HOSPITAL PRODUCTION FUNCTIONS

DISSertation

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

The Ohio State University
1973

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ACKNOWLEDGMENTS

I wish to thank my adviser, Professor William Dewald, who interested me in health economics and guided me through the long process of writing a dissertation. Professor P.A.V.B. Swamy gave unselfishly of his time and effort during my initial attempts to formulate a research project. Comments by Professor Donald Parsons were valuable in helping me to develop my research strategy and encouragement was offered to me throughout the time I worked on this project by Professor Jeffrey Caswell.

My wife, Marilyn, was always cheerful and understanding during all the ups and downs of writing this dissertation. Without her help it would certainly have been more difficult.
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INTRODUCTION

Almost every attempt to determine the optimum size of a hospital has been conducted using cost analysis. This reliance on cost analysis is a result of the available data. Cost function analysis is valid for estimating returns to scale only if every firm in the hospital industry is a cost minimizer. The assumption of cost minimizing behavior for hospitals may be unjustified. Most theories of hospital cost inflation are based on behavioral assumptions which imply a non-optimum use of resources by the hospital.


My study will concentrate on three tasks: first, to examine the usefulness in production functions of several common hospital output indices, second, to estimate economies of scale in the production of hospital services (e.g., x-rays, laboratory procedures, routine days of patient care, etc.) and third, to determine the effect of graduate medical education programs on the production of hospital services.
In order to examine these issues a hospital production function is formulated and estimated. The formulation of a hospital production function requires the selection of a hospital output index. Various hospital output indices were examined with respect to their likelihood in defining an identical hospital production function across hospitals using my model of hospital production. I also developed, by modifying a method derived by Ernst R. Berndt and Laurits R. Christensen, a procedure to evaluate various functional forms for use as hospital production functions. Estimation of the


chosen functional form required the development of a hypothesis of hospital behavior. My hypothesis of hospital behavior is described in Chapter 3.

The plan of my study is as follows. Chapter 1 will discuss the problems involved in defining a single output production function for a multi-product firm. All previous attempts to estimate the multi-product hospital production function in terms of a single aggregate output have ignored the problem of aggregation. Newhouse recently observed,
However, the 'product' of a hospital may vary so much depending on the diseases its patients have that a 'simple' measure of patient days cannot accurately reflect the output of hospital. This is an aggregation problem which is inherent in the multi-product firm.

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The problem of defining a single output production function for a multi-product firm using my model of hospital production is mathematically identical to the problem of defining an aggregate production function for an industry of firms which produce a single homogenous product. Under less restrictive conditions on the explanatory variables than required by other hospital output indices I will show that consistent aggregation with an identical production function for each hospital is possible using a weighted measure of hospital services output index.

Chapter 2 includes the description of data. The type of data collected and my sample are discussed. Only non-teaching, nonprofit, non-federal, short-term hospitals are used in my estimation of a hospital production function. The use of only non-teaching hospitals allows me to ignore the problem of the medical education component of output. In order to define a more homogenous group of hospitals I eliminated many types of hospitals
from my sample. It is desirable to work with a homogenous group of hospitals so that the assumption made in cross section studies that the micro-coefficients across firms are equal is satisfied. If the micro-coefficients of different firms are not equal then the expected value of a cross section regression estimator is almost always a weighted average of corresponding and non-corresponding micro-coefficients and the expected value of the square of the multiple correlation coefficient is lowered. If the square


of the multiple correlation coefficient is high in a cross section study then this may indicate that the variation in the micro-coefficients across firms is not great. Of course, if panel data or temporal data is available it is desirable to perform a test of the hypothesis that different firms have the same regression coefficients.

Chapter 3 develops my hypothesis of hospital behavior. I hypothesize that hospitals are combination profit and capital maximizers. Evidence supporting these assumptions is presented and various reasons why hospitals might desire to acquire capital and earn a profit are discussed.

In Chapter 4 I define the variables in my hospital production function. Measurement error and bias inherent in any aggregation of inputs are discussed. I modify Berndt and Christensen’s method of selecting an appropriate functional form for a production relationship. Modification is necessary since Berndt and Christensen assume the data are generated by profit maximizing firms. Finally, the empirical results are included. These results tell us what functional form was chosen and give the estimates for returns to scale and the elasticity of substitution. A discussion of the various estimation techniques is included.

Chapter 5 compares the production of hospital services by teaching and non-teaching hospitals. My original sample is augmented by 16 teaching hospitals and a dummy variable is added to the production function to differentiate hospitals with zero, one or two teaching programs (intern or residency programs).

The last chapter gives a summary and the conclusions.
It should be noted that two problems which I discuss are common to cost function analysis. One of the major difficulties in estimating a hospital cost function is deciding how to measure output. An index of output must be selected for a firm which produces many services, each by a different production process, and where the prices of these services are not determined in a competitive market. Another problem which faces those who estimate cost functions hospitals is the determination of the functional form of the cost output relationship. Theil has asserted that economics should be the major factor in the decision. Yet,


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economic theory does not provide us with the functional form of a hospital cost function. In practice, convenience of estimation is the determining factor as Lave and Lave attest,

In applying our method for estimating cost functions of multi-product firms to the hospital industry, much of our investigation consisted of a search for significant coefficients. This search was made necessary because there is little knowledge of the functional form of the cost relationship.\(^*\)
CHAPTER I

A MODEL OF HOSPITAL PRODUCTION

In this chapter I specify a model of hospital production and make assumptions which insure consistent aggregation for any output index. A comparison is then made of the assumptions

1. It should be noted that consistency is a concept which applies to each hospital production function separately and implies nothing about the relationship between production functions of different hospitals.

necessary to insure that the aggregate production function defined by each index specifies an identical production function across hospitals. If a particular output index defines a consistent but very different aggregate production function across hospitals it should not be used in cross section studies to estimate economies of scale. It is the homogeneity of the defined aggregate production function across hospitals which will be examined for several hospital output indices.

The hospital produces laboratory tests, inhalation therapy, operations and other services. Yet, in all cost studies and the few production function studies of hospitals, a single index
has been used to measure output. John Johnston comments,

The fault here is not all the statistician's. In studying the relationship between cost and an output index, he is trying to force reality into the straightjacket of the single homogenous product firm of economic theory, for little progress has yet been made with the theory of the multi-product firm.²


Let $(1) H(y_1, y_2, \ldots, y_n, x_1, x_2, \ldots, x_m) = 0$ be the implicit production relationship of hospital where $y_i = i^{th}$ service and $x_j = j^{th}$ input used by the hospital. I assume $(1)$ can be broken down into the following $n$ relations:

$$
\begin{align*}
  y_1 &= f_1(x_{11}, \ldots, x_{1m}) \\
  y_2 &= f_2(x_{21}, \ldots, x_{2m}) \\
  & \vdots \\
  y_n &= f_n(x_{n1}, \ldots, x_{nm})
\end{align*}
$$

(2)

where $x_{ij} =$ amount of the $j^{th}$ input used to produce the $i^{th}$ service (e.g., x-ray, laboratory procedure, operations, deliveries, routine days of patient care).

This specification is not appropriate for most multi-product firms. For most firms, the level of technology used in producing one output will directly effect the production technique of another
output. In fact, in most multi-product firms the same equipment is used to produce different products. A food processor uses the same equipment and labor to produce sliced bologna and salami. An automobile manufacturer produces many types of cars, yet each goes through the same assembly line process. Different types of cars are characterized by different engines, suspensions and frames but it is obvious the production functions of all types of cars are dependent on the technology employed on the assembly line.

If we were to specify the individual production functions of each type of car, they would include a common parameter which represented the technology of the assembly line and the rate of production of other types of cars. It would include a measure of production of other types cars because the more different types of cars produced on an assembly line, the less efficient the production of any type car will generally become.

In our specification of hospital production (2) there are no common parameters: hence each production relationship is assumed to be a function only of the inputs employed in the production of the given output. The production of an x-ray and laboratory procedure are not produced by the same equipment or even personnel. Each is produced using different equipment, procedures and labor. The radiologist may have no effect on how the laboratory is operated or how routine days of patient care are produced. A good hospital with a highly skilled staff may influence the radiology
and laboratory procedures to see that higher quality services are produced. Yet, radiologists and pathologists generally have a great deal of autonomy and are able to run their respective areas as they desire.

We define an index of output to be some function of the hospital services provided \((3) Y = Y(y_1, y_2, \ldots, y_n)\). If we use patient days as our measure of output then \(Y = y_j\), where \(y_j\) is patient days. The assumption is often made when using a patient day measure of output that the intermediate services are provided in a fixed proportion to the number of patient days.

Since the amount of each input used to produce each service is not known, some method of combining the micro-inputs must be used (i.e., labor may be measured by the number of personnel producing all services or total labor cost). Let \(X_j = \) the amount of the \(j\)th input where \((4) X_j = X_j(x_{jj}, x_{nj})\).

We define a hospital production function as \((5) Y = F(x_1, x_2, \ldots, x_n)\). Whether this function is consistent depends upon how the output, inputs, and functional form are designated. The necessary and sufficient condition for the function (4) to be aggregated to the function (5) when the variables are free to take on all values are that for all \(r = 1, 2, \ldots, m\), and \(s = 1, 2, \ldots, n\)

\[
\frac{\partial^3 F}{\partial x_r \partial y_s \partial x_{sr}} = \frac{\partial Y}{\partial y_s} \frac{\partial x_{sr}}{\partial x_{sr}},
\]

(6)

I will examine one way of defining a hospital production function and will make assumptions about the form of the service production functions and behavior of the independent variables which will allow consistent aggregation.

4. Consistent aggregation is equivalent to condition (6). Condition (6) implies that the calculated increase in output per unit increase of any input is the same whether we use our aggregate production function or the definition of output and the microproduction functions. Consequently, if (6) holds then a knowledge of the aggregate production function and the values of the aggregate independent variables leads to the same value of output as knowledge of the functional form of the output index and the values of individual independent variables.

Define (7) \( Y = Y(y_1, y_2, \ldots, y_n) = AC_1y_1 + \ldots + AC_ny_n \), where \( AC \) = the average cost for all hospitals of producing the \( i^{th} \) service and (8) \( x_r = w_r(x_{r1} + x_{r2} + \ldots + x_{rn}) \) where \( w \) is the average cost of an unit of the \( r^{th} \) input. I assume that one dollar spent on an input yields an increase of one dollar in the cost weighted
output measure. This is not an identity since our weights are determined by the average cost of all hospitals in producing a service.

Examining (6) we get:

\[(9) \quad \frac{\partial Y}{\partial Y_s} = AC_s \quad \text{by (7)}\]

\[(10) \quad \frac{\partial X_r}{\partial X_{sr}} = W_r \quad \text{by (8)}\]

\[(11) \quad \frac{\partial Y}{\partial X_r} = 1 \quad \text{by (9)}\]

\[(12) \quad \frac{W_r}{C_s} = \frac{\partial X_r}{\partial Y} = \frac{1}{\frac{\partial X_r}{\partial X_{sr}}} \Rightarrow (6) \text{ holds.}\]

It is impossible to prove that a set of conditions for aggregation are or are not satisfied, but it is possible to determine under what assumptions (6) will hold under different specifications of the production function.

The assumptions made under this specification were harsh. Indeed, if these assumptions were true there would be no need to examine our formulation of the hospital production function. If all hospitals gained one dollar of cost weighted output from each dollar spent, then our production function would be linear. The assumptions made were sufficient to insure consistency but they
were not necessary. Using (7) and (8) we see the necessary conditions from (6) are:

\[
\frac{\partial F}{\partial x_{sr}} = \frac{C_s}{\partial y_s} \frac{1}{w_r}
\]

for \( r = 1, 2, \ldots, m \) and \( s = 1, 2, \ldots, n \)

Generally it will be impossible to evaluate (13) because of lack of knowledge of the values of \( x_{sr} \) and marginal product of \( x_{sr} \).

In this study we assume that all hospitals possess identical service production functions. This is analogous to the assumption that each individual possesses the same demand function when an aggregate demand function is being formulated. If we have consistent aggregation within each hospital, then each hospital will have the same production function if we aggregate using (7) and (8).

5. If \( \frac{\partial y}{\partial y_s} \), \( \frac{\partial x_{sr}}{\partial x_{sr}} \), are identical for each hospital (and by construction they are identical) and we aggregate using (7) and (8), then from consistency and identical service production functions we get \( \frac{\partial F}{\partial x_{sr}} \) for each \( r \) must be equal for all hospitals and \( F \) is identical for each hospital.

If we aggregate using (7) and (8) then consistent aggregation implies that the individual service production functions must be linear. The coefficients, however, may vary between service production functions. This assumption is unrealistic for our situation. It is unlikely that the production of hospital services
is accomplished in this manner (e.g. this implies that a x-ray may be produced without a x-ray machine). We might conclude that consistent aggregation using (7) and (8) is impossible. This is only true if the independent variables are allowed to vary randomly and take on any values.

A simple method of loosening the stringency of consistency conditions is to assume some type of relationship between the independent variables. Suppose we assume that the marginal rate of technological substitution between any two inputs is constant across service production functions. Consistency can now be attained if each service expansion path is linear and parallel to the expansion paths of all other services. Under these


restrictions if we define our output as in (7) we may find a function $F(X_1, \ldots, X_n)$ such that $F$ is homogenous of degree one and aggregation is consistent. There are three drawbacks in this


procedure: one, we are not able to aggregate using (8), two, it is difficult to defend the assumption that hospitals produce such that the MRTS between any two inputs producing any service is
constant, and three, the assumption that the expansion paths of each service production function are parallel is untenable since we are aggregating across non-homogenous units.

If we assume hospitals are profit-maximizers and operate in competitive input and output markets then they will operate at a point where the MRTS between any two inputs is constant across service production functions. This can be simply demonstrated since profit maximization implies:

\[(14) \frac{MP_{ik}}{MP_{jk}} \cdot P_1 = \frac{MP_{j}}{P_j} = P_k \]

where \(MP_{ik}\) is the marginal product of input \(k\) in producing output \(i\), \(P_1 = \) price of output \(i\), \(P_k = \) price of input \(k\)

Hence \(15) \frac{MP_{ik}}{MP_{jk}} = \frac{P_1}{P_j} = \frac{MP_{j}}{MP_{jk}} \quad \text{and} \quad (16) \frac{MP_{ie}}{MP_{ie}} = \frac{MP_{ie}}{MP_{ie}}

Assuming constancy between the MRTS for various services leads us no closer to finding restrictions on the independent variables which allow consistent aggregation using (7) and (8). Let us instead assume that (17)

\[X_{sr} = g_{sr}(X_{sr}) + h_{sr} X_r, \quad \sum g_{sr}(X_{sr}) = 0 \quad \text{and} \quad \sum b_{sr} = 1.\]

It is clear that consistent aggregation is now possible using any output index. (17) implies that a knowledge of the amount of an input used in a hospital allows us to determine the amount used to produce each service. Suppose that hospitals normally divide nursing time between different services in fixed proportions then
a knowledge of the total amount of nursing services provided a hospital in a week would allow us to determine the hours of nursing service provided in the delivery room for a week.

I have shown that the problem of defining a single output production function for multi-product firms such as hospitals is similar to defining an aggregate production function for an industry of firms which produce a single product. Just as consistency of an aggregate production function is important, so is consistency in the specification of a single product production function for a multi-product firm.

Although we have not proven a consistent production function exists for a hospital for any output index, we have shown what conditions are necessary for consistency. We have made two assumptions in our consistency argument: one, that the service production functions between hospitals are identical and two, that inputs are divided between services in the same constant proportions for all hospitals. The first assumption is more easily defended. If each hospital possesses the same technical knowledge and has the same access to qualified personnel, its service production function will be identical. The second assumption is more difficult to defend and even more difficult to test. A knowledge of the distribution of each input between services is needed. This information is not available.
A comparison of our cost weighted output index and other output indices can be made. If patient days is used as a measure of output we may ask whether an output index which excludes every hospital service except one can be meaningful in measuring hospital output. If other services vary proportionally with the number of patient days then there is no problem (see Appendix (A)). But if the amount of other services does not vary proportionally with patient days and in the same proportion for each hospital then there are no apriori grounds that justify the use of patient days to measure output rather than operations or laboratory tests. If the severity of cases treated is greater for large hospitals and large hospitals produce more ancillary services per patient day then patient days of care biases downwards the output of larger hospitals.

Another index of hospital output is cases treated. If cases treated is used as output index (17) is replaced by (18) \( Y = y(y_1, \ldots, y_n) \) cases treated. Suppose two hospitals produce identical amounts of each service yet one hospital treats patients with more severe ailments. Then the number of cases treated will be different for the same input vector and hence (18) is not a function. The problem of casemix variation is generally considered to be important and often some index of casemix variation is included in the independent variables. Let us define (19) \( Y^* = Y^*(y_1, \ldots, y_n, p_1, \ldots, p_k) \) where \( p_1, \ldots, p_k \) are the proportion of the cases treated in \( k \) different categories of disease classification. Instead of using (18)
as an output index to examine the hospital's production processes it might be more useful to estimate separately (19) as a sort of physician's production function. The estimation of (19) has never been attempted but John Rafferty discussed this type of function in a recent article, "If treatments of diagnoses were considered as outputs, with specific services as inputs, the resulting production function relationships would be invaluable both for hospital planning and for research on costs."


If we assume identical service production functions and that inputs are divided in the same constant proportions between services for all hospitals then our service weighted output production function defines an identical production function for each hospital. In order for (18) to define an identical production function for each hospital, in addition to the above assumptions we must assume that the function defined by (17) is identical for each hospital. These functions may be identical if we assume that the severity of cases treated by different hospitals are equal but this is a questionable assumption as John Rafferty has demonstrated. In order for patient days to define an identical
production function for each hospital, in addition to the assumptions made for the service weighted output index we must assume that the quantity of services determines the number of patient days and that this relationship is identical for each hospital. Using a service weighted output measure solves the problem of defining an identical output index function across hospitals by providing an output index which by construction is identical across hospitals.

The production of cases treated involves the use of physician services as well as hospital services. It might be useful to consider the production of cases treated as a two-level process. First, the hospital combines its inputs to produce hospital services and second, physicians combine these hospital services with their own labor to produce cases treated. The first level of this process can be examined by using my service weighted hospital production function and the second by a function resembling (18).
CHAPTER II

A DESCRIPTION OF THE DATA

The data which I use in my study have been taken from a portion of the medicare audit forms for fiscal year 1970 (Reimbursable Cost on the Departmental RCC Method or Combination Method). I acquired data on one third of Ohio’s non-teaching, short-term, non-federal hospitals. The hospitals in my sample were selected by choosing every third hospital from an alphabetical list of Ohio’s hospitals (excluding Cleveland area hospitals). I was not able to obtain data from the Hospital Statement of Reimbursable Cost.

The data on the cost of producing a particular service were not determined directly from departmental cost records. A method known as cost finding is used to distribute the cost of non-revenue producing centers to the revenue producing centers. In this method the cost of non-revenue producing departments is allocated to the revenue producing centers in order to determine which center the rate charged covers the full cost of producing a service (as opposed to the direct or departmental cost). The cost of operating a laboratory is greater than the cost reported by the
laboratory department because the laboratory receives service from other departments (e.g., the laundry and linen department clean the laboratory jackets and the engineering department provides electricity). The problem faced by the hospital when it tries to allocate its costs to various services is the same problem any multi-product firm is faced with when it attempts to derive the cost of producing a particular output.

This problem is analogous to that of determining how much of the output a particular input is responsible for producing. Suppose \( Q = AX^aY^b \) where \( X \) and \( Y \) are inputs and \( Q \) is the output. What portion of the output does \( X \) produce? If \( X = 0 \) then \( Q = 0 \) so in one sense \( X \) is responsible for producing all of \( Q \). It is obvious that this question is unanswerable in any meaningful way.

Assume a hospital's total cost is a function of two services in the following form: \( TX = aX + bX^2 + cY + dY^2 + eXY + k \) where \( TX \) is total cost and \( X, Y \) are services produced. What portion of the total cost is \( X \) responsible for? The cost of \( X \) clearly includes \( aX + bX^2 \) and part of \( eXY \). In this case if \( e \) is relatively small compared with \( a, b, c \) and \( d \) then if \( \frac{eXY}{2} \) is attributed to the cost of producing \( X \) then \( aX + bX^2 + \frac{eXY}{2} \) may be used as an estimate for the cost of producing \( X \) (this is still an approximation). The cost
of operating a hospital is not a linear sum of the various services it produces and hence it is not possible to clearly distinguish the cost of producing each service.

For reimbursement purpose the hospital must obtain some measure of the cost of producing a particular service. This is done by choosing an "appropriate statistical measures" to allocate the cost of non-revenue producing centers to revenue producing centers. The hospital initially determines which non-revenue producing department provides the most service to other departments and receives the least. Suppose the laundry and linen department is chosen. Then the number of pounds of laundry processed for each department over some sample period is calculated and is used as the statistical measure to distribute cost to all other departments. In turn, another non-revenue producing department is selected using the above procedure. This second department allocates its cost to all other departments except laundry and linen. In turn, a third department is chosen which allocates its cost to all departments except the first two. This procedure continues until there are no non-revenue producing departments remaining.

Suppose the total cost of a hospital is written:

\[ TX = aX + bX^2 + cY + dY^2 + eXY \]

where \( TX \) is total cost and \( X, Y \) are service outputs. Let us assume that the departmental cost of each service represents costs which are incurred solely in the production of
that service (i.e., departmental cost of service X is $aX + bX^2$).
The cost finding procedure would then effect only the distribution
of $(eXY)$ to X and Y. Departmental costs may be good estimates
of the cost incurred solely in the production of a given service.
Laboratory departmental cost includes salaries for laboratory
technicians and pathologists, and chemicals used to conduct
various tests.

The major difficulty with the cost finding technique is the
arbitrary manner in which non-revenue centers allocate their cost
to other centers. As I have already shown there may be no meaning-
ful way of distinguishing the cost of one output from another
when allocating the cost of a non-revenue producing center. Yet,
for some non-revenue producing centers whose cost functions are
approximately linear functions of their statistical measure the
cost finding technique is an appropriate method of allocating
cost. The more non-revenue producing centers whose cost functions
are linear functions of their statistical measure, the better
the approximation of the cost per unit of service provided by
the cost finding technique.

If we are trying to develop a weighting system for services
to define an aggregate output index, it is not clear that the
average cost per unit of service found after using the cost
finding method is superior to the average cost per unit compiled
from departmental data (direct costs). To the extent that the chosen
statistical measures make departmental cost functions linear functions of the service the cost finding method is superior. For example, if the cost of the processing each pound of laundry is $0.30 per pound, it follows that:

\[ TX_L = 0.3Z \]

where \( TX_L \) is the total cost of operating the laundry and linen department and \( Z \) is the number of pounds of laundry processed. The cost function may also be written as:

\[ TX_L = 0.3(Z_1 + Z_2 + \cdots + Z_n) \]

where \( Z_i \) is the number of pounds of laundry processed for the \( i \)th center. Allocating the cost of the laundry and linen department by pounds of laundry processed makes the weights (average cost per unit of service) better reflect the actual cost of producing each service. It is for this reason I have used average cost per unit of service determined from the cost finding technique for my weights rather than departmental cost per unit.
CHAPTER III

THE SUPPLY OF HOSPITAL SERVICES

It is the hypothesis of this study that hospital administrators act to maximize their capital facilities and profit. 1

1. I use profit here as total patient revenue less operating expense.

Most theories of hospital behavior discount the profit motive. "A hospital administrator can gain no kudos from paying large dividends; if he succeeds in earning a 'profit' and for him this is not a very important measure of success all he can do is plow it back," agrees Melvin Reder. There are, however, many 2


observers of hospital behavior who feel that capital accumulation is a major goal of the hospital. Melvin Reder further comments, "Because doctors can admit patients into only one or two hospitals, they have an incentive to become affiliated with hospitals which are as fully equipped as possible, so that they may treat hospitalized patients for as wide a range of ailments as their competence
(as they judge it) permits." Reder assumes hospitals desire to expand and modernize their facilities. Somers and Somers add, "Most observers of the long reimbursement negotiations agree that the hospital's drive to increase funds for capital formulation colored and complicated every major dispute on payments."


Verification of the tendency for hospitals to over-invest in capital equipment is given by the following, "According to the President's Commission on Heart Disease, Cancer and Stroke, 30 percent of the 777 hospitals equipped to do closed-heart surgery had no such cases in the year under study."

5. Ibid., p. 198.

Nonprofit hospitals do make substantial profits as Karen Davis points out, "Contrary to popular belief, therefore, hospitals do make substantial profits and these profits have been increasing
over time." Hospital administrators may have many reasons for desiring to earn a profit. They may be trying to gain money for capital investment. This is certainly the case when a hospital does not borrow significant amounts of money for capital expansion. With increased pressure from unionized workers for higher pay, hospital administrators may want more available money to meet future demands of the union. Profit may be used to hire more personnel or better personnel, or it may be invested. "There's no such thing as a nonprofit hospital," says Marilyn G. Rose, head of the Washington office of the National Health and Environmental Law Program, a federal legal aid program. "The money goes to the staff trustees, and administration through high salaries and various arrangements," she says. "Nonprofit hospitals
8. Ibid., p. A12.

had the most rapid increase in net income from $123 million in 1962 to $216 million in 1966. When depreciation expenses are added to net income, nonprofit hospitals make sizable rates of return...cash flow of nonprofit hospitals averaged 7.4 percent of revenues over the period," Karen Davis and Richard Foster point out. The data in the Davis and Foster study: which was


contracted by the Social Security Administration from the American Hospital Association, were obtained from audited reports on 462 hospitals from 1962-66.

My hypothesis of hospital behavior is similar to that proposed by Karen Davis. Karen Davis assumes that hospitals

maximize their cash flow (net revenue plus depreciation) in order to continuously expand their capital facilities. She defines short run costs as operating expenses, and net revenue as total revenue less short run costs. Cash flow is the increment of funds available for new investments. Cash flow is important in her hypothesis and she explains, "It is the hypothesis of this paper that hospitals neither borrow to any significant extent nor accumulate liquid or financial assets. All net patient revenue is invested in plant and equipment". Karen Davis' hypothesis

11. Ibid., p. 47.

is equivalent to a short-run profit maximization hypothesis for hospitals since capital is fixed in her analysis.

In my hypothesis of hospital behavior I allow for the possibility of capital expansion by means other than self financing and the accumulation of cash for purposes other than capital expansion. Karen Davis provides no evidence in her study to support her assumptions that the amount of money borrowed by hospitals is insignificant or that hospitals build up cash surpluses solely for investment in capital facilities. Irwin Wolkstein estimated that in 1964 there was a capital input for hospitals of
about $1.68 billion of which 38 percent came from philanthropy, 9 percent from federal grants, 26 percent from depreciation funds, 8 percent from excess of income over costs and 18 percent from all other sources including borrowing.


Data from the Hospital Economic Survey which is analyzed by Karen Davis and Paul Ginsburg is a good source of information on hospital capital financing. Using this data we find 40.3 percent of total investment by nonprofit, private, short-term, non-psychiatric hospitals came from grants, 23.3 percent from borrowing and 36.3 percent from internal funds for the year 1966. We find 36 percent of capital supported by internal funds compared to the 34 percent found by Wolkstein. In both instances a substantial portion (over 60 percent) of capital financing was estimated
to come from external sources.

The implications of my hypothesis for hospital behavior may be in the following manner. Let hospitals maximize utility subject to a production function where utility is a function of capital and profits.

(1) $U(K, \Omega)$ where $U$ is utility, $K$ is capital and $\Omega$ is profits

(2) $U_{\Omega} \frac{\partial \Omega}{\partial L} = 0$ where $U_{\Omega} = \frac{\partial \Omega}{\partial L}$ and $L$ is labor

(3) $U_{\Omega} \frac{\partial \Omega}{\partial K} + U_K = 0$

The conditions for maximization of (1) are given by (2) and (3). We note:

(4) $\frac{\partial \Omega}{\partial K} = \frac{\partial R}{\partial K} - \frac{\partial C}{\partial K}$

and (5) $\frac{\partial \Omega}{\partial L} = \frac{\partial R}{\partial L} - \frac{\partial C}{\partial L}$

where $R$ is revenue and $C$ is cost

(1) and (2) may be written as:

(6) $U_{\Omega} \left( \frac{\partial R}{\partial L} - \frac{\partial C}{\partial L} \right) = 0 \Rightarrow \frac{\partial R}{\partial L} - \frac{\partial C}{\partial L} = 0$

(7) $U_{\Omega} \left( \frac{\partial R}{\partial K} - \frac{\partial C}{\partial K} \right) + U_K = 0 \Rightarrow \frac{\partial R}{\partial K} - \frac{\partial C}{\partial K} = \frac{U_K}{U_{\Omega}}$

(6) implies that labor should be employed up to the point where its marginal revenue product equals the marginal cost. This condition is identical to that imposed by profit maximization. (7) implies that capital be used up to the point where its marginal revenue
product equal its marginal cost less the marginal utility of capital divided by the marginal utility of profit. Hence more capital is employed than would be under profit maximization. This implication of my hypothesis is supported by data from the Hospital Economic Survey, "Major equipment plant assets per day of hospital care increased by 53 percent over the period in non-profit hospitals ... five times as rapid an increase as in for-profit hospitals."


If the marginal utility of capital is zero then we have the same implications as the profit maximization hypothesis. The greater the marginal utility of capital the greater deviation from the least cost position.

FIGURE 1
PROFIT CAPITAL INDIFFERENCE CURVES
For each level of capital there exists a level of output at which profit is maximized. If we graph each level of capital with the largest possible profit at that capital level we obtain Figure 1. Since we assume utility is a function of capital and profit, indifference curves may be imposed on Figure 1. The hospital will strive towards producing at \((k_2, \Pi_2)\) to maximize utility.

From (6) and (7) we find:

\[
\frac{\partial R}{\partial X \partial L} = \frac{\partial R}{\partial L \partial K} = \frac{\partial C}{\partial L} - \frac{\partial C}{\partial K} \frac{U_k}{U_l} \quad \Rightarrow \quad \frac{\partial X}{\partial L} = \frac{\partial C}{\partial K} \frac{U_k}{U_l}
\]

The marginal rate of substitution equilibrium conditions of my hypothesis are different from those derived under cost minimization by a factor of \((\frac{U_k}{U_l})\). This occurs because one of the arguments in the hospital utility function is not a function of output. Nagesh Ravenkar and Murray Brown prove that if the arguments in a firm's utility function are themselves functions of output then the marginal rate of substitution equilibrium conditions are identical to those derived from the neo-classical, profit maximization model.

There are some hospital observers who feel increases in capital equipment will increase the demand for a hospital's services.


Capital accumulation will influence demand to the extent that doctors prefer affiliation with better equipped hospitals. Capital also affects demand to the extent that supply creates its own demand in the hospital industry. The demand curve in my preceding analysis may be considered a function of capital as well as price and examined in the same framework. For a given level of capital demand will now be greater but labor will still be hired up to the point where its marginal revenue product is equal to its marginal factor cost. Again, a capital profit curve may be drawn and utility is maximized at the tangent point of the highest indifference curve.

If capital is a factor in the demand for hospital services then:
(10) \[ R = P(K, X)X \] where \( p \) is price.

(11) \[ \frac{\partial R}{\partial K} = P \frac{\partial X}{\partial K} - X \left( \frac{\partial P}{\partial X} \frac{\partial X}{\partial K} - \frac{\partial P}{\partial K} \right) \]

(12) \[ \frac{\partial R}{\partial K} = (P + X \frac{\partial P}{\partial X}) \frac{\partial X}{\partial K} + X \frac{\partial P}{\partial K} \]

(13) \[ \frac{\partial R}{\partial K} = MR_X MP_K + X \frac{\partial P}{\partial K} \]

where \( MP_K \) is the marginal product of capital and \( MR_K \) is the marginal revenue of output.

Capital will now be hired beyond the point it would if capital were not included in the demand function by a factor of \( X \frac{\partial P}{\partial K} \).
CHAPTER IV

SPECIFICATION AND ESTIMATION OF A HOSPITAL PRODUCTION FUNCTION

Specification analysis is designed to aid in determining the effects of incorrectly specifying a relationship. Much of the work in specification analysis stems from Their's pioneering effort in examining linear aggregation. Zvi Griliches three years later


in 1957 stated, "The major restriction of our framework lies in the fact that we must assume that something is 'true' --- we must specify our 'true' equation.


The specification of a single output production function for a hospital, which is a multi-product firm leads us to a problem in aggregation theory. Unfortunately, aggregation bias is only easily defined if the underlying micro-relations are linear and we are aggregating over homogenous units. In this case, the micro-relations are not linear and I am not aggregating across
homogenous units (i.e., \( Y = \sum_{i=1}^{n} y_i \)).

In this chapter I will first specify the variables in my production function. Secondly, I will briefly discuss the problems involved in the aggregation of inputs and measurement errors in my output index. Thirdly, I will modify Berndt and Christensen's method, in order to examine a firm which is not a profit maximizer, which will allow me to determine the appropriate functional form for my hospital production function. Finally, I will present and discuss my empirical results and estimation techniques.

The measure of output used in this study is a cost weighted combination of patient days of routine services, x-rays, laboratory procedures, deliveries and operations. The weight associated with a particular service is the average cost for all hospitals in the sample of producing that service. The number of units of each service and cost of each service were obtained for the last page of the portion of the sample medicare audit form in Appendix (A). As discussed in Appendix (A) the proper cost of a patient day of routine services is found under Part 1 Special Services Costs.

The measure for labor services is total labor cost. This is used as a surrogate for labor services rather than number of personnel because of the diversity of labor services provided for the hospital. I assume that the higher the salary an employee receives the greater the increase in output per unit increase in
his services. If the number of personnel is used as a measure of labor, the labor services provided to larger hospitals may be underestimated. The average annual salary of larger hospitals was consistently and significantly greater than that of smaller hospitals for the 482 hospitals in the Hospital Economic Survey.

3. See Karen Davis and Richard Foster, op.cit., p. 86.

This may reflect the higher skill level of the employees in larger hospitals. However, when employees were divided into administrative, dietary, household and property, and professional patient care, no definite difference in skill levels was found between large and small hospitals.

4. See Karen Davis and Richard Foster, op.cit., p. 89.

It is my hypothesis that for this sample the average annual salary variation between large and small hospitals reflects varied levels of skilled personnel rather than salary differences for identical services. This hypothesis is supported by the fact that in nine geographic areas from which the sample was drawn there were small differences in average salary per employee, yet there were substantial differences when the hospitals were differentiated by size.
Capital services is initially measured by depreciation costs. Capital services is very difficult to measure because depreciation costs rarely consider the rate of use. A further difficulty is that there are several ways depreciation may be calculated. Historical cost is the basis for depreciation for all the hospitals in the sample. Yet, straight line, accelerated, or sum of digits methods may be used to measure depreciation. For these reasons I will also use number of beds and specialized facilities as capital measures.

Supplies is measured by the cost of supplies. This cost is determined by subtracting labor and capital cost from total cost. This category includes all drugs, dressing, and equipment that is consumed during the year. It includes supplies used to maintain and repair the hospital as well as supplies used directly to treat the patient.

My output measure should reflect the production of all services. Unfortunately, due to data limitations, certain services have been omitted (e.g., occupational, physical and inhalation therapy). Even though these activities are responsible for a small percentage of a hospital's total cost, their omission introduces a bias with respect to size. Larger hospitals are more likely to provide these services than smaller hospitals, hence my measure of output understates the output of larger hospitals. In recognition of this fact, the estimate of economies of scale
will be biased downward.

In any estimation of a production function there exists some aggregation over inputs. For example, the labor services provided by an orderly and the labor services provided by a housekeeper are different and simply adding the hours of work of each category together to develop a measure of unskilled labor will probably cause aggregation bias. The necessary condition for consistent aggregation is weak separability. In order to be able to measure the bias caused by incorrect aggregation a knowledge of the "true" form of the production function is needed. Also, a detailed knowledge of the values of the micro-inputs and the relationship between them is necessary.


If we do not assume anything about the "true" relationship we are unable to measure the aggregation bias but we are able to test for the existence of an input index (i.e., weak separability) using a method developed in a recent paper by Berndt and Christensen.


This method uses a theorem developed in an earlier paper by Berndt and Christensen which proves that weak functional separability exists in homogenous production functions if and only if the Allen partial elasticities of substitution (AES) are equal for inter-partition pairs. The equality of AES for inter-


partition pairs may be tested using linear relationships of the coefficients in the factor share equations of the translog production function. These tests can only be performed if we know the values of the inputs we are aggregating over to form the index. If we can measure only a few inputs it is senseless to exclude any of them and try to develop an input index for any group of them. Even if we test and determine that a consistent index exists for a group of inputs we have no way of determining the form of the index.

There has recently been a great deal of work with functions
which are able to take on arbitrary values for the elasticities of substitution. Laurits R. Christensen, Dale W. Jorgenson and Laurence J. Lau have developed a transcendental logarithmic production function which is able to attain an arbitrary set of pairwise elasticities of substitution at any point in the input space. Their expression for the transcendental logarithmic production function (translog for short) is:

\[
\text{ln}V = \text{ln} \lambda_0 + \sum_{i=1}^{m} \lambda_i \text{ln} X_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \psi_{ij} \text{ln} X_{ij},
\]

higher order terms

where \( V \) is output and \( X_i \) is the \( i \)th input. This function is not necessarily monotonic. The condition for monotonicity is:

\[
\frac{\partial \text{ln} V}{\partial \text{ln} X_i} = \lambda_i + \frac{1}{2} \sum (\psi_{ji} + \psi_{ij}) \text{ln} X_j > 0
\]

Berndt and Christensen in their 1971 paper on the translog function use this function to estimate substitutability patterns between capital and labor inputs in the U.S. form 1928-1968. The form of
their equation is:

\[ \ln V = L_0 + \sum_{i=1}^{m} \lambda_i \ln X_i + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} \gamma_{ij} \]

A Taylor series expansion the ln X's around the fixed point
\((\ln \bar{X}_1, \ln \bar{X}_2, \ldots, \ln \bar{X}_m)\) can be written as:

\[ \ln V = G(\ln X_1, \ldots, \ln X_m) + \sum \frac{\partial G}{\partial \ln X_i} (\ln X_i - \ln \bar{X}_i) + \frac{1}{2} \sum \sum \frac{\partial^2 G}{\partial \ln X_i \partial \ln X_j} (\ln X_i - \ln \bar{X}_i)(\ln X_j - \ln \bar{X}_j) \]

The translog function is just a generalized Taylor series expansion in m variables, truncated after the second order term for an arbitrary function of m variables. The translog function can be used to estimate any arbitrary function. If the translog function can be effectively estimated the restrictions on the translog parameters imposed by estimating an arbitrary function may be tested. Assume:

\[ \frac{\partial V}{\partial X_i} = P_i \]

\[ \frac{\partial \ln V}{\partial \ln X_i} = \frac{P_i X_i}{V} = M_i \]

where \(M_i\) is the proportion of total cost due to factor \(i\) and
and \( p_i \) = price of the \( i \)th input.

With efficient production the following marginal conditions appear for a production function with three inputs.

\[
(7) \quad M_K = \lambda_K + \psi_{kk} \ln K + \psi_{kl} \ln L + \psi_{ks} \ln S \\
(8) \quad M_S = \lambda_S + \psi_{sk} \ln K + \psi_{sl} \ln L + \psi_{ss} \ln S \\
(9) \quad M_L = \lambda_L + \psi_{lk} \ln K + \psi_{ll} \ln L + \psi_{ls} \ln S
\]

where \( K \) is capital, \( L \) is labor and \( S \) is supplies.

Symmetry and constant returns to scale implies:

\[
(10) \quad \sum_{i=1}^{3} \lambda_i = 1, \sum_{j=1}^{3} \psi_{ij} = 0, \sum_{i=1}^{3} \psi_{ij} = 0, \text{and} \sum_{i=1}^{3} \sum_{j=1}^{3} \psi_{ij} = 0
\]

These restrictions are not all independent. If off diagonal elements are set equal, column sum equal to zero and sum of the \( \lambda_i \)'s set equal to one, the restrictions are satisfied. If our equations have three inputs it will take six restrictions to set off diagonal elements equal, for restrictions to set column sums to zero, and one more restriction for the sum of the \( \lambda_i \)'s. There are twenty parameters and eleven restrictions; hence, there are nine free parameters.

The Cobb-Douglas function is a special case of the translog
function with $\psi_{ij}=0$ for all $i$ and $j$. In our case there are nine restrictions imposed on this form by the Cobb-Douglas assumption. The CES function written as follows has three free parameters.

\[ (11) \ln V = \ln \theta - \frac{1}{\rho} \ln \left( \delta_1 X_1^{-\rho} + \delta_2 X_2^{-\rho} + \delta_3 X_3^{-\rho} \right) \]

The three free parameters are $\rho$, $\delta_1$, and $\delta_3$. Berndt and Christensen examined the three input case of a CES function with constant returns to scale. The CES has three free parameters, hence nine restrictions. They include three symmetry and three row sum restrictions. The other three restrictions are

\[ (12) \sum_{i=1}^{3} \delta_i = 1, \lambda_k \psi_{SL} = \lambda_s \psi_{SL}, \lambda_s \psi_{EL} = \lambda_L \psi_{ES} \]

Uzawa's Cobb-Douglas function of a CES function may be written:

\[ (13) \ln V = \ln \theta + \beta_L \ln L - \frac{\beta_K}{\rho} \ln \left( \delta_K K^{-\rho} + \delta_S S^{-\rho} \right) \]

Uzawa's function has three free parameters $\rho$, $\beta_L$ and $\delta_L$ thus the second order approximation to the Uzawa form requires two restrictions in addition to three symmetry and three row sum restrictions:
\[ \psi_{EL} = \psi_{SL} = 0 \]

Sato's CES function of a CES function may be written:

\[ \ln V = \ln \theta - \frac{1}{\omega} \ln \left( \beta L^{\omega -1} + \beta K^{\omega -1} (\delta K^{\rho} + \delta S^{\rho})^{1/\rho} \right) \]

Sato's form has four free parameters \( \omega, \rho, \beta_L, \) and \( \delta_L \). Only one restriction in addition to three symmetry and three row sum conditions is needed. It is:

\[ \lambda K \psi_{SL} = \lambda S \psi_{KL} \]

It should be noted that the CES and Sato functions require non-linear restrictions on the set of simultaneous equations (7)-(9). In order to use the Theil three stage least square program I initially estimated the system subject to no restrictions and used the estimates for \( \lambda, \delta, \) and \( \lambda, \) to linearize the restriction. This is an arbitrary procedure so I estimated the system subject to no restrictions and used the estimates for \( \psi, \psi_S, \) and \( \psi, \) to linearize the restrictions. Approximately the same results were obtained using both procedures.

The above procedure developed by Berndt and Christensen may be modified for a firm which is a combination profit and capital
maximizer. From (6) and (7) of Chapter 3 we get:

(17) \( \frac{\delta R}{\delta L} - \frac{\delta C}{\delta L} = 0 \)

(18) \( \frac{\delta R}{\delta K} = \frac{\delta C}{\delta K} - \frac{U_k}{U_T} \)

as the equilibrium conditions for our combination profit and capital maximizer. Since my function has output measured in dollars (15) and (16) may be rewritten as:

(19) \( MP_L = P_L \)

(20) \( \frac{MP_K}{V} = \frac{P_K K}{V} - \frac{U_K}{U_T} \frac{K}{V} = M_K \)

where \( P_L \) is the price of labor, \( MP_L \) is the marginal product of labor, \( P_K \) is the price of capital and \( MP_K \) is the marginal product of capital. Now rearranging the above condition we get:

(21) \( \frac{MP_L}{V} = \frac{P_L}{V} = M_L \)

(22) \( \frac{MP_K}{V} = \frac{P_K K}{X} - \frac{U_K}{U_T} \frac{K}{V} = M_K \)

If we add supplies as an input we get the same marginal productivity condition on supplies as we have for labor. Note that the price variables are are all one in our example since labor,
capital and supplies are measured in dollars. The value of $M_L$
is the same as under the profit maximization hypothesis. In
order to estimate $M_K$ we must assume something about the form of
the utility function. Let us assume:

\[(23) \, U = \Upsilon \, (1-x) \equiv K^{(1-x)} \]

that profit is a constant percentage of output:

\[(24) \, \Upsilon = cV \text{ where } c \text{ is a constant.} \]

Now \[(25) \, \frac{U_k}{U_n} = \frac{\Upsilon (1-x)}{K^x} = \frac{cV(1-x)}{K^x} \]

and (6) may be written:

\[(26) \, \frac{P_k K}{V} - \frac{U_k}{U_n} \frac{K}{V} = \frac{P_k K}{V} - \frac{c(1-x)}{x} = M_k \]

I assume $c = .04$ and $(\frac{1-x}{x}) = 5.9$. Now (9) may be expressed as:

9. I have set $c = .04$ because the capital return ratio
(defined as net revenue plus interest expense divided by total
revenue) was about 4% in 1966. I set $x = 1/6$ because I assume that
hospitals place more importance on expanding capital equipment
than increasing profit. I reestimated Table 1 for $x = .2$ and $x = .4$
and the CES functional form was still the first function not
rejected.

\[(27) \, \frac{P_k K}{X} - (.2) \quad \text{and we can test the various restrictions imposed}
\]

by our production functions on this new system of simultaneous
equations.
It should be noted that Berndt and Christensen's method for selecting the form of the production function may only be modified for relatively few types of nonprofit maximizing behavior. In particular, only firms whose behavior yields marginal productivity conditions which independent of each other may be used. Otherwise, estimates of the marginal productivity of various inputs will be needed.

Initially the set of simultaneous equations (7) - (9) for profit maximizing behavior is estimated subject to the restrictions imposed on the coefficients by the various forms (CD, CES, Sato and Uzawa) and then the profit capital maximizing set of equation is used (substitute (27) for (9)). The results are presented in Table (1). The CES is the first form whose restrictions on the coefficients of each system are not rejected. It is useful to use the first acceptable form since the more general forms are cumbersome to work with.

Using a method developed by Martin Feldstein I have directly estimated a CES production function. If we write the CES


function as in (28) we can use Feldstein's two step method.
(28) \[ \nu_x = \sum c_i x_i \]

where \( x_i \) is the input and \( \nu \) is output.

The first step is to select arbitrary values for \( \rho \) and \( \gamma \) and run a regression on (28). The second step entails picking the combination of \( \rho \) and \( \gamma \) which yields the largest \( R^2 \).

**TABLE 1**

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<tr>
<th>Profit Max Hypothesis</th>
<th>Profit-Capital Hypothesis</th>
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<td>All Hospitals</td>
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<tr>
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<tr>
<td>CES</td>
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<td>Uzawa</td>
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<tr>
<td>Sato</td>
<td>F(7,123)</td>
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</table>

Note: Critical Points for 5% are:

- F(9,100) | 2.76
- F(8,100) | 2.98
- F(7,100) | 3.28
- F(9,200) | 2.73
- F(8,200) | 2.96
- F(7,200) | 3.24

From Table 2 we can see the maximum correlation coefficient between estimated and actual output for small hospitals was
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Large Hospitals (more than 125 beds)

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Small Hospitals (less than 125 beds)

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</tbody>
</table>

* .508 is the correlation coefficient between the estimated output and actual output using $\sigma = .2$ and $\psi = .4$. 
reached where returns to scale was 1.0 and elasticity of substitution is .4. The maximum for large hospitals was at \( y = 1.0 \) and \( \sigma = 8 \). This implies that the substitution possibilities are greater for larger hospitals. For all hospitals the maximum was reached \( y = 8 \) and \( \sigma = 8 \). There are no local maxima.

Further regressions were run using values of \( y \) between .8 and 1.0 (i.e., .80, .81, .82, ..., .99, 1.00) with the same five values of \( \sigma \). It was found that for all hospitals a maximum is found at \( y = .88 \) and \( \sigma = .8 \), at \( y = .96 \) and \( \sigma = .8 \) for small hospitals and at \( y = .87 \) and \( \sigma = .8 \) for large hospitals.

Alternative methods of estimating the CES function should be weighted according to the plausibility of the assumptions underlying each method. The direct method of estimation assumes the input values are exogenous hence the estimates are subject to a simultaneity bias. I have estimated the CES by another method which was proposed by Arrow et. al. (hereafter referred to as the ACMS method). The ACMS method makes three assumptions: first, constant returns to scale, second, that the marginal product of labor equals the wage rate and, third, that the wage rate is exogenous. We cannot estimate returns to scale using this method. The marginal productivity condition on labor is consistent with my profit capital maximization hypothesis. An exogenous wage rate is a plausible assumption for hospital workers who have skills which
are used in other industries (e.g., orderlies, repair men and clerks).

Using the above assumptions, the CES function may be written as:

\[(29) \frac{V}{L} = W \left( \frac{1}{1+\rho} \right) \left( (1-\lambda)^{-1} \right) \frac{1}{1+\rho} \]

where \(W\) is the wage rate

and \(V = (\lambda K^{-\rho} + (1-\lambda) L^{-\rho})^{1/\rho}\)

Rewriting (29) we obtain:

\[(30) \log \frac{V}{L} = \frac{1}{1+\rho} \log W + \frac{1}{1+\rho} \log (1-\lambda)^{-1} \]

In this method I use \(L\) to mean number of personnel and wage rate as the total labor cost divided by number of employees. The results were:

\[(31) \log \frac{V}{L} = 3.614 + 0.617 \log W \]

\[R^2 = 0.52 \text{ and } (0.721) \text{ is the standard deviation of the constant term.}\]

Again, the elasticity of substitution is less than one. It is not significantly less than one at the 5 percent level or significantly greater than zero at the 5 percent level.

The estimate for \(\sigma\) derived from the ACMS method is less than for the other methods of estimation I have employed. This may have resulted from the existence of decreasing returns to scale instead of constant returns to scale. If we do not assume constant returns to scale in the ACMS method then the efficient use of labor implies:
\[
(32) \log \frac{V}{L} = \log \omega + \frac{1}{\eta \eta} \log W + (1 - \sigma)(V - 1) \log V
\]

and the \( E(\sigma) = \sigma + \frac{1 - \sigma}{\eta} \beta \) where \( \beta \) is the expected value of the regression coefficient of \( \ln V \) on \( \ln W \).


Suppose we have decreasing returns to scale and an elasticity of substitution less than one and a positive correlation between output and the wage rate. Then the ACS estimate of the elasticity of substitution will be biased downward.

I have also estimated the CES function by using a Taylor series expansion. If a Taylor series expansion is taken and the non-linear higher order terms dropped the unknowns will appear in linear form.

Write (33) \( V = A(\alpha_1 K^{-\rho} + \alpha_2 L^{-\rho} + \alpha_3 S^{-\rho})^{\frac{1}{\rho}} = F(\alpha_1, \alpha_2, \alpha_3, A, \rho, v, K, L, S) \)

Expand about the point \((\alpha_{10}, \alpha_{20}, \alpha_{30}, A_0, \rho_0, v_0)\) and we get:

\[
(34) \quad F - F(\alpha_{10}, \alpha_{20}, \alpha_{30}, A_0, \rho_0, v_0) + \sum \alpha \frac{\partial F}{\partial \alpha_0} + \frac{\partial F}{\partial A_0} \alpha_0 + \frac{\partial F}{\partial \rho_0} \rho_0 + \frac{\partial F}{\partial v_0} v_0 = \sum \frac{2}{\alpha} \frac{\partial F}{\partial \alpha_0} + A \frac{\partial F}{\partial A_0} + \rho \frac{\partial F}{\partial \rho_0} + v_0 \frac{\partial F}{\partial v_0} + \text{higher order terms}
\]
Estimates of \( \psi_1, \psi_2, \psi_3, \rho \) and \( \gamma \) may be obtained and be used as initial values in the second iteration. Whether or not this method converges depends upon how close the first stage initial values are to the actual values. Often, with cross section data this method will not converge.


Starting with initial values of \( \rho = .2 \) and \( \gamma = 1.2 \) I obtained final values (after 20 iterations) of \( \rho = .385 \) and \( \gamma = .86 \). The final estimate for \( \gamma \) is very significant with a \( T \)-value of almost 16. The estimate for \( \gamma \) did converge with very small changes in \( \gamma \) from the 4th iteration on. The estimate of \( \gamma \) in the 4th iteration was .38271 whereas the 20th iteration yielded and estimate for \( \gamma \) of .87532. The stability of this estimate for returns to scale gives it more weight than the estimates for the remaining parameters.

An estimated value of .385 for \( \rho \) implies an elasticity of substitution of just over .7. This agrees with the estimates for found by Feldstein's and the ACS and procedures. The estimate of derived from this Taylor series method must be regarded with caution for two reasons. First, there are substantial changes
in the estimate of \( \rho \) for each iteration. Second, the value of has a \( T \) value of 1.6.

The estimates of \( \delta_3 \) are extremely unstable and has a \( T \) value of less than .1 on the 20th iteration. This implies \( \delta_3 \) has not reached its equilibrium value and that varying its first stage initial value might effect other other parameter estimates.

An alternative specification of this production function might include the number of beds and number of specialized facilities (as defined by the American Hospital) as measures of capital and number of personnel as a measure of labor. I have estimated, using the non-linear estimation previously described, this specification. The function to be estimated may be written:

\[
V = A(d_1 K_1^\rho + d_2 K_2^\rho + d_3 L^\rho)^{\frac{\rho}{1+\rho}}
\]

where \( V \) is weighted service output, \( K_1 \) = number of beds, \( K_2 \) = is number of specialized facilities as reported to the American Hospital Association and \( L \) is the number of personnel.

The results from estimating this equation are give in Table 3.

Each coefficient was found to be significant under this specification whereas only one coefficient was clearly significant under the previous specification. Returns to scale is still estimated to be slightly less than one, but not significantly less than one at the 5 percent level. The elasticity of substitution is estimated to be .896. \( \rho \) is significantly greater than zero under the previous specification. The estimate for
each coefficient stabilized on the 13th iteration. The negative coefficient for specialized facilities is probably a result of my exclusion of many of the specialized services in my output measure. A hospital which operates an intensive care unit will not increase its output (as I have measured output) yet will be using resources to operate the unit.

In order to examine how the estimated returns to scale varies with different output indices I have estimated the following function:

\[ V = A \left( d_1 K_1^{-\rho} + d_2 K_2^{-\rho} + d_3 L^{-\gamma} \right)^{\frac{1}{\rho}} \]

where \( V \) is patient days and the inputs are the same as in (35). The results of this estimation by the non-linear TSP program are reported in Table 4.

The negative coefficient for specialized facilities reflects the exclusion of specialized services in the output measure patient days. The number of patient days does not reflect the intensity of treatment (i.e., the use of auxiliary services and specialized facilities). For a given amount of personnel and beds the more specialized services a hospital provides the fewer patient days it will produce. Generally, in Table 4, there are very low T-scores (only one is significant). Patient days is clearly inferior, in this sense, to our weighted service output which had all coefficients significant and converged. Returns to scale
using patient days is higher than in any other specification and is not significantly less than unity at the 5 percent level. The elasticity of substitution is estimated to be just below .5 which is smaller than under previous specifications. The large standard error for \( \rho \) implies we cannot be confident about the estimate for \( \rho \) (and consequently \( \sigma \)).

The results of all the estimations are presented in Table 5. For each specification the estimates for returns to scale for all hospitals was between .88 and .93. Large hospitals had lower returns to scale than small hospitals in every case. Since our output measure is biased downward for larger hospitals we can not be sure how much of the difference in the estimated returns to scale for large and small hospitals reflects actual production differences. Consequently, all that can be stated is that our findings weigh against the hypothesis that hospitals possess significantly increasing returns to scale. It should be carefully noted that these findings apply only for the production of hospital services; they do not imply that there are no economies of scale in the production of cases treated.

The estimated elasticity of substitution was found to vary from .49 to .90 for all hospitals. The estimate for .49 may be discounted since it was derived from an estimate of which was not significant. If we ignore the estimate of .49 then the elasticity of substitution was found to vary between .62 and .90. I used a first
order Taylor series expansion for direct estimation methods in order to get an estimate for the standard deviation of $\sigma$.

13. See Appendix (B).

Larger hospitals had a higher elasticity of substitution in every estimation except one. These results tend to confirm the hypothesis that hospitals do not possess a Leontief type production function and that large hospitals have higher elasticities of substitution than small hospitals.
TABLE 3

TAYLOR SERIES RESULTS USING COST WEIGHTED SERVICE OUTPUT AND REAL INPUTS

<table>
<thead>
<tr>
<th>Right Hand Variable</th>
<th>Estimated Coefficient</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>.188</td>
<td>3.787</td>
</tr>
<tr>
<td>$p$</td>
<td>.113</td>
<td>75.858</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-.117</td>
<td>-7.66</td>
</tr>
<tr>
<td>$d_3$</td>
<td>.517</td>
<td>4.689</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>.897</td>
<td>10.237</td>
</tr>
</tbody>
</table>

a. $R^2 = .905$

TABLE 4

TAYLOR SERIES RESULTS USING PATIENT DAYS AND REAL INPUTS

<table>
<thead>
<tr>
<th>Right Hand Variable</th>
<th>Estimated Coefficient</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>.0009</td>
<td>.137</td>
</tr>
<tr>
<td>$p$</td>
<td>1.019</td>
<td>1.291</td>
</tr>
<tr>
<td>$d_2$</td>
<td>-.001</td>
<td>-.126</td>
</tr>
<tr>
<td>$d_3$</td>
<td>.071</td>
<td>.577</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>.929</td>
<td>11.663</td>
</tr>
</tbody>
</table>

a. $R^2 = .307$
### Table 5

**Estimation of the CES Function**

<table>
<thead>
<tr>
<th>Specification and Estimation Procedure</th>
<th>All Hospitals</th>
<th>Large Hospitals</th>
<th>Small Hospitals</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Output</strong></td>
<td>ζ  γ</td>
<td>ζ  γ</td>
<td>ζ  γ</td>
</tr>
<tr>
<td>cost weighted labor cost</td>
<td>.80 .88</td>
<td>.80 .87</td>
<td>.80 .96</td>
</tr>
<tr>
<td>service</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>capital cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>supplies cost</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feldstein's method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor series</td>
<td>.71 (.25)(.05)</td>
<td>.82 (.30)(.13)</td>
<td>.62 (.17)(.04)</td>
</tr>
<tr>
<td><strong>Cost weighted wage rate</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>service/number of personnel (i.e. labor ACHS cost/number of personnel)</td>
<td>.62 (.33)</td>
<td>.58 (.41)</td>
<td>.71 (.21)</td>
</tr>
<tr>
<td><strong>Cost weighted service</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of beds</td>
<td>.96 (.002)(.09)</td>
<td>.90 (.013)(.09)</td>
<td>.88 (.001)(.032)</td>
</tr>
<tr>
<td>number of facilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of personnel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor series</td>
<td>.49 (.79)(.08)</td>
<td>.62 (.96)(.08)</td>
<td>.40 (.61)(.06)</td>
</tr>
<tr>
<td><strong>Patient days</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of beds</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of facilities</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>number of personnel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor series</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. ζ: elasticity of substitution  
γ: economies of scale  
Number in parentheses is the standard deviation of the number above it.
CHAPTER V

A PRODUCTION COMPARISON OF TEACHING AND NON-TEACHING HOSPITALS

There are several operational and many proposed experiments designed to determine the effects of different reimbursement procedures on hospital behavior. Yet, when the data is gathered from these experiments the results are often inconclusive, and no one really knows how or where the hospital changed. What is available from any new experiments is simply more data on the cost of different procedures, amount of inputs, number of people served and services provided. Those people who advocate continued experiments using different reimbursement formulas and organizational structures often find it impossible to analyze and benefit from the data already generated from such experiments. Possibly, what is needed more than experiments in hospital reimbursement plans is a careful analysis of the behavioral and technical relationships of the hospital. In order to understand the effects on hospital charges, inputs used, and outputs produced it is necessary to have some knowledge of the structural relationships.

There have not been any detailed and carefully formulated models of hospital behavior which are useful in measuring the
effects of different incentives and regulations on the hospital. We have not been able to develop a model which can be estimated before and after experimentation to help us analyze the effects. Suppose a group of hospitals is reimbursed under a new reimbursement system. If we find after a year that these hospitals have a lower cost per case, higher cost per patient day, and generally higher costs for x-rays, laboratory tests and routine patient care, then we will have difficulty even in determining whether these hospitals are more or less efficient. Indeed, until we define our output, efficiency cannot be measured. If we use cost per case then the hospitals are more efficient; even if they have substituted simple for complex cases. If we use cost per patient day as our output measure these hospitals have been less efficient.

Although we cannot specify and estimate all the structural relationships within a hospital (we cannot even measure the marginal product of capital), we can specify and estimate part of the system. We can formulate and utilize production and cost functions for the hospital. Since the cost and production functions constitute only a portion of the system we must be careful in interpreting the changes in their coefficients resulting from various changes in reimbursement formulas or organizational shifts. Cost and production function analysis may be used not only to estimate economies of scale but to measure changes in production and cost relationships resulting from various experiments. Cost and
production function analysis is the sole methodological tool economists have to analyze the effects of these experiments.

In my first chapter I evaluated various output indices for their likelihood in defining an identical production functions across hospitals. I showed that a weighted measure of services is preferable to a weighted case measure or patient days. Later in the first chapter I suggested that the hospital production process be broken down into two steps. The first was the production of various services (x-rays, laboratory tests, etc.). The second was the production of cases treated by the physicians and the hospital services. Cost and production function analysis of each step will allow us to determine where changes have occurred and whether they have resulted from technical or economic adjustments.

It should be noted that cost functions only reflect production relationships (such as economies of scale) if each hospital is using the least cost combination of inputs. If the hospitals are not least cost producers then cost function analysis of different groups of hospitals reflect economic and technical differences.

A hospital which operates an intern or residency program not only provides education for the student but receives services from them. Each year there are not enough medical students graduating to fill all the available internships. Obviously, the hospital values the services provided by interns and residents. One way
to assess the cost of providing medical education is to estimate the cost difference between teaching and non-teaching hospitals in treating identical cases. David Salkever, in recent dissertation, using cases treated and the number of interns and residents as a medical education variable, estimated it cost a a hospital \$25,000 a year per year of physician education. He also found


that most of the increase resulted from the increased length of stay found in hospitals which ran medical education programs. Salkever states, "These elasticities .047 and .054 respectively indicated that almost all of the impact of medical education on average cost per case does in fact arise from its impact on length of stay."

2. Ibid., p. 86

David Salkever demonstrated that teaching hospitals spend more for treatment of comparable cases than non-teaching hospitals and that teaching hospitals have a longer average length of stay per case. I will attempt to determine how graduate medical education effects the production of hospital services. The increased
cost per case of teaching hospitals found by Salkever may have resulted from an inefficient utilization of hospital services in treating patients (my second level) or an inefficient production of hospital services (my first level) or both. If an insurer does not want to reimburse hospitals for education cost it should try to determine the effect of graduate medical education on the production of hospital services. It is very difficult for any insurer to refuse payment for an "overutilization" of hospital services since the hospital may always claim that the decision was medically warranted and helped produce "high quality" care. The fact that teaching hospitals are generally considered to provide higher quality care supports their argument. In any case, whether or not a particular service should have been ordered to treat a patient is a decision only physicians can make. It would be easier for an insurance company to reduce its payments to a hospital for "education" costs if it could be shown that the existence of a graduate medical education program caused an increase in the cost of providing certain routine hospital services.

Graduate medical education programs might cause an increase in the cost of producing a particular service because of an intern's or resident's inexperience or the hospital's efforts to help educate their interns and residents. An intern or resident may have a more difficult time collecting certain tissue samples for laboratory tests. Technicians in the laboratory or radiology department
might work more slowly in order to help the intern or resident learn about testing procedures.

I will estimate the effect of graduate medical education programs on the production of hospital services. I will estimate the effect first, by using a dummy variable representing the existence of intern and residency program and second, by using a variable representing the number of interns and residents. Since my original sample of Ohio hospitals included only non-teaching hospitals I will augment my sample with 16 teaching hospitals.

3. Data on the number of interns and residents for each teaching hospital were obtained from the Office of Research and Statistics of the Social Security Administration. Data on patient days, number of personnel, and number of facilities for the teaching hospitals were obtained from the Hospital Guide issue, August 1971.

I will estimate a function of the following form:

\[ Y = A (d_1 K_1^p + d_2 K_2^p + d_3 L^p) \cdot e^{\beta M} \]

where:
- \( K_1 \) is the number of beds
- \( K_2 \) is the number of facilities
- \( L \) is the number of personnel
- \( M \) is 0, 1, or 2 according to the presence of an intern and residency programs or
- \( M \) is the number of interns and residents and
- \( Y \) is the cost weighted service output measure.

I will estimate this equation in two ways: first, by the direct Taylor series approximation procedure previously described
with initial value $\beta = 0$ and second, by modifying the ACMS procedure. 4


The modified ACMS procedure makes use of the same assumptions as the ACMS method (constant returns to scale, efficient use of labor and an exogenous wage rate). To use the ACMS procedure we write our CES function as:

$$I = (d_1 K^{-\rho} + d_1 L^{-\rho})^{-\frac{1}{\rho}} \cdot C^{GM}$$

where all variables are as previously defined.

The marginal product of labor may be written:

$$\frac{\partial Y}{\partial L} = C^{BM} \left[ -\frac{1}{\rho} \right] \left( d_1 K^{-\rho} + d_1 L^{-\rho} \right)^{-\frac{1}{\rho} - 1} \cdot d_2 (-\rho) L^{-\rho - 1}$$

$$\frac{\partial Y}{\partial L} = \left( \frac{Y}{L} \right) \left( \frac{d_2}{C^{BM} \rho} \right)$$

Assuming efficient use of labor we get:

$$W = \left( \frac{Y}{L} \right)^{1+\rho} \left( \frac{d_2}{C^{BM} \rho} \right)$$

where $W$ is the wage rate and

$$\frac{Y}{L} = \left( \frac{1}{d_2} \right)^{1+\rho} W^{1+\rho} C^{BM} \frac{\rho}{1+\rho}$$

hence (7) \( \ln \left( \frac{Y}{L} \right) = \ln \left( \left( \frac{1}{d_2} \right)^{1+\rho} \right) + \frac{\rho}{1+\rho} \ln W + \frac{\rho}{1+\rho} \ln M \)
(7) may be written:

\[ \ln \frac{Y}{L} = Y_1 + Y_2 \ln w + Y_3 M \]

where \( \beta = \frac{Y_3}{1 - Y_2} \) and using a Taylor series expansion to linearize this we can derive an estimate for the variance of \( \beta \).

(9) \[ \text{Var}(\hat{\beta}) \approx \left( \frac{1}{1 - Y_2} \right)^2 \left( \beta^2 \text{Var}(\hat{Y}_1) + \text{Var}(\hat{Y}_2) + 2 \beta \text{Cov}(\hat{Y}_1, \hat{Y}_2) \right) \]

If there is no effect on the production of hospital services then \( \beta \) should turn out to be insignificant. If, on the other hand, interns and residents decrease the production of hospital services should be significantly negative. The results of the estimations are given in Table 6 and they indicate that \( \beta \) is less than zero for specifications and using both estimation procedures. The estimated elasticity of substitution in every case was within the range estimated for the previous specifications. These results are again biased against larger hospitals because of the omission of various specialized facilities in the output index. \( \beta \) in every case was significantly less than zero which implies that interns and residents do not significantly offset their direct costs to hospitals by increasing the production of services and may impose more costs in the hospitals by causing a less efficient production of services.
TABLE 6

ESTIMATED EFFECT OF GRADUATE MEDICAL EDUCATION PROGRAMS ON THE PRODUCTION OF HOSPITAL SERVICES

1. \( M = 0, 1, 2 \) depending upon the existence of an intern and residency program

Results

a. Modified ACMS procedure \( \ln \left( \frac{Y}{L} \right) = C_1 + C_2 \ln W + C_3 M \)

\[ C_1 = 1.21 \quad T-Stat = 4.71 \]

\[ C_2 = 0.782 \quad T-Stat = 5.86 \]

\[ C_3 = -0.008 \quad T-Stat = -3.87 \implies \beta = -0.037 \quad S.O. \beta = .013 \]

b. Non-linear Taylor series procedure

\[ \hat{\beta} = -0.014 \quad S.O. \hat{\beta} = .005 \]

\[ \hat{\sigma} = 0.023 \]

2. \( M \) = number of interns and residents

Results

a. Modified ACMS procedure \( \ln \left( \frac{Y}{L} \right) = D_1 + D_2 \ln W + D_3 M \)

\[ D_1 = 1.416 \quad T-Stat = 7.81 \]

\[ D_2 = 0.815 \quad T-Stat = 6.23 \]

\[ D_3 = -0.013 \quad T-Stat = -10.08 \implies \hat{\beta} = -0.070 \quad S.O. \hat{\beta} = .016 \]

b. Non-linear Taylor series procedure

\[ \hat{\beta} = -0.053 \quad S.O. \hat{\beta} = .018 \]

\[ \hat{\sigma} = 0.043 \]
In my study I have estimated economies of scale in hospital production and the effect of intern and residency programs on hospital production through the estimation of a hospital production function. In order to estimate a hospital production function an index of hospital output must be selected, an appropriate functional form specified and valid estimation techniques employed. These three tasks encompass most of my work through the first five chapters.

The selection of an appropriate measure of output is a problem encountered throughout medical economics. A hospital administrator may consider patient days as a good measure of output. Physicians might feel some weighted sum of services provided by the hospital to their patients is an appropriate measure of output. Society as a whole must view hospital services as an input into the production of health. The difficulty in the quantification of the product called health is at the root of the problem. To an economist who desires to estimate economies of scale in hospitals the appropriate measure of output may be the one which defines the least heterogeneous production function across hospitals. If in using a particular measure of output very different production functions occur for a cross section of hospitals then this measure of output will not be useful in estimating economies of scale. This occurs because a critical assumption made in cross section studies is that the functional form and the
corresponding micro coefficients of each hospital production function are identical across hospitals. I have made several assumptions about the separability of technologies and inputs in the production of the many hospital services in order to employ aggregation analysis to determine which measure of output is most likely to define a similar production function across hospitals. A weighted sum of services is found to be most likely to define a similar production relation across hospitals as opposed to a patient day or case treated measure. By most likely I mean that the assumptions made to insure that identical production functions across hospitals are defined for a weighted sum of services output index are necessary but not sufficient to insure identical production functions are defined across hospitals if we use a patient day or cases treated measure.

The selection of the appropriate form for the production relation was accomplished through the use of the derived equations of the transcendental logarithmic production function. \(^1\) By approximating

\[ \text{an arbitrary function in logarithms by its second order Taylor series expansion about zero the restrictions imposed by any functional form may be tested on the set of derived equations.} \] \(^2\) I have modified

\[ \text{Berndt and Christensen's method to allow for non-profit maximizing} \]
behavior (profit maximization is a necessary condition for the use of Berndt and Christensen's method). In particular, I have allowed for a combination profit capital maximization hypothesis. Using both the profit capital maximization hypothesis and the profit maximization hypothesis the CES function is not rejected. Through use of this modified version of Berndt and Christensen's method we can state that the CES function is an appropriate functional form for my hospital production function.

Now that a CES form and a service weighted index have been selected we must decide on how to estimate our hospital production function. Without data on input and output prices or a generally accepted theory of hospital behavior it is impossible to use simultaneous equation estimation techniques. Since each single equation technique of estimating a CES production function makes different behavioral assumptions which we can neither prove nor disprove I have employed three different methods of estimation: the ACHS method, Feldstein's two step procedure and the Taylor series (or linearization) method. Both the Feldstein two step and the Taylor series procedure assume that capital and labor are fixed. Efficient use of labor, an exogenous wage rate and constant returns to scale are all assumed by the ACHS procedure. Under all specifications and using each of the above estimation procedures we find that hospitals do not possess significant economies of scale.
Using the CES formulation with a service weighted output measure and an attached variable representing the number of interns and residents I have estimated the effect of interns and residents on the production of hospital services. Evidence is presented in chapter 5 which supports the hypothesis that interns and residents do not substantially increase the production of hospital services. In fact, evidence is included which suggests that teaching hospitals are less productive; that is, if we measure labor as the number of employees (excluding interns and residents) then teaching hospitals produce less output for a given set of inputs.

It is common in the literature to come across comments on the heterogeneity of patient days and cases treated at different hospitals. Certainly, a patient day in one hospital is not the same product as a patient day in a larger and more specialized hospital. Laboratory tests and x-rays do not define homogeneous units of output. The complexity of each may vary within and across hospitals. I have weighted all laboratory tests by a fixed number and x-rays by a fixed number. Further studies should try and disaggregate laboratory tests and x-rays according to some measure of cost or complexity. Future attempts at estimating returns to scale should gather data on more types of services (e.g. occupational, physical and inhalation therapy, intensive care units). My study defines a service weighted output measure composed of routine days of patient care, x-rays, laboratory tests, deliveries and operations.
It should be understood that no study will ever be able to measure hospital output to the satisfaction of all researchers. The best that can be done is to define criteria for selection of an output index and compare different indices according to these criteria.
APPENDIX (A)

I have regressed each service per patient day on a constant, number of beds, occupancy rate, and the number of specialized facilities. The results suggest that hospitals produce services per patient day in very different proportions and that occupancy rates, size (measured by number of beds) and complexity (measured by the number of services) are unable to explain this variation.
<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Operations Per Patient Day</th>
<th>Laboratory Tests Per Patient Day</th>
<th>Deliveries Per Patient Day</th>
<th>X-rays Per Patient Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>.529</td>
<td>-.362</td>
<td>.040</td>
<td>.347</td>
</tr>
<tr>
<td></td>
<td>(1.159)b</td>
<td>(-.265)</td>
<td>(2.456)</td>
<td>(.307)</td>
</tr>
<tr>
<td>Number of Beds</td>
<td>.005</td>
<td>-.001</td>
<td>-.000009</td>
<td>.0001</td>
</tr>
<tr>
<td></td>
<td>(1.122)</td>
<td>(-.937)</td>
<td>(-.630)</td>
<td>(.147)</td>
</tr>
<tr>
<td>Occupancy Rate</td>
<td>-.005</td>
<td>-.021</td>
<td>-.0002</td>
<td>.013</td>
</tr>
<tr>
<td></td>
<td>(-.083)</td>
<td>(1.142)</td>
<td>(-1.077)</td>
<td>(.857)</td>
</tr>
<tr>
<td>Number of Specialized Facilities</td>
<td>-.161</td>
<td>.130</td>
<td>-.0003</td>
<td>-.019</td>
</tr>
<tr>
<td></td>
<td>(-1.162)</td>
<td>(2.926)</td>
<td>(.068)</td>
<td>(-.533)</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>.031</td>
<td>.278</td>
<td>.012</td>
<td>.053</td>
</tr>
</tbody>
</table>

a. Number of facilities as measured by the American Hospital Association

b. 1.159 is the \( T \) - statistic of .529
This appendix describes how estimates for the standard deviation of $\sigma$ (i.e., elasticity of substitution) were derived. Write $\sigma$ as follows:

$$\sigma = \frac{1}{1+\rho}$$

Expanding about $\bar{\rho}$ using a first order Taylor series expansion we get:

$$\sigma = \frac{1}{1+\bar{\rho}} + \frac{1}{(1+\bar{\rho})^2} (\rho - \bar{\rho}) + \text{higher order terms} \quad \text{let} \quad \rho = \bar{\rho} \quad \text{then}$$

$$\text{Var} \sigma = \frac{1}{(1+\bar{\rho})^4} \text{Var} \rho = \text{Var} \rho$$

Hence, in each instance where the CES function was estimated directly we may use the standard deviation of $\rho$ as an estimate for the standard deviation of $\sigma$. 
BIBLIOGRAPHY


27. Dunlop, David. "The Development of an Output Concept for Analysis of Curative Health Services." Social Science and Medicine, 6 (1972), 373-385.


34. "Hospital Cost Variations and Case-Mix Differences" *Medical Care*, 3(April-June 1965), 95-103.


43. Kmenta, J. "On Estimation of the CRS Production Function"

44. Lave, Judith and Lester Lave. "The Extent of Role Differentiation

45. Lave, Judith and Lester Lave. "Hospital Cost Functions"

46. Lave, Judith R., Lester Lave and Lester Silverman. "A
    Proposal for Incentive Reimbursement for Hospitals" Forth-
    coming in Medical Care.

47. Mann, Judith and Donald Yett. "An Analysis of Hospital Costs:

48. Newhouse, Joseph "Toward a Theory of Nonprofit Institutions:
    An Economic Model of a Hospital" American Economic Review,
    60 (March 1970), 64-74.

49. Pauly, Mark V. "Efficiency, Incentives and Reimbursement

50. Rafferty, John. "Hospital Output Indices" Economic and

51. Rafferty, John "Patterns of Hospital Use: Analysis of Short
    Run Variations" Journal of Political Economy 79:1 (Jan.-

52. Reder, Melvin. "Some Problems in the Economics of Hospitals"

53. Salkever, David "A Microeconometric Study of Hospital Cost
    144-165.

54. Salkever, David. "Studies in the Economics of Hospital Costs"
    unpublished dissertation Harvard University, June 1970.

55. Sigmund, Robert. "Hospital Capital Funds: Changing Needs

56. Somers, Anne and Herman Somers. Medicare and the Hospitals:

57. Theil, Henri. Linear Aggregation of Economic Relations.


