LIQUIDITY CONSTRAINTS, INTERTEMPORAL SUBSTITUTION, AND THE TERM STRUCTURE OF INTEREST RATES

DISSERTATION

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To My Parents
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CHAPTER I

INTRODUCTION

Neoclassical economics places consumption as an ultimate goal of economic activities. Since an understanding of the nature of aggregate consumption is essential in studying macroeconomic structures, a number of economists have made efforts to investigate the nature of aggregate consumption.

Following the guiding principle of individual rationality in Neoclassical Macroeconomics, Friedman (1957) proposed the Permanent Income Hypothesis (PIH) where consumption is linked to future incomes in terms of the discounted present value of expected future incomes. The PIH implies that consumption depends on permanent income and does not respond to transitory income changes. This finding opened a new horizon of consumption theory by departing from the naive Keynesian consumption function. The PIH is based upon the assumption of perfect financial markets where an individual consumer can borrow and lend any amount he or she wishes at the same interest rate.

However, in the early 1960s, Rational Expectations economists began to crit-
icize the adaptive expectation formula that Friedman and P. Cagan (1956) introduced for empirical implementation of the PIH because the scheme is backward-looking for an optimal predictor for uncertain future incomes (Muth (1960, 1961)). Rational Expectations theory strictly applies the principle of Neoclassical rationality to the information set in that an individual is assumed to utilize all information available. Furthermore, in the famous Lucas' critique (1976), an optimal consumption rule is shown not to remain fixed when stochastic macroeconomic variables change and thus an individual is subject to reassessing the endogenously varying expected future incomes.

Incorporating the Rational Expectations scheme, in 1978, R. Hall proposed a famous Random Walk Theorem (RWT) of consumption in conjunction with the PIH. With simplifying assumptions on interest rates and preferences, he showed that the consumption change envisioned in the PIH follows a random walk with drift and thus cannot be predicted using prior information. He reported that postwar U.S. quarterly aggregate consumption data support his theorem with few exceptions.

Since Hall addressed this insightful regularity in consumption, there has been increasing awareness in the last decade about the sensitivity of consumption to contemporaneous income. A number of researchers have in fact documented the empirical results that consumption does respond to transitory income change more than is justified by the PIH. They are Mankiw (1981), Flavin (1981, 1985), Hall and
Mishkin (1982), Hayashi (1982, 1985a, 1985b), Mankiw and Shapiro (1985), Deaton (1987, 1989), Nelson (1987), Campbell (1987), Campbell and Deaton (1989), Zeldes (1989), Bayoumi and Koujianou (1989), Carrol and Summers (1989), and Campbell and Mankiw (1989, 1990). For instance, Flavin (1985) considered two possibilities of the empirical failure of the theorem: first, some consumers, who fail to prove credibility of repayment, for example, by lacking collateral, are liquidity-constrained in borrowing and thus they are systematically excluded in the loan market. In this case, these consumers are unable to fulfill their intertemporal optimization conditions and risk diversification. Second, auxiliary assumptions upon which the derivation of the theorem from the PIH is based may be violated. For example, consumers may be intertemporally myopic. Flavin used the unemployment rate as a proxy variable for liquidity constraints in her sensitivity tests of consumption to income, in order to see if consumers face liquidity constraints or are myopic. She went on to conclude that the empirical shortcoming of the random walk theorem results from the presence of liquidity constraints in borrowing markets.

However, C. Nelson (1987) indicated that a data detrending method that Flavin applied to the aggregate data manipulation is inappropriate. Reflecting the developing theory of the unit root problem on aggregate data as shown in Dickey (1976), Fuller (1976), Nelson and Kang (1981), Nelson and Plosser (1982), and Mankiw and Shapiro (1985), Nelson showed that detrended aggregate data pro-
duce spurious results if income follows a random walk. In this case, data do not reject the null hypothesis that consumers are liquidity-constrained if income follows a random walk. Postwar U.S. aggregate income is in general believed to follow a random walk. Therefore, Flavin's paper is never convincing concerning evidence for binding constraints in loan market.

Recently Campbell and Mankiw (1989, 1990) reported that rule-of-thumb consumers and forward-looking consumers coexist, where the term rule-of-thumb is used in broader sense than the term binding constraints in loan market in order to take into account the possibility that the auxiliary assumptions are violated, for instance, if consumers are myopic. They showed that, when both types of consumers are taken into account in a model, the empirical results are robust to alternative model specifications. As a result, they argued that Hall's model, which does not take account the rule-of-thumb consumers in terms of absence of income variable in the equation, are misspecified. The Campbell and Mankiw's investigations now cast again a serious question on the validity of the PIH.

Furthermore, several Panel Data studies have presented the evidence that a significant fraction of the population is liquidity-constrained: Hall and Mishkin's (1982), Hayashi (1985), Zeldes (1989), and Campbell and Zeldes (1990). According to Hall and Mishkin's study on food expenditure in the Panel Study of Income Dynamics, one fifth of the households in their sample represent liquidity-constrained consumers.
There are another group of economists who relaxed Hall’s assumption of the constancy of real interest rates. They included financial variables in their frameworks to reflect the intertemporal substitution of consumption in response to the change in returns from asset holdings (Mankiw (1981), Grossman and Shiller (1981), Hansen and Singleton (1983), Mankiw and Shapiro (1986), Grossman, Melino and Shiller (1987), Harvey (1988), Breeden, Gibbons and Litzenberger (1989), and Kandel and Stambaugh (1991)). The intertemporal optimality conditions for consumers imply that consumption variability is related to asset return variability and the magnitude of its responsiveness depends upon the degree of risk aversion in time-separable preferences. Using time-varying moments of consumption growth, Kandel and Stambaugh showed that the predictability of asset returns is affected primarily by intertemporal substitution. C. Harvey explained that long-term interest rates can affect one-period consumption growth through the term structure and reported that the term structure contains information about consumption movement. On the contrary, Hall (1988) and Campbell and Mankiw (1989, 1990) reported that intertemporal elasticity of substitution is in fact zero and statistically insignificant. According to Campbell and Mankiw, when they estimated a model specification that includes both interest rates and income, the coefficient of income change remains about 0.5 for the U.S. and statistically significant but that of interest rate becomes close to zero and insignificant. In addition to the significance of income in explaining consumption, Campbell and Zeldes (1990), based
upon household panel data, recently reported that the consumption of stockholders differs from that of non-stockholders in that the former is more volatile than the latter. This finding supports the fact that the implied coefficient of relative risk aversion may be lower if the consumption of non-stockholders is excluded from total consumption. They documented that the value of the coefficient for stockholders is one-third of all households. From this result they discussed the implications on the unsolved equity premium puzzle and draw a conclusion that exclusion of the consumption of non-stockholders may partially solve the equity premium puzzle.¹

The study of the liquidity constraints is, as implied so far in this sequence of developments in consumer theory, not merely a discussion of the market imperfections. It deals with the fundamental problems that question the validity of the Random Walk Theorem and consumer's rationality of forward-looking decision. If aggregate consumption is sensitive to an aggregate variable that represents in any form the prevailing liquidity constraints in financial markets, the liquidity constraints should be taken into account in order to explain aggregate consumption. Furthermore, when the liquidity constraint effect is incorporated in explaining aggregate consumption, it will to a great extent resonate to other macroeconomic variables and to policy implementation. For instance, a counter-cyclical tax policy turns out to be more effective than many have thought no matter how predictable the policy change is for the public.

This paper intends to develop an intertemporal consumption-portfolio model with plausible liquidity constraints and to conduct empirical tests for whether such constraints really exist in financial markets. For these purposes the liquidity constraints will be incorporated into the conventional Euler equation by means of an ad hoc assumption of income distribution. The resulting Euler equation will be a refined form of the conventional Euler equation and will be examined for excess sensitivity of aggregate consumption growth to an extracted liquidity growth factor. From the empirical results this paper will appraise the effects of the liquidity constraints.

It is tempting to extend the basic model in the literature to the multiperiod interval's specification in order to investigate whether consumers are possibly myopic and examine the effects of the liquidity constraints on the term structure of interest rates. In this paper, the effect on the risk premium will be emphasized in the multiperiod framework. By stacking the individual equations with different time intervals, this paper will present the tests for stability of the estimates over the time interval spectrum.

This paper is organized as follows: in chapter 2 a stochastic model with the liquidity constraints is laid out and its assumptions are discussed. The liquidity constraint factor is extracted from the modified Euler equation and examined. In chapter 3, the linearized modified Euler equation is compared to the linearized conventional Euler equation in terms of misspecification. In chapter 4, the effects
of the liquidity constraints on the term structure are discussed. In chapter 5, Generalized Methods of Moment procedures are applied to implement the empirical investigations. Data problems such as time-averaging and overlapping observations are discussed in detail before conducting the estimation procedures. In the actual empirical work, the models are specified in order to clarify two potentially competing theories in explaining the failure of Hall's Random Walk Theorem of aggregate consumption, liquidity constraints and intertemporal substitution. Finally, conclusions follow in chapter 6.
CHAPTER II

THE STOCHASTIC MODEL WITH LIQUIDITY CONSTRAINTS

Individual rationality in the framework of Neoclassical Macroeconomics takes the form of optimization of a representative agent's objective function subject to suitable constraints. In the context of a consumer's intertemporal planning, individual rationality appears in the form of the maximization of present discounted lifetime utility subject to a sequence of suitable constraints.

2.1 The Modified Euler Equation

2.1.1 The Assumptions

To begin with, consider a representative consumer $h$ who has the following objective function that may be expressed in terms of the present discounted lifetime value of utility of consumption in the future periods

$$E_t\tilde{U}(C_t^h, C_{t+1}^h, C_{t+2}^h, \ldots) = E_t \sum_{\tau=0}^{\infty} \beta^\tau U(C_{t+\tau}^h), \quad (2.1)$$
where \( E_t \) is the expectation operator conditional on information available at time \( t \), \( \hat{U}_t(\cdot) \) is the consumer \( h \)'s lifetime utility at time \( t \), \( U(\cdot) \) is the consumer's one-period concave utility function, \( C_t^h \) is (real) consumption of the consumer \( h \) in period \( t \), and \( \beta \) is the subjective discount factor of the consumer. In building the model in this paper, several assumptions will be introduced. The following are the assumptions behind the preference formulation (2.1).

**Assumption 1 (On Preferences)** The following are the assumptions on the consumer's preferences:

(a) **Infinite lifetime**: It is assumed that the representative agent \( h \) lives infinitely for the analysis of the benchmark model. Because of this assumption, any bequest motive is not affected by the possibility of death.\(^1\)

(b) **Intertemporal additive utility function**: The preference rankings between the arguments in the utility functions of any two periods are taken to be independent of the arguments' levels in any third period. Since time is irreversible, intertemporal additivity implies that previous decisions affect present ones and future choices only through the consumer's budget constraints.\(^2\)

\(^1\)The relaxation of this assumption of infinite lifetime plays a critical role in the analysis of the Ricardian Equivalence Theorem. O. Blanchard (1985), P. Evans (1988a, 1988b, 1989), and W. Buiter (1989) used the effective discount factor for finite horizon of lifetime, incorporating the possibility of agent's death into the regular discount factor. For models incorporating gifts and bequests, see J. Ingersoll (1987) and W. Buiter (1989).

\(^2\)To loosen the strictness of additivity, G. Mankiw (1985) introduces the durability of goods. He included the stock of durable goods as an argument in the utility function. Alternatively, habit formation is introduced by Spinnewyn (1979a,b), Muellbauer (1985), and G. Constantinides (1990). The utility is, in the habit formation theory, a function of the sluggish consumption like \( C_t - \sum \alpha_t C_{t-1} \), where \( \alpha_t \) is a measure of habit formation.
(c) *Consumption-and-leisure separable utility function:* To avoid complicated analysis, the utility function is consumption-and-leisure separable. This assumption enables us to ignore the possible connection between consumption and income.³

(d) *A quasi-concave and monotonic utility function:* The utility function is assumed to be quasi-concave and non-decreasing in each of its arguments. The degree of the concavity reflects the risk aversion of the agent. The monotonicity of the utility function implies that the agent’s attitude can be described always as “more is better.”⁴

(e) *Homogeneous agents with regard to their preferences:* All the consumers have identical and time-constant utility functions and discount factor β in equation (2.1). The constant rate of pure time preference, coupled with zero probability of death, ensures that the discount factor is constant for the infinite horizon lifetime. Aggregation over agents is possible with homogeneous preferences over agents and additional assumptions such as identical information and degree of risk aversion.

Next, discussion follows about suitable constraints that consumers would face. The basic idea for the constraints is that some heterogeneity of the agents with respect to their endowments is inevitably required for the existence of private loan markets. Whereas the consumers’ preferences are assumed to be homogeneous, the

³The primary purpose in this paper is to investigate the existence of liquidity constraints and their linkage to savings and portfolio decisions. For nonseparable utility functions over consumption and leisure, see G. Mankiw, J. Rotemberg, and L. Summers (1985).

⁴In the Expected Utility theory in the Microeconomics literature, the expression “more is better” is explained in terms of Stochastic dominance preference (see M. J. Machina, 1987).
consumers' endowments are assumed to be heterogeneous as shown in the following assumption.

Assumption 2 (On the Endowments) Endowment patterns of each agent over time are assumed to be heterogeneous.

This assumption may be fulfilled if either the initial endowments of each agent or the endowment 'innovations' of each agent are different over agents. Assumption 2 may establish an incentive in the private markets to borrow and lend (or to sell and buy securities) across agents in response to the agents' desire to maximize their objective function (2.1).  

2.1.2 The Constraints

Consider the economy where there are \( n \) kinds of assets, \( i = 1, 2, \ldots, n \). Let \( A^h_t \) denote the ex-payoff (i.e., ex-dividend or ex-coupon) value of the consumer \( h \)'s financial assets at the beginning of period \( t \), \( N(t, \tau) \) denote the consumer \( h \)'s holdings of \( \tau \)-maturity's asset \( i \) with price \( P_i(t, \tau) \) at time \( t \), \( D_i(t, \tau) \) denote his or her (real) after-tax payoff on \( \tau \)-maturity's asset \( i \) at time \( t \), which is paid just before the beginning of period \( t \), and \( t + 1 \leq \tau \). Let \( Y_t \) denote nonasset (labor)

---

5Intertemporal production opportunities are ignored in this paper. Thus, the endowment is the only source of information that gives us information about productivity in the economy. Another implication behind the ignorance of productivity is that intertemporal production opportunities equal to intertemporal market (or exchange) opportunities. For the discussions related to this, see Sargent (1987), T. Bewley (1980), and R. Townsend (1980).

6The asset holdings used here include both the holdings until the maturity \( \tau \) and rolling-over a sequence of bonds over \( \tau \).
income. Then, the consumer $h$'s budget constraints can be written as

$$A_t^h + Y_t^h - C_t^h \equiv W_t^h = \sum_{i=1}^{n} N_i^h(t, \tau)P_i(t, \tau). \quad (2.2)$$

and nonhuman wealth $W_t^h$, for the holding period one, evolves into

$$A_{t+1}^h = \sum_{i=1}^{n} N_i^h(t, \tau)\left[P_i(t+1, \tau-1) + D_i(t+1, \tau-1)\right]. \quad (2.3)$$

There are two types of the liquidity constraints that have been frequently discussed in the literature although there could be as many diverse forms of the liquidity constraints as there are different definitions of liquidity constraints. The first type involves potential borrowers being denied borrowing by lenders because of lack of credence on future repayment. This type of liquidity constraint will appear when the current indebtedness of borrowers is substantial or the net worth of borrowers is close to zero. The second type of liquidity constraint involves borrowing rates higher than lending rates. The differentials of interest rates will in this case widen as the borrowing amounts become larger.

Basiclly, the addition of one more constraint into the model implies that it may more closely reflect the unknown true imperfect financial markets if the constraint represents the market imperfection correctly. That is, it can contribute to the

---

7. The values in equation (2.2) are thought of as consumption-good denominated in order to avoid the market incompleteness because of the inclusion of $Y_t$ in the standard budget constraints.

8. Equation (2.2) is considered to be a simplified consumer's budget constraint, and it can be generalized in terms of various different terms to maturities of a bond. Thus, the RHS of equation (2.2) can be written as $A_t^h + Y_t^h - C_t^h \equiv W_t^h = \sum_{i=1}^{n} \sum_{\tau=1}^{\infty} N_i^h(t, \tau)P_i(t, \tau)$. However, equation (2.2) is used for notational simplicity.
relaxation of the *ad hoc* constraints in Neoclassical Macroeconomics. However, the constraints in themselves are often *ad hoc* too. For instance, recently, Zeldes (1989) discussed a liquidity constraint by defining it as a lower quantitative limit of indebtedness (net worth or net stock of assets). A lower bound on the net worth of nonhuman wealth takes the form of one-sided inequality

$$
\sum_{i=1}^{n} N_i^h(t, \tau) P_i(t, \tau) \geq -a, \quad i = 1, 2, \ldots, n,
$$

where the $a$ is a positive constant that represents the upper bound on indebtedness. The one-sided inequality in equation (2.4) means that, while the consumer is allowed to make loans (equivalently, buy securities), the consumer is banned from loan borrowing (equivalently, securities selling) at a limit, $-a$, of their net indebtedness. Once the consumer is under the ban of equation (2.4), the consumer’s future labor income cannot be exchanged for the consumer’s present consumption. In other words, under the Zeldes’ type of liquidity constraint, his or her future labor incomes become a nontradable asset and thus the consumer cannot optimize the intertemporal allocations of resources. The failure of intertemporal optimization by the consumer would incur a substantial loss of of his or her welfare. Campbell and Mankiw (1990), following Cochrane (1989), derived the formulae of utility loss by using the second order Taylor approximation and calculated the welfare loss values. They indicated that the welfare loss is larger if the constrained and the unconstrained consumers coexist in the economy than if the economy consists of
identical consumers who are some periods under the constraints and are not during other periods. In the Zeldes paper, however, Zeldes set the limit, , at zero for convenience. Then, equation (2.4) becomes

\[ \sum_{i=1}^{n} N_i^i(t, \tau) P_i(t, \tau) \geq 0, \quad i = 1, 2, \ldots, n. \tag{2.5} \]

An alternative to the Zeldes' type of quantitative restriction on loan borrowing is a differential between borrowing and lending rates (or equivalently, differentials in selling and buying asset prices). The spread between the two rates (or prices) may reflect the implicit cost of default risk and interest rate risk. Or the spread may reveal transaction costs, the imperfect liquidity of assets or even imperfect information. In this case, loan rates available to the constrained consumer are higher than the lending rates. The Zeldes' type of liquidity constraint can be, however, regarded as a special case of the borrowing and lending rate differentials. In other words, the quantitative limit of net worth in the Zeldes' type of liquidity constraint is conceptually equivalent to the infinite level of borrowing rates in the latter case. The model in this paper will use the Zeldes' type of liquidity constraint for the following reasons:

\(^8\)Campbell and Mankiw (1990) derived the welfare loss formula \( L \) in the case that two types of consumers coexist in the economy as \( L = \frac{\sigma}{2} \alpha Var(c_i - y_i) \), where \( \alpha \) is the coefficient of relative risk aversion, \( \theta \) is the fraction of disposable income that accrues to the rule-of-thumb consumers, and \( c \) and \( y \) are the logarithms of \( C \) and \( Y \), respectively, implying that the welfare loss is a function of consumption-income growth with a coefficient that consists of the fraction of constrained consumers and the degree of risk aversion (p.28).

\(^9\)The asymmetry of information in financial markets is one of the important causes of the market failure. For the effect of private information on the loans markets, see D. M. Jaffee and T. Russel (1970).
(a) The interest rates differentials type of liquidity constraint would inevitably bring endogenous borrowing rates. For instance, the borrowing rate may be regarded as a function of net borrowing or the accumulated borrower's credit history. This endogeneity of borrowing rates will produce a very complicated process. On the contrary, the Zeldes' type of liquidity constraint can be easily formulated in the model.

(b) The approach that is based on the rate differentials type of liquidity constraint requires more information on the shape of the production opportunities frontier such as the slopes of the marginal rates of transformation. It means that we need more assumptions about productivity. But this is not intended in this paper. Furthermore, even though the shape of the production opportunities curve is known, the concavity of the production transformation may cause consumers not to participate in loan markets and, instead, to internalize their intertemporal allocation of resources. If such is the case, the differences between the differential rates type and the Zeldes' type of liquidity constraints would be narrowed.  

(c) Finally, since the Zeldes' type of liquidity constraint is a special case of the rates differentials, we can treat the Zeldes' type of liquidity constraint as a benchmark of liquidity constraints.  

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11 For more discussion on the relevance of the Zeldes' type of liquidity constraint compared to the differential rates, see appendix A.

12 For details, see appendix B.
2.1.3 The Euler Equation in the Presence of Liquidity Constraints

Assume that no Ponzi game is allowed and a unique stationary rational expectations equilibrium exists. Then, the solutions to maximize the utility function equation (2.1) subject to equations (2.2), (2.3), and (2.5) can be found from an application of the Kuhn-Tucker-first-order conditions to the standard Bellman equation as follows.

In each period \( t \), for \( t \geq 0 \), the consumer seeks to obtain the value function, implicitly defined as

\[
V(A^h_t) = \max_{\{C^h_t\} \{N^h_t\}} U(C^h_t) + \beta E_t V(A^h_{t+1})
\]  

(2.6)

subject to constraints (2.2), (2.3), and (2.5).

Let \( \lambda_t \) be the Lagrange multipliers (known at time \( t \)) associated with the liquidity constraint factor (2.5). Then, the envelope condition gives

\[
\frac{\partial V(A^h_t)}{\partial N^h_t(t, \tau)} = \lambda_t P_i(t, \tau).
\]  

(2.7)

Then, the first-order conditions for this problem are, for time \( t \),

\[
U'(C^h_t) P_i(t, \tau) = 
\]

\[
\beta E_t U(C^h_{t+1}) [(P_i(t+1, \tau - 1) + D_i(t+1, \tau - 1)] + \lambda_t P_i(t, \tau),
\]  

(2.8)

\[
\sum_{i=1}^{n} N^h_i(t, \tau) P_i(t, \tau) \lambda_t = 0 \quad [\lambda_t \geq 0], \quad i = 1, 2, \ldots, n,
\]  

(2.9)

---

\(^{13}\)For the existence and uniqueness of the solutions for the nonlinear rational expectation equilibrium, see S. McCafferty and R. Driskill (1980).
and budget constraints (2.2) and (2.3).\footnote{The first-order conditions for the consumer's planning problem of \( j \)-period interval are the same expressions as in (2.8), (2.9) and budget constraints except that \( \beta \), \( t + 1 \) and \( r - 1 \) are replaced by \( \beta' \), \( t + j \) and \( r - j \), respectively.}

Here, the expression \([ \cdot \geq 0]\) in equation (2.9) denotes the usual complementary slackness condition. That is, if \( \sum_{i=1}^{n} N_i^h(t, r)P_i(t, r) > 0 \), the equation (2.9) means that \( \lambda_i = 0 \) and, if \( \sum_{i=1}^{n} N_i^h(t, r)P_i(t, r) = 0 \), it means that \( \lambda_i > 0 \). Also the second-order conditions for maximization are satisfied since the concavity of utility function ensures the bordered Hessian to be negative semidefinite. The \( \lambda_i \) is the shadow price of the liquidity constraints (2.5) and equals the gain in expected utility that would result if the liquidity constraints (2.5) were relaxed by one unit. The resulting Euler equation (2.8) must be satisfied for the consumers who are not liquidity-constrained but violated for those who are liquidity-constrained. In particular, let \( h = 1 \) and \( h = 2 \) represent the unconstrained and the constrained consumers, respectively. If the consumer maintains positive net assets in period \( t \), then equation (2.8) becomes

\[
U'(C_t^1)P_i(t, r) = \beta E_t U''(C_{t+1}^1)[P_i(t+1, r-1) + D_i(t+1, r-1)]. \tag{2.10}
\]

Equation (2.10) states that, in the absence of the liquidity constraints, the Euler equation is always satisfied. The marginal utility from consuming an extra unit today is equal to the marginal utility from waiting until future \( t + 1 \) to consume the extra amount. Expressed differently, in the absence of the liquidity constraints, the cost of buying a unit of asset \( i \) in terms of utility of today's foregone consumption
always equals the benefit of selling that asset in terms of the discounted expected utility of future consumption.

However, if the consumer fails to maintain positive net assets and reaches zero net assets in period \( t \), equations (2.8) and (2.9) imply

\[
U'(C_t^2)P_i(t, \tau) > \beta E_i U'(C_{t+1}^2)[P_t(t+1, \tau-1) + D_i(t+1, \tau-1)].
\] (2.11)

Equation (2.11) states that, in the presence of the liquidity constraints, the Euler equation fails to hold. In this case, a conventional empirical estimation procedure, which is based on (2.10), will produce implausible estimates and misleading inferences. The presence of the liquidity constraints, \( \sum_{i=1}^{n} N_i^2(t, \tau)P_i(t, \tau) = 0 \), results in a positive wedge \( \lambda \) between the two sides in equation (2.10). The one-sided inequality in equation (2.11) reflects the consumer's motive to dissave until his or her net stock of assets are zero. Let \( t' \) denote a future period when the consumer will eventually dispose all of his net asset holdings.\(^{15}\) Then

\[
\sum_{i=1}^{n} N_i^2(t', \tau)P_i(t', \tau) = 0, \quad i = 1, 2, \ldots, n \quad \text{and} \quad t' \leq t.
\] (2.12)

When the consumer reaches the portfolio (2.12), his budget constraint becomes

\[
C_t^2 = Y_t^2, \quad t' \leq t.
\] (2.13)

Equation (2.13) states that, in the presence of the liquidity constraints, once the consumer's net assets fall to zero at time \( t' \), his constrained consumption will

\(^{15}\)For one-period interval, \( t' = t + 1 \).
be financed by his contemporaneous labor income. Moreover, the condition of equation (2.13) will continue to rule the consumer's consumption until his future labor income rises above his consumption (by assumption no windfall gains and bequests are assumed).

2.2 Parameterization

Empirical tests of the existence and effects of the liquidity constraints generally use indirect methods based upon proxies and other measures with an assured relationship to actual liquidity constraints. This is because aggregate time series data for direct measures of the liquidity constraints are not typically available. In this paper, following Flavin (1981), Hayashi (1982), Jappelli and Pagano (1989), and recently Campbell and Mankiw (1989, 1990), I make a key assumption in this model about income distribution in the economy in order to link variables for an individual to aggregate variables.

Assumption 3 (Constant Share of Labor Income) The population in the economy consists of two distinct groups. In the first group the consumers are all liquidity-constrained and, as a group, receive a share $\theta$ of aggregate nonasset income; whereas, in the second group, the consumers are not liquidity-constrained and, as a group, receive a share $1 - \theta$ of aggregate nonasset incomes.\(^{16}\)

\(^{16}\)Flavin (1985) extended her early time series version of excess sensitivity tests. There, she considered the two reasons why the innovation in consumption is excessively sensitive to the innovation in income: first, liquidity constraints exist in the borrowing markets; second, myopic consumers exist in the sense that they believe that they face infinite of discount factors over the
To illustrate Assumption 3, look at figure 1. In the figure, curve A is an income-Lorenz curve for an economy which shows considerable inequality of income distribution. Curve B is a wealth-Lorenz curve in the presence of the liquidity constraints. The implication of Assumption 3 in this figure is that, if the share $s$ of aggregate income is fixed, the corresponding share of total population, say $x$, may vary as the income-Lorenz curve changes. In other words, the $x$ depends on the income distribution. Now, by the assumption, the consumers who, as a group, receive income share $\theta$ are all under the liquidity constraints. Therefore, the $x$ is the limit (zero) of net stock of assets for the constrained group. In addition, Assumption 3 implies that the wealth-Lorenz curve will expand to the right as the income-Lorenz curve moves to the right. Furthermore, the $x$ must be larger than the $\theta$ as long as the society reveals income inequality. Therefore, Assumption 3 implies that inequality of wealth distribution tends to increase more rapidly than that of income distribution.

Although Assumption 3 is ad hoc and restrictive, it has been often used because it is a simple and highly tractable representation of liquidity constraints. The future income profile and thus their marginal propensity to consume out of transitory income is non-zero. Thus the null hypothesis $H_0 : \theta = 0$ does not have a well-nested alternative hypothesis $H_1 : \theta \neq 0$ if the null hypothesis is to test the existence of the liquidity constraints. Flavin included the unemployment rate in the equation to take inference to the alternative hypothesis.

In this paper, in order to identify the nature of the relevant alternative hypothesis, I will make use of some information in the term structure of interest rates. For instance, if a considerable fraction of consumers are myopic, then the results from the overidentifying restriction in the empirical process over terms to maturities may be different from those who are rational in Muth sense.
underlying idea in the assumption is that the \( \theta \) is systematically determined and tends to be constant through time.\(^{17}\) The constant \( \theta \) does not imply that any members in one group cannot transfer into the other group. For example, the person who has been credit-rationed before would no longer be rationed as his income rises substantially. But, the constant \( \theta \) means that, in aggregation, the share \( \theta \) tends to be fixed in the sense that the inflows and the outflows of incomes in one group exactly offset each other.

It is assumed that discount factors \( \beta \) and preferences are the same across agents and through time, in order to arrive linear aggregation. Thus, with the constant \( \theta \), total labor income for group 2, \( Y^2_t \), can be written in terms of aggregate labor income

\[
Y^2_t = \theta Y_t, \quad 0 \leq \theta \leq 1.
\]  

(2.14)

The aggregate consumption identity is

\[
C_t = C^1_t + C^2_t.
\]  

(2.15)

The equations (2.13) and (2.14) imply that consumption for the group 2 is

\[
C^2_t = \theta Y_t.
\]  

(2.16)

\(^{17}\)Nonconstancy of the \( \theta \) may lead to quite different results and make the procedure of analysis complicated. For instance, along with a business cycle, the \( \theta \) may rise and fall. Literature on the sensitivity tests of aggregate consumption to disposable income is explicitly or implicitly based on Assumption 3 in this paper (Flavin (1981), Hayashi (1982, 1985), Zeldes (1989), Jappelli and Pagano (1989), Bayoumi and Koujianou (1989), and Campbell and Mankiw (1989, 1990)).
Substituting (2.16) into (2.15), one can obtain the equation expressed in terms of aggregate consumption and aggregate income

\[ C_t^1 = C_t - \theta Y_t. \]  

(2.17)

Equation (2.17) says that the consumption of the consumers who are unconstrained and thus whose Euler equation holds can be represented in terms of aggregate consumption and aggregate labor income. It is useful to express the unconstrained Euler equation in terms of the aggregate variables by combining the result above and the unconstrained Euler equation. In order to do this, return to the unconstrained Euler equation (2.10). For simplicity, define ex-payoff price of an asset \( i \) as \( V_{it} \equiv P_i(t, \tau) \) and its gross payoff of holding period one as \( V_{it+1} = [P_i(t+1, \tau - 1) + D_i(t+1, \tau - 1)] \), \( i = 1, 2, \ldots, n \). Thus, \( V_i \) is a \( n \)-dimensional column vector. The consumer is assumed to be rational in the Muth sense and the expectation in RHS of equation (2.10) will be formed optimally using all available information \( I_t \) at time \( t \). Then when (2.17) is substituted into (2.10), the unconstrained Euler equation (2.10) becomes

\[ U'(C_t - \theta Y_t) V_t = E[\beta U'(C_{t+1} - \theta Y_{t+1}) V_{t+1} | I_t]. \]  

(2.18)

It is noteworthy that, although the Euler equation (2.18) is now expressed in terms of aggregate variables, it does not include the shadow price of the liquidity constraints (the Lagrange multipliers associated with the liquidity constraints), which is determined by the entire intertemporal plan of the constrained consumer.
and thus whose closed solution form cannot be easily found. That is the reason that the equation (2.18) is thought of as a stepstone to construct a testable Euler equation.

Since $U'(C_t - \theta Y_t)$ and $V_t$ are known at time $t$, (2.18) can be rewritten as

$$E\left[\beta \frac{U'(C_{t+1} - \theta Y_{t+1}) V_{t+1}}{U'(C_t - \theta Y_t)} | I_t\right] = 1. \quad (2.19)$$

Equation (2.19) states that, ex ante, marginal rates of substitution $\beta U'(C_{t+1} - \theta Y_{t+1})/U'(C_t - \theta Y_t)$ and the reciprocal of marginal rates of transformation $V_{t+1}/V_t$ are equated. The ex post form of the above is

$$\xi'_{t+1} = \beta \frac{U'(C_{t+1} - \theta Y_{t+1}) V_{t+1}}{U'(C_t - \theta Y_t)} V_t - 1, \quad \text{where } E[\xi'_{t+1} | I_t] = 0. \quad (2.20)$$

Equation (2.20) says that forecasting errors $\xi_{t+1}$ should be orthogonal to any variable known in period $t$ or earlier. In other words, ex post, the two rates can differ because the realized variables are not perfectly foreseen. This discrepancy is due to the consumption and asset value innovations. To render equation (2.20) operational, assume that the utility function is of the constant relative risk aversion form

$$U(C_t) = \frac{(C_t)^{1-\alpha}}{1-\alpha} \quad \text{if } \alpha > 0, \quad \alpha \neq 1$$

$$= \ln C_t \quad \text{if } \alpha = 1. \quad (2.21)$$

where $\alpha$ is the Arrow-Pratt measure of relative risk aversion and is the reciprocal of the intertemporal elasticity of substitution. With a positive $\alpha$, the agent is risk
averse. With this utility function being employed, equation (2.20) becomes

\[
\beta \left( \frac{C_t - \theta Y_t}{C_{t+1} - \theta Y_{t+1}} \right) \left( \frac{V_{t+1}}{V_t} \right) = \xi_{t+1},
\]

(2.22)

where \( \xi_{t+1} = \xi'_{t+1} + 1 \). When the ratio of aggregate consumption is factored out, the equation (2.22) becomes

\[
\left( \frac{C_t}{C_{t+1}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+1}/C_{t+1}} \right)^\alpha \left( \frac{V_{t+1}}{V_t} \right) = \beta^{-1} \xi_{t+1},
\]

(2.23)

where \( \beta(C_t/C_{t+1})^\alpha \equiv S_{t+1} \) is the quasi-marginal rate of substitution between present and one-period-away future consumptions at time \( t \). \((1 - \theta Y_t/C_t)/(1 - \theta Y_{t+1}/C_{t+1}))^\alpha \equiv L_{t+1} \) is the liquidity constraint factor between present and one-period-away future and regarded as time-varying weights over the regular marginal rate of substitution between the time \( t \) and \( t + 1 \). Thus, \( S_{t+1} L_{t+1} \) represents the marginal rate of substitution of aggregate consumptions with a time-varying weight \( L_{t+1} \) between time \( t \) and \( t + 1 \). This modified version of the Euler equation reveals very important implications for the consumption-portfolio decision and the implied estimator of the coefficient of relative risk aversion when loan markets are imperfect. The presence of the liquidity constraint factor in the ordinary Euler equation clearly suggests that the joint determination of consumption growth and asset holdings is influenced by consumption-income growth. In line with this, the empirical results will be different from those in the absence of constraints in bor-

\[16\] The "quasi-" means that, in the presence of the liquidity constraints \( (\theta \neq 0) \), \( S_{t+1} \) is not the true marginal rate of substitution between present and one-period away future consumptions at time \( t \).
rowing markets. This is a very important finding since no results in the literature have explicitly identified this liquidity constraint factor. This factor may provide a better opportunity to understand the intertemporal resource allocations of consumers. For example, to assess the effect of the presence of the liquidity constraints in the ordinary optimal conditions for consumers, consider riskless rates of return \( r_{t+1} \) between time \( t \) and \( t + 1 \). Then, equation (2.23) for the riskless asset is

\[
\left( \frac{C_t}{C_{t+1}} \right)^\alpha \mathcal{L}_{t+1}(1 + r_{t+1}) = \beta^{-1} \xi_{t+1}.
\]  

(2.24)

On the contrary, Hall's (1978) Random Walk Hypothesis (RWH) or Martingale Hypothesis (MH) is based on the Euler equation (2.24) without the term \( \mathcal{L}_{t+1} \). That is,

\[
\left( \frac{C_t}{C_{t+1}} \right)^\alpha (1 + r_{t+1}) = \beta^{-1} \xi_{t+1},
\]  

(2.25)

which implies that, if \( \delta = (1/\beta) - 1 \) is the rate of pure time preference,

\[
E_t r_{t+1} \leq \delta, \text{ then } E_t \left[ \frac{C_{t+1}}{C_t} \right] \leq 1.
\]  

(2.26)

Expression (2.26) means that consumption will grow [fall] if the rate of return is greater [less] than the rate of pure time preference. This statement may be applicable to equation (2.24). In other words, the logical statement (2.26) does not change when \( \mathcal{L}_{t+1} \) is added in equation (2.25). However, it does matter to the parameters estimates \( \hat{\alpha} \) and \( \hat{\beta} \). It is obvious that the existence of the liquidity constraints in the financial markets would distort the optimal conditions that a rational agent must choose.
In the following section, the stochastic liquidity constraint factor in the U.S will be identified and its implication in the determination of asset pricing will be examined.

2.3 Time Series of the Liquidity Constraint Factor

It may be suggestive to look at the historical movement of the Liquidity Constraint Factors. Figure 2 shows the time-series of the liquidity constraint factors for the postwar U.S. when $\theta$ is set 0.25 and 0.50. A dotted horizontal line represents the value of the liquidity constraint factors when $\theta = 0$ or it can be regarded as a long-run value, unity, of the liquidity constraint factors. It shows that the short-run liquidity constraint factors rise and fall around the long-run value. The cyclical variability of the short run factor increases with the magnitude of $\theta$. High variability in early 1950s may reflect the effects of the Korean War and a one time social insurance benefit to World War II veterans. It is interesting to find that the major change in the Federal Reserve’s operational procedures in 1979 does not lessen credit-rationing to the household sector. This observation indirectly supports the Campbell and Mankiw’s (1989) empirical result that there was no major shift in $\theta$ in 1979 in their version of model. This finding is potentially important since it explains that the sources of the liquidity constraints are more likely moral hazard and adverse selection because of asymmetric information rather than institutional restrictions on interest rates. However, the first world oil shock appears
to some extent to affect the short-run liquidity constraint factors. High production cost at that time pulled down income and thus appeared in a temporary reduction in income-consumption growth. The stochastic movement of the short-run factors reflects the cyclical strength of the liquidity constraint conditions toward the consumers who fail to hold collateral. Also it may represent the welfare loss of constrained consumers when they fail to achieve intertemporal optimality. For further discussion, consider the next figure. Figure 3 represents time series which compares two measures of the postwar U.S. aggregate consumption growth: consumption when the constrained and the unconstrained consumers are assumed to coexist at the ratio of labor incomes accruing to both groups of 0.33 (a series of continuous lines); and consumption when only unconstrained consumers are assumed to exist and so the $\theta = 0$ (a series of dotted lines). As the figure clearly shows, the variability for $\theta = 0.33$ appears to be generally larger than that for $\theta = 0.00$. The difference in the consumption growth volatility reflects the fact that, as implied in the consumers' Euler conditions, consumption for constrained agents is financed only by current income while consumption for unconstrained agents is jointly determined with asset prices. This is because the volatility of asset prices is transmitted to the unconstrained consumption movement in the process of intertemporal reallocation when asset prices change. If it is true that both types of consumers coexist, it is highly possible that this is the reason that the estimates for the parameters, which are obtained from ordinary Euler condition, may appear
to be implausibly high. However, the differences of two consumption variabilities, as shown in figure 3, do not seem to explain sufficiently the implausibly high magnitude of the estimate for the coefficient of relative risk aversion. Therefore, an attempt to find a solution for the equity premium puzzle solely with the liquidity constraints will turn out to be unsatisfactory although it partially explains the appearance of the puzzle in the financial markets.
CHAPTER III
CRITICISM OF THE CONVENTIONAL APPROACH

In the previous chapter the modified Euler equation with liquidity constraints is derived and the liquidity constraint factor is identified. In this chapter it is explained why the modified Euler approach is better than a conventional approach in order to test the excess sensitivity of aggregate consumption. The conventional tests are based on the model that is specified in the form in which a variable representing income growth (precisely, the logarithm of the level of disposable income) is added in a permanent-income-envisioned Euler equation. However, from the view advanced in this paper, the conventional approach suffers from a misspecification problem. This chapter shows that the model that is based upon the modified Euler equation is more robust than the conventional model.
3.1 The Misspecification Problem

In order to focus on the discussion about the possible misspecification in the conventional tests, assume that the real interest rate is constant and known.\textsuperscript{1}

When the equation (2.25) is linearized by taking logarithms, it becomes the following equation that follows the martingale process

\[ c_t - c_{t-1} = \mu + \nu_t, \]

where \( c_t \) is logarithm of \( C_t \). Here, \( \mu \) is positive, zero, or negative as real interest rate is greater than, equal to, or less than the pure time preference. It is usually positive. The \( \nu_t \) is the consumption innovation and \( E_t[\nu_t] = 0 \). Equation (3.1) states that the deviation of consumption change from its constant mean cannot be predicted using all information available at time \( t \) or earlier. The test for the null hypothesis that rule-of-thumb consumers exist is usually carried out by adding a variable of change in (logarithm of) disposable income \( \Delta y_t \) in order to represent the consumption of rule-of-thumb consumers, \( \Delta c_t^2 = \theta \Delta y_t \).\textsuperscript{2} As it has appeared in the literature,

\[ \text{\textsuperscript{1}In the literature on the PIH, real interest rates are usually assumed to be constant and known. This assumption allows to ignore the effect of intertemporal substitution of consumptions with respect to interest rate changes. However, some authors considered variable and uncertain interest rates (G. Mankiw (1981), Grossman and Shiller (1981), Hansen and Singleton (1983), Mankiw and Shaprio (1986), Grossman, Meleino and Shiller (1987), Harvey (1988), Hall (1988), Breeden, Gibbons and Litzenberger (1988), Campbell and Mankiw (1988, 1990), and Kandel and Stambaugh (1991)).}
\]
\[ \text{\textsuperscript{2}Hall (1978) used a quadratic utility function to obtain linear relationships. Hansen and Singleton (1983) assumed a constant relative risk aversion utility function and lognormality for the conditional distribution of consumption.}
\]
\[ \text{\textsuperscript{2}In order to represent the consumption of rule-of-thumb consumers, most authors in the literature have used "disposable" income. However, (after net tax) "labor" income is the more accurate variable for this case.} \]
the $\theta$ is the fraction of (logarithm of) aggregate disposable income for the rule-of-thumb consumers’ disposable income. The “rule-of-thumb” means that they simply consume all of their disposable income, either because they are liquidity-constrained or because they are myopic in the sense that marginal propensity to consume out of transitory labor income is nonzero. Thus the conventional excess sensitivity tests of consumption is a form that merely includes $\theta \Delta y_t$ in equation (3.1):

$$\Delta c_t = \mu + \theta \Delta y_t + \nu_t,$$

and the test of the existence of rule-of-thumb consumers is to test the null hypothesis:

$$ \begin{align*}
H_0 & : \theta = 0 \quad \text{(nonexistence of the rule-of-thumb consumers)} \\
H_1 & : \theta \neq 0 \quad \text{(the existence of the rule-of-thumb consumers or myopic consumers or both)}
\end{align*} $$

However, in this paper, I argue that equation (3.2) must be a wrong specification for sensitivity tests. The reason why it must be so appears obvious when the modified Euler equation (2.24) is log-linearized under the same assumptions as used to arrive at the equation (3.2). That is, the correct specification as a counterpart of (3.2) is

$$\Delta c_t = \mu + \theta \{(Y_t/C_t) - (Y_{t-1}/C_{t-1})\} + \nu_t,$$

or

$$\Delta c_t = \mu + \theta \Delta(Y_t/C_t) + \nu_t.$$  \hspace{1cm} (3.3)

The second term in RHS of (3.3) is the linearized version of the liquidity constraint factor of $L_t$. It comes directly from the ‘intact’ Euler equation (2.24) that always
holds, even in the presence of the liquidity constraints. The adjective 'intact' means that each term of equation (2.24) is not distorted after linearization and, accordingly, does not lose its original nature or its original information. However, the $\Delta y_t$ is not relevant variable for the liquidity constrained consumption under constant relative risk aversion. Thus, for the excess sensitivity tests of aggregate consumption to transitory labor income, equation (3.3) should be used instead of (3.2).

In general, any empirical study that is based on a type of Euler equation might often neglect the point that the equation after approximation will lose its original characteristics: for instance, its curvature or its denominator (or normalizer). Therefore, when a research model is based on a type of Euler equation, a more careful approximation procedure would be required.

3.2 The Unit Root Problem

Following Hall (1978), Flavin (1981) used level of income and consumption in order to test excess sensitivity to income. In this section the problem will be discussed with a level specification for the conventional model and a log-linear specification for the model developed in this paper in order to explain the method that the conventional approach used and to apply the method for the model here. Recall that the assumption that the utility form is of constant relative risk aversion gives the result that the liquidity constraint appears in the form of (3.3). Because the
level representation $\Delta Y_t$ in equation (3.2) is not necessarily in the information set at time $t-1$ or earlier, it can be correlated with the innovation $\nu_t$. It is, therefore, necessary to divide $\Delta Y_t$ into its forecastable and unforecastable parts on the past information set. Suppose aggregate labor income $Y_t$ follows the AR(1) process

$$Y_t = \rho_0 + \rho_1 Y_{t-1} + \epsilon_t, \quad \text{where} \quad E_{t-1} \epsilon_t = 0.4$$ (3.4)

Subtracting $Y_{t-1}$ from both sides of (3.4), Flavin obtained

$$\Delta Y_t = \rho_0 + (\rho_1 - 1) Y_{t-1} + \epsilon_t. \quad (3.5)$$

Plugging (3.5) back into level expression of (3.2), she had the following equation

$$\Delta C_t = \mu' + \pi Y_{t-1} + \nu_t, \quad (3.6)$$

where $\pi = \theta(\rho_1 - 1)$ and $\nu_t = \epsilon_t + \theta \nu_t$ is now uncorrelated with $Y_{t-1}$.

For the estimation here, Hayashi (1987) suggested two estimation methods. The first one is to estimate the $\theta$ and the $\rho$s by the instrumental variables tech-

4Masakiw and Shapiro (1985) considered even the cases in which (a) disposable income follows more general nonstationary process of $IMA(1,1)$:

$$Y_t = Y_{t-1} + \epsilon_t - \phi \epsilon_{t-1},$$

If $\phi = 0$, then it is a random walk. If $\phi = 1$, then it is white noise. For any $\phi$ less than one, income is nonstationary; and (b) it follows borderline stationary process: it is the case where $\rho_1$ of the equation (3.4) is very close to unity. They concluded that tests of the PIH that assume $Y_t$ is stationary around a deterministic trend are biased toward rejection if income follows a nonstationary, or barely stationary process. As a result, they cast doubt on the conclusion that consumption is excessively sensitive to income. Moreover, Ball (1989) pointed out that Flavin's models, based in the hypothesis that real income obeys a stable stochastic process, are subject to Lucas' criticism. It means that the stochastic process of income is a result of the interaction of all the actors in the economy and is not a deep structural characteristic of the consumer alone (p.155).
nique with the known $Y_{t-1}$ as an instrument for $\Delta Y_t$. The other method is to estimate the parameters in the model from equations (3.4) and (3.6) by the multivariate regression with the cross-equation restrictions that the same autoregression coefficient $p_1$ appears in both equations. These two different estimates of $\theta$ are, according to Hayashi, numerically identical to each other. Moreover, the test statistic for the hypothesis $\theta = 0$ in the multivariate regression is numerically identical to the $t$ statistic in the IV estimation.

Flavin's (1981) estimate of $\theta$ was based on detrended quarterly U.S. data on nondurables and disposable income. She obtained the results that $\theta$ was so large that almost all of aggregate consumption was attributable to rule-of-thumb consumers. However, if disposable income (or labor income) is a random walk ($p_1 = 1$ in (3.4)), the use of detrended data results in bias of the test toward rejection of the null hypothesis $H_0 : \theta = 0$ (Hayashi (1987) and Mankiw and Shapiro (1985)).

Since Flavin's tests were reported, however, some economists have reported that postwar U.S. data on nondurables and disposable income follow a random walk (DeLong and Summers (1984), and Mankiw and Shapiro (1985)). According to Hayashi, the coefficient of $Y_{t-1}$, $\pi = \theta(p_1 - 1)$ is zero no matter what the value of $\theta$ is (p.105). G. Mankiw and M. Shapiro (1985) precisely point out that Flavin's (1981) test statistics are inadequate in the presence of a unit root, e.g., $p_1 = 1$, for the following reasons. First, one is more likely to believe in the existence of a deterministic trend, which actually does not exist. Second, inappropriate
detrending can produce spurious cycles even if actual data do not have cyclical properties at all (p. 165).

The same method that Flavin used is now applied to equation (3.3). The term $\Delta(Y_{t-1}/C_{t-1})$ in (3.3) may be correlated with the innovation $\nu_t$ since it is not necessarily in the information set at time $t - 1$ or earlier. To extract the forecastable parts in $\Delta Y_t$, suppose that the ratio $(Y_{t-1}/C_{t-1})$ follows the AR(1) process

$$(Y_t/C_t) = \rho_0 + \rho_1(Y_{t-1}/C_{t-1}) + \epsilon_t,$$  \hspace{1cm} (3.7)

where $E_{t-1}\epsilon_t = 0$.

Plugging (3.7) into equation (3.3), one can obtain

$$\Delta c_t = \mu' + \pi(Y_{t-1}/C_{t-1}) + \nu'_t,$$ \hspace{1cm} (3.8)

where $\pi = \theta(\rho_1 - 1)$ and $\nu'_t = \epsilon_t + \theta \nu_t$ is now uncorrelated with $Y_{t-1}$.

This equation (3.8) has the same form as equation (3.6) has except that the term $Y_{t-1}$ in (3.6) is now replaced by the $(Y_{t-1}/C_{t-1})$. The underlying coefficients of (3.8) are the same as those of (3.6). Figure 4 is shown to compare the performance of $\Delta(Y_{t-1}/C_{t-1})$ and $\Delta Y_t$ as instrumental variables. $\Delta Y_t$ may have better predicting power than $\Delta(Y_{t-1}/C_{t-1})$ may have because at a glance the former appears to have higher variability than the latter does. As previously shown in figure 3, however, since the range of consumption growth is $-0.02$ to $0.02$, the variability of $\Delta(Y_{t-1}/C_{t-1})$ is almost the same level as that of consumption growth. This
observation implies that $\Delta(Y_{t-1}/C_{t-1})$ may be a good instrumental variable for estimation although the final judgment on its eligibility as a good instrumental variable must await until the individual variable's predictive power is examined later.

The discussion so far implies that, when the deep structural parameters such as $\theta$ and $\rho$ in the model are estimated, the conventional sensitivity test approach will directly confront the unit root problem if the unit root problem exists. It seems that the presence of the unit root in the instrumental variable in the modified Euler equation may not result in such serious problems as in the conventional Euler equation if the instrumental variables are constructed appropriately. However, in the empirical procedures, for example, Generalized Methods of Moment procedures, the instrumental variables in the model are assumed to be stationary.
CHAPTER IV

THE EFFECTS OF LIQUIDITY CONSTRAINTS ON THE TERM STRUCTURE

4.1 The Effects on the Risk Premium

In this chapter it is intended to investigate the effect of the liquidity constraints on the term structures of interest rates. Suppose the consumer will hold a safe asset Aᵢ until maturity n of that asset. The liquidity constraint factor denotes as

\[ \mathcal{L}_{t+n} = \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+n}/C_{t+n}} \right)^{\alpha} \]

Here, with the condition that \( Y_t/C_t \neq Y_{t+n}/C_{t+n} \), if \( \theta = 0 \) (no liquidity constraints exist), then \( \mathcal{L}_{t+n} = 1 \) and, if \( \theta \neq 0 \), then \( \mathcal{L}_{t+n} \neq 1 \). \(^2\) It is assumed that no call or put options are available. Let \( R_{1t} \) and \( R_{nt} \) denote the gross rates of return for

\(^1\)When the consumer will hold the assets until the maturity n of those assets, the resulting Euler equation is

\[ E\left[ \beta^n \left( \frac{C_t}{C_{t+n}} \right)^{\alpha} \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+n}/C_{t+n}} \right)^{\alpha} \left( \frac{Y_{t+n}}{V_t} \right) | \mathcal{I}_t \right] = 1. \]

\(^2\)Since \( 0 < (C_t/C_{t+n})^\alpha = (C_t/C_{t+n})^\alpha \mathcal{L}_{t+n} \), one can know that \( \mathcal{L}_{t+n} > 0 \).
the short- and the long-term of that bond $A_t$, respectively. At the beginning of
time $t$, the returns are known and risk free: for example, as with Treasury bills.
In other words, at time $t$, $R^{-1}_{it}$ and $R^{-1}_{nt}$ are the price $V_{jt}$ of perfectly sure claims
$V_{jt+1}$ and $V_{jt+n}$ to one unit of consumption at time $t + 1$ and $t + n$, respectively.
For the short-term bond, $V_{jt} = R^{-1}_{it}$ and $V_{jt+1} = 1$. Then, the Euler equation (in
footnote (1)) becomes

$$R^{-1}_{it} = E_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^{\alpha} L_{t+1} \right]. \quad (4.2)$$

For the long-term bonds of period $j = n$,

$$R^{-1}_{nt} = E_t \left[ \beta^n \left( \frac{C_t}{C_{t+n}} \right)^{\alpha} L_{t+n} \right]. \quad (4.3)$$

Let $L^{-j}$ denote the lead operator by $j$ periods and $L^0 = 1$. Then, using the
expression that $L_{t+n} = L_{t+1}(L^{-1}L_{t+1})(L^{-2}L_{t+1}) \ldots (L^{-n-1}L_{t+1})$, equation (4.3)
can be written as

$$R^{-1}_{nt} = E_t \left[ \prod_{j=1}^{n} \beta \left( \frac{C_{t+j-1}}{C_{t+j}} \right)^{\alpha} (L^{-j}L_{t+1}) \right]. \quad (4.4)$$

Applying the law of iterated expectation, equation (4.4) can be written as

$$R^{-1}_{nt} = E_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^{\alpha} L_{t+1} \prod_{j=2}^{n} E_{t+j-1} \beta \left( \frac{C_{t+j-1}}{C_{t+j}} \right)^{\alpha} L^{-j}L_{t+1} \right]. \quad (4.5)$$

By the definition of conditional covariance, equation (4.5) can be rewritten as

$$R^{-1}_{nt} = E_t \left[ \beta \left( \frac{\sigma_t}{\sigma_{t+1}} \right)^{\alpha} L_{t+1} \right] E_t \left[ \prod_{j=1}^{n-1} R_{it+j}^{-1} \right] + COV_t \left[ \beta \left( \frac{\sigma_t}{\sigma_{t+1}} \right)^{\alpha} L_{t+h} \prod_{j=1}^{n-1} R_{it+j}^{-1} \right]. \quad (4.6)$$
Substituting (4.2) into (4.5), one obtains

\[ R^{-1}_{nt} = R^{-1}_{nt} E_t \prod_{j=1}^{n-1} R^{-1}_{t+j} + COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right) \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha L_{t+h} \prod_{j=1}^{n-1} R^{-1}_{t+j} \right]. \] (4.7)

For example, if the long term is now two periods \( n = 2 \), then equation (4.7) becomes

\[ R^{-1}_{2t} = R^{-1}_{1t} E_t R^{-1}_{1t+1} + COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha L_{t+h} R^{-1}_{t+1} \right]. \] (4.8)

As seen in equation (4.7), the stochastic liquidity constraint factor affects the term structure of interest rates through the term premium. To examine this more precisely, one can decompose the conditional covariance, which is now denoted as \( \Omega_{t+1} \),

\[ \Omega_{t+1} \equiv COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha L_{t+h}, R^{-1}_{t+1} \right], \quad \text{or} \]

\[ \Omega_{t+1} \equiv COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha, R^{-1}_{t+1} \right] E_t \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha L_{t+h} + COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha R^{-1}_{t+1}, L_{t+h} \right] \]

\[ -COV_t \left[ \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha, L_{t+1} \right] E_t R^{-1}_{t+1}. \] (4.10)

For proof see Appendix C. This is a fundamental risk premium formula with the liquidity constraint factor. To see a further interesting implication, define \( S_{t+1} \equiv \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha \) as the regular quasi-marginal rate of substitution between period \( t \) and \( t + 1 \) (in absence of the liquidity constraints), \( f^{-1}_{t+1} \equiv E_t R^{-1}_{t+1} \) as reciprocal of forward rate, and \( E_t \xi^{-1}_{t+1} \) as the least-square residual that by construction satisfies \( E_t \xi^{-1}_{t+1} = 0 \). Then, \( \xi^{-1}_{t+1} \equiv \beta \left( \frac{C_t}{C_{t+1}} \right)^\alpha R^{-1}_{t+1} - 1 \) is the least-squares one-step-ahead
forecast error from information dated \( t \) and earlier. Then, risk premium formula (4.10) becomes

\[
\Omega_{t+1} = \text{COV}_t \left[ S_{t+1}, R_{t+1}^{1} \right] E_t \xi_{t+1} + \text{COV}_t \left[ \xi_{t+1}, \xi_{t+1}^{1} \right] \xi_{t+1}^{1} - \text{COV}_t \left[ S_{t+1}, \xi_{t+1}^{1} \right] f_{t+1}^{1}.
\] (4.11)

The effect of the liquidity constraints on the risk premium can be, according to (4.12), explained by three components. The first component reflects the effect of liquidity constraint factor \( \xi_{t+1} \) on the risk premium as a time-varying weight on the measure of risk \( \text{COV}_t \left[ S_{t+1}, R_{t+1}^{1} \right] \) of that asset (in absence of the liquidity constraints) when the conditional expected liquidity constraint factor will diverge from its long-run value, say \( \bar{\xi} \). The next two components reflect the effects of the liquidity constraints on the risk premium when the liquidity constraint factor is correlated with the forecasting error and with the marginal rate of substitution.

To see this in more detail, \( E_t[\xi_{t+1}] \) can be written as

\[
\xi_{t+1} = E_t \xi_{t+1} + \zeta_{t+1}, \quad i = s \text{ and } \ell, (4.12)
\]

where the \( s \) represents a short-term while the \( \ell \) represents a very long-term. \( \zeta_{t+1} \) is \( i \)-step-ahead forecast error from information dated \( t \) and earlier and \( E_t \zeta_{t+1} = 0 \) for \( i = s \) and \( \ell \). It is also assumed that \( E_t[\xi_{t+1} \zeta_{t+1}] = 0 \), for \( i = s \) and \( \ell \).

Assume that the time series of the liquidity constraint factor \( \xi_{t+1} \) is covariance stationary and its long-run value \( \bar{\xi} \) is equal to unity:
\[ \bar{\mathcal{L}} = E_t \mathcal{L}_{t+t} = 1, \]  
(4.13)

and

\[ \mathcal{L}_{t+t} = 1 + \zeta_{t+t}. \]  
(4.14)

The expected short run liquidity constraint factor \( E_t[\mathcal{L}_{t+t}] \), conditional on \( I_t \), can be divided into two parts: its long run value \( \bar{\mathcal{L}} \) and a (nonstochastic) cyclical trend term \( g_t \) deviated from its long run equilibrium value. Then,

\[ E_t \mathcal{L}_{t+t} = 1 + g_t, \]  
(4.15)

and

\[ \mathcal{L}_{t+t} = 1 + g_t + \zeta_{t+t}. \]  
(4.16)

Then, one can find the effect of the liquidity constraint factor on the (short-run) risk premium. Plugging (4.15) and (4.16) into (4.10), one can have

\[ \Omega_{t+t} = COV_t \left[ S_{t+t}, R_{s,t+t}^{-1} \right] [1 + g_t] + COV_t \left[ \xi_{t+t}, \zeta_{t+t} \right] \]

\[ -COV_t \left[ S_{t+t}, \zeta_{t+t} \right] f_{s,t+t}^{-1}. \]  
(4.17)

The RHS of equation (4.17) can be decomposed into the two parts of risk premium in the presence of the liquidity constraints: the risk premium \( \Omega^c_{t+t} \) in absence of the liquidity constraints and an additional risk premium \( \Omega^p_{t+t} \) due to the presence of risk.

\[ \text{The assumption that long-run value } \bar{\mathcal{L}} \text{ is unity is based upon historical observations that short-term values of liquidity constraint factor over the postwar period have been fluctuating around unity, as seen in figure 2.} \]
of the liquidity constraints. That is,

$$\Omega_{t+s} = \Omega_{t+s}^\circ + \Omega_{t+s}^*$$

(4.18)

where

$$\Omega_{t+s}^\circ \equiv \text{COV}[S_{t+s}, R_{s:t+s}^{-1}],$$

and

$$\Omega_{t+s}^* \equiv \text{COV}[S_{t+s}, R_{s:t+s}^{-1}] g_{t+s} + \text{COV}[s_{s:t+s}, \zeta_{t+s}] - \text{COV}[S_{t+s}, \zeta_{t+s}] s_{s:t+s}^{-1}.$$

Equation (4.18) suggests that the presence of the liquidity constraints will add to the risk premium by $$\Omega_{t+s}^*$$ . The sign of the $$\Omega_{t+s}^*$$ depends upon the signs and relative values of individual components in the $$\Omega_{t+s}^\circ$$ . However, a careful examination of equation (4.16) gives the following analysis on the effects of the liquidity constraints.

**Proposition 1 (Cyclical Effect of Liquidity Constraints on Risk)**: If the liquidity constraint factor (as defined in equation (4.1)) is above [below] its long-run equilibrium value unity, ceteris paribus, the risk premium of any risky asset tends to decrease [increase].

**Proof**: From equation (4.15), if $$E_t C_{t+s} > \bar{C}$$ , then $$g_t > 0$$ . Since the risk premium $$\Omega_{t+s}^\circ$$ of a risky asset in absence of the liquidity constraints is always negative, its risk premium in the presence of liquidity constraints will fall as $$g_t$$ rises. In the same way, if $$E_t C_{t+s} > \bar{C}$$ , then $$g_t < 0$$ . With a negative risk premium $$\Omega_{t+s}^\circ$$ of a risky asset in absence of the liquidity constraints, its risk premium in the presence of the liquidity constraints will rise as $$g_t$$ falls. Q.E.D.
Proposition 2 (Risk Premium on Risk-Free Asset) In the presence of liquidity constraints, a risk premium usually exists on any risk-free asset.

Proof: A risk-free asset in the case of no liquidity constraints has zero risk premium and \( \Omega_{t+1}^r = 0 \). However, from equation (4.18), its risk premium under liquidity constraints is not zero as long as the last two terms do not cancel each other. The implication from Propositions 1 and 2 is that changes in the income and wealth distribution can disturb the asset pricing mechanism through changes in the risk premium regardless of whether the change is forecastable. However, the effect of volatility of disturbances \( \xi_{t+1} \) in the liquidity constraint factor on the risk premium is uncertain. To show this, let \( \xi'_{t+1} \) be disturbance terms larger than the \( \xi_{t+1} \) and have the following linear relationship:

\[
\xi'_{t+1} = \sigma \xi_{t+1}, \quad \sigma < 1,
\]

(4.19)

where the \( \sigma \) is a constant and reflects heteroskedasticity of \( \xi_{t+1} \). Then, the risk premium under the higher liquidity factor volatility is

\[
\Omega_{t+1}^r = \Omega_{t+1}^r g_t + COV_t[\xi_{t+1}, \xi'_{t+1}] - COV_t[S_{t+1}, \xi'_{t+1}] f_{t+1}^{-1}.
\]

(4.20)

Plugging (4.19) into (4.20) and using the property of covariance that \( COV[\sigma X, Y] = \sigma COV[X, Y] \), one can obtain

\[
\Omega_{t+1}^r = \Omega_{t+1}^r g_t + \sigma \left( COV_t[\xi_{t+1}, \xi_{t+1}] - COV_t[S_{t+1}, \xi_{t+1}] f_{t+1}^{-1} \right).
\]

(4.21)
Since the sign of the second term of (4.21) is not known, the effect of \( \Delta \sigma \) on the risk premium in the presence of liquidity constraints cannot be determined. However, equation (4.21) suggests that the large disturbances in the liquidity constraint factor may potentially result in a high risk premium. Incorporated is the autoregressive conditional heteroskedacity in mean (ARCH-M) effects in this model, one can explain that, if the history of the \( \sigma \) showed that the high value of \( \sigma \) has persisted, the \( g_t \) tends to rise. In addition, as N. Mark (1985) pointed out, sample information available at time \( t \) can affect factors in equation (4.11) and thus the factors in (4.11) can be time-varying and in turn the risk premium \( \Omega_{t+} \) can be time-varying.

4.2 The Implications for the Risk Premium

Consider again equation (1.23). Before the parameters in (1.23) are estimated, it is convenient to express equation (1.23) by holding-period returns. With the same holding periods of any two assets (1.23) can be modified to find a testable equation of excess holding returns form.

Consider an asset whose term to maturity is \( \tau \). It is noted that term to maturity for any stock is considered to be infinite like consols. Investors may hold any asset over a desired period \( j \) shorter than \( \tau \). So \( 0 < j \leq \tau \). Denote its price at time \( t \) as \( P(t, \tau) \). The term of any asset steadily shrinks through time: for example, a three-month bond becomes a two-month bond after one month and a one-month
bond after two months. Its price and payoff such as coupons and dividends after $j$ periods are $P(t + j, \tau - j)$ and $D(t + j, \tau - j)$, respectively. The gross return (one plus the rate of return) of any asset $i$ is the ratio of the gross payoff $V_{it+\tau}$ at maturity and its today's ex-payoff price $V_{it}$. The gross return can be thus expressed as

$$R_i(t, \tau) = \frac{V_{it+\tau}}{V_{it}},$$

$$= \frac{P_i(t + \tau, 0) + D_i(t + \tau, 0)}{P_i(t, \tau)}.$$  \hspace{1cm} (4.22)

Let $H_i(t, j, \tau)$ denote $j$-period holding return of asset $i$ whose term to maturity is $\tau$ and will become $\tau - j$ after $j$ periods from time $t$. Then, the $j$-period holding return for asset $i$ can be expressed as

$$H_i(t, j, \tau) = \frac{V_{it+j}}{V_{it}},$$

$$= \frac{P_i(t + j, \tau - j) + D_i(t + j, \tau - j)}{P_i(t, \tau)}. \hspace{1cm} (4.23)$$

Then, the $j$-period excess return $e_{it+j}(\tau_1, \tau_2)$ between two assets 1 and 2 whose terms to maturity are $\tau_1$ and $\tau_2$ is

$$e_{it+j}(\tau_1, \tau_2) = H_1(t, j, \tau_1) - H_2(t, j, \tau_2).$$ \hspace{1cm} (4.24)

It is noted that stocks are treated as consols with maturity $\tau = \infty$.

Consider the ex ante form of (1.23). When any two assets, say assets 1 and 2 in the vector of gross return between time $t$ and $t+j$ among the asset vector in
are chosen, their $j$-holding period returns are
\[
E\left[ \beta^i \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha H_1(t, j, \tau_1) | I_t \right] = 1, \quad (4.25)
\]
and
\[
E\left[ \beta^i \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha H_2(t, j, \tau_2) | I_t \right] = 1. \quad (4.26)
\]
Subtracting (4.26) from (4.25) and cancelling $\beta^i$, one derives an equation of excess holding returns form
\[
E\left[ \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha \epsilon_{t+j}(\tau_1, \tau_2) | I_t \right] = 0. \quad (4.27)
\]
This is the general modified Euler equation of excess holding return form between any two different assets (or portfolio) with any different terms to maturity. Equation (4.27) can be estimated by using any nonlinear least square estimation with relevant distribution assumptions. For the estimation of deep structural parameters in this model, Hansen’s (1982) and Hansen and Singleton’s (1982) Generalized Methods of Moments will be used in this paper. GMM allows the ex post forecasting errors in equation (4.27) to be serially correlated, correlated with variables in the equation or in other equations in the system of the model, as well as to be conditionally heteroskedastic.

Consumption CAPM (CCAPM) of Lucas (1978) and Breeden (1979) seeks to illuminate the fundamental relation between the consumption plan and the portfolio plan. According to CCAPM, an asset is risky if its payoff (or excess return)
has a negative conditional covariance between the marginal rate of substitution of aggregate consumption and the excess return of that asset. The excess of the return of an asset over that of the riskless asset's return is considered to be a risk premium of that asset. Let \( r_{f_{t+1}} \) denote the (one-period-after-tax real) interest rate of a riskless asset \( f \), then its price \( V_{f_t} \) today is

\[
V_{f_t} = 1/P_f(t, 1) = 1/(1 + r_{f_{t+1}}),
\]

(4.28)

and its certain gross payoff \( V_{f_{t+1}} \) in period \( t+1 \) in terms of one unit of consumption goods in period \( t+1 \) is

\[
V_{f_{t+1}} = 1.
\]

(4.29)

Then, the one-period holding return of a riskless asset \( f \) is

\[
H_f(t, 1, t+1) = R_f(t, 1) = V_{f_{t+1}}/V_{f_t} = 1 + r_{f_{t+1}}.
\]

(4.30)

The modified Euler equation in the form of (one-period-holding) excess return between the riskless asset \( f \) whose maturity is \( \tau_f = 1 \) and any other asset \( j \) whose maturity is \( \tau_j \) is

\[
E\left[ \left( \frac{C_t}{C_{t+1}} \right)^\sigma \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+1}/C_{t+1}} \right)^\sigma e_{j_{t+1}}(\tau_j)|I_t \right] = 0,
\]

(4.31)

where \( e_{j_{t+1}}(\tau_j) = H_j(t, j, \tau_j) - H_f(t, j, \tau_f) \).

Let \( S'_{t+1} \equiv \beta^{-1}S_{t+1} = (C_t/C_{t+1})^\sigma \) and \( L_{t+n} = \left( \frac{1 - \theta Y_{t+n}/C_{t+n}}{1 - \theta Y_{t+1}/C_{t+1}} \right)^\sigma \). Then, a compact form of (4.31) is, for any asset \( j \),

\[
E\left[ S'_{t+1} L_{t+n} e_{j_{t+1}}|I_t \right] = 0.
\]

(4.32)
Decomposing the covariance in equation (4.32) conditional on $I_t$, one derives

$$
E[e_{jt+1}|I_t] = -\frac{COV[S_{t+1}^{'}L_{t+1}, e_{jt+1}|I_t]}{E[S_{t+1}^{'}L_{t+1}|I_t]}, \quad \text{or}
$$

$$
= -\frac{COV[S_{t+1}^{'}L_{t+1}, e_{jt+1}|I_t]}{E[S_{t+1}^{'}|I_t]E[L_{t+1}|I_t] + COV[S_{t+1}^{'}L_{t+1}|I_t]} \quad \text{(4.33)}
$$

The $E[\cdot]$s are always positive. The negative sign in the RHS implies that the risky asset $j$ is a good hedge against future uncertainty. The liquidity constraint factor $L_{t+1}$ must be stochastic if the liquidity constraints are binding and the ratio of (aggregate) consumption and income is stochastic. Equation (4.33) states that, conditional on $I_t$, the expected excess return or risk premium of holding an asset $j$ depends upon (a) the expected covariance between the stochastic ($\beta$-undiscounted) marginal rate of substitution and excess return, (b) the expected covariance between the stochastic ($\beta$-undiscounted) marginal rate of substitution and the corresponding weight of $L_{t+1}$, and (c) the ($\beta$-undiscounted) expected marginal rate of substitution and expected $L_{t+1}$. In the absence of the liquidity constraints, the term $L_{t+1}$ is unity and hence the $COV[S_{t+1}^{'}L_{t+1}|I_t]$ is zero. In the presence of the liquidity constraints, the term $L_{t+1}$ is stochastic and hence $COV[S_{t+1}^{'}L_{t+1}|I_t]$ is an additional determinant in risk premium. Discussion of the implications follows:

**Proposition 3 (Consumption Income Ratio and Risk)** The risk premium of holding a risky asset will increase, ceteris paribus, if the $COV[S_{t+1}^{'}L_{t+1}|I_t]$ is negative and becomes larger in absolute value; whereas, the risk premium of holding a
risky asset will decrease, ceteris paribus, if the $COV[S'_{t+1}, L_{t+1}|I_t]$ is positive and becomes larger.

Proof: Straightforwardly, a risky asset means that $COV[S'_{t+1}, L_{t+1}, e_{jt+1}|I_t] < 0$. As shown in figure 5, if $COV[S'_{t+1}, L_{t+1}|I_t] < 0$, the larger the value of $-COV[S'_{t+1}, L_{t+1}|I_t]$, the higher risk premium is. If $COV[S'_{t+1}, L_{t+1}|I_t] > 0$, the lower the value of $COV[S'_{t+1}, L_{t+1}|I_t]$, the higher the risk premium is. Q.E.D.

Proposition 3 indicates that liquidity constraints may be a nontrivial factor in the asset pricing mechanism. Along with the cycles of the relative growth rate of aggregate noncapital income to that of aggregate consumption, the risk premium may rise or fall in the presence of liquidity constraints.

Proposition 4 (Financial Regulation and Risk Premium) The risk premium of a risky asset will increase, ceteris paribus, when the income-consumption ratio $(Y/C)$ rises [falls] under the lower [higher] degree $\theta$ of the liquidity constraints (for example, possibly because of financial deregulation [reregulation]).

Proof: Differentiating the liquidity constraint factor $L_{t+1}$ in equation (4.31) with respect to $\theta$ and using approximation of $\ln(1+x) \approx x$, one can obtain

$$\frac{\partial L_t}{\partial \theta} = \left[aL_t \Delta(Y_t/C_t)\right]^{-1}. \quad (4.34)$$

From (4.34), as $\theta$ falls, $E[L_{t+1}|I_t]$ will rise if $\Delta(Y/C) > 0$ and, as $\theta$ rises, $E[L_{t+1}|I_t]$ will rise if $\Delta(Y/C) < 0$. Q.E.D.
The implication of Proposition 4 is that an increasing degree of liquidity constraints, because of financial deregulation, tends to result in a higher risk premium when the growth of aggregate income falls behind the growth of aggregate consumption that is expected to be much smoother than aggregate income. Or a decreasing degree of liquidity constraints, because of financial deregulation, tends to result in a higher risk premium when the growth of aggregate income exceeds the growth of aggregate consumption.
CHAPTER V

EMPIRICAL PROCEDURES

5.1 Derived Orthogonality Conditions

Consider the equation (4.25) for bonds at any maturity regardless of whether the maturity is smaller or larger than the holding period:

\[ E \left[ \beta^i \left( \frac{C_i}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_{i+j}}{1 - \theta Y_{t+j}} \right)^\alpha H(t,j,\tau|\tau) \right] = 1, \quad \forall j, \quad (5.1) \]

where \( H(t,j,\tau) \) is \( j \)-period holding return of a bond maturing at \( \tau \). If the maturity is smaller than the holding period, \( H(t,j,\tau) \) represents rolling over returns over \( j \) periods. Theoretically, it is possible to employ any combination of equations of different maturity in order to make a system and then conduct an empirical procedure. A system estimation approach with multiple equations, based upon this model, would produce a highly efficient estimator but it would incur prohibitive costs. When instrumental variable estimation procedures are applied, the overidentifying restrictions are more likely to be violated as the number of equations in a system increases. This is because the higher number of equations must accompany an ever higher number of orthogonality conditions when there are multiple
instrumental variables chosen and at time time the number of parameters to estimate remains unchanged or modestly increases. In practice, it is found that a single equation estimation can produce reasonably efficient estimates with good instruments. Accordingly, this paper will conduct empirical procedures based upon a single equation for return from holding a bond at any maturity. Consider the rate of return \( r_{t+j} \) of \( j \)-period holdings of any term bond. Then equation (5.1) can be rewritten as

\[
E \left[ \beta^j \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha (1 + r_{t+j}) \right] = 1, \quad \forall j. \tag{5.2}
\]

It is assumed that consumption growth, the liquidity constraint factor growth, and returns are stationary and jointly lognormally distributed. The linearized \( \text{a posteriori} \) form of the above equation with approximation is

\[
\left( \frac{C_{t+j}}{C_t} - 1 \right) = \mu_j + \theta \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right) + \sigma r_{t+j} + \nu_{t+j}, \quad \forall j, \tag{5.3}
\]

where \( \sigma = 1/\alpha \) is intertemporal elasticity of substitution,

\[
\mu_j = \frac{\alpha}{2} \ln \beta + \frac{\alpha}{2} (\sigma_v^2 + \sigma_r^2) + \frac{\alpha}{2} \sigma_r^2 + \frac{\alpha}{2} \sigma_v^2 - \frac{\alpha}{2} (\sigma_r^2 + \sigma_v^2) \text{ is assumed constant, and}
\]

\( \nu_{t+j} \) are disturbance terms.\(^1\)

Under the lognormality assumption, intertemporal optimality conditions in the

---

\(^1\text{Proof. When both sides of (5.2) are taken in natural logarithm, it becomes}
\)

\[
\ln E_t \left[ \beta^j \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha (1 + r_{t+j}) \right] = E_t \left[ \beta^j \left( \frac{C_t}{C_{t+j}} \right)^\alpha \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right)^\alpha (1 + r_{t+j}) \right] + \frac{\alpha}{2} \ln \beta + \alpha E_t \ln \left( \frac{C_t}{C_{t+j}} \right) + \frac{\alpha}{2} \ln \left( \frac{1 - \theta Y_t/C_t}{1 - \theta Y_{t+j}/C_{t+j}} \right) \ln (1 + r_{t+j}) + \frac{\alpha}{2} \sigma_r^2 + \frac{\alpha}{2} \sigma_v^2 + \frac{\alpha}{2} \sigma_r^2 - \frac{\alpha}{2} (\sigma_r^2 + \sigma_v^2) = 0, \quad \forall j, \text{ which can be approximated and rearranged to arrive (5.3). Barsky (1985) suggests that heteroskedasticity may result in the rejection of the model.}\]
presence of the liquidity constraints appear as a linear relation of aggregate consumption growth, the liquidity constraint factor growth, and \textit{ex post} return rates from bonds over the holding period. Any researcher may argue that model fitting requires \textit{ex ante} return rates that reflect the consumer's decision time to the future, for example, C. Harvey (1988). However, as Mankiw (1981) pointed out, when instrumental variable estimation procedures are used for empirical implementation, the procedures support the appearance of \textit{ex post} return rates. According to Mankiw (1981), the fitted values of \textit{ex post} return rates from the first stage appear similar to \textit{ex ante} return rates on the ground that the fitted values are a projection of return rates onto the instrument set. The fitted return rates are not necessarily required to be the same as the actual forecasted values by consumers since consistency involving instrumental variables estimation (IV) procedures is not based upon the actual forecasted values. This point is an advantage of IV procedures, especially for the model estimation such as (5.3). It means that a researcher does not have to find actual forecasted values of the return rates with the burden of looking for their accuracy. It is noteworthy that IV procedures allow a researcher to rely on a subset of an unknown information set and it is a matter of the efficiency of the estimator whether a subset of instruments represent the unknown information set well.

As will be discussed in the next section, however, it turns out that data imperfection causes the disturbance terms in (5.3) to be serially correlated and thus the
disturbance terms to be correlated considerably with the RHS variables. Consequently, this data problem jeopardizes the IV procedures. For circumventing the problem, Hansen's (1982) generalized methods of moment (GMM) procedures will be used in empirical estimations. The GMM procedures allow this kind of serial correlation and conditional heteroskedasticity.

For illustrative purpose for the time-being, assume that there are no data problems. If no data problems are involved in the empirical procedures, the $j$-period ahead forecasting errors $\nu_{t+j}$ should be orthogonal to the predetermined variables at time $t$. This can be expressed in the form of

$$E\left[\nu_{t+j} \left( \frac{C_t}{C_{t+j}}, \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right), r_{t+j} \right) \mid I_t \right] = 0, \quad \forall j. \quad (5.4)$$

If an instrument set $z_t$ is a subset of information set $I_t$, it follows that

$$\text{COV}\left[\nu_{t+j} \left( \frac{C_t}{C_{t+j}}, \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right), r_{t+j} \right), z_t \mid I_t \right] = 0, \quad \forall j, \quad (5.5)$$

hence the orthogonality conditions

$$E\left[\nu_{t+j} \left( \frac{C_t}{C_{t+j}}, \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right), r_{t+j} \right) \otimes z_t \mid I_t \right] = 0, \quad \forall j, \quad (5.6)$$

where $\otimes$ is the Kronecker product, must on the data.

Another point that I shall discuss is that the purpose of the null hypothesis that $H_0 : \theta = 0$ is to test the existence of rule-of-thumb consumers, but not to test the existence of liquidity-constrained consumers. Since the rule-of-thumb is defined more broadly than liquidity-constrained is, the alternative hypothesis
is not well defined. For instance, the alternative hypothesis that $H_1 : \theta \neq 0$ tests the existence of myopic consumers (a violation of Rational Expectation), the existence of liquidity constrained consumers, or both. This nesting problem will be confronted in GMM procedures when overidentifying restrictions are rejected. In order to identify which is the cause of the null hypothesis to be rejected among the alternatives, I will estimate the parameter $\theta$ over the different interval spectra and conduct quasi-likelihood ratio tests.

5.2 Applications of the GMM Procedures

Hall's (1978) Random Walk Theorem for consumption is based upon the assumption that real interest rates are constant. Since then, a growing number of researchers have reported that data do not support the theorem. As possible reasons that Hall's theorem fails in the data: the existence of liquidity-constraints in loans markets (Flavin (1981), Hayashi (1982), Zeldes (1989), Campbell and Mankiw (1989, 1990), Campbell and Zeldes (1990)); Myopic agent (Flavin (1983)); Stochastic real interest rates (Mankiw (1981), Michener (1984), Christiano (1987)); and durability in consumer goods (Mankiw (1985), B.Bernanke (1985)). The models with separable preferences in this paper can easily accommodate the issue of the constant real interest rate in conjunction with liquidity constraints. Thus, following Campbell and Mankiw (1989), I will also see if Hall's argument that intertemporal elasticity of substitution is close to zero or in fact zero is robust to
the appearance of the liquidity constraints. Define the \( j \)-period ahead forecasting error as

\[
h(z_{t+j}, \gamma) \equiv \nu_{t+j},
\]

(5.7)

where \( x_{t+j}' = \left\{ \left( \frac{C_{t+j}}{C_t}, \Delta(\frac{Y_{t+j}}{C_{t+j}}) \right) \right\} \) and \( \gamma' = \theta \)

with constant real interest rates;

\[
\left\{ \left( \frac{C_{t+j}}{C_t}, \Delta(\frac{Y_{t+j}}{C_{t+j}}), r_{t+j} \right) \right\} \) and \( \gamma' = (\theta, \sigma) \)

with stochastic real interest rates.

The orthogonality conditions can be rewritten as

\[
E[h(x_{t+j}, \gamma) \otimes z_t | I_t] = 0, \quad \forall j.
\]

(5.8)

Then, the sample counterpart \( G_T \) of (5.8) can be expressed as

\[
G_T(\gamma) = \frac{1}{T} \sum_{t=1}^{T} [h(x_{t+j}, \gamma) \otimes z_t], \quad \forall j,
\]

(3.9)

which is made as close to zero as it is possible by choosing the \( \gamma \). Hansen's (1982) GMM procedures solve this problem by minimizing the quadratic criterion form

\[
J_T(\gamma) = G_T(\gamma)'W_T^*G_T(\gamma),
\]

(5.10)

where \( W_T^* \) is an optimal weighting matrix that is symmetric and positive definite.

The search for an optimal weighting matrix is very important for the models in this paper since the forecasting errors in the orthogonality conditions are to a great extent serially correlated because of data imperfections. The following section will discuss in detail on how to determine the weighting matrix for actual empirical implementation.
The sample size $T$ times the resulting minimized value of the quadratic form $J_T$ is asymptotically distributed $\chi^2$ under the null hypothesis with degrees of freedom equal to the number of orthogonality conditions minus the number of parameters. This $\chi^2$ statistic provides a test of overidentifying restriction. When the model is overidentified, the remaining linearly independent orthogonality conditions are set to zero if the model restrictions are true. This is the idea behind the overidentifying restriction test.

5.3 Data Sources and Data Constructions

The empirical results reported in this paper are based on quarterly data from 1953:I through 1986:IV. Data on the U.S. consumption and labor income are constructed from the Citibase tape. Data on interest rates come from H.McCulloch's zero coupon yield curves series (1987, 1990). Like Campbell and Mankiw, the sample period before 1953 is truncated in order to avoid the disrupted influence of the one-time National Service Life Insurance benefits to World War II veterans and the Korean War. Also truncating the sample period by 1982:IV allows the empirical results in this paper to be compared with Campbell and Mankiw's results.

Aggregate consumption is (seasonally adjusted) nondurable ($GCN82$) plus services ($GCS82$) less clothing and shoes ($GCNC82$) in 1982 dollars.\footnote{Definitions in parentheses are the code names in the Citibase tape.} Clothing and shoes are excluded in the measure of aggregate consumption because, with
one-quarter separable preference in the model, these categories represent *de facto* durables and, by doing so, the data noise might be reduced. The price index is the implicit deflator associated with the measure of consumption.

Net aggregate labor income is constructed in the following way: Nominal labor income \( W \) is defined as wage and salary disbursement (\( GW \)) plus other labor income (\( GPOL \)). In addition, nominal nonlabor income \( NW \) is the sum of rental income (\( GPRENJ \)), personal dividend income (\( GPDIV \)), and personal interest income (\( GPINT \)). Let \( \kappa \) represent the fraction of \( W/(W + NW) \). Net tax payment \( T \) is personal contribution for social insurance (\( GPSIN \)) plus \( \kappa \) times personal tax and nontax payments (\( GPTX \)) less transfer payments (\( GPT \)). Then, net labor income with a fraction \( \kappa \) of proprietors' income (\( GPROJ \)) is \( W + \kappa GPROJ - T \), which is converted into real terms by deflating using the consumer price index. Consumption and net labor income are divided by the mid-quarter population (\( GPOP \)) to arrive at per capita measures.

Holding period returns are constructed from the McCulloch zero coupon bond yield series. For real holding period returns, the implicit price deflator for consumption is used. To illustrate in detail, let \( P_d(t, \tau) \) denote the pure discount bond price with maturity \( \tau \) at time \( t \). The real returns of zero-coupon-bearing bond, a

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*Transfer payments include government unemployment insurance, benefit, veterans benefits, and government retirement benefits.

*The McCulloch data include most of marketable U.S. government bills, note, and bonds. According to McCulloch, the data were obtained by fitting a discount function, the present value of a future dollar, with a cubic spline. Tax-induced bias was removed during his curve-fitting (1975b)
promise by the issuer to pay to the holder one dollar at the specified date $\tau$, is

$$
\left( \frac{1}{P_d(t, \tau)} \right) / \left( \frac{P_t}{P_\tau} \right),
$$

(5.11)

where $P_t$ denotes the implicit price deflator for consumption at time $t$. The continuously compounded yield to maturity $r_d(t, \tau)$ is given by

$$
-(1/\tau) \ln P_d(t, \tau).
$$

(5.12)

Then, the logarithm of real return to maturity $\tau$ at time $t$ is

$$
\tau r_d(t, \tau) - \ln(P_\tau/P_t).
$$

(5.13)

In the same way, the logarithm of real return to maturity $\tau - j$ at time $t + j$ is

$$
(\tau - j) r_d(t + j, \tau - j) - \ln(P_{\tau+j}/P_t).
$$

(5.14)

The real holding period returns $H_d(t, j, \tau)$ of bond maturing $\tau$ from $t$ to $t + \tau$ is, therefore,

$$
h_d(t, j, \tau) = \ln(P_{\tau+j}/P_t),
$$

(5.15)

where

$$
h_d(t, j, \tau) = \begin{cases} 
\tau r_d(t, \tau) - (\tau - j) r_d(t + j, \tau - j), & \text{if } 0 \leq j \leq \tau, \\
\sum_{k=0}^{j-1} m r_d(t + km, t + km + m), & \text{if } 0 \leq \tau \leq j,
\end{cases}
$$

where $m = \tau - t$ and $s = \max\{\text{integer}(\frac{\tau - t}{m})\}$. The monthly yields over the quarter are averaged to match with quarterly-averaged consumption and the marginal tax rate for the interest rates is assumed to be 0.3.
5.4 Instrumental Variables and the Weighting Matrix under Data Problems

The stationarity and the ergodicity of both data and instruments are a part of Hansen's (1982) sufficient conditions for the GMM estimator to converge almost surely (p.1032). In this section, with regard to these points, time-series of the data will be examined.

Table 1 presents the sample statistics and the first twelve sample autocorrelations in the case of a one-period interval. As seen in the table, the sample autocorrelations of consumption growth damp out after lag 4 with an exception of an aberrant value at lag 8. This suggests that the consumption growth series are stationary. The memory of the series turns out to be strong over the first three quarters and reflects data noise in the observations. A series in row 2 shows log-approximated values of liquidity constraint factor growth in row 3 and will be used for the liquidity constraints factor growth. Its sample autocorrelations indicate that the series is not serially correlated with its past observations and is stationary. Also its sample characteristics remarkably mimic the before-approximated series' sample characteristics.

The sample means and sample standard deviations of holding period returns over one quarter appear to rise as the bond maturity increases. The sample kurtosis of the returns is quite high, suggesting that the sample distribution of the returns

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*Sample statistics and autocorrelations based on nondurables only (ND) are qualitatively not different from those based on the NDS.*
might be fat-tailed. Is such, it would be expected that the estimates are unstable under the maintained assumption of lognormality.

When the variables of multiperiod intervals are sampled at a frequency of one period, the neighboring observations themselves are overlapped by as much as the difference of the interval less one in every interval. Thus the strength of memory that is observed in one period specification tends to increase approximately as much as the difference. Sample statistics and sample autocorrelations are displayed in table 2 for two period specification and in tables A.1 through A.4 in appendix D for multiperiod specifications. The tables show that generally the strength of past memory tends to rise as interval increases. For example, as seen in table 2, the sample autocorrelations of two-period consumption growth damp out after lag 5.

The intertemporal optimality of an agent must be reflected in an ordinary stochastic Euler equation. In a stochastic environment, rationality of an agent requires that the agent utilize all information available. An important merit of the Euler equation approach is that it does not require a complete and explicit specification of the entire economy, for example, preferences, productivity, and stochastic forces from exogenous variables. In IV estimation procedures, an instrumental variable set is formed as a subset of the information pool that a rational agent can obtain and learn. There might be many potential candidates available for an instrument and, theoretically, any stationary variable in the information set is an

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*A large sample is required for this conclusion, though.*
eligible instrument. Thus, a choice problem may be confronted. An axiom for this is looking for the IVs that are not correlated with the disturbance terms and, at the same time, have good forecasting power in explaining the RHS variables in the equation. The lagged values of the variables in the equation are usually the first choice among the possible IVs since the consumer's optimal conditions have held over the past and thus must contain information about the IVs. But the literature has included other lagged variables that do not appear in the Euler equation when they are believed to fulfill the axiom of IV choice. Since the modified Euler equation in this paper has a component of income in the liquidity constraint factor, the liquidity constraint factor may be considered to reflect a channel of productivity shocks as well. It seems that a set made of the lagged values of the variables in the modified Euler equation could well represent an IV set and thus the IVs chosen in this paper solely come from the Euler equation.

Before turning to the actual estimation, it is necessary to discuss the lag timing of the instruments in relation to the data problem. The orthogonality condition in GMM procedures requires that any IV be not correlated with the disturbance term $\nu_{t+1}$. If data have problems such as time-averaging, aggregation measurement error, and even durability in service flow, these problems will make GMM procedures inappropriate. As Working (1960) pointed out, when data which are strongly believed to follow a random walk are time-averaged over the interval, time-averaging will result in a spurious first-order auto- and cross-correlation in the constructed
data set.\textsuperscript{7} In addition to the averaging problem, the observed stochastic variable follows a first-order moving average MA(1) process when it includes a component of white noise error, mostly caused by measurement with random errors or created by unspecified preference shocks. A property of the MA(1) process is that its memory is only one period long.\textsuperscript{8} Furthermore, when the variable is \textit{de facto} durable consumer good over its interval, with the assumption of constant interest rates, it also follows a MA(1) process.\textsuperscript{9}

Because of the data problems listed above, the first lag is not qualified as an instrument. The relevant method that circumvents the problems is taking the lag at time $t - 2$ and earlier. This method legitimates the GMM procedures but the recent information loss may not be trivial. The information loss by this method is very high if financial markets are efficient. In this case, the forecasting


\textsuperscript{8}To illustrate this, consider the logarithm of observed consumption $C_t$,

$$\ln \tilde{C}_t = \ln C_t + \zeta_t,$$

where $\zeta_t$ is the white noise measurement error. Then consumption growth can be shown as

$$\Delta \ln \tilde{C}_{t+1} = \Delta \ln C_{t+1} + (\zeta_{t+1} - \zeta_t),$$

from which the autocorrelation function $\rho_j$ can be derived as

$$\begin{cases} 
\rho_1 = -0.5 \\
\rho_j = 0, \quad \text{for } j > 1.
\end{cases}$$

\textsuperscript{9}See Campbell and Mankiw (1990).
power would drastically decrease. Furthermore, the information loss must be even more severe for multiperiod interval specification since the instruments should be predetermined. For example, the most recent acceptable lag of consumption growth \( \left( \frac{C_{t+j}}{C_t} \right) \) of \( j \)-period interval is \( \left( \frac{C_{t-2}}{C_{t-j-2}} \right) \) rather than \( \left( \frac{C_{t+j-2}}{C_{t-2}} \right) \).

Next, it is necessary to discuss the question of how many lags are desired. In principle, adding an extra recent lag can help increase forecasting power of the IV set if it can summarize its history well. However, as the number of instruments marginally increases, the number of the orthogonality conditions will increase by the number of its increase times the number of the system's equations. Consequently, with a fixed number of parameters, the overidentifying restrictions tend to be more frequently rejected, in addition to which computational cost increases. Thus, it is required that the number of instruments should be carefully determined. Furthermore, as the lag length excessively increases, the marginal increase in forecasting power will be actually negative.

Nelson and Startz (1990), using a simple equation, showed that, if the IV is weakly correlated with the explanatory variable, the small sample distribution of the IV estimator tends to concentrate not around the true value but around an inverse value of the feedback from the dependent variable to the explanatory variable, and also that of the variance of residuals tends to concentrate around a value that reflect the strength of the feedback.\(^{10}\) They suggested two approaches

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\(^{10}\)For detail, see Nelson and Startz (1990)
of diagnostic checking against the concentration phenomenon. The first is to apply
the inequality criterion that they designed and examine whether the criterion is
satisfied under the obtained sample correlation and the sample size. The second is
to examine the significance of the first-stage regression. However, their diagnostic
checking methods will not work well when there are multiple explanatory variables.
Since their suggestions are the only theory available in the literature for checking
the relevance of IV, I will use the second checking method.

It is important to specify correctly the weighting matrix for the GMM pro-
cedures when data problems cause disturbance terms to be not independently
distributed. Newey and West (1987) showed that, based on an initial suboptimal
choice of a weighting matrix, an optimal weighting matrix $W^*_T$ can be computed
in the following form such that

$$ W^*_T = \left[ W_T(0) + \sum_{k=1}^{m} \omega(k)[W_T(k) + W_T(k)'] \right]^{-1}, \quad \forall j, \quad (5.16) $$

where

$$ W_T(k) = \left( 1/T \right) \sum_{t=k+1}^{T} \left[ h(x_{t+j}, \gamma_T) \otimes z_t \right] \left[ h(x_{t+j}, \gamma_T) \otimes z_t \right]' \left( \omega(k) = 1 - k/(m + 1) \right), $$

is the modified Bartlett weights to smooth the sample autocovariance function and make the weighting matrix positive definite, and the bound $m$ is a maximum lag length of sample autocovariances.\textsuperscript{11} The bound $m$ is equal to the order of the MA disturbance terms $\nu_{t+j}$.

\textsuperscript{11} The weighting matrix for the first step is based on starting values rather than it is a full rank identity matrix since the former method, in which autocorrelation is corrected, gave better results in terms of low criterion values than the latter method did.
When the time interval in variable specification is $j$ periods and actual observations have one-period frequency, the $j$-period prediction errors will be overlapped to the extent that the interval exceeds the sampling frequency. Then, the disturbances are likely to follow a high-order MA process. Dunn and Singleton (1986) reported that $j$-period holding returns follow a MA($j - 1$). For example, when the time interval is two periods and the sampling frequency is one period, the disturbance terms follow a MA(1).

The data overlapping problem is a drawback of the multiperiod specification. However, the multiperiod specification has a positive aspect. That is, as the interval increases, the data of one-period sampling frequency will become those measured at points in time. This aspect will mitigate the time-averaging problem in data. Furthermore, as C. Harvey (1988) indicated, if published data suffer the problem of stochastic measurement error in variables, the signal to noise ratio would be much higher in the case of multiperiod specification than that of one-period specification since the measurement errors in one period data are more likely to cancel out when the time interval increases. This effect would occur most often in panel data that typically suffer large measurement errors in sampling, and thus the multiperiod model specification would be more appealing for panel data studies. Finally, the effect of a high signal-to-noise ratio might mitigate durability problem in time-separable preferences. There are de facto durables such as clothing and shoes in the category of nondurables. The aggregate consumption constructed
for empirical study here excludes these de facto durables from nondurables. However, this exclusion cannot make the data immune from the durability problem since a significant portion of nondurables and services generate a time-discounted flow of utility service over time. A good example for the durability of consumption is a last happy vacation on a Bahama Islands beach. The happy memory there has lasted long over many quarters or even over years. The durability in consumption is obviously a major drawback of separable preferences. The multiperiod specification may also help reducing this problem although it cannot eliminate the problem.

Hansen and Singleton (1983) showed that, with the underlying assumption that no data problems exist, the bound \( m \) in (5.16) is equal to \( j - 1 \) for \( j \)-period specification. However, as discussed so far, if data problems exist and the data problems add an MA(\( q \)) process in disturbance terms, the bound \( m \) now becomes \( j + q - 1 \). The sample autocorrelation of one-period specification, in which \( j = 1 \) and thus no moving average effect from multiple periods exists, shows that the consumption series follows a MA(\( q = 3 \)) process. Accordingly the bound \( m \) is set to \( j + 2 \) for the actual weighting matrix.\(^{12}\) Therefore, the actual weighting matrix used here can be written as

\[
W_T^* = \left[ W_T(0) + \sum_{k=1}^{j+2} \omega(k)[W_T(k) + W_T(k)'] \right]^{-1}, \quad \forall j
\]  

\(^{12}\)Newey and West (1987) suggested that the bound \( m \) is a cubic root of sample size. However, it cannot be applied in the specification here.
For example, when the time interval is one, it is

$$W_T^2 = \left[ W_T(0) + \sum_{i=1}^3 \omega(k) [W_T(k) + W_T(k)'] \right]^{-1}$$

where \( \omega(k) = 1 - k/3 \).

5.5 Empirical Results

For practical empirical work, similar to the approach used by Campbell and Mankiw (1989, 1990), three different specifications of the model are used: first, a simple specification is formulated in which only the liquidity constraint growth appears in the RHS of the equation, under the assumption that real interest rates are constant, as suggested by Hall; second, a simple specification is formulated in which only the interest rates or the rates of holding period returns appear in the RHS of the equation under the assumption that no liquidity constraints exist in the loan market; third, a specification is formulated in the joint examination form in which both the liquidity constraint growth and the real interest rates appear in the RHS of the equation. In each model specification, a one-period specification is examined with different IV sets and multiperiod specification models are examined with the same IV set. The IVs employed in each model are chosen directly from the equation since it is believed that the first-order conditions themselves for consumers' intertemporal optimal allocation can reveal information on the future optimal conditions and may even reflect productivity shocks through the channel of the liquidity constraints. In the procedures to attempt to examine the forecasting power of each IV set, the adjusted \( R^2 \) is obtained by applying vector autoregressive
estimation and a Wald test is applied. Then, the model parameters are estimated by using the Hansen's GMM procedures and the overidentifying restrictions are tested.

5.5.1 The Case of Constant Interest Rates

Consider the modified Euler equation in the case that real interest rates are constant. This specification investigates whether Hall's RWH failure with the aggregate data can be attributed to the fact that a fraction of the population is liquidity-constrained or myopic. The empirical results of the multiperiod specification are expected to provide grounds for judgment on whether consumers are liquidity-constrained or myopic.

One-Period Specification

In the case of constant real interest rates, the equation (5.3) in section 1 becomes

\[
\left( \frac{C_{t+1}}{C_t} - 1 \right) = \mu_1 + \theta \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) + \nu_{t+1},
\]

(5.18)

where \( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) \equiv \frac{Y_{t+1}}{C_{t+1}} - \frac{Y_t}{C_t} \) is the liquidity factor growth. Table 3 reports the empirical results for this case.\(^{13}\) The first column in the table displays the instrument

\[^{13}\text{There was no difference in estimation results and inferences between the this form of equation and the form of equation before approximation,}\]

\[
\Delta \ln C_{t+1} = \mu_1 + \theta \Delta \ln(1 - \frac{Y_{t+1}}{C_{t+1}}) + \nu_{t+1}.
\]

Also there was no difference between \( z_t - 1 \) or \( z_t \) and \( \ln z_t \), as the form of IV since \( z \) is very small number. A constant term is always included in the instrument set. The GAUSS program was used for the GMM procedures and the Newey and West's (1987) maximum lag length in
sets. The second and third columns show the adjusted $R^2$ statistics of the consumption growth equation and the liquidity factor growth equation, respectively. These statistics were obtained by applying a vector autoregressive estimation. The values in parentheses under these statistics are the marginal significance levels of the statistics. The marginal significance levels were obtained by applying a Wald test on the null hypothesis that all coefficients except the constant term are jointly zero. These are presented to check the forecasting power of the IVs.\footnote{This approach comes from Campbell and Mankiw's (1989, 1990) unpublished working papers.} This consideration responds to the problem that, as Nelson and Startz (1988) indicated, IV procedures are vulnerable to weak explanatory power of IVs, coupled with a small sample size. The following two columns report the GMM estimates of parameters for the constant term and the liquidity constraint factor growth term. The asymptotic standard errors are reported in parentheses under the estimates. The final three columns display the statistics of overidentifying restriction tests. The reason that the maximum lag lengths of IVs are 4 or 6 is that the results of preliminary estimations indicated that, when an additional lag of IVs is included in the instrument set, the prediction power of the instruments increases and reaches its maximum at the lag length around 4 to 6 and then decreases. 

As seen in the first two rows, the lags of consumption growth fairly well predict the weighting matrix is 3. The weighting matrix for the first step estimation is obtained by using starting values rather than it is (non-autocorrelation-corrected) identity matrix since the former matrix produced better results in terms of low criterion values than the latter matrix did. The starting values are 0.0 for $\rho_1$ and 0.3 for $\theta$, respectively.
their own growth. The adjusted $R^2$s are 5.7 percent for lags 2 through 4 and 7.1 percent for lags 2 through 6 and statistically significant at conventional levels when a Wald test is conducted, suggesting that consumption does not follow a random walk. Campbell and Mankiw (1989) reported a similar forecasting powers (2.4 percent for lags 2 through 4 and 8.1 percent for lags 2 through 6). The lags of consumption growth also predict well the liquidity constraint factor growth. The adjusted $R^2$s in this case are 7.1 percent for lags 2-4 and 10.6 percent for lags 2-6. However, the lags of the liquidity constraint factor growth do not predict their own growth for lags 2-6 but predict well their own growth for lags 2-4 by as much as 9.5 percent, indicating that the forecasting power of the lags reaches its maximum at around lag 4 or 5 and falls as the number of the lags increases. It is unclear at what lag the maximum of forecasting power occurs in Campbell and Mankiw's report since they did not indicate this point. The only thing that a reader can know from their paper is that the forecasting power of each IV set increases as the number of IVs increases up to lag 6.

An interesting finding from columns 2 and 3 is that past consumption growth predicts the liquidity constraint growth more powerfully than the past liquidity constraint growth predicts itself. This finding, which is equivalent to Campbell and Mankiw's finding in which they considered income growth in place of the liquidity constraint growth here, is very suggestive of the possibility of the existence of rule-of-thumb consumers. In particular, this causality implies that a sig-
significant fraction of rational consumers have voluntarily or involuntarily reflected every consumption-denominated income change (the liquidity constraint change) in their consumption while consumption-denominated income follows a borderline stationarity and thus is unable to predict itself. The last two rows display the results when the lags of both IVs are used. The forecasting performance of the IV set is better than that of the IV sets in the first four rows. The adjusted $R^2$ is almost the sum of that in the two IV sets above, suggesting that the IV set in these rows will provide reliable estimation results and protect them from the problems of small sample.

As shown in the last two rows in the columns 4 and 5, the estimates of 0 are 0.37 for lags 2-4 and 0.27 for lags 2-6, and are four times as large as their asymptotic standard errors. The results mean that 27 to 37 percent of aggregate labor income accrues to consumers who are liquidity-constrained or myopic. The results of the overidentifying restriction tests show no evidence against the model. Therefore, the first model in this study strongly supports the hypothesis that two groups of consumers exist in the U.S. economy: the permanent income consumers and the rule-of-thumb consumers. Another potential important finding is that the estimates of 0 seem to be closer to the magnitude of 0 that Hall supposed to be in the U.S. economy (20 percent) than those in Campbell and Mankiw papers.

\[15\] It has a root close to the unit circle. Mankiw and Shapiro (1985) discussed on borderline stationarity. In such case, the asymptotic theory justifying standard test procedures is misleading for small sample size (p.170).
Campbell and Mankiw (1989, 1990) reported, in their cross country study, that a 50 percent of the U.S. consumers are in the category of the rule-of-thumb consumers and even that 100 percent of French consumers are in this category. Furthermore, the estimates in this study are close to those in many household panel data studies, those estimates being around 20 to 30 percent.

Multiperiod Specification

The multiperiod specification provides an opportunity to investigate whether the results in the one-period specification are robust to long-interval alternative specifications. In addition to this merit, the multiperiod specification can identify the cause of the empirical failure of the RWH. Another possible reason for the empirical failure of Hall's RWH is myopic behavior of consumers. Of the two most likely reasons for excess sensitivity of consumption to income, while the liquidity-constrained consumers cannot optimally execute their intertemporal schedules, myopic consumers lack the ability to look forward.

M. Flavin (1985) included the unemployment rate as a proxy for liquidity constraints and drew a conclusion that U.S. consumers are not myopic but liquidity-constrained. However, the multiperiod approach can directly test whether consumers are myopic by investigating and comparing the estimation results of each time interval. The multiperiod version of equation (5.18) is

$$\left( \frac{C_{t+j}}{C_t} - 1 \right) = \mu_j + \theta \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right) + \nu_{t+j},$$

(5.19)
where $\Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right) \equiv \frac{Y_{t+j}}{C_{t+j}} - \frac{Y_t}{C_t}$ is the liquidity factor growth between $t$ and $t+j$. Table 4 reports the estimation results. The first column displays the time interval $j$, where $j$s are one quarter, two quarters, three quarters, one year, two years, and three years. The maximum lag length on the weighting matrix are $j + 2$ in order to correct serial correlation in disturbance terms.

As the interval increases, the prediction power of the IV set for consumption growth quickly falls and becomes statistically insignificant while that for the liquidity constraint factor growth does not change and remains significant at conventional significance levels. There are two possible explanations for this phenomenon: the first is that data problems such as overlapping observation and time-averaging problems are responsible for this result. But this explanation is not appealing because the result that the loss of forecasting power of IVs is increasingly larger as the time interval widens is incompatible with the fact that the signal-noise ratio becomes higher, and thus data problems should not be expected to be most serious in long-periods. The second is that the random walk property of consumption increasingly appears as the time interval increases. Especially when the interval is longer than two years, the adjusted $R^2$ becomes zero and negative. This suggests that the IVs cannot predict consumption growth. Therefore, the empirical results and inferences for intervals longer than two years must be cautiously interpreted. However, since the IVs appear to be able to predict the RHS variables well, the presence of the random walk property itself in the first step estimation may not
jeopardize the GMM estimation procedures.

As shown in the fourth column, the estimates $\hat{\mu}_j$ of the constant terms become larger as the time interval increases and are statistically highly significant. This suggests that $j$ and the variances and covariances in the constant terms tend to rise as the interval increases. The estimates $\hat{\theta}$ of the fraction of income that is received by liquidity-constrained consumers are shown as statistically significant at conventional significance levels with the exception of only three years. The magnitude of $\hat{\theta}$ ranges 27.6 percent to 30.4 percent if the intervals longer than two years are excluded and 19.3 percent to 42.2 percent for all intervals. These results are remarkable since the estimates for one quarter to one year are quite reasonable in magnitude.

The results of the overidentifying restriction tests indicate that the model is never rejected. There is no evidence of myopic behavior on the part of U.S. consumers in that the results of the tests over all intervals reject the model. Especially, if consumers were myopic, then consumers could not achieve their long term allocation of resources optimally and thus it would be expected that the model for short intervals would not be rejected but that for the long intervals would be rejected.
5.5.2 The Case of Stochastic Interest Rates

One-Period Specification

Stochastic interest rates have been received much attention over last decade in explaining the consumption CAPM in the literature: Lucas (1978), Breeden (1979), Mankiw (1981), Grossman and Shiller (1982), Hansen and Singleton (1983), Campbell (1986), Dunn and Singleton (1986), Mankiw and Shapiro (1986), Breeden, Gibbons and Litzenberger (1986), Harvey (1988), Wheatley (1988).\footnote{Binder and Deaton (1985) paid attention to other variables such as wealth shocks and inflation shocks in explaining the intertemporal decision of consumers.} For instance, Michener (1984) explains consumption responsiveness to the interest rate change in a general equilibrium framework. C. Harvey (1988) argues that expected real interest rates forecast consumption growth better than lagged consumption or real stock returns in consumption CAPM. On the contrary, Hall (1988) still insists that there is little or no information in real interest rates by which to predict consumption growth. Thus, for empirical study of the sensitivity of consumption to interest rates, interest rates are now assumed to be stochastic. Before turning to the more general model that contains both liquidity constraint growth and the interest rates in RHS of the equation, consider a model that contains only real interest rates under the assumption that no liquidity constraints exist in loan markets. The relevant equation for one-period interval is now

\[
\frac{C_{t+1}}{C_t} - 1 = \mu_1 + \sigma r_{t+1} + \nu_{t+1}, \tag{5.20}
\]
where $r_{t+j}$ is *ex post* real interest rates and $\sigma$ is the intertemporal elasticity of substitution for permanent income consumers. The nominal interest rates are the average yields three-month zero-coupon bonds over one quarter.\(^1\) The IVs are in this case the lags of either consumption growth and/or real interest rates, and the first recent lag is $t - 2$ for all IVs.

Table 5 reports the empirical results. As shown in the second and the third columns, the IVs predict consumption growth and interest rate equations well. All of the estimates $\hat{\sigma}$ of the intertemporal elasticities of substitution are statistically significant at conventional significance levels. When both IVs are used, the estimates are around 0.158 to 0.198, implying that the coefficient of intertemporal elasticity of substitution is not high. However, the high chi-square statistics in the overidentifying restriction tests suggest the model is rejected at the 5.5 percent significance level for lags 2-4 and at the 10.5 percent level for lags 2-6.

**Multiperiod Specification**

The multiperiod version of equation (5.20) is

$$\left( \frac{C_{t+j}}{C_t} - 1 \right) = \mu_j + \sigma r_{t+j} + \nu_{t+j}, \quad j = 1, 2, 3, 4, 8, \text{ and } 12,$$

where $r_{t+j}$ is now *ex post* real rate of returns from holding bonds from time $t$ to time $t + j$.

\(^1\)The interest rates are time-averaged since the consumption in the LHS of the equation is time-averaged.
It is important to specify correctly timing of the IVs. What is required for adequate IVs is to move the lags of the variable back into the past in order for all lags to be predetermined. In particular, the most recent lag of the nominal rate of j-holding period returns as an IV at time \( t \) should have been \( t - j - 2 \) despite the loss of important information. The reason for this is that the ex post inflation rate during \( t - 2 \) and \( t - j - 2 \) should be associated with the nominal rate of \( t - j - 2 \). Unfortunately, the information loss is very high since the financial markets are efficient. In order to circumvent the problem of the information loss in the exchange of correct timing of the IVs, I conduct an estimation by using as the IVs the lags prior to \( t - 2 \) of the nominal rates and the lags prior to \( t - 2 \) of the ex post inflation rate separately. This will prevent much of the recent information loss on the nominal rates.

Table 6 reports the empirical results with these IVs. The estimates \( \hat{\mu}_j \) of the constant terms monotonically rise as the interval increases, reaffirming that \( j \) and the variances in the constant terms become larger as the holding periods increases. The estimates \( \hat{\sigma} \) of the intertemporal elasticity of substitution are all statistically significant and range 0.06 to 0.24. Now the empirical results so far would indicate that consumption growth is to some extent sensitive to interest rates changes. However, this conclusion turns out to be wrong when both IVs, the liquidity constraint factor and interest rates, are used at the same time.
5.5.3 Liquidity Constraints With Stochastic Returns

When the liquidity constraint factor growth is reintroduced in the earlier specification, it becomes

\[
\left( \frac{C_{t+j}}{C_t} - 1 \right) = \mu_j + \theta \Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right) + \sigma r_{t+j} + \nu_{t+j}. \tag{5.22}
\]

The empirical results for a one-period interval \( j = 1 \) are reported in table 7. As shown in the two bottom rows, the estimates \( \hat{\theta} \) of the fraction of the labor income accruing to the liquidity-constrained consumers are 40.6 percent for the lags 2-4 and 31.1 percent for the lags 2-6. These estimates are more than three times the corresponding asymptotic standard errors. However, when liquidity constraint growth is reintroduced, the estimates \( \hat{\sigma} \) of intertemporal elasticities of substitution turn out now to be statistically insignificant at the 10 percent significance level and close to zero in magnitude. The results of the overidentifying restriction tests indicate that the model is never rejected at the conventional significance levels.

Table 8 reports the empirical results with the IV set that includes the lags prior to \( t - 2 \) of the nominal return rate and the lags prior to \( t - 2 \) of the post inflation rate. The adjusted \( R^2 \)'s for consumption growth equation of one quarter through one year suggest that the consumption does not follow a random walk. Those of intervals two and three years show that the IVs do not predict consumption growth well. The adjusted \( R^2 \)'s for the liquidity constraint growth equation and the real interest rate equation indicate that the IVs to a great extent predict the
RHS variables in the equation, indicating that there are no problems in the GMM procedures. Columns 5 through 7 report the estimation results. The estimates $\mu_j$ of the constant terms increase as the time interval increases, as observed in the earlier results in this paper. The estimates $\hat{\theta}$ of the liquidity constraints are all statistically significant at conventional significance levels and ranges from 0.237 to 0.380, suggesting that a 23.7 to 38 percent of aggregate labor income accrues to liquidity-constrained consumers. However, when liquidity constraint growth is reintroduced, the estimates $\sigma$ of intertemporal elasticities of substitution turn out now to be generally statistically insignificant and close to zero in magnitude, supporting the reports of Hall (1988) and Campbell and Mankiw (1989, 1990). The results of overidentifying restriction tests do not reject the model over the intervals, suggesting that U.S. consumers are not myopic.

The findings in this empirical work are striking not only because they tell us the evidence of existence of liquidity constraints in U.S. financial markets but also because they explain why some authors like Harvey (1988) obtained the empirical results that the interest rates can explain aggregate consumption. Like Campbell and Mankiw (1989, 1990), I claim that a model is misspecified if it does not incorporate liquidity constraints in the consumption equation. Furthermore, it is misspecified if the liquidity constraints are represented by Campbell and Mankiw's type of income growth under the preference form of constant relative risk aversion. As fully shown in the derivation of the modified Euler equation, the combination
of the consumption of constrained and unconstrained consumers in the permanent income envisioned equation cannot reflect the optimality condition in aggregate sense. This fact can be detected in the linearized Euler equation where the denominator, consumption, in the liquidity constraint factor, the income-consumption ratio, is ignored. The empirical results in this paper suggest that, if denominator of the liquidity constraint factor is ignored, the income change generally overstates the effects of the liquidity constraints in explaining the consumption change.

5.5.4 Test For the Intertemporal Stability of the Parameters

The evidence reported in multiperiod specifications indicates that some U.S. consumers are under liquidity constraints but not myopic. There are variations in magnitude of the estimates over the time horizon, however. If the parameters are different over the time horizon, the model in this paper has a fundamental flaw. Accordingly, it is required to test whether the parameters of different intervals are the same. Since the estimates of intertemporal elasticity of substitution turn out to be statistically insignificant and economically close to zero, as shown in table 8, I conduct a test for the stability of the intertemporal elasticity to the liquidity constraint growth over the time horizon. It can be done by setting the following null hypothesis

\[ H_0 : \theta_1 = \theta_2 = \theta_3 = \ldots = \theta_{12}. \]
Relevant empirical methods for the tests above include a Wald test, a Lagrange Multiplier test, and a likelihood ratio test. Although the likelihood ratio test requires two different iterations for the minimum criteria of both the maintained and the alternative hypotheses, the quasi-likelihood ratio test is used since the test can be easily implemented by extending the GAUSS programs that were employed in the earlier estimations in this paper.

Unfortunately, when the null hypothesis above was tested, the resulting estimates in the estimation process turned out to be extremely sensitive to the starting values. That is, different starting values generated different estimates. The reason that the estimates in this case appear unstable seems to result from the fact that there are too many parameters to estimate in the process of nonlinear iterations. It was found that an iteration process fails to converge to a global extremum if the number of parameters exceeds five. Because of this technical problem in the iteration process, the null hypothesis above is inevitably split into many sub-hypotheses in order to reduce the number of parameters in a stacked system. Thus, each null hypothesis now is expressed as a pair of two parameters of different time intervals:

\[ H_0 : \theta_i = \theta_j, \text{ for all } i \text{ and } j. \]

Thus actual empirical procedures are carried out by combining two Euler equations out of the six Euler equations to form a stacked system. Thus there are four parameters including two constant terms in a system under the alternative hypothesis. Then the redefined null hypothesis can be tested by forming the following
vector

\[ h_t(\gamma) \equiv \left( \begin{array}{c} \left( \frac{C_{t+i}}{C_t} - 1 \right) - \mu_i - \theta_i \left( \frac{X_{t+i}}{C_{t+i}} - \frac{X_t}{C_t} \right) \\ \left( \frac{C_{t+i}}{C_t} - 1 \right) - \mu_j - \theta_j \left( \frac{X_{t+i}}{C_{t+i}} - \frac{X_t}{C_t} \right) \end{array} \right) \otimes z_t, \] (5.23)

where \( j = 1, 2, 3, 4, 8, 12 \) and \( i \neq j \),

and \( z_t \) is an instrument set that consists of lags 2-6 of one-period consumption growth and the liquidity constraint growth. Thus there are twenty orthogonality conditions, three parameters under the null hypothesis, and four parameters under the alternative hypothesis, so that the number of the overidentifying restrictions are seventeen for the null hypothesis and sixteen for the alternative hypothesis and thus the degrees of freedom for a quasi-likelihood ratio test is unity.

The quasi-likelihood ratio test requires two minimization iterations for the null and the alternative hypotheses. With notation that \( G_r \) is the sample counterpart of the system above, \( q \) is the number of parameter restrictions, \( T \) is the sample size, \( r \) denotes 'restricted', and \( ur \) denoted 'unrestricted', the quasi-likelihood ratio test statistic,

\[ \Delta \chi^2(q) = T[G_r(\gamma)'W_{ur}G_r(\gamma) - G_{ur}(\gamma)'W_{ur}G_{ur}(\gamma)], \] (5.24)

has an asymptotic (central) \( \chi^2 \) distribution with degrees of freedom \( q = 1 \) under the null hypothesis. For an interval pair \((i, j)\) of Euler equations, the maximum lag length in the weighting matrix for a stacked system is that of a longer interval \( j + 2 \), where \( j > i \).
The empirical results for this are displayed in table 9, where the first row and the first column denote the intervals $i$- and $j$-quarters. The first three rows in each $(i, j)$ denote the two estimates of the constant terms for intervals $i$ and $j$ and the estimates of the liquidity constraint growth parameters under the null hypotheses. The asymptotic standard errors are shown in parentheses below each estimate. The following two rows in each $(i, j)$ show the statistics of the quasi-likelihood ratio tests, coupled with their marginal significance levels in parentheses.

First of all, the results from the quasi-likelihood ratio tests indicate that the parameter restrictions over the pairs of intervals cannot be rejected at conventional significance levels with only one outlier of a pair of $(3,4)$. This result strongly suggests that, under the assumption of constant real interest rates, the unknown parameters are the same over the time intervals. It further implies that the prevailing liquidity constraints in the loan markets are not different with respect to the time horizon. The results of the overidentifying restriction tests for each null and alternative hypothesis, which are not reported in the table because of limited space, suggest that all of the individual model cannot be rejected at conventional significance levels. Therefore, the intertemporal consumption elasticity to the liquidity constraint growth is consistent over the consumers' intertemporal planning horizon.

According to the table 9, the estimates $\hat{\theta}$ are in the range of 17 percent to 30.7 percent and statistically all are significant at conventional significance levels. The
arithmetical average of the estimates over the time intervals is 21 percent. This average is not quite representative of the unknown true value, however.\textsuperscript{18} It means that 21 percent of U.S. aggregate labor income accrues to consumers who fail to hold assets and are subject to liquidity constraints. As a result, they finance their consumption by all their contemporaneous labor income. It further implies that, because of failure to optimize their intertemporal resource allocation, their welfare loss must be substantial.

An additional discussion is that, for a given interval \( i \), the estimates \( \hat{\mu}_i \) and \( \hat{\mu}_j \) of the constant terms in a pair of \( (i, j) \) tend to diverge as the time interval \( j \) increases. For example, for the one-quarter interval in the second and the fourth rows, the \( \hat{\mu}_1 \) becomes smaller and at the time \( \hat{\mu}_j \) becomes larger until two years as the \( j \) increases. It is unclear why the constant terms in the pairs diverge. One possible explanation is that it might reflect the fact that the iteration process in the GAUSS program does not work well when the number of model parameters is not small. This fact is similar to the aforementioned point that the estimates are extremely sensitive to the starting values if the number of parameters is more than five. If the iteration process is very sensitive to the dimension of the parameter space, the iteration process is often insensitive to searching for a global value and tends to stop at a local value.

\textsuperscript{18}The desired method is to find the estimate under the null hypothesis that all of the parameters are the same. Unfortunately, this estimation procedure involves seven parameters including six constant terms in the stacked system and thus, as mentioned earlier, it causes the estimates to be extremely sensitive to the starting values and the resulting estimates are never reliable.
This chapter provides very important empirical findings: evidence for the existence of liquidity constraints in U.S. loan markets; why some authors like Harvey (1988) obtained the possibly wrong empirical results that interest rates can explain aggregate consumption; and U.S. consumers are not myopic. My argument on the model specification is basically in line with Campbell and Mankiw (1989, 1990): a model must be misspecified if it does not incorporate liquidity constraints in the consumption equation. However, the model type in Campbell and Mankiw and many others in the literature on the excess sensitivity of aggregate consumption to disposable income is subject to another inadvertent misspecification if liquidity constraints are represented by income growth under the preference form of constant relative risk aversion. They reported overstated estimates for the excess sensitivity of consumption to income since they did not take into account the denominator, consumption, in the liquidity constraint factor in the process of aggregation over different groups of consumers.
CHAPTER VI

CONCLUSION

This paper investigates intertemporal consumption-portfolio behavior and conducts an empirical investigation on the effects of consumers' liquidity constraints and intertemporal substitution to shed light on (1) the excess sensitivity of aggregate consumption to contemporaneous liquidity factors and to holding period returns over the time-interval spectrum, (2) consumers' myopic intertemporal decisions, and (3) the stability of the estimates over different time-intervals.

If considerable liquidity constraints prevail in the economy, the conventional Consumption-Capital-Asset-Pricing Model may have quasi-marginal rates of substitution between today and tomorrow. By an ad hoc assumption of income distribution in an economy, the liquidity constraint factors are identified and extracted from the Euler equation for the agents who are not under the liquidity constraints in terms of aggregate consumption and aggregate labor income.

In the theoretical part of this paper, some notable findings are as follows:

(a) In the short run, the liquidity factor can affect the term structures of interest
rates via risk premia. In the period when the cyclical trend locates below [or above] its long-run equilibrium value, unity, the risk premium of a risky asset tends to increase [or decrease].

(b) In the presence of liquidity constraints, a risk premium usually exists on any risk-free asset.

(c) Combining results (a) and (b), it is inferred that changes in income and wealth distribution can disturb the asset pricing mechanism through the risk premium regardless of whether the changes in income and wealth are forecastable.

(d) The effect of volatility of the liquidity constraint factor on the risk premium is uncertain, but high disturbances of the liquidity constraint factor may potentially result in high risk premia.

(f) If the quasi-marginal rate of substitution and the liquidity constraint factor are correlated, conditional on the information set of the present or earlier, the risk premium of an asset will change.

(g) As the degree of liquidity constraints changes, the risk premium depends upon the direction of movement in the aggregate income-consumption ratios.

(h) It is argued that the conventional models for the sensitivity tests of consumption to income may possibly have a problem of misspecification because the previous models were constructed in a way that the income term is added in to the Euler equation, which, by the way, no longer fulfills the optimal condition. The model in this paper, however, does not have this sort of misspecification problem.
The consumption variability in the case that the liquidity constraint factor is introduced is higher than in the case envisioned by the Permanent Income Hypothesis. This observation on the aggregate data supports the recent claim by Campbell and Mankiw that the consumption of stockholders is more volatile than that of nonstockholders. From this finding, it can be inferred that the degree of risk aversion is relatively low when the liquidity constraint factor is incorporated in explaining aggregate consumption and accordingly the unsolved equity premium puzzle can be partially explained by this model, although it cannot be completely solved.

From the intertemporal orthogonality conditions in the form of holding period returns, Hansea's Generalized Methods of Moment scheme is applied to estimate the model with quarterly U.S. data for the last four decades. The empirical results show that 21 percent of aggregate labor income accrues to the liquidity-constrained population, indicating that the estimates in the model in this paper are much close to those in Hall and Mishkin's (1982) suggestion and the panel data studies. The tests of the overidentifying restrictions cannot reject the model and U.S. consumers are not myopic. Finally, the results of the stacked estimation imply that the model estimates are not different over the time-interval spectrum. Although the empirical results in this paper are striking in explaining the sensitivity of consumption to the liquidity constraints, it does not necessarily follow that Hall's random walk property of consumption is invalid. It does suggest that a potentially important group
of consumers whose consumption growth is very sensitive to income-consumption growth exist in the U.S. economy.
### Table 1: (One-Period) Sample Autocorrelations
(1953:I–1986:IV)

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<th>Means</th>
<th>Std.Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
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<tr>
<td>(1)</td>
<td>( \frac{Y_{t+1} - Y_t}{C^t} )</td>
<td>.0047</td>
<td>.0053</td>
<td>-.402</td>
<td>.929</td>
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<tr>
<td>(2)</td>
<td>( \Delta Y_{t+1}^{(c+1)} )</td>
<td>-.0013</td>
<td>.0108</td>
<td>.810</td>
<td>2.139</td>
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<tr>
<td>(3)</td>
<td>( \frac{1 - \gamma Y_{t+1}^{e(c+1)} \theta = .33}{1 - \gamma e^{Y_{t+1}^{e(c+1)}}} )</td>
<td>1.0007</td>
<td>.0057</td>
<td>.758</td>
<td>1.940</td>
</tr>
<tr>
<td>(4)</td>
<td>( r_{2m} )</td>
<td>.0396</td>
<td>.0226</td>
<td>.966</td>
<td>.702</td>
</tr>
<tr>
<td>(5)</td>
<td>( B_{3m} B_{6m} - r_{3m} )</td>
<td>.0034</td>
<td>.0075</td>
<td>1.645</td>
<td>8.300</td>
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<tr>
<td>(6)</td>
<td>( H_{3m} B_{6m} - r_{3m} )</td>
<td>.0039</td>
<td>.0144</td>
<td>1.152</td>
<td>6.221</td>
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<tr>
<td>(7)</td>
<td>( H_{3m} B_{1y} - r_{3m} )</td>
<td>.0040</td>
<td>.0212</td>
<td>.816</td>
<td>4.441</td>
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<td>(8)</td>
<td>( \ln(P_{t+1} / P_t) )</td>
<td>.0112</td>
<td>.0078</td>
<td>.912</td>
<td>.443</td>
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*Data for Nondurables plus services (ND5) and Labor income come from the Citibase tape. Yields data come from McCulloch's time structure of interest rates (1987). A marginal tax rate used for all of the financial variables is 0.3. The number of time series observations is 136.

*Std.Dev. are standard deviations.

*Skewness of normal distribution equals zero.

*Kurtosis of normal distribution equals three.

\[ \Delta Y_{t+1}^{(c+1)} = \frac{Y_{t+1}^{(c+1)}}{C^t} - \frac{Y_t}{C^t} \]

*Yields of three-months zero coupon bonds; but do not differ from the logarithm of one plus these yields.

\[ H_{3m} B_{6m} - r_{3m} = h_{3m}(t, I, t+I) - h_{3m}(t, I, t+1) \] in the text. That is, a quarter excess holding period return between six months and three months bonds.

\[ H_{3m} B_{6m} - r_{3m} = h_{3m}(t, I, t+1) - h_{3m}(t, I, t+1) \] in the text. That is, a quarter excess holding period return between nine months and three months bonds.

\[ H_{3m} B_{1y} - r_{3m} = h_{1y}(t, I, t+1) - h_{3m}(t, I, t+1) \] in the text. That is, a quarter excess holding period return between one year and three months bonds.
Table 2: (Two-Periods) Sample Autocorrelations
(1953:I–1986:IV)

<table>
<thead>
<tr>
<th>Row</th>
<th>Variables</th>
<th>Means</th>
<th>Std.Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>$C_{t+2} - 1$</td>
<td>.0095</td>
<td>.0083</td>
<td>-.353</td>
<td>.290</td>
</tr>
<tr>
<td>(2)</td>
<td>$\Delta C_{t+2}$</td>
<td>-.0027</td>
<td>0.0159</td>
<td>.352</td>
<td>-.257</td>
</tr>
<tr>
<td>(3)</td>
<td>$\gamma_t - \gamma_{t+2}$</td>
<td>1.0014</td>
<td>.0084</td>
<td>-.318</td>
<td>-.206</td>
</tr>
<tr>
<td>(4)</td>
<td>$\gamma_t - \gamma_{t+2}$</td>
<td>0.0408</td>
<td>0.0229</td>
<td>.910</td>
<td>.536</td>
</tr>
<tr>
<td>(5)</td>
<td>$H_{em}B_{gm} - r_{em}$</td>
<td>.0019</td>
<td>.0052</td>
<td>.0001</td>
<td>.250</td>
</tr>
<tr>
<td>(6)</td>
<td>$H_{em}B_{1yr} - r_{em}$</td>
<td>.0022</td>
<td>.0103</td>
<td>.069</td>
<td>1.898</td>
</tr>
<tr>
<td>(7)</td>
<td>$r_{em} - H_{em}R_{3m}$</td>
<td>.0017</td>
<td>.0038</td>
<td>1.645</td>
<td>8.296</td>
</tr>
</tbody>
</table>

Sample Autocorrelations

<table>
<thead>
<tr>
<th>Row</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_5$</th>
<th>$p_6$</th>
<th>$p_7$</th>
<th>$p_8$</th>
<th>$p_9$</th>
<th>$p_{10}$</th>
<th>$p_{11}$</th>
<th>$p_{12}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>.64</td>
<td>.27</td>
<td>.26</td>
<td>.11</td>
<td>-.04</td>
<td>.05</td>
<td>.83</td>
<td>-.13</td>
<td>-.09</td>
<td>.04</td>
<td>.07</td>
<td>.00</td>
</tr>
<tr>
<td>(2)</td>
<td>.51</td>
<td>-.06</td>
<td>-.23</td>
<td>-.37</td>
<td>-.30</td>
<td>-.09</td>
<td>-.04</td>
<td>-.04</td>
<td>.08</td>
<td>.15</td>
<td>.10</td>
<td>.06</td>
</tr>
<tr>
<td>(3)</td>
<td>.52</td>
<td>-.05</td>
<td>-.23</td>
<td>-.38</td>
<td>-.32</td>
<td>-.10</td>
<td>-.04</td>
<td>-.03</td>
<td>.09</td>
<td>.15</td>
<td>.09</td>
<td>.05</td>
</tr>
<tr>
<td>(4)</td>
<td>.95</td>
<td>.90</td>
<td>.88</td>
<td>.83</td>
<td>.78</td>
<td>.73</td>
<td>.68</td>
<td>.66</td>
<td>.63</td>
<td>.60</td>
<td>.56</td>
<td>.54</td>
</tr>
<tr>
<td>(5)</td>
<td>.48</td>
<td>.01</td>
<td>.11</td>
<td>.12</td>
<td>-.01</td>
<td>-.14</td>
<td>-.20</td>
<td>-.05</td>
<td>.09</td>
<td>.04</td>
<td>.05</td>
<td>.07</td>
</tr>
<tr>
<td>(6)</td>
<td>.50</td>
<td>.05</td>
<td>.17</td>
<td>.17</td>
<td>.02</td>
<td>-.11</td>
<td>-.14</td>
<td>-.03</td>
<td>.06</td>
<td>.02</td>
<td>-.04</td>
<td>-.07</td>
</tr>
<tr>
<td>(7)</td>
<td>.17</td>
<td>-.15</td>
<td>.19</td>
<td>.01</td>
<td>-.07</td>
<td>.03</td>
<td>-.20</td>
<td>-.01</td>
<td>.17</td>
<td>.04</td>
<td>-.06</td>
<td>-.05</td>
</tr>
</tbody>
</table>

* $\Delta (\gamma_{t+2}) = \gamma_{t+2} - \gamma_t$.

Yields of six-months zero coupon bonds; but do not differ from the logarithm of one plus these yields.

$H_{em}B_{gm} - r_{em} = h_{em}(t, t+1; +2) - h_{em}(t, t+1; +2)$ in the text. That is, two quarters excess holding return between nine months and six months bonds.

$H_{em}B_{1yr} - r_{em} = h_{1yr}(t, t+1; +2) - h_{em}(t, t+1; +2)$ in the text. That is, two quarters excess holding return between one year and six months bonds.

$r_{em} - H_{em}R_{3m} = h_{em}(t, t+1; +2) - h_{3m}(t, t+1; +2)$ in the text. That is, two quarters excess holding return between six months bond and three-months bond rolling over.
Table 3: GMM Results of Sensitivity Tests
(1953:1–1982:IV)

\[
\left( \frac{C_{it+1}}{C_{i}} - 1 \right) = \mu_1 + \theta \Delta \left( \frac{Y_{it+1}}{C_{it+1}} \right) + \nu_{t+1}
\]

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Forecasting Powers</th>
<th>Estimates</th>
<th>Overidentifying Restriction Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{C_{it+1}}{C_{i}} - 1 )</td>
<td>( \Delta \left( \frac{Y_{it+1}}{C_{it+1}} \right) )</td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{C_{it+1}}{C_{it}} )</td>
<td>.057</td>
<td>.106</td>
<td>.0054</td>
</tr>
<tr>
<td>Lags 2-6 of ( \frac{C_{it+1}}{C_{it}} )</td>
<td>(.009)</td>
<td>(.000)</td>
<td>(.0006)</td>
</tr>
<tr>
<td>Lags 2-4 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) )</td>
<td>.039</td>
<td>.095</td>
<td>.0048</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) )</td>
<td>(.033)</td>
<td>(.000)</td>
<td>(.0005)</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{C_{it+1}}{C_{it}} ), Lags 2-4 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) )</td>
<td>.028</td>
<td>.042</td>
<td>.9048</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) ), Lags 2-6 of ( \frac{C_{it+1}}{C_{it}} )</td>
<td>(.117)</td>
<td>(.055)</td>
<td>(.0005)</td>
</tr>
<tr>
<td>Lags 2-6 of ( \frac{C_{it+1}}{C_{it}} ), Lags 2-6 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) )</td>
<td>.092</td>
<td>.143</td>
<td>.9047</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{it+1}}{C_{it}} \right) ), Lags 2-6 of ( \frac{C_{it+1}}{C_{it}} )</td>
<td>(.006)</td>
<td>(.014)</td>
<td>(.0004)</td>
</tr>
</tbody>
</table>

* A constant term is included both in the instrument sets and the regressor sets.
* The Gauss program for GMM Estimation was used. The Newey and West’s (1987) lag is 3 in order to correct autocorrelation in weighting matrix. Starting values are 0.0 for \( \mu_1 \) and 0.3 for \( \theta \).
* Overidentifying Restrictions tests at the degrees of freedom \( DF \). \( MSL \) are marginal significance levels.
Table 4: Multiperiod Sensitivity Tests

\[
\begin{align*}
\left( \frac{C_{t+j}}{C_t} - 1 \right) &= \mu_j + \theta \left( \frac{Y_{t+j}}{C_{t+j}} - \frac{Y_t}{C_t} \right) + \nu_{t+j} \\
\text{Instruments:} & \quad \left( \frac{C_{t-6}}{C_{t-3}}, \ldots, \frac{C_{t-6}}{C_{t-7}}, \right. \\
& \quad \left. \left( \frac{Y_{t-6}}{C_{t-3}} - \frac{Y_{t-6}}{C_{t-7}}, \ldots, \frac{Y_{t-6}}{C_{t-6}} - \frac{Y_{t-7}}{C_{t-7}} \right) \right) \\
\end{align*}
\]

\[j = 1, 2, 3, 4, 8, \text{ and } 12.\]

<table>
<thead>
<tr>
<th>Model Intervals (j)</th>
<th>Forecasting Powers</th>
<th>GMM Estimates</th>
<th>QR Tests</th>
<th>(\chi^2) (DF = 9)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{C_{t+j}}{C_t} - 1)</td>
<td>(\Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right))</td>
<td>(\mu_j)</td>
<td>(\hat{\theta})</td>
</tr>
<tr>
<td>One Quarter (j = 1)</td>
<td>.010</td>
<td>.001</td>
<td>.0044</td>
<td>.299</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td>(.0004)</td>
<td>(.127)</td>
<td>(.219)</td>
</tr>
<tr>
<td>Two Quarters (j = 2)</td>
<td>.095</td>
<td>.198</td>
<td>.0090</td>
<td>.290</td>
</tr>
<tr>
<td></td>
<td>(.017)</td>
<td>(.0008)</td>
<td>(.120)</td>
<td>(.459)</td>
</tr>
<tr>
<td>Three Quarters (j = 3)</td>
<td>.075</td>
<td>.167</td>
<td>.0151</td>
<td>.276</td>
</tr>
<tr>
<td></td>
<td>(.041)</td>
<td>(.000)</td>
<td>(.168)</td>
<td>(.974)</td>
</tr>
<tr>
<td>Four Quarters (j = 4)</td>
<td>.088</td>
<td>.122</td>
<td>.0200</td>
<td>.304</td>
</tr>
<tr>
<td></td>
<td>(.023)</td>
<td>(.005)</td>
<td>(.145)</td>
<td>(.796)</td>
</tr>
<tr>
<td>Eight Quarters (j = 8)</td>
<td>.001</td>
<td>.109</td>
<td>.0428</td>
<td>.422</td>
</tr>
<tr>
<td></td>
<td>(.432)</td>
<td>(.009)</td>
<td>(.153)</td>
<td>(.991)</td>
</tr>
<tr>
<td>Twelve Quarters (j = 12)</td>
<td>-.060</td>
<td>.128</td>
<td>.0554</td>
<td>.193</td>
</tr>
<tr>
<td></td>
<td>(.958)</td>
<td>(.003)</td>
<td>(.184)</td>
<td>(.917)</td>
</tr>
</tbody>
</table>

\(\Delta \left( \frac{Y_{t+j}}{C_{t+j}} \right) = \frac{Y_{t+j}}{C_{t+j}} - \frac{Y_t}{C_t}\), for all j.

Note: The Newey and West's lags for j-period(s) are \(j + 2\) in order to correct autocorrelations. Starting values are 0.0 for \(\mu_j\) and 0.3 for \(\hat{\theta}\). For other references, see table 3.
Table 5: GMM Results of Intertemporal Substitution

\[
\left( \frac{C_{t+1}}{C_t} - 1 \right) = \mu_1 + \sigma r_{t+1} + \nu_{t+1}
\]

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Forecasting Power</th>
<th>GMM Estimates</th>
<th>OR Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( (\frac{C_{t+1}}{C_t} - 1) )</td>
<td>( r_{1,t+1} )</td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{C_t}{C_{t-1}} )</td>
<td>(.069)</td>
<td>(.943)</td>
<td>(.0072)</td>
</tr>
<tr>
<td>Lags 2-6 of ( \frac{C_t}{C_{t-1}} )</td>
<td>(.079)</td>
<td>(.034)</td>
<td>(.0056)</td>
</tr>
<tr>
<td>Lags 2-4 of ( r_{1,t} )</td>
<td>(.043)</td>
<td>(.297)</td>
<td>(.0052)</td>
</tr>
<tr>
<td>Lags 2-6 of ( r_{1,t} )</td>
<td>(.061)</td>
<td>(.285)</td>
<td>(.0052)</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{\nu_{t-1}^*}{\nu_{t-1}} )</td>
<td>(.081)</td>
<td>(.423)</td>
<td>(.0051)</td>
</tr>
<tr>
<td>Lags 2-6 of ( \frac{\nu_{t-1}^*}{\nu_{t-1}} )</td>
<td>(.077)</td>
<td>(.416)</td>
<td>(.0045)</td>
</tr>
</tbody>
</table>

*\( r_{1,t} \) is ex post real yield of zero coupon bond maturing at one quarter after time \( t \);

Note: The Newey and West's maximum lag length is 3 in order to correct autocorrelation.

Starting values are 0.9 for \( \mu_1 \) and 0.9 for \( \sigma \). For other references see Table 3.
Table 6: Multiperiod Intertemporal Substitution Tests (1953.I-1986.IV)

\[
\frac{C_{t+j}}{C_t} - 1 = \mu_j + \sigma r_{j,t+j} + \nu_{t+j}
\]

Instruments:

\[
\begin{pmatrix}
\frac{C_{t-6}}{C_t}, \ldots, \frac{C_{t-1}}{C_t}, \\
\frac{C_{t-12}}{C_t}, \ldots, \frac{C_{t-6}}{C_t}, \\
\pi_{t-1}, \ldots, \pi_{t-6}
\end{pmatrix}
\]

\(j = 1, 2, 3, 4, 8, \text{ and } 12.\)

<table>
<thead>
<tr>
<th>Model Intervals (j)</th>
<th>Forecasting Powers</th>
<th>GMM Estimates</th>
<th>OR Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{C_{t+j}}{C_t} - 1)</td>
<td>(r_{j,t+j})</td>
<td>(\hat{\mu}_j)</td>
</tr>
<tr>
<td>One Quarter (j = 1)</td>
<td>.164 (.001)</td>
<td>.416 (.000)</td>
<td>.0037 (.0005)</td>
</tr>
<tr>
<td>Two Quarters (j = 2)</td>
<td>.191 (.000)</td>
<td>.452 (.000)</td>
<td>.0067 (.0010)</td>
</tr>
<tr>
<td>Three Quarters (j = 3)</td>
<td>.186 (.000)</td>
<td>.461 (.000)</td>
<td>.0105 (.0014)</td>
</tr>
<tr>
<td>Four Quarters (j = 4)</td>
<td>.169 (.001)</td>
<td>.465 (.000)</td>
<td>.0125 (.0023)</td>
</tr>
<tr>
<td>Eight Quarters (j = 8)</td>
<td>.057 (.111)</td>
<td>.453 (.000)</td>
<td>.0255 (.0041)</td>
</tr>
<tr>
<td>Twelve Quarters (j = 12)</td>
<td>.074 (.066)</td>
<td>.483 (.000)</td>
<td>.0341 (.0034)</td>
</tr>
</tbody>
</table>

*\(i_{j,t}\) are yield of zero coupon bond maturing at \(j\) quarter(s) after time \(t\) and \(\pi_t\) is ex post one-period inflation rate.

Note: The Newey and West’s maximum lag lengths for \(j\)-period(s) are \(j + 2\) in order to correct autocorrelations. Starting values are 0.0 for \(\mu_j\) and 0.0 for \(\sigma\). For other references see Table 3.
Table 7: GMM Results of Sensitivity Tests with Stochastic Returns
(1953:I-1986:IV)
\[
\left( \frac{C_{t+1}}{C_t} - 1 \right) = \mu_1 + \theta \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) + \sigma r_{t+1} + \nu_{t+1}
\]

<table>
<thead>
<tr>
<th>Instruments</th>
<th>Forecasting Power</th>
<th>GMM Estimates</th>
<th>OR Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \frac{C_{t+1}}{C_t} )</td>
<td>( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>( r_{t+1} )</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{C_t}{C_{t-2}} )</td>
<td>.069</td>
<td>.129</td>
<td>.043</td>
</tr>
<tr>
<td>Lags 2-6 of ( \frac{C_t}{C_{t-2}} )</td>
<td>.072</td>
<td>.095</td>
<td>.034</td>
</tr>
<tr>
<td>Lags 2-4 of ( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>.029</td>
<td>.130</td>
<td>-.016</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>.040</td>
<td>.075</td>
<td>-.027</td>
</tr>
<tr>
<td>Lags 2-4 of ( r_{t+1} )</td>
<td>.043</td>
<td>.628</td>
<td>.297</td>
</tr>
<tr>
<td>Lags 2-6 of ( r_{t+1} )</td>
<td>.061</td>
<td>.027</td>
<td>.285</td>
</tr>
<tr>
<td>Lags 2-4 of ( \frac{Y_{t+1}}{C_{t+1}} )</td>
<td>.088</td>
<td>.181</td>
<td>.933</td>
</tr>
<tr>
<td>Lags 2-4 of ( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>.003</td>
<td>.000</td>
<td>.124</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>.107</td>
<td>.120</td>
<td>.005</td>
</tr>
<tr>
<td>Lags 2-6 of ( \Delta \left( \frac{Y_{t+1}}{C_{t+1}} \right) )</td>
<td>.010</td>
<td>.000</td>
<td>.387</td>
</tr>
</tbody>
</table>

\( r_{t+1} \) is ex post real yield of zero coupon bond maturing one quarter after time \( t \);

Note: The Newey and West's maximum lag length is 3 in order to correct autocorrelation.
Starting values are 0.0 for \( \mu_1 \), 0.3 for \( \theta \), and 0.0 for \( \sigma \). For other references see table 3.
Table 8: Multiperiod GMM Results of Sensitivity Tests with Stochastic Returns
(1953:1-1986:IV)

\[
\left( \frac{C_{t+i}}{C_t} - 1 \right) = \mu_j + \theta \left( \frac{Y_{t+i}}{C_{t+i}} - \frac{Y_t}{C_t} \right) + \sigma r_{j,t+j} + \nu_{t+j}
\]

Instruments:

\[
\begin{pmatrix}
\frac{C_{t-8}}{C_{t-9}}, \ldots, \frac{C_{t-1}}{C_{t-7}};
\frac{Y_{t-8}}{C_{t-9}} - \frac{Y_{t-1}}{C_{t-7}}), \ldots, \frac{Y_{t-1}}{C_{t-1}} - \frac{Y_{t-7}}{C_{t-7}}),
\tau_{j,t-2}, \ldots, \tau_{j,t-1},
\pi_{t-2}, \ldots, \pi_{t-1}
\end{pmatrix}
\]

<table>
<thead>
<tr>
<th>Model Intervals</th>
<th>Forecasting Powers</th>
<th>Estimates</th>
<th>OR Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\frac{C_{t+i}}{C_t} - 1)</td>
<td>(\Delta \left( \frac{Y_{t+i}}{C_{t+i}} \right))</td>
<td>(r_{j,t+j})</td>
</tr>
<tr>
<td>One Quarter ((j = 1))</td>
<td>.184</td>
<td>.131</td>
<td>.439</td>
</tr>
<tr>
<td>Two Quarters ((j = 2))</td>
<td>.210</td>
<td>.153</td>
<td>.463</td>
</tr>
<tr>
<td>Three Quarters ((j = 3))</td>
<td>.216</td>
<td>.138</td>
<td>.466</td>
</tr>
<tr>
<td>Eight Quarters ((j = 8))</td>
<td>.056</td>
<td>.096</td>
<td>.485</td>
</tr>
<tr>
<td>Twelve Quarters ((j = 12))</td>
<td>.034</td>
<td>.290</td>
<td>.539</td>
</tr>
</tbody>
</table>

Note: Refer to table 3, 4 and 6.
Table 9: Quasi-Likelihood Ratio Tests of Stacked Models

\[ H_0 : \theta_i = \theta_j \text{ for the Model: } \left( \frac{C_{i+1}(\theta_i)}{C_{i+1}(\theta_j)} - 1 \right) = \mu_{(i,j)} + \theta_{(i,j)} \Delta \left( \frac{C_{i+1}(\theta_i)}{C_{i+1}(\theta_j)} \right) + \nu_{i+1(i,j)} \]

<table>
<thead>
<tr>
<th>Intervals (i and j)</th>
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| 2 Quarters          | \( \mu_6 \) | \( \mu_7 \) | \( \mu_8 \) | \( \mu_9 \) | \( \mu_{10} \) |
| (se)                | (.001)     | (.001)     | (.001)     | (.001)     | (.001)      |
| \( \mu_j \)        | (.001)     | (.001)     | (.002)     | (.005)     | (.006)      |
| (se)                | (.001)     | (.001)     | (.002)     | (.005)     | (.006)      |
| \( \theta \)       | .234       | .226       | .215       | .297       |             |
| (se)                | (.080)     | (.085)     | (.084)     | (.078)     |             |
| \( \Delta \chi^2 \) | 1.377      | .988       | 2.74       | 2.54       |             |
| mal                 | (.241)     | (.321)     | (.601)     | (.614)     |             |

| 3 Quarters          | \( \mu_1 \) | \( \mu_2 \) | \( \mu_3 \) | \( \mu_4 \) | \( \mu_5 \) |
| (se)                | (.009)     | (.010)     | (.010)     | (.008)     |             |
| \( \mu_j \)        | (.001)     | (.002)     | (.002)     | (.002)     |             |
| (se)                | (.001)     | (.001)     | (.002)     | (.002)     |             |
| \( \theta \)       | .013       | .026       | .026       | .226       |             |
| (se)                | (.022)     | (.005)     | (.005)     | (.008)     |             |
| \( \Delta \chi^2 \) | 4.224      | .997       | 1.920      |             |             |
| mal                 | (.040)     | (.319)     | (.432)     |             |             |

| 4 Quarters          | \( \mu_6 \) | \( \mu_7 \) | \( \mu_8 \) | \( \mu_9 \) | \( \mu_{10} \) |
| (se)                | (.014)     | (.002)     | (.003)     | (.008)     | (.008)      |
| \( \mu_j \)        | (.002)     | (.002)     | (.003)     | (.008)     | (.008)      |
| (se)                | (.002)     | (.002)     | (.002)     | (.008)     | (.008)      |
| \( \theta \)       | .238       | .258       | .258       | .210       | (.091)      |
| (se)                | (.071)     | (.070)     | (.071)     | (.070)     | (.070)      |
| \( \Delta \chi^2 \) | .077       | 1.220      | .930       | (.023)     | (.006)      |
| mal                 | (.781)     | (.165)     | (.954)     | (.091)     | (.064)      |

| 8 Quarters          | \( \mu_1 \) | \( \mu_2 \) | \( \mu_3 \) | \( \mu_4 \) | \( \mu_5 \) |
| (se)                | (.003)     | (.003)     | (.003)     | (.003)     | (.003)      |
| \( \mu_j \)        | (.008)     | (.008)     | (.008)     | (.008)     | (.008)      |
| (se)                | (.003)     | (.003)     | (.003)     | (.003)     | (.003)      |
| (se)                | (.091)     | (.091)     | (.091)     | (.091)     | (.091)      |
| \( \Delta \chi^2 \) | .093       | .093       | .093       | .093       | .093        |
| mal                 | (.954)     | (.954)     | (.954)     | (.954)     | (.954)      |

Notes: \( \Delta \chi^2 (DF=1) = \chi^2(H_0) - \chi^2(H_1) \). mal is marginal significance level.
Figure 1: Income and Wealth Lorenz Curves in the Presence of Liquidity Constraints
Figure 2: Historical Movement of Liquidity Factors

\( \theta = 0.50 \)

\( \theta = 0.25 \)
Figure 3: Consumption Growth with Liquidity Factors
1947.1 - 2002.4
Figure 5: Short-Run Risk Premium under the Liquidity Constraint Factor Cycle
Appendix A

Consumer’s Internalization

For purposes of illustration, only two periods of lifetime will be considered in figure A.1. DABF is the consumer’s production possibilities frontier; curves I, II, and III are a family of the consumer’s indifference curves; and GH is the market opportunity line in absence of the liquidity constraints. A rational individual will choose the optimal point C. However, if the consumer cannot borrow, his or her actual market opportunity line will shrink to GAEP. In this situation the consumer chooses the constrained equilibrium point E if he has his own production opportunity set like ODF by transforming the production possibilities from point A to point E. In this constrained optimum, $MRT = MRS = 1 + r$, where MRT is the marginal rate of transformation in productions, MRS is the marginal rate of substitution in consumptions, and $r$ is the market lending rate. If the consumer ignores (or does not utilize) the production opportunities, then the constrained equilibrium is now the point A where the MRS is not equal to the slope of the market price line GH. When the consumer cannot or will not utilize the production
Figure A.1: Liquidity Constraint and Consumer’s Internalization
possibilities, he or she may incur the loss of utility.

When the borrowing rates diverge from the lending rates as 1J does in figure A.1, the market opportunity line will shrink to GABJ. Therefore, in the presence of differentials of borrowing and lending rates, the consumer's optimal behavior will lead him to choose the point E. This is the same result as if the consumer were credit-rationed but has his own production opportunities. Because of the concavity of the production opportunities line DEF there are two different Lypzinski expansion paths with two different interest rates. In figure A.1, $LL_A$ and $LL_B$ denote the different Lypzinski expansion paths along which the production opportunities frontier will be tangent to the lending budget line GH and to the borrowing budget line IJ, respectively. As long as the optimal point is located within the zone between the $LL_A$ and the $LL_B$, the consumer will not participate in the private loan markets and may, instead, internalize his or her intertemporal allocation of resources.
Appendix B

The Effect of Transitory Income

Consider the effect of transitory income on consumption by use of figure A.2. $GAEBO$ is the market opportunity set in the presence of endogenous borrowing rates type of liquidity constraint. These borrowing rates will increase as borrowing increases. This can be explained in terms of an increase in the slope of the tangent line on the segment $AEB$ of the market opportunity line with the movement from point $A$ toward point $B$. In this case, the consumer's optimal consumption-investment plan is to consume today as much as $a_1$ and at the same time to borrow today as much as $\alpha a_1$. When the consumer is instead credit-rationed at the limit of net worth $a_0$, the optimal position is on the point $A$ of today's consumption $a_0$. When current labor income increases by $AA'$, the consumer's budget line $GH$ will shift to $G'H'$ and, accordingly, the market opportunity set will expand to $G'A'EB'O$. As a result, the current consumption will rise by $AA'$ if the consumer is credit-rationed at the limit of $a_0$ and will rise by less than $AA'$ if he or she faces increasing borrowing rates as the net borrowing increase. Let $\Delta C_0$ and
Figure A.2: The Effect of Transitory Income
ΔC'_o denote the change in the current consumption when the consumer faces the credit rationing and endogenous borrowing rates, respectively. From figure A.2, it is known that ΔC'_o = (1 - α)ΔC_o if current and future consumption is normal, where 0 < α < 1. By introducing a constant α, the analysis on the Zeldes' type of liquidity constraints can be applied to the endogenous borrowing rates type of liquidity constraints.
Appendix C

Proof of $\Omega_{t+1}$

For convenience, let $A \equiv \beta(C_t/C_{t+1})$, $B \equiv R_{s,t+1}$, and $L = L_{t+1}$. Then when the conditional covariance is decomposed, the risk premium can be expressed as follows:

$$\Omega_{t+1} = COV_t[AL, B]$$

$$= E_t[ALB] - E_t[AL]E_t[B]$$


Appendix D

Tables for Multiperiod Sample Autocorrelations
Table A.1: (Three-Periods) Sample Autocorrelations
(1953:I–1986:IV)

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<th>Kurtosis</th>
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\[ \Delta \left( \frac{Y_{t+3}}{C_{t+3}} \right) = \frac{Y_{t+3}}{C_{t+3}} - \frac{Y_t}{C_t}. \]

*Yield of nine-months zero coupon bond; \[ H_{5m}B_{1yr} - r_{5m} = h_{1yr}(t, 3, t+3) - h_{5m}(t, 3, t+3). \] That is, three quarters excess holding period return between one year and nine months bonds.

*\[ r_{5m} - H_{5m}R_{5m} = h_{5m}(t, 3, t+3) - h_{5m}(t, 3, t+3). \] That is, three quarters excess holding return between nine months bond and three-months bond rolling over.

*\[ r_{5m} - H_{5m}R_{5m} = h_m(t, 3, t+3) - h_{5m}(t, 3, t+3). \] That is, three quarters excess holding return between nine months bond and a sequence of six- and three-months bond rolling over.
Table A.2: (Four-Periods) Sample Autocorrelations
(1953:I–1986:IV)

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\(\Delta \left( \frac{Y_{t+4}}{C_{t+4}} \right) = \frac{Y_{t+4}}{C_{t+4}} - \frac{Y_t}{C_t}\).

*Yields of one-year zero coupon bonds.

*\(H_{1 yr} B_{3 yr} - r_{1 yr}\) = \(h_{3 yr}(t, t+4) - h_{1 yr}(t, t+4)\) in the text. That is, one year excess holding period return between two years and one year bonds.

*\(H_{1 yr} B_{3 yr} - r_{1 yr}\) = \(h_{3 yr}(t, t+4) - h_{1 yr}(t, t+4)\) in the text. That is, one year excess holding period return between three years and one year bonds.

*\(r_{1 yr} - H_{1 yr} R_{6 m}\) = \(h_{6 m}(t, t+4) - h_{6 m}(t, t+4)\) in the text. That is, the one year excess yield between one year bond and six months bonds rolling over.
Table A.3: (Eight-Periods) Sample Autocorrelations
(1953:1-1986:IV)

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$a. \Delta \left( \frac{Y_{t+2}}{C_{t+2}} \right) = \frac{Y_{t+2}}{C_{t+2}} - \frac{Y_t}{C_t}$

$^b Yields of two-years zero coupon bonds.$

$^c B_{3yr} - r_{2yr} = h_{3yr}(t, \delta, t+\delta) - h_{2yr}(t, \delta, t+\delta)$ in the text. That is, two years excess holding period return between three years and two years bonds.

$^d r_{2yr} - B_{3yr} R_{6m} = h_{3yr}(t, \delta, t+\delta) - h_{6m}(t, \delta, t+\delta)$ in the text. That is, the two years excess yield between two-years bond and six-months bonds rolling over.

$^e r_{2yr} - B_{1yr} R_{1yr} = h_{3yr}(t, \delta, t+\delta) - h_{1yr}(t, \delta, t+\delta)$ in the text. That is, the two years excess yield between two-years bond and one-year bonds rolling over.
### Table A.4: (Twelve-Periods) Sample Autocorrelations
(1953:1–1986:IV)

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*a $\Delta (\frac{Y_{i+12}}{C_{i+12}}) = \frac{Y_{i+12}}{C_{i+12}} - \frac{Y_i}{C_i}$

*b Yields of three-years zero coupon bonds.

c $H_{3Yr} - B_{1Yr} - r_{3Yr} = k_{3Yr}(t, 12, t+12) - k_{3Yr}(t, 12, t+12)$ in the text. That is, three years excess holding period return between four years and three years bonds.

d $r_{2Yr} - H_{2Yr} - R_{1Yr} = k_{2Yr}(t, 12, t+12) - k_{2Yr}(t, 12, t+12)$ in the text. That is, the three years excess yield between three-years bond and one-year bond rolling over.

e $r_{2Yr} - H_{2Yr} - R_{3Yr+1Yr} = k_{2Yr}(t, 12, t+12) - k_{2Yr}(t, 12, t+12)$ in the text. That is, the three years excess yield between three-years bond and a combination of two-years and one-year bonds rolling over successively.
Bibliography


[38] Evans, Paul, "Finite Horizons, Infinite Horizons, and Stock Prices," working paper, Ohio State University, (December 1989).


