Lane Change Maneuver Quantification on a Freeway
Based on Vehicle Reidentification

Dissertation

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Abstract

Traffic congestion and associated delays have become a serious problem over much of the world. To mitigate traffic congestion, it is essential to better understand the factors that cause traffic delays. It has long been recognized that Lane Change Maneuvers (LCMs) are a critical factor in traffic flow theory, and LCMs could be a contributing factor to traffic congestion. However, research on LCMs has been limited by the fact that there are no efficient methods to collect the number of LCMs in the field. The collection of LCM data currently requires labor-intensive efforts to extract the information from film or video. Image processing technologies are starting to help in this task, but for obtaining accurate LCM data the labor demands remain high. To meet the need for LCM data, this work develops an approach for LCM quantification. Specifically, this approach estimates the number of vehicles entering a lane (Nen) and the number of vehicles exiting a lane (Nex) separately. This approach is compatible with existing vehicle detectors, and it only requires data collected at two detector stations to estimate the number of LCMs between them.

The proposed approach for LCM quantification employs recent advances in Vehicle Reidentification (VRI), a process to match a vehicle observation at one detector station to an observation of the same vehicle at another station. Building off of previous studies, this work develops a more robust VRI algorithm that is compatible with conventional loop detectors. This VRI algorithm is tested over several highway links. The test results
show that this VRI algorithm is able to reidentify long vehicles even when the traffic conditions change between free flow and congestion.

The VRI results yield the difference of $N_{en}$ and $N_{ex}$ between a pair of consecutive reidentified vehicles. The VRI results can also be used to estimate the lower bounds and upper bounds on $N_{en}$ and $N_{ex}$. Thus, the difference between $N_{en}$ and $N_{ex}$ is determined, and the values of $N_{en}$ and $N_{ex}$ are constrained to lie between their lower bounds and upper bounds. Based on these conditions, an approach to estimate $N_{en}$ and $N_{ex}$ is developed and three variants are proposed. A vehicle trajectory data set is used to evaluate the performance of the proposed approach for LCM quantification, since vehicle trajectory data are one of the few sources that could provide ground truth LCM information. The data set does not include loop detector data that can be used for VRI. Therefore, the VRI results are simulated to be consistent with the empirical performance of the proposed VRI algorithm. The evaluation results show that the proposed approach for LCM quantification looks promising for estimating $N_{en}$ and $N_{ex}$, although further testing on additional data sets is necessary.

The approach for LCM quantification could eventually be used to estimate the number of LCMs from conventional loop detector data, thereby providing new insight into travel patterns between lanes and the resulting impacts.
Dedication

Dedicated to my parents, Tongxiang Wang and Bingmei Xia
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CHAPTER 1 INTRODUCTION

Traffic congestion and associated delays have become a serious problem over much of the world. Traffic congestion will worsen over time if the prevailing trends of increasing travel demand and diminishing construction resources (e.g., funds or land) continue. To maximize the efficiency of the existing transportation infrastructure and reduce these delays, it is essential to improve traffic control and management. This objective could be served by improving traffic flow theory and the fundamental understanding of the factors that cause traffic congestion.

Lane Change Maneuvers (LCMs) are a critical factor in traffic flow theory. It has long been recognized that LCMs can influence the relationships underlying traffic flow theory (see, e.g., Wang and Coifman, 2008). A large number of LCMs will likely disturb the common assumption of stationary traffic conditions. LCMs to or from intervening ramps will preclude vehicle conservation between detector locations (see, e.g., Ahn et al., 2008). The effects of LCMs are often ignored in traffic flow theory. This omission can lead to errors between theory and practice that are often discounted as simply noise. As Mauch and Cassidy (2002) noted, “marked improvements in traffic flow theories will likely come by incorporating lane-changing effects.” It is also suspected that LCMs could be a source of traffic congestion. The empirical research has shown that LCMs reduce capacity at bottlenecks (Coifman et al., 2003b) and introduce extra delay on congested
freeway segments (Coifman et al. 2006, Ahn and Cassidy 2007).

Despite the important impacts of LCMs, research on LCMs has been limited by a shortage of field data. Some studies developed LCM models without a basis on empirical data, e.g., Gazis et al. (1962). Some studies relied on microscopic traffic simulation programs (instead of empirical field data) to conduct the numerical analysis, e.g., Michalopoulos et al. (1984). Other studies collected field data, however, the data were usually collected over a limited period of time (typically for one-hour or less), and the entire data set was used for model calibration with no data left for validation, e.g., Munjal and Hsu (1973) and Chang and Kao (1991).

Collecting LCM data is a demanding task that requires both spatial and temporal coverage. Currently there are no efficient methods to collect data on the times and locations of LCMs. Most field studies of LCMs rely on film or video and require labor-intensive efforts to extract the LCM information. Image processing technologies are starting to help in this task, but the labor demands remain high. Even with these semi-automated image processing techniques, occlusion and oblique viewing angles limit the distance that one camera or video recorder can cover, and a vehicle traveling at free flow speed will likely be out of view within a few seconds.

Recent advances in vehicle data collection enable new ways to collect LCM data. In particular, advances in Vehicle Reidentification (VRI) are promising. VRI is the process of matching a vehicle observation at one detector station to an observation of the same vehicle at another station. Several technologies have been used for VRI, as will be reviewed in Section 2.1. Among the various technologies, VRI based on conventional loop detectors is attractive because it can reidentify vehicles using the existing traffic
surveillance infrastructure. Based on such a VRI technology, Coifman (2003) demonstrated an approach to estimate inflow, the difference between the number of vehicles entering a lane (Nen) and the number of vehicles exiting a lane (Nex).

Motivated by the ability to estimate inflow based on VRI results, the study presented in this dissertation seeks to develop an approach to estimate the specific values of Nen and Nex between a pair of consecutive reidentified vehicles, which is called LCM quantification. In this study, VRI results are used to estimate the lower bounds and upper bounds on Nen and Nex. Therefore, the difference between Nen and Nex is determined by inflow, and the values of Nen and Nex are constrained to lie between their lower bounds and upper bounds. Based on these conditions, an approach to estimate Nen and Nex is developed and three variants are proposed to provide estimates of these values. It is worth noting that, in this work, both VRI and LCM quantification are applied on a per lane basis, i.e., the results in one lane are derived without using any information from the other lanes.

A vehicle trajectory data set is used to evaluate the performance of the proposed approach for LCM quantification. The evaluation results show that the proposed approach for LCM quantification looks promising for estimating Nen and Nex, although further testing on additional data sets is necessary. It is worth noting that the present approach does not consider the LCMs caused by vehicles that both enter and exit, or both exit and enter, the subject lane over the study segment. The inability to account for these types of LCMs could limit the applications of the proposed approach.

The approach for LCM quantification could eventually be used to estimate the number of LCMs from conventional loop detector data, thereby providing new insight
into travel patterns between lanes and the associated impacts. Compared to collecting LCM information from film or video, this approach is less accurate. However, this approach does not require the entire segment to be under direct surveillance, and it can easily provide the estimated number of LCMs with much less effort. This approach could benefit other studies that are currently limited by a shortage of LCM data, and could also be used in traffic control to reduce traffic delays.

The rest of this dissertation is organized as follows. In Chapter 2, previous studies on VRI and LCMs are reviewed. The potential applications of the estimated number of LCMs generated from the proposed LCM quantification approach are also discussed to motivate the research in this dissertation by showing how it could benefit other studies or practices.

In Chapter 3, a new VRI algorithm based on the matching of long vehicles is presented. This algorithm combines the strengths of two previous VRI algorithms and is more robust to the change of traffic conditions.

In Chapter 4, the inflow constraint, and the lower bounds and the upper bounds on Nen and Nex are developed based on the VRI results. The Assumed Percentage Method is proposed to estimate the number of LCMs that meet the inflow constraint and fall between the lower and upper bounds. Two variants of the Assumed Percentage Method are then proposed to improve the LCM quantification results: the Short Gap Method and the Long Vehicle Method. These two methods seek to apply the LCM information from samples believed to produce more accurate LCM estimates to samples believed to produce less accurate LCM estimates.
In Chapter 5, the performance of the VRI algorithm proposed in Chapter 3 and the LCM quantification methods proposed in Chapter 4 are evaluated. The proposed VRI algorithm is evaluated using several loop detector data sets. The evaluation results show that the proposed VRI algorithm can reidentify long vehicles even when the traffic conditions change between free flow and congestion. A vehicle trajectory data set is used to evaluate the performance of the proposed approach for LCM quantification. Since this data set does not include loop detector data that can be used for VRI, the VRI results are simulated to be consistent with the empirical performance of the proposed VRI algorithm from Chapter 3. The evaluation results show that the proposed approach for LCM quantification looks promising for estimating Nen and Nex. Further testing on additional data sets is necessary because the present study is only based on a 15-minute data set.

In Chapter 6, the research is summarized and its contributions are discussed. Several directions for future work are also suggested.
CHAPTER 2 LITERATURE REVIEW AND MOTIVATION

This chapter reviews several topics central to the research in this dissertation, namely, Vehicle Reidentification (VRI), Lane Change Maneuver (LCM) modeling, and applications that would benefit from LCM quantification. As introduced in Chapter 1, VRI is the basis for the proposed approach for LCM quantification. The review of the previous studies on LCMs demonstrates the importance of developing an effective approach to collect LCM information from empirical field observations. To meet the need for LCM data, this dissertation develops an approach for LCM quantification to estimate the number of LCMs. The potential applications of the estimated number of LCMs are then discussed to motivate the research in this dissertation by showing how it could benefit other studies or practices.

This chapter is organized as follows. First, Section 2.1 reviews the literatures in VRI. Next, Section 2.2 reviews the previous studies on LCMs. Finally, Section 2.3 discusses the potential applications of the estimated number of LCMs.

2.1 Vehicle Reidentification

As noted previously, vehicle reidentification (VRI) is the process of matching a vehicle observation at one detector station to an observation of the same vehicle at another station. VRI can provide traffic conditions on a link between two detector stations (e.g., loop detectors or video cameras), such as link travel time. Because VRI
yields the times that a given vehicle passes two or more detector stations, the link travel time is simply the difference between these observation times. This link travel time includes the impacts of any change in traffic state between the stations.

In conventional traffic surveillance strategies, link travel time is estimated based on the local speed measurements collected at detector stations along the freeway under the assumption that these point measurements are representative of the link spanned by the detector stations. This assumption is not valid in many cases, for example, an incident may occur between two stations and cause a delay that is not observed at either station. Thus, the point detector measurements are not representative of the link conditions. The discrepancy will persist until the queue eventually propagates to the upstream station, thereby either leading to a long latency or requiring a high density of detector stations. Once the queue reaches the upstream station, a new problem arises: how to interpolate between congested measurement at the upstream station and free flowing measurement at the downstream station. The link travel time depends on where the bottleneck is between the two stations, a feature that cannot be extracted from the local speed measurements.

Therefore, compared to the estimated link travel time in conventional traffic surveillance strategies, the link travel time from VRI provides better information on the traffic conditions between the stations. Such improved link travel time can be used to improve traffic delay detection and provide better traveler information. Several systems and technologies have been used for VRI and they will be discussed in this section.

2.1.1 Vehicle Reidentification Based on Emerging Technologies

Several emerging technologies have been proposed to reidentify vehicles, such as video (MacCarley 1998, MacCarley 2001, Huang and Russell 1997), laser (Cheng et al.
2001, Cheng et al. 2008), magnetic (Kwong et al. 2009), radar (Kuhne, 1991) and ultrasonic (Izumi et al. 2000) based detectors. They usually use specialized hardware to extract vehicle signatures, for example, the locations of the optical features, color, vehicle length, width and height. The reported reidentification performance varies across the different methods. For example, MacCarley (2001) showed that the video based detector system could correctly reidentify about 94% of the vehicles observed over 2 hours on a freeway segments about 0.4 miles long. Kwong et al. (2009) reidentified about 70% of the vehicles over half an hour on a 0.9-mile-long arterial with 6 intersections.

Some researchers have used inductive loop detectors to extract inductive signature data for vehicle reidentification and classification. Inductive signatures are obtained using specialized sensor hardware to capture the time series change of inductance in the loop caused by passing vehicles. These curves contain a wealth of information including maximum magnitude of inductance, vehicle length, and the distribution of the ferromagnetic mass over the vehicle. Different types and classes of vehicles provide different characteristic detuning curves. These detuning curves were first used for vehicle classification (Reijmers, 1979). Pfannerstill (1984 and 1989) used pattern recognition principles and correlation methods in VRI and proposed the idea to match platoons of vehicles rather than individual vehicles. Lexicographic optimization was introduced by Sun et al. (1999) for VRI based on vehicle detuning curves. Multi-objectives were optimized to find the optimum detuning curves pair. Since they were not identifying vehicle sequences, their algorithm is more suitable for the case when sequences of vehicles were not well preserved from the upstream station to the downstream station. The paper presented the vehicle reidentification results based on data collected from a
1.2-mile-long freeway segment with no ramps between the detector stations. The flow was moderate at 1000 vehicles per hour per lane. There were only 128 vehicles in the data set. The presented vehicle reidentification rate ranged from 61% for trucks, 75% for cars and station wagons to 100% for SUV and semi-trailers (probably due to the small number of vehicles in these two classes). Abdulhai and Tabib (2003) also used the vehicle signature detuning curves but improved the accuracy using new pattern-nearness distance measures.

In recent years, Automatic Vehicle Identification (AVI) and Automatic License Plate Recognition (ALPR) have been deployed in VRI. AVI systems are most commonly deployed for toll collection and access control. They consist of a machine-readable tag with a unique ID, mounted on the participating vehicles, most commonly employing Radio Frequency Identification (RFID) though other technologies have also been used. When the vehicles equipped with tags pass an AVI reader installed along the roadway, the ID information is transmitted to the reader and recorded with a time stamp. The matching process then finds time stamp entries from two tag reader stations for the same vehicle based on the ID information. ALPR seeks a similar identification process, but uses machine vision to read the conventional license plate, thereby eliminating the need for a dedicated tag.

Once deployed, the unique tag ID or license plate number makes VRI via AVI and ALPR trivial. The VRI results can be used to calculate the link travel time. Much research has been conducted to use AVI or ALPR system to obtain accurate travel time information under different conditions, such as Dion and Rakha (2006), Sherali et al. (2006) and Bertini et al. (2005). In practice, some operating agencies started to use the
AVI system for link travel time measurement to support traffic management and information provision. The TranStar system (TranStar, 2009) in Houston is the first system in the United States to apply AVI technology for monitoring traffic conditions (Levine and McCasland, 1994). Examples of other existing AVI systems in the US include the TransGuide system in San Antonio (TransGuide, 2009), and the Transmit system (Transmit, 2009) in the New York/New Jersey metropolitan area.

AVI and ALPR systems are also becoming more and more important in the estimation of origin-destination demand, which is an essential input for transportation planning (e.g., Zhou and Mahmassani, 2006; Dixon and Rilett, 2002; and Van der Zijpp, 1997). Since ALPR uses the license plates, it is also used for traffic law enforcement, unattended parking lots, security control of restricted areas, and congestion pricing (e.g., Change et al. 2004).

Although the unique vehicle ID from AVI and ALPR greatly simplify VRI, there are many drawbacks. From a traffic-monitoring standpoint AVI and ALPR provide link travel time information, but they do not provide local conditions (e.g., local speeds at the detector stations), which are also important inputs for traffic surveillance and control. If the systems are not already deployed for toll collection, then their use would require new investment. There is always a risk that AVI and ALPR could compromise personal privacy. Finally, AVI requires public participation to carry the tags; all vehicles without an AVI tag will go unobserved. Thus, depending on market penetration, AVI may only provide information on a small portion of vehicles in the traffic stream.

All of the systems reviewed in this section require new hardware in the field to measure the passing vehicles' signatures. In most cases, these VRI systems have only
been installed on a few locations and evaluated over a small time period. Therefore, it would be premature to assume the reported performance is typical of the given system.

2.1.2 Vehicle Reidentification Based on Conventional Loop Detectors

Despite recent advances of new detection systems, conventional loop detectors remain the most widely used vehicle detection technology. Conventional loop detectors only report the presence or absence of vehicles, from which, it is possible to measure or estimate vehicle speed and length (see, e.g., Section 3.2.1). The research group at the Ohio State University has developed various techniques to use the resulting vehicle lengths as the only vehicle feature to reidentify vehicles. This signature is much simpler than those used by the other VRI techniques and is readily available from the existing loop detectors\(^1\) as well as many emerging detectors that emulate loop detectors. But a given vehicle length is hardly unique and often the lengths are subject to considerable noise, so the challenge is overcoming these difficulties.

Coifman (2003) presented an algorithm to reidentify vehicles over the range from free flow through the onset of congestion with the goal of identifying the onset of congestion rapidly from existing loop detectors. Only long vehicles (vehicle length longer than 23 ft) were considered since they are more distinct in terms of vehicle length and typically have fewer possible matches than the shorter vehicles do. Although this paper used dual loop detectors to demonstrate the algorithm, Coifman and Banerjee (2002) showed that the algorithm can also be deployed with single loop detectors. Once congestion sets in, this

\(^1\) It is worth noting that while vehicle length can be measured or estimated for every vehicle passing a loop detector station, for archaic reasons, most operating agencies typically discard the individual vehicle data immediately after calculating aggregate measures.
algorithm ceases reidentifying vehicles until free flow periods return. By excluding the short vehicles, this algorithm only sought matches for approximately 10% of the vehicle fleet.

Separately, another algorithm was developed to reidentify vehicles during congested periods. The basic idea is to match vehicle sequences instead of individual vehicles since a sequence of vehicle lengths rapidly becomes distinct even in the presence of length measurement errors. The algorithm uses vehicles of all lengths, not just the long vehicles. The impact of LCMs in some simple situations is addressed in the algorithm to form a longer vehicle sequence. The basic algorithm was thoroughly documented in Coifman (1998) and Coifman and Cassidy (2002), and later was improved in Coifman and Ergueta (2003) by introducing four independent tests to eliminate false possible matches. The example based on field data showed that the algorithm in Coifman and Ergueta (2003) correctly matched 86% of the vehicles between two detector stations about 1800 feet apart with no ramps between them. Of the remaining vehicles, only 3% were incorrectly matched while the remaining 11% were simply labeled as "unmatchable". For another detector station pair that was also 1800 feet apart but with an off ramp in between the stations, the reidentification rate was between 35% and 65% depending on the lane. The remaining vehicles were labeled as "unmatchable". This second example used approximately 13,500 vehicles and the researchers did not generate ground truth matches, instead, they evaluated the performance in terms of travel time.

Coifman and Krishnamurthy (2007) combined much of the functionality of the two previous approaches into a single algorithm. This algorithm indexes vehicles by the resulting travel time instead of arrival number, so it allows for much greater reordering
among vehicles. It can match distinct vehicles (long vehicles) even when the vehicle changes lane. The algorithm was demonstrated separately with both single and dual loop detectors. It has an operating range from free flow until the link speed drops below 20mph. More importantly, the algorithm is more robust to the influence of LCMs and can be applied to challenging conditions, e.g., across a major merge or major diverge. The paper showed that about 40% of long vehicles or 5% of all vehicles observed at the downstream station were reidentified over links about 1 mile long using dual loop detector data.

Common to all of these algorithms is the idea of reidentifying distinct vehicles (or sequences) while ignoring more common vehicles that could easily be mismatched. Each paper uses a slightly different technique for finding the distinct vehicles, but in each case, the selection is based on features locally observable at a loop detector station. Therefore, rather than having a high error rate, the algorithms do not report a match for 30%-97% of the vehicles. Although this reidentification rate is not as great as those described in Section 2.1.1, e.g. MacCarley (2001) and Kwong et al. (2009), the reidentification accuracy is high and the algorithms are sufficient for travel time measurement. The general approach is attractive because it is compatible with the existing loop detector infrastructure.

2.1.3 Discussion

As described in Chapter 1, VRI is a key element in the proposed LCM quantification. Although various technologies promise higher reidentification rates than enumerated in Section 2.1.2 from conventional loop detectors, the present LCM quantification work seeks to be applicable to the existing loop detector infrastructure. Furthermore, the fact remains that an increase in the accuracy of VRI would greatly decrease the complexity of
LCM quantification. To this end, this dissertation builds on the ideas of VRI from conventional loop detectors and proposes a new VRI algorithm in an attempt to improve the accuracy and reidentification rate, which is presented in Chapter 3.

2.2 Studies on Lane Change Maneuvers

Most of the previous studies on LCMs focused on the development of mathematical models to model or simulate LCM behavior, or study the relationship between LCM and other traffic flow characteristics. The literature review of these studies is important because it shows what studies could utilize the estimated number of LCMs generated from this study. These models are classified as continuum models, discrete models, empirically-derived aggregate models and microscopic models. The following sections review each of the four classes of models in turn.

2.2.1 Continuum Models

In continuum models traffic is assumed to behave as a compressible fluid that obeys the continuity equation (see, e.g. Lighthill and Whitham 1955, Richards 1956). Continuum models of LCM usually focus on the oscillation or perturbation of traffic density as a result of LCMs. These models are commonly further classified as linear models and nonlinear models.

One of the pioneering models dealing with macroscopic LCMs was proposed by Gazis et al. (1962). This paper investigated the interchange of traffic density between lanes caused by LCMs and focused on how the change in density would oscillate. The proposed model yielded criteria of stability of such density oscillations in two or more lanes although no empirical results were given to support the argument. The model
assumes that there exists some equilibrium density (independent of speed) in each lane, i.e., the assumed density acceptable to the drivers, which might differ from the current density. The rate of exchange between two neighboring lanes (i.e., LCM) is then assumed to be proportional to how much each of those lanes differs from their equilibrium density. Along with all subsequent models based on similar assumptions, this model is called a linear model. Linear models study the exchange of flow between neighboring lanes so they can only yield the difference of the number of entering vehicles (Nen) and the number of exiting vehicles (Nex), i.e., the net inflow, between two neighboring lanes without differentiating between these two numbers.

Munjal and Pipes (1971) proposed another linear model to study the propagation in space and time of the perturbed densities initiated by an on-ramp flow based on the principles of continuum models. As an improvement, they considered the density oscillation as functions of time and distance, the latter of which was neglected in Gazis’s model. The derivative equations were listed and solutions for two- and three-lane freeways were given. Like Gazis et al. (1962), Munjal and Pipes (1971) did not include any empirical validation. Then Munjal et al. (1971) applied the model from Munjal and Pipes (1971) to examine the disturbances in density caused by a lane-drop on a multi-lane freeway. The result indicated that the effect of dropping the lane would produce waves of perturbation density in all lanes, which traveled upstream with a constant speed and the perturbation exponentially decayed with distance. The theoretical analysis was validated using aerial photographic data, from which flow, density and speed on a set of points along the freeway were obtained.
The aforementioned models were extended by Michalopoulos et al. (1984) to take into account generation or loss of vehicles introduced at on-ramps or off-ramps. Michalopoulos et al. (1984) also proposed a new model to include the road width as an additional dimension and another more complex model to take into account the acceleration of vehicles in an attempt to fit the un-congested traffic conditions better. Despite the solid theoretical work, the numerical results based on a microscopic simulation program did not show a significant improvement over the preceding models. The authors did not include any validation using empirical data.

Nonlinear models, on the other hand, assume a non-linear dependency. They were first proposed by Oliver and Lam (1965). Their model assumed the number of LCMs from lane $i$ to lane $i+1$ was proportional to the square of density in lane $i$ times the difference of jam density and density in lane $i+1$. It defined the number of LCMs from lane $i$ to lane $i+1$ and from lane $i+1$ to lane $i$ as two different variables, so this model can yield the number of entering vehicles (Nen) separately from the number of exiting vehicles (Nex). Due to a lack of field data, Oliver and Lam (1965) did not include any empirical validation.

Chang and Gazis (1975) compared the linear model by Gazis et al. (1962) and the nonlinear model by Oliver and Lam (1965). They used aerial photographic data collected from a 0.75 mile 3-lane freeway segment when the traffic was free flowing. The results showed that the nonlinear model performed slightly better than the linear model.

2.2.2 Discrete Models

Several researchers have approached LCMs as a discrete process. Worral et al. (1970) divided the freeway sections into subsections and the number of LCMs ($X_{ij}$) from lane
$i$ to its adjacent lane $j$ in subsection $K$ during consecutive intervals $t$ was modeled as a Poisson process. They made the following assumptions:

1. LCMs were independent of one another, with equal probability of occurrence for all vehicles.
2. $X_{ij}^{Kt} \geq 0$ if $|i - j| = 1$; $X_{ij}^{Kt} = 0$ if $|i - j| \neq 1$.
3. $X_{ij}^{Kt}$ followed the Poisson distribution with parameter $\lambda_{ij}^K$, the average number of LCMs between lanes $i$ and $j$ within subsection $K$ during unit time, which may depend on the flow or density.
4. The probability of a vehicle changing lanes in subsection $K$ was only a function of its lane in subsection $K - 1$ and of the lane into which the change would be made.

Based on these assumptions, LCMs were described as a finite Markov process that defined a probability transition matrix $T$ within subsection $K$, which was further assumed to be independent of traffic condition and subsection $K$. The initial state vectors for an $m$ lane freeway were defined as $a(o) = [a(o)_1, \ldots, a(o)_i, \ldots, a(o)_m]$ where $a(o)_i$ was the probability of a vehicle occupying lane $i$ as it entered the first section of the studied segment on a highway. A series of hypothetical examples were provided in the paper to show how to apply the Markov process to obtain the frequency and the pattern of LCMs under varying geometric and traffic conditions. The proposed model assumed that the probability transition matrix was measurable, and was constant across subsections and intervals. However, it is difficult to get the probability transition matrix from the field data since it requires knowing the numbers of LCMs from one lane to another for all lane pairs.
This discrete Markov model was extended to a continuous-time semi-Markov model by Abella et al. (1976). In addition to the transition matrix, Abella et al. (1976) required a holding time matrix to describe how long a vehicle would stay in its original lane before changing lanes. Similar to the discrete Markov model, this model is limited in application because of the difficulty obtaining the two matrices.

Sheu (1999) presented a nonlinear discrete model to estimate real-time lane-changing fractions (the proportion of vehicles in a given lane involved in LCMs in a given time step) and lane densities on a segment of freeway between the upstream and downstream detector stations. Different from most of the other LCM-related models, this model used lane traffic counts and occupancies collected from the upstream and downstream detectors, instead of using data collected from the freeway segment. Sheu (1999) introduced an extended Kalman Filter to the discrete model of LCMs, which also differentiated this study from previous work. The model took traffic counts and occupancies as inputs and specified six types of discrete-time state variables. The model consisted of recursive equations, measurement equations and boundary constraints. The outputs were the estimated lane-changing fractions and lane densities of the studied section at each discrete time point.

The model developed by Sheu (1999) was tested on 1 hour of field data collected on a four-lane mainline segment in California, which included traffic count and occupancy from the point detectors, as well as density and the number of LCMs\(^2\), sampled every 10 seconds. The density was about 70 vehicles per mile but the traffic condition was not provided in the paper. Half of the data were used for model calibration and the other half

\(^2\) Sheu (1999) did not indicate clearly how density and the number of LCMs were obtained.
were used for evaluation. Despite the solid and complicated theories used to develop the model, the paper did not show convincing results for the estimation of lane-changing fraction. Only one lane-changing fraction was estimated in each lane, without differentiating the entering vehicles and the exiting vehicles. The estimated lane-changing fraction did not seem to follow the trend of observed values. This study claimed that 97.5% of the estimations errors were located within the range of -0.05 and 0.05, but most of the actual observed lane-changing fractions varied between 0 and 0.05. Sheu (1999) regarded the presented results as preliminary since more data were necessary to adequately evaluate the proposed method.

Several studies compared the discrete models with the continuum models based on field data. Munjal and Hsu (1973) studied and compared the linear model by Gazis et al. (1962), the nonlinear model by Oliver and Lam (1965) and the discrete model by Worral et al. (1970). The objective was to evaluate the validity of these models with aerial photographic data. The data were collected from a 3-lane segment on the Long Island Expressway in New York that was grade- and curvature-free with no nearby on- and off-ramps, when the space-mean speed was almost 60 mph. Almost an hour of vehicle trajectory data were obtained from aerial photographic data after careful data cleaning, such as correction of optical distortion. The density and the number of LCMs for each lane and for each 3-min interval were calculated for model calibration. The unknown parameters of the linear and nonlinear models, as well as the probability transition matrix of the discrete model, were estimated. The number of LCMs was then calculated by using the models with estimated parameters and compared to the observed values. After the statistical comparisons, they concluded that most of the results were good and the three
models were ranked in decreasing order of validity: the nonlinear model, the discrete model and the linear model. However, the unknown parameters in this study were estimated based on the collected data and then applied back to the same set of data to estimate the number of LCMs. Although the primary objective was to evaluate the validity of these models, the fact that they used the same data set for calibration and validation precludes extending the results to other situations. In fact the authors speculated that the unknown parameters are density-dependent and thus, the estimated parameters were not readily transferable to other traffic conditions.

2.2.3 Empirically-derived Aggregate Models

Empirically-derived aggregate models of LCMs are not directly based on an underlying continuum or discrete theory. Instead, they directly study the relationship between the dependent variables that describe LCMs and the independent variables that are usually traffic flow characteristics, such as the difference of density or speed between adjacent lanes. The dependent and independent variables are all aggregate measurements of LCMs or traffic flow characteristics, therefore such models are aggregate models. The earliest empirically-derived aggregate model was developed by Worrall and Bullen (1970). They studied LCMs on multilane highways based on photographic data collected at 30 different freeway segments in Chicago under various traffic conditions (average speed varied from 10 mph to 50 mph). The paper did not indicate the duration of the data collection or the method of data reduction. Lane-changing frequency was used as the dependent variable in the regression analysis, which was defined to be the number of vehicles leaving the original lane (the lane a lane-changing vehicle leaves) along a given length of road and over a given time period. The independent variables were the flow and
speed difference between adjacent lanes. The paper focused on 3-lane highway segments and presented linear regression models for 4 different LCMs: from center lane to right lane, right to center, center to left and left to center.

Chang and Kao (1991) also studied the regression model of LCMs. The data were collected from two 4-lane freeway segments in Texas, which were away from any off-ramps or on-ramps. Each test site was observed using time-lapse video recorders for one hour when the traffic was not congested. The pooled data set from these two 4-lane freeway segments was used in the development of the models. The variables were represented with their average values over 5-min intervals. Two variables to describe LCMs were defined: lane-changing frequency and fraction. The lane-changing frequency was defined in the same way as in Worrall and Bullen (1970). The lane-changing fraction was the ratio of the number of vehicles leaving the original lane and the total number of observed vehicles in the original lane. An exploratory analysis was conducted first to investigate the interrelations between these two LCM-related variables and key traffic flow variables, for example, average lane density, flow, speed, average headway, variance of headway, flow ratio between adjacent lanes, density ratio between adjacent lanes, and the speed ratio between adjacent lanes. The result indicated that the average headway and the variance of headway were the best explanatory variables; however, they were highly correlated and only one could be included in the model. Similarly, the average lane flow and density were also good candidates for lane-changing models but were highly correlated.

Chang and Kao (1991) applied the results of the exploratory analyses to construct lane-changing frequency and fraction models in the form of a Poisson regression model
and a binary logistic regression model, respectively. The parameters for both models were estimated based on the standard maximum likelihood method, which yielded expected signs for all parameters and achieved reasonable levels of fit. The prediction results based on the lane-changing frequency model showed that approximately 65% of the total observations had less than 20% prediction errors (the differences between the predicted and observed values divided by the observed values) and more than 90% of the observations fell within the 40% prediction errors. However, like Munjal and Hsu (1973), Chang and Kao (1991) used the same data set for calibration and validation.

In spite of the validation shortcomings, Chang and Kao (1991) provided a good template for empirical analysis of LCMs. They considered different explanatory variables and different forms of regression models. The authors realized that the reported results were based only on limited observations from two segments of uncongested freeways, which may not be sufficient to represent the lane-changing characteristics under other conditions, such as LCMs on heavily congested freeways or near ramps. In addition, most of the explanatory variables are difficult to collect in practice, e.g., the traffic flow variables were collected using video recorders covering the entire segment, not from the discrete detector stations. Furthermore, LCM information was used in the definition of some explanatory variables, such as the "Average Flow Ratio", which used the number of LCMs between the subject lane and neighboring lanes. A model that includes input variables that are functions of LCMs cannot be used to estimate the unobserved LCMs.

2.2.4 Microscopic (Disaggregate) Models

Continuum models, discrete models and empirically-derived aggregate models seek to study the aggregated LCM behavior over some region of time and space. On the other
hand, microscopic models try to capture the individual drivers' LCM behavior, therefore are disaggregate models. These models are a key component of microscopic traffic simulation. Microscopic traffic simulation is becoming more and more important for both researchers and traffic engineers, since it is an effective tool in replicating traffic conditions and evaluating the transportation network performance under various options. Currently there exist many commercial microscopic traffic simulation programs, such as VISSIM, CORSIM, PARAMICS and SIMTRAFFIC.

Gipps (1986) introduced one of the earliest LCM models intended for micro-simulation tools. The paper focused on the hierarchy of the decisions that a driver has to make before changing lanes on arterials. The driver’s behavior fell into three patterns depending on the distance to this intended turn. While the turn was remote, the driver concentrated on maintaining his desired speed. When the turn was in middle distance, the driver started to ignore the opportunities to improve his speed that involve changing lanes in the wrong direction. Once he reached the lane most appropriate for his turn, the driver tended to remain in the lane. The transition between these behavior patterns was blurred and the model was not sensitive to the precise location of the boundaries. A LCM would be executed if particular combinations of criteria were met. The criteria included the existence of traffic signals, transit lanes, obstructions, and the presence of heavy vehicles since the model was intended to cover the urban driving situation. Gipps (1986) validated the model simply by plotting the simulated vehicle trajectories and checking if they were consistent with expectations.

Gipps’ model has been adopted by several microscopic simulators such as MITSIM and SITRAS. In MITSIM (Yang and Koutsopoulos 1996, Ahmed et al. 1996), a LCM
was implemented in three steps: (a) checking if a change is necessary and defining the type of change: mandatory or discretionary; (b) selecting the desired lane; and (c) executing the LCM if the gap is acceptable. LCMs were classified as either mandatory or discretionary. Mandatory Lane Changes (MLC) were performed to connect to the next link, bypass a downstream lane blockage, avoid entering a restricted lane, and respond to lane-use signs and variable message signs. Discretionary Lane Changes (DLC) referred to cases in which drivers change lane in order to increase speed, bypass a heavy vehicle or avoid the lane connected to a ramp. A driver selected the desired lane based on several factors such as prevailing traffic conditions, lane changing regulations and lane connections. In order to execute a LCM, the driver accessed the positions and speeds of vehicles in the target lane (i.e., the lane that the lane-changing vehicle enters) and decided whether the gap was sufficient for a LCM, which was described by a gap acceptance model.

Yang and Koutsopoulos (1996) applied MITSIM using a 4-hour data set obtained from 10 detector stations on a 5 mile freeway segment. The observed speed varied from 20 mph to 70 mph, with most of the observations falling between 50 mph and 70 mph. The simulated flow and average speed were aggregated over 10-min intervals and compared to the observed values. Comparison results showed that the root mean square error (the square root of the average squared difference between simulated and observed values) was 156 vph for flow and 5.8 mph for speed.

In SITRAS (Hidas, 2002), Gipp’s model was extended to include “co-operative” LCM, in which vehicles in the target lane respond to the lane change request from a lane-changing vehicle by slowing down to create sufficient space for the LCM. No field data
were collected to validate the model but the hypothetical examples suggested that incident situations could be simulated more realistically by considering “co-operative” LCMs.

Most microscopic (disaggregate) LCM models classified the maneuvers as either MLC or DLC. As a result, trade-offs between these considerations were prohibited. Toledo et al. (2003b) presented a model to overcome this limitation by integrating mandatory and discretionary considerations into a single utility model. The relative importance of these considerations varied depending on explanatory variables such as the distance to the off-ramp. Parameters in the lane utility functions and gap acceptance models were estimated using detailed vehicle trajectory data, which were collected from a 0.6-mile-long freeway segment with two off-ramps and one on-ramp. 442 vehicles were observed in this data set and the observed speed varied from 1 mph to 56 mph, with a mean of 35 mph. The estimation results suggested that the trade-offs between mandatory and discretionary considerations were important. Toledo et al. (2005) further proposed a generalized LCM model to explicitly incorporate the choice of target lane, which does not necessarily have to be an adjacent lane. This model was validated based on vehicle trajectory data and yielded better prediction results than the model in Toledo et al. (2003b) both in terms of travel speed and distribution of vehicles among lanes. Toledo et al. (2007) extended the work and developed a model to integrate the various decisions, such as acceleration, lane changing and gap acceptance. Two case studies were presented and they used vehicle trajectory data collected from two different locations. The results showed that the integrated model performed better than the independent models in both case studies.
2.2.5 Discussion

Most of the LCM-related models require careful calibration before being used in practice. The calibration work could be demanding, since it needs a large amount of LCM field data. For example, discrete models usually require the estimation of the probability transition matrix. The constant transition matrix assumption seems inappropriate since LCM patterns likely depend on the geometry, traffic conditions and population of drivers. However, if the transition matrix has to be estimated in such great detail, it would need a large amount of LCM field data. On the other hand, currently there are no efficient methods to collect LCM data. In almost all of the previous studies that utilized LCM field data, the LCM information was extracted from aerial photographs or video, which is still a labor intensive task even with the help of image-processing technologies.

The difficulties in collecting LCM data have limited the research on LCMs. Some studies developed mathematical models without any empirical support, such as Gazis et al. (1962). Some studies relied on microscopic traffic simulation programs to conduct the numerical analysis, such as Michalopoulos et al. (1984). Some studies collected field data but used all of the data for model calibration with no extra data left for validation, such as Munjal and Hsu (1973) and Chang and Kao (1991). Some studies used small samples of LCM field data that can only represent limited traffic conditions or locations. For example, both Munjal and Hsu (1973) and Chang and Kao (1991) studied the uncongested freeway segments that were far from any ramps, and they both noted that the LCM behavior might be quite different for congested traffic and in the vicinity of ramps.

The importance of developing techniques to provide LCM information has been recognized and some work has been done to address it. For example, Sheu (1999)
introduced a discrete model to extract real-time information of lane-changing fractions based on conventional point detector data. This paper is the only paper found in this literature review that conducted LCM quantification based on collectible data. Unfortunately, in Sheu (1999) only one lane-changing fraction was estimated in each lane without differentiating the entering vehicles and the exiting vehicles, and the estimation results did not seem to follow the trend of the observed values.

The review of the previous studies on LCMs demonstrates the importance of developing an effective approach to collect LCM information from empirical field observations. If such an approach is developed, it could be used to support the studies and practices that are currently limited due to a shortage of LCM field data. Section 2.3 discusses four applications that could benefit from the approach to collect LCM field data.

**2.3 Potential Applications that Would Benefit from Lane Change Maneuver Quantification**

As introduced in Chapter 1, the LCM quantification approach presented in this dissertation could eventually be used to estimate the number of LCMs from conventional loop detector data. If the estimated number of LCMs is sufficiently accurate, the LCM quantification approach could be used to improve existing traffic applications and enable new applications. Section 2.2 reviewed a variety of studies that rely on LCM field data. All these studies could likely benefit from the LCM quantification approach. This section shows how the LCM quantification approach could be used, and four potential applications are discussed. Since the fidelity of any application depends on the measurement resolution, the resolution of the estimated number of LCMs is discussed in Section 2.3.1, before any of the specific applications. Four applications are then discussed,
namely LCM modeling, microscopic traffic simulation, traffic flow theory, and traffic management, in Section 2.3.2 to Section 2.3.5, respectively.

2.3.1 Resolution of the Estimated Number of Lane Change Maneuvers

As introduced in Chapter 1, the proposed LCM quantification approach seeks to estimate the number of LCMs between a pair of consecutive reidentified vehicles (bounds in time and space) and between the upstream and downstream stations used for VRI (bounds in space). Therefore, the estimated number of LCMs generated from the proposed LCM quantification approach is an aggregate number subject to temporal resolution and spatial resolution of the sample.

The temporal resolution of the estimated number of LCMs is determined by the headway between two consecutive reidentified vehicles, which depends on the adopted VRI algorithm. Many VRI algorithms reviewed in Section 2.1 can be used to yield the estimated number of LCMs, since they are compatible with the proposed LCM quantification approach. The literature review in Section 2.1 found the reidentification rates of existing VRI algorithms vary from about 5% (e.g., Coifman and Krishnamurthy, 2007) to more than 90% (e.g., MacCarley, 2001). The large range of reidentification rate comes from the fact that these VRI algorithms are based on different technologies (e.g., using conventional loop detectors or video based detector system) and different strategies (e.g., matching only long vehicles or matching all vehicles). The average headway between successive reidentified vehicles is simply the reciprocal of the product of the reidentification rate and flow. For example, with a flow of 1000 vph (vehicles per hour), the average headway between reidentified vehicles should range from 4 seconds (90% reidentification rate) to 72 seconds (5% reidentification rate).
In the case of the present work, Chapter 3 develops a new VRI algorithm that will be used for LCM quantification. As will be shown in Section 5.1, the new VRI algorithm matched approximately 4% of the vehicles observed at the downstream station in the empirical studies, and on average a vehicle was reidentified every 60 seconds for the study data set. Of course, the individual headway between two consecutive reidentified vehicles may be higher or lower than 60 seconds, but the temporal resolution of the estimated number of LCMs based on the proposed VRI algorithm is on the order of 60 seconds.

The spatial resolution of the estimated number of LCMs is determined by the distance between detector stations, which is typically on the order of 0.5 miles to 2 miles in practice. The proposed LCM quantification approach considers each lane independently. Therefore, it can estimate the number of LCMs in each lane, instead of simply the aggregate number of LCMs across all lanes. Although the proposed LCM quantification approach can estimate the number of vehicles entering a lane and the number of vehicles exiting a lane separately, it does not provide information about which lane the entering vehicles come from or which lane the exiting vehicles go to. If this issue is not resolved, it could limit the use of the estimated number of LCMs in some applications, e.g., as discussed in Section 2.3.2.

2.3.2 Lane Change Maneuver Models

LCM models enable traffic engineers to effectively evaluate operation strategies and highway designs. Revisiting various models in Section 2.2, the proposed LCM quantification approach could be used to provide the number of LCMs in the development of empirical LCM models. It could also be used in the parameter estimation
of theoretical LCM models.

The empirically-derived aggregate LCM models in Section 2.2.3 directly model the empirical relationship between the dependent variables that describe LCMs and the independent variables (usually traffic flow characteristics). Therefore, LCM information is crucial to the development of such models. However, due to the limited amount of development data available, the previous studies are constrained to limited traffic conditions or locations. For example, Chang and Kao (1991) studied uncongested freeway segments that were far from any ramps. They noted that LCM behavior might be quite different for congested traffic and traffic in the vicinity of ramps. The proposed LCM quantification approach is able to yield the estimated number of LCMs under different traffic conditions and from different locations. These estimates can then be used to calculate the lane-changing frequency and the lane-changing fraction (defined in Section 2.2.3), which were the dependent variables in Chang and Kao (1991). The temporal resolution of the estimated number of LCMs should be sufficient for such a study, since the variables used in Chang and Kao (1991) were aggregated over 5-min intervals. The proposed LCM quantification approach thus could extend Chang and Kao’s analysis to a much larger number of facilities.

The proposed LCM quantification approach could also be used in the parameter estimation of some continuum models discussed in Section 2.2.1. The parameter estimation of the linear models (e.g., Gazis et al. 1962) requires density and the difference of the entering and exiting vehicles (i.e., inflow) over the study segment in each time interval. Density can be estimated based on loop detector data (e.g., Coifman, 2003b), and inflow can be provided by the proposed LCM quantification approach.
Similarly, the parameter estimation of the nonlinear models (e.g. Oliver and Lam 1965) requires the density and the number of LCMs from one lane to the other over the study segment in each time interval. As noted in Section 2.3.1, the proposed LCM quantification approach does not provide information about which lane the entering vehicles come from or which lane the exiting vehicles go to. Therefore, the estimated number of LCMs can only be applied to a two-lane roadway if it is used in the parameter estimation of a nonlinear model. The temporal and spatial resolution of the estimated number of LCMs should be sufficient for these parameter estimations. For example, Munjal and Hsu (1973) estimated the parameters of a linear model and a nonlinear model based on data aggregated over 3-min intervals over a study segment that was about 1.25 miles long.

It is challenging to use the proposed LCM quantification approach in the parameter estimation of the discrete models discussed in Section 2.2.2. For example, some discrete models require the estimation of the probability transition matrix, e.g., Worral et al. (1970). Different probability transition matrices should be estimated for different traffic conditions and locations. Determining such a vast set of matrices would need a large amount of LCM field data in each subsection during each time interval. Such data could likely be provided by the proposed LCM quantification approach. However, the spatial resolution of the estimated number of LCMs is a limiting factor, since the subsections are usually short. For example, Munjal and Hsu (1973) used subsections that were 200 feet long.

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3 Note that future research on integrating LCM information across lanes (as discussed in Section 6.2.2) may overcome this limitation.
2.3.3 Microscopic Traffic Simulation

Microscopic traffic simulation is becoming an increasingly important tool for both research and practice, since a well-calibrated microscopic model can replicate traffic conditions and be used to evaluate the transportation network performance under various options. The estimated number of LCMs from the proposed LCM quantification approach could be used in the calibration and validation of the microscopic traffic simulation models.

Although disaggregate data (e.g., vehicle trajectory data) are ideal for the calibration of microscopic traffic simulation, they are costly and difficult to collect. In most practical applications only aggregate traffic measurements are available, and model calibration is sometimes performed using only aggregate data. The basic idea when using aggregate data in microscopic model calibration is to find the parameters that minimize the deviation between observed and simulated measurements. The parameters are typically estimated using an iterative approach that re-estimates each parameter subset until convergence (e.g., Toledo et al., 2003a). The parameters can also be estimated simultaneously (e.g., Balakrishna et al., 2007). In either case, the aggregate data are usually flows, speeds and headways collected by loop detectors. However, none of these traffic measurements are directly related to LCMs, so the LCM component of the given microscopic traffic simulation model might not be well calibrated at the end of the process. The proposed LCM quantification approach can estimate the number of LCMs from conventional loop detectors. It might prove to be beneficial to use the estimated number of LCMs among the aggregate data used for microscopic model calibration. In such a case, the number of LCMs from the simulation could be aggregated to any time
interval between any two locations on the simulated facility to be consistent with the
temporal and spatial resolution of the estimated number of LCMs from the proposed
LCM quantification approach.

Similarly, the proposed LCM quantification approach could be used in the validation
of microscopic traffic simulation models by checking the number of LCMs from the
simulation against the estimated number of LCMs. Some microscopic models were only
validated by comparing the simulated and the observed flow and average speed, with no
direct validation on the LCM component of the model, e.g., Yang and Koutsopoulos
(1996). Some microscopic models did directly validate the LCM component by
comparing the simulated and observed distribution of vehicles among lanes, e.g., Toledo
et al. (2003b). However, when a microscopic model is applied to other locations, it might
need further validation to check if the model performs well at the new locations. In either
case, the proposed LCM quantification approach promises to be an easier way to provide
the number of LCMs for the validation of microscopic LCM models.

2.3.4 Traffic Flow Theory and Traffic Congestion

To maximize the efficiency of the existing transportation infrastructure and reduce
the delays, it is essential to improve traffic control and management. This objective could
be served by improving traffic flow theory and the fundamental understanding of the
factors that cause traffic congestion.

It has long been suspected that LCMs are a contributing source of traffic congestion
(e.g., Cassidy and Mauch 2001, Coifman et al. 2003b, Wang and Coifman 2008), but
nontrivial work needs to be done to quantify or model such an impact. The estimated
number of LCMs from the proposed LCM quantification approach could be used in the
research of the impact of LCMs on traffic congestion. For example, Coifman et al. (2006) defined the concept of delay caused by LCMs, and studied its relationship with the number of LCMs. Only a half-hour of trajectory data were used in Coifman et al. (2006) due to the limitation of the available data sets. The temporal and spatial resolution of the estimated number of LCMs from the proposed LCM quantification approach should be sufficient for such studies, since the number of LCMs used in Coifman et al. (2006) is aggregated over about 2-min intervals on a 0.3-mile-long freeway segment.

The proposed LCM quantification approach could also be used to study the spatial impact of LCMs at weaving sections. At present weaving sections are taken to start and end at discrete locations. Presumably most of the weaving related LCM in the outside lane occurs close to the ramps. The impacts of these in (or out) flows will extend to inside lanes as one moves further downstream (or upstream), e.g., an entering vehicle may take a few miles to move all the way over to the inside lane. Therefore, a discrete start and end of a weaving section is likely a poor fit for the LCMs caused by the weave. Furthermore, the region of influence likely depends on the traffic conditions. The proposed LCM quantification approach could be used to estimate the number of LCMs between each pair of adjacent detector stations near a weaving section to capture the spatial impact of the weave. Obviously, the spatial resolution of the estimated number of LCMs could limit the accuracy of such studies. If the loop detector stations are too far apart, it may simply be infeasible to determine where the LCMs occur with sufficient precision to be beneficial.

2.3.5 Traffic Control and Management

The number of LCMs estimated from the proposed LCM quantification approach could be used in the theoretical studies of LCM modeling, microscopic traffic simulation
and traffic flow theory, as discussed in Section 2.3.2 to Section 2.3.4. These theoretical studies could in turn lead to better traffic control and management strategies in practice.

The proposed LCM quantification approach could be used in ramp metering, where a ramp meter controls the rate of vehicles entering the freeway, i.e., at the metering rate. Typically ramp metering is employed to increase the mainline throughput, improve travel time reliability, reduce traffic delay, and decrease the accident rate. The metering rate is intended to maximize freeway throughput. It can be adjusted based on mainline traffic condition measurement, which typically is occupancy (the percentage of time in which a loop detector is occupied by a vehicle). But if studies in Section 2.3.4 can model the impact of LCMs on the drop of highway capacity, it may prove to be effective to set the metering rate in response to the number of LCMs in real time. The temporal resolution of the estimated number of LCMs should be sufficient for this application, since the metering rate usually dose not change rapidly.

The proposed LCM quantification approach may also prove to be beneficial for incident detection by looking for periods where the LCM pattern differs significantly from the typical pattern, e.g., a large increase in the number of LCMs across all lanes may be indicative of a traffic accident. The proposed LCM quantification approach estimates the number of LCMs in each lane, which can be summed to yield the total number of LCMs across all lanes. Such detection strategy could be used with other incident detection strategies for a more accurate detection than either method offers alone.
CHAPTER 3 A NEW VEHICLE REIDENTIFICATION ALGORITHM

Vehicle Reidentification (VRI) plays a critical role in the Lane Change Maneuver (LCM) quantification algorithm that will be proposed in Chapter 4. Many VRI algorithms based on emerging detector technologies have been proposed (as enumerated in Section 2.1.2). However, VRI based on loop detectors is attractive because it can be implemented with the existing detector infrastructure. Such a VRI approach would then require a smaller investment and promise a quicker deployment. The research group at The Ohio State University has developed various VRI algorithms, e.g., Coifman and Cassidy (2002) and Coifman and Krishnamurthy (2007). This chapter presents the latest advances of the VRI research at the Ohio State University.

The proposed VRI algorithm combines the strengths of two previous algorithms. On the one hand, borrowing ideas from Coifman and Krishnamurthy (2007), the new algorithm exploits the fact that consecutive long vehicles usually exhibit similar time to traverse the link. On the other hand, borrowing ideas from Coifman and Cassidy (2002), the new algorithm checks the vehicle arrival numbers at the detector stations. The contribution of this new algorithm is to develop a more effective method to search for true matches from the possible matches (as described in Section 3.2.2 and Section 3.2.3). Compared to the earlier algorithms, the new algorithm is more robust to the change of traffic conditions. This algorithm also uses new constraints to remove the false positives (as described in Section 3.2.4). The numerical evaluation of this new VRI algorithm is
presented in Chapter 5 based on loop detector data collected from I-80 in California. The empirical performance of this VRI algorithm is then used in the evaluation of the LCM quantification methods in Chapter 5.

This chapter is organized as follows. First, the previous algorithms are briefly reviewed in Section 3.1. Next, the new algorithm is described in detail in Section 3.2.

3.1 Introduction and Background

The research group at The Ohio State University has developed various algorithms to reidentify vehicles based on loop detectors. The only vehicle feature used for VRI in these algorithms is measured (or estimated) vehicle length. All of the vehicle length measurements include a degree of uncertainty, so each length is recorded as a feasible range rather than a discrete value. Typically, these lengths are very noisy. To surmount the noise, additional information about the traffic stream is employed, namely arrival order and arrival time at the detector stations. This section reviews the two existing algorithms that serve as the foundation for the new VRI algorithm presented in Section 3.2.

3.1.1 Vehicle Reidentification Based on Vehicle Arrival Number

Coifman and Cassidy (2002) presented an algorithm to reidentify vehicles between two loop detector stations based on vehicle arrival number. The basic idea is to index vehicles within a given lane by arrival number at each station. After a vehicle passes the downstream station, a set of feasible matches at the upstream station is established based on the following two rules. First, to ensure positive travel time a feasible match must arrive at the upstream station before the arrival of the studied downstream vehicle at the
downstream station. Second, the total number of feasible matches shall not exceed the jam density of the link between the upstream and downstream stations. If the downstream vehicle stayed in the lane and was measured correctly, its true match will be in this upstream set of feasible matches. Of course the vehicle could have changed lanes or was measured incorrectly, resulting in no true match for that downstream vehicle. To find the matches, the downstream vehicle feasible length range is compared to the feasible length range for each of the upstream vehicles. Whenever an upstream vehicle length range intersects with the downstream vehicle length range, a "possible match" is recorded and indexed by arrival number at both stations; otherwise, "no match" is recorded. In either case, a typical downstream vehicle will have many possible matches. The set of possible matches for each downstream vehicle is stored in a Vehicle Match Matrix (VMM) with the row indicating the downstream vehicle arrival number and the column indicating the difference between the upstream and downstream vehicle arrival number, i.e., the “upstream offset”.

Intuitively, the possible but incorrect matches, i.e., "false positives", are randomly distributed over the VMM. If vehicles maintain their order between stations, the true matches should manifest themselves as sequences of possible matches in the same column. In other words, false positives will typically form shorter sequences while true matches will usually form longer sequences. Although the algorithm does not attempt to match vehicles that have changed lanes, these LCMs impact the relative order of the vehicles that remain in the lane. A step is designed to correct for a few simple LCMs that would cause the sequence of true matches to shift columns in the VMM. Three maneuvers are considered, a vehicle exiting a lane, a vehicle entering a lane and a vehicle
entering a lane while another exiting the same lane. For each new sequence of possible matches, the algorithm checks to see if it can be linked to an earlier sequence via one of the maneuvers. If it can be linked, the algorithm joins the two sequences in a modified sequence, which contains at most two distinct sequences by definition. For each row, the algorithm keeps the possible match in the longest modified sequence as the best match and discards all other possible matches for that row.

Once all active sequences have ended in a given row, the algorithm selects the upstream vehicle corresponding to longest sequence as the match. At this point in the algorithm, each downstream vehicle has at most one match. Some of these remaining matches are false positives and a cleanup step is employed to remove many of the false positives. The basic ideas underling this cleanup step are as follows. First, an upstream vehicle should only have one matched downstream vehicle. Second, vehicles should travel at a reasonable speed. Third, consecutive true matches should fall in a small range of upstream offset to avoid large discontinuities between consecutive travel time measurements. The matches that meet these additional criteria are then regarded as the final matches.

This algorithm has two key limitations. First, it tries to reidentify all vehicles. Due to the limited sampling rate of conventional loop detectors (typically 60 Hz), length measurement resolution drops significantly at free flow speeds and most passenger vehicles become indistinguishable from one another. Therefore, the algorithm is limited to congested conditions (speeds under 45 mph). Second, since this algorithm indexes vehicles by arrival number and searches for vehicle sequences in the VMM, it can only tolerate a moderate amount of reordering due to LCMs or detector errors.
3.1.2 Vehicle Reidentification Based on Travel Time

To overcome the first limitation of the algorithm reviewed in Section 3.1.1, subsequent VRI algorithms have explicitly limited the set of vehicles considered. These algorithms seek distinct vehicles that have fewer possible matches (i.e., long vehicles). Long vehicles range roughly from 20 feet to 80 feet, which is much larger than the 5 or 10 feet range spanned by the passenger vehicles. Therefore, it is still possible to differentiate long vehicles from one another even if the loop detector resolution leads to a large feasible length range for a given long vehicle at free flow speeds. The second limitation of the algorithm has been addressed by indexing upstream vehicles by travel time rather than the upstream offset.

Coifman and Krishnamurthy (2007) described the most recent algorithm (preceding this work) in that evolution. They ignored all downstream vehicles with measured length below a threshold length. The long vehicles that remain are renumbered in their arrival order, as if the discarded short vehicles had not passed. Similar to Coifman and Cassidy (2002), if an upstream vehicle’s feasible length range intersects the downstream vehicle’s feasible length range, the upstream vehicle is regarded as a possible match. Possible matches are stored in Travel Time Matrix (TTM). As before, downstream vehicle number indexes the rows. However, the columns are now indexed by the resulting travel time for the possible matches, rounded to the nearest integer second.

To find the final matches, Coifman and Krishnamurthy (2007) exploited the fact that consecutive long vehicles usually exhibit similar time to traverse the link. As a result, over several rows the true matches manifest themselves among all possible matches by a higher density than the false positives. False positives have a lower density in TTM since
they have random traversal times. This algorithm seeks to identify the possible match in the current row closest to the dense region over several preceding rows (i.e., earlier downstream vehicles) in the TTM.

The major advantages of this algorithm are twofold. First, the use of long vehicles allows VRI under free flow conditions. Second, because this algorithm matches vehicles based on resulting travel time without considering the upstream vehicle arrival numbers, it can accommodate much greater reordering among vehicles. As a result, the Coifman and Krishnamurthy (2007) algorithm can be used to match from one lane downstream to many lanes upstream. This algorithm was tested under challenging conditions, including a segment with a major merge and a major diverge between the two detector stations. However, by not considering the upstream vehicle arrival numbers, this algorithm misses the opportunity to remove some of the false positives in the final matches.

### 3.2 Description of the New Vehicle Reidentification Algorithm

Section 3.1 reviewed two preceding VRI algorithms. This section presents a new VRI algorithm intended to improve upon these preceding algorithms. Like Coifman and Krishnamurthy (2007), the algorithm only uses long vehicles and constructs a TTM. However, instead of finding the dense areas in the TTM, the algorithm looks for long sequences of possible matches from the TTM in a manner that is similar to that used in Coifman and Cassidy (2002). Unlike the Coifman and Cassidy (2002) algorithm that assumed the upstream offset typically stays constant for several vehicles, the proposed algorithm assumes that consecutive travel times are usually similar among the true (but unknown) matches. The algorithm builds sequences in the TTM and eventually selects the longest sequence, subject to a few steps to remove the false positives.
3.2.1 Data from Loop Detectors

Vehicle length can be measured from a dual loop detector, in which two consecutive detectors are laid within tens of feet of each other. When a vehicle passes over a dual loop detector, it records four transitions, i.e., the turn-on and turn-off times at each loop, as shown in Figure 3.1. Two measurements of the vehicle length can be obtained by:

\[ L_1 = \frac{S}{TT_r} \times OT_1 \]  
\[ L_2 = \frac{S}{TT_f} \times OT_2 \]

where,

- \( L_1 \) is length measurement #1,
- \( S \) is the separation of two loops,
- \( OT_1 \) is the on-time (the time duration the loop detector detects the presence of a vehicle) at the first loop,
- \( TT_r \) is the time difference of turn-on times (rising edges) at two loops,
- \( L_2 \) is length measurement #2,
- \( OT_2 \) is the on-time at the second loop, and
- \( TT_f \) is the time difference of turn-off times (falling edges) at two loops.

\[ \text{Occasionally a vehicle will change lanes over the dual loop detector or one of the loop detectors malfunctions, resulting in only one of the loop detectors responding to the vehicle. Such unmatched pulses can be identified and they are excluded from subsequent steps.} \]
Figure 3.1 One vehicle passing over a dual loop detector, (a) the two detection zones and the resulting measurement from the detectors, (b) the associated turn-on and turn-off transitions at each detector (adapted from Coifman and Cassidy, 2002)
The vehicle length from Equation 3.1 or Equation 3.2 is called the effective vehicle length. The effective vehicle length is equivalent to the physical vehicle length plus the length of the loop detector. The pairing in Equations 3.1 and 3.2 is deliberate. Note that in Figure 3.1 $OT_1$ and $TT_1$ are measured roughly concurrently. Similarly, $OT_2$ and $TT_2$ are measured roughly concurrently.

Since the controller samples the loop detectors at 60Hz, the measurements in Equations 3.1 and 3.2 are accurate to $\pm 1/60$ seconds at best. To capture this resolution constraint, the measurement uncertainty is defined as the range spanned by $L_1$ and $L_2$ after including $\pm 1/60$ seconds in $TT_r$, $TT_f$, $OT_1$ and $OT_2$. In this way, each effective vehicle length is recorded as a range rather than a discrete value. The method to define the effective length range described in Krishnamurthy (2004) is used in this study.

Like Coifman and Krishnamurthy (2007), the present algorithm only searches for possible matches for the long vehicles at the downstream station. A vehicle is defined as a long vehicle if the midpoint of its effective length range is greater than the long vehicle threshold. The threshold is set at the 90th percentile of the midpoints of all the effective length ranges seen at the station. This present study ignores all vehicles passing the downstream station with the midpoints of the effective length ranges below the long vehicle threshold, and renumbers the remaining downstream long vehicles by their arrival.

Alternatively, vehicle length can be estimated at a single loop detector station, in which there is only one inductive loop detector per lane. In this case, vehicle length is just the product of on-time for the studied vehicle and estimated vehicle velocity. Key to this single loop detector application is making accurate speed estimates. Traditional single loop detector speed estimation techniques are too noisy to estimate vehicle length reliably, but Coifman et al. (2003a) and Coifman and Kim (2009) have demonstrated non-traditional methods to estimate speed and length from conventional single loop detectors with sufficient accuracy to allow VRI. Since a single loop detector can provide only one length estimate, the length range is defined as 0.8 and 1.2 times the estimated length.
order. Only long vehicles passing the upstream station are considered as the candidates for the downstream long vehicles. These candidates are renumbered in the same way as the downstream long vehicles. As noted previously, compared to the shorter passenger vehicles, the relatively infrequent observation of the long vehicles and the large range of effective lengths both greatly reduce the number of false positives. Since only long vehicles are studied, unless otherwise stated, the upstream vehicle and downstream vehicle used in Section 3.2 refer to upstream long vehicle and downstream long vehicle, respectively.

3.2.2 Creation of the Travel Time Matrix

The possible matches are identified in much the same fashion as discussed in Section 3.1.2, with two adjustments. First, in Coifman and Krishnamurthy (2007), only the downstream long vehicles are defined. These downstream long vehicles are defined as the vehicles whose lower bounds of the vehicle length ranges are greater than the long vehicle threshold. The upstream vehicle candidates are not constrained to long vehicles. However in the proposed algorithm, both the downstream and the upstream long vehicles are defined. The upstream vehicle candidates can only be upstream long vehicles because this algorithm cleans the reidentification results by checking the arrival order of long vehicles at both stations, as will be discussed in Section 3.2.4. Second, while Coifman and Krishnamurthy (2007) rounded link travel time to integer seconds, in this algorithm the link travel times are not rounded.

The difference between downstream and upstream passage time for a possible match is used to calculate the corresponding link travel time, which in turn is used as the index to the abscissa of the Travel Time Matrix (TTM). Each downstream vehicle will typically
have several possible matches, and the resulting link travel time for each of the possible matches are stored in the TTM. Figure 3.2 shows an example TTM from a pair of dual loop detector stations separated by approximately one mile. In this figure, each point represents a possible match for a given downstream vehicle (ordinate) with a certain link travel time (abscissa).

Figure 3.2 Example of a Travel Time Matrix

The maximum link travel time shown in Figure 3.2 is 250 seconds, which corresponds to a link speed of approximately 14 mph. Possible matches with link travel times greater than 250 seconds are also considered by the algorithm, but they are not shown in the figure to improve clarity. The real minimum link speed considered depends on the jam density, as used in Coifman and Cassidy (2002). Figure 3.2 shows that it is
possible for a downstream vehicle to have multiple possible matches (multiple points in its row) or to have no possible matches (no point in its row). The latter is caused by LCMs or detector errors.

3.2.3 Method to Find the Longest Sequences

The section seeks to separate the true matches from the false positives in a TTM based on two assumptions. The first assumption is that the false positives are randomly scattered over the TTM. A false positive emerges when the effective length range of a long vehicle observed at the downstream station intersects with the effective length range of another long vehicle observed at the upstream station. Since such an upstream long vehicle could arrive at the upstream station at any time, the resulting link travel time of the false positive could be any value. Therefore, the false positives should be randomly scattered over the TTM. The second assumption is that the link travel times of consecutive long vehicles should be similar. Such an assumption is reasonable if the entire link is free flowing or the entire link is congested, since each vehicle in this link experiences similar traffic conditions. As the traffic condition changes from free flow to congestion on a link, the upstream moving wave propagates with a speed on the order of 10 mph (Daganzo and Lin, 1993), and consecutive long vehicles will have a different travel time only over the portion of the link that was congested for one vehicle, and uncongested for the other. As long as the headway between two long vehicles is less than a few minutes, they will experience a similar travel time over the majority of the link. This argument is also true as the traffic condition changes from congestion to free flow on a link.
Based on these two assumptions, the data points corresponding to the true matches will tend to fall in a narrow vertical band in a TTM, whereas the false positives will be scattered randomly. The band will drift to the right (longer travel time) as the link gets more congested and to the left (shorter travel time) as the link gets less congested. Although the true matches are unknown a priori, this band will manifest itself in the TTM. Too many false positives will obscure the band. Avoiding this phenomenon is one of the reasons why only the long vehicles (vehicles longer than 90% of the vehicles observed at a station) are considered in the present study.

To find the vertical band in the TTM, the algorithm connects a given possible match from a downstream long vehicle to the possible matches that have similar travel times and are from the immediately preceding downstream long vehicle. Sequences of varying lengths emerge. For each downstream vehicle, the data points in the longest sequences are regarded as more possible matches and retained for the cleanup step. This section presents the method to find the longest sequences.

Formalizing the process, the link travel time of the $n$th possible match for vehicle $i$ in the TTM is denoted as $T_{i}^{n}$. Similarly, the $m$th possible match for vehicle $i - 1$ is denoted as $T_{i-1}^{m}$. If $|T_{i}^{n} - T_{i-1}^{m}|$ is less than a predefined threshold (termed the “close threshold”), the link travel times for these two possible matches are considered to be close, and they are joined to form a sequence of two possible matches. The possible match corresponding to $T_{i-1}^{m}$ is called a parent of the possible match corresponding to $T_{i}^{n}$. This process is repeated for all the possible matches in the TTM. Longer sequences then emerge. For example, if $|T_{i+1}^{k} - T_{i}^{n}|$ is less than the close threshold, the $k$th possible match for vehicle $i + 1$ is added to the sequence of $i - 1$ and $i$, which now becomes a
sequence of three. However, if $|T_{i+1}^k - T_i^n|$ is greater than the close threshold but $|T_{i+1}^k - T_i^l|$ is less than the close threshold, the $k$th possible match for vehicle $i+1$ and the $l$th possible match for vehicle $i$ will form a new sequence of two possible matches, separate from the vehicle sequence of $i-1$ and $i$. At this stage the sequences do not have to be mutually exclusive. A given possible match can have more than one parent, which may feed the same earlier parent or diverge to separate sequences.

Figure 3.3 shows the results after this process is applied to the TTM in Figure 3.2 using a close threshold of 10 seconds. The close threshold was selected after limited number of trials on different values. The value of 10 seconds is used because it yields more final matches and less incorrect matches for the development data sets (presented in Section 5.1.1). The constant close threshold makes sense at high speeds, e.g., if two consecutive vehicles have link speed of 60 mph and 55 mph over a 1-mile-long segment, the link travel times are 60 seconds and 65 seconds respectively, a difference of 5 seconds. But as speeds drop, the relative impact of a unit change in link speed on travel time increases. If, instead, the two consecutive vehicles have link speeds of 10 mph and 5 mph over the 1-mile-long segment (the same absolute difference of 5 mph as before), the link travel times are now 360 seconds and 720 seconds, respectively. The 360 seconds difference falls far outside the 10 seconds used for the close threshold. While the true difference in travel time can become quite large at higher travel times, the false positives for a given downstream vehicle should be randomly scattered throughout the entire range of travel times in the TTM. Keeping the close threshold constant throughout the entire range of travel times, as what is used in the present study, will cause the algorithm to reject more true matches when travel times are large. But increasing the close threshold at
higher travel times will result in more false positives being accepted by the algorithm. It is left to future research to optimize the tradeoff between the close threshold and travel time, and possibly develop a means to replace the constant with a function of the segment length or posted speed limit (see Section 6.2.1). At present, the algorithm simply finds shorter sequences of true matches in congestion than it does at free flow speed.

In Figure 3.3, some possible matches have very small link travel times. The small link travel times imply unreasonably high link speeds between the stations. The possible matches with corresponding link speed of 80mph or higher are not considered in this step to avoid their interference with the search for the longest sequences. The results in Figure 3.3 show that most of the linked possible matches fall in the range of 50 seconds to 70 seconds.

![Figure 3.3 Example Travel Time Matrix with all close possible matches linked](image-url)
LCMs or detector errors could break a long vehicle sequence into several short vehicle sequences. To control for such an impact, the algorithm next checks to see if a vehicle sequence ending at vehicle $i$ can be joined to another vehicle sequence starting from vehicle $i + 2$. The link travel time of a possible match for vehicle $i$ that is in a vehicle sequence ending at vehicle $i$ is denoted as $T_{i}^{end}$. Similarly, the link travel time of a possible match for vehicle $i + 2$ that is in a vehicle sequence starting from vehicle $i + 2$ is denoted as $T_{i+2}^{start}$. If $|T_{i+2}^{start} - T_{i}^{end}|$ is less than the close threshold, the two sequences can be jointed to form a longer sequence. By allowing a sequence to skip over vehicle $i + 1$, the algorithm is robust to single LCMs. However, if there is a gap of two or more consecutive downstream vehicles, the vehicle sequence will terminate$^6$. For each downstream vehicle, only the possible matches corresponding to the longest sequences are retained for the next cleanup step and all other possible matches are discarded (they are no longer the parent or child for adjacent downstream vehicles).

To explicitly identify the longest sequences, a Vehicle Sequence Matrix (VSM) is defined and used to store the vehicle sequence information. Each row of a VSM represents a possible match, a data point in TTM. Unlike the TTM, the columns of the VSM consist of the ID number of a possible match, sequence length information and parents information. If a possible match for vehicle $i$ is in a vehicle sequence and is not a parent for any possible matches of vehicle $i + 1$ or $i + 2$, this vehicle sequence cannot be joined to any other vehicle sequences and the column recording the length of the longest

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$^6$ Obviously this idea could be extended to skipping two downstream vehicles, but the longer the gap the greater the probability that it is simply between random false positives. Balancing these issues, the present algorithm only allows for single vehicle gaps to reduce the risk for the false positives to form a long vehicle sequence.
sequence will be updated. Since a possible match may be the parent to more than one branch, smaller values will be changed but any pre-existing larger values will be retained. After all sequences passing through the possible matches for a given downstream vehicle have terminated, the longest sequence is found (or sequences if there are more than one).

Figure 3.4 shows the results after this process is applied to the TTM in Figure 3.3. All vehicle sequences shorter than the longest sequence for each downstream vehicle are removed in Figure 3.4. The possible matches that are in the remaining vehicle sequences are shown with circles. Figure 3.4 shows that after this step, a downstream vehicle could still have several possible matches retained, e.g., downstream vehicle 1450. These possible matches are in different longest sequences that have equal sequence length.

![Figure 3.4 Example Travel Time Matrix after removing the short vehicle sequences and retaining only the longest vehicle sequences](image)

Figure 3.4 Example Travel Time Matrix after removing the short vehicle sequences and retaining only the longest vehicle sequences.
Compared to the Maximum Density Matrix (MDM) in Coifman and Krishnamurthy (2007), which tries to find the densest area in TTM, the Vehicle Sequence Matrix (VSM) is more robust to changes of link travel time. The MDM strategy considers the density of possible matches that yield slightly different travel time for the given possible match. In contrast, the VSM strategy directly links possible matches with similar link travel times, since it is assumed the travel times of true matches are roughly constant. Therefore, in the case of rapidly changing link travel time, such as the on-set of traffic congestion, the VSM will typically do a better job following the changing trend in the true travel time.

3.2.4 Cleanup of the Vehicle Reidentification Results

Several problems can remain after extracting the longest sequences in the previous step. False positives might be included in the longest sequences; occasionally a sequence of false positives might be longer than true sequences (especially if there is a gap of two or more vehicles in the true matches). In addition, a downstream vehicle might be matched to several upstream candidates, or vice versa. This section seeks to detect and correct as many of these problems as possible to enhance the VRI performance.

The basic idea in this cleanup step is to constrain the upstream offset of the remaining possible matches to change in a logical manner. As introduced in Section 3.1.1, the upstream long vehicle offset is defined as the difference between the downstream and upstream long vehicle arrival numbers. This offset could be different for the vehicles in a vehicle sequence due to LCMs. More subtly, some vehicles with lengths near the long vehicle threshold may be counted as long vehicles at one station but not at the other station. Such cases are equivalent to a single LCM for each occurrence. Considering the possible change of the upstream long vehicle offset, two constraints are proposed.
The first constraint, the within-sequence constraint, limits the change of upstream long vehicle offset within a vehicle sequence. For example, if the longest sequence from the method described in Section 3.2.3 had downstream vehicles 17, 18 and 19 matched respectively to upstream vehicles 2, 3 and 5, the upstream long vehicle offsets for these three vehicles are 15, 15 and 14. Such offsets are consistent with one another considering that upstream vehicle 4 might have left the lane between the upstream and downstream stations. However, if instead downstream vehicles 17, 18 and 19 are matched respectively to upstream vehicles 2, 3 and 12, the upstream long vehicle offsets are 15, 15 and 7. The last offset is very different from the offsets of its two predecessors. This offset could only occur if 8 consecutive upstream vehicles leave the lane or are not correctly detected as long vehicles at the downstream station. Such conditions would be rare in reality. In such a case, the algorithm removes the last possible match and searches for another possible match for downstream vehicle 19 in the TTM that meets the close threshold and the within-sequence constraint. If such a possible match can be found, it is added. Otherwise, downstream vehicle 19 does not have any possible matches retained. In the current algorithm, the within-sequence constraint does not allow more than one long entering vehicle or more than one long exiting vehicle between two consecutive long through vehicles in a vehicle sequence. Although the within-sequence constraint could remove true matches, most of the removed matches should be false positives.

The second constraint is called the between-sequence constraint, since it specifies the maximum acceptable range for the change of upstream long vehicle offsets between two sequences. The between-sequence constraint depends on the number of unmatched vehicles between two consecutive sequences. In the current algorithm, the between-
sequence constraint assumes that no long vehicles in one sequence can overtake a long vehicle in another sequence between the upstream and downstream stations. For example, if the last downstream vehicle in a longest sequence is 10 and the first downstream vehicle in the consecutive longest sequence is 15, and they are matched respectively to upstream vehicles 5 and 6, such matches meet the between-sequence constraint since it is possible that the downstream vehicles 11 to 14 are entering vehicles or the upstream station fails to detect them. However, matches with downstream vehicles 10 and 15 matched to upstream vehicles 5 and 4, respectively, fail to meet the between-sequence constraint because upstream vehicle 5 would have to overtake upstream vehicle 4. When two consecutive vehicle sequences do not meet the between-sequence constraint, all the possible matches in the sequence with fewer vehicles will be removed. Like the within-sequence constraint, most of the matches removed by the between-sequence constraint should be false positives.

In the step to apply the within-sequence and between-sequence constraints, if a downstream vehicle has more than one possible match, only the possible match that yields consistent upstream long vehicle offsets is retained to ensure that each downstream vehicle is matched to no more than one upstream vehicle.

At the end of the cleanup process, the algorithm checks if an upstream vehicle is matched to more than one downstream vehicle. If this is the case, only the candidate downstream vehicle that can yield consistent upstream long vehicle offsets is selected as the final match. Finally, all sequences with length of two vehicles are removed.

Figure 3.5 shows the final matches (circles) superimposed on the original possible matches (dots) in the TTM after applying the two upstream long vehicle offset constraints.
to the results in Figure 3.4. Many possible matches are discarded by the time the final matches are determined in Figure 3.5, even though they fell in the longest sequences shown in Figure 3.4. Obviously some true matches have been discarded after applying the two constraints. The constraints used in the cleanup step should be a deliberate choice in the trade-off between the quantity of the final matches and the accuracy of the final matches. Future research can try to propose new constraints to remove as many as false positives, while remain as many as true matches.

Figure 3.5 Example Travel Time Matrix with final matches
CHAPTER 4 LANE CHANGE MANEUVER QUANTIFICATION BASED ON VEHICLE REIDENTIFICATION

In Chapter 3, a new Vehicle Reidentification (VRI) algorithm is proposed. This chapter seeks to employ the results from VRI to estimate the number of Lane Change Maneuvers (LCMs) in a given time-space region. The basic idea is that VRI can provide a constraint on the difference between the number of entering vehicles (Nen) and the number of exiting vehicles (Nex), which is called the inflow constraint. VRI can also provide the lower bound and the upper bound of Nen and Nex.

In this chapter, the Assumed Percentage Method is proposed to generate the point estimates of the number of LCMs that meet the inflow constraint and fall between the lower and upper bounds. Two variants of the Assumed Percentage Method are also proposed, and they are expected to generate better LCM quantification results. They are called the Short Gap Method and the Long Vehicle Method. These two methods seek to apply the LCM information calculated from samples believed to have more accurate LCM estimates to samples believed to otherwise have less accurate LCM estimates. The numerical evaluations of the three LCM methods are presented in Chapter 5.

This chapter is organized as follows. First, the inflow constraint and the lower and upper bounds are discussed in Section 4.1. Next, the Assumed Percentage Method for LCM quantification is proposed in Section 4.2 to obtain the point estimates of Nen and Nex based on the constraints discussed in Section 4.1. Finally, two variants of the
Assumed Percentage Method, Short Gap Method and Long Vehicle Method are presented in Section 4.3 and Section 4.4, respectively.

4.1 Constraints on the Number of Entering and Exiting Vehicles

This study seeks to employ the results from VRI to estimate the number of LCMs in a given time-space region in a manner that is compatible with existing vehicle detectors. To illustrate the time-space region in which the number of LCMs is estimated, Figure 4.1 shows a hypothetical example.

![Figure 4.1 A hypothetical example to show the time-space region in which the number of LCMs is to be estimated](image_url)
Figure 4.1 is a time-space diagram of a single lane on a multi-lane freeway segment over a certain time period. The freeway segment spans a pair of adjacent detector stations denoted with horizontal lines, and labeled as upstream and downstream stations. The bold curves labeled a1 and a2 represent trajectories of two consecutive vehicles that pass both stations and are reidentified by VRI. The lighter curves (b1-b5) denote other vehicles that pass without being reidentified either because they change lanes between the stations (e.g., b1, b3 and b5), or because the VRI algorithm fails to reidentify them although they pass both stations in the subject lane (e.g., b2 and b4). Loop detector data only record vehicle passage times at each station. If a vehicle is reidentified, the passage times at the upstream and downstream stations are associated with one another. Although vehicle trajectories are shown in Figure 4.1 to illustrate the relationships, they are not available from the loop detector data.

The following study seeks to estimate the number of entering vehicles (Nen) and the number of exiting vehicles (Nex) in the time-space region between the upstream and downstream stations and between the trajectories of a pair of consecutive reidentified vehicles, i.e., the shaded area in Figure 4.1. In this example the true Nen is 1 (represented by trajectory b1) and the true Nex is 2 (represented by trajectory b3 and b5).

This section discusses the constraints on Nen and Nex. Section 4.1.1 defines the inflow constraint and introduces the method to estimate inflow. Next, Section 4.1.2 proposes the lower and upper bounds for Nen and Nex.

4.1.1 Inflow Constraint and the Method to Estimate Inflow from Vehicle Reidentification

Coifman (2003b) showed that VRI can yield inflow. Inflow is defined as the difference of Nen and Nex between a pair of consecutive reidentified vehicles. Thus, the
The difference of the estimated $N_{en}$ and $N_{ex}$ should be equal to the inflow estimated from VRI. Such a constraint will be called the inflow constraint and is given by:

\[
inflow = N_{en} - N_{ex}
\]  

(4.1)

where,

- \( \text{inflow} \) is the inflow between the trajectories of a pair of consecutive reidentified vehicles,
- \( N_{en} \) is the number of entering vehicles between the trajectories of a pair of consecutive reidentified vehicles, and
- \( N_{ex} \) is the number of exiting vehicles between the trajectories of a pair of consecutive reidentified vehicles.

Coifman (2003b) demonstrated the method to estimate inflow using loop detectors and VRI results on a freeway segment between two detector stations. To summarize the method, consider the hypothetical example illustrated in Figure 4.2, which uses the same notation as in the previous example.
In Figure 4.2, two vehicles are reidentified, and many more vehicles pass without being matched. Vehicle a1 passes the upstream station at time $t_1$ and the downstream station at time $t_3$. Similarly, vehicle a2 passes the upstream and downstream stations at times $t_2$ and $t_4$, respectively. In Figure 4.2, the trajectories of the non-reidentified vehicles are not shown. No matter reidentified or not, every time a vehicle passes the upstream or downstream station, a sequential number is assigned to the vehicle, which is called the arrival number. Based on Coifman (2003b), inflow can be estimated by:
\[
\text{inflow}(t_3, t_4) = (N_d(t_4) - N_d(t_3)) - (N_u(t_2) - N_u(t_1))
\] (4.2)

where,

\text{inflow}(t_3, t_4) \text{ is the inflow between the trajectories of a pair of consecutive reidentified vehicles that pass the downstream station at times } t_3 \text{ and } t_4,

\(N_d(t_4)\) is the (sequential) arrival number of the vehicle that passes the downstream station at time \(t_4\),

\(N_d(t_3)\) is the (sequential) arrival number of the vehicle that passes the downstream station at time \(t_3\),

\(N_u(t_2)\) is the (sequential) arrival number of the vehicle that passes the upstream station at time \(t_2\), and

\(N_u(t_1)\) is the (sequential) arrival number of the vehicle that passes the upstream station at time \(t_1\).

In Figure 4.2, between the pair of consecutive reidentified vehicles, two vehicles are seen at the upstream station and one vehicle is seen at the downstream station. Therefore the inflow is -1 based on Equation 4.2. While it is possible that only one vehicle leaves the lane, there are many combinations of LCMs that could result in this same inflow, e.g., two vehicles could exit and one vehicle could enter. This example illustrates the fact that inflow does not allow a unique determination of the number of LCMs. However, given inflow, the solution set of possible Nen and Nex values is greatly reduced. Figure 4.3 shows the set of feasible values when the inflow is -1.
Figure 4.3 Set of feasible Nen and Nex values when inflow is -1

Equation 4.2 can be re-written to highlight that inflow is the difference of the number of vehicles observed at the downstream station and the number of vehicles observed at the upstream station between a pair of consecutive reidentified vehicles. That is:

\[
\text{inflow}(t_3, t_4) = (N_d(t_4) - N_d(t_3)) - (N_u(t_2) - N_u(t_1)) \\
= (N_d(t_4) - N_d(t_3) - 1) - (N_u(t_2) - N_u(t_1) - 1) \\
= N_{dn}^{\text{gap}}(t_3, t_4) - N_{up}^{\text{gap}}(t_3, t_4) 
\]

where,

\( N_{dn}^{\text{gap}}(t_3, t_4) \) is the number of vehicles observed at the downstream station between a pair of reidentified vehicles that pass the downstream station at times \( t_3 \) and \( t_4 \), and

\( N_{up}^{\text{gap}}(t_3, t_4) \) is the number of vehicles observed at the upstream station between a pair of reidentified vehicles that pass the downstream station at times \( t_3 \) and \( t_4 \).
To simplify the notation, the time stamps of Equation 4.3 will be omitted unless needed for clarity, thus:

\[ \text{inflow} = N_{\text{dn}}^{\text{pop}} - N_{\text{up}}^{\text{pop}} \quad (4.4) \]

4.1.2 Lower Bound and Upper Bound on the Number of Entering and Exiting Vehicles

Figure 4.3 illustrates that Equation 4.1 constrains the possible pair-wise solutions of (Nen, Nex) to positive integer values, shown with circles on the straight line. Although the feasible solution space is reduced from two dimensions to one, the number of feasible pair-wise solutions is still infinite. To address this problem, this section develops bounds on Nen and Nex.

Since neither Nen nor Nex can be negative, it is trivial to define lower bounds based on Equation 4.1. The lower bound for Nen is \( \max(0, \text{inflow}) \), and the lower bound for Nex is \( \max(0, -\text{inflow}) \).

Theoretically, the concrete upper bounds for Nen and Nex do not exist because of two special types of LCMs. The first type is caused by “pass through vehicles”. A pass through vehicle is a vehicle that makes multiple LCMs to pass through the subject lane between the upstream and downstream stations. A pass through vehicle will not be observed at the upstream or downstream station in the subject lane, but will be observed in the lanes in which the vehicle passes the detector stations. For example, consider the case where lane 3 is the lane of interest and a vehicle changes lane from lane 2 to lane 3 and from lane 3 to lane 4 between the upstream and downstream stations. Such a vehicle, which would be observed at the upstream station of lane 2 and the downstream station of
lane 4, is called a pass through vehicle for lane 3. Each pass through vehicle maneuver will introduce one entering LCM and one exiting LCM in the subject lane, although the vehicle is not observed at either of the stations in the subject lane. The second special type of LCM that precludes upper bounds for Nen and Nex is caused by “back and forth vehicles”. A back and forth vehicle is a vehicle that changes its lane to another lane then back to the original lane between the upstream and downstream stations. Depending on which lane is under study, a back and forth vehicle will either be observed at both stations or not be observed at either of the stations in the subject lane. Similar to pass through vehicle maneuvers, each back and forth vehicle maneuver will introduce one entering LCM and one exiting LCM in the subject lane.

The concrete upper bounds for Nen and Nex do not exist because of pass through and back and forth vehicles. For example, consider the case where there are two consecutive reidentified vehicles with no other vehicles observed between them at the upstream or downstream station. There could still exist many pass through or back and forth vehicles between these two reidentified vehicles, which contribute to the total number of LCMs.

To define an upper bound for Nen and Nex, the present study does not account for the LCMs caused by pass through or back and forth vehicles. This restriction is equivalent to assuming that no single vehicle can enter and exit, or exit and enter, the subject lane between the upstream and downstream stations. However, such an assumption does not mean the pass through vehicles are not considered at all. In such a case, the pass through vehicles are still accounted for in the original and target lanes, since they are exiting vehicles in the original lane and entering vehicles in the target lane. It is left to future

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7 Note that many of the studies discussed in Section 2.2 either implicitly or explicitly made a similar assumption.
research to estimate the number of LCMs that accounts for the LCMs caused by pass through or back and forth vehicles, as discussed in Section 6.2.2.

Under the assumption being made, the upper bounds for Nen and Nex could be defined. Nen is bounded by the number of vehicles observed at the downstream station between the reidentified vehicles, i.e., \( N_{dn}^{\text{gap}} \). Similarly, Nex is bounded by the number of vehicles observed at the upstream station between the reidentified vehicles, i.e., \( N_{up}^{\text{gap}} \).

Returning to the example shown in Figure 4.1, there are four non-reidentified vehicles between vehicles a1 and a2 at the upstream station. Under the assumption being made, these are the only vehicles that can leave the lane between vehicles a1 and a2, so Nex can be no larger than 4. Similarly, the upper bound of Nen is 3 because only three vehicles pass the downstream station between vehicles a1 and a2.

Once the lower and upper bounds are determined, the solution space of Nen and Nex becomes finite, and has to meet the following constraints:

\[
\begin{align*}
\max(0, \text{inflow}) & \leq Nen \leq N_{dn}^{\text{gap}} \quad (4.5a) \\
\max(0, -\text{inflow}) & \leq Nex \leq N_{up}^{\text{gap}} \quad (4.5b)
\end{align*}
\]

### 4.2 Assumed Percentage Method

Assuming that the upper bounds defined in Section 4.1.2 hold, Nen and Nex must fall between the bounds defined by Equation 4.5. To obtain a point estimate, the Assumed Percentage Method simply assumes where the estimates fall between the bounds, which is described by Percentage in Bounds (\( PB \)). \( PB \) is defined by:
\[ P_{Ben} = \frac{N_{en} - L_{Ben}}{U_{Ben} - L_{Ben}} \times 100\% \]  \hspace{1cm} (4.6.a)

\[ P_{Bex} = \frac{N_{ex} - L_{Bex}}{U_{Bex} - L_{Bex}} \times 100\% \]  \hspace{1cm} (4.6.b)

where,

\[ \text{\( P_{Ben} \)} \text{ is the Percentage in Bounds for } N_{en}, \]

\[ L_{Ben} \text{ is the lower bound for } N_{en}, \text{ i.e., } \max(0, \text{inflow}), \]

\[ U_{Ben} \text{ is the upper bound for } N_{en}, \text{ i.e., } N_{dn}^{\text{gap}}, \]

\[ \text{\( P_{Bex} \)} \text{ is the Percentage in Bounds for } N_{ex}, \]

\[ L_{Bex} \text{ is the lower bound for } N_{ex}, \text{ i.e., } \max(0, -\text{inflow}), \text{ and} \]

\[ U_{Bex} \text{ is the upper bound for } N_{ex}, \text{ i.e., } N_{up}^{\text{gap}}. \]

Obviously, if \( N_{en} \) falls within its lower bound and upper bound, \( P_{Ben} \) should be a value between 0% and 100%, and similarly for \( P_{Bex} \). Rearranging these equations, one can get:

\[ N_{en} = L_{Ben} + P_{Ben} \times (U_{Ben} - L_{Ben}) \]
\[ = \max(0, \text{inflow}) + P_{Ben} \times \left( N_{dn}^{\text{gap}} - \max(0, \text{inflow}) \right) \]  \hspace{1cm} (4.7.a)

\[ N_{ex} = L_{Bex} + P_{Bex} \times (U_{Bex} - L_{Bex}) \]
\[ = \max(0, -\text{inflow}) + P_{Bex} \times \left( N_{up}^{\text{gap}} - \max(0, -\text{inflow}) \right) \]  \hspace{1cm} (4.7.b)
Based on Equation 4.7, two PB (PBen and PBex) are needed to estimate Nen and Nex. The following analysis shows that to meet the inflow constraint, PBen must be equal to PBex so that only one PB is needed in the Assumed Percentage Method.

When inflow > 0, Equation 4.7 yields:

\[
Nen - Nex = \text{inflow} + PBen \times (N_{up}^{gap} - \text{inflow}) - PBex \times N_{up}^{gap}
\]

\[
= \text{inflow} + PBen \times N_{up}^{gap} - PBex \times N_{up}^{gap}
\]

\[
= \text{inflow} + N_{up}^{gap} \times (PBen - PBex)
\] (4.8)

When inflow < 0, Equation 4.7 yields:

\[
Nen - Nex = PBen \times N_{dn}^{gap} + \text{inflow} - PBex \times (N_{up}^{gap} + \text{inflow})
\]

\[
= \text{inflow} + PBen \times N_{up}^{gap} - PBex \times N_{up}^{gap}
\]

\[
= \text{inflow} + N_{dn}^{gap} \times (PBen - PBex)
\] (4.9)

The only way these equations can be resolved with Equation 4.1 is when PBen is equal to PBex, that is:

\[
PBen = PBex
\] (4.10)

Henceforth PB will be used in place of PBen and PBex. Therefore, Equation 4.7 can be rewritten to yield:
\[ N_{\text{en}} = \max(0, \text{inflow}) + PB \times \left( N_{\text{gap}}^{\text{up}} - \max(0, \text{inflow}) \right) \] (4.11.a)

\[ N_{\text{ex}} = \max(0, -\text{inflow}) + PB \times \left( N_{\text{gap}}^{\text{up}} - \max(0, -\text{inflow}) \right) \] (4.11.b)

In Equation 4.11, \text{inflow} can be estimated via Equation 4.2 or 4.4, and \( N_{\text{gap}}^{\text{up}} \) and \( N_{\text{gap}}^{\text{dn}} \) can be obtained from loop detector data. However, \( PB \) is still unknown. Therefore, the performance of the estimates depends on the accuracy of the assumed \( PB \) used in Equation 4.11. The Assumed Percentage Method simply assumes or asserts a value for \( PB \) within its range, which is between 0\% and 100\%. Thus, the equation used by the Assumed Percentage Method to estimate \( N_{\text{en}} \) and \( N_{\text{ex}} \) is as follow:

\[ \hat{N}_{\text{en}} = \max(0, \text{inflow}) + PB \times \left( N_{\text{gap}}^{\text{up}} - \max(0, \text{inflow}) \right) \] (4.12.a)

\[ \hat{N}_{\text{ex}} = \max(0, -\text{inflow}) + PB \times \left( N_{\text{gap}}^{\text{up}} - \max(0, -\text{inflow}) \right) \] (4.12.b)

where,

\( \hat{N}_{\text{en}} \) is the estimated number of entering vehicles between a pair of consecutive reidentified vehicles,

\( \hat{N}_{\text{ex}} \) is the estimated number of exiting vehicles between a pair of consecutive reidentified vehicles, and

\( PB' \) is the assumed \( PB \) value.

The assumed \( PB \) value reflects the confidence in the lower bound versus the upper bound. That is, a \( PB \) value smaller than 50\% shows that one is more confident in the lower bound, and a \( PB \) value greater than 50\% shows that one is more confident in the
upper bound. Appendix A formulates the optimal PB for the Assumed Percentage Method. Such formulation reveals the factors that affect the optimal PB and could be used to narrow the range of PB (currently it is 0% to 100%) in the Assumed Percentage Method. However, it is still an important future work to develop a model that can help determine the assumed PB used for the Assumed Percentage Method. Such a study would likely require extensive calibration, and a large amount of field data with ground truth that is not currently available.

The evaluation results of the Assumed Percentage Method are shown in Section 5.3. While it is shown that the Assumed Percentage Method yields good results if the assumed PB is close to the optimal PB, as the assumed PB is set further from the optimal PB, the performance of the Assumed Percentage Method degrades significantly. To address this issue, two variants of the Assumed Percentage Method are proposed: the Short Gap Method and the Long Vehicle Method. They seek to apply the LCM information calculated from samples believed to have more accurate LCM estimates to samples believed to have less accurate LCM estimates.

### 4.3 Short Gap Method

When no vehicle is reidentified over a long time period, the gap (i.e., the time-space region between a pair of consecutive reidentified vehicles) is called a long gap; otherwise, the gap is called a short gap. For long gaps, the upper bound defined in Equation 4.5 ($N_{dn}^{gap}$ and $N_{up}^{gap}$) and the interval size are usually large. Therefore, a given error in the assumed PB leads to larger errors from Equation 4.12 during the long gaps than during the short gaps. For example, if the best PB is 30% but 50% is used in the estimation, this 20% difference will lead to much larger errors in the estimation of the number of LCMs
In long gaps than in short gaps. In other words, for a short gap, even if a poor assumed $PB$ is used in the Assumed Percentage Method, the resulting estimates will not be very different from the observed values because the interval is narrow. As a result, compared to the estimates in long gaps, the estimates in short gaps are less sensitive to the choice of assumed $PB$ in the Assumed Percentage Method.

The estimates from short gaps are expected to be more accurate too. Consider the following example: in a long gap, several vehicles are further reidentified and this long gap then is divided into a group of short gaps. Because of the additional reidentified vehicles, the group of short gaps have higher reidentification rate and contain more information than the long gap alone. Given the same assumed $PB$, the sum of estimated number of LCMs from the short gaps is different from that from the long gap alone, and it is expected that the former is more likely to be closer to the observed value since the additional reidentified vehicles provide extra information.

Therefore, if it is assumed that the LCM patterns in the short gaps are representative of those in the long gaps, the LCM information from the estimates in the short gaps can be used to improve the estimates in the long gaps, and this approach is called the Short Gap Method.

The basic idea of the Short Gap Method is to apply the Assumed Percentage Method to estimate the number of LCMs in the short gaps, and to apply the LCM information from the previous short gaps to estimate the number of LCMs in the long gaps. To this end, Lane Change Maneuver Rate (LCMR) is defined to describe the LCM information from the short gaps. It is the rate of entering vehicles as a function of downstream flow, or the rate of exiting vehicles as a function of the upstream flow, that is:
\[ LCMR_{en} = \frac{N_{en}}{\text{the number of vehicles observed at the downstream station}} \quad (4.13.a) \]

\[ LCMR_{ex} = \frac{N_{ex}}{\text{the number of vehicles observed at the upstream station}} \quad (4.13.b) \]

where,

\[ LCMR_{en} \] is the LCMR for entering vehicles, and

\[ LCMR_{ex} \] is the LCMR for exiting vehicles.

In the Short Gap Method, in order to reduce the impact of transient behavior in short gaps, the LCMR calculated from the previous short gaps over a certain time period (e.g., from all short gaps in the previous 15 minutes) is used as the LCMR for a long gap. The number of LCMs for the given long gap is then estimated by inverting Equation 4.13 and finding the product of the number of non-reidentified vehicles and the LCMR, which yields \( \hat{N}_{en} \) (estimated \( N_{en} \) following the analysis of entering vehicles) and \( \hat{N}_{ex} \) (estimated \( N_{ex} \) following the analysis of exiting vehicles). However, two adjustments are needed to this pair of estimates. First, it is possible that \( \hat{N}_{en} \) or \( \hat{N}_{ex} \) is out of the bounds defined in Equation 4.5. Therefore, when \( \hat{N}_{en} \) or \( \hat{N}_{ex} \) is less than the lower bound, the lower bound is used as the estimate. Similarly, when \( \hat{N}_{en} \) or \( \hat{N}_{ex} \) is greater than the upper bound, the upper bound is used. Second, such pair of estimates does not necessarily meet the inflow constraint in Equation 4.1, an important constraint obtained from VRI. To pair up with \( \hat{N}_{en} \) to meet the inflow constraint, another estimated \( N_{ex} \) can be obtained by subtracting inflow from \( \hat{N}_{en} \), which is denoted as \( \hat{N}_{ex} \) (estimated \( N_{ex} \) following the analysis of entering vehicles). Similarly \( \hat{N}_{en} \)
(estimated Nen following the analysis of exiting vehicles) is obtained by adding $\hat{N}_{ex}^e$ and inflow to pair up with $\hat{N}_{ex}^e$. To apply the information both in $LCMR_{en}$ and in $LCMR_{ex}$, and meet the inflow constraint, the average of the two estimated number of entering vehicles ($\hat{N}_{en}^e$ and $\hat{N}_{en}^e$) is taken as the final estimate of Nen; and the average of the two estimated number of exiting vehicles ($\hat{N}_{ex}^e$ and $\hat{N}_{ex}^e$) is taken as the final estimate of Nex.

It is possible for a gap to be a short gap for entering vehicles but a long gap for exiting vehicles (or vice versa). In such cases, the Assumed Percentage Method is applied to obtain a pair of estimated Nen and Nex in the short gap; and the LCMR and the inflow constraint are applied to obtain another pair of estimated Nen and Nex in the long gap. The average of two estimated Nen and the average of two estimated Nex are taken as the final estimates. If a gap is a short gap for both entering and exiting vehicles, only the Assumed Percentage Method is applied. In summary, entering and exiting vehicles are examined separately in the Short Gap Method.

The Short Gap Method can be described by the following process.

(1) Between the $i$th pair of consecutive reidentified vehicles, if the number of non-reidentified vehicles observed at the downstream station is smaller than the threshold of short gaps, go to Step (2), otherwise go to Step (3).

(2) The given gap is a short gap for entering vehicles. Apply the Assumed Percentage Method described in Section 4.2 to get $\hat{N}_{en}^e_i$ by:

$$\hat{N}_{en}^e_i = \max(0, inflow_i) + PB \times \left( \frac{N_{gap}^{en}}{\max(0, inflow_i)} \right)$$  \hspace{1cm} (4.14)
Update $\text{LCMR}_{en}'$ to include the estimated $\hat{\text{N}}_{en}^{\text{en}}$. $\text{LCMR}_{en}'$ is the LCMR for entering vehicles calculated based on all short gaps in previous $t$ minutes and the formula is as follows:\(^8\)

$$\text{LCMR}_{en}' = \frac{\sum_{j} \hat{\text{N}}_{en}^{\text{en}}}{\sum_{j} \left( N_{\text{dn},i}^{\text{pop}} + 1 \right)} \quad (4.15)$$

Go to Step (4).

(3) The given gap is a long gap for entering vehicles. Calculate the estimated $\hat{\text{N}}_{en}$ in this long gap based on the gap size for this reidentified vehicle pair and the most current $\text{LCMR}_{en}'$ by:

$$\hat{\text{N}}_{en}^{\text{en}} = (N_{\text{dn},i}^{\text{pop}} + 1) \times \text{LCMR}_{en}' \quad (4.16)$$

To ensure that the estimates are within the lower bound and upper bound defined in Equation 4.5, if $\hat{\text{N}}_{en}^{\text{en}}$ from Equation 4.16 is less than the lower bound, replace this $\hat{\text{N}}_{en}^{\text{en}}$ with the lower bound (i.e., max(0, inflow));

similarly if $\hat{\text{N}}_{en}^{\text{en}}$ is greater than the upper bound, replace this $\hat{\text{N}}_{en}^{\text{en}}$ with the

---

\(^8\) $N_{\text{dn},i}^{\text{pop}}$ uses the same superscript and subscript as in Equation 4.3, which means the number of vehicles observed at the downstream station in the gap between a pair of consecutive reidentified vehicles. $N_{\text{dn},i}^{\text{pop}} + 1$ is used to avoid zero denominators, that is, the smallest feasible value for the denominator is 1, which also ensures the sum of denominators is equal to the total number of vehicles observed at detector stations over a given time period. Similar notations are used in Equations 4.18, 4.23 and 4.26.
upper bound (i.e., $N_{\text{up},i}^{\text{cap}}$). In any event, proceed to Step (4).

(4) Calculate estimated number of exiting vehicles following the inflow constraint in Equation 4.1, that is $\hat{\text{Nex}}_{i}^{\text{ex}} = \hat{\text{Nen}}_{i}^{\text{ex}} - \text{inflow}_{i}$. Go to Step (5).

(5) Between the $i$ th pair of consecutive reidentified vehicles, if the number of non-reidentified vehicles observed at the upstream station is smaller than the threshold of short gaps, go to Step (6), otherwise go to Step (7).

(6) The given gap is a short gap for exiting vehicles. Apply the Assumed Percentage Method described in Section 4.2 to get $\hat{\text{Nex}}_{i}^{\text{ex}}$ by:

$$\hat{\text{Nex}}_{i}^{\text{ex}} = \max(0, -\text{inflow}_{i}) + PB\times(N_{\text{up},i}^{\text{cap}} - \max(0, -\text{inflow}_{i}))$$

(4.17)

Update $\text{LCMR}_{\text{ex}}$ to include the estimated $\hat{\text{Nex}}_{i}^{\text{ex}}$. $\text{LCMR}_{\text{ex}}$ is the LCMR for exiting vehicles calculated based on all short gaps in previous $t$ minutes and the equation is:

$$\text{LCMR}_{\text{ex}} = \frac{\sum_{j} \hat{\text{Nex}}_{j}^{\text{ex}}}{\sum_{j} (N_{\text{up}, j}^{\text{cap}} + 1)}$$

(4.18)

Go to Step (8).

(7) The given gap is a long gap for exiting vehicles. Calculate the estimated $\text{Nex}$ in this long gap based on the gap size for this reidentified vehicle pair and the most current $\text{LCMR}_{\text{ex}}$ by:
\[ \hat{N}_{\text{ex}}^{i} = (N_{\text{up},i}^{\text{gap}} + 1) \times LCMR \_ex' \] (4.19)

To ensure that the estimates are within the lower bound and upper bound defined in Equation 4.5, if \( \hat{N}_{\text{ex}}^{i} \) from Equation 4.19 is less than the lower bound, replace this \( \hat{N}_{\text{ex}}^{i} \) with the lower bound (i.e., \( \max(0, -\text{inflow}) \)); if \( \hat{N}_{\text{ex}}^{i} \) is greater than the upper bound, replace this \( \hat{N}_{\text{ex}}^{i} \) with the upper bound (i.e., \( N_{\text{up},i}^{\text{gap}} \)). In any event, proceed to Step (8).

(8) Calculate estimated number of entering vehicles following the inflow constraint in Equation 4.1, that is \( \hat{N}_{\text{en}}^{i} = \hat{N}_{\text{ex}}^{i} + \text{inflow}_i \). Go to Step (9).

(9) Take the average of the two estimates on the number of entering vehicles, \( \hat{N}_{\text{en}}^{i} \) and \( \hat{N}_{\text{ex}}^{i} \), as the final estimate on the number of entering vehicles by:

\[ \hat{N}_{\text{en}} = \frac{(\hat{N}_{\text{en}}^{i} + \hat{N}_{\text{ex}}^{i})}{2} \] (4.20)

Similarly, take the average of the two estimates on the number of exiting vehicles, \( \hat{N}_{\text{ex}}^{i} \) and \( \hat{N}_{\text{en}}^{i} \), as the final estimate by:

\[ \hat{N}_{\text{ex}} = \frac{(\hat{N}_{\text{ex}}^{i} + \hat{N}_{\text{en}}^{i})}{2} \] (4.21)

Go to Step (10).
(10) Move to the \(i+1\)th pair of consecutive reidentified vehicles and return back to Step (1).

4.4 Long Vehicle Method

The VRI algorithm presented in Chapter 3 is based on the matching of long vehicles. If all normal length vehicles are omitted and only long vehicles are considered, the vehicle reidentification rate is much higher. As the VRI algorithm reidentifies more of the passing vehicles, the gap between a pair of consecutive reidentified vehicles is shorter and the upper bound is lower, so the solution bounds will be tighter. In such a case, a given error in the assumed \(PB\) leads to smaller errors in the estimated number of LCMs. In addition, the proposed VRI algorithm presented in Chapter 3 could match more than 40\% of the long vehicles observed at the downstream station, as will be shown in Section 5.1. Therefore, sometimes the upper bound could even be equal to the lower bound. That is, the bounds lead to a deterministic estimate of the number of LCMs made by long vehicles, which will not be affected by the assumed \(PB\). As a result, compared to considering the entire fleet directly as in the Assumed Percentage Method and the Short Gap Method, the estimates for long vehicles is less sensitive to the choice of assumed \(PB\). If it is assumed that the LCM patterns of long vehicles are representative of the entire fleet (or are related by a constant scale factor), the LCM information from the estimates for the long vehicles can be used to estimate the number of LCMs made by all vehicles. In such a case, the estimates for all vehicles is also less sensitive to the assumed \(PB\), which is a favorable feature especially before an effective method is developed to provide a good assumed \(PB\).
The Long Vehicle Method is proposed in this section. Similar to short gaps in the Short Gap Method, long vehicles in the Long Vehicle Method provide LCM information, which is used in the estimation of the number of LCMs made by all vehicles based on the assumption that the LCM patterns of long vehicles are representative of the entire fleet.

The basic idea of the Long Vehicle Method is to estimate the number of LCMs made by all vehicles based on the historic Lane Change Maneuver Rate (LCMR) for long vehicles over the previous \( t \) minutes. Two LCMRs for long vehicles are used, LCMR for long entering vehicles \((LCMR_{en}^L)\) and LCMR for long exiting vehicles \((LCMR_{ex}^L)\). They are defined in the similar way as in Equation 4.13 but the numerator is calculated from the Assumed Percentage Method when applied strictly to the long vehicles. That is, omit all non-long vehicles and assign sequential numbers only to long vehicles observed at the detector stations, then follow the Assumed Percentage Method described in Section 4.2 to obtain the estimated number of long entering and exiting vehicles. Similarly, the denominator only includes long vehicles. The LCMR for long vehicles are the moving average for the previous \( t \) minutes to reduce the impact of transient behavior in the long vehicles. For example, if a platoon of school buses change lanes together because they are going on the same field trip, the LCM behavior will be amplified when scaled up to the entire fleet if moving average is not used.

The number of LCMs made by all vehicles is estimated as the product of the number of vehicles (of all lengths) and the historic LCMR for long vehicles from previous gaps, which yields the estimated \(N_{en}\) based on \(LCMR_{en}^L\) and the estimated \(N_{ex}\) based on \(LCMR_{ex}^L\). Similar to the approach used in the Short Gap Method, two adjustments are needed. First, these two estimates need to be adjusted to be within the bounds defined in

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Equation 4.5 if necessary. Second, these two estimates need to be adjusted to meet the inflow constraint listed in Equation 4.1.

The Long Vehicle Method can be described by the following process.

1. Between the $i^{th}$ pair of consecutive reidentified vehicles, estimate the number of long entering vehicles ($\tilde{N}_{en}^{L,i}$) by applying the Assumed Percentage Method strictly to long vehicles, that is:

$$\tilde{N}_{en}^{L,i} = \max(0, inflow_i^L) + PB \times \left( N_{en,\text{gap}}^{L,i} - \max(0, inflow_i^L) \right)$$

(4.22)

In Equation 4.22, the superscript $L$ means long vehicles and all variables are obtained by considering only long vehicles, i.e., omitting all non-long vehicles. For example, $N_{en,\text{gap}}^{L,i}$ is the number of long vehicles observed at the downstream station in the gap between a pair of consecutive reidentified vehicles. Go to Step (2).

2. Update $LCMR_{en}^{L,i}$ to include the estimated $\tilde{N}_{en}^{L,i}$ from Equation 4.22. $LCMR_{en}^{L,i}$ is the LCMR for long entering vehicles calculated based on all gaps in previous $t$ minutes as follows:

$$LCMR_{en}^{L,i} = \frac{\sum_j \tilde{N}_{en}^{L,j}}{\sum_j (N_{en,\text{gap}}^{L,j} + 1)}$$

(4.23)

Go to Step (3).
(3) Calculate the estimated number of LCMs made by all vehicles between the \( i \) th pair of consecutive reidentified vehicles, \( \hat{N}en_i^{en} \), based on the gap size for this reidentified vehicle pair and the most current \( LCMR \_ en^{L-i} \) by:

\[
\hat{N}en_i^{en} = (N_{gap,i} + 1) \times LCMR \_ en^{L-i}
\]  

(4.24)

To ensure that the estimates are within the lower bound and upper bound defined in Equation 4.5, if \( \hat{N}en_i^{en} \) from Equation 4.24 is less than the lower bound, replace this \( \hat{N}en_i^{en} \) with the lower bound value (i.e., \( \max(0, \text{inflow}) \)); similarly if \( \hat{N}en_i^{en} \) is greater than the upper bound, replace this \( \hat{N}en_i^{en} \) with the upper bound value (i.e., \( N_{gap,i} \)). In any event, proceed to Step (4).

(4) Calculate estimated number of exiting vehicles following the inflow constraint of Equation 4.1, that is \( \hat{Nex}_i^{en} = \hat{N}en_i^{en} - \text{inflow}_i \). Go to Step (5).

(5) Between the \( i \) th pair of consecutive reidentified vehicles, estimate the number of long exiting vehicles (\( \hat{N}ex_i^{L,ex} \)) by applying the Assumed Percentage Method strictly to long vehicles, that is:

\[
\hat{N}ex_i^{L,ex} = \max(0,-\text{inflow}_i^L) + PB \times \left( N_{gap,i}^{L} - \max(0,-\text{inflow}_i^L) \right)
\]  

(4.25)

In Equation 4.25, the superscript L means long vehicles and all variables are obtained by considering only long vehicles, i.e., omitting all non-long vehicles.
For example, \( N_{up}^{L,gap} \) is the number of long vehicles observed at the upstream station in the gap between a pair of consecutive reidentified vehicles.

Go to Step (6).

(6) Update \( LCMR_{ex}^{L,t} \) to include the estimated \( \hat{N}_{ex}^{L,ex} \) from Equation 4.25.

\( LCMR_{ex}^{L,t} \) is the LCMR for long exiting vehicles calculated based on all gaps in previous \( t \) minutes as follows:

\[
LCMR_{ex}^{L,t} = \frac{\sum_j \hat{N}_{ex}^{L,ex}_j}{\sum_j (N_{up,j}^{L,gap} + 1)}
\]  

(4.26)

Go to Step (7).

(7) Calculate the estimated number of LCMs made by all vehicles between the \( i \) th pair of consecutive reidentified vehicles, \( \hat{N}_{ex}^{L,i} \), based on the gap size for this reidentified vehicle pair and the most current \( LCMR_{ex}^{L,t} \) by:

\[
\hat{N}_{ex}^{L,i} = (N_{up,i}^{gap} + 1) \times LCMR_{ex}^{L,t}
\]  

(4.27)

To ensure that the estimates are within the lower bound and upper bound defined in Equation 4.5, if \( \hat{N}_{ex}^{L,i} \) from Equation 4.27 is less than the lower bound, replace this \( \hat{N}_{ex}^{L,i} \) with the lower bound value (i.e., \( \max(0, -\text{inflow}) \)); similarly if \( \hat{N}_{ex}^{L,i} \) is greater than the upper bound, replace this \( \hat{N}_{ex}^{L,i} \) with the
upper bound value (i.e., \( N_{\text{gap}}^{\text{up},i} \)). In any event, proceed to Step (8).

(8) Calculate estimated number of entering vehicles following the inflow constraint of Equation 4.1, that is \( \hat{\text{\(N\)}}^{en}_{i} = \hat{\text{\(N\)}}^{ex}_{i} + \text{inflow}_i \). Go to Step (9).

(9) Take the average of the two estimates of the number of entering vehicles, \( \hat{\text{\(N\)}}^{en}_{i} \) and \( \hat{\text{\(N\)}}^{ex}_{i} \), as the final estimate of the number of entering vehicles by:

\[
\hat{\text{\(N\)}}_{i}^{en} = \frac{(\hat{\text{\(N\)}}^{en}_{i} + \hat{\text{\(N\)}}^{ex}_{i})}{2}
\]  

(4.28)

Similarly, take the average of the two estimates of the number of exiting vehicles, \( \hat{\text{\(N\)}}^{en}_{i} \) and \( \hat{\text{\(N\)}}^{ex}_{i} \), as the final estimate of the number of exiting vehicles by:

\[
\hat{\text{\(N\)}}_{i}^{ex} = \frac{(\hat{\text{\(N\)}}^{en}_{i} + \hat{\text{\(N\)}}^{ex}_{i})}{2}
\]  

(4.29)

Go to Step (10).

(10) Move to the \( i+1 \)th pair of consecutive reidentified vehicles and return back to Step (1).
CHAPTER 5 EVALUATION OF THE VEHICLE REIDENTIFICATION ALGORITHM AND THE LANE CHANGE MANEUVER QUANTIFICATION METHODS

The performance of the Vehicle Reidentification (VRI) algorithm proposed in Chapter 3 and the Lane Change Maneuver (LCM) quantification methods proposed in Chapter 4 are evaluated in this chapter. The numerical evaluation of the proposed VRI algorithm is based on several loop detector data sets collected from I-80 in California. The evaluation results show that the proposed VRI algorithm can reidentify long vehicles even when the traffic conditions change between free flow and congestion.

Chapter 4 proposed three LCM quantification methods to estimate the number of LCMs. These methods can be applied with many VRI algorithms, including the one developed in Chapter 3 that is based on the matching of long vehicles. However, the numerical VRI results presented in this chapter are not used to validate the proposed LCM quantification methods, since they do not include the ground truth LCM information. Instead, a vehicle trajectory data set is used. This vehicle trajectory data set does not include loop detector data that can be used for VRI, so it is also used to simulate VRI results that are consistent with the empirical performance of the proposed VRI algorithm from Chapter 3. The evaluation results show that the proposed approach for LCM quantification looks promising for estimating $N_{en}$ and $N_{ex}$, although further testing on additional data sets is necessary.
Chapter 5 is organized as follows. First, the numerical evaluation of the proposed VRI algorithm is presented in Section 5.1. Second, the data set and the performance measures used to evaluate the LCM quantification methods are described in Section 5.2. Then the three LCM quantification methods are evaluated: the Assumed Percentage Method in Section 5.3, the Short Gap Method in Section 5.4, and the Long Vehicle Method in Section 5.5. A comparison of the three LCM quantification methods follows in Section 5.6.

5.1 Numerical Evaluation of the Vehicle Reidentification Algorithm

This section shows the numerical evaluation of the proposed VRI algorithm over different segments of the Berkeley Highway Laboratory (BHL). The BHL data set is described in Section 5.1.1. The detailed VRI results for one station pair are shown in Section 5.1.2. The VRI results for three other station pairs are summarized in Section 5.1.3.

5.1.1 Data Description

BHL is a 2.7-mile-long segment of I-80 in west Berkeley and Emeryville, CA (Coifman et al., 2000). The schematic of the study segment is shown in Figure 5.1. All of the stations are equipped with dual loop detectors in all lanes. There is an on-ramp just upstream of station 3, and an off-ramp between station 5 and station 6. The loop detector data were collected from westbound I-80 on July 15th, 2003 from 1:00 to 24:00.

To facilitate the comparison with the VRI results presented in Coifman and Krishnamurthy (2007), the proposed VRI algorithm is first applied to lane 3 between station 2 and station 5, the same pair studied by Coifman and Krishnamurthy (2007). The
VRI results for this station pair are presented in detail in Section 5.1.2. Three other station pairs are also studied in this numerical evaluation, namely, station 3 to station 5, station 2 to station 6, and station 2 to station 7. These station pairs are selected to examine the performance of the proposed VRI algorithm when the distance between stations changes. The smallest distance among these pairs (including the station 2 to station 5 pair) is 0.7 miles (station 3 to station 5) and the largest distance is 1.5 miles (station 2 to station 7). VRI results for these station pairs are summarized in Section 5.1.3.

![Figure 5.1 Schematic of the study segment in the westbound I-80, not to scale (adapted from Coifman and Krishnamurthy 2007, BHL 2010)](image)

5.1.2 Numerical Evaluation Based on a Data Set from I-80

The VRI algorithm proposed in Chapter 3 is implemented between station 2 (upstream station) and station 5 (downstream station). The long vehicle threshold is 23
feet. Figure 5.2 shows the resulting Travel Time Matrix (TTM) after the process described in Section 3.2.2 is implemented. In fact the example shown in Section 3.2 uses the same data set as studied in this section, and Figure 3.2 is simply a detail from Figure 5.2. Similar to Figure 3.2, the maximum travel time shown in Figure 5.2 is 250 seconds. Possible matches with link travel times greater than 250 seconds are also considered by the algorithm, but they are not shown in Figure 5.2 to improve clarity.

Figure 5.2 Travel Time Matrix between station 2 and station 5 in lane 3

Figure 5.2 shows that more than 3,500 long vehicles are observed at the downstream station during the 23 hours of data collection. Three groups of downstream long vehicles exhibit a high density of possible matches with link travel time close to 60 seconds (i.e., at or near free flow speed), roughly: long vehicles 0 to 500, 1,100 to 2,500 and 3,300 to
3,600. The downstream long vehicle number increases monotonically with the time of day. As will be shown shortly, these groups correspond to the off-peak periods.

Meanwhile, for long vehicles 500 to 1,100, the link travel time for the dense region of possible matches increases from 60 seconds to 100 seconds and decreases back to 60 seconds. This period corresponds to the morning peak hours. At first glance the dense region of possible matches for long vehicles 2,500 to 3,300 is not clearly discernible. This period corresponds to the more congested afternoon peak hours.

Figure 5.3 is analogous to Figure 3.4, showing the results after searching for the longest sequences described in Section 3.2.3. For clarity, Figure 5.3(a) shows the original possible matches (dots) and the possible matches (circles) in the longest sequences, while Figure 5.3(b) shows just the sequence links at the same scale as Figure 5.3(a). Following Section 3.2.3, the close threshold used to join two possible matches in a sequence is defined to be a constant 10 seconds. The trends in travel times in Figure 5.3 are largely consistent with the discussion of Figure 5.2, with one notable exception. The travel time trends during the period of heavier congestion (i.e., long vehicles 2,500 to 3,300) begin to emerge in Figure 5.3. However, there are still many incorrect vehicle sequences left in Figure 5.3, scattered randomly throughout the TTM. The methods from Section 3.2.4 are then used to clean the results and determine the final matches.
Figure 5.3 Travel Time Matrix after removing the short vehicle sequences and retaining only the longest vehicle sequences between station 2 and station 5 in lane 3, (a) possible matches that are in the longest sequences are circled, (b) possible matches that are in the longest sequences are linked together
Figure 5.4 shows the results after the cleanup step described in Section 3.2.4 is implemented. In Figure 5.4, the final matches (circles) are superimposed on the original possible matches (dots) in the TTM. Each downstream vehicle has at most one match in this matrix. Compared to Figure 5.3 the results are much cleaner, and the trend over the day looks reasonable.

Figure 5.4 Final matches superimposed on the Travel Time Matrix between station 2 and station 5 in lane 3

It is a challenging task to validate the individual VRI results on a vehicle level basis. It would be prohibitively labor intensive to generate the ground truth over such a long time period (23 hours), unless one had a machine readable vehicle tag (e.g., Automatic Vehicle Identification and Automatic License Plate Recognition as presented in Section 2.1.1). But such vehicle tags are not available for the study data set.
In the present study, the proposed VRI algorithm is validated by comparing the resulting link speed and the local speed at the detector stations. The resulting link speed is calculated as the distance between the stations divided by the link travel time. The local speed at each of the dual loop detector stations is calculated as the distance between the two loops divided by the time difference of turn-on times of the two loops (see Figure 3.1 and Equation 3.1 for details). The emergent trends in link speed must be consistent with traffic flow theory. In detail, when the entire segment is free flowing or congested, the link speed and local speed should be close to each other since the entire segment is under similar traffic conditions. When the traffic conditions change from free flow to congestion due to a bottleneck downstream of the study segment, the downstream station local speed will drop first. As the queue grows into the segment, the local speed will drop at points further up the segment. At the same time, the link travel time will increase and the link speed will decrease gradually. If the queue passes the upstream station and the entire segment is within the queue, the upstream station local speed will drop. The process will reverse as the queue recedes.

To this end, Figure 5.5 shows the resulting link speeds for the final matches from Figure 5.4, superimposed on the time series local speeds (30-second harmonic average) measured at the upstream and downstream stations. These local speeds are from the vehicles of all lengths, not just the long vehicles. Figure 5.5(a) shows the local speeds at the upstream station, separated from the local speeds at the downstream station, which are shown in Figure 5.5(b). One can clearly see the drop in local speeds during the morning and evening peak periods. The local speeds show some spikes in Figure 5.5 due to noise, which is typical of 30-second average speed measurements from loop detectors.
Figure 5.5 Comparison of the link speeds from Vehicle Reidentification and the local speeds of all vehicles at the detector stations (a) upstream station (station 2); (b) downstream station (station 5)
Figure 5.5 shows that except for the morning (between approximately 1:00 and 5:00) and the evening (between approximately 20:00 and 24:00), the resulting link speeds from the VRI algorithm are consistent with the local speeds at two stations, i.e., the link speeds fall between or are close to the upstream and downstream station local speeds. The morning and evening free flow periods will be discussed shortly. Figure 5.6 shows the details of four one-hour time periods from Figure 5.5. Figure 5.6(a) compares the link speeds and the local speeds at both stations between 13:00 to 14:00 when the traffic is free flowing at both stations. Figure 5.6(b) is a similar figure for time period between 17:00 and 18:00 when the traffic is congested at both stations. The maximum value on the y-axis in Figure 5.6(b) is only 40 mph, different from other plots in Figure 5.6, to show more details. Both Figure 5.6(a) and Figure 5.6(b) show that when the entire segment is under similar traffic conditions, the link speeds from the proposed VRI algorithm and the local speeds at the detector stations are close, as one would expect based on the preceding discussion.

Figure 5.6(c) shows the transition period from free flow to congestion between 14:00 p.m. and 15:00 p.m. In Figure 5.6(c), the link speeds are above the downstream station local speeds and below the upstream station local speeds as a queue grows upstream through the segment. The link speed averages conditions over the entire link, which falls somewhere between the speeds observed at either bounding station. Likewise, the reverse order of events is seen around 19:00 p.m. in Figure 5.6(d) when the queue dissipates: the upstream station local speed recovers first, then the link speed, then the downstream station local speed. Once more, the link speed falls between the upstream and downstream station speeds during the transition. Therefore, the trends of the resulting
link speed over different time periods (except for the morning and evening) are as one would expect from traffic flow theory, as discussed earlier.

Figure 5.6 Detailed comparisons of the link speeds from Vehicle Reidentification and the local speeds of all vehicles at the detector stations (a) free flow time period; (b) congestion time period; (c) transition period from free flow to congestion; (d) transition period from congestion to free flow

Figures 5.5 and 5.6 also show that the proposed VRI algorithm can reidentify vehicles in challenging conditions. It is challenging to reidentify vehicles in heavily congested traffic, since the constant close threshold (10 seconds) will cause many true sequences to be rejected by the VRI algorithm due to valid but large jumps in consecutive vehicles' travel times. Figures 5.5 and 5.6 show that the proposed VRI algorithm reidentifies some vehicles even when the local speeds are between 10 mph and 20 mph, and the resulting
link speeds are consistent with the local speeds at both stations. As a comparison, the VRI algorithm proposed in Coifman and Krishnamurthy (2007) stops reidentifying vehicles when the resulting link speed is below 20 mph. It is also challenging to reidentify vehicles during transition periods, since traffic is usually unstable and the link travel time can rapidly change. The results in Figures 5.5 and 5.6 demonstrate that the proposed VRI algorithm can track the evolving link speed as the traffic conditions change between free flow and congestion. Figures 5.6(c) and 5.6(d) show that the resulting link speeds seem reasonable during the transition periods.

Figure 5.5 shows that the resulting link speeds from the VRI algorithm are on general lower than the local speeds at two stations in the morning (between approximately 1:00 and 5:00) and the evening (between approximately 20:00 and 24:00) free flow periods. The local speeds shown in Figure 5.5 are from the vehicles of all lengths while the VRI algorithm only reidentifies long vehicles. It is suspected that long vehicles travel slower than the other vehicles during the morning and evening free flow periods. It is also suspected that the link speeds from the VRI algorithm should be consistent with the local speeds of long vehicles during these periods. To this end, Table 5.1 compares the median local speeds of all vehicles, the median local speeds of long vehicles, and the median link speeds of long vehicles for each of the three periods when the traffic is free flowing.9

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9 The median is used instead of the mean to reduce the impact of transient disturbances in the speed measurements from loop detectors.
Table 5.1 shows that during the midday free flow period the local speeds are similar between the long vehicles and all vehicles; and the link speed is consistent with the local speeds at the detector stations. During the morning and evening free flow periods, the local speeds of long vehicles are significantly lower than those of all vehicles, since the one-sided two-sample t-test with unequal variance (Rice, 2006) shows P-values of $10^{-7}$ or smaller. However, the link speed remains consistent with the local speeds of long vehicles at both stations. It is also shown that the local and link speeds of long vehicles are consistent across all three free flow periods. In other words, as one might expect, the VRI algorithm yields the travel time of the long vehicles.

During the middle of the day when flows are moderately heavy, there is less opportunity for faster short vehicles to overtake slower long vehicles, so the speed of all vehicles is similar to the speed of long vehicles. But as shown in the bottom of Table 5.1,
during the morning and evening periods the flow drops to about 25% and 60% of the
midday flows, respectively, allowing the faster short vehicles to overtake the slower long
vehicles. Therefore, the VRI algorithm will underestimate the link speed of all vehicles
during low flow free flow periods. However, the algorithm still clearly shows that the
traffic is free flowing. This discrepancy between the speeds of long vehicles and all
vehicles should only be evident during low flow free flow periods, since the short
vehicles will be constrained by the long vehicles both in moderate to high flow free flow
conditions and in congestion (see, e.g., the preceding discussion about these periods in
terms of Figure 5.6).

Figure 5.7 revises the comparison from Figure 5.5, and compares the link speeds from
the proposed VRI algorithm and the local speeds of long vehicles. The local speed shown
in Figure 5.7 is the 180-second harmonic average of the local speeds of long vehicles
(instead of 30-second used in Figure 5.5) to ensure that each time interval has several
speed measurements. Except for the fact that the local speeds are now lower in the
morning and evening free flow periods, the trends are similar to those observed in Figure
5.5. Figure 5.7 shows that the link speeds are similar to the local speeds of long vehicles
throughout the entire day (excluding the transition periods), which indicates that the
proposed VRI algorithm performs well for the study data set.
Figure 5.7 Comparison of the link speeds from Vehicle Reidentification and the local speeds of long vehicles at the detector stations (a) upstream station (station 2); (b) downstream station (station 5)
The VRI results shown in this section can be compared to the results presented in Coifman and Krishnamurthy (2007), since they are based on the same data set. The comparison indicates that two VRI algorithms show similar trends over the day. However, the proposed VRI algorithm matches more vehicles during the congested time periods and during the transition periods between free flow and congestion. Therefore, the proposed VRI algorithm seems more robust to the change of traffic conditions. In total, the proposed VRI algorithm matches 12% more vehicles than the VRI algorithm in Coifman and Krishnamurthy (2007).

5.1.3 Numerical Evaluation Based on More Data Sets from I-80

The VRI algorithm was applied to other detector station pairs in the BHL. Like Section 5.1.2, the long vehicle threshold is 23 feet. The close threshold is set to 10 seconds, no matter what the distance between the station pair is. Figures 5.8 to 5.10 compare the link speeds from VRI and the local speeds of long vehicles for the following station pairs: station 3 to station 5, station 2 to station 6, and station 2 to station 7, respectively. The last pair, station 2 to station 7, has the longest distance between stations at about 1.5 miles with one on-ramp and one off-ramp between them. These three figures show that the VRI results are similar to those presented in detail in Section 5.1.2, and the proposed VRI algorithm performs well for the study data sets. Although there are a few obviously incorrect matches, most of the final matches are consistent with the local speeds, and the algorithm continues to find matches during the transition periods between free flow and congestion.
Figure 5.8 Comparison of the link speeds from Vehicle Reidentification and the local speeds of long vehicles at the detector stations (a) upstream station (station 3); (b) downstream station (station 5)
Figure 5.9 Comparison of the link speeds from Vehicle Reidentification and the local speeds of long vehicles at the detector stations (a) upstream station (station 2); (b) downstream station (station 6)
Figure 5.10 Comparison of the link speeds from Vehicle Reidentification and the local speeds of long vehicles at the detector stations (a) upstream station (station 2); (b) downstream station (station 7)
Table 5.2 enumerates the reidentification rate for all four station pairs discussed so far. The “percentage of long vehicles that are reidentified” is defined as the number of final matches divided by the number of downstream long vehicles, and it varies from 20.1% to 43.9%, depending on the distance between detector stations. The “percentage of vehicles that are reidentified”, defined as the number of final matches divided by the total number of downstream vehicles in all lengths, varies between 3.4% and 5.9%. These percentages show a decreasing trend as the distance between the stations increases, which is expected since fewer vehicles will pass both stations in the subject lane and the travel time difference of two consecutive long vehicles will vary over a larger range when the stations are further apart. Although only a very small portion of the total vehicle fleet is identified (roughly 4%), this rate is sufficient for LCM quantification as will be shown in the following sections.

Table 5.2 Summary of Vehicle Reidentification results in lane 3

<table>
<thead>
<tr>
<th>Station pair (distance)</th>
<th>Num of final matches</th>
<th>Num of downstream long vehicles</th>
<th>Num of downstream vehicles</th>
<th>Percentage of long vehicles that are reidentified</th>
<th>Percentage of vehicles that are reidentified</th>
</tr>
</thead>
<tbody>
<tr>
<td>3~5 (3,590 feet)</td>
<td>1572</td>
<td>3578</td>
<td>26586</td>
<td>43.9%</td>
<td>5.9%</td>
</tr>
<tr>
<td>2~5 (5,068 feet)</td>
<td>1429</td>
<td>3578</td>
<td>26586</td>
<td>39.9%</td>
<td>5.4%</td>
</tr>
<tr>
<td>2~6 (6,758 feet)</td>
<td>1126</td>
<td>3324</td>
<td>25748</td>
<td>33.9%</td>
<td>4.4%</td>
</tr>
<tr>
<td>2~7 (7,814 feet)</td>
<td>838</td>
<td>4164</td>
<td>24811</td>
<td>20.1%</td>
<td>3.4%</td>
</tr>
</tbody>
</table>
5.2 Data Set and Performance Measures Used in the Numerical Evaluation of the Lane Change Maneuver Quantification Methods

The VRI results from Section 5.1 could be used by the LCM quantification methods proposed in Chapter 4 to estimate the number of LCMs. However, the estimation results cannot be used to evaluate the LCM quantification methods, since the data sets used in Section 5.1 do not include the ground truth LCM information. Vehicle trajectory data are one of the few sources that could provide such information. It would be ideal to use a data set that includes both loop detector data and ground truth LCM information to evaluate the proposed LCM quantification methods. However, no such data sets are currently available. Therefore, the present study uses vehicle trajectory data, and simulates the VRI results to be consistent with the empirical performance of the new VRI algorithm proposed in Chapter 3.

The vehicle trajectory data used in the present study are provided by the Federal Highway Administration’s (FHWA) Next Generation SIMulation (NGSIM) project (FHWA, 2008). This NGSIM data set is described in Section 5.2.1. Based on this NGSIM data set, the VRI results are simulated in Section 5.2.2 and the ground truth LCM information is summarized in Section 5.2.3. The performance measures used to evaluate the proposed LCM quantification methods are developed in Section 5.2.4.

5.2.1 Data Description

The vehicle trajectory data were collected on a segment of interstate I-80 in Emeryville, California (immediately north of Oakland) between 4:00 p.m. and 4:15 p.m. on April 13, 2005. Figure 5.11 shows a schematic diagram of the study segment.
Figure 5.11 Schematic of study segment on I-80, not to scale (adapted from CS, 2005)
The data were collected using seven video cameras mounted on a 30-story building adjacent to I-80. Vehicle trajectory data were transcribed from the video data using a semi-automated process that automatically detected and tracked vehicles from the video images. The resulting data set includes vehicle ID, vehicle length, speed, lateral position, longitudinal position, lane identification, preceding vehicle ID and following vehicle ID for every vehicle within the field of view, at 1/10th of a second.

The study segment is approximately 1,650 feet long. The Powell Street on-ramp is near the start of the study segment and the Ashby Avenue off-ramp is just downstream of the end of the study segment. The segment was congested for the 15 minutes when the data were collected. The space mean speeds were 30 mph, 20 mph, 21 mph, 15 mph, 14 mph and 14 mph for lane 1 to lane 6, respectively.

5.2.2 Simulation of Vehicle Reidentification Results

As noted in Chapter 4, VRI results can be used to provide the inflow constraint, the lower bounds and the upper bounds of Nen and Nex. The new VRI algorithm proposed in Chapter 3 can generate the VRI results. However, vehicle trajectory data are used in the present study to provide the ground truth LCM information, which is crucial to the evaluation of LCM quantification methods. Therefore, the VRI results are simulated based on the vehicle trajectory data.

In the present study, the VRI results are simulated by specifying which vehicles are reidentified in a manner consistent with the empirical performance of the VRI algorithm from Section 5.1. Since this simplified process does not yield any incorrect matches, the simulated VRI results are more accurate than what can be achieved by the VRI algorithm presented in Chapter 3. However, this simplified process also eliminates confounding
factors that could otherwise obscure the performance of the proposed LCM quantification methods. It is an important future work to study the impact of the errors in VRI results on the proposed LCM quantification methods (see Section 6.2.2).

To simulate the VRI results, the upstream and downstream stations are defined. In the present study, the upstream station is assumed to be located at 200 feet into the study segment and the downstream station at 1,400 feet, as shown in Figure 5.11. The instant a given vehicle passes one of these simulated stations is interpolated between the times of the observations just before and after the station.

The simulated VRI results specify which vehicles are reidentified at the simulated upstream and downstream stations. Since the VRI algorithm proposed in Chapter 3 only seeks to match long vehicles in the same lane, only long through vehicles could possibly be reidentified (a through vehicle is any vehicle that is observed in the same lane at the upstream and downstream stations). It is thus necessary to assume a “percentage of reidentified long through vehicles”, which is used to determine how many long through vehicles are reidentified in the simulation. The “percentage of reidentified long through vehicles” shows the proportion of the reidentified vehicles in the reidentifiable vehicles, and therefore is a more transferable performance measure than other performance measures. Unfortunately, it is not possible to obtain the “percentage of reidentified long through vehicles” based only on the loop detector data, since the number of through vehicles is not available. Therefore, no empirical values on the “percentage of reidentified long through vehicles” are available from Section 5.1. To solve this issue, this study assumes that the VRI algorithm proposed in Chapter 3 can match 70% of the long through vehicles. The resulting VRI results are used to calculate the performance
measures that are available from Section 5.1 to check the reasonableness of this assumption. This check, which will be presented shortly, indicates that the 70% assumption is reasonable.

In the present study, long through vehicles are selected randomly at a target selection rate of 70%. These selected vehicles are regarded as reidentified vehicles. Table 5.3 shows the simulated VRI results for lane 2 to lane 5. Lane 1 (a HOV lane) and lane 6 (an auxiliary lane) are not considered in the present study because of their special restrictions and geometries. To be consistent with the earlier studies in Section 5.1, long vehicles are defined to be the vehicles that are longer than 90% of the vehicles observed at the upstream and downstream stations. Different long vehicle thresholds are used in each lane and are shown in the first row of Table 5.3. Lane 3 has a different vehicle length distribution, and the long vehicle threshold is 6 feet longer than those in other lanes. These thresholds are defined based on the physical vehicle lengths provided in the NGSIM data. The proportions of long vehicles in Table 5.3 are not exactly 10% because the long vehicle thresholds are rounded to the nearest integers. The “percentage of reidentified long through vehicles” is calculated as the “number of reidentified (long) vehicles” divided by the “number of long through vehicles”. All the values of the “percentage of reidentified long through vehicles” are close to the target value of 70%, but may be slightly smaller or larger due to the random selection process.

---

10 The loop detectors from Chapter 3 report effective vehicle length, which is typically on the order of 6 feet longer than the physical vehicle length.
Table 5.3 Summary of the simulated Vehicle Reidentification results

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long vehicle threshold (feet)</td>
<td>18</td>
<td>24</td>
<td>18</td>
<td>18</td>
</tr>
<tr>
<td>Number of long vehicles at upstream station</td>
<td>23</td>
<td>42</td>
<td>27</td>
<td>23</td>
</tr>
<tr>
<td>Number of vehicles at upstream station</td>
<td>333</td>
<td>283</td>
<td>315</td>
<td>298</td>
</tr>
<tr>
<td>Number of long vehicles at downstream station</td>
<td>23</td>
<td>40</td>
<td>24</td>
<td>21</td>
</tr>
<tr>
<td>Number of vehicles at downstream station</td>
<td>383</td>
<td>304</td>
<td>321</td>
<td>322</td>
</tr>
<tr>
<td>Number of through vehicles</td>
<td>310</td>
<td>219</td>
<td>228</td>
<td>189</td>
</tr>
<tr>
<td>Number of long through vehicles</td>
<td>20</td>
<td>39</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>Number of reidentified (long) vehicles</td>
<td>14</td>
<td>24</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Percentage of reidentified long through vehicles</td>
<td>70%</td>
<td>62%</td>
<td>61%</td>
<td>73%</td>
</tr>
<tr>
<td>Percentage of long vehicles that are reidentified</td>
<td>61%</td>
<td>60%</td>
<td>46%</td>
<td>52%</td>
</tr>
<tr>
<td>Percentage of vehicles that are reidentified</td>
<td>3.7%</td>
<td>7.9%</td>
<td>3.4%</td>
<td>3.4%</td>
</tr>
</tbody>
</table>

As mentioned above, the resulting VRI results are used to calculate the performance measures of the VRI algorithm, and are compared to the empirical values listed in Table 5.3. Such a comparison provide a check on the reasonableness of the assumption on the “percentage of reidentified long through vehicles”.

Two performance measures are checked. The first performance measure is called “percentage of long vehicles that are reidentified”. It is calculated as the “number of reidentified (long) vehicles” divided by the “number of long vehicles at downstream station”. A comparison of Table 5.2 and Table 5.3 shows that the values of the “percentage of long vehicles that are reidentified” in Table 5.3 are higher than the
corresponding values in Table 5.2. Such performance is reasonable because the segment in Table 5.3 is only 1,200 feet long while the shortest segment in Table 5.2 is longer than 3,300 feet. As discussed in Section 5.1, usually more long vehicles can be reidentified on shorter segments based on the proposed VRI algorithm. The second performance measure is called “percentage of vehicles that are reidentified”. It is calculated as the “percentage of long vehicles that are reidentified” times the proportion of long vehicles (calculated as “number of long vehicles at downstream station” divided by the “number of vehicles at downstream station”). A comparison of Table 5.2 and Table 5.3 shows that most values of the “percentage of vehicles that are reidentified” in Table 5.3 are smaller than the corresponding values in Table 5.2. Such comparison implies that this performance measure based on the simulated VRI results is actually smaller than the empirical values. Therefore, the simulated VRI results do not overstate the performance of the proposed VRI algorithm. In summary, the assumed 70% for the “percentage of reidentified long through vehicles” seems reasonable. The VRI results shown in Table 5.3 will be used throughout the following sections in Chapter 5.

5.2.3 Observed Number of Lane Change Maneuvers

The observed number of LCMs will be compared to the estimated number of LCMs to evaluate the performance of the proposed LCM quantification methods. The NGSIM data set provides lane information for each vehicle every 1/10\textsuperscript{th} of a second. It should be straightforward to obtain the observed number of LCMs based on trajectory data: whenever the lane information for a vehicle changes, there is a LCM.

However, a close look at the NGSIM data reveals that several vehicles seemingly change lanes from one lane to another and back to the original lane within a very short
time period, e.g., less than 0.3 seconds. Since the NGSIM lane information was determined based on the lateral position of the front center of the vehicle with respect to the left-most edge of the section, it is possible for a vehicle straddling two lanes to cause such “flicker”. The precision limit in lateral position also contributes to such noise. To eliminate these discretization errors in the present study, the lane information is cleaned based on the following rule. If a vehicle changes lane from one lane to another and back to the original lane within 1 second, this pair of LCMs is considered to be false and will not be counted in the observed number of LCMs. The lane information is updated accordingly. For example, if a vehicle changes lane from lane 2 to lane 3, and then back to lane 2 within 1 second, both LCMs will be discarded and the lane information is updated so that the vehicle remains in lane 2 for the entire 1 second. Following this rule, the studied NGSIM data set has 66 false LCMs and 955 true LCMs. That is, about 7% of LCMs are considered false and will not be counted towards the observed number of LCMs.

The proposed LCM quantification methods seek to estimate \( N_{en} \) and \( N_{ex} \) between a pair of consecutive reidentified vehicles. Therefore, the observed \( N_{en} \) and \( N_{ex} \) should be counted between each pair of consecutive reidentified vehicles based on the simulated VRI results presented in Section 5.2.2. The observed number of LCMs in the 15-minute data set is summarized in Table 5.4.
Table 5.4 Observed number of LCMs between the first and the last reidentified vehicles in the study data set

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observed Nen</td>
<td>62</td>
<td>78</td>
<td>69</td>
<td>118</td>
</tr>
<tr>
<td>Observed Nex</td>
<td>20</td>
<td>59</td>
<td>60</td>
<td>91</td>
</tr>
<tr>
<td>Number of vehicles at upstream station ( N_{up} )</td>
<td>275</td>
<td>259</td>
<td>224</td>
<td>253</td>
</tr>
<tr>
<td>Number of vehicles at downstream station ( N_{dn} )</td>
<td>317</td>
<td>278</td>
<td>233</td>
<td>280</td>
</tr>
<tr>
<td>Number of pass through vehicles ( N_p )</td>
<td>8</td>
<td>27</td>
<td>20</td>
<td>29</td>
</tr>
<tr>
<td>Number of back and forth vehicles ( N_{bf} )</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>7</td>
</tr>
</tbody>
</table>

Table 5.4 shows the sum of the observed Nen (or Nex) between each pair of consecutive reidentified vehicles in a given lane, which is equivalent to the total number of Nen (or Nex) between the first and the last reidentified vehicles. The “number of vehicles at upstream station” and the “number of vehicles at downstream station” are also counted between the first and the last reidentified vehicles. Therefore, these numbers are smaller than the corresponding numbers in Table 5.3, which are counted for the entire 15-minute data set.

The observed number of LCMs in Table 5.4 does not include the LCMs caused by pass through or back and forth vehicles, since the present study does not account for pass through or back and forth vehicles (as noted in Section 4.1.2) in the subject lane. To evaluate the impact of not counting pass through or back and forth vehicles in the subject lane, Table 5.4 also shows the “number of pass through vehicles \( N_p \)” and the “number
of back and forth vehicles ($N_{bf}$)”. For example, there are 8 pass through vehicles and 2 back and forth vehicles between the first and the last reidentified vehicles in lane 2. If the LCMs caused by such vehicles are counted in the observed number of LCMs, the observed Nen in lane 2 will be 72, a 16% increase from its current value (62). The results from all four lanes indicate that there are some pass through vehicles in the study segment, which is believed to be related to the location of the data collection site. The data collection site is a few hundred feet downstream of a major merge of three freeways (I-80, I-580, and I-880) and about three miles upstream of a major diverge of two freeways (I-80 and I-580). The number of pass through vehicles is further increased by the HOV vehicles from the Powell ramp attempting to change to lane 1 (a HOV lane) as quickly as possible. As noted in Section 4.1.2, a pass through vehicle is still counted as an exiting vehicle in the original lane, and counted as an entering vehicle in the target lane. On the contrary, back and forth vehicles are not counted at all in the observed number of LCMs. Fortunately there are only 18 back and forth vehicles in all four lanes. It is quite possible that some of these back and forth vehicles are simply longer duration occurrences of the “flicker” mentioned above, i.e., arising from data reduction errors rather than real vehicle maneuvers. Therefore, the observed Nen and Nex shown in Table 5.4 still reflect the majority of the LCMs in the study segment.

Based on the observed number of LCMs in Table 5.4, Table 5.5 summarizes the LCM fractions. The LCM fraction is the ratio of the number of entering vehicles to the total number of vehicles at the downstream station, or the ratio of the number of the exiting vehicles to the total number of vehicles at the upstream station. The LCM fractions in Table 5.5 help understand the level of LCM activities in different lanes. It will be shown
in Section 5.6.2 that LCM fraction might be a factor that impacts the value of optimal $PB$ for the LCM quantification methods.

In Table 5.5, the “percentage of downstream vehicles that are entering vehicles” is calculated as the “observed $N_{en}$” divided by the “number of vehicles at downstream station ($N_{dn}$)”, both are from Table 5.4. Similarly, the “percentage of upstream vehicles that are exiting vehicles” in Table 5.5 is calculated as the “observed $N_{ex}$” divided by the “number of vehicles at upstream station ($N_{up}$)”. The numbers in Table 5.5 show that lane 2 has the lowest LCM fraction and lane 5 has the highest LCM fraction. The LCM fraction in lane 3 and lane 4 are similar to each other and between those for lane 2 and lane 5.

<table>
<thead>
<tr>
<th>Percentage of downstream station vehicles that are entering vehicles</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>20%</td>
<td>28%</td>
<td>30%</td>
<td>42%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Percentage of upstream station vehicles that are exiting vehicles</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>7%</td>
<td>23%</td>
<td>27%</td>
<td>36%</td>
<td></td>
</tr>
</tbody>
</table>

5.2.4 Performance Measures

Three performance measures are defined in this section to evaluate and compare the LCM quantification methods presented in Section 4.2 to Section 4.4. Consider the example shown in Figure 5.12, where a VRI algorithm reidentifies $M + 1$ vehicles over a time period. A LCM quantification method will yield an estimated $N_{en}$ and an estimated
Nex between each pair of consecutive reidentified vehicles. Thus there are $M$ estimated
Nen and $M$ estimated Nex in Figure 5.12. To quantify and evaluate the performance of a
LCM quantification method over a time period, all estimates should be considered and
compared to the observed values. Three performance measures are defined.

![Diagram showing Lane Change Maneuver quantification over a time period](image)

**Legend:**
- Trajectories of reidentified vehicles
- Passage of non-reidentified vehicles at detector stations

**Figure 5.12** A diagram to show Lane Change Maneuver quantification over a time period

The first performance measure (DIFF) is the sum of the difference between the
estimated values and the observed values over the $M$ pairs of consecutive reidentified
vehicles. **DIFF** is defined as:
\[ DIFF = \sum_{i=1}^{M} \left( \hat{Nen}_i - Nen_i \right) = \sum_{i=1}^{M} \left( \hat{Nex}_i - Nex_i \right) \] (5.1)

where,

- **DIFF** is the difference between the estimated and observed total number of LCMs over the study time period,
- **\( M \)** is the total number of reidentified vehicle pairs over the study time period,
- **\( \hat{Nen}_i \)** is the \( i \)th estimated \( Nen \), that is, the estimated number of entering vehicles between the \( i \)th and \( i + 1 \)th reidentified vehicles,
- **\( Nen_i \)** is the \( i \)th observed \( Nen \), that is, the observed number of entering vehicles between the \( i \)th and \( i + 1 \)th reidentified vehicles,
- **\( \hat{Nex}_i \)** is the \( i \)th estimated \( Nex \), that is, the estimated number of exiting vehicles between the \( i \)th and \( i + 1 \)th reidentified vehicles, and
- **\( Nex_i \)** is the \( i \)th observed \( Nex \), that is, the observed number of exiting vehicles between the \( i \)th and \( i + 1 \)th reidentified vehicles.

In Equation 5.1, the second equation holds because it can be shown based on Equation 4.1 (i.e., inflow = \( Nen - Nex \)) that:

\[ \hat{Nen}_i - Nen_i = \hat{Nex}_i - Nex_i \] (5.2)

The measure **DIFF** reflects the difference between the estimated and observed total number of LCMs over a time period. A **DIFF** value closer to 0 indicates better performance (less biased).
The second performance measure is Mean Absolute Error (MAE), which is defined as follows:

\[
MAE = \frac{\sum_{i=1}^{M} |\hat{N}_{en_i} - N_{en_i}|}{M} = \frac{\sum_{i=1}^{M} |\hat{N}_{ex_i} - N_{ex_i}|}{M} \tag{5.3}
\]

Similar to Equation 5.1, the second equation in Equation 5.3 holds because of Equation 5.2. The measure MAE reflects the average magnitude of the difference between the estimated and the observed number of LCMs. Unlike DIFF, MAE does not allow positive and negative errors in different samples to offset each other to achieve a small value. In addition, MAE reflects the average difference between a pair of consecutive reidentified vehicles, whereas DIFF is the total difference over a time period. A MAE value closer to 0 indicates better performance.

The third performance measure is Percentage Relative Error (PRE), which is defined as follows:

\[
PRE_{-nen} = \frac{\sum_{i=1}^{M} (\hat{N}_{en_i} - N_{en_i})}{\sum_{i=1}^{M} N_{en_i}} \times 100\% = \frac{DIFF}{\sum_{i=1}^{M} N_{en_i}} \times 100\% \tag{5.4.a}
\]

\[
PRE_{-nex} = \frac{\sum_{i=1}^{M} (\hat{N}_{ex_i} - N_{ex_i})}{\sum_{i=1}^{M} N_{ex_i}} \times 100\% = \frac{DIFF}{\sum_{i=1}^{M} N_{ex_i}} \times 100\% \tag{5.4.b}
\]
Unlike \textit{DIFF} and \textit{MAE}, the value of \textit{PRE} is not the same for \textit{Nen} and \textit{Nex}. Therefore, \textit{PRE} \_ \textit{Nen} (\textit{PRE} for \textit{Nen}) and \textit{PRE} \_ \textit{Nex} (\textit{PRE} for \textit{Nex}) are defined separately in Equation 5.4. The measure \textit{PRE} reflects the relative difference (or error) between the estimated and the observed total number of LCMs over a time period. It is determined not only by \textit{DIFF}, but also by the magnitude of the observed number of LCMs. A \textit{PRE} value closer to 0 indicates better performance.

As will be illustrated in the following sections, the studies based on the NGSIM data show that in most cases all three performance measures yield consistent trends: a LCM quantification method that has a \textit{DIFF} value close to zero also tends to have \textit{MAE} and \textit{PRE} values close to zero. However, it is possible that these three performance measures do not yield the same comparison result when they are used to compare LCM quantification methods. In such cases, which performance measure plays the most important role depends on the purpose of the estimates. For example, if the main purpose is to get an accurate estimate of the total number of entering and exiting vehicles over a time period, \textit{DIFF} will be a good measure. If it is desired to use the individual estimates between each pair of consecutive reidentified vehicles, \textit{MAE} should be considered. If it is desired to compare the performance of a LCM quantification method on two study segments with different number of LCMs, \textit{PRE} could be used. Obviously, additional performance measures could be defined for other purposes. For example, it might be interesting to show the total absolute error instead of the mean absolute error.

\textbf{5.3 Numerical Evaluation of the Assumed Percentage Method}

This section evaluates the performance of the Assumed Percentage Method proposed in Section 4.2, based on the data set described in Section 5.2.1. The Assumed Percentage
Method is applied to the simulated VRI results shown in Section 5.2.2. The estimation results are compared to the observed number of LCMs shown in Section 5.2.3. The performance measures defined in Section 5.2.4 are used to evaluate the performance of the Assumed Percentage Method.

5.3.1 The Assumed Percentage Method with Assumed PB of 50%

As discussed in Section 4.2, Equation 4.12 will be used by the Assumed Percentage Method to estimate the number of LCMs between a pair of consecutive reidentified vehicles. In Equation 4.12, only one variable, \( PB \), is unknown. The Assumed Percentage Method simply assumes a value for \( PB \) between its range (0% and 100%). As discussed in Section 4.2, the assumed \( PB \) value reflects the confidence in the lower bound versus the upper bound. When no other information is available, \( PB \) is assumed to be 50%. In such a case, the point estimate of Nen and Nex is the midpoint of the corresponding bounds. Although the observed Nen and Nex are integers, Equation 4.12 cannot guarantee integer estimates. To facilitate the following mathematical analysis, non-integer estimates are allowed in this study.

The Assumed Percentage Method was applied to each of the four lanes with \( PB \) set to 50%. As an example, Figure 5.13 shows the estimation results between each pair of consecutive reidentified vehicles in lane 4. A total of 11 long vehicles are reidentified in this lane over the 15-minute time period (as per the simulation results in Table 5.3). Therefore, there are 10 pairs of consecutive reidentified vehicles. For each pair, Equation 4.12 is used to calculate the estimated Nen and Nex. Figure 5.13(a) compares each of the estimated Nen and the observed Nen obtained from the vehicle trajectory data. The x-axis, “sequential order of estimation”, indicates the order of each estimate. For example, the
“sequential order of estimation” of 1 indicates that the corresponding estimate is the first estimate over the study time period. Figure 5.13(b) shows the same information for Nex. Figure 5.13 shows that the Assumed Percentage Method yields estimates that follow the trends evident in the observed values. However, the estimates are consistently higher than the observed values, which implies that the assumed $PB$ of 50% is too high. Section 5.3.2 studies the performance of the Assumed Percentage Method with different $PB$ values.

Figure 5.13 Estimation of the number of LCMs in lane 4 based on the Assumed Percentage Method with assumed $PB$ of 50%
To evaluate the performance of the Assumed Percentage Method with $PB$ set to 50%, the performance measures defined in Section 5.2.4 are calculated. The results are shown in Table 5.6. As discussed in Section 5.2.4, $DIFF$ and $MAE$ are the same for $Nen$ and $Nex$, so Table 5.6 does not differentiate between $Nen$ and $Nex$ for these two performance measures.

Table 5.6 Performance measures of the Assumed Percentage Method with assumed $PB$ of 50%

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DIFF$</td>
<td>112</td>
<td>66</td>
<td>50</td>
<td>32</td>
</tr>
<tr>
<td>$MAE$</td>
<td>8.6</td>
<td>3.0</td>
<td>5.0</td>
<td>3.6</td>
</tr>
<tr>
<td>$PRE_Nen$ (%)</td>
<td>181</td>
<td>85</td>
<td>72</td>
<td>29</td>
</tr>
<tr>
<td>$PRE_Nex$ (%)</td>
<td>560</td>
<td>112</td>
<td>83</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 5.6 shows that the estimated values are much higher than the observed values in all four lanes. For example, $PRE\_Nen$ shows that in lane 2, the estimated total number of entering vehicles during the 15 minutes is about 2 times (181%) higher than the observed value. It is even worse for the total number of exiting vehicles in lane 2 (the $PRE\_Nex$ is 560%). Similarly, the values for $DIFF$ and $MAE$ are poor. However, these results do not mean that the Assumed Percentage Method could not work well. The performance of the Assumed Percentage Method depends on how close the assumed $PB$ is to the best $PB$, as will be shown in Section 5.3.2. While assuming $PB$ of 50% is not perfect, the accuracy may be sufficient in some cases.
5.3.2 Sensitivity Analysis of the Assumed Percentage Method

Section 5.3.1 showed the performance of the Assumed Percentage Method with PB fixed at 50%. This section expands the analysis to all possible values of PB. Such analysis is called sensitivity analysis. The sensitivity analysis reveals the best and worst performance for any fixed PB, and it shows the sensitivity of the method to the PB value used as an input. The sensitivity analysis of the Assumed Percentage Method is conducted by varying PB from 0% to 100% with a step size of 10%. The performance measures defined in Section 5.2.4 are calculated for each PB value used in this sensitivity analysis. They are shown in Table 5.7 to Table 5.10, respectively for lane 2 to lane 5.

Table 5.7 Sensitivity analysis based on the Assumed Percentage Method in lane 2

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-18</td>
<td>8</td>
<td>34</td>
<td>60</td>
<td>86</td>
<td>112</td>
<td>137</td>
<td>163</td>
<td>189</td>
<td>215</td>
<td>241</td>
</tr>
<tr>
<td>MAE</td>
<td>1.4</td>
<td>1.3</td>
<td>2.7</td>
<td>4.6</td>
<td>6.6</td>
<td>8.6</td>
<td>10.6</td>
<td>12.6</td>
<td>14.6</td>
<td>16.5</td>
<td>18.5</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-29</td>
<td>13</td>
<td>55</td>
<td>97</td>
<td>139</td>
<td>181</td>
<td>221</td>
<td>263</td>
<td>305</td>
<td>347</td>
<td>389</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-90</td>
<td>40</td>
<td>170</td>
<td>300</td>
<td>430</td>
<td>560</td>
<td>685</td>
<td>815</td>
<td>945</td>
<td>1075</td>
<td>1205</td>
</tr>
</tbody>
</table>

Table 5.8 Sensitivity analysis based on the Assumed Percentage Method in lane 3

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-44</td>
<td>-22</td>
<td>0</td>
<td>22</td>
<td>44</td>
<td>66</td>
<td>88</td>
<td>110</td>
<td>132</td>
<td>154</td>
<td>176</td>
</tr>
<tr>
<td>MAE</td>
<td>1.9</td>
<td>1.1</td>
<td>0.6</td>
<td>1.1</td>
<td>2.0</td>
<td>3.0</td>
<td>3.9</td>
<td>4.8</td>
<td>5.8</td>
<td>6.7</td>
<td>7.7</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-56</td>
<td>-28</td>
<td>0</td>
<td>28</td>
<td>56</td>
<td>85</td>
<td>113</td>
<td>141</td>
<td>169</td>
<td>197</td>
<td>226</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-75</td>
<td>-37</td>
<td>0</td>
<td>37</td>
<td>75</td>
<td>112</td>
<td>149</td>
<td>186</td>
<td>224</td>
<td>261</td>
<td>298</td>
</tr>
</tbody>
</table>
Table 5.9 Sensitivity analysis based on the Assumed Percentage Method in lane 4

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-53</td>
<td>-32</td>
<td>-12</td>
<td>9</td>
<td>29</td>
<td>50</td>
<td>71</td>
<td>91</td>
<td>112</td>
<td>132</td>
<td>153</td>
</tr>
<tr>
<td>MAE</td>
<td>5.3</td>
<td>3.4</td>
<td>1.5</td>
<td>1.2</td>
<td>2.9</td>
<td>5.0</td>
<td>7.1</td>
<td>9.1</td>
<td>11.2</td>
<td>13.2</td>
<td>15.3</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-77</td>
<td>-46</td>
<td>-17</td>
<td>13</td>
<td>42</td>
<td>72</td>
<td>103</td>
<td>132</td>
<td>162</td>
<td>191</td>
<td>222</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-88</td>
<td>-53</td>
<td>-20</td>
<td>15</td>
<td>48</td>
<td>83</td>
<td>118</td>
<td>152</td>
<td>187</td>
<td>220</td>
<td>255</td>
</tr>
</tbody>
</table>

Table 5.10 Sensitivity analysis based on the Assumed Percentage Method in lane 5

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-83</td>
<td>-60</td>
<td>-36</td>
<td>-13</td>
<td>11</td>
<td>34</td>
<td>57</td>
<td>81</td>
<td>104</td>
<td>128</td>
<td>151</td>
</tr>
<tr>
<td>MAE</td>
<td>8.3</td>
<td>6.1</td>
<td>3.9</td>
<td>1.7</td>
<td>1.4</td>
<td>3.5</td>
<td>5.7</td>
<td>8.1</td>
<td>10.4</td>
<td>12.8</td>
<td>15.1</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-70</td>
<td>-51</td>
<td>-31</td>
<td>-11</td>
<td>9</td>
<td>29</td>
<td>48</td>
<td>69</td>
<td>88</td>
<td>108</td>
<td>128</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-91</td>
<td>-66</td>
<td>-40</td>
<td>-14</td>
<td>12</td>
<td>37</td>
<td>63</td>
<td>89</td>
<td>114</td>
<td>141</td>
<td>166</td>
</tr>
</tbody>
</table>

From Tables 5.7 to 5.10, one can determine the worst performance in each lane across the range of possible PB values. For example, Table 5.7 shows that the worst MAE is 18.5 in lane 2, which occurs when PB of 100% is used. This worst MAE value is more than two times of the MAE value in lane 2 when PB of 50% is used (8.6 based on Table 5.7).

Tables 5.7 to 5.10 also show the best performance in each lane when PB is varied from 0% to 100%. The PB value that yields the smallest (in magnitude) DIFF in a sensitivity analysis will be called the best PB. The best PB will also tend to yield the smallest (in magnitude) MAE and PRE for the study data set, since Tables 5.7 to 5.10 show that the three performance measures are consistent with one another in each lane.
That is, the $PB$ value that minimizes the magnitude of $DIFF$ in a given lane also minimizes the magnitude of $MAE$ and $PRE$ for that lane. Table 5.11 summarizes the best $PB$ from Tables 5.7 to 5.10. These best $PB$ values are obtained using a step size of 10% in the sensitivity analysis. Obviously, if a different step size is used, the best $PB$ could be different. It is clear in Table 5.11 that different lanes have different best $PB$, varying from 10% to 40%. Table 5.11 also shows that if the best $PB$ is used as the input, the Assumed Percentage Method can yield good results. For example, in each of the four lanes, the $MAE$ is around 1 when the best $PB$ is used, which implies that on average each estimated Nen or Nex is within about one vehicle of the observed value.

Table 5.11 Summary of the best $PB$ from the sensitivity analysis with a step size of 10% based on the Assumed Percentage Method

<table>
<thead>
<tr>
<th>Best PB value</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>8</td>
<td>0</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>MAE</td>
<td>1.3</td>
<td>0.6</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>13</td>
<td>0</td>
<td>13</td>
<td>9</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>40</td>
<td>0</td>
<td>15</td>
<td>12</td>
</tr>
</tbody>
</table>

The performance measures shown in Table 5.11 indicate the overall performance of the Assumed Percentage Method over many samples (i.e., pairs of consecutive reidentified vehicles). The present study also checks the performance of the Assumed Percentage Method in each sample when the best $PB$ (as per Table 5.11) is used. The results reveal that the best $PB$ also tend to have good individual estimates between each
pair of consecutive reidentified vehicles. For example, Figure 5.14 shows the individual estimates in lane 4 when the best $PB$ (30%) is used. Compared to the estimates in Figure 5.13 where $PB$ is set to 50%, the estimates in Figure 5.14 are much closer to the observed values.

![Graph showing entering and exiting vehicles](image)

Figure 5.14 Estimation of the number of LCMs in lane 4 based on the Assumed Percentage Method with assumed $PB$ of 30%

For each pair of consecutive reidentified vehicles, it can be easily shown that there exists a $PB$ that can yield the estimated $N_{en}$ and $N_{ex}$ equal to the observed values. Such a $PB$ will be called the optimal sample $PB$. The value of the optimal sample $PB$ can vary from one pair of consecutive reidentified vehicles to the next. Figure 5.14 implies that for
the studied 15-minute data set, the optimal sample $PB$ does not appear to change rapidly, since one fixed $PB$ (30% in this case) can yield good results over the 15-minute time period. However, it is likely that the optimal sample $PB$ in a lane could change significantly over a long time period or when the traffic condition changes. It is left for future study to investigate the change of optimal sample $PB$, since such investigation requires a data set collected from a much longer time period.

The preceding analysis reveals that the performance of the Assumed Percentage Method depends greatly on the value of $PB$ used as an input. It is still an open question as to how to deduce the best $PB$ given just loop detector data. However, the preceding analysis indicates that if an appropriate $PB$ is used, the estimates based on the Assumed Percentage Method not only have good performance over a time period, but also have good performance between each pair of consecutive reidentified vehicles.

5.4 Numerical Evaluation of the Short Gap Method

This section evaluates the performance of the Short Gap Method proposed in Section 4.3. Similar to the analysis shown in Section 5.3, the analysis shown in this section is based on the data set described in Section 5.2. The performance of the Short Gap Method with assumed $PB$ of 50% is presented in Section 5.4.1. The results of the sensitivity analysis based on the Short Gap Method are shown in Section 5.4.2.

5.4.1 The Short Gap Method with Assumed PB of 50%

The detailed procedures of the Short Gap Method were described in Section 4.3. The basic idea of the Short Gap Method is to apply the Assumed Percentage Method to estimate the number of LCMs in the short gaps, and to apply the LCM information from
the previous short gaps to estimate the number of LCMs in the long gaps. When the Assumed Percentage Method is applied in the short gaps, Equations 4.14 and 4.17 are used. Similar to Section 5.3.1, the assumed $PB$ values in Equations 4.14 and 4.17 are set to be 50% in this section. Non-integer estimates for $Nen$ and $Nex$ are allowed to facilitate the following mathematical analysis.

All four lanes are studied based on the Short Gap Method with assumed $PB$ of 50% for short gaps (used in Equations 4.14 and 4.17). In this study, the threshold for short gaps is chosen to be 10 vehicles (including vehicles in all lengths). This threshold is applied to all four lanes. According to Equations 4.15 and 4.18, the Lane Change Maneuver Rate (LCMR), $LCMR_{en'}$ and $LCMR_{ex'}$, are calculated based on the estimates from the short gaps in the previous $t$ minutes. However, the study data set is only 15 minutes long and the VRI simulation results only yield a limited number of short gaps. Therefore, the LCMR used in the present study is actually calculated based on the estimates from all preceding short gaps in the data set. In other words, $t$ is not a fixed value in this study. $t$ is increasing as the process moves to the next gap, but is always less than 15 minutes. As a result, the earlier gaps use less historic information than the later gaps, due to the limited duration of the data set.

As an example, the estimation results from the Short Gap Method for lane 4 are shown in Figure 5.15. These results can be compared with the corresponding estimates from the Assumed Percentage Method in Figure 5.13. Based on the simulated VRI results from Section 5.2.2, the second, third, sixth and tenth gaps are short gaps, and the Assumed Percentage Method is applied. Therefore, the estimates in these short gaps are exactly the same as those shown in Figure 5.13. The first gap is a long gap. However, no
historic data are available for the first gap, so the Assumed Percentage Method is used. The fourth gap (a long gap) applies the LCMR information from the second and third gaps (short gaps), and the estimated Nen is 20.5, as shown in Figure 5.15(a). This estimated Nen is closer to the observed value (15) than the corresponding estimate (23.9) based on the Assumed Percentage Method shown in Figure 5.13(a). The advantage of the Short Gap Method is more obvious in the ninth gap (using LCMR information from 3 short gaps). The estimated Nen is 31.0 based on the Assumed Percentage Method (in Figure 5.13(a)) and 25.6 based on the Short Gap Method (in Figure 5.15(a)), whereas the observed Nen is 16.

Figure 5.15 Estimation of the number of LCMs in lane 4 based on the Short Gap Method with assumed $PB$ of 50% for short gaps
As discussed in Section 4.3, the final estimated Nen is the average of the estimated Nen following the analysis of entering vehicles ($\hat{N}en^{en}$) and the estimated Nen following the analysis of exiting vehicles ($\hat{N}en^{ex}$). A closer look at the results, which are not shown here, reveals that $\hat{N}en^{en}$ is different from $\hat{N}en^{ex}$ when the gap is not a short gap for both entering and exiting vehicles. Either of the two estimates could be closer to the observed value, but without knowing which estimate is absolutely better than the other, the average of two helps minimize the expected error in the estimate. The results show the same trend for the estimated Nex.

As noted earlier, there is not enough historical data for the first several gaps. To partially address this issue, the Short Gap Method is applied again using a fixed LCMR. The fixed LCMR is calculated using Equation 4.15 or Equation 4.18 based on all short gaps in the 15-minute data set, instead of all short gaps in the previous $t$ minutes. Such an approach violates the temporal order of the estimates, since the estimates in later short gaps can impact the estimate in an earlier long gap. However, this approach ensures that all estimates in the long gaps are based on a 15-minute historical LCMR. Such a change only impacts the estimates in long gaps. Its impact is bigger for the earlier long gaps than for the later long gaps, since the later long gaps have already used a similar LCMR. The results show that by using the LCMR from all of the short gaps, the first estimated Nen is decreased from 18.0 in Figure 5.15(a) to 12.6 (the observed value is 8); and the ninth estimated Nen is decreased from 25.6 in Figure 5.15(a) to 25.3 (the observed value is 16). However, this approach cannot be applied if the temporal order of the gaps cannot be violated (e.g., when applied in real time) and its advantage will be smaller if enough historical data are already available for each gap.
In the end, the present study focuses on the Short Gap Method that uses the LCMR strictly from the preceding short gaps and does not violate the temporal order of the estimates. To evaluate the performance of the Short Gap Method with assumed $PB$ of 50%, the performance measures defined in Section 5.2.4 are calculated for each of the lanes, as shown in Table 5.12. All performance measures of the Short Gap Method have improved compared to those of the Assumed Percentage Method in Table 5.6. An explicit comparison between the Short Gap Method and the Assumed Percentage Method is presented and discussed in Section 5.6.

Table 5.12 Performance measures of the Short Gap Method with assumed $PB$ of 50% for short gaps

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>89</td>
<td>46</td>
<td>36</td>
<td>10</td>
</tr>
<tr>
<td>MAE</td>
<td>6.8</td>
<td>2.1</td>
<td>3.6</td>
<td>1.2</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>144</td>
<td>59</td>
<td>52</td>
<td>8</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>445</td>
<td>78</td>
<td>60</td>
<td>11</td>
</tr>
</tbody>
</table>

5.4.2 Sensitivity Analysis of the Short Gap Method

Section 5.4.1 discussed the performance of the Short Gap Method with an assumed $PB$ of 50% for short gaps. Similar to Section 5.3.2, a sensitivity analysis of the Short Gap Method can be conducted by choosing all possible values of $PB$ and tabulating the performance. To facilitate the comparison with the results in Section 5.3.2, the sensitivity analysis in this section changes the assumed $PB$ in short gaps from 0% to 100% with a
step size of 10%. The change of assumed $PB$ directly affects the estimates in short gaps, and then indirectly affects the LCMR and the estimates in long gaps. The results are shown in Table 5.13 to Table 5.16, one table for each lane. The sensitivity analysis of the Short Gap Method and the Assumed Percentage Method are compared and discussed in Section 5.6.

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-15</td>
<td>3</td>
<td>24</td>
<td>45</td>
<td>67</td>
<td>89</td>
<td>110</td>
<td>132</td>
<td>154</td>
<td>175</td>
<td>197</td>
</tr>
<tr>
<td>MAE</td>
<td>1.2</td>
<td>1.6</td>
<td>2.1</td>
<td>3.5</td>
<td>5.1</td>
<td>6.8</td>
<td>8.5</td>
<td>10.1</td>
<td>11.8</td>
<td>13.5</td>
<td>15.2</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-24</td>
<td>5</td>
<td>39</td>
<td>73</td>
<td>108</td>
<td>144</td>
<td>177</td>
<td>213</td>
<td>248</td>
<td>282</td>
<td>318</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-75</td>
<td>15</td>
<td>120</td>
<td>225</td>
<td>335</td>
<td>445</td>
<td>550</td>
<td>660</td>
<td>770</td>
<td>875</td>
<td>985</td>
</tr>
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</table>

Table 5.14 Sensitivity analysis based on the Short Gap Method in lane 3

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-24</td>
<td>-12</td>
<td>2</td>
<td>16</td>
<td>31</td>
<td>46</td>
<td>61</td>
<td>77</td>
<td>92</td>
<td>107</td>
<td>122</td>
</tr>
<tr>
<td>MAE</td>
<td>1.1</td>
<td>0.7</td>
<td>0.5</td>
<td>0.9</td>
<td>1.5</td>
<td>2.1</td>
<td>2.7</td>
<td>3.4</td>
<td>4.0</td>
<td>4.7</td>
<td>5.3</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-31</td>
<td>-15</td>
<td>3</td>
<td>21</td>
<td>40</td>
<td>59</td>
<td>78</td>
<td>99</td>
<td>118</td>
<td>137</td>
<td>156</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-41</td>
<td>-20</td>
<td>3</td>
<td>27</td>
<td>53</td>
<td>78</td>
<td>103</td>
<td>131</td>
<td>156</td>
<td>181</td>
<td>207</td>
</tr>
</tbody>
</table>
Table 5.15 Sensitivity analysis based on the Short Gap Method in lane 4

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-35</td>
<td>-21</td>
<td>-6</td>
<td>8</td>
<td>22</td>
<td>36</td>
<td>51</td>
<td>65</td>
<td>79</td>
<td>93</td>
<td>108</td>
</tr>
<tr>
<td>MAE</td>
<td>3.5</td>
<td>2.2</td>
<td>0.9</td>
<td>1.2</td>
<td>2.2</td>
<td>3.6</td>
<td>5.1</td>
<td>6.5</td>
<td>7.9</td>
<td>9.3</td>
<td>10.8</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-51</td>
<td>-30</td>
<td>-9</td>
<td>12</td>
<td>32</td>
<td>52</td>
<td>74</td>
<td>94</td>
<td>114</td>
<td>135</td>
<td>157</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-58</td>
<td>-35</td>
<td>-10</td>
<td>13</td>
<td>37</td>
<td>60</td>
<td>85</td>
<td>108</td>
<td>132</td>
<td>155</td>
<td>180</td>
</tr>
</tbody>
</table>

Table 5.16 Sensitivity analysis based on the Short Gap Method in lane 5

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-66</td>
<td>-57</td>
<td>-44</td>
<td>-27</td>
<td>-9</td>
<td>10</td>
<td>29</td>
<td>47</td>
<td>66</td>
<td>84</td>
<td>103</td>
</tr>
<tr>
<td>MAE</td>
<td>6.6</td>
<td>5.8</td>
<td>4.6</td>
<td>3.2</td>
<td>1.6</td>
<td>1.2</td>
<td>2.9</td>
<td>4.7</td>
<td>6.6</td>
<td>8.4</td>
<td>10.3</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-56</td>
<td>-48</td>
<td>-37</td>
<td>-23</td>
<td>-8</td>
<td>8</td>
<td>25</td>
<td>40</td>
<td>56</td>
<td>71</td>
<td>87</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-73</td>
<td>-63</td>
<td>-48</td>
<td>-30</td>
<td>-10</td>
<td>11</td>
<td>32</td>
<td>52</td>
<td>73</td>
<td>92</td>
<td>113</td>
</tr>
</tbody>
</table>

5.5 Numerical Evaluation of the Long Vehicle Method

This section evaluates the performance of the Long Vehicle Method proposed in Section 4.4. Similar to the analysis shown in Section 5.3 and Section 5.4, the analysis shown in this section is based on the data set described in Section 5.2. The performance of the Long Vehicle Method with assumed PB of 50% is presented in Section 5.5.1. The results of the sensitivity analysis are shown in Section 5.5.2.

5.5.1 The Long Vehicle Method with Assumed PB of 50%

The detailed procedures of the Long Vehicle Method were described in Section 4.4. The basic idea of the Long Vehicle Method is to apply the Assumed Percentage Method
to estimate the number of LCMs made by long vehicles, and to apply the LCM information from the long vehicles to estimate the number of LCMs made by all vehicles. When the Assumed Percentage Method is applied to the long vehicles, Equations 4.22 and 4.25 are used. Similar to Section 5.3.1 and Section 5.4.1, the assumed $PB$ values in Equations 4.22 and 4.25 are set to be 50% in this section. Non-integer estimates for $Nen$ and $Nex$ are allowed to facilitate the following mathematical analysis.

All four lanes are studied based on the Long Vehicle Method with assumed $PB$ of 50% for long vehicles (used in Equations 4.22 and 4.25). Similar to the LCMR in the Short Gap Method, the LCMR for long vehicles, $LCMR_{en L}$ and $LCMR_{ex L}$, are actually calculated based on the estimates from all preceding gaps in the data set. In such a case, $t$ is increasing as the process moves to the next gap but is always less than 15 minutes. As a result, the earlier gaps use less historic information than the later gaps, due to the limited duration of the data set.

As an example, the estimation results from the Long Vehicle Method for lane 4 are shown in Figure 5.16. These estimates are closer to the observed values than the corresponding estimates from the Assumed Percentage Method shown in Figure 5.13 and the Short Gap Method in shown Figure 5.15. In Figure 5.16, the largest difference between the estimated and the observed number of LCMs is less than 3, with the average difference of 1.
Similar to Section 5.4.1, a closer look at the results show that the average of the estimated \( \hat{N}_{en} \) following the analysis of entering vehicles (\( \hat{N}_{en}^{en} \)) and the estimated \( \hat{N}_{en} \) following the analysis of exiting vehicles (\( \hat{N}_{en}^{ex} \)) helps minimize the expected error in the estimates. The Long Vehicle Method is also repeated using a fixed LCMR. The fixed LCMR is calculated using Equation 4.23 or Equation 4.26 based on all gaps in the 15-minute data set, instead of all gaps in the previous \( t \) minutes. The results show that after using the LCMR from all of the gaps, the Long Vehicle Method has similar performance as what is shown in Figure 5.16. The maximum change of the estimated \( N_{en} \) is only 0.6 and the sum of all changes is only 0.3.

Figure 5.16 Estimation of the number of LCMs in lane 4 based on the Long Vehicle Method with assumed \( PB \) of 50% for long vehicles
In the end, the present study focuses on the Long Vehicle Method that uses the LCMR strictly from the preceding gaps and does not violate the temporal order of the estimates. To evaluate the performance of the Long Vehicle Method with assumed \( PB \) of 50\%, the performance measures defined in Section 5.2.4 are calculated for each of the lanes, as shown in Table 5.17. All performance measures of the Long Vehicle Method have improved compared to those of the Short Gap Method in Table 5.12 and the Assumed Percentage Method Table 5.6. An explicit comparison among the Long Vehicle Method, the Short Gap Method and the Assumed Percentage Method is presented and discussed in Section 5.6.

Table 5.17 Performance measures of the Long Vehicle Method with assumed \( PB \) of 50\% for long vehicles

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>12</td>
<td>-5</td>
<td>-1</td>
<td>-32</td>
</tr>
<tr>
<td>MAE</td>
<td>1.6</td>
<td>0.7</td>
<td>1.0</td>
<td>3.3</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>19</td>
<td>-6</td>
<td>-1</td>
<td>-27</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>60</td>
<td>-8</td>
<td>-2</td>
<td>-35</td>
</tr>
</tbody>
</table>

5.5.2 Sensitivity Analysis of the Long Vehicle Method

Section 5.5.1 discussed the performance of the Long Vehicle Method with an assumed \( PB \) of 50\% for long vehicles. Similar to Section 5.3.2 and Section 5.4.2, a sensitivity analysis of the Long Vehicle Method can be conducted by choosing all possible values of \( PB \) and tabulating the performance. To facilitate the comparison with
the results in Section 5.3.2 and Section 5.4.2, the sensitivity analysis in this section changes the assumed PB for long vehicles from 0% to 100% with a step size of 10%. The change of assumed PB directly affects the estimated number of LCMs made by long vehicles, and then indirectly affects the LCMR and the estimated number of LCMs made by all vehicles. The results are shown in Table 5.18 to Table 5.21, one table for each lane. The sensitivity analysis of the Long Vehicle Method, the Short Gap Method and the Assumed Percentage Method are compared and discussed in Section 5.6.

Table 5.18 Sensitivity analysis based on the Long Vehicle Method in lane 2

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-8</td>
<td>-5</td>
<td>-1</td>
<td>3</td>
<td>7</td>
<td>12</td>
<td>17</td>
<td>22</td>
<td>27</td>
<td>33</td>
<td>38</td>
</tr>
<tr>
<td>MAE</td>
<td>0.9</td>
<td>1.0</td>
<td>1.1</td>
<td>1.2</td>
<td>1.4</td>
<td>1.6</td>
<td>2.0</td>
<td>2.3</td>
<td>2.6</td>
<td>3.0</td>
<td>3.4</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-13</td>
<td>-8</td>
<td>-2</td>
<td>5</td>
<td>11</td>
<td>19</td>
<td>27</td>
<td>35</td>
<td>44</td>
<td>53</td>
<td>61</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-40</td>
<td>-25</td>
<td>-5</td>
<td>15</td>
<td>35</td>
<td>60</td>
<td>85</td>
<td>110</td>
<td>135</td>
<td>165</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 5.19 Sensitivity analysis based on the Long Vehicle Method in lane 3

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-40</td>
<td>-34</td>
<td>-28</td>
<td>-21</td>
<td>-13</td>
<td>-5</td>
<td>4</td>
<td>13</td>
<td>22</td>
<td>32</td>
<td>41</td>
</tr>
<tr>
<td>MAE</td>
<td>1.8</td>
<td>1.5</td>
<td>1.3</td>
<td>1.1</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.8</td>
<td>1.1</td>
<td>1.5</td>
<td>1.9</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-51</td>
<td>-44</td>
<td>-36</td>
<td>-27</td>
<td>-17</td>
<td>-6</td>
<td>5</td>
<td>17</td>
<td>28</td>
<td>41</td>
<td>53</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-68</td>
<td>-58</td>
<td>-47</td>
<td>-36</td>
<td>-22</td>
<td>-8</td>
<td>7</td>
<td>22</td>
<td>37</td>
<td>54</td>
<td>69</td>
</tr>
</tbody>
</table>
Table 5.20 Sensitivity analysis based on the Long Vehicle Method in lane 4

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-37</td>
<td>-32</td>
<td>-25</td>
<td>-17</td>
<td>-10</td>
<td>-1</td>
<td>7</td>
<td>16</td>
<td>24</td>
<td>33</td>
<td>42</td>
</tr>
<tr>
<td>MAE</td>
<td>3.8</td>
<td>3.4</td>
<td>2.7</td>
<td>2.0</td>
<td>1.3</td>
<td>1.0</td>
<td>1.2</td>
<td>1.7</td>
<td>2.4</td>
<td>3.3</td>
<td>4.2</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>-54</td>
<td>-46</td>
<td>-36</td>
<td>-25</td>
<td>-14</td>
<td>-1</td>
<td>10</td>
<td>23</td>
<td>35</td>
<td>48</td>
<td>61</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-62</td>
<td>-53</td>
<td>-42</td>
<td>-28</td>
<td>-17</td>
<td>-2</td>
<td>12</td>
<td>27</td>
<td>40</td>
<td>55</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 5.21 Sensitivity analysis based on the Long Vehicle Method in lane 5

<table>
<thead>
<tr>
<th>PB</th>
<th>0%</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>60%</th>
<th>70%</th>
<th>80%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIFF</td>
<td>-66</td>
<td>-62</td>
<td>-55</td>
<td>-49</td>
<td>-41</td>
<td>-32</td>
<td>-22</td>
<td>-12</td>
<td>-3</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>MAE</td>
<td>6.7</td>
<td>6.2</td>
<td>5.6</td>
<td>5.0</td>
<td>4.2</td>
<td>3.3</td>
<td>2.4</td>
<td>1.5</td>
<td>1.4</td>
<td>2.0</td>
<td>2.7</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>-73</td>
<td>-68</td>
<td>-60</td>
<td>-54</td>
<td>-45</td>
<td>-35</td>
<td>-24</td>
<td>-13</td>
<td>-3</td>
<td>8</td>
<td>19</td>
</tr>
</tbody>
</table>

5.6 Comparison of the Three Lane Change Maneuver Quantification Methods

Section 5.3 to Section 5.5 evaluated three LCM quantification methods, the Assumed Percentage Method, the Short Gap Method and the Long Vehicle Method. All three methods seek to estimate the number of LCMs by using an assumed $PB$. The performance of a LCM quantification method depends on how close the assumed $PB$ is to the optimal $PB$. As $PB$ is varied within its feasible range, 0% to 100%, the results can reveal both the worst and the best performance of a given method, as well as the sensitive of the method to $PB$. Such sensitivity analysis was conducted individually for each method in Section 5.3.2, Section 5.4.2 and Section 5.5.2 with a step size of 10% in each
of the four lanes. The sensitivity analysis results are further studied and compared in this section.

The optimal $PB$ likely depends on static factors, such as roadway geometry and the location relative to entrances and exits. It also likely depends on variable factors, such as traffic conditions, weather conditions, time of day and day of week. Ideally the assumed $PB$ should be an output from a model based on the study of the relationship between the optimal $PB$ and other factors. However, such study requires extensive calibration and a large amount of field data, which is currently not available.

In the mean time, since the a priori choice of an accurate $PB$ is resource intensive, it is preferable to develop a LCM quantification method that is robust to the assumed $PB$. Such a LCM quantification method should have reasonable worst and best performance, and should be less sensitive to $PB$. Therefore, Section 5.6.1 and Section 5.6.2 compare the worst and the best performance, respectively, for all three methods. Section 5.6.3 studies the sensitivity of the methods to $PB$. In the absence of an easy way to identify the optimal $PB$, a $PB$ of 50% is suggested. Section 5.6.4 compares the performance of the three proposed methods when $PB$ is set to 50%. Finally this section closes with a discussion in Section 5.6.5.

5.6.1 The Worst Performance from the Sensitivity Analysis

The performance is regarded as the worst if the magnitude of a performance measure is the largest when $PB$ is varied from 0% to 100%. The sensitivity analysis in Section 5.3.2, Section 5.4.2 and Section 5.5.2 shows that the assumed $PB$ that yields the worst $DIFF$ also tends to yield the worst $MAE$ and $PRE$. It also shows that the assumed $PB$ that yields the worst performance is either 0% or 100%. Table 5.22 tabulates the worst
performance based on the sensitivity analysis for all three proposed methods. It is shown in Table 5.22 that the worst performance for the Long Vehicle Method is much better than that for the Short Gap Method, which in turn is better than that for the Assumed Percentage Method. For example, the worst $DIFF$ from the Long Vehicle Method is only 1/6 to 1/2 of those from the other two methods.

Figures 5.17 and 5.18 compare the worst $PRE_{\_Nen}$ and $PRE_{\_Nex}$, respectively. Figure 5.17 shows that the worst $PRE_{\_Nen}$ in lane 2 is almost 400% for the Assumed Percentage Method, but is just a little bit over 50% for the Long Vehicle Method. In lane 5, the worst $PRE_{\_Nen}$ for the Long Vehicle Method is -56% (according to Table 5.22), which is still smaller in magnitude than that for the Short Gap Method (87% according to Table 5.22) and the Assumed Percentage Method (127% according to Table 5.22). The comparison of $PRE_{\_Nex}$ shows the similar results in Figure 5.18.
Table 5.22 Comparison of the worst performance based on the sensitivity analysis for all three methods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>241 18.5 389 1205</td>
<td>197 15.2 318 985</td>
<td>38 3.4 61 190</td>
<td>176 7.7 226 298</td>
<td>122 5.3 156 207</td>
<td>41 1.9 53 69</td>
<td>153 15.3 222 255</td>
<td>108 10.8 157 180</td>
<td>42 4.2 61 70</td>
</tr>
<tr>
<td>3</td>
<td>41 1.9 53 69</td>
<td></td>
<td></td>
<td>150 16.7 127 165</td>
<td></td>
<td></td>
<td>103 10.3 87 113</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-66 6.7 -56 -73</td>
<td></td>
<td></td>
<td>153 15.3 222 255</td>
<td></td>
<td></td>
<td>108 10.8 157 180</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.17 Comparison of the worst Percentage Relative Error for Nen based on the sensitivity analysis for all three methods

Figure 5.18 Comparison of the worst Percentage Relative Error for Nex based on the sensitivity analysis for all three methods
The comparison of Figures 5.17 and 5.18 shows that the worst \( PRE_{\_Nen} \) is smaller in magnitude than the worst \( PRE_{\_Nex} \) in a given lane for a given method. The reason is that \( DIFF \) is the same for \( Nen \) and \( Nex \) (based on Equation 5.2), but the observed number of entering vehicles is greater than the observed number of exiting vehicles in all four lanes in the study data set, as shown in Table 5.4.

In Figure 5.17, the worst \( PRE_{\_Nen} \) for the Long Vehicle Method stays around 60%. Therefore, the estimated total number of entering vehicles (over the 15-minute time period) from the Long Vehicle Method is within 60% of the observed values even in the worst case. Figure 5.18 shows that the worst \( PRE_{\_Nex} \) for the Long Vehicle Method is about 200% for lane 2, and around 70% for the other three lanes. Such performance might be better than that of some previous studies. For example, Sheu (1999) is the only study found in the literature review of Chapter 2 that conducted LCM quantification based on loop detector data. Sheu (1999) compared the estimated and observed lane-changing fractions (without differentiating the entering and exiting vehicles) over each 5-minute time period. The figures in Sheu (1999) showed that in some samples the observed lane-changing fraction was 10 times larger than the estimated value.

5.6.2 The Best Performance Based on the Optimal PB

In Section 5.3.2, the best \( PB \) is defined to be the \( PB \) value that yields the smallest (in magnitude) \( DIFF \) in a sensitivity analysis. Table 5.11 shows that the best \( PB \) varies from 10% to 40% for the Assumed Percentage Method. Obviously, if a different step size is used, the best \( PB \) could be different. That is, the best \( PB \) and the corresponding best performance are affected by the step size used in the sensitivity analysis. The sensitivity analysis conducted in earlier sections is based on a step size of 10%. If a smaller step was
used, the best $DIFF$, $MAE$ and $PRE$ could be closer to 0. Therefore, the sensitivity analysis in Section 5.3.2, Section 5.4.2 and Section 5.5.2 cannot be directly used to show the best performance of the three LCM quantification methods. On the other hand, the limitation of step size is not an issue for the worst performance, since the worst performance occurs only when the assumed $PB$ is 0% or 100% for the study data set. Therefore, the sensitivity analysis can be used in Section 5.6.1 to compare the worst performance.

As discussed in Appendix A, the $PB$ value that leads to $DIFF$ value of 0 can be used as the optimal $PB$. Appendix A shows the formula to calculate the optimal $PB$ for the Assumed Percentage Method. No such formula is available for the Short Gap Method and the Long Vehicle Method. However, if $DIFF$ is plotted as a function of $PB$ based on the sensitivity analysis, the intercept can be taken as the optimal $PB$. Figures 5.19 to 5.21 plot $DIFF$ versus $PB$ across lanes, one figure for each method. The solid points in Figures 5.19 to 5.21 represent the resulting optimal $PB$. Table 5.23 summarizes the values of the optimal $PB$ for all three methods in all four lanes.
Figure 5.19 The change of DIFF when the assumed PB is varied from 0% to 100% for the Assumed Percentage Method.
Figure 5.20 The change of \textit{DIFF} when the assumed \textit{PB} is varied from 0\% to 100\% for the Short Gap Method.
Figure 5.21 The change of DIFF when the assumed PB is varied from 0% to 100% for the Long Vehicle Method

Table 5.23 Summary of the optimal PB for all three methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assumed Percentage Method</td>
<td>7</td>
<td>20</td>
<td>26</td>
<td>35</td>
</tr>
<tr>
<td>Short Gap Method</td>
<td>8</td>
<td>19</td>
<td>25</td>
<td>46</td>
</tr>
<tr>
<td>Long Vehicle Method</td>
<td>22</td>
<td>55</td>
<td>52</td>
<td>83</td>
</tr>
</tbody>
</table>

Table 5.23 shows that for a given method, the optimal PB is different in different lanes. For example, the optimal PB for the Assumed Percentage Method is 7% in lane 2, which is much smaller than that in lane 5 (35%). The optimal PB in lane 3 and lane 4 are
similar (20% and 26% respectively). The trends shown in Table 5.23 are consistent with those for the LCM fractions in Table 5.5. Therefore, LCM fraction might be a factor that impacts the value of optimal $PB$. If some indication of the general LCM fractions can be measured or inferred (e.g., proximity to a weaving section), it would likely prove beneficial to adjust the assumed $PB$ accordingly.

Based on the definition of optimal $PB$, the $DIFF$, $PRE_{Nen}$ and $PRE_{Nex}$ are all equal to 0 when the optimal $PB$ is used. However, in such a case, the $MAE$ could be different from 0 and they are plotted in Figure 5.22. All $MAE$ shown in Figure 5.22 are less than or equal to 1.6, with most of $MAE$ smaller than 1.0. There is no obvious trend indicating which method is better. Therefore, the performance of all three methods seems similar when the optimal $PB$ is used.

![Figure 5.22 Comparison of the Mean Absolute Error from all three methods when the optimal PB is used](image-url)
5.6.3 Sensitivity Analysis on PB

This section compares the sensitivity of the three LCM quantification methods to PB. All three methods are applied to a given lane and their performance measures are compared when PB is varied from 0% to 100%. Thus, according to Table 5.23, the span of PB includes both the cases where the assumed PB is close to the optimal PB, and the cases where the assumed PB is far from it. The three performance measures defined in Section 5.2.4 are studied and they show similar trends. As an example, Figures 5.23 to 5.26 compare MAE from all three methods, one figure for each lane. In these figures, the solid points represent the optimal PB, obtained from Section 5.6.2.
Figure 5.23 Comparison of the Mean Absolute Error across methods for lane 2 when PB is varied from 0% to 100%
Figure 5.24 Comparison of the Mean Absolute Error across methods for lane 3 when $PB$ is varied from 0% to 100%
Figure 5.25 Comparison of Mean Absolute Error across methods for lane 4 when PB varies from 0% to 100%
Figure 5.26 Comparison of the Mean Absolute Error across methods for lane 5 when $PB$ is varied from 0% to 100%

Figures 5.23 to 5.26 show that at the same distance away from the optimal $PB$, the slope of the curve (i.e., the derivative) for the Long Vehicle Method is smaller than that for the Short Gap Method, which is in turn smaller than that for the Assumed Percentage Method. For example in lane 2, when the assumed $PB$ is 20% larger than the optimal $PB$ (i.e., the assumed $PB$ minus the optimal $PB$ is 20%), the slopes are about 3, 16 and 20 for the Long Vehicle Method, the Short Gap Method and the Assumed Percentage Method, respectively. Thus the Long Vehicle Method is less sensitive to the a priori choice of $PB$ than the other two methods, which is consistent with the expectation discussed at the beginning of Section 4.4. Alternatively, to achieve similar performance, the Long Vehicle
Method allows for a larger offset in the assumed $PB$ from the optimal $PB$ than the other two methods. Based on Figures 5.23 to 5.26, Table 5.24 summarizes the range of assumed $PB$ that can yield $MAE$ less than 2. For example, Table 5.24 shows that in lane 2, any assumed $PB$ between 0% and 15% can yield $MAE$ less than 2 if the Assumed Percentage Method is used. It is clear from Table 5.24 that the range for the Long Vehicle Method is wider than those for the other two methods in all four lanes. As a result, without any further information about the optimal PB, it is more likely that an assumed $PB$ will fall within such a range when the Long Vehicle Method is used.

Table 5.24 Comparison of the range of assumed $PB$ that can yield Mean Absolute Error less than 2

<table>
<thead>
<tr>
<th>Assumed Percentage Method</th>
<th>Short Gap Method</th>
<th>Long Vehicle Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range (%), Length of the range</td>
<td>Range (%), Length of the range</td>
<td>Range (%), Length of the range</td>
</tr>
<tr>
<td>Lane 2 [0, 15], 15</td>
<td>[0, 16], 16</td>
<td>[0, 61], 61</td>
</tr>
<tr>
<td>Lane 3 [0, 40], 40</td>
<td>[0, 48], 48</td>
<td>[0, 100], 100</td>
</tr>
<tr>
<td>Lane 4 [17, 34], 17</td>
<td>[11, 38], 27</td>
<td>[30, 74], 44</td>
</tr>
<tr>
<td>Lane 5 [29, 43], 14</td>
<td>[37, 55], 18</td>
<td>[64, 89], 25</td>
</tr>
</tbody>
</table>

5.6.4 Comparison of the Performance when $PB$ is set to 50%

In the absence of an easy way to identify the optimal $PB$, a $PB$ of 50% is suggested. Therefore, this section compares the performance of the three proposed methods when $PB$ is set to 50%.
Figures 5.27 to 5.30 compare the residual (i.e., estimated value minus the observed value) in each gap (between each pair of consecutive reidentified vehicles) from all three methods, one figure for each lane. According to Equation 5.2, the residual for $N_{en}$ is equal to that for $N_{ex}$, so the results shown in Figures 5.27 to 5.30 do not differentiate between $N_{en}$ and $N_{ex}$. Figures 5.27 to 5.29 show that the residuals from the Long Vehicle Method are typically much closer to 0 than the other two methods in lane 2 to lane 4. The Short Gap Method performs the best in lane 5, as shown in Figure 5.30.

![Figure 5.27 Comparison of the residual between each pair of consecutive reidentified vehicles in lane 2 when $PB$ is set to 50%](image)
Figure 5.28 Comparison of the residual between each pair of consecutive reidentified vehicles in lane 3 when $PB$ is set to 50%.

Figure 5.29 Comparison of the residual between each pair of consecutive reidentified vehicles in lane 4 when $PB$ is set to 50%.
Table 5.25 summarizes the number of gaps in which a given method yields the smallest (in magnitude) residual. Whenever two or three methods tie for the smallest (in magnitude) residual in a gap, they are all counted. Thus, the sum across methods is larger than the total number of gaps in Table 5.25. Based on Table 5.25, the Long Vehicle Method yields the smallest (in magnitude) residual for 46 out of 56 gaps in all four lanes, or equivalently 82% of the gaps. In lane 2 to lane 4, the Long Vehicle Method yields most smallest (in magnitude) residuals (varying from 87% to 92%). In lane 5, the Long Vehicle Method and the Short Gap Method tie in terms of the number of smallest (in magnitude) residuals.
Table 5.25 Summary of the residual comparison when $PB$ is set to 50%

<table>
<thead>
<tr>
<th></th>
<th>Total number of gaps</th>
<th>Number of gaps in which the smallest (in magnitude) residual is from</th>
<th>Percentage of gaps in which the smallest (in magnitude) residual is from</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Assumed Percentage Method</td>
<td>Short Gap Method</td>
</tr>
<tr>
<td>Lane 2</td>
<td>13</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lane 3</td>
<td>23</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Lane 4</td>
<td>10</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lane 5</td>
<td>10</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Sum</td>
<td>56</td>
<td>11</td>
<td>15</td>
</tr>
</tbody>
</table>

At the aggregate level, Figure 5.31 compares the $MAE$ when $PB$ is set to 50%. The Long Vehicle Method has the smallest $MAE$ in lanes 2 through 4, while the Short Gap Method has the smallest $MAE$ in lane 5. This observation is consistent with the results in Figures 5.27 to 5.30 and Table 5.25. As one would expect, the $MAE$ in Figure 5.31 are larger than those in Figure 5.22 where the optimal $PB$ is used.
Figure 5.31 Comparison of the Mean Absolute Error from all three methods when PB is set to 50%

5.6.5 Discussion

PB is an important input to the three proposed LCM quantification methods. However, developing a method to collect or estimate a good assumed PB likely requires extensive calibration and a large amount of field data, which are not currently available. Therefore, the present study focuses on developing a LCM quantification method that is robust to the assumed PB. The analysis in Section 5.6 shows that the Long Vehicle Method is more robust to the assumed PB for the study data set than the other two methods.

Section 5.6.1 shows that the worst performance for the Long Vehicle Method is much better than that for the other two methods. On the other hand, Section 5.6.2 shows that the best performance for all three methods is similar. The Long Vehicle Method is also less sensitive to PB, as shown in Section 5.6.3. Therefore, the range of the assumed PB that
can achieve the given performance based on the Long Vehicle Method is wider than those based on the other two methods. As a result, without any further information about the optimal $PB$, it is more likely that an assumed $PB$ will fall within such a range when the Long Vehicle Method is used.

Given the limited amount of data for development and validation, it is left to future research to identify the optimal $PB$. Currently it is recommended to use an assumed $PB$ of 50% in the absence of additional information. Section 5.6.4 shows that when $PB$ is set to 50%, the Long Vehicle Method yields the smallest (in magnitude) residual for 46 out of 56 gaps in all four lanes, or equivalently 82% of the gaps. If additional information is available, the assumed $PB$ can use values other than 50%. For example, Section 5.6.2 shows that the optimal $PB$ depends on the LCM fractions. Therefore, the assumed $PB$ for a highway segment that is believed to have a higher LCM fraction, e.g., in a weaving section, can use a value greater than 50%. Similarly, the assumed $PB$ for a highway segment that is believed to have a lower LCM fraction, e.g., on a very congested segment where it is difficult to make LCMs, can use a value smaller than 50%.

One of the reasons for the good performance of the Long Vehicle Method is that the VRI results used in this study are simulated based on the VRI algorithm proposed in Chapter 3, which only matches long vehicles. Thus, the reidentification rate of long vehicles is much higher than that of all vehicles. However, if a different VRI algorithm was used, such as the one in Coifman and Cassidy (2002) that matches vehicles in all lengths, conceivably the Long Vehicle Method could lose its advantage. In such a case, the Short Gap Method might be a good choice because the estimated number of LCMs in short gaps should still be better than those in long gaps. In addition, the advantages of the
Long Vehicle Method seen in this chapter may diminish or disappear if the LCM patterns of long vehicles are not correlated with that of the entire fleet, e.g., upstream of a diverge the trucks tend to travel towards one destination and the commuters to the other. It is left to future research to study the impact of such discrepancies after additional data sets are collected.
CHAPTER 6 CONCLUSIONS AND FUTURE WORK

This chapter summarizes the dissertation, and discusses the future work warranted by this research. Section 6.1 summarizes the dissertation and its contributions. Section 6.2 then outlines the directions for future work to improve the proposed Vehicle Reidentification (VRI) algorithm and the Lane Change Maneuver (LCM) quantification methods.

6.1 Summary and Conclusions

Lane Change Maneuvers (LCMs) are an important factor in traffic flow theory. However, the impacts of LCMs are often neglected because it is resource intensive to collect empirical LCM data. Field collection is time and labor intensive, even with the help of image processing technology, e.g., as employed in the NGSIM data reduction (Alexiadis et al., 2004). In response to the need for LCM quantification, this dissertation proposes an approach to estimate the number of vehicles entering a lane (Nen) and the number of vehicles exiting a lane (Nex) separately. The proposed approach is compatible with existing vehicle detectors, and it only requires data collected at two detector stations to estimate the number of LCMs between them.

The proposed LCM quantification approach employs recent advances in Vehicle Reidentification (VRI), a process to match a vehicle observation at one detector station to an observation of the same vehicle at another station. While the proposed LCM
quantification approach is compatible with many existing VRI algorithms, a new VRI algorithm was developed in this dissertation for two reasons. First, the new VRI algorithm is based on conventional loop detectors that are widely used in practice. LCM quantification built on such a VRI algorithm can thus be quickly implemented with a relatively small investment. Second, the new VRI algorithm is designed to combine the strengths from two preceding VRI algorithms that are based on conventional loop detectors. The new algorithm is more robust than either of the predecessors. Borrowing ideas from Coifman and Krishnamurthy (2007), the new algorithm exploits the fact that consecutive long vehicles usually exhibit similar time to traverse the link. Borrowing ideas from Coifman and Cassidy (2002), the new algorithm checks the vehicle arrival numbers at the detector stations. Different from the two preceding algorithms, this new algorithm develops a more effective method to search for true matches from the possible matches, and incorporates new constraints to remove false positives. Therefore, the new VRI algorithm is more robust to the change of traffic conditions than the two preceding algorithms. LCM quantification built on such a VRI algorithm can thus yield the estimated number of LCMs in different traffic conditions.

The new VRI algorithm was tested over four overlapping segments of I-80 in California using dual loop detector data collected on July 15th, 2003 from 1:00 to 24:00. The VRI results demonstrated that about 40% of long vehicles observed at the downstream station were reidentified over segments on the order of one mile long. The algorithm was validated by comparing the resulting link speeds from VRI and the local speeds at the detector stations. It was shown that the trends of the resulting link speeds over different time periods are consistent with the local speeds of long vehicles when the
traffic state in the segment is roughly homogeneous (whether it is free flow or congested). When the traffic state in the segment is not homogeneous, the VRI results are consistent with what one would expect from traffic flow theory. Therefore, the proposed VRI algorithm performs well for the study data sets.

Based on the VRI results, the difference in the number of vehicles entering the lane (Nen) and the number of vehicles exiting the lane (Nex) between a pair of consecutive reidentified vehicles can be calculated. This difference, which is called inflow, provides a constraint on the possible values of Nen and Nex. The lower bounds for Nen and Nex are defined based on the facts that neither of them can be negative and their difference should be equal to the inflow. The upper bounds for Nen and Nex are developed following the assumption that no single vehicle both enters and exits the lane (or vice versa) between the upstream and downstream stations. Thus, the VRI results provide the inflow constraint, as well as the lower and upper bounds on Nen and Nex. These values are used by the proposed LCM quantification approach. It is worth noting that, in this work, both VRI and LCM quantification are applied on a per lane basis, i.e., the results in one lane are derived without using any information from the other lanes.

The basic LCM quantification method was termed the Assumed Percentage Method. The Assumed Percentage Method employs the upper and lower bounds on Nen and Nex. The critical step in the Assumed Percentage Method is to estimate where Nen and Nex fall within the bounds, i.e., to determine the value of the assumed “Percentage in Bounds” (PB). Two variants of the Assumed Percentage Method were also developed to improve the LCM quantification results, namely, the Short Gap Method and the Long Vehicle Method. These two variants seek to calculate the Lane Change Maneuver Rate (LCMR)
from samples believed to produce more accurate LCM estimates and apply these calculated LCMR to samples believed to produce less accurate LCM estimates.

In the Short Gap Method, if the number of all vehicles between two consecutive reidentified vehicles is small enough (short gaps), the Assumed Percentage Method is used to estimate the number of LCMs and LCMR. Otherwise, the number of LCMs is estimated as the product of the number of vehicles between the reidentified vehicles and the LCMR calculated from the previous short gaps. The Long Vehicle Method is motivated by the fact that the reidentification rate is much higher for long vehicles than it is for the general vehicle fleet. In the Long Vehicle Method, the Assumed Percentage Method is applied strictly to long vehicles. The estimates from the long vehicles are used to calculate the LCMR, which in turn are used to estimate the number of LCMs made by all vehicles.

The Assumed Percentage Method, the Short Gap Method, and the Long Vehicle Method were evaluated using a 15-minute vehicle trajectory data set collected from a segment of I-80 that is about 1,600 feet long. The vehicle trajectory data set is one of the few sources that include the ground truth LCM information, which is crucial to the evaluation of LCM quantification methods. Since the vehicle trajectory data set does not include loop detector data that can be used for VRI, the VRI results are simulated to be consistent with the empirical performance of the proposed VRI algorithm.

It was shown that the performances of the Assumed Percentage Method, the Short Gap Method, and the Long Vehicle Method depend on how close the assumed $PB$ is to the optimal $PB$. The present study did not conduct a study to estimate the optimal $PB$, since such a study requires extensive calibration and a large amount of field data, which
is currently not available. In general the optimal $PB$ is unknown. Therefore, it is preferable to employ a LCM quantification method that is robust to the errors arising from a poor choice of the assumed $PB$.

Sensitivity analyses were conducted for the Assumed Percentage Method, the Short Gap Method, and the Long Vehicle Method by varying the assumed $PB$ between 0% and 100%. It was seen that all three methods can yield similarly good results if one is fortunate to choose an assumed $PB$ that is close to the unknown optimal $PB$. When the assumed $PB$ is far from the optimal $PB$, the worst case performance of the Long Vehicle Method is much better than that of the other two methods. In the worst case, the difference between the estimated and observed total number of entering vehicles over the 15-minute time period from the Long Vehicle Method was only 1/6 to 1/2 of those from the other two methods. It appears that the Long Vehicle Method is more likely to yield good estimates for a given assumed $PB$. When $PB$ was set to 50%, the Long Vehicle Method yielded the smallest residual for 46 out of 56 estimates in all four lanes, or equivalently 82% of the estimates. The Long Vehicle Method also had the smallest Mean Absolute Error ($MAE$) in three of the four studied lanes when $PB$ was set to 50%.

This dissertation made two major contributions. First, a new VRI algorithm was developed. Second, the approach of LCM quantification based on VRI was initiated, and three variants were proposed to estimate $Nen$ and $Nex$ separately.

The VRI algorithm developed in this dissertation is more robust to the change of traffic conditions than the two preceding algorithms upon which it is built. The VRI algorithm in Coifman and Cassidy (2002) can only be applied when the resulting link speeds are less than 45 mph. The VRI algorithm in Coifman and Krishnamurthy (2007)
allows VRI from free flow conditions down to an average link speed of 20 mph. The new VRI algorithm developed in this dissertation proposed a method to search for true matches from the possible matches. This method enables the new VRI algorithm to reidentify vehicles in challenging conditions.

Although future research could improve the performance of the new VRI algorithm, it was shown that the new VRI algorithm was able to reidentify vehicles when the traffic conditions change between free flow and congestion. It was also shown that the new VRI algorithm reidentified vehicles even when the resulting link speeds were below 20 mph. Such features are important because the LCM quantification methods based on the new VRI algorithm can thus yield the estimated number of LCMs in different traffic conditions. As a VRI algorithm, this new VRI algorithm can also be applied in traffic surveillance and accident detection by monitoring the change of link speed.

The proposed LCM quantification approach is important because it provides one of the first viable means to estimate the number of LCMs over extended distances. Despite the important impacts of LCMs, the literature review indicated that research on LCMs has been limited by a shortage of field data. The collection of LCM data currently requires labor-intensive efforts to extract the information from film or video. Image processing technologies are starting to help in this task, but for obtaining accurate LCM data the labor demands remain high. The approach presented in this dissertation is the first known effort to estimate Nen and Nex using data from conventional loop detectors. Since conventional loop detectors remain the most widely used vehicle detection technology, this approach requires a small investment and promises a quick deployment.

Compared to manually collecting LCM information from film or video, this approach is
less accurate. However, this approach does not require the entire segment to be under
direct surveillance, and it can easily provide the estimated number of LCMs with much
less effort.

The performance of the proposed LCM quantification methods depends on how close
the assumed $PB$ is to the optimal $PB$. Although the present study did not provide insight
on choosing a good value of $PB$ for practical implementation, it was shown that the Long
Vehicle Method could yield reasonable results even in the worst case. For example, the
estimated total number of entering vehicles (over the 15-minute time period) from the
Long Vehicle Method is within 60% of the observed values in the worst case scenario.
The evaluation of the proposed LCM quantification methods was based on the simulated
VRI results in which only about 4% of vehicles observed at the downstream stations were
reidentified. The accuracy of the estimated $N_{en}$ and $N_{ex}$ could be much better if the
proposed LCM quantification methods were applied to other VRI algorithms with much
higher reidentification rate (e.g., MacCarley, 2001 with reidentification rate of 94% using
video based detector system).

Although some limitations remain and more research is needed, the proposed LCM
quantification approach could eventually be used to yield the estimated number of LCMs
from conventional loop detector data. The estimated number of LCMs from the proposed
LCM quantification approach could potentially be used to improve traffic flow theory
and the understanding of the mechanisms underlying traffic congestion. It could also be
used in the calibration and validation of LCM models and microscopic traffic simulation
models. Ultimately, the estimated numbers of LCMs could potentially be used in traffic
control and management to reduce traffic delay, decrease accident rates, and increase
efficiency of the existing transportation infrastructure. Section 2.3 elaborated on the potential applications of the estimated number of LCMs.

6.2 Future Work

This section outlines several directions for future work to improve the VRI algorithm and the LCM quantification approach proposed in this dissertation.

6.2.1 Future Work on the Vehicle Reidentification Algorithm

Two parameters were used in the proposed VRI algorithm. The parameter values were selected after limited number of trials on different values. It would be beneficial to investigate how the performance of the proposed VRI algorithm is affected by these parameters. The first parameter is the long vehicle threshold. In this study, long vehicles are defined as vehicles longer than 23 feet, which corresponds roughly to the 90th percentile vehicle length observed in the study data set. It is expected that if the long vehicle threshold is increased, the VRI algorithm will match fewer vehicles because the algorithm will be seeking matches for fewer vehicles. However, the final matches may be more likely to be true matches because the disturbance from the false positives will be smaller. Similarly, if the long vehicle threshold is decreased, more vehicles will be matched, but it is likely that more of the final matches will be incorrect. When the long vehicle threshold is low enough, the false positives will obscure the trend of true matches, and the algorithm will not perform well. It would be interesting to increase or decrease the long vehicle threshold and study the performance of the proposed VRI algorithm.

The second parameter used in the proposed VRI algorithm is the close threshold. The link travel times for two possible matches are considered to be “close” if they differ by
less than 10 seconds. A larger threshold will lead to more false positives being accepted by the algorithm, and a smaller threshold will lead to rejecting more true possible matches. As with the long vehicle threshold, it would be interesting to investigate the sensitivity of VRI results to the close threshold.

The VRI algorithm could be further improved by studying the following two ideas. The first idea is to investigate the benefits of using a vehicle sequence length\(^{11}\) that is weighted by the constituent vehicles. For example, longer vehicles (or otherwise more unique vehicles) would have higher weights in the calculation of the sequence length. The second idea is to replace the constant close threshold with a function of the segment length, posted speed limit and geometric features.

Additional work to validate the proposed VRI algorithm would be beneficial. The following two validation methods could be applied, depending on the availability of the data. The first method is to compare the link travel time estimated from VRI results against the observed link travel time. The observed link travel time can be collected by probe vehicles, or by Automatic Vehicle Identification (AVI), as discussed in Section 2.1.1. The second method is to validate the VRI results on a vehicle level basis. Such validation requires a data set that includes both the raw loop detector data and a system that can produce ground truth of vehicle reidentification.

6.2.2 Future Work on the Lane Change Maneuver Quantification Approach

In the present study, vehicle trajectory data were used to simulate the VRI results by assuming that about 70% of long through vehicles can be reidentified, i.e., the Long Vehicle Reidentification Rate (LVRIR) is 70%. The 70% value is roughly consistent with

\(^{11}\) See Section 3.2.3 for the definition of vehicle sequence and how its length is calculated.
the performance of the VRI algorithm presented in Section 5.1. The performance of the LCM quantification methods depends directly on the VRI results. Therefore, it would be valuable to investigate the sensitivity of the performance of the LCM quantification methods to LVRIR. One could fix the first and last reidentified vehicles in the data set and change the number of reidentified vehicles between them. This approach indirectly changes the LVRIR. Such analysis would show if the performance shown in Chapter 5 is dependent on the assumed 70% rate used in the empirical study.

Another parameter that should be tested is the short gap threshold used in the Short Gap Method: a gap is regarded as a short gap if there are fewer than ten non-reidentified vehicles between a pair of consecutive reidentified vehicles. Similar to the assumed 70% LVRIR, it would be valuable to investigate the sensitivity of the performance of the Short Gap Method to the short gap threshold.

It is also important to study the impact of the reidentification errors to the performance of the LCM quantification methods. Currently vehicle trajectory data were used to simulate the VRI results, and all the reidentified vehicles have correct matches. However, VRI algorithms could produce some incorrect matches, which lead to invalid bounds and inflow constraint in the LCM quantification. Based on the trajectory data set, a reidentification error could be introduced by incorrectly matching a downstream long vehicle to an upstream long vehicle. There are two challenges in the study of the impact of the reidentification errors. First, it is challenging to realistically replicate the reidentification errors observed from the empirical study. Second, it is challenging to define the observed number of LCMs in the presence of reidentification errors. In the present study, no reidentification errors were introduced and the observed number of
LCMs is obtained by counting the number of LCMs in the time-space region bounded by the trajectories of reidentified vehicles and the locations of the upstream and downstream stations. However, such a method to count the observed number of LCMs cannot be applied after reidentification errors are introduced, since the trajectory of the incorrect match does not exist.

This dissertation studied the performance of the proposed LCM quantification methods based on a 15-minute trajectory data set when the traffic was congested. It would be beneficial to extend the present study to other trajectory data sets to study the performance of the proposed LCM quantification methods in different traffic conditions at different locations. NGSIM (FHWA, 2008) includes two other 15-minute trajectory data sets collected from the same segment on the same day as the one studied in the dissertation. Another 45-minute trajectory data set (segmented into three 15 minute periods) provided by NGSIM was collected from U.S. Highway 101 (Hollywood Freeway) in Los Angeles, California in 2005. In 1983, Turner-Fairbank Highway Research Center (TFHRC) also collected several trajectory data sets from various locations in the United States (Smith, 1985) that are available to the public.

An important improvement to the proposed LCM quantification methods would be developing a model that can help determine an appropriate \( PB \) to use for LCM quantification. The model would likely come from the study of the relationship between the optimal \( PB \) and other factors, such as traffic conditions, the presence or absence of ramps, the LCM frequency, the performance of the VRI algorithm and whether the lane is an inside lane, middle lane, or outside lane. Such a study would likely require extensive calibration and a large amount of field data, which is not currently available. As a starting
point, Section 5.6.2 showed the impact of LCM fractions to the optimal $PB$. Appendix A also formulated the optimal $PB$ for the Assumed Percentage Method. It was shown that the number of through vehicles is an important factor that affects the optimal $PB$.

The present study sought to estimate the number of LCMs without counting the LCMs caused by vehicles that both enter and exit, or both exit and enter a lane between the upstream and downstream stations, i.e., pass through and back and forth vehicles. However, pass through vehicles are omitted only in the lane that they pass through. They are still counted as exiting vehicles in their original lanes and as entering vehicles in the target lanes. The present study considers each lane independently. Future work should focus on integrating LCM information across lanes to deduce that an exit from the original lane corresponds to an entrance to the target lane. If the original lane and the target lane are not contiguous, the lane between them must have been passed through. The challenge to integrate LCM information across lanes is that the vehicles are not reidentified concurrently in different lanes, so the number of LCMs estimated from the present study is for different time intervals in different lanes. To include the number of LCMs caused by pass through and back and forth vehicles, the future research can also try to develop a model to directly estimate the rates of vehicles that undertake multiple LCMs between detector stations. As a starting point, Coifman et al. (2006) proposed a method to estimate delay caused by LCMs within a given lane relative to the situation in which no LCMs had taken place. Such a study has the potential to estimate the number of LCMs that includes the impact of pass through and back and forth vehicles.

The performance of the proposed LCM quantification methods was evaluated based on the proposed VRI algorithm that matches long vehicles using loop detector data. The
LCM quantification methods have implications for many other VRI algorithms as well. For example, Wang and Coifman (2007) applied the Assumed Percentage Method to the simulated VRI results based on a platoon-matching VRI algorithm. Therefore, the proposed LCM quantification methods and the basic ideas could be tested based on the VRI results from different VRI algorithms in the future studies.
Reference


Appendix A Unbiased PB for the Assumed Percentage Method

The unbiased $PB$ is defined as the $PB$ value that yields $DIFF$ of 0. That is, when the unbiased $PB$ is used, the estimated total number of LCMs over a given time period is equal to the observed value. $PRE$ is also equal to 0 with the unbiased $PB$. Based on Section 5.3.2, such $PB$ usually has small (in magnitude) $MAE$ values. Therefore, the unbiased $PB$ can be used as the optimal $PB$, although the optimal $PB$ can be defined in other ways depending on the definition of “optimal”. For example, the optimal $PB$ could also be defined as the $PB$ value that yields the smallest $MAE$ over a given time period.

This appendix focuses on the formulation of the unbiased $PB$ for the Assumed Percentage Method. Such formulation reveals the factors that affect the optimal $PB$ and could be used to narrow the range of $PB$ (currently it is 0% to 100%) in the Assumed Percentage Method.

Assuming that a VRI algorithm reidentifies $M + 1$ vehicles on a segment over a time period, so there are $M$ pairs of consecutive reidentified vehicles. The estimated total number of entering vehicles is:
\[ \hat{N}_{en}^T = \sum_{i=1}^{M} \hat{N}_{en_i} \]  \hspace{1cm} (A.1)

\[ = \sum_{i=1}^{M} \left( \max(0, \text{inflow}_i) + PB \times (N_{\text{gap},i}^{\text{up}} - \max(0, \text{inflow}_i)) \right) \]

\[ = \sum_{i=1}^{M} \left( \max(0, \text{inflow}_i) + PB \times \left( \sum_{i=1}^{M} N_{\text{gap},i}^{\text{up}} - \sum_{i=1}^{M} \max(0, \text{inflow}_i) \right) \right) \]

\[ = \sum_{\text{inflow} \geq 0} \text{inflow}_i + PB \times \left( N_{dn}^T - M - 1 - \sum_{\text{inflow} \geq 0} \text{inflow}_i \right) \]

\[ = N_{\text{inflow}^+} + PB \times \left( N_{dn}^T - N_{\text{inflow}^+} - M - 1 \right) \]

where,

\( \hat{N}_{en}^T \) is the estimated total number of entering vehicles over a time period,

\( M \) is the number of reidentified vehicle pairs over a time period, so \( M + 1 \) is the number of reidentified vehicles over a time period,

\( N_{dn}^T \) is the total number of vehicles observed at the downstream station over a time period, and

\( N_{\text{inflow}^+} \) is the sum of positive inflows over a time period, i.e., \( N_{\text{inflow}^+} = \sum_{\text{inflow} \geq 0} \text{inflow}_i \).

The present study does not account for LCMs caused by pass through or back and forth vehicles. Therefore, the vehicles observed at the downstream station are either through vehicles or entering vehicles. The observed \( N_{en} \) would then be equal to the number of vehicles observed at the downstream station minus the number of through vehicles. That is:
\[
N'\text{en}^T = N_{dn}^T - N_{\text{through}}^T
\]  
(A.2)

where,

\(N'\text{en}^T\) is the observed total number of entering vehicles over a time period, and 
\(N_{\text{through}}^T\) is the total number of through vehicles over a time period.

To force the estimated total number of entering vehicles \((\hat{N}'\text{en}^T)\) in Equation A.1 to be equal to the observed total number of entering vehicles \((N'\text{en}^T)\) in Equation A.2, the unbiased \(PB\) must be:

\[
P\text{Ben}\star = \frac{N_{dn}^T - N_{\text{inflow}^+} - N_{\text{through}}^T}{N_{dn}^T - N_{\text{inflow}^+} - M - 1}
\]  
(A.3)

where,

\(P\text{Ben}\star\) is the unbiased \(PB\) following the analysis on the number of entering vehicles.

Similar analysis on the number of exiting vehicles leads to a second form for the unbiased \(PB\) as follows:

\[
P\text{Ex}\star = \frac{N_{up}^T - N_{\text{inflow}^-} - N_{\text{through}}^T}{N_{up}^T - N_{\text{inflow}^-} - M - 1}
\]  
(A.4)

where,

\(P\text{Ex}\star\) is the unbiased \(PB\) following the analysis on the number of exiting vehicles,

\(N_{up}^T\) is the total number of vehicles observed at the upstream station over a time period, and
$N_{inflow-}$ is the sum of the magnitude of negative inflows over a time period, i.e.,

$$N_{inflow-} = \sum_{inflow_i < 0}^{-inflow_i} .$$

Further investigation shows:

$$N_{dn}^T - N_{up}^T = \sum_{i=1}^{M} inflow_i$$

$$= \sum_{inflow_i \geq 0} inflow_i + \sum_{inflow_i < 0} inflow_i$$

$$= \sum_{inflow_i \geq 0} inflow_i - \sum_{inflow_i < 0} -inflow_i$$

$$= N_{inflow+} - N_{inflow-}$$

Therefore, $N_{dn}^T - N_{inflow+} = N_{up}^T - N_{inflow-}$ and the right side of Equation A.3 is equivalent to the right side of Equation A.4. The unbiased $PB$ are thus the same for Nen and Nex, that is:

$$PB_{en}^* = PB_{ex}^*$$

(A.6)

Henceforth $PB^*$ will be used in place of $PB_{en}^*$ and $PB_{ex}^*$. Table A.1 shows the unbiased $PB$ from Equation A.3 and the corresponding performance measures based on the Assumed Percentage Method in all four lanes. These results are based on the data set presented in Section 5.2.
Table A.1 Summary of the unbiased $PB$ for the Assumed Percentage Method

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Unbiased PB</td>
<td>7%</td>
<td>20%</td>
<td>26%</td>
<td>35%</td>
</tr>
<tr>
<td>DIFF</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>MAE</td>
<td>1.1</td>
<td>0.6</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>PRE_Nen (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PRE_Nex (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

The comparison of Table A.1 and Table 5.11 shows that the performance of the Assumed Percentage Method based on the unbiased $PB$ is better than that based on the best $PB$ from the sensitivity analysis. In Table A.1, the DIFF and PRE are all 0 and the MAE are at least as good as those in Table 5.11. If the step size in sensitivity analysis is small enough, the best $PB$ from sensitivity analysis should be equal to the unbiased $PB$.

The estimates using the unbiased $PB$ in lane 4 (26%) are plotted in Figure A.1, as a comparison to Figure 5.13 where the assumed $PB$ is 50%, and to Figure 5.14 where the assumed $PB$ is 30%. As one would expect, Figure A.1 shows that the results are slightly better than Figure 5.14 and much better than Figure 5.13. Once again, the results show that if a good $PB$ is selected, the Assumed Percentage Method can yield good estimates on the number of LCMs.
Figure A.1 Estimation of the number of LCMs in lane 4 based on the Assumed Percentage Method with assumed $PB$ of 26%.

Although the results based on the unbiased $PB$ look good, it is not possible to obtain the value based strictly on the observations from the loop detector stations. In Equation A.3, $N_{dn}^T$, $N_{inflow}^+$, and $M$ can be obtained from loop detector data but $N_{through}^T$ cannot.

The value of Equation A.3 is that it shows the impact factors of the unbiased $PB$. In addition, Equation A.3 can be used to calculate the corresponding unbiased $PB$ if it is possible to get the range of the total number of through vehicles ($N_{through}^T$). $PB$ is a new concept proposed in this dissertation. It could be difficult to determine an assumed $PB$. However, it might be possible to provide the range of $N_{through}^T$ based on experience,
segment location, traffic information, lane information, or even the performance of the VRI algorithm, as the example shown below. In such a case, Equation A.3 can be used to covert the range of $N_{through}^T$ to the range of assumed $PB$. To ensure the range of assumed $PB$ to be between 0% and 100%, the assumed total number of through vehicles should be between $M + 1$ and $N_{dn}^T - N_{inflow+}$ based on Equation A.3.

In the present study, the long vehicles account for about 10% of all vehicles observed at the downstream station. It has been assumed that the adopted VRI algorithm can match about 70% of long through vehicles. Based on these values, it is assumed that the adopted VRI algorithm can match a range of 3% to 15% of through vehicles. Other numbers for the range could also be used. These two numbers are just examples used for illustration purposes. Table A.2 shows the range of $N_{through}^T$ based on the assumed performance of VRI algorithm, and the corresponding range of the unbiased $PB$ in lane 2 to lane 5. In Table A.2, the largest $N_{through}^T$ is calculated based on the assumption that the VRI algorithm can match at least 3% of through vehicles. However, all largest $N_{through}^T$ in Table A.2 are greater than $N_{dn}^T - N_{inflow+}$, so the resulting $PB$ are all negative. In Table A.2, 0% is used to replace all the negative values for $PB$. On the other hand, $PB$ based on the smallest $N_{through}^T$ are all less than 100%. Therefore, the range of $PB$ is narrowed after the assumed range of $N_{through}^T$ is considered. For example, Table A.2 shows the range of $PB$ is now 0% to 70% in lane 4, instead of 0% to 100% as what is used before. In such a case, the worst (biggest in magnitude) $DIFF$ is 91 based on Table 5.9. However, if the range of $N_{through}^T$ is not provided and the assumed $PB$ has to be chosen from 0% to 100%,
the worst \( DIFF \) is 153 based on Table 5.9. Therefore, it is possible to use the formula of the unbiased \( PB \) for the Assumed Percentage Method to narrow the range of feasible \( PB \), which helps choose an assumed \( PB \) closer to the optimal value.

Table A.2 The range of unbiased \( PB \) given the range of total number of through vehicles over a time period

<table>
<thead>
<tr>
<th></th>
<th>Lane 2</th>
<th>Lane 3</th>
<th>Lane 4</th>
<th>Lane 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{dn}^T )</td>
<td>317</td>
<td>278</td>
<td>233</td>
<td>280</td>
</tr>
<tr>
<td>( N_{inflow}^+ )</td>
<td>44</td>
<td>34</td>
<td>16</td>
<td>35</td>
</tr>
<tr>
<td>( M +1 )</td>
<td>14</td>
<td>24</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>Largest ( N_{through}^T ) based on the assumption that the VRI algorithm can match at least 3% of through vehicles</td>
<td>467</td>
<td>800</td>
<td>367</td>
<td>367</td>
</tr>
<tr>
<td>PB based on largest possible ( N_{through}^T )</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>Smallest ( N_{through}^T ) based on the assumption that the VRI algorithm can match at most 15% of through vehicles</td>
<td>93</td>
<td>160</td>
<td>73</td>
<td>73</td>
</tr>
<tr>
<td>PB based on smallest possible ( N_{through}^T )</td>
<td>69%</td>
<td>38%</td>
<td>70%</td>
<td>73%</td>
</tr>
</tbody>
</table>