An Investigation of Corporate Borrowing Strategies

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in the Graduate School of The Ohio State University

By

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ACKNOWLEDGEMENTS

I express gratitude to my advisor, Dr. René Stulz, for the guidance provided throughout the dissertation. I am also indebted to the other members of my committee, Dr. Stephen Buser and Dr. Anthony Sanders, for their comments and suggestions. Finally, a word of appreciation for insightful discussions and criticism is due to Dr. Warren Bailey, Dr. Francis Longstaff and Dr. Tom George.
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Introduction

A corporation that seeks external funds in the form of debt typically has a range of options vis-a-vis its borrowing strategies. For example, the firm can borrow short-term and roll-over the debt until a desired future date, or alternatively it can borrow long-term - a maturity strategy decision; also, given the maturity of the loan, the firm can borrow at a fixed coupon or it can borrow at a floating coupon tied to the level of interest rates - a coupon strategy decision. This dissertation presents two related essays on corporate borrowing strategies; it investigates separately and for different sets of market imperfections, maturity and coupon debt strategies.

Borrowing strategies are important if they influence the probability distribution of the total cash-flows of the firm. Modigliani and Miller (MM; 1958) lay out the conditions under which the value of the firm is independent of its financing policy. They show that if there are no taxes, no contracting costs, and if the investment policy of the firm is held constant, then the choice of financing policy doesn't affect the total cash-flows of the firm.¹ The MM irrelevancy proposition postulates that if borrowing strategies affect the value of the firm, they must do so via its

¹ Fama (1978) provides a thorough discussion on the irrelevance propositions.
impact on taxes, on contracting costs, or on the firm’s investment policy. One class of contracting costs that may lead to a breakdown of MM irrelevance propositions are those costs stemming from asymmetric information. When firm’s insiders have superior information about the firm’s prospects than investors in the market, the choice of financing instruments may affect insiders’ wealth. Hence, with asymmetric information insiders may prefer one borrowing strategy over another.

It is also well known that, the presence of risky debt in the firm’s capital structure, generates another class of contracting costs – agency costs of debt. With risky debt outstanding, stockholders have incentives to deviate from the investment and operating policy that maximizes the value of the firm and take actions that opportunistically expropriate wealth from bondholders.

The first essay analyses the coupon strategy decision in a environment in which (1) management is systematically better informed about their own firms than investors in the market, and (2) there are agency costs of debt. We develop and test a signalling model in which firms with favorable inside information select the coupon strategy with the largest agency costs of debt. The model helps us to understand how interest rate uncertainty interacts with asymmetric information and agency costs of debt to determine optimal financial policies.

As MM point out, taxes may also make capital structure decisions matter. The second essay investigates debt maturity strategies in a world in which the only market imperfection is taxation. We explore the
relevancy of debt maturity decisions when agents decide, simultaneously, about leverage and debt maturity structure. Taking leverage as given, we characterize optimal maturity strategies for investors and corporations and derive equilibrium relative prices between future dollars to be delivered with alternative strategies.

Linkages between maturity and coupon decisions are discussed in both essays. The first essay examines the implications of extending the insiders financing opportunity set to short-term debt. The second essay contrasts the effects of taxation on agents' choice between rolling-over short-term debt and issuing or purchasing long-term (fixed rate) debt with the effects of taxation on the choice of coupon strategy.
Part I

Asymmetric Information and the Corporate Choice
between Fixed and Floating Rate Debt

1. Introduction

It is well established in the corporate finance literature that in a world with asymmetric information, the capital structure may signal to the market the quality of the firm. The working hypothesis in this literature is that firm insiders possess private information about the firm, and that they use this private information in their self interest. With investors unable to verify the quality of inside information, the securities issued by the firm command a price consistent with the average firm quality, and there is a wealth redistribution from high to low quality firms. Rational investors, however, can sometimes reliably infer the quality of the firm from capital structure decisions. The literature presents non-dissipative signalling equilibria in which revelation of inside information is achieved with no deadweight loss, and dissipative signalling equilibria in which a costly signal is required for credible revelation of private information.

An example of a non-dissipative signalling solution for the asymmetric information problem is provided by Ross (1977). This author
argues that the amount of debt in the firm capital structure may constitute a credible signal for management's assessment of the quality of the firm. Managers, whose compensation contract is public information, incur a pre-specified penalty in the event of bankruptcy. Since an addition in leverage implies a larger increase in the expected value of the penalty for managers with less favorable information, the market infers the quality of the firm from the observation of the amount of debt in the firm's capital structure; a firm with "good" inside information supports more debt than an otherwise identical firm with "bad" inside information. The equilibrium is non-dissipative because the penalty incurred by managers in the bankrupt states is a pure incentive cost and not a deadweight cost.

Flannery (1986), in contrast, gives an example of a dissipative signalling equilibrium. He develops a model in which insiders use the debt maturity strategy to signal private information about firm quality. If firms incur in transaction costs when they borrow funds, the model shows that a separating equilibrium may emerge; in this equilibrium firms with "good" inside information sell short-term debt while firms with "bad" private information issue debt with longer maturities. Refinancing costs of short-term debt play the pivotal role. Low quality firms select long-term debt financing, because what they gain from mimicking high-quality firms and pooling in short-term debt is not large enough to compensate for the refinancing costs associated with a strategy of rolling over short-term debt. High quality firms, on the other hand, issue short-term debt.
For these firms, the refinancing costs of a roll-over strategy are small compared to what they lose from being pooled with low quality firms. The signalling equilibrium is of the dissipative type because high quality firms spend real resources in the form of refinancing costs to credibly convey their information to investors.

The first part of the dissertation explores the case in which firms' insiders use the debt coupon strategy to signal private information. By the debt coupon strategy we refer to the selection of a functional dependence of the contractual coupons on realizations of the level of the short-term interest rate. The analysis shows that the decision on the nature of the contractual coupons does sometimes convey information to the market. The investigation of the informational flows associated with the coupon structure of debt contributes to the literature on asymmetric information and capital structure, and in particular it complements the results obtained by Flannery.

We consider a framework in which insiders borrow funds to finance an investment opportunity about which they have private information. Contrary to Flannery, we abstract from the debt's maturity strategy and investigate instead how asymmetric information impacts on how insiders structure the contractual coupon payments on the borrowed funds. With investors unable to distinguish "Good" from "Bad" projects, bonds sell at a price that reflects the average project quality. To avoid selling underpriced bonds and redistributing wealth to insiders with low quality projects, insiders with high quality projects seek to signal their private
information. We address the issue of whether is feasible for the debt coupon decision to credibly convey private information on project quality.

With debt financing corporations incur agency costs. Jensen and Meckling (1976) define an agency relationship as

a contract under which one or more persons the principal(s) engage another person (the agent) to perform some service on their behalf which involves delegating some decision making authority to the agent (p. 308).

These authors also define agency costs as the sum of the monitoring expenditures incurred by the principal, the bonding expenditures incurred by the agent, and the residual loss. The residual loss refers to the reduction in welfare experienced by the principal due to the divergence between the agent’s actions and those which would maximize the welfare of the principal.

The relationship between equityholders and bondholders fits the definition of an agency relationship. With risky debt outstanding, principals are both equityholders and bondholders since the value of the stakes of the two classes of claimants is affected by managerial actions. Equityholders, however, run the firm and perform simultaneously the role of agent.² Agency costs arise because equityholders, who maximize the value of their claim on the firm, take opportunistic actions that

² This description of the relationship between stockholders and bondholders abstracts from all other agency relationships prevalent in the firm. In particular, it assumes that the incentives of stockholders and managers can be costlessly aligned.
expropriate wealth from the bondholders and reduce the value of the firm. For example, Myers (1977) shows that the stockholders will underinvest, i.e. they will pass up valuable investment opportunities, if the benefits accrue to the bondholders. Black and Scholes (1973), and Smith and Warner (1979), also point out that stockholders may exchange low-risk for high risk assets. In the words of Smith and Warner:

If a firm sells bonds with the stated purpose of engaging in low variance projects and the bonds are valued at prices commensurate with that low risk, the value of the stockholders' equity rises and the value of the bondholders' claim is reduced by substituting projects which increase the firm's variance rate (p. 118).

The stockholders' incentive to increase the riskiness of the cash-flows is detrimental for the value of the firm if low-risk projects or assets are substituted for high risk ones with a negative net present value. This is the case, when the increase in risk is sufficiently large to cause the value of the equity to rise in spite of the drop in firm's value.

Bondholders can limit stockholders' opportunistic actions by incurring monitoring costs. Stockholders can also curtail the potential for moral hazard by incurring bonding costs, i.e., by committing against undertaking selected courses of action. Although monitoring and bonding activities reduce the conflict between stockholders and bondholders, they do not totally eliminate it; some opportunistic actions remain open to equityholders which entails a residual loss in the value of the firm. At the time the debt is issued, rational bondholders shift to the stockholders the present value of the monitoring costs plus the present
value of the wealth transfers resulting from opportunistic actions not controlled by monitoring and bonding activities. In addition, stockholders bear the cost of the residual loss in the value of the firm. Therefore, when a firm floats debt, stockholders have an incentive to design the debt contract in a way to minimize the agency costs of debt.

In our model insiders affect the present value of the agency costs of debt by choosing among alternative coupon strategies. We show that there exist a set of values for the exogenous parameters in the model for which a dissipative signalling equilibrium is obtained. In this equilibrium, insiders with high quality projects reveal their private information by specifying a contractual relationship between the promised coupons and the realizations of the level of the short-term interest rate, so that they increase the present value of the agency costs of debt. On the other hand, low quality firms choose the coupon strategy that minimizes agency costs. The signal is credible because only for high quality firms it is economical to issue debt with a suboptimal structure of coupons and bear higher agency costs of debt. By observing the variance of interest rates, the covariance between the value of the firm and interest rates\(^3\), and the coupon structure of the debt issue, market participants infer the quality of the insider's private information.

Although the analysis focuses on the choice between fixed and floating rate debt, the choice of the contractual coupon on the debt needs

\(^3\)An estimate for the value of the covariance can be obtained from the observation of the type of business the firm operates.
not to be restricted to these two cases. Intermediate cases which span the full spectrum of interest rate risk exposure that range between the two polar cases of floating and fixed rate bonds, are available by attaching to a floating rate debt contract special provisions that limit the scope of variation of the contractual coupons (e.g. coupon floors and ceilings, and drop-lock features). Thus, even when the quality of firm is highly heterogeneous, the contractual coupon may still be a feasible mechanism for firms to signal their inside information.

We test the signalling model against the hypothesis that the coupon decision is simply motivated by the concern to hedge against incurring large agency costs. According to this competing hypothesis - the "hedging" hypothesis - firms always choose the coupon strategy that minimizes the present value of the agency costs of debt. Hence, the "hedging" hypothesis implies that the coupon strategy decision does not convey information about the quality of the firm. We report empirical results that provide partial support for the signalling model. We examine the relationship between the stock price response to announcements of fixed and floating rate debt and the volatility of short-term interest rates at the date of the announcement. The data shows that as the volatility of the interest rate increases, stock prices tend on average to react more positively to announcements of fixed than of floating rate debt. This evidence is consistent with the signalling model predictions; because an increase in interest rate volatility increases the expected costs of financial distress on fixed rate debt vis-a-vis floating rate debt, the model
predicts that as the uncertainty about future interest rates grows, the set of high quality firms that signal inside information with fixed rate debt expands. We also investigate the relationship between the stock price response to announcements of floating and fixed rate debt and the covariance between the interest rate and the value of the issuing firm. One implication of the signalling model is that a high quality firm is more likely to signal with fixed rate debt if the covariance between its value and the level of the short-term interest rate is large. This is so, since an increase in the covariance reduces the expected costs of financial distress on floating rate debt vis-a-vis on fixed rate debt. However, we cannot reject the hypothesis that the covariance between the value of the firm and the interest rate has no differential effect on stock returns at the announcement of fixed and floating rate debt.

The first part of the dissertation is organized as follows. This section closes with an examination of floating rate bonds. In Section 2, we develop the model and characterize the pooling and separating equilibria. Section 3 summarizes the testable hypotheses. Section 4 describes the sample, outlines the empirical procedures, and presents the evidence. Finally, section 5 offers concluding remarks.
1.1. Floating Rate Notes

Cox, Ingersoll, and Ross (CIR;1980) report that floating rate corporate notes were originally introduced in the U.S. in 1974, and that a total of $1.3 billion of these instruments were sold by corporations in the same year. The innovation of variable income debt securities came as a response to the new economic environment that emerged in the mid 1970s. This new environment, characterized by rapid inflation and high and volatile interest rates, underscored the interest rate risk associated with fixed income securities. Floating rate notes gave the opportunity to corporations to sell a debt vehicle with a lower sensitivity to interest rate movements, and at the same time to substitute a one time transaction cost for the repeated transaction costs associated with a strategy of rolling over short-term debt.

In a typical floater, the date at which the coupon rate applicable to the next coupon payment is determined (i.e. the reset date), is prior to the coupon payment date. Sometimes coupon payment dates coincide with the reset dates for the next coupon payment, but that is not usually the case. Another key element of a "floater" is the particular short-term or intermediate term interest rate index to which the floating coupon payment is tied. Examples are 3 and 6-months Treasury Bills yields, LIBOR, and commercial paper rates. A coupon formula specifies the form of dependence of the coupons on realizations of the interest rate index. Often the next coupon is constructed as an arithmetic averaging of the quoted yields for the index from two or three dates immediately prior to the previous coupon
date, plus a premium that accounts for the issuer credit risk. This premium is pre-specified at the time the issue is sold, and either remains constant over the life of the note, or declines according to a predetermined schedule.

Besides those contractual provisions which can be routinely found in all corporate bond issues (e.g. callability and convertibility provisions), floating rate notes often include special contractual features that limit the instrument's interest rate risk immunization potential. Examples are ceilings and floors on the index interest rate, drop-lock features, and exchange privileges. The effect of these contractual features is to create a hybrid debt security with an intermediate exposure to interest rate risk. It is therefore possible to achieve any desired exposure to interest rate risk by appropriately adjusting the contractual provisions on a floating rate note.

The valuation of floating rate loan contracts has been examined by CIR, and Ramaswamy and Sundaresen (1986). CIR focus on how coupon formulas can be constructed to immunize the value of the contract against shifts in interest rates. In particular, they show that for a default-free bond with discrete coupon payments, there are specifications of the coupon formula on the interest rates that make the bond sell at par at ex-coupon

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4 When the reference rate hits a prespecified minimum, the instrument ceases to float and becomes a fixed income bond with a predetermined coupon and maturity.

5 With an exchange privilege the holder is allowed to exchange his original security (i.e. the floating rate note) for a fixed rate note with a prespecified coupon and maturity.
dates. Further, although these bonds carry interest rate risk between coupon dates, their dynamics between coupons are identical to that of a Treasury Bill maturing at next coupon payment date. However, for bonds in which the reset date is prior to the coupon payment date this feature is generally lost. The exception is when the reset date for the each coupon payment coincides with payment date for the preceding coupon payment. The authors also show, that although it is possible to construct an interest payment schedule for a risky console that totally immunizes the value of the console against interest rate risk, the coupon formula is generally firm specific. They also point out that these coupon formulas, even for a risky console bond, tend to be very complex and that simpler formulas commonly used in actual floating rate notes don’t eliminate interest rate risk. Ramaswamy and Sundaresan extend the work of CIR in several directions: they value default-free floaters in which the coupon formula is an average of past rates, they examine the effects of different contractual provisions on the valuation of default-free floaters (e.g. floors and ceilings on the coupon rates and drop-lock features\(^6\)), and they provide additional insight on valuation of floaters subject to default risk.

\(^6\) The instruments cease to float when the base rate hits a prespecified minimum, at which point the instruments become fixed rate notes.
2. The Model

2.1. Assumptions and Notation

We analyze the firm's financing decision between fixed and floating rate debt in the context of a two period model in which there is uncertainty both with respect to the value of the firm and to the level of the short-term interest rate. At time $t=0$ entrepreneurs are endowed with indivisible, nontransferable real investment opportunities, which require a capital investment $I$ that exceeds their personal wealth. To undertake the projects, entrepreneurs are constrained to raise an exogenous amount $D$ ($0 \leq I$), either with a two period fixed or floating rate coupon debt issue. All projects' cash-flows occur at time $t=2$ after which firms are disbanded. The present value of the projects' cash-flows $V_t$ (i.e. the value of the firm) follows a binomial process with a deterministic component. In each period $V_t$ either grows at a rate $a+a'$ with probability $\frac{1}{2}$ or grows by $a-a'$ with probability $\frac{1}{2}$ ($a>0, a'>0$).\(^7\) Similarly, the process for the price of one period default-free discount bonds $Q(t,t+1)$ is binomial with a deterministic component. In each period $Q(t,t+1)$ either grows at rate $b+b'$ with probability $\frac{1}{2}$ or decreases at rate $b-b'$ with probability $\frac{1}{2}$ ($b>0, b'>0$).

Assume that the structure of the economy is such that the one-period expected returns on every asset at time $t$ are equal to the certain return

\(^7\) A represents the required rate of return on the value of the firm. Also note that, provided some regularity conditions are met, the binomial process for the value of the firm converges to geometric Brownian motion as both $a$ and the discrete time intervals converge to zero.
on default-free discount bonds maturing at time \( t+1 \).\(^8\) This entails that

\[
\alpha = \frac{1}{Q(t, t+1)} \quad \text{(1)}
\]

\[
\beta = \frac{Q(t, t+2)}{Q(t, t+1)^2} \quad \text{(2)}
\]

where \( Q(t, t) \) is the price of a default-free discount bond at date \( t \) which matures at time \( t \). Since in general realizations of the interest rate are not independent from the projects' returns we need more notation. Let \( p_{uu}, p_{ud}, p_{du}, p_{dd} \) denote respectively, the probability that \( Q(t, t+1) \) goes down and \( V_t \) goes up, the probability that \( Q(t, t+1) \) goes up and \( V_t \) goes down, and finally the probability that both \( Q(t, t+1) \) and \( V_t \) go down.\(^9\) This notation implies that \( \frac{1}{2}=p_{uu}+p_{ud} \) and \( \frac{1}{2}=p_{uu}+p_{du} \). Casting all probabilities as a function of \( p_{uu} \) yields:

\[
p_{uu} = p_{uu} \quad \text{(3)}
\]

\[
p_{ud} = \frac{1}{2} - p_{uu} \quad \text{(4)}
\]

\[
p_{du} = \frac{1}{2} - p_{uu} \quad \text{(5)}
\]

\[
p_{dd} = p_{uu} \quad \text{(6)}
\]

with \( 0 \leq p_{uu} \leq \frac{1}{2} \).

Figure 1 shows a tree diagram depicting the time path of firms' value and the price of one period default-free discount bonds. Parameters \( a' \) and \( b' \) have been rescaled to \( b=[Q(0,1)^2/Q(0,2)]b' \) and \( a=Q(0,1)a' \).

---

\(^8\) Investors exhibit local risk neutrality. See for example Cox/Ingersoll/Ross (1981).

\(^9\) Note that \( p_{uu}+p_{ud}+p_{du}+p_{dd}=1 \).
We turn now to the informational structure in the market. Information about $p_{w0}$, a, b, Q(0,1), and Q(0,2) is publicly available. However, at date $t=0$ there is asymmetric information with respect to the value of the investment opportunities. While investors only know that there are two types of firms - they know that $\theta$ percent of all firms seeking external finance have $V_0 = V_0(g)$ ("Good" firms) and $(1-\theta)$ percent have $V_0 = V_0(b) < V_0(g)$ ("Bad" firms) - entrepreneurs know whether their own firm is of the "Good" or "Bad" type. The inside information about firms' values is revealed at time $t=1$.

At date $t=0$ entrepreneurs raise $D$ either by selling fixed or floating rate two period bonds. We assume that the debt is sold at par, and that both types of bonds promise two coupons: an intermediate coupon $c_1$ at date $t=1$, and a final coupon $c_2$ jointly with the principal at date $t=2$. For the floating rate bonds let the coupons be computed in the following way:

\[c_1 = \frac{1}{Q(0,1)+\pi} D\]  

(7)

\[c_2 = \frac{1}{Q(1,2)+\pi} D\]  

(8)

where $\pi$ is an endogenous default premium which is set at date $t=0$ when the bonds are sold. According to (7) and (8) the coupon payment date lags the coupon reset date by one period (i.e. the coupon to be paid at date $t$ is determined at date $t-1$ based on the prevalent price of the short-term default-free discount bond $Q(t-1,t)$). For fixed rate debt, the promised

\[\pi\] is endogenous since we assumed that the bonds sell at par at date $t=0$.\footnote{\footnotetext}
coupons are constant, i.e.

\[ c_1 = c_2 = cD \quad (9) \]

where \( c \) is an endogenous constant coupon rate which is determined at date \( t=0 \). The debt is sold with a covenant that prohibits additional debt issues of senior (or equivalent) rank prior to date \( t=2 \).

Since all cash-flows from the projects occur at date \( t=2 \), additional external funds have to be raised at date \( t=1 \) to fund the intermediate coupon. Entrepreneurs sell equity at \( t=1 \) and pay out the coupon \( c_1 \) if and only if the value of the entrepreneurs' claim on the firm \( S_1 \) exceeds the coupon \( c_1 \); otherwise they default on the coupon payment and let the firm go bankrupt. To make the problem non-trivial it is required that both forms of debt financing entail (at least for the low quality firms) default risk. Provided that this minimal requirement is satisfied, the specification of the default states is arbitrary and it doesn't affect the qualitative implications of the model. For simplicity, we choose to analyze the case in which the default states are independent from firm quality and funding strategy. We assume that the parameters of the model are such that, regardless of project quality and borrowing strategy, (1) no firm defaults on the intermediate coupon \( c_1 \) and, (2) the firms' values of \( V_0(1-a)^2/Q(0,2)(1+b) \) and \( V_0(1-a)^2/Q(0,2)(1-b) \) at date \( t=2 \) are insufficient to repay \( D+c_2 \). Appendix A contains a set of sufficient conditions that guarantees this assumption to hold.
2.2. Promised Coupons with Asymmetric Information

When lenders cannot distinguish between "Good" and "Bad" firms they charge a coupon so that the bonds reflect the average firm quality. Let us first investigate the value of the coupon rate $c$ when entrepreneurs sell fixed rate debt. Since the debt is sold at par and the proportions in the market of "Good" and "Bad" firms are $\theta$ and $1-\theta$, the constant coupon rate $c$ is implicitly given by

$$D = \theta D_0^F(g) + (1-\theta) D_0^F(b) = \theta Q(0,1) E[D_1^F(g)] + (1-\theta) Q(0,1) E[D_1^F(b)] =$$

$$= \theta Q(0,1) \left[ cD + \frac{Q(0,2)}{Q(0,1)} D(1+c) (1-b)(1-p_{uu}) + (1+b)(\frac{1}{2}p_{uu}) + \frac{1}{2}(1/Q(0,1)) V_0(g)(1-a^2) \right] +$$

$$+ (1-\theta) Q(0,1) \left[ cD + \frac{Q(0,2)}{Q(0,1)} D(1+c) (1-b)(1-p_{uu}) + (1+b)(\frac{1}{2}p_{uu}) + \frac{1}{2}(1/Q(0,1)) V_0(b)(1-a^2) \right]$$

(10)

Solving (10) for $c$ yields

$$c = \frac{D[1-Q(0,2)\frac{1}{2}[(1-b)(1-p_{uu}) + (1+b)(\frac{1}{2}p_{uu})]] - \frac{1}{2} E[V_0](1-a^2)}{D[Q(0,1)+Q(0,2)\frac{1}{2}[(1-b)(1-p_{uu}) + (1+b)(\frac{1}{2}p_{uu})]]}$$

(11)

where $E[V_0] = \theta V_0(g) + (1-\theta) V_0(b)$.
On the other hand, if firms issue floating rate debt the default premium \( \pi \) comes out of the following equation:

\[
D = \theta D_0^A(g) + (1-\theta)D_0^A(b) = \theta Q(0,1)E[D_1^A(g)] + (1-\theta)E[D_1^A(b)] = \\
\theta \left[ Q(0,1)D(1-Q(0,1)) + Q(0,2)[\lambda(1-b)(1-p_{uw})D(1-Q(0,1)) + Q(0,1)] + \frac{1}{2}(1+b)(\lambda p_{uw}) D(1-Q(0,2)(1+b)) \right] + \frac{1}{2}V_0(g)(1-a)^2 + \\
(1-\theta) \left[ Q(0,1)D(1-Q(0,1)) + Q(0,2)[\lambda(1-b)(1-p_{uw})D(1-Q(0,1)) + Q(0,1)] + \frac{1}{2}(1+b)(\lambda p_{uw}) D(1-Q(0,2)(1+b)) \right] + \frac{1}{2}V_0(b)(1-a)^2 
\]

(12)

Solving (12) for the default premium \( \pi \) we obtain

\[
\pi = \frac{DQ(0,1)[1-\lambda(1-p_{uw})-\frac{1}{2}(\lambda p_{uw}) - \frac{1}{2}(1-a)^2E[V_0]]}{DQ(0,1)+DQ(0,2)[(1-b)(1-p_{uw})+(1+b)(\lambda p_{uw})]} 
\]

(13)

where again \( E[V_0] = \theta V_0(g) + (1-\theta)V_0(b) \).

The coupon rate \( c \) in (11) and the default premium \( \pi \) in (13) lie between those values of \( c \) and \( \pi \) that fully informed investors would charge for "Good" and "Bad" firms.
2.3. The Valuation of the Entrepreneurs' Equity

This subsection ascertains the insiders valuation of fixed versus floating borrowing strategies when investors cannot distinguish high from low quality firms. Let $S_0^A$ and $S_0^F$ denote respectively, the entrepreneurs' valuation of their equity claim at date $t=0$ under floating ($A$=adjustable) and fixed ($F$=fixed) rate debt financing. Assume that the entrepreneurs keep their claim on the firm in their portfolios.$^{11}$

For $S_0^F$ we have

$$S_0^F = Q(0,1)E[S_i^F] =$$

$$= Q(0,1)\left\{(\hat{z}-p_{wu})\left[\frac{V_0(1+a)-Q(0,2)(1-b)D(1+c)}{Q(0,1)} - cD\right] +

+ p_{wu}\left[\frac{V_0(1+a)-Q(0,2)(1+b)D(1+c)}{Q(0,1)} - cD\right]\right\} +

\frac{\hat{z}}{2}(V_0(1-a^2)-Q(0,2)(1-b)D(1+c)) - cD]\right\} +

+ (\hat{z} - p_{wu})\left[\frac{\frac{\hat{z}}{2}(V_0(1-a^2)-Q(0,2)(1+b)D(1+c))}{Q(0,1)} - cD}\right]\right\}$$

substituting $c$ from (11) and rearranging yields

$^{11}$ Myers/Majluf(1984) show that with asymmetric information debt always dominates equity as a vehicle to raise external funds. If entrepreneurs try to sell their equity a "lemons" equilibrium obtains, in which only the "Bad" firms issue stock. See also Flannery(1986) footnote 7.
$$S_0^F = V_0 - D + \frac{1}{2}p_{uu}(1-a)^2[E(V_0) - V_0] + \frac{1}{2}p_{uu}(1-a)^2[E(V_0) - V_0]$$

(15)

The entrepreneurs' valuation of their claim at date $t=0$ can be decomposed in two components: A component $V_0 - D$ which reflects the value that the equity claim would command if investors had the ability to assess the quality of firms (i.e. the "full revelation" component), and a second component which accounts for the transfer in wealth from "Good" to "Bad" firms due to the mispricing of debt (i.e. the "wealth redistribution" component). Rewrite (15) as

$$S_0^F = S_r^F + S_m^F$$

(16)

with

$$S_r^F = V_0 - D$$

(17)

$$S_m^F = \frac{1}{2}(1-a)^2[E(V_0) - V_0]$$

(18)

where $S_r^F$ and $S_m^F$ represent respectively, the "full revelation" and the "wealth redistribution" components of the entrepreneurs' claim on the firm with fixed rate debt financing. Then, for a "Good" type firm $S_m^F(g) < 0$ and for a "Bad" type firm $S_m^F(b) > 0$. Moreover, since investors on average receive adequate compensation on the debt securities they buy

$$\theta S_m^F(g) + (1-\theta)S_m^F(b) = 0$$

(19)

Alternatively, when firms borrow at a floating rate entrepreneurs value their claim at date $t=0$, $S_0^A$, according to
\[ S_0^A = Q(0,1)E[S_t^A] = \]
\[ = Q(0,1)\left\{ \frac{V_0}{Q(0,1)} - \frac{\pi Q(0,2)(1-b) - 1-Q(0,1)}{Q(0,1)} + \frac{1}{2}p_{uu}\left[ \frac{V_0}{Q(0,1)} - \frac{\pi Q(0,2)(1-b) - 1-Q(0,1)}{Q(0,1)} \right] + \frac{1}{2}V_0(1-a) + \frac{\pi Q(0,2)(1+b) - 1-Q(0,1)}{Q(0,1)} \right\} \]

Substituting \( \pi \) from (13) and rearranging yields

\[ S_0^A = V_0 - D + \frac{1}{2}(1-a^2)[E(V_0) - V_0] + \frac{1}{2}p_{wu}(1-a^2)[E(V_0) - V_0] \]  

Again, the entrepreneurs' valuation of their claim on the firm has two components: The "full revelation" component \( V_0 - D \), and an adjustment that reflects the distance between the "full revelation" value and the average value assigned by the market (i.e. the "wealth redistribution" component).

Rewrite (21) as

\[ S_0^A = S_r^A + S_m^A \]  

with

\[ S_r^A = V_0 - D \]  

\[ S_m^A = \frac{1}{2}(1-a^2)[E(V_0) - V_0] \]
where \( S_r^A \) and \( S_w^A \) represent respectively, the "full revelation" and the "wealth redistribution" components under floating rate debt financing. It follows that \( S_r^A(g) < 0, S_w^A(b) < 0 \), and

\[
\theta S_r^A(g) + (1-\theta) S_w^A(b) = 0
\]  

(25)

Notice that the "wealth redistribution" component is equal for the two borrowing strategies. This result follows from the assumption made with respect to the "default" states. Had we assumed credit risk to differ between fixed and floating financing, the extent of mispricing on fixed and floating rate bonds would also had differed. The mispricing would have been more severe for the borrowing strategy with the largest default risk.

2.4. Agency Costs of Debt

A number of authors have shown that with risky debt outstanding, the policy of maximization of the value of the equity claim does sometimes diverge from the policy of maximization of the value of the firm (e.g. Myers (1977), and Smith and Warner (1979)). So far the analysis has avoided this problem by assuming away interim investment decisions at date \( t=1 \). Suppose on the contrary that at date \( t=1 \) entrepreneurs face a set of discretionary actions (which are only costly observable by outsiders) which impinge on the projects's cash-flows to be received at date \( t=2 \). These may include for example, the option to raise additional funds to
complete the project (or to expand the project), or the option to choose among different mutually exclusive variants of the project. In this framework, the process for the value of the firm \( V_t \) ought to be reinterpreted as the process for the present value of the cash-flows to the firm to be received at date \( t=2 \) under the investment policy that maximizes the value of the firm.

The essence of the agency costs of debt can be captured by focusing on monitoring costs. The extension to the general case in which agency costs include in addition to monitoring costs, bonding costs incurred by stockholders, and residual loss (i.e. the residual reduction in the value of the firm due to the divergence between stockholders' investment decisions and those decisions that maximize the value of the firm) complicates the analysis without changing the implications of the model. We assume that at date \( t=1 \) when the value of the firm is "down", it is optimal for bondholders to spend resources to monitor and enforce the value maximizing investment policy. Since bondholders shift the present value of the monitoring costs to the entrepreneurs at date \( t=0 \) when they purchase the bonds, it is in the entrepreneurs' interest to provide for monitoring services in the least costly way. We assume that at date \( t=0 \) entrepreneurs enter in a contract with a third party to purchase monitoring services at date \( t=1 \) (e.g. with a firm that operates in a competitive market for monitoring services). The third party monitors entrepreneurs' compliance with the value maximizing investment policy in the interim period \( t=1 \) and bills the firm for these services. To raise the
funds required to pay the monitoring bill at date t=1 entrepreneurs sell part of their equity claim on the firm.\textsuperscript{12}

Assume that monitoring and enforcement costs are a decreasing convex function of the value of the entrepreneur equity claim. This assumption captures the fact that given the value of the firm, (1) the gains to entrepreneurs from opportunistic actions are larger the lower is the value of their claim on the firm, and (2) that as the value of the entrepreneurs' equity claim approaches zero the gains from opportunistic actions increase at an increasing rate.\textsuperscript{13} Let L represent the monitoring and enforcement costs incurred by the third party at date t=1. When the value of the firm is "up" at date t=1 the debt is default-free and therefore there is no need to monitor stockholders actions (i.e. L=0). On the other hand, when the value of the firm is "down" the monitoring and enforcement costs are positive, decreasing, and convex in the value of the entrepreneurs' equity claim. Therefore, with $V_t=V_0(1-a)/Q(0,1)$

$$L=L(S_t); \quad (L\geq 0, L'<0, L''>0)$$

(26)

To assess the effect of the borrowing strategy on monitoring costs it is useful to rank the values of the entrepreneurs' equity claim at date t=1 in the "downstate" realization of the value of the firm. For any given firm quality the following holds:

\textsuperscript{12}The contract is binding in the sense that it is costly for the entrepreneurs to renege ex-post on their obligation to purchase monitoring services at t=1.

\textsuperscript{13}Also the set of profitable opportunistic actions is likely to expand.
\[ S_F(V_t^d, Q(1,2)^u) < S_A(V_t^d, Q(1,2)^u) < S_A(V_t^d, Q(1,2)^d) < S_F(V_t^d, Q(1,2)^d) \] (27)

where \( V_t^d = V_0(1-a)/Q(0,1) \), \( Q(1,2)^u = Q(0,2)(1+b)/Q(0,1) \), and \( Q(1,2)^d = Q(0,2)(1-b)/Q(0,1) \). The ranking in (27) is a consequence of the fact that the equity of firms with floating rate debt outstanding has a lower interest rate risk than the equity of firms with fixed rate debt outstanding. With floating rate financing entrepreneurs reduce the sensitivity of the equity claim to shifts in the price of short-term default-free discount bonds.

Since \( L \) is a convex function of \( S_t \), the ranking in (27) establishes that for any given firm quality, an increase in the volatility of the price of short-term discount bonds \( Q(t,t+1) \) raises the monitoring costs associated with a fixed rate borrowing strategy vis-a-vis the monitoring costs associated with a floating rate borrowing strategy. The ranking in (27) also implies that for any firm quality, an increase in the probability that the value of the firm and the price of discount bonds move in the same direction \( p_{uw} \) reduces the monitoring costs associated with fixed rate bond financing vis-a-vis the costs of floating rate financing.

If the value of the firm \( V_t \) and the price of one period discount bonds \( Q(t,t+1) \) are not positively correlated (i.e. \( p_{uw} \leq \frac{1}{2} \)), floating rate bond financing yields lower expected monitoring costs irrespective of the quality of the firm. In contrast, when \( Q(t,t+1) \) and \( V_t \) are perfectly positively correlated (i.e. \( p_{uw} = \frac{1}{2} \)), "Good" and "Bad" type firms reduce the
expected monitoring costs by borrowing at a fixed rate. However, for values of $p_{uu}$ such that $\lambda p_{uu}^{\lambda}$ the borrowing strategy that minimizes the expected monitoring costs may depend on the unobservable value of the firm $V_0$. We constrain the function $L$ such that the same borrowing strategy minimizes monitoring costs across the firms' quality spectrum; assume that the following inequality is satisfied:

$$\frac{L[S_i^F(V_i^{d}(g),Q(1,2),c)]-L[S_i^A(V_i^{d}(g),Q(1,2),\pi)]}{L[S_i^{F}(V_i^{d}(b),Q(1,2),c)]-L[S_i^{A}(V_i^{d}(b),Q(1,2),\pi)]} \geq 0 \quad (28)$$

2.5. Equilibrium with Symmetric Information

When investors share at date $t=0$ the entrepreneurs' information about the quality of the investment opportunities, there is no mispricing of the firms' securities. With symmetric information, bonds sell at a price consistent with the quality of the firm and therefore, the "wealth redistribution" component of the value of the entrepreneur's equity claim collapses to zero (i.e. $S_m^F=S_m^A=0$). If in addition there are no costs associated with monitoring and enforcing the value maximizing investment policy at date $t=1$, then the choice of coupon strategy is a matter of indifference for entrepreneurs. However, if it is costly to monitor and enforce stockholders actions, the funding strategy is a matter of concern. As far as the choice between fixed and floating rate debt financing is

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14A sufficient condition for (28) to hold is that $L[S_i^{F}(V_i^{d},Q(1,2),c)]-L[S_i^{A}(V_i^{d},Q(1,2),\pi)]$ is homogeneous in $V_i^{d}$. 

concerned, entrepreneurs select the debt strategy that provides the best hedge against the risk of incurring a large monitoring bill. We call this rationalization of the choice between fixed and floating rate debt financing the "hedging" hypothesis.

The implications of the "hedging" hypothesis in terms of the optimal borrowing strategy depend on the values of the parameters of the model. For a given level of the short-term interest rate $1/Q(0,1)$, an increase in the variance of the price of one period bonds (i.e. an increase in $b$ since $\text{Var}Q(1,2)=Q(0,1)^2b$) increases the expected monitoring costs associated with fixed rate borrowing vis-a-vis the costs associated with floating rate borrowing. On the other hand, given the variance of the value of the firm $\text{Var}(V_t)=V_0^2a$ and the variance of the price of one period discount bonds $\text{Var}[Q(1,2)]=Q(0,1)^2b$, an increase in the covariance between the value of the firm and the price of one period discount bonds (i.e. an increase in $p_{uu}$ since $\text{Cov}[V_t,Q(1,2)]=V_0a(Q(0,1)b(4p_{uu}-1))$) raises the expected monitoring costs associated with floating rate bonds vis-a-vis the costs associated with fixed rate debt. Hence, the "hedging" hypothesis predicts that (1) an increase in the volatility of the short-term interest rate raises the number of firms that borrow at an adjustable rate, and (2) that cross-sectionally, there is a positive relationship between the odds of borrowing at a fixed rate and the covariance between the value of the firm and the short-term interest rates.
2.6. Equilibrium with Asymmetric Information

Consider first the case in which it is costless to monitor and enforce the value maximizing investment policy at date $t=1$. With $L=0$, high quality firms are unable to reveal to the market their favorable inside information. The "Bad" type firms mimic the "Good" type firms' financing decision, regardless on whether "Good" firms sell fixed or floating rate notes. This is so, because it is costless for "Bad" type firms to replicate the "Good" firms funding strategy, and because "Bad" type firms are better-off when they cannot be distinguished from "Good" type firms. Thus, only a pooling equilibrium is feasible. Entrepreneurs of high quality firms decide either to pool with fixed or floating rate financing, depending on which of the two strategies minimize the wealth transfer to low quality firms. Since the entrepreneurs' private information only affects lenders' assessment of the value of debt at date $t=0$ via the bankrupt states, "Good" type firms sell the debt security with the lowest default risk. In our model floating and fixed rate debt have the same credit risk and thus, entrepreneurs of "Good" type firms are indifferent between the two funding strategies. Formally, high quality firms are indifferent between fixed and floating rate bonds because $S^F(g) = S^A(g)$, i.e. because the wealth transfer from high to low quality firms due to the mispricing of the debt at date $t=0$ is the same for the two borrowing strategies.

Consider now the case in which entrepreneurs sell equity to pay for monitoring services at date $t=1$ when the value of the firm is "down"; the
cost of these services at date $t=1$ is characterized according to (26). Assume that the cost of monitoring services does not alter the default states; the value of the entrepreneurs' equity at date $t=1$ is large enough to pay for monitoring services (in addition for paying for the interim coupon on the bonds) regardless of firm quality and coupon strategy.

Table 1 shows the relation between firms' borrowing strategies and the payoffs to the firms' insiders. Each entry in the table reports the increase in the benefits to entrepreneurs above the "full revelation" value $V_{p-D}$. The characterization of equilibria depends on which borrowing strategy minimizes the expected monitoring costs for each firm quality. Inequality (28) guarantees that the borrowing strategy that minimizes expected monitoring costs is independent from the unobservable quality of the firm. Hence, depending on the parameters of the model, either (i) both "Good" and "Bad" type firms minimize expected costs of monitoring with floating rate debt or (ii) both "Good" and "Bad" type firms minimize expected costs of monitoring with fixed rate debt. We examine case (i). The extension to case (ii) is straightforward.

Case (i): "Good" and "Bad" type firms minimize expected monitoring costs with floating rate debt financing

As we have already pointed out, when the value of the firm and the price of short-term discount bonds are not positively correlated (i.e. $p_{uw} < \frac{1}{2}$), floating rate debt financing minimizes the expected value of the monitoring bill irrespective of firm quality. For $\frac{1}{2} < p_{uw} < 1$ case (i) is
more likely to occur the larger is the volatility of the price of short-term discount bonds (i.e. the larger is b), and/or the smaller is the positive correlation between the value of the firm and the price of short-term discount bonds (i.e. the closer $p_{ww}$ is to $\delta$).

When both "Good" and "Bad" type firms reduce their expected monitoring costs with floating rate bonds, equilibrium cannot occur at the top-left or bottom-left cells of Table 1. If "Good" firms borrow at a floating rate, then low quality firms are better-off by issuing floating rate bonds and pooling with high-quality firms. With this strategy "Bad" firms reduce their expected monitoring costs and at the same time extract some wealth from high quality firms (i.e. $S^A(b) > 0$). Thus, the bottom-left cell of Table 1 is irrelevant. Equilibrium at the top-left cell is also unfeasible. If "Bad" firms sell fixed rate bonds, "Good" firms always choose floating-rate bonds and reveal their true type. Given that "Bad" firms sell fixed rate debt, an issue of floating rate bonds by "Good" type firms eliminates the wealth redistribution to low quality firms and also reduces the expected monitoring costs.

We are left with the cells on the right of Table 1. These correspond to two feasible equilibria: a pooling equilibrium in floating rate debt, and a separating equilibrium in which "Good" type firms issue fixed rate bonds and "Bad" type firms sell floating rate debt. We investigate now the conditions that determine which of the two equilibria will emerge.
The separating equilibrium occurs if the following two conditions are simultaneously met:\textsuperscript{15}

\[
S_m^A(g) - Q(0,1)[p_{uw}[L^A(V_1^d(g),Q(1,2)^d,\pi(pool)) - L^F(V_1^d(g),Q(1,2)^d,c(g))] + 
+ (1-p_{uw})[L^A(V_1^d(g),Q(1,2)^u,\pi(pool)) - L^F(V_1^d(g),Q(1,2)^u,c(g))]] < 0 \quad (29)
\]

\[
S_m^F(b) - Q(0,1)[p_{uw}[L^F(V_1^d(b),Q(1,2)^d,c(pool)) - L^A(V_1^d(b),Q(1,2)^d,\pi(b))] + 
+ (1-p_{uw})[L^F(V_1^d(b),Q(1,2)^u,c(pool)) - L^A(V_1^d(b),Q(1,2)^u,\pi(b))]] < 0 \quad (30)
\]

\textsuperscript{15}The definition of signalling equilibrium follows Acharya (1988). Consider the manager of a firm who chooses between two possible actions (signals), s=1(low) and s=h(high), h>1, after receiving private information. The manager’s private information is a draw from a distribution which is known to investors. The space of the manager’s private information is partitioned into message m=G ("Good" firm) and m=B ("Bad" firm). Let p(m) be the investors’ prior probability belief that the manager’s message is m(m=G,B); let p(s|m) be the probability that the manager issues signal s(s=h,1), given message m(m=G,B); and let q(m|s) be the investors’ posterior probability belief (i.e. after observing the signal s issued by the manager) that the manager’s private information is m. Finally, assume that managers and investors are rational and maximize wealth. Then, a signalling equilibrium consists of:

(a). Investors’ beliefs: The investors believe with probability \(q(G|h)=q(B|h)=1\) that managers issue signal h(1) if their private information is G(B); that is q(G|h)=q(B|h)=1.

(b). Manager’s strategy: The manager signals h(1) if and only if his or her information is G(B); that is p(h|G)=p(1|B)=1.

(c). (a) and (b) above are together consistent with Bayes updating.

This definition uses the concept of sequential equilibrium. As Acharya points out, it is similar to the definition of Riley (1979) for a continuous signal. It also ensures that the probability beliefs of investors, given the strategies of entrepreneurs, are consistent with Bayes rule.
Otherwise, firms pool in floating rate debt.

Condition (29) states that when "Bad" type firms sell floating-rate bonds, the gains to high quality firms from selling fixed rate debt and separating from low quality firms (i.e. the elimination of the mispricing of the bonds) exceeds the loss (i.e. the additional monitoring costs). Condition (30) states that when "Good" type firms sell fixed rate debt, it is not economical for "Bad" type firms to increase the expected monitoring costs to replicate the borrowing strategy of high quality firms. The separating equilibrium is of the dissipative type. High quality firms are required to send a costly signal to the market to credibly separate from low quality firms. The market infers the quality of the firms from their willingness to take unnecessary monitoring costs.

To further illustrate the nature of the equilibrium consider the case in which \( L(S_t) \) is defined as

\[
L(S_t) = \begin{cases} 
K & S_t \leq 5 \\
0 & \text{otherwise}
\end{cases}
\]  

(31)

Assume that the interest rate uncertainty parameter \( b \) is large compared to the firm quality differential \( V_0(g) - V_0(b) \) such that

\[
S_t^F(V_1^d(g), Q(1,2)^u, c(g)) < S_t^A(V_1^d(b), Q(1,2)^u, \pi(b))
\]  

(32)

Then, choose \( \delta \) such that
\[ S_F^F(V_1, q, Q(1, 2), c(g)) < S_A^A(V_1, Q(1, 2), \pi) \] (33)

According to (31), (32) and (33), with fixed rate debt financing all firms spend \( K \) in monitoring services at date \( t=1 \) when the value of the firm is "down" and the price of discount bonds is "up". In contrast, with floating rate financing no firm purchases monitoring services at date \( t=1 \). Thus, both "Good" type and "Bad" type firms minimize the present value of monitoring costs by issuing floating rate debt at time \( t=0 \). Also notice that the level at which the threshold \( \delta \) is set implies that high quality firms do not enjoy a comparative advantage over low quality firm in the form of lower costs of monitoring; for any coupon strategy and informational equilibrium prevailing at date \( t=0 \) the present value of these costs is equal for high and low quality firms. As we shall see later on, a consequence of this particular assumption is that when high quality borrowers outnumber low quality borrowers (i.e. \( \theta > \bar{\theta} \)), there is no set of values for the parameters of the model such that a signalling equilibrium occurs.

With monitoring costs defined according to (31), (32) and (33), the separating equilibrium is reached when

\[ S_F^F(b) - Q(0, 1)(\frac{1}{2} - p_{uu}) < 0 \] (34)
\[ S_A^A(g) + Q(0, 1)(\frac{1}{2} - p_{uu}) < 0 \] (35)

Substituting \( S_F^F(b) \) and \( S_A^A(g) \) from (18) and (24) and rearranging we obtain
\[ V_0(b) > V_0(g) - \frac{4Q(0,1)(\frac{1}{2} - p_{uw})K}{(1-a)^2 \theta} \]  (36)

\[ V_0(b) < V_0(g) - \frac{4Q(0,1)(\frac{1}{2} - p_{uw})K}{(1-a)^2 (1-\theta)} \]  (37)

Conditions (36) and (37) are stated graphically in Figure 2 for a value of \( \theta < \frac{1}{2} \). Region I corresponds to the separating equilibrium, and both Regions II and III correspond to the pooling equilibrium. First, note that when the fixed monitoring costs \( K \) are equal to zero Region I vanishes and Regions II and III merge into a single region. This is consistent with our earlier result in which we argued that when it is costless to monitor and enforce the value maximizing investment policy at date \( t=1 \), the separating equilibrium is not feasible. Second, if \( \theta > \frac{1}{2} \) the separating equilibrium is not feasible regardless of the values of \( V_0(g), V_0(b), Q(0,1), p_{uw}, a, \) and \( L \). With \( \theta > \frac{1}{2} \) the two conditions (36) and (37) cannot be simultaneously satisfied. When it is optimal for "Good" type firms to signal with fixed rate debt and separate, it is also optimal for "Bad" type firms to sell fixed rate debt and replicate the borrowing strategy of "Good" firms. Alternatively, when it is optimal for "Bad" type firms not to issue fixed rate bonds and mimic the borrowing strategy of "Good" type firms, it is neither optimal for "Good" type firms to sell fixed rate bonds and separate. Finally, if \( \theta < \frac{1}{2} \) it is always possible to obtain a separating equilibrium by appropriately scaling the value of the monitoring cost \( K \). If the original pooling equilibrium occurs in Region II, then by reducing
the value of K Region I moves upward squeezing Region II. If the original pooling equilibrium is in Region III, an increase in K moves Region I downward filing the space previously occupied by Region III.

2.7. Interest Rate Swaps

In our model, firms with favorable inside information signal by selecting the borrowing strategy with the larger present value in agency costs. Investors infer the quality of the firm from the insider willingness to bear additional agency costs of debt. Clearly, if once the bonds have been issued it is feasible for insiders to neutralize the cost of the signal, the choice between fixed and floating rate debt cannot credibly reveal private information.

With interest rate swaps corporations transform fixed rate liabilities into floating rate liabilities and vice-versa. With an ex-post swap, insiders can therefore reverse the coupon strategy and reduce the agency costs of debt. It is the bondholders, however, who benefit from the ex-post reduction in agency costs. Once the debt has been issued, any financing decision that aligns insiders's goals with firm value maximization entails a windfall gain to bondholders; it lowers monitoring costs and in addition, it reduces the expected value of the losses stemming from opportunistic actions taken by the insider. Since bondholders are rational, they recognize that the insider lacks the incentive to reverse the coupon strategy. Therefore, the ability of firms to take ex-post interest rates swap positions doesn't undermine the
credibility of the signal.

Furthermore, it may be costly to the insider to take a swap position. For example, there are transaction costs associated with negotiating, implementing, and monitoring a swap that effectively converts long-term fixed rate obligations into long-term floating rate obligations (and vise-versa). This is specially true for smaller firms in countries with less developed swap markets. Also, as pointed out by Cooper and Mello (1988), if the swap default rule is such that swap payments are made provided that the net payer is solvent, an equilibrium swap (i.e. a swap with a zero net present value for the counterpart) entails a wealth transfer from stockholders to bondholders.\footnote{The intuition behind this result is the following: given that the swap is subordinated to the debt, the swap can never expropriate cash-flows from the bondholders; if, however, swap cash-flows in favor of the firm occur while the firm is insolvent to pay the debt, there is a gain to bondholders.} The magnitude of the wealth transfer depends on parameters such as the variance of the value of the firm ($\sigma_v$), the variance of interest rates ($\sigma_r$), and the covariance between interest rates and the value of the firm ($\sigma_{vr}$). If the firm’s outstanding bonds pay a fixed coupon, the wealth transfer associated with a swap in which the firm pays floating interest in exchange for fixed interest is positively related to all three parameters $\sigma_v$, $\sigma_r$ and $\sigma_{vr}$. Hence, with both the variance of the interest rates and the covariance between interest rates and the value of the firm large – so that the agency costs of debt with fixed rate financing exceed those with floating rate financing and
high quality firms signal with fixed rate debt - the cost for stockholders of converting fixed rate financing into floating rate with an ex-post swap is also large.

2.8. Expanding the Financing Opportunity Set

Because our focus is on the impact of asymmetric information on the debt coupon strategy, we restricted the financing choices available to the entrepreneur to fixed and floating two period debt. In this section we briefly discuss how the signalling model is extended to the case in which the firms' financing opportunity set includes short-term debt (i.e. one period bonds) and equity financing.

Suppose that in addition to fixed and floating rate two period bonds, entrepreneurs can also fund the project with a strategy of rolling over one period bonds. With no agency costs of debt, a pooling equilibrium is obtained in which all firms sell the security whose value is least affected by private information. As Flannery shows, this security is typically short-term debt. The rationale is that the divergence in realizations of firms' values between "Good" and "Bad" type firms grows as we move further into the future; the price of bonds that risk to go into default in period \( t=1 \) is less affected by inside information about the value of the firm than the price of bonds that risk to go into default in period \( t=2 \). On the other hand, if there are agency costs associated with risky debt in the firm's capital structure, a signalling equilibrium may occur. To send a credible signal to the market "Good" type firms are
required to demonstrate willingness to increase the agency costs of debt. Thus, high quality firms choose a financing strategy that departs from the policy of minimizing the expected agency costs of debt. In contrast, low quality firms sell the debt security that minimizes the agency costs of debt. Several authors have shown that firms reduce the agency costs of debt by shortening the maturity of the debt (e.g. Myers (1977), and Barnea, Haugen and Senbet (1980)). If this is the case, then in the signalling equilibrium low quality firms sell short-term and high quality firms sell either fixed or floating rate long-term bonds. High quality firms’ choice of coupon strategy depends on how favorable is their inside information. The higher is the quality of the firm the more a firm gains from separating from low quality firms. An insider with a very profitable project is willing to send a more costly signal to the market. On the other hand, the incentives of low quality firms to pool are stronger the higher is the quality of "Good" type firms. Consider for example, a "Good" type firm of exceptional quality; unless it selects a borrowing strategy with substantial agency costs, low quality firms are better off by replicating the funding strategy and pooling. It follows that, everything else the same, "Good" type firms that separate by selling long-term bonds with a coupon strategy that reduces expected agency cost of debt are of lower quality than "Good" type firms that choose a coupon strategy that increases the expected agency costs of debt. This analysis suggests the following two implications: first, that long-term debt financing conveys better news about the quality of the firm than
short-term debt financing;\textsuperscript{17} second, that in a framework in which insiders decide simultaneously about the maturity and the coupon structure of debt, the debt coupon decision may still be informative about the quality of the firm.

We now examine the case in which the financing opportunity set available to insiders includes equity financing, and fixed and floating rate two period debt financing. As Myers and Majluf show, if there are no agency costs of debt firms pool in debt financing. This is so because the market value of debt is less sensitive to the value of the investment opportunity than the market value of equity and therefore, the mispricing of debt is not as extensive as the mispricing of equity. Hence, high quality firms minimize the wealth loss due to underpricing by funding the project with debt. Moreover, they select the coupon strategy with the lowest default risk. "Bad" type firms are constrained to follow a similar funding strategy or otherwise they reveal themselves as the low quality firms. If however there are agency costs of debt, a signalling equilibrium may emerge. In such an equilibrium, low quality firms choose equity financing and high quality firms select debt financing. Low quality firms sell equity since with equity financing there are no agency costs of debt. High quality firms sell either fixed or floating rate bonds. The analysis made earlier in this section regarding high quality firms' borrowing strategy in the case in which the firm selects simultaneously the maturity

\textsuperscript{17}Flannery predicts exactly the opposite; in his signalling model high quality firms roll-over short-term debt while low quality sell long-term debt.
and the coupon structure of the bonds, applies directly here. Firms that select a coupon strategy that increases the agency costs of debt signal higher quality than firms that select a coupon strategy that minimizes expected agency costs. Thus, we may conclude that when the sources of external funding available to the firm include equity and debt financing, the debt coupon strategy can still convey private information about the quality of the firm.

3. Review and Summary of Hypotheses

We have argued that the decision to issue fixed versus floating rate debt conveys to the market inside information about the firm. We developed a model in which an insider chooses between fixed and floating rate debt to fund an investment opportunity. Both borrowing strategies entail agency costs of debt. At the time the bonds are issued, insiders have private information about the value of the project. Although investors don't know as much about the value of the project as insiders, they have the ability to identify which borrowing strategy minimizes the agency costs of debt. Firms with favorable inside information can sometimes use this ability to signal to the market their private information. In the separating equilibrium, "Good" type firms issue the debt security with the largest agency costs of debt, and "Bad" type firms select the coupon strategy that minimizes agency costs of debt. For high quality firm it pays to bear higher agency costs and reveal their private information to the market. On the other hand, low quality firms prefer to avoid unnecessary agency
costs and let their true type be revealed.

What is required for the results to hold is that the ability of investors to identify the coupon strategy that minimizes the agency costs of debt is not impaired by their lack of information about firms' quality. In a general context, it requires the market to have information about the covariance between firms' value and interest rates, about the volatility of interest rates, and about the volatility of firms' value (parameters \( p_w, a, \) and \( b \) in our model) and further, that this information is sufficient to determine for each individual firm, the debt strategy that minimizes the agency costs of debt.

In the separating equilibrium high quality firms signal their private information by selecting the coupon strategy that entails the largest agency costs of debt. The model predicts that the announcement of a bond issue with a coupon structure that increases the firm's expected agency costs triggers a more positive stock price response than a similar issue which features a contractual coupon that minimizes agency costs. The expected stock price response to an announcement of a debt issue with a particular structure of contractual coupons depends on firm specific parameter (i.e. the covariance between the value of the firm and the interest rates), and market parameters (i.e. the variance of interest rates). The model predicts that cross-sectionally, firms with a larger covariance between interest rates and firm value tend to exhibit larger abnormal stock returns at the announcement of fixed than floating rate debt issues. The model also predicts that an increase in the volatility
of short-term interest rates has a larger positive impact on abnormal stock returns at the announcement of fixed than of floating rate debt issues. This is so, since an increase in interest rate volatility and/or an increase in the covariance between the interest rate and the value of the firm raises the expected agency costs on fixed rate debt vis-a-vis floating rate bonds and therefore, it increases the number of high quality firms that signal with fixed rate debt.

In contrast with the "signalling" hypothesis, the "hedging" hypothesis asserts that firms always choose the coupon strategy that best hedges against incurring large agency costs of debt. According to the "hedging" hypothesis, the coupon decision does not convey any information about the quality of the firm. This competing hypothesis predicts no systematic difference between stock returns at the announcement of bond issues with different coupon strategies. In addition, it predicts that the volatility of interest rates and the covariance between firm's value and interest rate are unrelated to stock returns at the announcement of fixed and floating rate bond issues.

4. Sample and Empirical Procedures

4.1. Sample

To test the competing hypotheses a sample of floating and fixed rate corporate debt issues was collected. The Registered Offering Statistics (ROS) tape of the Securities and Exchange Commission (SEC) was used as the primary data source. The ROS tape contains information on public corporate
debt issues made in the U.S. market between 1977 and 1987. In particular, the tape supplies information on the principal, maturity, and filing date for each bond issue, and reports whether the issue carries a floating or a fixed rate coupon.

There is a total of 381 public issues of non-convertible, non-secured floating rate debt reported in the tape, out of which 162 are sold by companies listed in either the New York Stock Exchange (NYSE) or in the American Stock Exchange (AMEX). For the 162 debt issues made by firms listed in either the NYSE or the AMEX the Wall Street Journal Index (WSJI) and the Dow Jones News Retrieval Service (DJNRS) were searched for announcement dates. The DJNRS is a continuously updated electronic database containing selected articles published in the Wall Street Journal, appearing over the Dow Jones News Service (the "Broadtape"), or published in Barron's since June 1979. The DJNRS was searched for data on announcement dates by using key phrases. When news announcing a floating rate debt issue appeared simultaneously in the WSJI and the DJNRS the earliest report was selected.\textsuperscript{18} After searching the WSJI and the DJNRS for announcement dates and screening for any announcement of multiple securities issues (i.e. news which report that a company is selling some other securities simultaneously with floating rate debt), the sample was further screened for missing stock returns. An observation was dropped out of the sample if there were more than 30 missing daily stock returns over

\textsuperscript{18}Typically, the earliest report is in the DJNRS. That occurs because the DJNRS covers other sources of information besides the Wall Street Journal.
the 140 days period preceding the announcement day. The floating rate debt sample was extended to 7 additional observations by repeating the selection procedure for floating rate debt filings made by wholly owned subsidiaries of companies listed either in the NYSE or the AMEX.

The sample of clean announcements of floating rate debt issues had a total of 66 observations. However, out of the 66 observations 7 consisted of announcements of issues of exchangeable floating rate debt. These are floating rate notes in which, at the option of the issuer, the floating rate coupon can be converted into a pre-determined fixed rate coupon. These 7 observations were excluded from the floating rate debt sample.

To construct the fixed rate debt sample 313 fixed rate debt public issues were selected from the RDS tape. The population included all non-convertible, non-secured, non-shelf registration fixed rate debt SEC filings, for which no other security issue by the same company was simultaneously filed with the SEC. For every year except 1980, 1981, and 1982, 20 observations were randomly sampled from this population in the ROS tape. For the years of 1980, 1981, and 1982 five observations were randomly sampled for every month. If there were fewer than five observations in the population for any particular month then all the observations in that month were sampled. The sampling procedure gave special weight to issues made in 1980, 1981, and 1982, since these were the years within the sampling period in which the interest rate fluctuated.
more widely.\textsuperscript{18} For each of the 313 fixed rate debt filings the WSJI and the Wall Street Journal (WSJ) were searched for announcement dates. An observation was retained if the announcement date could be identified, and further if no contaminating information was released over a window of 5 days centered around the announcement date (e.g. announcement of multiple securities issues by the same company). Finally, after screening the sample for missing stock returns\textsuperscript{20}, there was a final count of 171 observations.

Table 2 reports the distribution over time of the floating and fixed rate bond sample. Table 3 contains selected summary statistics and the SIC classification breakdown for the two samples. Inspection of Table 3 reveals that floating rate debt carries on average a shorter maturity than fixed rate debt. Further, the floating rate debt sample is dominated by depository institutions, in particular by large money center banks.\textsuperscript{21} This accounts for the fact that the average asset size of the firm issuing floating rate debt is about four times the average asset size of the firm

\textsuperscript{18}The purpose of this sample design was to increase the power of the tests concerning the hypothesis on the volatility of interest rates and the covariance between interest rates and the value of the firm.

\textsuperscript{20}Identical screening procedure to the floating rate debt sample. An observation was excluded from the sample if there were more than 30 missing daily stock returns over the 140 days preceding the announcement day.

\textsuperscript{21}5 issues by Citicorp, 2 issues by Bankamerica Corp, 2 issues by Chase Manhattan Corp, 3 issues by Wells Fargo & Co, 1 issue by Manufacturers Hanover Corp, and 1 issue by Chemical New York Corp.
selling fixed rate debt.

4.2. Event Study Results

This section reports event study results for the fixed and floating rate debt samples. Although the "signalling" hypothesis has no implications on how the coupon strategy ought on average to affect stock returns at the announcement of a debt issue, the "hedging" hypothesis predicts that there should be no systematic difference between the stock price response to announcements of fixed and floating rate debt issues.

To determine stock returns at the announcement of a bond issue standard event study methodology was employed. Abnormal stock return for company i on day t \((AR_{it})\) was defined as

\[
AR_{it} = R_{it} - a_i - b_i R_{mt}
\]  

(38)

where \(R_{it}\) is the return for the common stock of firm i on day t, \(R_{mt}\) is the return for the CRISP equally weighted market index on day t, and \(a_i\) and \(b_i\) are the ordinary least squares coefficients of the market model estimated over the period -140/-21 days preceding the announcement day. Daily average abnormal returns (AAR) over the event window were computed as

\[
AAR_t = \frac{1}{N} \sum_{t=-20}^{+20} AR_{it}
\]  

(39)

Average cumulative abnormal returns over the trading interval \(T1,T2\) (ACAR\(_{T1,T2}\)) are obtained by summing the AAR\(_t\)'s over the interval \(T1,T2\). The test statistics for ACAR\(_{T1,T2}\) is based on average standardized cumulative abnormal returns.
\[ \text{ASCAR}_{T1,T2} = \frac{T2}{\sum_{t=T1}^N} \left( \frac{\sum_{i=1}^N (R_{it} - a_i - b_i R_{mt})}{s_{it}} \right) \]  

(40)

with

\[ s_{it}^2 = s_i^2 \left[ 1 + \frac{(R_{mt} - \mu)^2}{\sum_{k=1}^L (R_{mk} - \mu)^2} \right] \]  

(41)

where \( s_i^2 \) is the residual variance from the market model regression, \( L \) is the number of observations during the estimation period, \( R_{mk} \) is the return on the market for day \( k \) during the estimation period, \( R_{mt} \) is the return on the market for day \( t \) during the observation period, and \( \mu \) is the average return on the market for the estimation period. Assuming that individual abnormal returns are normal and independent across \( t \) and across securities, the statistics \( Z_{T1,T2} \) follows a unit normal distribution

\[ Z_{T1,T2} = \left[ \frac{N}{(T2-T1+1)} \right]^{\frac{1}{2}} \sum_{t=T1}^{T2} \text{ASCAR}_t \]  

(42)

Table 4 presents a time series of average common stock abnormal returns (AAR\(_t\)) centered around the announcement date (day 0), for both the floating and fixed rate debt sample. The floating rate sample excludes announcements of exchangeable bond issues and announcements of actual offerings. Column (1) identifies the trading day relative to the announcement day. Columns (2) and (3) report average daily abnormal returns (AAR) and average daily cumulative abnormal returns (ACAR) for the floating rate debt sample. Columns (5) and (6) contain the same statistics.
for the fixed rate debt sample. Table 5 presents average cumulative abnormal returns (ACAR_{t1,t2}) and the associated Z statistic over selected trading intervals.

The event study results for the fixed rate debt sample are in line with existing evidence. For a sample of 150 non-convertible fixed rate debt issues, Dann and Mikkelsen (1984) document a marginally significant two-day announcement period stock return of -.37% (t-value = -1.76). Mikkelsen and Partch (1986) report a two-day announcement period stock return of -.23% with a Z statistic of -1.4 (sample size = 171). Finally, Eckbo (1986) finds a -.06% (Z statistic = -.44) two-day period average stock return at the first public announcement of a non-convertible fixed rate debt issue (sample size = 459). For our sample of 171 observations the two-day announcement period average stock return is -.47% with a Z statistic of -1.62. As in Dann and Mikkelsen, the null hypothesis that the average two-day announcement period abnormal stock return equals zero is not rejected at the 0.05 significance level, but is rejected at the 0.10 level of significance.

For the floating rate debt sample we also find a marginally significant negative stock price response. The two-day announcement period average abnormal stock return is -.615% (Z statistic = -1.73). While for the fixed rate sample we find that 56% of the observations exhibit negative two-day announcement period AR, there are 19 (68%) announcements of floating rate debt issues that trigger negative two-day period stock returns. The data indicates that on average the coupon strategy doesn’t
have any effect on abnormal stock returns at the announcement of a debt issue. This evidence is consistent both with the hedging and the signalling hypotheses.

This result contrasts with the evidence reported by Kim (1987). For a sample of 213 non-convertible Eurobond (US dollar) issues Kim finds a +.39% (Z=3.13) two-day announcement period abnormal stock return. When he breaks down his sample into fixed rate issues (183 observations) and floating rate issues (30 observations) he documents respectively a +.46% (Z=3.38; 39.9% with negative returns) and a -.05% (Z=-.03; 53.3% with negative returns) two-day announcement period abnormal stock returns. Although Kim does not test the hypothesis that the two-day announcement period abnormal stock return for the fixed rate and the floating rate subsamples are equal, his figures suggest that the market responds on average more positively to announcements of fixed than of floating rate Eurobond issues. This result is inconsistent with the hedging hypothesis.

In summary, tests on average abnormal returns at the announcement of fixed and floating rate bond issues offer weak evidence against the hedging hypothesis. However, since the signalling hypothesis cannot be contradicted by inspection of average abnormal returns, these tests lack the power to differentiate the two competing hypotheses.
4.3. Regression Results

4.3.1. Results Concerning Interest Rate Volatility

In this subsection we assess the effect of interest rate volatility on stock returns at the announcement of fixed and floating rate debt. While the "signalling" hypothesis predicts that interest rate volatility to be positively related to the differential between stock returns at the announcement of fixed and floating rate debt, the "hedging" hypothesis predicts no relationship.

Data on short term interest rates was gathered from the Fama files in the CRSP tape. The CRSP Risk Free Rates File, one of the Fama files contained in the CRSP tape, reports monthly observations of nominal one and three month risk free rates from 1925 to 1987. Three yields are provided based on the bid, asked and average prices. Yields are continuously compounded 365 day rates. The one and three months series are constructed by selecting the Treasury Bill with closest to respectively, 30 and 90 days to maturity. The 3 months Treasury Bill yield series based on the average price was selected. To proxy for the anticipated volatility of interest rates at the announcement of a debt issue the standard deviation of the innovations in the interest rate series was estimated over the 12 months preceding the announcement date. Table 6 shows monthly series for the 3-months T. Bill yield and for the interest rate volatility proxy over the sampling period. Figure 3 plots quarterly series for both the 3-month and the 1-month T. Bill yields and interest rate volatility proxies based on the 3-months and 1-month T. Bill yields.
Table 7 contains estimates of the parameters of the regression of abnormal stock returns at announcement date on the proxy for the volatility of interest rate. Abnormal stock returns at the announcement of a debt issue were computed as the sum of the abnormal return on the announcement day (day 0) and the abnormal stock return on the day immediately preceding the announcement day (day -1). The top of the table reports regression coefficient estimates for the full sample whereas the bottom part reports estimates for a sample that excludes all depository institutions. To account for heteroscedasticity the White estimation procedure was followed. The White estimation procedure gives unbiased and consistent parameter estimates in the presence of heteroscedastic residuals without requiring any parametrization of the variance of the error term.

For the full sample the hypothesis that the market perception of the interest rate volatility has no effect on stock returns at the announcement of a floating rate debt issue (i.e. \( H_0: \beta_2=0 \)), and the hypothesis that the effect of interest rate volatility on abnormal stock returns does not differ between announcements of fixed and floating rate debt (i.e. \( H_0: \beta_3=0 \)), are both rejected at standard significance levels. These results are at odds with the predictions of the "hedging" hypothesis. In contrast, the evidence supports the "signalling" hypothesis. Table 7 reports a positive and significant estimate for \( \beta_3 \) which indicates that, at the announcement of a debt issue, the market perception of the volatility of the interest rate affects differentially
stock returns for fixed and floating rate bonds. The information content of the choice between floating and fixed rate debt appears thus to gain importance when investors anticipate high and volatile interest rates. The evidence indicates that indeed, as the anticipated volatility of interest rates grows, stock returns tend to respond more positively at the announcement of fixed rate debt issues than at the announcement of floating rate debt issue.

The parameters estimates are also in line with the "signalling" model predictions for the non-depository institutions sample. For this sub-sample the hypothesis that the market perception of the interest rate volatility has no effect on stock returns at the announcement of a floating rate debt issue (i.e. $H_0: \beta_2 = 0$), and the hypothesis that the effect of interest rate volatility on abnormal stock returns does not differ between announcements of fixed and floating rate debt (i.e. $H_0: \beta_3 = 0$), are also rejected at standard significance levels. Thus, the results are robust to the exclusion of depository institutions from the sample.

According to Table 2 no observations on floating rate debt issues can be found between March 1980 and May 1981. Inspection of the monthly time series for the interest rate volatility proxy in Table 6, reveals that it is during this time period that the proxy records its largest values. To control for the lack of observations on floating rate bond announcements during the periods of largest interest rate uncertainty (i.e. for the higher value range of the regressor variable), a few complementary regressions were run. Panel (a) of Table 8 contains
regression statistics for a sample that excludes all observations of fixed rate debt issues between March 1980 and May 1981. Panel (b) reports the same statistics for a sample that excludes all observations on fixed rate debt for which the volatility of interest rate proxy takes a value in excess of the largest value recorded by the proxy in the floating rate debt sample. Finally, Panel (c) provides regression statistics for a sample that is restricted to those months for which there are both observations on fixed and floating rate debt issues announcements.

The estimates and the t-statistics for the parameter $\beta_3$ in Table 8 indicate that the signalling model prediction concerning the differential effect of interest rate uncertainty on stock returns at the announcement of fixed and floating rate debt issues, collects empirical support across the three sub-samples. All the three panels report significant positive estimates for $\beta_3$. Thus, the results do not appear to be driven by a lack of observations on floating rate debt announcements over the higher range of interest rate volatility.

Another concern is to ascertain whether the sign and significance of the coefficient estimate $\beta_3$ traces to the differential effect of interest rate uncertainty on stock returns at the announcement of fixed and floating rate debt issues. For example, it might be argued that the level of short-term interest rates is an omitted variable which is positively correlated with the interest rate volatility proxy. The coefficient is picking up its sign and significance not because there is an interest rate volatility effect, but rather because there is a positive
association between the interest rate volatility proxy and the level of short-term interest rates. To test the alternative hypothesis that it is the level rather than the volatility of short-term interest rates that explains the divergence between stock returns at the announcement of fixed and floating rate bond issues, we run a regression that included in the set of independent variables both the interest rate volatility proxy and the annualized yield on 3-months Treasury Bills at the announcement month. Summary statistics pertaining to this regression are reported in Table 9. According to the coefficient estimates and t-statistics in Table 9, the level of short-term interest rates performs poorly in differentiating abnormal stock returns at the announcements of floating and fixed rate debt issues. In contrast, the interest rate volatility proxy retains strong explanatory power. These results suggest that it is indeed the volatility and not the level of short-term interest rate, that jointly with the observation of the coupon rate of the bonds, conveys information to investors about the quality of the firm.\footnote{Although the sample correlation between the interest rate volatility proxy and the level of the short-term interest rate is 0.6, the results of Table 8 are not driven by multicolinearity. We run a two step estimation of the coefficients of STDBILL and D*STDBILL in which we first orthogonalize abnormal stock returns with respect to the level of the short-term interest rate and then regressed the residuals on the interest rate volatility proxy. The estimated coefficients (t-statistics) are $-0.015275(-1.989)$ and $0.0173807(2.127)$.}

Finally, we checked a few additional factors for systematic effects on the differential in the stock price response to announcements of fixed
and floating rate debt issues. Table 10 presents the results of the regression of abnormal stock returns on the interest rate volatility proxy, and on the maturity and relative size of the debt issue. Maturity is defined as the number of years between the maturity year and the announcement year. Relative size was computed as the principal of the issue divided by the market value of the firm's outstanding common stock evaluated 20 days before the announcement day.

Except for the coefficient estimate of $\beta_2$ and $\beta_3$, the hypotheses that $\beta_1=0$ cannot be rejected at the 10% confidence level for every individual coefficient estimate. The hypotheses $\beta_2=0$ and $\beta_3=0$ are rejected at 5%. In summary, the data shows that neither the maturity nor the relative size of the issue help differentiate abnormal stock returns at the announcement of fixed and floating rate debt issues.

4.3.2. Results Concerning the Covariance between Interest Rates and the Value of the Firm

A second set of implications of the "signalling" and the "hedging" hypotheses concerns the effect of the covariance between interest rates and the value of the firm on the differential between stock returns at the announcement of floating and fixed rate debt issues. The "signalling" model predicts a positive effect and the "hedging" hypothesis predicts no effect.

To construct a proxy for the co-movement between interest rates and the value of the firm we first used COMPSTAT industrial tape data. The
COMPSTAT industrial tape provides financial, statistical, and market information on several thousand non-bank companies. In particular, it supplies quarterly accounting data since 1978 for firms listed in the NYSE and the AMEX. For each individual firm in the floating and fixed rate debt sample we collected from the tape a time series of quarterly observations for the item operating income before depreciation. This accounting item represents net sales less cost of goods sold and selling, general, and administrative expenses before deducting depreciation, amortization and depletion. To the extent that before tax cash-flows from operations correlate positively with the value of the firm, the accounting figures for the operating income before depreciation will also correlate positively with the value of the firm. However, because the relationship between operating income before depreciation and the value of the firm is likely to be rather noisy, a long time series of observations on operating income before depreciation and short-term interest rates is required in order to construct a sample covariance that will yield a suitable instrument for the true covariance. On the other hand, the longer we extend the estimation period the larger the probability that the instrumental variable will be affected by non-stationarities. We selected a 13 quarters estimation period. A debt issue was dropped out of the sample if there was any missing quarter of data on operating income before depreciation over the 13 quarters preceding the announcement date.\textsuperscript{23} A

\textsuperscript{23}Notice that, because the COMPSTAT quarterly data only starts at 1978, the screening criteria immediately excluded from the sample all debt issues made before 1980.
matching time series (i.e. with quarterly observations) for the short-term interest rate was constructed by averaging the monthly observations on the 3-months Treasury bill yields reported in the CRSP Fama files within each individual quarter. Finally, a proxy for the anticipated covariance between interest rates and the value of the firm at the announcement of a debt issue was calculated by estimating the sample covariance between the innovations in the operating income before depreciation quarterly series and the innovations in the matched interest rate quarterly series over the 13 quarters preceding the announcement date. The screening procedure yielded 13 observations of floating rate debt issues and 51 observations of fixed rate debt issues. Figure 4 shows the distributions of estimated covariances and correlations for the fixed and floating rate debt samples.

Table 11 reports summary statistics for the regression of abnormal stock returns at the announcement of a debt issue on the proxies for interest rate volatility and for the covariance between interest rates and the value of the firm. Again, the abnormal stock return at the announcement of a debt issue was computed as the sum of the abnormal return on the announcement day (day 0) and the abnormal stock return on the day immediately preceding the announcement day (day -1). Inspection of table 11 reveals that the regression fails to detect a covariance effect. Although a positive coefficient estimate for the variable $D_t \times \text{COVINCTBIL}_{it}$ indicates that the covariance between the value of the firm and short-term interest rates has a more positive effect on stock returns at the
announcement of fixed then of floating rate debt issues, the low t-statistic for the coefficient imply that the hypothesis that the covariance has no differential effect on stock returns at the announcement of fixed and floating rate debt issues cannot be reject. Thus, the covariance results lend support to the "hedging" hypothesis.

Several explanations may account for the lack of significance of the covariance effect. First, the reduced number of observations of floating rate debt issues (13 observations) makes it hard to distinguish the covariance effect between the fixed and the floating rate debt samples. Second, the covariance proxy used in the estimation is likely to be a poor instrument for the true covariance. With poor instrumental variables, parameter estimates are imprecise. Third, even if data on the true covariances were available, the correlations between interest rates and firm's value exhibited by most firms are not likely to present enough variation to elicit precise parameter estimates. With little cross-sectional variation in the correlation between interest rates and firms' values, coefficient estimates for the covariance proxy tend to exhibit large standard deviations and low t-statistics.

To deal with the small sample size and the poor instrumental variables explanations we used stock returns market data to compute a second proxy for the covariance between the value of the firm and the short-term interest rate. Monthly stock returns market data was collected from the CRSP tape (Monthly Stock Returns File) for all firms in the sample listed in the NYSE. The second proxy for the anticipated covariance
between interest rates and the value of the firm at the announcement of a debt issue was then calculated by estimating the sample covariance between the innovations in the stock returns monthly series and the innovations in the matched interest rate monthly series over the 48 months preceding the announcement date. Figure 5 shows the distributions of the new set of estimated covariances and correlations for the fixed and floating rate debt samples.

Summary statistics for the regression of abnormal stock returns on this covariance proxy are contained in Table 12. Again we fail to detect a covariance effect. This time though, it is unlikely that the failure to find a covariance effect is attributed to the small size of the floating rate sample (50 observations). Nonetheless, this second instrument for the true covariance has problems of its own. Firms borrow funds on an on-going basis. Every time they sell debt—and choose between fixed and floating rate financing—they affect the covariance between the value of the equity and interest rates. The signalling hypothesis predicts that except for the high quality firms in the separating equilibrium, firms select the coupon strategy that best hedges the value of the equity. If the firm has hedged the value of the equity over the period preceding the announcement of a new debt issue, then the covariance proxy yields a biased (biased towards zero) estimate of the covariance between the value of the firm and interest rates. Similarly, if the firm has signalled favorable inside information over the period preceding the announcement of a new debt issue, the covariance proxy is a biased (towards 1 in absolute value)
estimate of the covariance between the value of the firm and the interest rate. To the extent that the size of the bias (measurement error) is correlated to the proxy, the coefficient estimator of the covariance proxy is inconsistent.

At any rate, the argument that the absence of a covariance effect in the data is a result of a lack of sample variation in the true covariances between the interest rate and the value of the firm, remains a compelling one. The case can be made that for those firms in the sample, the correlations are not sufficiently different from each other to elicit a covariance effect. This suggests that the investors reaction to the corporate choice between fixed and floating rate debt is primarily based on marketwide conditions prevailing at the time of the announcement (i.e. the volatility of the short term interest rates), and that the importance of firm specific factors (e.g. the covariance between the value of the firm and the interest rate) is only of a second order of magnitude.

4.4. Matched Sample Non-Parametric Results

Because of the wide variation in firm characteristics, debt issue characteristics (other than being at a fixed or at a floating rate), and on the timing of the issues, it is desirable to select observations for the two samples in such a way that the fixed rate and the floating rate samples will resemble each other as closely as possible. Furthermore, regardless of whether or not there are systematic differences between the two samples, the wide variation among firms and debt issues implies that
the probability distribution of the stock price reaction will be different for each observation in the sample. Hence, when we take into consideration the different characteristics of firms and debt issues which appear in each of the samples, the stock response for either sample cannot be considered to form a random sample of observations from some common distribution. It follows that a large number of observations is required before it is appropriate to apply asymptotic results; standard parametric methods only become good approximations with larger sample sizes.

4.4.1. Matched Sample

To construct a matched floating-rate/fixed-rate sample we started by discarding the existing fixed rate debt sample. Then, for each announcement of a floating rate debt issue in the existing floating rate debt sample (59 observations) we looked for a matched fixed rate debt issue announcement. The matching criteria was as following:

(i) Timing; a clean fixed rate debt issue announcement had to be found within a 6 months period centered around a floating rate debt issue announcement date.

(ii) Industry classification; matched fixed rate debt issue announcements had to be made by a firm in the same industry as the firm announcing the floating rate debt issue.²⁴

²⁴Firms were classified by industry SIC codes: Extractive(99<SIC<150), Manufacturing(199<SIC<400), Transportation(399<SIC<480), Communication(479<SIC<490), Electric, Gas & Water(429<SIC<500), Depository Institutions (SIC=602, 603, 671), Financial - Non Banking (599<SIC<700 except SIC=602, 603, 671), and Commercial & Other (0<SIC<100, 149<SIC<180,
(iii) Maturity: Whenever more than one match for a floating rate debt issue announcement satisfying (i) and (ii) above could be identified, the one with a maturity closer to the maturity of the floating rate issue was selected.

Finally, we computed for every pair the difference in the 2-day period abnormal stock return between the fixed and the floating rate debt issue announcements.

4.4.2. Non-Parametric Results

Tables 13 and 14 report Sign and Wilcoxon Signed-Rank test results for the full matched sample and selected matched subsamples. Before we interpret the numbers in these tables it is important to recognize that although the Wilcoxon test is a more powerful test than the Sign test to determine the equality of medians between two populations, it requires the ancillary assumption that the distribution of the difference in abnormal stock returns is symmetric about zero. Rejection of $H_0$ could occur due to the lack of symmetry.

According to Table 13 except for the high interest rate volatility subsample all $p$-values are above 10% and the null cannot be rejected. For the high interest rate volatility subsample the $p$-value of 8% indicates some weak evidence that with the market anticipating volatile interest
rates the median of the differences in abnormal returns is positive. This conclusion is strongly supported by the Wilcoxon Signed-Rank test; for the high interest rate volatility subsample, Table 14 reports a p-value of .2%. These numbers contrast with the low interest rate volatility subsample results; the p-values for the Sign and the Wilcoxon Signed-Rank test of equality of medians are respectively 65% and 56%. The evidence lends support to the idea that in a market with volatile interest rates fixed rate debt issue announcements convey better news about the quality of the firm than floating rate debt issue announcements.

The Wilcoxon Signed-Rank test results for the Banking and the Manufacturing subsamples suggest the following observation: while the choice between fixed and floating rate debt does not on average signal information for a manufacturing firm (p-value 55%), the selection of fixed over floating rate debt financing conveys on average good news for a banking firm (p-value 3%). If we believe that the value of a banking firm tend to be more positively correlated with the interest rate than the value of a manufacturing firm, than this result is consistent with the covariance effect predicted by the signalling hypothesis.25

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25Sweeney and Warga (1986) fit a two-index model (i.e. a market portfolio index and an interest rate index) to stock returns on different industry portfolios; individual stocks are assigned to different industry portfolios based on the first 2-digit SIC codes. For the last five years of their sampling period (i.e. 1975-1979), they find non-significant interest rate betas for all industries except for utilities. For our sampling period, we constructed equally weighted portfolios of deposit-institutions' stocks (SIC=602,603,671) and manufacturing firms' stocks (SIC=200-399) and evaluated the correlation between the portfolios' monthly returns and contemporaneous changes on 3-months T-Bill's yields; we could not reject the hypothesis that the correlation is the same for the two portfolios.
It is also worth noting that for the full sample we obtain a Wilcoxon Signed-Rank test p-value of 4%. The test indicates that the market reacts on average more favorably to announcements of fixed than of floating rate debt, a result that conflicts with the hedging hypothesis.

Table 15 presents Mann-Whitney tests results. We test the hypothesis that two random variables come from a common distribution against the alternative hypothesis that one random variable is stochastically smaller(larger) than the other. The evidence from these tests further supports the prediction of the signalling model that an increase in interest rate volatility increases the number of firms with favorable inside information that signal with fixed rate debt; when we partition the paired sample into high and low interest rate volatility subsamples, we find that the difference in abnormal returns between (paired) announcements of fixed and floating rate debt issues is stochastically larger in the high interest rate volatility regime than in the low interest rate volatility regime(p-value of .75%). The Mann-Whitney tests also document that the market tends to react more positively to the preferment of fixed over floating rate debt financing for a banking than for a manufacturing firm. Again, to the extent that the market perceives

Footnote 25 (continued)

Although Kane and Unal (1988) report evidence that the return generating process for the depository institutions' industry undergoes significant change between 1975-1985, they point out that interest rate betas are fairly stable during the same period. Research on the stability of interest rate betas for other industries over our sampling period is, to our knowledge, inexistent.
banking firms as exhibiting larger covariances between firms' value and interest rates than manufacturing firms, we may interpret this result as indicative of the elusive covariance effect predicted by the signalling model.

Finally, we computed Spearman correlations between differences in abnormal returns for each pair and interest rate volatility. Table 16 presents correlation coefficients for the full sample, and the banking and manufacturing subsamples. It also presents the p-values for tests of independence (versus $H_0$: positive correlation). For both the full sample and the manufacturing subsample the difference in abnormal returns appears to be positively correlated with interest rate volatility. In contrast, there is no evidence of positive (or negative) correlation for the banking subsample.

It is important to point out that the evidence on the interest rate volatility and the covariance effects involving banking firms ought to be interpreted with caution as being in favor of or against the signalling model. We don't have any claim that the model ought to apply to depository institutions. The analysis of the choice between fixed and floating rate debt for a banking firm requires among other things to model deposits and loans (e.g. loans and deposits can themselves have fixed or floating rates).
5. Conclusion

This part of the dissertation develops a signalling model that shows that when insiders have private information about the value of the firm, the choice between fixed and floating rate debt conveys information about the quality of the firm. The differential effects of fixed and floating rate debt financing on the magnitude of agency cost of debt, imply that there exists a separating equilibrium in which high quality firms signal by selecting the coupon strategy with the largest expected costs. The model shows that, ceteris paribus, an increase in interest rate volatility raises the agency costs of fixed rate debt financing vis-a-vis the agency costs of floating rate financing. Accordingly, high quality firms are more likely to signal with fixed rate debt when the uncertainty about future interest rates is large. The model also shows that, ceteris paribus, an increase in the covariance between interest rates and the value of the firm raises the agency costs of fixed rate debt financing vis-a-vis the agency costs of floating rate financing. Consequently, the model also predicts that high quality firms with a large covariance between interest rates and the value of the firm are more likely to signal with fixed rate debt.

Empirical results on abnormal stock returns at the announcement of fixed and floating rate debt issues provide partial support for the model. With the exception of firms in the banking industry, we find evidence that the market perception of interest rate volatility affects differentially stock returns at the announcement of corporate issues of fixed and
floating rate debt. At high levels of interest rate volatility, the market reacts more positively to announcements of fixed than of floating rate debt issues. Thus, as investors become more uncertain about the future interest rates, the willingness (and ability) of the firm to sell fixed rate debt and take larger interest rate risk, conveys better news to the market compared to the choice of floating rate debt financing. However, a second prediction of the signalling model, i.e. that firms with a large covariance between interest rates and firm's value are more likely to signal high quality with fixed rate debt, is not validated by the data. The data show no relationship between covariance proxies and differences in abnormal stock returns at the announcements of fixed and floating rate debt issues. The failure to detect a significant covariance effect may trace to the poor instrumental variable used for the true covariance, to the lack of variability in the true correlations between interest rates and firm’ values exhibited by the companies in the sample, or yet to the tests’ low power to reject the null.
Figure 1
Uncertainty about the value of the firm, $V_t$, and about the price of one-period default-free bonds, $Q(t,t+1);[0 \leq a \leq 1, 0 \leq b \leq 1]$. 
Figure 2
Bond market equilibria ($\theta < \frac{1}{2}$). Region I: separating equilibrium
Region II and Region III: pooling equilibrium
Figure 3
Treasury Bills' yields and yield volatility
quarterly series over the sampling period.

rate3m: 3-months T. Bill annualized yield.
rate1m: 1-month T. Bill annualized yield.
stdbill3m: standard deviation of the innovations in the
3-months T. Bill annualized yields monthly series estimated
over the 12 months preceding the current observation.
stdbill1m: standard deviation of the innovations in the
1-month T. Bill annualized yields monthly series estimated
over the 12 months preceding the current observation.

Note: Each quarterly observation is obtained by averaging
the monthly observations within the quarter.
Figure 4

Distribution of covariances and correlations between operating income before depreciation and interest rates for firms issuing fixed (51 observations) and floating (13 observations) rate debt.

covinctbill: covariance between the innovations in the accounting item operating income before depreciation quarterly series and the innovations in the matched 3-months T. Bill's yields quarterly series estimated over the 13 quarters preceding the announcement date.
corrinctbill: correlation between the innovations in the accounting item operating income before depreciation quarterly series and the innovations in the matched 3-months T. Bill's yields quarterly series estimated over the 13 quarters preceding the announcement date.
Figure 5

Distribution of covariances and correlations between stock returns and interest rates for firms issuing fixed (120 observations) and floating (50 observations) rate debt.

covstkrtrtbill: covariance between the innovations in the stock returns monthly series and the innovations in the matched 3-months T. Bill's yields monthly series estimated over the 48 months preceding the announcement date.

crrstkrtrtbill: correlation between the innovations in the stock returns monthly series and the innovations in the matched 3-months T. Bill's yields monthly series estimated over the 48 months preceding the announcement date.
Table 1
Benefits to the entrepreneur over the full revelation equity value \( V_0 - D \) accruing from alternative borrowing strategies.

<table>
<thead>
<tr>
<th></th>
<th>Fixed rate debt</th>
<th>Floating rate debt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>( s^b_v(g) )</td>
<td>(-Q(0,1)(p_{uw})L^F[V_1^{d}(g),Q(1,2)^d,\pi(g))])</td>
<td>(-Q(0,1)(p_{uw})L^F[V_1^{d}(g),Q(1,2)^d,\pi(g))])</td>
</tr>
<tr>
<td></td>
<td>(+\delta_{p_{uw}}L^F[V_1^{d}(g),Q(1,2)^d,\pi(g)])</td>
<td>(+\delta_{p_{uw}}L^F[V_1^{d}(g),Q(1,2)^d,\pi(g)])</td>
</tr>
<tr>
<td></td>
<td>Bad</td>
<td>Bad</td>
</tr>
<tr>
<td>( s^b_v(b) )</td>
<td>(-Q(0,1)(p_{uw})L^F[V_1^{d}(b),Q(1,2)^d,\pi(b))])</td>
<td>(-Q(0,1)(p_{uw})L^F[V_1^{d}(b),Q(1,2)^d,\pi(b))])</td>
</tr>
<tr>
<td></td>
<td>(+\delta_{p_{uw}}L^F[V_1^{d}(b),Q(1,2)^d,\pi(b)])</td>
<td>(+\delta_{p_{uw}}L^F[V_1^{d}(b),Q(1,2)^d,\pi(b)])</td>
</tr>
<tr>
<td></td>
<td>Good</td>
<td>Good</td>
</tr>
<tr>
<td>( s^a_v(g) )</td>
<td>(-Q(0,1)(p_{uw})L^A[V_1^{d}(g),Q(1,2)^d,\pi(g)]))</td>
<td>(-Q(0,1)(p_{uw})L^A[V_1^{d}(g),Q(1,2)^d,\pi(g)]))</td>
</tr>
<tr>
<td></td>
<td>(+\delta_{p_{uw}}L^A[V_1^{d}(g),Q(1,2)^d,\pi(g)])</td>
<td>(+\delta_{p_{uw}}L^A[V_1^{d}(g),Q(1,2)^d,\pi(g)])</td>
</tr>
<tr>
<td></td>
<td>Bad</td>
<td>Bad</td>
</tr>
<tr>
<td>( s^a_v(b) )</td>
<td>(-Q(0,1)(p_{uw})L^A[V_1^{d}(b),Q(1,2)^d,\pi(b)]))</td>
<td>(-Q(0,1)(p_{uw})L^A[V_1^{d}(b),Q(1,2)^d,\pi(b)]))</td>
</tr>
<tr>
<td></td>
<td>(+\delta_{p_{uw}}L^A[V_1^{d}(b),Q(1,2)^d,\pi(b)])</td>
<td>(+\delta_{p_{uw}}L^A[V_1^{d}(b),Q(1,2)^d,\pi(b)])</td>
</tr>
</tbody>
</table>

\( c(\text{pool}) \): coupon rate on fixed rate debt when investors cannot distinguish "Good" from "Bad" type firms (expression (14) in the text).
\( \pi(\text{pool}) \): default premium on floating rate debt when investors cannot distinguish "Good" from "Bad" type firms (expression (16) in the text).
\( c(g) = c(\theta=1) \); \( c(b) = c(\theta=0) \); \( \pi(g) = \pi(\theta=1) \); \( \pi(b) = \pi(\theta=1) \).
<table>
<thead>
<tr>
<th>Time period (month/year)</th>
<th>Floating rate debt</th>
<th>Fixed rate debt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/71 2/71 3/71 4/71 5/71 6/71 7/71 8/71 9/71 10/71 11/71 12/71</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>1 0 1 0 3 0 0 1 0 1 1 2</td>
</tr>
<tr>
<td>1/72 2/72 3/72 4/72 5/72 6/72 7/72 8/72 9/72 10/72 11/72 12/72</td>
<td>0 0 0 0 0 0 1 0 0 0 0 0</td>
<td>1 0 2 0 2 0 0 0 0 1 1 0</td>
</tr>
<tr>
<td>1/73 2/73 3/73 4/73 5/73 6/73 7/73 8/73 9/73 10/73 11/73 12/73</td>
<td>0 0 0 0 0 0 1 0 1 1 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/80 2/80 3/80 4/80 5/80 6/80 7/80 8/80 9/80 10/80 11/80 12/80</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/81 2/81 3/81 4/81 5/81 6/81 7/81 8/81 9/81 10/81 11/81 12/81</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/82 2/82 3/82 4/82 5/82 6/82 7/82 8/82 9/82 10/82 11/82 12/82</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/83 2/83 3/83 4/83 5/83 6/83 7/83 8/83 9/83 10/83 11/83 12/83</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/84 2/84 3/84 4/84 5/84 6/84 7/84 8/84 9/84 10/84 11/84 12/84</td>
<td>1 0 2 0 2 1 1 1 1 0 0 1</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/85 2/85 3/85 4/85 5/85 6/85 7/85 8/85 9/85 10/85 11/85 12/85</td>
<td>1 1 1 1 2 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/86 2/86 3/86 4/86 5/86 6/86 7/86 8/86 9/86 10/86 11/86 12/86</td>
<td>2 1 0 1 1 2 0 0 3 3 0 1</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/87 2/87 3/87 4/87 5/87 6/87 7/87 8/87 9/87 10/87 11/87 12/87</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
<tr>
<td>1/88 2/88 3/88 4/88 5/88 6/88 7/88 8/88 9/88 10/88 11/88 12/88</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0</td>
</tr>
</tbody>
</table>
Table 3
SIC breakdown of debt issues and selected summary statistics for the floating and fixed rate bond sample

<table>
<thead>
<tr>
<th>Floating rate debt</th>
</tr>
</thead>
<tbody>
<tr>
<td># of observations: 59</td>
</tr>
<tr>
<td>SIC classification:</td>
</tr>
<tr>
<td>Extractive</td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Transportation</td>
</tr>
<tr>
<td>Communication</td>
</tr>
<tr>
<td>Electric, Gas &amp; Water</td>
</tr>
<tr>
<td>Banks, Bank Holding Co, and Thrifts</td>
</tr>
<tr>
<td>Financial excluding Banks, Bank Holding Co, and Thrifts</td>
</tr>
<tr>
<td>Commercial and Other</td>
</tr>
<tr>
<td>Average maturity(days): 3891</td>
</tr>
<tr>
<td>Average principal(millions): 137.2</td>
</tr>
<tr>
<td>Average asset size of the issuing firm(millions): 16833.1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fixed rate debt</th>
</tr>
</thead>
<tbody>
<tr>
<td># of observations: 171</td>
</tr>
<tr>
<td>SIC classification:</td>
</tr>
<tr>
<td>Extractive</td>
</tr>
<tr>
<td>Manufacturing</td>
</tr>
<tr>
<td>Transportation</td>
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<tr>
<td>Communication</td>
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<td>Electric, Gas &amp; Water</td>
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<tr>
<td>Banks, Bank Holding Co, and Thrifts</td>
</tr>
<tr>
<td>Financial excluding Banks, Bank Holding Co, and Thrifts</td>
</tr>
<tr>
<td>Commercial and Other</td>
</tr>
<tr>
<td>Average maturity(days): 5817</td>
</tr>
<tr>
<td>Average principal(millions): 100</td>
</tr>
<tr>
<td>Average asset size of the issuing firm(millions): 4468.3</td>
</tr>
</tbody>
</table>
Table 4

Common stock daily average abnormal returns (AARₜ) and daily average cumulative abnormal returns (ACARₜ) for 41 trading days around the announcement dates of U.S. public issues of floating (59 events) and fixed (171 events) rate bonds.

<table>
<thead>
<tr>
<th>Trading day</th>
<th>AARₜ</th>
<th>ACARₜ</th>
<th>AARₜ</th>
<th>ACARₜ</th>
</tr>
</thead>
<tbody>
<tr>
<td>-20</td>
<td>-0.024%</td>
<td>-0.24%</td>
<td>0.008%</td>
<td>0.008%</td>
</tr>
<tr>
<td>-19</td>
<td>-0.022</td>
<td>-0.046</td>
<td>-0.129</td>
<td>-0.130</td>
</tr>
<tr>
<td>-18</td>
<td>-0.011</td>
<td>-0.057</td>
<td>-0.227</td>
<td>-0.247</td>
</tr>
<tr>
<td>-17</td>
<td>0.534</td>
<td>0.477</td>
<td>0.109</td>
<td>0.238</td>
</tr>
<tr>
<td>-16</td>
<td>0.089</td>
<td>0.585</td>
<td>-0.137</td>
<td>-0.374</td>
</tr>
<tr>
<td>-15</td>
<td>0.143</td>
<td>0.709</td>
<td>0.034</td>
<td>-0.340</td>
</tr>
<tr>
<td>-14</td>
<td>-0.146</td>
<td>0.563</td>
<td>-0.257</td>
<td>-0.597</td>
</tr>
<tr>
<td>-13</td>
<td>0.227</td>
<td>0.783</td>
<td>-0.244</td>
<td>-0.841</td>
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<tr>
<td>-12</td>
<td>0.150</td>
<td>0.939</td>
<td>-0.019</td>
<td>-0.851</td>
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<tr>
<td>-11</td>
<td>-0.156</td>
<td>0.784</td>
<td>-0.063</td>
<td>-0.314</td>
</tr>
<tr>
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<td>0.182</td>
<td>0.665</td>
<td>0.196</td>
<td>-0.719</td>
</tr>
<tr>
<td>-9</td>
<td>0.560</td>
<td>1.526</td>
<td>0.083</td>
<td>-0.635</td>
</tr>
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<td>-0.143</td>
<td>1.383</td>
<td>0.271</td>
<td>-0.364</td>
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<td>-0.111</td>
<td>0.635</td>
<td>-0.005</td>
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<td>-0.073</td>
<td>0.562</td>
<td>-0.121</td>
<td>-0.391</td>
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<tr>
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Average cumulative abnormal returns (ACAR_{T1T2}) and Z statistics (Z_{T1T2}) over selected trading day intervals.

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Table 6
Three-months Treasury Bills’ yields and yield volatility monthly series over the sampling period.

rate3m: 3-months T. Bill annualized yield.
stdbil3m: standard deviation of the innovations in the 3-months T. Bill annualized yields monthly series estimated over the 12 months preceding the current observation.

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</tr>
<tr>
<td>8712</td>
<td>5.790</td>
<td>0.470117</td>
</tr>
</tbody>
</table>
Table 7
Summary statistics of the regression of abnormal stock returns at the announcement of debt issues on the volatility of the interest rate.

AR_{it}: Abnormal stock return for firm i at the announcement of a debt issue at date t.
D_i: Dummy variable that takes the value 1 if firm i sells fixed rate and the value 0 if it sells floating rate debt.
STDITBILL_t: Sample standard deviation of the innovations in 3-months T. Bill yields monthly series estimated over the 12 months preceding announcement date t.

<table>
<thead>
<tr>
<th>Regression</th>
<th>β₀</th>
<th>β₁</th>
<th>β₂</th>
<th>β₃</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Full sample</td>
<td>.005054</td>
<td>-.014477</td>
<td>-.016392</td>
<td>.021064</td>
<td>.0322</td>
</tr>
<tr>
<td># observations</td>
<td>(.832)</td>
<td>(-2.090)</td>
<td>(-2.137)</td>
<td>(2.581)</td>
<td></td>
</tr>
<tr>
<td>Floating rate: 58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fixed rate: 171</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| (b) Non-depository institutions sample | .00704 | -.017031 | -.01860 | .024377 | .0340 |
| # observations      | (.790) | (-1.774) | (-1.677) | (2.120) |       |
| Floating rate: 31   |        |         |         |        |       |
| Fixed rate: 162     |        |         |         |        |       |

(t-statistics in parentheses)
Table 8
Summary statistics, for selected sub-samples, of the regression of abnormal stock returns at the announcement of a debt issue on the volatility of the interest rates.

Panel (a): Sub-sample that excludes all observations on fixed rate debt issues announced between March 1980 and May 1981.
Panel (b): Sub-sample that excludes all observations on fixed rate debt issues' announcements for which the interest rate volatility proxy takes a value in excess of the largest value recorded by the proxy in the floating rate debt sample.
Panel (c): Sub-sample that excludes all observations on fixed and floating rate debt issues occurring in months for which there are no simultaneous announcements of fixed and floating rate bond issues.

AR_{it}: Abnormal stock return for firm i at the announcement of a debt issue at date t.
D_i: Dummy variable that takes the value 1 if firm i sells fixed rate and the value 0 if it sells floating rate debt.
STDTTBILL_t: Sample standard deviation of the innovations in 3-months T. Bill yields monthly series estimated over the 12 months preceding announcement date t.

\[ AR_{it} = \beta_0 + \beta_1 D_i + \beta_2 STDTTBILL_t + \beta_3 (D_i \ast STDTTBILL_t) + u_{it} \]

<table>
<thead>
<tr>
<th>Regression</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) # observations</td>
<td>0.005054</td>
<td>-0.15052</td>
<td>-0.016392</td>
<td>0.021855</td>
<td>0.0337</td>
</tr>
<tr>
<td>Floating rate: 59</td>
<td>(.851)</td>
<td>(-2.153)</td>
<td>(-2.188)</td>
<td>(2.527)</td>
<td></td>
</tr>
<tr>
<td>Fixed rate: 128</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) # observations</td>
<td>0.005054</td>
<td>-0.014371</td>
<td>-0.016392</td>
<td>0.020223</td>
<td>0.0230</td>
</tr>
<tr>
<td>Floating rate: 50</td>
<td>(.854)</td>
<td>(-2.061)</td>
<td>(-2.195)</td>
<td>(2.340)</td>
<td></td>
</tr>
<tr>
<td>Fixed rate: 127</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) # observations</td>
<td>0.002279</td>
<td>-0.017228</td>
<td>-0.014897</td>
<td>0.021942</td>
<td>0.0439</td>
</tr>
<tr>
<td>Floating rate: 40</td>
<td>(.285)</td>
<td>(-1.602)</td>
<td>(-1.687)</td>
<td>(1.900)</td>
<td></td>
</tr>
<tr>
<td>Fixed rate: 46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(t-statistics in parentheses)
Table 9
Summary statistics of the regression of abnormal stock returns at the announcement of a debt issue on the volatility of the interest rate, and on the level of short-term interest rates.

\[ \text{AR}_{it} = \beta_0 + \beta_1 D_i + \beta_2 \text{STDTBILL}_t + \beta_3 (D_i \times \text{STDTBILL}_t) + \beta_4 \text{TBILL}_t + \beta_5 (D_i \times \text{TBILL}_t) + u_{it} \]

<table>
<thead>
<tr>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00117</td>
<td>-.013536</td>
<td>-.015176</td>
<td>.020658</td>
<td>.000476</td>
<td>-.000037</td>
<td>.0338</td>
</tr>
</tbody>
</table>

# of observations
Floating rate: 59
Fixed rate: 171

(t-statistics in parentheses)
Table 10
Summary statistics of the regression of abnormal stock returns at the announcement of a debt issue on the volatility of the interest rate, and on the maturity and relative size of the issue.

$AR_i$: Abnormal stock return for firm $i$ at the announcement of a debt issue at date $t$.
$D_i$: Dummy variable that takes the value 1 if firm $i$ sells fixed rate and the value 0 if it sells floating rate debt.
$STD(T)BILT_i$: Sample standard deviation of the innovations in 3-months T.Bill yields estimated over the 12 months preceding announcement date $t$.
$MATUREY_{i}$: Number of years between the maturity year and the announcement year for debt issue $i$.
$SIZE_i$: Principal divided by the market value of the outstanding common stock 20 days before the announcement day for firm $i$.

---

Model: $AR_i = \beta_0 + \beta_1 D + \beta_2 STD(T)BILT_i + \beta_3 (D_i * STD(T)BILT_i) + \beta_4 MATUREY_i + \beta_5 (D_i * MATUREY_i) + \beta_6 SIZE_i + \beta_7 (D_i * SIZE_i) + u_{it}$

<table>
<thead>
<tr>
<th></th>
<th>$\beta_0$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$\beta_6$</th>
<th>$\beta_7$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.0037</td>
<td>-.0118</td>
<td>-.0147</td>
<td>.0181</td>
<td>-.0002</td>
<td>.0003</td>
<td>.0080</td>
<td>-.0160</td>
<td>.04</td>
</tr>
<tr>
<td></td>
<td>(.36)</td>
<td>(-.99)</td>
<td>(-1.76)</td>
<td>(2.05)</td>
<td>(-.27)</td>
<td>(.49)</td>
<td>(.67)</td>
<td>(-1.20)</td>
<td></td>
</tr>
</tbody>
</table>

# of observations
Floating rate: 54
Fixed rate: 167

(t-statistics in parentheses)
Table 11
Summary statistics of the regression of abnormal stock returns at the announcement of a debt issue on the volatility of the interest rate, and on the covariance between interest rates and operating income.

AR_{it}: Abnormal stock return for firm i at the announcement of a debt issue at date t.
D_{i}: Dummy variable that takes the value 1 if firm i sells fixed rate and the value 0 if it sells floating rate debt.
STDDBILL_{t}: Sample standard deviation of the innovations in 3-months T.Bill yields monthly series estimated over the 12 months preceding announcement date t.
COVINTBILL_{it}: Sample covariance between the innovations in the operating income before depreciation quarterly series for firm i and the innovations in the matched 3-months T.Bills' yields quarterly series estimated over the 13 quarters preceding the announcement date t.

Model: \[ AR_{it} = \beta_0 + \beta_1 D_{i} + \beta_2 \text{STDDBILL}_{t} + \beta_3 (D_{i} \times \text{STDDBILL}_{t}) + \beta_4 \text{COVINTBILL}_{it} + \beta_5 (D_{i} \times \text{COVINTBILL}_{it}) + u_{it} \]

<table>
<thead>
<tr>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.00347</td>
<td>-.020823</td>
<td>-.013329</td>
<td>.019876</td>
<td>.0000041</td>
<td>.000042</td>
<td>.0455</td>
</tr>
<tr>
<td>(.234)</td>
<td>(-1.279)</td>
<td>(-.827)</td>
<td>(1.161)</td>
<td>(.088)</td>
<td>(.373)</td>
<td></td>
</tr>
</tbody>
</table>

# of observations
Floating rate: 13
Fixed rate: 51

(t-statistics in parentheses)
Table 12
Summary statistics of the regression of abnormal stock returns at the announcement of a debt issue on the volatility of the interest rate, and on the covariance between interest rates and stock returns.

\[ \text{AR}_{it} = \beta_0 + \beta_1 D_t + \beta_2 \text{SDTTBILL}_t + \beta_3 (D_t \times \text{STDTBILL}_t) + \beta_4 \text{COVSTKRTBILL}_{it} + \beta_5 (D_t \times \text{COVSTKRTBILL}_{it}) + u_{it} \]

<table>
<thead>
<tr>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03186</td>
<td>-0.014598</td>
<td>-0.012139</td>
<td>0.021143</td>
<td>0.095517</td>
<td>-0.01586</td>
<td>0.0616</td>
</tr>
<tr>
<td>(0.484)</td>
<td>(-1.900)</td>
<td>(-1.417)</td>
<td>(2.291)</td>
<td>(.653)</td>
<td>(-0.095)</td>
<td></td>
</tr>
</tbody>
</table>

\# of observations: 60
Floating rate: 60
Fixed rate: 120
(t-statistics in parentheses)
**Table 13**

**Matched sample Sign tests**

\[ H_0: \Pr(\text{DIF}>0) = \frac{1}{2} \]
\[ H_1: \Pr(\text{DIF}>0) > \frac{1}{2} \]

**DIF**: Difference between the abnormal stock return at the announcement of a fixed rate debt issue and the abnormal stock return at the announcement of a matched floating rate debt issue.

**VARTBILL**: Sample variance of the innovations in the 3-months T.Bill yields monthly series estimated over the 12 months preceding the announcement date of the debt issue in the pair which occurs at the latest date.

<table>
<thead>
<tr>
<th>Sample</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>.23*</td>
</tr>
<tr>
<td>47 pairs</td>
<td></td>
</tr>
<tr>
<td>High interest rate volatility subsample(VARTBILL&gt;.7)</td>
<td>.08</td>
</tr>
<tr>
<td>19 pairs</td>
<td></td>
</tr>
<tr>
<td>Low interest rate volatility subsample(VARTBILL&lt;.7)</td>
<td>.65*</td>
</tr>
<tr>
<td>28 pairs</td>
<td></td>
</tr>
<tr>
<td>Banking firms subsample</td>
<td>.41</td>
</tr>
<tr>
<td>18 pairs</td>
<td></td>
</tr>
<tr>
<td>Manufacturing firms subsample</td>
<td>.69</td>
</tr>
<tr>
<td>17 pairs</td>
<td></td>
</tr>
</tbody>
</table>

* Large sample approximation
Table 14
Matched sample Wilcoxon Signed-Rank tests

**H₀; Pr[DIF>0] = 1/2**
**H₁; Pr[DIF>0] > 1/2**

**DIF:** Difference between the abnormal stock return at the announcement of a fixed rate debt issue and the abnormal stock return at the announcement of a matched floating rate debt issue.

**VARTBILL:** Sample variance of the innovations in the 3-months T.Bill yields monthly series estimated over the 12 months preceding the announcement date of the debt issue in the pair which occurs at the latest date.

<table>
<thead>
<tr>
<th>Sample</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample</td>
<td>.04*</td>
</tr>
<tr>
<td>47 pairs</td>
<td></td>
</tr>
<tr>
<td>High interest rate volatility subsample</td>
<td>.002</td>
</tr>
<tr>
<td>VARTBILL &gt; .7</td>
<td></td>
</tr>
<tr>
<td>19 pairs</td>
<td></td>
</tr>
<tr>
<td>Low interest rate volatility subsample</td>
<td>.56*</td>
</tr>
<tr>
<td>VARTBILL &lt; .7</td>
<td></td>
</tr>
<tr>
<td>28 pairs</td>
<td></td>
</tr>
<tr>
<td>Banking firms subsample</td>
<td>.03</td>
</tr>
<tr>
<td>18 pairs</td>
<td></td>
</tr>
<tr>
<td>Manufacturing firms subsample</td>
<td>.55</td>
</tr>
<tr>
<td>17 pairs</td>
<td></td>
</tr>
</tbody>
</table>

* Large sample approximation
Table 15
Matched sample Mann-Whitney tests

\[ H_0: \Pr\{\text{DIF}<a|\text{DIF} \in \mathcal{Q}_0\} = \Pr\{\text{DIF}<a|\text{DIF} \in \mathcal{Q}_1\} \]
\[ H_1: \Pr\{\text{DIF}<a|\text{DIF} \in \mathcal{Q}_0\} > \Pr\{\text{DIF}<a|\text{DIF} \in \mathcal{Q}_1\} \]

DIF: Difference between the abnormal stock return at the announcement of a fixed rate debt issue and the abnormal stock return at the announcement of a matched floating rate debt issue.

VARTBILL: Sample variance of the innovations in the 3-months T.Bill yields monthly series estimated over the 12 months preceding the announcement date of the debt issue in the pair which occurs at the latest date.

<table>
<thead>
<tr>
<th>Conditioning Set</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mathcal{Q}_0) = Low interest rate volatility(VARTBILL&lt;.7)</td>
<td>.0075</td>
</tr>
<tr>
<td>28 pairs</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{Q}_1) = High interest rate volatility(VARTBILL&gt;.7)</td>
<td></td>
</tr>
<tr>
<td>19 pairs</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{Q}_0) = Manufacturing</td>
<td>.0778</td>
</tr>
<tr>
<td>17 pairs</td>
<td></td>
</tr>
<tr>
<td>(\mathcal{Q}_1) = Banking</td>
<td></td>
</tr>
<tr>
<td>18 pairs</td>
<td></td>
</tr>
</tbody>
</table>
Table 16
Matched sample non-parametric tests of independence

$H_0$: DIF independent from VARTBILL
$H_1$: DIF positively correlated with VARTBILL

DIF: Difference between the abnormal stock return at the announcement of a fixed rate debt issue and the abnormal stock return at the announcement of a matched floating rate debt issue.

VARTBILL: Sample variance of the innovations in the 3-months T.Bill yields monthly series estimated over the 12 months preceding the announcement date of the debt issue in the pair which occurs at the latest date.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Spearman Correlation (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full sample 47 pairs</td>
<td>.32 (.029)</td>
</tr>
<tr>
<td>Banking firms subsample 18 pairs</td>
<td>.12 (.645)</td>
</tr>
<tr>
<td>Manufacturing firms subsample 17 pairs</td>
<td>.59 (.013)</td>
</tr>
</tbody>
</table>
Part II

Taxes and the Maturity Structure of Corporate Debt

1. Introduction

The effect of taxation on the optimal capital structure of the firm has been a topic of longstanding interest in corporate finance. The literature has thoroughly examined the tax consequences of the leverage decision. The same cannot, however, be said about the tax consequences of the debt maturity decision.

Brennan and Schwartz (1978) show that, if the interest tax savings of debt cease upon bankruptcy, the optimal leverage ratio is a function of debt maturity. In the absence of transaction costs, the optimal policy is to issue and redeem debt continuously so that bankruptcy is avoided while the tax savings are still enjoyed.

Boyce and Kalotay (1979) argue that when borrowers and lenders are differentially taxed, both parties enjoy benefits from suitably structuring the maturity of the loan. For example, if a firm faces a higher marginal corporate tax rate than lenders and the term structure is upward sloping, then there are mutual gains from extending the maturity of the loan.
Brick and Ravid (1985) demonstrate that the debt maturity strategy has no tax implications provided that both interest and the opportunity loss on the notes are tax deductible.¹ The result holds even in the absence of a Miller-type equilibrium in the corporate debt market. The symmetry in the tax treatment of the cash-flows (i.e. interest and price variation) guarantees the irrelevance proposition and makes the result robust to personal taxes considerations. The authors point out though, that under the current tax code in which the opportunity loss of the debt is not tax deductible, personal taxes are instrumental in determining a Miller-type irrelevance result. The argument is identical to the one put forward by Miller (1977). Firms gain from accelerating interest deductions. In contrast, investors minimize the present value of their tax bill by slowing down taxable interest payments. In the absence of personal taxes, corporations prefer long-term debt for an upward sloping yield curve and short-term debt for a downward sloping yield curve. However, with personal taxes, investors require higher promised yields on those securities in which they face a personal tax disadvantage. In equilibrium, the yield differentials across different maturities exactly offset their corporate tax advantages. Each individual corporation is indifferent about the debt maturity structure and there is an equilibrium aggregate supply for each debt maturity.

¹ A note with a tax deductible opportunity loss is a security in which price appreciation is treated as deductible interest expense whereas price declines are treated as taxable interest income.
In a recent paper, Lewis (1990) examines the effects of taxation on financial policy in a multiperiod context. On the demand side, Lewis assumes no short-sale restrictions on investors; since the absence of arbitrage with no short-sell restrictions requires marginal tax rates to be equal across investors, Lewis finds that in equilibrium there are no tax induced security clienteles (i.e. every investor is indifferent about the allocation of their portfolios between taxable and tax-exempt vehicles and about the maturity of their taxable-bonds' holdings). On the supply side, Lewis considers a framework in which firms decide simultaneously about leverage and debt maturity structure. He argues that (1) debt maturity is irrelevant for firm value, and (2) that leverage is relevant for firm value although the level at which it maximizes firm value is in general not unique.

This study starts by addressing the issue of the relevancy of the debt maturity structure. We show that debt maturity is relevant for investors if personal marginal tax rates exhibit cross-sectional variation. We also show that, for corporations, leverage and maturity decisions are substitutes to manage interest tax shields. Given the substitutability between leverage and debt maturity, it follows that the maximization of the tax subsidy from debt financing is consistent with multiple combinations of maturity and leverage strategies. The particular leverage-maturity strategy that will ultimately prevail depends on the costs associated with managing debt tax shields with maturity decisions
versus leverage decisions\(^2\) (or yet, the costs associated of managing tax shields externally via the market for tax shields).

The remaining of this study takes a focus similar to Brick and Ravid and investigates how, for a given leverage, investors and corporations decide about maturity strategies. The study of the impact of taxation on the maturity structure of corporate debt complements the results of Barnea, Talmor and Haugen (1987), who constrain the maturity of available debt instruments to one-period bonds and solve for optimal leverage strategies.

Our analysis focuses on two maturity strategies: a strategy of rolling-over short-term bonds and a strategy of issuing (holding) long-term bonds. At the investor level, the analysis shows that in a tax environment with heterogeneous constant marginal personal tax rates, there are distinct maturity preferences. At the firm level, we demonstrate that Miller irrelevance — with respect to the debt maturity decision — is upset once we allow for non-debt tax shields and idiosyncratic earnings streams.

Firms differ in their intertemporal structure of non-debt tax shields; there are firms which expect non-debt tax shields to grow and firms which expect non-debt tax shields to decline. In order to reduce the odds of losing interest tax shields, firms match large interest tax shields with low non-debt tax shields and vise-versa. For example, if a firm expects a rising schedule of non-debt tax shields, it selects a

\(^2\) For example, leverage and debt maturity are likely to affect differentially agency costs of debt; the firm will select the leverage-maturity strategy that minimizes agency costs of debt.
maturity strategy that accelerates interest deductions (e.g. it chooses to roll-over short-term debt if the term structure of corporate debt is downward sloping, and to sell long-term debt if it is upward sloping).

Corporations are also heterogeneous with regard to their earnings expectations. Some firms expect their earnings to improve while others expect them to deteriorate. Since tax shields in excess of operating income are (at least partially) lost, an efficient management of interest tax shields postulates that corporations concentrate large (low) interest deductions in periods of high (low) earnings. When the term structure is upward sloping, firms faced with earnings crunches roll-over short-term bonds while those faced with gluts borrow long-term. The opposite holds if the term structure is downward sloping.

The time profile of operating risk also affects the maturity strategy. Corporations select the maturity strategy that concentrates large (low) interest deductions in periods of low (high) operating risk. This "timing" of interest deductions implies that with an upward (downward) sloping yield curve, firms whose uncertainty about future cash-flows grows rapidly with the time-horizon of the cash-flows, rely more heavily on long-term (short-term).

Moreover, the optimal debt maturity depends on the stochastic association between the firm's cash-flows and corporate interest rates. To illustrate the argument, consider a firm A whose cash-flows from operations are strongly positively associated with corporate interest rates. This firm prefers to roll-over short-term debt to a larger extent
than a similar firm, B, whose cash-flows from operations are independent from corporate interest rates. Although an heavier emphasis on roll-over debt increases the uncertainty about future interest deductions for both firms, for firm A it reduces the uncertainty about future taxable income.

At the aggregate level we examine how the aggregate demand and supply for the two debt maturity strategies adjust until they become mutually consistent. From the model developed we derive predictions relating the difference in the equilibrium returns on alternative maturity strategies, to the slope of the yield curve, the expected direction of change in short-term interest rates, and the covariance between the short-term taxable and tax-free interest rates.

The second part of the dissertation closes by contrasting the effects of taxation on the choice between short-term and long-term (fixed rate) debt with those on the choice between long-term floating and fixed rate debt. Under the assumption of default-free debt, we argue that the two problems are identical. The reason is that, with no default risk, a strategy of rolling-over short-term bonds produces the same after-tax cash-flows to equityholders and bondholders as a strategy of issuing long-term floating rate bonds. Default risk, however, may make the cash-flow streams to diverge. We discuss how the periodic reassessment of default premia in the roll-over strategy and the loss of tax shields in the bankrupt states lead to a divergence between the after-tax cash-flows from rolling-over short-term and from selling floating rate bonds.
The rest of this study is organized as follows. Section 2.1 states the assumptions and notation. Section 2.2 discusses the relevancy of the maturity decision. In section 2.3, we study the debt maturity decisions of investors and corporations holding leverage constant. Subsection 2.3.3 shows that a Miller irrelevance result is obtained when corporations make full use of interest deductions. Subsection 2.3.4 considers the general case and investigates the properties of the equilibrium at the aggregate level. Subsection 2.3.5 examines the individual firm optimal debt maturity decision. Subsection 2.3.6 confirms and extends the results to the case of stochastic interest rates. Section 2.4 discusses the tax consequences of the choice between fixed and floating rate debt. Finally, Section 3 presents concluding remarks.

2. The Model

2.1. Assumptions and Notation

Consider an economy in discrete time, which lasts for N periods and has a single homogeneous consumption good. Let $P(t,T)$ and $Q(t,T)$ represent, respectively, the prices at time $t$ of tax exempt and taxable default-free discount bonds that pay $1$ at date $T$. Make the additional assumptions:

(a) personal tax code: For each investor $i$, $\tau_i$ denotes a constant marginal personal tax rate on interest (taxable) income. Investors are differentially taxed with personal tax brackets ranging between zero (tax exempt investors) and $\tau^\#$(maximum personal tax rate). The
personal tax rate on equity income is zero.

(b) corporate tax code: Firms face a positive constant marginal corporate tax rate of $\tau_c$ on positive taxable income, and face a zero rate on negative taxable income. Interest payments are tax deductible, and so are non-cash charges such as accounting depreciation. There are no carrybacks and carryforwards provisions (CB-CF)\(^3\) and no market for unused tax shields; all tax shields in excess of operating income are lost.

(c) financing policy of the firm: firms fund their operations with equity and taxable discount bonds.

(d) imputed interest on discount bonds: since a discount bond pays no interim interest, the periodic interest is imputed. The imputed interest on a discount bond at date $t$ is equal to the change in the book value of the bond between $t$ and $t-1$; the book value at date $t$ of a bond which matures at $T$ and which was previously transacted (for the last time) at date $t'(t'<t)$ for $Q(t',T)$, is defined as

$$Q(t',T)[1/Q(t',T)][(t-t')/(T-t')]\) \tag{1}$$

Thus, the book value of a discount bond grows at a (compounded) rate equal to the yield on the bond prevailing at the date the bond was last transacted. Imputed interest is taxed at the personal level and is tax deductible at the corporate level.

\(^3\)The incorporation of CB-CF in the model doesn't change any of the results. As DeAngelo and Masulis (1980) point out, CB-CF provisions reduce, but not eliminate, the expected value of the corporate tax shield loss on the marginal unit of debt.
(e) Agents are wealth maximizers, exhibit locally risk neutral preferences, and tax arbitrage is not permitted.

2.2. Relevance of Debt Maturity Structure

This section deals with Lewis' assertion that, for tax reasons alone, the maturity structure of corporate debt is irrelevant.

To see the relevance of the debt maturity decision from the investor's perspective, consider an arbitrary investor with marginal tax rate \( \tau_i \). Given our assumption about preferences, the investment policy is independent of realized wealth and is entirely motivated by tax reasons. At each date \( t \), the investor allocates his/her investment funds among tax-exempt securities (i.e. tax-exempt discount bonds and equity) and taxable discount bonds of different maturities. Since the investor chooses the security with the highest after-tax one-period expected return, the portfolio policy in each period is characterized by a portfolio "tax-status" decision - invest either all or none of the available funds in taxable bonds - and a taxable portfolio maturity decision - choice of the maturity of the corporate bonds where to invest all the funds.\(^4\)

Let the investment opportunities in corporate bonds be limited to one-period bonds. For simplicity, assume that corporate debt is default free. The investor computes the difference in after tax one-period returns between a tax exempt and a taxable investment

\(^4\)Investments in tax-exempt bonds of different maturities have the same tax consequences and therefore, the maturity decision with respect to tax-exempt investments is irrelevant.
\[ Z_{t+1} = \frac{1}{P(t,t+1)} - \frac{1 - \tau_i(1 - Q(t,t+1))}{Q(t,t+1)} \]  

and purchases tax-exempts if \( Z_{t+1} > 0 \), and one-period corporate bonds if \( Z_{t+1} < 0 \). The investment policy can therefore be characterized by a vector of state contingent binary variables of the form

\[ \Sigma_t = \{I_{O_1}(Z_{0_1}), I_{1_1}(Z_{1_1}), \ldots, I_{N-1_1}(Z_{N-1_1})\} \]

where \( I_{t_i} \) takes the value one (i.e. invest only in tax-exempt securities) if \( Z_{t_i} > 0 \), and zero (i.e. invest only in one-period corporate bonds) otherwise.

Assume that the ratio between taxable and tax-exempt short-term interest rates evolves slowly over time, i.e., assume that there exists \( \varepsilon_t \) small such that

\[ \Pr\{|[P(t,t+1)/Q(t,t+1)] - [P(t+1,t+2)/Q(t+1,t+2)]| < \varepsilon_t\} = 1 \]

which implies that portfolio "tax-status" policies exhibit a degree of persistence. This assumption guarantees that there are states of nature, \( Z_{t_i} \), for which investor \( i \) knows with certainty that his/her next period portfolio "tax-status" policy will be identical to the current one.

Consider now those states of the nature satisfying \( Z_{t+1} < 0 \) and \( \Pr[Z_{t+1} < 0|Z_t] = 1 \). For those states we open to the investor the opportunity to purchase and hold until maturity 2-period corporate bonds. To demonstrate the relevance of the maturity decision we need to show that there are circumstances in which individual \( i \) chooses to invest in to 2-period bonds rather than to roll-over one-period bonds.

At date \( t \), the after-tax one-period expected return associated with purchasing 2-period bonds, \( Q(t,t+2) \), and holding them until maturity is
\[ R_{t+1} = \frac{\text{EP}(t+1,t+2)[1-\tau_t(1-\text{Q}(t,t+2)\frac{1}{2})]-\tau_t[\text{Q}(t,t+2)\frac{1}{2}-\text{Q}(t,t+2)]}{\text{Q}(t,t+2)} \]  

(5)

where \( E \) is the expectations operator. On the other hand, the after-tax one-period expected return associated with purchasing one-period bonds, \( \text{Q}(t,t+1) \), and rolling them over at date \( t+1 \) is

\[ S_{t+1} = \frac{\text{EP}(t+1,t+2)[(1/\text{Q}(t+1,t+2)) - \tau_t[(1/\text{Q}(t+1,t+2))-1]] - \tau_t(1-\text{Q}(t,t+1))}{\text{Q}(t,t+1)} \]  

(6)

The investor computes the difference between the after-tax one-period expected returns on the two investments, \( S_{t+1} - R_{t+1} \), and invests in 2-period bonds if the difference is negative and in 1-period bonds if it is positive. Suppose that the expected values at date \( t+1 \) of the price of one-period tax-exempt bonds, \( P(t+1,t+2) \), and of the ratio of prices of tax-exempt and taxable one-period bonds, \( P(t+1,t+2)/Q(t+1,t+2) \), are equal to their current values. In this case, \( S_{t+1} - R_{t+1} \) is

\[ S_{t+1} - R_{t+1} = (1-\tau_t)P(t,t+1)[[1/Q(t,t+1)^2]-[1/Q(t,t+2)^2]] + \tau_t[P(t,t+1)-1][[1/Q(t,t+1)]-[1/Q(t,t+2)^2]] \]  

(7)

which is equal to zero if the yield curve for taxable bonds is flat (i.e. \( Q(t,t+1)=Q(t,t+2)^\frac{1}{2} \)). With \( d[S_{t+1} - R_{t+1}]/dQ(0,2)>0 \), the investor selects two-period bonds if the yield curve is upward sloping while with \( d[S_{t+1} - R_{t+1}]/dQ(0,2)<0 \), he/she purchases two-period bonds if the yield curve is downward sloping.

Inspection of \( S_{t+1} - R_{t+1} \) indicates the existence of bond maturity clienteles. Investors in low tax brackets find the before-tax one-period
expected return differentials among alternative bond maturities strategies sufficiently attractive to choose strategies with a tax disadvantage. On the other hand, investors in high tax brackets select bond maturity strategies which are more advantageous from the point of view of taxes. The existence of maturity clientele implies that the maturity structure of corporate debt is at least relevant at the aggregate level. Even if the debt maturity decision is irrelevant at the firm level, the fact that individuals have distinct preferences for alternative bond maturities, establishes that there is an equilibrium aggregate level of debt for each different maturity. This result contrasts with the prediction of Lewis (1990), who argues that the maturity structure of corporate debt is irrelevant at the aggregate level. Lewis assumes that (1) every investor faces an identical tax function and (2) that individuals have equal access to financial markets and can short sell without restriction. To prevent tax arbitrage, assumptions (1) and (2) imply that in equilibrium investors must face identical marginal tax rates. Consequently, in equilibrium the tax consequences associated with alternative maturity strategies are the same for all investors and maturity clientele cannot exist. Finally, since in Lewis' framework corporations are indifferent about bond maturity decisions, the equilibrium aggregate maturity structure of corporate debt is indeterminate. In summary, from the individual investor's perspective, the relevancy of the debt maturity decision hinges on whether, in equilibrium, investors face identical or heterogeneous marginal tax rates.
Lewis also claims that the debt maturity decision is irrelevant for corporations. He shows that the financial policy only affects firm value through the aggregate promised interest payments on the debt. His argument is that, given an optimal aggregate stream of promised interest payments, the debt maturity decision is irrelevant. Although his argument is correct, the assertion that the debt maturity decision is irrelevant is misleading. The following discussion clarifies what in Lewis paper is meant by the irrelevancy of the maturity decision.

As Lewis points out, the financial policy of the firm in a multiperiod context is characterized by a debt issuance strategy: a mapping of a sequence of realizations of states of nature into a sequence of debt issuance decisions. The debt issuance decision in each period consists of a state contingent leverage decision - choice of the current market value of debt - and a state contingent debt maturity decision - allocation of the current market value of debt among bonds of different maturities. The debt issuance strategy consists therefore, of a leverage and a maturity strategy. These two strategies jointly determine a state contingent stream of promised interest payments. Lewis’ argument amounts to asserting that for any maturity strategy, it is possible to devise a leverage strategy that produces an optimal stream of state contingent promised interest payments. It is in this sense that maturity decisions are irrelevant. Thus, it is not the case that maturity decisions do not affect the promised stream of interest payments (and therefore, the value of the firm). The case is, that for any change in the maturity strategy
that disturbs a sequence of optimal promised interest payments, it is possible to readjust the leverage strategy so that the original sequence of promised interest payments is restored. Consequently, there are multiple combinations of maturity and leverage strategies consistent with the same optimal stream of state contingent promised interest payments.

The fact that both the leverage and maturity decisions affect promised interest payments, suggests that it may be feasible to specify the leverage strategy arbitrarily, and find a maturity strategy that produces an optimal sequence of state contingent promised interest payments. If this is true, then we might as well reverse Lewis' assertion that leverage is relevant and debt maturity irrelevant, and claim instead that leverage is irrelevant and maturity is relevant.

Consider a firm at date \( t=0 \) that operates for two periods after which it liquidates all the assets. The value of the firm is maximized by a sequence of state contingent promised interest payments of the form

\[
\begin{align*}
&c^*_1, c^*_2(s_1) \\
\end{align*}
\]  

where \( s_1 \) is the state of nature at date \( t=1 \). For each possible state of nature at date \( t=1 \) there is a corresponding realization of the price of one-period corporate bonds, \( Q(1,2) \). Assume that there are 2 possible states of nature at date \( t=1 \), \( s_a \) and \( s_b \), and that the corresponding realizations of \( Q(1,2) \) are \( Q_a(1,2) \) and \( Q_b(1,2) \). Also assume that the financing opportunity set available to the firm is as follows: at date \( t=0 \) the firm can sell equity and one and two-period bonds; at date \( t=1 \) it can sell additional equity, buy back existing shares, repurchase outstanding
two-period bonds and issue new one-period bonds. For simplicity assume that all debt is default free.

The optimal financial policy is any vector of state contingent leverage and maturity decisions \( \{D_0, m_0, D_1(s_a), m_1(s_a), D_1(s_b), m_1(s_b)\} \) consistent with the optimal sequence of state contingent optimal promised interest payments \( c^*_1, c^*_2(s_1) \). At date \( t \) for state \( s_t \), the leverage decision consists of choosing the market value of the outstanding debt \( D_t(s_t) \), while the maturity decision, \( m_t(s_t) \), consists of allocating \( D_t(s_t) \) between one and two period bonds. The optimal financial policy solves

\[
c^*_1 = m_0D_0 \left\{ \frac{1}{Q(0,1)} - 1 \right\} + (1-m_0)D_0 \left\{ \frac{1}{Q(0,2)} - 1 \right\}
\]

\[
c^*_2(s_a) = m_1(s_a)D_1(s_a) \left\{ \frac{1}{Q_a(1,2)} - 1 \right\} + (1-m_1(s_a))D_1(s_a) \left\{ \frac{1}{Q(0,2)} \right\} \frac{Q_a(1,2)}{Q_a(1,2)}
\]

\[
c^*_2(s_b) = m_1(s_b)D_1(s_b) \left\{ \frac{1}{Q_b(1,2)} - 1 \right\} + (1-m_1(s_b))D_1(s_b) \left\{ \frac{1}{Q(0,2)} \right\} \frac{Q_b(1,2)}{Q_b(1,2)}
\]

subject to the constraints

\[
(1-m_1(s_a))D_1(s_a) \left\{ \frac{1}{Q_a(1,2)} \right\} \leq (1-m_0)D_0 \left\{ \frac{1}{Q(0,2)} \right\}
\]

\[
(1-m_1(s_b))D_1(s_b) \left\{ \frac{1}{Q_b(1,2)} \right\} \leq (1-m_0)D_0 \left\{ \frac{1}{Q(0,2)} \right\}
\]

\[
0 \leq D_0 \leq V_0^*, \quad 0 \leq D_1(s_a) \leq V_1^*(s_a), \quad 0 \leq D_1(s_b) \leq V_1^*(s_b)
\]

\[
0 \leq m_0 \leq 1, \quad 0 \leq m_1(s_a) \leq 1, \quad 0 \leq m_1(s_b) \leq 1
\]

where \( V_0^* \) is the maximum value of the firm at date \( t=0 \), and \( V_1^*(s_a) \) and
\( V_i^*(s_b) \) are respectively, the maximum value of the firm at date \( t=1 \) under state \( s_a \) and state \( s_b \). Constraints (12) and (13) state that the face value of outstanding 2-period bonds at date \( t=1 \) in either state \( s_a \) or \( s_b \), cannot exceed the face value of the 2-period bonds sold at date \( t=0 \).

From equations (9), (10), and (11), we can see that the effect of maturity decisions on promised interest payments depends at date \( t=0 \), on the slope of the yield curve \([Q(0,1)-Q(0,2)]\), and at date \( t=1 \), on the difference between the current price of one-period bonds and the current book value of two-period bonds \([Q_j(1,2)-Q(0,2)], j=a,b\). If the yield curve is flat at date \( t=0 \), then changing maturity \((m_0)\) has no effect on the promised interest at \( t=1 \), \( c_i \); in this case the maturity decision at date \( t=0 \) doesn't matter. However, with a non flat yield curve the maturity decision influences \( c_i \). Furthermore, the steeper is the slope of the yield curve the larger the impact of the maturity decision on \( c_i \). At date \( t=1 \), the maturity decision in state \( s_j \) \((j=a,b)\), \( m_1(s_j) \), doesn't matter if \( Q_j(1,2)-Q(0,2) \leq 0 \). With \( Q_j(1,2)-Q(0,2) \neq 0 \) different from zero, however, the maturity decision affects the promised interest at date \( t=2 \), \( c_2 \). An increase \( Q_j(1,2)-Q(0,2) \) in absolute value, enhances the ability of the maturity decision to affect promised interest payments.

The discussion above implies that, with a non flat yield curve at date \( t=0 \) and with the price of new one period bonds at date \( t=1 \) different from the book value of two period bonds, there is a set of (multiple) feasible leverage strategies\(^5\), \( Q_0 \), for which it is possible to find

\(^5\)A feasible leverage strategy is a strategy that satisfies (14).
maturity strategies that solve (9), (10), and (11), subject to the constraints (12), (13), and (15). The size of this set increases with the slope of the yield curve at date $t=0$, $[Q(0,1)-Q(0,2)]$, and with the magnitude (in absolute value) at date $t=1$ of the difference between the current price of one-period bonds and the current book value of two-period bonds $[Q_j(1,2)-Q(0,2)]$, $j=a,b$. Similarly, there is a set of (multiple) feasible maturity strategies, $Q_m$, for which there are leverage policies that solve (9), (10), and (11), subject to (12), (13), and (14). The optimal sequence of state contingent promised interest payments can therefore, be achieved either by fixing the maturity strategy arbitrarily within set $Q_m$ and solving for the leverage strategy, or by fixing the leverage strategy arbitrarily within set $Q_o$ and solving for the maturity strategy.

In summary, leverage decisions and debt maturity decisions are substitutes in managing corporate debt tax shields. It is therefore important to study how, for a given leverage strategy, corporations manage interest tax shields with maturity decisions. Furthermore, firms are more likely to face restrictions on their ability to change leverage than on

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*A feasible maturity strategy is a strategy that satisfies (15).*

*There may exist maturity strategies for which a feasible leverage strategy consistent with $[c_1^*,c_2^*(s_1^*),c_2^*(s_2^*)]$ cannot be found. For example, suppose that the yield curve at date $t=0$ is upward sloping (i.e. $Q(0,2)<Q(0,1)$), and that $V^s_0[[1/Q(0,1)]-1]<c_1^*<V^s_0[[1/Q(0,2)]-1]$. Then, there exists a value $\beta$ ($0<\beta<1$), such that for a maturity decision $m_0<\beta$ there is no feasible leverage decision that yields the optimal promised interest payment at date $t=1$, $c_1^*$. 
their ability to change the maturity of the debt. Smith and Warner (1979), report that often bond covenants go beyond restricting the issuance of debt of equivalent or higher priority, to restrict the firm's right to issue any additional debt (e.g. issuance of new debt is subject to aggregate dollar limitations). Limitations on the flexibility of the leverage decision further underscore the importance of studying how corporations manage debt tax shields with maturity decisions.

2.3. Taxation and Debt Maturity Structure

In the economy described in section 2.1., equilibrium prices are characterized by the distributions of all future spot bond prices (taxable and tax-exempt), the current yield curve for taxable bonds, plus the current short-term (one-period) tax-exempt interest rate. At the equilibrium prices, investors' savings, portfolio "tax-status", and taxable portfolio maturity policies are consistent with firms' investment, leverage and debt maturity policies. Further, the market for spot tax-exempt bonds of different maturities clears.

The remaining part of this study investigates the effect of taxation on debt maturity decisions, given investors' savings and portfolio "tax-status" policies, and given firms' investment and leverage policies. Specifically, we analyze how firms and investors choose between

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Note that since tax-exempt bonds have no tax implications, the assumption of local risk neutrality implies that the current yield curve for tax-exempt bonds can be obtained from the distributions of future spot bond prices and the current short-term tax-free interest rate.
rolling-over short-term bonds and issuing (holding) long-term bonds in the
tax environment described in section 2.1.. We solve for the equilibrium
relative prices between future before-tax dollars to be delivered with the
two maturity strategies, taking all other prices as exogenous. We start
by assuming that interest rates evolve deterministically. Section 2.3.6.
relaxes this assumption and evaluates the effect of interest rate
uncertainty on maturity strategies.

2.3.1. Demand for Long-Term versus Short-Term Taxable Bonds

In this section we examine the tax motivations behind investors
decision to allocate funds between matched\(^9\) versus roll-over lending
strategies. Consider at date \(t=0\), an individual who invests in taxable
bonds for two periods. Assume the yield curves for both tax-exempt and
taxable bonds to be exogenously given. Define

\[ \delta = \frac{Q(0,1)Q(1,2)}{Q(0,2)} \]  \hspace{1cm} (16)

as the relative price between $1 before taxes at date \(t=2\) to be delivered
with a roll-over and a matched lending strategies.

\[ \text{Proposition I} \]

If individual \(i\) has a positive constant marginal personal tax rate
\(\tau_i\), then in order for he/she to be indifferent between the matched
and the roll-over lending strategy the following is required:

\[ \text{By a matched lending strategy we refer to the purchase of a bond}
\text{with a maturity that matches the investor's holding period.} \]
a) $\delta < 1$ if the term structure for taxable bonds is downward sloping.
b) $\delta > 1$ if the term structure for taxable bonds is upward sloping.
c) $\delta = 1$ if the term structure for taxable bonds is flat.

Proof in appendix B.

Proposition I states that investors who pay personal taxes on interest income require a premium on the before-tax return to the lending strategy with the larger present value in taxes. When the term structure is upward sloping, the schedule of taxable (imputed) interest payments associated with the roll-over strategy rises at a faster rate than the schedule associated with a matched lending strategy. The opposite is true when the term structure is downward sloping. Due to the time value of money, investors minimize the present value of their tax bill by selecting the strategy with the steepest schedule of interest payments (i.e. the strategy that slows down imputed interest payments). Thus, to compensate for the differential in tax bills, investors require a premium on the before tax returns to the matched lending strategy when the term structure is upward sloping, whereas they require a premium on the before tax return to the roll-over borrowing strategy when the term structure is downward sloping.

Let $\delta_i$ represent the value of the relative price, $\delta$, in (16) that makes investor $i$ indifferent between a roll-over and a matched lending strategy.
Proposition II

Cross-sectionally, the effect of an increase in the marginal tax rate, $\tau_i$, on $\delta_i$ is as follows:

i. $d\delta_i/d\tau_i = 0$ if the yield curve of taxable bonds is flat.

ii. $d\delta_i/d\tau_i > 0$ if the yield curve of taxable bonds is upward sloping.

iii. $d\delta_i/d\tau_i < 0$ if the yield curve of taxable bonds is downward sloping.

Proof in appendix B.

Proposition II establishes that investors in higher tax brackets require a larger premium on the before-tax return to the lending strategy with a tax disadvantage. Propositions I and II enable us to characterize the aggregate demand for the two alternative investment strategies. Function $D(\delta)$ in Figures 6 and 7 represents the aggregate demand for two-period bonds. Panels (a), (b), and (c) plot function $D(\delta)$ for an upward, flat, and downward yield curve. On the horizontal axis, variable $p$ represents the percentage of the total face value of debt to be paid out at date $t=2$ in the form of two-period bonds (hence, $1-p$ represents the percentage to be paid out at date $t=2$ with the roll-over strategy). The vertical axis measures the relative price $\delta$.

When the term structure of taxable bonds is flat, the schedule of (imputed) interest payments is independent from the maturity strategy. Since the two strategies have the same tax consequences, $\delta=1$, and every investor is indifferent about debt maturity. However, when the term structure of taxable bonds is either upward or downward sloping, the two
strategies entail different tax consequences for investors in positive tax brackets. When the term structure is upward sloping, taxable investors face a tax disadvantage with two-period bonds. Tax exempt investors are willing to hold two-period bonds without any premium (segment 0 to 1). To increase the value of \( p \) beyond 1, it is required to induce taxable investors to hold two-period bonds. Therefore, the relative price \( \delta \) has to rise above 1. Further, since investors in higher tax brackets require larger premia on the lending strategy with a tax disadvantage, \( p \) can only increase by steadily raising \( \delta \). The reverse occurs for a downward sloping yield curve.

As in Boyce and Kalotay, our analysis implies the existence of tax induced bond maturity clienteles. Investors in high tax brackets concentrate their taxable bond holdings in maturities with a low tax penalty while individuals in low tax brackets invest in maturities with stiffer tax penalties.

2.3.2. Supply of Long-Term versus Short-Term Taxable Bonds

In this section we investigate the tax consequences for corporations from different maturity strategies. We consider the simplified framework in which, at date \( t=0 \), corporations borrow for two periods and choose between rolling-over one period taxable bonds and issuing two period bonds; consequently, after \( t=2 \) every firm is all equity financed. The yield curves for both tax-exempt and taxable bonds are again assumed to be exogenously given.
Without loss of generality, normalize the face value of debt to be retired at date \( t=2 \) to 1. The decision variable at date \( t=0 \) is the percentage of the total face value of debt to mature at date \( t=2 \) in the form of two period bonds \( \rho \) (thus, \( 1-\rho \) matures in the form of one period bonds). For a given firm, define the following variables:

\[ X_t = \text{operating income in period } t, \ t=1,2,\ldots,N; \ F_t(\ ) \text{ and } f_t(\ ) \text{ are respectively the c.d.f. and p.d.f. of } X_t; \ X_t \text{ independent from } X_t, \text{ for } t>t' \text{ or } t<t'; \ f_t(\ ) \text{ symmetric with } 0 \leq X_t \leq Q_t. \]

\[ L_t = \text{corporate tax deductions resulting from non-cash charges such as accounting depreciation in period } t, \ t=1,2,\ldots,N. \]

Assume that corporate debt is default-free. At date \( t=2 \) firms sell equity to finance the repayment of debt; at date \( t=1 \) firms refinance any outstanding one period bonds with new one period bonds. All operating income net of corporate taxes is distributed as cash dividends.

Under this set of assumptions the value of the firm at date \( t=0 \), \( V_0 \), is given by

\[ V_0 = E_0 + \rho Q(0,2) + (1-\rho)Q(0,1)Q(1,2) \]  \hspace{1cm} (17)

where \( E_0 \) is the time \( t=0 \) market value of equity. To evaluate \( E_0 \) we examine the cash-flows accruing to equityholders. These are contained in Table 17.

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10As in Dammon & Semet (1988) we choose not to incorporate into the model investment tax credits. As these authors point out, tax credits have the same qualitative implications for the financing decisions of the firm as depreciation. Also, the Tax Reform Act of 1986 eliminated the investment tax credit.
Define
\[ s_1 = [L_1 + p(Q(0,2)^{-}Q(0,2)) + (1-p)Q(1,2)(1-Q(0,1))] \]  \hspace{1cm} (18)
\[ s_2 = [L_2 + p[1-Q(0,2)^{-}] + (1-p)[1-Q(1,2)]] \]  \hspace{1cm} (19)

as the total tax shields available to the firm at dates \( t=1 \) and \( t=2 \). The market value of equity at date \( t=0 \) can now be written

\[ E_0 = P(0,1)\{E(X_t) - \tau_c \int_{s_1}^{a_1} X_t f_t(X_t) dX_t + \tau_c s_1[1-F_1(s_1)] + \}
\[ + P(0,2)\{E(X_t) - \tau_c \int_{s_2}^{a_2} X_t f_t(X_t) dX_t + \tau_c s_2[1-F_2(s_2)] - 1\}
\[ + \sum_{t=3}^{N} P(0,t)\{E(X_t) - \tau_c \int_{L_t}^{a_t} X_t f_t(X_t) dX_t + \tau_c L_t[1-F_t(L_t)] \} \]  \hspace{1cm} (20)

The optimal \( p^* \) solves the first order condition (FOC), \( dV_0/dp = 0 \).

Differentiating (17) with respect to \( p \), and using (18), (19) and (20), we obtain

\[ dV_0/dp = P(0,1)\tau_c [Q(0,2)^{-}Q(0,2)-Q(1,2)(1-Q(0,1))] \]
\[ +(1-F_1[L_1 + p(Q(0,2)^{-}Q(0,2)) + (1-p)Q(1,2)(1-Q(0,1))]) + \]
\[ +P(0,2)\tau_c [Q(1,2)-Q(0,2)]\{1-F_2[L_2 + p[1-Q(0,2)^{-}] + (1-p)(1-Q(1,2))]\} + \]
\[ +Q(0,2)-Q(0,1)Q(1,2)=0 \]  \hspace{1cm} (21)
2.3.3. Miller Irrelevance

In a Miller framework tax shields are always fully utilized. Technically, we have

\[ F_1(s_1) = F_2(s_2) = 0 \]  \hspace{1cm} (22)

i.e., the probability at date \( t = 1, 2 \) of tax shields to exceed operating income is zero. Using (22) in (21), and substituting \( Q(1,2) = \delta Q(0,2)/Q(0,1) \) from (16) yields

\[
dV_0/dp = P(0,1)\tau_c(Q(0,2)\delta - Q(0,2) - \delta Q(0,2)/Q(0,1))(1-Q(0,1)) + \\
+ P(0,2)\tau_c(\delta Q(0,2)/Q(0,1) - Q(0,2)\delta) + Q(0,2) - \delta Q(0,2) = 0 \hspace{1cm} (23)
\]

The FOC (23) is independent of \( p \). It is also independent of firm specific parameters. Consequently, the choice of maturity strategy is irrelevant for each individual firm. Furthermore, the value of the relative price that makes firms indifferent between the two borrowing strategies (i.e. the value of \( \delta \) that solves (23)), is the same across all firms. Thus, firms exhibit identical perfectly elastic supply schedules for the percentage of the total face value of debt to mature at date \( t = 2 \) in the form of two-period bonds, \( p \).

The characterization of equilibrium at the aggregate level depends on the value of the relative price, \( \delta \), that solves corporations’ FOC (23). If the term structure of taxable bonds is flat (i.e. \( Q(0,2)\delta = Q(0,1) \)), \( \delta = 1 \) solves (23) and therefore, \( p \) is perfectly elastic at the relative price \( \delta = 1 \). Since on the demand side, \( p \) is also perfectly elastic at relative prices \( \delta = 1 \), functions \( \delta = \delta(p) \) on the demand and supply side totally
overlap. Consequently, for a flat term structure the value of p at the aggregate level is indeterminate. For an upward term structure (i.e. \(Q(0,2) > Q(0,1)\)), \(\delta > 1\) makes FOC (23) positive. Because the function \(dV_p/dp\) is monotonically decreasing in \(\delta\), it takes \(\delta > 1\) to solve the FOC (23). Hence, p is perfectly elastic at relative prices \(\delta > 1\). The interplay of aggregate borrowing and lending strategies determines an equilibrium level for p. By the same argument, for a downward sloping term structure p is perfectly elastic at the relative price \(\delta < 1\), and there is an equilibrium aggregate level of p. Figure 6 shows the Miller equilibrium for an upward, flat, and downward yield curve.

(a) Upward sloping term structure \([Q(0,1) > Q(0,2)]\)  
(b) Flat term structure \([Q(0,1) = Q(0,2)]\)  
(c) Downward sloping term structure \([Q(0,1) < Q(0,2)]\)

**Figure 6**
Miller equilibrium

p = value of the face value of debt to mature at date t=2 in the form of matched bonds. 
\(\delta = \) relative price between $1 before taxes to be delivered at date t=2 with a rollover and a matched strategy. 
D = Demand for matched bonds; S = Supply of matched bonds. 
1 = Demand for matched bonds by tax-exempt investors.
In summary, when firms have the ability to fully utilize all debt tax shields (or to market any unused tax shield), a Miller type irrelevance result is obtained with respect to the maturity structure of debt at the individual firm level. For a non-flat term structure there is an aggregate equilibrium structure of debt maturity while for a flat term structure the aggregate maturity structure is indeterminate. The intuition behind these results was first discussed by Brick and Ravid. With a flat term structure, interest payments sequences are independent from maturity strategies; it follows that debt maturity decisions are irrelevant both for corporations and investors and thus, the debt maturity structure is indeterminate at the aggregate level. For a non-flat term structure, the assumption that corporations fully use all tax shields implies that differences in the corporate tax benefits among different borrowing strategies are fully offset by differentials in before-tax required rates of return; consequently, corporations are indifferent about debt maturity. On the demand side, a non-flat yield curve implies the existence of bond maturity clienteles. At the aggregate level, corporations supply different clienteles and there is an equilibrium structure of debt maturity.

2.3.4. Interior Optimal Debt Maturity Structure

Consider the general case in which there is a positive probability that the firm will not be able to fully utilize all tax shields (i.e. \( F_1(s_1), F_2(s_2) \geq 0 \)). The next four propositions characterize the supply of two-period bonds.
Proposition III
If $Q(0,1) \geq P(0,2) \tau_c$ and $f_t(x) \leq F_t(x)$, then for every individual firm the supply of two-period bonds, $p^*(\delta)$, is a continuous non-positive function of the relative price $\delta$.
Proof in appendix B.

Proposition IV
If the yield curve is flat (i.e. $Q(0,2)^{\frac{1}{2}} = Q(0,1)$) and $Q(0,1) \geq P(0,2) \tau_c$, then both individual and aggregate supplies of two-period bonds, $p^*$, are perfectly elastic at relative prices $\delta = 1$.
Proof in appendix B.

Proposition V
If the term structure is upward sloping (i.e. $Q(0,2)^{\frac{1}{2}} < Q(0,1)$) and $Q(0,1) \geq P(0,2) \tau_c$, then no firm will supply short-term bonds at a relative price $\delta < 1$, and no firm will supply two-period bonds at a relative price $\delta \geq Q(0,1)/Q(0,2)^{\frac{1}{2}}$.
Proof in appendix B.

Proposition VI
If the term structure is downward sloping (i.e. $Q(0,2)^{\frac{1}{2}} > Q(0,1)$) and $Q(0,1) \geq P(0,2) \tau_c$, then no firm will supply two-period bonds at a relative price $\delta > 1$, and no firm will supply short bonds at a relative price $\delta \leq Q(0,1)/Q(0,2)^{\frac{1}{2}}$.
Proof is the same as for proposition V.
Figure 7 plots the aggregate demand and supply for two-period bonds for an upward sloping, a flat, and a downward sloping term structure. The horizontal axis measures the percentage of the total face value of corporate debt to mature at date $t=2$ in the form of two-period bonds $p$, and the vertical axis measures the relative price $\delta$.

With a flat term structure, both demand and supply are perfectly elastic at relative prices $\delta=1$. This result is already familiar to us and requires no further discussion. For a upward sloping yield curve investors in positive tax brackets require $\delta>1$ to hold two-period bonds. Firms, on the other hand, require $\delta>1$ to supply short-term bonds. Unless there are

![Graphs](image)

(a) Upward sloping term structure \([Q(0,1)>Q(0,2)]\)
(b) Flat term structure \([Q(0,1)=Q(0,2)]\)
(c) Downward sloping term structure \([Q(0,1)<Q(0,2)]\)

**Figure 7**
Equilibrium with non-debt tax shields and idiosyncratic earnings.

$p\%$ of the face value of debt to mature at date $t=2$ in the form of matched bonds.

$\delta$=relative price between $\$1$ before taxes to be delivered at date $t=2$ with a roll-over and a matched strategy.

$D$=Demand for matched bonds; $S$=Supply of matched bonds.

$1$=Demand for matched bonds by tax-exempt investors.
enough tax-exempt investors to hold all the corporate debt supply, the
equilibrium value of p at the aggregate level is an interior solution
between zero and one. Further, from proposition V we know that the market
clears at relative prices δ such that 1<δ<Q(0,1)/Q(0,2). Thus, with an
upward sloping yield curve, equilibrium entails a premium on the
before-tax return to the matched lending strategy. In addition, for every
individual firm there is an optimal interior solution for the debt
maturity decision (i.e. 0<p<1). At the optimal level of p*, the benefit
from switching one additional dollar from short to two-period bonds and
accelerating interest deductions is equal to the market determined premium
on long-term debt. The interior optimal solution exists because there is
a constant marginal personal tax disadvantage on long-term debt while a
positive probability of loss in tax shields implies that the expected
marginal corporate tax benefit declines as firms switch additional dollars
from roll-over to long-term bonds and accelerate debt tax deductions. The
converse argument applies to a downward sloping yield curve. In this case
the market determines a premium on the before-tax return to the roll-over
lending strategy.

The result that there is an optimal interior maturity structure for
corporations, is an extension of DeAngelo and Masulis result on optimal
interior leverage decisions. In DeAngelo and Masulis framework, interior
leverage is optimal because there is a trade-off between the declining
corporate tax subsidy from debt financing and the constant personal tax
advantage to equity financing. In our model, leverage is kept constant,
and the optimal interior debt maturity decision is a consequence of the trade-off between the declining corporate tax subsidy from selling debt with a maturity strategy that accelerates interest deductions and the constant personal tax advantage of selling debt with a maturity strategy that slows down taxable interest payments.

2.3.5. Idiosyncratic Debt Maturity Structures

We know that for a non-flat term structure of taxable bonds there is an interior optimal solution for the debt maturity decision at the individual firm level. An obvious question to raise is whether it is possible to characterize borrowing strategies cross-sectionally. The next proposition extends DeAngelo and Masulis result on the effect of tax shields substitutes on leverage, to the effect on debt maturity. Specifically, it relates the intertemporal structure of non-debt tax shields to optimal maturity strategies.

Proposition VII

Let \( L_2 = L_{1g} \). Then for an upward sloping yield curve (i.e. \( Q(0,2)<Q(0,1) \)) \( dp^*/dg>0 \), while for a downward sloping yield curve (i.e. \( Q(0,2)>Q(0,1) \)) \( dp^*/dg<0 \).

Proof in appendix B.

The intuition is straightforward: firms seek to utilize all tax shields; to achieve this goal firms match (intertemporally) large debt tax
shields with small non-debt tax shields and vise-versa. When period \( t=2 \) non-debt tax shields increase compared to period \( t=1 \) non-debt tax shields, firms shift debt tax shields from period \( t=2 \) to period \( t=1 \). In other words, firms respond to an increase in the rate of growth of non-debt tax shields by slowing down the rate of growth of debt tax shields (i.e. by accelerating interest tax-deductions). With a upward sloping term structure firms accelerate interest payments by selling two-period bonds while with a downward sloping term structure firms accelerate interest payments by rolling-over short-term bonds. Proposition VII implies that for an upward sloping term structure of taxable bonds, there is a positive cross-sectional association between the expected rate of growth in non-debt tax shields and the weight of long-term debt in the firm's total debt portfolio. For a downward sloping term structure it implies a negative cross-sectional association.

The following two propositions relate idiosyncratic earning streams to optimal maturity strategies.

**Proposition VIII**

Assume \( Q_1=Q_2=Q \), and let \( F_2(a)=F_1(a)+S(a,\mu) \) with \( S(0,\mu)=0, S(Q,\mu)=0 \), and \( dF_2/da+dS/da\geq0 \). Further, let \( ds/d\mu\geq0 \) such that an increase in \( \mu \) makes the random variable \( X_2 \) stochastically larger. Then for an upward sloping yield curve (i.e. \( Q(0,2)^{\uparrow}<Q(0,1) \)) \( dp^*/d\mu<0 \), while for a downward sloping yield curve (i.e. \( Q(0,2)^{\downarrow}>Q(0,1) \)) \( dp^*/d\mu>0 \).

Proof in appendix B.
Efficient management of debt tax shields dictates that firms structure the debt maturity so that large interest deductions occur in periods when high operating income is expected. Firms that expect operating income to grow (i.e. for large values of $\mu$) make a more efficient use of tax shields by slowing down the tax deductible (imputed) interest payments. In contrast, firms that expect a stable or declining stream of operating income maximize the tax subsidy from debt financing by accelerating the tax deductible (imputed) interest payments.

Proposition VIII states that the structure of the firm's earnings stream is related to debt maturity. More specifically, it asserts that for an upward (downward) sloping yield curve there should be a negative (positive) cross-sectional association between the expected rate of growth in earnings and the weight of long-term debt in the firm's total debt portfolio. A result in a similar vein is presented by Barnea, Haugen and Talmor. These authors show that, in a multiperiod framework in which firms choose periodically between equity and short-term debt financing, there exists a cross-sectional relationship between an individual firm's (operating) earnings-value ratio and leverage. When most of the value of the firm is accounted by current earnings (as compared to future earnings), firms tolerate more current interest payments. With the maturity structure of debt restricted to short-term debt, firms increase current interest payments by increasing leverage. In contrast, our model holds leverage constant and hence, an increase in current interest payments is achieved by readjusting the debt portfolio towards maturities
that accelerate interest payments.

Proposition IX

Let \( \beta \) be a mean preserving spread operator on random variable \( X_2 \); that is, if \( \beta' > \beta \) then \( F_2(a, \beta') = F_2(a, \beta) + S(a) \) with \( \int_{-\infty}^{\infty} S(u) \, du = 0 \), and \( \int_{-\infty}^{\infty} S(u) \, du = 0 \). Further assume that the operator \( \beta \) preserves the symmetry of the density \( f_2(\cdot) \). Then, depending on the slope of the yield curve for taxable bonds, the following holds:

(i) Flat yield curve; \( dp^*/d\beta = 0 \).

(ii) Upward sloping yield curve; \( dp^*/d\beta > 0 \) if \( s_2 < E(X_2) \), and \( dp^*/d\beta < 0 \) if \( s_2 > E(X_2) \).

(iii) Downward sloping yield curve; \( dp^*/d\beta > 0 \) if \( s_2 > E(X_2) \), and \( dp^*/d\beta < 0 \) if \( s_2 < E(X_2) \).

Proof in appendix B.

To maximize the present value of all tax shields firms structure the time profile of interest payments in such a way as to reduce the probability of losing tax shields. Suppose that a firm expects \( t=2 \) operating income in excess of tax shields (\( E(X_2) > s_2 \)). If \( t=2 \) operating income becomes riskier, firms reoptimize the borrowing strategy by shifting interest deductions from \( t=2 \) to \( t=1 \). With an upward (downward) sloping yield curve firms accelerate interest deductions by switching to long-term (short-term) debt. Suppose now, that the firm expects at date \( t=2 \) operating income to be less than tax shields (\( E(X_2) < s_2 \)). In this case,
an increase in the riskiness of the t=2 cash-flows from operations increases the probability that tax shields will be utilized. Hence, firms reoptimize by shifting interest deductions from t=1 to t=2. With a upward (downward) sloping yield curve this implies readjusting the debt portfolio towards short-term (long-term) debt.

Proposition IX predicts a systematic relationship between operating risk and the weight of long-term debt in the firm’s debt portfolio. The sign of the relationship depends on the shape of the yield curve for corporate debt, and on whether firms are expected to generate enough operating earnings to utilize tax shields. Barnea, Haugen and Talmor find a similar result for the relationship between leverage and operating risk. They argue that leverage and operating risk are related, with a sign that depends on the magnitude of the tax-induced differential return between corporate debt and equity. When the differential is low, firms are highly levered and consequently, accumulate tax shields in excess of expected operating income. An increase in operating risk increases the odds of ex-post utilization of tax shields, which in turn makes optimal for firms to increase leverage. With the differential large, leverage is low and firms expect operating income to exceed tax shields. In this case, an increase in operating risk increases the odds of an ex-post loss of tax shields and therefore, leads firms to reduce leverage.
2.3.6. Stochastic Interest Rates

We focus now on the implications of relaxing the assumption of deterministic interest rates. The purpose of this section is to confirm earlier results, and to obtain insights on how uncertainty about interest rates affects maturity strategies.

Rather than assuming the yield curves for taxable and tax-exempt bonds to be exogenously given, we now take as exogenous the current spot prices of one-period bonds (taxable and tax-exempt), plus the distributions of all future spot prices of one-period bonds (taxable and tax exempt). With exogenous yield curves, future spot bond prices adjust until borrowing and lending strategies are mutually consistent. This is a reasonable way to proceed if interest rates are deterministic. However, with stochastic interest rates it is more convenient to assume as exogenous the distributions of all future spot one-period bond prices (and the current spot), and let the yield curves endogenously adjust. Again, we start by examining how an individual who holds taxable bonds up to date $t=2$, decides today ($t=0$) between a matched versus a roll-over lending strategy. The relative price between the two lending strategies is redefined as

$$\delta = \frac{Q(0,1)EQ(1,2)}{Q(0,2)}$$  \hspace{1cm} (24)$$

where $E$ represents the expectations operator.
Proposition X

If individual $i$ has a positive constant marginal personal tax rate, $\tau_i$, then in order for he/she to be indifferent between the matched and the roll-over lending strategies the following is required:

i. $\delta < 1$ if $[1 - EP(1,2)][EQ(1,2) - Q(0,1)][EQ(1,2)] - Cov[P(1,2), Q(1,2)] > 0$

ii. $\delta > 1$ if $[1 - EP(1,2)][EQ(1,2) - Q(0,1)][EQ(1,2)] - Cov[P(1,2), Q(1,2)] < 0$

iii. $\delta = 1$ if $[1 - EP(1,2)][EQ(1,2) - Q(0,1)][EQ(1,2)] - Cov[P(1,2), Q(1,2)] = 0$

Proof in appendix B.

The case of stochastic interest rates differs from the case of deterministic interest rates in two respects. First, with a roll-over strategy the taxable (imputed) interest on the bonds at time $t=2$ is uncertain; second, the price at date $t=1$ of one-period after-tax cash-flows, $P(1,2)$, is also uncertain. The larger the covariance between these two uncertain quantities (i.e. the smaller the $\text{Cov}[Q(1,2), P(1,2)]$ is), the larger the present value of the taxes to be paid with a roll-over strategy. Intuitively, this is so because for a low $\text{Cov}[Q(1,2), P(1,2)]$, the taxable interest on roll-over bonds tends to be large (low) when the discounting rate is low (high). Let us call this the "covariance" effect. In addition, expectations about the future spot price, $Q(1,2)$, have an effect on the present value of taxes. This second effect is similar to the one identified in the case of deterministic interest rates. We showed that a downward sloping yield curve entails a personal tax advantage to the matched strategy while an upward sloping curve entails a personal tax
advantage to the roll-over strategy. The "slope of the yield curve" effect translates in the present context into an "expected change in interest rates" effect. If the price of one-period taxable bonds is expected to increase (i.e. \( EQ(1,2) - Q(0,1) > 0 \)), then there is a tax benefit to the matched strategy. On the other hand, if the price of one-period taxable bonds is expected to decrease (i.e. \( EQ(1,2) < Q(0,1) \)), then there is a tax benefit to the roll-over strategy. Expression

\[
[1 - EP(1,2)] [EQ(1,2) - Q(0,1)] [EQ(1,2)] \text{Cov}[P(1,2), Q(1,2)]
\]

adds up the two effects for the matched strategy. If the two effects net out to be zero, then the two strategies have the same tax consequences and the investor is indifferent at relative price \( \delta = 1 \). If the two effects net out to be positive (negative), then there is a tax advantage (disadvantage) to the matched strategy and the investor requires \( \delta < 1 \) (\( \delta > 1 \)) to invest in roll-over (matched) bonds.

The next proposition extends Proposition II to the case of stochastic interest rates. Define \( \delta_i \) as the relative price that makes investor \( i \) indifferent between the matched and the roll-over strategy.

**Proposition XI**

Cross-sectionally, the effect of an increase in the personal marginal tax rate, \( \tau_i \), on \( \delta_i \) is as follows:

1. \( \frac{d\delta_i}{d\tau_i} = 0 \) if

\[
[1 - EP(1,2)] [EQ(1,2) - Q(0,1)] [EQ(1,2)] \text{Cov}[P(1,2), Q(1,2)] = 0
\]

2. \( \frac{d\delta_i}{d\tau_i} > 0 \) if
\[
[1-EP(1,2)][EQ(1,2)-Q(0,1)^d[EQ(1,2)]^d]-Cov[P(1,2),Q(1,2)]<0
\]

\[d\delta_i/dt_i<0 \text{ if} \]

\[
[1-EP(1,2)][EQ(1,2)-Q(0,1)^d[EQ(1,2)]^d]-Cov[P(1,2),Q(1,2)]>0
\]

Proof in appendix B.

We turn now to the supply side; we retain the simplified framework in which corporations borrow for two periods and decide on the allocation of their debt portfolio between one-period and two-period bonds. As before, the face value of the debt maturing at \(t=2\) is normalized to 1.

Table 18 contains the cash-flows to equityholders. At date \(t=1\), to keep the face value of the debt maturing at date \(t=2\) equal to 1, the following is required: if \(Q(1,2)>EQ(1,2)\), corporations pay an additional cash dividend equal to the difference between the value of the newly issued one-period bonds (with face value \(1-p\)) and the face value of maturing bonds; if \(Q(1,2)<EQ(1,2)\), the proceeds from the sale of new one-period bonds aren’t large enough to pay for maturing bonds, and corporations cover the difference with an equity issue.

The value of the firm at date \(t=0\), \(V_0\), is again given by expression (17). The value of the tax shields at dates \(t=1\) and \(t=2\) are redefined as

\[
k_1 = [L_1+p[Q(0,2)^d-Q(0,2)]+(1-p)EQ(1,2)[1-Q(0,1)]]
\]

\[
k_2[Q(1,2)] = [L_2+p[1-Q(0,2)^d]+(1-p)[1-Q(1,2)]]
\]

The value of the equity at date \(t=0\), \(E_0\), is given by
\[ E_0 = P(0,1) \left\{ \int_0^{k_1} X_1 f_1(X_1) dX_1 + \int_{k_1}^{q_1} [X_1 - \tau_c(X_1 - k_1)] f_1(X_1) dX_1 \right\} + \\
+ P(0,1) \left\{ \int_0^1 \int_0^{q_2} P(1,2) X_2 h(.) dX_2 dP(1,2) dQ(1,2) - \\
- \int_0^1 \int_0^{q_2} P(1,2) \tau_c [X_2 - k_2] [Q(1,2)] h(.) dX_2 dP(1,2) dQ(1,2) \right\} + G(.) \] (27)

where \( h(.) \) is the joint density of \( P(1,2), Q(1,2) \) and \( X_2 \), and \( G(.) \) is an expression which is independent of \( p \). Substituting (27) in (17) and differentiating the value of the firm with respect to \( p \) yields the FOC

\[ \frac{dV_0}{dp} = P(0,1) \tau_c [Q(0,2)^3 - Q(0,2) - EQ(1,2)[1 - Q(0,1)]][1 - F_1(k_1)] + \\
+ P(0,1) \tau_c \int_0^1 \int_0^{q_2} P(1,2) [Q(1,2) - Q(0,2)]^3 h(.) dX_2 dP(1,2) dQ(1,2) + \\
+ Q(0,2) - Q(0,1) EQ(1,2) = 0 \] (28)

To verify that Miller irrelevance is also obtained with stochastic interest rates, assume that tax shields are always fully utilized, i.e., assume that

\[ F_1(k_1) = 0 \] (29)

\[ \int_{k_2[Q(1,2)]}^{q_1} h(.) dX_2 = w[P(1,2), Q(1,2)] \] (30)
where \( w[P(1,2), Q(1,2)] \) is the bivariate density of \( P(1,2) \) and \( Q(1,2) \). Use (29) and (30) in the FOC (28), and substitute \( Q(0,2) = (1/5)Q(0,1)EQ(1,2) \)

\[
dV_0/dp = P(0,1)\tau_c\{[EP(1,2)-1][EQ(1,2)-\delta^2Q(0,1)\delta[EQ(1,2)]\delta] + \text{Cov}[P(1,2), Q(1,2)] + [Q(0,1)EQ(1,2)(1-\delta)/5](1-P(0,1)\tau_c)\} = 0 \quad (31)
\]

As expected, parameter relationships that make one strategy attractive to investors, also make the strategy unattractive to corporations. While for investors a large covariance between tax-exempt and taxable one-period bond prices (i.e. high interest payments tend to be associated with high discounting rates and vice-versa) reduces the present value of the taxes to be paid with a roll-over lending strategy, for corporations it reduces the present value of the interest deductions. The same reasoning applies to the expected change in one-period taxable bond prices. Expectations of higher interest rates imply simultaneously a personal tax advantage and a corporate tax disadvantage to the roll-over strategy.

As in the case of deterministic interest rates, firms’ FOC (31) is independent of \( p \) and firm specific parameters. The Miller result of irrelevancy of the debt maturity decision is again obtained with the individual and aggregate supplies of matched bonds being perfectly elastic at the same relative price \( \delta \). However, the value of \( \delta \) that clears demand and supply depends now on the sign of

\[
[EP(1,2)-1][EQ(1,2)-Q(0,1)\delta[EQ(1,2)]\delta] + \text{Cov}[P(1,2), Q(1,2)] \quad (32)
\]
If (32) equal to zero, then $\delta = 1$ makes (31) equal to zero and the demand and aggregate supply totally overlap at the relative price $\delta = 1$. With (32) positive (negative), $\delta = 1$ makes (31) larger (smaller) than zero; since $d^2V_o/dp^2\delta < 0$, it takes $\delta > 1$ ($\delta < 1$) to make (31) equal to zero and therefore, demand and supply clear at a relative price $\delta > 1$ ($\delta < 1$). Figure 8 shows the Miller equilibrium with stochastic interest rates for (32) negative, equal to zero, and positive.

It is interesting to note that if taxable interest rates are expected to remain constant (i.e. $EQ(1,2) = Q(0,1)$), the before-tax 2-period

\begin{figure}
\centering
\begin{subfigure}[b]{0.32\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8a}
\caption{$\Sigma < 0$}
\end{subfigure}
\hspace{0.02\textwidth}
\begin{subfigure}[b]{0.32\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8b}
\caption{$\Sigma = 0$}
\end{subfigure}
\hspace{0.02\textwidth}
\begin{subfigure}[b]{0.32\textwidth}
\centering
\includegraphics[width=\textwidth]{figure8c}
\caption{$\Sigma > 0$}
\end{subfigure}
\caption{Miller equilibrium with stochastic interest rates.}
\end{figure}

$p$ is the face value of debt to mature at date $t=2$ in the form of matched bonds.
$\delta$ is the relative price between $1$ before taxes to be delivered at date $t=2$ with a roll-over and a matched strategy.
$D$ is demand for matched bonds; $S$ is supply of matched bonds.
$1$ is demand for matched bonds by tax-exempt investors.
$\Sigma = [1 - EP(1,2)][EQ(1,2) - Q(0,1)^2(EQ(1,2))^2] - Cov[P(1,2), Q(1,2)].$
return on a matched lending strategy commands a premium over the (expected) before-tax 2-period return on a roll-over strategy. This is so, since the "covariance" effect implies that investors have a natural preference for roll-over bonds. To induce investors to hold matched bonds and forego the beneficial "covariance" effect, firms are required to bid up the return to the matched lending strategy.

In the general case corporations face the risk of being unable to utilize all tax shields. To characterize the aggregate supply curve for the general case, we examine how the individual firm readjust its maturity strategy in response to a change in the relative price δ. The following proposition is equivalent to proposition III.

Proposition XII
If Q(0, 2) ≥ P(0, 2)τ_c^2, f_1(x) ≤ F_1(x),

\[ \int_0^{k_2[Q(1,2)]} h(x_2, Q(1,2), P(1,2))dx_2 \geq h(k_2[Q(1,2)], Q(1,2), P(1,2)) \]

for all P(1, 2) and Q(1, 2) such that 0 ≤ Q(1,2) ≤ P(1,2) ≤ 1, then for every individual firm the optimal p is a continuous non-positive function of the relative price δ.

Proof in appendix B.

In contrast with deterministic interest rates, stochastic interest rates imply that individual supply curves can never be perfectly elastic.

\[ ^{11} \text{We are implicitly assuming that Cov}[P(1,2), Q(1,2)] > 0. \]
An easy way to confirm this statement is by inspection of the firms' FOC in (28); there are no values for Q(0,1), Q(0,2) and EQ(1,2), that satisfy the FOC independently of the decision variable \( p \). It follows that the debt maturity decision is always relevant to individual firms. In other words, individual supply curves for two-period bonds always slope downwards. Figure 9 plots the aggregate demand and supply schedules for two-period bonds as a function of the relative price \( \delta \).

We close the section by evaluating a comparative static that was unfeasible with deterministic interest rates; we study how the stochastic association between the firm's cash-flows, \( X_2 \), and the price of one-period

\[
\delta
\]

\[
\begin{array}{cc}
\delta & D \\
1 & S \\
p^* & p \\
1 & 1
\end{array}
\]

(a) \( \Sigma < 0 \)

\[
\begin{array}{cc}
\delta & D \\
1 & S \\
p^* & p \\
1 & 1
\end{array}
\]

(b) \( \Sigma = 0 \)

\[
\begin{array}{cc}
\delta & D \\
1 & S \\
p^* & p \\
1 & 1
\end{array}
\]

(c) \( \Sigma > 0 \)

Figure 9
Equilibrium with non-debt tax shields, idiosyncratic earnings, and stochastic interest rates.

- \( p \equiv \% \) of the face value of debt to mature at date \( t=2 \) in the form of matched bonds.
- \( \delta \equiv \) relative price between \$1 \) before taxes to be delivered at date \( t=2 \) with a roll-over and a matched strategy.
- \( D \equiv \) Demand for matched bonds; \( S \equiv \) Supply of matched bonds.
- \( \Sigma \equiv [1 - EP(1,2)][EQ(1,2) - Q(0,1)\hat{\epsilon}EQ(1,2)] - Cov[P(1,2), Q(1,2)]. \)
bonds, \( Q(1,2) \), affects the optimal borrowing strategy. To keep the problem tractable we make the simplifying assumptions:

- The price of one-period taxable and tax-exempt bonds are perfectly positively correlated, i.e., \( P(1,2) = qQ(1,2) \), with \( q \geq 1 \) and \( 0 \leq P(1,2) \leq 1 \).
- The marginal densities of \( X_2 \), \( f_x(.) \), and \( Q(1,2) \), \( f_q(.) \), are symmetric (about \( \mu_x \) and \( \mu_q \)) with \( 0 \leq x_2 \leq q_x \) and \( q_- \leq Q(1,2) \leq q_+ \).

Under these assumptions FOC (28) is rewritten

\[
\frac{dV_q}{dp} = P(0,1)\tau_c \{ Q(0,2) - Q(0,2) - EQ(1,2)[1 - Q(0,1)] \} [1 - F_1(k_1)] + \\
P(0,1)\tau_c \int_{q_-}^{q_+} \int_{k_2[Q(1,2)]} Q(1,2) [Q(1,2) - Q(0,2)] z(.) dx_2 dq(1,2) + \\
Q(0,2) - Q(0,1)EQ(1,2) = 0
\]

(33)

where \( z(.) \) is the bivariate density of \( X_2 \) and \( Q(1,2) \).

We want to conceptualize an experiment that enhances a positive stochastic association between the two random variables, \( X_2 \) and \( Q(1,2) \), and at the same time preserves the two marginal distributions. One possible way to proceed is to parametrize the joint density \( z(.) \) as a bivariate normal\(^{12}\) and differentiate the FOC (33) with respect to the covariance

\(^{12}\)Assuming a bivariate lognormal makes the problem intractable; this is so because lower limit of integration of \( X_2, k_2[Q(1,2)] \), is a linear function of \( Q(1,2) \). For a bivariate lognormal to be a feasible approach requires \( k_2[Q(1,2)] \) to be of the form \( k_2[Q(1,2)] = qQ(1,2)^\theta \).
parameter. This approach presents two problems. First, it forces bond prices to take values that make little economic sense (i.e. negative values, and values greater than one); second, it restricts the inferences to a particular parametric family. We choose instead a more general approach that builds on the work of Rothchild/Stiglitz(1970).

Consider the following transformation on the bivariate density \( z(.) \):

Define \( a, b, a', b', c, d, c', d' \) and \( K \), such that \( r_{q-} < b < a < \mu_q < b' < a' < r_{q+} \), \( 0 < d < c < \mu_x < d' < c' < \Omega_2 \), \( a' - b' = a - b \), \( c' - d' = c - d \), \( b' - \mu_q = -a' - \mu_{q-} \), \( d' - \mu_x = -c' - \mu_{x-} \), and \( 0 < K \leq (X_2, Q(1,2)) \) for \( b' \leq Q(1,2) \leq a', d \leq X_2 \leq c \) and \( b \leq Q(1,2) \leq a, d' \leq X_2 \leq c' \). Make \( b' - \mu_q \) large so that \( a < Q(0,2) < b' \). Let \( s[X_2, Q(1,2)] \) be a step function defined by

\[
s[X_2, Q(1,2)] = \begin{cases} 
-K \text{ for } b' \leq Q(1,2) \leq a', d \leq X_2 \leq c \\
-K \text{ for } b \leq Q(1,2) \leq a, d' \leq X_2 \leq c' \\
+K \text{ for } b' \leq Q(1,2) \leq a', d' \leq X_2 \leq c' \\
+K \text{ for } b \leq Q(1,2) \leq a, d \leq X_2 \leq c \\
0 \text{ otherwise }
\end{cases}
\]

(34)

It is easy to verify that

\[
\int_{r_{q-}}^{r_{q+}} \int_{0}^{\Omega_2} s[X_2, Q(1,2)] dX_2 dQ(1,2) = 0,
\]

(35)

\[
\int_{r_{q-}}^{r_{q+}} s[X_2, Q(1,2)] dQ(1,2) = 0
\]

(36)

\[
\int_{0}^{\Omega_2} s[X_2, Q(1,2)] dX_2 dQ(1,2) = 0,
\]

(37)

so that if we add \( s[X_2, Q(1,2)] \) to the bivariate density, \( z(.) \), we obtain a function that is itself a bivariate density, and has marginal densities identical to the marginal densities of \( z(.) \) (i.e. \( f_x(.) \), and \( f_q(.) \)). The
effect of adding a function like \( s(.) \) to \( z(.) \), is to shift probability mass from the ranges of (high \( Q(1,2) \)-low \( X_2 \)) and (low \( Q(1,2) \)-high \( X_2 \)) to the ranges of (high \( Q(1,2) \)-high \( X_2 \)) and (low \( Q(1,2) \)-low \( X_2 \)). Thus, the transformed density \( z(.) + s(.) \) features a stronger positive stochastic association between the random variables \( X_2 \) and \( Q(1,2) \). Figure 10 shows a projection of function \( s(.) \) on the \([X_2, Q(1,2)]\) plane. It also shows function \( Q(1,2)[Q(1,2) - Q(0,2)]^2 \) which is inside the double integral in FOC (33), and line \( k_z[Q(1,2)] = L_z + p[1-Q(0,2)] + (1-p)[1-Q(1,2)] \) that defines

\[
Q(1,2)^2 - Q(1,2)Q(0,2)^2
\]

**Figure 10**

Effect of increasing the stochastic association between firm's operating income, \( X_2 \), and the price of one-period bonds, \( Q(1,2) \).
the range of integration. We analyze the effect of a transformation $z'(.) = z(.)+s(.)$ on the optimal maturity strategy, $p$, for the case in which the firm expects to fully utilize all tax shields (i.e. $\mu_r > k_2 [\mu_q]$). Since $\Gamma_{q,t} \leq 1$, $L_2 \geq 0$ and $0 \leq p \leq 1$, it follows that line $k_2 [Q(1,2)]$ satisfies

$$0 \leq k_2 [\Gamma_{q,t}] < k_2 [a'] < k_2 [b'] < k_2 [a] < k_2 [b] < k_2 [\Gamma_{q-}] < q_2$$

(38)

A geometric analysis of Figure 10 reveals that regardless of $Q(0,2)^\dagger$,

$$\int_{q-}^{q+} \int_{k_2 [Q(1,2)]} Q(1,2) [Q(1,2) - Q(0,2)^\dagger] s(.) dX_2 dQ(1,2) \geq 0$$

(39)

which implies that when we add function $s(.)$ to the bivariate density $z(.)$, the value of the maturity structure parameter, $p$, that solves FOC (33) now yields $dV_0/dp \geq 0$. Therefore, it is optimal for the firm not to decrease the proportion of two-period bonds, $p$. Moreover, since (39) is strictly positive if $k_2 [b] > c$, it is optimal to increase $p$ if the shift in probability mass isn't all contained within the range of integration. This result is intuitive; the effect of adding function $s(.)$ to the bivariate density $z(.)$ is to make low (high) realizations of $Q(1,2)$ more likely to be associated with low (high) realizations of $X_2$; this implies that the interest deductions on the roll-over portion of the firm's debt portfolio, tend now to be more negatively associated with operating cash-flows. This increases the risk that the firm will not be able to fully utilize its (costly) interest tax shield. The way to reduce this risk is to lower the weight of roll-over bonds in the firm's debt portfolio.
2.4 The choice between fixed and floating rate bonds

It is interesting to compare the tax consequences of a strategy of
rolling-over short-term bonds to those of a strategy of issuing (holding)
long-term floating rate bonds. If the two strategies produce the same
after-tax cash-flows to equityholders and bondholders, then agents are
indifferent between the two, and the choice between rolling-over short-
term debt and issuing (holding) long-term debt does not differ from the
choice between issuing (holding) long-term floating and fixed rate debt.

With a roll-over strategy firms periodically refinance maturing
short-term bonds with new short-term bonds. The yield on newly issued
bonds reflects the currently prevailing default-free short-term interest
rate plus a default premium that accounts for the current credit
worthiness of the firm. In contrast, interest reset formulas in floating
rate bonds typically account only for changes in the level of the default-
free interest rate; the default-premium (i.e. the spread over the default-
free interest rate) is set at the time the debt is issued and remains
constant until maturity. Tax reasons alone, make this difference between
short-term and floating rate debt relevant to the firm. To illustrate the
point, suppose that improvements (deteriorations) in credit worthiness
tend to be accompanied by higher (lower) current operating cash-flows. In
this case there is a tax advantage to floating rate debt. This is so,
since when interest payments are insensitive to credit risk, operating
income is more positively associated with interest deductions and
therefore, the probability of losing interest tax shields is reduced.
The tax treatment of bankruptcy may as well lead firms to choose one strategy over the other. Brennan and Schwartz demonstrate that if corporate debt tax shields are (at least partially) lost upon bankruptcy, the choice of leverage and maturity strategies involve a trade-off between the tax savings in the non-bankrupt states and the loss in tax shields in the bankrupt states. When firms decide simultaneously about leverage and debt maturity, the optimal policy is to roll-over short-term debt and readjust leverage continuously to the point at which the probability of bankruptcy is zero. In the context of our model in which leverage is kept constant, however, the loss of tax shields in the bankrupt states entails a corporate tax advantage to long-term floating rate debt. A firm that rolls-over short-term debt and keeps the face value of newly issued bonds equal to the face value of maturing bonds, is more likely to default over an extended period of time than a similar firm that sells long-term floating rate debt. To avoid bankruptcy with a roll-over strategy, the value of the firm has to exceed the sum of the principal and one interest payment, every time the firm refinances. On the other hand, with long-term floating rate debt the sequence of hurdles that trigger bankruptcy consists entirely of interest payments.\textsuperscript{13} Furthermore, the periodic readjustment of default premium in a roll-over strategy makes default even more likely; the fact that higher premia are required on newly issued

\textsuperscript{13}Brennan and Schwartz assume that with long-term debt financing, the firm goes bankrupt at an interim interest payment date if the value of the firm is less than the par value of the debt. This assumption is at odds with corporate law; under the law, bankruptcy can only be triggered if the firm defaults on a current obligation.
bonds when the value of the firm is low, makes the sequence of default hurdles more difficult to pass.

All these complications are avoided in the model developed in this study. Since we assumed corporate debt to be default-free, (1) there are no default premia implications, and (2) the tax consequences of bankruptcy don't matter. Therefore, all the results presented earlier with respect to the effect of taxation on the choice between short-term and long-term debt apply equally to the choice between long-term fixed and floating rate debt. With no default risk, it is always possible to find a schedule of interest reset dates and a coupon reset formula, that makes the after-tax cash-flows associated with floating rate debt financing identical to those associated with the rolling-over short-term debt.

3. Conclusion

This study has examined the tax implications of debt maturity policies. We find that multiple configurations of leverage and debt maturity are consistent with maximization of the tax subsidy from debt financing. The analysis indicates that leverage and debt maturity are substitute financing decisions for managing corporate tax shields. Holding leverage constant, we then investigate how firms manage tax shields with maturity decisions. We show that a Miller type irrelevancy result with respect to maturity strategy is obtained if firms face no risk of losing tax shields. When firms have the ability to fully utilize all tax shields (or to market any unused tax shields), differences in corporate tax
benefits among maturity strategies are exactly offset by personal-tax induced differentials in before-tax required rates of return; consequently, maturity decisions have no effect on firm value and are irrelevant for corporations. The value of the firm, however, is affected by debt maturity structure if there is a possibility of loss of tax shields. In this case, we find that maturity decisions are effective in managing tax shields and that optimal maturity strategies are related to firm characteristics.
Table 17
Cash-flow accruing to equityholders as a function of realizations of operating income $X_t$

<table>
<thead>
<tr>
<th>Realization of $X_t$</th>
<th>Cash-Flow to equityholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=1$</td>
<td>$X_1$</td>
</tr>
<tr>
<td>$0 \leq x_1 \leq L_1 + p[Q(0,2)\frac{1}{2}-Q(0,2)] + (1-p)Q(1,2)[1-Q(0,1)]$</td>
<td>$X_1 - \tau_c(X_1 - L_1 - p[Q(0,2)\frac{1}{2}-Q(0,2)] - (1-p)Q(1,2)[1-Q(0,1)])$</td>
</tr>
<tr>
<td>$t=2$</td>
<td>$X_2$</td>
</tr>
<tr>
<td>$0 \leq x_2 \leq L_2 + p[1-Q(0,2)\frac{1}{2}] + (1-p)[1-Q(1,2)]$</td>
<td>$X_2 - \tau_c(X_2 - L_2 - p[1-Q(0,2)\frac{1}{2}] - (1-p)[1-Q(1,2)])$</td>
</tr>
<tr>
<td>$t \geq 3$</td>
<td>$X_t$</td>
</tr>
<tr>
<td>$0 \leq x_t \leq L_t$</td>
<td>$X_t$</td>
</tr>
<tr>
<td>$x_t \geq L_t$</td>
<td>$X_t - \tau_c(x_t - L_t)$</td>
</tr>
</tbody>
</table>
Table 18
Cash-flow accruing to equityholders as a function of realizations of operating income $X_t$, and the price of one period bonds $Q(t,t+1)$.

<table>
<thead>
<tr>
<th>Realization of $X_t$, $Q(t,t+1)$</th>
<th>Cash-Flow to equityholders</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t=1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_1 \leq L_1 + p[Q(0,2)\bar{d}-Q(0,2)] + (1-p)EQ(1,2)[1-Q(0,1)]$</td>
</tr>
<tr>
<td></td>
<td>$X_1 - \tau_c(X_1 - L_1 - p[Q(0,2)\bar{d}-Q(0,2)]$</td>
</tr>
<tr>
<td></td>
<td>$-(1-p)EQ(1,2)[1-Q(0,1)] + (1-p)[Q(1,2) - EQ(1,2)]$</td>
</tr>
<tr>
<td></td>
<td>$X_1 \leq L_1 + p[Q(0,2)\bar{d}-Q(0,2)] + (1-p)EQ(1,2)[1-Q(0,1)]$</td>
</tr>
<tr>
<td></td>
<td>$X_1 - \tau_c(X_1 - L_1 - p[Q(0,2)\bar{d}-Q(0,2)]$</td>
</tr>
<tr>
<td></td>
<td>$-(1-p)EQ(1,2)[1-Q(0,1)] + (1-p)[Q(1,2) - EQ(1,2)]$</td>
</tr>
<tr>
<td></td>
<td>$X_1 \geq L_1 + p[Q(0,2)\bar{d}-Q(0,2)] + (1-p)EQ(1,2)[1-Q(0,1)]$</td>
</tr>
<tr>
<td></td>
<td>$X_1 - \tau_c(X_1 - L_1 - p[Q(0,2)\bar{d}-Q(0,2)]$</td>
</tr>
<tr>
<td></td>
<td>$-(1-p)EQ(1,2)[1-Q(0,1)] + (1-p)[Q(1,2) - EQ(1,2)]$</td>
</tr>
<tr>
<td>$t=2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_2 \leq L_2 + p[1-Q(0,2)\bar{d}] + (1-p)[1-Q(1,2)]$</td>
</tr>
<tr>
<td></td>
<td>$X_2 - \tau_c(X_2 - L_2 - p[1-Q(0,2)\bar{d}]$</td>
</tr>
<tr>
<td></td>
<td>$-(1-p)[1-Q(1,2)]$</td>
</tr>
<tr>
<td>$t=\infty$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$X_t \leq L_t$</td>
</tr>
<tr>
<td></td>
<td>$X_t - \tau_c(X_t - L_t)$</td>
</tr>
</tbody>
</table>
Appendix A.

In this appendix we develop a set of sufficient conditions on the model developed in the first part of the dissertation, that guarantee that (i) at date t=1 "Good" and "Bad" quality firms do not default on the coupon payments of both floating and fixed rate debt. (ii) at date t=2 "Good" and "Bad" type firms default both on fixed and floating rate debt when the value of the firm $V_2$ is either $V_0(1-a)^2/Q(0,2)(1+b)$ or $V_0(1-a)^2/Q(0,2)(1-b)$.

Before proceeding we introduce some notation. Let $c(g)$ and $c(b)$ denote the fixed rate coupons under full information for the "Good" and "Bad" type of firms. Let $c(pool)$ be the fixed rate coupon under a pooling equilibrium. Similarly, let $\pi(g)$ and $\pi(b)$ represent the risk premiums on floating rate debt for the "Good" and "Bad" quality firms under full information, and let $\pi(pool)$ represent the risk premium on floating rate debt under a pooling equilibrium.¹

Let us start with (ii). Assume that

$$a > b$$

Then, regardless of the firm's quality,

¹Note that $c(g) < c(pool) < c(b)$ and $\pi(g) < \pi(pool) < \pi(b)$.

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\[ V_0(1-a^2)/Q(0,2)(1+b) > V_0(1-a)^2/Q(0,2)(1-b). \quad (2) \]

For a "Good" type firm with fixed rate debt outstanding (ii) holds iff

\[ V_0(g)(1-a^2)/Q(0,2)(1+b) > D+c(pool) \quad \text{and} \quad V_0(g)(1-a)^2/Q(0,2)(1-b) < D+c(g) \quad (3) \]

For a "Bad" type firm with fixed rate debt outstanding (ii) holds iff

\[ V_0(b)(1-a^2)/Q(0,2)(1+b) > D+c(b) \quad \text{and} \quad V_0(b)(1-a)^2/Q(0,2)(1-b) < D+c(pool) \quad (4) \]

(3) and (4) collapse to

\[ V_0(b)(1-a^2)/Q(0,2)(1+b) > D+c(b) \quad \text{and} \quad V_0(g)(1-a)^2/Q(0,2)(1-b) < D+c(g) \quad (5) \]

For a "Good" type firm with floating rate debt outstanding (ii) holds iff

\[ \frac{V_0(g)(1-a^2)}{Q(0,2)(1+b)} > D\left\{\frac{Q(0,1)}{Q(0,2)(1-b)} + \pi(pool)\right\} \quad \text{and} \quad \frac{V_0(g)(1-a)^2}{Q(0,2)(1-b)} < D\left\{\frac{Q(0,1)}{Q(0,2)(1+b)} + \pi(g)\right\} \quad (6) \]

For a "Bad" type firm with floating rate debt outstanding (ii) holds iff

\[ \frac{V_0(b)(1-a^2)}{Q(0,2)(1+b)} > D\left\{\frac{Q(0,1)}{Q(0,2)(1-b)} + \pi(b)\right\} \quad \text{and} \quad \frac{V_0(b)(1-a)^2}{Q(0,2)(1-b)} < D\left\{\frac{Q(0,1)}{Q(0,2)(1+b)} + \pi(b)\right\} \]
\[ \frac{V_0(b)(1-a)^2}{Q(0,2)(1-b)} < \frac{Q(0,1)}{Q(0,2)(1+b)} \] (7)

Sufficient conditions for (6) and (7) are

\[ \frac{V_0(b)(1-a^2)}{Q(0,2)(1+b)} > \frac{Q(0,1)}{Q(0,2)(1-b)} \]
and

\[ \frac{V_0(g)(1-a^2)}{Q(0,2)(1-b)} < \frac{Q(0,1)}{Q(0,2)(1+b)} \] (8)

Because \( D[\frac{Q(0,1)}{Q(0,2)(1-b)}] > D(1+c(b)) \) and \( D[\frac{Q(0,1)}{Q(0,2)(1+b)}] < D(1+c(g)) \),

(11) holds as long as (8) is satisfied.

We turn now to (i). At date t=1 firms default if the value of the equity is less than the coupon due at that time. The assumption that \( a > b \) guarantees that, for both "Good" and "Bad" type firms and for both fixed and floating rate borrowing strategies, the value of the equity at date t=1 when the value of the firm is "up" exceeds the value of the equity when the value of the firm is "down" regardless of the realization of the price of one period default-free bonds. Then, for "Good" type firms with fixed rate debt outstanding (i) holds iff

\[ S_i^P[V_i = V_0(g)(1-a)/Q(0,1), Q(1,2) = Q(0,2)(1+b)/Q(0,1), c = c(pool)] > c(pool) \] (9)
where $S^F_t$ denotes the value of the stock at date $t=1$ under fixed rate financing. On the other hand, for a "Bad" type firm borrowing at a fixed rate (i) holds iff

$$S^F_t[V_1 = V_0(b)(1-a)/Q(0,1), Q(1,2) = Q(0,2)(1+b)/Q(0,1), c = c(b)] > c(b) \quad (10)$$

Because $c(b) > c(pool)$ condition (9) is fulfilled provided (10) is satisfied. For "Good" quality firms with floating rate debt outstanding (i) holds iff

$$\frac{V_0(g)(1-a)}{Q(0,1)}, \frac{Q(0,2)(1+b)}{Q(0,1)}, [\pi = \pi(pool)] > D\left[\frac{1-Q(0,1)}{Q(0,1)} + \pi(pool)\right] \quad (11)$$

where $S^A_t$ represents the value of the equity at date $t=1$ under floating (A=adjustable) rate financing. For "Bad" type firms borrowing funds at a floating rate (i) holds iff

$$\frac{V_0(b)(1-a)}{Q(0,1)}, \frac{Q(0,2)(1+b)}{Q(0,1)}, [\pi = \pi(b)] > 0\left[\frac{1-Q(0,1)}{Q(0,1)} + \pi(b)\right] \quad (12)$$

If (12) holds then (11) must also hold. Thus, only (12) is required. In summary, sufficient conditions for (i) and (ii) to hold are given by (1), (8), (10), and (12). Expressions for $c(g)$, $c(b)$, $\pi(g)$, and $\pi(b)$ are given in equations (11) and (13) in section 2.2. of the first part of the dissertation.²

²$c(g)$ is obtained by substituting $\theta = 1$ in equation (11) and $c(b)$ by substituting $\theta = 0$, whereas $\pi(g)$ is obtained by substituting $\theta = 1$ in equation (13) and $\pi(b)$ by substituting $\theta = 0$. 
Appendix B

This appendix contains proofs for the propositions presented in the second part of the dissertation.

Proof of Proposition I

Let \( \text{NPV}_i(M) \) be investor’s i net present value at date \( t=0 \) of \$1 before taxes to be delivered at date \( t=2 \) with a matched lending strategy (i.e. purchasing \( Q(0,2) \)). Let \( \text{NPV}_i(R) \) represent investor’s i net present value at date \( t=0 \) of \$1 before taxes to be delivered at date \( t=2 \) with a roll-over lending strategy (i.e. by purchasing \( Q(0,1)Q(1,2) \)). Expressions for \( \text{NPV}_i(M) \) and \( \text{NPV}_i(R) \) are:

\[
\text{NPV}_i(M) = P(0,2) - P(0,1)\tau_1 [Q(0,2)^{\delta} - Q(0,2)] - P(0,2)\tau_1 [1 - Q(0,2)^{\delta}] - Q(0,2) \tag{1}
\]

\[
\text{NPV}_i(R) = P(0,2) - P(0,1)\tau_1 Q(1,2) [1 - Q(0,1)] - P(0,2)\tau_1 [1 - Q(1,2)] - Q(0,1)Q(1,2) \tag{2}
\]

Compute \( \text{NPV}_i(M) - \text{NPV}_i(R) \) and substitute \( Q(1,2) = \delta Q(0,2)/Q(0,1) \)

\[
\text{NPV}_i(M) - \text{NPV}_i(R) =
\]

\[
= P(0,1)[Q(0,2)^{\delta}/Q(0,1)]\tau_1 [\delta Q(0,2)^{\delta}(1 - Q(0,1)) - Q(0,1)(1 - Q(0,2)^{\delta})] +
\]

\[
+ P(0,2)[Q(0,2)^{\delta}/Q(0,1)]\tau_1 [Q(0,1) - \delta Q(0,2)^{\delta}] - Q(0,2)(1 - \delta) \tag{3}
\]

\( \text{NPV}_i(M) - \text{NPV}_i(R) \) is monotonically increasing in \( \delta \), i.e.,

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\[
d[\text{NPV}_i(M)-\text{NPV}_i(R)]/d\delta = \\
[Q(0,2)/Q(0,1)]\tau_i[P(0,1)-P(0,2)]-Q(0,2)[P(0,1)\tau_i-1] > 0 
\]

(4)

Substituting \(\delta=1\) in (3) yields

\[
\text{NPV}_i(M)-\text{NPV}_i(R)\big|_{\delta=1} = \tau_i[Q(0,2)^{\frac{3}{2}}/Q(0,1)][Q(0,2)^{\frac{1}{2}}-Q(0,1)][P(0,1)-P(0,2)] 
\]

(5)

Investor \(i\) chooses the lending strategy with the largest net present value. If the term structure of taxable bonds is flat (i.e. \(Q(0,2)^{\frac{3}{2}}=Q(0,1)\)) then (5) is equal to zero and \(\delta=1\) makes investor \(i\) indifferent between the two lending strategies. If the term structure of taxable bonds is upward sloping (i.e. \(Q(0,2)^{\frac{3}{2}}<Q(0,1)\)) then (5) is negative and \(\delta>1\) is required to make investor \(i\) indifferent between the two strategies. Finally, if the term structure of taxable bonds is downward sloping (i.e. \(Q(0,2)^{\frac{3}{2}}>Q(0,1)\)) (5) is positive and it takes \(\delta<1\) to make investor \(i\) indifferent between the two investments. Q.E.D.

Proof of Proposition II

Define \(A_i\) as

\[
A_i = \text{NPV}_i(M)-\text{NPV}_i(R) = \\
=P(0,1)[Q(0,2)^{\frac{3}{2}}/Q(0,1)]\tau_i[\delta Q(0,2)^{\frac{3}{2}}(1-Q(0,1))-Q(0,1)(1-Q(0,2)^{\frac{1}{2}})] + \\
+P(0,2)[Q(0,2)^{\frac{3}{2}}/Q(0,1)]\tau_i[Q(0,1)-\delta Q(0,2)^{\frac{1}{2}}]-Q(0,2)(1-\delta) 
\]

(6)

A value of \(A_i\) equal to zero guarantees that investor \(i\) is indifferent between the two lending strategies. Set \(A_i=0\) and solve for \(\delta\)
\[ \delta = \frac{Q(0,1) \tau_i[P(0,1) - P(0,2)] + Q(0,2) [1 - \tau_i P(0,1)]}{Q(0,2) [P(0,1) - P(0,2)] + Q(0,1) [1 - \tau_i P(0,1)]} \]  

(7)

If the term structure is upward sloping (i.e. \(Q(0,2) < Q(0,1)\)) \(d\delta/d\tau_i > 0\), whereas if the term structure is downward sloping \(d\delta/d\tau_i < 0\). Q.E.D.

Proof of Proposition III

Upon substitution of \(Q(1,2) = \delta Q(0,2)/Q(0,1)\) in the FOC (21) in the second part of the dissertation we obtain

\[
dV_o/dp = P(0,1) \tau_c [Q(0,2) - Q(0,1)] \{ \delta Q(0,2)/(Q(0,1) - 1) + Q(0,1)(1-Q(0,2)) \} + \{1-F_1[L_1 + p(Q(0,2) - Q(0,1)) + (1-p)\delta(Q(0,2)/Q(0,1)) + Q(0,1)) \} + P(0,2) \tau_c [Q(0,2)/Q(0,1)] (\delta Q(0,2) - Q(0,1)) \} + (1-p)(1-\delta Q(0,2)/Q(0,1)) + Q(0,2)(1-\delta) = 0 \]  

(8)

The continuity of the function \(p^* = p^*(\delta)\) on \(\delta\) is established by noting that the implicit function \(dV_o/dp(p^*, \delta) = 0\) is continuous in both \(p^*\) and \(\delta\). To compute \(dp^*/d\delta\) use the implicit derivative rule. Accordingly,

\[
dp^*/d\delta = [d^2 V_o/dpd\delta]/[d^2 V_o/dp^2] \]  

(9)

where all derivatives are evaluated at the point \(p = p^*\). Since \(p^*\) maximizes \(V_o\), \(d^2 V_o/dp^2 \geq 0\) and the sign of \(dp^*/d\delta\) is the same as the sign of \(d^2 V_o/dpd\delta\). Compute \(d^2 V_o/dpd\delta\). Differentiating the FOC (8) with respect to \(\delta\) and collecting terms gives
\[ \frac{d^2 V_0}{dpd\delta} = P(0,1)\tau_c [Q(0,1) - 1] \]
\[ + \{1 - F_1(s_1) + f_1(s_1)(1-p)(Q(0,2) - Q(0,1) - Q(1,2)(1-Q(0,1))) + \]
\[ + P(0,2)\tau_c \{1 - F_2(s_2) + f_2(s_2)(1-p)(Q(1,2) - Q(0,2) - Q(0,1))\} - Q(0,1) \] (10)

Inspection of (10) reveals that regardless of the value of \( p^* \), \( \frac{dp^*}{d\delta} \) is non-positive if \( Q(0,1) \geq P(0,2)\tau_c \) and \( f_c(x) \leq F_c(x) \). Q.E.D.

Proof of Proposition IV
Substituting \( Q(0,2) - Q(0,1) \) in FOC (8) yields
\[ \frac{dV_0}{dp} = P(0,1)\tau_c Q(0,1)(1-\delta)(1-Q(0,1))[1-F_1(s_1)] + \]
\[ + P(0,2)\tau_c Q(0,1)(1-\delta)(1-F_2(s_2)) + Q(0,1)^2(1-\delta) \] (11)

For \( \delta = 1 \), (11) is equal to zero regardless of the value of \( p \). For \( \delta > 1 \), (11) is negative if \( Q(0,1) \geq P(0,2)\tau_c \) and therefore, \( p^* = 0 \) for every individual firm in the economy. Finally, for \( \delta < 1 \), (11) is positive if \( Q(0,1) \geq P(0,2)\tau_c \) and thus, \( p^* = 1 \) for every individual firm in the economy. Q.E.D.

Proof of Proposition V
Using \( Q(0,2) - Q(0,1), \delta < 1 \), and \( Q(0,1) \geq P(0,2)\tau_c \) in the FOC (8) implies \( \frac{dV_0}{dp} > 0 \) and therefore, \( p^* = 1 \) for every individual firm. Using \( Q(0,2) - Q(0,1), \delta = Q(0,1)/Q(0,2) \) and \( Q(0,1) \geq P(0,2)\tau_c \) in FOC (8) yields \( \frac{dV_0}{dp} < 0 \) and thus, \( p^* = 0 \) for every individual firm. Finally, because \( \frac{d^2 V_0}{dpd\delta} \leq 0 \) (see proposition III), \( \frac{dV_0}{dp} < 0 \) and \( p^* = 0 \) for \( \delta = Q(0,1)/Q(0,2) \). Q.E.D.
Proof of Proposition VII

To examine the effect of \( g \) on the firm's optimal debt maturity use again the implicit derivative rule on the FOC (8)

\[
dp*/dg = -\left[\frac{d^2V_o/dp^2}{(dp^2)}\right]/\left[\frac{d^2V_o/dp^2}{(dp^2)}\right]
\]

where all derivatives are evaluated at the point \( p=p^* \). Since \( p^* \) maximizes \( V_o \), \( d^2V_o/dp^2 \leq 0 \) and the sign of \( dp^*/dg \) is the same as the sign of \( d^2V_o/dp^2 \).

Compute \( d^2V_o/dp^2 \). Substituting \( L_2=L_1g \) in FOC (8), differentiating with respect to \( g \), and collecting terms gives

\[
\left[d^2V_o/dp^2\right] = P(0,2)\left\{\tau_c\left[Q(0,2)^{\frac{1}{2}}/Q(0,1)\right]\left[5Q(0,2)^{\frac{1}{2}}-Q(0,1)\right]\right\}
\]

\[
\{f_2[L_2+p(1-Q(0,2)^{\frac{1}{2}})+(1-p)(1-5Q(0,2)/Q(0,1)))]\}
\]

For an upward sloping yield curve the equilibrium relative price \( \delta \) is such that \( 1 \leq \delta \leq Q(0,1)/Q(0,2)^{\frac{1}{2}} \) and therefore, (13) is positive. For a downward sloping yield curve the equilibrium relative price is such that \( 1 \geq \delta \geq Q(0,1)/Q(0,2)^{\frac{1}{2}} \) and therefore, (13) is negative. Q.E.D.

Proof of Proposition VIII

Rewrite the FOC (8) as

\[
dV_o/dp = P(0,1)\tau_c\left[Q(0,2)^{\frac{1}{2}}/Q(0,1)\right]
\]

\[
\left[5Q(0,2)^{\frac{1}{2}}(Q(0,1)-1)+Q(0,1)(1-Q(0,2)^{\frac{1}{2}})\right][1-F_1(s_1)]+
\]

\[
+P(0,2)\tau_c\left[Q(0,2)^{\frac{1}{2}}/Q(0,1)\right]\left[5Q(0,2)^{\frac{1}{2}}-Q(0,1)\right][1-F_2(s_2)]+Q(0,2)(1-\delta)=0
\]
Substitute $F_2(a)=F_1(a)+S(a, \mu)$ in (14)

\[
\begin{align*}
\frac{dV_0}{dp} &= P(0,1)\tau_c [Q(0,2)\hat{Q}(Q(0,1)) \\nonumber \\
&[\delta Q(0,2)\hat{Q}(Q(0,1)-1)+Q(0,1)(1-Q(0,2)\hat{Q})][1-F_1(s_1)]+ \\
&+P(0,2)\tau_c [Q(0,2)\hat{Q}(Q(0,1))(\delta Q(0,2)\hat{Q}-Q(0,1)) \\
&[1-F_1(s_2)-S(s_2, \mu)]+Q(0,2)(1-\delta)=0
\end{align*}
\] (15)

To compute $dp^*/d\mu$ use the implicit derivative rule the FOC (15),

\[
\frac{dp^*}{d\mu} = -\frac{[d^2V_0/dpd\mu]/[d^2V_0/dp^2]}{d^2V_0/dp^2} \tag{16}
\]

where all derivatives are evaluated at the point $p=p^*$. Since $p^*$ maximizes $V_0$, $d^2V_0/dp^2 \leq 0$ and the sign of $dp^*/d\mu$ is the same as the sign of $d^2V_0/dpd\mu$. Differentiating (15) with respect to $\mu$ yields

\[
\frac{d^2V_0}{dpd\mu} = -P(0,2)\tau_c [Q(0,2)\hat{Q}(Q(0,1))(\delta Q(0,2)\hat{Q}-Q(0,1))(dS/d\mu) \tag{17}
\]

With an upward sloping yield curve the equilibrium relative price $\delta$ is such that $1 \leq \delta \leq Q(0,1)/Q(0,2)$ and therefore, (17) is negative. With a downward sloping yield curve the equilibrium relative price is such that $1 \geq \delta \geq Q(0,1)/Q(0,2)$ and therefore, (17) is positive. Q.E.D.
Proof of Proposition IX

Substituting $F_2(s_2)=F_2(s_2,\beta)$ in (14) gives the expression

$$dV_0/dp = P(0,1)\tau_c [Q(0,2)^{1/2}/Q(0,1)]$$
$$+ P(0,2)^{1/2}Q(0,1)-1+Q(0,1)(1-Q(0,2)^{1/2})\cdot [1-F_1(s_1)]$$
$$+ P(0,2)^{1/2}Q(0,1)\cdot [Q(0,2)^{1/2}-Q(0,1)]\cdot [1-F_2(s_2,\beta)]+Q(0,2)(1-\delta)=0 \quad (18)$$

To compute $dp^*/d\beta$ use the implicit derivative rule in the FOC (18)

$$dp^*/d\beta = -[d^2V_0/dp^2]/[d^2V_0/dp^2] \quad (19)$$

where all derivatives are evaluated at $p=p^*$. Since the function $V_0$ is at a maximum at $p=p^*$ the sign of $dp^*/d\beta$ is the same as the sign of $d^2V_0/dp^2$. Evaluate $d^2V_0/dp^2$

$$d^2V_0/dp^2 = -P(0,2)^{1/2}Q(0,1)^{1/2}-Q(0,1)S(s_2) \quad (20)$$

From the definition of a mean preserving spread we know that there exists a value $c$ such that $S(a)>0$ if $a<c$, $S(a)<0$ if $a>c$, and $S(c)=0$. Since the spread operator $\beta$ preserves both the mean and the symmetry of the distribution, it follows that the value $c$ must be equal to $E(X_2)$. For a flat yield curve (i.e. $Q(0,2)^{1/2}=Q(0,1)$) the equilibrium relative price, $\delta$, is equal to 1 and (20) collapses to zero regardless of the value of $S(s_2)$; for an upward sloping yield curve (i.e. $Q(0,2)^{1/2}<Q(0,1)$) the equilibrium relative price, $\delta$, is such that $1\leq \delta \leq Q(0,1)/Q(0,2)^{1/2}$ and therefore, the sign of (20) is positive when $s_2\leq E(X_2)$ and negative when $s_2\geq E(X_2)$;
finally, for a downward sloping yield curve (i.e. $Q(0,2) < Q(0,1)$) the equilibrium relative price, $\delta$, is such that $1 > \delta > Q(0,1)/Q(0,2)$ and therefore, the sign of (20) is negative when $s_2 \leq E(X_2)$ and positive when $s_2 \geq E(X_2)$. Q.E.D.

Proof of Proposition X

Investor $i$'s net present value of $\$1$ before taxes at date $t=2$ to be delivered with a matched and a roll-over lending strategies are respectively

$$NPV_i(M) = P(0,1)E\{P(1,2)[1-\tau_i(1-Q(0,2)])\} - P(0,1)\tau_i[Q(0,2)-Q(0,2)] - Q(0,2)$$

$$NPV_i(R) = P(0,1)E\{P(1,2)[1-\tau_i(1-Q(1,2)])\} - P(0,1)\tau_i EQ(1,2)[1-Q(0,1)] - Q(0,1)EQ(1,2)$$

Computing $NPV_i(M) - NPV_i(R)$ and using $Q(0,2) = (1/\delta)Q(0,1)EQ(1,2)$ we get

$$NPV_i(M) - NPV_i(R) =$$

$$= P(0,1)\tau_i [E(1,2) - Q(0,1)\delta^{-1}EQ(1,2)] + EQ(1,2)Q(0,1)[(\delta-1)/\delta][1-P(0,1)\tau_i]$$

$$NPV_i(M) - NPV_i(R)$$ is monotonically increasing in $\delta$, i.e.,

$$d(NPV_i(M) - NPV_i(R))/d\delta = [d(NPV_i(M) - NPV_i(R))/d\delta^{-1}][d\delta^{-1}/d\delta] =$$

$$=[P(0,1)\tau_i [EP(1,2) - 1]Q(0,1)\delta^{-1}EQ(1,2)] - 2\delta^{-2}Q(0,1)EQ(1,2)[1-P(0,1)\tau_i]$$

$$[d\delta^{-1}/d\delta] > 0$$
If \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]=0\), \(\delta=1\) makes (23) equal to zero and the lending strategy is irrelevant.

If \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]>0\), \(\delta=1\) makes (23) positive, and \(\delta<1\) is required to make (23) equal to zero. Finally, if \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]<0\), \(\delta=1\) makes (23) negative, and it takes \(\delta>1\) to drive (23) to zero. Q.E.D.

Proof of Proposition XI

Using expression (23) compute \(d[\text{NPV}_i(M)-\text{NPV}_i(R)]/dt_i\)

\[
d[\text{NPV}_i(M)-\text{NPV}_i(R)]/dt_i = \\
= P(0,1)[[1-\text{EP}(1,2)][\text{EQ}(1,2)-\delta^{-1}Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]]-
-\text{Cov}[P(1,2),Q(1,2)][(\delta-1)/\delta]P(0,1) \tag{25}
\]

If \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]=0\), then from proposition X, \(\delta=1\) and (25) is zero.

If \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]>0\), then from proposition X, \(\delta<1\). In order for (23) to be equal to zero with \(\delta<1\), \([1-\text{EP}(1,2)][\text{EQ}(1,2)-\delta^{-1}Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]>0\) and hence, (25) is negative.

Finally, if \([1-\text{EP}(1,2)][\text{EQ}(1,2)-Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]<0\), proposition X entails \(\delta>1\). If (23) is to be equal to zero with \(\delta>1\), then \([1-\text{EP}(1,2)][\text{EQ}(1,2)-\delta^{-1}Q(0,1)^\text{\textregistered}[\text{EQ}(1,2)]-\text{Cov}[P(1,2),Q(1,2)]>0\), and hence (25) is positive. Q.E.D.
Proof of Proposition XII

The continuity of the function \( p^* = p^*(\delta) \) on \( \delta \) is established by noting that the implicit function \( dV_0/dp(p^*, \delta) = 0 \) is continuous in both \( p^* \) and \( \delta \). To compute \( dp^*/d\delta \) use the implicit derivative rule on FOC (28) in the second part of the dissertation. Accordingly,

\[
dp^*/d\delta = -\frac{d^2V_0/dpd\delta}{d^2V_0/dp^2}
\]  

(26)

where all derivatives are evaluated at the point \( p = p^* \). Since \( p^* \) maximizes \( V_0 \), \( d^2V_0/dp^2 \leq 0 \) and the sign of \( dp^*/d\delta \) is the same as the sign of \( d^2V_0/dpd\delta \). Compute \( d^2V_0/dpd\delta \). Substitute \( Q(0,2) = (1/\delta)Q(0,1)EQ(1,2) \) in the FOC (28) in the second part of the dissertation and differentiate with respect to \( p \)

\[
d^2V_0/dpd\delta = [d^2V_0/dpd\delta^{-\frac{1}{2}}][d\delta^{-\frac{1}{2}}/d\delta] = \\
= \left[ P(0,1)\tau_c\delta^2 \{ [Q(0,2)^{-\frac{1}{2}}-2Q(0,2)][1-F_1(k_1)]- \\
- [Q(0,2)^{-\frac{1}{2}}-2Q(0,2)]f_1(k_1)p[Q(0,2)^{-\frac{1}{2}}-Q(0,2)-EQ(1,2)(1-Q(0,1))]ight] + \\
+ pQ(0,2)^{-\frac{1}{2}} \int_{0}^{1} \int_{0}^{1} P(1,2)Q(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) - \\
- pQ(0,2)^{-\frac{1}{2}} \int_{0}^{1} \int_{0}^{1} P(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) - \\
- pQ(0,2)^{-\frac{1}{2}} \int_{0}^{1} \int_{0}^{Q_2} P(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) + \\
+ 2\delta^2Q(0,2)) [d\delta^{-\frac{1}{2}}/d\delta]
\]  

(27)

Rearrange \( [d^2V_0/dpd\delta^{-\frac{1}{2}}] \) in (27) in the following way
\[ \frac{d^2 V_o}{dp d\delta^{-2}} = \]
\[ = P(0,1)\tau_c \delta^4 [Q(0,2)\frac{4}{2} - Q(0,2)] \]
\[ \{ - F_1(k_i) - f_1(k_i) p [Q(0,2)\frac{4}{2} - Q(0,2) - EQ(1,2)(1 - Q(0,1))] \} - \]
\[ - P(0,1)\tau_c \delta^4 Q(0,2)\{ - F_1(k_i) - f_1(k_i) p [Q(0,2)\frac{4}{2} - Q(0,2) - EQ(1,2)(1 - Q(0,1))] \} + \]
\[ + \delta^4 Q(0,2) + P(0,1)\tau_c \delta^4 \]
\[ \{ pQ(0,2)\frac{4}{2} \int_0^1 \int_0^1 P(1,2)Q(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) \}
\[ - pQ(0,2)\frac{4}{2} \int_0^1 \int_0^1 P(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) \}
\[ - pQ(0,2)\frac{4}{2} \int_0^1 \int_0^1 \int_0^2 P(1,2)h[k_2(Q(1,2),Q(1,2),P(1,2)]dP(1,2)dQ(1,2) \}
\[ + \delta^4 Q(0,2) \] (28)

There are eight terms in expression (28). Since \( f_i(k_i) \leq F_i(k_i) \) and \( f_i \) is a symmetric density, \( f_i(k_i) \leq 1 - F_i(k_i) \). Hence,
\[ 1 - F_i(k_i) - f_i(k_i) p [Q(0,2)\frac{4}{2} - Q(0,2) - EQ(1,2)(1 - Q(0,1))] \geq 0 \] (29)
which implies that the first four terms in (28) have a net positive effect. Assumption
\[ \int_0^{k_2(Q(1,2)]} h(x_2, Q(1,2), P(1,2))dx_2 \geq h(k_2[Q(1,2)], Q(1,2), P(1,2)) \] (30)
guarantees that the sum of the sixth and seventh term is larger then

\[-P(0,1)\tau_c 5^2 pQ(0,2)^4 EP(1,2)\]  \hspace{1cm} (31)

Use \(P(0,2)=P(0,1)EP(1,2)\) in (31). Assumption \(Q(0,2)\geq P(0,2)^2 \tau_c^2\) implies that the sum of the last three terms in (28) is positive. Finally, since the fifth term in (28) is positive, \([d^2 V_0/dp d\delta^{-1}]\) is non-negative and thus, \(d^2 V_0/dp d\delta\) is non-positive. Q.E.D.
References


