EFFECTS OF ICE AND MECHANICAL LOADS
FOR THREE SPECIES OF TREES

A THESIS

Presented in Partial Fulfillment of the Requirements for the Degree Master
of Science

by

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# TABLE OF CONTENTS

ACKNOWLEDGEMENTS ...................................................... ii

TABLE OF CONTENTS ...................................................... iii

LIST OF TABLES .......................................................... v

LIST OF FIGURES ........................................................ vii

INTRODUCTION ............................................................ 1

LITERATURE REVIEW ..................................................... 4
  Temperature ............................................................. 4
  Moisture content ...................................................... 5
  Wood defects and decay .............................................. 6
  Branch surface area .................................................. 6
  Crotch angle ............................................................ 7
  Wood fiber ............................................................... 8
  Specific gravity ........................................................ 9

MATERIALS AND METHODS ............................................... 11
  Deflection by distributed loads on silver maple ............... 11
  Deflection by concentrated loads on silver maple .......... 17
  Surface area of the branch ........................................ 21
  External force on branches for three species ............... 23
  Necrosis on silver maple .......................................... 29
RESULTS AND DISCUSSIONS ........................................ 30

The comparisons of ice loads and concentrated loads on silver maple ........................................ 30
Crotch necrosis in silver maple ........................................ 34
The comparisons of branch strength for three species of trees ........................................ 37
I. Pin oak ..................................................................... 37
II. Silver maple .......................................................... 44
III. Japanese zelkova ..................................................... 47
IV. Overall comparisons ............................................... 50

SUMMARY .................................................................. 53
LITERATURE CITED .................................................... 56
LIST OF TABLES

Table 1. Branch diameter, crotch angle and average of ice thickness information on silver maple...........12

Table 2. The analysis of variation for differences in deflection among concentrated loads and ice loads at four different locations on silver maple...........30

Table 3. Correlation coefficients ($R^2$) and associated significance levels with the best fitting lines describing deflection caused by distributed loads at four locations.................................32

Table 4. The comparisons of crotch angle, specific gravity and position of failure for three species of trees.................................................................38

Table 5. Breaking force information and branch parameters of 20 pin oak branches chosen at random.................................................................41

Table 6. Breaking force information and branch parameters of 20 silver maple branches chosen at random.................................................................46
Table 7. Breaking force information and branch parameters of 20 Japanese zelkova branches chosen at random. 49
LIST OF FIGURES

Fig. 1. The branch was fitted to the post with 2 bolts (1.5 cm in diameter) spaced 18 cm apart.............13

Fig. 2. The tape was held vertically by a plumb bob........15

Fig. 3. The technique used to measure the deflection caused by an ice load. Original position A1, A2, A3 and A4 were the loading points. Deflection is shown by the dotted lines with B1, B2, B3 and B4 representing points A1, A2, A3 and A4 in their respective deflected locations. Branch sections are bolted to the pole at 2 locations.............16

Fig. 4. A schematic diagram of the technique used to measure the deflection caused by a concentrated load at 4 points (A1, A2, A3 and A4 respectively). Deflection is shown by the dotted lines with B1, B2, B3 and B4 representing points A1, A2, A3 and A4 in their respective deflected locations. Branch sections are bolted to the pole at two locations..19
Fig. 5. A detailed diagram of the forces illustrated in Fig. 4. The weight indicated on the scale was only half of the actual weight to which the branch was subjected. .......................... 20

Fig. 6. The technique of measuring branch surface area. Branches were cut into a series of cylinders and the surface area of each cylinder was calculated and added to obtain the total surface area.............. 22

Fig. 7. A schematic diagram illustrates a concentrated load added at 20% of the horizontal distance from the crotch to the end of the branch. The electronic scale was powered by the trunk battery............ 24

Fig. 8. A diagram depicts the forces applied to branches in Equation 3. The procedure was used to calculate bending stress for the branches which broke at the crotch. F is external force. L_1 is the horizontal distance from the crotch to the loading point....... 26

Fig. 9. A diagram depicts the forces applied to branches in Equation 4. This technique was used if the branch broke somewhere between the crotch and the loading point........................................... 28

Fig. 10. This regression line predicts the relationships between crotch angle and crotch condition for silver maple.................................................... 36
Fig. 11. a. This Japanese zelkova branch broke by the combination of bending, tensile and shear stress..............................................39
b. This Japanese zelkova branch broke right at the crotch..............................................39

Fig. 12. Most pin oak branches broke when bent below the horizontal line..............................40

Fig. 13. The comparisons between shear and bending stress. For a cross section, the shear stress is zero at both top and bottom, and is maximum in the central part. Bending stress is zero in the center and maximum at both top and bottom..........................43

Fig. 14. Most silver maple branches broke when bending down toward the horizontal line..................45

Fig. 15. Most Japanese zelkova branches broke before bending to the horizontal line..................48
INTRODUCTION

Winter snow damage in trees is a serious problem in the United States. Mechanical injury caused by excessive loads of ice or snow can be seen in many areas. For example, severe snow loads causing a 31.5% average canopy loss in Siberian elm (Ulmus pumila) have been reported (5). Pirone reported that the weight of an ice coated twig was approximately 40 times that of the twig alone after the ice had melted (23). This excessive weight usually causes branch or twig breakage. For example, in Wooster, Ohio, on December 1-2, 1974, 56 cm of heavy, wet snow were measured. The snow adhered to the branches of many trees and weighted them down causing very severe breakage (10). The breakage was so bad that many trees had to be replanted. The fractured ends of branches are also a good place for disease or insect entry.

There has been much research concerned with the
physiological injuries of trees from low temperature (15,18,20,24), but very few researchers have been concerned with the physical breakage due to overloading by ice or snow.

Physical breakage or injury varies by species, size and shape of the trees. Generally speaking, the amount of physical breakage on a tree during the winter may be influenced by genetics and the environment to which the tree is subjected. Although genetic factors vary between and among species, they are important parameters essential to the investigation of wood strength. These environmental and genetic factors are: temperature, moisture content, wood defects and decay, surface area of the branch, crotch angle, wood fiber and specific gravity. The species, bark texture and branch size (diameter) might also be important.

In this study, the relationships among the branch strength of various species, branch surface area, crotch angle and branch size were tested. Two experiments were conducted. One compared the influence of natural loads caused by ice on branches and artificial loads which were imposed by adding weight to branches. The purpose was to find a location on the branch that could be
loaded by artificial weights (a concentrated load), and cause a deflection similar to the natural load of ice on branches (a distributed load).

The second experiment measured the force required to break branches. The objective of this experiment was to investigate the relationships among the sizes of branches, crotch angles and species. Trees that can resist storms have stronger branch structure and can hold more snow, wind or ice loads. This kind of tree can be suggested as a good city tree to resist storm loading.
LITERATURE REVIEW

It is generally known that some wood is stronger than other wood (25). The physical properties contributing to branch strength will be discussed separately.

Temperature

When temperature increases, the strength of wood decreases; conversely when temperature decreases, the strength of wood increases (16). With an estimated moisture content of about 12%, there will be an increase of 0.0033-0.005% of wood strength for each 0.52°C decrease in temperature below 21°C (13). The reverse is also noted and the magnitude of the change is the same. Strength is thus measured at a standard temperature of 21°C.
Moisture Content

Moisture content of wood is the weight of water in wood expressed as percentage of the oven dry weight of wood (25). Greenwood is often defined as wood in which the cell walls are completely saturated with water (13). In general, the average percent moisture of greenwood is 30% water (13). Specific gravity depends on the moisture content of wood, thus, changes in moisture content affect the structural properties of wood (16). Drying greenwood to 12% moisture content may double the strength of the wood, and drying to 5% moisture content may triple the wood strength (16).

Actual changes in structural properties do not begin to show until the moisture content is less than the "fiber saturation point" (16). The fiber saturation point is the moisture content at which cell walls are completely saturated and where water is not found in cell cavities (13). Specific gravity is constant during the drying process until the fiber saturation point is reached at about 30% moisture content (15). Below this moisture content, the volume of wood decreases with concurrent increases in specific gravity and structural properties.
Wood Defects and Decay

Occasionally, the branch has physical defects, such as a place where insects or diseases have damaged the tissue (21). Even a hole made by birds or incompletely healed branch stubs are defects which will cause the branch to crack at that point if enough pressure is applied. Some species such as maple, hickory and white ash are often damaged by woodpeckers (13). These holes or cracks will also be a very good entry point for insects and diseases which further weaken the branch. Decay acts on either the cellulose or lignin components of wood (21) which weakens branches to considerable extent.

Branch Surface Area

From a physical point of view, the internal resistance to deformation of a body is called stress (27). Stress is the external force divided by the area which carries the force. Ice or snow adheres to the branch by surrounding the branch surface. Therefore, increasing the branch surface area increases the snow or ice loads which can accumulate on the branch. The largest stress which can develop under severe loading conditions may be far above the maximum load that branches can support if the branch
surface area is high.

**Crotch Angle**

In general, a limb will break without the crotch splitting, but sometimes the crotch will fail first (19). The crotch angle is an important factor in determining the strength of the branch (19). The narrower the crotch angle, the weaker the union is if other factors remain constant (19). Therefore, the v-shaped crotch of a branch is undesirable. When the crotch is narrow and the growth is rapid, the cortches, phloem and cambium are forced together (8). In such cases, the bark (cortex) between the crotch is subjected to considerable pressure by the increasing diameter growth of the branch. This results in the death of parts of the bark which are forced together and tightly squeezed (8). There is no actual union in the crotch between the branches because of the necrotic tissue and this weakens the crotch.

Another very important factor in the strength of the branch is the interruption of wood continuity and change in the direction of wood fiber (13). The burial of a branch base is done by secondary xylem. When branch
xylem is imbedded into the wood of the trunk, the branch is strongly attached to the old wood of the trunk. All the tissue within the cambium is imbedded into the wood of the tree trunk when new layers of secondary xylem grow. The cambium is moved outward by the new xylem. A distortion of the xylem occurs when the base of the branch is stripped of its phloem (8). The region where the trunk and branch connect also forms many thick-walled parenchyma cells which are not fastened tightly (8). This flaw may cause the branch to break when pressure is applied.

When cambium of a main trunk extends into a branch, two types of burial can be observed. One is a loose knot which occurs where the cambium encircles the dead branch. The burial of the dead branch is poorly attached. This is one of the factors that may cause the crotch angle to be a weak point. On the other hand, a tight knot is formed when the caubium encircles a live branch. This kind of knot is structurally stronger than a loose knot.

Wood Fiber

Wood is composed of many different cell fibers. Chemists have shown that the cell walls of woody plants
are composed of approximately 50% cellulose, 25% lignin, 24% hemicellulose and extractable materials and less than 1% inorganic salts (2). The cellulose can break into finer microfibrils (4).

All cellulose fibers have the same structure (2). They are connected together by the middle lamella which primarily contains lignin. Lignin deposition is considered as a filling or cementing process (16). The bonding of lignin links the fiber to the middle lamella which in turn resists the external force that can separate the fibers (16).

Specific Gravity

In order to compare species, specific gravity is used as a standard reference, rather than density (9,13). Usually, foresters use the specific gravity of wood based on the oven dry weight and the volume at a specified moisture content. The reference density of water is at 4°C where the density of water is 1.0000 g per cc..

The specific gravity of wood gives a reliable measure of the amount of wood substance in any wood (16). The strength of a given wood species is proportional to the
specific gravity of that species. When specific gravity increases, the strength of wood increases (13). Strength value within a species may vary somewhat from the average value of that species, thus the relationship is only an estimate (16). This also suggests potential for selecting individuals within a species. Species which have greater specific gravity should have less storm damage if other factors remain constant.
MATERIALS AND METHODS

Deflection by Distributed Loads on Silver Maple

Five major branches with an attached trunk were chosen at random from the nursery of the Department of Horticulture at The Ohio State University in December, 1981. Crotch angle and branch diameter were measured by a protractor and diameter tape respectively (Table 1). Each trunk had 2 holes (1.5 cm in diameter) drilled 18 cm apart through the trunk. A solid wood post (2 m height, 10 cm wide) was fitted with two 1.4 cm bolts spaced 18 cm apart (Fig. 1). This allowed the trunk to be bolted vertically.

Branch weight was measured after drilling. It was then bolted to the post and a level was used to adjust its position to the vertical. The horizontal distance was measured from the junction of the stem and the main branch (i.e. crotch angle) to the tip of the branch.
Table 1. Branch diameter, crotch angle and average of ice thickness information on silver maple.

<table>
<thead>
<tr>
<th>Branch diameter (cm)</th>
<th>Crotch angle (degrees)</th>
<th>Average of ice thickness (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>30</td>
<td>.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.91</td>
</tr>
<tr>
<td>2.3</td>
<td>60</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.25</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.16</td>
</tr>
<tr>
<td>2.6</td>
<td>28</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.66</td>
</tr>
<tr>
<td>2.9</td>
<td>85</td>
<td>.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.99</td>
</tr>
<tr>
<td>4.4</td>
<td>60</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.60</td>
</tr>
</tbody>
</table>
Fig. 1. The branch was fitted to the post with 2 bolts (1.5 cm in diameter) spaced 18 cm apart.
This distance was divided equally into 5 segments. The locations were marked on the branch by white paint. A 15 m. tape was laid horizontal directly below the main branch. Vertical distance from the branch to the horizontal tape was measured at the four marked points by another 15 m. tape. This tape was held vertically by a plumb bob (Fig. 2).

This experiment was done at temperatures below freezing. A very fine water spray was used to wet the branch. Because of the low temperature and small water particles, the water froze when sprayed on the branch. After a random amount of ice formed, the vertical distance from the branch to the fixed tape was measured again at each point (Fig. 3). The diagram of this experiment is shown in Fig. 3.

The wood post was solid enough so that the total weight of the ice and branch didn't cause any deflection of the post. The difference between the distance from the branch to the horizontal tape before and after ice formation was the deflection caused by ice.
Fig. 2. The tape was held vertically by a plumb bob.
Fig. 3. The technique used to measure the deflection caused by an ice load. Original position
A1, A2, A3 and A4 were the loading points.
Deflection is shown by the dotted lines
with B1, B2, B3 and B4 representing points A1, A2, A3 and A4 in their respective deflected locations. Branch sections are bolted to the pole at 2 locations.
Deflection = height before ice loading - height after ice loading

(1)

After measuring the deflection at these four points, the branch was removed from the post and the weight of the ice and branch was measured. The weight of the ice and branch minus the branch weight was the net weight of the ice. This was the cause of the deflection of the branch.

The addition of ice to the branch was repeated at least 4 times for each branch before the branch was used in the next experiment. This was to determine the deflection caused by the concentrated load.

Deflection by Concentrated Loads on Silver Maple

The same five branches used to determine deflection with an ice load were used to test the deflection caused by a weight that was concentrated at the four points marked on the branch.
Again, the branch was bolted to the post (Fig. 4). A cement block of about 40 Kg with a built-in vertical board (80 cm height, 20 cm wide) was used as a dead weight. A pulley was attached to the branch at the specified places and a rope was run through the pulley, with one end connected to the scale and the other to a fixed hook (Fig. 4). External force was applied by the pulley on the rope and was measured by the Chatillon Model 776 scale (Fig. 4). A detailed diagram in force applied is shown in Fig. 5. Because the external force was applied by two sections of rope, the weight that the scale indicated was only half of the actual weight to which the branch was subjected (Fig. 5).

Weight was added to the branch at location one until the same deflection from the ice loading test was achieved. This was to compare the ice load and concentrated load at location one. This procedure was repeated for all 4 locations for the five branches.

An F-test was used to analyze the differences between concentrated loads and ice loads at four different locations on the branch. An analysis of variance was performed. Stepwise regression analysis was then used to construct a least square line when the R-square was
Fig. 4. A schematic diagram of the technique used to measure the deflection caused by a concentrated load at 4 points (A1, A2, A3 and A4 respectively). Deflection is shown by the dotted lines with B1, B2, B3 and B4 representing points A1, A2, A3 and A4 in their respective deflected locations. Branch sections are bolted to the pole at two locations.
Fig. 5. A detailed diagram of the forces illustrated in Fig. 4. The weight indicated on the scale was only half of the actual weight to which the branch was subjected.
greater than 50.

**Surface Area of the Branch**

*Acer saccharinum* (silver maple), *Quercus palustris* (pin oak) and *Zelkova serrata* (Japanese zelkova) branches were measured. Five branches of each species were cut into sections as shown in Fig. 6 so that a uniform shape could be obtained. These small sections were assumed to have a uniform cylindrical shape. The average diameter and length for each section was measured. The surface area of each branch was calculated as follows (Equation 2):

\[
\text{Surface Area} = \pi \sum \left( D_1^2 L_1 + D_2^2 L_2 + \ldots + D_n^2 L_n \right)
\]

(2)

Where:

- \( L \) = the length of the branch section
- \( D \) = the average diameter of the branch section
Fig. 6. The technique of measuring branch surface area. Branches were cut into a series of cylinders and the surface area of each cylinder was calculated and added to obtain the total surface area.
External Force on Branches for Three Species

Pin oak, silver maple and Japanese zelkova were used in this study. Small size trees of Japanese zelkova were generously provided by Mr. Bill Richards from Circleville, Ohio. Small size trees of pin oak and silver maple were obtained from the nursery of the Department of Horticulture at The Ohio State University. Large size trees of these three species were generously provided by Slemons Nursery Company, Plain City, Ohio. An Electroscale Weightmeter Model DR-102 and a load cell, manufactured by Absco Scale Inc., Columbus, Ohio, were used to measure the force added to the branch. The electronic scale and the load cell were loaned to us by the U.S.D.A. Forest Service. Different branch diameters and crotch angles were chosen at random, measured and recorded. A cable was tied to a point, which was 1/5 of the total length of the horizontal distance from the crotch angle to the end of the branch, via a pulley to the hand winch (puller) (Fig. 7). This was because the ice load and concentrated loads gave similar deflections at this location (20% of the total length of the horizontal distance from the crotch angle to the end of the branch) on silver maple.
Fig. 7. A schematic diagram illustrates a concentrated load added at 20% of the horizontal distance from the crotch to the end of the branch. The electronic scale was powered by the trunk battery.
The hand winch was connected to the load cell. This load cell was then tied to a nearly fixed object (Fig. 7). Force was applied to the limb by the hand winch until the branch started to break. The maximum reading when the branch started to break was recorded in kg. The horizontal distance from the crotch angle to the cable, which was vertical to the ground, was measured. These procedures were repeated for all three species with a total of 20 branches per species for the large trees and 50 branches per species for the smaller trees.

If the branch broke at the crotch, the bending stress at the crotch was calculated in Newton per square cm (N.T./cm²) as follows (Fig. 8):

\[
S_b = \frac{Mc}{I} = \frac{(FCos\theta\frac{l}{2})r/2}{\pi r^4} = \frac{(FCos\theta L_1/Cos\theta)r/2}{\pi r^4} = \frac{2FL_1}{\pi r^3}
\]

(3)
Fig. 8. A diagram depicts the forces applied to branches in Equation 3. The procedure was used to calculate bending stress for the branches which broke at the crotch. F is external force. L₁ is the horizontal distance from the crotch to the loading point.
Where:

\[ S_b = \text{bending stress} \]

\[ C = \text{the distance from the netural surface to the top of the branch} \]

\[ = \text{branch radius} \]

\[ M = \text{moment} \]

\[ I = \text{the moment of inertia of the cross-sectional area} \]

\[ F = \text{external force (N,T.)} \]

\[ L_1 = \text{the horizontal distance from the crotch to the loading point (m)} \]

If the branch broke somewhere between the crotch and the loading point, the bending stress was calculated as follows (Fig. 9):

\[ S_b = \frac{2FL_2}{\pi r^3} \quad (4) \]
Fig. 9. A diagram depicts the forces applied to branches in Equation 4. This technique was used if the branch broke somewhere between the crotch and the loading point. $F$ is the external force. $L_2$ is the horizontal distance from the broken point to the loading position.
Where:

\[ F = \text{external force (N.T.)} \]
\[ L_2 = \text{the horizontal distance from the broken point to the loading position (m)} \]
\[ r = \text{the distance from the neutral surface to the top of the branch (cm)} \]
\[ = \text{the radius of the broken point of the branch (cm)} \]

Again, the stepwise regression analysis with maximum R-square was used to build the best fitting line for the moment required to cause failure of the branch or crotch.

Necrosis on Silver Maple

Fifty small size silver maple branches and 20 large size branches (70 branches total) were chosen for the experiment. After the branches were broken, the presence or absence of necrosis was recorded. If the branch didn't break at the crotch, the crotch was assumed to contain no necrotic tissue.
RESULTS AND DISCUSSIONS

The Comparisons of Ice Loads and Concentrated Loads on Silver Maple

To compare the concentrated weights at four specified locations of a branch with that of ice load causing the same deflection, an F-test at the 10% level of confidence interval was used. The results show significant differences among the four locations (Table 2).

Table 2. The analysis of variation for differences in deflection among concentrated loads and ice loads at four different locations on silver maple.

<table>
<thead>
<tr>
<th>Source of variation</th>
<th>Degree of freedom</th>
<th>Sum of square</th>
<th>Mean square</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Location</td>
<td>3</td>
<td>1005.5</td>
<td>335.2</td>
<td>12.6*</td>
</tr>
<tr>
<td>Error</td>
<td>53</td>
<td>1410.32</td>
<td>26.2</td>
<td></td>
</tr>
</tbody>
</table>

*: significant at the 10% level of confidence interval
This analysis shows that weights concentrated at four different locations and ice load weights cause significantly different deflections. Only a concentrated load applied to the branch at location 1 caused a deflection similar to the corresponding ice load. Location 1 is 20% of the horizontal distance from the crotch to the tip of the branch. At locations 2, 3 and 4, the concentrated load caused more deflection than the corresponding ice weight.

Regression analysis was used to build a best fitting line (Table 3). The relationships between concentrated and ice loads is not significant for locations 1 and 2. For locations 3 and 4, the best fitting lines are given for concentrated and ice loads as shown in table 3.

Since there was no significant difference between deflections caused by ice loads and the concentrated loads which were added to location 1, the data were combined. R-square statement was used in an effort to define the genetic and environmental factors as a function of deflection. Eighty-seven percent of the variation in deflection (\( R^2 = .87 \)) was explained by four factors: crotch angle, the interaction of branch diameter and crotch angle, branch diameter and the weight
Table 3. Correlation coefficients ($R^2$) and associated significance levels with the best fitting lines describing deflection caused by distributed loads at four locations.

<table>
<thead>
<tr>
<th>Locations</th>
<th>Type of weight</th>
<th>Best fitting line</th>
<th>$R^2$</th>
<th>Significance level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Conc.</td>
<td>$-2.8 + 9<em>wt - .01</em>wt^2$</td>
<td>.07</td>
<td>N.S.</td>
</tr>
<tr>
<td></td>
<td>Dist.</td>
<td>$1.2 + 8<em>wt - .02</em>wt^2$</td>
<td>.08</td>
<td>N.S.</td>
</tr>
<tr>
<td>2</td>
<td>Conc.</td>
<td>$26.8 - 4.3<em>wt + .3</em>wt^2$</td>
<td>.14</td>
<td>N.S.</td>
</tr>
<tr>
<td></td>
<td>Dist.</td>
<td>$3.2 + 5<em>wt - .43</em>wt^2$</td>
<td>.31</td>
<td>N.S.</td>
</tr>
<tr>
<td>3</td>
<td>Conc.</td>
<td>$-25.1 + 9.8<em>wt - .4</em>wt^2$</td>
<td>.86</td>
<td>.0001*</td>
</tr>
<tr>
<td></td>
<td>Dist.</td>
<td>$7.3 + .03*wt^2$</td>
<td>.84</td>
<td>.0001*</td>
</tr>
<tr>
<td>4</td>
<td>Conc.</td>
<td>$-14 + 12.6<em>wt - .5</em>wt^2$</td>
<td>.84</td>
<td>.0001*</td>
</tr>
<tr>
<td></td>
<td>Dist.</td>
<td>$13.0 + 1.7*wt^2$</td>
<td>.69</td>
<td>.0004*</td>
</tr>
</tbody>
</table>

*: significant at the 10% level of confidence interval

Conc.: concentrated load
Dist.: distributed load (ice load)
wt: weight added to the branch
N.S.: non-significant at the 10% level of confidence interval
expressed as the square of weight(g) per square cm. A regression equation with these four factors resulted in the best equation (Equation 5).

\[
\text{Deflection} = 9.8 - 0.9\times C.A. + 0.2\times D.C.A. + 480\times A^2 + 46.9\times 1/D^3
\]

Where;

\[C.A. = \text{crotch angle (degrees)}\]
\[D.C.A. = \text{the interaction of branch diameter and crotch angle}\]
\[A^2 = \text{the square of weight per square cm (g/cm}^2)^2\]
\[D = \text{branch diameter (cm)}\]

This model shows that the narrower the crotch, the larger the deflection for a given weight. Equation 5 also suggests that the more surface area that a branch has the less the deflection which results from a given weight.

Besides crotch angle, branch diameter, the interaction of branch diameter and crotch angle and the square of weight per square cm, there are other parameters which influence branch strength, such as, temperature, specific gravity, moisture content and physical defects. In this study, because the same species (silver maple) was used
and sampled at the same time and the same place, and because the moisture content and specific gravity were assumed to be equal the effects of these parameters were not considered. This experiment was carried out at temperatures below freezing (about -10 to 0°C). According to the Forest Products Laboratory, this would result in less than 0.09% change in branch strength during the testing time (13). This was considered as negligible. The five samples didn't have any known defect or decay that would seriously weaken the branch. Thus, effects from defects and decay, moisture content, temperature and specific gravity were assumed to be essentially equal and of no consequence.

Crotch Necrosis in Silver Maple

Twenty six percent of the branches showed necrotic tissue in the crotch. Most of the necrotic tissue appeared in narrow crotches which ranged from 17 to 55 degrees. Using the stepwise regression with maximum R-square to model this, the best fitting line is as follows (Equation 6):

\[
\text{Necrosis in the crotch} = 0.04 + 0.06 \times \text{C.A.} - 0.0004 \times \text{C.A.}^2
\]

(6)
Where:

C.A. = the crotch angle
Necrotic crotch = 2
Healthy crotch = 1

Only 41.5% of variation in crotch angle necrosis can be explained by the model. Although the R-square is not very high, it is significant. Fig. 10 is a graphic representation of the line for the equation. A crotch angle of 35 degrees or less in silver maple will result in at least a 50% chance of necrotic tissue occurring in the crotch. Some wider angles such as 55 degrees will still show necrotic tissue. This might be due to other injuries such as winter injury, disease, insect or mechanical injuries, instead of growth pressure forcing tissue collapse as suggested by MacDaniels (19). Crotch angles greater than 70 degrees appear to show increased chances of damage (Fig. 10). This is again likely to be due to mechanical injuries.
Fig. 10. This regression line predicts the relationships between crotch angle and crotch condition for silver maple.
The Comparisons of Branch Strength for Three Species of Trees

I. Pin Oak

Pin oak had a wider average crotch angle (76°) than silver maple (48°) and Japanese zelkova (28°) (Table 4).

Thirty percent of the branches of pin oak broke at the crotch while 70% of the branches broke between the crotch and the loading point (Table 4). Fifteen percent of the breakage occurred at branch defects. If branches didn't break at the crotch, the breakage seemed to be caused by the combination of bending, shear and tensile stress (Fig. 11). The branch then tore back to the main trunk if additional external force was added. An interesting phenomenon was observed. Because of the wide crotch, most of the branches didn't break until the branch bent well below the horizontal line (Fig. 12).

Weight needed to cause failure in the branch, branch diameter and angle of attachment are given (Table 5). For branches that broke right at the crotch, the bending
Table 4. The comparisons of crotch angle, specific gravity and position of failure for three species of trees.

<table>
<thead>
<tr>
<th>Species</th>
<th>Average of crotch angle</th>
<th>$S_d$</th>
<th>Specific gravity</th>
<th>Position of breakage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Crotch</td>
</tr>
<tr>
<td>pin oak</td>
<td>78.5</td>
<td>5.2</td>
<td>.75</td>
<td>30%</td>
</tr>
<tr>
<td>silver maple</td>
<td>47.7</td>
<td>4.3</td>
<td>.44</td>
<td>90%</td>
</tr>
<tr>
<td>Japanese zelkova</td>
<td>28.3</td>
<td>4.0</td>
<td>z</td>
<td>5%</td>
</tr>
</tbody>
</table>

$x$: based on oven dry weight and volume at 12% moisture content (1).

$y$: the standard deviation of the crotch angle.

$z$: no information source.
Fig. 11.a This Japanese zelkova branch broke by the combination of bending, tensile and shear stress.

Fig. 11.b This Japanese zelkova branch broke right at the crotch.
Fig. 12. Most pin oak branches broke when bent below the horizontal line.
Table 5. Breaking force information and branch parameters of 20 pin oak branches chosen at random.

<table>
<thead>
<tr>
<th>Position of failure</th>
<th>Crotch angle (degrees)</th>
<th>Branch diameter (cm)</th>
<th>Breaking force (N.T.)</th>
<th>Distance (m)</th>
<th>Bending stress (N.T./cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch</td>
<td>10</td>
<td>3.3</td>
<td>428.26</td>
<td>.30</td>
<td>19.69x</td>
</tr>
<tr>
<td>Branch</td>
<td>12</td>
<td>3.8</td>
<td>1029.00</td>
<td>.20</td>
<td>14.60y</td>
</tr>
<tr>
<td>Crotch</td>
<td>15</td>
<td>6.9</td>
<td>2962.54</td>
<td>.40</td>
<td>19.99y</td>
</tr>
<tr>
<td>Crotch</td>
<td>25</td>
<td>5.1</td>
<td>1340.64</td>
<td>.54</td>
<td>19.99y</td>
</tr>
<tr>
<td>Crotch</td>
<td>25</td>
<td>6.1</td>
<td>1617.00</td>
<td>.67</td>
<td>24.50x</td>
</tr>
<tr>
<td>Branch</td>
<td>35</td>
<td>6.4</td>
<td>3434.90</td>
<td>.21</td>
<td>14.01x</td>
</tr>
<tr>
<td>Branch</td>
<td>35</td>
<td>7.8</td>
<td>4166.00</td>
<td>.39</td>
<td>40.18y</td>
</tr>
<tr>
<td>Crotch</td>
<td>43</td>
<td>4.6</td>
<td>2142.28</td>
<td>.35</td>
<td>38.22y</td>
</tr>
<tr>
<td>Crotch</td>
<td>45</td>
<td>6.9</td>
<td>2619.54</td>
<td>.86</td>
<td>35.28x</td>
</tr>
<tr>
<td>Branch</td>
<td>50</td>
<td>5.6</td>
<td>2129.54</td>
<td>.65</td>
<td>40.18y</td>
</tr>
<tr>
<td>Crotch</td>
<td>55</td>
<td>5.1</td>
<td>3122.28</td>
<td>.50</td>
<td>59.78x</td>
</tr>
<tr>
<td>Branch</td>
<td>55</td>
<td>8.9</td>
<td>7029.54</td>
<td>.57</td>
<td>28.42x</td>
</tr>
<tr>
<td>Branch</td>
<td>65</td>
<td>11.9</td>
<td>17818.72</td>
<td>.57</td>
<td>30.67x</td>
</tr>
<tr>
<td>Branch</td>
<td>75</td>
<td>11.2</td>
<td>17928.12</td>
<td>.68</td>
<td>36.26x</td>
</tr>
<tr>
<td>Branch</td>
<td>75</td>
<td>4.6</td>
<td>2917.46</td>
<td>.31</td>
<td>47.04x</td>
</tr>
<tr>
<td>Branch</td>
<td>78</td>
<td>3.6</td>
<td>1890.42</td>
<td>.27</td>
<td>54.88x</td>
</tr>
<tr>
<td>Branch</td>
<td>85</td>
<td>3.5</td>
<td>2371.60</td>
<td>.35</td>
<td>98.98x</td>
</tr>
<tr>
<td>Branch</td>
<td>100</td>
<td>4.6</td>
<td>2739.10</td>
<td>.43</td>
<td>65.66x</td>
</tr>
<tr>
<td>Branch</td>
<td>110</td>
<td>3.9</td>
<td>2289.28</td>
<td>.24</td>
<td>48.02x</td>
</tr>
<tr>
<td>Branch</td>
<td>115</td>
<td>6.4</td>
<td>6860.00</td>
<td>.32</td>
<td>42.34</td>
</tr>
</tbody>
</table>

x: breakage caused by the combination of bending, shear and tensile stress.

y: bending stress (N.T./cm²)

z: the horizontal distance from broken position to loading point.
stress was calculated. For a cross section, the shear stress is zero at both top and bottom, and is maximized in the central part (Fig. 13). On the contrary, the bending stress is maximized at both top and bottom, and is zero in the center. Cracks often showed from the upper side of the branch. At that point, shear stress is zero, therefore, we only calculated the bending stress and ignored the shear stress which was considered negligible.

Branches that didn't break at the crotch had a stronger crotch than the branch itself. The bending stress for this kind of breakage was calculated too. One phenomenon noted was that most wide branches didn't break at the crotch but failure along the branch (Table 5).

In order to make the results comparable, the bending stress was calculated using the distance from crotch to loading point ($L_1$) times 2 times the weight required to cause failure ($F$) and divided by the cube of the branch radius ($r$) times $\pi$ (Equation 3) (26) were computed when the crotch failed. For branches that didn't break at the crotch, the bending stress was determined as the distance from broken point to loading point ($L_2$) times the weight that was required to cause failure ($F$) times 2 and divided by the cube of the radius at the broken position and $\pi$ (Equation 4) (26). Arranging the results in order of width of crotch angle
Fig. 13. The comparisons between shear and bending stress. For a cross section, the shear stress is zero at both top and bottom, and is maximum in the central part. Bending stress is zero in the center and maximum at both top and bottom.
from the narrowest (15 degrees) to the widest (50 degrees), it seems that there is a tendency for the narrowest crotch to take less bending stress.

II. Silver Maple

Silver maple had a 48° average crotch angle which is intermediate between the pin oak (77 degrees) and Japanese zelkova (28 degrees) (Table 4). Ninety percent of the silver maple branches broke at the crotch and only 2 out of 20 branches broke at a flaw along the branch. If the branch didn’t break at the crotch, the combination of shear, tensile and bending stress appeared to break the branch. The breakage didn’t tear along the limb to the main trunk if additional force was added continuously as pin oak did. Most of the branches broke when the branch approached the horizontal line (Fig. 14).

Arranging the results in order of increasing crotch angles, bending stress increased with wider crotch angles (Table 6). From the previous study in silver maple, we know that crotch angles below 35 degrees are likely to show necrotic areas. Therefore, the crotch angles were grouped into two groups: 0-40 degrees and 45-90 degrees. There was a more than 200% increase in allowable bending stress when crotch angle increased from less than 40
Fig. 14. Most silver maple branches broke when bending down toward the horizontal line.
Table. 6. Breaking force information and branch parameters of 20 silver maple branches chosen at random.

<table>
<thead>
<tr>
<th>Position of failure angle (degrees)</th>
<th>Crotch</th>
<th>Branch diameter (cm)</th>
<th>Breaking force (N.T.)</th>
<th>Distance (m)</th>
<th>Bending stress (N.T./cm²)</th>
<th>Bending stress average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crotch 12</td>
<td>2.7</td>
<td>552.72</td>
<td>.12</td>
<td>17.15³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 20</td>
<td>3.6</td>
<td>948.64</td>
<td>.22</td>
<td>19.40³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 35</td>
<td>9.1</td>
<td>2277.54</td>
<td>1.58</td>
<td>33.71³</td>
<td>19.88</td>
<td></td>
</tr>
<tr>
<td>Crotch 40</td>
<td>8.3</td>
<td>993.72</td>
<td>.34</td>
<td>21.56³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 40</td>
<td>5.1</td>
<td>890.82</td>
<td>.76</td>
<td>25.97³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 40</td>
<td>6.4</td>
<td>1915.90</td>
<td>.30</td>
<td>11.10³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Branch 40</td>
<td>7.9</td>
<td>3176.18</td>
<td>.30</td>
<td>9.80³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 45</td>
<td>2.3</td>
<td>806.54</td>
<td>.10</td>
<td>33.71³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 50</td>
<td>3.8</td>
<td>1318.10</td>
<td>.24</td>
<td>29.40³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 50</td>
<td>7.4</td>
<td>1879.64</td>
<td>.85</td>
<td>20.05³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 50</td>
<td>9.4</td>
<td>3318.10</td>
<td>1.22</td>
<td>24.76³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Branch 55</td>
<td>3.1</td>
<td>657.58</td>
<td>.24</td>
<td>26.95³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 60</td>
<td>3.8</td>
<td>1180.90</td>
<td>.30</td>
<td>32.93³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 62</td>
<td>6.6</td>
<td>2485.28</td>
<td>.41</td>
<td>56.64³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 65</td>
<td>3.3</td>
<td>684.04</td>
<td>.36</td>
<td>34.79³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 65</td>
<td>4.6</td>
<td>801.64</td>
<td>.88</td>
<td>37.04³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 70</td>
<td>3.3</td>
<td>1799.28</td>
<td>.17</td>
<td>43.31³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 75</td>
<td>3.6</td>
<td>2249.10</td>
<td>.17</td>
<td>41.75³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 75</td>
<td>3.0</td>
<td>1808.10</td>
<td>.39</td>
<td>119.68³</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Crotch 90</td>
<td>7.4</td>
<td>2209.90</td>
<td>1.41</td>
<td>39.10³</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

x: breakage caused by the combination of bending, shear and tensile stress.
y: bending stress (N.T./cm²)
z: the horizontal distance from broken position to loading point.
degrees to more than 40 degrees.

Only 2 out of 20 branches (10%) didn’t break at the crotch. This suggests that silver maple had a weak crotch with branches stronger than the crotches.

III. Japanese Zelkova

Japanese zelkova had the narrowest average crotch angle (28 degrees) of the three species studied. Only one out of twenty branches broke at the crotch even though the crotch angle is narrow by silver maple standards. Eight of them (40%) broke at a little node and the others broke at flaws along the branch. Moreover, if the branch did not break at the crotch, the break tore to the main trunk if additional external force was applied.

Japanese zelkova broke before the branch was bent down to the horizontal line (Fig. 15). This might be because of the very narrow crotch.

Only one branch broke at the crotch. The bending stress shows the same tendency as in pin oak and silver maple. The allowable stress increases as the crotch angle grew wider (Table 7).
Fig. 15. Most Japanese zelkova branches broke before bending down to the horizontal line.
Table 7. Breaking force information and branch parameters of 20 Japanese zelkova branches chosen at random.

<table>
<thead>
<tr>
<th>Position of failure</th>
<th>Crotch angle (degrees)</th>
<th>Branch diameter (cm)</th>
<th>Breaking force (N.T.)</th>
<th>Distance (m)</th>
<th>Bending stress (N.T./cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch 5</td>
<td>5</td>
<td>5.8</td>
<td>886.90</td>
<td>.55</td>
<td>11.62x</td>
</tr>
<tr>
<td>Branch 18</td>
<td>5.8</td>
<td>1269.10</td>
<td></td>
<td>.27</td>
<td>8.92 ©</td>
</tr>
<tr>
<td>Branch 25</td>
<td>5.8</td>
<td>1269.10</td>
<td></td>
<td>.25</td>
<td>13.72x</td>
</tr>
<tr>
<td>Branch 25</td>
<td>3.8</td>
<td>586.04</td>
<td></td>
<td>.15</td>
<td>17.84x</td>
</tr>
<tr>
<td>Branch 30</td>
<td>2.8</td>
<td>513.52</td>
<td></td>
<td>.10</td>
<td>12.35x</td>
</tr>
<tr>
<td>Branch 30</td>
<td>4.6</td>
<td>1191.68</td>
<td></td>
<td>.22</td>
<td>13.72x</td>
</tr>
<tr>
<td>Branch 35</td>
<td>6.9</td>
<td>8018.36</td>
<td></td>
<td>.24</td>
<td>11.76x</td>
</tr>
<tr>
<td>Branch 35</td>
<td>4.8</td>
<td>1025.28</td>
<td></td>
<td>.15</td>
<td>13.92x</td>
</tr>
<tr>
<td>Branch 35</td>
<td>5.6</td>
<td>1269.10</td>
<td></td>
<td>.65</td>
<td>17.05x</td>
</tr>
<tr>
<td>Crotch</td>
<td>7.6</td>
<td>2254.00</td>
<td></td>
<td>.52</td>
<td>25.68y</td>
</tr>
<tr>
<td>Branch 35</td>
<td>11.9</td>
<td>5594.82</td>
<td></td>
<td>.10</td>
<td>21.95x</td>
</tr>
<tr>
<td>Branch 40</td>
<td>4</td>
<td>672.28</td>
<td></td>
<td>.33</td>
<td>24.40x</td>
</tr>
<tr>
<td>Branch 40</td>
<td>4.1</td>
<td>1003.52</td>
<td></td>
<td>.57</td>
<td>27.24x</td>
</tr>
<tr>
<td>Branch 40</td>
<td>4.8</td>
<td>908.46</td>
<td></td>
<td>.16</td>
<td>24.30x</td>
</tr>
<tr>
<td>Branch 50</td>
<td>3.0</td>
<td>806.54</td>
<td></td>
<td>.17</td>
<td>22.93x</td>
</tr>
<tr>
<td>Branch 55</td>
<td>4.8</td>
<td>2935.10</td>
<td></td>
<td>.38</td>
<td>24.70x</td>
</tr>
<tr>
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<td>2240.28</td>
<td></td>
<td>.16</td>
<td>31.06x</td>
</tr>
<tr>
<td>Branch 60</td>
<td>5.6</td>
<td>2145.31</td>
<td></td>
<td>.52</td>
<td>33.81x</td>
</tr>
<tr>
<td>Branch 65</td>
<td>4.6</td>
<td>1795.36</td>
<td></td>
<td>.36</td>
<td>33.81x</td>
</tr>
<tr>
<td>Branch 68</td>
<td>2.3</td>
<td>408.66</td>
<td></td>
<td>.12</td>
<td>20.48x</td>
</tr>
</tbody>
</table>

x: breaksage caused by the combination of bending, shear and tensile stress

y: bending stress

z: the horizontal distance from broken position to loading point

49
IV. Overall Comparisons

The crotch angle is an important parameter in branch strength and determines in part the magnitude of a concentrated load which 3 branch species can tolerate. Pin oak had the widest average crotch angle of species studies. This would explain at least in part why the pin oak branch is stronger. Silver maple had crotches which are weaker than the branch as 90% of branches broke at the crotch. Japanese zelkova had crotches which are stronger than the branches. Forty percent of the Japanese zelkova branches broke at a node on the branch. These frequent nodes might be one of the reasons that the branches of zelkova are weaker. As a result of the weaker branches, the crotches appear strong.

For pin oak, branch strength was dependent on the crotch angle. The wider the crotch, the more likely that the branch would fail rather than the crotch. The narrower the crotch, the more likely the crotch was to fail. From the literature review, the specific gravity is dependent on the species. Thus, it seems that the species is the determining factor for branch strength as specific gravity varies greatly with species.

Branches take more bending stress when crotch angle
increases. If the branches were stronger than the crotch, the failure would occur at the crotch rather than the branch.

The same parameters that influenced the moment can be found in all three species; therefore, a regression equation using species as a dummy variable allows us to compare these three species (Equation 7).

\[
\text{Moment} = -199 + 268*b_1 + 48.8*b_2 + 2.9*D.C.A. + 127.8*Diam - 34.3*D.S.Q. \tag{7}
\]

Where:
- \( b_1 = 0 \) and \( b_2 = 0 \) when silver maple appeared
- \( b_1 = 1 \) when pin oak appeared while \( b_2 = 0 \)
- \( b_2 = 1 \) when Japanese zelkova appeared while \( b_1 = 0 \)
- D.C.A. = the interaction of branch diameter and crotch angle
- D.S.Q. = the square of branch diameter (cm)
In this regression line, 69.9% of variation in moment can be explained ($R^2 = .69$). It is obvious from the above equation that the pin oak is much stronger than Japanese zelkova which is in turn, stronger than silver maple. Specific gravity may well be a major contributor to this difference among species.
SUMMARY

The deflection caused by ice and artificial weights concentrated on four points of a branch were compared. Weights added to the branch at 20% of the horizontal distance from the crotch to the tip of the branch had the similar deflection as an ice load distributed on the branch. Four important factors: crotch angle, the interaction of branch diameter and crotch angle, branch diameter and the square of weight per square cm can explain 87% of variation in deflection ($R^2 = .87$). This model equation shows that the narrower the crotch, the larger the resulting deflection for a given weight. It also shows that the more surface area that a branch has, the less the deflection. For a given species, the larger the branch diameter, the less the deflection.

One very interesting thing is that a very narrow crotch usually had some necrotic tissue in the crotch on
silver maple. Twenty-six percent of 70 silver maple branches were found to have necrotic areas in the crotch. Regression analysis indicated that a crotch angle of 35 degrees or less in silver maple will result in at least 50% chance of necrosis tissue being present in the crotch.

Concentrated loads required to cause failure of pin oak, silver maple and Japanese zelkova branches were compared. The results indicate that species is the determining factor for branch strength.

The overall comparisons for these three species are as follows:

1. The average crotch angle in descending order is pin oak (75°), silver maple (47°) and Japanese zelkova (28°).
2. All three species have the same tendency for bending stress required to cause failure of the crotch.
3. Silver maple tended to have crotches which were weaker than the branch as 90% broke at the crotch. Japanese zelkova had weaker branches than the crotch as 95% broke along the branch itself. Pin oak branches...
depended on the crotch angle to predict where failure would occur. Narrow crotch angles tended to fail at the crotch while wider crotches tended to be stronger than the branch.
LITERATURE CITED


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