.

Rather than decrease the body’s vertical displacement in the sagittal plane, foot and knee interaction promotes a smoother trajectory in the vicinity of two intersecting compass-gait arcs.
Ankle dorsiflexion and plantarflexion, together with appropriate knee flexion, are responsible for the disappearance of inflexion points at intersecting arcs of translation.

The above five determinants are concerned with motion in the major plane of progression, and, as such, become the focal point of our investigations here. Lateral pelvic displacement affects only the net displacement in the transverse plane. Its contribution to the major ground reactions is therefore assumed negligible.

1.2 LINE MOVEMENT CONTROL HYPOTHESIS

An idea advocating the execution of movement via a stimulus-response process is known as the 'peripheral hypothesis'. This assumes that sensory feedback from muscle receptors is responsible for generating the movement pattern. Since it essentially involves the description of a closed-loop mechanism for movement execution, it fits well into the framework of engineering control theory. State feedback, for example, represents the system input as a function of the sensed state. However, the major physiological objection to such an hypothesis is that the human nervous system is comparatively slow in the face of faster (ballistic-type) movements. A skilled pianist rapidly positions his fingers according to their desired movements. In the time taken for sensory feedback to reach the central nervous system (CNS), the resulting sensations perceived, and finally appropriate action taken, any subsequent control effected by the muscles will clearly be outdated.
Consequently, the idea of a motor program has been proposed to explain the execution of movement in terms of neural impulses sent to appropriate muscles in a specific sequence, timing and force pattern. The implementation of such preprogrammed neuromuscular activity is essentially open-loop, meaning that it is dispatched from the higher centers, and carried out uninfluenced by peripheral feedback.

While the above 'central hypothesis' adequately describes fast and well-structured movements, it fails to explain the implementation of more demanding tasks. For example: movements to a set target position are performed more accurately at slower speeds, where sufficient time is available for modifications to take place.

A more general description of limb movement control involves an integration of the above hypotheses (Chase, 1965). In effect, the control exerted is closed-loop, where the preprogrammed movement pattern is initiated and serves as a reference signal, against which the actual response is compared. Proprioceptive feedback is utilized on-line to correct deviations from the desired trajectory. In the context of control theory, this represents a model-reference scheme, employing a comparator to detect error during movement, and a controller to eliminate it. This form of control has often been utilized in simulations of normal walking (Hemami, 1985).

While no clear-cut answers exist concerning the applicability of each hypothesis, one conclusion remains firm. Given that humans select different strategies for movement based on their past experience, the particular control scheme is dependent upon the type
of movement exercised. Movements perfected through learning are likely to fall within the realm of open-loop control.

1.3 MODELING

Current understanding of normal human walking is largely a result of experimentally-based investigations. In fact, the elements constituting a complete framework for gait are, by necessity, experimentally derived. However, furtherance of knowledge is often hindered by technological limitations. The study of human gait is no exception. For example: ground reaction forces and center of pressure locations were not accurately detectable until the advent of a force platform (Cunningham, 1950). Similarly, there remain some functional characteristics of walking which state-of-the-art techniques cannot extract. Individual muscle forces, for example, are not measurable non-invasively. They must, instead, be deduced from indirect methods such as optimization. In the face of such shortcomings, one must seek alternative procedures for tackling unanswered questions. An increasingly popular strategy involves mathematical modeling.

By definition, modeling a physical system amounts to describing its behavior via suitably chosen mathematical relations. However, no mathematical model can ever replicate a given system's behavior. It is rarely possible to completely characterize complex, nonlinear system and/or environment properties, and if so, their description in mathematical terms is, almost always, only approximate. Moreover, model complexity, particularly in the early stages of an
investigation, often hinders physical understanding. Thus, simple models are usually preferred at the outset; and only with the development of basic understanding can model complexity proceed.

A general aim of this thesis is to explain, in terms of mechanisms defining normal gait, the shape of the ground reaction forces generated. Focus is entirely upon the lower extremity, where the human musculoskeletal system is represented by a set of articulated, rigid links (the skeletal system), and its actuators (the muscles). The approach to modeling this extraordinarily complex system involves finding an acceptable compromise between simplicity and accuracy. Simplicity is demanded by the need to minimize problems associated with the control of open-loop unstable systems. Model accuracy, on the other hand, is quantitatively assessed by comparison with measured limb displacements and ground reactions for a given subject.

1.4 PROBLEM FORMULATION AND SCOPE

The motivation for undertaking this investigation is a more complete understanding of the dynamics of normal human walking. For the single support phase, the aim is to identify those determinants responsible for the shape of the ground reaction forces. More specifically, we seek to establish the dynamic contribution from each gait determinant to the overall inertia forces generated. This objective is driven by the inadequacy of current experimental methodology to accurately pinpoint the relative influence of these
mechanisms during gait. That is, prediction of ground reactions from measured limb displacements is insufficiently accurate to contend with the above aims. The essential goal in simulating double support is an identification of the mechanisms determining the observed force distribution.

At the heart of our approach lie two key ideologies. These are implicitly linked, and form the foundation of the work presented. Firstly, the term simulation infers a direct dynamics solution. In this case, input joint moments or muscle forces are supplied, and the equations of motion, representing a (simplified) mathematical model of the musculoskeletal system, are integrated for the limb displacements and velocities. It is this procedure that presents an alternative to predicting ground reaction forces from gait data.

In conjunction with the above is an idea that stable locomotion (walking) is achievable on the basis of open-loop control. Here we borrow the concept of a motor pattern, representing preprogrammed neuromuscular activity, to initiate and sustain limb movement. These movement patterns are recognised by the model as a set of applied muscle forces or joint moments, delivered by the higher centers (the brain or spinal cord) in a specific sequence and timing.

The first, and most important, step towards accomplishing the above goals is an effective means of simulating gait. A general technique for simulating movement of the lower extremity is developed in the next chapter. Both open and closed kinematic chains are treated, employing an iterative Newton-Euler formulation to
numerically generate the dynamic equations of motion. The algorithms given offer a powerful alternative to manual derivation, and therefore set the scene for increasing model complexity.

With regard to the single support phase, a variety of models are systematically examined. In Chapter 3, the inadequacy of the inverted single pendulum is first proven, followed by a discussion of the control problems evident during stance phase. This latter feature is illustrated using an inverted double pendulum. Only subsequent to developing such fundamental understanding can model complexity proceed; and it does so through the construction of first planar, and subsequently three-dimensional models for single support.

An important limitation of any planar model is its inability to accurately account for pelvic influence. Chapters 4 and 5 aim to rectify this by assessing the significance of pelvic list and transverse pelvic rotation within the framework of the ground reactions generated. Chapter 4 formulates a three-dimensional model for normal walking, consisting of all five gait determinants relevant to motion in the sagittal plane. Chapter 5 then disassembles this model in search of those mechanisms dominating the dynamic response of the lower extremity. In parallel with this, certain features of pathological gait are explored.

Finally, a model for the double support phase of normal walking is formulated in Chapter 6. In the event of a closed kinematic chain, system redundancy injects considerable complexity into the modeling process, and one of two possible methods are available for negotiating
a solution. The most obvious entails eliminating all redundant
degrees of freedom until the inertia matrix is again made square.
Alternatively, retaining all variables yields an infinity of possible
solutions, in which case one is selected based on some criterion of
optimality. The approach expounded in Chapter 2, and demonstrated in
Chapter 6, is the former. In closing, Chapter 7 summarizes the more
important findings of this research and indicates suitable directions
for future exploration.
CHAPTER II
A NUMERICAL METHOD FOR SIMULATING THE DYNAMICS OF HUMAN WALKING

2.1 INTRODUCTION

For any reasonably complicated model of the human lower extremity (one possessing three or more links), manually deriving the equations of motion is a time consuming process irrespective of the procedure involved. In the case of a single model (Beckett and Chang, 1968; Chow and Jacobson, 1971; Onyshko and Winter, 1980; Mochon and McMahon, 1980), this task, though tedious, remains manageable. It quickly becomes an intolerable limitation, however, if a variety of models are to be investigated. Under these circumstances, it is imperative to seek a more general strategy for producing the required equations.

This chapter presents a general technique for simulating limb movement during human walking. It is based upon an iterative Newton-Euler formulation for solving the inverse dynamics problem, a method commonly employed in the kinetic analysis of manipulators (Orin et al., 1979; Luh et al., 1980). The two algorithms presented enable a three dimensional simulation of the lower limb over one complete cycle. The first deals with the simpler case of an open kinematic chain. Here one end of the linkage is free to move in space, while the other end remains fixed to some known frame of reference (the
ground). This then describes the single support phase of human walking, where the stance leg supports the body while the swing leg executes a step. The second algorithm relates to the double support phase; here both feet are constrained to form a closed kinematic chain.

The foundation of each algorithm is an iterative formulation of the kinematic and dynamic equations of motion. Specifically, the equations for single support are generated numerically using a technique applicable to any open kinematic chain, where no external forces act beneath the swing leg. This method is subsequently extended to the double support phase by including the appropriate position, velocity and acceleration constraint equations. Straightforward elimination then allows a solution of the unknown ground reactions.

The power of this method rests with its potential in a clinical environment. Since the equations of motion are generated iteratively, the technique is highly conducive to model alteration. It may therefore prove a valuable diagnostic tool when examining the effects of joint motion loss during pathological gait.

2.2 A NUMERICAL METHOD

The iterative Newton-Euler dynamics algorithm, originally suggested by Stepanenko and Vukobratovic (1976) and further developed by various investigators (Orin et al. 1979; Luh et al., 1980), was utilized mainly in the solution of the control or inverse problem for
manipulators. That is, given the motion of each link, the joint forces and moments are computed. In a later paper, Walker and Orin (1982) extended its use to the direct dynamics or simulation problem. Here, assuming input joint moments are known, the equations of motion are integrated for link angular displacements and velocities.

In this chapter, Walker and Orin's technique is applied to simulate the motion of the lower limb during single support. An algorithm unique to the double support phase is then developed which partitions the redundant and nonredundant equations of motion using the kinematic equations of constraint.

Figure 2.1 shows two rigid links joined by a simple revolute joint. The notation chosen is that due to Craig (1986). The angular velocity and acceleration of the \( i \)th link are written with respect to a frame attached to the \( i \)th joint. Then, for the linear velocity and acceleration of both the \( i \)th joint and the \( i \)th link's center of mass, the former angular quantities are used, together with the appropriate position vectors. Again, all quantities are conveniently expressed in the \( i \)th reference frame. Having computed the relevant kinematic terms, a straightforward application of Newton's second law produces the inertial force acting at the \( i \)th link's center of mass. Euler's equation is then used to give the inertial torque acting about the center of mass of link \( i \).

Appendix C presents the iterative Newton-Euler dynamics algorithm using either rotational or prismatic joints. The derivation of these equations is not repeated here, but the notation is explained in the
nomenclature preceding the text. For the inverse problem, the joint forces and torques are calculated by commencing at the base (or link zero) and progressing outwards along the chain, computing first the inertial forces and torques present at each link's center of mass. Subsequent to reaching the outermost link, an inwards sweep then produces the required kinetic quantities. This algorithm remains the heart of the technique used to generate and solve the equations of motion for an open chain.

For any general (open or closed-loop) kinematic linkage, the dynamic equations of motion can be written in the form:

\[ [H(\theta)] \dot{\mathbf{\theta}} + \dot{C}(\theta, \mathbf{\theta}) \cdot \dot{\mathbf{\theta}} + \mathbf{G}(\theta) + [K(\theta)]^T \mathbf{\kappa} = \mathbf{T} \]  \hspace{1cm} (2.1)

The meaning and dimension of each term is given in the nomenclature. In the special case of single support, since the foot of the swinging limb is not in contact with the ground, the vector of end-effector constraint forces \( \mathbf{k} \) is zero. Therefore, the equations of motion simplify to:

\[ [H(\theta)] \dot{\mathbf{\theta}} + \dot{C}(\theta, \mathbf{\theta}) \cdot \dot{\mathbf{\theta}} + \mathbf{G}(\theta) = \mathbf{T} \]  \hspace{1cm} (2.2)

If the terms including only the coriolis and gravity effects (the bias vector) are given by

\[ \mathbf{b} = \dot{C}(\theta, \mathbf{\theta}) \cdot \dot{\mathbf{\theta}} + \mathbf{G}(\theta) \]  \hspace{1cm} (2.3)

then, equation (2.2) can be solved for the required accelerations simply by:

\[ [H(\theta)] \dot{\mathbf{\theta}} = \mathbf{T} - \mathbf{b} \]  \hspace{1cm} (2.4)
or \( \Theta = [H(\Theta)]^{-1}(I - b) \)  \( (2.5) \)

The inverse of the inertia matrix, \([H(\Theta)]^{-1}\), will always exist since \([H(\Theta)]\) is positive definite. In fact, depending on the coordinate system chosen, \([H(\Theta)]\) can also be made symmetric. Therefore, if the equations of motion are expressed in the form of equation 2.2, simply inverting the inertia matrix will produce the required limb accelerations. Any common numerical integration routine (a fourth order Runge-Kutta scheme, for example) may then be employed to update the motion at the next time interval.

Numerically generating the bias vector \( b \) in equation (2.2) is straightforward. Here all joint accelerations are made zero, and the resulting torques are computed using the Newton-Euler inverse dynamics algorithm. Hence, the equation solved is:

\[
\dot{x} = C(\Theta, \Theta) \cdot \Theta + G(\Theta)
\]

\( (2.6) \)

since \( \Theta = 0 \) everywhere.

Less obvious, however, is a technique which creates the inertia matrix \([H(\Theta)]\). One possible method involves forcing the acceleration of the \( i^{th} \) link to be unity, and that of all remaining links zero. In this event, the torques computed from the Newton-Euler algorithm (with all velocity and gravity terms now set to zero) represent the \( i^{th} \) column of the inertia matrix. Repeating this procedure for each link then generates the remaining entries of the inertia matrix. Therefore, the equation now solved is:

\[
\dot{x} = [H(\Theta)] \cdot \Theta
\]

\( (2.7) \)
where $\theta_i$ for the $i^{th}$ link is unity, and that for all other links is zero. Also, $\theta = g(\theta) = 0$ in equation (2.2).

In summary, generating the equations of motion (at each time step) for single support amounts to calling the Newton-Euler algorithm once to compute the bias vector $b$, followed by calling it as many times as there are degrees of freedom defining the lower limb. In the general case of $n$ degrees of freedom, the inverse dynamics algorithm is called a total of $n+1$ times. With a set of external joint moments ($\tau$) specified, the accelerations are calculated directly from equation (2.5).

At the instant of heel-strike, to commence the double support phase, the foot of the previously air-borne leg becomes constrained. Assuming a no-slip condition for level walking, the constraint surface is a horizontal plane (the ground) resulting in the loss of, at most, three degrees of freedom. Substituting the kinematic constraint equations into the equations of motion for single support (open chain), produces a set of redundant equations which are a function only of the nonredundant coordinates. Also, since the end-effector constraint forces $k$ (the interaction forces under the foot constrained by heel-strike) remain unknown, there are effectively as many unknowns as there are redundant equations to solve. The obvious path, therefore, is an elimination of these unknown ground reactions, followed by a solution of the remaining nonredundant equations of motion.
While the above proposition is certainly viable, it may ultimately prove unsuccessful in the face of the ground reactions generated. An examination of the actual ground forces produced during normal walking (Inman, 1966) suggests that the total vertical load is transferred linearly in time from one leg to the other. However, this trend is not as prevalent in the horizontal direction. In fact, a number of studies (McGhee et al., 1976; Morecki et al., 1981) have reported the inadequacy of this linear hypothesis when predicting the horizontal ground reactions. In any event, a thorough study of the force-distribution problem is beyond the scope of the present chapter.

In the interest of a general algorithm for double support, the approach forwarded involves directly eliminating the unknown ground reactions without recourse to assumptions involving force distribution. The key to this algorithm is the decomposition of the open chain equations into redundant and nonredundant systems. The method is logically set forth in what follows.

Step 1: Make a judicious choice of those coordinates (degrees of freedom) to be eliminated. This is an important step since an incorrect choice may lead to an unstable solution of the kinematic constraint equations. For example, when the angle between adjacent limbs becomes small (nearly zero), large velocities and accelerations result. Those coordinates to be eliminated are labeled as redundant, all others being nonredundant. Form the position constraint equations, expressing the redundant coordinates in terms of the nonredundant set:
\[ \Theta_r = f(\Theta_{nr}) \]  \hspace{1cm} (2.8)

Now form the velocity constraint equations, again isolating the redundant velocities, thus:
\[ \begin{array}{l}
\dot{s} = [K(\Theta)] \dot{\Theta} = 0 \\
\end{array} \]  \hspace{1cm} (2.9)

where \( \dot{s} \) is the foot velocity vector.

or,
\[ \begin{bmatrix}
    K_a(\Theta) & K_b(\Theta)
\end{bmatrix} \begin{bmatrix}
    \ddot{\Theta}_r \\
    \ddot{\Theta}_{nr}
\end{bmatrix} = 0 \]  \hspace{1cm} (2.10)

which leads to
\[ \dot{\Theta}_r = - [K_a(\Theta)]^{-1} [K_b(\Theta)] \dot{\Theta}_{nr} \]  \hspace{1cm} (2.11)

Next, form the vector of redundant accelerations in terms of the nonredundant set, as well as all position and velocity quantities:
\[ \ddot{s} = [K(\Theta)] \ddot{\Theta} + [K_c(\Theta, \Theta)] \dot{\Theta} = 0 \]  \hspace{1cm} (2.12)

where \( \ddot{s} \) is the foot acceleration vector. Then, substituting from equation (2.10):
\[ \begin{bmatrix}
    K_a(\Theta) & K_b(\Theta)
\end{bmatrix} \ddot{\Theta}_r + [K_b(\Theta)] \ddot{\Theta}_{nr} + [K_c(\Theta, \Theta)] \dot{\Theta} = 0 \]  \hspace{1cm} (2.13)

and therefore,
\[ \ddot{\Theta}_r = -[K_a(\Theta)]^{-1} \left[ [K_b(\Theta)] \ddot{\Theta}_{nr} + [K_c(\Theta, \Theta)] \dot{\Theta} \right] \]  \hspace{1cm} (2.14)

**Step 2:** Partition the equations of motion into redundant and nonredundant sections. First, let
\[ [H(\Theta)] \ddot{\Theta} = \begin{bmatrix}
    H_1(\Theta) & H_2(\Theta)
\end{bmatrix} \begin{bmatrix}
    \ddot{\Theta}_r \\
    \ddot{\Theta}_{nr}
\end{bmatrix} \]  \hspace{1cm} (2.15)
Substitute equation (2.14) into (2.15) to get:

\[ [H(\theta)] \ddot{\theta} = \begin{bmatrix} H_1(\theta) & H_2(\theta) \end{bmatrix} \]

\[ \begin{bmatrix} -[K_a(\theta)]^{-1} [K_b(\theta)] \theta_{nr} - [K_c(\theta)] \theta_c \end{bmatrix} \]

\[ = \theta_{nr} \]

which, upon rearrangement, gives:

\[ [H(\theta)] \ddot{\theta} = \begin{bmatrix} H_2(\theta) - [H_1(\theta)] [K_a(\theta)]^{-1} [K_b(\theta)] \end{bmatrix} \theta_{nr} \]

\[- [H_1(\theta)] [K_a(\theta)]^{-1} K_c(\theta, \theta) \]

\[ (2.16) \]

Therefore, equation (2.1) becomes:

\[ \begin{bmatrix} H_2(\theta) - [H_1(\theta)] [K_a(\theta)]^{-1} [K_b(\theta)] \end{bmatrix} \theta_{nr} \]

\[- [H_1(\theta)] [K_a(\theta)]^{-1} K_c(\theta, \theta) + b + [K(\theta)]^T k = \tau \]

or

\[ [H_T(\theta)] \ddot{\theta}_{nr} + b_T(\theta, \theta) + [K(\theta)]^T k = \tau \]

(2.17)

where

\[ [H_T(\theta)] = [H_2(\theta)] - [H_1(\theta)] [K_a(\theta)]^{-1} [K_b(\theta)] \]

\[ b_T(\theta, \theta) = -[H_1(\theta)] [K_a(\theta)]^{-1} K_c(\theta, \theta) + b \]

Now decompose equation (2.17) as follows:

\[ [H_r(\theta)] \ddot{\theta}_r + b_r(\theta, \theta) + [K_r(\theta)] k = \tau_r \]

(2.17a)

and

\[ [H_{nr}(\theta)] \ddot{\theta}_{nr} + b_{nr}(\theta, \theta) + [K_{nr}(\theta)] k = \tau_{nr} \]

(2.17b)

where equations (2.17a) and (2.17b) are respectively, the first r rows and last (n-r) rows of the n equations of motion. Note that here,
\[ \dot{b}_T(\theta, \dot{\theta}) = \begin{bmatrix} \dot{b}_r(\theta, \dot{\theta}) \\ \vdots \\ \dot{b}_{nr}(\theta, \dot{\theta}) \end{bmatrix} \]

\[ [K(\theta)]^T \cdot \dot{k} = \begin{bmatrix} [K_r(\theta)] \\ -[K_{nr}(\theta)] \end{bmatrix} \cdot \dot{k} \]

and \[ T = \begin{bmatrix} \dot{T}_r \\ \dot{T}_{nr} \end{bmatrix} \]

**Step 3:** Solve for the unknown ground reactions \( \dot{k} \) under the previously unconstrained limb. The redundant set (equation 2.17a) directly gives:

\[ \dot{k} = [K_r(\theta)]^{-1} \cdot \begin{bmatrix} \dot{T}_r - [H_r(\theta)] \dot{\theta}_{nr} \end{bmatrix} \quad (2.18) \]

**Step 4:** Eliminate the vector \( \dot{k} \) and solve for the required nonredundant accelerations. Substituting equation \( (2.18) \) into equation \( (2.17b) \) and rearranging gives:

\[ \ddot{\theta}_{nr} = [H_{nr}'(\theta)]^{-1} \cdot \begin{bmatrix} \ddot{T}_{nr} - \dot{b}_{nr}(\theta, \dot{\theta}) \end{bmatrix} \]

where:

\[ [H_{nr}'(\theta)] = [H_{nr}(\theta)] - [K_{nr}(\theta)] \cdot [K_r(\theta)]^{-1} \cdot [H_r(\theta)] \]

\[ b_{nr}'(\theta) = b_{nr}(\theta) - [K_{nr}(\theta)] \cdot [K_r(\theta)]^{-1} \cdot \dot{b}_r(\theta, \dot{\theta}) \]

and \[ \ddot{T}_{nr} = T_{nr} - [K_{nr}(\theta)] \cdot [K_r(\theta)]^{-1} \cdot \dot{T}_r \quad (2.19) \]

As with the inertia matrix \([H(\theta)]\) for the open chain, \([H_{nr}'(\theta)]\) for the closed chain will always be positive definite (but not symmetric). Therefore, it is also always invertible.
The above steps define the equations necessary for a solution of the nonredundant accelerations. The algorithm for numerically generating the closed-chain equations of motion is based on the above, and is presented in Figure 2.2.

2.3 AN EXAMPLE

Planar simulations of both single and double support phases of normal level walking are used to demonstrate the above algorithms. The attractiveness of the method is that it requires only a specification of the geometric parameters defining the structure of each model. These, together with the externally applied joint moments and initial conditions, serve as inputs to a computer program. In this manner, the flexibility of re-structuring a particular model is maintained, as this ultimately involves only a change in the geometric parameters.

2.3.1 Single support

Figure 2.3 shows a model for simulating human walking, from the instant of opposite toe-off to heel-off. A damped spring, placed between the hip joint and stance leg ankle, simulates the flexion-extension characteristic of the knee. The torso mass is lumped entirely at the hip which is represented as a simple revolute joint.

The swing thigh and shank masses are lumped at their respective centers of gravity, while the foot is represented by a point mass lumped at the distal end of the shank. The model's geometric
Figure 2.2: An algorithm for generating the equations of motion for double support
parameters are derived using the convention given in Appendix B. In the case of a revolute joint, $\theta_i$ is the joint variable, all other parameters ($a_i$, $\alpha_i$ and $d_i$) remaining constant. A prismatic or sliding joint on the other hand, has $d_i$ as its joint variable.

Returning to Figure 2.3, the displacement of the spring is modeled via a prismatic joint, while the ankle, hip and swing knee are all simple revolutes. Table 2.1 presents the geometric parameters for this four degree of freedom system. Using these, together with an appropriate set of external joint moments and initial conditions (Appendix G), the equations of motion are generated and solved numerically using the open chain (single support) technique described above.

The second region of single support involves the period from heel-off to opposite heel-strike. The model now representing the lower limb is shown in Figure 2.4. The stance leg pivots about its toes, assumed to be a revolute joint. Also, both ankle and stance knee joints are simple revolutes. To circumvent problems arising from excessive dynamic coupling, the swing leg is shown detached from the stance limb. The model, therefore, consists of a three degree of freedom (all revolute joints) stance leg, separated from a two degree of freedom (revolute joints for the hip and knee) swing leg. The geometric parameters for this model are as indicated in Table 2.2. All limbs have their mass appropriately lumped at the link’s center of gravity, while the torso is again treated as an entity lumped at the hip.
Figure 2.3: Planar model for single support depicting stance knee flexion - extension

Figure 2.4: Planar model for single support depicting foot and knee interaction
Table 2.1: Geometric link parameters for the model of Figure 2.3. Note that link #3 models the effect of the torso lumped at the hip.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>-90</td>
<td>$d_2$</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$z_4$</td>
<td>0</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
</tbody>
</table>

Table 2.2: Geometric parameters for the model of Figure 2.4.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$z_1$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$z_2$</td>
<td>0</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$z_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>6</td>
<td>$z_4$</td>
<td>0</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
</tbody>
</table>
Figure 2.5 shows the motion of the lower limb during single support. For the model of Figure 2.3, the equivalent angular displacements of the stance thigh and shank were calculated using the appropriate position constraint equations, together with the linear displacement of the spring. These limb displacement patterns are found to compare favorably with experimental data recorded for a normal adult male (Rahmani, 1979).

2.3.2 Double support

A five-link planar model for double support is given in Figure 2.6. Only the limb about to enter swing phase has the effect of a foot included. All joints are revolutes, including the hip which defines the position of the torso mass. The equivalent unconstrained model possesses a total of five degrees of freedom. However, at the instant of heel-strike, two kinematic equations (in the planar case) define the position of the constrained foot. Therefore, only three (nonredundant) coordinates are required to completely specify the system.

In Figure 2.6, the two redundant coordinates chosen are those specifying the position of the previously unconstrained limb ($\theta_4$ and $\theta_5$). These are then eliminated via the position, velocity and acceleration constraint equations (equations 2.8, 2.11 and 2.14). The externally applied moments are given in Appendix G, and the simulated motion is shown in Figure 2.7. Due to the absence of a pelvis, the planar model is incapable of producing a step length consistent with
Figure 2.5: Planar simulation of the single support phase of normal, level walking
the subject's measured limb displacements. Therefore, the motion synthesized in Figure 2.7 cannot coincide with that observed experimentally.

Contrary to normal gait, hip vertical displacement increases substantially (more than 5 cm) subsequent to heel-strike. Changing either the input joint moments or initial conditions has not rectified this anomaly. This result, we believe, is related to the model's high sensitivity to a moment applied at the knee of the leading extremity. To obtain a less exaggerated hip displacement during double support, an increase in front-limb knee flexion is required. One means of effecting this is a larger knee moment; but simple step or ramp inputs to increase knee flexion have led to an excessive lowering of the body's center of gravity.

As a possible remedy, we propose a passively controlled knee (a damped spring) to reproduce the desired flexion characteristic of this joint. The equations given in Appendix C readily allow for such a replacement. It is also an attractive solution since it encourages continuity of the single and double support models. In any event, the present chapter emphasizes a method for numerically generating the equations of motion, in which case the details relating to a solution of the double support problem are peripheral.

2.4 DISCUSSION

The algorithms presented allow the construction of any general, three-dimensional model of the lower extremity. A limitation of the technique, however, is that upper-body dynamics cannot accurately be
Figure 2.6: Planar model for double support walking.

Figure 2.7: Planar simulation of the double support phase of normal, level walking.
accounted for. This, unfortunately, involves the complications of a branched kinematic chain. Therefore, strictly speaking, the method is applicable only to serial linkages where the torso is represented either as a point mass lumped at the pelvic level, or a body rigidly affixed to the pelvis.

An attractive feature of the algorithms is their capacity for incorporating devices such as a damped spring. In the case of stance knee flexion-extension (Figure 2.3), rotation at the ankle and linear spring displacement are effected via a cylindrical joint. That is, a revolute and prismatic joint placed in series. The passive action of the damped spring is included merely by specifying a force that varies linearly with both spring displacement (i.e. a stiffness element) and velocity (i.e. a damper).

Since the computational requirements of this method exceeds that involving a solution of the closed-form analytic equations, the elapsed time during simulation is proportionately larger. In numerically generating the equations of motion, a significant proportion of time is spent creating the inertia matrix (since this involves calling the Newton-Euler algorithm as many times as there are degrees of freedom). However, this rapidly becomes justifiable when weighed against the effort demanded by manual derivation; particularly as the degree of model complexity increases. In opposition, however, is the loss of mathematical insight inherited by most numerically-based techniques. For example, an analytic representation of the equations of motion allows one to assess the degree of coupling imbedded in the model. Numerical computation offers no such luxury.
Summarizing, the algorithms presented in this chapter offer both a powerful and systematic means of simulating lower limb movement during human walking. In fact, in their current form, they may also be applied to various other activities, including running and jumping. A most important feature of the technique is its adaptation to model dismemberment, and subsequent reconstruction. By eliminating the pains of manual derivation, it is particularly attractive from a clinical standpoint, readily permitting a variety of alterations to models for both normal and pathological gait.
CHAPTER III
A PLANAR MODEL FOR SINGLE SUPPORT

3.1 INTRODUCTION

A prerequisite to understanding those mechanisms which govern the individual characteristics of normal level walking is a study of the ground reactions generated, recorded variations of limb positions and orientations, and the temporal distribution of phasic muscular activity. However, such information, collectively, cannot reveal the relative contribution of individual gait determinants. Force plate measurements, for example, reflect the total sum of all body segment interactions. One requires, instead, a more subtle appreciation of the dynamics of limb movement, and its concomitant effect on the shape of the ground reaction forces. With this in mind, two fundamentally opposing approaches become apparent.

The first has its foundations in analysis, and employs an inverse dynamics solution to predict ground reaction forces based on experimentally recorded limb motion. The other, the subject of the present chapter, is one of synthesis. In this case, attention is turned to the direct dynamics problem where both muscular moments and initial conditions are supplied, and the equations of motion (representing a mathematical model of the physical system) are

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integrated for segmental angular velocities and displacements.

The principal determinants of normal human walking were first defined by Saunders et al. (1953). These factors, determining the observed pathway of the body's center of gravity, were derived purely from kinematic considerations. Pelvic list and rotation, stance knee flexion-extension, foot and knee mechanisms and lateral pelvic displacement all contribute, in varying proportions, to the smooth sinusoidal trajectory of the body's center of mass. A result of this interaction is also to produce a cyclic variation in the whole-body velocity and acceleration. While simple geometric arguments suffice to explain the position variation of the center of gravity in time, more detailed procedures are necessitated for comprehending the presence of inertia forces during locomotion.

Central to attempts at modeling human locomotion is the theme of dynamic stability. A common ploy involves specifying the joint torque as the sum of an open-loop bias component, and a correction term based on state feedback (Gubina et al., 1974; Hemami and Farnsworth, 1977). Inevitably, simulating a motion presumes the existence of a reference model. Stability about some desired trajectory is assured via appropriate placement of the system poles.

Such models assume, incorrectly, that the nervous system is capable of functioning at a speed sufficient to allow a constant transfer of information from the periphery to higher motor centers (spinal cord or the brain). McMahon and Greene (1979) have pointed out that, at least in human running, the leg can be represented as a
rack-and-pinion element (regulating muscle stiffness) in series with a damped spring. In this way, the leg musculature is assumed to function passively. In fact, the overall reflex stiffness of man's leg during running is reported to lie in the range 73-117 kN/m.

Various investigators have used the minimum energy hypothesis as a basis for a solution to the synthesis problem. Among these are the well-known efforts of Beckett and Chang (1968) and Chow and Jacobson (1971). The former authors offer a direct dynamics solution, specifying joint moment profiles on the basis of a least work criterion. By dealing solely with the swing phase, they avoid problems of stability. Alternatively, Chow and Jacobson synthesize a motion, and the corresponding inputs, using optimal control theory. In each case, however, constraints are introduced in the interests of a tractable problem. These take the form of assumed trajectories for the hip and/or ground reactions.

Several simulation studies have also been undertaken without recourse to energy matters. Mochon and McMahon (1980a) present a ballistic model of the swing phase of level walking. The muscle forces generated during double support are assumed to create the swing phase initial conditions. In this event, the motion during single support remains unforced and the model correctly predicts the observed natural swing time for a human leg. In a similar venture, Mena et al. (1981) derive the minimum set of gait variables constituting normal swing leg motion. The most significant variables cited are the
initial thigh velocity, a knee constraint and an active moment applied to the foot.

Finally, as an extension to their earlier ballistic model, Mochon and McMahon (1980b) investigate the effect of various determinants on the time history of the vertical and horizontal ground reactions. They conclude that neither heel rise nor stance knee flexion contribute to the second peak in the vertical ground reaction. Instead, they offer pelvic list and ankle plantarflexion as the mechanisms responsible for the variation in vertical force during single support.

A primary objective of this thesis is to identify those determinants characterizing the dynamics of human walking. Specifically, it is hoped to uncover those mechanisms responsible for the observed variation in the horizontal and vertical ground reactions, and further, quantify the significance of each with respect to the final outcome. To this end, an assessment of the ground reactions will play a major role. In the present chapter, only the single support phase of normal walking will be considered. In addition, focus will be upon the lower extremity, the model being constrained entirely to the major (sagittal) plane of progression.

3.2 PRELIMINARY CONSIDERATIONS

The aim here is to synthesize the time course of limb movement in the lower extremity, together with the ground reaction forces during single support. As a beginning, however, we examine two simple
models: the inverted single and double pendulum. First, the inadequacy of the inverted single pendulum as a model for normal walking is brought to light. Then, the control problems pertaining to the stance phase are exposed via a model one level higher in complexity, the inverted double pendulum. To quantitatively assess each model's accuracy, its results are compared with actual experimental data taken from a normal adult male (Rahmani, 1979).

3.2.1 The inverted single pendulum

The inverted pendulum has often attracted the attention of those striving to comprehend the basic mechanics and control of human gait. Cavagna et al. (1976) have used it to explain the interchange between the whole body's kinetic and potential energy, while Mochon and McMahon (1980a) conclude that it is a necessary constituent of the 'simplest representation of walking'.

Figure 3.1 shows the vertical and horizontal ground reactions for an inverted single pendulum based upon the subject's body mass of 69 kg. The initial conditions are chosen to attain a natural speed of 1.3 m/s and step length 0.75m. An external ankle moment is required to prevent the pendulum from collapsing subsequent to midstance. Also shown are the subjects experimentally obtained ground reactions. Clearly, the difference between the actual and predicted vertical force is marked. However, the model appears to be successful in generating a reaction in the horizontal direction. Even though the pendulum's motion is forced by the presence of an ankle moment, the
Figure 3.1: Ground reactions predicted by the inverted single pendulum. Solid line indicates experimental result for normal walking. (a) vertical and (b) horizontal.
outcome is similar to that derived from a ballistic model (Mochon and McMahon, 1980a).

To carry the above a step further, it is necessary to define the conditions under which the pendulum predicts the correct vertical ground reaction. The details are supplied in Appendix D, the key result being that the pendulum must accelerate through midstance to allow the vertical force to drop below body weight. Consequently, an ankle moment that initially accelerates the center of mass in the horizontal direction is required. The result is an increase in both the body’s kinetic and potential energy.

This situation is, of course, contrary to that observed in normal level walking. The response given in Figure 3.1 indicates an increasing vertical force during midstance since the rate of vertical acceleration increases with time (in the downward or negative direction). Alternatively, the conditions derived in Appendix D contain the necessary restriction of a decreasing whole-body vertical acceleration.

As a means of reinforcing the above, it is convenient to draw from simple control theory. The equation of motion for an inverted single pendulum is:

\[ m_b l^2 \ddot{\theta} - m_b g l \sin \theta = \tau \]  \hspace{1cm} (3.1)

where \( m_b \), \( l \) and \( \theta \) are the mass, length and angle of the pendulum, respectively. Using a small angle assumption, the corresponding linear model is:

\[ m_b l^2 \ddot{\theta} - m_b g l \theta = \tau \]  \hspace{1cm} (3.2)
With simple position feedback for stability, the required ankle moment must be of the form:
\[ \tau = -(k + m_b g \ell) \dot{\theta} \]  
(3.3)

Implementing this control law, equation (3.1) becomes:
\[ m_b \ell^2 \ddot{\theta} + k \theta = 0 \]  
(3.4)

The ankle moment, given as a linear function of the angle, is analogous to a torsional spring placed at this joint. Stability is ensured via an appropriate choice of the spring stiffness \( k \), since this governs the position of the system eigenvalues. Figure 3.2 shows the vertical and horizontal forces generated for \( k = 3 \) kNm/rad. It substantiates the above claim, revealing that the vertical force has moved more into line with the expected result; only, however, at the expense of the horizontal ground reaction. Apparently the requirement of raising the vertical force beyond the level of body weight initially, and thereafter decreasing it, is incompatible with the constraints imposed by the single pendulum and the horizontal ground force produced during normal walking.

3.2.2 The inverted springy pendulum

As the center of gravity progresses in time, the distance between it and the center of pressure changes. Ultimately, it is this characteristic which determines the observed vertical acceleration of the body's center of mass. Therefore, we require a model that not only accounts for this facet of locomotion, but also retains the response of the inverted pendulum in the horizontal direction. Such a
Figure 3.2: Ground reactions predicted by inserting a torsional spring at the base of the inverted single pendulum. (a) vertical and (b) horizontal
device is termed the inverted 'springy' pendulum.

A well-known result is the sinusoidal displacement produced by a simple spring-mass model. Further, the vibration-induced force transmitted to the base is also sinusoidal, fluctuating about body weight (as defined by the static equilibrium condition of the system). Therefore, a mass supported by a spring, in the form of an inverted pendulum, is an appealing proposition. The horizontal and vertical ground reactions generated by such a model are shown in Figure 3.3, indicating that both are now in general agreement with the experimental results. The initial conditions are identical to those employed for the single pendulum, with the exception of that relating to the spring; this being chosen on the basis of the whole-body vertical displacement at the instant of opposite toe-off. The equations of motion are presented in Appendix F, but may also be generated using the method outlined in Chapter 2.

3.2.3 The inverted double pendulum

A logical progression from the above is an examination of the inverted double pendulum. This device offers just one gait determinant: stance knee flexion-extension. In doing so, it is potentially capable of delivering the required whole-body vertical acceleration. Focus here is upon the stringent control requirements at the knee, specifically in the region from opposite toe-off to heel-off.

If the mass of the upper extremity and swing leg are lumped at
Figure 3.3: Ground reactions predicted by the inverted 'springy' pendulum. (a) vertical and (b) horizontal
Figure 3.4: The inverted double pendulum
the hip, then a simple model for single support is that of Figure 3.4. The inertial parameters of the shank and thigh are those for a normal adult male taken from Rahmani (1979) (see Appendix A). Not surprisingly, the large mass at the hip dominates the dynamic response of the system. Consequently, the motion of the lower limb is highly sensitive to any moment applied at either the ankle or knee joint. This, in turn, necessitates a finer control at the knee, to both stabilize this joint and impart the required vertical acceleration.

The equations of motion for the double pendulum are given in Appendix F, but, again, may be arrived at via the technique in Chapter 2. If the ankle and knee moments are represented by simple step or ramp inputs in time, difficulties in controlling the model are immediately apparent. These are due to the model's sensitivity to a change in the externally applied joint moments. Figure 3.5 characterizes this behavior by presenting the response of the lower limb to a change in first the ankle, and then knee moment. As indicated, changes of less than five percent in the applied moment, at either joint, cause significant variations in the concomitant motion.

Since the model's sensitivity to changing inputs is quite pronounced, any form of open-loop control is likely to enjoy only limited success. In Figure 3.5, a set of ankle and knee moments are applied that produce an acceptable motion (Case 1). However, they fail to generate the desired ground reactions. In fact, after much effort, attempts at constructing a simple set of inputs to produce the required ground forces and limb displacements were abandoned. A
Figure 3.5: Limb displacements for varying ankle and knee moments applied to the inverted double pendulum. Dashed line indicates experimental result. (a) thigh angle and (b) shank angle.
general conclusion, therefore, is that (at least in the initial region of single support) the required extension of the knee joint is a function unreceptive to crude inputs, implemented open-loop.

To pursue the control problems associated with an inverted double pendulum, the idea of placing torsional springs at the ankle and knee is now explored. One justification for this is a plot of the measured knee angle versus calculated joint moment. The results indicate a straight-line fit, the gradient signifying a torsional spring stiffness (Cappozzo, 1986). Figure 3.6 shows the ground reactions generated for four cases in which the motion appears acceptably normal.

In Case 1, only a knee spring is incorporated. The stiffness chosen is one which provides the largest possible rate of extension, without inducing knee hyperextension. That is, a spring constant larger than 120 Nm/rad produces knee hyperextension during midstance. To bring the horizontal force into closer agreement with the experimental result, Case 2 considers an additional spring placed at the ankle. Again, the knee stiffness is held at its maximum permissible value. In either case, the vertical force exhibits the same trend. It commences well below the level of body weight and thereafter, either increases or decreases slightly.

Forcing the vertical ground reaction to commence at a level above body weight requires a larger value of knee stiffness. Case 3 presents the effect of a 150 Nm/rad spring at the knee. The model, however, is ultimately uncontrollable for any choice of ankle
Figure 3.6: Ground reactions for the inverted double pendulum. Cases 1 to 3 relate to torsional springs at the ankle and knee. Case 4 is the result of placing a linear spring between the ankle and hip joint. (a) vertical and (b) horizontal
stiffness. The accelerations created by this large moment (necessary for producing the required vertical ground force) result in rapid knee extension, a motion which cannot be corrected by simple position feedback.

With the failure of a linear torsional spring to characterize the behavior of the knee joint, a more elaborate form of position feedback is now sought. In this instance, an expression for the externally applied knee moment is derived on the basis of a fictitious linear spring placed between the ankle and hip joints. The effect of such a device is to replace the knee joint with a nonlinear torsional spring, the knee torque now expressed as a nonlinear function of both ankle and knee angle. The derivation of this expression is presented in Appendix E. Case 4 of Figure 3.6 is the response when such a spring is placed at the knee, the ankle moment now being zero.

By an appropriate choice of spring stiffness and free length, the vertical force can be made to commence at a level above that predicted earlier (Case 1 and 2). Again, however, the limit of an acceptable solution (in terms of stability) is quickly reached. Any further increase in either spring stiffness or free length exhibits model instability in the guise of knee hyperextension. Once more, the stringent demand of an upwards vertical acceleration imposed upon the double pendulum, and ultimately the knee joint, requires a rather finely tuned control system capable of firstly meeting the above requirement, and then acting to prevent instability encouraged by its
former action. Note here that damping has not shown itself to be a remedy.

Finally, the results of Figure 3.6 incorporate measured angular displacements and velocities of the shank and thigh at the instant of opposite toe-off. Perturbing these parameters means an appropriate adjustment in spring stiffness and/or spring free length. The outcome, however, in light of the problems discussed above, remains unchanged. The conclusions arrived at are not merely dependent on the particular set of initial conditions chosen. They are, instead, more generally concerned with the structure of the equations of motion; and more particularly with the inertial coupling present.

3.3 MODEL FORMULATION

The foundation of our solution to the synthesis problem is the belief that walking is the result of preprogrammed muscle activity, represented by a collection of learned input (joint moment) patterns. The main contention is that the actions responsible for limb movement are, by nature, gross. While, in the light of the preceding results, this may not be sustained by the knee joint during stance phase, it proves a successful concept when addressing the functions of the two major power producers, the ankle and hip (Zarrugh, 1981).

In the case of the swing leg, for example, numerous studies have shown its behavior to resemble that of an upright double pendulum, with both gravity and inertia providing important energy-saving utilities (Saunders et al., 1947; Beckett and Chang, 1968; Mochon and
McMahon, 1980; Mena et al., 1981). The hip flexors and knee extensors show activity early in the swing phase, after which they are silenced to allow gravity to perform. Then, in preparation for heel-strike, the hamstrings are activated to decelerate the rapidly advancing shank (Cappozzo, 1976; Inman et al. 1953). On this basis, ramp input moments are applied at the hip and knee for approximately the first ten percent of swing time. These are then withdrawn, to utilize both gravity and inertia of the limb. Finally, prior to heel-strike, a knee flexor moment (in the form of a step input) is applied to restrain the shank.

The above is, of course, an over-simplified view of the function muscles serve during gait. It neglects, for example, their role as antagonists, built into the system for protection. Also, assuming the combined effect of muscle action to be a relatively uncomplicated function is, in itself, a gross simplification. Muscles, taken individually, are dynamically nonlinear; and moreover, their action about a joint creates time-varying moment arms that are complicated functions of the adjacent joint angles (Hatze, 1980). Nevertheless, in keeping with the overall goal of elucidating those mechanisms responsible for the observed ground reactions, the models presented below, incorporating the above ideas, seem to be an acceptable starting point.

In constructing a model that simulates the movement of the lower extremity during single support, two individual regions are delineated. These are firstly, opposite toe-off to heel-off, followed
by heel-off to opposite heel-strike. The following sections describe models pertaining to each phase.

3.3.1 Phase I: A model for stance knee flexion-extension

At the instant of opposite toe-off, to commence single support, the stance foot may be considered to pivot about the heel. Then, at approximately seven percent of cycle time, foot-flat occurs. As the entire foot is planted firmly on the ground, a moment is created about the ankle joint to restrain the forward progression of the shank. Electromyographic studies have shown that this action is largely due to activity of the soleus and post-tibial muscles (Perry, 1967). The initial region of single support (opposite toe-off to heel-off) is modeled using a three-link, four degree of freedom serial chain (Figure 3.7). By definition of Saunders et al. (1953), this model offers just one gait determinant: stance knee flexion-extension. It does so by simulating the function of the stance knee with a passive, linear, damped spring.

The behavior of the knee is modeled in this fashion since the spring circumvents the difficulty of controlling, either actively or passively, a model which more strictly includes the effects of the stance shank and thigh. Moreover, the idea of the knee behaving as a spring during walking, as has shown to be the case in running (McMahon and Greene, 1979), is an attractive one. The primary aim here is to learn whether stance knee flexion-extension is capable of imparting the necessary vertical acceleration to the body's center of mass. To
Figure 3.7: Planar model for single support (phase I) depicting stance knee flexion – extension

Figure 3.8: Planar model for single support (phase II) depicting foot and knee interaction
this end, the damped spring serves as an adequate replacement. In Figure 3.7, the torso is treated as a lumped mass located at the hip joint, while the swing shank and thigh have their masses individually apportioned. The body segment parameters used are those pertaining to a normal adult male (Rahmani, 1979). They are summarized in Appendix A.

3.3.2 Phase II: A model for foot and knee interaction

The second region of single support commences with heel rise, and terminates at opposite heel-strike. In this region, a sudden burst of activity from gastrocnemius and the peroneals, together with the already active posterior tibialis and soleus, produce a growing moment at the ankle prior to opposite heel-strike (Perry, 1967). This steadily increasing ankle moment becomes a candidate for the second peak in the vertical ground reaction.

This region of single support, therefore, is modeled using a five-link serial chain. Figure 3.8 now shows the inclusion of a stance foot, the torso mass again being lumped at the hip. Two additional gait determinants are now evident: foot and knee mechanisms. Kinematically, these are concerned with smoothing out the path of the body's center of gravity in the sagittal plane (Saunders et al., 1953). Consequently, the model of Figure 3.8 accounts for three of the total six gait determinants.

Once more, the equations of motion defining the above models are most easily generated using the algorithms of Chapter 2. However,
these equations have also been derived via the more conventional Newtonian approach (Appendix F), and serve as a useful check.

3.4 RESULTS

3.4.1 The influence of swing leg dynamics

The results of Figures 3.1 and 3.2 clearly indicate the inadequacy of the inverted single pendulum as a model for normal level walking. However, these conclusions only relate to the whole-body when considered as a single lumped mass. By virtue of this model's limitations, one is left wondering whether the swing leg possesses a centripetal acceleration sufficient to explain the observed variation in the vertical ground reaction.

Our previous studies of the inverted single and double pendulum suggest that the swing leg alone cannot force the vertical reaction to commence above body weight. The key to this requirement is a sufficiently large vertical acceleration of the body's center of mass. This, in turn, can only be provided by an effective limb length adjustment between the center of pressure and the body's center of mass. However, the swing leg, representing some seventeen percent of total body mass, may contribute significantly to the ground reactions generated during midstance phase. In fact, Cappozzo et al. (1976) have shown that, at least in relation to mechanical energy transfer, the swing leg does play a major role during walking.

Figure 3.9 presents a three-link model to study the effects of swing leg dynamics. Its structure is identical to the model submitted
Figure 3.9: Three-link model to investigate swing leg dynamics
by Mochon and McMahon (1980a) for ballistic walking with knee flexion. The equations of motion are given in Appendix F.

Joint moments, in the form of either simple step or ramp inputs, are applied at the hip and swing knee joints. In addition, an ankle moment is provided at the stance limb identical to that for the inverted single pendulum. (The absence of a restraining moment leads to an excessive vertical hip displacement and step length). The initial angular displacement and velocity of the swing thigh and shank are specified from experimental gait data. Those belonging to the kneeless stance leg are chosen according to the position and progressional velocity of the body's center of mass.

Figure 3.10 gives the resulting ground reactions, together with the motion of the swing knee. Clearly, the addition of a swing leg has made little impression on the result predicted by the single pendulum (Figure 3.1). In this regard, our findings agree with Mochon and McMahon's ballistic model (1980b). The important conclusions here are that firstly, swing leg dynamics makes a relatively insignificant contribution to the ground reactions generated during normal walking. This, therefore, directs one's attention to the function of the stance limb. Secondly, the results of Figure 3.10 are encouraging since crudely constructed, intermittent inputs are shown to be adequate for controlling the motion of the swing leg. This, however, is not totally surprising since, when considered independently, the swing leg behaves as an open-loop stable system (gravity is the restoring force).
Figure 3.10: Ground reactions and motion predicted by the three-link model. (a) vertical force, (b) horizontal force, and (c) knee angle.
3.4.2 The effect of stance knee Flexion-Extension

Adding the effect of stance knee flexion-extension (Figure 3.7), produces a vertical ground reaction more closely aligned with the experimental trace. Figure 3.11 presents the predicted ground reactions from opposite toe-off to heel-off. With an effective stance leg stiffness of 12 kN/m and spring free length of 0.953m, the vertical reaction is now seen to commence above the level of body weight. Thereafter, it dips sharply, in agreement with the measured result, to reach a minimum at approximately seventy percent of body weight.

The frequency with which the knee extends is dictated by the stiffness of the spring, while the free length and initial conditions determine the amplitude of its displacement. The inclusion of damping has been found a necessity as it slows the rate of knee extension during midstance phase. Also, its presence has little, if any, effect on the vertical reaction generated initially. It does, however, maintain the force at a slightly higher level during midstance. The damping ratio employed for the results of Figure 3.11 is 0.4. Interestingly, this is lower than the value of 0.57 quoted during running. Additionally, a spring stiffness of 12 kN/m falls well below the range of 73-117 kN/m predicted for running (McMahon and Greene, 1979). The displacement of the spring is translated into an effective angular displacement of the stance shank and thigh (see Figure 3.13) by enforcing the position constraint equations.

The above values of spring stiffness and free length produce the
Figure 3.11: Ground reactions predicted by model for stance knee flexion – extension. (a) vertical and (b) horizontal.
closest agreement with the measured ground reactions. In fact, a parameter sensitivity analysis shows the vertical ground reaction to be mostly influenced by the spring stiffness, free length and initial conditions. It is relatively insensitive to either the inputs or initial conditions of the swing leg. This, we believe, is the result of a rather weak coupling between the radial and rotational motions of the stance limb (see the equations in Appendix F).

Changing either the spring's free length or initial velocity represents a change in the amplitude of the resulting displacement, and hence the vertical ground force. For a normal gait pattern, with spring stiffness held constant, there is a limit as to the choice of spring free length. That is, any value exceeding this limit results in knee hyperextension. Values well below the limit produce knee flexion rather than extension, and a relatively flat vertical reaction commencing below body weight. Alternatively, changing the spring stiffness, for a constant free length, alters the periodicity with which the vertical force fluctuates. A value of 12 kN/m reproduces the frequency exhibited by the force plate measurement.

In summary, it has not been possible to specify a set of parameters capable of producing both the desired force at the instant of opposite toe-off, and an amplitude of the vertical reaction that agrees more closely with the result of Figure 3.11.
3.4.3 The effect of foot and knee interaction

From the instant of heel-rise, the foot has an important kinematic function in determining the pathway of the knee and ultimately, the position of the body's center of gravity. A patient with a flail ankle, for example, will exhibit a motion approaching the compass gait (Inman et al., 1981). The linkage of Figure 3.8 is a five-link serial chain. However, simulations employing such a model have emphasized the high degree of coupling present. Any moment applied to decelerate the swinging limb severely affects the motion of the stance leg. Clearly, such a model is an unsatisfactory representation of the human skeletal system since it assumes both hips are joined by a single revolute joint (a constraint imposed by planar motion).

To rectify this, the motions of the swing and stance limbs have been decoupled. In Figure 3.8, the response of the stance leg is modeled via a three-link serial chain, the torso and swing leg mass being lumped at the hip. The motion of the uncoupled swing leg is now derived by first computing the hip trajectory (resulting from stance leg dynamics), and then applying moments to appropriately decelerate the limb. The justification for this lies with our previous conclusions concerning the influence of swing leg dynamics on the overall ground reactions generated.

Figure 3.12 shows the ground reactions predicted by the above model. Also reproduced, to complete the single support phase, are the
ground forces generated by the previous model for stance knee flexion-extension. The corresponding limb displacement patterns are given in Figure 3.13, where the model's results are compared with actual segment orientations. Figure 3.14 then presents the input joint moments responsible for this motion. The demarcations between the single and double support phases are as indicated.

For the model effecting foot and knee interaction, crudely constructed inputs are again applied. The ankle moment varies linearly with time, while simple step inputs are specified for the knees and hip. The results, however, are not as promising as those obtained for the initial region of single support. While the vertical ground reaction is in reasonable agreement, differing by no more than ten percent, the horizontal force diverges markedly from the measured result.

This anomaly, we feel, is related to the limitations of a planar model. Generating a sufficient thrust in the vertical direction demands a threshold of ankle moment; an insufficient ankle moment will not produce the required vertical acceleration of the body's center of mass. In attempting to satisfy this requirement, a larger-than-normal push is exerted in the direction of progression. Consequently, an excessive whole-body horizontal acceleration is created, with no action taken to suppress it.

The ramp function employed for the ankle moment produces a vertical reaction that exceeds body weight prior to heel-strike (Figure 3.12). Changing either the intercept (the value of torque at
Figure 3.12: Ground reactions predicted by model for foot and knee interaction. Note that results for phase I are included. (a) vertical and (b) horizontal.
Figure 3.13: Limb displacements predicted by the planar models for single support. (a) Shank, (b) thigh and (c) knee angle
Figure 3.14: Input joint moments applied to the planar models for single support
the instant of heel-off) or gradient of this moment, results in a vertical shift in the ground reaction and/or variation in the rate at which it increases. Choosing, for example, an intercept slightly lower than that indicated by Figure 3.12, but maintaining the gradient at its previous value, results in a vertical force commencing below the level shown at heel-off. Furthermore, a much lower value will fail to produce heel-rise. Observations such as these reinforce the contention that the second peak in the vertical ground reaction is a consequence of a growing moment exerted at the ankle.

In passing, we note that any change in the ankle moment must necessarily be compensated by appropriate adjustments at the stance knee (if the gait pattern is to remain normal). In addition, any moment exerted at the stance knee bears a comparatively small influence on the vertical ground reaction. The above findings are not particularly startling in the light that the ankle is well documented to be one of the major power producers of the lower extremity (Inman et al., 1981; Zarrugh, 1981).

A primary concern of the knee is to control the motion of the body's center of mass, rather than provide power for its progression. That is, while the ankle delivers the necessary vertical thrust, the knee becomes responsible for preventing both hyperextension and an excessive vertical displacement of the whole-body (its response must be a controlled flexion). Hence, within the constraint of a normal gait pattern, the knee moment alone cannot sufficiently decelerate the
body's center of gravity to produce a more reasonable horizontal reaction than that of Figure 3.12.

The model is also sensitive to the initial limb angular displacements and velocities. For example, increasing the initial foot velocity requires a smaller ankle moment to maintain the above results. The sensitivity to both inputs and initial conditions is largely a consequence of the model's open-loop instability. Changing any of these parameters will produce a different result; but, within the confines of a normal gait pattern, the three-link stance limb model has succeeded in elucidating the distinct functions of the ankle and knee.

Finally, within the restrictions of specifying inputs no more complex than linear time-varying functions, Figure 3.12 is the closest match between the model's behavior and that observed experimentally. Modifying the ankle moment (in the form of a higher degree polynomial, for example) may result in better agreement, but it does not detract from our central conclusion. What remains clear is the failure of an insufficient ankle moment to produce a vertical reaction that exceeds body weight prior to opposite heel-strike.

3.5 DISCUSSION

This chapter has aimed to firstly create, and secondly implement, a methodology geared towards understanding the dynamics of normal human walking. At the heart of this lies an idea that limb movement results from preprogrammed neuromuscular activity, accepted
by the model as a set of grossly constructed inputs. The approach to solving the synthesis problem is conceptually simple, though not altogether obvious from a functional standpoint. That is, open-loop instability discourages the implementation of preprogrammed (input) motor patterns.

Any realistic model of the stance limb (one other than the inverted single pendulum) brings with it problems of limb movement control. This is a characteristic shared by all open-loop unstable systems. To overcome such difficulties, past attempts have either imposed motion constraints (Chow and Jacobson, 1971; Mochon and McMahon, 1980b) or, generate control torques based upon assumed limb trajectories (Hemami and Farnsworth, 1977). In formulating the approach presented here, a major objective has always been to avoid a priori assumptions concerning both ground reactions and limb motion.

3.5.1 Limitations of the model

In synthesizing a motion for the single support phase, the above models have approximated, to an acceptable degree, the vertical ground reaction. The model representing foot and knee interaction, however, offers poor agreement in the direction of progression. This presumably is due to the limitations of a planar assumption.

The results of the three-link model for swing leg dynamics (Figure 3.10) indicates a rather insignificant contribution to the vertical ground reaction generated. However, the constraint of a single revolute hip joint immediately assumes a higher degree of
coupling than that evident during walking. Muscular action involved in stabilizing and powering the stance limb is clearly independent of that necessary to progress the swing leg. Hence, an important limitation of this study is the absence of a pelvis.

To alleviate the problem of excessive coupling, each hip joint could possess, at least, two degrees of freedom. The motion of the pelvis will then be described in three dimensions. By introducing transverse pelvic rotation, such a model might also bring better agreement in the horizontal ground reaction. With the ankle moment still providing the necessary vertical thrust (subsequent to heel-off), the action of the gluteus maximus and adductor muscles may be incorporated as mechanisms for controlling pelvic motion in the transverse plane.

The vertical ground reaction due to stance knee flexion-extension and foot and knee interaction argues for the importance of each to the whole-body vertical acceleration. At this stage, however, their significance to the overall dynamics of gait cannot fully be understood. A more complete assessment necessitates an addition of the two remaining determinants, pelvic list and transverse pelvic rotation. Intuitively, one would not expect transverse pelvic rotation to contribute significantly in the vertical direction. Pelvic list, however, is a direct consequence of gravity's action on the body's center of mass. The effect of the abductor muscles must, therefore, be attended to more closely.
CHAPTER IV

A THREE-DIMENSIONAL MODEL FOR SINGLE SUPPORT: NORMAL GAIT

4.1 INTRODUCTION

A long-standing issue in the study of normal human walking, as yet unresolved, is the identification of those mechanisms responsible for the observed variation in the ground reaction forces. Previous attempts at isolating these determinants involve both experimental (Sutherland et al., 1980), and analytical (Mochon and McMahon, 1980; Siegler et al., 1982) strategies.

Sutherland et al. (1980) aimed at assessing the contribution of the ankle plantarflexor muscles during normal level walking. By inserting a tibial nerve block to paralyze this muscle group, ground reaction forces and limb displacements were measured and compared to their normal patterns. The results indicated a significant decrease in the vertical ground reaction, subsequent to tibial nerve block, during the second half of stance phase.

Mochon and McMahon (1980) formulated a planar model for ballistic walking based on the assumption that muscle action during double support creates a desired set of initial conditions for the ensuing swing phase. The objective here was to identify the mechanisms responsible for the vertical ground reaction during single support.
However, to circumvent control problems involving open-loop unstable systems, motion constraints were imposed on individual limbs. Pelvic list and ankle plantarflexion were then forwarded as factors responsible for the first and second peaks, respectively.

In a similar study, Siegler et al. (1982) constructed a mechanical model to simulate the stance phase of walking. With each leg modeled by a damped spring, again only a set of initial conditions were specified, and the equations of motion integrated for both limb displacements and ground reaction forces. While the predicted ground reactions showed consistency with experimental results, the model’s simplicity prevented any detailed explanation of the influence of individual gait determinants.

Most recently, Hurmuzlu and Moskowitz (1986) investigated the stability properties of bipedal locomotion using a simple double-inverted pendulum. Assuming a functional relationship between the hip and ankle angles, phase-plane portraits of the generalized coordinates and velocities were constructed to study the behavior of the system. Their results indicated that impact of the swing leg with the ground enabled the system to avoid certain unstable trajectories, and was therefore the major factor contributing to stable locomotion.

In Chapter 3, a planar model for single support was employed to demonstrate an open-loop control hypothesis for walking. Specifically, grossly constructed inputs (joint moments) were shown to produce stable locomotion. The present chapter is a direct extension of this work. While the previous model contained three of the six
major determinants of gait, the three-dimensional model presented here possesses a total of five: stance knee flexion-extension, foot and knee interaction, pelvic list and transverse pelvic rotation. Also, rather than sustaining limb movement by means of an applied moment, a more realistic approach involving muscle action is given. The Newton-Euler algorithm, previously used to numerically generate the dynamic equations of motion (Chapter 2), is now modified to account for the presence of individual muscle groups. This is realized through the action of appropriate forces upon specific limbs, directed along the known muscle lines of action.

A central objective of this (and subsequent) work is an identification of those determinants responsible for the presence of inertia forces during walking. The approach must, by necessity, be one of synthesis rather than analysis. In particular, assessing the contribution of individual determinants on the basis of experimentally recorded limb motion is a fruitless endeavor. Previous attempts at predicting ground reactions from gait data (Hardt, 1978; Zarrugh, 1980) have reported reasonable success. However, the issue attended to here necessitates a decomposition of the whole-body trajectory into its constituent parts; these being composed of the motion attributed to individual gait determinants. Also, due to the nature of such mechanisms (pelvic list, for example, varies by no more than 5 degrees), kinematic measurement followed by numerical differentiation is a process fraught with uncertainty. Therefore, ground reactions
predicted from an inverse dynamics solution will be, at best, unreliable.

The present chapter is concerned with synthesizing the limb displacements and corresponding ground reactions for the single support phase of normal walking. Also, the effects of input variation (muscle forces and/or joint moments) are analyzed. Chapter 5 then extends this work by firstly evaluating the effects of joint motion loss on the resulting ground reactions, and then quantifying the degree of compensatory action during pathological gait.

4.2 MODEL DEVELOPMENT

This section describes the three-dimensional model proposed. Also given are details pertaining to the applied inputs, as well as a description of the numerical technique used to generate the equations of motion.

4.2.1 Physical description of the model

Figure 4.1 shows a three-dimensional model for simulating the single support phase of normal human walking. The link parameters defining its structure are given in Table 4.1. In the initial region, from opposite toe-off to heel-off, the model possesses a total of six degrees of freedom as it pivots about the stance ankle (the effect of the foot is neglected). A massless damped spring, placed between the hip and ankle, reproduces the flexion-extension characteristic of the stance knee during this portion of the walking cycle. The reasons for
Figure 4.1: Three-dimensional model for single support.
Table 4.1: Geometric link parameters for the three-dimensional model in Figure 4.1.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$l_1$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$l_2$</td>
<td>0</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>$l_3$</td>
<td>90</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>-90</td>
<td>$l_5$</td>
<td>$\theta_6$</td>
</tr>
<tr>
<td>7</td>
<td>$l_6$</td>
<td>0</td>
<td>0</td>
<td>$\theta_7$</td>
</tr>
</tbody>
</table>

utilizing such a device relate to the fine control demanded by the knee initially, and are discussed in detail elsewhere (see Chapter 3). It is worth mentioning, however, that an actuator wrapped around some pulley arrangement located in the vicinity of the knee, might represent a more realistic model of quadriceps activity. Nevertheless, the damped spring, while not a physiological analog, manages to deliver the necessary whole-body vertical acceleration, together with the desired control at the knee.
To include the effect of pelvic list, hip abductor muscle activity must also be taken into account. The muscles comprising this group (gluteus medius, gluteus minimus and tensor fasciae latae) are known to be active during the initial region of stance (Perry, 1967; Inman et al., 1953), and are therefore incorporated into the model of Figure 4.1.

During the second phase of single support, from heel-off to opposite heel-strike, the effect of the stance foot is included (Figure 4.1), so that the model now becomes a seven degree of freedom serial chain. In Chapter 3, it was shown that a growing ankle moment produced the second peak in the vertical ground reaction. To account for this, ankle plantarflexor activity is simulated via a force originating below the knee and terminating at the heel (see Figure 4.1). Though this force only represents the action of a one-joint muscle (typically the soleus), it has proved successful in generating the required ankle moment. Also, models incorporating the action of two-joint muscles (for example, the gastronemius) have encountered serious control problems while attempting to produce the flexion characteristic of the knee prior to heel-strike. Finally, the contribution of pelvic list is again included via the hip abductor muscles which remain active subsequent to heel-off (Perry, 1967).

The body segment parameters used are those for a young adult male apportioned according to Dempster (Williams and Lissner, 1977). These are presented in Appendix A. The mass of the torso is shown lumped at the pelvic level, leading to a neglect of its inertial properties. Cappozzo (1982), in calculating the energy levels associated with
lumping the torso mass at the pelvis as opposed to its more accurate location, has shown, at least for walking at normal speeds (up to 1.5m/s), that such an approximation remains justifiable.

4.2.2 Model inputs

The basis upon which the inputs are applied is an idea that limb movement during walking is controlled open-loop. As such, the inputs specified (see Figure 4.5) represent a collection of learned motor patterns, dispatched from higher centers located in the brain or spinal cord.

In the initial region of single support, the value of spring stiffness selected dictates the frequency with which the body vibrates. Therefore, a value of 25 kN/m is specified since this reproduces the frequency with which the vertical reaction oscillates about body weight. Also, in order for this force to initially exceed body weight without inducing knee hyperextension, a spring free length of 0.831 m and damping 500 Ns/m are chosen. To simulate the activity of the hip abductor muscles, a simple ramp input of force (that is, a muscle force varying linearly in time) is applied at two distinct levels. Initially, to impart a large vertical acceleration to the pelvis, a high-level force is specified. This is subsequently decreased to avoid excessive pelvic list. Thereafter a level of force is maintained only to stabilize the limb under gravity's action.

Subsequent to heel-off, ankle plantarflexor activity is represented by an active force, again in the form of a simple ramp. Simultaneously, a stabilizing moment is exerted at the stance knee by
way of an undamped torsional spring with stiffness 100 Nm/rad and free
length 0.355 radians. A justification for utilizing this device rests
with the observed linear relationship between knee joint moment and
angular displacement (Cappozzo, 1986). Also, pelvic list is
controlled via the hip abductor muscles as they continue to generate a
linearly varying force with time. The magnitudes of these applied
muscle forces are specified in Appendix G.

Throughout the single support phase, transverse pelvic rotation
is controlled via a damped torsional spring located at the stance hip
(Figure 4.1). Such a choice is dictated by simplicity, together with
the fact that step or ramp-type inputs do not adequately control
horizontal rotation. Initially, a spring of stiffness 50 Nm/rad and
damping 20 Nms/rad is employed. Subsequent to heel-off, these
parameters are updated to 10 Nm/rad and 30 Nms/rad respectively. In
this case, values of spring stiffness and damping are consistent with
the frequency of pelvic motion in the transverse plane.

Finally, crudely constructed joint moments control the swinging
limb's motion. This is conceivable if the swing leg is likened to an
open-loop stable system (that is, an upright double pendulum). In
particular, to simulate the activity of the hip flexors, a ramp of
torque is applied to the thigh subsequent to toe-off (see Figure 4.5).
Then, in accordance with well documented muscle activity (Inman et
al., 1981), it is removed after some ten percent of cycle time.
Finally, to prepare for heel-strike, decelerating step inputs are
applied at both hip and knee to simulate hamstring activity.
4.2.3 The solution technique

In Chapter 2, we presented a general method for numerically generating the dynamic equations of motion for any serial-type linkage (open or closed chain), using an iterative Newton-Euler formulation. Now, only the open-chain (single support case) will be addressed, and the following development presents a systematic means of modifying the above algorithm to include the action of specific muscles.

To develop the most general algorithm for computing the joint forces and torques (given the limb motion and applied muscle forces), consider the free body diagram of Figure 4.2. Since any number of muscle forces may act upon a given link $i$, these are represented in Figure 4.2 by $F_{ji}$ and $F_{ki}$ where $j, k = 1, m$ and $i = 1, n$.

A summation of the forces acting upon link $i$ gives:

$$i_{f_i} + (-i_{f_{i+1}}) + i_{F_{ji}} + i_{F_{ki}} = i_{F_i}$$  \hspace{1cm} (4.1)

from which:

$$i_{f_i} = i_{F_i} + i_{R_{i+1}} + i_{f_{i+1}} - i_{F_{ji}} - i_{F_{ki}}$$  \hspace{1cm} (4.2)

Also, summing moments about the center of mass of link $i$ gives:

$$i_{n_i} + (-i_{n_{i+1}}) + (i_{p_{ci}}) \times i_{f_i}$$
$$+ \left[ -i_{p_{ci}} - i_{p_{ji}} \right] \times i_{F_{ji}} + \left( i_{p_{ki}} - i_{p_{ci}} \right) \times i_{F_{ki}}$$
$$+ (i_{p_{i+1}} - i_{p_{ci}}) \times (-i_{f_{i+1}}) = i_{N_i}$$  \hspace{1cm} (4.3)

and rearranging:

$$i_{n_i} = i_{N_i} + i_{R_{i+1}} + i_{f_{i+1}} + i_{p_{ci}} \times i_{F_i} - i_{p_{ji}} \times i_{F_{ji}}$$
$$- i_{p_{ki}} \times i_{F_{ki}} + i_{p_{i+1}} \times i_{R_{i+1}} + i_{f_{i+1}}$$  \hspace{1cm} (4.4)

Equations (4.2) and (4.4) are the modified Newton-Euler equations containing additional terms arising from the action of muscle forces $F_{ji}$ and $F_{ki}$. Note that since $j, k = 1, m$ there are a total of $2m$
Figure 4.2: Free body diagram of the $i^{th}$ link.
muscle forces acting on each link with their respective moment arms defined by $P_{ji}$ and $P_{ki}$.

To solve the direct dynamics problem for any open kinematic chain (single support case), consider now the dynamic equations of motion of the form:

$$\mathbf{\ddot{\theta}} + \mathbf{\dot{C}(\theta, \dot{\theta})} \cdot \mathbf{\dot{\theta}} + \mathbf{G(\theta)} = \mathbf{\tau}$$  \hspace{1cm} (4.5)

However, accounting for the presence of the muscle forces, an additional term $\mathbf{F_m(\theta)}$ is inserted so that:

$$\mathbf{\ddot{\theta}} + \mathbf{\dot{C}(\theta, \dot{\theta})} \cdot \mathbf{\dot{\theta}} + \mathbf{G(\theta)} + \mathbf{F_m(\theta)} = \mathbf{\tau}$$  \hspace{1cm} (4.6)

If the coriolis and gravity terms are represented by a bias vector given by:

$$\mathbf{b} = \mathbf{\dot{C}(\theta, \dot{\theta})} \cdot \mathbf{\dot{\theta}} + \mathbf{G(\theta)}$$  \hspace{1cm} (4.7)

then, equation (4.6) becomes:

$$\mathbf{\ddot{\theta}} + \mathbf{\dot{b}} + \mathbf{F_m(\theta)} = \mathbf{\tau}$$  \hspace{1cm} (4.8)

The inertia matrix $[H(\theta)]$ and bias vector $\mathbf{b}$ are numerically generated in the manner described previously (Chapter 2), with all muscular forces $F_{ji}$ and $F_{ki}$ set to zero. The vector of muscle forces $\mathbf{F_m(\theta)}$ is formed simply by setting $\dot{\theta} = \dot{\theta} = G(\theta) = 0$, and calling the modified Newton-Euler algorithm (Appendix C) once. In this event, from equation (4.8):

$$\mathbf{F_m(\theta)} = \mathbf{\tau}$$  \hspace{1cm} (4.9)

so that the calculated vector of joint torques $\mathbf{\tau}$ is equal to the contribution from the muscle forces alone.
Having formed the individual terms of equation (4.8), the vector of joint accelerations (at each time step) is found directly from:

\[ \Theta = [H(\Theta)]^{-1} \left[ \begin{array}{c} T - B - F_m(\Theta) \end{array} \right] \]  

(4.10)

Numerical integration is then employed to update the link displacements and velocities (\( \Theta \) and \( \dot{\Theta} \)) at the next time interval.

### 4.3 Results

To assess the model's accuracy, its results are quantitatively compared with gait data obtained from the Newington Children's Hospital in Connecticut (Tashman, 1987). Both force and motion information were collected, for a normal young adult male, at a rate of 30 samples/s, the pelvic displacements being determined from markers placed at each anterior superior iliac spine (ASIS). In order to specify a complete set of initial conditions, limb angular displacements were numerically differentiated via a dynamic programming filter (Dohrmann, 1986) to obtain the required angular velocities.

Figure 4.3 presents the horizontal and vertical ground reactions generated by the three-dimensional model for single support. The corresponding limb displacement patterns are given in Figure 4.4, while Figure 4.5 defines the applied joint moments. The boundaries separating the single and double support phases are as indicated.

The predicted vertical and horizontal reactions are validated by the force plate measurements. At the instant of opposite toe-off the vertical ground force lies below the level of body weight, after which
Figure 4.3: Ground reactions predicted by three-dimensional model for normal walking. Solid line indicates experimental result. (a) vertical and (b) horizontal
Figure 4.4: Limb displacements predicted by three-dimensional model. Dashed line indicates experimental result. (a) shank, (b) thigh, (c) knee, (d) pelvic list, and (e) transverse pelvic rotation.
Figure 4.4 (Continued)
Figure 4.5: Input joint moments applied to three-dimensional model
it rises quickly to just exceed that value. Then, as midstance is approached, it decreases to a level approximately 85 percent of body weight. This first peak in the vertical reaction, while not precisely reaching the value obtained experimentally, is the maximum force generated by the model consistent with the vertical displacements of both the pelvis and stance leg. For example, increasing the force present in the hip abductors produces not only a larger peak, but also an excessive vertical displacement of the body’s center of mass (that is, increased pelvic list). Alternatively, increasing either the stiffness or free length of the stance leg spring also results in higher levels of vertical force, but brings with it stance knee flexion rather than extension during midstance phase.

At the instant of heel-off the vertical reaction begins rising, and continues to do so until opposite heel-strike (Figure 4.3). Again, this trend is consistent with that observed experimentally. During this time the force in the hip abductor muscles is applied mainly to stabilize the pelvis in the vertical direction. Note the controlled lowering of the pelvis in Figure 4.4 as opposite heel-strike is approached. As a result, the vertical ground reaction during the latter region of single support is almost entirely the result of a force present in the ankle plantarflexors. A ramp of force generated by this muscle group produces a steadily increasing ankle moment that accelerates the body’s center of mass upwards.

In the transition region of single support (heel-off), the horizontal reaction of Figure 4.3 exhibits a slight discontinuity. This apparently is the result of discontinuities in the applied
inputs. In the initial region of single support, a small ramp-like joint moment is exerted at the ankle to simulate the restraint of the plantarflexors on shank forward progression. At the instant of heel-off, however, a sudden discontinuity exists in this moment (Figure 4.5) due to the pretension of the ankle plantarflexors prior to muscle contraction. The result of such 'model discontinuities' is a sudden change in the limb's acceleration. In the case of the ankle moment, this discontinuity is transmitted directly to the ground reaction generated in the horizontal direction. However, as shown by Figure 4.4, the individual limb orientations are smooth and in reasonable agreement with experimental results. At this point, we note that the experimentally determined pelvic displacements are less reliable than those pertaining to the shank and thigh.

4.3.1 Model sensitivity

To validate the generality of the model, a sensitivity analysis was undertaken. In this study, the applied inputs are varied by twenty percent above and below the values previously specified (Figure 4.5). In the initial region of single support, both the damped spring and hip abductor muscle forces are changed and the resulting effects on limb displacements and ground reactions observed. In the second phase, the moment supported by the torsional spring at the knee, together with the hip abductor muscle force, are both varied independently. It should be noted that on changing one input parameter, all others are maintained at their previous levels.
4.3.1.1 The effect of a changing stance leg spring force

Figure 4.6 presents the effects of a change in both spring stiffness and damping, from the time of opposite toe-off to heel-off. Increasing the stiffness and damping by twenty percent amounts to a larger initial vertical reaction, but also introduces knee hyperextension. On the other hand, a decrease in these parameters produces an excess of knee flexion initially, resulting in a lower initial vertical force. Interestingly, these results suggest that the desired vertical reaction may be generated via a time-varying stiffness. That is, initially a large value of stiffness is required to impart the necessary acceleration to the body's center of mass. Subsequently, this must be decreased (perhaps linearly in time) to ensure that knee hyperextension is prevented.

4.3.1.2 The effect of a changing hip abductor muscle force

In relation to stability, the motion of the pelvis does not appear overly sensitive to a twenty percent change in the hip abductor muscle force. As anticipated, increasing this force produces a larger amount of pelvic list, together with a correspondingly larger initial vertical reaction. Decreasing it, however, still produces an upward acceleration of the body's center of mass, but to a much lesser extent. Hence the vertical reaction generated is initially lower. Somewhat surprisingly, the motion of the stance knee remains relatively unchanged in each case.
Figure 4.6: Variation in stance leg spring force. Solid line indicates experimental result. Spring force specified in Figure 4.5 indicated by ---, twenty percent lower force ---, and twenty percent higher force --.--.--. (a) vertical force, and (b) knee angle
4.3.1.3 The effect of a changing knee moment during foot and knee interaction

The model's response is clearly most sensitive to a change in this parameter. With the plantarflexor muscles exerting a growing moment at the ankle subsequent to heel-off, the control provided by the knee is crucial during this period. It is the responsibility of the muscles surrounding this joint to produce a controlled degree of flexion in preparation for double support. An undamped torsional spring is used to achieve this behavior. Increasing the value of this linear stiffness results in excessive knee flexion and ankle plantarflexion, together with an abnormal lowering of the body's center of gravity. Conversely, decreasing this parameter leads to model instability, since the moment supported by the knee is now insufficient to accommodate the large thrust supplied by the ankle plantarflexors. The result, therefore, is knee hyperextension.

4.3.1.4 The effect of a changing ankle plantarflexor muscle force

At the instant of heel-off, a sufficient ankle moment is required to ensure continuity of the vertical reaction. This condition is apparently incompatible with any requirement for a continuous ankle moment (Figure 4.5). Therefore, to maintain continuity in the vertical reaction, only the gradient of the specified plantarflexor muscle force is varied. Since this translates into a variation in ankle moment inclination, the result is obvious. Increasing the gradient of the applied force results in a steeper vertical ground
reaction as heel-strike is approached. Decreasing it, leads to the generation of a proportionately lower force.

As a final remark, it is noted that a twenty percent variation in the moments applied to the swing leg make little, if any, impression on the ground reactions generated. Within these limits, the model's dynamic response appears to be relatively insensitive to swing leg behavior. This, in fact, supports an earlier finding (see Chapter 3) regarding swing leg influence on the dynamics of normal level walking.

4.4 DISCUSSION

The motivation for undertaking a three-dimensional simulation of normal human gait was an assessment of pelvic influence on the ground reactions generated. Our results indicate that pelvic list is not as dominant a factor as previously suggested (Mochon and McMahon, 1980). This is further reinforced by the predictions of a planar model presented in Chapter 5. In the absence of pelvic list, the vertical reaction is shown to approximate the normal pattern. While the hip abductor muscles, to some extent, actively contribute to the presence of the initial peak in the vertical ground force, their major concern involves stabilizing the pelvis. Consequently, the work presented here has succeeded in identifying mechanisms which dominate the dynamic response of the lower extremity during single support - stance knee flexion-extension and foot and knee interaction.

Contrary to the results predicted by a planar model (Chapter 3), the horizontal ground reaction now compares more favorably with given force plate data (Figure 4.3). We attribute this to the inclusion of
transverse pelvic rotation. On approaching heel-strike, the stance hip accelerates quite rapidly in the direction of progression. By decelerating it, a torsional spring located at this joint prevents the body's center of mass from following such a path. As a result, the horizontal ground force is held to within twenty percent of body weight.

The input joint moments applied to the ankle, knee and hip exhibit discontinuities at the time of heel-off (Figure 4.5). These arise from simplifications assumed during model construction, and, as such, have no physiological analog. In the case of the stance knee, a moment discontinuity arises from the use of two independent devices (a linear damped spring, followed by an undamped torsional spring) to generate the required inputs. Neither of these represent an attempt to model actual muscle behavior. A more conspicuous discontinuity is exhibited at the ankle. This is purely a result of forcing continuity upon the vertical ground reaction. Also, increasing the ankle moment during the initial region of single support (to achieve continuity), rapidly produces knee hyperextension.

In reality, to produce continuously varying limb segment accelerations, the applied joint moments should themselves be continuous. Ultimately the movement of a limb is subject to muscle action and since a variety of muscles surround individual joints, it is feasible that the forces developed by individual muscles be discretely timed. In fact, this behavior is well documented by electromyographic results obtained for normal walking on the level (Perry, 1967; Inman et al., 1953). The net result of such action,
however, should be a smoothly varying joint torque. In this sense, the model of Figure 4.1 is driven by a set of 'artificial' inputs. Nonetheless, these have served well to illustrate the concept of an open-loop control for normal walking, and elucidate the contribution of individual gait determinants during single support.

Figure 4.5 is the justification for our belief that normal walking is implementable open-loop. The fact that stable locomotion is achievable on this basis argues for the existence of such a control scheme, as opposed to one requiring motion feedback. It also correlates with the presence of unavoidable delays in the neural control system. For example, the shortest time for information to travel from peripheral sensors in the human arm to higher motor centers in the central nervous system is only 70 milliseconds. Loop transmission delays, however, are typically in the range from 100 to 150 milliseconds (Hogan, 1985). Therefore, the implementation of feedback for limb movement control (Hemami et al., 1980) remains questionable; particularly in the face of such highly learned activities as normal walking.
5.1 INTRODUCTION

In Chapter 4, a three-dimensional model for the single support phase of normal human walking was presented. By applying a set of grossly constructed muscle forces and/or joint moments, the synthesized limb displacements and ground reactions were shown to agree well with their experimental counterparts. By means of further extension, this open-loop control hypothesis is now applied to the single support phase of pathological gait.

Saunders et al. (1953) defined the fundamental determinants of normal human walking, and examined their relationship to pathological gait. Within the framework of measured ground reactions and calculated segmental energy levels, the effect of joint motion loss was evaluated. However, given that force plate records convey only the total sum of all body segment interactions, the degree to which remaining determinants compensate for a particular loss cannot be deciphered. Nevertheless, qualitative assessments of compensatory action are provided by examining the effect of absent determinants on the pathway of the whole body’s center of mass. For example, complete ankle loss is found to be compensated by 'slight modifications in the
behavior of the knee and hip. Alternatively, knee immobilization requires excessive pelvic list to effect a nearly normal vertical displacement of the body's center of gravity. While such descriptions permit clinical diagnoses, they do little to quantify the dynamic contribution of individual determinants during abnormal gait. One means of developing such insight is a synthesis approach. With this methodology, models reflecting the absence of certain gait determinants are constructed, and, given suitable input patterns, both limb displacements and ground reactions are synthesized.

Within the framework of measured ground reactions, several investigators (Cunningham, 1950; Jacobs et al., 1972; Knirk et al., 1978; Jarrett et al., 1980) have aimed at pathological gait assessment. Cunningham (1950) presents both vertical and horizontal ground reactions for a variety of abnormal walkers, all wearing lower limb prostheses (both above-knee and below-knee amputees). Trends in these results indicate that while below-knee subjects, in general, produce near-normal ground reactions in the vertical direction, above-knee subjects do not. In this latter group, vertical reactions are found to barely reach the level of body weight during single support.

Rather than merely comparing the resultant ground forces generated during pathological gait, Knirk et al. (1978) integrate force plate data to obtain an approximate trajectory of the body's center of mass. However, pathological gait assessment is again qualitative, the framework for a clinical evaluation now being the displacement of the body's center of gravity.
Some investigations (Mansour et al., 1982; Winter, 1978) have also aimed at a detailed energetic analysis of abnormal gait. Mansour et al. (1982) construct a three-dimensional twelve-segment model of the human body to calculate the segmental mechanical energy variations during walking. Using an energy correlation coefficient to assess the exchange of potential and kinetic energy (both within and between segments), these authors qualitatively correlate the energy changes during gait with the degree of pathological involvement. For example, subjects exhibiting neuromuscular disorders indicate a lower correlation coefficient, suggesting the presence of agonist and antagonist muscle activity which consume additional metabolic energy.

Finally, in a study directed only at the swing phase of gait, Mena et al. (1981) simulate the motion of a simple three-link model to characterize the minimum set of variables required for near-normal swing leg motion. While the limb is shown to possess a ballistic character, a knee constraint and active ankle moment are found necessary for normal walking. However, for abnormal hip trajectories of the swing leg due to pathologically induced stance leg motion, the model predicts knee hyperextension as heel-strike is approached. Compensating for such abnormalities requires larger joint moments, resulting in increased energy consumption.

The major aim of the present investigation is twofold: firstly, to evaluate the effect of an absent determinant on the overall ground reactions generated; and secondly, to define the compensatory actions necessary for producing near-normal reaction forces. The methodology
involves systematically removing individual determinants from the three-dimensional model for normal gait presented in Chapter 4. In the former instance, the applied inputs are adjusted only to retain stable locomotion, while calculated ground reactions indicate contributions from individual gait determinants. Conversely, to assess the degree of compensatory action required, inputs are appropriately applied to effect the desired whole-body acceleration.

5.2 MODELS FOR PATHOLOGICAL GAIT

In this chapter four separate models are used to study pathological gait, each a subset of that previously constructed for normal walking (Chapter 4). In all cases, the mass of the torso is lumped at the pelvic level and body segment parameters are chosen in accordance with the previous model. The link parameters defining each model's structure are presented in Appendix B.

Each determinant contributing to the vertical acceleration of the body's center of mass is systematically removed. Firstly, the absence of ankle plantarflexor activity is modeled by excluding tibial restraint in the initial region of single support, and neglecting any contribution from foot and knee interaction prior to opposite heel-strike. The structure of this model is identical to that developed in Chapter 4, except for removal of the stance foot. It possesses a total of six degrees of freedom, and exhibits only three gait determinants: stance knee flexion-extension, pelvic list and transverse pelvic rotation. This, therefore, replicates the
circumstances created by Sutherland et al. (1980) in their examination of ankle plantarflexor influence during normal walking. With the absence of this muscle group (a flail ankle), the foot is assumed to remain flat throughout single support phase. Consequently, the damped spring, producing stance knee flexion-extension in the initial region of single support, is retained to the point of heel-strike. Also, in keeping with the model for normal gait, pelvic list is controlled via the action of the hip abductor muscles, while a torsional spring inserted at the hip controls transverse pelvic rotation. Additionally, simple ramp and step inputs of torque, applied at discrete intervals, produce the desired motion of the swinging limb (see Figure 5.2).

A logical progression from the above is a removal of both stance knee flexion-extension and foot and knee interaction. In this event, the stance limb is represented as a rigid member (the knee joint locked at zero degrees), so that the original single support model now degenerates to one comprising five degrees of freedom. The remaining determinants are those associated only with the pelvis (pelvic list and transverse pelvic rotation).

In direct contrast, if relative pelvic displacements are assumed zero, all remaining limb motion is constrained to the sagittal plane. Such a (planar) model was previously studied in Chapter 3. It exhibits all three 'planar' gait determinants (stance knee flexion-extension in the initial region, and foot and knee interaction subsequent to heel-off), and possesses at most five degrees of
freedom. Again, the initial flexion-extension characteristic of the stance knee is reproduced using a linear damped spring, while ankle plantarflexor activity (during the latter region of single support) is represented in the form of a muscle force originating below the knee and terminating at the heel.

Finally, a model in which all six major gait determinants are absent is considered. Again the stance limb is modeled as a rigid link pivoting about the ankle. With only the motion of the swing leg coupled to it, the model now possesses just three degrees of freedom. Therefore, only the inputs applied to the hip and knee of the swing leg are of relevance, and no resisting moment is effected at the ankle to simulate the behavior of a flail joint.

5.3 THE CONTRIBUTION OF INDIVIDUAL GAIT DETERMINANTS

Figure 5.1 shows the ground reactions and knee joint motion resulting from a model that neglects the effect of ankle plantarflexor activity. The most striking difference between this behavior and that observed during normal walking relates to the shape of the vertical reaction generated. An insufficient moment exerted by the flail ankle has led to a marked reduction in vertical force prior to heel-strike. The horizontal reaction, however, remains reasonably consistent with the result given in Chapter 4, showing no significant deviation from the force plate measurement. In addition, a noticeable feature of stance knee motion is a reduced amount of extension as heel-off is approached. These predictions are all well supported by findings from
Figure 5.1: Model predictions for walking in the absence of ankle plantarflexor activity. Solid line indicates experimental result for normal gait. (a) vertical force, (b) horizontal force, (c) knee angle
Figure 5.2: Input joint moments for model neglecting ankle plantarflexors
Sutherland et al.'s (1980) study, where ankle plantarflexor activity was silenced via a tibial nerve block. As a result, a significant reduction in the second peak of the vertical ground reaction was observed, together with increased ankle dorsiflexion and a loss of stance knee extension during midstance phase. The joint moments applied to the above model are presented in Figure 5.2. Note that the moment supported by the ankle is now zero.

Figure 5.3 presents the vertical reaction for a model exhibiting only pelvic list and transverse pelvic rotation. In this case, maintaining a level of hip abductor activity comparable to that present during normal gait leads to a vertical reaction which lies well below body weight throughout the single support phase. Here, the absence of stance knee flexion-extension and foot and knee interaction has meant a significant decrease in the overall vertical force generated. The discontinuity exhibited in Figure 5.3 arises from discrete levels of force applied by the hip abductors to control pelvic list (see Appendix G). At this point, note that, in each of the above cases, the hip abductor muscles are activated only to produce coronal pelvic rotations comparable to that observed during normal walking (not greater than five degrees).

In the interest of conserving space, the input joint moments producing Figure 5.3 have not been presented. These inputs are similar to those of Figure 5.2, with the exception of the knee moment during stance phase. In this case, with the stance knee locked at zero degrees, no active moment is supported at this joint.
Figure 5.3: Vertical ground reaction for model with pelvic list and transverse pelvic rotation only

Figure 5.4: Vertical ground reaction predicted by planar model
The vertical ground reaction generated by the planar model (stance knee flexion-extension and foot and knee interaction only) is shown in Figure 5.4. Again, the applied joint moments are not presented since these are similar to their counterparts producing normal gait. One minor difference, however, is that active moments are no longer exerted at the stance hip due to pelvic motion constraints. Here, the vertical force is seen to comply acceptably well with the measured result (Figure 5.4). The first peak rises to a level marginally below body weight, and thereafter, in the presence of ankle plantarflexor activity, the vertical reaction again exceeds body weight.

The above results clearly advance stance knee flexion-extension as the mechanism responsible for the first peak in the vertical ground force. In its absence, pelvic list alone cannot deliver the necessary vertical acceleration, without simultaneously producing an abnormal whole-body vertical displacement. Moreover, in the latter region of single support, the influence of pelvic list further diminishes. The planar model also forwards foot and knee interaction (via the ankle plantarflexors) as the mechanism dominating the vertical acceleration of the body's center of gravity prior to heel-strike. During this period, the hip abductor muscles are concerned primarily with stabilizing the pelvis under gravity's action, rather than actively contributing to the whole-body vertical acceleration.

Finally, Figure 5.5 presents the vertical ground reaction predicted in the absence of all six gait determinants. This result is
Figure 5.5: Vertical ground reaction for model with all determinants missing

similar to that previously derived for an inverted single pendulum (see Chapter 3). Here, the force commences at a level well below body weight, reaches a maximum during midstance, and finally decreases prior to heel-strike. The input joint moments applied to the swing leg are identical to that shown in Figure 5.2. Again, no moment is effected at the ankle to simulate a flail joint.
5.4 COMPENSATORY MECHANISMS DURING PATHOLOGICAL GAIT

In the absence of foot and knee interaction, prior to heel-strike, an appropriate hip abductor force must be sustained to force the vertical reaction back beyond body weight. The result of such action is shown in Figure 5.6; a second peak is now successfully generated, but at the expense of excessive pelvic list (the pelvis is tilted sharply upwards prior to heel-strike). In addition, the knee exhibits a normal amount of flexion (approximately five degrees), while swing leg and transverse pelvic motion both remain relatively undisturbed. The input joint moments are given in Figure 5.7, which, in comparison to Figure 5.2, clearly shows the larger force imparted to the pelvis via the hip abductor muscles.

Figure 5.8 represents an attempt to produce both peaks in the vertical reaction from a gait possessing only pelvic determinants (pelvic list and transverse pelvic rotation). With the absence of stance knee flexion-extension in the initial region of single support (see Figure 5.3), a much larger force is now required by the hip abductors to deliver the necessary vertical acceleration. This force is then quickly removed (hence the appearance of a discontinuity) to avoid unreasonably large vertical displacements of the body's center of mass. However, even with the decelerating effect of this lower-level abductor muscle force, subsequent application of a sharply increasing hip torque (to obtain the second peak in the vertical reaction) leads to a maximum pelvic displacement of twenty degrees. Once again, the joint moments producing the results of Figure 5.8 are
Figure 5.6: Compensatory action of hip abductors in the absence of ankle plantarflexors. (a) Vertical reaction and (b) pelvic list
Figure 5.7: Input joint moments for model with hip abductor compensation in the absence of ankle plantarflexors
Figure 5.8: Compensatory action of hip abductors when ankle plantarflexors and stance knee flexion - extension are absent. (a) vertical reaction, and pelvic list
not shown. These are, for the most part, identical to those presented previously (Figures 5.2 and 5.7), with the exception of the above noted comments regarding applied hip abductor muscle forces. The magnitude of this applied muscle force, however, is given in Appendix G.

Finally, the effect of raising stance leg spring stiffness is shown in Figure 5.9. With pelvic determinants removed (the planar model), spring stiffness is increased by almost a factor of two to reproduce the first peak in the vertical ground reaction. Note that damping is now totally absent. As evidenced by the knee motion (Figure 5.9), an increased stiffness translates into a higher 'bounce' frequency of the body's center of mass. Therefore, by appropriately selecting leg stiffness and damping (these values are indicated in Appendix G), a desired vertical acceleration can be delivered to the body's center of gravity without inducing knee hyperextension. To achieve this, Figure 5.9 presents the behavior of an undamped spring specifically chosen to promote early knee flexion (prior to heel-off). Thereafter, foot and knee interaction again carries the vertical reaction beyond body weight. During this period, however, in order to retain stability, the knee must be forced back into extension under severe activity from the ankle plantarflexor muscles.

5.5 DISCUSSION

The model for normal walking presented in Chapter 4 possessed five of the six principal determinants of gait, and generated ground
Figure 5.9: Compensatory action of stance knee flexion - extension when hip abductors are absent. (a) vertical reaction, and (b) knee angle
reactions similar to experimentally derived results. However, a simpler planar model exhibiting only stance knee flexion-extension and foot and knee interaction also predicted a near-normal pattern in the vertical direction, with the exception of a slightly depressed first peak (Figure 5.4). This result confirms an earlier finding given in Chapter 3. The horizontal reaction, however, was found to diverge from the experimental result subsequent to heel-off. This is also in agreement with our previous work, where the anomaly was explained by the absence of transverse pelvic rotation. The planar model, therefore, accurately reflects the contributions of stance knee flexion-extension and foot and knee interaction to the overall dynamics of normal, level, human walking. Figure 5.4 clearly indicates that the attainment of a normal gait pattern is most attributable to the presence of these three determinants. In support of this are the results of Figures 5.1, 5.3 and 5.5 which systematically show the effect created by their absence. The variation of the vertical reaction about body weight (due to inertial forces) is immediately destroyed, and with it a smooth sinusoidal translation of the body’s center of gravity.

In evaluating the degree of compensation during pathological gait, the ground reaction forces served as a framework for comparison. The justification for this lies with the fact that the total ground reaction is a reflection of the displacements undergone by the body's center of gravity in time. That is, dual integration of the whole-body acceleration produces position. Therefore, the ground reactions
generated during walking are indicative of the body's net displacement, and moreover, of the energy variations over one cycle (Cappozzo et al., 1976). Consequently, they serve as a tool for evaluating the 'quality' of one's gait.

Our simulations of pathological gait have revealed that a loss of ankle motion may successfully be compensated by exaggerated displacements at alternate joints; in particular, increased activity in the hip abductor muscles controlling pelvic list. In this case, maximum pelvic displacements are within 8 to 10 degrees. While this is excessive, it is a commonly observed phenomenon that the ground reaction forces generated by below-knee amputees are almost identical to the normal characteristic (Cunningham, 1950; Saunders et al., 1953). In other words, the center of gravity is able to retain a smooth sinusoidal pathway at a presumably low energy cost.

In contrast to the above, the loss of both ankle and knee presents a more severe problem (Figure 5.8). Now, to achieve the desired vertical acceleration, coronal pelvic displacements in the region of twenty degrees are necessitated. The energy demanded by such levels of acceleration is apparently greater than that which the body can afford. Consequently, above-knee amputees exhibit vertical ground reactions that deviate markedly from the normal pattern (Cunningham, 1950; Saunders et al., 1953).

As a final remark, we note that a deeper understanding of the role played by compensatory mechanisms may be sought through a consideration of segmental energy variations over one cycle. The
models for normal and pathological gait submitted in Chapters 4 and 5 are useful implements for assessing the effect of joint motion loss, as well as the degree of compensatory action adopted. Further, they readily permit a complete mechanical energy analysis to be undertaken in support of experimental gait data.
CHAPTER VI

A PLANAR MODEL FOR DOUBLE SUPPORT

6.1 INTRODUCTION

An aspect of human walking to receive relatively little attention is the distribution of foot forces during double limb support. The problem is characterized by a redundancy in the dynamic equations of motion. Specifically, the number of unknown muscle moments and ground reactions exceeds the number of governing equations. The underspecified nature of this problem admits an infinite number of solutions, unless some assumption is made to remove the indeterminacy. Alternatively, one may endure the redundancy, and attempt to synthesize a set of ground reactions and limb displacements on the basis of appropriately chosen muscle forces. It is the latter course which is taken in this chapter.

Previous investigations of double support have focused almost entirely upon the inverse dynamics problem. That is, given the motion of individual limb segments, the corresponding ground reactions are predicted. The challenge here is in overcoming the system's redundant structure. In essence, assuming no force plates are available, the system equations are indeterminate since the ground reactions under one limb always remain unknown. These are an additional set of
constraint forces which cannot be eliminated.

To remove this indeterminate condition, McGhee et al. (1976) advocated a linear hypothesis for the ground reaction forces and ankle torques. They assumed that the total ground reaction and combined ankle torque are transferred linearly in time from one leg to the other. In this event, the system equations are returned to determinacy. However, the ground reactions predicted did not fully conform with force plate measurements. In fact, the horizontal force estimates were entirely incorrect.

Following McGhee's (1976) lead, Zarrugh (1981) imposed constraints on both the ground reaction forces and center of pressure during double support. Specifically, the horizontal reactions are constrained by a 'symmetry' assumption, while the center of pressure is forced to move linearly forward in time. Again, however, discrepancies in the predicted ground reactions are evident. In addition, the results are not quantitatively supported by comparisons with actual forces exerted during walking.

Scattered attempts at solving the direct dynamics or synthesis problem have also appeared. Siegler et al. (1982) formulate very simple planar and three-dimensional models for double support. With the effective limb length variation modeled using linear damped springs, the equations of motion are integrated, in time, for the displacement of the whole body's center of mass. The resulting horizontal and vertical ground reactions are shown to compare favorably with experimental results for normal walking on the level.
However, the effects of such major assumptions as a massless lower extremity are not considered.

Townsend and Tsai (1976) develop a more complex mathematical model for bipedal (robot) climbing, descending and level walking. Rather than assuming the input actuator torques, gait motion algorithms are nominated for a solution to the synthesis problem. In particular, constant angular accelerations are found suitable for the limb segments of the lower extremity, with their magnitudes chosen by trial and error. A certain gait is considered acceptable if it is stable, controllable, and obeys specific feasible motion constraints. Otherwise, the coefficients and/or initial conditions of the locomotion algorithm are varied, and an alternative solution sought.

More recently, Vaughan et al. (1982) re-examined the activity of stair climbing, but now with direct correspondence to human locomotion. Since the under-determined nature of a closed kinematic chain lends itself to mathematical optimization, these investigators formulate a solution to three different closed-loop problems: walking up stairs, vertical jumping, and cartwheeling. In all cases, the objective function minimized is the sum of the joint moments squared. Also, the vertical ground reactions are linearly apportioned, while the horizontal force under one limb is assumed some fraction of the total body weight. While predictions of ground reactions are reasonably successful, those relating to the points of application are not.
In their studies of quadrupedal running, Goldberg and Raibert (1987) explore the relationship between body motion and symmetric leg actuation during single and double support phases. Their motivation is in understanding symmetry as a tool for simplifying the control of running machines. To this end, leg symmetry during both single and double support is assessed by enforcing symmetric body motion. No attempt is made at synthesizing either a gait pattern or the ground reactions. Instead, mathematical proofs are given to show that symmetric body motion during single support demands symmetric leg actuation, but that redundancy during double support allows asymmetric actuation to compensate for asymmetric leg motion.

Due to the recent surge of interest in legged robots and multi-arm manipulators, considerable attention has been given to modeling and control of constrained dynamic systems. Since the double support phase of human walking falls into this category, it becomes pertinent to consider such works.

Oh and Orin (1986) formulate a technique for deriving the dynamic equations of motion for multiple closed-chain robotic mechanisms. With each arm being a six degree of freedom manipulator, the equations of motion are numerically generated using an iterative Newton-Euler formulation. The equivalent open-chain equations are first derived, and later modified by enforcing the governing (kinematic) constraint equations at each end-effector. Introducing these constraints returns the system to determinacy. Solution of the resulting set of linear
simultaneous equations then produces the unknown joint accelerations and tip constraint forces and moments.

A more general, rather interesting, method of generating constrained system dynamic equations is expounded by McClamroch (1986). The closed-chain equations (including end-effector contact force) and kinematic constraints constitute a determinate, but singular, system of differential equations. To retain solvability, a reduction transformation is employed to eliminate the unknown contact force and convert the system into a set of non-singular, linear, differential equations.

Negotiating a similar path, Hemami and Wyman (1979a) develop a control architecture for closed-chain mechanisms, with specific reference to bipedal locomotion in the frontal plane. State feedback is used to achieve dynamic stability (about specified reference trajectories), and simultaneously maintain the desired kinematic constraints. By writing the unknown contact forces as a function of both state and input, the original dimensionality of the system is preserved.

In a companion paper (1979b), the above authors extend their previous work by controlling (implicitly) the forces of constraint under one limb. Employing a previous formulation of these unknown forces (a function of state and input), force control is implemented open-loop. That is, constraint maintenance does not necessitate force feedback. However, the proposed algorithm does require the control forces to be a priori specified.
An alternative approach to force distribution control is presented by Orin and Oh (1981). To make allowance for the underspecified nature of closed kinematic chains, a linear programming problem is formulated to optimize a weighted combination of energy consumption and 'load balancing'. The particular example considered is a six-legged vehicle driven in a tripod gait. The force distribution is derived by limiting the magnitude of torque developed at each (rotary) actuator, as well as constraining each leg to remain in contact with the terrain (a no-slip constraint imposed in the horizontal plane). However, since the solution technique is computationally expensive, its practical implementation, in the light of real-time control, is limited.

While the above propositions for force control are, in essence, indirect, several architectures encompassing active force control have also appeared. These are, by nature, more general; and have been successfully implemented in legged locomotion systems (Klein and Briggs, 1980), as well as single and multi-arm manipulator structures (Whitney, 1977; Salisbury, 1980; Raibert and Craig, 1981). In all cases, active control involves direct force and position sensing.

For navigation over unstructured terrain, Klein and Briggs (1980) sense and actively control the ground forces generated by a six-legged robot. When either foot position or force is controlled independently, the vehicle exhibits poor terrain adaptation. Combining these two, however, permits adequate obstacle negotiation. The control system is built on the concept of active compliance.
Here, both end-effector position and force errors are regulated in cartesian space, and subsequently resolved into commanded joint velocities.

Similar control strategies have been proposed and implemented in simple manipulation tasks. Whitney (1977) presents a linear force feedback scheme for controlling contact forces during assembly operations. Again end-effector forces sensed in cartesian coordinates are used to generate velocity modification commands. As such, this is a velocity control scheme; but has the desired effect of controlling interaction forces.

Salisbury (1980) introduces the concept of active stiffness control, also implemented in cartesian coordinates expressed in the end-effector reference frame. Controlling the hand 'nominal' position permits simultaneous position and force control. This is essentially achieved by controlling the 'apparent stiffness' of the hand.

Finally, Raibert and Craig (1981) propose a more formal architecture for hybrid position/force control during manipulation. In this formulation, accurate control of both end-effector and force trajectories is made possible through a controller that partitions these variables, and controls each independently.

In this chapter, the force distribution problem during double support is resolved without recourse to any form of active position or force control. Within this context, the term active refers to either direct sensing during closed-loop feedback control, or energy input to the system from external sources. The approach to synthesizing a set
of ground reactions and limb displacements is conceptually simple, yet has built into it a fundamental understanding of the mechanics of walking. This understanding is transmitted through a model which accurately depicts the dynamic response of the lower extremity during double support. As outlined shortly, the solution technique exercised stems largely from an algorithm previously developed (see Chapter 2) for the dynamic synthesis of closed-chain linkages.

6.2 The Model

Arriving at a model that successfully embraces the dynamics of walking involves firstly identifying those determinants most crucial to the production of inertia forces. In developing a suitable model for the double support phase, we rely heavily upon our previous dealings with single support, as well as any physical intuition gathered from personal observations.

In Chapter 3, a planar model was found to adequately reproduce the ground reactions during single support. This model possessed three of the total six gait determinants: stance knee flexion-extension, and foot and knee interaction. In effect, these mechanisms were found to dominate the dynamic response of the lower extremity during single limb stance. A reasonable starting point, therefore, would be to incorporate these determinants in our present attempts at simulating the double support phase.

Reliance on physical intuition is exposed through the belief that the primary objective of double support is an energy-efficient
transportation of the body’s center of mass, coupled with a smooth transference of load from one limb to the other. Essentially, the limb preparing for swing phase pushes (via the calf muscles), while that accepting an increasing proportion of load does so via knee flexion.

Figure 6.1 presents a planar model for simulating double support. With each joint modeled as a frictionless revolute, the lower extremity is represented by a five degree of freedom serial chain. The link parameters defining the model’s structure are given in Table 6.1. The entire torso mass is lumped at the hip, while that of the lower limb is apportioned according to the segmental parameters of a young adult male (Appendix A). Since the limb about to enter swing phase is constrained, the mechanism qualifies as a closed kinematic chain.

Imposing hard kinematic constraints at both feet implicitly assumes that frictional forces are sufficient to prevent slippage. If the inertia forces in the horizontal direction are not contained within the friction cone defining a no-slip condition, then the above assumption is, of course, unrealistic. However, in synthesizing a set of ground reactions, the horizontal forces are always compared (quantitatively) with experimental observations. Given that the model’s predictions conform, the frictional forces can be considered capable of maintaining the imposed kinematic constraints.

A major simplification proposed by Figure 6.1 is the neglect of pelvic determinants during double support. With respect to the ground
Figure 6.1: Planar model for double support
Table 6.1: Geometric link parameters for the model of Figure 6.1.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
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<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$z_1$</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$z_2$</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$z_4$</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>$z_5$</td>
<td>0</td>
<td>$\theta_6$</td>
</tr>
</tbody>
</table>

reactions generated, the consequences of such an assumption are not considered here. For this, a three-dimensional model, incorporating both pelvic list and transverse pelvic rotation, is required. While the solution technique presented is directly extendable to these more complex models, the challenge of controlling them (to produce stable locomotion) is considerably greater. In any event, as will be shown, the planar model manages to produce an acceptable force distribution. It therefore presents itself as a justifiable starting point.
The primary function of the limb responsible for weight acceptance (herein referred to as the leading extremity) is provision of adequate knee flexion under load. To accomplish this, a linear actuator, in the form of a damped spring, is placed between the hip joint and ankle (Figure 6.1). With stiffness and damping constants of 50 kN/m and 900 Ns/m respectively, this device has succeeded in reproducing the flexion characteristic of the knee. In addition, it is attractive from the viewpoint of providing continuity between the present model and that previously developed for single support (Chapter 3). Moreover, alternatives such as a torsional spring are less suited to controlling motion at the knee. In short, the force generated by a linear spring passively controls knee flexion by simulating the restraint provided by the quadriceps muscles.

In contrast, the limb preparing for swing phase (herein referred to as the trailing extremity) indicates major muscle activity in both the ankle plantarflexors and quadriceps (Perry, 1967). A crucial function of the hip flexors is to prevent knee hyperextension. Under a severe thrust from the ankle plantarflexors, both the knee and hip are forced into extension. If such motion goes unimpeded, the model will eventually enter an unstable region. To counter this, Figure 6.1 shows a muscle force acting (on link 4) to flex the hip. This muscle is activated for approximately thirty percent of double support phase, sufficient to produce the required knee flexion. Note also that only a small moment is exerted at the knee, so that, for all practical purposes, it undergoes passive flexion. This finding is consistent
with electromyographic results concerning muscle activity surrounding the knee joint (Perry, 1967).

To simulate a decreasing moment exerted at the ankle (Inman et al., 1981), a ramp input of force is generated by the plantarflexors. The action of this group is approximated in Figure 6.1 by a muscle originating below the knee and inserting at the heel. Meanwhile, no moment is supported by the ankle of the leading extremity.

Finally, an important attribute of the model, not immediately apparent, is the specification of those (redundant) coordinates to be determined from a solution of the kinematic constraint equations. In Figure 6.1, these coordinates have been nominated as $\theta_5$ and $\theta_6$. Consequently, the nonredundant coordinates are taken as $\theta_1$, $\theta_2$ and $\theta_4$. The repercussions of such a choice become evident only during simulation.

Choosing to eliminate, for example, links 1 and 2 instead, does not advance as satisfactory a result as that predicted by the model of Figure 6.1. An important ingredient of the force distribution solution is the rate of knee flexion during weight acceptance ($\theta_2$ and its derivatives). It is crucial, therefore, to maintain some measure of control over this variable during double support. Choosing $\theta_4$, $\theta_5$ and $\theta_6$ as the (nonredundant) coordinates to be determined from numerical integration, will, to a large extent, obstruct this requirement.

In other words, given that the ground reactions generated by the leading extremity are a strong function of that limb's knee joint motion, determining $\theta_1$ and $\theta_2$ from the kinematic constraint equations
is an undesirable approach. Furthermore, since the ground reactions generated by the trailing extremity are largely a result of a moment exerted at the ankle, a more thoughtful strategy involves nominating \( \theta_5 \) and \( \theta_6 \) as the nonredundant coordinates. This, then, is the thinking behind the structure of Figure 6.1, particularly as it relates to the solution technique developed below.

### 6.3 CLOSED-CHAIN EQUATIONS

In Chapter 2, we presented a technique for numerically generating the dynamic equations of motion for closed-chain mechanisms. This method was founded upon an iterative formulation to first compute the kinematic equations via an outwards sweep (starting from the base), followed by an inwards sweep to form the Newton-Euler dynamic equations. Finally, the governing kinematic constraints were imposed to eliminate the redundant ground reactions, and hence convert the under-determined system to a set of reduced, determinate, differential equations.

In what follows, the above algorithm is re-applied with minor modifications to account for the presence of muscle forces rather than joint moments. The closed-chain dynamic equations have the form:

\[
\dot{L}(\theta) J(\theta) \ddot{\theta} + C(\theta, \dot{\theta}) \dot{\theta} + G(\theta) + F_m(\theta) + [K(\theta)]^T \cdot k = \tau
\]  

(6.1)

and may be generated using the algorithm given in Appendix C. If the bias vector is defined by:

\[
b = C(\theta, \dot{\theta}) \dot{\theta} + G(\theta)
\]  

(6.2)

then equation (6.1) simplifies to:
\[ L[H(\theta)] \dot{\theta} + b + F_m(\theta) + [K(\theta)]^T \mathbf{k} = \mathbf{0} \] (6.3)

Partitioning the inertia matrix \([H(\theta)]\) into sub-matrices \([H_1(\theta)]\) and \([H_2(\theta)]\), and proceeding exactly as outlined in Chapter 2, an overall inertia matrix and bias vector are defined thus:

\[ [H_T(\theta)] = [H_2(\theta)] - [H_1(\theta)] [K_a(\theta)]^{-1} [K_b(\theta)] \] (6.4)

\[ b_T(\theta) = -[H_1(\theta)] [K_a(\theta)]^{-1} K_c(\theta, \dot{\theta}) + b \] (6.5)

Replacing \([H(\theta)]\) and \(b\) in equation (6.3) with their counterparts (equations (6.4) and (6.5)) gives:

\[ [H_T(\theta)] \dot{\theta}_{nr} + b_T(\theta) + F_m(\theta) + [K(\theta)]^T \mathbf{k} = \mathbf{0} \] (6.6)

Equation (6.6) is the set of closed-chain equations containing only the vector of nonredundant joint accelerations \(\dot{\theta}_{nr}\). It now remains to eliminate the unknown ground reactions \(\mathbf{k}\), and subsequently solve the determinate system of closed-chain equations.

Decomposing equation (6.6) into redundant and nonredundant sets gives:

\[ [H_r(\theta)] \dot{\theta}_{nr} + b_r(\theta) + F_r(\theta) + [K_r(\theta)]^T \mathbf{k} = \mathbf{0} \] (6.7)

\[ [H_{nr}(\theta)] \dot{\theta}_{nr} + b_{nr}(\theta) + F_{nr}(\theta) + [K_{nr}(\theta)]^T \mathbf{k} = \mathbf{0} \] (6.8)

The redundant ground reactions are found from equation (6.7), thus:

\[ \mathbf{k} = [K_r(\theta)]^{-1} \left[ \mathbf{0} - [H_r(\theta)] \dot{\theta}_{nr} - b_r(\theta) - F_r(\theta) \right] \] (6.9)

Back-substitution then yields the closed-chain dynamic equations:

\[ [H_{nr}'(\theta)] \dot{\theta}_{nr} + b_{nr}'(\theta) + F_{nr}'(\theta) = \mathbf{0} \] (6.10)

where

\[ [H_{nr}'(\theta)] = [H_{nr}(\theta)] - [K_{nr}(\theta)] [K_r(\theta)]^{-1} [H_r(\theta)] \]

\[ b_{nr}'(\theta) = b_{nr}(\theta) - [K_{nr}(\theta)] [K_r(\theta)]^{-1} b_r(\theta) \]
\[ \mathbf{F}_{nr}'(\mathbf{q}) = \mathbf{F}_{nr}(\mathbf{q}) - [\mathbf{K}_{nr}(\mathbf{q})] \mathbf{K}_{r}(\mathbf{q})^{-1} \mathbf{F}_{r}(\mathbf{q}) \]

\[ \mathbf{\tau}_{nr}'(\mathbf{q}) = \mathbf{\tau}_{nr}(\mathbf{q}) - [\mathbf{K}_{nr}(\mathbf{q})] \mathbf{K}_{r}(\mathbf{q})^{-1} \mathbf{\tau}_{r}(\mathbf{q}) \]

Finally, inverting the closed-chain inertia matrix \( [\mathbf{H}_{nr}'(\mathbf{q})] \) and rearranging terms in equation (6.10) produces the required nonredundant joint accelerations \( \mathbf{\theta}_{nr}' \). Note that \( [\mathbf{H}_{nr}'(\mathbf{q})] \) and \( [\mathbf{K}_{nr}(\mathbf{q})] \) are positive definite; therefore, they are both always invertible.

6.4 RESULTS

To validate the model, predictions of both ground reactions and limb displacements are compared with experimentally observed results (Tashman, 1987). Model simulation commences at heel-strike, with estimates of the required initial conditions obtained from numerical differentiation using a dynamic programming filter (Dohrmann, 1986).

Figure 6.2 exhibits the horizontal and vertical ground reactions generated during double support. The corresponding segment rotations are given in Figure 6.3, while Figure 6.4 defines the input joint moments producing this motion.

Both the vertical and horizontal ground reactions relate favorably with force plate data. A uniform transfer of load is observed in the vertical direction, where the force carried by the trailing extremity rapidly decreases, while that associated with weight acceptance increases at approximately the same rate. Simultaneously, the horizontal force distribution predicted by the model is consistent with that observed during walking. While the leading extremity experiences retardation, the trailing extremity
Figure 6.2: Ground reactions predicted by the model. Solid line indicates experimental result. (a) vertical and (b) horizontal.
Figure 6.3: Limb displacements predicted by the model. (a) shank, (b) thigh, and (c) knee angle.
Figure 6.4: Input joint moments applied to the double support model.
accelerates the body’s center of mass in the direction of progression. In this manner, the force-distribution solution exposes a zero interaction force characteristic (Waldron, 1986) in the horizontal direction. That is, the horizontal reactions generated by the closed-chain reflect the system’s natural, energy-efficient ability to prevent the lower limbs from ‘fighting’ one another.

The key to producing an acceptable force distribution is controlled knee flexion in the leading extremity, together with an appropriate moment (one that decreases approximately linearly in time) at the trailing ankle. Interestingly, the participation of each mechanism is felt simultaneously during double support, as opposed to their separate appearance during the single support phase (see Chapter 3).

The five-link planar model, we believe, is the simplest representation of double support that successfully delivers the expected limb displacements and ground reactions (Figures 6.2 and 6.3). Moreover, it embodies some degree of physical understanding. The simplest conceivable model for double support is a three-link planar mechanism, which, upon constraint, transforms into the well-known four-bar linkage. In this depiction, neither limb possesses a knee. The trailing extremity, however, retains its ability to exert an ankle moment between the foot and rigid shank-thigh member.

Our dealings with such a model have shown it to be ineffectual insofar as demonstrating an acceptable force distribution. While it remains capable of commanding the required ankle moment (responsible
for a decreasing vertical reaction beneath the trailing extremity),
the absence of fore-limb knee flexion produces an acceptable result.

Introducing a knee joint to the leading extremity results in a
model one level higher in complexity: a four-link planar mechanism,
with only two degrees of freedom in its constrained state. The
mechanisms crucial to a satisfactory force-distribution (an ankle
moment powering the trailing extremity and fore-limb knee flexion) are
now present. However, from a kinematic, and ultimately energetic,
standpoint, this model is also an inappropriate portrayal of walking.
The sole purpose of rear-limb knee flexion is preparation for the
ensuing swing phase. Removal of this determinant coincides with a
significant increase in energy expenditure, a result which manifests
itself in excessive pelvic list (to provide foot clearance during
swing phase) (Saunders et al., 1953). Issues such as these reinforce
the claim that a five-link planar representation successfully
integrates both model simplicity and accuracy.

6.4.1 Model sensitivity

An important indication of any model's usefulness is its
sensitivity to parameter variation. To assess this criterion, the
nominal values of the applied muscle forces were adjusted
(arbitrarily) by twenty percent. Specifically, the spring force
restraining fore-limb knee flexion, ankle plantarflexor force, and
rear-limb hip flexor muscle force were all singularly, and
independently, varied. In addition, the effect of initial conditions
(limb angular displacements and velocities) on model response was also analyzed.

The results revealed a heavy dependence on both fore-limb initial knee joint velocity and rear-limb hip flexor muscle force. Knee angular velocity directly influences spring compressional velocity subsequent to heel-strike, so that the model, understandably, is sensitive to this parameter. A value of 1.0 rad/s, derived from experimental gait data, produces the results of Figures 6.2 and 6.3. Decreasing this initial velocity preserves fore-limb knee flexion, while introducing thigh extension rather than flexion. Vertical hip displacement, therefore, increases. As a consequence, the angular displacements of the trailing extremity also deviate from their experimental counterparts, particularly that of the shank and foot. This, in turn, is due purely to the imposed kinematic constraint defining a fixed step length.

Alternatively, increasing the initial knee joint velocity amounts to exaggerated fore-limb knee flexion, and hence an abnormal lowering of the body's center of mass. Moreover, under continued action from the ankle plantarflexors and quadriceps, the trailing extremity prematurely enters swing phase (that is, the vertical ground reaction becomes tensile).

The parameter sensitivity analysis also stressed the importance of rear-limb quadriceps activity during double support. A twenty percent reduction in hip flexor muscle force maintains adequate rear-limb knee flexion, and the predicted ground reactions remain unchanged. A proportionate increase in this muscle's activity,
however, brings model instability, as large vertical forces are generated by the trailing extremity in the presence of excessive knee flexion.

Coupled with the above effects is the model’s response to a changing ankle plantarflexor muscle force. The initial magnitude of the resulting ankle moment determines the peak value of vertical force generated by the trailing extremity. Lower values of muscle force produce (initially) lower vertical ground reactions. More importantly, if the previous level of hip flexor muscle force is maintained, excess knee flexion results in a premature transition to swing phase (tensile vertical ground forces generated beneath the trailing extremity). Alternatively, increasing ankle plantarflexor force produces a lesser degree of rear-limb knee flexion, unless, of course, hip flexor muscle force is appropriately adjusted.

Finally, somewhat surprisingly, the model is relatively insensitive to any variation in spring force controlling fore-limb knee flexion. In fact, changing spring stiffness by as much as fifty percent produces no visible signs of either model instability or deviation from the ground reactions predicted in Figure 6.2. This behavior, we believe, is a consequence of specifying a spring free length that coincides with initial spring displacement at heel-strike; a choice dictated by an initially low vertical ground reaction beneath the leading extremity. Achieving this, in turn, requires a low initial value of spring force. In this respect, the ground reaction generated by the leading extremity is initially independent of spring stiffness; but, then, this dependence grows as spring compression
increases. Therefore, spring stiffness, and hence force, plays an increasingly important role toward the latter stages of double support.

6.5 DISCUSSION

The muscle simulating quadriceps activity in the trailing extremity (Figure 6.1) serves a dual purpose. First, it more realistically models the mechanism responsible for hip flexion, in comparison with an applied joint moment. Second, and more importantly, it circumvents a limitation inherited by most planar representations of the lower extremity.

Within the context of a closed-chain mechanism, applying a moment at the hip to achieve rear-limb knee flexion has the undesirable side effect of also producing fore-limb knee extension. This is purely a result of modeling the hip as a (single degree of freedom) revolute joint. (By Newton's law, equal and opposite moments act on adjacent limbs joined together at the hip). The muscle attached to link 4 in Figure 6.1 successfully provides thigh flexion without transmitting a moment to neighbouring limbs. In this sense, it conveniently decouples the motion of the lower limbs and avoids the need for a more complicated, three-dimensional model.

In reality, the lower limbs are, to a large extent, decoupled through the pelvis. Muscular action producing rear-limb motion should not adversely affect fore-limb stability. In particular, moments arising from applied muscle forces about the hip are reacted against by the torso. Therefore, more accurate, but also more complicated,
depictions of double support must incorporate trunk behavior. Such representations, however, introduce problems associated with modeling and control.

With respect to the former, proper inclusion of trunk dynamics necessitates a detailed knowledge of branched kinematic chains. In this event, the numerical technique forwarded in this chapter becomes obsolete. Alternatively, the torso may be included in the form of a body rigidly attached to the pelvis. The lower limbs are then decoupled if each hip is modeled as a single revolute joint. Note that, again, motion is constrained to the sagittal plane. Unfortunately, simulations incorporating such a model have encountered serious problems relating to trunk motion control. In light of this, the inclusion of a muscle to effect rear-limb hip flexion offers a reasonably uncomplicated, and therefore attractive, solution.

The applied muscular moments of Figure 6.4 sustain an idea that stable locomotion is achievable without closed-loop control. This concept was also the basis of earlier work dealing with the single support phase (Chapters 3, 4 and 5). In essence, an open-loop control hypothesis for walking is built upon the belief that 'learned' movement patterns are the result of preprogrammed neuromuscular activity. Under this assumption, muscular inputs, in a specific sequence, timing and force pattern, are applied within the framework of desired (experimentally measured) limb displacements and ground reactions; the major contention being that limb movement is not the result of feedback, either from higher motor centers in the central nervous system, or from local (spinal) mechanisms.
Evidence supporting preprogrammed patterns of neuromuscular activity stem from studies of hopping movements in man (Melville-Jones and Watt, 1977a), as well as landing from unexpected falls (Melville-Jones and Watt, 1977b). Results from such works have led to conclusions that 'the entire act is programmed and dispatched from higher centers as a single entity...., the correct timing and sequence of muscle contraction having been learned through previous experience.'

In addition to the concept of preprogrammed motor patterns for open-loop limb movement control, our solution to the force-distribution problem advocates the idea of passive compliance during walking. This feature resembles early attempts at simulating manipulation tasks using open-loop position/force control strategies (Nevins and Whitney, 1978).

The analogy is more readily apparent if the lumped torso mass (Figure 6.1) is considered the end-effector, while the trailing extremity constitutes an open-chain manipulator. The presence of a leading extremity is then accounted for by appropriately 'choosing' its behavior to model the (fictitious) interaction between the body's torso and ground. That is, a compliance representing the overall response of the leading extremity effectively simulates this interaction, in much the same way that a set of passive springs models the interaction between a gripper and held part during assembly operations (Nevins and Whitney, 1977).

The damped spring in Figure 6.1 passively controls trunk (end-effector) position, and hence regulates, passively, the force
transmitted to the ground. In this sense, the passive compliance
functions as an open-loop controller of torso (end-effector) position.
As we have previously remarked, the ground reactions are relatively
insensitive to the stiffness (compliance) chosen. Damping, however,
is an essential ingredient which serves to slow the rate of knee
flexion during the latter stages of double support.

6.5.1 Limitations of the model

The mechanisms responsible for approximating the desired force
distribution have been identified as an ankle moment exerted at the
trailing extremity, and a controlled rate of fore-limb knee flexion.
In being able to generate the required ground reactions, these
determinants produce the necessary net acceleration of the body's
center of mass.

These conclusions, while perhaps instinctively comprehensible,
are limited by the constraints of a planar model. In particular, a
major characteristic of the double support phase is the observed
lowering of the pelvis to the side of the limb approaching toe-off.
This motion is controlled primarily by the abductor muscles (Inman et
al., 1981). Additionally, the trailing extremity imparts a propulsive
force to the pelvis in the forward direction which initiates
transverse pelvic rotation about the opposite hip. This, combined
with the action of the leading extremity, subjects the pelvis to two
strongly opposing forces, which tend to 'whip' the limb around in the
horizontal direction. This sequence of actions is well timed with
heavy adductor muscle activity (Inman et al., 1953).
If pelvic motion, in unison with the remaining displacements of the lower limb, accurately reflect the motion of the body's center of mass (a justifiable assumption for level walking at speeds up to 1.5 m/s), then transverse pelvic rotation and pelvic list become suitable candidates for influencing the ground reactions during double support. The planar model given here neglects any contribution from such mechanisms, and, in this respect, the conclusions offered are restrictive. In short, the greatest objection to the model of Figure 6.1 is the coincidence of whole-body and hip trajectories. In reality, while vertical hip displacement of the leading extremity increases, the body's center of mass undergoes a lowering in the vertical direction (Cappozzo et al., 1976).

The limb displacement patterns provided in Figure 6.3 show a reasonable conformity with experimental gait data. However, it is noted that the rear-limb orientations, particularly that of the shank, deviate somewhat from the expected results. This, we believe, is again a manifestation of the planar restriction. As opposed to the above, however, the underlying reasons are now kinematic in origin.

Given that the step length during normal level walking is fixed (0.432m for this subject), the simulated limb displacements of the planar model must, at every instant, satisfy this kinematic constraint, as well as experimentally observed segment orientations. Since the present model has not offered a better agreement than that depicted in Figure 6.3, it appears that these two requirements are incompatible within the restrictions imposed by the sagittal plane. Introducing the determinant of transverse pelvic rotation accounts,
from a kinematic standpoint, for the separation of each hip joint. A consequence of this additional degree of freedom is greater flexibility in orienting the trailing extremity. This, we feel, will allow closer agreement between simulated and experimental rear-limb displacement patterns.
Chapter VII
CONCLUSIONS AND RECOMMENDATIONS

A general method for simulating human walking has been forwarded based on the Newton-Euler inverse dynamics algorithm for serial chains. As opposed to manually deriving the equations of motion, this numerically-based technique is an attractive proposition with regard to time investment. It is therefore the foundation on which model complexity rests. Clearly, our studies of normal and pathological gait would not have been possible without a methodology conducive to both model dismemberment and reconstruction. This feature remains the pivot of all work presented here.

The starting point of our dealings with single support was the inverted single pendulum. The inability of this device to adequately represent normal walking was conclusive. Rather than predicting a whole-body vertical acceleration that decreases with time, the constraints imposed by the pendulum produce a totally inconsistent result. Here the vertical ground reaction commences well below body weight, and progressively increases through midstance phase.

The concept of a 'springy' pendulum served to illustrate one most important characteristic of human walking: limb length adjustment during stance phase. In particular, it paved the way for an
identification of those determinants responsible for the observed levels of vertical acceleration. The inverted double pendulum, while a logical choice for demonstrating stance knee flexion-extension, unfortunately failed to sustain a workable solution. In fact, quite unexpectedly, it brought to the fore some curious aspects of knee joint control.

Within the framework of an open-loop control hypothesis for walking, grossly constructed joint moments applied to a double pendulum created problems that eventually proved insurmountable. To circumvent this, overall leg musculature was represented by a damped spring which successfully reproduced the observed flexion-extension characteristic of the knee. The attractiveness of this device led to its retention in all models describing single support.

Simulations involving planar models served to identify stance knee flexion-extension and foot and knee interaction as determinants successively responsible for the first and second humps in the vertical ground reaction. In addition, they exposed the distinct functions of the ankle and knee during single support phase. The thrust provided by a growing ankle moment prior to heel-strike is consistent with an increasing vertical reaction during this period, as well as independent observations placing this joint among the major power producers of the lower extremity. Conversely, the knee belongs to that realm responsible for fine tuning system response, and behaves primarily as an energy absorber. An important limitation of such models, however, is their neglect of pelvic motion. In this respect,
conclusions derived from planar simulations were limited by their inability to assess individual contributions to overall acceleration levels.

Adding transverse pelvic rotation and pelvic list realized a three-dimensional model capable of more accurately predicting ground reaction forces during normal gait. Specifically, transverse pelvic rotation seemed responsible for bringing a better agreement in the horizontal reaction generated. The vertical force also moved more into line with experimental observations, particularly during midstance phase, as pelvic list was controlled through hip abductor action. Nevertheless, an inescapable conclusion was the dominance of first stance knee flexion-extension, and then foot and knee interaction. Pelvic list was ascertained not to be a significant contributor to the observed vertical acceleration of the body's center of mass. While the hip abductors, to some extent, contributed actively, their ultimate function involved securing pelvic stability.

To conclude investigations of single support, mechanisms compensating for joint motion loss during pathological gait were explored. Complete loss of the ankle was successfully compensated by increased hip abductor muscle activity. In this case, maintenance of normal ground reactions necessitated pelvic displacements up to ten degrees. This served to reinforce previous experimental observations that below-knee amputees commonly produce ground reactions identical to the normal characteristic, at reasonably competitive energy costs. In contrast, precluding both ankle and knee rotation poses a more
serious threat to the amputee. Here, to sustain desired levels of vertical acceleration, coronal pelvic displacements up to twenty degrees are demanded. Consequently, above-knee amputees often generate ground reactions markedly different from the norm.

Finally, to complete the walking cycle, simulations of double support phase were conducted. Central to producing an acceptable force distribution was a controlled knee flexion in the leading extremity, coupled with an appropriate ankle moment to power the trailing limb. A five link planar model, embracing the above mechanisms, was submitted as the simplest representation of walking based on its ability to appropriately distribute foot forces, as well as permit proper gait execution subsequent to double support. With respect to the latter, incorporating knee flexion in the trailing limb anticipates the need for toe clearance during swing phase.

7.1 RECOMMENDATIONS

A rather important, and reasonably straightforward, means of further validating model response entails an assessment of the mechanical energy over one complete cycle. In essence, this will explicitly verify synthesized limb angular velocities in terms of calculated rotational kinetic energy. Total energy levels, comprising potential and kinetic components, should then be qualitatively checked against experimental findings cited in the literature (Cappozzo et al., 1976). Among the more important features to be confirmed by the model include a significant energy increase during double support
phase, together with decreasing total energy for the period of single support.

Since any thorough mechanical energy analysis should consider motion in all three directions, it first becomes necessary to formulate an appropriate three-dimensional model for double support. In short, this involves the addition of a pelvic link to the model submitted in Chapter 6. Under the conditions of both pelvic list and transverse pelvic rotation, three kinematic constraints now define the closed-chain mechanism. An added complication is the allowance of transverse pelvic rotation about each hip during double stance. This is necessary only to ensure that limb motion remains primarily in the direction of progression. Inclusion of these determinants will also allow a more complete assessment of pelvic influence during this phase of walking.
BIBLIOGRAPHY


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APPENDIX A

BODY SEGMENT PARAMETERS

A.1 SUBJECT 1:

The following are segment parameters used for all models given in Chapters 2 and 3. The data pertains to a normal, adult male taken from Rahmani (1979).

Foot:  mass = 1.67 kg
        length = 0.150 m
        distance of link center of mass from distal end = 0.076 m
        moment of inertia = 0.001 kgm$^2$

Shank: mass = 3.37 kg
        length = 0.473 m
        distance of center of mass = 0.287 m
        moment of inertia = 0.051 kgm$^2$

Thigh: mass = 7.020 kg
        length = 0.441 m
        distance of center of mass = 0.253 m
        moment of inertia = 0.084 kgm$^2$

Torso: mass = 45.10 kg
A.2 SUBJECT 2:

The following are segment parameters used for all models given in Chapters 4, 5 and 6. The data is that belonging to a normal, adult male weighing 745 N (Tashman, 1987).

Foot: \[ \text{mass} = 1.07 \text{ kg} \]
\[ \text{length} = 0.125 \text{ m} \]
\[ \text{distance of link center of mass from distal end} = 0.087 \text{ m} \]
\[ \text{moment of inertia} = 0.001 \text{ kgm}^2 \]

Shank: \[ \text{mass} = 3.429 \text{ kg} \]
\[ \text{length} = 0.394 \text{ m} \]
\[ \text{distance of mass center} = 0.224 \text{ m} \]
\[ \text{moment of inertia} = 0.051 \text{ kgm}^2 \]

Thigh: \[ \text{mass} = 7.390 \text{ kg} \]
\[ \text{length} = 0.418 \text{ m} \]
\[ \text{distance of mass center} = 0.238 \text{ m} \]
\[ \text{moment of inertia} = 0.084 \text{ kgm}^2 \]

Pelvis: \[ \text{mass} = 52.42 \text{ kg} \ (\text{lumped mass of torso}) \]
\[ \text{length} = 0.199 \text{ m} \]
\[ \text{distance of mass center} = 0.0995 \text{ m} \]
APPENDIX B

GEOMETRIC LINK PARAMETERS

Defining the structure of a linkage to input to the computer program amounts to specifying four parameters for each link. These are the link length $a_i$, link twist $\alpha_i$, link offset $d_i$ and joint displacement $\Theta_i$. Figure B-1 shows link frames attached to adjacent links, connected by a revolute joint. Using the convention outlined by Craig (1986), and referring to Figure B.1, the link parameters are found from the following definitions:

. $a_i$ is the distance from joint axis $\hat{z}_i$ and $\hat{z}_{i+1}$, measured along the link axis $\hat{x}_i$

. $\alpha_i$ is the angle formed between the joint axes $\hat{z}_i$ and $\hat{z}_{i+1}$, measured about the link axis $\hat{x}_i$

. $d_i$ is the distance from link axis $\hat{x}_{i-1}$ to $\hat{x}_i$, measured along the joint axis $\hat{z}_i$

. $\Theta_i$ is the angle formed between the link axes $\hat{x}_{i-1}$ and $\hat{x}_i$, measured about the joint axis $\hat{z}_i$

B.1 GEOMETRIC PARAMETERS FOR MODELS DEFINING PATHOLOGICAL GAIT

The following tables define the link parameters for the four pathological models described in Chapter 5.
Table B.1. Geometric link parameters for the model indicating no ankle plantarflexor activity.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_1$</td>
<td>0</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>$\lambda_2$</td>
<td>90</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-90</td>
<td>$\lambda_4$</td>
<td>$\theta_5$</td>
</tr>
<tr>
<td>6</td>
<td>$\lambda_5$</td>
<td>0</td>
<td>0</td>
<td>$\theta_6$</td>
</tr>
</tbody>
</table>

Table B.2. Geometric link parameters for the model without ankle plantarflexor activity and stance knee flexion-extension.

<table>
<thead>
<tr>
<th>Link #</th>
<th>$a_{i-1}$</th>
<th>$\alpha_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
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<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>$\lambda_1$</td>
<td>90</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>90</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>-90</td>
<td>$\lambda_3$</td>
<td>$\theta_4$</td>
</tr>
<tr>
<td>5</td>
<td>$\lambda_4$</td>
<td>0</td>
<td>0</td>
<td>$\theta_5$</td>
</tr>
</tbody>
</table>
Table B.3. Geometric link parameters for the model without pelvic determinants (planar model for single support).

<table>
<thead>
<tr>
<th>Link #</th>
<th>(a_{i-1})</th>
<th>(a_{i-1})</th>
<th>(d_i)</th>
<th>(\theta_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(\theta_1)</td>
</tr>
<tr>
<td>2</td>
<td>(\theta_1)</td>
<td>0</td>
<td>0</td>
<td>(\theta_2)</td>
</tr>
<tr>
<td>3</td>
<td>(\theta_2)</td>
<td>0</td>
<td>0</td>
<td>(\theta_3)</td>
</tr>
<tr>
<td>4</td>
<td>(\theta_3)</td>
<td>90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>-90</td>
<td>(\theta_4)</td>
<td>(\theta_5)</td>
</tr>
<tr>
<td>6</td>
<td>(\theta_5)</td>
<td>0</td>
<td>0</td>
<td>(\theta_6)</td>
</tr>
</tbody>
</table>

Table B.4. Geometric link parameters for the model with no determinants of gait.

<table>
<thead>
<tr>
<th>Link #</th>
<th>(a_{i-1})</th>
<th>(a_{i-1})</th>
<th>(d_i)</th>
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<tr>
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<td>0</td>
<td>(\theta_1)</td>
</tr>
<tr>
<td>2</td>
<td>(\theta_1)</td>
<td>90</td>
<td>0</td>
<td>(\theta_2)</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>-90</td>
<td>(\theta_2)</td>
<td>(\theta_3)</td>
</tr>
<tr>
<td>4</td>
<td>(\theta_3)</td>
<td>0</td>
<td>0</td>
<td>(\theta_4)</td>
</tr>
</tbody>
</table>
APPENDIX C

SUB-ALGORITHMS FOR GENERATING THE EQUATIONS OF MOTION

The notation used to develop the following is that due to Craig (1986). All symbols are defined in the nomenclature preceding the text.

C.1 THE NEWTON-EULER INVERSE DYNAMICS ALGORITHM

For an N link model with only revolute joints, the external moments are calculated as follows:

Moving outwards from the base ($i = 0, N-1$):

\[ \omega_{i+1}^+ = i+1^R \omega_i + \Theta_{i+1}^+ \]  \hspace{1cm} (C.1)

\[ \omega_{i+1}^+ = i+1^R \omega_i + i+1^R \omega_i \times \Theta_{i+1}^+ i+1^R \]  \hspace{1cm} (C.2)

\[ v_{i+1}^+ = i+1^R ( i \omega_i \times p_{i+1}^+ + \omega_i \times ( i \omega_i \times p_{i+1}^+) \]  \hspace{1cm} (C.3)

\[ v_{i+1}^+ = i+1 \omega_{i+1}^+ \times i+1 p_{i+1}^+ \]  \hspace{1cm} (C.4)
\[ i+1 F_{i+1} = m_{i+1} \cdot v_{c_{i+1}} \]
\[ i+1 N_{i+1} = c_{i+1} \cdot i+1 \cdot \omega_{i+1} \]
\[ + i+1 \omega_{i+1} \times c_{i+1} \cdot i+1 \omega_{i+1} \]  \hspace{1cm} (C.6)

Moving inwards from the swing foot leg \((i = N \text{ to } l)\):  
\[ i_{f_i} = i_{i+1 R} \cdot i+1 f_{i+1} + iF_i \]
\[ i_{n_i} = i_{N_i} + i_{i+1 R} \cdot i+1 n_{i+1} + iP_i \times iF_i \]
\[ + iF_{i+1} \times i^{i+1 R} \cdot i+1 F_{i+1} \]
\[ \tau_i = i_{n_i}^T \cdot i \cdot z_i \]  \hspace{1cm} (C.9)

For the inclusion of a prismatic joint, the following replace equations (C.1), (C.2), (C.3) and (C.9) respectively.

\[ i+1 \omega_{i+1} = i+1 R \cdot i \omega_i \]  \hspace{1cm} (C.10)

\[ i+1 \omega_{i+1} = i+1 R \cdot i \omega_i \]  \hspace{1cm} (C.11)

\[ i+1 v_{i+1} = i+1 R \cdot (i \omega_i \times i p_{i+1} + i \omega_i \times (i \omega_i \times i p_{i+1}) \]
\[ + i v_i ) + 2. i+1 \omega_{i+1} \times d_{i+1} \cdot i+1 z_{i+1} \]
\[ + d_{i+1} \cdot i+1 z_{i+1} \]  \hspace{1cm} (C.12)

\[ \tau_i = i_{f_i}^T \cdot i \cdot z_i \]  \hspace{1cm} (C.13)

where \( \tau_i \) in equation (C.13) now represents a force along link \( i \), rather than a moment about the \( i^{th} \) joint.
C.2 AN ALGORITHM FOR COMPUTING THE JACOBIAN

Equation (2.9) gives the velocity of the constrained foot, subsequent to heel-strike. That is,

\[ \mathbf{\dot{s}} = \mathbf{K(\dot{\theta})} \mathbf{\dot{\theta}} \]

where \([\mathbf{K(\dot{\theta})}]\) is the Jacobian which transforms the joint rates to a linear velocity vector of the foot. In solving for the redundant velocities \(\dot{\theta}_r\) using equation (2.11), we are first required to create the Jacobian \([\mathbf{K(\dot{\theta})}]\). The following is used when all joints are revolute:

Moving outwards from the base (i = 0 to N-1):

\[
\begin{align*}
\mathbf{\dot{\omega}}_{i+1} & = \mathbf{i}^{i+1 \mathbf{R}} \mathbf{i}^{\mathbf{\dot{\omega}}} \mathbf{\dot{\theta}}_{i+1} + \mathbf{i}^{i+1 \mathbf{R}} \mathbf{\dot{z}}_{i+1} \\
\mathbf{\dot{v}}_{i+1} & = \mathbf{i}^{i+1 \mathbf{R}} (\mathbf{i}^{\mathbf{v}} + \mathbf{i}^{\mathbf{\omega}} \times \mathbf{i}^{\mathbf{p}_{i+1}}) 
\end{align*}
\]

(C.14)

(C.15)

and for link N only:

\[ \mathbf{n}_{\mathbf{v}_{\mathbf{f}}_{\mathbf{n}}} = \mathbf{n}_{\mathbf{v}} + \mathbf{n}_{\mathbf{w}} \times \mathbf{n}_{\mathbf{p}_{\mathbf{f}}_{\mathbf{n}}} \]  

(C.16)

which gives the velocity of a chosen point on the constrained foot, with respect to link N's reference frame. Finally, for the velocity of such a point expressed relative to the base, we have:

\[ \mathbf{o}_{\mathbf{v}_{\mathbf{f}}_{\mathbf{n}}} = (\mathbf{o}_{\mathbf{1}^{\mathbf{R}}} \mathbf{R} \mathbf{2}^{\mathbf{3}} \mathbf{R} \mathbf{3}^{\mathbf{4}} \mathbf{R} \cdots \mathbf{n}^{-1} \mathbf{R}) \cdot \mathbf{n}_{\mathbf{v}_{\mathbf{f}}_{\mathbf{n}}} \]

or

\[ \mathbf{o}_{\mathbf{v}_{\mathbf{f}}_{\mathbf{n}}} = \mathbf{o}_{\mathbf{n}} \cdot \mathbf{n}_{\mathbf{v}_{\mathbf{f}}_{\mathbf{n}}} \]  

(C.17)

For any joint which is prismatic, we substitute the following for equations (C.14), (C.15) and (C.16) respectively:

\[ \begin{align*}
\mathbf{\dot{\omega}}_{i+1} & = \mathbf{i}^{i+1 \mathbf{R}} \mathbf{i}^{\mathbf{\dot{\omega}}} \\
\mathbf{\dot{v}}_{i+1} & = \mathbf{i}^{i+1 \mathbf{R}} (\mathbf{i}^{\mathbf{v}} + \mathbf{i}^{\mathbf{\omega}} \times \mathbf{i}^{\mathbf{p}_{i+1}}) + \mathbf{i}^{i+1 \mathbf{R}} \mathbf{\dot{z}}_{i+1}
\end{align*} \]

(C.18)

(C.19)
and if link N is prismatic, then:

\[ v_{fn} = v_n + \omega_n \times p_{fn} + d_n \cdot \dot{z}_n \]  

(C.20)

C.3 FORMING THE VECTOR \( \mathbf{K}_c \) IN THE ACCELERATION CONSTRAINT EQUATIONS

The following algorithm forms the vector \( \mathbf{K}_c \), defined in equation (2.13), for the case where all joints are revolutes.

Outwards from the base (\( i = 0 \) to \( N-1 \)):

\[ i+1 \omega_{i+1} = i+1 R_i \omega_i + \dot{\theta}_{i+1} \cdot z_{i+1} \]  

(C.21)

\[ i+1 \omega_{i+1} = i+1 R_i \omega_i + i+1 R_i \omega_i \times \dot{\theta}_{i+1} \cdot z_{i+1} \]  

(C.22)

\[ i+1 v_{i+1} = i+1 R_i ( \omega_i \times i p_{i+1} \) \]  

\[ + \omega_i \times ( i \omega_i \times i p_{i+1} ) + \dot{v}_i \]  

(C.23)

where \( \ddot{v}_0 = 0 \) in equation (C.23). Then, for link N only, the foot acceleration with respect to link N's reference frame is:

\[ n \cdot v_{fn} = n \cdot v_n + n \cdot \omega_n \times p_{fn} + n \omega_n \times ( n \omega_n \times p_{fn} ) \]  

(C.24)

Finally, with respect to the base, we have:

\[ o \cdot v_{fn} = ( o R_1 R_2 R_3 \ldots \ldots \ldots n-1 R ) n v_{fn} \]

or \( o \cdot v_{fn} = o R_n v_{fn} \)  

(C.25)

For a prismatic joint, replace equations (C.21), (C.22), (C.23) and (C.24) by the following:

\[ i+1 \omega_{i+1} = i+1 R_i \omega_i \]  

(C.26)

\[ i+1 \omega_{i+1} = i+1 R_i \omega_i \]  

(C.27)

\[ i+1 v_{i+1} = i+1 R_i ( i \omega_i \times i p_{i+1} \) \]  

\[ + \omega_i \times ( i \omega_i \times i p_{i+1} ) \]  

(C.28)
\[ + i \dot{v}_i \) + 2 \cdot i+1 \omega_{i+1} \times d_{i+1}. \cdot i+1 \hat{z}_{i+1} \] (C.28)

and if link N is prismatic:

\[ n \cdot \dot{v}_{fn} = n \cdot \dot{v}_n + n \cdot \omega_n \times p_{fn} + n \cdot \omega_n \times (n \cdot \omega_n \times n \cdot p_{fn}) \]
\[ + 2 \cdot n \cdot \omega_n \times d_n. \cdot n \hat{z}_n \] (C.29)

C.4 THE MODIFIED NEWTON-EULER INVERSE DYNAMICS ALGORITHM

The following algorithm computes the joint moments given the limb displacements, velocities, accelerations and applied muscle forces.

Link velocities and accelerations are first computed from the stance foot (base) outwards to the end-effector (swing foot). Next, the modified Newton-Euler equations are applied to compute the joint forces and moments from the end-effector back to the base.

Outwards \((i = 0, N-1)\):

\[ i+1 \omega_{i+1} = i+1 \omega_i + \theta_{i+1} \cdot i+1 \hat{z}_{i+1} \] (C.30)

\[ i+1 \cdot \omega_{i+1} = i+1 \omega_i + i+1 \omega_i \times \theta_{i+1} \cdot i+1 \hat{z}_{i+1} \]
\[ + \theta_{i+1} \cdot i+1 \hat{z}_{i+1} \] (C.31)

\[ i+1 \cdot v_{i+1} = i+1 \cdot R_i. (i \cdot \omega_i \times i \cdot p_{i+1} + i \cdot \omega_i \times (i \cdot \omega_i \times i \cdot p_{i+1}) \]
\[ + i \cdot v_i) \] (C.32)

\[ i+1 \cdot \omega_{i+1} \times i+1 \cdot \omega_{i+1} + i+1 \omega_{i+1} \times (i+1 \omega_{i+1} \times i+1 \cdot \omega_{i+1}) \]
\[ + i+1 \cdot v_{i+1} \] (C.33)
\( i+1 F_i + 1 = m_i + 1 \nu_i + 1 \)  \hspace{1cm} (C.34)

\( i+1 N_i + 1 = c_i + 1 I_i + 1 \)  \hspace{1cm} (C.35)

\( + i+1 \omega_i + 1 x c_i + 1 I_i + 1 \omega_i + 1 \)

**Inwards** \((i = N, 1)\):

\( i f_i = \frac{i}{F_i} + i + 1 R i+1 f_i + 1 - i f_j - i f_k \)  \hspace{1cm} (C.36)

\( i n_i = i N_i + i + 1 R i+1 n_i + 1 + i p_i c_i x f_i \)

\( + i p_i + 1 x i + 1 R i+1 f_i + 1 - i p_j x f_j \)

\( - i p_k i f_k \)  \hspace{1cm} (C.37)

\( \tau_i = i n_i \cdot i Z_i \)  \hspace{1cm} (C.38)
APPENDIX D
A PROOF FOR THE INVERTED SINGLE PENDULUM

The aim is to show that the inverted single pendulum is incapable of simultaneously producing the form of the vertical and horizontal ground reaction found during normal, level walking.

Figure D.1 shows the inverted single pendulum model, with the total body mass lumped at it’s free end. The acceleration of the body’s center of gravity is defined by:

\[
x = -l \sin \theta \ddot{\theta}^2 + l \cos \theta \ddot{\theta} \\
y = -l \cos \theta \ddot{\theta}^2 - l \sin \theta \ddot{\theta}
\]  

(D.1)  
(D.2)

At the instant of opposite toe-off, the horizontal reaction \( F_x \) is required to have positive slope and the vertical reaction \( F_y \), negative slope. Differentiating equations (D.1) and (D.2) gives:

\[
x = -l \cos \theta \dot{\theta}^3 - 3l \sin \theta \dot{\theta}^2 \ddot{\theta} + l \cos \theta \dddot{\theta} \\
y = l \sin \theta \dot{\theta}^3 - 3l \cos \theta \dot{\theta}^2 \ddot{\theta} - l \sin \theta \dddot{\theta}
\]  

(D.3)  
(D.4)

Then, solving for \( \dddot{\theta} \) and \( \ddot{\theta} \) gives:

\[
\dddot{\theta} = \frac{\cos \theta \dddot{x} - \sin \theta \dddot{y}}{3l^2} \\
\ddot{\theta} = \frac{1}{3l \theta} \left( \sin \theta \dddot{x} + \cos \theta \dddot{y} \right)
\]  

(D.5)  
(D.6)

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Figure D.1: The inverted single pendulum

Now impose the conditions that \( x > 0 \) for the slope of \( F_x \) to be positive, and \( y < 0 \) for a negative slope of \( F_y \). Then, from equation (D.6), this leads to \( \theta > 0 \) always, since \( \theta \) is initially negative and less than 90 degrees, while \( \dot{\theta} \) is always positive. Therefore, for the ground reactions to commence in the manner of that observed during normal walking, the pendulum must accelerate initially. In fact, the requirement that \( y \) be less than zero, at least to the time of midstance, means that the pendulum must be accelerated during this
entire region of single support. Consequently, the horizontal ground reaction commences in the wrong direction, since the center of gravity is being accelerated rather than decelerated.

Finally, using the conditions $x > 0$ and $y < 0$ in equation (D.5), a relationship may be obtained for the single pendulum to produce the correct vertical ground reaction force. For $\Theta > 0$, the following must hold:

$$\cos\Theta \cdot x + \dot{\Theta}^3 > \sin\Theta \cdot y$$  \hspace{2cm} (D.7)

Alternatively, the case of $\Theta < 0$ requires:

$$\cos\Theta \cdot x + \dot{\Theta}^3 < \sin\Theta \cdot y$$  \hspace{2cm} (D.8)

Now, the condition $\Theta > 0$ is not desirable since this means that the rate at which the pendulum accelerates is increasing. Therefore, the relationship sought is that of equation (D.8). This equation, however, makes no further contribution to the central conclusion stated above, since it implicitly involves the condition that $\ddot{\Theta} > 0$. It merely defines the condition under which the vertical ground reaction can dip below the level of body weight, as midstance is approached.
APPENDIX E

A NONLINEAR KNEE TORQUE

Figure E.1 shows the inverted double pendulum model, with a linear spring attached between the ankle and hip joints. The geometry involved is also presented and the meaning of all symbols is given in the nomenclature.

The equivalent knee torque may be calculated from:

Knee torque = Spring force \times Moment arm

or

\[ \tau_k = k( a - z ) \cdot r_k \]  \hspace{1cm} \text{(E.1)}

The aim is to find an expression for the torque as a function of the shank and thigh angles, \( \theta_2 \) and \( \theta_3 \) respectively. The hip coordinates are given by:

\[ x = l_2 \cdot \sin \theta_2 + l_3 \cdot \sin(\theta_2 + \theta_3) \] \hspace{1cm} \text{(E.2)}

\[ y = l_2 \cdot \cos \theta_2 + l_3 \cdot \cos(\theta_2 + \theta_3) \] \hspace{1cm} \text{(E.3)}

Therefore,

\[ \tan \theta_k = \frac{x}{y} = \frac{l_2 \cdot \sin \theta_2 + l_3 \cdot \sin(\theta_2 + \theta_3)}{l_2 \cdot \cos \theta_2 + l_3 \cdot \cos(\theta_2 + \theta_3)} \]

and

\[ \theta_k = \tan^{-1} \left( \frac{l_2 \cdot \sin \theta_2 + l_3 \cdot \sin(\theta_2 + \theta_3)}{l_2 \cdot \cos \theta_2 + l_3 \cdot \cos(\theta_2 + \theta_3)} \right) \] \hspace{1cm} \text{(E.4)}

Hence,
Figure E-1: (a) The inverted double pendulum with a linear spring inserted between the ankle and hip joint. (b) Geometry for derivation of nonlinear knee moment.
\[ r = \frac{l_2 \cdot \sin \theta_2 + l_3 \cdot \sin(\theta_2 + \theta_3)}{\sin \theta_k} \]  
(E.5)

Also,  
\[ \theta_k' = \theta_k - \theta_2 \]  
(E.6)

so that

\[ r_k = l_2 \cdot \sin \theta_k' \]  
(E.7)

Finally, substituting equations (E.5) and (E.7) into (E.1) gives:

\[ \tau_3 = k \cdot (a - \frac{l_2 \cdot \sin \theta_2 + l_3 \cdot \sin(\theta_2 + \theta_3)}{\sin \theta_k}) \cdot (l_2 \cdot \sin(\theta_k - \theta_2)) \]  
(E.8)

where \( \theta_k \) is given by equation (E.4). Equation (E.8), therefore, defines a knee torque which is a highly nonlinear function of the limb angles \( \theta_2 \) and \( \theta_3 \). It is the expression used to produce the ground reactions given as Case 4 in Figure 3.6.
APPENDIX F

EQUATIONS OF MOTION FOR PLANAR MODELS

F.1 INVERTED DOUBLE PENDULUM:

The equations of motion for the model of Figure 3.4 are:

\[(I_2 + m_2 \ell_2^2 + m_3 \ell_2^2 + m_1 \ell_2^2 + m_3 \ell_2 r_3 C_3 + m_1 \ell_2 r_3 C_3).\dot{\theta}_2 + (m_3 \ell_2 r_3 C_3 + m_1 \ell_2 r_3 C_3).\dot{\theta}_3 - (m_3 \ell_2 r_3 S_3 + m_1 \ell_2 r_3 S_3)(\dot{\theta}_2 + \dot{\theta}_3)^2
- (m_2 r_2 + m_3 \ell_2 + m_1 \ell_2)gS_2 = \tau_2 - \tau_3 \]  \hspace{1cm} (F.1)

\[(I_3 + m_2 \ell_3^3 + m_1 \ell_3^3 + m_3 \ell_2 r_3 C_3 + m_1 \ell_2 r_3 C_3)\dot{\theta}_2 + (I_3 + m_3 \ell_3^2 + m_1 \ell_3^2)\ddot{\theta}_3 + (m_3 \ell_2 r_3 S_3 + m_1 \ell_2 r_3 S_3).\dot{\theta}_2 - (m_3 r_3 + m_1 \ell_3)gS_{23} = \tau_3 \]  \hspace{1cm} (F.2)

F.2 MODEL FOR SWING LEG DYNAMICS:

For the model of Figure 3.9, the equations of motion are:

\[(I + m_r^2 + m_1 \ell_r^2 + m_4 \ell_r^2 + m_5 \ell_r^2 + m_4 \ell r_4 C_4 + m_5 \ell r_4 C_4 + m_5 \ell r_5 C_{45}).\ddot{\theta}
+ (m_4 \ell r_4 C_4 + m_5 \ell r_4 C_4 + m_5 \ell r_5 C_{45}).\dot{\theta}_4 + (m_5 \ell r_5 C_{45}).\dot{\theta}_5
- (m_4 \ell r_4 S_4 + m_5 \ell r_4 S_4)(\dot{\theta} + \dot{\theta}_4)^2
- (m_5 \ell r_5 S_{45})(\dot{\theta} + \dot{\theta}_4 + \dot{\theta}_5)^2) \]
\[-(m_r + m_4\ell + m_5\ell_5 \cdot g \cdot \sin \theta = \tau - \tau_4 \quad (F.3)\]

\[(I_4 + m_4r_4^2 + m_5r_5^2 + m_4r_4S_4 + m_5r_5S_4 + m_4r_5S_5).\ddot{\theta} \]
\[+ (I_4 + m_4r_4^2 + m_5r_5^2 + m_5r_5S_5).\ddot{\theta}_4 + (m_5r_4^2S_5).\ddot{\theta}_5 \]
\[+ (m_4r_4S_4 + m_5r_5S_4).\dddot{\theta} \quad - (m_5r_4S_5).(-\ddot{\theta}_4 + \ddot{\theta}_5) \]
\[-(m_4r_4 + m_5r_5).g \cdot \sin(\theta + \theta_4) + \tau_4 = \tau_5 \quad (F.4)\]

\[(m_5r_4S_4 + m_5r_5S_5 + I_5 + m_5r_5^2).\dddot{\theta} \]
\[+ (m_5r_5S_5 + m_5r_5^2 + I_5).\ddot{\theta}_4 + (I_5 + m_5r_5^2).\ddot{\theta}_5 \]
\[+ (m_5r_5S_4).\dddot{\theta} \quad - (m_5r_5S_5).(-\ddot{\theta}_4 + \ddot{\theta}_5) \]
\[-m_5r_5g \cdot \sin(\theta + \theta_4 + \theta_5) = \tau_5 \quad (F.5)\]

**F.3 MODEL FOR STANCE KNEE FLEXION-EXTENSION:**

For the model of Figure 3.7, the equations of motion are:

\[(m_4r_4S_4 + m_5r_5S_4 + m_5r_5S_4).\dddot{\theta} \]
\[+ (m_4r_4S_4 + m_5r_5S_4 + m_5r_5S_4).\ddot{\theta}_4 \]
\[+ (m_5r_5S_4).\dddot{\theta}_5 \quad - (m + m_4 + m_5).z \]
\[+ (m_4r_4C_4 + m_5r_5C_4).z \quad \dddot{\theta} \]
\[+ (m_4r_4C_4 + m_5r_5C_4).(-\ddot{\theta}_4 + \ddot{\theta}_5) \]
\[+ (m_5r_5C_4).(-\ddot{\theta}_4 + \ddot{\theta}_5) \quad - (m + m_4 + m_5).g \cdot \cos \theta \]
\[+ k.(a - z) = 0 \quad (F.6)\]

\[(m_4r_4S_4 + m_5r_5S_4 + m_5r_5^2)\]
+ I + m_4 z r_4^2 C_4 + m_5 z l_4 C_4 + m_5 z r_5 C_{45}) \cdot \theta
+ (m_4 z r_4 C_4 + m_5 z l_4 C_4 + m_5 z r_5 C_{45}) \cdot \theta_4
+ (m_5 z r_5 C_{45}) \cdot \theta_5 + (2m_z (z-r) + 2m_4 z + 2m_5 z) \cdot z \cdot \theta
- (m_4 z r_4 S_4 + m_5 z l_4 S_4) \cdot (\dot{\theta} + \dot{\theta}_4)^2
- (m_5 z r_5 S_{45}) \cdot (\dot{\theta} + \dot{\theta}_4 + \dot{\theta}_5)^2
- (m_z (z-r) + m_4 z + m_5 z) \cdot g \cdot \sin \theta = \tau - \tau_4 \tag{F.7}

(I_4 + m_4 r_4^2 + m_5 l_4^2 + m_4 z r_4 C_4 + m_5 z l_4 C_4 + m_5 z r_5 C_{45}) \cdot \theta
+ (I_4 + m_4 r_4^2 + m_5 l_4^2 + m_5 z r_4 C_{45}) \cdot \theta_4 + (m_5 z r_5 C_{45}) \cdot \theta_5
- (m_4 r_4 S_4 + m_5 l_4 S_4) \cdot z + (2m_4 r_4 C_4 + 2m_5 l_4 C_4) \cdot z \cdot \theta
+ (m_4 z r_4 S_4 + m_5 z l_4 S_4) \cdot \dot{\theta}^2 - (m_5 z r_5 S_{45}) \cdot (\dot{\theta} + \dot{\theta}_4 + \dot{\theta}_5)^2
- (m_4 r_4 + m_5 l_4) \cdot g \cdot \sin (\theta + \theta_4) = \tau - \tau_5 \tag{F.8}

(m_5 r_5 C_{45} + m_5 z r_5^2 C_{45} + m_5 z l_5 C_{45} + I_5) \cdot \theta
+ (m_5 r_5^2 + I_5 + m_5 z r_5 C_{45}) \cdot \theta_4
+ (m_5 r_5^2 + I_5) \cdot \theta_5 - (m_5 z r_5 S_{45}) \cdot z + (2m_5 z r_5 C_{45}) \cdot z \cdot \theta
+ m_5 r_5 \cdot g \cdot \sin (\theta + \theta_4 + \theta_5) = \tau_5 \tag{F.9}

F.4 MODEL FOR FOOT AND KNEE INTERACTION:

The equations of motion for the model of Figure 3.8, with the stance and swing leg's uncoupled, are:
Stance knee:

\[ (I_1 + m_1 \ell_1^2 + m_2 \ell_1^2 + m_3 \ell_1^2 + m_T \ell_1^2 + m_1 r_1 C_2 + m_3 \ell_1^2 C_2 + m_3 \ell_1^2 C_3 + m_{T1} \ell_1^2 C_3 + m_{T3} \ell_1^2 C_3) \dot{\theta}_1 \]
\[ + (m_2 \ell_2 r C_2 + m_3 \ell_2 r C_2 + m_T \ell_2 r C_2 + m_1 \ell_3 r C_2 + m_3 \ell_3 r C_2 + m_T \ell_3 r C_2) \dot{\theta}_2 \]
\[ + (m_3 \ell_3 r C_2 + m_T \ell_3 r C_2) \dot{\theta}_3 \]
\[- (m_2 \ell_2 r S_2 + m_3 \ell_2 r S_2 + m_T \ell_2 r S_2) (\dot{\theta}_1 + \dot{\theta}_2) \]
\[- (m_3 \ell_3 r S_3 + m_T \ell_3 r S_3) \dot{\theta}_1 (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \]
\[- (m_1 r_1 + m_2 \ell_1 + m_3 \ell_1 + m_T \ell_1) g \sin \theta_1 = \tau_1 - \tau_2 \quad (F.10) \]

\[ (m_2 \ell_2^2 + m_3 \ell_2^2 + m_T \ell_2^2 + I_2 + m_2 \ell_2 r C_2 + m_3 \ell_2 r C_2 + m_T \ell_2 r C_2 + m_3 \ell_2 r C_3 + m_T \ell_2 r C_3 + m_3 \ell_2 r C_3) \dot{\theta}_1 \]
\[ + (m_2 \ell_2 r C_2 + m_3 \ell_2 r C_2 + m_T \ell_2 r C_2 + m_3 \ell_2 r C_3 + m_T \ell_2 r C_3) \dot{\theta}_2 \]
\[ + (m_3 \ell_3 r C_3 + m_T \ell_3 r C_3) \dot{\theta}_3 \]
\[ + (m_2 \ell_2 r S_1 + m_3 \ell_2 r S_2 + m_T \ell_2 r S_2) \dot{\theta}_1 \]
\[- (m_3 \ell_3 r S_3 + m_T \ell_3 r S_3) (\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3) \]
\[- (m_2 \ell_2 + m_3 \ell_2 + m_T \ell_2) g \sin (\theta_1 + \theta_2) = \tau_2 - \tau_3 \quad (F.11) \]

\[ (m_3 \ell_3 r C_23 + m_3 \ell_3 r C_3 + m_T \ell_3 r C_23 + m_T \ell_3 r C_23 + m_T \ell_3 r C_3) \dot{\theta}_1 \]
\[ + (m_3 \ell_3 r C_3 + m_T \ell_3 r C_3 + m_3 \ell_3 r C_3 + m_T \ell_3 r C_3 + I_3) \dot{\theta}_2 \]
\[ + (m_T \ell_3^2 + m_3 \ell_3^2 + I_3) \dot{\theta}_3 \]
\[ + (m_T \ell_3^2 + m_3 \ell_3^2 + I_3) \dot{\theta}_3 \]
\[- (m_3 \ell_3 + m_T \ell_3) g \sin (\theta_1 + \theta_2 + \theta_3) = \tau_3 \quad (F.12) \]
Uncoupled swing leg:

\[
\begin{align*}
\dot{\theta}_4 &= (I_4 + m_5 r_4^2 + m_5 r_4 C_5) \ddot{\theta}_4 + (m_5 r_4 C_5) \dddot{\theta}_5 \\
&- m_5 r_4 S_4 (x - \dot{x}_4 \dot{\theta}_4 - r_4 C_4 \dot{\theta}_4^2) - r_5 C_{45} \dot{\theta}_5^2 \\
&+ m_5 r_4 C_4 (y - \dot{y}_4 \dot{\theta}_4 - r_4 S_4 \dot{\theta}_4^2) \\
&- m_4 r_4 S_4 (x - r_4 C_4 \dot{\theta}_4^2) + m_4 r_4 C_4 (y - r_4 S_4 \dot{\theta}_4^2) \\
&+ (m_4 r_4 + m_5 r_4) g \sin \theta_4 = \tau_4 - \tau_5
\end{align*}
\]

(F.13)

\[
\begin{align*}
\dot{\theta}_5 &= (I_5 + m_5 r_5^2 + m_5 r_5^2) \ddot{\theta}_5 + (I_5 + m_5 r_5^2) \dddot{\theta}_5 \\
&+ m_5 r_5 C_{45} (y - \dot{y}_4 \dot{\theta}_4 - r_5 S_{45} \dot{\theta}_5^2) \\
&- m_5 r_5 S_{45} (x - \dot{x}_4 \dot{\theta}_4 - r_5 C_{45} \dot{\theta}_5^2) \\
&- m_5 r_5 g \sin (\theta_4 + \theta_5) = \tau_5
\end{align*}
\]

(F.14)
APPENDIX G

SPECIFICATION OF APPLIED INPUTS

G.1 MUSCLE FORCE SPECIFICATION

To completely specify a muscle force as an input to the model, both its magnitude and line of action must be given. This is most easily accomplished using vector analysis. Consider the muscle force $F_{ji}$. That is, the force exerted by the $j^{th}$ muscle on the $i^{th}$ link. Its three components are given by:

$$F_{ji} = F_{jix} \hat{i} + F_{jiy} \hat{j} + F_{jiz} \hat{k}$$
$$= F_{ji} \cos \omega_{ji} \hat{i} + F_{ji} \cos \beta_{ji} \hat{j} + F_{ji} \cos \gamma_{ji} \hat{k} \quad (G.1)$$

where $\cos \omega_{ji}$, $\cos \beta_{ji}$ and $\cos \gamma_{ji}$ are the direction cosines of the force $F_{ji}$. These can be determined by calculating the appropriate muscle length which, in turn, is most conveniently arrived at using the $4 \times 4$ position transformations (Chapter 2). As an example, take the $j^{th}$ muscle acting between links $i$ and $i-1$. Then,

$$\text{muscle length} = \text{mag} ( \hat{0}p_i - \hat{0}p_{i-1} )$$
$$= \text{mag} ( ( \hat{0}p_i - \hat{0}p_{i-1} )_x \hat{i} + ( \hat{0}p_i - \hat{0}p_{i-1} )_y \hat{j} + ( \hat{0}p_i - \hat{0}p_{i-1} )_z \hat{k} ) \quad (G.2)$$
where \( ^{0}p_{i} \) = distance from base (link 0) to link \( i \)
\[
^{0}p_{i} = ^{0}_{1}T \cdot ^{2}_{1}T \cdot \ldots \cdot ^{i-1}_{i}T \cdot ^{i}_{i}p_{i}
\]
and \( ^{0}p_{i-1} = ^{0}_{1}T \cdot ^{2}_{1}T \cdot \ldots \cdot ^{i-2}_{i-1}T \cdot ^{i-1}_{i-1}p_{i-1} \)

Here, \( ^{i}_{i}p_{i} \) and \( ^{i-1}_{i-1}p_{i-1} \) are the known attachment points of each muscle, specified in the coordinate frame of the relevant link.

Therefore, the direction cosines are immediately calculated from:
\[
\cos \alpha_{ji} = \frac{ ^{0}p_{i} - ^{0}p_{i-1} }{ \text{mag}( ^{0}p_{i} - ^{0}p_{i-1} ) } \quad (G.3)
\]
and similarly for \( \cos \beta_{ji} \) and \( \cos \gamma_{ji} \).

Specifying the magnitude of the muscle force \( F_{ji} \) in equation (G.1) and substituting equations like (G.3), the force \( F_{ji} \) becomes completely known. Note however that this force has been specified relative to the ground reference frame. To utilize the modified Newton-Euler equations of Appendix C, it must first be rotated back into the local coordinate frame of link \( i \). That is:
\[
^{i}_{j}F_{ji} = ^{i}_{i-1}R \cdot ^{i-1}_{i-2}R \cdot \ldots \cdot ^{2}_{1}R \cdot ^{1}_{1}R \cdot ^{0}_{0}F_{ji} \quad (G.4)
\]

G.2 INPUT JOINT MOMENTS FOR PLANAR MODELS

For the model of Figure 2.3, the parameters pertaining to the input moments are:
\[
\tau_{1} = 241. t \quad 0 < t < 0.27 \text{ s}
\]
\[
k = 12 \text{ kN/m}
\]
\[
C_{D} = 700 \text{ Ns/m}
\]
\[
\tau_{4} = 900. t \quad 0 < t < 0.05 \text{ s}
\]
\[ \tau_5 = -100t \quad 0 < t < 0.05 \text{ s} \]

For the model of Figure 2.4, the inputs are:

\[ \tau_1 = 0.0 \]
\[ \tau_2 = 200(t-0.27) + 65 \quad 0.27 < t < 0.43 \text{ s} \]
\[ \tau_3 = 14 \quad 0.27 < t < 0.43 \text{ s} \]
\[ \tau_4 = 0.0 \]
\[ \tau_5 = -5.0 \quad 0.38 < t < 0.43 \text{ s} \]

For the double support model given in Figure 2.6, the input joint moments are:

\[ \tau_1 = 0.0 \]
\[ \tau_2 = -500(t-0.43) + 80 \quad 0.43 < t < 0.58 \text{ s} \]
\[ \tau_3 = 20 \quad 0.43 < t < 0.58 \text{ s} \]
\[ \tau_4 = 0.0 \]
\[ \tau_5 = -1.0 \quad 0.43 < t < 0.58 \text{ s} \]

G.3 MUSCLE FORCES FOR 3-D SIMULATION OF NORMAL GAIT

The following are actual muscle forces applied to the model of Figure 4.1 resulting in the joint moments given in Figure 4.5.

Hip abductor muscles:

\[ F_{ab} = 600t + 1050 \quad 0 < t < 0.1 \text{ s} \]
\[ F_{ab} = 100t + 750 \quad 0.1 < t < 0.3 \text{ s} \]
\[ F_{ab} = 700t + 850 \quad 0.3 < t < 0.46 \text{ s} \]

Ankle plantarflexor muscles:

\[ F_{ap} = 2300t + 1400 \quad 0.3 < t < 0.46 \text{ s} \]
Note that these are the applied forces corresponding only to those muscles directly included in the model. The remaining inputs are joint moments applied as shown in Figure 4.5.

G.4 INPUT PARAMETERS FOR SIMULATIONS OF PATHOLOGICAL GAIT

G.4.1 Contribution of individual gait determinants

The following define the inputs applied to the models of Chapter 5 in assessing the contribution of individual gait determinants to the dynamics of normal walking. For an explanation regarding the choice of each parameter see Chapter 4.

Ankle plantarflexors:

\[ F_{\text{ap}} = 2300 \cdot t + 400 \quad 0.3 < t < 0.46 \text{s} \]

Muscle origin: At the stance shank, just below the knee joint

Muscle insertion: At the stance foot, 50 mm from ankle joint on the side opposite to that containing the lumped foot mass

Note that the muscle attachment positions are chosen on the basis of providing suitable moment arms about which the muscle forces may develop appropriate joint moments.

Stance leg linear spring (for \( 0 < t < 0.3 \text{s} \)):

\[ k_\zeta = 25 \text{ kN/m} \]

\[ C_\zeta = 500 \text{ Ns/m} \]

\[ A_\zeta = 0.831 \text{ m} \]
Hip abductors:
\[
F_{ab} = \begin{cases} 
600t + 1050 & 0 < t < 0.1 \text{ s} \\
100t + 750 & 0.1 < t < 0.3 \text{ s} \\
700t + 750 & 0.3 < t < 0.46 \text{ s} 
\end{cases}
\]

Muscle origin: At the pelvis, 100mm from stance hip on the side opposite to that containing the lumped torso mass.

Muscle insertion: At the stance thigh, 100mm below the hip joint.

Stance knee torsional spring for planar model (for \(0.3 < t < 0.46\) s):
\[
k_t = 100 \text{ Nm/rad} \\
C_t = 0.0 \text{ Nms/rad} \\
A_t = 0.42 \text{ rad}
\]

Torsional spring for transverse pelvic rotation:
\[
k_t = \begin{cases} 
50 \text{ Nm/rad} & 0 < t < 0.3 \text{ s} \\
10 \text{ Nm/rad} & 0.3 < t < 0.46 \text{ s} \\
20 \text{ Nms/rad} & 0 < t < 0.3 \text{ s} \\
30 \text{ Nms/rad} & 0.3 < t < 0.46 \text{ s} 
\end{cases} \\
A_t = 4.9 \text{ rad}
\]

G.4.2 Compensatory mechanisms for pathological gait

To evaluate the degree of compensation required during pathological gait, the following muscle forces and model constants were used to replace those given above.

No ankle plantarflexors:
\[
F_{ab} = \begin{cases} 
600t + 1050 & 0 < t < 0.1 \text{ s} \\
100t + 750 & 0.1 < t < 0.3 \text{ s} \\
6000t + 750 & 0.3 < t < 0.46 \text{ s} 
\end{cases}
\]
No ankle plantarflexors and no stance knee flexion-extension:
\[
F_{ab} = \begin{cases} 
4000.0t + 1050 & 0 < t < 0.1 \text{ s} \\
100.0t + 750 & 0.1 < t < 0.3 \text{ s} \\
6000.0t + 750 & 0.3 < t < 0.46 \text{ s} 
\end{cases}
\]

No pelvic determinants (planar model):

Stance leg spring (for \(0 < t < 0.3\text{s}\)):
\[
\begin{align*}
    k_{\lambda} &= 45 \text{ kN/m} \\
    C_{\lambda} &= 0.0 \text{ Ns/m} \\
    A_{\lambda} &= 0.821 \text{ m} 
\end{align*}
\]

Ankle plantarflexors:
\[
F_{ap} = \begin{cases} 
3000.0t + 1300 & 0.3 < t < 0.46 \text{s} 
\end{cases}
\]

Stance knee torsional spring (for \(0.3 < t < 0.46\text{s}\)):
\[
\begin{align*}
    k_t &= 100 \text{ Nm/rad} \\
    C_t &= 0.0 \text{ Nms/rad} \\
    A_t &= 0.34 \text{ rad} 
\end{align*}
\]

G.5 INPUT PARAMETERS FOR DOUBLE SUPPORT MODEL

The following define muscle forces applied to the planar double support model given in Chapter 6 (Figure 6.1):

Ankle plantarflexors:
\[
F_{ap} = 11000.0t - 1500 \quad 0 < t < 0.14 \text{s}
\]

Hip flexors:
\[
F_{hf} = 50 \quad 0 < t < 0.05 \text{s}
\]