DEVELOPMENT OF A PROCEDURE FOR THE ANALYSIS OF LOAD DISTRIBUTION, STRESSES AND TRANSMISSION ERROR OF STRAIGHT BEVEL GEARS

A Thesis

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ABSTRACT

The thesis aims to develop a procedure for the creating of geometry and analysis of straight bevel gears. The gear geometry is generated from standard AGMA equations that use Tregold’s approximation to create planar involute teeth. Contact lines are based on the rolling action of the pitch cones and a finite element model is used to compute the compliance of the straight bevel gear pair. The program analyzes the gear pair for the transmission error and the load distribution and may be used to evaluate the contact stresses. The effect of the microgeometry modifications on the model is studied to optimize the contact pattern and noise excitations. The program results are compared with the results from the finite element gear analysis software, CALYX.
To my parents
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TABLE OF CONTENTS

ABSTRACT ........................................................................................................................................ ii

ACKNOWLEDGMENTS ................................................................................................................. iv

LIST OF TABLES ......................................................................................................................... x

LIST OF FIGURES ..................................................................................................................... xi

CHAPTER 1 INTRODUCTION .......................................................................................... 1

  1.1 Introduction ..................................................................................................................... 1

  1.2 Objective ....................................................................................................................... 2

  1.3 Literature Review .......................................................................................................... 3

  1.4 Overview ....................................................................................................................... 4

CHAPTER 2 STRAIGHT BEVEL GEOMETRY ............................................................. 5

  2.1 Introduction ..................................................................................................................... 5

  2.2 Dimension Sheet ........................................................................................................... 5

  2.3 Geometry Design Factors ........................................................................................... 11

    2.3.1 Depth Factor .......................................................................................................... 12

    2.3.2 Clearance Factor .................................................................................................... 12
CHAPTER 2 TOOTH GEOMETRIC MODEL AND TOOTH SURFACE

2.3.3 Addendum Factor ................................................................................................ 12
2.3.4 Thickness Factor ............................................................................................... 13
2.4 Topland Thickness ............................................................................................... 14
2.5 Tooth Surface ....................................................................................................... 16
2.6 Root Geometry ...................................................................................................... 18
2.7 Contact Line ......................................................................................................... 22
2.8 Comparison of tooth surface coordinates ......................................................... 23
2.9 Summary .............................................................................................................. 27

CHAPTER 3 CALCULATION OF TOOTH COMPLIANCE MATRIX AND LOAD
DISTRIBUTION .............................................................................................................. 28

3.1 Introduction ........................................................................................................... 28
3.2 Compliance .......................................................................................................... 29
  3.2.1 Classical LDP Plate Method ........................................................................ 29
  3.2.2 Finite Element Method ................................................................................ 29
3.3 Mesh Generator ................................................................................................... 31
3.4 Stiffness Calculation ........................................................................................... 37
  3.4.1 Boundary Conditions ................................................................................... 41
  3.4.2 Compliance .................................................................................................. 41
3.5 Compliance along the contact line .................................................................... 43
3.6 Loaded Tooth contact Analysis .......................................................................... 44
3.7 Summary ................................................................................................................. 47

CHAPTER 4 STRAIGHT BEVEL LOAD DISTRIBUTION PROGRAM ..................... 48

4.1 Introduction ............................................................................................................. 48

4.2 Program Interface .................................................................................................... 48

   4.2.1 Input Module .................................................................................................... 49

   4.2.2 Output Module .................................................................................................. 52

      4.2.2.1 Transmission Error .................................................................................... 53

      4.2.2.2 Mesh Stiffness ........................................................................................... 54

      4.2.2.3 Tooth forces .............................................................................................. 55

      4.2.2.4 Zone of Contact ......................................................................................... 56

      4.2.2.5 Output Details ........................................................................................... 57

      4.2.2.6 Contact Stress, Load and Load Intensity Distribution .............................. 58

      4.2.2.7 Multitorque Plots ...................................................................................... 59

4.3 Summary ................................................................................................................. 60

CHAPTER 5 MICROGEOMETRY MODIFICATION ................................................... 61

5.1 Introduction ............................................................................................................. 61

5.2 Microgeometry Modification .................................................................................. 62

      5.2.1 Profile Modification ......................................................................................... 62

      5.2.2 Lead Modification ........................................................................................... 66
**LIST OF TABLES**

Table 2.1 Dedendum angle equations ................................................................................................. 9
Table 5.1 Different cases of tip relief modification ............................................................................ 65
Table 6.1 Gear pair used for the comparison between the Straight bevel LDP and CALYX .......................................................................................................................... 87
Table 6.2 Percentage deviation of the contact stress from CALYX ................................................ 94
LIST OF FIGURES

Figure 2.1  Input Templates (a) Factors Input (b) Dimension Sheet Input......................... 6
Figure 2.2 Bevel gear (a) Nomenclature (b) Transverse section (A-A)[16]...................... 7
Figure 2.3 Tooth Taper Types .......................................................................................... 11
Figure 2.4  Constant Clearance ....................................................................................... 13
Figure 2.5 Development of virtual spur gear along the transverse plane .................. 16
Figure 2.6 Coordinate system for tooth surface coordinates ........................................ 18
Figure 2.7 Elliptical root profile ...................................................................................... 19
Figure 2.8 Surface model of straight bevel gear ............................................................. 21
Figure 2.9 Contact line for different depthwise taper (a) Equal contact ratio standard
depth (b) AGMA standard depth (c) Uniform depth (c) Duplex depth or Tilted root line23
Figure 2.10 Surface comparison of spherical involute and planar involute (inch)........... 25
Figure 2.11 Surface comparison of octoid and planar involute (inch) ......................... 26
Figure 3.1 Surface of the element for compliance calculation ....................................... 30
Figure 3.2 Finite element mesh inputs ............................................................................. 31
Figure 3.3 20 noded isoparametric element .................................................................... 33
Figure 3.4 Layer of the rim section ................................................................................... 33
Figure 3.5 Dense and Sparse layer of tooth mesh, respectively ........................................ 34
Figure 3.6  Finite element sector with the tooth ............................................................... 35
Figure 3.7 Sectional view of the gear sector with the tooth.............................................. 36
Figure 3.8 Skyline storage approach [10] ..................................................................... 41
Figure 3.9 Direction cosines of the normal to the involute ............................................... 42
Figure 3.10 Point of intersection of the normal with the middle plane ......................... 44
Figure 3.11 Moment balance between the applied torque and forces acting on the gear .46
Figure 4.1 Geometry Input - Factors ............................................................................. 50
Figure 4.2 Geometry Input - Dimension sheet ................................................................. 50
Figure 4.3 Input module - Program control ................................................................. 51
Figure 4.4 Input module – Torque ............................................................................ 52
Figure 4.5 Transmission error plots .......................................................................... 53
Figure 4.6 Mesh stiffness plots .................................................................................. 54
Figure 4.7 Tooth forces ............................................................................................. 55
Figure 4.8 Zone of contact ........................................................................................ 56
Figure 4.9 Output details .......................................................................................... 57
Figure 4.10 Contact stress distribution .................................................................... 58
Figure 4.11 Load distribution .................................................................................... 59
Figure 4.12 Multitorque plots ................................................................................... 60
Figure 5.1 Ideal straight bevel gear contact pattern[26] ............................................. 61
Figure 5.2 Input frame for profile modification ......................................................... 63
Figure 5.3 Sample profile modifications (a) Tip Relief (b) Root Relief (c) Profile Crown (d) Pressure Slope ................................................................. 64
Figure 5.4 Modification plots for the tip relief cases in Table 5.1 ......................... 66
Figure 5.5 Input frame for lead modification ............................................................. 67
Figure 5.6 Sample lead modification (a) lead crown (b) lead slope (c) end modification 68
Figure 5.7 Input frame for bias modification ............................................................. 69
Figure 5.8 Sample bias modification ....................................................................... 69
Figure 5.9 Gear geometry for the microgeometry modification analysis .............. 70
Figure 5.10 Tooth shape and contact zone of sample gear pair (a) Standard depth (b) Uniform depth ........................................................................................................ 72
Figure 5.11 Unmodified –gear pair 1 ............................................................................ 73
Figure 5.12 First iteration - only profile modification, gear pair 1 ......................... 74
Figure 5.13 Second iteration - profile and lead modification, gear pair 1 ................. 75
Figure 5.14 Final iteration, gear pair 1 ..................................................................... 76
Figure 5.15 Lines of contact of gear pair 1 ............................................................... 77
Figure 5.16 Transmission error plot of uniform depth gear with similar amount of linear and parabolic modification ................................................................. 78
Figure 5.17 Unmodified - gear pair 2 ............................................................... 79
Figure 5.18 Profile Modification – gear pair 2 ............................................... 80
Figure 5.19 Profile and Lead Modification – gear pair 2 .............................. 81
Figure 6.1 Coordinate system and reference cone in CALYX [33] .................. 85
Figure 6.2 Sample CALYX input file ................................................................ 85
Figure 6.3 Gear mesh model in CALYX .......................................................... 86
Figure 6.4 Load across the face width for one mesh position for sample gear pair 2 at 2500 lbf-in ............................................................................................................. 89
Figure 6.5 Harris plot for transmission error (a) Sample gear pair 1 (b) Sample gear pair 2 (c) Sample gear pair 3 ....................................................................... 90
Figure 6.6 Load distribution at different mesh position at a torque of 1 lbf-in for sample gear pair 3 ............................................................. 91
Figure 6.7 Sample gear pair 1 - Transmission error plots .............................. 95
Figure 6.8 Sample gear pair 1 - Tooth force plots .......................................... 96
Figure 6.9 Sample gear pair 1 - Contact stress with edge loading .................. 97
Figure 6.10 Sample gear pair 1 - Contact stress without edge loading ................ 98
Figure 6.11 Sample gear pair 2- Transmission error plots .............................. 99
Figure 6.12 Sample gear pair 2 - Tooth force plots ...................................... 100
Figure 6.13 Sample gear pair 2 - Contact stress with edge loading ................. 101
Figure 6.14 Sample gear pair 2 - Contact Stress without edge loading ............. 102
Figure 6.15 Sample gear pair 3 - Transmission error plots ............................. 103
Figure 6.16 Sample gear pair 3 - Tooth force plots ........................................ 104
Figure 6.17 Sample gear pair 3 - Contact stress with edge loading ................. 105
Figure 6.18 Sample gear pair 3 - Contact stress without edge loading ............. 106
CHAPTER 1 INTRODUCTION

1.1 Introduction

The straight bevel gear that is widely used in automotive differential is the simplest form of bevel gear, transmits torque through intersecting shafts. The design of a straight bevel gear requires effective tools to create the overall geometry and microgeometry and then predict load distribution and contact stress of the gear pair. The straight bevel gears have tooth thickness and tooth height that increase across the face width of the gears resulting in non-uniform load distribution a factor that affects the stresses in the gear. In addition, for noise and vibration control, one wishes to control the transmission error of the gear pair.

The current programs developed at Gear and Power Transmission Research Laboratory performs the load distribution and transmission error analysis of spur, helical and hypoid gears. The lack of an application dedicated to the design and analysis of straight bevel gears that are common in automotive differentials has motivated this current study.
1.2 Objective

The work reported in this thesis develops a design tool for straight bevel gears. The main goals are to generate the dimensions of the gear and to perform torque analysis to estimate the load distribution and contact stress of the gear pair. The procedure will in many ways mimic the performance of the Gear lab’s LDP program for spur and helical gears. The gear geometry is generated from standard AGMA equations and Tregold’s approximation of designing bevel gears using the equivalent virtual spur gears. The theoretical surface geometry with an involute flank and elliptical root is generated in the program and is appropriate for automotive differentials which are widely manufactured by forging process that has the flexibility to manufacture the desired tooth surfaces independent of the cutter geometry and cutting methods. The study develops a contact routine to generate contact points across the face width at different mesh position and creates a finite element model to compute the compliance of the straight bevel gear pair. The program analyses the gear pair to solve the load distribution and the transmission error of the gear pair and a post processing procedure evaluates contact stresses. The effect of the microgeometry modifications on the load distribution and the transmission error is also developed in the thesis. The program is developed with the focus of maintaining the computational speed and simplicity of the current Windows LDP program for spur and helical gears, a program that has features for analyzing multiple design geometries at the preliminary stage of the gear design procedure.
1.3 Literature Review

The work done on bevel gears involved straight bevel, spiral bevel and hypoid gears. Tsai and Chin[1] proposed a model to generate the geometry of the straight and spiral bevel gears using the spherical involute which is created from the envelope of the tangents to the base cone of the gear. The straight bevel gears manufactured by the two tool generation process are defined by the octoid tooth form using the crown rack and the tool kinematics. The formulation of this surface was presented in the work of Hedge, et al [2] and Chang and Tsay [3]. Al-Daccak, et al [4] proposed an approach for exact spherical involute on the spherical surface and Figliolini and Angeles [5] formulated an algorithm for generation of exact spherical involute and octoid bevel gears. The above research was mainly focused on developing the geometry for further analysis of the gears. Li, et al[6] proposed a new method to perform the static analysis of straight and helical bevel gears by finite element methods. Elkholy et al [7] proposed a model to predict the load distribution and tooth stress for straight bevel gears using Tregold’s approximation by discretizing the gear into finite number of spur gears and the work was extended to include the manufacturing error in the model for load distribution[8]. Sentoku and Itou[9] performed the study on transmission error of straight bevel gears of Gleason type including the effect of shafts and manufacturing errors.

The current work is aimed at developing a full finite element model for the analysis of straight bevel gears for the prediction of load distribution, transmission error and stress and also to include the effect of surface deviations in the load distribution. The finite element model and the compliance calculation is based on the work for Talbot[10], Prabhu [11] for thin rim spur gears and Wilcox, et al[12] for bevel and hypoid gear for
calculating the stress. Seireg and Conry [13] work on elastic bodies in contact and Windows LDP[14] is the basis for the solver for the load distribution of gears.

1.4 Overview

The introduction of the thesis and the objective of the work are stated in this chapter. Chapter 2 explains the generation of the geometry of straight bevel gears from the AGMA equations, the surface generation of the gears and the contact line development used for the load distribution analysis. Chapter 3 explains the finite element compliance matrix formation and the load distribution solver. Chapter 4 gives the overview of the program developed in the thesis and the visual graphic interface is explained. Chapter 5 explains the microgeometry modification applied to the tooth and the microgeometry effects on the contact pattern and noise characteristics of the gear. Chapter 6 shows the results and the comparison with the CALYX program. The conclusions and the future recommendations for the program are stated in Chapter 7. The thesis plots all the results using the English system of units.
CHAPTER 2 STRAIGHT BEVEL GEOMETRY

2.1 Introduction

This chapter discusses the generation of the straight bevel gear geometry used in this thesis. The dimension sheet of the straight bevel gear at the three different sections of the face width defining the tooth proportions is generated in the program. The dimensions of the gear define the parameters for the tooth surface and root profile of the gear that is generated for each gear pair along with the dimension sheet. The chapter explains the development of the geometry, surface coordinates and contact lines for straight bevel gears.

2.2 Dimension Sheet

The straight bevel gear tooth geometry is calculated based on AGMA standard “ANSI/AGMA 2005 – D03 Design Manual for Bevel gear” [15] which provides equations for the layout of the straight bevel, spiral bevel and hypoid gears. The gear parameters are chosen by the designer based on the application or existing designs. The program has two types of input templates as shown in Figure 2.1. The first approach requires the specification of basic geometry parameters, tooth taper type and a set of factors defining the tooth proportions. This methodology is the start for the new gear design development where the designer could use the AGMA suggested factors for the
first iteration of the design. This method requires minimum inputs for the straight bevel gear design. The procedure for the choice of inputs is provided in the AGMA standards. The factors may be modified based on previous experience or different strength balance in the further iterations. The factors and their effects are explained later in this chapter.

The second approach incorporates more involved geometry details like outside diameter, tooth thickness, working depth and whole depth. This method can be a start for designs with an existing dimension sheet. The program calculates input geometry factors used in the previous method from the user dimension inputs, thereby allowing one to switch between the two input modules. This approach accurately recreates the existing dimension sheets.

![Figure 2.1 Input Templates (a) Factors Input (b) Dimension Sheet Input](image-url)

**Figure 2.1** Input Templates (a) Factors Input (b) Dimension Sheet Input
The equations in the standard provide the dimensions at the mean and outer section of the gear. The tooth thickness and chordal addendum are calculated in the transverse plane of the gear. The layout of the bevel gears that defines the terminology is shown in Figure 2.2.

Figure 2.2 Bevel gear (a) Nomenclature (b) Transverse section (A-A)[16]
The bevel gear teeth used here are generated from the base cone that is similar to the base cylinder for parallel axis gears. The tooth cross section of the gears increases linearly as one progresses from toe to heel while the tooth proportion change is determined by the type of tooth taper. The different depthwise tapers are explained below and Table 2.1 lists the dedendum angle formulae for each case.

**Standard Depth:**

In this case, the tooth depth changes proportional to the cone distance of the gear. The root and pitch cone of the gear intersect at the pitch apex while the face line meets the axis below the pitch apex. This is not a typical standard depth taper as the three cones (pitch cone, root cone, face cone) do not intersect at the same point as shown in the Figure 2.3a. The small tilt in the face cone results in a higher contact ratio at the heel compared to the toe of the gear. Most straight bevel gears are designed using a depthwise taper similar to the standard taper. However, many bevel gears in axle application have heavy chamfers that adjust the zone of contact.

**Uniform Depth:**

In this case, the tooth depth of the gear is maintained constant across the face width of the gear, giving a narrow topland at the toe of the gear. The root cone and face cone are parallel to the pitch cone (Figure 2.3b) and the dedendum angle of both the pinion and the gear are zero. The uniform depth causes a longer tooth at the toe increasing contact ratio of the toe compared to a standard depth taper. Uniform depth have seldom been used for straight bevel gears, but are used extensively when face
hobbing spiral bevel and hypoid gears. In Chapter 5 of the thesis a constant depth tooth will be evaluated and it will be analyzed and compared to a standard depth tooth.

**Duplex Depth and Tilted Root Line:**

These tapers involve the tilt of the root line about the mean point resulting in a shorter toe and a longer heel section of the gear (Fig 2.3c). The tilt in the root line changes the space width of the gear and pinion and the face angle of the mate which causes larger topland thickness at the toe and vice versa at the heel section of the gear.

### Table 2.1 Dedendum angle equations

<table>
<thead>
<tr>
<th>Depthwise Taper</th>
<th>Dedendum Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard</strong></td>
<td>$\Sigma \delta_s = \arctan \left( \frac{b_p}{A_m} \right) + \arctan \left( \frac{b_G}{A_m} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\delta_p = \arctan \left( \frac{b_p}{A_m} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\delta_G = \Sigma \delta_s - \delta_p$</td>
</tr>
<tr>
<td><strong>Uniform Depth</strong></td>
<td>$\Sigma \delta_U = 0$ ; $\delta_p = \delta_G = 0$</td>
</tr>
<tr>
<td><strong>Duplex</strong></td>
<td>$\Sigma \delta_D = \left( \frac{90.0}{P_d A_o \tan \phi \cos \psi} \right) \left( 1 - \frac{A_m \sin \psi}{r_c} \right)$</td>
</tr>
<tr>
<td></td>
<td>$\delta_p = \Sigma \delta_D \frac{a_G}{h}$</td>
</tr>
<tr>
<td></td>
<td>$\delta_G = \Sigma \delta_D - \delta_p$</td>
</tr>
<tr>
<td><strong>Tilted Root Line</strong></td>
<td>$\Sigma \delta_T = \Sigma \delta_D$ or 1.3$\Sigma \delta_s$, whichever is smaller</td>
</tr>
<tr>
<td></td>
<td>$\delta_p = \Sigma \delta_T \frac{a_G}{h}$</td>
</tr>
<tr>
<td></td>
<td>$\delta_G = \Sigma \delta_T - \delta_p$</td>
</tr>
</tbody>
</table>
where,

\( \Sigma \delta \) - Sum of dedendum angle, suffix S-standard depth, U-uniform depth, D-duplex depth, T- tilted root line

\( \delta_p, \delta_g \) - Pinion, gear dedendum angle

\( \phi \) - Pressure angle

\( \psi \) - Spiral angle

\( A_m \) - Mean cone distance

\( A_o \) - Mean cone distance

\( P_d \) - Diametral pitch

\( a_p, a_g \) - Pinion, gear addendum

\( b_p, b_g \) - Pinion, gear dedendum

\( h \) - Working depth

\( r_c \) - Cutter radius

**Custom Depth:**

This is a generic version of the tooth tapers, where the tooth depth is defined by the dedendum angle. Any of the first three schemes can be specified using this taper but more importantly, one can create tooth tapers not achieved by other three. The straight bevel gear with a constant clearance defines the addendum angle equal to the dedendum angle of the mate which calculates the tooth dimensions of the gear. The program then accurately recreates the dimension sheet of a non standard tooth taper.
2.3 Geometry Design Factors

The taper shows the variation of the tooth cross section across the face width. The four factors shown in Fig 2.1 from the AGMA standard [15] define the tooth proportion at the mean section of the gear. The factors can either be the defaults recommended in the AGMA standard based on calculations from gear input parameters or user defined to satisfy other design requirements.
2.3.1 Depth Factor

The depth factor is directly proportional to the working depth of the gear at the mean section. The mean working depth is normalized to the module/diametral pitch. The suggested factor for the straight bevel gear is 2 (similar to the standard parallel axis gears).

\[ Working \ Depth = Depth \ factor \times Module \ or \ \frac{Depth \ factor}{Diametral \ Pitch} \]  

(2.1)

2.3.2 Clearance Factor

The absolute clearance of the gear is constant for the entire length of the face width. This factor defines the required clearance as a proportion of mean working depth. The suggested clearance factor for a straight bevel gear is 0.125 which gives clearance 0.25 times the mean module for a standard depth factor. The constant clearance shows the root line of the gear and the face of the mate are parallel to each other. This ensures the addendum angle of the gear is equal to the dedendum angle of the pinion as shown in Figure 2.4

2.3.3 Addendum Factor

This factor apportions the mean working depth of the gear into addendum of the gear and pinion at the mean section. Bevel gears are typically designed with longer addendum on the pinion to avoid undercut. The factor is equivalent to profile rack shift of spur gears. This value typically varies from zero to unity where the extremes will give full recess or full approach action depending on which element is driving. The case of equal addendums on both members is achieved with a value of 0.5.
2.3.4 Thickness Factor

The thickness factor \( k_4 \) defines the tooth normal circular thickness at the mean section. The AGMA suggested factor calculated from the equation (2.2) is based on equal stress on both of the members or a factor defined for a different strength balance between the gears can be used.

\[
k_4 = -0.088 + 0.092m_G - 0.004m_G^2 + 0.0016(n - 30)(m_G - 1)
\]  

(2.2)

where,

\( n \) - Number of pinion teeth

\( m_G \) - Equivalent 90 ratio

The AGMA standard equations [15, 17] define bevel gear layout dimensions and the tooth proportions at the mean section and certain variables at the outer section of the
gear. The dimensions are completed at the toe and heel section by projecting linearly from the cone apex.

2.4 Topland Thickness

The topland thickness is the tooth thickness defined on the transverse plane at the face cone of the bevel gear. The AGMA standard “Calculation of Bevel gear Top Land and Guidance on Cutter Edge Radius” [17] presents the guidelines for the topland thickness calculation. In this thesis and in the AGMA calculation the topland thickness is calculated by the tooth thickness formulation [18] of involute spur gears, using virtual spur gears created by Tregold’s approximation.

\[
t_R = R_R \left[ \frac{t_s}{R_s} + 2(\text{inv} \Phi_s - \text{inv} \Phi_R) \right]
\]  

(2.3)

where,

- \(t_R, t_s\) - Tooth thickness
- \(R_R, R_s\) - Radius
- \(\Phi_R, \Phi_s\) - Pressure angle

The virtual involute spur is defined for each layer of the face width along the transverse plane using the pitch diameter, outside diameter, root diameter and pressure angle. The number of teeth on the virtual involute[19] is defined by equation (2.4). The axis of rotation of the virtual gears is shown by line S in Figure 2.5 and line B represents the rotational axis of the actual straight bevel gear. The hatched section in the figure is an enlarged representation of the virtual spur gear tooth whose face width is assumed to be infinitesimal.
\[ n_v = \frac{n}{\cos \gamma} \]
\[ N_v = \frac{N}{\cos \Gamma} \]  (2.4)

where,

\( n_v, N_v \) - Number of pinion, gear teeth on virtual spur gear

\( n, N \) - Number of pinion, gear teeth

\( \gamma, \Gamma \) - Pitch angle of pinion, gear

The mean normal tooth thickness at the pitch line is calculated using equation (2.5) and the tooth thickness for toe and heel are calculated using the similar formula with the corresponding section dimension. The topland tooth thickness is calculated from this using equation (2.3).

\[ t_n = p_m \cos \psi - T_n \]
\[ T_n = (0.5 p_m \cos \psi) - (a_p - a_g) \tan \phi - \frac{k_4 \cos \psi}{P_{dm}} \]  (2.5)

where,

\( P_{dm} \) - Mean diametral pitch

\( p_m \) - Mean circular pitch

\( t_n, T_n \) - Mean normal tooth thickness of pinion, gear
2.5 Tooth Surface

In this thesis, the bevel gear tooth surface is a mathematically generated surface that is independent of the cutter geometry or generating action of cutter. This kind of approach is useful for forged gears that are net shaped with a die. The forging process doesn’t require a cutting tool with relative motion between tool and blank. The dies are essentially the inverse of the final part developed from the solid model of the gear. Thus,
one develops the 3D model of the gear along with the dimension sheet. This encourages the development of a mathematical surface as opposed to a cutter generated surface.

The tooth surface here is defined using planar involutes. The involute tooth surface is simple in geometry and in ease to develop surface coordinates of the teeth. Using Tregold’s approximation of designing the bevel gear using a virtual spur gear at the back cone surface, the tooth surface is developed at each section of the gear along the transverse plane. The geometry parameters of each section like base radius \( r_b \), half tooth angle \( \theta \) and roll angle \( \varepsilon \) define the tooth surface. The flank surface is generated by unwrapping action from the base circle of the \( X'Y' \) plane.

\[
\begin{align*}
x' &= (r_b \cos \varepsilon + \varepsilon \sin \varepsilon) \cos \theta - (r_b \sin \varepsilon - \varepsilon \cos \varepsilon) \sin \theta \\
y' &= (r_b \cos \varepsilon + \varepsilon \sin \varepsilon) \sin \theta + (r_b \sin \varepsilon - \varepsilon \cos \varepsilon) \cos \theta
\end{align*}
\] (2.6)

The tooth flank coordinates generated from the start of the active profile (SAP) to the outside diameter create one half of the tooth. The other half is created by mirroring the tooth coordinates about the centerline of the tooth (\( X' \) axis); the \( X'Y' \) plane is defined for each section with the origin at the intersection of transverse plane and the gear axis \( Z \). The coordinates from \( X'Y' \) are transformed into the global \( XYZ \) system (Figure 2.6) that has an origin at the pitch apex of the gear. The \( Z \text{Corr} \) is the distance from the pitch apex of the gear to the origin of \( X'Y' \).

\[
\begin{align*}
x &= x' \cos \gamma \\
y &= y' \\
z &= -x' \sin \gamma + Z \text{Corr}
\end{align*}
\] (2.7)
2.6 Root Geometry

The development of a root is also an analytical surface independent of the cutter corner radius. The root of the gear is developed by fitting an elliptical curve between the tooth flank and the root circle for each layer of the face width. The ellipse passes through two points in the plane and tangent to the tooth flank at the SAP and at the root circle defined by half tooth angle of the gear. An ellipse of the form (2.8) depending on four parameters \((h,k,A,B)\) is used.

\[
\frac{(x-h)^2}{A} + \frac{(y-k)^2}{B} = 1
\]  

(2.8)
The root profile is generated in the transverse plane of the gear (Fig 2.7). The coordinates of point A \((x_1,y_1)\) at the SAP is determined using an involute equation and point B \((x_2,y_2)\) on the root circle is calculated from root radius and the half tooth angle. The involute profile on the gear are extended below the SAP to a point called true involute form (TIF) either to avoid the tip interference of the gear due to tooth deflection or misalignment or to fit a specific root shape (circular root). The point A should be an independent point located by the radius which, when defined below the base radius of the involute, gives a straight sided tooth below the base circle. The program currently uses the SAP to define the point A and the slope of the tangent to involute \((m_1)\) at A and the slope of the tangent to the root circle\((m_2)\) at point B are calculated. Using the four
constraints the parameters of the ellipse are solved by equation (2.9) calculated using Maple [20].

\[
A = \frac{(m_1 + m_2)(x_1 - x_2)(-y_1 - x_2m_2 + x_1m_2 + y_2)(x_1m_1 + y_2 - y_1 - x_2m_1)}{((-x_1m_1 + x_1m_2 - 2y_1 - x_2m_2 + 2y_2 + x_1m_1)(2m_1m_2x_1 - 2m_1x_2m_2 + y_2y_m1 - m_2y_1 + m_2y_2 - y_1m_2))}
\]

\[
B = \frac{(m_1 + m_2)(-y_2 + y_1)(-y_1 - x_2m_2 + x_1m_2 + y_2)(x_1m_1 + y_2 - y_1 - x_2m_1)}{((-x_1m_1 + x_1m_2 - 2y_1 - x_2m_2 + 2y_2 + x_1m_1)(2m_1m_2x_1 - 2m_1x_2m_2 + y_2y_m1 - m_2y_1 + m_2y_2 - y_1m_2))}
\]

\[
h = \frac{(m_1m_2x_1^2 + x_m2y_2 - y_1x_m2 - m_1y_1x_2 - m_2x_2m_2 + x_1y_2m_1)}{(2m_1m_2x_1 - 2m_1x_2m_2 + y_2y_m1 - m_2y_1 + m_2y_2 - y_1m_2)}
\]

\[
k = \frac{(m_1x_1y_1 - m_1y_ax_2 - y_1^2 + x_1m_2y_2 - x_2m_2y_2 + y_2^2)}{(-x_1m_1 + x_1m_2 - 2y_1 - x_2m_2 + 2y_2 + x_1m_1)}
\]

The coordinates of the root are calculated using the parametric form of the ellipse (Eq. 2.10) where the root angle (\(\alpha\)) varies from \(\text{SRootAng}\) to \(\text{ERootAng}\) from equation 2.11.

\[
x' = \sqrt{A} \cos \alpha + h
\]
\[
y' = \sqrt{B} \sin \alpha + k
\]

\[
\text{SRootAng} = a \cos \left( \frac{x2 - h}{\sqrt{A}} \right)
\]

\[
\text{ERootAng} = a \cos \left( \frac{x1 - h}{\sqrt{A}} \right)
\]
The root profile on the other side of the gear is generated by mirroring the shape about the tooth center line. The coordinates are finally transformed into the global XYZ coordinate system. This completes the generation of the single tooth profile, which can be rotated about the Z axis to create the surface model of the complete gear. The coordinates of the complete gear are used in MATLAB [21] to create the surface model (Figure 2.8).

Figure 2.8 Surface model of straight bevel gear
2.7 Contact Line

The contact line is defined by a set of points that occur on both the pinion and the gear surface at a given mesh position. The motion of the straight bevel gear can be represented by pure rolling action of the pitch cone surfaces. As the gear rolls through the mesh, the conical surfaces of the tooth flank come in contact. For an equal contact ratio, standard depth straight bevel gear the pitch cone, face cone and root cone meet at the pitch apex of the gear. The contact lines of these gears are tangent to the flank surface with an extension of the lines always intersecting the pitch apex as shown in Figure 2.9a.

The contact zone of the gear will increase or decrease for a change in depthwise taper. The contact lines at the tip are truncated across the face width, leading to different contact ratios at the toe and heel. The contact line development starts from the end of higher contact ratio. In the cases of standard depth, duplex depth and tilted root line gears with inner dedendum less than the outer dedendum, the contact line development starts at the heel section which has higher contact ratio and vice versa for the case of uniform depth gears. In the program, the number of mesh positions to be analyzed is defined by the number of POSCONS using a recommended minimum is 21 positions across one base pitch (one mesh cycle). The radius and the position along the face width locate the contact points. The radius of curvature at each contact point is calculated in this routine for contact stress estimation. The contact line patterns for different depthwise tapers are shown below.
Figure 2.9 Contact line for different depthwise taper (a) Equal contact ratio standard depth (b) AGMA standard depth (c) Uniform depth (d) Duplex depth or Tilted root line

2.8 Comparison of tooth surface coordinates

The mathematical model for the tooth surface of straight bevel gears based on an involute at each section plane is developed in this thesis. Other common bevel gear geometries are the spherical involute, a mathematical surface generated based on gearing kinematics and involute geometry and the octoid tooth form defined using a gear generation mechanism and cutting tool geometry. Both are formulated in the literature and are compared here to the virtual involute gear used in this thesis. The development of spherical involute by the envelope of the tangent planes to the base cone was proposed in [1]. But the octoid tooth form generated by machining is different from the spherical
involute; the surface geometry of the gears generated by a two tool generation process using two straight sided tools has been formulated by Hedge, et al [2]. Furthermore, the mathematical model of an exact spherical involute from the fundamental involute geometry that lies on the spherical surface was developed by Al-Daccak, et al [4]. The recent work on modeling the spherical involute and octoid bevel gear [5] is used in this study to compare it with the surface developed in this thesis.

The formulation developed in [5] is implemented in MATLAB. The spherical involute, based on the pitch angle, pressure angle, cone distance, face angle and rotating plane angle, was unwrapped from the base cone of the straight bevel gear while the octoid form required an additional parameter of flat flank angle of the octoid crown rack. The expression of the spherical involute and the octoid surface are utilized to generate the respective tooth surface and the planar involute tooth surface generated in the thesis was recreated to align the coordinate systems of this thesis with that of the paper [5]. The gear for the comparative study has a 14 teeth pinion with module 4.535, pitch angle 19.7°, face angle 24.59° and pressure angle 20° and the flat flank angle of 25° for the octoid tooth form used. The surface coordinates generated for all three forms were exported into Unigraphics format to generate the surfaces and measure the 3D surface deviation between the surfaces using the deviation gage analysis tool. The surface deviations of the planar involute from spherical involute and octoid were performed and the results are shown in inch units. Figure 2.10 shows the deviation of the planar involute from the spherical involute surface, the surface deviation varies from 1E-5 to 4.05E-4 in. The surfaces have lesser deviation closer to the base cone and the deviation increases as one moves out to the tip of the gear.
Figure 2.10 Surface comparison of spherical involute and planar involute (inch)
Figure 2.11 Surface comparison of octoid and planar involute (inch)
Figure 2.11 shows the difference between the octoid and the planar involute where the deviation between the surfaces are higher than observed with the spherical involute with range from 1.34E-3 in to 1.83E-2 in. The increase in deviation for the octoid is due to the straight sides of the crown rack that is used for the tooth generation and increases with the change in flank angle of the crown rack. The effect of these surfaces could be considered in the program developed in this thesis as tooth deviations from the planar involutes.

2.9 Summary

The program generates the straight bevel gear geometry dimension sheet based on the user inputs along the surface coordinates required for the finite element mesh generator followed by the contact line development. This chapter shows a comparison of the tooth surface with the common bevel gear tooth surface. The planar involute used in this thesis has coordinates quite similar to those of the spherical involute, but its coordinates differ a lot from the octoid surface. The next chapter will use the gear data generated here to compute the compliance of the gear.
CHAPTER 3 CALCULATION OF TOOTH COMPLIANCE MATRIX  
AND LOAD DISTRIBUTION

3.1 Introduction

The tooth compliance matrix is the deflection of the tooth centerline due to a unit load applied at a specific point on the tooth. The load distribution solver requires the dimensions of the straight bevel gear geometry and gear tooth compliance. The dimension sheet and the 3D surface model of the straight bevel gear are developed as discussed in the previous chapter and the compliance of the straight bevel gear is calculated from the finite element approach. The first step for the compliance calculation is to create the finite element mesh of the gear using a mesh generator that creates both the nodal coordinates and the element connectivity of the mesh. The mesh creation is followed by a stiffness formulation to calculate the displacements at the nodes and saves the list of nodes for their displacement solution. The compliance matrix is formed by interpolating the deflection due to the unit load at the contact point. This chapter explains the procedure for mesh generation and formulation of the compliance matrix developed by Prabhu [11] and Talbot [10] that is then used to develop the compatibility equations that are solved with the equilibrium equations to compute the load distribution.
3.2 Compliance

The total mesh compliance $[C]$ of the gear tooth pair consists of tooth bending and shear $[C_{B+S}]$ and base motion $[C_{BR}]$ of each tooth obtained from finite element analysis and the Hertzian compliance $[C_{HZ}]$ computed separately with the Weber equation [22].

$$[C] = [C_{B+S}]^p + [C_{BR}]^p + [C_{B+S}]^g + [C_{BR}]^g + [C_{HZ}]$$  \hspace{1cm} (3.1)

3.2.1 Classical LDP Plate Method

An initial study of the compliance calculation of the straight bevel gears with linearly increasing tooth thickness and tooth height across the face width using the tapered plate approach of Windows LDP on the virtual spur gear at three sections (toe, mean and heel) was carried out. The results for different sample gear pairs showed an absence of simple correlation with the finite element calculation leading to the use of the more involved and accurate method of finite elements for compliance calculation.

3.2.2 Finite Element Method

The finite element approach discretizes the complex domain into nodes and elements whose individual stiffness on assembly generates the global stiffness of the domain. The compliance of the tooth model is generated by computing the deflection at a point due to a unit load. The unit load is applied on the discrete points of the contact lines and the deflection is computed at a point where the normal to the tooth surface (F-surface) at the contact point intersects the tooth middle surface (U-surface). The displacements at the mid-plane of the loaded teeth are considered to avoid singularities at the point load. The surfaces for the compliance calculation as shown in Figure 3.1 are opposite sides of the same set of finite elements.
The finite element approach for the compliance calculation for spur and helical gears with the uniform tooth cross-section over the face width was developed in the thesis by Prabhu [11] and programmed for the Thin rim Windows LDP for spur and helical gears by Talbot [10]. The straight bevel gear developed using section wise involutes uses the same methodology, where the compliance calculations used in [10, 11] are applied for the virtual spur gear at any section. The finite element compliance calculation involves the following three main steps:

1. Mesh generator

2. Stiffness calculation

3. Compliance along the contact line.
3.3 Mesh Generator

The mesh generator creates a 3D 20 noded isoparametric finite element mesh of the straight bevel gear tooth center line. The mesh is generated with a user controlled mesh generator whose inputs (Figure 3.2) are number of elements in the face width direction, number of elements in the rim and number of elements along the tooth height. The geometric parameters like outside diameter, root diameter, number of teeth and tooth thickness are taken from the geometry calculations. The number of elements in the angular direction is fixed at 4 for the rim and 2 for the tooth; this reduces the complexity of locating the middle plane of the tooth.

Figure 3.2 Finite element mesh inputs
The gear is generated in sectors, a sector being a segment that circumferentially repeats itself to create the total gear. Each sector subtends an angle of \( \frac{2\pi}{N} \) radians. The gear rim is generated for the entire 360° while the number of sectors with teeth, \( m = 2*\text{ceiling}[\text{Max Contact Ratio}] - 1 \) is based on the maximum contact ratio. This ensures the inclusion of the compliance of adjacent teeth due to load sharing of the gears. For a gear with a theoretical contact ratio of 1.65, there is one additional tooth on either side of the loaded teeth. The mesh generation is a two step process. The first step generates the rim sector of the gear with the elliptical root shape and next the tooth sector is created with the involute profile.

The rim sector is defined from the inner rim radius to the SAP of the tooth with the rim thickness being 1.5 times the tooth height. The root profile is modeled as a part of the rim. The nodes are generated in sections across the face width with the node layers switching between the dense and sparse layers of 20 noded isoparametric elements. The 20 noded isoparametric element model (Figure 3.3) has a layer of 8 nodes and a layer of 4 nodes adjacent to each other in any direction, these layers are called the dense and sparse layer of an element, respectively. The node numbers start from the lower most radial section (Figure 3.4) and then increase in the angular direction of the sector after which the nodes at the next radial level are generated. The nodes of the next layer along the face width are generated in a similar manner. The node numbering starts from the toe end of the gear. The nodal coordinates, element connectivity of the rim and the nodes connected to the shaft are saved for stiffness computation.
Figure 3.3 20 noded isoparametric element

Figure 3.4 Layer of the rim section
The tooth sector mesh is generated next with two elements in the angular direction. The tooth is generated with the involute profile resulting in nearly conjugate surfaces. The mesh of the tooth is also performed in the transverse section. The mesh generator starts from the toe end of the gear with the node number starting from the SAP radius on the tooth flank, increases in the angular direction and then into the radial direction. The nodes in the face width direction are switched between the dense and sparse layer as shown in Figure 3.5.

![Figure 3.5 Dense and Sparse layer of tooth mesh, respectively](image)

In the sectors with the tooth (Figure 3.6), the tooth and the rim sector are combined. The node number for the sector starts from the shaft nodes increasing first in the angular direction and then in the radial direction. The node numbering scheme of the sector with the tooth is shown in Figure 3.7.
Figure 3.6 Finite element sector with the tooth
Figure 3.7 Sectional view of the gear sector with the tooth
3.4 Stiffness Calculation

The stiffness of the gear used for the compliance calculation is computed using the routines developed by Talbot [10] for thin rim Windows LDP, uses nodal coordinates, elemental connectivity and the shaft nodes that are generated from the mesh generator and then generates the elemental stiffness matrix and assembles it into a global stiffness matrix using a skyline approach. The element formulation matrices and the global stiffness matrix storage approach are next explained.

The element used for the finite element mesh is a 20 noded isoparametric quadratic element that represents the curved boundary more accurately than a lower order element. In isoparametric elements both the geometry and displacement are given by the same shape function. The displacement or location at any point of the element is given by

\[ \{x\} = \sum_{i=1}^{20} N_i \{x_i\} \]

\[ \{u\} = \sum_{i=1}^{20} N_i \{u_i\} \]

(3.2)

The shape functions \( N_i \) for the 20 noded isoparametric elements are

\[ N_i = \frac{1}{8} (1 + \xi_0)(1 + \eta_0)(1 + \rho_0)(\xi_0 + \eta_0 + \rho_0 - 2) \quad i = 1-8 \]

\[ N_i = \frac{1}{4} (1 - \xi^2)(1 + \eta_0)(1 + \rho_0) \quad i = 9,11,13,15 \]

\[ N_i = \frac{1}{4} (1 + \xi_0)(1 - \eta^2)(1 + \rho_0) \quad i = 10,12,14,16 \]  
\[ N_i = \frac{1}{4} (1 + \xi_0)(1 + \eta_0)(1 - \rho^2) \quad i = 17,18,19,20 \]  
\[ \xi_0 = \xi_0^i; \eta_0 = \eta_0^i; \rho_0 = \rho_0^i \]
The element stiffness matrix is given by

\[ [K_e] = \int_V [B]^T [D] [B] \, dV \]  

(3.4)

here \([B]\) is strain displacement matrix and \([D]\) is the material property matrix.

\[
[B] = \begin{bmatrix}
N_{1,x} & 0 & 0 & \ldots & N_{20,x} & 0 & 0 \\
0 & N_{1,y} & 0 & \ldots & 0 & N_{20,y} & 0 \\
0 & 0 & N_{1,z} & \ldots & 0 & 0 & N_{20,z} \\
N_{1,y} & N_{1,x} & 0 & \ldots & N_{20,y} & N_{20,x} & 0 \\
0 & N_{1,z} & N_{1,y} & \ldots & 0 & N_{20,z} & N_{20,y} \\
N_{1,z} & 0 & N_{1,x} & \ldots & N_{20,z} & 0 & N_{20,x}
\end{bmatrix}
\]

(3.5)

where \(N_{i,x} = \frac{\delta N_i}{\delta x}\)

\[
[D] = \begin{bmatrix}
d_1 & d_2 & d_2 & 0 & 0 & 0 \\
d_2 & d_1 & d_2 & 0 & 0 & 0 \\
d_3 & d_2 & d_1 & 0 & 0 & 0 \\
0 & 0 & 0 & d_3 & 0 & 0 \\
0 & 0 & 0 & 0 & d_3 & 0 \\
0 & 0 & 0 & 0 & 0 & d_3
\end{bmatrix}
\]

where

\[
d_1 = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}
\]

\[
d_2 = \frac{E\nu}{(1+\nu)(1-2\nu)}
\]

\[
d_3 = \frac{E}{2(1+\nu)}
\]

(3.6)

\(E\) is elastic modulus and \(\nu\) is Poisson’s ratio
Using the chain rule, the partial derivative of the shape function with respect to the Cartesian coordinate is given by equation (3.7) and the Jacobian matrix \([J]\) by equation (3.8).

\[
\begin{bmatrix}
N_{i,\xi} \\
N_{i,\eta} \\
N_{i,\rho}
\end{bmatrix} = [J]^{-1}
\begin{bmatrix}
N_{i,\xi} \\
N_{i,\eta} \\
N_{i,\rho}
\end{bmatrix} \tag{3.7}
\]

where,

\[
[J] =
\begin{bmatrix}
\frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\
\frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\
\frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} & \frac{\partial z}{\partial \rho}
\end{bmatrix}
\begin{bmatrix}
N_{i,\xi} & \ldots & N_{20,\xi} \\
N_{i,\eta} & \ldots & N_{20,\eta} \\
N_{i,\rho} & \ldots & N_{20,\rho}
\end{bmatrix}
\begin{bmatrix}
x_i \\
y_i \\
z_i
\end{bmatrix}
\tag{3.8}
\]

The element stiffness matrix is evaluated using 3 x 3 x 3 Gauss quadrature.

\[
[K_e] = \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} [B]^T [D] [B] \det [J] d\xi d\eta d\rho \tag{3.9}
\]

The integrand in the above equation is just a function of \(\xi, \eta, \rho\). Rewriting the above equation and applying the integration scheme gives:

\[
\int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} f(\xi, \eta, \rho) d\xi d\eta d\rho = \sum_{k=1}^{3} \sum_{j=1}^{3} \sum_{i=1}^{3} W_k W_j W_i f(\xi_i, \eta_j, \rho_k) \tag{3.10}
\]

The above expression calculates the symmetric element stiffness matrix using 27 Gauss points. The elements above the main diagonal are calculated and the rest of the matrix is populated by reflecting the matrix. The elements of the matrix are computed
node wise. The following equations define the entries of the elemental stiffness matrix for a node pair:

\[
\begin{bmatrix}
  k_{ij} \\
  \end{bmatrix} = [B_i]^T [D] [B_j] \\
\begin{bmatrix}
  k_{11} & k_{12} & k_{13} \\
  k_{21} & k_{22} & k_{23} \\
  k_{31} & k_{32} & k_{33} \\
\end{bmatrix}
\]

\[
\begin{align*}
  k_1 &= N_{i,x} \frac{d_1}{d_4} N_{j,x} + N_{i,y} \frac{d_2}{d_4} N_{j,y} + N_{i,z} \frac{d_3}{d_4} N_{j,z} \\
  k_2 &= N_{i,x} \frac{d_2}{d_4} N_{j,y} + N_{i,y} \frac{d_1}{d_4} N_{j,x} + N_{i,z} \frac{d_3}{d_4} N_{j,z} \\
  k_3 &= N_{i,z} \frac{d_1}{d_3} N_{j,z} + N_{i,y} \frac{d_2}{d_3} N_{j,y} + N_{i,x} \frac{d_3}{d_3} N_{j,x} \\
  k_{12} &= N_{i,x} \frac{d_2}{d_2} N_{j,y} + N_{i,y} \frac{d_2}{d_2} N_{j,y} \\
  k_{13} &= N_{i,x} \frac{d_3}{d_3} N_{j,z} + N_{i,z} \frac{d_3}{d_3} N_{j,z} \\
  k_{21} &= N_{i,y} \frac{d_2}{d_2} N_{j,x} + N_{i,x} \frac{d_2}{d_2} N_{j,x} \\
  k_{23} &= N_{i,z} \frac{d_2}{d_3} N_{j,z} + N_{i,y} \frac{d_2}{d_3} N_{j,z} \\
  k_{32} &= N_{i,z} \frac{d_3}{d_3} N_{j,y} + N_{i,y} \frac{d_3}{d_3} N_{j,y} \\
\end{align*}
\]

The elemental stiffness matrices are then assembled into the global stiffness matrix and stored using a Skyline approach as two one dimensional arrays. This approach truncates the zeros of the sparse stiffness matrix and makes use of the symmetric property to save only the upper diagonal matrix. The first array stores the matrix values of each column starting from the first non-zero entry of the column and going up to the main diagonal including the zeros between them. The second array saves the index for the principal diagonal of the matrix. The approach reduces the memory requirement for saving the stiffness matrix (Figure 3.8).
3.4.1 Boundary Conditions

The boundary condition of the gear is to fix all three degrees of freedom of the shaft nodes generated from the mesh generator. All the other external nodes of the mesh are assumed to be free.

3.4.2 Compliance

The application of computing compliance from the finite element deflection by applying unit load on the surface nodes for bevel and hypoid gear is formulated in [12]. This is the most computationally time consuming procedure. A unit load is applied at each tooth contacting surface of the gear tooth and the corresponding displacement solution constitutes each load case. The magnitude of the load is $10^6$ lbs, which ensure
better accuracy of compliance and ease of transmission error calculation in micro inches. The load is applied normal to the involute surface at the node points on the tooth surface. The direction cosines of the normal are given by equation 3.12. The compliance values of all of the load cases are saved in a file for compliance computation at the contact line for different mesh positions. This same compliance matrix need not be recomputed when the load distribution is calculated for different load torques or microgeometries, thus saving significant computer time.

\[
\begin{align*}
N_x &= \sin \theta \cdot \cos \gamma \\
N_y &= \cos \theta \\
N_z &= \sin \theta \cdot \sin \gamma
\end{align*}
\]  

(3.12)

Figure 3.9 Direction cosines of the normal to the involute
3.5 Compliance along the contact line

The compliance coefficient $C_{ij}$ is defined as the displacement at a point $i$ due to the load at $j$ (Figure 3.10). The load points $j$ on the $F$ surface are the discrete points on the contact line and point $i$ on the $U$ surface is the intersection of the normal at $j$ with the middle plane of the tooth. Since both the points lie on the face of an element, the interpolation of the point is carried out on a 2D surface with the 8 nodes of the element. The contact points on the tooth surface are transformed into the natural coordinates of the finite elements ($\xi, \eta$). The compliance matrix generation for the spur gear [10, 11] uses the tooth end section for $\xi$ calculation for all contact points as the tooth cross section remains the same and $\eta$ from the face width position. The compliance matrix for straight bevel gears gets more involved where the section corresponding to each contact point is defined for the $\xi, \eta$ calculation. The contact point face width position defines the section dimensions such as base diameter, outside diameter and tooth thickness used to calculate the $\xi$ coordinate of the contact point. In the current work, the tooth is modeled with the involute profile and the elliptical root for the compliance computation while straight sided tooth is modeled in the tapered plate model for spur and helical gears in Windows LDP. The interpolation scheme remains the same with the modification of the direction cosines of the normal tooth force.
3.6 Loaded Tooth contact Analysis

The number of gear teeth in contact is based on the theoretical contact ratio. The contact analysis does not consider the influence of the deflected tooth due to the load; however, this deflection is extremely small, so the change is felt to be negligible. The solver needs the compliance matrix, initial separation and radius vector computed for the contact lines. The total compliance matrix is the sum of the compliance matrix of the pinion, the gear and the Hertzian compliance. The program assumes conjugacy between

Figure 3.10 Point of intersection of the normal with the middle plane
the two involute surfaces giving us the zero initial separation vector in the absence of microgeometry modification.

The solution for load distribution is based on Conry & Seireg’s [13] work on elastic bodies in contact. The compatibility equation and the equilibrium equation are solved simultaneously to obtain the load distribution across the face width. For the compatibility equation, the total deflection of the gear plus the initial separation must be equal to the rigid body motion of the gear.

\[ [C]\{F_N\} + \{I\} \geq \{R_a\} \theta_a \]

where

\[ [C] \text{— Total compliance of the gear} \]
\[ \{F\} \text{— Normal load vector} \]
\[ \{I\} \text{— Initial separation vector} \]
\[ \{R_a\} \text{— Radius vector} \]
\[ \theta_a \text{— Rigid body rotation} \]

The equilibrium equation balances the external torque \( T_a \) to the moments due to the applied forces.

\[ \{F_T\}^T \{R_a\} = T_a \]

where

\[ \{F_T\} \text{— Tangential load vector} \]
Figure 3.11 Moment balance between the applied torque and forces acting on the gear

The program uses the simplex solver used in the Windows LDP [14] for the load distribution. The solving routine has been modified to account for the change in the moment arm across the face width when compared to the spur gear moment arm. For the straight bevel gear (Figure 3.11) the moment arm is no longer the base radius and
becomes a radius vector computed in the axial plane of the gear. On solving the equations 3.13 and 3.14, the load distribution and rigid body rotation of the gear set are determined.

3.7 Summary

The finite element compliance calculation for straight bevel gears has all of the routines for mesh generation, stiffness computation and the solver, making it independent of a commercial finite element analysis program. The run time for the finite element calculation depends on the number of nodes on the model which depends on the user inputs to the mesh tool. The program computes the compliance once for a given geometry of the pinion or gear reducing the run time for multi torque and microgeometry modifications analysis. The program has the basic configuration of Windows LDP with appropriate modifications to several LDP routines.
CHAPTER 4 STRAIGHT BEVEL LOAD DISTRIBUTION

PROGRAM

4.1 Introduction

The program developed in this thesis is an independent design tool for straight bevel gears using Compaq Visual FORTRAN [23] for all analytical computations of dimension sheet, finite element compliance, load distribution and stress calculation. Microsoft Visual Basic [24] is used for designing the user input interface, output result plots and file operations at the front end of the program. The algorithms for development of all the FORTRAN codes were explained in the previous chapters. The program’s front end is developed using two forms, one for the input and dimension sheet calculation and the next for the display of the results. The chart plotting routines and the table layout of the WindowsLDP [14] have been used in the straight bevel program with necessary modifications wherever applicable. This chapter explains the user input modules, torque controller, program control and results plots in the output module of the program.

4.2 Program Interface

The program interface is categorized into two parts, the input module and the output module, where the first includes the entire user input controls like geometry,
program control, microgeometry and torque and the result plots are shown in the output module of the program. The microgeometry module is explained in the next chapter.

4.2.1 Input Module

The main function of the input module is to get user inputs to generate the dimension sheet of the gear using either one of two geometry input methods. The geometry tab includes the input form for the two methods, the finite element mesh tool, the dimension sheet table for three sections and axial view of the gears. The module defines the unit system (English or Metric) used for the analysis, which can be switched back and forth while in any part of the program execution. The unit system option is provided at the top left corner of the input module. The computation for the dimension sheet is initiated using the Initial Calc command button on the top right of the form. Figure 4.1 shows the factors input module with the dimension sheet displayed at the right and Figure 4.2 displays the dimension sheet geometry input with the axial plane diagram of the pinion and gear. This diagram also shows the schematic of the topland of the tooth which warns the designer of a narrow topland or pointed tooth. The third tab next to the dimension sheet shows the axial plane view of the gear assembled at the perfect mounting distance.

The next tab of the input module, the program control (Figure 4.3) defines user input for the mesh analysis like number of mesh positions to be analyzed, multiplier factor for points across the face width and the base pitch. The program saves input, output and the compliance files using the filename specified under the file frame. The program generates a file for tooth space coordinates and coordinate measuring machine (CMM) flank coordinates to be used in other programs. The tooth space coordinates can
Figure 4.1 Geometry Input - Factors

Figure 4.2 Geometry Input - Dimension sheet
be opened up in Unigraphics [25] to generate the tooth space which can be cut from the blank and repeating it around the axis to generate the total gear. The CMM file is generated with user defined grid points and clearance defined by percentage of the tooth height and face width. The coordinates generated for both of these files fixes the origin at the pitch apex in the coordinate system mentioned in Chapter 2.

The torque tab (Figure 4.4) is similar to that of LDP that defines the torque for the current analysis and the chart for the Harris plot of transmission error.
4.2.2 Output Module

The program performs the compliance calculations and the load distribution for all of the input torques once the SolveLDP command button is pressed. All the routines for the computation are written in FORTRAN. The results from the program are post processed for plotting charts and for the display of a summary table of the data in the output module. The output module displays several outputs at the torque picked from the drop down list box in the top of the output module. The output module has an interactive feature of displaying the results of the mouse position over the output charts at the bottom.
of the tabs. The output module is influenced from the chart plot routines and the layout of WindowsLDP.

4.2.2.1 Transmission Error

The transmission error tab (Figure 4.5) has two plots and a summary table. The first chart plots the pinion transmission error in µrad against the mesh position and second one shows up the first five harmonics of the Fourier series of transmission error. The summary table displays the peak to peak, average, minimum and maximum transmission error and the magnitude of transmission error harmonics.

Figure 4.5 Transmission error plots
4.2.2.2 Mesh Stiffness

The mesh stiffness at a mesh position is computed by computing the local stiffness at each contact point and summing them up using springs in parallel. The maximum, minimum, average, peak to peak and harmonics are computed (Figure 4.6).

Figure 4.6 Mesh stiffness plots
4.2.2.3 Tooth forces

The tooth forces are the summation of the load across the face width. The first chart plots the load on each tooth which goes through the mesh cycle for each mesh position and the second shows the tooth forces of one tooth as it comes in contact and leaves over the mesh cycle. There is a table to represent the theoretical contact ratio and the actual contact ratio of the gear pair (Figure 4.7).

![Figure 4.7 Tooth forces](image-url)
4.2.2.4 Zone of Contact

This selection plots the load distribution over the zone of contact (Figure 4.8). The plots can be rolled through the mesh positions to see the load distribution progression. The y-axis of the plot can be scaled to the radius of the pinion or the length of contact of the virtual spur gears. The load distribution in conjugate bevel gears is not uniform across the face width because of the variation of the tooth stiffness resulting in a higher load at the heel compared to the toe of the gear.

Figure 4.8 Zone of contact
4.2.2.5 Output Details

This selection tabulates all the results shown in the output module (Figure 4.9) for a mesh position selected using the slide bar on the top. The table lists the position of contact points, roll angle, radius of curvature, load values, deflection, contact stress and the microgeometry modification of each contact point at the given mesh position.

Figure 4.9 Output details
4.2.2.6 *Contact Stress, Load and Load Intensity Distribution*

These are the 3D plots used to represent the results over the flank surface of the tooth. The chart has face width along the X-axis, the pinion/gear roll angle along the Y-axis and the results on the Z axis. The charts are generated with the approximation of the data over the flank surface thus one has to use a higher multiplier factor to reduce the ripples in the face width direction and mesh position for the roll angle directions. The 2D contour plots and scale of the legends are changed by the settings on the right of the chart.

Figure 4.10 Contact stress distribution
Figure 4.11 Load distribution

4.2.2.7 Multitorque Plots

This plot shows the effect of torque of contact stress, load, peak to peak transmission error etc.
4.3 Summary

The straight bevel load distribution program is a useful design tool to calculate the vital outputs like load distribution and contact stress which are required for the assessment of gear performance. The program scope could be enhanced by adding the computation of bending stress, flash temperature, etc. and by enhancing the graphic utility of the program.
CHAPTER 5 MICROGEOMETRY MODIFICATION

5.1 Introduction

The straight bevel gear generated using the theoretical involute surface has to be modified by tip/root relief and lead modification to achieve the ideal tooth contact (Figure 5.1) that minimizes edge contact and reduces transmission error (TE). The changes applied on the tooth profile and in the face width direction are called microgeometry modifications. This chapter explains the various modifications applied to the tooth surface and their effect on contact patterns and transmission error.

Figure 5.1 Ideal straight bevel gear contact pattern[26]
5.2 Microgeometry Modification

The tooth surface deviation from the theoretical is input using the microgeometry modification module. The microgeometry module is developed from the current Windows LDP [27] using the similar terminology and modification used for spur/helical gears where the modifications are applied normal to the involute surface of the straight bevel gear. A positive modification implies removal of material from the theoretical surface. The different types of modifications that can be applied are

1. Profile Modification
2. Lead Modification
3. Bias Modification

5.2.1 Profile Modification

The tooth modification specified along the involute profile in the transverse plane is called a profile modification. The profile modification is approximated either using linear or parabolic shapes. The different modifications in the profile direction are tip relief, root relief, profile crowning and pressure angle slope. The tooth of the gear enlarges as one moves from the toe to the heel, necessitating a variation of the profile modification in the face width direction. The program requires inputs for the profile modification at the toe and heel and then assumes a linear variation to determine the value of the modification at any given layer across the face width.

The tip relief requires the roll angle at the start of modification and the modification value for the tip of the gear (Figure 5.2). The program assumes zero modification at the start roll angle and then varies either linearly or parabolically to the
tip based on the method chosen. It is possible to enter both the parabolic and a linear approximation simultaneously. The cumulative tip modification is calculated by summing both modifications. The program also allows two sets of inputs for tip and root relief so that one may have one broader relief and another that is much shorter. The root relief is similar to the tip relief. It applies involute profile modification from the given roll angle to the SAP roll angle. There is quick guide table for listing of roll angles at the bottom of inputs.

Figure 5.2  Input frame for profile modification
The next type of profile modification is profile crowning. In this modification, a circular crown is applied over the involute profile from the SAP to the tip of the gear. The modification is zero at the tooth center and reaches the given value at both the tip and SAP of the gear (Figure 5.3). The pressure angle slope error is a linear profile modification from the SAP to tip. The positive pressure angle slope gives a linear modification with zero at SAP and the given value at the tip. The total profile modification is calculated as sum of all the above modification.

Figure 5.3 Sample profile modifications (a) Tip Relief (b) Root Relief (c) Profile Crown (d) Pressure Slope
In the case of tip and root relief, the start roll angle and the modification value at any point across the face width is determined using the linear fit. Thus one has to take care in entering the modification values. To illustrate the approximation, three cases of tip relief shown in the Table 5.1 are discussed. In case 1, modification of the toe is alone specified which generates modification values in the zone below the 30 deg roll angle as the start roll angle for the modification varies from 30 to zero (Figure 5.4). This will be an undesired case if one intend for just a tip relief. The case 2 could be modification where there is a linearly decreasing tip relief of 0.002 in. at the toe to 0.000 in. at the heel. To create a uniform tip relief of 0.002 in., one has to specify all the four parameters as shown in case 3.

<table>
<thead>
<tr>
<th>Table 5.1 Different cases of tip relief modification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
</tr>
<tr>
<td>Toe</td>
</tr>
<tr>
<td>First Roll Angle at Start of EAP Mod: (deg)</td>
</tr>
<tr>
<td>First Parabolic EAP Mod Magnitude: (in.)</td>
</tr>
<tr>
<td>First Linear EAP Mod Magnitude: (in.)</td>
</tr>
</tbody>
</table>
5.2.2 Lead Modification

The modification of the tooth flank in the face width direction is called lead modification. The lead modifications can be lead crown, lead slope and lead modification at the toe or heel using the linear or parabolic approximation. In lead modification the toe end is referred as $x = 0$, the heel as $x = FW$ and any point in the lead direction is referred by the distance from the toe end of the gear.
The lead crown is circular modification applied along the tooth face width that has the lead apex at the center of the face width with the relief at both ends of the tooth (Figure 5.6). The lead slope is a linear modification in the lead direction. A positive lead slope modification gives a positive modification at the heel starting from zero at the toe. The approximate modification at the end of the tooth is mentioned with the start of modification distance and the modification magnitude at the end of the tooth (Figure 5.5). The program interpolates from the zero modification at the start distance to the desired value at the ends. When a lead crown with offset lead apex is desired, the parabolic approximation can be applied at both ends with the start distance at the required apex point.

![Figure 5.5 Input frame for lead modification](image-url)
5.2.3 Bias Modification

Bias modification essentially provides twist to the tooth surface. The modification is defined using the modification at the four corner points of the tooth surface (Figure 5.7). The program linearly interpolates the error both in the profile and lead direction. A sample modification is shown in Figure 5.8
Figure 5.7 Input frame for bias modification

![Figure 5.7 Input frame for bias modification](image)

Figure 5.8 Sample bias modification

![Figure 5.8 Sample bias modification](image)
5.3 Effects of microgeometry modifications

The microgeometry modifications are applied to reduce the edge contact and peak to peak transmission error of the gear set. The previous section discussed the capabilities of the program and procedure to apply modifications to the tooth design. The effect of the profile and lead modification and the method to optimize the contact pattern and peak to peak transmission error are shown for two sample gear pairs (Figure 5.9). The first gear is a standard depth gear having contact characteristics similar to the spur gear; the second sample is the uniform depth gear whose contact develops gradually through the mesh similar to the helical gears.

![Gear geometry for the microgeometry modification analysis](image)

**Figure 5.9** Gear geometry for the microgeometry modification analysis
The tip/edge loading of the gear is one of the major concerns that are to be avoided in good gear design. The load up in the tooth increases the root bending stress increasing the chance of tooth breakage failure. The profile modifications to avoid tip contact create relief at the tip or/and root of the gear allowing the contact to be in the central zone of the tooth profile. The peak to peak transmission error harmonics causing the noise excitations of the gear pair are reduced by tip relief chosen from the transmission error curve which represents the tooth deflection under load. The shaft angle errors caused by misalignments and manufacturing tolerances shift the loading to the edges of the tooth. The sensitivity of shaft angle errors is reduced by concentrating the load at the central toe region by lead modification with the apex of the lead crown offset towards the toe end of the gear in order to shift the center of contact from the heel to the toe side of the tooth.

In the following example, a sample gear pair at an operating torque at 700 lbf-in is modified to reduce the peak to peak transmission error and edge loading. Figure 5.11 shows the load distribution and TE plot of the unmodified gear. The first iteration is to reduce the peak to peak transmission error by profile modifications (Figure 5.12) like tip relief and root relief starting from the approximate highest and lowest point of single tooth contact roll angle and pressure slope error. This is aimed at flattening the hump in the transmission error curve at the double tooth contact zone. The peak to peak TE is reduced from 643 µrad to 173 µrad, with the load distribution going across the entire face width without any lead modification. The lead crown with the crown apex offset towards the toe centralizes the load at the modified torques but spreads over the tooth at higher torques. A reduction in the first harmonic of transmission error was observed after the
lead modification (Figure 5.13). The peak to peak transmission error goes up to 352 µrad in this case. To centralize the load and decrease the peak to peak transmission error the profile modification values on the gear are increased (Figure 5.14). This modification balances the peak to peak TE and load distribution. The peak to peak TE for the final iteration is 300 µrad and the first harmonic of the transmission error is 70 µrad.

Figure 5.10 Tooth shape and contact zone of sample gear pair (a) Standard depth (b) Uniform depth
Figure 5.11 Unmodified – gear pair 1
Figure 5.12 First iteration - only profile modification, gear pair 1
Figure 5.13 Second iteration - profile and lead modification, gear pair 1
Figure 5.14  Final iteration, gear pair 1
The sample gear pair 1 when modified from the standard taper to a uniform depth to generate the gear tooth resulted in negative topland at the toe end of the pinion and narrow topland for the gear due to the increase in the tooth height. The mean working depth of the gear is therefore decreased using the depth factor to give a positive topland at the pinion toe but decreases the clearance that is then increased using the clearance factor to match the clearance of sample gear pair 1. The thickness factor is changed to balance the topland at the toe of the pinion and gear and generate a tooth form to be analyzed. The sample gear pair 2 is also optimized for an operating torque of 700 lbf-in. Figure 5.17 shows the unmodified gear load distribution and the transmission error plots with peak to peak transmission error at 604 µrad. It is interesting to note that the contact pattern shows the single tooth contact region along the face width and shows the much higher contact ratio at the toe than the heel. In the TE plot, there are no sudden shifts due to changes in tooth contact because of the gradual increase in the length of the contact.
lines. This gradual shift could be one of the reasons that face hobbed teeth tend to be less noise sensitive than are face milled gears.

The profile modification of tip/root relief is applied along with the pressure angle slope error. The tip relief at the heel was kept close to the edge to just avoid edge loading as the contact ratio at the heel is nearly 1. In Figure 5.18, the modification values were iterated to reach a peak to peak transmission error of 150 µrad with the first harmonic of 32 µrad. The load in this case was high on the heel and the corner contact on the toe of the gear is removed. The next step was to modify the gear in the lead direction to centralize the load. With the lead crown and modified profile modification (Figure 5.19), further reduction in peak to peak transmission error to 140µrad was obtained with the first harmonic of 32µrad. In order to go any lower with the TE values, much like with parallel axis gears, it would be necessary to apply an inverse profile crown to the single tooth pair contact region and to change the shape of the modification in the two tooth pair contact region.

Figure 5.16 Transmission error plot of uniform depth gear with similar amount of linear and parabolic modification
Figure 5.17 Unmodified - gear pair 2
Figure 5.18 Profile Modification – gear pair 2
Figure 5.19 Profile and Lead Modification – gear pair 2
5.4 Load Distribution Analysis

Microgeometry modifications are independent of the load applied and they are modeled as deviations from the theoretical involute surface. The modifications are included in the load distribution calculation as initial separations which are computed at the contact points to complete the initial separation vector $\{f\}$. The load distribution equation 3.12 is updated and resolved for all the mesh positions. The compliance matrix calculations remain the same. This matrix is retrieved from the file thus saving computational time in running various modifications on the gear.

5.5 Summary

Microgeometry modifications are always used in the design of gears to avoid corner contact and to optimize the contact pattern and transmission error. The effects of microgeometry modification and an example for the microgeometry modification have been explained in this chapter.
CHAPTER 6 COMPARISON OF RESULTS WITH CALYX

6.1 Introduction

The straight bevel gear program developed in this thesis uses the finite element approach and approximate contact analysis to solve for load distribution and transmission error of the gear pair. The comparative study of the program with the CALYX program developed by Dr. Sandeep Vijayakar [28], a customizable finite element analysis based contact solver for gears widely used for the analysis of spur and helical gears, is done for the validation of the results. The CALYX program creates the 3D finite element mesh of the complete gear pair for the compliance calculation and a surface contact approach is used for the contact line development [29, 30]. CALYX is used in the previous comparative study of Thin Rim Windows LDP and Hypoid Analysis Program developed at The Ohio State University Gear Lab [10, 31]. The comparison of the transmission error, tooth forces and contact stresses between the two programs is shown in this chapter.
6.2 CALYX Module

At the start of this project, CALYX, a generic finite element gear solver which can solve the load distribution for most type of gears did not have a specific input module for straight bevel gears. Therefore a development of an intermediate input routine using C++ compiler [32] to define the straight bevel gear was performed.

The main parameters required for the definition of the gear in CALYX were the tooth flank, root, normal and set of cones defining the gear (Figure 6.1). The cones are represented by a line in the $RZ$ plane in the form $A_1R + A_2Z + B = 0$. The tooth surface coordinates defined by the transverse section radii, tooth thickness, number of teeth and pitch angle and the normal are generated using the equations developed in Chapter 2. The inputs of the intermediate routine are generated from the straight bevel LDP as an output file which is read into the CALYX program to create the parameters for the mesh generator. The sample input file is shown in Figure 6.2. The routines used along with the CALYX mesh generator create the mesh file to solve for the load distribution using the CALYX solver where the number of time steps and time step increment are defined. The preprocessor generates the 3D mesh and cut sections of the gears (Figure 6.3) and the post processor plots the load distribution and stresses on the tooth surface of the gears.
Figure 6.1 Coordinate system and reference cone in CALYX [33]

SAMPLE INPUT FILE

<table>
<thead>
<tr>
<th>Gear Cone Parameters</th>
<th>Pinion Cone Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Z.78571 -1.00000 10.93023</td>
<td>Back Cone (Az, Bz, E) -0.35897 -1.00000 3.93086</td>
</tr>
<tr>
<td>-2.78571 -1.00000 7.93047</td>
<td>Back Cone (Az, Bz, E) -0.35897 -1.00000 2.98638</td>
</tr>
<tr>
<td>0.40933 -1.00000 -0.04426</td>
<td>Base Cone (Az, Bz, E) 8.21948 -1.00000 -0.202568</td>
</tr>
<tr>
<td>0.31895 -1.00000 0.04030</td>
<td>Base Cone (Az, Bz, E) 2.18506 -1.00000 0.035245</td>
</tr>
<tr>
<td>9.68479 6.27592 7.06736</td>
<td>Base Radius 1.24001 1.07934 0.01038</td>
</tr>
<tr>
<td>10.12085 6.76415 7.39749</td>
<td>SAD Radius 1.26201 1.09939 0.02531</td>
</tr>
<tr>
<td>10.40567 6.50975 7.57035</td>
<td>Outside Radius 1.60305 1.30120 1.15933</td>
</tr>
<tr>
<td>0.40931 0.40067 0.33400</td>
<td>Base Thickness 0.36475 0.32470 0.23400</td>
</tr>
<tr>
<td>9.36275 0.64227 7.29179</td>
<td>Root Radius 1.12524 1.03872 0.07216</td>
</tr>
<tr>
<td>39</td>
<td>Number of Teeth 14</td>
</tr>
<tr>
<td>1.22615</td>
<td>Pitch Angle 0.34465</td>
</tr>
<tr>
<td>1.00000</td>
<td>Face Width 1.00000</td>
</tr>
</tbody>
</table>

Figure 6.2 Sample CALYX input file
Figure 6.3 Gear mesh model in CALYX
6.3 Test Matrix

The comparative study between straight bevel LDP and CALYX is performed for three gear sets (Table 6.1). The gear sets are analyzed at three different torques (1000, 2500 and 5000 lbf-in). The tooth flank and the root geometry remains the same between the two programs and the gears have no microgeometry modifications. The first gear set was modified from the spiral bevel gear of the AGMA standard and the second gear is taken from the Gear Handbook [16] and the third set is from an automotive application.

<table>
<thead>
<tr>
<th></th>
<th>Gear Set 1</th>
<th>Gear Set 2</th>
<th>Gear Set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pinion Teeth</td>
<td>14</td>
<td>16</td>
<td>10</td>
</tr>
<tr>
<td>Number of Gear Teeth</td>
<td>39</td>
<td>49</td>
<td>14</td>
</tr>
<tr>
<td>Diametral Pitch (1/in)</td>
<td>5.6</td>
<td>5.08</td>
<td>5.9</td>
</tr>
<tr>
<td>Pressure Angle (deg)</td>
<td>20</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>Face Width (in)</td>
<td>1 in</td>
<td>1.575 in</td>
<td>0.5641 in</td>
</tr>
<tr>
<td>Shaft Angle (deg)</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Type of Taper</td>
<td>Standard</td>
<td>Standard</td>
<td>Standard</td>
</tr>
</tbody>
</table>

Table 6.1 Gear pair used for the comparison between the Straight bevel LDP and CALYX
The outputs that are compared between the two programs are transmission error, tooth forces and maximum contact stress. The transmission error comparison relates the mesh stiffness calculation between the two programs. The contact stress is chosen for its correlation to the load distribution and radius of curvature calculations. The tooth forces show the comparison of the total load at each mesh position which shows the transition from the single to double tooth pair contact.

6.4 Results

The Bevel LDP program considers conjugacy of the planar involute surfaces of straight bevel gears for the load distribution. The sample gears are analyzed in CALYX which has a generic approach for searching for contact between the surfaces. Three very low torques, 0.1, 1 and 10 lbf-in are used to generate the Harris plot for transmission error (Figure 6.5). If the gears are conjugate one would expect zero TE at zero torque. In fact each torque level here is low relative to the gear rating of about 5000 lbf-in. The lowest torque plot indicates the non conjugacy and as torque is increased the deflection due to the tooth stiffness shifts the plots. When the TE’s are converted to the linear units using the mean base radius, one gets no load values from 0.000012 in. for case a to 0.000086 in. for case c. In these gears the tooth deflections at the rated load are at least 10 times the no load TE values, thus indicating that the gears are nearly conjugate at no load. The transmission error plot shows the tooth surface has deviation similar to profile crowning and pressure angle slope modification. The load distribution in CALYX (Figure 6.6) shows the presence of twist on the lead direction of the gear surface. The load moves from toe to the heel as we roll them in mesh. These observations were made at torques 0.1 lbf-in and 1 lbf-in, but at a torque increases to 10 lbf-in, the load goes across the
entire face width again indicating near conjugacy. An analysis of the example gear pair 2 at a load of 2500 lbf-in for loaded TE and load distribution, one can observe the effect of corner contact results in higher effective contact ratio and the load distribution across the tooth increases a bit from toe to heel (Figure 6.4). The latter is to be expected since the tooth stiffness increases at a higher rate than the radius change from toe to heel.

Figure 6.4 Load across the face width for one mesh position for sample gear pair 2 at 2500 lbf-in
Figure 6.5 Harris plot for transmission error (a) Sample gear pair 1 (b) Sample gear pair 2 (c) Sample gear pair 3
Figure 6.6 Load distribution at different mesh position at a torque of 1 lbf-in for sample gear pair 3
6.4.1 Transmission Error

The test gears are analyzed over one base pitch with 15 time steps/mesh positions at three torques. The transmission error curve at three torques and the peak to peak transmission error comparison between the programs are shown in Figure 6.7, Figure 6.11 and Figure 6.15. The trend of the transmission error curve from both programs is nearly the same for all of the test cases. The variation between the two programs can be seen at the transition from the single tooth contact to the double tooth contact. The BevelLDP program doesn’t predict the early/corner contact due to the loading on the gears resulting in the shorter zone of double tooth contact while CALYX more accurately captures the corner contact which is more prominent at higher torques of the study in the transmission error curves. The differences of the peak to peak transmission error values between the two programs are small. For the sample gear pair 3, we find there is a small shift in the mean transmission error but the peak to peak TE which is a major noise excitation compare well between CALYX and the straight BevelLDP.

6.4.2 Tooth Forces

The tooth forces plot (Figure 6.8) is the summation of the load across the face width for each mesh position. This shows the load on one tooth from the instant it comes in contact until it leaves contact. This shows the variation of the load on one tooth and shows the transition from the single tooth pair contact to multiple teeth in contact. The trend of the results from both programs shows close resemblance. The same observation made in transmission error curves can also be made with the tooth force plot.
6.4.3 Contact Stress

The contact stress plot is the plot of the maximum contact stress at the given mesh position against the position along the mesh cycles. The results from the straight bevel LDP considers the stress at different mesh position with the consideration of the loading at the tip however early contact cannot be predicted. CALYX provides the contact stress with or without the stress due to the tip loading. The inclusion of tip loading considers the corner load of the gear tip with the pinion and the load due to the early contact resulting in high stresses at tip of the tooth being in contact. However, in practice these large corner contact stresses may be greatly reduced through the use of tip relief, Figure 6.9 for sample gear pair 1, the contact stresses are compared with the edge loading stresses from CALYX which seems to follow the trend and magnitude over most of the mesh cycle except at the tip load condition. The results from the CALYX in the single tooth pair contact region are higher than the straight bevel LDP values. In Figure 6.10, when stresses from without edge loading case are compared, the stresses from CALYX drop down to zero at the tip. The similar effect was observed even for the pinion tip loading but all the plots shown below this effect in the stress calculation. The similar comparisons for the other gear samples are also shown in Figure 6.11 to Figure 6.18. In general, the agreement between the two methods is amazingly good, in plot trend and amplitude. The percentage deviation of the contact stress from the CALYX at the single tooth contact zone and double tooth contact zone is shown in the Table 6.2.
Table 6.2 Percentage deviation of the contact stress from CALYX

<table>
<thead>
<tr>
<th>TORQUE (lbf-in)</th>
<th>Single Tooth Zone (%)</th>
<th>Double tooth Zone (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SAMPLE GEAR PAIR 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>8.46</td>
<td>2.71</td>
</tr>
<tr>
<td>2500</td>
<td>7.65</td>
<td>1.92</td>
</tr>
<tr>
<td>5000</td>
<td>2.39</td>
<td>5.02</td>
</tr>
<tr>
<td><strong>SAMPLE GEAR PAIR 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>-0.27</td>
<td>5.92</td>
</tr>
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<td>5000</td>
<td>7.13</td>
<td>4.43</td>
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</tbody>
</table>
Figure 6.7 Sample gear pair 1 - Transmission error plots
Figure 6.8  Sample gear pair 1 - Tooth force plots
Figure 6.9 Sample gear pair 1 - Contact stress with edge loading
Figure 6.10  Sample gear pair 1 - Contact stress without edge loading
Figure 6.11 Sample gear pair 2 - Transmission error plots
Figure 6.12  Sample gear pair 2 - Tooth force plots
Figure 6.13  Sample gear pair 2 - Contact stress with edge loading
Figure 6.14 Sample gear pair 2 - Contact Stress without edge loading
Figure 6.15  Sample gear pair 3 - Transmission error plots
Figure 6.16  Sample gear pair 3 - Tooth force plots
Figure 6.17  Sample gear pair 3 - Contact stress with edge loading
Figure 6.18  Sample gear pair 3 - Contact stress without edge loading
6.5 Summary

The results comparison between the straight bevel load distribution program and CALYX gives an initial validation to the program for the sample gear pair with unmodified surfaces as the tooth is loaded. The straight bevel gear has increasing load at the heel due to the stiffer tooth. The program helps in the study of the load distribution to optimize the contact pattern and the stress on the gear and transmission error to reduce the noise of the gear pair by minimizing the peak to peak TE.
CHAPTER 7 CONCLUSIONS AND FUTURE WORK

7.1 Conclusion

The program developed in the research of this thesis is for the study of the design and analysis of straight bevel gears. The program starts with the dimension sheet calculations and gear parameters required for the load distribution analysis followed by an approximate contact line development algorithm considering conjugacy of the planar involute straight bevel gears. The tooth compliance is calculated by an accurate finite element approach. The program’s computation time, one of the considerations was thought of in the finite element mesh generator to optimize the straight bevel gear model and reduce the compliance calculation time. The finite element compliance of the gear and the Hertzian compliance are summed to solve for the load distribution and the transmission error of the straight bevel gear. The results from CALYX were found to follow the trend and the magnitude of the results predicted by the straight bevel gear load distribution program. The program can be effectively used for the study of contact pattern and minimizing the transmission error using the microgeometry modifications. The microgeometry results actually can work for any of the common bevel gear geometries (octoid, spherical involute, etc) that have tooth dimensions similar to the planar involute.
7.2 Future Work

The program has a lot of scope for further development and some of the recommendations are:

- Calculation of the bending stress, a vital component for the study of bending failure of straight bevel gear teeth, using either a 2D boundary element like that in LDP or a finite element approach. The finite element approach would require a new mesh template with relatively fine mesh at the root compared to the current one that has just one element at the root of the gear.

- Net forged gears are found to have tooth shapes different from the theoretical tooth used in this study. The analysis of the actual forged gears needs to consider things like tooth chamfer at the toe and web stiffener beams at the heel. The program currently has the capability to truncate the contact lines of the tooth without any change in stiffness which requires the mesh generator to be modified to analyze the gears with the tooth chamfers. The current model can include the web across the entire tooth height using the truss elements. The feature can be enhanced using 8 noded plane elements to have better approximation of the webs and variable web height to study the effect of web height on load distribution and perform material cost reduction study.

- To perform experimental validation of the program for the sample gear set with microgeometry modifications.
• Study the effect of misalignments (E,P,G errors) in the gear pair by considering them as virtual center distance change and deviations between the surfaces in the line of action and off line of action directions.

• The current modifications applied to the tooth model were not sufficient enough to zero the peak to peak transmission error. The program requires higher order tip/root relief to achieve this. The model also requires a capability to include external inputs measured from the inspection machine.

• To compute fluid film thickness and temperature distribution parameters that are computed for the spur and helical gears in WindowsLDP.

• Development of the contact line using the surface approach separations instead of the current approximate approach of assuming conjugacy.

• Enhance the root shape to allow other shapes and the extension of the SAP starting point to a point farther down the tooth root.

• Since the base geometry for spiral bevel gears has been done, one could recreate the finite element mesh generator for spiral bevel gears and this program could be readily applied to this gear type.
REFERENCES


[28] "CALYX," Advanced Numerical Solution LLC.


