THE EFFECT OF ALTERNATE INFORMATION STRUCTURES ON PROBABILITY REVISIONS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1970

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CHAPTER I

STATEMENT OF THE PROBLEM

Accounting researchers are attempting to discover the effect of information systems on human behavior. This research is designed to examine the impact of two types of information systems on human behavior. The types are single and joint information systems. If an information system, F, includes an information system, f, the information system, F, is a joint system and the information system, f, is a single system. For example, the accounting information system which includes historical and current cost accounting systems is a joint system. The historical and current cost systems are each single systems.

Several bases presently exist for suggesting that no user should find a joint information system inferior to the single systems included in the joint system. These bases are presented in Chapter II. As an additional means of considering the impact of information systems on behavior, this research studies, in an experimental context, the possibility that a confusion effect is present when a user employs the joint information system that is not present when a single system is used. In the experiment, subjects are asked to make intuitive probabilistic estimates on the basis of messages generated
from the two systems. These intuitive estimates are compared to probabilities derived through the application of Bayes theorem, considered to be an optimal form of estimation. It is hypothesized that the differences between Bayesian and intuitive estimates will be greater for users of the joint information system because of a confusion effect.

The experimental setting and type of subject also are varied to determine whether a confusion effect is present in a number of experimental situations.

As is described in detail in later chapters, the experimental results do not refute the hypothesized confusion effect. In addition, the statistical evidence indicates an interaction of the type of experimental setting and type of subject.

Previous Accounting Research Dealing with the Impact of Information Systems on Human Behavior

Accounting researchers have used a variety of approaches in examining the effect of information systems on behavior. Dyckman and Bruns compare decisions made under alternate inventory valuation methods; their studies indicate that users of different accounting

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methods do not necessarily make different decisions. Jensen asks stock analysts to evaluate prospectuses of two hypothetical companies using different depreciation and inventory valuation methods; his results suggest that alternate depreciation methods and alternate inventory valuation methods can change analysts' evaluations of a company's potential. Livingstone examines rate making behavior of public utility commissions; he hypothesizes that a learning set may enable commissions to incorporate changes in accounting methods in their rate making decisions.

Mock compares one information system to a more timely information system; he computes an expected theoretical difference between profits of companies under the two systems and finds that this computation is less than an observed experimental difference between

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The current research examines probability revision behavior based on data generated by single and joint information systems. Chapter II contains a detailed description of the type of information systems used in this study. The remainder of this chapter is devoted to a discussion of the research assumptions and hypotheses. The major sections of this discussion are: (1) messages from information systems; (2) the relationship between messages and probability; (3) research assumptions concerning individual behavior; and (4) the design of the experiment.

**Messages from Information Systems**

Messages from an information system discriminate among the underlying set of objects on which the system operates. The ring of an alarm clock discriminates among times of day; a stop sign discriminates among possible operations of an automobile; a book cover discriminates among possible contents of the volume.

Numerical messages from more complex information systems similarly discriminate among an underlying set of objects. The mileage indicator discriminates among possible distances a vehicle has traveled; grade point averages discriminate among possible sets of grades.

If an information system associates a single message with each object of an underlying set (although the same message may be
associated with different objects), the manner in which the system's 
messages discriminate the underlying set is described by a partition 
of the underlying set. The partition is a collection of subsets of 
the underlying set, and each subset in the collection consists of 
all objects which have a particular message. For instance, if the 
underlying set were minutes of the day, the ring of the alarm clock 
would partition the set into two subsets. One subset would consist 
of all the minutes of the day except the minute at which the alarm 
sounds; the other subset would consist only of the minute of the day 
at which the alarm sounds. 6

A complex example is the case in which the underlying set con-
sists of businesses and the information system associates a profit 
figure with each business. The partition of the underlying set con-
tains many subsets; each of these subsets contains businesses which 
have a particular profit figure.

The Relationship between Messages 
and Probability

If it is assumed that there is a probability distribution on 
the underlying set of objects, then the probability of receiving a 
particular message can be derived; it is the sum of the probabilities 
of those objects for which the information system produces the parti-
cular message. For instance, assume that the underlying set of ob-

6 The concept of a partition is developed extensively in the 
first chapter of Yuji Ijiri, The Foundations of Accounting Measure-
jects is possible rolls of a die. If the probability is one-sixth that a particular face of the die will be rolled, and the information system states whether the number rolled is above or below 3.5, then the probabilities can be derived for receiving the two messages: "above 3.5" and "below 3.5." Both probabilities are 1/2 (1/6 + 1/6 + 1/6).

The importance of the relationship between messages and the probability distribution on the underlying set of objects is that the probability of a particular outcome given the message can be derived. For instance, in the case of the die, the message that the roll is "above 3.5" reduces the set of possible outcomes to \{4, 5, 6\} and the probability of the outcome, 4, given the message that the number rolled is "above 3.5" is 1/3.

In the case in which the information system is an accounting system, Beaver argues that individuals may develop probability distributions of stock prices given information from annual earnings announcements.

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Research Assumptions Concerning Individual Behavior

It is assumed in this research that (1) individuals develop personal probabilities of events given messages from information systems and that (2) individuals can develop these probabilities intuitively. The critical words in the assumptions are "personal" and "intuitively." Personal means that probabilities belong to individuals and are not necessarily universal. Intuitively means that probability is not determined by explicitly considering and manipulating a mathematical formula.

Some writers question the validity of the first assumption. The two points of view, as presented in Fellner,9 will be described briefly, but no attempt will be made to justify either position.

The set of writers who do not accept the notion of personal probabilities are called frequentists; they argue that the concept of probability is only applicable to restricted sets of physical events like the flipping of a coin. These events are distinguished because they are repeatable and because there is no appreciable difference between repetitions of these events. The probability associated with this type of event often is called "objective" probability.

A second set of writers holds that probability is applicable to an individual's opinions and feelings concerning non-repeatable types of events. For example, these writers believe that it is pos-

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sible for a person to assign his personal probability that the Dow Jones Industrial Average will increase in the coming year even though every year is different. It is entirely possible that one person's probability concerning a particular event will differ from another person's probability. This type of probability usually is called "subjective."

The basis for the assumption that estimates of probability are made intuitively is research that suggests that people can make probabilistic estimates without making computations. Peterson and Beach describe experiments in which subjects are asked to estimate means, variances, correlation coefficients, and other statistical properties of probability distributions. The experimental evidence indicates that subjects are capable of accurately estimating many of these statistical properties intuitively.\(^\text{10}\)

Several experimenters have examined subjects' abilities to make probabilistic estimates after subjects have received a piece of data. These probabilistic estimates are made within the widely used experimental paradigm which follows.

A subject is shown two bags, \(B_1\) and \(B_2\). \(B_1\) contains \(p_1\) percent red poker chips and \(1 - p_1\) percent white poker chips. \(B_2\) contains \(p_2\) percent red poker chips and \(1 - p_2\) percent white poker chips.

\(^{10}\)Peterson and Beach, "Men as an Intuitive Statistician," pp. 29-46.
The experimenter flips a coin and chooses $B_1$ or $B_2$ on the basis of the way the coin lands (heads or tails). The subject does not know the result of the coin flip or which bag is chosen. The subject does know $p_1$ and $p_2$ and that the bag was chosen by flipping a coin. The experimenter draws a chip from the bag chosen and shows the subject the chip. On the basis of his knowledge of the contents of the bags, the subject estimates the probability that the chip was drawn from $B_1$.

Subjects' estimates are compared to a probability computed using Bayes theorem, which in this specific case is:

$$P(B_1 | D) = \frac{P(D | B_1) P(B_1)}{\sum_{i=1}^{2} P(D | B_i) P(B_i)}$$

where

- $B_i$ denotes Bag $i$
- $D$ denotes the datum, the observed poker chip
- $P(B_i | D)$ denotes the probability of $B_i$ given the datum
- $P(D | B_i)$ denotes the probability of the datum given $B_i$
- $P(B_i)$ denotes the prior probability of choosing $B_i$

If prior to receiving the datum, the subject's probability that $B_1$ is the chosen bag is equated to 1/2, then the difference between the subject's estimate after receiving the datum and 1/2 represents the subject's change in probability that $B_1$ is the chosen bag.

---

11 A more general description of Bayes theorem is given in Chapter II.
Similarly, if \( P(B_1) \) is equated to \( 1/2 \), then the difference between \( P(B_1 \mid D) \) and \( 1/2 \) would represent the change in probability resulting from the application of Bayes' theorem.

When these two differences are compared, the sign of the difference is usually the same; however, the magnitude of the Bayesian difference usually exceeds the magnitude of the subject's difference. This phenomenon is often called conservatism.

The Design of the Experiment

In this research, Bayes' theorem is used to examine probabilistic estimates of subjects based on messages from information systems. In addition, several variables that may affect these estimates are simultaneously examined.

A subject considers a set of objects, \( X \), which has a probability distribution. The set is divided into two mutually exclusive and exhaustive subsets, \( T_1 \) and \( T_2 \). Each object in \( X \) can be represented by a 4-tuple of numbers. An object is drawn from the set \( X \), and an information system operates on the object. The subject is shown the message which the information system associates with the object. On the basis of his knowledge of the probability distribution of the objects in the underlying set and his knowledge about the information system, the subject estimates the probability that the object with the particular message was drawn from subset, \( T_1 \).

The experimenter varies the number of information systems with which the subject may deal, the experimental setting, and the
type of subject.

There are single and joint information systems used in the experiment. The single information system associates a single number with an object; the joint information system, which includes the single system, associates two numbers with an object.

The experimental setting is manipulated by varying the underlying set of objects, \( X \), by varying the subsets, \( T_1 \) and \( T_2 \), of \( X \), and by varying the information systems. In one case the underlying set, \( X \), consists of types of businesses. The coordinates of the 4-tuples which represent the types of businesses indicate particular dimensions of the types of businesses considered. The coordinates indicate (1) units of sales in a given year; (2) units in beginning inventories in the given year; (3) units purchased for inventories in the given year; and (4) whether the type business is in the subset, \( T_1 \), the subset of \( X \) which has stock price increases in the year following the given year, or the subset \( T_2 \), the subset of \( X \) which does not have stock price increases in the year following the given year.

The single information system, a simplified historical cost accounting system, associates a single profit figure with each type of business. The joint information system, a combination of the simplified historical cost accounting system and a simplified current cost accounting system, associates two profit figures with each type of business. In the second experimental setting, the underlying set \( X \) consists of cubes each of which has three blank sides, one red side, one yellow side, and one blue side. The coordinates of the 4-tuples
that represent the cubes indicate particular dimensions of the cubes considered. The coordinates indicate (1) the number on the red side of the cube; (2) the number on the yellow side of the cube; (3) the number on the blue side of the cube; and (4) whether the cube is in subset, T₁, the subset of cubes in Tumbler 1, or T₂, the subset of cubes in Tumbler 2. The single information system, a simple algebraic statement, associates a single number with each cube. The joint information system, which includes the single system just described, associates two numbers with each cube.

There are two sets of subjects used in the experiment. The first set of subjects is a group of businessmen in The Ohio State University Executive Development Program. The second set of subjects is a group of students in The Ohio State University Undergraduate Business Administration Program.

**Summary**

This chapter contains a description of the nature of information systems and the probabilistic context within which information systems are examined. In particular, an experiment designed to explore probabilistic revision behavior is described, and the experimental variables are discussed. Later chapters contain the mathematical structure of the experiment, the theoretical significance of the manipulated variables, a complete description of the actual experiment, a discussion of the statistical model, the analysis of the data, and the conclusions.
CHAPTER II

THE MATHEMATICAL STRUCTURE

The purposes of this chapter are to detail the mathematical structure underlying this investigation of the effect of information systems on human behavior and to state the rationale for comparing revision behavior based on messages received from single information systems to revision behavior based on messages received from joint information systems.

The Underlying Mathematical Structure

The brief description in Chapter I of a typical probability revision experiment indicates that Bayes theorem, the probability a fair coin lands heads, and the percentages of colored poker chips in two bags constitute the mathematical bases of such an experiment. When information systems are introduced into a probability revision experiment, the mathematical structure is expanded.

The mathematical structure underlying this experiment is the combination of 1) the probability model underlying the experiment, 2) the information systems underlying the experiment, and 3) Bayes theorem modified to reflect the impact of messages received from single and joint information systems on probability estimates. Set
theory will be used to develop the structure.\footnote{An excellent discussion of set theory is presented in Ijiri, The Foundations of Accounting Measurement, pp. 167-83.}

The Probability Model Underlying the Experiment

It is assumed that \( X \) stands for a finite set of objects of interest and \( P \) stands for a probability function defined on \( X \), i.e.,

\[
0 \leq P(x) \quad \text{for all } x \in X
\]

and

\[
\sum_{x \in X} P(x) = 1.
\]

For any subset \( A \) of \( X \), the function \( Pr \) is defined by

\[
Pr(A) = \sum_{x \in A} P(x) \quad \text{where } A \subseteq X \quad \text{(meaning, } A \text{ is a subset of } X).\]

In this discussion, any subset \( A \) of \( X \) is also considered to be an event, and \( Pr(A) \) represents the probability of event \( A \) occurring.\footnote{It can be shown that \( Pr \) is a probability set function on \( X \). See, Robert V. Hogg and Allen T. Craig, Introduction to Mathematical Statistics (2nd ed.; New York: The MacMillan Company, 1969), pp. 16-18. (Hereafter referred to as Mathematical Statistics.)}

The probability of an event \( A \) occurring given that an event \( B \) occurs is denoted by \( Pr(A|B) \) and defined by the equation

\[
Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)} \quad \text{assuming } Pr(B) \neq 0
\]

where \( A \cap B = \{x|x \in A \text{ and } x \in B\} \).\footnote{The notation \( \{x|Z(x)\} \) will be used frequently. It refers to all objects in an underlying set which have property \( Z \). Thus, \( \{x|x > 4 \text{ and } x \text{ is real}\} \) denotes the subset of all real numbers greater than 4. \( \{x|x \in A \text{ and } x \in B\} \) denotes the subset of all objects that are in subset \( A \) and in subset \( B \).}
The Information Systems Underlying the Experiment

Throughout this study, the only information systems to be considered will be those that can be characterized as a function whose domain \( X \) is a finite set of possible objects and whose range is a finite set of possible messages. Not all information systems are functions. For instance, Ijiri describes a representational system which is a relation but not a function.\(^4\) The single and joint information systems used in this study are special cases of the general systems Ijiri describes.

**Single information systems**

A single information system is defined to be a function, \( f \), which assigns a single message, \( m \), to an \( x \in X \). Letting \( M \) stand for the set of all possible messages, we have

\[
f: X \rightarrow M.
\]

The partition of \( f \) on \( X \)

Consider a specific message, \( m \); there may be many objects in \( X \) to which \( f \) assigns the message, \( m \). The subset of \( X \) to which \( f \) assigns the message \( m \) is denoted by \( \{ x \in X | f(x) = m \} \). The set of all such subsets of \( X \) is the partition of \( X \) formed by \( f \):

\[
\{ x \in X | f(x) = m \} \forall m \in M.
\]

A partition of a set \( X \) can be defined in the following manner. "A partition of a set \( X \) is a disjoint collection \( \mathcal{C} \) of nonempty and

distinct subsets of $X$ such that each member of $X$ is a member of some member of $\mathcal{R}$.

Since the union of sets in
\[
\{ \{ x \mid f(x) = m \} \mid m \in M \} \cup \{ \{ x \mid f(x) = m' \} \mid m' \in M \}
\]
is equal to $X$, and since
\[
\{ x \mid f(x) = m \} \cap \{ x \mid f(x) = m' \} = \emptyset \quad \text{for } m \neq m',
\]
the collection of sets, $\{ \{ x \mid f(x) = m \} \mid m \in M \}$ meets the requirements of the above definition. $\emptyset$ is the null set.

**Example 1.** Suppose $X = \{1, 2, 3\}$ and $M = \{0, 1\}$, and $f$ is denoted by
\[
f(1) = 0, \quad f(2) = 0, \quad f(3) = 1.
\]
Then
\[
\{ x \mid f(x) = 0 \} = \{1, 2\}
\]
\[
\{ x \mid f(x) = 1 \} = \{3\}
\]
and
\[
\{ \{ x \mid f(x) = m \} \mid m \in \{0, 1\} \} = \{\{1, 2\}, \{3\}\}.
\]
Thus, $\{\{1, 2\}, \{3\}\}$ represents the partition of $X$ formed by $f$.

**Joint information systems**

If $f$ and $g$ represent two single information systems such that $f \colon X \to M$ and $g \colon X \to W$, then a joint information system is defined by:

---


6$\{x_1, x_2, \ldots, x_n\} = \{ x \mid x = x_1 \text{ or } x = x_2 \ldots \text{ or } x = x_n \}$
\( F: X \rightarrow M \times W \) such that \( F(x) = (f(x), g(x)) \) for all \( x \in X \).\(^7\)

The partition of \( X \) formed by \( F \)

The subset of \( X \) to which \( F \) assigns the message, \((m, w)\), is denoted by

\[ \{ x | f(x) = m \text{ and } g(x) = w \} \].

The partition of \( X \) formed by \( F \) is

\[ \{ \{ x | f(x) = m \text{ and } g(x) = w \} | (m, w) \in M \times W \} \].

**Example 2.** Assume \( X, f \) and \( M \) are as given in Example 1.

\( W = \{0, 1\} \) and \( g \) is denoted by

\[ g(1) = 1, \; g(2) = 0, \; g(3) = 0. \]

Then

\[ \{ x | f(x) = 0 \text{ and } g(x) = 0 \} = \{2\} \]
\[ \{ x | f(x) = 0 \text{ and } g(x) = 1 \} = \{1\} \]
\[ \{ x | f(x) = 1 \text{ and } g(x) = 0 \} = \{3\} \]
\[ \{ x | f(x) = 1 \text{ and } g(x) = 1 \} = \emptyset \text{ where } \emptyset \text{ is the null set} \]

and

\[ \{ \{ x | f(x) = m \text{ and } g(x) = w \} | (m, w) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\} \} \]
\[ = \{ \{2\}, \{1\}, \{3\}, \emptyset \}. \]

Thus, \( \{ \{2\}, \{1\}, \{3\}, \emptyset \} \) is the partition of \( X \) formed by \( F \).

**Bayes Theorem Modified to Reflect the Impact of Messages Received from Single and Joint Information Systems**

**Bayes theorem**

If \( H_1 \) and \( H_2 \) are subsets of \( X \) such that \( H_1 \cap H_2 = \emptyset \) and

\[ M \times W = \{(m, w) | m \in M \text{ and } w \in W \}. \]
\[ H_1 \cup H_2 = X \text{ where } H_1 \cup H_2 = \{ x \mid x \in H_1 \text{ or } x \in H_2 \} \], and if \( A \) is a subset of \( X \), then

\[
Pr(H_1 \mid A) = \frac{Pr(A \mid H_1)Pr(H_1)}{\sum_{i=1}^{2} Pr(A \mid H_i)Pr(H_i)} \quad \text{assuming } \sum_{i=1}^{2} Pr(A \mid H_i)Pr(H_i) \neq 0.
\]

The above equality is Bayes’ theorem.\(^8\)

To modify Bayes’ theorem, it is necessary to establish the probability of receiving a message, \( m \in M \), from a single information system, \( f \), and the probability of receiving a message, \( (m, w) \in M \times W \), from a joint information system, \( F \).

The probability of receiving a message \( m \in M \) from a single information system, \( F \)

If \( m \) is a member of \( M \), the probability of receiving the message \( m \) is equivalent to the probability that a member of \( X \) is drawn from the subset whose elements all have the message \( m \), i.e., \( \{ x \mid f(x) = m \} \). The probability of receiving a message \( m \) is given by

\[
Pr(\{ x \mid f(x) = m \}) = \sum_{x \in \{ x \mid f(x) = m \}} P(x)
\]

Example 3

If \( P(1) = \frac{1}{4} \), \( P(2) = \frac{1}{4} \), \( P(3) = \frac{1}{4} \), and \( f \) is given as in Example 1, then

\[
Pr(\{ x \mid f(x) = 0 \}) = Pr(\{ 1, 2 \})
\]

\[
= \frac{1}{4} + \frac{1}{4}
\]

\[
= \frac{3}{4}.
\]

\(^8\)Bayes theorem is proved in Hogg and Craig, *Mathematical Statistics*, p. 54.
The probability of receiving a message \((m,w)\in M\times W\) from a joint information system, \(F\)

If \((m,w)\) is a member of \(M\times W\), the probability of receiving the message \((m,w)\) is equivalent to the probability that a member of \(X\) is drawn from the subset whose elements all have the message \((m,w)\), i.e., \(\{x|f(x) = m\text{ and } g(x) = w\}\). The probability of receiving the message \((m,w)\) is given by

\[
Pr(\{x|f(x) = m\text{ and } g(x) = w\}) = \sum_{x\in\{x|f(x) = m\text{ and } g(x) = w\}} P(x)
\]

Bayes theorem modified to reflect the impact of messages received from a single information system, \(f\)

Assuming that \(T_1\) and \(T_2\) denote two events of interest such that \(T_1 \cup T_2 = X\) and \(T_1 \cap T_2 = \emptyset\), Bayes theorem as it applies to a specific event, \(T_1\), and a specific message, \(m\), from a single information system, \(f\), is denoted by

\[
Pr(T_1|\{x|f(x) = m\}) = \frac{Pr(\{x|f(x) = m\}| T_1)Pr(T_1)}{\sum_{i=1}^{2} Pr(\{x|f(x) = m_i|T_1\})Pr(T_1)}
\]

\[
= \frac{Pr(\{x|f(x) = m_2\cap T_1\})}{\sum_{i=1}^{2} Pr(\{x|f(x) = m_i\cap T_1\})}
\]

\[
= \frac{\sum_{x\in\{x|f(x) = m_2\cap T_1\}} P(x)}{\sum_{i=1}^{2} \sum_{x\in\{x|f(x) = m_i\cap T_1\}} P(x)}
\]

Bayes theorem modified to reflect the impact of messages received from a joint information system, \(F\)

Assuming that \(T_1\) and \(T_2\) denote two events of interest such that
\[ T_1 \cup T_2 = X \text{ and } T_1 \cap T_2 = \emptyset, \] Bayes theorem as it applies to a specific event \( T_1 \) and a specific message \((m,w)\), from a joint information system \( \mathcal{F} \) is denoted by

\[
\frac{\sum P(x)}{\sum_{i=1}^{2} \sum P(x) \bigg| \sum P(x) = m \text{ and } g(x) = w \cap T_1}
\]

**The Rationale for Comparing Revision Behavior Based on Messages Received from Single Information Systems to Revision Behavior Based on Messages Received from Joint Information Systems**

In the first part of this chapter, the mathematical structure underlying the experiment is developed. The mathematical structure can be used to compare single and joint information systems.

Ijiri provides a basis for comparing single and joint information systems in terms of the relative imperfection of the two systems. \(^9\) Marschak provides a basis for comparing single and joint information systems in terms of the utility of the systems. \(^10\) Posner offers an information reduction theory that indicates that the comparison of probability revision behavior under single and joint information systems may be a basis for comparing the systems.

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Bases for Comparison Provided by
Ijiri and Marschak

To determine the relative imperfection of two types of information systems, Ijiri uses the concept "fineness of a partition." One information system has a finer partition than a second information system if every member in the partition of the first information system is a subset of some member in the partition of the second information system.\(^{11}\) If one information system has a finer partition than a second information system, the first system is less imperfect than the second system.\(^{12}\)

To demonstrate that a joint information system, \(F\), is finer than a single information system, \(f\), consider a member of the partition of \(X\) formed by \(F\): \(\{x \mid f(x) = m\text{ and } g(x) = w\}\). Let \(x_o \in \{x \mid f(x) = m\}\) and \(g(x) = w\). This implies that \(f(x_o) = m\) and \(g(x_o) = w\). It follows that \(x_o \in \{x \mid f(x) = m\}\). Thus, \(\{x \mid f(x) = m\} \subseteq \{x \mid f(x) = m\}\). In other words, \(F\) is finer than \(f\) and, thus, less imperfect than \(f\).

In information economics, Jacob Marschak suggests that if there are no costs to implementing the information system and if one information system is finer than a second, the utility of the finer system is at least as great as the utility of the less fine system. Such an approach indicates that for any individual, the utility of a joint information system, \(F\), is at least as great as the utility of a single

\(^{11}\)Since every subset is a member of itself, it follows that every partition is finer than itself.

information system, f.13

Posner's Information Reduction Theory

Although the above arguments support the use of the joint information system, F, the additional information provided by the joint system may confuse the system's user and hinder his ability to incorporate the information.

Posner's rationale provides the logical support for a confusion effect of a joint information system. His theory applies to information reduction tasks.14 The theory is that task difficulty is directly related to the amount of information reduced. The following paragraphs describe 1) a typical experimental task and measurement techniques for the theory and 2) the application of the theory to the current experiment.

Typical information reduction tasks are classification and concept identification tasks.15 In classification tasks, for example, a subject knows he will observe one of several equally likely stimuli.


The subject observes a stimulus; he then announces to which of several equally likely classes he feels the stimulus belongs. Such an experiment is one for which the stimulus set may consist of people's names. The subject is asked to classify the names as male or female names.

In these tasks, the number of stimuli is frequently manipulated. It has been found that as stimulus uncertainty (the number of stimuli) increases, the task difficulty (measured by reaction time) increases.\(^{16}\)

**Information reduction measurement**

Typical measurements of psychological tasks like classification and concept identification are founded on information theory.\(^{17}\) The basic measurement is the following. If

\[
\{x_1, \ldots, x_t\}
\]

denotes a set of \(t\) objects,

\[
\{P(x_1), \ldots, P(x_t)\}
\]

denotes a set such that \(0 \leq P(x_i) \leq 1\) for all \(i\), and

\[
P(x_i)
\]

denotes the probability of drawing an object, \(x_i\), from the set \(x_1, \ldots, x_t\), then the average amount of information (uncertainty) remaining after knowing which object is drawn from the set is given by\(^{18}\)


\(^{17}\)For descriptions of the incorporation of information theory in psychology, see Garner, Uncertainty and Structure, pp. 310–37, and Fred Attneave, Applications of Information Theory to Psychology: A Summary of Basic Concepts, Methods and Results (New York: Holt, Rinehart and Winston, 1959).

\(^{18}\)All logarithms used in this chapter are logarithms to the base 2.
\[
\sum_{i=1}^{t} -P(x_i) \log P(x_i)
\]

Posner suggests that task difficulty is a function of the amount of information reduction in the task (i.e., uncertainty reduction). The amount of information reduction in a task is the difference between stimulus information (uncertainty) and response information (uncertainty). In classification tasks in which the probability of the \(i\)th stimulus being drawn is \(P(x_i) = 1/t\) for all \(t\) stimuli, the stimulus uncertainty is varied by varying \(t\). Stimulus uncertainty is given by

\[
\sum_{i=1}^{t} -P(x_i) \log P(x_i) = \sum_{i=1}^{t} -1/t \log 1/t
\]

\[= \log t\]

If the number of classes is held constant, response information is assumed constant for all possible \(t\). As \(t\) increases, \(\log t\) increases, thus the difference between stimulus and response information increases and task difficulty should increase.

A formula essential to the application of Posner's theory to the current Bayesian revision experiment follows. The proof of the formula is included in the appendix to this chapter. The formula will be used to demonstrate that implicit information reduction in a joint information system task is at least as great as implicit information reduction in a single information task.

Suppose \(\{A_1, \ldots, A_k\}\) and \(\{A_1, \ldots, A_d\}\) are collections of subsets of \(X = \{x_1, \ldots, x_t\}\) such that for each of the collections, the union of its members is \(X\), and the intersection of any two distinct
members of a particular collection is the empty set. Furthermore, every member of \( \{A_1, \ldots, A_j\} \) is a subset of some member of \( \{A_1, \ldots, A_k\} \). Then

\[
\sum_{i=1}^{k} \Pr(A_i) \sum_{j=1}^{N(i)} -P(x_{ij}) \log \left( \frac{P(x_{ij})}{\Pr(A_i)} \right) \geq \sum_{i=1}^{j} \Pr(A'_i) \sum_{j=1}^{N'(i)} -P(x_{ij}) \log \left( \frac{P(x_{ij})}{\Pr(A'_i)} \right)
\]

Ineq. (1)

where \( N(i) \) stands for the number of elements in \( A_i \); \( N'(i) \) stands for the number of elements in \( A'_i \); \( \{x_{i1}, \ldots, x_{iN(i)}\} \) is the set \( A_i \);

\( \{x_{i1}, \ldots, x_{iN'(i)}\} \) is the set of elements \( A'_i \); \( \Pr(A_i) = N(i) \sum_{j=1}^{P(x_{ij})} \)

\( \Pr(A'_i) = N'(i) \sum_{j=1}^{P(x_{ij})} \).

Application of Posner's theory to the current Bayesian revision experiment

The Posner theory can be applied to the current experiment by examining two probability revision formulas and the inequality above. Such an examination involves 1) the separation of the operations involved in making probability revisions; 2) the definition of a probability function on a set of interest, \( T_1 \); 3) the association of the structure of the probability experiment with Ineq. (1); 4) the measurement of average uncertainty after receiving a message from an information system; 5) the demonstration that average information reduction for the joint information system, \( F \), is at least as great as for the
single information system, f; and 6) a discussion of the implications for the experiment.

The separation of the operations involved in making probability revisions

The probability revision formula to determine the probability of an event, $T_1$, given message $m$ from a single information system, $f$: $X \rightarrow M$, is given by

$$
\frac{\sum P(x)}{x \in \{x | f(x) = m\} \cap T_1}
$$

$$
2 \sum_{i=1}^{2} \frac{\sum P(x)}{x \in \{x | f(x) = m\} \cap T_1}
$$

The probability revision formula to determine the probability of an event, $T_1$, given a message $(m, w)$ from a joint information system $F$: $X \rightarrow M \times W$ (F such that $F(x) = (f(x), g(x))$ where $g$: $X \rightarrow W$), is given by

$$
\frac{\sum P(x)}{x \in \{x | f(x) = m \text{ and } g(x) = w\} \cap T_1}
$$

$$
2 \sum_{i=1}^{2} \frac{\sum P(x)}{x \in \{x | f(x) = m \text{ and } g(x) = w\} \cap T_1}
$$

Although subjects do not explicitly use the probability revision formulas in the experiment, the formulas will be examined as if the revisions were made explicitly. The examination is meant to suggest a possible difference in task difficulty between single and joint information systems.

19 Supra, p. 19.

20 Supra, p. 20.
Each of the formulas can be separated into two sequences of operations. The first sequence consists of information reductions, and the second sequence consists of arithmetical manipulations. For information system $f$, the information reduction involves reducing sets $T_1$ and $T_2$ to those members which could have the message $m$. For the information system $F$, the information reduction involves reducing sets $T_1$ and $T_2$ to those members which could have the message $(m,w)$. For both systems, the arithmetical manipulations consist of two summations followed by a division.

It is assumed that the arithmetical manipulations possess the same level of difficulty for both the single and joint systems, $f$ and $F$; however, it will be suggested that the difficulty of information reduction may differ.

The definition of a probability function on the set of interest, $T_1$

To establish the amount of information reduction under the single and joint information systems, a conditional probability function will be defined on $T_1$, where $T_1$ is a subset of interest of $X$, a set which has a probability function $P$ defined on it.

Let $\{x_1, \ldots, x_t\}$ denote the set of objects, $T_1$, and define $P'(x_i)$ by the equation

$$P'(x_i) = \frac{P(x_i)}{Pr(T_1)}$$

where $x_i \in T_1$.

It can be shown that $P'$ is a probability function defined on $\{x_1, \ldots, x_t\} = T_1$. Furthermore, for a subset of $\{x_1, \ldots, x_t\}$, say
\[ A_j = \{x_{j1}, \ldots, x_{jN(j)}\} \] with an associated set of conditional probabilities \( P'(x_{j1}), \ldots, P'(x_{jN(j)}) \), the conditional probability of event \( A_j \) occurring is given by

\[ \text{Pr}'(A_j) = \sum_{i=1}^{N(j)} P'(x_{ji}) \]

where \( x_{ji} \) is the \( i \)th member of \( A_j \).

The association of the structure of the probability experiment with the elements of Ineq. (1)

If the set of \( k \) messages in the single information system, \( f \), is labeled \( \{m_1, \ldots, m_k\} \) and the set of \( s \) messages in the joint information system, \( F \), is labeled \( \{(m,w)_1, \ldots, (m,w)_s\} \), the sets of \( A_j \) and \( A'_j \) are defined by

\[ A_j = \{x_1 f(x_1) = m_j\} \]

and

\[ A'_j = \{x_1 f(x_1) = (m,w)_j\} \]

it can be shown that \( \{A_1, \ldots, A_k\} \) and \( \{A'_1, \ldots, A'_s\} \) are both collections of subsets of \( \{x_1, \ldots, x_t\} \) such that for each of the collections, the union of its members is \( X \), and the intersection of any two distinct members of a particular collection is the empty set. Furthermore, every member of \( \{A'_1, \ldots, A'_s\} \) is a subset of some member of \( \{A_1, \ldots, A_k\} \)
\{A_1, \ldots ,A_k\}. 21

Let \(N(i)\) stand for the number of elements in \(A_i\) and \(N'(i)\) stand for the number of elements in \(A'_i\). Let \(x_{i1}, \ldots ,x_{iN(i)}\) denote the elements in \(A_i\) and \(x'_{i1}, \ldots ,x'_{iN'(i)}\) denote the elements in \(A'_i\).

The measurement of average uncertainty after receiving a message from single and joint information systems

It is assumed that a person has temporarily restricted his consideration to the set of objects, \(T_1\), consisting of objects \(\{x_1, \ldots ,x_t\}\) so that the probability associated with \(x_e\), a member of \(T_1\), is

\[P'(x_e) = \frac{P(x_e)}{\Pr(T_1)}\]

If a message \(m_1\) is received (i.e., \(x_e \in \{x_d \mid f(x_d) = m_1\}\)), then the condi-

\[\text{To demonstrate that the union of } \{A_1, \ldots ,A_k\} \text{ is}

\(\{x_1, \ldots ,x_t\}\) assume \(x_0 \in \{x_1, \ldots ,x_t\}\); then, \(f(x_0) = m_j\) for some \(m_j \in \{m_1, \ldots ,m_k\}\). Thus, \(x_0 \in \{x_1\mid f(x_1) = m_j\}\). Thus, \(x_1 \in A_j\). Furthermore, if \(x_0 \in A_j\), then \(x_0 \in \{x_1, \ldots ,x_t\}\). We conclude that the union of members of \(\{A_1, \ldots ,A_k\}\) is \(X\).

To demonstrate that the intersection of any two distinct members of \(\{A_1, \ldots ,A_k\}\) is the empty set, assume that there exist two sets, \(A_e, A_j\), such that \(e \neq j\), and assume each has a common element \(x_0\). Then, \(x_0 \in \{x_i \mid f(x_i) = m_e\}\) and \(x_0 \in \{x_i \mid f(x_i) = m_j\}\). This implies that \(f(x_0) = m_e\) and \(f(x_0) = m_j\) where \(e \neq j\). It could be then concluded that since \(m_e \neq m_j\), \(f\) is not a function, which is a contradiction. Thus, the intersection of any two members of \(\{A_1, \ldots ,A_k\}\) is the empty set.

A similar proof could be used for \(A_1, \ldots ,A_s\).

To show that every member of \(\{A_1, \ldots ,A_k\}\) is a subset of some member of \(\{A_1, \ldots ,A_s\}\), let \(A_j\) denote a member of \(\{A_1, \ldots ,A_k\}\). Then let \(x_0\) denote a member of \(A_j\). It follows that \((m,w)_j = f(x_0) = (f(x_0), g(x_0))\). Thus, \(f(x_0) = m\) where \(m\) is some member of \(\{m_1, \ldots ,m_k\}\), say \(m_e\), and hence \(x_0 \in \{x_i \mid f(x_i) = m_e\}\). Thus every member of \(\{A_1, \ldots ,A_k\}\) is a subset of \(\{A_1, \ldots ,A_k\}\).
tional probability associated with $x_e$ given the message $m_1$ becomes

$$\frac{P'(x_e)}{Pr'(A_1)}.$$ 

Since $x$ is an $x_{ij} \in A_1$, the uncertainty after receiving the message $m_1$ becomes

$$\sum_{j=1}^{N(i)} \frac{-P'(x_{ij})}{Pr'(A_1)} \log \left[ \frac{P'(x_{ij})}{Pr'(A_1)} \right].$$

On the average, the amount of uncertainty after receiving a message from a single information system is

$$\sum_{i=1}^{k} Pr'(A_i) \sum_{j=1}^{N(i)} \frac{-P'(x_{ij})}{Pr'(A_i)} \log \left[ \frac{P'(x_{ij})}{Pr'(A_i)} \right].$$

where the probability of receiving message $m_1$ from the single information system is denoted by

$$\sum_{j=1}^{N(i)} P'(x_{ij}) = Pr'(A_1).$$

On the average, the amount of uncertainty after receiving a message from a joint information system is

$$\sum_{i=1}^{s} Pr'(A_i) \sum_{j=1}^{N'(i)} \frac{P'(x_{ij})}{Pr'(A_i)} \log \frac{P'(x_{ij})}{Pr'(A'_i)}.$$ 

where the probability of receiving message $(m, w)_1$ from the joint information system is

$$\sum_{j=1}^{N'(i)} P'(x_{ij}) = Pr'(A'_1).$$
The demonstration that average information reduction for the joint information system, $F$, is at least as great as for the single information system, $f$

Applying Ineq. (1) to the preceding statements which describe the average amount of uncertainty, we see that the average uncertainty after receiving a message from a single information system, $f$, is greater than the average uncertainty after receiving a message from the joint information system, $F$.

$$\sum_{i=1}^{k} \Pr'(A_i) \sum_{j=1}^{N(i)} \frac{-P'(x_{ij})}{\Pr'(A_i)} \log \frac{P'(x_{ij})}{Pr'(A_i)} \geq$$

$$\sum_{i=1}^{s} \Pr'(A_i') \sum_{j=1}^{N'(i)} \frac{-P'(x_{ij})}{\Pr'(A_i')} \log \frac{P'(x_{ij})}{Pr'(A_i')}$$

Ineq. (2)

Multiplying both sides of Ineq. (2) by $-1$ and adding

$$\sum_{r=1}^{t} -P'(x_r) \log P'(x_r)$$

(the amount of uncertainty before receiving a message) to both sides of Ineq. (2), we have

$$\sum_{r=1}^{t} -P'(x_r) \log P'(x_r) - \sum_{i=1}^{k} \Pr'(A_i) \sum_{j=1}^{N(i)} \frac{-P'(x_{ij})}{\Pr'(A_i)} \log \frac{P'(x_{ij})}{Pr'(A_i)} \leq$$

Ineq. (3)

$$\sum_{r=1}^{t} -P'(x_r) \log P'(x_r) - \sum_{i=1}^{s} \Pr'(A_i') \sum_{j=1}^{N'(i)} \frac{-P'(x_{ij})}{\Pr'(A_i')} \log \frac{P'(x_{ij})}{Pr'(A_i')}$$

From Ineq. (3) we find that
\[ \sum_{i=1}^{k} \sum_{j=1}^{N(i)} \frac{-p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_i)} - \sum_{j=1}^{N'(1)} \frac{p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_i)} \leq \sum_{i=1}^{s} \sum_{j=1}^{N'(i)} \frac{-p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_{i'})} \]

From Ineq. (4), we find that

\[ \sum_{i=1}^{k} \sum_{j=1}^{N(i)} \frac{-p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_i)} - \sum_{j=1}^{N'(1)} \frac{p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_i)} \leq \sum_{i=1}^{s} \sum_{j=1}^{N'(i)} \frac{-p'(x_{ij}) \log p'(x_{ij})}{Pr'(A_{i'})} \]

Ineq. (5)

Inequality (5) suggests that the average amount of information reduction (uncertainty prior to receiving a message from an information system less uncertainty after receiving the message) is always at least as great for the joint information system as for the single information system.

Implications of applying Posner's theory to the current Bayesian revision experiment

The purpose of relating Posner's information reduction theory to the current experiment involving probability revision behavior is to indicate that explicit performance of the experimental task never requires more information reduction when a single information system is used than when a joint information system is used.
In both tasks involving single information systems and tasks involving joint information systems, the subject considers first a subset $T_1$ of an underlying set $X$ and then a subset $T_2$ of an underlying set $X$. The information reduction in both single and joint information tasks involves eliminating those members of $T_1$ which do not yield a particular message received and eliminating those members of $T_2$ which do not yield a particular message received. When the subject's attention is focused on $T_1$ the information (uncertainty) prior to receiving the message is measured by

$$\sum_{i=1}^{t} -P'(x_i) \log P'(x_i)$$

where $P'(x_i) = P(x_i)/Pr(T_1)$ and $x_i \in T_1$.

Measures of average information (uncertainty) after a message is received are developed for joint and single information systems. When subset $T_1$ is the subset of interest, the amount of information reduction $\overline{\Delta}$ the difference between information (uncertainty) prior to receiving a message and information (uncertainty) after receiving a message in the explicit performance of the single information task never exceeds the information reduction in the explicit performance of the joint information system task. An identical result can be shown for $T_2$.

Posner's theory is that as the amount of information to be reduced increases, the difficulty of the task increases. Applying this theory indicates that task difficulty for explicit performance of the probability revision task using the single information system never ex-
ceeds task difficulty for explicit performance of the probability revision task using the joint information system. Although subjects do not explicitly compute their probability estimates, it is hypothesized that differences in the degree of task difficulty in the explicit performance of the task may be reflected in the intuitive performance of the task by a confusion effect.

Summary

In this chapter, the mathematical structure underlying the experiment was developed. The mathematical structure has been used to indicate Ijiri's and Marschak's bases for selecting a joint over a single information system. Also, the mathematical structure has been used to predict that in the current probability revision experiment there may be a confusion effect present when the joint information system is used that is not present when the single information system is used.

In Chapter III, the two additional variables manipulated in the experiment --- the nature of the underlying set $X$ and the type of subject --- will be discussed.
APPENDIX TO CHAPTER II

The proof of Ineq. (1) is presented here because it is necessary for the conclusion that task difficulty on the average is at least as great under a joint information system, \( F \), as under a single information system, \( f \). The author is not aware of the existence of the proof in other literature.

The proof requires several lemmas. After the lemmas are proved, the major theorem is presented.

**Lemma 1**

Let \( 0 \leq P(x_r) \leq 1 \) for all \( r = 1, \ldots, t \) and assume

\[
\sum_{r=1}^{t} P(x_r) \leq 1.
\]

It will be shown that

\[
\left( \sum_{r=1}^{t} P(x_r) \right)^{t} \geq \prod_{r=1}^{t} \left[ P(x_r) \right]^{P(x_r)}
\]

Another way to state the lemma is

\[
(P(x_1) + P(x_2) + \ldots + P(x_t)) \geq \left[ P(x_1) \right]^{P(x_1)} \left[ P(x_2) \right]^{P(x_2)} \cdots \left[ P(x_t) \right]^{P(x_t)}
\]
Proof of Lemma 1

\[ P(x_1) + P(x_2) + \ldots + P(x_r) \geq P(x_1) \]
\[ P(x_1) + P(x_2) + \ldots + P(x_r) \geq P(x_2) \]
\[ \vdots \]
\[ P(x_1) + P(x_2) + \ldots + P(x_r) \geq P(x_r) \]

\[ \left[ P(x_1) + P(x_2) + \ldots + P(x_r) \right] \begin{bmatrix} P(x_1) \end{bmatrix} \geq \begin{bmatrix} P(x_1) \end{bmatrix} \begin{bmatrix} P(x_1) \end{bmatrix} \]
\[ \left[ P(x_1) + P(x_2) + \ldots + P(x_r) \right] \begin{bmatrix} P(x_2) \end{bmatrix} \geq \begin{bmatrix} P(x_2) \end{bmatrix} \begin{bmatrix} P(x_2) \end{bmatrix} \]
\[ \vdots \]
\[ \left[ P(x_1) + P(x_2) + \ldots + P(x_r) \right] \begin{bmatrix} P(x_r) \end{bmatrix} \geq \begin{bmatrix} P(x_r) \end{bmatrix} \begin{bmatrix} P(x_r) \end{bmatrix} \]

\[ \begin{bmatrix} P(x_1) + P(x_2) + \ldots + P(x_r) \end{bmatrix} \begin{bmatrix} P(x_1) \end{bmatrix} \begin{bmatrix} P(x_2) \end{bmatrix} \ldots \begin{bmatrix} P(x_r) \end{bmatrix} \begin{bmatrix} P(x_r) \end{bmatrix} \begin{bmatrix} P(x_r) \end{bmatrix} \]
\[ \geq \begin{bmatrix} P(x_1) \end{bmatrix} \begin{bmatrix} P(x_1) \end{bmatrix} \begin{bmatrix} P(x_2) \end{bmatrix} \begin{bmatrix} P(x_2) \end{bmatrix} \ldots \begin{bmatrix} P(x_r) \end{bmatrix} \begin{bmatrix} P(x_r) \end{bmatrix} \begin{bmatrix} P(x_r) \end{bmatrix} \]

Q.E.D.

Corollary

Suppose \( \{x_1, \ldots, x_r\} \) is separated into \( k \) mutually exclusive and exhaustive subsets. By mutually exclusive and exhaustive it is meant that the union of these subsets is \( \{x_1, \ldots, x_r\} \), and the intersection of any two of the subsets is the empty set. Denote the \( j \)th subset by \( A_j \). Denote the members of \( A_j \) by \( \{x_{j1}, x_{j2}, \ldots, x_{jN(j)}\} \) where \( N(j) \) denotes the number of members of \( A_j \).

It then follows from Lemma 1 that
\[
\begin{pmatrix}
N(1) \\
N(2) \\
\sum_{i=1}^{N(k)} p(x_{1i}) + \sum_{i=1}^{N(k)} p(x_{2i}) + \ldots + \sum_{i=1}^{N(k)} p(x_{ki})
\end{pmatrix}
\begin{pmatrix}
N(1) \\
N(2) \\
\sum_{i=1}^{N(k)} p(x_{1i}) + \sum_{i=1}^{N(k)} p(x_{2i}) + \ldots + \sum_{i=1}^{N(k)} p(x_{ki})
\end{pmatrix}^{-1}
\]
\[
\begin{pmatrix}
N(j) \\
\sum_{i=1}^{N(k)} p(x_{ki})
\end{pmatrix}
\geq
\begin{pmatrix}
k \\
\sum_{j=1}^{N(k)} \sum_{i=1}^{N(j)} p(x_{ki})
\end{pmatrix}
\]
\[
\begin{pmatrix}
N(1) \\
\sum_{i=1}^{N(1)} p(x_{1i})
\end{pmatrix}
\begin{pmatrix}
N(2) \\
\sum_{i=1}^{N(2)} p(x_{2i})
\end{pmatrix}
\ldots
\begin{pmatrix}
N(k) \\
\sum_{i=1}^{N(k)} p(x_{ki})
\end{pmatrix}
\]
\[
\begin{pmatrix}
\text{Pr}(A_1) \\
\text{Pr}(A_1')
\end{pmatrix}
\begin{pmatrix}
\text{Pr}(A_2) \\
\text{Pr}(A_2')
\end{pmatrix}
\ldots
\begin{pmatrix}
\text{Pr}(A_k) \\
\text{Pr}(A_k')
\end{pmatrix}
\]

**Lemma 2**

Suppose \(\{A_1, \ldots, A_k\}\) and \(\{A_1', \ldots, A_s'\}\) are two mutually exclusive and exhaustive sets of subsets of \(\{x_1, \ldots, x_t\}\). Furthermore, suppose every member in \(\{A_1', \ldots, A_s'\}\) is a subset of some member of \(\{A_1, \ldots, A_k\}\). It will be shown that every member of \(\{A_1, \ldots, A_k\}\) which is not in the null set can be expressed as the union of subsets in \(\{A_1', \ldots, A_s'\}\).

Let \(A_j\) denote such a member of \(\{A_1, \ldots, A_k\}\). Since \(A_j\) is not empty, it must contain some elements, say \(\{x_{j1}, \ldots, x_{j N(j)}\}\). Now each \(x_{ji}\) must be in \(\{x_1, \ldots, x_t\}\) since \(A_1, A_2, \ldots, A_k\) are subsets of
\{x_1, \ldots, x_n\}. Then, since \(\{A'_1, \ldots, A'_s\}\) is exhaustive, each of the elements in \(\{x_{j1}, \ldots, x_{jN(j)}\}\) must be in some subset of \(\{A'_1, \ldots, A'_s\}\).

Let us label the sets \(A''_{j1} \ldots A''_{ju}\) to which \(x_{j1}, \ldots, x_{jN(j)}\) belongs.

We claim that

\[
A''_{j1} \cup A''_{j2} \ldots \cup A''_{ju} = A_j.
\]

To demonstrate this, it must be shown that

\[
(A''_{j1} \cup A''_{j2} \ldots \cup A''_{ju}) \subseteq A_j \quad \text{and that} \quad A_j \subseteq (A''_{j1} \cup A''_{j2} \ldots \cup A''_{ju}).
\]

Now \(A_j \subseteq (A''_{j1} \cup A''_{j2} \ldots \cup A''_{ju})\), because of the manner in which the subsets, \(A''_{j1} \ldots A''_{ju}\) were chosen. Suppose it were possible for some set \(A''_{j1}\) to have a member which is not in \(A_j\). The \(A''_{j1}\) could not be a subset of any member of \(\{A_1, \ldots, A_k\}\). We conclude that

\[
A_j = (A''_{j1} \cup \ldots \cup A''_{ju}).
\]

Q.E.D.

**Lemma 3**

Suppose \(\{A_1, \ldots, A_k\}\) and \(\{A'_1, \ldots, A'_s\}\) are two mutually exclusive and exhaustive sets of subsets of \(\{x_1, \ldots, x_n\}\), a subset of the range of a probability function. Furthermore, suppose every member of \(\{A'_1, \ldots, A'_s\}\) is a subset of some member of \(\{A_1, \ldots, A_k\}\). For a specific member of \(\{A_1, \ldots, A_k\}\) we know from Lemma 2 that

\[
A_j = (A''_{j1} \cup \ldots \cup A''_{ju})
\]

where \(A''_{j1} \ldots A''_{ju}\) are as selected in Lemma 2. It will be shown that

\[
\sum_{i=1}^{N(j)} p(x_{ji}) \log \Pr(A_j) \geq \sum_{i=1}^{u} \sum_{r=1}^{N''(i)} p(x_{ir}) \log \Pr(A''_{ji}) \quad \text{Ineq. (6)}
\]

where \(N(j)\) stands for the number of elements in \(A_j\), and \(N''(i)\) stands
for the number of elements in $A_{j1}'$. Also, $\{x_{j1}^1, \ldots, x_{jN(j)}^1\}$ is the set of elements in $A_j$ and $\{x_{i1}', \ldots, x_{iN''(i)}\}$ is the set of elements in $A_{j1}'$.

The expression on the left side of Ineq. (6) may be rewritten:

$$\sum_{i=1}^{N(j)} P(x_{j1}) \log Pr(A_j) = \left[ \log Pr(A_j) \right] \sum_{i=1}^{N(j)} P(x_{j1})$$

Since

$$\sum_{i=1}^{N(j)} P(x_{j1}) = Pr(A_j)$$

we have

$$\sum_{i=1}^{N(j)} P(x_{j1}) \log Pr(A_j) = Pr(A_j) \log Pr(A_j).$$

By examining the expression on the right side of Ineq. (6), we find that

$$\sum_{i=1}^{N''(i)} \sum_{r=1}^{N''(i)} P(x_{ir}) \log Pr(A_{j1}')$$

$$= \sum_{i=1}^{u} \log Pr(A_{j1}') \sum_{r=1}^{N''(i)} P(x_{ir})$$

Since

$$\sum_{r=1}^{N''(i)} P(x_{ir}) = Pr(A_{j1}')$$

we have

$$\sum_{i=1}^{u} \sum_{r=1}^{N''(i)} P(x_{ir}) \log Pr(A_{j1}') = \sum_{i=1}^{u} Pr(A_{j1}') \log Pr(A_{j1}').$$

From the corollary to Lemma 1, we know that
\[
\left[ \Pr(A_j) \right] \left[ \Pr(A_j') \right] = \left[ \sum_{i=1}^{u} \frac{\Pr(A''_{ij})}{\sum_{i=1}^{u} \Pr(A''_{ij})} \right] \geq \\
\Pr(A''_{j1}) \Pr(A''_{j2}) \ldots \Pr(A''_{ju})
\]

The preceding statement implies that
\[
\log \Pr(A_j) \geq \log \left[ \Pr(A''_{j1}) \ldots \Pr(A''_{ju}) \right] \\
= \log \Pr(A''_{j1}) + \log \Pr(A''_{j2}) + \ldots + \log \Pr(A''_{ju})
\]

Using the fact that \( \log a^b = b \log a \), we conclude

\[
\Pr(A_j) \log \Pr(A_j) \geq \\
\Pr(A''_{j1}) \log \Pr(A''_{j1}) + \ldots + \Pr(A''_{ju}) \log \Pr(A''_{ju}) \quad Q.E.D.
\]

**The Major Theorem**

Suppose \( \{ A_1, \ldots, A_k \} \) and \( \{ A'_1, \ldots, A'_s \} \) are two sets of mutually exclusive and exhaustive subsets of \( \{ x_1, \ldots, x_t \} \). Furthermore, suppose every member of \( \{ A'_1, \ldots, A'_s \} \) is a subset of some member of \( \{ A_1, \ldots, A_k \} \).

The theorem which will be demonstrated is

\[
\sum_{i=1}^{k} \frac{N(i)}{\Pr(A_i)} \sum_{j=1}^{N(i)} \frac{-P(x_{ij})}{\Pr(A_i)} \log \frac{P(x_{ij})}{\Pr(A_i)} \geq \\
\sum_{i=1}^{s} \frac{N'(i)}{\Pr(A'_i)} \sum_{j=1}^{N'(i)} \frac{P(x_{ij}')}{\Pr(A'_i)} \log \frac{P(x_{ij}')}{\Pr(A'_i)}
\]
In this instance, \( N(i) \) stands for the number of elements in \( A_i \); \( N'(i) \) stands for the number of elements in \( A'_i \); \( \{x_{i1}, \ldots, x_{iN'(i)}\} \) is the set of elements in \( A_i \); and \( \{x_{i1}, \ldots, x_{iN'(i)}\} \) is the set of elements in \( A'_i \).

The theorem is important if \( \{A_1, \ldots, A_k\} \) are thought of as categories of \( \{x_1, \ldots, x_n\} \). The relevance of the theorem can be established in the following manner.

If we are told that an object belongs to a particular category, say \( A_i \), the uncertainty concerning what the actual object is, after being told what category the object is in, is denoted by

\[
N(i) \sum_{j=1}^{N(i)} \frac{-P(x_{ij})}{Pr(A_i)} \log \frac{P(x_{ij})}{Pr(A_i)}
\]

On the average, by knowing the category, the uncertainty will be

\[
\sum_{i=1}^{k} \frac{Pr(A_i) N(i)}{\sum_{j=1}^{N(i)} \frac{-P(x_{ij})}{Pr(A_i)} \log \frac{P(x_{ij})}{Pr(A_i)}}
\]

This theorem relates to the quality of a set of categories and suggests that if every member in a set of categories is a subset of a member of a second set of categories, then the average uncertainty, knowing from which category an object comes, is less for the first set of categories than for the second set of categories.

From Lemma 3, we know that

\[
\sum_{i=1}^{N(j)} P(x_{ij}) \log Pr(A_j) \geq \sum_{i=1}^{u} \sum_{r=1}^{N''(i)} P(x_{ir}) \log Pr(A''_{ij})
\]
where $N(j)$ stands for the number of elements in $A_j$; $N''(i)$ stands for the number of elements in $A_{j1}''$, the $i$th subset of $j$ in $\{A_1', \ldots, A_s'\}$. Furthermore, $\{x_{11}, \ldots, x_{iN''(i)}\}$ is the set of elements in $A_{j1}''$.

The statement above implies that

\[(a) \quad \sum_{j=1}^{k} \sum_{i=1}^{N(j)} p(x_{ji}) \log \Pr(A_j) \leq \sum_{j=1}^{k} \sum_{i=1}^{N''(i)} \sum_{r=1}^{u} p(x_{ir}) \log \Pr(A''_{j1}) \]

Now

\[(b) \quad \sum_{j=1}^{k} \sum_{i=1}^{u} \sum_{r=1}^{N''(i)} p(x_{ir}) \log \Pr(A''_{j1}) = \sum_{j=1}^{s} \sum_{i=1}^{N'(j)} p(x_{ji}) \log \Pr(A'_j) \]

To understand this, note that every member in the triple summation above is the probability in one member of an element in $\{A_1', \ldots, A_s'\}$ times the log of the probability of the member it is in. This implies that it is a member of the second summation.

Furthermore, a term in the sum cannot occur more than once since this would imply that some $A'_j$ is a subset of more than one $A_j$.

A similar statement can be made for the double summation. We conclude that they must be equivalent. Thus, from (a) and (b), we have

\[\sum_{j=1}^{k} \sum_{i=1}^{N(j)} p(x_{ji}) \log \Pr(A_j) \geq \sum_{j=1}^{s} \sum_{i=1}^{N'(j)} p(x_{ji}) \log \Pr(A'_j)\]

\[\Rightarrow \sum_{j=1}^{k} \sum_{i=1}^{N(j)} p(x_{ji}) \log p(x_{ji}) + \sum_{j=1}^{k} \sum_{i=1}^{N(j)} p(x_{ji}) \log \Pr(A_j) \geq \sum_{j=1}^{s} \sum_{i=1}^{N'(j)} p(x_{ji}) \log p(x_{ji}) + \sum_{j=1}^{s} \sum_{i=1}^{N'(j)} p(x_{ji}) \log \Pr(A'_j)\]
\[ \Rightarrow - \sum_{j=1}^{k} \sum_{i=1}^{N(j)} p(x_{ji}) \log p(x_{ji}) - \sum_{j=1}^{s} \sum_{i=1}^{N'(j)} p(x_{ji}) \log \frac{p(x_{ji})}{\Pr(A_j)} \]

Making the transformation \( i=j \) and multiplying and dividing by \(-\Pr(A_i)\) and \(\Pr(A'_i)\), we have

\[ \sum_{i=1}^{k} \Pr(A_i) N(i) \sum_{j=1}^{\Pr(A_i)} -p(x_{ij}) \log \frac{p(x_{ij})}{\Pr(A_i)} \geq \]

\[ \sum_{i=1}^{s} \Pr(A'_i) N'(i) \sum_{j=1}^{\Pr(A'_i)} -p(x_{ij}) \log \frac{p(x_{ij})}{\Pr(A'_i)} \]

Q.E.D.
CHAPTER III

TWO VARIABLES: THE EXPERIMENTAL SETTING AND THE TYPE OF SUBJECT

This chapter examines the experimental variables: the experimental setting and the type of subject. The experimental setting is varied by interchanging the objects in the set \( X \), by interchanging the subsets of \( X \) of interest, \( T_1 \) and \( T_2 \), and by interchanging the information systems.

There are two experimental settings. In the first setting, the objects in the set \( X \) are hypothetical businesses, and the information systems are simplified accounting systems that assign a given year's profit figure to the businesses. The subsets of \( X \) of interest are \( T_1 \), the subset of businesses whose stock prices increase in the year following the given year, and \( T_2 \), the subset of businesses whose stock prices do not increase in the year following the given year. In the second setting, the objects are cubes with three blank sides, one red side, one yellow side, and one blue side. Each of the colored sides of the cubes is imprinted with a number. The information systems are algebraic identities applied to the numbered sides of the cubes. The subsets of \( X \) that are of interest are subset \( T_1 \), the cubes in a tumbler labeled Tumbler 1, and subset \( T_2 \), the cubes in a tumbler labeled
Tumbler 2.

The types of subjects are businessmen in The Ohio State University Executive Development Program and students in The Ohio State University Undergraduate Business Administration Program.

The experimental setting and the type of subject may be related variables. An exploration of the possible relationship between the variables, a detailed presentation of the relationship between the types of experimental settings, and a prediction of the experimental outcomes resulting from the relationships of the variables follow.

The Possible Relationship between the Experimental Setting and the Type of Subject

To suggest a possible relationship between the experimental variables, the experimental setting and the type of subject, the complete experiment is diagramed in Table 1. The diagram is used to suggest that the type of subject and the type of experimental setting may interact. The theoretical support for the possible interactions also is examined.

The Diagram of the Experiment

Table 1 is a diagram of the experiment. In the diagram, the combination of a specific experimental setting and specific type of subject is called an experimental situation.
<table>
<thead>
<tr>
<th>Type of Setting</th>
<th>Hypothetical Businesses and Profit Figures</th>
<th>Cubes and Algebraic Identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students in the Ohio State University Undergraduate Business Administration Program</td>
<td>Single          Joint Info. vs. Info. System System</td>
<td>Single          Joint Info. vs. Info. System System</td>
</tr>
</tbody>
</table>

Table 1 indicates that in the four experimental situations, probability revision behavior based on messages from single information systems is compared to probability revision behavior based on messages received from joint information systems. The four situations are 1) businessmen in The Ohio State University Executive Development Program making probabilistic revisions based on profit figures generated by a hypothetical set of businesses; 2) businessmen in The Ohio State University Executive Development Program making probabilistic revisions based on numbers generated by applying algebraic identities to the numbered sides of cubes; 3) students in The Ohio State University Undergraduate Business Administration Program making probabilistic
revisions based on profit figures generated by a hypothetical set of businesses; and 4) students in The Ohio State University Undergraduate Business Administration Program making probabilistic revisions based on numbers generated by applying algebraic identities to the numbered sides of cubes.

The Interaction of the Experimental Setting and the Type of Subject

The hypothesis stated in Chapter II is that a confusion effect may be present when intuitive probability revisions are made based on messages from joint information systems. The development of the hypothesis is not dependent on the type of subject or the type of experimental setting; thus, the hypothesis should be applicable to any type of subject or setting. This research seeks confirmation of the hypothesis in four experimental situations in which the subjects and the setting are varied.

These situations are designed to manipulate the familiarity of the experimental setting for each type of subject. Experimental settings built using the business language of profits and stock price changes are assumed to be more familiar to businessmen subjects than experimental settings built using cubes and the abstract symbolic language of algebraic identities. Experimental settings built using cubes and the abstract symbolic language of algebraic identities are assumed to be more familiar to students than experimental settings built using the business language of profits and stock price changes. Although the student may be aware of the terms "business," "profit,"
and "stock price change," students have not dealt with these terms in a probabilistic setting.

The Theoretical Support for the Interaction of the Type of Subject and Type of Setting

Several researchers have suggested that the familiarity of the experimental setting may alter subjects' performances on a particular task. In regard to experiments designed to measure the impact of accounting information systems on human behavior, Birnberg and Nath suggest that lack of familiarity of the task can hinder the experimenter's ability to generalize his results to the "real world."\(^1\) Although this research does not deal with the "real world" problem, it does examine the degree to which the experimental settings' familiarity affects the validity of the hypothesis of the confusion effect.

Accounting researchers have considered familiarity to be an important variable; psychological researchers have suggested that familiarity of the setting of a probabilistic task may alter subjects' intuitive probability revisions.

DuCharme and Peterson have conducted two experiments which use height distributions of males and females. The heights are normally distributed for each sex. In the experiments, an attempt is made to determine if the amount of conservatism is affected by the familiarity of this task. DuCharme and Peterson find conservatism less than for traditional probability revision experiments using bags and poker

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chips.\textsuperscript{2}

DuCharme and Peterson suggest that the experimental result of reduced conservatism may be related to two aspects of familiarity. The first aspect is that the distributions are normal; the second aspect is that the distributions are defined on very familiar objects, heights.\textsuperscript{3} Thus, the reduced conservatism may be a reflection of the type of distribution or the type of object or both.

\textbf{A Detailed Presentation of the Relationships between Types of Experimental Settings}

In this experiment, the objects in the set $X$, the subsets of set $X$, and the information systems are manipulated to produce a familiarity effect upon subjects. To understand how the familiarity of the setting is varied, it is necessary to examine the relationships of the experimental settings more thoroughly.

For each of the experimental settings, a message from either a single or a joint information system has a counterpart in the other setting. For instance, a profit figure has a specified probability of occurrence and a specified probabilistic relationship to the event, stock price increase. The counterpart of the profit figure message is a specific number generated by applying an algebraic identity to the numbered cubes. The number has the same probability of occurrence as

\textsuperscript{2}Supra, pp. 8-10.

\textsuperscript{3}Wesley M. DuCharme and Cameron R. Peterson, "Intuitive Inference about Normally Distributed Populations," \textit{Journal of Experimental Psychology}, 78, No. 2, pp. 269-75. (Hereafter referred to as "Intuitive Inference.")
the profit figure. Furthermore, the number has the same probabilistic relationship to the event, being in Tumbler 1, as the profit figure has to the event, stock price increase.

To examine this system of counterparts, several relationships are considered: the relationship between cubes and businesses, the relationship between the simple accounting information systems and the algebraic identities, and the relationship between probabilities of messages.

The Relationship between Cubes and Businesses

Both settings can be represented by using 4-tuples. If $X'$ stands for the set of 4-tuples which represents hypothetical businesses, $X'$ will consist of 4-tuples such as $(x_1', x_2', x_3', x_4')$ where $x_1' \in \{200,000; 300,000; 400,000\}$, the possible levels of units of a given year's sales; $x_2' \in \{0; 100,000; 200,000\}$, the possible levels of units in beginning inventory for a given year; $x_3' \in \{100,000; 200,000; 300,000\}$, the possible levels of units of purchases for inventory for a given year; and $x_4' \in \{1, 2\}$, where $x_4'$ denotes whether the hypothetical business which the tuple represents has a stock price increase (1) or does not have a stock price increase (2) in the year following the given year. No tuples exist for which the units of sales exceed the

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4Subjects are told that the number of units of sales, the number in units of beginning inventories, and the number of units purchased for inventories are figures rounded to the nearest 100,000. Furthermore, subjects explicitly deal with 3-tuples related to the probability of a stock price increase or no stock price increase given the particular 3-tuple. However, implicitly, the subjects are dealing with 4-tuples as stated in this theoretical development.
sum of the units in beginning inventories and the units purchased for inventories.

For example, the tuple, \((200,000; 100,000; 200,000; 1)\), denotes a type of business that has 200,000 units of sales in a given year; 100,000 units in beginning inventories in a given year; and 200,000 units purchased for inventories in a given year; and has a stock price increase in the year following the given year.

The second experimental setting uses cubes as objects. If \(X''\) stands for the set of 4-tuples which represent cubes, \(X''\) will consist of 4-tuples such as \((x''_1, x''_2, x''_3, x''_4)\) where \(x''_1 \in \{2, 3, 4\}\), the set of possible numbers on the red side of the cube; \(x''_2 \in \{0, 1, 2\}\), the set of possible numbers on the yellow side of the cube; \(x''_3 \in \{1, 2, 3\}\), the set of possible numbers on the blue side of the cube; and \(x''_4 \in \{1, 2\}\), where \(x''_4\) denotes whether the cube is in Tumbler 1 (1) or is not in Tumbler 1 (2).\(^5\) No tuples exist in the experiment for which the numbers on the red side of the cube exceed the sum of the numbers on the yellow and blue sides of the cube.

For example, the tuple, \((2, 1, 2, 1)\), stands for a cube that has 2 on the red side, one on the yellow side, 2 on the blue side, and is in Tumbler 1.

The 4-tuples representing businesses and cubes are related by a one to one and onto mapping, \(W\), from the set of businesses to the set of cubes.

\(^5\) In this portion of the experiment, when it is presented to subjects, mnemonic labels are used such as \(R\) for the number on the red side of the cube, \(Y\) for the number on the yellow side of the cube, and \(B\) for the number on the blue side of the cube.
$W$ is denoted by

$$W(x_1', x_2', x_3', x_4') = (x_1'/100,000, x_2'/100,000, x_3'/100,000, x_4')$$

Furthermore, if $P'$ denotes the probability function used in the experiment on the set of 4-tuples in $X'$, and $P''$ denotes the probability function used in the experiment on the set of 4-tuples in $X''$, then the probability functions are related by the equation

$$P'(x_1', x_2', x_3', x_4') = P''(W(x_1', x_2', x_3', x_4'))$$

The equality says that the probability associated with a particular type of business is equal to the probability associated with a particular type of cube, namely the cube whose first three coordinates are the first three coordinates of the hypothetical business divided by 100,000 and whose last coordinate is equal to the last coordinate of the hypothetical business.

The Relationship between the Simplified Accounting Information Systems and the Algebraic Identities

When presented in the experiment, the simplified accounting information systems are developed in the typical language and format of an accounting text. These accounting information systems compute profits for the set of hypothetical businesses using the assumption that all sales are $4.00 per unit, all units in beginning inventories of all companies cost $2.00 per unit, all purchases for inventories for all companies cost $3.00 per unit, all operating expenses for all companies are $100,000, and the tax rate for all companies is .50.
The accounting information systems are simplified historical cost, $f_b$, and simplified current cost, $g_b$, systems which assign profit figures to the hypothetical set of companies. Specifically, $f_b$ is given by

$$f_b(x_1', x_2', x_3', x_4') = \frac{4x_1' - 2x_2' - 3(x_1' - x_2') - 100,000}{2}$$

and $g_b$ is given by

$$g_b(x_1', x_2', x_3', x_4') = \frac{4x_1' - 3x_1' - 100,000}{2}$$

The single accounting information system examined in the experiment is $f_b$ and the joint accounting information system examined is $F_b$, which is denoted by

$$F_b(x_1', x_2', x_3', x_4') = (f_b(x_1', x_2', x_3', x_4'), g_b(x_1', x_2', x_3', x_4')) = \left(\frac{4x_1' - 2x_2' - 3(x_1' - x_2') - 100,000}{2}, \frac{4x_1' - 3x_1' - 100,000}{2}\right)$$

When presented in the experiment, the information systems for cubes are developed using algebra. The systems are stated in terms of algebraic identities. Subjects are told that a value, called a Z value, can be computed for a cube and that the Z value is computed as follows:

$$Z = \frac{4R - 2Y - 3(R - Y) - 1}{2}$$

where $R$ denotes the number on the red side of the cube, and $Y$ denotes the number on the yellow side of the cube.

The Z values and an additional set of values called Q values define information systems on the set of 4-tuples in $x''$ that represent the

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$^6 f_b$ incorporates a first in first out inventory flow assumption.
cubes. These systems, called $f_c$ and $g_c$, associate numbers with cubes.

The functions are given by

$$f_c(x''_1, x''_2, x''_3, x''_4) = \frac{4x''_1 - 2x''_2 - 3(x''_1 - x''_2) - 1}{2}$$

and

$$g_c(x''_1, x''_2, x''_3, x''_4) = \frac{4x''_1 - 3x'' - 1}{2}$$

The single information system with respect to the cubes is $f_c$ and the joint information system is $F_c$ which is given by

$$F_c(x''_1, x''_2, x''_3, x''_4) = (f_c(x''_1, x''_2, x''_3, x''_4), g_c(x''_1, x''_2, x''_3, x''_4)) = \left( \frac{4x''_1 - 2x''_2 - 3(x''_1 - x''_2) - 1}{2}, \frac{4x''_1 - 3x''_1 - 1}{2} \right)$$

The major relationships between information systems defined on $X'$, the set of 4-tuples representing businesses, and $X''$, the set of 4-tuples representing cubes, are the following.

Recalling that

$$W(x'_1, x'_2, x'_3, x'_4) = (x'_1/100,000, x'_2/100,000, x'_3/100,000, x'_4)$$

it can be seen that

$$f_b(x'_1, x'_2, x'_3, x'_4) = \frac{4x'_1 - 2x'_2 - 3(x'_1 - x'_2) - 100,000}{2}$$

$$= (100,000) f_c(W(x'_1, x'_2, x'_3, x'_4))$$

Similarly,

$$F_b(x'_1, x'_2, x'_3, x'_4) = 100,000 F_c(W(x'_1, x'_2, x'_3, x'_4))$$

The algebraic statements denote that the profit figures assigned to a business $(x'_1, x'_2, x'_3, x'_4)$ is 100,000 times the number assigned to the
cube associated with the hypothetical business, \( W(x^1_1, x^1_2, x^1_3, x^1_4) \). A similar relationship is found for the joint systems \( F_b \) and \( F_c \).

The Relationship between Probabilities of Messages

If, (1) the set of messages associated with \( X' \) by \( f_b \) is \( M' \); (2) the set of messages associated with \( X'' \) by \( f_c \) is \( M'' \); (3) the probability of an event \( A' \) which is a subset of \( X' \) is denoted by

\[
Pr'(A') = \sum_{(x^1_1, x^1_2, x^1_3, x^1_4) \in A'} P'(x^1_1, x^1_2, x^1_3, x^1_4)
\]

and (4) the probability associated with an event \( A'' \), a subset of \( X'' \), is given by

\[
Pr''(A'') = \sum_{(x''_1, x''_2, x''_3, x''_4) \in A''} P''(x''_1, x''_2, x''_3, x''_4)
\]

it can be shown that

\[
\sum_{(x^1_1, x^1_2, x^1_3, x^1_4) \in \{x^1_1, x^1_2, x^1_3, x^1_4\} | f_b(x^1_1, x^1_2, x^1_3, x^1_4) = m'} P'(x^1_1, x^1_2, x^1_3, x^1_4) = m'/100,000
\]

\[
\sum_{(x''_1, x''_2, x''_3, x''_4) \in \{x''_1, x''_2, x''_3, x''_4\} | f_c(x''_1, x''_2, x''_3, x''_4) = m''} P''(x''_1, x''_2, x''_3, x''_4) = m''/100,000
\]

The equality states that for any message, \( m' \), in the setting which uses businesses and profit figures, it is possible to find an \( m'' \), namely \( m'/100,000 \), in the setting using cubes and numbers; therefore, the probability of receiving the message \( m' \) in the business setting is equal to the probability of receiving message \( m'' \) in the cube setting. A similar statement can be made for any \( m'' \in M'' \), and for a joint information system. Furthermore, it can be shown that the event, stock price
increase, in the business setting has the same probability of occurrence as the event, being in Tumbler 1, in the cube setting and that no stock price increase in the business setting has the same probability of occurring as the event, not being in Tumbler 1, in the cube setting.

In addition, it is possible to show that any probabilistic situation in X' can be duplicated in X''. Thus, the same probabilistic revision questions can be asked in both settings.

A Prediction of the Experimental Outcomes Resulting from the Relationships of the Experimental Setting and the Type of Subject

The examination of the differences between experimental settings permits an examination of the impact of familiarity on revision performance. It has been shown that it is possible to make the probabilistic tasks in each setting identical. When the problems are identical in terms of their probabilistic characteristics, there is the possibility that the familiarity to the subject of the problem statement may alter behavior. Wetherick provides an hypothesis which suggests that probability revisions of the particular types of subjects examined may be altered by the particular settings chosen. Wetherick hypothesizes that the age of a subject and the way in which a problem is stated may interact. Specifically, Wetherick suggests that with increased age a person's ability to handle abstract symbolic language tends to deteriorate. On the other hand, a person's ability to manipulate ordinary language increases with increased age. Wetherick suggests that
problems in concept attainment may be better suited to college students when they are stated symbolically rather than in ordinary language. As a person increases in age, Wetherick expects a crossover point at which concepts can be attained better when stated in ordinary language. Wetherick has examined only students and has found that problems stated in symbolic language were handled better than problems stated in ordinary language. 7

Wetherick's reasoning seems applicable to the current experiment. Age is a factor by which the subjects differ and the presentation of the problems differs in that the setting employing businesses and profit figures is built using ordinary language while the cube setting uses more abstract symbols. Thus, according to the Wetherick hypothesis, Bayesian revision behavior in one experimental situation will differ from Bayesian revision behavior in other experimental situations.

The Wetherick hypothesis does not suggest that within an experimental situation the confusion effect of a joint information system will be altered.

Summary

In this chapter, it is suggested that the experimental variables—the experimental setting and the type of subject—may interrelate because of the familiarity of the experimental setting to a

particular type of subject. The particular experimental settings involving cubes and hypothetical businesses are shown to possess the same probabilistic attributes so that the probabilistic questions in one setting can be duplicated in the other setting.

Finally, Wetherick's hypothesis relating age to language is used to predict the outcomes when the experimental setting and type of subject are manipulated.
APPENDIX TO CHAPTER III

This appendix describes the structure of the probability revision experiment in terms of subject procurement and assignment to experimental treatments; the experimental environment; and the conduct of the experiment.

The discussion of each major topic includes two parts: (1) a description of the particular experimental procedure and (2) a discussion of the reasons for the selection of the particular procedure.

The primary constraints on the experimental procedures were the practical problems associated with securing experimental subjects and the requirements of the statistical model. Two major assumptions of the statistical model are that (1) in assigning subjects within a specific subpopulation to the experimental treatments, a random process must underly the assignment and (2) the error in measuring one experimental unit must not be correlated with the error in measuring another experimental unit. The statistical model itself is described in detail in Chapter IV.

Subject Procurement

The experimenter was invited to present a discussion of "Subjective Probabilities" to the 79 participants in the two-week (August 24 to September 6, 1969) Ohio State University Executive Development
Program. The text of the presentation begins on Page 104 of this appendix. At the end of the presentation, the experimenter asked for 24 volunteers as subjects for the experiment. Twenty-four business executives volunteered.

Twenty-four undergraduate business administration students at The Ohio State University volunteered to participate in the experiment after a similar presentation (with minor changes in examples of subjective probability) was made before a large undergraduate management class.

The experimenter felt that the presentation could arouse a potential subject's curiosity which would serve as motivation to participate in the experiment. 8

Although curiosity is not often claimed as the major source of motivation in laboratory experiments, curiosity seemed to be an appropriate motivator in the present experiment for at least two reasons. First, the executives and the students were both part of a classroom environment, an environment in which curiosity is often a motivating device. Second, the more usual form of motivation in a laboratory experiment, a small monetary payment, seemed both ineffective and inappropriate for use with relatively highly paid executives. 9

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9 The median salary of the businessmen subjects was between $20,000 and $29,000 annually.
nique of arousing curiosity as a motivational tool is maintained throughout the experiment.

Assignment of Subjects to Experimental Treatments

Businessmen chose from among four days to participate in the experiment: 4-6 p.m., Monday through Thursday of the second week of the Executive Development Program. Student subjects chose from among the first four days of the week of November 9-14, 1969. Student subjects also could arrange a special experimental time.

The hours 4-6 p.m. were the only "free time" allowed to the executives between approximately 8 a.m. and 10 p.m. each day of the Executive Development Program, restricting the scheduling of the experimental sessions to these hours. Originally, the November experimental sessions for students were arranged to coincide with the 4-6 p.m. time period; however, several volunteer subjects had work, study or eating schedules which necessitated arranging other hours for participation in the experiment.

The businessmen who volunteered to participate were assigned by the experimenter, without the subject's knowledge, to particular experimental treatments according to the day for which the subject volunteered. Businessmen who signed up for the first day of the study were tested using the single information system with profit figures and hypothetical businesses; businessmen who signed up for the second day of the experimentation were tested using the single information system with cubes and algebraic identities; businessmen who signed up for the
third day were tested using the joint information system with profit figures and hypothetical businesses; and businessmen who signed up for the fourth day of experimentation were tested using the joint information system with cubes and algebraic identities.

This same procedure for assigning subjects to experimental treatments was used with student subjects tested the week of November 9-14. Students tested individually were randomly assigned to treatment combinations.

The restrictions on the availability of subjects (particularly the businessmen) permitted only two alternative methods of assignment of subjects to experimental treatments. The first method is described in the preceding paragraphs. The second method involved asking subjects to be available for the experiment on all four days during which the experiment could be given. The subjects would then be assigned to treatments using a random number table and be told which day to participate. Each method of assignment has limitations. The first method may produce a non-random assignment of subjects to treatments; the second may seriously restrict subject availability.10

It was assumed that there would be prohibitive loss of potential subjects had the second method of assignment been used. Therefore, the experimenter chose the first assignment procedure and made several assumptions which imply the random assignment required by the statistical

10 The Executive Development Program lasted only two weeks and any experimental trials with executives after the completion of the program seemed very difficult to arrange.
The first assumption is that for a given businessman participant, the time he was likely to sign up to participate would be independent of the times other businessmen participants would sign up. Most of the men in the Executive Development Program met each other for the first time on the day preceding the day on which they were asked to sign up for the experiment. It was felt that these casual acquaintanceships could have only a very minor effect on the time a particular businessman chose to participate in the experiment. Also, the choice to participate was an individual rather than a group choice so that a group preference was unlikely to affect the choice of time to participate. Additionally, most executives who participated in the experiment had only a minute or two between the request for volunteers and actual sign up to consider the day on which to participate. The experimenter felt, therefore, that any previous arrangements a potential subject might have made for use of the time during which the experiment was presented would involve commitments to persons not in the Executive Development Program. For instance, a businessman might have arranged to meet his family or confer with a business associate outside the program during this time.

In addition to the assumption concerning independence, the experimenter made several assumptions which imply that the probability an executive would sign up for a specific experimental day was equal to

\[ \text{\footnote{In Chapter IV, it will be shown that the responses of all volunteer subjects were not used in the statistical process. The assumptions presented here apply with equal force to the original set of volunteers and the restricted set.}} \]
the probability that he would sign up for any other experimental day. To develop this assumption, the following notation is used.

Let $V$ stand for the event that a subject signs up for the experiment.

$V_i$ stand for the event that a person volunteers for day $i$.

$B_i$ stand for the event that a participant has prescheduled an activity which prohibits participation on day $i$.

The occurrence of an experimental participant volunteering for day $i$ is equivalent to both his not having scheduled an activity that prohibits participation and his volunteering for day $i$. The probability of interest is

$$\Pr(V_i \mid \overline{B}_i, V)$$

It was assumed that if $i, j$ are two days on which the participant did not have prescheduled activities during the experimental times, then the probability that a participant would assign himself to day $i$ is equivalent to the probability that he would assign himself to day $j$, or,

$$\Pr(V_i \mid \overline{B}_i, V) = \Pr(V_j \mid B_j, V) \text{ for all } i, j$$

This is equivalent to assuming that a person would just as soon relinquish his unscheduled "free time" to participate in the experiment on day $i$ as relinquish the time on day $j$ to participate in the experiment.

It was also assumed that the executive participants were just as likely not to have scheduled a meeting on day $i$ as day $j$. This assumption implies that scheduled activities such as a business meeting...
arranged prior to the program with associates not enrolled in the program were just as likely to occur on day i as day j, assuming that the scheduled events were independent. Thus, we have

$$\Pr(B_i | V) = \Pr(B_j | V) \text{ for all } i,j$$

Using these assumptions, it is concluded that for all i,j

$$\Pr(V_i, B_i | V) = \frac{\Pr(V_i \cap B_i \cap V)}{\Pr(V)}$$

$$= \frac{\Pr(V_i | B_i \cap V) \Pr(B_i \cap V)}{\Pr(V)}$$

$$= \Pr(V_i | B_i \cap V) \Pr(B_i | V)$$

$$= \Pr(V_j | B_j \cap V) \Pr(B_j | V)$$

$$= \Pr(V_j, B_j | V)$$

That is, the probability a participant signs up for day i is equivalent to the probability that he signs up for day j.

To examine these assumptions, the experimenter compiled information on a number of characteristics of the individuals involved in the experiment. These factors were examined as potential biasing factors in the assignment of subjects to treatments. These data (presented in Chapter V) do not indicate that any of these characteristics represents a biasing factor in the assignment of subjects to experimental treatments.

Similar assumptions were made concerning student subjects. Because the time between request for student volunteer subjects and actual sign up was very brief (one or two minutes), it was assumed that choice
of sign up time would be likely to be influenced by out-of-class commitments such as an appointment with an advisor or employment obligations rather than in-class relationships and that a student's decision to participate at a particular time would be independent of any other student's decision.

As with the businessmen, it was also assumed that the probability a student would sign up for a specific experimental day was equal to the probability that he would sign up for any other experimental day.

To examine these assumptions about student subjects, the experimenter collected information about a number of characteristics of students who participated in the study. These data (presented in Chapter V) indicate no bias with respect to the characteristics examined.

The Experimental Environment

During the experiment, subjects sat at two six-foot tables facing the experimenter. In front of each subject were a copy of the experimental instructions, a stack of file cards numbered 1 to 100, two pens and an ashtray. Separating the subjects were wooden screens. The purpose of the screens was to eliminate intercorrelations of measurement error that may occur if facial contact is permitted between two subjects. Facial expressions often reflect attitudes,\(^\text{12}\) and the experimenter wished to prevent one subject's attitude toward an experimental subject.

question from influencing another subject's attitude. The only person visible to each subject was the experimenter.

Of course, since all subjects could see the experimenter, it was possible, if the experimenter appeared differently to different subjects, to have intercorrelations of measurement error due to experimenter influence. The experimenter attempted to keep any such effect constant across treatments by dressing similarly for each experimental session and by demonstrating a seriousness of approach at all times. Thus, if there was an experimenter effect, it was constant from experimental treatment to experimental treatment.

The experimenter stood at a desk approximately 20 feet in front of the subjects. On the experimenter's desk were four sets of poker chips, two bags marked Bag 1 and Bag 2, a set of large cardboard rectangles (8" x 3" in the case of the single information systems and 8" x 6" in the case of the joint information systems) upon which black numerals two inches in height were inscribed. During the experimental sessions in which cubes were used, there were two tumblers on the desk; each of the tumblers contained cubes colored and numbered on three sides. A wooden screen was placed at the front of the experimenter's desk to prevent subjects from seeing objects on the desk before these objects were used in the experiment. Behind the experimenter was a blackboard and beside the experimenter was a stand on which the cardboard rectangles were placed.

13 Supra
The Conduct of the Experiment

Introduction to the Experiment

After subjects seated themselves, the experimenter gave a brief introduction in which he thanked subjects for coming and tried to encourage subjects' attentiveness to the experiment. The experimenter suggested that the particular experiment was designed to examine a single aspect of each participant's behavioral repertoire and that the examination was important as an attempt to build a scientific understanding of the way in which people relate to information systems. The experimenter also stated that he hoped he would not be the only person who gained an understanding from the particular experiment. The experimenter suggested that it had been his experience in previous studies that subjects making probability estimates seemed to indicate that they learn more about their behavior than the experimenter does.

Although the experimenter does not question the truth of these statements, he did not make them for their truth value. Rather he believed that the statements would make subjects curious about what was to follow.

Finally, the experimenter told subjects that it would be possible to discuss any insights they had made into their own estimating procedures immediately following the experiment. The experimenter asked that all questions be saved until the conclusion of the session to prevent one subject from altering another subject's participation.
The Body of the Experiment

There was the possibility that the experimenter's own statements might produce intercorrelations of measurement error. Any probability revision experiment involves acquainting a subject with the response mode, the nature of the information system, and the underlying system of probabilities. Since the experimenter's dissemination of this information necessarily affects the subject's response, the experimenter attempted to hold this effect constant across treatments. In attempting to hold the experimenter effect constant, the experimenter made the format of all experimental sessions identical. The format was:

(1) introduction to the experiment

(2) discussion of subjective probabilities and response mode

(3) discussion of the experimental situation

(4) precise statement of the experimental task

The first two items of the format were identical for all experimental sessions. The third and fourth items discussed identical topics with respect to different experimental conditions. For example, the topics covered in the third area of the treatment involving single information systems with hypothetical businesses and profit figures were the underlying set of objects, the nature of the information system and the underlying probability system. The same topics were covered in the experimental treatment using a single information system with cubes and algebraic identities and for the remaining two experimental treatments. Thus, to keep the experimenter effect constant, the experimenter varied the instructions only when it was necessary to de-
scribe a factor particular to the experimental treatment in question.

The written instructions for the four experimental treatments, as presented to the subjects, begin on page 73. There were written and oral portions for each set of instructions. The experimenter and the subject each had the written portions of the experimental instructions before them. The experimenter read the instructions aloud and the subject followed on his set of written instructions.

In two portions of the instructions, the experimenter made oral presentations. The first oral presentation, entitled "Review of the Basic Notions of Objective and Subjective Probability," described the traditional distinction made between the two types of probability. The method of developing probabilities using a utility theory also was discussed.

In the second oral presentation, entitled, "The Importance of Information in a Decision Making Task," two aspects of information were discussed. First, it was suggested that individuals rarely receive perfect information and several examples of imperfect information were given. Second, it was suggested that the value of information may differ at different times and examples of this phenomenon were discussed.

The experimental instructions were formulated using a basic component of programmed instruction. The component involves asking subjects a question and giving subjects immediate feedback about their answer. After a subject has answered a question, the experimenter discusses the meaning of the subject's answer so that the subject can assess whether he is answering the precise question asked. Following
the feedback, the same type of question is asked again and feedback is given a second time. For example, a subject was asked to assess points on a personal cumulative distribution function. After each written assessment, the nature of the subject's response was explained to him in light of how the experimenter would interpret the response. Following the explanation, the subject was asked to assess a second point on his cumulative distribution function and a second explanation was offered.

The programmed instruction approach was used as a reasonably quick way through which subjects learn the nature of the experiment and the experimenter gains insight into how well the subject has understood the experimenter.

The subjects were asked not to change answers after recording them. Such a procedure was instituted as a means of determining the degree to which a given subject understood the experimental instructions. The procedure is described in greater detail in Chapter V.

Following the reading of the experimental instructions, subjects were presented a sample of 100 preordered messages and were asked to record a probability estimate based on each message on 5" x 3" index cards. The first 10 messages were presented every 45 seconds. The remaining 90 messages were presented every 20 seconds.

After every fifth message, the experimenter announced whether the element of the underlying set came from the set of interest. For

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14 Berelson and Steiner, Human Behavior, pp. 149-50.
example, the experimenter might state after the 40th estimate a subject made in the experiment involving hypothetical businesses and profit figures that: "The company with this profit figure \( \sqrt{ \) points to the profit figure \( \sqrt{ \) in 1967 had a stock price increase in 1968."

It commonly occurs that task difficulty can relate to at least two areas, accuracy and length of time taken to respond. Thus, if accuracy is held constant, more task difficulty implies more time taken to respond. On the other hand, if time is held constant, less accuracy will occur for the more difficult task. In this experiment, time was held constant across all tasks.\(^{15}\)

Most experiments in human performance involve some sort of feedback to the subject from the environment in which the subject performs. The purpose of the feedback is to motivate the subject by increasing his knowledge of and interest in the experimental environment.\(^{16}\) Experiments examining subjective probability invariably use some form of feedback. The nature of the feedback usually is either a specific statement of what the Bayesian revision is or a statement about which underlying hypothesis generated the datum; e.g., the chip was drawn from Bag i.\(^{17}\) This latter type of feedback is termed nominal feedback.


\(^{16}\)Ibid.

In this experiment, the experimenter did not wish to tell subjects precisely what the Bayesian revision was since a datum would appear often in the experiment, and feedback of the actual Bayesian revision might dictate later responses. Thus, the nominal form of feedback was chosen.

There is no clear criterion concerning what percent of feedback is optimal. On the basis of an article by Goldstein, Emmanuel and Howell, the experimenter used 20 percent feedback since he felt that no more than 20 percent would be needed to inform the subject about the environment. 18

The Experimental Instructions

Introductory Section 19

1) Review of the basic notions of Objective and Subjective probability

2) Questions dealing with probabilities

In this section, we will ask questions to give you a feel for stating answers in terms of probabilities. Please try to answer the questions even if you feel awkward using probabilities at first.

After you have answered some of the questions, we will discuss the meaning of your answers. You may feel that you want to change some of your answers after our discussion of their meaning. Do not change an answer you have given since


19The same introductory section was used in all sets of experimental instructions.
your understanding of the meaning of the answer is more important than the answer itself.

(2A) Consider the set of all the people who presently live within the city limits of Columbus, Ohio. It would be possible to go out and count the people and come up with an exact population figure. Instead we want you to estimate a number so that you feel there is a 50% chance that the actual population of Columbus is above the number you estimate and a 50% chance that the actual population is below the number you estimate.

Your answer to (2A)________________________________________

Your answer to (2A) indicates that you believe that it is equally likely that the actual population of Columbus is greater than or less than the number you estimated in (2A). In other words, the chances that the actual population of Columbus is more than the number you estimate are equal to the chances that the actual population of Columbus is less than the number you estimated.

We can graph your answer as follows.

50% chance the actual population of Columbus is less than your estimate 50% chance the actual population of Columbus is more than your estimate

__________________________________________________________

Your Estimate

(2B) Reconsider the set of all people living in Columbus, Ohio. Estimate a number such that you feel that there is a 25% chance that the actual number of people in Columbus is less than the number you estimate and a 75% chance that the actual number is more than the number you estimate.

(2B)_______________________________________________________

A graph of this answer would be:

25% chance the actual population is less than your answer 75% chance the actual population is more than your answer

__________________________________________________________

Your Estimate
Your answer to (2B) should be less than your answer to (2A) since your answer to (2B) indicates that you think there is a 75% chance that the actual number of people living in Columbus is more than your answer to (2B), and your answer to (2A) indicates that you feel there is a 50% chance that the actual number of people is more than your answer to (2A). Since a 75% chance is greater than a 50% chance, you should feel it is more likely that the actual number will be above (2B) than above (2A); thus, (2B) should be less than (2A).

(2C) Reconsider the set of all people living in Columbus, Ohio, and estimate a number such that you feel there is a 75% chance the actual number is less than the number you estimate and a 25% chance that the actual number is more than the number you estimate.

(2C)______________________________

Again, let's graph your answer.

75% chance the actual population is less than your answer 25% chance the actual population is more than your answer

______________________________

Your estimate

Your answer to (2C) should be greater than your answers to (2A) and (2B) since you are saying that you feel there is a 75% chance the actual number will be below your answer to (2C), a 50% chance the actual number will be below your answer to (2A) and a 25% chance the actual number will be below your answer to (2B). Thus, you should feel that the chances the actual number will be less than the answer you gave to (2C) are greater than the chances the actual number will be less than the answers you gave to (2A) and (2B).

(2D) Now consider a new set, the set of all cities which will have rain tomorrow. Estimate a temperature such that you feel one-half of the cities which will have rain tomorrow will have a high temperature today above the number you estimate and one-half of the cities which will have rain tomorrow will have a high temperature below the number you estimate.

(2D)______________________________
Your answer to (2D) indicates that you feel that of those cities which will have rain tomorrow, 50% will have a high temperature today above the number you estimated and 50% will have a high temperature today below the temperature you estimated.

A graph of this answer would be:

50% of the cities which will have rain tomorrow

50% of the cities which will have rain tomorrow

__________________________

__________________________

Your estimate

Temperatures

(2E) Reconsider the set of cities which will have rain tomorrow. Estimate a temperature such that you feel that 90% of the cities which will have rain tomorrow will have a high temperature today above the temperature you estimate and 10% of the cities which will have rain tomorrow will have a high temperature today below the number you estimate.

(2E)_____________________________________________________

The temperature you estimated as an answer to (2E) should be less than the temperature you estimated in (2D).

(2F) Reconsider the set of cities which will have rain tomorrow. Estimate a temperature such that you feel 90% of the cities which will have rain tomorrow will have a high temperature below the number you estimate and 10% will have a high temperature above the number you estimate.

(2F)_____________________________________________________

3) The Importance of Information in a Decision Making Task

Discussion by the experimenter

4) The type of probabilistic answer to be used in the remainder of the experiment

In this experiment we are going to ask you to state your probabilistic answers in terms of odds. Not all people mean the same thing when they use odds, and your interpretation of odds may be somewhat different from that of the experimenter.
Let's consider an example. Suppose we had an urn, and suppose the urn had 80 red marbles (R) and 20 black marbles (B) in it.

In this study we will say that the odds of drawing a red from the urn are "4-1". This notation means that for every time we draw four red marbles, we expect to draw one non-red marble. We realize that you may not have thought of odds in this way before. Please adopt this approach to odds for the remainder of the experiment so that the experimental results will be meaningful.

Using the same urn which has 80 red and 20 black marbles, we can also talk about the odds of not drawing a red. These odds are 1-4, the same as the odds of drawing a black. This means that for every time we expect to draw a non-red, we expect to draw four reds.

The expression of odds, then, is composed of two numbers and a dash. If the number before the dash exceeds the number after the dash, the event in question is more likely to happen than not to happen. If the number after the dash exceeds the number before the dash, the event in question is more likely not to happen than to happen.

Suppose we had an urn with 25 red marbles (R) and 75 black marbles (B).

What are the odds a red will be drawn?

*20

(4A) ____________

What are the odds a non-red will be drawn?

(4B) ____________

\[\text{The experimenter will discuss the answers to (4A) and (4B)\}]

20 Answers in blanks preceded by an asterisk were later graded by the experimenter; see Chapter IV, pp.
5) Sample questions to be answered in odds

We will now consider a set of examples in which you will be asked to state odds. You see before you two bags. In the bags the experimenter will put poker chips. In Bag 1, the experimenter will put 9 red poker chips and 1 white poker chip. In Bag 2, the experimenter will put 1 red poker chip and 9 white poker chips.

Let's display the information in the following way.

<table>
<thead>
<tr>
<th></th>
<th>Bag 1</th>
<th>Bag 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red Poker Chips*</td>
<td>90</td>
<td>10</td>
</tr>
<tr>
<td>White Poker Chips*</td>
<td>10</td>
<td>90</td>
</tr>
</tbody>
</table>

*Listed in Percentages

Now the experimenter will flip a coin and on the basis of the coin flip will choose one of the bags. If the coin lands heads, the experimenter will choose Bag 1; if the coin lands tails, the experimenter will choose Bag 2. You will not see the flip of the coin and you will not see which bag is chosen. Then, the experimenter will draw a chip from the bag and show you the chip. On the basis of what you know about the contents of the bags, you are to estimate the odds that the chip was drawn from Bag 1. Remember that if the number to the left of the dash in the odds estimate exceeds the number to the right of the dash, you feel it is more likely that the chip was drawn from Bag 1 than that it was not drawn from Bag 1. On the other hand, if the number to the right of the dash exceeds the number to the left of the dash, you feel it is more likely that the chip was not drawn from Bag 1 than that it was drawn from Bag 1.

A chip will now be drawn. Estimate the odds that the chip was drawn from Bag 1.

(5A)____________________

[The experimenter will state which bag the chip came from]

The chip will now be replaced and the coin will be flipped. On the basis of the coin flip, the experimenter will again choose one of the bags and draw a chip from it. Look at the chip and on the basis of what you know about the composition of the two bags, estimate the odds that the chip was drawn from Bag 1.
The experimenter will state from which bag the chip was drawn.

Now consider two different sets of poker chips. In this situation, we will have a greater variety of colors of chips to examine. In Bag 1 we will put 7 red poker chips, 4 white poker chips, 2 gold poker chips, 2 green poker chips, 1 purple poker chip, and 4 blue poker chips. In Bag 2 we will put 3 red poker chips, 4 white poker chips, 1 gold poker chip, 3 green poker chips, 5 purple poker chips, and 4 blue poker chips.

Information about the contents of the bags is shown below.

<table>
<thead>
<tr>
<th></th>
<th>Bag 1</th>
<th>Bag 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red Poker Chips*</td>
<td>35</td>
<td>15</td>
</tr>
<tr>
<td>White Poker Chips</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Gold Poker Chips</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>Green Poker Chips</td>
<td>10</td>
<td>15</td>
</tr>
<tr>
<td>Purple Poker Chips</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Blue Poker Chips</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

*Listed in Percentages

The experimenter will flip a coin and then choose a bag on the basis of the coin flip. You will not see the flipping of the coin or the bag which is chosen. The experimenter will then draw a chip from the bag chosen and show you the chip that was drawn. On the basis of the chip drawn and what you know about the composition of the bags, you are to estimate the odds that the chip was drawn from Bag 1.

The chip will now be drawn. On the basis of the chip drawn and the composition of the bags, estimate the odds that the chip came from Bag 1.

The experimenter will state which bag the chip came from.

The chip will be replaced, the coin flipped again, a bag chosen on the basis of the coin flip, and another chip drawn from the bag chosen.

On the basis of the chip drawn and your knowledge of the composition of the bags, estimate the odds that the chip was drawn from Bag 1.
The experimenter will state which bag the chip came from. The chip will be replaced, the coin flipped again, a bag chosen on the basis of the coin flip, and a chip drawn from the bag chosen.

On the basis of the chip drawn and your knowledge of the composition of the bags, estimate the odds that the chip was drawn from Bag 1.

Instructions Concerning Single Information Systems Used with Hypothetical Businesses and Profit Figures

6) Probability statements based on accounting information

We are going to ask you to develop a probabilistic opinion on the basis of accounting information. The accounting information will be a profit figure like those published in the Wall Street Journal or local newspapers. For instance, the Columbus Dispatch recently published:

**JACOBSEN PROFITS DOWN**

The Jacobsen Manufacturing Co. reported net income for the period was $1,270,768 as compared with $1,367,322 last year.

The profit figures we will show you belong to a particular set of companies. The major asset of every company in the set is inventory. Each company in the set carries only one product in its inventory and that product is the same for all companies. Furthermore, the set has been restricted to include only those companies which have the same size plant. An example of a set of companies like the ones with which we will be dealing is a set of wine distributing companies with the same cubic feet of storage space per company. Another example of the kind of company with which we will be dealing is a set of grain storage companies with 100,000 bushels capacity of storage. Each of these sets of companies, the wine distributing set and the grain storage set, has a major asset in terms of inventories and each company in each set carries the same product in inventory.
The type of answer

We will ask you to respond to our questions in probabilistic terms. We would never expect you to feel certain about an event simply because you knew a company's profit figure. We do feel is that your knowledge of a company's profit figure may help you form an opinion concerning the chances that a particular type of event might happen concerning the company.

The nature of the problem

Since every company in the set is about the same size and since each company's major asset is inventory, the only significant way to distinguish between companies in the set is in terms of how well they can sell and how well they manage their inventories. By managing inventories we mean (1) their ability to judge when to purchase units for inventory and (2) their ability to achieve a respectable inventory turnover rate.

During any period of time three factors that indicate the quality of performance of these companies would be (1) the quantity of units of sales, (2) the quantity of units in beginning inventories, and (3) the quantity of units purchased for inventories.

We have rounded the quantity of units sold, the quantity of units in beginning inventories, and the quantity of units purchased for inventories to the 100,000's. For instance, if we had a company with 215,000 units in sales, 110,000 units in beginning inventories, and 125,000 units purchased for inventories during a particular period of time, we would round sales to 200,000, units in beginning inventories to 100,000 and purchases for inventory to 100,000.

Suppose one of the companies in the set had 325,000 units in sales, 195,000 units in beginning inventory, and 85,000 units in purchases for inventory.

(6A) What would sales be rounded to?

* (6A) __________________

(6B) What would beginning inventory be rounded to?

* (6B) __________________

(6C) What would purchases be rounded to?

* (6C) __________________
We will consider activities of our particular set of companies during the year 1967. After we round the three quantities for each company for 1967, there were three possible levels of sales -- 200,000; 300,000; and 400,000. There were three possible levels of beginning inventories -- 0; 100,000; and 200,000; and there were three possible levels of purchases during the year 1967 -- 100,000; 200,000; and 300,000.

We will represent a particular type of company with the notation

\[(300,000; 200,000; 100,000)\]

where the first number (300,000) represents the quantity of units of sales, the second number (200,000) represents the quantity of units in beginning inventories, and the third number (100,000) represents the quantity of purchases for inventories during the year 1967.

We emphasize that (300,000; 200,000; 100,000) represents a type of company and that there may be many companies of this type. For instance, a company with 335,000 units in sales, 195,000 units in beginning inventory, and 110,000 units purchased will be of this type. Also, a company with 285,000 units in sales, 220,000 units in beginning inventory, and 75,000 units purchased for inventory will be of this type.

(6D) What does the notation \[(400,000; 100,000; 300,000)\] mean?

* (6D)

(6E) Name two companies that could be of the type \[(400,000; 100,000; 300,000)\].

(6E)

Prices and profit figures

During the year 1967, the selling price per unit was $4.00. The cost per unit of beginning inventory was $2.00, and the cost per purchase for inventories was $3.00. Since all companies were the same size, each had the same operating expenses of $100,000. The tax rate for each company was .50.

It is possible to compute a profit figure for each type of company. A profit figure will be computed under the assumption that the first units in inventory are the first units sold. Thus, we can compute a profit figure for a company of the type...
<table>
<thead>
<tr>
<th>Sales</th>
<th>Beginning Inventories</th>
<th>Purchases for Inventories</th>
</tr>
</thead>
<tbody>
<tr>
<td>$300,000</td>
<td>200,000</td>
<td>200,000</td>
</tr>
</tbody>
</table>

Sales revenues would be $4.00 \times 300,000 = $1,200,000

Cost of goods sold would be determined in the following way:

- 200,000 units sold at $2.00
- 100,000 units sold at $3.00

Thus, cost of goods sold would be $2.00 \times 200,000 + $3.00 \times 100,000 = 700,000

Gross Margin = 500,000
Operating Expenses = 100,000
Profits before taxes = 400,000
Taxes = 0.50 \times 400,000 = 200,000
Profits after taxes = 200,000

We would like you to try profit calculations for two companies.

First calculate a profit figure for a company of the type (200,000; 100,000; 300,000)

*

Now calculate a profit figure for a company of the type (400,000; 100,000; 300,000)

*

The profit figures we will be dealing with will range from $50,000 to $250,000.

The experimenter will discuss the answers after all profit calculations have been made.
We examined stock prices for this set of companies for the year 1968 and divided the total set of companies into two subsets: (1) the subset which had stock price increases during 1968 and (2) the subset which did not have stock price increases during 1968. One-half of the companies in the total set belonged to the subset which had stock price increases during 1968 and one-half belonged to the subset which did not have stock price increases during 1968.

Each subset had at least one of each type of company. Thus, there was at least one company of the type (200,000; 0; 200,000) in the subset that had stock price increases in 1968 and at least one company of the type (200,000; 0; 200,000) in the subset that did not have stock price increases in 1968.

In the table on the following page we have to the left of the table a particular type of company. In the column below stock price increase we have the percentage of companies of that particular type which had stock price increases in 1968. To the right of the first percentage we have listed the percentage of companies of that type in the subset of companies which did not have stock price increases in 1968.

Please examine the table carefully.
<table>
<thead>
<tr>
<th>Sales</th>
<th>Beginning Inventory</th>
<th>Purchases for Inventory</th>
<th>Percentage of companies of this type in the subset, stock price increase</th>
<th>Percentage of companies of this type not in the subset, stock price increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>(200,000; 0; 300,000)</td>
<td>(200,000; 300,000)</td>
<td>3</td>
<td>12%</td>
<td>11%</td>
</tr>
<tr>
<td>(200,000; 100,000; 100,000)</td>
<td>(200,000; 100,000)</td>
<td>8</td>
<td>12%</td>
<td>13%</td>
</tr>
<tr>
<td>(200,000; 100,000; 300,000)</td>
<td>(200,000; 300,000)</td>
<td>3</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>(200,000; 200,000; 100,000)</td>
<td>(200,000; 200,000)</td>
<td>3</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>(200,000; 200,000; 300,000)</td>
<td>(300,000; 300,000)</td>
<td>10</td>
<td>9%</td>
<td>11%</td>
</tr>
<tr>
<td>(300,000; 100,000; 200,000)</td>
<td>(300,000; 300,000)</td>
<td>5</td>
<td>5%</td>
<td>13%</td>
</tr>
<tr>
<td>(300,000; 100,000; 300,000)</td>
<td>(300,000; 300,000)</td>
<td>3</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>(300,000; 200,000; 200,000)</td>
<td>(300,000; 300,000)</td>
<td>2</td>
<td>2%</td>
<td>12%</td>
</tr>
<tr>
<td>(300,000; 200,000; 300,000)</td>
<td>(300,000; 300,000)</td>
<td>2</td>
<td>2%</td>
<td>3%</td>
</tr>
<tr>
<td>(400,000; 100,000; 300,000)</td>
<td>(400,000; 200,000)</td>
<td>12</td>
<td>3%</td>
<td>2%</td>
</tr>
<tr>
<td>(400,000; 200,000; 200,000)</td>
<td>(400,000; 300,000)</td>
<td>12</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>(400,000; 200,000; 300,000)</td>
<td></td>
<td>11</td>
<td>2%</td>
<td>2%</td>
</tr>
</tbody>
</table>

Thus, for instance, in the year 1968, three percent of the companies that had stock price increases were (200,000; 0; 200,000) companies and twelve percent of the companies that did not have stock price increases were (200,000; 0; 200,000) companies.

(6F) What percent of the subset of companies that had stock price increases in 1968 were (300,000; 0; 300,000) companies?

* * *

The Experimental Task

We are now going to ask you to assume that the date is January 2, 1968, and that you have just received the percentage information in the preceding table from a reputable person who knows something about this particular set of companies. Such a person would be a financial analyst or a security analyst.
The table would represent this person's estimates for the year 1968.

Earlier this week, the experimenter drew 100 companies from the entire set of companies in which we are interested. He then computed the profit figure for each company for the year 1967 after units of sales, beginning inventories, and purchases for inventories were rounded to 100,000's. He then recorded each company's profit figure and noted whether the company had a stock price increase in 1968.

On the basis of the profit figure we show you, we want you to assign odds that the company which has this profit figure will have a stock price increase during 1968. Remember, it is now January 2, 1968.

We ask you to make no computations with paper and pencil. We are interested only in your estimate of odds not in a "right answer." You may, if you wish, refer to the way in which a profit figure is calculated.

It may take several trials before you become accustomed to stating the odds that a company will have a stock price increase. For this reason, we will ask you to make 100 odds estimates.

Many of the estimates will be for the same profit figure since many of the companies in the entire set had the same profit figure after units of sales, beginning inventories, and purchases for inventories were rounded to 100,000's. Also, it was possible for the same company to be drawn more than once since we replaced a company back in the set after it was drawn. The profit figures with which we will be dealing will range from $50,000 to $250,000.

We ask you only to make your best estimate of odds on the basis of the percentage information you have received and on the basis of the profit figure we will show you. We will give you 45 seconds for the first 10 odds estimates and 20 seconds for the following 90 odds estimates. We have limited the time for each estimate because we are interested in studying instances when you must deal quickly with limited information.

After every fifth profit figure, we will tell you from which subset the company with that profit figure was drawn. Sometimes, you may find that you have assigned high odds to the event stock price increase when, in fact, the company with the particular profit figure was not drawn from the subset stock price increase. At other times, you may find that you
have assigned low odds to the event stock price increase when in fact, the company did come from the subset stock price increase.

Remember that in estimating odds, if the number preceding the dash is greater than the number following the dash, this means that you think that the company with the particular profit figure is more likely to have a stock price increase during '76 than not to have a stock price increase. On the other hand, if the number following the dash exceeds the number preceding the dash, this means that you think that it is more likely that the company will not have a stock price increase than have a stock price increase.

As the experimenter shows you the profit figures, on the cards you have estimate the odds that this profit figure belongs to a company in the subset of companies which will have stock price increases during 1968. Record each estimate on a different card.

Instructions Concerning Joint Information
Systems Used with Hypothetical Businesses and Profit Figures

6) Probability statements based on accounting information

We are going to ask you to develop a probabilistic opinion on the basis of accounting information. The accounting information will be a profit figure like those published in the Wall Street Journal or local newspapers. For instance, the Columbus Dispatch recently published:

**J A C O B S E N P R O F I T S D O W N**

The Jacobsen Manufacturing Co. reported net income for the period was $1,270,768 as compared with $1,367,322 last year.

The profit figures we will show you belong to a particular set of companies. The major asset of every company in the set is inventory. Each company in the set carries only one product in its inventory and that product is the same for all companies. Furthermore, the set has been restricted to include only those companies which have the same size plant. An example of a set of companies like the ones with which we will be dealing is a set of wine distributing companies with the same cubic feet of storage space per company. Another example of the kind of company with which we will be dealing is a set of
grain storage companies with 100,000 bushels capacity of storage. Each of these sets of companies, the wine distributing set and the grain storage set, has a major asset in terms of inventories and each company in each set carries the same product in inventory.

The type of answer

We will ask you to respond to our questions in probabilistic terms. We would never expect you to feel certain about an event simply because you knew a company's profit figure. What we do feel is that your knowledge of a company's profit figure may help you form an opinion concerning the chances that a particular type of event might happen concerning the company.

The nature of the problem

Since every company in the set is about the same size and since each company's major asset is inventory, the only significant way to distinguish between companies in the set is in terms of how well they can sell and how well they manage their inventories. By managing inventories we mean (1) their ability to judge when to purchase units for inventory and (2) their ability to achieve a respectable inventory turnover rate.

During any period of time three factors that indicate the quality of performance of these companies would be (1) the quantity of units of sales, (2) the quantity of units in beginning inventories, and (3) the quantity of units purchased for inventories.

We have rounded the quantity of units sold, the quantity of units in beginning inventories, and the quantity of units purchased for inventories to 100,000's. For instance, if we had a company with 215,000 units in sales, 110,000 units in beginning inventories, and 125,000 units purchased for inventories during a particular period of time, we would round sales to 200,000, units in beginning inventory to 100,000, and purchases for inventory to 100,000.

Suppose one of the companies in the set had 325,000 units in sales, 195,000 units in beginning inventory, and 85,000 units purchased for inventory.

(6A) What would sales be rounded to?

* (6A) ______________
(6B) What would beginning inventories be rounded to?

* (6B) __________________________

(6C) What would purchases be rounded to?

* (6C) __________________________

We will consider activities of our particular set of companies during the year 1967. After we rounded the three quantities for each company for 1967, there were three possible levels of sales -- 200,000; 300,000; and 400,000. There were three possible levels of beginning inventories -- 0; 100,000; and 200,000; and there were three possible levels of purchases during the year 1967 -- 100,000; 200,000; and 300,000.

We will represent a particular type of company with the notation

   (300,000; 200,000; 100,000)

where the first number (300,000) represents the quantity of units of sales, the second number (200,000) represents the quantity of units in beginning inventories, and the third number (100,000) represents the quantity of purchases for inventories during the year 1967.

We emphasize that (300,000; 200,000; 100,000) represents a type of company and that there may be many companies of this type. For instance, a company with 335,000 units in sales, 195,000 units in beginning inventory, and 110,000 units purchased will be of this type. Also, a company with 285,000 units in sales, 220,000 units in beginning inventory, and 75,000 units purchased for inventory will be of this type.

(6D) What does the notation (400,000; 100,000; 300,000) mean?

* (6D) __________________________

(6E) Name two companies that could be of the type (400,000; 100,000; 300,000).

(6E) __________________________
Prices and Profit Figures

During the year 1967, the selling price per unit was $4.00; the cost per unit of beginning inventory was $2.00, and the cost per purchase for inventories was $3.00. Since all of the companies in which we are interested were the same size, each had the same operating expenses of $100,000. The tax rate for each company was .50.

In this study we will be interested in two types of profit figures for each company. The first type will be computed under the assumption that the first units in inventory are the first units sold. We will call this type of profit figure a regular profit figure. It is regular in the sense that many companies compute this type of profit figure. Thus, we can compute a regular profit figure for a company of the type

Sales | Beginning Inventory | Purchases for Inventory
(300,000; 200,000; 200,000)

Sales revenues would be $4.00 x 300,000 $1,200,000
Cost of goods sold computed under the assumption that the first units in inventory are the first units sold would be
200,000 units at $2.00 = $400,000
100,000 units at $3.00 = 300,000 700,000
Gross Margin 500,000
Operating Expenses 100,000
Regular profits before taxes 400,000
Taxes .50 x $400,000 200,000
Regular profits after taxes $ 200,000

We would like you to try calculating regular profit figures for two companies.

First calculate a regular profit figure for a company of the type
(200,000; 100,000; 300,000)
Now calculate a regular profit figure for a company of the type

\[(400,000; 100,000; 300,000)\]

* 

The regular profit figures we will be dealing with will range from $50,000 to $250,000.

The second type of profit figure will be computed under the assumption that expenses of a firm are figured at the prevailing market price for those expenses. We will call this type of profit figure a current profit figure. We would compute a current profit figure for a company of the type \((300,000; 200,000; 200,000)\) in the following way.

Sales Revenues would be \(4.00 \times 300,000 \quad \$1,200,000\)

Cost of goods sold computed under the assumption that expenses are figured at the prevailing market price would be \(3.00 \times 300,000 \quad 900,000\)

Gross Margin \(300,000\)

Operating Expenses \(100,000\)

Current profits before taxes \(200,000\)

Taxes \(.50 \times \$200,000 \quad 100,000\)

Current profits after taxes \(\$100,000\)

We would like you to try calculating current profit figures for two companies.

First calculate a current profit figure for a company of the type

\[(200,000; 100,000; 300,000)\]
Now calculate a current profit figure for a company of the type

(400,000; 100,000; 300,000)

* 

The current profit figures we will be dealing with will range from $50,000 to $150,000.

Stock prices

We examined the stock prices for our particular set of companies for the year 1968 and divided the total set of companies into two subsets: (1) the subset of companies which had stock price increases during 1968 and (2) the subset of companies which did not have stock price increases during 1968. One-half of the companies in the total set belonged in the subset of companies which had stock price increases during 1968 and one-half of the companies in the total set belonged in the subset of companies which did not have stock price increases during 1968.

In the table on the following page we have listed each type of company in the total set and the percentages of each type of company in the two subsets -- the subset of companies which had stock price increases in 1968 and the subset of companies which did not have stock price increases during 1968.

Please examine the table on the following page. The table shows, for instance, that three percent of the subset of companies which had stock price increases during 1968 were (200,000; 0; 200,000) companies and twelve percent of the subset of companies which did not have stock price increases during 1968 were (200,000; 0; 200,000) companies.

(6F) What percent of the subset of companies which had stock price increases during 1968 were (300,000; 0; 300,000) companies?

* 

(6F) ____________________

At least one of each type of company in the total set was in each subset. Thus there was at least one company of the type (200,000; 0; 200,000) in the subset of companies which had stock price increases in 1968 and at least one of the same type of company in the subset of companies which did not have stock price increases in 1968.
The Experimental Task

Earlier this week, the experimenter drew 100 companies from the entire set of companies in which we are interested. He then computed the regular and current profit figures for each company for the year 1967 after units of sales, beginning inventory, and purchases for inventory were rounded to 100,000's. Next, he recorded each company's current and regular profit figures and noted whether the company had a stock price increase in 1968.

We are now going to ask you to assume that the date is January 2, 1968. You have just received the percentage information in the preceding table from a reputable person who knows something about the particular set of companies in which we are interested. Such a person would be a financial analyst or a security analyst. The table is this person's estimates about the companies for the year 1968.
On the basis of the regular and current profit figures we show you, we want you to assign odds that the company which has these profit figures will have a stock price increase in 1968. Remember it is January 2, 1968.

We ask you to make no computations with paper and pencil. We are interested only in your estimate of odds, not in a "right" answer. You may, if you wish, refer to the way in which regular and current profit figures are calculated.

Many of the estimates will be for the same regular and current profit figures since many of the companies in the total set have the same profit figures after units of sales, beginning inventory, and purchases for inventory were rounded to 100,000's. Also, it was possible for the same company to be drawn more than once since we replaced a company back in the set after it was drawn. The regular profit figures with which we will be dealing will range from $50,000 to $250,000. The current profit figures with which we will be dealing will range from $50,000 to $150,000.

We ask you only to make your best estimate of odds on the basis of the percentage information you have received and on the basis of the profit figures we will show you. We will give you 45 seconds for the first 10 odds estimates and 20 seconds for the following 90 odds estimates. We have limited the time for each estimate because we are interested in studying instances when you must deal with limited information quickly.

After every fifth regular and current profit figure, we will tell you from which subset the company with those profit figures was drawn. Sometimes, you may find that you have assigned high odds to the event that the company was drawn from the subset of companies which had stock price increases when, in fact, it was not. At other times, you may find that you have assigned low odds to the event that the company with the profit figures we show you was drawn from the subset of companies which had stock price increases when, in fact, it was.

Remember that in estimating odds, if the number preceding the dash is greater than the number following the dash, this means that you think that the company with the particular profit figures is more likely to have a stock price increase during 1968 than not to have a stock price increase. On the other hand, if the number following the dash exceeds the number preceding the dash, this means that you think that it is more likely that the company will not have a stock price increase than that it will have a stock price increase.
As we show you the profit figures, on the cards you have estimate the odds that these profit figures belong to a company in the subset of companies which will have stock price increases during 1968. Record each estimate on a different card.

Instructions Concerning Single Information Systems Used with Cubes and Algebraic Identities

6) Estimating odds

In this portion of the experiment, we are going to use two tumblers. Each of the tumblers contains cubes, and each cube in the tumblers has one red side, one yellow side, one blue side, and three blank sides.

Let's draw a cube from one of the tumblers. The red side will have one of three numbers on it. The three possible numbers are 2, 3, and 4. The yellow side will have one of three numbers on it. The three possible numbers are 0, 1, and 2. And the blue side will have one of three numbers on it. The three possible numbers are 1, 2, and 3.

We will represent the cubes in a special way; for example (3,2,1).

The notation has this meaning. The first number (3) is the number on the red side. The second number (2) is the number on the yellow side, and the third number (1) is the number on the blue side.

From now on we will denote a cube with this notation. Thus, (2,0,1) refers to a cube with 2 on the red side, 0 on the yellow side, and 1 on the blue side.

(6A) What type of cube would (3,0,3) represent?

* Z values

We are now going to take a cube and perform some arithmetic operations with the numbers on the three sides of the cube. We will be computing what we call a Z value. A Z value can be computed in the following way.

Let R stand for the number on the red side of the cube.
Let Y stand for the number on the yellow side of the cube.

Let B stand for the number on the blue side of the cube.

We compute a Z value with the following formula:

\[
\frac{4R - 2Y - 3(R-Y) - 1}{2}
\]

The formula says, "Multiply four times the number on the red side; subtract two times the number on the yellow side; subtract three times the number on the red side minus the number on the yellow side; subtract one; and divide by two."

For example, let's consider a (2,1,3) cube. The number on the red side, R, is 2. The number on the yellow side, Y, is 1, and the number on the blue side, B, is 3. Using the formula to compute the Z value we have:

\[
Z = \frac{4R - 2Y - 3(R-Y) - 1}{2}
\]

\[
= \frac{(4 \times 2) - (2 \times 1) - (3 \times (2-1)) - 1}{2}
\]

\[
= \frac{8 - 2 - 3 - 1}{2}
\]

\[
= \frac{2}{2}
\]

\[
= 1
\]

One, then, is the Z value of cube (2,1,3).

(6B) Compute the Z value for the cube (4,1,3).

* The experimenter will discuss the computation of the Z value for cube (4,1,3).
(6C) Now compute a Z value for the cube (3,2,2).

*  

The Z values we will be dealing with will range from 0.5 to 2.5.

The contents of the tumblers

There is at least one cube of each type in each tumbler. Thus, in Tumbler 1 there is at least one cube of the type (2,0,2) and in Tumbler 2 there is at least one cube of the type (2,0,2).

In the list on the following page, we describe the contents of the two tumblers in terms of the percentage of a particular type of cube in each tumbler. For instance, the list shows you that three percent of the total composition of Tumbler 1 is cubes of the type (2,0,2) and twelve percent of the total composition of Tumbler 2 is cubes of the type (2,0,2).

Examine the list on the following page.

(6D) What percent of the cubes in Tumbler 2 are (3,0,3) cubes?

* (6D) ___________________
<table>
<thead>
<tr>
<th>Red Side</th>
<th>Yellow Side</th>
<th>Blue Side</th>
<th>Percentage of this type of cube in Tumbler 1</th>
<th>Percentage of this type of cube in Tumbler 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 0, 2)</td>
<td>3</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2, 0, 3)</td>
<td>2</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2, 1, 1)</td>
<td>8</td>
<td>12</td>
<td></td>
<td></td>
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The Experimental Task

Earlier this week, the experimenter drew 100 cubes from the tumblers in the following manner. First he flipped a coin. If the coin landed heads, he drew a cube from Tumbler 1. If the coin landed tails, he drew a cube from Tumbler 2. After a cube was drawn, the experimenter computed the cube’s Z value. He then recorded the Z value of the cube and the tumbler from which it came. Next, he replaced the cube in its tumbler.

We will show you the Z values the experimenter recorded. Each time we show you a Z value, we want you to estimate the odds that the cube which has that Z value was drawn from Tumbler 1. Many of your estimates will be based on the same Z value since there are many cubes in each tumbler with the same Z value. The Z values will range from 0.5 to 2.5.

We ask you to make no computations with paper and pencil. We are interested only in your probabilistic opinion and not in a "right" answer. You may, if you wish, refer to the way in which Z values are computed.
Estimate the odds a particular cube came from Tumbler 1 on the basis of what you know about the composition of the tumblers and the Z value of the cube. We will give you 45 seconds to make each of your first 10 estimates and 20 seconds to make each of the remaining 90 estimates. We have limited the time for each estimate because we are interested in studying instances when you must deal with limited information quickly.

After every fifth Z value, we will tell you whether the cube with that Z value was drawn from Tumbler 1. Sometimes you may find that you have assigned high odds to the event that the cube with a particular Z value was drawn from Tumbler 1 when, in fact, it was not. At other times, you may find that you have assigned low odds to the event that the cube was drawn from Tumbler 1 when, in fact, it was. Remember, we are interested only in your best estimate.

We have given you cards numbered from 1 - 100. Record a single estimate on each card.

Keep in mind that in estimating odds, if the number preceding the dash is greater than the number following the dash, this means that you feel the cube with the particular Z value is more likely to have come from Tumbler 1 than not to have come from Tumbler 1. On the other hand, if the number following the dash exceeds the number preceding the dash, this means that you think that it is more likely that the cube did not come from Tumbler 1 than that it did come from Tumbler 1.

Instructions Concerning Joint Information Systems Used with Cubes and Algebraic Identities

6) Estimating odds

In this portion of the experiment, we are going to use two tumblers. Each of the tumblers contains cubes, and each cube in the tumblers has one red side, one yellow side, one blue side, and three blank sides.

Let's draw a cube from one of the tumblers. The red side will have one of three numbers on it. The three possible numbers are 2, 3, and 4. The yellow side will have one of three numbers on it. The three possible numbers are 0, 1, and 2. And the blue side will have one of three numbers on it. The three possible numbers are 1, 2, and 3.

We will represent the cubes in a special way; for example (3,2,1)
The notation has this meaning. The first number (3) is the number on the red side. The second number (2) is the number on the yellow side, and the third number (1) is the number on the blue side.

From now on we will denote a cube with this notation. Thus, (2,0,1) refers to a cube with 2 on the red side, 0 on the yellow side, and 1 on the blue side.

(6A) What type of cube would (3,0,3) represent?

Z values

We are now going to take a cube and perform some arithmetic operations with the numbers on the three sides of the cube. We will be computing what we call a Z value. A Z value can be computed in the following way.

Let R stand for the number on the red side of the cube.

Let Y stand for the number on the yellow side of the cube.

Let B stand for the number on the blue side of the cube.

We compute a Z value with the following formula.

\[
\frac{4R - 2Y - 3(R-Y) -1}{2}
\]

The formula says, "Multiply four times the number on the red side; subtract two times the number on the yellow side; subtract three times the number on the red side minus the number on the yellow side; subtract one; and divide by 2."

For example, let's consider a (2,1,3) cube. The number on the red side, R, is 2. The number on the yellow side, Y, is 1, and the number on the blue side, B, is 3. Using the formula to compute the Z value we have:

\[
Z = \frac{4R - 2Y - 3(R-Y) -1}{2}
\]

\[
= \frac{(4 \times 2) - (2 \times 1) - (3 \times (2-1)) -1}{2}
\]

\[
= \frac{8 - 2 - 3 - 1}{2}
\]
\[ \frac{2}{2} = 1 \]

One, then, is the Z value of cube \((2,1,3)\).

(68) Compute the Z value of cube \((4,1,3)\).

* 

(The experimenter will discuss the computation of the Z value for the cube \((4,1,3)\).)

(6C) Now compute a Z value for the cube \((3,2,2)\).

* 

The Z values we will be dealing with will range from 0.5 to 2.5.

Q values

We will now perform a slightly different series of arithmetic operations on the numbers on the three sides of the cube. We will be computing what we call a Q value. A Q value is computed in the following way.

Let \( R \) stand for the number on the red side of the cube.

The formula for computing a Q value is then

\[ \frac{4R - 3R - 1}{2} \]

The formula means, "Multiply four times the number on the red side; subtract three times the number on the red side; subtract one; and then divide by two."
For instance, let's consider a (2,1,3) cube. The number on the red side, $R$, is 2. Using the formula to compute the $Q$ value we have:

$$Q = \frac{(4 \times 2) - (3 \times 2) - 1}{2}$$

$$= \frac{8 - 6 - 1}{2}$$

$$= \frac{1}{2}$$

which is the $Q$ value of cube (2,1,3).

Compute the $Q$ value of cube (4,1,3)

* 

Compute the $Q$ value of cube (3,2,2).

* 

The $Q$ values we will be dealing with will range from 0.5 to 1.5.

The contents of the tumblers

There is at least one of each type of cube in each tumbler. Thus, in Tumbler 1 there is at least one cube of type (2,0,2) and in Tumbler 2 there is at least one cube of the type (2,0,2).

In the list on the following page, we describe the contents of the two tumblers in terms of the percentage of a particular type of cube in each tumbler. For example, the list shows you that three percent of the total composition of Tumbler 1 is cubes of the type (2,0,2) and twelve percent of the total composition of Tumbler 2 is cubes of the type (2,0,2).
Examine the list below.

(6D) What percent of the cubes in Tumbler 2 are (3,0,3) cubes?

* (6D)______________

<table>
<thead>
<tr>
<th>Red Side</th>
<th>Yellow Side</th>
<th>Blue Side</th>
<th>Percentage of this type of cube in Tumbler 1</th>
<th>Percentage of this type of cube in Tumbler 2</th>
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The Experimental Task

Earlier this week, the experimenter drew 100 cubes from tumblers in the following manner. First he flipped a coin. If the coin landed heads, he drew a cube from Tumbler 1. If the coin landed tails, he drew a cube from Tumbler 2. After a cube was drawn, the experimenter computed the cube's Z and Q values. He then recorded the Z and Q values of the cube and the tumbler from which the cube was drawn. Next, he replaced the cube in its tumbler.

We will show you the Z and Q values the experimenter recorded. Each time we show the Z and Q values of a cube, we want you to estimate the odds that the cube which has those Z and Q values was drawn from Tumbler 1. Many of your estimates will be based on the same Z and Q values since there are many cubes in each
tumbler with the same Z and Q values. The Z values with which we will be dealing will range from 0.5 to 2.5. The Q values with which we will be dealing will range from 0.5 to 1.5.

We ask you to make no computations with paper and pencil. We are interested only in your probabilistic opinion and not a "right" answer. You may, if you wish, refer to the way in which Z and Q values are computed.

Estimate the odds that a particular cube came from Tumbler 1 on the basis of what you know about the composition of the tumblers and the Z and Q values of the cube. We will give you 45 seconds to make each of the first 10 odds estimates and 20 seconds for each of the remaining 90 odds estimates. We have limited the time for each estimate because we are interested in studying instances when you must deal with limited information quickly.

After every fifth Z and Q value, we will tell you whether the cube with these Z and Q values was drawn from Tumbler 1. Sometimes, you may find that you have assigned high odds to the event that the cube with particular Z and Q values was drawn from Tumbler 1, when, in fact, it was not. At other times, you may find that you have assigned low odds to the event that the cube was drawn from Tumbler 1, when, in fact, it was drawn from Tumbler 1. Remember, we are interested only in your best estimate.

We have given you cards numbered from 1-100. Record a single estimate on each card.

Keep in mind that in estimating odds, if the number preceding the dash is greater than the number following the dash, this means that you feel that the cube with the particular Z and Q values is more likely to have come from Tumbler 1. On the other hand, if the number following the dash exceeds the number preceding the dash, this means that you think that it is more likely that the cube did not come from Tumbler 1 than that it did come from Tumbler 1.

Presentation Made Prior to Request for Volunteer Subjects

The following presentation was made to participants in The Ohio State University Executive Development Program just prior to requesting volunteers to participate in the probability experiment. A similar presentation (with minor changes in examples) was made before a large
undergraduate business administration class at The Ohio State University just prior to requesting students to volunteer to participate in the experiment.

I'm going to talk with you about uncertainty. I won't spend time defining terms since I believe you deal with uncertainty every day of your life. What I am going to discuss with you are ways to make uncertainty a little more manageable. We are going to discuss and utilize tools that have recently been developed. Surprisingly enough, these tools are not computers, nor do they exist only in the minds of obscure mathematicians. These tools are, right now, part of you. You haven't been asked to demonstrate them previously because we didn't know that the demonstration of these capabilities would be helpful, either to you as executives or to researchers like myself.

We can begin by examining exactly how you have handled uncertainty previously. For instance, if the problem concerned a new employee, you may have waited to give the employee a test or asked for references before making the decision to hire or not to hire. However, suppose the decision couldn't wait. For instance, suppose there was stiff competition for this particular potential employee and that he must be given a clear cut decision on your part quickly or he will accept employment elsewhere.

In such a situation in which you have uncertainty that cannot be definitely resolved, you may become fatigued, worried, and generally plagued by the distresses which affect modern day executives.

One thing you can do, of course, is avoid making the decision, but then you will not be much better off because you will be worried about what might have happened if you had made a decision.

Thus, it becomes clear what we mean by uncertainty. It is nothing more than the fact that many times you feel unsure of yourself and your situation. Of course, I can't show you how to eliminate your uncertainty, but I am going to help you try to deal with it a little more systematically. At least we are going to deal with uncertainty to the point that you will discuss it a little better with other people. The ability to discuss uncertainty may not seem important to you, but if you can effectively communicate your doubts, this fact may pave the way to problem solutions you may not previously have imagined.
For instance, in a business situation a good statistician may be able to take your systematic feelings of uncertainty and show you why one action you might take could be more reasonable than another. I emphasize the fact that it would be your uncertainty and not his or some other person's uncertainty that would be incorporated in the problem solution. This fact alone should make suggestions you receive from statisticians a little more appealing to you.

The tool which we are going to use to help you extract your uncertainty is called "subjective probability." Although the word probability may seem a little strange to you, the word subjective shouldn't -- since by this word we only mean your personal and inner feelings. Thus, subjective probability is not something we are going to give you but rather something we are going to extract from you. What we are asking you to do is merely employ a facility that we feel you use intuitively all the time, but have never been asked to verbalize.

Let's get at just what we mean by probability. A probabilistic phenomenon that all of us are aware of is the flipping of a coin. If asked the chances that a head will occur on the flip of a coin, most of us will say the odds are one to one, or in other words, that the probability is one-half that the coin will come up heads. Mathematically, these statements are the same. One to one odds means that for every head which occurs, one tail will occur.

Thus, if we were to take a coin and flip it a great number of times and compute the ratio of the number of heads to the number of tails, we would find that this ratio would be one to one or something very close to it.

On the other hand, if we say that probability is one-half, we are looking at a slightly different ratio. That ratio is the number of heads to the total number of flips. If we were to compute this ratio after a large number of flips of the coin, we could expect it to be one-half or something very close.

Such a probability we are all quite familiar with and we distinguish it by giving it the name "objective" probability. You may ask, "why?" Well, for one thing, a coin is a physical object and most people are willing to agree on what a coin is and what constitutes heads or tails. Thus, we have a feeling that there is "something really there" that generates this probability. A second reason that we distinguish this probability as being "objective" is that we can take the coin and repeat the tosses a great number of times. It is the fact that we can repeat the tossing, this pseudo-experiment with the coin, that gives this probability the meaning of objective. In fact, probability in
this instance is nothing more than the ratio of the heads to the
total tosses that we get after a large number of tosses.

This same notion of probability is used to assign probabi-
lities to the roll of a die. For instance, the probability of
getting a two on the die is one-sixth because in a large number
of throws of the die we would expect the ratio of the number of
two's to the total number of rolls of the die to be somewhat
close to one-sixth. This notion of objective probability can
help us arrive at "subjective" probability.

First, let's consider an instance in which the probability
or odds you would associate with an event would be subjective.
Suppose I would ask you, "What is the probability that the Dow
Jones Industrial Average will increase next year?" And suppose
you said that the odds are one to one or, in other words, that
the probability is one-half that the average will go up. Now
this probability is not exactly like the earlier probability.
The stock market is not identical to a coin nor can we flip the
market over and over until we are at some mathematical state-
ment of odds. Also, while we might have thought it reasonable
to suggest that the time of a particular flip of a coin would
not affect the outcome, we cannot really say that we are in-
different to time in the case of the stock market. After all,
was are fairly certain such things as Presidential messages and
successful space explorations can affect people's feelings about
a particular stock. Thus, the time that the probability is esti-
mated may be very important.

There are, of course, many, many events with this kind of
uncertainty. We talk about the uncertainty of the outcome of
discussing an employee's work with him, that is, will a discus-
sion lead to better or worse performance in the future. Or we
could talk about the uncertainty concerning the effect of a new
incentive program on the efficiency of plant operations. In
neither of these instances will we be concerned with things like
the flip of a coin. You cannot have complete information. In
each case, the action of the people involved could very well de-
termine the outcome.

Now this discussion makes the odds of one to one or the pro-
bability of one-half assigned to the Dow Jones Industrial Aver-
age increase look rather tenuous, and possibly we should recon-
sider making a judgement at all.

The means I offer to get us out of the trap I have tried so
hard to put us in are not complicated. We will employ what I
call the "just like" hypothesis. If we make this assumption, I
think we can make the probability assignment of one-half more
meaningful. Suppose I have a prize, say $100,000. In the first
case you get $100,000 if the Dow Jones Industrial Average goes up next year and nothing if it goes down, or stays the same. Or you can get $100,000 if I flip a coin and it comes up heads and nothing if it comes up tails. If you don't care which of these bets you take, if you feel you stand as good a chance of winning the prize with either bet, then we say that the probability the Dow Jones Average will go up is "just like" the probability of getting heads. Thus, we assign the probability one-half to the Dow Jones Average going up.

From this we can see that we have a way of getting at probability if we can think of an "objective" experiment such as a coin flip or a roll of a die about which we can have the same feelings of uncertainty that we have about the subjective event.

At this point it may be apparent that we will have trouble handling events where subjective probabilities are not able to be related to a two-sided coin or a six-sided die. For this reason we introduce the notion of urns. Imagine, if you will, an urn containing red and black marbles. What we want to do is determine what the composition of the urn must be so that you would be just as willing to bet on a red being drawn from the urn as you would be to bet on the occurrence of an event to which you can assign a subjective probability.

As an example of a particular event, let's consider asking the question, "What are the chances that a pay increase will lead to 10 percent increase in efficiency?" If you say you are indifferent between receiving a prize from drawing a red from an urn with 60 red marbles and 40 black marbles and receiving the same prize if a pay increase will lead to a 10 percent increase in efficiency, then we say your subjective probability that a pay increase will lead to a 10 percent increase in efficiency is 60 percent.

Now let's try you out. The first question we will ask is "What are the chances your company will have at least a 10 percent increase in profits next year?"

If you aren't sure, try to think of an urn with a hundred marbles. Assume that you will receive $100,000 if a red is drawn and nothing if a black is drawn. Now try to think just how many reds you would need to have in the urn so that you would be just as willing to bet on a red coming up and winning the prize as you would be to bet that your company will have at least a 10 percent increase in profits and winning the prize.

Now let's consider another question. What are the chances your company will have at least a 20 percent increase in profits in the coming year?
Again, if you are having trouble stating this probability, try to think of what the composition of an urn would have to be such that you are indifferent between getting a prize contingent upon drawing a red and getting the same prize if your company had a 20 percent profit during the coming year.

Now we will ask some more difficult questions. First, I'll give you an example of the kind of question I will ask, followed by a sample answer and an explanation.

The question is, "Give a number such that the chances the real number of corporations in Ohio is above that number are equal to the chances that the real number of corporations in Ohio is below that number."

If you were to answer, "20,000," we would interpret this answer to mean that you think that the probability is one-half that the real number of corporations in Ohio is above 20,000; in other words, that there is a 50 percent chance that the real number of corporations is above 20,000 and a 50 percent chance that the real number is below 20,000.

Graphically, we have

\[
\begin{array}{c|c|c}
\text{0} & \text{20,000} \\
\hline
\text{50\% chance} & \text{50\% chance}
\end{array}
\]

Using the urn analogy, we mean that you are indifferent between receiving $100,000 contingent upon a red being drawn from an urn containing 50 red and 50 black marbles and receiving $100,000 if we were to go out and count the number of corporations in Ohio and find out that there are fewer than 20,000.

Another type of question we can ask is, "Give a number such that you think there is a 75 percent chance that the actual number of corporations in Ohio is above that number and a 25 percent chance that the real number of corporations is below that number." If you answered, 100,000, we would interpret this answer to mean that there is a one-quarter probability that the actual number is below 10,000 and a three-quarter probability that it is above 10,000.

Or, in terms of urns, you would be indifferent about receiving a prize contingent upon a red marble being drawn from an urn containing 75 red marbles and 25 black marbles and receiving a prize if the real number of corporations in Ohio turned out to be greater than 10,000.
Now suppose we ask you to estimate a profit figure for your company next year. Can you estimate a figure such that there is a 50 percent chance that the true figure will be above the figure you give and a 50 percent chance that the true figure will be below the figure you give?

Can you estimate a profit figure so that there is a 75 percent chance that next year's profit figure will be above the number you give and a 25 percent chance that it will be below the number you give?

Can you estimate a figure such that there is a 75 percent chance that the true profit figure will be below the number you give and a 25 percent chance that the true figure will be above the number you give?

Can you estimate a profit figure so that there is a 90 percent chance that next year's profit figure will be above the number you give and a 25 percent chance that it will be below the number you give?

Can you estimate a figure such that there is a 90 percent chance that the true figure will be below the number you give and a 10 percent chance the true profit figure will be above the number you estimate?

What I have tried to do here tonight is show you that probability is not impossible to deal with—that, in fact, it is a way of dealing with uncertainty. If you use this technique, you may gain some insight into stating uncertainties. Obviously, I have not been able to give you a complete approach to problems involving uncertainty. As a matter of fact, researchers are just beginning to explore some of the possible applications of the tool I have discussed with you. My particular interest stems from the fact that I am an accountant and I feel that by employing the notion of subjective probabilities, accountants will be able to help executives like you in decision making efforts.

I have constructed an experiment in which I hope you will be willing to participate. The experiment involves statements of probability, and these statements will be statements of your personal probability estimates. The experiment is in no way meant to exploit you but rather it is intended to help us help you make better decisions by showing us how to present you with better information. The experiment will take about an hour and one half and will be run with at least four groups of six people. If you would be willing to participate in the experiment, please sign one of the forms that are available. Thank you for your time and attention.
CHAPTER IV

MEASUREMENT ALTERNATIVE AND

THE STATISTICAL MODEL

In this chapter, the experimental measurement and the statistical model are discussed. These topics are interrelated. The choice of a measurement leads to the use of a particular statistical model; the choice of a particular model requires that the process of collecting measurements not violate the assumptions of the statistical model.

The chapter begins with the consideration of the measurement alternatives, the criteria applied to the choice of a measurement, and the selection of a particular measurement. The discussion then shifts to consideration of the nature of the statistical inquiry and a description of the statistical model. Finally, attempts to measure the ability of the data to meet the requirements of the statistical model are discussed.

Measurement Alternatives

The major candidates to measure each subject's behavior are (1) a set of accuracy ratios, (2) a set of absolute values of differences between the subject's estimate and the Bayesian estimate, (3) an average over all of the subject's accuracy ratios, and (4) an average over the set of all absolute values of differences between the subject's
estimates and the Bayesian estimates.

A Set of Accuracy Ratios

The accuracy ratio is the most often used measurement in Bayesian revision research.\textsuperscript{1} To examine this measurement, let $H_1$ and $H_2$ denote two mutually exclusive and exhaustive hypotheses. Let $Pr_s(H_i|D)$ stand for the subject's posterior probability, the probability of $H_i$ after the subject has seen the datum. The datum can be any message from either a single or a joint information system.

If the subject is intuitively using Bayes' theorem, he arrives at $Pr_s(H_i|D)$ in the manner described in the following equation:

$$Pr_s(H_1|D) = \frac{Pr_s(D|H_1) Pr_s(H_1)}{2 \sum_{i=1} Pr_s(D|H_i) Pr_s(H_i)}$$

Similarly, for $H_2$

$$Pr_s(H_2|D) = \frac{Pr_s(D|H_2) Pr_s(H_2)}{2 \sum_{i=1} Pr_s(D|H_i) Pr_s(H_i)}$$

Dividing $Pr_s(H_1|D)$ by $Pr_s(H_2|D)$, assuming $Pr_s(H_2|D) \neq 0$ yields

$$\frac{Pr_s(H_1|D)}{Pr_s(H_2|D)} = \frac{Pr_s(D|H_1) Pr_s(H_1)}{Pr_s(D|H_2) Pr_s(H_2)}$$

Taking the logarithm of both sides of the equation yields\textsuperscript{2}

\textsuperscript{1}David A. Schum, "Prior Uncertainty and Amount of Diagnostic Evidence as Variables in a Probabilistic Inference Task," \textit{Organizational Behavior and Human Performance}, 1, 1966, pp. 31-54. (Hereafter referred to as "Prior Uncertainty.")

\textsuperscript{2}In this chapter, the logarithms have as a base the number e.
\[
\log \frac{\Pr_s(H_1|D)}{\Pr_s(H_2|D)} = \log \frac{\Pr_s(D|H_1)}{\Pr_s(D|H_2)} + \log \frac{\Pr_s(H_1)}{\Pr_s(H_2)}
\]

Experimentally, \(\Pr_s(H_2|D)\) is usually inferred by using the fact that \(H_1\) and \(H_2\) are mutually exclusive and exhaustive and computing \(1 - \Pr_s(H_1|D)\); also, subjects are often told that the prior probability of each hypothesis is \(1/2\), i.e.,
\[
\Pr_s(H_1) = \Pr_s(H_2) = 1/2
\]
which implies
\[
\log \frac{\Pr_s(H_1)}{\Pr_s(H_2)} = 0
\]
Thus, the \(\log \frac{\Pr_s(D|H_1)}{\Pr_s(D|H_2)}\) is inferred from knowledge of the subject's behavior; \(\log \frac{\Pr_s(D|H_1)}{\Pr_s(D|H_2)}\) is called the inferred likelihood ratio of the subject. It should be noted that when logs of both sides of the equation are not taken, \(\frac{\Pr_s(H_1|D)}{\Pr_s(H_2|D)}\) can be inferred by using the same facts about the subject's behavior. The quantity \(\frac{\Pr_s(D|H_1)}{\Pr_s(D|H_2)}\) is called the subject's inferred likelihood ratio.

On the basis of knowledge about the data input of the experiment, the Bayesian likelihood ratio and the Bayesian loglikelihood ratio can be computed for a datum. The Bayesian inferred loglikelihood ratio is denoted by
\[
\log \frac{\Pr_B(D|H_1)}{\Pr_B(D|H_2)}
\]
Determining an accuracy ratio for a particular datum consists of finding the subject's inferred loglikelihood ratio and dividing it by the Bayesian loglikelihood ratio for the same datum. Thus, if $\text{A.R.}$ denotes the accuracy ratio,

$$\text{A.R.} = \frac{\log \frac{\Pr_S(D|H_1)}{\Pr_S(D|H_2)}}{\log \frac{\Pr_B(D|H_1)}{\Pr_B(D|H_2)}}$$

It should be noted that the less the subject deviates from Bayes, the closer the subject's accuracy ratio will be to one.

Using the accuracy ratio approach, a set of measurements for each subject can be compiled. The set consists of accuracy ratios of those messages for which there exists a message with the same Bayesian likelihood ratio for each subject.

**A Set of Absolute Values of the Differences between the Subject's Posterior Estimate and the Bayesian Posterior Estimate**

The second type of measurement is the absolute value of the difference between the subject's posterior estimate and the Bayesian posterior estimate, i.e.,

$$\left| \Pr_S(H_1|D) - \Pr_B(H_1|D) \right|$$

A set of these absolute values is computed for each subject. The set is determined by those messages for which there exists a message with the same Bayesian likelihood ratio for each subject.

**The Average Accuracy Ratio**

The average accuracy ratio involves finding all the accuracy
ratios for the subject, weighting the accuracy ratio by its probability of occurrence and summing over the weighted ratios.

Using the notation of Chapter II, the measure for the single system would be

\[
\sum_{m \in M} \frac{\log \frac{\Pr_S(\{x|f(x) = m\} \mid T_1)}{\Pr_S(\{x|f(x) = m \mid T_2)}}}{\log \frac{\Pr_B(\{x|f(x) = m\} \mid T_1)}{\Pr_B(\{x|f(x) = m \mid T_2)}}
\]

where \(\Pr_S(\{x|f(x) = m\} \mid T_1)\) is the subject's inferred likelihood ratio with respect to message \(m\) and \(\Pr_B(\{x|f(x) = m\} \mid T_1)\) is the Bayesian likelihood ratio derived from the data inputs of the experiment.

Using the notation of Chapter II, the average accuracy ratio of a subject in the joint information system would be

\[
\sum_{(m,w) \in M \times X} \frac{\log \frac{\Pr_S(\{x|f(x) = m, g(x) = w\} \mid T_1)}{\Pr_S(\{x|f(x) = m, g(x) = w \mid T_2)}}}{\log \frac{\Pr_B(\{x|f(x) = m, g(x) = w\} \mid T_1)}{\Pr_B(\{x|f(x) = m, g(x) = w \mid T_2)}}
\]

where \(\Pr_S(\{x|f(x) = m, g(x) = w\} \mid T_1)\) is the subject's likelihood ratio with respect to message \((m,w)\) and \(\Pr_B(\{x|f(x) = m, g(x) = w\} \mid T_1)\) is the Bayesian likelihood ratio with respect to message \((m,w)\).

The Average Absolute Value of the Differences

The average absolute value of the differences is computed by tak-
ing the absolute value of the difference between the subject's pos-
terior estimate and the Bayesian posterior estimate for each message 
and weighting each absolute value by the message's probability of occur-
rence. The sum of these weighted absolute values is taken to be the 
measurement for the subject.

Using the notation of Chapter II, this measure for the single 
information system would be

\[ \sum_{m \in M} \text{Pr}(\{x|f(x)=m\}) \left| \text{Pr}_s(T_1|x=f(x)=m) - \text{Pr}_B(T_1|x=f(x)=m) \right| \]

where \( \text{Pr}_s(T_1|x=f(x)=m) \) is the subject's stated posterior probability 
and \( \text{Pr}_B(T_1|x=f(x)=m) \) is the Bayesian posterior probability.

The average absolute value of the differences for the joint in-
formation system, using the notation of Chapter II, would be

\[ \sum_{(m,w) \in M \times W} \text{Pr}(\{x|f(x)=m, g(x)=w\}) \left| \text{Pr}_s(T_1|x=f(x)=m, g(x)=w) - \text{Pr}_B(T_1|x=f(x)=m, g(x)=w) \right| \]

where \( \text{Pr}_s(T_1|x=f(x)=m, g(x)=w) \) is the subject's stated posterior pro-
bability for message \((m,w)\) and \( \text{Pr}_B(T_1|x=f(x)=m, g(x)=w) \) is the Bayes-
ian posterior estimate for the message \((m,w)\).

**Evaluation of Measurement Alternatives**

The major criteria by which the candidates to measure each sub-
ject's behavior have been evaluated are: (1) sensitivity of the measure-
ment to the experimental hypothesis, (2) the degree to which the measure-
ment introduces uncontrolled variation into the statistical process, 
and (3) knowledge about the probability distribution of the experimental 
measurement.
It will be shown that all of the proposed measurements fail to meet at least one of the criteria. The discussion will then center on attempts which can be made to minimize the measurement's faults in specific experimental paradigms. Some inadequacies of the measurements are not correctible in any experimental setting and would preclude meaningful measurement of an experimental hypothesis. One of the measurements, the average absolute value of the differences between the subject's and the Bayesian posterior estimate, can be used to perform a meaningful test of the hypothesis and attempts can be made to minimize its failure to meet other criteria.

A Set of Accuracy Ratios

The first measure examined on the bases of the criteria for measurement evaluation is the subject's set of accuracy ratios. The set consists of only those accuracy ratios for which there exists a message with the identical Bayesian loglikelihood ratio for each subject.

The positive attributes of the measure are that (1) knowledge about the probability distribution of the measurement exists and (2) the measurement does not introduce uncontrolled variation into the statistical process.

In a number of previous experiments, the probability distribution of the union of all sets of accuracy ratios is assumed to have been generated by a multivariate normal distribution. If the results

of the earlier experiments are to be incorporated in the present study, the experimental hypothesis can be tested using the statistical technique, the parametric analysis of variance.4

In addition to the fact that the set of accuracy ratios has a widely accepted distribution, the measure does not introduce uncontrolled variance into the statistical process. Only specific accuracy ratios of a subject are used—those accuracy ratios which have a Bayesian loglikelihood ratio in common with a Bayesian loglikelihood of every other subject. Thus, a Bayesian loglikelihood ratio unique to a particular type of subject does not determine any accuracy ratio used in the statistical process.

The negative characteristics of the measurement all relate to the measure's insensitivity in testing one experimental hypothesis. Since the experimental hypothesis stated in Chapter II indicates that the average difficulty in estimation will be at least as great for the joint system as for the single system, the process of selecting only particular ratios and comparing them will not generate a sensitive test for this hypothesis. The measurement will not be testing an average; it will be testing a number of individual items.5

---


5If a four-way analysis of variance were used as suggested earlier, a weighting system would be used to compare the main effects of the two information systems; however, in such a case, the messages would all be given equal weights and would not be weighted by their probability of occurrence.
Under the experimental hypothesis, the appropriate weights for determining the average amount of information reduction (and, hence, task difficulty) are probabilities of occurrences of particular messages. The set of accuracy ratios measurement technique not only ignores the weighting system, but excludes some of the messages as well.

Another problem which leads to data exclusion using the set of accuracy ratios is the explosion of the accuracy ratio when prior probabilities of two mutually exclusive and exhaustive hypotheses are 1/2 and when the Bayesian posterior estimate approaches 1/2. As the Bayesian posterior estimate approaches 1/2, the Bayesian loglikelihood ratio approaches zero and the accuracy ratio approaches ±∞. It would appear that accuracy ratios based on zero or near zero loglikelihood ratios would have to be eliminated from consideration by the experimenter.

A Set of Absolute Values of the Differences between the Subject's Posterior Estimate and the Bayesian Posterior Estimate

Like the set of accuracy ratios, the set of absolute values of the differences between the subject's posterior estimate and the Bayesian posterior estimate is a selective measurement and excludes certain messages, namely those which do not yield a Bayesian posterior probability for which there is an identical counterpart for every subject. Because of this exclusion, the measurement is insensitive to the hypothesis concerning task difficulty.

Like the set of accuracy ratios, the set of absolute values does
not introduce uncontrolled variation into the experiment; only those absolute values which have identical Bayesian loglikelihood ratios for each subject are admitted.

Unlike the set of accuracy ratios, the set of absolute values has no known distribution, and a statistical test may lack the efficiency which exists when the correct distribution is known. Unlike the set of accuracy ratios, the set of absolute values does not explode when the Bayesian loglikelihood ratio is at or near zero.

The Average Accuracy Ratio

Since a linear combination of elements from a multivariate normal is itself a multivariate normal, it follows that the set of average accuracy ratios of subjects will be distributed as a multivariate normal. Under this probability distribution, the appropriate statistical test for examining experimental differences is the parametric analysis of variance.

Unlike the set of accuracy ratios and the set of absolute values of the differences, the average accuracy ratio is not restrictive and includes all of the estimates generated from the system (as long as no Bayesian loglikelihood ratio is at or near zero). Thus, the average accuracy ratio is a more inclusive measure and seems to be more sensitive to the experimental hypothesis about task difficulty.

The average accuracy ratio can introduce uncontrolled variance.

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6For a demonstration, see H.A. Scheffé, The Analysis of Variance (New York: Wiley & Sons, 1960), Appendix 5. (Hereafter referred to as The Analysis of Variance.)
into the statistical process, and this variation may influence the
test of the hypothesis that the joint information system tends to pro-
duce at least as great a divergence from the Bayesian estimate as the
single information system. The variation arises if the distribution of
the likelihood ratios differs in both systems.

A discussion of the importance of the uncontrolled variation
and an indication of the process by which the uncontrolled variation
leads to a false confirmation of the hypothesis follows.

The most general finding of Bayesian revision research is that
subjects tend to be conservative.\(^7\) Although conservatism is the most
frequent finding, it is not all pervasive over the set of all Bayesian
likelihood ratios. For Bayesian likelihood ratios at or near one, sub-
jects have often made estimates close to the Bayesian posterior esti-
mate and have occasionally exceeded the Bayesian estimate. As the
Bayesian likelihood ratio departs from one (for instance, 1/4 or 4/1)
changes in subjects' probabilities become less than changes in Bayesian
probabilities, and as the likelihood ratio departs markedly from one,
changes in subjects' probabilities become much less than changes in
Bayesian probabilities.\(^8\)

The results of previous experimentation indicate that it is en-
tirely possible to get a difference between the joint information sys-
tem and the single information system simply because the distribution

\(^7\)Supra, p. 8-10

\(^8\)DuCharme and Peterson, "Intuitive Inference," pp. 269-75.
of the likelihood ratios of the joint system is different from the distribution of the likelihood ratios of the single system.

As an example, consider the set of events

\[ \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \]

and suppose the discrete probability distribution of these events is given by

\[
\begin{align*}
P(x_1) &= 5/48 \\
P(x_2) &= 1/48 \\
P(x_3) &= 1/12 \\
P(x_4) &= 5/12 \\
P(x_5) &= 5/24 \\
P(x_6) &= 1/24 \\
P(x_7) &= 1/48 \\
P(x_8) &= 5/48
\end{align*}
\]

Let the event \( A \) be denoted by the set \( \{x_1, x_3, x_5, x_7\} \) and let \( \overline{A} \) be denoted by the set \( \{x_2, x_4, x_6, x_8\} \).

Furthermore, suppose that there are two information systems. The single information system has two messages. The first message, \( m_1 \), states whether the event is a member of the set \( \{x_1, x_2, x_3, x_4\} \) and the second message, \( m_2 \), states whether the event is a member of the set \( \{x_5, x_6, x_7, x_8\} \).

The information system which is combined with the single information system to form the joint system has two messages. The first message, \( w_1 \), states whether the event is a member of the set
\{ x_1, x_2, x_5, x_6 \}; the second message, w_2, states whether the event is a member of the set \{ x_3, x_4, x_7, x_8 \}.

Assume that an experiment using the information systems is performed. The experimenter flips a coin and chooses set A or set \( \overline{A} \) on the basis of the way in which the coin lands (heads or tails). The experimenter tells the subject the message and asks the subject to estimate the probability that the message came from set A. The experiment is varied by including a single information system for some subjects and a joint information system for other subjects.

In this hypothetical experiment, the Bayesian likelihood ratio for each system would be as follows.

**Single information system**

| Message \( (m_i) \) | Likelihood ratio \( \frac{\Pr_B(\{x | f(x) = m_j^i \} | A)}{\Pr_B(\{x | f(x) = m_j^i \} | \overline{A})} \) |
|----------------------|-------------------------------------------------|
| \( m_1 \)           | \( \frac{9}{21} \)                              |
| \( m_2 \)           | \( \frac{11}{7} \)                              |

**Joint information system**

| Message \( (m_i, w_j) \) | Likelihood ratio \( \frac{\Pr_B(\{x | f(x) = m_i, g(x) = w_j \} | A)}{\Pr_B(\{x | f(x) = m_i, g(x) = w_j \} | \overline{A})} \) |
|---------------------------|-------------------------------------------------|
| \( (m_1, w_1) \)         | \( \frac{5}{1} \)                              |
| \( (m_1, w_2) \)         | \( \frac{1}{5} \)                              |
| \( (m_2, w_1) \)         | \( \frac{5}{1} \)                              |
| \( (m_2, w_2) \)         | \( \frac{1}{5} \)                              |

Every likelihood ratio in the distribution of the joint system's like-
lihood ratios differs from 1 by more than at least one of the members of the single system. Thus, it would be expected that there would be a greater deviation from Bayesian estimates when subjects' estimates are made on the basis of messages from the joint system than when made on the basis of messages from the single system.

A major inadequacy with the average accuracy ratio is that, even though a true difference between the subject's and the Bayesian estimates may exist, the average accuracy ratio can fail to uncover the difference.

As an illustration, assume that a single information system generates equally likely messages, \( m_1, m_2 \). Further assume that for an event T, the Bayesian likelihood ratios are as follows:

<table>
<thead>
<tr>
<th>Message</th>
<th>Bayesian Likelihood Ratio</th>
</tr>
</thead>
</table>
| \( m_1 \) | \[
\frac{\Pr_B(\{x|f(x)=m_1\}T)}{\Pr_B(\{x|f(x)=m_1\}\bar{T})} = 2
\] |
| \( m_2 \) | \[
\frac{\Pr_B(\{x|f(x)=m_2\}T)}{\Pr_B(\{x|f(x)=m_2\}\bar{T})} = 5
\] |

Assume that the joint information system yields messages \((m_1,w_1), (m_2,w_2), (m_2,w_3)\), and assume that the probability of \((m_1,w_1)\) occurring is 1/2, the probability of \((m_2,w_2)\) occurring is 1/4, and the probability of \((m_2,w_3)\) occurring is 1/4. Further assume that the following Bayesian likelihood ratios exist for event T:

<table>
<thead>
<tr>
<th>Message</th>
<th>Bayesian Likelihood Ratio</th>
</tr>
</thead>
</table>
| \((m_1,w_1)\) | \[
\frac{\Pr_B(\{x|f(x)=m_1, g(x)=w_1\}T)}{\Pr_B(\{x|f(x)=m_1, g(x)=w_1\}\bar{T})} = 2
\] |
Message \hspace{1cm} \text{Bayesian Likelihood Ratio}

\[(m_2, w_2)\]
\[
\frac{\Pr_B(f(x)^{m_2}, g(x)=w_2J | T)}{\Pr_B(f(x)^{m_2}, g(x)=w_2J | T)} = \frac{5}{1}
\]

\[(m_2, w_3)\]
\[
\frac{\Pr_B(f(x)^{m_2}, g(x)=w_3J | T)}{\Pr_B(f(x)^{m_2}, g(x)=w_3J | T)} = \frac{5}{1}
\]

Assume that a probability estimation experiment has been performed; that the subject was successfully measured on both systems; and that for the single information system his inferred likelihood ratios were 3/1 and 4/1 for messages \(m_1\) and \(m_2\) respectively. His inferred likelihood ratios were 3.1/1 for all three messages in the joint system.

It is clear that when holding the likelihood ratio fixed in both systems, the subject performs closer to Bayes in the single system. However, in computing average accuracy ratios, it is found that for the single system, the average accuracy ratio is 1.223, while the average accuracy ratio for the joint system is 1.167, indicating that, on the average, the subject is closer to Bayes under the joint system than under the single system.

The Average Absolute Value of the Differences

Like the set of absolute values, the distribution of the average absolute value of the differences is not known; therefore, the test of the experimental hypothesis may not be as efficient as when the distribution is known. Furthermore, like the average accuracy ratio, the measurement can add uncontrolled variation to the experimental setting.

The important attribute of this measurement, which makes it
superior to the other measurements, is that the average absolute value of the differences is sensitive to the experimental hypothesis concerning task difficulty since it uses all of the data and makes the differences from the Bayesian estimate additive. In addition, the deviations are weighted by the system described in Chapter II.

The Selection

On the basis of the evaluation of the measurement alternatives, the average absolute value of the differences was chosen for use in this study. Such a choice leads to the use of the nonparametric analysis of variance.

The problem of uncontrolled variation associated with the average absolute value of the differences measurement is attacked by attempting to keep the distribution of the likelihood ratios similar. For instance, the first moment of the distribution of the likelihood ratios used in the single information system is 1.5924, the second moment is 3.4353, the third moment is 9.3980, and the fourth moment is 35.6389. For the joint system, the first moment is 1.6714, the second is 3.2244, the third is 9.9640, and the fourth is 35.3971.

The Statistical Model

The statistical model of a measurement from the experiment includes no assumption about the distribution of the error in measurement. This model leads to tests of experimental hypotheses using the nonparametric analysis of variance.
An oddity of the approach is that a parametric statistical test, the parametric analysis of variance, approximates the results of the nonparametric test. Since many of the necessary extensive tables do not exist for the complex higher order types of nonparametric analyses of variance dictated by such experiments as the present one, the parametric test equivalent is used as a replacement.  

In the present experiment, all of the statistical tests which are desirable when the nonparametric technique is used cannot be performed. Specifically, the technique and tables for testing the three-way interactions are not available. For this reason, the equivalent parametric technique is used. The general reasoning underlying this approach is presented in an article by Eddington. Numerous criticisms have been made of the approach, among them the one found in Bradley's Distribution Free Statistical Tests. A defense of the

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9 For a discussion see, Scheffé, The Analysis of Variance, Chapter 10.


11 In his text, Distribution Free Statistical Tests, Bradley describes a test using the two-way analysis of variance; however, a three-way higher order analysis is not mentioned. In a personal communiqué, Bradley suggested that the main effects and the first order interactions of the three-way design be tested using the method of his text. For the higher order interactions, Bradley suggests using the equivalent parametric test. These tests have been performed and a one-tail test of significance using Bradley's procedure yields the same results in terms of the main effects and first order interactions as the two-tailed test described in the section, "The Model Underlying the Parametric Test," of this paper. For further information on the test, see, James V. Bradley, Distribution Free Statistical Tests (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1968), pp. 87-145.
approach and a basis for the development of this section is found
in Scheffe. 12

The development of the remainder of this chapter is as follows. first, the model underlying the nonparametric test of the hypotheses
is described. Second, the equivalent model underlying the parametric
test is described, and the appropriate statistical tests are developed. Last, the attempts to meet the requirements of the model which under-
lies the nonparametric tests are discussed.

Model Underlying the Nonparametric Test

In the present experiment several factors vary: the type of
information system, the nature of the setting, and the type of subject.

The type of information system is called factor A. Factor A
has two levels: one level is the single information system; the second
is the joint information system. The nature of the setting is called
factor B. Factor B has two levels: one level is cubes and algebraic
identities; the second is hypothetical businesses and profit figures.
Factor C, the type of subject, also has two levels: businessmen parti-
cipating in The Ohio State University Executive Development Program
and students in The Ohio State University Undergraduate Business Ad-
ministration Program.

Before subjects are assigned to treatment combinations, it
is possible that any member of a specific subpopulation will be ex-
amined under any treatment combination of factor A and factor B. The

12 Scheffe, The Analysis of Variance, Chapter 10.
set of possibilities will be called the set of potential measurements.

The potential experimental measurements are members of the set \( \{y_{ijkl}\} \). \( y_{ijkl} \) stands for the potential measurement of the 1th member of the kth subpopulation if the subject is examined under the ijth treatment combination. Thus, \( y_{1,2,2,15} \) stands for the measurement of the 15th student when the student is observed using the single information system with hypothetical businesses and profit figures. Note that not all potential measurements are actual measurements since \( y_{1,1,1,15} ; y_{1,1,2,15} ; \ldots ; y_{2,2,2,15} \) are all potential measurements but only one is an actual measurement.

A potential measurement consists of two parts: the true response of the 1th member of the subpopulation k if he is observed under treatment combination ij and the error in observing the 1th member of the subpopulation k under the ijth treatment combination. The true response is denoted by \( \eta_{ijkl} \). The error is denoted by \( u_{ijkl} \); therefore,

\[
y_{ijkl} = \eta_{ijkl} + u_{ijkl}
\]

\( \eta_{ijkl} \) is a constant and \( u_{ijkl} \) is a random variable with mean 0, so that

\[
E(y_{ijkl}) = \eta_{ijkl}
\]

where \( E \) denotes the expected value operator.

A new set of definitions follows. Let \( L_k \) stand for the number of subjects in the kth subpopulation; then
\[ \eta_{ijk.} = \sum_{l=1}^{L_k} \frac{\eta_{ijkl}}{L_k} \]

That is, \( \eta_{ijk.} \) is the mean over all subjects' true responses in subpopulation \( k \) when this specific subpopulation is observed under the \( ij \) treatment combination.

\[ \eta_{..jk} = \frac{\sum_{i=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{2L_k} \]

\[ \eta_{i..k} = \frac{\sum_{j=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{2L_k} \]

\[ \eta_{ij..} = \frac{\sum_{j=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{(L_1+L_2)} \]

\[ \eta_{..k..} = \frac{\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{4L_k} \]

\[ \eta_{ijk..} = \frac{\sum_{i=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{4(L_1+L_2)} \]

\[ \eta_{i...} = \frac{\sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{L_k} \eta_{ijkl}}{4(L_1+L_2)} \]

The main effect of the \( i \)th level of factor A, \( a_i \), is the difference between the mean of the true response of all subjects poten-
tially observed under the ith condition, $\eta_{i...}$, and the overall mean, $\eta_{...}$, i.e.,

$$a_i = \eta_{i...} - \eta_{...}$$

Similarly, $\beta_j = \eta_{.j...} - \eta_{...}$ and $\gamma_k = \eta_{..k...} - \eta_{...}$

$a_{ij}$ is the combined effect of the ith level of A and the jth level of B. It is found by removing the main effects $\{a_i, \beta_j\}$ and the overall mean, $\eta_{...}$, from $\eta_{ij...}$, i.e.,

$$a_{ij} = \eta_{ij...} - a_i - \beta_j - \eta_{...} = \eta_{ij...} - \eta_{i...} - \eta_{.j...} + \eta_{...}$$

Similarly,

$$a_{ik} = \eta_{i.k...} - \eta_{i...} - \eta_{..k...} + \eta_{...}$$

$$\beta_{jk} = \eta_{.jk...} - \eta_{.j...} - \eta_{..k...} + \eta_{...}$$

$a_{ijk}$ is the combined effect of the ith level of A, the jth level of B, and the kth level of C. It is the mean, $\eta_{ijk...}$, less the interactions $a_{ij}, a_{ik}, \beta_{jk}$, the main effects $(a_i, \beta_j, \gamma_k)$, and the overall mean, $(\eta_{...})$, i.e.,

$$a_{ijk} = \eta_{ijk...} - a_{ij} - a_{ik} - \beta_{jk} - a_i - \beta_j - \gamma_k - \eta_{...} = \eta_{ijk...} - \eta_{i...} - \eta_{.k...} - \eta_{.jk...} + \eta_{i...} + \eta_{..k...} - \eta_{...}$$

The individual effect, $\xi_{ijkl}$, is the difference between the true response of the lth subject of the k subpopulation observed under treatment combination ij; $\eta_{ijkl}$; and the mean of the set of observations of the kth subpopulation when examined under the treatment combination, ij; i.e.,
\[ \xi_{ijkl} = \eta_{ijkl} - \eta_{ijk}. \]

Using these definitions and letting \( \mu = \eta \ldots \), a potential score becomes the following:

\[ y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + a\beta_{ij} + a\gamma_{ik} + \beta\gamma_{jk} + a\beta\gamma_{ijk} + \xi_{ijkl} + u_{ijkl} \]

One assumption which allows a test of the experimental hypotheses is that the \( \xi_{ijkl} \) are constant across the experimentally manipulated treatment combinations, i.e.,

\[ \xi_{ijkl} = \xi_{kl}, \text{ for all } ij, \]

so that no matter what the treatment combination, the individual effect does not vary.

The second assumption which allows a test of the experimental hypotheses is that the \( u_{ijkl} \) are independent of the treatment combinations and that the distribution of error is the same across different treatments, i.e.,

\[ u_{ijkl} = u_{kl}, \text{ for all } ij \]

Furthermore, the \( u_{kl} \) are expected to be independent across subjects so that

\[ E(u_{kl} | \mu_{k'1},) = E(\mu_{k'1}) \ E(\mu_{k'1}) \]

for either \( k \neq k' \) or \( l \neq l' \).

Adjusting the model to reflect the assumption yields

\[ y_{ijkl} = \mu + \alpha_i + \beta_j + \gamma_k + a\beta_{ij} + a\gamma_{ik} + \beta\gamma_{jk} + a\beta\gamma_{ijk} + \xi_{kl} + u_{kl} \]
The standard statistical questions are about the main effects and interactions. The standard hypotheses are:

\[ H_0: \quad a_i = 0 \text{ for all } i \]
\[ H_0: \quad \beta_j = 0 \text{ for all } j \]
\[ H_0: \quad \gamma_k = 0 \text{ for all } k \]
\[ H_0: \quad a\beta_{ij} = 0 \text{ for all } ij \]
\[ H_0: \quad a\gamma_{ik} = 0 \text{ for all } ik \]
\[ H_0: \quad \beta\gamma_{jk} = 0 \text{ for all } jk \]
\[ H_0: \quad a\beta\gamma_{ijk} = 0 \text{ for all } ijk \]

Tests can be legitimately performed only if a random process underlies the assignment of subjects in subpopulations of the experimental treatments.

The derivation of the tests of significance level involves taking the actual measurements that occur (as opposed to the potential measurements) and finding a set of permutations of the actual measurements which are equally likely under the hypothesis being tested (e.g., \( a_i = 0 \) for all \( i \)).

For each of the permutations, a statistic is computed. For example, in the case of testing the hypothesis that \( a_i = 0 \) for all \( i \), the statistic is

\[
S = \frac{\zeta \sum_{i=1}^{2} (y_{i...} - y_{...})^2}{\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{T} \sum_{t=1}^{T} (y_{ijkt} - y_{ijk.})^2}
\]
where the dot notation indicates finding a mean as before, \( T \) is the number of observations in each \( ijk \) treatment combination and \( \zeta \) is any positive constant.\(^\text{13}\)

\( S^* \), a particular \( S \), is chosen which has the property of being the lowest \( S \) for which there are at least \( 100 - \alpha \) percent of the members of the set of permutations which have \( S \) values less than \( S^* \). The test then consists of rejecting the null hypothesis if the actual \( S \) observed is greater than \( S^* \).

The Model Underlying the Parametric Test

The model underlying the nonparametric test and the testing procedure have been discussed. The model underlying the nonparametric test represents the model which is the basis of the experiment and attempts have been made to meet the assumptions of the model. The model underlying the parametric test can be used to approximate the results of the nonparametric technique.

Let \( y_{ijkt} \) denote the actual set of measurements (not the potential set) taken in the experiment.

\[ i = 1, 2 \]
\[ j = 1, 2 \]
\[ k = 1, 2 \]
\[ t = 1, \ldots, T \] where \( T \) is the number of measurements taken

\(^{13}\)This is not the typical measurement found in statistics books (e.g., Bradley). The difference results from the fact that scores in these texts have been transformed to ranks and the statistic, \( S \), has been transformed by a positive monotonic transformation. The results of both tests are, however, identical. For a general discussion, see Scheffe, The Analysis of Variance, Chapter 9.
under each \( ijk \) treatment combination. (Here an equal number of observations are assumed per cell to facilitate presentation and because the experimenter randomly eliminated members of cells with larger numbers of observations per cell to arrive at an equal number per cell.)

The model of a measurement has two parts which can be represented as

\[
y_{ijkl} = \eta_{ijk} + u_{ijkl}
\]

\( u_{ijkl} \) is a measurement error and random variable which is assumed to have mean 0, variance \( \sigma_u^2 \), a normal distribution, and is assumed to be independent of all other \( u_{ijkl} \).

Using these assumptions, definitions of \( \eta_{...}; \eta_{i...}, \) etc. can be developed, and similarly, definitions of \( a_i, \beta_j, \) etc. can be developed which yield

\[
y_{ijkl} = a_i + \beta_j + \gamma_k + \alpha_i \beta_j + \alpha_i \gamma_k + \beta_j \gamma_k + \alpha_i \beta_j \gamma_k + u_{ijkl}
\]

Under this model, the same experimental questions as those stated in the discussion of the model underlying the nonparametric test exist. Tests are arrived at using the least squares technique and the assumptions of normality, equal variance, and independence of error terms.

In the case of testing the assumption, all \( a_i = 0 \), the statistic computed is similar to that described for the nonparametric case. The statistic is
\[
\frac{\sum(y_{1..} - y_{...})^2}{\sum(y_{ijk} - y_{ij..})^2 / 8(T-1)}
\]

The statistic has an F distribution with 1 and 8(T - 1) degrees of freedom. The hypothesis that all \(a_i = 0\) is rejected at the \(a\) significance level if the computed statistic is greater than \(F_{a,1,8(T-1)}\) where \(F_{a,1,8(T-1)}\) is a number such that the probability the specified F distribution is greater than \(F_{a,1,8(T-1)}\) is equal to \(a\).

The numerator of the statistic is called the sum of squares of factor A (SSA); the denominator is the sum of squares error divided by its degrees of freedom. All of the tests to be performed using this technique are listed in the table on the following page (Table 2).

The table\(^{14}\) indicates the F tests that are performed on the experimental data. The experimenter predicted that there would be a significant A main effect and a significant BC interaction effect, but made no predictions concerning the other effects.

Attempts to Meet the Requirements of the Model

The means used by the experimenter to meet the requirements of the model follow. The relevant model is the model underlying the non-parametric test. The requirements of this model influence the manner

\(^{14}\) Here, as in a number of presentations, the only distinction between a main effect and a significance level is that the main effect has a subscript and the significance level does not.
TABLE 2.—F Tests Performed on the Data

<table>
<thead>
<tr>
<th>Effects</th>
<th>Sum of Squares</th>
<th>df</th>
<th>MS</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$\sum_{i=1}^{2} (y_i \ldots - \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{i=1}^{2} (y_i \ldots - \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_A}{MS_E}$</td>
</tr>
<tr>
<td>B</td>
<td>$\sum_{j=1}^{2} (y_{j} \ldots - \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{j=1}^{2} (y_{j} \ldots - \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_B}{MS_E}$</td>
</tr>
<tr>
<td>C</td>
<td>$\sum_{k=1}^{2} (y_{k} \ldots - \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{k=1}^{2} (y_{k} \ldots - \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_C}{MS_E}$</td>
</tr>
<tr>
<td>AB</td>
<td>$\sum_{i=1}^{2} \sum_{j=1}^{2} (y_{ij} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{j} \ldots + \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{i=1}^{2} \sum_{j=1}^{2} (y_{ij} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{j} \ldots + \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_{AB}}{MS_E}$</td>
</tr>
<tr>
<td>AC</td>
<td>$\sum_{i=1}^{2} \sum_{k=1}^{2} (y_{ik} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{k} \ldots + \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{i=1}^{2} \sum_{k=1}^{2} (y_{ik} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{k} \ldots + \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_{AC}}{MS_E}$</td>
</tr>
<tr>
<td>BC</td>
<td>$\sum_{j=1}^{2} \sum_{k=1}^{2} (y_{jk} \ldots - \bar{y}<em>{j} \ldots - \bar{y}</em>{k} \ldots + \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{j=1}^{2} \sum_{k=1}^{2} (y_{jk} \ldots - \bar{y}<em>{j} \ldots - \bar{y}</em>{k} \ldots + \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_{BC}}{MS_E}$</td>
</tr>
<tr>
<td>ABC</td>
<td>$\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} (y_{ijk} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{j} \ldots - \bar{y}_{k} \ldots + \bar{y} \ldots)^2$</td>
<td>1</td>
<td>$\frac{\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} (y_{ijk} \ldots - \bar{y}<em>{i} \ldots - \bar{y}</em>{j} \ldots - \bar{y}_{k} \ldots + \bar{y} \ldots)^2}{2}$</td>
<td>$\frac{MS_{ABC}}{MS_E}$</td>
</tr>
<tr>
<td>Error</td>
<td>$\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{L} (y_{ijkl} - \bar{y}_{ijkl})^2$</td>
<td>$\frac{\sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{2} \sum_{l=1}^{L} (y_{ijkl} - \bar{y}_{ijkl})^2}{8(T-1)}$</td>
<td></td>
<td>137</td>
</tr>
</tbody>
</table>
in which data is collected and a response of the subject is obtained.

The first assumption of the model is the implicit assumption that the overall mean, \( \eta \), and the means of individual treatment combinations are stable during the measurement process. This assump-
the experimenter to use only 30 percent of each subject's experimental trials in data analysis.

In reality, it is expected that a number of experimental trials would represent a learning situation for subjects and that it would require a number of trials before subjects became accustomed to the ex-
perimental environment. After these learning trials, it was anticipated that subjects' estimates would stabilize.

In a number of pretrials, the experimenter asked pretrial sub-
jects how many trials of the experiment were necessary for them to be-
come accustomed to the experimental environment. Pretrial subjects' answers and their estimates indicated that their behavior stabilized after the 70th trial. On this basis, the experimenter decided to use only the last 30 probability estimates of subjects.

To examine the possibility of a general learning effect during the first 70 trials and to examine the possibility of a learning ef-
fact in the individual ijk treatment conditions during the last 30 trials, nine tests were devised. The tests differ since the first examines all of the subjects simultaneously while tests two through nine examine subjects in the ijk treatment combinations simultan-
eously.
Learning is considered to be indicated by a subject's performing closer to Bayesian standards over time. The experimenter, therefore, expects a tendency to behave in a more Bayesian manner over a number of trials.

To determine whether a learning effect exists, a nonparametric trends test is used. A similar approach, utilizing a parametric technique, was used by Gettys and Martin. This test is built upon the binomial distribution.

The method of examining the data for a trend over the first 70 trials is as follows. First, after computing the absolute value of the difference between the subject's posterior estimate and the Bayesian posterior estimate for each of the 70 estimates, every ten of these absolute values are averaged. With these seven averages of absolute values per subject, the data are arranged in the following order:

\[ x_{1,1}, x_{1,2}, \ldots, x_{1,N}, x_{2,1}, x_{2,2}, x_{2,N}, \ldots, x_{7,1}, \ldots, x_{7,N} \]

where \( x_{i,j} \) represents the \( i \)th mean of the \( j \)th subject. If there is no trend then the probability of \( x_{i,j} \) being greater than \( x_{i',j} \) for any \( i \neq i' \) is .50. To perform the test, these differences were computed:

\[ x_{5,1} - x_{1,1}, x_{5,2} - x_{1,2}, \ldots, x_{5,N} - x_{1,N}, x_{6,1} - x_{2,1}, \ldots, x_{6,N} - x_{2,N}, \ldots, x_{7,1} - x_{3,1}, \ldots, x_{7,N} - x_{3,N} \]

15 For a discussion of the test, see, Bradley, *Distribution Free Statistical Tests*, pp. 174-78.
If there is no trend, the number of minus signs should tend to be equal to the number of plus signs. The assumption of no trend is rejected if the actually observed number of differences with minus signs is likely to occur α100 percent or less of the time when drawing 3N times from the binomial distribution.

The eight tests for trend of the individual ijk treatment combinations in the last 30 trials are slightly different. In this case, the absolute value of the difference between the subject's posterior estimate and the Bayesian posterior estimate is arranged over every three trials. The 10T measurements of the T people in the ijkth treatment combinations are arranged in the following order:

\[ z_{1,1}', z_{1,2}', \ldots, z_{10,1}', \ldots, z_{10,T} \]

where \( z_{i,j} \) represents the ith average of the jth subject. The following set of differences is developed:

\[ z_{6,1} - z_{1,1}', \ldots, z_{6,T} - z_{1,T}', z_{7,1} - z_{2,1}', \ldots, z_{10,T} - z_{5,T} \]

The test described previously is then performed on the 5T differences.

Another assumption of the model is that for all \( i,j \)

\[ \xi_{ijkl} = \xi_{kl} \]

Although no measurement of the extent to which this assumption may be violated can be made since the experimenter did not test all subjects on all treatments, an attempt was made to minimize the occurrence of a deviation from this assumption. It was assumed that a difference of individual effects would result from administering the
experiment differently in different ijk treatment combinations. To avoid this, the experimenter's apparel and tone, the experimental format, the physical distance between the experimenter and subjects, the mode of response, and the length of time allowed for response were held constant in all ijk treatment combinations.\textsuperscript{16}

The two assumptions of the model relating to $u_{ijkl}$ are that $u_{ijkl}$ are uncorrelated across subjects and that the $u_{ijkl}$ do not differ in distribution across treatment conditions. Rather than test the data for deviations from these assumptions, the experimenter attempted to control the factors that would give rise to deviations from the assumptions. Attempts to avoid intercorrelations of measurement error are described in the appendix to Chapter III.\textsuperscript{17} Attempts to keep the measurement error constant, if the subjects were exposed to all treatments, were as follows.

It was assumed that the distribution of the $u_{ijkl}$ would vary across the ij treatment combinations if the subject failed to understand the experiment. Lack of understanding would imply a higher probability of measurement error. To assess understanding, the experimenter graded questions asked in the experimental instructions completed by each subject. He required at least 85 percent correct

\textsuperscript{16}For a more complete discussion, see Supra, pp. 69-71.

\textsuperscript{17}Supra, pp. 69-71.
answers on those questions which related directly to the experimental question, i.e., understanding odds, computing profit figures, understanding the probability table, understanding the symbolic representations of companies and cubes. These questions are stated in the set of experimental instructions found in the appendix to Chapter III. The questions actually graded are marked with an asterisk (*).

Another means used to assess understanding of the experimental environment was post-experiment interviews with the subjects. Some subjects indicated that they forgot the meaning of the experimental response during the running of the experiment. The experimenter examined the estimates made by these subjects and found several estimates which indicated that an error in interpretation did occur. These subjects' answers were not used in the testing procedure.

In addition to attempting to keep the distribution of individual effects constant across the ij treatment conditions, the experimenter attempted to control the variance of the error distribution. This variance was controlled by taking the average of probabilistic estimates for a message over the last 30 trials to determine the probability estimates of a subject for a specific message. Thus, if there were three probability estimates for a specific message after the 70th trial and these estimates were 2/3, 3/4, and 5/6, then 3/4 would be taken as the estimate of the subject for the particular message. A subject's estimates might not be identical for a particular message even though a learning effect was not occurring because subjects tend to add variation to estimates since there is a utility to variation. The experi-
menter assumed that the variation would just as likely be an addition as a reduction from the true estimate of the subject and took the mean of the subject's responses as the method to reduce the variation effect.\textsuperscript{18}

Another requirement of the model which the experimenter attempted to satisfy was the random assignment of subjects to treatments. A model believed to underly the assignment process is presented in the appendix to Chapter III.\textsuperscript{19} In addition to the model, several characteristics of each subject in a subpopulation were collected to determine whether a bias in the assignment process could be suspected. Characteristics of executives included salary level, age, general area of specialization (e.g., sales, production, or finance), years of education beyond high school, and employer. Characteristics of students included grade point average, years of education beyond high school, subject area, and local address.

Two tests were used to detect whether a bias was present: The Kruskal-Wallis one-way analysis of variance and the Kolmogorov Smirnov test.\textsuperscript{20}

\textsuperscript{18} A review of repetitive tasks in which subjects have added variation is in William E. Scott, Jr., "The Behavioral Consequences of Repetitive Task Design," in Readings in Organizational Behavior and Human Performance, ed. by L.L. Cummings and William E. Scott (Homewood, Ill.: Richard D. Irwin and the Dorsey Press, 1969), pp. 42-59. Although the tasks are not of the type Scott describes, they are repetitive and the experimenter expected the subjects to produce some form of variation.

\textsuperscript{19} Supra, pp.

\textsuperscript{20} See Bradley, Distribution Free Statistical Tests, pp. 129-34 and 296-303.
The Kruskal-Wallis one-way analysis of variance can be used to examine non-randomness of ranked data such as age and grade point average. The Kolmogorov Smirnov test can be used to determine whether the cumulative distribution of a specific attribute varies from the expected distribution of the attribute when a random process occurs. The tests is not extremely sensitive since only a few subjects fall in each category and since the test, when applied to discrete distributions, tends to be conservative. The test was used to examine such attributes as area of specialization, salary range, years of education, and major subject area.

Summary

In this chapter, the interrelated topics of experimental measurement and the statistical model have been discussed. In summary, it might be helpful to relate exactly how the data are to be treated. In a chronological order, the following system is used.

1) The individual responses are examined to determine the degree to which the experimental instructions have been understood.

2) The ijk treatment combinations are made equal in size first by eliminating those subjects with the least number of valid responses to sample questions in the experimental instructions and then by randomly eliminating other subjects to equalize the ijk treatment combinations.

3) The remaining data are examined to determine if a bias has entered into the assignment process and tests for non-random
assignment are performed.

4) For each probabilistic estimate of each subject, the absolute value of the difference between the subject's posterior estimate and the Bayesian posterior estimate is computed.

5) A test for a learning trend is applied to the first 70 estimates of subjects, and a test for learning is applied to the last 30 estimates of subjects in the ijk treatment combinations.

6) The average estimate for a specific message over the last 30 estimates is subtracted from the Bayesian estimate for each of the messages, and the absolute value of the difference is recorded.

7) The absolute values are weighted by the probability of the occurrence of the message, and a sum of the weighted absolute values is recorded for use as a measurement of the subject.

8) The subject's measurements are examined using an analysis of variance technique.
CHAPTER V

DATA ANALYSIS

This chapter includes four major sections: (1) the results of the analysis of variance test, (2) the data relating to learning effects, (3) examinations for biasing factors in the sampling process, and (4) a discussion of deviations from the planned experimental procedure as presented in the appendix to Chapter III. An interpretation of the data and test results is presented in Chapter VI.

The Analysis of Variance Test

The basic measurement used for the analysis of variance test, the average absolute values of the differences between the Bayesian and the subjects' posterior estimates, are presented in the following table. Four results per cell of the table are recorded after (1) grading the experimental instructions, (2) eliminating responses of those subjects who did not understand the instructions, and (3) randomly eliminating measurements of subjects until there was an equal number of measurements in each treatment condition.
<table>
<thead>
<tr>
<th>Setting with</th>
<th>Single Information System</th>
<th>Joint Information System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students*</td>
<td>Businessmen*</td>
</tr>
<tr>
<td>Cubes and Algebraic Identities</td>
<td>.13250</td>
<td>.08119</td>
</tr>
<tr>
<td></td>
<td>.03499</td>
<td>.12835</td>
</tr>
<tr>
<td></td>
<td>.20070</td>
<td>.08169</td>
</tr>
<tr>
<td></td>
<td>.09175</td>
<td>.13818</td>
</tr>
<tr>
<td>Setting with Hypothetical Business and Profit Figures</td>
<td>.13717</td>
<td>.05412</td>
</tr>
<tr>
<td></td>
<td>.08081</td>
<td>.10953</td>
</tr>
<tr>
<td></td>
<td>.07706</td>
<td>.06299</td>
</tr>
<tr>
<td></td>
<td>.08119</td>
<td>.09753</td>
</tr>
</tbody>
</table>

*In all tables, "Students" refers to students in The Ohio State University Undergraduate Business Administration Program and "Executives" refers to executives in The Ohio State University Executive Development Program.

Table 4 summarizes the data in terms of the mean of the absolute value of the differences for specific combinations of different factors of the experiment.
TABLE 4.—Cell Means of the Average Absolute Value of the Differences between the Subjects' and Bayesian Posterior Estimates

<table>
<thead>
<tr>
<th>Setting</th>
<th>Single Information System</th>
<th>Joint Information System</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students</td>
<td>Businessmen</td>
</tr>
<tr>
<td>Setting with Cubes and Algebraic Identities</td>
<td>.11498</td>
<td>.10735</td>
</tr>
<tr>
<td>Setting with Hypothetical Businesses and Profit Figures</td>
<td>.09406</td>
<td>.08104</td>
</tr>
</tbody>
</table>

In Tables 5, 6, and 7, marginal means for specific levels of the experimental factors are presented. First, the marginal means for levels of information systems are presented; next, marginal means for the different experimental settings are presented; last, marginal means for the different types of subjects are presented.
TABLE 5.--Marginal Means for Different Levels of the Factor, Type of Information System

Single Information System  Joint Information System
0.09936  0.14389

TABLE 6.--Marginal Means for Different Levels of the Factor, Type of Experimental Setting

Setting Using Cubes and Algebraic Identities  Setting Using Hypothetical Businesses and Profit Figures
0.12126  0.12199

TABLE 7.--Marginal Means for Different Levels of the Factor, Type of Subject

Students  Businessmen
0.13115  0.11210

Table 8 contains the analysis of variance results. The second column is the sum of squares (SS) for each factor. The type of information system is factor A; the type of setting is factor B; and the type of subject is factor C. The degrees of freedom (df) are in the third column. The F ratios (F) are in the fifth column. The probability (p), the lowest tabled probability at which the null hypothesis of no difference can be refuted, is in the sixth column.
TABLE 8.--Analysis of Variance Source Table*

<table>
<thead>
<tr>
<th>Factors</th>
<th>SS</th>
<th>df</th>
<th>Mean Squares</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>.01587</td>
<td>1</td>
<td>.01587</td>
<td>6.20</td>
<td>.025</td>
</tr>
<tr>
<td>B</td>
<td>.00000</td>
<td>1</td>
<td>.00000</td>
<td>0.00</td>
<td>1.000</td>
</tr>
<tr>
<td>C</td>
<td>.00290</td>
<td>1</td>
<td>.00290</td>
<td>1.13</td>
<td>.500</td>
</tr>
<tr>
<td>AB</td>
<td>.00475</td>
<td>1</td>
<td>.00475</td>
<td>1.86</td>
<td>.250</td>
</tr>
<tr>
<td>AC</td>
<td>.00061</td>
<td>1</td>
<td>.00061</td>
<td>.23</td>
<td>1.000</td>
</tr>
<tr>
<td>BC</td>
<td>.01362</td>
<td>1</td>
<td>.01362</td>
<td>5.32</td>
<td>.050</td>
</tr>
<tr>
<td>ABC</td>
<td>.01190</td>
<td>1</td>
<td>.01190</td>
<td>4.65</td>
<td>.050</td>
</tr>
<tr>
<td>Error</td>
<td>.06139</td>
<td>24</td>
<td>.00256</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>.11104</td>
<td>31</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>


Data Relating to Learning Effects

In this section, the data relate to two types of learning effects. First, data related to the learning effect that may occur during the first 70 trials of the experiment is presented. Next, data relating to the possibility of a learning effect within the last 30 trials are presented.

Table 9 contains the averages of the first seven means across all subjects.
### TABLE 9.---Average Mean of Absolute Differences during Stated Trials for all subjects

<table>
<thead>
<tr>
<th>Trials</th>
<th>Average Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>.1578</td>
</tr>
<tr>
<td>11-20</td>
<td>.1375</td>
</tr>
<tr>
<td>21-30</td>
<td>.1275</td>
</tr>
<tr>
<td>31-40</td>
<td>.1125</td>
</tr>
<tr>
<td>41-50</td>
<td>.1141</td>
</tr>
<tr>
<td>51-60</td>
<td>.1171</td>
</tr>
<tr>
<td>61-70</td>
<td>.1114</td>
</tr>
</tbody>
</table>

To compute the statistic to examine the presence of a learning effect, a specific set of means was computed. For each trial, for each subject, the absolute value of the difference between the subject's posterior estimate and the Bayesian posterior estimate was computed. Means were then computed over every ten trials for the first 70 trials of each subject. Under the null hypothesis of no learning effects over the first 70 trials, the probability the fifth average of a subject exceeds the first average of a subject is equal to the probability that the first average exceeds the fifth average. Similar statements can be made for the second and fifth averages and the third and seventh averages. Thus, if the differences of the first average from the fifth average, the second average from the sixth and the third from the seventh are taken, the negative signs of the differences should be distributed as a binomial distribution with mean 10 and
variance 14.\(^1\)

Table 10 denotes the number of positive and negative differences observed in the 96 samples taken.

**TABLE 10.**—Number of Positive and Negative Differences in a Test for Trend

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>69</td>
</tr>
<tr>
<td>(p = .001)</td>
<td></td>
</tr>
</tbody>
</table>

In this case, \(p\) denotes a lower bound as the probability, 69 or more negative differences, will occur under the null hypothesis that a negative difference is just as likely as a positive difference.\(^2\)

The tests for learning effects within individual treatment combinations over the last 30 trials are similar to the previous test except that means are taken over every three trials and only 20 observations are made to develop the test under every treatment combination.

There are three parts to the presentation of the results of each of the eight treatment combinations. The first part relates the averages of the three trial means across subjects. The second part

\(^1\)The variance of a binomial distribution is \(n \cdot r \cdot q\) where \(n\) denotes sample size, \(r\) denotes the probability of a success (in this case a negative difference) and \(q\) denotes the probability of a failure. In this case, \(n = 96, q = .5, r = .5\).

\(^2\)Bradley, Distribution Free Statistical Tests, p. 332.
relates the total number of positive, negative, and zero differences after the first average of the subject is subtracted from the sixth, the second average from the seventh, etc. The third part of each presentation states the exact probability that the actual number of negative differences or more will occur under the null hypothesis that a positive difference is just as likely to occur as a negative difference.

TABLE 11.—Averages for Businessmen Examined on the Single Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1001</td>
</tr>
<tr>
<td>74-76</td>
<td>.1251</td>
</tr>
<tr>
<td>77-79</td>
<td>.1109</td>
</tr>
<tr>
<td>80-82</td>
<td>.1421</td>
</tr>
<tr>
<td>83-85</td>
<td>.1273</td>
</tr>
<tr>
<td>86-88</td>
<td>.0989</td>
</tr>
<tr>
<td>89-91</td>
<td>.1314</td>
</tr>
<tr>
<td>92-94</td>
<td>.1564</td>
</tr>
<tr>
<td>95-97</td>
<td>.1107</td>
</tr>
<tr>
<td>98-100</td>
<td>.1107</td>
</tr>
</tbody>
</table>

TABLE 12.—Positive, Negative, and Zero Differences for Businessmen Examined on the Single Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 10 or more negative differences occurring from a sample of
17 (zero differences eliminated, a method suggested by Bradley\(^3\)) is \(.3145.4\)

**TABLE 13.**—Averages for Students Examined on the Single Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.0875</td>
</tr>
<tr>
<td>74-76</td>
<td>.1422</td>
</tr>
<tr>
<td>77-79</td>
<td>.1497</td>
</tr>
<tr>
<td>80-82</td>
<td>.1715</td>
</tr>
<tr>
<td>83-85</td>
<td>.1437</td>
</tr>
<tr>
<td>86-88</td>
<td>.1338</td>
</tr>
<tr>
<td>89-91</td>
<td>.1572</td>
</tr>
<tr>
<td>92-94</td>
<td>.1701</td>
</tr>
<tr>
<td>95-97</td>
<td>.1274</td>
</tr>
<tr>
<td>98-100</td>
<td>.1688</td>
</tr>
</tbody>
</table>

**TABLE 14.**—Positive, Negative, and Zero Differences for Students Examined on the Single Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 6 or more negative differences occurring from a sample of 19 (zero differences eliminated) is .9682.

\(^3\)Ibid., p. 52.

**TABLE 15.**--Averages for Businessmen Examined on the Single Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.0827</td>
</tr>
<tr>
<td>74-76</td>
<td>.0790</td>
</tr>
<tr>
<td>77-79</td>
<td>.0906</td>
</tr>
<tr>
<td>80-82</td>
<td>.0542</td>
</tr>
<tr>
<td>83-85</td>
<td>.1038</td>
</tr>
<tr>
<td>86-88</td>
<td>.0763</td>
</tr>
<tr>
<td>89-91</td>
<td>.1188</td>
</tr>
<tr>
<td>92-94</td>
<td>.0838</td>
</tr>
<tr>
<td>95-97</td>
<td>.0769</td>
</tr>
<tr>
<td>98-100</td>
<td>.0996</td>
</tr>
</tbody>
</table>

**TABLE 16.**--Positive, Negative, and Zero Differences for Businessmen Examined on the Single Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of eight or more negative differences occurring from a sample of 18 (zero differences eliminated) is .7966.
TABLE 17.—Averages for Students Examined on the Single Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1527</td>
</tr>
<tr>
<td>74-76</td>
<td>.1348</td>
</tr>
<tr>
<td>77-79</td>
<td>.1356</td>
</tr>
<tr>
<td>80-82</td>
<td>.1223</td>
</tr>
<tr>
<td>83-85</td>
<td>.1197</td>
</tr>
<tr>
<td>86-88</td>
<td>.1295</td>
</tr>
<tr>
<td>89-91</td>
<td>.1270</td>
</tr>
<tr>
<td>92-94</td>
<td>.0751</td>
</tr>
<tr>
<td>95-97</td>
<td>.1395</td>
</tr>
<tr>
<td>98-100</td>
<td>.1271</td>
</tr>
</tbody>
</table>

TABLE 18.—Positive, Negative, and Zero Differences for Students Examined on the Single Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 10 or more negative differences occurring from a sample of 19 (zero differences eliminated) is .5000.
TABLE 19.--Averages for Businessmen Examined on the Joint Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1594</td>
</tr>
<tr>
<td>74-76</td>
<td>.2028</td>
</tr>
<tr>
<td>77-79</td>
<td>.1344</td>
</tr>
<tr>
<td>80-82</td>
<td>.1922</td>
</tr>
<tr>
<td>83-85</td>
<td>.1679</td>
</tr>
<tr>
<td>86-88</td>
<td>.1591</td>
</tr>
<tr>
<td>89-91</td>
<td>.1780</td>
</tr>
<tr>
<td>92-94</td>
<td>.1621</td>
</tr>
<tr>
<td>95-97</td>
<td>.1426</td>
</tr>
<tr>
<td>98-100</td>
<td>.1341</td>
</tr>
</tbody>
</table>

TABLE 20.--Positive, Negative, and Zero Differences for Businessmen Examined on the Joint Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Positive D1 Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>12</td>
<td>0</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 12 or more negative differences occurring from a sample of 20 is .2518.
TABLE 21.--Averages for Students Examined on the Joint Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1241</td>
</tr>
<tr>
<td>74-76</td>
<td>.1474</td>
</tr>
<tr>
<td>77-79</td>
<td>.0990</td>
</tr>
<tr>
<td>80-82</td>
<td>.1624</td>
</tr>
<tr>
<td>83-85</td>
<td>.1030</td>
</tr>
<tr>
<td>86-88</td>
<td>.1146</td>
</tr>
<tr>
<td>89-91</td>
<td>.0912</td>
</tr>
<tr>
<td>92-94</td>
<td>.1030</td>
</tr>
<tr>
<td>95-97</td>
<td>.0864</td>
</tr>
<tr>
<td>98-100</td>
<td>.1017</td>
</tr>
</tbody>
</table>

TABLE 22.--Positive, Negative, and Zero Differences for Students Examined on the Joint Information System with Cubes and Algebraic Identities

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>13</td>
<td>0</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 13 or more negative differences occurring from a sample of 20 is .1316.
TABLE 23.--Averages for Businessmen Examined on the Joint Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1200</td>
</tr>
<tr>
<td>74-76</td>
<td>.2785</td>
</tr>
<tr>
<td>77-79</td>
<td>.1181</td>
</tr>
<tr>
<td>80-82</td>
<td>.0727</td>
</tr>
<tr>
<td>83-85</td>
<td>.0991</td>
</tr>
<tr>
<td>86-88</td>
<td>.1626</td>
</tr>
<tr>
<td>89-91</td>
<td>.1080</td>
</tr>
<tr>
<td>92-94</td>
<td>.1364</td>
</tr>
<tr>
<td>95-97</td>
<td>.0852</td>
</tr>
<tr>
<td>98-100</td>
<td>.1130</td>
</tr>
</tbody>
</table>

TABLE 24.--Positive, Negative, and Zero Differences for Businessmen Examined on the Joint Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 11 or more negative differences occurring from a sample of 20 is .4119.
TABLE 25.--Averages for Students Examined on the Joint Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Trials</th>
<th>Averages</th>
</tr>
</thead>
<tbody>
<tr>
<td>71-73</td>
<td>.1380</td>
</tr>
<tr>
<td>74-76</td>
<td>.2398</td>
</tr>
<tr>
<td>77-79</td>
<td>.1961</td>
</tr>
<tr>
<td>80-82</td>
<td>.2587</td>
</tr>
<tr>
<td>83-85</td>
<td>.1900</td>
</tr>
<tr>
<td>86-88</td>
<td>.1960</td>
</tr>
<tr>
<td>89-91</td>
<td>.2485</td>
</tr>
<tr>
<td>92-94</td>
<td>.2469</td>
</tr>
<tr>
<td>95-97</td>
<td>.1988</td>
</tr>
<tr>
<td>98-100</td>
<td>.2030</td>
</tr>
</tbody>
</table>

TABLE 26.--Positive, Negative, and Zero Differences for Students Examined on the Joint Information System with Hypothetical Businesses and Profit Figures

<table>
<thead>
<tr>
<th>Positive Differences</th>
<th>Negative Differences</th>
<th>Zero Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

Under the hypothesis of equally likely differences, the probability of 10 or more negative differences occurring from a sample of 20 is .5881.
Examinations of Potential Biasing Factors in the Sampling Process

In this section, some potential biasing factors are examined. As mentioned in the appendix to Chapter III, the experimenter assumed a random process underlay the assignment of subjects in the businessman and student subpopulations to the experimental treatments. If a random process does underly the method of assignment, it should be impossible to find a characteristic or attribute which distinguishes which subjects within a subpopulation are assigned to which treatments. Such a characteristic or attribute will be called a biasing factor.

Two types of tests are involved in examining potential biasing factors. For ranked data, the Kruskal-Wallis test is used. This approach assumes all permutations of the data are equally likely. The test involved is based on the sum of the squares of the overall mean of the data from the individual treatments of each permutation. The probability derived from this test shows how likely a sum of squares as great as or greater than the one actually observed is under the assumption that all permutations of the data are equally likely, i.e., random sampling.

The second type of test used is the Kolmogorov-Smirnov test which is only an approximate test since the distribution involved is a discrete distribution. For these tests, the treatment combinations to which businessmen and students have been assigned have been num-

---

5See Von Mise's definition of random assignment in Fellner, Probability and Profits.
bered. Treatment combination 1 is the single information system with cubes and algebraic identities. Treatment combination 2 is the single information system with hypothetical businesses and profit figures. Treatment combination 3 is the joint information system with cubes and algebraic identities, and treatment combination 4 is the joint information system with hypothetical businesses and profit figures.

Let \( x \) denote a random variable with cumulative distribution given by

\[
F(x) = \begin{cases} 
0 & \text{for } x < 1 \\
1/4 & \text{for } 1 \leq x < 2 \\
1/2 & \text{for } 2 \leq x < 3 \\
3/4 & \text{for } 3 \leq x < 4 \\
1 & \text{for } 4 \leq x 
\end{cases}
\]

For any characteristic the cumulative probability of that characteristic being assigned to particular treatment combinations under a random assignment process is denoted by the above cumulative distribution function.

To compute the Kolmogorov-Smirnov statistic, the actual sampled cumulative distribution of the characteristic is compared to the hypothesized distribution. The largest absolute deviation between the hypothesized and sampled cumulative distribution is the statistic. The probability reported states how likely the observed absolute difference is to occur when random assignment is assumed.

In this section exact probabilities are given rather than statements that the result is or is not significant at some signifi-
cance level. Thus, rather than merely attempting to refute the randomness assumption, an attempt is made to find out whether the observed phenomenon is likely to occur under the null hypothesis of random assignment.

Potential Biasing Factors for Businessmen

The first factor examined is the age of the businessmen. In Table 27, the ages of the businessmen appear in the numbered treatment combinations in which they occur.

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>29</td>
<td>36</td>
<td>35</td>
</tr>
<tr>
<td>39</td>
<td>42</td>
<td>44</td>
<td>39</td>
</tr>
<tr>
<td>40</td>
<td>43</td>
<td>44</td>
<td>48</td>
</tr>
<tr>
<td>47</td>
<td>48</td>
<td>49</td>
<td>49</td>
</tr>
<tr>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
</tr>
<tr>
<td>39.75</td>
<td>40.50</td>
<td>43.25</td>
<td>43.00</td>
</tr>
</tbody>
</table>

The Kruskal Wallis $H$ computed on the data\(^6\) is 1.79. The probability of obtaining this $H$ or greater is approximately .6149 under the null hypothesis of random sampling.\(^7\)

---

\(^6\)Bradley, *Distribution Free Statistical Tests*, p. 131.

\(^7\)The probability of this $H$ is equivalent to the sum of squares probability discussed previously. It is an approximate probability since $H$ is approximately chi square with 3 df. The probability was found in the *Handbook of Mathematical Functions*, p. 978.
The characteristics of businessmen upon which the Kolmogorov Smirnov test is applied include salary level, area in the firm (sales, production, general management), and years of education beyond high school. Also, information about the businessmen's places of employment is stated, although no tests are performed on these data.

Businessmen fall into three salary levels (actual salaries were not obtainable): $15,000 - $19,000; $20,000 - $29,000; and $30,000 - $49,000.

TABLE 28.--Distribution of Salary Levels across Treatment Combinations in $1,000's

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
<td>(15 - 19)</td>
<td>(15 - 29)</td>
</tr>
<tr>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
</tr>
<tr>
<td>(30 - 49)</td>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
<td>(20 - 29)</td>
</tr>
</tbody>
</table>

Each of the salary levels is regarded as an attribute. The attribute, (15 - 19) salary level, has the following sample distribution; treatment combination 1: .1667, treatment combination 2: .3333, treatment combination 3: .6667, treatment combination 4: 1.000.

When compared to the hypothetical distribution found on page 162, the maximum absolute value of the difference between the sample distribution and the hypothesized distribution is

.5000 - .3333 = .1667
The probability of obtaining this deviation or greater is approximately .42 under the null hypothesis of random assignment of businessmen to treatment combinations. 8

For the attribute, (20 - 29) salary level, the maximum absolute deviation from the hypothesized distribution is .0555. The probability of obtaining this deviation or greater is approximately .99 under the null hypothesis of random assignment.

For the attribute, (30 - 49) salary level, the maximum absolute deviation is .750. The probability of obtaining this deviation or greater is .500 under the null hypothesis of random assignment. 9

The next biasing factor examined for businessmen is the general area of specialization within the firm. Three broad categories were developed: Sales, Production, and General Management. Under the category, Sales, fall people whose title contained either the words sales or distribution or marketing. Under the category, Production, are individuals whose title contained the words engineer, plant or production. Under the category, General Management, are people whose titles did not contain sales or production terms, whose immediate

8The probability given here and all of the Kolmogorov Smirnov probabilities later (except those calculated directly) were developed from Z.W. Birnbaum, "Numerical Tabulation of the Distribution of Kolmogorov's Statistic for Finite Sample Size," Journal of the American Statistical Association, 47 (1958), pp. 425-41.

9This is an exact probability not found in the tables. Note that if there is a sample of one, it will fall into one of four treatment combinations. The possible maximum deviations are .750, .500, .500, and .750. Thus, .750 or greater has a .500 chance of occurring under the null hypothesis.
superiors were either general managers or presidents, and whose subordinates were either in both sales and production or in neither sales nor production.

**TABLE 29.**—Distribution of Areas of Specialization across Treatment Combinations

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>Sales</td>
<td>Sales</td>
<td>Sales</td>
</tr>
<tr>
<td>Production</td>
<td>Production</td>
<td>Production</td>
<td>Production</td>
</tr>
<tr>
<td>Production</td>
<td>Production</td>
<td>Production</td>
<td>Production</td>
</tr>
<tr>
<td>General Management</td>
<td>General Management</td>
<td>General Management</td>
<td>General Management</td>
</tr>
</tbody>
</table>

For the attribute, Sales, the absolute value of the maximum deviation between the hypothesized and sampled distributions is .100. In a sample of five, the approximate probability this deviation or greater will occur under the null hypothesis is .96. For the attribute, Production, the absolute value of the maximum deviation is .0715, and the approximate probability this deviation or greater will occur is .99 under the null hypothesis. For the attribute, General Management, the maximum deviation is zero, and the exact probability of this deviation or greater is 1.00 under the null hypothesis.

Next, the factor, years of education beyond high school, was examined. There are five possible levels: one, two, three, four, and
five years beyond high school.

TABLE 30.--Years of Education beyond High School for Businessmen across Treatment Combinations

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

The maximum deviation of the absolute value of the difference between the hypothesized and sample distribution for the attribute, four years of education beyond high school, is .137. Under the null hypothesis of random sampling, the approximate probability of the occurrence of this deviation or greater is .33. The maximum deviation for the attribute, five years of education beyond high school, is .500. The exact probability of the occurrence of this deviation or greater under the null hypothesis of random sampling is .500. The attributes, one, two, or three years of education beyond high school, would yield maximum deviations which have the exact probability of occurrence of either .500 or 1.00.

The other attribute of businessmen examined is the company for which the individual businessman subject works. It was found that all but two executives worked for different companies. The two who worked in the same company were examined under different treatment combinations.
Potential Biasing Factors of Students

The first student factor examined is the grade point hour ratio of students where the highest possible ratio is 4.000. The distribution of student grade point hour ratios is presented in Table 31.

TABLE 31.—Grade Point Hours of Students across Treatment Combinations

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.000</td>
<td>2.219</td>
<td>2.139</td>
<td>2.125</td>
</tr>
<tr>
<td>2.105</td>
<td>2.272</td>
<td>2.350</td>
<td>2.235</td>
</tr>
<tr>
<td>2.912</td>
<td>2.446</td>
<td>2.387</td>
<td>2.393</td>
</tr>
<tr>
<td>3.556</td>
<td>2.900</td>
<td>3.873</td>
<td>3.396</td>
</tr>
<tr>
<td>Average</td>
<td>Average</td>
<td>Average</td>
<td>Average</td>
</tr>
<tr>
<td>2.643</td>
<td>2.459</td>
<td>2.687</td>
<td>2.537</td>
</tr>
</tbody>
</table>

The Kruskal Wallis H computed for this data is .21. The probability of occurrence of this H or greater is approximately .97 under the null hypothesis of random assignment.

The factors to which the Kolmogorov-Smirnov test relates are the year in school of students who were subjects and the academic major of student subjects.

Student subjects were either juniors or seniors at The Ohio State University. The distribution is in Table 32.
TABLE 32.—Year in School of Students across Treatment Combinations

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
</tr>
<tr>
<td>Junior</td>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
</tr>
<tr>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
</tr>
<tr>
<td>Junior</td>
<td>Senior</td>
<td>Senior</td>
<td>Senior</td>
</tr>
</tbody>
</table>

For the attribute, Senior, the absolute value of the maximum deviation of the hypothesized distribution from the actual sampled distribution is .058. The approximate probability of such a deviation or greater occurring under the null hypothesis of random sampling is .99. For the attribute, Junior, the maximum deviation is .416 and the approximate probability of this occurrence is .4391.

The academic majors of the student subjects were Marketing, Finance, Industrial Production, Insurance, and Personnel Management.

The maximum absolute of the deviation between the sampled and the hypothesized distribution for the attribute, Marketing, is .125. This deviation or greater has probability .99 of occurring under the assumption of random sampling. The maximum deviation for the attribute, Industrial Production is .500. This deviation has probability .500 of occurring. The probability of observing maximum deviations as great or greater than those observed for the attributes, Insurance and Personnel Management, is 1.00.
TABLE 33.--Distribution of Academic Majors across Treatment Combinations

<table>
<thead>
<tr>
<th>Treatment Combination 1</th>
<th>Treatment Combination 2</th>
<th>Treatment Combination 3</th>
<th>Treatment Combination 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marketing</td>
<td>Marketing</td>
<td>Marketing</td>
<td>Marketing</td>
</tr>
<tr>
<td>Marketing</td>
<td>Marketing</td>
<td>Finance</td>
<td>Marketing</td>
</tr>
<tr>
<td>Finance</td>
<td>Marketing</td>
<td>Industrial Production</td>
<td>Finance</td>
</tr>
<tr>
<td>Insurance</td>
<td>Finance</td>
<td>Industrial Production</td>
<td>Personnel Management</td>
</tr>
</tbody>
</table>

Additionally, the experimenter examined the home addresses of student subjects. None of the students had the same local address from which the experimenter assumed that none of the student subjects lived in the same dormitory, house, apartment building, or fraternity house.

**Deviation from the Planned Experimental Procedure**

During the administration of the experiment, there was one departure from the planned experimental procedure, as discussed in the appendix to Chapter III. The departure arose when a businessman subject left the room in which the experiment was being given. The man left on the eighteenth trial of the experimental treatment using the joint information system with cubes and algebraic identities. The subject was written 1-1 odds for some messages and no estimates for
other messages. When the experimenter talked with the subject after the completion of the experiment, the subject indicated that he did not understand the instructions (his graded instructions showed .625 correct answers), that he had had only four hours sleep the previous night, and that if he had continued the experiment, he would have recorded all odds estimates as 1-1.

When the subject left, he was visible to the other subjects who were participating in the particular experimental treatment. As the subject left, he said he did not feel that he could continue the experiment and the experimenter thanked him for his participation. The experimenter assumed that the subject's behavior did not alter the behavior of the remaining subjects. The sparse amount of data relating to this occurrence is included in Chapter VI.

**Summary**

Related in this chapter are the results of the analysis of variance test, the data concerning learning effects, the examinations for biasing factors in the sampling process, and a deviation from the planned experimental procedure.

The interpretations of the data and conclusions are presented in Chapter VI.
CHAPTER VI

CONCLUSIONS

In this chapter several conclusions are offered concerning the effects of the variables manipulated experimentally. The conclusions are subject to limitations imposed by the experimental procedure. The limitations are discussed to form a basis for additional experiments built on the present experimental paradigm.

No conclusion is offered concerning the ability to generalize the results of the experiment. The experiment was conducted with a selected set of subjects, a selected set of experimental settings, a selected set of information systems, a selected mode of response, and a selected order of message presentation. The generality of the study is in the theory, not in the experiment. The primary purpose of the experiment is to instantiate the theory and to support possible areas of theory modification.

The statistical questions in the experiment relate only to the observations and potential observations of individuals actually involved in the experiment. The statistical questions do not extend beyond these populations. Basically, the statistical questions ask whether the observed differences in experimental treatment conditions are due solely to measurement error.
Conclusions about the Experimental Effects

The conclusions about the experimental effects are that, within the experiment, (1) the different information systems did affect the average absolute value of the difference between the Bayesian and the subjects' posterior estimates; (2) the interaction of the type of subject and type of setting did produce an overall effect on the average absolute value of the difference between the Bayesian and the subjects' posterior estimates; and (3) the type of information system, the type of subject, and the type of setting interacted to produce an effect on the average absolute value of the difference between the Bayesian and the subjects' posterior estimates.

These conclusions are subject to several limitations. First, it cannot be irrevocably established that the requirements of the statistical model are met. Any test of the requirements involves additional assumptions and further testing of the new assumptions requires additional assumptions, so that an infinite regression is involved in establishing whether the requirements of the statistical model are met.

A number of attempts were made to meet the requirements of the model in the design and administration of the experiment, and a small amount of data has been collected to consider whether particular violations of the model did occur.

One possible violation examined is that parameters of the statistical model, assumed to be constant, may change during the measurement process. The only type of change in the parameters considered has
been that of a potential learning effect which occurs on the 100 actual experimental trials. The possibility of eliminating the observed experimental differences through additional trials is of interest. The elimination of differences in additional trials not only would refute the theory, but also might lead to an examination of the degree of difficulty of learning in various experimental treatment conditions. For example, an initial joint information system effect may be eliminated over time; however, the elimination of the effect may require a vast number of trials.

The measurement of learning effects did not relate to all parameters of the statistical model. Possible individual learning effects which could alter $\xi_{kl}$'s, the differences of the experimental measurement due to individual differences, were not examined. Only general learning effects which could alter the overall mean of the responses, $\eta_{...}$, and the individual means, $\eta_{ijk}$'s, have been considered.

Tests for learning effects which may alter $\eta_{...}$ were performed on the first 70 trials. The tests did not indicate an absence of learning effect on these experimental trials, and subjects in general did come closer to the Bayesian estimates during the first 70 trials.

On the other hand, tests of learning effects which could alter the $\eta_{ijk}$'s of individual treatment combinations showed no indication of a strong learning phenomenon over the last 30 trials (with the pos-
sible exception of students examined on the joint information system with cubes and algebraic identities). It should be mentioned that the test of individual treatment means was less sensitive than the test of the overall learning effect during the first 70 trials since three trial averages were used to examine learning effects in relation to ijk treatment combinations and ten trial averages were used to examine an overall learning effect.

The examination of potential biasing factors was also limited. For all subjects, there are an infinite number of characteristics which may alter the randomness of assignment of subjects in subpopulations to the experimental treatments. Several obvious and accessible characteristics were examined, and there was no evidence to indicate a biasing effect in the assignment of subjects to experimental treatments. However, the Kolmogorov Smirnov tests administered to the data are somewhat conservative and can involve an overstatement of probability when applied to discrete distributions.

Additionally, it is possible that the requirements of the statistical model were violated when a subject left the experiment. Such an exit may influence the distribution of the measurement error and may alter the effects of individual differences across treatment combinations. In observing the remaining subjects after one subject left the experiment, the experimenter did not see any changes in facial expression or deviations in the manner in which the remaining subjects continued participation in the experiment. The experimenter himself attempted to remain calm as the subject left the experiment so that
the experimenter's reaction to the departure of the subject would not adversely influence the remaining subjects' responses.

Averages over ten trials of the absolute value of the difference between Bayesian and subjects' posterior estimates were taken before and after the 18th trial, the trial on which the subject left the experiment. The averages for the four subjects whose estimates were used in the statistical tests were .2678, .0855, .2678, and .2006 before the 18th trial and .2314, .1667, .1804, and .2550 after the 18th trial. It is not believed that these differences reflect any alteration in the remaining subjects' performance after the subject left the experiment. Of course, the possibility of an effect does exist, and lack of an effect can be established only through future replications of the experiment.

The Effect of Alternate Information Systems
on the Average Absolute Value of the Difference between the Bayesian and the Subject's Posterior Estimate

In all but one experimental situation,¹ the average absolute value of the difference between the Bayesian and the subject's posterior estimates is greater under the joint information system than under the single system. To consider the one case in which the average absolute value under the joint system was slightly less than the average absolute value under the single system, Scheffe's method of post hoc experimental examinations was employed. The method indicates that

¹Supra, p. 148.
under the assumption that the means of the single and joint information systems of students examined using cubes and algebraic identities are the same, the probability of a squared difference between the means as great as or greater than the one observed is less than .500 but more than .250. Thus, the difference between the two experimental observations appears to be largely the effect of measurement error rather than a true difference between experimental combinations.

It might be stated that the observed difference between the average absolute values under the single and joint information systems may result not because subjects were confused but rather because they chose to ignore one of the information systems.

If subjects using the joint information system did ignore one of the information systems and made estimates on the basis of a single system only, then the difference between the subjects exposed to the single system and subject exposed to the joint system should be eliminated by the following procedure.

Let $P_{B|R_1}(T_1|x; f(x) = m, g(x) = w_j)$ denote the Bayesian posterior estimate when the Bayesian estimate is made as if information was restricted to information system 1.

Let $P_{B|R_2}(T_1|x; f(x) = m, g(x) = w_j)$ denote the Bayesian posterior estimate when the Bayesian estimate is made as if information was restricted to information system 2.

---

was restricted to information system 2.

For each subject, the following is computed.

\[
\sum_{(m, w) \in M \times W} \Pr_{S}(T_1 \mid \{x \mid f(x) = m, g(x) = w\}) \left| \frac{\Pr_{S}(T_1 \mid \{x \mid f(x) = m, g(x) = w\})}{\Pr_{B1R1}(T_1 \mid \{x \mid f(x) = m, g(x) = w\})} \right|  \\
\min \left\{ \sum_{(m, w) \in M \times W} \Pr_{S}(x \mid f(x) = m, g(x) = w) \left| \frac{\Pr_{S}(T_1 \mid \{x \mid f(x) = m, g(x) = w\})}{\Pr_{B1R2}(T_1 \mid \{x \mid f(x) = m, g(x) = w\})} \right| \right\}
\]

When the minimum is computed for each subject, the average minimum should approach the estimate of the single information system because the subject is now treated as if he is making estimates on the basis of messages from a single system. In the experiment, the average of the minimums was .12722, a number less than that of the joint system but still greater than the average of the absolute values observed for the single system, .09936.

**Effects of Interactions of the Type of Subject and Type of Setting**

There is an apparent overall interaction of subject and setting effect on the average absolute value of the difference between the Bayesian and the subject's estimates. It is possible that the nature of the measurement may be the source of the apparent interaction effect. Thus, although the measurement acts in a desirable way, it increases when the subject's deviation from the Bayesian estimate increases, the rate of change in the measurement with respect to the rate of change of the subject from Bayes may be so high that it is the rate
of change of the measurement which produces an interaction result. Such an effect can be eliminated by finding a positive monotonic transformation of the experimental observations.

In the present experiment, it is important to establish that a positive monotonic transformation does not exist which will eliminate an interaction effect.

To indicate that such a transformation does not exist, consider the means of each experimental treatment when averages are taken over the types of information systems.

<table>
<thead>
<tr>
<th>Cubes and Algebraic Identities</th>
<th>Students</th>
<th>Businessmen</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>.11015</td>
<td>.13236</td>
</tr>
<tr>
<td>Hypothetical Businesses and Profit Figures</td>
<td>.15215</td>
<td>.09183</td>
</tr>
</tbody>
</table>

If these tabulated parameter estimates are taken as the actual parameters of the underlying distribution, then, according to Scheffe, the observed interaction effect cannot be eliminated by a positive monotonic transformation since the sign of the difference between the first and second numbers of the rows of the table is different for
different rows of the table.  

Effects of the Interaction of the Type of Information System, the Type of Subject, and the Type of Setting on the Average Absolute Value of the Difference between the Bayesian and Subjects' Posterior Estimates

An unpredicted result of the experiment was a three-factor interaction of the type of subject, the type of setting, and the type of information system. Like the two factor interaction, it is possible that the interaction effect is merely a result of the measurement and that the effect can be eliminated by a positive monotonic transformation of the observations.

To suggest that interactions are not able to be eliminated, a heuristic approach provided by Winer is used. The approach is a graphing procedure.

![Figure 1: Single Information System](image)

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3Ibid., pp. 95-98.
The vertical axis of Figure 1 is the variable, average absolute value of the difference between the subjects' and the Bayesian posterior estimates. The horizontal axis treats the variable type of setting as if it were continuous. The lines of the graph connect points which represent students and businessmen examined under the single information system.

A similar graph is developed below for the joint information system.

![Graph showing average absolute value of difference between subjects' and Bayesian estimates.](image)

**Figure 2.** Joint Information System

On the following page, the two graphs are compared to the graph of the average taken across information systems.
Average Absolute Value of the Difference between Bayesian and Subjects' Estimates

Figure 3.—Comparison of Graphs
The heuristic rule applied to these graphs is that the three-way interaction is not able to be eliminated by a positive monotonic transformation if a positive monotonic transformation cannot be applied to each line of the individual information system graphs to arrive at a line parallel to the equivalent line in the graph labeled, "Average across Information Systems." It is easily seen that any positive monotonic transformation of the line labeled students in the first graph will yield a negatively sloping graph which cannot parallel the positively sloping graph of the line labeled students in the graph entitled, "Average across Information Systems." Thus, it does not appear that the measured interaction effect is able to be eliminated by any positive monotonic transformation of the experimental observations.

Theory Modification and Extensions

In previous sections of this chapter, the experimental limitations have been examined. The following discussion reexamines the theory in light of empirical evidence.

It does not appear that the theory that task difficulty (as represented by the average absolute value of the difference between the Bayesian and the subjects' posterior estimates) is at least as great in the single information system as in the joint information system has been refuted by the experimental evidence. However, the experiment demonstrates some present limitations of the theory. It seems important to develop a means of distinguishing when the joint information
system produces a greater departure from the Bayesian theorem than
the single system and when the joint information system effect tends
to be equivalent to the single information system effect. An im-
portant variable in making this distinction is the familiarity of the
setting to the subject. Thus, as familiarity is decreased, the de-
partures from Bayesian revisions should increase.

A second important variable, not studied here, may be the
relative fineness of the joint information system.\textsuperscript{4} Such a factor
would be closely related to the theory of task difficulty expressed
in Chapter II. It could be predicted that as the partition of the
joint system becomes finer, the task becomes more difficult, and,
hence, deviations from Bayesian revisions increase.

With regard to the theory concerning an interaction of type
of subject and type of setting, it appears that the interaction effect
may be affected by the nature of the information system. In general,
it might be predicted that task difficulty would have an intermediary
effect on the interaction and that as task difficulty increases, the
interaction would become prevalent.

\textsuperscript{4}Supra, p. 20.
BIBLIOGRAPHY

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