IMPLEMENTATION USING SIMPLE MECHANISMS:
SOME THEORETICAL RESULTS AND APPLICATIONS TO ACCOUNTING

DISSERTATION

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CHAPTER I

INTRODUCTION

The impact of accounting on managerial incentives is well-recognized. For example, Hatfield (1909, p. 295) states that one of the four primary purposes of cost accounting is to check the efficiency of factory management. According to Simon et al. (1954, p.27), one of the three uses of accounting numbers by the controller's office is that of attention-directing (the other uses are score-card keeping and problem-solving): "the greatest significance of attention-directing accounting data lies in the information they transmit independently of operating supervisors." Horngren and Foster (1991, p.426) point out that the designer of a management control system must be aware that "managers may not have incentives to communicate their information in an honest, complete, and timely way ... some operating managers -- individually or in collusion -- may withhold important information from management." Preventing (implicit) collusion in reporting can be a particularly challenging task and is the topic of this dissertation.

The possibility of collusion has been demonstrated in various applied agency settings. For example, Antle (1982, 1984) extends the basic owner-manager agency setting to include a strategic auditor. Antle's main result is that if the owner designs optimal contracts subject only to the usual Bayes-Nash incentive compatibility and individual rationality constraints, the agents' subgame will possess multiple Bayes-Nash equilibria. Moreover, the equilibrium preferred by the principal is, from the agents' perspective, Pareto-dominated by another equilibrium. This subgame or collusion problem
has been studied in other accounting settings. For example, Amershi and Cheng (1987) study the managerial control systems and collusion in a joint production setting.

Although useful in having provided a precise description of the problem of collusion, neither Antle (1982, 1984) nor Amershi and Cheng (1987) provide results on whether or not the principal can design more sophisticated incentive schemes that prevent collusion. This is one of the important questions addressed in the economics literature on implementation. The question is can the principal (planner) design some incentive scheme (mechanism) under which the agents' subgame has a unique equilibrium and under which the equilibrium outcomes are as prescribed by a given allocation rule. To be more precise, given a behavioral principle and an allocation rule that maps states to outcomes, the following question is posed. Can a planner, who does not know the state, design a mechanism under which the equilibrium outcomes of the game played by the agents are identical to the outcomes prescribed by the given allocation rule? In other words, can the planner design a mechanism that implements the given allocation rule?

Early work in the implementation literature focused on dominant strategy implementation. For the most part, this literature arrived at impossibility results (e.g., Gibbard 1973; Green and Laffont 1979; Hurwicz 1972; Satterthwaite 1975). Much of the subsequent literature has focused on Nash (e.g., Maskin 1977; Saijo 1988) and Bayes-Nash implementation (e.g., Jackson 1991; Palfrey and Srivastava 1987, 1989; Postlewaite and Schmeidler 1986).

The sufficiency results on Nash and Bayes-Nash implementation rely on constructive proofs without relating the constructed mechanisms to observed practices (institutions). In fact, these theoretical mechanisms appear to bear little resemblance to actual institutions. Recent work in the accounting literature has begun to explore the possibility of using mechanisms with characteristics that can be interpreted in terms of
observed practices to replicate the performance of mechanisms constructed in the economics literature (see, for example, Rajan 1991, 1992). In particular, Rajan (1992) designs a mechanism for a setting almost identical to that of Demski and Sappington (1984). Rajan's (1992) mechanism, which employs a fixed overhead allocation scheme, replicates the performance obtained by the mechanism of Ma, Moore, and Turnbull (1988). (Ma, Moore, and Turnbull's (1988) mechanism was designed specifically for Demski and Sappington's (1984) setting.) Unfortunately, Rajan's (1992) mechanism has the undesirable property that the agents' best-reply correspondences are not well-defined. Under the Ma, Moore, and Turnbull (1988) mechanism, the agents' best-reply correspondences are well-defined.

Rajan's work is not alone in being subject to this criticism. The mechanisms constructed in Palfrey and Srivastava (1987, 1989) and Postlewaite and Schmeidler (1986) employ a tail-chasing construction (in particular, an integer game) under which the agents' best-reply correspondences are not well-defined. Much of the remaining literature ignores mixes strategies (e.g., Jackson 1991; Maskin 1977; Saito 1988). As Jackson (1992) points out, ignoring mixed strategies seems unsatisfactory for a literature whose goal is to account for all equilibria, and a tail-chasing construction seems unsatisfactory in that it relies on the failure of the (Bayes) Nash solution concept to predict the play of certain types of games (e.g., integer games).

Recently, the implementation literature has focused on characterizing allocation rules that can be implemented by mechanisms that are not subject to the above criticisms. For example, Abreu and Matsushita (1990, 1992) show that a fairly large class of allocation rules can be approximately ("virtually") implemented via the iterative removal of strictly dominated strategies. Their mechanisms are appealing in that they (1) are finite and, hence, not subject to the tail-chasing criticism and (2) employ a solution concept that is
weaker than the Bayes-Nash equilibrium concept. A unique strategy combination that survives the iterative removal of strictly dominated strategies is not only a unique Bayes-Nash equilibrium in both pure and mixed strategies but also a strict equilibrium and a unique rationalizable strategy combination in the sense of Bernheim (1984) and Pearce (1984). Abreu and Matsushima's (1990, 1992) mechanisms have the undesirable property that both the size of the message space and the number of iterations of eliminating strictly dominated strategies are increasing in the closeness of the approximation. A mechanism with a large message space is undesirable in that more complicated mechanisms are implicitly more costly to design and use and less likely to induce the desired behavior than are simple mechanisms (Mount and Reiter 1974).

The fact that Abreu and Matsushima's mechanisms are relatively complicated should not be surprising: their results apply to a large class of environments. On the other hand, one might hope for relatively simple mechanisms within the context of more applied settings (Palfrey and Srivastava 1991). These simpler mechanisms would then (presumably) be easier to relate to observed practices. This dissertation adopts the behavioral principle of Abreu and Matsushima (1990, 1992) -- the iterative elimination of strictly dominated strategies -- and explores the possibility of constructing simple mechanisms that achieve implementation in more structured settings. The mechanisms constructed in this dissertation achieve implementation in a small number of iterations of eliminating strictly dominated strategies (two or less). As Kreps (1990) points out, the iterative elimination of strictly dominated strategies is particularly persuasive as a solution concept when the number of iterations is small.

The starting point is the multi-agent adverse selection model of Demski and Sappington (1984). As mentioned above, the Ma, Moore, and Turnbull (1988) mechanism, which was designed specifically for Demski and Sappington's (1984) setting,
is not subject to Jackson's (1992) criticisms of the general results in the implementation literature. However, the Ma, Moore, and Turnbull (1988) mechanism is subject to the criticism of employing a large message space (Demski, Sappington, and Spiller 1988). In Chapter 2, a simple, deterministic mechanism is constructed that approximately implements the second-best solution in Demsiki and Sappington's setting; the second-best solution is the allocation rule that maximizes the principal's expected utility subject to the usual Bayes-Nash incentive compatibility and individual rationality constraints.

There are two important features of Demsiki and Sappington's (1984) model which facilitate the construction of simple mechanisms. First, in their model, all goods are private and there are no aggregate budget-balancing constraints. This sort of extreme separability in the agents' environments is heavily exploited in Chapter 2. Chapter 3 takes a closer look at the role of separability in facilitating the construction of simple mechanisms.

The second feature of Demsiki and Sappington's (1984) setting which is exploited is the assumption that the agents' preferences satisfy the single crossing property. Although the single crossing property was originally introduced because of its intuitive interpretation within the context of applied models, it has recently been shown to be useful in solving implementation problems. The work of Mookherjee and Reichelstein (1992) shows that in principal-agent settings in which the risk-neutral agents' preferences satisfy the single crossing property and in which the agents have uncorrelated private information, Bayes-Nash incentive compatible allocation rules that satisfy a monotonicity condition can be equivalently implemented in dominant strategies. Chapter 4 extends Chapter 2's result to more general settings in which the agents' preferences satisfy the single crossing property and unifies and extends some existing results in the literature. In contrast to Mookherjee and Reichelstein, the results in Chapter 4 allow for both risk aversion and correlated information.
Demski, Sappington, and Spiller (1988) study a principal-multi-agent setting in which the agents are risk-neutral. A contracting friction arises between the principal and the agents because of bankruptcy constraints on the payments the principal makes to the agents and because of an information asymmetry that arises either prior to contracting (i.e., a typical adverse selection setting) or immediately following contracting (i.e., a post-contractual hidden information setting). They leave the issue of implementation in their adverse selection setting unaddressed.

Chapter 5 of this dissertation provides the following results on Demski, Sappington, and Spiller's (1988) adverse selection model. First, the second-best solution can be equivalently implemented in dominant strategies. In other words, without decreasing her expected utility, the principal can design a mechanism under which the Bayes-Nash incentive compatibility constraints are replaced by dominant strategy incentive compatibility constraints. Second, at an arbitrarily small cost, the principal can replace the dominant strategy incentive compatibility constraints with strict dominant strategy incentive compatibility constraints. Although the results in Chapter 5 are closely related to those in Mookherjee and Reichelstein (1992), they differ in that Mookherjee and Reichelstein's (1992) model does not incorporate bankruptcy constraints on the agents' payments.

Chapters 6 studies a two-agent extension of Antle and Eppen's (1985) single-agent model of capital budgeting. Antle and Eppen's single-agent analysis shows that resource rationing, an ex post inefficiency, can be an ex ante efficient response to an information asymmetry. In accounting textbooks, resource rationing is often described as a comparison of positive net present value projects with the outcome being that only the best projects are undertaken. Antle and Eppen's single-agent analysis, although useful in explaining the existence of rationing, does not (and was not intended to) explain relative project ranking as the means by which resources are rationed. Chapter 6's multi-agent extension of Antle and
Eppen's model provides such an explanation -- the second-best solution exhibits a demand for relative project ranking. As in Demski and Sappington (1984), the second-best solution suffers from a subgame problem. Unlike Demski and Sappington (1984), when information is objective (e.g., the agents' costs are perfectly correlated), the second-best solution cannot be implemented in Bayes-Nash equilibrium. This impossibility result is driven by capacity constraints on production and implies that any collusion-preventing mechanism that achieves implementation in Bayes-Nash (or any weaker solution concept) necessarily involves rationing above and beyond that called for by the second-best solution. A simple rationing mechanism is presented that achieves implementation in iteratively strictly undominated strategies.

The remainder of this dissertation is organized into six chapters. Chapters 2 - 6 are as described above. Chapter 7 discusses this dissertation's contributions and limitations and provides some suggestions for future research.
CHAPTER II

A SIMPLER MECHANISM THAT STOPS AGENTS FROM CHEATING

2.1 Introduction

Demski and Sappington (1984) study incentive schemes in a principal-two-agent model of a firm subject to adverse selection. An agent's type (low or high) is a productivity parameter of the technology he is being hired to operate. Each agent privately observes his own type before contracting with the principal, and the agents' types are imperfectly correlated. Demski and Sappington assume that the only communication from the agents to the principal is the agents' choice of outputs. Further, they consider only direct mechanisms: each agent communicates his private information by choosing one of two outputs. The strategy combination that has each agent choosing low output when his type is low and high output when his type is high will be referred to as truth-telling.

If the principal expects the agents to reveal their private information, truth-telling must be a (Bayes-Nash) equilibrium in the agents' subgame, i.e., truth-telling must be incentive compatible. However, when the principal limits herself to direct mechanisms, incentive compatibility is not sufficient: the principal's utility-maximizing, incentive compatible direct mechanism (the second-best solution) produces a lying equilibrium in which both agents always choose the low output. Moreover, from the agents' perspective, the lying equilibrium Pareto-dominates the truth-telling equilibrium. Demski and Sappington's solution to the multiple equilibria or "subgame" problem is to add costly constraints to the principal's program. They design a direct mechanism that makes truth-
telling a dominant strategy for one agent and a best-response to truth-telling for the second agent.

In the context of Demski and Sappington's model, Ma, Moore, and Turnbull (1988) (MMT) design an indirect mechanism under which truth-telling is a unique equilibrium, and the equilibrium payoffs are those prescribed by the second-best solution, i.e., the MMT mechanism exactly implements the second-best solution in Bayes-Nash equilibrium. The message space of one agent (agent A) is expanded to include non-type messages or "flags"; each flag corresponds to one of an infinite number of lying mixed strategies of the other agent (agent B). Agent A is provided with strict incentives to turn agent B in for deviating from the truth-telling equilibrium if and only if agent B is actually deviating from the truth-telling equilibrium. This approach has been generalized in Mookherjee and Reichelstein (1990). As Demski, Sappington, and Spiller (1988) point out, the MMT approach is implicitly costly in that their mechanism employs a large message space; although there are only four possible states in Demski and Sappington's model, the MMT mechanism employs an infinite message space.

By exploiting the structure of Demski and Sappington's setting (e.g., the Spence-Mirrlees condition), this chapter develops an alternative to the MMT approach for dealing with the subgame problem. A mechanism with the following properties is constructed: (1) agent A chooses one of three outputs, while agent B chooses one of two outputs, (2) truth-telling is the unique strategy combination that survives the iterative removal of strictly dominated strategies, and (3) the equilibrium outputs are those prescribed by the second-best solution, while the equilibrium payments do not exceed those prescribed by the second-best solution by any more than an arbitrarily small, positive amount \( \epsilon \). Hence, this chapter's mechanism is said to approximately implement the second-best solution via the iterative removal of strictly dominated strategies. It is also shown that this chapter's
mechanism employs a message space that is smaller than the message space of any
mechanism that exactly implements the second-best solution in Bayes-Nash equilibrium.

The remainder of this chapter is organized into two sections. Section 2.2 presents
the model, and Section 2.3 presents the results.

2.2 Model

A risk-neutral principal owns two productive technologies, A and B. The principal
has other uses of her time and chooses to hire two agents, agent A and agent B, to operate
the technologies. The value of each technology \( x^i = X^i(a^i, \theta^i) \), \( i = A, B \), depends on both
the unobservable effort \( a^i \) exerted by agent i and a random productivity parameter (agent i's
type) \( \theta^i \in \{ \theta^i_1, \theta^i_2 \} \). \( \theta^i_2 \) places agent i in a more productive setting than does \( \theta^i_1 \):
\[ X^i(a^i, \theta^i_2) > X^i(a^i, \theta^i_1) \] for all \( a^i \). Each technology exhibits decreasing returns to effort:
\[ X^a_i(\cdot) > 0 \textrm{ and } X^a_{aa}(\cdot) < 0 \]. Agent i privately observes \( \theta^i \) before contracting with the
principal. \( \phi \) is the joint probability distribution on \((\theta^A, \theta^B)\), where \( \phi_{km} \) denotes the
probability that \( \theta^A = \theta^A_k \) and \( \theta^B = \theta^B_m \), \( k, m = 1, 2 \). Denote by \( p_{km}^i \) the conditional
probability that \( \theta^i = \theta^i_m \), given that \( \theta^i = \theta^i_k \). The joint and conditional probability
distributions are assumed to be common knowledge among the agents and the principal.
The agents' productivity parameters are assumed to be positively but imperfectly correlated.
That is, \( p_{kk}^i > p_{km}^i \), \( k \neq m \), \( k, m = 1, 2 \), \( i = A, B \).

Agent i's preferences over wealth \( R^i \) and effort \( a^i \) are represented by a von
Neumann-Morgenstern utility function \( u^i(R^i, a^i) = u^i(R^i) - V^i(a^i) \). The agents are risk
averse and dislike effort increasingly, i.e., \( U^i(\cdot) > 0 \), \( U^{ii}(\cdot) < 0 \), \( V^i(\cdot) > 0 \), and \( V^{ii}(\cdot) > 0 \).
Let \( D^i(x^i, \theta^i) = V^i(\tilde{a}^i) \), where \( \tilde{a}^i \) solves \( x^i = X^i(\tilde{a}^i, \theta^i) \). In other words, \( D^i(x^i, \theta^i) \) is agent
i's disutility from producing \( x^i \) when his type is \( \theta^i \). It follows from the above assumptions
on \( X^i \) and \( V^i \) that \( D^i(x^i, \theta^i_2) < D^i(x^i, \theta^i_1) \) for all \( x^i \), \( D^{ii}_x(\cdot) > 0 \), and \( D^{ii}_{xx}(\cdot) > 0 \). Assume also
that $D^i(x^i, \theta^i_2) < D^i(x^i, \theta^i_1)$ for all $x^i$. The last assumption is commonly referred to as the single crossing property.

Agent $i$ privately observes $\theta^i$, $i = A, B$.

The principal offers agent $i$ a choice of $x_1$ or $x_2$ and payments of $R_{11}, R_{12}, R_{21}, R_{22}$.

Agent $i$ chooses $x_1$, $x_2$, or rejects the contract. (Assume both accept.)

Agent $i$'s utility is $U^i(R^i) - D^i(x^i, \theta^i)$.

The principal's utility is $x^A + x^B - R^A - R^B$.

FIGURE 1. Time-Line.

The principal's contracting problem is to maximize her expected utility (2.1) subject to the following constraints. First, the individual rationality constraints (2.2) and (2.3) require that the contract be sufficiently attractive to each agent -- the contract must provide agent $i$ with at least his reservation utility $\bar{U}$. Second, the truth-telling constraints (2.4) and (2.5) require that truth-telling be incentive compatible. The Revelation Principle (e.g., Dasgupta, Hammond, and Maskin 1979; Myerson 1979) justifies examination of the truth-telling equilibrium. The principal's contracting problem (P-BN) is as follows.
\[(P\text{-BN}) \quad \text{Max } \phi_{11}(2x_1-2R_{11})
+ (\phi_{12}+\phi_{21})(x_1+x_2-R_{12}-R_{21}) + \phi_{22}(2x_2-2R_{22}) \quad (2.1)
\]
\[x_1, x_2, R_{11}, R_{12}, R_{21}, R_{22} \]

s.t.
\[p_{11}U(R_{11}) + p_{12}U(R_{12}) - \text{D}(x_1, \theta_1) \geq \bar{U} \quad (2.2),
\]
\[p_{21}U(R_{21}) + p_{22}U(R_{22}) - \text{D}(x_2, \theta_2) \geq \bar{U} \quad (2.3),
\]
\[p_{11}U(R_{11}) + p_{12}U(R_{12}) - \text{D}(x_1, \theta_1) \geq \]
\[p_{21}U(R_{21}) + p_{22}U(R_{22}) - \text{D}(x_2, \theta_1) \quad (2.4), \text{ and}
\]
\[p_{21}U(R_{21}) + p_{22}U(R_{22}) - \text{D}(x_2, \theta_2) \geq \]
\[p_{21}U(R_{11}) + p_{22}U(R_{12}) - \text{D}(x_1, \theta_2) \quad (2.5).
\]

Denote by \(K^* \equiv (x_1^*, x_2^*, R_{11}^*, R_{12}^*, R_{21}^*, R_{22}^*)\) the solution to \((P\text{-BN})\). We refer to \(K^*\) as the second-best solution. The second-best solution is characterized in Demski and Sappington (1984) and more completely in MMT. In particular, it has been shown that \((2.2)\) and \((2.5)\) are binding, \((2.4)\) is not binding, \(R_{11}^* > R_{12}^*\), and \(R_{21}^* = R_{22}^* = R_2^*\). From this characterization, it follows that the strategy combination that has both agents always choosing \(x_1^*\) (i.e., a lying strategy combination) is also an equilibrium in the agents' subgame. Moreover, from the agents' perspective, the lying equilibrium is also a strict equilibrium and Pareto-dominates the truthful equilibrium.

2.3 Results

This chapter's main result is that the multiple equilibria problem created by the second-best solution can be remedied by offering one agent a third level of output.

**Proposition 2.1:** For any \(\varepsilon > 0\), there exists an output \(x_3\) and payments \(R_{11}, R_{12}, R_{13}, R_{21}, R_{22}, R_{23}, R_{31},\) and \(R_{32}\) such that:
(i) if agent A chooses his output from the set \( \{ x_1^*, x_2^*, x_3 \} \) and agent B chooses his output from the set \( \{ x_1^*, x_2^* \} \), truth-telling is the unique strategy combination that survives the iterative removal of strictly dominated strategies, and

(ii) \( R_{km} - R_{km}^* \leq \varepsilon, k,m = 1,2 \).

All proofs are presented in Appendix A. The key step of the proof of Proposition 2.1 is adding a strategy for agent A that strictly dominates the lying equilibrium strategy but not the truthful one. This is accomplished by offering agent A an additional output choice, \( x_3 \). \( x_3 \) is chosen to be greater than \( x_1^* \), and the payments associated with \( x_3 \) are chosen so that:

1. agent A prefers \( x_3 \) to \( x_1^* \) if his type is high (regardless of agent B's choice of output),
2. agent A prefers \( x_1^* \) to \( x_3 \) if his type is low (regardless of agent B's choice of output).

Such a construction is possible by the single crossing property and is useful in that it induces agent A to reveal his type to the principal: a choice of \( x_2^* \) or \( x_3 \) indicates that A's type is high, while a choice of \( x_1^* \) indicates that A's type is low. The problem with the above construction is that if A is paid \( R_2^* \) when he chooses \( x_2^* \), he prefers \( x_3 \) to \( x_2^* \) when his type is high, given that B plays the truth-telling strategy. In order to remedy this problem, the payment associated with \( x_2^* \) is increased. Since agent A's utility function is continuous in wealth, it is possible to pick payments associated with \( x_3 \) that satisfy (1) and (2) above for any \( \varepsilon > 0 \), where setting \( R_2 = R_2^* + \varepsilon \) ensures that A will pick \( x_2^* \) instead of \( x_3 \) when his type is high, given that B plays the truth-telling strategy.

Proposition 2.1 states that the second-best solution can be approximately implemented via the iterative removal of strictly dominated strategies. In other words, truth-telling is obtained as the unique strategy combination that survives the iterative removal of strictly dominated strategies, and the equilibrium allocations are arbitrarily close to those prescribed by the second-best solution. A unique strategy combination that
survives the iterative removal of strictly dominated strategies is also a unique Bayes-Nash equilibrium, a unique strict equilibrium, and a unique rationalizable strategy combination in the sense of Bernheim (1984) and Pearce (1984). According to Kreps (1980, pp.390-410), implementation via the iterative removal of strictly dominated strategies is particularly persuasive when the number of iterations is small, as is the case for this chapter's mechanism.

If the iterative removal of strictly dominated strategies is fixed as the solution concept, the best that can be done with a finite mechanism is to approximately implement the second-best solution. (This claim follows directly from the fact that under the second-best solution, the strategy that has an agent always choosing the low output weakly dominates the truth-telling strategy.) Moreover, Proposition 2.1's mechanism employs the smallest message space of any mechanism that achieves such implementation.

As promised, Proposition 2.1's mechanism has a smaller message space than the MMT mechanism's message space. However, the MMT mechanism is (presumably) not the only mechanism that exactly implements the second-best solution in Bayes-Nash equilibrium. (A mechanism is said to exactly implement the second-best solution in Bayes-Nash equilibrium if, under that mechanism, the agents' subgame has a unique Bayes-Nash equilibrium in both pure and mixed strategies and if the equilibrium allocations are as prescribed by the second-best solution.) This raises the following question. Does Proposition 2.1's mechanism employ a smaller message space than the message space of any mechanism that exactly implements the second-best solution in Bayes-Nash equilibrium? Proposition 2.2 answers this question affirmatively.
Proposition 2.2: The second-best solution cannot be exactly implemented in Bayes-Nash equilibrium by any finite mechanism that offers additional outputs to only one of the agents.

Thus, it can be said that Proposition 2.1's mechanism approximately implements the second-best solution in both Bayes-Nash equilibrium and via the iterative removal of strictly dominated strategies. It is the minimal-sized mechanism that achieves this objective. Exact implementation (using a finite mechanism) in Bayes-Nash equilibrium would require additional strategies for both agents and, hence, a larger message space; exact implementation via the iterative removal of strictly dominated strategies (if possible at all) would require an infinite (and unbounded) mechanism.

Both this chapter and the papers of Abreu and Matsushima (1990, 1992) contain results on approximate or "virtual" implementation via the iterative removal of strictly dominated strategies. By relaxing the performance standard from exact to approximate implementation, Abreu and Matsushima substantially broaden the class of implementable social choice functions. Moreover, Abreu and Matsushima's mechanisms are not subject to Jackson's (1992) powerful critique of earlier results in the implementation literature that either ignore mixed strategies or rely on a "tail-chasing" construction.

When compared to Abreu and Matsushima's mechanisms, this chapter's mechanisms (one for each approximation) are simpler in two ways. First, this chapter's mechanisms have the property that the closeness of the approximation does not affect the message space; each mechanism employs the same, small message space. Abreu and Matsushima's mechanisms have the property that the size of the message space is increasing in the closeness of the approximation; as the approximation becomes arbitrarily close, the message space becomes arbitrarily large. Second, this chapter's result relies on
deterministic outcome functions; Abreu and Matsushima's results rely on randomized outcome functions. The improvements of this chapter's mechanisms over Abreu and Matsushima's mechanisms are at the cost of generality -- Demski and Sappington's (1984) model is a very special case of the model presented in Abreu and Matsushima (1990). It might be worthwhile to explore the extent to which simple mechanisms can be constructed for other settings (e.g., more general settings in which the Spence-Mirrlees condition is satisfied).
CHAPTER III
SEPARABLE ENVIRONMENTS

3.1 Introduction

The purpose of this chapter is to understand the role separability plays in deriving the previous chapter's main result. The remainder of this chapter is organized as follows. Section 3.2 presents the notation and some definitions. Section 3.3 studies approximate implementation in separable complete information environments, and Section 3.4 studies approximate implementation under a stronger separability assumption in incomplete information environments.

3.2 Preliminaries

Let $A$ be the set of simple lotteries (outcomes) over an arbitrary allocation set $X$. Denote by $N = \{1, \ldots, n\}$, $n \geq 2$, the set of agents. Agent $i$'s preferences are over $A$, depend on his type, $\theta^i \in \Theta^i = \{\theta^i_1, \ldots, \theta^i_1\}$, and have a von Neumann-Morgenstern expected utility representation, $U^i: A \times \Theta^i \rightarrow \mathbb{R}$. The state space $\Theta = \prod_{i=1}^{n} \Theta^i$. Note that attention is restricted to finite environments.

When state $\theta = (\theta^1, \ldots, \theta^n)$ is realized, agent $i$, $i \in N$, privately observes $\theta^i$. Each agent has beliefs about the underlying state. The beliefs arise from the application of Bayes rule to a common prior probability distribution satisfying $p(\theta) \geq 0$ for all $\theta \in \Theta$, and $\sum_{\theta \in \Theta} p(\theta) = 1$. Let $p^i(\theta^i) = \sum_{\theta^i \in \Theta^i} p(\theta^{-i} \theta^i)$, where $\Theta^{-i}$ denotes $\prod_{j \neq i} \Theta^j$, and, without loss of generality, assume $p^i(\theta^i) > 0$ for all $\theta^i$ and all $i$. The beliefs of type $\theta^i$ of agent $i$ are represented by the conditional probability distribution $q^i(\theta^{-i} | \theta^i)$ over $\Theta^i$: 
\[ q^i(\theta^i \mid \theta^i) = \frac{p(\theta^i, \theta^i)}{p^i(\theta^i)} \]. These beliefs are common knowledge among the agents.

Note the assumption that \( U^i \) depends on \( \theta \) only through \( \theta^i \). Hence, an agent’s preferences are completely characterized by his own type, and any information that one agent has about another agent is because of correlation in the agents’ types. The information structure assumed here is typical of principal-agent settings and implies that Abreu and Matsushima’s (1990) measurability condition is satisfied. Measurability and incentive compatibility are sufficient conditions for an allocation rule to be approximately implementable via the iterative removal of strictly dominated strategies. The goal of the chapter is to identify additional domain restrictions that ensure that such implementation can be accomplished by simple mechanisms.

The goals of the planner (e.g., an employer, a regulator, or the agents themselves before the state is realized) are represented by an allocation rule \( f : \Theta \to A \), which specifies for any realization \( \theta \in \Theta \) the desired outcome \( f(\theta) \). (The planner knows the prior \( p(\theta) \), but not the type \( \theta^i \) of any agent.) In order to implement \( f \), the planner designs a message space \( M \) and an outcome function \( g : M \to A \), where \( M = \prod_{i=1}^n M^i \). A (pure) strategy of agent \( i \) is a function \( s^i : \Theta^i \to M^i \). Denote by \( S^i \) the set of all strategies available to agent \( i \).

Let \( S = \prod_{i=1}^n S^i \). (\( M, g \)) is referred to as a mechanism or game form.

Let \( V^i(g, s^{-i}, s^i, \theta^i) = \sum_{\theta^i \in \Theta^i} U^i(g(s^{-i}(\theta^{-i}), s^i(\theta^i), \theta^i)) q^i(\theta^{-i} \mid \theta^i) \) denote agent \( i \)’s expected utility when the outcome function is \( g \), the other agents choose the strategies \( s^{-i} \), agent \( i \) chooses the strategy \( s^i \), and agent \( i \) observes \( \theta^i \).

**Definition 1:** A strategy \( s^i \) is strictly dominated with respect to \( S \) if there exists \( s^i \in S^i \) and \( \theta^i \in \Theta^i \) such that for all \( s^{-i} \in S^{-i} \) and all \( \theta^i \in \Theta^i \),

\[ V^i(g, s^{-i}, s^i, \theta^i) \geq V^i(g, s^{-i}, s^i, \theta^i), \text{ and } V^i(g, s^{-i}, s^i, \theta^i) > V^i(g, s^{-i}, s^i, \theta^i). \]
Definition 2: Let \( S_t = \{ s \in S_{t-1} : \text{for all } i \in N, s^i \text{ is not strictly dominated with respect to } S_{t-1} \} \), \( t = 1, 2, 3, ..., \) where \( S_0 = S \).

Definition 3: A strategy combination \( s \) is iteratively strictly undominated if \( s \in S_* \), where \( S_* = \bigcap_{t=1}^{\infty} S_t \).

Definition 4: Let \( id^i : \Theta^i \rightarrow \Theta^i \) be the identity map, i.e., the truth-telling strategy. An allocation rule \( f \) is incentive compatible if \( V_i(f, id^i, id^i, \theta^i) \geq V_i(\hat{f}, id^i, s^i, \theta^i) \) for all \( s^i \in S^i \), all \( \theta^i \in \Theta^i \), and all \( i \in N \).

Definition 5: An allocation rule \( f \) is exactly implementable via the iterative removal of strictly dominated strategies if there exists a mechanism \( (M, g) \) such that for all \( \theta \in \Theta \), there exists a unique \( s_*(\theta) \in S_*(\theta) \) and \( g(s_*(\theta)) = f(\theta) \).

Definition 6: An allocation rule \( f \) is approximately implementable via the iterative removal of strictly dominated strategies if for any \( \epsilon \in [0, 1) \), there exists a (randomized) mechanism \( (M, g) \) such that for all \( \theta \in \Theta \), there exists a unique \( s_*(\theta) \in S_*(\theta) \) and \( g(s_*(\theta)) = f(\theta) \) with probability \( 1 - \epsilon \) or greater.

3.3 Complete Information

This section assumes that the agents possess complete information. Here, agent \( i \)'s strategy is a function \( s^i : \Theta \rightarrow M^i \), and a strategy \( s^i \) is strictly dominated with respect to \( S \) if there exists \( s^i \in S^i \) and \( \theta' \in \Theta \) such that for all \( s^{-i} \in S^{-i} \) and all \( \theta \in \Theta \), \( U^i(g(s^{-i}(\theta), s^i(\theta)), \theta^i) \geq U^i(g(s^{-i}(\theta'), s^i(\theta)), \theta^i) \), and \( U^i(g(s^{-i}(\theta'), s^i(\theta')), \theta^i) > U^i(g(s^{-i}(\theta'), s^i(\theta')), \theta^i) \).
The following domain restrictions are useful in constructing simple mechanisms.

(A1) Separability in punishments: For any allocation rule $f$ and for any $a \in A$ and any $J \subseteq N$, there exists an $a^I \in A$ such that $U^j(a^I, \theta^j) < U^j(f(\theta), \theta^j)$ for all $\theta^j \in \Theta^j$, all $\theta \in \Theta$, and all $j \in J$, while $U^i(a^I, \theta^i) = U^i(a, \theta^i)$ for all $\theta^i \in \Theta^i$ and all $i \in NJ$.

(A2) Value distinction: Different types induce different preference orderings, and agent $i$ is never indifferent over all elements of $A$.

(A1) is similar to the separability assumption in Jackson, Palfrey, and Srivastava (1991) in that it enables the planner to punish any group of agents so that they are worse off than they would have been under any outcome prescribed by the given allocation rule, while all other agents are unaffected. Further, (A1) ensures that any allocation rule can be extended to yield an equivalent incentive compatible allocation rule, even for the two-agent case. (A2) allows the planner to construct a function that induces each agent to truthfully reveal his type. These assumptions are satisfied by many (finite) economic environments such as pure exchange economies, all economies with a transferable private good such as public good environments, and principal-agent settings.

Assuming $n \geq 3$, our setting is a special case of the setting examined in Abreu and Matsushima (1990). Hence, any allocation rule can be approximately implemented via the iterative removal of strictly dominated strategies using finite mechanisms. Proposition 3.1 shows that the additional structure of our setting, i.e., the availability of large punishments, facilitates approximate implementation using mechanisms that are not only finite but also simple. Further, our result holds for the $n = 2$ case.

**Proposition 3.1:** Assume (A1) and (A2) and that each agent observes $\theta$. Then (1) any allocation rule is approximately implementable via the iterative removal of strictly
dominated strategies, and (2) each of the mechanisms (one for each ε) employs a message space \( M = Θ \times Θ \).

All proofs are provided in Appendix B. The mechanism constructed in the proof of Proposition 3.1 has many of the same desirable features that the mechanism in Jackson, Palfrey, and Srivastava (1991) possesses. In particular, the mechanism is simple and intuitive (each agent reports on his own and one neighbor's preferences), applies to the two-agent case, and is immune to mixed strategies. In contrast to Jackson, Palfrey, and Srivastava (1991), this chapter employs a weaker solution concept (iteratively strictly undominated strategies) and achieves only approximate implementation. Further, our mechanism is a revelation mechanism and is strictly dominance solvable, while their mechanism is not a revelation mechanism and is only weakly dominance solvable.

3.4 Incomplete Information

Attention is now shifted to an incomplete information setting. Our goal is to develop mechanisms with the desirable properties described above that can be used in incomplete information settings. The setup here is the same as in the previous section, except that each agent privately observes \( Θ^i \). Further, the extremely strong assumption that there are only private goods and that there are no aggregate budget balancing constraints is made. Let \( A^i, i \in N \), be the set of simple lotteries over agent i's allocation set \( X^i \). Agent i's preferences are over \( A^i \) and depend on his type, i.e., \( U^i : A^i \times Θ^i \rightarrow R \). The following assumptions replace assumptions (A1) and (A2).

(A3) **No aggregate budget-balancing:** \( f : Θ \rightarrow \prod_{i=1}^{n} A^i \).

(A4) **Individual value distinction:** Different types induce different preference orderings, and agent i is never indifferent over all elements of \( A^i \).
(A3) implies that the range of the allocation rule is the product of $n$ (one for each agent) sets. Hence, the model applies primarily to principal-agent settings in which the principal has enough resources not to be constrained by an ex post budget-balancing condition. (A4) is an adaptation of (A2) to this section's model.

**Proposition 3.2:** Assume (A3) and (A4) and that each agent privately observes $\Theta^i$. Then (1) any incentive compatible allocation rule is approximately implementable via the iterative removal of strictly dominated strategies, and (2) each of the mechanisms (one for each $\epsilon$) employs a message space $M = \Theta \times \Theta$.

Proposition 3.2 adapts the result of Glover (1992) to an incomplete information setting.
CHAPTER IV
ON THE USEFULNESS OF THE SINGLE CROSSING PROPERTY
IN CONSTRUCTING SIMPLE MECHANISMS

4.1 Introduction

The single crossing property has been used extensively in adverse selection models (e.g., Mirrlees 1976; Spence 1980; Stiglitz 1982) because of its intuitive interpretation within the context of specific models. More generally, it guarantees that a sufficient condition for self-selection is that each agent prefers his own bundle to those intended for his adjacent neighbors. Hence, this condition limits the number of constraints that need to be considered in solving an adverse selection problem (Cooper (1984)). Recently, it has been shown that the single crossing property is also useful in dealing with implementation problems. Mookherjee and Reichelstein (1992) study an adverse selection principal-agent model and show that the single crossing property makes it possible for the principal to equivalently and uniquely implement Bayes-Nash incentive compatible allocation rules that satisfy a (strict) monotonicity condition in weakly dominant strategies. Their result relies on risk neutrality and on the agents' preference characteristics being uncorrelated. This chapter considers an adverse selection principal-agent model which allows for risk preferences other than risk neutrality and for correlation in the agents' characteristics.

Given a behavioral principle, an information structure, and an allocation rule, the implementation literature asks the following question. Can a principal, who does not know the agents' characteristics, design a mechanism (contract) under which the equilibrium outcomes of the game played by the agents are identical to the outcomes prescribed by the
given allocation rule? The behavioral assumption made throughout this chapter is that agents limit themselves to strategies that survive the iterative elimination of strictly dominated strategies. A unique strategy combination that survives the iterative removal of strictly dominated strategies is not only a unique Bayes-Nash equilibrium in both pure and mixed strategies but also a strict equilibrium and a unique rationalizable strategy combination in the sense of Bernheim (1984) and Pearce (1984). Following Postlewaite and Schmeidler (1986), information structures can be classified into two categories. First, when any n-1 of the agents have complete information, where n is the number of agents, information is said to be nonexclusive. Second, agents may possess exclusive information.

By exploiting the single crossing property, it is shown that (1) when information is nonexclusive, any allocation rule can be exactly implemented by a direct mechanism, and (2) in the presence of exclusive information, any Bayes-Nash incentive compatible allocation rule can be approximately implemented by deterministic mechanisms with small message spaces. This chapter also provides examples of settings studied in the literature to which our results apply, thereby unifying and extending previous results.

The solution concept employed throughout this chapter, the iterative elimination of strictly dominated strategies, is the same as that used in Abreu and Matsushima (1990, 1992). In their more general setting, Abreu and Matsushima (1990, 1992) achieve approximate implementation using randomized mechanisms. Their mechanisms have the property that the size of the message space employed and the number of iterations of eliminating strictly dominated strategies required to achieve implementation depends on the closeness of the approximation; as the approximation becomes arbitrarily close, the size of the message space and the number of iterations required to achieve implementation becomes arbitrarily large. Note that in contrast, by exploiting the structure of our model, we are able
to achieve implementation with either direct mechanisms or mechanisms with small message spaces (simple mechanisms) that achieve implementation in only two rounds of iteratively eliminating strictly dominated strategies. As Kreps (1990) points out, the iterative elimination of strictly dominated strategies is particularly persuasive as a solution concept when the number of iterations is small. The construction of a mechanism with a small message space is consistent with viewing a mechanism that employs a small message space as (implicitly) less costly than a mechanism that employs a large message space (Mount and Reiter (1974)).

The results described above are derived in a private goods setting. In the last part of the chapter, a simple (binary) public goods setting is studied.

The remainder of the chapter is organized into three sections. Section 4.2 presents preliminaries. Section 4.3 presents our main results. Section 4.4 discusses extensions of our results to public goods settings.

4.2 Preliminaries

We study an adverse selection model in which a principal contracts with n agents. Denote by \( N = \{1,...,n\} \) the finite set of agents, \( n \geq 2 \). Agent i's preferences are characterized by a parameter \( \theta^i \in \Theta^i = \{\theta_1^i, ..., \theta_k^i\} \subseteq \mathbb{R} \) and are represented by a von Neumann-Morgenstern utility function, \( u^i : Y^i \times X^i \times \Theta^i \rightarrow \mathbb{R} \). We assume that agent i likes \( x^i \in X^i \) and dislikes \( y^i \in Y^i \), i.e., \( \frac{\partial u^i(\cdot, \theta^i)}{\partial x^i} > 0 \) and \( \frac{\partial u^i(\cdot, \theta^i)}{\partial y^i} < 0 \). Further, we assume that the agents' preferences satisfy the single crossing property, i.e.,

\[
\text{if } \theta^i, \theta'^i \in \Theta^i \text{ and } \theta^i < \theta'^i, \text{ then } \frac{\partial u^i(\cdot, \theta^i)/\partial y^i}{\partial u^i(\cdot, \theta'^i)/\partial x^i} < \frac{\partial u^i(\cdot, \theta'^i)/\partial y^i}{\partial u^i(\cdot, \theta^i)/\partial x^i}.
\]

The single crossing property states that an agent's preferences are revealed by the slope of his indifference curves.
A preference profile is a vector $\theta = (\theta^1,...,\theta^n)$. Let $\Theta = \Theta^1 \times \cdots \times \Theta^n$. Each agent has beliefs about the underlying preference profile. The beliefs arise from the application of Bayes rule to a common prior probability distribution satisfying $p(\theta) \geq 0$ for all $\theta \in \Theta$, and $\sum_{\theta \in \Theta} p(\theta) = 1$. It is convenient to denote the set of possible preference profiles by $\Theta^* = \{ \theta \in \Theta \mid p(\theta) > 0 \}$. When preference profile $\theta = (\theta^1,...,\theta^n)$ is realized, agent $i$, $i \in N$, privately observes $\pi^i(\theta)$. $\pi^i(\theta)$ is an element of $\Pi^i$, a partition of $\Theta^*$, and defines the set of preference profiles which agent $i$ believes may be the true preference profile.

Given our focus on the single crossing property, it seems reasonable to assume that each agent knows at least his own preference characteristic, i.e., $\pi^i(\theta) \subseteq \{ \theta^* \in \Theta^* \mid \theta^*i = \theta^i \text{ and } p(\theta^*) > 0 \}$. Let $p^i(\pi^i(\theta)) = \sum_{\theta^* \in \pi^i(\theta)} p(\theta^*)$. The beliefs of agent $i$ who has observed $\pi^i(\theta)$ are represented by the conditional probability distribution $q^i(\theta^*i \mid \pi^i(\theta))$ over $\Theta^i$:

$$q^i(\theta^*i \mid \pi^i(\theta)) = \frac{p(\theta^*i,\theta^i)}{p^i(\pi^i(\theta))}.$$

These beliefs are common knowledge.

The goals of the principal are represented by an allocation rule $f: \Theta^* \rightarrow Y \times X$, where $Y = Y^1 \times \cdots \times Y^n$ and $X = X^1 \times \cdots \times X^n$, which specifies for any realization $\theta \in \Theta^*$ the desired outcome $f(\theta) = (y(\theta), x(\theta))$. Let $f^i$ be the $i$th component of $f$, i.e., $f = (f^1,...,f^n)$. The principal knows the common prior but not the preference characteristic $\theta^i$ of any agent. In order to implement $f$, the principal designs a message space $M$ and an outcome function $g: M \rightarrow Y \times X$, where $M = M^1 \times \cdots \times M^n$. $(M,g)$ is referred to as a mechanism or contract. A (pure) strategy of agent $i$ is a function $s^i: \Pi^i \rightarrow M^i$. Denote by $S^i$ the set of all strategies available to agent $i$ and $S^1 \times \cdots \times S^n$ by $S$.

Let $U^i(g,s^i,s^i,\theta) = \sum_{\theta^* \in \pi^i(\theta)} u^i(g(s^i(\pi^i(\theta^*)),s^i(\pi^i(\theta)),\theta^i)q^i(\theta^*i \mid \pi^i(\theta))$ denote agent $i$'s expected utility when the outcome function is $g$, the other agents choose the strategies $s^i$, agent $i$ chooses the strategy $s^i$, and agent $i$ observes $\pi^i(\theta)$.
In the following definitions, fix a mechanism \((M,g)\) arbitrarily.

**Definition 1:** A strategy \(s^i\) is strictly dominated with respect to \(S\) if there exists \(s^i \in S^i\) and \(\theta' \in \Theta\) such that for all \(s^i \in S^i\) and all \(\theta \in \Theta\), \(U^i(g, s^i, s'^i, \theta) \geq U^i(g, s'^i, s^i, \theta)\), and \(U^i(g, s^i, s'^i, \theta') > U^i(g, s^i, s^i, \theta')\).

**Definition 2:** Let \(S_t = \{s \in S_{t-1} \mid \text{for all } i \in N, s^i \text{ is not strictly dominated with respect to } S_{t-1}\}\), \(t = 1,2,3,...\), where \(S_0 = S\).

**Definition 3:** A strategy combination \(s\) is iteratively strictly undominated if \(s \in S_*\), where \(S_* = \bigcap_{t=1}^{\infty} S_t\).

**Definition 4:** Let \(\text{id}^i : \Pi^i \rightarrow \Pi^i\) be the identity map, i.e., the truth-telling strategy. An allocation rule \(f\) is incentive compatible if \(U^i(f, \text{id}^i ; \text{id}^i, \theta) \geq U^i(f, \text{id}^i, s^i, \theta)\) for all \(s^i \in S^i\), all \(\theta \in \Theta\), and all \(i \in N\).

**Definition 5:** The mechanism \((M,g)\) is said to exactly implement an allocation rule \(f\) via the iterative removal of strictly dominated strategies if for all \(\theta \in \Theta\), there exists a unique \(s_*(\theta) \in S_* (\theta)\) and \(g(s_*(\theta)) = f(\theta)\).

**Definition 6:** An allocation rule \(f\) can be approximately implemented via the iterative removal of strictly dominated strategies if for any \(\varepsilon > 0\), there exists a (deterministic) mechanism \((M,g)\) such that for all \(\theta \in \Theta\), there exists a unique \(s_*(\theta) \in S_* (\theta)\) and \(d(g(s_*(\theta)), f(\theta)) \leq \varepsilon\), where \(d\) is the usual Euclidean distance metric.

4.3. Results

In this section, we present two results on implementation via the iterative removal of strictly dominated strategies. First, when information is nonexclusive, it is possible for the principal to design a direct mechanism that exactly implements any allocation rule. In
the presence of exclusive information, however, exact implementation of (incentive compatible) allocation rules is not always possible. Our second result is that in the presence of exclusive information, any incentive compatible allocation rule can be approximately implemented by simple, deterministic mechanisms. In both cases, implementation is achieved in only two rounds of iteratively eliminating strictly dominated strategies.

A. Nonexclusive Information

Information is nonexclusive if any n-1 of the agents, by pooling their information, can determine the preference profile. Assumption A.1 formalizes this definition.

Assumption A.1: Information is said to be nonexclusive if \( \bigcap_{i \in N(i)} \pi'(\theta) = \{\theta\} \) for all \( \theta \in \Theta \) and all \( i \in N \).

Nonexclusive information encompasses complete information as a special case. In complete information settings and using a solution concept stronger than Nash, Jackson, Palfrey, and Srivastava (1991) and Sjöström (1991a, 1991b) identify domain restrictions under which simple mechanism achieve exact implementation. By exploiting the single crossing property, Proposition 4.1 shows that it is possible to use direct mechanisms to achieve exact implementation using a solution concept weaker than Nash. The mechanism that achieves exact implementation in Proposition 4.1 employs a message space \( M = \Pi^1 \times \cdots \times \Pi^n \) (a direct mechanism).

Proposition 4.1: Assume A.1. Then any allocation rule can be exactly implemented via the iterative removal of strictly dominated strategies by a direct mechanism.

Proofs of all propositions are provided in Appendix C. The idea behind the mechanism used to achieve implementation in Proposition 4.1 is as follows. Agent i's
allocation is determined by the other agents' messages unless agent i's report of his own characteristic is inconsistent with agent j's (for some j ≠ i) report about agent i's characteristic. In this case, agent i's allocation is perturbed. By exploiting the single crossing property, these perturbations are chosen so that agent i always has strict incentives to report his own characteristic truthfully. Further, agent j is punished when his report of agent i's characteristic is inconsistent with agent i's report of his own characteristic. Hence, given that all agents report their own characteristics truthfully, each agent has strict incentives to report all other agents' characteristics truthfully.

Proposition 4.1 applies a variety of economic settings that have been studied in the literature. We provide three examples in which preferences satisfy the single crossing property and information is nonexclusive. In the first example, a principal contracts with two division managers who operate in perfectly correlated environments. In the second example, a principal contracts with a worker and his boss; the worker and the boss each have complete information. In the third example, a principal contracts with n agents; each agent knows his own characteristic and the ex-post distribution of characteristics.

Example 1: Sappington and Demski (1983) study an adverse selection setting in which a risk neutral principal contracts with two risk averse agents. Agent i, i = A, B, is hired to operate technology i; the output of technology i (y^i) is a function of the effort (a^i) exerted by agent i and a binary productivity parameter (θ^i ∈ {θ^i_1, θ^i_2}). Each agent's output is observable to all parties. Agent i alone observes θ^i, but θ^A and θ^B are assumed to be perfectly correlated. The principal knows only the common prior on (θ^A, θ^B). Agent i's monetary compensation (x^i) depends on both his output and the output of the other agent. Agent i's preferences over monetary compensation and effort are represented by a von Neumann-Morgenstern utility function, which can be reformulated in terms of preferences
over monetary compensation and output as follows: \( u^i(x^i, y^i, \theta^i) = W^i(x^i) - D^i(y^i, \theta^i) \). The agents are risk averse and dislike effort increasingly, i.e., \( W^u() > 0, W^m() < 0, D^i() > 0, \) and \( D^i_{y^i}() > 0 \). A higher realization of \( \theta^i \) implies a more productive environment, i.e., \( D^i(y^i, \theta^i_{2}) < D^i(y^i, \theta^i_{1}) \) for all \( y^i \). They also assume the single crossing property, i.e., \( D^i_{y^i}(y^i, \theta^i_{2}) < D^i_{y^i}(y^i, \theta^i_{1}) \) for all \( y^i \). The principal maximizes expected net production (production less payments to the agents).

Sappington and Demski (1983) show that the principal can exactly implement the first-best (full information) allocation rule in strict dominant strategies. Their result is related to Proposition 4.1. Proposition 4.1 achieves implementation in two rounds of iteratively eliminating strictly dominated strategies. However, given that the agents' environments are perfectly correlated, the direct mechanism in Proposition 4.1 can be adapted to achieve exact implementation in strict dominant strategies. To see this, note that the first round of eliminating strictly dominated strategies results in each agent truthfully revealing his own preference characteristic. When the agents' preference characteristics are perfectly correlated, the agents need only report their own preference characteristics.

**Example 2**: Consider a hierarchical adverse selection setting in which a principal contracts with both a worker (agent A) and his boss (agent B). All parties are risk-neutral. Agent A is hired to produce an output \( y \), which is a function of the effort, \( a \), exerted by agent A and a productivity parameter, \( \theta \in \{\theta_1, ..., \theta_z\} \). Prior to contracting, the agents each observe \( \theta \) and are asked to submit reports on \( \theta \). The principal knows only the common prior on \( \theta \). Agent A's preferences over monetary compensation \( x_A \) and output \( y \) are represented by a von Neumann-Morgenstern utility function \( u^A(x^A, y, \theta) = x^A - D(y, \theta) \). Agent A dislikes effort increasingly, i.e., \( D_{y}(\cdot) > 0 \) and \( D_{y^2}(\cdot) > 0 \). Assume that a higher realization of \( \theta \) implies a more productive environment, i.e., \( D(y, \theta_{2}) < D(y, \theta_{1}) \) for all \( y \). Also assume the
single crossing property, i.e., $D(y, \theta_2) < D(y, \theta_1)$ for all $y$. Agent B's preferences are over monetary compensation: $u^B(x^B) = x^B$. The firm cannot be efficiently rented to agent A because of the timing of the information asymmetry and cannot be efficiently rented to agent B because of limited liability constraints on his payments, i.e., $x^B \geq K$. The principal maximizes expected net production subject to the individual rationality and Bayes-Nash incentive compatibility constraints for both the agents and the limited liability constraints on agent B's payment. This example is similar to the "informationally balanced" setting of Demski and Sappington (1989).

Since complete information implies nonexclusive information and the other assumptions of our model are satisfied, it follows from Proposition 4.1 that the first-best allocation rule can be exactly implemented by a direct mechanism in two rounds of iteratively eliminating strictly dominated strategies.

**Example 3:** The setup here is the same as in Example 1 except that the principal contracts with $n$ agents. Also, the agents' productivity parameters are not perfectly correlated, but the agents know the ex-post distribution of productivity parameters. In other words, they know the number of agents with any given productivity parameter. The principal knows only the common prior distribution of the productivity parameters and not the ex-post distribution of the productivity parameters.

This information structure implies nonexclusive information, and the other assumptions of our model are satisfied. Hence, the first-best allocation rule can be exactly implemented by a direct mechanism in two rounds of iteratively eliminating strictly dominated strategies. This example is similar to that of Piketty (1992). Piketty (1992) studies the theory of optimal income taxation in which a principal (planner) seeks to implement the first-best allocation rule using a generalized taxation scheme.
B. Exclusive Information

We now consider settings in which agents possess exclusive information. We assume that each agent observes only his characteristic. This information structure is typical of adverse selection principal-agent settings and is formalized in Assumption A.2.

Assumption A.2: \( \pi^i(\theta) = \{ \theta^* \in \Theta^* | \theta^{*i} = \theta^i \text{ and } p(\theta^*) > 0 \} \) for all \( \theta \in \Theta \) and all \( i \in N \).

We begin by providing an example in which Assumption A.2 is satisfied.

Example 4: Demski and Sappington (1984) study an adverse selection setting identical to that of Sappington and Demski (1983), described in Example 1, except that the agents' productivity parameters are positively (but imperfectly) correlated. Since each agent privately observes his productivity parameter (and the correlation is not perfect), Assumption A.2 is satisfied. Demski and Sappington (1984) show that the second-best solution (i.e., the direct mechanism that maximizes expected net production subject to the agents' individual rationality and Bayes-Nash incentive compatibility constraints) suffers from a subgame problem. In particular, the strategy combination that has both agents always reporting \( \theta^i \) is a Bayes-Nash equilibrium in the agents' subgame and, from the agents' perspective, Pareto-dominates the truthful equilibrium. Ma, Moore, and Turnbull (1988) show that an indirect mechanism exactly implements the second-best solution in Bayes-Nash. As Demski, Sappington, and Spiller (1988) point out, the Ma, Moore, and Turnbull (1988) mechanism is implicitly costly in that their mechanism employs a large message space; although there are only four possible preference profiles in Demski and Sappington's model, Ma, Moore, and Turnbull's mechanism employs an infinite message space. In the context of Demski and Sappington's model, Chapter 2 designs a simple,
indirect mechanism that approximately implements the second-best solution via the iterative removal of strictly dominated strategies.

Here, the second-best solution cannot be exactly implemented via the iterative removal of strictly dominated strategies. This follows from the fact that under the second-best solution, the strategy that has an agent always reporting the low productivity parameter weakly dominates the truth-telling strategy.

In Proposition 4.2, we use the single crossing property to construct simple, deterministic mechanisms that achieve approximate implementation via the iterative removal of strictly dominated strategies. Proposition 2.1 is a special case of Proposition 4.2.

Proposition 4.2: Assume A.2. Then (1) any incentive compatible allocation rule can be approximately implemented via the iterative removal of strictly dominated strategies, and (2) each of the implementing mechanisms (one for each $\epsilon$) employs a message space $M = \Theta \times \Theta$.

The idea behind the mechanisms used to achieve implementation in Proposition 4.2 is as follows. Each agent submits two messages on his own type. The principal uses the two messages submitted by agent i in the following fashion. In order to determine the $\epsilon$-neighborhood of agent i's allocation, the principal uses agent i's second message and all other agents' first messages. In order to determine agent i's actual allocation within that neighborhood, the principal uses only agent i's first message. The single crossing property makes it possible for the principal to choose allocations within each of the $\epsilon$-neighborhoods that induce truth-telling, i.e., any strategy that has an agent sometimes reporting his first message falsely is strictly dominated by a strategy that is otherwise the same except that it has that agent always reporting his first message truthfully. The delicate part of the construction is ensuring truth-telling across $\epsilon$-neighborhoods, i.e., ensuring that given that
all first messages are truthful, any strategy that has an agent sometimes reporting his second message falsely is strictly dominated by the strategy that has that agent always reporting his second message truthfully.

The nature of approximate implementation achieved in Proposition 4.2 is in contrast to the usual notion of approximate implementation, which relies on randomized mechanisms (see, for example, Abreu and Matsushima (1990, 1992), Abreu and Sen (1991), and Matsushima (1988)). The use of a randomized mechanism leads to an outcome that is with probability 1 - ε the same as the outcome prescribed by the given allocation rule; with probability ε, the outcome could be very different from the outcome prescribed by the given allocation rule. The use of deterministic mechanisms in Proposition 4.2 ensures that with probability 1, the outcome is close in the physical sense to the outcome prescribed by the allocation rule.

4.4. Extensions to Public Goods Settings

Up to this point, our analysis has relied on the absence of public goods. In this section, we discuss extensions of our results to public goods settings in which the agents' preferences satisfy the single crossing property.

If large punishments are available, which our framework implicitly assumes, Proposition 4.1 can be extended to public goods settings in which information is nonexclusive. The punishment agent j receives for disagreeing with agent i's truthful report of agent i's characteristic must be large enough to outweigh any benefit agent j might receive by lying about agent i's characteristic. In Proposition 4.1, only small punishments were needed since, in the absence of public goods, agent j is unaffected by agent i's allocation.
Extending Proposition 4.2 to public goods settings seems more problematic. It is hoped that the following example might provide some insight as to how to construct simple mechanisms for public goods settings in the presence of exclusive information.

*Example 5:* In this example, we study an adverse selection setting that is similar to that of Demski and Sappington (1984) except that in our setting the two agents are hired to operate a single technology. The output of the technology \( y \) is a function of the effort \( (a^A, a^B) \) exerted by the agents and the agents' binary productivity parameters \( \theta^A \in \{\theta^A_1, \theta^A_2\} \) and \( \theta^B \in \{\theta^B_1, \theta^B_2\} \), i.e., \( y = Y(a^A, a^B, \theta^A, \theta^B) \). Agent \( i \) alone observes \( \theta^i \), but \( \theta^A \) and \( \theta^B \) are assumed to be positively (but imperfectly) correlated. The principal knows only the common prior \( p \) on \( (\theta^A, \theta^B) \). Denote by \( p_{km} \) the joint probability that \( \theta^A = \theta^A_k \) and \( \theta^B = \theta^B_m \). Let \( q(\theta^A_k | \theta^B_m) \) denote the conditional probability that \( \theta^i = \theta^A_k \), given \( \theta^i = \theta^B_m \). The contract specifies a level of output \( y \) and monetary compensation to the agents \( (x^A, x^B) \), both of which depend on the agents' reports of their productivity parameters. Let \( y_{km} \) and \( x^i_{km} \) denote the level of output and payment to agent \( i \) when agent \( A \) reports \( \theta^A_k \) and agent \( B \) reports \( \theta^B_m \). Agent \( i \)'s preferences over monetary compensation \( x^i \) and output \( y \) are represented by a von Neumann-Morgenstern utility function: \( u^i(x^i, y, \theta^i) = W^i(x^i) - D^i(y, \theta^i) \). The agents are risk averse and dislike effort increasingly, i.e., \( W^i(\cdot) > 0, W''^i(\cdot) < 0, D^i_y(\cdot) > 0, \) and \( D''^i_y(\cdot) > 0 \). A higher realization of \( \theta^i \) implies a more productive environment, i.e., \( D^i(y, \theta^i_2) < D^i(y, \theta^i_1) \) for all \( y \). We also assume the single crossing property, i.e., \( D'^{i}_{y}(y, \theta^i_2) < D'^{i}_{y}(y, \theta^i_1) \) for all \( y \). The principal maximizes expected net production (production less payments to the agents) subject to the usual individual rationality (each agent has a reservation utility of \( U^i \)) and Bayes-Nash incentive compatibility constraints. Contracting frictions arise in this adverse selection model because of risk aversion.
The second-best solution solves following program, Program (P-BN).

\[
\max_{y,x} \sum_{k=1}^{2} \sum_{m=1}^{2} p_{km} [y_{km} - x^A_{km} - x^B_{km}]
\]

\[
q(\theta^B_k | \theta^A_k)[x^A_{kk} - D^A(y_{kk}, \theta^A_k)] + q(\theta^B_m | \theta^A_k)[x^A_{km} - D^A(y_{km}, \theta^A_k)] \geq \bar{U}^A
\]  \hspace{1cm} (4.1)

\[
q(\theta^A_m | \theta^B_m)[x^B_{mm} - D^B(y_{mm}, \theta^B_m)] + q(\theta^A_m | \theta^B_k)[x^B_{km} - D^B(y_{km}, \theta^B_m)] \geq \bar{U}^B
\]  \hspace{1cm} (4.2)

Denote the solution to (P-BN) by \((y^*_1, y^*_2, y^*_1, y^*_2, x^A_{11}, x^A_{12}, x^A_{21}, x^A_{22}, x^B_{11}, x^B_{12}, x^B_{21}, x^B_{22})\). The following observation partially characterizes the solution. The proof of the observation follows arguments similar to those in Demski and Sappington (1984) and Ma, Moore, and Turnbull (1988).

**Observation:** Agent A’s optimal contract satisfies the following:

(i) \(x^A_{11} > x^A_{12}\),

(ii) \(x^A_{21} = x^A_{22} = x^A_{22}\),

(iii) \(q(\theta^B_1 | \theta^A_1)[W^A(x^A_{11}) - D^A(y^*_1, \theta^A_1)] + q(\theta^B_2 | \theta^A_1)[W^A(x^A_{12}) - D^A(y^*_2, \theta^A_1)] = \bar{U}^A\),

(iv) \(W^A(x^A_{11}) - q(\theta^B_1 | \theta^A_1)D^A(y^*_1, \theta^A_2) - q(\theta^B_2 | \theta^A_1)D^A(y^*_2, \theta^A_2) = q(\theta^B_1 | \theta^A_2)[W^A(x^A_{11}) - D^A(y^*_1, \theta^A_2)] + q(\theta^B_2 | \theta^A_2)[W^A(x^A_{12}) - D^A(y^*_2, \theta^A_2)]\), and

(v) \(q(\theta^B_1 | \theta^A_1)[W^A(x^A_{11}) - D^A(y^*_1, \theta^A_1)] + q(\theta^B_2 | \theta^A_1)[W^A(x^A_{12}) - D^A(y^*_2, \theta^A_1)] > W^A(x^A_{21}) - q(\theta^B_1 | \theta^A_1)D^A(y^*_1, \theta^A_1) - q(\theta^B_2 | \theta^A_1)D^A(y^*_2, \theta^A_1)\).

As in Demski and Sappington (1984), while truth-telling is an equilibrium in the agents’ subgame, there exist other nontruthful equilibria. In contrast to Demski and
Sappington (1984), the nontruthful equilibria may include not only the strategy combination that has both agents always reporting that their productivity parameters are low but also the strategy combination that has both agents always reporting that their productivity parameters are high. By exploiting the single crossing property, Proposition 4.3 shows that the subgame problem that arises can be dealt with by simple, deterministic mechanisms.

**Proposition 4.3:** There exist mechanisms which each employ a message space

\[ M = \{ \{ \theta_1^A, \theta_2^A, L, H \}, \{ \theta_1^B, \theta_2^B \} \} \]

that approximately implement the second-best solution via the iterative removal of strictly dominated strategies.

The key step of the proof of Proposition 4.3 is adding strategies for agent A that strictly dominate the lying equilibrium strategies, but not the truthful strategy. This is accomplished by offering agent A two additional messages, L and H. The allocations associated with L and H are chosen so that: (1) agent A prefers to report H instead of \( \theta_1^A \) and \( \theta_2^A \) instead of L if his productivity parameter is \( \theta_2^A \) (regardless of agent B's message), and (2) agent A prefers to report \( \theta_1^A \) instead of H and L instead of \( \theta_2^A \) if his productivity parameter is \( \theta_1^A \) (regardless of agent B's message). Such a construction is possible by the single crossing property and is useful in that it induces agent A to reveal his productivity parameter to the principal: a report of \( \theta_2^A \) or H indicates that agent A's productivity parameter is high, while a report of \( \theta_1^A \) or L indicates that agent A's productivity parameter is low. The problem with the above construction is that if agent A is paid \( x^*_A \) when he reports \( \theta_2^A \), he prefers to report H instead of \( \theta_2^A \) when his productivity parameter is high, given that agent B plays the truth-telling strategy. In order to remedy this problem, the payment associated with a report of \( \theta_2^A \) is increased. Since agent A's utility function is continuous in wealth, it is possible to pick payments associated with \( \theta_2^A \) that satisfy (1) and
(2) above for small $\varepsilon > 0$, where setting $x^A_2 = x^*_A + \varepsilon$ ensures that agent A will report $\theta^A_2$ instead of H when his productivity parameter is high, given that agent B plays the truth-telling strategy. Similar care must be taken to ensure that agent A prefers to report $\theta^A_1$ instead of L when his productivity parameter is $\theta^A_1$, given that agent B plays the truth-telling strategy. $y_{L2}$ is chosen sufficiently greater than $y_{L1}$ and $y_{H2}$ is chosen sufficiently greater than $y_{H1}$ so that agent B prefers the truth-telling strategy to any of the lying strategies, given that agent A reports either $\theta^A_2$ or H when his productivity parameter is $\theta^A_2$ and either $\theta^A_1$ or L when his productivity parameter is $\theta^A_1$.

Proposition 4.3, we believe, provides an interesting result. However, the contribution comes in the context of a very specific (binary) example. It would be interesting to see whether or not Proposition 4.3 can be extended to more general settings.
CHAPTER V

A SECOND LOOK AT DEMSKI, SAPPINGTON, AND SPILLER'S MODEL

5.1 Introduction

Demski and Sappington (1984) study a model in which a single principal contracts with two agents whose productivity parameters (types) are correlated. They assume that each agent privately observe his type prior to contracting (referred to as the (PI) case). They show that (1) if the agents are risk-neutral, then the first-best solution is feasible, and (2) if the agents are risk averse, a non-trivial multiple equilibria problem arises in the agents' subgame under the second-best solution. The second-best solution is a direct mechanism (each agent reports only his own type) under which truth-telling by each agent is constrained to be a best response to truth-telling by the other agent. The subgame problem is particularly troublesome for the principal since there exists a non-truthful (Bayes-Nash) equilibrium that, from the agents' perspective, Pareto dominates the truthful equilibrium. Demski and Sappington's (1984) approach to solving the subgame problem is to make truth-telling a dominant strategy for one agent and to make truth-telling a best response to truth-telling for the other agent. These results continue to hold when the information asymmetry arises subsequent to contracting (referred to as the (SI) case).

Demski, Sappington, and Spiller (1988) demonstrate that the results differ when the contracting friction arises because of bankruptcy constraints instead of risk-aversion. They show that (1) even with risk-neutral agents, the first-best solution is not generally feasible, (2) a subgame problem does not arise in the (SI) case, and (3) a subgame problem does
arise in the (PI) case. Hence, in the later case, "the question of implementation via direct mechanisms remains an open and delicate one (Demski, Sappington, and Spiller 1988)."

In this chapter, we address the issue of implementation via direct mechanisms in the (PI) case studied in Demski, Sappington, and Spiller (1988). We show that (1) without decreasing her expected utility, the principal can design a direct mechanism that replaces the Bayes-Nash incentive compatibility constraints with dominant strategy incentive compatibility constraints, i.e., the second-best solution can be equivalently implemented in dominant strategies, and (2) at an arbitrarily small cost, the principal can design a direct mechanism that makes truth-telling a strict dominant strategy for each agent. As a corollary to (2), truth-telling is obtained as a unique Bayes-Nash equilibrium in both pure and mixed strategies.

Note that in contrast to Demski and Sappington (1984), the imposition of dominant strategy incentive compatibility constraints is costless in our setting. However, similar to Demski and Sappington (1984), the principal in our setting can strengthen the dominant strategy constraints to strict dominant strategy constraints at an arbitrarily small cost. The most general characterization of conditions under which it is possible to equivalently implement Bayesian incentive compatible allocation rules in dominant strategies can be found in Mookherjee and Reichelstein (1992). Their analysis, however, does not incorporate bankruptcy constraints on the agents' payments; hence, their results cannot be directly applied to our setting.

The remainder of this chapter is organized as follows. Section 5.2 presents the model, and Section 5.3 presents the results.
5.2 Model

The model presented here is the same as the adverse selection model studied in Demski, Sappington, and Spiller (1988). A risk-neutral principal owns two productive technologies, A and B. The principal has other uses of her time and chooses to hire two agents, agent A and agent B, to operate the technologies. The output produced by technology i, i = A,B, is denoted \( x^i = X^i(a^i,\theta^i) \), where \( a^i \) is the unobservable effort exerted by agent i and \( \theta^i \in \{\theta^i_1, \theta^i_2\} \) is the productivity parameter of technology i (type of agent i). \( \theta^i_2 \) places agent i in a more productive setting than does \( \theta^i_1 \): \( X^i(a^i,\theta^i_2) > X^i(a^i,\theta^i_1) \) for all \( a^i \).

Each technology exhibits decreasing returns to effort: \( X^i_a(\cdot) > 0 \) and \( X^i_{aa}(\cdot) < 0 \). Agent i privately observes \( \theta^i \) before contracting with the principal. \( \phi \) is the joint probability distribution on \( (\theta^A, \theta^B) \), where \( \phi_{km} \) denotes the probability that \( \theta^A = \theta^A_k \) and \( \theta^B = \theta^B_m \), \( k,m = 1,2 \). Denote by \( p(\theta^i_m | \theta^i_k) \) the conditional probability that \( \theta^i = \theta^i_m \), given that \( \theta^i = \theta^i_k \).

The joint and conditional probability distributions are assumed to be common knowledge among the agents and the principal. The agents' productivity parameters are assumed to be positively but imperfectly correlated, i.e., \( p(\theta^i_k | \theta^i_k) > p(\theta^i_m | \theta^i_m), k \neq m \); \( k,m = 1,2; i,j = A,B \).

Agent i's preferences over wealth \( R^i \) and effort \( a^i \) are represented by a von Neumann-Morgenstern utility function \( u^i(R^i, a^i) = R^i - V^i(a^i) \). The agents are risk-neutral and dislike effort increasingly, i.e., \( V^i_a(\cdot) > 0 \) and \( V^i_{aa}(\cdot) > 0 \). Let \( D^i(x^i, \theta^i) = V^i_\theta(\bar{a}^i) \), where \( \bar{a}^i \) solves \( x^i = X^i(\bar{a}^i, \theta^i) \). In other words, \( D^i(x^i, \theta^i) \) is agent i's disutility from producing \( x^i \) when his type is \( \theta^i \). It follows from the above assumptions on \( X^i(\cdot) \) and \( V^i(\cdot) \) that \( D^i(x^i, \theta^i_2) < D^i(x^i, \theta^i_1) \), \( D^i_\theta(\cdot) > 0 \), and \( D^i_{xx}(\cdot) > 0 \) for all \( x^i \). Assume also that \( D^i_\theta(x^i, \theta^i_2) < D^i_\theta(x^i, \theta^i_1) \) for all \( x^i \). The last assumption is commonly referred to as the single crossing property.
The principal asks agent $i$ to communicate his type $\theta^i$ by choosing an output from the set $\{x^i_1, x^i_2\}$. If agent $i$ chooses $x^i_1$, he is sending a message to the principal that $\theta^i = \theta^i_1$, and if he chooses $x^i_2$, he is sending a message to the principal that $\theta^i = \theta^i_2$. The strategy that has agent $i$ choosing $x^i_1$ when $\theta^i = \theta^i_1$ and $x^i_2$ when $\theta^i = \theta^i_2$ is referred to as truth-telling. Denote by $R^i_{km}$ the payment the principal makes to agent $i$ if agent $A$ produces $x^A_k$ and agent $B$ produces $x^B_m$. Figure 2 clarifies the sequence of events.

Agent $i$ privately observes $\theta^i$, $i = A, B$.

The principal offers agent $i$ a choice of $x^i_1$ or $x^i_2$ and payments of $R^i_{11}, R^i_{12}, R^i_{21}, R^i_{22}$.

Agent $i$ chooses $x^i_1$, $x^i_2$, or rejects the contract. (Assume both accept.)

Agent $i$'s utility is $R^i - D^i(x^i, \theta^i)$. The principal's utility is $x^A + x^B - R^A - R^B$.

FIGURE 2. Sequence of Events

The principal's contracting problem is to maximize her expected utility (5.1) subject to the following constraints. First, the individual rationality constraints (5.2 and 5.3) require that the contract be sufficiently attractive to each agent -- the contract must provide agent $i$ with at least his reservation utility $\bar{U}^i$. Second, the bankruptcy constraints (5.4 and 5.5) require that the payment to agent $i$ never fall below a pre-specified level, $M^i$, if agent $i$ abides by the terms of the contract. It is assumed that $M^i < \bar{U}^i$. (The results presented in this chapter continue to hold if the bankruptcy constraints are on the agents' utility levels instead of payments.) Third, the Bayesian incentive compatibility constraints (5.6 and 5.7) require that agent $i$ be willing to truthfully report $\theta^i$, given that agent $j$ is reporting $\theta^j$ truthfully. The Revelation Principle (e.g., Dasgupta, Hammond, and Maskin 1979; Myerson 1979) justifies examination of the truth-telling equilibrium. The principal's contracting problem (P-BN) is as follows.
\[
\begin{align*}
\max_{R, x} & \sum_{k=1}^{2} \sum_{m=1}^{2} \phi_{km} \left[ x_k^A + x_m^B - R_{km}^A - R_{km}^B \right] \\
&
p(\theta_k^B | \theta_k^A) R_{kk}^A + p(\theta_m^B | \theta_k^A) R_{km}^A - D^A(x_k^A, \theta_k^A) \geq \bar{U}^A \quad k \neq m; k,m = 1,2 \\
&
p(\theta_m^A | \theta_m^B) R_{mm}^B + p(\theta_k^A | \theta_m^B) R_{km}^B - D^B(x_m^B, \theta_m^B) \geq \bar{U}^B \quad k \neq m; k,m = 1,2 \\
&
R_{km}^A \geq M^A \quad k,m = 1,2 \\
&
R_{km}^B \geq M^B \quad k,m = 1,2
\end{align*}
\]

Denote by \( K^* \equiv \{ x_{i_1}^A, x_{i_2}^B, R_{11}^A, R_{12}^B, R_{21}^A, R_{22}^B \}_{i=A,B} \) the solution to (P-BN). We refer to contract \( K^* \) as the second-best solution.

### 5.3 Results

In this section, we begin by characterizing the first-best solution and the second-best solution.

Under the first-best solution (when the agents' types are publicly observable), each agent's expected utility is held to his reservation level, \( \bar{U}^i \), and the output produced by each agent is such that its marginal value is equal to the agent's marginal disutility of effort. Thus, under the first-best solution, \( D_k^i(x_k^i, \theta_k^i) = 1 \) for \( i = A,B \) and \( k = 1,2 \). In the ensuing discussion, \( x_k^i \) will denote the first-best output level for agent \( i \) when \( \theta_i = \theta_k^i \). We now present two observations, which are restatements of Propositions 1 and 4 in Demski, Sappington, and Spiller (1988). (For ease of exposition, we characterize results for agent A only.)
**Observation 5.1:** The first-best solution is a feasible solution to (P-BN) if and only if

\[
[U^A - MA] - \frac{\phi_{12} - \phi_{21}}{[\phi_{11} + \phi_{12}] + \phi_{22}]} = \frac{\phi_{21}}{\phi_{21} + \phi_{22}} D^A(x^A_1, \theta^A_1) - \frac{\phi_{11}}{\phi_{11} + \phi_{12}} D^A(x^A_1, \theta^A_2) \tag{5.8}
\]

**Observation 5.2:** When the condition in Observation 5.1 does not hold, the solution to (P-BN) has the following features:

(i) \( R^{*A}_{11} > R^{*A}_{12} = MA \) \hspace{1cm} (5.9);

(ii) \[ \frac{\phi_{21}}{\phi_{21} + \phi_{22}} R^{*A}_{21} + \frac{\phi_{22}}{\phi_{21} + \phi_{22}} R^{*A}_{22} = D^A(x^A_1, \theta^A_1) - D^A(x^A_2, \theta^A_2) \]

\[ = \frac{\phi_{21}}{\phi_{21} + \phi_{22}} R^{*A}_{11} + \frac{\phi_{22}}{\phi_{21} + \phi_{22}} R^{*A}_{12} - D^A(x^A_1, \theta^A_1) \tag{5.10} \]

(iii) \( x^*_1 < x^*_1; x^*_2 = x^*_2 \) \hspace{1cm} (5.11).

It follows from Observation 5.2 that the strategy combination that has agent i always choosing \( x^*_1 \) is also an equilibrium in the agents' subgame. Further, from the agents' perspective, this non-truthful equilibrium Pareto dominates the truthful equilibrium.

Our first proposition shows that it is costless for the principal to design a direct mechanism that replaces the Bayesian incentive compatibility constraints by the following dominant strategy incentive compatibility constraints (i.e., the principal can equivalently implement the second-best solution in dominant strategies).

\[ R^A_{km} - D^A(x^A_k, \theta^A_k) \geq R^A_{nm} - D^A(x^A_n, \theta^A_k) \hspace{1cm} k \neq n; \ k,m,n = 1,2 \tag{5.12} \]

\[ R^B_{km} - D^B(x^B_m, \theta^B_m) \geq R^B_{nm} - D^B(x^B_n, \theta^B_m) \hspace{1cm} m \neq n; \ k,m,n = 1,2 \tag{5.13} \]

**Proposition 5.1:** By setting \( x^A_1 = x^*A_1, x^A_2 = x^*A_2 \), \( R^A_{11} = R^*A_1, R^A_{12} = R^*A_2 \),

\[ R^A_{21} = R^*A_1 + D^A(x^*A_2, \theta^A_2) - D^A(x^*_1, \theta^A_2), \text{ and } R^A_{22} = R^*A_2 + D^A(x^*A_2, \theta^A_2) - D^A(x^*_1, \theta^A_2), \]

the second-best solution can be equivalently implemented in dominant strategies.
The proof of Proposition 5.1 is provided in Appendix D. Although Proposition 5.1 characterizes the second-best solution so that truth-telling is a dominant strategy for each agent, truth-telling is still Pareto-dominated by the non-truthful equilibrium that has each agent always choosing \( x_{1}^{*i} \). However, Proposition 5.2 shows that the non-truthful equilibrium can be easily eliminated by increasing agent i's payments when he chooses \( x_{2}^{*i} \) by an arbitrarily small, positive amount.

**Proposition 5.2:** For any \( \varepsilon, 0 < \varepsilon < [DA(x_{2}^{*A}, \theta_{1}^{A}) - DA(x_{1}^{*A}, \theta_{1}^{A})] - [DA(x_{2}^{*A}, \theta_{2}^{A}) - DA(x_{1}^{*A}, \theta_{2}^{A})] \), setting \( x_{1}^{A} = x_{1}^{*A}, x_{2}^{A} = x_{2}^{*A}, R_{11}^{A} = R_{11}^{*A}, R_{12}^{A} = R_{12}^{*A}, R_{21}^{A} = R_{11}^{*A} + DA(x_{2}^{*A}, \theta_{1}^{A}) - DA(x_{1}^{*A}, \theta_{2}^{A}) + \varepsilon \), and \( R_{22}^{A} = R_{12}^{*A} + DA(x_{2}^{*A}, \theta_{2}^{A}) - DA(x_{1}^{*A}, \theta_{2}^{A}) + \varepsilon \) makes truth-telling a strict dominant strategy.

The upper bound on \( \varepsilon \) ensures that the increase in \( R_{21}^{i} \) and \( R_{22}^{i} \) does not induce a low-type agent to choose \( x_{2}^{i} \) instead of \( x_{1}^{i} \).

If the principal were content with a unique Bayes-Nash equilibrium, the explicit \( \varepsilon \)-cost could be avoided by using the augmented revelation approach of Ma, Moore, and Turnbull (1988) and Mookherjee and Reichelstein (1990). However, as Demski, Sappington, and Spiller (1988) point out, the use of an indirect mechanism (an augmented revelation mechanism) would come at an implicit cost of an increased message space. In contrast, we solve the subgame problem using a direct mechanism and employing a solution concept that is weaker than Bayes-Nash.
CHAPTER VI

CAPITAL BUDGETING AND RELATIVE PROJECT RANKING
IN AN ASYMMETRIC INFORMATION SETTING

6.1 Introduction

Capital budgeting has been the subject of extensive investigation in many disciplines. Much of this literature analyzes the advantages of alternative methods of evaluating a project. The benchmark is net present value (discounted cash flows). Reasons to consider other methods such as accounting rates of return, payback period, and adding a risk premium to the discount rate are discussed. These reasons often have to do with uncertainty about future cash flows and potential risk aversion of management or owners. This line of work emphasizes a classical setting, wherein the organization is characterized by costless elicitation of an informed manager's private information.

In contrast, the purpose of this chapter is to study capital budgeting in a setting where emphasis is on control over project selection. The control problem arises because the manager has private information about characteristics of his pet project and has incentives to misrepresent this information. Loosely speaking, this study explores the potential for game-playing in a capital budgeting setting and derives an optimal mechanism for controlling project selection.

This chapter explores aspects of the capital budgeting decision which are studied in industrial organization. A key question is the importance of separating the proposal and implementation of projects from their ratification (Fama and Jensen 1983; Williamson 1983), which is a common feature of capital budgeting settings in practice. Fama and
Jensen (1983) further suggest that managers' incentives to deviate from those of owners can be mitigated by developing a control system that exploits the common information among managers, using them as mutual monitors.

In this chapter, we formally demonstrate a benefit of using managers as mutual monitors in a capital budgeting setting. We further demonstrate that attempts to exploit managers as mutual monitors introduces an additional control problem that may be addressed with a simple modification to capital budgeting procedures.

We model the firm as consisting of three risk neutral individuals: a representative owner and two divisional managers. The model is an extension of the single-manager model in Antle and Eppen (1985), wherein a manager's only input is his or her information about a project's profitability. (See also Antle and Fellingham (1990) and Fellingham and Young (1990).) It is assumed that the manager has no funds of his or her own, which roughly approximates the idea that the owner is someone who has better access to capital markets than a manager. Thus, the role of the owner is to supply funds. A manager has incentives to obtain project funding because he or she has taste for slack, defined as the difference between the amount funded by the owner and the amount needed to cover the cost of the project. The owner's objective is to maximize ex ante expected profits. The owner generally finds it optimal to restrict production in order to limit a manager's informational rents by committing to a hurdle rate that is greater than the firm's cost of capital. That is, equilibrium project selection in the presence of an information asymmetry trades off the costs of capital rationing and organizational slack.

Capital rationing in practice sometimes involves more elaborate procedures than simply setting a hurdle rate higher than the firm's cost of capital. For example, some firms may establish a procedure wherein proposed projects are ranked and only a pre-specified
number of projects are accepted, or only a pre-specified amount of resources is invested in total.

Our results include a description of how an optimal control mechanism (a set of capital budgeting procedures) uses the private information of the two managers in deciding which projects get approved. We demonstrate that optimal capital budgeting procedures may require a comparison of managers' claims regarding the profitability of their projects, i.e., mutual monitoring. In fact, we show that there are cases in which capital budgeting procedures are valuable only because they facilitate comparison of managers' projects -- independent treatment of managers would make their budgets valueless for purposes of project evaluation by the owner.

In our model, resource rationing does not result because of a desire to encourage competition but is a consequence of the owner's ability to use one manager's budget to better trade off slack and rationing for the other manager. The presence of correlation makes a manager's honest communication regarding his project informative about the other manager's project. There exist benefits from using capital budgeting procedures that compare projects if the rate of return on one manager's project is sufficiently correlated with the rate of return on the other manager's project.

We also demonstrate that under positive correlation, the optimal contract between top management and divisional managers involves relative project ranking. In particular, if one manager claims a low rate of return and the other a high rate of return, only the latter project is funded.

Top management's use of relative project ranking induces a second incentive problem wherein the managers' subgame includes an equilibrium that encourages the managers to coordinate their budgeting behavior in a way that is detrimental to the principal. However, we demonstrate that the managers can be dissuaded from such
coordination by a simple mechanism which provides a little extra slack to a manager who proposes a project with a high rate of return. It is only in addressing this subgame problem that capital budgeting procedures are modified so as to encourage competition.

The remainder of the paper is organized as follows. Section 6.2 presents the model. Section 6.3 presents benchmark solutions. Section 6.4 describes our results on mutual monitoring and relative project ranking. Section 6.5 provides a numerical example, Section 6.6 is devoted to multiple equilibria concerns, Section 6.7 presents extensions, and Section 6.8 concludes.

6.2 Model

A risk-neutral owner (principal) runs two divisions. Because the owner has other uses of her time, the divisions are managed by risk-neutral managers (agents), denoted A and B. Each manager is responsible for providing information to the owner about potential projects. The information is then used by the owner to decide which projects are approved. This process is meant to be typical of capital budgeting procedures within the firm.

Two features create contracting frictions. First, each manager privately learns the rate of return on his own project prior to the project approval decision. The owner never observes or verifies any of the managers' private information. Second, we assume that each manager obtains all investment funds from the owner.

The sequence of events is as follows. First, the owner offers take-it-or-leave-it contracts to the managers. Second, subsequent to contracting, each manager i privately learns his project's rate of return, denoted by \( \rho^i > 0 \). This timing is consistent with the notion that managers' proximity to operations makes them better informed than their superiors.
Third, the managers submit budgets to the owner. We assume that if $y^i$ dollars are invested, manager $i$'s project earns $y^i(1+\rho^i)$ dollars of revenue, i.e., constant returns to scale. This permits a linear programming formulation of the capital budgeting problem. It is convenient to let $c^i$ denote the minimum amount of investment needed to produce one dollar of revenue, i.e., $c^i = \frac{1}{1 + \rho^i}$. Our constant returns to scale assumption implies that $c^i x^i$ dollars must be invested to produce $x^i$ dollars of revenue. We further assume $c^i \in \{c^i_L, c^i_H\}$, where $0 < c^i_L < c^i_H < 1$. Thus, project $i$'s rate of return is greater if $c^i_L$ rather than $c^i_H$ is realized. Henceforth, low (high) costs and high (low) rates of return are synonymous. We denote the budgets of manager A and B by $\hat{c}^A$ and $\hat{c}^B$, respectively. For existence of an optimal contract, we assume capacity is constrained, i.e., $x^i \leq X$.

Although at the time of contracting neither the owner nor the managers observe $c^i$, there is a common knowledge (prior) probability distribution on the costs. Denote by $p_{km}$ ($k,m = L,H$) the joint probability that $c^A = c^A_k$ and $c^B = c^B_m$, $p_{km} > 0$, $\sum p_{km} = 1$. Further, denote by $p^i_k$ the marginal probability that $c^i = c^i_k$. It will be useful to interpret some of our results in terms of the sign of the correlation coefficient, which is the same as the sign of:

$$p_{LL} p_{HH} - p_{LH} p_{HL}.$$  

The contracts offered by the owner consist of budget-contingent investment and revenue amounts for each manager. The contract specifies: (1) the amount of resources (investment), denoted $y^i_{km}$, to be transferred to manager $i$ at the beginning of the period and (2) the amount of production (revenue), denoted $x^i_{km}$, manager $i$ must return to the owner at the end of the period if budgets of $\hat{c}^A = c^A_k$ and $\hat{c}^B = c^B_m$ are submitted. Revenue is jointly observable and belongs to the owner. Manager $i$ bears the cost $c^i x^i_{km}$ of producing revenue of $x^i_{km}$ and consumes any slack $y^i_{km} - c^i x^i_{km}$. The owner retains the residual $x^A_{km} + x^B_{km} - y^A_{km} - y^B_{km}$. We assume a discount rate of 0. (A nonzero discount rate could be
incorporated into our analysis but would not change the qualitative implications.) Note that if the owner herself observed the costs, she would accept all projects, since even high cost projects have a positive net present value. Figure 1 summarizes the sequence of events.

![Sequence of Events Diagram]

**FIGURE 3.** Sequence of Events

The owner's contracting problem is to maximize her expected residual (6.1) subject to the following constraints. First, the individual rationality constraints (6.2 and 6.3) require that the contract be sufficiently attractive to each manager; the contract must provide each manager with at least his reservation utility, denoted $\bar{U}_j \geq 0$. Second, since the revelation principle (Myerson 1979) applies to our model, without loss of generality, we assume that the contract is written as one which induces each manager to truthfully budget his project's cost, given that the other manager's budget is truthful (6.4 and 6.5). Third, the bankruptcy constraints (6.6 and 6.7) require that the contract respect the managers' lack of resources. Fourth, the output feasibility constraints (6.8) reflect that there is an upper bound $X$ on revenue. The nonnegativity (6.9) are part of the standard linear programming formulation.
The owner's problem (P-BN) is as follows.

\[
\max_{x_{km}^i, y_{km}^i} \sum_{i=A,B} \sum_{k=L,H} \sum_{m=L,H} P_{km} [x_{km}^i - y_{km}^i]
\]

(6.1)

\[
\sum_{k=L,H} \sum_{m=L,H} P_{km} [y_{km}^A - c_k^A x_{km}^A] \geq \bar{U}^A
\]

(6.2)

\[
\sum_{k=L,H} \sum_{m=L,H} P_{km} [y_{km}^B - c_m^B x_{km}^B] \geq \bar{U}^B
\]

(6.3)

\[
\sum_{m=L,H} \frac{P_{km}}{P_{kL} + P_{kH}} [y_{km}^A - c_k^A x_{km}^A] \geq \sum_{m=L,H} \frac{P_{km}}{P_{kL} + P_{kH}} [y_{nm}^A - c_k^A x_{nm}^A] \quad k,n = L,H
\]

(6.4)

\[
\sum_{k=L,H} \frac{P_{km}}{P_{Lm} + P_{Hm}} [y_{km}^B - c_m^B x_{km}^B] \geq \sum_{k=L,H} \frac{P_{km}}{P_{Lm} + P_{Hm}} [y_{kn}^B - c_m^B x_{kn}^B] \quad m,n = L,H
\]

(6.5)

\[
y_{km}^A - c_k^A x_{km}^A \geq 0 \quad k,m = L,H
\]

(6.6)

\[
y_{km}^B - c_m^B x_{km}^B \geq 0 \quad k,m = L,H
\]

(6.7)

\[
x_{km}^i \leq \mathcal{X} \quad k,m = L,H
\]

(6.8)

\[
x_{km}^i, y_{km}^i \geq 0 \quad k,m = L,H
\]

(6.9)
The truth telling constraints reflect that when each manager learns his own cost he revises his beliefs concerning the other manager's costs. The extent of such belief revision is critical in determining whether comparison of the managers' reports is optimal. Note that the owner's program (P-BN) identifies a revelation mechanism under which truth-telling is an (Bayes-Nash) equilibrium in the managers' subgame -- truth by one manager is a best response to truth by the other manager. The solution to this program is referred to as the second-best solution.

6.3 Benchmark Solutions

We present two different benchmark solutions. First, we characterize the full information solution and show that it is generally unattainable. Second, we present the solution if the owner constrains herself to treating the managers independently, i.e., she offers the managers optimal single-agent contracts.

Full information Solution

The full information solution calls for maximum production in both low and high cost projects, (i.e., \(x_{km}^i = X, k, m = L, H\)) and providing manager \(i\) with expected slack of \(\bar{U}^i\). With risk neutral managers, this solution can be achieved if project \(i\) can be rented to manager \(i\) for an amount equal to its expected profit minus \(\bar{U}^i\). This contract does not violate the bankruptcy constraints if \(\bar{U}^i\) is greater than \(U_{\text{max}}^i \equiv p_L^i (c_H^i - c_L^i)X\). We henceforth restrict \(\bar{U}^i\) to be strictly less than \(U_{\text{max}}^i\).

When \(\bar{U}^i\) is less than \(U_{\text{max}}^i\), the owner can achieve the full information solution only by making the amount of resources contingent on the manager's budget. For example, if \(\bar{U}^i = 0\), the owner optimally sets \(x_{km}^i = X\) and \(y_{km}^i = c_k^i X\). However, when
the cost is privately observed by the manager, the full information solution is infeasible, since it violates the truth-telling constraints.

Single-Agent Solution

One may characterize the optimal single-agent solution by adding to the owner's program (P-BN) constraints that require that the managers be treated independently. These additional constraints would state that each manager's investment and revenue amounts may depend only on his own report.

Proposition 6.1 characterizes optimal single-agent contracts. (For expository purposes, throughout the paper we characterize optimal contracts for manager A only; symmetric optimal contracts for manager B can be characterized by replacing A by B, \( p_{LH} \) by \( p_{HL} \), and \( p_{HL} \) by \( p_{LH} \).)
**Proposition 6.1:** Table 1 characterizes optimal single-agent contracts.

**TABLE 1**

Optimal Single-Agent Contracts

<table>
<thead>
<tr>
<th>Rationing</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{p_H^A}{p_L^A} \leq \frac{c_H^A - c_L^A}{1 - c_H^A}$</td>
<td>$\frac{p_H^A}{p_L^A} \geq \frac{c_H^A - c_L^A}{1 - c_H^A}$</td>
</tr>
<tr>
<td>$x_{LL}^A = x_{LH}^A = x_L^A$</td>
<td>$x_{HH}^A = x_{HL}^A = x_H^A$</td>
</tr>
<tr>
<td>$y_{LL}^A = y_{LH}^A = y_L^A$</td>
<td>$y_{HH}^A = y_{HL}^A = y_H^A$</td>
</tr>
<tr>
<td>$c_L^A x + \frac{\tilde{U}_A^A}{p_L^A}$</td>
<td>$\frac{\tilde{U}_A^A}{p_L^A(c_H^A - c_L^A)}$</td>
</tr>
<tr>
<td>$c_H^A x$</td>
<td>$c_H^A x$</td>
</tr>
</tbody>
</table>

**Proof:** See Appendix E.

Similar to Antle and Eppen (1985), optimal single-agent contracts involve resource rationing or slack. If $\tilde{U}_i^i = 0$, the bankruptcy constraints dominate the individual rationality constraints; therefore, our post-contractual information asymmetry model and Antle and Eppen's pre-contractual information asymmetry model yield identical optimal contracts. In the $\tilde{U}_i^i = 0$ case, Rationing, when compared to Slack, allows the owner to reduce manager A's expected slack by $p_L^A (c_H^A - c_L^A) X$ but implies foregone expected net profit of $p_H^A (1 - c_H^A) X$. Hence, Rationing is preferred to Slack when $\frac{p_H^A}{p_L^A} \leq \frac{c_H^A - c_L^A}{1 - c_H^A}$. Note that Rationing involves rejecting positive net present value projects.

If $\tilde{U}_i^i$ is positive, the difference between the pre-contractual and post-contractual information asymmetry models is the optimality of partial production in the post-contractual
information asymmetry setting. The reason the optimal contract in the post-contractual information asymmetry setting exhibits partial production is as follows. Suppose the owner were to offer the contract $x_L^A = X, y_L^A = c_L^A X, x_H^A = 0, y_H^A = 0$. This contract provides manager $A$ with zero slack and hence violates his individual rationality constraint if $V_A > 0$. In this case, a feasible option for the owner is to increase both $y_L^A$ and $y_H^A$ by $V_A$. This scheme satisfies the individual rationality constraint as well as the truth-telling constraints. However, it can be improved on by setting $y_L^A = c_L^A X + V_A / p_L^A$ and $y_H^A = 0$ and increasing $x_H^A$ by an amount so that the truth-telling constraint when $c^A = c_L^A$ is binding. This approach holds the agent to his reservation utility level and increases expected production; hence, the owner's expected residual is increased.

In both the post-contractual and pre-contractual information asymmetry cases, the single-agent analysis provide insights into why a firm might set its hurdle rate higher than its cost of capital. The textbook explanation of how resources are rationed involves not only setting a hurdle rate higher than the cost of capital but also revising the hurdle rate based on the budgets submitted by project managers. In other words, resource rationing is related to the notion of relative ranking of projects (Bierman et al. 1990; Davidson et al. 1985; Hilton 1991; Horngren and Foster 1991). In the next section, provide an economic motivation for the relative ranking of projects.

6.4 Optimal Bayes-Nash Contracts and Relative Project Ranking

In the spirit of the interpretation that the managers are consuming slack, we begin by assuming each manager's reservation utility is 0. Later, we investigate the role of the individual rationality constraints on the owner's contracting problem.

Before characterizing the second-best solution, we introduce the following definition of relative project ranking.
Definition: The contract offered to manager A is said to exhibit relative project ranking if when manager B’s budget indicates his project is better than A’s, B’s is funded and A’s is not.

Proposition 6.2 characterizes optimal two-agent contracts.

**Proposition 6.2:** Table 2 characterizes the second-best solution.

**TABLE 2**

<table>
<thead>
<tr>
<th>Contract N</th>
<th>Rationing</th>
<th>Contract P</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>x^A_{LL}</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>y^A_{LL}</td>
<td>c^A_HX</td>
<td>c^A_LX</td>
<td>c^A_HX</td>
</tr>
<tr>
<td>x^A_{HL}</td>
<td>X</td>
<td>0</td>
<td>c^A_HX</td>
</tr>
<tr>
<td>y^A_{HL}</td>
<td>c^A_HX</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>x^A_{LH}</td>
<td>X</td>
<td>X</td>
<td>c^A_HX</td>
</tr>
<tr>
<td>y^A_{LH}</td>
<td>c^A_LX</td>
<td>c^A_LX</td>
<td>c^A_HX</td>
</tr>
<tr>
<td>x^A_{HH}</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>y^A_{HH}</td>
<td>0</td>
<td>c^A_LX</td>
<td>c^A_HX</td>
</tr>
</tbody>
</table>

**Proof:** See Appendix E.

The Slack and Rationing contracts in Proposition 6.2 are equivalent to the optimal single-agent contracts in Proposition 6.1 -- the level of investment in manager A’s project depends only on manager A’s budget. We now provide intuition for why the owner may want to deviate from these contracts.
Contract P can be optimal whether the optimal single-manager contract is Slack or Rationing. For example, suppose that Slack is optimal for manager A in a single-agent setting, i.e., \( p_H^A \geq p_L^A \frac{c_H^A - c_L^A}{1 - c_H^A} \). Contract P is then optimal if manager B's claim that his own cost is low causes sufficient downward revision in the owner's conditional probability that A's cost is high. With sufficient revision, Rationing becomes optimal for manager A when B budgets a low cost, i.e., \( \frac{p_{HL}}{p_{HL} + p_{LL}} \leq \frac{p_{LL}}{p_{HL} + p_{LL}} \frac{c_H^A - c_L^A}{1 - c_H^A} \). Of course, for Slack to remain optimal when B budgets a high cost, we need: \( \frac{p_{HH}}{p_{HH} + p_{LH}} \geq \frac{p_{LL}}{p_{HH} + p_{LH}} \frac{c_H^A - c_L^A}{1 - c_H^A} \).

These conditions are stated in terms of conditional probabilities, and are equivalent to the conditions in Proposition 6.2 that make Contract P optimal.

The intuition provided above focused on how the owner's beliefs about one manager were affected by the other manager's budget. Implicit in the explanation is that the managers are playing the truth-telling equilibrium designed by the owner. While the contracts described in Proposition 6.2 are written so that truth-telling is a dominant strategy, multiple equilibria are still a potential problem. The problem of multiple equilibria in the managers' subgame is addressed in Section 6.6.

The discussion so far has focused on Contract P. Corollary 6.2.1 states that Contract P is optimal only if there is positive correlation between the managers' costs, and Contract N is optimal only if there is negative correlation.

**Corollary 6.2.1:** A necessary condition for Contract P to be an optimal contract is positive correlation in the managers' costs. A necessary condition for Contract N to be an optimal contract is negative correlation.

**Proof:** From Proposition 6.2, we obtain: \( \frac{p_{HH}}{p_{LH}} \geq \frac{p_{HL}}{p_{LL}} \); cross-multiplication implies positive correlation.

\( \square \)
Corollary 6.2.1: A necessary condition for Contract P to be an optimal contract is positive correlation in the managers' costs. A necessary condition for Contract N to be an optimal contract is negative correlation.

Proof: From Proposition 6.2, we obtain: \( \frac{P_{HH}}{P_{LH}} \geq \frac{P_{HL}}{P_{LL}} \); cross-multiplication implies positive correlation.

Notice that Contract N seems a little perverse in that it states that manager A's project is fully funded if his project is ranked worse than B's, but if his budget is ranked equivalent to B's (both budget high cost), it is turned down. Further, the optimality of Contract N depends on the presence of negative correlation, which may be atypical of capital budgeting settings within the firm. *We henceforth assume nonnegative correlation.*

Assuming the managers' environments are ex ante identical, Contract P satisfies the definition of relative project ranking. We now turn our attention to the relative project ranking interpretation of Contract P. Under Contract P, if manager B budgets a lower cost than A, then B's project is fully funded and A's project is rejected. However, if both managers budget a high cost or a low cost, then both projects are fully funded. Relative project ranking can be interpreted as a special form of relative performance evaluation, which has been discussed elsewhere in the principal-agent literature (e.g., Baiman and Demski 1980; Dye 1992; Holmstrom 1982).

Corollaries 6.2.2(i) and 6.2.2(ii), which follow directly from Proposition 6.2, focus on relative project ranking.

Corollary 6.2.2: (i) If the managers' costs are perfectly correlated, then the optimal contract exhibits relative project ranking and provides the owner and managers with the full information payoffs.
(ii) If the managers' costs are uncorrelated, then a twofold replication of the optimal single-agent contract is optimal, i.e., either Slack or Rationing is optimal.

Corollary 6.2.2 first states that, when the managers' cost environments are perfectly correlated, then Contract P, which exhibits relative project ranking, provides the owner and managers with full information payoffs. Second, if the managers' cost environments are uncorrelated, the best that the owner can do is to treat the managers independently.

6.5 Example

We now illustrate our main result with a numerical example. The example highlights the fact that if the managers' costs are sufficiently correlated, it is optimal for the owner to determine each project's funding based on the relative ranking of the managers' projects. Thus, from the owner's perspective, relative project ranking improves on a twofold replication of optimal single-agent contracts.
### TABLE 3
An Example Demonstrating the Benefit of Relative Project Ranking

<table>
<thead>
<tr>
<th>Panel A. The Example:</th>
<th>Panel B. Optimal Payoffs:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The cost structure: $c_L = 0.7$, $c_H = 0.8$;</td>
<td>Full information payoffs</td>
</tr>
<tr>
<td>Maximum revenue: $X = 100$;</td>
<td>Owner's expected residual = 50</td>
</tr>
<tr>
<td>Correlation: $r = 0.6$;</td>
<td>Manager i's expected slack = 0</td>
</tr>
<tr>
<td>Joint probabilities: $p_{LL} = p_{HH} = 0.4$, $p_{LH} = p_{HL} = 0.1$;</td>
<td>Single-agent payoffs</td>
</tr>
<tr>
<td>Reservation utilities: $\bar{U}^i = 0$.</td>
<td>$2 \times$ owner's expected residual = 40</td>
</tr>
<tr>
<td></td>
<td>Manager i's expected slack = 5, $i = A,B$</td>
</tr>
<tr>
<td></td>
<td>Two-agent Bayes-Nash payoffs</td>
</tr>
<tr>
<td></td>
<td>Owner's expected residual = 44</td>
</tr>
<tr>
<td></td>
<td>Manager i's expected slack = 1, $i = A,B$</td>
</tr>
</tbody>
</table>

When the owner observes the costs of the projects, both low cost and high cost projects are fully funded, and the project manager is given just enough resources to cover the cost of production. The owner's expected residual is $2X[p_L^i(1-c_L^i) + p_H^i(1-c_H^i)] = 50$, and each manager's expected slack is 0.

When each manager privately observes his own project's cost and the owner treats the managers independently, the owner prefers to offer the Slack contract to manager $i$ since $p_H^i = 1 \geq \frac{c_H^i - c_L^i}{1 - c_H^i} = .5$. The owner's expected residual is $2X(1-c_H^i) = 40$, and each manager's expected slack is 5.

When each manager privately observes his project's cost and the owner exploits the correlation in the managers' environments, she offers Contract P to each manager. Under Contract P, when the managers' budgets agree (both budget high or both budget low), the two projects are approved. However, when one manager's budget makes his project appear better than the other's, the better project is approved and the other is rejected.
Project ranking as part of the capital budgeting procedure is useful in mitigating the asymmetric information problem. Intuitively, when one manager budgets his costs are low, the owner revises her beliefs downwards that the other manager's cost is high from .5 to .2. This revision makes Rationing more desirable to the owner since
\[ \frac{p_{HL}}{p_{LL}} = \frac{1}{4} < \frac{c_H^A - c_L^A}{1 - c_H^A} = .5. \]
Under Contract P, the expected residual is 44, and each manager's expected slack is 1.

It is interesting to note that since Slack is the optimal single-agent contract, the investment and revenue amounts are independent of the budgets submitted by the managers. Budgeting becomes valuable only if the managers are used as mutual monitors. Also note that positive correlation is necessary but not sufficient for Contract P to improve upon Slack. For example, if instead \( p_{HH} = p_{LL} = .3 \) and \( p_{HL} = p_{LH} = .2 \), the marginal probability that one manager's cost is \( c_H \) is again .5. This probability is revised downwards to .4 if the other manager budgets low, but the revision is not sufficient to make Contract P optimal, i.e.,
\[ \frac{p_{HL}}{p_{LL}} = \frac{2}{3} > \frac{c_H^A - c_L^A}{1 - c_H^A} = .5. \]

6.6 Using Competition to Discourage Coordination

Before accepting the qualitative implications of the second-best solution, one should be confident that the managers will indeed play the truth-telling equilibrium. Although the second-best solution was characterized so that truth-telling is a dominant strategy in the managers' subgame, there exists a second dominant strategy (non-truthful) equilibrium. This non-truthful equilibrium has both managers always reporting a high cost. Moreover, the non-truthful equilibrium Pareto dominates (from the managers' perspective) the truth-telling equilibrium. Multiple agent settings often lead to this problem, whether the equilibrium concept is Bayes-Nash or dominant strategies (Antle 1982; Dasgupta, Hammond and Maskin 1979; Demski and Sappington 1984; Demski, Sappington, and
Spiller 1988). One may be concerned that the managers would coordinate to choose the non-truthful equilibrium.

The following proposition shows that a slight adjustment to Contract P ensures that truth-telling is a strict dominant strategy equilibrium.

**Proposition 6.3:** At an arbitrarily small cost, the principal can ensure that truth-telling is a strict dominant strategy equilibrium in the managers' subgame.

**Proof:** For any \( \varepsilon, 0 < \varepsilon < (c_H^A - c_L^A) X \), the following contract provides manager A with strict incentives to budget truthfully, no matter how manager B budgets; i.e., truthful budgeting is a strict dominant strategy for A.

\[
\begin{align*}
x_{LL}^A &= X \\
x_{LH}^A &= X \\
y_{LL}^A &= c_L^A X + \varepsilon \\
y_{LH}^A &= c_H^A X - (c_H^A - c_L^A) \frac{\varepsilon}{2} \\
x_{HL}^A &= 0 \\
y_{HL}^A &= 0 \\
x_{HH}^A &= X - \varepsilon \\
y_{HH}^A &= c_H^A (X - \varepsilon)
\end{align*}
\]

The upper bound on \( \varepsilon \) is needed to ensure that manager A will not prefer to budget a low cost when his cost is high and manager B is budgeting a low cost. Assuming a similar contract is offered to manager B, the effect on the owner's expected residual is:

\[
p_{LL} (-2\varepsilon) + p_{HH} (-\varepsilon) \left[ (1-c_H^A) + (1-c_H^B) \right] + \frac{\varepsilon}{2} \left[ p_{LH} (c_H^A - c_L^A) + p_{HL} (c_H^B - c_L^B) \right],
\]

which is greater than \(-2\varepsilon\).

The important characteristic of the mechanism presented in the proof of Proposition 6.3 is that it exploits the single crossing property, which is satisfied in our model, by making each manager's production level decreasing in his own budget. To do this, the owner commits to a small amount of rationing when both managers budget a high cost. Hence, rationing has two roles. First, in the underlying second-best solution, it enables the
owner to efficiently trade off managerial slack and forgone production. Second, it enables the owner to deal with the multiple equilibria problem.

A little rationing when both managers budget a high cost is only part of the description of Proposition 6.3's contract. What is important is that the small amount of rationing allows the owner to use competition among the managers to get them to deviate from what was the non-truthful equilibrium; considering only truth-telling and the high-high strategies, it puts the managers in a Prisoners' Dilemma. Since truthful budgeting is a strict dominant strategy equilibrium, it is also a unique Bayes-Nash equilibrium. Because the equilibrium is a strict dominant strategy, the mechanism is robust with respect to both the sequence of budgeting and the manager's beliefs about each other.

If the owner is satisfied with a unique Bayes-Nash equilibrium and the correlation between costs is imperfect, truth-telling can be achieved by simply paying each manager a small bonus for budgeting a low cost. This bonus approach leads to truth-telling after two iterations of eliminating strictly dominated strategies. The bonus approach will not work when correlation is perfect.

The fact that a direct mechanism can be used to implement an allocation rule that is arbitrarily close to the second-best solution in strict dominant strategies is not surprising, given the work of Mookherjee and Reichelstein (1992). In the context of a model more general than ours in which the single crossing property is satisfied, Mookherjee and Reichelstein show that any Bayes-Nash incentive compatible allocation rule can be equivalently implemented in dominant strategies if and only if a monotonicity condition is satisfied; equivalent and unique implementation is possible if and only if a stronger monotonicity condition is satisfied. In our setting, the monotonicity condition implies (for manager A) that $x_{LL}^A \geq x_{HL}^A$ and $x_{LH}^A \geq x_{HH}^A$; the stronger monotonicity condition implies strict inequalities. Thus, given an allocation rule that satisfies the weak but not the strong
monotonicity condition and under which the agents' subgame has multiple dominant strategy equilibria, the natural approach seems to be to try to uniquely implement some nearby allocation rule that satisfies the stronger monotonicity condition.

If the managers' costs are imperfectly correlated, an alternative approach would be to design an indirect mechanism of the type presented in Ma, Moore, and Turnbull (1988) that uniquely implements the given allocation rule. As Demski, Sappington, and Spiller (1988) point out, the use of an indirect mechanism is implicitly costly in that it relies on a larger message space than does a direct mechanism. Intuitively, a larger message space means that reports are more complicated and hence more costly to formulate, transmit, receive, and interpret.

As the following proposition demonstrates, an indirect mechanism is of no avail when the managers' costs are perfectly correlated.

**Proposition 6.4:** If the managers' costs are perfectly correlated, the second-best solution cannot be exactly implemented in Bayes-Nash equilibrium by any mechanism.

**Proof:** By Theorem 3.2 of Mookherjee and Reichelstein (1990), it suffices to show that the second-best solution cannot be exactly implemented by any augmented revelation mechanism.

If a revelation mechanism's equilibrium outcomes are to coincide with those prescribed by the second-best solution, it must be true that \( x^i(c_k^A, c_k^B) = X \) and \( y^i(c_k^A, c_k^B) = c_k^i X, k = L, H, i = A, B \). Moreover, if truth-telling is to be incentive compatible, it must be true that \( y^A(c_k^A, c_k^B) - c_k^A X^A(c_k^A, c_k^B) \geq y^A(c_m^A, c_k^B) - c_m^A X^A(c_m^A, c_k^B) \) and \( y^B(c_k^A, c_k^B) - c_k^B X^B(c_k^A, c_k^B) \geq y^B(c_k^A, c_m^B) - c_k^B X^B(c_k^A, c_m^B) \). Under any such revelation mechanism, the strategy combination that has each manager always reporting \( c_k^i \) is a Bayes-Nash equilibrium.
If an augmented revelation mechanism is to exactly implement the second-best solution, it must be true that a message of one of the two managers upsets the given non-truthful equilibrium without upsetting the truthful equilibrium. Without loss of generality, suppose this is true for manager A, and denote the message that is used to upset the non-truthful equilibrium by $m^A$. In order for $m^A$ to upset the non-truthful equilibrium, it must be true that

$$y^A(m^A,c^B_H) - c^A_Lx^A(m^A,c^B_H) > (c^A_H - c^A_L)X$$  \hspace{1cm} (1).

In order for $m^A$ not to upset the truthful equilibrium, it must be true that

$$y^A(m^A,c^B_H) - c^A_Lx^A(m^A,c^B_H) < 0$$  \hspace{1cm} (2).

But (1) and (2) imply that $x^A(m^A,c^B_H) > X$, which violates (OF). \hspace{1cm} Q.E.D.

As the proof suggests, Proposition 6.4 is driven by the output feasibility constraint. Hence, in the case of perfect correlation, the owner is forced to rely on a mechanism of the type presented in the proof of Proposition 6.3, i.e., to rely on a commitment to underproduction. This result applies to other models as well. For example, it applies to Sappington and Demski (1983) in the presence of a sufficiently small output feasibility constraint.

Our result is consistent with Palfrey and Srivastava's (1991, p.67) comment on Bayes-Nash implementation that "one suspects (and hopes) that in many specific applications, relatively simple mechanisms, resembling ones we actually observe in practice, might work." We believe that when the agents' preferences satisfy the single crossing property, an approach similar to that taken in Proposition 6.3 is useful in finding such simple mechanisms even when the underlying second-best solution cannot be equivalently implemented in dominant strategies. (Chapters 2-4 support this conjecture.)
6.7 Extensions

The Role of Managers’ Reservation Utilities

So far we have assumed that each manager’s reservation utility is 0. We now discuss the solution to the owner’s problem when one or both manager’s reservation utility may be greater than 0 (recall that we have assumed nonnegative correlation).

Proposition 6.5: Allowing for positive reservation utilities, Table 4 characterizes the second-best solution.

<table>
<thead>
<tr>
<th></th>
<th>RPR1</th>
<th>RPR2</th>
<th>RPR3</th>
<th>RPR4</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_{HL}^A)</td>
<td>(x)</td>
<td>(x)</td>
<td>(x)</td>
<td>(x)</td>
<td>(X)</td>
</tr>
<tr>
<td>(y_{HL}^A)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X + (c_H^A - c_L^A) x_{HL}^A)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X + (c_H^A - c_L^A) x_{HL}^A)</td>
<td>(c_H^A X)</td>
</tr>
<tr>
<td>(x_{HL}^A)</td>
<td>0</td>
<td>(\frac{\hat{U}^A}{p_{HL}(c_H^A - c_L^A)} - \frac{p_{HL} X}{p_{LL}})</td>
<td>0</td>
<td>(\frac{\hat{U}^A}{p_{HL}(c_H^A - c_L^A)} - \frac{p_{HL} X}{p_{LL}})</td>
<td>(X)</td>
</tr>
<tr>
<td>(y_{HL}^A)</td>
<td>0</td>
<td>(c_H^A X)</td>
<td>0</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
</tr>
<tr>
<td>(x_{HH}^A)</td>
<td>(x)</td>
<td>(x)</td>
<td>(x)</td>
<td>(x)</td>
<td>(X)</td>
</tr>
<tr>
<td>(y_{HH}^A)</td>
<td>(c_H^A X + \frac{\hat{U}^A}{p_{HL}})</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
</tr>
<tr>
<td>(x_{HH}^A)</td>
<td>(\frac{\hat{U}^A}{p_{HL}(c_H^A - c_L^A)})</td>
<td>(x)</td>
<td>(x)</td>
<td>(X)</td>
<td>(X)</td>
</tr>
<tr>
<td>(y_{HH}^A)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
<td>(c_H^A X)</td>
</tr>
</tbody>
</table>
Proposition 6.4 can be proved along the same lines as Proposition 6.2. If \( \bar{U}^A = 0 \), then RPR1 reduces to Rationing in Proposition 6.2; further, RPR3 and Slack are identical to Contract P and Slack, respectively in Proposition 6.2. Note that when \( \bar{U}^A > 0 \), optimal contracts exhibit relative project ranking under a wider range of parameter values than when \( \bar{U}^A = 0 \). In particular, under the conditions corresponding to RPR1 or RPR2, optimal contracts exhibit relative project ranking when \( \bar{U}^A > 0 \); when \( \bar{U}^A = 0 \) optimal contracts do not exhibit relative project ranking.

We now compare the Rationing contract in Proposition 6.1 and the RPR1 Contract in Proposition 6.4. If \( \bar{U}^A > 0 \) and correlation is positive, the RPR1 contract strictly increases the principal's expected residual as compared to the Rationing contract. The reason for this is as follows. Partial production in the Rationing contract occurs if manager A budgets high cost. On the other hand, partial production in the RPR1 contract occurs only if both manager A and manager B report a high cost. Doing so when the managers' cost environments are positively correlated enables the principal to increase expected production while maintaining the same expected payment to the manager.

Corollaries analogous to 6.2.1 and 6.2.2 also hold when \( 0 < \bar{U}^i < U^i_{\text{max}} \); also, the subgame problem can be dealt with by slightly modifying the mechanism presented in the proof of Proposition 6.3.

*Timing of the Information Asymmetry*

Throughout the paper we have assumed that the information asymmetry arises subsequent to contracting. If the asymmetry were to arise prior to contracting (adverse selection), then the individual rationality constraints in program (P-BN) would be replaced by the following constraints:
\[ \sum_{m=L,H} \frac{P_{km}}{P_{kL} + P_{kH}} [y_{km} - c_k x_{km}] \geq \bar{U}^A \quad k = L,H \quad (6.10) \]

\[ \sum_{k=L,H} \frac{P_{km}}{P_{kL} + P_{kH}} [y_{km}^B - c_{km}^B] \geq \bar{U}^B \quad m = L,H \quad (6.11) \]

In the adverse selection setting, a subgame problem again arises and can be dealt with in a similar fashion to the way in which it was dealt with in Proposition 6.3. Also, the qualitative implications of the second-best solution in the adverse selection setting are similar to those obtained in our post-contractual information asymmetry setting.

6.8 Conclusion

We explore a model in which an endogenous demand for relative project ranking arises. The results are derived in a multi-agent model in which the managers' cost environments are correlated. We show that it is possible for the owner to exploit the correlation to design contracts that improve on twofold replications of optimal single-agent contracts. One implication of our results is that the value of capital budgeting procedures may be increasing in the number of managers and the correlation in the managers' cost environments. In particular, as our example demonstrates, communication (budgeting) may be valueless in a single-agent setting but valuable in a two-agent setting simply because it permits the relative ranking of projects. It is interesting to note that the qualitative results were derived in a multi-agent model in which multiple equilibria problems can be easily dealt with. However, our results are derived in a limited (two-agent, two-cost) setting. It might be interesting to extend the results to a more general setting.
CHAPTER VII
CONTRIBUTION, LIMITATIONS, AND
SUGGESTIONS FOR FUTURE RESEARCH

This dissertation makes contributions to two literatures. First, from the perspective of the implementation literature, it identifies characteristics of settings in which allocation rules can be implemented by simple mechanisms. Second, from the perspective of the accounting literature, it provides insights into commonly observed practices such as relative performance evaluation in executive compensation and relative project ranking in capital budgeting.

The major limitation of this work is that although the mechanisms presented here are simple and intuitive, not enough effort is made to compare them to observed practices. Following Rajan (1991, 1992), this appears to be a fruitful avenue for future research. For example, since Rajan's (1992) fixed overhead allocation mechanism has the undesirable property that the agents' best-response correspondences are not well-defined, it seems reasonable to inquire as to the existence of a fixed overhead allocation scheme that replicates the performance of Rajan's (1992) mechanism but under which the agents' best-reply correspondences are well-defined.

One insight that can be gleaned directly from the previous chapters' results is that within the context of a specific applied model, a collusion-preventing mechanism can often be constructed that is an extension of the underlying second-best solution. In contrast to more general mechanisms that typically involve an elaborate scheme of having agents turn in other agents for anticipated deviations from the truth-telling equilibrium, the mechanisms
presented in this dissertation simply give agents additional opportunities to reveal their own private information. For example, the second-best solution in Demski and Sappington's (1984) model removes common uncertainty from the agents' environments by using relative performance evaluation. The collusion preventing mechanism developed for Demski and Sappington's setting in Chapter 2 continues to be essentially a relative performance evaluation mechanism: an agent who outproduces the other agent receives higher pay.

The remainder of this chapter is devoted to three extensions of the previous chapters' results. First, the importance of verifiability in implementation problems is discussed. Second, a role for managerial forecasts is developed. Third, a participative budgeting mechanism is presented. These extensions provide examples of links between simple theoretical mechanisms and observed practices.

The Importance of Verifiable Information: Suppose a principal contracts with a manager and an auditor. Because of limitations on the amount of information the owner can process, the owner has to choose between one of two information systems. Under the first information system, the manager is asked to report the historical cost of an asset and the auditor is asked to verify the manager's claim. Under the second information system, the manager and the auditor are each asked to report a subjective appraisal of the market value of the asset. Under either information system, the manager has incentives to overreport the value of the asset and the auditor must exert costly effort to acquire the necessary information. Under the first information system, there is no room for disagreement between the auditor and manager since the historical cost of an asset is objective in nature. Under the second system, even if the auditor and manager act as the owner intends, their appraisals may differ since appraisals are more subjective in nature. One of the results in
Chapter 4 of this dissertation is that when information is non-exclusive, implicit collusion is relatively easy to prevent. The importance of non-exclusive information is that it allows the principal (owner) to compare reports submitted by the agents (the auditor and the manager). This ability to compare the agents' reports is present if the principal employs the first information system but not present if the principal employs the second system. In other words, one advantage of historical cost reports over market value appraisals is that it is easier to deter collusion in reporting when the reported information is objective rather than subjective in nature.

All of the previous chapters' results were derived in the context of pre-contractual information asymmetry (adverse selection) models. The notion of verifiability, however, is arguably more consistent with a post-contractual information asymmetry (moral hazard) setting. As the following example demonstrates, verifiability has an important role in moral hazard settings as well.

Example 1: A risk neutral owner contracts with two risk averse agents, a manager (M) and an auditor (A). The contractual arrangement is subject to moral hazard. In particular, the owner would like to make payments to the agents (s^M and s^A) that would induce each of them to exert high effort (e^M = e^A = e_H). In the absence of an incentive to do otherwise, the manager and auditor would each prefer to exert low effort (e^M = e^A = e_L). Agent i, i = M, A, has preferences over wealth and effort that are represented by a von Neumann-Morgenstern utility function u_i(s^i,e^i) = \sqrt{s^i} - V_i(e^i), where V_i(e_H) > V_i(e_L) \geq 0. Denote \sqrt{s^i} by t^i. Agent i has a reservation utility level of U^i. After the manger (auditor) takes his private act, he privately observes a signal x (y). There is a third signal, z, which is publicly observable. The manager and auditor are asked to simultaneously submit reports of x and y, denoted by \hat{x} and \hat{y}, respectively. The contract the owner offers agent i specifies the amount of a signal/report-contingent utility transfer t_{jkm}^i, when \hat{x} = j, \hat{y} = k, and z = m.
The owner chooses both the contract and one of the two information systems presented in Table 5 so as to minimize the expected payments to the agents, subject to the usual individual rationality and (Bayes-Nash) incentive compatibility constraints.

**TABLE 5. Two Information Systems**

**Information System 1**

<table>
<thead>
<tr>
<th>e^{\text{MA}_X}</th>
<th>1</th>
<th>2</th>
<th>\text{x'y}</th>
<th>1</th>
<th>2</th>
<th>\text{x'z}</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_L</td>
<td>.75</td>
<td>.25</td>
<td>.75</td>
<td>.25</td>
<td></td>
<td>.75</td>
<td>.25</td>
<td></td>
</tr>
<tr>
<td>e_H</td>
<td>.25</td>
<td>.75</td>
<td>.25</td>
<td>.75</td>
<td></td>
<td>.25</td>
<td>.75</td>
<td></td>
</tr>
</tbody>
</table>

**Information System 2**

<table>
<thead>
<tr>
<th>e^{\text{MA}_X}</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>\text{x'y}</th>
<th>1</th>
<th>2</th>
<th>\text{x'z}</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>e_L</td>
<td>.4</td>
<td>.2</td>
<td>.1</td>
<td>.3</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>.75</td>
<td>.25</td>
</tr>
<tr>
<td>e_H</td>
<td>.2</td>
<td>.2</td>
<td>.1</td>
<td>.5</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>.75</td>
<td>.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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<td>3</td>
<td>.4</td>
<td>.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>.4</td>
<td>.6</td>
</tr>
</tbody>
</table>
Information System 1 provides subjective or unverifiable information, since the agents' reports may disagree even if they are both acting as the owner intends. Information System 2, on the other hand, provides objective or verifiable information, since the agents' reports cannot disagree if the agents are acting as the owner intends.

The second-best (Bayes-Nash) solution is presented in Table 6.

TABLE 6
The Second-Best Solution

<table>
<thead>
<tr>
<th>Information System 1</th>
<th>Information System 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t^M_{11} = 205 )</td>
<td>( t^A_{11} = 310 )</td>
</tr>
<tr>
<td>( t^M_{11z} = 205 )</td>
<td>( t^A_{11z} = 310 )</td>
</tr>
<tr>
<td>( t^M_{121} = 205 )</td>
<td>( t^A_{121} = 70 )</td>
</tr>
<tr>
<td>( t^M_{122} = 205 )</td>
<td>( t^A_{122} = 70 )</td>
</tr>
<tr>
<td>( t^M_{211} = 157 )</td>
<td>( t^A_{211} = 190 )</td>
</tr>
<tr>
<td>( t^M_{212} = 265 )</td>
<td>( t^A_{212} = 190 )</td>
</tr>
<tr>
<td>( t^M_{221} = 265 )</td>
<td>( t^A_{221} = 270 )</td>
</tr>
<tr>
<td>( t^M_{222} = 277 )</td>
<td>( t^A_{222} = 270 )</td>
</tr>
</tbody>
</table>

Denote by \( k \) the expected revenue (the owner's expected payoff before deducting payments to the agents) when the agents play the truth-telling (and hard working) equilibrium in their subgame. If the agents act as the owner intends, the owner's expected payoff under Information System 1 is \( k - 129,882.5 \) and under Information System 2 is \( k - 130,400 \). Before counting on such a payoff, the owner must be sure that the agents will indeed play the truth-telling equilibrium. This seems doubtful since, from the agents' perspective, the truth-telling equilibrium is Pareto-dominated by a non-truthful equilibrium
under either information system. This entices the agents into coordinating their behavior (implicit collusion). In the case of Information System 2, the owner can costlessly ensure that the truth-telling equilibrium is Pareto-undominated by making $t^M$ depend on $z$ as in Table 7.

**TABLE 7**

Producing a Pareto-Undominated Equilibrium under Information System 2

<table>
<thead>
<tr>
<th>$t^M_{111}$ = 160</th>
<th>$t^M_{311}$ = 159</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^M_{112}$ = 160</td>
<td>$t^M_{312}$ = 161</td>
</tr>
<tr>
<td>$t^M_{121}$ = 311</td>
<td>$t^M_{321}$ = 310</td>
</tr>
<tr>
<td>$t^M_{122}$ = 309</td>
<td>$t^M_{322}$ = 310</td>
</tr>
<tr>
<td>$t^M_{211}$ = 160</td>
<td>$t^M_{411}$ = 159</td>
</tr>
<tr>
<td>$t^M_{212}$ = 160</td>
<td>$t^M_{412}$ = 161</td>
</tr>
<tr>
<td>$t^M_{221}$ = 311</td>
<td>$t^M_{421}$ = 310</td>
</tr>
<tr>
<td>$t^M_{222}$ = 309</td>
<td>$t^M_{422}$ = 310</td>
</tr>
</tbody>
</table>

Under Information System 1, producing a Pareto-undominated equilibrium necessitates having the manager's compensation depend only on $x$ and $z$; optimally, the owner sets $t^M_{1 km} = 205$, $t^M_{2 k1} = 175$, and $t^M_{2 k2} = 295$. This is costly, leaving the owner with an expected payoff of $k$ - 131,300.

In summary, Information System 1 is preferred to Information System 2 before multiple equilibria concerns are addressed, while Information System 2 is preferred to Information System 1 after such concerns are addressed. Hence, one benefit of verifiable information is that it is easier to deter collusion in reporting when the reported information is objective (verifiable) rather than subjective in nature.
Management Forecasts

Example 2: Assume the same setup as in Example 1, except that at a cost of \( c \) the owner can modify Information System 1 to allow for a management forecast of \( z, \hat{z} \). Denote by \( t_{ijk\text{mn}}^i \) the signal/report-contingent utility transfer made to agent \( i \) when the manager reports \( \hat{x} = j \) and \( \hat{z} = n \), the auditor reports \( \hat{y} = k \), and the owner observes \( z = m \). With the addition of a management forecast, the payment scheme presented in Table 8 ensures that the truth-telling equilibrium is Pareto-undominated and (before subtracting \( c \)) produces an expected payoff for the owner that is arbitrarily close to \( k - 129882.5 \).

TABLE 8

Producing a Pareto-Undominated Equilibrium under Information System 1

<table>
<thead>
<tr>
<th>Manager</th>
<th>Auditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{1111} )</td>
<td>205 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1112} )</td>
<td>205 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1121} )</td>
<td>205 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1122} )</td>
<td>205 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1211} )</td>
<td>205 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1212} )</td>
<td>205 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1221} )</td>
<td>205 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{1222} )</td>
<td>205 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2111} )</td>
<td>157 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2112} )</td>
<td>157 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2121} )</td>
<td>265 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2122} )</td>
<td>265 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2211} )</td>
<td>265 + ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2212} )</td>
<td>265 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2221} )</td>
<td>277 - ( \varepsilon )</td>
</tr>
<tr>
<td>( t_{2222} )</td>
<td>277 + ( \varepsilon )</td>
</tr>
</tbody>
</table>
Hence, as long as $c < k - 129882.5 - (k - 130400) = 517.5$, the owner prefers the modified Information System 1 to Information System 2.

If the owner insists on a unique equilibrium (not just a Pareto-undominated equilibrium), the payment scheme presented in Table 9 ensures that truth-telling is the only strategy combination that survives two rounds of iteratively eliminating strictly dominated strategies.

**TABLE 9**

Producing a Unique Equilibrium under Information System 1

<table>
<thead>
<tr>
<th>Manager</th>
<th>Auditor</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{1111}$</td>
<td>$205 + 2\varepsilon$</td>
</tr>
<tr>
<td>$t_{1112}$</td>
<td>$205 - 2\varepsilon$</td>
</tr>
<tr>
<td>$t_{1121}$</td>
<td>$205 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{1122}$</td>
<td>$205 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{1211}$</td>
<td>$205 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{1212}$</td>
<td>$205 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{1221}$</td>
<td>$205 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{1222}$</td>
<td>$205 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{2111}$</td>
<td>$157 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{2112}$</td>
<td>$157 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{2121}$</td>
<td>$265 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{2122}$</td>
<td>$265 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{2211}$</td>
<td>$265 + \varepsilon$</td>
</tr>
<tr>
<td>$t_{2212}$</td>
<td>$265 - \varepsilon$</td>
</tr>
<tr>
<td>$t_{2221}$</td>
<td>$277 - 3\varepsilon$</td>
</tr>
<tr>
<td>$t_{2222}$</td>
<td>$277 + 3\varepsilon$</td>
</tr>
</tbody>
</table>
A Participative Budgeting Mechanism:

Consider again the model of Dems & Sappington (1984), except now allow for an arbitrary, finite number of agents and cost parameters, and the output of each agent depends on both agents’ reports. The mechanism used in Proposition 4.2 can be applied.

1. Agent $i$, $i \in N$, privately observes his cost parameter, $\theta^i \in \Theta^i = \{\theta_1^i, \ldots, \theta_T^i\}$.
2. The principal offers contracts.
3. Agent $i$ submits $m_1^i, m_2^i \in \Theta^i$.
4. $m$ determines agent $i$’s output, $x^i$, and pay, $R^i$.

**FIGURE 4.** Proposition 4.2’s Mechanism

Unfortunately, relating the above mechanism to observed practices seems difficult. In particular, what interpretation should be given to the agents being asked to report their cost parameters twice? Fortunately, a slight reformulation yields a more intuitive mechanism.

1. Agent $i$, $i \in N$, privately observes $\theta^i \in \Theta^i = \{\theta_1^i, \ldots, \theta_T^i\}$.
2. The principal offers contracts.
3. Agent $i$ submits $m^i \in \Theta^i$.
4. $m$ determines agent $i$’s profit target, $\gamma^i$.
5. Agent $i$ produces $x^i(m) + \eta^i$, $\eta^i \in \{0, \ldots, \eta_{T-1}^i\}$, and is paid $R^i(m, \eta^i) = s^i(m) + b^i(m) \cdot \eta^i - p^i(m, \eta^i)$.

**FIGURE 5.** A Participative Budgeting Mechanism

A feature of the budgeting mechanism that bears some resemblance to observed budgeting practices is that the mechanism exhibits a sort of relative performance evaluation.
in the budgeting process. A feature of the mechanisms that seems to bear little resemblance to observed practices is that at equilibrium, agents just meet their profit targets ($\eta^1 = 0$). It is hoped that in a model that allows for residual uncertainty, a mechanism more closely resembling observed practices might be found.
APPENDIX A

Proof of Proposition 2.1: Set \( R_{11} = R_{11}^* + \varepsilon', R_{12} = R_{12}^* + \varepsilon', \) and \( R_{21} = R_{22} = R_2 = R_2^* + \varepsilon'' \), where \( \varepsilon' \) and \( \varepsilon'' \) are chosen between 0 and \( \varepsilon \) so that

\[
U(R_2) - D(x_2^*, \theta_1) < U
\]
\[
(1), \quad \text{and}
\]
\[
U(R_2) - D(x_2^*, \theta_2) >
\]
\[
\text{p}_{21} U(R_{11}) + \text{p}_{22} U(R_{12}) - D(x_1^*, \theta_2)
\]
\[
(2).
\]

Since (2.2) is binding, (2.4) is not binding, the solution to (P-BN) satisfies (2.5), and agent \( i \)'s preferences are continuous in \( R_i \), such \( \varepsilon' \) and \( \varepsilon'' \) exist.

Pick \( x_3 > x_1^* \), \( R_{31} > R_{11} \), and \( R_{32} > R_{12} \) so that

\[
U(R_{31}) - D(x_3, \theta_2) > U(R_{11}) - D(x_1^*, \theta_2)
\]
\[
(3),
\]
\[
U(R_{32}) - D(x_3, \theta_2) > U(R_{12}) - D(x_1^*, \theta_2)
\]
\[
(4),
\]
\[
U(R_{31}) - D(x_3, \theta_1) < U(R_{11}) - D(x_1^*, \theta_1)
\]
\[
(5),
\]
\[
U(R_{32}) - D(x_3, \theta_1) < U(R_{12}) - D(x_1^*, \theta_1)
\]
\[
(6), \quad \text{and}
\]
\[
U(R_2) - D(x_2^*, \theta_2) >
\]
\[
\text{p}_{21} U(R_{31}) + \text{p}_{22} U(R_{32}) - D(x_3, \theta_2)
\]
\[
(7).
\]

(3) - (6) rely on the single crossing property, while (7) relies on the assumption that the agents' utility functions are continuous in wealth. The continuity assumption allows us to pick \( R_{31} \) and \( R_{32} \) satisfying (3) - (6) sufficiently small that (7) is also satisfied.
Set $R_{13} = R_{12}$ and $R_{23} = R_2$. Denote by $x_0$ the choice by an agent to refuse his contract. Set $R_{10} = R_{11}$, $R_{20} = R_2$, and $R_{30} = R_{31}$.

It remains to be shown that truth-telling is the unique strategy combination that survives the iterative removal of strictly dominated strategies. This will be shown in six steps.

**Step 1:** A will not choose $x_2^*$ or $x_3$ when $\theta^A = \theta_1$, and B will not choose $x_2^*$ when $\theta^B = \theta_1$. This is by equations (1), (5), and (6).

**Step 2:** A will not choose $x_1^*$ when $\theta^A = \theta_2$. This is by (3) and (4).

**Step 3:** Agent $i$, $i = A, B$, will not refuse the contract when $\theta^i = \theta_2$. This is by the choice of $R_2$.

**Step 4:** Agent $i$, $i = A, B$, will not refuse the contract when $\theta^i = \theta_1$. This is by Step 1 and the choice of $R_{10}$.

**Step 5:** B will not choose $x_1^*$ when $\theta^B = \theta_2$. This is by Steps 2 and 3, the fact that $R_{13} = R_{12}$, and (2).

**Step 6:** A will not choose $x_3$ when $\theta^A = \theta_2$. This is by Steps 3 and 5 and (7). Q.E.D.

**Proof of Proposition 2.2:** From Selten (1975), we know that the agents' finite subgame has a perfect equilibrium. We also know that every perfect equilibrium is undominated. Under the second-best solution, each agent's truth-telling strategy is dominated by the strategy that has him always choosing $x_1$. By making additional strategies available to one agent but not the second agent, we can ensure that truth-telling is undominated for the second agent, but truth-telling will still be dominated for the first agent. Hence, truth-telling cannot be the unique equilibrium in the agents' subgame. Q.E.D.
APPENDIX B

Proof of Proposition 3.1: Let \( M^i = \Theta^i \times \Theta^{i+1} \), and write \( m^i \) as \((m^i_1, m^i_2)\). Abreu and Matsushima (1990) show that (A2) implies that there exists an outcome function \( t^i: \Theta^i \to A \) such that for all \( \theta^i \in \Theta^i \) and all \( \theta^{i+1} \in \Theta^{i+1} \setminus \{\theta^i\} \), \( U^i(t^i(\theta^i), \theta^{i+1}) > U^i(t^i(\theta^i), \theta^i) \).

Choose \( t^i \) as defined above, and fix an allocation rule \( f \). Suppose \( f(m_2) = a \). For all \( J \subseteq N \), choose \( a^j \) as in (A1).

Define \( h: M \to A \) as follows.

\[
h(m) = \begin{cases} 
  a & \text{if } m^i_2 = m_1^{i+1} \text{ for all } i \in N, \\
  a^j, \text{ where } J = \{j | m^i_2 \neq m_1^{i+1}\}, & \text{otherwise}.
\end{cases}
\]

For any \( \varepsilon \in [0,1) \), define \( g: M \to A \) by

\[
g(m) = \frac{\varepsilon}{n} \sum_{i=1}^{n} t^i(m^i) + (1-\varepsilon) h(m).
\]

We now verify that \((M, g)\) is the required mechanism. From agent \( i \)'s perspective, his first message affects him only through \( t^i \). Hence, any strategy that has agent \( i \) reporting his first message falsely is strictly dominated by a strategy that is otherwise the same except that it has agent \( i \) reporting his first message truthfully. Further, given that all agents' first messages are truthful, \( g \) provides agent \( i \) with strict incentives to report his second message truthfully, since he is severely punished if his second message is not identical to agent \( i+1 \)'s first message. Q.E.D.
The following lemma will be used in the proof of Proposition 3.2.

**Lemma:** There exists an outcome function \( \tilde{t}^i : \Theta^i \rightarrow A^i \) such that for all \( \theta^i \in \Theta^i \) and all \( \theta^i \in \Theta^i \setminus \{ \theta^i \} \), \( U^i(\tilde{t}^i(\theta^i), \theta^i) > U^i(t^i(\theta^i), \theta^i) \).

**Proof of the Lemma:** This lemma is identical to the lemma in Abreu and Matsushima (1990), except that \( \tilde{t}^i \) maps on to the set of simple lotteries over \( X^i \) rather than to the set of simple lotteries over \( X \). (A5) implies that for all \( \theta^i \in \Theta^i \) and all \( \theta^i \in \Theta^i \setminus \{ \theta^i \} \), there exist lotteries \( b^i, c^i \in A^i \) such that \( U^i(b^i, \theta^i) > U^i(c^i, \theta^i) \) and \( U^i(b^i, \theta^i) < U^i(c^i, \theta^i) \).

Let \( B^i \) be a finite subset of \( A^i \), such that for all \( \theta^i \in \Theta^i \) and all \( \theta^i \in \Theta^i \setminus \{ \theta^i \} \), there exist \( b^i \in B^i \) and \( c^i \in B^i \) which satisfy the above inequalities. Let \( J = |B^i| \), and \( \beta_j = (J-j+1)/(1+\ldots+J) \). For all \( \theta^i \in \Theta^i \) and all \( j \in \{1,\ldots,J\} \), let \( b_j^i(\theta^i) \) be lotteries which satisfy \( \{b_j^i(\theta^i) : j \in \{1,\ldots,J\}\} = B^i \), and \( U^i(b_j^i(\theta^i), \theta^i) > U^i(b_{j+1}^i(\theta^i), \theta^i) \) for all \( j = 1,\ldots,J-1 \). The required function \( \tilde{t}^i \) is defined by \( \tilde{t}^i(\theta^i) = \sum_{j=1}^{J} \beta_j b_j^i(\theta^i) \). Q.E.D.

**Proof of Proposition 3.2:** Let \( M^i = \Theta^i \times \Theta^i \), and write \( m^i \) as \( (m_1^i, m_2^i) \). We know from the above lemma that there exists an outcome function \( \tilde{t}^i : \Theta^i \rightarrow A^i \) such that for all \( \theta^i \in \Theta^i \) and all \( \theta^i \in \Theta^i \setminus \{ \theta^i \} \), \( U^i(\tilde{t}^i(\theta^i), \theta^i) > U^i(t^i(\theta^i), \theta^i) \).

Choose \( \tilde{t}^i \) as defined above, and fix an incentive compatible allocation rule \( f \). For any \( \varepsilon \in [0,1) \), define \( g^i \) by
\[
g^i(m) = (\varepsilon/2)\tilde{t}^i(m_1^i) + (\varepsilon/2)\tilde{t}^i(m_2^i) + (1-\varepsilon)f^i(m_1^i, m_2^i).
\] By (A4), \( g = (g^1, \ldots, g^n) \) is an outcome function.

We now verify that \( (M, g) \) is the required mechanism. Since \( g^i \) uses agent \( i \)'s first message only in \( \tilde{t}^i \), any strategy that has agent \( i \) reporting his first message sometimes falsely is strictly dominated by a strategy that is the same except that it has agent \( i \) always reporting his first message truthfully. Further, given that all agents' first messages are
truthful, $g^i$ provides agent $i$ with strict incentives to report his second message truthfully.

This follows from the construction of $t^i$ and the incentive compatibility of $f$. Q.E.D.
APPENDIX C

Proof of Proposition 4.1: Let $M^i = \Pi^i$. Fix an allocation rule $f = (y,x)$. Because information is nonexclusive it is possible to construct for any $i \in N$, a function $h^i : M^i \to Y^i \times X^i$ s.t. $h^i(\pi^i(\theta)) = f^i(\theta)$ for all $\theta \in \Theta$. Further, the single crossing property and the openness of $Y^i \times X^i$ enables us to construct a function $t^i : Y^i \times X^i \times M^i \to Y^i \times X^i$ s.t.

$$t^i(h^i(m^i), m^i) = h^i(m^i) \text{ if for all } \theta \in m^i \text{ and all } \tilde{\theta} \in \bigcap_{j \in N \setminus \{i\}} m^j, \tilde{\theta}^i = \theta^i \quad (1),$$

and

$$u^i(t^i(h^i(m^i), \theta^i) \theta^i) > u^i(t^i(h^i(m^i), m^i), \theta^i) \text{ and } (t^i(h^i(m^i), m^i)) \in Y^i \times X^i$$

for all $\theta^i \in \Theta^i$, all $m^i \in M^i$, all $m^i \in M^i$ s.t. for some $\tilde{\theta} \in m^i$, $\tilde{\theta}^i \neq \theta^i \quad (2)$.

Pick any $p' > 0$. Let $p^i(m) = p'$ if for some $\tilde{\theta} \in m^i$, $j \in N \setminus \{i\}$, $\theta \in m^j$, $\tilde{\theta}^j \neq \theta^j$.

Let $p^i(m) = 0$, otherwise. Let $g^i : M \to Y^i \times X^i$ be defined by $g^i(m) = t^i(h^i(m^i), m^i) - (0, p^i(m))$, and let $g = (g^1, ..., g^n)$.

We now verify that $(M, g)$ is the required mechanism. Implementation is achieved in two rounds of iteratively eliminating strictly dominated strategies. In the first round, by (2), any strategy that has agent $i$ sometimes reporting $\theta^i$ falsely is strictly dominated by a strategy that is otherwise the same except that it has agent $i$ always reporting $\theta^i$ truthfully. Given that all agents report their own preference characteristics truthfully, the $p$ component of $g$ ensures that in the second round, any strategy that has agent $i$ sometimes reporting $\theta^i$ falsely is strictly dominated by a strategy that is otherwise the same except that it has agent $i$ always reporting $\theta^i$ truthfully, since he is severely punished if his report on $\theta^i$ is inconsistent with (does not contain) agent $j$'s report on $\theta^i$. By (1), the implementation achieved is exact.

Q.E.D.
Proof of Proposition 4.2: Let $M^i = \Theta^i \times \Theta^i$, and write $m^i$ as $(m^i_1,m^i_2)$ and $M$ as $M_1 \times M_2$. Fix an incentive compatible allocation rule $f$. By the openness of $Y \times X$, there exists an $\eta > 0$ such that for all $\theta \in \Theta$, $d((y,x),f(\theta)) < \eta$ implies $(y,x) \in Y \times X$. Fix $m^i_1$ and $m^i_2$.

Now, we exploit the single crossing property. Suppose $m^i_2 = \theta^i_k$ and $f^i(m^i_1,m^i_2) = (y^i,x^i)$. Let $\delta = \min\{\epsilon/2n, \eta/2n\}$. For now, choose $\delta(k)$ arbitrarily from the interval $(0,\delta]$.

Pick $y^i(k) \in (y^i, y^i + \delta(k))$ and $x^i(k) \in (x^i, x^i + \delta(k))$ such that

\[ u^i(y^i(k),x^i(k),\theta^i_k) > u^i(y^i,x^i,\theta^i_k) \quad (1), \]
\[ u^i(y^i,x^i,\theta^i_h) > u^i(y^i(k),x^i(k),\theta^i_h) \quad \text{for all } h < k \quad (2). \]

For any $h < k$, pick $y^i(h) \in (y^i, y^i(h+1))$ and $x^i(h) \in (x^i, x^i(h+1))$ such that

\[ u^i(y^i,h),x^i(h),\theta^i_h) > u^i(y^i, x^i(h), \theta^i_h) \quad (3), \]
\[ u^i(y^i, x^i(h)+1, \theta^i_h) > u^i(y^i(h+1), x^i(h)+1, \theta^i_h) \quad (4), \]
\[ u^i(y^i(h+1),x^i(h+1),\theta^i_{h+1}) > u^i(y^i(h),x^i(h),\theta^i_{h+1}) \quad (5). \]

For any $h > k$, pick

\[ y^i(h) \in (y^i(h-1), y^i + \delta(k)) \text{ and } x^i(h) \in (x^i(h-1), x^i + \delta(k)) \text{ such that} \]

\[ u^i(y^i(h),x^i(h),\theta^i_h) > u^i(y^i(h-1), x^i(h-1), \theta^i_h) \quad (6), \]
\[ u^i(y^i(h-1),x^i(h-1),\theta^i_{h-1}) > u^i(y^i(h),x^i(h),\theta^i_{h-1}) \quad (7). \]

Given a $(m^i_1,m^i_2)$, the above scheme defines a function $t^i_{(m^i_1,m^i_2)} : M^i \rightarrow Y^i \times X^i$, where $t^i_{(m^i_1,m^i_2)}(m^i) = (y^i(k),x^i(k))$ if $m^i_1 = \theta^i_k$.

Now, let $\delta(z) = \delta$. For $k < z$, choose $\delta(k)$ sufficiently small (but positive) so that for all $h > k$,

\[ \sum_{\theta^{-i} \in \Theta^{-i}} u^i(\theta^{-i}, \theta^i_h) q(\theta^{-i}|\theta^i) > \sum_{\theta^{-i} \in \Theta^{-i}} u^i(\theta^{-i}, \theta^i_h, \theta^i_{h+1}) q(\theta^{-i}|\theta^i) \quad (8). \]

Define $g^i : M \rightarrow Y^i \times X^i$ by $g^i(m) = t^i_{(m^i_1,m^i_2)}(m^i)$. Let $g = (g^1, ..., g^n)$. 
We now verify that \((M, g)\) is the required mechanism. Equations (4) - (7) imply that any strategy that has an agent sometimes reporting his first message falsely is strictly dominated by the same strategy, except that it has that agent always reporting his first message truthfully. Given that all agents report their first messages truthfully, equation (8) (which relies on equations (1) - (3)) and the incentive compatibility of \(f\) imply that any strategy that has an agent sometimes reporting his second message falsely is strictly dominated by the same strategy, except that it has that agent always reporting his second message truthfully. Q.E.D.

**Proof of Proposition 4.3:** We assume that both agents accept their contracts. Our proof can be modified to incorporate the possibility of either one or both of the agents not accepting their contracts. Fix any \(\epsilon > 0\). Let \(y_{km} = y_{km}^*\), \(k, m = 1, 2\). Set \(x_{11}^i = x_{11}^*, x_{12}^i = x_{12}^*, x_{21}^i = x_{21}^*, \) and \(x_{22}^i = x_{22}^* + \epsilon\), where \(\epsilon\) is chosen between 0 and \(\epsilon\) and small enough so that

\[
W^A(x_{12}^i) - q(\theta_1^A | \theta_2^A)D^A(y_{21}, \theta_2^A) - q(\theta_2^A | \theta_2^A)D^A(y_{22}, \theta_2^A) > q(\theta_1^A | \theta_2^A)[W^A(x_{11}^A) - D^A(y_{11}, \theta_2^A)] + q(\theta_2^A | \theta_2^A)[W^A(x_{12}^A) - D^A(y_{12}, \theta_2^A)] \quad (1),
\]

\[
W^B(x_{22}^i) - q(\theta_1^B | \theta_2^B)D^B(y_{12}, \theta_2^B) - q(\theta_2^B | \theta_2^B)D^B(y_{22}, \theta_2^B) > q(\theta_1^B | \theta_2^B)[W^B(x_{11}^B) - D^B(y_{11}, \theta_2^B)] + q(\theta_2^B | \theta_2^B)[W^B(x_{12}^B) - D^B(y_{12}, \theta_2^B)] \quad (2),
\]

and

\[
q(\theta_1^A | \theta_1^B)[W^B(x_{11}^i) - D^B(y_{11}, \theta_1^B)] + q(\theta_2^A | \theta_1^B)[W^B(x_{12}^i) - D^B(y_{12}, \theta_1^B)] > q(\theta_1^A | \theta_1^B)[W^B(x_{11}^A) - D^B(y_{11}, \theta_1^B)] + q(\theta_2^A | \theta_1^B)[W^B(x_{12}^A) - D^B(y_{12}, \theta_1^B)] \quad (3).
\]

By Observation (iv) and (v), such an \(\epsilon\) exists.

Pick \(y_{H1} > y_{11}, y_{H2} > y_{12}, x_{H1}^A > x_{11}^i, \) and \(x_{H2}^A > x_{12}^i\) so that

\[
W^A(x_{H1}^i) - D^A(y_{H1}, \theta_2^A) > W^A(x_{11}^i) - D^A(y_{11}, \theta_2^A) \quad (4),
\]

\[
W^A(x_{H2}^i) - D^A(y_{H2}, \theta_2^A) > W^A(x_{12}^i) - D^A(y_{12}, \theta_2^A) \quad (5),
\]

\[
W^A(x_{H1}^i) - D^A(y_{H1}, \theta_1^A) < W^A(x_{11}^i) - D^A(y_{11}, \theta_1^A) \quad (6),
\]

\[
W^A(x_{H2}^i) - D^A(y_{H2}, \theta_1^A) < W^A(x_{12}^i) - D^A(y_{12}, \theta_1^A) \quad (7),
\]
\begin{align*}
W^A(x^A_{21}) - q(\Theta^B_1 | \Theta^A_1)D^A(y_{21}, \Theta^A_1) - q(\Theta^B_2 | \Theta^A_2)D^A(y_{22}, \Theta^A_2) > \\
q(\Theta^B_1 | \Theta^A_1)[W^A(x^A_{11}) - D^A(y_{11}, \Theta^A_1)] + q(\Theta^B_2 | \Theta^A_2)[W^A(x^A_{12}) - D^A(y_{12}, \Theta^A_2)]
\end{align*} (8).

Equations (4) - (7) rely on the single crossing property, while (8) relies on (1) and the assumption that the agents' utility functions are continuous in wealth. This allows us to pick $x^A_{H1}$, $x^A_{H2}$, $y_{H1}$, and $y_{H2}$ satisfying (4) - (7) sufficiently small that (8) is also satisfied.

Using the single crossing property, we pick $y_{H2}$ enough larger than $y_{H1}$ so that the following equations are satisfied.

\begin{align*}
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)] > \\
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)]
\end{align*} (9), and

\begin{align*}
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)] > \\
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)]
\end{align*} (10).

Pick $y_{L1} < y_{21}$, $y_{L2} < y_{22}$, $x^A_{L1} < x^A_{11}$, and $x^A_{L2} < x^A_{12}$ so that

\begin{align*}
W^A(x^A_{L1}) - D^A(y_{L1}, \Theta^A_1) > W^A(x^A_{11}) - D^A(y_{21}, \Theta^A_1)
\end{align*} (11),

\begin{align*}
W^A(x^A_{L2}) - D^A(y_{L2}, \Theta^A_1) > W^A(x^A_{12}) - D^A(y_{22}, \Theta^A_2)
\end{align*} (12),

\begin{align*}
W^A(x^A_{L1}) - D^A(y_{L1}, \Theta^A_2) < W^A(x^A_{11}) - D^A(y_{21}, \Theta^A_2)
\end{align*} (13),

\begin{align*}
W^A(x^A_{L2}) - D^A(y_{L2}, \Theta^A_2) < W^A(x^A_{12}) - D^A(y_{22}, \Theta^A_2)
\end{align*} (14), and

\begin{align*}
q(\Theta^B_1 | \Theta^A_1)[W^A(x^A_{11}) - D^A(y_{11}, \Theta^A_1)] + q(\Theta^B_2 | \Theta^A_2)[W^A(x^A_{12}) - D^A(y_{12}, \Theta^A_2)] > \\
q(\Theta^B_1 | \Theta^A_1)[W^A(x^A_{11}) - D^A(y_{11}, \Theta^A_1)] + q(\Theta^B_2 | \Theta^A_2)[W^A(x^A_{12}) - D^A(y_{12}, \Theta^A_2)]
\end{align*} (15).

Equations (11) - (14) rely on the single crossing property, while (15) relies on Observation (v) and the assumption that the agents' utility functions are continuous in wealth. This allows us to pick $x^A_{L1}$, $x^A_{L2}$, $y_{L1}$, and $y_{L2}$ satisfying (11) - (14) sufficiently large that (15) is also satisfied.

Using the single crossing property, we pick $y_{L2}$ enough larger than $y_{L1}$ so that the following equations are satisfied.

\begin{align*}
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)] > \\
q(\Theta^B_1 | \Theta^A_1)[W^B(x^B_{11}) - D^B(y_{11}, \Theta^B_1)] + q(\Theta^B_2 | \Theta^A_2)[W^B(x^B_{12}) - D^B(y_{12}, \Theta^B_2)]
\end{align*} (16), and
q(θ₁^A | θ₁^B)[W^B(x_{L1}^B) - D^B(y_{L1}, θ₁^B)] + q(θ₂^A | θ₁^B)[W^B(x_{21}^B) - D^B(y_{21}, θ₁^B)] >

q(θ₁^A | θ₁^B)[W^B(x_{L2}^B) - D^B(y_{L2}, θ₁^B)] + q(θ₂^A | θ₁^B)[W^B(x_{22}^B) - D^B(y_{22}, θ₁^B)]  \quad (17)

It remains to be shown that truth-telling is the unique strategy combination that survives the iterative removal of strictly dominated strategies. This will be shown in three steps.

**Step 1:** Agent A will not report θ₂^A or H when θ^A = θ₁^A. This relies on equations (6), (7), (11), and (12). Also, agent A will not report θ₁^A or L when θ^A = θ₂^A. This relies on equations (4), (5), (13), and (14).

**Step 2:** Agent B will not report θ₂^B when θ^B = θ₁^B. This relies on Step 1 and equations (3), (10), and (17). Also agent B will not report θ₁^B when θ^B = θ₂^B. This relies on Step 1 and equations (2), (9), and (16).

**Step 3:** Agent A will not report L when θ^A = θ₁^A nor will he report H when θ^A = θ₂^A. This is by Step 2 and equations (8) and (15).

Q.E.D.
APPENDIX D

Proof of Proposition 5.1: Denote by (P-DS) the principal's program when the Bayesian incentive compatibility constraints in (P-BN) are replaced by the dominant strategy incentive compatibility constraints.

We need to show that contract \( K = \{ x_1^A, x_2^A, R_{11}^A, R_{12}^A, R_{21}^A, R_{22}^A \} \), as defined in the statement of Proposition 5.1, provides the principal with the same expected utility that she obtained under contract \( K^* \) and also that contract \( K \) satisfies the constraints in (P-DS).

To see that \( K \) provides the principal with the same expected utility as the solution to (P-BN), note that \( x_1^A = x_{*1}^A \), \( x_2^A = x_{*2}^A \), and

the principal's expected transfer to agent A under (P-DS) =

\[
\phi_{11}R_{11}^A + \phi_{12}R_{12}^A + \phi_{21}R_{21}^A + \phi_{22}R_{22}^A =
\]

\[
\phi_{11}R_{11}^A + \phi_{12}R_{12}^A + \phi_{21}[R_{11}^A + D^A(x_{*1}^A, \theta_{*2}^A) - D^A(x_{*1}^A, \theta_{*2}^A)] + \phi_{22}[R_{12}^A + D^A(x_{*2}^A, \theta_{*2}^A) - D^A(x_{*1}^A, \theta_{*2}^A)] =
\]

the principal's expected transfer to agent A under (P-BN).

The second equality follows from Observation 5.2 (ii).

From the above, it follows that \( K \) satisfies the individual rationality constraints. Further, since \( D^A(x_{*2}^A, \theta_{*2}^A) - D^A(x_{*1}^A, \theta_{*2}^A) \geq 0 \) and \( K^* \) satisfies the bankruptcy constraints, it follows that \( K \) also satisfies the bankruptcy constraints. By the choice of the payments, contract \( K \) satisfies (5.12) when \( \theta^A = \theta_{2}^A \). By the choice of the payments and the single crossing property, contract \( K \) satisfies (5.12) when \( \theta^A = \theta_{1}^A \).

Q.E.D.
APPENDIX E

Proof of Proposition 6.1: Writing the program (P-BN) for manager A alone and substituting constraints that the owner treats the managers independently, i.e., $x_{km}^A = x_{kn}^A = x_k^A$ and $y_{km}^A = y_{kn}^A = y_k^A$, $k, m, n = L, H$, yields the optimal single-agent contract for manager A. Denote the marginal probability that manager A’s costs are low by $p_L = p_{LL} + p_{LH}$ and the marginal probability that manager A’s costs are high by $p_H = p_{HH} + p_{HL}$, respectively. For notational convenience, we suppress the superscript A. The primal program is given below. The dual variables are indicated in parentheses next to each constraint.

The primal program is as follows:

$$\begin{align*}
\text{Max} & \quad [p_L(x_L y_L) + p_H(x_H y_H)] \\
& \quad x_L, x_H, y_L, y_H \\
\text{ST:} & \quad p_L c_L x_L + p_H c_H x_H - p_L y_L - p_H y_H \leq -\bar{U} \quad (w_R) \\
& \quad - c_L x_L - y_L \leq 0 \quad (w_{B1}) \\
& \quad c_H x_H - y_H \leq 0 \quad (w_{B2}) \\
& \quad c_L x_L - c_L x_H - y_L + y_H \leq 0 \quad (w_{C1}) \\
& \quad - c_H x_L + c_H x_H + y_L - y_H \leq 0 \quad (w_{C2}) \\
& \quad x_L, x_H \leq X \quad (w_{F1}, w_{F2}) \\
& \quad x_L, x_H, y_L, y_H \geq 0
\end{align*}$$

The dual program is as follows:

$$\begin{align*}
\text{Min} & \quad [w_{F1} + w_{F2}] X - \bar{U} w_R \\
& \quad w_R, w_{B1}, w_{B2}, w_{C1}, w_{C2}, w_{F1}, w_{F2} \\
\text{ST:} & \quad p_L c_L v_R + c_L w_{B1} + c_L w_{C1} - c_H w_{C2} + w_{F1} \geq p_L (x_L)
\end{align*}$$

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\[
\begin{align*}
PHCHWR + CHWB2 - CLWC1 + CHWC2 + WF2 & \geq PH \quad (x_H) \\
- PLWR - WB1 & \geq -PL \quad (y_L) \\
- PHWR - WB2 + WC1 - WC2 & \geq -PH \quad (y_H)
\end{align*}
\]

It can be verified that, within the specified region of the exogenous parameters, the primal and the dual solutions below are feasible and provide the same value when substituted into each program's objective function. Hence, the primal and dual solutions are both optimal. (See Corollary 1 of Luenberger 1973, p. 71.)
### TABLE 10

Optimal Single-Agent Contracts (Proof)

<table>
<thead>
<tr>
<th>Rationing</th>
<th>Slack</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>(p_H \leq \frac{c_H - c_L}{c_H} )</strong></td>
<td><strong>(p_H \geq \frac{c_H - c_L}{c_H} )</strong></td>
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<tr>
<td>(X_L)</td>
<td>(X)</td>
</tr>
<tr>
<td>(Y_L)</td>
<td>(c_H X)</td>
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<tr>
<td>(x_H)</td>
<td>(X)</td>
</tr>
<tr>
<td>(y_H)</td>
<td>(c_H X)</td>
</tr>
</tbody>
</table>

| \(w_R\)                               | \(0\)                              |
| \(w_{B1}\)                            | \(0\)                              |
| \(w_{B2}\)                            | \(1\)                              |
| \(w_{C1}\)                            | \(p_L\)                            |
| \(w_{C2}\)                            | \(0\)                              |
| \(w_{F1}\)                            | \(p_L(1 - c_L)\)                   |
| \(w_{F2}\)                            | \(p_H(1 - c_H) - p_L(c_H - c_L)\) |

Q.E.D.

**Proof of Proposition 6.2:** The primal program corresponding to the owner contracting with manager A is given below (Program (Q)). Note that we suppress the individual rationality constraints since with \(U^i = 0\), the bankruptcy constraints dominate the individual rationality constraints. A symmetrical problem corresponding to the owner contracting with manager B can be written. For notational convenience, we suppress the superscript A.
in the program and introduce the notation \( q_{km} = \frac{p_{km}}{p_{kl} + p_{kh}} \). The dual variables are indicated in parentheses next to each constraint.

\[
\text{Max} \quad [p_{ll}(x_{ll} - y_{ll}) + p_{lh}(x_{lh} - y_{lh}) + p_{hl}(x_{hl} - y_{hl}) + p_{hh}(x_{hh} - y_{hh})]
\]
\[
x_{ll}, x_{lh}, x_{hl}, x_{hh}, y_{ll}, y_{lh}, y_{hl}, y_{hh}
\]

\[
\text{ST:}
\]
\[
c_{l}x_{ll} - y_{ll} \leq 0 \quad (w_{b1})
\]
\[
c_{l}x_{lh} - y_{lh} \leq 0 \quad (w_{b2})
\]
\[
c_{h}x_{hl} - y_{hl} \leq 0 \quad (w_{b3})
\]
\[
c_{h}x_{hh} - y_{hh} \leq 0 \quad (w_{b4})
\]
\[
 q_{ll}c_{l}x_{ll} + q_{lh}c_{l}x_{lh} - q_{ll}c_{h}x_{hl} - q_{lh}c_{h}x_{hh} - q_{ll}y_{ll} - q_{lh}y_{lh} + q_{hl}y_{hl} + q_{hh}y_{hh} \leq 0 \quad (w_{c1})
\]
\[
 q_{lh}c_{l}x_{ll} - q_{hh}c_{h}x_{lh} + q_{lh}c_{h}x_{hl} + q_{hh}c_{h}x_{hh} + q_{hl}y_{ll} + q_{hh}y_{lh} - q_{hl}y_{hl} - q_{hh}y_{hh} \leq 0 \quad (w_{c2})
\]
\[
x_{ll}, x_{lh}, x_{hl}, x_{hh}, y_{ll}, y_{lh}, y_{hl}, y_{hh} \geq 0
\]

The dual program is as follows:

\[
\text{Min} \quad X[w_{f1} + w_{f2} + w_{f3} + w_{f4}]
\]
\[
w_{b1}, w_{b2}, w_{b3}, w_{b4}, w_{c1}, w_{c2}, w_{f1}, w_{f2}, w_{f3}, w_{f4}
\]

\[
\text{ST:}
\]
\[
c_{l}w_{b1} + q_{ll}c_{l}w_{c1} - q_{lh}c_{h}w_{c2} + w_{f1} \geq p_{ll} \quad (x_{ll})
\]
\[
c_{l}w_{b2} + q_{lh}c_{h}w_{c1} - q_{hh}c_{h}w_{c2} + w_{f2} \geq p_{lh} \quad (x_{lh})
\]
\[
c_{h}w_{b3} - q_{ll}c_{l}w_{c1} + q_{hl}c_{h}w_{c2} + w_{f3} \geq p_{hl} \quad (x_{hl})
\]
\[
c_{h}w_{b4} - q_{lh}c_{h}w_{c1} + q_{hh}c_{h}w_{c2} + w_{f4} \geq p_{hh} \quad (x_{hh})
\]
\[
- w_{b1} - q_{ll}w_{c1} + q_{hl}w_{c2} \geq - p_{ll} \quad (y_{ll})
\]
\[
- w_{b2} - q_{lh}w_{c1} + q_{hh}w_{c2} \geq - p_{lh} \quad (y_{lh})
\]
\[
- w_{b3} + q_{ll}w_{c1} - q_{hl}w_{c2} \geq - p_{hl} \quad (y_{hl})
\]
\[
- w_{b4} + q_{lh}w_{c1} - q_{hh}w_{c2} \geq - p_{hh} \quad (y_{hh})
\]
\[
w_{b1}, w_{b2}, w_{b3}, w_{b4}, w_{c1}, w_{c2}, w_{f1}, w_{f2}, w_{f3}, w_{f4} \geq 0
\]
Again it can be verified that, within the specified region of the exogenous parameters, the primal and the dual solutions below are feasible and provide the same value when substituted into each program's objective function. Hence, the primal and dual solutions are both optimal.

**TABLE 11**

The Second-Best Solution (Proof)

<table>
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<tr>
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<th>Rationing</th>
<th>Contract P</th>
<th>Slack</th>
</tr>
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<td>( \frac{\text{PHL}}{\text{PLL}} \leq \frac{c_H - c_L}{1 - c_H} )</td>
<td>( \frac{\text{PHL}}{\text{PLL}} \geq \frac{c_H - c_L}{1 - c_H} )</td>
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<td>( c_L X )</td>
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</tr>
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Q.E.D.
LIST OF REFERENCES


