A NUMERICAL ANALYSIS OF FROST
FORMATION UNDER FORCED CONVECTION

A Thesis
Presented in Partial Fulfillment of the Requirements
for the Degree Master of Science

by
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1970

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ACKNOWLEDGEMENTS

I wish to express my sincere appreciation to Professor Charles F. Sepsy, Mechanical Engineering Department, Ohio State University, for his assistance and guidance during the preparation of this thesis. I am greatly indebted to my wife, Elma, for her sustained patience and encouragement during my graduate studies as well as for much assistance in the preparation of the thesis, and to my parents for their guidance, encouragement and support throughout my academic career.
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<tr>
<td>A</td>
<td>surface area; ft²</td>
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<tr>
<td>A'</td>
<td>constant determined empirically for the heat of sublimation (see Equation 20); btu/lbm-°F</td>
</tr>
<tr>
<td>AC</td>
<td>coefficient of the diffusion difference equation (see Equation 40c); sec⁻¹</td>
</tr>
<tr>
<td>AT</td>
<td>coefficient of the energy difference equation (see Equation 34c); sec⁻¹</td>
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<td>coefficient of the energy difference equation (see Equation 26a); dimensionless</td>
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<td>Al</td>
<td>coefficient of the momentum difference equation (see Equation 10c); sec⁻¹</td>
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<td>B'</td>
<td>constant determined empirically for the heat of sublimation (see Equation 20); btu/lbm</td>
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<td>BC</td>
<td>coefficient of the diffusion difference equation (see Equation 40b); sec⁻¹</td>
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<td>BT</td>
<td>coefficient of the energy difference equation (see Equation 34b); sec⁻¹</td>
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<td>coefficient of the energy difference equation (see Equation 26b); dimensionless</td>
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<td>-----------------------------------------------------------------------------</td>
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<tr>
<td>Bl</td>
<td>coefficient of the momentum difference equation (see Equation 10b); sec⁻¹</td>
</tr>
<tr>
<td>C</td>
<td>concentration of H₂O vapor in air expressed as mole fraction; dimensionless</td>
</tr>
<tr>
<td>cp</td>
<td>specific heat of air; btu/lbm-°F</td>
</tr>
<tr>
<td>CC</td>
<td>coefficient of the diffusion difference equation (see Equation 40a); sec⁻¹</td>
</tr>
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<td>CT</td>
<td>coefficient of the energy difference equation (see Equation 34a); sec⁻¹</td>
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<tr>
<td>CT'</td>
<td>coefficient of the energy difference equation (see Equation 26c); dimensionless</td>
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<tr>
<td>Cl</td>
<td>coefficient of the momentum difference equation (see Equation 10a); sec⁻¹</td>
</tr>
<tr>
<td>D</td>
<td>binary diffusion coefficient for water vapor in air; ft²/sec</td>
</tr>
<tr>
<td>DC</td>
<td>coefficient of the diffusion difference equation (see Equation 40d); sec⁻¹</td>
</tr>
<tr>
<td>DEL</td>
<td>incremental change in frost thickness; ft</td>
</tr>
<tr>
<td>DT</td>
<td>coefficient of the energy difference equation (see Equation 34d); °F/sec</td>
</tr>
<tr>
<td>DL</td>
<td>coefficient of the momentum difference equation (see Equation 10d); ft/sec²</td>
</tr>
<tr>
<td>e_solid</td>
<td>enthalpy of frost; btu/lbm</td>
</tr>
<tr>
<td>e_vapor</td>
<td>enthalpy of H₂O vapor; btu/lbm</td>
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EC  intermediate variable used in the solution of the diffusion equation (see Equation 42); dimensionless

ET  intermediate variable used in the solution of the energy equation (see Equation 36); dimensionless

ET' intermediate variable used in the solution of the energy equation (see Equation 28); dimensionless

El intermediate variable used in the solution of the momentum equation (see Equation 11); dimensionless

F_1 coefficient in series expression for the stream function; sec^{-1}

f_1 universal parameter = F_{1}'/(u_1 \nu)^{1/9}; dimensionless

F_3 coefficient in series expression for the stream function; ft^{-1}sec^{-1}

F_5 coefficient in series expression for the stream function; ft^{-3}sec^{-1}

FC intermediate variable used in the solution of the diffusion equation (see Equation 42); dimensionless

FT intermediate variable used in the solution of the energy equation (see Equation 36);
$\mathcal{F}_T$ intermediate variable used in the solution of the energy equation (see Equation 28); °F

$\mathcal{F}_l$ intermediate variable used in the solution of the momentum equation (see Equation 11); ft/sec

$\alpha_c$ conversion factor in Newton's Second Law of Motion; lbm-ft/lbf-sec²

$\mathcal{H}_c$ convective heat transfer coefficient; btu/sec-ft²-°F

$\mathcal{H}_c$ thermal conductance of the frost layer; btu/sec-ft²-°F

$\mathcal{H}_d$ mass transfer coefficient; lbm/sec-ft²

$J_{a*}$ molar flux relative to the molar-average velocity; lb-moles/ft²-sec

$k$ air thermal conductivity; btu/sec-ft-°F

$k_f$ frost thermal conductivity; btu/sec-ft-°F

$M_{\text{air}}$ molecular weight of air; dimensionless

$M_{\text{H}_2\text{O}}$ molecular weight of $\text{H}_2\text{O}$; dimensionless

$M_{\text{H}_2\text{O}}$ mass flux of $\text{H}_2\text{O}$ vapor; lbm/sec-ft²

$M_{\text{CHK}}$ the upper limit of "m" to be used by the computer, corresponding to the point of separation; dimensionless

$N_a$ molar flux of $\text{H}_2\text{O}$ vapor relative to stationary co-ordinates; lb-moles/ft²-sec

$N_b$ molar flux of dry air relative to stationary co-ordinates; lb-moles/ft²-sec
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tr>
<td>NU</td>
<td>nusselt number = (\frac{2R H_C}{k}); dimensionless</td>
</tr>
<tr>
<td>p</td>
<td>pressure; lbf/ft(^2)</td>
</tr>
<tr>
<td>PH(_{20})</td>
<td>partial pressure of water vapor; lbf/ft(^2)</td>
</tr>
<tr>
<td>QCND</td>
<td>heat flux through the frost layer by conduction; btu/sec-ft(^2)</td>
</tr>
<tr>
<td>QCNV</td>
<td>sensible heat flux to the frost surface by convection; btu/sec-ft(^2)</td>
</tr>
<tr>
<td>QSTOR</td>
<td>net rate of energy addition to the frost layer as sensible heat, i.e., resulting in temperature rise within frost layer; btu/sec-ft(^2)</td>
</tr>
<tr>
<td>QSUB</td>
<td>thermal energy release rate at the frost surface due to sublimation; btu/sec-ft(^2)</td>
</tr>
<tr>
<td>R</td>
<td>radius of cylinder; ft</td>
</tr>
<tr>
<td>RF</td>
<td>total frost thickness; ft</td>
</tr>
<tr>
<td>s</td>
<td>reduced distance = (y/\Delta y); dimensionless</td>
</tr>
<tr>
<td>T</td>
<td>temperature; °F</td>
</tr>
<tr>
<td>TMO</td>
<td>intermediate value of the frost surface temperature as calculated by the program in the iteration procedure; °F</td>
</tr>
<tr>
<td>U</td>
<td>tangential velocity within the boundary layer; ft/sec</td>
</tr>
<tr>
<td>u</td>
<td>velocity at outer edge of boundary layer; ft/sec</td>
</tr>
<tr>
<td>(u_1)</td>
<td>coefficient in series expression for the external flow velocity; sec(^{-1})</td>
</tr>
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</table>
$u_3$  coefficient in series expression for the external flow velocity; $\text{ft}^{-2}\text{-sec}^{-1}$

$u_5$  coefficient in series expression for the external flow velocity; $\text{ft}^{-4}\text{-sec}^{-1}$

$U_A$  tangential velocity used in portion of program to develop initial conditions, characterized by small step intervals; $\text{ft/sec}$

$V$  normal velocity within the boundary layer; $\text{ft/sec}$

$V_A$  normal velocity used in portion of program to develop initial conditions, characterized by small step intervals; $\text{ft/sec}$

$x$  coordinate parallel to surface; $\text{ft}$

$y$  coordinate normal to surface; $\text{ft}$

**Greek Symbols**

$\alpha$  thermal diffusivity $k/\rho c_p$; $\text{ft}^2/\text{hr}$

$\eta$  distance parameter used in the solution of stagnation flow $= y(u_1/v)^{1/2}$; dimensionless

$\theta$  angle from stagnation point; radians

$\mu$  air viscosity; $\text{lbf-sec}/\text{ft}^2$

$\nu$  kinematic viscosity; $\text{ft}^2/\text{sec}$

$\rho$  air density; $\text{lbm}/\text{ft}^3$

$\rho_f$  frost density; $\text{lbm}/\text{ft}^3$
Subscripts

1 conditions at the tube surface

m mth grid point in boundary layer along x co-
ordinate starting with m = 1 at the
initial station

n nth grid point in boundary layer along y co-
ordinate starting with n = 0 at the
surface

NM the maximum value of m to be analyzed

NN the maximum value of n to be analyzed

NT number of incremental layers of frost to be
calculated by program

0 conditions at the frost-air interface

∞ free stream conditions
ABSTRACT

An analytical method was developed to predict the transient process of frost formation on a cold surface with moist air flowing over the cold surface in forced convection. The process of frost formation involves simultaneous heat and mass transfer. The transfer phenomena was calculated by solution of the momentum, thermal and diffusion boundary layer equations, along with the continuity equation, at several intervals in time. Each such solution was obtained for new boundary conditions created by the developing frost layer.

The boundary conditions were coupled in that the partial pressure of H$_2$O vapor at the surface was required to correspond to the vapor pressure of water at the surface temperature.

The solution to the boundary layer equations were obtained on a high speed digital computer, using an implicit finite-difference scheme.

The analytical method was tested by comparing predicted frost growth rate and effective frost conductivity with experimentally measured quantities. Although this was done only for the case of a cylinder with flow normal
to the axis of the cylinder, the method is developed in such a way that it can be applied for an arbitrary geometry provided the external flow field can be described, either from potential flow theory or experimental results. An additional limitation requires that the surface does not contain a sharp edge. Neither does the method apply after boundary layer separation occurs or after the boundary layer transition from the laminar to the turbulent regime. The latter restriction does not exclude turbulence in the external flow field.
CHAPTER 1
INTRODUCTION

The detrimental effects of frost formation on heat transfer to cold surfaces are well known. The resulting degradation of system performance has prompted considerable investigation into the nature of heat and mass transfer processes occurring during the formation of frost as well as investigations into the properties of frost. The results of these investigations generally indicate that the process of frost formation is complicated because of the following factors:

1) The boundary layer solution is a transient one. That is, as the frost layer grows in thickness, the thermal resistance of the frost layer changes both with respect to time and position. As the thermal resistance changes, the surface temperature of the frost layer changes with time and position and consequently the partial pressure of water vapor at the surface also changes. This changes the structure of the thermal and diffusion boundary layers, resulting in changes in the heat and mass transfer rates with time, particularly in the portion of the cylinder where sharp temperature gradients exist along the surface (ie., $\left| \frac{\partial T}{\partial x} \right| \rightarrow 0$).
2) The frost properties are not constant but vary continuously during the development of the frost layer, although a quasi steady state condition is eventually achieved. The variations in frost properties are quite significant. It was reported in Reference (4) that the density varied from approximately 6 to 36 lbm/ft$^3$ as the temperature varied from about 18$^\circ$F to 32$^\circ$F. The thermal conductivity has been reported in Reference (9), to vary from about 0.06 to 0.40 btu/hr-ft-$^0$F, where the range of conductivities correspond to the range of densities given above.

3) The process of frost formation involves simultaneous heat and mass transfer. For the most part, problems dealing with forced convection were analyzed using empirical heat and mass transfer data for flow past a circular cylinder. Therefore, the available solutions are restricted accordingly.

Both experimental and analytical studies have been reported to date on the subject of frost formation. However, the analytical studies have primarily been based on steady state, empirical correlations for heat and mass transfer and have not taken into account variations in frost properties. Thus they have not been completely satisfactory for interpreting the experimental data obtained.

This thesis represents an attempt to improve the
analytical model by taking into consideration the transient nature of the boundary layer solution as well as the variations in frost properties with time and position. In addition, the local heat and mass transfer rates are obtained by solution of the boundary layer equations, thus eliminating the need for empirical transport data. It should therefore be relatively easy to obtain solutions for different geometries.
CHAPTER 2
LITERATURE SURVEY

Several analysis have been performed for frosting conditions with varying degrees of complexity and detail.

Kamei et al. (10) used values for heat and mass transfer coefficients, obtained from their experimental results, to show that the Colburn analogy held for the conditions which they investigated. The experimental work was performed on a cylinder with air flow parallel to the axis in an annular passage.

Chung and Algren (11) performed an elaborate analysis in which they used a known analytical solution for the local Nusselt number around the circumference of a cylinder and used the Colburn analogy to obtain the Sherwood number. Variation in the surface temperature and Reynolds number with time were considered, making it a transient problem.

However, the Nusselt number variation around the cylinder was based on an isothermal surface. Therefore, changes in the average surface temperature around the cylinder were taken into consideration but temperature gradients along the surface could not be provided for.

It was also assumed that the frost conductivity was constant.
Sparrow and Spalding (13) analyzed sublimation occurring between parallel plates. The plates are considered to be insulated on the outside so that the heat of sublimation is supplied by the sensible heat of the gas stream. The channel flow is further simplified by assuming slug flow with no velocity boundary layer. Consequently, no normal velocity component developed as would be the case if a boundary layer developed as flow progressed down the channel. A solution is obtained for the above system in closed form. The solution is obtained for the coupled boundary conditions indicated above.
CHAPTER 3
ANALYSIS

The analysis involves the numerical solution of the three boundary layer equations, the momentum, energy and diffusion equations, plus the continuity equation. The solution proceeds stepwise around the circumference of a cylinder starting near the stagnation point and proceeding to the point of separation. The solutions make possible the calculation of the local rate of frost formation based on the resultant mass diffusion rates at a given instant of time. This rate of formation is then multiplied by some increment of time to obtain the projected frost thickness at the end of that increment of time.

The computerized solution of the boundary layer equations is accomplished in two subroutines. One provides the solution of the momentum and continuity equation; the other subroutine provides the solution of the energy and diffusion equations. These two subroutines are utilized by the main program which provides the required boundary and initial conditions, calls upon the subroutines to provide the solutions to the boundary layer equations at a given instant of time, calculates the rate of frost formation and projects the new frost thickness at each step in time. The development of the subroutines and main program will be discussed in that order.
3.1 Solution of the Momentum and Continuity Equations

The equations of momentum and continuity are solved in subroutine "CALCV", providing values for the normal and tangential velocities throughout the boundary layer. The momentum and continuity equations are written as difference equations which approximate Prandtl's boundary layer equations. The derivation of the latter equations is well known and can be found in Reference (2). The equations are presented below for easy reference.

The momentum equation:

\[
\frac{\rho}{\varepsilon_c} \left[ \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} \right] = - \frac{dp}{dx} + \mu \frac{\partial^2 U}{\partial y^2} \tag{1}
\]

The continuity equation:

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{2}
\]

3.2 Coordinate System

The above equations are written using the rectangular coordinate system even though the system under consideration is cylindrical in nature. An order of magnitude analysis was used in Reference (2) to show that the use of rectangular coordinates results in negligible error provided the radius of curvature of the wall is large compared to the boundary layer thickness and provided there are no rapid
changes in the radius of curvature such as occur near a sharp edge. These conditions are met in all cases analyzed in this effort.

3.3 Boundary Conditions

The surface is assumed to be impermeable with no normal velocity component at the surface. The velocity due to diffusion is neglected. The maximum normal velocity at the surface, due to diffusion, was calculated to be approximately 0.002 ft/sec. This was considered to be negligible in terms of its effect on the transport processes.

As usual, a non-slip condition is applied at the surface so that the tangential velocity component is zero at all points on the surface.

The external flow field was described using both potential flow and empirical data. The effect of each is discussed in Chapter 5.

It should be noted that, as the frost layer develops, the shape differs from that of a circular cylinder. The effect of this change could be taken into account provided the external flow field is calculated from potential flow theory. In general, it cannot be adjusted where empirical external flow velocity data is used. The program developed for this effort did not take the non-circular shape into account in calculating the external velocity. For the usual frost thickness encountered, deviations from the circular
shape should not have a significant effect.

3.4 Initial Conditions

Basically, the initial tangential and normal velocities for a point near the stagnation point are obtained from a truncated Blasius series. It will be shown later, however, that the velocities so obtained were somewhat modified in order to obtain a relatively smooth solution for the first few stations downstream of the initial station.

From Reference (5), the external flow may be described by the series:

\[ u = u_1 x + u_3 x^3 + u_5 x^5 + \ldots \]

where \( u_1, u_3, u_5, \ldots \) are not a function of \( x \). Therefore, for small \( x \), \((x=1)\):

\[ u = u_1 x \]

For potential flow around a cylinder:

\[ u = 2u_\infty \sin \theta \]

For small \( \theta \), \( \sin \theta \approx x/R \), therefore:

\[ u = 2u_\infty \frac{x}{R} \]

Thus for small \( \theta \), i.e., points near the stagnation point, the series expression for the external velocity may be
reduced to:

\[ u = u_1 x \]

with:

\[ u_1 = \frac{2u_0}{x} \]

Using Blasius' method of solution, the tangential and normal velocity components within the boundary layer may be shown (see Reference (5)) to be of a form:

\[ U = F_1 x + F_5 x^5 + F_6 x^9 + \ldots \]

\[ V = F_1 - 3F_5 x^3 - 5F_6 x^7 + \ldots \]

where:

\[ F' = \frac{DF}{Dx} \]

Again, by restricting the region to values of \( x < 1 \), the terms containing \( x \) to a power greater than one may be neglected (\( F_1, F_5, F_6 \) etc. not a function of \( x \)) and the series reduces to:

\[ U = F_1' x \]

\[ V = F_1 \]

10
Also from Reference (5):

\[ F_1 = \left[ f_1(\eta) \right] u_1 \nu^{\frac{1}{8}}, \quad \eta = \sqrt[8]{\frac{u_1}{\nu}} \]

\[ F_1' = \frac{\partial F}{\partial y} = \frac{\partial}{\partial y} \left[ f_1(\eta) \left( u_1 \nu^{\frac{1}{8}} \right) \right] \]

\[ = u_1 f_1'(\eta) \]

Therefore:

\[ \frac{u}{u_1} \frac{f_1(\eta)}{x} = f_1'(\eta) \quad (3) \]

\[ V = -\left[ \frac{2u_1 \omega}{R} \right] \frac{1}{8} f_1(\eta) \quad (4) \]

The functions \( f_1'(\eta) \) and \( f_1(\eta) \) have been evaluated by several investigators and are tabulated in both References (2) and (5). The values used in this thesis were taken from Reference (2) which, in turn, modified slightly the results found in Reference (5) with more recent data taken from H. Görler.

A non-linear interpolating subroutine was developed to interpolate tables of \( f_1(\eta) \) and \( f_1'(\eta) \) so that values of \( f_1(\eta) \), \( f_1'(\eta) \) may be obtained for any arbitrary distance from the cylinder, i.e., for any value of \( \eta \).

A check on the above method for obtaining the initial
conditions was made by calculating the initial tangential velocity profile at $2^\circ$ from the stagnation point, as described above, and comparing this with a computer solution which was obtained by supplying similar initial conditions at a point $0.5^\circ$ from the stagnation point and which calculated new velocity profiles using $0.5^\circ$ step intervals so that the solution appears to have smoothed out by $2^\circ$.

The results are illustrated in Figure (9). It can be seen from Figure (9) that the shape of the tangential velocity curve is approximately correct. However, the magnitude as predicted by Equation (3) runs consistently high, (about 18% at the edge of the B.L.). This was due to the difference in the external flow velocity as calculated from potential flow theory and as obtained from empirical data. Equation (3) was subsequently modified by dividing the right hand side by 1.3, giving Equation (3a).

$$\frac{U + f_1(U)}{U} = \frac{1}{1.3}$$  \[ (3a) \]

The resulting velocity profile is also shown in Figure (9). The numerical factor used in Equation (3a) was 1.3 which provided very good results near the wall, whereas a factor of 1.18 would have yielded good agreement at the outer edge of the boundary layer. It was decided that it would be more important to provide the proper values of $\frac{\partial U}{\partial y}$.
near the wall than to match the external flow field.

To further improve the initial conditions used for the analysis, initial tangential and normal velocity profiles are calculated from Equations (3a) and (4) respectively, for a point upstream of the first station required by the analysis. The program then calculates subsequent velocity profiles (see section 3.6) at several intermediate stations so that by the time the first station is reached, the velocity profiles have developed in such a way that they satisfy the momentum and continuity equations as well as the boundary conditions at the initial station.

It should be noted that Equations (3a) and (4) are valid only for a circular cylinder in plane flow. Other configurations, such as flow over a flat plate, would require different initial conditions.

3.5 Finite Difference Approximations

The difference quotients used in this analysis are the same as those used in Reference (6) for the Crank-Nicolson type difference equations. These are presented for reference along with the associated lowest order error terms. The nomenclature is defined by the grid sketched below.
Difference Quotients and associated lowest order error terms

\[
\frac{\partial H}{\partial x} = \frac{H_e - H_b}{\Delta x}
\]  \hspace{1cm} (5)

\[
\text{error} = (\Delta x)^2 \frac{\partial^3 H}{\partial x^3} + \ldots.
\]

\[
\frac{\partial H}{\partial y} = \frac{(H_e + H_f - H_d)}{4(\Delta y)}
\]  \hspace{1cm} (6)

\[
\text{error} = (\Delta y)^2 \frac{\partial^3 H}{\partial y^3} + (\Delta x)^2 \frac{\partial^3 H}{\partial x^3 \partial y} + \ldots.
\]

\[
\frac{\partial^2 H}{\partial y^2} = \frac{(H_e - 2H_b + H_c + H_d - 2H_e + H_f)}{2(\Delta y)^2}
\]  \hspace{1cm} (7)

\[
\text{error} = \frac{(\Delta x)^2}{8} \frac{\partial^4 H}{\partial x^2 \partial y^2} + \frac{(\Delta y)^2}{12} \frac{\partial^4 H}{\partial y^4} + \ldots.
\]
The above difference quotients evaluate the given derivative within some error whose approximate magnitude is indicated by the lowest order term or terms. The total error is, of course, given by an infinite series of progressively higher order terms. All of the above difference quotients are evaluated for the point "k".

Substituting the difference quotients defined by Equations (5), (6) and (7) into Equation (1), the momentum equation is obtained in difference form and is presented as Equation (8).

\[
\begin{aligned}
&\left[ U_k, n \right] \left[ \frac{U_{m+1,n} - U_{m,n}}{\Delta x} \right] \\
&+ \left[ V_k, n \right] \left[ \frac{U_{m,n+1} - U_{m,n} + U_{m+1,n+1} - U_{m+1,n-1}}{4(\Delta y)} \right] \\
&= \left[ \frac{g_c}{\rho} \right] \left[ \frac{dp}{dx} \right]_k + \left[ \frac{g_c \mu}{\rho} \right] \left[ \frac{U_{m,n-1} - 2U_{m,n} + U_{m,n+1}}{2(\Delta y)^2} \right] + U_{m+1,n-1} - 2U_{m+1,n} + U_{m+1,n+1} \\
&\quad /2(\Delta y)^2 \quad (8)
\end{aligned}
\]

The quantities \( U_k, n, V_k, n \) are unknown and therefore make Equation (8) non-linear. To linearize the equation, the terms \( U_k, n \) and \( V_k, n \) are replaced by \( U_m, n \) and \( V_m, n \) respectively. This procedure assumes that \( \frac{U_k, n}{U_m, n} \) and \( \frac{V_k, n}{V_m, n} \) are nearly unity. This condition will generally be satisfied if the step sizes are small and \( U_m, n, V_m, n \) are not zero.
Rewriting the momentum equation:

\[
\begin{bmatrix} U_{m+1,n-1} \\ U_{m,n} \end{bmatrix} \left[ \frac{-V_{m,n}- \frac{\xi \sigma \mu}{4\Delta y}}{2\rho(\Delta y)^2} \right] + \begin{bmatrix} U_{m+1,n} \end{bmatrix} \left[ \frac{U_{m,n} + \frac{\xi \sigma \mu}{\Delta x}}{\rho(\Delta y)^2} \right] \\
+ \begin{bmatrix} U_{m+1,n+1} \end{bmatrix} \left[ \frac{V_{m,n} - \frac{\xi \sigma \mu}{4\Delta y}}{2\rho(\Delta y)^2} \right] = \begin{bmatrix} \frac{-\xi \sigma}{\rho} \end{bmatrix} \left[ \frac{dp}{dx} \right]_m \\
+ \begin{bmatrix} U_{m,n} \end{bmatrix} \left[ \frac{U_{m,n-1} - U_{m,n+1}}{4\Delta y} \right] + \begin{bmatrix} U_{m,n} \end{bmatrix}^2 \\
+ \begin{bmatrix} \frac{\xi \sigma \mu}{\Delta x} \end{bmatrix} \begin{bmatrix} 2U_{m,n} - 2U_{m+1,n} + U_{m,n+1} \end{bmatrix}
\]

(8a)

Substituting the differentials defined by Equations (5) and (6) into the continuity Equation (2), the continuity equation is obtained in difference form and is presented as Equation (9).

\[
\begin{bmatrix} U_{m+1,n} - U_{m,n} + U_{m+1,n-1} - U_{m,n-1} \end{bmatrix} = \\
- \begin{bmatrix} V_{m,n} - V_{m,n-1} + V_{m+1,n} - V_{m+1,n-1} \end{bmatrix}
\]

(9)

Unfortunately, it was found in running the program that this form of the continuity equation resulted in sustained oscillations of the calculated normal velocity.
These oscillations are illustrated in Figure (8). The oscillations were apparently initiated because the initial conditions did not match the computer solution in the neighborhood of the initial station. It should be noted that the initial condition used in Figure (8) was substantially in error in that the absolute viscosity was inadvertently used in Equation (4) rather than the dynamic viscosity. Although this error was corrected prior to running the analysis, the data in Figure (8) serves to indicate that incompatibility of the initial conditions with respect to solutions of the difference equations can cause sustained oscillations in the solution. Since one cannot, in general, expect to develop initial conditions which are perfectly compatible with the solutions to the difference equations, it was decided to attempt to rewrite the continuity equation in such a way that any perturbations introduced by the initial conditions would not be propagated. It was found that this could be accomplished by changing the form of the continuity equation as described in the following. The resulting improvement is shown graphically in Figure (8). The change in the continuity equation can be rationalized as follows. The form of the continuity difference equation, as expressed in Equation (9), essentially averages the difference term $\Delta y$ at two stations, $m$ and $m+1$ as $\frac{\Delta y}{\Delta Y}$ illustrated in the sketch below.
The procedure for finding the normal and tangential velocities for station "m+1", given those quantities at station "m", requires the solution of the momentum equation first to find the tangential velocity at station "m+1".

The values of the tangential velocity at stations "m" and "m+1" \((U_m, U_{m+1})\) are used to calculate the difference \(\frac{\Delta U}{\Delta x}\) for station "0". This value of \(\frac{\Delta U}{\Delta x}\) is equated to the average \(\frac{\Delta V}{\Delta y}\), i.e.,

\[
\frac{\Delta V}{\Delta y}_{0} = \frac{\Delta U}{\Delta x} = \frac{\Delta V}{\Delta y}_{m} + \frac{\Delta V}{\Delta y}_{m+1}
\]

Since \(\frac{\Delta U}{\Delta x}\) has been determined from the solution of the momentum equation and \(\frac{\Delta V}{\Delta y}_{m}\) known:
\[ \frac{\Delta V}{\Delta y} \bigg|_{m+1} = 2 \left( \frac{\Delta U}{\Delta x} - \frac{\Delta V}{\Delta y} \right) \bigg|_{m} \]

Thus it may be seen that if the initial conditions are incompatible and that \( \frac{\Delta V}{\Delta y} \bigg|_{m} \) is perturbed by an amount \( \delta V \) then the quantity \( \frac{\Delta V}{\Delta y} \bigg|_{m+1} \) will be perturbed by an amount \( -\delta V \) and that the error is thus propagated from one station to the next. On the basis of the above, it was decided to modify the continuity equation by replacing the right hand side of Equation (9) with:

\[ \frac{V_{m+1,n} - V_{m+1,n-1}}{\Delta y} \]

The corresponding lowest order error term(s) went from:

\[ \text{error} = \frac{(\Delta y)^2}{6} \frac{\delta^3 V}{\delta y^3} + \frac{(\Delta x)^2}{8} \frac{\delta^3 V}{\delta x^2 \delta y} \]

to

\[ \text{error} = \frac{(\Delta y)^2}{6} \frac{\delta^3 V}{\delta y^3} + \frac{(\Delta x)}{2} \frac{\delta^2 V}{\delta x \delta y} \]

Thus the error in the \( \delta V / \delta y \) term is increased from a second order term in \( \Delta x \) to a first order term. However, the term \( \frac{\delta}{\delta x} \left[ \frac{\delta V}{\delta y} \right] \) is small so that the error term still is small.
Solutions obtained prior to this change were compared to solutions obtained after the change. The results are shown in Figure (8) by plotting the normal velocity calculated at a constant distance from the wall for various angles. It is apparent that changing the continuity equation greatly reduced the magnitude of the oscillations. The continuity equation, as used in the analysis, is given as Equation (9a).

\[ V_{m+1,n} = V_{m+1,n-1} - \left[ \frac{\Delta V}{\Delta x} \right] U_{m+1,n-1} - U_{m,n} \]

\[ + U_{m+1,n-1} - U_{m,n-1} \]  \hspace{1cm} (9a)

3.6 Solution of the Difference Equations

The following method of solution is discussed in detail in Reference (7).

Referring to Equation (8a), define the following quantities:

\[ C_{ln} = \frac{V_{m,n} + \mu g_c}{4 \Delta y} \frac{\rho}{2 \rho (\Delta y)^2} \]  \hspace{1cm} (10a)

\[ B_{ln} = \frac{U_{m,n}}{\Delta x} \frac{\mu g_c}{\rho (\Delta y)^2} \]  \hspace{1cm} (10b)

\[ A_{ln} = -\frac{V_{m,n}}{4 \Delta y} \frac{\mu g_c}{2 \rho (\Delta y)^2} \]  \hspace{1cm} (10c)
\[
D_{1n} = \left[ -\frac{g_c}{\rho} \right] \left[ \frac{dp}{dx} \right] + \left[ \frac{v_{m,n}}{4 \Delta y} \right] \left[ u_{m,n-1} - u_{m,n+1} \right] \\
+ \left[ \frac{U_{m,n}}{\Delta x} \right]^2 \left[ \frac{g_c}{2 \rho (\Delta y)^2} \right] \left[ u_{m,n-1} - 2u_{m,n} + u_{m+1,n} \right] \tag{10a}
\]

Substituting the definitions from Equations (10) into Equation (8a):

\[
\left[ -A_{1n} \right] \left[ u_{m+1,n+1} \right] + \left[ B_{1n} \right] \left[ u_{m+1,n} \right] - \left[ C_{1n} \right] \left[ u_{m+1,n-1} \right] = D_{1n} \tag{8b}
\]

Define \( E_{1n} \) and \( F_{1n} \) as follows:

\[
u_{m+1,n} = \left[ E_{1n} \right] \left[ u_{m+1,n+1} \right] + F_{1n} \tag{11}
\]

From the boundary conditions:

\[
u_{m+1,0} = 0 \tag{12}
\]

If the boundary condition given in Equation (12) is to be satisfied in Equation (11) for all \( u_{m+1,1} \) then:

\[
E_{10} = F_{10} = 0 \tag{13}
\]

A recursive relationship may be developed for \( E_{1n} \) and \( F_{1n} \) as follows.
From Equation (11):

\[ U_{m+1,n-1} = [E_{n-1}][U_{m+1,n}] * F_{n-1} \]

Substituting in Equation (8b):

\[ [-A_{n}][U_{m+1,n+1}] + [B_{n}][U_{m+1,n}] \]
\[ + [-C_{n}][E_{n-1}] + U_{m+1,n} * F_{n-1} = D_{n} \] (11a)

or:

\[ U_{m+1,n} = \frac{[A_{n}][U_{m+1,n+1}] + [C_{n}][F_{n-1}] * D_{n}}{B_{n} - [C_{n}][E_{n-1}]} \] (11b)

Equating the right hand sides of Equations (11) and (11b):

\[ E_{n} = \frac{A_{n}}{B_{n} - [C_{n}][E_{n-1}]} \] (14)

\[ F_{n} = D_{n} + [C_{n}][F_{n-1}] \frac{B_{n} - [C_{n}][E_{n-1}]}{B_{n} - [C_{n}][E_{n-1}]} \] (15)

It is shown in Reference (7) that the quantity \( E_{n} \) lies between 0 and 1 for all "n" and that \( F_{n} \) is reasonably bounded for all "n".

The method of solution then is to start with \( n=1 \) using
Equations (14) and (15) and the boundary condition, Equation (13), solve for $E_{1_l}$ and $F_{1_l}$. Once the values $E_{1_l}$ and $F_{1_l}$ are known, Equations (14) and (15) can be used to calculate $E_{2_l}$ and $F_{2_l}$ and the process repeated until $E_{n_l}$ and $F_{n_l}$ have been found for all "n". With all $E_{n_l}$ and $F_{n_l}$ known and $U_{m+1,n+1}$ known at the edge of the boundary layer (see section 3.3), one can solve for $U_{m+1,n}$ by application of Equation (11). This procedure can be repeated, moving across the boundary layer to the surface.

Once the tangential velocity has been calculated for the next station ($m+1$), Equation (9a) is then used to calculate the normal velocities for that station. This completes the solution of the momentum and continuity equations at the station ($m+1$) and the calculation proceeds to the next station.

3.7 Solution of the Energy Equation

The energy equation to be solved is the well-known boundary layer equation for constant specific heat, constant thermal conductivity and incompressible flow whose derivation can be found in Reference (2).

The equation is given below for easy reference:

$$\rho c_p \left[ U \frac{\partial T}{\partial x} - V \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2}$$

(16)
3.8 Boundary Conditions

The temperature at the outer edge of the thermal boundary layer is assumed to be constant and equal to the free stream temperature ahead of the cylinder. The surface temperature is required to satisfy an energy balance on the frost layer as described below. The energy balance cannot be performed explicitly so that an iteration procedure is required whereby a surface temperature is assumed and the energy balance made to calculate a new surface temperature; the calculated temperature is taken as the new assumed surface temperature and the process repeated until the calculated surface temperature equals the assumed value within a specified error.

Referring to the frost element below:

\[
\begin{align*}
Q_{\text{CNV}} & \uparrow T_{\infty} \\
\downarrow & \\
\quad T_0 \\
\downarrow Q_{\text{CNV}} & \\
\downarrow Q_{\text{CND}} & \\
\quad T_1
\end{align*}
\]

An energy balance on the frost element yields Equation (17).

\[Q_{\text{STOR}} = Q_{\text{CNV}} + Q_{\text{SUB}} - Q_{\text{CND}} \quad (17)\]
To simplify the analysis, the \( Q_{\text{STOR}} \) term is taken to be zero. This would seem to be justified in that a relatively small mass undergoes small changes in temperature at any point in the frost layer. Consequently, the rate of energy addition should be quite small compared to the heat transfer terms \( Q_{\text{CNV}} \) and \( Q_{\text{CND}} \). With this simplification, Equation (17) reduces to Equation (17a).

\[
Q_{\text{CNV}} + Q_{\text{SUB}} - Q_{\text{CND}} = 0
\]  

(17a)

As a first step, \( Q_{\text{CNV}} \) is calculated by solving the thermal boundary layer equation (see section 3.11), using an assumed frost surface temperature.

The solution to the thermal boundary layer equation gives the temperature profile across the boundary layer. The resulting temperature gradient near the surface can be used to obtain the heat transfer rate to the surface using Fourier's law (see section 3.18). The heat transfer coefficient \( H_0 \) can then be readily calculated as:

\[
H_0 = \frac{Q_{\text{CNV}}}{(T_\infty - T_0)}
\]

\( Q_{\text{CND}} \) is calculated using the well-known form of Fourier's law for one dimensional heat transfer in cylindrical coordinates, as given by Equation (18).
\[ \text{QCOND} = h_0(T_0 - T_1) \] (18)

Where \( h_0 \) is the thermal conductance of the frost layer and is given by (see Section 3.17):

\[ h_0 = \frac{1}{(R + kF)^{j+1}} \sum_{j} \left[ \frac{\ln[(R + kF)^{j+1}/(R + kF)_j]}{kF, j} \right] \]

\( Q_{\text{SUB}} \) is the energy given up as \( \text{H}_2\text{O} \) goes from a vapor to a solid stage. This term is given by Equation (19).

\[ Q_{\text{SUB}} = m_{\text{H}_2\text{O}}(e_{\text{vapor}} - e_{\text{solid}}) \] (19)

To facilitate the solution of Equation (17a), the enthalpy difference \( (e_{\text{vapor}} - e_{\text{solid}}) \) is assumed to be a linear function of temperature (see Equation (20)). Data was taken from Reference (3) at 0°F and 30°F and used to determine the constants in Equation (20).

\[ Q_{\text{SUB}} = m_{\text{H}_2\text{O}}(A'T_0 + B') \] (20)

The constants \( A' \) and \( B' \) were found to be:

\[ A' = -0.04667 \]

\[ B' = 1220.1 \]
Therefore, Equation (20) becomes:

\[ Q_{SUB} = M_{H_2O}(-0.04667 T_o + 1220.1) \]  \hspace{1cm} (20b)

The solution to the diffusion equation (see section 3.16), gives the concentration profile across the boundary layer. The resulting concentration gradient near the surface can be used to obtain the mass transfer rate to the surface using Fick's law of diffusion (see section 3.17). The mass transfer coefficient can then be readily calculated as:

\[ H_d = \frac{M_{H_2O}}{(C_\infty - C_o)} \]

In this analysis, the thermal resistance of the tube wall is neglected. Therefore, the term \( Q_{CND} \) is calculated on the resistance of the frost layer only and the temperature \( T_1 \) is taken to be the temperature of the refrigerant. Substituting the above expressions for \( Q_{CNV}, Q_{CND} \) and \( Q_{SUB} \) into Equation (17a):

\[ H_c(T_\infty - T_o) + H_d(C_\infty - C_o)(A'T_o + B') \]

\[ - h_c(T_o - T_1) = 0 \]  \hspace{1cm} (21)

Simplifying and rearranging:
\[ h_c(T_0 - T_1) = \frac{H_d(C_\infty - C_0)(A'T_0 + B') + H_c(T_\infty - T_0)}{H_c - H_d(C_\infty - C_0)A'T_0 + H_c} \]  \hspace{1cm} (21a)

Solving for \( T_0 \):

\[ T_0 = \frac{h_c T_1 + H_d(C_\infty - C_0)B' + H_c T_\infty}{h_c - H_d(C_\infty - C_0)A'T_0 + H_c} \]  \hspace{1cm} (22)

To summarize, the procedure is to assume a value for \( T_0 \) and calculate the corresponding values of \( H_c \) and \( H_d \) as indicated above. \( h_c \) is calculated from the frost thickness and conductivity. \( T_0 \) can then be calculated from Equation (22). If the calculated value of \( T_0 \) differs appreciably from the assumed value of \( T_0 \), the calculated value for \( T_0 \) can then be used as the next assumed value. This procedure was found to converge rapidly to within 0.1 degrees Fahrenheit. When successive values of \( T_0 \) agree within 0.1 degrees, then \( T_0 \) is taken as the correct boundary condition.

3.9 Initial Conditions

Two different temperature distributions, i.e., two initial conditions were examined.

The first trial was to assume that the first term of the energy equation, Equation (16), is negligible due to the small magnitude of the term \( U \), the tangential velocity. The term \( U \) goes to zero at the stagnation point whereas \( V \), the normal velocity, does not go to zero. This condition
was implemented by solving Equation (23) at the first station.

\[
\frac{V \partial T}{\partial y} = \frac{\alpha \partial^2 T}{\partial y^2}
\]  

(23)
The method of solution is similar to that used in section 3.6. Therefore, the method will be described only briefly here.

The difference equation associated with Equation (23) is presented as Equation (24).

\[
T_{1,n+1} - 2T_{1,n} + T_{1,n-1} - \left[ V_{1,n} \left[ \frac{\Delta y}{2a} \right] \right] T_{1,n+1} - T_{1,n} = 0
\]

(24)

Rearranging the terms results in Equation (25):

\[
\left[ T_{1,n+1} \left[ V_{1,n} \Delta y - 1 \right] \right] + 2T_{1,n} + \left[ T_{1,n-1} \left[ -V_{1,n} \Delta y - 1 \right] \right] = 0
\]

(25)
The following definitions are useful:

\[
AT'_n = 1 - \frac{V_{1,n} \Delta y}{2a}
\]

(26a)

\[
BT'_n = 2
\]

(26b)

\[
CT'_n = 1 + \frac{V_{1,n} \Delta y}{2a}
\]

(26c)
Equation (25) may be rewritten as Equation (27), using the above definitions:

\[
[-AT'_n][T_{1,n+1}] + [BT'_n][T_{1,n}] - [CT'_n][T_{1,n-1}] = 0 \quad (27)
\]

Defining \( ET'_n \) and \( FT'_n \) in the same manner as \( E_{1n} \) and \( F_{1n} \) in Equation (11), an expression for \( T_{1,n} \) may be written as:

\[
T_{1,n} = [BT'_n][T_{1,n+1}] + FT'_n \quad (28)
\]

The boundary condition is given as \( T_{1,0} = T_0 \).

As before, the boundary condition implies:

\[
ET'_0 = 0 \quad (29a)
\]

\[
FT'_0 = T_0 \quad (29b)
\]

Writing \( ET'_n \) and \( FT'_n \) in terms of \( AT'_n, BT'_n \) and \( CT'_n \):

\[
ET'_n = AT'_n - \frac{BT'_n - [CT'_n][ET'_n-1]}{BT'_n - [CT'_n][ET'_n-1]} \quad (30)
\]

\[
FT'_n = \frac{[CT'_n][FT'_n-1]}{BT'_n - [CT'_n][ET'_n-1]} \quad (31)
\]

The quantities \( ET'_n \) and \( FT'_n \) may be solved for all \( n \) using Equations (30), (31) and the boundary conditions, Equations (29a), (29b).
tions (29). With \( E'T'_n \), \( FT'_n \) and \( T_{1,n} \) known at the edge of the thermal boundary layer, \( T_{1,n} \) can be found recursively, starting at the edge of the thermal boundary layer and proceeding to the wall. Data is plotted in Figure (3) showing the data generated using the initial conditions as obtained from above. It can be seen that the first Nusselt's number is about 5% high but that the solution rapidly approaches the "true solution" as given in Reference (2).

Another approach was considered which employed the principle of similarity between the tangential velocity and temperature profiles applied to the initial station. The tangential velocity distribution was obtained as outlined in Section 3.4.

The principle of similarity may be stated mathematically as Equation (32).

\[
\begin{bmatrix}
T_{1,n} - T_{1,0} \\
T_{1,NN} - T_{1,0}
\end{bmatrix} = \begin{bmatrix}
U_{1,n} - U_{1,0} \\
U_{1,NN} - U_{1,0}
\end{bmatrix}
\] (32)

In Equation (32), the subscript "1" was used for the first subscript since it is the initial station for which a temperature distribution is required.

Data obtained by using the principle of similarity is presented in Figure (4). The Nusselt's number at the initial station is about 70% high. Although the first approach
yields much better initial values, the similarity approach offers a distinct advantage in that it should be a much more general procedure since setting $U \frac{dT}{dx} = 0$ in the energy equation could only be expected to supply reasonable values in the case of stagnation flow. Therefore, the decision was made to use the principle of similarity to obtain an approximate initial condition and improve the approximate solution as described below. A comparison of the initial temperature profiles, as obtained by the two methods, is illustrated in Figure (6).

For the following, the initial station will be taken to mean the first station for which frost formation is predicted, i.e., the station nearest the stagnation point. To improve the initial conditions obtained by the application of the principle of similarity, Equation (32) was used to establish an approximate temperature profile for a point upstream of the initial station. The energy equation is then solved implicitly, beginning with the initial conditions supplied at the upstream point and progressing to the initial station. The method of solution is very similar to the method by which the momentum equation was solved in Section 3.6. The reader is also referred to Section 3.11 where the solution of the energy equation is discussed.

There are, however, three major differences in technique employed from the upstream point to the initial station: 1) the step size is reduced from $2^0$ intervals to 32
\( \frac{1}{2} \)° intervals, 2) the surface temperature \( T_{m,0} \) is considered constant for all "m", and 3) the temperature at each station is averaged with the corresponding temperature from the previous station (at the same "n") in order to smooth oscillations which otherwise might occur. The third difference may be expressed as:

\[
T_{m+1,n} = \frac{T_{m+1,n \text{ (CALC)}} + T_{m,n}}{2}
\]

The averaging technique proved to be effective in eliminating the oscillations which are evident in Figure (4). The combined techniques, using the principle of similarity along with the application of the initial conditions upstream of the initial station, provided satisfactory initial conditions at the initial station. This is supported in Figure (5) where data, obtained by the combined techniques, is plotted along with data from Reference (2).

3.10 Finite Difference Approximations to the Energy Equation

The difference quotients of Equations (5), (6) and (7) are substituted into Equation (16), to obtain Equation (33).

\[
\begin{bmatrix}
U_{m,r} \\
V_{m,n} \\
T_{m,n+1}-T_{m,n-1}+T_{m+1,n}-T_{m+1,n-1}
\end{bmatrix}
\begin{bmatrix}
\frac{T_{m+1,n-T_{m,n}}}{\Delta x} \\
\frac{T_{m,n+1}-T_{m,n-1}+T_{m+1,n}-T_{m+1,n-1}}{4 \Delta y}
\end{bmatrix}
\]

\[
= \frac{k}{\rho c_p (\Delta y)^2} \begin{bmatrix}
T_{m,n-1}-2T_{m,n}+T_{m,n+1} \\
+T_{m+1,n-1}-2T_{m+1,n}+T_{m+1,n+1}
\end{bmatrix}
\]

(33)
3.11 Solution of the Difference Equations

The solution of the energy difference equation proceeds in the same way as the solution of the momentum equation in Section 3.6.

The following definitions are made for the sake of convenience:

\[
CT_n = \frac{V_{m,n}}{4\Delta y} + \frac{k}{2\rho c_p (\Delta y)^2} \]

\[
BT_n = \frac{U_{m,n}}{\Delta x} + \frac{k}{\rho c_p (\Delta y)^2} \]

\[
AT_n = -\frac{V_{m,n}}{4\Delta y} + \frac{k}{2\rho c_p (\Delta y)^2} \]

\[
DT_n = \left[ \frac{k}{\rho c_p^2 (\Delta y)^2} \right] \left[ T_{m,n-1} - 2T_{m,n} + T_{m,n+1} ight] 
+ T_{m+1,n-1} - 2T_{m+1,n} + T_{m+1,n+1} \]

Substituting the definitions from Equations (34) into Equation (33) and rearranging gives Equation (35).
\[
\begin{bmatrix}
-AT_n \\
\end{bmatrix}
\begin{bmatrix}
T_{m+1,n+1}
\end{bmatrix}
+ \begin{bmatrix}
BT_n \\
\end{bmatrix}
\begin{bmatrix}
T_{m+1,n}
\end{bmatrix}
+ \begin{bmatrix}
-CT_n \\
\end{bmatrix}
\begin{bmatrix}
T_{m+1,n-1}
\end{bmatrix}
= DT_n 
\]

(35)

Defining \( ET_n \) and \( FT_n \) similar to the definitions of \( E\alpha_n \) and \( F\alpha_n \) in Equation (11):

\[
T_{m+1,n} = \begin{bmatrix}
ET_n
\end{bmatrix}
\begin{bmatrix}
T_{m+1,n+1}
\end{bmatrix}
+ FT_n
\]

(36)

The boundary condition is:

\[
T_{m+1,0} = T_0
\]

Recursive relationships can be derived for \( ET_n \) and \( FT_n \) and are presented as Equations (37).

\[
ET_n = \frac{AT_n}{BT_n - \begin{bmatrix}
CT_n \end{bmatrix}\begin{bmatrix}
ET_{n-1}
\end{bmatrix}} 
\]

(37a)

\[
FT_n = \frac{DT_n - \begin{bmatrix}
CT_n \end{bmatrix}\begin{bmatrix}
FT_{n-1}
\end{bmatrix}}{BT_n - \begin{bmatrix}
CT_n \end{bmatrix}\begin{bmatrix}
ET_{n-1}
\end{bmatrix}} 
\]

(37b)

Equations (37) and (36) can be solved in that order to obtain the temperature distribution throughout the boundary layer, in the same way that the tangential velocity was found in Section 3.6.
3.12 Solution of Diffusion Equation

The diffusion equation is identical in form to the energy equation and its method of solution is directly parallel. Therefore, the procedure is outlined only briefly.

The differential equation is presented as Equation (38).

\[
\frac{U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y}}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \tag{38}
\]

This equation is developed in Reference (8).

3.13 Boundary Conditions

The concentration of H₂O vapor at the outer edge of the diffusion boundary layer is assumed to be constant and equal to the concentration of H₂O vapor in the free stream ahead of the cylinder.

The water vapor concentration at the frost surface is taken to be that of saturated air at the surface temperature. It is known that for dilute solutions the log of the vapor pressure of the minor constituent is inversely proportional to the absolute temperature of the mixture. In equation form:

\[
P = e^{\frac{A+B}{T}} \tag{39}
\]

A plot of the vapor pressure of water and temperature
is presented as Figure (10). It can be seen that for the range of interest the points can be represented quite well by a straight line on a semi-log graph which verifies the validity of Equation (39). A line was constructed through the points of Figure (12) and the slope and intercept of that line are used to calculate the constants A and B of Equation (39). The resulting equation is presented as Equation (39a).

\[ P_{H_2O} = 144 e^{\frac{19.986-11.304}{T+460}} \]  \hspace{1cm} (39a)

With the partial pressure of water vapor known, the concentration of water in mole fraction is obtained by application of Dalton's law.

3.14 Initial Conditions

Since the differential equations for the thermal boundary layer and the diffusion boundary layer are identical in form, the results of the investigation on initial conditions for the thermal boundary layer were applied to the diffusion boundary layer also. Therefore, the initial condition was obtained by applying the principle of similarity at a point upstream of the initial station and solving the diffusion boundary layer equation at several intervals from that point to the initial station. This approach was described in more detail in Section (3.9).
3.15 Finite Difference Approximations to the Diffusion Equation

The difference quotients of Equations (5), (6) and (7) are substituted into Equation (38) to obtain Equation (38a).

\[
\begin{bmatrix}
U_{m,n} & \frac{C_{m+1,n} - C_{m,n}}{\Delta x} \\
\end{bmatrix}
\]

\[
+ \begin{bmatrix}
V_{m,n} & \frac{C_{m,n+1} - C_{m,n-1} + C_{m+1,n} - C_{m-1,n-1}}{4\Delta y}
\end{bmatrix}
\]

\[
=D \begin{bmatrix}
\frac{C_{m,n-1} - 2C_{m,n} + C_{m,n+1} + C_{m+1,n-1} - 2C_{m+1,n} + C_{m+1,n+1}}{2(\Delta y)^2}
\end{bmatrix}
\]

(38a)

3.16 Solution of the Difference Equation

The following definitions correspond to those made in Equations (34).

\[
CG_n = \frac{V_{m,n}}{4\Delta y} + \frac{D}{2(\Delta y)^2}
\]

(40a)

\[
BG_n = \frac{U_{m,n}}{\Delta x} + \frac{D}{(\Delta y)^2}
\]

(40b)

\[
AC_n = -\frac{V_{m,n}}{4\Delta y} + \frac{D}{2(\Delta y)^2}
\]

(40c)
\[ \text{DC}_n = \frac{D}{2(\Delta y)^2} \left[ C_{m,n-1} - 2C_{m,n} + C_{m,n+1} \right] \]

(40a)

Rearranging the terms in Equation (38a) and substituting the definitions from Equations (40) yields Equation (41).

\[ \left[ -\mathcal{A}_n \right]\left[ C_{m+1,n+1} \right] + \left[ \mathcal{B}_n \right]\left[ C_{m+1,n} \right] - \left[ \mathcal{C}_n \right]\left[ C_{m+1,n-1} \right] \]

\[ = \text{DC}_n \]

(41)

Defining \( \mathcal{E}_n \) and \( \mathcal{F}_n \) similar to the definitions of \( \text{El}_n \) and \( \text{Fl}_n \) in Equation (11):

\[ C_{m+1,n} = \left[ \mathcal{E}_n \right]\left[ C_{m+1,n+1} \right] + \mathcal{F}_n \]

(42)

The boundary condition is obtained readily from Equation (39a):

\[ C_{m+1,0} = \frac{144}{14.698} e^{19.966 - \frac{11.304}{T_{m+1}c^460}} \]

(43)

Recursive relationships have been derived for \( \mathcal{E}_n \) and \( \mathcal{F}_n \) and are given in Equations (44).

\[ \mathcal{E}_n = \frac{\mathcal{A}_n}{\mathcal{B}_n - \left[ \mathcal{C}_n \right]\left[ \mathcal{E}_{n-1} \right]} \]

(44a)
\[
F_{Cn} = \frac{D_{Cn} + \left[ C_{Gn} \right] F_{Cn-1}}{B_{Cn} - \left[ C_{Gn} \right] E_{Cn-1}}
\] (44b)

Equations (44) and (48) can be solved in that order to obtain the concentration profile throughout the boundary layer as was done for the momentum and energy equations.

3.17 Frost Model

The thermal resistance of the frost layer is required in order to perform the heat balance indicated in Section 3.8 and hence supply the required boundary condition, i.e., the surface temperature. The analytical model developed for the frost layer is a relatively simple one which, it was hoped, would serve to demonstrate the potential of the over-all method. The model may be extended as fundamental information on frost structure becomes available.

The important properties of the frost layer for this analysis are the frost density and frost conductivity. The rate of frost growth (change in thickness per unit time) is obviously inversely proportional to the frost density for a given rate of mass transfer to the surface. Since much of the experimental data concerning frost formation is presented in terms of the rate of growth of the frost layer under various conditions, the test of any analytical model would be likely to include a comparison of predicted and measured growth rates.

Also, in order to correctly evaluate the frost sur-
face temperature (boundary condition discussed in Section 3.8) the frost thickness and conductivity are required in order to obtain the thermal resistance of the frost layer.

The model chosen for this analysis is described with the aid of the following sketch.

The surface of the tube is shown as having a radius \( R_1 \). At some short period of time after frost formation begins, there is a layer of frost having an outer radius \( R_2 \) and at a later time \( R_3 \) and so on. The time intervals are chosen to be sufficiently short that the properties of the frost deposited during that time interval may be approximated as constant. In other words the properties
are considered constant from R₁ to R₂ with a step change at R₂; constant from R₂ to R₃, and etc. The problem is made two dimensional by allowing a variation in frost properties with the angle "theta". Circumferential heat transfer within the frost layer was neglected since the total thickness was always small compared to the circumference, i.e., $(R₃-R₁) \ll (2\pi R₁)$. The heat balance for each angular segment is treated as a series of cylindrical segments having a variable thickness and conductivity. Another sketch illustrates one of these segments.

![Diagram of frost layer with temperature difference and adiabatic surface](image)

The equation for the rate of heat transfer per unit area from the frost surface to the cylinder surface is given as Equation (45).

$$Q = A \left[ \frac{T₀ - T₁}{R₃ \left( \frac{\ln(R₂/R₁) + \ln(R₃/R₂)}{k₁ + k₂} \right)} \right]$$  \hspace{1cm} (45)
As more layers of frost are added, more terms of the form $\frac{\ln(R_{i+1}/R_i)}{k_i}$ will appear in the denominator of Equation (45). For the general case, with "$i$" layers, Equation (45) may be written as Equation (45a)

$$Q = Ah_c (T_0 - T_i)$$

(45a)

The quantity $h_c$ is the thermal conductance of the frost layer and is given by:

$$h_c = \frac{1}{R_{i+1}} \left( \frac{\ln(R_{i+1}/R_i)}{1 - \frac{\ln(R_{i+1}/R_i)}{k_i}} \right)$$

The above neglects any change in the frost properties at a given position with respect to time. In other words, the frost is considered to be in steady state after formation. It has been well established that vapor diffusion does occur within the frost layer due to the vapor pressure gradient which is a consequence of the temperature gradient. However, the diffusion process is a relatively slow process and it was concluded in Reference (11) that the diffusion process has little effect on changing the local frost density or thermal conductivity with time (although the diffusion of vapor does play an important role in the thermal conductivity, it is apparently fairly constant.
with time). The above conclusion is limited to the first 60 to 80 minutes of frost formation. However, the time intervals considered in this thesis are substantially less and therefore the assumption of invariant (with respect to time) properties is felt to be justified.

The remaining problem then is to predict the properties at a given point in the frost layer. This problem has been simplified in that a good correlation between density and conductivity has been developed and may be found in References (9) and (12). For this thesis it was decided to express the conductivity as a function of density. This function was given in Reference (12) as Equation (46).

\[ k_f = 0.0140 + 0.00668 \rho_f + 0.000175 \rho_f^2 \quad (46) \]

Thus the problem is now reduced to predicting the density at a given point.

Several variables have been suggested in the literature as having an apparent effect on the density of frost. These include surface temperature, air velocity and humidity ratio. However, it would seem to this author that these are all secondary variables in the sense that they are all properties external to the frost layer and could not uniquely define properties within the frost layer itself. Consistent with this concept, primary variables would be defined as ones which uniquely define conditions
at the surface of the frost layer. Two variables which might be considered primary variables would be the frost surface temperature (to be distinguished from tube surface temperature) and the rate of mass transfer to the surface. To summarize, it will be assumed that the way in which the H₂O crystals form at the surface of the frost layer must be related to conditions existing at the surface of the frost layer itself, i.e., to the primary variables and not directly to what has been designated the secondary variables. As a first step, it was assumed that the most important primary variable is the surface temperature. There was inferred in Reference (4), a correlation of surface density versus time. The density was calculated from the measured rate of change of thickness of the frost layer and the calculated rate of diffusion of water vapor to the surface. Although the density was estimated as a function of time, the surface temperature history was not known so that the relationship between temperature and density cannot be inferred directly. However, the surface temperature at the beginning of the run was given as 19.6°F while the surface temperature at the end of the run could be expected to be approximately 32°F. The initial and final densities were given as about 5.6 and 37.5 lbm/ft³. Based on the above rather sketchy information, the relationship assumed for this analysis was taken to be that illustrated by the solid line in Figure (14).
After having completed most of the computer analysis, further data was obtained which supported the assumed temperature dependency. A portion of the data from Reference (9) was presented as the measured frost thickness, total weight and lapsed time. The data was used by that author to calculate the average density by dividing the weight of the entire layer by the volume, with the volume based on the total thickness of the frost layer. The data was re-evaluated for this analysis, assuming the frost layer to be built up of various layers, each layer having a different density. Hence the net change in weight between consecutive readings was divided by the change in volume, based on the change in frost thickness during the same interval of time. The resulting data is also plotted in Figure (14) for comparison to the correlation used in this analysis. All the data tabulated in Reference (9) is included in Figure (14) with the following exceptions. The data in Reference (9) represents several runs. The first and last points taken for each run have been discarded. The first points were discarded because the author of Reference (9) indicated they were generally felt to be unreliable due to measurement difficulties. The end points were also discarded because they generally exhibited a pronounced drop in apparent density. It is felt that this may represent the onset of significant diffusion within the frost layer, transferring mass from the outer toward the inner
surface. This is supported in that the last reading would generally indicate the onset of a quasi-steady state condition. Also, it was indicated in Reference (11) that diffusion within the frost layer may become significant after 60 to 80 minutes. The runs recorded in Reference (9) generally exceeded two hours in length.

Also, in several instances where two consecutive readings showed large discrepancies in the calculated density, the changes in weight and volume were evaluated from the beginning of the first time interval to the end of the second time interval and the average density for both time intervals thus obtained. This approach was felt to be justified in that the differences obtained for one time interval, were small quantities so that the quotient \( \frac{\Delta \text{mass}}{\Delta \text{volume}} \) could result in a large error if one of the readings were slightly in error.

The resulting points were submitted to a curve fit, using the method of least squares and assuming a linear equation for the range from \( T = 25.8^\circ F \) to \( T = 32.0^\circ F \). The density was taken to be constant at 15.6 lbm/ft\(^3\) for \( T < 25.8^\circ F \). The resultant best fit curve is also shown in Figure (14). The best fit curve was also used as input data for one computer analysis. The results are discussed in Chapter (5).

It should be noted that two points appear in Figure (14) that were not included in the curve fitting procedure.
These points appear in the upper left hand corner of Figure (14). These points obviously represent large deviations from the pattern shown by the other data. Both points were obtained in the same run. It was observed that the air velocity was approximately two times the air velocity for the next highest run, possibly resulting in the transition from a laminar to a turbulent boundary layer. Changes in velocity between other runs (all at lower velocities) did not show a consistent velocity effect.

It was considered probable that either the data was in error or a different mechanism was in effect, such as the possibility of a turbulent boundary layer. Therefore, the data was plotted but was not included in the curve fitting procedure.

3.18 Calculation of the Dimensionless Transport Parameters

The Nusselt's number is calculated by numerical differentiation of the temperature profile, using the first four points from the wall. Using the Newton forward-difference formula (4th degree) to approximate temperature as a function of distance from the surface, \( T(y) \), results in Equation (47):

\[
T(s) = T(0) + \left[ \frac{s}{1} \right] \Delta T(0) + \left[ \frac{s}{2} \right] \Delta^2 T(0) \\
\quad + \left[ \frac{s}{3} \right] \Delta^3 T(0) + \left[ \frac{s}{4} \right] \Delta^4 T(0)
\]

(47)

where \( \binom{s}{n} \) is the binomial function and \( s = y / \Delta y \). Thus
$s = 1$ implies $y = \Delta y$, $s = 2$ implies $y = 2 \Delta y$, etc. Also, in Equation (47), $\Delta^n T(0)$ is the $n^{th}$ forward difference of $T(0)$. The derivative of $T$ with respect to $y$, $\frac{dT}{dy}$, evaluated at the wall results in Equation (48).

$$\frac{dT}{dy} = \frac{1}{\Delta y} \begin{bmatrix} \left[ \frac{1}{6} \right] T_m, 0 - 3 T_m, 1 - \left[ \frac{3}{2} \right] T_m, 2 - \left[ \frac{1}{3} \right] T_m, 3 \end{bmatrix}$$  \hspace{1cm} (48)

Fourier's law for heat transfer normal to the wall is:

$$Q_{CNV} = -k \frac{dT}{dy}$$  \hspace{1cm} (49)

Substituting Equation (48) into Equation (49) and simplifying:

$$Q_{CNV} = \frac{k}{6 \Delta y} \begin{bmatrix} 11 T_m, 0 - 18 T_m, 1 - 9 T_m, 2 - 2 T_m, 3 \end{bmatrix}$$  \hspace{1cm} (50)

letting $f(T) = 11 T_m, 0 - 18 T_m, 1 - 9 T_m, 2 - 2 T_m, 3$:

$$Q_{CNV} = \left[ \frac{k}{6 \Delta y} \right] [f(T)]$$  \hspace{1cm} (50a)

By definition of the convective heat transfer coefficient:

$$-Q_{CNV} = h_c (T_\infty - T_1)$$  \hspace{1cm} (51)

The left hand term is preceded by a minus sign to be
consistent with the sign convention used in Equations (50) and (50a). The Nusselts number is defined by Equation (52).

\[ \frac{\text{Nu} \cdot \text{Nu}}{\text{Re}} = \frac{2R_{\text{f}} C}{k} \]  

(52)

Substituting Equation (50a) and (51) into Equation (52):

\[ \text{Nu} = 2R_{\text{f}}(T) \frac{6 \Delta y(T_1 - T_\infty)}{6 \Delta y(T_1 - T_\infty)} \]  

(52a)

The mass transfer rate is obtained from Equation (53):

\[ J_{\alpha}^* = -\rho D \frac{\partial C}{\partial y} \]  

(53)

This is the equation for diffusion of one species (gas) through a second gas with the second gas being stagnant. This condition is met in the present case since the diffusion rate is sought in a direction normal to the wall and the normal velocity goes to zero at the wall. Equation (53) may be found in Reference (8). By definition:

\[ J_{\alpha}^* = N_{\alpha} - C(N_{\alpha} + N_{\beta}) \]

Also, \( N_{\beta} = 0 \) since the second species (air) is not removed at the surface. Substituting the above definition into Equation (53) results in Equation (53a):

\[ N_{\alpha}(1-C) = \rho D \frac{\partial C}{\partial y} \]  

(53a)
To get the mass flow rate, the molar flow rate is multiplied by the molecular weight of H2O. Therefore, multiplying both sides of Equation (53a) by the molecular weight of H2O gives Equation (54):

\[ \frac{\dot{M}_{\text{H}_2\text{O}}}{\dot{M}_{\text{air}}^0} \rho_d \frac{\partial C_{\text{m},3}}{(1-C) \partial y} \]  \(54\)

The quantity \( \frac{\partial C_{\text{m}}}{\partial y} \) is evaluated in the same way as \( \frac{\partial T}{\partial y} \) above:

\[ \frac{\partial C_{\text{m}}}{\partial y} = \frac{1}{\Delta y} \left[ -\left( \frac{11}{5} \right) C_{\text{m},0} + 3 C_{\text{m},1} + 3 C_{\text{m},2} + \left( \frac{1}{3} \right) C_{\text{m},3} \right] \]  \(55\)

Since \( \frac{\partial C}{\partial y} \) is evaluated at the surface, the term \((1-C)\) in Equation (54) is also evaluated at the surface. Substituting Equation (55) into Equation (54):

\[ \frac{\dot{M}_{\text{H}_2\text{O}}}{\dot{M}_{\text{air}}^0} = \left[ \frac{\rho}{(1-C_{\text{m},0})} \right] \left[ \frac{D}{6 \Delta y} \right] \left[ 11 C_{\text{m},0} - 18 C_{\text{m},1} + 9 C_{\text{m},2} \right] \]  \(56\)

\[ -2 C_{\text{m},3} \left[ \frac{\dot{M}_{\text{H}_2\text{O}}}{\dot{M}_{\text{air}}} \right] \]

The usual definition for the mass transfer coefficient is given in Equation (57).

\[ -\frac{\dot{M}_{\text{H}_2\text{O}}}{H_d} = \left[ C_{\infty} - C_1 \right] \left[ \frac{\dot{M}_{\text{air}}}{\dot{M}_{\text{H}_2\text{O}}} \right] \]  \(57\)

51
The left hand side of Equation (57) is negative, consistent with the sign conventions previously adopted.

The Lewis number is defined in Equation (58):

\[ \text{Le} = \frac{(H_c)}{(H_d)(c_p)} \]  

(58)
CHAPTER 4
DISCUSSION OF RESULTS

The solutions of the three boundary layer equations were tested for accuracy by obtaining solutions for an isothermal, cylindrical surface and comparing the solutions with accepted analytical and empirical correlations.

4.1 Momentum and Continuity Equation

Solutions were obtained for flow normal to the axis of a circular cylinder. Two solutions were obtained, one for an external flow field as predicted by potential flow theory and another solution was obtained for an external flow field obtained from experimental data. Both external flow fields are plotted for reference in Figure (1). The curve representing the empirical data is extrapolated as a smooth curve beyond the point of separation to avoid imposing an artificial point of separation on the computer solution. The predicted point of separation also serves as a check on the accuracy of the computer solution. The empirical data plotted in Figure (1) was taken from data reproduced in Reference (1).

The point of separation was calculated to be 108.2 degrees from the stagnation point for a potential external field and 53.3 degrees for the empirical external flow field. The commonly accepted points of separation are
108.8 degrees and 62 degrees respectively. It was felt that the calculated points of separation were sufficiently close for the purposes of this analysis.

A further check on the program was made by comparing velocity profiles across the boundary layer, as calculated by the program, with similar profiles as published in Reference (2). The external flow was calculated from potential flow theory in both cases. Representative data shown in Figure (2) was tabulated before plotting to avoid biasing the data. The tabulated points are shown in Figure (2) to indicate the accuracy with which the data from Reference (2) could be read. It should be noted that the grid used for the computer program was finer than indicated in Figure (2) since only the even numbered points were plotted \((n=0,2,4,\text{etc.})\) to improve the clarity of the figures. This comparison was made at the angular positions \((\theta=20,40,60,80,90\text{ and }100\text{ degrees from the stagnation point})\). A deviation appeared at 80 degrees, as may be seen in Figure (2), where the calculated velocity distribution shows a less sharp bend in the "knee" of the curve than is indicated by the data from Reference (2). Approximately the same degree of deviation is in evidence for \(\theta=90\text{ and }\theta=100\text{ degrees as is seen for }\theta=80\text{ degrees. The discrepancy is not considered to be serious since it is relatively small in magnitude and occurs primarily at a distance from the surface greater than 0.0012\text{ ft.} The points used to calculate the transport
processes are less than 0.0004 ft. from the surface.

4.2 Energy Equation

The solution of the energy equation was tested by obtaining a solution for an isothermal, cylindrical surface. The velocity distributions used in the solution were obtained by solution of the momentum and continuity equations for the empirical external flow field. The results are presented in Figure (5) as the parameter $\frac{\text{NU}}{\sqrt{\text{Re}}}$, along with an accepted analytical solution found in Reference (2) and several experimental points, also from Reference (2). It may be seen from Figure (5) that the present solution does diverge somewhat from the accepted solution, from about 60 degrees to the point of separation. It seems likely that this divergence is due to the external flow field imposed as a boundary condition. Unfortunately, the empirical external flow data used for this program could not be read from Reference (1) with a high degree of accuracy, especially from about 60 degrees where a rapid change in acceleration occurs. The data used was read as carefully as possible and smoothed. The results could undoubtedly be improved by adjusting the external flow velocity from 60 degrees to the point of separation. However, this would certainly bias the results and therefore the external flow field was unaltered.

It is also seen from Figure (5) that the calculated quantity $\frac{\text{NU}}{\sqrt{\text{Re}}}$ decreases rapidly from about 70 degrees
compared to the analytical results reported in Reference (2). However, the experimental results drop off even more rapidly than that calculated in the present analysis. It is not known what external flow values were imposed on the solution from Reference (2). Therefore, the above results were considered acceptable in view of the uncertainty associated with the external flow velocity used in this analysis as well as that used in other analytical solutions.

4.3 Diffusion Equation

Having established the acceptability of the solution to the energy equation, the solution to the diffusion equation was checked by calculating the Lewis Number at each station and comparing the calculated Lewis Number with an experimentally determined Lewis Number obtained from Reference (3). The results are plotted in Figure (7). It should be noted that the experimental value for the Lewis Number is an average value whereas the calculated Lewis Number is a local value. However, since the calculated Lewis Numbers are relatively constant, it is felt that this comparison verifies the correctness of the solution to the diffusion equation.

4.4 Effect of Surface Temperature Gradients on Solutions To Energy and Diffusion Equation.

As the frost layer develops, the frost surface temperature varies around the circumference of the cylinder as shown for representative time intervals in Figure (25).
The quantity $NU/\sqrt{Re}$ was also plotted for the same time intervals. The results are presented as Figure (23). It can be seen that, at the point of separation, the quantity $NU/\sqrt{Re}$, as calculated for a non-isothermal surface, was 54% higher than the same parameter calculated for an isothermal surface.

The Lewis Number was also plotted for two time intervals and is shown as Figure (24). The increase in the Lewis Number, at the point of separation, is about 26% for a non-isothermal surface. Therefore, if the diffusion rate were obtained using Colburn's analogy and using the established solution for the heat transfer coefficient, based on an isothermal surface, the actual diffusion rate divided by the calculated diffusion rate would be:

$$\frac{H_d(\text{actual})}{H_d(\text{calc})} = \frac{1.58}{1.23} = \frac{1.23}{1.28}$$

In other words, the simplified solution would give a diffusion rate which would be about 19% low at the point of separation.

4.5 Comparison of Theoretical and Experimental Rates of Frost Formation

The most meaningful comparison of predicted and observed growth rates would be possible if the observations were made at various positions around the cylinder. Such data is rare although one such observation is reported in
Reference (4). The analysis was performed under the same conditions of air temperature, air velocity, cylinder diameter, humidity ratio, and tube surface temperature which were reported for the experimental observation. The analysis was performed two times for different correlations of frost temperature versus frost density as discussed in Section 3.17. The first analysis was based on a very approximate correlation which was obtained from limited information found in Reference (4). The results are shown in Figures (15) and (16) for data near the stagnation point and point of separation respectively. The second analysis was based on more complete frost density data reported in Reference (9). The interpretation of this data was discussed in Section 3.17 in some detail. The results of this analysis are shown in Figures (21) and (22) for data near the stagnation point and the point of separation respectively. The results shown in the four Figures, (15), (16), (21) and (22) are felt to be satisfactory, especially considering the uncertainty associated with the correlation of frost density as a function temperature.

A qualitative check of the analytical model was made by calculating the frost growth rate for different wall temperatures with other conditions remaining the same. The predicted effect of wall temperature is shown in Figure (17). This may be compared with a similar correlation obtained in Reference (12) and reproduced as Figure (26).
The data from Reference (12) was obtained for flow over a flat plate so that a quantitative comparison would not be meaningful. The calculated data is for a position near the stagnation point on a cylinder. However, the trends and the general shape of the curves are strikingly similar.

Another qualitative check of the analytical model was made by calculating the frost growth rate for different humidity ratios with other conditions remaining the same. The predicted effect of the humidity ratio is shown in Figure (18). This may be compared with a similar correlation obtained in Reference (12) and reproduced as Figure (27). Again, the data obtained in Reference (12) is for flow over a flat plate so that a meaningful quantitative comparison cannot be made. However, the qualitative comparison appears to be satisfactory in this case also. The higher humidity ratio results in higher rates of frost formation in both cases. The data from Reference (12) began to diverge significantly after about ten minutes. Unfortunately, the computer run was terminated at about the ten minute point. The runs were not repeated in the interest of economy although it should be emphasized that there was no computer limitation on the extent of data generated. In any case, it appears that the analytical solutions are also beginning to diverge after about ten minutes.

A third qualitative check of the analytical model was made by calculating the frost growth rate for different air
free stream velocities with other conditions remaining the same. The predicted effect of the free stream velocity is shown in Figure (19). This may be compared with a similar correlation from reference (12), obtained for flow over a flat plate, which is reproduced as Figure (28). The qualitative comparison again appears to be satisfactory. The indicated effect of velocity appears to be small in both cases. The higher velocities result in slightly higher rates of frost formation in both cases for the first ten minutes. The data from reference (12) converged again after approximately forty minutes, whereas the calculations terminated after about ten minutes so that it is not known whether the analytical results would have converged.

The effective frost thermal conductivity was obtained from the standard equation for heat transfer through a hollow, homogeneous cylinder with inner radius \( r_1 \) and outer radius \( r_0 \):

\[
Q_{CND} = \frac{k_{eff}(T_0 - T_1)}{r_0 \ln(r_0/r_1)}
\]

The above equation may be readily solved for \( k_{eff} \) since the other quantities have been evaluated in the analysis.

The effective conductivity was evaluated for two analysis wherein the humidity ratio was changed between runs but with the other conditions remaining the same. The
results are plotted in Figure (20). These results may be compared with a similar study made in Reference (12) using data obtained from a flat plate. Data from Reference (12) is reproduced as Figure (29).

Again the results appear to be reasonably similar. In both cases the higher humidity ratio resulted in a higher effective conductivity, and in both cases the difference was quite significant.

The analytically predicted effect of humidity ratio on the effective thermal conductivity of the frost layer tends to support the concept of the roles of the primary and secondary variables as proposed in Section (3.17).
CHAPTER 5
CONCLUSIONS

The following conclusions may be drawn from this investigation.

(1) The implicit numerical procedure, employed to obtain solutions to the boundary layer equations, gave satisfactory results as determined by the comparison of calculated results to known analytical solutions and empirical data.

(2) The numerical solution of the boundary layer equations can be combined with a relatively simple frost model which was shown to predict frost formation rates that agreed reasonably well with experimental values.

(3) The method lends itself to the use of a more comprehensive frost model which might take into account primary variables other than surface temperature as the basic data becomes available.

(4) The method can readily be used for analysis of problems involving arbitrary geometries.

(5) It was shown that the use of analytical or empirical correlations based on an isothermal surface can be considerably in error if applied under frosting conditions.
APPENDIX A - COMPUTER PROGRAM DESCRIPTION

A.1 Computer Program Description

The momentum and continuity equations are solved in one of two subroutines which is identified as "CALCV". The energy and diffusion equations are solved in the second subroutine which is identified as "CALCTD". The main program contains the basic logic required to execute the program, provides initial and boundary conditions and loads the necessary constants. The two subroutines and the main program will be discussed individually.

A.2 Subroutine "CALCV"

The subroutine "CALCV" is called by the main program with the main program supplying the quantities $U_m^n$ and $V_m^n$ for all "n" as well as the boundary conditions $U_{m+1,0}$, $U_{m+1,NN}$ and $V_{m+1,0}$. The subroutine then returns to the main program the quantities $U_{m+1,n}$ and $V_{m+1,n}$, for $n = 1$ to $n = NN-1$, and $V_{m+1,NN}$. A simplified flow diagram for "CALCV" is presented as Figure(11). The procedure is as follows:

1) Block 1 - The subroutine in turn calls the function "CUB" which interpolates external velocity data,
given the angular position, to obtain the external velocity corresponding to that position. The tangential velocity, \( U_{m+1,NN} \), at the outer edge of the boundary layer is equated to the external flow velocity.

2) Block 2 - An iteration procedure varies the index "n" from 1 to \( NN-1 \). The point \( n = 0 \) corresponds to a point on the surface, while \( n = NN \) corresponds to the outer edge of the boundary layer.

3) Block 3 - The quantities \( Al_n, Bl_n, Cl_n, Dl_n, El_n \) and \( Fl_n \) are evaluated from Equations (10a), (10b), (10c), (10d), (13), (14) and (15), as discussed in Section 3.6, as "n" goes from 1 to \( NN-1 \).

4) Block 4 - Another iteration procedure, similar to Block 2, varies the index "n" from \( n = NN-1 \) to \( n = 0 \).

5) Block 5 - The tangential velocity, \( U_{m+1,n} \), is calculated recursively as "n" varies from \( NN-1 \) to zero.

6) Block 6 - The normal velocity at the surface is set equal to zero.

7) Block 7 and 8 - An iteration procedure varies "n" from \( n = 1 \) to \( n = NN \). The normal velocity, \( V_{m+1,n} \), is calculated for each "n" from Equation (9a).

8) Blocks 9 and 10 - A test is made for flow separation. The variable "ISEP" remains zero until flow separation is reached. This is determined when \( U_{m+1,1} \), the tan-
tential velocity adjacent to the surface, becomes
less than or equal to zero. Upon return of control
to the main program, ISEP is checked for zero and
subsequent operations executed accordingly.

A.3 Subroutine "CALCTD"

The subroutine "CALCTD" is called by the main program
with the main program supplying the quantities \( T_{m,n} \) and
\( C_{m,n} \) for all "n" as well as the boundary conditions \( T_{m+1,0}, \)
\( T_{m+1,NN}, C_{m+1,0} \) and \( C_{m+1,NN} \). The subroutine then returns
to the main program the quantities \( T_{m+1,n} \) and \( C_{m+1,n} \) for
\( n = 1 \) to \( n = NN-1 \). A simplified flow diagram for "CALCTD"
is presented as Figure (12). The procedure is as follows:

1) Block 1 - \( ET_0 \) and \( FT_0 \), defined in Equation (36), are
   set.

2) Blocks 2 and 3 - An iteration procedure varies the
   index "n" from \( n = 1 \) to \( n = NN-1 \) while the quanti-
ties \( AT_n, BT_n, CT_n, DT_n, ET_n \) and \( FT_n \) are calculated using
Equations (34a), (34b), (34c), (34d), (37a) and (37b).

3) Block 4 - The temperature at the outer edge of the
   boundary layer, \( T_{m+1,NN} \), is set equal to the free
   stream temperature.

4) Blocks 5 and 6 - An iteration procedure varies the
   index "n" from \( n = 1 \) to \( n = NN-1 \) while \( T_{m+1,n} \) is
   calculated using Equation (36).

5) Block 7 - The variables \( EC_0 \) and \( FC_0 \), defined in Equa-
tion (36) are evaluated.
6) Blocks 8 and 9 - An iteration procedure varies the index "n" from \( n = 1 \) to \( n = \text{NN} - 1 \) while the quantities \( AC_n, BC_n, CC_n, DC_n, EC_n \) and \( FC_n \) are calculated using Equations (40a), (40b), (40c), (40d), (44a) and (44b).

7) Block 10 - The concentration of water vapor at the outer edge of the boundary layer, \( C_{m+1,N} \), is set equal to the free stream concentration.

8) Blocks 11 and 12 - An iteration procedure varies the index "n" from \( n = 1 \) to \( n = \text{NN} - 1 \) while the quantity \( C_{m+1,n} \) is calculated recursively using Equation (42).

9) Block 13 - The Nusselt's number, rate of diffusion of water vapor to the surface, Lewis number, and the heat and mass transfer coefficients are calculated using the equations given in Section (3.17). Control is then returned to the main program.

A.4 Main Program

A simplified flow diagram for the main program is presented as Figure (13). The procedure is as follows:

1) Block 0 - Constants such as properties, free stream variables and constants which appear in the various equations are loaded.

2) Blocks 1 and 2 - The initial velocities \( U(1,n) \) and \( V(1,n) \) are obtained as described in Section (3.4).

3) Block 3 - Calculations are performed in an iteration procedure with index "K" varied from \( K = 1 \) to \( K = 2 \). Calculations performed with \( K = 1 \) provide the inter-
mediate quantities $U_{Am,n}$ and $V_{Am,n}$. Calculations with $K = 2$ evaluate the velocities $U_{m,n}$ and $V_{m,n}$, using the quantities $U_{ANM,n}$ and $V_{ANM,n}$ as the initial conditions $U_{1,n}, V_{1,n}$.

4) Block 4 - Calculations are performed in an iteration procedure with index "m". The condition $m = 1$ corresponds to the point at which the analysis begins near the stagnation point i.e., the initial station. The condition $m = NM$ corresponds to the point at which the analysis terminates. For $K = 1$, the limit $NM$ is set at the minimum value required to assure that a satisfactory initial condition is obtained. For $K = 2$, $NM$ is set sufficiently high to insure that the analysis will reach the point of separation, at which time the analysis will terminate as described previously.

5) Block 5 - The main program calls the subroutine "CALCV", while supplying the appropriate boundary conditions corresponding to the position $(m+1)$. The subroutine "CALCV" returns the velocities $U_{m,n}$ and $V_{m,n}$.

6) Block 6 - The program checks the variable "ISEP". If "ISEP" is one, control is transferred out of the iteration procedure and the variable "NM" is equated to the value of "m" at that point.

7) Block 7 - Control is passed to Block 8 provided "K" is equal to 1.

8) Blocks 8 and 9 - A nested iteration procedure varies
the index "m" from \( m = 1 \) to \( m = NM+1 \) in the outer loop
and the index "n" from \( n = 0 \) to \( n = NN \) in the inner loop.

9) Block 10 - The quantities \( U_{m,n} \) and \( V_{m,n} \) are saved by
equating \( UA_{m,n} \) and \( VA_{m,n} \) to \( U_{m,n} \) and \( V_{m,n} \) respectively.
These quantities represent the values obtained in the
development of the initial conditions (since \( K = 1 \))
and will be used subsequently in the calculation of
the initial temperature and concentration profiles.

10) Blocks 11, 12 and 13 - The variable "m" is set at
\( m = NM+1 \). An iteration procedure is employed as Block
12, and the quantities \( U_{1,n} \) and \( V_{1,n} \) are equated to
the quantities \( U_{m,n} \) and \( V_{m,n} \) respectively, providing
the initial conditions to be used in the calculations
with \( K = 2 \). This completes the first pass through
the iteration procedure defined by Block 3. Thus "k"
is changed to 2. This time, control passes from
Block 7 to Block 14.

11) Block 14 - An iteration procedure is employed with
index NT. The index "NT" is varied from 1 to any
desired number, depending on how many times the oper-
tor wishes to calculate incremental additions of
frost.

12) Block 15 - This is an iteration procedure, similar in
function to Block 3. When \( K = 1 \), the program develops
initial values for temperature and H\textsubscript{2}O concentration,
Tl,n and Cl,n, respectively, as "n" varies from n = 1 to n = NN-1.

13) Block 17 - If k = 1, control goes to Block 17. Block 17 defines an iteration procedure with index "i" ranging from 1 to 10. The upper limit of "i" is arbitrary and designates the number of trials allowed to achieve the initial conditions.

14) Blocks 18 and 19 - This is an iteration procedure similar to Block 2 in that the initial temperature and concentration profiles are calculated as described in Sections (3.9) and (3.14) respectively.

15) Blocks 20 and 21 - An iteration procedure is used with the index "m" varying from m = 2 to m = NM. The limit "NM" represents a somewhat arbitrary number and is chosen to be no larger than necessary to assure the development of the initial values Tl,n and Cl,n. In Block 21, the subroutine "CALCTD" is called and returns the quantities Tm,n and Cm,n.

16) Block 22 - When the limit (m = NM) is reached in Block 20, control goes to Block 22 where the variable TMO is equated to the temperature T_{NM,0}.

17) Block 23 - The difference in magnitude of the quantities TMO and T_{m,0} is calculated. If the absolute value of the difference is greater than some arbitrary value (0.10°F was used in this program), T_{m,0} is set equal to the variable TMO and control is returned to
Block 17 where the procedure is repeated. If the absolute value of the difference is less than some arbitrary value, control goes to Block 31.

18) Block 31 - An iteration procedure varies the index "n" from \( n = 0 \) to \( n = NN \). In the iteration procedure the quantities \( T_1,n,C_1,n,H_{c,1},H_{d,1} \) and \( Nu_1 \) are calculated. Control then goes back to Block 15 where the index "k" is increased from 1 to 2.

19) Block 16 - Since \( k = 2 \), control goes to Block 24.

20) Block 24 - An iteration procedure varies the index "m" from \( m = 2 \) to \( m = DCHK+1 \) as the angular position varies from near the stagnation point to the point of separation.

21) Block 25 - A heat balance is performed to solve for the surface temperature as outlined in Section (3.8).

22) Block 26 - An iteration procedure varies the index "I" from \( I = 1 \) to \( I = 10 \). The value placed on the upper limit is arbitrary as it serves to limit the number of attempts to converge on a solution. If satisfactory accuracy is not achieved in the given number of attempts, a caution statement is printed out and calculation proceeds. In this work, the upper limit was set at ten and convergence was always achieved.

23) Block 28 - Control is passed to subroutine "CALCIND" with the calling parameters \((U,V)\). Temperature and \( H_2O \) concentration data are returned for the station "m".
24) Block 29 - A new surface temperature is calculated based on the new boundary layer temperature and concentration gradient data calculated in Block 28.

25) Block 30 - The new surface temperature is compared with the previously calculated surface temperature. If the values agree within some specified difference, control is passed to Block 24 and calculations proceed to the next station. The last values for the surface temperature and the boundary layer temperature and H₂O concentration data are taken as the solution at the station "m".

If the consecutive surface temperatures differ by more than the specified value (the limit was set at 0.1°F for this work), control is returned to Block 26 where the procedure is repeated, using the most recently calculated surface temperature as the starting point.

When the calculations proceed to the point of separation, control is passed to Block 33.

26) Blocks 33 and 34 - This iteration procedure has "m" as the index, with the calculations again proceeding from the region of the stagnation point to the point of separation as "m" varies from m = 1 to m = MCHK+1. Based on the rate of diffusion of water vapor to the surface as calculated above, and a frost density corresponding to the calculated surface temperature,
the volume of frost added to the surface in some small increment of time is calculated. When the calculations have proceeded to the point of separation, control is passed to Block 15.

27) Block 15 - Since \( K = 2 \), control reverts back to Block 14.

28) Block 14 - The procedure is repeated, adding a new layer of frost around the circumference of the cylinder from the stagnation point to the point of separation.
APPENDIX B - GRAPHS

The following is a list of figures designated in the text:

Figure 1----External Flow Velocity for U∞ = 10
Figure 2----Tangential Velocity Profiles
Figure 3----Local Heat Transfer Rates Calculated From
Initial Condition $\frac{\partial T}{\partial x} = 0$
Figure 4----Local Heat Transfer Rates Calculated Using
the Principle of Similarity as the Initial
Condition
Figure 5----Local Heat Transfer Coefficients Calculated
With the Initial Condition Obtained From the
Combined Technique Described in Section 3.9
Figure 6----Temperature Profiles Representing Various
Initial Conditions
Figure 7----Calculated Local Lewis Number Compared to
Average Value From Reference (3)
Figure 8----Calculated Normal Velocity at the Outer Edge
of the Boundary Layer
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Figure 11----Flow Diagram for Subroutine to Calculate Velocity Gradients in Boundary Layer

Figure 12----Flow Diagram for Subroutine to Calculate Temperature and Concentration Gradients in the Boundary Layer

Figure 13----Flow Diagram for Main Program

Figure 14----Frost Temperature-Density Correlation

Figure 15----Comparison of Experimental and Analytical Results Near the Stagnation Point Using Frost Density from Reference (4)

Figure 16----Comparison of Experimental and Analytical Results Near the Point of Separation Using Frost Density from Reference (4)

Figure 17----Effect of Wall Temperature on Frost Thickness-Time History

Figure 18----Effect of Humidity Ratio on Frost Thickness-Time History

Figure 19----Effect of Air Velocity on Frost Thickness-Time History

Figure 20----Effect of Humidity Ratio on Frost Thermal Conductivity

Figure 21----Comparison of Experimental and Analytical Results Near the Stagnation Point Using Frost Density from Reference (9)
Figure 22----Comparison of Experimental and Analytical Results Near the Point of Separation Using Frost Density from Reference (9)

Figure 23----Local Heat Transfer Rates Calculated for Different Surface Temperature Distributions

Figure 24----Local Lewis Number Calculated for Different Surface Temperature Distributions

Figure 25----Surface Temperature Variation with Time

Figure 26----Correlation of Plate Surface Temperature and Frost Growth Rate as Published in Reference (12)

Figure 27----Correlation of Humidity Ratio and Frost Growth Rate as Published in Reference (12)

Figure 28----Correlation of Air Velocity and Frost Growth Rate as Published in Reference (12)

Figure 29----Correlation of Humidity Ratio and Frost Thermal Conductivity as Published in Reference (12)
FIGURE 1 - EXTERNAL FLOW VELOCITY FOR $u_\infty = 10$
FIGURE 2 - TANGENTIAL VELOCITY PROFILES
Figure 3 - Local heat transfer rates calculated from initial condition $\partial T/\partial x = 0$
DATA CALCULATED WITH THE TEMPERATURE PROFILE SIMILAR TO THE VELOCITY PROFILE AND SAME BOUNDARY LAYER THICKNESS AS THE INITIAL CONDITION

THEORETICAL CURVE FROM REFERENCE 2

FIGURE 4 - LOCAL HEAT TRANSFER RATES CALCULATED USING THE PRINCIPLE OF SIMILARITY AS THE INITIAL CONDITION
Figure 5 - Local heat transfer coefficients calculated with the initial condition obtained from the combined technique described in Section 3.9.
Figure 6 - Temperature profiles representing various initial conditions.
LEWIS NUMBER FROM REFERENCE 3 BASED ON MEAN FILM TEMPERATURE

○ CALCULATED LEWIS NUMBER

FIGURE 7 - CALCULATED LOCAL LEWIS NUMBER COMPARED TO AVERAGE VALUE FROM REFERENCE 3
Figure 8 - Calculated normal velocity at the outer edge of the boundary layer
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FIGURE 12-FLOW DIAGRAM FOR SUBROUTINE TO CALCULATE TEMPERATURE AND CONCENTRATION GRADIENTS IN THE BOUNDARY LAYER
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- Points calculated from data presented in Reference 9
- Used in analysis: curve fit data from Reference 9
- Used in analysis: maximum and minimum limits from Reference 4
FIGURE 15 - COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS NEAR THE STAGNATION POINT USING FROST DENSITY FROM REFERENCE 4
FIGURE 16 - COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS NEAR THE POINT OF SEPARATION USING FROST DENSITY FROM REFERENCE 4

CONDITIONS:
- $R = 1$ INCH
- $T_{\text{AIR}} = 84^\circ F$
- $T_{\text{WALL}} = 18.7^\circ F$
- $V_{\text{AIR}} = 12.3$ FT/SEC
- $W = 0.01455$ LBW/LBA

EXPERIMENTAL DATA FROM REFERENCE 4

CALCULATED DATA
FIGURE 17 - EFFECT OF WALL TEMPERATURE ON FROST THICKNESS-TIME HISTORY

○ CONDITIONS "A" R = 1 INCH, T\text{AIR} = 84°F, T\text{WALL} = 18.7°F, U\text{AIR} = 12.3 FT/SEC, W = 0.01455 LBW/LBA

□ CONDITION "B" SAME AS CONDITION "A" EXCEPT T\text{WALL} = -18.0°F

FROST THICKNESS - INCHES

TIME - MINUTES
FIGURE 18 - EFFECT OF HUMIDITY RATIO ON FROST THICKNESS - TIME HISTORY

- CONDITIONS "B"
  R = 1 INCH, $T_{AIR} = 84^\circ F$, $T_{WALL} = -18.0^\circ F$, $U_{AIR} = 12.3$ FT/SEC
  $W = 0.01455$ LBW/LBA

- CONDITIONS "D"
  SAME AS CONDITIONS "B"
  EXCEPT $W = 0.00900$ LBW/LBA
FIGURE 19 - EFFECT OF AIR VELOCITY ON FROST THICKNESS-TIME HISTORY

conditions "A" R = 1 INCH, T_air = 84° F, T_wall = 18.7° F, V_air = 12.3 FT/SEC
W = 0.01455 LBW/LBA

conditions "C" same as conditions "A" except V_air = 6.1 FT/SEC
EFFECTIVE FROST CONDUCTIVITY - BTU/HR-FT-°F

○ CONDITION IS "B"
R = 1 INCH, T_{AIR} = 84°F,
T_{WALL} = -18.0°F, U_{AIR} = 12.3 FT/SEC,
W = 0.01455 LBW/LBA

□ CONDITIONS "D"
SAME AS CONDITIONS "B"
EXCEPT W = 0.00900 LBW/LBA

FIGURE 20 - EFFECT OF HUMIDITY RATIO ON FROST THERMAL CONDUCTIVITY
FIGURE 21-COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS NEAR THE STAGNATION POINT USING FROST DENSITY FROM REFERENCE 9

CONDITIONS: \( R = 1 \) INCH
- \( T_{\text{AIR}} = 84^\circ \text{F} \)
- \( T_{\text{WALL}} = 18.7^\circ \text{F} \)
- \( V_{\text{AIR}} = 12.3 \text{ FT/SEC} \)
- \( W = 0.01455 \text{ LBW/LBA} \)
FIGURE 22—COMPARISON OF EXPERIMENTAL AND ANALYTICAL RESULTS NEAR THE POINT OF SEPARATION USING FROST DENSITY FROM REFERENCE 9

CONDITIONS:
- \( R = \text{1 inch} \)
- \( T_{\text{AIR}} = 84^\circ \text{F} \)
- \( T_{\text{WALL}} = 18.7^\circ \text{F} \)
- \( V_{\text{AIR}} = 12.3 \text{ FT/SEC} \)
- \( W = 0.01455 \text{ LBW/LBA} \)
FIGURE 23 - LOCAL HEAT TRANSFER RATES CALCULATED FOR DIFFERENT SURFACE TEMPERATURE DISTRIBUTIONS
FIGURE 24 - LOCAL LEWIS NUMBER CALCULATED FOR DIFFERENT SURFACE TEMPERATURE DISTRIBUTIONS
Figure 25 - Surface Temperature Variation with Time

- □ CALCULATED AT 0.34 MINUTE
- ▲ CALCULATED AT 1.30 MINUTE
- ○ CALCULATED AT 2.45 MINUTE
FIGURE 27 - CORRELATION OF HUMIDITY RATIO AND FROST GROWTH RATE AS PUBLISHED IN REFERENCE (12)
Figure 28 - Correlation of air velocity and frost growth rate as published in Reference (12).
Figure 29 - Correlation of Humidity Ratio and Frost Thermal Conductivity as Published in Reference (12)
APPENDIX C - REFERENCES


