THE ACTION POINT MODEL OF THE
DRIVER-VEHICLE SYSTEM

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By

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* * * * *

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CHAPTER I
INTRODUCTION

This dissertation is concerned with qualifying and quantifying one aspect of a man-machine system. The man-machine system consists of the driver-automobile complex; and the subject of interest of this system is the longitudinal control of the automobile by the human under the restraint that the driven automobile is confined to one traffic lane under dense traffic conditions.

In a problem of this nature, the usual approach is to consider the human as merely a mechanical control element performing a particular task. This approach is usually meaningful and expeditious, but in view of the complexities of the task, it was felt that it would be a mistake to adopt a purely deterministic and mechanical attack. Instead, a more fundamental psychophysical approach was used in the hope that the fundamental results would not only explain the longitudinal control problem, but also be applicable to other phases of the vehicular control problem and possibly even to control problems encountered in other man-machine systems.
This study is divided into three phases. The first phase is concerned with the initial formalization of a mathematical model of the driver-vehicle complex based on experimental evidence obtained from the driving simulator. In the second phase, results from a velocity threshold experiment performed on the simulator are used to obtain two general velocity threshold equations which describe the driver's sensitivity to relative velocity. Finally, in the third phase, the results of the velocity threshold study in phase two are correlated with the model formalized in phase one to give a mathematical model for the driver-vehicle system. This model is called the action point model.
CHAPTER II
THE TWO-CAR PROBLEM

A. Introduction

The automobile driver of today is subjected to many different traffic situations. Probably the most frequent situation is the one where the driver is forced to follow another vehicle. Little is known about the driver's behavior when he is in this driving situation. In order to learn more about the behavior of the driver in this situation, a two-car simulator was designed and built. An analog computer was made an integral component of the simulator, a feature which was found to be invaluable. This simulator was the main tool used in the study of driver behavior in the two-car problem. Figure 1 shows the complete simulator. Complete details of the simulator may be found in Reference (1). A block diagram is given in Fig. 2.

B. Nomenclature

The variables defined here will be used throughout the remainder of the dissertation. Figure 3 illustrates the physical variables involved in the two-car problem. The instantaneous
Fig. 2 -- Block diagram of automobile simulator
Fig. 3--Variables of the two-car problem

velocities of the lead vehicle and rear vehicle with respect to the roadways are designated by the velocity vectors \( v_1 \) and \( v_2 \) respectively. Similarly, the instantaneous accelerations of the lead vehicle and rear vehicle with respect to the roadway are represented by the acceleration vectors \( a_1 \) and \( a_2 \) respectively. The relative velocity and relative acceleration between the two vehicles are given by \( v \) and \( a \) respectively, where \( v=v_1 - v_2 \) and \( a=a_1 - a_2 \). The headway \( h \) is defined to be the distance from the eyes of the driver in the rear vehicle to the rear bumper of the lead vehicle and is positive in keeping with the definitions of \( v \) and \( a \).

C. The Steady State Condition

Investigation of the two-car problem was initiated with a simulator study of the characteristics of the man-machine complex. This man-machine complex consisted of the driver in the rear vehicle with the physical state of the lead vehicle serving as the primary input to this complex.
The lead vehicle was controlled by an analog computer so that it traveled at a constant speed of $V_1$ mph. Instructions to the driver of the rear vehicle were that he follow the lead vehicle at what he considered to be a minimum safe headway. While driving under this closely coupled condition, the driver was able to maintain the velocity of the rear vehicle at a value closely approximating the velocity of the lead vehicle. Consequently, this condition was defined to be the steady state situation.

D. The Phase Plane Trajectory

Poincaré(2), while investigating the relative motion of the earth, moon, and sun, developed new methods for studying celestial mechanics. These methods were characterized by the use of the phase plane. A phase plane is a plane on which different states or phases of a physical system can be mapped out. By definition, the $y$ axis of the phase plane represents the time derivative of the dependent system variable while the $x$ axis denotes the dependent variable. Time is the independent variable of the system. For instance, if a two body system was under study, then the separation between the bodies would be the dependent variable while the relative velocity between the bodies would be the time derivative of the dependent variable. If the time derivative of the dependent variable is plotted versus the dependent
variable on the phase plane, then the resulting curve is termed a phase portrait or trajectory. In general a given system has many possible trajectories. Since the topological configuration of the trajectories is directly related to the state or phase of the system, it is then a simple matter to determine which values of the system parameters will cause system instability(3). Only autonomous systems, describable by a second order differential equation, are generally analyzed using the phase plane technique.

Occasionally the differential equation will have a special solution which is represented by one unique closed trajectory in the phase plane. Poincaré called this closed trajectory a limit cycle. Each trajectory, beginning sufficiently near a limit cycle, either approaches it asymptotically or recedes from it asymptotically as the time $t \to \infty$. If each nearby trajectory approaches the limit cycle as $t \to \infty$, then the limit cycle is said to be stable. If on the other hand each nearby trajectory recedes from the limit cycle as $t \to \infty$, then the limit cycle is said to be unstable. At present, the phase plane technique is a powerful tool in the study of non-linear control systems.

It must be remembered particularly when studying the response of the driver, that the conditions arising in the various situations are not confined to those generally imposed when using the phase plane representation. For this reason, trajectories, which in the restricted
or classical case do not cross except at special points called singular points, can cross at any point in the phase plane. These variations from the classical characteristics of trajectories are due to the fact that the driver's response is not deterministic and the driver cannot be represented exactly by an analytic second order differential equation.

Phase plane trajectories were used to study driver behavior under the steady state condition of the car following situation. Here, a phase plane trajectory is obtained by plotting the relative velocity $v$ versus the headway $h$. It will now be shown that phase plane trajectories can be readily determined analytically if certain restrictions are imposed upon the system variables.

If the relative acceleration between the two vehicles is equal to a constant $A$, then the relative motion of the two vehicles can be completely described by the following equation

$$ h = \frac{1}{2} At^2 + v_0 t + h_0 $$

where $v_0$ is the initial relative velocity

$h_0$ is the initial headway, and

$t$ is time.

On differentiating with respect to time, Eq. (1) becomes
\[
\frac{dh}{dt} = v = At + v_o.
\]

Equation (2) can be rewritten as

\[
t = \frac{v - v_0}{A}.
\]

Substitution of Eq. (3) into Eq. (1) yields

\[
h = \frac{v^2}{2A} + K
\]

where

\[
K = h_o - \frac{v_o^2}{2A}.
\]

Equation (4) is that of the phase plane trajectory for the specific case of constant relative acceleration. Note that this equation is that of a parabola. The phase plane trajectories for both A positive and negative are shown in Fig. 4. In each case the arrows point in the direction of increasing time. Note that the displacement of the trajectory along the h axis is a function of \(h_o\), \(v_o\), and A.

E. Results of Phase Plane Studies

Barbosa(4), in studying Herman's(5) and other equations for the car following situation was able to show with the aid of the simulator, that these equations gave only an approximate fit to the observed data of the human driver under limited variations of the variables. Under
Fig. 4--Phase plane trajectories for constant acceleration

the restraint of these limitations he found that a large class of functions would give the same approximate fit.

In his attempts to develop a more general model, Barbosa studied the phase plane trajectories obtained from steady state car following experiments. A typical phase plane trajectory is shown in Fig. 5.

Note that the trajectory crosses itself at several points. Classically, trajectories do not cross except at singular points.

In analyzing phase plane trajectories, Barbosa found certain portions of these trajectories to be approximately parabolic. For
Fig. 5--Typical phase plane trajectory

example, portions A-B and B-C of the trajectory shown in Fig. 5 are very nearly parabolic. Since the equation of a phase plane trajectory is parabolic under the condition of constant relative acceleration, it follows that the relative acceleration in region A-B is approximately equal to a constant positive value. Similarly, the relative acceleration in region B-C is approximately equal to a constant negative value.

Since the velocity of the lead vehicle in the steady state car-following situation was always a constant, it follows that the driver was braking at a constant value of deceleration in region A-B and accelerating at a constant value of acceleration in region B-C. On this basis, Barbosa proposed the decision point model.
F. The Decision Point Model

The decision point model is based on hypothetical decisions made by the driver while following a lead vehicle at close range. Figure 5 can be used to illustrate the model. At point 1 on the trajectory, Barbosa postulated that the driver made a decision to decelerate at some constant rate. After a delay of $\tau_1$ seconds, the driver changes his acceleration from a positive value to a negative value. This action takes place at point A. When point 2 is reached, the driver makes a decision to switch his acceleration to some positive value. After a delay of $\tau_2$ seconds, the driver takes action and switches his acceleration to some positive value. This action occurs at point B. When point C is reached the driver has completed one cycle of operation. Classically, the cycle of operation just described is not a limit cycle but for purposes of this discussion, it has been found advantageous to refer to it as a limit cycle.

The driver completes many such cycles in following a vehicle at close range. A number of these cycles are illustrated in Fig. 6. Notice that the limit cycles drift around the approximate equilibrium point H. The periods of the limit cycles vary from 5 to 20 seconds.

However, the decision point model as proposed by Barbosa leaves many aspects of the two-car problem unexplained. Some of these aspects are:
Fig. 6--Typical phase plane trajectory

(i) On what information or stimuli does the driver base his decisions?

(ii) What values of acceleration does the driver choose in going through his limit cycles and why?

(iii) What is the magnitude of the delay between the time the driver makes a decision to the time that he takes action?

(iv) What causes the limit cycles to drift from the equilibrium point located on the h axis?

(v) How is the driver able to maintain stability?

The preceding shortcomings of the decision point model served as the primary reason for continuation of the research on the
development of a more refined car-following model. This research was initiated by extending the concept of the phase plane to a "second order" phase plane and then examining the driver’s trajectory in this new domain.

G. The \( a-v \) Phase Plane Representation

Trajectories in which \( a \) is plotted as a function of \( v \) were used in an attempt to obtain support for the decision point model. An \( a-v \) trajectory is simply a plot of the relative acceleration \( a \) versus the relative velocity \( v \). If the \( v-h \) plot is considered as a first order phase plane trajectory, then the \( a-v \) plot can be thought of as a second order phase plane trajectory.

A typical \( a-v \) trajectory is shown in Fig. 7. If the small variations in acceleration are removed, then this trajectory appears as shown in Fig. 8. One rectangular trajectory on the \( a-v \) plane corresponds to one limit cycle on the \( v-h \) plane. In both cases the arrows point in the direction of increasing time.

A complete cycle of operation on the \( a-v \) plane is similar to that on the \( v-h \) plane. At point \( 1 \) the driver decides to change acceleration. After a delay of \( \tau_1 \) seconds, the driver takes action at point \( A \) and switches his acceleration to some negative value at point \( A' \). The switch from \( A \) to \( A' \) is almost instantaneous. The driver then
Fig. 7--Typical a-v trajectory

Fig. 8--Idealized a-v trajectory
decelerates at a constant rate until point 2 is reached. Here the driver makes a decision to change acceleration again. After a delay of $\tau_2$ seconds the driver takes action at point B and switches his acceleration to a positive value at point B'. The switch from B to B' is almost instantaneous. When point C is reached, the driver has completed one cycle of operation. In following a vehicle at close range, a driver completes many such rectangular trajectories. A number of these are shown in Fig. 9.

![Diagram of a-v trajectory](image)

**Fig. 9--Typical a-v trajectory**

The preceding discussion of the cycle of operation on a-v plane clearly indicates that the driver abruptly switches from acceleration to deceleration and vice versa and generates a rectangular a-v trajectory. Consequently, this discussion supports the acceleration
switching predicted by the decision point model. Using the decision point model as a basis, a new model termed the action point model will now be proposed.

H. The Action Point Model

At the present time it is not possible to pinpoint where a driver makes a decision on either his v-h trajectory or his a-v trajectory. However, the point at which the driver takes action is clearly evident from the a-v trajectory. That is, the action point is defined as the point where the acceleration changes sign. Since the determination of the action point is so readily available from the a-v trajectory, it was decided to develop a model around these points rather than the decision points which are presently indeterminable. This model has been termed the action point model.

An investigation of a-v trajectories for different steady state velocities of the lead vehicle revealed that the positions of the action points were dependent on the steady state velocity. A study was then initiated to obtain a more basic understanding of the decision-action process.
CHAPTER III
VELOCITY THRESHOLDS IN CAR-FOLLOWING

A. Introduction

Velocity thresholds are important parameters in driver behavior studies. A driver is constantly concerned about his own velocity as well as the velocity of the vehicle he is following. Of the different velocities involved in car-following, it is safe to assume that the relative velocity between the two vehicles is most important. This is so because it is the magnitude and sense of this velocity which determines whether or not the two vehicles will eventually collide as well as the severity of the collision if it does result. It is the threshold of this velocity that has been investigated. The term "velocity threshold" will be defined as that relative velocity which the driver can just detect with a certain fixed probability at a given headway for a given presentation time of the relative velocity. Here headway will be taken as the distance from the driver's eyes to the rear bumper of the lead vehicle. A great deal of classical research has been conducted on the velocity threshold and some of the findings will be presented in the next section.
B. Historical Review of the Motion Threshold

Exner (6) in 1875 made the first significant contribution to the general subject of motion thresholds. He postulated that an observer with eyes stationary can discriminate motion directly by perception or indirectly by inference. Since that time much research has been conducted on motion thresholds. Brown (7) has summarized all of the preceding studies on motion thresholds into the following basic theory of motion.

An observer judges motion on the relative motion of objects plus perception of eye and head movements. This judgment is less complex if the eye and head are stationary. If an observer fixates upon a moving object on a stationary visual field then his detection of its movement depends primarily on its luminance and the speed at which the object is moving.

If the observer has unlimited time in which to discriminate the movement, he may infer movement on the basis of successive observations of the object’s changes in position. This inference is a function, in part, of the exposure time and the reference objects present.

Now if the observer has limited time for observation, it is found that he cannot discriminate motion if the motion speed is less than
some lower limit. Brown defines this lower limit as the lower speed threshold. As the motion speed is increased above the lower threshold, the motion is then directly perceived. This discrimination is primarily dependent upon the observer receiving a constant amount of energy from the object (i.e., the Bunsen-Roscoe law). The Bunsen-Roscoe law is based on a certain level of photochemical activity being attained within the retina. If the motion speed is further increased, then a velocity is reached such that there is no discrimination of motion. This upper limit is defined to be the upper speed threshold. For motion speeds exceeding the upper speed threshold, the observer senses only a stationary stimulus pattern or a blur.

The Bunsen-Roscoe law is valid for speeds in the neighborhood of the lower speed threshold. In its most general form, the Bunsen-Roscoe law states that the product of the intensity and duration of exposure must be constant in order to give a certain visual effect. In this context the visual effect is perception of motion. However, the Bunsen-Roscoe law does not hold when the exposure time exceeds some critical value. This critical value is in the order of a 1/10 of a second. For exposure times greater than the critical value, the intensity or power is the important factor. That is, the intensity must now be some constant value if motion is to be perceived for exposure times greater than the critical value.
It has been pointed out that reference objects are used by the observer in determining whether or not motion takes place. Liebowitz (8), at the University of Wisconsin, has shown that a reference definitely affects an observer's motion threshold. For a short exposure duration of 0.125 seconds, he found that a reference did not affect the motion threshold of an observer. However, if a reference is introduced when the exposure duration is 16 seconds, the threshold velocity was found to diminish by 50%. Liebowitz also used his data to support the hypothesis that motion is discerned at slow speeds by observation of change of position.

Much of the classical research on motion thresholds has been conducted under experimental conditions far removed from the real world. For instance the moving object was usually minute in size and totally lacking character (e.g., a spot of light). Also the test subject usually viewed this minute object with monocular vision. Lastly, the visual field on which the object moved was usually stationary and also lacking in character. These conditions are very rarely found in the real world.

To appreciate the complexity of an image in the real world, consider the situation faced by a driver following a lead vehicle. The moving object that he is most concerned with, of course, is the lead
vehicle. This lead vehicle is of appreciable size and has characteristics which are non-uniform. Surrounding the lead vehicle is a visual field which is not stationary or uniform. Lastly, the driver is using binocular vision. The common highway situation described above will be the subject under study in this chapter. This study will be centered on the threshold of the relative motion between the lead and rear vehicles as viewed by the driver of the rear vehicle. In the following section, the subject of environmental control will be discussed.

C. Control of the Experimental Environment

A driver following another vehicle is presented with many time varying cues, most of which affect his driving behavior. One cue is the time variation of the light intensity enveloping the lead vehicle, scenery, and roadway. Another is the time variation in the scenery, road conditions, and traffic flow. His most important cue is the state of the lead vehicle that he is following. For instance, the acceleration of the lead vehicle (whether negative or positive) has a definite effect on the behavior of the driver in the following vehicle. In studying this driver's behavior, an attempt must be made to control his physical environment as well as the state of the lead vehicle. This was accomplished by using a simulator \(^1\). However, it must be remembered that a simulation is always an approximation and consequently any
results obtained from simulator studies are also approximate. A description of the optics of the simulator is given in the following section.

D. Simulator Optics

A block diagram of the optical system of the simulator is shown in Fig. 10. Figure 11 shows a ray diagram of how the image of the lead vehicle is processed by the closed circuit T.V. system. For simplicity, only a horizontal line element of the lead vehicle has been considered as the source image.

The various variables shown in Fig. 11 were defined as follows

\( w_s \) = width of lead vehicle (toy car)

\( h_s \) = simulator headway

\( \theta_s \) = horizontal camera angle

\( K \) = linear amplification of closed circuit T.V. system

\( K_b \) = width of T.V. image of lead vehicle

\( q \) = distance from eyes of driver in rear vehicle to T.V. screen

\( \alpha_s \) = horizontal visual angle.

In the real world the car following situation can be represented as shown in Fig. 12. Here \( \alpha \) is the real world visual angle, \( w \) is the width of the lead vehicle, and \( h \) is the distance from the eyes of the driver in the rear vehicle to the rear bumper of the lead vehicle.
Fig. 10--Block diagram of simulator optical system

Fig. 11--Ray diagram of closed circuit T.V. system

Fig. 12--Ray diagram of car following situation
The problem now arises of relating the physical variables in the real world to the physical variables in the simulator. This interrelation is derived in the following section.

E. Transformation from the Simulation to the Real World

If the results of the simulation are to be useful, then the variables in the simulation must vary exactly as the corresponding variables in the real world. If this situation does not exist, then a transformation must be made to convert the simulator results such that they will be valid in the real world. It was found necessary to make a transformation of this type because at a given headway the simulation visual angle $\alpha_s$ was not equal to the real world visual angle $\alpha$.

As a first step in the derivation of this transformation, consider the horizontal camera angle $\theta_s$. All angles in the simulator were sufficiently small such that the small angle approximations were valid. Therefore, from Fig. 11 it follows that

$$\theta_s = \frac{w_s}{h_s} = \frac{b}{p}.$$  

Also

$$\alpha_s = \frac{Kb}{q} = \frac{Kp\theta_s}{q} = K_1 \theta_s \quad \text{or} \quad \alpha_s = K_1 \frac{w_s}{h_s}$$

From Fig. 12 it is evident that $\alpha = \frac{w}{h}$. 
The scale factor relating lengths in the real world to lengths in the simulation is $K_2$. Hence $w_s = K_2 w$. Now if $\alpha_s$ is to be equal to $\alpha$, then it is a simple matter to show that the following relation must exist between the headways in the real world and the simulation

$$h_s = K_1 K_2 h.$$  

This means that the simulator headway $h_s$ can be converted to the corresponding headway in the real world simply by dividing by $K_1 K_2$. Also the relative velocity $v_s$ in the simulation is related to the relative velocity $v$ in the real world by the equation

$$v_s = K_1 K_2 v$$

where $v_s = \dot{h}_s$ and $v = \dot{h}$.

F. Description of Experiment

In the automobile simulator, the driver of the following vehicle observes the state of the lead vehicle by a T.V. monitor. The driver's vehicle is automatically controlled so that his only task is to observe the lead vehicle. The state of the lead vehicle is controlled by the analog computer. A block diagram of the control arrangement is shown in Fig. 13.
Initially the headway is at some value \( h_0 \) and the velocities \( v_1 \) and \( v_2 \) are set equal to 50 mph. As a result the initial relative velocity \( v \) is zero. This set of conditions is termed the steady state. Now if \( v_1 \) is changed by an amount \( \Delta v_1 \), \( v \) will also change by the same amount since \( v = v_1 - v_2 \). This change in relative velocity will result in a corresponding change of \( \Delta h \) in the headway.

Three male subjects with normal vision were hired for the experiment. All subjects had driving experience and ranged in age from 20 to 28 years. The experiment was performed in a darkened
room. In performing the experiment, the following procedure was used.

G. Procedure

Initially both vehicles' velocities were adjusted such that steady-state conditions existed. Each subject was then instructed to observe his monitor when the command "observe" was given. A fraction of a second later a step change in velocity $\Delta v$ of duration equal to $T_p$ was imparted to the lead vehicle. After this presentation the subject was given the command "record". On this command he recorded whether or not he had detected any motion of the lead vehicle during the presentation time $T_p$. If he had detected motion during this interval he was also to record the sense of the motion (i.e., was the relative velocity positive or was it negative). After the subjects had recorded their observations, the lead vehicle was returned to its original steady state. Another run was then made. The sequence of events as well as the change in the variables $v$ and $h$ are shown in Fig. 14. Fifty such runs were made at one setting. For the presentation times used, this took about 15 minutes. After a rest period of 5 minutes another 50 runs were made and so on until a total of 200 runs were completed. A rest period was necessary to minimize fatigue. Experimentation was restricted to 3 hours in the morning and 3 in the afternoon. With this
schedule a period of 2 months was required to complete the experimenta-
tion (this period included down time). A total of 84,000 runs were made.

The subjects were presented with nine different incremental velocities $\Delta v$ for a given set of steady state conditions. These velocities had positive, negative, and zero magnitudes and were presented to the driver in a random sequence. That is, the largest negative incremental velocity was denoted by 1, the next largest by
2, \ldots, zero velocity was represented by 5, \ldots, the second largest positive velocity by 8, and the largest positive velocity by 9. The velocities to be used were then randomly selected by using the random sequence of the digits 1 through 9 shown in Fig. 15. One thousand runs were made for each combination of steady state headway $h_o$ and presentation time $T_p$. For statistical purposes each block of 200 runs was considered as one set of runs. Hence, for each combination of $h_o$ and $T_p$ there are five sets. Simulator steady state headways of 15.7, 28.7,
41.7 and 61.2 cms were used. Since \( K_1 = 0.647 \) and \( K_2 = 0.343 \) cms/ft then it follows from the equation \( h_s = K_1 K_2 h \) that the corresponding real world headways are 70.7, 129, 188 and 276 feet. The presentation times \( T_p \) were 0.31, 0.50, 1.00, 1.50, 2.00, 3.00 and 5.00 seconds. Results obtained from the experiment will be reported upon in the following section.

H. Results

1. Velocity perception curve

The 5 sets of runs for a specific headway \( h_0 \) and presentation time \( T_p \) were used to plot 5 velocity perception curves. These curves are plots of the probability of velocity detection versus the incremental velocity \( \Delta v \). The probability of velocity detection for a given velocity increment was determined by dividing the total number of runs in which the subject detected motion correctly (i.e., the subject had to not only determine if motion had taken place but also the sense of the motion), by the total number of runs and expressing the result as a percentage. From hereon the incremental velocity \( \Delta v \) will be referred to as \( v \).

Figure 16 shows the 5 velocity perception curves for subject A for a headway of 129 ft and a presentation time \( T_p \) of 1.5 sec. Notice there is some variation between the several curves. These curves were then averaged to give one velocity perception curve per subject for
Fig. 16--Velocity perception curves
each combination of headway and presentation time. The three velocity perception curves so obtained were then averaged to give the average velocity perception curve. These average velocity perception curves as well as the velocity perception curves of each subject are shown for two experimental values of $h_0$ and $T_p$ in Figs. 17 and 18. The other values of $h_0$ and $T_p$ used, yielded similar curves.

2. **Confidence limits**

Confidence limits were calculated for the average velocity perception curve in order to determine the degree of variability of this curve. Student's $t$ was used to determine the confidence limits. Limits of 90% were considered to be practical and consequently were used.

In general, Student's $t$ is defined by the equation

$$t = \frac{\bar{x} - \mu}{\frac{S_x}{\sqrt{n}}}$$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

and

$$S_x = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n(n-1)}}$$

The variable $X_i$ denotes the i-th subject's average probability of detection of motion for a given increment of velocity. For $n = 3$ (3 subjects) and 90% confidence, the variable $t$ must satisfy the following inequality.
$h_0 = 129$ Ft.
$T_p = 0.5$ sec.

Fig. 17--Average velocity perception curves
Fig. 18--Average velocity perception curves
\[ -2.35 \leq t \leq +2.35 \]

The value of \( \mu \) calculated using Eqs. (5) and (6) will lie between two limits for a given incremental velocity. These limits are the 90\% confidence limits of the average velocity perception curve and two typical sets of limits are shown in Figs. 17 and 18. The threshold velocity then was determined from the average velocity perception curve.

3. **Threshold velocity characteristics**

The 50 percentile threshold velocity is defined as that velocity for which the subject has a 50\% probability of detection for a given set of steady state conditions. For the remainder of this discussion, the 50 percentile threshold velocity will be simply called the threshold velocity. Note that there will be both a positive and a negative threshold velocity for a given set of steady state conditions.

Figure 19 shows the relation between the positive threshold velocity \( v_t \) and the headway \( h \) where \( T_p \) is a parameter. In general, the velocity threshold decreases as \( T_p \) increases. The equations of the curves shown in Fig. 19 have the general form \( v_t = K h^n \). The constants \( K \) and \( n \) are dependent on \( T_p \) and have the values shown in Table I.
Fig. 19—Positive velocity threshold characteristics

TABLE I

<table>
<thead>
<tr>
<th>$T_p$ (sec)</th>
<th>$K$ (mph)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>$38.2 \times 10^{-4}$</td>
<td>1.41</td>
</tr>
<tr>
<td>0.50</td>
<td>$13.9 \times 10^{-4}$</td>
<td>1.53</td>
</tr>
<tr>
<td>1.00</td>
<td>$21.9 \times 10^{-4}$</td>
<td>1.35</td>
</tr>
<tr>
<td>1.50</td>
<td>$3.11 \times 10^{-4}$</td>
<td>1.66</td>
</tr>
<tr>
<td>2.00</td>
<td>$16.4 \times 10^{-4}$</td>
<td>1.31</td>
</tr>
<tr>
<td>3.00</td>
<td>$10.3 \times 10^{-4}$</td>
<td>1.33</td>
</tr>
<tr>
<td>5.00</td>
<td>$8.09 \times 10^{-4}$</td>
<td>1.31</td>
</tr>
</tbody>
</table>
Figures 20 and 21 show the relation between the negative velocity threshold $v_t$ and the headway $h$ where $T_p$ is a parameter. Figure 20 is a log-log plot while Fig. 21 is a semi-log plot. From the shape of the curves in these two figures, it is evident that for $T_p = 1$ and 2 seconds $v = Kh^n$ while for $T_p = 0.31, 0.50, 1.50, 3.00$ and $5.00$ seconds $v_t = K_1 e^{mh}$. For $T_p = 1$ and 2 seconds, the constant $K$ has values of $9.65 \times 10^{-3}$ mph and $4.46 \times 10^{-3}$ mph respectively, while $n$ has values of 1.05 and 1.07 respectively. The values of $K_1$ and $m$ are also dependent on $T_p$ and this dependence is shown in Table II.

TABLE II

<table>
<thead>
<tr>
<th>$T_p$(sec)</th>
<th>$K_1$ (mph)</th>
<th>$m$(ft$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31</td>
<td>1.34</td>
<td>$6.68 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.50</td>
<td>0.771</td>
<td>$7.77 \times 10^{-3}$</td>
</tr>
<tr>
<td>1.50</td>
<td>0.280</td>
<td>$7.44 \times 10^{-3}$</td>
</tr>
<tr>
<td>3.00</td>
<td>0.171</td>
<td>$7.14 \times 10^{-3}$</td>
</tr>
<tr>
<td>5.00</td>
<td>0.142</td>
<td>$6.99 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

The 90% confidence limits for the positive and negative threshold velocities as functions of $h$ are shown in Figs. 22 through 35.

The preceding three subsections have been concerned with the presentation of the macroscopic results of the velocity threshold experiment. At this point the problem of relative motion thresholds will be investigated in a microscopic manner by applying physiological techniques.
Fig. 20--Negative velocity threshold characteristics
Fig. 21--Negative velocity threshold characteristics
Fig. 22--Confidence limits of positive velocity threshold characteristic

Fig. 23--Confidence limits of positive velocity threshold characteristic
Fig. 24--Confidence limits of positive velocity threshold characteristic

Fig. 25--Confidence limits of positive velocity threshold characteristic
Fig. 26--Confidence limits of positive velocity threshold characteristic

Fig. 27--Confidence limits of positive velocity threshold characteristic
Fig. 28--Confidence limits of positive velocity threshold characteristic

Fig. 29--Confidence limits of negative velocity threshold characteristic
Fig. 30--Confidence limits of negative velocity threshold characteristic

Fig. 31--Confidence limits of negative velocity threshold characteristic
Fig. 32--Confidence limits of negative velocity threshold characteristic

Fig. 33--Confidence limits of negative velocity threshold characteristic
Fig. 34--Confidence limits of negative velocity threshold characteristic

Fig. 35--Confidence limits of negative velocity threshold characteristic
I. The Psychophysical Law

Psychophysics is concerned with the relations between the physical world and the sensational world. In particular, the psychophysicist is interested in the relation between a physical stimulus and the resultant psychological sensation. If the sensation is denoted by \( \psi \) and the stimulus by \( \phi \), then the psychophysicist is interested in the general function \( \psi = f(\phi) \). This functional relationship is the psychophysical law. Weber (9) made the first significant contribution to psychophysics by proposing the law \( \Delta \phi = K \phi \). This law is sometimes written as \( \Delta \phi / \phi = K \) where \( K \) is the j.n.d. (just noticeable difference). Physically this law states that if the human observing \( \phi \) just detects a change in \( \phi \) (i.e., a just noticeable difference) then the change \( \Delta \phi \) was of a magnitude such that

\[
\frac{\Delta \phi}{\phi} = K.
\]

This law holds over a range of values of \( \phi \).

Fechner (10) then proposed that the sensation is related to the stimulus by the equation

\[
\Delta \psi = CK
\]

where \( C \) is a proportionality constant and \( \psi \) ranges over the whole continuum under consideration. By substituting for \( K \) and integrating, it is possible to arrive at Fechner's law \( \psi = C \ln \frac{\phi}{\phi_0} \) as follows:
\[ \Delta \psi = CK = \frac{C \Delta \phi}{\phi} \quad \psi = C \ln \phi + C_1 \]

when \( \psi = 0 \) then \( C_1 = -C \ln \phi_0 \).

Hence \( \psi = C \ln \frac{\phi}{\phi_0} \).

This law is known as the classical psychophysical law and has been found to hold in what may be termed small-stimulus cases.

Recently Stevens (11) has proposed that \( \Delta \psi \neq C \frac{\Delta \phi}{\phi} \) over all values of \( \psi \) but that instead \( \frac{\Delta \psi}{\psi} = C_1 \frac{\Delta \phi}{\phi} \). This appears to be a more reasonable assumption. By integration it is possible to obtain Stevens' law which is

\[ \psi = C_2 \phi^{C_1} \]

According to Stevens, this is the true psychophysical law. It does appear to hold for many different types of stimuli under large-stimulus conditions.

Weber's law will now be applied to the velocity threshold data in the hopes of determining a more fundamental insight into the psychophysical process of velocity perception.
J. Justification of Weber's Law

In general Weber's law states that \( \frac{\Delta \phi}{\phi} = K \) where \( \phi \) is the stimulus variable and \( K \) is a constant. If \( \alpha \) is taken to be the stimulus variable then the ratio \( \frac{\Delta \alpha}{\alpha} \) should be constant over the continuum of \( \alpha \) if Weber's law is to hold. The incremental change in visual angle \( \Delta \alpha \) is that angular change which must take place before the observer can perceive motion 50\% of the time for a given presentation time of the incremental change.

The variable \( \Delta \alpha \) was calculated from the experimental data by first calculating \( \Delta h \) where

\[
\Delta h = 1.466 v_t T_p
\]

if \( v_t \) has units of mph and \( T_p \) is expressed in seconds. Since it is known that \( \alpha = \frac{W}{h} \) (section E), it follows that

\[
\Delta \alpha = -\frac{W}{h^2} \Delta h.
\]

The width \( W \) of the lead vehicle was taken to be 7 feet so that consequently \( \Delta \alpha = 401 \frac{\Delta h}{h^2} \) degrees if \( \Delta h \) and \( h \) are expressed in feet.

Figure 36 shows the variation of \( \frac{\Delta \alpha}{\alpha} \) with \( \alpha \) where \( T_p \) is a parameter and the angular change \( \Delta \alpha \) is positive (headway increasing). If \( \Delta \alpha \) is negative (headway decreasing) then the ratio \( \frac{\Delta \alpha}{\alpha} \) varies with \( \alpha \) as shown in Fig. 37. It is evident from Figs. 36 and 37 that Weber's law is approximately valid for each value of \( T_p \). The
Fig. 36--Weber's law ($\phi = +\alpha$)

Fig. 37--Weber's law ($\phi = -\alpha$)
dependence of $\frac{\Delta \alpha}{\alpha}$ on the presentation time $T_p$ is shown in Figs. 38 and 39. These figures indicate that the Weber ratio $\frac{\Delta \alpha}{\alpha}$ increases approximately linearly with $T_p$. To further illustrate the approximate validity of Weber's law, $\Delta \alpha$ was plotted versus $\alpha$. The resultant curves are approximately linear and are shown in Figs. 40 and 41.

Since Weber's law is approximately valid for $\phi = \alpha$, it would appear that the change in visual angle is the stimulus that the observer is using in the perception of velocity. This conclusion cannot be drawn as Weber's law is not unique in this psychophysical problem. The following discussion clearly shows why this is the case.

Since $\alpha = \frac{W}{h}$ and $\Delta \alpha = -\frac{W}{h^2}\Delta h$ it follows that $\frac{\Delta \alpha}{\alpha} = -\frac{\Delta h}{h}$.

It has been experimentally shown that $\frac{\Delta \alpha}{\alpha} \approx K$. Consequently $\frac{\Delta h}{h} \approx -K$. This equation is simply Weber's law for the case where $\phi = h$ and seems to indicate that $h$ is the stimulus variable.

The area $A$ of the lead vehicle image on the T.V. monitor of the simulator can also be considered as a stimulus variable which obeys Weber's law. Since $A = ab$, it follows that $\Delta A = a\Delta b + b\Delta a$ as shown in Fig. 42. Experimentally $b = 0.708a$ so that $\Delta b = 0.708\Delta a$. Consequently $\Delta A = 1.416a\Delta a$. 
Fig. 38 -- Dependence of $\frac{\Delta \alpha}{\alpha}$ on $T_p$

Fig. 39 -- Dependence of $-\frac{\Delta \alpha}{\alpha}$ on $T_p$
Fig. 40--Weber's law ($\phi = +\alpha$)

Fig. 41--Weber's law ($\phi = -\alpha$)
Since $\alpha_S = \frac{a}{l}$ then $\Delta \alpha_S = \frac{\Delta a}{l}$. Also $\alpha_S = \alpha$ so that $\Delta \alpha_S = \Delta \alpha$.

This means that $a = \frac{Wl}{h}$ and $\Delta a = -\frac{Wl}{h^2} \Delta h$. As a result

$$A = 0.708 \frac{W^2 l^2}{h^2}$$

and

$$\Delta A = 1.416 \frac{W^2 l^2}{h^3} \Delta h.$$ It follows that

$$\frac{\Delta A}{A} = -\frac{2\Delta h}{h} = 2K.$$ Again this equation is Weber's law and the stimulus now appears to be the area $A$. In the real world the image of the lead vehicle on the observer's retina would have an area directly proportional to $A$. 

Fig. 42--Ray diagram of lead vehicle image
The above analysis clearly shows that Weber's law is not unique for this particular psychophysical problem. Mathematically it has been shown that the stimulus variable can be the visual angle \( \alpha \), the headway \( h \), or the area \( A \).

The psychophysical law will now be applied to the velocity threshold data.

K. Application of the Psychophysical Law

It has been shown that Weber's law is not unique for the velocity threshold problem under study. Consequently, Weber's law cannot be used to determine which stimulus is used by an observer in the perception of motion. An attempt will now be made to apply the psychophysical law to the velocity threshold problem in the hope of obtaining a general equation for velocity perception as well as the stimulus variable.

Assume that the stimulus \( \phi \) is the visual angle \( \alpha \). The corresponding sensation \( \psi \) is, of course, the threshold velocity \( v_t \) where \( v_t \) is defined as that velocity which the observer can just detect 50% of the time. Fechner's law then states that the relation between \( v_t \) and \( \alpha \) should be of the following form

\[
(7) \quad v_t = K \ln \frac{\alpha}{\alpha_0}.
\]
A plot of $\alpha$ versus plus and minus $v_t$ on semi-log graph paper is shown in Figs. 43 and 44. If Eq. (7) is to be valid then the curves shown in Figs. 43 and 44 should be straight lines. Since these curves are not straight lines, it follows that Fechner's law does not hold for $\phi = \alpha$. Stevens' law, on the other hand, states that the relation between $v_t$ and $\alpha$ should be of the form

$$v_t = K\alpha^n$$  

Figures 45 and 46 show plots of plus and minus $v_t$ versus $\alpha$ on log-log graph paper. If Eq. (8) is to be valid then the curves in Figs. 45 and 46 should be straight lines with slope $n$. Examination of Figs. 45 and 46 shows that this is not the case, and consequently Stevens' law also does not hold for $\phi = \alpha$. Thus, it has been shown that neither Fechner's or Stevens' law holds for the experimental data on the velocity threshold if the visual angle alone is taken as the stimulus $\phi$.

Now assume that the stimulus $\phi$ is the time rate of change of the visual angle $\alpha$. That is $\phi = \frac{d\alpha}{dt}$. Since $\alpha = \frac{W}{h}$ it follows that

$$\frac{d\alpha}{dt} = -\frac{W}{h^2} v_t$$ as $\frac{dh}{dt} = v_t$. Weber's law states that $\frac{d\phi}{\phi} = K$. 

Fig. 43--Fechner's law ($\phi = +\alpha$)
Fig. 44--Fechner's law ($\phi = -\alpha$)
Fig. 45—Stevens' law ($\phi = +\alpha$)
Fig. 46--Stevens' law ($\phi = -\alpha$)
Therefore \( d\phi = d \left( \frac{d\alpha}{dt} \right) = \left( \frac{d^2 \alpha}{dt^2} \right) dt \).

The second derivative \( \frac{d^2 \alpha}{dt^2} \) can be easily evaluated and has the value
\[ \frac{d^2 \alpha}{dt^2} = \frac{2Wv_t^2}{h^3} \]
as the acceleration \( \frac{dv_t}{dt} = 0 \). It is now possible to apply Fechner's law which in differential equation form is
\[ dv_t = \frac{C \left[ \frac{d^2 \alpha}{dt^2} \right]}{\frac{d\alpha}{dt}} dt \].

On substituting for \( \frac{d^2 \alpha}{dt^2} \) and \( \frac{d\alpha}{dt} \) and realizing that \( h = h_0 + v_t t \), the following differential equation results:
\[ \frac{d^2 v_t}{dt} = \frac{-2C v_t}{h_0 + v_t t} \].

This differential equation has the solution
\[ T_p = C_2 \epsilon - \frac{1}{2C} v_t g(v_t, h_0) \]
where
\[ C_2 g(v_t, h_0) = \left[ C_1 - \frac{h_0}{2C} \left( \frac{ln v_t + \frac{1}{2C} \frac{v_t}{1!} \frac{v_t^2}{8C^2 2!} + \frac{v_t^3}{24C^3 3!} + \cdots } \right) \right] \].

Since \( \epsilon - \frac{1}{2C} v_t \) is the dominant term, then the solution may be simplified to the approximate form
\[ T_p = C_3(h) \epsilon - \frac{1}{2C} v_t \quad (9) \].
Figures 47 and 48 show plots of $T_p$ versus plus and minus $v_t$ on semi-log graph paper. Since the curves in Figs. 47 and 48 are not straight lines it follows that Eq. (9) does not hold. It is also possible to apply Stevens' law which in differential equation form is

$$\frac{d v_t}{v_t} = C \left[ \frac{d^2 \alpha}{dt^2} \right] dt$$

On substituting for $\frac{d^2 \alpha}{dt^2}$ and $\frac{d \alpha}{dt}$ and realizing that $h = h_0 + v_t t$, the following differential equation results

$$\frac{d v_t}{dt} = -2C \frac{v_t^2}{h_0 + v_t t}$$

The solution of this equation has the form

$$T_p = \left( \frac{h_0}{2C - 1} \right) v_t^{-1} + C_1 v_t^{-1/2C}$$

If $2C \approx 1$, then this equation can be simplified to the following approximate form

$$v_t = K_2 (h) T_p^{-n}$$

where $n = 2C$.

Figures 49 and 50 show plots of plus and minus $v_t$ versus $T_p$ on log-log graph paper. The method of least squares was used to obtain the best linear fit among the data points for each headway. Since the
Fig. 47--Fechner's law ($\phi = -\frac{d\alpha}{dt}$)
Fig. 48--Fechner's law ($\phi = -\frac{d\alpha}{dt}$)
Fig. 49 -- Stevens' law \( \phi = + \frac{d\alpha}{dt} \)

Fig. 50 -- Stevens' law \( \phi = - \frac{d\alpha}{dt} \)
curves in these figures are straight lines and have slopes approximately equal to 1, it follows that Stevens' law holds if the stimulus is taken as \( \frac{d\alpha}{dt} \). The positive velocity threshold equation was determined to be \( v_t = K_2 \frac{T_p}{T_p - 0.743} \) while the negative velocity threshold equation was determined as \( v_t = K_2 T_p^{-0.811} \).

It is also evident that \( K_2 = K_2(h) \). Plots of \( K_2 \) versus \( h \) on semi-log graph paper (Fig. 51) and log-log graph paper (Fig. 52) reveals that \( K_2 = 0.00187 h^{1.38} \) for positive velocity thresholds while for negative threshold velocities \( K_2 = 0.47 e^{-0.00705h} \). In both cases the method of least squares was used to obtain the best linear fit among the data points.

Consequently the perception of threshold velocity by an observer can be described by the following invariant equations if the velocity is presented to the driver in the form of \( v_t(t) - v_t(t + T_p) \) at some initial static headway \( h \) where \( v_t(t) \) is a constant.

\[
\text{(10) } v_t^{\text{positive}} v_t \frac{T_p}{T_p - 0.743} h^{-1.38} = 0.00187
\]

\[
\text{(11) } v_t^{\text{negative}} v_t \frac{T_p}{1 - 0.811 e^{-0.00705h}} = 0.47
\]

In these equations \( h \) is expressed in feet, \( T_p \) is in seconds and \( v_t \) is in mph. These equations are valid only for the range of variable values given below.
Fig. 51 -- Determination of $K_2(h)$

Fig. 52 -- Determination of $K_2(h)$
0.2 mph ≤ + \( v_t \) ≤ 11.6 mph

0.2 mph ≤ \( v_t \) ≤ 8.7 mph

0.31 sec ≤ \( T_p \) ≤ 5 sec.

71 ft. ≤ \( h \) ≤ 276 ft.

The invariance of the general velocity threshold equations (Eqs. (10) and (11)) was checked by calculating the left hand portions of these equations for all of the experimental data and then plotting the result as a function of \( T_p \), \( h \), and \( v_t \). Figures 53 through 58 show these plots. It is evident from these plots that the general velocity threshold equations are in fact, almost invariant in that there is no consistent trend of the data points with these variables. This argument for invariance is further strengthened by the fact that the deviations for Eqs. (10) and (11) are only 13.7% and 17.5% respectively.

L. Effect of Presentation Time on Stimulus Magnitude

The velocity threshold stimulus \( \frac{\Delta a}{\Delta t} \) was plotted against \( T_p \) to graphically determine the dependence of the stimulus magnitude on the presentation time. Determination of \( \frac{\Delta a}{\Delta t} \) was made by first determining \( \Delta a \) and then differentiating with respect to time. Since \( \alpha_s = \frac{K_1 W_s}{h_s} \) it follows that \( \Delta \alpha_s = \frac{-K_1 W_s^2}{h_s} \Delta h_s \). On differentiating with respect to time, dropping the negative sign and transforming \( h \), the following equation results
Fig. 53--Dependence of the positive general velocity threshold equation on $T_p$

Fig. 54--Dependence of the positive general velocity threshold equation on $h$
Fig. 55--Dependence of the positive general velocity threshold equation on $+v_t$

Fig. 56--Dependence of the negative general velocity threshold equation on $T_p$
Fig. 57--Dependence of the negative general velocity threshold equation on $h$

Fig. 58--Dependence of the negative general velocity threshold equation on $-v_t$
\[
\frac{\Delta \alpha_s}{\Delta t} = \frac{\Delta \alpha}{\Delta t} = \frac{W_s \, v_t}{K_2 \, h^2}
\]

On substituting values for \( W_s \) (2.40 cms) and \( K_2 \) (0.343 cm/ft), the equation for \( \frac{\Delta \alpha}{\Delta t} \) becomes

\[
\frac{\Delta \alpha}{\Delta t} = 588 \, \frac{v_t}{h^2} \frac{\text{degrees}}{\text{second}}
\]

In this equation \( v_t \) is measured in mph and \( h \) in feet.

Figures 59 and 60 show the resultant functional relationship between \( \frac{\Delta \alpha}{\Delta t} \) and \( T_p \). Examination of these curves reveals that the stimulus magnitude decreases approximately exponentially as the presentation time is increased. This result is the one that is expected from psychophysical considerations. That is, if the presentation time of a stimulus is increased, it follows that the magnitude of the required stimulus should decrease as the subject has a longer time in which to process the information in the stimulus.
Fig. 59--Effect of presentation time on stimulus magnitude (v positive)
Fig. 60--Effect of presentation time on stimulus magnitude (v negative)
CHAPTER IV

THE ACTION POINT MODEL

A. Introduction

In Chapter II the problem of car-following was defined and discussed. An analysis was made of the steady state condition by using phase plane trajectories. This analysis led to the discussion of Barbosa's decision point model which was found to be inadequate in many respects. In attempting to justify this model it was found advantageous to use the second order phase plane trajectory. A preliminary analysis of car following using these trajectories pointed the way to the development of the action point model framework. In this analysis it was found that the relative velocity was one of the fundamental variables of the action point model.

In Chapter III a detailed experimental statistical analysis was made of the threshold of the relative velocity. The most important result of this analysis was the determination of two general velocity threshold equations. With these equations it is but a simple matter to determine a person's velocity threshold (whether positive or negative) for a given headway and presentation time.
In this chapter, an attempt will be made to use the velocity threshold data of Chapter III to give support as well as a stronger basis for development of the action point model proposed in Chapter II.

B. The Action Point

The discussion in Chapter II has shown that the action point exists and is readily determinable from the a-v trajectories. In order to gain more knowledge about the nature of the action points, an experiment was performed in car-following on the simulator for different steady state velocities of the lead vehicle. Steady state velocities of 30, 50, and 70 mph were used and the instructions to the driver of the rear vehicle were that he drive at what he considered to be a minimum safe headway. Both the a-v and v-h trajectories were recorded.

The resultant a-v trajectories were then used to determine the unnormalized approximate action point density as a function of the relative velocity v. These density functions are shown in Figs. 61, 62, and 63. Notice that as the steady state velocity increases, the relative velocity for which the action point density is a maximum also increases. This velocity will be designated by $v_m$. The average headway $h_m$ was also found to increase with $v_1$ at the rate of 1 foot per each mph. Since the maximum action point density occurs at some value of $v(v_m)$ for a given lead vehicle velocity, it appears that the action point is somehow
$V_1 = 30 \text{ mph}$

$h_m = 31 \text{ Ft.}$

Fig. 61--Action point density function $V_1 = 30 \text{ mph}$
Fig. 62—Action point density function $V_1 = 50$ mph.

$V_1 = 50$ mph

$h_m = 49$ Ft.
Fig. 63--Action point density function $V_1 = 70$ mph

$V_1 = 70$ m.p.h.

$h_m = 70$ Ft.
related to one particular value of relative velocity for a given velocity of the lead vehicle.

Plots of the unnormalized approximate action point density as a function of the relative acceleration $a$ are given in Figs. 64, 65, and 66. These figures indicate that the maximum action point density occurs at a value of $a$ ($a_m$) which does not vary appreciably as $v_1$ is increased.

If the action point density functions are normalized with respect to both $a$ and $v$, then two probability hills will result if these normalized probability density functions are plotted against $a$ and $v$ (i.e., three co-ordinate axes must be used). If only the $a$ and $v$ co-ordinates are used, then probability contours will result. These are shown in Fig. 67 where $v_1$ is some constant. The numbers within the contours denote the probability of an action point occurring within the contours if the driver of the rear vehicle is following a lead vehicle at a minimum safe headway. Figure 68 shows the orientation of experimental data points on the $a$-$v$ plot when $v_1 = 30$ mph. Note that the distribution of the data points is approximately as expected from Fig. 67. It is evident from this discussion that the location of the action point in time and space is essentially the result of a statistical process and that consequently it is impossible to predict the location of an action point on other than a statistical basis.
Fig. 64--Action point density function $V_1 = 30$ mph
Fig. 65--Action point density function $V_1 = 50$ mph
Fig. 66—Action point density function $V_1 = 70$ mph
Fig. 68—Action point distribution

- $V_i = 30$ mph
- $x = \text{Action Point}$

- $a (g)$
- $v (mph)$
C. Correlation of the Threshold Velocity with the v-h Trajectory

In the preceding section it was shown experimentally that the peak action point density occurred at one specific value of v (v_m) for a given velocity of the lead vehicle. Comparison of v_m and h_m with threshold velocity data indicated that v_m was very close to being equal to v_t for h equal to h_m. This result then meant that the driver was probably basing his decision to take action, on his threshold velocity for the particular headway at which he was traveling. In other words the driver had to first realize or sense that the relative velocity was some value other than zero before he could make a decision and take corrective action.

Figure 69 shows the threshold velocity v_t as a function of h for several values of T_p. Notice that for most values of T_p the positive threshold velocity is greater than the negative threshold velocity for the same headway. This asymmetry is shown very clearly in Fig. 70.

For simplicity, assume that the driver operates on only one set of threshold curves as shown in Fig. 71. Further, assume that initially the driver is at point A on his v-h trajectory and traveling such that v = -k and a = 0. At point B the driver's trajectory intersects the negative velocity threshold curve. After a short perception delay, the driver is able to perceive the velocity. This occurs at point C and this
Fig. 69—Velocity threshold characteristics
Fig. 71--The perception-decision-action process
point is consequently called the velocity perception point. After another delay (during which the driver processes the data obtained at C), the driver makes his decision at point D (the decision point). Following another delay interval, the driver finally takes action at point E (action point) by decelerating at \(-k_1 g\), and goes into a parabolic trajectory. When point F on the positive velocity threshold curve is reached, it is postulated that the driver goes through a perception-decision-action process similar to the one that he underwent in going from point B to E. Finally at point I he takes action and accelerates at \(+k_1 g\). When his trajectory intersects the negative threshold curve at J, the driver again goes through his sequence of perception, decision, and action until he reaches point M. Hence in this fashion the driver completes one cycle of his trajectory. In going through one cycle of the trajectory the driver has used the perception-decision-action process two times. This process may seem highly complicated but it is felt the human driver can easily perform the task in an automatic manner.

Figure 72 shows the effect of different initial conditions on the resultant v-h trajectory. If initially the driver is at point A or point \(A_1\), then his resultant v-h trajectory will slowly drift to the right (headway increasing) as the number of completed cycles increase. This means that the above trajectories are unstable. In the real world
Fig. 72--Stability of calculated v-h trajectories
the driver would find his headway slowly increasing if he was using the process outlined above to follow a lead vehicle. However, if the driver is initially at points $A_2$, $A_3$ or $A_4$, then his resultant $v$-$h$ trajectories are stable as they do not drift to the right or the left. The conditions for stability of the limit cycles will now be determined.

D. Stability Analysis of the Limit Cycle

Stability of the limit cycle is dependent on four variables. These variables are acceleration ($A_2$), deceleration ($A_1$), delay time ($\Delta t$), and threshold velocity. In this context a stable limit cycle will be defined as one whose net drift per cycle ($\Delta h$) is zero.

Figure 73 shows a typical limit cycle which will be used to obtain a general expression for the net drift per cycle ($\Delta h$). In this derivation it is assumed that the driver uses constant values of acceleration and deceleration. Consequently, the equation of the $v$-$h$ trajectory is given by $h = \frac{v^2}{2A}$ and it follows that:

$$\Delta h_2 = \frac{(b + \Delta v_2)^2}{2A_2} = \frac{1}{2A_2} \left[ b^2 + 2b\Delta v_2 + \Delta v_2^2 \right]$$

$$\Delta h_1 = \frac{(a + \Delta v_1)^2}{2A_1} = \frac{1}{2A_1} \left[ a^2 + 2a\Delta v_1 + \Delta v_1^2 \right]$$

where $a$ and $b$ are the ordinates of the positive and negative intersection points respectively, and $\Delta v_1$ and $\Delta v_2$ are the resultant changes
Fig. 73--The limit cycle
in relative velocity due to movement along the trajectory from the intersection point to the action point. These variables are illustrated in Fig. 73.

It can be shown that $\Delta v_1 = A_1 \Delta t_1$ and $\Delta v_2 = A_2 \Delta t_2$ where $\Delta t_1$ is the delay time between the threshold intersection point 1 and the action point 2. Similarly $\Delta t_2$ is the delay time between the threshold intersection point 3 and the action point 4. Since $\Delta h = 2(\Delta h_2 - \Delta h_1)$, it finally can be shown that the general expression for the net drift per cycle is:

$$\Delta h = \left( \frac{b^2}{A_2} - \frac{a^2}{A_1} \right) + 2(b \Delta t_2 - a \Delta t_1) + (A_2 \Delta t_2^2 - A_1 \Delta t_1^2).$$

Now it has been found experimentally that the driver on the average uses constant levels of acceleration and deceleration in steady state car following. In addition, if the assumption is made that the delay time $\Delta t_1 = \Delta t_2 = \Delta t$, then the following case for Eq. (12) results.

**Case 1**

$A_2 = A_1 = A$ \hspace{1cm} $\Delta t_2 = \Delta t_1 = \Delta t$

With the above restrictions Eq. (12) becomes

$$\Delta h = \left( \frac{b^2 - a^2}{A} \right) + 2\Delta t(b-a)$$

If $b > a$ then $\Delta h > 0$ and the net drift is to the left as $\Delta h_2 > \Delta h_1$.

This situation is illustrated in Fig. 74. On the other hand if $b < a$ then $\Delta h < 0$ and the net drift is to the right as $\Delta h_2 < \Delta h_1$. This condition is shown in Fig. 75.
Fig. 74--Unstable limit cycle ($b > a$)
Fig. 75.—Unstable limit cycle ($b < a$)

- $b < a$
- $\Delta h_2 < \Delta h_1$
If $b = a$ then $\Delta h = 0$ and stability results. This is the stability condition for Case I and is shown in Fig. 76. It is also possible that $\Delta h$ can be made zero by setting $\frac{b^2 - a^2}{A} = 2 \Delta t(a-b)$. If simplified this equation becomes $\Delta t = \frac{(a+b)}{2A}$ and it is immediately evident that this condition for stability is physically unrealizable as the delay time must be negative.

The situation where $A_2 = A_1 = A$ and $b = a$ will now be examined.

**Case II** \[ A_2 = A_1 = A \quad b = a \]

Under these conditions Eq. (12) becomes

$$\Delta h = 2a(\Delta t_2 - \Delta t_1) + A(\Delta t_2^2 - \Delta t_1^2)$$

If $\Delta t_2 > \Delta t_1$ then $\Delta h > 0$ and consequently the net drift is to the left as shown in Fig. 77. However, if $\Delta t_2 < \Delta t_1$ then $\Delta h < 0$ and as a result the net drift is to the right as shown in Fig. 78. Stability results if $\Delta t_2 = \Delta t_1$ as $\Delta h = 0$. The resultant stable limit cycle is the same as shown in Fig. 76 because $\Delta v_1 = \Delta v_2 = \Delta v$ if $\Delta t_2 = \Delta t_1$. It is also possible that stability may exist if

$$2a(\Delta t_2 - \Delta t_1) = -A(\Delta t_2^2 - \Delta t_1^2)$$

Upon simplification, this equation is reduced to $A = \frac{-2a}{\Delta t_2 + \Delta t_1}$. Again this is a physically unrealizable stability condition as $A$ is defined to be positive for both acceleration and deceleration.
Fig. 76--Stable limit cycle
Fig. 77—Unstable limit cycle ($\Delta t_2 > \Delta t_1$)

$\Delta v_1$

$\Delta h_1$

$\Delta h_2$

$\Delta v_2$

$\Delta h_1 > \Delta h_2$

$\Delta t_2 > \Delta t_1$
Fig. 78--Unstable limit cycle ($\Delta t_2 < \Delta t_1$)
The situation where $\Delta t_2 = \Delta t_1$ and $b = a$ will now be examined.

**Case III** $\Delta t_2 = \Delta t_1$  
$b = a$

With these restrictions Eq. (12) becomes

$$
\Delta h = a^2 \left( \frac{1}{A_2} - \frac{1}{A_1} \right) + \Delta t_1^2 (A_2 - A_1).
$$

If $A_2 \neq A_1$ it is impossible to determine whether $\Delta h$ is positive or negative without actually knowing the values of all the variables involved. However, if $A_2 = A_1$ then stability results as $\Delta h = 0$ and the resultant limit cycle is as shown in Fig. 76. It is also possible to achieve stability by setting

$$
a^2 \left( \frac{1}{A_2} - \frac{1}{A_1} \right) = - \Delta t_1^2 (A_2 - A_1).
$$

This yields the equation

$$
a^2 = A_1 A_2 \Delta t_1^2
$$

which is a physically realizable stability condition. The resultant stable limit cycle is shown in Fig. 79.

From the analysis of Cases I, II, and III it is evident that stability is only possible under two sets of conditions. The first set of stability conditions requires that $A_2 = A_1$, $\Delta t_2 = \Delta t_1$, and $b = a$ while the second set requires that $\Delta t_2 = \Delta t_1$, $a = b$, and $a^2 = A_1 A_2 \Delta t_1^2$. 

\[ A_1 \neq A_2 \]
\[ \Delta h = 0 \]

Fig. 79--Stable limit cycle \((A_1 \neq A_2)\)
Reference to Fig. 72 reveals that the stable limit cycles are stable because the first set of stability conditions discussed above are satisfied.

A stability analysis will now be made where more than one set of positive and negative threshold curves are used by the driver. Since no restrictions were applied on the presentation times of the velocity thresholds in the above development, it follows that the stability criteria derived will be applicable.

E. Effect of Velocity Sampling Time on the v-h Trajectory

It does not seem reasonable to expect that the driver will use only one fixed visual sampling time in executing his v-h trajectory. Instead it is postulated that he uses several visual sampling times in a random manner. By using several sampling rates he might be able to achieve stability. To determine if this may actually be the case, four different values of $T_p$ were chosen and the corresponding threshold curves plotted on the phase plane as shown in Fig. 80. Point A was chosen as the starting point for the trajectory. The particular threshold curve that the driver used to initiate his perception-decision-action process was randomly selected by tossing coins. Equal delay times, acceleration, and deceleration were used. The v-h trajectory was graphically determined and is shown in Fig. 80.
Fig. 80--Calculated v-h trajectory
Examination of this trajectory shows that it is unstable. That is, the trajectory drifts to the right and to the left as time increases and finally terminates (as far as the calculation is concerned), at a value of $h$ which is less than the value at the starting point $A$. Note that the instability of this trajectory is not as severe as that of the unstable trajectories of Fig. 72, where each succeeding limit cycle drifts further and further to the right.

Figure 81 shows the resultant trajectory, if the trajectory is started at a value of $h$ equal to 135 feet. Again the trajectory oscillates to the right and to the left and finally terminates (as far as the calculation is concerned), at a value of $h$ (255 feet) which is far removed to the right of the starting point. The degree of instability is much greater in this case, than that shown in Fig. 80. In Fig. 81 the average overall drift per cycle was 13.7 feet to the right while the average overall drift per cycle in Fig. 80 was only 1.93 feet to the left.

It may be mathematically shown that instability will result from using the four threshold curves. The trajectories shown in Figs. 80 and 81 were graphically determined under the restraints of one delay time ($T_d = 1$ sec) and a constant acceleration and deceleration. With these restrictions, the condition for a stable limit cycle has been shown in Section D to be that the positive
Fig. 81--Calculated v-h trajectory
threshold intersection velocity "a" be equal to the negative threshold intersection velocity "b". This stability requirement is further complicated by the fact that the velocities "a" and "b" do not occur at the same headway.

Reference to Fig. 70 shows that the least headway for which the positive threshold velocity is always greater than the negative threshold velocity (for a given $T_p$) is 140 feet. Figure 70 also shows all the possible trajectories that the driver may use if the trajectory initially crosses the h axis at $h = 145$ feet. The positive threshold intersection points are denoted by the squares while the negative threshold intersection points are denoted by the dots. Since the threshold curves increase monotonically with headway, it is evident that the relative orientation of the threshold intersection points would not change appreciably if the driver initially crossed the h axis at some headway greater than 145 feet. Note that for $h > 155$ feet, the threshold intersection points for $T_p = 0.31$ seconds become non-existent. For this reason these points will not be included in the following calculation.

Let the positive threshold intersection velocity for $T_p = T_{pn}$ be represented by $x_n$ and let $y_n$ represent the corresponding negative threshold intersection velocity. The subscript $n$ can take on any of the integer values 1 through 4. In this development
\( T_{p_1} = 0.31 \text{ sec.}, \ T_{p_2} = 0.50 \text{ sec.}, \ T_{p_3} = 1 \text{ sec.}, \) and \( T_{p_4} = 2 \text{ sec.} \)

Upon assuming an equal probability of selection for each velocity threshold curve and examining all the possible trajectories shown in Fig. 70, it follows that the following inequalities hold:

\[
\begin{align*}
    x_2 &< y_2 & x_2 &> y_3 & x_2 &> y_4 \\
    x_3 &< y_2 & x_3 &> y_3 & x_3 &> y_4 \\
    x_4 &< y_2 & x_4 &< y_3 & x_4 &> y_4
\end{align*}
\]

Since a net drift to the right will occur if \( x_n > y_m \), then the probability of drift to the right \( [p(x_n > y_m)] \) can be directly determined from the above inequalities to be \( 6/9 \). Likewise, the probability of drift to the left \( [p(x_n < y_m)] \) is \( 3/9 \). Therefore since \( p(x_n > y_m) > p(x_n < y_m) \) it follows that the net drift will be to the right for the region of the phase plane where \( h > 140 \text{ feet} \).

Now consider the region between \( h = 105 \text{ feet} \) and \( h = 140 \text{ feet} \). Examination of all of the trajectories possible in this region (see Fig. 70) reveals that the following inequalities hold:

\[
\begin{align*}
    x_1 &= y_1 & x_1 &> y_2 & x_2 &> y_3 & x_2 &> y_4 \\
    x_2 &< y_1 & x_2 &= y_2 & x_2 &> y_3 & x_2 &> y_4 \\
    x_3 &< y_1 & x_3 &< y_2 & x_3 &< y_3 & x_3 &> y_4 \\
    x_4 &< y_1 & x_4 &< y_2 & x_4 &< y_3 & x_4 &> y_4
\end{align*}
\]

Now \( p(x_n > y_m) = 7/14 \) and \( p(x_n < y_m) = 7/14 \).
Since the probabilities are equal, it appears at first glance that the net drift should be zero. However, careful consideration of the physical problem will yield the result that the trajectory will probably oscillate around the initial starting headway.

If the regions from \( h = 105 \) feet to \( h = 140 \) feet and \( h > 140 \) feet are treated as a whole, it follows from the probability analysis that the trajectories in this region should drift to the right. This result is in agreement with the direction of drift graphically determined for the same region in Fig. 81.

Lastly, consider the region between \( h = 60 \) feet and \( h = 105 \) feet. Reference to Fig. 70 indicates that the following inequalities hold in this region for the possible trajectories shown in Fig. 70:

\[
\begin{align*}
    x_1 < & y_1 & x_1 > & y_2 & x_1 > & y_3 & x_1 > & y_4 \\
    x_2 < & y_1 & x_2 < & y_2 & x_2 > & y_3 & x_2 > & y_4 \\
    x_3 < & y_1 & x_3 < & y_2 & x_3 < & y_3 & x_3 > & y_4 \\
    x_4 < & y_1 & x_4 < & y_2 & x_4 < & y_3 & x_4 > & y_4
\end{align*}
\]

From these inequalities it follows that \( p(x_n > y_m) = 9/16 \) and \( p(x_n < y_m) = 7/16 \). Since neither of these probabilities is substantially greater than \( 1/2 \) it is difficult to state whether the trajectory should drift to the right or to the left. The only conclusion that can be drawn from the probabilities is that the trajectory will probably oscillate around the initial starting headway.
Reference to Fig. 80 indicates that this is approximately the behavior of the trajectory in this region of the phase plane.

In conclusion, it may be said that the probability analysis described above is sufficiently accurate to predict whether or not the v-h trajectory will drift to the right, or left, or be oscillatory.

F. Solution of the Drift Problem

The discussion in the preceding sections has shown that drift of headway due to instability of the v-h trajectory can be expected if the driver uses a perception-decision-action process while driving. Simulator results of car-following were then examined to possibly obtain support for the theory that this drift does exist. Many examples of drift were found in the results and two typical examples are shown in Figs. 82 and 83. It was concluded that drift does exist in car-following.

The results of the simulator study of car-following were then carefully examined to see how the driver coped with the drift problem. It was found that the driver solved the problem by employing a rather simple technique. When the driver detected that his headway had reached too large a value because of drift, he would correct the situation by going from an accelerating trajectory into a trajectory of zero acceleration at some negative relative velocity.
Fig. 82—Example of drift in actual v-h trajectory
Fig. 83—Example of drift in actual v-h trajectory
instead of switching to a decelerating trajectory. Since his relative velocity was negative his trajectory was parallel to the h axis and moving in a negative sense such that his headway was decreasing. The resultant trajectory is shown in Fig. 84. When the driver had cancelled out the headway change due to drift, he would go into a decelerating trajectory and start another series of trajectory cycles. Experimental evidence of this phenomenon is given in Figs. 85 and 86.

There is another method that the driver could use in overcoming drift. This technique is based on using unequal accelerations and decelerations. The resulting stable limit cycle is identical to the second type of stable limit analytically determined in Section D. Up to this time equal accelerations and decelerations have been used in analytically determining the a-v and v-h trajectory. Experimental studies have shown that the drivers tested, in general, used equal accelerations and decelerations. The action point density functions also indicated that the value of acceleration and deceleration used was in the order of 0.02g and this value is roughly the acceleration threshold of a human. If the driver uses a smaller deceleration than acceleration, and if he chooses the values of acceleration and deceleration correctly, then his resultant trajectories will be such that he will be able to maintain a fairly constant mean headway.
Fig. 84. Compensation for drift
Fig. 85--Example of drift compensation in actual v-h trajectory
Fig. 86--Example of drift compensation in actual v-h trajectory
An experimental example of this technique is shown in Fig. 87. Of six people tested only one person exhibited evidence of using the above technique to maintain a more or less constant headway.

An attempt will now be made to determine the velocity sampling time of the driver.

G. Velocity Sampling Time of the Human Driver

In this context the velocity sampling time will be defined as the time for which a velocity must be viewed by a subject before that subject can just detect motion 50% of the time. That is, the velocity sampling time may be equal to the presentation time \( T_p \) and then again it may only be some fraction of \( T_p \). For purposes of this discussion, it will be assumed that the velocity sampling time equals \( T_p \). In the conduction of the velocity threshold experiment, the subject was presented with a step change in relative velocity. However in actuality, the driver is usually exposed to relative velocities which are either slowly increasing or decreasing. This fact further complicates the problem of determining the velocity sampling time of the driver by applying the threshold data of Chapter III.

An estimate can be made of the velocity sampling time by using the following technique. For a given headway, determine the
Fig. 87—Example of drift compensation in actual v-h trajectory $(A_1, A_2)$. 

Legend:
- $h(F_1)$
- $i = 0.5$
- $i = 1.0$
- $i = 2.0$
- $T_p = 0.31$
most likely position of occurrence of the action point in the upper right hand plane of the phase plane. For a speed of 70 mph, it follows from Fig. 63 that this position has the co-ordinates \( h = 70 \text{ ft}, v = 1.25 \text{ mph} \). Reference to Fig. 66 shows that the average acceleration to be expected is 0.015 g. The resultant trajectory is shown in Fig. 88. Trajectories for steady state velocities of 30 and 50 mph are also shown. Now assume that the perception-decision-action process requires 1, 2, 3, or 4 seconds for completion. If the process time is 4 seconds, it follows from Fig. 88 that the intersection point lies below the \( h \) axis and consequently this is greater than the maximum process time possible. Further examination of the trajectories shown in Fig. 88 reveals that the maximum process time possible is approximately 2.5 seconds. If the process time is taken as 2.5 seconds, it follows from Fig. 88, that the corresponding presentation time is 2.0 seconds. Therefore, an estimate of the driver velocity sampling time is 2.0 seconds.

At this point the results of this chapter will be summarized in the form of a qualitative description of the action point model,
Fig. 88 -- Velocity sampling time of human driver
H. The Action Point Model

The action point model is a statistical mathematical model of the driver-vehicle complex and is based on the threshold sensitivities of the human to the physical variables of relative velocity and acceleration. This model would indicate that the driver uses the following mode of operation in performing the task of following a lead vehicle at close range.

Suppose the driver has been forced into the situation where he has to follow another vehicle in his lane due to heavy traffic. As the driver approaches the lead vehicle, he will judge the traffic situation and pick a resultant headway \( h_o \) at which he would like to follow the lead vehicle. This headway is a function of the steady state velocity \( v_1 \) and can be expressed by the equation \( h_o = K v_1 \). However, the headway is also influenced by the particular state of mind of the driver as well as other factors. The constant \( K \) may be considered as a sensitivity coefficient which is proportional to the mental attitude of the driver. If the driver is in a hurry, then \( K \) will be small and the resultant \( h_o \) also small. However, if the driver is traveling leisurely, then \( K \) will be large and \( h_o \) will also be large.

Once \( h_o \) is determined, the driver will oscillate about this headway as time progresses. His oscillation is described by the
v-h trajectory. The model suggests that the driver uses a perception-decision-action process in generating his v-h trajectory. This process is initiated as soon as the driver's trajectory crosses a particular velocity threshold curve. These curves are not perfectly defined as their characteristics are probabilistic rather than deterministic. At the present time it is felt that the selection of a particular curve is dependent on the existing environmental conditions as well as their past history, and that the selection process is probably highly overlearned and thus automatic in nature. That is, the subject is not aware of either the stimulus or response. After crossing the threshold curve, the driver must visually sample the relative velocity before he is able to perceive relative motion. After a short judgment time, the driver makes a decision to either accelerate or decelerate, depending on whether his relative velocity is positive or negative. After another delay, the driver finally takes action and either accelerates or decelerates. The value of acceleration chosen by the driver is a little above threshold because it is postulated that the driver uses the acceleration sensation as feedback to inform him that he is actually accelerating.

The perception-decision-action process time is made up of three separate process times. In order of occurrence, these times are the perception time, the judgement time, and finally the
reaction time. To further complicate the issue, it is felt that the magnitude of the sub-process times are statistical in nature.

Due to the variability of the positive and negative threshold velocity curves, the process times as well as the acceleration and deceleration, it was found that the driver's trajectory was unstable. As a consequence the driver's headway increases. When the headway reaches a value sufficiently large to disturb the driver (i.e., the driver has picked a certain value of headway at which to follow), the driver decides to correct the situation and takes action by first accelerating until he attains a negative relative velocity. When this occurs, he reduces his acceleration to zero. As his relative velocity is negative, the driver slowly decreases his headway to his desired headway $h_0$. Upon reaching this headway, his trajectory again goes into a limit cycle about the headway $h_0$. After a time dependent on the randomness of the perception-decision-action process used by the driver in generating each limit cycle, the driver's trajectory again drifts away from his desired headway. The driver again corrects the situation by using the method described above.

It is seen from the preceding discussion that this model is statistical as well as highly non-linear. Consequently, it is concluded that the car-following situation cannot be completely described
by a simple non-linear continuous equation such as has been pro-
posed by some workers in this field.
CHAPTER V
CONCLUSIONS

A preliminary analysis of the car-following problem indicated that Barbosa's decision point model described the problem more fully than any other model so far proposed. However, a more detailed examination of the decision point model revealed that it left many aspects of the car-following problem unexplained. In justifying the decision point model, the a-v trajectory was defined and employed. Utilization of the a-v plane definitely established the existence of the action point. The nature of the statistical variation of the action point should be investigated more thoroughly than was possible in this work in which only the average position was considered.

Results from the velocity threshold experiment yielded the driver's threshold velocity for different values of headway and presentation time. In applying psychophysical techniques to the velocity threshold data, it was determined that the time rate of change of the visual angle was the velocity stimulus. In addition, Stevens' psychophysical law was shown to be applicable. This law
then served as the basis for the derivation of two general velocity
threshold equations which interconnect the relative velocity threshold,
headway, and presentation time.

Correlation of the threshold velocity data with the phase plane
trajectory was accomplished. This correlation made possible the
proposal of the perception-decision-action process which is the
basis of the action point model. The existence of the perception
point was definitely established because it was shown that the
postulated perception points all fell within the region of the phase
plane occupied by the velocity threshold curves. Consequently, it
appears as if the driver is definitely using velocity threshold data
to initiate a process which culminates in an action being taken.
Since the existence of both the action point and the perception point
has been established, the problem now remains of determining
the nature of the intervening process. This process has one
unique feature in that both the initial and final points are known.
Consequently, the investigation of the process can be initiated at
either the perception point or the action point. In this work, the
process has been postulated to be a perception-decision-action
process. However, more work is required to determine the
characteristics of this intervening process.
A stability analysis of the phase plane limit cycle yielded the criteria for stability as well as an indication of the technique the driver uses to maintain stability. This analysis also indicated that the driver can behave either in a cognizant or non-cognizant manner while following a lead vehicle. It is postulated that the driver is following in a non-cognizant manner when his trajectory is drifting and in a cognizant manner when he has discovered the drift and is correcting for it. An estimate of the driver's velocity sampling time was also made. Lastly, the action point model was described in detail.

It is felt that the perception-decision-action process is a fundamental concept in man-machine systems and not one that is just characteristic of the driver-automobile complex. This concept and its application is considered to be a powerful technique applicable to any man-machine system, because it is essentially a process which relates the physical variables in the real world to the resultant sensation or response of the human in the sensational world. In the general case of a man-machine system, the perception-decision-action process can be defined as that process utilized by a man in the perception and judgement of input stimuli from the real world to form a basis for a decision that results in action being taken by the man on the machine which in
turn changes the input stimuli to some desired condition. In application of this technique, most difficulty will be experienced in determining the relevant stimuli and the resultant actions. The determination of the characteristics of the particular perception-decision-action process involved, will also be an arduous task.
REFERENCES


AUTOBIOGRAPHY

I, Ernest Peter Todosiev, was born in Rossburn, Manitoba, Canada, July 14, 1936. I received my secondary-school education in the public schools of Welland, Ontario and my undergraduate training at Queen's University, Kingston, Ontario, which granted me the Bachelor of Science degree in electrical engineering in 1959. In 1960 I was granted the degree of Master of Applied Science from the University of Toronto, Toronto, Ontario. Between 1960 and the present I held the position of research assistant and finally research associate at the Antenna Laboratory of the Electrical Engineering Department of The Ohio State University. While at the Antenna Laboratory I completed the requirements for the Doctor of Philosophy degree.