A COMPARISON OF THE ACCOUNTING AND
INTERNAL RATES OF RETURN OF FIRMS
WITH NON-NEGATIVE GROWTH RATES AND INFINITE LIVES

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * *

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ACKNOWLEDGMENTS

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VITA

Personal

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LIST OF SYMBOLS

\( g \) = the annual growth rate in the real gross investment of the firm.

\( k \) = the tax rate on firm income.

\( p \) = the rate of change per year in all prices.

\( r \) = the real internal rate of return of all projects acquired by the firm.

\( n \) = the useful life of all firm projects.

\( q_i^j \) = the real cash flow (before payment for taxes and raw materials) provided at the end of the first project's \( i \)-th year \((i=1,2,\ldots,n)\) given that the firm adopts accounting policy \( j \). The dollar flow is measured in terms of the price level which prevailed at the end of year zero.

\( h \) = the parameter which defines the relationship between the common dollar cash flows (before payments for raw materials and taxes) of firm projects in their \( i \)-th and \((i+1)\)-th year. Specifically, the real cash flows of the firm's first project at the end of the project's \( i \)-th and \((i+1)\)-th year are related as follows:

\[
q_{i+1}^j = b q_i^j = b^i q_i^j \quad \text{for} \quad (i=1,2,\ldots,n-1)
\]

\( x \) = the parameter which defines the relationship between the common dollar cash flow (before payments for taxes and raw materials) of firm projects at the end of the projects' \((i+1)\)-th year and the common dollar amount of raw material purchases required by the projects' at the end of their \( i \)-th year \((i=0,1,2,\ldots,n-1)\). Specifically, the parameter \( x \) is such that the dollar amount of raw material required to be purchased by the firm for the first project at the end of the project's \( i \)-th year is:

\[
x q_{i+1}^j (1+p)^i \quad \text{for} \quad (i=0,1,2,\ldots,n-1) \quad \text{and} \quad 0 \leq x \leq 1.
\]

\( \hat{g} \) = \( 1+g \).

\( \hat{p} \) = \( 1+p \).

\( \hat{r} \) = \( 1+r \)
\[ \alpha = \left\{ \frac{n(n+1)\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)^2 - 2k[1-\hat{\gamma}^n\hat{\rho}^n - n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)]]}{[1-\hat{\gamma}^n\hat{\rho}^n - n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)]]} \right\} \]
\[ \hat{\alpha} = [1-\hat{\gamma}^n\hat{\rho}^n - n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)] \]
\[ \hat{\gamma} = [1-\hat{\gamma}^n - n\hat{\gamma}^n(1-\hat{\gamma})] \]
\[ \hat{\psi} = \left[ (n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n) - k[1-\hat{\gamma}^n\hat{\rho}^n)] \right] \]
\[ \hat{\chi} = \left[ k(1-\hat{\gamma})(1-\hat{\gamma}^n\hat{\rho}^n) - \hat{\rho}^n(1-\hat{\gamma}^n)(1-\hat{\gamma}^n) \right] \]
\[ \hat{\gamma} = \left[ \frac{(n+1)(1-\hat{\gamma}^n\hat{\rho}^n) - 2[1-\hat{\gamma}^n\hat{\rho}^n - n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)]]}{[1-\hat{\gamma}^n\hat{\rho}^n - n\hat{\gamma}^n\hat{\rho}^n(1-\hat{\gamma}^n)]]} \right\} \]
\[ \hat{\gamma} = (1-k + k\hat{x} - \hat{x}) \]
\[ \hat{\alpha} = (1-k + k\hat{x} - \hat{x}) \]

\( I_t = \) the nominal cost of depreciable assets purchased by the firm at the end of year \( t \) (\( t=0,1,2,\ldots \)). \( I_t \) is also the gross investment made by the firm at the end of \( t \).

\( R_t^j = \) the accounting earnings of the firm in year \( t \) before depreciation expense, raw material charged to expense, and tax expense given that the firm adopts accounting policy \( j \) (also called net revenue of the firm in year \( t \)).

\( FM_t^j = \) the dollar amount of raw material charged to expense on the firm's financial report in year \( t \) given that the firm adopts accounting policy \( j \).

\( TM_t^j = \) the dollar amount of raw material charged to expense on the firm's tax return in year \( t \) given that the firm adopts accounting policy \( j \).

\( MP_t^j \) = the nominal dollar amount of raw material purchases made by a firm at the end of year \( t \) given that the firm adopts accounting policy \( j \).

\( FD_t^j = \) the dollar amount of depreciation expense on the firm's financial report in year \( t \) given that the firm adopts accounting policy \( j \).

\( TD_t^j = \) the dollar amount of depreciation expense on the firm's tax return in year \( t \) given that the firm adopts accounting policy \( j \).
\( DT_t^j \) = the dollar amount of deferred tax expense on the firm's financial report in year \( t \) given that the firm adopts accounting policy \( j \).

\( A_t^j \) = the accounting earnings reported by the firm in year \( t \) given that the firm adopts accounting policy \( j \).

\( N_t^j \) = the accounting book value of long-lived depreciable assets at the beginning of year \( t \) given that the firm adopts accounting policy \( j \).

\( MI_t^j \) = the accounting book value of raw material inventory at the beginning of year \( t \) given that the firm adopts accounting policy \( j \).

\( DTL_t^j \) = the dollar amount of the deferred tax liability on the firm's financial report at the beginning of year \( t \) given that the firm adopts accounting policy \( j \).

\( BV_t^j \) = the accounting book value of the firm's owners' equity at the beginning of year \( t \) given that the firm uses accounting policy \( j \).

\( ARR_t^j \) = the accounting rate of return of the firm in year \( t \) given that the firm adopts accounting policy \( j \).
INTRODUCTION

The topic of project and organizational unit evaluation has filled many pages. A measure of return on investment is frequently recommended as a measure of profitability or efficiency which can be helpful in this evaluation process. Vatter has observed that: "As a short-cut way to express concisely the financial efficiency or desirability of projects or operations, the rate of return is a most appealing measurement device."¹ Vatter further argues that the basic simplicity of a rate of return measure makes the measure both popular and susceptible to misinterpretation.² I believe another reason that a measure of return on investment is susceptible to misinterpretation is that in some situations it is possible to calculate more than one measure but in other situations only one measure can be calculated.

Table 1
Cash Flows and Accounting Earnings of a Proposed Investment

<table>
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<tr>
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<th>Year 0</th>
<th>Years 1-5</th>
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<tr>
<td>Cash flows of investment</td>
<td>-$3791.00</td>
<td>$1000.00</td>
</tr>
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<td>Depreciation expense</td>
<td>--</td>
<td>753.20</td>
</tr>
<tr>
<td>Accounting earnings</td>
<td>--</td>
<td>$241.89</td>
</tr>
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²Ibid., p. 682.
Table 1 is an example of a situation in which it is possible to calculate two measures of return on investment. The situation depicted in Table 1 is one in which a decision-maker is trying to determine the profitability of an investment project which costs $3791 and returns $1000 at the end of each subsequent year for five years. Two measures of return on investment can be calculated in this situation: (1) the internal rate of return (IRR) and (2) the accounting rate of return (ARR). The IRR of the project is 10% while the ARR of the project is 12.8%. The IRR of the project was calculated by finding that number \( r \neq -1 \), such that:

\[
\sum_{i=0}^{n} C_i \ (1+r)^{-i} = 0
\]

where:

- \( C_i \) is the cash flow associated with the project at the end of the year \( i \) (\( C_i \geq 0 \) means there is a cash inflow at the end of year \( i \) and \( C_i < 0 \) means there is a cash outflow at the end of year \( i \)) and
- \( n \) is the useful life of the machine.

The ARR of the project was calculated by dividing the average annual accounting earnings by the average book value of the project. While ARR and IRR are derived by different methods, they are both called rates of return. Is rate of return more properly measured by the ARR or the IRR?

In the analysis of proposed capital projects, the ARR is criticized...
because it does not consider the time value of money. Consequently, the ARR is labeled as an incorrect method for assessing the profitability of projects. The IRR, on the other hand, does consider the time value of money and is, therefore, considered to provide a better measure of project profitability than the ARR.\(^4\) Thus, in the evaluation of project profitability the ARR and IRR can both be calculated but the IRR is judged to be a better measure for this purpose.

When we turn our attention away from the assessment of project profitability to the assessment of firm profitability, we run into a problem of a different kind. We find that only one measure of rate of return is available. The ARR of a firm is easily calculated by using information produced by the firm's accounting system. The IRR of a firm, however, cannot be calculated unless the future cash flows of the firm are known. An estimate of firm IRR based upon estimates of the firm's future cash flows is impractical since it has been shown that the number of periods for which estimates of cash flows must be made is extremely large.\(^5\)

\(^4\)There is a controversy over whether net present value or the internal rate of return is the theoretically correct method to evaluate investment proposals. For an excellent discussion of this topic (which is beyond the scope of this paper) see: J. Hirschleifer, "On the Theory of the Optimal Investment Decision," *The Management of Corporate Capital*, Ezra Solomon (Editor), (New York: The Free Press, 1959), pp. 205-228.

Thus, the only measure of a firm's rate of return that is available is the firm's ARR.

Because the ARR is the only measure of rate of return available, it is often used as an indicator of a firm's long-run profitability. For example, Harcourt cites several studies in which economists have used the ARR as a measure of firm profitability. The assumption that is made in these studies is that if the ARR of firm A is greater than the ARR of firm B then the long-run profitability of firm A is also greater than that of firm B. Ezra Solomon argues that the ARR is a poor indicator of firm profitability. He has also argued that the IRR is a better measure of the firm's profitability than the ARR. Furthermore, Solomon has constructed models of firms in which both firm IRR and ARR have been calculated. Within the context of these models it is demonstrated that if firm A has a greater ARR than firm B then it does not necessarily follow that firm A

---


8 Ibid., p. 234. Solomon, however, seems to have tempered his position in a later article. In Ezra Solomon, "Alternative Rate of Return Concepts and Their Implications for Utility Regulation," The Bell Journal of Economics and Management Science, Spring 1978, pp. 65-81, he argues that the IRR is different — rather than better — than the ARR.
will also have a higher IRR than firm B.\textsuperscript{9} Therefore, the results of any study on firm profitability which makes such an assumption should be viewed with some skepticism.

The work of Solomon and others\textsuperscript{10} has shown that the ARR of a firm is not equal to the IRR of the firm. The disparity between firm ARR and firm IRR has caused concern among users of accounting information. The concern among users of accounting information is evidenced by a statement made by an executive of International Harvester Company. In explaining why a large amount of cash is allocated to the divisions for determining their investment base, he stated that:

...it fulfills the useful purpose of compensating for the fact that the asset base has been overly reduced by a conservative write-off policy. It has been found to be equitable among several divisions, and it offers a general assurance that the calculated divisional book yield (ARR) is fairly close to the true return (IRR).\textsuperscript{11}

\textsuperscript{9} Ibid., p. 236. Solomon shows that two firms with the same IRR (10\%) can exhibit individual accounting rates of return of 10.79\% and 12.10\%.

\textsuperscript{10} For example, see:

Thus, the disparity between ARR and IRR has led the management of International Harvester Company to establish a method of determining divisional investment which attempts to reduce or eliminate that disparity. The reason advanced for this attempt to eliminate the difference between ARR and IRR is explained by the following passage.

When we think of the disparity between book return (ARR) and true return (IRR) as used by the general management of a business in measuring the results of the various segments of that business, almost anyone would concede the importance of making sure that it is a fair measurement in order to prevent unwarranted conclusions as to the relative performance of divisional managers as well as to check on the 'true return' yardstick as a valid way of indicating the comparative investment worth of individual investment projects. Because of this, it is important to trace the divisional results as shown by the two concepts, and to determine how within practical limits, they may be removed. One basic responsibility of general management is to allocate assets of the business to the most productive and profitable channels. For this task the true return concept is perhaps the best that could be devised.12 (emphasis added)

Thus, the general management of at least one divisionalized firm uses ARR to approximate IRR because they believe that the IRR is a more useful measure of firm profitability for some purposes. Consequently, research which increases our knowledge on the relationship between firm ARR and firm IRR will be beneficial to these users of accounting information.

12 Ibid., pp. 44-45.
A certain group of accounting theorists is also interested in the relationship between firm ARR and firm IRR. Greenball has observed that "...many students of accounting and finance hold that the accounting rate of return should, ideally, equal the internal rate of return."\textsuperscript{13} Anton,\textsuperscript{14} Bierman,\textsuperscript{15} Reynolds,\textsuperscript{16} and Solomon\textsuperscript{17} all appear to hold this view. It is well-known that the ARR of a firm depends in part upon the accounting methods used by the firm. Some accounting methods will, under a given set of circumstances, produce an ARR which is closer to firm IRR than other methods. Therefore, one way to choose among alternative accounting methods is to choose that set of methods whose resultant ARR is closest to the IRR of the firm. In order to be able to use the relationship between ARR and IRR as a basis for choice among sets of accounting methods much additional research on the relationship needs to be done.


\textsuperscript{14,15,16} These authors suggest the use of a depreciation method which would result in the equivalence of ARR and IRR, see: Hector R. Anton, "Depreciation, Cost Allocation, and Investment Decisions," \textit{Accounting Research}, April 1956, pp. 117-134.


\textsuperscript{17} Ezra Solomon, "Return on Investment....", op. cit., pp.233-234.
The purpose of this dissertation is to study the relationship between firm ARR and firm IRR under sets of conditions not examined by prior researchers. The results of the study will benefit those users of accounting information who use the ARR as an approximation of firm IRR and that group of accounting theorists who believe firm IRR is a standard by which the ARR should be judged.

In Chapter 1 a review of the methodology and results of prior research will be made in order to place the present research in its proper perspective.
CHAPTER 1

REVIEW OF THE METHODOLOGY AND RESULTS OF PRIOR RESEARCH

The relationship between firm IRR and firm ARR cannot be studied unless a situation can be found in which both firm ARR and firm IRR can be calculated. As indicated in the introduction to this paper, it is not possible to get a direct measure of the IRR of an on-going firm. We therefore cannot study the relationship between the ARR and IRR of an on-going firm. We can, however, develop models of firms in which firm IRR is known and in which firm ARR can be calculated. We can then study the relationship between firm ARR and IRR within the context of these models. This is the procedure which has been used in many prior studies. It is the purpose of this chapter to examine this methodology and to review the results of the prior studies.

Methodology

The immediate problem we face when we want to study the relationship between firm ARR and firm IRR is essentially this: How can we study the relationship between something observable (the ARR) and something unobservable (the IRR)? An excellent statement of and answer to this question is given in the following quotation.

---

1Parts of this chapter rely heavily on material contained in Livingstone and Salamon, op. cit., pp. 199-201.
We are somewhat in the position of Plato's man in the cave who can see shadows on the ground outside, but who cannot directly see the objects which cast the shadows. Is the length of an observable shadow always a correct indication of the height of the actual object or are there systematic biases due to some other factor or factors? Like Plato's man, we can seek at least a partial answer by trying a controlled experiment. In our case, we can experiment with simulated models of companies which have a known built-in true yield [IRR]. We can thus observe how the book-yield [ARR] behaves under varying conditions.\(^2\)

In all of the research to be reviewed here the relationship between the ARR and IRR was studied by the method indicated in the above quotation. Essentially this methodology involved the construction of models of firms which operated in environments of certainty. A firm consisted of a collection of independent projects which had identical characteristics (e.g., the projects had the same useful life, the same IRR, the same pattern of cash flows, and the same working capital requirements). The collection of projects was assembled by annual gross investment whose amount was determined either by:

1. hypothesizing an arbitrary amount of initial investment and then assuming this amount grew at a constant rate or

2. hypothesizing an arbitrary amount of initial investment and then assuming that there was a constant rate of reinvestment of firm cash flows.

In all of the prior studies, the assumption was then made that the IRR of the firm was the same as the IRR of the firm's projects. Then

based upon the characteristics of the firm (e.g., the growth rate in gross investment, accounting methods used), the environment (e.g., tax rate, rate of change in prices), and the projects (e.g., cash flow patterns, IRR, useful life, salvage value, and working capital required), an expression for the firm's [ARR] was derived. In most of the models developed by prior researchers, the ARR of the firm became constant after a period of time equal in length to the useful life of the firm's projects. Any conclusions reached by the prior researchers about the relationship between firm ARR and firm IRR have been based upon an analysis of this "steady-state" expression for the ARR.

The Relationship Between the IRR of Firm Projects and the IRR of the Firm

One aspect of the research methodology described above needs further examination. This aspect is the assumption that firm IRR is equal to the IRR of firm projects. Carlson, Greenball, Harcourt, Livingstone and Salamon, and Solomon all make this assumption implicitly, if not explicitly, in their work. Solomon makes perhaps the clearest explicit statement of this assumption in the following quotation:

---

3 Robert S. Carlson, op. cit.

4 M.N. Greenball, "Appraising Alternative...", op. cit.

5 G.C. Harcourt, op. cit.

6 J.L. Livingstone and G.L. Salamon, op. cit.

7 Ezra Solomon, "Alternative Rate of Return Concepts...", op. cit. and "Return on Investment...", op. cit.
However, logic tells us that if a company holds many projects, each of which individually yields a DCF rate [IRR] \( r \), then the company as a whole will also be generating a DCF rate [IRR] \( r \) on its total portfolio. This is true regardless of the pace at which the projects have been acquired over time. \(^8\)

Since this assumption is critical to the legitimacy of the modeling methodology used by the researchers mentioned above, it is imperative that we delve into it more deeply.

The procedure we will use to investigate this assumption is simply to apply a common definition of firm IRR to the models used by prior researchers. In order to make this application we first need to develop an accurate representation (i.e., a model) of the models used by the prior researchers \(^9\) which incorporates all the relevant characteristics of their models. In order to get an accurate representation of the models developed by prior researchers, we need to develop a model in which all prices change at a constant rate \( (p) \) per year, in which the annual rate of growth in gross investment \( (g) \) is constant, and in which all projects acquired by the firm have the same useful life \( (n) \), IRR \( (r) \), and cash flow pattern. We now turn to this task.

First, for the sake of notational convenience, we define:

---

\(^8\) Ezra Solomon, "Alternative Rate of Return Concepts...", op. cit., p. 68.

\(^9\) In order to avoid the frequent repetition of several authors' names, we will use the term "prior researchers" to denote the persons whose work is cited in footnotes three through seven of this chapter.
\[ \hat{g} = 1 + g, \]
\[ \hat{p} = 1 + \hat{p}, \text{ and} \]
\[ \hat{r} = 1 + r. \]

Next, we designate the point of time at which the firm acquired its first project as the end of year zero. We assume that the firm acquires one project at the end of each year and that the cash inflows (outflows) generated (required) by firm projects occur only at discrete points of time which are one year apart. We, then, define \( C_i^j \) as the real cash flow (i.e., it is measured in terms of the price level which prevails at the end of year zero) of the project acquired by the firm at the end of year \( j \), \( i \) years after the project's purchase (i.e., the flow is received by the firm at the end of year \( j+i \)) where \( (i = 0, 1, 2, \ldots, n) \) and \( (j = 0, 1, 2, \ldots) \). Given that the rate of change in all prices is \( p \) per year, the definition above means that the nominal cash flows of the first project acquired by the firm is given by the \( n \)-tuple \( (C_0^0, C_1^0 \hat{p}, C_2^0 \hat{p}^2, \ldots, C_n^0 \hat{p}^n) \). If \( C_i^j \) is greater than zero this means that the project brings cash into the firm and if \( C_i^j \) is less than zero this means that the project takes cash from the firm. We assume as did the prior researchers whose work we are modeling that \( C_0^j \) is negative and that \( C_i^j \) is non-negative for all \( i \neq 0 \) and positive for at least one \( i \neq 0 \). We define the absolute value of \( C_i^j \) to be the amount of real gross investment made by the firm at the end of year \( j \). We further assume that there is a unique number \( r \neq -1 \) such that:

\[
(\text{I}) \sum_{i=0}^{n} (C_i^0 \hat{p}) \hat{p}^{-i} \hat{r}^{-i} = \sum_{i=0}^{n} C_i^0 \hat{r}^{-i} = 0
\]
This means that we assume that the first project acquired by the firm has a known and unique real IRR \( r \). For the sake of clarity, we designate this specific value of \( r \) to be \( r_1 \) and let \( \hat{r}_1 = 1 + r_1 \).

Within this representation of the models developed by prior researchers, the nominal cash flows of projects other than the first project will be derived from the nominal cash flows of the first project in a simple manner. Since we want to incorporate a rate of growth in real gross investment of \( g \) per year into our model, we must have that \( C_0^j = C_0^0 g^j \). Since we want to incorporate a rate of change in all prices of \( p \) per year into our model, we must have that the nominal amount of dollars which must be paid by the firm to acquire a project at the end of year \( j \) be \( C_0^0 p^j g^j \). Consequently, if we want the project acquired by the firm at the end of year \( j \) to have the same real IRR as the first project acquired by the firm, we must find the nominal cash flows of the project acquired by the firm at the end of the year \( j \) by multiplying the appropriate nominal cash flows of the firm's first project by the factor \( p^j g^j \). This means that we can represent the nominal cash flows of the project acquired by the firm at the end of the year \( j \) by the n-tuple \((C_0^0 p^j g^j, C_1^0 p^j g^j, C_2^0 p^{j+2} g^j, \ldots, C_n^0 p^{j+n} g^j)\).

We have now developed a representation of the models used by prior researchers to study the relationship between firm ARR and firm IRR. This representation contains all of the relevant characteristics of the models developed by prior researchers (e.g., a constant rate of change in
prices, constant rate of growth in real gross investment, identical project characteristics) and this representation will allow us to make a direct calculation of firm IRR by using a common definition of firm IRR.

For the cases in which the cash transactions between the firm and its owners occur only at the ends of years, the IRR of the firm is commonly defined as that number \( r \neq -1 \) which solves:

\[
(2) \quad \sum_{i=0}^{T} D_i \left( \frac{r}{p} \right)^i = 0.
\]

In this equation, \( D_i \) is defined as the nominal cash flow at the end of year \( i \) from (to) the firm to (from) its owners if \( D_i \) is greater (less) than zero, \( p \) is the annual rate of change in all prices and \( T \) is the date of firm liquidation. In order to apply the definition of firm IRR given in equation (2) to the work done by prior researchers, we need to make a reasonable assumption regarding the length of firm life or search for the assumption regarding firm life which appears to be implicit in their work.

No specification of firm life is made by any of the prior researchers. However, all of them have investigated cases of constant growth in gross investment and have not specified that this growth ever stops. It, therefore, appears they have been working with firms of infinite life, at least implicitly. We, consequently, will assume that \( T \) is equal to infinity in our application of equation (2) to their work.

Given the above definitions of the nominal cash flows of the firm's projects and assuming that the firm holds a zero cash balance, we can write the nominal stream of cash transactions between the firm and its
owners as:

\[ D_0 = C_0 \]

\[ D_1 = C_1 \hat{p} + C_0 \hat{p} \hat{g} \]

\[ D_2 = C_2 \hat{p}^2 + C_1 \hat{p} \hat{g} + C_0 \hat{p} \hat{g} \]

\[ \ldots \]

\[ D_n = C_n \hat{p}^n + C_{n-1} \hat{p}^{n-1} \hat{g} + C_{n-2} \hat{p}^{n-2} \hat{g} + \ldots + C_0 \hat{p} \hat{g} \]

\[ D_{n+j} = \hat{p}^j D_n. \]

For any \( m \) such that \( 0 \leq m \leq n \), we can write that:

\[ D_m = \hat{p}^m \sum_{i=0}^{m} C_{m-i} \hat{p}^{m-i} \hat{g}^{i-1} \]

The assumption made by prior researchers that the IRR of the firm is equal to the IRR of firm projects means that they are assuming that the specific value of \( r \) which solves equation (2) is \( r_1 \) the IRR of all firm projects. In order to check whether this assumption holds, we can simply substitute \( r_1 \) for \( r \) in the left-hand side of equation (2) and determine whether the equality holds. By making this substitution, we can write the left-hand side of equation (2) as:

\[ \sum_{i=0}^{\infty} D_i \hat{r}_1^{-i} \hat{p}^{-i} = \sum_{i=0}^{n} D_i \hat{r}_1^{-i} \hat{p}^{-i} + \sum_{i=n+1}^{\infty} D_i \hat{r}_1^{-i} \hat{p}^{-i} \]
\[
= \sum_{i=0}^{n} D_i \left( \hat{r}_1 \hat{p} \right)^{-i} + D_n \left( \hat{r}_1 \hat{p} \right)^{-n} \sum_{i=1}^{\infty} \left( \hat{g} / \hat{r}_1 \right)^i \]

Substituting for \( D_i \) and \( D_n \) from equation (3) into the right-hand side of equation (4), we have:\(^{10}\)

\[
\sum_{i=0}^{\infty} D_1 \hat{r}_1^{-i-1} \hat{p}^{-1} = \sum_{i=0}^{n} \frac{p^i}{n} \sum_{j=0}^{i} C_{i-j} g_j \hat{r}_1^{-i-1} \hat{p}^{-1} + \sum_{i=0}^{n} \frac{p^i}{n} \sum_{j=0}^{i} C_{i-j} g_j \sum_{i=1}^{\infty} \left( \hat{g} / \hat{r}_1 \right)^i \]

\[
= \sum_{i=0}^{n} \sum_{j=0}^{i} C_{i-j} g_j \hat{r}_1^{-i-1} + \frac{1}{\hat{r}_1^2} \sum_{j=0}^{n} C_{n-j} g_j \sum_{i=0}^{\infty} \left( \hat{g} / \hat{r}_1 \right)^i \]

\[
= \sum_{i=0}^{n} \sum_{j=0}^{i} C_{i-j} g_j \hat{r}_1^{-i-1} + \frac{\hat{g}}{\hat{r}_1} \sum_{j=0}^{n} C_{j} g_j \sum_{i=0}^{\infty} \left( \hat{g} / \hat{r}_1 \right)^i \]

In order to carry our analysis of equation (5) any further, we must proceed by cases. The first case is the case of \( g \neq r_1 \). This first case will have the two sub-cases of \( g > r_1 \) and \( g < r_1 \). The second case is the case of \( g = r_1 \).

Case 1 (\( g \neq r_1 \))

For the case of \( g \neq r_1 \), we can rewrite equation (5) as:\(^{11}\)

\(^{10}\) For a proof of the equality between line two and line three of equation (5) see note (1) of the mathematical appendix.

\(^{11}\) For a proof of the first line of equation (5) see note (2) of the mathematical appendix.
\[ \sum_{i=0}^{\infty} D_i^{-1} \hat{p}^{-1} = \sum_{i=0}^{n} C_i^{-1} \left( \sum_{i=0}^{n} \frac{\hat{g}^{n+1-i} \cdot g^{n+1-i}}{(\hat{r}_1 - \hat{g})} \right) + \frac{\hat{g}}{\hat{r}_1} \sum_{j=0}^{n} \sum_{i=0}^{\infty} C_j^0 \hat{g}^{-i} \sum_{i=1}^{\infty} \left( \frac{\hat{g}}{\hat{r}_1} \right)^i \]

(6) = \frac{\hat{r}_1}{(\hat{r}_1 - \hat{g})} \sum_{i=0}^{n} C_i^0 \hat{g}^{-i} - \frac{\hat{g}}{\hat{r}_1} \sum_{j=0}^{n} \sum_{i=0}^{\infty} C_j^0 \hat{g}^{-i} + \frac{\hat{g}}{\hat{r}_1} \sum_{j=0}^{n} \sum_{i=0}^{\infty} C_j^0 \hat{g}^{-i} \sum_{i=0}^{\infty} \left( \frac{\hat{g}}{\hat{r}_1} \right)^i

Substituting for \( \sum_{i=0}^{\infty} C_i^0 \hat{g}^{-i} \) from equation (1) into equation (6) and rearranging terms, we have:

\[ \sum_{i=0}^{\infty} D_i^{-1} \hat{p}^{-1} = \left[ (\hat{g}/\hat{r}_1) \sum_{i=0}^{n} C_i^0 \hat{g}^{-i} \right] - \left[ \sum_{i=1}^{\infty} \left( \frac{\hat{g}}{\hat{r}_1} \right)^i \right] \frac{\hat{g}}{\hat{r}_1 - \hat{g}} \]

Since we have assumed that there is a unique \( r_1 \) which solves equation (1) and since we are now examining the case of \( g \neq r_1 \), we have that the first term in square brackets on the right-hand side of equation (7) is not equal to zero. In order to proceed with the analysis of equation (7), we must now resort to our sub-cases. We first examine the case of \( g > r_1 \). If \( g > r_1 \), then \( \sum_{i=0}^{\infty} \left( \frac{\hat{g}}{\hat{r}_1} \right)^i \) increases without bound. This means that the expression \( \sum_{i=0}^{\infty} D_i^{-1} \hat{p}^{-1} \) approaches positive or negative infinity (depending upon the sign of \( \sum_{i=0}^{n} C_i^0 \hat{g}^{-i} \)) and, of course, does not equal zero. Therefore, for the case of \( g > r_1 \), the IRR of the firm does not equal the IRR of firm projects and the assumption to the contrary made by prior researchers is incorrect. We now turn to an investigation of the sub-case of \( g < r_1 \).

If \( g < r_1 \), then:
\[ \sum_{i=1}^{\infty} (\hat{g}/\hat{r})^{-1} = \hat{g}/(\hat{r} - \hat{g}). \] We therefore, can rewrite equation (7) as:

\[ \sum_{i=0}^{\infty} D_i \hat{r}_1^{-i} \hat{p}_i^{-i} = \left[ (\hat{g}/\hat{r})_1^n \sum_{i=0}^{n} C_i^0 \hat{g}^{-i} \right] \begin{bmatrix} \hat{g} \overline{r}_1^{-\hat{g}} & -\hat{g} \\ \overline{r}_1^{-\hat{g}} & \overline{r}_1^{-\hat{g}} \end{bmatrix} = 0. \]

This means that whenever \( g < r_1 \) the IRR of the firm is equal to the IRR of all firm projects. We now turn to an investigation of case (2) or \( g = r_1 \).

Case 2 \( (g = r_1) \)

If \( g = r_1 \), we have from equation (1) that:

\[ \sum_{j=0}^{n} C_j^0 \hat{g}^{-j} = 0. \]

Substituting this result back into equation (5), we have:

\[ \sum_{i=0}^{\infty} D_i \hat{r}_1^{-i} \hat{p}_i^{-i} = \sum_{i=0}^{n} \sum_{j=0}^{n-i} C_i^0 \hat{r}_1^{-i} \hat{g}^j \hat{r}_1^{-j} = \sum_{i=0}^{n} C_i^0 \hat{r}_1^{-i} (n + 1 - i) \]

\[ = (n+1) \sum_{i=0}^{n} C_i^0 \hat{r}_1^{-i} - \sum_{i=0}^{n} iC_i^0 \hat{r}_1^{-i} = - \sum_{i=1}^{n} iC_i^0 \hat{r}_1^{-i}. \]

If we make the reasonable assumption that \( \hat{r}_1 > 1 \) (i.e., \( \hat{r}_1 > 0 \)) then \( \sum_{i=1}^{n} iC_i^0 \hat{r}_1^{-i} \) is positive since we are assuming that \( C_i^0 \geq 0 \) for all \( i \geq 1 \) and \( C_i^0 > 0 \) for at least one \( i > 1 \). This means \( \sum_{i=0}^{\infty} D_i \hat{r}_1^{-i} \hat{p}_i^{-i} < 0 \) and thus \( \hat{r}_1 \) is not the IRR of the firm when \( g = r_1 \).

Our investigation into the assumption made in the work in prior researchers that the firm IRR is equal to the IRR of the firm projects can be summarized as follows. First, we developed a representation of the models used by the prior researchers who have studied the relationship between firm ARR and firm IRR. Then, in an examination of this representation, we found that if the growth rate in gross investment \( (g) \) is
greater than or equal to the IRR of all firm projects \((r_1)\) then the IRR of the firm is not equal to the IRR of all firm projects. Next, we found that the IRR of the firm was equal to the IRR of all firm projects whenever the growth rate in gross investment \((g)\) is less than the IRR of all firm projects \((r_1)\). We can, therefore, conclude that the modeling methodology used by prior researchers is a legitimate means of investigating the relationship between firm ARR and firm IRR whenever the growth rate in gross investment is less than the IRR of all firm projects.

The finding that the IRR of the firm does not equal the IRR of the firm projects whenever \(g \geq r_1\) is significant in light of the fact that prior researchers have found the case of \(g = r_1\) to have such interesting characteristics. Most of the prior researchers whose work is being investigated here have found that (in the absence of price level changes) the ARR of the firm is equal to \(r_1\) whenever \(g = r_1\). The significance of this finding has considerably less impact when we realize that \(r_1\) is not the IRR of the firm.

In fairness to the work done by prior researchers, we must state that the finding that firm IRR does not equal the IRR of firm projects whenever \(g \geq r_1\) depends heavily upon our assumption of an infinite firm life. However, in fairness to ourselves, we must state that the assumption of an infinite firm life appears to be implicit in their work. Furthermore, even if it is assumed that the prior researchers were working with models of a firm of finite life \((T)\), the IRR of the firm is not necessarily
equal to the IRR of firm projects unless we make particular assumptions about how the firm does liquidate.  

This concludes our investigation of the methodology utilized by prior researchers. We now turn to a review of the results produced by their efforts.

Results of Prior Research

The research results to be reviewed here are considered to be a representative sample of the prior research into the relationship between firm ARR and firm IRR. A number of articles have been written on the relationship between the ARR and IRR of projects but these will not be reviewed. In this section, the discussion will center on the scope and results of prior research since the methodology used by all prior researchers was similar and was examined in the previous section of this chapter.

12 For example, let us assume that the firm acquires its last project at the end of year T-1. If the IRR of this firm is to be equal to the IRR of all firm projects, then the liquidating dividend the firm must pay to its owners at the end of year T must be equal to the present value (at the end of year T) of the future cash flows of the firm projects whose useful lives would normally have extended past the end of year T. Furthermore, this present value must be calculated by using \( r_1 \) as the interest rate.

13 For example, see:


Ezra Solomon\textsuperscript{14} divided his pioneering study of the relationship between firm ARR and firm IRR into two distinct but not different cases—the case in which the firm's annual gross investment was constant and the case in which the firm's annual gross investment grew at a constant rate, \( g \). The cases are not different since the constant annual investment case is the constant growth case with \( g=0 \). In the zero growth model, Solomon found that the ARR-IRR relationship was affected by the length of project life, the pattern of project cash flow, capitalization policy, depreciation method, and price level changes. In the growth model, Solomon studied the effect of the growth rate on the relationship between ARR and IRR for firms whose projects were depreciated on a straight-line basis and which generated level cash flows.

Carlson\textsuperscript{15} extended the work of Solomon in several ways. First, he studied the effects in the zero growth and growth models of capitalization policy, depreciation method, length of project life, salvage value of projects, investment time lags, cash flow pattern, income taxes, and price level changes on the ARR-IRR relationship. This part of his study was conducted under the assumption that the firm was a collection of projects which all had the same life and IRR. Second, Carlson extended the work of Solomon by formulating expressions for ARR in terms of model parameters for the cases in which the firm had:

\textsuperscript{14} Ezra Solomon, "Return on Investment...", op. cit.
\textsuperscript{15} Robert Carlson, op. cit.
(1) invested in projects with different lives and same IRR,

(2) invested in projects with different internal rates of return and same life, and

(3) invested in current assets as well as depreciable assets.

The work of Carlson and Solomon seemed to have a similar purpose. Vatter has characterized Solomon's study as one in which the purpose was to prove that the ARR "was bad." 16 This purpose (as opposed to the purpose of determining the characteristics of the ARR as an estimator of the IRR) influenced both of these authors' studies in definite ways. Both authors showed that many variables affected the relationship between the ARR and the IRR but neither attempted to build a model which contained all of the variables simultaneously. For example, once it had been shown that the cash flow pattern of the projects did affect the relationship between firm ARR and IRR, only projects with level cash flows were considered in the remainder of their analyses. Including variables in a model one-at-a-time does allow us to determine which variables affect the relationship being modeled but obviously it does not enable us to determine how a variable included in the model now will interact with one that is included later.

Harcourt 17 divided his study of the relation of firm ARR to firm IRR

16 William J. Vatter, op. cit., p. 684.
17 G.C. Harcourt, op. cit.
into two main cases each of which contained two sub-cases. His two main cases, like those of Solomon and Carlson, were that of either constant annual investment or constant growth in annual investment. Harcourt also assumed that the firm operated in a certain and taxless environment. The first sub-case considered by Harcourt was that in which the firm invested solely in projects with the same IRR, life, and cash flow pattern. The second sub-case was one in which the firm invested in financial assets (i.e., securities) as well as in the regular projects. For each of these four models, Harcourt formulated expressions for ARR in terms of IRR, cash flow pattern, project life, and the growth rate in gross investment. He reported separate expressions for the straight-line depreciation and 150% declining balance depreciation methods. One of the important differences between the Harcourt study and those previously reviewed was that in the Harcourt study each expression for ARR contained all of the parameters under study. Thus, Harcourt was able to examine the joint effect that asset life, cash flow pattern, investment growth, and IRR had on the relationship between ARR and IRR.

M.N. Greenball\(^{18}\) studied the after tax relationship between ARR and IRR for firms which operated in a certain environment and which invested in projects which had the same life, cash flow pattern, and IRR. Annual gross investment was presumed to grow at a constant rate per year.

\(^{18}\) M.N. Greenball, "Appraising Alternative...", op. cit.
The projects were depreciated on the sum-of-year digits (SYD) method for tax purposes. Greenball then formulated expressions for firm ARR under three different circumstances:

1. the firm used straight-line depreciation for financial reporting purposes and recognized deferred income taxes,

2. the firm used straight-line depreciation for financial reporting purposes but did not recognize deferred income taxes, and

3. the firm used SYD depreciation for financial reporting purposes.

The results of Greenball's study indicate that the bias of ARR as an estimate of IRR depended upon the project cash flow pattern, IRR, and the growth rate in gross investment. Specifically, Greenball found that when the growth rate in gross investment was positive and less than the IRR of the projects, then the ARR of the firm went from being significantly higher than the IRR to significantly lower than the IRR as the cash flow pattern of the projects was varied from being sharply increasing to sharply decreasing.\(^{19}\) Thus, the relationship between ARR and IRR is extremely sensitive to changes in the cash flow pattern of the projects. It was also found that the bias of ARR as an estimate of IRR was rather insensitive to changes in the useful life of the projects and the tax rate.\(^{20}\)

Livingstone and Salamon\(^ {21}\) have studied the relationship between IRR and ARR for firms which operated in a certain, taxless environment.

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\(^{19}\) Ibid., p. 274.

\(^{20}\) Ibid., p. 275.

\(^{21}\) John Leslie Livingstone and Gerald L. Salamon, op. cit.
and which invested in projects that had the same life, cash flow pattern, and IRR. In this model, the continued existence of the firm was generated by a constant rate of reinvestment of firm cash flows rather than by a constant growth in gross investment. Livingstone and Salamon found that in their model the bias of ARR as an estimate of IRR depended upon the values of the cash flow pattern and reinvestment rate parameters. They found that the bias was relatively insensitive to changes in the values of IRR and asset life. They also showed that a constant rate of reinvestment tended toward a constant gross investment growth rate, and vice versa. Since the reinvestment rate and growth rate seem to be different sides of the same coin, study of the relationship between ARR and IRR may proceed by using either of these mechanisms for generating the continued existence of the firm without any loss of generality.

This concludes our review of the results of specific studies on the relationship between the ARR and IRR of a firm. The purpose of this review has been two-fold. First, we wanted to point out that prior researchers have shown that a number of items separately affect the relationship between firm ARR and IRR. The discovery of the individual items which affect this relationship was the primary objective of the studies by Solomon and Carlson. Second, we wanted to point out that studying the effect of variables on the relationship between firm ARR and IRR by building and analyzing models which incorporate variables one-at-a-time does not enable us to determine how the particular
variable incorporated will interact with those not incorporated. This is an important point since Greenball, Harcourt, and Livingstone and Salamon have shown that considerable interaction effects among the variables do exist. Consequently, when we study the relationship between firm ARR and IRR by building models of firms it is important that these models be comprehensive.

In the next chapter, we will construct models of firms in order to study the relationship between firm IRR and firm ARR. The development of these models extends prior research in this area by including several variables which have been shown to separately affect the relationship between firm ARR and IRR in the models simultaneously.
CHAPTER II

THE DEVELOPMENT OF MODELS OF THE RELATIONSHIP BETWEEN
THE ACCOUNTING AND THE INTERNAL RATES OF RETURN OF A FIRM

The main tasks of this chapter are to develop models of firms which
have known internal rates of return and to derive the accounting rates of
return for these firms. However, before we proceed with these tasks,
we need to discuss the scope of our study and the assumptions which
will be made in the development of the firm models. We now turn to a
discussion of these topics.

Scope

In order to relate the scope of the present work to the work done by
others, we present the following list of items which are known (from the
results of prior research) to affect the relationship between the ARR and
IRR of a firm:\textsuperscript{1}

1. the set of accounting methods used by the firm.

2. the rate at which prices increase.

3. the fraction of each investment outlay expensed.

4. the time lag in years between the initial investment and the
   first cash inflow.

\textsuperscript{1}This list is an amended version of a similar list which appears
in Carlson, \textit{op. cit.}, pp. 80-81.
5. the salvage value portion of each investment outlay.
6. the fraction of each investment outlay committed to current assets.
7. the productive life of each asset.
8. the corporate income tax rate.
9. the rate of growth in new investment outlays.
10. the level of firm IRR.
11. the cash flow pattern of each asset.

One of the purposes of this dissertation will be to study how eight of these eleven variables affect the relationship between firm ARR and IRR. The other variables (those numbered 3, 4, and 5) will be held constant. Specifically, the fraction of each investment outlay not capitalized will be zero, the time lag between the initial investment outlay and the first cash inflow will be one year, and the salvage value of all long-lived depreciable assets will be zero. We are holding these variables constant not because they are thought to be unimportant but because if we allowed them to vary they would complicate and perhaps confuse a model which is already very complex. Furthermore, by concentrating our attention on the other eight variables, we will encounter several problems of interest to accountants, which have not been encountered by prior researchers. For example, prior researchers have not constructed a model which contains both price level changes and purchases of raw materials (a specific element of working capital). As soon as we construct a model which includes both of these items, we are forced to
make a cost-flow assumption in order to calculate the amount of raw material charged to expense. In the models to be developed later in this chapter, we will include two cost-flow assumptions — first-in, first-out (FIFO) and last-in, first-out (LIFO). This has not been done before by other researchers.

The set of accounting methods which will be modeled in this study is larger than that included in any of the prior studies. We will develop sixteen different expressions of the ARR of a firm. Each different expression corresponds to a different accounting policy that the firm may adopt. An accounting policy is defined as the set of accounting methods which the firm selects from the alternative methods which are available. In this paper, we will examine the straight-line (SL) and sum-of-years-digits (SYD) depreciation methods, the first-in, first-out (FIFO) and the periodic last-in, first-out (LIFO) inventory methods, the flow-through and normalization treatments of deferred taxes, and price-level unadjusted and adjusted financial statements. We assume that the firm will use the same inventory methods in preparing its financial and tax report. We also assume that all items on the firm’s tax report will not be adjusted for changes in the price-level. We define $\text{ARR}_t^j$ as the ARR of a firm in year $t$ given that the firm adopts accounting policy
$j$ where $j = 1, 2, \ldots, 16$. The particular set of accounting methods which correspond to each accounting policy $j$ is given in Table 2.

### Table 2

The Combination of Accounting Methods Included in Each Accounting Policy

<table>
<thead>
<tr>
<th>Accounting Policy $j$</th>
<th>Periodic Inventory Method Book and Tax</th>
<th>Depreciation Method Book</th>
<th>Tax</th>
<th>Deferred Tax Treatment</th>
<th>Price level Treatment of Financial Statements</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>FIFO</td>
<td>SL</td>
<td>SL</td>
<td>Not applicable</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>2</td>
<td>LIFO</td>
<td>SL</td>
<td>SL</td>
<td>Not applicable</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>3</td>
<td>FIFO</td>
<td>SYD</td>
<td>SYD</td>
<td>Not applicable</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>4</td>
<td>LIFO</td>
<td>SYD</td>
<td>SYD</td>
<td>Not applicable</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>5</td>
<td>FIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Flow-through</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>6</td>
<td>FIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Normalization</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>7</td>
<td>LIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Flow-through</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>8</td>
<td>LIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Normalization</td>
<td>Unadjusted</td>
</tr>
<tr>
<td>9</td>
<td>FIFO</td>
<td>SL</td>
<td>SL</td>
<td>Not applicable</td>
<td>Adjusted</td>
</tr>
<tr>
<td>10</td>
<td>LIFO</td>
<td>SL</td>
<td>SL</td>
<td>Not applicable</td>
<td>Adjusted</td>
</tr>
<tr>
<td>11</td>
<td>FIFO</td>
<td>SYD</td>
<td>SYD</td>
<td>Not applicable</td>
<td>Adjusted</td>
</tr>
<tr>
<td>12</td>
<td>LIFO</td>
<td>SYD</td>
<td>SYD</td>
<td>Not applicable</td>
<td>Adjusted</td>
</tr>
<tr>
<td>13</td>
<td>FIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Flow-through</td>
<td>Adjusted</td>
</tr>
<tr>
<td>14</td>
<td>FIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Normalization</td>
<td>Adjusted</td>
</tr>
<tr>
<td>15</td>
<td>LIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Flow-through</td>
<td>Adjusted</td>
</tr>
<tr>
<td>16</td>
<td>LIFO</td>
<td>SL</td>
<td>SYD</td>
<td>Normalization</td>
<td>Adjusted</td>
</tr>
</tbody>
</table>

---

We note that $t$ will often be used as a subscript in the definition of certain terms. The subscript $t$ denotes a time period of one year which ends at time point $t$ in some terms while in other terms the subscript $t$ denotes the time point $t$. The particular meaning of the subscript $t$ in association with a particular term is made clear when the term is defined. We will also find it convenient later in this paper to use the phrases "year $t$" and "beginning of year $t". The phrase "in year $t$" means in the time period of one year which ends at time point $t$. The phrase "at the beginning of year $t$" denotes the time point $t-1$. 

It is important to note that an accounting policy specifies the measurement methods that the firm will use for tax reporting as well as the methods it will use for financial reporting. For example, we see from Table 2 that the firm which adopts accounting policy 5 uses the FIFO inventory method and the SYD depreciation method for tax reporting and uses the FIFO inventory method, the SL depreciation method, the flow-through treatment of deferred taxes, and price-level unadjusted data for financial reporting.

An examination of Table 2 reveals that certain combinations of book and tax accounting methods have not been considered in this dissertation. We have not considered an accounting policy which combines the SL depreciation method for tax reporting and the SYD depreciation method for financial reporting nor have we considered an accounting policy which combines the FIFO inventory method for tax reporting and the LIFO inventory method for financial reporting. These combinations have not been considered because it is doubtful that any firm would adopt such combinations in the circumstances to be examined in this dissertation. We have not considered an accounting policy which combines the LIFO inventory method for tax reporting and the FIFO inventory method for financial reporting since this combination of methods is prohibited by the Internal Revenue Service.

The set of accounting policies depicted in Table 2 is broader than
any set developed by prior researchers. In order to develop the sixteen expressions for $ARR_t^j$ ($j=1,2,\ldots,16$), we made certain simplifying assumptions. We now turn to a discussion of these assumptions.

Assumptions

In order to develop models of firms which have known internal rates of return and derive expressions for the accounting rates of return for these firms, we made several simplifying assumptions about the firm and its environment. The main ones are listed below.

1. The firm is assumed to have an infinite life and is a collection of projects.\(^3\)

2. The firm acquires one project at the end of each year $t$ ($t=0,1,2,\ldots$). Each project has the same IRR, same useful life, same pattern of cash flows, same working capital requirements, and the same salvage value (zero).

3. Each project is composed of a long-lived depreciable asset and related working capital. The working capital is assumed to be raw material and the amount of raw material purchased and paid for at the end of year $i$ is consumed in year $i+1$.

4. All cash payments and receipts are made at the end of time periods of equal length (assumed to be years).

5. The corporate income tax rate is constant through time. Taxable income is based upon data unadjusted for changes in the price level. The firm will use the same inventory method for tax and financial reporting.

6. All prices change at an annual rate of $p$.

7. The annual growth rate of real gross investment of the firm ($g$) is constant, less than the IRR of firm projects, and non-negative.\(^4\)

\(^3\)Assumptions (1) and (7) together mean that firm IRR will be the same as the IRR of firm projects (see Chapter 1, pages 11-20).

\(^4\)For an explanation of why we have restricted our attention to the case of non-negative growth, see pages 45-47.
8. Firm accounting income is equal to the positive cash flows generated by the projects less depreciation expense, raw material charged to expense, current tax expense, and deferred tax expense (i.e., the firm has no general or indirect costs).

9. All expectations are realized.

Some additional assumptions which pertain to particular aspects of the models are made later in this chapter. These assumptions are not listed above because they are understood best as they naturally arise in the development of the models. We now turn to the main task of this chapter — the development of the models.

**Development of the Models**

A definition of all the key symbols used in this section is given in the preliminaries of this dissertation. In addition each symbol will be defined as it is encountered in our development of the models. In order to begin this development we need to define the following terms:

- \( r \) = the real (i.e., price level adjusted) internal rate of return of all projects acquired by the firm.

- \( n \) = the useful life of all firm projects.

- \( q^i_j \) = the real cash flow (before payment for taxes and raw materials) provided at the end of the first project's \( i \)-th year (\( i=1,2,\ldots,n \)), given that the firm adopts accounting policy \( j \). The dollar flow is measured in terms of the price level which prevailed at the end of year zero.
\[ b = \text{the parameter which defines the relationship between the real cash flows (before payments for raw materials and taxes) of firm projects in their } i\text{-th and } (i+1)\text{-th year. Specifically, the real cash flows of the firm's } (i+1)\text{-th year are related as follows:} \]
\[ q_{i+1}^j = b q_i^j = b^j q_i^j \]
\[ \text{for } (i=1,2,\ldots,n-1), \; b > 0. \]

\[ p = \text{the rate of change per year in all prices.} \]

\[ x = \text{the parameter which defines the relationship between the common dollar cash flow (before payments for taxes and raw materials) of firm projects at the end of the projects' } (i+1)\text{-th year and the common dollar amount of raw material purchases required by the projects at the end of their } i\text{-th year } (i=0,1,2,\ldots,n-1). \]
Specifically, the parameter \( x \) is such that the nominal dollar amount of raw material required to be purchased by the firm for the first project at the end of the project's \( i\text{-th} \) year is:
\[ x q_{i+1}^j (1+p)^i \]
\[ \text{for } (i=0,1,2,\ldots,n-1) \text{ and } 0 < x < 1. \]

\[ g = \text{the annual growth rate in the real gross investment of the firm.} \]

\[ k = \text{the tax rate on firm income.} \]

\[ I_t = \text{the nominal cost of depreciable assets purchased by the firm at the end of year } t \; (t=0,1,2,\ldots). \]
For purposes of this paper, \( I_t \) is also defined to be the gross investment made by the firm at the end of \( t \).

Therefore, we have that:
\[ I_t = (1+p) (1+g) I_{t-1} = (1+p)^t (1+g)^t I_0. \]

---

5 Defining gross investment as the amount the firm must spend to acquire a project was an alternative which was considered. If we used this definition, the amount of gross investment made by the firm at the end of year zero would be \( I_0 + x q_1^j \). At the end of year one, it would be \( (I_0 + x q_1^j) (1+p) (1+g) \). It should be noted, however, that at the end of year one the firm must also purchase raw material of \( x q_2^j (1+p) \) for the first project. Therefore, if we used this definition of gross investment, some raw material purchases would be classified as gross investment while other raw material purchases would not. This inconsistent treatment of raw material purchases was the reason that this definition of gross investment was rejected.
For the sake of convenience, we make the following additional definitions:

\[ \hat{p} = 1 + p, \]
\[ \hat{g} = 1 + g, \text{ and} \]
\[ \hat{r} = 1 + r. \]

At this point an explanation of why the real cash flows (before payments for taxes and raw materials) of the firm's first project \( (q^1) \) depends upon the accounting policy is needed. As we have previously mentioned, an accounting policy specifies both the tax reporting and financial reporting methods which will be used by the firm. Since the scope of our study encompasses accounting policies for which the tax reporting methods used by the firm are not the same, we are presented with a problem in the formulation of our model.\(^6\) The nature of this problem is whether to concentrate our attention on situations in which the cash flows before payments for taxes and raw materials is the same for all accounting policies. If we restrict our attention to situations in which the cash flows before payments for taxes and raw materials is the same for all accounting policies then the cash flows after payments for taxes and raw materials and the IRR of the firm's first project depend upon the accounting policy.

\(^6\)The nature of this modeling problem is such that it prohibits us from making any statements about the relative magnitudes of the differences between the ARR and IRR of two firms whenever the tax reporting methods used by the firms are not the same. For a discussion of this problem, see Chapter III.
used by the firm. This approach to the formulation of the model would make the presentation of the results of an examination of the model quite complex and was therefore rejected. The other approach to the formulation of the model is to restrict our attention to situations in which the IRR of firm projects is the same for all accounting policies. This approach will facilitate the presentation of our results but also means that the cash flows before payments for taxes and raw materials of firm projects will not be the same for all accounting policies. We have chosen to take this second approach to the formulation of the model and therefore have that the cash flows before payments for taxes and raw material purchases depend upon the accounting policy adopted by the firm.

We can now turn our attention to the development of expressions for \( a_{j}^{T} \) \( (j=1, 2, \ldots, 16) \). The procedure to be used is described below:

1. For each accounting policy, find expressions for the cash flows of the first project acquired by the firm.

2. For each accounting policy, find the expression which relates the initial investment, subsequent cash flows, and real IRR of the first project acquired by the firm.

\[ 7 \text{Steps 1 and 2 are necessary because many of the expressions developed in steps 3 and 4 contain terms which can be eliminated by using expressions developed in steps 1 and 2.} \]
3. For each accounting policy, develop an expression for the accounting income reported by the firm in the year which ends at time $t$. In order to complete step (3) it is necessary to develop for each accounting policy an expression for:

(a) Accounting income in year $t$ before depreciation expense, raw material charged to expense, current income tax expense, and deferred income tax expense. Hereafter, we define this expression to be the net revenue of the firm in year $t$.

(b) Depreciation expense in year $t$.

(c) Raw material expense in year $t$.

(d) Current income tax expense in year $t$.

(e) Deferred income tax expense in year $t$.

4. For each accounting policy find an expression for the book value of firm equity at the beginning of year $t$. In order to accomplish this step, it is necessary to find for each accounting policy an expression for:

(a) Book value of long-lived depreciable assets at the beginning of year $t$.

(b) Book value of raw material at the beginning of year $t$.

(c) Book value of the deferred tax liability at the beginning of year $t$.

5. For each accounting policy, divide the result obtained in step (3) by the result of step (4).

The reader will note that the definition of firm ARR used in this paper is accounting income in year $t$ divided by the book value of owners' equity at the beginning of year $t$. Two comments need to be made regarding this particular definition. First, we note that beginning of period book values and not an average of beginning and ending book values are used in the denominator of this calculation. The only reason
for using an average of beginning and ending book values would be if firm investment took place uniformly throughout the year. In our model, firm investment takes place only at discrete points in time. We, therefore, use beginning of period book values because it is consistent with the usual calculation of firm ARR. Second, we note that the book value of owners' equity rather than the book value of firm assets is used to calculate firm ARR. Both methods of calculating firm ARR are mentioned in accounting literature. The use of total assets in the calculation of firm ARR is often advocated as a means of measuring management's efficiency in the use of firm assets. The use of owners' equity in the calculation of firm ARR is advocated as a means by which the residual owners of the firm may assess the profitability of their investment. Since the purpose of this paper is to study the relationship between firm ARR and a measure of long-run firm profitability called the internal rate of return under conditions of certainty, I believe it is more reasonable to use owners' equity rather than total assets in the calculation of firm ARR.

Now that we have established all of the ground rules necessary for the development of the expressions for $\text{ARR}_t^j$ we can begin with the first two steps of the five steps given above.
The Derivation of the Relationship Between the Initial Investment, Subsequent Cash Flows, and the Real IRR of the First Project

In order to derive the relationship between the initial investment, the subsequent cash flows, and the real IRR of the first project acquired by the firm, we will proceed as follows: First, for each accounting policy (or group of accounting policies for which the cash flows of the firm's first project will be equal), we will derive expressions for the nominal cash flows after payments for taxes and raw materials of the firm's first project at the end of each year $i$ of the project's life ($i=0,1,2,\ldots,n$). We note that the nominal cash flows after payment for taxes and raw materials depend upon the accounting policy used by the firm only to the extent that the given policy affects the amount of taxes that must be paid. It is important to remember that we are assuming that taxable income will be based upon nominal amounts rather than price level adjusted amounts. Consequently, the expressions for the nominal cash flow after payment for taxes and raw materials of the firm's first project at the end of year $i$ will be the same for all accounting policies which utilize the same accounting methods for tax reporting. Second, we will write and simplify the equation for the real IRR of the first project acquired by the firm in order to obtain the expression which relates the initial investment, subsequent cash flows, and the real IRR of the firm's first project.

In order to aid the reader's understanding of the development of
these expressions, we have presented in Table 3 a schedule of the nom-
inal cash flows of the first project acquired by the firm for specific para-
meter values and each accounting policy considered in this dissertation.

Accounting Policies 1 and 9

Accounting policy indices 1 and 9 indicate that the firm uses the
FIFO inventory method and SL depreciation method for tax reporting.
Consequently, the cash flows of the first project acquired by the firm
which uses either of these policies will be the same.

The nominal cash inflow generated by the firm's project after pay-
ments for taxes and raw materials at the end of the project's i-th year
(i=1,2,...,n-1) is:

\[
q_i^j \hat{p}_i^j - k \left[ q_i^j \hat{p}_i^j - I_0/n - xq_i^j \hat{p}_i^{i-1} \right] - xq_i^j \hat{p}_i^j.
\]

This expression can be explained as follows:

- \(q_i^j \hat{p}_i^j\) is the nominal cash inflow before payments for taxes and raw
  materials at the end of the first project's i-th year.
- \(k \left[ q_i^j \hat{p}_i^j - I_0/n - xq_i^j \hat{p}_i^{i-1} \right]\) is the nominal tax payment which
  must be paid at the end of the first project's i-th year. In this expression,
  \(I_0/n\) is the amount of SL depreciation expense in year \(i\). Given our as-
  sumption that the quantity of raw material purchases in year \(h\) is con-
  sumed in year \((h+1)\), we have that \(xq_i^j \hat{p}_i^{i-1}\) is the nominal dollar amount
  of raw material consumed in year \(i\) under the FIFO cost-flow assumption.

- \(xq_i^j \hat{p}_i^j\) is by definition the nominal payment for raw material made at
Table 3
The Nominal Cash Flows of the First Project Acquired by the Firm
For b=1, k=4, n=3, p=0.03, r=10, x=3, and l_0=$1200

<table>
<thead>
<tr>
<th>Accounting Policy</th>
<th>1 and 9 Year</th>
<th>2 and 10 Year</th>
<th>3, 5, 6, 11, 13, and 14 Year</th>
<th>4, 7, 8, 12, 15, and 16 Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Nominal cash flows before payments for taxes and raw materials</td>
<td>$883.04</td>
<td>$909.53</td>
<td>$936.82</td>
<td>$875.22</td>
</tr>
<tr>
<td>(2) Raw material taken as a tax deduction</td>
<td>257.20</td>
<td>264.91</td>
<td>272.86</td>
<td>262.57</td>
</tr>
<tr>
<td>(3) Depreciation taken as a tax deduction</td>
<td>400.00</td>
<td>400.00</td>
<td>400.00</td>
<td>400.00</td>
</tr>
<tr>
<td>(4) Taxable income (1) - (2) - (3)</td>
<td>225.84</td>
<td>244.63</td>
<td>263.96</td>
<td>212.65</td>
</tr>
<tr>
<td>(5) Payments for taxes</td>
<td>90.34</td>
<td>197.85</td>
<td>105.58</td>
<td>85.06</td>
</tr>
<tr>
<td>(6) Payments for raw materials</td>
<td>264.91</td>
<td>272.86</td>
<td>-</td>
<td>262.57</td>
</tr>
<tr>
<td>(7) Nominal cash flows after payments for taxes and raw materials (1) - (2) - (3)</td>
<td>527.79</td>
<td>538.82</td>
<td>831.29</td>
<td>527.59</td>
</tr>
<tr>
<td>(8) Original outlay</td>
<td>$1457.20</td>
<td>$1454.92</td>
<td>$1452.33</td>
<td>$1450.22</td>
</tr>
<tr>
<td>(9) Present value of common dollar net cash flows in years 1, 2, and 3 at time zero (discount rate = r)</td>
<td>$1457.20</td>
<td>$1454.92</td>
<td>$1452.33</td>
<td>$1450.22</td>
</tr>
</tbody>
</table>
the end of the first project's $i$-th year.

The nominal cash inflow generated by the first project after taxes at
the end of the first project's $n$-th year is:

$$q_n p^t = k \left[ q_n p^n - \left( \frac{I_0}{n} \right) - xq_i p^{n-1} \right].$$

The cash payment required at the end of year zero to acquire the
first project is $I_0 + xq_i^1$.

Using the above expression for the nominal cash flows of the first
project, we can calculate its real IRR by finding that number $r \neq -1$
which solves:

$$I_0 + xq_i^1 = \sum_{i=1}^{n-1} \frac{q_i^1 p_i - k \left[ q_i^1 p_i - \left( \frac{I_0}{n} \right) - xq_i^1 p^{i-1} \right] - xq_i p^i}{r p} + \frac{q_n p^n - k \left[ q_n p^n - \left( \frac{I_0}{n} \right) - xq_i p^{n-1} \right]}{p r}.$$

By substituting $b_i^{i-1} q_i^1$ for $q_i^1$ and rearranging terms, we have that:

$$I_0 = q_i^1 \left[ 1 - k + k x p^{-1} - x r \right] \sum_{i=1}^{n} b_i^{i-1} p^{-1} + \left( \frac{k I_0}{n} \sum_{i=1}^{n} r^{-1} p^{-1} \right)$$

For the sake of notational convenience, we let:

$$\chi = \left( 1 - k + k x p^{-1} - x r \right)$$

(9) $\chi = \left( 1 - k + k x p^{-1} - x r \right)$
We also note that:

(10) \[ \sum_{i=1}^{n} b_i^{1-L_i-1} = \frac{\hat{\omega} b_n}{r^n (r-b)} \] provided \( \hat{r} \neq b \), and

(11) \[ \sum_{i=1}^{n} r_i y_i = \frac{1-r^n p^n}{r^n p^n (1-rp)} \] provided \( r_p \neq 1 \).

Substituting these three results back into the equation above, we have that:

\[ I_0 = q^j_1 \chi \left[ \frac{r^n - b^n}{r^n (r-b)} \right] + k I_0 \left[ \frac{1-r^n p^n}{n r^n p^n (1-rp)} \right] \]

\[ = \frac{q^j_1 \chi (r^n - b^n) n p^n (1-rp)}{(r-b) \left[ n r^n p^n (1-rp) - k (1-r^n p^n) \right]} \]

For the sake of notational convenience, we define:

(12) \[ \Psi = n r^n \hat{p} (1-rp) - k (1-r^n p^n) \]

\[ \Omega \]

For the case in which \( \hat{r} = b \), \[ \sum_{i=1}^{n} b_i^{1-L_i-1} = n/b \]. Since there is no general expression for \[ \sum_{i=1}^{n} b_i^{1-L_i-1} \] which covers both the cases \( b=\hat{r} \) and \( b \neq \hat{r} \), we have to choose between these two cases if we are to avoid having multiple expressions for many of the model elements. In this and other similar situations, we have chosen to include the case decisions result in a significant restriction in the scope of our model and consequently, we note these decisions as they arise but do not refer to them later.
Making this substitution into the above equation, we have:

\[
\begin{align*}
I_0 &= \left( q_i^j \left( f^n - b^n \right) n^n \left( 1 - \hat{r} \hat{p} \right) \left( f - b \right) \right) \\
&= \left( q_i^j \left( f^n - b^n \right) n^n \left( 1 - \hat{r} \hat{p} \right) \left( f - b \right) \right)
\end{align*}
\]

Accounting Policies 2 and 10

Accounting policy indices 2 and 10 indicate that the firm uses the periodic LIFO inventory and SL depreciation methods for tax reporting. Consequently, the cash flows of the first project acquired by the firm which uses either of these policies will be the same.

The nominal cash inflow generated by the first project after payments for raw materials and taxes at the end of the first project's i-th year (i=1,2,...,n-1) is:

\[
(14) \quad q_i^j \hat{p}^j = k \left[ q_i^j \hat{p}_i^j - \left( l_0/n \right) - xq_i^j \hat{p}_i^j \right] = xq_i^j \hat{p}_i^j
\]

This expression is the same as expression (8) except for the last term in square brackets (i.e., \( xq_i^j \hat{p}_i^j \)). The term \( xq_i^j \hat{p}_i^j \) represents the amount of the reduction in taxable income from raw material consumption associated with the first project at the end of the project's i-th year under the periodic LIFO inventory method. The lengthy explanation of this small term is given below.

The calculation of the dollar amount of raw material consumption under any inventory method involves the multiplication of quantities and prices. In the models developed in this paper, we are assuming
that the quantity of raw material purchased at the end of year \( i-1 \) is consumed in year \( i \). The problem is then one of finding the price (or prices) to associate with this quantity. If we can assume that the quantity of raw material purchased by the firm at the end of year \( t \) is greater than or equal to the quantity of raw material purchased at the end of year \( t-1 \), the problem of finding the price to associate with the raw material quantity consumed by the firm in year \( t \) would be quite straightforward under the periodic LIFO inventory method. We would simply multiply the quantity of raw material consumed by the firm in year \( t \) (i.e., the quantity it purchased at the end of year \( t-1 \)) by the price which prevailed at the end of year \( t \). The price which prevailed at the end of year \( t \) is \( \hat{p} \) times the price which prevailed at the end of year \( t-1 \). Therefore, in order to derive the nominal dollar amount of inventory consumed by the firm in year \( t \) under the periodic LIFO cost-flow assumption, we would simply multiply \( \hat{p} \) by the nominal dollar amount of raw material purchased by the firm at the end of year \( t-1 \). If, however, we cannot assume that the quantity of raw material purchased by the firm at the end of year \( t \) is greater than or equal to the quantity of raw material purchased at the end of year \( t-1 \), the problem of finding the price to associate with the quantity of raw material consumed by the firm in year \( t \) under the periodic LIFO inventory method is quite complex. We would have to associate at least two different prices with the quantity of raw material consumed in year \( t \) in
order to determine the nominal dollar amount of raw material consumed by the firm in year $t$ under the periodic LIFO inventory method. This means that we would have at least two expressions for the nominal dollar amount of raw material consumed in year $t$ under the periodic LIFO inventory method if we tried to model both of these situations. In order to avoid having two expressions for this single element of our model, we decided to model either the situation in which the quantity of raw material purchased by the firm at the end of year $t$ is greater than or equal to the quantity purchased at the end of year $t-1$ or the situation in which the quantity of raw material purchased by the firm at the end of year $t$ is less than the quantity purchased at the end of year $t-1$. The situation in which the quantity of raw material purchased by the firm at the end of year $t$ is greater than or equal to (less than) the quantity purchased at the end of year $t-1$ corresponds to the case in which the growth rate in real gross investment is greater than or equal to (less than) zero. Since the case of non-negative growth is probably more prevalent and is of more interest to accountants than the case of negative growth (i.e., decay) we have chosen to restrict our attention to case of non-negative growth. So far in our discussion of the periodic LIFO inventory method, we have concentrated on determining the total nominal dollar amount of raw material that is consumed by the firm in year $t$. We have not yet touched upon the topic of determining the nominal dollar amount of raw material consumption at the end of the
first project's i-th year under the periodic LIFO inventory method. We know that our model of the firm is such that the firm is just an aggregation of identical projects. Given our assumption of a non-negative growth rate, we also know that the nominal dollar amount of raw material consumed by the firm in year t under the periodic LIFO cost-flow assumption is found by multiplying the nominal amount of raw material purchased by the firm at the end of year t-1 by the factor Ș. In order to have consistency within our model, we should use a similar method of calculation to determine the nominal dollar amount of raw material consumed at the end of the first project's i-th year. Consequently, we have that the amount of raw material associated with the firm's first project that is consumed at the end of the project's i-th year under the periodic LIFO inventory method is given by \( xq^j_1 \hat{p}^i \).

The nominal cash inflow generated by the first project after taxes at the end of the first project's n-th year is:

\[
q^j_n \hat{p}^n - k \left[ q^j_n \hat{p}^n - (I_0 / n) - xq^j_1 \hat{p}^n \right].
\]

The cash payment required at the end of year zero to acquire the first project is \( I_0 + xq^j_1 \).

Using the above expressions for the nominal cash flows of the first project, we can calculate its real IRR by finding that number \( r \neq -1 \) which solves:

\[
I_0 + xq^j_1 = \sum_{i=1}^{n-1} \frac{q^j_i \hat{p}^i}{\hat{p}^i} - k \left[ q^j_i \hat{p}^i - (I_0 / n) - xq^j_1 \hat{p}^i \right] - xq^j_i \hat{p}^i + xq^j_{i+1} \hat{p}^i + \ldots
\]
\[
q^j_n - k \left[ \frac{q^j_n - (I_0/n) - xv^j_n}{n^{n_p/n}} \right] = \\
\sum_{i=1}^{n} b^{i-1} \hat{r}^{-i} + (kI_0/n) \sum_{i=1}^{n} \hat{r}^{-i} \hat{p}^{-i}
\]
By rearranging terms and substituting \(b^{i-1} q^j_1\) for \(q^j_1\), we have that:
\[
I_0 = q^j_1 (1-k + kx - \hat{x}) \sum_{i=1}^{n} b^{i-1} \hat{r}^{-i} + (kI_0/n) \sum_{i=1}^{n} \hat{r}^{-i} \hat{p}^{-i}
\]
For notational convenience, we define:
\[(15) \quad \Delta = (1-k + kx - \hat{x})\]
By making the appropriate substitutions from equations (10), (11), and (15) we have:
\[
I_0 = q^j_1 \left[ \frac{\Delta (b^n \hat{r}^n)}{n^{n_p/n}} \right] + kI_0 \left[ \frac{1-\hat{r}^n}{n^{n_p/n}} \right]
\]
\[
= \frac{q^j_1 \Delta (b^n \hat{r}^n) n^{n_p/n} (1-\hat{p})}{(b-\hat{r}) [n^{n_p/n} (1-\hat{p})]} \quad (15)
\]
By substituting for \(\Psi\) from equation (12) into the above equation, we have:
\[(16) \quad I_0 = \frac{q^j_1 \Delta (b^n \hat{r}^n) n^{n_p/n} (1-\hat{p})}{(b-\hat{r}) \Psi}\]
Accounting Policies 3, 5, 6, 11, 13, and 14.
All of these accounting policies use the FIFO inventory and SYD depreciation methods for tax reporting.
The amount of SYD depreciation charged to expense in the \(i\)-th year \((i=1,2,\ldots,n)\) of the first project's life is given by \([2(n+1-i)/n(n+1)]I_0\). The only difference between the expressions for the nominal cash flows
for the first project under accounting policies 3, 5, 6, 11, 13, and 14 and the expressions for the nominal cash flows of the first project under accounting policies 1 and 9 is the term in the expressions relating to tax depreciation. Using this fact and equation (8), we can write the nominal cash inflow after payments for project's i-th year (i=1, 2, ..., n-I) as:

\[ q_{i}^{j} p^{i} - k \left\{ q_{i}^{j} p^{i} - 2 \left[ \frac{(n+1-i)}{n(n+1)} \right] - xq_{i}^{j} p^{i-1} \right\} - xq_{i+1}^{j} p^{i} . \]

Similarly, the cash inflow at the end of the first project's n-th year is:

\[ q_{n}^{j} p^{n} - k \left\{ q_{n}^{j} p^{n} - \left[ \frac{2}{0} \right] - xq_{n}^{j} p^{n-1} \right\} . \]

The cash payment required at the end of year zero to acquire the first project is \( I_{0} + xq_{1}^{j} \).

Using these expressions for the nominal cash flows of the first project, we can find its IRR by finding that number \( r \neq -1 \) which solves:

\[
I_{0} + xq_{1}^{j} = \sum_{i=1}^{n-1} q_{i}^{j} p^{i} - k \left\{ q_{i}^{j} p^{i} - 2I_{0} \left[ \frac{(n+1-i)}{i i p^{i}} \right] - xq_{i}^{j} p^{i-1} \right\} - \sum_{i=1}^{n-1} \frac{q_{i}^{j} p^{n} - k \left\{ q_{n}^{j} p^{n} - \left[ \frac{2I_{0}}{n(n+1)} \right] - xq_{n}^{j} p^{n} \right\}}{\frac{1}{i i p^{n}}} \]

By rearranging terms and making the appropriate substitutions from equations (9) and (10), we have:

\[
I_{0} = q_{1}^{j} \frac{b^{n} - b^{n}}{r^{n} - b} + \left[ \frac{2kI_{0}}{0(n(n+1))} \right] \sum_{i=1}^{n} (n+1-i) r^{i} p^{i} .
\]
It is easily shown that:

\[ (17) \sum_{i=1}^{n} (n+1-i) \hat{r}^{-i} \hat{p}^{-i} = \frac{1-\hat{r}^{n} \hat{p}^{n}}{\hat{r}^{n} \hat{p}^{n}} - \frac{nr \hat{r}^{n} \hat{p}^{n} (1-\hat{r} \hat{p})}{(1-\hat{r} \hat{p})^2}, \]

provided \( \hat{r} \hat{p} \neq 1 \).\(^9\) Making this substitution in the equation above and rearranging terms, we have:

\[ I_0 = \frac{q^j_1 \hat{r}^{n} - b^n}{(r-b) \left\{ \frac{n(n+1) \hat{r}^{n} \hat{p}^{n}}{(1-\hat{r} \hat{p})^2} - 2k \left[ 1-\hat{r}^{n} \hat{p}^{n} - nr \hat{r}^{n} \hat{p}^{n} (1-\hat{r} \hat{p}) \right] \right\}} \]

For the sake of notational convenience, we let

\[ (18) \alpha = \left\{ \frac{n(n+1) \hat{r}^{n} \hat{p}^{n}}{(1-\hat{r} \hat{p})^2} - 2k \left[ 1-\hat{r}^{n} \hat{p}^{n} - nr \hat{r}^{n} \hat{p}^{n} (1-\hat{r} \hat{p}) \right] \right\} \]

Substituting \( \alpha \) in the equation above, we have:

\[ I_0 = \frac{q^j_1 \hat{r}^{n} - b^n}{(r-b) \alpha} \]

Accounting Policies 4, 7, 8, 12, 15, and 16

All of these accounting policies use the LIFO inventory and SYD depreciation methods for tax purposes. Consequently, we can write the nominal cash inflow after payments for taxes and raw materials at the end of the first project's \( i \)-th year \((i=1,2,\ldots,n-1)\) as:

\[ q^j_{1,i} \hat{p}^i - k \left\{ q^j_{1,i} \hat{p}^i - 2I_0 \left[ \frac{(n+1-i)}{n(n+1)} \right] - x_{1,i} \hat{p}^i \right\} - x_{1,i+1} \hat{p}^i. \]

The cash payment required at the end of year zero to acquire the first project is \( I_0 + x_{1}^j \).

\(^9\) See note (3) of the mathematical appendix for a proof of this equality.
Therefore, the real IRR of the first project is that number \( r \neq -1 \)

which solves:

\[
I_0 + xq^j = \sum_{i=1}^{n-1} \frac{q^j \hat{p}^i}{\hat{r}^i \hat{p}^i} - k \left\{ \frac{q^j \hat{p}^i}{\hat{r}^i \hat{p}^i} - \frac{2I_0}{\hat{r}^i \hat{p}^i} \left[ \frac{n+1-i}{n(n+1)} \right] - xq^j \right\}
\]

\[
- \sum_{i=1}^{n-1} \frac{xq^j \hat{p}^i}{\hat{r}^i \hat{p}^i} + \frac{q^j \hat{p}^n}{\hat{r}^n \hat{p}^n} - k \left\{ \frac{q^j \hat{p}^n}{\hat{r}^n \hat{p}^n} - \frac{2I_0}{\hat{r}^n \hat{p}^n} \left[ \frac{n}{n(n+1)} \right] - xq^j \right\}
\]

By rearranging terms and making the appropriate substitutions from equations (10), (15), (17), and (18), we have

\[
(20) \quad I_0 = \frac{q^j \triangle (b^{n+1}_t) \hat{p}^n n(n+1)(1-\hat{r}^n)^2}{(b-\hat{r}) \propto}
\]

We have now developed the relationship between the initial investment, subsequent cash flows, and the real IRR of the first project acquired by the firm. We now turn our attention to developing the expressions needed to calculate the accounting income reported by the firm in year \( t \).

The Derivation of the Accounting Income Reported By the Firm

The development of expressions for the accounting income of the firm for the year which ends at time point \( t \) given accounting policy \( j(A^j_t) \) requires the development of expressions for:

1. The net revenue of the firm for the year which ends at time point \( t \) given accounting policy \( j(R^j_t) \).

2. The depreciation expense on the firm’s financial report for the year which ends at time point \( t \) given accounting policy \( j(FD^j_t) \).

3. The depreciation expense on the firm’s tax return for the year which ends at time point \( t \) given accounting policy \( j(TD^j_t) \).

4. The expense associated with the consumption of raw material.
on the firm's financial report for the year which ends at time point \( t \) given accounting policy \( j (FM^j_t) \).

(5) The expense associated with the consumption of raw material on the firm's tax return for the year which ends at time point \( t \) given accounting policy \( j (TM^j_t) \).

(6) The amount of deferred tax expense on the firm's financial report for the year which ends at time point \( t \) given accounting policy \( j (DT^j_t) \).

Given these definitions of the above expressions, we can define \( A^j_t \) as:

\[
A^j_t = R^j_t - FD^j_t - FM^j_t - DT^j_t - k (R^j_t - TM^j_t - TM^j_t)
\]

We now begin the development of the expressions needed to calculate \( A^j_t \). We note that in the development which follows we will be restricting our attention to years \( t \) such that \( t \) is greater than or equal to \( n \).

In order to aid the reader's understanding of the expressions developed in this section we have prepared a numerical example in which the accounting income of the firm in year \( n \) is calculated. The results of this example are depicted in Table 4.

The Net Revenue of the Firm in Year \( t \)

The net revenue of the firm in year \( t \) given accounting policy \( j (R^j_t) \) depends upon the nominal cash flows received from each of the firm's projects at the end of year \( t \). For \( t \geq n \), \( R^j_t \) is composed of the \( n \)th nominal cash flow (before payments for taxes and raw materials) of the project acquired by the firm at the end of year \((t-n)\), the \((n-1)\)-th flow of the project acquired by the firm at the end of year \((t-n+1)\), etc. Therefore, for \( t \geq n \) we can write:
Table 4
Accounting Income of the Firm in Year n
For \( b = 1, g = 0.03, k = 0.4, n = 3, p = 0.03, r = 0.10, x = 0.3, \) and \( l_0 = 1200 \)

<table>
<thead>
<tr>
<th>Accounting Policy</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<th>11</th>
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<th>13</th>
<th>14</th>
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<tbody>
<tr>
<td>Net revenue</td>
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<td>2869.97</td>
<td>2840.65</td>
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<td>2840.65</td>
<td>2817.06</td>
<td>2840.65</td>
<td>2817.06</td>
<td>2840.65</td>
<td>2817.06</td>
<td>2840.65</td>
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<td>2840.65</td>
<td>2817.06</td>
<td>2840.65</td>
<td>2817.06</td>
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<td>Depreciation expense on financial report</td>
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<td>1274.56</td>
<td>1299.67</td>
<td>1299.67</td>
<td>1274.56</td>
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<td>1364.31</td>
<td>1351.00</td>
<td>1351.00</td>
<td>1351.00</td>
<td>1351.00</td>
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<tr>
<td>Raw material charged to expense on financial report</td>
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<td>860.99</td>
<td>827.43</td>
<td>845.11</td>
<td>827.43</td>
<td>827.43</td>
<td>845.11</td>
<td>845.11</td>
<td>868.68</td>
<td>860.99</td>
<td>852.25</td>
<td>845.11</td>
<td>852.25</td>
<td>852.25</td>
<td>845.11</td>
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<td>713.55</td>
<td>672.25</td>
<td>738.66</td>
<td>738.66</td>
<td>627.39</td>
<td>697.38</td>
<td>675.92</td>
<td>657.98</td>
<td>624.09</td>
<td>607.64</td>
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<tr>
<td>Net revenue</td>
<td>2895.60</td>
<td>2869.97</td>
<td>2840.65</td>
<td>2817.06</td>
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<td>2840.65</td>
<td>2817.06</td>
<td>2840.65</td>
<td>2817.06</td>
</tr>
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<td>1274.56</td>
<td>1299.67</td>
<td>1299.67</td>
<td>1274.56</td>
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<td>1299.67</td>
<td>1299.67</td>
</tr>
<tr>
<td>Raw material charged to expense on tax return</td>
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<td>860.99</td>
<td>827.43</td>
<td>845.11</td>
<td>827.43</td>
<td>827.43</td>
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<td>845.11</td>
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<td>713.55</td>
<td>672.28</td>
<td>713.55</td>
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<td>672.28</td>
<td>672.28</td>
<td>777.66</td>
<td>734.42</td>
<td>713.55</td>
<td>672.28</td>
<td>713.55</td>
<td>672.28</td>
<td>713.55</td>
<td>672.28</td>
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<td>285.42</td>
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<td>268.91</td>
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<td>268.91</td>
<td>285.42</td>
<td>285.42</td>
<td>268.91</td>
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<td>-</td>
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<td>-</td>
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<td>-</td>
</tr>
<tr>
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<td>293.77</td>
<td>285.42</td>
<td>268.91</td>
<td>285.42</td>
<td>295.46</td>
<td>268.91</td>
<td>278.95</td>
<td>311.06</td>
<td>293.77</td>
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<td>295.46</td>
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<td>352.04</td>
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</table>
\[(21) \quad R^j_t = q_n^j p^j g^{t-n} \hat{p}^t \hat{p}^{t-n} + q_{n-1}^j p^{n-1} g^{t-n+1} \hat{p}^{t-n+1} + q_{n-2}^j p^{n-2} g^{t-n+2} \hat{p}^{t-n+2} + \ldots + q_{1}^j p g^{t-1} \hat{p}^{t-1} \]

The general term in equation (21) \(q_h^j \hat{p}^h g^{t-h} \hat{p}^{t-h}\) can be explained as follows:

\(q_h^j \hat{p}^h g^{t-h} \hat{p}^{t-h}\) is the nominal cash flow before payments for taxes and raw materials of the project acquired by the firm at the end of year \(h\) years after the project was acquired.

\(q_h^j \hat{p}^h g^{t-h} \hat{p}^{t-h}\) is the cash flow before payment for taxes and raw materials generated at the end of year \(t\) by the project which the firm acquired at the end of year \((t-h)\).

We can re-write equation (21) as:

\[R^j_t = \hat{p}^n \sum_{i=1}^{n} q_i^j \hat{g}^{t-i} = \hat{p}^t \hat{g}^{t} q_1^j \sum_{i=1}^{n} b_{i-1}/\hat{g}_i\]

\[= q_1^j \hat{p}^t \hat{g}^{t-n} (\hat{g}^n - \hat{p}^n) / (\hat{g} - \hat{b}) \quad \text{provided} \quad \hat{g} \neq \hat{b}.\]

At this point we note that \(R^j_t\) depends upon the cash flows generated by firm projects. However, the relationship between \(R^j_t\) and the cash flows of firm projects is the same for all accounting policies. Therefore, equation (22) holds for \(j=1,2,\ldots,16\).

Next we turn our attention to the development of expressions for the amount of depreciation expense on the firm's financial report for the year which ends at time point \(t\) given accounting policy \(j\) \((PD^j_t)\). Depreciation Expense on the Firm's Financial Report For Year \(t\)

Accounting Policies 1,2,5,6,7,8

Accounting policy indices 1,2,5,6,7,8 indicate that the firm reports
figures unadjusted for changes in the price level and uses the SI depreciation method for financial reporting. Consequently, we have for \( t : n \) and \( j=1, 2, 5, 6, 7, 8 \) that:

\[
FD_t^j = \frac{(1/n)}{\left( I_{t-1} + I_{t-2} + \ldots + I_{t-n} \right)} = \frac{\sum_{i=1}^{n} (\hat{g}^t)^{-1}}{I_0 (\hat{g}^t)^{t/n}} = I_0 (\hat{g}^t)^{t/n} (1-\hat{g}^n) \frac{1}{n\hat{g}^n \hat{p}^n (1-\hat{g}^t)} \quad \text{provided } \hat{g}^t \neq 1.
\]

(23)

Accounting Policies 9, 10, 13, 14, 15, 16

Accounting policies indices 9, 10, 13, 14, 15, 16 indicate that the firm reports figures adjusted for price level change and uses the SI depreciation method for financial reporting. For firms which issue financial reports adjusted for changes in the price level, we assume that all reported figures in year \( t \) will be stated in terms of the price level which prevails at the end of year \( t \). Therefore, for \( j=9, 10, 13, 14, 15, 16 \) and for \( t : n \), we have:

\[
FD_t^j = \frac{(1/n)}{\left( I_{t-1} + \hat{p}^2 I_{t-2} + \hat{p}^3 I_{t-3} + \ldots + \hat{p}^n I_{t-n} \right)} = \frac{\sum_{i=1}^{n} \hat{g}^{-1}}{I_0 \hat{g}^t I_0 (1-\hat{g}^n) \frac{1}{n\hat{g}^n \hat{p}^n (1-\hat{g})}} \quad \text{provided } \hat{g} \neq 1.
\]

Accounting Policies 3 and 4

Accounting policy indices 3 and 4 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the SYD depreciation for financial reporting. Therefore, for \( j=3, 4 \) and for \( t : n \),
we have:

$$PD_t^j = \left[ \begin{array}{c} \frac{1}{n(n+1)} \end{array} \right] \sum_{i=1}^{n} 2(n+1-i) I_{t-1}$$

$$= \left[ \begin{array}{c} \frac{1}{n(n+1)} \end{array} \right] \sum_{i=1}^{n} (n+1-i) \hat{g}^{-i-1} p$$

(25)

Now,

$$\sum_{i=1}^{n} (n+1-i) \hat{g}^{-i-1} p = \frac{1-\hat{g}^n p^n}{(1-\hat{g}^n p^n)^2} \hat{g}^{n+1} p^n \hat{g}^{n+1} (1-\hat{g}^n p^n)$$

provided $\hat{g}^n p^n \neq 1$.

For notational convenience, we define:

$$\beta = 1-\hat{g}^n p^n - n \hat{g}^n p^n (1-\hat{g}^n p^n)$$

Consequently,

$$\sum_{i=1}^{n} (n+1-i) \hat{g}^{-i-1} p = \beta \left[ \sum_{i=1}^{n} \hat{g}^n p^n (1-\hat{g}^n p^n)^2 \right]$$

Substituting this last result into equation (25), we have:

$$PD_t^j = \frac{2 \left[ \frac{1}{n(n+1)} \right] \beta}{n(n+1) \hat{g}^n p^n (1-\hat{g}^n p^n)^2}$$

Accounting Policies 11 and 12

Accounting policy indices 11 and 12 indicate that the firm reports figures adjusted for changes in the level of prices and uses the SYD depreciation method for financial reporting. Therefore, for $j=11$ and 12 and for $t \geq n$, we have:

---

10 See note (3) of the mathematical appendix for a proof of this equality.
\[ \text{FD}_t^i = \left(\frac{1}{n(n+1)}\right) \sum_{i=1}^{n} 2 (n+1-i) \hat{p}_t^i \hat{g}_{t-1}^{i} \]

\[ = \left[ 2 \int_0^{1} \hat{g}_t^t \hat{g}_t^t / n(n+1) \right] \sum_{i=1}^{n} (n+1-i) \hat{g}_{t-1}^{i} \]

But,

\[ \sum_{i=1}^{n} (n+1-i) \hat{g}_{t-1}^{i} = \frac{1-\hat{g}^n - n\hat{g}^n}{(1-\hat{g})^2 \hat{g}^n} \quad \text{provided } \hat{g} \neq 1. \]

For the sake of notational convenience, we define:

\[ \mathcal{E} = 1-\hat{g}^n - n\hat{g}^n (1-\hat{g}). \] Consequently, we have:

\[ \sum_{i=1}^{n} (n+1-i) \hat{g}_{t-1}^{i} = \frac{\mathcal{E}}{(1-\hat{g})^2 \hat{g}^n} \quad \text{provided } \hat{g} \neq 1. \]

Substituting this result into equation (28), we have:

\[ \text{FD}_t^i = \frac{2 \int_0^{1} \hat{g}_t^t \hat{g}_t^t \mathcal{E}}{n(n+1) \hat{g}^n (1-\hat{g})^2} \]

We have now developed expressions for \( \text{FD}_t^i \) for \( j=1,2,\ldots,16 \) and all \( t \geq n \). We now turn our attention to finding tax depreciation in year \( t \) given accounting policy \( j \) (TD\(_t^j\)).

Depreciation Expense on the Firm's Tax Return For Year \( t \)

We note again that we are assuming that taxable income will be based upon nominal dollar amounts and not price level adjusted amounts.

Accounting Policies 1, 2, 9, 10

Accounting policy indices 1, 2, 9, and 10 indicate that the firm uses

\[ \text{u} \quad \text{See note (3) of the mathematical appendix for a proof of this equality.} \]
the SL depreciation method for tax reporting. Therefore, for \( t \geq n \) and \( j = 1, 2, 9, \) and 10, we have (see equation (23)):

\[
TD_t^j = \frac{\frac{\prod j \hat{g} \hat{t} \hat{p} (1 - \hat{g} \hat{p}^n)}{n \hat{g}^n \hat{p}^n (1 - \hat{g} \hat{p})}}{\text{provided } \hat{g} \hat{p} \neq 1.}
\]

Accounting Policies 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16

All of these accounting policies use the SYD depreciation method for tax reporting. Therefore, for \( t \geq n \) and \( j = 3, 4, 5, 6, 7, 8, 11, 12, 13, 14, 15, 16, \) we have (see equation (27)):

\[
TD_t^j = \frac{\frac{2 \prod j \hat{p} \hat{t} \hat{g} (1 - \hat{g} \hat{p})}{n (n+1) \hat{g}^n \hat{p}^n (1 - \hat{g} \hat{p})^2}}{\text{provided } \hat{g} \hat{p} \neq 1.}
\]

We now turn our attention to the derivation of the expense associated with the consumption of raw material in year \( t \) given accounting policy \( j \).

Raw Material Charged to Expense on the Firm's Financial Report in Year \( t \)

In order to derive expressions for the dollar amount of raw material charged to expense on the firm's financial report in year \( t \) given accounting policy \( j \) (\( FM_t^j \)), we will find it helpful to have an expression for the dollar amount of raw material purchased by the firm at the end of year \( t \) given accounting policy \( j \) (\( MP_t^j \)). We now turn to this task.

First, we need to remember that the nominal dollar amount of raw material purchased by the firm will be the sum of the nominal dollar amounts of raw material purchased by the firm for each of the projects that it holds. We also need to remember that \( q_p g p^h \) is the nominal cash flow before payments for taxes and raw materials gener-
ated at the end of year \( t \) by the project which the firm acquired at the end of year \( (t-h) \) for \( t \geq n-1 \) and \( h=(1, \ldots, n) \). By definition, of the parameter \( x \), we have that \( x_{n} \beta^{t-h} g^{t-h} \) is the nominal amount of raw material purchases made by the firm at the end of year \( t-1 \) for the project which the firm acquired at the end of year \( t-h \). Using this information, we have that for \( t \geq (n-1) \):

\[
MP_{t}^{j} = \sum_{i=1}^{n} x_{i} \beta^{t} q_{i}^{j} \frac{\sum_{i=1}^{n} p_{i}^{j-1}}{q_{i}^{j-1}}
\]

\[
= \left[ \sum_{i=1}^{n} \beta^{t-n+1} q_{i}^{j} \left( g^{n-b} \right) \right] / (g-b)
\]  

(33)

At this point, we note from equation (22) that:

\[
R_{t+1}^{j} = \frac{\beta^{t-n+1} q_{t}^{j} (g^{n-b})}{(g-b)}.
\]  

(34)

By comparing equations (33) and (34) we see that:

\[
MP_{t}^{j} = x \frac{R_{t+1}^{j}}{\hat{\beta}}.
\]  

(35)

We now turn our attention to the development of expressions for \( FM_{t}^{j} \).

Accounting Policies 1, 3, 5, and 6

Accounting policy indices 1, 3, 5, and 6 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the FIFO inventory method. Since we are assuming that the quantity of raw material purchased at the end of year \( t-1 \) is consumed in year \( t \), we have that the dollar amount of raw material charged to expense on
the firm's financial report in year $t$ under the FIFO inventory method is
the dollar amount of raw material purchased at the end of year $t-1$.

Therefore, for all $t \geq n$ and $j=1,3,5,6$, we have (see equation (33)):

$$FM_t^j = MP_{t-1}^j = x^{t-1}_{-1} g^{t-n} q_1^j (g^n - b^n) / (g^n - b)$$

By comparing equations (35) and (36), we have for $t \geq n$ and $j=1,3,5,6$ that:

$$FM_t^j = x R_t^j / \hat{p}$$

Accounting Policies $2,4,7,8,9,10,11,12,13,14,15,16$

The accounting policy indices considered in this section indicate that the firm either (1) reports figures unadjusted for changes in the level of prices and uses the LIFO inventory method or (2) reports figures adjusted for changes in the level of prices. For both of these reporting practices the expression for $FM_t^j$ is the same for $t \geq n$.

Given that:

(1) the quantity of raw materials purchased at the end of year $t-1$

is consumed in year $t$, and

(2) the dollar amount of raw material purchased at the end of year $t-1$ is stated in terms of the price level which prevailed at the end of year $t-1$,

we have for all accounting policies that report price-level adjusted figures (i.e., for $j=9,10,11,\ldots,16$) that:

$$FM_t^j = \hat{p} MP_{t-1}^j$$

Remembering that we are assuming that the quantity of raw material
purchased at the end of year $t$ is greater than or equal to the quantity purchased at the end of year $t-1$, we have under the LIFO inventory method that:

$$\text{FM}_t^j = \hat{p} M_{t-1}^j.$$ 

Therefore, we have that for $j=2,4,7,8,\ldots,16$ and $t \geq n$, (see equation (33)):

$$\text{FM}_t^j = x^t g^{t-n} q_1^j (g^n - b^n) / (g-b).$$

We note that equation (38) implies that the dollar amount of raw material charged to expense is the same under the FIFO and LIFO cost-flow assumptions for all accounting policies which report price-level adjusted figures.\textsuperscript{12}

It will be helpful to have an expression which relates $R_t^j$ and $\text{FM}_t^j$ for $j=2,4,7,8,9,\ldots,16$. By comparing equations (22) and (39), we see that for $j=2,4,7,8,9,\ldots,16$ and for $t \geq n$:

$$\text{FM}_t^j = x^t R_t^j.$$ 

We now turn our attention to finding the expressions for the dollar amount of raw material charged to expense on the firm's tax return in the year $t$ given accounting policy $j$ ($\text{TM}_t^j$).

\textsuperscript{12} Within our model all prices change in the same proportion as the change in the general price level. Under this set of circumstances, Drebin has shown that the inventory cost flow assumption makes no difference in the amounts charged to expense and the amounts carried in inventory whenever price-level adjusted statements are prepared. Thus, the implication of our equation (36) is consistent with prior research findings. See:

Raw Material Charged to Expense on the Firm's Tax Return in Year $t$

Accounting Policies $1, 3, 5, 6, 9, 11, 13, 14$

Accounting policy indices $1, 3, 5, 6, 9, 11, 13, 14$ and $14$ indicate that the firm uses the FIFO inventory method for financial reporting (either adjusted or unadjusted for changes in the level of prices in order to determine the amount of raw material taken as a tax deduction in year $t$. Therefore, for $j=1, 3, 5, 6, 9, 11, 13, 14$ and for $t \geq n$, we have (see equation (36)):

$$T_{M_{t}^{j}} = x_{p}^{t-1} q_{l}^{t-n} (g_{n}^{n} - b^{n}) / (g-b).$$

Accounting Policies $2, 4, 7, 8, 10, 12, 15, 16$

Accounting policy indices $2, 4, 7, 8, 10, 12, 15, 16$ indicate that the firm uses the LIFO inventory method for tax purposes. Since taxable income is based upon nominal amounts, we have that for $j=2, 4, 7, 8, 10, 12, 15, 16$ and $t \geq n$, we have (see equation (38)):

$$T_{M_{t}^{j}} = x_{q}^{j} p_{l}^{t} q_{l}^{t-n} (g_{n}^{n} - b^{n}) / (g-b).$$

We now turn to the task of finding the amount of deferred tax shown as an expense on the firm's financial report for the year ended at time point $t$ given accounting policy $j$ ($D_{t}^{j}$).

The Amount of Deferred Income Tax Charged to Expense on the Firm's Financial Report in Year $t$

Accounting Policies $1, 2, 3, 5, 7, 9, 10, 11, 12, 13, 15$

Accounting policy indices $1, 2, 3, 5, 7, 9, 10, 11, 12, 13, 15$ indicate that either the firm (1) uses the same depreciation method for tax and financial reporting or (2) uses the SL depreciation method for financial
reporting, the SYD depreciation method for tax reporting, and the flow-
through method of accounting for deferred taxes. Therefore, for these
accounting policies, we have:

\[(42) \quad D_{i}^{j} = 0.\]

Accounting Policies 6, 8, 14, 16

Accounting policy indices 6, 8, 14, 16 indicate that the firm uses the
SL depreciation method for financial reporting, the SYD depreciation
method for tax reporting, and that the firm records the effect of deferred
taxes. Specifically, the firm will charge to expense in year \(t\) an amount
equal to the difference between the taxes that it would have had to pay
at the end of year \(t\) if it had used the SL depreciation method for tax
reporting and the amount of taxes that it actually had to pay at the end
of year \(t\).

The amount of deferred tax expense on the firm's financial report
in year \(t\) given accounting policy \(j\) \((j=6, 8, 14, 16)\) is the tax rate \(k\)
times the difference between price-level unadjusted SYD depreciation
in year \(t\) and price-level unadjusted SL depreciation in year \(t\). Using
this information, and equations (23) and (27) we have that for \(j=6, 8, 14,
16\) and for \(t \geq n:\n
\[(43) \quad D_{i}^{j} = k \left\{ \left[ \hat{\alpha}^{n+1} \frac{\hat{\alpha}^{n+1}}{n(n+1)} \frac{\hat{\alpha}^{n+1}}{(1-\hat{\alpha}^{n+1})^{2}} \right] - \left[ I_{0} \frac{\hat{\alpha}^{t}}{\hat{\alpha}^{t} (1-\hat{\alpha}^{n+1})} \right] \right\} \right\} \]
\[\left[ I_{0} \frac{\hat{\alpha}^{t}}{\hat{\alpha}^{t} (1-\hat{\alpha}^{n+1})} \right] \]
\[ \begin{align*}
&= k I_0^{g^t} \left[ - \frac{2 \beta}{n(n+1) (1-\hat{g}^n)^2} \left( 1 - \frac{\hat{g}^n \hat{g}^n}{\hat{g}^n \hat{g}^n} \right) \right] \\
& \quad \text{When we begin to derive the expressions for the accounting income of the firm in year } t \text{ given accounting policy } j \ (A_t^j), \text{ it will be convenient to note that for } j=6,8, \text{ and } t \geq n:\n& \quad (44) \quad D_T^1 = k(TD_t^j - FD_t^j).
\end{align*} \]

We now turn to the task of developing expressions for \( A_t^j \).

**Accounting Income of the Firm in Year**

For all accounting policies considered in this dissertation, we have:

\[ \begin{align*}
& (45) \quad A_t^j = R_t^j - FD_t^j - FM_t^j - k(R_t^j - TD_t^j - TM_t^j) - DT_t^j.
\end{align*} \]

**Accounting Policy 1**

For accounting policy 1, we have for \( t \geq n \):

\[ \begin{align*}
& FD_t^1 = TD_t^1 \quad (\text{from equations (23) and (31)}),\\
& FM_t^1 = TM_t^1 \quad (\text{from equations (36) and (40)}), \text{and}\\
& DT_t^1 = 0 \quad (\text{from equation (42)}).
\end{align*} \]

Therefore, we can write equation (45) as:

\[ \begin{align*}
& A_t^1 = (1-k) \left( R_t^1 - FD_t^1 - FM_t^1 \right).
\end{align*} \]

Substituting for \( R_t^1, FD_t^1, \) and \( FM_t^1 \) from equations (22)(23), and (37), and rearranging terms, we have:

\[ \begin{align*}
& A_t^1 = (1-k) \hat{g}^t \left[ \left( \hat{p} - \frac{1}{n} \frac{\hat{g}^n - \hat{p}^n}{\hat{g}^n \hat{g}^n} \right) g_1^1 - \frac{l_0 (1-\hat{g}^n \hat{g}^n)}{n \hat{g}^n \hat{g}^n (1-\hat{g}^n)} \right].
\end{align*} \]

Now substituting for \( l_0 \) from equation (13), we have:
\[
A_t^1 = (1-k) \hat{p}^t \hat{g}^t q_1^1 \left[ \frac{(\hat{g}^n - b^n)}{\hat{g}^n (\hat{g}-b)} \psi - \frac{\psi (\hat{g}^n - b^n) (1-\hat{g}^n) (1-\hat{g}^n)}{\hat{g}^n (1-\hat{g}^n) (\hat{g}-b) \psi} \right].
\]

\[
(46) = (1-k) \hat{p}^t \hat{g}^t q_1^1 \times \\
\left[ \frac{(\hat{g}^n - b^n)}{\hat{g}^n (\hat{g}-b)} (\hat{g}-b) \psi - \frac{\psi (\hat{g}^n - b^n) (1-\hat{g}^n) (1-\hat{g}^n)}{\hat{g}^n (1-\hat{g}^n) (\hat{g}-b) \psi} \right].
\]

Accounting Policy 2

For accounting policy 2, we have that for \( t \geq n \) that:

\[
FD_t^2 = TD_t^2 \quad \text{(from equations (23) and (31))},
\]

\[
FM_t^2 = TM_t^2 \quad \text{(from equations (38) and (41))},
\]

\[
DT_t^2 = 0 \quad \text{(from equation (42))}.
\]

Making these substitutions in equation (45), we have:

\[
A_t^2 = (1-k) (R_t^2 - FD_t^2 - FM_t^2).
\]

Now substituting for \( R_t^2, FD_t^2, \) and \( FM_t^2 \) from equations (22) and (23) and (39) and rearranging terms, we have:

\[
A_t^2 = (1-k) \hat{p}^t \hat{g}^t \left[ \frac{(1-x)(\hat{g}^n - b^n)}{\hat{g}^n (\hat{g}-b)} \psi - \frac{I_0 (1-\hat{g}^n) \hat{p}^n}{n\hat{g}^n \hat{g}^n (1-\hat{g}^n) \psi} \right].
\]

Substituting for \( I_0 \) from equation (16) and rearranging terms, we have:

\[
(47) \left[ \frac{(1-x)(b^n - \hat{g}^n)}{\hat{g}^n (b-\hat{g}) (b-\hat{g}) \psi - \frac{(b^n - \hat{g}^n) (1-\hat{g}^n)}{\hat{g}^n (b-\hat{g}) (b-\hat{g}) \psi}} \right] \]

Accounting Policy 3
For accounting policy 3, we have for $t \geq n$:

\[ FD_t^3 = TD_t^3 \quad \text{(from equations (27) and (32))}, \]

\[ FM_t^3 = TM_t^3 \quad \text{(from equations (36) and (40))}, \]

\[ DT_t^3 = 0 \quad \text{(from equation (42))}, \]

\[ FM_t^3 = xR_t^3 \hat{p} \quad \text{(from equation (37))}. \]

Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_t^3 = (1-k) \left[ R_t^3 \hat{p} - FD_t^3 \right] . \]

Substituting for $R_t^3$ from equation (22) and for $FD_t^3$ from equation (27) and rearranging terms, we have:

\[ A_t^3 = (1-k) \left[ -\frac{\hat{p}-x}{\hat{p}} \hat{p}^t \hat{q}^t \left( \hat{g}^n - \hat{b}^n \right) \hat{q}^3 - \frac{2I_0^3 \hat{p}^t \hat{q}^t \hat{g}^n \hat{b}^n}{n(n+1) \hat{g}^n \hat{b}^n \left( 1 - \hat{g}^n \right)^2} \right] . \]

Substituting for $I_0$ from equation (19) and rearranging terms, we have:

\[ A_t^3 = (1-k) \hat{p}^t \hat{q} \left( \hat{g}^n \hat{b}^n \left( 1 - \hat{g}^n \right)^2 \right) X

\[ \left[ \frac{\hat{p}-x}{\hat{p}} \left( \hat{g}^n - \hat{b}^n \right) \left( \hat{g}^n \hat{b}^n \left( 1 - \hat{g}^n \right)^2 \right) X - \frac{2I_0^3 \hat{p}^t \hat{q}^t \hat{g}^n \hat{b}^n}{n(n+1) \hat{g}^n \hat{b}^n \left( 1 - \hat{g}^n \right)^2} \right] . \]

Accounting Policy 4

For accounting policy 4, we have for $t \geq n$:

\[ FD_t^4 = TD_t^4 \quad \text{(from equations (27) and (32))}, \]

\[ FM_t^4 = TM_t^4 \quad \text{(from equations (38) and (41))}, \]
\[ DT_t^4 = 0 \quad \text{(from equation (42))}, \quad \text{and} \]

\[ FM_t^4 = xR_t^4 \quad \text{(from equation (39))}. \]

Making these substitutions into equation (45), we have that for \( t \geq n: \)

\[ A_t^4 = (1-k) \left( R_t^4 (1-x) - FD_t^4 \right). \]

Substituting for \( R_t^4 \) from equation (22) and for \( FD_t^4 \) from equation (27), we have:

\[ A_t^4 = (1-k) p^t q_t^4 \left[ \frac{q_t^4}{g_t^n} \frac{(g_t^n - b_t^n)(1-x)}{g_t^n (g_t^n - b_t^n)} - \frac{2I_0}{n(n+1)} \left( \frac{2}{1-g_t^p} \right)^2 \frac{\hat{\beta} n \hat{\beta} n}{g_t^n (b_t^n)} \right] . \]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[ A_t^4 = (1-k) p^t q_0^4 X \]

\[ \left( A_t^4 \right) = \frac{(1-x)}{(b_t^n - \hat{\beta}^n)} \frac{(1-\hat{\beta}_t^2 (b_t - \hat{\beta}) \alpha - 2(b_t - \hat{\beta}) (b_t^n - \hat{\beta}^n) (1-\hat{\beta}_t^2) \frac{\hat{\beta}_t}{g_t^n (b_t - \hat{\beta})^2 (b_t - \hat{\beta}) \alpha}}{g_t^n \frac{2}{1-g_t^p} n \frac{2}{1-g_t^p} \frac{\hat{\beta} n \hat{\beta} n}{g_t^n (b_t^n)}} \]

Accounting Policy 5

For accounting policy 5, we have that:

\[ FM_t^5 = TM_t^5 \quad \text{(from equations (36) and (40))}, \]

\[ DT_t^5 = 0 \quad \text{(from equation (42))}, \]

\[ FM_t^5 = xR_t^5 / \hat{\beta} \quad \text{(from equation (37))}. \]

Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_t^5 = (1-k) (\hat{\beta} - x) R_t^5 / \hat{\beta} + k TD_t^5 - FD_t^5. \]

Substituting for \( R_t^5 \) from equation (22), \( TD_t^5 \) from equation (32), and
From equation (23), we have:

\[ A_t^5 = \left( 1 - k \right) \frac{n^t \hat{q}^t \hat{q}^t (\hat{g}^t - b)^n}{\hat{p} g^n (g - b)} + \left( \frac{2 \hat{p}(\gamma \hat{q}^t \hat{p}^t \hat{p})}{n(n+1) \frac{\hat{g}^n \hat{p}^n}{(1 - \hat{g} \hat{p})^2}} \right) \]

By substituting for \( l_0 \) from equation (19) and rearranging terms,

we have:

\[ A_t^5 = \left( 1 - k \right) \frac{n^t \hat{q}^t \hat{q}^t (\hat{g}^t - b)^n}{\hat{p} g^n (g - b)} \left( \frac{2 \hat{p}(\gamma \hat{q}^t \hat{p}^t \hat{p})}{n(n+1) \frac{\hat{g}^n \hat{p}^n}{(1 - \hat{g} \hat{p})^2}} \right) \]

(50) \( (1 - k) \left( \hat{p} - \hat{x} \right) \left( \hat{g}^n - b^n \right) \left( 1 - \hat{g} \hat{p} \right)^2 \left( \hat{p} - b \right) \alpha \)

\[ \hat{p}(\hat{g} - b)^n \left( \hat{g}^n - b^n \right) \left( 1 - \hat{g} \hat{p} \right)^2 \left[ 2 \left( k \hat{p} - (n+1) \right) \frac{\hat{g}^n \hat{p}^n}{(1 - \hat{g} \hat{p})^2} \right] . \]

Accounting Policy 6

For accounting policy 6, we have for \( t \geq r \):

\[ F M_t^6 = T M_t^6 \quad \text{from equations (36) and (40)}, \]

\[ F M_t^6 = x R_t^6 / \hat{p} \quad \text{from equation (37)}, \]

\[ D I_t^6 = k(T D_t^6 - F D_t^6) \quad \text{from equation (44)}. \]

Making these substitutions in equation (45) and rearranging terms,

we have:

\[ A_t^6 = (1 - k) \left( R_t^6 \left( \hat{p} - \hat{x} / \hat{p} \right) - F D_t^6 \right) \]

Substituting for \( R_t^6 \) from equation (22) and for \( F D_t^6 \) from equation
(23), we have:

\[ A_t^6 = (1-k)^6 \beta_t \hat{\beta}^t \gamma \chi \left( \frac{(\hat{\beta}-\chi)(\hat{\beta}^n-b^n)}{\beta^n(\hat{\beta}-b)} \right) - I_0 (1-\hat{\beta}^n) \frac{\beta^n(\hat{\beta}^n-b^n)}{n^\beta^n(\hat{\beta}^n-b^n)} \]

Substituting for \( I_0 \) from equation (19) and rearranging terms, we have:

\[ A_t = (1-k)^6 \beta_t \hat{\beta}^t \chi \left( \frac{(\hat{\beta}-\chi)(\hat{\beta}^n-b^n)}{\beta^n(\hat{\beta}-b)} \right) \]

\[ \times \left( \frac{(\hat{\beta}-\chi)(\hat{\beta}^n-b^n)}{\beta^n(\hat{\beta}-b)} \right) \]

(51)

Accounting Policy 7

For accounting policy 7, we have for \( t \geq n \):

\[ FM_t^7 = TM_t^7 \quad \text{(from equations (38) and (41))}, \]

\[ DT_t^7 = 0 \quad \text{(from equation (42))}, \]

\[ FM_t^7 = xR_t^7 \quad \text{(from equation (30))}. \]

Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_t^7 = (1-k)^6 (1-x)R_t^7 - FD_t^7 + kTD_t^7. \]

Substituting for \( R_t^7 \), \( FD_t^7 \), and \( TD_t^7 \) from equations (22), (23), and (32), we have:

\[ A_t^7 = \hat{\beta}^t \gamma \chi \left( \frac{(1-k)^6 (1-x)(\hat{\beta}^n-b^n) \chi}{\hat{\beta}^n(\hat{\beta}-b)} \right) - I_0 (1-\hat{\beta}^n) \frac{\beta^n(\hat{\beta}^n-b^n)}{n^\beta^n(\hat{\beta}^n-b^n)} + \frac{2kI_0 \beta}{n(n+1)(\hat{\beta}^n-b^n)} \]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:
\[ A_t^7 = \left[ \frac{\hat{q}_0^{7} \hat{p}^{7} \hat{t}^{7} \hat{t}_{\infty}}{\hat{g}^{7} (b-\hat{g}) (b-\hat{\gamma})(1-\hat{\gamma} \hat{p})^{2} \infty} \right] \times \]

(52) \[ \sum_{(1-k)(1-x)(b^{n}-\hat{g}^{n})(1-\hat{\gamma}^{n} \hat{p})^{2} (b-\hat{\gamma}) \infty} + \Delta \left( b^{n-\hat{g}^{n}} (1-\hat{\gamma} \hat{p})^{2} (b-\hat{g}) \right) [2k \beta - (n+1)(l-\hat{g} \hat{p})(l-\hat{\gamma}^{n} \hat{p})] \]

Accounting Policy 8

For accounting policy 8, we have:

\[ FM_t^8 = TM_t^8 \quad \text{(from equations (38) and (41))}, \]

\[ DT_t^8 = k(TD_t^8 - FD_t^8) \quad \text{(from equation (44))}, \]

\[ FM_t^8 = xR_t^8 \quad \text{(from equation (39))}. \]

Making these substitutions in equation (45) and rearranging terms, we have:

\[ A_t^8 = (1-k)(R_t^8(1-x) - FD_t^8). \]

Substituting for \( R_t^8 \) and \( FD_t^8 \) from equations (22) and (23), we have:

\[ A_t^8 = (1-k)\hat{p}^{7} \hat{t}^{7} \hat{q}^{7} \left[ (1-x)(\hat{g}^{7} - b^{7})^{-1} q^{7} - \frac{\hat{g}^{7} (1-\hat{g}^{7} \hat{p}^{7})}{n \hat{g}^{7} \hat{g}^{7}(1-\hat{g}^{7} \hat{p}^{7})} \right]. \]

Substituting for \( L_0 \) from equation (20) and rearranging terms, we have:

\[ A_t^8 = (1-k)\hat{p}^{7} \hat{t}^{7} q_{1}^{7} \times \]

(53) \[ \frac{(1-x)(b^{n}-\hat{g}^{n})(1-\hat{\gamma}^{n} \hat{p}) - (b^{n}-\hat{g}^{n})(1-\hat{\gamma}^{n} \hat{p}) (b-\hat{g})(n+1)(1-\hat{\gamma}^{n} \hat{p})^{2} \Delta}{\hat{g}^{7} (b-\hat{g}) (b-\hat{\gamma})(1-\hat{\gamma} \hat{p})^{2} \infty} \]

Accounting Policy 9

For accounting policy 9, we have that for \( t \geq n \):

\[ FM_t^9 = xR_t^9 \quad \text{(from equation (39))}, \]
\[ TM_t^9 = xR_t^9/p \quad (\text{from equations (22) and (41)}) \, , \text{ and} \]
\[ DT_t^9 = 0 \quad (\text{from equation (42)}) \, . \]

Making these substitutions in equation (45) and rearranging terms, we have:
\[ A_t^9 = R_t^9 (1-k + kxp^{-1} - x) + kTD_t^9 - FD_t^9 . \]

For the sake of notational convenience, we define:
\[ \Theta = (1-k + kxp^{-1} - x) \, . \]

Substituting for \( R_t^9 \), \( FD_t^9 \), and \( TD_t^9 \) from equations (22), (24), and (31) and rearranging terms, we have:
\[ A_t^9 = g_t^9 \left\{ \frac{q_t^9 (a^n b^n)}{\hat{g}^n (a-b)} \Theta \right\} + \frac{I_0 [(1-\hat{g}^n)(1-\hat{g}^n) - (1-\hat{g}_t^m)(1-\hat{g}_t^m)]}{n_a^m \hat{P}^m (1-\hat{g}_t^m) (1-\hat{g}_t^m)} \]

For the sake of notational convenience, we define:
\[ \phi = k(1-\hat{g}) (1-\hat{g}^n \hat{P}^n) - \hat{P}^n (1-\hat{g}) (1-\hat{g}^n) \, . \]

Substituting for \( \phi \) from equation (55) and for \( I_0 \) from equation (13) and rearranging terms, we have
\[ A_t^9 = q_t^9 \left\{ g_t^9 \right\} \, x \]

\[ \frac{(b^n-\hat{g}^n)}{(b-\hat{g}) \hat{g}^n (1-\hat{g}) (1-\hat{g})} \Phi + \frac{\phi (b^n-\hat{g}^n)}{(b-\hat{g}) \hat{g}^n (1-\hat{g}) (1-\hat{g})} \Psi \]

Accounting Policy 10

For accounting policy 10, we have for \( t = n \) that:
\[ PM_t^{10} = TM_t^{10} \quad (\text{from equations (38) and (41)}) \, , \]
\[ DT_t^{10} = 0 \quad (\text{from equation (42)}) \, , \text{ and} \]
\[ \text{FM}_{t}^{10} = xR_{t}^{10} \quad \text{(from equation (39)).} \]

Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_{t}^{10} = (1-k) (1-x) R_{t}^{10} + kTD_{t}^{10} - FD_{t}^{10}. \]

Substituting for \( R_{t}^{10} \), \( TD_{t}^{10} \), and \( FD_{t}^{10} \) from equations (22), (31), and (24) and rearranging terms, we have:

\[ A_{t}^{10} = \hat{p}^{t} g \left[ \frac{(1-k)(1-x)(\hat{g}^n-b^n)q^{10}}{g^n(g-b)} \right] + \frac{10}{n} \left[ \frac{(1-q^n)(1-q^n)(1-q^n)(1-q^n)}{n \hat{g}^n \hat{p}^n (1-\hat{g}^n)(1-\hat{g}^n)} \right]. \]

Substituting for \( I_0 \) from equation (16) and for \( \phi \) from equation (55), and rearranging terms, we have:

\[ A_{t}^{10} = q^{10} \hat{p}^{t} g \left[ \sum \right]. \]

\[ \left[ \frac{(1-k)(1-x)(b^n - \hat{g}^n)(b^n - \hat{g}^n)(b^n - \hat{g}^n)(b^n - \hat{g}^n)(b^n - \hat{g}^n)(b^n - \hat{g}^n)(b^n - \hat{g}^n)}{\hat{g}^n (b-\hat{g})(1-\hat{g}^n)(1-\hat{g}^n)(b-\hat{g})(1-\hat{g}^n)(b-\hat{g})(1-\hat{g}^n)(b-\hat{g})(1-\hat{g}^n)(b-\hat{g})} \right]. \]

Accounting Policy II

For accounting policy II, we have for \( t \geq n \) that:

\[ \text{FM}_{t}^{11} = \hat{p} \text{TM}_{t}^{11} = xR_{t}^{11} \quad \text{(from equations (38), (39), and (40)), and} \]

\[ \text{DT}_{t}^{11} = 0 \quad \text{(from equation (42)).} \]

Making these substitutions into equation (45), we have:

\[ A_{t}^{11} = (1-k - k \hat{g}^{n-1} - x) R_{t}^{11} + kTD_{t}^{11} - FD_{t}^{11}. \]

Substituting for \( \Theta \), \( R_{t}^{11} \), \( TD_{t}^{11} \), and \( FD_{t}^{11} \) from equations (54), (22), (32) and (30), and rearranging terms, we have:

\[ A_{t}^{11} = \hat{p}^{t} g \left[ \sum \right]. \]

\[ \left[ \frac{\phi (\hat{g}^n - b^n)q^{11}}{\hat{g}^n (g-b)} \right] + \frac{2k \phi I_0}{n(n+1)(1-\hat{g}^n)^2 \hat{g}^n \hat{p}^n} - \frac{2 \phi I_0}{n(n+1)\hat{g}^n \hat{p}^n (1-\hat{g}^n)^2}. \]
Substituting for $I_0$ from equation (19) and rearranging terms, we have:

$$
\lambda_t^{11} = \frac{p_t^t q_t^{11}}{q_t^{n(b-\delta)}(1-\delta)^2(1-\theta)^2(b-\gamma)-\infty} \times
$$

$$(58) \left( b^n - \delta^n \right) (1-\delta)^2 (1-\theta)^2 (b-\gamma) \Theta \infty + 2(b^n - \delta^n)(1-\delta)^2 (b-\gamma) \Theta \left[ k \delta (1-\delta)^2 - \delta^n (1-\delta)^2 \right] \lambda_t^{12}$$

Accounting Policy 12

For accounting policy 12, we have for $t \geq n$ that:

$$FM_t^{12} = TM_t^{12} \quad \text{from equations (38) and (41)},$$

$$FM_t^{12} = xR_t^{12} \quad \text{from equation (39)},$$

$$DF_t^{12} = 0 \quad \text{from equation (42)}.$$

Making these substitutions into equation (45) and rearranging terms, we have:

$$A_t^{12} = (1-k)(1-x)R_t^{12} + kTD_t^{12} - FD_t^{12}.$$  

Substituting for $R_t^{12}$, $TD_t^{12}$, and $FD_t^{12}$ from equations (22), (32), and (30) and rearranging terms, we have:

$$A_t^{12} = \frac{p_t^t g_t^{12}}{q_t^{n(b-\delta)}(1-\delta)^2(1-\theta)^2(b-\gamma)-\infty} \times
$$

$$\left( b^n - \delta^n \right) (1-\delta)^2 (1-\theta)^2 (b-\gamma) \Theta \left[ \frac{2k_0 \delta}{n(n+1)} - \frac{2k_0}{n(n+1)} \right]$$

Substituting for $I_0$ from equation (17) and rearranging terms, we have:

$$A_t^{12} = \left[ \frac{12 t^{12}}{q_t^{n(b-\delta)}(1-\delta)^2(1-\theta)^2(b-\gamma)-\infty} \right] x.$$
\[ (55) \quad \left\{ (1-k)(1-x)(b^n - \hat{a}_n^p) (1-\hat{a}_n)^2 (1-\hat{a})^2 (b-\hat{a}) \right\} \hat{c} + 2(b^n - \hat{a}_n^p) (1-\hat{a}_p)^2 (b-\hat{a}) \Delta \left[ \beta (1-\hat{a})^2 - \hat{a}_n (1-\hat{a}_p)^2 \right] \]

Accounting Policy 13

For accounting policy 13, we have for \( t \geq n \) that:
\[ F_{t}^{13} = pT_{t}^{13} = xR_{t}^{13} \quad (\text{from equations (38), (39), and (40)}) \quad \text{and} \quad D_{t}^{13} = 0 \quad (\text{from equation (42)}). \]

Making these substitutions into equation (45) and rearranging terms, we have:
\[ A_{t}^{13} = (1-k) + kxR_{t}^{13} - x) \hat{R}_{t}^{13} - F_{t}^{13} + kT_{t}^{13}. \]

Substituting for \( \hat{R}_{t}^{13}, F_{t}^{13}, \) and \( T_{t}^{13} \) from equations (52), (22), (24), and (30) are rearranging terms, we have:
\[ A_{t}^{13} = \hat{B}_{13}^{\omega} \left[ \left. \frac{\Theta ((b-n-\hat{a}_n^p) \hat{B}^n (b-\hat{a})^n)}{\hat{a}_n (b-\hat{a})^n} \right] - \frac{I_{n} (1-\hat{a})}{n^2 (1-\hat{a})} + \frac{2k \beta (1-\hat{a})}{n (n+1) \hat{a}_n^2 (1-\hat{a}_p)^2 \hat{a}_n^2} \right]. \]

Substituting for \( I_{0} \) from equation (15) and rearranging terms, we have:
\[ A_{t}^{13} = \left[ \frac{\hat{B}_{13}^{\omega} + \hat{B}_{13}^{\omega}}{\hat{a}_n (b-\hat{a})^n (1-\hat{a}_p)^2 (b-\hat{a})} \right] X \]

\[ (60) \quad \left\{ \Theta (b-n-\hat{a}_n^p) (1-\hat{a}_p)^2 (b-\hat{a}) \right\} \hat{c} + 2(b^n - \hat{a}_n) (1-\hat{a}_p)^2 (b-\hat{a}) \Delta \left[ \beta (1-\hat{a})^2 - \hat{a}_n (1-\hat{a}_p)^2 \right] \]

Accounting Policy 14

For accounting policy 14, we have for \( t \geq n \) that:
\[ F_{t}^{14} = pT_{t}^{14} = xR_{t}^{14} \quad (\text{from equations (38), (39), and (40)}). \]
Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_{t}^{14} = (1-k + k\hat{\gamma} - 1 - x) R_{t}^{14} - FD_{t}^{14} + kTD_{t}^{14} - DT_{t}^{14}. \]

Substituting for \( \Theta \), \( R_{t}^{14} \), \( FD_{t}^{14} \), \( TD_{t}^{14} \), and \( DT_{t}^{14} \) from equations (54), (22), (24), (32), and (43) and rearranging terms, we have:

\[ A_{t}^{14} = \hat{p}^{\gamma} t \left[ \frac{\Theta (\hat{\gamma})^{n} - b^{n}}{g^{n} (b^{-g})} \right] + \frac{I_{0} k (1-\hat{\gamma}) (1-\hat{\gamma})^{n} p^{n} - b^{n} (1-\hat{\gamma})^{n} (1-\hat{\gamma})^{n} (1-\hat{\gamma})^{n}}{n (1-\hat{\gamma})^{n} c^{n} p^{n} (1-\hat{\gamma})^{n}} \]

Substituting for \( I_{0} \) and \( \hat{p} \) from equations (19) and (55) into the above and rearranging terms, we have:

\[ A_{t}^{14} = \hat{p}^{\gamma} t \frac{t^{14}}{g^{n} (b^{-g}) (1-\hat{\gamma}) (b^{-f})} \times \]

\[ (61) \left\{ \Theta (\hat{\gamma}^{n} - b^{n}) (1-\hat{\gamma}) (1-\hat{\gamma}) (b-\hat{\gamma}) \right\} + (n+1) \frac{(b^{-g}) (b^{-r}) (1-\hat{\gamma})^{2} \phi \chi^{2}}{2} \]

Accounting Policy 15

For accounting policy 15, we have for \( t \geq n \) that:

\[ FM_{t}^{15} = TM_{t}^{15} = xR_{t}^{15} \] (from equations (38), (39), and (41)) and

\[ DT_{t}^{15} = 0 \] (from equation (42)).

Making these substitutions into equation (45) and rearranging terms, we have:

\[ A_{t}^{15} = (1-k) (1-x) R_{t}^{15} - FD_{t}^{15} + kTD_{t}^{15}. \]

Substituting for \( R_{t}^{15} \), \( FD_{t}^{15} \), and \( TD_{t}^{15} \) from equations (22), (24), and (32), and rearranging terms, we have:
\[ A_t^{15} = \frac{\hat{p}_t^{\hat{p}_t}}{a_t^{\hat{p}_t}} \left[ \frac{(1-k)(1-x)(\beta^n - b^n)q_t^{15}}{\sigma^n (b - \beta)} - I_0^{n+1}(1-\delta^n) + \frac{2k\hat{p}_t}{n(n+1)(1-\delta^n)(1-\delta\beta)^2} \right]. \]

Substituting for \( I_0 \) from equation (20) and rearranging terms,
we have:
\[
A_t^{15} = \left[ \frac{\hat{p}_t^{\hat{p}_t} q_t^{15}}{\sigma^n (b - \beta)(1-\delta^n)(1-\delta\beta)^2 (b - \hat{p})} \right] X
\]

(62) \[
\left( (l-k)(1-x)(b^n - \hat{p}^n)(1-\delta^n)(1-\delta\beta)^2 (b - \hat{p}) \right) \Delta \left[ (n+1)^\delta^n (1-\delta^n)(1-\delta\beta)^2 - 2k\hat{p}_t \right] \right].
\]

Accounting Policy 16

For accounting policy 16, we have for \( t \geq n \) that:
\[
FM_t^{16} = TM_t^{15} - xk_t^{16} \quad \text{(from equations (38), (39), and (40)).}
\]

Making these substitutions into equation (45) and rearranging terms, we have:
\[
A_t^{16} = (1-k)(1-x) R_t^{16} - FD_t^{16} + kTD_t^{16} - DT_t^{16}.
\]

Substituting for \( R_t^{16}, FD_t^{16}, TD_t^{16} \), and \( DT_t^{16} \) from equations (22),
(24), (32), and (43) and rearranging terms, we have:
\[
A_t^{16} = \frac{\hat{p}_t^{\hat{p}_t}}{a_t^{\hat{p}_t}} \left[ \frac{(1-k)(1-x)(\beta^n - b^n)q_t^{16}}{\sigma^n (b - \beta)} + \frac{I_0^{n+1}(1-\delta^n)(1-\delta^n\delta^n - \hat{p}^n(1-\delta^n)(1-\delta\beta}}{n(1-\delta^n)(1-\delta\beta)\delta^n} \right].
\]

Substituting for \( \Phi \) and \( I_0 \) from equations (55) and (20) and rearranging terms, we have:
\[
A_t^{16} = \frac{\hat{p}_t^{\hat{p}_t} q_t^{16}}{\sigma^n (b - \beta)(1-\delta\beta)(1-\delta\beta) (b - \hat{p})} X
\]
\[
\begin{align*}
(63) \quad & \left(1 - x\right) \left(1 - k\right) \left(b^n - \hat{g}^n\right) \left(1 - \frac{\hat{g}}{b}\right) \left(1 - \frac{\hat{g}}{b}\right) \left(1 - \frac{\hat{g}}{b}\right) \\
& + \left(n + 1\right) \left(b - \hat{g}\right) \left(b^n - \hat{g}^n\right) \left(1 - \frac{\hat{g}}{b}\right) \left(1 - \frac{\hat{g}}{b}\right) \left(1 - \frac{\hat{g}}{b}\right)
\end{align*}
\]

This then concludes our development of expressions for the accounting income of the firm in year \( t \) for all accounting policies considered in this dissertation. We now turn our attention to the development of expressions for the accounting book value of owners' equity at the beginning of year \( t \).

The Derivation of the Accounting Book Value of Owners' Equity

The development of expressions for the accounting book value of owners' equity at the beginning of year \( t \) given accounting policy \( j \) \((B'V^j_t)\) requires the development of expressions for:

1. The accounting book value of the firm's long-lived depreciable assets at the beginning of year \( t \) given accounting policy \( j \) \((N_t^j)\).
2. The accounting book value of the firm's raw material inventory at the beginning of year \( t \) given accounting policy \( j \) \((M_t^j)\).
3. The accounting book value of the firm's deferred tax liability at the beginning of year \( t \) given accounting policy \( j \) \((DTL_t^j)\).

We now turn to the task of developing these expressions. In order to aid the reader's understanding of our development of these expressions, we have prepared a numerical example in which the accounting book value of owners' equity at the beginning of year \( n \) is calculated. The results of this example are depicted in Table 5.
<table>
<thead>
<tr>
<th>Raw material inventory</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
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<tbody>
<tr>
<td>843.38</td>
<td>812.24</td>
<td>827.43</td>
<td>797.26</td>
<td>827.43</td>
<td>827.43</td>
<td>797.26</td>
<td>858.68</td>
<td>860.95</td>
<td>852.26</td>
<td>838.40</td>
<td>852.26</td>
<td>852.26</td>
<td>838.40</td>
<td>838.40</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-lived depreciable assets (net)</td>
<td>2599.33</td>
<td>2599.33</td>
<td>2187.21</td>
<td>2187.21</td>
<td>2599.33</td>
<td>2599.33</td>
<td>2599.33</td>
<td>2728.63</td>
<td>2728.63</td>
<td>2285.01</td>
<td>2728.63</td>
<td>2728.63</td>
<td>2728.63</td>
<td>2728.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total assets</td>
<td>3342.71</td>
<td>311.57</td>
<td>3014.64</td>
<td>2984.47</td>
<td>3426.76</td>
<td>3426.76</td>
<td>3396.59</td>
<td>3396.59</td>
<td>3597.31</td>
<td>3580.62</td>
<td>3137.27</td>
<td>3123.41</td>
<td>3580.89</td>
<td>3567.03</td>
<td>3567.03</td>
<td></td>
</tr>
<tr>
<td>Deferred tax liability</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>164.83</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>Owners' equity</td>
<td>3342.71</td>
<td>311.57</td>
<td>3014.64</td>
<td>2984.47</td>
<td>3426.76</td>
<td>3426.76</td>
<td>3396.59</td>
<td>3396.59</td>
<td>3597.31</td>
<td>3580.62</td>
<td>3137.27</td>
<td>3123.41</td>
<td>3580.89</td>
<td>3567.03</td>
<td>3397.20</td>
<td></td>
</tr>
</tbody>
</table>

Table 5
Book Value of Owners' Equity at the Beginning of Year n
For b=1, g=.03, k=.4, n=3, p=.03, r=.10, x=.3, and I₀=1200
The Accounting Book Value of Long-lived Depreciable Assets

Accounting Policies 1, 2, 5, 6, 7, 8

Accounting policy indices 1, 2, 5, 6, 7, and 8 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the SL depreciation method for financial reporting. Therefore, for \( j = 1, 2, 5, 6, 7, 8 \), and for \( t \geq n \), we can write:

\[
N_t^j = \frac{n}{n} I_{t-1} + \frac{n-1}{n} I_{t-2} + \ldots + \frac{1}{n} I_{t-n}
\]

\[
= \frac{\hat{t}^n t^t I_0}{n} \sum_{i=1}^{n} (n+1-i) \hat{p}_i^{-1}
\]

But,

\[
\sum_{i=1}^{n} (n+1-i) \hat{p}_i^{-1} = \frac{1 - \hat{p}_n \hat{p}^{n} - \hat{p}_2 \hat{p}^{n} (1-\hat{p})}{\hat{p}_n \hat{p}^n (1-\hat{p})^2} \quad \text{provided} \quad \hat{p} \neq 1
\]

\[
= \frac{\hat{p}}{\hat{p}_n \hat{p}^n (1-\hat{p})^2} \quad \text{(see equation (26) ).}
\]

Substituting this result back into the equation above, we have:

\[
(64) \quad N_t^j = \frac{\hat{t}^n t^t I_0 \hat{p}}{n \hat{p}_n \hat{p}^n (1-\hat{p})^2}
\]

Accounting Policies 9, 10, 13, 14, 15, 16

Accounting policy indices 9, 10, 13, 14, 15, and 16 indicate that the firm reports figures adjusted for changes in the level of prices and uses the SL depreciation method for financial reporting. For these accounting policies, all statement of earnings items in year \( t \) are

\[13\] See note (3) of the mathematical appendix.
stated in terms of the price level which prevails at the end of year \( t \).

In order that the calculation of firm ARR be made in common dollars, we will also express the book value of firm equity in terms of the price level which prevails at the end of year \( t \). Therefore, for \( j = 9, 10, 13, 14, 15, 16 \) and for \( t \geq n \), we have:

\[
N_t = \frac{n}{n} t_{t-1} \hat{p} + \frac{n-1}{n} t_{t-2} \hat{p}^2 + \ldots + \frac{1}{n} t_{t-n} \hat{p}^n
\]

\[
= \frac{\hat{p} t_{t} t_{0}}{n} \sum_{i=1}^{n} \frac{(n+1-i) \hat{g}^{-1}}{i}
\]

But,

\[
\sum_{i=1}^{n} \frac{(n+1-i) \hat{g}^{-1}}{i} = \frac{1 - \frac{\hat{g}^n}{\hat{g}^{n-1}} (1 - \hat{g})}{\hat{g}^n (1 - \hat{g})} \quad \text{provided } \hat{g} \neq 1. \quad (see \text{equation (29)})
\]

Substituting this result into the equation above, we have:

\[
(65) \quad N_t^j = \frac{\hat{p} t_{t} t_{0}}{n \hat{g}^n (1 - \hat{g})^z} E
\]

Accounting Policies 3 and 4

Accounting policy indices 3 and 4 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the SYD depreciation method for financial reporting.

It is an easy matter to show that if an asset has a useful life of \( n \) years then the book value of that asset after it has been depreciated for \( i \) years under the SYD depreciation method is:

\[1^{4}\text{ See note (3) of the mathematical appendix.}\]
\[
\begin{bmatrix}
(n-j)(n-j+1) \\
n(n+1)
\end{bmatrix}
\text{asset cost-salvage value}
\]

Therefore, for \( j = 3, 4 \) and \( t \geq n \), we can write that:

\[
N_j^t = \frac{(n-0)(n-0+1)}{n(n+1)} \frac{I}{t-1} + \frac{(n-1)(n-1+1)}{n(n+1)} \frac{I}{t-2} + \ldots + \frac{(1)(2)}{n(n+1)} \frac{I}{t-n}
\]

\[
= \sum_{i=0}^{n-1} \frac{1}{n(n+1)} \frac{I}{i^t} \left( n-i \right) \left( n-i+1 \right) g^{-i} p^{-i}
\]

But, \( \sum_{i=0}^{n-1} (n-i)(n-i+1)g^{-i}p^{-i} = \frac{2n - n^2}{2(n-1)!} p^{-n} (1-\delta_P) \frac{p^n}{g^n (1-\delta_P)^3} \)

provided \( \delta_P \neq 1 \).\(^{15}\) Substituting from equation (26) into the equation above, we have:

\[
\sum_{i=0}^{n-1} (n-i)(n-i+1)g^{-i}p^{-i} = \frac{2n - n^2}{2(n-1)!} p^{-n} (1-\delta_P) \frac{p^n}{g^n (1-\delta_P)^3}
\]

Substituting this last result into equation (66), we have:

\[
N_j^t = \frac{1}{n(n+1)} \frac{I}{t^t} \left[ \frac{2n - n^2}{2(n-1)!} p^{-n} (1-\delta_P) \frac{p^n}{g^n (1-\delta_P)^3} \right]
\]

Accounting Policies 11 and 12

Accounting policy indices 11 and 12 indicate that the firm reports figures adjusted for changes in the level of prices and uses the SYD depreciation method for financial reporting. Using the ideas of the previous section, we can write for \( j = 11, 12 \) and \( t \geq n \) that:

\[^{15}\text{See note (4) of the mathematical appendix.}\]
\[ N_t^j = \frac{(n-0)(n-0+1)}{n(n+1)} \sum_{i=0}^{t-1} \frac{p^i}{n(n+1)} + \frac{(n-1)(n-1+1)}{n(n+1)} \sum_{i=2}^{t-n} \frac{p^i}{n(n+1)} \]

Substituting from equation (29) into the above, we have:

\[ (68) \quad N_t^j = \frac{1}{n(n+1)} \left[ \frac{1}{\bar{g}^{t-t+1}} \sum \frac{2(1-\bar{g}^n - n\bar{g}^n(1-\bar{g}) - n(n+1)\bar{g}^n(1-\bar{g})^2}{\bar{g}^{n-t+1}} \right] \]

This concludes our development of expressions for the accounting book value of long-lived depreciable assets at the beginning of year \( t \) for all accounting policies to be considered in this dissertation. We will now develop expressions for the accounting book value of raw material inventory at the beginning of year \( t \) given accounting policy \( j (M_t^j) \).

The Accounting Book Value of Raw Material Inventory

In order to understand the expressions developed below, we must remember that the quantity of raw material purchased by the firm at the end of year \( (t-1) \) is consumed in year \( t \). This means that the quantity of inventory on hand at the beginning of year \( t \) is equal to the quantity purchased at the end of year \( t-1 \). Our problem is to find the price to

---

See note (4) of the mathematical appendix.
associate with this quantity.

**Accounting Policies 1,3,5,6**

Accounting policy indices 1, 3, 5, and 6 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the FIFO inventory method. Under the FIFO inventory method, the dollar amount of raw material inventory on hand at the beginning of year $t$ is the dollar amount purchased at the end of year $t-1$. Consequently, we have for $j=1,3,5,6$, and $t \geq n$, we have:

$$M_t^j = MP_{t-1}^j.$$  

(69)  

$$= \frac{e^{t-1}a^n (a^{n-b^n})^{-1}}{a^{n(b-n)}}$$  

from equation (33).

**Accounting Policies 9,10,11,12,13,14,15,16**

Accounting policy indices 9, 10, 11, 12, 13, 14, 15, and 16 indicate that the firm reports figures adjusted for changes in the level of prices and uses either the FIFO or LIFO inventory method. We know that the quantity of inventory on hand at the beginning of year $t$ is the quantity of raw material purchased at the end of year $t-1$. We know that the dollar amount of raw material purchased by the firm at the end of year $t-1$ given accounting policy $j$ ($MP_{t-1}^j$) is the product of the quantity purchased and the price which prevailed at the end of year $t-1$. Therefore, for any accounting policy which indicates that the firm reports price level adjusted figures, (i.e., for $j=9,10,11,12,13,14,15,16$), we have:
\[ M_{t}^{j} = \hat{p}M_{t-1}^{j}. \]

By using equation (33) we have for \( t \geq n \) that:

\[
(70) \quad M_{t}^{j} = \frac{x_{t}^{j} - \sum_{i=1}^{n} g_{i}^{j} (g_{t}^{j} - b_{t}^{j}) a_{1}^{j}}{g_{t}^{j} (g - b)}. 
\]

Accounting Policies 2, 4, 7, 8

Accounting policy indices 2, 4, 7, 8 indicate that the firm reports figures unadjusted for changes in the level of prices and uses the LIFO inventory method.

For any nominal inventory method, we know that:

\[ M_{t}^{j} = \sum_{i=0}^{t-1} M_{t}^{j} - \sum_{i=1}^{t-1} F_{t}^{j}. \]

Under the LIFO inventory method we have for \( g \geq 0 \) and \( t \geq 1 \) that:

\[ F_{t}^{j} = \hat{p}M_{t}^{j}. \]

Therefore, we can write that:

\[
(71) \quad M_{t}^{j} = \sum_{i=0}^{t-1} M_{t}^{j} - \hat{p} \sum_{i=0}^{t-2} M_{i}^{j}.
\]

In order to derive an equation for \( M_{t}^{j} \) for \( t \geq n \) in terms of our parameters we must proceed to find an expression in terms of the model parameters for

\[ \sum_{i=0}^{t-2} M_{t}^{j}. \]

We note that for \( t \geq n \) we have:
(72) \[ \sum_{i=0}^{t-2} MP_i^j + \sum_{i=n-1}^{t-2} MP_i^j = \sum_{i=0}^{n-2} MP_i^j \]

We know that for \( j = 2, 4, 7, 8 \):

\[
\begin{align*}
MP_0^j &= xq_1^j \hat{p}^0 \\
MP_1^j &= xq_2^j \hat{p} + xq_1^j \hat{p}q \\
MP_2^j &= xq_3^j \hat{p}^2 + xq_2^j \hat{p}^2 q + xq_1^j \hat{p}^2 q^2 \\
&\vdots \\
MP_h^j &= xq_{h+1}^j \hat{p}^h + xq_h^j \hat{p}^h q + xq_{h-1}^j \hat{p}^h q^2 + \ldots + xq_1^j \hat{p}^h q^h \\
&= xq_1^j \hat{p}^h \sum_{i=1}^{h+1} q_j^i \hat{p}^{h+1-i} q_i^{\hat{a}^i-1} \\
&= xq_1^j \hat{p}^h \sum_{i=1}^{h+1} q_j^i \hat{p}^{h+1-i} \hat{a}^{i-1} q_i^{g} \\
&= xq_1^j \hat{p}^h \sum_{i=1}^{h+1} (g/b)^{i-1} q_i^{g} \\
\end{align*}
\]

Therefore, we have for \( h < (n-1) \) that:

\[
MP_h^j = \left[ xq_1^j \hat{p}^h \sum_{i=1}^{h+1} q_j^i \hat{p}^{h+1-i} \hat{a}^{i-1} q_i^{g} \right] \\
\]

\[
= \left[ xq_1^j \hat{p} \]_{(b-g)} \left[ \hat{a}^{h+1} q \hat{a}^{h+1} q \hat{a}^{h+1} q \right] \\
\]

Now we have that:

\[
\sum_{i=0}^{n-2} MP_i^j = \left[ xq_1^j \hat{p} \]_{(b-g)} \sum_{i=0}^{n-2} \left( \hat{a}^{i+1} q i + i \hat{a}^{i+1} q \right) \\
\]
\[
\begin{align*}
(73) & \quad \frac{\sum_{i=0}^{t-2} \mathcal{M}_p^j}{b-g} \left[ \frac{\mathcal{N}_b(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_b)} - \frac{\mathcal{N}_d(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_d)} \right] \\
& \quad = \left[ \frac{\sum_{i=0}^{t-2} \mathcal{M}_p^j}{b-g} \right] \left[ \frac{b(1-\mathcal{N}_b)(1-p^{n-1})}{(1-p\mathcal{N}_b)} - \frac{\mathcal{N}_d(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_d)} \right]
\end{align*}
\]

From equation (33), we have that for \( j=2, 4, 7, 8 \) and \( t \geq n-1 \):

\[
\mathcal{M}_p^j = \left[ \frac{\sum_{i=0}^{t-1} \mathcal{M}_p^j}{b-g} \right] \left[ \frac{\mathcal{N}_b(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_b)} - \frac{\mathcal{N}_d(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_d)} \right]
\]

We note that for \( t \geq n (n-1) \) that:

\[
\begin{align*}
\mathcal{M}_p^j & = \sum_{i=0}^{t-1} \mathcal{M}_p^j \\
& = \sum_{i=0}^{t-1} \left[ \frac{\mathcal{N}_b(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_b)} - \frac{\mathcal{N}_d(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_d)} \right]
\end{align*}
\]

Therefore,

\[
\begin{align*}
\mathcal{M}_p^j & = \sum_{i=0}^{t-1} \left[ \frac{\mathcal{N}_b(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_b)} - \frac{\mathcal{N}_d(1-p)(1-n^{1-n})}{(1-p\mathcal{N}_d)} \right]
\end{align*}
\]

Substituting our results from equations (73) and (74) into equation (72) we have:
Substituting this last result and the expression for \( M_{t-1} \) from equation (33) into equation (71), we have that for \( t \geq n-1 \) and \( j=2,4,7,8 \):

\[
M_t^j = \left[ \frac{x g^j}{b-g} \right] \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right] - p \left[ x g^j \right] \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right] \\
- p \left[ x g^j \right] \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right] \\
- \frac{\hat{p} \hat{n} - n \hat{n}}{g-b} \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right]
\]

(75)

\[
= \left[ \frac{x g^j}{b-g} \right] \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right] - p \left[ x g^j \right] \left[ \frac{\hat{a} - n-1 \hat{n}}{g-b} \right] \left[ \frac{\hat{p} \hat{n} - n \hat{n}}{1-p \hat{n} \hat{g}} \right] X
\]

For notational convenience, we let \( y \) be equal to the numerator of the final term of equation (75) which is in square brackets. By expanding terms, we have that:

\[
y = b - b \hat{p} \hat{n} + \hat{a} \hat{n} \hat{n} - \frac{\hat{a} - n-1 \hat{n}}{g-b} - \frac{\hat{a} - n-1 \hat{n}}{g-b} + \hat{a} \hat{n} \hat{n} - b \hat{g} \hat{p} \hat{n} + p \hat{n} \hat{b}
\]

\[
= b - b \hat{p} \hat{n} + \hat{a} \hat{n} \hat{n} - \frac{\hat{a} - n+1}{g-b} + \hat{a} \hat{n} \hat{n} - b \hat{g} \hat{p} \hat{n} + p \hat{n} \hat{b}
\]

\[
= b - b \hat{g} + \hat{a} \hat{n} \hat{n} - \frac{\hat{a} - n+1}{g-b}
\]

\[
= b - b \hat{g} + \hat{a} \hat{n} \hat{n} (g-b)
\]

\[
= (b - b \hat{g}) (g-b)
\]

Substituting this result back into equation (75), we have that for \( t \geq n-1 \)
and \( j = 2, 4, 7, 8 \):

\[
M^j_t = \hat{p}^t g^t q^j \left[ \frac{x p (1 - \gamma) (b - \beta)}{\beta g (b - \gamma) (1 - \beta \gamma)} - \frac{x q^j}{(b - \gamma)} \right] \left[ \frac{(1 - \beta \gamma)}{(1 - \beta b) (1 - \beta \gamma)} \right]
\]

(76)

This concludes our development of expressions for the accounting book value of raw material inventory at the beginning of year \( t \) for all of the accounting policies to be considered in this dissertation. We will now develop expressions for the accounting book value of the firm's deferred tax liability at the beginning of year \( t \) given accounting policy \( j \) \( (DTL^j_t) \).

The Accounting Book Value of the Deferred Tax Liability

For those accounting policies in which deferred tax expense is recorded, a related credit account must also be recorded. In this dissertation, this credit account is interpreted to be a liability account and its amount will be deducted from the book value of firm assets in order to derive the accounting book value of firm equity.

The interpretation of the credit account as a long-term liability is explained as follows. We recall that there are three interpretations of this credit account. The interpretations are that the credit account is (1) a liability, (2) a deferred credit, or (3) a contra-asset. The deferred credit approach was rejected because the whole concept of
deferred credits implies that the firm's statement of financial position is meaningless. According to this view the statement of financial position is nothing more than a storehouse for future income debits and credits. The contra-asset approach was rejected because it is not consistent with the valuation procedures followed in accounting. The contra-asset approach represents an application of a theoretically appealing valuation rule to only one aspect of one kind of asset. Accounting book values do not generally decline or increase as expectations about their future cash benefits decline or increase. Therefore, by the process of elimination, we chose to interpret the credit account arising from the recognition of deferred tax expense as a long-term liability account.

The accounting problem of deferred taxes arises in this dissertation because the firm uses one depreciation method for financial reporting and another depreciation method for tax reporting. The amount of deferred tax expense in year t is found by multiplying the expected tax rate times the difference between the tax depreciation in year t and the amount of tax depreciation that would be deductible in year t had the firm used the same method of depreciation for tax reporting as it did for financial reporting. Within the context of this dissertation, this means that the amount of deferred tax expense in year t is the current tax rate (which is expected to prevail for all periods in the future) times the difference between SYD and SL depreciation in year t where
both depreciation amounts are measured in dollars unadjusted for changes in the price level. This means that the dollar amount of the deferred tax liability at the beginning of year \( t \) is equal to the tax rate times the difference between SYD and SL accumulated depreciation at the beginning of year \( t \). This amount can be determined as follows:

Let:

\[ AD^j_t = \text{the amount of accumulated depreciation at the beginning of year } t \text{ } (t \leq n) \text{ given accounting policy } j. \]

Then, for any \( j \), we have that \( AD^j_t \) is such that:

\[ N_t^j = \sum_{i=1}^{n} \frac{1}{t-1} - AD^j_t. \]

Rearranging terms, we have that:

\[ AD^j_t = \sum_{i=1}^{n} \frac{i}{t-1} - N_t^j. \]

We know that:

\[ AD_t^1 = \text{the amount of accumulated depreciation at the beginning of year } t \text{ given that the firm uses the SL depreciation and reports figures unadjusted for changes in the price level.} \]

\[ AD_t^3 = \text{the amount of accumulated depreciation at the beginning of year } t \text{ given that the firm uses the SYD depreciation method and reports figures unadjusted for changes in the price level.} \]

Now we can write that:
\[ k \left[ AD_t^3 - AD_t^1 \right] = k \left[ \left( \sum_{i=1}^{n} I_{t-i} - N_t^3 \right) - \left( \sum_{i=1}^{n} I_{t-i} - N_t^1 \right) \right] \]

\[ = k \left[ N_t^1 - N_t^3 \right] \]

Now by substituting for \( N_t^1 \) and \( N_t^3 \) from equations (64) and (66) and rearranging terms, we have:

\[ k \left[ AD_t^3 - AD_t^1 \right] = \frac{p^t g^t k I_0}{n g^n \hat{p} (1-\hat{g} \hat{p})^2} \left\{ \frac{\beta}{n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})} - \frac{2 \hat{g} - n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})^2}{n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})^3} \right\} \]

\[ = \left( \frac{k I_0 p^t g^t}{n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})^3} \right) \left( (n+1) (1-\hat{g} \hat{p}) (1-\hat{g} \hat{p}) - 2 \hat{g} \right) \]

For the sake of notational convenience, we define:

(77) \[ \lambda = (n+1) (1-\hat{g} \hat{p})(1-\hat{g} \hat{p}) - 2 \hat{g} \]

Therefore, we have that:

(78) \[ k \left[ AD_t^3 - AD_t^1 \right] = \frac{k p^t g^t \lambda I_0}{n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})^3} \]

Now for \( t \geq n \) and \( j=6, 8 \), we have that:

(79) \[ DTL_t^j = \left( \frac{k p^t g^t \lambda I_0}{n (n+1) g^n \hat{p} (1-\hat{g} \hat{p})^3} \right) \]

At this point it is important to note that the amount \( k \left[ AD_t^3 - AD_t^1 \right] \) given in equation (79) is interpreted in this dissertation as the amount of a liability at the beginning of year \( t \). Since it is a liability it is stated in terms of the price level which prevails at the beginning of
year \( t \). Consequently, to find the amount of the deferred tax liability for those accounting policies which utilize figures adjusted for changes in the level of prices, we simply multiply the right-hand side of equation (79) by \( \hat{p} \).\(^{17}\) For \( t \geq n \) and \( j = 14, 16 \), we have:

\[
\text{DTL}_t^j = \frac{k \hat{p}^{t+1} \hat{p}^t}{n(n+1)g_1^j \beta^n (1-\hat{p})^2}
\]

We note that for \( j = 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 15 \), \( \text{DTL}_t^j = 0 \).

We have now developed all the expressions needed to find the book value of owners' equity at the beginning of year \( t \) given accounting policy \( j \) (\( \text{BV}_t^j \)).

The Accounting Book Value of Owners' Equity

For any accounting policy \( j \), we have that:

\[
\text{BV}_t^j = N_t^j + \text{MI}_t^j - \text{DTL}_t^j.
\]

Accounting Policy 1

Substituting \( N_t^1, \text{MI}_t^1 \), and \( \text{DTL}_t^1 \) from equations (64), (69), and (81)

\(^{17}\) The question arises at this point of whether or not to include the purchasing power gain (loss) on this liability in firm income. There is a controversy over whether or not to include purchasing power gains on long-term liabilities in firm income. While we believe that our liability interpretation of this credit account is the best choice, we also realize that this interpretation is somewhat tenuous. We have consequently chosen not to include the purchasing power gain on this liability in firm income.
into equation (82), we have for $t \geq n$:

$$B^1 V_t = \frac{\hat{\alpha}^t \hat{a}^t I_0 \hat{p}}{n \hat{g}^n \hat{p}^n (1-\hat{g} \hat{b})^{2}} + \frac{x \hat{p}^{t-1} \hat{g}^{n-b} n-b \hat{q}_1}{\hat{g}^n (\hat{g}-\hat{b})}$$

Substituting for $I_0$ from equation (13) and rearranging terms, we have:

$$B^1 V_t = q_1 \hat{p}^t \hat{g}^t \left[ \frac{\hat{x}^n-b n ((1-\hat{g} \hat{b}) \hat{\psi} \hat{\phi})^2}{\hat{p}^n (1-\hat{g} \hat{b})^2 (1-\hat{g} \hat{b})^2} + \frac{x (\hat{g}-b)^{(n-b)} (1-\hat{g} \hat{b}) \hat{p} (\hat{g}-b)^{(n-b)} (1-\hat{g} \hat{b}) \hat{p}}{(1-\hat{g} \hat{b}) (1-\hat{g} \hat{b})^2 (1-\hat{g} \hat{b})^2} \right]$$

(83)

Accounting Policy 2

Substituting $N^2_t$, $M^2_t$, and $DT^2_L$ from equations (64), (76), and (81) into equation (82), we have for $t \geq n$:

$$B^2 V_t = \frac{\hat{\alpha}^t \hat{a}^t I_0 \hat{p}}{n \hat{g}^n \hat{p}^n (1-\hat{g} \hat{b})^2} + \frac{x (1-\hat{g} \hat{b}) (b \hat{g}-\hat{a} \hat{n}) \hat{p}^t \hat{g}^t \hat{q}_1}{\hat{g}^n (b-\hat{g}) (1-\hat{g} \hat{b})} - \frac{x \hat{p} (1-\hat{g} \hat{b} b \hat{g} \hat{b}) \hat{g}^2}{(1-\hat{g} \hat{b}) (1-\hat{g} \hat{b})}$$

Substituting for $I_0$ from equation (16) and rearranging terms, we have:

$$B^2 V_t = q_1 \hat{p}^t \hat{g}^t \left[ \frac{1}{\hat{p}^n (1-\hat{g} \hat{b})^2 (1-\hat{g} \hat{b})^2} \right] - x \frac{(1-\hat{g} \hat{b}) (b \hat{g}-\hat{a} \hat{n}) \hat{p}^t \hat{g}^t \hat{q}_1}{\hat{g}^n (b-\hat{g}) (1-\hat{g} \hat{b})} - \frac{x \hat{p} (1-\hat{g} \hat{b} b \hat{g} \hat{b}) \hat{g}^2}{(1-\hat{g} \hat{b}) (1-\hat{g} \hat{b})} \hat{p}^t \hat{g}^t \hat{q}_1$$

(84)
Accounting Policy 3

Substituting the expressions for $N_t^3$, $M_t^3$, and $DTL_t^3$ given in equations (67), (69), and (81) into equation (82), we have:

$$BV_t^3 = \frac{I_0 p^*_t q^*_t [2 \hat{p} - n(n+1)\hat{q}^n\hat{p}^n(1-\hat{q}^p)^2]}{n(n+1)\hat{q}^n\hat{p}^n(1-\hat{q}^p)^3} + \frac{x(\hat{q}^n-b^n)(1-\hat{q}^p)^3}{\hat{q}^n(\hat{g}-b)}$$

Substituting for $I_0$ from equation (19) and rearranging terms, we have:

$$BV_t^3 = \left[ \frac{p^*_t q^*_t q^4}{\hat{p}^n(\hat{g}-b)(b-\hat{q}^p)(1-\hat{q}^p)^3} \right] x$$

$$\frac{\hat{q}^n-b^n}{\hat{q}^n(\hat{g}-b)}$$

(85) $$\frac{\hat{p}^n(\hat{g}-b)(\hat{q}^n-b^n)(1-\hat{q}^p)^2}{\hat{q}^n(\hat{g}-b)(1-\hat{q}^p)^3} + x(\hat{q}^n-b^n)(1-\hat{q}^p)^3$$

Accounting Policy 4

Substituting the expressions for $N_t^4$, $M_t^4$, and $DTL_t^4$ given in equations (67), (76), and (81) into equation (82), we have:

$$BV_t^4 = \frac{I_0 p^*_t q^*_t [2 \hat{p} - n(n+1)\hat{q}^n\hat{p}^n(1-\hat{q}^p)^2]}{n(n+1)\hat{q}^n\hat{p}^n(1-\hat{q}^p)^3} + \frac{\hat{p}^n(\hat{g}-b-n(\hat{q}^n-b^n)q^4}{\hat{g}^n(\hat{g}-b)(1-\hat{q}^p)^3}$$

$$- \frac{x(\hat{p}^n-b^n)q^4}{(\hat{p}^n-b^n)}$$

Substituting for $I_0$ from equation (20) and rearranging terms, we have:

$$BV_t^4 = \left[ \frac{p^*_t q^*_t q^4}{\hat{g}^n\hat{q}^n(\hat{g}-b)(b-\hat{g})(1-\hat{q}^p)} \right] x$$

$$\frac{(\hat{p}^n-b^n)(b-n(\hat{q}^n-b^n)(1-\hat{q}^p)^2)}{(1-\hat{q}^p)^2(b-\hat{g})\hat{q}^n(\hat{g}-b)(1-\hat{q}^p)^3}$$

(86) $$\frac{\hat{p}^n-b^n}{\hat{q}^n(\hat{g}-b)} x[\hat{q}^n(b-n(\hat{q}^n-b^n)(1-\hat{q}^p)^2) + (1-\hat{q}^p)^2(b-\hat{g})\hat{q}^n(\hat{g}-b)(1-\hat{q}^p)^3]$$
Accounting Policy 5

Substituting the expressions for $N_t^5$, $M_t^5$, and $DTL_t^5$ given in equations (64), (69), and (81) into equation (82), we have:

$$BY_t^5 = \frac{p^t \lambda_0}{n \hat{g}^n \hat{p}_n (1-\hat{g})_t} + \frac{x \hat{p}_n \hat{q}_n (b-n \hat{q})_t q_5^5}{\hat{p}_n (b-\hat{g})}$$

Substituting for $I_0$ from equation (19) and rearranging terms, we have:

$$BY_t^5 = \left[ \frac{p^t \lambda_0}{n \hat{g}^n \hat{p}_n (1-\hat{g})_t} \right] \times \left[ \frac{x \hat{p}_n \hat{q}_n (b-n \hat{q})_t q_5^5}{\hat{p}_n (b-\hat{g})} \right]$$

(87)

$$\left[ (b-n \hat{q})_t (n+1) (1-\hat{g})_t^2 \hat{g}(b-\hat{g}) \hat{q} \lambda + x(b-n \hat{q})_t (1-\hat{g})_t^2 (b-\hat{g}) \lambda \right]$$

Accounting Policy 6

Substituting the expressions for $N_t^6$, $M_t^6$, and $DTL_t^6$ given in equations (64), (69), and (79) into equation (82), we have:

$$BY_t^6 = \frac{p^t \lambda_0}{n \hat{g}^n \hat{p}_n (1-\hat{g})_t^2} + \frac{x \hat{p}_n \hat{q}_n (b-n \hat{q})_t q_6^6}{\hat{p}_n (b-\hat{g})} \times \frac{1}{n(n+1) \hat{g}^n \hat{p}_n (1-\hat{g})_t^3}$$

Substituting for $I_0$ from equation (19) and rearranging terms, we have:

$$BY_t^6 = \left[ \frac{p^t \lambda_0}{n \hat{g}^n \hat{p}_n (1-\hat{g})_t^2} \right] \times \left[ \frac{x \hat{p}_n \hat{q}_n (b-n \hat{q})_t q_6^6}{\hat{p}_n (b-\hat{g})} \right]$$

(88)

$$\left[ (b-n \hat{q})_t (1-\hat{g})_t^2 \hat{g}(b-\hat{g}) \hat{q} \lambda (n+1) (1-\hat{g})_t \hat{q} - k \lambda \right]$$

$$+ x(b-n \hat{q})_t (b-\hat{g}) (1-\hat{g})_t^3 \lambda \hat{q}$$
Accounting Policy 7

Substituting the expressions for \( N_7, \) \( M_1, \) and \( DTL_t^7 \) given in equations (64), (76), and (81) into equation (82), we have:

\[
BV_t^7 = \frac{\hat{p}^t \hat{q}^t I_0 \hat{p}}{n \hat{p} \hat{n} (1-\hat{p})^2} + \frac{x \hat{p}^t \hat{q}^t (1-\hat{p}) (b^n \hat{n} b_n) q_1^7} {\hat{p}^n (b-\hat{q}) (1-\hat{p})} - \frac{x \hat{p}^t (1-\hat{p}) b^n b_n) q_1^7} {\hat{p}^n (b-\hat{q}) (1-\hat{p})}
\]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[
BV_t^7 = \left[ \frac{\hat{p}^t \hat{q}^t I_0 \hat{p}}{\hat{p}^n (b-\hat{q}) (1-\hat{p})} \right] - x \frac{\hat{p}^t \hat{q}^t (1-\hat{p}) (b^n \hat{n} b_n) q_1^7} {\hat{p}^n (b-\hat{q}) (1-\hat{p})}
\]

(89)

\[
x(b^n \hat{n} b_n) \alpha [(1-\hat{q}) (1-\hat{p}) b^n (b^n \hat{n} - \hat{p}^n \hat{q}^n - \hat{p}^n \hat{q}^n \hat{q}^t b^n b_n)]
\]

Accounting Policy 8

Substituting the expressions for \( N_8, \) \( M_1, \) and \( DTL_t^8 \) given in equations (64), (76), and (79), into equation (82), we have:

\[
BV_t^8 = \frac{\hat{p}^t \hat{q}^t I_0 \hat{p}}{n \hat{p} \hat{n} (1-\hat{p})^2} + \frac{x \hat{p}^t \hat{q}^t (1-\hat{p}) (b^n \hat{n} b_n) q_1^8} {\hat{p}^n (b-\hat{q}) (1-\hat{p})} - \frac{x \hat{p}^t (1-\hat{p}) b^n b_n) q_1^8} {\hat{p}^n (b-\hat{q}) (1-\hat{p})}
\]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[
BV_t^8 = \left[ \frac{\hat{p}^t \hat{q}^t I_0 \hat{p}}{\hat{p}^n (b-\hat{q}) (1-\hat{p})^2} \right] - x \frac{\hat{p}^t \hat{q}^t (1-\hat{p}) (b^n \hat{n} b_n) q_1^8} {\hat{p}^n (b-\hat{q}) (1-\hat{p})}
\]

(90)

\[
x(b^n \hat{n} b_n) \alpha [(1-\hat{q}) (1-\hat{p}) b^n (b^n \hat{n} - \hat{p}^n \hat{q}^n - \hat{p}^n \hat{q}^n \hat{q}^t b^n b_n)]
\]
Accounting Policy 9

Substituting the expressions for $N^9_t$, $M^9_t$, and $DLT^9_t$ given in equations (65), (70), and (81) into equation (82), we have:

$$ BV_t^9 = \frac{\hat{p}^t \hat{g}^t I_0 \hat{E}}{n^g(n+1)(1-\hat{q})^2} + \frac{x^{\hat{p}^t \hat{g}^t (b-n \hat{q}^n) g^9_i}}{\hat{g}^n(b-\hat{q})} $$

Substituting for $I_0$ from equation (13) and rearranging terms, we have:

$$ BV_t^9 = \hat{p}^t \hat{g}^t q^9_i \left[ -\frac{x(b-n \hat{g}^n)(b-\hat{r})(1-\hat{q})^2 \psi + \hat{N}(b-\hat{g})(1-\hat{r})(b-n \hat{q}^n) \chi}{\hat{g}^n(b-\hat{q})(b-\hat{r})(1-\hat{q})^2 \psi} \right] $$

Accounting Policy 10

Substituting the expressions for $N^{10}_t$, $M^{10}_t$, and $DLT^{10}_t$ given in equations (65), (70), and (81) into equation (82), we have:

$$ BV_t^{10} = \frac{\hat{p}^t \hat{g}^t I_0 \hat{E}}{n^g(n+1)(1-\hat{q})^2} + \frac{x^{\hat{p}^t \hat{g}^t (b-n \hat{q}^n) g^{10}_i}}{\hat{g}^n(b-\hat{q})} $$

Substituting for $I_0$ from equation (16) and rearranging terms, we have:

$$ BV_t^{10} = \hat{p}^t \hat{g}^t q^{10}_i \left[ -\frac{x(b-n \hat{g}^n)(1-\hat{q})^2 (b-\hat{r}) \psi + \hat{N}(b-\hat{g})(1-\hat{r})(b-n \hat{q}^n) \chi}{\hat{g}^n(b-\hat{q})(b-\hat{r})(1-\hat{q})^2 \psi} \right] $$

Accounting Policy 11

Substituting the expressions for $N^{11}_t$, $M^{11}_t$, and $DLT^{11}_t$ given in equations (68), (70), and (81) into equation (82), we have:

$$ BV_t^{11} = \frac{\hat{p}^t \hat{g}^t (2 \hat{E}(n+1)(1-\hat{q})^2)}{n(n+1) \hat{g}^n(n+1)(1-\hat{q})^3} + \frac{x^{\hat{p}^t \hat{g}^t (b-n \hat{q}^n) g^{11}_i}}{\hat{g}^n(b-\hat{q})} $$

Substituting for $I_0$ from equation (19) and rearranging terms, we
have:

\[ BV^1_{t} = \begin{bmatrix} \frac{\hat{q}^1 \hat{t}^1 q_{11}}{\hat{q}^1 (b-\hat{q}) (b-\hat{r}) (1-\hat{q})^3} \end{bmatrix} X \begin{bmatrix} x(bn-\hat{q}n)(1-\hat{q})^3(b-\hat{r}) \alpha + \hat{p}^n(b-\hat{q})(1-n+1) \hat{q}^n(1-\hat{q})^2 \end{bmatrix} \]

(93)

Accounting Policy 12

Substituting the expressions for \( N^1_{t} \), \( M_{t}^1 \), and \( DTL_{t}^1 \) given in equations (68), (70), and (81) into equation (82), we have

\[ BV^1_{t} = \frac{I_0 \hat{p}^t \hat{q}^t [2 \xi - n(n+1) \hat{q}^n(1-\hat{q})^2]}{n(n+1) \hat{q}^n(1-\hat{q})^3(1-n+1) \hat{q}^n(1-\hat{q})^2} + \frac{x \hat{p}^t \hat{q}^t (bn-\hat{q}n)(1-\hat{q})^3(b-\hat{r}) \alpha}{\hat{q}^n(b-\hat{q})} \]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[ BV^1_{t} = \frac{\hat{p}^t \hat{q}^t [2 \xi - n(n+1) \hat{q}^n(1-\hat{q})^2]}{n \hat{q}^n(1-\hat{q})^2} + \frac{x \hat{p}^t \hat{q}^t (bn-\hat{q}n)(1-\hat{q})^3(b-\hat{r}) \alpha}{\hat{q}^n(b-\hat{q})} \]

(94)

Accounting Policy 13

Substituting the expressions for \( N^1_{t} \), \( M_{t}^1 \), and \( DTL_{t}^1 \) given in equations (65), (70), and (81) into equation (82), we have:

\[ BV^2_{t} = \frac{\hat{p}^t \hat{q}^t I_0 \xi}{n \hat{q}^n(1-\hat{q})^2} + \frac{x \hat{p}^t \hat{q}^t (bn-\hat{q}n)(1-\hat{q})^3}{\hat{q}^n(b-\hat{q})} \]

Substituting for \( I_0 \) from equation (19) and rearranging terms, we have:

\[ BV^2_{t} = \frac{\hat{p}^t \hat{q}^t [2 \xi - n(n+1) \hat{q}^n(1-\hat{q})^2]}{\hat{q}^n(b-\hat{q})(b-\hat{r}) (1-\hat{q})^2} + \frac{x \hat{p}^t \hat{q}^t (bn-\hat{q}n)(1-\hat{q})^3(b-\hat{r}) \alpha + (n+1) \hat{q}^n(b-\hat{q})(1-n+1) \hat{q}^n(1-\hat{q})^2 \xi}{\hat{q}^n(b-\hat{q})(b-\hat{r})(1-\hat{q})^2} \]

(95)
Accounting Policy 14

Substituting the expressions for \( N_t^{14} \), \( M_t^{14} \), and \( DTL_t^{14} \) given in equations (65), (70), and (80) into equation (82), we have:

\[
BV_t^{14} = \frac{p^t q^t I_0 \xi}{ng^{n}(1-\hat{\theta})^2} + \frac{x^t g^t (b^n - \hat{\theta} n) q^{14}}{g^n (b-\hat{\theta})} - \frac{I_0 g^t p^{t+1} \lambda_k}{n(n+1)g^n q^n (1-\hat{\theta} \hat{\rho})^3}
\]

Substituting for \( I_0 \) from equation (19) and rearranging terms, we have:

\[
BV_t^{14} = \left[ \frac{p^t q^t g^{14}}{g^n (b-\hat{\theta}) (b-\hat{\rho}) (1-\hat{\theta})^2 (1-\hat{\theta} \hat{\rho})^3} \right] \cdot x
\]

(96) \( c(x(b^n - \hat{\theta} n)(1-\hat{\theta})^2 (1-\hat{\theta} \hat{\rho})^3 (b-\hat{\theta}) \right. + \left. (b-\hat{\theta}) (1-\hat{\rho})^2 (b^n - \hat{\theta} n) \gamma ((n+1)g^n (1-\hat{\theta} \hat{\rho})^3 \xi - \hat{\rho} (1-\hat{\theta})^2 \lambda k) \right)

Accounting Policy 15

Substituting the expressions for \( N_t^{15} \), \( M_t^{15} \), and \( DTL_t^{15} \) given in equations (65), (70), and (81) into equation (82), we have:

\[
BV_t^{15} = \frac{p^t q^t I_0 \xi}{ng^{n}(1-\hat{\theta})^2} + \frac{x^t g^t (b^n - \hat{\theta} n) q^{15}}{g^n (b-\hat{\theta} n)}
\]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[
BV_t^{15} = \hat{p}^t q^t q^{15} \left( \frac{x(b^n - \hat{\theta} n)(1-\hat{\theta})^2 (b^n - \hat{\theta} n) \gamma ((n+1)g^n (b-\hat{\theta}) (1-\hat{\rho})^2 (b^n - \hat{\theta} n) \Delta \right)}{g^n (b-\hat{\theta}) (b-\hat{\rho}) (1-\hat{\theta})^2}
\]

Accounting Policy 16

Substituting the expressions for \( N_t^{16} \), \( M_t^{16} \), and \( DTL_t^{16} \) given in equations (65), (70), and (80) into equation (82), we have:
\[ BV_t^{16} = \frac{\hat{B}^t \hat{A}^t I_0 \hat{A}^n C}{n^2 n(1-g)^2} + \frac{\chi n^1 \hat{A}^t (n \hat{A}^n) Q_0}{(1-g)^2} - \frac{\chi n^2 \hat{A}^t I_0}{n(n+1)^2 n^2 n^2 (1-g)^2} \]

Substituting for \( I_0 \) from equation (20) and rearranging terms, we have:

\[ BV_t^{16} = \left[ \frac{\hat{A}^t \hat{A}^t Q_0}{\hat{A} n^1 (b-\hat{A}) (1-\hat{A})^2 (1-\hat{A}^n)^2} \right] X \]

\[ \approx \frac{1}{\hat{A} n^1 (b-\hat{A}) (1-\hat{A})^2 (1-\hat{A}^n)^2} \left[ \chi (b-\hat{A}) (1-\hat{A})^2 (1-\hat{A}^n)^3 (b^n - \hat{A}^n) + \right. \]

\[ \left. (b-\hat{A}) (1-\hat{A})^2 (b^n - \hat{A}^n) \right] \Delta \left[ (n+1)^2 n^2 (1-\hat{A}^n)^3 C - \hat{A} n^2 (1-\hat{A})^2 J \right] \]

We have now developed all the expressions needed in order to find expressions for the accounting rate of return of the firm in year \( t \) given accounting policy \( j \) (\( ARR_t^j \)). We now turn to this task.

The Accounting Rate of Return of the Firm

For any accounting policy \( j \), we have:

\[ ARR_t^j = \frac{A_t^j}{BV_t^j} \]

Accounting Policy 1

Substituting the expressions for \( A_t^1 \) and \( BV_t^1 \) from equations (46) and (83) into equation (99), and rearranging terms, we have:

\[ ARR_t^1 = \left[ \frac{(1-k) (1-\hat{A}^n)}{\chi (\hat{A} - b) (1-\hat{A}^n)^2 (\hat{A}^n - b^2)^2)} \right] X \]

\[ \left[ (b-\hat{A}) (\hat{A} - b)^2 (\hat{A}^n - b)^3 \right] \]

Accounting Policy 2

Substituting the expressions for \( A_t^2 \) and \( BV_t^2 \) from equations (47)
and (84) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_t^2 = (1-k)(1-\hat{g})p (1-b \hat{p}) x \\
(1-x)(b \hat{r} (1-\hat{g})b \hat{n} - g \hat{n}) \Psi - (b-\hat{g})(1-\hat{p})(r \hat{n} - \hat{r}) (1-\hat{g} \hat{n} \hat{p} \hat{n}) \Delta 
\]

\[ (101) \]

\[
x(b-\hat{r})(1-\hat{g} \hat{p}) \Psi [(1-b \hat{p})(1-\hat{g})(b \hat{n} - g \hat{n}) - \hat{r} \hat{g} \hat{n} \hat{p} \hat{n} (b-\hat{g})(1-b \hat{n} \hat{p} \hat{n})] 
\]

Accounting Policy 3

Substituting the expressions for \(A_t^3\) and \(B_t^3\) given in equations (48)

and (85) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_t^3 = (1-\hat{g} \hat{p})(1-k) [(\hat{p}-\hat{x})(r-b)(1-\hat{g})b \hat{n}] \alpha \\
2 \hat{p} (\hat{p}-\hat{x})(1-\hat{g} \hat{p})^2 (g \hat{n} - b \hat{n}) \Psi \\
(102)
\]

\[
x(b-r)(1-\hat{g} \hat{p})^3 (g \hat{n} - b \hat{n}) \]

Accounting Policy 4

Substituting the expressions for \(A_t^4\) and \(B_t^4\) given in equations (49) and (86) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_t^4 = (1-\hat{p} \hat{b})(1-\hat{g} \hat{p})^2 (1-k) [(1-x)(b \hat{n} - g \hat{n})(1-\hat{g} \hat{p}) (b \hat{r} - \hat{g}) \alpha - \\
2(b-\hat{g})(1-\hat{r} \hat{p})^2 (b \hat{n} - r \hat{n}) \Psi \Delta \\
(103)
\]

\[
x(b-\hat{g})(1-\hat{p} b)(1-\hat{r} \hat{p})^2 (b \hat{n} - r \hat{n}) \Delta [2 \hat{p} - n(n+1) \hat{g} \hat{n} \hat{p} \hat{n} (1-\hat{g} \hat{p})^2] + \\
x(b-\hat{r})(1-\hat{g} \hat{p})^2 \alpha [(1-\hat{g})(1-\hat{p} b)(b \hat{n} - g \hat{n}) - \hat{r} \hat{g} \hat{n} \hat{p} \hat{n} (b-\hat{g})(1-\hat{p} \hat{n} \hat{b} \hat{n})] 
\]

Accounting Policy 5

Substituting the expressions for \(A_t^5\) and \(B_t^5\) given in equations (50) and (87) into equation (99) and rearranging terms, we have:
\[ \text{ARR}_t^5 = \frac{\left(1-k\right)\left(\hat{p}-\hat{x}\right)\left(\hat{r}-\hat{b}\right)\left(1-\hat{g}\hat{p}\right)^2\left(\hat{g}^n-b^n\right)}{\left(2k\hat{p} - (n+1)\left(1-\hat{g}\hat{p}\right)\left(1-\hat{g}^n\hat{p}^n\right)\right)} + \]

\[ \frac{\hat{p}^2\left(b^n-b^n\right)\left[2k\hat{p} - (n+1)\left(1-\hat{g}\hat{p}\right)\left(1-\hat{g}^n\hat{p}^n\right)\right]}{\left(n+1\right)\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\left(1-\hat{r}^n\hat{p}^n\right)\hat{p}} \]

\[ \text{Accounting Policy 6} \]

Substituting the expressions for \( A_t^6 \) and \( BV_t^6 \) given in equations (51) and (88) into equation (99) and rearranging terms, we have:

\[ \text{ARR}_t^6 = \left(1-k\right)\left(1-\hat{g}\hat{p}\right)^2 \left(\hat{p}^2\left(b^n-b^n\right)\left[2k\hat{p} - (n+1)\left(1-\hat{g}\hat{p}\right)\left(1-\hat{g}^n\hat{p}^n\right)\right] \right) + \]

\[ \frac{\left(n+1\right)\hat{p}^2\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\left(1-\hat{r}^n\hat{p}^n\right)\hat{p}}{\left[n(n+1)\left(1-\hat{g}\hat{p}\right)\hat{p} - k\lambda\right]} \]

\[ \text{Accounting Policy 7} \]

Substituting the expressions for \( A_t^7 \) and \( BV_t^7 \) given in equations (52) and (89) into equation (99) and rearranging terms, we have:

\[ \text{ARR}_t^7 = \left(1-\hat{p}\hat{b}\right)\left(1-k\right)\left(1-\hat{x}\right)\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\hat{p} \]

\[ + \frac{\hat{b}^2\left(1-\hat{g}\hat{p}\right)^2\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\hat{p}}{\left[2k\hat{p} - (n+1)\left(1-\hat{g}\hat{p}\right)\left(1-\hat{g}^n\hat{p}^n\right)\right]} \]

\[ \text{Accounting Policy 8} \]

Substituting the expressions for \( A_t^8 \) and \( BV_t^8 \) given in equations (53) and (90) into equation (99) and rearranging terms, we have:

\[ \text{ARR}_t^8 = \left(1-k\right)\left(1-\hat{p}\hat{b}\right)^2\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}\hat{p}\right)^2\left(1-\hat{x}\right)\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\hat{p} \]

\[ \left(n+1\right)\left(b^n-b^n\right)\left[\hat{b}^2\left(1-\hat{g}\hat{p}\right)^2\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\hat{p}\right] \]

\[ + \frac{\hat{b}^2\left(1-\hat{g}\hat{p}\right)^2\left(1-\hat{r}\hat{p}\right)^2\left(1-\hat{g}^n\hat{p}^n\right)\hat{p}}{\left[n(n+1)\left(1-\hat{g}\hat{p}\right)\hat{p} - k\lambda\right]} \]

\[ \text{Accounting Policy 9} \]
Accounting Policy 9

Substituting the expressions for $A^9_t$ and $BV^9_t$ given in equations (56) and (91) into equation (99) and rearranging terms, we have:

$$ARR^9_t = \sum \frac{(1+\hat{g})}{(1-\hat{g}^n)} x \left\{(1-\hat{g})(1-\hat{p})(1-\hat{g}^n)(b^n-\hat{g}^n) \Theta \psi \right\} + \left\{(b-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \phi \Delta \right\} \frac{x(b-\hat{g})(1-\hat{g})^2 (b^n-\hat{g}^n) \psi + \hat{p}^n (b-\hat{g})(b^n-\hat{g}^n) \epsilon \Delta}{\left\{(b-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \phi \Delta \right\}}$$

Accounting Policy 10

Substituting the expressions for $A^{10}_t$ and $BV^{10}_t$ given in equations (57) and (92) into equation (99) and rearranging terms, we have:

$$ARR^{10}_t = \sum \frac{(1+\hat{g})}{(1-\hat{g}^n)} x \left\{(1-k)(1-x)(1-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \psi \right\} + \left\{(b-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \phi \Delta \right\} \frac{x(b-\hat{g})(1-\hat{g})^2 (b^n-\hat{g}^n) \psi + \hat{p}^n (b-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \epsilon \Delta}{\left\{(b-\hat{g})(1-\hat{p})(b^n-\hat{g}^n) \phi \Delta \right\}}$$

Accounting Policy 11

Substituting the expressions for $A^{11}_t$ and $BV^{11}_t$ given in equations (58) and (93) into equation (99) and rearranging terms, we have:

$$ARR^{11}_t = \sum \frac{(1+\hat{g})}{(1-\hat{g}^n)^2} x \left\{(b-\hat{g})(1-\hat{g})^2 (1-\hat{p})^2 (b^n-\hat{g}^n) \Theta \infty \right\} + \left\{2(b-\hat{g})(1-\hat{p})^2 (b^n-\hat{g}^n) \chi [k(1-\hat{g})^2 \hat{p}^n (1-\hat{g}^n) \Theta \infty \right\} \frac{x(b-\hat{g})(1-\hat{g})^3 (b^n-\hat{g}^n) \infty + \hat{p}^n (b-\hat{g})(1-\hat{p})^2 (b^n-\hat{g}^n) \chi [2 \epsilon-n(n+1)\hat{g}^n(1-\hat{g})^2 \right\}}{\left\{2(b-\hat{g})(1-\hat{p})^2 (b^n-\hat{g}^n) \chi [k(1-\hat{g})^2 \hat{p}^n (1-\hat{g}^n) \Theta \infty \right\}}$$

Accounting Policy 12

Substituting the expressions for $A^{12}_t$ and $BV^{12}_t$ given in equations
(59) and (94) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_{t}^{12} = \left( \frac{(1-\hat{g})}{(1-\hat{g} \hat{p})} \right)^{2} \times \left( (1-k)(1-x)(1-\hat{g}) \right)^{2} (1-\hat{g} \hat{p})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle + 2 (b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \right] \\
\times \left( k(1-\hat{g})^{2} \hat{p} \hat{n} \hat{g} \hat{p}^{n} (1-\hat{g} \hat{p})^{2} \triangle \right) \\
\triangle \]

(111) \( \triangle \)

\[
\hat{p}^{n}(b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \left( 2 \xi - n(n+1) \hat{g}^{n}(1-\hat{g})^{2} \right) \\
\triangle \]

Accounting Policy 13

Substituting the expressions for \( A_{t}^{13} \) and \( B_{t}^{13} \) given in equations

(60) and (95) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_{t}^{13} = \left( \frac{(1-\hat{g})}{(1-\hat{g} \hat{p})} \right)^{2} \times \left( (b \hat{f})(1-\hat{g}) \right)^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \Theta \triangle + \\
\left( (b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \right] \\
\times \left( 2k(1-\hat{g}) \hat{p} \hat{n} \hat{g} \hat{p}^{n} (1-\hat{g} \hat{p})^{2} \triangle \right) \\
\triangle \]

(112) \( \triangle \)

\[
\hat{p}^{n}(b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle + \left( n+1 \right) \hat{p}^{n}(b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \\
\triangle \]

Accounting Policy 14

Substituting the expressions for \( A_{t}^{14} \) and \( B_{t}^{14} \) given in equations

(61) and (96) into equation (99) and rearranging terms, we have:

\[
\text{ARR}_{t}^{14} = (1-\hat{g})(1-\hat{g} \hat{p})^{2} \left( (b \hat{f})(1-\hat{g}) \right)^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \Theta \triangle + \\
\left( (b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \right] \\
\times \left( 2k(1-\hat{g}) \hat{p} \hat{n} \hat{g} \hat{p}^{n} (1-\hat{g} \hat{p})^{2} \triangle \right) \\
\triangle \]

(113) \( \triangle \)

\[
\hat{p}^{n}(b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \left( n+1 \right) \hat{p}^{n}(b \hat{g})(1-\hat{g})^{2} (b \hat{n} \hat{g} \hat{p}^{n}) \triangle \\
\triangle \]

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Substituting the expressions for \( A_{t}^{15} \) and \( B_{t}^{15} \) given in equations

(62) and (97) into equation (99) and rearranging terms, we have:
\[
\text{ARR}^{15}_t = \frac{(1-\hat{g})}{(1-\hat{g}^\hat{p})} \times \left\{ (1-k)(1-x)(1-\hat{g})(b-\hat{r})(1-\hat{g}^\hat{p})^2 (b^n - \hat{g}^n) + \right.
\]
\[
(b-\hat{g})(1-\hat{r}^\hat{p})^2 (b^n - \hat{g}^n) \Delta \left[ 2k(1-\hat{g})\hat{p} - (n+1)\hat{p}^n(1-\hat{g}^\hat{p})^2 (1-\hat{g}^n) \right] \frac{1}{\Delta E} \]
\[
\left\{ x(b-\hat{r})(1-\hat{g})^2 (b^n - \hat{g}^n) + (n+1)\hat{p}^n(b-\hat{g})(1-\hat{g}^\hat{p})^2 (b^n - \hat{g}^n) \Delta E \right\}
\]

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Substituting the expressions for \( A_t^{16} \) and \( B_t^{16} \) given in equations (63) and (98) into equation (99) and rearranging terms, we have:

\[
\text{ARR}^{16}_t = \frac{(1-\hat{g})}{(1-\hat{g}^\hat{p})} \times \left\{ (1-k)(1-x)(b-\hat{r})(1-\hat{g})(b^n - \hat{g}^n) + \right.
\]
\[
(n+1)(b-\hat{g})(1-\hat{r}^\hat{p})^2 (b^n - \hat{g}^n) \phi \Delta E \frac{1}{\Delta E} \]
\[
\left\{ x(b-\hat{r})(1-\hat{g})^2 (b^n - \hat{g}^n) + \right.
\]
\[
(b-\hat{g})(1-\hat{r}^\hat{p})^2 (b^n - \hat{g}^n) \Delta \left[ (n+1)\hat{p}^n(1-\hat{g}^\hat{p})^3 \xi - \hat{p}^k(1-\hat{g})^2 \lambda \right] \}
\]

We have now completed the development of models of firms with known internal rates of return and the derivation of the accounting rates of return for these firms. In Chapter III, our attention will be focused on an analysis of the expressions of \( \text{ARR}^{15}_t \) that we have developed in this chapter.

Before we turn to Chapter III, some comments on the model developed in this chapter are in order. The model is more comprehensive than any of those developed by prior researchers. The model includes more accounting policies as well as more parameters than previous models. While an analysis of this more comprehensive model will increase our knowledge of the relationship between firm APR and IRR, the analysis is much more difficult than with a less comprehensive
model. We have, therefore, chosen to conduct our analysis of the expressions developed in this chapter by a means of a simulation. We can take two views of this approach. The short view is that the analysis of the simulation results will give us some insights into the relationship between firm ARR and IRR under a variety of circumstances. In this view the analysis of the simulation results is the logical last step in this dissertation. The second and long run view of the simulation results is that they will provide insights into the relationship between firm ARR and IRR which will give direction to a direct mathematical analysis of the expressions developed in this chapter. In other words, the knowledge of the behavior of the expressions developed in this chapter gained through an analysis of the simulation may provide us with insights that will allow us to work directly with these complex expressions and thereby derive results of a more general nature. Both of these viewpoints should be kept in mind during our discussion of the simulation results.
CHAPTER III

ANALYSIS OF THE RELATIONSHIP BETWEEN THE ACCOUNTING
AND INTERNAL RATES OF RETURN OF A FIRM

The purpose of this chapter is to examine the relationship between
firm ARR and firm IRR by examining the properties of the expressions of
$\text{ARR}^j_t$ ($j=1,2,\ldots,16$) and $t=4n$ which were developed in Chapter II. The
complex nature of these expressions necessitates that this examination
be conducted by means of a simulation. In essence, the simulation
involves the calculation of the value of the differences ($\text{ARR}^j_t - r$) for
($j=1,2,\ldots,16$) and selected values of the parameters $b,g,k,n,p,r,$ and $x$.

As we had previously mentioned, the projects of two different firms
which utilize different tax reporting methods will generate different cash
flows before payments for taxes and raw materials when the projects have

---

We note that $\text{ARR}^j_t$ for $t\geq n$ depends upon $t$ only when $j=2,4,7,8$ (see
equation 100-115). For these accounting policies, as $t$ increases without
bound, $\text{ARR}^j_t$ increases but approaches a finite constant asymptotically.
The value of the constant is, of course, a function of $b,g,k,n,p,r,$ and $x$.
By conducting a sensitivity analysis, we found that $\text{ARR}^j_t$ converges to
this constant rapidly. For example, we found that

$$j = 2,4,7,8 \quad \max_{\text{ARR}^j_{4n} - \text{ARR}^j_{3n}}$$

was .004 for the following values of the other parameters: $b=.7, k=1.0, l.3$
$g=.08; k=.48; n=10; p=.07; r=.15; \text{ and } x=.3$. We, therefore, chose to set
t=4n since this will result in representative values of $\text{ARR}^j_t$ for $j=2,4,7,8$
and all $t \geq 4n$. 

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the same IRR. For an example of this fact see Table 3. This fact raises
the question of whether we can say anything about the relative magnitude
of the differences between the ARR and IRR of two firms which use differ-
ent tax reporting methods. The answer to this question is that we will
not say anything about these relative magnitudes. The reason is that
firms which use different accounting policies may be different on two
dimensions, not one dimension. In this dissertation, the two dimensions
are the tax reporting methods which affects firm cash flows and the
financial reporting methods which has no direct effect on firm cash
flows. We will therefore restrict our comments on the relative difference
between firm ARR and IRR to only those accounting policies which accom-
pass the same tax reporting methods. For example, we will reach con-
clusions by comparing |ARR_1 - r | and |ARR_9 - r | because accounting poli-
cies 1 and 9 both use the same tax reporting methods. Now that we have
made this warning regarding how we can interpret the results of the simu-
lation, we turn our attention to describing the nature of the simulation.

The specific values of the parameters which were used in the simu-
lation are:

\[ b = .7, .9, 1.0, 1.12, 1.3 \]
\[ g = .03, .08, .14 \]
\[ k = .30, .48, .52 \]
\[ n = 5, 10, 20 \]
\[ p = .03, .07, .11 \]
\[ r = .05, .10, .15, .20 \]
\[ x = 0, .3, .6 \]

Values of \((ARR_t^j - r)\) were calculated for all combinations of the values of these parameters except for the cases in which \(g\) is greater or equal to \(r\). For each combination of the values of these parameters, ranks were assigned to accounting policies based upon the magnitude of the absolute value of \((ARR_t^j - r)\). For example, the accounting policy index for \(j\) for which the absolute value of \((ARR_t^j - r)\) was the smallest was assigned the rank of one. As explained above we cannot compare the accounting rates of return for firms which use different tax reporting methods. This means that the comparison of the ranks assigned to \(|ARR_t^j - r|\) can only be compared for those accounting policies \(j\) which incorporate the same tax reporting methods. For example, it is meaningful to compare the ranks of accounting policies 1 and 9 but not the ranks of accounting policies 1 and 2.

We can now state the purpose of this chapter more precisely. The purpose is to determine:

1. how the sign, magnitude, and rank of the differences \((ARR_t^j - r)\) depend upon the values of the parameters \(b, g, k, n, p, r,\) and \(x\), and

2. how the sign, magnitude, and rank of the differences \((ARR_t^j - r)\) change as the values of the parameters \(b, g, k, n, p, r,\) and \(x\) change.

In order to simplify the discussion of the simulation results, we
have designated specific values of the parameters as "central" values. We will first analyze the sign, magnitude, and rank of the differences \((\text{ARR}_t^j - r)\) for the central values of the parameters and then determine how these attributes of the differences change as the values of the individual parameters change from the central value. The following values of the parameters have been designated as the central values: \(b=1.0, \ g=.08, \ k=.48, \ n=10, \ p=.07, \ r=.15, \) and \(x=.3\). The values of the differences \((\text{ARR}_t^j - r)\) (times 100) for the central values of the parameters are given in Table 6.

**Table 6**

Differences Between Firm ARR and IRR
For Accounting Policy \(j\) and the Central Values of the Parameters
\(b=1.0, \ g=.08, \ k=.48, \ n=10, \ p=.07, \ r=.15, \) and \(x=.3\)
Cell Values Are Measured in Per Centage Points
Ranks of the Differences Are Given in Parentheses

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<th>Accounting Policy</th>
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There are several interesting observations that can be made from a study of Table 6. The most obvious observation is that while each accounting rate of return is larger than the IRR of the firm, the accounting rates of return based upon price-level adjusted data (accounting policies 9, 10, \ldots , 16) are significantly closer to firm IRR than the accounting rates of return based upon price-level unadjusted data (accounting policies 1, 2, \ldots , 8). What is somewhat surprising about this result is that each ARR based upon price-level adjusted data is closer to firm IRR than the corresponding ARR which is based upon price-level unadjusted data. We shall find that this result holds for almost all parameter values examined.\footnote{2}{The reader may confirm this finding by examining Tables 7, 8, 9, and 10. We note that exceptions only occur when b=.7 (i.e., the cash flow pattern of firm projects is sharply decreasing).} For those accounting theorists who believe that we should select among alternative accounting measurement methods based upon how well their respective accounting rates of return approximate firm IRR, this result provides some evidence in support of preparing financial statements which have been adjusted for changes in the level of prices.

The organization of the remainder of this chapter is described below. We will initially separate our discussion of accounting rates of return based upon price-level unadjusted data from our discussion of accounting rates of return based upon price-level adjusted data. Our reason for doing so is that, as mentioned above, in the vast majority of cases
considered in this dissertation, the magnitude of the difference between firm ARR and IRR is significantly larger when the ARR is based upon price-level unadjusted data than when ARR is based upon price-level adjusted data. We will first concentrate our attention on the relationship between firm ARR and IRR for those accounting policies which report price-level unadjusted data (i.e., indices \(j = 1, 2, \ldots, 8\)). Second, we will examine the relationship between firm ARR and IRR for those accounting policies which report price-level adjusted data (i.e., indices \(j = 9, 10, \ldots, 16\)). The examination which will be made in these two parts of this chapter will proceed by examining the differences \((ARR_t^j - r)\) for a large proportion of the parameter value combinations considered in the simulation. These differences are displayed in Tables 7, 8, 9, and 10. Table 7 shows how the differences are affected by changes in the values of \(b, p,\) and \(g\) given that the values of \(k, n, r,\) and \(x\) are at their central values. Table 8 shows how the differences are affected by changes in the values of \(n, r,\) and \(x\) given that the values of \(b, g, k,\) and \(p\) are at their central values.

Prior research has shown that the behavior of the difference between firm ARR and IRR is significantly different when \(b\) is less than one than when \(b\) is greater than or equal to one. We have therefore prepared Table 9 which shows how the differences \((ARR_t^j - r)\) are affected by changes in the values of \(n, r,\) and \(x\) when \(g, k,\) and \(p\) are at their central values and \(b\) is equal to 0.7. By comparing Tables 8 and 9, we will be
able to determine whether the effect that changes in the values of \( n, r, \)
and \( x \) have on the differences \( \text{ARR}_t^j - r \) depends upon the value of \( b \).

Table 10 shows how the values of the differences are affected by
changes in the values of \( k, b, \) and \( y \) given that \( g, n, r, \) and \( x \) are at their
central values. Thus, an examination of Tables 7, 8, 9, and 10 will
allow us to determine how the values of the differences are affected
by changes in each of our model parameters.

The third and last section of this chapter will examine the properties
of those situations in which the magnitude of the difference between
\( \text{ARR} \) and \( \text{IRR} \) is smaller for price-level unadjusted accounting rates of
return than for price-level adjusted accounting rates of return. Table II
depicts those combinations of parameter values for which the magnitude
of the difference \( \text{ARR}_t^j - r \) \( (j=1, 2, \ldots, 8) \) is not greater than the magni-
tude of the difference \( \text{ARR}_t^{j+8} - r \).
Table 7

Differences Between Firm ARR and IRR For Accounting Policy j
and For Parameter values k = .48, n = 10, r = 15, and x = .3

Cell Values Are Measured In Per Centage Points

Rank of the Difference Are Given In Parentheses

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| <strong>A</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>B</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>C</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>D</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>E</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>F</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>G</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>H</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>I</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>J</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>K</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>L</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>M</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>N</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>O</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>P</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>Q</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>R</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>S</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>T</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>U</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>V</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>W</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>X</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>Y</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |
| <strong>Z</strong>          |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |         |</p>
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The Relationship Between Firm IRR and Price-Level Unadjusted ARR

By examining Tables 7, 8, 9, and 10, we see that when firm ARR is calculated on the basis of price-level unadjusted data (i.e., indices j=1,2,...,8), firm ARR is greater than firm IRR for (almost) all values of the parameters considered in this study. The importance of this observation is that for most parameter value combinations considered in the simulation price level, unadjusted accounting rates of return are larger than firm IRR no matter what other accounting methods the firm uses for financial or tax reporting.

We now turn our attention to a discussion of how the firm's depreciation method and treatment of deferred taxes influences the differences between firm ARR and firm IRR. In order to be sure that we are isolating the effect that the firm's selection of a particular depreciation method and a particular treatment of deferred taxes has on the relationship between firm ARR and firm IRR, we must be sure to compare those differences (ARR_j^t - r) only for the cases in which the firm's tax reporting methods are the same. If we compare the differences (ARR_3^t - r), (ARR_5^t - r), and (ARR_6^t - r), we will be comparing the relationship between firm ARR and IRR for firms which report figures unadjusted for changes in the price level, which use the FIFO inventory method, and which:

1) use SYD depreciation for tax and financial reporting (j=3)
(2) Use SL depreciation for financial reporting, SYD depreciation for tax reporting, and which do not recognize deferred taxes (j = 5).

(3) Use SL depreciation for financial reporting, SYD depreciation for tax reporting, and which do recognize deferred taxes (j = 6).

If we compare the differences $(\text{ARR}_t - r)$, $(\text{ARR}_t - r)$, and $(\text{ARR}_t - r)$, we will be comparing the relationship between firm ARR and firm IRR for firms which report figures unadjusted for changes in the price level, which use the LIFO inventory method, and which:

(1) Use SYD depreciation for tax and financial reporting (j = 4),

(2) Use SL depreciation for financial reporting, SYD depreciation for tax reporting, and do not recognize deferred taxes (j = 7),

(3) Use SL depreciation for financial reporting, SYD depreciation for tax reporting, and do recognize deferred taxes (j = 8).

By using Table 6 to make both of these comparisons, we find that there is consistency in the ranking of these alternatives for the central parameter values. Specifically, we find that:

Rank (5) < Rank (6) < Rank (3) and

Rank (7) = Rank (8) = Rank (4).

This means that when a firm uses the SYD depreciation method for tax reporting and the model parameters are at their central values then the magnitude of the difference between firm ARR and IRR will be smallest if the firm uses the SL depreciation method and the flow-through treatment of deferred taxes. In these circumstances, the firm ARR next closest to firm IRR is produced by the SL depreciation for financial reporting and the normalization method of deferred taxes. The ARR produced by SYD
depreciation for financial and tax reporting is furthest from firm IRR.

We must remember that we have been restricting the scope of our discussion of depreciation methods and deferred tax treatments to those accounting policies which use SYD depreciation for tax reporting and which use price-level unadjusted data for financial reporting. We have also restricted our discussion to the case of the central values of the model parameters. However, when we examine Tables 7, 8, 9, and 10, we see that the order of the ranks assigned to depreciation methods and deferred tax treatments is the same as that given above for almost all parameter values considered in the simulation.

We now turn our attention to an examination of how changes in the values of the model parameters affect the magnitude of the differences \( (ARR_t^j-r) \). By examining Tables 7 and 10, we find that as the values of \( b \) and \( p \) increase the magnitude of the differences \( (ARR_t^j-r) (j=1,2,...,8) \) increase for all values of the parameters \( g, k, n, r, \) and \( x \) given in these tables. We see, however, that the effect on the magnitude of the differences \( (ARR_t^j-r) (j=1,2,...,8) \) of changes in the values of the parameters \( g, k, n, r, \) and \( x \) are not independent of the values of \( b \) or the accounting policy \( j \).

For example, from Table 8, we note that the effect of increases in the inventory parameter \( x \) depend upon the inventory method used by the firm. As \( x \) increases from 0.0 to 0.6, the differences \( (ARR_t^j-r) \) decrease and increase when \( j=2,4,7,8 \), i.e., when the firm uses the LIPO inventory method. However, from Table 9, we note that as \( x \) increases
from 0.0 to 0.6, the differences $\text{ARR}_t^j - r$ increase for $j = 1, 2, ..., 8$. The only difference between Tables 8 and 9 is that in Table 8 $b$ is equal to 1.00 while in Table 9 $b$ is equal to .7. We, therefore, see that the effect of changes in $x$ on the differences $\text{ARR}_t^j - r$ ($j = 1, 2, ..., 8$) depends upon $b$ and the specific accounting policy $j$. As another example, we find that, in the majority of situations, as $n$ increases, the differences $\text{ARR}_t^j - r$ increase (decrease when $b$ is greater than or equal to less than one) (see Tables 8 and 9). The reader can determine in a similar manner that the effect of changes in the parameters $g, k,$ and $r$ on the differences $\text{ARR}_t^j - r$ ($j = 1, 2, ..., 8$) also depend upon $b$ and in some cases the accounting policy $j$.

These findings confirm the finding by Greenball that the value of the cash flow parameter is crucial to the relationship between firm ARR and IRR. However, these findings conflict with one of the conclusions reached by Carlson. Carlson found that increases in real growth reduces firm ARR relative to firm IRR whenever prices increase. We see from Table 7, however, that increases in $g$ cause firm ARR to increase relative to firm IRR whenever $b$ is equal to .7. I believe that the reason for these conflicting results is that most of Carlson's analysis was carried

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4 R.S. Carlson, op. cit., p. 81.
out for the case of level project cash flows (i.e., b=1). For this case (b=1), we also observe that increases in g cause firm ARR to decrease relative for firm IRR.

This concludes our investigation of the relationship between firm ARR and IRR when ARR is based upon price-level unadjusted data. We now turn to an investigation of the differences (ARR\textsubscript{t}^j - r) (j=9,10,...,16). The Relationship Between Firm IRR and Price-Level Adjusted ARR

The purpose of this section is to examine the relationship between firm ARR and IRR when the ARR is based upon price-level adjusted data. This examination will be made by examining the differences (ARR\textsubscript{t}^j - r) (j=9,10,...,16) as they are given in Tables 7, 8, 9, and 10. As previously mentioned, we find that |ARR\textsubscript{t}^j - r| is smaller for j=9,10,...,16 than for j=1,2,...,8 for most of the circumstances we have examined. We also note that while price-level unadjusted ARR is greater than IRR, price-level adjusted ARR is sometimes greater than IRR and sometimes less than IRR.

We now turn to the task of determining the effect that the firm's depreciation and deferred tax method has on the relationship between firm ARR and IRR for those cases in which the firm's tax reporting methods are the same. If we compare the differences (ARR\textsubscript{t}^{11} - r), (ARR\textsubscript{t}^{13} - r), and (ARR\textsubscript{t}^{14} - r), we will be comparing the relationship between firm ARR and IRR for firms which report figures adjusted for changes in the price level, which use the FIFO inventory method, and which:
(1) use SYD depreciation for financial and tax reporting (j=11);

(2) use SL depreciation for financial reporting, SYD depreciation for tax reporting, and which do not recognize deferred taxes (j=13);

(3) use SL depreciation for financial reporting, SYD depreciation for tax reporting, and which do recognize deferred taxes (j=14).

If we compare the differences \( (ARR^{12}_t - r) \), \( (ARR^{15}_t - r) \), and \( (ARR^{16}_t - r) \), we will be comparing the relationship between firm ARR and IRR for firms which report figures adjusted for changes in the price level, which use the LIFO inventory method, and which:

(1) use the SYD depreciation method for financial and tax reporting (j=12);

(2) use SL depreciation for financial reporting, SYD depreciation, tax reporting, and which do not recognize deferred taxes (j=15);

(3) use SL depreciation for financial reporting, SYD depreciation for tax reporting and which do recognize deferred taxes (j=16).

Using either of these comparisons and examining Tables 7, 9, and 10, we find that in most situations the magnitude of the difference between firm ARR and IRR is smallest for the firm which uses SYD depreciation for financial and tax reporting when the cash flow parameter \( b \) is equal to .7 (j=11,12). The magnitude of the difference between firm ARR and IRR is largest for firms which use SL depreciation for financial reporting and SYD depreciation for tax reporting when \( b = .7 \). Whether the flow-through or normalization treatment results in the smaller difference in this case depends upon the values of \( g, p, n, \) and \( r \). As the reader can tell there appears to be considerable interaction among these latter
variables and it is difficult to make any general statements about the flow-through and normalization methods for the case of $b = .7$.

When $b$ is greater than or equal to one we find ourselves in a similar situation. We find that the magnitude of the difference between firm ARR and IRR is largest for firms which use SYD for financial and tax reporting ($j=11,12$). The magnitude of the difference is smallest for firms which use SL depreciation for financial reporting and SYD depreciation for tax reporting ($j=13,14,15,16$). However, we find that in some cases the magnitude is smallest for the flow-through treatment of deferred taxes while in other cases the magnitude is smallest for the normalization treatment of deferred taxes. In other words, the rank assigned to the policies which use flow-through or normalization depend upon the parameters $g, n, p, r, x$, and there is very little that can be said about this topic which is general. This concludes our examination of how the accounting policy $j$ affects the relationship between price-level adjusted ARR and firm IRR.

We now turn to an examination of how the differences $(\text{ARR}_t^j - r)$ ($j=9,10,\ldots,16$) are affected by changes in the values of the model parameters.

By examining Tables 7,8,9, and 10 we find that changes in the value of the cash flow pattern parameter $b$ have an obvious effect on the differences $(\text{ARR}_t^j - r)$, $j=9,10,\ldots,16$. First, we notice that when $b = .7$ the sign of the differences are all negative. When $b = 1.3$ the sign of the differences are all positive. This means that as $b$ increases,
\(\text{ARR}^j_t\) \((j=9,10,\ldots,12)\) also increases. We, therefore, can conclude that the value of \(b\) is critical in determining the sign of the differences \((\text{ARR}^j_t - r)\) when \(j=9,10,\ldots,16\). This finding is in contrast to our finding that the sign of \((\text{ARR}^j_t - r)\) \((j=1,2,\ldots,8)\) is almost always positive and does not depend upon the value of \(b\).

We again find that the effect of changes in the values of some of the parameters \(g, k, n, p, r,\) and \(x\) \((\text{ARR}^j_t - r)\) \((j=9,10,\ldots,16)\) depend upon the value of \(b\). However, we also find that the effect of changes in any one of these parameters does not seem to depend upon the values of the others, and in some cases the effect does not depend upon the value of \(b\). We will summarize the effect that changes in the values of \(g, k, n, p, r,\) and \(x\) have on the differences by first examining those parameters whose effect on the differences depend upon the value of \(b\) (i.e., \(k, n, p,\) and \(r\)). We will then examine those parameters whose effect on the differences does not depend on the value of \(b\).

If \(b\) is equal to .7, we find for \(j=9,10,\ldots,16\) that:

1. As \(k\) increases from .40 to .60, \(\text{ARR}^j_t\) increases which means that \(|\text{ARR}^j_t - r|\) decreases since for this case \(\text{ARR}^j_t\) is less than \(r\) (see Table 10).

2. As \(n\) increases from 5 to 20, \(\text{ARR}^j_t\) decreases and \(|\text{ARR}^j_t - r|\) increases (see Table 9).

3. As \(p\) increases from .03 to .11, \(\text{ARR}^j_t\) decreases which means that \(|\text{ARR}^j_t - r|\) increases (see Tables 7 and 10).
(4) As \( r \) increases from .10 to .20, \( \text{ARR}_t^j \) decreases which means that \( |\text{ARR}_t^j - r| \) increases (see Table 9).

If \( b \) is greater than or equal to 1, we find for \( j=9,10,\ldots,16 \) that:

(1) As \( k \) increases from .40 to .60, \( \text{ARR}_t^j \) decreases which means that (in most cases) \( |\text{ARR}_t^j - r| \) also decreases (see Table 10).

(2) As \( n \) increases from 5 to 20, \( \text{ARR}_t^j \) increases. This means that \( |\text{ARR}_t^j - r| \) increases in most cases since \( \text{ARR}_t^j \) is greater than \( r \) when \( b \geq 1 \). There are some exceptions, however (see Table 8).

(3) As \( p \) increases from .03 to .11, \( \text{ARR}_t^j \) does not change very much except when \( j=14,16 \). For these cases, \( \text{ARR}_t^j \) decreases which means \( |\text{ARR}_t^j - r| \) decreases (see Tables 7 and 10).

(4) As \( r \) increases from .10 to .20, \( \text{ARR}_t^j \) increases. This means that \( |\text{ARR}_t^j - r| \) also increases in most cases (see Table 8).

The effect of changes in the values of the parameters \( g \) and \( x \) on the differences (\( \text{ARR}_t^j - r \)) do not seem to depend upon the value of \( b \).

As \( g \) increases from .03 to .14, we find that \( |\text{ARR}_t^j - r| \) decreases. This effect does not depend upon the sign of the differences (see Table 7).

As \( x \) increases from .0 to .6, we find that \( |\text{ARR}_t^j - r| \) decreases. This effect also does not depend upon the sign of the difference (see Tables 8 and 9).

This concludes our examination of the relationship between firm \( \text{ARR} \) and \( \text{IRR} \) for those cases in which the calculation of \( \text{ARR} \) is based
upon price-level adjusted data. Our justification for separating the analysis into two parts — the first in which ARR is based upon price-level unadjusted data and the second in which ARR is based upon price-level adjusted data — was that in most cases price-level adjusted accounting rates of return are closer to firm IRR than the corresponding price-level unadjusted accounting rate of return. We now turn to an examination of the circumstances in which this is not true.

**Cases in Which Price-Level Unadjusted ARR is Closer to Firm IRR than Price-Level Adjusted ARR**

Before we present the simulation results for those situations in which the inequality $|\text{ARR}_t^j - r| \leq |\text{ARR}_t^h - r|$ (j=1,2,...,8 and h=j+8) does not hold, we review our prior findings to determine what we expect the characteristics of these situations to be. First, we remember that in most cases $\text{ARR}_t^j > r$ when j=1,2,...,8. We also found that $\text{ARR}_t^j$ is smallest when b=.7 and p=.03 given any specified values of the other parameters. We also found that when b is less than one, increases in the parameter n cause $\text{ARR}_t^j$ (j=1,2,...,8) to decrease. This means that we should expect $|\text{ARR}_t^j - r|$ (j=1,2,...,8) to be smallest for low b (b=.7), low p (p=.03) and large n (n=20). By examining Table 9 for the cases in which n=20, we find that $(\text{ARR}_t^j - r)$ (j=1,2,...,8) decreases as r increases from .10 to .20. This means that we should expect the difference between price-level unadjusted ARR and IRR to be smallest when b is small (.7), p is small (.03), n is large (20), and r is large (.20).
We now examine what we expect price-level adjusted ARR to be in this situation. Since b is equal to .7, we should find that $\text{ARR}_t^j$ is less than r for j=9,10,...,16. As previously mentioned, if b is less than one the magnitude of the difference $\frac{\text{ARR}_t^j}{\text{ARR}_t^j - r}$ will be largest for large n and large r. Since increases in g, k, and x cause the magnitude of the difference $(\text{ARR}_t^j - r)$ to decrease, the $\frac{\text{ARR}_t^j}{\text{ARR}_t^j - r}$ will be largest when g, k, and x are small.

We should therefore find that the magnitude of the difference between ARR and IRR will be smaller for price-level unadjusted ARR than price-level adjusted ARR when b is small, g is small, k is small, n is large, p is small, r is large, and x is small. In Table II, we have displayed the differences $(\text{ARR}_t^j - r)$ (times 100) for some of these cases. We find that for every one of the cases depicted in Table II that price-level adjusted ARR is further from firm IRR than price-level unadjusted IRR. However, one of the interesting characteristics of these situations if that they appear to be "uneconomic" or "irrational". By this I mean that the cases in which b=.7 and n is large are cases in which the cash flows in the latter years of the project's useful life are so small that the project would have a higher real IRR if the firm had abandoned the project at an earlier time. For example, for the case of accounting policy 1 and b=.7, n=20, x=.3, p=.03, g=.03, r=.20, and k=.4, it can be shown that if the firm can abandon the project at no cost at the end of its fourth
### Table II

**Difference Between Firm A and IB For Accounting Policy J**

And for Parameter values $b = .07$, $c = .33$, and $c = .20$

Cell Values Are Measured in Per Cent Points

Ranks of the Difference Are Given in Parentheses

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<th>0.20</th>
<th>0.15</th>
<th>0.10</th>
<th>0.05</th>
<th>0.03</th>
<th>0.01</th>
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<td>0.05</td>
<td>0.03</td>
<td>0.01</td>
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<td>2.0</td>
<td>3.0</td>
<td>4.0</td>
<td>5.0</td>
<td>6.0</td>
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year, then the project would have a real IRR in excess of .22. This means that for this case, the choice of a useful life of 4 years results in a project IRR which is greater than the IRR produced by a choice of a 20-year useful life. These statements are not intended to imply that projects with long lives cannot possess a declining pattern of cash flows. The intent of these statements is to point out that projects with long lives and a severely (exponentially) declining pattern of cash flows are projects for which useful life should be shorter than physical life.

While we have not investigated enough specific examples to make any broad generalizations that cover all cases, it does appear that many of the situations in which price-level unadjusted ARR is closer to IRR than price-level adjusted ARR are situations in which the firm's choice of a useful life is inconsistent with the IRR criterion of project selection.

Summary

We can summarize the findings of this chapter on the relationship between firm ARR and IRR as follows.

1. When ARR is based upon price-level unadjusted data, we find that for most combinations of parameter values considered in this study that:

(a) \( ARR_1 \) is greater than the IRR of the firm;

(b) the magnitude of the difference between firm ARR and IRR is larger than when ARR is based upon price-level adjusted data;

(c) whenever the firm uses the SYD depreciation method for tax
reporting the magnitude of the difference between firm ARR and IRR will be the smallest when the firm uses the SL depreciation method and the flow-through treatment of deferred taxes for financial reporting;

(d) the magnitude of the difference between firm ARR and IRR increases as b and p increase for all accounting policies and all values of the other parameters;

(e) the effect of changes in the values of g, k, n, r, and x on the magnitude of the difference between firm ARR and IRR depends upon the value of b and the accounting policy j.

2. When ARR is based upon price-level adjusted data, we find that for most combinations of parameter values considered in this study that:

(a) the sign of the difference \( \text{ARR}_t^j - r \) depends upon the value of the cash flow pattern parameter b. When \( b = .7 \), \( \text{ARR}_t^j - r \) is negative for all \( j = 9, 10, \ldots, 16 \). When \( b = 1.3 \), \( \text{ARR}_t^j - r \) is positive for all \( j = 9, 10, \ldots, 16 \). For intermediate values of b, \( \text{ARR}_t^j - r \) is positive for some j and negative for other j;

(b) the magnitude of the difference between firm ARR and IRR is smaller than when ARR is based upon price-level unadjusted data;

(c) little can be said in general about how the firm's choice of book and tax depreciation methods and treatment of deferred taxes affect the magnitude of the difference between firm ARR and IRR;
(d) the magnitude of the difference between firm ARR and IRR decreases as g and the inventory parameter x increase;

(e) the effect of changes in the parameters k, n, p, and r on the magnitude of the difference between firm ARR and IRR depend upon the value of b (but not on the accounting policy j).

3. For those situations in which price-level unadjusted ARR is closer to r than price-level adjusted ARR, it appears that the firm's choice of a useful life for its projects is inconsistent with the IRR criterion for project selection.
CHAPTER IV

SUMMARY, CONCLUSIONS, AND SUGGESTIONS FOR FUTURE RESEARCH

Summary and Conclusions

The purpose of this paper has been to investigate the relationship between firm ARR and IRR for a class of positive-growth, infinite-lived firms. The methodology that has been used to conduct this investigation is similar to the methodology used by several researchers who have studied this topic in the past. This methodology involves the construction of models of firms whose IRR is known and whose ARR can be derived. This methodology incorporates the assumption that firm IRR is equal to the IRR of all firm projects. We have investigated this assumption and found that it is true only when the growth rate in firm gross investment is less than the IRR of all firm projects.

The research conducted here extends prior research by including more parameters in our model than have been included previously. The inclusion of more parameters in the model not only makes the model more general but also allows us to examine the effect that alternative accounting methods not considered by prior researchers have on the relationship between firm ARR and IRR. We have analyzed this expanded model by conducting a simulation. The major findings of this analysis were:
1. When ARR is based upon price-level unadjusted data, we find that for most combinations of parameter values

(a) $ARR_j^t$ is greater than the IRR of the firm;

(b) the magnitude of the difference between firm ARR and IRR is larger than when ARR is based upon price level adjusted data;

(c) whenever the firm uses the SYD depreciation method for tax reporting the magnitude of the difference between firm ARR and IRR will be the smallest when the firm uses the SL depreciation method and the flow-through treatment of deferred taxes for financial reporting;

(d) the magnitude of the difference between firm ARR and IRR increases as $b$ and $p$ increase for all accounting policies and all values of the other parameters;

(e) the effect of changes in the values of $g, k, n, r, \text{and } x$ on the magnitude of the difference between firm ARR and IRR depends upon the value of $b$ and the accounting policy $j$.

2. When ARR is based upon price-level adjusted data, we find that for most combinations of parameter values considered in this study that:

(a) the sign of the difference $(ARR_j^t - r)$ depends upon the value of the cash flow pattern parameter $b$. When $b=.7$, $(ARR_j^t - r)$ is negative for all $j=9,10,\ldots,16$. When $b=1.3$, $(ARR_j^t - r)$ is positive for all $j=9,10,\ldots,16$. For intermediate values of $b$, $(ARR_j^t - r)$ is positive for some $j$ and negative for other $j$;
(b) the magnitude of the difference between firm ARR and IRR is smaller than when ARR is based upon price-level unadjusted data;
(c) little can be said in general about how the firm's choice of book and tax depreciation methods and treatment of deferred taxes affect the magnitude of the difference between firm ARR and IRR;
(d) the magnitude of the difference between firm ARR and IRR decreases as q and the inventory parameter x increase;
(e) the effect of changes in the parameters k, n, p, and r on the magnitude of the difference between firm ARR and IRR depend upon the value of b (but not on the accounting policy j).

3. For those situations in which price-level unadjusted ARR is closer to r than price-level adjusted ARR, it appears that the firm's choice of a useful life for its projects is inconsistent with the IRR criterion for project selection.

These findings are significant to those people who use firm ARR as an estimate of firm IRR and to those people who believe that we should select among alternative accounting methods based upon how well the resulting accounting rates of return approximate the IRR of the firm. In the next section, however, we shall discuss the limitations of our findings and suggest ways in which these limitations may be removed.
Suggestions for Future Research

The findings of our study depend on and are limited by the assumptions we made in the development of the model. Future research on the relationship between firm ARR and IRR could proceed by relaxing any of these assumptions. For example, one of the assumptions of our study was that all prices change at the same rate and this rate is the same in every year. This means that in our model there is no difference between accounting data adjusted for changes in the general price level and accounting data adjusted for changes in specific prices. If the model were extended in such a way that different prices could change at different rates then the question of whether accounting data adjusted for specific price changes or adjusted for changes in the general price level results in a firm ARR closer to firm IRR could be investigated.

A more comprehensive and less manageable simulation study could be made in which several of our assumptions would be relaxed at once. For example, a simulation model could be constructed in which the firm invested in projects of different lives and which had different rates of

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1 A similar research project was proposed by Ezra Solomon in; Ezra Solomon, "Alternative rate of return concept...," op. cit., p. 78.
return, in which the firm's investment pattern was cyclical, and in which different prices changed in different cyclical patterns. Such a model would certainly be more realistic than the model developed here and its construction is now feasible. The problem in conducting such a study, however, is that there is no apparent way in which to analyze the output from such a model. If we can develop a method by which the output from such a model could be analyzed then we can achieve a significant addition to our knowledge of the relationship between firm ARR and IRR.

Our study has been complicated by the fact that the accounting policy adopted by the firm incorporates both the tax and financial reporting methods used by the firm. This complication has been compounded in our study because we have only examined cases in which the firm's inventory method is the same for financial and tax reporting. This has meant that we cannot compare the magnitude of the differences between firm ARR and IRR for two firms which use different inventory methods because the firms differ on two dimensions rather than one dimension. Future research can be conducted to circumvent this problem. This future research would proceed by first specifying a single set of tax

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reporting methods that all firms under study would use. For example, if we restricted the scope of this proposed research to positive growth firms and to periods of inflation, then the tax reporting methods that the firm should adopt would be the LIFO inventory method and the SYD depreciation method. Then, we could examine how the relationship between ARR and IRR is affected by the set of accounting methods that the firm adopts for financial reporting purposes. The research could then make statements about the relative magnitudes of the differences between firm ARR and IRR since the tax reporting methods used by all firms examined would be the same.
APPENDIX A
MATHEMATICAL NOTES

The purpose of this appendix is to prove some of the less obvious equalities in the dissertation that are too lengthy to prove in the body of the text.

Note (1)

In equation (5) of this study, we state that:

\[
\sum_{i=0}^{n} \sum_{j=0}^{i} C_{1-j} g^{j-1} r_{1}^{n} + \left(1/r_{1}^{n}\right) \sum_{j=0}^{n} C_{n-j} g^{j} \sum_{i=1}^{\infty} \left(g/r_{1}^{i}\right) = \sum_{i=0}^{n} \sum_{j=0}^{n-i} C_{1-j} g^{j-1} r_{1}^{n} + \left(1/r_{1}^{n}\right) \sum_{j=0}^{n} C_{n-j} g^{j} \sum_{i=1}^{\infty} \left(g/r_{1}^{i}\right)
\]

The proof of this equality will proceed in two steps. The first step is to prove that:

\[(116) \quad \sum_{i=0}^{n} \sum_{j=0}^{i} C_{1-j} g^{j-1} r_{1}^{n} = \sum_{i=0}^{n} \sum_{j=0}^{n-i} C_{i} g^{j} r_{1}^{n-i}
\]

The second step is to prove that:

\[(117) \quad \left(1/r_{1}^{n}\right) \sum_{j=0}^{n} C_{n-j} g^{j} \sum_{i=1}^{\infty} \left(g/r_{1}^{i}\right) = \left(1/r_{1}^{n}\right) \sum_{j=0}^{n} C_{n-j} g^{j} \sum_{i=1}^{\infty} \left(g/r_{1}^{i}\right)
\]

The proof of equation (116) involves writing out the terms of the double sum on the left-hand side of the equation and simply rearranging terms:

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\[
\sum_{i=0}^{n} \sum_{j=0}^{n} c_{i-j}^{0} \dot{r}_{1}^{j} \dot{r}_{1}^{-i} = (\dot{\lambda}/r_{1})^{0} \left[ C_{0}^{1,0} \right] + (\dot{\lambda}/r_{1})^{1} \left[ C_{1}^{1,0} g^{0} + C_{0}^{0} g^{1} \right] + (\dot{\lambda}/r_{1})^{2} \left[ C_{2}^{1,0} g^{0} + C_{1}^{0} g^{1} + C_{0}^{0} g^{2} \right] + (\dot{\lambda}/r_{1})^{3} \left[ C_{3}^{1,0} g^{0} + C_{2}^{0} g^{1} + C_{1}^{0} g^{2} + C_{0}^{0} g^{3} \right] + \ldots + (\dot{\lambda}/r_{1})^{n} \left[ C_{n}^{1,0} g^{0} + C_{n-1}^{0} g^{1} + \ldots + C_{0}^{0} g^{n} \right] + (\dot{\lambda}/r_{1})^{1+n} \left[ 1 + (\dot{\lambda}/r_{1})^{2} + \ldots + (\dot{\lambda}/r_{1})^{n} \right] + (\dot{\lambda}/r_{1})^{1+n} \left[ 1 + (\dot{\lambda}/r_{1})^{2} + \ldots + (\dot{\lambda}/r_{1})^{n} \right] + \ldots + (\dot{\lambda}/r_{1})^{1+n} \left[ 1 + (\dot{\lambda}/r_{1})^{2} + \ldots + (\dot{\lambda}/r_{1})^{n} \right] + \ldots
\]

The proof of equation (117) involves writing out the terms of the finite sum on the left-hand side of the equation and rearranging terms:

\[
(\dot{\lambda}/r_{1})^{n} \sum_{j=0}^{n} c_{n-j}^{0} \dot{g}^{j} \sum_{i=1}^{\infty} (\dot{\lambda}/r_{1})^{i} = (\dot{\lambda}/r_{1})^{n} \sum_{i=1}^{\infty} (\dot{\lambda}/r_{1})^{i} \left[ C_{n-i}^{0} g^{i} + C_{n-1-i}^{0} g^{i+1} + \ldots + C_{0}^{0} g^{n} \right] = (\dot{\lambda}/r_{1})^{n} \sum_{i=1}^{\infty} (\dot{\lambda}/r_{1})^{i} \left[ C_{n-i}^{0} g^{i} + C_{n-1-i}^{0} g^{i+1} + \ldots + C_{0}^{0} g^{n} \right] = (\dot{\lambda}/r_{1})^{n} \sum_{j=0}^{n} c_{j}^{0} \dot{g}^{j} \sum_{i=1}^{\infty} (\dot{\lambda}/r_{1})^{i}
\]
In equation (6), we state that if \( g \neq r_1 \), then:

\[
D_{1} \mathbf{r}^{-1} \mathbf{p}^{-1} = \sum_{i=0}^{n} C_{i,1}^{0} \left( \frac{r_{n+1-i}}{r_{1}} - \frac{g^{n+1-i}}{(r_{1} - g)} \right) + \\
\sum_{i=0}^{n} \frac{r_{n}}{r_{1}} C_{j,1}^{0} \sum_{i=1}^{\infty} \left( \frac{g}{r_{1}} \right)^{i}
\]

The only difference between this equation and the last line of equation (5) is the first term on the right-hand side of each equation.

Therefore, to prove this equality, we have to prove that if \( g \neq r_1 \):

\[
\sum_{i=0}^{n} \frac{r_{n}}{r_{1}} C_{i,1}^{0} \sum_{j=0}^{n-i} \left( \frac{g^{n-i-j}}{r_{1}^{n-i-j}} \right) = \sum_{i=0}^{n} C_{i,1}^{0} \left( \frac{r_{n+1-i}}{r_{1}} - \frac{g^{n+1-i}}{(r_{1} - g)} \right)
\]

This can be proven as follows:

\[
\sum_{i=0}^{n} \sum_{j=0}^{n-i} C_{i,1}^{0} \frac{g^{n-i-j}}{r_{1}^{n-i-j}} = \sum_{i=0}^{n} C_{i,1}^{0} \left( \frac{r_{n+1-i}}{r_{1}} - \frac{g^{n+1-i}}{(r_{1} - g)} \right)
\]

For the sake of notational convenience, we let:

\[
y = \left( \frac{g}{r_{1}} \right)^{i} \quad \text{and} \quad S_{n} = \sum_{j=0}^{n-i} y^{i}
\]

We can write, \( S_{n} \) as:

\[
S_{n} = 1 + y + \ldots + y^{n-i} \quad \text{Therefore,}
\]

\[
yS_{n} = y + y^{2} + \ldots + y^{n-i+1} \quad \text{Therefore,}
\]
\((1 - y) S_n = 1 - y^{n-i+1} \) and \( S_n = \frac{1 - y^{n-i+1}}{(1 - y)} \)

By substituting \((\hat{g}/\hat{r}_1^i)\) for \(y\) and rearranging terms, we have:

\[ S_n = \frac{\hat{r}_1^{n+1-i} - \hat{g}^{n+1-i}}{\hat{r}_1^n \cdot (\hat{r}_1 - \hat{g})} \]

Substituting \(S_n\) into equation (118), we have:

\[ \sum_{i=0}^{n} \sum_{j=0}^{n-i} C_i^0 \hat{r}_1^{-i} \hat{g}_j \hat{r}_1^{-j} = \sum_{i=0}^{n} C_i^0 \hat{r}_1^{-i} \left[ \frac{\hat{r}_1^{n+1-i} - \hat{g}^{n+1-i}}{\hat{r}_1^n \cdot (\hat{r}_1 - \hat{g})} \right] \]

Note (3)

In equation (17), we state that:

\[ \sum_{i=1}^{n} (n+1-i) \hat{r}_1^{-i} \hat{r}_1^{-i} = \frac{1 - \hat{r}^n - \hat{r}_1^n \hat{n} \hat{r}_1^n (1 - \hat{r}_1 \hat{r}_1)}{\hat{r}_1^n \hat{n} (1 - \hat{r}_1 \hat{r}_1)^2} \] provided \(\hat{r} \neq 1\).

The proof of this equality is as follows: For the sake of notational convenience, let:

\[ y = \hat{r}_1 \hat{r}_1 \]

\[ S_n = \sum_{i=1}^{n} (n+1-i) y^{-i} \]

Then, \( S_n = \frac{n}{y} + \frac{(n-1)}{y^2} + \ldots + \frac{1}{y^n} \) and

\[ \frac{1}{y} S_n = \frac{n}{y^2} + \frac{n-1}{y^3} + \ldots + \frac{1}{y^{n+1}} \]

\[ (1 - \frac{1}{y}) S_n = \frac{n}{y} - \frac{1}{y^2} - \frac{1}{y^3} - \ldots - \frac{1}{y^{n+1}} \]
\[
\frac{n}{y} - \frac{1}{y} \sum_{i=1}^{n} y^{-i}
\]

\[
= \frac{n}{y} - \frac{1}{y} \left[ \frac{1-y^n}{y^n(1-y)} \right] \quad \text{provided } 1 \neq y.
\]

Multiplying both sides of this equation by \(y\), we have:

\[
(y-1) S_n = n - \frac{1-y^n}{y^n(1-y)}
\]

Dividing both sides of this equation by \((y-1)\) and rearranging terms, we have:

\[
S_n = \frac{1-y^n}{y^n(1-y)^2} - \frac{n}{y^n(1-y)} = \frac{1-y^n}{y^n(1-y)^2} - \frac{ny^n}{y^n(1-y)^2}(1-y).
\]

By substituting \(\hat{\gamma}\) for \(y\) into the above equation, we have equation (17).

On pages 57 and 80, we state that:

\[
\sum_{i=1}^{n} (n+1-i) \hat{\gamma}^{-i} = \frac{1-\hat{\gamma}^{n+1} - n\hat{\gamma}^{n}(1-\hat{\gamma})}{\hat{\gamma}^{n+1}(1-\hat{\gamma})^2} \quad \text{provided } \hat{\gamma} \neq 1.
\]

This equality is proven the same as above except that this time we let \(\hat{\gamma} = \gamma\).

On pages 58 and 81, we state that:

\[
\sum_{i=1}^{n} (n+1-i) \hat{\gamma}^{-i} = \frac{1-\hat{\gamma}^{n+1} - n\hat{\gamma}^{n}(1-\hat{\gamma})}{\hat{\gamma}^{n+1}(1-\hat{\gamma})^2} \quad \text{provided } \hat{\gamma} \neq 1.
\]

This equality is proven the same as above except that this time we let \(\hat{\gamma} = \gamma\).
Note (4)

On page 82, we state that:

\[
\sum_{i=0}^{n-1} \frac{(n-i)(n+1)}{pq^{n-p}} y_i = \frac{2[1-q^n p^n - nq^n p^n (1-gp)] - n(n+1)q^n p^n (1-gp)^2}{g^n-1 p^n-1 - (1-gp)^3}
\]

provided \( g_p \neq 1 \). The proof of this equality is as follows. For the sake of notational convenience, we let:

\[
y = g_p,
\]

and

\[
S_n = \sum_{i=0}^{n-1} (n-i)(n+1) y_i^{-1}.
\]

Then,

\[
S_n = \frac{n(n+1)}{y^0} + \frac{(n-1)(n)}{y^1} + \frac{(n-2)(n-1)}{y^2} + \ldots + \frac{1(2)}{y^{n-1}}
\]

\[
\frac{1}{y} S_n = \frac{n(n+1)}{y} + \frac{(n-1)(n)}{y^2} + \ldots + \frac{1(2)}{y^{n-1}}
\]

\[
(1 - 1) S_n = \frac{n[n+1 - (n-1)]}{y} + \frac{(n-1)[n-(n-2)]}{y^2} + \ldots + \frac{2}{y^n} - \frac{n(n+1)}{y^0}
\]

\[
= \frac{2n}{y} + \frac{2(n-1)}{y^2} + \frac{2(n-2)}{y^3} + \ldots + \frac{2}{y^n} - \frac{n(n+1)}{y^0}
\]

\[
= 2 \sum_{i=1}^{n} (n+1-i) y^{-i} - n(n+1)
\]

Now from note (2) of this appendix, we know that:

\[
\sum_{i=1}^{n} (n+1-i) y^{-i} = \frac{1-y^n - ny^n (1-y)}{y^n (1-y)^2} \quad \text{provided } y \neq 1.
\]

Substituting this result back into the above, we have:

\[
\frac{1 - 1}{y} S_n = \frac{2[1-y^n - ny^n (1-y)]}{y^n (1-y)^2} - n(n+1)
\]
Multiplying both sides of this equation by \( y \) and then dividing by \((1-y)\), we have:

\[
S_n = \frac{2[1-y^n-ny^n(1-y)]}{y^{n-1}(1-y)^3} - \frac{n(n+1)y}{(1-y)}
\]

By putting terms over a common denominator, we have:

\[
S_n = \frac{2[1-y^n-ny^n(1-y)] - n(n+1)y^n(1-y)^2}{y^{n-1}(1-y)^3}
\]

By substituting \( \hat{g} \) for \( y \) in the above equation, we have arrived at the desired conclusion.

On page 83, we state that:

\[
\sum_{i=0}^{n-1} \frac{(n-i)(n-i+1)\hat{g}^{-i}}{\hat{g}^{n-1}(1-\hat{g})^3} = \frac{2[1-\hat{g}^n-\hat{g}^n(1-\hat{g})]}{\hat{g}^{n-1}(1-\hat{g})^3} - \frac{n(n+1)\hat{g}^n(1-\hat{g})^2}{(1-\hat{g})^3}
\]

provided \( \hat{g} \neq 1 \). The proof of this equation is the same as that given above except we define \( y = \hat{g} \).
APPENDIX B

SIMULATION PROGRAM

INTEGER I, J, N, N1, M, XX(31), YY(31), XY(31, 31), ZZ
DIMENSION XX(31), YY(31), XY(31, 31), ZZ

REAL X, Y, Z

PARAMETER (EX = 1, N = 30, M = 150)

INTEGER N, M

COMMON X, Y, Z, EX, N, M

DIMENSION XX(31), YY(31), XY(31, 31), ZZ

DATA X, Y, Z / 0.0, 0.0, 0.0 /
BIBLIOGRAPHY

Books


Articles and Periodicals


**Unpublished Materials**

