ECONOMETRIC STUDIES OF RATIONAL CONSUMPTION DECISIONS
WITH LIQUIDITY CONSTRAINTS AND STOCHASTIC LABOR INCOME

DISSERTATION

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the Degree Doctor of Philosophy in the Graduate
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By

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* * * * *

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Approved by
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To the Loving Memory of My Grandmother

Fang-Lan Shen

(沈芳兰)

(1901-1992)
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# TABLE OF CONTENTS

DEDICATION .................................................. ii

ACKNOWLEDGMENTS .......................................... iii

VITA ........................................................... iv

LIST OF TABLES ............................................. vi

LIST OF FIGURES .......................................... viii

<table>
<thead>
<tr>
<th>CHAPTER</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>II. CONSUMPTION, INCOME AND LIQUIDITY CONSTRAINTS: EVIDENCE FOR NINETEEN COUNTRIES</td>
<td>5</td>
</tr>
<tr>
<td>2.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>2.2 The Permanent Income Hypothesis and the Alternative $\lambda$-Model</td>
<td>13</td>
</tr>
<tr>
<td>2.3 Model Specification and Some Econometric Issues</td>
<td>20</td>
</tr>
<tr>
<td>2.4 The Data</td>
<td>30</td>
</tr>
<tr>
<td>2.5 Individual Estimation Results for Nineteen Countries</td>
<td>36</td>
</tr>
<tr>
<td>2.6 System Estimation Results for Pooled Data from All Nineteen countries</td>
<td>42</td>
</tr>
<tr>
<td>2.7 Concluding Remarks</td>
<td>47</td>
</tr>
<tr>
<td>III. CONSUMPTION DECISIONS WITH STOCHASTIC LABOR INCOME TESTING THE IMPLICATIONS OF AN APPROXIMATE SOLUTION</td>
<td>49</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>49</td>
</tr>
<tr>
<td>3.2 The Permanent Income Hypothesis with Stochastic Labor Income</td>
<td>53</td>
</tr>
<tr>
<td>3.3 Testing Precautionary Saving in the Approximate Consumption Function</td>
<td>63</td>
</tr>
<tr>
<td>3.4 Empirical Results and Discussion</td>
<td>73</td>
</tr>
<tr>
<td>3.5 Concluding Remarks</td>
<td>85</td>
</tr>
<tr>
<td>Appendix to Chapter III</td>
<td>88</td>
</tr>
</tbody>
</table>
IV. COINTEGRATION OF CONSUMPTION AND DISPOSABLE INCOME:
   EVIDENCE FROM PANEL DATA OF TWELVE OECD COUNTRIES 91

4.1 Introduction .......................... 91
4.2 RE-PIH: Its Cointegration Implications .... 94
4.3 Review of Econometric Methods .......... 97
4.4 The Data ................................ 105
4.5 Empirical Results ....................... 107
4.6 Monte Carlo Simulations ................ 110
4.7 Concluding Remarks .................... 113

BIBLIOGRAPHY ............................ 153


LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Summary Statistics for $\Delta \ln C_t$ $\Delta \ln Y_t$ $\frac{\Delta \ln Y}{C_{t-1}^t}$$\frac{\Delta \ln Y}{C_{t-1}^t}$ No GS</td>
</tr>
<tr>
<td>2.2</td>
<td>Summary Statistics for $\Delta \ln C_t$ $\Delta \ln Y_t$ $\frac{\Delta \ln Y}{C_{t-1}^t}$$\frac{\Delta \ln Y}{C_{t-1}^t}$ With GS</td>
</tr>
<tr>
<td>2.3</td>
<td>Estimation of $\Delta \ln C_t=\mu+\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}$ $\frac{\Delta \ln Y}{C_{t-1}^t}-\mu\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}+\nu_t$ No GS : GMM</td>
</tr>
<tr>
<td>2.4</td>
<td>Estimation of $\Delta \ln C_t=\mu+\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}$ $\frac{\Delta \ln Y}{C_{t-1}^t}-\mu\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}+\nu_t$ With GS : GMM</td>
</tr>
<tr>
<td>2.5</td>
<td>Estimation of $\Delta \ln C_t=\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}$ $\frac{\Delta \ln Y}{C_{t-1}^t}+(1-\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}})(\mu+\sigma_t)+\nu_t$ No GS : GMM</td>
</tr>
<tr>
<td>2.6</td>
<td>Estimation of $\Delta \ln C_t=\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}}$ $\frac{\Delta \ln Y}{C_{t-1}^t}+(1-\lambda_{\frac{\Delta \ln Y}{C_{t-1}^t}})(\mu+\sigma_t)+\nu_t$ With GS : GMM</td>
</tr>
<tr>
<td>2.7</td>
<td>Comparison of $\mu$ and $\lambda$ with the Sample Mean of Six Variables</td>
</tr>
<tr>
<td>2.8</td>
<td>System Estimation of $\Delta \ln C_{it}=\mu_{i}+\lambda_{\frac{\Delta \ln Y_{it}}{C_{i,t-1}^t}}-\mu_{i}\lambda_{\frac{\Delta \ln Y_{it}}{C_{i,t-1}^t}}+\nu_{it}$ E-NL3SLS</td>
</tr>
</tbody>
</table>
2.9 System Estimation of
\[
\Delta \ln C_{it} = \mu_i + (\phi_i + \sum_{k=1}^{6} \phi_{ki}) \Delta \ln Y_{i,t-1} + \phi_i + \sum_{k=1}^{6} \phi_{ki} \Delta \ln Y_{i,t-1} + \nu_{it}
\]
E-NL3SLS

2.10 Decomposition of \( \lambda \) to Various Factors
Associated with Liquidity Constraints

3.1 GMM Estimation of \( \Delta \ln C_t = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \epsilon_t \)

3.2 GMM Estimation of \( \Delta \ln C_t = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \lambda \frac{W_t}{C_{t-1}} - \lambda(1+\beta)(1-\alpha) \frac{W_{t-1}}{C_{t-1}} + \epsilon_t \)

3.3 GMM Estimation of \( \frac{\Delta C}{W_{t-2}} = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{W_{t-2}} \right) + \epsilon_t \)

3.4 GMM Estimation of \( \frac{\Delta C}{W_{t-2}} = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{W_{t-2}} \right) + \lambda \frac{W_t}{W_{t-2}} - \lambda(1+\beta)(1-\alpha) \frac{W_{t-1}}{W_{t-2}} + \epsilon_t \)

3.5 GMM Estimation of the System \( \Delta \ln C_t = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \epsilon_t \)
and \( r_t = r + u_t \)

3.6 GMM Estimation of the System
\( \Delta \ln C_t = (\beta - \alpha) \lambda + \frac{\alpha(r-\beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \lambda \frac{W_t}{C_{t-1}} - \lambda(1+\beta)(1-\alpha) \frac{W_{t-1}}{C_{t-1}} + \epsilon_t \)
and \( r_t = r + u_t \)
3.7 GMM Estimation of the System
\[ \Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \left[ \frac{\alpha (r - \beta)}{r} \right] \frac{Y_t - W_t}{C_{t-1}} + \varepsilon_t \]
and \[ \Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + \nu_t \]

3.8 GMM Estimation of the System
\[ \Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \left[ \frac{\alpha (r - \beta)}{r} \right] \left( \frac{Y_t - W_t}{C_{t-1}} + \lambda(\frac{W_t}{C_{t-1}}) - \lambda(1 + \beta)(1 - \alpha) \frac{W_{t-1}}{C_{t-1}} \right) + \varepsilon_t \]
and \[ \Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + \nu_t \]

3.9 The Standard Error of Consumption Innovation Resulted from a Unitary Labor Income Innovation
\[ \alpha \sum \frac{1}{1 + \beta^i} (E_t - E_{t-1}) W_{t+i} = \alpha \cdot [P \left( \frac{1}{1 + \beta} \right)]^{-1} \]
and \[ (1 - \lambda) \alpha \sum \frac{1}{1 + \beta^i} (E_t - E_{t-1}) W_{t+i} = (1 - \lambda) \cdot \alpha \cdot [P \left( \frac{1}{1 + \beta} \right)]^{-1} \]

4.1 Dickey-Fuller, Augmented Dickey-Fuller and Phillips-Perron Test:
Per Capita Private Disposable Income \( Y_t \)

4.2 Dickey-Fuller, Augmented Dickey-Fuller and Phillips-Perron Test:
Per Capita Private Consumption \( C_t \)

4.3 Estimation of \( Y_t = \phi + \Pi C_t + \varepsilon_t \) : OLS

4.4 Engle-Granger, Augmented Engle-Granger and Phillips-Ouliaris
Test for Cointegration \( E_t \)

4.5 Testing Unit Root in Disposable Income with Panel Data

4.6 Testing Unit Root in Consumption with Panel Data

4.7 Testing Cointegration between Consumption and Disposable Income
with Panel Data

4.8 Results of Monte Carlo Simulations
for Finite Sample Critical Values
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 Consumption Growth Rate and Income Growth Rate for Australia, Austria, Belgium and Canada</td>
<td>139</td>
</tr>
<tr>
<td>2.2 Consumption Growth Rate and Income Growth Rate for Denmark, Finland, France and Germany</td>
<td>140</td>
</tr>
<tr>
<td>2.3 Consumption Growth Rate and Income Growth Rate for Greece, Ireland, Italy and Japan</td>
<td>141</td>
</tr>
<tr>
<td>2.4 Consumption Growth Rate and Income Growth Rate for Luxembourg, The Netherlands, Norway and Sweden</td>
<td>142</td>
</tr>
<tr>
<td>2.5 Consumption Growth Rate and Income Growth Rate for Switzerland, UK and US</td>
<td>143</td>
</tr>
<tr>
<td>2.6 Consumption Growth Rate and Income Growth Rate for Australia, Austria, Belgium and Canada</td>
<td>144</td>
</tr>
<tr>
<td>2.7 Consumption Growth Rate and Income Growth Rate for Finland, Germany, Greece and Japan</td>
<td>145</td>
</tr>
<tr>
<td>2.8 Consumption Growth Rate and Income Growth Rate for Norway, Switzerland, UK and US</td>
<td>146</td>
</tr>
<tr>
<td>2.9 Scatterplot of $\lambda_i$ and $(\Delta \ln Y)_i$, $i=1,\ldots,19$</td>
<td>147</td>
</tr>
<tr>
<td>2.10 Scatterplot of $\lambda_i$ and $r_i$, $i=1,\ldots,19$</td>
<td>147</td>
</tr>
<tr>
<td>2.11 Scatterplot of $\lambda_i$ and $(\Delta \ln (POP))_i$, $i=1,\ldots,19$</td>
<td>148</td>
</tr>
<tr>
<td>2.12 Scatterplot of $\lambda_i$ and UNEMP$_i$, $i=1,\ldots,19$</td>
<td>148</td>
</tr>
<tr>
<td>2.13 Scatterplot of $\lambda_i$ and $s_i$, $i=1,\ldots,19$</td>
<td>149</td>
</tr>
</tbody>
</table>
2.14 Scatterplot of $\lambda_i$ and $F_i$, $i=1,\ldots,19$  

3.1 Real Interest Rate (3-Month T-Bill Rate, 1953:2-1984:4)  

4.1 $Y_{it}$ Plotted Against $Y_{i,t-1}$  

4.2 $C_{it}$ Plotted Against $C_{i,t-1}$  

4.3 $|E_{it}|$ Plotted Against $|E_{i,t-1}|$
CHAPTER I

INTRODUCTION

The question of how consumers make their consumption decisions is of fundamental importance in economics. Because it is closely related to saving, consumption behavior has direct influence on capital accumulation in the economy. It therefore has a great impact on the growth and efficiency of the economy in the long-run. In the short run, because aggregate consumption is a major component of the gross national product, it has always occupied a central role in explaining aggregate economic fluctuations.

An important theory describing consumption is the rational expectation-permanent income hypothesis. It assumes that consumers are fully rational and forward looking, making their consumption decisions in order to maximize their life-time utility. This theory not only has significant public policy implications, it also provides methodological guidance to other models in economics that are based on the principle of intertemporal utility maximization. Some examples are the consumption asset pricing model, the real business cycle model and the endogenous growth model.
Two implications of the rational expectation-permanent income hypothesis have received the most attentions from economists: consumption should approximately follow a random walk; consumption should be cointegrated with disposable income. Recent studies have found that the first implication is violated in both aggregate and panel data, while the second implication yields conflicting conclusions when it is tested with data from different countries.

For the violation of the random walk hypothesis of consumption, two theories have been advanced to explain the negative results in the literature: imperfection of the capital market; and the precautionary savings of consumers in response to labor income uncertainty. For the conflicting results of cointegration between consumption and disposable income, they are likely due to the small power of the tests that those conclusions are based upon. More powerful tests may yield a robust conclusion.

The primary objectives of this dissertation are to test the validity of the claims that liquidity constraints and precautionary saving explain the violation of the random walk hypothesis of consumption, to provide empirical measurement of their importance if those claims are indeed relevant, and to test whether consumption is cointegrated with disposable income using panel data.

The dissertation consists of three essays, each constituting a chapter in the rest of this dissertation. In chapter II, I use the national accounts data from nineteen OECD countries to test whether liquidity constraints exist and to measure their importance if
evidence is found that they do exist. This is accomplished by nesting a fraction of current income consumers in the permanent income model. The generalized method of moments used in estimation overcomes various econometric difficulties involved. The results of the estimation reveal that in all countries a significant fraction of consumers do not behave in accordance with RE-PIH. Furthermore, by using the panel feature of the data, the magnitude of the fraction of the current income consumers is found to be indeed related to the prevalence of liquidity constraints. Specifically, it is significantly lower in countries with higher private saving rates and lower unemployment rates.

In chapter III, I use an approximate solution to model optimal consumption when the representative consumer faces labor income uncertainty. This approximate consumption function is based on Zeldes' numerical solution to the optimal consumption problem with CRRA utility and stochastic labor income. Unlike the certainty equivalence solution, this model assumes that the consumer discounts expected future labor income at a rate higher than the real interest rate. It therefore takes into consideration the precautionary savings of the consumer. The implications of the approximate consumption function, with and without the liquidity constrained consumers, are tested using quarterly US data. The evidence lends support to the claims of the approximate consumption function, particularly when liquidity constrained consumers are included. The empirical results of this chapter also imply that current consumption should be Granger
caused by variables in the lagged information set. Meanwhile, consumption should be smoother than labor income, even when the latter follows an integrated process. It therefore rationalizes the empirical anomalies documented in the consumption literature.

In chapter IV, I use the national accounts data from twelve OECD countries to test the nonstationarity of consumption and disposable income as well as their cointegration, which is implied by the RE-PIH. I found that when residual based cointegration tests are performed on individual time series data of each country, one cannot reject the null hypothesis that consumption and disposable income are not cointegrated. In comparison, when the data are pooled as a panel with time series and cross sections, the evidence strongly rejects the null hypothesis of no cointegration. It therefore reveals that the results obtained from the data of each individual country are due to the small sample sizes that they are based upon, and that consumption and disposable income are cointegrated as implied by the rational expectation-permanent income hypothesis.
CHAPTER II

CONSUMPTION, INCOME AND LIQUIDITY CONSTRAINTS:

EVIDENCE FOR NINETEEN COUNTRIES

2.1 Introduction

The relationship between consumption and income has long been a focus of interest among economists, since it is an integral part of many macroeconomic debates. Different perceptions of this relationship will often result in vastly different conclusions in many macroeconomic issues. For example, in considering the government stabilizing policies, the effectiveness of a temporary tax cut on aggregate demand will be perceived as substantially different according to different understandings of this relationship.

The traditional Keynesian model assumes current consumption to be determined primarily by the current level of disposable income. In particular, it is not significantly affected by the flow of expected future incomes. Under these assumptions, current consumption responds directly to current income to the full extent of the marginal propensity to consume. Consequently, fiscal policy such as a temporary tax cut is believed to be effective in affecting aggregate demand.
The permanent income hypothesis\(^1\), however, assumes individual household's current consumption to be based on its permanent income, which depends not only on its current income, but on the flow of expected future incomes as well. A temporary tax cut affects consumption and thus aggregate demand only so much as it affects permanent income, which will be changed little by an increase in current income alone. Fiscal policy is therefore believed to be ineffective for stabilization purposes in this framework.

In the past decade, numerous papers have appeared to test the empirical validity of the permanent income hypothesis (Hall (1978), Flavin (1981,1985), Hayashi (1982,1985a,1985b), Hall and Mishkin (1982), Mankiw (1981, 1982), Mankiw and Shapiro (1985), Bernanke (1984, 1985), Blinder and Deaton (1985), Campbell (1987), West (1988), Campbell and Deaton (1989), Zeldes (1989), Runkle (1991), etc). These studies differ in many regards. In terms of data sources, some studies used aggregate time series data, in particular, the post war U.S. data; others panel data, notably the Panel Study of Income Dynamics (PSID). In terms of consumption measurements, some used only nondurable goods and services; others only durable goods, or durable goods as well as nondurable goods and services. Furthermore, some of the studies assumed an \textit{a priori} constant real interest rate, others included the real interest rate as a variable in the consumption

\(^1\) Friedman (1957), henceforth PIH.
regression. A majority of these studies leads toward the rejection of the strict form of PIH.

One noticeable strand of the literature on this topic is closely related to the recent debate on whether aggregate macroeconomic time series variables should be characterized as integrated processes. Deaton (1987) first pointed out that if the labor income is indeed a difference stationary process, then the PIH fails in this context because it implies a consumption process that should be more volatile than it appears to be in the actual data. Quah (1990) provides an explanation to this apparent contradiction by orthogonally decomposing a difference stationary income process into permanent and transitory parts. He assumes that the consumer observes the different parts of the income process that an econometrician is not aware of.

Another strand of the literature tests the PIH using the "Euler equation" approach, which is to test the time series restrictions implied by the PIH. In this context, most of the studies found that


4. It is often referred to as "Deaton's Paradox" in the literature.

5. See also Christiano and Eichenbaum (1990).
besides the lagged consumption itself, other variables also have forecasting power for consumption. This evidence is inconsistent with the implication of PIH that consumption should be approximately a random walk.

While the tests in the empirical literature, especially those based on the Euler equation, have the rational expectation-permanent income hypothesis (RE-PIH) as their null, they are usually made possible only under one or more of the following maintained hypotheses: (1) Capital markets are perfect, in the sense that consumers can lend or borrow against future income flow at the market interest rates without quantitative limit; (2) Households' utility functions are intertemporally separable; (3) Momentary utility is separable with respect to different consumption goods. In particular, it is separable between durable consumption, and nondurable and service consumption; (4) In the tests based on aggregate time series data, aggregation over all agents is trivialized by looking merely at the behavior of a representative agent.

These maintained hypotheses, although virtually indispensable for practical purposes, pose serious challenges for the interpretation of the empirical rejections of RE-PIH reported in the literature. If the failures are indeed due to the assumptions of economic agents forming expectations rationally and making consumption decisions based on intertemporal utility maximization, validity of many conclusions based on applications of RE-PIH is questionable. Furthermore, certain doubts are cast on the credibility of other theories built upon the
hypotheses of rational expectation and intertemporal optimization. On
the other hand, the empirical failures may well have been the result
of other maintained hypotheses in the model. For example, if the
capital market is imperfect and consumers are liquidity constrained,
consumption change is likely to exhibit sensitivity to income change
that is beyond what can be predicted by the RE-PIH. The assumption of
intertemporal separability of utility also appears to be problematic,
as well as the simple aggregation within the representative agent
paradigm. Although the implications of these assumptions are not easy
to formulate and be separated from those of the null, it is
conceivable that they could jeopardize the empirical tests of RE-PIH.
Among all the maintained hypotheses, the consequences of liquidity
constraints have received the most scrutinies of researchers.

In a series of papers, Campbell and Mankiw (1989, 1990, 1991)
proposed a simple model that nests both the permanent income
hypothesis and the effect of liquidity constraints. In that model, they assume that a fraction $\lambda$ of disposable income accrues to
individuals ("rule-of-thumb" consumers) who consume their current
disposable income, while the remainder $1-\lambda$ accrues to individuals who consume their permanent income. The advantage of this simple nesting
model is that it is capable of synthesizing various findings in the

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6. Henceforth, the $\lambda$-model. Similar ideas were actually explored, although to much less an extent, by Hall (1978), Flavin (1981),
Hall and Mishkin (1982), Hayashi (1982) e.t.c.
literature. For example, regardless of whether income process contains a unit root or not, consumption, when characterized by the $\lambda$-model, may be smoother than both disposable income itself and the permanent income associated with it. It also explains why consumption can be Granger caused by variables other than itself, for example, by savings.

With estimation and testing approaches robust against various difficulties in the literature, Campbell and Mankiw (1991) estimated $\lambda$ in the model for six countries. Their results show that in most of those countries, $\lambda$'s are significantly different from zero, and are different from each other as well. Their explanation for the results is that consumers in those countries are liquidity constrained. Moreover, differences of $\lambda$'s reflect the fact that, in those countries, the extent of the liquidity constraints consumers face are different. For example, if $\lambda$ is to be interpreted as the fraction of consumers who are liquidity constrained, as suggested in their

7. They are US 0.351 (0.117); UK 0.203 (0.092); Canada 0.225 (0.107); France 0.401 (0.028); Japan 0.035 (0.366); Sweden 0.357 (0.173). The numbers in the parentheses are standard errors.

8. Japan is the exception.

9. Their empirical estimates of the $\lambda$'s, in fact, cannot be interpreted as the fraction of the disposable income that accrues to those (liquidity constrained consumers) who consume their current disposable income, or in a representative agent paradigm, for that matter, the fraction of the "rule-of-thumb" consumers in

(Footnote continues on next page)
papers, we may conclude that economies with higher $\lambda$'s are the ones with more liquidity constrained consumers. This explanation, although being most plausible in this context, may not be the only one that can account for their results. Other features of the $\lambda$-model, such as aforementioned intertemporal separability of utility and aggregation across consumers, may also have caused their empirical results. Further study is therefore called for to investigate whether the $\lambda$ in the nesting model is indeed an index of the liquidity constraints in those countries.

In this chapter I address two issues. First, I extend the $\lambda$-model to data from nineteen OECD countries, and provide more evidence of the relationship between consumption change and the contemporaneous income change in those countries. This exercise confirms the robustness of the characterization of consumption by the $\lambda$-model. Second, by pooling the data from all nineteen countries, I investigate whether the $\lambda$ estimates obtained in this chapter are associated with

(Footnote continued from previous page)

the economy. This is because they estimated the regression equation

$$\Delta \ln C_t = \mu + \lambda \Delta \ln Y_t + \varepsilon_t.$$  

The $\lambda$ in the above equation is, strictly speaking, the fraction of total consumption that is consumed by the "rule-of-thumb" consumers. Therefore, this $\lambda$ is different from the one estimated in equation (2.5) of this paper, which preserves its interpretation as the fraction of the disposable income that accrues to the "rule-of-thumb" consumers.
the liquidity constraints prevailing in those countries. By decomposing the $\lambda$ into various parts that may be associated with the liquidity constraints, I estimate the model with a panel composed of those nineteen OECD countries. Pooled data allow the effects on $\lambda$ from the various parts to be identified.

The rest of this chapter is organized as follows. Section 2.2 reviews the model of RE-PIH, and lays out the alternative $\lambda$-model. A modification is made on the empirical implementation of the $\lambda$-model suggested in Campbell and Mankiw (1989, 1990, 1991). This allows the $\lambda$ in the regression equation to preserve its interpretation as the fraction of disposable income that accrues to those "rule-of-thumb" consumers.\textsuperscript{10} Section 2.3 discusses model specification and points out some econometric issues that are associated with the estimation of the model. Section 2.4 discusses the data used in this chapter. Section 2.5 estimates the $\lambda$-model individually for nineteen OECD countries. Section 2.6 pools the data from the nineteen countries and the model is estimated in the full information context. A test is performed there to see whether the results obtained in section 2.5 can indeed be attributed to the different extent of the liquidity constraints that consumers face in those countries. Concluding remarks of this chapter constitute section 2.7.

\textsuperscript{10} This, however, makes it not as straightforward to explain the "excess smoothness" and "excess sensitivity" of the consumption time series. But it can still be done by invoking the conditional expectations and iterated projections arguments.
2.2 The Permanent Income Hypothesis and the Alternative λ-Model

2.2.a The Permanent Income Hypothesis

The infinitely-lived representative consumer is assumed to maximize the expected value of an intertemporally separable utility function, and to be subject to an intertemporal budget constraint. Specifically, in period $t$, the consumer solves the following problem:

$$\max_{C_t, A_t} E_t \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} U(C_{t+i})$$

s.t. $C_t + A_t = W_t + (1+r)^{A_t}_{t-1}$

and

$$\lim_{i \to \infty} \frac{1}{(1+r)^i} A_{t+i} = 0$$

where

$E_t$ = mathematical expectation conditional on all information available in period $t$;

$\rho$ = rate of subjective time preference;

$r$ = real rate of interest, assumed to be constant over time. When it is written as $r_t$ with a time subscript, it is regarded as a variable;

$U(\cdot)$ = momentary utility function, $U' > 0$, $U'' < 0$;

$C_t$ = consumption in period $t$;

$W_t$ = labor income in period $t$;
$A_t =$ asset holding (not including human capital) in period $t$;

The intertemporal budget constraint in the model indicates that consumers can lend or borrow against future income flow at the market interest rate without quantitative limit, namely, capital markets are perfect. The transversality condition that the present value of the household's asset holding is zero in the infinite future guarantees insolvency will not arise for this individual.

The first order conditions for the above problem are

$$U'(C_{t-1}) = \frac{1+r}{1+r}E_{t-1}U'(C_t)$$

$t = 2, 3, \ldots$

These conditions reflect the fact that while being on the optimal consumption path, the consumer should not be able to increase expected utility by consuming one unit less today, increasing one unit of asset holding, and then increasing consumption tomorrow by $1+r$ units.

Under some simplifying assumptions about the utility function in the model\textsuperscript{11}, the $t$-th period F.O.C. can be stated as

$$E_{t-1}C_t = \mu + C_{t-1}$$

\textsuperscript{11. The assumptions are either $U(\cdot)$ is quadratic with $r=\rho$, this leads to the simplified F.O.C. of $E_t C_{t+1} = C_t$. Or, $U(\cdot)$ exhibits constant absolute risk aversion, i.e., $U(C_t) = \frac{-1}{\theta} \exp(-\theta C_t)$, $\theta > 0$. And conditional on the information known to them in period $t$, households' consumption in each period $t+i$ are normally distributed with a variance that depends only on $i$ (but not on $t$), then $E_t C_{t+1} = \mu + C_t$.}
or,

$$\Delta C_t = \mu + \varepsilon_t$$  \hspace{1cm} (2.1)$$

where \( \varepsilon_t \) is a random disturbance term such that \( E_{t-1} \varepsilon_t = 0 \).

Because aggregate time series data on consumption and disposable income have their mean change as well as innovation variance grow with their levels, they seem to be closer to log-linear than linear. In empirical analysis, it is therefore more plausible to model the log of consumption as following equation (2.1). This can also be justified if we assume \( U \) exhibits constant elasticity of intertemporal substitution, i.e.,

$$U(C_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma},$$

and conditional on the information known to them in period \( t \), households' consumption in each period \( t+i \) are log-normally distributed with a variance that depends only on \( i \) (but not on \( t \)).

\[\text{12. As will be discussed in section 2.3.1, it appears to be more realistic in empirical analysis to model } \varepsilon_t \text{ as being MA(1), namely } \varepsilon_t = \mu + \varepsilon_{t-1}, \]

where \( \varepsilon_t \) is a white noise, i.e., \( E(\varepsilon_t) = 0 \), \( E(\varepsilon_t^2) = \sigma^2 \), and \( E(\varepsilon_s \varepsilon_t) = 0 \) for \( s \neq t \). At this moment, however, we just consider \( \varepsilon_t \) to be a white noise error term.
Under these assumptions, we have\textsuperscript{13}

$$\Delta ln C_t = \mu + \varepsilon_t,$$ (2.2)

where $E_{t-1}\varepsilon_t = 0$.

Equations (2.1) and (2.2) are linear (and log-linear) approximations of the time series restrictions implied by the RE-PIH. They indicate that consumption in period $t$ should not be forecasted by any variable, dated $t-1$ or earlier, other than $C_{t-1}$, i.e., consumption is a martingale with respect to the consumer's information set.

The derivations of equations (2.1) and (2.2) above are based on the assumption of a constant real interest rate. If this assumption is relaxed, i.e., if we consider $r_t$ to be stochastic, then the first order conditions of the consumer's intertemporal optimization problem will be

$$U'(C_{t-1}) = \left(\frac{1}{1+\rho}\right)E_{t-1}[(1+r_t)U'(C_t)] \quad t = 2, 3, \ldots$$

If we assume the consumer's momentary utility is of the constant elasticity of intertemporal substitution type, and consumption as well as real interest rate are of log-normal distribution, we then have

$$\Delta ln C_t = \mu + \sigma r_t + \varepsilon_t,$$ (2.3)

\textsuperscript{13} I have used $\mu$ and $\varepsilon_t$ to denote, in various contexts, a constant term and a random disturbance term respectively, so long as it will not cause any confusion. This, however, should not be interpreted as their being the same in all occasions. Indeed, they should be different under different assumptions about the model, and certainly so when in different models.
where $E_{t-1}e_t = 0$, and $\sigma = \frac{1}{\gamma}$ is the elasticity of intertemporal consumption substitution for the permanent income consumers.

2.2.b The Alternative $\lambda$-Model

Following Campbell and Mankiw (1989, 1990, 1991), I nest the RE-PIH by assuming a fraction $\lambda$ of disposable income in the economy accrues to individuals (the "rule-of-thumb" consumers) who consume their current disposable income, while the remainder $1-\lambda$ accrues to individuals who are permanent-income consumers. More specifically, let $Y_t$ be the total disposable income in period $t$. If $Y_{1t}$ represents the part of the $Y_t$ that accrues to the "rule-of-thumb" consumers, then

$$Y_{1t} = \lambda Y_t.$$ 

Denote by $C_{1t}$ and $C_{2t}$ the consumption in period $t$ of the "rule-of-thumb" consumers and that of the permanent-income consumers respectively, we have, for the "rule-of-thumb" consumers,

$$\Delta C_{1t} = \Delta Y_{1t} = \lambda \Delta Y_t,$$

while for the permanent income consumers,

$$\Delta C_{2t} = \mu + \varepsilon_t.$$

Considering $C_t = C_{1t} + C_{2t}$, we then have

$$\Delta C_t = \Delta(C_{1t} + C_{2t}) = \Delta C_{1t} + \Delta C_{2t} = \lambda \Delta Y_t + (\mu + \varepsilon_t).$$
Formulated in levels, the nesting model that includes a fraction of the "rule-of-thumb" consumers can therefore be specified as

\[ \Delta C_t = \mu + \lambda \Delta Y_t + \varepsilon_t, \]

(2.4)

where \( \varepsilon_{t-1} \varepsilon_t = 0. \)

For the empirical analysis in the rest of this chapter, I assume consumer's momentary utility function is of the constant elasticity of intertemporal substitution type. In this case, according to equation (2.2), the F.O.C. for the permanent income consumers can be approximated as

\[ \Delta \ln C_{2t} = \mu + \varepsilon_t. \]

For the "rule-of-thumb" consumers, since \( C_{1t} = Y_{1t} = \lambda Y_t, \)

\[ \Delta \ln C_{1t} = \Delta \ln (\lambda Y_t) = \Delta (\ln \lambda + \ln Y_t) = \Delta \ln Y_t. \]

Considering \( C_t = C_{1t} + C_{2t}, \) we then have

\[ \Delta \ln C_t = \frac{\Delta C_t}{C_{t-1}} = \frac{\Delta (C_{1t} + C_{2t})}{C_{t-1}} = \frac{\Delta C_{1t} + \Delta C_{2t}}{C_{t-1}} \]

\[ \approx \frac{C_{1t-1} \Delta \ln C_{1t} + C_{2,t-1} \Delta \ln C_{2t}}{C_{t-1}} \]

\[ = \lambda \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t + \left[ 1 - \lambda \frac{Y_{t-1}}{C_{t-1}} \right] (\mu + \varepsilon_t) \]

\[ = \mu + \lambda \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t - \mu \lambda \frac{Y_{t-1}}{C_{t-1}} + \left[ 1 - \lambda \frac{Y_{t-1}}{C_{t-1}} \right] \varepsilon_t. \]
\[ \Delta \ln C_t = \mu + \lambda(\frac{Y_{t-1}}{C_{t-1}}) \Delta \ln Y_t - \mu(\frac{Y_{t-1}}{C_{t-1}}) + \nu_t, \]

where \( \nu_t = (1 - \lambda(\frac{Y_{t-1}}{C_{t-1}})) \epsilon_t \) and \( E_{t-1} \nu_t = (1 - \lambda(\frac{Y_{t-1}}{C_{t-1}})) E_{t-1} \epsilon_t = 0. \)

The parameter \( \mu \) in equation (2.5) is the expected consumption growth rate of the permanent income consumers. For the "rule-of-thumb" consumers, their consumption growth rate is a variable, which is equal to the growth rate of disposable income in the economy.

In the case of a stochastic real interest rate, the \( \lambda \)-model becomes

\[ \Delta \ln C_t = \lambda(\frac{Y_{t-1}}{C_{t-1}}) \Delta \ln Y_t + (1 - \lambda(\frac{Y_{t-1}}{C_{t-1}}))(\mu + \sigma r_t + \epsilon_t) \]

---

14. In the above derivation, I have used the fact

\[ \Delta \ln C_t = \ln(\frac{C_t}{C_{t-1}}) = \ln(1 + \frac{C_t - C_{t-1}}{C_{t-1}}) \]

\[ = \frac{C_t - C_{t-1}}{C_{t-1}} - \frac{1}{(1+\eta)^2} \cdot (\frac{C_t - C_{t-1}}{C_{t-1}})^2 \]

where \( 0 < \eta < \frac{C_t - C_{t-1}}{C_{t-1}} \), and the last equality follows from a second order Taylor expansion of \( \ln(1+x) \) at 1. Ignoring the second term, we then have

\[ \Delta \ln C_t \approx \frac{C_t - C_{t-1}}{C_{t-1}} \]

For practical purposes, we can be reasonably sure that \( \frac{C_t - C_{t-1}}{C_{t-1}} \) approximates \( \Delta \ln C_t \) with an error no more than 0.0025. Similarly, we have \( \Delta c_{1t} \approx c_{1,t-1} \Delta \ln c_{1t} \), \( \Delta c_{2t} \approx c_{2,t-1} \Delta \ln c_{2t} \).
\[ y_{t+1} = \mu + \lambda t^{-1} \Delta \ln Y_t + \sigma t^{-1} - \mu \lambda t^{-1} - \lambda \sigma t^{-1} + v_t \quad (2.6) \]

In this case, the expected consumption growth rate of the permanent income consumers is \( \mu \) plus the product of \( \sigma \) and the expected real interest rate, where \( \sigma \) is the elasticity of intertemporal consumption substitution.

2.3 Model Specification and Some Econometric Issues

2.3.a Model Specification and Estimation for Individual Countries

Equation (2.5) is nonlinear, but only in its parameters. One way of proceeding is to estimate the unrestricted linear model

\[ \Delta \ln C_t = \mu + \lambda t^{-1} \Delta \ln Y_t + \delta t^{-1} + v_t, \]

and then test the nonlinear over-identifying restriction \( \delta = -\mu \lambda \). The estimates from the unrestricted model, however, will have lower power. Considering that equation (2.5) is a well specified nonlinear model, I therefore estimate the parameters directly in the restricted model. This does not impose substantial difficulty in achieving estimation convergence, since it is a relatively simple nonlinear model.

In estimating equation (2.5), several other issues need to be taken into consideration. First, the error term \( v_t \) in equation (2.5), which equals \( 1 - \lambda t^{-1} \epsilon_t \), is likely to be heteroskedastic since
\( \frac{Y_{t-1}}{C_{t-1}} \) varies over time. Furthermore, \( \nu_t \) may have a first order moving average structure as well\(^{15}\). An MA(1) error term may arise from these features of the model: (a) Durability of consumption goods (Mankiw 1982), when we use consumption expenditure, which includes durable goods, as our consumption measure; (b) Time aggregation or time averaging of data (Working (1960)), which will induce a first order moving average error structure even when the original series can be characterized as a random walk; (c) White noise measurement error in the consumption data; and (d) White noise "transitory consumption" caused by, say, preference shocks. In both (c) and (d), white noise error in the level of consumption will become MA(1) in the first order condition.

Second, if \( \nu_t \) has an MA(1) structure, it must be correlated with the regressors \( \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t \) and \( \frac{Y_{t-1}}{C_{t-1}} \), although it should be uncorrelated with these variables when they are lagged at least twice. Equation (2.5) therefore cannot be estimated by GLS, since the estimates will be inconsistent.

Campbell and Mankiw (1987, 1990, 1991) suggest that the instrumental variables approach be used under a similar circumstance. This approach appears to be appropriate in the current situation as

\(^{15}\) See footnote 12.
well. A NL2SLS estimation with appropriately chosen instruments, for example, will produce consistent estimates for the parameters of the model. However, when $\nu_t$ is MA(1), the conventional standard errors obtained from NL2SLS are inconsistent. Consequently, statistical inferences based on them are invalid. This problem can be resolved by using an approach suggested by Newey and West (1985). Their approach yields standard errors that are heteroskedasticity and autocorrelation consistent.

Another method, which is the one I use in this chapter, is the generalized method of moments (GMM) (Hansen (1982), Hansen and Singleton (1982)). GMM is appropriate in estimating this model since $E_{t-2} \nu_t = 0$, so that the orthogonality conditions

$$E(Z_{t-2} \nu_t) = 0,$$

should hold on the data, where $Z_{t-2}$ is a constant or any variable that belongs to $\Theta_t$, the consumer's information set at $t-2$. It yields parameter estimates and their standard errors that are consistent even when the error term is correlated with the regressors, and is heteroskedastic and serially correlated.

The above discussed two methods of estimation are both consistent. However, they are likely to produce different results in

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16. GMM should be based on $E_{t-2} \nu_t = 0$ in this case because the conditional expectation $E_{t-1} \nu_t$ will not be zero given that $\nu_t$ is MA(1).
a finite sample. In fact, GMM estimates are obtained by using an (iterative) two step NL2SLS. The optimal weighting matrix used in performing the second step NL2SLS is constructed with residuals from the first step unweighted NL2SLS. Therefore, GMM is likely to yield more efficient estimates than a simple NL2SLS.

The selection of instrumental variables from $\Omega_{t-2}$ is based on the following considerations: (i) The instrumental variables should be stationary, because the asymptotic theory justifying the statistical inferences is valid only when the time series variables used in the estimations are stationary; (ii) The instrumental variables should be correlated with the regressors as much as possible.

The specific instrumental variables used in the empirical analysis of this chapter are: constant, twice to fourth lagged $\Delta lnC_t'$, $\frac{Y_{t-1}}{C_{t-1}}\Delta lnY_t$, and $\frac{Y}{C_t}$. ($r_t$ is also used when estimating equation (2.6)). Differencing of $lnC_t$ and $lnY_t$ avoids the problem that consumption and disposable income processes may contain a unit root for the US (Nelson and Plosser (1982)) and other countries (Kormendi and McGuire (1990)). The lagged disposable income to consumption
The ratio $\frac{Y_t}{C_t}$ is included because, as shown by Campbell (1987), $\ln Y_t - \ln C_t$, which is a stationary variable\(^{17}\), Granger causes income change.

An asymptotic test of the over-identifying restrictions, which are imposed on the model by the fact that there are more orthogonality conditions than parameters to be estimated, is also provided in Hansen (1982). The test, which is based on the $\chi^2$ distributed $J$-statistic with degrees of freedom equaling to the number of moment conditions less the number of estimated parameters, is also used in this chapter to verify the plausibility of the $\lambda$-model.

2.3.b Model Specification and System Estimation for Pooled Data from Nineteen Countries

Let $C_{it}$ and $Y_{it}$ denote the private consumption and private disposable income in period $t$ of a representative consumer in country $i$, where $t = 1, \ldots, T$ and $i = 1, \ldots, N^{18}$. According to the $\lambda$-model, we have for country $i$,

$$\Delta \ln C_{it} = \mu_i + \lambda_i \left(\frac{Y_{i,t-1}}{C_{i,t-1}}\right) \Delta \ln Y_{it} - \mu_i \lambda_i \left(\frac{Y_{i,t-1}}{C_{i,t-1}}\right) + v_{it} \quad (2.7)$$

\(^{17}\) $\frac{Y_t}{C_t}$ will be stationary if $\ln Y_t - \ln C_t$ is strictly stationary.

\(^{18}\) In our data, $i = 1, \ldots, 19$, $t = 1960, \ldots, 1988$. 
\[
\begin{align*}
  y_{it} - &\frac{\gamma_{it-1}}{C_{i,t-1}} \Delta \ln y_{it} - \frac{\gamma_{i,t-1}}{C_{i,t-1}} \mu_i + \lambda_i + v_{it} \\
  &= f_i (x_{it}, \beta_i) + v_{it}, \quad t = 1, \ldots, T.
\end{align*}
\]

where \( \mu_i \) is the expected consumption growth rate of the permanent income consumers and \( \lambda_i \) is the fraction of "rule-of-thumb" consumers in country \( i \). \( v_{it} \) is the error term associated with \( t \)-th period variables in country \( i \). I assume \( v_{it} = u_{it} + \theta_i u_{i,t-1} \), where \( u_{it} \) is a serially uncorrelated random error with mean zero. \( \theta_i \) is a parameter that measures the durability of average consumption goods in country \( i \).

Because the countries studied in this chapter are all OECD members, it is reasonable to expect that there are significant integration of economic activities among them. Reflected in our model, this can be represented by correlation of the error term among all the countries, namely,

\[
E(v_{it}v_{jt}') = \Sigma_{ij}, \text{ for } i, j = 1, \ldots, N,
\]

where \( \Sigma_{ij} \) is a TXT matrix. If we assume the durability of average goods in all countries are the same, then we can parameterize \( \Sigma_{ij} \) as \( \sigma_{ij} \), where \( \sigma_{ij} = E(u_{it}u_{jt}) \), and \( V = [v_{ij}]_{TXT} \) is a band matrix, i.e.,

\[
\begin{align*}
  v_{ij} &= 1 + \theta^2, \quad \text{if } i = j; \\
  &= \theta, \quad \text{if } |i - j| = 1; \\
  &= 0, \quad \text{if } |i - j| > 1;
\end{align*}
\]
A stacked equation for all countries can be written as,

\[ Y = F(X, \beta) + \nu \]

where \( Y, F, \) and \( \nu \) are \( NT \times I \) matrices and \( \beta \) is a \( 2N \times I \) matrix. They are defined as

\[ Y = (\Delta \text{Inc}_1, \ldots, \Delta \text{Inc}_1T, \Delta \text{Inc}_21, \ldots, \Delta \text{Inc}_2T, \ldots, \Delta \text{Inc}_{N1}, \ldots, \Delta \text{Inc}_{NT})' \]

\[ F = (f_{11}, \ldots, f_{1T}, f_{21}, \ldots, f_{2T}, \ldots, f_{N1}, \ldots, f_{NT})' \]

\[ \beta = (\beta_1, \beta_2, \ldots, \beta_N)' = (\mu_1, \lambda_1, \mu_2, \lambda_2, \ldots, \mu_N, \lambda_N)' \]

\[ \nu = (\nu_{11}, \ldots, \nu_{1T}, \nu_{21}, \ldots, \nu_{2T}, \ldots, \nu_{N1}, \ldots, \nu_{NT})' \]

and \( E(\nu \nu') = (\sigma_{ij})_{N \times I} \) is the variance-covariance matrix of the stacked equations. It is an \( NT \times NT \) matrix.

Since the error term, \( \nu \), is correlated with the regressors \( X \), this model cannot be estimated by the nonlinear seemingly unrelated regressions (NLSUR) method because the parameter estimates will be inconsistent. I therefore estimate the model using the estimated nonlinear three stage least least square (E-NL3SLS) \(^{19}\). Because the instrumental variables I use in the system estimation are all from \( \Omega_{t-2} \) and therefore uncorrelated with \( \nu \), it guarantees the consistency of the model estimation by E-NL3SLS.

An E-NL3SLS estimation can be described as being consisted of the following steps: step 1, for \( i = 1, \ldots, N \), \( \Delta \text{Inc}_i = f_i(X_i, \beta_i) + \nu_i \).
estimated by NL2SLS with instrumental variables \( W \). We denote the parameter estimates obtained in this step as \( \hat{\beta}_i \). Step 2, \( \Sigma_{ij} \), which is parameterized as \( \sigma_{ij}^2 \), is estimated through the residuals obtained from the previous step. Namely,

\[
\hat{\sigma}_{ij} = \frac{1}{T} \left[ \Delta \ln c_i - f_i(x_i, \hat{\beta}_i) \right] \left[ \Delta \ln c_j - f_j(x_j, \hat{\beta}_j) \right], \quad i, j = 1, \ldots, N
\]

\[
\hat{\theta} = \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( \frac{1}{T} \sum_{t=2}^{T} \left[ \Delta \ln c_i, t - f_i(x_i, t, \hat{\beta}_i) \right] \left[ \Delta \ln c_j, t - f_j(x_j, t, \hat{\beta}_j) \right] \right).
\]

Step 3, \( \beta^* \), the parameter estimates of \( \beta = (\beta_1, \ldots, \beta_N) \) by E-NL3SLS, is estimated by minimizing

\[
(Y - f(X, \beta))' W (W' \hat{\Sigma}^{-1} W)^{-1} W' (Y - f(X, \beta))
\]

where \( \hat{\Sigma} = (\hat{\sigma}_{ij})_{N \times N} \) is an \( NT \times NT \) matrix.

The above discussed system estimation technique can be applied in two ways in this chapter: First, it is used to estimate the \( \lambda \)-model, so that the precision of the parameter estimates can be improved because of the cross country correlation of the error terms. More importantly, it is used to investigate the relation between the \( \lambda \) in all countries and the liquidity constraints believed to exist in those countries.

Intuitively, it is clear that if indeed the fraction \( \lambda \) of the "rule-of-thumb" consumers in the model are different across countries because the liquidity constraints consumers face in these countries are different, we should expect them to depend on the following factors:
(i) Labor market characteristics such as the unemployment rate in those countries. It is plausible to expect that economies with high unemployment rates will have more liquidity constrained consumers than economies with low unemployment rates.

(ii) The private saving rate in those countries. It is conceivable that more people will be liquidity constrained in a country with a low private saving rate than in a country with a high one.

(iii) For a given private saving rate, the concentration of the savings within the population. A small fraction of the population with each saving a large amount and a large fraction with each saving a moderate amount may result in the same private saving rate in the economy, but there will be more liquidity constrained consumers in the former case than in the latter. This implies that demographic variables such as population growth rate and the proportion of young people (say from 20 to 34) in the country should have explanatory power for \( \lambda \). The population growth rate matters in this case because the fraction of the young households will be larger in a fast growing population than in a slow one. The young workers are more likely to be liquidity constrained because of the hump-shaped age-earning profiles in market economies.

(iv) The expected growth rate of disposable income. A high expected growth rate of disposable income will result in a steeper expected age-earning profile for the consumers in the economy. They are consequently liquidity constrained for a longer time because the
period when their actual disposable income is below their optimal consumption path will be longer.

(v) Expected real interest rates. Depending on the magnitude of intertemporal substitution, a higher expected real interest rate may provide incentive for consumers to substitute future consumption for current consumption, thereby reducing the likelihood of being liquidity constrained when they are young.

Let \( Z_{0it} = 1 \), and \( Z_{kit} \), where \( k = 1, \ldots, K \), \( i = 1, \ldots, N \), \( t = 1, \ldots, T \), be the \( t \)-th period observation in country \( i \) of the \( k \)-th factor that may have explanatory power for \( \lambda_i \). The variable coefficient model for any one country,

\[
\Delta \ln C_t = \mu + (\sum_{k=0}^{K} \phi_k Z_{kt}) \Delta \ln Y_t - \mu (\sum_{k=0}^{K} \phi_k Z_{kt}) \mu_t + \nu_t
\]

seems to be the appropriate model to estimate in order to recover the effects of \( Z_k \)'s on \( \lambda \). However, because of the limited number of time series observations, especially when the data are annual, this approach will yield results that do not have much power. It is therefore necessary to take advantage of the panel feature of the \( N \) countries available. A more appropriate approach in this case is first to do a panel transformation on \( Z_{kit} \) for \( k = 1, \ldots, K \). Suppose the transformed data are denoted as \( \bar{Z}_{kit} \), we can then apply the E-NL3SLS to the system

\[
\Delta \ln C_{it} = \mu_i + (\sum_{k=0}^{K} \phi_{ki} \bar{Z}_{kit}) \Delta \ln Y_{it} - \mu (\sum_{k=0}^{K} \phi_{ki} \bar{Z}_{kit}) \mu_{it} + \nu_{it}
\]
where \( i = 1, \ldots, N, \ t = 1, \ldots, T. \)

Significance tests can be performed on the \( \phi_{ki} \) coefficients of this model to verify whether the \( k \)-th factor (\( k = 1, \ldots, K \)) has an effect on the fraction of the liquidity constrained consumers in country \( i \). Cross equation tests of, say, \( \phi_{ki} = \phi_{kj} \) for \( i \neq j \) and \( \phi_{k1} = \phi_{k2} = \ldots = \phi_{kN} \) can be performed to see if the effect of \( Z_k \) on \( \lambda \) are the same across countries.

2.4 The Data

Data for the following nineteen OECD countries are used in the empirical analysis: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DEN), Finland (FIN), France (FRA), Germany (GER), Greece (GRE), Ireland (IRE), Italy (ITA), Japan (JPN), Luxembourg (LUX), the Netherlands (NET), Norway (NOR), Sweden (SWE), Switzerland (SWI), the United Kingdom (UKD), and the United States (USA).

Data on the following variables are obtained from OECD National Accounts: Main Aggregates (1990). They are annual from 1960 to 1988 for all nineteen countries. (The names in the parentheses are the ones used in the data tapes from which they are extracted.)

(a) Private final consumption expenditure, valued at current prices (POPC);

(b) National disposable income, valued at current prices (NDI);
(c) General government consumption expenditure, valued at current prices (POGC);
(d) Mid-year estimates of population (POP);
(e) Price indices of private final consumption expenditure (IDXPRIPC, 1985 = 100);
(f) Price indices of gross domestic product (IDXPRIGDPE, 1985 = 100);
(g) Price indices of general government consumption expenditure (IDXPRIGC, 1985 = 100);
Annual data from 1960 to 1988 on
(h) General government net saving, valued at current prices (SAVGG);
are available for twelve countries from OECD National Accounts: Detailed Tables (1990). These twelve countries are Australia, Austria, Belgium, Canada, Finland, Germany, Greece, Japan, Norway, Switzerland, the United Kingdom\(^\text{20}\), and the United States.

Annual data on nominal interest rates are taken from International Monetary Fund: Yearbook of International Financial Statistics. They are available from 1961 to 1988 for the following ten countries: Belgium, Canada, France, Germany, Greece, Japan, the Netherlands, Switzerland, the United Kingdom and the United States.

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\(^{20}\) The 1988 SAVGG value for the United Kingdom is not available in the 1990 edition of OECD National Accounts: Detailed Table. It is available in the 1991 edition and that value is therefore used.

Annual data on the unemployment rate are taken from Annex 1.6 of Layard, Nickell and Jackman (1991). The data are available from 1960-1988 for all countries except Greece and Luxembourg. Their data are taken from OECD: Economic Outlook, Dec. 1990 and earlier issues. All unemployment rates are standardized except for those of Austria, Denmark, Ireland and Sweden. For more details, see Annex 1.6 of their book.

The data on age distribution of population for all countries are from United Nations: Population Studies No. 122 (1991). It provides age distribution of population at every five years age intervals from 0-80+. The observations are every five years from 1950 to 1985, with medium, high and low projections from 1990 to 2025. The sample mean of young workers ratios (20-34 to 20-64) are constructed using 1955-1985 observations plus the medium projection for 1990.

Real per capita private final consumption expenditure ($C_t$) is constructed by dividing nominal private final consumption expenditure (POPC) by both the price index of private final consumption expenditure (IDXPRIIPC) and mid-year estimates of population (POP). Real per capita private disposable income ($Y_t$) is constructed by dividing real private disposable income by the mid-year estimates of population (POP), where the real private disposable income is real national disposable income less real general government disposable
income. General government disposable income is the sum of general government consumption expenditure (POGC) and general government saving (SAVGG). Because data on general government saving (SAVGG) are available for only 12 of the 19 countries for the entire sample period (1960-1988). I use two measures of the private disposable income. The first measure is as what is described above, but this measure is only available for twelve countries. The second measure is constructed by setting the general government disposable income equal to general government consumption expenditure. This measure can be constructed for all 19 countries over the sample period with the available data.

The second measure, which is an approximation, may not change the empirical results substantially because: (1) SAVGG is relatively small compared to POGC; (2) If the government saving is used by the government to obtain physical or financial assets, and the government uses its asset income to make transfer payments or provide public consumption goods to the private sector, then we may expect an increase in the savings of the government will result in an equal amount increase in the expected present value of the future government.

21. Real national disposable income and real general government disposable income are obtained, respectively, by dividing nominal national disposable income (NDI) by price indices of gross domestic product (IDXPRIGDPE), and dividing nominal general government disposable income by price indices of general government consumption expenditure (IDXPRIGC).
consumption expenditures, when the government balances its budget.\footnote{The government budget balance condition is in fact irrelevant in this case, if we assume consumers are Ricardian, so that they will treat budget deficits as equivalent to future tax increases (Evans (1988,1991a)). Therefore, the proposition that increases in government saving are equivalent to expected present value of future government consumption increases is independent of whether the government budget is balanced or not.} From a dynamic perspective, the government's consumption increases in the current period are not due only to its current savings, but also its past accumulated savings, then its current savings increase will actually be approximately equal to its contemporaneous consumption increase as long as SAVGG does not change substantially over time. Under this scenario, private disposable income should conceptually be equal to national disposable income minus only general government consumption expenditure. The second measure is therefore approximately correct, while the first measure is over subtracted by SAVGG which has already had its representation in POGC. In practice, the second measure is an upper bound for private disposable income while the first measure is a lower bound. In the empirical analysis of this chapter, I list both the results when SAVGG is not taken into consideration and those when it is taken into consideration in the construction of private disposable income.

Another concern with the data used in this chapter is the difference between the concepts of consumption and consumption expenditures. The model I have discussed involves consumption, which
is different from the consumption expenditure provided in the data, particularly with respect to durable goods. Ideally, I should include expenditures on nondurable goods and services, plus the flow of services from the stock of durable goods. This problem, however, is reduced to some extent by the following features of the data and the estimation: (i) Annual data are used instead of data from more frequent observations (quarterly, monthly). It can reasonably be assumed that consumption is close to consumption spending in a given year even though some goods will last longer than one year; (ii) Aggregate data also helps to reduce the differences between consumption and consumption expenditure. The unusually high consumption spending of one household in a particular year, when it makes a durable consumption purchase, is likely to be offset in the aggregate data by the low spending of others that have made durable consumption purchases earlier; (iii) Any additional effects of durability may be captured by the MA(1) structure I have assumed in the specification of the model.

There are certain merits of using annual aggregate total consumption spending that are worth mentioning. By using total consumption spending, one allows the possibility that consumers, if liquidity constrained, may adjust durable consumption (or nondurable and service consumption) disproportionately when their current income
changes. This possibility is not entertained when only either nondurable and service consumption or durable consumptions are included. Therefore, in some sense, using total consumption in estimating the $\lambda$-model reduces the problem associated with the assumption of utility being separable among goods.

Using data with less frequent observations (annual, rather than quarterly or monthly) yields a smaller sample size, but makes consumption expenditure closer to consumption. Besides, it makes time separability of utility function more realistic. Furthermore, the seasonality problem, which exists in monthly or quarterly data, will be minimized in annual data.

2.5 Individual Estimation Results for Nineteen Countries

Table 2.1 and 2.2 list some basic sample statistics for the variables in equation (2.5). The results in Table 2.1 are the ones when government saving is not considered in constructing the private disposable income, while those in Table 2.2 are obtained when government saving is considered for the twelve countries that have data on them over the sample period.

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23. One of the components, say durables, may change more in case of a current income change to the extent that it will lead to an exaggerated estimate of $\lambda$, if based only on durable goods (Caroll and Summers (1989)). On the other hand, an estimate of $\lambda$, if based only on nondurables and services, may lead to a smaller estimate of $\lambda$ than it really is.
The average growth rate for private consumption expenditure \( c \) ranges from around 2% (Denmark, Sweden and Switzerland) to almost 5% (Japan), while the average growth rate for private disposable income ranges from about 2% or below (Australia, Denmark, Sweden, Switzerland and UK) to about 5.5% (Japan) in Table 2.1. In Table 2.2, after government saving is taken into consideration, the average growth rate for private disposable income increases to some extent (from about 0.1% to 0.7%) for all countries except the United Kingdom, for which it decreases slightly. This indicates that for those 11 countries, government saving rate has decreased over time. Column 5 in the two tables list the sample mean of \( y_t \) to \( c_t \) ratios, from which we can see, for example, Luxembourg and Switzerland have the highest values. This reveals that the private saving rates in these two countries are, on average, higher than those of the other countries.

In Table 2.1 and 2.2, I compare the sample variances of \( \Delta \ln c_t \) and \( \Delta \ln y_t \) by listing their ratio after column 4. In both tables, this ratio is smaller than 1 for all countries, except for Norway in Table 2.2. These results agree with the intuition that consumption process is smoother than the income process. For reference purpose, I also compare the sample variances of \( \Delta \ln y_t \) and \( \frac{y_{t-1}}{c_{t-1}} \Delta \ln y_t \) by listing their ratio after column 8. In both tables, this ratio is smaller than 1 for all countries, except for Japan in Table 2.2.
The comparison of sample variances of $\Delta \ln C_t$ and $\Delta \ln Y_t$ can be seen more clearly from the time series plots in Figure 2.1 to Figure 2.8. Regardless of whether government saving is considered in the measurement of private disposable income, $\Delta \ln C_t$ appears to be smoother than $\Delta \ln Y_t$. This fact is especially obvious in the cases of Luxembourg and Switzerland, while the contrary is true in the case of Ireland.

Table 2.3 and 2.4 list the results of GMM estimation of equation (2.5), using a constant and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t$

and $\frac{Y_t}{C_t}$ as instruments. The results in Table 2.3 for nineteen countries are obtained when government saving is not considered in constructing the private disposable income, while those in Table 2.4 are obtained when government saving is considered for the twelve countries that have the necessary data.

Columns 1, 2 and 3 provide the estimated coefficient $\mu$, its heteroskedasticity autocorrelation robust standard error, and its significance level. $\mu$ is expected consumption growth rate of the

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24. Since the model I have estimated is a nonlinear one, this significance level is based on a $\chi^2$ distributed asymptotic test with 1 degree of freedom rather than the conventional $t$-test, although they should be very close. The same significance levels are used in other tables of chapter 2.
permanent income consumers. Except for Austria and Belgium in Table 2.3, and Belgium, Germany and Japan in Table 2.4, all estimates are significant. For all the significant estimates, they are in reasonable agreement with actual experiences, only the estimate for Ireland appear to be at odds with intuition. An interesting phenomenon to notice here is that in all countries except Finland, the Netherlands, and US (with UK only in Table 2.4), the expected consumption growth rates of the permanent income consumers are smaller than the average consumption growth rates of the "rule-of-thumb" consumers. This indicates that the variance of the consumption distribution among the population in those countries tends to decrease over time.

Columns 4, 5 and 6 provide the estimate of $\lambda$ (the fraction of the "rule-of-thumb" consumers), its heteroskedasticity autocorrelation robust standard error, and its significance level. All estimates are significant and between 0 and 1. Luxembourg and Switzerland have small and insignificant $\lambda$ estimates, while Ireland and Belgium have large and significant $\lambda$ estimates. These are consistent with what we have observed in Table 2.1 and 2.2, namely, high $Y_t$ to $C_t$ ratios and small sample variances of $C_t$ relative to that of $Y_t$ in Luxembourg and Switzerland, and large sample variances of $C_t$ relative to that of $Y_t$ in Ireland. It is also noticeable that Germany, Japan and US have relatively large $\lambda$ estimates in Table 2.4. This may be due to the moderately low expected interest rate in those countries, or the fact
that in the early part of the sample period, Japan and Germany had relatively low per capita income. As for the US, it may be associated with the large and rapidly growing government debt and budget deficits in the later part of the sample period.

Column 7 and 8 list the test statistics for the over-identifying restrictions imposed on the model by the fact that there are more orthogonality conditions than the parameters to be estimated. The \( J \)-statistic is asymptotically distributed as \( \chi^2 \) with 8 degrees of freedom, because there are 10 orthogonality conditions and 2 parameters estimated. The results in both tables indicate that the \( J \)-statistics are never significant at conventional levels, confirming the plausibility of the \( \lambda \)-model in characterizing consumption in all nineteen countries.

Tables 2.5 and 2.6 list the results of GMM estimation of equation (2.6), using a constant and twice to fourth lagged \( \Delta \ln C_t \), \( \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t \), \( \frac{Y_t}{C_t} \) and \( r_t \) as instruments. The results in Table 2.5, for the ten countries with data available on \( r_t \), are obtained when government saving is not considered in constructing the private disposable income, while those in Table 2.6 are obtained when government saving is considered for the eight countries that have data both on government savings and real interest rates over the sample period.

The estimates of \( \mu \) and \( \lambda \) in these tables are again very reasonable. All countries, except Greece, Switzerland and UK, have
larger $\mu$ estimates in column 1 than those in Tables 2.3 and 2.4. This is because the expected consumption growth rate of the permanent income consumers is $\mu + \sigma E(r_t)$. Therefore, the results on $\mu$ are consistent with the $\sigma$ estimates in column 7, which are significantly negative in those countries for which $\mu$ has increased. For UK, $\mu$ has decreased because $\sigma$ is significantly positive, while for Greece and Switzerland it is because they have insignificant $\sigma$ estimates.

The $\lambda$ estimates for Belgium and Germany in these two tables have decreased from those in Tables 2.3 and 2.4. This reveals that the large estimates of $\lambda$ in Table 2.3 and 2.4 for those two countries are likely due to the relatively low real interest rates they experienced during the sample period.

Most of the $\sigma$ estimates in Table 2.5 and 2.6 are either significantly negative or insignificant. Only UK has significantly positive $\sigma$. Moreover, all estimates have absolute values smaller than one. The $\sigma$ has the interpretation of intertemporal elasticity of consumption substitution. The results here imply that the momentary utility function is convex in eighteen countries, which is a contradiction with the consumer's maximizing behavior we have assumed\textsuperscript{25}.

\textsuperscript{25} Hall (1988) also obtained a negative significant estimate of $\sigma$ for US. He found that the negative sign of $\sigma$ was not sensitive to the choice of instruments in $\hat{r}_{t-2}$. The aggregation of interest rate in his paper was constructed so that it was consistent with the

(Footnote continues on next page)
Finally, the \( J \)-statistics for the over-identifying restrictions in Table 2.5 and 2.6 are all insignificant, with even higher significance levels than those in Table 2.3 and 2.4 for a majority of the countries. This shows that it is as plausible, if not more so, to characterize consumption in all countries by a model with variable interest rate, rather than one with a constant real interest rate.

2.6 System Estimation Results for Pooled Data from All Nineteen Countries

The individual estimation results that I reported in the previous section support the claim that private consumption behavior can be characterized by the \( \lambda \)-model, particularly when the real interest rate is assumed to be a constant. The results also appear to be robust with respect to whether government saving is considered in the construction of private disposable income. Alternative results from full information estimation of the system composed of equation (2.5) for all the countries involved, which I have listed in Table 2.8, lead to the same conclusions. These results reveal that, in virtually all

(Footnote continued from previous page) consumption data. The conclusions he drew from the negative \( \sigma \) was that the intertemporal substitution of consumption was rather insignificant.

\( \Box \)
of the countries in the sample, some consumers are not behaving in accordance with the permanent income hypothesis. Whether this fraction of "rule-of-thumb" consumers in an economy is related to the liquidity constraints in that country is the question that I will pursue in this section.

In Table 2.7, I have listed in columns 1 and 2 the estimates of $\mu$ and $\lambda$ obtained in Table 2.3, where real interest rates are considered to be constant, and government saving is not considered in the private disposable income measurement. I have also listed in columns 3 to 8 the sample means of the unemployment rate ($UNEMP$, column 3), the private saving rate ($S/Y$, column 4), the population growth rate ($\Delta ln(POP)$, column 5), the fraction of the population aged 20-34 in the population of ages 20-64 ($F$, column 6), the growth rate of the real per capita private disposable income ($\Delta lnY$, column 7), and the real interest rate ($r$, column 8). Values of $UNEMP$ for Greece, as well as $UNEMP$ and $r$ for Luxembourg, are not available from the data sets I have used. The private disposable income measure used in both $S/Y$ and $\Delta lnY$ does not take government saving into consideration, in order to be consistent with the values of $\mu$ and $\lambda$ listed in column 1 and 2.

Scatterplots of $\lambda_i$ paired with $UNEMP_i$, $(S/Y)_i$, $(\Delta ln(POP))_i$, $F_i$, $(\Delta lnY)_i$, and $r_i$ are shown in Figure 2.9 to Figure 2.14 for $i=1,\ldots,19$, in order to see if there is any relationship between $\lambda$ and the sample means of those variables that may be associated with liquidity.
constraints. From those scatterplots, $\lambda_i$ appears to be positively correlated with $UNEMP_i$ and $F_i$, and negatively correlated with $S_i$, which is consistent with my analysis in section 2.3.b.

More formally, I have regressed $\lambda_i$ on those variables and the results are as the following:

$$\lambda_i = 0.295 + 0.049UNEMP_i$$
$$R^2 = 0.281$$
$$\lambda_i = 0.818 - 2.197(S_i)$$
$$R^2 = 0.381$$
$$\lambda_i = 0.489 - 2.134(\Delta ln(POP))_i$$
$$R^2 = 0.001$$
$$\lambda_i = -0.273 + 1.934F_i$$
$$R^2 = 0.034$$
$$\lambda_i = 0.443 + 1.134(\Delta lnY)_i$$
$$R^2 = 0.002$$
$$\lambda_i = 0.464 + 2.308x_i$$
$$R^2 = 0.038$$

The coefficients of $UNEMP_i$ and $(S_i)_i$ in the above regressions are positive and negative, respectively, and are highly significant. Each of the two regressions explains about 30% of the variation in $\lambda_i$. These results are in accordance with the scatterplots, and are consistent with our intuitions. The positive coefficients of $F_i$ and $(\Delta lnY)_i$ are also intuitively reasonable, but they are both insignificant.
The problem with these primitive checks is clearly the small sample size that they are based upon, particularly when the distribution of the error term is not known. Obviously, it will be rather inaccurate to base inferences of the relationship between λ and liquidity constraints only on the information contained in the sample means of the variables involved. In order to obtain more reliable results, it is necessary to take advantage of the panel feature of the data. In the rest of this section, I therefore perform system estimation of the model with pooled data from all nineteen countries.

I first estimate the following restricted model using the E-NL3SLS,

\[
\Delta \ln C_{i,t} = \mu_i + (\phi_0 + \sum_k \phi_k \bar{Z}_{ki}) (\frac{Y_{i,t-1}}{C_{i,t-1}}) \Delta \ln Y_{i,t} - \mu_i + (\phi_0 + \sum_k \phi_k \bar{Z}_{ki}) (\frac{Y_{i,t-1}}{C_{i,t-1}}) + \nu_{i,t}
\]

where \( \bar{Z}_{ki} \) is the sample mean (with respect to t) of variable k in country i, where \( k=1,...,6, i=1,...,19 \). I impose the cross equation restrictions that \( \phi_k = \phi \) for all \( i=1,...,19, k=0,...,6 \) to reduce the number of estimated parameters and obtain more powerful statistical inferences. The instruments are a constant and \( \Delta \ln C_{i,t-2} \) for \( i=1,...,19 \). The results from this estimation are listed in Table 2.9.

Columns 1 to 3 list the estimated coefficient \( \mu_i \), its standard error, and the \( t \)-statistic for the significance test. The results are very close to those in Table 2.8. Column 4 list the adjusted \( R^2 \)'s for each equation, based on the coefficients obtained by the E-NL3SLS,
which are above 40% for all countries except Australia, Austria, Luxembourg and Norway.

Table 2.9 also lists the estimates of \( \hat{\phi}_k \) \((k=0,\ldots,6)\), their standard errors, and the t-statistics for their significance tests. \( \hat{\phi}_{ki} \) \((k=1,\ldots,6, i=1,\ldots,19)\) has the interpretation of contribution of factor \( Z_k \) to \( \lambda_i \), the fraction of the consumers in country \( i \) who are liquidity constrained. From the table, it is clear that higher growth rate of disposable income, higher unemployment rate, and larger fraction of young population result in a larger fraction of the liquidity constrained consumers. On the other hand, higher private saving rate and higher population growth rate contribute to a smaller fraction of the liquidity constrained consumers. The effect of real interest rate on \( \lambda \) is insignificant.

The relative importance of contribution of factor \( Z_{ki} \) on \( \lambda_i \) \((k=1,\ldots,6, i=1,\ldots,19)\) can be seen more clearly from Table 2.10, where I list the decomposition of \( \lambda_i \) to \( \hat{\phi}_0 \), and \( \hat{\phi}_{kZ_{ki}}, k=1,\ldots,6 \).

Overall, the extent of contribution on \( \lambda \) from demographic variables.

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26. Intuitively, these two variables should be positively correlated. A high population growth rate results in a large fraction of the young population, while to maintain a large fraction of the young population, it entails a high population growth rate. Consequently, it is more informative to look at the combined effects from both \( F \) and \( \Delta \ln(Pop) \), rather than from each of the two individually.
$F$ and $\Delta ln(POP)$ is by far the largest, while those from $\Delta lnY$, $S/Y$, and $UNEMP$ are comparable. The effect from $r$ is negligible. These results make more explicit explanations of situations, say Japan, where private saving rate and disposable income growth rate are both high.

2.7 Concluding Remarks

In this chapter, I have estimated a simple extension of the PIH to include some current disposable income consumers. The results obtained from data for 19 OECD countries indicate that this simple nesting model is a reasonable characterization of private consumption behavior in virtually all nineteen countries. Evidence of contemporaneous correlation between the growth rate of private consumption and the growth rate of private disposable income have apparently revealed that at least some consumers in those economies do not behave according to the RE-PIH. More importantly, the degree of the correlation between current consumption change and current income change varies substantially across countries. This variation of $\lambda$ is undoubtedly related to the different extent of liquidity constraints consumers face in those countries.

It is not entirely clear, however, why consumers should set their consumption equal to their current disposable income, even if they are liquidity constrained. In fact, Beaton (1991) has shown that the optimization behavior of a liquidity constrained consumer is to accumulate some moderate amount of assets as a buffer stock to smooth
consumption for periods with a low level of income. This is especially true when the income process has little serial correlation.

Considering the results obtained in this chapter and Deaton (1991), it is therefore imperative to have a well constructed microeconomic model to explain why some consumers set their consumption equal to their current disposable income. In doing this, the effects of liquidity constraints, the statistical characteristics of the income processes and precautionary and other motives of saving should all be considered.
CHAPTER III

CONSUMPTION DECISIONS WITH STOCHASTIC WAGE INCOME:

TESTING THE IMPLICATIONS OF AN APPROXIMATE SOLUTION

3.1 Introduction

The simple life cycle/permanent income model described in most macroeconomic texts assumes that individuals determine their current consumption according to the sum of their financial assets and the present discounted value of their expected future labor income. This proposition is generally valid when labor income is nonstochastic. However, in the case of stochastic labor income, as being assumed in most circumstances, it is true only when some stringent assumptions on the momentary utility function and/or the labor income process of the consumers are maintained. This is because the underlying intertemporal optimization problem of a representative consumer cannot be explicitly solved with arbitrary uncertain labor income and concave utility.

A special case which validates the simple consumption decision rule is when the momentary utility is assumed to be quadratic. Quadratic utility is justified to some extent because it can be considered as a local approximation to the underlying utility of the
consumer. The explicit consumption function obtained in this special case is often called the certainty equivalence solution, since only the first moments of the stochastic labor income appear in it. Indeed, since Hall (1978), most studies of the permanent income model have relied on the certainty equivalence solution, which has the important implication that consumption should be approximately a random walk. Rejections of the consumption random walk in a majority of the studies, with either aggregate or panel data, are often interpreted as evidence of the prevalence of liquidity constraints.

Although it is simple to handle for empirical studies, the quadratic utility function is implausible, because it implies that consumers have increasing absolute risk aversion. That is, consumers are supposed to be willing to pay more to avoid a given lottery as their consumptions increase. Casual empiricism suggests that this is not a realistic description of rational consumers' behavior under uncertainty.

There is thus a need to investigate the more realistic case of decreasing (or at least, nonincreasing) absolute risk aversion. In fact, recent studies of the effect of government budget deficit on consumption has proven that explicit modelling of labor income uncertainty is fruitful. It generates new insights when consumers are assumed to have decreasing absolute risk aversion utility function (Barsky, Mankiw and Zeldes (1986), Kimball and Mankiw (1989)). These new developments also shed light on the prospect of studies of consumption with decreasing absolute risk aversion utility and
stochastic labor income. A necessary condition for utility to exhibit nonincreasing absolute risk aversion is marginal utility being convex, or the third derivative of the utility being positive. As a result, optimal consumption should generally be less relative to the certainty equivalence level, reflecting their prudence or precautionary savings (Leland (1968), Sandmo (1970) and Dreze & Modigliani (1972)).

The biggest disadvantage of considering this type of utility function is that in even very simple cases, a closed form solution does not exist. To allow for any empirical work, other restrictive assumptions would have to be imposed. An example of this is given by Caballero (1990, 1991). Caballero demonstrated that if utility is characterized by the constant absolute risk aversion, and both consumption and labor income innovations are i.i.d., then optimal consumption is the certainty equivalence level less a constant. The constant reflects the consumer's precautionary savings. Although his studies have emphasized precautionary savings, the explicit solution is reached at the sacrifice of realism because of his additional assumptions.

In this chapter I study a compromise solution that is empirically tractable, yet consider the effects of labor income uncertainty. Based on a numerical simulation by Zeldes (1989), I investigate an approximate consumption function in which the consumer is assumed to discount the uncertain future labor income at a rate higher than the real interest rate. The implications of that consumption function is then tested with quarterly US data. Evidence suggests that this
consumption function, particularly when we assume that a fraction of the consumers are liquidity constrained, is a good characterization of the aggregate US data. The claims of Zeldes's simulation are also supported by the data.

The primary purpose of this chapter is to provide a starting point for probing the usefulness of this approximate consumption function in empirical studies. Because the assumptions are on the parameters of the consumption function, we do not have to assume whether the consumer's utility function is in one form or another. Different characteristics of the utility function are not our primary concern here, although there might be a correspondence between the parameters of the utility function and those of the approximate consumption function.

The rest of this chapter is organized as follows: Section 3.2 lays out the general intertemporal optimization problem that underlies the permanent income hypothesis. Two tractable cases of the general problem are when labor income is nonstochastic and when utility is quadratic. Both cases provide the bases for the usual claim that consumption should be proportional to financial assets and present value of expected future income. This section also presents several results of Zeldes' simulation. An approximate consumption function is formulated based on his simulation results. Section 3.3 derives some implications of the approximate consumption function. Some relevant econometric issues and the data are also discussed there. Section 3.4 presents and discusses the empirical results. In particular, the
results are used to explain the two empirical anomalies in the consumption literature. Section 3.5 concludes the chapter and proposes some possible future researches in this direction.

3.2 The Permanent Income Hypothesis with Stochastic Labor Income

3.2.a The General Intertemporal Consumption Problem of a Representative Consumer

The infinitely-lived representative consumer is assumed to maximize the expected value of an intertemporally separable utility function, and to be subject to an intertemporal budget constraint. Specifically, in period $t$, the consumer solves the following problem:

$$\max E_t \sum_{i=0}^{\infty} \left( \frac{1}{1+\rho} \right)^i U(C_{t+i})$$

(3.1)

s.t. $C_{t+i} + A_{t+i} = W_{t+i} + (1+r)A_{t+i-1}, i = 0, 1, 2, ...$

(3.2)

and

$$\lim_{i \to \infty} \left( \frac{1}{1+r} \right)^i A_{t+i} = 0$$

(3.3)

where

$E_t =$ mathematical expectation conditional on all information available in period $t$;

$\rho =$ rate of subjective time preference;

$r =$ real rate of interest, assumed to be constant over time;

$U(\cdot) =$ momentary utility function, assumed to be monotonically
increasing, concave and continuously differentiable to at least the third order;

\[ C_t = \text{consumption in period } t; \]

\[ W_t = \text{labor income in period } t; \]

\[ A_t = \text{nonhuman wealth in period } t; \]

The intertemporal budget constraints in the model indicate that consumers can lend or borrow against future income flow at the market interest rate without quantitative limit, namely, capital markets are perfect. The transversality condition that the present value of the household’s asset holding is zero in the infinite future guarantees insolvency will not arise for this individual.

The first order conditions for the above problem are

\[ U'(C_t) = \left( \frac{1+r}{1+\rho} \right) E_t U'(C_{t+1}), \quad t = 1, 2, 3, \ldots \]  \hspace{1cm} (3.4)

These conditions reflect the fact that while being on the optimal consumption path, the consumer should not be able to increase expected utility by consuming one unit less today, increasing one unit of asset holding, and then increasing consumption tomorrow by \(1+r\) units.
3.2.b A Special Case: Nonstochastic Labor Income

Consider the simple case when labor income is variable over time but nonstochastic\textsuperscript{27}. In addition, we assume that the time discount rate is equal to the real rate of return from nonhuman wealth. In this case, the first order conditions are

\[ U'(c_t) = U'(c_{t+1}) \quad t = 1, 2, 3, \ldots \]  \hspace{1cm} (3.5)

Because \( U \) is concave, marginal utility is monotonically decreasing. The F.O.C.'s in (3.5) therefore hold if and only if

\[ c_t = c_{t+1} \quad t = 1, 2, 3, \ldots \]  \hspace{1cm} (3.6)

Equation (3.2), the consumer's intertemporal budget constraints, can be combined to be written as

\[ \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} c_{t+i} = (1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} w_{t+i} \]  \hspace{1cm} (3.7)

From (3.6) and (3.7), we get

\[ c_t = \frac{r}{1+r} (1+r) A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} w_{t+i} \]  \hspace{1cm} (3.8)

We conclude, in this special case of nonstochastic labor income, that the optimal consumption decision of a representative consumer is to follow a flat consumption path. The consumer uses the capital market, which is assumed to be perfect, to shield consumption from

\textsuperscript{27}. Alternatively, we can assume that the consumer has perfect foresight while letting labor income be stochastic.
changing over time in the presence of the ups and downs of the labor income stream. The optimal levels of consumption, from time $t$ on, are equal to the annuity value of the sum of the $t$-th period nonhuman wealth, $(1+r)A_{t-1}$, and human wealth, $\sum_{i=0}^{\infty} \frac{1}{1+r}^i W_{t+i}$. The latter is the present discounted value, as of period $t$, of all future (variable but certain) earnings.

A fact worth of noting here is that the consumption function derived in equation (3.8) is independent of whichever form of the momentary utility we assume it to be, as long as it is concave. In particular, it includes the special cases of quadratic, constant absolute risk aversion and constant relative risk aversion utilities that are frequently used in the literature.

3.2.c The General Case: Stochastic Labor Income — Certainty

Equivalence and Precautionary Savings

When future labor incomes are uncertain, there is generally no closed form solution to the consumer's intertemporal optimization problem stated in equations (3.1), (3.2) and (3.3). An exception to that is when the momentary utility function is quadratic

$$\varphi(C_t) = \frac{1}{2}(C^* - C_t)^2$$

where $C^*$ is the bliss level of consumption. In this case, and with the additional assumption of $\rho = r$, equation (3.4) can be written as
\[ C_t = E_t C_{t+1}, \quad t = 1, 2, 3, \ldots \]

The consumer's optimal level of consumption in period \( t \) can hence be written as

\[ C_t = \frac{1}{1+r} (1+r) A_{t-1} + \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i E_t W_{t+i} \]  

(3.9)

i.e., the optimal level of consumption in period \( t \) should be the annuity value of the sum of the \( t \)-th period nonhuman wealth, \((1+r)A_{t-1}\), and expected human wealth, \( \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i E_t W_{t+i} \), where the latter is the present discounted value of expected future labor income.

The solution in equation (3.9) is usually called the "certainty equivalence" level of consumption when the labor income is stochastic. As long as utility is quadratic, this formulation of optimal consumption is independent of the stochastic properties of the labor income involved, aside from its first moments. In particular, it is independent of the second or higher moments of the labor income innovations, and of whether the labor income follows an integrated process or a stationary process.

An important implication of the certainty equivalence solution is that consumption in period \( t \) should not be forecasted by any variable, dated \( t-1 \) or earlier, other than the \((t-1)\)-th period consumption itself, namely, consumption is a martingale with respect to the consumer's information set. This fact has been a fundamental theme in the literature on the studies of consumption. Since Hall (1978),
numerous papers have appeared, either to test the validity of that implication, or to obtain estimates of important parameters, e.g., the consumer's attitude toward risk, by exploiting that implication.

An unattractive feature of the quadratic utility is that it implies increasing absolute risk aversion, that is, the consumer is assumed to be willing to pay more to avoid a given lottery as his or her consumption increases. A moment of reflection, as well as casual empiricism, suggests that a utility exhibiting decreasing absolute risk aversion is a more realistic description of rational consumers' behavior under uncertainty.

When consumers have decreasing absolute risk aversion utility, their optimal consumption will be less than the certainty equivalence level, and their consumption profile over time will also be steeper (see, for example, Leland (1968), Sandmo (1970) and Dreze & Modigliani (1972)). Caballero (1990) has shown that when the utility is characterized by the constant absolute risk aversion\(^{28}\), consumption should be the certainty equivalence level less a constant. The constant, which measures the precautionary savings, depends on the coefficient of absolute risk aversion and the riskiness of the labor income. For the aggregate labor income risks, it apparently depends

\(^{28}\) And when certain restrictive assumptions are imposed on both the consumption and income processes. This point is not explicitly emphasized when he derived the simple result that the precautionary saving can be represented by a constant.
on both the variance of the labor income shocks and the persistence of the labor income process.

Zeldes (1989) provides a numerical solution to the optimal consumption problem with momentary utility exhibiting constant relative risk aversion. The results of his simulations show that, when labor income is uncertain, optimal consumption deviates from the corresponding certainty equivalence level in, among others, the following ways:

(i) The marginal propensity to consume out of wealth, when labor income is uncertain, is consistently larger than that under the certainty equivalence. The difference between the two is especially significant when the amount of the (certain) nonhuman wealth is low relative to the expected future (uncertain) labor income.

(ii) The expected growth rate of consumption, when labor income is uncertain, is consistently higher than that under the certainty equivalence. The difference between the two also depends on the amount of the (certain) nonhuman wealth relative to the expected future (uncertain) labor income.

Based on these simulation results, Zeldes concluded that in determining optimal consumption levels, current assets play a much more important role than risky future labor incomes. Empirical studies, if based on the certainty equivalence, are consequently inadequate. For example, "excess sensitivity" of consumption to transitory income and "excess growth" of consumption, which appear to be contradictory to the optimal consumption behavior under certainty
equivalence, may actually be consistent with optimal consumption behavior which takes labor income uncertainties into account.

Zeldes also suggested a possible remedy of the consumption function for the purpose of empirical studies. That is, to put a weight \( \chi(\cdot) \) less than one on human wealth before adding it to nonhuman wealth, and let \( \alpha \), the marginal consumption out of total wealth (which includes nonhuman wealth and present discounted and risk adjusted future labor incomes), to be a free parameter. \( \alpha \) is possibly larger than its counterpart in the certainty equivalence case, reflecting a larger marginal propensity to consume out of wealth which is consistent with the result obtained in the simulation. After these adjustments, the approximate consumption function can be written as

\[
C_t = \alpha [(1+r)A_{t-1} + \chi(\cdot) \sum_{i=0}^{\infty} \frac{1}{(1+r)^i} E_t W_{t+i}]
\]  

(3.10)

where \( \chi(\cdot) \) is a function of, among other variables, the ratio of nonhuman wealth to expected future labor income. The higher the nonhuman wealth is relative to the expected future labor income, the less risky the consumer feels, and the higher \( \chi(\cdot) \) should be.

3.2.d Alternative Approximate Consumption Function with Stochastic Labor Income

For the purpose of empirical studies, the approximate consumption function (3.10) suggested by Zeldes is a valid compromise between tractability for practical purposes and unavailability of closed form
solution when the utility is CRRA. It respects the optimization principle to the extent of discounting uncertain present value of expected future labor incomes more than it does with the current assets. An added justification is offered by the fact that any first order deviation of the choice variable from the optimality, as long as it is feasible in the sense of satisfying the life-time budget constraint, will result in a utility loss only of second order in terms of the deviation of the choice variable (Cochrane (1989)).

Equation (3.10), however, is most appropriate in describing situations when the labor income risks in all future periods are the same, regardless how far in the future they are. For example, labor income is a constant (the unconditional mean) plus a random shock term which is independently and identically distributed, as was assumed by Zeldes in his simulations. In practice, however, labor income is usually considered to be an integrated process (Mankiw and Shapiro (1985)). Indeed, per capita real disposable labor income of US can well be characterized by the ARIMA(1,1,0) process

$$\Delta w_t = 8.2 + 0.442 \Delta w_{t-1} + \epsilon_t$$  \hspace{1cm} (t-statistics are in parenthesis)

$$\sigma_{\epsilon} = 25.2$$

29. Especially when aggregate data are used in the empirical analysis.

When the labor income process has a unit root, shocks to the labor income will be permanent. Consequently, the further away in the future, the more risky is the expected future labor income perceived by the consumer. This suggests that a more realistic way to approximate the consumption function is to discount expected future labor income more when it is further away from the current period. I therefore take the following consumption function as the starting point of my empirical analysis,

$$C_t = a((1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} E W_{t+i})$$  \hspace{1cm} (3.11)

The discount factor $\frac{1}{(1+\beta)^i}$ in the above expression can be thought of as being composed of two parts, i.e., $(\frac{1}{1+\beta})^i = (\frac{1}{1+r})^i \cdot (1-p)^i$, where $(\frac{1}{1+r})^i$ is used to discount expected $i$-th period labor income to its present discounted value, while $(1-p)^i$ is a risk discount factor of the expected $i$-th period labor income because it is more risky than those of the earlier periods, given that labor income is an integrated process. When the consumer perceives substantial risk in his or her labor income, we may expect $\beta$ to be significantly greater than $r$, while $p$, which equals $\frac{\beta-r}{1+\beta}$, to be significantly greater than zero. In particular, $\beta$ should be a function of the ratio of nonhuman wealth to expected future labor income. The lower the nonhuman wealth is relative to the expected future labor income, the more risk the consumer perceives, and the larger $\beta$ should be relative to $r$. 
The formulation of consumption function as in equation (3.11) was also emphasized by Hayashi (1982) for the purpose of dealing with stochastic labor income. We motivate the formulation from a different perspective, because the simulations of Zeldes that I rely on did not appear until the rapid development of computing technology recently made them possible. Hayashi’s empirical results are also different from what I get in this chapter. I believe this is due to the short data set that he used in his empirical analysis.

3.3 Testing Precautionary Saving in the Approximate Consumption Function

3.3.a First Order Implication of the Approximate Consumption Function

Similar to the testing of permanent income hypothesis under certainty equivalence, we cannot estimate equation (3.11) directly because the expected values of future labor income are unobservable. In order to use the exiting techniques of estimating and testing rational expectation models, I derive first order implications of the approximate consumption function and test the time series restrictions imposed on the data by those conditions. Parameter estimates of $\alpha$ and $\beta$ are also obtained from those conditions. Based on these estimates, the hypotheses of a modified version of the Zeldes’ (and Hayashi’s) model are subsequently tested against those of the certainty
equivalence. Namely, the alternative hypotheses that $\beta > r$ and $\alpha > \frac{r}{1+r}$ are tested against the null hypotheses that $\beta = r$ and/or $\alpha = \frac{r}{1+r}$.

Lagging equation (3.11) one period, multiplying both sides of the lagged equation by $1+\beta$, subtracting the resulting equation from equation (3.11), and rearranging yields\(^{31}\)

$$\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \frac{\alpha (r - \beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \alpha \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} \left( E_t - E_{t-1} \right) \left( \frac{W_t}{C_{t-1}} \right)$$

(3.12)

where $Y_t$ is the total disposable income in period $t$, which includes both labor income and asset income.

3.3.b First Order Implication: Nesting Liquidity Constraints

Previous studies of the permanent income hypothesis based on certainty equivalence indicate that the assumption of some consumers being bound by liquidity constraints is well supported by evidence in the US and most OECD countries (Campbell and Mankiw (1989, 1990, 1991), Chapter II of this dissertation). Following the hypothesis in that literature, I nest liquidity constraints by assuming a fraction $\lambda$ of total disposable labor income in the economy accrues to the liquidity constrained consumers (type 1) who consume their current disposable labor income, while the remainder $1-\lambda$ accrues to

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\(^{31}\) See Appendix for detailed derivation.
individuals who are permanent-income consumers (type 2), setting their consumption level according to equation (3.11). More specifically, let $W_t$ be the total disposable labor income in period $t$. If $W_{1t}$ represents the part of the $W_t$ that accrues to the type 1 consumers, while the rest accrues to the type 2 consumers, then

$$C_{1t} = W_{1t} = \lambda W_t$$

$$C_{2t} = \alpha[(1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{1+\beta^i} E(W_{t+1})]$$

$$= \alpha[(1+r)A_{t-1} + (1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta^i} E(W_{t+1})]$$

where $C_{1t}$ and $C_{2t}$ are the consumption in period $t$ of the two types of consumers respectively. Since the type 1 consumers are liquidity constrained, I assume they have no assets so that all assets in the economy are held by type 2 consumers. The type 2 consumers' $t$-th period budget constraint is

$$A_t + C_{2t} = (1+r)A_{t-1} + W_{2t} = (1+r)A_{t-1} + (1-\lambda)W_t$$

hence the $t$-th period aggregate budget constraint is

$$A_t + C_t = (A_t + C_{2t}) + C_{1t}$$

$$= (1+r)A_{t-1} + (1-\lambda)W_t + \lambda W_t = (1+r)A_{t-1} + W_t$$

Total consumption is the sum of consumption by both types of consumers

$$C_t = C_{1t} + C_{2t} = \lambda W_t + \alpha[(1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{1+\beta^i} E(W_{t+i})] \quad (3.13)$$
Lagging equation (3.13) one period, multiplying both sides of the
lagged equation by 1+β, subtracting the resulting equation from
equation (3.13), and rearranging yields \(^{32}\)

\[
\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \left( \frac{\alpha (r-\beta)}{r} \right) \left( \frac{Y_t - W_t}{C_t} \right) + \lambda \frac{W_t}{C_t} - \lambda (1+\beta)(1-\alpha) \frac{W_{t-1}}{C_{t-1}} \\
\quad + \alpha (1-\lambda) \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) \frac{W_{t+i}}{C_{t-1}}
\]

(3.14)

3.3.c Cointegration Implication of the Approximate Consumption
Function

Besides the first order implications derived above, the
approximate consumption function (3.11) has other implications. In
particular, it implies cointegration of consumption, capital income,
and labor income.

A set of variables \(x_1, x_2, \ldots, x_n\) are cointegrated (of order
(1,1)) if each is difference stationary, and there is at least one
nonzero linear combination \(a_1 x_1 + a_2 x_2 + \ldots + a_n x_n\) of them that is
stationary. \((a_1, a_2, \ldots, a_n)\) is called the cointegrating vector.

Campbell (1987) showed that if consumption follows the certainty
equivalence characterization of equation (3.9), then

\(^{32}\) See Appendix for detailed derivation.
\[ Y_t - C_t = -\sum_{i=1}^{\infty} \left( \frac{1}{1+r} \right)^i E_t \Delta W_{t+i} \] 

(3.15)

Because \( \Delta W_{t+i} \) for \( i = 1, 2, \ldots \) are stationary, and the coefficients \( \left( \frac{1}{1+r} \right)^i \) sum to \( \frac{1}{r} \), the right hand side should also be stationary. This implies that total disposable income and consumption are cointegrated with cointegrating vector \((1, -1)\).

One interesting fact worth noticing here is that if we include some liquidity constrained consumers to whom a fraction \( \lambda \) of the disposable labor income is attributed, then

\[ Y_t - C_t = -(1-\lambda) \sum_{i=1}^{\infty} \left( \frac{1}{1+r} \right)^i E_t \Delta W_{t+i} \] 

(3.16)

Namely, consumption should be cointegrated with total disposable income with cointegrating vector \((1, -1)\), regardless of whether and how many consumers are liquidity constrained. In another word, when consumption follows the certainty equivalence solution, testing cointegration between \( Y_t \) and \( C_t \) cannot identify whether some consumers are liquidity constrained or not.

For the case of consumption assumed to follow equations (3.11) and (3.13), we get after some manipulation

\[ \alpha(\frac{1+r}{x^2})(rA_{t-1}) + \alpha(\frac{1+\beta}{\beta}) W_t - C_t = -\alpha(\frac{1+\beta}{\beta}) \sum_{i=1}^{\infty} \left( \frac{1}{1+\beta} \right)^i E_t \Delta W_{t+i} \] 

(3.17)

and
\[
\alpha \left( \frac{1+r}{r} \right) \left( r \Lambda_{t-1} \right) + (1-\lambda) \alpha \left( \frac{1+\beta}{\beta} \right) + \lambda \right) w \right] - C_t = -(1-\lambda) \alpha \left( \frac{1+\beta}{\beta} \right) \sum_{i=1}^{\infty} \left( \frac{1}{1+\beta} \right)^t \Delta w_{t+i} \\
(3.18)
\]

These equations imply that consumption, capital income, and labor income are cointegrated when consumption follows the approximate solutions we proposed earlier. The cointegrating vectors are, however, different from those of the certainty equivalence consumption function. In particular, liquidity constraints should affect the cointegrating vector. The larger the \( \lambda \), the larger is the coefficient of the wage in the cointegration relationship \(33\).

Testing of these cointegration relations, because there are two unknown parameters in the cointegrating vector, is complicated and deserves careful discussion of the econometric techniques involved. They are not tested in this chapter. I will address this issue in a separate paper in the near future.

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\[ \text{\footnotesize 33. Equations (3.15) and (3.16) are, respectively, special cases of equations (3.17) and (3.18) with } \alpha = \frac{-r}{1+r} \text{ and } \beta = r. \text{ Furthermore, equations (3.18) and (3.16) become equations (3.17) and (3.15) if we let } \lambda = 0. \]
3.3.4 Estimation and Testing: Some Econometric Issues

In order to estimate the parameters in equations (3.12) and (3.14), I rewrite them in the more succinct regression forms as

\[
\Delta \text{ln}C_t = (\beta - \alpha - \alpha \beta) + \frac{\alpha (r - \beta)}{r} \left( \frac{Y_t - w_t}{C_t - 1} \right) + \varepsilon_t
\]  
(3.12a)

and

\[
\Delta \text{ln}C_t = (\beta - \alpha - \alpha \beta) + \frac{\alpha (r - \beta)}{r} \left( \frac{Y_t - w_t}{C_t - 1} \right) + \lambda \left( \frac{w_t}{C_t - 1} \right) - \lambda (1 + \beta) (1 - \alpha) \left( \frac{w_{t-1}}{C_{t-1}} \right) + \varepsilon_t
\]  
(3.14a)

where \( \varepsilon_t = \alpha \sum_{i=0}^{\infty} \left( \frac{i}{1 + \beta} \right)^i \left( \frac{w_{t+i}}{C_{t+i}} \right) \) and

\[
\varepsilon_t = \alpha (1 - \lambda) \sum_{i=0}^{\infty} \left( \frac{i}{1 + \beta} \right)^i \left( \frac{w_{t+i}}{C_{t+i}} \right).
\]

Both regression equations are nonlinear, but only in their parameters. One way of proceeding is to estimate the unrestricted linear models

\[
\Delta \text{ln}C_t = A + B \cdot \left( \frac{Y_t - w_t}{C_t - 1} \right) + \varepsilon_t
\]  
(3.12b)

and

\[
\Delta \text{ln}C_t = A + B \cdot \left( \frac{Y_t - w_t}{C_t - 1} \right) + C \cdot \left( \frac{w_t}{C_t - 1} \right) + D \cdot \left( \frac{w_{t-1}}{C_{t-1}} \right) + \varepsilon_t
\]  
(3.14b)

Because the parameters in equation (3.12a) are exactly identified, I could estimate \( A \) and \( B \) in equation (3.12b) first, and
then solve for \( \alpha \) and \( \beta \) from the estimates of \( A \) and \( B \) to obtain their point estimates. For equation (3.14a), because it is overidentified, the nonlinear overidentifying restrictions involved should be tested if I estimate the linear equation (3.14b). Considering the fact that equation (3.12a) and (3.14a) are well specified and are relatively simple nonlinear models, I therefore estimate the parameters directly by the nonlinear model (3.12a) and restricted nonlinear model (3.14a). An advantage of doing so is that I can perform statistical inferences directly on the parameters, rather than just obtain their point estimates, since I will also obtain the standard errors of the parameter estimates.

In estimating equations (3.12a) and (3.14a), several other issues need to be taken into consideration. First, the error terms are contemporaneous with the regressors, and consequently may be correlated with them. Moreover, because

\[
\sum_{i=0}^{\infty} \left( \frac{1}{1+\beta^i} \right)^2 (E_t - E_{t-i}) W_{t+i}
\]

likely to be heteroskedastic, \( e_t \) and \( E_t \) may be so too even though after being scaled by \( C_{t-1} \) the heteroskedasticity involved can be reduce to some extent. \(^{34}\)

\(^{34}\) The error terms may also have a first order moving average structure. This is possibly due to the following facts: (i) there are measurement errors in \( e_t \), \( Y_t \), \( W_t \), (ii) there are white noise transitory consumption caused by preference shocks, (iii) time aggregation or time averaging of data (Working (1960)). These effects turn out to be unimportant for data used in this chapter. See section 3.4.a for more details.
Because of these difficulties, the conventional method of nonlinear least square cannot be used here because it will yield inconsistent estimates. The method I use in this chapter is the generalized method of moments (GMM, see Hansen (1982), Hansen and Singleton (1982)). It can yield parameter estimates and standard errors that are consistent even when the error term is correlated with the regressors and is heteroskedastic and serially correlated.

As in the estimation of many other rational expectation models, the appropriateness of GMM in this occasion is based on the fact that $E_{t-1}e_t = 0$ and $E_{t-1}E_t = 0$, which follows from the law of the iterated projections. The orthogonality conditions

$$E(Z_{t-1}e_t) = 0 \text{ and } E(Z_{t-1}E_t) = 0$$

should therefore hold on the data, where $Z_{t-1}$ is a constant or any variable that belongs to the consumer's information set at $t-1$. When there are more instruments (hence more orthogonality conditions) than the estimated parameters, GMM also provides an asymptotic test of the overidentifying restrictions imposed on the model. The test statistic $J$ has a $\chi^2$ distribution with degrees of freedom equaling the number of moment conditions less the number of estimated parameters.

3.3.e The Data

Seasonally adjusted quarterly US data are used in this chapter to estimate (3.12a) and (3.14a). The data run from 1953:2 to 1984:4 and
are constructed by Blinder and Deaton (1985) from the U.S. National Income and Product Account. They made several sensible adjustments to the data, which can be found in both their paper and that of Campbell (1987)\textsuperscript{35}. The series I use from their data set are real per capita total disposable income, real per capita disposable labor income, and real per capita consumption of nondurables and services. The consumption measure I use in estimation is real per capita consumption of nondurables and services divided by 0.7855. The scale factor 0.7855 is the sample mean of the ratio of total consumption spending to those on nondurables and services.

The real interest rate is constructed from the three month treasury bill rate in the secondary market and the consumer price index of all items by urban consumers. Both are obtained from the CITIBASE data tape (FYGM3 for treasury bill rate, FUNEW for CPI).

\textsuperscript{35} They are, briefly, (i) removing the 1975 tax rebate from the disposable income series, (ii) subtracting consumer interest payments to business from NIPA disposable income series, (iii) adding personal nontax payments to state and local governments to both disposable income and the consumption series, (iv) treating clothing and shoes as durables, (v) dividing proprietors’ income and personal income taxes, which are not done in NIPA, to capital income and labor income according the ratio of the two in the rest of the economy, (vi) deducting social insurance contributions from labor income, and (vii) consumer spending deflator used to construct real per capita data are adjusted in the same way as consumption.
3.4 Empirical Results and Discussion

3.4.a Empirical Results

With a constant and twice to fourth lagged $\Delta lnC_t$ and $\frac{Y_t - W_t}{C_{t-1}}$ as instruments, equation (3.12a) is estimated using generalized method of moments. When (3.12a) is estimated, the real rate of interest $r$ in it is fixed as a constant. Eight different values are assigned to $r$, ranging from 0.10 % per quarter to 1.25 % per quarter\(^{36}\). This covers a wide range of possible values for real interest rate, including the sample mean of the real 3-month treasury bill rates for the 1953:2–1984:4 sample period, which is around 0.25 % per quarter. The results of the estimation are listed in Table 3.1.

The claim that consumers discount expected future labor income at a higher rate than the real interest rate is supported by the results. \(\beta\) estimates are greater than $r$ for all eight possible $r$ values at less than 1 % significance level. \(\alpha\), the marginal propensity to consume out of total wealth (nonhuman wealth plus present discounted and risk adjusted future labor incomes), is also estimated to be greater than \(\frac{r}{1+r}\) for all eight values of $r$, with significance levels all below or around 5 %. All these are consistent with the simulation results

36. Or from 0.4 % per annum to 5.0 % per annum.
obtained by Zeldes (1989). The $J$-statistics in the table is asymptotically distributed as $\chi^2$ with 5 degrees of freedom, because there are 7 moment conditions and 2 parameters to be estimated\(^\text{37}\). An unsatisfactory result in estimating equation (3.12a) is that the $J$-statistic is too large (significant at only slightly more than the 5% level) to accept the model, even though $\alpha$ and $\beta$ estimates turn out to be quite reasonable.

The fact that equation (3.12a) does not provide a good fit to the data is understandable, because we have not taken the behavior of liquidity constrained consumers into account. By nesting the behavior of the liquidity constrained consumers, equation (3.14a) may conceivably increase its goodness of fit relative to that of equation (3.12a). The results of GMM estimation of equation (3.14a), using the same eight values of $r$ as in Table 3.1, and a set of instruments including a constant and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_t - W_t}{C_t}$, $\frac{W_t}{C_{t-1}}$, $\frac{W_t}{C_{t-1}}$

and $\frac{W_t}{C_t}$, are listed in Table 3.2.

Estimates of $\alpha$ and $\beta$ for various values of $r$ are slightly lower than their counterparts in Table 3.1, after the liquidity constrained consumers are taken into account. Nevertheless, $\beta$ is still

\(^{37}.\) Because equation (3.12a) is exactly identified, estimations with different $r$ values all produce the same $J$-statistic, because the underlying linear model (3.12b) is the same for all estimations.
significantly greater than \( r \) for all values of \( r \). This confirms the hypothesis that consumers, as long as they are not liquidity constrained, discount expected future labor income at a higher rate than the real interest rate. The marginal propensity to consume out of total wealth \( \alpha \) is still estimated to be greater than \( r \) for all values of \( r \), but not significantly so at conventional levels. The liquidity constrained consumers are estimated to consume about one third of the total disposable labor income in the economy. This result is consistent with both the findings in the other relevant studies and with our casual observation. Furthermore, the \( J \)-statistic, being insignificant in this case, indicates that modelling aggregate consumption behavior by equation (3.14a) cannot be rejected by the data.

Two problems that deserve some more scrutinies are the following: First, although I have been conservative in estimating equation (3.12a) and (3.14a) by treating the error terms \( e_t \) and \( \varepsilon_t \) as MA(1) \(^{38}\), and using variables lagged at least twice as instruments, the fact that

\[
e_t = \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+i} \right) (E_t - E_{t-1}) \left( \frac{\varepsilon_{t+i}}{C_{t-1}} \right)
\]

and

\(^{38}\) See footnote 8.
\[ \epsilon_t = \alpha(1-\lambda) \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} (E_t - E_{t-1}) W_{t+i} \]

does not strictly imply \( E_{t-2}(\epsilon_t) = 0 \) and \( E_{t-2}(\epsilon_t) = 0 \), because \( C_{t-1} \) may be correlated with \( \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} (E_t - E_{t-1}) W_{t+i} \) from the perspective of period \( t-2 \).

To make sure that results in Table 3.1 and 3.2 are free of the above problem, I divide the equations \(^{39}\)

\[ \Delta C_t = (\beta - \alpha - \alpha\beta) C_{t-1} + \frac{\alpha(r-\beta)}{r} (Y_t - W_t) + \alpha \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} (E_t - E_{t-1}) W_{t+i} \]

and

\[ \Delta C_t = (\beta - \alpha - \alpha\beta) C_{t-1} + \frac{\alpha(r-\beta)}{r} (Y_t - W_t) + \lambda W_t - \lambda(1+\beta)(1-\alpha) W_{t-1} \]

\[ + \alpha(1-\lambda) \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} (E_t - E_{t-1}) W_{t+i} \]

by \( W_{t-2} \) instead of \( C_{t-1} \). The resulting equations are therefore

\[ \frac{\Delta C_t}{W_{t-2}} = (\beta - \alpha - \alpha\beta) \frac{C_{t-1}}{W_{t-2}} + \frac{\alpha(r-\beta)}{r} \frac{Y_t - W_t}{W_{t-2}} + \epsilon_t \quad (3.12c) \]

and

\[ \frac{\Delta C_t}{W_{t-2}} = (\beta - \alpha - \alpha\beta) \frac{C_{t-1}}{W_{t-2}} + \frac{\alpha(r-\beta)}{r} \frac{Y_t - W_t}{W_{t-2}} + \lambda \frac{W_t}{W_{t-2}} - \lambda(1+\beta)(1-\alpha) \frac{W_{t-1}}{W_{t-2}} + \epsilon_t \]

\[ \quad (3.14c) \]

---

39. They are the intermediate results (equations (3.A6) and (3.A11)) of the derivations contained in the appendix.
where

\[
e_t' = \alpha \sum_{i=0}^{\infty} \frac{1}{1+\beta'} \left( E_t - E_{t-1} \right) \frac{W_{t+i}}{W_{t-2}} \quad \text{and} \quad E_t' = \alpha(1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta'} \left( E_t - E_{t-1} \right) \frac{W_{t+i}}{W_{t-2}}.
\]

Both \( e_t' \) and \( E_t' \) are truly orthogonal to the consumer's information set as of period \( t-2 \).

The results of estimating equations (3.12c) and (3.14c) are listed in Tables 3.3 and 3.4, which lead to the similar conclusions we have obtained earlier from Tables 3.1 and 3.2. This tells us that either the error terms hardly have any first order moving average structure\(^{40}\), or the correlation between \( C_{t-1} \) and \( \sum_{i=0}^{\infty} \frac{1}{1+\beta'} \left( E_t - E_{t-1} \right) W_{t+i} \), from the perspective of period \( t-2 \), is too small to matter.

The second problem deserving some more discussions is that in all of the estimations so far, I have treated \( r \) as a fixed constant. Although various different values have been assigned to \( r \), no explicit relationships have been established between the data on real interest rates and the \( r \) used in those previous estimations. This could be problematic when the hypotheses posed in this chapter are tested. If there are large sample errors associated with the estimate of real interest rate, we may not be able to reject the null hypotheses of \( \alpha = 

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\(^{40}\) Possibly because all the contributing effects approximately cancel with each other.
\( \frac{r}{1+r} \) and \( \beta = r \) in favor of the alternative hypotheses of \( \alpha > \frac{r}{1+r} \) and \( \beta > r \).

To address and clarify this issue, I use the following two equations, (3.19) and (3.20), to estimate the values of \( r \). Equation (3.19) is based on the assumption that the observed real interest rate is a constant (unconditional) mean plus a random disturbance term \(^{41}\). If we denote the before tax real three month treasury bill rate \(^{42}\) as \( r_t \), we then have

\[
r_t = r + u_t
\]

(3.19)

where \( u_t \) is the error term associated with \( r_t \).

The second equation that characterizes the real interest rate is based on the \((t-1)\)-th period budget constraint

\[
C_{t-1} + A_{t-1} = (1+r)A_{t-2} + W_{t-1}
\]

Substitute the \( A_{t-1} \) and \( A_{t-2} \) terms in the above equation by \( \frac{1}{r}(Y_t - W_t) \)

and \( \frac{1}{r}(Y_{t-1} - W_{t-1}) \) respectively, and rearranging yields

\[
\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1})
\]

\(^{41}\) This will be proven, by both the results of estimation and data plot, to be a poor approximation.

\(^{42}\) I have also experimented with the real after tax three month treasury bill rate. The resulting estimate of \( r \) turned out to be a negative number.
If we assume that there are measurement errors in $Y$, $W$ or $C$, we then have the regression equation

$$
\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + v_t
$$

(3.20)

where $v_t$ is the error term associated with the above equation.

Equation (3.20) is a behavioral relationship which identifies the underlying real interest rate $r$ on which the consumers' consumption decisions are based. Incidentally, it is independent of whether some consumers are liquidity constrained, and of how many consumers are consuming their current labor income because of the liquidity constraints.

The results of jointly estimating equations (3.12a) and (3.19) by GMM, using a constant and twice to fourth lagged $\Delta\ln C_t$ and $\frac{Y_t - W_t}{C_{t-1}}$ as instruments, are listed in Table 3.5. Quarterly real interest rate $r$ is estimated to be around 0.175%. The null hypotheses can be rejected at the conventional significance level. The $J$-statistic for the overidentifying restrictions is barely insignificant at the 5% level. The large $J$-statistic here is likely due to the fact that equation (3.19) is not an accurate approximation of the behavior of real interest rate$^{43}$. This fact can be clearly seen from the time series

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43. Of course, it may also result from the exclusion of liquidity constrained consumer in equation (3.12a). The results of Table 3.6 discussed in the next paragraph, however, does not support this conjecture.
plot of $r_t$ in Figure 3.1, especially for the 1970's and 1980's.

Similar results are obtained when liquidity constrained consumers are included in the model, namely, when equations (3.14a) and (3.19) are jointly estimated by GMM. Quarterly real interest rate $r$ is estimated to be around 0.256%. The fraction of the liquidity constrained consumers, $\lambda$, is estimated to be around 19%. The null hypotheses can also be rejected at the conventional significance level. The $J$-statistic for the overidentifying restrictions is significant at the 5% level, probably because of the same reason discussed above.

Joint estimation of equations (3.12a) and (3.20), and equations (3.14a) and (3.20), by GMM yields substantially higher estimate of $r$, as listed in Table 3.7 and Table 3.8 respectively. This is no doubt because the real rate of return on nonhuman wealth is risky. The hypotheses that $\alpha = \frac{r}{1+r}$ and $\beta = r$ can be rejected decisively by the results in Table 3.7, and less so for those in Table 3.8. The $J$-statistics for the overidentifying restrictions are insignificant at any conventional level for both estimations, reflecting an improved goodness of fit for equation (3.20) relative to that for equation (3.19).
3.4.6 Discussion of the Empirical Results

The empirical finding that consumers, because of their prudence or decreasing absolute risk aversion, discount expected future labor income at a higher rate than the real interest rate has important implications in explaining the two most important empirical anomalies in the consumption literature, namely, the excess sensitivity and excess smoothness puzzles (Deaton (1987), Campbell and Deaton (1989) and West (1988)).

First, because the approximate consumption function implies

\[ C_t = (1-\alpha)(1+\beta)C_{t-1} + \alpha(r-\beta)A_{t-1} + \varepsilon'_t \]  

(3.21)

consumption in period \( t \) can be Granger caused by the consumer's asset holding in period \( t-1 \). This appears to be consistent with Hall's (1978) finding that lagged stock prices have predictive power for consumption.44 Furthermore, because \( rA_{t-1} = Y_t - W_t' \), any variable that Granger causes \( Y_t \) and/or \( W_t' \) may also Granger cause \( C_t \). Adding the existence of liquidity constrained consumers only reinforces the arguments above.

Second, because the error term in equation (3.21) can be written as

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44. The effects of stock prices on consumption change have different signs for the different lags, though. The coefficients on the first and third lags are positive, but those on the second and fourth lags are negative.
\[
\varepsilon_t' = \alpha \sum_{i=0}^{\infty} \frac{1}{1+\beta} i (\varepsilon_{t-i} - E_{t-1}) W_{t+i},
\]

the variance of the consumption innovation can be expressed in the form of \(^{45}\)

\[
\{\alpha \cdot [P(\frac{1}{1+\beta})]^{-1}\}^2 \cdot \text{var}(\varepsilon_t),
\]

where \(P(L)W_t = \varepsilon_t\) is the autoregressive representation of the univariate labor income process. Under the hypothesis that real per capita disposable labor income follows the ARIMA(1,1,0) process of

\[\Delta W_t = 8.2 + 0.442 \Delta W_{t-1} + \varepsilon_t\]

so that,

\[P(L) = 1 - 1.442 \cdot L + 0.442 \cdot L^2\]

the standard error of the consumption innovation for a unitary labor income innovation can be calculated from point estimates of \(\alpha\) and \(\beta\) by

\[\frac{\alpha(1+\beta)}{\beta(0.588+\beta)}\].

When the liquidity constrained consumers are included, the standard error of the consumption innovation should be

\[\frac{(1-\lambda)\alpha(1+\beta)}{\beta(0.588+\beta)}\].

For reference purpose, if labor income follows a random walk, namely,

\[P(L) = 1\]

See the derivation in Flavin (1981) for the certainty equivalence case. The similar conclusion of \(\alpha \cdot [P(\frac{1}{1+\beta})]^{-1}\) can be derived for the alternative solution presented here.
\[ W_t = W_{t-1} + \varepsilon_t \quad \text{or} \quad p(L) = 1 - L \]

then the standard errors of the consumption innovation for a unitary labor income innovation will be \( \frac{\alpha(1+\beta)}{\beta} \) for the case of no liquidity constraints, and \( \frac{(1-\lambda)\alpha(1+\beta)}{\beta} \) for the case when some consumers are liquidity constrained.

Values of these expressions for various point estimates of \( \alpha, \beta \) and \( \lambda \) obtained in the previous regressions are listed in Table 3.9. Whenever liquidity constrained consumers are included, values of \( \frac{(1-\lambda)\alpha(1+\beta)^2}{\beta(0.588+\beta)} \) and \( \frac{(1-\lambda)\alpha(1+\beta)}{\beta} \) (in columns 4, 5, 8 and 9) never exceed 1 for all values of \( r \) listed in the table (including those estimated in Tables 3.8) and are rarely more than 0.5 for realistic values of \( r \). For models that do not include liquidity constrained consumers, values of \( \frac{\alpha(1+\beta)^2}{\beta(0.588+\beta)} \) (in columns 2 and 3) exceed 1.0 only when \( r \) is higher than 0.75 \% per quarter (or 3 \% per annum). For the more realistic values of \( r \) (less than 2 \% per annum), it is always less than 1.0. For the case when labor income is a random walk, \( \frac{\alpha(1+\beta)}{\beta} \) (in columns 6 and 7) is again always smaller than 1, and mostly around or below 0.5 for the most plausible values of \( r \).

The outcomes of these calculations reveal that, contrary to the case of certainty equivalence solution, the approximate consumption decision rule proposed in this chapter predicts that consumption should be smoother than the disposable labor income even when the
latter can be characterized as an integrated process. This prediction is confirmed by the evidence in the aggregate consumption and disposable labor income data of the US, as long as the real interest rate is not unusually high. In other words, after I modify the optimal consumption so that consumers will, because of their prudence, discount expected future uncertain labor income at a higher rate than the real interest rate, the "excess smoothness" paradox suggested by Deaton (1987) will no longer exist. Adding the existence of the liquidity constrained consumers reduces the standard error of the consumption innovation to an even smaller magnitude of \( \alpha \cdot (1 - \lambda) \cdot \left[ P \left( \frac{1}{1+\beta} \right) \right]^{-1} \).

The point here is clearly different from that of Quah (1990). His paper shows that an integrated labor income process can be decomposed into various combinations of permanent and and transitory parts which preserve the dynamic property of the univariate labor income process. At least one of those decompositions will give rise to a smooth consumption process if the consumer can recognizes the two parts of the labor income. His arguments rationalizes the smoothness of aggregate consumption even when the hypothesis that consumption follows the certainty equivalence solution is maintained. My point, however, simply argues that Deaton's paradox may not arise in the scenario of this chapter.
3.5 Concluding Remarks

In this chapter, I used an approximation to model the optimal consumption of a representative consumer when he or she faces uncertain labor income. This approximate consumption function is based on the numerical solution (of Zeldes) to the optimal consumption problem with CRRA utility and stochastic labor income, which takes into consideration the precautionary savings of the consumer. It assumes that the consumer discounts expected future labor income at a rate higher than the real interest rate. The further in the future labor income is, the more it should be discounted. The first order implications of the approximate consumption function, with and without the liquidity constrained consumers, are tested using quarterly U.S. data. The evidence lends support to the claims of the approximate consumption function, particularly when liquidity constrained consumers are included.

Even though the model is not an exact solution to the intertemporal optimization problem, it appears to be a promising way of getting around the difficulties involved in obtaining a closed form solution when utility is of the general decreasing absolute risk aversion type. For example, the "excess sensitivity" and "excess smoothness" puzzles are easily understandable in this model, even without resorting to the assumption that the income process is composed of permanent and transitory parts which are observed only by the consumer but not the econometrician. An important purpose of this
chapter is to provide a starting point for probing the usefulness of this model for empirical purposes.

Many other issues concerning both the validity of the model and its application, if it is valid, are not addressed in this chapter. I take those unaddressed issues as part of my research agenda. First, the cointegration implication of the model described in section 3 can be tested both on US and international (say, OECD countries) data. Second, \( \beta \) should be a function of the ratio of current assets to expected future labor income. Moreover, it may depend on the risk involved in the labor income, which in turn depends on the variance of the labor income and the persistence of the labor income process.46 As for the idiosyncratic risks, it may depend on, among others, the marginal tax rate on labor income and the profession the consumer is engaged in.

These implications can, in principle, be tested both on international data and panel data. Difficulties may arise, nevertheless, when one uses disaggregated data. For example, some characteristics of consumers are not controllable in constructing a sample. More specifically, a consumer may appear to be in a more risky situation than others, but he may not discount future labor income more simply because he is less prudent and less risk averse.

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46. This can be measured, say, by the sum of the coefficients of the moving average representation for \( \Delta W_t \) as used in Campbell and Mankiw (1987), or by the variance ratio proposed by Cochrane (1988).
than the others. The fact that he is in that situation could be the result of self selection. The selection bias of this kind needs careful treatment should disaggregate data be used in testing theories involving precautionary savings e.t.c.

If the theory proves to be a good description of consumption, a good application of it is to calculate aggregate wealth accumulation in a life-cycle version of the model. A calculation of asset holdings, as the difference between the certainty equivalence consumption and that predicted by the approximate model, by different age groups will give us an idea how much wealth is accumulated due to the prudence or the precautionary saving of the consumers.
Appendix to Chapter III

a. First Order Implication with No Liquidity Constrained Consumers

Consumption in period $t$ is

$$C_t = \alpha(1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i}E_t W_{t+i}$$  \hspace{1cm} (3.A1)

Lag the above equation one period, we get consumption in period $t-1$ as

$$C_{t-1} = \alpha(1+r)A_{t-2} + \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i}E_{t-1} W_{t-1+i}$$ \hspace{1cm} (3.A2)

Multiply both sides of the $C_{t-1}$ expression by $1+\beta$, we get

$$(1+\beta)C_{t-1} = \alpha((1+\beta)(1+r)A_{t-2} + \sum_{i=1}^{\infty} \frac{1}{(1+\beta)^i}E_{t-1} W_{t+i-1} + (1+\beta)W_t$$

$$= \alpha((1+\beta)(1+r)A_{t-2} + \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i}E_{t-1} W_{t+i} + (1+\beta)W_{t-1})$$  \hspace{1cm} (3.A3)

Subtract (3.3) from (3.1)

$$C_t - (1+\beta)C_{t-1} = \alpha(1+r)A_{t-1} - (1+\beta)(1+r)A_{t-2} - (1+\beta)W_{t-1}$$

$$+ \alpha \sum_{i=0}^{\infty} \frac{1}{(1+\beta)^i} (E_t - E_{t-1}) W_{t+i}$$

$$= \alpha(\beta A_{t-1} + (1+\beta)A_{t-1} - (1+\beta)(1+r)A_{t-2} - (1+\beta)W_{t-1})$$
\[ + \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i} \]

\[ = \alpha(\beta - \alpha \beta) A_{t-1} + \alpha(1+\beta) C_{t-1} + \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i} \]

(3.A4)

which is

\[ C_t = (1-\alpha)(1+\beta) C_{t-1} + \alpha(\beta - \alpha \beta) A_{t-1} + \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i} \]

(3.A5)

or

\[ \Delta C_t = (\beta - \alpha - \alpha \beta) C_{t-1} + \alpha(\beta - \alpha \beta) A_{t-1} + \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (E_t - E_{t-1}) W_{t+i} \]

(3.A6)

so

\[ \Delta \ln C_t = \frac{\Delta C_t}{C_{t-1}} = \frac{(\beta - \alpha - \alpha \beta)}{C_{t-1}} + \frac{A_{t-1}}{C_{t-1}} + \frac{\alpha}{\frac{1}{1+\beta}} \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i \frac{(E_t - E_{t-1}) W_{t+i}}{C_{t-1}} \]

\[ = (\beta - \alpha - \alpha \beta) + \frac{\alpha(\beta - \alpha \beta)}{r} \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i \frac{(E_t - E_{t-1}) W_{t+i}}{C_{t-1}} \]

(3.A7)

b. First Order Implication with Liquidity Constrained Consumers

\[ C_t = \lambda W_t + \alpha_i (1+r) A_{t-1} + (1-\lambda) \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i E_{t} W_{t+i} \]

(3.A8)

Lag (3.11) one period and multiply 1+\beta on both sides, we get

\[ (1+\beta) C_{t-1} = \lambda (1+\beta) W_{t-1} + \alpha_i (1+\beta) (1+r) A_{t-2} \]
\[ C_t - (1+\beta) C_{t-1} = \lambda W_t - \lambda (1+\beta) W_{t-1} + \alpha (1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta} \frac{1}{i} (E_t - E_{t-1}) W_{t+i} + (1-\lambda) W_{t-1} \]  

(3.10)

Subtract (3.12) from (3.11), we have

\[ C_t - (1+\beta) C_{t-1} = \lambda W_t - \lambda (1+\beta) W_{t-1} + \alpha (1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta} \frac{1}{i} (E_t - E_{t-1}) W_{t+i} + (1-\lambda) W_{t-1} \]

(3.11)

or

\[ \Delta C_t = (\beta - \alpha - \alpha \beta) C_{t-1} + \alpha (r-\beta) A_{t-1} + \lambda W_t - \lambda (1+\beta) (1-\alpha) W_{t-1} \]

\[ + \alpha (1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta} \frac{1}{i} (E_t - E_{t-1}) W_{t+i} \]  

(3.12)

and therefore

\[ \Delta n C \frac{\Delta C_t}{C_{t-1}} = (\beta - \alpha - \alpha \beta) \frac{W_t - W_{t-1}}{C_{t-1}} + \frac{Y_t - W_t}{C_t} + \lambda \left( \frac{W_t}{C_{t-1}} \right) \]

\[ - \lambda (1+\beta) (1-\alpha) \left( \frac{W_{t-1}}{C_{t-1}} \right) + \alpha (1-\lambda) \sum_{i=0}^{\infty} \frac{1}{1+\beta} \frac{1}{i} (E_t - E_{t-1}) \left( \frac{W_{t+i}}{C_{t-1}} \right) \]  

(3.13)
CHAPTER IV

COINTEGRATION OF CONSUMPTION AND DISPOSABLE INCOME:

EVIDENCE FROM PANEL DATA OF TWELVE OECD COUNTRIES

4.1 Introduction

One of the implications of the rational expectation-permanent income hypothesis is that consumption and disposable income are cointegrated. Campbell (1987), using tests that are developed in Engle and Granger (1987), concludes that this implication holds on the quarterly aggregate data of the U.S. He finds that although Phillips-Perron test could not reject the nonstationarity of the disposable income and consumption processes, an augmented Dickey-Fuller test rejects the existence of a unit root in the residuals obtained from the least square regression of disposable income on consumption. Campbell and Clarida (1987) confirm those conclusions with the quarterly aggregate time series data of Canada.

These tests, however, may yield different conclusions if they are applied to data from other countries. This is because the unit root tests in the literature have rather weak power when they are applied to individual time series of moderate length, particularly when the alternative is close to the null. Indeed, Attfield, Demery and Duck
(1990), using quarterly aggregate time series data of the U.K.\footnote{Their paper, which is a critique of MacDonald and Speight (1989), corrected an error of data construction in the latter.}, find that Phillips-Perron test strongly rejects (at the 1% level) the nonstationarity of the disposable income process. They also find no evidence (at even the 10% level) against a unit root in the residuals of the regression between disposable income and consumption with ADF test of 1 or 4 lags. Campbell and Clarida (1987) indicate that, for the United Kingdom, the unit root hypothesis for the disposable income is rejected at the 5% level, and the hypothesis of no cointegration between consumption and disposable income can only be rejected at the 10% level.\footnote{The data used in Campbell and Clarida (1987) are different from those in Attfield, Demery and Duck (1990). The former run from 55:1 to 84:4 while the latter run from 66:1 to 86:3. Campbell and Clarida also made some adjustments on their data as la Blinder and Deaton (1985).}

The conflicting results documented in those literature reveal that empirical findings of nonstationarity of both consumption and disposable income, as well as cointegration between them, may simply be due to the weak power of the unit root tests that they are based upon. In order to get more robust results, one must use tests of greater power.

In this chapter I use the national accounts data from twelve OECD countries to test the nonstationarity of consumption and disposable
income as well as the cointegration of them that is implied by the RE-PIH. The econometric method used in this chapter is based on the theory of testing unit roots with panel data developed by Levin and Lin (1992). The advantage of using panel data is that a substantial gain in the power of the test can be obtained, even with moderate number of cross sections and time series observations.

The findings of this chapter confirm that consumption and disposable income are cointegrated. They also provide comparisons between the conclusions obtained using only individual time series data and those obtained using pooled data of time series and cross sections. Specifically, I find that residual based cointegration tests cannot reject the hypotheses that consumption and income are not cointegrated when performed on individual time series data of each country. Nevertheless, when the data are pooled as a panel with time series and cross sections, the evidence strongly rejects the hypothesis of no cointegration between consumption and disposable income.

The rest of this chapter is organized as the following. In section 4.2, I briefly review the cointegration of consumption and disposable income as an implication of the RE-PIH. I also show that this cointegration relationship should hold regardless of whether some consumers are liquidity constrained. In section 4.3, I first briefly review some unit-root tests and residual based cointegration tests widely used in the literature and then review the econometric methods of testing unit roots and cointegration with panel data. Data used in
this chapter are described in section 4.4. Empirical results are presented in section 4.5. Section 4.6 contains the results of Monte Carlo simulations for the finite sample distributions of the test statistics used in statistical inferences. The conclusion of the chapter follows in section 4.7.

4.2 RE-PIH: Its Cointegration Implications

The starting point of this chapter is the consumption function under rational expectation-permanent income hypothesis; i.e.,

\[ C_t = \beta [(1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^{i}}E_{t}W_{t+i}] \]  

(4.1)

where \( C_t \), \( W_t \) and \( A_t \) are respectively, consumption, labor income and asset holding in period \( t \). The real interest rate \( r \) is assumed to be a constant and equal to the consumer's subjective time discount rate. \( E_t \) is the mathematical expectation conditional on all information available to the consumer in period \( t \). \( \beta \) is the marginal propensity to consume for a unitary increase in the consumer's permanent income.

Following Campbell (1987), if consumption can be described by equation (4.1), it should be cointegrated with disposable income. This can be seen from the following,

\[ \beta \left( \frac{1+r}{r} \right) Y_t - C_t = \beta \left( \frac{1+r}{r} \right) Y_t - \beta [(1+r)A_{t-1} + \sum_{i=0}^{\infty} \frac{1}{(1+r)^{i}}E_{t}W_{t+i}] \]

\[ = \beta \left( \frac{1+r}{r} \right) (W_t + rA_{t-1}) - \beta (1+r)A_{t-1} - \beta \sum_{i=0}^{\infty} \frac{1}{(1+r)^{i}}E_{t}W_{t+i} \]
\[ 
= \beta \sum_{i=0}^{\infty} \frac{1}{1+r}^i w_t - \beta \sum_{i=0}^{\infty} \frac{1}{1+r}^i E_t w_{t+i} \\
= -\beta \sum_{i=0}^{\infty} \frac{1}{1+r}^i (E_t w_{t+i} - w_t) \\
= -\beta E_t \{ \sum_{i=1}^{\infty} (\frac{1}{1+r})^i \sum_{j=1}^{\infty} \Delta w_{t+j} \} \\
= -\beta E_t \{ \sum_{j=1}^{\infty} (\frac{1}{1+r})^j \Delta w_{t+j} \} \sum_{i=0}^{\infty} (\frac{1}{1+r})^i \\
= -\beta (\frac{1+r}{r}) \sum_{i=1}^{\infty} (\frac{1}{1+r})^i E_t \Delta w_{t+i} \\
(4.2) 
\]

If \( \Delta w_{t+i} \) for \( i = 1, 2, \ldots \) are covariance stationary, then the right hand side of equation (4.2) should also be stationary since the coefficients \( (\frac{1}{1+r})^i \) sum to \( \frac{1}{r} \). This implies that disposable income and consumption are cointegrated with cointegrating vector \( (1, -\beta(\frac{1+r}{r})) \).

In the special case when \( \beta = -\frac{r}{1+r} \), equation (4.2) has a straightforward economic interpretation. Under the RE-PIH, consumption equals the annuity value of the permanent income. Saving will forecast declines in future labor income, because only when the current income is high relative to the permanent income will a rational consumer save, and this indicates that expected income in the future will decline. Similarly, dissaving forecasts increase in future labor income, because only when the current income is low relative to the permanent income will a rational consumer dissave, and
this indicates that expected income in the future will be higher than its current level.

A similar result can be obtained if \( \beta = \frac{r}{1+r} \), and a fraction \( \lambda \) of total disposable labor income in the economy accrues to the liquidity constrained consumers (type 1) who consume their current disposable labor income, while the remainder \( 1-\lambda \) accrues to individuals who are permanent income consumers (type 2), setting their consumption level according to PIH. Under these assumptions,

\[
C_{1t} = W_{1t} = \lambda W_t
\]

\[
C_{2t} = \left( \frac{r}{1+r} \right) \left( (1+r) A_{t-1} + \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i E_t W_{2, t+i} \right)
\]

\[
= \left( \frac{r}{1+r} \right) \left( (1+r) A_{t-1} + (1-\lambda) \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i E_t W_{t+i} \right)
\]

where \( W_{1t} \) and \( W_{2t} \) represent the parts of \( W_t \) that accrue to the two types of consumers in period \( t \), and \( C_{1t} \) and \( C_{2t} \) are the consumptions in period \( t \) of the two types of consumers.

Total consumption is the sum of consumption by both types of consumers

\[
C_t = C_{1t} + C_{2t} = \lambda W_t + \left( \frac{r}{1+r} \right) \left( (1+r) A_{t-1} + (1-\lambda) \sum_{i=0}^{\infty} \left( \frac{1}{1+r} \right)^i E_t W_{t+i} \right)
\]

Similar manipulations as in (4.2) give rise to the following condition

\[
Y_t - C_t = - (1-\lambda) \sum_{i=1}^{\infty} \left( \frac{1}{1+r} \right)^i E_t \Delta W_{t+i}
\]  

(4.3)
As a result, consumption should be cointegrated with total disposable income regardless of whether and how many consumers are liquidity constrained. Therefore, one need not be concerned with whether households are liquidity constrained.

4.3 Review of Econometric Methods

Numerous papers in the literature since Nelson and Plosser (1982) suggest that most aggregate macroeconomic time series should be modeled as integrated processes. In particular, Mankiw and Shapiro (1985) indicate that consumption and disposable income in the U.S. can well be characterized as integrated time series.\(^{49}\) In the first of the following two subsections, I briefly review the methodology of some unit-root tests and residual-based cointegration tests. In the second subsection, I review the methodology of testing unit roots with panel data recently developed by Levin and Lin (1992).

4.3.a Testing Unit Roots and Cointegration with Individual Time Series Data

A leading and most commonly used approach of testing unit roots in univariate time series is the Dickey-Fuller test (Fuller (1976) and

\(^{49}\) For international evidence of unit roots in macroeconomic time series, see Kormendi and McGuire (1990).
Dickey and Fuller (1979, 1981)). In its simplest form, the Dickey-
Fuller test for a unit root in $z_t$ is performed by least square
estimation of the model
\[ \Delta z_t = \delta_t + \rho z_{t-1} + \varepsilon_t \]  (4.4)
where $\delta_t$ is a deterministic term, and $\varepsilon_t$ is assumed to be i.i.d.
normal with mean 0 and variance $\sigma^2$. The null hypothesis of
\[ H_0: \rho = 0 \]
is then tested against the alternative hypothesis of
\[ H_1: \rho < 0. \]

Under the null hypothesis that $z_t$ has a unit root, the $t$-
statistic for $\rho$ does not, however, follow the standard distribution.
The conventional critical values for $t$-distribution therefore cannot
be used to evaluate the significance level of a $t_\rho$ obtained in the
regression above. Furthermore, the distribution of the $t_\rho$ depends on
whether deterministic terms such as a constant and/or a linear time
trend are included in the regression. Fuller (1976, p. 373 Table
8.5.2) provides critical values of the asymptotic distribution for $t_\rho$
obtained with Monte Carlo simulation, for the cases when (i) no
deterministic regressors are included, (ii) only a constant term is
included, and (iii) both a constant term and a linear time trend are
included. Those critical values are used in this chapter to test unit
roots in the consumption and disposable income time series of various
countries.
In the more general cases when $\varepsilon_t$ is serially correlated, two approaches are used to modify the simple unit root test described above so that the asymptotic distribution for $t_\rho$ will not depend on those serial correlations. The first is the augmented Dickey-Fuller (ADF) test suggested by Dickey and Fuller (1981) and Said and Dickey (1984). This parametric correction for serial correlation which is made by including an appropriately selected number of lagged differences of $z_t$ in equation (4.4); i.e., one estimates

$$\Delta z_t = \rho z_{t-1} + \sum_{i=1}^{p} \theta_i \Delta z_{t-i} + \varepsilon_t,$$

(4.5)

where the $\theta$'s are parameters. The asymptotic distribution of the $t$-statistic for $\rho$ obtained in equation (4.5) is the same as the one obtained from equation (4.4). Moreover, the asymptotic properties of $t_\rho$ in the more general cases when deterministic terms such as a constant and/or a linear time trend are included in equation (4.4) also carry over to the ADF test.

The second approach is the test suggested by Phillips (1987) and Phillips and Perron (1988). The Phillips-Perron test uses a nonparametric approach to correct for the serial correlation in $\varepsilon_t$ so that the resulting test statistic is free of the parameters characterizing the serial serial correlation. Because it has the same asymptotic distribution as the Dickey-Fuller test, the critical values tabulated by Fuller (1976) can be used for statistical inference.
The unit-root tests described above also provide the basis for the residual-based tests of cointegration that are most widely used in the empirical literature. In the simplest two-variable case, $X_t$ and $Y_t$ are said to be cointegrated if each is integrated of order one but some linear combination of them is stationary\(^{50}\). The cointegration relationship provides a description of the long-run economic relationships between the two integrated variables.

Engle and Granger (1987) suggested several tests of cointegration using a two-step procedure. They also tabulated the critical values of these tests in a finite sample of 100 by Monte Carlo simulation. Based on a comparison of power among several tests, they recommended the approach consisting of the following two steps\(^{51}\): First, the cointegrating vector is consistently\(^{52}\) estimated by the cointegrating regression

\[
Y_t = \phi + \Pi X_t + z_t
\]

\(^{50}\) For a more general definition of cointegration with more than two variables, and each may be an integrated process of order greater than one, see Engle and Granger (1987).

\(^{51}\) These tests are usually called Engle-Granger tests if no lags are included and augmented Engle-Granger tests if lags are included. See MacKinnon (1990).

\(^{52}\) The least square estimation of the cointegrating vector is actually "super consistent". See Stock (1987).
The second step is an augmented Dickey-Fuller test applied to the
residuals \( \hat{z}_t \) obtained in the first step,

\[
\Delta z_t = \rho z_{t-1} + \sum_{i=1}^{Q} \theta_i \Delta z_{t-i} + \epsilon_t \tag{4.7}
\]

Engle and Yoo (1987) extended the model to the test of cointegration
among \( N \) variables where \( N \geq 2 \) and tabulated the critical values for a
variety of sample sizes.

A second residual-based test for cointegration is proposed by
Phillips and Ouliaris (1990), who suggest to use the Phillips-Perron
test in the second step instead of the ADF test. This test is
asymptotically equivalent to the ADF test, although their finite
sample distributions may be different. Critical values for sample
size 500 are provided by Phillips and Ouliaris (1990).

For critical values of the augmented Dickey-Fuller test and
augmented Engle-Granger test at 1%, 5% and 10% sizes, MacKinnon (1990)
provides extensive tabulation of the response surface regressions
obtained from Monte Carlo experimentation. From the response
surfaces, the critical values for any given sample size can be easily
computed. They can be used to test for unit roots in the individual
time series and for whether the series are cointegrated.
4.3.b Testing Unit Roots and Cointegration with Panel Data

Consider a group of $N$ countries observed annually over $T$ years. We assume that for each country $i$, $Z_{it}$ follows the simplest case of an AR(1) in $t$, and has a country specific effect $\eta_i$, i.e.

$$Z_{it} = \rho Z_{i,t-1} + \eta_i + \varepsilon_{it} \quad i=1,2,\ldots,N; \quad t=1,2,\ldots,T. \quad (4.8)$$

Assume first that $\varepsilon_{it}$ are i.i.d. with $E(\varepsilon_{it}) = 0$ and $\text{Var}(\varepsilon_{it}) = \sigma^2$ for all $(i,t) \in \{1, 2, \ldots, N\} \times \{1, 2, \ldots, T\}$, and that $E|\varepsilon_{it}|^{2+\lambda} < +\infty$ for some $\lambda > 0$ for all $i$ and $t$. Estimation of (8) by the fixed effect model yields

$$\hat{\rho} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (Z_{it} - \bar{Z}_i)(Z_{i,t-1} - \bar{Z}_i)}{N \sum_{i=1}^{N} \sum_{t=1}^{T} (Z_{i,t-1} - \bar{Z}_i)^2}$$

where

$$\bar{Z}_i = \frac{1}{T} \sum_{t=1}^{T} Z_{it}$$

Levin and Lin (1992) proved that under the null hypothesis of $\rho = 1$, $T\sqrt{\hat{\rho}}(\hat{\rho} - 1) - \frac{\sqrt{N} \mu_{7T}}{\mu_{8T}}$ is asymptotically distributed as normal with mean 0 and variance 10.2, and $\sqrt{1.25}(t \hat{\rho} - \frac{\sqrt{N} \mu_{7T}}{\sqrt{\mu_{8T}}})$ is asymptotically
distributed as standard normal when $N \to \infty$ and $T \to \infty$. Furthermore, if
\[
\sqrt{N} \to 0, \text{ then } T \sqrt{N} (\hat{\rho} - 1) + 3 \sqrt{N} \text{ is asymptotically distributed as normal with mean 0 and variance 10.2, while } \sqrt{1.25} t_\rho + \sqrt{1.875} \sqrt{N} \text{ is asymptotically distributed as standard normal. In other words, the presence of individual specific fixed effects causes the unit root test statistics to have non-central asymptotic normal distribution with a variance smaller than 1. The downward shift of the mean of this distribution is proportional to } \sqrt{N}.

Levin and Lin have also proved that those asymptotic properties are independent of whether a constant intercept, a time trend, or time specific fixed effects are included in the model, and of what the values of the individual specific effects are. By Monte Carlo simulation, Levin and Lin (1992) tabulated critical values for the purpose of testing unit root in finite samples of various $N$ and $T$.

In the more general case when $\varepsilon_{it}$ are serially correlated, Levin and Lin suggest either to include $\Delta Z_{i,t-k}$ ($k = 1, 2, \ldots, K$) to correct for those serial correlation (à la the Augmented Dickey-Fuller test),

\[53. \] The definitions for $\mu_{7T}$ and $\mu_{8T}$ are
\[
\mu_{7T} = E\left( \frac{1}{T} \sum_{t=1}^{T} \left( Z_{i,t-1} - \bar{Z}_i \right) \varepsilon_{it} \right) = -\frac{1}{2} + o(T)
\]
\[
\mu_{8T} = E\left( \frac{1}{T^2 \sigma^2} \sum_{t=1}^{T} \left( Z_{i,t-1} - \bar{Z}_i \right)^2 \right) = \frac{1}{6} + o(T)
\]
or to correct for the serial correlation directly by nonparametric approach, using certain window sizes and lag length (a la the Phillips-Perron test). It is noted in their paper that the asymptotic properties of \( \hat{\rho} \) under the null will be independent of the serial correlations in \( \epsilon_{it} \) after these corrections.

In this chapter, a general model as the following is utilized to test unit roots and cointegration

\[
\Delta z_{it} = \delta_0 + \delta_1 t + \eta_i + v_t + \rho \Delta z_{i,t-1} + \sum_{j=1}^{R} \theta_j \Delta z_{i,t-j} + \epsilon_{it}
\]

where \( i = 1, 2, \ldots, N, \ t = 1, 2, \ldots, T. \ \delta_1 t \) is a linear time trend and \( v_t \) is the time specific fixed effect. The \( \rho \) in this model is estimated by a panel transformation of the data, which subtracts the individual mean and time specific mean from all variables involved in the above equation, followed by an estimation of

\[
\tilde{\Delta z}_{it} = \tilde{\rho} \tilde{z}_{i,t-1} + \sum_{j=1}^{R} \theta_j \tilde{\Delta z}_{i,t-j} + \tilde{\epsilon}_{it}
\]

where variables with a tilde are those after the panel transformation.

The asymptotic distribution of the \( t \)-statistic for \( \hat{\rho} \) follows that of the non-central normal described above since it is not affected by inclusion of \( \delta_0, \delta_1 t, v_t \) and \( \Delta z_{i,t-j} (j = 1, \ldots, R) \). The critical values for the finite sample size of the data set used in this chapter is obtained by Monte Carlo simulation.
4.4 The Data

Data for the following twelve OECD countries are used in my empirical analysis: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Finland (FIN), Germany (GER), Greece (GRE), Japan (JPN), Norway (NOR), Switzerland (SWI), the United Kingdom (UKD), and the United States (USA). The data necessary for this study are not generally available for the other OECD countries.

Data on the following variables are obtained from the computer tapes of OECD National Accounts: Main Aggregates (1990) and OECD National Accounts: Detailed Tables (1990). They are annual and from 1960 to 1988 for all twelve countries. (The names in the parentheses are the ones used in the data tapes from which they are extracted.)

(a) Private final consumption expenditure, valued at current prices (POPC);
(b) National disposable income, valued at current prices (NDI);
(c) General government consumption expenditure, valued at current prices (P0GC);
(d) Mid-year estimates of population (POP);
(e) Price indices of private final consumption expenditure
    (IDXPRIPC, 1985 = 100);
(f) Price indices of gross domestic product (IDXPRIGDPE, 1985 = 100);
(g) Price indices of general government consumption expenditure
    (IDXPRIGC, 1985 = 100);
(h) General government net saving, valued at current prices
\( \text{(SAVGG)} \);

Real per capita private final consumption expenditure \( (C_t) \) is constructed by dividing nominal private final consumption expenditure \( (POPC) \) by both the price index of private final consumption expenditure \( (IDXPRIPC) \) and mid-year estimates of population \( (POP) \). Real per capita private disposable income \( (Y_t) \) is constructed by dividing real private disposable income by the mid-year estimates of population \( (POP) \), where the real private disposable income is real national disposable income less real general government disposable income\(^{54}\).

A potential problem with the data used in this chapter exists because of the difference between consumption and consumption expenditure. This problem, however, is reduced to certain extent by the following features of the data. (i) Annual data are used instead of data from more frequent observations (quarterly, monthly). It is reasonable to assume that consumption is close to consumption

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\(^{54}\) Real national disposable income and real general government disposable income are obtained, respectively, by dividing nominal national disposable income \( (NDI) \) by price indices for gross domestic product \( (IDXPRIGDP) \), and dividing nominal general government disposable income by price indices for general government consumption expenditure \( (IDXPRIGC) \), where the nominal general government disposable income is the sum of general government consumption expenditure \( (PUGC) \) and general government saving \( (SAVG) \).
expenditure in a year’s time, even though some goods will last longer than one year. (ii) Aggregation across households also helps to reduce the extent of the difference between consumption and consumption expenditure. The unusually high consumption spending of some households in a particular year, when they make durable consumption purchases, are likely to be offset in the aggregate data by the low spending of others that have made durable consumption purchases earlier. In the rest of this chapter, I will take private consumption expenditure as private consumption without further explanation.

4.5 Empirical Results

Table 4.1 and 4.2 list the results of Dickey-Fuller tests, Augmented Dickey Fuller tests and Phillips-Perron tests for unit roots in per capita private disposable income and per capita private consumption series for twelve OECD countries. The ADF tests include one to three lags of $\Delta Y_t$ in Table 4.1, and one to three lags of $\Delta C_t$ in Table 4.2. In both tables, one to fourth order Newey-West (1985) corrections are used in the Phillips-Perron test statistics. Furthermore, all tests are performed on the data both for the cases with and without a linear time trend.

The results in both tables are overwhelmingly in favor of the unit root hypothesis in disposable income and consumption. With only one exception, the values of all test statistics in Table 4.1 are
insignificant at the 10% level. In Table 4.2, only one test statistic
is significant at the 5% level and only two are significant at the 10%
level. Appreciably more of these test statistics should have been
significant merely by chance: three at the 5% level and six at the 10%
level in each table.

Table 4.3 lists the results of the cointegrating regression for
all twelve countries. The residuals of those regressions are
subsequently used in testing cointegration between consumption and
disposable income. The results of Engle-Granger test, Augmented
Engle-Granger test and Phillips-Ouliaris test are listed in Table 4.4.
One to three lags of $\Delta t$ are included in the Augmented Engle-Granger
test, while one to fourth order Newey-West (1985) corrections are used
in the Phillips-Ouliaris test.

The results in Table 4.4 indicate that only in three of the
twelve countries is there strong evidence against no cointegration
between consumption and disposable income. The EG test and AEG test
with 1 lag are significant at the 5% level for Greece and U.S., while
the EG test for Austria, AEG test with 2 lags for Greece and AEG test
with 4 lags for US are significant at the 10% level. As for the
Phillips-Ouliaris test, Austria, Greece and U.S. are all significant
at the 5% level, regardless of the number of lags included in the
Newey-West correction. These results conform well with the
conflicting ones documented by Campbell (1987), Campbell and Clarida
(1987) and Attfield, Demery and Duck (1990), thus providing strong
motivation for investigating the issues using panel data.
Preliminary scatterplots offer even more intuitive appeal for using panel data. Scatterplots of $Y_{it}$ on $Y_{i,t-1}$, $C_{it}$ on $C_{i,t-1}$, $|\varepsilon_{it}|$ on $|\varepsilon_{i,t-1}|$ are shown in Figure 4.1, 4.2 and 4.3 respectively. Evidently, the scatterplot for disposable income forms a very tight 45 degree line as does the one for consumption. By contrast, the scatterplot for the residuals from the regression is much flatter.

Formal evidence of unit roots in consumption and disposable income, and their cointegration, are provided in Table 4.5, 4.6 and 4.7. In Table 4.5,

$$\Delta \tilde{Y}_{it} = \rho \Delta \tilde{Y}_{i,t-1} + \sum_{j=1}^{R} \Theta_j \Delta \tilde{Y}_{i,t-j} + \tilde{\varepsilon}_{it}$$

is estimated with $R = 1, 2, 3$ and 4. Similar panel estimation is applied to consumption and the residual of the cointegrating regression, with their results presented in Table 4.6 and 4.7 respectively.

The estimates of the $\rho$'s in all three regressions are listed in the first rows of each table, with the $t_\rho$ statistics for them listed in the parenthesis immediately following. These $t_\rho$'s have been appropriately corrected for loss of the degree of freedom due to panel transformation of the original data. It is obvious that $t_\rho$'s for disposable income are all negative, two of the $t_\rho$'s for consumption are negative while the other two are positive. More importantly, all of them have small absolute values.
The $t_p$'s for the residuals of the cointegrating regressions are, however, all negative with large absolute values. According to the Monte Carlo simulation of Levin and Lin (1992), the critical values of $t_p$ for $T = 25$ and $N = 10$ are $-5.94$ for the 1% significance level, $-5.42$ for 5%, and $-5.14$ for 10%. By these critical values, we cannot reject the unit root hypothesis in either consumption or disposable income at any significance level. We can, nevertheless, strongly reject the hypothesis of no cointegration between the two.

The finite sample critical values of Levin and Lin (1992) used above are obtained in a model with no lagged difference terms. Furthermore, because the residuals are generated by the cointegrating regression, unit root hypothesis will be rejected too often if the critical values for raw data are used to test for cointegration based on the generated residuals. For these reasons, I will use Monte Carlo experiments to get finite sample critical values in the next section.

4.6 Monte Carlo Simulations

Three sets of Monte Carlo simulations are performed in this chapter to get the finite sample distribution of the test statistics. The critical values, which are obtained in models calibrated to the data employed in this chapter, are then used to make more appropriate inferences in the unit root and cointegration tests with panel data.
In the first simulation, I obtain the critical values for testing unit roots in disposable income with panel data. This is achieved by first estimating the model under the null hypothesis that $\rho = 0$, i.e.,

$$\Delta \bar{y}_{it} = \sum_{j=1}^{R} \theta_j \Delta \bar{y}_{i,t-j} + \bar{u}_{it}'.$$

The null model above, with its coefficients $\theta_j$'s calibrated to the data set of this chapter, are used along with 10,000 draws of samples to obtain simulated data. Each draw of the $\bar{u}_{it}$'s come from a standard normal random number generator. The simulated data, after appropriate panel transformation, are then used to estimate the alternative model

$$\Delta \bar{y}_{it} = \rho \bar{y}_{i,t-1} + \sum_{j=1}^{R} \theta_j \Delta \bar{y}_{i,t-j} + \bar{\varepsilon}_{it}'.$$

The results of the $t_\rho$'s provide an approximate distribution of the statistic for testing unit roots with panel data. Column 1 in Table 4.8 summarizes the results of this simulation for the particular case of $R = 1$. In the simulation, starting values, $\bar{y}_{i,1}$ and $\bar{y}_{i,2}'$ are taken to be zero.

In the second simulation, I obtain the critical values for testing unit root in consumption with panel data, with the same method used in the first simulation. The results for the case when $R = 1$ are summarized in column 2 of Table 4.8.

In the third simulation, I obtain the critical values for testing cointegration between consumption and disposable income with panel
data. This is achieved by first estimating the model for consumption under the nonstationary null hypothesis that \( \rho = 0 \), i.e.,
\[
\Delta \tilde{C}_{it} = \sum_{j=1}^{R} K_j \Delta \tilde{C}_{i,t-j} + \tilde{u}_{it}
\]
and the model for cointegrating residual under the nonstationary null hypothesis that \( \rho = 0 \), i.e.
\[
\Delta \tilde{\epsilon}_{it} = \sum_{j=1}^{S} \tilde{\epsilon}_j \Delta \tilde{C}_{i,t-j} + \tilde{v}_{it}
\]
The null models above, with their coefficients \( K_j \)’s and \( \tilde{\epsilon}_j \)’s calibrated to the data set of this chapter, are used along with 10,000 draws of samples to obtain simulated data. Each draw of the \( \tilde{u}_{it} \)’s and \( \tilde{v}_{it} \)’s, assumed to be independent of each other, come from a standard normal random number generator. Note that the simulated data for \( Y_{it} \) are produced through the cointegrating regressions listed in Table 4.3.

The simulated data for \( Y_{it} \) and \( C_{it} \) are first used to obtain the cointegrating residuals. These residuals are then appropriately subtracted by their individual as well as time specific means. Finally they are used to estimate the alternative model
\[
\Delta \tilde{\epsilon}_{it} = \rho \tilde{\epsilon}_{i,t-1} + \sum_{j=1}^{S} \tilde{\epsilon}_j \Delta \tilde{\epsilon}_{i,t-j} + \tilde{\omega}_{it}
\]
A total of 10,000 sets of data provide an approximate distribution of the $t_p$ statistic for testing cointegration with panel data. The results are listed in column 3 of Table 4.8.

The critical values listed in Table 4.8 are not very far from those of Levin and Lin (1992), especially those for testing unit roots in the raw data. The critical values for testing cointegration are, as expected, generally larger in absolute value. However, we can still comfortably reject the null of no cointegration between consumption and disposable income at all conventional significance levels.

4.7 Concluding remarks

In this chapter, I use the National Accounts data from twelve OECD countries to test the nonstationarity of consumption and disposable income, as well as the cointegration of them that is implied by the RE-PIH. With panel data, more powerful unit root tests are not able to reject the unit root hypothesis for both consumption and disposable income. Furthermore, I found that when residual based cointegration tests are performed on individual time series data of each country, one cannot reject the null hypothesis that consumption and disposable income are not cointegrated. In comparison, when the data are pooled as a panel with time series and cross sections, the evidence strongly rejects the null hypothesis of no cointegration. I therefore conclude that the results from individual countries are due
to the small sample sizes that they are based upon, and that consumption and disposable income are cointegrated as implied by the rational expectation-permanent income hypothesis.
### Table 2.1
Summary Statistics for \( \Delta lnC_t \), \( \Delta lnY_t \), \( \frac{Y_t - 1}{C_{t-1}} \Delta lnY_t \), and \( \frac{Y_t - 1}{C_{t-1}} \)

<table>
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<tr>
<th></th>
<th>( \Delta lnC_t )</th>
<th>( \Delta lnY_t )</th>
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<th>( \frac{Y_t - 1}{C_{t-1}} \Delta lnY_t )</th>
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<td>( \bar{\sigma^2} )</td>
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<td>( (2) )</td>
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</table>

**Note:**

Government saving is not considered in constructing private disposable income in this table.
Table 2.2 Summary Statistics for $\Delta \ln C_t$, $\Delta \ln Y_t$, $\frac{Y_{t-1}}{C_{t-1}}$ $\Delta \ln Y_t$, $\frac{Y_{t-1}}{C_{t-1}}$ $\Delta \ln Y_t$

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<th>$\Delta \ln Y_t$</th>
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<th>$\frac{Y_{t-1}}{C_{t-1}}$</th>
<th>$\Delta \ln Y_t$</th>
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<td>$\sigma^2$</td>
<td>$\bar{\mu}$</td>
<td>$\sigma^2$</td>
<td>$\bar{\mu}$</td>
</tr>
<tr>
<td>AUS</td>
<td>2.20 %</td>
<td>3.8E-4</td>
<td>2.37 %</td>
<td>11.3E-4</td>
<td>.33</td>
</tr>
<tr>
<td>AUT</td>
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<td>3.2E-4</td>
<td>3.76 %</td>
<td>6.0E-4</td>
<td>.53</td>
</tr>
<tr>
<td>BEL</td>
<td>2.75 %</td>
<td>3.7E-4</td>
<td>3.08 %</td>
<td>5.1E-4</td>
<td>.72</td>
</tr>
<tr>
<td>CAN</td>
<td>2.83 %</td>
<td>4.0E-4</td>
<td>3.22 %</td>
<td>6.2E-4</td>
<td>.64</td>
</tr>
<tr>
<td>FIN</td>
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<td>6.9E-4</td>
<td>3.22 %</td>
<td>16.2E-4</td>
<td>.43</td>
</tr>
<tr>
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<td>3.00 %</td>
<td>3.6E-4</td>
<td>2.84 %</td>
<td>3.9E-4</td>
<td>.92</td>
</tr>
<tr>
<td>GRE</td>
<td>3.81 %</td>
<td>6.7E-4</td>
<td>4.43 %</td>
<td>18.8E-4</td>
<td>.36</td>
</tr>
<tr>
<td>JPN</td>
<td>4.73 %</td>
<td>8.4E-4</td>
<td>5.70 %</td>
<td>15.0E-4</td>
<td>.56</td>
</tr>
<tr>
<td>NOR</td>
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<td>6.8E-4</td>
<td>2.93 %</td>
<td>6.3E-4</td>
<td>1.08</td>
</tr>
<tr>
<td>SWI</td>
<td>1.97 %</td>
<td>2.5E-4</td>
<td>1.98 %</td>
<td>5.9E-4</td>
<td>.42</td>
</tr>
<tr>
<td>UKD</td>
<td>2.33 %</td>
<td>4.7E-4</td>
<td>2.03 %</td>
<td>11.2E-4</td>
<td>.42</td>
</tr>
<tr>
<td>USA</td>
<td>2.34 %</td>
<td>3.2E-4</td>
<td>2.44 %</td>
<td>4.5E-4</td>
<td>.70</td>
</tr>
</tbody>
</table>

Note:
Government saving is considered in constructing private disposable income in this table.
Table 2.3 Estimation of $\Delta \ln C_t = \mu + \lambda \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t - \mu \lambda \frac{Y_{t-1}}{C_{t-1}} + \nu_t : \text{GMM}$

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\text{se}(\mu)$</th>
<th>$p$-value($\mu$)</th>
<th>$\lambda$</th>
<th>$\text{se}(\lambda)$</th>
<th>$p$-value($\lambda$)</th>
<th>J-stat(8)</th>
<th>$p$-value(J)</th>
</tr>
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<tbody>
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<td>AUS</td>
<td>1.85 %</td>
<td>0.0045</td>
<td>0.00 %</td>
<td>0.520</td>
<td>0.184</td>
<td>0.48 %</td>
<td>5.36</td>
<td>71.86 %</td>
</tr>
<tr>
<td>AUT</td>
<td>2.05 %</td>
<td>0.0177</td>
<td>24.45 %</td>
<td>0.685</td>
<td>0.103</td>
<td>0.00 %</td>
<td>8.33</td>
<td>40.18 %</td>
</tr>
<tr>
<td>BEL</td>
<td>-6.80 %</td>
<td>0.1005</td>
<td>49.85 %</td>
<td>0.892</td>
<td>0.021</td>
<td>0.00 %</td>
<td>5.19</td>
<td>73.68 %</td>
</tr>
<tr>
<td>CAN</td>
<td>2.70 %</td>
<td>0.0042</td>
<td>0.00 %</td>
<td>0.446</td>
<td>0.116</td>
<td>0.01 %</td>
<td>6.95</td>
<td>54.25 %</td>
</tr>
<tr>
<td>DEN</td>
<td>1.60 %</td>
<td>0.0047</td>
<td>0.07 %</td>
<td>0.500</td>
<td>0.077</td>
<td>0.00 %</td>
<td>9.12</td>
<td>33.21 %</td>
</tr>
<tr>
<td>FIN</td>
<td>3.81 %</td>
<td>0.0051</td>
<td>0.00 %</td>
<td>0.450</td>
<td>0.077</td>
<td>0.00 %</td>
<td>6.25</td>
<td>61.97 %</td>
</tr>
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<td>3.14 %</td>
<td>0.0026</td>
<td>0.00 %</td>
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<td>0.074</td>
<td>0.00 %</td>
<td>9.63</td>
<td>29.22 %</td>
</tr>
<tr>
<td>GER</td>
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<td>0.0050</td>
<td>0.00 %</td>
<td>0.462</td>
<td>0.098</td>
<td>0.00 %</td>
<td>9.70</td>
<td>28.66 %</td>
</tr>
<tr>
<td>GRE</td>
<td>3.86 %</td>
<td>0.0044</td>
<td>0.00 %</td>
<td>0.464</td>
<td>0.062</td>
<td>0.00 %</td>
<td>5.84</td>
<td>66.54 %</td>
</tr>
<tr>
<td>IRE</td>
<td>-53.77 %</td>
<td>0.2516</td>
<td>3.26 %</td>
<td>0.908</td>
<td>0.004</td>
<td>0.00 %</td>
<td>5.73</td>
<td>67.70 %</td>
</tr>
<tr>
<td>ITA</td>
<td>3.97 %</td>
<td>0.0031</td>
<td>0.00 %</td>
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<td>0.064</td>
<td>0.00 %</td>
<td>7.97</td>
<td>43.64 %</td>
</tr>
<tr>
<td>JPN</td>
<td>2.75 %</td>
<td>0.0075</td>
<td>0.02 %</td>
<td>0.584</td>
<td>0.051</td>
<td>0.00 %</td>
<td>4.74</td>
<td>78.50 %</td>
</tr>
<tr>
<td>LUX</td>
<td>2.57 %</td>
<td>0.0022</td>
<td>0.00 %</td>
<td>0.049</td>
<td>0.083</td>
<td>55.38 %</td>
<td>6.85</td>
<td>55.24 %</td>
</tr>
<tr>
<td>NET</td>
<td>2.86 %</td>
<td>0.0101</td>
<td>0.45 %</td>
<td>0.580</td>
<td>0.108</td>
<td>0.00 %</td>
<td>8.46</td>
<td>39.02 %</td>
</tr>
<tr>
<td>NOR</td>
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<td>0.00 %</td>
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<td>0.107</td>
<td>0.06 %</td>
<td>7.04</td>
<td>53.26 %</td>
</tr>
<tr>
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<td>1.81 %</td>
<td>0.0025</td>
<td>0.00 %</td>
<td>0.334</td>
<td>0.104</td>
<td>0.12 %</td>
<td>8.82</td>
<td>35.78 %</td>
</tr>
<tr>
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<td>0.0026</td>
<td>0.00 %</td>
<td>0.109</td>
<td>0.108</td>
<td>31.12 %</td>
<td>7.70</td>
<td>46.33 %</td>
</tr>
<tr>
<td>UKD</td>
<td>2.28 %</td>
<td>0.0049</td>
<td>0.00 %</td>
<td>0.379</td>
<td>0.114</td>
<td>0.09 %</td>
<td>7.90</td>
<td>44.30 %</td>
</tr>
<tr>
<td>USA</td>
<td>2.88 %</td>
<td>0.0034</td>
<td>0.00 %</td>
<td>0.443</td>
<td>0.089</td>
<td>0.00 %</td>
<td>7.19</td>
<td>51.67 %</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is not considered in constructing private disposable income in this table.
2. Instruments are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t$, $\frac{Y_t}{C_t}$.
3. The heteroskedasticity and autocorrelation consistent standard errors in the table are obtained as in Newey and West (1989).
4. $J$ is asymptotically distributed as $\chi^2$ with 8 degrees of freedom since there are 10 orthogonality conditions and 2 parameters to be estimated.
Table 2.4 Estimation of $\Delta \ln C_t = \mu + \lambda \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t - \mu \lambda \frac{Y_{t-1}}{C_{t-1}} + \nu_t$ : OMM

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>se($\mu$)</th>
<th>p-value($\mu$)</th>
<th>$\lambda$</th>
<th>se($\lambda$)</th>
<th>p-value($\lambda$)</th>
<th>J-stat(9)</th>
<th>p-value(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>2.06 %</td>
<td>0.0017</td>
<td>0.00 %</td>
<td>0.233</td>
<td>0.052</td>
<td>0.00 %</td>
<td>6.01</td>
<td>64.59 %</td>
</tr>
<tr>
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<td>1.98 %</td>
<td>0.0039</td>
<td>0.00 %</td>
<td>0.404</td>
<td>0.066</td>
<td>0.00 %</td>
<td>6.89</td>
<td>54.90 %</td>
</tr>
<tr>
<td>BRL</td>
<td>1.72 %</td>
<td>0.0349</td>
<td>62.29 %</td>
<td>0.764</td>
<td>0.261</td>
<td>0.35 %</td>
<td>4.88</td>
<td>77.01 %</td>
</tr>
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<td>0.00 %</td>
<td>0.391</td>
<td>0.152</td>
<td>1.01 %</td>
<td>9.70</td>
<td>28.66 %</td>
</tr>
<tr>
<td>FIN</td>
<td>3.70 %</td>
<td>0.0082</td>
<td>0.00 %</td>
<td>0.478</td>
<td>0.153</td>
<td>0.17 %</td>
<td>6.01</td>
<td>64.65 %</td>
</tr>
<tr>
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<td>0.1367</td>
<td>51.51 %</td>
<td>0.862</td>
<td>0.028</td>
<td>0.00 %</td>
<td>6.57</td>
<td>58.36 %</td>
</tr>
<tr>
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<td>0.0060</td>
<td>0.00 %</td>
<td>0.423</td>
<td>0.033</td>
<td>0.00 %</td>
<td>9.51</td>
<td>30.11 %</td>
</tr>
<tr>
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<td>0.0192</td>
<td>79.35 %</td>
<td>0.706</td>
<td>0.066</td>
<td>0.00 %</td>
<td>6.10</td>
<td>63.60 %</td>
</tr>
<tr>
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<td>0.203</td>
<td>0.30 %</td>
<td>3.60</td>
<td>89.17 %</td>
</tr>
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<td>0.00 %</td>
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<td>0.158</td>
<td>97.07 %</td>
<td>6.44</td>
<td>59.76 %</td>
</tr>
<tr>
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<td>0.0031</td>
<td>0.00 %</td>
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<td>0.071</td>
<td>0.02 %</td>
<td>5.03</td>
<td>75.47 %</td>
</tr>
<tr>
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<td>0.0107</td>
<td>2.19 %</td>
<td>0.710</td>
<td>0.170</td>
<td>0.00 %</td>
<td>8.16</td>
<td>41.79 %</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is considered in constructing private disposable income in this table.
2. Instruments are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t$, $\frac{Y_{t-1}}{C_{t-1}}$.
3. The heteroskedasticity and autocorrelation consistent standard errors in the table are obtained as in Newey and West (1989).
4. $J$ is asymptotically distributed as $\chi^2$ with 8 degrees of freedom since there are 10 orthogonality conditions and 2 parameters to be estimated.
Table 2.5 Estimation of $\Delta \ln C_t = \lambda(\frac{Y_{t-1}}{C_{t-1}}) \Delta \ln Y_t + [1 - \lambda(\frac{Y_{t-1}}{C_{t-1}})](\mu + \sigma r_t) + \nu_t$ : GMM

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>se($\mu$)</th>
<th>p-value($\mu$)</th>
<th>$\lambda$</th>
<th>se($\lambda$)</th>
<th>p-value($\lambda$)</th>
<th>$\sigma$</th>
<th>se($\sigma$)</th>
<th>p-value($\sigma$)</th>
<th>$J$-stat(10)</th>
<th>p-value($J$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEL</td>
<td>3.83%</td>
<td>0.0040</td>
<td>0.00%</td>
<td>0.580</td>
<td>0.048</td>
<td>0.00%</td>
<td>-0.788</td>
<td>0.188</td>
<td>0.00%</td>
<td>7.17</td>
<td>70.89%</td>
</tr>
<tr>
<td>CAN</td>
<td>3.89%</td>
<td>0.0017</td>
<td>0.00%</td>
<td>0.405</td>
<td>0.024</td>
<td>0.00%</td>
<td>-0.423</td>
<td>0.042</td>
<td>0.00%</td>
<td>5.82</td>
<td>82.98%</td>
</tr>
<tr>
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<td>0.00%</td>
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<td>0.073</td>
<td>0.00%</td>
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<td>0.09%</td>
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<td>35.58%</td>
</tr>
<tr>
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<td>4.74%</td>
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<td>0.00%</td>
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<td>0.147</td>
<td>0.00%</td>
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<td>79.33%</td>
</tr>
<tr>
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<td>0.00%</td>
<td>0.561</td>
<td>0.062</td>
<td>0.00%</td>
<td>-0.553</td>
<td>0.336</td>
<td>10.03%</td>
<td>7.26</td>
<td>70.07%</td>
</tr>
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<td>0.0060</td>
<td>0.00%</td>
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</tr>
<tr>
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<td>0.176</td>
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<td>0.03%</td>
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<td>61.91%</td>
</tr>
<tr>
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<td>5.54%</td>
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<td>0.074</td>
<td>0.00%</td>
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<td>0.371</td>
<td>54.30%</td>
<td>10.54</td>
<td>39.44%</td>
</tr>
<tr>
<td>UKD</td>
<td>1.94%</td>
<td>0.0038</td>
<td>0.00%</td>
<td>0.179</td>
<td>0.108</td>
<td>9.73%</td>
<td>0.214</td>
<td>0.093</td>
<td>2.12%</td>
<td>7.92</td>
<td>63.63%</td>
</tr>
<tr>
<td>USA</td>
<td>3.43%</td>
<td>0.0034</td>
<td>0.00%</td>
<td>0.442</td>
<td>0.078</td>
<td>0.00%</td>
<td>-0.206</td>
<td>0.133</td>
<td>12.24%</td>
<td>9.56</td>
<td>48.01%</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is not considered in constructing private disposable income in this table.
2. Instruments are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t$, $\frac{Y_t}{C_t}$, $r_t$.
3. The heteroskedasticity and autocorrelation consistent standard errors in the table are obtained as in Newey and West (1989).
4. $J$ is asymptotically distributed as $\chi^2$ with 10 degrees of freedom since there are 13 orthogonality conditions and 3 parameters to be estimated.
Table 2.6: Estimation of $\Delta \ln C_t = \frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t + (1 - \frac{Y_{t-1}}{C_{t-1}}) (\mu + \sigma r_t) + \nu_t$ : GMM

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>se($\mu$)</th>
<th>p-value($\mu$)</th>
<th>$\lambda$</th>
<th>se($\lambda$)</th>
<th>p-value($\lambda$)</th>
<th>$\sigma$</th>
<th>se($\sigma$)</th>
<th>p-val($\sigma$)</th>
<th>J-stat(10)</th>
<th>p-value(J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEL</td>
<td>3.21%</td>
<td>0.0034</td>
<td>0.00%</td>
<td>0.401</td>
<td>0.115</td>
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<td>0.122</td>
<td>1.52%</td>
<td>6.51%</td>
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</tr>
<tr>
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<td>0.00%</td>
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<td>0.099</td>
<td>0.00%</td>
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<td>1.19%</td>
<td>10.62%</td>
<td>38.82%</td>
</tr>
<tr>
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<td>0.00%</td>
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<td>0.04%</td>
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<td>76.53%</td>
</tr>
<tr>
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<td>2.82%</td>
<td>0.0069</td>
<td>0.00%</td>
<td>0.455</td>
<td>0.052</td>
<td>0.00%</td>
<td>-0.216</td>
<td>0.242</td>
<td>37.08%</td>
<td>9.90%</td>
<td>44.90%</td>
</tr>
<tr>
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<td>0.065</td>
<td>0.00%</td>
<td>0.356</td>
<td>0.562</td>
<td>52.72%</td>
<td>6.37%</td>
<td>78.36%</td>
</tr>
<tr>
<td>SWI</td>
<td>1.71%</td>
<td>0.0042</td>
<td>0.00%</td>
<td>0.334</td>
<td>0.081</td>
<td>0.00%</td>
<td>-0.054</td>
<td>0.214</td>
<td>80.09%</td>
<td>9.52%</td>
<td>48.33%</td>
</tr>
<tr>
<td>UKD</td>
<td>1.94%</td>
<td>0.0027</td>
<td>0.00%</td>
<td>0.277</td>
<td>0.078</td>
<td>0.04%</td>
<td>0.204</td>
<td>0.071</td>
<td>0.40%</td>
<td>6.36%</td>
<td>78.43%</td>
</tr>
<tr>
<td>USA</td>
<td>6.43%</td>
<td>0.0800</td>
<td>42.16%</td>
<td>0.823</td>
<td>0.116</td>
<td>0.00%</td>
<td>-1.794</td>
<td>2.963</td>
<td>54.50%</td>
<td>7.41%</td>
<td>68.65%</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is considered in constructing private disposable income in this table.
2. Instruments are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_{t-1}}{C_{t-1}} \Delta \ln Y_t$, $\frac{Y_{t-1}}{C_{t-1}} r_t$.
3. The heteroskedasticity and autocorrelation consistent standard errors in the table are obtained as in Newey and West (1989).
4. $J$ is asymptotically distributed as $\chi^2$ with 10 degrees of freedom since there are 13 orthogonality conditions and 3 parameters to be estimated.
<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\lambda$</th>
<th>UNEMP</th>
<th>$S/Y$</th>
<th>$F$</th>
<th>$\Delta \ln(POP)$</th>
<th>$\Delta \ln Y$</th>
<th>$r$</th>
</tr>
</thead>
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<tr>
<td>AUS</td>
<td>1.85%</td>
<td>0.520</td>
<td>4.44%</td>
<td>10.92%</td>
<td>40.81%</td>
<td>1.61%</td>
<td>2.08%</td>
<td>0.60%</td>
</tr>
<tr>
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<td>0.685</td>
<td>1.98%</td>
<td>15.99%</td>
<td>37.08%</td>
<td>0.27%</td>
<td>3.26%</td>
<td>1.23%</td>
</tr>
<tr>
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<td>-6.80%</td>
<td>0.892</td>
<td>5.88%</td>
<td>9.23%</td>
<td>37.09%</td>
<td>0.27%</td>
<td>2.90%</td>
<td>0.95%</td>
</tr>
<tr>
<td>CAN</td>
<td>2.70%</td>
<td>0.446</td>
<td>6.85%</td>
<td>11.29%</td>
<td>42.56%</td>
<td>1.32%</td>
<td>3.11%</td>
<td>2.20%</td>
</tr>
<tr>
<td>DEN</td>
<td>1.60%</td>
<td>0.500</td>
<td>4.74%</td>
<td>12.28%</td>
<td>37.54%</td>
<td>0.41%</td>
<td>1.73%</td>
<td>3.50%</td>
</tr>
<tr>
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<td>3.81%</td>
<td>0.450</td>
<td>3.45%</td>
<td>15.61%</td>
<td>39.82%</td>
<td>0.39%</td>
<td>2.91%</td>
<td>4.85%</td>
</tr>
<tr>
<td>FRA</td>
<td>3.14%</td>
<td>0.432</td>
<td>4.70%</td>
<td>14.39%</td>
<td>38.58%</td>
<td>0.72%</td>
<td>2.81%</td>
<td>1.31%</td>
</tr>
<tr>
<td>GER</td>
<td>3.28%</td>
<td>0.462</td>
<td>2.91%</td>
<td>17.89%</td>
<td>36.99%</td>
<td>0.36%</td>
<td>2.36%</td>
<td>1.40%</td>
</tr>
<tr>
<td>GRE</td>
<td>3.86%</td>
<td>0.464</td>
<td>NA</td>
<td>19.38%</td>
<td>38.61%</td>
<td>0.66%</td>
<td>3.70%</td>
<td>-0.97%</td>
</tr>
<tr>
<td>IRE</td>
<td>-53.77%</td>
<td>0.908</td>
<td>8.48%</td>
<td>9.17%</td>
<td>39.18%</td>
<td>0.79%</td>
<td>2.31%</td>
<td>1.71%</td>
</tr>
<tr>
<td>ITA</td>
<td>3.97%</td>
<td>0.416</td>
<td>4.98%</td>
<td>17.60%</td>
<td>38.24%</td>
<td>0.48%</td>
<td>3.41%</td>
<td>0.93%</td>
</tr>
<tr>
<td>JPN</td>
<td>2.75%</td>
<td>0.584</td>
<td>1.81%</td>
<td>16.78%</td>
<td>41.95%</td>
<td>0.98%</td>
<td>5.48%</td>
<td>2.38%</td>
</tr>
<tr>
<td>LUX</td>
<td>2.57%</td>
<td>0.049</td>
<td>NA</td>
<td>32.04%</td>
<td>36.80%</td>
<td>0.61%</td>
<td>3.81%</td>
<td>NA</td>
</tr>
<tr>
<td>NET</td>
<td>2.86%</td>
<td>0.580</td>
<td>4.82%</td>
<td>20.40%</td>
<td>41.01%</td>
<td>0.90%</td>
<td>2.28%</td>
<td>0.75%</td>
</tr>
<tr>
<td>NOR</td>
<td>2.17%</td>
<td>0.366</td>
<td>2.06%</td>
<td>21.12%</td>
<td>37.03%</td>
<td>0.57%</td>
<td>2.84%</td>
<td>3.33%</td>
</tr>
<tr>
<td>SWE</td>
<td>1.81%</td>
<td>0.334</td>
<td>1.72%</td>
<td>9.26%</td>
<td>35.54%</td>
<td>0.43%</td>
<td>2.02%</td>
<td>1.41%</td>
</tr>
<tr>
<td>SWI</td>
<td>1.89%</td>
<td>0.109</td>
<td>0.79%</td>
<td>22.62%</td>
<td>38.56%</td>
<td>0.77%</td>
<td>1.85%</td>
<td>-2.23%</td>
</tr>
<tr>
<td>UKD</td>
<td>2.28%</td>
<td>0.379</td>
<td>5.64%</td>
<td>11.62%</td>
<td>36.57%</td>
<td>0.31%</td>
<td>2.09%</td>
<td>1.12%</td>
</tr>
<tr>
<td>USA</td>
<td>2.88%</td>
<td>0.443</td>
<td>5.98%</td>
<td>9.52%</td>
<td>40.85%</td>
<td>1.11%</td>
<td>2.17%</td>
<td>2.35%</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is not considered in constructing private disposable income in this table.
2. $\mu$ and $\lambda$ estimates are taken from Table 2.3.
3. UNEMP is the sample mean of unemployment rate.
4. $S/Y$ is the sample mean of private saving rate.
5. $\Delta \ln(POP)$ is the sample mean of population growth rate.
6. $F$ is the sample mean of the fraction of young population (age 20-34) in the total working age population (age 20-64).
7. $\Delta \ln Y$ is the sample mean of growth rate of private disposable income.
8. $r$ is the sample mean of real interest rate.
Table 2.8 System Estimation of $\Delta \ln C_{it} = \mu_i + \lambda_i (C_{i,t-1}) \Delta \ln Y_{it} - \mu_i \lambda_i (C_{i,t-1}) + \nu_{it}$, $i=1, \ldots, 19$ : E-NL3SLS

<table>
<thead>
<tr>
<th></th>
<th>$\mu$</th>
<th>$\text{se}(\mu)$</th>
<th>$t(\mu)$</th>
<th>$\lambda$</th>
<th>$\text{se}(\lambda)$</th>
<th>$t(\lambda)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>2.17%</td>
<td>0.0045</td>
<td>4.82</td>
<td>0.257</td>
<td>0.051</td>
<td>5.03</td>
</tr>
<tr>
<td>AUT</td>
<td>2.92%</td>
<td>0.0051</td>
<td>5.72</td>
<td>0.350</td>
<td>0.055</td>
<td>6.35</td>
</tr>
<tr>
<td>BEL</td>
<td>2.86%</td>
<td>0.0063</td>
<td>4.56</td>
<td>0.489</td>
<td>0.057</td>
<td>8.63</td>
</tr>
<tr>
<td>CAN</td>
<td>2.81%</td>
<td>0.0044</td>
<td>6.42</td>
<td>0.473</td>
<td>0.032</td>
<td>14.59</td>
</tr>
<tr>
<td>DN</td>
<td>1.77%</td>
<td>0.0089</td>
<td>2.00</td>
<td>0.528</td>
<td>0.034</td>
<td>15.69</td>
</tr>
<tr>
<td>FIN</td>
<td>3.97%</td>
<td>0.0075</td>
<td>5.31</td>
<td>0.539</td>
<td>0.029</td>
<td>18.75</td>
</tr>
<tr>
<td>FRA</td>
<td>2.92%</td>
<td>0.0029</td>
<td>10.23</td>
<td>0.326</td>
<td>0.043</td>
<td>7.62</td>
</tr>
<tr>
<td>GER</td>
<td>3.38%</td>
<td>0.0062</td>
<td>5.48</td>
<td>0.428</td>
<td>0.029</td>
<td>14.53</td>
</tr>
<tr>
<td>GRE</td>
<td>3.75%</td>
<td>0.0052</td>
<td>7.27</td>
<td>0.337</td>
<td>0.035</td>
<td>9.74</td>
</tr>
<tr>
<td>IRE</td>
<td>1.43%</td>
<td>0.0178</td>
<td>0.80</td>
<td>0.639</td>
<td>0.072</td>
<td>8.90</td>
</tr>
<tr>
<td>ITA</td>
<td>4.31%</td>
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<td>7.45</td>
<td>0.499</td>
<td>0.035</td>
<td>14.08</td>
</tr>
<tr>
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<td>0.0085</td>
<td>3.46</td>
<td>0.544</td>
<td>0.042</td>
<td>12.97</td>
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<td>LUX</td>
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<td>7.34</td>
<td>0.043</td>
<td>0.033</td>
<td>1.30</td>
</tr>
<tr>
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<td>0.0089</td>
<td>3.27</td>
<td>0.493</td>
<td>0.037</td>
<td>13.51</td>
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<tr>
<td>NOR</td>
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<td>0.0090</td>
<td>2.55</td>
<td>0.368</td>
<td>0.067</td>
<td>5.52</td>
</tr>
<tr>
<td>SWE</td>
<td>1.84%</td>
<td>0.0067</td>
<td>2.73</td>
<td>0.496</td>
<td>0.068</td>
<td>7.26</td>
</tr>
<tr>
<td>SWI</td>
<td>1.77%</td>
<td>0.0040</td>
<td>4.41</td>
<td>0.316</td>
<td>0.031</td>
<td>10.07</td>
</tr>
<tr>
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<td>4.41</td>
<td>0.414</td>
<td>0.046</td>
<td>9.05</td>
</tr>
<tr>
<td>USA</td>
<td>2.57%</td>
<td>0.0036</td>
<td>7.08</td>
<td>0.369</td>
<td>0.032</td>
<td>11.51</td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is not considered in constructing private disposable income in this table.
2. Instruments are $C$ and $\Delta \ln C_{i,t-2}$ for $i=1, \ldots, 19$.
3. The standard errors are heteroskedasticity and autocorrelation consistent.
Table 2.9 System Estimation of

\[ \Delta \ln C_{it} = \mu_i + (\phi + \Sigma \phi_k \zeta_{kt}) \Delta \ln Y_{it-1} + (\sum_{k=1}^{6} \phi_k \zeta_{kt} + \phi_k \zeta_{kt-1}) + \psi_{it} \]

\text{for } i = 1, \ldots, 19 : E-WL3SLS

<table>
<thead>
<tr>
<th></th>
<th>( \mu )</th>
<th>( \text{se}(\mu) )</th>
<th>( \bar{t} )</th>
<th>( R^2 )</th>
<th>( \text{se} )</th>
<th>( t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>2.21 %</td>
<td>0.0037</td>
<td>5.92</td>
<td>31.56 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AUT</td>
<td>2.74 %</td>
<td>0.0072</td>
<td>3.82</td>
<td>33.92 %</td>
<td>z0</td>
<td>-0.879</td>
</tr>
<tr>
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<td>2.49 %</td>
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<td>41.56 %</td>
<td>z1</td>
<td>3.683</td>
</tr>
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<td>2.69 %</td>
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<td>7.72</td>
<td>74.96 %</td>
<td>Z2</td>
<td>0.012</td>
</tr>
<tr>
<td>DEN</td>
<td>1.65 %</td>
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<td>2.33</td>
<td>69.65 %</td>
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<td>-35.923</td>
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<tr>
<td>FIN</td>
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<td>5.37</td>
<td>74.33 %</td>
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</tr>
<tr>
<td>FRA</td>
<td>3.05 %</td>
<td>0.0033</td>
<td>9.20</td>
<td>57.27 %</td>
<td>Z5</td>
<td>0.016</td>
</tr>
<tr>
<td>GER</td>
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<td>0.0054</td>
<td>5.62</td>
<td>47.40 %</td>
<td>Z6</td>
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</tr>
<tr>
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<td>6.69</td>
<td>65.26 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IRE</td>
<td>1.86 %</td>
<td>0.0102</td>
<td>1.82</td>
<td>41.33 %</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ITA</td>
<td>4.50 %</td>
<td>0.0063</td>
<td>7.14</td>
<td>64.82 %</td>
<td>z0</td>
<td>CONSTATE</td>
</tr>
<tr>
<td>JPN</td>
<td>3.53 %</td>
<td>0.0067</td>
<td>5.27</td>
<td>77.61 %</td>
<td>z1</td>
<td>( \Delta \ln Y )</td>
</tr>
<tr>
<td>LUX</td>
<td>1.74 %</td>
<td>0.0069</td>
<td>2.51</td>
<td>3.97 %</td>
<td>z2</td>
<td>( \Delta \ln (POP) )</td>
</tr>
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<td>3.99</td>
<td>47.50 %</td>
<td>z3</td>
<td>( \Delta \ln (POP) )</td>
</tr>
<tr>
<td>NOR</td>
<td>2.30 %</td>
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<td>3.02</td>
<td>28.52 %</td>
<td>z4</td>
<td>S/Y</td>
</tr>
<tr>
<td>SWE</td>
<td>1.77 %</td>
<td>0.0047</td>
<td>3.78</td>
<td>40.68 %</td>
<td>z5</td>
<td>UNEMP</td>
</tr>
<tr>
<td>SWI</td>
<td>1.79 %</td>
<td>0.0034</td>
<td>5.34</td>
<td>45.92 %</td>
<td>z6</td>
<td>F</td>
</tr>
<tr>
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<td>0.0070</td>
<td>8.88</td>
<td>48.13 %</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0.0034</td>
<td>7.46</td>
<td>67.27 %</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes:
1. Government saving is not considered in constructing private disposable income in this table.
2. Instruments are \( C \) and \( \Delta \ln C_{i,t-2} \) for \( i = 1, \ldots, 19 \).
3. The standard errors are heteroskedasticity and autocorrelation consistent.
4. \( \bar{z}_{ki} \) is the sample mean (with respect to \( t \)) of variable \( z_k \) in country \( i \), where \( k = 1, \ldots, 6 \), \( i = 1, \ldots, 19 \).
Table 2.10 Decomposition of $\lambda$ to Various Factors Associated with Liquidity Constraints

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\phi}_1 \bar{Z}_1$</th>
<th>$\hat{\phi}_2 \bar{Z}_2$</th>
<th>$\hat{\phi}_3 \bar{Z}_3$</th>
<th>$\hat{\phi}_4 \bar{Z}_4$</th>
<th>$\hat{\phi}_5 \bar{Z}_5$</th>
<th>$\hat{\phi}_6 \bar{Z}_6$</th>
<th>$\lambda = \hat{\phi}<em>0 + \sum</em>{k=1}^6 \hat{\phi}_k \bar{Z}_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>0.077</td>
<td>-0.000073</td>
<td>-0.577</td>
<td>-0.047</td>
<td>0.074</td>
<td>1.512</td>
<td>0.160</td>
</tr>
<tr>
<td>AUT</td>
<td>0.120</td>
<td>-0.000150</td>
<td>-0.096</td>
<td>-0.068</td>
<td>0.033</td>
<td>1.373</td>
<td>0.483</td>
</tr>
<tr>
<td>BEL</td>
<td>0.107</td>
<td>-0.000116</td>
<td>-0.099</td>
<td>-0.039</td>
<td>0.098</td>
<td>1.374</td>
<td>0.561</td>
</tr>
<tr>
<td>CAN</td>
<td>0.114</td>
<td>-0.000270</td>
<td>-0.476</td>
<td>-0.048</td>
<td>0.114</td>
<td>1.577</td>
<td>0.402</td>
</tr>
<tr>
<td>DEN</td>
<td>0.064</td>
<td>-0.000429</td>
<td>-0.148</td>
<td>-0.052</td>
<td>0.079</td>
<td>1.391</td>
<td>0.454</td>
</tr>
<tr>
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<td>0.107</td>
<td>-0.000594</td>
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<td>-0.067</td>
<td>0.058</td>
<td>1.475</td>
<td>0.552</td>
</tr>
<tr>
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<td>0.103</td>
<td>-0.000160</td>
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<td>-0.061</td>
<td>0.078</td>
<td>1.429</td>
<td>0.412</td>
</tr>
<tr>
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<td>0.087</td>
<td>-0.000172</td>
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<td>-0.076</td>
<td>0.049</td>
<td>1.370</td>
<td>0.419</td>
</tr>
<tr>
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<td>0.000119</td>
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<td>-0.083</td>
<td>N/A</td>
<td>1.430</td>
<td>0.368</td>
</tr>
<tr>
<td>IRE</td>
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<td>-0.039</td>
<td>0.142</td>
<td>1.451</td>
<td>0.475</td>
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<tr>
<td>ITA</td>
<td>0.126</td>
<td>-0.000114</td>
<td>-0.173</td>
<td>-0.075</td>
<td>0.083</td>
<td>1.417</td>
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</tr>
<tr>
<td>JPN</td>
<td>0.202</td>
<td>-0.000291</td>
<td>-0.351</td>
<td>-0.072</td>
<td>0.030</td>
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<tr>
<td>LUX</td>
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<td>N/A</td>
<td>-0.221</td>
<td>-0.137</td>
<td>N/A</td>
<td>1.363</td>
<td>0.267</td>
</tr>
<tr>
<td>NET</td>
<td>0.084</td>
<td>-0.000092</td>
<td>-0.322</td>
<td>-0.087</td>
<td>0.081</td>
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<tr>
<td>NOR</td>
<td>0.104</td>
<td>-0.000408</td>
<td>-0.206</td>
<td>-0.090</td>
<td>0.034</td>
<td>1.372</td>
<td>0.334</td>
</tr>
<tr>
<td>SWE</td>
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<td>-0.000172</td>
<td>-0.155</td>
<td>-0.040</td>
<td>0.029</td>
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<td>0.346</td>
</tr>
<tr>
<td>SWI</td>
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<td>0.000273</td>
<td>-0.278</td>
<td>-0.097</td>
<td>0.013</td>
<td>1.429</td>
<td>0.256</td>
</tr>
<tr>
<td>UKD</td>
<td>0.077</td>
<td>-0.000137</td>
<td>-0.111</td>
<td>-0.050</td>
<td>0.094</td>
<td>1.355</td>
<td>0.486</td>
</tr>
<tr>
<td>USA</td>
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<td>-0.398</td>
<td>-0.041</td>
<td>0.100</td>
<td>1.513</td>
<td>0.375</td>
</tr>
</tbody>
</table>

Notes:
1. Estimates of $\hat{\phi}_k$, $k=0,...,6$ are from Table 2.9,
   \[ \hat{\phi}_0 = -0.879; \hat{\phi}_1 = 3.683; \hat{\phi}_2 = -0.012; \hat{\phi}_3 = -35.923; \hat{\phi}_4 = -0.426; \hat{\phi}_5 = 0.016; \hat{\phi}_6 = 3.704. \]
2. $\bar{Z}_k$, $k=1,...,6$ is the sample mean (with respect to $t$) of variable $Z_k$.
   $Z_1$ to $Z_6$ are, respectively, $\Delta \ln Y$, $\Delta$, $\Delta \ln (POP)$, $S/Y$, $UNEMP$ and $F$.  

124.
Table 3.1 Estimation of $\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \frac{\alpha(r - \beta)}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \epsilon_t : \text{GMM}$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\alpha$</th>
<th>se($\alpha$)</th>
<th>p-value($\alpha &gt; \frac{r}{1+r}$)</th>
<th>$\beta$</th>
<th>se($\beta$)</th>
<th>p-value($\beta &gt; r$)</th>
</tr>
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<tbody>
<tr>
<td>0.10%</td>
<td>0.241%</td>
<td>0.085%</td>
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<td>0.248%</td>
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</tr>
<tr>
<td>0.15%</td>
<td>0.343%</td>
<td>0.115%</td>
<td>4.720%</td>
<td>1.162%</td>
<td>0.278%</td>
<td>0.014%</td>
</tr>
<tr>
<td>0.20%</td>
<td>0.437%</td>
<td>0.141%</td>
<td>4.628%</td>
<td>1.258%</td>
<td>0.305%</td>
<td>0.026%</td>
</tr>
<tr>
<td>0.25%</td>
<td>0.526%</td>
<td>0.164%</td>
<td>4.584%</td>
<td>1.349%</td>
<td>0.328%</td>
<td>0.041%</td>
</tr>
<tr>
<td>0.50%</td>
<td>0.923%</td>
<td>0.253%</td>
<td>4.648%</td>
<td>1.754%</td>
<td>0.421%</td>
<td>0.144%</td>
</tr>
<tr>
<td>0.75%</td>
<td>1.272%</td>
<td>0.318%</td>
<td>4.857%</td>
<td>2.114%</td>
<td>0.490%</td>
<td>0.269%</td>
</tr>
<tr>
<td>1.00%</td>
<td>1.596%</td>
<td>0.370%</td>
<td>5.090%</td>
<td>2.450%</td>
<td>0.547%</td>
<td>0.400%</td>
</tr>
<tr>
<td>1.25%</td>
<td>1.902%</td>
<td>0.414%</td>
<td>5.321%</td>
<td>2.770%</td>
<td>0.595%</td>
<td>0.532%</td>
</tr>
</tbody>
</table>

$J\text{-stat}(5) = 10.622$ \hspace{1cm} p-value($J$) = 5.941%

Notes:
1. Quarterly real interest rate $r$ is taken to be a fixed constant with different values listed in column 1.
2. The values of $r$, $\alpha$ and $\beta$ in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.
3. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta \ln C_t$ and $\frac{Y_t - W_t}{C_{t-1}}$.
4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
5. The $J$-statistic is asymptotically distributed as $\chi^2$ with 5 degrees of freedom, because there are 7 moment conditions and 2 parameters to be estimated.
Table 3.2 Estimation of $\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + [\frac{\alpha (r - \beta)}{r}] (\frac{Y_t - W_t}{C}) + \frac{W_t}{C} - \lambda (1 + \beta) (1 - \alpha) (\frac{C}{t-1}) + \epsilon_t; \text{ GMM}$

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\alpha$</th>
<th>se($\alpha$)</th>
<th>p-value($\alpha &gt; \frac{r}{1+r}$)</th>
<th>$\beta$</th>
<th>se($\beta$)</th>
<th>p-value($\beta &gt; r$)</th>
<th>$\lambda$</th>
<th>se($\lambda$)</th>
<th>p-val($\lambda$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10%</td>
<td>0.180%</td>
<td>0.077%</td>
<td>14.978%</td>
<td>0.945%</td>
<td>0.243%</td>
<td>0.025%</td>
<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
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<tr>
<td>0.15%</td>
<td>0.261%</td>
<td>0.106%</td>
<td>14.757%</td>
<td>1.027%</td>
<td>0.271%</td>
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<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
</tr>
<tr>
<td>0.20%</td>
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<td>0.131%</td>
<td>14.630%</td>
<td>1.104%</td>
<td>0.296%</td>
<td>0.113%</td>
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<td>0.0777</td>
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<tr>
<td>0.25%</td>
<td>0.410%</td>
<td>0.153%</td>
<td>14.563%</td>
<td>1.178%</td>
<td>0.318%</td>
<td>0.176%</td>
<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
</tr>
<tr>
<td>0.50%</td>
<td>0.746%</td>
<td>0.236%</td>
<td>14.615%</td>
<td>1.521%</td>
<td>0.404%</td>
<td>0.578%</td>
<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
</tr>
<tr>
<td>0.75%</td>
<td>1.053%</td>
<td>0.296%</td>
<td>14.877%</td>
<td>1.835%</td>
<td>0.467%</td>
<td>1.011%</td>
<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
</tr>
<tr>
<td>1.00%</td>
<td>1.343%</td>
<td>0.343%</td>
<td>15.169%</td>
<td>2.135%</td>
<td>0.518%</td>
<td>1.419%</td>
<td>0.317%</td>
<td>0.0777</td>
<td>0.000</td>
</tr>
<tr>
<td>1.25%</td>
<td>1.622%</td>
<td>0.381%</td>
<td>15.452%</td>
<td>2.424%</td>
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<td>0.0777</td>
<td>0.000</td>
</tr>
</tbody>
</table>

$J$-stat(10) = 12.243 $p$-value($J$) = 26.913%

Notes: 1. Quarterly real interest rate $r$ is taken to be a fixed constant with different values listed in column 1.

2. The values of $r$, $\alpha$ and $\beta$ in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.

3. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_t - W_t}{C}$, $\frac{W_t}{C}$ and $\frac{W_t}{t-1}$.  

4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).

5. The $J$-statistic is asymptotically distributed as $\chi^2$ with 10 degrees of freedom, because there are 13 moment conditions and 3 parameters to be estimated.
Table 3.3 Estimation of \( \frac{\Delta C_t}{W_{t-2}} = (\beta - \alpha - \alpha \beta) \frac{C_{t-1}}{W_{t-2}} + \frac{\alpha (r - \beta)}{r} \frac{Y_t - W_t}{W_{t-2}} + \epsilon_t : \text{GMM} \)

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \alpha )</th>
<th>se(( \alpha ))</th>
<th>( p )-value(( \alpha \times \frac{r}{1+r} ))</th>
<th>( \beta )</th>
<th>se(( \beta ))</th>
<th>( p )-value(( \beta &gt; r ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10%</td>
<td>0.205%</td>
<td>0.102%</td>
<td>15.209%</td>
<td>0.955%</td>
<td>0.266%</td>
<td>0.064%</td>
</tr>
<tr>
<td>0.15%</td>
<td>0.294%</td>
<td>0.138%</td>
<td>14.910%</td>
<td>1.045%</td>
<td>0.302%</td>
<td>0.151%</td>
</tr>
<tr>
<td>0.20%</td>
<td>0.377%</td>
<td>0.169%</td>
<td>14.735%</td>
<td>1.130%</td>
<td>0.333%</td>
<td>0.264%</td>
</tr>
<tr>
<td>0.25%</td>
<td>0.456%</td>
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<td>1.210%</td>
<td>0.361%</td>
<td>0.393%</td>
</tr>
<tr>
<td>0.50%</td>
<td>0.815%</td>
<td>0.301%</td>
<td>14.618%</td>
<td>1.576%</td>
<td>0.470%</td>
<td>1.100%</td>
</tr>
<tr>
<td>0.75%</td>
<td>1.137%</td>
<td>0.376%</td>
<td>14.859%</td>
<td>1.907%</td>
<td>0.549%</td>
<td>1.762%</td>
</tr>
<tr>
<td>1.00%</td>
<td>1.439%</td>
<td>0.435%</td>
<td>15.147%</td>
<td>2.219%</td>
<td>0.613%</td>
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<tr>
<td>1.25%</td>
<td>1.727%</td>
<td>0.484%</td>
<td>15.432%</td>
<td>2.513%</td>
<td>0.667%</td>
<td>2.854%</td>
</tr>
</tbody>
</table>

\( J \)-stat(\( \beta \)) = 14.994 \( p \)-value(\( J \)) = 5.926

Notes:

1. Quarterly real interest rate \( r \) is taken to be a fixed constant with different values listed in column 1.
2. The values of \( r \), \( \alpha \) and \( \beta \) in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.
3. Instruments used in estimation are \( C \) and twice to fourth lagged \( \Delta In C_t \) and \( \frac{Y_t - W_t}{C_{t-1}} \).
4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
5. The \( J \)-statistic is asymptotically distributed as \( \chi^2 \) with 8 degrees of freedom, because there are 10 moment conditions and 2 parameters to be estimated.
Table 3.4 Estimation of 
\[ \frac{\Delta C_t}{W_t} = (\beta - \alpha_0_0) \left( (\frac{C_{t-1}}{W_{t-2}})(\frac{1}{1+r}) + \frac{\alpha(1-\beta)}{W_{t-2}} \right) + \lambda \left( \frac{W_t}{W_{t-2}} - 1 \right) \]

<table>
<thead>
<tr>
<th>$r$</th>
<th>$\alpha$</th>
<th>se($\alpha$)</th>
<th>p-value($\alpha &gt; \frac{\beta}{1+r}$)</th>
<th>$\beta$</th>
<th>se($\beta$)</th>
<th>p-value($\beta &gt; r$)</th>
<th>$\lambda$</th>
<th>se($\lambda$)</th>
<th>p-val($\lambda$)</th>
</tr>
</thead>
<tbody>
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<td>0.185%</td>
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<td>13.971%</td>
<td>0.959%</td>
<td>0.266%</td>
<td>0.062%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
<td>0.15%</td>
<td>0.268%</td>
<td>0.108%</td>
<td>13.785%</td>
<td>1.043%</td>
<td>0.295%</td>
<td>0.124%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
<td>0.20%</td>
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<td>0.133%</td>
<td>13.686%</td>
<td>1.122%</td>
<td>0.321%</td>
<td>0.202%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
<td>0.25%</td>
<td>0.420%</td>
<td>0.156%</td>
<td>13.644%</td>
<td>1.197%</td>
<td>0.343%</td>
<td>0.289%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.762%</td>
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<td>13.775%</td>
<td>1.546%</td>
<td>0.433%</td>
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<td>0.75%</td>
<td>1.072%</td>
<td>0.305%</td>
<td>14.084%</td>
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<td>0.498%</td>
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<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
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<td>0.353%</td>
<td>14.411%</td>
<td>2.167%</td>
<td>0.551%</td>
<td>1.709%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
<tr>
<td>1.25%</td>
<td>1.647%</td>
<td>0.393%</td>
<td>14.719%</td>
<td>2.459%</td>
<td>0.595%</td>
<td>2.105%</td>
<td>0.343%</td>
<td>0.0647</td>
<td>0.000</td>
</tr>
</tbody>
</table>

$J$-stat(13) = 13.642 \quad \text{p-value}(J) = 39.954 \%$

Notes:
1. Quarterly real interest rate $r$ is taken to be a fixed constant with different values listed in column 1.
2. The values of $r$, $\alpha$ and $\beta$ in the first panel are measured in percentage per quarter, while those in the last panel are their corresponding values measured in percentage per annum.
3. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta ln C_t$, $Y_t - W_t$, $\frac{W_t}{W_{t-2}}$, and $\frac{W_t}{C_{t-1}}$. $\frac{W_t}{C_{t-1}}$ and $\frac{W_t}{C_t}$.
4. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
5. The $J$-statistic is asymptotically distributed as $\chi^2$ with 13 degrees of freedom, because there are 16 moment conditions and 3 parameters to be estimated.
Table 3.5 System Estimation of $\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \left( \frac{\alpha (r - \beta)}{r} \right) \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \epsilon_t$

and $r_t = r + u_t$ : GMM

<table>
<thead>
<tr>
<th></th>
<th>s.e.</th>
<th>t-stat</th>
<th>significance level</th>
</tr>
</thead>
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<td>0.175%</td>
<td>4.374</td>
<td>0.001%</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.684%</td>
<td>5.090</td>
<td>0.000%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.297%</td>
<td>7.154</td>
<td>0.000%</td>
</tr>
</tbody>
</table>

$J$-statistic(11) = 18.325, p-value($J$) = 7.434%

Notes:
1. Initial values for the parameters in estimation are $r = 0.2\%$, $\alpha = 0.2\%$ and $\beta = 0.2\%$.
2. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta \ln C_t$ and $\frac{Y_t - W_t}{C_{t-1}}$.
3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
4. The $J$-statistic is asymptotically distributed as $\chi^2$ with 11 degrees of freedom, because there are 14 moment conditions and 3 parameters to be estimated.
Table 3.6 System Estimation of $\Delta \ln C_t = (\beta - \alpha) + \left[ \frac{\alpha (r - \beta)}{r} + \frac{\lambda}{C_{t-1}} \right] Y_t - \frac{W_t}{C_t} + \frac{W_{t-1}}{C_{t-1}} + \epsilon_t$

and $r_t = r + u_t$ : GMM

<table>
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<tr>
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<th>t-stat</th>
<th>significance level</th>
</tr>
</thead>
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<td>0.000 %</td>
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<tr>
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<td>7.324</td>
<td>0.000 %</td>
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<tr>
<td>$\beta$</td>
<td>0.293 %</td>
<td>7.148</td>
<td>0.000 %</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.0669</td>
<td>2.846</td>
<td>0.442 %</td>
</tr>
</tbody>
</table>

J-statistic(22) = 49.628  p-value(J) = 0.066 %

Notes:
1. Initial values for the parameters in estimation are $r=0.2\%$, $\alpha=0.2\%$, $\beta=0.2\%$ and $\lambda=0.3$.
2. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_t - W_t}{C_{t-1}}$, $\frac{W_t}{C_{t-1}}$ and $\frac{W_{t-1}}{C_{t-1}}$.
3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
4. The J-statistic is asymptotically distributed as $\chi^2$ with 22 degrees of freedom, because there are 26 moment conditions and 4 parameters to be estimated.
Table 3.7 System Estimation of $\Delta lnC_t = (\beta - \alpha \bar{\beta}) + \frac{\alpha(r - \bar{\beta})}{r} \left( \frac{Y_t - W_t}{C_{t-1}} \right) + \epsilon_t$

and $\Delta Y_t - \Delta W_t = r(Y_{t-1} - C_{t-1}) + \nu_t : GMM$

<table>
<thead>
<tr>
<th></th>
<th>s.e.</th>
<th>t-stat</th>
<th>significance level</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.259%</td>
<td>7.138</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>4.897%</td>
<td>0.429%</td>
<td>11.402</td>
</tr>
<tr>
<td>$\beta$</td>
<td>7.043%</td>
<td>0.684%</td>
<td>10.299</td>
</tr>
</tbody>
</table>

$J$-statistic(23) = 21.942, p-value($J$) = 52.375%

Notes:
1. Initial values for the parameters in estimation are $r = 0.2\%$, $\alpha = 0.2\%$ and $\beta = 0.2\%$.
2. Instruments used in estimation are $C$ and twice to fourth lagged $Y_t - W_t, \Delta Y_t - \Delta W_t$ and $Y_t - C_t$.
3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
4. The $J$-statistic is asymptotically distributed as $\chi^2$ with 23 degrees of freedom, because there are 26 moment conditions and 3 parameters to be estimated.
Table 3.8 System Estimation of $\Delta \ln C_t = (\beta - \alpha - \alpha \beta) + \frac{\gamma - \tilde{w}_t}{r} + \lambda(\frac{\tilde{w}_t}{C_{t-1}} - \lambda(1 + \beta)(1 - \alpha)(C_{t-1}) + \epsilon_t$

and $\Delta Y_t - \Delta \tilde{w}_t = r(Y_{t-1} - C_{t-1}) + \nu_t$ : GMM

<table>
<thead>
<tr>
<th></th>
<th>s.e.</th>
<th>t-stat</th>
<th>significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td>1.695 %</td>
<td>0.259 %</td>
<td>6.541 %</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.156 %</td>
<td>0.469 %</td>
<td>4.594 %</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.886 %</td>
<td>0.612 %</td>
<td>4.712 %</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.241</td>
<td>0.0580</td>
<td>4.150 %</td>
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</tbody>
</table>

$J$-statistic(34) = 27.691  p-value(J) = 76.913 %

Notes:
1. Initial values for the parameters in estimation are $r=0.2 \%$, $\alpha=0.2 \%$, $\beta=0.2 \%$ and $\lambda=0.3$.
2. Instruments used in estimation are $C$ and twice to fourth lagged $\Delta \ln C_t$, $\frac{Y_t - \tilde{w}_t}{C_{t-1}}$, $\frac{\tilde{w}_t}{C_{t-1}}$, $\Delta Y_t - \tilde{w}_t$ and $Y_t - C_t$.
3. The standard errors in the table are heteroskedasticity and autocorrelation consistent, which are obtained according to Newey and West (1989).
4. The $J$-statistic is asymptotically distributed as $X^2$ with 34 degrees of freedom, because there are 38 moment conditions and 4 parameters to be estimated.
\[
\alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (e_{t+i} - e_{t+i-1}) w_{t+i} = \alpha \cdot [P(\frac{1}{1+\beta})]^{-1}
\]

\[
(1-\lambda) \alpha \sum_{i=0}^{\infty} \left( \frac{1}{1+\beta} \right)^i (e_{t+i} - e_{t+i-1}) w_{t+i} = (1-\lambda) \cdot \alpha \cdot [P(\frac{1}{1+\beta})]^{-1}
\]

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>(\frac{\alpha(1+\beta)^2}{\beta(0.588+\beta)})</th>
<th>(\frac{(1-\lambda)\alpha(1+\beta)^2}{\beta(0.588+\beta)})</th>
<th>(\frac{\alpha(1+\beta)}{\beta})</th>
<th>(\frac{(1-\lambda)\alpha(1+\beta)}{\beta})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10%</td>
<td>0.389</td>
<td>0.222</td>
<td>0.230</td>
<td>0.132</td>
</tr>
<tr>
<td>0.15%</td>
<td>0.504</td>
<td>0.296</td>
<td>0.298</td>
<td>0.175</td>
</tr>
<tr>
<td>0.20%</td>
<td>0.593</td>
<td>0.356</td>
<td>0.352</td>
<td>0.211</td>
</tr>
<tr>
<td>0.25%</td>
<td>0.666</td>
<td>0.406</td>
<td>0.396</td>
<td>0.241</td>
</tr>
<tr>
<td>0.50%</td>
<td>0.899</td>
<td>0.573</td>
<td>0.535</td>
<td>0.340</td>
</tr>
<tr>
<td>0.75%</td>
<td>1.030</td>
<td>0.670</td>
<td>0.614</td>
<td>0.399</td>
</tr>
<tr>
<td>1.00%</td>
<td>1.116</td>
<td>0.736</td>
<td>0.667</td>
<td>0.439</td>
</tr>
<tr>
<td>1.25%</td>
<td>1.178</td>
<td>0.783</td>
<td>0.706</td>
<td>0.468</td>
</tr>
</tbody>
</table>

0.175% (From Table 3.5) 0.586 0.349
0.256% (From Table 3.6) 0.576 0.344
1.845% (From Table 3.7) 1.210 0.744
1.695% (From Table 3.8) 0.974 0.584

Notes:

1. The values of \(\frac{\alpha(1+\beta)^2}{\beta(0.588+\beta)}\) in columns (2) and (3) are calculated according to the point estimates of \(\alpha\) and \(\beta\) listed in Tables 3.1 and 3.3.

2. The values of \(\frac{\alpha(1+\beta)}{\beta}\) in columns (6) and (7) are calculated according to the point estimates of \(\alpha\) and \(\beta\) listed in Tables 3.1 and 3.3.

3. The values of \(\frac{(1-\lambda)\alpha(1+\beta)^2}{\beta(0.588+\beta)}\) in columns (4) and (5) are calculated according to the point estimates of \(\alpha\), \(\beta\) and \(\lambda\) listed in Tables 3.2 and 3.4.

4. The values of \(\frac{(1-\lambda)\alpha(1+\beta)}{\beta}\) in columns (8) and (9) are calculated according to the point estimates of \(\alpha\), \(\beta\) and \(\lambda\) listed in Tables 3.2 and 3.4.
### TABLE 4.1

Dickey-Fuller, Augmented Dickey-Fuller and Phillips-Perron Test:
Per Capita Private Disposable Income ($Y_t$)

Twelve Countries, 1960-1988, Annual

<table>
<thead>
<tr>
<th></th>
<th>DF and ADF Tests</th>
<th>Phillips-Perron Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lags</td>
<td>Lags</td>
</tr>
<tr>
<td></td>
<td>Trend 0 1 2 3</td>
<td>1 2 3 4</td>
</tr>
<tr>
<td>AUS</td>
<td>-1.50 -1.91 -1.54 -1.25</td>
<td>-1.61 -1.58 -1.60 -1.62</td>
</tr>
<tr>
<td></td>
<td>N -1.19 -1.23 -1.50 -1.21</td>
<td>Y -1.22 -1.30 -1.30 -1.29</td>
</tr>
<tr>
<td>AUT</td>
<td>0.21 -0.19 -0.32 -0.51</td>
<td>0.20 0.24 0.19 0.17</td>
</tr>
<tr>
<td>BEL</td>
<td>-0.06 -0.24 -0.37 -0.55</td>
<td>-0.02 -0.01 -0.00 -0.01</td>
</tr>
<tr>
<td></td>
<td>N -2.12 -1.70 -1.81 -1.68</td>
<td>Y -2.23 -2.30 -2.33 -2.39</td>
</tr>
<tr>
<td>CAN</td>
<td>1.17 0.57 0.96 0.62</td>
<td>1.11 1.20 1.29 1.38</td>
</tr>
<tr>
<td></td>
<td>N -2.28 -2.31 -2.35 -2.49</td>
<td>Y -2.50 -2.48 -2.44 -2.40</td>
</tr>
<tr>
<td>FIN</td>
<td>-0.03 0.15 0.04 0.35</td>
<td>-0.04 0.04 0.13 0.22</td>
</tr>
<tr>
<td></td>
<td>N -2.59 -3.10 -2.56 -2.29</td>
<td>Y -2.88 -2.83 -2.73 -2.64</td>
</tr>
<tr>
<td>GER</td>
<td>0.31 -0.14 -0.30 -0.66</td>
<td>0.27 0.28 0.29 0.30</td>
</tr>
<tr>
<td></td>
<td>N -2.00 -2.31 -2.25 -2.33</td>
<td>Y -2.28 -2.34 -2.37 -2.37</td>
</tr>
<tr>
<td>GRE</td>
<td>-1.11 -1.16 -1.88 -1.66</td>
<td>-1.18 -1.26 -1.28 -1.29</td>
</tr>
<tr>
<td></td>
<td>N -1.48 -1.33 -0.98 -1.14</td>
<td>Y -1.57 -1.50 -1.52 -1.55</td>
</tr>
<tr>
<td>JPN</td>
<td>-0.86 -1.08 -1.54 -1.75</td>
<td>-0.90 -0.94 -0.92 -0.92</td>
</tr>
<tr>
<td></td>
<td>N -1.17 -1.17 -0.78 -1.44</td>
<td>Y -1.30 -1.25 -1.37 -1.39</td>
</tr>
<tr>
<td>NOR</td>
<td>0.88 0.55 0.95 0.90</td>
<td>0.86 0.95 1.00 1.14</td>
</tr>
<tr>
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<td>N -1.62 -2.33 -2.03 -2.72</td>
<td>Y -1.87 -1.87 -1.87 -1.78</td>
</tr>
<tr>
<td>SWI</td>
<td>-1.07 -0.87 -1.26 -1.04</td>
<td>-1.11 -1.15 -1.17 -1.19</td>
</tr>
<tr>
<td></td>
<td>N -2.25 -2.32 -1.96 -2.32</td>
<td>Y -2.46 -2.38 -2.39 -2.36</td>
</tr>
<tr>
<td>UKD</td>
<td>0.55 0.05 0.37 0.90</td>
<td>0.41 0.54 0.77 1.04</td>
</tr>
<tr>
<td></td>
<td>N -2.17 -3.56* -2.05 -2.46</td>
<td>Y -2.56 -2.50 -2.33 -2.16</td>
</tr>
<tr>
<td>USA</td>
<td>0.06 -0.09 0.09 -0.05</td>
<td>0.01 0.04 0.10 0.18</td>
</tr>
</tbody>
</table>

**Notes:**
1. Critical values for DF test, ADF test and Phillips-Perron test are (for a sample size 25, Fuller (1976)): With a constant only: 1% level -3.75; 5% level -3.00; 10% level -2.63. With a constant and a linear trend: 1% level -4.38; 5% level -3.60; 10% level -3.24.
2. (*) significant at the 10% level, (**) significant at the 5% level.
<table>
<thead>
<tr>
<th></th>
<th>DF and ADF Tests</th>
<th>Phillips-Perron Tests</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Trend 0 1 2 3</td>
<td>Lags 1 2 3 4</td>
<td></td>
</tr>
<tr>
<td>AUS N</td>
<td>-1.10 -1.70 -1.52</td>
<td>-1.63</td>
<td>-1.09</td>
</tr>
<tr>
<td></td>
<td>-1.03 -1.67 -0.78</td>
<td>-0.89</td>
<td>-1.28</td>
</tr>
<tr>
<td>Y</td>
<td>-1.03 -1.67 -0.78</td>
<td>-0.89</td>
<td>-1.22</td>
</tr>
<tr>
<td>AUT N</td>
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<td>-0.32</td>
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<tr>
<td>Y</td>
<td>-2.03 -1.66 -1.53</td>
<td>-1.78</td>
<td>-2.09</td>
</tr>
<tr>
<td>BEL N</td>
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<td>-0.76</td>
<td>-0.44</td>
</tr>
<tr>
<td>Y</td>
<td>-1.12 -1.48 -1.57</td>
<td>-2.46</td>
<td>-1.38</td>
</tr>
<tr>
<td>CAN N</td>
<td>1.02 0.13 0.13 0.10</td>
<td></td>
<td>0.84</td>
</tr>
<tr>
<td>Y</td>
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<td>-2.75</td>
<td>-2.10</td>
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<td>FIN N</td>
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<td></td>
<td>0.66</td>
</tr>
<tr>
<td>Y</td>
<td>-1.34 -2.04 -1.62</td>
<td>-2.74</td>
<td>-1.74</td>
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<td>-0.40</td>
</tr>
<tr>
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<td>-2.01</td>
<td>-1.84</td>
</tr>
<tr>
<td>GRE N</td>
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<td>-2.50</td>
<td>-1.80</td>
</tr>
<tr>
<td>Y</td>
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<td>-1.34</td>
<td>-0.36</td>
</tr>
<tr>
<td>JPN N</td>
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<td>-1.03</td>
<td>-0.72</td>
</tr>
<tr>
<td>Y</td>
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<td>-1.91</td>
<td>-1.64</td>
</tr>
<tr>
<td>NOR N</td>
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<td></td>
<td>-0.27</td>
</tr>
<tr>
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<td>-1.49</td>
<td>-1.73</td>
</tr>
<tr>
<td>Y</td>
<td>-1.35 -2.09 -1.40</td>
<td>-1.59</td>
<td>-1.61</td>
</tr>
<tr>
<td>UKD N</td>
<td>2.88 1.69 1.91 2.07</td>
<td></td>
<td>2.51</td>
</tr>
<tr>
<td>Y</td>
<td>1.15 -0.17 0.62 0.50</td>
<td></td>
<td>0.62</td>
</tr>
<tr>
<td>USA N</td>
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<tr>
<td>Y</td>
<td>-1.94 -3.32* -2.43</td>
<td>-2.47</td>
<td>-2.39</td>
</tr>
</tbody>
</table>

Notes: 1. Critical values for DF test, ADF test and Phillips-Perron test are (for a sample size 25, Fuller (1976)): With a constant only: 1% level -3.75; 5% level -3.00; 10% level -2.63. With a constant and a linear trend: 1% level -4.38; 5% level -3.60; 10% level -3.24.
2. (*) significant at the 10% level, (**) significant at the 5% level.
### TABLE 4.3
Estimation of $y_t = \phi + \pi t + \varepsilon_t$ : OLS
Twelve Countries, 1960-1988, Annual

<table>
<thead>
<tr>
<th>Country</th>
<th>$\hat{\phi}$</th>
<th>$\hat{\tau}(\hat{\phi})$</th>
<th>$\hat{\pi}$</th>
<th>$t(\hat{\pi})$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1.12</td>
<td>29.59</td>
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<tr>
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<td>1.34</td>
<td>84.24</td>
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<tr>
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<td>1.23</td>
<td>58.45</td>
</tr>
<tr>
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<td>-1434.62</td>
<td>-6.56</td>
<td>1.30</td>
<td>50.79</td>
</tr>
<tr>
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<td>1.19</td>
<td>1.03</td>
<td>33.39</td>
</tr>
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<td>67.90</td>
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<td>99.72</td>
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<tr>
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<td>-1769.58</td>
<td>-1.33</td>
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<td>0.87</td>
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<td>1.07</td>
<td>29.90</td>
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<td>350.15</td>
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<td>1.08</td>
<td>69.28</td>
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</table>

### TABLE 4.4
Engle-Granger, Augmented Engle-Granger and Phillips-Ouliaris Test
for Cointegration ($\hat{\varepsilon}_t$)
Twelve Countries, 1960-1988, Annual

<table>
<thead>
<tr>
<th>EG and AEG Tests</th>
<th>Phillips-Ouliaris Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laggs</td>
<td>Laggs</td>
</tr>
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</tr>
<tr>
<td>1</td>
<td>2 3</td>
</tr>
</tbody>
</table>

#### EG and AEG Tests

<table>
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<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUS</td>
<td>-2.39</td>
<td>-2.20</td>
<td>-2.20</td>
<td>-1.64</td>
</tr>
<tr>
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<td>-3.19</td>
<td>-2.93</td>
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<td>-1.88</td>
<td>-1.89</td>
</tr>
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<td>-2.54</td>
</tr>
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<td>-3.71**</td>
<td>-3.49*</td>
<td>-3.03</td>
</tr>
<tr>
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<td>-2.10</td>
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<td>-1.50</td>
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<td>-1.82</td>
<td>-2.13</td>
</tr>
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<td>-2.34</td>
<td>-1.69</td>
<td>-1.44</td>
</tr>
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#### Phillips-Ouliaris Tests

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<th>4</th>
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<td>-2.00</td>
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<td>-2.21</td>
<td>-2.20</td>
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</tr>
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<td>-3.70**</td>
<td>-3.63**</td>
</tr>
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<td>-2.47</td>
<td>-2.37</td>
</tr>
<tr>
<td>NOR</td>
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<td>-2.67</td>
<td>-2.61</td>
<td>-2.59</td>
</tr>
<tr>
<td>SWI</td>
<td>-3.12</td>
<td>-3.03</td>
<td>-3.06</td>
<td>-3.06</td>
</tr>
<tr>
<td>UKD</td>
<td>-1.87</td>
<td>-1.89</td>
<td>-1.77</td>
<td>-1.59</td>
</tr>
<tr>
<td>USA</td>
<td>-3.90**</td>
<td>-3.85**</td>
<td>-3.83**</td>
<td>-3.83**</td>
</tr>
</tbody>
</table>

**Notes:**
1. Critical values for EG test, AEG test and Phillips-Ouliaris test are: 1% level (-4.33, -4.35, -4.37); 5% level (-3.57, -3.58, -3.59); 10% level (-3.20, -3.21, -3.21). The three critical values at each significance level corresponds to sample sizes of 27, 26 and 25 or inclusion of lags 1, 2 and 3 of $\hat{\varepsilon}_t$.
2. (*) significant at the 10% level, (**) significant at the 5% level.
### TABLE 4.5
Testing Unit Root in Disposable Income with Panel Data

<table>
<thead>
<tr>
<th>lags of $\Delta \tilde{y}_{it}$ included</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{y}_{i,t-1}$</td>
<td>-0.0029(-0.90)</td>
<td>-0.0035(-1.09)</td>
<td>-0.0049(-1.71)</td>
<td>-0.0045(-1.50)</td>
</tr>
<tr>
<td>$\Delta \tilde{y}_{i,t-1}$</td>
<td>0.7434(18.98)</td>
<td>0.4805(8.69)</td>
<td>0.2716(5.39)</td>
<td>0.2287(3.74)</td>
</tr>
<tr>
<td>$\Delta \tilde{y}_{i,t-2}$</td>
<td>0.3589(6.50)</td>
<td>0.0854(1.63)</td>
<td>0.0798(1.48)</td>
<td></td>
</tr>
<tr>
<td>$\Delta \tilde{y}_{i,t-3}$</td>
<td></td>
<td>0.5712(11.33)</td>
<td>0.5494(10.13)</td>
<td></td>
</tr>
<tr>
<td>$\Delta \tilde{y}_{i,t-4}$</td>
<td></td>
<td></td>
<td>0.0689(1.11)</td>
<td></td>
</tr>
</tbody>
</table>

### TABLE 4.6
Testing Unit Root in Consumption with Panel Data

<table>
<thead>
<tr>
<th>lags of $\Delta \tilde{c}_{it}$ included</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{c}_{i,t-1}$</td>
<td>-0.0003(-0.11)</td>
<td>-0.0003(-0.11)</td>
<td>0.0000(0.02)</td>
<td>0.0008(0.31)</td>
</tr>
<tr>
<td>$\Delta \tilde{c}_{i,t-1}$</td>
<td>0.8389(24.79)</td>
<td>0.6206(10.94)</td>
<td>0.4923(9.05)</td>
<td>0.4366(7.05)</td>
</tr>
<tr>
<td>$\Delta \tilde{c}_{i,t-2}$</td>
<td>0.2781(4.84)</td>
<td>0.0122(0.20)</td>
<td>0.0150(0.24)</td>
<td></td>
</tr>
<tr>
<td>$\Delta \tilde{c}_{i,t-3}$</td>
<td></td>
<td>0.4537(8.23)</td>
<td>0.3954(6.33)</td>
<td></td>
</tr>
<tr>
<td>$\Delta \tilde{c}_{i,t-4}$</td>
<td></td>
<td></td>
<td>0.1074(1.71)</td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 4.7
Testing Cointegration between Consumption and Disposable Income with Panel Data

<table>
<thead>
<tr>
<th>lags of $\Delta \xi_{it}$ included</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi_{i,t-1}$</td>
<td>-0.5217(-9.11)</td>
<td>-0.6561(-10.00)</td>
<td>-0.6109(-8.01)</td>
<td>-0.6100(-7.00)</td>
</tr>
<tr>
<td>$\Delta \xi_{i,t-2}$</td>
<td>0.2488(4.25)</td>
<td>0.3453(5.46)</td>
<td>0.3243(4.59)</td>
<td>0.3289(3.95)</td>
</tr>
<tr>
<td>$\Delta \xi_{i,t-3}$</td>
<td></td>
<td>0.2936(4.17)</td>
<td>0.2300(3.30)</td>
<td>0.2166(2.80)</td>
</tr>
<tr>
<td>$\Delta \xi_{i,t-4}$</td>
<td></td>
<td></td>
<td>-0.1021(-1.57)</td>
<td>-0.1164(-1.57)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.0561(0.84)</td>
</tr>
</tbody>
</table>

### TABLE 4.8
Results of Monte Carlo Simulations for Finite Sample Critical Values

<table>
<thead>
<tr>
<th></th>
<th>Disposable Income</th>
<th>Consumption</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>01-%ile</td>
<td>-5.65</td>
<td>-5.59</td>
<td>-6.55</td>
</tr>
<tr>
<td>05-%ile</td>
<td>-5.07</td>
<td>-4.99</td>
<td>-5.92</td>
</tr>
<tr>
<td>10-%ile</td>
<td>-4.76</td>
<td>-4.66</td>
<td>-5.58</td>
</tr>
<tr>
<td>25-%ile</td>
<td>-4.23</td>
<td>-4.14</td>
<td>-5.03</td>
</tr>
<tr>
<td>Median</td>
<td>-3.66</td>
<td>-3.57</td>
<td>-4.44</td>
</tr>
<tr>
<td>75-%ile</td>
<td>-3.11</td>
<td>-3.01</td>
<td>-3.82</td>
</tr>
<tr>
<td>90-%ile</td>
<td>-2.61</td>
<td>-2.52</td>
<td>-3.19</td>
</tr>
<tr>
<td>95-%ile</td>
<td>-2.32</td>
<td>-2.24</td>
<td>-2.82</td>
</tr>
<tr>
<td>99-%ile</td>
<td>-1.80</td>
<td>-1.69</td>
<td>-2.06</td>
</tr>
<tr>
<td>Maximum</td>
<td>-0.84</td>
<td>-0.27</td>
<td>0.75</td>
</tr>
</tbody>
</table>
Figure 2.1: Consumption Growth Rate and Income Growth Rate for Australia, Austria, Belgium and Canada, No GS
(Solid Line Is Income Growth Rate, Dotted Line Is Consumption Growth Rate)
Figure 2.2: Consumption Growth Rate and Income Growth Rate for Denmark, Finland, France and Germany, No GS
(Solid Line Is Income Growth Rate, Dotted Line Is Consumption Growth Rate)
Figure 2.5: Consumption Growth Rate and Income Growth Rate for Switzerland, UK and US. No GS (Solid line is Income Growth Rate, Dotted line is Consumption Growth Rate)
Figure 2.6: Consumption Growth Rate and Income Growth Rate for Australia, Austria, Belgium and Canada, With GS
(Solid Line Is Income Growth Rate, Dotted Line Is Consumption Growth Rate)
Figure 2.7: Consumption Growth Rate and Income Growth Rate for Finland, Germany, Greece and Japan, With GS (Solid Line Is Income Growth Rate, Dotted Line Is Consumption Growth Rate)
Figure 2.8: Consumption Growth Rate and Income Growth Rate for Norway, Switzerland, UK and US, With GS (Solid Line Is Income Growth Rate, Dotted Line Is Consumption Growth Rate)
Figure 2.9: Scatterplot of $\lambda_i$ and $(\Delta \ln Y)_i$, $i = 1, \ldots, 19$.

Figure 2.10: Scatterplot of $\lambda_i$ and $r_i$, $i = 1, \ldots, 19$. 
Figure 2.11: Scatterplot of $\lambda_i$ and ($\Delta \ln(POP)$)$_i$, $i = 1, \ldots, 19$.

Figure 2.12: Scatterplot of $\lambda_i$ and $UNEMP_i$, $i = 1, \ldots, 19$. 
Figure 2.13: Scatterplot of $\lambda_i$ and $\left(\frac{S}{Y}\right)_i, i=1,\ldots,19$.

Figure 2.14: Scatterplot of $\lambda_i$ and $F_i, i=1,\ldots,19$. 
Figure 3.1: Real Interest Rate (3-Month T-Bill Rate, 1953:2-1984:4)
Figure 4.1: $Y_t$ Plotted Against $Y_{t-1}$, For $i = 1, \ldots, 12, t = 2, \ldots, 29$

Figure 4.2: $C_t$ Plotted Against $C_{t-1}$, For $i = 1, \ldots, 12, t = 2, \ldots, 29$
Figure 4.3: $|\hat{e}_i|$ Plotted Against $|\hat{e}_{i,t-1}|$, For $i = 1, \ldots, 12, t = 2, \ldots, 29$
BIBLIOGRAPHY


Christiano, Lawrence J. and Martin Eichenbaum, "Unit Roots in GNP: Do We Know, and Do We Care?". Carnegie-Rochester Conference Series on Public Policy, Spring 1990, Vol. 32, pp. 7-62.


