STATIC AND DYNAMIC NONCOOPERATIVE FISCAL POLICY GAMES
AMONG MAJOR INDUSTRIAL COUNTRIES

DISSERTATION

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By

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CHAPTER I

INTRODUCTION

The last ten years have witnessed increasing talks and efforts for economic policy coordination among industrial countries. After the switch to a flexible exchange rates system, many economists thought that the need for cooperative economic policies would diminish. This would be true because, for example, the "n-1 problem" would not be effective any more. However, due basically to increasing integration of the financial markets around the world, industrial countries are more interdependent than ever and discussions about more cooperation in the design of macroeconomic policies are becoming a hot topic among politicians and economists. The increasing interest to policy coordination was motivated by a steep increase in the U.S. budget deficit starting in 1981, an unprecedented appreciation of the real value of the dollar in the first half of the 1980s and a reversion of the U.S. current account from a surplus in 1981 to an unprecedented annual deficit that averaged over $100b in the 1982-1989 period.

Several attempts have been made to analyze monetary policy and exchange rate coordination among policymakers of industrial countries using a game-theoretic approach. The motivation for these attempts stems from the increasing efforts by the governments of the U.S., Japan, and Germany to intervene in the foreign exchange markets in a
cooperative way. These efforts culminated in the Plaza meeting in September 1985 (where the common objective was to stop the appreciation of the dollar) and the Louvre Accord in February 1987 (where the objective was to terminate the continuous fall of the dollar). Despite the increasing number of research papers dedicated to monetary policy games, there have not been many theoretical works dealing with the subject of fiscal policy coordination. And more important, there have not been many attempts that incorporate explicitly in a game-theoretic framework the relationship between the current account balance and the budget deficit. The motivation for analyzing fiscal policy games is based on the attempts to coordinate fiscal policies as exemplified by the Bonn agreement of 1978 and the continuous talks for adjustment of the fiscal deficits in major industrial countries in the mid 1980s. This paper deals exclusively with fiscal policy interdependence in a two-country framework. However, since fiscal policy coordination is very difficult to sustain, for reasons that will be explained later, the paper follows a noncooperative approach. That is, accepting the fact that fiscal policies are set according to government decisions that are made in parliaments and not by relatively independent Central Bankers, we model the strategic interactions between the fiscal authorities in a noncooperative manner. The cooperative solution is briefly mentioned as a benchmark case.

The paper deals with both static and dynamic (or differential) noncooperative games. First, it analyzes a static (one shot) policy game between two major industrial countries (e.g., U.S. and W. Germany) that could explain the existence of very active (both contractionary and expansionary) national fiscal policies and large current account
disequilibria during the 1980s. The macroeconomic framework that is employed in order to model the interdependence between the two countries is a two-country version of the fixed-price, flexible exchange rates Mundell-Fleming model. Second, the model is then extended to incorporate private investors' forward looking expectations and their relationship to policy announcements. This is done by assuming that the uncovered interest parity condition is true. It is shown that fiscal policy is time-inconsistent since the government has an incentive to renege on its preannounced policy. This framework could possibly explain the increasing value of the U.S. dollar in the 1988-1989 period in a game-theoretic framework. It is also shown that if the government has the appropriate commitment technology to precommit against the private sector, its welfare will increase. A simplifying (though not restrictive) assumption made is that the two countries are symmetric as far as the loss functions and the transmission effects are concerned. Third, explicit structural dynamics are introduced in the model by considering the stock of foreign assets that adjusts over time in response to fiscal policy changes in the two countries. This implies that a differential game can be analyzed where different equilibrium strategies can be chosen by the two players according to the information structure assumed in the game. In this paper, we assume that each player reacts to the current state of the economy according to a feedback equation and, therefore, a feedback or closed loop Nash equilibrium is derived. This equilibrium strategy is subgame perfect since it will be chosen even if the players are off the equilibrium path. In general, differential games are difficult to solve because closed-form solutions do not always exist.
However, for quadratic-linear games (i.e., games where the objective function is quadratic and the constraints are linear) a linear solution can be found. Due to the complexity of the model a numerical solution is provided based on a set of parameter values.

The paper is organized as follows: Chapter II provides a broad survey of macroeconomic (both fiscal and monetary) policy games, discusses the literature on the gains from policy coordination and lists some arguments justifying why cooperation is useful. Chapter III analyzes the basic static fiscal policy game using a two-country Mundell-Fleming model. The Nash noncooperative solution is derived and compared with the cooperative solution. In addition, a Stackelberg (or leadership) solution is derived. Chapter IV presents the two-period game played among private investors and the two governments. This game is a natural extension of the previous game but it is more realistic in that the private sector forward looking expectations about the stance of fiscal policy are explicitly incorporated. Chapter V presents the differential game and derives a numerical closed loop (subgame perfect) Nash solution. Finally, Chapter VI summarizes the results of the paper and provides some ideas for future extensions.
CHAPTER II

REVIEW OF THE LITERATURE

There are two ways to approach problems of interaction between different countries. The first is the approach of cooperation proposed by Cooper. He argues that as the degree of interdependence among countries increases the independent pursuit of national policies may not lead to a desirable situation for the set of countries involved. The second is the optimizing approach introduced by Niehans (1968). According to this approach, policy targets are substitutable and the values that the instruments take are derived from a maximization of a welfare function that depends on deviations of the targets from their desired values. Niehans used a fixed price, fixed exchange rates model with one instrument (monetary policy) and two policy targets: income and the balance of payments. He concluded that the Nash noncooperative solution is inferior to the cooperative solution that lies on the Pareto-efficient contract curve. The reason is that the noncooperative solution can lead to a bias toward recession or inflation depending on each country’s preferences towards a balance of payments surplus or deficit. Monetary expansion is a public good (public bad) whenever each country desires a balance of payments surplus (deficit).

The first attempt to analyze policy interdependence using explicit strategic interactions was made by Hamada [(1976), (1985)]. Applying
the monetary approach to the balance of payments, he used a fixed exchange rate, flexible prices model where each of the two countries has one policy instrument (monetary policy) and two targets (inflation and the amount of international reserves). He concluded that when international reserves are a "public bad", (i.e., there is oversupply relative to their desired quantity) each country increases its domestic credit to defend itself against reserve accumulation. The result is higher world inflation (i.e., the Nash noncooperative solution results in an inflationary bias). In a similar manner, a deflationary bias results whenever international reserves are considered a "public good".

Canzoneri and Gray (1985) examine the case of two governments that act in a flexible exchange rates environment and use one instrument (monetary growth) to minimize a loss function that depends on output deviations from its natural rate and inflationary expectations. They analyze monetary policy games that the two countries play (in response to a common external shock) under three different specifications of the spillover effects:

First, in the case of negative spillover effects, the Nash solution leads to an expansionary bias (through competitive depreciations) as countries run more expansionary policies than what are consistent with fixed rates. The Nash noncooperative solution leads to a contractionary bias in the case of positive spillover effects (locomotive world). In both cases a fixed exchange rate system leads to Pareto improvement over the Nash solution. Finally, in an asymmetric case where country 1's monetary policy causes positive spillover effects while country 2's policy causes negative spillover effects, the result is that there is a contractionary bias for country 1 and an
expansionary bias for country 2. In this case, fixed exchange rates do not lead to Pareto improvement over the Nash solution.

Oudiz and Sachs (1984) analyze fiscal and monetary policy interdependence in a two-country model and show that the transmission effects depend on some structural parameters (e.g., degree of international asset substitutability, degree of wage indexation in each country, and the interest elasticity of money demand). Then, they point out that, (using the optimizing approach) at least in theoretical terms, the noncooperative Nash equilibrium is Pareto inferior to the cooperative solution (except in the case where the number of targets equals the number of policy instruments). Several cases are examined and among them the case where each country uses two instruments (fiscal and monetary policy) to attain three targets (output, inflation, and the current account balance). One of their conclusions dealing with fiscal policy is that, if policymakers prefer a larger current account surplus and expansionary fiscal policy worsens the current account, noncoordinated fiscal policies will not be as expansionary as a Pareto optimal equilibrium would require. But the most significant contribution of their paper is that they try to measure the gains from policy coordination among the U.S., Germany, and Japan. Using two large-scale econometric models, (the Federal Reserve MCM model and the Japanese EPA model) they find that the gains from a coordinated expansion are around 1/2 % of GNP per year for each country. These estimates though are based on some strong assumptions: The world is assumed to include only the above mentioned countries, and the policymakers are assumed to know the "true" model of the world economy as well as to have perfect information about the actions taken in other
countries. Other examples of empirical simulation models that find that the Nash and cooperative solutions are not very far apart include Oudiz and Sachs (1985) and Canzoneri and Minford (1988) where, using a static two-country disinflation game, the authors find that, even though the spillover effects of monetary policy are large in the Liverpool model, the cooperative solution is only 1.5% less deflationary than the Nash. They also find that the gains from cooperation depend on the character and the size of the initial shock.

Frankel (1988, p. 2-3) argues that the biggest obstacle to policy coordination is uncertainty that takes the three following forms: (i) Where does the initial position of the domestic country lie relative to the optimum values of the target variables? (ii) What are the correct weights to put on the various possible target variables? (iii) What is the correct model of the economy?

Frankel and Rockett (1988) improve on Oudiz and Sachs (1984) by considering the case where policymakers do not agree on the "true" model of the world economy. In this case, policy coordination lowers welfare in most of the cases. Arguing that this approach implies irrationality on behalf of the policymakers (since they do not recognize the existence of model uncertainty), Ghosh and Masson (1988) show that if model uncertainty is taken into account it is more likely that policy coordination improves welfare. The intuition behind this result is that, under uncertainty with respect to policy parameters, the policymakers will be more conservative in their use of the policy instruments as Brainard (1967) claimed using a framework about domestic economic policy. Canzoneri and Henderson (1988) claim that the gains
from cooperation are understated in the above mentioned studies. The reason is that these studies rule out any long run trade off between inflation and unemployment. If, however, ongoing policy conflicts that take the form of, for example, conflicting current account targets are introduced, the gains from policy coordination will be more significant.

Even if policymakers agree on the true model, cooperation can be counterproductive. A result of game theory states that when only some (but not all) players in a game cooperate, all players can be worse off than in a situation where nobody cooperates. Rogoff (1985), using a two-country version of the Barro-Gordon (1983) model, shows that cooperation between the monetary authorities of two countries may be suboptimal if cooperation cannot be extended to the game that takes place within each country between the monetary authorities and private agents. This result is based on the assumption that the objective function of the authorities differs from that the private agents maximize and is due to the fact that the cooperative equilibrium is time inconsistent. In Rogoff's model there is a distortion in the labor market (e.g., income taxation) that leads to an employment target for the society that is bigger than the employment target for the wage setters. Each central bank has two objectives (an employment target and an inflation target) and one instrument (the money supply). In the noncooperative case, each central bank's willingness to expand output by increasing the money supply is constrained by the higher inflation that is caused by the depreciation of the domestic currency. In the cooperative case where the two central banks agree to expand their money supplies uniformly, this constraint is not in effect. Therefore,
the wage setters, expecting higher inflation, set higher nominal wages and as a result, monetary policy cooperation exacerbates the inflation bias that the domestic distortion causes.

Oudiz and Sachs (1985) analyze a dynamic game of monetary policy where each country inherits inflation and is at full employment. In their model, the price level and the exchange rate are the state variables that evolve over time. According to the precommitment policy, each country promises a tighter monetary policy in order to affect exchange rate expectations and lead to appreciation of the domestic country and lower inflation. However, this policy is time inconsistent. Cooperation is better than the noncooperative case because the competitive appreciations under noncooperation lead to lower output and the same inflation rate (i.e., there is a deflation bias under noncooperation). Their most important result is that without cooperation, precommitment is counterproductive. The reason is that both countries reduce their money supplies and the exchange rate does not change.

Kehoe (1989) presents a two-period optimizing model of fiscal policy where cooperation among governments reduces welfare relative to the noncooperative solution even though benevolent governments maximize the utility functions of their residents. His analytical framework is a two-country version of Fischer's (1980) optimal taxation model. In the first period, consumers use their endowment to make a consumption and savings decision. The government in each country sets proportional taxes on capital (that is mobile internationally) and labor (that is immobile) in order to finance the second period exogenous government spending. Then, consumers decide in which country to invest. In the
second period, consumers decide how much to consume and work and governments collect tax revenues. In the Nash solution, tax rates on capital are zero because otherwise the low-capital taxing country gets all the savings. In the cooperative solution, both governments tax capital fully in order to avoid labor market distortions. The result is zero savings and a deterioration in welfare.

Other models that deal with fiscal policy coordination include those by Chang (1988) and Tabellini (1990). Chang presents an optimizing overlapping generations model of the international coordination of fiscal deficits. Assuming no Ricardian Equivalence, perfect international capital mobility, and a benevolent government that maximizes the utility of all the current and future generations, he shows, in a dynamic game framework, that the coordination of fiscal deficits implies a lower budget deficit and a higher welfare level relative to the case of noncooperation. An increase in the home country’s budget deficit increases the world interest rate and, therefore, the foreign government’s future debt service burden. This leads to higher taxes for the foreign generations and, therefore, lower welfare. The magnitude of the negative externality that the Markov Nash strategy implies can be weakened through the international cooperation of fiscal deficits. Chang also concludes that the divergence between the Markov Nash and the cooperative solution becomes larger as the number of countries increases.

Tabellini (1990) uses a two-period optimizing, symmetric model where the policymaker is not a benevolent social planner since he/she does not take into account the welfare of the future policymakers or the citizens. Since each policymaker is subject to electoral
uncertainty, there is a bias towards overaccumulation of public debt. This bias can exacerbate in the case of international policy coordination leading to the conclusion that cooperation reduces welfare. A large domestic fiscal deficit in the first period increases the price of the domestic good relative to the foreign good (i.e., improves the terms of trade). The opposite is true for the second period. In addition, for a given supply, a higher public consumption in the first period implies a lower private consumption. This means that the real interest rate is higher which increases the marginal cost of issuing public debt for the domestic government. However, the marginal cost of issuing public debt does not change when both countries raise their deficits since there are no terms of trade effects. In this case, the bias towards budget deficits becomes larger.
CHAPTER III
A GAME-THEORETIC APPROACH TO FISCAL POLICIES

III.1. Motivation
In the 1980s economists were (and still are) concerned about the sharp real appreciation of the dollar (from 1980 through the first quarter of 1985 the real appreciation of the dollar against other currencies was close to 40% and it was even higher against the German mark and the Japanese yen) and the huge increase in the U.S. budget deficits (the fiscal deficit increased from 1% of the nominal GNP in 1981 to 3.5% in 1982 and it remained at the same level for the next four years). Some economists [Feldstein (1986a), (1986b), Helliwell (1989)] argued that the dollar appreciation and the increase in the U.S. current account deficit were results of expansionary (current and expected) U.S. fiscal policy combined with contractionary fiscal policy of other major industrial countries.¹ Feldstein (1986b) supports his claims by providing econometric evidence: A one percentage point increase in the ratio of future U.S. budget deficits to GNP decreases

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¹ There is another strand in the literature that argues that budget deficits do not have any positive effects on the real exchange rate. This result is based on theoretical models that assume Ricardian Equivalence (Barro (1974)). Evans (1986) presented empirical evidence in favor of this result. He also showed that budget deficits do not affect the current account (Evans (1988)).
the real dollar-mark exchange rate by about thirty percentage points². The real dollar appreciation is then responsible for the deterioration of the U.S. trade and current account deficits after allowing for some temporary J-curve effects. Even though much has been said about the importance of coordinating fiscal policies among the major industrial countries, there has not been much research that tries to model the fiscal interactions among these countries and to discuss what scenarios might arise in response to noncooperative games. Chang (1988), Kehoe (1989), and Tabellini (1988) represent attempts to model fiscal policy games but they do not address the relationship between the budget deficit and the current account deficit. The reasons for concentrating on fiscal policies in the present study are the following:

(1) One of the objectives of this paper is to explain the existence of the high U.S. current account disequilibria in the 1980s and the effect of the U.S. monetary policy on the current account is theoretically uncertain. This is because a contractionary monetary policy improves the current account (through a reduction in real spending) but it also worsens the current account through a dollar appreciation. Fiscal policy, on the other hand, has an unambiguous

² It should be mentioned that Feldstein (1986b) did not include the budget deficits of other European countries in his regressions. This point is made by Dornbusch in his comment to Feldstein's paper. On the other hand, Hutchison and Throop (1985) examine the relationship between the multilateral U.S. real exchange rate and the U.S. and foreign budget deficits. They find that a rise in U.S. fiscal deficits leads to a multilateral real dollar appreciation while a rise in foreign deficits leads to a real dollar depreciation.
effect on the current account (under a sufficient degree of capital mobility) since both the real spending effect and the exchange rate effect (under sufficient capital mobility) tend to move the current account in the same direction. Empirical evidence [Bryant et al. (1988)] shows that this is the case for the U.S.

(2) The Plaza-Louvre attempts for cooperation explicitly incorporated agreements for expansionary fiscal policy in W. Germany and Japan and reductions in the U.S. budget deficits.

III.2. A Two-Country Model of Policy Interdependence

There are various models one can choose from to analyse policy interdependence: the Mundell-Fleming (MF) model, the monetary model, the portfolio-balance model, the Dornbusch overshooting model or an optimizing overlapping generations model. The monetary model makes the simplifying assumption that there is full employment in the goods market. The portfolio-balance model [e.g., Branson et al. (1977)] is a static model with money and government bonds serving as wealth. In this model an increase in the stock of government bonds that arises from a higher budget deficit might appreciate or depreciate the domestic currency depending on whether the substitution or the wealth effect is stronger. In a two-country framework, a domestic fiscal expansion is positively transmitted to the foreign country, but the long run effect is larger than the short run effect due to stock-flow interactions that take place over time. The reason this model is not used here is that it becomes rather complex in a two-country framework. The Dornbusch model introduces exchange rate and price dynamics into
the Mundell-Fleming model. The transmission effects are similar to those in the MF model but a distinction between the short and the long run is necessary due to the sticky-prices assumption. OG models have been analyzed for two-country models (Persson (1985)). These models could be used to present future extensions. This leaves us with the MF model that is used in this paper. Even though this is a static model that lacks microfoundations for the behavior of the private and public sectors, it is still considered to be the workhorse of open-economy macroeconomic models since it finds considerable validity by the empirical evidence.

The two-country fixed price Mundell-Fleming model under flexible exchange rates can be written as follows:

\[ y = D + \alpha_1 y - \alpha_2 i + \alpha_3 e - \alpha_4 y + \alpha_5 y^* \]  \hspace{1cm} (1)

\[ M = \beta_1 y - \beta_2 i \]  \hspace{1cm} (2)

\[ y^* = D^* + \alpha_1 y^* - \alpha_2 i^* - \alpha_3 e + \alpha_4 y - \alpha_5 y^* \]  \hspace{1cm} (3)

\[ M^* = \beta_1 y^* - \beta_2 i^* \]  \hspace{1cm} (4)

\[ \alpha_3 e - \alpha_4 y + \alpha_5 y^* + \beta_3 (i - i^*) = 0 \]  \hspace{1cm} (5)

Starred variables refer to the foreign country. All variables, except for the interest rates and the budget deficits, are measured in logs. \( y, i, \) and \( M \) stand for domestic income, nominal interest rate and the money supply respectively. The exchange rate \( e \) is defined as the number of units of domestic currency per unit of foreign currency. Eq. (1) represents the goods market equilibrium for the home country. \( D \) stands for a fiscal policy parameter (e.g., the budget deficit) while
all the other parameters are positive. Since $\alpha_3$ is positive, the
Marshall-Lerner condition is assumed to be true. Eq. (3) represents
the analogous equation for the foreign country where $D^*$ is the budget
deficit of the foreign country. Eqs. (2) and (4) are the money-market
equilibria conditions for the domestic and foreign country
respectively. Finally, eq. (5) reflects the balance of payments
equilibrium for the two-country economy. It states that under flexible
exchange rates the sum of the current and the capital account must add
to zero. The assumption that the investment income balance is zero is
made. The capital account is a function of the interest rate
differential between domestic and foreign assets. Since the objective
of the paper is the coordination of fiscal policies, we will assume
that $M = M^* = 0$.
The system of eqs. (1)-(5) can be solved with respect to the 5
endogenous variables $y, y^*, i, i^*, e$ as functions of the exogenous
variables $D$ and $D^*$. The solution is given in appendix A.

The current account deficit for the home country is determined by
the following equation [which is equation (8A) of the appendix plus a
shock]:

$$CA_t = \eta (D_t - D^*_t) + \delta e_t + x_t \quad \delta < 0$$  (6)

where $CA_t$ is the current account deficit, and $x_t$ is an exogenous demand
shock (e.g., a change in consumer preferences from domestic to foreign
goods) that reduces the demand for domestic output and increases the
demand for the foreign output ($x_t > 0$). The demand shock is introduced
because it gives rise to a policy conflict and the externalities that arise allow us to use strategic behavior modelling. Equation (6) implies that an increase in the budget deficit affects the current account deficit directly, through an increase in income, and indirectly, through a decline in $e_t$.

The exchange rate is assumed to be flexible and is determined according to equation (7A) derived in the appendix and presented below:

$$e_t = \gamma (D_t^* - D_t) \quad (7)$$

An increase in the home budget deficit relative to the foreign budget deficit tends to appreciate the domestic currency through its positive effect on the domestic interest rate. Substituting eq. (7) into (6) and rearranging we get:

$$CA_t = (\eta - \gamma \delta)(D_t - D_t^*) + x_t \quad (8)$$

According to this equation the following are true: First, an expansionary domestic fiscal policy affects adversely the domestic current account\(^3\). Second, an expansionary fiscal policy in the foreign country is transmitted positively to the domestic country by decreasing its current account deficit. The mechanism is the following: First, an expansion in the budget deficit abroad increases foreign income and, therefore, imports improving the domestic current

---

3. The result that higher budget deficits lead to higher current account deficits can also be derived using an OCG model (Persson (1985)). In this case, though, this result hinges on the assumption of lack of Ricardian Equivalence.
account balance. In addition, under a sufficient degree of capital mobility, the domestic currency depreciates and, therefore, the home current account balance improves even more.

III.3. A Two-Country One-Shot Game

It is assumed that all players have full information about the objectives of the others. Suppose that the domestic government is interested in minimizing the deviations of its budget deficit and its current account deficit from their desired levels. Including a budget deficit target in the loss function of the domestic country (U.S.) can be justified as follows: As Sargent and Wallace (1981) argue, a game of chicken is played between different decentralized branches of the U.S. government (Congress, Administration, Federal Reserve). As a result of this game, a desired federal deficit might reflect: (i) a large desired level of government expenditures by the Congress, and (ii) a small desired level of taxes by the Administration. Barro (1979) argues that fiscal deficits (or surpluses) are desirable because they can be used to smooth tax distortions optimally over time in order to minimize the deadweight losses of taxation. In addition, fiscal deficits can be used in order to redistribute income over time. Another model that might be used to explain the desirability of a budget deficit on the part of the government is the "political business

4. The view that the instruments of fiscal policy (government spending and taxes) are considered by political leaders as targets (or domestic objectives) for the U.S. is also shared by C. Schultze (1988).
cycle" model analyzed by Tabellini (1988). Persson and Svensson (1989) and Alesina and Tabellini (1990) provided a similar model. Tabellini assumes that the government that is in office in the current period faces a given probability of being replaced next period by another party that prefers a different composition of public spending. Since, if the other party is elected, the incumbent will not receive any utility, a bias for a budget deficit is introduced. The incorporation of the current account deficit target in the loss function can be explained as follows: Changes in the current account balance can affect the income distribution since some industries (e.g., steel, automobile, textiles and apparel) are more dependent than others on foreign competition. If the government authorities, responding to the resulting political pressures, try to attain a specific income distribution (using protectionist measures among other factors) in order to satisfy their constituents, then, they choose a specific target value for the current account balance.\(^5\)

Assuming a quadratic loss function, (which implies that (i) larger deviations from objectives matter more, (ii) positive and negative deviations from objectives matter equally, and (iii) linear policy rules) we have:

\[
L = \frac{1}{2} (\omega D_t^2 + \sigma CA_t^2)
\]  

---

\(^5\) Baldwin (1989) argues that due to the large increase in the U.S. trade deficit in the early 1980s, the U.S. government has placed a great degree of attention to the value of the current account deficit and has changed the course of its trade policy by increasing the magnitude of protectionist measures imposed on imports.
where \( L \) is a quadratic loss function and \( \alpha, \beta \) are the nonnegative weights assigned to the deviations from the targets (normalized to zero for simplicity).

Assuming a symmetric model, the analogous equations for the foreign country are:

\[
CA^*_t = \eta(D^*_t - D_t) - \delta e_t - x_t \quad \delta < 0 \quad (6^*)
\]

\[
e_t = \gamma(D^*_t - D_t) \quad (7^*)
\]

\[
CA^*_t = (\eta - \gamma \delta)(D^*_t - D_t) - x_t \quad (8^*)
\]

\[
L^* = \frac{1}{2}(\omega D^*_t + \beta CA^*_t) \quad (9^*)
\]

where \( CA^*_t \) is the current account surplus for the foreign country. It is assumed here that the two current account targets are not conflicting. If there were a target conflict, (i.e., the sum of the current account bliss points were different than zero) then the gains from cooperation would be even larger. According to Tinbergen's theory of economic policy, to attain \( n \) independent targets policymakers need at least \( n \) policy instruments. In this model each country has one policy instrument available (the budget deficit) and two targets, (the budget deficit target and the current account target) and, therefore, it faces a conflict of targets. For example, for a given shock that affects adversely the current account in the home country, (and favorably the current account in the foreign country) a decrease in the home country's budget deficit raises welfare because it offsets the shock, but it also lowers welfare because it reduces the budget deficit below the desired level.
III.4. The Nature of the Policy Game

To analyze strategic policy interactions between two countries one could use a cooperative or a noncooperative approach. The case of the prisoners' dilemma which is often used in the theory of games makes the distinction between the two approaches clear. In the repeated prisoners' dilemma game the only subgame perfect equilibrium is the noncooperative solution where each player confesses. In this case cooperation cannot take place because communication between the prisoners is not allowed. But in the case of policy coordination among different countries this lack of communication does not exist. How can then the fact that policy coordination does not happen in practice be explained? There are four answers to this question:

(a) Cooperative agreements cannot be sustained in general because players have incentives to renege on announced policies. For example, in the case of a Stackelberg leadership equilibrium (without a binding precommitment), the leader might have an incentive to renege on his/her announced policy because this would increase his/her welfare by moving on his/her reaction function. Also, an efficient cooperative equilibrium cannot be sustained without the existence of binding agreements (agreements enforced by a third party and whose violation would be so costly that cheating is unthinkable). This is because at least one player can improve his welfare by breaking the agreement.

(b) Since there is usually disagreement about the "true" model of the economy or the mechanism of spillover effects, countries cannot agree on the appropriate policy to be followed. Relevant to this is the fact that there is imperfect information in regards to whether each
country adheres to its domestic policy goal. For example, given a deviation from a desired outcome under a cooperative agreement, can the involved countries find out whether this deviation is due to a violation of the cooperative agreement or an external shock? For the countries involved in the agreement to be able to answer this question "objective indicators" should be established that can be used in monitoring policy compliance (i.e., policymakers should express their goals in terms of quantifiable economic aggregates). As an example consider the case where a coordinated policy required each country to enact an anti-inflationary monetary policy. Officials should first select as an objective indicator a particular interest rate or a monetary aggregate to follow. Also they should choose an indicator of inflation. Unless countries decide on some objective indicators on variables like inflation, money supply, interest rates, or budget deficits, cooperative agreements are difficult to sustain. (e.g., Smithsonian agreement).

(c) Cooperation is difficult to apply when some policy instruments are treated as targets. (e.g., in the present model the budget deficit is itself a target). As an example, consider the proposal by Williamson that argues in favor of a coordinated attempt to keep the dollar within a target zone. This proposal would be difficult to enforce since, during the Reagan Administration, a decline in the U.S. budget deficit would be objected and presumably the Fed would be unwilling to share the burden of the exchange rate adjustment (since this would require an expansionary monetary policy).
(d) In the case of fiscal policy, in particular, the lengthy legislative process that is required for the enactment of a fiscal policy change inhibits any attempts for short run policy coordination. This problem is not so important in the case of monetary policy because in many industrial countries (e.g., Germany) monetary policy is determined by relatively independent Central Banks.

III.5. The Cooperative Efficient Equilibrium

The cooperative equilibrium is given first as a benchmark. This is the Pareto efficient equilibrium where no country can be better off without making the other country worse off. Under cooperation, the equilibrium takes place in the contract curve at the point where the loss ellipse for each country is tangent to the 45 degrees line (see fig. 1). At this point the slope of the 45 degrees line \( \frac{dD_t}{dD^*_t} \) is equal to the slope of the loss ellipse \( \left( \frac{\partial L}{\partial D^*_t} / \frac{\partial L}{\partial D_t} \right) \). This condition, along with the requirement that along the 45 degrees line \( D_t = -D^*_t \), implies (after some manipulation) that the budget deficit at the cooperative equilibrium is equal to:

\[
D^E_t = -\left( \frac{2\beta(\eta-\gamma\delta)}{[\alpha+4\beta(\eta-\gamma\delta)^2]} \right) x_t < 0
\]

(remember that the budget deficit target was normalized to zero. Therefore, the negative sign in the above equation implies that the budget deficit is lower than its target level or that the budget surplus is above its target). Without a shock, both countries set their budget deficits at their desired level (i.e., zero) and the
equilibrium takes place at the origin. Point B is the bliss point for the home country. (Substituting $D_t=0$ and $D_t^* = x_t/\eta - \gamma \delta$ into the loss function implies that $L=0$). The intuition behind this result is that at point B, $D_t$ is at its desired level (i.e., zero) and the increase in $D_t^*$ helps offset the current account shock that tends to increase the domestic current account deficit (through its effect on $e$). Because of the symmetry of the model, $B^*$ is the bliss point for the foreign country. The efficient equilibrium is given by point E in fig. 1. This equilibrium is time inconsistent (i.e., not credible) in a multiperiod framework because, as it can be shown using the above model, the home country can increase its welfare by reneging on the cooperative agreement and choosing its budget deficit on its reaction function. A fiscal policy that announces a budget deficit for the next period that is not optimal from the second period's point of view is said to be time inconsistent. The time-inconsistency problem associated with the above game can be described as follows: Once the U.S. sets the budget deficit target, it has an incentive to renege on its promises about its fiscal policy in order to keep the budget deficit at the desired level. Foreign countries expect this failure of the U.S. government to stick to its promises and, therefore, they do not help the dollar depreciate sufficiently (i.e., they follow a tight

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6. A time consistent equilibrium is always subgame perfect because time consistency is a less stringent concept than perfection.
fiscal policy keeping their interest rates low). As a result, a noncooperative solution arises. This is consistent with the view that the rise in the value of the dollar in the pre-1985 period was widely attributed to the increase in the U.S. interest rates relative to the foreign interest rates (Tanzi (1988)). In other words, the foreign countries expect that if the dollar depreciates, the U.S. government will increase again its budget deficit, and hence the current account will deteriorate again going back to its "natural" level which is determined by the desired budget deficit. This is an example of the time-inconsistency problem in an open-economy framework: Before the foreign country chooses its fiscal policy, (and hence its effect on the value of the dollar) the U.S. argues that it is not going to increase its budget deficit above the agreed upon level. If the foreign country cooperates and, therefore, follows an expansionary fiscal policy, the dollar starts depreciating and the U.S. current account deficit improves. But, after the foreign country has contributed to the dollar's depreciation, the U.S. has an incentive to increase its budget deficit towards the desired level. By doing that, the U.S. does not impose an excessive burden on the current account deficit. Therefore, the U.S. optimal policy is time inconsistent. The above game represents the noncoordinated policy. To attain the cooperative solution, (which implies a lower budget deficit and a lower value of the dollar and, therefore, a higher welfare level) the U.S. has to obtain some credibility that it is not going to renege on its promises.
III.6. The Nash Noncooperative Solution

In an one-shot game the game is played only once and, therefore, neither past nor future confrontations between the players influence their current behavior. In the Nash noncooperative game each player chooses his strategy treating as parametric the strategies of the other players. When a Nash equilibrium has been attained, no player can improve his welfare by unilaterally changing his policy. In this model each country chooses its budget deficit to minimize its loss function for a given budget deficit policy of the other country. The domestic country's reaction function is derived by substituting eq. (8) into (9):

\[ L = \frac{1}{2} \{ \alpha D_t^2 + \beta [ (\eta - \gamma \delta) (D_t - D_t^*)^2 ] \} \]

(10)

Setting the first derivative of \( L \) with respect to \( D_t \) equal to zero and rearranging gives the domestic country's reaction function:

\[ D_t = \frac{\beta (\eta - \gamma \delta)^2}{(\alpha - \beta (\eta - \gamma \delta)^2)} D_t^* - \frac{\beta (\eta - \gamma \delta)^2}{(\alpha + \beta (\eta - \gamma \delta)^2)} x_t \]

(11)

A similar minimization procedure for the foreign country gives the foreign country's reaction function:

\[ D_t^* = \frac{\beta (\eta - \gamma \delta)^2}{(\alpha + \beta (\eta - \gamma \delta)^2)} D_t + \frac{\beta (\eta - \gamma \delta)^2}{(\alpha + \beta (\eta - \gamma \delta)^2)} x_t \]

(12)

The Nash solution for \( D_t \) and \( D_t^* \) is derived by solving the system of equations (11) and (12):

\[ D_t^N = -\left( \frac{\beta (\eta - \gamma \delta)^2}{\alpha + 2\beta (\eta - \gamma \delta)^2} \right) x_t \]

(13)

\[ D_t^{*N} = \left( \frac{\beta (\eta - \gamma \delta)^2}{\alpha + 2\beta (\eta - \gamma \delta)^2} \right) x_t \]

(14)
As expected, $D^N_t < 0$, $D_*^N > 0$, and $D^N_t = -D_*^N$.

The Nash equilibrium is presented graphically in figure 1 by point N. $L_N^*$, $L_N^*$ are the loss ellipses at the Nash equilibrium for the home and the foreign country respectively. They show the level of the loss for each country.

The Nash equilibrium is characterized by the fact that each player ignores the beneficial external effect of its fiscal policy on the other player’s current account deficit. Therefore, both fiscal policies are not as active as they would be under a case of internalization of externalities. In other words, $D^E_t > D^N_t$. This can be explained more clearly in the following way: With $x_t > 0$, the home country current account deficit increases while the foreign current account deficit decreases by the same amount since the shock affects the two countries in equal but opposite ways. The home country reduces its budget deficit, while the foreign country increases its budget deficit, in order to bring the current account deficit back to their desired levels. Each change in the budget deficit has a positive external effect on the other country, but each country does not take into account this effect when considering the stance of its fiscal policy. As a result, the budget deficits under a Nash equilibrium do not change by as much as they should in response to a current account shock. Finding the losses for each country under the Nash and the efficient equilibria and comparing them, it can be shown that their difference (i.e., the gains from coordination) is equal to
\[ \frac{1}{2} \alpha \beta \sigma_t^2 \left[ \alpha^2 \beta^2 (\eta - \gamma \delta)^2 + \alpha^4 \beta (\eta - \gamma \delta)^2 \right] > 0. \] These gains are positively related to \( \alpha, \beta, \eta, \gamma, \delta, \) and \( x_t \) (that is, the gains from policy coordination are higher, the higher the weights assigned to the targets, the higher the value of the transmission parameters and the higher the value of the demand shock).

III.7. The Stackelberg Leadership Solution

Suppose now that the home country is a Stackelberg leader which means that it commits to set its budget deficit at a specific level, while the foreign country reacts according to its reaction function. Notice that for the Stackelberg equilibrium to take place it is necessary for the leader to precommit. But unless enforced by a third party, this precommitment is not credible since the home country is not on its reaction function. In other words, the Stackelberg solution is time inconsistent. To derive the Stackelberg equilibrium, the home country minimizes its loss function subject to the reaction function of the foreign country: i.e.,

\[
\min_{D_t^*} L = \frac{1}{2} \left\{ \alpha D_t^2 + \beta [ (\eta - \gamma \delta)(D_t - D_t^*) + x_t ]^2 \right\}.
\]

s.t. eq. (12)

The substitution of the constraint into the loss function and the setting of the first derivative with respect to \( D_t \) equal to zero implies:

\[
D_t^S = -\frac{\alpha \beta (\eta - \gamma \delta)}{\left\{ \alpha + \beta (\eta - \gamma \delta)^2 \right\}^2 + \alpha \beta (\eta - \gamma \delta)^2} x_t
\]
A comparison of the budget deficit under the Nash equilibrium with that under the Stackelberg equilibrium implies that $D^S_t > D^N_t$. The Stackelberg equilibrium is represented by point S in fig. 1. It corresponds to a higher welfare level for the home country and a lower welfare level for the foreign country compared to the Nash equilibrium. (This can be seen by looking at the loss ellipses at point S). The intuition is as follows: The home country realizes that if it does not reduce its budget deficit by as much as under the Nash equilibrium (i.e., if it goes to point S), the foreign country will have to increase its budget deficit by more than in the Nash solution and, therefore, the home country will be better off since it can reduce its current account deficit without having to decrease its budget deficit by a significant amount. On the other hand, the foreign country is worse off since it has to increase its budget deficit (away from its desired level) by even more than under the Nash solution. Since welfare improves for the leader, both countries would prefer to be leaders. This implies an instability for the Stackelberg game.

III.8. Interpretation and Limitations

The objective of the above model was to provide a fiscal policy game that could explain: (i) Why the U.S. fiscal stance in the 1980s was not as contractionary as it would be optimal from a world point of view and, (ii) why, as a consequence of (i) above, the U.S. current account deficit was bigger than what would be optimal from a world point of view. If the domestic government places a high weight on the budget deficit target, then the budget deficit reduction in response to
the demand shock would be very small. This could explain the reluctance by the U.S. government to reduce its budget deficit appreciably in the 1980s.

The above framework suffers from some weaknesses:
1) It uses a static model where exchange rate expectations play no role and the exchange rate does not depend on expected budget deficits. This assumption will be relaxed in section III below.
2) It assumes a symmetric model of transmission effects and a symmetric game. Asymmetry could be introduced in many ways: (a) Different weights assigned to the targets by the two countries (e.g., the U.S. could assign a larger weight to its budget deficit target than the foreign country). This case is described below. (b) The values of the parameters in the structural model could be assumed to be different in the two countries.

III.9. The Asymmetric Game

This part describes the asymmetric case where the foreign country assigns a higher weight to its current account balance target (and, therefore, a lower weight to its budget deficit target) than the domestic country does. This case represents the situation after 1987 with Japan and W. Germany willing to pursue somewhat more expansionary fiscal policies and the U.S., influenced by national priorities, unwilling to reduce the expansionary stance of its fiscal policy (see paper by Grober, in Guth (1988)). Let the loss function of the foreign country be:

\[ L^* = \alpha' D_t^2 + \beta' CA_t^2 \quad \alpha' < \alpha, \beta' > \beta \]

The Nash solution for the asymmetric game is given by the following:
\[ D^A_t = -\left( \alpha' \beta (\eta - \gamma \delta) / \left[ \alpha \alpha' + (\alpha \beta' + \alpha' \beta)(\eta - \gamma \delta)^2 \right] \right) x_t \]  
\[ (13') \]
\[ D'^{A*}_t = \left( \alpha \beta' (\eta - \gamma \delta) / \left[ \alpha \alpha' + (\alpha \beta' + \alpha' \beta)(\eta - \gamma \delta)^2 \right] \right) x_t \]  
\[ (14') \]

As expected, \( |D^A_t| < |D'^{A*}_t| \). Comparing the Nash solutions under the cases of symmetry and asymmetry it is found that \( D^A_t > D^N_t \) and \( D'^{A*}_t > D'^{N*}_t \).

III. 10. Gains from cooperation under real and monetary shocks.

This section analyzes the gains from fiscal policy coordination under different types of exogenous shocks. More specifically, it examines two types of shocks:

(a) Real shocks (e.g., an exogenous increase in private investment) that affect the goods markets in the two countries in a symmetric way and

(b) Monetary shocks (e.g., an increase in money demand) that affect the money markets in the two countries in a symmetric way.

However, only symmetric shocks of different size for the two countries are examined, the reason being that equal-size symmetric shocks do not have any effects on the policy targets due to the symmetry imposed on the structural models for the two countries. The objective is to determine the Nash noncooperative and the cooperative solutions and the welfare gains attained through fiscal policy coordination in a static, two-country, Mundell-Fleming model, under the effects of real and monetary shocks respectively. Then, it can be determined whether the gains from fiscal policy coordination are greater under real or monetary shocks.
(a) **Real Shocks**

The static, two-country Mundell-Fleming model with real shocks can be written as follows:

\[
y = \alpha_0 + \alpha_1 y - \alpha_2 i + \alpha_3 e - \alpha_4 y + \alpha_5 y^* + u_1 \tag{15}
\]

\[
M = \beta_1 y - \beta_2 i \tag{16}
\]

\[
y^* = \alpha_0^* + \alpha_1^* y^* - \alpha_2^* i^* - \alpha_3^* e + \alpha_4^* y - \alpha_5^* y^* + u_2 \tag{17}
\]

\[
M^* = \beta_1^* y^* - \beta_2^* i^* \tag{18}
\]

\[
\alpha_3 e - \alpha_4 y + \alpha_5 y^* + \beta_3 (i - i^*) = 0 \tag{19}
\]

All variables are as defined previously. \( M \) and \( M^* \) are set equal to zero for simplicity. \( u_1 \) and \( u_2 \) represent the real shocks in countries 1 and 2 respectively. The solution of the above system for the endogenous variables \( y, y^*, i, i^*, e \) (as functions of the fiscal policy variables and the exogenous shocks) and the subsequent substitution of these variables into eq. (5) determines the value of the current account surplus:

\[
CA = - (\beta_3 \beta_1 \lambda^*/\pi) (D-D^*) + A (u_1 - u_2) \tag{20}
\]

where \( A = - \beta_3 \beta_1 \lambda^*/\pi \)

and \( \lambda^* = \lambda - \beta_1 \beta_3 > 0 \) and \( \pi > 0 \)

\( \pi \) and \( \lambda \) are defined in Appendix A. Equation (20) above implies that the real shocks affect the current account in equal but opposite ways. Therefore, if \( u_1 = u_2 \), the domestic current account (as well as the
foreign current account) is unaffected. This result can be justified by the fact that the two economies are symmetric and therefore each shock affects both the domestic and the foreign economy in the same way.

The Nash and the Cooperative solutions

The domestic policymaker tries to minimize the following loss function:

\[ L = \frac{1}{2} \left( \alpha D^2 + b \left[ A (D - D^*) + A(u_1 - u_2) \right]^2 \right) \]

The solution for the Nash noncooperative equilibrium implies that

\[ D_N^R = -\beta A^2 (u_1 - u_2) / (\alpha + 2\beta A^2) \leq 0 \text{ if } u_1 \geq u_2 \]

Setting \(-\partial L / \partial D^*) / \partial L / \partial D = -1\), the following cooperative solution is derived:

\[ D_C^R = -2\beta A^2 (u_1 - u_2) / (\alpha + 4\beta A^2) \leq 0 \text{ if } u_1 \geq u_2 \]

Calculating the losses under the cooperative solution and subtracting from the losses under the Nash solution, the following gains from coordination are derived:

\[ L_N - L_C = \alpha^2 B^2 A^4 (u_1 - u_2)^2 / (\alpha + 2\beta A^2)^2 (\alpha + 4\beta A^2) \]

(b) Monetary Shocks

The system of equations (15)-(19) described in the previous section is modified as follows: The real shocks in eqs. (15) and (17) are replaced by the monetary shocks \( v_1 \) and \( v_2 \) in eqs. (16) and (18) respectively:
\[ M = \beta_1 y - \beta_2 i + v_1 \quad (16') \]

\[ M^* = \beta_1 y^* - \beta_2 i^* + v_2 \quad (18') \]

Setting \( M=M^*=0 \) as above and solving the system of eqs. (15), (16'), (17), (18'), and (19) in a similar fashion, the following equation for the current account is derived:

\[ CA = A (D-D^*) - B (v_1 - v_2) \]

where \( B = -(\beta_3/\beta_2) [1-\beta_1^\lambda'(\alpha_2+2\beta_3)/\pi] > 0 \)

Again, because of the symmetry assumption imposed in the model, the current account is not affected by equal-size monetary shocks.

**Nash and Cooperative solutions**

In a similar manner as under real shocks, the following solutions are derived:

\[ D^M_N = \{\beta_1 B (v_1 - v_2)\} / (\alpha+2\beta A^2) \]

\[ D^M_C = 2\beta^2 A^2 B (v_1 - v_2) / (\alpha+4\beta A^2) \]

The gains from cooperation are:

\[ L_N - L_C = \alpha^2 B^2 A^2 (v_1 - v_2)^2 / (\alpha+2\beta A^2)^2 (\alpha+4\beta A^2) > 0 \]

It would be interesting to determine whether the gains from fiscal policy coordination are greater under real or monetary shocks. Intuitively, we would expect them to be greater under real shocks. Assuming that real and monetary shocks are equal in magnitude (i.e., \( u_1=v_1 \) and \( u_2=v_2 \)) it is found that the gains are greater under real
shocks only if $\beta_1 > 1 - \alpha_1$, i.e., if the income elasticity of money demand exceeds one minus the income elasticity of consumption. Since this result is likely to occur in reality, our beliefs are verified.

III. 11. Exchange Rate Systems and the Gains from Fiscal Policy Coordination

This section examines the importance of the exchange-rate regime in determining the welfare gains derived from the international coordination of fiscal policies. Someone would expect that fiscal policy coordination would be more productive, the closer the exchange rate system to a perfectly flexible exchange rates regime. The reason is that under purely flexible exchange rates the international transmission of fiscal policies is more significant due to the change in the exchange rate that constitutes one of the channels the domestic fiscal policy is transmitted abroad. As shown below, this is actually the case for sensible values of the model parameters: Fiscal policy coordination is more desirable under flexible exchange rates. The model used is similar to the one employed earlier. The only difference is that the assumption of perfect capital mobility is made. Two versions of the model are examined below: A model under fixed exchange rates and a model under flexible exchange rates.

A Flexible Exchange Rates Model

\[ y = D + \alpha_1 y - \alpha_2 i + \alpha_3 e - \alpha_4 y + \alpha_4 y^* \]  
\[ M = \beta_1 y - \beta_2 i \]  

(21)  
(22)
\[ y^* = D^* + \alpha_1 y^* - \alpha_2 i^* - \alpha_3 e + \alpha_4 y - \alpha_4 y^* \]  
(23)

\[ M^* = \beta_1 y^* - \beta_2 i^* \]  
(24)

\[ i = i^* \]  
(25)

Eqs. (21)-(24) have been defined previously. Equation (25) is a consequence of the assumption that the domestic and foreign assets are perfect substitutes. The above system has 5 eqs. in 5 unknowns: \( y, y^*, i, i^* \) and \( e \). The system of eqs. (21)-(24) can be solved for \( y, y^* \), and \( e \) as functions of the budget deficits and the money supplies in the two countries:

\[ y = (\beta_2 / \beta_1 \lambda) D + (\beta_2 / \beta_1 \lambda) D^* + [1 / \beta_1 - \beta_2 (1-\alpha_1) / \lambda \beta_2^2] M - \beta_2 (1-\alpha_1) / \lambda \beta_1^2 M^* \]

\[ y^* = (\beta_2 / \beta_1 \lambda) D + (\beta_2 / \beta_1 \lambda) D^* + [1 / \beta_1 - \beta_2 (1-\alpha_1) / \lambda \beta_2^2] M^* - \beta_2 (1-\alpha_1) / \lambda \beta_1^2 M \]

\[ e = -[\beta_2 (1-\alpha_1) + \beta_1 \alpha_2] / \lambda \alpha_3 \beta_1 D + [\beta_2 (1-\alpha_1) + \alpha_2 \beta_1] / \lambda \alpha_3 \beta_1 D^* \]

\[ \mu / \lambda \alpha_3 \beta_1^2 (M-M^*) \]

where \( \lambda = 2 [\beta_2 (1-\alpha_1) + \beta_1 \alpha_2] / \beta_1 \)

and \( \mu = \beta_2 (1-\alpha_1)^2 + \beta_1 (1-\alpha_1) \alpha_2 + 2 \alpha_4 [\beta_1 \alpha_2 + \beta_2 (1-\alpha_1)] \)

The equations for the income levels above imply that fiscal policy is effective in affecting the output levels, even though capital is perfectly mobile internationally. This result is due to the fact that we have employed a two-country model where each country has a large size that can affect the foreign variables. As is well known, in a small country model, as Mundell's seminal work has shown, fiscal policy
is completely ineffective under perfect capital mobility and perfectly flexible exchange rates. Substituting the values of \( y, y^* \), and \( e \) into the equation for the current account balance we derive the following equation:

\[
CA = \alpha_3 e + \alpha_4 (y^* - y) = [\beta_2 (1-\alpha_1) + \alpha_2 \beta_1] / \lambda \beta_1 (D^* - D) \cdot (M - M^*) \cdot (u / \lambda \beta_1^2 - \alpha_4 / \beta_1) + u
\]

\[
= \gamma (D^* - D) + u \tag{26}
\]

where \((M - M^*)\) has been set equal to zero, \( \gamma = [\beta_2 (1-\alpha_1) + \alpha_2 \beta_1] / \lambda \beta_1 \) and \( u \) is a positive current account shock that has been added to the current account.

**A Fixed-Exchange Rates Model**

The model under fixed exchange rates is identical to the system of eqs. (21)-(25). However, the endogenous variables are now \( y, y^*, i, i^*, \) and \( M^* \). This is the dollar standard that prevailed during the Bretton Woods era: the foreign country intervenes in the foreign exchange market in order to keep the dollar exchange rate fixed and, therefore, its money supply becomes endogenous. The solution of the above system for the endogenous variables (assuming that the exchange rate is equal to 1 and, therefore, its log value is zero) implies:

\[
y = [k \beta_2 D + \beta_2 \alpha_4 D^*] / E
\]

\[
y^* = [ \beta_2 \alpha_4 - \alpha_2 \beta_1 ] D + (\beta_1 \alpha_2 + k \beta_2) D^*] / E
\]

where \( k = 1-\alpha_1 + \alpha_4 > 0 \).
and \( E = \beta_2 k^2 + \alpha_4 \alpha_2 \beta_1 + \beta_1 \alpha_2 k - \beta_2 \alpha_4^2 > 0 \)

To derive the above equations, \( M \) has been set equal to zero for simplicity. The equation for \( y^* \) shows that an increase in the domestic budget deficit can increase or decrease foreign output depending on whether \( \beta_2 \alpha_4 \) is greater or smaller than \( \alpha_2 \beta_1 \). This result is due to the fact that a higher domestic budget deficit appreciates the domestic currency and forces the foreign country to endogenously adjust its money supply in a downward direction in order to stabilize the exchange rate. The substitution of \( y \) and \( y^* \) into the equation for the current account balance leads to:

\[
CA = [\alpha_4 (k \beta_2 + \beta_1 \alpha_2 - \beta_2 \alpha_4) / E] (D^* - D) + u = \delta (D^* - D) + u \tag{27}
\]

where \( \delta = [\alpha_4 (k \beta_2 + \beta_1 \alpha_2 - \beta_2 \alpha_4)] / E \)

According to this equation, an expansionary domestic fiscal policy worsens the domestic current account while an expansionary foreign fiscal policy improves the domestic external balance.

A comparison of equations (26) and (27) shows that \( \gamma > \delta \), that is, an expansionary domestic fiscal policy has a greater effect on the current account under flexible rates than under fixed exchange rates.

Nash, Cooperative Games and the Gains from Fiscal Policy Coordination

(a) Flexible exchange rates

The noncooperative Nash solution is derived by minimizing the domestic loss function assuming a given budget deficit for the foreign country:
\[ \text{Min } L = 1/2 \left[ \alpha D^2 + \beta CA^2 \right] \text{ with respect to } D \]

Substituting eq. (26) into the loss function and setting \( \partial L/\partial D = 0 \) yields the following solution:

\[ D_N^* = \beta \gamma u / (\alpha + 2 \beta \gamma^2) > 0 \]

An analogous procedure for the foreign country leads to a similar solution:

\[ D_N^* = -\beta \gamma u / (\alpha + 2 \beta \gamma^2) < 0 \]

The cooperative solution is given by the following equations:

\[ D_C = 2 \beta \gamma u / (\alpha + 4 \beta \gamma^2) \text{ and } D_C^* = -2 \beta \gamma u / (\alpha + 4 \beta \gamma^2) \]

It can easily be seen that the efficient solution implies more active fiscal policies than the Nash solution. Substituting the above solutions into the loss function the losses for the Nash and cooperative solutions can be determined and then compared to each other. This comparison gives the gains from coordination:

\[ L_N - L_C = \alpha^2 (\beta \gamma u)^2 / \pi' > 0 \]

where \( \pi' = (\alpha + 2 \beta \gamma^2)^2 (\alpha + 4 \beta \gamma^2) \), \( L_N \) stands for the welfare losses under a Nash solution and \( L_C \) stands for the losses under a cooperative solution.

(b) **Fixed Exchange Rates**

The minimization of the same loss function (following the substitution of the current account eq. (27) into the loss function) with respect to the domestic budget deficit (assuming a given deficit for the foreign player) provides the Nash solution:
\[ D_N = \frac{\beta \delta u}{(\alpha + 2\beta \delta^2)} \quad \text{and} \quad D_N^* = -\frac{\beta \delta u}{(\alpha + 2\beta \delta^2)} \]

The cooperative equilibrium is derived by setting \(\partial L/\partial D^* = \partial L/\partial D\):

\[ D\_C = \frac{2\beta \delta u}{(\alpha + 4\beta \delta^2)} \quad \text{and} \quad D\_C^* = -\frac{2\beta \delta u}{(\alpha + 4\beta \delta^2)}. \]

As expected, the budget deficit under the cooperative solution is greater than that under the Nash solution.

As previously, it can be shown that:

\[ L_N - L_C = \frac{\alpha^2(\beta \delta u)^2}{\pi''} > 0 \]

where \(\pi'' = (\alpha + 2\beta \delta^2)^2 (\alpha + 4\beta \delta^2)\).

Next, the gains from coordination will be compared for the two exchange rate regimes. The comparison is facilitated by the fact that \(\gamma = 1/2\) (for any parameter set) and \(\delta < \gamma\). Under the above implications of the model, it can be shown, after some algebraic manipulations, that for some sensible parameter values, the gains from fiscal policy coordination are greater under flexible exchange rates. Setting \(\alpha = \beta\), the difference in the gains is equal to the following expression:

\[ \gamma^2(1+20\delta^4+16\delta^6) - \delta^2(1+20\gamma^4+16\gamma^6) \]

For the following parameter values, among others, the above expression is positive, i.e., the coordination gains are greater under flexible exchange rates:

\[ 0.7 < \alpha_1 < 0.8, \quad \alpha_2 = 1, \quad 0.1 < \alpha_4 < 1, \quad \beta_1 = 0.5, \quad \beta_2 = 0.15. \]
CHAPTER IV
INTERACTIONS AMONG PRIVATE INVESTORS AND GOVERNMENT POLICIES

IV.1. Introduction

Games played between the government sector and the private sector have been analyzed extensively in the macroeconomics literature. (Their significance derives from the Lucas' critique that the expectations formed by the private agents with respect to policy instruments are important for the formulation of macroeconomic policy). The Phillips curve monetary policy game played between the monetary authority and the private agents (workers) is the most well known of these games. In this game the government sector acts strategically against the private atomistic agents while these agents do not behave strategically\(^7\). Kydland and Prescott (1977) that discussed the issue of rules (precommitment) versus discretion (consistent policy) concluded that rules are always welfare-improving relative to the time-consistent policy. Since the rules case is not credible, (due to the incentive that the monetary authority has to cheat on its announced inflation rate and improve welfare) the inferior time-consistent policy is derived. The time-inconsistency problem in this model is due to a

\(^7\) Even though many studies model the private sector as a passive player, Cooley and Smith (1989) examine the case where the government sector cooperates with the private agents that become an active player in the process.
labor market distortion. Barro and Gordon (1983) introduced reputation effects into the Kydland-Prescott framework and they showed that (for a sufficiently low discount rate) a reputational equilibrium that is superior to the time-consistent solution can be derived.

Horn and Persson (1988) analyze a game played between an exchange rate setting policymaker and a wage setting trade union. In this model both players act strategically. The exchange rate policy of the government is time-inconsistent: After the trade union has set the economy-wide nominal wage rate, the policymaker has an incentive to devalue the currency in order to increase the international competitiveness of the traded goods (and therefore employment). The trade union anticipates this response and, therefore, it sets a high nominal wage rate. The result is a "devaluation-wage spiral" and a loss in welfare due to an inflationary bias. The authors make a case for a fixed exchange rate system (e.g., Bretton Woods) that would make the exchange rate policy of the government credible and would result in the precommitment (rules) solution.

Other examples of this kind of games include the optimal taxation game (Fischer, 1980) where the government avoids the creation of distortions by taxing only capital in the second period. Therefore, the policymaker reneges on his announced policy (in the first period) of taxing both capital and labor income in the second period.

Another very recent example is the case of trade policy in the presence of imperfect markets presented by Staiger and Tabellini (1989). In their model, tariff policy is used to offset a domestic distortion. Since the production decisions take place before the
setting of the tariff policy, the government does not consider the production-side distortion effects of the tariff. (These effects take place because producers expect the forthcoming tariff policy). The authors conclude that rules (precommitment) are better than discretion and that government commitments can be enforced by international agreements (e.g., the GATT).

Matsuyama (1990) presents an infinite horizon, perfect information trade liberalization game of timing that is played between the domestic government and a domestic firm. The government contemplates about providing temporary protection to the domestic firm. However, the strategy of optimal temporary protection is not effective because it is not credible. This is shown by establishing that this strategy is not renegotiation-proof. Maskin and Newbery (1990) analyze a two-period import tariff game played between the domestic government and domestic oil producers. They show that the open loop equilibrium is not credible (time consistent) except for one borderline case. Waller (1990) constructs a discount window borrowing game that is played between the monetary authority and a commercial bank. Excessive bank borrowing can be penalized by the monetary authority through harassment (e.g., denial of discount loans in the future). Waller examines both a game of perfect information and a game of imperfect information where the bank does not know the utility function of the monetary authority. In the latter case, harassment might occur as the central bank tries to create a reputation of being tough.

The above papers deal with games played between a single policymaker and the private sector. Several authors have analyzed
games that are played simultaneously among two governments (or policymakers) and the private sector. In this framework, some paradoxical results have emerged: (1) Economic policy coordination can be counterproductive if policymakers have not solved the credibility problems they face against their private sectors and (2) when policymakers cannot coordinate their policies, precommitment with respect to the private sector can reduce social welfare. Section IV.5 below reviews this literature and presents a fiscal policy game that fits in this area.

IV.2. The Model

This section introduces the game that is played between the policymakers and private investors. A multiperiod model is applied to model the interactions between the government sectors and the private sector. This model differs from that used earlier in two respects: First, it assumes perfect substitution between domestic and foreign assets and second, it introduces private investors' forward looking expectations about the fiscal policy stance. Private investors' demand for assets in the foreign exchange market depends on their beliefs about the stance of the future fiscal policy (the size of the budget deficits) in the two countries. Therefore, the current value of the exchange rate depends on these expectations as well as on the current value of the domestic and foreign budget deficits. It should be

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8. Branson (1985) argues that one of the factors accounting for the U.S. dollar appreciation in the early 1980s may have been the anticipation of high future U.S. budget deficits.
noted that individual investors do not set their expectations strategically. It is true, though, that the collective actions of private agents have a strategic effect on the policy choice made by the government. However, it is not necessary that the atomistic private investors cooperate in setting their expectations. In order to introduce exchange rate expectations into the model, it is assumed that the uncovered interest parity condition (UIP) holds:\(^9\):

\[ i_t = i_t^* + E_t e_{t+1} - e_t \]  (5'')

where \(E_t\) is the expectation operator based on information available at time \(t\). Eq. (5'') reflects the assumption of perfect substitutability between domestic and foreign assets. The use of the UIP condition implies that the exchange rate is determined according to the asset market approach. The solution of the model\(^10\) (i.e., system of equations (1)-(4) and (5'')) for the exchange rate (described in appendix B) implies that:

\[ e_t = e + (1/A) \sum_{j=0}^{\infty} (B/A)^j E_t (D_t^* - D_{t+j}) \]  (28)

where

---

9. This condition is based on the assumption that investors are risk neutral and, therefore, that there is no risk premium. The evidence on the UIP remains controversial. The testing of the plausibility of UIP is equivalent to the testing that the forward exchange rate is an unbiased predictor of the future expected spot rate. For evidence in favor of the existence of an exchange risk premium, see Fama (1984), Hsieh (1984), and Hodrick and Srivastava (1984).

10. This model is similar to that used by Genberg and Swoboda (1989). The difference is that Genberg and Swoboda use the fixed-output, flexible-prices version of the model.
\[ A = \frac{(2\alpha_3 \beta_1 + \beta_2 - \alpha_1 \beta_2 + \alpha_2 \beta_1 + 2\alpha_4 \beta_2)}{\beta_1} \]
\[ B = \frac{\beta_2 - \alpha_1 \beta_2 + \alpha_2 \beta_1 + 2\alpha_4 \beta_2}{\beta_1} \]
\[ e = E_t e_{t+h} \] is the long-run exchange rate.

In equation (28) above, \((B/A) = (1/1+C) \cdot 1\) is the discount factor and \(C = 2\alpha_3 \beta_1 / B \geq 0\).

According to this equation, an expected lower domestic (U.S.) budget deficit implies that the U.S. interest rates are expected to decline and, therefore, the dollar is expected to depreciate. This leads immediately to a dollar depreciation. The solution of the model for the current account deficit (described in appendix B) implies that:

\[ CA_t = e + \frac{(\alpha_4 / (c + \alpha_4))(D_t - D_t^*)}{(1/A)\alpha_3((c - \alpha_4) / (c + \alpha_4))} \sum_{j=0}^{\infty} (B/A)^j E_t (D_{t+j} - D_{t+j}^*) \] (29)

where \(c = 1 - \alpha_1 + \alpha_4 + \alpha_2 \beta_1 / \beta_2 > 0\).

Suppose now that a one-time current account shock \(x_t \geq 0\) takes place at the beginning of period \(t\). Assume for the moment that the foreign policymaker is passive (i.e., \(D_t^* = D_{t+1}^* = \ldots = D_{t+\infty}^* = 0\)). Then, the game that is played between the domestic policymaker and the private investors can be described as follows:

The domestic policymaker first observes the shock at the beginning of period \(t\), and then sets the budget deficit for the current period \(t\) at \(D_t\) and promises to set the budget deficit for the future periods at the
lower levels $D_{t+1}, D_{t+2}, \ldots$. If the policymaker's precommitment is credible, the assumption of rational expectations implies that $E_t D_{t+j} = D_{t+j}$, $j = 1, \ldots, \infty$. In this case, the domestic government acts as a Stackelberg leader while private investors act as Stackelberg followers. However, if the government does not have the commitment technology to credibly precommit to the private sector, the government's announcements are not credible because the optimal ex ante policy differs from the optimal ex post policy. (These policies will be defined in the next section).

IV.3. Equilibrium with Precommitment

Assume for simplicity a two-period version of the above model. (The closed-form solution cannot be found for the infinite-horizon optimization problem because the objective function becomes non time separable following the substitution of the constraint (i.e., eq. (29))). To find the equilibrium with precommitment, the two-period loss function of the domestic government is minimized with respect to the current and next period budget deficits subject to the reaction function of the investors (i.e., $E_t D_{t+1} = D_{t+1}$) and eq. (29):

$$\min \ L = \frac{1}{2} [\omega D_t^2 + \beta(\gamma + x_t)^2] + \frac{1}{2} d (\omega D_{t+1}^2 + \beta \gamma D_{t+1})$$

s.t. $E_t D_{t+1} = D_{t+1}$

eq. (29)

where $d$ is the discount factor.

The solution gives
\[
D_t = \left\{ \frac{-\alpha \beta d(K+M) - \beta^2 d(K+M)^3}{\alpha \beta (KB/A)^2 + d [\alpha + \beta (K+M)^2]^2} \right\} x_t
\]

\[
D_{t+1} = \left\{ \frac{-\alpha \beta (KB/A)}{\alpha \beta (KB/A)^2 + d [\alpha + \beta (K+M)^2]^2} \right\} x_t
\]

where \( K = (1/A) \alpha_3 [(c - \alpha_4)/(c + \alpha_4)] > 0 \)

and \( M = \alpha_4/(c + \alpha_4) > 0 \)

Substituting the optimal values of \( D_t, D_{t+1} \) in the objective function gives the value of the losses:

\[
L^P = 1/2 \left\{ \frac{\alpha \beta [\alpha + \beta (K+M)^2]}{\alpha \beta (KB/A)^2 + d [\alpha + \beta (K+M)^2]^2} \right\} x_t^2
\]

As mentioned previously, an expected reduction of the U.S. budget deficit leads to a current dollar depreciation (and, therefore, to an improvement of the U.S. current account balance) because it influences private investor behavior. The domestic government realizes that and, therefore, it announces (at time \( t \)) a lower future budget deficit in order to induce a favorable change to its current account balance. This is the ex ante optimal policy for the government. However, in the next period \( t+1 \) (after the private investors' choices have been made) the policymaker has an incentive to renege on his promise and set \( D_{t+1} = 0 \) (i.e., leave the budget deficit fixed at its target level).

This is the ex post optimal policy. The divergence between the ex ante

11. Oudiz and Sachs (1985) also present a game where the government tries to influence the private sector's exchange rate expectations by announcing its future policy actions. However, the policymaker uses monetary policy to attain an inflation and an output target.
and ex post optimal policies (i.e., the time-inconsistency problem\(^\text{12}\)) is due to the fact that the ex ante optimal policy leads to a second best rather than a first-best outcome. This fact arises from the lack of policy instruments on the part of the policymaker. The government tries to increase the set of available instruments by changing its fiscal policy unexpectedly (i.e., through a fiscal surprise). This game can be motivated by the situation in the 1988-1989 period: U.S. announcements about reductions in the U.S. budget deficit (according to the targets set by the Gramm-Rudman Act) were not credible and private investors expecting high U.S. budget deficits (and high real interest rates) were pushing the real value of the dollar up. Therefore, foreign investors were willing to finance the U.S. budget and current account deficits through the increasing capital inflow in the U.S. The U.S. fiscal policy could become more credible by incorporating the proposal for a balanced-budget policy in the Constitution as the proponents of this policy have advocated.

IV.4. The Time-Consistent Policy

If there is no mechanism (e.g., the institutional framework) through which the home policymaker can credibly precommit to reduce the future budget deficits, the time-consistent policy is obtained. In this case, the solution is derived by minimizing backwards in time: at time \(t+1\), the optimal value of the budget deficit is zero (derived from

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the minimization of the second-period loss function). An intuitive reason for setting $D_{t+1}=0$ in the time-consistent policy is the following: By assumption there is no current account shock in the second period. Hence, a lower budget deficit (i.e., $D_{t+1}<0$) in the next period leads to a deviation of next period's deficit from its target value without providing any benefit to the policymaker. At time $t$, (given that $D_{t+1}=0$) the minimization of the two-period loss function with respect to $D_t$ gives:

$$D_t = -[\beta(K+M)/\alpha+\beta(K+M)^2] x_t < 0$$

The losses in this case are equal to:

$$L^T = 1/2 \left[ \alpha\beta/\alpha+\beta(K+M)^2 \right] x_t^2$$

The comparison of the losses in the two cases implies that $L^T > L^P$. The intuition is that under precommitment the policymaker has two policy instruments rather than one as in the time-consistency case.

IV.5. Interactions among the Two Policymakers and the Private Agents

Models that belong to this category are those by Rogoff (1985), Van der Ploeg (1988), Kehoe (1989), Canzoneri and Henderson (1988a), Tabellini (1990), and Devereux (1990).

A taxonomy of all the possible cases that can be analyzed is given in the table below:
Relation between the policymakers

<table>
<thead>
<tr>
<th>Cooperation</th>
<th>Noncooperation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Precommitment</td>
<td>A</td>
</tr>
<tr>
<td>No precommitment</td>
<td>C</td>
</tr>
</tbody>
</table>

Rogoff (1985) was the first to show that cooperation between two policymakers can be counterproductive if the policymakers have not solved the time-inconsistency problem that they face against the private sector (i.e., losses under case C are greater than those under case D). Carraro and Giavazzi (1988) have criticized Rogoff's result arguing that the cases of cooperation and noncooperation cannot be compared because the noncooperative equilibrium is not subgame perfect. Kehoe (1989) derived the same conclusion with Rogoff using an extension of the Fischer (1980) taxation model in a two-country framework. Tabellini (1990) also showed, in a model where policymakers are not Pigovian but maximize the utility of their constituents, that cooperation can be counterproductive. The reason is that the fiscal deficit bias, that exists due to the positive probability that the incumbent will not be reelected next period, becomes higher with cooperation. Van der Ploeg (1988) used a general equilibrium model of two monetary economies but contrary to Rogoff he assumed a long run trade-off between inflation and output. The government tries to maximize the utility of the representative consumer by levying distortionary taxes on production and imposing "inflation taxes" in
order to finance the production of public goods. He concludes that policy coordination, when policymakers cannot precommit to the private sector, can be counterproductive because it leads to lower real money balances held by the consumers (that appear explicitly in the utility function) even though distortionary taxes are zero. Canzoneri and Henderson (1988a) used a two-country, two-period model where workers sign two-period wage contracts. In their model, credible announcements about tomorrow's money supplies affect today's employment and price index. The effect on today's CPI takes place through a change in today's real exchange rate. The two-country game arises in response to a world productivity shock. If both countries credibly precommit to reduce next period's money supply, the real exchange rate is not affected and, therefore, today's CPI is not affected either. These commitments are counterproductive since they generate losses (next period employment and CPI decrease) but do not provide any gains (i.e., losses under case B are greater than those under case D). Devereux (1990) uses a two-period, two-country model where strategic interactions are due to the interest rate effects of the government policy. He shows that precommitment can reduce welfare in both countries if cooperation is not possible.

Next, the assumption (made in the previous section) that the foreign government is passive will be relaxed and, therefore, four different cases can be analyzed:

(i) The case where the two governments do not cooperate but the home policymaker precommits to the investors to reduce his budget deficit
while the foreign policymaker precommits to increase his budget deficit.

(ii) The case of cooperation between the two governments and precommitment.

(iii) The case of noncooperation without precommitment.

(iv) The case of cooperation without precommitment.

Then, the losses for the domestic policymaker for each of the above cases can be computed and compared to each other.

IV.5.1. Nash Equilibrium With Precommitment

In this case the two policymakers play a noncooperative game against each other but each policymaker credibly precommits to the private investors to change his budget deficit. The first-order conditions derived from the minimization of the home policymaker's two-period loss function with respect to $D_t$ and $D_{t+1}$ (setting $E_tD_{t+1} = D_{t+1}$ due to precommitment and $D_t = -D_t^*$, $D_{t+1} = -D_{t+1}^*$ due to the symmetry assumption) imply the following solution:

$$D_t^{NP} = \frac{-[\beta \delta (K+H)[\alpha + 2\beta (K+H)]]}{[2\alpha \beta (K+B/A)^2 + d[\alpha + 2\beta (K+H)]^2]} \kappa_t < 0$$

$$D_{t+1}^{NP} = \frac{-[\alpha \beta (K/B/A)^2 + d[\alpha + 2\beta (K+H)]^2]}{[2\alpha \beta (K+B/A)^2 + d[\alpha + 2\beta (K+H)]^2]} \kappa_t < 0$$

According to the above solution, the home policymaker reduces his budget deficit in the current period and promises to reduce his deficit in the next period. The opposite is true for the foreign policymaker. Substituting the above solutions into the loss function the following expression for the losses is derived:
\[ L^{NP} = \alpha \beta d \left[ \alpha + 2 \beta (K+M)^2 \right]^2 \left[ \alpha + \beta (K+M)^2 \right] + \alpha \beta (KB/A)^2 \left[ \alpha + 4 \beta (K+M)^2 \right] x_t^2 / \left[ 2 \alpha \beta (KB/A)^2 + d \left( \alpha + 2 \beta (K+M)^2 \right)^2 \right] \]

IV.5.2. Cooperation With Precommitment

This is the case where the policymakers decide cooperatively about the stance of their fiscal policy and, therefore, the externalities arising from the independent setting of fiscal deficits are internalized. As in case 1 above, the policymakers make promises about their future budget deficits. The solution is found by setting \( \partial L / \partial D_t^* = \partial L / \partial D_t \) and \( \partial L / \partial D_{t+1}^* = \partial L / \partial D_{t+1} \) and is given by the following equations:

\[ D_t^{CP} = -\{2 \beta (K+M) \left[ \alpha + 4 \beta (K+M)^2 \right] / \{4 \alpha \beta (KB/A)^2 + d \left[ \alpha + 4 \beta (K+M)^2 \right]^2 \} \} x_t < 0 \]

\[ D_{t+1}^{CP} = -\{2 \alpha \beta (KB/A) / \{4 \alpha \beta (KB/A)^2 + d \left[ \alpha + 4 \beta (K+M)^2 \right]^2 \} \} x_{t+1} < 0 \]

The losses for this case are:

\[ L^{CP} = \{\alpha \beta d \left[ \alpha + 4 \beta (K+M)^2 \right] / \{4 \alpha \beta (KB/A)^2 + d \left[ \alpha + 4 \beta (K+M)^2 \right]^2 \} \} x_t^2 \]

IV.5.3. Nash Equilibrium Without Precommitment

If the two policymakers cannot make binding agreements with the investors with respect to the future fiscal deficits the time-consistent equilibrium is derived where the solution is found by minimizing backwards in time: First, the second-period loss function is minimized with respect to the second-period budget deficit for each country. This results to \( D_{t+1} = D_{t+1}^* = 0 \). Since there is no demand
shock in the second period, the fiscal deficits should be unchanged. Then, taking into account the above results, the two-period loss function is minimized with respect to $D_t$ leading to

$$D_{t}^{NT} = -\{\beta(K+M) / [\alpha^2 + 2\beta(K+M)^2]\} x_t < 0$$

The losses for this case are:

$$L_{t}^{NT} = \{\alpha\beta / [\alpha^2 + 2\beta(K+M)^2]\} x_t^2$$

IV.5.4. **Cooperation Without Precommitment**

The solution is again derived by minimizing backwards. First, using the second period loss function, $\partial L / \partial D_{t+1}^*$ is set equal to $\partial L / \partial D_{t+1}^*$ [since $-(\partial L / \partial D_{t+1})/(\partial L / \partial D_{t+1}^*) = -1$]. This leads to the optimal values of the budget deficits for the second period:

$$D_{t+1} = D_{t+1}^* = 0.$$ Taking this result into account, and setting $\partial L / \partial D_t = \partial L / \partial D^*$ (using the two-period loss function) the solution for the first period is derived:

$$D_{t}^{CT} = -\{2\beta(K+M) / [\alpha^2 + 4\beta(K+M)^2]\} x_t < 0$$

The losses for this case are:

$$L_{t}^{CT} = \{\alpha\beta / [\alpha^2 + 4\beta(K+M)^2]\} x_t^2$$

IV.5.5. **Discussion**

Comparing the losses under the first two cases, it can be shown, after some algebraic manipulations, that the losses under cooperation are lower than those under noncooperation when policymakers are able to
precommit because the external effects of the policymakers' actions are taken into account in the former case. This result is similar to that obtained by Canzoneri and Henderson (1988a) in a different context.

A more interesting comparison is that of precommitment and time consistency under cooperation (cases (ii) and (iv)). It is found that precommitment reduces losses. This result is similar to that derived by Rogoff in a two-country model of an expectational Phillips curve and is explained by the fact that under precommitment each policymaker has two policy instruments instead of one.

Comparing the losses in cases (iii) and (iv), it is found that the losses under the cooperative solution are smaller than those under the Nash solution. This result verifies the fact that cooperation always makes both countries better off and is opposite to Rogoff's result that cooperation can be counterproductive when policymakers have not solved the credibility problem they face against the private sector. The intuition for our result is as follows: In the noncooperative case (without precommitment) an expansionary fiscal bias is caused because the domestic country does not reduce its budget deficit in the second period as it promised initially. Under cooperation, however, this expansionary fiscal bias is smaller because of the positive spillover effect the expansionary foreign fiscal policy has on the domestic current account balance.

The comparison of cases (i) and (iii) above gives that losses under precommitment are lower than those under time consistency. This result is opposite to that obtained by Canzoneri and Henderson. In their model precommitments (as explained above) are counterproductive
while in the present case they increase welfare. However, the case of noncooperation between the policymakers and precommitment is not very realistic since it seems that if the policymakers can precommit to the private agents they should be able to precommit against each other. Also, it can be argued that (Van der Ploeg, 1988) if policymakers can coordinate their policies, they should be able to precommit against the private sector. Therefore, the possibility of economic policy coordination being counterproductive that has been raised by several researchers should be considered as a theoretical curiosum. However, as Van der Ploeg argues, it might be easier to sustain cooperation through punishment strategies or reputational equilibria than establish precommitment against the private sector.

Finally, comparing the losses under cases (ii) and (iii), it is found that cooperation accompanied with precommitment improves welfare relative to the noncooperative time-consistent case. This result is consistent with intuition since both the ability to precommit and the possibility of cooperation tend to increase social welfare.

IV.5.6. A Numerical Example

Assume the following parameter values: $\alpha_1 = 0.8$, $\alpha_2 = 1$, $\alpha_3 = 0.1$, $\alpha_4 = 0.2$, $\beta_1 = 0.5$, $\beta_2 = 0.15$, $\delta = 0.05$, $\alpha = 0.5$, $\beta = 0.5$. Then, the losses for the four cases examined above are as follows:
Relation between the policymakers

<table>
<thead>
<tr>
<th>Relation between</th>
<th>Precommitment</th>
<th>Cooperation</th>
<th>Noncooperation</th>
</tr>
</thead>
<tbody>
<tr>
<td>the government</td>
<td></td>
<td>$L^C = 0.3867$</td>
<td>$L^N = 0.416$</td>
</tr>
<tr>
<td>and the private</td>
<td></td>
<td>$L^T = 0.475$</td>
<td>$L^NT = 0.479$</td>
</tr>
</tbody>
</table>

No precommitment

(time consistency)

These results are consistent with the preceding discussion and lead to the following welfare ranking (in order of decreasing welfare):

Cooperation with Precommitment
Nash with Precommitment
Cooperation without Precommitment
Nash without Precommitment
CHAPTER V
A DIFFERENTIAL GAME OF FISCAL POLICY

V. 1. An Introduction to Dynamic Games

The one-shot game analyzed in the previous chapters implied that sovereign policymaking leads to inefficiency (except for the cases mentioned in Chapter II). This result can change significantly if the previous game is extended to a sequence of one-shot games (i.e., a dynamic game). Dynamic games take the form of repeated or differential games.

A repeated game (or supergame)\(^{13}\) represents a repeated play of the one-shot game in which each player’s choice of strategy is a function of the past behavior of others and payoffs are a function of current actions only. Repeated games can be finitely or infinitely repeated.

For infinitely repeated games, trigger mechanisms (that reward good behavior and punish bad behavior in the sense that if either player fails to cooperate, both players play noncooperatively for a certain number of time periods thereafter) will always support a

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13. Some authors define supergames as repeated games where the game played every period differs from the game played earlier because the payoff matrix changes every period. One special case of supergames is the time-dependent game where current period payoffs depend on current as well as past actions.
cooperative equilibrium even if the game is noncooperative (under the condition that the future is not discounted too heavily). This result is known as the "Folk Theorem" of repeated games and it states that in an infinitely repeated game with low enough discount rates, any individually rational outcome\(^\text{14}\) can arise as a Nash equilibrium (Fudenberg and Maskin (1986)). It follows that sufficiently farsighted parties could always achieve the first best policies on the efficient frontier. In this case, sovereign policymakers can achieve the cooperative outcome without precommitment (i.e., to use the words of Canzoneri and Henderson [1988] "sovereign policymaking is not bad"). One problem with trigger strategies is that they give rise to a multiplicity of Nash solutions (for a critique of trigger strategies as well as of the game theoretic approach to economics see Fisher (1989)).

For finitely repeated games, and under the assumption of full information, however, the trigger mechanisms do not work. The reason is that during the last period of the game both players will play Nash since their bad behavior cannot be punished (or their good behavior cannot be rewarded). By backward induction, we conclude that the Nash solution will result each period. For finitely repeated games trigger strategies can work only in two cases:

(i) The case of incomplete information where each policymaker does not know the identity of the opponent and, therefore, he/she has an incentive to manipulate his/her reputation. The equilibrium concept

\[^{14}\text{An individually rational outcome is one that gives each player at least as much utility as she can receive by her own actions.}\]
applied is that of a "reputational" equilibrium and it has been applied in the Barro-Gordon inflation game by Backus and Driffill (1985) and Barro (1986).

(ii) The one-shot game has more than one Nash solutions but one of them dominates the others. This is not true for the prisoners' dilemma game because there is only one Nash equilibrium, but it is true for many other games. According to Friedman (1985), the appropriate trigger strategy is to reward good behavior in early periods with the good Nash solution in the last period. This equilibrium strategy is subgame perfect.

Recently, game theorists have criticized trigger strategies arguing that they might introduce a noncredible threat. To avoid these kinds of threats a refinement of the Nash equilibrium concept has been suggested: the Renegotiation-Proof equilibrium (introduced by Farrell and Maskin (1987) and Pearce (1987)) that restricts the set of possible Nash equilibria because it includes only those that are self enforcing. For the case of the prisoners' dilemma, for example, Farrell and Maskin argue that after the players have reverted to the noncooperative solution (due to the trigger strategy) they might renegotiate and, since bygones are bygones, decide that they can do better by playing the cooperative solution.

15. One equilibrium is said to strictly dominate another equilibrium if each player receives a larger payoff at the first equilibrium than he receives at the second.
Another form of dynamic games is that of a differential game where the game that is repeated differs from the game played earlier because structural dynamics are explicitly modelled. This happens by including a state variable that defines the current state of the game and whose value depends on the actions of each of the agents in the past. The open loop and closed loop Nash solutions, as well as the analogous Stackelberg solutions can be derived for this game. In the present model, structural dynamics can be introduced by the fact that the foreign asset holdings change through time. The following section analyzes a two-country dynamic (differential) game of fiscal policy.

V. 2. The Differential Game

Recently, economists have made an increasing use of applications of dynamic game theory to models of economic policy interdependence. An extensive survey of the literature is presented in Pohjola (1986). Applications of dynamic games to macroeconomic policy are given in Buiter and Marston (1985), Hamada (1985), and Van der Ploeg and de Zeeuw (1989). McMillan (1986) discusses some applications on the problem of extracting exhaustible resources. Applications of dynamic game theory to duopolistic competition problems are examined in the papers by Fershtman and Kamien [(1987), (1990)], and Driskill and McCafferty [(1989a), (1989b)]. This section introduces a noncooperative, two-country, dynamic, continuous-time fiscal policy game. Continuous-time games are known in the game theory literature as differential games. They apply dynamic optimization techniques (i.e., the value function approach or the Hamiltonian approach) to situations
with two or more players. For a theoretical presentation of
differential games see Starr and Ho (1969), Basar and Olsder (1982) and
Mehlmann (1988). The problem analyzed here deals with the game played
between two sovereign policymakers and abstracts from the interactions
between the government and the private sector. The structural model
used differs from that employed earlier in that it incorporates
explicit dynamics and it consists of equations (30)-(35) given below:

\[(IS) \quad y = D + \alpha_1 y - \alpha_2 i + \alpha_3 e - \alpha_4 y + \alpha_4 y^* \quad (30)\]

\[(LM) \quad M = 0 = \beta_1 y - \beta_2 i \quad (31)\]

\[(IS^*) \quad y^* = D^* + \alpha_1 y^* - \alpha_2 i^* - \alpha_3 e + \alpha_4 y - \alpha_4 y^* \quad (32)\]

\[(LM^*) \quad M^* = 0 = \beta_1 y^* - \beta_2 i^* \quad (33)\]

Net Stock Demand for foreign assets:

\[Z = -\beta (i - i^* - e) \quad (34)\]

Balance-of-Payments (BOP) Equilibrium:

\[\alpha_3 e - \alpha_4 y + \alpha_4 y^* - \dot{Z} = 0 \quad (35)\]

Equations (30)-(33) are identical to those used in the static model in
the third chapter of the paper. The coefficients on \(y\) and \(y^*\) are equal
due to the symmetry assumption. Also, since we abstract from monetary
policy considerations, the money supply levels were set equal to zero.
Equation (34) expresses the stock net demand for foreign assets as a
function of the interest rate differential and the expected
appreciation (that is equal to the actual appreciation due to the
assumption of perfect foresight) of the domestic currency. It implies
imperfect substitutability between domestic and foreign assets with $\beta$ a parameter measuring the degree of capital mobility. This equation can be derived from a mean-variance analysis with investors maximizing the expected value of their wealth (see Black, 1985). Equation (35) states that, under purely flexible exchange rates, the current account surplus is equal to the capital accumulation (i.e., the net change in the stock of foreign assets).

The system of (30)-(33) can be solved for $(y - y^*)$ as a function of the exchange rate and the budget deficits:

$$y - y^* = \frac{(D - D^*) + 2\alpha_3 e}{(\pi + \alpha_4)}$$

(21')

where $\pi = 1 - \alpha_1 + \alpha_4 + \beta_2 \alpha_1 / \alpha_2 > 0$

Substituting this equation into the expression for the current account surplus (i.e., $\alpha_3 e - \alpha_4 y + \alpha_4 y^*$), the following equation is derived:

$$T = \{\alpha_3 (\pi - \alpha_4) e - \alpha_4 (D - D^*)\} / (\pi + \alpha_4)$$

where $T$ is the current account surplus.

Solving the system of equations (30)-(35) for the exchange rate (see appendix C) the following second-order differential equation is derived:

$$\ddot{e} = 2(\beta_1 / \beta_2) (\alpha_3 / (\pi + \alpha_4)) \dot{e} + (\alpha_3 / \beta) [(\pi - \alpha_4) / (\pi + \alpha_4)] e -$$

$$- [\alpha_4 / (\beta (\pi + \alpha_4))] (D - D^*) + [\beta_1 / \beta_2 (\pi + \alpha_4)] (\dot{D} - D^*)$$

(36)

The above differential equation can be transformed into a coupled system of two first-order differential equations:

$$\dot{e} = -x$$

(37)
\[
\dot{x} = [2(\beta_1/\beta_2) (\alpha_3/(\pi+\alpha_4))] x - (\alpha_3/\beta) [(\pi-\alpha_4)/(\pi+\alpha_4)] e + \\
+ [\alpha_4/\beta(\pi+\alpha_4)] (D-D^*) - [\beta_1/\beta_2(\pi+\alpha_4)] (\dot{D}-\dot{D}^*) \quad \text{or}
\]

\[
\dot{x} = \gamma_1 x - \gamma_2 e + \gamma_3 (D-D^*) - \gamma_4 (\dot{D}-\dot{D}^*) \quad (38)
\]

where \[
\gamma_1 = 2(\beta_1/\beta_2) (\alpha_3/(\pi+\alpha_4))
\]

\[
\gamma_2 = (\alpha_3/\beta) [(\pi-\alpha_4)/(\pi+\alpha_4)]
\]

\[
\gamma_3 = \alpha_4/\beta(\pi+\alpha_4)
\]

\[
\gamma_4 = \beta_1/\beta_2(\pi+\alpha_4)
\]

The domestic country tries to set the growth rate of the domestic budget deficit in order to minimize an infinite-horizon quadratic loss function that depends on:

(a) deviations of the "level" of the budget deficit from its target level

(b) deviations of the current account balance from its target level

and

(c) deviations of the growth rate of the budget deficit from its target level. For simplicity, the desired levels for the three policy targets are normalized to zero. The introduction of the growth rate of the budget deficit in the loss function deserves some discussion:

First, from a technical perspective, the exclusion of this policy target from the objective function leads to a bang-bang control where the value of the control variable (i.e., the growth rate of the budget deficit) fluctuates from one extreme value to another (e.g., from \(-\infty\) to \(+\infty\)) but it never takes on intermediate values. This problem is caused by the fact that the Hamiltonian (to be defined next) is linear with
respect to the control (see Intriligator). Since extreme fluctuations of the growth rate of budget deficits are never observed in reality but policymakers adjust their deficit gradually towards its desired level, the incorporation of this target variable in the objective function is justified. Second, a budget deficit growth rate target can reflect the willingness to attain long term objectives that can take the form of an expansion or contraction in the size of the public sector over time and the provision of social programs.

More formally, the above optimization problem faced by the domestic policymaker can be expressed as follows:

\[
\begin{align*}
\min L \{ e(t), x(t), D(t), D^*(t), \dot{D}, \dot{D}^* \} &= \frac{1}{2} \int_0^\infty e^{-\delta t} \{ kD^2 + \lambda \dot{x}^2 + \nu \dot{D}^2 \} \, dt = \\
\dot{D} &= \frac{1}{2} \int_0^\infty e^{-\delta t} \{ kD^2 + \lambda [\alpha_3 (\pi-\alpha_4)/(\pi+\alpha_4)] e - [\alpha_4/(\pi+\alpha_4)] (D-D^*)^2 + \nu \dot{D}^2 \} \, dt \\
\text{s.t. (37), (38)}
\end{align*}
\]

\[ \dot{D} = u \quad \text{and} \]

\[ \dot{D}^* = f(e, x, D, D^*) \]

In the above problem, \( e, x, D, \) and \( D^* \) are the state variables that evolve through time, \( \dot{D} \) is the control, and \( \delta \) is the common discount rate. An analogous problem holds for the foreign country. The combination of the two optimization problems allows one to use the game-theoretic approach to macroeconomic policy. Since the objective function is quadratic and the state equations are linear, (i.e., a linear-quadratic game) the above problem has a closed-form solution.
V. 3. The Solution Concepts

The above game belongs to the category of nonzero-sum differential games, that is, continuous-time dynamic games that incorporate structural dynamics: the game played at each time period is different from the game played earlier because the state of the system changes through time in response to the players' actions in previous periods. In the present game, structural dynamics are introduced by the use of the stock of foreign assets. To analyze a nonhierarchical dynamic policy game (i.e., a game where all players move simultaneously) one can make use of different equilibrium concepts that are based on different assumptions regarding the use of information by the players. The most widely used strategies that depend on different information structures are the open-loop and the closed-loop (memoryless) strategies (see Basar and Olsder, Ch. 6).

For the present game these strategies are defined as follows:

\[
\dot{D} = f(t, x_0, e_0, D_0, D_0^*), \quad t \in [0, \infty) \quad \text{(Open loop)}
\]

\[
\dot{D} = f(t, x_t, e_t, D_t, D_t^*), \quad t \in [0, \infty) \quad \text{(Closed loop memoryless)}
\]

When the players follow an open-loop (or path) strategy, they announce and commit to a path of the control variables that depends on time and the initial state only. Therefore, for this strategy to make sense the ability of the players to precommit to a policy is necessary. When the players follow a closed-loop (memoryless) strategy they announce their control laws that specify the dependence of the control variables on time and the current state of the system. This type of strategies satisfies Bellman's principle of optimality. In the stabilization
theory in macroeconomics the open loop strategies are sometimes referred to as rules strategies (e.g., Friedman's fixed money supply growth rule) while the closed loop strategies are called decision strategies. However, the interpretation is not exactly correct since an open loop strategy does not imply necessarily a fixed value for the control. One additional strategy space is that of memory strategies: The information set consists of time and the whole history of the state variables. However, these strategies are difficult to use because they lead to multiple Nash equilibria, a problem known as informational nonuniqueness. Setting $\dot{D} = u$ and $D^* = u^*$, the following definition applies:

**Definition:** A Closed-Loop Nash Equilibrium is a pair of closed-loop strategies $(\tilde{u}, \tilde{u}^*)$ such that:

$$L(\tilde{u}, \tilde{u}^*) \leq L(u, u^*),$$

$$L^*(\tilde{u}, \tilde{u}^*) \leq L^*(u, u^*)$$

for every $u$, $u^*$ and for every possible initial condition $\{t(0), e(0), x(0), D(0), D^*(0)\}$. Since the above inequalities hold for every possible initial condition, the closed-loop Nash equilibrium is subgame perfect (i.e., credible) because the equilibrium conditions are satisfied in every subgame of the original game. In contrast, in the open-loop Nash equilibrium the above inequalities hold only for one initial condition. Therefore, in general, the open loop Nash equilibrium is not subgame perfect. However, for some classes of games, called perfect games, the open loop Nash equilibrium degenerates to the closed loop equilibrium and hence is subgame perfect (see
Mehlmann (1988), p. 129-139). Since subgame perfection is a stronger requirement than time consistency (due to the fact that it holds in both the equilibrium and the nonequilibrium paths), the above solution is also time consistent. For the linear-quadratic problem described above Basar and Olsder (1982, p.p. 287-88) show that the solutions express the controls as linear functions of the state variables (i.e., the game admits of a linear strategy). However, it remains an unresolved problem whether other solutions (linear or nonlinear) exist. In addition, since this is an infinite horizon game, the equilibrium strategy derived from the solution will be autonomous, that is, independent of time.

The optimization problem for the domestic policymaker can be rewritten as follows:

\[
\min L = \frac{1}{2} \int_0^\infty e^{-\delta t} \left[ kD^2 + \lambda T^2 + \nu D^2 \right] dt =
\]

\[
\frac{1}{2} \int_0^\infty e^{-\delta t} \left\{ kD^2 + \lambda \left\{ \alpha_3 (\pi - \alpha_4)/(\pi + \alpha_4) \right\} e - \left\{ \alpha_4/(\pi + \alpha_4) \right\} (D - D^*) \right\}^2 + \nu D^2 \right\} dt
\]

s.t. \( \dot{D} = u \)

\[
\dot{e} = -x
\]

\[
\dot{x} = \gamma_1 x - \gamma_2 e + \gamma_3 (D - D^*) - \gamma_4 (\dot{D} - D^*)
\]

\[
\dot{D}^* = M_1^* e + M_2^* x + M_3^* D^* + M_4^* D
\]

(feedback equation)

Theorem: Let

\[
\dot{e} = -x
\]

\[
\dot{x} = \gamma_1 x - \gamma_2 e + \gamma_3 (D - D^*) - \gamma_4 (\dot{D} - D^*)
\]

\[
\dot{D} = M_1 + M_2 e + M_3 x + M_4 D + M_5 D^*
\]

\[
(39)
\]
\[
D^* = M_1^* + M_2^* e + M_3^* x + M_4^* D + M_5^* D
\] (40)

where \( M_i^* \), and \( M_i^* , i = 1, \ldots, 5 \) are the solutions to the following system of equations: (Note that the \( M_i^* \)'s do not appear in the system of equations due to the symmetry assumptions imposed later).

\[
M_1 = (1/z_1) \left[ z_0 M_1 (\lambda_3 + \lambda_4) - z_2 M_1 (t_3 + t_4) - z_2 M_1 (M_4 + M_5)^2 - z_4 M_1 (M_4 + M_5) \right] \] (26')

\[
M_2 = (1/z_1) \left[ z_0 N_2 - z_2 \delta_2 - z_3 t_2 - z_4 \delta_2 - z_6 N_2 - z_7 P_2 + 2z_8 \theta_1 M_3 - z_9 \theta_1 \right] \] (27')

\[
M_3 = (1/z_1) \left[ z_0 N_4 - z_2 \lambda_1 - z_3 t_1 - z_4 \delta_1 - z_6 N_1 - z_7 P_1 - z_8 (-\theta_1 - 2\theta_2 M_3) \right] \] (28')

\[
M_4 = (1/z_1) \left[ z_0 N_6 - z_2 \lambda_3 - z_3 t_3 - z_4 \delta_3 - z_5 N_3 - z_7 P_3 + z_8 \theta_2 (M_4 - M_5) + z_9 \theta_2 \right] \] (29')

\[
M_5 = (1/z_1) \left[ z_0 N_7 - z_2 \lambda_4 - z_3 t_4 - z_4 \delta_4 + z_6 N_3 + z_7 P_3 - z_8 \theta_2 (M_4 - M_5) - z_9 \theta_2 \right] \] (30')

where \( z_i, i = 0, \ldots, 9, \delta_j, \lambda_j, t_j, j = 1, \ldots, 4, N_k, k = 1, \ldots, 7, \theta_1, \theta_2, P_1, P_2, P_3 \) are defined in appendix D and where the system of the four differential equations above satisfies the Routh-Burwick stability conditions (to be defined later). Then, the solution \((\dot{D}, D^*)\) represents a stable, closed-loop Nash equilibrium for the dynamic game defined above.

**Proof:**

The objective is to show that the stipulated strategies (i.e., the feedback equations (39) and (40)) satisfy the Pontryagin necessary conditions for each optimization problem. The current value Hamiltonian for the domestic country is defined as follows:
\[ H = \frac{1}{2} \left( \kappa D^2 + \lambda \left[ \frac{\alpha_3 (\pi - \alpha_4)}{(\pi + \alpha_4)} \right] e - \frac{\alpha_4}{(\pi + \alpha_4)} \right)(D - D^*)^2 + \nu D^2 \right) - \mu_1 x + \mu_2 (\gamma_1 x - \gamma_2 e + \gamma_3 (D - D^*) - \gamma_4 (u - D^*)) + \mu_3 u + \mu_4 (M_1^* + M_2^* e + M_3^* x + M_4^* D^* + M_5^* D) \]

where \( \mu_1, \mu_2, \mu_3 \) and \( \mu_4 \) are the costate variables.

The Pontryagin necessary conditions\(^{16} \) are as follows:

\[ \frac{\partial H}{\partial u} = uu - \mu_2 \gamma_4 + \mu_3 = 0 \quad (41) \]

\[ -\frac{\partial H}{\partial e} = -\lambda \theta_1 \mu_2 (\gamma_2 - \gamma_4 M_2^*) - \mu_4 M_2^* = \dot{\mu}_1 - \delta \mu_1 \quad (42) \]

\[ -\frac{\partial H}{\partial x} = \mu_1 - \mu_2 (\gamma_1 + \gamma_4 M_3^*) - \mu_4 M_3^* = \dot{\mu}_2 - \delta \mu_2 \quad (43) \]

\[ -\frac{\partial H}{\partial D} = -\kappa D + \lambda \theta_2 \mu_2 (\gamma_3 + \gamma_4 M_5^*) - \mu_4 M_5^* = \dot{\mu}_3 - \delta \mu_3 \quad (44) \]

\[ -\frac{\partial H}{\partial D^*} = -\lambda \theta_2 \mu_2 (\gamma_3 - \gamma_4 M_4^*) - \mu_4 M_4^* = \dot{\mu}_4 - \delta \mu_4 \quad (45) \]

Imposing symmetry in equations (39) and (40) and taking into account the fact that \( e \) and, therefore, \( x \) affects the growth rates of the deficit in opposite ways, the following is true:

\[ M_1 = M_1^*, \quad M_2 = -M_2^*, \quad M_3 = M_3^*, \quad M_4 = M_4^*, \quad \text{and} \quad M_5 = M_5^*. \]

Using these symmetry conditions, the feedback equation for the foreign country (eq. (40)) can be written as:

\[ D^* = M_1^* e - M_2^* x + M_4^* D^* + M_5 D \quad (26''') \]

---

16. Since the objective function is strictly convex, the necessary conditions are also sufficient for minimizing the loss function. (see Arrow and Kurz, Ch. II, p. 43-45)
Imposing symmetry, the system of the first-order conditions (41)-(45) can be rewritten as:

\[ \frac{\partial H}{\partial \theta} = uu - \nu_2 \gamma_4 + \nu_3 = 0 \]  \hspace{1cm} (46)

\[ -\frac{\partial H}{\partial e} = -\lambda \theta_1 + \nu_2 (\gamma_2 + \gamma_4 \gamma_2) + \mu_4 M_2 = \dot{\mu}_1 - \delta \mu_1 \]  \hspace{1cm} (47)

\[ -\frac{\partial H}{\partial x} = \mu_1 - \nu_2 (\gamma_1 - \gamma_4 \gamma_3) + \mu_4 M_3 = \dot{\mu}_2 - \delta \mu_2 \]  \hspace{1cm} (48)

\[ -\frac{\partial H}{\partial D} = -kD + \lambda \theta_2 - \mu_2 (\gamma_3 + \gamma_4 \gamma_5) - \mu_4 M_4 = \dot{\mu}_3 - \delta \mu_3 \]  \hspace{1cm} (49)

\[ -\frac{\partial H}{\partial D^*} = -\lambda \theta_2 + \mu_2 (\gamma_3 - \gamma_4 \gamma_4) - \mu_4 M_4 = \dot{\mu}_4 - \delta \mu_4 \]  \hspace{1cm} (50)

The system of equations (46)-(50) can be manipulated algebraically in order to derive an equation of the form:

\[ \dot{D} = \tilde{M}_1 + \tilde{M}_2 \ e + \tilde{M}_3 \ x + \tilde{M}_4 \ D + \tilde{M}_5 \ D^* \]  \hspace{1cm} (51)

where \( \tilde{M}_1, \tilde{M}_2, \tilde{M}_3, \tilde{M}_4, \) and \( \tilde{M}_5 \) are complicated functions of the structural parameters of the model and the feedback parameters \( \mu_i, \ i = 1, \ldots, 5 \) (for a description of the way the model was solved, see appendix E). Setting \( M_i = \tilde{M}_i, \ i = 1, \ldots, 5 \), a system of five nonlinear equations in the M's arises (i.e., equations (26')-(30') above) that can be solved numerically for the M's by using the Newton-Raphson method of approximating nonlinear systems of equations. Since this method requires the derivation of the matrix of the analytical first-order derivatives and since these derivatives are very difficult to determine analytically, we found the value of the derivatives numerically. If the solution for the M's satisfies the Routh-Hurwitz conditions for stability then the chosen strategies are also stable. Q.E.D.
In order to derive the Routh-Hurwitz stability conditions for the above system of the differential equations, we first substitute equations (39) and (26') into equation (38) to derive

\[ \dot{x} = c_1 x + c_2 e + c_3 D + c_4 D^* \]  \hspace{1cm} (52)

where

\[ c_1 = \gamma_1 \gamma_4 M_2 \]

\[ c_2 = -2 \gamma_4 \gamma_2 M_2 \]

\[ c_3 = \gamma_3 - \gamma_4 (M_4 - M_5) \]

\[ c_4 = -c_3 \]

The system of equations (37), (39), (52), and (26') is used to derive the characteristic equation \(|pI - C| = 0\), where \(I\) is the identity matrix and the 4×4 matrix \(C\) is as follows:

\[
C = \begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 \\
  -1 & 0 & 0 & 0 \\
  M_3 & M_2 & M_4 & M_5 \\
  -M_3 & -M_2 & M_5 & M_4 \\
\end{bmatrix}
\]

Expanding the characteristic equation yields the following fourth-degree equation:

\[ \rho^4 + \rho^3 \kappa_1 + \rho^2 \kappa_2 + \rho \kappa_3 + \kappa_4 = 0 \]

where

\[ \kappa_1 = -2 M_4 - c_1 \]

\[ \kappa_2 = M_4^2 - M_5^2 + M_3 (c_4 - c_3) + 2 c_1 M_4 + c_2 \]

\[ \kappa_3 = -2 c_2 M_4 + c_2 (c_4 - c_3) + c_1 (M_5^2 - M_4^2) + c_3 M_3 (M_4 + M_5) - c_4 M_3 (M_4 + M_5) \]

\[ \kappa_4 = c_2 (M_4^2 - M_5^2) - M_2 c_3 (M_4 + M_5) + M_2 c_4 (M_4 + M_5) \]
where the c's are as defined earlier.

According to the Routh-Hurwicz stability conditions for the above characteristic equation, (Takayama (1985), p. 310) the following four determinants have to be positive:

\[
\begin{vmatrix}
  k_1 & 1 & 0 \\
  k_1 & 1 \\
  k_3 & k_2 & k_1 \\
  k_3 & k_2 \\
\end{vmatrix}
\begin{vmatrix}
  k_1 & 1 & 0 & 0 \\
  k_3 & k_2 & k_1 & 1 \\
  k_4 & k_3 & k_2 \\
  k_4 \\
\end{vmatrix}
\]

In order to verify whether the above conditions are satisfied, first we have to solve the model numerically. The next section includes the results obtained by the numerical simulations of the model.

V. 4. **Numerical solution**

In order to solve the model numerically arithmetic values must be assigned to the parameters. These values are taken from the empirical macroeconomics and international economics literature. Savage (1978) reports that the interest elasticity of investment for the U.S. is between 0.5 and 1. Goldfeld (1973) presented money demand elasticities in the following ranges: \(0.19 < \beta_1 < 0.68\), and \(0.019 < \beta_2 < 0.16\) where \(\beta_1\) is the income elasticity of money demand and \(\beta_2\) is the interest elasticity of money demand. The lower limits in the above inequalities refer to the short run elasticities while the upper limits refer to the long run elasticities. Macroeconomists usually assign a value of 0.2-0.3 to the elasticity of domestic income with respect to the foreign income.
Based on this estimate, our model implies that $\alpha_4$ (the income elasticity of the current account) is around 0.1. Carlozzi and Taylor (1985) report a value of 0.1 for the elasticity of domestic income with respect to the exchange rate. We chose a value of 0.1 for the coefficient $\alpha_3$. Based on the above empirical results, the values assigned to the parameters are as follows (these parameters represent elasticities since all the variables (except for the interest rates and the budget deficits) are measured in logs):

$\alpha_1$ (Income Elasticity of Consumption) = 0.8

$\alpha_2$ (Interest semielasticity of investment) = 1.0

$\alpha_3$ (Exchange rate Elasticity of the Current account) = 0.1

$\alpha_4$ (Income Elasticity of the Current account) = 0.2

$\beta_1$ (Income elasticity of money demand) = 0.5

$\beta_2$ (Interest semielasticity of money demand) = 0.15

$\beta$ (coefficient of capital mobility) = 20

$\delta$ (rate of time preference) = 0.05

$\kappa$ (weight assigned to the budget deficit target) = 0.3

$\lambda$ (weight assigned to the current account deficit target) = 0.3

$\upsilon$ (weight assigned to the budget deficit growth rate target) = 0.4

A number of different sets of initial values for the $M$'s was used in order to solve the model numerically. However, only one of the solutions obtained is satisfying all of the stability conditions. This does not prove that this solution is unique, but it does show that there exists at least a stable solution to this dynamic game. This solution is the following:
\( M_1 = 0.00, \ M_2 = -0.035, \ M_3 = 0.7177, \ M_4 = -0.3582, \ M_5 = -0.4830 \)

Convergence was obtained after 55 iterations. Based on this solution, the feedback equation for the domestic country is as follows:

\[ \dot{D} = -0.035 \ v + 0.7177 \ x - 0.3582 \ D - 0.4830 \ D^* \]

For the above values for the \( M \)'s, the Routh-Hurwicz conditions are satisfied since the values of the four above-mentioned determinants are: 1.763, 1.434, 0.086, and 0.0000279.

V. 5. Interpretation of the results

An interpretation of the signs of the above feedback parameters is as follows: An increase in the rate of depreciation of the domestic currency (\( v \)) leads to an improvement of the current account and, therefore, the domestic policymaker should respond by increasing the rate of growth of the budget deficit (i.e., a positive coefficient for \( M_3 \)) in order to change the current account balance towards its initial level which is also the target level. In addition, a higher domestic budget deficit will lead to a lower foreign fiscal deficit (and a further beneficial effect on the current account balance) due to the negative sign of the feedback parameter \( M_5 \).

An explanation of the sign of \( M_2 \) is more subtle: A depreciation of the domestic currency improves the domestic current account balance but this improvement is not as significant as under an increase in the rate of growth of the exchange rate. (This interpretation is also justified by the fact that the absolute value of the coefficient \( M_2 \) is
smaller than that of the coefficient $M_3$). In this case, the domestic policymaker should, according to the prescriptions of this game, reduce the growth rate of the budget deficit so that he still attains a smaller increase in the "level" of the budget deficit that helps to counteract the initial change in the current account balance.

An explanation of the coefficients $M_4$ and $M_5$ is now in order: An increase in the "level" of the domestic budget deficit leads to a deterioration of the current account balance and the domestic government responds by decreasing the growth rate of the budget deficit. The decline in the domestic budget deficit will tend to improve the current account balance and it will also elicit a favorable response from the foreign country that takes the form of a higher foreign budget deficit. This is true since $M_5$ is negative. However, according to our solution, $M_4$ and $M_5$ are both negative. The sign of $M_5$ can be explained as follows: An increase in the foreign budget deficit improves the domestic current account. The domestic government reduces the growth rate of the budget deficit but still this does not mean that the "level" of the deficit will be lower. Actually, one would expect that the "level" of the deficit should be increasing in order to shift the current account balance towards its desired level and invoke a decrease in the foreign budget deficit.

In order to test whether the numerical solution obtained above is sensitive to "small" changes in the parameter values used in the solution, the simulation procedure was repeated for parameter values in the neighborhood of the original set. In all cases, the solution was identical to the stable solution reported previously.
Since the solution that we obtained satisfies the Routh-Hurwitz conditions for stability, the following should be true: starting from any arbitrary initial values for the state variables of the system \( e(0), x(0), D(0), \) and \( D^*(0) \), the numerical solution for the system of the differential equations should lead to convergence to the steady state solution. In order to verify that this is actually the case, the steady state solution was calculated first. This was done by setting all the time derivatives in equations (23), (25), (26'), and (37) equal to zero. This led to \( e = x = D = D^* = 0 \). Then, the system of these four equations was solved numerically. For all the sets of arbitrary initial values that we tried, the time path of the state variables was converging towards the steady state solution. The speed of adjustment was slow but the convergence was monotonic (after the first 37 iterations). It should be mentioned that, during the adjustment, \( D \) and \( D^* \) were approaching the zero steady state values from the positive and the negative quadrant respectively. This is due to the fact that the feedback equations are asymmetric in the sense that the exchange rate (and, therefore, \( x) affects the two control variables in equal but opposite ways. Table 1 lists the main results obtained by the numerical solution of the system:
### TABLE 1

Convergence of the State Variables towards their Steady State Values

<table>
<thead>
<tr>
<th>Initial values for e, x, D, D^*</th>
<th>Solution for the state variables</th>
<th>Number of Iterations</th>
<th>Min. Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>e, x, D, D^*</td>
<td>e  x   D   D^*</td>
<td>459</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0179 0.00009 0.0042 -0.0042</td>
<td>583</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0179 0.00009 0.0042 -0.0042</td>
<td>748</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0017 0.00009 0.0042 -0.0042</td>
<td>873</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0017 0.00009 0.0042 -0.0042</td>
<td>998</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0017 0.00009 0.0042 -0.0042</td>
<td>1162</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.1</td>
<td>0.00017 9.9E-6 4.2E-5 -4.2E-5</td>
<td>1287</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.2</td>
<td>0.00017 9.9E-6 4.2E-5 -4.2E-5</td>
<td>1412</td>
<td>0.00001</td>
</tr>
<tr>
<td>0.5</td>
<td>0.00017 9.9E-6 4.2E-5 -4.2E-5</td>
<td>1576</td>
<td>0.00001</td>
</tr>
</tbody>
</table>
The first of the cases listed in the above table is described graphically by figures 2-5. Figure 6 plots the time path for the trade deficit given the time paths for the exchange rate and the budget deficits. The column entitled Number of Iterations shows the number of iterations that is necessary for the state variables to converge to the steady state. The last column of the above table shows the minimum loss which is the criterion for convergence for the system of differential equations. The loss is defined as the maximum among the absolute values of the difference between two consecutive values of the state variables. The meaning of Minimum Loss = 0.0001, for example, is that the computer program stops iterating as long as the loss calculated becomes smaller than the prespecified minimum loss. It is clear from the above table that, the more stringent the criterion for convergence, (i.e., the smaller the value for the minimum loss), the larger the number of iterations performed, and the closer the solution to the steady state. It is also clear that the rate of convergence is relatively slow since the number of the necessary iterations is quite large.

V. 6. The Open-Loop Solution

When players follow open-loop strategies, they announce the path of their controls for all the periods of the game based only on the initial state of the system. This means that the feedback parameters
are all zero since the players do not condition their actions on the current state of the economy. Since the equilibrium strategies in this case are not subgame perfect, we will not derive the open-loop Nash equilibrium. However, we can easily find the steady state solution for this case. Forming the Hamiltonian, finding the first-order conditions and setting all time derivatives equal to zero leads to $e = x = D = D^*_\ell = 0$. Therefore, the steady state open loop Nash solution is identical to the steady state closed loop Nash solution. This result is due to the fact that in the steady state open-loop solution the costate variables are all equal to zero.
CHAPTER VI
CONCLUSIONS AND FUTURE EXTENSIONS

This paper has tried to fill a gap in the macroeconomics literature by focusing on the strategic interactions of the fiscal authorities that take into account the relationship between the current account deficit and the budget deficit. A noncooperative approach was followed since (a) cooperation between sovereign policymakers in general is difficult to enforce and (b) fiscal policy instruments are set according to decisions made in parliamentary meetings and reflect primarily domestic considerations.

In the third chapter, a very simple, two-country, fixed-price, flexible-exchange rates Mundell-Fleming model was used to analyze a static, one-shot, symmetric policy game that arises due to an exogenous current account shock. Since there is a positive externality involved, it is expected that if the two policymakers act noncooperatively, the resulting outcome will be Pareto inferior. This was shown by comparing the Nash noncooperative equilibrium with the Pareto optimal cooperative outcome. The gains from cooperation were, therefore, explicitly determined as as function of the policymakers' preferences and the model's parameters. However, due to the problems of enforcement of the cooperative solution, the cooperative outcome was derived only as a benchmark case. It was also shown, that it is important to distinguish
between real and monetary shocks because their effects on the current account balance are different. As expected, it was derived that the gains from fiscal policy coordination are greater under real shocks than under monetary shocks. Another extension to the simple model was the comparison of the gains from fiscal policy coordination under different exchange rate regimes.

In the fourth chapter, the model was expanded to a multiperiod framework and private investor expectations about the future stance of fiscal policy in the two countries were shown to be important factors for the determination of the exchange rate. In this framework, a game played between the policymakers and the investors in the foreign exchange market was analyzed. One of the results derived was that fiscal policy can become time-inconsistent if the government authorities do not have enough policy instruments in order to achieve the first-best outcome. To avoid inferior solutions, (i.e., the time-consistent solution) the ability of the policymakers to credibly precommit to the private agents with respect to their future fiscal policy should be enhanced. This game is relevant for the recent experience since the increasing value of the dollar in the 1988-89 period might reflect (according to the prescriptions of the above game) that the U.S. fiscal policy announcements were not credible. Since strategic interactions take place over time, rather than instantaneously, the one shot games presented in chapters III and IV did not capture any of the interesting dynamics.

The fifth chapter presented a differential game where each policymaker chooses the rate of growth of his budget deficit in order
to minimize an infinite horizon quadratic loss function. The closed-loop (feedback) Nash equilibrium in linear strategies was derived for this game using a numerical procedure. It was shown that there is at least one stable solution to this game. This solution is subgame perfect and, therefore, credible.

Several extensions can be introduced to the basic one-shot static model examined in chapter III:

First, by assuming asymmetric information with respect to the realization of an exogenous shock, another Nash equilibrium can be added to the set of the solutions. With two Nash equilibria, if the game is repeated a finite number of times, using a trigger strategy, (as explained in the introduction to chapter V) it can be shown that a cooperative outcome can be sustained for some periods during the game.

Second, the two-country static game can be extended to the three-country case with some interesting implications. More specifically, if some type of asymmetry is introduced into the three-country structural model, it can been shown that cooperation between two of the countries can lead to a lower welfare level depending on the reaction of the third country. This result is another application of the general proposition that the creation of a coalition can be counterproductive depending on the response of the players that do not participate in the coalition. Canzoneri and Henderson (1988, Ch. III) provide an application of this result analyzing a coalition between two countries (where the size of each one is one half of the size of the third country) that play a Nash game against the third country.

For the differential game of Chapter V, future work would involve the derivation of the cooperative equilibrium and the open loop-Nash
equilibrium. This would allow the determination of the time paths of the state variables under these equilibria and, therefore, a comparison with the time paths that were derived for the closed-loop Nash solution.
Figure 1

Cooperative, Nash, and Stackelberg Equilibria
Figure 3
Time path of the growth rate of the exchange rate
Figure 4
Time path of the domestic budget deficit
Figure 5
Time path of the foreign budget deficit
This appendix gives the reduced-form solution to the structural model described by eqs. (1)-(5) in the text:

\[ y = D \beta_2 \lambda/\pi + D^* \beta_1 \beta_2 \beta_3/\pi \]  \hspace{1cm} (A.1)

where \( \lambda = \beta_2 (1-\alpha_1) + \beta_1 (\alpha_2 + \beta_3) > 0 \) and \( \pi = \lambda^2 (\beta_1 \beta_3)^2 > 0 \)

\[ y^* = D \beta_1 \beta_2 \beta_3/\pi + D^* \beta_2 \lambda/\pi \]  \hspace{1cm} (A.2)

\[ i = D \beta_1 \lambda/\pi + D^* \beta_3^2 \beta_3/\pi \]  \hspace{1cm} (A.3)

\[ i^* = D \beta_1 \beta_3/\pi + D^* \beta_1 \lambda/\pi \]  \hspace{1cm} (A.4)

\[ e = D \{ \beta_2 \alpha_4 \lambda - \alpha_5 \beta_1 \beta_2 \beta_3 - \beta_3 [\beta_1 \lambda - \beta_1^2 \beta_3]/\alpha_3 \pi + \]

\[ D^* \{ \alpha_4 \beta_1 \beta_2 \beta_3 - \alpha_5 \beta_2 \lambda - \beta_3 [\beta_1 \beta_3 - \beta_1 \lambda]/\alpha_3 \pi \} \]  \hspace{1cm} (A.5)

Current account surplus = \( D[(\alpha_5 \beta_1 \beta_2 \beta_3 - \alpha_4 \beta_2 \lambda)/\pi] + \]

\[ D^* [(\alpha_3 \beta_2 \lambda - \alpha_4 \beta_1 \beta_2 \beta_3)/\pi] + \alpha_3 e \]  \hspace{1cm} (A.6)

Under a symmetry assumption, (i.e., \( \alpha_4 = \alpha_5 \)) it can be shown that the coefficient of \( D \) in the current account equation above is negative and, therefore, the coefficient of \( D^* \) is positive. Under the same assumption about symmetry and high capital mobility, (i.e., high value
of $\beta_3$) it can also be shown that the coefficient of $D$ in the exchange rate equation is negative while the coefficient of $D^*$ is positive. Eqs. (A.5) and (A.6) above can be written as follows:

$$e_t = \gamma(D^*_t - D_t)$$

(A.7)

$$CA_t = \eta(D_t - D^*_t) + \delta e \quad \delta < 0$$

(A.8)

where

$$\gamma = \frac{\alpha_4 \beta_1 \beta_2 \beta_3 - \alpha_5 \beta_2 \lambda - \beta_3 (\beta_1^2 \beta_3 - \beta_1 \lambda)}{\alpha_3 \pi} > 0$$

$$\eta = \frac{\alpha_5 \beta_2 \lambda - \alpha_4 \beta_1 \beta_2 \beta_3}{\pi} > 0$$

$$\delta = -\alpha_3 < 0$$

$CA_t$ stands for the current account deficit. Eqs. (A.7) and (A.8) above are the equations used in the model in the text.


APPENDIX B

Derivation of eq. (28):

Subtracting equations (2) and (4) leads to:

\[ \theta_1(y-y^*) = \beta_2(i-i^*) \]  \hspace{1cm} (B.1)

Subtracting equations (1) and (3) leads to:

\[ y-y^* = (D-D^*) + \alpha_1(y-y^*) - \alpha_2(i-i^*) + 2\alpha_3 e - 2\alpha_4(y-y^*) \]  \hspace{1cm} (B.2)

Inserting equation (B.1) into (B.2) and using equation (5') leads to:

\[ e_t [2\alpha_3 + B] = (D^*-D) + E_e t e_{t+1} \]  \hspace{1cm} (B.3)

Solving eq. (B.3) for the exchange rate using forward recursion leads to eq. (28) in the text.

Derivation of eq. (29):

Using equations (1)-(4), the following solutions for \( y \) and \( y^* \) are derived:

\[ y = \left[ D + \alpha_4 D^* + \alpha_3 e(c-\alpha_4) \right] / (c^2 - \alpha_4^2) \]

\[ y^* = \left[ \alpha_3 e(\alpha_4 - c) + \alpha_4 D^* + cD^* \right] / (c^2 - \alpha_4^2) \]

Substituting these solutions into the equation for the current account deficit \[ \alpha_4(y-y^*) - \alpha_3^* e \] and doing some algebraic manipulations yields equation (29) in the text.
APPENDIX C

Differentiating equation (34) with respect to time leads to $\dot{z} = -\beta(i - i^* - e^*)$. Using this equation, along with equations (35) and (21'), leads to

$$\alpha_3e - \alpha_4(D - D^* + 2\alpha_3e) / (\pi + \alpha_4) + \beta(i - i^* - e^*) = 0. \quad \text{(C.1)}$$

Substituting equations (31) and (33) into equation (21') and time differentiating leads to

$$i - i^* = \left(\frac{\beta_1}{\beta_2}\right)(D - D^* + 2\alpha_3\dot{e}) / (\pi + \alpha_4) \quad \text{(C.2)}$$

Combining equations (C.1) and (C.2) leads to the second-order differential equation (eq. (36)) in the text.
This appendix defines the variables used in the system of equations (26')-(30'):
\[ \theta_1 = \alpha_3/(\pi+\alpha_4) \]
\[ \theta_2 = \alpha_4/(\pi+\alpha_4) \]
\[ \delta_1 = -M_1 + H_2 \gamma_1 + H_3 (M_4 - M_5 - 2 \gamma_3) \gamma_4 \]
\[ \delta_2 = -\gamma_2 H_3 + H_2 (M_4 - M_5 - 2 \gamma_4) \gamma_3 \]
\[ \delta_3 = \gamma_3 H_3 + H_4 (M_4 - M_3 \gamma_4) + H_5 (M_5 + M_3 \gamma_4) \]
\[ \delta_4 = -M_3 \gamma_3 + H_4 (M_4 - M_3 \gamma_4) + H_4 (M_5 + M_3 \gamma_4) \]
\[ \eta_1 = \gamma_1^2 + \gamma_2 - 2 \gamma_1 \gamma_4 + 2 M_3 (\gamma_4 \gamma_1 + \gamma_3) \]
\[ \eta_2 = -\gamma_1 \gamma_2 - 2 \gamma_4 \delta_2 + 2 M_2 (\gamma_4 \gamma_1 + \gamma_3) \]
\[ \eta_3 = \gamma_1 \gamma_3 - \gamma_4 \delta_3 + \gamma_4 \delta_4 + (M_4 - M_5) (\gamma_4 \gamma_1 + \gamma_3) \]
\[ t_1 = -M_2 \gamma_1 + H_3 \eta_1 + \delta_1 (M_4 - M_5) + 2 M_2 M_3 \gamma_4 \]
\[ t_2 = M_2 \gamma_2 + H_3 \eta_2 + \delta_2 (M_4 - M_5) + 2 M_2^2 \gamma_4 \]
\[ t_3 = -\gamma_3 H_2 + \gamma_3 H_3 + \delta_3 H_4 + \delta_3 H_5 + M_2 \gamma_4 (M_4 - M_5) \]
\[ t_4 = \gamma_3 H_2 - \gamma_3 H_3 + \gamma_4 H_4 + M_5 \delta_3 - H_2 \gamma_4 (M_4 - M_5) \]
\[ \lambda_1 = t_1 \gamma_1 - t_2^2 + M_3 (t_3 - t_4 - 2 t_1 \gamma_4) \]
\[ \lambda_2 = t_1 \gamma_2 + M_2 (t_3 - t_4 - 2 t_1 \gamma_4) \]
\[ \lambda_3 = t_1 \gamma_3 + M_4 (t_3 - t_1 \gamma_4) + M_5 (t_4 + t_1 \gamma_4) \]
\[
\lambda_4 = -t_1 \gamma_3 + M_3 (t_3 - t_1 \gamma_4) + M_4 (t_4 + t_1 \gamma_4)
\]
\[
N_1 = -\theta_1 \eta_1 - 2 \theta_2 t_1
\]
\[
N_2 = -\theta_1 \eta_2 - 2 \theta_2 t_2
\]
\[
N_3 = -\theta_1 \eta_3 - \theta_2 (t_3 - t_4)
\]
\[
N_4 = \lambda_1 \gamma_1 - \lambda_2 + M_3 (\lambda_3 - \lambda_4 - 2 \lambda_1 \gamma_4)
\]
\[
N_5 = -\lambda_1 \gamma_2 + M_2 (\lambda_3 - \lambda_4 - 2 \lambda_1 \gamma_4)
\]
\[
N_6 = \lambda_1 \gamma_3 + M_4 (\lambda_3 - \lambda_4 + M_5 (\lambda_4 + \lambda_1 \gamma_4)
\]
\[
N_7 = -\lambda_1 \gamma_3 + M_5 (\lambda_3 - \lambda_4 + M_4 (\lambda_4 + \lambda_1 \gamma_4)
\]
\[
P_1 = -\theta_1 \gamma_1 - 2 \theta_2 \delta_1 + 2 M_3 \theta_1 \gamma_4
\]
\[
P_2 = \theta_1 \gamma_2 - 2 \theta_2 \delta_2 + 2 M_2 \theta_1 \gamma_4
\]
\[
P_3 = -\theta_1 \gamma_3 - \theta_2 (\delta_3 - \delta_4) + \theta_1 \gamma_4 (M_4 - M_5)
\]
\[
z_0 = -V/LAN
\]
\[
z_1 = (Y/N) - (VH/\gamma_4) - (U/N) (Z_4'/K/A + VW_3) + (HV/\gamma_4 C) + (IK/\gamma_4 CA)
\]
\[
z_2 = (V/LAN) (B - 3 \delta - E' + M_4)
\]
\[
z_3 = -(K/LAN) - (VW_2'/N) - (HV/\gamma_4 CA) - (VZ_3'/U/NA)
\]
\[
z_4 = (RZ_4'/AN) + (VW_3'/N) + (U/N) (K/LA + VW_2) + (HV(2 \delta - B)/\gamma_4 CA)
\]
\[
z_5 = -(U/Y/N) - HK(\delta - B)/\gamma_4 CA
\]
\[
z_6 = \lambda \theta_2 /LAN,
\]
\[
z_7 = (\lambda \alpha_4 W/N) - (\lambda \theta_2 U/LAN)
\]
\[
z_8 = (\lambda \alpha_4 X'/N) - (\lambda \alpha_4 WU/N) - (H\lambda \theta_2 /\gamma_4 CA)
\]
\[
z_9 = \lambda \theta_2 - (\lambda \alpha_4 X' U/N) - (H\lambda \theta_2 /\gamma_4 C)
\]
\[
U = \delta - H_4 - HL/\gamma_4 C,
\]
\[
Z_4' = (1/L)(\delta - B + E') + (H/\gamma_4 C)
\]
\[ W = \frac{1}{L}(\pi+\alpha_4)(P-E'/A)-(H/\gamma_4(\pi+\alpha_4)CA) \]
\[ W_3 = \left(\frac{C}{L}\right)(G/\gamma_4+E'/\gamma/C)-(H/\gamma_4)(1-\gamma/C) \]
\[ W_2 = \left(\frac{1}{L}\right)(\gamma-E'(2\delta-B)/A)+(H/\gamma_4CA)(B-2\delta) \]
\[ Y = \left(\frac{K}{L}\right)(C-(E'/A)(\delta-B))-(HK/\gamma_4CA)(\delta-B) \]
\[ X' = \left(\frac{1}{(\pi+\alpha_4)}\right)-(PH/C \gamma_4(\pi+\alpha_4))-(1/L(\pi+\alpha_4))(C+E'P) \]
\[ N = \delta-M_3-(\delta/L/\gamma_4C)-(M_5C/L)-E' \]
\[ P = -\delta/A+(\beta_1 \theta_1/\beta_2 A \alpha_4)+(B/A)-1 \]
\[ C = (E+\delta B-E'B)/A-(H/\gamma_4) \]
\[ L = (\gamma_4 M_2/A)+\delta+(M_5 B/A)-(\delta-M_4) \]
\[ \gamma = (E/A)+\delta(B-\delta)/A+GB/\gamma_4 A-H/\gamma_4 \]
\[ A = -M_5-\gamma_4 M_3, \quad B = F-G/\gamma_4, \quad E' = \delta-G/\gamma_4 \]
\[ E = \gamma_2+\gamma_4 M_2, \quad F = \gamma_1-\gamma_4 M_3, \quad G = \gamma_3+\gamma_4 M_5, \quad H = \gamma_3-\gamma_4 M_4 \]
APPENDIX E

This appendix describes the necessary steps in order to derive equation (51) in the text:

1. Solving eq. (46) for $\mu_2$ leads to the following equation

$$\mu_2 = f(\mu_3, \text{the control, state variables, parameters})$$  \hspace{1cm} (E.1)

where $f$ denotes functional form.

2. Substituting (E.1) into eq. (47) yields:

$$\dot{\mu}_1 = f(\mu_1, \mu_3, \mu_4, (.))$$  \hspace{1cm} (E.2)

where $(.)$ stands for the control, state variables, and parameters.

3. Substituting (E.1) into eq. (48) leads to

$$\dot{\mu}_3 = f(\mu_1, \mu_3, \mu_4, (.))$$  \hspace{1cm} (E.3)

4. Substituting eq. (E.1) into eq. (49) leads to

$$\dot{\mu}_3 = f(\mu_3, \mu_4, (.))$$  \hspace{1cm} (E.4)

5. Setting (E.3) equal to (E.4) leads to

$$\mu_1 = f(\mu_3, \mu_4, (.))$$  \hspace{1cm} (E.5)

6. Substituting eq. (E.3) into eq. (E.2) leads to

$$\dot{\mu}_4 = f(\dot{\mu}_3, \mu_3, \mu_4, (.))$$  \hspace{1cm} (E.6)
(7) Substituting eq. (E.4) into eq. (E.6) leads to

\[ \dot{\mu}_4 = f(\mu_3, \mu_4, (\cdot)) \]  \hspace{1cm} (E.7)

(8) Substituting eq. (E.1) into eq. (50) leads to

\[ \dot{\mu}_4 = f(\mu_3, \mu_4, (\cdot)) \]  \hspace{1cm} (E.8)

(9) Setting (E.7) equal to (E.8) yields

\[ \mu_3 = f(\mu_4, (\cdot)) \]  \hspace{1cm} (E.9)

(10) Substituting eqs. (E.1) and (E.9) into eq. (E.4) leads to

\[ \dot{\mu}_4 = f(\mu_4, (\cdot)) \]  \hspace{1cm} (E.10)

(11) Substituting eq. (E.9) into eq. (E.8) leads to

\[ \dot{\mu}_4 = f(\mu_4, (\cdot)) \]  \hspace{1cm} (E.11)

(12) Setting (E.10) equal to (E.11) leads to the solution for \( \mu_4 \):

\[ \mu_4 = f(\cdot) \]  \hspace{1cm} (E.12)

(13) Substituting eq. (E.12) into eq. (E.9) yields the solution for \( \mu_3 \):

\[ \mu_3 = f(\cdot) \]  \hspace{1cm} (E.13)

(14) Substituting eqs. (E.12) and (E.13) into eq. (E.9) leads to a 5th-order difference equation in \( \dot{D} \). Making continuous use of the feedback equation in order to express the higher-order time derivatives of \( \dot{D} \) as a function of the state variables leads to eq. (51) in the text.
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