THE ALLOCATION MECHANISM OF AUDITS:
AN EXPERIMENTAL APPROACH

Dissertation
Presented in Partial Fulfillment of the Requirements for
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*** ***
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CHAPTER 1

INTRODUCTION

The interface of the market for auditing services and audit market characteristics has not been an area closely examined, but will be studied here by examining the effects of audit market characteristics on audit allocations. The decision facing an auditor will be examined analytically. A non cooperative game theoretic model is introduced to examine bidding strategies of auditors. An economic experiment is performed that examines actual bidding strategies.

Competitive microeconomic analysis tends to ignore the allocation system. Sellers and buyers are both assumed to know the price and are assumed to be so numerous that any one of them can have no effect on price. Given this market, the firm operates as if it were faced by a horizontal (infinitely elastic) demand curve. Any price above the market price drops demand to zero for the individual firm. Any price below the market price and the firm has all demand.

Given a perfectly competitive market, price must equal long run marginal cost which equals long run average total cost. Given costless entry, the equilibrium long-run position requires that every firm earn zero economic profits. If profits are higher than this point, entry will take place into the market and if profits are lower exit will take place. The two sources of equilibrium are: first,
profit maximization (P = MC) and second, zero economic profits (P = ATC). Prices above or below this point are defeated by market forces, and the allocation (market) mechanism may not be of interest.

The audit market differs markedly from this competitive analysis. For any client, only one audit is performed. Given that the audit is up for bid, the 'price' of the audit is not known. Consequently, the assumption of unlimited demand at the going price does not hold. And with prices unknown, auditors bid against each other for one client the allocation mechanism may have an effect on market outcomes. Plott (1982) and Robinson (1985) argue that market institutions may have substantial effects on market outcomes. The analytic and empirical economic literature has examined many competing allocation mechanisms (Engelbrecht-Wiggans, 1980; Smith, 1982). Allocation mechanisms with audit market characteristics inserted will be studied.

Game theory has shown that different outcomes can be expected given different rules of the game and solution concepts. If binding agreements can be made then the solution should use cooperative game theory. If binding agreements cannot be made then non-cooperative game theory may better describe the audit environment. As Shubik (1982) has pointed out, one of the key elements of social engineering is to frame the rules of the game so that there will be optimal non-cooperative equilibria. This goal is one of implementing self-policing systems.

As game theory and related empirical work has shown, the rules of allocation may have a significant effect on outcomes. Different allocation methods and audit market characteristics will be studied.
The audit market characteristic of interest here is the higher cost of the initial audit. Arens and Loebbecke (1981) give three reasons for these higher costs: first, all balance sheet accounts of a permanent nature must be verified; second, all initial balance sheet accounts must be verified; and third, descriptions and testing needs to be performed because of the unfamiliarity with the client's operations. These costs are assumed to be sunk non-recurring costs. Any auditor obtaining the audit would have to incur these costs in the initial audit. Therefore, the continuing engagement of audits is not an independent event since the incumbent auditor has cost advantages over rivals, ceteris paribus.

The exact allocation system used in the audit market has not been examined systematically. Many audit allocations use the first-price sealed-bid auction. This auction tenders sealed bids from each auditor. The auditor with the lowest bid wins the auction at the bid price. If the client bargains with several auditors, the allocation method may more resemble the progressive auction. The progressive auction tenders decreasing bids from the auditors till only one auditor remains with the lowest price. Other allocation systems have been argued to be more efficient than the first-price auction. Demand revealing auctions (i.e. progressive and second-price auctions) have been cited as methods which guarantee relative efficient results. Audit market characteristics may modify the efficiency of these other systems. Arguably audit market characteristics may extend to other auction markets. Plott (1982) argues that market institutions have a substantial influence on market outcomes and may outweigh market
concentration and relative firm size in determining the characteristics of the market. The market institution (i.e. allocation method) may have an impact on the market structure and efficiency.

Most economic auction modeling assumes that the auctions are independent events. In other words the winner of the auction in one period has no advantage in succeeding periods. This may not hold in the audit market because of the higher costs of the initial audit. In the second period the incumbent has cost advantages over rival auditors, all else being equal. Bidding strategies in the first-price sealed bid auction under higher cost for the initial audit will be examined. Comparison of the first-price system with other allocation methods will be made.

The auction systems and the audit market will be examined using experimental methods. As with most real world phenomenon, the bidding for audit clients is a very complex problem. Many factors enter into the decision to select a certain audit firm (i.e. price, reputation, expertise, social issues etc.). In the proposed research, only the pricing aspects of the audit will be examined. Other factors could be entered into a model of the audit market by using a multi-component bid function, however the added complexity makes this unappealing for a first attempt at modeling the allocation of audits.

Fellingham and Newman (1985) introduce a game theoretic model of the auditor's decision after the auditor has already been employed by the auditor. Given that the fee has already been set, their model tries to minimize cost. This model takes one step back and examines the allocation of audits and therefore looks at the fees of audits.
Cost minimization is assumed in this study, and the auditors try to maximize revenues. As with the Fellingham and Newman study the bidding for audit clients appears to be game theoretic in nature. Since only one audit is performed per year per client, the winning auditor is the only auditor that can make profits from the engagement.

PURPOSE OF THE STUDY

The study's overall objective is to investigate the effects of audit market characteristics on allocations (bidding) of audits. Prior analytical research has shown that a competitive environment and up front costs should lead to bidding below cost (DeAngelo, 1981a). The up front fixed costs may lead to inefficiencies in the demand revealing single period mechanisms.

A brief outline of the more specific research objectives follows:

1) To examine analytically two period auctions for sensible Nash equilibrium points.

2) To determine the effects of audit market characteristics on bidding patterns under the current allocation mechanism.

3) To compare empirically independent single-period auctions and multi-period dependent auctions.

4) To compare the current allocation system to demand revealing mechanisms under audit market characteristics.

5) To determine the efficiency of demand revealing mechanisms under audit market characteristics.

6) To determine bidding strategies in dependent auctions.

The first objective will be accomplished by using a model of auctions proposed by Vickrey (1961) and used by many other authors for
analyzing bidder behavior (i.e. Milgrom and Weber, 1981). This model posits cost differences among the bidders. This model will be extended to a two period model. Three of the most common auction forms (first-price, second-price and progressive auctions) will then be examined for possible Nash equilibrium points. While a Nash equilibrium point is necessary for a rational outcome, it does not have to be sufficient or unique. Thus, only 'sensible' equilibria will be discussed. The Nash equilibrium will have to be last period rational. Final periods can be seen as sub-games of the first period. The progressive and second-price auctions have sensible multi-period Nash strategies. Using expected utility theory, the different factors that determine the bid will also be examined.

The second objective will be addressed by examining the data from a series of first-price auctions with the initial fixed cost inserted. This test will determine if the bidders bid below cost in the initial period. Previous research has shown that bidding below cost should occur in a competitive environment (DeAngelo, 1981a). The first objective will establish that in this experimental market bidding below cost does in fact occur. This objective also includes analysis of efficiency and bidding patterns of the first-price auction.

The third objective will address the comparisons of dependent and independent auctions. Since single period auctions were not run in this study the comparison will be between previous single period research and the two period model used in this study. Consequently, statistical analysis would be difficult to use because of the differences in parameter values between this study and the previous work.
But the first and second auctions in the two period model could be compared. The second period is essentially a one period auction. Thus, comparison of the first and second period may shed light on the differences between the independent and dependent auction models. This objective will be examined using nonparametric tests in differences in location. In particular, the tests will determine if the price in the first and second period are different. Many other auctions also are dependent. For example, oil lease bidding is usually assumed to be an independent auction, however acquired expertise and information about other tracts may make the auctions dependent. Therefore, this study may address issues beyond the audit market.

The fourth objective will compare the first-price auction and the demand revealing auctions (progressive and second-price auctions). This will be accomplished by comparing experimental bidding patterns under the three auction methods. This objective will be examined by comparing bids and prices among the three allocation mechanisms. Various tests will be conducted, however the primary test will be conducted using nonparametric techniques for a difference in location. The nonparametric tests do not use the magnitude of the differences. Therefore the effects of parameter values on the statistical tests are minimized. In the following section, the demand revealing processes will be hypothesized to give more efficient results than the first-price auction.

The fifth objective will compare the efficiency of results of the three auction mechanisms. These results will be compared with one period model research. In past experimental research, only
percentages have been reported with no statistical tests. The same will be followed here because of the low incidence of inefficiency.

The Investigation

A series of experiments were conducted to test the hypotheses generated in this study. The experiments were conducted from July through September, 1986. The experimental design used in this study permits direct consideration of the research objectives and statistical testing of the related hypotheses. The test results are as follows:

1) In the first period of the first-price auction, bidding was consistently below cost.

2) The winning price in the first period of three auction methods consistently below cost of the winning bidder.

3) The first period price of the first-price auction was consistently below the price of the progressive and second-price auction.

4) The second-period first-price auction gave a consistently lower price than the other two auction methods, while the progressive and second-price auctions appeared equivalent.

5) The progressive and second-price auctions gave higher profits to the bidder than the first-price auction.

6) The first-price auction was not as efficient as the second-price or progressive auction.

Contributions of the Study

The contributions of this study can be classified under two headings: analytical results and empirical results.
Analytical Results

This model extends a model prevalent in the economic literature termed the 'private valuations model.' This means that each auditor has a certain cost for producing the audit. In Chapter II, these cost differences are shown to be documented in much of the audit market research. This model is applied to the audit market through incumbent cost deflation.

This work extends the model proposed by DeAngelo by inserting cost differences and the allocation mechanism. The model proposed in this dissertation is best analyzed by non-cooperative game theory. The outcomes of this market may depend on the allocation mechanism.

If the audit market is best characterized by the first-price auction then there is an explicit trade-off between the probability of winning and profit. A higher bid leads to higher profit but at the expense of a decreased probability of winning.

Certain desirable properties of the progressive auction are lost under dependent auctions. However, a sensible Nash equilibrium strategy is derived. The game in the progressive auctions forces each auditor to calculate the lowest amount he/she would be willing to bid. This bid may or may not be called for.

The economic literature has derived a 'one shot' auction that has the same bidding strategy as the progressive auction. As will be shown in Chapter III, the progressive and second-price auctions still have the same strategies with dependent auctions and incumbent cost deflation.
Empirical Results

This study is the first to test auctions in a multi-period setting. This study will show that even with a time period as short as two periods, bidders will bid below cost. The first-price auction will be shown to have consistently lower profits than the other two allocation mechanisms, which may explain why audit clients select this auction mechanism over other possible systems. An empirical investigation of the auction mechanism shows that the progressive and second-price auction are not as stable as in the one period results. Rather than a dominant strategy, the weaker Nash equilibrium may lead to more variability in the results.

The progressive and second-price auction are shown to be for the most part isomorphic. The overall profits to the auditors is identical. The two period results of this experiment are also consistent with one period results of previous experimental research if the two period results are aggregated.

Chapter Summary

The allocation of audits has not been an area systematically examined by accounting researchers. There appear to be two different approaches for studying this issue: survey study or experimental laboratory markets focusing on audit market characteristics. The first approach would yield the best insight into bidding strategies, but the method is hampered by difficulty in obtaining meaningful data, the lack of cost data and the bias inherent in a survey study.
Use of experimental markets avoids the problems of the survey study. The use of the experimental market is rather a recent phenomenon. The recent emphasis on experimental research may be explained by various factors. First, the lack of databases makes certain interesting questions unanswerable. In this case, actual bidding of audit firms does not exist. Second, the actual market may have unknown parameter values. For example, in this study, cost data on audits would be impossible to determine. Third, extraneous forces may make it difficult to isolate the factors of interest. For example, it has been shown that audit turnover increases at a time of bankruptcy. This type of turnover cannot be classified as an inefficiency. A controlled market eliminates turnover of this kind when it is not of interest.

While the experimental market does overcome these important problems, it is not without its own drawbacks. The main difficulty with the experimental market is external validity. The problem of correct parameter values is of serious concern. In this experiment, the absence of unique parameter values makes it difficult to insert correct values. The use of surrogate subjects in a simplified setting is also of concern.

This research replicates several results of the one-period auction. The final period in a finite auction will always be a one-period model. The basic structure of the auction is used from the economic literature introduced initially by Vickrey (1961). This makes it possible to compare these results with prior analytical and empirical results.
Organization of the Study

The next chapter presents the literature review. The auditing market literature is reviewed in order to better motivate the dissertation. A detailed discussion of DeAngelo's model is given in order to contrast her model with the model proposed in this study. This is followed by a review of the analytical economic literature which attempts to provide a foundation for the audit model. The subsequent section reviews the empirical auction literature. On the basis of the literature review, Chapter III develops an analytical framework for testing and the related hypotheses. Chapter IV specifies the experimental design and the statistical tests used to analyze the results. Chapter V presents the results of the research. Chapter VI, the final chapter, summarizes the study in terms of the results of the hypotheses and their relations to the research objectives. The last chapter concludes with a discussion of the limitations of the study, as well as areas for future research.
CHAPTER II

LITERATURE REVIEW

The accounting and economic literature relevant to a study of audit allocations and audit market characteristics is reviewed. The purpose is to justify the research objectives, to review previous work in related areas to enable the reader to gain a proper perspective for evaluating this study, and to identify problems in prior work which have been addressed in this study.

The chapter is organized into four sections. The first section reviews relevant auditing market literature. This section reviews important variables to better motivate the rest of the study. The second section examines a accounting structure model. Because of the importance of the work of DeAngelo (1981a) this work is reviewed at length. This analysis explains in a very simplified setting the economic rationale for bidding below cost. This section is essential for understanding the theoretical development in chapter III.

The third section broadly reviews the economic literature on auctions. Since audits are usually allocated by sealed-bids, auctions are appropriate vehicles to study the allocation of audits. The theoretical economic literature has been a very fruitful area. The Stark and Rothkopf (1979) bibliography includes approximately 500 works on the subject of auction models. Given the extensive work in
the area only the articles directly dealing with the proposed study will be reviewed. The review of the literature will include models with many buyers and one seller. There is a symmetry between this model and the model with many sellers and one buyer, which is used in this study.

The fourth and final section review previous empirical auction studies. The analysis deals with the comparison of single period auction mechanisms. The empirical work has shown that the auction method may have an impact on price distribution and efficiency.

AUDITING MARKET LITERATURE

This section discusses the work relating to audit market characteristics in order to place this work within the auditing research and to identify important variables in studying the audit market.

The first issue is the nature of supply and demand for auditing services. Examination of the nature of demand for auditing will first be analyzed. There are at least four distinct hypotheses put forth for the demand for auditing. The first is the monitoring hypothesis. Jensen and Meckling (1978) argue that an uncontrolled manager of a firm has a motive to divert firm resources to his/her own personal use (perquisites). The owners realizing the motivation of the manager to divert funds have incentives to invest in monitoring activities. One information source available to the owners is the firm's financial statements. In order to verify these statements the owners may attempt to have third party specialists verify these statements. This monitoring activity then brings about demand for auditing services.
The second hypothesis for the demand for auditing is the information content available in verified financial statements. Wallace (1985) gives three major benefits to be derived from this information: reduction of risk, improvement in decision making and earnings of trading profits. Given risk aversion, the audit may decrease the uncertainty of reported financial statements. More accurate information may help both the internal firm and external users. Auditing may produce more accurate information through discovery of errors and also the incentive given to firms to be more careful in financial statement preparation.

The last information reason for auditing is the possibility of trading gains. The likelihood of managers or any other party of making excessive gains is directly related to the privacy of the information. If the information is not private, then superior gains cannot be expected. For this reason, excessive trading gains is not a very satisfying explanation.

The third hypothesized demand for auditing is the insurance hypothesis (Wallace, 1985). In this explanation of the audit the auditor is hypothesized to have 'deep pockets'. Under the securities act, the auditor and auditee are jointly and severally liable to third parties given that there are losses due to reliance on defective financial statements. The pervasiveness of auditor litigation is a fact documented in the accounting literature (Carpenter and Strawser, 1984). The ability to shift responsibility for reported data to the auditor lowers the expected loss to managers and third parties.
Given larger rewards and increased litigation, the insurance explanation for the auditor becomes more important.

One last demand for the auditor may be induced demand from government agencies. The securities acts guarantee that all public companies will have audited statements. This guarantees auditing for public firms, however auditing was demanded prior to the securities acts and therefore auditing cannot be explained by regulation alone. This hypothesis may include third party insistence on audited financial statements, that the auditee finds in its best interest to comply. For example, a bank may insist on audited financial statements before approving a loan.

In summary, there are four hypotheses for the demand for auditing: the monitoring hypothesis, the information hypothesis, the excess trading hypothesis and the legislated hypothesis. These hypotheses are not mutually exclusive and depending on the company and circumstances, different hypotheses may explain the reason for auditing. The demand for auditing services is assumed for the work in this dissertation. Therefore, the audit is assumed to be mandated by the government.

In order to understand the auditing market, the nature of the supply of auditing services must also be examined. The supply of auditing services has been an area not thoroughly researched probably due to the lack of public information. Ng (1978) discusses three major characteristics that distinguish audits from other consumption commodities. The first is the unobservability of the audit. Unobservability refers to the fact that current and potential stockholders
cannot determine the probability of error detection by the auditor. Current and potential stockholders find it difficult to judge the quality of an audit.

The second characteristic of the supply of audits is the non-additivity of the probability of detection across different audit engagements. For a normal good, individual supply is added to obtain aggregate supply. However, the probabilities of detection should not be added in order to determine the supply of audits. The audit engagement is a heterogeneous product. There are two major implications to non-additivity. First, supply measures cannot be output based but must be input based. Therefore cost of the audit becomes an important factor in determining supply. Second, non-additivity suggests that the audit fee charged may be based on the cost of inputs used by the auditor on the engagement.

The last characteristic is non-privacy. If one person makes use of the audit function through the audited report, another person is not precluded from using the audited statements. Consequently, an audit has attributes comparable to a public good. While infinite persons can make use of the audited financial statements, informed individuals are not necessarily indifferent to making the statements 'public'. One of the demands for audits was the possibility of trading gains; in order to obtain gains, there must be privacy of the information. The non-privacy of the audit, gives the standard free rider problem discussed in the economic literature. The free-rider problem may be one reason that the securities acts require audits for public companies.
Given the arguments provided by Ng, costs of the audits are an important characteristic in the audit market. Elliott and Korpi (1978), Simunic (1980) and Wallace (1985) list different factors that may affect the cost of the audit which in turn affect the fees charged for the audit. The first factor is the legal environment of the audit. Characteristics of this factor include the proportion of public versus private clients in an auditor's portfolio, auditee size, financial distress of the auditee and the loss experience of the auditor. The second factor is the internal characteristics of the auditee. Certainly a strong internal control system will lead to less auditing than a weak internal control system, everything else held constant. Reliance on the internal control system will lead to reduced work in the substantive test phase. The third factor is the economies enjoyed by staff specialization. This factor is probably obtained through two different means: large staff size and industry specialization. Through sheer audit staff size, specialization of the staff may take place and this should lead to economies. The second means is industry specialization. Through industry specialization, the cost of an additional audit in the same industry may drop because the start up costs of being in that industry have already been incurred. Simunic argues that industry specialization is one of the major determinants of audit cost.

The auditee also has some bearing on audit cost through internal control and specification of the extent of auditing. Generally accepted auditing standards give a minimum amount of auditing, the auditee can contract for additional testing.
Kinney (1986) argues that firms tend to have different audit technologies. Most firms have their own method of conducting the audit that may differ significantly from other firms. For example, Arthur Andersen utilizes TFA (transaction flow analysis), Touche Ross TRAP (Touche Ross Audit Process) and Deloitte Haskins and Sells STAR (statistical technique for analytical review). Per Kinney, certain firms are more statistically based in their auditing while other firms rely more on professional judgement. These different technologies may lead to consistent voting and lobbying positions with the Financial Accounting Standards Board (FASB). These different technologies also lead to different cost structures in performing the audit. For example, sample size will almost always be different when using a statistical model when compared with professional judgement. Therefore, divergent audit technologies also lead to cost differences among the auditors. Cost differences among auditors is one of the basic premises of this dissertation and that these cost differences exist appears well documented.

The next issue is the extent of the audit market. In the United States, there are hundreds of audit firms and the extent of the market is an important issue. Dopuch and Simunic (1981) have argued that the audit market is segmented and that all audit firms do not compete against each other. An important factor determining the respective market is auditor size. In examining auditor concentration percentages, the percentage of firms audited by the Big Eight increases as auditee size increases. If the audit market is defined as the Fortune 500 firms, virtually all firms are audited by the Big Eight
accounting firms. (In 1976, for firms with sales in excess of 500 million, 95% of the firms were audited by Big Eight firms). Dopuch and Simunic argue that the Big Eight firms compete in different markets than non Big Eight firms. At least for the large clients, the respective auditor pool appears to come from Big Eight firms. This gives an upper bound on the number of auditors involved in bidding for a large client.

Palmrose (1984) examined the association between agency cost variables and the use of quality-differentiated auditors. Two factors of quality differentiation were employed: Big Eight versus non Big Eight and industry specialization versus non-specialists. Her findings were mixed, with the size of the client being the most important explanatory variable. This finding is consistent with the hypothesis that the Big Eight and non Big Eight firms operate in different markets.

DeAngelo (1981b) puts forth a hypothesis for the concentration statistics found for audit clients. She argues that audit firm size has a collateral aspect because larger firms have a larger future quasi-rent stream (i.e. more clients and billings) to lose than smaller audit firms. Therefore given identical audit technologies, large audit firms are perceived by the market as being more independent from the client. This hypothesis also supports the idea of a segmented market. Given the previous results, the large audit firms are apparently serving a different market than the small audit firms. For experimental testing, bounds are provided for the number of auditors bidding on an engagement.
Another important characteristic of the audit market is the nature of the market (i.e. competitive versus oligopolistic). Given the interest shown by the government in the late seventies, this became an important issue. The lack of data and competitive market benchmarks made it difficult to determine the nature of the audit market. The Big Eight firms as a group have been accused of monopolizing the audit market (Staff Study, 1977). The evidence of the Staff Study relies on concentration statistics and anecdotal evidence. The allegations tend to rely on the concentration doctrine (Scherer, 1981). The problem in addressing this issue is the lack of data about auditors' fees and costs.

In a rather elaborate design, Simunic (1980) tried to address this issue through a test of pricing differences between Big Eight and non-Big Eight accounting firms. This study employed a regression analysis and had as the dependent variable audit fees. Controlling for other differences between audit firms, the independent variable of interest was a dummy variable taking on a value of one for a Big Eight firm and a zero for a non-Big Eight firm. The regression coefficient was found to not differ significantly from zero. Therefore, the hypothesis of price competition prevailing throughout the audit market cannot be rejected. In fact, the coefficient on this variable was negative, indicating that Big Eight firms price lower than non-Big Eight firms. These results suggest that Big Eight firms may have scale economies in their production functions that they pass on to the client in the form of lower fees. However, without knowing cost functions it is difficult to assess the earning of supranormal
profits. Given the results of this study and anecdotal evidence provided by auditors that the audit market has become increasingly competitive in the past years, (Deloitte Haskins & Sells, 1985) competition in pricing is assumed. This means that audit firms do not coordinate their pricing (bid) policies. Not rejecting the hypothesis of price competition leads to selecting a non-cooperative model as relevant to the audit market. If price collusion is present in the audit market, then a cooperative model may better explain the audit market.

Baber et. al. (1986) examine the fees charged by audit firms and turnover in North Carolina Counties. They found that political and financial factors all explained the fees charged by the firms. Also, fees tended to drop when a new auditor was employed.

The state government put out guidelines that the counties were encouraged to follow. The first guideline was that the counties were expected to put their audits out for bid yearly. Second, the auditors were to be changed every four to five years. The counties were also encouraged not to accept bids for multi-year contracts and none were found to exist. This work assumes costless auctions (for both parties) therefore auctions will be held every period. Some audit allocations are bid yearly. However, while auctions may not be held explicitly every period, they are held implicitly. If the client feels the incumbent price is too high, the client will put the audit up for bid.

Fellingham and Newman (1985) examine the interaction of the auditor and client in an audit setting. Using a game theoretic approach and a Nash equilibrium solution, they examine the effects of auditor
strategies on client strategies and vice versa. They indicate that the main contribution of their study is to allow the audit to affect the behavior of the auditee. The game begins with the auditor either choosing high audit level or low audit level (A1 or A2). The authors assume allocation of auditors before the game begins. Since the price of the audit has already been determined, the auditor's game is cost minimization and not profit maximization. The high audit level guarantees knowledge of client actions however is more costly. After the first move by the auditor, the client can choose either high effort or low effort (E1 or E2). High effort decreases the probability of there being a material breach in the client records. The client selects effort level without knowing the preceding action taken by the auditor. Therefore, he/she is in an information set. High effort decreases the probability of a material error but is more costly to the client. The last move in the game is by the auditor with choice of audit report: qualified or not qualified and the respective payoffs are given. After the game is shown in strategic form, the paper inserts ad hoc values and solves for Nash equilibrium strategies for the auditor and client. While the use of game theory in the auditing environment is a good step, there are problems with the analysis. While the authors admit the model is stylized, whether the client moves after the audit work is completed is uncertain. Therefore, the sequencing of the game appears to be incorrect. The game is also a one period model. Therefore, any effects of previous audits or strategies is ignored. Given no audit turnover, this game becomes an infinite game. As the authors point out, the fees charged in this game
are ignored. The game is formulated as one that minimizes expected cost (Kinney, 1975a, 1975b). The fee is assumed to be fixed and predetermined. This dissertation attempts to examine the game that is played in bidding for audit clients. Just as Fellingham and Newman ignored the effects of bidding on their game, the game of the audit report is ignored in the bidding game in this study. Ex-ante, the auditor determines the game, given by Fellingham and Newman, for cost minimization; this expected value is the realization of his/her cost. Therefore, the game of fee maximization is examined in this study, with cost minimization assumed.

The allocation mechanism and allocation of audits is an area that has not been closely addressed. The actual mechanism used in this allocation has not been determined and probably is not unique. For example, government bidding on the average probably uses a one-shot first-price auction. Other audits allocations may use a progressive auction by tendering multiple bids. The outcomes of this issue are studied analytically and empirically.

The first important premise of this dissertation is the assumption of price competition in the audit market which underlies the remainder of the study. Auditors are assumed not to collude in determining their pricing policies. The second important premise is the assumption of differing costs. Therefore auditors face different cost schedules.
Accounting Structure Model

One of the main articles dealing with audit pricing, DeAngelo (1981a) attempts to explain low-balling (i.e. bidding below cost) within a competitive environment. In this model, the incumbent auditor's costs drop relative to other potential auditors. Any first time auditor must incur fixed start-up costs in the first period that do not have to be expended in later periods. DeAngelo also argues that there is a cost to the client of switching auditors. Switching auditors is assumed to generate a negative signal to the market. These sunk costs enable the incumbent auditor to charge quasi-rents in future periods. If the environment is competitive, the present value of the expected rent stream will be equal to the initial cost minus the bid. If the quasi-rent stream is a positive amount, the initial bid will be below cost.

Define $A_1$ to be the cost of the initial audit. Let $A$ represent the cost of the continuing audit. Let $K$ represent the initial start up cost to the auditor so that $A + K = A_1$. Let $F_1$ represent the audit fees for period 1. Let $CS$ represent the cost to the client of switching auditors. Let $r$ represent the discount rate that discounts all future fees and costs. If we assume perfect competition then economic profit must equal zero. Therefore, the following equation holds:

$$F_1 + F/r = A_1 + A/r$$  (1)

Assume that the initial audit has been completed and that the incumbent auditor is deciding the fee to charge the client so that the auditor maximizes profit and still retains the client. In order to
do this he/she must bid lower than the discounted cost of the rival. Therefore the following equation must hold:

$$F + F/r \geq A + K + A/r + CS$$  \hspace{1cm} (2)

The left side of the inequality is current fees plus the present value of future fees. The right side of the equation is equal to current cost of the audit plus the discounted cost of all future costs plus the cost of switching auditors. If this inequality did not hold, then rival auditors could price below the incumbent. Solving equation (2) for $F$:

$$F \geq A + r(CS + K) / (1 + r)$$  \hspace{1cm} (3)

Assume that the auditor charges the highest possible fee, then equation (3) becomes an equality. Since $CS > 0$ and $K > 0$ then $F > A$. Given the assumption of perfect competition, the following equation holds:

$$0 = (F - A_1) + (F - A) / r$$  \hspace{1cm} (4)

Since $F - A > 0$ then $F1 - A1 < 0$ and bidding below cost in the initial period takes place. Apparently bidding below cost in the initial period does not affect the future economic rent stream.

While DeAngelo explains bidding below cost in a very simple setting, there are some weaknesses in the analysis. The first weakness is the assumption of homogeneous costs. If this assumption is relaxed, the result is difficult to determine. If cost differences are sufficiently large, bidding below cost may not take place. The second weakness is the assumption of perfect knowledge and rationality of the auditors. The auditors know exactly all costs and fees
present and future for themselves and rivals. The last criticism is
the lack of initial conditions. Given the symmetry of the model and
the perfect knowledge and rationality of the participants, the selec-
tion of the initial auditor is indeterminate. Given that all bidders
are homogeneous, the selection must be made by lottery or some other
random process.

Analytic Auction Models

Engelbrecht-Wiggins (1980) classifies auctions into four major
components. This classification scheme will be used in discussing
the major results in this literature that relate to the audit allo-
cation.

1. PLAYERS - A player or strategic bidder is one whose bidding
function is unspecified by the model. Non-strategic bidders' strate-
gies are assumed in the model. Standard decision-theoretic auction
models employ only one strategic bidder. The behavior of the non-
strategic bidders is part of the state of nature. In this study, all
participants are considered strategic bidders. This assumption makes
it difficult to derive unique maximum strategies. Each auditor knows
his/her own preference ordering, and knows the probability distribu-
tion on cost of his/her rivals.

2. OBJECTS - The most commonly studied object is a single unit.
The true characteristics of the object may be known or unknown. If
the characteristics are known, the auction may differ on information
signals or value (utility) functions of the respective players. In
this study, the object will be a single unit. The characteristics of
the object will be known but the bidders will differ on value (cost). While the audit market may differ on additional characteristics (i.e. information) this will be suppressed.

3. PAYOFF FUNCTION - The payoff function of the auction determines the distribution of the auctioned objects depending on the different player's bids. In auctions this function determines the worth and who obtains the auctioned item(s). Occasionally this function specifies fees for entering the auction. In this study, the payoff function is a simple one, paying off to the winner price minus cost. Consistently in single unit auctions, the unit is awarded to the bidder submitting the highest bid; subject to the bid being above the reservation price. However, there are many other payoff functions studied in the literature.

4. BIDDING STRATEGIES - Many different bidding strategies are possible in auction models. Various max-min models have been used. However, in most multi-bidder models Nash equilibrium strategies are sought. Nash strategies are in equilibrium if each player uses a strategy which, for the strategies used for the remaining rivals, maximizes the expected value (utility) of the outcome. In most analytical work, players are assumed bidding strategies and the solution is found by solving or by simulation. In empirical work, the bidding strategies of all participants is difficult to determine. The question of whether the Nash points would be discovered or found appealing is a legitimate question.

The seminal work on single period single object auctions is by Vickrey (1961). Vickrey used a very simplified setting with one
seller and many buyers. His analysis does not include a reservation price or bidding costs. The analysis assumes a symmetric, with respect to information, buyers with one object up for bid.

The first auction examined is the progressive auction. In this auction, bids are announced until no further bids are tendered. The last outstanding bid transacts the unit. The expected price for this auction is approximately at the value placed on the object by the second highest bidder. Given rational behavior, the bidding should stop at the value placed by the second highest bidder as after this point is reached only one bidder has incentive to bid. This result is Pareto-optimal because the bidder who values the object the most will obtain the item.

The other type of auction discussed in the literature is the so-called "Dutch Auction". In this auction, the price starts at an arbitrarily high price and the price is dropped in a descending sequence. The first and only bid tendered concludes the transaction. Vickrey states that this auction is technically a game. If a bidder bids full value, he/she maximizes the chance of winning but at a zero gain. By allowing the bidding to continue, players increase their profit but at the expense of the probability of winning. Each bidder must attempt to balance these two factors in determining his/her bid strategy.

Individual values for the object are drawn from the uniform distribution scaled on (0,1), Vickrey assumed that all bidders realize that these values are obtained from this uniform probability distribution, but do not know the respective values given to all the players.
This analysis assumes a value maximizer so any risk preferences are ruled out.

Vickrey explored a Nash equilibrium for the Dutch auction and compares this bidding scheme with the dominated solution of the progressive auction. He finds that both auctions lead to the same expected price.

Vickrey next examined the competitive sealed-bid tenders. He states:

"Actually the usual practice of calling for the tender of bids on the understanding that the highest bid will be accepted and executed in accordance with its own terms is isomorphic with the Dutch auction."

The Dutch and first-price auction lead to gaming among the participants. This gaming may lead to results that are not Pareto-optimal. This leads Vickrey to generate a sealed-bid auction that is analogous to the progressive auction. Vickrey states:

"... the required procedure is to ask for bids on the understanding that the award will be made to the highest bidder but on the basis of the price set by the second highest bidder."

This second-price sealed-bid auction system should lead bidders to bid their full value or full valuation. If the bidder bids more than value, there is an increased probability of winning the auction but this marginal increased probability represents a loss. On the other hand, by bidding less than full value, the probability of winning the auction decreases but the award amount stays fixed at the second highest bid. Therefore, the dominant strategy is to bid full value in a second-price sealed-bid auction. Risk preferences and bidding strategies of rival players should not change this dominant solution.
In the first-price sealed-bid auction, in order for a player to maximize profit, he/she must not only be concerned with his/her own bid, but also the bidding strategies of the other participants. This may cost the participants a considerable amount as they attempt to gather information on market conditions. The second-price sealed-bid auction makes this activity unnecessary as the dominant strategy is to bid full value regardless of other bidding strategies.

Milgrom and Weber (1982) have titled this model the private valuations model. Most of the existing theoretical literature on auctions uses this model. Another model found in the literature is the common value model. In this model, bidders make independent estimates of the common value where the estimates are drawn from the same underlying distribution. This model is assumed to capture auctions such as oil lease bidding. In this model, bidders have information on the worth of the auctioned item. Other things being equal, the bidder with the highest estimate will bid the highest amount. This may lead to the winner's curse phenomenon. On average, the winner will have overestimated the true value of the asset.

While both models introduce uncertainty, they differ on how the uncertainty affects the bidders. The private valuation model has random valuations drawn independently from the same underlying distribution. The uncertainty facing the bidder is the valuation placed on the asset by rival bidders. The bidder knows his/her valuation with certainty. Under the common value model, all bidders have the same unknown valuation. Estimation of the value is possible by information signals, which are usually modeled as independent draws from a statistical distribution.
Milgrom and Weber assume a general model that has as special cases the private valuation and common value models. The general model includes intermediate models between these two models. A major assumption of this general model is that informational variables and valuations are affiliated. Roughly, affiliation means that large values for some of the variables make the other variables more likely to be large than small. Again, the assumption of risk neutral bidders is needed. The first major result is that the Dutch and first-price auction are still strategically identical.

Also, when bidders are uncertain about their value estimates, the progressive auction will lead to a higher price than the second-price auction. Finally, when bidders are uncertain about their value estimates, the second-price auction generates a higher average price than the first-price auction.

One note should be made about the progressive auction assumed by Milgrom and Weber. In their progressive auction, the bidders know the bid price and number of bidders remaining in the game and the exit point of each bidder. The number of bidders still bidding is an important informational variable, that may not exist in many real world auctions.

If risk aversion is allowed, no clear comparison can be made between the auctions. Most analytical work on auctions does not hold under risk aversion. The difficulties of modeling auctions with an unknown utility function makes any type of solution intractable. Certain work has tried to get around this by assuming constant absolute risk aversion.
Auctions may be defined as games with incomplete information as defined by Harsanyi (1967). The dichotomy of cost or information asymmetries has driven most of the theoretical auction literature. As Holt (1979) states, even the first-price auction is an approximation to the real world. In many auctions, some type of bargaining takes place besides the sealed-bids. Is this auction a first-price auction or maybe some other allocation system (i.e. progressive)? Additionally, in almost all auction work, non-price considerations are absent. Previous economic work and this study will be more relevant to auctions with price considerations only.

Holt (1979) is one of the few studies to have both symmetric but unknown costs and symmetric information. In this model, the bidders differ on possible alternative profits of the bid project is not won. In this model, the winner of the auction is the bidder with the lowest alternative profit. A Nash equilibrium strategy is found that is very similar to the marginal analysis found in the other papers. As with all previous work, a closed form solution cannot be found without risk neutrality or constant risk aversion.

Rob (1986) develops a model that combines optimal auction theory and incentive contracting. His model is assumed to deal with bidding such as defense contracting. His goal in the paper is to select the most efficient producer (notice cost differences) and then to induce socially maximizing research and development costs.

Underlying much of the work on auctions is the assumption of independence of the auctions. The economics literature has noted that the assumption of independence is at best an approximation to the
real situation. Possibly, dependent auctions can be modeled as one large independent auction but much of the sequential decision making is lost in this formulation. Oren and Rothkopf (1975) modeled a sequence of auctions with one strategic bidder as a dynamic programming problem. Remaining players are assumed to react to the bidding in a prescribed manner. According to Eglebrecht-Wiggin's (1980), sequences of auctions with more than one player are much more difficult to analyze; there are no results in this area.

Empirical Auction Results

This section will review single period, single unit auctions. There has been extensive work examining the efficiency and allocation effects of different auction methods. Coppinger, Smith and Titus (1980), CST, compared the four different auction methods in an experimental setting. The valuations given to the subjects were determined in two different fashions. In the first method the subjects were given valuations that differed by a constant and ran sequentially from the highest to lowest valuation. The other method determined valuations by drawing them independently from a uniform distribution.

The following were the major results from the study. First, the progressive price did not differ significantly from the optimal price. The optimal price is defined as the lowest point where supply equals demand (i.e. the valuation of the second highest bidder). The major reason that the optimal price was not met was because bids are raised in noninfinitesimal amounts. Secondly, the second-price auction also did not differ significantly from the optimal price. There
was a learning curve detected in the second-price experiments where the dominant strategy was not obvious immediately. CST questions whether any meaningful one-shot observations can be made on processes characterized by a dominant strategy equilibrium.

Third, the second-price and progressive auctions appear to be isomorphic if allowances are made for the learning curve and certain technical differences.

Fourth, prices in the first-price sealed-bid auction tend to be higher than the optimal price. The Dutch and first-price systems do not appear to be isomorphic.

Fifth, if efficiency is measured by the percentage of sales that are Pareto-optimal (i.e. the bidder with the highest valuation obtains the item), the progressive auction is the most efficient. The progressive auction is relatively free from strategic considerations. The second-price auction is the next most efficient auction and with the learning curve omitted may be as efficient as the progressive auction. The first-price auction is the third most efficient market auction.

Cox, Robertson and Smith (1982), CRS, extended the work of CST with a more complete experimental design. CRS did not examine the progressive auction because of the evidence that this auction gave the optimal price. Pooling across all experiments they found that the mean price under the second-price auction was below the Dutch auction which is less than the mean price in the first-price auction.

The experiment was run with groups of different sizes in order to measure the impact of the number of bidders. The group sizes tested
started at three group members and ran to nine. A risk averse model described bidding strategies for all groups except those with three group members. The authors hypothesized that groups of three may induce collusion.

Efficiency measured by the percentage of total gains from exchange that are actually realized is greatest in the second-price auction. Next highest is the first-price auction and the Dutch auction is the lowest.

Chapter Summary

The first section of this chapter examine the audit market literature for important characteristics in addressing the allocation of audits. The nature of demand and supply of audits was introduced. The audit market was assumed to be competitive and this means that the non-cooperative game theoretic model is the most relevant for the audit market. The large audit firms tend to operate in different markets than the small firms. The study by Fellingham and Newman (1985) that introduced game theory to a audit setting was also introduced.

The second section of this chapter discussed the model introduced by DeAngelo. The importance of the up front cost on bidding was discussed. This section showed that competition guaranteed bidding below cost if costs are identical. While the model met its stated purpose, the model still is very simplified as discussed in this chapter.

The next section discussed single period analytical results. While this research uses very powerful mathematical techniques, the
results and settings are still at an infant state. The possible asymmetries between bidders were discussed along with possible strategies. This work still rests on the assumption of risk neutrality. The lack of dependent auction results was noted. However, although many authors realize that auctions are not independent events, assume a one-shot auction because of tractability.

The final section reviewed prior empirical auction literature. The discussion highlighted the major results that will be compared to the results in this study.

The purpose of this chapter has been to motivate this study, as well as to provide a basis or perspective from which to analyze and evaluate the study. Previous research has shown that further investigation of the auction market may yield interesting insights into market behavior. The chapter has discussed both empirical and analytical models provided by auction research to date, and has suggested potentially fruitful areas for extension.
Chapter III
Auction Theory and Hypotheses

This chapter introduces auction theory into the audit market by making the auctions dependent through incumbent cost deflation. Before the theory is discussed, an audit framework is introduced, highlighting the variables discussed in this section. This framework will attempt to highlight important points in this process. Next the theory of bidding in auctions is considered. As was previously noted, there is an extensive literature on this subject (Engelbrecht-Wiggins, 1980). Theoretical and empirical results have shown that the method of allocation may have an influence on the results of the market. This section examines the characteristics of the first-price auction and compares it with other possible allocation systems. The modeling in this paper is not meant to describe all audit allocations. Every audit auction has its own structure and characteristics. This theory will give insights into the nature and efficiency of the audit allocation question. The third section concerns the theory of a single object, single-period auctions. One period models are examined because the multi-period models build on these models and, for comparison with the multi-period models. Both risk neutral and risk averse models are considered. The next section examines multi-period
auctions that have dependencies between the auctions. The three auction mechanisms of interest are examined for possible Nash equilibrium points. The last section of the study details the research variables of interest and the resulting hypotheses.

Audit Allocation Framework

This section attempts to examine the important variables in the audit allocation and identify the variables examined in this study. Figure 1 presents a framework for examining the allocation of audits. The framework focuses on the auditor's characteristics and the client's perception of these characteristics. Each auditor brings to the auction decision his/her particular mix of personal attributes including cost, risk preferences, expertise and reputation.

Many factors may potentially influence the bid price that an auditor quotes a potential client. The first identified factor is the perception of future profit. The higher the expected profit in succeeding periods the lower the bid in the initial period, ceteris paribus. The second factor is cost. The higher the cost the higher the bid in the initial period. The third factor, risk aversion may also influence the bids. The last factor is the perception of desirability. The first three factors will be discussed here.

Bid price is not the only relevant variable in the decision of auditor allocation. Other hypothesized variables include expertise, reputation and social factors. Quality of staff and related clients are both hypothesized to impact on perceived expertise and reputation. Social considerations, while important are very difficult to
Figure 1

CLIENT/AUDITOR ALLOCATION PROCESS

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Bid Price

Client Perception of Auditor

Client Decision
observe or model. The client decision in this dissertation will focus explicitly on cost of the audit. Given the literature review, cost differences appear to be an important difference between auditors. While this is not the only relevant parameter, in a first attempt at modeling the allocation process this will be the only variable considered. Arguably all differences can be denominated as cost differences. Bid price, expertise, reputation and social skills make up the auditor's parameter values. It is these values that are observed by a potential client. After observing these values from the possible auditor pool, then the client selects an auditor.

Single Period Auctions

In order to understand multiperiod auctions, single period auctions will first be explored. In any finite auction sequence, the last period is a single period auction. In order to model the auction process certain assumptions have to be made about the utility functions of the auditors. The audit setting is deliberately simplified in order to highlight the variables of interest. Assume that the utility of bidding is zero. This rules out any transaction or bidding costs. Assume that the only utility derived from the auction is the money outcome. Assume auctions are costless to hold and enter. In the seminal piece in this area, Vickrey (1961) assumed a value maximizer. Given current research, this assumption is suspect; however, certain insights into the market are obtained by keeping this assumption. Reed (1982) states that expected value models are reasonably good predictors of human decision making. This analysis
will examine one buyer with N sellers. In the market with many sellers each seller is trying to sell a good with value (cost) to them. This value will be considered the cost of producing the unit (audit). Assume that the audit environment is price competitive and the auditors offer from a client point of view a homogeneous product.

Three typical auction methods: progressive, first-price and second-price have already been introduced. In the progressive auction, the price should stop an epsilon (E) below the second lowest cost. Because of this trait, most authors have used the second lowest cost as the Pareto-optimal price. Assume rational behavior, defined as Nash equilibrium behavior, and that the costs come from a rectangular probability distribution on the interval (0,1) and that all participants are aware of how costs are determined. This assumption is a typical one made in the economic auction literature. Feinstein, Block and Nold (1985) give another possible determination of cost. In their paper, cost of player 1 is \( C_1 = X + Z_1 \) where \( X \) is the mean cost for all players and \( Z_1 \) is distributed \( N(0, s^2) \). Both cost assumptions are random variables. For simplicity, costs uniformly distributed on (0,1) were assumed.

First Price Auctions

As previously discussed, the first-price auction gives rise to possible gaming. In this case, the Nash equilibrium bid for each auditor is defined by:

\[
B_{1d} = 1 - \left[ \frac{(N-1)/N}{(1-C_1)} \right]
\]  

(5)
where:

\[ N = \text{number of participants in the game, and} \]
\[ C_i = \text{the cost of auditor } i. \]

This bidding strategy meets the definition of a Nash equilibrium point. If all auditors follow this bidding strategy, the auditor with the lowest cost will win the auction which is a sufficient condition for Pareto-optimality.

To calculate the expected price, the expected value of a first order statistic must be calculated. The first order statistic is equivalent to the expected cost of the low cost auditor. This value is calculated by solving the following expression:

\[
E(C) = \int_0^1 C N(1-C)^{N-1} \, dC \tag{6}
\]

where:

\[ E(C) = \text{Expected cost of low cost auditor.} \]
\[ E(C) = \frac{1}{(N+1)} \]

Inserting this value into equation (5) for \( C_i \) gives the value of the expected price. The expected price is equal to:

\[
E(P) = \frac{2}{(N+1)} \tag{7}
\]

Therefore, as \( N \) becomes large, the expected winning bid price drops. This is logical given the rectangular probability distribution on the \( C_i \).
Progressive Auction

In the progressive auction, the bidding should stop an epsilon (E) below the cost of the second cost auditor. Ignoring the E difference, assume that the winning bid price will be at the second lowest cost point. To find the expected price of this type of auction is equivalent to finding the expected value of a second order statistic. Redefine \( C \) to represent the second lowest cost. The probability that \( C \) falls between \( C \) and \( C-dC \) is given by the expression:

\[
P(C) = N(N-1)(1-C)^{N-2}C dC. \tag{8}
\]

The term \( (1-C)^{N-2}C dC \) gives the probability that \( (N-2) \) auditors fall above \( C \). The price in this auction will be at \( C \), therefore, the expected price will be determined by the following equation:

\[
E(P) = \int_{0}^{1} CN(N-1)(1-C)^{N-2}C dC \tag{9}
\]

Equation (10) is derived by integrating equation (9) by parts:

\[
E(P) = 2/(N+1) \tag{10}
\]

\( \star \) The expected price for both auctions is identical. The second-price auction is analytically isomorphic to the progressive auction.

Under the assumptions of Nash equilibrium behavior, the variance of the price and the gain is wider under the first-price auction compared with the progressive and second-price auctions.
Utility Maximization

The expected utility hypothesis holds that an auditor chooses his/her bid in order to maximize his/her expected utility of the money income gained from participating in the auction. The progressive and second-price auctions have solutions that should not depend on player's expectations or risk preferences. The price transacted should be at the cost placed by the second lowest seller (dominant strategy).

Let $u_i$ represent the concave utility of income for the $i$th auditor. The utility from winning this auction is equal to $u_i(b_i-C_i)$. Let $f_i(b_i)$ be the subjective probability perceived by auditor $i$ that he/she will win the auction given bid $b_i$. Let "" refer to the differentiation operator. Thus, the expected utility of bidding $b_i$ is equal to:

$$U_i = f_i'(b_i)u_i'(b_i-C_i).$$ (11)

If the necessary conditions for differentiation are assumed, then a solution can be obtained for the optimal bid, $b^*$.

$$0 = U_i''(b^*) = f_i''(b^*)u_i'(b^*-C_i) + f_i(b^*)u_i'(b^*-C_i).$$ (12)

Assume that the necessary second order conditions hold:

$$0 > U''(b^*)$$
The past two equations along with the implicit function theorem imply that there exists a differentiable function such that:

\[ b^* = \phi(C_i) \]

\[ \phi(C_i) = C_i + u^{-1} [-u'(b*-C_i)f(b*)/f'(b*)]. \]  

(13)

Under utility maximization hypothesis, \((C_i)\) is auditor \(i\)'s bidding function or bidding strategy. Since \(f'(b^*)\) is strictly less than zero, the term inside the bracket in equation (13) is greater than zero. Given this result, the optimal bid by auditor \(i\) is greater than his/her cost on the object. The first-price sealed bid auction is not a demand revealing process. The bid is a function of cost plus the embedded shading. Riley and Samuelson (1981) explicitly calculate the Nash function with its embedded shading. The optimal bid by bidder \(i\) depends on \(u_i\) and \(f_i\). The terms \(u_i\) and \(f_i\) are influenced by risk preferences and expectations of other auditors' bids. Since these two quantities may differ among auditors, the lowest bid may not be tendered by the auditor having the lowest cost. However, if risk preferences and expectations are identical, this auction will result in a Pareto-optimal allocation. These two conditions are not likely met in actual practice.

Cox, Roberson and Smith (1982) examine an equilibrium model that allows bidders to differ in their attitude toward risk. Assume that the bidders have utility of income equal to \(y^{rl}\), where \(y\) is income and \(rl\) is a random variable scaled on \((0,1)\) with probability distribution equal to \(A\). Each auditor will know his/her risk parameter but only knows the probability distribution on his/her rivals. In this case, the Nash equilibrium bid is:
\[ b_1 = 1 - \frac{(N-1)C_1}{(N-1-1)} \]  

(14)

If \( A \) equals one then the auditors are risk neutral, and the bidding function is equivalent to equation (5). Using a strong first-order stochastic dominance ordering on the probability of the winning price under the the risk averse and risk neutral models, the mean price under a risk averse model is lower than the mean price under a risk neutral model. The mean price should be lower under the first-price auction compared with the progressive and second-price auction if auditors are risk averse.

Conclusion - Single-period Auctions

Many of the one period models assume fixed strategies by the \( N-1 \) non-strategic bidders with only one strategic bidder making decisions. These models cannot be tested when all market participants can make decisions. The key results are that the first-price auction is a game and the second price and progressive auctions are not. Academics and others (Friedman, 1957; Engelbrecht-Wiggins, 1980) have called for replacing the first-price auction with the progressive or second-price auction. The reason for this is clear when the auctions are independent.

Under risk neutrality, the expected price under the three auctions is identical. This does not mean the actual resulting price will be the same under any cost assignment. However, under risk aversion, the first-price auction should result in a lower price than the other two auction mechanisms. This leads to the hypotheses that in the final period of a multi-period sequence, the first-price auction
will have a lower price than the other two auction mechanisms. If there are dependencies between the auctions, the characteristics of these models may break down. Thus, implementing the single period non-gaming auctions may not result in an efficiency improvement.

**Multiperiod Auctions**

The assumption of independence of auctions appears to be rather restrictive assumption for many auctions and for the audit market in particular. The audit market is assumed to entail fixed cost upon entry. These fixed costs give rise to a dependency among the auctions as the incumbent auditor holds an advantage, ceteris paribus. In addition to the initial cost advantage, DeAngelo (1981a) also suggests that there is a cost to the client in switching auditors. In the author's experience, there is also a start-up cost to the client. The client is also involved in the initial work of the new auditor. This dependency eliminates the dominant solution of the progressive and second-price auction. This section will first look for Nash equilibrium points within the three auctions. The Nash equilibrium strategy will also have to be last period rational. The Nash equilibrium strategy will, at a minimum, have to give positive profits to the winner in the last period. Making negative profits in the last period would be irrational because zero profits can always be generated by not bidding. A two period model will first be discussed and will then be extended to a T period model. The basic assumptions from the one period model hold (i.e. cost determination etc.). The winner of the auction in period one has his/her cost
deflate by (1-θ). After discussing Nash points, the computational calculations that must be made by the auditors if the Nash strategy is not implemented will be discussed (Riley and Samuelson, 1981). For the discussion on the computation in an auction, a two period model will be assumed.

Define the following terms:

\( P_m(b) \): Subjective probability of outcome \( m \) in first auction given bid \( b \).

\( P_{nm}(b2) \): Subjective probability of outcome \( n \) in second auction given outcome \( m \) in first period and bid \( b2 \).

\( m, n = 1 \): auction won.

\( m, n = 0 \): auction lost.

\( D_l \): initial cost for auditor \( l \).

\( 1 - \theta \): proportional decrease in cost if incumbent

\( C_l \): recurring costs for incumbent auditor \( l \).

\( T \): Number of dependent periods in an auction.

\( b \): bid in period one.

\( b2 \): bid in period two given a win in period one.

\( b^* \): bid in period two given a loss in period one.

\( N \): number of auditors.

\( 0 DJ = C_j \)

In these models, discounting will be ignored. While discounting adds some realism, the basic results do not change by discounting future profits. Assume that the winning auditor and price is revealed after each auction. Both variables are known in real world audits. All other assumptions from the one period model will hold for the multi-period model.
Progressive Auction

Before modeling the progressive auction, basic characteristics of the progressive auction will be noted. In a progressive bid auction, ex-post first period results are mutually satisfactory to all the auditors. This means that all bids have been exhausted, therefore, no bidder would change the outcome. This does not guarantee Pareto-optimality (as compared with the one period model) as auditors may have different perceptions of the possible future economic rent stream from being the incumbent and different risk preferences. The first-price auction does not guarantee that the results are mutually satisfactory. An auditor may wish to rebid once the results of the auction are announced but will not be allowed to do so.

Given the use of the progressive auction, the necessary strategy formulation is not to determine one bid, but to determine the lowest amount that an auditor is willing to bid. Therefore in the formulation of the problem, the auditors are in a multiple move bidding game within each period. Therefore the price is always at the second lowest reservation bid. The auditor with the lowest cost has advantages over the other auditors and therefore needs to devise a strategy that guarantees nonnegative profits to himself/herself and zero profits to everyone else. If any other auditor wins the bidding, he/she must earn negative profits. However, before the bidding starts the low cost auditor is not known by the market participants. Each auditor must devise a strategy as if they were the low cost auditor. The game is formulated in the following program:
Max: \([(F11-D1) + (F21-C1), 0] \) \( (15) \)

subject to:

\[(F1j-Dj) + (F2j-Cj) < 0 \quad \text{for} \ D1 < Dj \] \( (15a) \)
\[(F1j-Dj) < 0 \quad \text{for} \ D1 < Dj \] \( (15b) \)
\[(F2j-Dj) < 0 \quad \text{for} \ D1 < Dj \] \( (15c) \)

where:

\[ Fmi = \text{The lowest acceptable bid for auditor } i \text{ in period } m. \]

Equation (15) gives the formulation of the problem and there are no probabilities in the formulation because of the mutually satisfying principle. Constraint (15a) states that winning the audit for the two periods by an auditor with higher cost guarantees negative profits. Constraints (15b) and (15c) state that an auditor with higher costs cannot make positive profit for any period of the auction. Since all auditors are strategic participants, a Nash equilibrium will be sought.

In a two period model, the game must be solved backwards. This guarantees that the solution is last period rational. The second period can be thought of as a subgame. In order for a Nash equilibrium strategy to be rational, the outcome in the second period must be rational (give positive profit). Assume that the first period has taken place and bidding in the second period is about to start. Therefore, the equation to maximize becomes: Max \([(F2i-Ci), 0] \) for the incumbent. Bidding below cost in the second period gives a loss, therefore, as in the one period model the price should stop at the cost of the second lowest cost auditor. If the low cost auditor won
the auction in the first period, his/her profit would be equal to \( D_2 - C_1 \) (where 2 is the second lowest cost bidder and 1 is the lowest). Given this profit in the second period, the low cost auditor must devise a bidding strategy in the first period that solves equation (15).

Given profit of \( D_2 - C_1 \), the auditor can bid to \( D_1 - D_2 + C_1 \) and still have nonnegative overall profit. The solution to equation (15) is for all auditors to bid to their respective \( C_1 \) in the initial period and to their \( D_1 \) in the second period. This Nash strategy guarantees profits to the low cost auditor and zero profits to everyone else. If any non-low-cost auditor deviates from this Nash strategy and wins the auction, he/she must have negative profits.

The progressive auction will now be examined for possible Nash points in a dependent auction sequence larger than two. In a Nash equilibrium solution auditor \( i \) with initial costs of \( D_1 \) and repetitive costs of \( C_i \) would be to be willing to bid as low as \( C_i \) for periods 1 through \( T-1 \). In period \( T \), the incumbent should be willing to bid to \( C_1 \), all other auditors to their respective \( D_1 \)’s.

If auditor \( i \) follows this strategy, he/she guarantees certain beneficial outcomes. The first outcome is that any auditor that has higher costs will guarantee losses if he/she obtains the auction. This result can easily be verified. Assume that auditor \( i \) follows the preceding Nash strategy. Assume that auditor \( j \)’s costs are an epsilon higher than the cost of auditor \( i \). Assume that auditor \( j \) obtains the audit for the full auction horizon. To accomplish this, player \( j \) would not be following the Nash strategy as he/she will have
to bid below \( C_j \) in order to win the auction. Player \( j \)'s loss is equal to:

\[
L_j = [C_i-D_j] + (T-2)[C_i-C_j] + [D_i-C_j]
\]  \hspace{1cm} (16)

The first term represents the loss to auditor \( j \) in the first period. The second term represents the recurring loss for periods 2 through \( T-1 \). The last term gives the expected gain in the last period. Since \( D_j > D_i \) and \( C_j > C_i \) the last period gain does not even offset the first period loss. Since the first two terms in equation (16) are negative for auditor \( j \), he/she could not have positive profits for any sequence of wins and losses in the dependent auction sequence. The best auditor \( j \) can do is break even.

If all players follow this Nash strategy, the profit to the low cost bidder \( i \) is equal to:

\[
\pi_i = [(C_j-D_i)] + (T-2)[(C_j-C_i)] + [D_j - C_i]
\]  \hspace{1cm} (17)

The last term in this equation does not drive the result. This auction could go to infinity with the same Nash strategy. The second term in equation (17) is positive and in contrast to equation (16) the low cost auditor begins to recoup the first period loss in the second period.

In contrast to the one period model, bidding a Nash strategy does not guarantee a positive outcome against all other possible bidding strategies and therefore the strategy is not dominant. For example,
assume again that auditor 1 has the lowest cost and auditor j and auditor k have the second and third lowest cost. Assume that auditors j and k bid the Nash strategy for the full auction horizon. However, in the first period, auditor 1 bids D_1 and then after the first period he/she follows the Nash strategy. Auditor j will win the auction in the first period and auditor 1 will win the auction for periods 2 through T. In period one, auditor j will have a loss equal to (C_k - D_j). By following this strategy, auditor 1 does not maximize his/her gain. If auditor j does not change his/her bidding strategy following the first period loss, then there is no apparent reason for auditor 1 to bid this strategy. However, if bidding the Nash strategy auditor j still takes a loss, this auditor may revise his/her bidding strategy making it possible for auditor 1 to have a greater gain in periods 2 through T.

The next step in the analysis will examine the expected profit in the progressive auction if all auditors follow the Nash strategy. Given the basic assumptions, the expected value of a first order and second order statistic must be calculated. From the one period model, the expected values of a first and second order statistic are equal to 1/(N+1) and 2/(N+1). Inserting these values into equation (16), the expected profit for the auction is equal to:

\[ E(\pi) = \frac{[2\theta/(N+1) - 1/(N+1)] + (T-2)\theta[2/(N+1) - 1/(N+1)]}{2/(N+1) - \theta/(N+1)} \]  

or

\[ E(\pi) = \frac{[(T-1)\theta+1]}{(N+1)} \]  

This equation reduces to:

\[ E(\pi) = \frac{[(T-1)\theta+1]}{(N+1)} \]  

(19)
There are a few points to make about equation (19). First, as $N$ increases, the expected profit for the winning auditor decreases. To determine the expected profit per auditor before the bidding starts, divide equation (19) by $N$. Therefore, the expected profit per auditor decreases with $N^2$. This points to a possible tradeoff. The buyer would want to maximize the number of sellers because the expected selling price drops. However, as auditors are added the expected profit decreases rapidly and the motivation to take the auction seriously decreases rapidly. Engelbrecht-Wiggens (1980) discusses the situation of having non-serious bidders. The second result is that the expected profit of the low cost seller increases with an increase in the number of dependent auctions.

**Computational Model - Progressive Auction**

If an auditor does not realize the above Nash strategy, the auditor will have to make certain calculations in order to determine his/her bid. Assume a two period model, that the bid is at $b^{**}$ and that auditor 1 does not have the outstanding bid. Assume the next bid increment would bring the bid down to $b$. For simplicity, assume that the auditor is a value maximizer. The following condition must hold in order for auditor i to enter the bid $b$:

$$
EV = (b-D1) + \int_{b2>C1} P11(b2)(b2-C1)db2 \geq \int_{b*>D1} P10(b*)(b*-D1)db* \geq 0 \quad (20)
$$

where:

$$
P11(b2) = \text{perceived probability distribution on } b2 \text{ given a win in period one.}
$$
\[ P_{10}(b^*) = \text{perceived probability distribution on } b^* \text{ given a loss in period one.} \]

A probability is not tied to \((b-Di)\) because if auditor 1 bids \(b\) and they are assured of winning the auction subject to no other auditor bidding lower. Therefore, the real task in this auction is to try to determine the future quasi-rent stream.

Assume that the middle term in equation (20) is zero. This is plausible since if an auditor did not win the auction in the first period, then he/she knows that the incumbent's costs have dropped relative to his/her costs. Apparently the probability of winning in this situation has to be close to zero. In the empirical portion of the paper, no turnover took place in the progressive auction lending credence to the fact that this term is zero. If this assumption is made then an auditor will bid down to the point that the following equality holds:

\[
(b-Di) + \int_{b \geq C1}^1 P_{11}(b^2)(b^2 - Ci)db^2 = 0
\]

or equivalently

\[
b = Di - \int_{b \geq C1}^1 P_{11}(b^2)(b^2 - Ci)db^2
\]  

(21)

If rational behavior is assumed (i.e. winning bidder does not bid below cost in the second period), the integral is only valid for \(b^2 \geq Ci\), therefore, the integral does not sum negative amounts. Since the integral is greater than or equal to zero, the auditor would be willing to bid below cost. The multi-period progressive auction may lead to bidding below cost. If the probability distribution, \(P_{11}\), sums to one on the interval \(Ci\) to one in the second period, the
uncertainty in the second period for the incumbent is not whether he/she will win but rather the winning amount. Given that turnover is zero in the progressive auction, this is identical to stating that P11 sums to one.

The result for a utility maximizer is the same as a value maximizer but the expressions for any value functions must be replaced by utility functions. The rest of the results follow directly from the value maximizer calculation.

The progressive auction first period bid is a function of cost and the expected profit in the second period. As will be shown later, this auction will have less uncertainty (fewer unknown variables) than the other auctions.

Second-price Auction

The second price auction has the same Nash strategy as the progressive auction. If all auditors bid their cost C1, for the auction horizon, then any auditor cannot improve his/her income by bidding another strategy. By bidding this strategy, the same characteristics of the progressive auction are captured by the second-price auction.

Computational Model - Second-price Auction

If this strategy is not understood, the calculation for the auditors in the second-price auction is complicated. The computational solution to this auction is difficult to determine as the actual selling price cannot be determined from a winning bid. The winning bid
only determines who was the winner of the auction, not the price. Therefore, the auditors will each have probability distributions on the possible selling price.

Given a two period model, auditor 1 must maximize the following equation:

\[
EV = P1(b)\left[ \int_{b_r}^{b_2} P(b_r) (b_r - D1) db_r + P11(b_2) \left( \int_{b_2}^{1} P(b_1) (b_1 - C1) db_1 \right) \right] + \\
P0(b) [P10(b^*)] \int_{b^*}^{1} P(b^*) [b^* - D1] db^*.
\]  

(22)

where:

- \( b_r \) : the second lowest bid in the first period.
- \( b_1 \) : the second lowest bid in the second period given a win in the first period.
- \( b^* \) : the second lowest bid in the second period given a loss in the first period.

Again, assume that \( P10 \) is sufficiently close to zero so that the second term is zero. For tractability, assume that the bid in the second period does not depend on the bid in the first period. Let \( d \) refer to the differentiation operator. Define the following terms:

\[
i = \int_{b_r}^{1} P(b_r) [b_r - D1] db_r + P11(b_2) \left( \int_{b_2}^{1} P(b_1) (b_1 - C1) db_1 \right)
\]

(23)

\[
T' = d\left( \int_{b_r}^{1} P(b_r) [b_r - D1] db_r \right)/db
\]

(24)

If this were a one period model, \( T' \) would be the only equation of interest. As can be seen, equation (24) is maximized by bidding \( D1 \). This term is also maximized by bidding \( D1 \) in the multi-period model, but this must be balanced by the possible gains in the next period.
Assuming the conditions for differentiability, the following equation must hold for a maximum:

$$\frac{d \text{EV}}{db} = P1'^i + P1'T' = 0. \quad (25)$$

Since $P1' < 0$, then for equation (25) to hold $T' > 0$. The result that $T'$ must be greater than zero has implications for the possible bid $b$. For any bid $b$ above $D$, $T'$ is negative as the amount gained has to be positive for any bid above $D$. Therefore, for $T'$ to be greater than zero, $b$ has to be less than $D$. This result guarantees bidding below cost in the second-price auction in the first period. However, bidding below cost in the second-price auction has a different interpretation than bidding below cost in the first-price auction. The second-price bid does not give the transacting price and therefore there is more incentive to bid below cost in the second-price auction, ceteris paribus. This result does not guarantee pricing below cost as the following example will demonstrate. Assume there are two auditors with costs uniformly distributed on $(200, 700)$. Assume one auditor is given the cost value of 250, while the other obtains 650. In the first period, the first auditor will bid 125 and the second will bid 325 (given Nash strategies). The first bidder will win at a price of 325 which is not below the cost of the winning bidder.

While the expected bids are of interest, the real issue is the price pattern that arises. Direct comparisons between the respective auctions are difficult because of the problems in determining the perceived probabilities and utilities of the auditors. If the Nash
strategies are apparent, price outcomes are identical between the progressive and second-price auction.

Prior single period research has shown that the progressive and second-price auction tend to the same price. Therefore, in a two period model, the second period is expected to price at the same point for both auction mechanisms. However, the first period does not have a dominant strategy as the second period and if the Nash strategy is not apparent, then as was previously shown the second-price auction has more computational parameters and may give a higher price given risk averse auditors.

First-price Auction

Analyzing the first price auction with respect to bidding strategies and Nash equilibrium points is very complicated. The Nash equilibrium solution will have to deal with a concept that will be called the 'disclosure principle'. If price is public knowledge, pure Nash strategies make the incumbent's cost known to all auditors after the first period. When rival auditors know the incumbent's cost and bidding strategy, they may be able to formulate a strategy that has a positive profit. This type of problem may be solved by the incumbent using a random strategy. However, deriving a possible randomized strategy is beyond the scope of this paper. (Randomized strategies can be indirectly tested by empirically observing the bidding of auditors, however bids appeared stable and not random.) As Shubik (1982) has pointed out, mixed strategies in non-zero sum games are difficult to justify. In a constant-sum case, the use of mixed
strategies can be justified on defensive grounds. In the general case, an advantage to an opponent need not imply a disadvantage to yourself and in many cases the opposite may hold. This leads Shubik to state "... the equilibrium point in pure strategies is the really significant non-cooperative solution concept."

For a beginning analysis, assume auditors are restricted to pure strategies. The importance of price revelation after the first period will now be discussed. In the second period, the non-incumbent auditors know the cost and strategy of the incumbent. For an extreme initial case, assume the incumbent auditor assumes all other auditors bid their respective costs in the second period (this gives zero profits to non-incumbent bidders). Non-incumbent auditors should not bid below their costs in the second period. If this is the case, the incumbent auditor maximizes the following equation:

\[ E(\pi) = \left( \frac{1-b_1}{1-D_1} \right)^{N-1} (b_1 - C_1) \]  

(26)

where:

\[ b_1 = \text{bid of bidder 1}. \]

Solving equation (26) for a maximum yields the following optimal bid:

\[ b^* = \left( \frac{1}{N} \right) - \left( C_1 / N \right) + C_1 \]

(27)

However, this bidding strategy, even though it is conservative, is not credible for the following reason. As soon as the incumbent follows this strategy, any other auditor that has a cost value between \( D_1 \) and \( \left( \frac{1}{N} \right) - \left( C_1 / N \right) + C_1 \) will bid an epsilon below \( \left( \frac{1}{N} \right) - \left( C_1 / N \right) + C_1 \). If the incumbent realizes this strategy of
nonincumbent auditors, then he/she bids two epsilon below this point and wins the auction with certainty. However, this contradicts his/her first strategy. Any strategy that yields a bid higher than \( D_1 \) in the second period is not credible. Therefore, the only credible (Nash equilibrium) strategy is for the incumbent auditor to bid \( D_1 \) in the second period.

The non-incumbent auditors may use a mixed strategy, however, a mixed strategy does not appear credible because the non-incumbent auditor must find an optimal bid point given the incumbent's strategy. Once this point is found, the probability of bidding it has to be one. Therefore, the only credible strategy for the incumbent is to bid his/her \( D_1 \) in the second period. Given this bid in the second period, the second period gain is equal to \( D_1 \).

The characteristics of the Nash equilibrium in the one period model gives insights into possible pure Nash strategies in the multi-period model. If all rival auditors follow the Nash strategy, and auditor \( i \) must select his/her strategy, auditor \( i \) must maximize the following equation with respect to his/her bid \( b_i \): 

\[
E(p) = (1-b_i)^{N-1} \left[ 1-((N-1)(1-b_i)/N - D_1) \right]
\]  

Equation (28) is maximized by bidding \( 1-((N-1)(1-D_1)/N) \). 

\((1-D_1)^{N-1}\) represents the probability of having the lowest \( D_i \). If all players follow a pure Nash strategy in a multi-period model, the probability of winning does not change. In order for the Nash strategy to hold in a multi-period model, the gain for the winner must
equal \((1-D_1)/N\) (which is derived from equation (28)). This expected gain has the trait that it is not a function of \(T\).

There are a family of Nash equilibrium points that meet the criterion of the gain equaling \((1-D_1)/N\) in the multi-period model. However, many of these solutions run into difficulties from the disclosure principle. Because of the sequential nature of the multi-period model, information may be revealed that gives rival bidders advantages over the incumbent auditor.

If the gain is equal to \((1-D_1)/N\) in the two period model, the opening bid in the first period is equal to:

\[
b_1 = ((1-D_1)/N) + C_1 \tag{29}
\]

This bid is a Nash equilibrium point given the assumptions of this auditor bidding game. To prove this, assume each auditor follows this bidding strategy, and auditor 1 must now select his/her strategy, he/she must maximize:

\[
(1-b_1)^{N-1} \left[ ((1-b_1)/N + b_1 - D_1) + (D_1 - C_1) \right] \tag{30}
\]

As with equation (28), this function is maximized by inserting the true \(D_1\) for the respective \(b_1\). This Nash strategy gives the same expected profit as the independent auction model discussed in the previous section.

While pure Nash strategies can be found, they are not appealing for several reasons: first, the expected gain is not a function of the number of periods; second, many models are last period driven; and last, the disclosure of incumbent cost.
Hypothetically, the incumbent auditor will attempt a strategy that gives a gain in periods 2 through T. Presumably, a small gain will be attempted in periods 2 through T-1. In period T, the incumbent should attempt a larger gain as his/her rivals have no incentive to bid below their initial cost. A few conservative and ad hoc strategies have been worked out but are omitted.

The sealed bid auction is a very complex institution. The strategies are very difficult to determine. For example, the strategy of a bidder in the second period who did not obtain the auction in the first period is not obvious. Under a Nash equilibrium strategy, he/she might as well not rebid as he/she knows that they are not the low cost producer. However, if this assumption is relaxed then the bidders may try to bid a little lower to see if they were close to having the low bid. A higher bid is also possible as the future economic rent stream has decreased.

Computational Model - First-price Auction

The first-price auction will now be examined in order to determine the relevant computational decision parameters. In a two period model, auditor I will try to maximize the following function:

\[ EV = P_1(b)[(b-D_1) + P_{II}(b_2)(b_2-C_i)] + P_0(b)[P_{IO}(b^*)(b^*-D_1)] \]  \( (31) \)

To solve for the optimal \( b \), differentiate \( EV \) with respect to \( b \) and suppressing the \( b \) notation:

\[ \frac{dEV}{db} = P_1'(F) + P_1 + P_0'(R) = 0 \]  \( (32) \)
where:

\[ F = [(b-D1) + P11(b2-C1)] \]

\[ R = [P10(b2'-D1)]. \]

Solving for the optimal \( b^* \) produces the following expression:

\[ b^* = D1 - [P11(b2-C1) + (P1+P0')R/P1'] . \] (33)

Therefore, an auditor will bid below cost (\( D1 \)) if certain conditions hold. Namely, \( P11(b2-C1) > (P1+P0')R/P1' \). Remember that \( P1 + P0 = 1 \). Therefore, \( |P1'| = P0' \) and \( P11(b2-C1) > (P1/P1'+1)R \). Assume that \( R \) is close to zero for the following reason; if an auditor bid below \( D1 \) in the first period and still lost the auction he/she knows that one rival's costs have dropped. In the second period, this auditor will not bid below \( D1 \) because after the second period there is no future economic rent stream. Thus, \( P10(b2) \) subject to \( b2 > D1 \), has to be close to zero. If this assumption is made, the following expression is valid:

\[ \frac{dEV}{db} = P1'[((b-D1) + P11(b2-C1)] + P1 = 0. \]

Solving for \( b^* \):

\[ b^* = D1 - P11(b2-C1) - P1/P1'. \] (34)

\( P1' < 0 \), therefore if \( P11(b2-C1) > P1'/P1 \) the bidder will bid below cost. DeAngelo (1981a) ignored the gaming inherent in the first-price auction, but even when this mechanism is added, apparently the auditors will still bid below cost. Further, this is only a two period model and if the total profit is a function of \( T \), then for a
longer dependent time period the incentive to bid below cost is even greater. As can be seen from equation (34), the optimal bid is a function of $P_1$ and $P_1'$. The progressive auction was not a function of these quantities.

Equation (34) is very similar to the optimal bid found in Attanasi (1974). $P_1/P_1'$ is referred to as the competitive advantage fee (CAF). The following analysis will help in interpreting equation (34). Let $f(p_i)$ represent the probability distribution of winning for player $i$. Let $F(p_i)$ represent the cumulative distribution. Given this definition, the following equation will hold:

$$P_1(b_i) = \int_{p_i=b_i}^1 f(p_i) dp_i = 1 - F(p_i) \quad (35)$$

$$P_1'(b_i) = -f(p_i)$$

Inserting these values into equation (34), the following condition will hold:

$$f(p_i)/[1-F(p_i)] = (1/b* - D_1) + P_11(b_2-C_1) \quad (36)$$

The preceding equation shows the optimal bid as equating the proportional increase in the probability of losing with the proportional increase in expected value (utility).

This past analysis has assumed that $db_2/db = 0$, that is, the bid in auction 2 is unaffected by the bid in auction 1. There are several reasons that this might hold. In order to determine the amount to bid in the first period, the auditor must calculate the amount he/she expects to win in the second period. Expected profit in period 2 is both a function of the amount bid and the perceived probability of
winning. The lower \( b \) is with respect to \( D_1 \) the more the auditor must recoup in the second period. However, the lower the bid, the less information obtained on the distribution of possible bids, because the greater the interval of other rivals' bids. The wider the interval, the more uncertain the winning auditor is of the distribution of other possible bids. If an auditor bids close to \( D_1 \) and wins he/she will probably bid quite high in the next period as he/she reasonably believes there is no one close. However, by doing this he/she minimizes the probability of winning the auction. There are two opposing influences on the auditor.

If this assumption is relaxed, the solution to the problem becomes more difficult. Auditor 1 must then solve the following derivative in order to find a maximum:

\[
\frac{dP_1}{db} \left[ (b - C_1) + P_1 (b - C_2) \right] + P_1 \left[ 1 + \frac{dP_1}{db} (b - C_1) \right] + \frac{[P_1 \frac{db}{db}]}{[P_1 \frac{db}{db}]} (33)
\]

The assumption of \( \frac{db}{db} < 0 \) may be sensible in that the bidder has less to recoup in period two if he/she makes a higher bid in period one. However, the assumption that \( \frac{db}{db} > 0 \) may also hold, because of the information effect. By bidding higher, the interval of possible other bids is reduced. This reduced interval may make the auditor bid a higher bid. The first-price auction first period bid is a function of the same quantities as the progressive auction plus the probability of winning in the first period.
Costly Auctions

The analysis in previous sections examines the simplest possible case because cost is the only difference between the auditors and the auctions are assumed costless. If these assumptions are relaxed, the attributes and strategies of the auction also change. Figure 2 details the results when the importance of price is modified and the auctions are no longer costless. Box 1 is the situation examined in this paper.

In boxes 3, 4, 5 and 6, the decision facing the client is included. With costly auctions, the client must determine if he/she wants to hold an auction. Presumably, the client has the choice of holding an auction and the auction method.

In Figure 2, the auction is classified according to price importance and the cost of the auction. If price is important and the auction is costly to the client but not to the auditor, an auction

FIGURE 2

The Classification of Audits by Cost and Price Importance

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will only be held when the expected sum of discounted price plus the cost of the auction is lower under holding an auction compared with the expected sum of discounted price when an auction is not held. In this case, an auction is not held every period but only when the condition is met. All auditors should enter the market because the auction is costless.

If price is not of major importance, some of the auctions are difficult to implement. The second-price auction is probably intractable as a suitable second-price cannot be found. The progressive auction is still a viable allocation system. In this case, the client would have an indifference mapping for all auditors and would select the auditor that is willing to bid the highest on this mapping. While some low price importance auctions may take place in the real world, these auctions are not directly addressed in this study.

If price is the main consideration and the auction is costly to both parties, the behavior of the participants of the auction again changes. The client may decide not to hold an auction every period. In periods when an auction is not held, the progressive and second-price auctions are no longer feasible systems. Given this problem, the progressive auction will be modified. The modified progressive auction will use the progressive auction at each bid auction, and in between auctions the incumbent auditor will bid one price.

Figure 3 gives a flowchart of the decisions to be made by the client and auditors. At the first auction, auditors must decide if they want to enter bids. For this to hold, the expected utility of bidding must be greater than the utility of not bidding. The utility
FIGURE 3

Flowchart of Auditor/Client Decisions
With Costly Auctions

1: Auditor decision to enter.
2: Bids tendered from auditors.
3: Auditor chosen by client.
4: Last period? (Mandatory rotation, bankruptcy)
5: Incumbent bid for next period.
6: Client decision to accept incumbent bid.
7: Last period?
of not bidding is assumed to be zero. Therefore, as long as the expected utility from winning is greater than the utility decrease from costs to enter the market, auditors will enter.

The next step is for the entered auditors to submit bids. Any of the previously described auction methods could be used. Once bids are tendered to the client, he/she must select an auditor. Once an auditor is selected the sale is consummated and the period is completed.

At the beginning of the next period, the incumbent submits a price for the audit. The client may accept this price or hold another auction. If the client accepts the price given by the incumbent, the sale is consummated with no need for holding an auction. For the client to hold another auction the expected cost of retaining the incumbent must be greater than the expected cost of changing (this includes cost of bids and cost of holding the auction). If the client rejects the price given by the incumbent, a new auction is called and the process starts over from box 1.

In this game, a pure Nash strategy for the progressive auction is probably intractable. Since the decision processes of the client are unknown, the auditors cannot even be sure of how the game will evolve. Thus an auditor may bid below his/her cost, C1, in the initial period and still have a positive profit.

The strategy of the incumbent auditor at the time of an auction will now be analyzed. At this auction, the incumbent can adjust his/her bid with respect to his/her price from the previous period. If this auditor were making high profits, on the surface reasonably
the bid price will drop. However, there is a danger in doing this because once this reputation is established, the client base of this auditor may also attempt to get the incumbent to lower his/her audit fee. For this reason, presumably the incumbent will not drop his/her bid price.

Given this reason, the client can expect that the incumbent will not bid an excessively different price. Given this result, the client should go out to bid only if he/she will change auditors. With the knowledge that a call for an auction guarantees a change in auditors, all rival auditor will take the auction seriously increasing N.

The client also has incentives not to select the incumbent at the time of an auction. If the client holds an auction and consistently selects the incumbent, the number of auditors entering the market will decrease. The expected low bid decreases with a decrease in the number of auditors. In the author's limited experience whenever a client goes out for bid, the Incumbent may be asked to bid but seldom wins the auction. As can be seen, the game becomes difficult to solve when costs are tied to auctions. For this reason and because not all audit auctions are costless, the cost of auctions is suppressed. An important extension to this study would be to insert these costs.

Conclusion

The multi-period theory has compared the three auction systems. The progressive and second-price auction have simple Nash strategies. The first-price auction also has a Nash strategy, but the profit was
not a function of T. Examining the calculations made by the auditor, the calculation made in the progressive auction was the simplest. The calculation in the first-price auction was very similar to the tradeoffs found in the economic auction literature.

While game theory gives structure to the problem, there is no guarantee that actual subjects will follow the 'solution'. As Shubik (1982) has pointed out, a dynamic theory of games is still in its early stage of development. Problems have been encountered with modeling and with the selection of a solution concept. For example, even in a simple Prisoner's Dilemma game, empirical evidence has shown that subjects do not play perfect equilibria. As Shubik has pointed out, personality, limited memory and limited ability to plan or process data all may have an effect on outcomes.

In the model, kept very simple for reasons of tractability, some of the assumptions may not always hold. One assumption is the zero cost of bidding. If this were not true, the solution to the progressive and second-price auction breaks down. Possibly there would only be one bidder after the initial period. In this case, the transaction price would always be the client's reservation price. In the first-price auction, the transacting price might not be at the reservation price or only be one bidder would be left. This may explain why the client would prefer the first-price auction.

In this model, the buyer is always assumed to select the low bid. This rules out any gaming by the buyer. By not selecting the low bid, the buyer may be attempting to drive the price even lower.
Research Variables

Given the preceding theory, the research variables of interest will now be examined. This study focuses on pricing aspects of audits and therefore, price is one variable. Prices are derived from bids from the auditors and therefore, bids are another variable of interest. Given cost differences among the auditors, the last measure of interest will be efficiency. The first three segments of this section will discuss the three major research variables and their variants. The research variables are bids, prices and efficiency.

Bids

The first variable to be examined is the actual bids from the experimental market. For the first-price and second-price auction, the bids were tendered from each subject, therefore the bids were known to the experimenter. The progressive auction was more difficult to determine. While record was kept of the actual bidding sequence, because only one bidder could bid at each bid increment the limit bid of each bidder was not obtained. Because of this deficiency, certain tests were not performed on the progressive auction. Also, the second-price auction did not give a true limit price as the bid does not represent the actual expected transaction price. Therefore, interpretation of this bid must be done very carefully. The first-price auction does represent the actual bid price for an auction.
Price

The major hypotheses of this study deal with the resulting price given audit market characteristics and different allocation methods. Given the rules of the allocation method, the bids are combined in a predetermined manner in order to determine the winner and price. The resulting price from an auction is unambiguous, but the price variable will have to be transformed in order to address certain questions.

To make comparisons between the auctions, some type of base price or 'optimal' price is needed. Vickrey (1961) has shown that in the one period progressive auction the bidding should stop at the second lowest valuation with the person with the lowest valuation transacting the unit. This result is Pareto-optimal. Using this as their reason, CST use this second lowest valuation point as the 'optimal' price in their analysis.

Because of the desirable properties of this second price and the theoretical development, the same optimal price will be used in this experiment. The second-price will be calculated in the following fashion. In the first period, the optimal price will be the deflated second cost point. In the second period, this price will be the second cost point. Given the Nash strategy discussed earlier, this should be the Nash pricing points. Obviously, in the second period the second cost bidder may be different than the second cost bidder in the first period.
Efficiency

The last measured variable of interest will be efficiency. Given the low efficiency found in past similar experimental work, presumably efficiency will be difficult to test.

Efficiency in these experimental markets is not analogous to the measure of efficiency in a standard economic sense. In the typical analysis, an efficient outcome is defined as the maximization of consumer and producer surplus, and has both a price and volume measure. In these markets the number of units transacted is set at one. Therefore price is not a measure of efficiency. A higher price increases producer surplus at a direct expense to consumer surplus. However, for the addition of consumer and producer surplus to be maximized, the low cost producer must transact the unit. If the low cost producer does not transact the unit, the outcome of the auction cannot be efficient. However, given the gaming inherent in some auctions, an efficient outcome is not guaranteed. There has been an extensive literature on the development of demand revealing mechanisms that attempts to eliminate gaming (Loeb, 1977).

There are two distinct types of inefficiency in this market. The first inefficiency, called turnover inefficiency, is the more serious given the parameters of this experiment. This inefficiency takes place when a different bidder wins the bidding in the second period than the first period. The other inefficiency, cost inefficiency, is not as serious as turnover inefficiency. Cost inefficiency takes place when a non-low cost bidder wins the auction for both (all) periods. Total inefficiency is a summation of these two inefficiencies.
Research Hypotheses

Now that the research variables have been discussed, the specific research hypotheses can be detailed. (All hypotheses will be stated in the null form.) As explained previously, the overall research objective is to investigate the effect of up front costs on bidding strategies and auction mechanisms. This broad objective breaks down into several specific hypotheses.

As was shown in the theory section, under all auction methods, there is a strong incentive to bid below cost. The first hypothesis relates to bidding below cost in the initial period. Given up front costs, a competitive environment and a possible future quasi rent, bidders should bid below cost in the initial period. A positive finding would support the model suggested by DeAngelo (1981a) and show her model to be robust with respect to auction methods and cost differences. Because ilmit bids are not tendered under the progressive auction, this auction system will not be tested for bids below cost.

H1: Bidders bid below cost in the initial period.

Since the bids of the first-price auction are price bids, if all auditors bid below cost then the resulting price has to be below the cost of the winning auditor. In the second-price auction and progressive auction, bids are not price bids and therefore depending on the bid of the second lowest bidder, the price result may or may not be below cost.
H2: The winning price is below cost of the winning auditor.

The next hypothesis compares the number of bids below cost between the three auction mechanisms. If first-price bids are below cost, this guarantees a below price outcome. Both the second-price and progressive auction bids are dependent on the distribution of prices. The wider the gap between the lowest cost and second lowest cost, the higher is the expected price given the same bid by the lowest auditor, ceteris paribus. Therefore while the expected price may be below cost in the second price and progressive auctions depending on the distribution of cost, the actual price may or may not be below cost. Because of the dependence of price on the distribution of costs, presumably the second-price auction and progressive auction will have significantly more prices above cost than the first-price auction.

H3: All three auction mechanisms lead to the same number of prices below cost in the initial period.

The fourth hypothesis compares the prices under the three auction mechanisms. This hypothesis compares the resulting first period price. As was detailed in the theory section, presumably the progressive and second-price auction will price at the deflated second cost point. The solution to the first-price auction is difficult to determine. If results of previous empirical work holds, then the first-price auction should price at a lower price than the other two auction mechanisms, with the progressive and second-price auctions pricing at the same points.
H4: In the first period, all prices between the three auctions will be equal.

The second period becomes a one period auction with probable cost advantages to the incumbent. In this auction, the previous empirical testing is directly relevant. However, there are three distinct differences between this second period and the model tested previously in CRS. First difference is the many buyers and one seller of the previous work and the one buyer and many sellers of this auction. Second is the deflation of cost so that costs are not random in the same sense as the previous work. The last difference is the information content of knowing the winning bidder and bid in the first period. Under risk neutrality, all prices should be at the same point. Given risk aversion and the prior experimental testing, presumably the first-price auction will price lower than the second-price and progressive auction with the latter two auctions pricing the same. Given the dominant strategies of the progressive auction and second-price auction in the single period, presumably these auctions will price at the second cost point.

H5: In the second period, the transacting price will be the same between the three auctions.

The next hypothesis deals with the profits under the three allocation mechanisms. Profit is analogous to price results if auctions are efficient. If the price of the first-price auction is consistently below the price of the other two mechanisms, presumably the profit will also be lower. Given the same strategies of the progressive and second-price auction, presumably the profit will be equivalent.
H6: Profit is equal among the auctions.

Efficiency will be tested under the three concepts of inefficiency: turnover, cost and total inefficiency. Given the lack of a Nash strategy and the gaming inherent in the first-price auction, presumably this auction will be more inefficient than the other two mechanisms. This result would be consistent with the results of CST and CRS.

Since the progressive auction guarantees results that are mutually satisfying, presumably turnover inefficiency will equal zero. Because limit bids are tendered for all but the winner under the progressive auction, presumably this auction will have lower cost, turnover and total inefficiency. The second-price auction has the same Nash strategy as the progressive auction but this strategy may not be as obvious. Given this trait and previous experimental results, presumably inefficiency of the second-price auction will be between the first-price and progressive auction.

H7: Turnover inefficiencies will be constant among all bidders.

Cost inefficiencies result when a non-low-cost auditor wins the auction for both auctions. This inefficiency is not as costly as the turnover inefficiency, given the parameters of this experiment. The progressive and second-price auctions may show this inefficiency if strategies differ and the low cost and second low cost auditors are sufficiently close together. Therefore, hypothetically cost inefficiency will be constant among the three auction types.

H8: Cost inefficiencies will be constant among the auditors.
Total inefficiency is just a summation of the first and second inefficiencies.

**H9:** Total inefficiency will be constant among the auditors.

The next issue to address is turnover. Again, turnover leads to a turnover inefficiency. Because of the lack of a dominant strategy in the second period of the first-price auction, presumably this auction mechanism will have more turnover than the other two systems.

**H10:** Turnover will be constant among the three methods.

The last analysis will deal with bidding strategies. The hypotheses will be more driven by description than 'hard' theory. While costs for all bidders either hold constant or drop, if all bidders understand the bidding strategy presumably bids will rise between the first and second period even though the cost of the winning bidder does the exact opposite.

**H11:** Prices and bids will fall between the first and second period.

The last research objective examines individual bidding strategies. The additional analysis will consist of two parts. The first section will compare single-period auction results with the two period results of this paper. Similar and dissimilar outcomes will be noted and the reasons for the differences will be given. Included will be comparison of Nash strategies outcomes with actual outcomes. The hypotheses are as follows:
H12: In the first period, the three auction mechanisms price at the Nash progressive auction price.

H13: In the second period, the three auction mechanisms price at the Nash progressive auction price.

H14: Under the three auction mechanisms, actual profits are equivalent to profits if the optimal price were followed.

The second part will examine individual bidding strategies. Comparisons will be made for both the first and second periods with the Nash equilibrium strategies noted in Chapter III. Following this examination, regression equations will be constructed for the first and second-price auctions. After examining the fit of the models, a simulation of the regression equation is run on the actual cost data used in this experiment.
Chapter IV

DESCRIPTION OF THE RESEARCH METHODOLOGY

The overall objective of this study is to investigate the effects of up front costs on bidding strategies. The specific research objectives, as well as methodological techniques used in this research were motivated by a study of the literature reviewed in Chapter II and the theory development in Chapter III. To reiterate, the specific objectives addressed by this study include:

1) To examine analytically two period auctions for sensible Nash equilibrium points.

2) To determine the effects of audit market characteristics on bidding patterns under the current allocation mechanism.

3) To compare empirically independent single-period auctions and multi-period dependent auctions.

4) To compare the current allocation system with demand revealing mechanisms.

5) To determine the efficiency of different auction institutions with dependent auctions.

This chapter presents the experimental methodology used to investigate these research objectives. In order to facilitate discussion of the specific methodological details, this chapter is organized into major sections. Since the study employs experimental economic techniques, the first section discusses the strengths and weaknesses...
of this methodology. There are two different methods to investigate the issue of audit market characteristics and allocation:

1) Examination of actual bidding data, and

2) Experimental investigation.

While the first method yields the most convincing evidence on how the actual audit market works, this approach is hampered by the difficulty of obtaining data, non-controllable variables, unknown parameters and unknown parameter values.

The researcher can avoid these problems by studying the effects of audit market characteristics on bidding patterns and independence through controlled economic experiments. In order to explain why a laboratory experiment was chosen rather than a field study the first section of this chapter briefly discusses the advantages and disadvantages of each approach.

The second section details general objectives that must be met in order to have a controlled experimental market. The third section discusses issues that deal directly with the experiments used in this study, and the experimental design employed in this study. The statistical techniques used in this investigation are then described in the final section.

FIELD VERSUS EXPERIMENTAL STUDY

The field study of data analysis has the strong advantage of using actual data. External validity problems are minimized as the data is directly from the market of interest. However, a field study does have distinct disadvantages in this study. The first
disadvantage is the problem of data availability. The author is not aware of a database that has auditor bidding data on individual clients. Given the lack of a database, a potential researcher would have to develop his/her own database. This database would probably have to build using questionnaire data. Given the historically low response rate (Simunic, 1980) and self-selection problems inherent in a survey study, a sufficient and viable database would be difficult to develop.

Second, unknown parameter values may be encountered. In order to address the issue in this study, the cost of the audit must be known. The cost of an audit would be difficult to obtain or reconstruct. Auditing firms are not willing to divulge this sensitive information. Third, internal control is at risk because of possible confounding factors. Factors that are not of primary interest may affect outcomes and dilute the effects of factors of interest. For example, one potential factor in the audit allocation is reputation. This factor is not of interest here. Therefore, by analyzing real data reputation may confound the result of factors of interest.

Fourth, studying actual market data precludes examining other potential market mechanisms. For example, the effects of bidding below cost on independence cannot be ascertained unless bidding below cost is not allowed. The 'what if' questions cannot be addressed by studying the existing market.

Experimental economic techniques make it possible to gather insights into the auditing market without actually obtaining field data. This technique does not suffer from the problems discussed with the field study.
The next section of the paper discusses concerns and objectives of experimental economic techniques. Certain criticisms of the technique along with principles for insuring a controlled experimental market are discussed in depth in the next section.

General Issues

Smith (1982) states that control over preferences is one of the major distinguishing elements of the laboratory experiment over other methods of economic inquiry. Individual values can be chosen in a specified way. This control is exercised by setting up the market so that monetary rewards and a property rights system induce monetary value on the outcomes. In order to achieve these objectives, the experiment must satisfy several conditions which shall be called precepts. They are not self-evident truths and therefore are not labeled axioms. Certainly all precepts can only be validated empirically. Smith gives five precepts in order to control preferences. The first four deal with the internal validity of the experiment, the last deals with external validity.

a. Nonsatiation: Given a costless choice between two alternatives, identical (i.e., equivalent) except that the first yields more of a reward medium (for example, U.S. currency) than the second, the first will always be chosen (i.e., preferred) over the second, by an autonomous individual. Hence, utility, \( U(V) \), is a monotone increasing function of the monetary reward, \( U' > 0 \), where \( V \) is dollars of currency.

Nonsatiation is equivalent to the economic assumption that more is chosen (preferred) to less. This assumption appears trivial but must be satisfied in order to control for preferences.
Certain criticisms of the nonsatiation precept have been made. The first is that there may be subjective costs or benefits in the auction itself. For example, a subject may infer costs in thinking up a strategy for bidding. Another criticism is that the bidders may attach value to the auction beyond the monetary value. Thus the subjects may receive utility in just winning the game. This is why many experiments that do not pay for outcomes but only "make believe" pay still get results that are similar to actual pay results. This also explains why a person might try hard to win a board game such as monopoly. Chow (1983) is a good example of an experiment that does not pay the subjects, but asks the subjects to act as if they were getting paid. Thus, it appears the gaming effect reinforces the monetary income. Many decisions in the real world also have a mixture of monetary and nonmonetary attributes.

Another criticism of nonsatiation is that the utility of subjects may not be independent. The independence of utility functions is not only assumed by experimental work but is assumed in standard microeconomic analysis. If independence is not met, I's induced demand will not be independent of J's. The classic work by Siegel and Fouraker (1960) on bilateral monopoly effectively controlled this problem by giving incomplete information. Under this condition, subjects are given their own payoff contingencies but not that of their rivals. This leads to the next precept:

b. Privacy: Each subject in an experiment is given information only on his/her own payoff alternatives.

Induced privacy would be an important condition to have in the experimental market aside from the guaranteeing of independence of
utility functions. In the real world most rivals do not know, to differ- 
ent degrees, the utility functions or cost functions of their ri-
vals. Privacy appears to be a pervasive characteristic of many market
institutions especially the audit market because of the limited dis-
closure of the audit firm.

c. Saliency: Individuals are guaranteed the right to claim a re-
ward which is increasing in the goods outcomes of an ex-
periment; Individual property rights in messages, and how
messages are to be translated into outcomes are defined by
the institution of the experiment.

In the laboratory, value is induced on the messages by paying for
certain outcomes. In connection with control over preferences
another concern is that the outcomes may not be enough to guarantee
control. Dominance is another precept has been put forth to answer
the question of insufficient motivation.

d. Dominance: The reward structure dominates any subjective
costs or values associated with participation in the activi-
ties of the experiment.

The best way to guarantee this precept is to give monetary pay-
offs that are judged to be high by the subjects. Again, a high pay-
off guarantees this precept but the precept is probably guaranteed
with no monetary payoffs.

Value is induced on messages by the institution that relates mes-
sages to valuable outcomes. Nonsatiation and saliency are sufficient
conditions for a microeconomy, however, the market may not be 'con-
trolled'. By not being controlled, the experiment may have interde-
pendent utility functions and 'high' nonmonetary outcomes. Dominance
and privacy control the experiment so that important subjective costs
and other tastes from the social economy are minimized (i.e.
invidious, altruistic and egalitarian tastes). These precepts guarantee that real economic agents receive real messages and try to obtain real outcomes. These precepts guarantee that economic theories that try to model elementary behavior can be rigorously tested. The theory itself abstracts from a variety of human experiences. The laboratory because it uses actual subjects provides a richer environment than the theory itself. Because of this, the laboratory can also falsify theories.

The next question is, to what populations can the results be generalized? This last precept deals with the external validity of the experiment. Since economic theory has been motivated by field settings, the results of the experiment should be applicable to the real world. But these concerns do not deal with the validity of the internal experiment. A condition addressing the transferability of results is called parallelism:

e. Parallelism: Propositions about the behavior of individuals and the performance of institutions that have been tested in laboratory microeconomies apply also to nonlaboratory microeconomies where, ceteris paribus, conditions hold.

Parallelism generalizes the important conjecture that "as far as we can tell, the same physical laws prevail everywhere." The data of most sciences are derived from experimental methods. All sciences using the experimentation must address the question of parallelism. For example, much of the scientific progress of astronomy depends on the maintained hypothesis that the nature of gases studied in the laboratory have application to stellar and planetary environments. In biology, parallelism means that if rats have a reaction to a
substance, then the likelihood is increased that humans may have similar reactions.

If institutions (i.e. auction methods) make a difference, parallelism hypothesizes, in an economy, that the rules make a difference and, therefore, the incentives of the institution make a difference. Ceteris paribus, parallelism states that the incentive effects of different bidding rules are qualitatively the same; if rule $x$ produces a different location in bids than rule $y$ in one market then it will do the same in other markets. The more narrowly defined is the supposed parallelism the more narrowly defined must be the ceteris paribus conditions across the economies. However, as Smith (1982) has pointed out, if one is interested in testing a theory which assumes that bidders bid so to maximize expected utility, most samples of agents that are not saturated in the goods of the experiment are sufficient to initiate a program of research.

The experiments, in this dissertation, address the audit market with price competition. This experiment parallels quite well audit allocations that put a heavy emphasis on the price of the audit. (Many government audits are awarded only on price.) As price diminishes in importance, the apparent parallelism of this experiment with the audit market decreases.

The question of relevance (external validity) can surface in many different forms. One criticism is that 'real businessman' do not behave as the subjects in experiments. Since these experiments deal with the audit market, the real issue deals with the parallelism between auditors and student subjects. Certainly, the students cannot
perform all complex activities performed by auditors. This study
deals with bidding strategies in a simplified setting. As long as
the subjects attempt to maximize utility (profits), the results of
the experimental market should not fail to parallel the audit market
because of choice of subject.

In the experimental literature, an experiment such as this was
tested among many subjects and payment levels (Plott, 1982). No sub-
ject pool differences that bear on the reliability of the experiment
were detected. Plott (1982) argues that if persons understand the ex-
periment and attempt to win profits, the choice of subjects is not
critical. The drive for maximization of profits should lead to the
same outcome. Students were selected for two major reasons: ease of
acquisition and level of payment. The probability of getting a num-
ber of auditors together for an extended period appeared to be low
(zero). While students were also difficult to obtain, the number of
subjects required was satisfied.

One of the precepts discussed earlier was dominance. Dominance
guaranteed that payment dominated any subjective costs of bidding.
Since auditors make a higher salary and have a higher asset base than
the typical college student, higher payments would be required to dom-
inate subjective bidding costs with auditors than with students.
Given a fixed amount of money, student subjects would maximize the
number of observations. For the preceding reasons, students were
selected. Student subjects were recruited from the accounting honors
program at The Ohio State University.
Pilot studies of the experiment demonstrated that the level of training of the student may affect outcomes. In particular, it was noted that the student with statistical training tended to outperform the student who did not have this training. Because of this finding, students of similar training were selected for the actual experiment.

Experimental Design

The next section of the study will deal with specific experimental design issues that will have to be addressed in this study. These issues will parallel the issues that were addressed in the CRS and CST (1982) study. These issues will be stated as objectives.

Plott (1980) gives laboratory procedures that are important to address in order to insure the internal validity of the experiment. The first proposition is that the procedures must be formulated so that the experiment can be replicated. One important aspect of the experimental method is replication. In considering this, parameter values and conditions must be assessed in order to determine their effect on the outcome. If parameter values drive the results then the results may not be robust among other parameter values. Procedures and parameters should be considered carefully before the experiments are conducted.

The second proposition deals with possible bias introduced by the experimenter. There appears to be a widespread belief that experimenters will/can influence the results of the experiment. Whether this criticism is valid or not, the experimenter must design procedures that minimize the possibility for such influences.
Plott (1980) describes the usual procedures in conducting an experiment. These procedures have been developed in order to insure the preceeding propositions and proper experimental control. Usually, instructions are explained orally and in written form. The language in the instructions are fairly standard. Words such as "competition", "collusion" and "maximizing" are not used in order not to suggest some theory or expectation that might influence subject behavior.

After recruitment and assembly of the subjects, the instructions are given orally and in written form. Instructions are given for the accounting of profits. A test section may be given in order to insure that the instructions were understood. Questions about the experiment are answered honestly and fully. If questions are asked that are dealt with in the instructions, the relevant portion of the instructions are repeated. Reference to actual markets is avoided in order not to induce expectations regarding behavior. Payment is made in private at the end of the experiment.

The first objective is control of procedures so that the experiments are all run in the same fashion. This is a standard procedure to guarantee that differences found in the experiment are due to behavorial differences and not differences in running the experiments. There were three different persons leading the experiments. All leaders received the same instructions and materials. The students were all given the same written instructions.

The second objective is to determine the effect of different parameter values on experimental outcomes. There are three major parameters that can be manipulated, the number of rival bidders, the
number of time periods in the dependent auction and the deflation of cost number. CRS (1982) found that results differed for groups of size three compared with larger groups in an independent auction experiment. Apparently, noncooperative behavior may break down when group size is three. Because of the tradeoff between statistical power and a variation in the number of bidders given a fixed number of auctions, group size was selected at four. This size should avoid problems of collusion. In conversation with auditors (and the author's personal experience), this is a reasonable number. Prior research has shown that this number of subjects does induce competition (CRS, 1982).

Another parameter of interest is the number of periods in the auction sequence. Ex ante, it would appear the longer the period, the lower the bid in the initial period. For this reason, time period may be an important variable, however, it may only change the magnitude of the results and not the results per se. Again, given a fixed number of auctions, there is a tradeoff of statistical power of the tests and changing the length of the dependent auction sequence. Given that there is no 'right' parameter value, a dependent auction sequence of two was selected. This choice maximized the power of the statistical tests. Given the preliminary nature of this work and the lack of correct period values, statistical power was selected over variation in the parameter value. (The pilot studies tested longer dependent sequences. No differences were found between the pilot's results and the results of this experiment.)
The last parameter value of interest is the deflation of incumbent cost. In discussing this value with auditors and examining the auditing literature, this value is client and auditor specific and no consistent estimate exists. Therefore, an ad hoc value of 50 percent was selected. This value should not affect the results, except through magnitude changes. Since most statistical tests are nonparametric, the effects of magnitude on statistical tests are minimized.

One issue that leads to a problem of parameter value formulation is the difference among auditors in a bidding situation. For example, assume that a client is about to go public for the first time. If all auditors have identical expectations, cost structures and utility functions then the same bid should be tendered from all auditors (DeAngelo solution). In practice, this rarely (if ever) takes place. Therefore, there must be cost or information differences among the auditors. For example, Kinney (1986) has shown that different auditors have different audit technologies. Different technologies should lead to different cost structures, since audit processes are different. Office size, related clients and other factors may result in cost differences among auditors. While other differences may exist, the driving difference in bids is probably cost. An estimate of the actual cost of a proposed audit is impossible to obtain. Therefore, there is a problem of operationalizing the cost differences of auditors in a bidding experiment. Again, there are no correct cost difference values for all audits. In order to stay consistent with prior experimental work, and because auditors probably are unaware of rival's cost function and view it as random, random
generation of costs was selected. A random cost drawn from an uniform distribution was selected. A fictitious commodity, marks, was introduced that was exchanged at a rate of 1 mark to 1 cent. Cost values were independently drawn from the interval 200 to 700 marks with equal probability at each integer on this interval. A random number generator from a BASIC program was used in the generation of cost values.

The third objective is to provide an experimental design so that paired comparisons of the treatment effects of the different auction institutions is possible and use different treatment switchover sequences on paired subject groups. This objective is a standard experimental procedure to reduce error. Switchovers in the experiment are the sequencing of different auctions within the experimental design. Paired comparisons increase the power of the test while switchovers increase the credibility of the claim that any measured differences are attributable to differences in the auction institutions and not to particular differences in the experimental groups. Define P, 1 and 2 to refer to the progressive, first and second price auction, respectively. Consequently, all experiments consist of 30 auctions (60 periods). Each auction consists of a two period dependent sequence. One sequence was the running of ten progressive auctions followed by 10 first-price auctions and concluded with 10 second-price auctions (P12). This sequence was paired with the auction sequence 12P and 2P1. The identical cost sequence randomly drawn was given to the three subject groups. Therefore, the P in P12 would have the same cost values as the 1 in 12P and the 2 in 2P1. The combination of P12
with the same cost values will be called a set. The experimental design is shown in Figure 4.

The following describes in detail the steps followed in this experiment. Before the experiment was conducted, a pilot study was carried out to insure that the experiment was properly designed. In this pilot, eight subjects (accounting honors students) conducted the three auction methods over different dependent period lengths. From this pilot, the length of the period sequence changed the magnitude of results but not the basic results of interest. Because of the pilot, certain minor modifications were made to the experiment and the instructions.

A total of 12 subjects participated in a design consisting of 3 experimental sessions with 4 bidders. In each session, a demand for selling was induced on subjects by promising to pay a cash payment of auction price minus individual cost. The costs were selected randomly from the interval (200, 700) with equal probability at each point. Each subject’s earnings were accumulated and paid in cash. Subjects were given their own cost values and were seated so that this information would remain private.

In 1985, the author wrote to Vernon Smith requesting information and instructions on the experiments run in the CRS and CST studies. A detailed response was received. From this information and the instructions in the appendix of the CRS study, the written instructions for this study were compiled. By doing this, key terms that may bias the experiment were omitted (Plott, 1982). The instructions and record sheets used for bidder one in the set P12 are shown in Appendix.
A. While the basic flow of the instructions was the same, certain technical differences were inserted. The main technical difference was the rules of the progressive auction. The rules of the progressive auction needed to be stated so that multiple bids were received. If multiple bids were not received then the auction may become a first-price auction. In order to insure this the bidding was required to start at 600. This amount was high enough so that multiple bids were received. In addition, the next bid increment had to be at least within 20 marks of the outstanding bid.

On the day of the experiment, the students were randomly assigned to three separate classrooms. First, demographic information was obtained from each subject. No difference in demographics was detected among the groups. The instructions were read orally and given in
written form to the subjects. No talking was allowed while the experiment was in progress. At the end of the instructions, all remaining questions were answered. The instructions included record sheets for all the experiments. Included in these record sheets were the cost values for each period, with the deflated costs calculated. After questions were answered, a short test was administered to ensure that the subjects understood the auction rules. No mistakes on the test were found.

After the test, the first set of experiments were conducted. The progressive auction was conducted orally. In the progressive auction, winning bidder and price are public knowledge. In addition, at the end of each period the winning bidder and price were noted on a blackboard at the front of the classroom.

The first-price auction was conducted via index cards. On each card was the bidder number, auction number and period number. At the start of each auction, the bidders were asked to enter bids on index cards. In the first-price auction, after bids were tendered the winning bidder and price were written on the blackboard. After the bids were collected from the subjects, the index card were kept by the experimenter.

The same process held true for the second-price allocation method in a sequence. After the index cards were collected, the winning price and bidder were noted on the blackboard. After the first set of experiments, the instructions for the second set were given. No test was given for the second set of the experiment as the rules were identical except for the allocation system change. After completion
of the second allocation method, Instructions for the third allocation method were given and the third period was conducted. After the third set of the experiment a short debriefing questionnaire was given, marking the end of the experiment.

Statistical Analysis

Testing the above hypotheses require various statistical procedures. After a brief discussion of the advantages of nonparametric statistical procedures, this section proceeds to the statistical tests about which inferences are drawn about each of the hypotheses. The presentation includes a justification for using each test, plus a brief explanation of the test, including the underlying assumptions.

Although assumptions vary from test to test, nonparametric procedures do not require normally distributed data. Nonparametric procedures have the additional advantage of being relatively insensitive to outliers. This property arises from the fact that nonparametric tests generally use the rank, or ordinal properties (rather than cardinal properties) of the sample observations. Finally, Hollander and Wolfe (1973) show that nonparametric procedures are more powerful than their normal-theory counterparts when data are not normally distributed, and are nearly as powerful when data do follow the normal distribution.

Analysis of the First Hypothesis. For the reasons outlined above, most of the analyses from which statistical inferences concerning the research hypotheses are drawn consist of nonparametric procedures. The first hypothesis, that bidding will be below cost in
the first period, may be written in the null form as:

\[ H_0: B_1 - C_1 \geq 0 \]

or in the alternative form as:

\[ H_1: B_1 - C_1 < 0 \]

where:

- \( B_1 \) = bid of bidder 1.
- \( C_1 \) = cost of bidder 1.

In order to test this hypothesis under the progressive auction, a limit bid would have to be obtained from each bidder. Since only one bid can be made at each price point, this information is impossible to be obtained. Therefore, the progressive auction will be excluded from analysis under this hypothesis. Since the bid in the second-price auction is not a bid on the transaction price, it is difficult to determine if the bidder is expecting a price below cost. The second-price analysis of bids will be included however the difficulty of interpretation of the bid must be remembered.

The hypothesis will be tested under the first-price and second-price auction. The first-price auction is the auction typically used in the audit market, and the bid is unambiguous. This hypothesis will be tested using the nonparametric binomial test. The null hypothesis will be that bids are as likely to be above cost as below cost (i.e. \( p = .5 \)). This null hypothesis is very conservative against the alternative. The purpose of the statistical analysis is to determine if the probability of bidding below cost is equal to fifty percent. The procedure is based on the following model:
\[ P = P_0 \]

where \( P_0 \) is some specified number, \( 0 < P_0 < 1. \)

Given this model, the first hypothesis may be restated in the null and alternative form as follows:

\[ H_0: P(B_{1} - C_{1}) = .5 \]

\[ H_1: P(B_{1} - C_{1}) < .5 \]

The binomial test employs the following assumptions:

1) The outcome of each trial can be classified as a success or a failure.

2) The probability of success denoted by \( p \), remains constant from trial to trial.

3) The \( n \) trials are independent.

The binomial test employs the number of successes as the test statistic. If the statistic exceeds a cutoff point, the null hypothesis will be rejected. The advantage of this test lies in its weak assumptions, but reduced power is the price of these less restrictive assumptions. The test only considers whether the outcome is a success or failure; it does not consider the relative magnitude of the bid.

**Analysis of the Second Hypothesis.** The second hypothesis which is an extension of the first hypothesis is that the resulting price will be below cost in the first period. In null form, the hypothesis may be written:

\[ H_0: \text{Price} - \text{Cost} > 0. \]

or in the alternative form as:

\[ H_1: \text{Price} - \text{Cost} < 0. \]

This hypothesis will be tested on all three auction mechanisms. A rejecting of the null hypothesis would lend support to the fact
that with cost differences and up front cost bidders do find it in
t heir best interest to bid below cost. In the first-price auction,
if all bidders bid below cost, then the resulting price will be below
cost of the winning bidder. However, if only one bidder does not bid
below cost and wins the auction, the resulting price will not be be-
low cost. In the second-price and progressive auction, the resulting
price is dependent on the distribution of costs. If the lowest and
second lowest cost are disperse the resulting price may not be below
cost.

The binomial test again will be used so that the hypothesis be-
comes:

\[ H_0: P(P_{i-C}) = 0.5 \]
\[ H_1: P(P_{i-C}) < 0.5 \]

**Analysis of the Third Hypothesis.** The third hypothesis will com-
pare the three auction mechanisms for the number of prices below
cost. The null hypothesis will be:

\[ H_0: \sum A = \sum B = \sum C \]
or in the alternative form as:

\[ H_0: \sum A < \sum B = \sum C \]

where:

A = 1 if \( P-C < 0 \), 0 otherwise for the first-price auction.
B = 1 if \( P-C < 0 \), 0 otherwise for the second-price auction.
C = 1 if \( P-C < 0 \), 0 otherwise for the progressive auction.

The data form binary success or failure outcomes. Three separate
2 x 2 contingency tables will be constructed to test for differences
in the summation of number of prices below cost. The hypothesis will
be tested using the chi-square statistic. The chi square is a measure of the significance of the association between two variables. Therefore, this test will examine the association between the number of prices below cost and auction type. The chi square statistic is not a measure of the degree of association because the statistic is dependent on sample size. For this reason the cross-product ratio will also be calculated and tested for significance. This statistical procedure has the same assumptions as the binomial test namely, outcomes are either success or failure, probabilities remain constant and trials are independent.

**Analysis of the Fourth and Fifth Hypothesis.** The fourth hypothesis compares the relative price in the first period of the three auction mechanisms. The null hypothesis is:

\[ H_0: P_P = P_1 = P_2 \]

or in the alternative form:

\[ H_1: P_1 < P_P = P_2 \]

Again the data will be compared among the three different auctions. Between any two auctions, the data form paired replicates. The first-price auction can be viewed as the 'pre-treatment' variable, and the other two auctions can be viewed as 'post-treatment' variables. In comparing the second-price auction and the progressive auction, the order of pre and post is irrelevant. The purpose of the statistical analysis is to determine whether a shift in location has occurred due to application of the 'treatment', the use of different auction methods. The Fisher sign test is a nonparametric procedure appropriate for analyzing a shift in location of paired replicates.
The procedure is based on the following model:

\[ Z_1 = \text{Price}_i - \text{Price}_j \]

\[ Z_1 = T + e_1 \]

where:

\( T \): unknown treatment effect, in this case, the change in price due to the allocation change, and

\( e_1 \): unobservable random variable.

Given this model, the fourth hypothesis can be restated in the null and alternative forms as:

- **H0**: \( T = 0 \)
- **H1**: \( T \neq 0 \).

The Fisher sign test employs the following assumptions:

1) the \( e \)'s are mutually independent,

2) each \( e \) comes from a continuous population (not necessarily the same one) that has median zero so that

\[ P(e_1 < 0) = P(e_1 > 0) = \frac{1}{2} \]

In the sign test, the number of positive Z's, (or differences between any two auction methods) are counted and if this number exceeds a cutoff point, the null hypothesis that \( T = 0 \) will be rejected. The advantage of this test is the same as the binomial test with less restrictive assumptions but also less power. Again, the test only considers whether the Z's are positive or negative, it considers neither the ranking nor relative magnitude of the Z's.

The fifth hypothesis is the same as the fourth hypothesis except that it is the second period. The direction of the alternative hypothesis is the same as the first period hypothesis.

**Analysis of the Sixth Hypothesis.** The sixth hypothesis test for profit differences among the auctions. Given the same cost bidder
winning the auction among the three methods, price results should translate directly into identical profit results:

\[ H_0 : D = E = F \]

or in the alternative form:

\[ H_1 : D < E = F \]

where:

- \( D \): profits for first-price auction
- \( E \): profits for second-price auction
- \( F \): profits for progressive auction.

The hypothesis will be tested using the Fisher's sign test. The hypothesis will be tested in the same manner as hypothesis four and five.

**Analysis of Hypotheses Seven through Ten.** Hypotheses seven through ten all deal with efficiency of the respective auction methods. Given the low incidence of inefficiency, standard statistic analysis cannot be performed. The sample size would have to be significantly enlarged in order to get proper power of the statistical tests. Given this result, this study will follow the reporting method of CRS and CST and report percentages. Even though statistical tests are not possible, certain insights into efficiency can be observed in the data.

**Additional Analysis.** Additional analysis will be performed examining individual bidding strategies. The first examination will examine the pattern of bids between the first and second period. Even though the cost of the winning bidder drops between the first and second period, presumably, the bid will rise between the first and
second period for all bidders. This will be tested using the binomial nonparametric test. The null hypothesis is that the second period price is just as likely to be above or below the first period price.

The Nash equilibrium points of the deflated second cost point in the first period and second cost point in the second period will be compared with actual price results. This analysis will examine the results across sets and will employ t-tests.

Regressions will be run on the first-price and second-price auction. Given the results of the regressions, a simulation will be run examining the outcomes on actual cost data realizations.

Chapter Summary

The chapter began with a discussion of the strengths and weaknesses of experimental economic research. The next section examined important precepts that must be considered in this research. Since the tests employ experimental economic techniques, the next two sections discuss important design variables of this approach. The second section discussed the exact experimental procedures used in this study. The final section explained the advantages of nonparametric statistics, justified the selection of particular statistical procedures, and explained the underlying assumptions. The statistical analysis of the data obtained for the study, together with the interpretation of the results, is presented in the following chapter.
CHAPTER V

STATISTICAL ANALYSIS AND INTERPRETATION
OF THE RESULTS

The purpose of this chapter is to present the results of the investigation described in the previous chapter. The chapter consists of two presentations. The first section details the results of the analysis of price results from the three auction methods. Specifically, some of the key analytical results from DeAngelo are examined to see if they hold in these experimental markets. This section includes a four part analysis on bids, prices, profits and efficiency. Both the first and second periods are examined for regularities. The second section examines individual bidding patterns, including a comparison of actual prices with Nash prices. Regressions are run to gain insights into bidding behavior. Given the strength of the regression results, a simulation is run on the cost data to determine the effects of distribution of cost on price outcomes. The progressive auction is not included in the simulation because limit bids are not obtained. Potential explanations for the observed patterns are sought, and comparison with the results of Chapter III are given.
The data is shown in Appendix C. Before the hypotheses are tested, the price data are shown in graphical form, in order to highlight the differences between the auction mechanisms. Each graph is particular to a certain set and period. Three sets and two periods yield six resulting graphs. Each set has ten dependent auction sequences, so that the horizontal axis numbers the ten auction sequence. The vertical axis displays price. Each graph comparison point has identical cost values given to the subjects and sequencing within the experiment, therefore, the prices can be compared directly.

Figures 5, 7 and 9 plot the price for each of the experimental sessions in the first period among the three sets. Therefore, each comparison in the graphs had the same cost value given to the subjects. Inspection of the three graphs points to certain regularities found within the data. With respect to the first period and the low price, no definite pattern emerges. But regarding the high price, apparently the second-price auction tends to give a fairly consistent higher price than the other two auction mechanisms in the first period.

Figures 6, 8 and 10 detail results of the second period among the three sets. In these graphs, there is little doubt that the first-price auction, with this particular parameter values, is a distinct pricing system from the other two auction mechanisms. The first-price auction tends to give a lower price in the second period.
Figure 5
Auction Prices - Period 1, Set 1

○ Progressive  + First  ○ Second
Figure 6
Auction Prices - Period 2, Set 1

○ Progressive  + First  ◊ Second
Figure 7
Auction Prices – Period 1, Set 2

- Progressive
- First
- Second
Figure 8

Auction Prices - Period 2, Set 2

- □ Progressive
- + First
- ◇ Second
Figure 9
Auction Prices - Period 1, Set 3

- Progressive
- First
- Second
Figure 10

Auction Prices - Period 2, Set 3

- Progressive
- First
- Second
Hypothesis 1

The first hypothesis can be stated in the null and alternative forms, respectively:

H0: First period bidding is random.
H1: First period bidding is systematically below cost.

As was noted earlier, this hypothesis will only be tested on the first-price and second-price auctions. The reason is that in the progressive auction the reservation bids of all bidders are not obtained. Of 120 possible first period bids in the first-price auction, 112 resulted in bids below cost or 93.3%. If the null hypotheses assumes bidding above cost as likely as bidding below cost, then the binomial test can be used with the null that $p = .5$. The large sample approximation of the binomial test gives a test statistic of $Z = 9.49$ which is highly significant ($p < .0001$). The null hypothesis of random first-price auction bidding can be rejected and it appears that bidders do bid consistently below price in the first-price auction. This finding supports the model proposed by DeAngelo even when costs are not identical among the bidders and the first-price auction is inserted as the allocation mechanism.

In the second-price auction, only one bidder in the initial period did not bid below cost. Therefore, of 120 first period bid points, 119 were bids that were below cost. The null hypothesis of random second-price auction bidding can be rejected and apparently bidders do bid consistently below cost. Again, interpretation of this result is difficult because the second-price auction bid is not a price bid.
Bidding below cost appears robust even though the time period is only of a two period duration. No learning curve with respect to bidding below cost was detected. In the first period on the first auction, all bids were below cost. Given the parameter values of this experiment, there is little doubt that bidding below cost does take place.

Hypothesis 2

The second hypothesis can be stated in the null and alternative forms, respectively:

H0: In the first period of all auctions, the winning price is not below the cost of the winning bidder.

H1: In the first period of all auctions, the winning price is below the cost of the winning bidder.

The second hypothesis will be tested under all three auction mechanisms. Bidding below cost in the first-price auction guarantees that the winning price will be below cost. Bidding below cost in the second-price auction does not guarantee prices below cost since the price is dependent on the second bid. In this experiment, there are a total of 30 first period prices under the three auction mechanisms. The results of the number of prices below cost are shown in Table 1. As can be seen, the first-price auction resulted in all 30 first period prices being below cost of the winning bidder. The second-price auction resulted in 20 of 30 prices being below cost of the winning bidder. The progressive auction gave 27 prices below cost.

Using the binomial test with the null hypothesis of equal probability of the price being below or above cost, the test of price being below cost is highly significant in all three auction methods. Using
Table 1

STATISTICAL EVIDENCE BEARING ON HYPOTHESIS 2

Binomial Test

<table>
<thead>
<tr>
<th>Auction Method</th>
<th>Number of Price Below Cost (N=30)</th>
<th>Large Sample Test Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-price</td>
<td>30</td>
<td>5.48</td>
<td>p &lt; .001</td>
</tr>
<tr>
<td>Second-price</td>
<td>20</td>
<td>1.83</td>
<td>p = .034</td>
</tr>
<tr>
<td>Progressive</td>
<td>27</td>
<td>4.38</td>
<td>p &lt; .001</td>
</tr>
</tbody>
</table>

the large sample approximation (which is approximately normal) the test statistics of the first-price, second-price and progressive auctions were 5.48, 1.83 and 4.38, respectively. The p-values being p < .001, p = .0336 and p < .001, respectively.

This presents evidence that the resulting prices under the three auction mechanisms are consistently below cost. Apparently, bidding below cost is to be expected given audit market characteristics (i.e. up front costs). This bidding below cost results in price being consistently below cost for the three auction mechanisms. The absence of bidding below cost may be the result of collusion, and given that this experiment induces competition, the initial price should be below cost.

Hypothesis 3

The third hypothesis can be stated in the following null and alternative forms, respectively:
H0: The three auction methods lead to the same number of prices below cost in the initial period.

H1: The three auction mechanisms do not lead to the same number of price below cost in the initial period.

Table 2 presents evidence concerning the comparison of the three auction mechanisms. Three two by two contingency tables and the associated chi squared statistics are shown. The first-price and progressive auction are shown not to differ significantly as measured by the chi square statistic ($\chi^2 = 1.403$). The second-price and first-price auctions differ significantly ($\chi^2 = 9.72, p < .001$). As will be discussed later in this chapter, the second-price auction's price is highly dependent on the distribution of cost. Given a certain cost structure, the resulting price may not be below cost even though the bidding strategies of all bidders is constant over all trials.

The progressive and second-price auction association is moderately significant ($\chi^2 = 3.535, .05 < p < .1$). The reason these auctions differ is unknown, given the same Nash solution. The sequential nature of the progressive auction versus the one-shot bid of the second-price auction appears to lead to different strategies.

As previously mentioned, the chi-square statistic is a good measure of the significance of association but does not measure the degree of association. The odds ratio is one measure of the degree of association. Odds ratios were calculated on all three 2 x 2 tables. However, the problem of having zero entries in cells was encountered. To address the problem, the approximation suggested by Mosteller (1968) was used. The odds ratio comparing the first-price and second-price ratio was found to be 31.24, therefore, the
Table 2  
STATISTICAL EVIDENCE BEARING ON HYPOTHESIS 3  
Chi Square Test

**Panel A**

2 x 2 CONTINGENCY TABLE: FIRST-PRICE AND PROGRESSIVE

<table>
<thead>
<tr>
<th></th>
<th>PRICES BELOW COST</th>
<th>PRICES NOT BELOW COST</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRST-PRICE</td>
<td>30</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>PROGRESSIVE</td>
<td>27</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>TOTAL</td>
<td>57</td>
<td>3</td>
<td>60</td>
</tr>
</tbody>
</table>

CHI-SQUARE: 1.403  
ODDS RATIO: 7.76

**Panel B**

2 x 2 CONTINGENCY TABLE: FIRST-PRICE AND SECOND-PRICE

<table>
<thead>
<tr>
<th></th>
<th>PRICES BELOW COST</th>
<th>PRICES NOT BELOW COST</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIRST-PRICE</td>
<td>30</td>
<td>0</td>
<td>30</td>
</tr>
<tr>
<td>SECOND-PRICE</td>
<td>20</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>TOTAL</td>
<td>50</td>
<td>10</td>
<td>60</td>
</tr>
</tbody>
</table>

CHI-SQUARE: 9.72  
p < .005  
ODDS RATIO: 31.24

**Panel C**

2 x 2 CONTINGENCY TABLE: PROGRESSIVE AND SECOND-PRICE

<table>
<thead>
<tr>
<th></th>
<th>PRICES BELOW COST</th>
<th>PRICES NOT BELOW COST</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROGRESSIVE</td>
<td>27</td>
<td>3</td>
<td>30</td>
</tr>
<tr>
<td>SECOND-PRICE</td>
<td>20</td>
<td>10</td>
<td>30</td>
</tr>
<tr>
<td>TOTAL</td>
<td>47</td>
<td>13</td>
<td>60</td>
</tr>
</tbody>
</table>

CHI-SQUARE: 3.535  
.05 < P < .1  
ODDS RATIO: 4.5
second-price auction is 31.24 times as likely to be found with the initial price not being below cost. The odds ratio for comparison of the progressive and second-price auction was found to be 4.5. The comparison is analogous, the second-price auction is 4.5 times as likely to have a price above cost. In summary, the second-price auction was the most likely to have price above the cost of the winning bidder. This auction was followed by the progressive and first-price auctions.

Hypothesis 4:

$H_0$: In the first period, the resulting price among the three auction methods is equivalent.

$H_1$: In the first period, the resulting price among the three auction methods is not equivalent.

Table 3 presents evidence concerning the comparison of price among the three auction methods. Each auction was compared pairwise in the first period. In order to compare the resulting price, the Fisher sign test was used. In comparing the first-price and progressive auctions in the first period, the progressive auction gave a higher price 12 times, the first-price auction 16 times with two ties. The large sample approximation of the Fisher sign test gave a statistic of .7559 which is not significant. This provides evidence that in the first period the progressive and first-price auction price at the same point. Since this test ignores magnitudes, further evidence bearing on price will be presented in the next section.

In comparing the first-price and second-price auctions, the first-price auction gave a higher price in 7 auctions and the second-price auction gave a higher price in 21 auctions with 2 ties. The large
Table 3

STATISTICAL EVIDENCE BEARING ON HYPOTHESIS FOUR
Fisher Sign Test

<table>
<thead>
<tr>
<th></th>
<th>Progressive</th>
<th>First-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>12</td>
<td>16</td>
<td>2</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = .7559
p-value = n.s.

<table>
<thead>
<tr>
<th></th>
<th>First-price</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>7</td>
<td>21</td>
<td>2</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = 2.64
p-value = .0041

<table>
<thead>
<tr>
<th></th>
<th>Progressive</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>7</td>
<td>22</td>
<td>1</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = 2.8
p-value = .0026

Sample approximation of the Fisher sign test gave a statistic of 2.64 which has a p value of .0041. This gives evidence that the first-price and second-price auction price at distinct points. The first-price auction tends to price at a lower price in the initial period. This may explain the reason that business enterprises select this auction method over other possible auction designs. For example, most government bidding in Utah is a one shot first-price auction. Given this method and the results of this experiment, the government can
expect a lower price using this method. This may also explain the
pervasiveness of the first-price auction in the real world.

Comparison of the second-price auction with the progressive auc-
tion resulted in the price being higher in the second-price auction
in 22 of the auctions, the progressive auction gave a higher price in
7 of the auctions and there was one tie. The large sample approxima-
tion of the Fisher sign test gave a test statistic of 2.8 and a
p-value of .0026. This would indicate that the second-price and pro-
gressive auction are not isomorphic in the initial period. This re-
sult runs contrary to the one period results found in CST and CRS.
The second-price auction appears to price at a consistently higher
price in the initial period. This points to the fact that for the
results of CST and CRS to hold the auctions must be independent.
Learning has been noted in certain economic experiments. For this
reason, the statistical tests were also performed on the second half
of the auctions in each set. The results were comparable with the
results for the complete data set.

In summary, the auction methods do not appear to converge to the
same price. While the Fisher sign test ignores magnitude, apparently
there are consistencies within the data. The first-price and pro-
gressive auction do converge to the same price and thus appear to be
isomorphic. However, the second-price auction consistently prices
above the progressive and first-price auction. This result is con-
trary to one period results which found that the progressive and
second-price auction were isomorphic and the first-price auction
priced at a higher (lower) price.
Hypothesis 5

The fifth hypothesis can be stated in the null and alternative as follows:

H0: in the second-period, the resulting price among the three auction methods is equivalent.

H1: in the second-period, the resulting price among the three auction methods is not equivalent.

Table four presents evidence concerning the resulting price among the three auction methods in the second period of the experiment. The results of the second period coincide very closely with the results of the one period auction of CRS and CST. The second-period of the auction is very similar analytically to an independent auction discussed in Chapter IV. The finding that the empirical results are very similar to the independent auction is reassuring that the participants of the auctions understood the rules of the auctions and that the results can be replicated.

In comparing the progressive auction and the first-price auction, the progressive auction gave a higher price in 29 auctions with one tie. The large sample approximation of the Fisher sign test gave a test statistic of 10.78 with a p-value less than .0001. This gives strong evidence that the progressive auction gives a higher price than the first-price auction. This result is consistent with the independent auction results.

In comparing the first-price auction and the second-price auction, the first-price auction gave a higher price in 2 of the auctions with the second-price auction giving a higher price in the remaining auctions. The large sample approximation of the Fisher
Table 4

STATISTICAL EVIDENCE BEARING ON HYPOTHESIS FIVE
Fisher Sign Test

<table>
<thead>
<tr>
<th></th>
<th>Progressive</th>
<th>First-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>29</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Fisher Test Statistic</td>
<td>10.78</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>&lt; .0001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>First-price</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>2</td>
<td>28</td>
<td>0</td>
</tr>
<tr>
<td>Fisher Test Statistic</td>
<td>4.74</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>&lt; .0001</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Progressive</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of prices higher (N = 30)</td>
<td>11</td>
<td>16</td>
<td>3</td>
</tr>
<tr>
<td>Fisher Test Statistic</td>
<td>.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>n.s.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The Fisher sign test was 4.74 with the p-value being less than .0001. Again, this provides strong evidence that the second-price auction gives a higher price in the second period.

In comparing the second-price and progressive auctions, the second-price auction gave a higher price in 11 auctions with the progressive auction giving a higher price in 16 auctions with three ties. The large sample approximation of the Fisher sign test gave a value of .68 which is not significant. This presents evidence that
the progressive and second-price auction are isomorphic in the second period.

In summary, the following results were found in the second-period of the experiment. The progressive auction and the second-price were isomorphic with the first-price auction pricing at a lower price. CST and CRS found that the progressive and second-price auction were isomorphic with the first-price auction giving a higher price. Given that these experiments test the opposite end of the market, the results are consistent.

Hypothesis 6

The sixth hypothesis can be stated in the null and alternative forms as follows:

H0: The three auction methods give the same profits to the bidders.

H1: The three auction methods do not give the same profits to the bidders.

The tests for profit can be very involved because there are many possible ways to test for differences in profit. In this section, two measures will be described: among sets and among auctions. The first measure will only be descriptive in nature. The second measure was chosen because the two dependent periods are tied together in one auction and, therefore, profit is better measured in a total auction rather than a one period measure. Therefore, profit to a bidder is not one period independent but the combination of two periods.

First, the total profits among the three sets will be given. In the first set, the progressive auction gave the highest profit followed by the second-price auction and first-price auction. In the
second set, second-price auction gave the highest profit followed by the progressive and first-price auction. The third set was identical to the second set, with the second-price auction giving the highest profit followed by the progressive auction and first-price auction. These results are consistent with the price results detailed in the preceding two hypotheses. The first-price auction gave the lowest or equal to the lowest price and this translated to lower profits.

The next test details the pairwise comparison of the auction. The Fisher sign test was used to test for a difference in profits. This test gives evidence whether the results of the profits are consistent among auctions or whether the profits might be driven by a few outliers. The results of this test are detailed in Table 5. In comparing the progressive and first-price auction, the progressive auction gave the higher profit in 24 of the periods, the first-price auction gave the higher price in the remaining 6 auctions. The large sample approximation of the Fisher sign test gave a value of 3.29 and the p-value was equal to .001 (two tailed).

In the pairwise comparison of the first-price and second-price auction, the first-price auction gave a higher price in 5 auctions with the remaining 25 auctions giving higher profits to the second-price auction. The large sample approximation of the Fisher sign test gave a test statistic of 3.65 which is significant at the p-value less than .0001.

The last pairwise comparison is between the progressive and second-price auction. The progressive auction gave a higher profit in 14 of the auctions and the second-price auction gave a higher profit.
Table 5

STATISTICAL EVIDENCE BEARING ON HYPOTHESIS SIX
Fisher Sign Test

Number of profits higher (N = 30)

<table>
<thead>
<tr>
<th>Progressive</th>
<th>First-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>6</td>
<td>0</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = 3.29
p-value = .005

Number of profits higher (N = 30)

<table>
<thead>
<tr>
<th>First-price</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>25</td>
<td>0</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = 3.65
p-value < .0001

Number of profits higher (N = 30)

<table>
<thead>
<tr>
<th>Progressive</th>
<th>Second-price</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>16</td>
<td>0</td>
</tr>
</tbody>
</table>

Fisher Test Statistic = .68
p-value = n.s.

In 16 of the auctions, the large sample approximation to the Fisher sign test gave a value of .365 which is significant at a p-value of .2483.

The results of the profit tests are consistent with the results of hypotheses 4 and 5. The progressive auction and second-price auction appear to be isomorphic with respect to profits. The profits generated under each system appear to be equivalent. The first-price auction gives consistently lower profits to the bidders. This result is logical, given the consistently lower price found in hypotheses 4
and 5. This result is also consistent with the independent auction experimental literature. The audit market typically uses the first-price auction.

Hypothesis 7, 8 and 9

The seventh, eighth and ninth hypotheses can be stated in the following null and alternative forms:

$H_0$: The turnover (cost, total) inefficiency among the three auction mechanisms is equivalent.

$H_1$: The turnover (cost, total) inefficiency among the three auction mechanisms is not equivalent.

Because of the rather low incidence of inefficiency found within the auctions, there are no statistical tests presented, only descriptive statistics. Price does not have efficiency effects only allocation effects. The only measure of efficiency is that the low cost auditor does not win the auction (produce the audit). Inefficiency in this section will be measured in points. The difference in points between the efficient solution and the actual outcome will be reported given both periods in the dependent sequence.

Given the parameter values used in this experiment, turnover inefficiency was found to be a very costly inefficiency. Turnover inefficiency is defined as the changing of auditor between the first and second period. Out of 90 possible possible turnover occasions, turnover only occurred four times. Of these four turnovers, three occurred in the first-price auction and one occurred in the second-price auction. The progressive auction never incurred a turnover inefficiency. Given the added uncertainty found within the first-price auction, this was the anticipated result. The second-price auction
was expected to have the second highest incidence of turnover because the Nash equilibrium point may not be obvious as the progressive auction. While the expected order was consistent with the actual order, because of low incidence of turnover no statistical tests were performed. The four turnover inefficiencies resulted in an inefficiency of 1,023 points, or an average of 255.7 per turnover inefficiency. The low turnover rate helps explain why turnover in the audit market is relatively low. The characteristics of the audit market make it unattractive to the client to change auditors.

The second type of inefficiency is called cost inefficiency. This inefficiency occurs when a non-low-cost auditor wins the auctions for both periods in the dependent auction. The progressive auction was found to incur this inefficiency seven times, the first-price auction four times and the second-price auction three times. The progressive auction cost inefficiency totaled 183 points for an average of 26.1 per inefficiency. This interprets as two bidders with cost values that are very close together and the one with the second lowest cost wins the auction. In comparing the averages, obviously the turnover inefficiency is more serious than the cost inefficiency on a per average basis.

The total points of inefficiency for the first-price auction totaled 135 points, this gives an average of 33.7 per cost inefficiency. The second-price auction gave total points of 396, or an average of 132 per inefficiency. As can be seen from the total per averages, the second-price auction per average is considerably higher than the other two averages. In examining the second-price auction, one cost
Inefficiency totaled 336 points. This is an outlier in the data and the reason for this one high cost inefficiency is difficult to explain. If this one outlier is deleted, the second-price auction has two cost inefficiencies for a total of 60 points. The average per inefficiency with the outlier deleted is 30 points. The deletion makes the average of the second-price auction in line with the averages of the other two auction mechanisms.

The last measure to discuss is the total efficiency of the auction mechanisms. This measure is simply an addition of turnover and cost inefficiency. The lowest total inefficiency was the progressive auction with a total of 183 points. The second lowest total was the second-cost auction with a total of 549 points (if the one outlier is discarded the total is 213 points which is very close to the progressive auction). The auction with the highest inefficiency was the first-price auction with a total of 1,005 points. This finding is consistent with the work of CRS that found the progressive auction to be the most efficient followed by the second-price auction and the first-price auction to be the most inefficient.

In conclusion, apparently the first-price auction is the least efficient system with respect to turnover inefficiency. Because of the importance of turnover inefficiency, this results in greater total inefficiency. With respect to cost inefficiencies, the three auction mechanisms are difficult to separate, they all appear to be fairly equal with respect to this inefficiency.
Section II

The next section of the paper compares the bidding strategies discussed earlier in the study with the outcomes found in this experiment. The first analysis examines price patterns between the two auction periods.

Rising Prices

Even though incumbent cost decreases between the two period, price is anticipated to rise between the two periods. Given the inherent trade off between probability of winning and the profit, the first-price auction will be expected to show less (in number) of price increases than the other two auctions. The Nash equilibrium of the progressive and second-price auction guarantees a price rise between the two periods, because the first period price is at the deflated second cost point and the second period cost point is at the second cost point. The results are shown in Table 6. The progressive auction price rose in 29 out of 30 auctions with one tie. The first-price auction price rose in 19 of the auctions with three ties. The second-price auction price rose in 22 of the auctions with one tie. This finding lends support to the result of Baber et. al. (1986) that prices tend to rise in periods following the start of a new auditor.

Nash Prices

To begin discussion of the Nash strategy, the second period of the auctions will be discussed. As was previously discussed, the second period of the progressive and second-price auction have the
same dominant strategies as the independent auction. The first period outcomes are irrelevant except for the determination of the cost values. In this case, the Nash equilibrium price for the progressive and second-price auctions is equal to the cost value of the second lowest cost bidder. The second-price auction has the dominant strategy of cost bidding. If all auditors follow this strategy, the price outcome will be at the cost of the second lowest cost bidder.

The results of the comparison of the Nash equilibrium points and the actual price results are shown in Tables 7 and 8. Table 8 shows the results of this comparison for the second period. The progressive auction averaged 23.3 points below the second cost point. The second-price auction averaged 51.3 points below this point and the first-price auction averages 113.3 points below this point. These results are consistent with the results of hypothesis five.

As can be seen, the progressive auction does not differ significantly from the Nash point in any of the three sets. This gives evidence that the dominant strategy of the progressive auction is realized in the second period. While the prices do not differ significantly from the Nash point, the price is consistently negative. The reason for this is difficult to determine. In discussing strategies with subjects after the experiment, most subjects realized that the winner in the first period would win the auction in the second period. In order to influence future bidding, non incumbent bidders bid below cost noting that they would not have to take the loss but might affect future bidding patterns of the incumbent. Thus, by keeping incumbent's profits low, the incumbent in future periods may not bid
Table 6

STATISTICAL EVIDENCE BEARING ON RISING PRICES
Binomial Test

**Progressive Auction**

<table>
<thead>
<tr>
<th>Period Price Higher</th>
<th>First</th>
<th>Second</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>29</td>
<td>1</td>
</tr>
</tbody>
</table>

Binomial Test = 5.38  
$p < 0.001$

**First-price Auction**

<table>
<thead>
<tr>
<th>Period Price Higher</th>
<th>First</th>
<th>Second</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8</td>
<td>19</td>
<td>3</td>
</tr>
</tbody>
</table>

Binomial Test = 2.17

**Second-price Auction**

<table>
<thead>
<tr>
<th>Period Price Higher</th>
<th>First</th>
<th>Second</th>
<th>Ties</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>22</td>
<td>1</td>
</tr>
</tbody>
</table>

Binomial Test = 2.79
Table 7
EVIDENCE CONCERNING NASH BIDDING

PERIOD 1

Auction Type

<table>
<thead>
<tr>
<th>Set</th>
<th>Progressive</th>
<th>First-price</th>
<th>Second-price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean 46.6 **</td>
<td>38.6 **</td>
<td>77.1 **</td>
</tr>
<tr>
<td></td>
<td>Std 27.1</td>
<td>30.9</td>
<td>36.9</td>
</tr>
<tr>
<td>2</td>
<td>Mean 19.2</td>
<td>29.4 **</td>
<td>77.5 **</td>
</tr>
<tr>
<td></td>
<td>Std 45.5</td>
<td>52.7</td>
<td>29.8</td>
</tr>
<tr>
<td>3</td>
<td>Mean 34.7 **</td>
<td>40.3 **</td>
<td>81.7 **</td>
</tr>
<tr>
<td></td>
<td>Std 25.9</td>
<td>25.8</td>
<td>45.4</td>
</tr>
</tbody>
</table>

t-test: * = p < .05  
t-test: ** = p < .01
Table 8  
EVIDENCE CONCERNING NASH BIDDING  

PERIOD 2  
Auction Type  

<table>
<thead>
<tr>
<th>Set</th>
<th>Progressive</th>
<th>First-price</th>
<th>Second-price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-25</td>
<td>-115.3 **</td>
<td>-75.1 **</td>
</tr>
<tr>
<td></td>
<td>60.1</td>
<td>95.9</td>
<td>65.8</td>
</tr>
<tr>
<td>2</td>
<td>-24.2</td>
<td>-122.8 **</td>
<td>-78.0 **</td>
</tr>
<tr>
<td></td>
<td>40.1</td>
<td>95.9</td>
<td>77.6</td>
</tr>
<tr>
<td>3</td>
<td>-21.3</td>
<td>-103.9 **</td>
<td>.4</td>
</tr>
<tr>
<td></td>
<td>38.8</td>
<td>79.7</td>
<td>56.7</td>
</tr>
</tbody>
</table>

* t-test: *= p < .05  
** t-test: ** = p < .01  

as low in the first period. In an independent auction, this strategy would not be plausible since bidding below cost may result in a loss.  
The first-price auction consistently priced below the second cost point. This point is probably not a Nash point in the first-price auction because of the information differences between this auction and the independent auction. The overall average of -115 points shows that the price is consistently below the second price point. This outcome has been explained by risk aversion (CRS, 1982).  
The second-price auction has the same dominant strategy as the progressive auction; however, two of the three sets have significant
differences from the dominant point. The reason for this may be that the participants may again be attempting to keep incumbents profits low so that the future bidding of the incumbents may be affected. This gaming was not found in the independent auctions. This result is easily explained. In the independent auction, the cost values of your opponents is completely unknown. Therefore, bidding below cost is very dangerous as the cost value of your rivals is unknown. However, in the dependent two sequence auction, the bidders know that the cost of the winner has been deflated. This deflation almost guarantees the winning of the second period by the Incumbent. Therefore, in order to affect future incumbent bidding, profits of the Incumbent are minimized by bidding below cost by the non-incumbent bidders.

The variance of the differences is consistently larger under the first-price auction. This would argue that the strategies under the first-price auction may not be as consistent as strategies under the other two auction mechanisms. This result is consistent with the findings of CST, where the first-price independent auction also had the larger variance.

The first period pattern is very consistent. All average prices are positive. This interprets as the price being consistently above the deflated second cost point. The progressive auction averaged 43.5 points above this cost point. The first-price auction averaged 36.2 points above the cost point and the second-price auction averaged 78.7.

The progressive auction and first-price auction both had one set where the difference from zero was not significant. This non
significant set is the same set for both auction methods. The second-price auction consistently priced above the 'optimal' price. This is consistent with the hypothesis that either this point was not realized or that the Nash strategy was perceived as being too risky. As was shown in Chapter III, bidding this strategy does not guarantee non-negative profits against all other bidding strategies. In examining variances of the difference between the actual price and the second cost point, no consistent pattern was detected between the three auction institutions.

Table 9 combines Tables 7 and 8 so that the overall second cost points are compared. As can be seen, the total first-price auction prices are consistently below the addition of the second cost points. The progressive and second-price auction both had one set that the difference was significantly positive (price was above the combined second cost point). This provides evidence that the second-price and progressive auction both tended to price at the second cost point in total. The first-price auction had a consistently larger standard deviation than the other two auction mechanisms.

Regression and Simulation Analysis

This subsection analyzes bidding strategies by examining the bidding data with regression analysis. The regressions were of the form of bid as a function of cost. Therefore, bid was the dependent variable and cost was the independent variable. The progressive auction cannot be analyzed in this format because limit bids are not tendered. Accordingly, only the first-price and second-price auction will be analyzed under the regression analysis. The regression analysis
Table 9

STATISTICAL EVIDENCE BEARING ON OVERALL PRICES
T-tests

<table>
<thead>
<tr>
<th>Set</th>
<th>Progressive</th>
<th>First-price</th>
<th>Second-price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mean 55.6*</td>
<td>-77.1*</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>Std 51.9</td>
<td>112.4</td>
<td>50.9</td>
</tr>
<tr>
<td>2</td>
<td>Mean -5.4</td>
<td>-93.1*</td>
<td>-.9</td>
</tr>
<tr>
<td></td>
<td>Std 80.5</td>
<td>146.0</td>
<td>62.8</td>
</tr>
<tr>
<td>3</td>
<td>Mean 13.3</td>
<td>-63.5*</td>
<td>82.1*</td>
</tr>
<tr>
<td></td>
<td>Std 38.5</td>
<td>101.5</td>
<td>91.9</td>
</tr>
</tbody>
</table>

* = p < .05

Is run on both auction methods and both periods resulting in four regressions. In addition to the regressions analyzed in this section, more regressions were run that used dummy variables for the different bidders. Nothing of interest was noted in these regressions. One problem with these regressions was the rather small sample size. There are some outliers in the data that need explanation. In some cases, bidders with high cost values in the second period would bid 700, knowing that they would not be involved in the auction results. These regression points are outliers and skew the residuals. Engelbrecht-Wiggins (1980) note the existence on non-serious bidders and their bids in these auctions. These points were deleted and the regressions analysis performed, however, the basic results did not change and, therefore, these points are included.
Table 10 reports the results of the regression for the first period under both auction methods. The F-tests for all regressions are highly significant (F value < .0001). The adjusted $r^2$ for the first-price auction first period is .8628. The intercept term is -1.53 and not significant (does not differ significantly from zero) and the intercept term is .8085 with a highly significant t-value ($t < .0001$). This result argues that bidders tended to bid roughly 80% of cost in the initial period. This bidding pattern guarantees bidding below cost.

Panel A shows the results of the regression of cost on price for the second-price auction in the first period. Again, the regression is very significant with the F test for the regression less than .0001. Again, the intercept is not significant with a value of -9.03. The slope coefficient has a value of .7628 and is very significant. Therefore, bidders tended to bid 76% of cost in the first period. On average, the second-price expected price in the first period was equal to $(-9.027 + [500/(N+1) + 200] \times .7628)) = 295.91$. The expected low cost in the first period was equal to $(500/(N+1) + 200) \times 300$. This shows that there is weak support for prices below cost on average. The second-price auction had 20 auctions that were below cost and ten that were above cost.

Analysis of the second period will now be presented. The results are shown in Panel B. In this regression, the intercept was significant and equal to 89.63. The slope coefficient was also significant and equal to .8275. The slope coefficient of the two regressions are almost identical. The difference between the two strategies is
Table 10
REGRESSION RESULTS OF COST ON BIDS

<table>
<thead>
<tr>
<th>Panel A - FIRST PERIOD</th>
<th>Coefficient</th>
<th>Estimate</th>
<th>Sig Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-price</td>
<td>B0</td>
<td>-1.53</td>
<td>N.S.</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>.8085</td>
<td>.0001</td>
</tr>
<tr>
<td>Adj. R^2 = .8628</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second-price</td>
<td>B0</td>
<td>-9.03</td>
<td>N.S.</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>.7628</td>
<td>.0001</td>
</tr>
<tr>
<td>Adj. R^2 = .8125</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B - SECOND PERIOD</th>
<th>Coefficient</th>
<th>Estimate</th>
<th>Sig Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>First-price</td>
<td>B0</td>
<td>89.63</td>
<td>.0001</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>.8275</td>
<td>.0001</td>
</tr>
<tr>
<td>Adj. R^2 = .837</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Second-price</td>
<td>B0</td>
<td>67.16</td>
<td>.0029</td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td>.7446</td>
<td>.0001</td>
</tr>
<tr>
<td>Adj. R^2 = .6312</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

mostly a change in location (intercept). Also this regression equation shows that as cost becomes larger, auditors bid closer to cost over the relevant range.

The second-price auction shows the same result: about the same slope but now a significant intercept. The slope coefficient gives a value of .7446 with an intercept of 67.16. This regression guarantees bidding below cost for any cost value above 202. This argues that the incumbent on average bids above cost while nonincumbents bid
below cost. This is consistent with the hypothesis that non incumbents attempted to minimize incumbent profits subject to not winning the auction in the second period.

The last analysis in this section was a simulation that used the regression equations found in the preceding analysis on the actual cost data. This analysis gives insights into the effects of realized cost distributions on price and auction outcomes. For example, in the first-price auction in the first period and first-price auction, the cost values for the first set were 526, 264, 314 and 606. Given the regression equation for the first period of bid = -1.53 + .8628(cost), the simulated bids for this auction are approximately 423, 211, 252 and 488. The price for this auction would be at 211. If the same cost data were simulated under the second-price auction in the first period, the regression equation bid = -9.027 + .7628(cost) would give bid values equal to 392, 192, 230 and 453. The resulting price under the second-price auction would be 230. Given the cost values in this first period, the first-price auction gives a lower price.

The simulation was run on cost values for the complete experiment. Of 30 first period auctions, the first-price auction gave a higher price in seven auctions and the second-price auction gave a higher price in 23 of the auctions, which is very close to the actual result of the first-price auction having the higher price in seven auctions, the progressive having a higher price in 21 auctions with two ties. If the null hypothesis assumes each auction as equally likely to give the lower price (i.e. p = .5), the large sample
approximation of the Fisher Sign Test gives a value of 2.92 which gives a p-value of .0009.

An analysis of the simulated bids and price comparisons points to conditions that must be present in order for the first-price auction to give a higher price. When the first-price auction gives the higher price, the average difference between the first-price and second-price auction is 12.86 points and the difference when the second-price auction gives the higher price is 63.13 on the simulated bids. In examining the cost data for auctions when the first-price auction gave the higher price, the average difference between the lowest cost point and the second lowest cost point was 7.71. If the second-price auction gave the higher simulated price, the average difference between the two auctions was equal to 108.3.

This difference points to the fact that if the lowest and second lowest cost points are 'close' together then the second-price auction gives the lower price. If these cost points are divergent, the first-price auction gives a lower price. The author calculated the needed difference between these values and found that for bids at 170 and 200 the needed difference at 170 was 24.24 while the needed difference at the low bid of 200 was 25.08. Therefore, at the cost value of 249.26, the first-price auction gives a bid value of 200. As long as the second-price auction second cost point is at 274.34 or less, the second-price auction gives a lower price. However, this bounds the difference between prices between the two auctions at 18.89 (i.e. difference between the price of the first-price auction and the lowest price possible in the second price auction). This shows that if
the second-price auction gives the lower price, the price difference between these auctions is rather small. On the other hand, if the first-price auction gives the lower price the upper bound on the difference is greater, since large differences between the low cost point and second lowest cost point give a relatively high second-price auction price.

A simulation was also run with the regressions obtained for the second period. The results were consistent with the empirical results. The effects of the distribution of cost on price outcomes had very little effect in contrast with the simulation of the first period.

Chapter Summary

This chapter presents the analysis of data obtained from the research investigation described in chapter IV. The data enabled the testing of the hypotheses which were directly related to the research objectives. The evidence presented in the study suggests that:

1) The DeAngelo model is robust with respect to cost differences and bidders consistently bid below cost in the initial period. The question of allocation is addressed with the majority of auctions won by the low cost (efficient) bidder.

2) The overall prices and profits of the first-price auction are below the prices and profits of the progressive and second-price auction. Therefore, from the buyer's point of view, the first-price auction would be preferred because of lower prices. This may argue why many auction bids are one shot first-price bids, ceteris paribus.

3) The progressive and second-price auction appear isomorphic in profits and second period prices. If the outcome of the progressive auction are desired, the second-price auction may be used.
4) If efficiency is measured in points, the progressive auction is the most efficient followed by the second-price and first-price auctions. The first-price auction appears to give lower prices, profits and efficiency than the other auction mechanisms.

5) All prices were above the Nash equilibrium point in the first period and below the point in the second period, but not all differences were significantly different from zero. This may argue that risk aversion impacts on bidding strategies.

A secondary analysis provided insights into the bidding strategies of the bidders and the effects of realized cost values on prices. The data suggest that the difference between the two lowest cost points has a very pronounced effect on the progressive and second-price auction. The first-price auction is unaffected by the actual realization of the cost values.

The next chapter includes a summary of the results, and a discussion of the conclusions and implications which can be derived from the results.
CHAPTER VI
SUMMARY AND CONCLUSIONS

This chapter summarizes the research undertaken, provides a more detailed discussion of the interpretations, conclusions and implications of the results, discusses the limitations of the study, and suggestions for future research.

The chapter consists of five sections. The first section summarizes the research described in the first four chapters. This summary provides the basis for drawing, in the second section, conclusions from the results. Implications with respect to the auditing market and economic theory are discussed in the third section. The fourth section enumerates the limitations of the research and discusses the possible implications of these limitations on the validity of the study's results. Finally, in section five suggestions for future research are provided.

Summary

The market for auditing services has been an area that has increasingly been analyzed. Many persons concerned with the auditing profession have voiced concern over the market. Many current researchers have examined the audit market using field data. The
problems of using field data has been noted by researchers working in the area (i.e. responses bias, unknown auditor costs etc.). Experimental economic methods provide another avenue to investigate the audit market.

The allocation method of the auditing market has not been closely addressed in the academic accounting literature. The allocation institution is probably not unique and for this reason several types of institutions were analyzed. The results of the study suggest that allocation mechanisms do make a difference on prices and efficiency and that the model proposed by DeAngelo is descriptive of the results of this experiment. Further, apparently subjects use stable strategies among the auction methods. The relative cost differences among the subjects explained part of the auction results.

Chapter II reviewed the literature relating to this experiment. The first section reviewed the auditing market literature. This section attempted to place this research within the work that has been done previously. As was documented in section I, researchers have noted the lack of research on the allocation (profit maximization) of audits. The second section of the literature review surveyed, in depth, the model proposed by DeAngelo (1981a). The third section reviewed the economic auction literature. As was noted, no game theoretic multi-period models have been developed; however, a few decision theoretic models had been developed using dynamic programming. Engelbrecht-Wiggins (1980) note that this is an area that needs to be researched. The last section in Chapter II surveys the independent experimental economic literature. This is done for theory and comparison with the results of multi-period models.
Because of the lack of multi-period game theoretic models, the third chapter attempted to examine the theory behind multi-period auctions. The first section first examined risk neutral Nash equilibrium independent auctions. The reason for setting up these models was two fold: first, the multi-period auctions build on this theory and, secondly, for comparison with the multi-period auction. The next section of the theory chapter examined the multi-period auctions for Nash equilibrium strategies. As was shown in this section, certain auction institutions could be shown to have Nash equilibrium strategies. All auction institutions were examined for the computational parameters that would have to be made if the Nash strategy was not found. The last section of this chapter presented the research variables of interest and the related hypotheses. The hypotheses were developed from the examination of the single and multi-period auction theory.

Chapter IV presented the research methodology used in this investigation and the statistical tests that were employed testing the research hypotheses. The statistical tests were described and justified in this section of the paper.

Chapter V presented the analysis and interpretations of the data obtained in the study. The results of the hypothesis tests are summarized in Table 11. Additional analyses included investigation of:

1) comparison of first and second period results with the price outcomes expected under the Nash strategy of the progressive auction.

2) comparison of the profit of the auction systems with the profit expected under the Nash strategy,

3) a regression analysis of the effects of cost on the predicted bid, and
**Table 11**  
Summary of Hypothesis Tests

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Result</th>
<th>Model and Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$: First period bidding is not below cost (first-price and second-price only).</td>
<td>Rejected</td>
<td>Bids below cost Binomial Test</td>
</tr>
<tr>
<td>$H_2$: Price is not below cost of winning auditor.</td>
<td>Rejected</td>
<td>Prices below cost Binomial Test</td>
</tr>
<tr>
<td>$H_3$: The three auction methods lead to the same number of prices below cost.</td>
<td>Rejected</td>
<td>1 &lt; 2 Chi-Square Test F vs S .005 F vs P .7559 F vs S .0026</td>
</tr>
<tr>
<td>$H_4$: In the first period, the resulting price among the three auctions is identical.</td>
<td>Rejected</td>
<td>P = 1 &lt; 2 Fisher Sign Test F vs S .0041 F vs P .0000 P vs S .0026</td>
</tr>
<tr>
<td>$H_5$: In the second period, the resulting price among the three auctions is identical.</td>
<td>Rejected</td>
<td>1 &lt; 2 = P Fisher Sign Test F vs S .0000 F vs P .0000 P vs S .0026</td>
</tr>
<tr>
<td>$H_6$: The profits to the auditor are the same under the three auction methods.</td>
<td>Rejected</td>
<td>1 &lt; 2 = P Fisher Sign Test F vs S .0000 F vs P .0050 P vs S .0026</td>
</tr>
</tbody>
</table>
| $H_7$: The turnover inefficiency among the three auctions is the same. | Rejected | 1 > 2 = P .
Table 11 (continued).

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Result</th>
<th>Model and Significance Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_8: ) The cost inefficiency among the three auctions is the same.</td>
<td>Accepted</td>
<td></td>
</tr>
<tr>
<td>( H_9: ) Total inefficiency among the three auctions is the same.</td>
<td>Rejected ( 1 &gt; 2 = P )</td>
<td></td>
</tr>
<tr>
<td>( H_{10}: ) The price does not rise between the two periods.</td>
<td>Rejected Price Rises</td>
<td>Binomial Test</td>
</tr>
<tr>
<td>( H_{11}: ) The price in the first period is at the 'optimal' price.</td>
<td>Rejected</td>
<td>T-test</td>
</tr>
<tr>
<td>( H_{12}: ) The price in the second period is at the 'optimal' price.</td>
<td>Rejected</td>
<td>T-test</td>
</tr>
<tr>
<td>( H_{13}: ) The total price for the auction is different than the total price given a Nash Strategy.</td>
<td>Rejected for ( P ) and ( S )</td>
<td>Accepted for ( F )</td>
</tr>
</tbody>
</table>

\( F = \) First-price Auction  
\( S = \) Second-price Auction  
\( P = \) Progressive Auction
4) a simulation of the expected outcomes given the regression analysis and the actual cost parameters.

The results of this analysis are not summarized here. Rather, they are combined with the results of the hypothesis tests, and the next two sections present an interpretation of the conclusions and their implications.

Conclusions

In a strict sense, the following conclusions apply only to the subject pool sampled. However, one of the main tenants of experimental economics is replication. Given the consistent experimental results of the study, apparently the results can be replicated.

As with all experimental economic work, the parallelism of the work is of great concern. The author attempted to maximize the external validity of the experiment without sacrificing the internal validity. Given the auditing literature reviewed in the literature review apparently many audit allocations have attributes that are similar to this experiment.

The conclusions suggested by this research are organized according to the research hypotheses:

First, Second and Third Research Hypotheses

The first research hypothesis examines the model developed by DeAngelo (1981a) that explains bidding below cost in the initial period. The government and small firms argued that this bidding below cost was anticompetitive and, therefore, should be discontinued. DeAngelo demonstrated that bidding below cost was to be expected
given up front costs and a competitive environment. The model in this dissertation expands on the work by DeAngelo by adding the allocation mechanisms and introducing cost differences among the auditors. Both of these changes decrease the incentive to bid below cost. However, even with both of these changes in an experimental market, auditors still bid below cost. This finding was consistent with both the first-price and second-price auctions.

The progressive auction could not be tested for bidding below cost because limit bids were not tendered for all market participants; however, this auction institution would also be expected to consistently lead to bidding below cost.

One assumption of these experimental markets is that the markets were competitive. Given the nature of the auditing market, whether the market is competitive is difficult to determine because the game between the auditor and client is a zero sum game. A higher audit fee must be paid by the client and vice versa. Because of this attribute, the experiment was carefully designed to insure a competitive environment. The number of subjects was carefully selected so as to parallel the actual audit market and to induce competition. Privacy of cost values and profits was enforced and no talking was allowed during the experiment.

The second hypothesis dealt with the price being below cost. If all bids tendered under the first-price auction are below cost then this guarantees that the resulting price of the winning auditor will be below cost since the first-price auction requires that the bids be price bids. The first-price auction was always found priced below
the cost of the winning bidder. This result was so pervasive that there is little doubt that this does take place. The progressive and second-price auction also priced below cost on the majority of occasions.

While bids may be below cost in the second-price auction, this does not guarantee that the price will be below the cost of the winning auditor. The price of this auction is heavily dependent on the distribution of cost. If the distribution gives a wide difference between the low cost and second low cost auditor, the result may be that the price is not below cost. This auction priced below cost in 66.67% of the auctions that were run. This result was significant, and, thus, the second-price auction also tended to price below the cost of the winning bidder. As was shown later in the study, the reason 33.33% of the auctions did not result in price below cost was because of the distribution of cost.

The simulation in Chapter V pointed to the fact that the resulting price under the progressive and second-price auction was highly dependent on the distribution of costs. If the lowest cost and the second lowest cost were sufficiently far apart, there was a probability that the resulting price would not be below the cost of the winning auditor. The results show that the second-price auction leads to significantly less prices below cost than the first-price auction. The number of prices below cost in the progressive auction were in between the first and second-price auction methods.
Fourth Research Hypothesis

The fourth hypothesis compared the resulting price among the three auction institutions in the first period. The theory in Section III found that a 'sensible' Nash equilibrium existed for the progressive and second-price auction and was the same strategy for both auction mechanisms. If the low cost auditor followed this Nash equilibrium in the game, he/she was guaranteed non-negative profits and negative profits to anyone else who won the auction. The first-price auction did not lend itself to finding 'sensible' Nash equilibriums.

As was expected, the first-price and second-price auction did not price at the same points. The second-price auction consistently priced at a higher price than the first-price auction. This finding is consistent with the independent testing performed by CRS and CRT. A mild surprise in this study was the finding that the second-price and progressive auction priced at significantly different prices in the first period. As was shown in the computational model, if the subject does not observe the Nash equilibrium, the second-price auction has more computational parameters (therefore more unknowns). Given risk aversion, this may lead to higher bidding in the second-price auction compared with the progressive auction. It is left to future research to test this hypothesis.

Fifth Research Hypothesis

The fifth hypothesis compared the resulting price among the three auctions in the second period. The theory examined previous experimental results and resulted in the hypotheses that the progressive and second-price auction should give a higher price and the
first-price auction should give a lower price. The reason for this is as follows: the second-price and progressive auctions have dominant solutions in the final period that state the price should stop at the cost value of the second lowest auditor. The price of the first-price auction was found to be below this second cost point.

The findings of the experiment strongly supported the conclusion of the experiments of CST and CRS. The first-price auction was found to consistently price below the price of the other two auction mechanisms. The progressive auction and second-price auction did not differ significantly with respect to price.

Sixth Research Hypothesis

The tests of hypothesis six compared the profits of the three auction institutions. Given that profits are equal to price minus costs, profits are a total of the price results given the same auditor winning the auction. The profits were compared in two fashions: across sets and across individual auctions. Given the small number of sets, no statistical testing was performed on this measure of profits. The following order of total profits was found within the sets: for set 1, progressive, second-price and first-price; for set 2, second-price, progressive and first-price; and for set 3, second-price, progressive and first-price. The first-price auction gave consistently lower profits than the other two auction mechanisms.

The second comparison was a period by period comparison where the auctions were compared pairwise and tested under the Fisher sign test. The first-price auction gave consistently lower auction by auction profits than the other two auction methods. The second-price
and progressive auction did not differ significantly, meaning that
the finding of a different price in the first period did not result
in a significantly different profit.

Seventh, Eighth and Ninth Research Hypotheses

Hypotheses 7, 8, and 9 all dealt with the inefficiency of the
three auction mechanisms. Given that the inefficiency is dependent
on the parameters of the model, absolute inefficiency is difficult to
measure. Therefore, the auction mechanisms were only compared among
themselves and not according to some idealized benchmark. Inefficien-
cy in this market is only measured by the cost of not having the low
cost auditor produce the audit. Three different types of inefficien-
cy were introduced: turnover, cost and total.

Turnover inefficiency occurs when the first period auditor and
the second period auditor are different auditors. Given the para-
meters of this model, a different auditor in the two periods almost
guaranteed a greater inefficiency than if the incumbent continued the
audit. The first-price auction had three turnovers, the second-price
auction had one and the progressive auction had no turnovers. This
provides evidence that the first-price auction is more likely to
result in turnover inefficiency.

The second inefficiency was called cost inefficiency. This re-


results when a non-low-cost auditor wins the auction for both periods
there is no turnover). Depending on the distribution of cost, this
inefficiency is relatively minor when compared with the turnover in-
efficiency. The progressive auction incurred this inefficiency seven
times, the second-price auction four times and the first-price
auction three times. Again, given the rather low incidence of cost inefficiency, no statistical tests were performed.

Total inefficiency is a summation of cost and turnover inefficiency. The progressive auction was found to have the lowest total inefficiency (as measured in the monetary unit, marks) followed by the second-price auction and the first-price auction. This finding was expected given the previous experimental work (CRS and CRT), however, the reason was different. In CRS and CST, the progressive and second-price auctions had dominant solutions. In this experiment, this strategy was downgraded to Nash equilibrium strategies.

**Bidding Strategies**

These three hypotheses compare the results of the actual auctions with the 'optimal' prices given by the Nash strategies of the progressive and second-price auctions. The difference of the actual prices and optimal price was tested using t-tests to see if the price difference differed from zero. The first analysis compared the Nash strategy with the actual price outcome in the first period. All differences were positive implying that the price was consistently above the Nash price in the first period. Only one difference was not significant and that was the progressive auction second set. This appears to argue that the subjects found the Nash strategy too risky and is consistent with the results found in period two.

Next, the Nash price was compared with the price resulting from the actual bid data in the second period. The price was consistently below the Nash price. The differences were consistently negative implying that the price was below the cost of the second cost subject,
however, many results were not significant. While the progressive auction was consistently negative, none of the differences were different from zero. This may argue that once the price dropped below the second cost point, the second low cost subject dropped out very quickly but not before his/her value was reached. The negative coefficient is difficult to explain because it is not rational to bid below cost in the second period if the auctions are treated as independent. In discussing the auctions with the subjects, some did not treat the auctions independently. They noted that lower incumbent profits in one period may affect the incumbent in future auctions.

The CRS and CST studies did not find this gaming because in the one-period independent auction to bid below cost was not sensible since there is a non-trivial probability that subject may win the auction but at a loss. This strategy will not affect the future strategies of rival subjects since their strategy is a dominant strategy no matter what the strategy of rival subjects.

Two of the three second-price auctions were significantly different from zero. The last set actually had a small positive coefficient. The first-price auction consistently priced below the Nash equilibrium price. All averages were consistently below zero and negative. This result is consistent with hypothesis five.

The comparison of the overall price is very interesting in that some of the differences between the first and second period wash against themselves so that the total price under the Nash strategy is very close to the actual total price.
A regression was run with the dependent variable being bid price and the independent variable being cost. The regressions showed a very significant relationship. With the regression data, a simulation was run with the regression results on the actual cost data. The results were very similar to the actual results of the experiment. The pricing among the two auctions was very dependent on the distribution of cost.

**Implications**

The Implications of the above conclusions are discussed in this section. This study is the first attempt to analyze the allocation of audits. As previous audit research has noted, a main difficulty of analyzing the audit market is the lack of data. One of the main variables that is unknown is the cost of the audit. Because of the lack of these data points, an experimental approach was chosen. This dissertation is one of the first experiments that attempt to model the audit environment. One of the strengths of the experimental approach is the knowledge of parameter values. For example, in this study, the cost of the audit was known by the experimenter.

The first implication is the modeling done in this study that analyzes the auction methods in a game theoretic fashion. To the author's knowledge, dependent auctions has not been analyzed in a game theory approach, even though some attempts have been made using decision theory. The analysis finds that the dominant solutions of the progressive auction and the second-price auction are lost under dependent auctions. However, Nash equilibrium strategies that are
last period rational are found. The risk neutral first-price auction Nash equilibrium is also lost under dependent auctions. Following Riley and Samuelson (1981), the calculations that must be made in deciding a strategy are derived for the three auction methods. The progressive auction has the least number of parameters that must be imputed in making the bid. A there is a direct tradeoff between the profit and the probability of winning in the first-price auction. The higher an auditor is willing to bid the higher is the profit given a win but the lower the probability of the win. This attribute of the first price auction makes the strategy of the auditor a very important consideration. In the progressive and second-price auction, the tradeoff between the probability of winning and the winning price in the first-price auction is not made. With these auction mechanisms, the auditors must determine the lowest acceptable price.

Three auctions were tested in the experimental markets. The first-price auction was tested because presumably this was the auction typically used in the audit market. Certainly for many auctions (i.e. government bidding) this allocation system is undoubtedly used. The progressive auction was tested for two reasons. First, auctions theory has shown that this auction has desirable properties for the independent auction (i.e. dominant strategy). Second, some of the audit allocations may follow this auction given multiple bidding and some of the multiple contacts prospective auditors make with a potential audit client. The secondprice auction was the last auction tested because a one-shot auction that had the same attributes as the progressive auction was desired to be tested.
The analysis first examined the attributes of the experimental market. Bidding below cost was a consistent attribute of all markets. The deflation of incumbent cost appears to guarantee bidding below cost in an experimental market. Competition appears to guarantee bidding below cost and CAR should really be concerned if bidding below cost did not take place.

The first-price auction gave a consistently lower profit than the other two auction mechanisms. It is hypothesized that risk aversion and the explicit tradeoff between profit and the probability of winning leads the auditor to a lower price in the first-price auction. This may explain client preference of the first-price auction over other allocation institutions (especially one with a high emphasis on price).

Consistent with other work, the rank of efficiency followed the following ordering: progressive, second-price and first-price auction. It is hypothesized that the tradeoffs between efficiency and profits noted in Vickrey (1961) and Loeb (1977) are found here. While incentive compatible systems can be found, they appear more costly to operate to the client. In this case, implementing the second-price or progressive auction leads to higher profit.

The analytical literature and the previous empirical literature have failed to note the effects of actual cost distributions on auctions outcomes. Since Nash equilibrium strategies are sought, no matter what the cost outcomes are, the analytical literature has ignored this issue (this literature is only interested in expected price). However, the actual realized cost distribution is critical to price
outcomes of the progressive and second-price auction. Since the first-price auction price is independent of other auditor's bids, the actual cost distribution has no effect on the price of this auction. However, the price of the other two auctions was very dependent on the cost of the auditors. In particular, the closer the cost of the second lowest and lowest cost auditor, the lower was the price, ceteris paribus. Wide cost dispersions between these two auditors led to higher prices.

**Limitations**

This section discusses the limitations of the study which may confound or invalidate the conclusions and their implications. Given the preliminary nature of this work, the limitations are extensive.

One of the severe problems with trying to model the allocation of audits is the lack of a unique allocation method. Each audit allocation has its own circumstances and attributes. For example, many government audits are decided by sealed bid with the winning auditor being the one who bids the lowest cost. On the other hand, many public client audit allocations have a negotiation process (progressive auction). Attributes such as social factors are difficult to model and for this reason the only factor introduced in the model was cost. This difference is not non-trivial as most of the auditing literature points to the cost differences between auditors.

A second weakness is that the work of the auditor was ignored. Fellingham and Newman (1985) using a game theoretic model showed the interdependence between the auditor's and client's strategy. This
game was ignored in the present work and this game was assumed to be already solved by the auditor and client before the bidding started in order to determine the cost values.

One concern is the parallelism of the study. As with all experimental work, the tie between the laboratory and the real world needs to be addressed. One concern is the effect of the parameters of the experiment on the outcomes. Parameters were carefully selected in order to mimic the audit environment as closely as possible. As was discussed in the dissertation, changing parameter values should only change the magnitude of results and not the results per se (this was verified in the pilot studies). One problem with the parameter values is the lack of unique parameter values. Every audit allocation has its own unique parameter values. Therefore, the values were chosen so as to maximize statistical power, given a constraint of money to pay the subjects. However, in a strict sense the results only hold for the parameter values used in this experiment.

The use of students as surrogates for auditors is also a factor of concern. Given that the students understood the experiment and that they were nonsatiated in the goods of the experiment (money), this use of surrogates did not affect the results of the experiment. Given the consistent results across the sets, apparently the results of this experiment are can be replicated.

Further Extensions

Given the preliminary nature of this work there are many possible extensions to this work. The first extension is to vary parameter
values to ascertain the change in magnitude on the results. Varying dependent time periods, cost deflation values and number of auditors bidding would be interesting. The pilot study had longer time periods than two and it was found that some inefficiencies appeared to increase, however, the basic results did not change. CRS found that different models of human decision making explain bidding strategy in groups of three compared with larger groups. They explain this difference to cooperative bidding strategies in groups of three and non cooperative behavior in larger groups. Checking to see if this result can be replicated in this dependent auction sequence would be interesting.

Another possible extension is to fuse the model proposed by Fellingham and Newman with the model proposed here, and to note if the bidding strategy appeared to affect future auditing work. There might be a tie between future auditing work and the bid price in the initial period. For example, the auditor may have the choice of two prices ($H_p$ or $H_1$) and two effort levels ($H_e$ and $H_1$) and the choice of audit report. This game could be solved using non-cooperative game theory.

Other factors facing the auditor, such as reputation, could be inserted using a multi-component bid function. In this model, the auditor would be rated on factors besides cost.

As was discussed in this dissertation, the decision facing the client could be inserted by making the auctions costly to the client. Therefore, after the initial audit, the client would decide if an auction should be held. Costs for auction could also be inserted for
auditors, and, therefore, they could decide if they wanted to enter the auction.

Another extension already undertaken by the author is to study the effects of the bid on future opinion decisions. Using a model proposed by Tversky and Kahnemann (1979), bidding below cost may change the point of reference of the auditor (framing) and that a different decision may be made than if the initial bid were not below cost. An important assumption in this modeling is that the selection of audit opinion be seen as a selection of a gamble. Given the binary choice of an audit opinion (qualified or not qualified), the selection of a gamble seems very plausible.
INSTRUCTIONS

SEALED BID TENDERS

This is an experiment in the economics of market decision making. The instructions are simple, and if you follow them carefully, you may earn an amount of money which will be paid to you in cash.

This experiment will consist of 30 periods and within each period there will be a two auction sequence. At the beginning of the period, each of you will attempt to sell one unit of commodity. For each auction only one unit of commodity can be sold. Your cost values represent the amount that I will deduct from the sales price if you win the auction. The cost value can be considered the production cost of producing the unit. To sum up Price received - Cost value = Profit (Loss). For each period (two auction sequence), your cost values will hold constant with one exception; if you win the auction in auction 1 your cost will drop by 50% in auction 2. Let me give you an example of one period. In the first auction, all sellers’ resale values will be the initial amount. The winner of the first auction will have his/her cost value drop by 50% in the second period. The other sellers’ cost will hold constant. For the next period, we will repeat the cycle just described.

Next, I want to explain how the cost values were determined. We will be dealing in a fictitious currency called oompa loompas. The exchange rate for dollars to oompa loompas is $0.01 for 1.0, where
""'m' designates oompas loompas. The possible cost values range on the
interval ("200, "700). On this interval, a random number generator
was used to determine the cost values with equal probability for each
point on the interval ("200, "700). Thus, you have an equally likely
chance of receiving a cost value between "200 and "700. Furthermore,
the chance of you being assigned any particular value in this range
is not changed if that value was assigned earlier to you or to anot-
er participant. It is therefore possible for you to get the same
cost value for different auction periods or for two participants to
have the same value in the same auction. All participants will have
their cost values assigned in this manner. You will be given your
own cost value but you will not know the exact cost value of the
other sellers. After each of the auctions the winning bidder and
amount will be given. This will be the only information reported.
Your individual record sheet has your cost values for the respective
periods.

The following are the exact auction rules:

a) In auction periods 1-10, I will open the floor for bids. You
will determine how you will bid by referring to your profit sheet.
Please call your bid by referring to your seller number. For ex-
ample; if you are seller 3 and you wish to bid "450 -- bid by saying,
"3 bids "450." I will hold each bid for a few moments until a lower
bid is put to the floor. The person with the lowest bid will be
awarded the right to sell the unit of commodity. All bids will start
at "600 and the largest bid decrement is "20.

Are there any questions?
To insure that the rules of the auction are understood by all market participants, please answer the following questions:

In period 1 there are three bidders: A, B, and C. Their respective costs are $200, $250, and $300. Assume that bidder B wins the first auction at a price of $220. What are the profits (losses) for the three bidders in the first auction?

A:_____  B:_____  C:_____

What are the respective cost values for the bidders in auction 2?

A:_____  B:_____  C:_____

Assume in auction two that bidder B again wins at a price of $190. What are the respective profits (losses) for the three bidders?

A:_____  B:_____  C:_____

Instead of bidder B winning in the first auction, assume that bidder A obtains the auction at a price of $220. What then would be the respective profits (losses) for the three bidders?

A:_____  B:_____  C:_____

You will determine your profit by subtracting the price sold from your cost value in your record sheet. Carefully consider your strategy for each two auction sequence. This will be paid to you in cash. You are not to reveal your cost values or profits, nor to speak to any other participant while the experiment is in progress.

If you have a question that you feel was not adequately addressed please raise your hand. Your earnings may suffer if you proceed into the marketplace without understanding the rules!
### RECORD SHEET
Progressive
Seller No. 1

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<tr>
<th>PERIOD</th>
<th>1</th>
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<tr>
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<table>
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<tr>
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<td></td>
</tr>
<tr>
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<td>434</td>
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<td>307</td>
<td>324</td>
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<tr>
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<tr>
<td>AUCTION</td>
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<tr>
<td>PROFIT (LOSS)</td>
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</tbody>
</table>
b) In periods 11–20, each of you will be asked to submit a bid on one unit of commodity. Each of you should determine your bid by referring to your record sheet where your cost values are given. Each unit you are able to sell in a period will give you a profit (loss) equal to selling price minus your cost. The transaction price in this first auction will be to the lowest bidder at the bid price. Sealed bids will be collected from each seller. You will enter your bids on the index cards provided. On each index card is a sequence of three numbers. The top number refers to the bidder number. The second number is the period number. The last number is the auction number. The index cards will be collected after you have entered your bid. The winning bidder and bid will then be announced. Are there any questions?
# Record Sheet

**First Price Auction**  
**Seller No. 1**

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<tr>
<td><strong>My Bid</strong></td>
<td>:</td>
<td>:</td>
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</tr>
<tr>
<td><strong>Winning Bid</strong></td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
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<tr>
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<td><strong>Cost If Incumbent</strong></td>
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</tr>
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<tr>
<td>BID</td>
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<tr>
<td>WINNING</td>
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<tr>
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<td>DIFF</td>
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<tr>
<td>PROFIT (LOSS)</td>
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</tr>
<tr>
<td>PROFIT (LOSS)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
c) In periods 21-30, the same procedures will be followed subject to the following exception: The item is awarded to the lowest bidder at a price equal to the second lowest bidder. Again, sealed bids will be collected from each bidder.
RECORD SHEET
Second Price Auction
Seller No. 1

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<td>MY BID</td>
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<tr>
<td>WINNING BID</td>
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<td>SECOND BID</td>
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<tr>
<td>PROFIT (LOSS)</td>
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<td>WINNING BID</td>
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_______. "Auctions and Bidding Games," in Recent Advances In Game Theory, Princeton University, 1962.

