Optical Beam Steering using a MEMS-driven White Cell

Masters’ Thesis

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Abstract

The concept of beam steering is a common problem in many engineering applications. Electronic beam steering methods are the focus of several research endeavors due to the lack of moving parts and the quick response time. Optical beam steering furthers the capabilities of steering by reducing electromagnetic interference and lowering power demands, many previously developed solutions have a strong dependence on wavelength when steering. The goal of this research was to develop an electronic and optical beam steering platform which would be able to steer an optical beam accurately over small angles, and without dependence on wavelength. A MEMS-driven White Cell was the foundation for the development of a solution. It is shown this platform is capable of steering accurately over small angles, and through demagnification can steer over angles of several degrees. Diffraction and a lack of phase preservation were problems inherent to the design, and while diffraction could be limited through the use of flatter pixels on the MEMS structure, phase preservation would require a different steering platform.
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1 Introduction

The goal of this research is to develop, design, and demonstrate a device for the steering of an optical beam using Optical True Time Delay (OTTD) with a MEMS-driven White cell. The concept follows from the steering of phased array antennas where each of several antenna elements emits a variation of a wide beam pattern, and by adding these emissions the resulting front is a beam steered in a specific and desired direction. In this design, we apply the principle of phased array antennas to optical emitters instead of radio frequency emitters, to steer an optical beam instead of a radio frequency beam.

1.1 Mechanical Steering Methods

There are many ways to achieve this type of steering an optical beam; all of these types can fall into one of two categories: mechanical or electronic. Mechanical steering is the most straightforward approach. Typical mechanical steering solutions involve tilting or rotating a mirror which directs an optical beam to a desired location. One such solution is shown in Figure 1.1 where a laser beam comes from a stationary point and interacts with a mirror, indicated by Y, which rotates to redirect the beam to a desired point [10].
Mechanical methods, however, tend to be slow, imprecise, and can require significant maintenance. Electronic beam steering hopes to provide a reliable solution to all these problems.

1.2 **Steering using Phased Array Antenna Principles**

The beam steering application in this thesis centers on the principles of Phased Array Antennas and uses an electronic method to steer rather than a mechanical method. Electronic beam steering is further broken down into two categories. The first approach is phase-shifting. Beam steering solutions that use phase-shifting focus around a central concept: using a series of emitters that emit an identical electromagnetic wave, each with some added phase, Figure 1.2. This approach takes advantage of the fact that the electromagnetic fields emitted by each emitter element in an array will sum coherently to create a flat wave front traveling in a specific direction.
Phased Array Antennas have some desired characteristics such as high beam agility, a low cross-sectional area to scan a set space, and they do not contain any moving parts on their own, although often the array is rotated mechanically.

The far-field behavior determines the overall behavior of the phased array. The far-field electric field, $E_f$, is determined by two factors the element factor, $EF$, and the array factor, $AF$.

$$E_f = AF \cdot EF$$  \hspace{1cm} (1.1)

Here the element factor is the far-field pattern created by a single emitting element. The array element is determined by the pattern of emitting elements in the overall PAA.

![Figure 1.2 - Phased Array Antenna [11]](image)

To demonstrate this concept we will consider the example in Figure 1.2 [11]. In this example the PAA will be a uniform, linear array of ideal, lossless antennas that emit uniformly in all directions. In this example each antenna emits a beam with a multiple of a constant phase-delay, $\Delta\Phi$, and of unit amplitude, so in the figure $A_n = 1$
for all $n$. The distance between the antennas, $d$, is also fixed. The array factor is given by [12]

$$AF = \sum_{i=0}^{N-1} e^{i(jkd \cos \theta + \theta_s)}$$

$$AF = e^{\left( \frac{N-1}{2} \right)(jkd \cos \theta + \theta_s)} \left[ \frac{\sin \left( \frac{N}{2} (kd \cos \theta + \theta_s) \right)}{\sin \left( \frac{kd \cos \theta + \theta_s}{2} \right)} \right]$$

$$|AF| = \left[ \frac{\sin \left( \frac{N\psi}{2} \right)}{\sin \left( \frac{\psi}{2} \right)} \right]$$

In Equation (1.2) $\theta_s$ is the steering angle, $\theta$ is the angle of observation, $k$ is the angular wavenumber, $N$ is the number of emitting elements, and $d$ is the distance between each antenna. The phase shift, $\psi$, is defined to be $(kd \cos \theta + \theta_s)$.

As shown in Figure 1.3, the array factor produces one sharp peak with smaller side-lobes; it is a periodic function in that every integer multiple of $\pi$, another “main lobe,” also called a grating lobe, occurs. Larger $N$ increases the distance between the main lobe and the grating lobes, as well as decreases the angular thickness of the peak.

An example array factor and element factor can be seen in Figure 1.3a. Here AF is normalized by dividing by $N$, chosen to be 20. The element is a rectangular aperture, for which the element factor, the far-field energy pattern of the beam emitted by a single antenna, is a simple sinc function.
Figure 1.3 - (a) Example Element and Array Factors, (b) The resulting Far-Field Intensity pattern

Now the overall pattern produced is the element factor multiplied by the array factor. If \( N \) is large enough such that the distance between the peaks of the array factor is wider than the element factor, the result is a single peak, as shown in Figure 1.3b.

With the far-field pattern developed, the next idea is how steering plays its part. From equation 1.2 the phase shift needed on the \( n \)th element to steer the overall beam by \( \theta_s \) is:

\[
\Psi_n = -nk \sin \theta_s \quad (1.3)
\]

Figure 1.4a shows the normalized array factor as the steering angle varies. The solid black line is the unsteered beam; the dashed red line shows the array beam steered by 45°. A key development is that as \( \theta_s \) increases, the locations of the peaks in AF shift. This limits how far a given PAA can steer, because if the steering angle is too great it is possible to get two peaks visible in the far-field, Figure 1.4b, or the central peak can move entirely out of the element factor envelope.
Phase shifting produces a steered beam, but causes a dependence on wavelength called “beam squint.” The wavenumber

\[ k = \frac{2 \pi}{\lambda} \]  

(1.4)

is dependent on the wavelength, \( \lambda \), of the input beam, so comparison to equation (1.3) shows that to steer a beam with a range of frequencies by the same angle requires different phase shifts for each wavelength. The result is that individual wavelengths are steered by different angles. To see this, we substitute equation (1.4) into equation (1.3) and solve for wavelength, \( \lambda \).

\[ \Psi_n = -n \frac{2 \pi}{\lambda} d \sin \theta_S \]

\[ \lambda = -n \frac{2 \pi}{\Psi_n} d \sin \theta_S \]  

(1.5)

Now we have a direct relationship between wavelength, \( \lambda \), and steering angle, \( \theta_S \).

Taking the derivative:
\[ \Delta \lambda = -n \frac{2\pi}{\Psi_n} d \cos \theta_S (\Delta \theta_S) \quad (1.6) \]

We now have a functional relationship between change in steering angle and change in wavelength. If we substitute in the functional form of our phase shift from equation (1.5), we obtain a simple function relating change in wavelength to change in steering angle.

\[ \Delta \theta_S = \tan \theta_S \left( \frac{\lambda}{\Delta \lambda} \right) \quad (1.7) \]

This gives us a mathematical characterization of “beam squint.” For a given phase shift, \( \Psi_n \), a specific wavelength, \( \lambda \), sees a specific steering angle, \( \theta_S \), and as the wavelength diverges, \( \Delta \lambda \), the steering angle does as well, \( \Delta \theta_S \), resulting in the separation of the output by wavelength. This can be detrimental for systems that rely on multiple wavelengths, as the phase shift has to vary accordingly.

True Time Delay, as described in the next section, proposes to fix this problem of “beam squint” by creating time-based delays instead of phase delays.

### 1.3 Time Delay and True Time Delay

True time delay techniques are ones that steer a given electromagnetic signal by delaying the emitting elements in time. As we saw in the last section by strictly shifting phase we develop dependence between steering angle and wavelength. If we take this same phase shift for one wavelength and correlate it to a time delay, this time delay will provide the correct steering angle for all wavelengths [8].

One such approach to time delay adjusts the properties of the dielectric materials that an electromagnetic wave is interacting with [7, 8]: the piezoelectric transducer
(PET) is shown in Figure 1.5. Here different bias voltages applied to the PET will move it up or down, changing the air gap between the microstrip and the perturber, changing the electric permittivity of the that space. Since propagation speed of an electromagnetic wave is based on this electric permittivity, the PET can induce a time delay. In this case the electrical signal is then carried onto an antenna with a delay.

![Figure 1.5 - Piezoelectric Transducer](image)

This solution and others of its type all share similar problems, namely that cost and loss both increase rapidly as the maximum frequency of a signal or resolution increases, they often require high amounts of power, and their switching speed is slow [8], and while it does not add delay through phase, the delay does depend on the wavelength of the RF signal.

This is where true time delay solution come into play, many involve switches on the transmission line that force the signal to take a longer path or pass through more electronics, such as inductors and capacitors or transmission lines and cables.

Another option to this problem is to use optical true time delay. In optical true time delay the electrical signal is modulated onto a laser source, where the carrier and signal can be delayed using any one of a number of optical true time delay
techniques, and after that the optical signal can be passed through a photodetector and the original electrical signal can be recovered. This provides a multitude of advantages primarily resistance to electromagnetic interference, potential for much lower power demands, quicker switching speeds, and lower losses. These advantages apply to optical phase-shifting as well. Both phase-shifting and true time delay in the optical domain has undergone extensive research.

1.4 Other Research

As discussed in the previous section there are advantages to using optical true time delay. We would like to establish a basis for this research by looking at previous work done in the optical domain, both in phase delay and true time delay.

1.4.1 Electrowetting microprisms

Electrowetting microprisms (EMPs) are an optical approach to electronic steering of optical beams using the optical principle of a prism. As an electromagnetic wave passes from air to a medium of a different refractive index and then back to air the overall angle of the beam is manipulated by Snell’s Law [2].

EMPs take advantage of this principle by creating a modular prism using an aqueous solution responding to the presence of an electric field, Figure 1.6 [2]. The aqueous solution is placed in a channel with two walls of fluoropolymer-coated silicon separated by glass of a desired width. A voltage is introduced to the silicon; this forces charges of the opposite sign to gather on one end of the aqueous solution forming an electric field across the fluoropolymer material. By manipulating the
voltage levels at each side, the aqueous solution can be made to form a prism, Figure 1.6.

\[ \delta = \alpha - \varphi + \sin^{-1}\left[(n^2 - \sin^2 \alpha)^{1/2} \sin \varphi - \cos \varphi \sin \alpha\right] \]  

(1.6)

The index of refraction of the material, \( n \), is assumed to be >1, and \( \alpha \) is the angle of incidence. The apex angle, \( \varphi \), of the prism created by the aqueous solution is the controlled element.

This approach is not without its detractions. First is the voltage requirements, which to steer over the proposed range of 14° requires >70 volts. Second is the switching speed, which is on the order of milliseconds, although it should be noted that the switching speed is proportional to the square root of the density-volume product, so smaller prisms switch faster. Another problem is that a liquid creates the deflection, so this setup is not resistant to disturbances caused by motion, and would most certainly have to be vacuum-sealed to prevent interference with the liquid by outside pollutants such as moisture, air, or dirt [2].

1.4.2 Liquid-crystal blazed-grating beam deflector

Liquid crystal methods are popular answers to beam steering due to their low driving voltage, low cost, and maturity of liquid crystal technology. One of the areas
of research is liquid-crystal blazed-grating beam deflection. Typical liquid crystal applications such as displays take advantage of the liquid crystal material’s ability to change the polarization of incident light as the orientation of the molecules change in the presence of an electric field. These solutions work differently, instead using the liquid crystal to manipulate the refractive index [4].

![Diagram](image)

**Figure 1.7 - Liquid-crystal blazed-grating beam deflector [4]**

In Figure 1.7, indium tin oxide (ITO) coated glass is used to create a cavity for two types of material in the device substrate and to electrically drive the liquid crystal. Liquid crystal (LC) material is used as a beam deflecting medium. A polymethyl methacrylate (PMMA) forms a blazed grating of a certain period is grown on the ITO glass. The refractive index of the liquid crystal is dependent on an input electric field. Here when no electric field is present there is a strong diffraction produced by the difference between the refractive indices of the LC and the PMMA resulting in deflection, referred to as the OFF state. When an electric field is present, the ON state, the refractive index does not vary and the LC-PMMA can be considered an optically flat plate [4].
This process is not without its problems. First is since the design functions in an “ON” and “OFF” state, there are only two angles available to each deflector. Cascading $N$ deflectors with varying PMMA periods can result in $2^N$ possible angles [4]; this also increases losses. Secondly, the setup is dependent on the input angle and polarization, working for only one input angle and one polarization, so these elements need to be controlled to steer a beam.

### 1.4.3 Liquid-crystal phased-array beam steering

![Figure 1.8 - One-dimensional Liquid-crystal reflection-mode beam steerer](image)

Liquid-crystal blazed-grating beam deflectors represent a non-traditional approach to using liquid crystals to steer beams. Figure 1.8 shows a more traditional approach, which is to create a phase profile within the liquid crystal to create phase delays identical, in concept, to phase-shifting [3]. Here a substrate with a set number of electrodes for each element of the phase profile and another mirrored electrically-
grounded plane are used to create a cavity of constant thickness for the liquid crystal. The electrodes are used to apply a periodic, ramping phase profile in the liquid crystal. This creates a variable phase delay array because as the electric field changes, the orientation of the liquid crystal molecules changes, altering the refractive index of the liquid crystal [3].

When the incident light interacts with the liquid crystal it receives a phase delay, this phase delay is equivalent, in concept, to phase-shifting in an antenna array (PAA) and the beam is steered according to

\[ \sin \theta = \frac{\lambda}{\Lambda} \]  \hspace{1cm} (1.6)

where \( \lambda \) is the wavelength of the beam, \( \theta \) is in steering angle, and \( \Lambda \) is the repeat distance, or period, of the 0-to-2\( \pi \) phase ramp [3].

This method of using phase delays to steer an optical beam is very common, and there have been multiple iterations of this method researched [5]. They all, however, share many common downfalls. First, as equation 1.6 shows and as with all deflectors using phase delay, the wavelength of the incident light determines the steering angle, so beams consisting of multiple wavelengths will be spread out at different angles. Second, as with anything involving liquid crystal, the polarization of the incident light is important to the behavior of the liquid crystal, so it does need to be controlled [3, 5].

It is possible to get the equivalent of true time delay out of liquid crystal phase shifters by making the liquid crystal cavity between substrate and the reflective surface much thicker; this would allow for more than one wavelength of delay as the light passes through the liquid crystal. These approaches are unreasonable due to
high losses, high power consumption, and high switching speeds associated with the thicker liquid crystal, and the fabrication difficulties with making the phase shifter with such a thick cavity [7].

1.4.4 True time delay White Cells

There are several true time delay approaches to beam steering using the White cell. One of these approaches uses a Microelectromechanical system (MEMS) driven White Cell to bounce beams back and forth between three mirrors. One of the mirrors is a MEMS micromirror array, which can redirect the incident beam down different paths. Each path is of a different length, resulting in delays introduced by the physical distance the beam has to travel between each bounce. The number of bounces and the paths that each bounce takes determines the overall delay experienced, and the resulting steering angle.

There are two different types of White cells used so far for producing optical true time delay for RF beams, with the division being the delay type. There are polynomial cells, such as the linear or quadratic cell, where the delay is proportional to the number of bounces raised to a power [6]. There are exponential cells, such as the binary cell, where the delay is proportional to some number raised to the power of the number of bounces [13].

There are benefits to using the White cell for RF beam steering. The fact that it is a free-space design using TTD allows for multiple beams to circulate simultaneously in the White cell, each of which can be controlled independently via the MEMS. Some solutions can offer very fine angular resolution, with delays added on the order
of picoseconds, which is necessary for applications where steering accuracy is paramount.

The White cell methods to date have been designed for radio frequencies, with delays on the order of picoseconds and nanoseconds, which are appropriate for radio waves. In the case of optical beams, where the wavelength, $\lambda$, is a thousand times smaller, the delays must be a thousand times shorter.

In this thesis we sought to adapt the White cell to produce delays appropriate for optical beams. Our solution will seek to steer an input optical beam in a cell without multiple cell arms, instead using the ability of a MEMS pixel to piston, thereby creating very small path changes on each bounce in a single cell. Because this is TTD, we will eliminate beam squint, while taking advantage of the simplicity and modular nature of this approach.

### 1.5 Organization of this Thesis

The organization of this thesis is as follows. Chapter 2 will introduce the basis of the proposed solution to beam steering, the White cell. It will explain the physics behind the original White cell as well as how replacing one mirror with a Microelectromechanical System (MEMS)/field-lens combination creates a useable cell.

Chapter 3 will introduce how the cell can steer an input beam. Chapter 4 will give an overview of the design process of the apparatus to meet desired specifications. Chapter 5 will contain the experimental results of the MEMS driven White Cell in beam steering.
Chapter 6 will conclude the thesis as well as provide potential avenues whereby this method can be further researched. Appendices will contain extra information necessary to the research including the necessary ray matrices as well as MATLAB code used in the design.
2 The White Cell

Our proposed solution to steering optical beams using true time delay is based on the White Cell, designed by John U. White in 1942 [14]. The original purpose of the White Cell was for spectroscopy, where a specific type of light could be passed through a material a specific and easily adjustable number of times. Since its introduction, the White cell has seen uses elsewhere in photonics, including previous true time delay, TTD, solutions. Its appeal comes from its simplicity in design, allowance of parallel operation, and adjustable nature, even post construction.

2.1 The optical phenomenon of the White Cell

![Figure 2.1 - The White Cell](image)
The White Cell, shown in Figure 2.1, consists of three spherical mirrors with the same radii of curvature under specific spatial conditions and two additional mirrors. The field mirror, mirror F, and the objective mirrors, mirror A and B, all have a curvature of $R$ and are set that same distance, $R$, apart. The centers of curvature of the objective mirrors are set some distance, $d$, apart and the mirrors themselves are placed such that the center of curvature of the field mirror is between them. The two additional mirrors are the input and output turning mirrors, ITM and OTM, which are responsible for providing an input light beam into the cell and bouncing the output light beam out of the cell.

Figure 2.2 shows the operation of the White Cell. The input beam is brought into the cell by the input turning mirror. The beam diverges as it travels to mirror B. The plane of the ITM is imaged by the first objective mirror, mirror B in this example, onto the field mirror, with the image landing a set distance, $r_B$ away from the center of curvature of the first objective mirror, $CC_B$. This represents one bounce.
Figure 2.2 - The bounce process in a White Cell, (a) is the development of the first bounce as the ITM is imaged to a point on the field mirror, (b) the field mirror reflects the first image and it expands outward to the next mirror, (c) development of the second bounce

As Figure 2.2 goes on to show, the first bounce is now placed on the field mirror and is some distance, \( r_A \), away from the second objective mirror’s center of curvature, \( CC_A \). This spot acts as a new object for mirror A. It diverges out toward
the mirror A and is reflected and re-imaged down to a spot on the opposite side of the field mirror a distance \( r_A \) away from \( CC_A \). This represents another bounce, and from this point mirror B sees a new object some distance away from its center of curvature and continues. The process of bounces continues until mirror A images the beam onto the output turning mirror, OTM. These spots on the field mirror are referred to as the “spot pattern.”

It is important to note the field mirror also treats the light from each of the objective mirrors as an object and re-images it on the opposite side of its center of curvature, \( CC_F \). This is where the other objective mirror needs to be placed. This symmetry allows bouncing to continue.

This logic creates the two imaging conditions necessary for the White Cell to operate properly. The first is that the field mirror always images back onto itself through either objective mirror. The second imaging condition is that either objective mirror is imaged onto the other the field mirror.

### 2.2 The Spot Pattern

Figure 2.3a shows the spot pattern as seen on the surface of the field mirror. The centers of curvature for the objective mirrors, A and B, are indicated on the mirror face as \( CC_A \) and \( CC_B \). The simplest case considered is a “one-dimensional” case where we consider a single spot. Here a beam enters the White Cell and creates a single spot on the ITM, bounces a set number of times, and exits as a single spot on the OTM. As the beam bounces in the White cell it alternates across the centers of curvature, coming closer together on each bounce, until it reaches half of the total number of bounces and then begins to move further apart until eventually the last
spot is imaged on the OTM and exits the White Cell. This can be seen more clearly in Figure 2.3b, which shows the case of a beam introduced into the cell off the center axis.

**Figure 2.3** - Example spot patterns, (a) simple one-dimensional case where the input and output turning mirrors line up, (b) introducing an offset to the bounce locations

The numbers on the side of the spots indicate which bounce the spot represents, and the overall number of spots is determined by the distance between the centers of curvature, \(d\), and the dimensions of the field mirror. The larger \(d\) is, the fewer spots there will be, and the larger the field mirror is, the more spots there will be.

**Figure 2.4** - Example of a multi-dimensional spot pattern
This simple case can be extended to “multi-dimensional” cases to include multiple input beams, Figure 2.4, each with a different offset from the center line. The figure shows a one dimensional array of input beams, with two specific inputs highlighted. Here each beam forms a unique spot pattern based on its offset, that does not interfere with other beams’ spot patterns, so multiple beams can circulate within the White Cell simultaneously, giving the White Cell the capability of being highly parallel in its applications.

2.3 Extending the White Cell to Beam Steering

The White Cell, however, can not steer beams on its own. To create a system that is capable of beam steering, the field mirror will be replaced with a micro-electro-mechanical system, MEMS, device and a field lens with a focal length of \( f \). The MEMS will have reflective pixels, so it will be capable of reproducing the field mirror’s reflective property. The field lens will be designed such that it satisfies the field mirror’s imaging conditions by making \( f \) equal to one half the radius of curvature, \( R \), of the field mirror. This is a modification of a common optical property where by a flat mirror and a lens can be placed such that they are optically equivalent to a spherical mirror. Once this modification is properly made, the MEMS driven White Cell will operate identically to the White Cell in the previous section.

The system will be able to steer a beam with the assistance of the MEMS. In this case, the MEMS pixels are capable of a “piston” action moving up and down with respect to the surface of the MEMS. By extending or retracting these pixels a specific shape can be created on the MEMS surface delaying each pixel-sized portion of the beam by a specified amount each bounce. The multiple bounces in the White cell
allow the beam to be influenced by the MEMS on each bounce, amplifying the stroke by the number of bounces and thus increasing the steering angle. This process will be further examined in the next chapter as the design methodology is solidified.
3 Steering with the MEMS-driven White Cell

The previous chapter introduced the concept of the White Cell and the necessary optical conditions to be satisfied, as well as how exactly we can control the device using a Microelectromechanical System, MEMS, to displace the beam. This chapter looks to solidify the concepts behind beam steering in our generalized design.

3.1 Beam Steering

As described in the previous chapter our White Cell will have the field mirror replaced with a MEMS and a field lens. The MEMS has small square mirrored pixels approximately 150 microns on each side, which can change their height by approximately half a micron relative to the surface of the MEMS to shape the wave front of the beam in a controllable manner. This displacement is a true time delay element to steer the input beam in an identical manner to Phased Array Antennas by treating each pixel as an antenna or emitter and the height of that pixel as its delay.
If an \( N \times N \) array of pixels is considered on the face of the MEMS as being a fundamental patch or pattern of pixels, we can treat this section as the new face of the beam after a bounce, as shown in Figure 3.1. As the figure shows, an input beam comes in and reflects from each pixel, each pixel being at a different height reflects the wavefront at a different point in time, each individual reflection interferes constructively to approximate a flat wave front traveling at a new angle, \( \theta \). This can be simplified by treating the pixel faces themselves as a flat front. To define \( \theta \) we can construct a triangle with the array of pixels.
Figure 3.2 - Physical view of one row of an example MEMS

Figure 3.2 shows one row, $1 \times N$, of the array, $N \times N$, considered. Along with the physical number of pixels two other elements are of importance to defining this angle. These are the pixel pitch, $a$, and fill factor, $b$, both related back to the pixel size, $p$.

$$b = \frac{P}{a}$$

(3.1)

We can now treat a row of length $Na$, as the first side of the triangle in Figure 3.1. We now need to look at the other side of the triangle. In the case of Figure 3.2, we are only looking at one bounce, in which case the spatial difference between wavefronts is $2S$, where $S$ is the maximum stroke of the piston, and the factor of 2 arises because the beam makes a round trip. When we have $m$ bounces we can treat the height of the triangle as the sum of the effects of all bounces, in which case the overall displacement can be defined in equation 3.1.

$$D = 2mS$$

(3.2)

In this case $D$ represents the spatial difference between the incident wavefront and the output wavefront.
The overall steering angle, $\theta$, can be defined using equation 3.3.

$$\theta = \tan^{-1}\left( \frac{D}{Na} \right) \quad (3.3)$$

For the purposes of demonstration it is also necessary to define the width of the beam as it expands out in space, equation 3.4 [1], where $\lambda$ is the wavelength of light. The far-field diffraction angle is approximated by [1]

$$\delta\theta = \frac{\lambda}{Na} \quad (3.4)$$

To effectively demonstrate beam steering it is necessary that our steering angle, $\theta$, be larger than our beam width, $\delta\theta$. This will also play a part in designing the output optics. We will choose $\lambda = 633$ nm for the purposes of this thesis.

Before continuing it is important to note that $\theta$ defines the total angle by which the MEMS will steer the beam after propagating through the White Cell, but the beam will steer by a certain fraction of that angle on each bounce. The optics must be large enough to avoid clipping the beam, but to steer by tens of degrees in each direction; the optics will quickly become unreasonably large. However, for a fixed $D$, the desired angles can be obtained with a very small $a$, on the order of $1 \, \mu m$. The MEMS pixels are limited by processing to $a$ on the order of $100 \, \mu m$. To adjust for this, the beam will be de-magnified on output of the cell. This will keep the objective mirrors a reasonable size, and effectively reduce $a$ at the output to allow the proposed cell to steer over large angles.
3.2 The AO1K MEMS

The MEMS that will be used in this design is the AO1K MEMS provided by Sandia National Laboratories. As shown in Figure 3.3, it is capable of retracting pixels of a micromirror array.

![Figure 3.3 - A quadrant of the AO1K MEMS with an example ramp pattern](image)

There were two types of MEMS provided for this work; the first features a metalized mircomirror array and the second has an unmetalized micromirror array. The metalized MEMS has a much higher reflectivity, but it introduces a radius of curvature of 24 mm to the mirrored surface on each pixel. This will result in less loss of power as the optical beam reflects off the MEMS, but will result in higher diffraction losses and lower beam quality. This MEMS will be used to verify steering and losses. The unmetalized MEMS has very low reflectivity, but the pixels are flat. This will preserve beam quality, but will result in higher losses when the optical beam reflects off the MEMS. This MEMS will be used to verify demagnification.

The curvature of the pixels of the MEMS will introduce diffraction losses and will manipulate the structure of the output as seen in the next two chapters. Figure 3.4 shows how this curvature affects the far-field intensity profile of a single pixel. The
beam quality will improve with more pixels. We are provided pixels with a curvature of 24 mm.

Figure 3.4 - The far-field intensity profile as the radius of curvature of the MEMS pixels changes

This degradation in beam quality is important both inside the White Cell and while taking measurements of steering at the output. Inside the White Cell the lenses and mirrors must be larger to capture the same amount of power. As Figure 3.5 shows, a flat mirror (R=0m in the figure) needs to be only 6 degrees in diameter to keep losses below 1%, it must be several degrees larger to keep the same losses with curved MEMS pixels.
Figure 3.5 - Power kept by objective mirrors as radius of curvature of MEMS pixels changes

In the White Cell the beam encounters the curved pixels of the MEMS on each bounce. Figure 3.6 shows the effect multiple bounces has on the output pattern observed. Here we can see more bounces results in multiple peaks spread out over a larger area. This degradation will play a role in the remainder of this thesis and is the reason why a second MEMS was provided to verify demagnification, so as much of the beam quality can be preserved when the beam is magnified.

Figure 3.6 - Effect of multiple bounces on output pattern

\[ \lambda = 633, \ R = 24 \text{ mm, one bounce} \]

\[ \lambda = 633, \ R = 24 \text{ mm, four bounces} \]
3.3 Developing a White cell

Before developing our design further, it is necessary to figure out exactly how the MEMS should be positioned inside the White cell. The other question that needs to be answered is what the array size should be, how many bounces, and what method of getting an optical beam in and out of the White cell will be used. There are several trade-offs involved; a larger $N$ will steer by a smaller angle, but will have a smaller beam width as it diffracts. More bounces, $m$, will steer by a larger angle, but will introduce more aberrations in the beam itself.

It is important to note that certain elements of the design are considered fixed. We will assume a particular MEMS device, the Sandia National Laboratories piston-type micromirror array MEMS AO1K. This fixes the pixel size, pitch and fill factor. We are allowed to manipulate $N$ and $m$, which will change the value of $D$, but $S$ has fixed maximum value of 300 nm for our chosen device. There is also a fixed array of 32x32 pixels on which we can place patches of light (for the multiple bounces), so $N$ and $m$ must be chosen such that all the bounces fit within this space.

We also have to take into consideration how exactly optical beams will enter and exit the White cell, as different options affect the overall performance. The first potential solution, an auxiliary mounting as shown in Figure 3.7a, puts the ITM and OTM on a flat mirror to the side of the MEMS. There is also a direct mounting, Figure 3.7b, which mounts the ITM and OTM directly to the surface of the MEMS. Using an auxiliary mounting will be easier to construct and less costly than a direct mounting, but $S$ will be forced to 0 on half the bounces because those bounces must land on the auxiliary mirror instead of the MEMS. Direct mounting will require a
mirror to be constructed and mounted on the surface of the MEMS, but will allow us to take advantage of the stroke $S$ on every bounce.

Figure 3.7 - Examples of the two mounting types, auxilliary mounting (a) and direct mounting (b)

There is also the question of our capability to align the White cell. The pixels on the AO1K are 150 µm by 150 µm with a gap of approximately 2 µm between each pixel. These small distances make alignment without overlapping the beam patches on each bounce difficult. A solution is to allow for an unused row and column of pixels between each beam patch to create padding; this will change $N$ and $m$, and as a result change the overall steering angle, $\theta$, and our beam width, $\delta\theta$. Table 3.1 gives values to the beam width and maximum steering angle as the rows of padding and $N$ are varied.
Table 3.1 - Changes to Beam width and Steering angle with 1 row/column of padding

<table>
<thead>
<tr>
<th>N</th>
<th>Beam width</th>
<th>Steering angle</th>
<th>Steering angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2.11 mrad</td>
<td>26.6 mrad</td>
<td>53.28 mrad</td>
</tr>
<tr>
<td>4</td>
<td>1.05 mrad</td>
<td>13.3 mrad</td>
<td>26.6 mrad</td>
</tr>
</tbody>
</table>

The condition that $\theta > \delta \theta$ will always be satisfied. We choose $N = 15$ with one row/column of padding. Figure 3.8 provides an example placement of bounces for the chosen setup.

![Figure 3.8 - Placement of the bounces on the face of the MEMS in our setup](image)

3.4 The Input and Output Turning Mirrors

Figure 3.9 shows the input and output turning mirrors as they will be mounted to the MEMS; here we will have a glass roof prism with two reflective surfaces on two faces of the prism.
Figure 3.9 - The input/output turning mirror mounted to the MEMS

The prism will be built such that the width of the reflective surfaces matches the width of the MEMS micromirror pixel array; this is necessary for the four bounce setup shown in Figure 3.8. The tilt of the reflective surfaces is for ease in the placement of the input and output optics.
4 Design Methodology

In this chapter a complete account of the design methods and procedures from the White cell discussed in the previous section, this will include the design choices behind every component, the wavelength used to demonstrate and how it influenced the design choices, as well problems encountered and how changes were made to account for them.

4.1 The White Cell

Before we get into the final design of the White Cell we first need to figure out exactly how big the parts used need to be and where in space they need to be located. The most important part of this design process is that the two imaging conditions are satisfied:

1) The two objective mirrors must image onto each other through the MEMS with a magnification of -1.

2) The MEMS must image back onto itself through an objective mirror with a magnification of -1.

There are several items that need to be defined, Figure 4.1 shows the general design of our White Cell.
Figure 4.1 - Overview of the needed White Cell parameters

First it is important to note that some things need to be fixed before we can start the design. Because we desire the distance between the MEMS and Field Lens to be small, but still workable, we will fix this distance, $d_0$, in Figure 4.1, as approximately 25.4 mm. We will start the design with the Objective Mirrors and their diameters.

4.1.1 Diffraction in the White Cell

There is more to defining the physical characteristics of the objective mirrors than their radius of curvature and ensuring that the imaging conditions are satisfied. The size of the objective mirrors plays a role in determining the overall performance of the White Cell because of diffraction.
**Figure 4.2 - Diffraction**

Diffraction is a physical phenomenon that happens when any wave; in this case we are referring to the way in which electro-magnetic waves interact with an object, Figure 4.2. Diffraction develops as a result of the Huygens-Fresnel principle which states that the wave front of any wave emanating from a disturbance can be treated as the sum of a series of radial waves originating at a point along the disturbance [1]. In our case the MEMS can be seen as this disturbance to the optical beam in the White Cell.

More importantly there are two types of diffraction: near-field and far-field. Far-field refers to the way a wave looks at a sufficiently far distance.

\[ L \gg \frac{A}{\lambda} \]  

(4.1)

Here \( L \) is the distance from the disturbance, \( A \) is the area of the disturbance, and \( \lambda \) is the wavelength of our electromagnetic wave [1]. The Far-field energy pattern is simply the Fourier transform of the shape of the disturbance. In this case
we are considering a slit or square disturbance, so the far-field pattern will be a sinc function [1].

Near-field diffraction refers to how an energy profile of a wavefront will look sufficiently close to the disturbance; in this case we will consider it whenever we are not in Far-field conditions. The energy profile for Near-field is given by

$$E_p = \frac{e^{-j\pi/2}E_s}{\lambda} \int \int F(\theta) e^{jk(r+r')} \frac{1}{rr'} da$$  \hspace{1cm} (4.2)

Here $E_p$ is the energy pattern at the point of observation, $E_s$ is the energy pattern at the source, $r'$ is the radial distance the disturbance is from the source, $r$ is the radial distance the observation point is from the disturbance, $F$ is the attenuation factor, and $\lambda$ is the wavelength of our electromagnetic wave[1].

Figure 4.3 shows how a beam profile changes as it diffracts away from a slit, as the propagation distance $L$ grows larger the beam expands as mentioned in equation 3.4.

Figure 4.3 - A beam profile changing with diffraction. From [1].

For our design we are going to consider Far-field diffraction because the objective mirrors will be placed a sufficiently large distance away from the
MEMS. This simplifies calculations because the far-field diffraction pattern of any interference is the Fourier transform of the interference pattern \[1\], so to determine the energy profile seen on the White Cell mirrors we need only to take the Fourier transform of the profile of the MEMS. Figure 4.4 shows an example of what the far-field pattern will look like. The MEMS will be treated as a two-dimensional array of step-functions. This calculation, contained in Appendix A, will be done in MATLAB.

Figure 4.4 - Example far-field diffraction pattern from a square aperture \[1\]

The concept of diffraction will play a role in determining the diameter of the objective mirrors because we are going to look at how many “zeros” or “dark spots” are captured on the reflective portion of the objective mirrors and look at how that plays a role in the overall power kept on each bounce. This was done in MATLAB by calculating the Far-field diffraction pattern of the MEMS and looking at how much power is kept by a mirror of a certain diameter by multiplying this pattern by a
step function corresponding to that diameter. Figure 4.5 shows an example of this; here a mirror with a diameter of 10 degrees is used. Appendix A contains this code.

Figure 4.5 - The far-field diffraction pattern with the mirror profile super-imposed (a) and the results of this multiplication, which is the Far-field profile captured by the mirror (b).

Figure 4.6 contains the result of this calculation with several different mirror diameters and we can see that a mirror with a diameter of 7.5 degrees captures 99.2% of the information on each bounce; the mirrors used will be chosen to accept a 7.5 degree cone.

Next, before we can move on with our design, we need to introduce the concept of paraxial ray matrices, used to develop constraints on the White Cell.
4.1.2 Paraxial Ray Matrices

Use of paraxial ray matrices requires that the wavelength of the optical beam is much smaller than the optics it is interacting with and that the beams remain close to a “principle axis” so that $\sin \theta$ and $\tan \theta$ can be approximated as $\theta$. That is, angles are small. These matrices provide information on the location and angle of a beam relative to a principle axis. Shown in Figure 4.7 we can see this ray representation visually. Here $x$ is the point of origin of a given ray, relative to a reference axis, and $x'$ is the end-point of the ray, $\theta$ is the initial angle the ray travels at relative to this reference axis, and $\theta'$ is the final angle.
Figure 4.7 - Visual representation of a Paraxial Ray

This can be seen in equation 4.3. The matrix elements $A$, $B$, $C$, and $D$ are the elements which relate to the optical object the beam is encountering. For the purposes of this thesis there are two elements of the matrix that will be used to design: $A$ and $B$. Since we are considering imaging in the White Cell, the element $A$ indicates the magnification and the $B$ term will be 0 to ensure imaging conditions are met [1].

$$
egin{pmatrix}
    x' \\
    \theta'
\end{pmatrix} =
\begin{pmatrix}
    A & B \\
    C & D
\end{pmatrix}
\begin{pmatrix}
    x \\
    \theta
\end{pmatrix}
\tag{4.3}
$$

There are two matrix types used in this design. The first is the translation matrix, which relates to a distance traveled by an optical beam in a medium of constant refractive index, equation 4.4, here $n$ is the refractive index of the material and $d$ is the distance traveled by the ray [1].

$$
\begin{pmatrix}
    1 & d \\
    0 & n
\end{pmatrix}
\tag{4.4}
$$

In the case of the translation matrix we can see that $\theta' = \theta$ and that $x' = x + d*n^{-1}*\theta$. The next is the refractive matrix, equation 4.5, which relates to the change in the angle of a ray when it encounters a curved surface of a different
refractive index, such as the surface of a lens. Here \( n_1 \) is the refractive index of the first material, \( n_2 \) is the refractive index of the second material, and \( R \) is the second materials radius of curvature [1]. It is important to note that there is a third type used for reflection; it is a special case of refraction.

\[
\begin{pmatrix}
1 & 0 \\
-\frac{(n_2 - n_1)}{R} & 1
\end{pmatrix}
\]

(4.5)

Appendix B contains further analysis of the ray matrices used as well as all calculations done in this thesis.

The next step is to define the focal length of the field lens, the distance between the field lens and the objective mirror, and the radius of curvature for the objective mirror. Ray matrices will allow us to solve for all of them simultaneously by considering the entire optical system. A more detailed explanation of this is contained as well as a derivation of all formulas is in Appendix B.
The two imaging conditions needed for the White Cell, shown in Figure 4.8a, can be satisfied from this system:

1) The MEMS needs to image back onto itself through the objective mirror with a magnification of -1.

2) The objective mirrors need to image onto each other through the MEMS with a magnification of -1.

From our simplified system, Figure 4.8b, the second imaging condition can be satisfied by ensuring that the single objective mirror images back onto itself via the MEMS. Several ray matrices will be written representing all actions a ray will undergo in this system: traveling from the objective mirror to the field lens, going through the field lens, traveling from the field lens to the MEMS, and bouncing off
the MEMS (represented as a flat mirror) to travel back through the field lens and to the objective mirror. Each of these steps can be represented as one matrix and each of these matrices will be multiplied together to develop one ray matrix representing the entire action the ray undertakes traveling through the system. This derivation is contained in Appendix B.

The result is a matrix representation as seen earlier in equation 4.3. In the special case of imaging, when \( B = 0 \), the \( A \) element represents the magnification, which for us is set to -1. This ensures that the image at the start point is a reflection of the image at the end point. There is only one solution which simultaneously satisfies both imaging conditions:

\[
d_1 = f
\]

(4.6)

So we now have a constraint on our optical system that satisfies the second imaging condition, which requires the distance between the field lens and the objective mirror, \( d_1 \), and the focal length of the field lens, \( f \), be identical. We next need to develop a constraint for the first imaging condition. Here we will consider a ray leaving the MEMS traveling to the field lens, traversing through the field lens, traveling to the objective mirror, reflecting off the objective mirror and traveling back to the MEMS. Just as with the previous constraint each of the ray matrices representing these steps will be multiplied together to create one ray matrix representing the entire action, to ensure proper imaging the \( A \) element of this matrix needs to be -1 and the \( B \) element needs to be 0. There is only one solution that simultaneously satisfies both of these:
Here $R$ is the radius of curvature of the objective mirror, $d_1$ is the distance between the field lens and the objective mirror, and $d_0$ is the distance between the MEMS and the field lens.

We now have two constraints necessary for the imaging conditions of the White Cell to be satisfied. The radius of curvature of the objective mirror, $R$, can be chosen from a set of standard values from catalog parts as well as the focal length of the field lens, $f$, so once these parts are chosen the appropriate $d_0$ and $d_1$ can be found from equations 4.6 and 4.7. In this solution we will pick a focal length of 300 mm and a radius of curvature of the objective mirror of 324.2 mm from standard catalog parts.

The next step that needs to be done is to solve for the diameter of the field lens. Because the field lens and MEMS combination acts as our field mirror, we need to ensure that all optical information that bounces off the MEMS passes through the field lens so that our assumptions work and the imaging conditions will be satisfied. We have to take into consideration several factors before determining the necessary radius of the lens.
First we have to consider our mirror diameter, which determines the fraction of the far-field diffraction pattern kept by the objective mirrors. We can convert that angular diameter into millimeters since we know $R$. Second we have to consider the White Cell angle, $\beta$, Figure 4.9, which represents the angular separation of the mirrors and is defined in equation 4.8, where $D_s$ is the mirror diameter in millimeters, $Sep$ is the offset of the mirror from the principle axis, and $R$ is the radius of curvature of the mirrors.

$$\beta = \tan^{-1}\left(\frac{D_s - Sep}{R}\right) \quad (4.8)$$

Third we need to consider the angle at which the input and output turning mirrors are tilted. As Figure 4.10 shows, if our field lens is large enough to capture a ray on the very edge of the turning mirror traveling at the very edge of a spot under the most extreme angles we are considering in our design, then the field lens will be large enough that the imaging conditions we assumed throughout the design will always be satisfied.
Figure 4.10 - Visual representation of field lens diameter calculation

Equation 4.9 contains the ray matrix representation of the system and the ray being considered. Note that 0.325 radians corresponds to the most extreme angle considered, the sum of the angular diameter of the mirror and the angle of the ITM. As a result of the focal length of the field lens chosen and the objective mirrors chosen the distance between the MEMS and the field lens, $d_0$, needs to be 22.3mm. As equation 4.9 shows, a field lens with a radius greater than 11.6mm, or diameter greater than 23.2mm, will be sufficient for our design. A standard catalog part with a diameter of 50.8mm will be used as our field lens.

$$
\begin{bmatrix}
  x' \\
  \theta'
\end{bmatrix} =
\begin{bmatrix}
  1 & 22.3 \text{mm} \\
  0 & 1
\end{bmatrix}
\begin{bmatrix}
  4.325 \text{mm} \\
  0.325 \text{radians}
\end{bmatrix}
$$

We now have a complete set of constraints on our design and a series of parts were chosen to fit these constraints. Table 4.1 contains the physical descriptions of all the optics in the White Cell design as well as the distances used.
Table 4.1 - Physical descriptions of White Cell optics and design

<table>
<thead>
<tr>
<th>Optical Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEMS</td>
<td>Model: AO1K</td>
</tr>
<tr>
<td></td>
<td>-- Provided by Sandia</td>
</tr>
<tr>
<td></td>
<td>Maximum Stroke: 300 nm</td>
</tr>
<tr>
<td></td>
<td>Pixel Curvature: 24mm</td>
</tr>
<tr>
<td></td>
<td>Gold coated micromirrors</td>
</tr>
<tr>
<td>Field Lens</td>
<td>Diameter: 50.8mm, Positive Meniscus</td>
</tr>
<tr>
<td></td>
<td>Focal Length: 300mm</td>
</tr>
<tr>
<td></td>
<td>Material: BK-7, Grade A</td>
</tr>
<tr>
<td>Objective Mirrors</td>
<td>Diameter: 50.8mm, Spherical Mirror</td>
</tr>
<tr>
<td></td>
<td>Radius of curvature: 324.2 mm</td>
</tr>
<tr>
<td>$d_0$, distance from MEMS to field lens</td>
<td>22.3 mm</td>
</tr>
<tr>
<td>$d_1$, distance from field lens to</td>
<td>300 mm</td>
</tr>
<tr>
<td>objective mirrors</td>
<td></td>
</tr>
</tbody>
</table>

4.2 Input Optics

For our purposes the job of the input optics is to provide an optical beam of uniform illumination (a “clean” beam) to the White Cell at a desired location with a desired angle. Figure 4.11 shows the initial design for the input optics.

![Initial input optics design](Figure 4.11 - Initial input optics design)

The input laser will be a Melles Griot 5 mW HeNe laser. The energy profile of the output is unknown in this case and will contain noise, both of which are undesired in this case. To “clean” the beam we will use a set of optics to filter out the noise.
elements and provide a beam with a known energy profile. The first step necessary is to pass the original laser beam through a microscope lens; this will focus the beam down to a point a very short distance away from the end of the objective. From here that beam will pass through a small circular aperture, 25 µm, and this results in a known far-field diffraction pattern, shown as it is seen by the eye, Figure 4.12a, and with an accompanying energy profile, Figure 4.12b. All the noise elements will be filtered out by blocking the smaller peaks away from the center “Airy Disk” [1]. By just keeping this center peak we can create an optical beam with a smoothly varying intensity profile for our input into the cell.

By definition the central peak has a specific angular width [1],

\[
\sin \theta = \frac{1.22 \lambda}{d}
\]

(4.10)

Here \( \theta \) is the angular location of the first zero or dark spot, \( d \) is the size of the circular aperture, and \( \lambda \) is the wavelength of light. This can be used to determine how
far away our input optics need to be from the circular aperture so that our beam patch falls entirely in the evenly illuminated center.

The next step is to collimate the beam. If we consider light diffracting through space as a summation of rays traveling along all angles of diffraction, collimated light is a collection of parallel rays and, ideally, does not diffract as it travels. We need to collimate the beam to stop the diffraction caused by the clean-up process and create a plane wave of even intensity.

To collimate the beam we will use a single lens. To define the necessary parameters we use the thin lens approximation, equation 4.11. Here $d_o$ is the distance from the object to the lens, $d_i$ is the distance from the lens to the image, and $f$ is the focal length of the lens. Figure 4.13 shows this relationship visually.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$  \hfill (4.11)

Figure 4.13 - Thin lens approximation

For a collimated beam we can consider the image distance at an infinitely large distance away from the lens, in this case our distance to object, $d_o$, becomes our focal
length, \( f \). When this is taken into consideration our thin lens approximation becomes Figure 4.14.

![Figure 4.14 - Thin lens approximation of a collimated beam](image)

From this figure we get a simple relationship, equation 4.12, between the radius of the lens, \( R \), the focal length of the lens, \( f \), and the angle of diffraction, \( \theta \).

\[
\tan \theta = \frac{R}{f} \tag{4.12}
\]

We can use this relationship to define the minimum radius necessary for our collimating lens, if we consider the light at the 25 \( \mu \)m circular aperture a point source, by combining equations 4.10 and 4.12.

\[
\begin{align*}
\theta &= \sin^{-1} \frac{1.22 \lambda}{d} \\
\theta &= \tan^{-1} \frac{R}{f} \\
\tan^{-1} \frac{R}{f} &= \sin^{-1} \frac{1.22 \lambda}{d} \tag{4.13} \\
R &= \tan \left( \sin^{-1} \frac{1.22 \times 633 \eta m}{25 \mu m} \right) f
\end{align*}
\]
We end up with a linear relationship between the minimum lens radius necessary to collimate our center peak, $R$, and its focal length, $f$, in equation 4.14.

$$R = 0.03091 f$$  \hspace{1cm} (4.14)

In the case of our input optics a focal length <100 mm is desired and by this relationship any lens with a radius greater than 3.1 mm will be sufficient to collimate the central peak.

At this point we have a collimated optical beam with a well defined energy profile; the next step is to build a second aperture to act as the image plane for the input turning mirror. This aperture will be imaged to the ITM, so it should be exactly the size of our $N \times N$ array of pixels necessary for each bounce. As mentioned in section 3.2 $N = 15$; this correlates to a 2.25 mm square optical beam patch, so the aperture should be 2.25 mm by 2.25 mm. Then finally we will have an imaging lens to image the second aperture to the input turning mirror, as shown in Figure 4.15.

![Figure 4.15 - The 2nd aperture imaging to the Input Turning Mirror](image)

To define this relationship and the necessary characteristics for the imaging lens ray matrices will be used; Appendix B contains the derivations. The result is a focal length, $f$, of 158.7 mm, an object distance, $d_1$, of 303.1 mm and an image distance, $d_2$, of 304.8 mm, and the lens needs to have a radius of at least 11 mm.
This is where a design problem was encountered. The initial assumption was that the second aperture would cause the beam to diffract again thus allowing for the imaging conditions between the second aperture and the input turning mirror, as shown previously in Figure 4.15. As Figure 4.16a shows, the 2.25 mm square aperture is so large that the beam exiting the aperture is still essentially collimated. This prevents the imaging conditions necessary for the operation of the White Cell from being satisfied, as the objective mirrors are not imaged onto each other through the MEMS. Instead the situation in Figure 4.16b arises, where each objective mirror is not imaging the other with a magnification of -1.

Figure 4.16 - The imaging problems at the end of the input optics (a) and the imaging conditions violated in the White Cell (b)
To correct this, the imaging lens must be removed and replaced with another lens, referred to as the conditioning lens, and a new set of conditions must be developed to ensure proper imaging.

### 4.3 Modifications to the Input Optics

The new input optics, shown in Figure 4.17, include the change to the end of the optics necessary to satisfy the imaging conditions. Instead of using an imaging lens we will use a “conditioning lens” to change how the beam enters the White Cell.

![Figure 4.17 - Updated Input Optics](image)

The conditioning lens will image the second aperture onto the input turning mirror through a folding mirror, as shown in Figure 4.18. This will change the operation of the White Cell slightly from previous drawings. Earlier it was shown as light focusing to a point on the MEMS and expanding out to fill the objective mirrors A and B. Now, Figure 4.18, we have a patch of light illuminating a part of the MEMS and focuses to a point on the objective mirrors. The light between the field lens and MEMS is now collimated. Note that this change still satisfies the imaging conditions. The point on A images to the point on B, and a patch on the surface of the MEMS images to a new patch on the surface of the MEMS.
Because the conditioning lens needs to focus the light down to a point on the folding mirror, the folding mirrors need to be conjugate to the objective mirrors, and the light needs to be collimated by the field lens, the conditioning lens has to have a focal length equal to the field lens. Table 4.2 contains the physical descriptions of the finalized input optics.

Table 4.2 - Physical Description of Input Optics

<table>
<thead>
<tr>
<th>Optical Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium Neon Laser</td>
<td>$\lambda = 633 , \mu m$, Power = 5 $\mu W$</td>
</tr>
<tr>
<td>Beam Cleanup</td>
<td>Microscope Objective: 10X 25$\mu m$ pinhole aperture</td>
</tr>
<tr>
<td>Collimating Lens</td>
<td>Newport KPX088AR.14: 25.4 mm dia., 75.6 mm focal length</td>
</tr>
<tr>
<td>2nd Aperture</td>
<td>2.25 mm square aperture fabricated using a drill in a aluminum plate</td>
</tr>
<tr>
<td>Conditioning Lens</td>
<td>Newport KPX112AR.14: 25.4 mm dia., 300mm focal length</td>
</tr>
</tbody>
</table>
4.4 Combining the input optics with the White Cell

The input optics and the White Cell have been completely specified. To ensure proper operation we need to verify that the aperture at the end of the input optics is of even illumination and of the proper size: 2.25mm square. As Figure 4.19 shows, this is indeed the case.

![Figure 4.19 - The beam patch directly after the second aperture](image1)

The next step is to ensure that the aperture at the end of the input optics images properly onto the input turning mirror. The image on the face of the input turning mirror is pictured in Figure 4.20.

![Figure 4.20 - The optical beam patch on the face of the input turning mirror](image2)
It is important to note that there is degradation of the beam as it travels through space and on each bounce. This is caused by the fact that the White Cell does not preserve the phase; Figure 4.21 shows how the lack of preservation of the phase effects the development of an image. If we consider an object directly at the plane of a lens with a focal length $f$, then the Fourier transform of this object appears at its focal point. If we then observe it directly after a second lens of identical focal length placed a distance $f$ away of the focal point, such as the situation at the top of Figure 4.21, this second lens inverts this process [15]. While the intensity profile will be correct, however, the phase will not. This results in an image with blurred edges and a square array of intensity peaks, as shown in the simulation on the right side of the figure, performed by another member of our group. This situation of the object and image being close to the field lens is what we find in the White Cell.

If the object is moved a distance $f$ away, the phase is allowed to develop and the observation becomes a better approximation to an evenly illuminated square patch. If the observation is also moved a distance $f$ away, the object is reproduced exactly at the point of observation because the phase is allowed to develop completely, as seen at the bottom of Figure 4.21 [15]. This is the situation found in the Fourier Cell, and represents a possible point of improvement for further work.
Finally we need to ensure the bounces are properly lined up on the face of the MEMS; this is pictured in Figure 4.22 along with a key to show the proper order of the bounces as the beam travels through the White cell. This figure was developed by imaging a camera to the surface of the MEMS or turning mirror and taking data on the intensity patterns. The data was compiled below and arranged in proper sequence. As shown we do indeed have proper alignment of all the bounces from the input turning mirror to the output turning mirror. The figure also includes a key showing the proper order and the manner in which the images on the face of the MEMS inverts with each bounce. The blurred edges and complicated peak structure are undesirable but explained from the above discussion.
4.5 Output Optics

Now that we have the White Cell and input optics designed, we now have to look at the output optics. The output optics are used to de-magnify the beam so we can steer over larger angles.

The output optics takes a different design approach than mentioned in the previous chapters. We don’t have to worry about imaging conditions or how the beam needs to look, just the magnification factor of the optics. The approach taken here is to demagnify the aperture, in this case the pixel pitch, \( a \), by a factor of \( M \). As shown in equation 4.15, demagnifying the pixel patch increases the steering angle.

\[
a > Ma
\]

\[
\theta = \tan^{-1}\left(\frac{D}{Na}\right) < \theta = \tan^{-1}\left(\frac{D}{NMa}\right)
\]

\[ (4.15) \]
Figure 4.23 shows the output optics design. The principle behind the output optics is a microscope in reverse; the light will bounce off the output turning mirror, travel a certain distance, $d_o$, pass through the eyepiece lens, travel through free space to the microscope objective a specific distance referred to as the rail length, $l$, travel through the microscope objective to focus down to a point, and after which it will expand outward. After this point is where our observation plane will be, a certain distance away, $d_i$. The desire is that the angle seen at this plane is significantly larger than the angle seen at the output turning mirror.

![Figure 4.23 - Output Optics](image)

Table 4.3 contains the physical description of the chosen output optics.

<table>
<thead>
<tr>
<th>Optical Component</th>
<th>Physical Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eyepiece Lens</td>
<td>Nikon CFWN 10x/20 Magnification: 10x</td>
</tr>
<tr>
<td>Microscope Objective</td>
<td>Olympus A10 Numerical Aperture: 0.25 Magnification: 10x</td>
</tr>
</tbody>
</table>

A ray matrix representation of this system is not possible without schematics provided by Nikon and Olympus. Instead the system was calibrated using the apparatus shown in Figure 4.24, using a backlit ruling at a fixed working distance. The ruling, eyepiece lens, and microscope objective are set up along an optical rail. The eyepiece creates an intermediate image somewhere after the lens, and the
microscope objective uses that image as a second object and creates a second magnified image. This image can be measured with a ruler to determine the magnification factor, $M$. By varying the distance, $d_o$, a calibration curve can be obtained.

Figure 4.24 - The output optics calibration process, first the rail length is set (a), second we determine the magnification factor by comparing the output ruling with the input ruling (b)

Figure 4.25 contains the calibration data relating the working distance to the magnification factor associated with it. It will be used in the next chapter to determine the proper demagnification of the output beam.
In Figure 4.25, the “Distance from object” represents the distance from the eyepiece lens to the output turning mirror. There is no way of knowing exactly what this distance is with the given experimental setup, there is error introduced through measurement and the optics the beam patch must travel through. It was measured to be 250mm with an error of ±10 mm. Relating this to the calibration data, we should expect a magnification factor of 90 with an error of ±6.
5 Experimental Results and Descriptions

Contained in this chapter are the procedures and results of the experiment. First, we will outline the data taking process and provided a basis to ensure the data is correct. This will be done using a camera and an imaging lens. Second, we will provide the results of the experiment without any demagnification; this provides assurance that our design works. Third, we will verify that the demagnification process does work as intended. Finally, we will characterize the efficiency of our system and analyze where improvements could be made.

5.1 Data taking process

In order to verify the operation of our White Cell, we need to develop a process to take data to verify the optical beam steering. Figure 5.1 shows the output of this system without the demagnification output optics. Here the beam patch is reflected off the output turning mirror, passes through the field lens and focuses down to a point on a bending mirror. The bending mirror then reflects this beam toward our data taking setup. The idea is to image a specific plane in the output, referred to as the object plane in the figure.
Figure 5.1 - Output of the system without the output optics

Figure 5.2 shows that a camera is used to image a specific plane along the output path, referred to as the object plane in Figure 5.1 and Figure 5.2, using an imaging lens. The purpose is to use the camera to measure the deflection angle of the optical beam.

Figure 5.2 - Data taking setup

To make sure the camera is focused to the correct imaging plane, a ray matrix relationship will be developed. We will choose an imaging lens and consider rays leaving from the imaging plane, passing through the lens, and hitting the face of the camera. This system is contained in Appendix B. Table 5.1 contains the results of
this derivation. The numbers produced by this derivation were confirmed
eperimentally by backlighting the blade of an exacto knife. This provided an object
of a known size with a very sharp edge. All the data from the camera was measured
using the Spiricon Laser Beam Analyzer, which provides graphical data on the
intensity profile observed by the camera. The intensity profile is colored according to
one of 1048 levels with white being the most intense and black being no intensity.

Table 5.1- Physical Description of Data taking optics

<table>
<thead>
<tr>
<th>Optical Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Camera</td>
<td>Sony CCD XC-ST50</td>
</tr>
<tr>
<td></td>
<td>Serial Number:</td>
</tr>
<tr>
<td>Imaging Lens</td>
<td>Newport KBX061AR.14</td>
</tr>
<tr>
<td>$d_c$</td>
<td>$88.3$ mm</td>
</tr>
<tr>
<td>$d_i$</td>
<td>$175.388$ mm</td>
</tr>
<tr>
<td></td>
<td>$175.588$ mm</td>
</tr>
</tbody>
</table>

The second aspect of the data taking process involves the controller program
provided by Sandia National Laboratories to control the MEMS. This controller uses
a MATLAB program that connects to the MEMS via a USB Controller, and using
the graphical user interface we can select any one of a number of specific ramp
types, and each ramp has an associated steering angle.

The ramps are all labeled PXYZ where X represents the steepness of the ramp
across all bounces; YZ represents the overall direction steered. Figure 5.3 shows an
example ramp is labeled PQLR. The label “Q” means there is a single ramp across
the entire quadrant, and LR means the beam steers from left to right. It is important
to note that the ramps flip direction in each quadrant; this is by design. As mentioned
in chapter 4, the MEMS must image back onto itself with a magnification of -1. This
causes the image on the surface of the MEMS to invert every bounce. To ensure the steering is correctly applied on each bounce, the ramps must invert as well.

Figure 5.3 - An example ramp as seen through the graphical user interface

5.2 Results

Using the setup contained in Figure 5.2 and the process outlined in the previous section, we first measured the beam steering without demagnification.

Figure 5.4 shows an example output pattern seen by the camera in a steered and un-steered case, and relates the features of the beam used to verify the results shown in Table 5.2. This data was taken using a MEMS with a metalized micromirror array, serial number 11, so the radius of curvature is 24mm, but the micromirrors have high reflectivity.
Figure 5.4 - An example output pattern observed both un-steered (top) and steered (bottom)

The numbers in Table 5.2 relate to the peak of the output pattern as seen by the camera when the MEMS was flat (top) and when a ramp was applied (bottom). Ramp types are P, in which the ramp across each quadrant is 150 nm, PQ, in which the ramp across each quadrant is 300 nm, and P8, in which the ramp across each quadrant is twice as steep as it is with PQ, so there are two ramps of 300 nm with a reset in the middle.

<table>
<thead>
<tr>
<th>Direction Type:</th>
<th>Ramp Type:</th>
<th>Measured Angle:</th>
<th>Expected Value:</th>
<th>Error:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-to-Right</td>
<td>P</td>
<td>0.25 mrad</td>
<td>0.27 mrad</td>
<td>7.4%</td>
</tr>
</tbody>
</table>

Table 5.2- Experimental results without demagnification
The results in the table indicate that our solution is capable of steering up, down, left, and right with an average error of 4.5%. It is also possible for our setup to steer up and right at the same time, referred to as the cardinal direction north-west, and Table 5.3 contains these results. Here the average is slightly more, 6.1%.

Table 5.3- Experimental results without demagnification

<table>
<thead>
<tr>
<th>Direction:</th>
<th>Ramp Type:</th>
<th>Measured Angle:</th>
<th>Expected Value:</th>
<th>Error:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest</td>
<td>P</td>
<td>0.58 mrad</td>
<td>0.53 mrad</td>
<td>9.4%</td>
</tr>
<tr>
<td></td>
<td>P8</td>
<td>1.13 mrad</td>
<td>1.1 mrad</td>
<td>2.73%</td>
</tr>
<tr>
<td>Northeast</td>
<td>P</td>
<td>0.58 mrad</td>
<td>0.53 mrad</td>
<td>9.4%</td>
</tr>
<tr>
<td></td>
<td>P8</td>
<td>1.12 mrad</td>
<td>1.1 mrad</td>
<td>1.82%</td>
</tr>
<tr>
<td>Southwest</td>
<td>P</td>
<td>0.59 mrad</td>
<td>0.53 mrad</td>
<td>11%</td>
</tr>
<tr>
<td></td>
<td>P8</td>
<td>1.12 mrad</td>
<td>1.1 mrad</td>
<td>1.82%</td>
</tr>
<tr>
<td>Southeast</td>
<td>P</td>
<td>0.58 mrad</td>
<td>0.53 mrad</td>
<td>9.4%</td>
</tr>
<tr>
<td></td>
<td>P8</td>
<td>1.13 mrad</td>
<td>1.1 mrad</td>
<td>2.73%</td>
</tr>
</tbody>
</table>
During the experimental process the measured values were consistent, with negligible fluctuation during observation. The observed errors are likely caused by errors in the actual value of the ramps when the MEMS takes the required shape.

### 5.3 Verifying demagnification

The next step is to verify the beam can be properly de-magnified to steer over larger angles, as being able to steer over half a milliradian is not of practical use. To do this we will use a different camera setup, Figure 5.5. The camera is some set distance away from the focal point of the microscope objective at the end of the output demagnification optics. Here a separate MEMS was used with an un-metalized micromirror array, serial number 09. This reduces the curvature of the pixels, but also their reflectivity.

![Camera setup for measuring the demagnified optical beam](image)

**Figure 5.5 - Camera setup for measuring the demagnified optical beam**

Since we know from the previous section that our White Cell steers properly, we will assume that before the optical beam passes through the objective lens it has been steered by some angle, $\theta_i$, and that the camera should see a beam steered by another angle, $\theta_o$, following the relationship in equation 5.1, if the beam has undergone the correct demagnification.
\[
\theta_O = M \times \theta_I
\] (5.1)

The demagnification factor, \(M\), is known from the calibration data in section 4.5, but with some error, as discussed in section 4.5. Since we know the distance between the eyepiece lens and output turning mirror to be 250 mm with an error of ±10 mm, we should expect a magnification factor of 90 with an error of ±6. Figure 5.6 shows exactly how \(\theta_O\) will be measured. The camera’s observation plane is a known distance away from the focal point of the microscope objective, this is known to within ±0.1 mm, and the camera measures the distance a specific point on the output beam’s intensity pattern moves as the ramp is applied.

![Diagram](image)

**Figure 5.6 - Method of determining demagnified steering angle**

Figure 5.7 shows an example of the pattern seen by the camera un-steered and steered. Here the boxes indicate the matched regions of the intensity profile we are using to measure the distance steered. The measurement is taken by comparing the like sides of two correlating boxes. The average of these six values is taken to determine the measured distance steered; this gives us \(\theta_O\). As discussed in section
3.2, the use of the unmetalized MEMS was done to preserve as much of the beam quality as possible. When the MEMS with the metalized micromirror array is used, the degradation is so great that the boxes in this figure used to measure steering could not be drawn.

![Figure 5.7 - Demagnified output pattern un-steered (top) and steered (bottom)](image)

The output was observed to steer by 2.7 degrees or 47 mrad with an error of ± 1.15 mrad with the ramp PQRL, which has an expected angle of 0.53 mrad. This represents a magnification factor of 88 ± 3, which is within the expected magnification factor of 90 ± 6.

### 5.4 Efficiency and Analysis

The question of efficiency and potential improvements to our system is critical. Table 5.4 contains the optical average power of the beam measured at several points.
of interest in our system. These measurements were taken with a Newport photo-detector model 818-SL, and one measurement was taken every five seconds for one minute, then its average was computed.

### Table 5.4 - Power measured at several points in our White Cell

<table>
<thead>
<tr>
<th>Point in MEMS:</th>
<th>Average Power of the Optical Beam (µW):</th>
<th>Cumulative Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input:</td>
<td>5.130 ± 0.180</td>
<td>0 dB</td>
</tr>
<tr>
<td>1st Bounce:</td>
<td>4.897 ± 0.166</td>
<td>0.2 dB</td>
</tr>
<tr>
<td>2nd Bounce:</td>
<td>3.090 ± 0.121</td>
<td>2.2 dB</td>
</tr>
<tr>
<td>Output:</td>
<td>0.870 ± 0.104</td>
<td>7.7 dB</td>
</tr>
</tbody>
</table>

The power is going into our White cell on average is 5.130 µW on the input turning mirror. The power after the first bounce characterizes how much power is lost as it travels from the input turning mirror, hits the first White cell mirror, and travels back through the field lens, and finally is measured before reflecting off the face of the MEMS. This was measured with an average of 4.897 µW, which means a loss of 0.2 dB with respect to the average input power was observed. The power after the second bounce characterizes how much power was lost taking the same trip described previously plus the diffraction losses caused by the curvature of the MEMS pixels; this was measured as 3.090 µW, corresponding to a cumulative loss of 2.2 dB. Finally the power on the face of the output turning mirror was measured as 0.870 µW, so this is a cumulative loss of 7.7 dB.

As the drop from the first bounce to the second bounce shows, diffraction losses represent a major source of power loss in our system. If we consider the 0.2 dB loss between the input bounce and first bounce as a constant of the design, it would mean only 66.1% of our power is kept as a result of diffraction on each bounce. To
illustrate this, Figure 5.8 compares the power of the system with and without diffraction losses. As the figure shows if we consider the system without diffraction losses, the output of the system is at 4.066 µW, this represents a 1 dB loss.

![Figure 5.8 - Comparison of power kept by the White Cell with and without diffraction losses](image)

The diffraction losses are primarily caused by the curvature of the MEMS pixels; as discussed in section 3.2, the radius of curvature on the MEMS pixels and the multiple bounces we have in the system manipulates the output pattern to what we see in Figure 5.5. We would require larger optics to preserve the same amount of power as a MEMS with flat pixels. Figure 5.9 shows an example output pattern of an un-metalized MEMS, meaning the pixels are not as reflective as the AO1K MEMS, but the pixels are flat. This represents what the output of our system would look like if the MEMS pixels were not curved. Not only are we keeping more power with flat MEMS pixels, but we do not have drastic changes in beam quality.
Figure 5.9 - Example output pattern from an un-metalized MEMS
6 Conclusions

This thesis outlined the development of a MEMS-driven White Cell as a true time delay approach to beam steering. The designed system allows for steering in all directions. Steering without optical demagnification up to 1.2 milliradians was demonstrated, and in conjunction with the output optics, it is possible to steer up to 0.114 radians with the current output optics.

The overall performance of the design is not without room for improvement. The curvature of the MEMS pixels caused significant diffraction. The result is poor beam quality and large diffraction losses. As this thesis demonstrated, reducing this curvature results in less loss and greater beam quality.

The lack of phase preservation with imaging also represents a clear point for improvement, and this could be corrected by using a Fourier cell instead of a White cell, where a large circular lens with a focal length of \( f \) is placed in between of a MEMS and a set of objective mirrors, both of which are placed a distance \( f \) away from the lens. This allows the phase to fully develop and represents a possible next step.
References


A MATLAB Code

In this appendix we will show all MATLAB code used. As explained in Chapter 4, the code contained in this section is used for computing the diffraction pattern of the MEMS on a bounce. This was then used to compute the amount of power kept by our objective mirrors as their size increases. The first section of this Appendix will contain code as written for this research. The second section will contain the sections of unaltered code provided by Sandia National Laboratories.

A.1 Code used to define angular diameter of Objective Mirrors

```matlab
A=pixelpatch(4,48,1);
mircurve=localsphere(A,150e-6,0);

%This defines the pixelated area of the MEMS and their curvature.

[mircoords,fftcoords,mask,height]=square_array(A,mircurve,150e-6,1,2048);
wv_ln=1.55e-6;
%near_fld adds phases to all pixels
near_fld=mask.*(cos(2*pi/wv_ln*height)+(-1)^0.5*sin(2*pi/wv_ln*height));
far_int=fftshift(abs(fft2(near_fld))).^2;

figure
plot(far_int(1:1:2048,2048/2));

%At this point I have used provided code to compute a 2D FFT of the Far-Field diffraction pattern for this single square element
%I want to define the overall power provided as "100%" or 1
%To do this I will sum all the elements of far_int

S1 = sum(far_int);
P_total = sum(S1);

%P_total is the total power contained in the far-field
```
% I will define 10 matricies as the mirrors necessary to capture from 
% the first zero up to the 10th zero. We run into one 
% problem however and that is due the nature of the FFT function 
% it is necessary to estimate the location of each zero. This 
% will be done by another function "estimatezero" which will 
% find the locations of the minima in the "far_int" array.

I = estimatezero(far_int(1:1:2048,2048/2));

% For the case of 633 nm I is of length 44, meaning there are 22 
"lobes" along the center axis. Since this mirror was square 
% each other axis should merely be a projection of this center 
% axis and hence have the same number of zeros at the same 
% locations (although error is introduced since we are dealing 
% with estimates and are limited by the resolution of the FFT)

%M# represents the mirror itself, m# is the pixel dimension of the 
mirror

M1=zeros(2048,2048);m1=94;
M2=zeros(2048,2048);m2=182;
M3=zeros(2048,2048);m3=272;
M4=zeros(2048,2048);m4=360;
M5=zeros(2048,2048);m5=450;
M6=zeros(2048,2048);m6=538;
M7=zeros(2048,2048);m7=628;
M8=zeros(2048,2048);m8=716;
M9=zeros(2048,2048);m9=806;
M10=zeros(2048,2048);m10=894;

% Fill mirror will be used to create a circular mirror with radius r
M1=fillmirror(M1,m1);
M2=fillmirror(M2,m2);
M3=fillmirror(M3,m3);
M4=fillmirror(M4,m4);
M5=fillmirror(M5,m5);
M6=fillmirror(M6,m6);
M7=fillmirror(M7,m7);
M8=fillmirror(M8,m8);
M9=fillmirror(M9,m9);
M10=fillmirror(M10,m10);

% Ok, we now have 10 mirrors designed to catch the first 10 zeros. 
% We will take each mirror and multiply it by the 
% far-field diffraction pattern to see what power % each captured.

P1=far_int.*M1;
P2=far_int.*M2;
P3=far_int.*M3;
P4=far_int.*M4;
P5=far_int.*M5;
P6=far_int.*M6;
P7=far_int.*M7;
P8=far_int.*M8;
P9=far_int.*M9;
p10=sum(sum(P10));

%we now have the total power contained in each of the 10 mirrors, %question is now how much total was captured by each.

power_per_zero(1)=p1/P_total;
power_per_zero(2)=p2/P_total;
power_per_zero(3)=p3/P_total;
power_per_zero(4)=p4/P_total;
power_per_zero(5)=p5/P_total;
power_per_zero(6)=p6/P_total;
power_per_zero(7)=p7/P_total;
power_per_zero(8)=p8/P_total;
power_per_zero(9)=p9/P_total;
power_per_zero(10)=p10/P_total;

### A.2 Provided Code

Below is the set of functions provided by Sandia National Laboratories which is needed to execute the code provided in the previous section.

```matlab
function in=pixelpatch(numsides,npix,halfgap)

% makes an npix by npix square array of 0s and 1s to represent a
% segmented mirror
% element <npix from tip to tip
% usage: pixelpatch(numsides,npix,halfgap) where
% numsides=4 or 6
% npix=number of pixels used to represent the single element
% (<=FFTsize/elements)
% halfgap=the space between elements
% force npix to be even for now

mir_size=npix-halfgap;        % even number <npix

if numsides==4     % square pixels are easy
    in=zeros(npix,npix);
    in(halfgap+1:npix-halfgap,halfgap+1:npix-halfgap)=1;
end

if numsides==6     % hex mirror case
    x=(linspace(-npix/2,npix/2,npix)'*ones(1,npix))';
    y=rot90(x);
    t = ((0:1/6:1)')*2*pi;
    xv = sin(t)*mir_size/2;
    yv = cos(t)*mir_size/2;
    in = inpolygon(x,y,xv,yv);
end

figure
image(in*128)
axis image
```
function mircurve=localsphere(patch,mir_size,R)
% generates a spherical surface with radius R in meters across the
% surface of a micomirror defined by the patch
% usage:  mircurve=localsphere(patch,mir_size,R)
% where, patch-array representing a single element from pixelpatch
% mir_size=the mirror patch physical size in meters
% R=the radius of curvature to be imposed on the mirror surface

D=length(patch);   % patch needs to be a square array of 1s and 0s
scale_factor=mir_size/D;

% define spherical surface of a single element
x=(linspace(-D/2,D/2,D)'*ones(1,D))';
y=rot90(x);
x=x*scale_factor;
y=y*scale_factor;
if R==0 %R=0 signifies flat mirror (special case)
  z=zeros(D,D);
else
  if R<0
    z=(R+(R^2-x.^2-y.^2).^0.5);
  else
    z=abs(R-(R^2-x.^2-y.^2).^0.5);
  end
end
z=z.*patch;

% figure
% subplot(2,2,1); mesh(z)
% subplot(2,2,2); plot(z(D/2,:))
% subplot(2,2,3); mesh(z)
% zmax=max(max(z));
% subplot(2,2,4); image(z./zmax.*64); axis square

mircurve=z;
return

Function
[mircoords,fftcoords,mask,height]=square_array(patch,mircurve,mirsize,Mnum,FFTsize)

% makes an NxN square array of 0s and 1s to represent a square element
% segmented mirror array using patch = an array of 1s and
% 0s defining the shape of a single element mircurve = an
% array of heights (in meters) defining local mirror curvature
% mirsize = mirror size in meters
% Mnum = the number of mirrors on a side
% FFTsize = overall size of the FFT (i.e. 512, 1024, 2048)
% usage:
% [mircoords,fftcoords,mask,height]=square_array(patch,mircurve,mirsize,Mnum,FFTsize)
% returns
% mircoords = N^2 by 2 length array of mirror centers in physical dimensions
% fftcoords = like mirror coords but integer indices to mask and height arrays
%    mask = amplitude array
%    height = height of mirror surfaces

close all

D=length(patch);  % mirror cell size in pixels
scalefactor=mirsize/D

% compute coordinates for mirror centers, center mirror at
% origin noting that mirror size defines the longest dimension of the
% active region of the mirror patch and the maximum number of mirrors
% across can be defined by d=mirsize*cos(30 deg)
if Mnum/2==round(Mnum/2)  % even number of mirrors on a side
    mirnum=[1:1:Mnum)-(Mnum+1)/2;
    xm=(mirnum' * ones(1,length(mirnum)))';
    xm=xm*mirsize;
    ym=rot90(xm);
end
if Mnum/2~=round(Mnum/2)  % odd number of mirrors on a side
    mirnum=[1:1:Mnum)-(Mnum+1)/2;
    xm=(mirnum' * ones(1,length(mirnum)))';
    xm=xm*mirsize;
    ym=rot90(xm);
end

[a,b]=size(xm);
xm=reshape(xm,a*b,1);
MirrorNumberOnASide=length(xm)

ym=reshape(ym,a*b,1);

mircoords=[xm ym];
%mircoords;

figure; plot(mircoords(:,1),mircoords(:,2),'r*'); axis equal
%convert physical mircoords to fft indices
fftcoords=fix(mircoords/scalefactor + FFTsize/2);
%
% populate the mask array with patches and the height array with mircurve
mask=zeros(FFTsize,FFTsize);
height=zeros(FFTsize,FFTsize);
outofbounds=0;

%length(fftcoords)
if size(fftcoords)==[1,2]
    lo_x=fftcoords(1,1)-D/2; hi_x=lo_x+D-1;
    lo_y=fftcoords(1,2)-D/2; hi_y=lo_y+D-1;
    if hi_x<FFTsize & hi_y<FFTSize & lo_x>=1 & lo_y>=1
        mask(lo_x:hi_x,lo_y:hi_y)=mask(lo_x:hi_x,lo_y:hi_y)+patch';
    end
height(lo_x:hi_x,lo_y:hi_y)=height(lo_x:hi_x,lo_y:hi_y)+mircurve';
else
    outofbounds=1;
end
else
    for ii=1:length(fftcoords)
        lo_x=fftcoords(ii,1)-D/2; hi_x=lo_x+D-1;
        lo_y=fftcoords(ii,2)-D/2; hi_y=lo_y+D-1;
        if hi_x<FFTsize & hi_y<FFTsize & lo_x>=1 & lo_y>=1
            mask(lo_x:hi_x,lo_y:hi_y)=mask(lo_x:hi_x,lo_y:hi_y)+patch';
        end
        height(lo_x:hi_x,lo_y:hi_y)=height(lo_x:hi_x,lo_y:hi_y)+mircurve';
    end
    outofbounds=1;
end
end
if outofbounds==1
    fprintf('Mirrors lost due to overstepping FFT bounds\n')
end
% figure; image(mask*128); axis image;
% figure; imagesc(height); axis image;
return

function [I] = estimatezero(C)
N=size(C);
prev=C(1);
index=1;
for i=2:N-1
    if abs(C(i))>0
        if C(i)<prev & C(i)<C(i+1)
            I(index)=i;
            index=index+1;
        end
    end
    prev=C(i);
end
return

function [Mo]=fillmirror(Mi,mi)
upper=size(Mi);
center=upper/2;
Mo=Mi;
for m=1:upper(1)
    for n=1:upper(2)
        if sqrt((abs(m-center(1)))^2+(abs(n-center(2)))^2) <= mi/2;
            Mo(m,n)=1;
        end
    end
end
return
B Ray Matrices

In this appendix we will further outline the Ray Matrices explained in Chapter 4, as well as solidify the equations derived. As explained in Chapter 4, Ray Matrices represent optical problems as matrix algebra. The simple 2x2 matrices can be used to solve imaging conditions and where images will form in our White Cell. This process is shown in equation 8.1 [1]. There are four elements to the ray matrix, which are used to describe a particular type of action a ray undergoes. Here $x$ is the input ray height, with respect to a principle axis, $x'$ is the output ray height, $\theta$ is the input ray slopes, and $\theta'$ is the output ray slopes. This is shown visually in Figure 8.1.

$$\begin{bmatrix} x' \\ \theta' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} \quad (8.1)$$

There are several types of Ray Matrices, but our system will use the transition and reflection matrices, as our system only consists of refracting surfaces, such as lenses, and reflecting surfaces, such as mirrors and the MEMS.
To use these Ray Matrices, we make a set of assumptions. First is that the wavelength of light is much smaller than the size of any optics in our design. This is necessary so we can treat light as a ray instead of a wavefront with respect to any lenses or mirrors in our design. Since the smallest components of our design are on the order of millimeters and the wavelength of light we are using is on the order of nanometers, this assumption holds. Second is that all rays above are paraxial, or that the angles at which rays are traveling with respect to the principle axis are small. This is necessary so that in our ray matrices we can approximate \( \sin \theta \) and \( \tan \theta \) as \( \theta \). It is important to note that ray matrices also assume that all rays move in the same direction, so we will have to set up bounces along a straight line path, even though the mirrors reflect back [1].

There are two primary matrices we will use which will describe any action a ray can undergo in our system: translation and refraction. Translation, equation 8.2, describes a ray traveling a distance, \( d \), through a material of a constant index of refraction, \( n \) [1].

\[
\begin{pmatrix}
1 & d \\
\frac{1}{n} & 1
\end{pmatrix}
\]  

(8.2)

Refraction, equation 8.3a, describes a ray traveling from one material with an index of refraction, \( n_1 \), into a material with another index of refraction, \( n_2 \), with a radius of curvature, \( r \). Reflection can be considered a special case of refraction, shown in 8.3b, where \( R \) is the radius of curvature of the mirror [1].

\[
\begin{pmatrix}
1 & 0 \\
-\left(\frac{n_2 - n_1}{n_1}\right) & 1
\end{pmatrix}
\]  

(8.3a)
For any optical system the total effect it will have on any ray can be found by multiplying the ray matrix of each individual action together, going backward from output to input. We will show an example of this by deriving an important formula for the remainder of our derivations.

\[
\begin{pmatrix}
1 & 0 \\
\frac{2}{R} & 1
\end{pmatrix}
\]  

(8.3b)

**B.1 Thin Lens Equation**

![Thin Lens](image)

**Figure B.2 - Thin Lens**

The thin lens, Figure 8.2, is an important piece of our derivations, and will be used frequently. A thin lens is defined as two refracting surfaces of a specific index of refraction, with a negligible thickness between them. In the case of a bi-convex lens, as shown, there are two sides with equal radii of curvature, the only difference being the direction of the curvature. We will develop an overall system for this lens by looking at each surface and multiplying the two [1].

The first surface has a radius of \( r \) and a ray will pass from one index of refraction, \( n_1 \), to another, \( n_2 \). This can be represented as the matrix \( M_1 \).
The second surface has a radius of $r$ and a ray will pass from one index of refraction, $n_2$, to the original, $n_1$. This can be represented as the matrix $M_2$.

$$M_2 = \begin{pmatrix} 1 & 0 \\ -\frac{(n_2 - n_1)}{r} & 1 \end{pmatrix}$$

By multiplying these two matrices we get a new matrix $M_3$, which can be further simplified to a matrix only involving the focal length of the lens, $f[1]$.

$$M_3 = \begin{pmatrix} \frac{(n_1 - n_2)}{r} & 1 & 0 \\ -\frac{2(n_2 - n_1)}{r} & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

This shows an example of how a set of optics can be simplified down to one matrix, which can then be solved for under the constraints needed. This matrix will be used further in our derivations.

**B.2 Imaging Conditions in the White Cell**

There are two conditions in the White Cell that need to be satisfied for its proper operation. The first is that each objective mirror must image onto the other with a magnification of -1. The second is that the MEMS must image back into itself with a magnification of -1. This will be done by developing two different matrices to represent each imaging condition, setting the $A$ term of the matrix, the magnification, to be -1 and setting $B$ to be 0. This will ensure imaging is satisfied, and we will develop our constraints from there.
First we will ensure that each objective mirror images onto the other with a magnification of -1. For the purposes of our calculations we can treat the MEMS as a flat mirror, or a mirror with an infinite radius of curvature, looking at equation 8.3b this turns into the identity matrix. This system is shown in Figure 8.3, and the ray matrices representing it are in equation 8.4.

\[
\begin{pmatrix}
1 & d_1 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\frac{1}{f} & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & d_0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
\frac{1}{f} & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & d_1 \\
0 & 1
\end{pmatrix}
\]

\( (8.4) \)

This set of ray matrices simplifies down to one ray matrix, equation 8.5. First we require that the \( A \) term is -1, which develops the constraints in equation 8.6.

\[
f = d_1 \text{ or } d_0
\]

\( (8.6) \)

It is important to note that we have two different constraints here, but only one can be right in this case. When we set the \( B \) term to 0, we narrow this down to one constraint, equation 8.7.

\[
f = d_1
\]

\( (8.7) \)
Figure B.4 - Optical system for the second imaging condition

The second imaging condition requires that the MEMS images back onto itself through either objective mirror with a magnification of -1. This system is shown in Figure 8.4, and the ray matrices representing it are in equation 8.8.

\[
\begin{bmatrix}
1 & d_0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{bmatrix}
\begin{bmatrix}
1 & d_1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
\frac{1}{R} & 1
\end{bmatrix}
\begin{bmatrix}
1 & d_1 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-\frac{1}{f} & 1
\end{bmatrix}
\begin{bmatrix}
1 & d_0 \\
0 & 1
\end{bmatrix}
\]

(8.8)

This can be further simplified down to one ray matrix, equation 8.9.

\[
\begin{bmatrix}
-1 & 2(R(d_0 - d_1) - d_1^2) \\
0 & R \\
\end{bmatrix}
\begin{bmatrix}
1 & 0 \\
-1 & 1
\end{bmatrix}
\]

(8.9)

Using the requirements previously stated, we can set another constraint, equation 8.10.

\[
R = \frac{-d_1^2}{d_0 - d_1}
\]

(8.10)

It is important to note that these two constraints represent a good estimation, this is because the field lens was approximated as a thin lens, we can use the lensmaker equation [1] to relate the focal length to the design elements of our final lens. Figure 8.5 shows this, and equation 8.11 contains the equation.
Figure B.5 - An example lens

Here we assume the lens is in air, $n$ is the index of refraction of the lens, $T$ is the thickness at the center, $R_1$ is the radius of curvature of the first surface, $R_2$ is the radius of curvature of the second surface, and $f$ is the focal length of the lens.

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} - \frac{1}{R_2} + \frac{T(n - 1)}{nR_1R_2} \right)$$

(8.11)

There are several equations and several unknowns, so certain choices will be made out of catalog parts until a suitable lens can be found. Table 8.1 contains the chosen part’s specification.

It is important to note that the shape of the lens was chosen to attempt to match the shape of the wavefront leaving the MEMS. In the case of our original design, the assumption was made that the beams would be expanding as it reflected off the MEMS and converging as it reflected off the Objective mirrors. The meniscus lens was chosen so the angles between the wavefront and the surface of the lens are minimized, reducing aberrations.
Table B.1 - Physical Description of Field Lens

<table>
<thead>
<tr>
<th>Lens Specification</th>
<th>Physical Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lens</td>
<td>50.8mm BK7 Positive Meniscus Lens</td>
</tr>
<tr>
<td></td>
<td>Part No. LE1985-A</td>
</tr>
<tr>
<td>Focal Length</td>
<td>300.0 mm</td>
</tr>
<tr>
<td>Thickness at Center</td>
<td>5.1 mm</td>
</tr>
<tr>
<td>Index of Refraction</td>
<td>1.51509</td>
</tr>
<tr>
<td>R₁</td>
<td>279.1 mm</td>
</tr>
<tr>
<td>R₂</td>
<td>100.1 mm</td>
</tr>
</tbody>
</table>

B.3 Imaging Conditions on the Input

The input optics were changed from the original design, as discussed in Chapter 4. The necessary condition is the imaging of the input aperture on the input turning mirror. This system is shown in Figure 8.6.

![Figure B.6 - Optical system to ensure imaging on the input](image)

Equation 8.13 shows the set of ray matrices representing it.

\[
\begin{pmatrix}
1 & 0 & T \\
-\frac{(1-n)}{R₁n} & n & 0 \\
0 & 1 & \frac{T}{n}
\end{pmatrix}
\begin{pmatrix}
1 & 0 \\
\frac{(n-1)}{R₂} & n \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & d₄ \frac{1}{-1} & 0 \\
0 & 1 & f_i
\end{pmatrix}
\]

(8.13)

By setting our imaging conditions we develop a set of constraints, equation 8.14.
\[ f_i = f \]
\[ d_i = 2f \]  \hspace{1cm} (8.14)

**B.4 Imaging Conditions on the Data Taking Process**

There are two different setups used for the data taking process. The first is outlined in Figure 8.7 and was used to take images of the bounces on the face of the MEMS.

Equation 8.15 shows the set of ray matrices representing it.

\[
\begin{pmatrix}
1 & d_4 \\
0 & 1 \end{pmatrix} \begin{pmatrix}
1 & 0 \\
-\frac{1}{f_2} & 1 \end{pmatrix} \begin{pmatrix}
1 & d_3 \\
0 & 1 \end{pmatrix} \begin{pmatrix}
-\frac{1}{n} & 0 \\
\frac{n-1}{R_2} & 1 \end{pmatrix} \begin{pmatrix}
1 & T \\
0 & 1 \end{pmatrix} \begin{pmatrix}
-\frac{1}{n} & 0 \\
\frac{(1-n)}{R_i n} & 1 \end{pmatrix} \begin{pmatrix}
1 & d_0 \\
0 & 1 \end{pmatrix}
\]  \hspace{1cm} (8.15)

A set of constraints is developed from our imaging conditions, as shown in equation 8.16.

\[ f_2 = 0.497118(d_3 + 2.23594) \]
\[ d_4 = \frac{(d_3 - 598.854)f_2}{d_3 - f_2 + 300.041} \]  \hspace{1cm} (8.16)

The second system is used to take data on the output, and is somewhat more ambiguously defined. This process will be further explained in Appendix C, as no actual ray matrices were used.
C Alignment Procedures

This appendix will contain all alignment procedures for the White Cell. First will be
the input optics, second will be the White Cell itself, third will be the output optics, and
fourth will be the second camera setup used to take data.

C.1 Input Optics

There are several components to the input optics, and to ensure there alignment
the following steps were used:

1) The first step is to establish an optical axis using the input laser. This is
done using two apertures with some distance between them. The holes in
each aperture are arranged to form two points on a straight line in space.
The laser should pass through the center of both.

2) We now have an established axis with the laser; the next step is to ensure
the proper alignment of the microscope objective and pinhole aperture.
Using the apertures from the previous step, so we ensure the airy disk
diffractions along the same axis, the microscope objective should be moved
into the laser’s path.
   a. The pinhole objective moved into the microscope objective’s focal
      point.
3) If the aperture is not centered on the airy disk, move the microscope objective in the opposite direction the airy disk needs to move (so if the disk needs to go left, move the objective right), then repeat 2a.

4) We now have the airy disk centered on our optical axis. A 25.4 mm (1 inch) circle should be drawn on an optical card. The collimating lens should be placed approximately its focal length away from the pinhole aperture, centered along the optical axis using the airy disk, and parallel to the surface of the pinhole aperture, and the cue card should be placed some distance away from the collimating lens (~300mm was used).
   a. Manipulate the distance the collimating lens is from the pinhole aperture until the beam is collimated, using the optical card as a reference for the diameter of the lens.

5) Next the second aperture should be placed such that the center of the aperture overlaps the center of the airy disk.

6) The conditioning lens should be placed as close to the second aperture as possible, and the center of the conditioning lens should overlap the center of the second aperture.

7) Place two folding mirrors in space such that twice the focal length of the conditioning lens/field lens (they are equal) is between the conditioning lens and the field lens. This can be done as follows:
   a. Place the field lens in its proper location in space, without the MEMS behind it.
   b. Consider one mirror fixed, and place it close to the conditioning lens.
c. Place an optical card with a 2.25mm square box some distance away from the back of the field lens (~300 mm was used).

d. Move the second mirror such that it is at the focal length of the conditioning lens, so when the beam bounced off the first mirror comes to a point, and so that the beam bouncing off the second mirror passes through the field lens.

e. Move the second mirror until the beam passing through the field lens is collimated, using the optical card as a check.

8) Finally, place the MEMS in its proper location, and ensure that the two mirrors place the input beam on the input turning mirror.

   a. The tilt angles of the two mirrors should be manipulated such that the beam bouncing off the input turning mirror going out to the objective mirrors hits the center of the objective mirrors.

   b. This can be done by treating the beam bouncing off the input turning mirror as a ray. Use the second mirror to control its angle, and the first to control its position.

C.2 White Cell

There are several steps necessary to ensure the proper alignment of the White Cell.

1) Place the field lens in space at a fixed location.

2) The MEMS should be placed at the correct location directly behind the field lens.
3) To establish our optical axis, a separate laser will be used. First the laser should be placed at the proper height, such that its beam will pass through the center of the field lens.

4) Using two apertures as intersects for a straight line, place the laser such that it passes through both apertures, the field lens, and bounces off the MEMS.

5) Adjust the position of the MEMS so that the beam passing through the center of the field lens hits the MEMS at its center point.

6) Adjust the tilt of the MEMS until the center of the beam reflecting off it passes back through the same two apertures used in step 3.

   a. Since the AO1K has curved pixels, it is best to do this with an unreleased MEMS or a flat mirror placed in the MEMS mount.

We now have an established optical axis, and we know our field lens and MEMS are properly centered on it. The next step is to ensure the objective mirrors are aligned.

![Figure C.1 - Bounce Pattern](image)

7) First, the objective mirrors such that they are touching. Place the mirrors such that the point they are touching is centered on optical axis using the
laser from step 3. This will ensure the objective mirrors are centered on the same plane as the axis of the MEMS and field lens.

8) We need to ensure the objective mirrors are at the proper distance. First place each objective mirror such that it is at the focal point of the field lens, or where the reflection appears the smallest.
   a. The AO1K has curved pixels, again this may be easiest with an unreleased MEMS or flat mirror placed in the MEMS mount.

9) Begin to move the MEMS backwards from its current point. If the objective mirrors are placed properly, the optical beams passing through the field lens and moving toward the MEMS should be collimated. If moving the MEMS backwards does not change the size of the beams, they are properly placed.
   a. The objective mirrors should be moved if this is not the case, consider the field lens a fixed location.

10) We now need to place the objective mirrors so that the bounces properly form. First we will need to turn on the input optics so that the input beam hits the input turning mirror and is centered on the first objective mirror, as explained in section 9.1.

11) Using the bounce pattern in Figure 9.1 as a guide, manipulate the tilt angles of the first objective mirror until the beam reflected off it is placed where bounce 1 should be.

12) Using Figure 9.1 as a guide again, manipulate the tilt angles of the second objective mirror until the beam reflected off it is placed where bounce 2 should be.
13) At this point the two centers of curvature are properly placed on the MEMS, and the remainder of the bounce pattern, Figure 9.1, should form. Figure 9.2 shows this process visually.

Figure C.2 - Bounce Pattern placement guide

C.3 Output Optics

This section will outline the alignment as well as calibration procedures of the output optics.

C.3.1 Alignment

1) First, as described in the previous two sections, using two apertures as intersects for a straight line in space; a laser should be used to establish an optical axis. The two apertures should be placed far enough apart to encompass the entire output optics.

2) Place the eyepiece lens such that it is centered on the optical axis and in between the two apertures. The beam expanding outward of the eyepiece lens should be centered on the second aperture.
3) Now place the microscope objective centered on the optical axis and in between the two apertures. In the same manner as step 2, ensure that the beam expanding outward is centered on the second aperture.

4) We will now use the optical axis established and guide the beam coming off the output turning mirror to it. To do this, first place a folding mirror such that it reflects the output toward the output optics.

5) Adjust the folding mirror until the output goes through the first aperture.

C.3.2 Calibration

1) Using the first 3 steps in section 9.3.1, ensure that the output optics are all on the same optical axis.

   a. It is best to mount the output optics on translatable stages, because they will need to move along this axis.

2) First, set an objective of known size (a clear ruler was used), at a set distance away from the eyepiece lens. This object should be backlit with a light source.

3) Using an optical card, ensure that the light travels through the eyepiece lens properly.

4) Treating the eyepiece lens as a fixed point, adjust the microscope objective ensuring that the light is still traveling through it, and that an image forms properly out the output of the microscope objective.

5) Compare the objects known size with the size of the phenomena at the output. This is the magnification factor for an object at this distance away from the eyepiece lens.
C.4 Second Camera Setup

This section outlines the alignment and calibration of the second camera setup mentioned in Chapter 8. This second camera setup was used for taking data on the output of the White Cell.

C.4.1 Alignment

1) First establish an optical axis. Use two apertures spaced some distance apart as intersects for a straight line. Ensure that a laser passes through both apertures.

2) Place the camera, in this case a Sony XC-ST50 CCD, such that the beam from the previous step hits the center of the camera’s sensing area.

3) Place the imaging lens some distance away from the camera such that the beam from step 1 still hits the center of the camera’s sensing area.

   a. The distance chosen should be large enough so that when the output bounces off the output turning mirror, the features of the beam can be seen by the camera, but small enough that the camera can capture as much of the beam as possible.

C.4.2 Calibration

We now need to ensure we are properly conducting measurements. To do this we will find out where exactly in space our setup is imaging, and what the magnification factor is exactly.
1) Use an object of known size (a clear ruler for example) setup such that it is on the optical axis, can be kept at a fixed location, and is parallel to the plane of the camera’s sensing area.
   a. The laser used in the previous section can be used to ensure the object is on the optical axis.

2) Ensure that the object is properly backlit and that the camera’s sensing area can properly sense it.
   a. Spiricon LBA was used here to ensure the camera could properly sense it, and a simple LED light was used to backlight it.

3) Move the object along the axis until it comes to an image on the camera’s sensing area. Record this distance from the imaging lens, this is the object distance.
   a. This is when the object comes into focus and can clearly be seen in Spiricon LBA.

4) Measure the exact size of the object or the feature of interest using the camera, and compare it to the object or features actual size. This is the magnification factor for an object at this distance from the imaging lens.

Using the magnification factor and the object distance, we can measure exactly how much a beam is steered in our design.