AN ANALYSIS OF REAL ESTATE AND OTHER TAX-PREFERRED INVESTMENTS

DISSEPTION
Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

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* * * * *

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Finally, the completion of my studies and this thesis would have been much more difficult without the support of my wife, Gert.
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Chapter I

INTRODUCTION

Federal income tax laws are such that certain assets are given preferential tax treatment relative to other assets. Sources of preferential taxation include deferral of taxes, conversion of taxes to a capital gains tax rate, and partial or full exemption of taxes. Assets can differ by both the amount and number of different types of tax preference present. This differential tax treatment of certain assets creates economic incentives that differ from a world where either all assets are taxed equally or there are no federal income taxes.

The purpose of this study is to extend the literature which has examined the effects of differential taxation on an individual's portfolio choice and the relative prices of assets. The key issues which are examined in this study are summarized in the following sections.

The Nature of Tax Preference

In order to study the effect of differential taxation on asset prices and an individual's portfolio choice, it is first necessary to examine tax preference concepts. Chapter II identifies the main sources of income and the way they interact with the individual's tax function. The nature of tax preference is shown to be different for wage income as opposed to investment income. This is especially relevant to
understanding the role that assets such as real estate with artificial accounting losses play in the provision of "tax preference services."

Economic Incentives Created by Tax Preference

The value of tax preference to an individual is dependent on his overall tax situation, and usually increases with an investor's marginal ordinary income tax rate. This results in incentives for individuals to purchase or trade tax shelter. But an asset consists of more than just tax shelter. In general, an asset can be thought of as consisting of two components: 1) a cash-flow component which in the absence of any tax shelter would be fully taxable at the individual's ordinary income tax rate, and 2) a tax-shelter component which results in some amount of taxable income being sheltered from taxation at the investor's current ordinary income tax rate.

Many studies have considered the proportion of these two components as fixed for a given asset. This affects both the nature of an individual's portfolio choice and the nature of yield differentials for assets. Certainty models which have made this assumption include those of Bailey (1974), and Galper and Zimmerman (1977). Uncertainty models with the same assumption, but which focused only on the differential taxation of dividends and capital gains include Brennan (1970), Litzenberger and Ramaswamy (1979), and Long (1977).

By including differential taxation into the Sharpe (1964), Linter (1965), Mossin (1966) Capital Asset Pricing Model (CAPM), Brennan shows that higher dividend-yield stocks would have greater expected before-tax
returns due to the unfavorable taxation of dividends relative to capital gains. Each stock is assumed to have an exogenous amount of dividend cash flow. In effect, the investor purchases the tax-preference characteristic of a stock along with its stochastic characteristic which may have diversification benefits. This causes investors with different marginal tax rates to hold a different portfolio of risky assets. This result is also shown by Long (1977) who examines the effect of the differential taxation of dividends and capital gains on the after-tax mean-variance efficient portfolios held by investors in different tax brackets. It is shown that investors will generally not hold the same portfolio if they are in different tax brackets. Furthermore, the "market portfolio" may not be mean-variance efficient or held by any investor.

Long's specification of after-tax return for a security is the same as in Brennan's model. A conclusion of Long's paper is that a linear cross-sectional relationship would have to exist between dividend yields and before-tax returns for all assets if "taxes don't matter." i.e., investors all hold the same portfolio. Since this relationship is not observed in the market, Long concludes that taxes will affect portfolio choice.

Litzenberger and Ramaswamy (1979) also develop a model for the differential taxation of dividends and capital gains. Their model allows the ordinary income tax rate to be progressive and endogenous. The combination of the zero capital-gains tax rate and certain dividend payments results in taxable income (from dividends only) being nonstochastic.
The interesting difference between the Litzenberger and Ramaswamy model and the Brennan model is the inclusion of margin requirements and a limitation of interest deductions to the amount of dividend income. Investors (if any) who find the income constraint to be binding will prefer dividends to capital gains. With two classes of investors, each having a different preference for dividends vs. capital gains, a supply response could result in dividends having no effect on equilibrium expected before-tax returns even though they affect individual demand equations. A similar result is found in this study for excess depreciation. Because of the tax on "tax preference items", a class of investors could choose not to use accelerated depreciation even though it might be supplied at a effective price of zero.*

Chapter III discusses the fact that the rate-of-return model used in the above studies might not be appropriate for the differential taxation of dividends and capital gains due to: a) an exclusion of a portion of dividend income from taxation and b) the lack of incentive on the part of firms to supply dividends at a yield premium over capital gains. However, a similar rate-of-return model might be appropriate for an analysis of other types of differential taxation with different supply and demand characteristics. Therefore, a more general model is developed in this study and interpreted for a broader range of tax preference.

*As discussed in this study, a positive price for tax shelter could be reflected in a yield differential for an asset which contains the tax shelter, or a payment for the claim on the tax shelter itself e.g. by getting a greater allocation of depreciation from a limited partnership in exchange for receiving less cash flow.
Miller and Scholes (1978) show how the taxation of dividends over capital gains can be "laundered" by the use of debt. Their analysis assumes an effective price of zero for tax shelter. The price of tax shelter may not in fact be zero, and the strategy may not be efficient for dividend income since, as discussed previously, there may be no yield differential between dividends and capital gains. However, the lesson to be learned from Miller and Scholes is that debt can be used to "sell" taxable income which is equivalent to buying tax shelter. The fact that the price of tax shelter is not zero simply means that only investors above a certain tax bracket will purchase tax shelter.

The tax shelter could be purchased from investors in a lower tax bracket e.g. through leases or from an intermediary that repackages cash flows into components with different tax characteristics e.g. corporations and limited partnerships. This study points out the advantages of real estate as a means of using debt to sell taxable income.

Miller (1977), for example, points out that corporations use financing to supply individuals with debt which is fully taxable and stock which is assumed to be effectively tax exempt. Corporations are willing to supply debt at a higher yield since the interest is tax deductible at the corporate level. The higher yield for debt over stocks attracts lower tax rate investors who are not willing to purchase the tax-preference characteristics of stock as reflected in its lower before-tax yield.
This study discusses the way debt can be used to sell other types of taxable income. It becomes apparent that debt plays an important role in using real estate income property as a means of selling taxable income from other sources including personal service income. The equilibrium price for tax shelter and tax-preferred assets is then examined under more general assumptions regarding the purchase and sale of tax shelter. Under conditions of certainty, all assets will tend to equate at the same marginal tax rate that is determined by the price of tax shelter relative to the price of taxable cash flow. This is quite different from the models of Bailey, and Galper and Zimmerman, where a continuum of breakeven marginal tax rates separate assets with different amounts of tax preference.

Uncertain Cash Flows

When cash flows are uncertain, the nature of equilibrium also depends on whether the proportion of an asset's tax shelter relative to taxable cash flow is fixed or whether claims on these components can be traded separately. As shown in Chapter IV, when these claims are in fixed proportion, investors in different tax brackets hold different portfolios of risky assets. This is therefore a generalization of the results of Brennan, Litzenberger and Ramaswamy, and Long. However, as mentioned previously, the model might be more appropriate for other sources of differential taxation than dividends over capital gains.

The reason why investors hold different portfolios of risky assets in the model described above is because they must trade off between
tax-shelter services and diversification services. Given the market price of risk and tax preference, individuals adjust their portfolio composition such that after risk adjusted rates of return for all assets will equal the after-tax risk-free rate.

Clearly there are incentives for individuals or market intermediaries to decompose assets into these separate claims. This allows investors to make diversification decisions separately from tax-shelter decisions. The nature of equilibrium under these assumptions is discussed in Chapter IV and devices for separating these claims are discussed in Chapter VII.

**Endogenous Progressive Tax Rate Model**

Instead of assuming an exogenous marginal tax rate for each investor, it can be assumed that the tax rate is a function the investor's portfolio income. Fellingham and Wolfson (1978) developed a model which assumes the investor's tax rate is a progressive function of portfolio income. They show that risk-neutral investors who maximize expected after-tax wealth still require a premium to hold risky assets. Furthermore, investors find it optimal to hold diversified portfolios. This study extends the endogenous progressive tax model to include preferential taxation. This provides an alternative interpretation for the market price of tax preference.

**Uncertain Differential Taxation Model**

The models developed by Brennan, Litzenberger and Ramaswamy, and Long assume certainty with respect to the benefit of tax preference. However, the benefit of tax deferral to a capital-gains tax rate can
be reduced due to uncertainty as to the investor's future tax rate. The investment objectives of the individual may include retirement income, estate planning, and future consumption before retirement. Deferred income and capital gains will normally be recognized when the investment is sold. While the investor can time the sale of the asset to some extent to minimize taxes, several factors could lead to uncertainty as to the marginal tax rate which will apply when the investment is sold. First, the investor's exogenous nominal wage income could be affected by both uncertain inflation and changes in his real wage rate. Marginal tax rates have tended to rise for many individuals during recent years. Second, the investor will not know what his future tax rate will be. Third, there may be tax law changes which would affect the investor's future tax status.

The endogenous tax model mentioned earlier, which is developed in this study, captures some of the uncertainty as to future investment income since the tax rate is a function of this income. A more general model is also developed in this study which allows each investor's tax rate to be a random variable. It is then shown that investors hold different portfolios if their current tax rates differ, expected future tax rates differ, or the variance of future tax rates differ. The value of an asset's tax preference that involves either deferring income to a future ordinary income tax rate, or converting income to a future capital gains tax rate, will depend on investor's future tax rate distribution. The market price of tax preference will depend on the aggregate expected benefit of the deferral or conversion and the market's
aversion to the uncertain benefit of converting income to an uncertain future tax rate.

One result of this analysis is that different types of tax preference would not necessarily have the same market price. The value of deferral, for example, will depend on the distribution of future ordinary income tax rates, whereas the value of conversion will depend on the distribution of future capital-gains tax rates. The importance of separating the various components of tax preference becomes even more apparent when analyzing an investment such as real-estate income property which contains many sources of tax preference, the value of which depends on many characteristics of the investor's tax situation.

**Real-Estate Income Property**

Recent articles in the real-estate literature argue that the CAPM should be applied to real-estate valuation. These include Miles and Rice (1978), Gau and Kohlhepp (1978), Wofford and Moses (1978), and Young (1977). None of these articles considered whether the tax-preference characteristics of real estate might be relevant to its pricing. Since real estate has preferential taxation, the tax-preference components of the return needs to be considered when evaluating individual and market equilibrium.

The various sources of tax preference for real estate cannot be adequately captured by a single tax-preference variable. The sources of tax preference for real estate income property comes from: 1) conversion of income from the ordinary income tax rate to the capital-gains tax rate, due to the amount by which straight-line depreciation exceeds economic depreciation; 2) any nominal price increase due to inflation
is taxed at the capital-gains tax rate; 3) deferral of taxes due to the excess of accelerated depreciation over straight-line depreciation. This latter factor is deferral instead of conversion, due to "recapture of excess depreciation" at the ordinary income tax rate. Since excess depreciation is also a "tax-preference item," the benefits of the accelerated depreciation will also depend on the investor's other tax-preference items. Not only can there be a tax on the investor's tax-preference items, but tax-preference items can reduce the benefits of the maximum tax on personal-service income.

The benefits of capital gains from items 1 and 2 above can also be reduced by the alternative minimum tax which applies to capital gains and excess itemized deductions. This tax was instituted by the Revenue Act of 1978 which also eliminated capital gains as a tax-preference item.

Because of the many factors which affect the tax-preference components of real estate, the market price of the various tax-preference components can differ. This study examines the supply and demand characteristics of these tax-shelter components in order to see how the prices of each component might differ. As discussed previously, the demand for excess depreciation can be affected by the investor's ordinary income and tax-preference tax rates. There can also be differences in the supply of each characteristics and a limited degree of substitutability with other tax-preferred assets.

Thus, the market prices of the various tax-shelter components can differ substantially. If the price is zero, there will be no effect on
the asset's before-tax yield. The greater the price of a particular types of shelter, the greater the expected yield differential. In order to compare assets with differential taxation, or to properly value real-estate income property, it is necessary to have some knowledge as to the price of the various tax-shelter components, or all the relevant marginal tax rates (ordinary income, capital gains, tax preference, etc.) at which the asset equates with other assets on an after-tax basis.
Chapter II
TAX-PREFERENCE CONCEPTS

Introduction

Individuals receive income from their human and investment capital. The former comes primarily in the form of wages, while the latter is received through a variety of investment vehicles. In a world of tax neutrality, all forms of income would receive the same tax treatment. However, the federal income tax laws are such that certain income sources receive preferential tax treatment relative to other sources. This chapter is intended as an overview of the sources of differential taxation.

Investment vs. Wage Income

Figure 2-1 is a simplified diagram to show the treatment of various sources of income. Investment in human capital results in wage income. For tax purposes, this will normally be considered personal service income as opposed to investment income. This income will usually be taxed at the investor's ordinary income tax rate and be limited to a maximum tax rate of 50%. A limited amount of wage income can be placed in certain retirement plans which result in the income being taxed at some future point in time. This will normally be at retirement, although the individual can withdraw the funds early by paying a penalty in addition to recognizing the gain upon withdrawal. The nature of deferral will be discussed further in a later section.
Figure 2-1
Tax Treatment of Income
Investment income comes from a variety of real and financial assets. There are many reasons why investment income must be differentiated from wage income in understanding differential taxation. First, investment income can be taxed at rates up to 70% while wage income is normally limited to a 50% tax rate.* Second, there are many more alternatives available to the individual to shelter taxes on investment income. Third, there is an interaction between decisions to shelter investment income, and the taxation of wage income.

**Fully Taxable Investment Income**

Investment income that has no preferential tax treatment would be fully taxable as ordinary income. Interest income from corporate bonds would be one example. A portion of the income from investments such as real estate income property would also be fully taxable. Dividend income is also taxed as ordinary income. However, a certain amount of dividend income is exempt from taxation. For example, the first $100 of dividend income for a single individual is exempt from taxation. Thus dividend income has tax exempt and fully taxable components.

**Tax-Exempt Investment Income**

Interest income from state and municipal bonds are exempt from federal taxation. Thus, no taxes are explicitly paid on this income.

*The amount of wage income subject to this maximum tax rate can be reduced by tax preference items. This is discussed further in a later section.
The extent to which investors might implicitly be taxed through yield differentials is discussed in Chapter III. Any income from selling a municipal bond at a gain will be taxed as a short-term or long-term capital gain as discussed in the next section.

**Capital Gains**

Capital assets held for one year or longer are taxed as a long term capital gain. Under the current tax law 60% of the gain is excluded from taxation and the remaining 40% is taxed with other ordinary income. The Revenue Act of 1978 eliminated capital gains as a tax preference item.* However, an investor with a large amount of capital gains may be subject to an alternative minimum tax which applies to capital gains and excess itemized deductions.

Prior to the Revenue Act of 1978 there was an exclusion of only 50% of the capital gain, but there was also a maximum tax rate of 25 percent on the first $50,000 of capital gain, before consideration of the impact of any additional preference tax. Thus, the relationship between the treatment of capital gains and ordinary income was not often as simple as assuming a percentage of the ordinary income tax rate. It was more convenient to think in terms of capital gains tax rate as distinct from the ordinary income tax rate. However, under the current tax law, the effective tax on capital gains for individuals will in most cases be 40% of the ordinary income tax rate.

* Details of the minimum tax on tax-preference items is discussed in a later section.
Deferral

There are several ways to defer investment income. Deferred annuities can be purchased from insurance companies which provide tax-free compounding of interest. The earnings will still be taxed as ordinary income when withdrawn, but the investor's tax rate might be lower if the funds are withdrawn at retirement.

Tax on the gain from any price appreciation of a capital asset is normally recognized when the asset is sold rather than on an accrual basis. Thus capital gain income has both deferral and capital-gains benefits. Perhaps even more important, the individual has an option of recognizing this gain at a time when it is most beneficial, i.e., in a year when he has a relatively low taxable income.

Deferral of investment income can also come from depreciation of assets such as real estate. This is treated separately in a later section because of the complex but important interaction of this source of deferral with the individual's wage income.

Deferral of wage income is not as easy as deferral of investment income. Retirement plans such as Individual Retirement Accounts and Keogh plans allow only limited contributions. Furthermore, the individual is penalized for early withdrawal of funds, which reduces the value of an option to withdraw funds early when the individual is in a lower tax bracket or desires the cash.

Tax Losses

One way by which wage income can be deferred and even converted to capital gains treatment is through tax losses on investments such as
real estate, leasing, and other "deep shelters." These losses are
"artificial accounting losses" in the sense that the actual cash
income from the investment is usually positive. The taxable income
for the investment is negative however due to non-cash deductions such
as depreciation and amortization. The way in which these losses can
shelter wage income is discussed in the next section.

**Real Estate Income Property**

Real estate income property has many potential tax benefits.
Depreciation is allowed for tax purposes that generally exceeds
economic depreciation. Taubman and Rasche (1971) found that the
economic depreciation for real estate income property is substantially
less than the straight-line rate, and that the economic life exceeds
that allowed for tax purposes by 15 or more years. The difference
between straight-line depreciation and economic depreciation represents
tax shelter. Most real estate investments also can be depreciated at
a rate which exceeds straight-line depreciation. This so called excess
depreciation can result in additional tax benefits.

The extent of tax shelter from depreciation depends on the amount
of depreciation relative to the income before depreciation deductions.
The depreciation deduction is often sufficient to more than offset the
income from the real estate investment so that the resulting tax loss
can offset other investment income and/or wage income.

The nature of the tax benefit from depreciation ultimately depends
on the sale of the property. The accumulated depreciation deductions
are subtracted from the original cost basis for the property resulting in an adjusted basis. If the selling price of the property exceeds this adjusted basis, the resulting gain is treated as a capital gain. However, the excess of accelerated over straight-line depreciation is "recaptured" as ordinary income.

The net effect of the depreciation deductions and tax on the sale of the property can be broken down as follows:

1. The excess of straight-line depreciation over economic depreciation results in both deferral of income and conversion of income to a capital-gains tax rate.*

2. The excess of accelerated depreciation over straight-line depreciation results only in deferral of income due to recapture as ordinary income.

3. Any increase in price over the original cost basis would result in additional income which is deferred until sale of the property and treated as a capital gain.

The effect of tax losses due to depreciation is therefore actually deferral and/or conversion of income. This income will be income from the project and other investment or wage income. Thus, artificial accounting losses from investments such as real estate income property have the rather unique potential for deferral and conversion of wage

*Reference to a "capital gains tax rate" will in most cases actually mean taxation of the non-excluded portion at the ordinary income tax rate as discussed previously.
income. There is also no limit on the extent to which such tax losses can be used for this purpose. However, there are some additional tax considerations as discussed in the next section.

Costs of Tax Preference

Benefits of tax preference are not without cost. These additional costs are both explicit and implicit. Implicit costs include yield differentials for tax-preferred assets which represent the price of tax preference. This is discussed in Chapter III. Explicit costs include additional taxes, and transactions costs associated with buying tax-preferred assets. The penalty for early withdrawal of funds from a retirement account would be another explicit cost. Transactions costs will be discussed further in Chapter VIII.

Additional taxes from tax-preferred investments come in three ways:

1. Certain items such as excess depreciation for real estate discussed previously are considered "tax-preference items." The sum of all tax-preference items less an exclusion is subject to a minimum tax rate of 15%. The exclusion is the greater of $10,000 or one-half the individual's regular tax liability.

2. The amount of personal-service income subject to a maximum tax rate of 50% is reduced dollar-for-dollar by the amount of a taxpayer's tax preferences for the year.

3. An alternative minimum tax applies to the excluded portion of long-term capital gains and excess adjusted itemized
deductions, reduced by a $20,000 exemption. A tax rate which ranges from 10% to 25% is applied to the base and must be paid if it exceeds the sum of the regular income tax plus the regular minimum tax on tax-preference items.

Thus, the costs of tax avoidance increase with the amount of tax-preferred investments and the deepness of the shelter. It is also clear that investment capital is relatively easy to shelter as compared to personal-service income. An individual can have tax-exempt investment income by purchasing municipal bonds. However, exemption of wage income is very difficult.* Retirement plans provide deferral of limited amounts of wage income. Investments with artificial tax losses such as real estate can defer wage income and even convert this income to a capital-gains tax rate. But excess depreciation which deepens the shelter also has greater costs. Since excess depreciation is a tax-preference item, use of accelerated depreciation as an attempt to shelter personal-service income can: (1) result in the deferred personal-service income being taxed at a rate greater than 50% since the income re-captured at sale of the property is not considered personal-service income, (2) subject other personal-service income to a tax rate higher than 50%, and (3) result in an additional preference tax.

* The growth of the "underground economy" and barter clubs is an indication of increasing attempts to exempt wage income.
Chapter III

PREFERENTIAL TAXATION AND ECONOMIC INCENTIVES UNDER CERTAINTY

Introduction

Chapter II introduced the basic elements of preferential taxation and some of the devices through which tax preference is available. This chapter examines how tax preference can be modeled, and the economic incentives created by differential taxation.

After-tax Return Models

In order to gain more insight into the effect of preferential taxation on portfolio choice and market prices, it is convenient to develop models which express the after-tax return for assets with different amounts of preferential taxation. This chapter concentrates on single tax-preference variable models.

Conversion

In its most general form, differential taxation involves the conversion of income from the investor's current ordinary income tax rate to some other current or future tax rate. We can capture this concept by expressing the after-tax return for an asset as the sum of two components: (1) the after-tax yield that would apply if the proceeds were entirely taxable, and (2) an adjustment for the portion of the yield that is converted to a different tax rate. Thus, we can develop the following expression:
\[ R_j^T = R_j (1-t_o) + \delta_j (t_o - \gamma_n t_n), \]

where

- \( R_j^T \) - after-tax yield on asset \( j \),
- \( R_j \) - before-tax yield on asset \( j \),
- \( \delta_j \) - portion of yield which is converted to a different tax rate,
- \( t_o \) - the investor's current ordinary income tax rate,
- \( t_n \) - the tax rate which applies to the differentially taxed (converted) income.
- \( \gamma_n \) - present value adjustment which applies if the converted income is taxed in a future period.

The rate at which the sheltered or converted income is taxed can be a current or future ordinary income tax rate or a capital-gains tax rate. * Suppose, for example, that income is to be converted to a current capital-gains tax rate \( t_g \). Equation 1 now becomes

\[ R_j^T = R_j (1-t_o) + \delta_j (t_o - t_g). \]

An alternative way of expressing 2 would be

\[ R_j^T = R_j (1-t_g) - \delta_j (t_o - t_g), \]

where

\[ \delta_j^* = R_j - \delta_j. \]

* In most cases this will be 40% of the current or future ordinary income tax rate.
This is the model applied by Brennan (1970) and Long (1977) to the
differential taxation of dividends over capital gains. In their models,
$\delta^j$ would represent the dividend yield which is not converted to the
capital-gains tax rate. Thus, their models are also a special case of
differential taxation. There are many other special cases as discussed
in the following sections.

Deferral

Some investments e.g. single-premium deferred annuities, series E
bonds, and some retirement plans allow the investor to defer taxes, but
the income will eventually be taxed at the investor’s ordinary income
tax rate. If it is assumed that the future tax rate is certain and equal
to the current ordinary income tax rate, equation 1 becomes

$$R_j^T = R_j(1-t_o) + \delta_j t_0(1-\gamma_n).$$

(5)

Thus, the value of deferral in this special case depends entirely on the
length of time until the deferred income is recognized. The more general
conversion model of equation 1 discussed earlier allows the tax rate
which applies to the deferred income to differ from the current ordinary
income tax rate and is therefore less restrictive. It also allows for
the incorporation of tax rate uncertainty where the expected future
tax rate is a random variable even though its expected value may equal
the current ordinary income tax rate. This is discussed further in
chapter VI.

*Uncertainty is introduced into returns in chapter IV and into
future tax rates in chapter VI.
It would also be noted that as the deferral period increases, the benefit of deferral approaches the concept of exemption. This is discussed in the next section.

**Exemption**

Refering to the differential-tax model of equation 1, consider the following two cases:

1. the length of time until the converted income is recognized is quite long so that the present value factor \( \gamma_n \) approaches zero, or

2. the tax rate at which the gain is recognized \( t_n \) approaches zero.

The first case might be true for assets held for retirement income or estate planning. The second case might result from the investor having a very low effective capital-gains tax rate or a low future ordinary income tax rate. Furthermore, there is a multiplicative effect of both the length of deferral time increasing (\( \gamma \) approaching zero) and the future tax rate approaching zero.

For the two cases outlined above, equation 1 becomes

\[
R_j^T = R_j (1-t_0) + \delta_j t_0. \tag{6}
\]

Thus, equation 5 expresses the two special cases of differential taxation as discussed above. In general, equation 5 expresses the concept of exemption. The tax-preference variable \( \delta_j \) represents the portion of the yield which is exempt from taxation. A municipal bond, for example, would be represented by this model. If the bond were
purchased at a discount, only a portion of the yield would be tax-exempt. If the municipal was purchased at par, and/or not sold at a gain, the entire yield would be tax-exempt. This would be the case where $\delta_j = R_j$ in equation 5 so that $R_j^{T} = R_j$.

Another interesting aspect of the model expressed by equation 5 is that it can capture the full range of taxation. As discussed above, when $\delta_j = R_j$, asset $j$ is fully tax exempt. When $\delta_j = 0$, asset $j$ is fully taxable. The range of $\delta_j$ between zero and $R_j$ represents partial taxation. When $\delta_j$ exceeds $R_j$, the asset has losses which offset some of the investor's other taxable income. In this case the after-tax return will exceed the before-tax return. This is the case of deep shelter with accounting losses discussed in Chapter II.

**Two Security Equilibrium Model**

Suppose we have two securities, one which is fully taxable, and one which is tax-exempt. Bailey (1974) discusses how equilibrium would result for the two securities. This is illustrated in Figure 3-1.

![Figure 3-1](image-url)

**Two Security Equilibrium Model**
The after-tax rate of return for the fully taxable security decreases proportionally with the investor's marginal tax rate. That is, 
\[ R_T = R(1-t) \]. The before-tax return is of course the same as the after-tax return for the investor in a zero marginal tax bracket. For the tax-exempt security with no capital gain, the before- and after-tax return are the same to investors in all tax brackets.

If the fully taxable and tax-exempt security had the same pre-tax return clearly the tax-exempt security would dominate the fully taxable security. However, the higher an investor's marginal tax bracket, the more attractive the tax-exempt security is relative to the fully taxable security. The price of the tax-exempt security is thus bid up, driving down the before-tax (and after-tax) return.

In equilibrium, investors in tax brackets above the marginal tax rate \( t_M \) will find the tax-exempt security more attractive whereas investors in tax brackets below \( t_M \) will find the fully taxable security more attractive. Thus investors tend to specialize in either fully taxable or tax-exempt securities depending on their tax bracket. Only the marginal investor whose tax rate is \( t_M \) is indifferent between the two assets.

A closer look at this model shows the importance of constraints on short-selling by investors. Without such constraints, investors could make arbitrage profits by selling short the investment with the lower after-tax return and buying the investment with the higher after-tax return. Individuals would have an infinitely positive demand for the asset with the highest after-tax return and an infinitely negative
(short-sell) demand for the asset with the lower after-tax return. With a progressive tax structure, the income of individuals in lower tax brackets would rise as they bought the more fully taxed asset and sold short the tax-preferred asset. Income of individuals in higher tax brackets would fall as they sold short the more fully taxable asset and bought the more tax-preferred asset. Equilibrium would obtain only when everyone had the same marginal tax rate. Alternative ways of "selling" taxable income and purchasing tax shelter are discussed in later sections of this chapter.

**Multiple Security Model**

In general, there are many securities with differing amounts of taxation. Suppose, for example, in addition to the two securities described above, there is a real estate asset which is partially tax-sheltered. According to Bailey, equilibrium will now be as shown in Figure 3-2. For persons in marginal tax brackets below $t_A$, given by the intersection of the fully taxable investment and the partially tax-sheltered real estate investment return curves, the best available return comes from the fully taxable security. Above $t_B$, determined by the crossing of the partially tax-sheltered and fully taxable yield curves, investors find the tax-exempt security offers the highest return. Between $t_A$ and $t_B$, investors find the partially tax-sheltered security offers the highest after-tax return.

Thus, according to this model, for each asset with a different degree of taxation, there is a marginal tax rate at which investors prefer
this asset because it offers the highest after-tax return. There will be a different clientele for each class of assets that differs by tax preference. Investments with differing amounts of taxation or tax shelter can be thought of as being in different tax-shelter classes. In the model developed in equation 5, the amount of exemption \( \delta_j \) determines a continuum of tax-shelter classes whereas in the above example there are three tax-shelter classes. Different tax-shelter classes can be thought of as being separated by "breakeven marginal tax rates." This is discussed by Galper and Zimmerman (1977).

![Graph showing after-tax return vs. marginal tax rate with three categories: tax-exempt, partially taxed, and fully taxable.](image)

**Equilibrium Model With Breakeven Marginal Tax Rates**

Referring to Figure 3-2, the breakeven marginal tax rate \( t_A \) at which investors are indifferent between the fully taxable and partially taxed assets can be found. Using equation 5,

\[
R_1(1-t_A) = R_2(1-t_A) + d_2 t_A.
\]  

(7)
The marginal-tax rate $t_A$ can now be expressed as

$$t_A = \frac{R_2 - R_1}{R_2 - R_1 - d_2} .$$ (8)

The empirical work by Galper and Zimmerman shows that investors with higher pre-investment marginal tax rates tend to acquire disproportionate shares of industries which receive relatively favorable tax treatment and, in contrast, investors with lower marginal tax rates tend to acquire disproportionate shares of industries which have less favorable tax treatment. This is consistent with the Bailey's model. However, it is also consistent with a model where all assets equate at a single marginal tax rate. Investors in higher tax brackets would purchase assets with favorable tax treatment such as real estate, and through use of debt, leases, etc, trade tax liabilities with lower tax rate investors so that expected post investment marginal tax rates tend to equate. The implications of such an equilibrium are introduced in the next section. The way in which debt, leases, and other market devices might be used to trade tax-shelter claims is then discussed in the remainder of this chapter and in Chapter VII.
The Demand for Tax Shelter

An important consideration in the effect of tax shelter on portfolio choice is the ability of investors to purchase a claim on tax shelter that is separate from the claim on the before-tax cash flow. That is, in order to get a specific type of stochastic before-tax cash flow, must the investor also purchase the tax shelter component(s) of the asset, and vice versa? Or, can tax shelter be purchased and traded separately from the claim on the stochastic before-tax cash-flow component?

In order to examine the investor's demand for tax shelter, this section assumes that an investor has a choice between purchasing two claims. The first claim is on an asset which offers fully taxable cash flow. The second claim is on a quantity of tax shelter.

The investor's after-tax cash flow is expressed as follows:

\[ c^T = X_f p_f - f(x_f r_f - x_D \frac{1}{p_D}), \]  \hspace{1cm} (9)

or

\[ c^T = x_f \frac{1}{p_c} - f(x_f \frac{1}{p_c} - x_D \frac{1}{p_D}), \]  \hspace{1cm} (10)

where

- \( x_f \) - the portion of funds invested in the fully taxable asset;
- \( x_D \) - the portion of funds used to purchase tax shelter;
- \( p_c \) - represents the price paid at the beginning of the period to receive an amount of fully taxable cash income at the end of the period. The reciprocal of \( p_c \) is the rate of return;
\( P_D \) - represents the price for a unit of tax shelter. The tax shelter is assumed to reduce taxable income dollar for dollar;

\( f(*) \) - represents the tax function facing the investor. In this case it is assumed to be progressive. The quantity within the parenthesis is taxable income.

The after-tax cash flow received by the investor is thus determined by the amount of before-tax cash flow \( (X_f/P^T_c) \) and the amount of taxes determined by \( f(X_f/P^T_c - X_D/P_D) \). The investor can increase after-tax cash flow either by purchasing more cash flow (which is taxed) or by reducing taxes by purchasing more tax shelter. In equilibrium, the investor is indifferent between either alternative. Thus we can express equilibrium for the individual as follows:

\[
\frac{\partial C}{\partial X_f} = \frac{\partial C}{\partial X_D}, \quad \text{or}
\]

\[
\frac{(1 - f')}{P^T_c} = \frac{f'}{P_D}.
\]

In equation 12, \( f' \) represents the derivative of taxes with respect to taxable income which is of course the investor's marginal tax rate. The quantity \( (1 - f')/P^T_c \) represents the marginal after-tax benefit from purchasing more taxable cash flow whereas the quantity \( f'/P_D \) represents the marginal tax-reduction benefit from purchasing more tax shelter.

Another way of expressing equation 12 would be

\[
\frac{f'}{1-f'} = \frac{P_D}{P_c}.
\]
The quantity \[ f'(1-f') \] represents the marginal rate of substitution between additional taxable income and tax shelter. In equilibrium, this must equal the ratio of the price of tax shelter to the price of taxable cash flow.

These equilibrium conditions assume the investor has enough funds to achieve the optimal balance between taxable cash flow and tax shelter. Investors with insufficient investable funds might not reach this equilibrium, and would invest only in taxable cash flow. Investors would only purchase tax shelter when additional taxable income would cause their marginal tax rate to rise above the point where \[ f'(1-f') > P_D/P_C. \] Additional funds would then be allocated between taxable cash flow and tax shelter so that the investor's marginal tax rate would not rise any further. This is analogous to the investor purchasing only tax-exempt securities above the breakeven marginal tax rate \( t_m \) in Figure 3-1.

**Yield Differential for Tax-Preferred Assets**

The yield differentials between fully taxable and tax-sheltered assets can be explained in terms of the above model. The value of asset j can be expressed as

\[
V_j = C_j P_c + D_j P_d
\]

(14)

where \( C_j \) is the amount of taxable cash flow and \( D_j \) is the amount of tax shelter. The before-tax return for asset j is expressed as

\[
R_j = \frac{C_j}{V_j}
\]

(15)

Substituting \( V_j \) from (14) into (15),
\[ R_j = \frac{\frac{C_j}{P} + \frac{D_j}{P_D}}{C_j + \frac{D_j}{P_D}} = \frac{1}{P + \frac{D_j}{P_D}}. \]  

(16)

Equation 16 expresses the before-tax return for asset \( j \) as a weighted average of the price of before-tax cash flow and the price of tax shelter. Thus, the extent of yield differential for an asset due to tax preference depends on the price of tax shelter and the relative amount of tax shelter offered by the asset.

Equation 16 can be rewritten as

\[ R_j = \frac{1}{P_c} - D_j \frac{P_D}{P_c} \frac{R_j}{C_j}. \]  

(17)

Since \( \frac{R_j}{C_j} = \frac{1}{V_j} \), equation 17 becomes

\[ R_j = \frac{1}{P_c} - \frac{P_D}{P_c} \frac{D_j}{V_j}. \]  

(18)

The ratio \( D_j/V_j \) expresses the return on tax shelter relative to the total value of asset \( j \). Defining this return as \( d_j \), and using the equilibrium condition from 13, equation 18 becomes

\[ R_j = \frac{1}{P_c} - d_j \frac{t_m}{1-t_m}, \]  

(19)

or

\[ R_j = R_f - d_j \frac{t_m}{1-t_m}. \]  

(20)

Equation 20 can now be written as

\[ R_f (1-t_m) = R_j (1-t_m) + d_j t_m. \]  

(21)

Equation 21 states that at the marginal tax rate \( t_m \), the after-tax rate of return for an asset with fully taxable cash flow equals the after-tax rate of return for the tax-sheltered asset.
This result is illustrated in Figure 3-3.

An interesting difference between this analysis and that of Bailey and of Galper and Zimmerman described earlier is that all assets, regardless of the extent of differential taxation, are equated at the same marginal tax rate. It would appear from the diagram that an asset with partial tax preference would not be purchased since below $t^m$ the fully taxable asset has the highest after-tax return, whereas above $t^m$ the asset with the most tax shelter has the highest after-tax return. However, this analysis assumes investors can somehow purchase only the tax-shelter portion of the asset's return. Conceptually, this could be done by purchasing a partially tax-sheltered asset and either reselling the taxable portion of the cash flow or simultaneously short-selling a...

*Kane (1980) also suggests that arbitrage across assets with different degrees of tax-exemption result in yields for assets being equated at an exogenous effective marginal tax rate.
fully taxable asset with the same amount of taxable cash flow. Thus, this analysis assumes an asset can be decomposed into fully taxable cash flow and tax shelter, whereas Bailey's model assumes assets have fixed and non-separable tax preference components with constraints on short selling.

In this analysis, yield differentials are determined by the amount of tax shelter available from the asset and the price of tax shelter relative to the price of fully taxable cash flow. If the price of tax shelter is zero, there will be no yield differential due to tax preference. All yields would be equated at an effective marginal tax rate of zero, and no investor would have an effective marginal tax rate greater than zero since tax shelter could be freely used to offset taxable income.

**Demand for Multiple Tax-Shelter Components**

The results of the previous section can be generalized to multiple tax-shelter components with different market prices which interact differently with the individual's tax function. For example, assume the following types of income and tax shelter are available to the individual:

1) Ordinary income is taxed according to the function $f(\cdot)$ and has a price $P_f$;

2) Capital gain which is taxed according to the function $g(\cdot)$ and has a price $P_g$;

3) Pure tax shelter which reduces the ordinary income tax liability but which is also subject to a tax-preference tax function $p(\cdot)$ and has a price $P_D$. 
After-tax cash flow is expressed as follows:

\[ C^T = \frac{X_f}{p_f} + \frac{X_g}{p_g} - f \left( \frac{X_f}{p_f} - \frac{X_D}{p_D} \right) - g \left( \frac{X_g}{p_g} - \frac{X_D}{p_D} \right) \]

where \( X_f \) and \( X_g \) represent the amount of funds invested in assets with ordinary income, assets with capital gains, and tax-shelter claims respectively.

The individual now purchases ordinary income, capital gains, and tax shelter until

\[ \frac{1 - f'}{p_f} = \frac{1 - g'}{p_g} = \frac{f' - P'}{p_D} \]

Each term represents a ratio of the marginal after-tax benefit of an additional quantity of the income or tax shelter to its respective market price. These ratios must be equal for the individual to be in equilibrium. In the case of ordinary income and capital gains, the marginal benefit is the increase in after-tax income reflected by the complement of the marginal tax rates. For the pure tax shelter, the after tax benefit is only in reduction of taxes. However, the marginal decrease in ordinary income taxes is reduced by any marginal increase in tax-preference taxes.

This illustration is obviously an oversimplication of the complexities of the tax code. However, it illustrates potential tradeoffs the individual can face in structuring a portfolio with differentially taxed assets and tax-shelter claims.

The results of this section can be generalized to many types of tax preference with different prices. Some types of tax preference might be relatively costless, and used freely by all investors. Other
types of tax preference might be more costly, and used only by investors with a sufficient amount of taxable income. As discussed in Chapter II, there are different types of tax preference, and different devices through which they are available. As the type of shelter deepens, especially when personal service income can be sheltered, the supply of shelter seems more limited. This is discussed further in the remaining chapters.

Equilibrium for Corporate Securities

Miller (1977) extends Bailey's work to equilibrium for a fully taxable asset and a tax-exempt asset. In Miller's analysis corporate bonds are fully taxable and returns for the stock are assumed to be taxed at an effective rate of zero. Investors with non-zero tax rates will therefore require a higher before-tax yield for debt. Firms can deduct the interest on corporate debt at the corporate tax rate. Thus firms are willing to offer debt at a higher before-tax yield. The price of debt and equity in equilibrium will reflect the trade-off between the tax deduction of interest payments by the firm and the taxes paid by individuals.

Equilibrium in Miller's model results when the breakeven marginal tax rate is equal to the corporate tax rate. Since the marginal investor is indifferent between taxables and tax-exempts, the ratio of tax-exempt yields to taxable yields will be the complement of the corporate tax rate \(1-t_c\). This is shown in Figure 3-4.

By adding supply considerations, Miller thus specifies what the breakeven marginal tax rate is which separates fully taxable and tax-exempt assets in a two-asset world under certainty. The before-
Equilibrium for Corporate Securities

tax return for the tax-preferred asset is lower, and investors specialize, as shown by Bailey, Galper and Zimmerman.

In the context of the earlier analysis, investors with marginal tax rates above the corporate tax rate have purchased tax shelter from the firm. The lower before-tax yield for the stock reflects the price investors pay for the tax shelter. The firm supplies tax shelter by packaging its total cash flow (before financing) so that stock is available to investors willing to purchase tax shelter whereas debt is available to investors who are not willing to purchase the tax shelter at the market price. The cost to the firm of selling tax shelter through stock is the loss of the tax deduction at the corporate level for interest payments on debt.

Income to the corporation is normally taxed at the corporate tax rate. Since interest payments on debt are tax deductible, use of debt financing by the firm is, in a sense, a way of selling the taxable portion of the corporate income. Investors who are not willing to pay for tax shelter will be attracted to bonds. Investors who are willing to pay
for tax shelter through lower before-tax yields will be attracted to stock. These same investors might even use personal borrowings to further sell their other taxable income. This is discussed in the next section.

Leverage

Interest paid by individuals on debt is tax deductible. However, there are limitations on this tax deduction. First, interest deductions are limited to $10,000 plus net investment income. Investment income includes dividends, rental income, etc. but does not include capital gains. Interest on personal debt such as a car or personal residence is not covered by this limitation.

Second, individuals can not deduct interest on funds borrowed to purchase tax-exempt securities. The money does not have to be borrowed explicitly to purchase the tax exempt securities. Rather, the IRS looks for a relationship between borrowings for other purposes and the investor's tax-exempt portfolio.*

Third, limited partnerships and most deep tax-shelter investments other than real estate are subject to an "at risk" provision of the tax code which limits tax losses to the amount of funds an investor has "at risk." This includes equity but not non-recourse loans. Thus an investor with little equity in an investment might lose the tax-shelter benefits.

Real estate has two rather unique tax benefits with respect to leverage. First, real estate is specifically excluded from the "at risk" limitation. Second, rental real estate is generally considered

*For a recent case see CCH Federal Tax Guide Reports, No. 28, April 11, 1980, p. 8494.
property held in one's trade or business rather than investment property. It is therefore not subject to the interest-deduction limitation.

As discussed in the previous section, debt financing can be used by corporations to, in effect, sell taxable income. Individuals can also use debt to sell taxable income. The borrowings could then be used to purchase tax-preferred assets. If there is a yield differential for tax-preferred assets reflecting a positive price for tax shelter, then we would expect only investors above a certain marginal tax rate to sell taxable income and buy tax-preferred assets. It is possible that the price of certain types of tax shelter is close to zero so that most investors can benefit from some use of leverage, at least until constrained by the interest deduction limitation or some other constraint.

Miller and Scholes (1978) discuss the use of debt to "launder" the tax on dividend income. The additional borrowings are used to purchase either tax-deferred annuities or additional stock. The amount of debt can, in either case, be set so that the dividend income is exactly offset by interest deductions. The strategy outlined by Miller and Scholes can be interpreted as "selling" taxable dividend income, keeping the tax-preferred capital gains, and purchasing additional tax-preferred income. If there are no yield differentials between taxable and tax-preferred assets, this implies that tax shelter is costless (zero price) and the investor completely launders taxes. If there are yield differentials, then the strategy allows higher tax
rate investors to reduce taxable income and marginal tax rates until their marginal tax rate equals the effective price of tax shelter as discussed earlier.

This analysis can be related to the earlier discussion of equilibrium for tax-exempt and fully taxable assets. In Miller's model, stock was assumed to be effectively tax-exempt and debt was fully taxable. As shown in Figure 3-4, investors above a marginal tax rate equal to the corporate tax rate \( t_C \) would specialize in tax-exempts. These higher-tax-rate investors would not purchase debt. Taking this one step further, these same investors would want to issue debt by borrowing at the tax deductible interest rate. As long as the investor has other taxable income, use of debt is a way of selling taxable income. Furthermore, if it were not for the interest deduction constraints discussed earlier, the investor would continue to issue debt and buy tax-exempt securities until the investor had sold off all taxable income including personal service income.

This process is analogous to purchasing pure tax shelter as discussed previously. As the investor purchases tax shelter, his marginal tax rate will fall. Tax shelter is purchased until the investor's marginal tax rate is equal to the effective price of tax shelter. In Miller's model, the price of tax shelter is the complement of the ratio of the tax-exempt yield to the fully taxable yield, which is equal to the corporate tax rate. Thus no investor would have a marginal tax rate above the corporate tax rate.
Arbitrage of Tax-exempt Securities

The limitation on interest deductions for individuals borrowing to purchase tax-exempt securities was mentioned in the previous section. This limits an individual's ability to sell taxable income in order to purchase tax-exempt assets.* Individuals would therefore seek other vehicles such as real estate through which to purchase tax shelter.

Unlike individuals, commercial banks can issue tax-deductible debt and use the proceeds to purchase tax-exempt securities. Commercial banks might therefore dominate the municipal bond market. Fama (1977) argues that banks can earn arbitrage profits if the ratio of the tax-exempt municipal yield to the fully taxable (fully tax deductible) debt yield is not equal to the complement of the banks marginal tax rate. This is no different from individuals selling taxable income to purchase tax shelter. It suggests that banks may be more efficient in performing this action with respect to municipal bonds. A bank would use debt to sell taxable income until the relative benefit of adding tax shelter is equal to the relative cost of purchasing tax shelter. Thus the bank's marginal tax rate will tend to equate with the price of tax shelter implicit in the yield differential for tax-exempt securities. Banks also purchase other forms of tax shelter e.g. through leasing which is discussed in Chapter VII. Banks are prohibited from purchasing other types of tax shelter e.g. through real estate leasing.

*Although individuals can to some extent circumvent this constraint by borrowing more on other assets, this is not without the costs associated with greater audit risk.
In an efficient market, substitutes for municipal bonds with the same amount of tax preference and risk characteristics should sell for the same price. Thus we would expect a relationship between the price for the tax-shelter characteristics of tax-exempt securities, reflected by their yield differential, and the price for a similar amount of tax shelter offered through close substitutes. These substitutes, such as real estate, are not dominated by commercial banks. An important question is the extent to which various types of tax shelter are substitutes. This issue is discussed further in the next section and the remaining chapters of this study.

**Yield Differentials for Conversion**

As discussed in Chapter 2, 60% of a long-term capital gain is excluded from taxation, and the remainder is taxed at the ordinary income tax rate. Thus, in most cases, long-term capital gains consists of two portions: (1) a fully taxable portion which consists of 40% of the total gain, and (2) a tax-exempt portion consisting of 60% of the total gain. With this treatment of capital gains, under certain conditions a close substitute for capital gains can be created by the purchase of a fully taxable asset with a before-tax return \( R_f \) and a tax-exempt asset with a return \( R_o \). Assume there is only one type of tax preference that is priced, and all capital gains are taxed as discussed above. The proportion of funds to be placed in the fully taxable asset \( X_f \) would be

\[
X_f = \frac{.4}{\frac{.4}{R_f} + \frac{.6}{R_o}}.
\]

(24)
The before-tax yield on this portfolio would be

\[ R_c = \frac{1}{R_f + \frac{.6}{R_o}} \tag{25} \]

For example, assume a fully taxable asset has a before-tax return of 10% and a tax-exempt asset has a return of 5%. The proportion to be invested in the fully taxable asset is

\[ X_f = \frac{.4}{.10} = \frac{.4}{.10 + .05} = 25\%. \]

The remaining 75% would be placed in the tax-exempt asset.

If $1,000 is invested in this portfolio, 25% or $250 would result in a taxable return of $25. The other $750 in the portfolio would give a tax-exempt return of $37.50.

If the same $1,000 is placed directly in an asset with income taxed as a capital gain, the yield on this asset would be

\[ R_o = \frac{1}{.4 + \frac{.6}{.05}} = 6.25\%. \]

The $1,000 investment would result in a capital gain income of $62.50. Of this amount, 40% or $25 would be fully taxable and 60% of $37.50 would be excluded and therefore tax exempt.

According to this analysis, investors would appear to be indifferent between a capital gain, and a combination of a fully taxable and

*This relationship is linear in prices. That is, \( \frac{1}{R_o} = .4 \frac{1}{R_f} + .6 \frac{1}{R_f} \). Recall that the reciprocal of the yield is a price.
tax-exempt asset. Each alternative is taxed exactly the same way, regardless of the investor's tax rate. However, there are several reasons why this relationship might not hold. This analysis assumes all investors view capital gains as simply ordinary income with a fixed exclusion. This allows capital gains to be expressed in terms of a fully taxable and tax exempt asset independent of the investor's ordinary income tax rate, or other taxable income. However, capital gains can be treated differently by investors for two reasons: (1) an investor with a lot of capital gains can be affected by the alternative minimum tax as discussed in Chapter II, and (2) net long-term capital gains (losses) must be netted against net short-term capital losses (gains). Since this netting occurs before the exclusion, it can reduce or eliminate the net benefit of the long term capital gains exclusion.

Another key assumption in the above analysis is that there is only one type of tax preference: exemption. This allows yields to be expressed as a linear relationship in the amount of tax shelter. Conversion and exemption may in fact contain two different types of tax preference. For example, the yield differential between full taxation and conversion to capital gains may be small due to the nature of the supply of conversion, whereas the yield differential for assets with exemption as compared with those with conversion might be much greater. That is, the yield differential for tax-exempt assets might reflect mainly the incremental benefit of exemption over conversion.

It should also be noted that conversion usually involves an asset where the gain does not have to be recognized until the asset is sold.
Thus conversion usually includes a deferral element which might also be a distinct type of tax preference with a non-zero market price.

Finally, there are two distinct types of income which can be converted to capital-gains treatment as discussed in Chapter II. A combination of a fully taxable and tax-exempt asset might produce investment income similar in yield and taxation to all investors for investment income. However, investments with tax losses such as real estate can result in conversion of personal service income. This latter form of conversion may be much more valuable than the former.

**Differential Taxation of Dividends and Capital Gains**

An issue in the literature is the extent to which stocks which provide returns in the form of dividends would be priced differently from a stock which provided capital gains. There are several factors from the previous discussion which provide some insight into this question.

As pointed out in Chapter II, an amount of dividend income received by the individual is completely tax exempt.* Beyond this exemption, dividends are included in taxable income. A portion of dividend income is therefore taxed more favorably than capital gains whereas a portion is taxed less favorably. Thus it is not clear that corporations would need to offer a differential yield to attract investors to dividend yields. Furthermore, firms have no tax-related incentive to offer dividends as opposed to capital gains, as was the case.

*Corporations also receive a dividend exclusion of 85% of dividend income.
case for debt discussed earlier. Thus, it is quite conceivable that there is no differential yield for stocks which differ only in the amount of dividend yield.

It is interesting that if dividends are not priced as taxable income, then it would be inefficient to use interest to sell dividend income, as discussed earlier, if the yield on debt does reflect the full taxation (and deduction) of interest. However, debt could still be used to sell other forms of taxable income with yields which reflect full taxation. In fact, the most important role of debt may be in allowing higher-tax-rate investors to sell personal service income. Levering real estate, for example, increases potential tax losses which, as discussed earlier, can shelter other investment and wage income.

Conclusion

This chapter examined alternative models of the demand for and price of tax shelter and tax-preferred assets. Many factors were identified which have a bearing on the price of tax preference and the number of distinct types of tax preference. A key factor in the pricing of tax-preferred assets and the affect of tax preference on portfolio choice is the extent to which the tax-shelter claim can be separated from a claim on before-tax cash flow for tax-preferred assets. Incentives clearly exist for investors to trade tax-shelter claims and cash-flow claims. Thus there are also incentives for devices through which tax shelter can be sold and market intermediaries to aid in the repackaging and sale of these claims. The next 3 chapters examine the
effect of risk on the preceding analysis. Chapter VII discusses some of the devices and intermediaries through which tax-shelter claims can be traded.
Chapter IV
PREFERENTIAL TAXATION WITH UNCERTAIN CASH FLOW

Introduction

The nature of preferential taxation under certainty was examined in Chapter III. In that analysis, it was assumed that individuals maximize the expected return on their portfolios. If assets have relatively fixed tax-preference components, and short sales are restricted, individuals choose those assets with the highest after-tax return. That is, to purchase tax shelter the investor must purchase the asset with the desired amount of tax preference. The nature of yield differentials will also reflect this assumption. If, on the other hand, tax shelter can be traded separately from the taxable cash flow component of an assets return, the nature of portfolio choice and yield differentials will be quite different. Portfolios still differ as investors with different pre-investment marginal tax rates essentially purchase or sell tax shelter through short-selling, use of debt, and choosing the desired cash flow and tax preference claims offered through market intermediaries. In this case there will be an equilibrium marginal tax rate which reflects the market price of tax shelter. The yields on all assets will be equated at this marginal tax rate. An asset's price reflects the relative market price of tax shelter and the asset's relative amount of tax shelter.
The analysis in Chapter III did not consider uncertainty in either the asset's before-tax cash flow or the tax-preference benefit. This chapter introduces uncertainty into the cash-flow stream available from an asset. Chapter IV and V considers uncertain tax benefits.

Brennan (1970) analyzed the differential taxation of dividends and capital gains. In Brennan's model, the tax-preference variable was the dividend yield. Because dividends result in part of the stock's return being taxed at the ordinary income tax rate instead of the capital-gains tax rate, higher dividend yields result in a non-stochastic portion of the return being converted from the capital-gains tax rate to the ordinary income tax rate. Thus, in Brennan's model, a greater dividend yield results in less tax preference.

Brennan's results show that higher dividend-yield stocks would have greater expected before-tax returns due to the unfavorable taxation of dividends relative to capital gains. As discussed in Chapter III, it may not be appropriate to consider dividends as taxed unfavorably relative to capital gains, and that differential dividend yields may not affect relative prices. However, other elements of preferential taxation could affect the expected before-tax return and price of an asset.

Another implication of Brennan's model is that portfolio choice is affected by an individual's marginal tax rate. Long (1977) also showed that investors in different tax brackets see different portfolios as after-tax mean-variance efficient. Whereas his model was also developed
for the differential taxation of dividends vs. capital gains, we would expect his results to hold in a more general analysis of tax-preferred assets.

This chapter develops an after-tax version of the capital asset pricing model. The model uses a single tax-preference variable to capture the effect of perferential taxation. The model is developed under the assumption that asset's have a fixed tax-shelter component. The effect of dropping this assumption is then evaluated.

**Simplifying Assumptions**

The following simplifying assumptions are made in the development of the model in this chapter:

1. Investors have a single-period investment horizon.
2. Investors are concerned only with the mean and variance of their after-tax portfolio returns.
3. The amount of tax preference for the investment is known with certainty.
4. The investor's average and marginal tax rate is exogenously determined.
5. Risk-free borrowing and lending opportunities are available to investors at an exogenously determined rate. Any return from lending is subject to tax at the investor's marginal tax rate. Interest costs on borrowing are tax deductible.
6. The relative amount of tax preference available from each asset is fixed in proportion to the asset's cash flow.
**Notation**

- $w_i^i$ - the amount of the $i^{th}$ investor's initial wealth;
- $W_0 = \frac{1}{m} \sum_{i=1}^{m} w_i^i$ - the aggregate initial wealth of all investors in the market;
- $\bar{W}_i$ - the $i^{th}$ investor's random end-of-period wealth after taxes;
- $x_i^j$ - the amount of the $i^{th}$ investor's holding of the $j^{th}$ risky asset, where $j = 1, \ldots, n$ and $i = 1, \ldots, m$;
- $y_i^i$ - the amount of the $i^{th}$ investor's lending ($y_i^i > 0$) or borrowing ($y_i^i < 0$) at the risk-free nominal rate of interest;
- $r_f$ - the risk-free rate of interest;
- $x_i^f$ - the $i^{th}$ investor's after-tax borrowing (lending) rate;
- $\bar{r}_j$ - the random total before-tax nominal rate of return on the $j^{th}$ risky asset; with homogeneous expectations, the mean $\bar{r}_j$ and variance $\text{Var}(\bar{r}_j)$ are the same for all investors;
- $\bar{r}_i^j$ - the after-tax value of $\bar{r}_j$ to the $i^{th}$ investor;
- $d_j$ - the amount of return for asset $j$ that is tax-sheltered. If $d_j = 0$ the asset's return is completely taxable. If $d_j = \bar{r}_j$ the expected return is completely tax-sheltered;
- $t_i^i$ - the $i^{th}$ investor's average and marginal tax rate on taxable income from assets owned.

**Individual Equilibrium**

Using the model developed in equation 5 of Chapter 3, security's after-tax yield to any investor can be expressed as the sum of two components:
(1) the after-tax yield that would apply if the proceeds were entirely taxable and

(2) a credit for the portion of the yield that is tax-exempt:

\[ R^i_j = R^i_j (1-t^i) + d^i_t. \]  

(1)

The \( i \)th investor's real end-of-period wealth after taxes becomes

\[ \tilde{W}^i_1 = \sum_j x^i_j (1+\tilde{R}^i_j) + Y_1 (1+R^i_f), \]

(2)

where

\[ R^i_f = R^i_f (1-t^i). \]

(3)

Substituting for \( \tilde{R}^i_j \) from (1) and using the budget constraint that

\[ Y^i = \tilde{W}^i_0 - \sum_j x^i_j \]

we have

\[ \tilde{W}^i_1 = \sum_j x^i_j [(\tilde{R}^i_j - R^i_f) (1-t^i) + d^i_t] + \tilde{W}^i_0 [1 + R^i_f (1-t^i)]. \]

(4)

Taking the expected value and variance of these expressions for each investor establishes his or her mean-variance opportunities:

\[ E^i(\tilde{W}^i_1) = \sum_j x^i_j [(\tilde{R}^i_j - R^i_f) (1-t^i) + d^i_t] + \tilde{W}^i_0 [1 + R^i_f (1-t^i)], \]

(5)

\[ V^i(\tilde{W}^i_1) = (1-t^i)^2 \sum_j \sum_k x^i_j x^i_k \text{Cov}(\tilde{R}^i_j, \tilde{R}^i_k). \]

(6)

For the mean-variance investor the preference function is defined completely in terms of the first two moments in \( \tilde{W}^i_1 \). Each investor acts to maximize \( U(E^i, V^i) \) and first-order conditions are

\[ \frac{\partial U}{\partial E} \frac{\partial E^i}{\partial x^i_j} + \frac{\partial U}{\partial V} \frac{\partial V^i}{\partial x^i_j} = 0. \]

(7)
From equations 5, 6, and 7, we have

\[
\frac{\partial U}{\partial \theta} \left[ (\bar{R}_j - R_p)(1-t^i) + d_j t^i \right] + \frac{\partial U}{\partial V} \left[ 2(1-t^i)^2 \sum_k X_k^i \operatorname{Cov}(\bar{R}_j, \bar{R}_k) \right] = 0 .
\] (8)

We can rewrite equation 8 as

\[
\theta_i \left[ (\bar{R}_j - R_p)(1-t^i) + d_j t^i \right] = (1-t^i)^2 \sum_k X_k^i \operatorname{Cov}(\bar{R}_j, \bar{R}_k),
\] (9)

where

\[
\theta_i = -1/2 \frac{\partial U}{\partial \theta} / \frac{\partial U}{\partial V} = -1/2 \frac{\partial V}{\partial \theta}
\]

is the investor's marginal rate of substitution between mean and variance of end of period wealth. It is a measure of the investor's "audacity" or willingness to accept additional risk in exchange for a promised increase in portfolio return. The reciprocal of \( \theta_i \) is the individual's price for risk-bearing services.

Simplifying equation 9 we obtain

\[
\frac{\theta_i}{1-t^i} (\bar{R}_j - R_p) = \sum_k X_k^i \operatorname{Cov}(\bar{R}_j, \bar{R}_k) - \frac{\theta_i t^i}{(1-t^i)^2} d_j .
\] (10)

For a world with differential taxes, individual demand functions for a risky asset depend on the covariances with returns on other assets and the amount of tax shelter.

Equation 10 can be rewritten as

\[
\sum_k X_k^i \operatorname{Cov}(\bar{R}_j, \bar{R}_k) = \frac{\theta_i}{1-t^i} \left\{ (\bar{R}_j - R_p) + \frac{t_i}{1-t^i} d_j \right\} .
\] (11)

* The term "audacity" is due to Bierwag and Grove (1965). See also Chen and Kane (1978).
Writing the \( n \) simultaneous equations specified by (11) for each asset
in matrix form and solving for the individual's demand for each asset
we have:

\[
\begin{bmatrix}
X^*_1 \\
X^*_2 \\
\vdots \\
X^*_n
\end{bmatrix} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\
\vdots & \ddots & \ddots & \vdots \\
\sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
(R_1 - R_f) + d_1 \frac{t^i}{1-t^i} \\
\vdots \\
(R_n - R_f) + d_n \frac{t^i}{1-t^i}
\end{bmatrix} \frac{\theta^i}{1-t^i}.
\]

(12)

Note that if the individual's tax rates were all zero equation (12)
becomes

\[
\begin{bmatrix}
X^*_1 \\
X^*_2 \\
\vdots \\
X^*_n
\end{bmatrix} = \begin{bmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1n} \\
\vdots & \ddots & \ddots & \vdots \\
\sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
R_1 - R_f \\
\vdots \\
R_n - R_f
\end{bmatrix} \theta^i,
\]

(13)

which is the demand equation for the traditional CAPM. All individuals
now hold the same portfolio of risky assets scaled by their audacity \( \theta^i \).

We see from (12) that the individual's tax rate affects the relative
demands for each asset as well as his total demand for risky assets.
This is because the individual's tax rate affects the relative value of
the tax-preference return to total before-tax return for an asset. Thus
all individuals do not hold the same relative proportions of each asset
in their portfolio as implied by the traditional CAPM. High-tax-rate

*This assumes the variance-covariance matrix is non-singular.
investors hold proportionally more of the tax-sheltered security in their portfolio. Due to diversification benefits, both high- and low-tax-rate investors may hold positive quantities of assets with different amounts of tax preference. Whereas more specialization might increase expected after-tax portfolio return loss of diversification benefits could increase after-tax portfolio variance more than enough to offset the higher expected return.

Before-tax prices adjust in the market for a given supply of risky assets and supply of "tax shelter". The individual, however, takes expected excess returns as given and adjusts the weights of the assets in his portfolio to equate the after-tax expected excess return with his required after-tax risk premium. This can also be seen by writing equation 9 as

\[ \frac{\bar{R}_j (1-t^i)}{d_j} + \frac{(1-t^i)^2}{\bar{g}_i^j} \sum_k x_k \text{ Cov}(\bar{R}_j, \bar{R}_k) = R_f (1-t^i). \]

Individuals are in equilibrium when the after-tax risk-adjusted return is equal to the after-tax risk-free rate for all assets in their portfolio. Individuals in different tax brackets expect different after-tax returns, however, and hence the composition of their portfolios differ.

**Market Equilibrium**

By aggregating expression 10 over all investors and imposing market clearance we obtain the market equilibrium relationships between risk
and return for each risky asset:

$$\left( \bar{R}_j - R_f \right) \sum_i \frac{\theta_i}{1-t_i} = V_m \text{Cov}(\tilde{R}_j, \tilde{R}_m) - d_j \sum_i \frac{\theta_i t_i}{(1-t_i)^2}. \quad (14)$$

Equation 14 can be written as:

$$\bar{R}_j = R_f + \lambda_1 \text{Cov}(\tilde{R}_j, \tilde{R}_m) - \lambda_2 d_j.$$

The $\lambda$ parameters are interpreted as follows:

$$\lambda_1 = V_m \left[ \sum_i \frac{\theta_i}{1-t_i} \right]^{-1}$$

is the market's aggregate tax-adjusted price for market volatility risk;

$$\lambda_2 = \left[ \sum_i \frac{\theta_i t_i}{(1-t_i)^2} \right] \left[ \sum_i \frac{\theta_i}{(1-t_i)} \right]^{-1}$$

is the market's weighted-average before-tax yield-adjustment factor for securities that offer a tax advantage over fully taxable securities.

Thus the before-tax yield for asset $j$ is a linear function of two factors: (1) systematic risk and (2) tax preference. Securities with higher systematic risk will have higher expected returns. Securities with greater tax preference will have lower before-tax returns since high-tax-rate investors bid up the prices of tax-preferred investments.

When $t_i = 0$ for all investors, equation 2 becomes $\bar{R}_j = R_f + \lambda \text{Cov}(\tilde{R}_j, \tilde{R}_m)$ which is the traditional CAPM. The traditional CAPM would, of course, be valid if no securities offered tax preference, i.e., $d_j = 0$ for all securities, or if the market adjustment for tax shelter, $\lambda_2$ is zero. The size of $\lambda_2$ is an empirical question. In the next section, it is further shown that with homogeneous tax rates, the traditional CAPM holds as long as $\tilde{R}_j$ and $\tilde{R}_m$ are interpreted as after-tax returns.
Homogenous Tax Rates

If all investors are assumed to have the same tax rate, equation 14 becomes

$$\left( \tilde{R}_j - R_f \right) \frac{1}{1-t} \sum_i \theta_i = w_o \text{Cov}(\tilde{R}_j, \tilde{R}_m) - d_j \frac{t}{(1-t)^2} \sum_i \theta_i . \quad (16)$$

Multiplying both sides of equation 16 by \((1-t)^2\) and simplifying, we obtain

$$\left( \tilde{R}_j - R_f \right)(1-t) + d_j t = \lambda \left[ \text{Cov}(\tilde{R}_j(1-t), \tilde{R}_m(1-t)) \right] \quad (17)$$

where \(\lambda\) can be expressed as

$$\lambda = \frac{\left( \tilde{R}_m - R_f \right)(1-t) + d_j t}{\text{Var}[R_m(1-t)]} . \quad (18)$$

Expressions 17 and 18 simply represent the traditional CAPM stated in terms of after-tax cash flow. If the effective tax rate for all market participants is virtually the same but non-zero then the traditional CAPM applies if all cash flows are adjusted to be after tax.

Cash-Flow CAPM

Equation 15 can be stated in terms of cash flows instead of returns by using the following definitions:

$$\tilde{R}_j = \frac{\tilde{C}_j}{V_j} - 1 \quad \tilde{R}_m = \frac{\tilde{C}_m}{V_m} - 1 \quad d_j = \frac{D_j}{V_j} - 1 .$$

Substituting these definitions for returns into equation 15 results in the following cash flow model:

$$V_j = \frac{\tilde{C}_j - \lambda_1 \text{COV}(\tilde{C}_j, \tilde{R}_m) + \lambda_2 D_j}{1 + R_f} , \quad (19)$$
or

\[ V_j = \frac{C_j - \frac{\lambda_1}{V_m} \text{COV}(\tilde{C}_j, \tilde{C}_m) + \lambda_2 V_j}{1 + R_f} \]  

Equations 19 and 20 give the present value of a risky asset as the certainty equivalent of the expected before-tax cash flow adjusted for tax preference and discounted at the risk-free rate.

**Market Price of Risk-Bearing Services**

For the traditional CAPM, the market price of risk can be stated in terms of the excess return and variance of the market portfolio. In equilibrium, all assets plot on a line which can be specified in terms of the risk-free rate and the market portfolio. This section examines how the after-tax CAPM would be described relative to similar market portfolios.

Since equation 15 applies to all risky assets, it applies as well to the return for a portfolio composed entirely of all fully taxable securities, of tax-exempt securities, and the market portfolio. Any two of the above reference portfolios can be used to establish the \( \lambda \)'s in terms of market parameters.
Substituting the market portfolio into equation 15 and solving for \( \lambda_1 \) we have

\[
\lambda_1 = \frac{\overline{R}_m - R_f + \lambda_2 d_m}{(\sigma_m)^2}. \tag{21}
\]

Using this expression for \( \lambda_1 \) in equation 15 we have

\[
\overline{R}_j = R_f + [(\overline{R}_m - R_f) + \lambda_2 d_m] B_j^1 - \lambda_2 d_m B_j^2, \tag{22}
\]

where \( B_j^1 = \text{COV}(\overline{R}_j, \overline{R}_m)/(\sigma_m)^2 \) is the traditional beta for the single-beta CAPM. \( B_j^2 = d_j/d_m \) is the tax-sheltered return for security \( j \) relative to the tax-sheltered return for the market portfolio.

We can find \( \lambda_2 \) by applying equation 15 to the portfolio of fully taxable securities to obtain

\[
\lambda_1 = \frac{\overline{R}_T - R_f}{\sigma_{TM}}, \tag{23}
\]

where \( \overline{R}_T \) is the expected return on a portfolio of all fully taxable securities in the market and \( \sigma_{TM} \) is the covariance between this portfolio and the market portfolio. Equations 21 and 22 can now be used to solve for \( \lambda_2 \). We have

\[
\lambda_2 = \frac{(\sigma_m)^2}{\sigma_{TM}} \left[ \frac{\overline{R}_T - R_f}{\sigma_{TM}} - (\overline{R}_m - R_f) \right] d_m. \tag{24}
\]

Using the above equations for \( \lambda_1 \) and \( \lambda_2 \) we can express equation 15 as

\[
\overline{R}_j = R_f + (\overline{R}_T - R_f) \frac{(\sigma_m)^2}{\sigma_{TM}} B_j^1 - [(\overline{R}_T - R_f) \frac{(\sigma_m)^2}{\sigma_{TM}} - (\overline{R}_m - R_f)] B_j^2. \tag{25}
\]
Equation 25 provides a testable alternative to the single-beta CAPM. If the degree of tax-preference in a security is relevant to its pricing in the capital market, then specification 25 should result in a more reliable model than the single-beta CAPM.

A broader testing of the CAPM is implied by these results. The full spectrum of asset with respect to tax preference should be included. Such a test might find differences in taxation affect prices even though tests applied to the differential taxation of dividends and capital gains were inconclusive. Differences in the taxation of dividends as against capital gains represent only a small cross section of tax-preferred assets. Also, the effective differential taxation of dividends and capital gains might be less than thought for the reasons discussed in Chapter III.

Another interpretation of the above results is that at least two funds in addition to the risk-free rate are required to span the market. This follows from the fact that assets will fall on a tax-impacted security-market plane as opposed to a security-market line. Two possible reference portfolios are a portfolio of fully taxable assets and a portfolio of tax-exempt assets. Since the market portfolio contains all assets, it can substitute for one of the above reference portfolios.

Evaluating Alternative Assets

For the single tax-preference-variable model developed in this chapter, equilibrium expected before-tax returns for risky assets can be expressed as linear combination of (2) the covariance of the asset's before-tax returns on the "market portfolio" and (b) teh amount of
before-tax return which is sheltered from taxation at the investor's ordinary income tax rate. Thus, according to this model, assets in equilibrium will fall on a tax-impacted security-market plane, as shown in Figure 4-1, and cannot be compared on the basis of systematic risk alone. Only when all assets equate at the same breakeven marginal tax rate does the plane collapse to an after-tax security market line. Conditions under which this might occur are examined in the next section.

![Diagram](image)

**Figure 4-1**
**Security Market Plane**

The security market plane in Figure 4-1 can be contrasted with the security market line by taking slices through the plane for different amounts of tax preference. This is shown in Figure 4-2.
A fully taxable asset lies on the line where \( d=0 \). It should be noted that this is not the before-tax security market line of the traditional CAPM. The market price or risk and consequently the slope of the after-tax SML loci differ from the traditional CAPM. Since the market portfolio contains all securities including those with tax shelter \( R_m \), it does not plot on the \( d=0 \) line. Thus, the slope is less than \((R_m - R_f)/\sigma_m^2\) as discussed in the previous section.

Assets with increasing amounts of tax shelter \( (d > 0) \) plot along a line parallel to the \( d=0 \) line as shown in the diagram. It should be obvious that returns cannot be compared without considering the tax-preference component. For example, consider assets A, B, and C in Figure 4.3. Assets A and C would both be at equilibrium prices if their
tax-shelter components were zero and \( d' \) respectively, even though asset A has a higher expected return than C for the same amount of systematic risk. According to the traditional CAPM, however, asset A would appear underpriced, and asset C would appear overpriced. Asset B plots on the traditional CAPM. Yet, whether its return is an equilibrium expected return would depend on its tax-preference component.

![Figure 4-3: Comparing Alternative Assets](image)

**Supply Considerations**

Two important assumptions regarding the nature of the supply of assets was made for the model developed in this chapter. First, it was assumed that two assets with different amounts of tax shelter did not have identical stochastic cash-flow components. Second, it was assumed that each asset has a fixed proportion of tax shelter relative to the stochastic
before-tax cash flow. If the first assumption does not hold, then a long and a short position in the two assets would effectively separate the tax-shelter component. Similarly, if one security was risky debt, then an individual could also separate tax shelter by borrowing with the risky debt, using the proceeds to purchase the second asset. This is analogous to the use of debt discussed in Chapter III. Tax shelter might also be separated through other devices such as leases, or purchased from intermediaries such as limited partnerships. These devices and intermediaries are discussed further in Chapter VII.

Individuals purchase assets for both diversification benefits and tax-shelter benefits. If tax preference cannot be separated from the asset's stochastic cash flow, or purchased separately in the market, then investors are forced to make a tradeoff between diversification and tax preference.

If it is possible that an asset's return can be decomposed into a before-tax cash flow component and a tax-shelter component, investors will gain by either trading tax shelter or purchasing it as was the case for certainty discussed in Chapter III. For example, assume we have two investors with different effective tax rates, $t^1$ and $t^2$, where $t^1$ is less than $t^2$. Suppose investor 1 sells the tax-preference component of asset $j$, $D_j$, to investor 2 at a price $P_j$. Investor 1's wealth will increase by $P_j - D_j t^1$. Investor 2's wealth will increase by $D_j t^2 - P_j$. Thus both investors gain as long as

$$D_j t^1 < P_j < D_j t^2,$$
or

\[ t^1 < \frac{P_i}{P_j} < t^2. \]

In Chapter III it was shown that individuals are in equilibrium when

\[ \frac{t^i}{1-t^i} = \frac{P_D}{P_C}. \]

In this case \( \frac{P_D}{P_C} \) is reflected in \( \lambda_2 \), the market price of risk. This chapter developed \( P_D/P_C \) as an aggregation of \( t^i/1-t^i \) and audacities as discussed previously. The point is that investors would sell tax shelter if \( t^i/1-t^i < \lambda_2 \) and buy tax shelter if \( t^i/1-t^i > \lambda_2 \), where \( \lambda_2 \) represents the relative price of tax shelter, however determined.

Because the value of tax preference increases directly with the investor's marginal tax rate, both investors increase expected wealth by this trade. Furthermore, if it is assumed that the tax-preference component is nonstochastic, the variance of the investor's portfolio is unaffected. Clearly then there are incentives for tax shelter to be traded in the market. In fact, if assets are repackaged by market intermediaries, then it will not be necessary for trading among investors to take place. Individuals will purchase the desired amount of tax shelter as discussed in Chapter III. The effect of uncertainty in asset cash flows will still be to cause investors to diversify. But the diversification decision can be made separately from the decision to purchase tax shelter. As
discussed in Chapter III, when tax shelter can be purchased, investors will tend to have the same marginal tax rate determined by the market price of tax preference. If, in fact, all market participants achieve the same marginal tax rate, then, as discussed earlier, the traditional CAPM will hold with all returns calculated at this marginal tax rate. Before-tax yield differentials will still exist for assets with different amounts of taxation since it is the after-tax yields which are equated. Of course, if the price of tax shelter is virtually zero, then returns will be equated at a zero marginal tax rate and there would not be any before-tax yield differentials due to differential taxation.

Conclusion

The effect of tax preference and cash-flow uncertainty on an individual's portfolio choice and the relative prices of assets with different amounts of tax shelter depends on the nature of the supply of tax shelter. If the benefit of the tax preference can be obtained only by purchasing the asset, then one of the costs of tax avoidance is loss of diversification benefits. If assets which have tax preference can be costlessly decomposed by individuals or intermediaries into cash flow and tax-shelter claims, then the effect of tax preference is primarily to reduce the marginal tax rates of individuals to that determined by the market price of tax shelter. Individuals with different pre-investment tax rates will hold different claims on tax shelter, but diversification can be achieved independently from the decision to purchase tax shelter. As individuals tend toward the same marginal tax
rate (after purchasing or trading tax shelter) they will tend to hold identical claims on the stochastic cash-flow component of an asset's return and the traditional CAPM will hold on an after-tax basis.

The results of the uncertainty analysis of this chapter are analogous to that of certainty in Chapter III. Under certainty, if there are constraints on short selling, and tax-preference and cash-flow claims can not be separated, then a continuum of breakeven marginal tax rates separate assets with different amounts of tax shelter and investors in different tax brackets hold different assets. Under uncertainty, if tax shelter can not be purchased separately or effectively separated by short selling, use of debt, or other portfolio adjustments, then investors in different tax brackets hold different portfolios of risky assets and there is again no single tax rate at which all assets equate on an after-tax basis.

If tax-shelter and cash-flow claims can be separated, then under certainty as well as uncertainty, assets equate at the same marginal tax rate. Under uncertainty this implies that the traditional CAPM applies with returns calculated at the marginal tax rate which reflects the price of tax shelter. If tax shelter is costless, this marginal tax rate is zero, and there are no before-tax yield differentials due to tax preference. Finally, with separation of tax-shelter and cash-flow claims, there is separation of the decision to purchase tax shelter and risky assets. Investors can specialize in tax-shelter claims but hold the same portfolio of stochastic cash-flows claims.
The extent to which devices and intermediaries exist to repackage tax shelter is discussed further in Chapter VII. The next two chapters examine the affect of uncertain tax shelter benefits and multiple tax-preference components on portfolio choice and relative prices.
Chapter V

PROGRESSIVE TAX RATES AND UNCERTAIN CASH FLOW

Chapter IV explored the effect of introducing uncertainty into an asset's cash-flow claim. For simplicity it was assumed that each investor's marginal tax rate was exogenously determined. With a progressive tax structure, if tax-shelter claims can be traded separately from cash-flow claims, marginal tax rates will tend to equate as discussed in Chapter III.

Marginal tax rates are a function of the investor's taxable income. Since taxable income depends in part on the stochastic cash-flow income on the investor's portfolio, the marginal tax rate can itself be uncertain.

Fellingham and Wolfson (1978) showed that when investment returns are subject to a progressive income tax, risk-neutral investors find it optimal to hold diversified portfolios since this maximizes expected after-tax returns. In their model all assets were taxed equally.

This chapter explores the effect of a progressive income tax on the single tax-preference variable model when cash flows are stochastic but investors are risk neutral. After-tax returns are assumed to be generated by a model of the same form as equation one of Chapter IV. That is

\[ \tilde{R}_j^i = \tilde{R}_j (1-t^i) + d_j t^i, \]  
\[ \text{or} \]
\[ \tilde{R}_j^i = (\tilde{R}_j - d_j) (1-t^i) + d_j. \]  

(1)

(2)
Simplifying Assumptions

The following simplifying assumptions are made in the development of the model:

1. Investors have a single-period investment horizon.

2. Investors are neutral and act to maximize the expected value of after-tax portfolio returns.

3. Risk-free borrowing and lending opportunities are available to investors at an exogenously determined rate. This return is also subject to tax (tax-deductible in the case of borrowing).

4. All investors are subject to the same progressive tax, which is of the form \( t = a + bI \), where \( a \) is the tax rate which applies to the first dollar of income, \( b \) is the rate of progressivity of the tax rate, and \( I \) is the investor's taxable income from all assets.

5. The amount of tax preference an investment has is known with certainty.

Notation

In addition to the notation introduced in Chapter IV, the following notation is used:

- \( t_i \) - the \( i \)th investors average tax rate which will be a function of his total before-tax portfolio income;
- \( a \) - the tax rate which applies to the first dollar of an individual's income;
- \( b \) - the rate of progressivity of the tax rate;
- \( M \) - total number of individual investors.
**Individual Equilibrium**

We can write the \(i^{th}\) investor's after-tax wealth as

\[
\tilde{W}_1 \ = \ \sum_j X_i^j (1 + \tilde{R}_j^i) + Y_i (1 + R_f^i)
\]  
(3)

or

\[
\tilde{W}_1 \ = \ \sum_j X_i^j (1 + R_j^i) + (W_o^i - \sum_j X_i^j) (1 + R_f^i).
\]  
(4)

We can re-write equation 4 as

\[
\tilde{W}_1 \ = \ \sum_j X_i^j (R_j^i - R_f^i) + W_o^i (1 + R_f^i).
\]  
(5)

Writing equation 5 in terms of before-tax returns, the amount of tax sheltered return, and the investor's tax rate we have

\[
\tilde{W}_1 \ = \ \sum_j X_i^j [(R_j^i - d_j - R_f^i)(1 - \tau_i^j) + d_j] + W_o^i [1 + R_f^i (1 - \tau_i^j)].
\]  
(6)

The tax shelter variable \(d_j\) in equation 6 reduces the amount of before-tax return that is subject to tax. It must then be added back (since it is not a cash outflow) to obtain after-tax cash flow.

The investor's tax rate \(\tau_i^j\) depends on before-tax taxable portfolio income. Thus, we have

\[
\tau_i^j = a + b [\sum_k X_k^i (R_k - d_k - R_f)] + W_o^i R_f
\]  
(7)

Combining equations 6 and 7

\[
\tilde{W}_1 \ = \ \sum_j X_i^j \left\{ (R_j^i - d_j - R_f^i) [1 - (a + b \sum_k X_k^i (R_k - d_k - R_f^i) + b W_o^i R_f] + d_j \right\}
\]

\[
+ W_o^i \left\{ 1 + R_f^i [1 - (a + b \sum_k X_k^i (R_k - d_k - R_f^i) + b W_o^i R_f] \right\}
\]  
(8)

or

\[
\tilde{W}_1 \ = \ \sum_j X_i^j (R_j^i - d_j - R_f^i) K - b \sum_j X_i^j (R_j^i - d_j - R_f^i) \sum_k X_k^i (R_k - d_k - R_f^i)
\]

\[
- b \sum_j X_j^i (R_j^i - d_j - R_f^i) W_o^i R_f + W_o^i + KW_o^i R_f
\]

\[
- W_o^i R_f b \sum_k X_k^i (R_k - d_k - R_f^i) - (R_f^i W_o^i)^2 b + \sum_j X_j^i d_j
\]  
(9)
where $K = 1 - a$.

Taking the expected value of $g$

$$E(W^i_f) = \sum_j x^i_j (R_j - d_j - R_f) K - b \sum_j x^i_j \sum_k E[(\bar{R}_j - d_j - R_f)(\bar{R}_k - d_k - R_f)]$$

$$- b \sum_j x^i_j (\bar{R}_j - d_j - R_f) W^i_o R_f + W^i_o + kW^i_o R_f$$

$$- w^i_o R_f b \sum_k x^i_k (\bar{R}_k - d_k - R_f) - b(W^i_o R_f)^2 + \sum_j x^i_j d_j.$$  \hspace{1cm} (10)

The risk-neutral investor will act to maximize his expected after-tax wealth as given by equation 10. The first-order condition for wealth maximization is:

$$\frac{\partial E(W^i_f)}{\partial x^i_j} = (\bar{R}_j - d_j - R_f)K - 2b \sum_k x^i_k E[(\bar{R}_j - d_j - R_f)(\bar{R}_k - d_k - R_f)]$$

$$- 2b(\bar{R}_j - d_j - R_f) W^i_o R_f + d_j = 0.$$ \hspace{1cm} (11)

Equation 11 can be re-written as

$$(\bar{R}_j - d_j - R_f)(K - 2bW^i_o R_f) + d_j = 2b \sum_k x^i_k E[(\bar{R}_j - d_j - R_f)(\bar{R}_k - d_k - R_f)].$$ \hspace{1cm} (12)

Individual demand equations result from solving the $n$ simultaneous equations resulting from specifying the first-order condition 10 for each security $j$.

The result is:

$$\begin{bmatrix}
  x^i_1 \\
  \vdots \\
  x^i_n
\end{bmatrix} = \begin{bmatrix}
  E_{11} & \cdots & E_{1n} \\
  \vdots \\
  E_{n1} & \cdots & E_{nn}
\end{bmatrix}^{-1} \begin{bmatrix}
  (\bar{R}_1 - d_1 - R_f)(K - 2bW^i_o R_f) + d_1 \\
  \vdots \\
  (\bar{R}_n - d_n - R_f)(K - 2bW^i_o R_f) + d_n
\end{bmatrix}$$ \hspace{1cm} (13)

$$\frac{1}{2b}$$
where the elements $E_{jk}$ represent the product of the expected excess taxable returns for the $j^{th}$ and $k^{th}$ asset.

The demand for each asset depends on the investor's initial wealth and the amount of tax shelter the security has. All investors do not hold securities in the same relative proportions. This is because the investor's initial wealth affects his tax rate which in turn affects the relative value of the tax preference in each security.

If however, we assume no tax preference i.e., $d_j = 0$, for all securities, then the resulting demand equations are

$$
\begin{bmatrix}
X_1^i \\
X_2^i \\
\vdots \\
X_n^i \\
\end{bmatrix} = \begin{bmatrix}
E_{11} & E_{12} & \cdots & E_{1n} \\
E_{21} & E_{22} & \cdots & E_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
E_{n1} & E_{n2} & \cdots & E_{nn} \\
\end{bmatrix}^{-1} \begin{bmatrix}
-r_1 - r_f \\
r_2 - r_f \\
\vdots \\
r_n - r_f \\
\end{bmatrix} - \begin{bmatrix}
k \\
2b - W^i \omega_f \\
\end{bmatrix},
\tag{14}
$$

where the elements $E_{jk}$ in the second matrix represent the expected value of the product of the returns for the $j^{th}$ and $k^{th}$ asset. That is

$$E_{jk} = E[(\tilde{r}_j - r_f)(\tilde{r}_k - r_f)] = \sigma_{jk} + (\bar{r}_j - r_f)(\bar{r}_k - r_f).$$

Equation 14 implies that all investors will hold the same portfolio of risky asset scaled by the term $[k/2b - W^i \omega_f]$ or $\frac{1-a}{2b} - W^i \omega_f$.

If either the tax rate on the first dollar of income or the rate of progressivity increases, the investor holds proportionately less risky assets. Also if the investor's wealth $W^i_0$ rises, fewer risky assets are held. The higher the investor's initial wealth, the higher is the tax
rate and consequently the greater the tax effect. Consequently, the investor acquires more risk-free assets or borrows less of the risk-free asset.

Market Equilibrium

By aggregating equation 12 over all investors and imposing market clearance the market-equilibrium relationships are obtained.

\[
\sum_i \left[ (\tilde{R}_j - d_j - R_f) (K - 2bW_o^R_f) \right] + d_j = \sum_i \left\{ 2b \sum_k^i x_k^i \text{E}[(\tilde{R}_j - d_j - R_f) (\tilde{R}_k - d_k - R_f)] \right\}
\]

(15)

or using the definition of covariance

\[
\sum_i \left\{ (\tilde{R}_j - d_j - R_f) (K - 2bW_o^R_f) \right\} = \sum_i \left\{ 2b \text{COV}(\tilde{R}_j, \sum_k^i x_k^i \tilde{R}_k) \right\} + 2b \sum_k^i x_k^i (\tilde{R}_j - d_j) (\tilde{R}_k - d_k).
\]

(16)

Simplifying equation 16,

\[
(\tilde{R}_j - d_j - R_f) (MK - 2bW_o R_f) + Md_j = 2b \text{COV}(\tilde{R}_j, \sum_k^i x_k^i \tilde{R}_k) + 2b (\tilde{R}_j - d_j) (\sum_k^i x_k^i (\tilde{R}_k - d_k)),
\]

(17)

\[
(\tilde{R}_j - d_j - R_f) (MK + 2bW_o R_f) + Md_j = 2bV_m \text{COV}(\tilde{R}_j, \tilde{R}_m) + 2bV_m (\tilde{R}_j - d_j) (\tilde{R}_m - R_f - d_m),
\]

(18)

\[
(\tilde{R}_j - d_j - R_f) [MK - 2bW_o R_f - 2bV_m (\tilde{R}_m - R_f - d_m)] = 2bV_m \text{COV}(\tilde{R}_j, \tilde{R}_m) - Md_j.
\]

(19)

Equation 19 can be re-written as

\[
\tilde{R}_j = R_f + \lambda_1 \text{COV}(\tilde{R}_j, \tilde{R}_m) - \lambda_2 d_j,
\]

(20)
\[
\lambda_1 = \frac{1}{\frac{MK - W_o \hat{R}_f}{2bV_m} - \frac{\hat{R}_f - (R_{m} - R_{f} - d_m)}{V_m}}, \\
\lambda_2 = \frac{M - (MK - 2bW_o \hat{R}_f - (R_{m} - R_{f} - d_m))}{MK - 2bW_o \hat{R}_f - (R_{m} - R_{f} - d_m)} \frac{2bV_m}{bV_m}.
\]

Alternatively,
\[
\lambda_1 = \frac{2bV_m}{\psi}, \\
\lambda_2 = \frac{m - \psi}{\psi},
\]

where
\[
\psi = MK - 2bW_o \hat{R}_f - (R_{m} - R_{f} - d_m) \frac{2bV_m}{bV_m}.
\]

It should be noted that equation 20 reduces to the traditional CAPM when tax preference is ignored, i.e., \(d_j, j = 1 \ldots n\), and \(d_m\) equal zero. With tax preference, equation 20 is the same as equation 15 in Chapter IV for the heterogeneous tax model which assumed mean-variance efficiency.

This model assumed only that investors maximize the expected value of their after-tax portfolio returns. Yet the effect of a progressive income tax is to induce investors to hold diversified portfolios. The expected return on any risky asset is equal to the sum of three components: (1) the risk-free rate of interest; (2) a "risk premium" measured by the covariance of the asset's return with the market return; (3) an adjustment for tax preference which reduces the required before-tax return. Thus,
even when investors are risk-neutral, the effect of a progressive tax rate leads to the same result of investors desiring to hold well-diversified portfolios.

A curious feature of the market prices of risk and tax shelter in the market model of equation 20 is the influence of the number of investors in the market M. This can be interpreted as an indication of the distribution of wealth in the economy. In this model, an investor's taxes increase with his taxable income, which in turn depends on how much wealth the individual has to invest. The greater the number of individuals in the market, the lower the aggregate tax liability in the market.

The market price of risk and tax shelter does not have to be viewed as an aggregation of individual demand curves. The point of this model is simply to show the effect of a progressive income tax is to induce even risk-neutral investors to hold diversified portfolios. Another way of seeing this result is by a simplified illustration. Suppose the investor's tax rate \( t^i \) is proportional to the return on his portfolio such that \( \bar{r}^i = \bar{r}_p (1-t^i) = \bar{r}_p (1-br^i_p) \) (26)

or

\[
\bar{r}^i = \bar{r}_p - br^2_p.
\] (27)

Taking the expected value of equation 27 we have

\[
\bar{r}^i = \bar{r}_p - b \text{Var}(\bar{r}_p) - br^2_p.
\] (28)
We can rewrite equation 28 as

\[
\frac{\bar{r}^1_p}{\bar{r}_p} = \bar{r}_p (1-b\bar{r}_p) - b \text{Var}(\tilde{r}_p)
\]  

(29)

or

\[
\frac{\bar{r}^1_p}{\bar{r}_p} = \bar{r}_p (1-t^1) - b \text{Var}(\tilde{r}_p).
\]  

(30)

Equation 30 shows the expected after-tax portfolio return for the individual to consist of two components. The first term is the expected before-tax portfolio return multiplied by one minus the expected tax rate. The second term is the product of the rate of progressivity of the tax rate and the variance of the portfolio. Since this variance term is subtracted from the first term, investors act to minimize the variance of their portfolio for a given expected return.

Separating Tax-Shelter Components

As discussed in Chapter IV, the return model specified by equation 1 assumes the proportion of tax shelter is fixed relative to the stochastic cash-flow component. Suppose these tax-shelter and cash-flow claims can be traded separately in the market, but the other assumptions of this chapter are retained. The investor's stochastic end-of-period wealth can now be expressed as

\[
\tilde{W}_1 = \sum_j x^j \left( \tilde{R}_j - R_f \right) + (W^i_o - X_o)(1+R_f) - \tilde{T},
\]  

(31)

where \(X_o\) and \(X_j\) are the amount of funds invested in tax-shelter and cash-flow claims respectively. The tax liability \(\tilde{T}\) is equal to the product of the average tax rate and taxable income. That is,

\[
\tilde{T} = t \left\{ \sum_k x^k (\tilde{R}_k - R_f) + (W^i_o - X_o) (R_f - X_D) \right\}
\]  

(32)

*Note that \(\bar{R}_j\) now represents only the stochastic cash-flow component of asset j's return.
where $R_D$ is the "return" from investing in tax shelter. Since tax shelter reduces taxable income, the value of the tax shelter to the individual depends on his marginal tax rate. According to the progressive tax rate assumption, the investor's tax rate is

$$
\bar{t} = a + b \left\{ \sum_k \bar{X}_k (R_k - R_f) + (W_o - X_D) R_f - X_D R_D \right\}.
$$

Substituting 32 and 33 into 31,

$$
\bar{W}_1 = \sum_j X^i_j (R^i_j - R_f) + (W^i_o - X_D) (1 + R_f) - \left\{ a + b \left\{ \sum_k \bar{X}_k (R_k - R_f) + (W^i_o - X_D) R_f - X_D R_D \right\} \right\}.
$$

By proceeding in the same manner as before, the investor's demand equations now become

$$
\begin{bmatrix}
X_1 \\
\vdots \\
X_n
\end{bmatrix} =
\begin{bmatrix}
E_{11} & \cdots & E_{1n} \\
\vdots & \ddots & \vdots \\
E_{n1} & \cdots & E_{nn}
\end{bmatrix}^{-1}
\begin{bmatrix}
\frac{k}{2b} - W^i_o R_f + X_D (R^i_D - R_f) \\
\vdots \\
\end{bmatrix}.
$$

Investors now hold the same portfolio of stochastic cash flow claims. The investment in tax shelter $X_D$ only affects the scale of this investment. If no tax shelter can be purchased ($X_D = 0$), then equation 35 is identical to equation 14.

---

*The reciprocal of $R_o$ is the price of tax shelter $P_D$ as discussed in Chapter III. The product $X_D R_D$ is the total amount of tax shelter claims the investor will hold at the end of the period.*
Conclusion

The results of this model are similar to the exogenous tax-rate model of Chapter IV. Portfolio composition depends on the investor's initial wealth. This is because the investor's initial wealth affects his or her pre-investment tax rate. Investors still benefit from diversification, even though they are risk-neutral, because the variance of the before-tax portfolio return affects the expected after-tax portfolio return. Investors hold different portfolios unless they can trade cash flow and tax shelter claims separately.

As was the case in Chapter IV, the expected before-tax return on a risky asset is equal to the sum of three components: (1) the risk-free rate of interest, (2) a "risk premium" measured by the covariance of the asset's return with the market return, and (3) an adjustment for tax-preference which reduces the required before-tax return. However, this model provides an alternative interpretation for the reason taxes and tax preference affect portfolio choice and asset pricing.
Chapter VI
PREFERENTIAL TAXATION WITH UNCERTAIN TAX BENEFITS

Introduction

As discussed in Chapter I, the models developed by Brennan, Litzenberger and Ramaswamy, and Long assume certainty with respect to the benefit of tax preference. This assumption was also made in the single tax preference variable model developed in Chapter IV. The benefit of tax deferral and conversion to a capital-gains tax rate can be reduced, however, due to uncertainty as to the investor's future tax rate.

The investment objectives of the individual may include future consumption before retirement, retirement income, and estate planning. Deferred income and capital gains will normally be recognized when the investment is sold. While the investor can time the sale of an asset to some extent to minimize taxes, several factors could lead to uncertainty as to the marginal tax rate which will apply when the investment is sold. First, the investor's exogenous nominal wage income could be affected by both uncertain inflation and changes in his or her real wage rate. The average amount of personal income paid in federal taxes has risen from about 12% in 1964 to almost 18% in 1979. Second, the investor's actual future investment income will affect his future marginal tax rate. Third, tax-law changes can affect future marginal tax rates. Fourth, the date at which the investor will recognize the deferred income may also be uncertain. Thus, factors which affect an
asset's before-tax return may be quite different from those that affect individuals' tax rates. These tax rates will have stochastic properties that are not perfectly correlated with the return on the "market portfolio."

The endogenous progressive-tax model developed in Chapter V captures some of the variability of future tax rates since the tax rate was specified as a function of before-tax portfolio income. This chapter develops a more general model which allows each investor's tax rate to be a random variable.

An asset's after-tax return can be specified as the sum of two components: 1) the after-tax yield that would apply if the proceeds were entirely taxable, and 2) an adjustment for the portion of the yield that can be converted to an uncertain future tax rate. The uncertain future tax rate could be interpreted as either an ordinary-income or capital-gains tax rate. These are special cases of the concept of conversion to an uncertain future tax rate. Equation 1 capture this concept.

\[ \tilde{R}_j^T = \tilde{R}_j(1-t_o) + d_j(t_o - \gamma t_n) \]  \hspace{1cm} (1)

Developing the individual's ending wealth as before, we have

\[ \tilde{W}_i^T = \sum_j X_j^i [(\tilde{R}_j - R_f)(1-t_o) + d_j(t_o^{i-} - \gamma t_n^{i-})] + \tilde{W}_o^i|1 + R_f(1-t_o^{i-})] \]  \hspace{1cm} (2)
Taking the expected value and variance of (2):

\[ \mathbb{E}(\tilde{W}_1^i) = \sum_j X_j^i [(\bar{R}_j - R_f) (1-t_o^i) + d_j (t_o^i - \gamma t_n^i)] + W_o^i [1 + R_f (1-t_o^i)] \]

\[ \text{Var}(\tilde{W}_1^i) = (1-t_o^i)^2 \sum_j X_j^i \sum_k X_k^i \text{Cov}(\tilde{R}_j, \tilde{R}_k) + (\sum_j X_j^i d_j)^2 \gamma^2 \text{Var}(t_n^i) \]

\[ + 2 \sum_k X_k^i \sum_k X_k^i d_k \gamma (1-t_o^i) \text{Cov}(\tilde{R}_j, \tilde{t}_n^i) \]

First order conditions imply that:

\[ \theta_i \left\{ (\bar{R}_j - R_f) (1-t_o^i) + d_j (t_o^i - \gamma t_n^i) \right\} = (1-t_o^i)^2 \sum_k X_k^i \text{Cov}(\tilde{R}_j, \tilde{R}_k) \]

\[ + \sum_k X_k^i d_k \gamma^2 d_j \text{Var}(t_n^i) + \sum_k X_k^i d_j \gamma (1-t_o^i) \text{Cov}(\tilde{R}_j, \tilde{t}_n^i) \]

\[ + \sum_k X_k^i d_k \gamma (1-t_o^i) \text{Cov}(\tilde{R}_j, \tilde{t}_n^i) \]

We can re-write equation 5 as

\[ \frac{\theta_i}{1-t_o^i} (\bar{R}_j - R_f) + \frac{t_o^i - \gamma t_n^i}{(1-t_o^i)^2} d_j = \sum_k X_k^i \text{Cov}(\tilde{R}_j, \tilde{R}_k) \]

\[ + \sum_k X_k^i d_k \gamma^2 d_j \text{Var}(t_n^i/(1-t_o^i)) + \sum_k X_k^i d_j \gamma \text{Cov}(\tilde{R}_k, t_n^i/(1-t_o^i)) \]

\[ + \sum_k X_k^i d_k \gamma \text{Cov}(\tilde{R}_j, t_n^i/(1-t_o^i)) \].

\[ \text{(6)} \]

**Individual Equilibrium**

Equation 6 can be written as

\[ \sum_k X_k^i \left\{ \text{Cov}(\tilde{R}_j, \tilde{R}_k) + \text{Cov}(d_j \gamma, \frac{t_n^i}{1-t_o^i}), d_k \gamma \frac{t_1^i}{1-t_o^i} \right\} + \text{Cov}(\tilde{R}_k, \gamma d_j t_n^i/(1-t_o^i)) \]

\[ + \text{Cov}(\tilde{R}_j, \gamma d_k t_n^i/(1-t_o^i)) \right\} = \frac{\theta_i}{1-t_o^i} (\bar{R}_j - R_f) + \frac{t_o^i - \gamma t_n^i}{(1-t_o^i)^2} d_j. \]

\[ \text{(7)} \]
Equation 7 could be solved for the individual's demand for each security \( (X^i_j) \) as was done in Chapter IV. We can see from 7 that the demand will depend on several factors including (1) the covariance between the return for asset \( j \) and the return for all other assets in his portfolio, (2) the covariance between the uncertain tax shelter (deferral) component of asset \( j \) and that for all other assets in his portfolio, (3) the covariance between the return on asset \( j \) and the tax-shelter component on all other assets in his portfolio and (4) the covariance between the tax-shelter component of asset \( j \) and the returns on all other assets in his portfolio, (5) the asset's expected return, and (6) the asset's expected tax shelter component.

In general, investors will hold different portfolios if either current tax rates, the expected value of future tax rates, or the variance of the future tax rates differ. Thus, as in the models with certain tax rates, investors in different tax brackets see different portfolios as optimal. Investors still adjust their portfolios so that after-tax risk-adjusted returns are equal for all assets. However, the risk adjustment now must also include the effect of the uncertain future tax rate on the asset's tax-preference component.

Market Equilibrium

Aggregating equation 6 over all investors and imposing market equilibrium as before, we have

\[
\bar{R}_j - R_f = \sum_i \frac{\theta^i_j}{1-t_o} = \nu_m \text{Cov}(\bar{R}_j, \bar{R}_m) - d_j \sum_i \frac{t^i_n - t^i_o}{(1-t_o)^2} + d_j \sum_k X^i_k d_k \text{Var}(\gamma^i_n/(1-t_o))
\]

\[
+ d_j \sum_k X^i_k \text{Cov}(\bar{R}_k, \gamma^i_n/(1-t_o)) + \text{Cov}(\bar{R}_j, \sum_k X^i_k d_k \gamma^i_n/(1-t_o)),
\]

\[(8)\]
We can express equation 8 as

\[
\widehat{R}_j - R_f = \lambda_1 \text{Cov}(\tilde{R}_j, \tilde{R}_m) - \lambda_2 d_j + \text{Cov}(\tilde{R}_j, \tilde{D}_m)
\]

(9)

where

\[
\lambda_1 = V_m \left[ \sum_{i} \frac{\theta_i}{1-t_i^o} \right]^{-1},
\]

\[
\lambda_2 = \left[ \sum_{i} \frac{\theta_i}{1-t_i^o} \right]^{-1} \left[ \sum_{i} \frac{t_i^o - \gamma t_i^o}{(1-t_i^o)^2} - \sum_{i,k} \sum X_k^i e_k \text{Var} \left( \gamma \frac{\tilde{r}_n^i}{(1-t_i^o)} \right) - d_j \sum X_k^i \text{Cov} \left( \tilde{R}_k, \gamma \frac{\tilde{r}_n^i}{(1-t_i^o)} \right) \right],
\]

\[
\tilde{D}_m = \sum_{i,k} X_k^i \gamma d_k \tilde{r}_n^i / (1-t_i^o).
\]

According to equation 9, the expected excess return on asset \( j \) depends on

1. its systematic risk as measured by the covariance of the asset's return with the market portfolio, (2) an adjustment for tax shelter (deferral), and (3) the covariance of the asset's return with uncertain market tax shelter \( (D_m) \). This latter risk component reflects the uncertain tax rates for all market participants.

The two lambdas reflect the market's price for systematic risk and tax shelter. \( \lambda_2 \) itself reflects three distinct market factors as follows:

1. the market's aggregate expected tax rate. This reflect the expected tax benefit of asset \( j \);
2. the aggregate variance of future tax rates weighted by the tax shelter component of all other assets. This term actually
reflects the covariance of the tax shelter component of asset \( j \) with the tax-shelter component of all other assets in the market;

(3) the aggregate covariance of uncertain future tax rates with the returns on all other assets in the market. This term reflects asset \( j \)'s tax shelter component and the returns on all other assets.

Thus, the market price of tax preference will reflect the aggregate expected benefit of deferral or conversion and the market's aversion to the uncertain benefit of converting income to an uncertain future tax rate. It is interesting that whether the tax-shelter component \( d_j \) raises the asset's required return depends on the market price of deferral tax shelter \( \lambda_2 \) being positive. It is conceivable that the expected benefit of deferring taxes would be offset by the riskiness of deferring taxes. Also, expected increase in future tax rates would further reduce the market's perceived benefits of deferral.

**Conclusion**

This analysis was based on a general uncertain-conversion model. The future tax rate would differ from the current ordinary income tax rate due to either a difference in the future ordinary-income tax rate or conversion to a future capital gains tax rate. It follows that different types of tax preference will have a different market price. The value of deferral will depend on the distribution of future ordinary income tax rates. The value of conversion will depend on the distribution of future
capital-gains tax rates. In a more complex model, the deferral and conversion tax-preference components can be separated and made explicit. The importance of separating the various components of tax preference becomes even more apparent when analyzing an investment such as real estate income property which contains many sources of tax preference. The value of each component depends on many characteristics of the investor's tax function.

The effect of uncertain tax rates on individual portfolio choice and market equilibrium is, in a sense, similar to the effect of incorporating uncertain inflation in the CAPM.* Nominal rates or return depend on both uncertain real rates of return and uncertain inflation. These are two distinct sources of uncertainty. Similarly, after-tax rates of return depend on uncertain before-tax returns and uncertain tax rates. These are also two distinct sources of uncertainty.

In an even more complex framework, nominal after-tax returns depend on uncertain real before-tax returns, uncertain inflation, and uncertain tax rates. Furthermore, tax preference benefits may to some extent depend on uncertain inflation. This is incorporated into the model in chapter VII.

* See Chen (1976) and Chen and Boness (1975). Uncertain inflation is incorporated into this study in chapter VII.
Chapter VII

MULTIPLE TAX-PREFERENCE COMPONENTS:
REAL-ESTATE INCOME PROPERTY

Real Estate income property is an example of a tax-preferred investment that has several components of tax preference. Chapter II discussed the source of these various components, the interaction with personal service income, and the additional tax consequences which could reduce the tax benefits of real estate. Chapter III explored some of the ways in which different types of tax preference might affect portfolio choice and be priced in the market. The way in which cash flow uncertainty interacts with tax preference for a single tax-shelter variable model was then analyzed in Chapter IV. This chapter extends the model developed in Chapter IV to several tax-preference components. Tax preference is initially assumed to be a fixed proportion of cash flow. The effect of relaxing this assumption is then discussed.

Notation

In addition to the notation introduced in Chapter IV, the following notation is used:

\( C_N \) - After-tax cash flow if the investment was not tax-preferred i.e. fully taxable.

\( C_A \) - Actual after-tax cash flow.
I - Net Income to the property over the holding period.

\( D_e \) - Economic depreciation in the properties value over the holding period.

\( D_s \) - Amount of straight-line depreciation that would be allowed for tax purposes during the holding period.

\( D_t \) - Total tax depreciation allowed during the holding period.

\( t_p \) - The effective marginal tax rate for "tax-preference items."

**Income-Property Cash Flows**

To understand the nature of tax preference for a real-estate income property, it is useful to separate the cash flows into two components: (1) the after-tax cash flow that would result if the property were fully taxable and (2) the incremental cash flow that results from the tax-preference benefits of the investment.

**Actual After-Tax Cash Flows**

Tax depreciation for real-estate income property held as an investment or used in one's trade or business, differs depending on whether it is a new or existing property, and whether it is a residential or non-residential property. For all but existing commercial real-estate investments, accelerated depreciation is allowed which exceeds straight-line depreciation. Since the excess of tax depreciation over straight-line depreciation is recaptured at the ordinary-income tax rate at the end of the holding period, this tax rule results in deferral only.

Straight-line depreciation is usually calculated over a depreciable life much shorter than the economic life of the property. Furthermore, the rate of economic depreciation would typically be less than a straight-
line rate. Thus the excess of straight-line depreciation over economic
depreciation results in a conversion of income from the ordinary-income
tax rate to a capital-gains tax rate.

Inflation results in an increase in the nominal selling price
over that which would otherwise be expected. This gain is taxed at the
capital-gains rate. The actual after-tax cash flows for a real-estate
income property can be expressed as

\[ C_A = (1 - D_t)(1 - t_o) + D_t + P_1 - (P_1 - P_o)\gamma t_g - D_s \gamma t_g - (D_t - D_s)(\gamma t_o + t_p). \] (1)

The first term \((1 - D_t)(1 - t_o)\) gives income after taxes. The tax depreci-
ation \((D_t)\) is added back to get after-tax cash flow from income. The
remaining terms reflect sale of the property at \(P_1\) and the calculation of
taxes on the sale of the property. Any increase in price results in a
capital gain taxed at the capital-gains tax rate. Straight-line depre-
ciation taken during the holding period lowers the book value, and this
gain is also taxed at the capital gains tax rate. Finally, as discussed
previously, the excess of tax depreciation over straight-line depreciation
\((D_t - D_s)\) is recaptured at the ordinary income tax rate and thus results in
deferral only. Furthermore, as discussed earlier, excess depreciation is
a "tax-preference item". This tax is paid each year i.e. during the
period instead of upon sale of the property at the end of the period. The
trade off between deferral and the additional "tax preference" tax will
be discussed in a later section.

**Fully Taxable Cash Flows**

If real-estate income property were fully taxable, there would be no
benefit of deferring taxes or converting taxes to a capital-gains tax rate. The income and any gain (or loss) in value would be immediately reflected at the ordinary income tax rate. In this case, the after-tax cash flows would be as follows:

\[ C_N + I(1-t_o) + P_1 - (P_1 - P_o)t_o, \]  

or

\[ C_N + (I + P_1 - P_o)(1-t_o) + P_o. \]  

The after-tax holding-period yield is therefore

\[ R_N = \frac{(I + P_1 - P_o)(1 - t_o) P_o}{P_o} - 1, \]  

or

\[ R_N = \frac{I + P_1 - P_o}{P_o} (1 - t_o), \]  

or

\[ R_N = R_j (1 - t_o), \]  

where

\[ R_j = \frac{1 + P_1 + P_o}{P_o}. \]  

\( R_j \) is the familiar before-tax holding period return. Multiplying \( R_j \) by the complement of the ordinary income tax rate results in the after-tax holding period return \( R_N \), assuming full taxation of cash income.

Taking the difference between the actual cash flows in equation 1 and the fully taxable cash flows in equation 2 and using the fact that \( D_t = D_s + (D_t - D_s) \), we have

\[ C_A - C_N = D_s (t_o - \gamma t_g) + (D_t - D_s) \left\{ t_o (1-\gamma) - t_p \right\} + (P_1 - P_o)(t_o - \gamma t_g). \]
Equation 8 shows the incremental cash flows (actual vs. fully taxable) to consist of several components. First, straight-line depreciation \( (D_s) \) results in conversion of income from the ordinary income tax rate to the capital-gains tax rate.

Second, excess depreciation \( (D_t - D_s) \) results in deferral which is reduced and possibly completely offset by the marginal "tax preference" tax. For after-tax cash flows to increase as a result of excess depreciation, the condition that \( t_o(1-\gamma) > t_p \) must hold. Investors with a high marginal tax rate on "tax preference items" may find excess depreciation reduces their after-tax cash flows and would therefore prefer straight-line depreciation over accelerated depreciation. This trade-off will be reflected in an individual's portfolio choice and the property's market price to be discussed later.

Third, any increase in price results in conversion since this is taxed at the capital gains tax rate. A decrease in price would similarly reduce the conversion benefit of straight-line depreciation.

It is informative to decompose the nominal price change discussed above into two components. Assume that the selling price for the property is equal to the initial price plus any increase in price level due to inflation, less economic depreciation in the property. That is,

\[
P_l = P_o (1 + \pi) - D_e.
\]  

Substituting this into equation 8 and using the fact that

\[
D_s = D_e + (D_s - D_e), \text{ we have}
\]

\[
C_n - C_A = (D_s - D_e)(t_o - \gamma t_g) + (D_t - D_s) \left\{ t_o (1-\gamma) - t_p \right\} + \pi P_o (t_o - \gamma t_g).
\]
The excess of accelerated depreciation over straight-line depreciation has the same interpretation as before. For straight-line depreciation, however, it is the excess of straight-line over economic depreciation that results in conversion. Actual economic depreciation represents loss of capital and there is no tax benefit if tax depreciation is not greater than economic depreciation. Any nominal price increase due to inflation, however, is taxed. Since this inflationary gain is taxed at the ordinary income tax rate for fully taxable investments, there is a conversion tax-preference benefit for real estate relative to fully taxable investments.

Rate-of-Return Model

Combining equation 10 with equation 3 and expressing the after-tax return, we have

\[ R^t_j = R_j(1-t_o) + (\delta_j + \tau)(t_o - \gamma t_g) + d_j \left\{ t_o(1-\gamma) - t_p \right\}, \]  

(11)

where

\[ \delta_j = \frac{D_s - D_e}{P_o} \]  
is a measure of the tax benefit resulting from the excess of straight-line over economic depreciation;

\[ d_j = \frac{D_t - D_s}{P_o} \]  
is the excess of accelerated over straight-line depreciation (so called "excess depreciation").

Individual Equilibrium

The results from equation 11 can now be used to analyze individual portfolio choice where real-estate income property is one of the assets the investor can choose from. In order to include fully taxable as well as other tax-preferred assets in the same model, we must allow for the
fact that any nominal price increase due to inflation may be fully taxable or result in conversion as discussed earlier. If we let \( \alpha_j \) represent the percentage of any nominal price increase due to inflation that is taxed at the capital-gains tax rate, then equation 11 becomes

\[
R^c_j = R_j (1-t_o) + (\delta_j + \alpha_j \pi) (t_o - \gamma t_g) + d_j \left\{ t_o (1-\gamma) - \frac{t}{p} \right\}.
\]  

(12)

When \( \alpha_j = 1 \) equation 12 is the same as equation 11. For a fully taxable investment, on the other hand, \( \alpha_j = 0 \) reflects the fact that any increase in the nominal before-tax return \( R_j \) due to inflation would be fully taxed.

Since inflation has been incorporated in the return model of equation 12, it will be assumed that both the before-tax return \( R_j \) and the rate of inflation \( \pi \) are random variables. Furthermore, it is assumed that investors are concerned with real after-tax portfolio returns. Thus, in addition to the additional tax-preference variables, uncertain inflation is also introduced into the model. Inflation is treated as an additive component as was done by Chen and Boness (1975) and Chen and Kane (1978).

The real after-tax return for asset \( j \) can be expressed as

\[
\tilde{R}_j^T = R_j (1-t_o) + (\delta_j + \alpha_j \pi) (t_o - \gamma t_g) + d_j \left\{ t_o (1-\gamma) - \frac{t}{p} \right\} - \pi.
\]  

(13)

Real end-of-period after-tax random wealth will be

\[
\tilde{W}_1 = \sum_j X_j \left[ (\tilde{R}_j-R_f) (1-t_o) + (t_o - \gamma t_g) (\delta_j + \alpha_j \pi) + d_j \left\{ t_o (1-\gamma) - \frac{t}{p} \right\} \right] + \tilde{W}_0 [1 + R_f (1-t_o) - \pi].
\]  

(14)

\[
E(\tilde{W}_1) = \sum_j \tilde{W}_j \left[ (\tilde{R}_j-R_f) (1-t_o) + (t_o - \gamma t_g) (\delta_j + \alpha_j \pi) + d_j \left\{ t_o (1-\gamma) - \frac{t}{p} \right\} \right] + \tilde{W}_0 [1 + R_f (1-t_o) - \pi].
\]  

(15)
\[ v(\tilde{w}^1) = \sum_{j} \sum_{k} \sum_{l} \sum_{l'} x_{j}^1 x_{k}^1 (1-t_o^1)^2 \text{COV}(\tilde{r}_j, \tilde{r}_k) + \sum_{j} \sum_{k} \sum_{l} x_{j}^1 x_{k}^1 (t_o^1 - \gamma t_g^1)^2 \text{COV}(\alpha_j, \alpha_k) \]

\[ + (w_o^1)^2 \text{Var}(\tilde{\pi}) - 2 \sum_{j} x_{j}^1 \alpha_j (t_o^1 - \gamma t_g^1) w_o^1 \text{Var}(\tilde{\pi}) + 2 \sum_{j} \sum_{k} x_{j}^1 x_{k}^1 (t_o^1 - \gamma t_g^1) \]

\[ \times (1-t_o^1) \alpha_k \text{COV}(\tilde{r}_j, \tilde{\pi}) - 2 w_o^1 (1-t_o^1) \sum_{j} x_{j}^1 \text{COV}(\tilde{r}_j, \tilde{\pi}). \] (16)

Using the first order conditions we have:

\[ \theta^i_1 (\tilde{r}_j - R_f^i) (1-t_o^1) + (t_o^1 - \gamma t_g^1) (\delta_j + \alpha_j) \tilde{\pi} + d_j \left\{ t_o^1 (1-\gamma) - t_p^1 \right\} \]

\[ = \sum_{k} x_{k}^1 (1-t_o^1)^2 \text{COV}(\tilde{r}_j, \tilde{r}_k) - (t_o^1 - \gamma t_g^1) [w_o^1 - (t_o^1 - \gamma t_g^1) \sum_{k} \alpha_k x_{k}^1] \text{Var}(\tilde{\pi}) \alpha_j \]

\[- (1-t_o^1)(w_o^1 - (t_o^1 - \gamma t_g^1) \sum_{k} \alpha_k x_{k}^1) \text{COV}(\tilde{r}_j, \tilde{\pi}) + \alpha_j (t_o^1 - \gamma t_g^1) (1-t_o^1) \sum_{k} x_{k} \text{COV}(\tilde{r}_k, \tilde{\pi}) \] (17)

Equation 17 specifies the n simultaneous equations (one for each asset) that can be solved for the individual's demand for each asset. The individual's demand for each asset depends on the asset's before-tax return and tax-preference components, expected and uncertain inflation, the covariance between all assets in the portfolio, and the individual's tax rates.

Individuals still adjust their portfolio so that risk-adjusted after-tax rates of return are equal for each asset. Many factors now affect the tax adjustment, however. Given the market price of each tax-preference component, investors with different tax situations will have different positive or negative demands for different types of tax-preferred and fully taxable assets. An investor with a high marginal tax rate on tax-preference items, for example, might prefer existing residential or commercial income property to new residential or commercial income property.
This is because investors in new income property can use a more accelerated depreciation method. The market price of this tax-preference component (excess depreciation) may be positive, which would result in higher price for new property relative to existing property at the same level of risk. Since excess depreciation is a tax-preference item, however, investors with a high marginal tax rate on tax-preference items might not benefit from using the accelerated depreciation which is reflected in the market price of the new property investment. The determination of the market price for real estate income property is the subject of the next section.

**Market Equilibrium**

We can express equation 17 as

$$
\frac{\delta \hat{t}}{1-t_0^\hat{t}} (\tilde{R}_j - R_k) + \frac{\theta (t^{\hat{t}} - \gamma t^{\hat{t}})}{(1-t_0^{\hat{t}})^2} (\delta_j + \alpha_j)\bar{\pi} + \frac{\theta (t_0^{\hat{t}} (1-\gamma) - t_0^{\hat{t}} p)}{(1-t_0^{\hat{t}})^2} dj =
$$

$$
\sum_k X_k^{\hat{t}} \text{COV}(\tilde{R}_j, R_k) - \frac{(t^{\hat{t}} - \gamma t^{\hat{t}})}{(1-t_0^{\hat{t}})^2} [W_0^{\hat{t}}(t_0^{\hat{t}} - \gamma t^{\hat{t}}) \sum_k X_k^{\hat{t}}] \text{Var}(\bar{\pi}) = j
$$

$$
- \frac{1}{1-t_0^{\hat{t}}} [W_0^{\hat{t}}(t_0^{\hat{t}} - \gamma t^{\hat{t}}) \sum_k X_k^{\hat{t}}] \text{COV}(\tilde{R}_j, \bar{\pi}).
$$

Aggregating equation 18 over all investors and imposing market clearance, we have
\[
\sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \left( R_{j}^{i} - R_{x}^{i} \right) + \sum_{i} \frac{\theta_{i}^{i}(t_{o}^{i} - \gamma t_{i}^{i})}{(1-t_{o}^{i})^2} (j + \alpha_{j} \pi) + \sum_{i} \frac{\theta_{i}^{i}((1- \gamma) - t_{p}^{i})}{(1-t_{o}^{i})^2} d_{j} = \\
V_{m} \text{COV}(\tilde{R}_{j}, \tilde{R}_{m}) - \sum_{i} \frac{(t_{o}^{i} - \gamma t_{i}^{i})}{(1-t_{o}^{i})^2} \left[ W_{o}^{i} - (t_{o}^{i} - \gamma t_{i}^{i}) \sum_{k} \alpha_{k} x_{k}^{i} \right] \text{Var}(\pi) \alpha_{j} \\
- \sum_{i} \frac{W_{o}^{i} - (t_{o}^{i} - \gamma t_{i}^{i}) \sum_{k} \alpha_{k} x_{k}^{i}}{(1-t_{o}^{i})} \text{COV}(\tilde{R}_{j}, \pi) + \sum_{i} \frac{(t_{o}^{i} - \gamma t_{i}^{i})}{(1-t_{o}^{i})} \sum_{k} x_{k}^{i} \text{COV}(\tilde{R}_{k}, \pi) \alpha_{j}
\] (19)

Equation 19 can now be written as

\[
\tilde{R}_{j} - R_{x}^{i} = \lambda_{1} \text{COV}(\tilde{R}_{j}, \tilde{R}_{m}) - \lambda_{2} \delta_{j} - \lambda_{3} d_{j} - \lambda_{4} \alpha_{j} - \lambda_{5} \text{COV}(\tilde{R}_{j}, \tilde{\pi})
\] (20)

where

\[
\lambda_{1} = V_{m} \left[ \sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \right]^{-1};
\]

\[
\lambda_{2} = \left[ \sum_{i} \frac{\theta_{i}^{i}(t_{o}^{i} - \gamma t_{i}^{i})}{(1-t_{o}^{i})^2} \right] \left[ \sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \right]^{-1};
\]

\[
\lambda_{3} = \left[ \sum_{i} \frac{\theta_{i}^{i}((1- \gamma) - t_{p}^{i})}{(1-t_{o}^{i})^2} \right] \left[ \sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \right]^{-1};
\]

\[
\lambda_{4} = \left[ \sum_{i} \frac{(t_{o}^{i} - \gamma t_{i}^{i})}{(1-t_{o}^{i})^2} \left\{ W_{o}^{i} - (t_{o}^{i} - \gamma t_{i}^{i}) \sum_{k} \alpha_{k} x_{k}^{i} \right\} \text{Var}(\pi) - (1-t_{o}^{i}) \sum_{k} x_{k}^{i} \text{COV}(\tilde{R}_{k}, \pi) + \theta_{i}^{i} \pi \right\} \left[ \sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \right]^{-1};
\]

\[
\lambda_{5} = \left[ \sum_{i} \frac{W_{o}^{i} - (t_{o}^{i} - \gamma t_{i}^{i}) \sum_{k} \alpha_{k} x_{k}^{i}}{(1-t_{o}^{i})} \right] \left[ \sum_{i} \frac{\theta_{i}^{i}}{1-t_{o}^{i}} \right]^{-1}.
\]
If the tax and tax-shelter components of equation 19 are ignored, the model reduces to the CAPM with uncertain inflation.* If inflation is also ignored, the model reduces to the traditional CAPM.

Equation 20 shows the expected before-tax nominal return for any asset in equilibrium to be a linear combination of the risk-free rate, tax-shelter factors as discussed earlier, the asset's systematic risk and inflation risk. The \( \lambda \)'s express the market's aggregate willingness to trade off risk and tax shelter for expected before-tax nominal return.

The market price of the different tax-preference components will reflect the aggregate market benefit of the tax-preference component. The market price of excess depreciation, for example, will depend on the marginal tax rates on "tax-preference items" for market participants. The market price of deferral will depend on the effective capital-gains tax rate for market participants.

It is interesting that the expected value and variance of inflation is relevant to the pricing of real-estate income properties \((\alpha_j = 1)\) or any asset with an inflation-induced conversion \((\alpha_j > 0)\). A closer look at \( \lambda_4 \), the market price of inflation-induced deferred-conversion, shows it to be composed of three effects: (1) the expected inflation-induced conversion, (2) the covariance between the uncertain inflation-induced conversion on asset \( j \) and the return on all other assets in the market, and (3) the covariance between the uncertain inflation-induced conversion on asset \( j \) and that for all other assets. It is this latter factor that results in the variance of inflation being reflected in \( \lambda_4 \).

* See Chen (1976) and Chen and Boness (1975).
Note that the market price of the excess-depreciation tax-preference component $\lambda_3$ will depend on the aggregate trade off between deferral and the additional tax-preference item tax. Recall that the after-tax cash flows would increase for some investors but decrease for other investors depending on the deferral benefit relative to the additional tax-preference tax. Given these two clienteles it is likely that the supply of excess depreciation in the market would adjust such that the market price of excess depreciation is zero. Other factors affecting the supply and demand for each tax preference component is discussed in the next section.

Repackaging and Trading Tax-shelter Components

The model developed in this chapter assumed the proportion of each tax-preference component was fixed relative to the asset's cash flow. For example, in order to get the benefits of depreciation, the investor had to also purchase the stochastic cash flow component of the property's return.

As discussed in Chapter IV, there are incentives for investors to trade tax-preference benefits separately from before-tax cash flow. In the case of multiple tax-preference components, investors have the incentive to trade or purchase each type of tax preference that interacts differently with his or her tax function. Thus, we would expect devices and intermediaries through which tax preference can be traded or purchased. This section will discuss some of the ways in which tax preference might be separated from the asset's cash flow and traded in the market.
Debt Financing

The use of debt to repackage cash flow was discussed in Chapter III. Since interest on debt financing is tax deductible, interest payments offset cash income from a real-estate investment that would be fully taxable as ordinary income. As was also pointed out in Chapter III, the use of leverage and interest deductions for real estate is normally not limited by the interest-deduction limitation or the "at risk" provisions of the tax code. Thus it is not uncommon for real estate to be levered to the point where virtually all taxable cash flow is eliminated and depreciation deductions result in artificial accounting losses. The investor has essentially sold taxable income to the lender, and kept the tax shelter. In some cases, the financing is even structured such that not only can future income be sold, but the lender can share in the potential inflation-hedging characteristic of the real estate. This can be done in many ways, the most common being a "participation" or "equity kicker." For example, the loan terms may specify a fixed interest rate plus a percentage of income or cash flow in excess of a base amount. Thus as income for the property rises in the future, possibly due to inflation, the lender receives some or all of this increased income. Since participations are also tax deductible, the participation is essentially a way of selling uncertain future taxable income. Lenders receiving a participation often accept higher loan-to-value ratios, lower interest rates, and longer loan terms. This, of course, makes it even more attractive to the investor. Thus, the use of debt with a participation is a way of selling taxable income as well as
allowing the lender to participate in the property's "inflation risk."

Leasing

Miller and Upton (1976) show that, under conditions of uncertainty as well as certainty, owning a capital good and using it in a production process are two distinct economic activities. If the tax-preference characteristics of leases are ignored, then the decision of a firm to buy or lease is a matter of indifference. The equilibrium rental rate which the firm saves by buying instead of leasing, is exactly offset by the opportunity cost of the funds invested in the capital asset plus the expected depreciation of the machine. The opportunity cost will, of course, reflect the risk incurred.

By introducing a tax subsidy which may have a different value to the owner as against the user of capital, Miller and Upton show that there can be gains from specialization. Firms or individuals that gain the most from the tax subsidies in leasing will tend to lease to the firms or individuals who do not gain as much from the subsidy.

Suppose a firm has building being used for the production of income. If the firm has a relatively low marginal tax rate, it may be better off selling the tax-preference characteristics of owning the building to a higher-tax-rate investor. This can be accomplished through a sale-leaseback. The firm sells the building to a lessor and agrees to lease the same building for a specified number of years. The lessor now has the tax-shelter benefits of the real estate, which is paid for through a lease rate which the lessee finds more attractive than owning the building and financing in some other way.
Leasing is often used in a similar way by banks to purchase tax-shelter benefits from customers through equipment leasing. Equipment leasing has depreciation tax-shelter benefits similar to real estate. The bank may have a customer with low taxable income. If the bank can benefit from the tax-shelter benefits of owning the equipment, it may agree to lease the equipment to the customer at a rate which is less than it would charge for a loan on the same equipment.

The exchange of tax shelter through leasing can be done without necessarily exchanging risk. This is accomplished through "net" leases which require the lessee to pay all taxes, maintenance, insurance, and other necessary service costs of operating the asset. Thus it may be possible to sell tax shelter through a device such as a sale-leaseback without disturbing the stochastic nature of the investor's portfolio.

Another use of the sale-leaseback is to separate the ownership of the land underlying a building from the ownership of the building itself. Lower-tax-bracket institutions are often willing to purchase the land, leasing it to the building owner for a term exceeding the useful life of the building as determined for tax purposes. This permits the higher-tax-bracket building owner to retain that portion of the property which is 100% depreciable.

Real Estate Investment Trusts

A real estate investment trust (REIT) is a market intermediary which provides limited diversification and tax-shelter benefits. The REIT must invest primarily in real estate, and therefore can only diversify
across real estate assets. The dividend paid by an equity REIT can include distributions of ordinary income, capital gains, and cash attributable to depreciation. Shareholders pay no tax on this later portion of the dividend but instead reduce the cost basis of their shares to reflect the return of capital. However, a real estate investment trust cannot pass through tax losses to the investor to offset other taxable income. Thus, while an REIT is an attempt by the market to sell the tax-preference benefits of real estate, it is not a substitute for direct ownership.

**Limited Partnership**

The partnership has become the most frequently used form of organization for the tax-shelter enterprises. The tax advantage of the partnership is that the partners are permitted to offset artificial losses of the partnership against income from other sources. Furthermore, partnerships can, to some extent, allocate depreciation deductions among partners in a different proportion than the allocation of taxable cash flow. Thus the higher-tax-rate partner can be given more depreciation deductions whereas the lower-tax-rate partner can be given more taxable income. However, there is the risk that a special allocation of depreciation will not be allowed by the IRS if the allocation has no economic basis other than the avoidance or evasion of income tax. The IRS does not provide definitive guidelines as to how disproportionate these special allocations can be before they are considered to have the sole purpose of tax-avoidance.*

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*A complete discussion of rulings and cases regarding special allocations by limited partnerships can be found in Eck and Pyhrr (1978).
Thus, the partnership provides at least some potential for repackaging and selling tax-shelter benefits. A partner who receives more depreciation, for example, could "pay" for this additional allocation by agreeing to receive less cash flow. The limited partnership also allows partners to overcome the hurdle of personal liability. The general partner assumes this liability along with the responsibility for managing the business.

Real estate has a unique advantage with respect to the use of a partnership as a means of providing artificial losses to the partners. A partner may not deduct losses in excess of the basis of his partnership interest. For limited partnerships other than those engaged primarily in real estate, the partner's initial basis includes only the equity that the partner has "at risk" and does not include a pro-rata share of non-recourse liabilities. Since non-recourse debt can be included in the partner's basis for real-estate partnerships, this facilitates the use of tax losses which exceed the investor's cash investment in the partnership.

Conclusion

Real-estate income property has many tax-preference benefits. Some of the more important benefits are summarized below.

First, real-estate income property has depreciation tax-shelter benefits which result in both deferral of taxes and conversion of income to a capital-gains tax rate. More importantly, this shelter can result in the deferral and conversion of personal-service income in addition to income from the investment itself. Because of this "deepness" of
the shelter, other tax-preferred assets which offer deferral and conversion benefits, but not artificial accounting losses, are only partial substitutes for real estate.

Second, real-estate income property is usually classified as property used in one's trade or business. Thus borrowings on real-estate income property are business expenses and not subject to the interest-deduction limitation. This makes it easier to "sell" taxable income.

Third, real-estate income property is specifically exempted from the "at risk" rule which applies to limited partnerships. Deduction of tax losses are limited to a partner's basis in the partnership. For investments subject to the "at risk" rule, non-recourse debt cannot be included in the partner's basis. Since this does not apply to real estate, it is again easier to lever real estate to sell taxable income from the investment, leaving tax losses which can shelter personal service income.

Fourth, the use of real estate can be by a lease or direct ownership. Separation of the decision to use real estate from the decision to invest in real estate allows the user to "sell" the tax-shelter benefits through leases.

Thus, real-estate income property may have several tax-shelter benefits which are quite different from other tax-preferred assets. There are also distinct components to the tax-shelter characteristics of real estate. These components interact differently with the investor's tax function, and the demand for a particular component could differ from that for another component. For example, accelerated depreciation
results in deferral of taxes. Since accelerated depreciation is also a "tax-preference item", an investor who adds accelerated depreciation could find that any decrease in his ordinary-income tax liability is reduced by an increase in his or her "minimum tax" liability. Also, any direct reduction in taxable income due to losses from accelerated depreciation can be partially or fully offset by the reduction in benefits of the maximum tax on personal-service income.

This interaction of accelerated depreciation with the investor's taxable income is quite different than that of the straight-line depreciation component. Thus these two components of tax shelter are substitutes only to a limited extent. An investor could find additional straight-line depreciation reduces his tax liability whereas additional accelerated depreciation does not. As discussed in Chapter III, investors add each distinct component of tax shelter until at the margin the ratio of after-tax benefit to the market price is equal for each component. Of course, the market price for particular tax-shelter component such as accelerated depreciation could be zero. This implies investors would add accelerated depreciation to their portfolios until the marginal tax benefit is also zero. This would be the point where the investor's total tax liability would actually increase if more accelerated depreciation were added.

The before-tax yield for tax-preferred assets such as real estate reflects the price of those tax-shelter components which have a positive market price. However, empirical work attempting to compare returns for assets such as real estate and stocks has not adequately dealt with
the tax effect on yields. Differences in taxation are often ignored, or an arbitrary tax rate is assumed and after-tax returns are compared. The appraisal and valuation of investment real estate requires an appropriate recognition of differences in both risk and taxation.

The appropriate before-tax discount rate applied to real estate or any asset must clearly reflect the correct market prices for taxable cash flow, tax shelter, and risk. This could involve evaluation of several tax-shelter components and several risk components.

Conceptually, assets could be valued using either after-tax discount rates with the appropriate marginal tax rates specified, or before-tax discount rates with the appropriate yield adjustments to reflect the price of tax shelter. Traditional appraisal methods use "comparable" properties and thus avoid making adjustments for differences in taxation. In this case, there is no need for knowledge as to the prices of the tax-shelter components or the relevant marginal tax rates. If discount rates are to be derived from a broader capital market, either these prices or the marginal tax rates are necessary. In either case, care must be taken to recognize the amount of each type of tax preference present in the property. For example, there is a tendency to use only marginal ordinary-income and capital-gains tax rates. This results in accelerated depreciation always appearing more valuable than straight-line depreciation. However, to capture appropriately the price of excess depreciation requires inclusion of the marginal tax rate on tax-preference items.
Another difficulty in using after-tax discount rates is that all assets may not equate at the same set of marginal tax rates.* As discussed in Chapter's III and IV, only with separability of tax-shelter and cash-flow claims will assets tend to equate at the same tax rate. In the case of uncertain cash flows, investors over a range of tax rates could hold real estate due to diversification benefits. In the case of certainty, there would also be a continuum of breakeven marginal tax rates. Thus the marginal tax rate would only relate an asset to another asset in the next tax-shelter class. In summary, using risk- and tax-adjusted before-tax discount rates might be the most promising approach. Assets with varying amounts and types of tax preference would have to be evaluated to estimate the market price of each type of tax shelter reflected in yield differentials. These prices would in turn be used to arrive at a before-tax discount applicable for a particular real-estate investment which had a specific amount of each tax-shelter component.

*The phrase "set of marginal tax rates" refers to a marginal ordinary-income tax rate, capital-gains tax rate, tax-preference tax rate, etc.
Chapter VIII
SUMMARY AND CONCLUSION

Types of Tax Preference

This study examined a number of factors that would appear to be relevant to the relative prices of assets with differing amounts of tax preference, and the effect of tax preference on an individual's portfolio decision. Chapter II described ways in which different sources of income can affect the investor's current and future income tax. Some income such as long-term capital gains is partially excluded from taxation as ordinary income whereas other income such as interest from municipal bonds is completely exempt from taxation. However, other sources of tax-preferred income can not be described simply in terms of the degree of exclusion from taxation as ordinary income. For example, as income is deferred from taxation as current ordinary income through excess depreciation on real estate, the investor's ordinary-income tax liability may decrease whereas his or her minimum tax on "tax-preference items" may increase. Furthermore, these tax-preference items can reduce the benefits of the maximum tax on personal-service income. Even additional capital-gains exclusion can result in an alternative minimum tax after the investor has a sufficient amount of this types of tax-preferred income. Thus, different sources of tax preference have different effects on the investor's marginal tax liability.
Substitutability of Tax-Preference Vehicles

Tax-preference vehicles also differ in whether it is only the income from the vehicle itself which is less than fully taxed, or if other taxable income, especially personal-service income, can be sheltered. For example, stocks and tax-exempt securities can provide preferential taxation of income from that asset only. Real-estate income property and other deep shelters can also result in preferential treatment of personal-service income. Thus different sources of tax preference have limited substitutability.

For example, a certain amount of dividend income is tax-exempt, after which additional income is fully taxable. Therefore a limited amount of dividend income is a substitute for income from municipal bonds. Retirement plans defer personal-service income until retirement but the individual is limited in the amount which can be sheltered and is penalized for early withdrawal. Real-estate excess depreciation has the potential to defer an unlimited amount of personal-service income with the option of recognizing the income at an earlier point in time, perhaps when other taxable income is low due to losses actually incurred on other investments. But excess depreciation also results in potential alternative minimum taxes each year and reduction in benefits of the maximum tax on personal service income. Thus deferral of personal-service income via retirement plans and real estate are also limited substitutes.

The ability of the individual to use leverage to purchase different assets also differs. This limits the individual in using the interest deductions to "sell" taxable income and retain the tax shelter. For
example, capital-gains income does not increase the individual's investment income for the purpose of the interest-deduction limitation. The individual also cannot deduct interest on money borrowed to purchase tax-exempt bonds. Borrowing for real-estate income property, which is usually considered property "used in one's trade or business" even though it is investment property, is not limited by the interest-deduction limitation. Thus real estate has a comparative advantage in its potential to use leverage to separate the asset's return into fully taxable and tax-sheltered components.

Given the market prices for the different tax-shelter components, the individual allocates funds between different sources of income and tax shelter so that the after-tax risk-adjusted benefit of each is the same. The after-tax risk-adjusted benefit will depend on all the aspects of the individual's current and uncertain future tax function, and government-imposed constraints on the quantity of tax shelter that can be utilized. As the individual increases his attempt to circumvent these constraints, additional risks and costs associated with "audit risk", fines, and penalties are incurred.*

**The Price of Tax Preference**

If there are distinct sources of tax preference, then it is possible that each type would have a different market price, whether somehow purchased as pure shelter or through a particular asset which includes other cash income. In the latter case, the price of tax preference is

*One penalty is, of course, imprisonment which is a risk some individuals are less averse to than others.*
reflected in the before-tax yield calculated for the asset. As discussed in Chapter III, this yield is computed relative to a price which includes the purchase of both the cash-flow and tax-shelter claims for the asset.

The price of each tax-shelter component would, of course, reflect both the demand for, and supply of that particular component. Conversion of investment income to capital-gains treatment is available through many substitute vehicles and in many cases simply requires the investor to hold the asset for a year to be considered a long-term capital gain. Thus the price for conversion of investment income might be relatively low. The sources of exempt income may be less than for conversion. While state and local governments can issue tax-exempt securities, and continuously search for new reasons to do so, the federal government does attempt to limit this supply. The yield differential between tax-exempt and comparable fully taxable assets may therefore reflect a positive price for the benefits of exempting investment income. In fact, based on the previous discussion, most of this price may be for the incremental benefit of exemption over conversion.

Much of the research to determine the price of tax shelter has been directed at yield differentials for tax-exempt municipal bonds. The major investors in tax-exempt securities are high-income individuals, commercial banks, and casualty insurance companies. Changes in the yield for municipal bonds relative to taxable treasury bonds is often attributed to the changing demand for municipals from commercial banks. *

*For a discussion of how these yield differentials have changed over time, see Light and White (1979).
If banks were the dominant force in the market for municipal bonds, the yield ratio would tend to be determined by the bank's marginal tax rate as discussed in Chapter III. However, the demand for municipals from banks is volatile as credit conditions change and substitute tax-shelter devices such as leases are used. Throughout the 1970's commercial banks had been reducing their federal income taxes relative to their before-tax income. Through their holding companies, the large banks were becoming actively involved in leasing activities to provide tax shelter for the earnings of the subsidiary banks. Consequently, the need for tax-exempt income from municipal securities decreased. According to Light and White (1979), from 1972-1976, commercial banks purchased only 30 percent of the new tax-exempt securities, down from 68 percent the previous decade.

As more and more substitutes for tax-exempt income are available in the market, the price of this tax shelter would tend to fall. The ratio of long-term taxable yields to fully taxable yields has been rising and currently approaches 70 percent, which means after-tax yields equate at only a 30 percent marginal tax bracket.

Changes in the supply of municipal bonds must also be considered in an analysis of the pressures on the price for exemption as reflected in yield differentials. Municipalities have the incentive to issue tax-exempt bonds and invest the proceeds directly in otherwise equivalent taxable bonds as long as there is a yield differential. As the supply increased, the yield would eventually have to rise to attract
investors in lower tax brackets.* The supply would increase until the yield differential between taxable and tax-exempt securities was virtually zero for otherwise equivalent securities.

As expected, much legislation has been directed at limiting the supply of tax-exempt municipal securities. The Tax Reform Act of 1969 requires that interest on "arbitrage bonds" is taxable. An arbitrage bond is one where the proceeds are invested in other bonds, taxable government or corporate, providing a higher yield. Only if the proceeds are temporarily invested until needed for the bonds original purpose, are they not considered arbitrage bonds.

Interest on industrial-revenue bonds issued after April 30, 1968, is also taxable unless the bonds are used to fund certain exempt activities, or are part of an issue of one-million dollars or less and substantially all of the proceeds are used to acquire, construct, reconstruct or improve land or depreciable property.**

A recent attempt to increase the supply of tax-exempt municipal bonds is the "mortgage revenue bond." The proceeds of the bond issue are used to provide home mortgages to low and moderate income families at a below-market rate.***

It would appear that an equilibrium price for municipal securities relative to taxable securities, and consequently the price of tax

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* Banks would not add any more municipals when their taxable income became zero. In fact, even though the marginal tax rate is currently 46 percent for most banks, their average tax rate is closer to 10 percent.

** For a further discussion of this legislation, see Drollinger (1979).

*** For an example of an analysis of the effect of the increasing supply of mortgage revenue bonds, see Hendershott (1980).
shelter reflected in exemption of investment income, can only be evaluated in terms of the interaction of market and regulatory forces. The price for this tax shelter, of course, influences that of substitutes such as real estate and equipment leasing. These substitutes, especially sources of shelter for personal-service income, appear to be even more limited, as discussed previously. To provide this type of shelter, the investment must provide some deductions or artificial accounting losses which can offset some or all of the individual's personal service income. As discussed in Chapter II and illustrated in Chapter VII for real estate, this income is actually either deferred and/or converted to capital-gains treatment. This often involves a limited partnership ownership form which allocates income, depreciation, etc. to each partner. As discussed in Chapter VII, real estate is not nearly as restricted as other tax shelters in its potential to supply tax losses. But real estate is a real asset that must also have value in use as a productive capital asset as well as ownership value (including tax benefits). Thus, there is also a less elastic supply of shelter through real estate. This could result in a higher price for the tax-shelter benefits of real estate. But, as shown in Chapter VII, real estate has different elements of tax preference, and the relative amount of each may vary for different types of real estate. The rate of depreciation allowed for the property will depend on whether it is residential or commercial, and whether it is a new or existing building. For example, new residential buildings can be depreciated using a double declining-balance rate whereas existing commercial buildings are limited to straight line. But excess depreciation (over straight line)
interacts with taxable income quite differently from straight-line depreciation. It is quite possible that the market price for excess depreciation is zero, since at any positive price the use of excess depreciation may make some investors worse off. In fact, there are limited partnerships which own new residential buildings that use straight-line depreciation. They would perhaps be attractive to a clientele that had a high marginal tax rate on tax preference items (which includes excess depreciation).

Whereas the market price for excess depreciation may be quite low, the price for straight-line depreciation, especially from properties with artificial accounting losses, may be quite high. The excess of straight-line over economic depreciation can defer and convert other investment income and personal-service income to capital-gains treatment.

The relative prices of each tax-shelter component, the extent to which other tax-shelter devices become available, and the degree to which tax-shelter devices are substitutes, should determine the relative prices for tax-preferred assets. The price for a particular tax-preferred asset such as real estate can only be understood in relation to the broader market for tax-shelter vehicles.

Equilibrium for Tax Shelter

The supply of and demand for different tax-shelter components can be quite different and be reflected in different market prices. How then might the market reach equilibrium? Chapter III indicated that investors attempt to balance the benefits of purchasing more tax shelter
with the benefit of purchasing more taxable income. The investor whose pre-investment ordinary-income tax rate is high, for example, would tend to purchase more tax shelter than fully taxable cash flow. With a progressive tax structure, the investor's taxable income and marginal tax rate will fall. At a lower price for particular tax-shelter component (relative to the prices of other tax shelters and taxable cash flow) a greater quantity will be purchased.

At this lower price, the marginal tax rate at which tax shelter becomes attractive is lowered. This results in some investors purchasing tax shelter who could not previously benefit from the use of the tax shelter. The investors who were using the tax shelter will purchase more since they can lower their marginal tax rate even further.* Equilibrium results when the supply and demand for a particular tax-shelter type are equated. This equilibrium price could be close to zero for some types of tax shelter but quite high for others.

This is a generalization of the equilibrium described by Miller (1977) for corporate debt and stock, which was discussed in Chapter III. In Miller's analysis, investors could obtain one type of shelter, exemption, by purchasing stock. Fully taxable cash flow was purchased through debt. The supply price is, in this case, perfectly elastic, and is determined by the corporate tax rate since this reflects the tax deductibility of debt relative to equity.

* Some individuals, may not purchase more because they are on the borderline of triggering additional tax-preference taxes.
Trading Tax-Shelter Claims

Certain investments have tax-shelter components that are more beneficial to some investors than others, at a given price. Investors who want these tax-shelter benefits therefore seek ways of purchasing them from the market, or from investors for whom the market price exceeds the value of the shelter to them in terms of tax savings. Most assets can be considered as having a fully taxable cash-flow component and one or more tax-shelter components. The way in which investors structure their portfolios to achieve the desired level of taxable cash flow vs. tax shelter depends on 1) the degree of certainty of the cash flow and tax-preference benefits, and 2) the degree to which investors can either structure their portfolios to effectively buy or sell taxable cash flow and tax shelter, or market intermediaries can re-package and sell these claims separately.

In the case of certainty, buying an asset with tax shelter while simultaneously short-selling an asset with the same taxable cash flow would result in a portfolio of only tax shelter. Borrowing with tax-deductible interest payments is another way of selling taxable income. This was discussed in Chapters III and VII. The use of sale-leasebacks as a way of selling tax shelter was also discussed in Chapter VII. These methods of trading tax shelter require the investor to purchase an asset to get its tax-shelter component. Low-tax-rate investors specialize in taxable securities and would sell short tax-preferred assets if possible. Higher-tax-rate investors specialize in the tax-preferred assets while short selling taxable assets and using debt
to the extent possible to sell taxable income.

Assuming the cash-flow component of an asset's return is stochastic makes it more costly to specialize in specific assets. As discussed in Chapter IV, specialization can result in loss of the risk-reduction benefits of a well-diversified portfolio.

Thus there are incentives for a market intermediary to decompose an asset into a stochastic cash-flow claim and a tax-preference claim, and sell these components separately. Real estate investment trusts, limited partnerships, "dual funds" which allow investors to choose between the capital gains and dividend income of a mutual fund, and tax-exempt funds are means through which a claim with a certain cash flow and tax-preference characteristic can be purchased.

If investors could purchase stochastic cash-flow claims separately from tax-shelter claims without constraints, investors would tend to hold identical portfolios of claims on stochastic cash flows while specializing in tax-shelter claims. This gives investors the fullest diversification benefits and the most freedom to buy or sell tax-shelter claims. Thus, we would expect the market to continue to develop new devices to repack these claims.

 Costs of Tax Avoidance

If investors cannot effectively separate tax-shelter and cash-flow claims, the extent to which diversification benefits are given up to receive tax-shelter benefits is one cost of tax avoidance.

The more an individual attempts to shelter income, especially personal-service income, the more likely it is that additional costs will be incurred. Limited partnerships with tax losses are subject to a
high probability of audit, a risk which is even stated in the prospectus of some limited partnerships. Limited partnerships also involve substantial syndication fees, and tax-law changes are often aimed at reducing limited-partnership tax shelters. As shown in Chapter VI, the uncertainty of future tax-preference benefits, whether from tax-law changes, uncertain inflation, or the interaction with actual income or losses realized on investments, is reflected in an asset's risk premium and is therefore an additional cost of tax avoidance. The only thing that seems certain is that there will be tax-law changes (often dubbed "lawyers and accountants relief acts") resulting in a different set of market incentives for tax avoidance. In fact, the Internal Revenue Service partnership training material states: "Although certain business or industrial endeavors are generally associated with tax shelters, you can expect many new variations to replace those outmoded by recent laws, rulings, and court decisions."

Inflation and The Demand for Tax Shelter

The inflation of the 1970's has probably affected the demand for tax shelter for many reasons. First, by rapidly increasing nominal income, more households have been pushed into higher tax brackets. At a given price for tax shelter, it is likely that more investors now have pre-investment marginal tax rates which allow them to benefit from purchasing tax shelter through assets such as real estate and municipal bonds. Relatively high minimum-purchase requirements made it difficult for investors to hold a diversified portfolio of municipals. In

addition, many municipal bonds have limited financial disclosure and no active secondary market with readily available price quotations.

In 1976, legislation was passed which enabled the sale of mutual funds which invested in municipal bonds. Within a few months many different municipal-bond funds were founded. The passage of this legislation might be attributed in part to the demand by investors for a fund with both tax-shelter and diversification services.

A second effect of uncertain inflation is that the individual's future marginal tax rate, which is a function of nominal income, is itself uncertain. The effect of tax-rate uncertainty was modeled in Chapter VI. The recently proposed legislation to "index" tax schedules could be interpreted as a response to this increasing tax rate uncertainty. In fact, Lubell and Lavin (1980) show that past legislation has to some extent resulted in indexation of tax rates.

Robinson and Wrightsman (1980) suggest that tax-exempt securities have an advantage over other kinds of tax shelters since the investor is almost certainly assured of this shelter over the full life of any security he or she purchases. This is not true of such tax shelters as investments induced by oil depletion allowances or cash-flow real estate investments where a change of legislation could strip the shelter from those now holding it. Thus part of the tax-induced yield differential for municipal bonds could be a reflection of the lower tax-rate uncertainty.
Effective Tax Structure

The statutory ordinary-income tax rate schedule facing individuals is progressive. Yet with the many types of tax shelter available, it is possible that the effective tax structure is quite different. Miller and Scholes (1978) argue that individuals are taxed on consumption rather than investment, since it is relatively easy to defer investment income. However, it might not be as easy to defer personal-service income for the following reasons: 1) the limited amounts that can be placed in retirement plans, 2) the supply of shelters such as real estate which have artificial accounting losses may be more limited than shelters for investment income, 3) adding tax preferences which shelter personal-service income can result in a tax-preference tax, reduction in benefits of the maximum tax on personal-service income, and possibly an alternative minimum tax, 4) personal-service income that is deferred which would have been taxed at a maximum tax of 50% can be recaptured at a higher rate when recognized, since it is now investment income, 5) the interest-deduction limitation makes it more difficult to "sell" personal-service income, and 6) tax shelters which are structured to have artificial accounting losses which can shelter personal-service income usually involve limited partnerships which are costly to set up, have a high degree of audit risk, and are more susceptible to tax law changes.

Thus personal-service income appears to bear much of the burden of taxation. If the price of these shelters is relatively high, then
only individuals with sufficient taxable income will purchase them. The price of these shelters will, in turn, determine the maximum effective marginal tax rate that individuals pay. With marginal tax rates for most individuals rising due to inflationary increases in nominal income, coupled with the maximum effective tax rate determined by the price of shelters, it is likely that the marginal tax rate for most individuals is within a rather narrow range, and that the effective tax structure is much less progressive.

It is widely recognized that tax-exempt bonds are an inefficient way of subsidizing state and local governments. In the context of this study, municipalities only receive the "price" of shelter reflected by the marginal tax rate, whereas investors with higher pre-investment marginal tax rates can reduce their federal tax liability further than the amount paid for the shelter. Thus, tax shelter through tax-exempt bonds is not only a subsidy to state and local governments, but also serves to diminish the progressivity of the federal tax structure. Since it is well recognized that a pure subsidy would be more efficient if the goal is only to assist state and local governments, it might be concluded that it is also the intent of Congress to use tax shelters to make the tax structure less progressive than it appears to be from an examination of tax-rate schedules.

*As Kane (1980) has stated, "the huge gaps between statutory and effective tax rates are in an important sense politically optimal."
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