DYNAMIC STRATEGIC MONETARY POLICIES, THE TRADE BALANCE, AND INTERNATIONAL CAPITAL FLOWS

DISSERTATION

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To My Late Parents

And

My Wife
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CHAPTER I
INTRODUCTION

This paper investigates the welfare gains derived from the Nash, Consistent Conjectural Variations (CCV), Stackelberg, and Cooperative monetary policies for dynamic games, and describes how those gains vary according to the underlying economic structure. It generalizes the discussion of strategic policies in interdependent economies by allowing for the interaction of trade and capital flows, and continues the analysis of the behavior of policy makers and its welfare implications under closed-loop equilibrium strategies.

Interest in these problems, although not new, derives from two sources. The first is that much of the recent literature on the strategic analysis of optimal dynamic monetary policy in interdependent economies, such as the work of Canzoneri and Gray (1985), Oudiz and Sachs (1985), Basar, Turnovsky, and d’Orey (1986), and Turnovsky, Basar, and d’Orey (1988), has generally been confined to capital movements, ignoring trade flows entirely. Therefore, it is called the stock equilibrium approach. In this approach, the role of the exchange rate is to maintain a continuous portfolio balance among the existing stocks of financial capital. It plays no role in balancing the flow demand and supply for foreign exchange arising from
trade in goods and capital. This assumption greatly simplifies the analysis, but it does so at the expense of the interaction between trade and capital flows in the dynamic adjustment process (see, for example, Niehans, 1977 and Driskill, 1980, 1981).

The second source of interest is the increase in the study of closed-loop equilibrium strategies for dynamic games, such as the investigations by Kydland and Prescott (1977), Oudiz and Sachs (1985), and Turnovsky, Basar, and d'Orey (1988). The reason for this attention stems from the knowledge that the closed-loop equilibrium strategies are self-enforcing and have the property of subgame perfection. Since the gains from various strategic monetary policies must follow from the chosen path towards equilibrium and not in the final equilibrium itself, the closed-loop solution is the plausible equilibrium for characterizing the behavior of policy makers in the absence of precommitment.

The analysis shows that the ranking of welfare gains varies according to the underlying economic structure. This conclusion is perhaps obvious and innocent enough as applied to the evolution of observed exchange rate regimes (see, for example, Canzoneri and Gray, 1985), but it also has implications for the coordination of economic policies among nations.

The analytical framework used is a generalization of the Turnovsky, Basar, and d'Orey (1988) model which, by the assumption of imperfect capital substitutability, permits the interaction of trade and capital flows during the dynamic adjustment process.
The remainder of this paper is organized as follows. Chapter II presents a literature survey on the strategic elements inherent in monetary policy formation, the definitions of strategic equilibrium, the role these elements play in the welfare gains in terms of the nature of the inefficiency, and the empirical results of welfare comparisons. Chapter III presents a generalization of the Turnovsky, Basar, and d'Orey (1988) theoretical model. Chapter IV derives the closed-loop solutions for various strategic equilibria. The numerical analysis is conducted in Chapter V. Finally, we summarize the results in Chapter VI.
CHAPTER II
LITERATURE SURVEY

In interdependent economies, the monetary policy of one country may have spillover effects on other countries. Rational policy makers in one country may be expected to condition their actions on the policies pursued in other countries; therefore, policy making has unavoidable game aspects. These strategic elements inherent in monetary policy formation have long been recognized in the seminal work of Cooper (1968, 1969) and Hamada (1976). Both examined the inefficiency problem under fixed exchange rates. By using a fixed-price model, Cooper was able to demonstrate the speed of adjustment for restoring the equilibria can be altered by the incorporation of the spillover effects of macroeconomic policies. Hamada argued that the inefficiency problems arising from competition for depreciation in the flexible-price model can be resolved by a cooperation among countries or ameliorated by a Stackelberg leadership.

The recent research into the welfare issues of various strategic equilibria, such as Canzoneri and Gray (1985), Oudiz and Sachs (1985), Basar, Turnovsky, and d'Orey (1986), and Turnovsky, Basar, and d'Orey (1988), has explored different dimensions of such an efficiency problem inherent in monetary policy formation. It adopts the stock
equilibrium approach under flexible exchange rates and allows for forward-looking behavior of the exchange market participants. In addition, a sluggish response of commodity prices to the contemporary changes in demand occurs.

II.1 The Origins of Strategic Elements

The strategic elements inherent in policy decision-making are caused by the presence of monetary spillover effects. There are three major channels through which monetary policy can propagate abroad the positive or negative effects described by Canzoneri and Gray (1985).

The first channel of monetary policy is to directly affect interest rates through the asset markets, producing international capital flows and changing the asset market equilibrium and the real income of other countries. Under a flexible exchange rate regime with perfect capital mobility and substitutability, it is well-known that this monetary spillover in fixed-price models has negative effects on other countries' real income (see, for example, Mundell, 1968, Ch.18 and Fleming, 1962). This occurs because the lower real interest rate produces a reduction in the foreign real interest rate and, in turn, increases foreign money demand. In order to restore the asset market equilibrium, there is downward pressure on real income. For this reason, a depreciation policy that raises income at home and lowers income abroad is assailed as a beggar-thy-neighbor policy.
The second transmission channel of monetary policy operates directly through the demand for goods and therefore, depends on the existence of two or more goods in the model. A monetary expansion in one country appreciates the terms of trade of the other countries. The terms of trade appreciation that makes imports cheaper thus produces the expenditure-switching effect, which reduces the demand for home goods and raises real income. This generates the expenditure-increasing (Laursen-Metzler) effect, which increases the demand for home goods. Therefore, the combined effect on real income in other countries is generally ambiguous. The presence of positive spillover effects on real income requires that the impact of the terms of trade appreciation on the expenditure switching be less than the expenditure increasing effect. Thus, the goods demand channel generates spillover effects that may be positive across countries.

The third monetary transmission mechanism works through the labor market and depends on the response of wages and prices to the terms of trade effects occurring in the goods demand channel. As discussed above, an expansion in one country improves the terms of trade for goods produced by the other country. If nominal wages are linked to a price index that includes the price of imports as well as domestically produced goods, then the second country's real wage falls and its real income increases. Therefore, one country's monetary policy may generate a positive spillover on other countries' real incomes.
II.2 The Definition of Strategic Equilibrium

Specifically, in the analysis of strategic monetary policies, such as Canzoneri and Gray (1985), Basar, Turnovsky, and d'Orey (1986), and Turnovsky, Basar, and d'Orey (1988), there are three noncooperative duopoly (two player) games, Nash, Stackelberg, and Conjectural Variations. By a noncooperative game, we mean a game in which the players are not able to make binding agreements, except for the ones which are explicitly allowed by the rules of the game. Since in a noncooperative game binding agreements are not possible, the solution of such a game must be self-enforcing, i.e., it must have the property that, once it is agreed upon, nobody has an incentive to deviate. This implies that the solution to a noncooperative game must be an equilibrium if a pair of strategies has the property that no player can gain by unilaterally deviating from it.

The Nash equilibrium occurs as each player symmetrically chooses his action while taking as given the other's strategy, say, to minimize his welfare costs. Therefore, the Nash strategy for each player is defined as a pair of policy rules \( \{\bar{m}, \bar{m}^*\} \) (given the available information sets) such that no one would be able to improve his own welfare by deviating from this chosen strategy because the following relationship holds:

\[
V(\bar{m}, \bar{m}^*) \leq V(m, \bar{m}^*),
\]
where foreign variables are denoted with asterisks and domestic variables are unstarred. \( V \) is the welfare cost function of the home economy. \( m \) is the policy variable controlled by the policy makers of home economy; \( \tilde{m} \) represents the best response of the policy maker of the home economy to his rival’s policy.

In the Nash equilibrium, by definition, each player naively assumes no response on the part of his opponent to his own action. In fact, there is a case in which each player symmetrically assumes that the action of the other will adjust with respect to potential adjustments in his own action. This type of game is called the Conjectural Variations game. Additionally, when the conjectural variations is correct, Bresnahan (1981) called the solution the "Consistent Conjectural Variations" (CCV). Therefore, the CCV equilibrium is defined as a pair of strategies for each player such that:

\[
m = g(m) = \arg \min V(m, g^*(m)),
\]

\[
m^* = g^*(m) = \arg \min V^*(m^*, g(m)).
\]

Where \( g \) and \( g^* \) denote each player’s reaction function. The two cases specified above formulate the behavior of two players in a symmetrical manner. However, it is also possible that one player (the leader) is allowed to dominate the decision processes and the other
(the follower) is committed to a well-defined reaction in response to any particular policy the dominant player may choose. This game is called the Stackelberg leader-follower game, where the Stackelberg strategy for each player is a pair of policy rules \( \{m, m^*\} \) such that:

\[
m = \text{arg Min } V(m, g^*(m)),
\]

\[
m^* = g^*(m) = \text{arg Min } V^*(m^*, g(m)).
\]

Accordingly, the Stackelberg equilibrium is defined as there is a pair of policy rules \( \{\bar{m}, \bar{m}^*\} \) at each stage such that:

\[
V(\bar{m}, \bar{m}^*) \leq V(m, m^*),
\]

\[
V^*(\bar{m}, \bar{m}^*) \leq V^*(\bar{m}, m^*).
\]

As argued above, the equilibrium strategy possesses the self-enforcing property. Given that it is hard to assume that future policy makers can be bound by the policy chosen today (i.e., open-loop control), it is more sensible to presume that the policy maker at each stage chooses his own policy rule with the knowledge that the policy makers in future periods will have the freedom to change course and will have the incentive to do so. Precisely, closed-loop control characterizes
this behavior. As a result, while selecting the policy rule to mini-
mize the welfare cost for the remainder of the planning periods, the
rational policy maker maintains the perspective that succeeding policy
makers will also do so. The resulting closed-loop equilibrium
strategy, which is subgame perfect, is self-enforcing and has time
consistency (see Kydland and Prescott, 1977 and Oudiz and Sachs,
1985).

II.3 The Nature of Inefficiency

In interdependent economies, because of monetary spillovers,
rational policy makers in one country may be expected to condition
their actions on the policies pursued in other countries. The suc-
cessful pursuit of national economic objectives, therefore, becomes
much more difficult (see, for example, Cooper, 1968). Policy makers
of each country may engage either in noncooperative policy games or in
coordinated monetary policies. It is well-known that without direct
cooperation or side-payments, the resulting outcomes of noncooperation
are socially inefficient (see, for example, Cooper, 1969, Hamada,
1976). Noncooperative monetary policies may lead to over-contraction
or over-inflation.

For example, in a Nash game characterized by a positive spillover
of each country’s monetary policy, policy makers are faced with a
global disturbance that will, in the absence of corrective policy (the
flexible exchange rate case), produce unemployment in both the domestic and foreign economies. Policy responses are designed to balance the benefits of smaller employment losses against the costs of increased inflationary expectations. Both countries' policy makers opt for an expansionary monetary policy. Each is aware of the beneficial effect of the other player's expansion on its own employment and appropriately internalizes the information by expanding less than he would in the absence of the other player's response. However, neither player takes into account the beneficial impact of his own actions on the other player. As a result, the Nash solution does not sufficiently exploit the positive externalities associated with monetary policy.

In the Nash equilibrium, each player assumes no response on the part of his opponent to his own action. In fact, each player will respond in accordance with his reaction. The Nash equilibrium is therefore consistently wrong in predicting the response of the rival. While correcting the unsatisfactory feature of the Nash game, the CCV equilibrium in the case of positive monetary spillover results in a lower social welfare. This occurs because, while opting for an expansionary monetary policy, policy makers of each country conjecture that the other nation's policy would be less expansionary because each is aware of the beneficial impact of each country's expansionary policy on the home employment of the other. Nevertheless, both correctly conjecture each other's action and as a result, inflate less. Thus, the conjectural variations introduces caution and therefore produces a more gradual adjustment in the players' actions.
In the Stackelberg (leader-follower) game, the leader is aware of the beneficial impact of the follower's expansion on home employment and thus expands less. In response to the leader's action, and knowing the adverse effect of the leader's less expansionary monetary policy on home employment, the follower will expand more than he would in the Nash equilibrium. Although total welfare may be greater than in the Nash equilibrium, it is not evenly distributed. The follower suffers higher inflation and therefore, benefits less by taking most of the adjustment responsibility.

Analogously, in a game characterized by a negative monetary spillover, world welfare would be enhanced if the nations could agree to inflate less. In a game characterized by the positive spillover of one country's monetary policy and the negative spillover of the other's, both would be better off if the country with the positive spillover inflated more while the other country with the negative spillover inflated less. This leads to the resulting alternatives that would require either cooperation or a Stackelberg leadership game among sovereign monetary authorities.

II.3 Empirical Evidence

It is difficult to provide a positive empirical analysis of the policy notions above. In fact, the existing empirical literature is, at most, a verification of the theoretical propositions through
simulations of two-country models, which are either constructed as hypothetical numerical examples or estimated by actual data.

The early empirical (numerical) studies examined the effects of monetary policy under fixed exchange rates by computing the changes in official reserves, and were an indirect test of the validity of the monetary approach to the balance of payments. The results have been summarized in Hamada (1979). There was no direct welfare comparison of optimal monetary policy under various strategic equilibria until the work of Oudiz and Sachs (1985), Basar, Turnovsky, and d'Orey (1986), and Turnovsky, Basar, and d'Orey (1988). The common feature of these studies is the adoption of stock equilibrium models under flexible exchange rates. The forward-looking behavior on the part of the exchange market participants is allowed in both studies. In addition, the objective of policy makers is to minimize the intertemporal welfare cost of inflation and unemployment.

Oudiz and Sachs have compared the welfare gains derived from the noncooperative Nash and cooperative policies that raise society's welfare by systematically balancing, at each stage, the decreased unemployment loss against the cost of increased inflation. Given the larger weight assigned to the inflationary fluctuations, the monetary policies under Nash and Cooperation, in response to the depreciation of exchange rate, call for a real monetary contraction (often referred to as "leaning against the wind"). They also verified that the cooperative policy can yield a social welfare that is higher than the noncooperative Nash policy.
The comprehensive welfare analysis of dynamic monetary policies under various strategic equilibria can be found in Basar, Turnovský, and d’Orey (1986) and Turnovský, Basar, and d’Orey (1988). The purpose of their study is to examine the welfare aspects of time-consistent strategic equilibria through the steady-state rate of convergence of real exchange rates corresponding to numerical models with varying degrees of interdependence.

By assigning a larger weight on output fluctuations, the optimal monetary policy is characterized by a leaning with the wind approach for the Nash, CCV, and cooperative equilibria and by a leaning against the wind for the Stackelberg equilibrium. The depreciation policy to combat the increased unemployment requires a more expansionary monetary policy in Cooperation than in the Nash case, which in turn inflates more than under CCV. The policy of leaning against the wind in the Stackelberg approach arises from the dominance of the leader in the anti-inflation decision process. The follower responds to the leader’s monetary policy by adopting a more contractionary policy and therefore incurs the higher cost of unemployment by assuming more of the adjustment responsibility. However, under Cooperation, both countries will be better off by gradually inflating more.

It has been demonstrated that the long-run welfare ranking is fairly robust, while the short-run welfare gains under various strategic equilibria are highly parameter sensitive. Specifically, cooperation is the preferred equilibrium, followed by Stackelberg (leader), Nash, CCV, and Stackelberg (follower). In addition, the
steady-state rate of convergence is consistent with the welfare ranking. Independent of the numerical values of parameters, the rate has been shown to be faster under the closed-loop cooperation than the closed-loop Stackelberg, which in turn is faster than under the closed-loop Nash. In Basar, Turnovsky, and d'Orey (1986), the closed-loop Nash also converges more quickly than CCV. In the latter case, each policy maker, while taking account of his rival's behavior, maintains caution and a more gradual adjustment ensues.

II.4 Concluding Remarks

In the stock equilibrium approach, the role of the exchange rate is to maintain a continuous portfolio balance among the existing stocks of financial capital. Because it plays no role in balancing the flow demand and supply for foreign exchange arising from trade in goods and capital, the exchange rate response to a step change in the relative money supplies can overshoot and then monotonically adjust back to its long-run equilibrium value (see Dornbusch, 1976). The monotonic exchange rate dynamics during the dynamic adjustment period introduces uni-directional changes in the expected future exchange rate as a potentially important source of disturbance to the world economy and as a potentially important channel for the transmission of disturbances between national economies. As discussed previously, Canzoneri and Gray (1985) have argued that the evolution of the observed exchange-rate regimes could be characterized by rankings in
welfare gains, linked to the structure-dependent specifications of monetary spillover, under various strategic equilibria. By contrast, Basar, Turnovsky, and d'Orey (1986) and Turnovsky, Basar, and d'Orey (1988) have found that the ranking among the cooperative and non-cooperative equilibria is invariant to the underlying economic structure.

Since the stock equilibrium approach, with its assumption of perfect capital mobility and substitutability, considers the momentary equilibrium aspect of the flow market of foreign exchange, it leaves no room for the trade balance effects on the exchange rates. This approach greatly simplifies the analysis. However, it does so at the expense of the interaction between trade and capital flows during the dynamic adjustment process for the determination of the exchange rate. Since the gains from various strategic monetary policies must follow from the chosen path towards equilibrium and not in the final equilibrium itself, the gains during the dynamic adjustment phase are always the result of the interaction of trade and capital flows.

There has been a synthesis of the elasticity and monetary approaches to exchange rate determination, developed in Niehans (1977) and Driskill (1980, 1981). This approach assumes imperfect capital substitutability and generalizes the Dornbusch model by permitting the exchange rate to equilibrate the flow demand and supply for foreign exchange arising from trade in goods and capital. This has been referred to as the stock/flow equilibrium approach. In this approach, the exchange rate adjustment patterns are linked to the underlying economic structure and can be much more complex. The exchange rate
overshooting may stem from the interactions between trade and capital flows during the dynamic adjustment process; hence policymakers may be more complicated. While it is an empirical matter as to which approach can truly characterize the observed world economy, the implications of these stock/flow equilibrium models have not been examined in the strategic analysis of dynamic monetary policies. This paper fills this gap.
CHAPTER III
THE THEORETICAL FRAMEWORK

The analysis is a generalization of the Turnovsky, Basar, and d'Orey (1988) model, which is a direct extension of the Dornbusch (1976) model. There are four fundamental building blocks: a money market equilibrium condition, a commodity price-adjustment equation, a perfect capital mobility and substitutability condition, and a perfect-foresight assumption. In deviation form, it describes two identical economies, each specializing in the production of a distinct good and trading two bonds.

The key differences between this model and the Turnovsky-Basar-d'Orey model are the assumption of imperfect, rather than perfect, capital substitutability. Foreign variables are denoted with asterisks and domestic variables are unstarred.

III.1 The Perfect-Foresight Model

III.1.1 The Money Market

Assuming an equilibrium obtains in each period, the equilibrium conditions for the world money market are:
\[ M_t = P_t + \phi Y_t - \lambda i_t, \]  
(1)

\[ M_t^* = P_t^* + \phi Y_t^* - \lambda i_t^*, \quad \phi, \lambda > 0. \]  
(2)

The specifications of the demand for money are the same as in the Dornbusch analysis. \( M \) is the log of the money supply, \( P \) is the log of the price level, \( Y \) is the log of real income, which is measured as a deviation about its natural rate level, \( i \) is the nominal interest rate, \( \lambda \) is the interest rate semi-elasticity of the demand for money, and \( \phi \) is the income elasticity of the demand for money.

III.1.2 The Goods Market

In the goods markets, the demand-side determined real income of each country depends on the real rate of interest, the real income in the other country, and the real exchange rates. The corresponding effects for the two economies are identical, with the real exchange rate influencing demand in exactly offsetting ways. The functional forms are:

\[ Y_t = \theta Y_t^* - \sigma r_t + \omega q_t, \]  
(3)

\[ Y_t^* = \theta Y_t^* - \sigma r_t^* - \omega q_t, \quad 0 < \theta < 1, \text{ and } \sigma, \omega > 0. \]  
(4)
where \( q = S + P^* - P \) is the real exchange rate, \( S \) is the log of the exchange rate measured as the domestic currency price of foreign exchange, \( r = i_t - P_{t+1} + P_t \) is the real rate of interest. The parameters of the functions can be linked to the underlying economic structure (see, for example, Bhandari, 1985, ch. 11).

The price adjustment of domestic good, \( P_{t+1} - P_t \), is proportional to the one-period deviation of lagged real income about its natural rate level:

\[
P_{t+1} - P_t = \delta Y_t, \tag{5}
\]

\[
P^*_t - P^*_t = \delta Y^*_t \tag{6}
\]

\( \delta > 0. \)

The consumer price indexes, denoted by \( C \), are defined as a weight average of the prices of domestic and foreign goods:

\[
C_t = \omega P_t + (1 - \omega)(S_t + P^*_t), \tag{7}
\]

\[
C^*_t = \omega P^*_t - (1 - \omega)(S_t - P_t), \tag{8}
\]

\( 0.5 < \omega < 1. \)

This embodies the assumption that the proportion of consumption \( \omega \) spent on the respective home goods is the same in the two economies,
0.5 < \psi < 1. Thus, residents in both countries have a preference for their own good.

III.1.3 The Foreign Exchange Market

The third building block of the model is the foreign exchange market equilibrium condition. This condition is derived from a net demand for foreign assets function and a trade balance function. The net demand for foreign assets, \( B_t \), is assumed to be a linear function of the net yield:

\[
B_t = \eta(S_{t+1} - S_t + i_t^* - i_t), \quad \eta > 0.
\] (9)

Assuming that individuals fully adjust their actual asset stocks to their desired levels within each period, the net capital flows can be obtained by taking the first difference of \( B \).

The trade balance at time \( t \) is assumed to be a linear function of the log of the ratio of domestic to foreign real income and real exchange rates at time \( t \):

\[
T_t = -\beta(Y_t - Y_t^*) + \alpha q_t, \quad \beta, \alpha > 0,
\] (10)

III.1.4 The Intertemporal Objective Functions

Policy makers in these economies are assumed to have symmetrical objective functions
\[ J = \sum_{t=1}^{T} \left[ k y_t^2 + (1 - k)(c_{t+1} - c_t)^2 \right] \rho^{(t-1)}, \]  

(11)

\[ J^* = \sum_{t=1}^{T} \left[ k y_t^{*2} + (1 - k)(c_{t+1}^{*} - c_t^{*})^2 \right] \rho^{(t-1)}, \]  

(12)

\[ 0 < k < 1, \]

where \( k \) and \( 1 - k \) are the relative weights assigned to real income stability, and price stability, respectively.

III.2 The Reduced-Form Solutions

III.2.1 The Output and Price Solutions

Combining equations (1) - (6) and solving for \( Y_t, Y_t^*, P_t, \) and \( P_t^* \)

yields:

\[ Y_t = a_1 m_t + a_2 m_t^* + a_3 q_t, \]  

(13)

\[ Y_t^* = a_2 m_t + a_1 m_t^* - a_3 q_t, \]  

(14)

\[ P_{t+1} - P_t = \delta a_1 m_t + \delta a_2 m_t^* + \delta a_3 q_t, \]  

(15)
The equation:

\[ \frac{P^*_{t+1}}{P^*_t} = \delta a_2 m_t + \delta a_1 m^*_t - \delta a_3 q_t, \]  

where the lower case m and m* denote the real money stocks of domestic and foreign countries, respectively; and

\[ a_1 = \frac{\sigma}{2} \left[ \frac{1}{\lambda(1 - \theta - \delta \sigma) + \phi \sigma} + \frac{1}{\lambda(1 + \theta - \delta \sigma) + \phi \sigma} \right], \]

\[ a_2 = \frac{\sigma}{2} \left[ \frac{1}{\lambda(1 - \theta - \delta \sigma) + \phi \sigma} - \frac{1}{\lambda(1 + \theta - \delta \sigma) + \phi \sigma} \right], \]

\[ a_3 = \frac{\lambda \omega}{\lambda(1 + \theta - \delta \sigma) + \phi \sigma}. \]

It is assumed that \( 1 - \theta - \delta \sigma > 0 \), implying that the IS curve of the aggregate world economy is downward sloping. As in the Turnovsky, Basar, and d'Orey (1988) analysis, it follows that the stability of the world economy requires

\[ \frac{1}{\lambda(1 - \theta - \delta \sigma) + \phi \sigma} > \frac{1}{\lambda(1 + \theta - \delta \sigma) + \phi \sigma} > 0, \]

and

\[ a_1 > a_2 > 0. \]
III.2.2 The Exchange-Rate Solution

The foreign exchange market-clearing equation states that the net capital flows equal the net trade flows:

\[ T_t = \Delta B_t. \]

Therefore, the exchange rate can be solved from the following second-order difference equation,

\[ -\beta(Y_t - Y_t^+) + \alpha(S_t + P_t^* - P_t) \]

\[ = \eta(S_{t+1} - 2S_t + S_{t-1} + i_t^* - i_{t-1}^* - i_t + i_{t-1}). \quad (17) \]

Substitution for equations (1) - (2) and (5) - (8) yields

\[ q_{t+1} = c_0 q_t + c_1 q_{t-1} + b_0 (m_t - m_t^*) + b_1 (m_{t-1} - m_{t-1}^*), \quad (18) \]

where

\[ c_0 = \frac{\alpha}{\eta} + 2[1 - a_3(\delta + \frac{\beta}{\eta} - \frac{\phi}{\lambda})], \]

\[ c_1 = 2a_3(\delta - \frac{\phi}{\lambda}) - 1, \]

\[ b_0 = -\frac{1}{\lambda} - (a_1 - a_2)(\delta + \frac{\beta}{\eta} - \frac{\phi}{\lambda}), \]

\[ b_1 = \frac{1}{\lambda} + (a_1 - a_2)(\delta - \frac{\phi}{\lambda}). \]
III.2.3 The Rate of Inflation

Taking the difference in the cost of living equations (7) and (8), at two consecutive points in time and applying equations (13) - (16) and (18), the rates of inflation become

\[
C_{t+1} - C_t = \alpha_1 m_t + \alpha_2 m^*_t + \alpha_3 q_t
\]
\[+ \alpha_4 (m_{t-1} - m^*_{t-1}) + \alpha_5 q_{t-1}, \tag{19}\]

\[
C^*_t - C^*_t = \alpha_2 m_t + \alpha_1 m^*_t - \alpha_3 q_t
\]
\[- \alpha_4 (m_{t-1} - m^*_{t-1}) - \alpha_5 q_{t-1}, \tag{20}\]

where

\[
\alpha_1 = \delta a_1 + (1-\nu)b_0; \\
\alpha_2 = \delta a_2 - (1-\nu)b_0; \\
\alpha_3 = \delta a_3 + (1-\nu)(c_0-1); \\
\alpha_4 = (1-\nu)b_1; \\
\alpha_5 = (1-\nu)c_1. 
\]
III.3 The Dynamic Optimization Problem

The dynamic optimization problem faced by the two policy makers may now be summarized as

$$\min J = \sum_{t=1}^{T} \left[ kY_t^2 + (1 - k)(C_{t+1} - C_t)^2 \right] \rho(t-1), \quad (21)$$

subject to

$$Y_t = a_1 m_t + a_2^* m_t + a_3 q_t, \quad (22)$$

$$C_{t+1} - C_t = \alpha_1 m_t + \alpha_2^* m_t + \alpha_3 q_t$$
$$+ \alpha_4 (m_{t-1} - m_{t-1}^*) + \alpha_5 q_{t-1}, \quad (23)$$

and

$$\min J^* = \sum_{t=1}^{T} \left[ kY_t^2 + (1 - k)(C_{t+1}^* - C_t^*)^2 \right] \rho(t-1), \quad (24)$$

subject to

$$Y_t^* = a_2^* m_t + a_1^* m_t - a_3^* q_t, \quad (25)$$
\[ C_t^* - C_t^* = \alpha_2 m_t + \alpha_1 m_t^* - \alpha_3 q_t \]
\[ - \alpha_4 (m_{t-1} - m_{t-1}^*) - \alpha_5 q_{t-1}^* \]  
(26)

and where
\[ q_{t+1} = c_0 q_t + c_1 q_{t-1} + b_0 (m_t - m_t^*) + b_1 (m_{t-1} - m_{t-1}^*). \]  
(27)

The minimization in (21) and (24) are performed over the policy rules under different information patterns and different modes of decision making.

As a preliminary to the analysis, since the exchange rate is regarded as a free variable, substituting (22) and (23) into (21), and (25) and (26) into (24) enables us to express each country's objective function in terms of the control variables only for both countries in the finite-horizon context. The resulting expressions are

\[ V = \sum_{t=1}^{T} \left[ D_1 q_t^2 + 2D_2 q_t q_{t-1}^2 + D_3 q_{t-1}^2 + 2D_4 q_t m_t + 2D_5 q_{t-1}^2 m_t \right. \]
\[ + 2D_6 q_t m_t^* + 2D_7 q_{t-1}^2 m_t^* + 2D_8 m_t^2 + 2D_9 m_t (m_{t-1} - m_{t-1}^*) \]
\[ + 2D_{10}^* (m_{t-1} - m_{t-1}^*) + D_{11} m_t^2 + D_{12}^* m_t \]
\[ + 2D_{13} q_t (m_{t-1} - m_{t-1}^*) + 2D_{14} q_{t-1}^2 (m_{t-1} - m_{t-1}^*) \]
\[ + D_{15}^* (m_{t-1} - m_{t-1}^*)^2 \]  
(28)
\[ V^* = \sum_{t=1}^T \left[ D_1^* q_t^2 + 2D_2^* q_t q_{t-1} + D_3^* q_{t-1}^2 + 2D_4^* q_t m_t + 2D_5^* q_{t-1} m_t \
+ 2D_6^* q_t m_t + 2D_7^* q_{t-1} m_t + 2D_8^* m_t m_t^* + 2D_9^* (m_{t-1} - m_{t-1}^*) \
+ 2D_{10}^* m_{t-1} (m - m_{t-1}^*) + D_{11}^* m_t^2 + D_{12}^* m_t^2 \
+ 2D_{13}^* q_t (m_{t-1} - m_{t-1}^*) + 2D_{14}^* q_{t-1} (m_{t-1} - m_{t-1}^*) \
+ D_{15}^* (m_{t-1} - m_{t-1}^*)^2 \right]. \tag{29} \]

where

\[
D_1 = \gamma a_3^2 + (1-\gamma)\alpha_3^2 = D_1^*, \\
D_2 = (1-\gamma)\alpha_3 \alpha_5 = D_2^*, \\
D_3 = (1-\gamma)\alpha_5^2 = D_3^*, \\
D_4 = \gamma a_1 a_3 + (1-\gamma)\alpha_1 \alpha_3 = -D_4^*, \\
D_5 = (1-\gamma)\alpha_1 \alpha_5 = -D_5^*, \\
D_6 = \gamma a_2 a_3 + (1-\gamma)\alpha_2 \alpha_3 = -D_6^*, \\
D_7 = (1-\gamma)\alpha_2 \alpha_5 = -D_7^*, \\
D_8 = \gamma a_1 a_2 + (1-\gamma)\alpha_1 \alpha_2 = D_8^*, \\
D_9 = (1-\gamma)\alpha_1 \alpha_4 = -D_9^*, \\
D_{10} = (1-\gamma)\alpha_2 \alpha_4 = -D_{10}^*,
\]
\[ D_{11} = \gamma \alpha_1^2 + (1-\gamma)\alpha_1^2 = D_1^*, \]
\[ D_{12} = \gamma \alpha_2^2 + (1-\gamma)\alpha_2^2 = D_2^*, \]
\[ D_{13} = (1-\gamma)\alpha_3\alpha_4 = D_3^*, \]
\[ D_{14} = (1-\gamma)\alpha_4\alpha_5 = D_4^*, \]
\[ D_{15} = (1-\gamma)\alpha_4^2 = D_5^*. \]

If the state variable evolves according to (27), and is combined with the optimal monetary rules, the expressions for the finite-horizon welfare cost functions \( V \) and \( V^* \), as given by (28) and (29), provide a convenient framework for the application of the available optimal control theory on dynamic games for this two-country model. The solution technique can be found in Basar and Olsder (1982, ch. 6 and 7) and Basar (1986, ch.1). The following chapter provides a discussion of the closed-loop equilibrium solution for dynamic games.

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1. Note that it is readily acknowledged that the strategy chosen by a policy maker in a given period may influence the strategy choices made by various policy makers in the game in future periods. This type of game is termed a structure-dependent game (see Friedman, 1973).
CHAPTER IV
THE CLOSED-LOOP EQUILIBRIUM SOLUTIONS

The strategic equilibrium considered here includes Nash, Stackelberg, and CCV. They are compared to the cooperative equilibrium. This chapter attempts to derive the closed-loop equilibrium strategies for dynamic games. We consider a case in which the policy maker during a given time selects his own policy rule to minimize the welfare costs for the remainder of the planning periods while taking as given that future policy makers would behave accordingly. By this reasoning, the resulting equilibrium is a subgame perfect (time-consistent) equilibrium (see Oudiz and Sachs, 1985). Using the structure-dependent property to eliminate the equilibrium non-uniqueness that arises from the multiplicity of loop formation, the closed-loop strategy for each policy maker at each stage is assumed to be a pair of simple policy rules \( \{m_t, m_t^*\}, t = 1,2, \ldots , T \):

\[
m_t = f_t(m_{t-1}, m_{t-1}^*, q_t, q_{t-1}), \tag{30}
\]

\[
m_t^* = f_t^*(m_{t-1}, m_{t-1}^*, q_t, q_{t-1}). \tag{31}
\]
It is readily seen that (1) under this closed-loop information pattern, the strategy chosen by one policy maker in a particular time would not be influenced by the other policy maker operating in the same time-horizon; (2) the policy makers at each stage fully know the action taken at the previous stage. Therefore, it is called a strategy with memory. By utilizing the technique of optimal control theory, we can derive the monetary policy rule to obtain the corresponding equilibrium solution for such a finite horizon game. The following subsections derive the four closed-loop equilibrium strategies in turn.

IV.1 Nash Equilibrium Solution

The first equilibrium to be addressed is the noncooperative Nash equilibrium, using the so-called closed-loop information pattern dictated by (30) and (31). Each truncated dynamic game, as described by the strictly convex cost functions (28) and (29), is of the "linear-quadratic" type for which the Nash equilibrium solution is well known (see Basar and Olsder, 1982). Hence, the T-period Nash equilibrium can be obtained recursively. For a fixed-T period, the solution of the truncated dynamic game is unique, and linear in the current and lagged values of the states, admitting the expressions given below in Proposition IV.1. Its proof is provided in Appendix A, which utilizes the backward recursive technique given in Basar and Olsder (1982, Chapter 6).
**Proposition IV.1:** For the $T$-period dynamic game, the closed-loop Nash equilibrium solution is unique and is given by

\begin{equation}
  m_t = \alpha_t(m_{t-1}^* - m_{t-1}^*) + \beta_t q_t + \gamma_t q_{t-1},
\end{equation}

(32)

\begin{equation}
  m_t^* = \alpha_t^*(m_{t-1}^* - m_{t-1}^*) + \beta_t^* q_t + \gamma_t^* q_{t-1},
\end{equation}

(33)

\[ \tau = T - t; \ t = 1, 2, \ldots, T, \]

where $\tau$ represents the time remaining for the rest of the game and

\[ \alpha_t = \frac{d_{9,11,t} - d_{9,8,t}}{\Delta_t}, \]

\[ \alpha_t^* = \frac{d_{9,11,t}^* - d_{9,8,t}^*}{\Delta_t}, \]

\[ \beta_t = \frac{d_{4,11,t} - d_{4,8,t}}{\Delta_t}, \]

\[ \beta_t^* = \frac{d_{4,11,t}^* - d_{4,8,t}^*}{\Delta_t}, \]
\[
\gamma_\tau = \frac{d_{5, \tau} d_{11, \tau}^* - d_{5, \tau} d_{8, \tau}^*}{\Delta_\tau},
\]

\[
\gamma_\tau^* = \frac{d_{5, \tau} d_{11, \tau}^* - d_{5, \tau} d_{8, \tau}^*}{\Delta_\tau},
\]

and \( \Delta_\tau = d_{8, \tau} d_{8, \tau}^* - d_{11, \tau} d_{11, \tau}^* \).

Additionally,

\[
d_{1, \tau} = D_1 + \rho(c_0^2 \varepsilon_1, \tau-1 + 2c_0 \varepsilon_2, \tau-1 + \varepsilon_3, \tau-1),
\]

\[
d_{2, \tau} = D_2 + \rho(c_0 c_1 \varepsilon_1, \tau-1 + c_1 \varepsilon_2, \tau-1),
\]

\[
d_{3, \tau} = D_3 + \rho c_1^2 \varepsilon_1, \tau-1',
\]

\[
d_{4, \tau} = D_4 + \rho(c_0 b_0 \varepsilon_1, \tau-1 + b_0 \varepsilon_2, \tau-1 + c_0 \varepsilon_4, \tau-1 + \varepsilon_5, \tau-1),
\]

\[
d_{5, \tau} = D_5 + \rho(c_1 b_0 \varepsilon_1, \tau-1 + c_1 \varepsilon_4, \tau-1),
\]

\[
d_{6, \tau} = D_6 - \rho(c_0 b_0 \varepsilon_1, \tau-1 + b_0 \varepsilon_2, \tau-1 + c_0 \varepsilon_4, \tau-1 + \varepsilon_5, \tau-1),
\]

\[
d_{7, \tau} = D_7 - \rho(c_1 b_0 \varepsilon_1, \tau-1 + c_1 \varepsilon_4, \tau-1),
\]

\[
d_{8, \tau} = D_8 - \rho(b_0^2 \varepsilon_1, \tau-1 + b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1),
\]

\[
d_{9, \tau} = D_9 + \rho(b_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_4, \tau-1),
\]

\[
d_{10, \tau} = D_{10} - \rho(b_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_4, \tau-1),
\]

\[
d_{11, \tau} = D_{11} + \rho(b_0^2 \varepsilon_1, \tau-1 + 2b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1),
\]
\[ d_{12, \tau} = D_{12} + \rho(b_0^2 \varepsilon_1, \tau-1 + 2b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1), \]

\[ d_{13, \tau} = D_{13} + \rho(c_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_2, \tau-1), \]

\[ d_{14, \tau} = D_{14} + \rho c_1 b_1 \varepsilon_1, \tau-1, \]

\[ d_{15, \tau} = D_{15} + \rho b_1^2 \varepsilon_1, \tau-1, \]

\[ d_{1, \tau} = D_{1} + \rho(c_0^2 \varepsilon_1, \tau-1 + 2c_0 \varepsilon_2, \tau-1 + \varepsilon_3, \tau-1), \]

\[ d_{2, \tau} = D_{2} + \rho(c_0 c_1 \varepsilon_1, \tau-1 + c_1 \varepsilon_2, \tau-1), \]

\[ d_{3, \tau} = D_{3} + \rho c_1^2 \varepsilon_1, \tau-1, \]

\[ d_{4, \tau} = D_{4} - \rho(c_0 b_0 \varepsilon_1, \tau-1 + b_0 \varepsilon_2, \tau-1 + c_0 \varepsilon_4, \tau-1 + \varepsilon_5, \tau-1), \]

\[ d_{5, \tau} = D_{5} - \rho(c_1 b_0 \varepsilon_1, \tau-1 + c_1 \varepsilon_4, \tau-1), \]

\[ d_{6, \tau} = D_{6} + \rho(c_0 b_0 \varepsilon_1, \tau-1 + b_0 \varepsilon_2, \tau-1 + c_0 \varepsilon_4, \tau-1 + \varepsilon_5, \tau-1), \]

\[ d_{7, \tau} = D_{7} + \rho(c_1 b_0 \varepsilon_1, \tau-1 + c_1 \varepsilon_4, \tau-1), \]

\[ d_{8, \tau} = D_{8} - \rho(b_0^2 \varepsilon_1, \tau-1 + b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1), \]

\[ d_{9, \tau} = D_{9} - \rho(b_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_4, \tau-1), \]

\[ d_{10, \tau} = D_{10} + \rho(b_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_4, \tau-1), \]

\[ d_{11, \tau} = D_{11} + \rho(b_0^2 \varepsilon_1, \tau-1 + 2b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1), \]

\[ d_{12, \tau} = D_{12} + \rho(b_0^2 \varepsilon_1, \tau-1 + 2b_0 \varepsilon_4, \tau-1 + \varepsilon_6, \tau-1), \]

\[ d_{13, \tau} = D_{13} + \rho(c_0 b_1 \varepsilon_1, \tau-1 + b_1 \varepsilon_2, \tau-1), \]
\[ d_{14, \tau}^* = D_{14}^* + \rho c_1 b_1 e_{1, \tau-1}^* \]
\[ d_{15, \tau}^* = D_{15}^* + \rho b_1 e_{1, \tau-1}. \]

(35)

And,

\[ \varepsilon_{1, \tau} = q_{1, \tau} + 2 \beta_{\tau} q_{4, \tau} + 2 \beta_{\tau} q_{6, \tau} + 2 \beta_{\tau} \beta_{\tau} q_{8, \tau} \\
+ \beta_{\tau}^2 q_{11, \tau} + \beta_{\tau}^2 q_{12, \tau'} \]

\[ \varepsilon_{2, \tau} = q_{2, \tau} + \gamma_{\tau} q_{4, \tau} + \beta_{\tau} q_{5, \tau} + \gamma_{\tau}^* q_{6, \tau} + \beta_{\tau}^* q_{8, \tau} \\
+ (\beta_{\tau} \gamma_{\tau} + \gamma_{\tau} \beta_{\tau}^*) q_{8, \tau} + \beta_{\tau} \gamma_{\tau} q_{11, \tau} + \beta_{\tau} \gamma_{\tau} q_{12, \tau'} \]

\[ \varepsilon_{3, \tau} = q_{3, \tau} + 2 \gamma_{\tau} q_{5, \tau} + 2 \gamma_{\tau}^* q_{7, \tau} + 2 \gamma_{\tau} \gamma_{\tau}^* q_{8, \tau} \\
+ \gamma_{\tau}^2 q_{11, \tau} + \gamma_{\tau}^2 q_{12, \tau'} \]

\[ \varepsilon_{4, \tau} = q_{13, \tau} + \alpha_{\tau} q_{4, \tau} + \alpha_{\tau}^* q_{6, \tau} + (\alpha_{\tau} \beta_{\tau} + \alpha_{\tau} \beta_{\tau}^*) q_{8, \tau} \\
+ \beta_{\tau} q_{9, \tau} + \beta_{\tau} q_{10, \tau} + \alpha_{\tau} \beta_{\tau} q_{11, \tau} + \alpha_{\tau} \beta_{\tau}^* q_{12, \tau'} \]

\[ \varepsilon_{5, \tau} = q_{14, \tau} + \alpha_{\tau} q_{5, \tau} + \alpha_{\tau} q_{7, \tau} + (\alpha_{\tau} \gamma_{\tau} + \alpha_{\tau} \gamma_{\tau}^*) q_{8, \tau} \\
+ \gamma_{\tau} q_{9, \tau} + \gamma_{\tau} q_{10, \tau} + \alpha_{\tau} \gamma_{\tau} q_{11, \tau} + \alpha_{\tau} \gamma_{\tau}^* q_{12, \tau'} \]
\[ \epsilon_{6, \tau} = q_{15, \tau} + 2\alpha_{\tau}^* q_{8, \tau} + 2\alpha_{\tau} q_{9, \tau} + 2\alpha_{\tau}^* q_{10, \tau} + \alpha_{\tau}^2 q_{11, \tau} + \alpha_{\tau}^* q_{12, \tau}, \]

\[ \epsilon_{1, \tau} = q_{1, \tau} + 2\beta_{\tau}^* q_{4, \tau} + 2\beta_{\tau} q_{6, \tau} + 2\beta_{\tau}^* q_{8, \tau} + \beta_{\tau}^2 q_{11, \tau} + \beta_{\tau}^* q_{12, \tau}, \]

\[ \epsilon_{2, \tau} = q_{2, \tau} + \gamma_{\tau} q_{4, \tau} + \beta_{\tau}^* q_{5, \tau} + \gamma_{\tau} q_{6, \tau} + \beta_{\tau} q_{7, \tau} + (\beta_{\tau}^\gamma + \gamma_{\tau}^\beta) q_{8, \tau} + \beta_{\tau}^* q_{11, \tau} + \beta_{\tau}^\gamma q_{12, \tau}, \]

\[ \epsilon_{3, \tau} = q_{3, \tau} + 2\gamma_{\tau}^* q_{5, \tau} + 2\gamma_{\tau} q_{7, \tau} + 2\gamma_{\tau}^* q_{8, \tau} + \gamma_{\tau}^2 q_{11, \tau} + \gamma_{\tau}^* q_{12, \tau}, \]

\[ \epsilon_{4, \tau} = q_{13, \tau} + \alpha_{\tau}^* q_{4, \tau} + \alpha_{\tau} q_{6, \tau} + (\alpha_{\tau}^\beta + \alpha_{\tau}^\beta) q_{8, \tau} + \alpha_{\tau}^* q_{9, \tau} + \beta_{\tau}^* q_{10, \tau} + \alpha_{\tau}^* q_{11, \tau} + \alpha_{\tau}^* q_{12, \tau}, \]

\[ \epsilon_{5, \tau} = q_{14, \tau} + \alpha_{\tau}^* q_{5, \tau} + \alpha_{\tau} q_{7, \tau} + (\alpha_{\tau}^\gamma + \alpha_{\tau}^\gamma) q_{8, \tau} + \gamma_{\tau}^* q_{9, \tau} + \gamma_{\tau}^* q_{10, \tau} + \alpha_{\tau}^* q_{11, \tau} + \alpha_{\tau}^* q_{12, \tau}, \]

\[ \epsilon_{6, \tau} = q_{15, \tau} + 2\alpha_{\tau}^* q_{8, \tau} + 2\alpha_{\tau}^* q_{9, \tau} + 2\alpha_{\tau}^* q_{10, \tau}. \]
\[ + \alpha^* q_{11,\tau}^* + \alpha^* q_{12,\tau'}^* \]  

(36)

The backward recursive substitution of (32) and (33) into (28) and (29) yields the corresponding Nash equilibrium values for the cost-to-go expression at time t, denoted by \( V_\tau \), \( V_\tau^* \), and as follows:

\[
V_\tau = \varepsilon_1,\tau-1 q_t^2 + 2\varepsilon_2,\tau-1 q_t q_{t-1} + \varepsilon_3,\tau-1 q_t^2 + 2\varepsilon_4,\tau-1 q_t (m_{t-1} - m_{t-1}^*) \\
+ 2\varepsilon_5,\tau-1 q_{t-1} (m_{t-1} - m_{t-1}^*) + \varepsilon_6,\tau-1 (m_{t-1} - m_{t-1})^2,
\]

\[
V_\tau^* = \varepsilon^*_1,\tau-1 q_t^2 + 2\varepsilon^*_2,\tau-1 q_t q_{t-1} + \varepsilon^*_3,\tau-1 q_t^2 + 2\varepsilon^*_4,\tau-1 q_t (m_{t-1} - m_{t-1}^*) \\
+ 2\varepsilon^*_5,\tau-1 q_{t-1} (m_{t-1} - m_{t-1}^*) + \varepsilon^*_6,\tau-1 (m_{t-1} - m_{t-1})^2, \quad (37)
\]

IV.2 CCV Equilibrium Solution

In the Nash equilibrium, each agent assumes no response by his opponent to his own action. In fact, each policy maker responds in accordance with his own reaction curve. Therefore, the Nash equilibrium is consistently wrong in predicting the response of the rival. Recently, the CCV equilibrium was introduced into static game theory by Bresnahan (1981). This equilibrium assumes that in choosing his own strategy, each agent correctly anticipates the response of his rival. The solution therefore corresponds to a rational expectations
equilibrium. While debate on the CCV versions still continues, the players considered here are assumed to act simultaneously and more importantly, adopt the first-order approximation to the optimal CCV policy at each stage\(^2\). The second equilibrium we address is a dynamic version of CCV.

This generalization to a dynamic game utilizes the intrinsic closed-loop property of the extensive form description of the two-economy model (see Basar, 1986, ch.1). Its derivation follows a pattern similar to that of the closed-loop Nash solution, but at each stage a static CCV solution is obtained.

The following proposition presents the closed-loop CCV solution for a truncated version of the two-economy stochastic dynamic game model. Its proof is found in Appendix B.

**Proposition IV.2:** For the T-period dynamic game, the closed-loop CCV equilibrium is unique and is given by

\[
m_t^* = \alpha_t(m_{t-1}^* - m_{t-1}^*) + \beta_t q_t + \gamma_t q_{t-1}^*
\]

\[
m_t = \alpha_t(m_{t-1} - m_{t-1}^*) + \beta_t q_t + \gamma_t q_{t-1}^*
\]

\(^2\) The player's sequential reaction under the Bresnahan's static CCV has been criticized as a nonoptimal behavior (see, for example, Makowski, 1987). And in some cases where the players act simultaneously, the CCV equilibrium in an infinite regresssive model tends to be a Cournot (Nash) equilibrium (see, for example, Daugherty, 1985 and Basar, 1986, ch.1).
where

\[
\alpha_{\tau} = \frac{\left( -\frac{d_{8,\tau} + d_{12,\tau}x_{\tau}}{d_{11,\tau} + d_{8,\tau}x_{\tau}} \right) \left( d_{9,\tau} + d_{10,\tau}x_{\tau} \right) - d_{9,\tau} - d_{10,\tau}x_{\tau}}{(1 - x_{\tau}^{2})(d_{11,\tau} + d_{8,\tau}x_{\tau})},
\]

\[
\alpha^*_{\tau} = \frac{\left( -\frac{d_{8,\tau} + d_{12,\tau}x_{\tau}}{d_{11,\tau} + d_{8,\tau}x_{\tau}} \right) \left( d_{9,\tau} + d_{10,\tau}x_{\tau} \right) - d_{9,\tau}^* - d_{10,\tau}^*x_{\tau}}{(1 - x_{\tau}^{2})(d_{11,\tau} + d_{8,\tau}x_{\tau})},
\]

\[
\beta_{\tau} = \frac{\left( -\frac{d_{8,\tau} + d_{12,\tau}x_{\tau}}{d_{11,\tau} + d_{8,\tau}x_{\tau}} \right) \left( d_{4,\tau} + d_{6,\tau}x_{\tau} \right) - d_{4,\tau} - d_{6,\tau}x_{\tau}}{(1 - x_{\tau}^{2})(d_{11,\tau} + d_{8,\tau}x_{\tau})},
\]

\[
\beta^*_{\tau} = \frac{\left( -\frac{d_{8,\tau} + d_{12,\tau}x_{\tau}}{d_{11,\tau} + d_{8,\tau}x_{\tau}} \right) \left( d_{4,\tau} + d_{6,\tau}x_{\tau} \right) - d_{4,\tau}^* - d_{6,\tau}^*x_{\tau}}{(1 - x_{\tau}^{2})(d_{11,\tau} + d_{8,\tau}x_{\tau})},
\]

\[
\gamma_{\tau} = \frac{\left( -\frac{d_{8,\tau} + d_{12,\tau}x_{\tau}}{d_{11,\tau} + d_{8,\tau}x_{\tau}} \right) \left( d_{5,\tau} + d_{7,\tau}x_{\tau} \right) - d_{5,\tau} - d_{7,\tau}x_{\tau}}{(1 - x_{\tau}^{2})(d_{11,\tau} + d_{8,\tau}x_{\tau})},
\]
\[ \gamma_\tau = \frac{\{d_{12}^*, \tau + d_{12}^*, \tau x_\tau^2 \} - d_{5}^*, \tau x_\tau^2}{(1 - x_\tau^2)(d_{11}^*, \tau + d_8^*, \tau x_\tau^2)}. \] (40)

Therefore, as in the Nash case, the corresponding CCV equilibrium values for the cost-to-go expression at time \( t \) are as follows:

\[ V_\tau = \varepsilon_{1, \tau-1} q_t^2 + 2\varepsilon_{2, \tau-1} q_t q_t q_t q_{t-1} + \varepsilon_{3, \tau-1} q_t^2 q_{t-1} + 2\varepsilon_{4, \tau-1} q_t (m_{t-1} - m_{t-1}) \]
\[ + 2\varepsilon_{5, \tau-1} q_t (m_{t-1} - m_{t-1}) + \varepsilon_{6, \tau-1} (m_{t-1} - m_{t-1})^2, \]

\[ V_\tau^* = \varepsilon_{1, \tau-1} q_t^2 + 2\varepsilon_{2, \tau-1} q_t q_t q_t q_{t-1} + \varepsilon_{3, \tau-1} q_t^2 q_{t-1} + 2\varepsilon_{4, \tau-1} q_t (m_{t-1} - m_{t-1}) \]
\[ + 2\varepsilon_{5, \tau-1} q_t (m_{t-1} - m_{t-1}) + \varepsilon_{6, \tau-1} (m_{t-1} - m_{t-1})^2, \] (41)

where \( d_1, d_1^*, \varepsilon_1, \text{ and } \varepsilon_1^* \) are defined previously in (35) and (36). \( x_\tau \) is a solution for the quadratic equation:

\[ d_8^*, \tau x_\tau^2 + (d_{11}^*, \tau + d_{12}^*, \tau) x_\tau + d_8, \tau = 0, \]
\[ \tau = 0, 1, 2, \ldots, T. \] (42)

As shown in Appendix B, \( x_\tau \) is the slope of the static reaction functions, and is the same for both countries under the given symmetry assumptions. Given the relationship between the parameters of the
model, the quadratic equation (42) permits generically two real solutions and hence the number of equilibria grows with the number of periods.

IV.3 Stackelberg Equilibrium Solution

The Nash and CCV equilibria solutions considered above are a symmetric equilibrium concept in terms of the roles of the players in the game. Suppose now that one of the two policy makers (called the leader) has the power to dominate the decision process. Even in a model such as this, where the structures of the two economies are taken to be symmetric, this case is of interest. For example, the home economy can be taken to be experiencing a trade deficit like the United States, and the rest of the world are collectively having a corresponding trade surplus. The home economy intends to take the initiative by announcing its monetary policy seeking to restore an external balance with the rest of world. Such an asymmetry in the roles of the players leads to the Stackelberg solution, which assumes the home economy can enforce its policy throughout the duration of the game. However, the leader can enforce this policy only from one period to the next. This mode of play, which corresponds to the "closed-loop Stackelberg Solution," allows for a recursive derivation and satisfies the time consistency property. This is the equilibrium solution adopted in this subsection for the two-country model.
In the derivation of the closed-loop Stackelberg solution, we follow the recursive technique presented in Basar and Olsder (1982, Chapter 7), which is parallel to the derivation given in Appendix A for the Nash solution. The only difference is that now, at every stage a static Stackelberg game is solved, instead of a Nash game. Appendix C elucidates this difference in the derivation. Again, if we take the home country as the leader and the foreign country as the follower (superscripted by an asterisk), the main result for a $T$-period stochastic, dynamic game is presented in Proposition IV.3.

**Proposition IV.3:** For the $T$-period dynamic game, the closed-loop Stackelberg equilibrium solution is unique and is given by

$$m_t^* = \alpha^*_\tau (m_{t-1}^* - m_{t-1}) + \beta^*_\tau q_t^* + \gamma^*_\tau q_{t-1}^*,$$

(43)

$$m_t^* = \alpha^*_\tau (m_{t-1}^* - m_{t-1}) + \beta^*_\tau q_t^* + \gamma^*_\tau q_{t-1}^*,$$

(44)

$$\tau = 0,1,2,\ldots,T-1,$$

where

$$\alpha^*_\tau = \frac{d_{12}^* \frac{d_{8},\tau}{d_{11},\tau} - d_{10},\tau \frac{d_{8},\tau}{d_{11},\tau} - d_{8},\tau \frac{d_{9},\tau}{d_{11},\tau} + d_{9},\tau}{\Delta_\tau}$$
\[ \alpha^*_\tau = \frac{- (\alpha \frac{d^*_8, \tau + d^*_9, \tau}{d^*_{11, \tau}})}{d^*_{11, \tau}}, \]

\[ \beta^*_\tau = \frac{d_{12, \tau} \frac{d^*_8, \tau d^*_4, \tau}{d^*_{11, \tau}} - d_{8, \tau} \frac{d^*_4, \tau}{d^*_{11, \tau}} - d_{6, \tau} \frac{d^*_8, \tau}{d^*_{11, \tau}} + d_{4, \tau}}{\Delta^*_\tau}, \]

\[ \gamma^*_\tau = \frac{- (\beta \frac{d^*_8, \tau + d^*_4, \tau}{d^*_{11, \tau}})}{d^*_{11, \tau}}, \]

\[ \gamma^*_\tau = \frac{d_{12, \tau} \frac{d^*_8, \tau d^*_5, \tau}{d^*_{11, \tau}} - d_{8, \tau} \frac{d^*_5, \tau}{d^*_{11, \tau}} - d_{7, \tau} \frac{d^*_8, \tau}{d^*_{11, \tau}} + d_{5, \tau}}{\Delta^*_\tau}, \]

\[ \Delta^*_\tau = 2d_{8, \tau} \frac{d^*_8, \tau}{d^*_{11, \tau}} - d(\frac{d^*_8, \tau}{d^*_{11, \tau}})^2 - d_{11, \tau}. \]

And, as in the Nash case, the corresponding Stackelberg equilibrium values for the cost-to-go expression at time \( t \) can be expressed as follows:
\[ V_\tau = \epsilon_{1, \tau-1} q_t^2 + 2 \epsilon_{2, \tau-1} q_t q_{t-1} + \epsilon_{3, \tau-1} q_{t-1}^2 + 2 \epsilon_{4, \tau-1} q_t (m_{t-1}^* - m_{t-1})^2. \]

\[ V_\tau^* = \epsilon_{1, \tau-1} q_t^2 + 2 \epsilon_{2, \tau-1} q_t q_{t-1} + \epsilon_{3, \tau-1} q_{t-1}^2 + 2 \epsilon_{4, \tau-1} q_t (m_{t-1}^* - m_{t-1})^2. \]

where \( d_{i}s, d_{i}^* s, \epsilon_{i}s, \) and \( \epsilon_{i}^* s \) are defined previously in (35) and (36). Moreover, the follower's reaction function is derived as:

\[ m_t^* = \frac{1}{d_{11, \tau}} \left[ d_{8, \tau} m_t + d_{9, \tau} (m_{t-1} - m_{t-1}^*) + d_{4, \tau} q_t + d_{5, \tau} q_{t-1}^* \right], \]

where \( \frac{-d_{8, \tau}}{d_{11, \tau}} \) is the slope of the follower's reaction function.

IV.4 Cooperative Equilibrium Solution

The final equilibrium addressed is the cooperative equilibrium over time. The policy makers of the two economies are assumed to coordinate their monetary policies only from one period to the next.
only. In the derivation of the closed-loop cooperative solution, the
two players agree to cooperate by minimizing their joint cost func-
tion, $\bar{V}_\tau = V_\tau + V_\tau^+$ at each stage. Therefore, the closed-loop
cooperative equilibrium is defined as a pair of closed-loop coopera-
tive policy rules $(\tilde{m}_t, \tilde{m}_t^*)$, given by (30) and (31), such that:

$$\bar{V}_\tau(\tilde{m}_t, \tilde{m}_t^*) \geq \bar{V}_\tau(m_t, m_t^*) \geq \bar{V}_\tau(m_t, \tilde{m}_t^*) \geq \bar{V}_\tau(m_t, m_t^*).$$

The main result for a $T$-period stochastic, dynamic game is presented
below. Its proof is in Appendix D.

$$m_t = \alpha_\tau (m_{t-1} - m_{t-1}^*) + \beta_\tau q_t + \gamma_\tau q_{t-1}, \quad (48)$$

$$m_t^* = \alpha_\tau^* (m_{t-1} - m_{t-1}^*) + \beta_\tau^* q_t + \gamma_\tau^* q_{t-1}, \quad (49)$$

$$\tau = 0, 1, 2, \ldots, T-1,$$

where

$$\alpha_\tau = \frac{\bar{d}_{9, \tau} \bar{d}_{12, \tau} - \bar{d}_{10, \tau} \bar{d}_{8, \tau}}{\Delta_\tau},$$

$$\alpha_\tau^* = \frac{\bar{d}_{10, \tau} \bar{d}_{11, \tau} - \bar{d}_{9, \tau} \bar{d}_{8, \tau}}{\Delta_\tau},$$
\[ \beta_\tau = \frac{\bar{d}_4, \tau \bar{d}_{12}, \tau - \bar{d}_6, \tau \bar{d}_8, \tau}{\Delta_\tau}, \]

\[ \beta^*_\tau = \frac{\bar{d}_6, \tau \bar{d}_{11}, \tau - \bar{d}_4, \tau \bar{d}_8, \tau}{\Delta_\tau}, \]

\[ \gamma_\tau = \frac{\bar{d}_5, \tau \bar{d}_{12}, \tau - \bar{d}_7, \tau \bar{d}_8, \tau}{\Delta_\tau}, \]

\[ \gamma^*_\tau = \frac{\bar{d}_7, \tau \bar{d}_{11}, \tau - \bar{d}_5, \tau \bar{d}_8, \tau}{\Delta_\tau}, \] (50)

and \[ \Delta_\tau = \bar{d}_{8, \tau}^2 - \bar{d}_{11, \tau} \bar{d}_{12, \tau}; \] in addition, the \( d^*_i \)'s and \( d^*_i \)'s are given by:

\[ \bar{d}_1, \tau = d_1, \tau + d^*_1, \tau; \quad \bar{d}_2, \tau = d_2, \tau + d^*_2, \tau \]

\[ \bar{d}_3, \tau = d_3, \tau + d^*_3, \tau; \quad \bar{d}_4, \tau = d_4, \tau + d^*_4, \tau \]

\[ \bar{d}_5, \tau = d_5, \tau + d^*_7, \tau; \quad \bar{d}_6, \tau = d_6, \tau + d^*_6, \tau \]

\[ \bar{d}_7, \tau = d_7, \tau + d^*_5, \tau; \quad \bar{d}_8, \tau = d_8, \tau + d^*_8, \tau \]

\[ \bar{d}_9, \tau = d_9, \tau + d^*_10, \tau; \quad \bar{d}_{10}, \tau = d_{10}, \tau + d^*_9, \tau \]

\[ \bar{d}_{11}, \tau = d_{11}, \tau + d^*_12, \tau; \quad \bar{d}_{12}, \tau = d_{12}, \tau + d^*_11, \tau \]
\[ \dd_{13, \tau} = d_{13, \tau} + \dd_{13, \tau}^*; \quad \dd_{14, \tau} = d_{14, \tau} + \dd_{14, \tau}^* \]
\[ \dd_{15, \tau} = d_{15, \tau} + \dd_{15, \tau}^* \]

The corresponding cooperative equilibrium values for the cost-to-go expression at time \( t \) can be expressed as follows:

\[
V_{\tau} = \varepsilon_{1, \tau-1} q_{t-1}^2 + 2 \varepsilon_{2, \tau-1} q_{t-1} + \varepsilon_{3, \tau-1} \frac{q_{t-1}^2}{\tau} + 2 \varepsilon_{4, \tau-1} q_{t-1} (m_{t-1} - m_{t-1}^*) + 2 \varepsilon_{5, \tau-1} q_{t-1} (m_{t-1} - m_{t-1}^*)^2,
\]
\[
V_{\tau}^* = \varepsilon_{1, \tau-1} q_{t-1}^2 + 2 \varepsilon_{2, \tau-1} q_{t-1} + \varepsilon_{3, \tau-1} q_{t-1}^2 + 2 \varepsilon_{4, \tau-1} q_{t-1} (m_{t-1} - m_{t-1}^*) + 2 \varepsilon_{5, \tau-1} q_{t-1} (m_{t-1} - m_{t-1}^*)^2,
\]

(51)

IV.5 Stationary Equilibria

The stationary solutions are obtained by considering the limits, as \( \tau \to \infty \), of the solutions given by (32) and (33) in the case of closed-loop Nash game, and the analogous equations for the other games. The limits are defined as

\[
\lim_{\tau \to \infty} \alpha_{t} = \bar{\alpha}; \quad \lim_{\tau \to \infty} \alpha_{t}^* = \bar{\alpha}
\]
\[
\lim_{\tau \to \infty} \beta^*_\tau = \bar{\beta}; \quad \lim_{\tau \to \infty} \beta^*_\tau = \bar{\beta}
\]

\[
\lim_{\tau \to \infty} \gamma^*_\tau = \bar{\gamma}; \quad \lim_{\tau \to \infty} \gamma^*_\tau = \bar{\gamma},
\]

and are similar for \( \bar{d}_i, \bar{d}^*_i, \bar{e}_i, \) and \( \bar{e}^*_i \). The steady-state policy rules are given by

\[
m_t = \bar{\alpha}(m_{t-1} - m^*_{t-1}) + \beta q_t + \gamma q_{t-1}, \quad (52)
\]

\[
m^*_t = \bar{\alpha}^*(m_{t-1} - m^*_{t-1}) + \beta^* q_t + \gamma^* q_{t-1}, \quad (53)
\]

where \( \bar{\alpha}, \bar{\alpha}^*, \bar{\beta}, \beta^*, \bar{\gamma}, \) and \( \gamma^* \) are the solutions to the set of equations obtained by letting \( \tau \to \infty \) in (32) and (33) in the closed-loop Nash game. These constitute highly nonlinear coupled algebraic equations in \( \bar{d} \) and \( \bar{e} \), and the recursive procedure we have outlined provides a solution to these coupled equations.

The equilibrium steady-state path, obtained by finding the roots of the second-order difference equation system, (52) and (53), is given by
\[
\begin{bmatrix}
1 & 0 \\
-(\bar{B}-\bar{B}^*) & 1
\end{bmatrix}
\begin{bmatrix}
q_{t+1} \\
m_{t+1} - m^*
\end{bmatrix}
= 
\begin{bmatrix}
c_0 & b_0 \\
-\gamma & -\alpha - \bar{\alpha}^*
\end{bmatrix}
\begin{bmatrix}
q_t \\
m_{t} - m^*
\end{bmatrix}
+ 
\begin{bmatrix}
c_1 & b_1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
q_{t-1} \\
m_{t-1} - m^*_{t-1}
\end{bmatrix},
\]

(54)

Under the plausible conditions, it admits one stable and two complex conjugate roots. With the endogenous policy rules, \( m \) and \( m^* \), stability can be accomplished through appropriate adjustments in the policy variables. The unstable root may be eliminated from the system but it does not necessarily prevent the need for the exchange rate from undergoing endogenous jumps. Since the exchange rate reversion (overshooting or undershooting) during the adjustment phase may arise from the need to balance the overall trade and capital flows (see Niehans, 1977 and Driskill, 1981), an initial larger adjustment in the trade balance is likely to prevent this reversion. Our numerical simulations indicate that this condition is satisfied by our equilibrium solution candidates. Assuming the initial values for \( q, m, m^* \) are zero,

\[
\lim_{T \to \infty} V_{T,1} = \varepsilon_1 q_1^2,
\]

\[
\lim_{T \to \infty} V^*_{T,1} = \varepsilon^* q_1^2,
\]

both of which are finite.
CHAPTER V

NUMERICAL ANALYSIS

The equilibrium solutions derived previously show a high degree of nonlinearity in the parameter space. While not analytically tractable, the monetary policies and their welfare implications can be examined numerically for particular parameter values. This chapter provides such a numerical analysis. Any selected set of values is somewhat arbitrary, but the values have been selected to correspond, however crudely, to the empirical literature on policy analysis. The well-known phenomenon of the trade balance J-Curve effect (where the monetary expansion results in a worsening of the trade balance) has been chosen as the underlying numerical example (see Niehans, 1975 and Driskill, 1981). In addition, a higher value is assigned to the response of the international capital flows to its net yield, so that it makes the capital flow considerations more important in the flow market for foreign exchange.

Taking the flexible exchange rate case as a benchmark, now suppose that the real exchange rate is anticipated to appreciate by a unit in the next period and is assumed to clear the foreign exchange
market without intervention.\(^3\) Since the capital flow, by construction, responds to the appreciation in the real exchange rate, exchange market speculation produces \(\eta\) units of capital inflows, financing an equivalent amount of the trade deficit for the home economy; the opposite situation occurs abroad (hereafter the home country is called a deficit economy and the foreign country a surplus economy). The appreciation of the real exchange rate reduces the demand for domestic output and raises the demand for foreign output. With the real money stocks held constant in both economies, and with highly mobile capital, these changes in output will lead to a decrease in the domestic interest rate, accompanied by a rise in the foreign interest rate. The net effect of which causes the exchange rate appreciation of the domestic currency to increase, while the trade imbalance continues to worsen. The decrease in domestic output leads to a further decline in the prices of domestic output. This, together with the increase in the exchange appreciation, causes the overall rate of deflation to increase; precisely the opposite effects occur in the surplus economy.

While the absence of intervention is not desirable from the viewpoint of long-run welfare maximization, addressing the situation, in which there is recessionary pressure in the deficit economy and inflation in the surplus economy, has been described as the most difficult task among Meade’s (1951) conflicts of internal and external balance that face the policy makers of both economies. This occurs

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\(^3\) The cause of appreciation of home currency exchange is assumed to be the result of the misalignment of monetary policies between the home and foreign economies over the past.
because there is no combination of domestic policies in the two economies that will simultaneously restore both balances. If the real money stock were reduced in the surplus economy and increased in the deficit economy to obtain an internal balance, the external imbalance will worsen as the surplus economy's imports fall and the deficit economy's imports rise. By the same token, if the surplus economy increased the real money stock and the deficit economy reduced its real money stock to obtain an external balance, the inflationary pressure and recession would be made worsen in the respective economies.

V.1 The Moderate Trade Balance J-Curve Effect

The first case of the trade balance J-curve effect presented is the case in which the expenditure effect of the real exchange rate moderately dominates its substitution (switching) effect. Given high capital mobility, exchange rate appreciation is required to attract net capital inflows to equilibrate the trade deficit caused by the real money expansion. The base parameter set is chosen as follow

\[ \theta = 0.40, \sigma = 1.00, \omega = 1.00, \phi = 0.50, \lambda = 1.00, \delta = 0.70, \]
\[ \alpha = 1.00, \beta = 1.50, \eta = 3.00, \psi = 0.60, \rho = 0.90, k = 0.75. \]

The numerical results show that for a time horizon of 48 periods, the restrictions for the stationary equilibrium provided in Chapter IV are
satisfied. In other words, the stationary equilibria exist for various strategic games. Figures 1-5 illustrate the time paths for the real exchange rate, \( q \), the real money supply, \( m \), domestic output and inflation, \( y \) and \( \Delta C \), and the net accumulation of foreign assets, \( CTB \), for the closed-loop equilibria. Given the symmetry of the model, the effects on the two economies are identical, so that the time paths for \( m^* \), \( y^* \), and \( \Delta C^* \) are simply mirror images of \( m \), \( y \), and \( \Delta C \). The only exception is the Stackelberg case in which asymmetrical behavior of the policy makers is allowed. The four equilibria are discussed in turn.

![Figure 1](image_url)

**Figure 1**
The Time Path for Real Exchange Rate
Figure 2
The Time Path for Real Money Supply

Figure 3
The Time Path for Domestic Output
Figure 4
The Time Path for Overall Rate of Inflation

Figure 5
The Time Path for Cumulative Trade Balance
V.1.1 Nash equilibrium

Suppose that each policy maker follows a Nash strategy, using an intertemporal objective function. The optimal closed-loop policy in to the decrease in the relative price calls for a real monetary con- traction in the ensuing periods in the deficit economy and a corresponding expansion in the surplus economy. However, the sub- sequent policies for both economies are characterized by alternating policies of leaning against and leaning with the wind. By contrast, employing an one-period (static) objective function each policy maker, while following a Nash strategy, will adopt an expansionary monetary policy in the deficit economy and a contraction in the surplus economy. Therefore, neither the absence of intervention, nor the optimal short-run Nash policy of leaning against the wind is desirable from the viewpoint of long-run welfare maximization.

The incipient contractionary policy in the deficit economy causes a greater reduction of domestic output and a further deflation than in the benchmark situation. This reduces the exchange rate appreciation and hence the overall rate of domestic deflation relative to the benchmark policy. Precisely the opposite effects occur abroad. The smaller exchange rate appreciation, combined with the above changes in domestic and foreign output, leads to an increase in the real exchange rate, q, and an improvement of the trade balance. This mitigates the fluctuations in output and deflation. By the same reasoning, the

4. The coefficients of the static Nash policy are \( \bar{\alpha} = -\bar{\alpha}^* = -0.050 \), \( \bar{\beta} = \bar{\beta}^* = -1.136 \), \( \bar{y} = \bar{y}^* = 0.023 \).
subsequent expansion (contraction) in the deficit economy leads to an increase (decrease) in domestic output and the overall deflation and a deterioration (improvement) of the trade balance in the deficit economy. However, the magnitudes of the above movements, dictated by the time path for the real exchange rate near its long-run value, will be much smaller; correspondingly the opposite situation occurs in the surplus economy. As a result of implementing this closed-loop Nash policy, the real exchange rate follows a convergent path of dampened cycles with steadily declining welfare costs.

For the 48-period horizon illustrated in Figure 1, the steady-state coefficients of the optimal Nash policy rules and the steady-state characteristic roots of (54) are reported in Table 1 and 2, respectively. Each equilibrium solution admits one real root and one pair of complex conjugate roots. The real and image parts of the roots, in absolute terms, all fall outside the unit circle, demonstrating the strong property of convergence. The summation over \( \psi_j \), denoted by \( R \), represents the fraction of the total adjustment remaining after the first period. Under the closed-loop solution, it takes about twelve periods before the control law converges to its steady state. The speed of adjustment of the economy given by \( R = 1.053 \) along the optimal trajectory (54) implies that, instead of heading for its long-run value, the real exchange rate further appreciates around 5% more within the first period. In addition, the negative real root and the complex roots cause the real exchange rate to cyclically converge to its long-run value.
V.1.2 CCV Equilibrium

The Nash equilibrium is based on the assumption that the policy maker of each economy presumes that the policy maker of the other will not react to his actions. In the CCV solution, each policy maker correctly takes into account his opponent’s reaction. The slope of the short-run reaction function in the closed-loop CCV solution is given by solving the quadratic reaction function for $x_\tau$ in (42) as follows:

$$d_8, \tau x^2 + (d_{11}, \tau + d_{12}, \tau)x_\tau + d_8, \tau = 0. \quad (55)$$

In the initial period, we find $d_8 = -2.759$, $d_{11} = 5.400$, and $d_{12} = 3.645$, so that $x_0$ is 0.340 and positive for the two economies. The policy maker of either economy knows that if he follows the closed-loop Nash strategy of contracting (expanding) the real money supply, the other should expand (contract) less. This leads the deficit (surplus) economy to experience a larger recession (inflation). Hence, the optimal CCV strategy for the deficit economy calls for a smaller adjustment in the real money supply by choosing an initial expansion. The policy maker of the surplus economy under the closed-loop CCV equilibrium knows this and responds with an expansion, but with a smaller magnitude.

As a result, each policy maker, while taking account of his rival’s behavior, maintains caution and a more gradual adjustment
ensues. By the same reasoning above, this leads to an exacerbation of the fluctuations in output and the overall rate of deflation (inflation). The trade imbalance is widened relative to the Nash situation. Therefore, Basar, Turnovský, and d'Orey (1986) called this situation a beggar-thy-neighbor world. While the time path for \( q \), and hence for all other relevant variables in the two economies, follows similar previous time paths, everything occurs on a larger scale. This causes the output and overall deflation in the deficit economy to increase by more than it did in the closed-loop Nash policy, with the corresponding opposite situation occurring abroad. The trade imbalance between the two economies is widened. As a result, the closed-loop CCV equilibrium is outperformed by the closed-loop Nash equilibrium.

The steady-state coefficients of the optimal CCV policy rules are reported in Table 1. The steady-state characteristic roots, given in Table 2, indicates that, under the closed-loop solution, the speed of adjustment of the economy along the optimal trajectory (54), given by \( R = 1.202 \), implies that as in the Nash case, the exchange rate will further appreciate by 20% within the first period and then cyclically head for its long-run value because of the negative real root and of the complex roots. In addition, it takes more than 17 periods to get the control law converge to its steady-state value.
V.1.3 Stackelberg Equilibrium

Consider now a Stackelberg case where the deficit economy assumes the leadership role in a noncooperative world. The follower's short-run reaction function in the closed-loop solution is given by (47), which is

$$m_t^* = -\frac{1}{d_{11},\tau}[d_{8},\tau m_t^* + d_{9},\tau (m_{t-1}^* - m_{t-1}^*)$$

$$+ d_{4},\tau q_t^* + d_{5},\tau q_{t-1}^*].$$

(56)

In the initial period, we find $d_{8}^* = -5.409$ and $d_{11}^* = 7.267$, so that the slope value is now 0.744 and positive. This means that the surplus economy responds to a unit contraction in deficit economy's money supply, with a corresponding monetary contraction of 0.744. This leads to an exacerbation of the fluctuations in output and deflation in the deficit economy, which, given by the larger weight assigned to the output fluctuation, is a nonoptimal situation for the leader. Therefore, the leader's optimal strategy is to engage in a larger initial monetary expansion, thereby inducing an even larger corresponding expansion abroad by the follower, relative to the closed-loop Nash policy. Turnovsky, Basar, and d'Orey (1988) described this as a locomotive world. A monetary expansion in the deficit economy causes the domestic output to increase by more than it did in the closed-loop Nash situation. By the same reasoning, exchange rate appreciation of the domestic currency, and hence the
overall rate of domestic deflation, decreases relative to the benchmark policy. The smaller domestic nominal exchange rate appreciation, accompanied by the movements in domestic and foreign output, causes the real exchange rate, \( q \), to overshoot its long-run value. Thus, the trade imbalance is significantly reduced. As a result, while the implementation of the closed-loop Stackelberg policy brings the real exchange rate to a quickly converging path with a sharp decline in welfare costs, the welfare costs for the two economies are not evenly distributed.

Figures 6, 7, and 8 illustrate the comparison of the closed-loop time paths for the real money supply, domestic output, and inflation between the deficit and surplus economies. Pursuing the closed-loop Stackelberg policy, a world-wide expansion results and leads the deficit economy (leader) to experience smaller fluctuations in its output and rate of deflation. The opposite situation occurs in the surplus economy. By assigning its monetary policy on the internal balance initially, the deficit economy is able to impose most of the burden of external adjustment on the surplus economy (follower), forcing it to bear the bulk of the associated costs.
Figure 6
The Time Path for Real Money Supply

Figure 7
The Time Path for Domestic Output
The steady-state coefficients of optimal Stackelberg policy rules and the steady-state characteristic roots are given in Table 1 and 2, respectively. The characteristic roots show that, under the closed-loop solution, the speed of adjustment for the economy, along the optimal trajectory (54), is given by $R = 0.751$ and implies that around 25% of the adjustment is completed within the first period. Again, the negative real root and the complex roots reflect the cyclical convergence of the time path for the real exchange rate. It takes about 14 periods to get the control law to converge to its steady state.
Since the values of these parameters are invariant to which economy acts as the leader, neither economy will agree to be the follower or leader unless there is some other mechanism whereby leadership is determined and enforced.

V.1.4 Cooperative Equilibrium

In the noncooperative Nash policy, each economy's policy maker independently chooses his action given no response of the other, and therefore ignores the beneficial effect of his monetary policy on the other nation. However, under the cooperative policy, the policy makers of each economy are aware of this positive externality and, as a result, pursue a more aggressive monetary policy. The closed-loop cooperative policy allows the policy maker of the deficit (surplus) economy to pursue a lower (higher) output, which it is unable to obtain under the noncooperative closed-loop Nash and CCV policies. Compared to the closed-loop Nash, the closed-loop cooperative strategy requires a larger initial contraction in the deficit economy and a corresponding larger expansion in the surplus economy than was necessary in the noncooperative situation. Similar to the reasoning above, this leads to a greater mitigation of the fluctuations in output and deflation than occurred in the noncooperative situation.

The steady-state coefficients of the optimal cooperative policy are reported in Table 1. Under the closed-loop solution, it takes more than 10 periods before the control law converges to its steady state. The steady-state characteristic roots, given in Table 2, show
that the speed of adjustment of the economy along the optimal trajectory (54), is given by $R = 0.508$ and implies that around 49% of the adjustment is completed within the first period.

V.1.5 Welfare Gains And The Steady-State Rate of Convergence

The welfare gains derived from the various equilibrium solutions can be characterized by the behavior of the players' reaction and the convergence of the real exchange rate. The latter is the only source of disturbance to the world economy and hence governs the behavior of all other relevant variables. Using the equation (54) yields the time path for the real exchange rate as follows:

$$q_{t+1} = \theta_1 q_t - \theta_2 q_{t-1} - \theta_3 q_{t-2},$$

(57)

where the numerical values of parameter $\theta_j$ are given in Table 3. The parameter $\theta_j$'s reflect that how much the fraction of adjustment is remaining after the first, second periods, and so on. It turns out that the rankings of the steady-state rate of convergence are quite uniform:

$$\theta_i^{CCV} > \theta_i^N > \theta_i^S > \theta_i^C, \quad i = 1, 2, 3.$$ 

Note that those extra terms with the lags more than four periods are far less important to determine the speed of convergence and therefore are not shown in the equation (57).
This ranking is identical to the results obtained in Basar, Turnovsky, and d'Orey (1986) and Turnovsky, Basar, and d'Orey (1988). However, the major differences are that the time path for the real exchange rate under each equilibrium overshoots its long-run value in the presence of negative real roots and complex roots. Given no net accumulation of foreign assets, this overshooting is necessary to facilitate the reduction in the trade deficit of the home economy. In addition, the monetary policies in the dynamic adjustment periods alternate between leaning-against-the-wind and leaning-with-the-wind. The welfare gains derived from the closed-loop strategies, given in Figure 9, show that Stackelberg leadership is preferred, while being a Stackelberg follower is the worst equilibrium position. Between these extremes, we find that the cooperative equilibrium dominates Nash, which in turn is better than CCV. Again, this welfare ranking conforms to that obtained by Basar, Turnovsky, and d'Orey (1986) and Turnovsky, Basar, and d'Orey (1988).
Figure 9
The Time Path for Welfare Costs
### Table 1
**Steady-State Coefficients of Optimal Policy Rule**

\( T = 48 \)

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>( \alpha )</th>
<th>( \alpha^* )</th>
<th>( \beta )</th>
<th>( \beta^* )</th>
<th>( \gamma )</th>
<th>( \gamma^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>0.286</td>
<td>-0.286</td>
<td>0.036</td>
<td>-0.036</td>
<td>-0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>CCV</td>
<td>0.272</td>
<td>-0.272</td>
<td>-0.011</td>
<td>0.011</td>
<td>-0.156</td>
<td>0.156</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>0.207</td>
<td>-0.419</td>
<td>-0.243</td>
<td>-0.506</td>
<td>-0.118</td>
<td>0.239</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.336</td>
<td>-0.336</td>
<td>0.208</td>
<td>-0.208</td>
<td>-0.192</td>
<td>0.192</td>
</tr>
</tbody>
</table>

### Table 2
**Steady-State Characteristic Roots**

\( T = 48 \)

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>( \psi_1 )</th>
<th>( \psi_2 )</th>
<th>( \psi_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>-0.213</td>
<td>0.633 + 0.026i</td>
<td>0.633 - 0.026i</td>
</tr>
<tr>
<td>CCV</td>
<td>-0.204</td>
<td>0.538</td>
<td>0.877</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>-0.227</td>
<td>0.489 + 0.196i</td>
<td>0.489 - 0.196i</td>
</tr>
<tr>
<td>Cooperation</td>
<td>-0.222</td>
<td>0.365 + 0.212i</td>
<td>0.365 - 0.212i</td>
</tr>
</tbody>
</table>

### Table 3
**Steady-State Rate of Convergence**

\( T = 48 \)

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>( \theta_1 )</th>
<th>( \theta_2 )</th>
<th>( \theta_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>1.053</td>
<td>0.130</td>
<td>0.085</td>
</tr>
<tr>
<td>CCV</td>
<td>1.202</td>
<td>0.180</td>
<td>0.095</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>0.751</td>
<td>0.056</td>
<td>0.063</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.508</td>
<td>0.016</td>
<td>0.040</td>
</tr>
</tbody>
</table>
V.2 The Strong Trade Balance J-Curve Effect

We now assign a smaller value to the coefficient for the response of trade flows to the relative price so that the trade balance J-Curve effect is more apparent. Accordingly, the coefficient of the relative price effect on aggregate demand is smaller also. The smaller values can be considered the result of decreased economic interdependence. The base parameter set is chosen as follow:

\[
\begin{align*}
\theta &= 0.50, \quad \sigma = 1.00, \quad \omega = 0.50, \quad \phi = 0.50, \quad \lambda = 1.00, \quad \delta = 0.70, \\
\alpha &= 0.25, \quad \beta = 1.50, \quad \eta = 3.00, \quad \psi = 0.60, \quad \rho = 0.90, \quad k = 0.75.
\end{align*}
\]

The numerical results show that for a time horizon of 72 periods, which is longer than the previous case, the stationary restrictions required in IV.5 are satisfied. Therefore, stationary equilibria exist for various strategic games.

V.2.1 Steady-State Policy Rules

The steady-state coefficients of the optimal closed-loop policy rules are given in Table 3, but are different signs compared to the case of a moderate J-Curve trade balance effect. Additionally, as economic interdependence decreases, a larger adjustment is necessary. The lower degree of economic interdependence affects the short-run behavior of the policy makers in the closed-loop equilibrium. The steady-state coefficients of the optimal closed-loop policy are
provided in Table 4 and imply that the optimal closed-loop strategies for CCV and Stackelberg now call for an initial contraction. This differs from the initial policies pursued in the previous case, while Nash and Cooperation maintain the same policies as in the previous case. The reason for this difference stems from the changed nature of the short-run reaction functions. Under CCV, the slope of the short-run reaction function is given by $x_1$ in (42). In the initial period, we find $d_8^* = 1.254$, $d_{11}^* = 4.475$, and $d_{12}^* = 2.329$, so that the slope value is now -0.191 and is negative. In Stackelberg leadership, we find $d_6^* = 2.487$ and $d_{11}^* = 3.859$ in the initial period, so that the slope value is -0.644 and is negative also. Recall that both were positive in the previous case. As the short-run reaction function is negatively sloped, as was explained previously, the optimal closed-loop policy calls for an initial larger contraction in the deficit economy, accompanied by a corresponding less proportionate expansion in the surplus economy. As a result, the time path for $q$, and hence all other relevant variables in the two economies, follow time paths that differ from the earlier case. The results that obtain under Nash and Cooperation are similar to those determined earlier.

V.2.2 Steady-State Rate of Convergence

For this parameter set, the roots given in Table 5 imply that the optimal paths strongly converge. A detailed comparison shows that the absolute values of each real root and each real part of the conjugate
roots are in general larger than those in Table 2. This implies that the time horizons of adjustment are longer than those in the previous case. The ranking of the values of \( \theta_i \), given in Table 6, under the closed-loop Nash, CCV, Stackelberg, and Cooperative behavior differs from the earlier case. The ranking

\[
\theta_i^N > \theta_i^{CCV} > \theta_i^S > \theta_i^C, \quad i = 1, 2, 3.
\]

implies that Nash now is slower than CCV, followed by Stackelberg, which is slower than the cooperative equilibrium.

V.2.3 Steady-State Welfare Costs

Figure 10 illustrates the welfare costs for this parameter set. As noted in the earlier case, the steady-state rate of convergence, accompanied by the nature of the reaction functions, determines the pattern of welfare costs. By varying the values of specific parameters, the ranking of the welfare costs associated with the closed-loop equilibrium strategies is altered. On the one hand, we found the speed of convergence of real exchange rate is now faster under CCV than under Nash. On the other, the changed nature of the short-run function also alters the resulting winner and loser. The new ranking for this parameter set shows that the Stackelberg follower is now the best position and the position of Stackelberg leadership is
least preferred. Between these extremes, the cooperative equilibrium still dominates CCV, which in turn outperforms Nash.

Figure 10
The Time Path for Welfare Costs
**Table 4**  
**Steady-State Coefficients of Optimal Policy Rule**  
$T = 72$

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>$\alpha$</th>
<th>$\alpha^*$</th>
<th>$\beta$</th>
<th>$\beta^*$</th>
<th>$\gamma$</th>
<th>$\gamma^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>0.214</td>
<td>-0.214</td>
<td>0.033</td>
<td>-0.033</td>
<td>-0.157</td>
<td>0.157</td>
</tr>
<tr>
<td>CCV</td>
<td>0.223</td>
<td>-0.223</td>
<td>0.051</td>
<td>-0.051</td>
<td>-0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>0.358</td>
<td>-0.221</td>
<td>0.374</td>
<td>-0.048</td>
<td>-0.262</td>
<td>0.162</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.294</td>
<td>-0.294</td>
<td>0.214</td>
<td>-0.214</td>
<td>-0.215</td>
<td>0.215</td>
</tr>
</tbody>
</table>

**Table 5**  
**Steady-State Characteristic Root**  
$T = 72$

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>$\psi_1$</th>
<th>$\psi_2$</th>
<th>$\psi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>-0.222</td>
<td>0.833 + 0.083i</td>
<td>0.833 - 0.083i</td>
</tr>
<tr>
<td>CCV</td>
<td>-0.229</td>
<td>0.808 + 0.115i</td>
<td>0.808 - 0.115i</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>-0.118</td>
<td>0.507 + 0.830i</td>
<td>0.507 - 0.830i</td>
</tr>
<tr>
<td>Cooperation</td>
<td>-0.273</td>
<td>0.580 + 0.240i</td>
<td>0.580 - 0.240i</td>
</tr>
</tbody>
</table>

**Table 6**  
**Steady-State Rate of Convergence**  
$T = 48$

<table>
<thead>
<tr>
<th>Closed-Loop Equilibrium</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\theta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nash</td>
<td>1.444</td>
<td>0.330</td>
<td>0.156</td>
</tr>
<tr>
<td>CCV</td>
<td>1.388</td>
<td>0.297</td>
<td>0.152</td>
</tr>
<tr>
<td>Stackelberg</td>
<td>0.896</td>
<td>0.083</td>
<td>0.112</td>
</tr>
<tr>
<td>Cooperation</td>
<td>0.887</td>
<td>0.078</td>
<td>0.107</td>
</tr>
</tbody>
</table>
CHAPTER VI

OVERVIEW

The numerical analyses show that, in the presence of a strong trade balance J-Curve effect (a lower degree of economic interdependence), the optimal closed-loop strategies for changes in the exchange rate call for an initial leaning-with-the-wind policy and then a reversion to a leaning-against-the-wind policy during the dynamic adjustment periods. This implies that within multiperiod horizons the optimal policy for each economy requires that monetary policy be assigned on the external balance in early periods and on the internal balance for the remaining periods. The reason for this policy behavior stems from the larger weight assigned to output fluctuations than to inflation fluctuations. The intertemporal tradeoff arises from the nature of the time path for the real exchange rate, which is the only disturbance to the world economy. Since the exchange rate equilibrates the flow demand and supply for foreign exchange arising from trade in goods and capital, the faster rate of convergence of the time path of the exchange rate suggests a lower welfare cost relative to a flexible exchange rate. Therefore, the optimal intervention policy in the initial periods calls for a lean-
with-the-wind policy. This leads to the mitigation of an external imbalance. In future periods, the optimal intervention policy reverts to a lean-against-the-wind policy so that the internal imbalance can be narrowed over time. Therefore, the static version of Meade's conflicts of internal and external balance do not constitute as formidable a task that faces the policy makers of each economy.

As the trade balance J-Curve effect moderates, i.e., a higher degree of economic interdependence, the optimal closed-loop policies are altered. The numerical analysis shows that the optimal closed-loop policy requires an initial lean-against-the-wind policy for an economy under CCV and for the leader under Stackelberg, while the optimal strategy for Nash and Cooperation remains the same as before. As a result of the implementation of closed-loop policies at each stage, the policy maker under CCV or Stackelberg (leader) now assigns his monetary policy on the internal balance and imposes most of the burden of adjustment for external balance on the other economy.

For the closed-loop equilibrium solutions, negative real roots cause the real exchange rate overshoot its long-run value and produce a time path of cyclical convergence under Nash and CCV and of monotonic convergence under Stackelberg and Cooperation. Because the real exchange rate governs the behavior of the world economy, all other relevant variables follow accordingly.

Finally, for the two parameter sets, the ranking of the welfare gains varies according to the underlying economic structure. In our numerical examples, the size of the trade balance J-Curve effect
influences the ranking of welfare gains derived from various closed-loop equilibrium strategies. In particular, we obtain a ranking identical to the one in Basar, Turnovsky, and d'Orey (1986) and Turnvosky, Basar, and d'Orey (1988) for a moderate trade balance J-Curve effect. However, we also find a different ranking in the case of a strong trade balance J-Curve effect. The reason for this different ranking stems from the changed nature of the short-run reaction function for CCV and Stackelberg. The extent of economic interdependence alters the policy makers' response in one economy to the action of the other. Therefore, the intervention policy or the beggar-thy-neighbor policy that results should not be invariant to the economic structure as the current literature on dynamic strategic monetary policies suggests.
APPENDICES

A. Closed-Loop Nash Equilibrium Solution

The verification of the result of Proposition IV.1 follows Theorem 9 (p.271) of Basar and Olsder (1982). The solution is derived in two steps. The finite-horizon problem is solved for the last period, \( T \), and then it is solved for period \( t \) given the solution for period \( t+1 \). Finally, the proof is completed by the induction argument.

In period \( T+1 \), it is assumed that the real exchange rate has stabilized and the cost-to-go function \( V_{-1} \) and \( V_{-1}^{*} \) are equal to zero. Given \( q_T \), the policy makers of each economy choose the real money supplies to minimize the welfare costs (28) and (29) for \( t = T \). A necessary and sufficient condition for \( m_T \) and \( m_T^{*} \) to constitute solutions to these quadratic minimization problems is joint satisfaction of the following first-order conditions

\[
D_{11} m_T + D_{12} m_T^{*} + D_4 q_T + D_5 q_{T-1} + D_9 (m_{T-1} - m_{T-1}^{*}) = 0, \quad (A1)
\]

\[
D_{12}^{*} m_T + D_{11}^{*} m_T^{*} + D_4^{*} q_T + D_5^{*} q_{T-1} + D_9^{*} (m_{T-1} - m_{T-1}^{*}) = 0. \quad (A2)
\]

This set of first-order conditions admits a unique solution

\[
m_T = \alpha_0 (m_{T-1} - m_{T-1}^{*}) + \beta_0 q_T + \gamma_0 q_{T-1}, \quad (A3)
\]

\[
m_T^{*} = \alpha_0^{*} (m_{T-1} - m_{T-1}^{*}) + \beta_0^{*} q_T + \gamma_0^{*} q_{T-1}, \quad (A4)
\]

where

\[
\alpha_0 = \frac{d_{9,0}^{d_{11,0}} - d_{9,0}^{d_{8,0}}}{\Delta_0} ; \quad \alpha_0^{*} = \frac{d_{9,0}^{d_{11,0}} - d_{9,0}^{d_{8,0}}}{\Delta_0}
\]

\[
\beta_0 = \frac{d_{4,0}^{d_{11,0}} - d_{4,0}^{d_{8,0}}}{\Delta_0} ; \quad \beta_0^{*} = \frac{d_{4,0}^{d_{11,0}} - d_{4,0}^{d_{8,0}}}{\Delta_0}
\]
\[
\gamma_0 = \frac{d_5,0^d_{11,0} - d_5^* 0^d_{8,0}}{\Delta_0}; \quad \gamma_0^* = \frac{d_5^*,0^d_{11,0} - d_5^* 0^d_{8,0}}{\Delta_0}
\]  

(A5)

where \(d_{i,0} = D_i\) correspondingly and \(\Delta_0 = d_8,0^d_{8,0} - d_{11,0}^* d_{11,0}\).

The cost-to-go functions, \(V_0, V_0^*\), respectively denoted by \(V_0\) and \(V_0^*\), can be derived for period \(T\);

\[
V_0 = \varepsilon_{1,-1}^2 q_T^2 + 2\varepsilon_{2,-1} q_T q_{T-1}^* + \varepsilon_{3,-1} q_{T-1}^2 + 2\varepsilon_{4,-1} q_T (m_{T-1}^* - m_{T-1})^2 + 2\varepsilon_{5,-1} q_T (m_{T-1}^* - m_{T-1})^2 + \varepsilon_{6,-1} (m_{T-1}^* - m_{T-1})^2,
\]

(A6)

\[
V_0^* = \varepsilon_{1,-1}^* q_T^2 + 2\varepsilon_{2,-1}^* q_T q_{T-1}^* + \varepsilon_{3,-1}^* q_{T-1}^2 + 2\varepsilon_{4,-1}^* q_T (m_{T-1}^* - m_{T-1})^2 + 2\varepsilon_{5,-1}^* q_T (m_{T-1}^* - m_{T-1})^2 + \varepsilon_{6,-1}^* (m_{T-1}^* - m_{T-1})^2
\]

(A7)

where \(\varepsilon_{1,0}^*\)s are given by (35) for \(t = T\).

We now prove by induction that the unique solution set is given by (32) and (33). Assume that this is true for \(t = s + 1\) and we prove its validity for \(t = s\). The welfare cost for the remainder of the planning periods, i.e., the cost-to-go function, is expressed as

\[
V_{\tau}(m^*_t, m^*_t) = \min \left\{\pi_t(m^*_t, m^*_t, m^*_{t-1}, m^*_{t-1}, q_t, q_{t-1}) \right. \\
+ \rho V_{\tau-1}(m^*_{t+1}, m^*_{t+1}, m^*_t, m^*_{t-1}, q_{t+1}, q_t), \right\},
\]

(A8)

\[
V_{\tau}(m_t, m^*_t) = \min \left\{\pi_t^*(m_t, m^*_t, m^*_{t-1}, m^*_{t-1}, q_t, q_{t-1}) \right. \\
+ \rho V_{\tau-1}^*(m^*_{t+1}, m^*_{t+1}, m^*_t, m^*_{t-1}, q_{t+1}, q_t) \right\}.
\]

(A9)

Use of the unique relation between \(m^*_{t+1}\) and \(m^*_{t+1}\) given by (A3) and (A4) for \(t\) and backward recursive substitution of the state trajectory (27) into (A8) and (A9) yields the cost-to-go expression, \(V_{\tau}\) and \(V_{\tau}^*\), as follow

\[
V_{\tau} = [d_{1,\tau} q_t^2 + 2d_2,\tau q_t q_{t-1} + d_3,\tau q_{t-1}^2 + 2d_4,\tau q_t m_t]
\]
\[ + 2d_5, \tau q_{t-1}m_t + 2d_6, \tau q_t^*m_t + 2d_7, \tau q_{t-1}^*m_t + 2d_8, \tau m_t^*m_t \\
+ 2d_9, \tau m_t(m_{t-1} - m_t^*) + 2d_{10}, \tau^*m_t(m_{t-1} - m_t^*) + d_{11}, \tau m_t \\
+ d_{12}, \tau m_t^2 + 2d_{13}, \tau q_t(m_{t-1} - m_t^*) + 2d_{14}, \tau q_{t-1}(m_{t-1} - m_t^*) \\
+ d_{15}, \tau(m_{t-1} - m_t^*)^2 \]. \tag{A10} \\

\[ v_{\tau}^* = [d_1, \tau q_t^2 + 2d_2, \tau q_tq_{t-1} + d_3, \tau^*q_{t-1} + 2d_4, \tau q_t^*m_t \\
+ 2d_5, \tau q_{t-1}^*m_t + 2d_6, \tau q_t^*m_t + 2d_7, \tau q_{t-1}^*m_t + 2d_8, \tau^*m_t \\
+ 2d_9, \tau^*m_t(m_{t-1} - m_{t-1}^*) + 2d_{10}, \tau^*m_t(m_{t-1} - m_{t-1}^*) + d_{11}, \tau^*m_t \\
+ d_{12}, \tau^*m_t^2 + 2d_{13}, \tau q_t(m_{t-1} - m_{t-1}^*) + 2d_{14}, \tau q_{t-1}(m_{t-1} - m_{t-1}^*) \\
+ d_{15}, \tau(m_{t-1} - m_{t-1}^*)^2] \tag{A11} \\
\]

Minimization of (A10) and (A11) yields

\[ d_{11}, \tau m_t + d_{12}, \tau m_t^* = -d_4, \tau q_t - d_5, \tau q_{t-1} \\
- d_9, \tau(m_{t-1} - m_{t-1}^*), \tag{A12} \]

\[ d_{12}, \tau m_t + d_{11}, \tau m_t^* = -d_4, \tau q_t - d_5, \tau q_{t-1} \\
- d_9, \tau(m_{t-1} - m_{t-1}^*). \tag{A13} \]

The unique solution to (A12) and (A13) is stated by equations (32) and (33). It can be verified that (32) and (33) is the unique solution set to the closed-loop Nash equilibrium by letting \( \tau = 0 \), i.e., \( t = T \). By substituting (32) and (33) into (A10) and (A11), the cost-to-go expression, \( V_{\tau} \) and \( V_{\tau}^* \), can be further verified to be the analogous equations (37). An inductive argument then completes the proof of the main result.

**B. Closed-Loop CCV Equilibrium Solution**

In the proof of Proposition IV.2, I again follow the basic line of reasoning of Appendix A, with the static equilibrium concept of CCV. The static game under consideration now is described at time \( t \) by the pair of cost functionals

\[ V_{\tau}(m_t, m_t^*) = \min_{m_t} \left[ \pi_t(\cdot) + V_{\tau-1}(m_t, m_t^*) \right], \tag{B1} \]
\[ V^*_{\tau}(m_t^*, m_t^*) = \min_{m_t} \left[ \pi^*_t(.) + V^*_{\tau-1}(m_t^*, m_t^*) \right], \quad (B2) \]

which are quadratic in \( m_t \) and \( m_t^* \) as given by the equations (A10) and (A11). For such quadratic functions, the CCV solution is defined in terms of the relationships:

\[ \frac{\partial V_{\tau}}{\partial m_t} + \frac{\partial V_{\tau}}{\partial m_t^*} \cdot \frac{dm_t^*}{dm_t} = 0 \quad ; \quad \frac{\partial V_{\tau}}{\partial m_t^*} + \frac{\partial V_{\tau}}{\partial m_t} \cdot \frac{dm_t}{dm_t^*} = 0 \quad (B3) \]

along with two consistency conditions on the first-order response functions:

\[ \frac{dm_t^*}{dm_t} = x_{\tau}^* \quad ; \quad \frac{dm_t}{dm_t^*} = \frac{dm_t}{dm_t^*} \]

Let us assume for a moment that these first-order response functions are constants, i.e.,

\[ \frac{dm_t^*}{dm_t} = x_{\tau} \quad ; \quad \frac{dm_t}{dm_t^*} = x_{\tau}^* \]

then, (B3) is equivalent to

\[ \begin{align*}
(d_{11, \tau} + d_{8, \tau} x_{\tau}^*) m_t + (d_{8, \tau} + d_{12} x_{\tau}) m_t^* \\
= -(d_{9, \tau} + d_{10} x_{\tau})(m_{t-1} - m_{t-1}^*) - (d_{4, \tau} + d_{6} x_{\tau}) q_t \\
- (d_{5, \tau} + d_{7} x_{\tau}) q_{t-1}
\end{align*} \quad (B4) \]

\[ \begin{align*}
(d_{8, \tau} + d_{12} x_{\tau}^*) m_t + (d_{11} x_{\tau}^*) m_t^* \\
= (d_{9, \tau} + d_{10} x_{\tau})(m_{t-1} - m_{t-1}^*) - (d_{4} x_{\tau} + d_{6} x_{\tau}) q_t \\
- (d_{5} x_{\tau} + d_{7} q_{t-1}) \quad (B5) \end{align*} \]

which justifies the assumption made on \( \frac{dm_t^*}{dm_t} \) and \( \frac{dm_t}{dm_t^*} \). That is they are constants. Now, consistency in these derivatives dictates the following relationships:
\[
\begin{align*}
\chi_\tau &= -\frac{(d_8, \tau + d_{12}, \tau \chi_\tau)}{(d_{11}, \tau + d_8, \tau \chi_\tau)}; \quad \chi_\tau^* = -\frac{(d_8^*, \tau + d_{12}^*, \tau \chi_\tau^*)}{(d_{11}^*, \tau + d_8^*, \tau \chi_\tau^*)}
\end{align*}
\]

which involve two quadratic equations, one for \( \chi_\tau \) and another one for \( \chi_\tau^* \). Since we are dealing with a symmetric two-country model, an iterative argument proves that \( \chi_\tau = \chi_\tau^* \), where \( \chi_\tau \) satisfies

\[
d_8, \tau \chi_\tau^2 + (d_{11}, \tau + d_{12}, \tau) \chi_\tau + d_8, \tau = 0,
\]

which is precisely (42). Solving for \( m_t \) and \( m_t^* \) from (B4) and (B5) obtains

\[
m_t = \alpha_t (m_{t-1} - m_{t-1}^*) + \beta_t q_t + \gamma_t, \quad \text{(B6)}
\]

\[
m_t^* = \alpha_t^* (m_{t-1} - m_{t-1}^*) + \beta_t^* q_t + \gamma_t^* q_{t-1}^*, \quad \text{(B7)}
\]

where the coefficients are given by (40). Finally, substituting this solution into (A10) and (A11) arrives at equations (41). The proof is completed by making use of the standard inductive argument.

C. Closed-Loop Stackelberg Equilibrium Solution

In the proof of Proposition IV.3, I use Theorem 4 of Chapter 7 of Basar and Olsder (1982). The line of argument is similar to that presented previously for the closed-loop Nash solution. The main difference is that a stagewise game of the Stackelberg type is played instead of a Nash game. Hence,

\[
V_\tau = \min_{m_t} \left[ \pi_t(m_t, g_t^*(m_t), ..) + V_{\tau-1}(m_t, g_t^*(m_t), ..) \right], \quad \text{(C1)}
\]

\[
V_\tau^* = \min_{m_t^*} \left[ \pi_t(m_t, g_t^*(m_t), ..) + V_{\tau-1}^*(m_t, g_t^*(m_t), ..) \right], \quad \text{(C2)}
\]

where \( m_t \) is the argument of the minimization in (C1), and
\[ g_t^*(m_t) = \arg \min_{m_t} \left[ n_t^*(m_t, m_{t-1}^*) + \nu_t^*(m_t, m_{t-1}^*) \right] \]

is the optimal reaction of the follower, foreign economy.

For my problem, and under the assertion (A11), \( g_t^* \) can readily be determined to be

\[
m_t^* = -\frac{1}{d_{11, \tau}} \left[ d_{8, \tau} m_t + d_{9, \tau} (m_{t-1} - m_{t-1}^*) + d_{4, \tau} q_t + d_{5, \tau} q_{t-1}^* \right]. \tag{C3}
\]

After substituting (C3) into (C1) and minimizing the resulting quadratic expression over \( m_t \), we obtain

\[
m_t = \alpha^* (m_{t-1} - m_{t-1}^*) + \beta^* q_t + \gamma^* q_{t-1}, \tag{C4}
\]

\[
m_t^* = \alpha^* (m_{t-1} - m_{t-1}^*) + \beta^* q_t + \gamma^* q_{t-1}, \tag{C5}
\]

where the coefficients are given by (45). Then, the left-hand sides of (C1) and (C2) can be evaluated by substituting (C4) and (C5) and arriving at the analogous equations to (46). Thus verifies the assertion and the result of proposition IV.3 follows inductively.

D. Closed-Loop Cooperative Equilibrium Solution

Under cooperation, the minimization problem again becomes the standard optimal control exercise where a single controller minimizes the joint cost functionals, (A10) and (A11), at each stage. Let it be a simple linear sum of the quadratic cost functional of the two economies and assign equal weights to each country's cost function due to the symmetry property. The intertemporal minimization problem under consideration is described at time t by a single cost function

\[
V_\tau + V^*_\tau = \min_{m_t^*, m_t} \left[ d_{1, \tau} q_t^2 + 2d_{2, \tau} q_t q_{t-1} + d_{3, \tau} q_{t-1}^2 \right. \\
+ 2d_{4, \tau} q_t m_t + 2d_{5, \tau} q_{t-1} m_t + 2d_{6, \tau} q_t^* m_t + 2d_{7, \tau} q_{t-1}^* m_t \\
+ 2d_{8, \tau} m_t^* + 2d_{9, \tau} (m_{t-1} - m_{t-1}^*) + 2d_{10, \tau} m_t^* (m_{t-1} - m_{t-1}^*) \\
+ d_{11, \tau} m_t^2 + d_{12, \tau} m_t^* + 2d_{13, \tau} q_t (m_{t-1} - m_{t-1}^*)
\]
\[ + 2d_{14}, \tau q_{t-1}(m_{t-1} - m^*_{t-1}) + d(m_{t-1} - m^*_{t-1})^2 \], \quad (D1) \]

where

\[
\begin{align*}
\bar{d}_{1, \tau} &= d_{1, \tau} + d_{1, \tau}^*; & \bar{d}_{2, \tau} &= d_{2, \tau} + d_{2, \tau}^*; \\
\bar{d}_{3, \tau} &= d_{3, \tau} + d_{3, \tau}^*; & \bar{d}_{4, \tau} &= d_{4, \tau} + d_{6, \tau}^*; \\
\bar{d}_{5, \tau} &= d_{5, \tau} + d_{7, \tau}^*; & \bar{d}_{6, \tau} &= d_{6, \tau} + d_{4, \tau}^*; \\
\bar{d}_{7, \tau} &= d_{7, \tau} + d_{5, \tau}^*; & \bar{d}_{8, \tau} &= d_{8, \tau} + d_{8, \tau}^*; \\
\bar{d}_{9, \tau} &= d_{9, \tau} + d_{10, \tau}^*; & \bar{d}_{10, \tau} &= d_{10, \tau} + d_{9, \tau}^*; \\
\bar{d}_{11, \tau} &= d_{11, \tau} + d_{12, \tau}^*; & \bar{d}_{12, \tau} &= d_{12, \tau} + d_{11, \tau}^*; \\
\bar{d}_{13, \tau} &= d_{13, \tau} + d_{13, \tau}^*; & \bar{d}_{14, \tau} &= d_{14, \tau} + d_{14, \tau}^*; \\
\bar{d}_{15, \tau} &= d_{15, \tau} + d_{15, \tau}^*.
\end{align*}
\]

Minimizing the resulting quadratic expression over \( m_t \) and \( m^*_t \) yields the following first-order conditions,

\[
\begin{align*}
\bar{d}_{11, \tau} m_t + \bar{d}_{8, \tau} m^*_t &= -\bar{d}_{9, \tau}(m_{t-1} - m^*_{t-1}) - \bar{d}_{4, \tau} q_{t-1} \bar{d}_{5, \tau} q_{t-1}, \quad (D2) \\
\bar{d}_{8, \tau} m_t + \bar{d}_{12, \tau} m^*_t &= -\bar{d}_{9, \tau}(m_{t-1} - m^*_{t-1}) - \bar{d}_{6, \tau} q_{t-1} \bar{d}_{7, \tau} q_{t-1}. \quad (D3)
\end{align*}
\]

The solution to the system of equations is:

\[
\begin{align*}
m_t &= \alpha_{\tau}(m_{t-1} - m^*_{t-1}) + \beta_{\tau} q_t + \gamma_{\tau} q_{t-1}, \quad (D4) \\
m^*_t &= \alpha^*_{\tau}(m_{t-1} - m^*_{t-1}) + \beta^*_{\tau} q_t + \gamma^*_{\tau} q_{t-1}. \quad (D5)
\end{align*}
\]

where the coefficients are given by (50), respectively. Finally, substituting this solution into (D1) provides equations similar to (51), thereby completing the proof by making use of the standard inductive argument.
REFERENCES


