MONEY DEMAND AND THE MODERATE QUANTITY THEORY OF MONEY: AN EMPIRICAL INVESTIGATION

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

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* * * * *

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2000

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ABSTRACT

The Moderate Quantity Theory (MQT) of Money derives an adjustment process for the periods of disequilibrium between money supply and money demand from microfoundations. It specifies the inflation rate as a function of the disequilibrium between money supply and money demand as well as agents' expectations of next period's inflation.

We examine this theory as an alternative to other money demand models in the literature. First, agents' expectations of next period's inflation rate are modeled. A median unbiased estimator for the lag dependent variable in an augmented Dickey-Fuller regression is estimated that allows for a time varying order of integration. We find a unit root for the monthly U.S. inflation series from the mid 1970's to the mid 1980's. Before and after that time period, the U.S. inflation series is either stationary or nearly stationary.

The inflation series is pseudo-differenced using the median unbiased parameter estimate to give a stationary time series. An expanding window ARMA approach is used to estimate the inflation data generating process as new data points are added.

The MQT regressions give estimates of the speed of adjustment, the scale variable, and the opportunity cost parameters. The parameter estimates are significant and are the right sign. Income elasticities are about 0.5 for narrow measures of money, but increase as the definition of the monetary aggregate is broadened. The interest rate semi-elasticities remain about the same at -0.03 across all the monetary aggregates.

We compare the explanatory power of various monetary aggregates and find that M1 plus is the best monetary aggregate within the context of our model. Chow tests are used to examine the parameter stability in our regressions: and, unlike many other money demand models, there is little parameter
instability across the volatile time periods and various monetary aggregates. Possible endogeneity between the variables in the MQT regressions are also examined with little evidence of that problem.

The remainder of this work duplicates the efforts made on U.S. post-War monthly data to the OECD G7 countries for the time period 1960-1997 using the quarterly data series found in OECD Main Economic Indicators *Historical Statistics* 1960-1997.
Dedicated to my mother, Dolores, my father, Casimir, my wife, Jackie, and my daughter, Cassandra.
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CHAPTER 1

Introduction

1. Traditional Money Demand Models

A rich tradition exists in macroeconomics with respect to research on the demand for money. It has always been thought that a stable money demand function is a necessary condition for the formulation of effective monetary policy. Laidler (1980) notes that, “In the 1960’s, questions about the supply and demand for money and their interaction were at the very center of research in macroeconomics.” Research focused not only on the proper specification of a money demand function, but also on the importance of the supply of money in influencing the level of aggregate demand.

Seminal papers of Baumol (1952), Tobin (1956), Friedman (1959), Meltzer (1963), and Chow (1966) defined the money demand specification from theoretical foundations and specified adjustment processes for short run money market disequilibrium. They noted that an instantaneous adjustment of money demand to money supply is not observed in the data. In the long run there is money neutrality, but in the short-run money demand does not adjust to money supply immediately. In other words, the Quantity Theory of Money does not hold in the short run.

To accommodate this empirical regularity, monetary economists introduced a short-run adjustment mechanism to conventional money demand specifications. The partial-adjustment model (PAM) of money demand posits an adjustment equation where the difference between this period’s money holdings and last period’s money holdings is a function of the difference between the desired stock of money holdings this period and the actual stock of money held by agents last period.
In other words,

$$\ln M_i - \ln M_{i-1} = \mu(\ln M_i^* - \ln M_{i-1}) + \tau(\ln P_i - \ln P_{i-1})$$

(1.1)

where $M_i$ is the actual money stock held, $M_i^*$ is the desired money stock, and $P_i$ is a price index (see Goldfeld and Sichel (1990)). Equation 1.1 is then incorporated with a specification for money demand to produce an empirically testable proposition about both long run and short run money demand.

Research in this vein explained well U.S. money demand throughout the 1960's and early 1970's. Especially attractive about this approach was the 1) parsimonious description of money demand, 2) the stability of parameter estimates in the money demand specification, and 3) the predictability of future money demand. Simply put, this was an ideal situation for U.S. central bankers concerned with implementing effective monetary policy.

Unfortunately, this utopia did not last for long. Researchers began to find large prediction errors with conventional money demand specifications culminating in the famous “Missing Money Episode” of Goldfeld (1976) in the mid 1970's. Aply named, the episode found that conventional money demand equations were overestimating U.S. money demand on such a scale that the usefulness of these specifications were very doubtful. Moreover, the econometrics behind the estimation of money demand equations became suspect with the identification of spurious regressions by Granger and Newbold's (1974) noteworthy paper. With the recognition that many macroeconomic time series were most likely generated by I(1) processes (Nelson and Plosser, 1982) as well as the instabilities found in current versions of money demand models, macroeconomists began to question all aspects of the Partial Adjustment Model to money demand.

The tools to overcome the econometric difficulties with PAM specifications of money demand began to become available with the important work of Granger (1983), Engle and Granger (1987), and Johansen (1988). Those econometricians developed the concept of a cointegrating relationship between a set of economic variables in which a cointegrating vector could be estimated. This cointegrating vector are the parameters of a long run linear relationship between these economic variables in which the linear combination of these I(1) variables is I(0). In other words, a long run relationship between the economic
variables exists and these variables never deviate infinitely far apart even though the variance of each individual variable is infinite.

Since individual movements away from this long run relationship will always eventually revert back to equilibrium, a natural short-run dynamics specification for this system of cointegrated variables is their deviation from equilibrium. In the case of money demand, a cointegrating relationship might exist between a monetary aggregate, a scale variable like real GDP, and an opportunity cost variable like a short-term interest rate. Any deviation of these variables from their long run relationship would be an explanatory variable in a short-run dynamic specification of money demand. Granger's Representation Theorem stipulates this short-run specification formally. Any cointegrating relationship between a set of economic variables will have a short run dynamic specification of the Error Correction form (cf. Granger, 1983; and Engle and Granger, 1987).

A necessary condition for a short run dynamic representation of out-of-equilibrium behavior in this context, however, is a well-defined long run cointegrating relationship between the variables of interest. This is not always fulfilled in practice as conventional tests for the existence of a cointegrating relationship can suffer from a disturbing lack of power (cf. Stock and Watson, 1993; and Toda, 1994). Moreover, the short-run dynamics of an Error Correction Model are completely ad hoc in the sense that the structure of the dynamic specification is not specified by theory.

2. The Moderate Quantity Theory of Money

The Moderate Quantity Theory of Money (McCulloch (1980)) proposes a theoretically derived adjustment process for monetary disequilibria. The theory is similar in structure to the Quantity Theory of Money in that it presents prices as a function of the disequilibrium between desired money holdings and the actual money stock. But, prices are also a function of agents' expectations of future inflation rates. This addition to the Quantity Theory allows for the existence of non-zero inflation rates even when the money market is in equilibrium. Moreover, the Moderate Quantity Theory has the attractive feature

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1 Chapters 2, 4, and Appendix B go into much more detail on the Moderate Quantity Theory of Money.
of specifying a theoretical explanation for the adjustment of money demand and supply to equilibrium that does not require this adjustment to be instantaneous.

This incomplete adjustment process is crucial to the Moderate Quantity Theory because it allows for the non-neutrality of a money shock in the short run. The exact current price level is unknown to agents, so recent past prices are used in their decision rules. Prices will not adjust instantaneously then to a monetary shock. As consumers gain more information regarding output levels and prices, though, prices adjust to their long-run levels nominalizing a monetary shock. So, MQT retains the long-run neutrality of money that any monetary model of money must have.

3. Direction of this Research

This dissertation will focus on empirically testing the Moderate Quantity Theory (MQT) as an alternative to past and current models of money demand. The MQT model itself is not susceptible to the econometric difficulties of the Partial Adjustment Models or the ad hoc nature of the Cointegration/Error Correction Models. Chapter II will review thoroughly previous research in the money demand literature as well as present arguments and evidence why these models leave something to be desired. Estimation of PAM and cointegrating vectors in an Engle-Granger and Johansen sense are undertaken with the goal to illustrate their shortcomings. The chapter also examines the different directions the money demand literature has gone after the “Missing Money Episode” and whether these directions have lead to any resolutions of post 1970’s money demand instability.

Chapter III examines realistic models of agents’ inflation expectations with the goal of providing a proxy for anticipated inflation in the MQT specification. Throughout that chapter, the point is made that an examination of the monthly U.S. inflation series over the time period January 1959 to May 1999 indicates that the time series properties of the series are not time invariant. Specifically, there is strong evidence that the order of integration of the inflation series varies over time. This suggests that the proper way to model inflation expectations would be to estimate the order of integration of the inflation series directly instead of imposing an order of integration structure for the series across the whole time period. Recent econometric work with median unbiased estimators is used to properly model this phenomenon.
Once the proper order of integration for a specific time period is determined, the inflation series can be pseudo-differenced to obtain a covariance stationary series from which to model and generate agents’ next period inflation expectations. An appeal is made to Wold’s Decomposition Theorem in order to specify an ARMA model that adequately captures the data generating process behind the inflation series. From this ARMA model, next period expectations of inflation are computed. Repeating the process described above and taking advantage of each recently observed inflation data point, generates subsequent forecasts of next period’s expectations of inflation. This expanding window estimation process requires 485 estimates of the order of integration for the inflation series as well as a search of over 120 nested ARMA models with each new data point. In all, over 58,000 nested ARMA models are examined in the context of generating the full time series of inflation expectations.

Chapter IV synthesizes the efforts of the previous chapters to test the Moderate Quantity Theory. The results of Chapter II and the demonstrated existence of at least one cointegrating vector across the cointegrating space of a monetary aggregate, a scale variable, and an opportunity cost variable is used to provide evidence that the MQT regression is sound econometrically. Since the MQT regression is what is known as an unbalanced regression (Pagan and Wickens, 1989), it is important to show that the regressors in an unbalanced regression are cointegrated.

Once multiple avenues of evidence are given to this effect, the focus becomes the explanatory power of the MQT regressions, the estimates of the income elasticities and interest semi-elasticities, and the best monetary aggregate in the MQT context. The stability of the parameter estimates in the MQT regressions as well as whether endogeneity might be a problem are examined in detail as well.

Chapter V duplicates the analyses done in Chapter IV except the data examined is for the international G7 countries. Unfortunately, a scale variable like Personal Income does not exist for G7 countries other than the U.S. So, the analyses done in this chapter use data at quarterly horizons. Chapter VI concludes the dissertation and summarizes the main findings.
CHAPTER 2

Money Demand Models

1. Early Work

Friedman (1959) put forth the hypothesis that "... measured income will be highly sensitive in
the short period to changes in the nominal stock of money -- the short-run money multiplier will be large
and decidedly higher than the long-run money multiplier." Further work by Friedman builds on this
hypothesis to deliver the now well-accepted tenet that money is neutral in the long run. He also goes on
to espouse the view that money is non-neutral in the short run. These two principles, taken together,
imply that there is an adjustment mechanism that works its way through the economy neutralizing any
changes to aggregate real income. Friedman spells this adjustment process out by the noting that
holders of money cannot alter the nominal money stock directly. But, they can make the real amount of
money anything they want to by increasing or decreasing their expenditures thereby reducing or
increasing, respectively, their individual nominal cash balances. In the aggregate, this behavior leads to
an adjustment of the real quantity of money to the desired level. In his words, "...the nominal quantity of
money (is) determined primarily by conditions of supply, and the real quantity of money ... (is)
determined by the conditions of demand." His influential work sets other economists off toward
specifying a money demand function that has not only the scale and opportunity cost variables of Baumol
(1952) and Tobin (1958), but an adjustment process that connects the short and long run demands for
money together.

Chow (1966) notes that "...one of the major weaknesses in the available theoretical formulations
of demand functions for money seems to be the failure to distinguish between long-run or equilibrium
demand and short-run demand by introducing a mechanism for the adjustment of the actual stock of money to its equilibrium level." His influential work introduces just such an adjustment mechanism. His idea for an adjustment mechanism comes, coincidentally, from his own work on consumption. He recognizes a long-run and short-run aspect to money demand. For the long run component, the equilibrium money stock is a linear function of assets and the rate of interest.

\[ M_t^* = b_0 + b_1 A_t + b_2 R_t \]  

(2.1)

He posits an ad hoc process of money demand adjustment whereby the difference between the money stock in period t and the money stock in period t-1 is proportional to the difference in the equilibrium money stock in period t and the money stock in period t-1 as well as the difference in savings at time t and assets at time t-1. Specifically, the short-run mechanism for money demand adjustment to its long-run equilibrium level is

\[ M_t - M_{t-1} = c(M_t^* - M_{t-1}) + d(A_t - A_{t-1}) \]  

(2.2)

Combining the long-run money demand equation (2.1) and this short-run adjustment process (2.2), the money stock at time t is

\[ M_t = cb_0 + cb_1 A_t + cb_2 R_t + (1 - c)M_{t-1} + dY_t - dY_{pt} \]  

(2.3)

where \( Y_{pt} \) is permanent income and \( Y_t \) is current income.

These specifications allow Chow to investigate a number of different issues. One is the relative importance of permanent versus current income. Another is the value of the money demand income elasticity in the logarithmic version of 2.3. Using a definition of money as M1, Chow finds a unit income elasticity in his long-run money demand regressions (2.1). His findings here correspond closely to that of Meltzer (1963), except that Chow finds permanent income to be a better explanatory variable than wealth. Chow also finds that permanent income has more explanatory power than does current income and that the interest rate is a significant explanatory variable.

When Chow uses 2.3 to explain money demand, he finds that current income is a better explanatory variable of money demand. In other words, when short-run adjustments are allowed for, current income, through its influence on savings, is a better explanatory variable. He estimates a short-
run income elasticity of 0.36. It is Chow’s work, specifically his specification of an adjustment equation for money demand that greatly influences future work in this area. But, that future work does not come until Goldfeld’s seminal work in 1976. Until that time, the money demand literature lay dormant due to how well Friedman, Meltzer, and Chow had seemed to explain money demand.

Goldfeld’s path breaking article first noted an apparent shift in the money demand function that could not be explained away by final data revisions. Specifically, estimated money demand functions overestimated consumer’s demand for money starting in 1974 by about nine percent over the 1974:I to 1976:II period. Goldfeld noted, “Among other things that period saw the most severe recession of the postwar era; an extended bout of double-digit inflation; the highest interest rates in many years; and many institutional changes in the financial structure” (p.684). He went on to say that the failure of the empirical money demand relationship should not be surprising under those types of conditions. Nonetheless, it should prompt the question whether the conventional specification is adequate to cope with these types of macroeconomic phenomena.

Goldfeld presents evidence that overestimation of the demand for M1 during this period rests on an inability to estimate demand deposits out of the sample when considering the components of M1 separately. He surmises that a logical explanation for the “Missing Money episode” is the adoption of more sophisticated money management techniques in which the, heretofore, ignored transactions costs of money demand can be addressed. (He also investigates the use of different scale and opportunity cost variables with little resolution of the problem). He introduces “previous peak” variables meant to model the ratchet effect in the demand for money. Once a new money management technology is adopted, it will not necessarily be abandoned should expected interest rates fall. So, a ratio of the value of the current value for the variable to its previous peak value is entered as an explanatory variable in the money demand specification. This addition significantly reduces the missing money from the mid 1970’s, but this specification does not work well out of sample for periods other than the Missing Money episode. He concludes that the ratchet variables provide “…only a mirage of an explanation of the recent puzzle” (p.706). He concludes his work rather dismally by saying, “Insofar as the objective was an improved specification of the demand function for M1, capable of explaining the current shortfall in money demand, the paper is rather a failure” (p.725).
Goldfeld's ability to present the problem of an unstable money demand function during the 1974 to 1976 period and his inability to find a stable money demand specification spurred a wealth of research into this area. Besides not finding an adequate explanation and solution for the Missing Money episode, scholars were finding that, when data from the Missing Money episode was included in the money demand regression, the coefficient on the lagged dependent variable was implausibly large (implying slow adjustment back to equilibrium) and sometimes was greater than one (implying dynamic instability).

Judd and Scadding (1982) note that most economists, at that time, believed that the high inflation rates of the mid-1970's are what made that period different than the periods that preceded it. With these high inflation rates, there was much more incentive for innovations in the financial markets that would allow consumers and businesses to manage non-interest bearing deposits more efficiently. Yet, Judd and Scadding also note that two different lines of research arose from the Missing Money episode.

The first line of research concentrated on the well-received reason why the Missing Money episode was a different economic period than economic periods that preceded it. If the high rates of inflation were causing agents to economize and more efficiently manage their money holdings, then examining financial innovation and, to a lesser extent, deregulation in the financial services industry should lead to a solution to the Missing Money episode.

The second line of research focused instead on a reinvestigation of many of the pre-1973 issues that many had once thought were resolved. These issues included a reexamination of the appropriate monetary aggregates, scale and opportunity cost variables, and disequilibrium adjustment process used to estimate a money demand function. As part of this research agenda, the econometric techniques used to estimate a money demand function were also called into scrutiny.

2. Variables in Money Demand Specifications

1. Financial Innovation

In November 1974, commercial banks could offer interest bearing savings accounts to domestic governments. In 1975, banks could offer interest bearing savings accounts to businesses. These accounts
coupled with NOW (negotiable order of withdrawal) accounts changed the landscape with respect to M1. Since these types of financial instruments were not included in definitions of M1 prior to 1980, demand for these accounts was at the expense of transactions money balances. But, research by Porter, Simpson, and Mauskopf (1979, p.215) showed that only about 25% of the estimated money demand shortfall could be accounted for by the introduction of these interest-bearing accounts.

Greater efficiency in managing money balances was also thought to account for the overestimation of M1 money demand. This greater efficiency was thought to be driven by implicit compensation offered by banks in the form of credit and cash management services. Repurchase agreements (RPs), an agreement that involves the sale of a security with the promise to repurchase the security in the future at a predetermined price, cash management techniques (e.g., zero balance accounts and cash concentration accounts) afforded agents the ability to economize their money holdings, thereby reducing M1 demand. These low-cost techniques of converting other assets into money and reductions in the variance of cash flows should impact money demand directly by lowering transactions costs and reducing cash flow uncertainty, thereby lowering the demand for money. However, as was mentioned above, research by Goldfeld (1976) that introduced "previous peak" interest rates or income as proxies for changes in transactions costs was largely unsuccessful.

Another possibility to resolve the Missing Money episode is to add RPs to the conventional definition of M1. While this does help to explain the Missing Money episode, a valid concern is the amount of substitutability that RPs have with other more traditional components of M1. In fact, the definition of M2 now has RPs of less than $100,000 in it because conventional wisdom views RPs as closer substitutes for components of M2 rather than M1. Moreover, Farr, Porter, and Pruitt (1978) note that these financial innovation and cash management techniques took place in the business marketplace. Yet, when they estimated the household money demand function and the business money demand function, they found that in percentage terms the household demand function shifted almost as much over the 1974-6 period. Obviously, if financial innovation and cash management techniques were to blame for the money demand overestimation, a much more pronounced shift would have occurred in the business demand function vs. the household demand function.
The Missing Money episode also called into question the money demand specification, both in terms of its form and substance. That is, questions arose on whether the conventional money demand equation was well specified and whether the appropriate measure of money, scale, and opportunity cost were used.

2. Opportunity Cost

At issue in terms of interest rates is the inability to test more than one interest rate at a time due to their interdependence and, therefore, collinearity. Another issue is the appropriate opportunity cost for holding a component of the monetary aggregate. A transactions-based approach implies that money is primarily used as a medium of exchange and therefore is held for short periods of time. A short-term interest rate would be most appropriate under this paradigm. Goldfeld and Sichel (1990) note that researchers that ascribe to a transactions-based approach to money demand have used short-term yields on government securities, the yield on commercial paper, and/or the yield on savings deposits. A portfolio-approach views money as, primarily, a store of value that competes directly with other financial assets. In this case, the rates of return on longer-term assets, including equities and physical capital should be used.

Again, most vexing for the determination of the appropriate opportunity cost is the collinearity of interest rates. This makes the direct test of the most appropriate proxy for opportunity cost difficult.

Ideally, the opportunity cost of holding money should be the difference between some benchmark asset that is a feasible alternative to holding money and the money aggregates own rate of return. Barnett (1978) derived just such an explicit user cost formula to calculate the user cost for any monetary component. However, some components of a monetary aggregate have no explicit rate of return. Instead, these components might have some implicit rate of return that may include deposit fee waivers, gifts, or additional services that are received by deposit holders. These types of non-pecuniary services are inherently difficult to measure, so that an aggregate's own rate of return might be in calculable.

Perhaps worse is when different components within a monetary aggregate have different own rates of return. The issue then becomes the proper weights to use in the aggregation process. This, of
course, assumes that the components are similar enough to aggregate together in the first place. The difficulties with calculating the own rate of return for a monetary aggregate has lead most researchers to adopt the various types of proxies described above.

3. Monetary Aggregate

Researchers also thought that broader measures of money might solve the Missing Money episode. Goldfeld (1976) and Laidler (1980) found that M2 over the 1974-6 period was more stable than M1. In both cases, the prediction errors using M2 as the monetary aggregate versus M1 were much smaller. Both authors reasoned that the inclusion of small time and saving deposits into the aggregate accounted for the missing money. In fact, a study cited by Judd and Scadding (1982), Bennett et al. (1980) noted that the Federal Reserve's redefined monetary aggregate for M1 (as defined in Appendix A) plus small time and savings deposits had the best out-of-sample prediction performance.

At issue here is the appropriate components to include in the monetary aggregate. M1 is looked at as the transactions aggregate in the sense that components there should be liquid enough with which to conduct transactions. But, arguments can certainly be made to include what are now currently components of M2 in M1, e.g., retail purchase agreements (RPs). The question arises as to the degree of substitutability between money components. In essence, a monetary aggregate should contain those components that are perfect substitutes or nearly perfect substitutes.

Another problem with conventional monetary aggregate specifications is components in these monetary aggregates are simply summed together. Since the components within a monetary aggregate offer different degrees of monetary services or liquidity, Divisia index theory proposes that the components be differential weighted to reflect their varying degrees of moneyness.

Work starting with the seminal paper of Chetty (1969) examined the "nearness" of different monetary components by invoking a microtheoretic framework in which liquid assets were viewed as substitutes for money. Empirically, a utility function would have to be specified by which the elasticities of substitution would determine the degree of substitutability of a given asset for money. Chetty uses differential weights on the components and sums them together to aggregate them.
This aggregation method, however, does not recognize the work of Diewert (1976) on index number theory. In this work, Diewert proved that the Divisia index lies within his class of superlative index numbers. This does not invalidate a simple sum approach to aggregation. However, "...if the relative prices of the monetary components fluctuate over time, then neither the (simple sum) method or the Hicksian approach to aggregation will produce theoretically satisfactory definitions of money" (Barnett, Fisher, and Serletis, 1992). Barnett et al. notes that the reason these methods will not work in cases where the relative prices of the components fluctuate over time is because those aggregation methods incorrectly account for substitution effects. This makes the simple sum index suboptimal with the Divisia index being the appropriate alternative.

Barnett (1980, 1982) and Barnett, Offenbacher, and Spindel (1984) synthesized the work of Chetty and his ideas of aggregating monetary components by their degrees of moneyness or liquidity with the formal indexation theory work of Diewert to develop the Divisia monetary aggregates. Money is viewed in the Friedman (1956) sense as durable goods rendering monetary services flows. Money enters in to the representative consumer's utility function as an argument:

\[ u = U(c, L, f(x)) \]  \hspace{1cm} (2.4)

\[ s.t \ y = p^\top c + \pi^\top x + wL \]  \hspace{1cm} (2.5)

where \( c \) is a vector of the services of consumption goods, \( L \) is leisure, and \( x \) is a vector of monetary assets that provides monetary services. \( P \) is the vector of prices of \( c \), \( \pi \) is the vector of monetary asset user costs, and \( w \) is the price of leisure.

This approach has the appealing virtue that it derives monetary aggregates by application of microeconomic demand theory. This implies that the construction of monetary aggregates is consistent with the optimizing behavior of economic agents. A few questions arise, however, with this derivation of monetary aggregates. First, \( f(x) \), what is known as the aggregator function (Barnett, et al, 1992), requires a functional specification that is not trivial. As already noted, a problem with simple-sum monetary aggregates is that they treat components within the aggregate as perfect substitutes. The adoption of a general Cobb-Douglas function implies that the elasticity of substitution is one for each of the components within the monetary aggregate. Fisher's (1922) investigation of the statistical properties
of indices detailed his Ideal Index chosen on the basis of it possessing the largest number of satisfactory statistical properties. Diewert (1976, 1978) provided the link between index theory and the specification of an aggregator function by showing that the use of a number of well-known statistical indices is equivalent to using a specific functional form for the aggregator function. In essence, the adoption of Fisher’s Ideal Index implies the functional form of the aggregator function to be the square root of a homogenous quadratic function.

While the adoption of a particular statistical index implies the form of the aggregator function, this form is unknown a priori. Adoption of a particular statistical index based solely on the number of satisfactory statistical properties does not mean that the implied aggregator function approximates well the unknown aggregator function. Flexible functional forms can be used to approximate to a second order or higher the unknown aggregator function, so the common way to link the aggregator function to a statistical index is to specify the index that is equivalent to a specific flexible functional form. Often the Divisia index is chosen because it has a large number of satisfactory statistical properties and it is equivalent to the linearly homogenous and flexible form translog function. In fact, while the Divisia index is not equivalent to Fisher’s Ideal Index, Barnett (1980) has shown that the difference between the two is typically less than the round off error in monetary data.

The unknown aggregator function can be approximated well by a flexible functional form specification. And, this specification implies a certain statistical index. What is more problematic for the Divisia paradigm is the necessary property the aggregator function must have in the context of the representative agent’s utility function. The aggregator function must be weakly separable in the services of monetary assets. This is necessary because ultimately the consumer choice maximization problem is a two-stage optimization problem akin to Strotz (1957, 1959) and Gorman (1959). There the first stage is where the consumer allocates expenditures across the broad categories of consumption goods, leisure, and monetary services. The second stage allocates expenditures within each category. Without this weak separability condition, the optimal money holdings across monetary components cannot be determined. Yet, this separability condition is not an innocuous one and many researchers (Swafford and Whitney, 1988 and Fayyad, 1986) have not been able shows the separability condition holds empirically.
Another problem with the Divisia paradigm is with one of the key inputs into this consumer choice maximization problem, the user cost of each monetary component. As mentioned above, the user cost of a monetary component can be difficult even to approximate. This is because for some components there is no explicit own rate of return. Moreover, a benchmark rate of return for an alternative asset must be chosen without guidance from theory. This sometimes causes the user cost to be negative with the own rate of return exceeding the benchmark rate. While this can be ameliorated by setting all negative user costs to zero, ideally the user cost of each monetary component would be non-negative across time. With a zero user cost the implied monetary services for this asset are regarded unreasonably as zero. This also implies that the construction of user costs is not robust to the choice of a benchmark. Unfortunately, this choice of the appropriate benchmark is not determined by the theory.

There are also economists that do not view monetary assets as an appropriate argument in a utility function, namely because money, as a good, is not thought to bring utility to the user. Money is thought of as a medium exchange; necessary to finance expenditures but worthless in and of itself. Instead the appropriate modeling tool used to incorporate money into a general equilibrium model is often a cash-in-advance constraint that requires expenditures to be financed with money but ascribe no inherent value to it.

4. Scale Variable

The scale variable also depends on the theoretical approach one takes with respect to money demand. A transactions approach, as the name implies, uses a proxy for transactions as its scale variable. The commonly used scale variable here is GDP. A big drawback to using GDP is that it does not include financial transactions and transactions in existing goods. Another drawback is that all intermediate transactions are netted out of GDP. This implies that total transactions are proportional to GDP. Unfortunately, a direct measure of transactions does not exist and other proxies for transactions are saddled with drawbacks as well.

A portfolio approach examines money demand in the context of durable goods, securities, and other assets. Money is demanded for the services it provides, much like durable goods. If capital markets are perfect, or nearly so, loans can be extended at a given rate of interest and the demand for money will
be determined by its market price and income. Assuming that agents are making long-term decisions when they allocate assets to their portfolios, money demand will be determined, in part, by a long-term measure of income like permanent income or wealth.

Judd and Scadding (1982) note that a solution of this argument between current income and permanent income (wealth) will probably not resolve the periods of demand instability. This realization has lead researchers to 1) construct better measures of transactions or 2) disaggregate all transactions into various components. As mentioned above, GDP as a proxy for transactions does not count financial transactions, purchase of existing goods, or sales of intermediate goods. Lieberman (1977) used bank debits as a replacement for GDP. Goldfeld (1976) and Enzler, Johnson, and Paulus (1976) used the level of bank loans as a scale variable. In these cases, the effects of these different scale variables were small to insignificant and not very robust to different out-of-sample forecast periods.

More recently, Cramer (1986) and Corrado and Spindt (1993) have made attempts to measure transactions. Cramer breaks down the role of transactions in money demand by delineating current transactions, income transfers, capital transactions, and idle money. Of these four flows, only idle money (transfers that reflect the disposal of temporarily idle transaction balances by their conversion into time deposits or money market funds) is not a source of money demand. This is because the other three types of transactions require payment when they are consummated. Current transactions are constructed by adding together total sales and income disbursements and adjusting for income accruing directly from sales receipts. Because current transactions are defined with a heavy reliance on the National Income and Product Accounts, they are nearly proportional to aggregate income.

However, income transfers and capital transactions when added to current transactions and called total transactions are not proportional to aggregate income or GDP. But, the measurement of income transfers is low because private transfers are incalculable. Capital transactions and idle money are unobservable as well. Cramer estimates these unobservable transactions by a constant and a random variable in a model with bank transfers as a function of current transactions, demand deposits, and the nominal value of currency. He notes, though, that the specification for unobservable transactions is not very precisely determined and the overall results are "as incomplete and uncertain as the early amateur estimates of GNP." So, while his efforts do imply that total transactions, as estimated in his work, are
not proportional to the traditional scale variables used in money demand specifications, his results are preliminary and it is unclear that they would resolve periods of money demand instability like the Missing Money episode.

Other scholars have decomposed GDP into its various components under the rationale that the different components should generate different money demands. Mankiw and Summers (1986), for example, examined consumption as the appropriate scale variable. There they find that a nested test of consumption versus GDP finds consumption the better explanatory variable in a money demand regression. It still is not clear from the aforementioned work on the appropriate scale variable can explain away money demand instability.

After surveying the literature up to 1982, Judd and Scadding (1982) conclude two things. First, they think that the explanation of the Missing Money episode rests on innovations in financial arrangements. They recognize, however, that there are some questions that remain unanswered if one relies on this as an explanation for money demand instability in the 1974-76 period. One questions is why did the apparent shift in money demand occur in not only the business sector but the household sector as well? The cash management techniques and the financial innovations were concentrated in the business sector. A second question is why a money demand shift did not occur overseas when interest rates were not only high in the U.S. but everywhere else in the world as well?

Second, they note that a reopening of the pre-1973 agenda, in which the ingredients of the money demand function as well as its recipe were reexamined, did not solve the money demand instability problem of the mid-1970’s.

3. Partial Adjustment Models of Money Demand

Later work by monetary economists found the PAM money demand equation broke down during other periods. In the early 1980’s, the Great Velocity Decline saw money demand equations consistently underpredict money holdings. During this period, observed velocity fell markedly thereby giving this period’s money demand instability its name. In the mid 1980’s, money demand equations consistently overpredicted M1 money holdings giving the instability of this period the name, The M1 Explosion. This
period coincided with the introduction to the household sector of NOW and SuperNOW accounts. Again, in the early 1990's, money demand equations overpredicted money holdings.

By 1987, the investigations above into 1) financial innovations that might explain money demand instability and 2) inadequate proxies for variables in money demand specifications had not lead to a solution to the Missing Money episode or the Great Velocity Decline. Economists began to question the partial adjustment model's account of the adjustment process of the money stock to disequilibrium between supply and demand. Moreover, econometric methods became more sophisticated with respect to non-stationary series and long run relationships.

Table 2.1 contains the parameter estimates and standard errors of those estimates for a conventional money demand specification using U.S. monthly data from January 1959 to April 1999:

\[ \ln m_t = \alpha + \beta_1 \ln y_t + \beta_2 R_t + \beta_3 \ln m_{t-1} + \beta_4 \ln \left( \frac{P_t}{P_{t-1}} \right) + \epsilon_t \]  

(2.6)

where \( m_t \) is the real money stock, \( y_t \) is real personal income, \( R_t \) is the three month U.S. t-bill rate, and \( P_t \) is the CPI-U. Figures 2.1-2.4 present the times series of real M1, real M2, real personal income, and the nominal three-month T-bill rate.

Row one of the table contains a money demand model uncorrected for first order autocorrelation. Examining the \( \overline{R^2} \) for this regression along with the Durbin-Watson statistic, there are telltale signs of exactly the spurious regression that Granger and Newbold (1974) present in their seminal paper. They conclude “...if a regression equation relating economic variables is found to have strongly autocorrelated residuals, equivalent to a low Durbin-Watson value, the only conclusion that can be reached is that the equation is mis-specified, whatever the value of \( \overline{R^2} \) observed.” A low Durbin-Watson statistic can be ameliorated by imposing and estimating a first-order autocorrelation structure on the error term, as was done in row two of Table 2.1. However, this just suggests data mining as the error structure imposed is determined by the initial examination of the first regression.

Another serious econometric problem is, with each of the regressions in table 2.1, there is a very significant Ljung-Box Q statistic indicating higher order autocorrelation problems in the error term. It is well known that, in the presence of a lagged dependent variable, autocorrelation of the error term not only
makes for inefficient standard errors on the parameter estimates, but it also makes for inconsistent parameter estimates. With these two serious econometric problems, it certainly makes one wonder about the out-of-sample forecasting ability of this type of PAM equation. Yet, this was the standard estimating procedure for a PAM equation in the literature. It is of little surprise that money demand instability remains a serious and unresolved problem with these models.

Moreover, in every regression above, the coefficient on the lagged dependent variable is almost one and significantly different from zero. This suggests an implausibly slow adjustment process to monetary disequilibrium. But, more importantly, it strongly suggests the presence of a unit root in the M1 data series. Without the backbone of cointegration theory to support this type of regression, a regression of an I(1) variable on a set of explanatory variables is a nonsensical regression. And, until Engle and Granger (1987) developed cointegration theory, the best that could be hoped for in regressions like (2.6) was first-differencing the data series to make them stationary.

4. Long and Short-run Money Demand Relationships and Cointegration Theory

Using the concept of economic equilibria as a starting point for the development of cointegration theory, Engle and Granger (1987) point out that certain economic relationships will cause a linear combination of I(1) variables to have a finite variance and move about a constant mean. If $x_t$ is a vector of I(1) economic variables interrelated by some underlying economic relationship, then in equilibrium $\alpha^T x_t = 0$ where $\alpha$ is a vector of non-zero constants. This means that $z_t$, where $z_t = \alpha^T x_t$, will be I(0) even though $x_t$ is a linear combination of I(1) variables and, therefore, should likewise be I(1). The key observation is that the vector of economic variables, $x_t$, are interrelated and therefore no variable may drift infinitely far from the others. The economic variables are destined to move together by virtue of the fact of their mutual interdependence.

In the case of money demand, economic theory postulates a long-run relationship between a monetary aggregate, a scale variable, an opportunity cost variable, and possibly some measure of price inflation. Cointegration theory suggests that if these variables are I(1) and the economic theory
stipulating their relationship is correct, then there should exist at least one cointegrating vector, \( \alpha \), that makes a linear combination of these I(1) variables I(0). Granger and Engle suggest estimating this long-run relationship using OLS. They show that OLS will produce consistent estimates for all the parameters in the cointegrating regression, if a cointegrating relationship between the variables exists, because the difference between the true values of the cointegrating vector and the OLS-estimated values of the cointegrating vector vanish asymptotically. In fact, Stock (1994) shows that the parameter estimates in the cointegrating equation converge more rapidly to their probability limits than do the parameter estimates in a non-cointegrating OLS regression.

A number of researchers have examined U.S. money demand in a cointegration framework. The results of their studies are mixed. King, Plosser, Stock, and Watson (1991) examined the relationship between M2, a scale variable, and an opportunity cost variable. They found evidence of a cointegrating relationship for quarterly data from 1953-88 using Johansen’s (1988) maximum likelihood procedure. Likewise, Hafer and Jansen (1991) found evidence of a cointegrating relationship using M2 as the monetary aggregate and quarterly data from 1915-88. King et al. used the commercial paper rate and corporate bond rate in natural logs to proxy the opportunity cost variable. Hafer and Jansen used the t-bill rate in levels. However, when Hafer and Jansen examined M1 in this context, they did not find a cointegrating relationship.

Hoffman and Rasche (1991) also examined M1, but at a monthly frequency for 1953-88. They used a long and short-term interest rate as their opportunity cost variables and real personal income as the scale variable. Johansen’s ML procedure is used to estimate the cointegrating relationship. They find a cointegrating relationship with M1 as the monetary aggregate. Miller (1991), on the other hand, uses quarterly data from 1959-87 and finds that there is no cointegrating relationship between M1 and the other money demand variables. However, he does find that a cointegrating relationship exists using M2. In his study, he uses the Engle-Granger cointegration methodology to test all possible cointegrating regressions. His choice of the best cointegration model rests with the model that has the highest adjusted coefficient of determination.

procedure determines that there is a long-run cointegrating relationship between M1, real income, and a long-term interest rate. Lee (1992) examines money demand on a quarterly basis using data from 1953 to 1991. He uses the Engle-Granger procedure and Johansen’s ML procedure to find that a cointegrating relationship does exist between M1, GNP, and a t-bill rate. However, using Stock and Watson’s (1988) common trend test and Phillips and Ouliaris’ (1990) Z tests, he finds no evidence of a cointegrating relationship.

Miyao (1996) comes to a similar conclusion when he examines M2 at a quarterly frequency from 1959-93. He looks at possible cointegrating relationships between M2, a price level, real output, and a nominal interest rate. Using the Engle-Granger (1987) procedure, Stock and Watson’s (1988) common trends test, and Johansen’s (1988) ML procedure, Miyao finds that virtually none of his results supports a cointegration hypothesis. In fact, he examines a number of proxies for the scale and opportunity cost variables with little success. Ball (1998) examines M1 money demand over the period 1946-96. The data is annual with the scale variable of net national product and the opportunity cost variable as the commercial paper rate. Based on augmented Dickey-Fuller tests and the Johansen procedure, he finds a cointegrating relationship.

Table 2.2 summarizes the literature regarding money demand and whether a cointegrating relationship exists. As the previous paragraphs explain, the evidence for a cointegrating relationship in a money demand specification is mixed. In what follows, we examine both M1 and M2 monetary aggregates along with real personal income, the inflation rate, and the three-month t-bill rate. The monetary aggregates, real income, and inflation are in natural logs. The interest rate is in levels. This data is at a monthly frequency for the U.S. economy from January 1959 to May 1999. We use both the augmented Dickey-Fuller (1981) test and the Johansen (1988) ML test.

As a first step to estimating and testing a possible cointegrating relationship, it is necessary to confirm that all the variables in the cointegrating regression are non-stationary and integrated of the same order. Table 2.3 presents unit root tests of proxies for all the variables economic theory suggests are important in an explanation of money demand. (The monetary aggregates and income are real quantities in natural logs, the interest rate is nominal, and inflation is the gross rate). The unit root tests used are augmented Dickey-Fuller (1981) tests and Phillips and Perron (1988) tests. In implementing these unit
root tests, care was given to specify properly the number of lags in the ADF tests and to make sure the error terms in each of the unit root tests approximates a white noise series.

It is crucial to recognize that the ADF unit root tests require the error term to be IID. Data-based procedures were used to determine the lag length in the ADF tests to ensured whitened error terms. However, Choi (1990) notes that, if the errors have a MA component, long lagged autoregressions that try to account for autocorrelation bias the OLS estimate of the unit root toward one. This biases the unit root test statistic toward not rejecting a unit root hypothesis when, in fact, there is not a unit root. In these cases, a moving average (MA) component may be present in the error term that cannot be adequately captured by a AR(p) representation. The Phillips and Perron (1988) test relaxes the assumption of IID errors and, therefore, can accommodate possible MA representations in the error term.

In almost every case, each of the unit root tests does not reject the null hypothesis of a unit root. A necessary condition for the testing of a cointegrating relationship holds. Table 2.4 presents four different, estimated cointegrating equations:

\[
\ln m_i = \alpha + \beta_1 \ln y_i + \beta_2 R_i + \beta_3 \ln \left( \frac{P_i}{P_{i-1}} \right) + \epsilon_i \tag{2.7}
\]

and

\[
\ln m_i = \alpha + \beta_1 \ln y_i + \beta_2 R_i + \epsilon_i \tag{2.8}
\]

In each of the regressions, both monetary aggregates, M1 and M2, are used and each is examined with and without inflation as an explanatory variable. An important thing to note is that cointegration theory, through Granger’s (1983) Representation Theorem, specifies not only a linear long-run equilibrium equation, but also dynamic short-run adjustment in the form of an error correction model (ECM). So, it would be inappropriate to include a lagged dependent variable in these cointegrating regressions because the lagged monetary aggregate is posited in the PAM framework to be a specification for the short-run adjustment process. In the cointegrating regressions, it is unnecessary.

Since the parameter estimates in a cointegrated system converge very rapidly to their probability limits, autocorrelated errors are not a problem as the parameter estimates will still be superconsistent.
However, if a cointegrating relationship does not exist, the above regressions will be spurious in the Granger-Newbold sense. In that case, the standard errors will be highly misleading.

It is important then to conduct unit root tests on the residuals from the cointegrating regressions to determine the residuals’ order of integration. Specifically, if a cointegrating relationship exists between the economic variables, then the residuals from the cointegrating regression should be integrated of order zero. The residuals should have all the properties of a stationary series. Table 2.5 examines the residuals from the regressions in table 2.4.

It is important to note that the critical value for statistical significance at the 95% level of confidence is different from the critical values normally used in unit root tests. This is because, when the residual series is being tested for a unit root, the estimated errors are known to the researcher and not the actual errors. Ordinarily, a Dickey-Fuller table can be used, but, because one is working with estimates of the errors and not the actual errors, using Dickey-Fuller statistics would not account for the sampling error (and possible measurement error) in the residuals. Engle and Yoo (1987) provide tables for the appropriate test statistic to use in cases where estimates are used instead of the actual series. Those statistics are a function of the number of variables that appear in the equilibrium relationship and that is why the 5% critical value varies in Table 2.5 across regressions.

From Table 2.5, the evidence for a cointegrating relationship for U.S. money demand at a monthly frequency is not strong. In fact, in only two unit root tests out of the twenty-four can the null hypothesis of a unit root be rejected with a 90% level of confidence. This is taking into account the unknown ARMA structure in the unit root test residuals that has a significant amount of autocorrelation. Again, using ADF and PP tests that use the data to determine the appropriate number of AR lags or that are general enough to allow for MA error structures should allow for a whitening of the residuals from the unit root tests.

There are limitations to the Engle-Granger method of testing for a cointegrating relationship, however. Specifically, Engle-Granger cointegration theory requires a dependent variable be specified in the cointegrating equation. Without this normalization, a non-unique estimate of the cointegrating vector will result. This is because the OLS fit of a reverse regression will not give the reciprocal of the coefficient in the forward regression.
It is possible, though, that a cointegrating relationship could exist between a set of variables normalized in one way, but a cointegrating relationship would not exist between the same set of variables normalized another way. Since the economic relationship is between a set of economic variables and should not be dependent on an arbitrary normalization, the Engle-Granger procedure is problematic.

Moreover, as was already alluded to above, since the Engle-Granger procedure relies on a two-step estimation process, the error term from the cointegrating regression is an estimate, not the actual error. So, any estimation error introduced into the residual series from the cointegrating regression will be carried over to the estimation of the ECM.

Johansen (1988) and Johansen and Juselius (1990) proposed a procedure for estimating a cointegrating relationship that overcomes the aforementioned problems in the Engle-Granger approach. Specifically, Johansen's method does not require one to specify a dependent variable nor to estimate the long-run and short-run equations in two steps.

If $x_t$ is a vector of economic variables thought to be cointegrated, then a $p$th order VAR can be set up such that

$$x_t = \sum_{i=1}^{p} A_i x_{t-i} + \varepsilon_t$$

(2.9)

Johansen reparameterizes (2.9) as

$$\Delta x_t = \sum_{i=1}^{i=p+1} \Pi_i \Delta x_{t-i} - \Pi x_{t-p} + \varepsilon_t$$

(2.10)

where $\Pi = (I - \sum_{i=1}^{p} A_i)$. $\Pi$ is the matrix of eigenvalues such that its rank determines the number of cointegrating vectors for the system. If the rank of $\Pi$ is zero, then equation 2.10 corresponds to a traditional, differenced VAR. If the rank of $\Pi$ is non-zero and the system has rank, $r$, then there exists $r$ independent cointegrating vectors.

$\Pi$ can be estimated and the trace of $\Pi$ can be computed. Johansen (1988) and Johansen and Juselius (1990) propose two tests for determining the number of distinct cointegrating vectors and for
determining the number of alternative cointegrating vectors as r vs. r + 1. These two tests are the trace test and the lambda max test, respectively.

The trace test is:

$$\lambda_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i)$$

(2.11)

where $\hat{\lambda}_i$ is the ith ordered characteristic root of $\pi$. If the variables in $x_t$ are not cointegrated, the rank of $\pi$ will be zero and all the characteristic roots of $\pi$ will be one. Therefore, for each i, $\ln(1 - \hat{\lambda}_i)$ will be zero and the trace statistic will be zero.

The lambda max test statistic is:

$$\lambda_{max}(r, r + 1) = -T \ln(1 - \hat{\lambda}_{r+1})$$

(2.12)

The null is that the number of cointegrating vectors is r against the alternative of $r + 1$ cointegrating vectors. If the estimated value of the eigenvalues is close to zero, then $\lambda_{max}$ will be close to zero.

Since there is no need to specify a normalized variable, Johansen's procedure is used first to determine the number of cointegrating vectors among the money demand variables that are posited to be related by money demand theory. Recall the Engle-Granger procedure for determining the existence of a cointegrating relationship found almost no evidence that the null of no cointegration could be rejected. However, this does not mean that the variables are not cointegrated because results from the Engle-Granger procedure are sensitive to the normalization. Table 2.6 presents the results from Johansen's trace test for the existence of a cointegrating vector.

As can be seen from the table, the null of no cointegrating vector can be rejected at better than a 99% level of significance for M1. For M2, the evidence is not as strong, but there is still evidence of a cointegrating relationship as shown in Table 2.6. These results are also robust to the lag length specification. The longest lag length also shows that the error processes in the VAR have been whitened satisfactorily. However, it is possible that these results could be somewhat confounded by the possibility that nominal interest rates and inflation could be cointegrated. According to Fisher's equation, there may
be a linear cointegrating relationship between nominal interest rates and inflation, especially if real interest rates are I(0).

Table 2.7 presents the results from Johansen’s procedure on a monetary aggregate in real terms, a scale variable in real terms, and a nominal interest rate. It appears that eight lags of the variables are sufficient to whiten all the error terms. Again, a cointegrating vector exists with better than 95% confidence for M1. The evidence for M2 is a little less conclusive, but the null of no cointegrating can still be rejected at the 90% level of confidence. Both table 2.6 and 2.7 illustrate that a cointegrating relationship exists between the set of variables suggested by monetary theory to explain money demand. Moreover, care is taken to obtain whitened residual series in the Johansen ML estimation. However, there is not evidence, according to Table 2.5, that the cointegrating relationship exists with the monetary aggregate as the dependent (normalized) variable.

Stock and Watson (1993) noted that the Johansen ML has sample size distortions as the results are derived asymptotically. Moreover, they remark after running Monte Carlo simulations that Johansen estimates are highly sensitive to the choice of lag length. Toda (1994) also found severe finite sample bias via Monte Carlo simulations. These concerns, while serious, are less pertinent for our results above as we are examining a data set that approaches 500 observations. Moreover, we examine various lag lengths and found our results robust to different lag length specifications.

A couple of concerns that are more problematic do arise, though. First, we saw with the Engle-Granger methodology that there is not evidence for the existence of a cointegrating relationship if the monetary aggregate is the normalizing variable. However, much of the money demand literature specifies a money stock as a function of a number of explanatory variables. While this isn’t a criticism of the cointegration literature, it does question whether money demand theory and its empirical testing is suspect. Relying on cointegration theory also begs the question as to the causal relationship between the important variables in the money demand system.

Perhaps what is more damaging for cointegration theory is the arbitrary nature of the short-run adjustment process. Granger’s representation theorem simply states that a finite Wold decomposition exists that would approximate the first differences of a system variable when supplemented by an error correction term as an explanatory variable. In other words, an error-correction model (ECM) exists that
represent the short-run or out-of-equilibrium dynamics of the variable of interest. The dynamics of this ECM are not specified by cointegration theory or monetary theory in the case of money demand. This makes cointegration results liable to an omitted variable bias and other misspecification errors.

Moreover, it suggests that data mining might be a legitimate concern complete with potentially poor out-of-sample forecasting. These concerns lead one in the direction of a well-specified model that specifies a money demand relationship, but not necessarily one in which the monetary aggregate is the dependent variable. It also leads one to a process of adjustment to a monetary disequilibrium that is specified by a theoretical model.

5. The Buffer Stock Model of Money Demand

The buffer stock approach to money demand developed in response to the poor performance in the 1970’s of conventional money demand equations (Mizen, 1994, p. 25). In essence, a buffer stock model proposes that when an economic shock occurs, it is costly to adjust some variables in the short run. If the shock can be redistributed to a variable that is less costly to adjust, then the cost of adjustment to the shock can be minimized or at least spread out over time.

In the case of money demand, there is a portfolio of other assets that are fairly illiquid. The illiquid nature of these assets makes it more costly to adjust them in an agent’s portfolio. If another asset exists that can be adjusted relatively inexpensively, then it can take up monetary shocks when they occur. In buffer stock models, this instrument is the money stock. The money stock removes the impact of a shock by taking up the slack in order to meet an objective of stable illiquid asset holdings. If there is this buffer stock of money that can protect the holdings of more illiquid assets from frequent readjustment, then the asset portfolio can be readjusted slowly over time, in effect, reducing the cost of adjustment. Carr and Darby (1981), Laidler (1982, 1984) and Cuthbertson and Taylor (1987) are researchers that suggested this role of money in a buffer stock model.

2 Santomero and Seater (1981) propose a buffer stock model where the impetus for a slow adjustment process to money holdings is not due to the costly adjustment of illiquid assets, but instead due to agents’ aversion to allocating excess money holdings to “bad” or “rash” investments. In this case, there is a slow adjustment in money holdings while agents search for suitable illiquid assets in which to invest.
This monetary disequilibrium is suggested by both a buffer stock and a cointegration approach to money demand. Actual aggregate money holdings may differ from their long run targets in the short run. This certainly implies that, at the macroeconomic level, in the short run an economy can be off its aggregate demand for money function. Cointegration theory suggests no theoretical explanation of this short-run disequilibrium other than positing that it is some ad hoc linear function of the variables (and their lags) in the long run, cointegrating relationship. The buffer stock approach differs from a cointegration approach to money demand by offering an explanation of why there may exist a monetary disequilibrium in the short run. That explanation is often of an agent being subject to unforeseen variations in his income, varying availability of goods, and unexpected fluctuations in price (Laidler, 1984). Holding money enables an agent to endure the surprise variations in these variables at a lower cost than would otherwise be the case. So, a discrepancy between actual money holdings and long-run target money holdings can arise to prevent "...surprises in markets where the agent is a seller from impinging upon his buying activities, and vice versa" (Laidler, 1984).

Carr and Darby (1981) were some of the first researchers that operationalized the ideas behind the buffer stock approach to money demand into an empirically testable equation. They used Chow’s adjustment mechanism (2.2) for short-run money demand adjustment and amended a transitory income term and a money supply shock term to it. Their rational for these amendments is that if the money stock was, in fact, best thought of as a buffer stock then it would be most useful in that capacity when variations to transitory income and unexpected money shocks occurred. They specified the short-run money demand function as:

\[ m_t = \lambda m_{t-1} + (1 - \lambda)m_{t-1} + \beta y_t^T + \phi \hat{M}_t \]  

(2.13)

where \( y_t^T \) is the natural log of real transitory income and \( \hat{M}_t \) is the money supply shock. Combining (2.13) with a simple Cagan (1956) long run money demand function gives:

\[ m_t = \lambda y_0 + \lambda y_1 y_t^T + \lambda y_2 R_t + (1 - \lambda)m_{t-1} + \beta y_t^T + \phi \hat{M}_t + \epsilon_t \]  

(2.14)
where $y_i^P$ is the natural log of real permanent income. The expected natural log of $M_i$ is estimated by a univariate ARIMA process.

Carr and Darby go on to test whether an unexpected money shock impacts money holdings in a statistically significant way. Also important is whether real transitory income has any significant influence on money holdings. Since the Chow specification and therefore a PAM specification is nested within the Carr and Darby model, a finding of $\beta = \phi = 0$ is evidence for the PAM. They use quarterly data from 1957-76 for the G7 countries plus the Netherlands. Carr and Darby find that $\hat{M}_i$ statistically significantly influences money holdings in every country’s equation. While they do not find evidence that $y_i^T$ has a significant influence on money holdings, they conclude that a buffer stock approach to examining money demand is useful. They note that, without a buffer stock approach to money demand, a neo-Keynesian explanation of money demand requires interest rates to move by an implausibly large amount to relatively small, unexpected increases in money.

It is apparent from the Carr and Darby study that there are a number of significant econometric issues that need to be addressed. In light of our previous discussion, it is apparent here that the possibility exists for the problem of spurious regressions. An examination of Carr and Darby’s results indicates that the symptoms of a spurious regression are there in each country’s estimated equation, namely significant autocorrelation coupled with large adjusted coefficient of determination. Moreover, since a lagged dependent variable is present in (2.14), the presence of autocorrelation suggests that parameter estimates will not only be inefficient but inconsistent as well. Another issue is the inclusion of an estimated explanatory variable in (2.14). Since it is an estimate, it is necessary to correct the standard error for that parameter estimate (Pagan, 1984).

MacKinnon and Milbourne (1984) presents more criticisms of Carr and Darby’s results, however, these criticisms do not undermine the buffer stock approach per se as much as they undermine Carr and Darby’s work. In fact, Cuthbertson and Taylor (1987) modify Carr and Darby’s original approach to account for the econometric shortcomings to produce an acceptable model for narrow money in the United Kingdom.
However, a serious problem with the buffer stock approach that is harder to overcome is its reliance on an unmodeled adjustment process that returns money holdings to their long run equilibrium values. Laidler (1984) remarks that the buffer stock approach to money demand relies on price stickiness to prevent monetary disequilibria from instantaneously equilibrating. However, this price stickiness makes a buffer stock approach "...vulnerable to the usual charges of ad hocness in its reliance on unexplained wage-price stickiness" (Laidler, 1984).

Carr and Darby (1981), Cuthbertson and Taylor (1987), Cuthbertson (1988), and Mizen (1994) among others all assume a quadratic cost function in the same mold Chow (1966) did to derive buffer stock specifications. In Carr and Darby's case, the expectations of variables in the model were generated by backward-looking estimations. In the other cases, forward-looking variables were estimated and inserted into the cost of adjustment models. But, in either case, the cost of adjustment models did not derive from a base specification of agents' preferences. In that sense, the models did not come from microfoundations.

6. The Moderate Quantity Theory of Money Demand

The Moderate Quantity Theory of Money (McCulloch (1980)) proposes a theoretically derived adjustment process for monetary disequilibria. The theory is similar in structure to the Extreme Quantity Theory of Money in that it presents prices as a function of the disequilibrium between desired money holdings and the actual money stock. But, prices are also a function of agents' expectations of future inflation rates. This addition to the Extreme Quantity Theory allows for the existence of non-zero inflation rates even when the money market is in equilibrium. Moreover, the Moderate Quantity Theory has the attractive feature of specifying a theoretical explanation for the adjustment of money demand and supply to equilibrium that does not require this adjustment to be instantaneous.

The idea behind the Moderate Quantity Theory of Money (MQT) is that an excess supply of money causes economic agents to go to different markets to exchange money for goods. They do this

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3 Appendix B has a detailed derivation of the Moderate Quantity Theory of Money.
because the excess money depresses consumers' marginal utility for money. This has the effect, in consumer choice theory, that other goods and services become more desirable relative to money at the current level of prices for the average consumer. Only until prices adjust to accommodate the excess supply of money will equilibrium again be achieved. However, consumers do not have perfect information regarding all prices throughout the economy. So, when they compare the marginal utility of money (the price of money is normalized to be one) to the ratio of a good's marginal utility to its own price, they are using past prices for that given good as a proxy for what current prices are. This is not to imply that consumers have imperfect price information for a given good when they are in the market for that good. Once consumers enter the market for that good, they are able to ascertain the price of that good immediately. However, the average consumer cannot be in all markets at once. So, to the extent that consumers are comparing the marginal utilities of goods and money to each other simultaneously before making spending choices, they do not simultaneously know the prices of every good in every market for which they might make a purchase.

MQT is a theory of how prices adjust as a function of the disequilibrium between money supply and money demand as well as expectations of what the inflation rate will be. This price adjustment restores equilibrium in the money market by decreasing the real value of money through an increase in the price level. However, the price adjustment equation specified by the MQT will not be instantaneous, unlike the Extreme Quantity Theory of Money.

Mises (1953) explained the value of money in marginal utility terms, as one would for any ordinary commodity. Namely, agents interact to buy and sell goods and services until the price ratio of any two commodities is equal to the respective ratio of marginal utilities of those commodities. This is just the first order conditions of the general consumer optimization (choice) problem derived from a well-behaved (continuous, monotonic, concave, locally nonsatiated) utility function. In other words,

$$\frac{P_i}{P_j} = \frac{MU_i}{MU_j} \quad i \neq j$$  \hspace{1cm} (2.15)

where $P_i$ is the price of commodity $i$ and $MU_i$ is the marginal utility of commodity $i$. 

31
In Mises’ framework, money’s marginal utility was compared to other goods marginal utility using respective prices as deflators; and, as such, the quantity held of money was subject to the same optimization conditions as any other commodity. A money shock puts additional money balances in the hands of agents, but does not affect the existing quantities of other goods. This has the direct consequence that the marginal utility for money decreases. If money is the $i^{th}$ good in equation (2.15), then the equality no longer holds for money and any other good, $j$. Agents trading for relatively fewer goods with relatively more money pass around this excess supply of money in the economy. This has the effect that the price of any other good, $j$, is bid up while the value of money, $P_i$, is bid down. “Thus the increase of prices continues, having a diminishing effect, until all commodities, some to a greater and some to a lesser extent, are reached by it” (Mises (1953) p. 139).

McCulloch’s Moderate Quantity Theory takes the ideas first expounded by Mises and quantifies them in the form of a mathematical model. The model, in essence, is an economy in which agents receive cash flows from the production side of the economy that are stochastic. The uncertain cash flows are the result of unrelated microeconomic events that cause unforecastable deviations in production. But, these deviations in output are never so large as to cause discontinuous jumps in output from previous output levels. So, while consumers do not know the exact production levels of any given good in the economy, recent past levels of production are good proxies for current levels.

Since there is uncertainty in what is being produced in this economy, consumers will be uncertain as to the exact prices for goods as well as what consumers’ potential levels of consumption will be. Given that consumers use the decision rule illustrated in (2.15) to determine optimally their consumption, the uncertainty in both prices and consumption will translate into uncertainty surrounding the comparison of goods using this decision rule. Due to this uncertainty, consumers will use past values of prices and consumption to make decisions on current period consumption. This will allow for some stickiness in prices as consumers are unable to adjust immediately to stochastic production shocks due to the incomplete information they initially have about them.

This incomplete adjustment process is crucial to the Moderate Quantity Theory because it allows for the non-neutrality of a money shock in the short run. Since the exact current price level is unknown,
Recent past prices are used in the decision rule (equation (2.15)). So, prices do not adjust instantaneously to a monetary shock. As consumers gain more information regarding output levels, though, prices adjust to their long-run levels nominalizing a monetary shock. So, MQT retains the long-run neutrality of money that any monetary model of money must have.

Using an additively separable, CRRA utility function, a stochastic production function for each of \( n \) goods, and a specification for consumers’ price expectations, McCulloch (1980) provides us with what he refers to as the functional form of the Moderate Quantity Theory of Money

\[
\pi_t = \gamma (\ln m_{t-1} - \ln m^d_{t-1}) + \pi^a_{t-1} + c \frac{dz}{dt}
\]

(2.16)

where \( \gamma = \frac{n m_{t-1}}{w_{t-1}} \), \( n \) is the coefficient of relative risk aversion, \( w_{t-1} \) is consumers’ wealth, \( \pi^a \) is consumers’ expectations of next period’s inflation, and \( dz \) is a stochastic differential with mean zero. The variables, \( m_{t-1} \) and \( m^d_{t-1} \), are real money balances and a specification for real money demand, respectively.

Before going on to test this adjustment equation empirically, there are a few comments that should be made at this point. First, the derivation of this adjustment equation gives an economically meaningful definition of the adjustment parameter, \( \gamma \). The greater the coefficient of relative risk aversion (which is also in the case of the CRRA utility function the elasticity of marginal utility), the smaller the lag between current prices and past prices used as proxies, and the greater the fraction of money to overall wealth, the faster will be the adjustment of money supply to money demand. As in the Extreme Quantity Theory, the larger the difference between money supply and money demand, the faster is the adjustment between the two. And, the greater consumers’ expectations of future inflation rates, the faster will be the adjustment between money supply and money demand.

This adjustment back to equilibrium in the money market takes place, as specified by equation (2.16), as the real stock of money is revalued by \( t - 1 \) period prices. The readjustment of \( m_{t-1} \) brings closer the money supply to money demand in subsequent periods. This implies that a one-time money supply shock will have its greatest effect on prices initially; and, the effect will subsequently die down in
future periods as money supply and money demand become closer to their equilibrium value. In the long run, the growth rate of the nominal money stock will be internalized in the inflation rate making the real money stock grow at the same rate as money demand. Yet, due to the inflationary expectations of consumers, it is still possible to have an inflation rate different from zero even if the money market is in equilibrium.

7: Summary

This chapter reviews the literature on money demand models. Early work focused less on the potential econometric problems or functional form and more on the appropriate proxies to use for the scale and opportunity cost variables. Whether a transactions approach or a portfolio approach to money demand was the appropriate way to view the issue was the leading discourse.

The Missing Money Episode changed the landscape of the literature by leading to the realization that, regardless of which ideology one espoused, neither one adequate explained the money demand in the mid 1970's. This lead to Judd and Scadding's (1982) famous view that the Missing Money Episode split the subsequent literature into two separate lines of investigation. The first focused on financial innovation as the proximate cause of the money demand instability. The second opened up the pre-1973 research agenda in which the appropriate proxies to use in money demand specification took center stage. In either case, a satisfactory solution was not discovered.

Partial adjustment models incorporated Chow's (1966) views on a non-instantaneous adjustment to a monetary disequilibrium to the thoughts of Baumol (1952) and Tobin (1958) when the money market is in equilibrium. However, we show that the partial adjustment models suffer from a number of flaws as well as being ad hoc in nature. These flaws include succumbing to the Granger-Newbold (1974) spurious regression problem, high degrees of serial correlation, possibly suggesting misspecification, poor out-of-sample prediction power, and parameter instability.

Cointegration theory and error-correction models address the Granger-Newbold problem by postulating a cointegrating relationship between the regressand and the regressors in a money demand specification. If a cointegrating relationship exists between money, income, and interest rates, then
parameter estimates from the cointegrating equation are superconsistent and suffer from neither spurious correlation nor endogeneity. Moreover, Granger’s (1983) Representation Theorem states that, if this cointegrating relationship exists, then an error correction model exists that will most efficiently specify the short run dynamics of the system. We find, however, that following the Engle-Granger (1987) method of testing for a cointegrating relationship does not yield a cointegrating vector using U.S. monthly data from January 1959 to May 1999. Besides the error correction model being an ad hoc and arbitrary specification of the short run dynamics, the non-existence of a cointegrating relationship with the real money stock as the normalized variable leave this model of money demand unfeasible.

We last examine buffer stock models of money demand. These models postulate money as a buffer that allows agents to adjust their non-monetary asset holdings without having to so instantaneously and, therefore, costly in response to a macroeconomic shock. But, there are very little rigorous underpinnings to frictions that underlie a less than instantaneous adjustment to a shock. While it may be costly to adjust asset holdings, often these models do not specify these costs directly. Worse, little is said and less is derived about the speed of adjustment to economic shocks.

The Moderate Quantity Theory of Money (MQT) is an alternative view to these other money demand models. Unlike PAM’s, MQT does not suffer from those econometric difficulties. Unlike cointegrated ECM’s, MQT needs only show a cointegrating relationship exists between money, income, and interest rates without relying on money to be the normalizing variable. That a cointegrating vector exists is something that, using Johansen’s (1988) trace tests, we were able to show in this chapter. And, unlike all the other models considered here, the Moderate Quantity Theory of Money has an adjustment process to a monetary disequilibrium that has rigorous microeconomic foundations. The remainder of this work will examine the empirical validity of the Moderate Quantity Theory of Money.
CHAPTER 3

Inflation Expectations

1: The Price Level

The process by which inflationary expectations are generated also must be specified. Consumers’ inflationary expectations are based on prior price levels and price changes. But, the theoretical MQT model does not specify how expectations of next period’s inflation should be modeled. However, it is clear from the Moderate Quantity Theory of Money model that, if rational expectations are adopted as the method by which agents develop their expectations of future events, monetary policy will be neutral even in the short-run, i.e., there cannot be a longer than one period disequilibrium between money supply and money demand.

We adopt a univariate time series approach to proxying agents’ next period inflation expectations. This will allow for what appears to be a stylized fact of monetary policy, money is non-neutral in the short-run. At the monthly frequency, we use the Consumer Price Index as our aggregate prices time series. It is important to recognize that the CPI-U series is thought to have seriously mismeasured the housing component of the true cost of living prior to its 1983 revision (Bidarkota and McCulloch, 1999). The Bureau of Labor Statistics adjusts the CPI-U series for this measurement error starting in 1967. This recomputed series is the CPI-X. We construct a spliced time series of the CPI using the CPI-U from January 1950 to June 1967 and splicing it to the CPI-X from July 1967 to December 1982. We then splice this created series with the CPI-U from January 1983 through May 1999 to obtain what we call the CPI-UX series. While the CPI-UX series still suffers from the measurement error that plagued the original CPI-U series prior to 1983, we have corrected this measurement error from July 1967.
to December 1982 thereby making CPI-UX a better aggregate prices series than the CPI-U over the time period considered here.

The annualized monthly inflation series was constructed by computing the annualized differences of natural logarithms of the CPI-UX series. This monthly inflation series was then seasonal adjusted using the X11 seasonal adjustment algorithm. We used this monthly seasonally adjusted inflation series to develop next period forecasts of inflation. These next period forecasts were the proxy variable used for agents inflation expectations in the empirical tests of the Moderate Quantity Theory Model of Money Demand.

2: Inflation and Unit Roots

Examining monthly inflation annualized over the post-war period, it appears that inflation is a trending series with the possibility that the trend is time-varying (see Figure 3.1). It is also plausible that monthly inflation could move around a time varying mean. In either case, this suggests that, at least for certain parts of the sample period, monthly inflation could be a non-stationary process.

One way to model the inflation series in order to generate agents' next period expectations of inflation is to use first differenced inflation data. Given that the U.S. inflation series is, at most, an I(1) series, this transformation will ensure that we are modeling a stationary time-series with our time series models. However, first differencing has two significant drawbacks. First differencing results in the loss of long run information in the data. Also, first differencing can lead to inefficient parameter estimates if the assumption of a unit root is untrue. Moreover, many authors (Blough 1992; Cochrane, 1991; and Stock 1990) argue that the question of whether a series has a unit root or not is inherently unanswerable when dealing with a finite sample. In essence, unit root tests do not have the power to distinguish between a series with a unit root and a series with a near unit root.

We tested the inflation series using an augmented Dickey-Fuller (1981) unit root test and a Phillips and Perron (1988) Z-test with no time trend. The ADF t-test for the parameter estimate of the
first lag of the inflation series was -2.4025.\textsuperscript{4} The Phillips and Perron Z, statistic for the parameter estimate of the first lag of the inflation series was -9.4686.\textsuperscript{5} The 5\% critical value for both tests is -2.87. It is apparent that the null of a unit root cannot be rejected using the ADF test statistic, but the null can be rejected using the Phillips and Perron Z, statistic. While the conclusion of a unit root series would seem to hold from the early 1970's through the early to mid 1980's, examination of the inflation series prior to that would certainly suggest a stationary series (cf. Sargent, 1971). Also, examination of the series after the mid 1980's would suggest a stationary (or near stationary) series. Rather than rely on unit root tests to give a yes-no answer to the question of a unit root, it would seem better to try and estimate the parameter estimate on the first lag term in an ADF regression directly.

\textbf{3: A Median Unbiased Estimator for } \alpha

Work by Andrews (1993), Andrews and Chen (1994), Fuller (1996), and Fuller and Roy (1998) has suggested that the direct modeling of a unit root or near unit root process can be done by using median unbiased estimators. It is well known that the coefficient on the AR(1) term in an OLS autoregression will be biased downward as the true value of the estimator approaches one (Mariott and Pope (1954), Pantula and Fuller (1985), and Shaman and Stine (1988)). The same bias is evident when one uses an Augmented Dickey-Fuller test to estimate the sum of the AR coefficients from an AR(p) model. To correct for this bias, the authors calculate the bias contingent on the sample size and the true AR(1) parameter. The estimated parameter is then corrected by incorporating this bias.

In the case of Andrews (1993) and Andrews and Chen (1994), the model is written as a standard AR(p) model:

$$Y_t = \mu + \gamma_1 Y_{t-1} + \gamma_2 Y_{t-2} + \ldots + \gamma_p Y_{t-p} + \epsilon_t.$$ 

(3.1)

Rewriting (3.1) in Augmented Dickey-Fuller form gives

\textsuperscript{4} Eleven lagged difference terms in the ADF regression were used. That number of lagged difference terms was specified by using the Akaike Information Criteria to specify the best model, the Ljung-Box Q-statistic to specify the model until enough lags were used to insure no serial correlation in the ADF residuals, and the Campbell and Perron (1990) method of determining the optimal lag length by dropping the last lag term until the parameter estimate for the term is significant. All three methods chose the same lag length.\textsuperscript{5} The number of lags used in the spectral estimation window is 4.
\[ Y_t = \mu + \alpha Y_{t-1} + \phi_1 \Delta Y_{t-1} + \ldots + \phi_{p-1} \Delta Y_{t-p+1} + \epsilon_t \]  

(3.2)

where \( \alpha = \sum_{i=1}^{p} \gamma_i \). The more persistent the time series, \( Y_t \), the greater is the downward bias present in the OLS estimator for \( \alpha \).

In Andrews and Chen’s method, \( \alpha \) is estimated by OLS, but it is then adjusted up for this downward bias in cases where \( Y_t \) is generated by a unit root or near unit root process. Specifically, if \( m \) is the median of a random variable, \( Y \), then

\[ \Pr(Y \geq m) \geq \frac{1}{2} \quad \text{and} \quad \Pr(Y \leq m) \geq \frac{1}{2} \]

(3.3)

(3.4)

The definition of a median unbiased estimator for \( \alpha \) is an estimator where, if \( \alpha \) is the true parameter, the estimate \( \hat{\alpha} \) has \( \alpha \) as its median across the parameter space for \( \alpha \). The median unbiased estimator has the property that the probability of underestimation equals the property of overestimation. Moreover, the time series considered have an upper bound on \( \alpha \) of 1. This means that any time series \( Y_t \) is not an explosive time series. But, it also means that a mean unbiased estimator will also be biased downward due to the truncated parameter space for \( \alpha \).

To solidify this concept of a median unbiased estimator, suppose for a sample size of 50, the OLS estimate of \( \alpha \) is 0.90. Andrews (1993, Table II, p. 148) calculates the exact median unbiased estimate to be approximately 0.98. (This assumes a linear model with a constant, but no time trend). In this example, if the true \( \alpha \) is 0.98, sets of OLS regressions on that data would give 0.90 as its median value.

Andrews and Chen note that for AR(p) processes in (3.2) the median unbiased estimator, \( \hat{\alpha} \), is a function not only of \( \alpha \) but also of \( \phi_1 \ldots \varphi_{p-1} \) which are nuisance parameters in this case. Since, in
general, $\varphi_1 \ldots \varphi_{p-1}$ are unknown, Andrews and Chen method for AR(p) processes (3.2) yields only an approximate median unbiased estimator for $\alpha$.

To obtain an approximately median unbiased estimate for an AR(p) time series process as in (3.2), Andrews and Chen suggest an iterative process to calculate first the estimate of $\hat{\varphi}_1 \ldots \hat{\varphi}_{p-1}$ via OLS. Then treat $\hat{\varphi}_1 \ldots \hat{\varphi}_{p-1}$ as if they were the true values and compute $\hat{\alpha}$.

Correct for the bias inherent in the estimate of $\alpha$ and impose the bias-corrected estimate on a second stage OLS regression to determine a second round of estimates of $\varphi_1 \ldots \varphi_{p-1}$. The correction of the estimate of $\alpha$ takes the following form:

\[
\tilde{\alpha} = \begin{cases} 
1 & \text{if } \hat{\alpha} > m(1) \\
m^{-1}(\hat{\alpha}) & \text{if } m(-1) < \hat{\alpha} \leq m(1) \\
-1 & \text{if } \hat{\alpha} \leq m(-1)
\end{cases}
\]  

(3.5) (3.6) (3.7)

where $\tilde{\alpha}$ is the median unbiased estimate, $\hat{\alpha}$ is the OLS estimate, and $m(\cdot)$ is the median function which is defined as $m(\tilde{\alpha}) = \hat{\alpha}$. Values for $m(\tilde{\alpha})$ are generated by numerical simulation and are presented in Andrews (1993).

Once $\tilde{\alpha}$ is determined, treat it as if it was $\alpha$ and compute a second round of estimates for $\varphi_1 \ldots \varphi_{p-1}$. Andrews and Chen suggest continuing this process until convergence of $\tilde{\alpha}$ to $\alpha$ is achieved. Practically speaking, Andrews and Chen found all the time series they considered converged after at most four iterations of this procedure.

Fuller's (1996) and Fuller and Roy's (1998) median unbiased estimator for $\alpha$ in (3.2) is not susceptible to the nuisance parameters, $\varphi_1 \ldots \varphi_{p-1}$, inherent in the Andrews and Chen method. So, it does not require the iterative computational method to determine $\varphi_1 \ldots \varphi_{p-1}$ found in Andrews and Chen. This
is because the bias correction for \( \hat{\alpha} \) is a function only of \( \hat{\alpha} \), the sample variance of \( \hat{\alpha} \), and the \( t \)-statistic based on \( \hat{\alpha} \) not being different from one. Fuller and Roy (1998) also show that across much of the parameter space for \( \alpha \) the Fuller estimator has a smaller mean square error than does the Andrews and Chen estimator. It is for these two reasons that we adopt the Fuller (1996) estimator to estimate directly \( \alpha \) for the inflation data we examine here.

One can represent a stationary Gaussian autoregressive process by either its forward representation, \( Y_t + \sum_{j=1}^{p} \alpha_j Y_{t-j} = e_t \), or its backward representation, \( Y_t + \sum_{j=1}^{p} \alpha_j Y_{t+j} = v_t \), where \( e_t \) and \( v_t \) are serially uncorrelated random variables. One could construct a class of estimators that is a weighted representation of both the forward and backward representations of the series, \( Y_t \), and calculate these estimators by minimizing this representation with respect to these estimators. The full representation would be

\[
F(\alpha) = \sum_{t=p+1}^{n} w_t \left[ Y_t + \sum_{j=1}^{p} \alpha_j Y_{t-j} \right]^2 + \sum_{t=1}^{n-p} (1 - w_t) \left[ Y_t + \sum_{j=1}^{p} \alpha_j Y_{t+j} \right]^2.
\] (3.8)

This equation would be minimized with respect to \( \alpha \) where the expressions in brackets are just \( e_t \) and \( v_t \), respectively. If the weight, \( w_t \), is set to 1, then \( \alpha \) is the OLS estimator. If the weight, \( w_t \), is defined as

\[
w_t = \frac{1}{(n - 2p + 2)}(t - p) \quad t = 1, 2, \ldots, p,
\] (3.9)

\[
w_t = \frac{1}{(n - 2p + 2)}(t - p) \quad t = p + 1, p + 2, \ldots, n - p + 1,
\] (3.10)

\[
n - p + 2, n - p + 3, \ldots, n,
\] (3.11)

then the estimator is the weighted symmetric estimator. The weighted symmetric estimator has the same limiting distribution as the maximum likelihood estimator and the OLS estimator when the time series, \( Y_t \), is a stationary autoregressive process. In the case of all these estimators, however, they are biased in the vicinity of a unit root. Since the U.S. inflation series in Figure 3.1 suggests that assuming a unit root
governs the entire time series is a specious assumption to make, it is natural to adopt an estimator for $\alpha$ that is the least biased in the neighborhood of a unit root. Fuller and Roy (1998) show that a median unbiased weighted symmetric OLS estimator has a smaller bias across the parameter space of $\alpha$. The median unbiased weighted symmetric estimator also performs better than other estimators for processes with roots close to or equal to one (in absolute value).

Since the weighted symmetric estimator is still biased toward zero for non-stationary series, Fuller and Fuller & Roy's method requires that the weighted symmetric estimator be calculated and then adjusted to account for the downward bias. Specifically, (3.2) is estimated using weighted symmetric LS. Weights are created for each observation according to (3.9-3.11), and (3.8) is then minimized with respect to the parameters using the appropriate weights.\(^6\)

The $t$-statistic, $\hat{\tau}_1$, with the null hypothesis of $\alpha = 1$ is computed and $\hat{\alpha}_{WS}$ is adjusted according to the following formula:

$$
\hat{\alpha}_{WS} = \hat{\alpha}_{WS} + c(\hat{\tau}_1)(\hat{\alpha}_{WS})^{-1/2}
$$

where $\hat{\alpha}_{WS}$ is the WOLS estimate for $\alpha$ and $(\hat{\alpha}_{WS})^{-1/2}$ is the standard error of $\hat{\alpha}_{WS}$. $c(\hat{\tau}_1)$ is a function of the $t$-statistic, which ensures an approximately median unbiased estimate of $\alpha$ when the true value of $\alpha$ is one. This is because the median of $\hat{\tau}_1$ when $\alpha = 1$ is approximately -1.2.

$c(\hat{\tau}_1)$ is defined as:

$$
c(\hat{\tau}_1) = \begin{cases} 
-\hat{\tau}_1 & \text{if } \hat{\tau}_1 \geq -1.2 \\
0.035672(\hat{\tau}_1 + 7.0)^2 & \text{if } -7.0 < \hat{\tau}_1 < -1.2 \\
0 & \text{if } \hat{\tau}_1 \leq -7.0.
\end{cases}
$$

\(^6\) Fuller (1996, p. 416) details a method that uses Equation 3.2 to compute $\hat{\alpha}_{WS}$. That is the method that is used below.
4: Modeling U.S. Monthly Inflation with a Median Unbiased Estimator

Equation 3.2 in conjunction with the weights specified in Equations 3.9-3.11 are used to obtain a biased estimate of $\hat{\alpha}$. $\alpha_{M}$ is then adjusted using (3.12-3.15) to obtain a median unbiased estimate of $\alpha$. However, the number of lags, $p$, of $\Delta Y$ in 3.2 is not specified by theory. This presents an interesting problem. First, there is evidence of a fairly long “memory” to the post-war U.S. inflation series. In other words, inflation levels not only in the more recent but also in the more distant past tend to influence today’s inflation rate. An autocorrelation up to 121 lags for the annualized U.S. inflation series (January 1959 to April 1999) shows statistically significant autocorrelations exist for up to about 45 lags. This corresponds to roughly a 3 ½ to 4-year memory. This would imply that three to four years of $\Delta Y$ should be incorporated into the estimation of $\alpha$ in (3.2). However, using as many as fifty explanatory variables in (3.2) does not leave many degrees of freedom especially in the early years of the inflation series.

To overcome this problem, the lags for the inflation series are first averaged together across twelve-month periods starting with the seventh lag.²

$$Y_t = \mu + \alpha Y_{t-1} + \sum_{i=1}^{5} \varphi_i \Delta Y_{t-i} + \varphi_6 (Y_{t-6} - \frac{1}{12} \sum_{i=1}^{12} Y_{t-i}) + \varphi_7 (\frac{1}{12} \sum_{i=1}^{12} Y_{t-18+i}) + \varphi_8 (\frac{1}{12} \sum_{i=1}^{12} Y_{t-36+i}) + \epsilon_t$$  (3.16)

This effectively reduces every twelve lags into the average of twelve lags. Then, each averaged lag is differenced with another averaged lag. For a 3 ½ year lag window, this procedure, in effect, cuts the number of parameters estimated in (3.2) from forty-four to ten. We can account for the longer memory of the inflation series while still allowing for enough degrees of freedom to estimate (3.2) effectively. From this set of computations, we obtain a median unbiased estimator for $\alpha$ using (3.16) and (3.13-3.15) that is more efficient than the Andrews and Chen estimator for $\alpha$.

To estimate agents’ expectations of inflation, it is important to take the perspective of the econometrician who has data up to and including the current period. Since we are interested in generating a time series of agents’ expectations of inflation, it is important to adopt an expanding window
approach to estimating $\alpha$. In other words, $\alpha$ is estimated using past and present inflation rates up to
time $t$ in order to generate expectations of inflation at time $t+1$. And, as a new data point for the
observed inflation series at time $t+1$ is obtained, it is used and (3.16) is reestimated with this additional
piece of inflation information. This generates a time series of estimates for $\alpha$ in which the order of
integration of the inflation series is reestimated with each new piece of information.

Figure 3.2 presents the median unbiased WSLS estimate of $\alpha$ in the context of this expanding
window estimation process. As is apparent from that figure, monthly-annualized U.S. inflation is not a
non-stationary or even a nearly non-stationary series through the early 1970's. However, this changes
drastically beginning in early 1973. For basically the next 10 years, monthly U.S. inflation is a non-
stationary or nearly non-stationary series. After 1983, U.S. monthly inflation has moved from being a
non-stationary series to a nearly non-stationary series to what today looks like a stationary series. Figure
3.2 illustrates that assuming either a stationary or non-stationary process for monthly-annualized U.S.
inflation would not hold for large parts of the data period considered here.

It is important to note that if serial correlation in $e_t$ exists in (3.16) then not only will the
parameter estimate, $\hat{\alpha}_{WS}$, be inefficient, but it will also be inconsistent (Maddala and Rao, 1973). To
test for the presence of higher order serial correlation in $e_t$, a Lagrange multiplier test was used. Using
the estimated errors from the WSLS regression (Equation 3.16) and regressing them on a constant, six
lagged error terms, and the set of independent variables from the WSLS regression, the F statistic for this
regression was computed.

An F test was done on this errors regression to determine the joint significance of the
coefficients of the lagged error terms. The LM statistic was then computed which is the number of
lagged error terms ($p$) in the errors regression times the F value generated from the test for joint
significance. This computed statistic is distributed asymptotically as chi-squared with $p$ degrees of
freedom. The null hypothesis of no serial correlation from the (3.16) regression was not rejected in all.

---

7 This procedure admittedly loses some information contained in the lags that are averaged together.
Therefore, because there is such a large autocorrelation in the first six lags of the inflation series, these
lags enter in (3.2) as unaveraged differenced lags.
but four of the 485 expanding window regressions. This was taken as evidence that the lag length used to estimate (3.16) was appropriate.

5. Forecasting One-Month Ahead U.S. Inflation

In order to test the MQT, we must create one period ahead forecasts of inflation. The above method to estimate \( \alpha \) will ensure we are dealing with a stationary series once we impose the coefficient estimate of \( \alpha \) on subsequent models. Our ultimate goal, once we are dealing with a stationary series, is to create unbiased one period ahead forecasts. We appeal to Wold's Decomposition Theorem to represent best our covariance-stationary series. This means we will use parsimonious ARMA models in the Box-Jenkins (1994) mold to model the data then generate one-step ahead forecasts from these models. What is an important difference here, though, is the estimation of the order of integration of the data rather than imposing (or not imposing) a unit root process to monthly U.S. inflation.

To generate a forecast of annualized monthly inflation for January 1959, the beginning of the data set used to test the MQT, we use inflation rates based on the CPI-UX described above starting in January 1950. The set of median unbiased estimates, \( \hat{\alpha}_{WS}(t) \), of \( \alpha \) from Figure 3.2 are imposed on the data. In other words, a covariance stationary series is created by pseudo differencing the U.S. monthly inflation series as such:

\[
X_t = Y_t - \hat{\alpha}_{WS}(t)Y_{t-1}
\]

(3.17)

where \( Y_t \) is the U.S. monthly inflation series and \( \hat{\alpha}_{WS}(t) \) is the time \( t \) median unbiased WSLS estimate for \( \alpha \). This ensures a covariance stationary series with which to work and makes the application of Wold's Theorem appropriate.

This series is then estimated using up to an ARMA(10,10) model in which the degree of the model is determined by the Schwartz-Bayesian Criterion. Once the model is chosen and estimated for that span of data, a forecast is generated for next period's inflation rate. This forecast is used as agents' expectations of inflation at time \( t + 1 \) generated at time \( t \).
Computationally, the method described above is computer intensive. First, for the inflation series that goes from 1 to $T$ which corresponds to the data period January 1950 to May 1999, $\alpha$ is estimated from 1 to $t$ where $t$ goes from December 1958 to May 1999. This is 485 WSLS regressions where a new regression is run to update $\alpha_{WS}$ as each new observation is added to the expanding window.

Once $\alpha$ is estimated for data up to $t$, the pseudo-differenced series $\tilde{X}_t$ is created as in (3.17). A search of the Schwartz-Bayesian criterion all nested time series models up to and including ARMA(10,10). This entails a search of over 120 ARMA models for each next-period forecast. In other words, 485 data points are added, one-by-one, to the original inflation series that begins in January 1950 and generates the first forecast of expected inflation for January 1959. For each additional data point that is added to the time series, not only is $\alpha$ estimated anew, but so is the ARMA model that best represents $\tilde{X}_t$. This requires that over 58,000 ARMA models are estimated to generate the time series of next-period inflation expectations that is used to proxy anticipated inflation in the tests of the Moderate Quantity Theory. On a Pentium II, 400 Mhz processor, computation of the inflation expectations time series requires approximately 20 hours to complete.

The one-period ahead forecasts of inflation are intended to be unbiased forecasts. An examination of forecasted inflation for period $t$ with the actual inflation rate at period $t$ shows this to be the case. A Ljung-Box Q statistic for serial correlation was computed on the difference between actual inflation and the forecast of inflation for period $t$. That statistic, computed for 121 lags (1/4 of the total observations), was not significant at even a 90% level of confidence. This indicates that the differenced series is quasi-white noise, i.e., the inflation series has been modeled adequately. Figure 3.3 presents this differenced series. The series resembles a white noise series, which is a series that is not serial correlated and is integrated of order 0.

The above method for generating agents' expectations of next period's inflation rate, obviously, is a univariate time series approach in which past periods of inflation are used to model what future inflation is expected to be. This approach is not necessarily overrestrictive because past inflation and
changes in inflation incorporate other macroeconomic information. Moreover, while the expectations are generated using only past levels and changes in inflation, that is not to say the actual process by which inflation expectations are generated is not well approximated by this method.

The computational method to compute these expectations, while time consuming, is a feasible way in which the order of integration for the inflation series can be modeled directly over time. This is mere sound than imposing a I(0) or I(1) process on the entire inflation series, or parts of the inflation series. Using unit root tests that only allow a unit root or lack of a unit root to be decided ignore the near unit root behavior that the U.S. monthly inflation series seems to exhibit in certain parts of its history.

Moreover, the computational method allows for the pseudo-differenced inflation series to be parsimoniously represented by ARMA representations. According to Wold's Theorem, this representation is fundamental for any covariance-stationary series. With the expanding window approach to estimation, the forecasts of next period inflation are continually updated by new inflation information. And, the models that generate these forecasts are continually reestimated to allow for agents' expectations that may adjust quickly to new information or whose very structure may adjust over time as well. (Figure 3.4 plots the adjusted $R^2$ for this expanding window approach). In the next section, we compare the performance of these forecasts of next period inflation to other models of inflation found in the literature.

6. Other Methods of Forecasting Inflation

1. $P^*$ Models of Inflation

It could still be the case, however, that more efficient forecasts, i.e., forecasts that have smaller forecasting errors, can be generated by other methods of modeling inflation expectations. One method of inflation forecasting that uses the equation of exchange as its starting point, and thus has economic theory at its underpinnings, is the $P^*$ approach put forth by Hallman, Porter, and Small (1991). In that paper, they define the long run equilibrium price, $P^*$ as:

$$p^*_t = \frac{M_2^*V^*_t}{O^*_t}$$

(3.18)
where \( V^*_r \) is the long run equilibrium value of velocity and \( Q^*_r \) is the current value of potential real GDP. (3.18), of course, is the equation of exchange.

Hallman, Porter, and Small go on to propose what they call the "price-gap" model for U.S. inflation. They recognize that post World War II U.S. inflation has different time series properties than pre World War II inflation. However, they do not estimate, or even acknowledge, the process generating U.S. inflation could be vacillating between an I(0) and I(1) process over that time period. Instead, they test the inflation rate using augmented Dickey-Fuller (1981) tests and conclude that a unit root in inflation is present over the whole time period. (As noted above, direct estimation of the largest root of inflation gives different results). The evidence they find of a unit root from the ADF test leads them to suggest the following price-gap model (all variables in lower case are in natural logs):

\[
\Delta \pi_r = \beta_1 (P_{r-1} - P^*_r) + \beta_2 \Delta \pi_{r-1} + \beta_3 \Delta \pi_{r-2} + \beta_4 \Delta \pi_{r-3} + \beta_5 \Delta \pi_{r-4} + \varepsilon_r. \tag{3.19}
\]

Of course, this specification differences out valuable long run information that could possibly help explain inflation. But, Hallman, Porter, and Small find that their \( P^* \) approach to forecasting inflation is superior to other time series models and independent, private forecasting models based on smaller forecast root mean square errors.

To implement the price-gap model, Hallman, Porter, and Small note that M2 velocity has been roughly constant for the sub-period (1955-1988) that they consider. They, therefore, use the mean value of M2 velocity across the whole sub-period. They also need a calculation of potential output (GDP) for the economy. They use a method of using pooled preliminary labor market data as found in Braun (1990). From this \( P^*_r \) can be calculated and used in (3.19) to generate inflation forecasts.

To compare the Fuller (1996) and Fuller and Roy (1998) coupled with our expanding window approach to forecasting one-period ahead inflation, we compute the price gap model and generate monthly inflation forecasts from it. Specifically, we calculate monthly M2 velocity from January 1959 through April 1999. We first use Hallman, Porter, and Small's assumption that velocity is roughly constant. To calculate potential personal income, we use the one-sided version of the Hodrick-Prescott
(1997) filter that is meant to extract the business cycle components of a time-series. The resultant series is an estimate of potential income that can be used to create the proxy for $Q^*$. The forecast root meansquare error (FRMSE) and Theil’s (1961) U statistics are used to measure the accuracy of the forecasts. The Theil U statistic does not have the scaling problems that the FRMSE has, although scaling differences are not an issue here. Large values of Theil’s U statistic indicate poor forecasting performance. Table 3.1 displays the results of this exercise.

Figure 3.5 maps M2 velocity over the time period considered by Hallman, Porter, and Small as well as over the remainder of the 1990’s. While it seems that the constant M2 velocity assumption may hold into the early 1990’s, it is apparent that velocity has risen rather substantially for most of the 1990’s. Unit root tests around a constant term, both augmented Dickey-Fuller and Phillips-Perron, were computed and in no test were the null hypothesis of a unit root rejected. This lends evidence to the rejection of the constant M2 velocity assumption when the 1990’s are included in the estimation window.

Again, we used the Hodrick-Prescott filter to extract a smooth business cycle series for M2 velocity and used that to compute Hallman, Porter, and Small’s $P^*$. This modified $P^*$ (in natural logs) was then used in (3.19) to create forecasts of next month’s inflation rate. These forecasts were generated from January 1970 to April 1999 using the same expanding window procedure. Table 3.1 presents these results.

2. Interest Rate and Time Series Models

In the Hallman, Porter, and Small (1991) paper, the authors also note that Christiano (1989) puts forth another model with the claim that it does as well as the price-gap model. Christiano (1989) specifies an interest rate model of the form:

$$\Delta r_t = \alpha \Delta r_{t-1} + \sum_{i=1}^{4} \beta_i \Delta \pi_{t-i} + \varepsilon_t. \quad (3.20)$$

---

8 Using the one-sided version of the HP filter only requires past values of the data series for its computation, thereby making it an operationally feasible way to conduct forecasting exercises.

9 For the augmented Dickey-Fuller test the appropriate lag length was determined by an AIC criterion, a BS criterion, a Ljung-Box test, a LM test, and the Campbell-Perron (1991) method. For the Phillips-Perron test a lag length of four was used in the spectral estimation window.
Christiano uses the yield on a three month t-bill as the interest rate in (3.20). This model has the advantage that estimated inputs like potential output and long run equilibrium velocity are not inputs into the equation. However, Hallman, Porter, and Small (1991) show that their model forecasts inflation more accurately than does Christiano (1989). We used Christiano’s model to obtain inflation forecasts and the results are also in Table 3.1. We also use an ARIMA(0,1,1) for forecasts of the inflation rate. The rationale here is that the Hallman, Porter, and Small and the Christiano work specify a finite autoregressive lag length. We duplicate their work but the question remains whether an AR(4) specification is sufficient. With the ARIMA(0,1,1), the MA(1) component captures a possible infinite order AR process that may be underlying the data.\(^{10}\)

### 3. Fisher Equation Models

Fama (1975) suggested the use of the Fisher (1930) equation to generate forecasts of inflation. Specifically, the equation specifies the nominal interest rate as a linear function of the real interest rate and inflation. In Fama’s early work, he assumed that the real interest rate was constant and found that assumption provided good inflation forecasts for the 1953-71 period. Subsequent evidence presented by Hess and Bicksler (1975), Fama (1976), Carlson (1977), Nelson and Schwert (1977), Garbade and Wachtel (1978), and Fama and Gibbons (1982) all rejected the assumption of a constant real interest rate. These studies found instead that the expected real interest rate behaves as a random walk.

Fama and Gibbons (1984) recognizes that if the real interest rate behaves as a random walk, then changes in the observed, \textit{ex post} real interest rate can be modeled as a simple moving average model. In other words, the real \textit{ex post} rate can be written as:

\[
R_{t-1} - \pi_t = E_{t-1} r_{t-1} + \varepsilon_t \tag{3.21}
\]

where \(E_{t-1} r_{t-1}\) is the expected real interest rate and \(\varepsilon_t\) is the unexpected component of the real return.

\(^{10}\) Fama and Gibbons (1984) also argues for a ARIMA(0,1,1) model of inflation based on the sample autocorrelations for the inflation series in which there is a large first-order autocorrelation, but negligible higher order autocorrelation.
Then changes in the real return can be written as the following time series model:

\[
(R_{t-1} - \pi_t) - (R_{t-2} - \pi_{t-1}) = \Delta E_{t-1} R_{t-1} + \varepsilon_t - \varepsilon_{t-1}
\]  \hspace{1cm} (3.22)

where \( \Delta E_{t-1} R_{t-1} \) is the change in the expected real return. If \( ER_{t-1} \) is a random walk as many previous researchers have proposed, then \( \Delta E_{t-1} R_{t-1} \) is white noise. Therefore, the right hand side of (3.21) is white noise. If this is true, then the difference in real returns can be represented as a first order moving average process:

\[
(R_{t-1} - \pi_t) - (R_{t-2} - \pi_{t-1}) = u_t + \theta u_{t-1}.
\]  \hspace{1cm} (3.23)

A forecast of inflation can be derived by subtracting the \textit{ex ante} forecast of the real interest rate,

\[
(R_{t-1} - \pi_t) = (R_{t-2} - \pi_{t-1}) + u_t + \theta u_{t-1},
\]  \hspace{1cm} (3.24)

from the nominal interest rate observed at the end of period \( t - 1 \). So, if the expected real interest rate is a random walk, the inflation forecast generated by the Fisher equation and (3.23) forecast of the real interest rate should be very accurate.

An examination of Table 3.1 indicates that the Fuller-ARMA model does have a smaller FRMSE and Theil U Statistic than any of the other methods. The Fama-Gibbons model and the Hallman, Porter, and Small time-varying V2 price gap model for inflation forecast perform better than the Hallman, Porter, and Small constant V2 price-gap, Christiano, and simple ARIMA models.

4. Inflation Forecasts from Consumer Surveys

All of these models may be improved on, however, if agents’ expectations of future inflation are measured directly using survey questions and the corresponding data generated from the questions. We now examine inflation expectations of sampled household in the U.S. population gathered by the University of Michigan from their monthly survey of consumer confidence and inflation expectations. The Survey of Consumers is conducted by the Survey Research Center at the University of Michigan. Started in 1946 by George Katona (1975), the current monthly survey contains approximately fifty core questions dealing with consumers’ attitudes and expectations. The Survey is a nationally representative survey based on approximately 500 telephone interviews with adult men and women living in households
in the coterminous United States (Curin, 2000). For each monthly sample, an independent cross-section
sample of households is drawn. The respondents chosen in this drawing are then reinterviewed six
months later. The total sample for any one survey is normally made up of 60% new respondents and 40%
being interviewed for the second time. The method used to draw the national sample is random digit dial
(RDD) telephone sampling in which an equal probability sample of all telephone households is provided.
Monthly data on consumers' inflation expectations has been gathered of a random sample of U.S.
households since 1978.

Specifically, respondents are asked the following questions:
During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?

\(<1>\) UP
\(<2>\) DOWN
\(<3>\) SAME
\(<9>\) DK, NO OPINION

and

By what percent do you expect prices to go up/down, on the average, during the next 12 months?

Twelve-month inflation forecasts are generated directly from this data by aggregating the
individual household responses across that month's survey. Since these are forecasts of inflation twelve
months hence, the data with which we will be able to examine the accuracy of Michigan's inflation
forecasts is from 1979 to 1999.

To compare each of the inflation forecasting methods used above, we generate table 3.1 again,
but this time we use data from January 1979 to April 1999. This corresponds to as far back as the Survey
of Consumers goes for monthly inflation expectations data. Table 3.2 contains the FRMSE and the Theil
U statistic for each of the forecast methods.

In this table, as well as Table 3.1, the Fuller-ARMA expanding window method for forecasting
one-month ahead inflation is superior in terms of predictive power over the other forecasting methods.
There is twelve percent improvement in Fuller's method over the next best model when looking over the
January 1970 to April 1999 time period. And, there is a sixteen percent improvement over the next best method when looking at the January 1979 to April 1999 time period. In either case, the FRMSE and the Theil U statistic suggest the Fuller-ARMA method for forecasting one-month ahead inflation. This is the forecasting method that will be used for the remainder of this work to proxy agents’ expectations of next month’s inflation rate.

7: Summary

This chapter examined ways in which agents’ expectations could be modeled. This is an especially important issue in the context of the Moderate Quantity Theory because the formation agents’ inflation expectations gives rise to real short-term effects of a monetary expansion and a less than instantaneous price adjustment to a monetary expansion. An effort was made to create a clean composite price index from which to calculated U.S. inflation. The price index used was the CPI-UX.

The time series properties of U.S. inflation were also examined. It is apparent by examining a graph of U.S. post-War inflation that the time series is not everywhere I(0) or I(1). So, using first differenced data would not be an appropriate way to obtain a stationary time series.

Instead, recent work in the econometric literature on the estimation of a time series’ unit root directly is used. Once one recognizes that estimation of the unit root using OLS is not appropriate because of a downward bias on the parameter estimate as its true value approaches one, a correction for this bias can be used. The correction obtains a median unbiased estimator for the unit root parameter that allows for a possible time varying order of integration in the U.S. inflation series. We find that, in fact, a unit root process governs the U.S. inflation series from the mid 1970’s to the mid 1980’s. Before and after that time period, the U.S. inflation series is either stationary or nearly stationary.

Once the estimate of the parameter is obtained, the inflation series is differenced by the parameter estimate time the first lag of inflation series to give a stationary time series. An appeal to Wold’s decomposition theorem is used to generate one-period ahead forecasts of the U.S. inflation series. An expanding window approach is used in which up to an ARMA(10,10) model, selected by the Schwartz-Bayesian criterion, is used to estimate the data generating process. Only data up to time \( t \) is
used to generate a forecast of inflation in time \( t + 1 \). With each additional data point, the data generating process is reestimated and a forecast for next period is generated (c.f. Figure 3.6 for a comparison of the forecasts to the actual inflation rate).

Lastly, to judge the accuracy of these forecasts, a comparison of the forecast RMSE and the Theil \( U \) statistics is made among a set of other inflation forecasting models. Those models include the Hallman, Porter, and Small (1991) \( P^* \) model (with and without the constant M2 velocity assumption), Christiano’s (1989) interest rate model, an ARIMA(0,1,1) time series model, Fama and Gibbons’ (1984) real interest rate model, and consumer inflation expectations as measured by the University of Michigan’s Survey of Consumers. In each case, the time series model presented in this chapter outperforms the competing models. It is the inflation expectations generated by this model that are used in subsequent chapters.
CHAPTER 4

Empirical Tests of the Moderate Quantity Theory of Money$^{11}$

1. Specifying the Demand Equation

McCulloch's (1980) Moderate Quantity Theory of Money (MQT) specifies period $t$ inflation as a function of period $t-1$ real money supply, real money demand, expectations of period $t$ inflation, and independent microeconomic shocks.

$$\pi_t = \gamma (\ln m_{t-1} - \ln m^d_{t-1}) + \pi^a_{t-1} + c \frac{dz}{dt} \quad (4.1)$$

In order to make this equation amenable to empirical tests, we first specify the lag length between current period prices and past period prices to be monthly. As the MQT states, consumers' information about current period values for prices and output becomes better as the lag length decreases. So, lag lengths of less than a month for goods and services may more appropriately fit the informational lag with which consumers have to deal. But, aggregate rational data does not exist for many of the variables in equation 4.1 for horizons shorter than one month. The data available constrains us to study consumers' money demand and the underlying adjustment process at monthly frequencies or greater.$^{12}$

$^{11}$ The theoretical derivation of the Moderate Quantity Theory of Money is in Appendix B.

$^{12}$ It is conceivable that better data could exist on a micro-level that would allow for testing of MQT under less restrictive assumptions of informational lag length. Survey data could be used whereby random samples of consumers could be asked on a greater than monthly basis about their money holdings, household income, and the levels of prices. To our knowledge, this type of survey data does not exist at this short a frequency. (The Survey of Consumer Finances, conducted by the Federal Reserve Board, does ask survey questions that address at least some variables mentioned above). And the operational difficulties of collecting survey data at such short frequencies are formidable. Ideally, though, this type of survey data would be the most useful for testing the MQT from the standpoint of more realistic informational lag lengths.
Most studies of consumer money demand specify as the determinants of money demand variables derived from an inventory-theoretic model in the sense of Baumol (1952) and Tobin (1956). In order to test the MQT, a specification for \( m^d \) must be used. And, the MQT does not specify a particular form for money demand. In keeping with the literature, we will adopt an inventory-theoretic specification. Judd and Scadding (1982), Goldfeld and Sichel (1990), and chapter two of this dissertation present discussion of the problems that researchers have encountered by adopting the conventional Baumol-Tobin inventory theoretic framework. But, as an initial test of the MQT theory, adopting the inventory theoretic approach allows us to focus on the MQT under the very familiar surroundings of inventory theoretic framework. We can also examine the parameter estimates that we obtain from the MQT estimation against the well-known results derived under the Partial Adjustment Model (PAM) approach to money demand that also uses an inventory theoretic specification. So, we will specify money demand as in the Cagan (1956) as

\[
\ln m^d_t = \alpha_1 + \alpha_2 \ln y_t + \alpha_3 R_t. \tag{4.2}
\]

This is the well-known form of money demand that constrains the elasticity of money demand with respect to aggregate income to be constant, but allows for a non-constant elasticity of money demand with respect to the interest rate.\(^{13}\)

Money demand theory does not specify the appropriate variables to use in the equation (4.2) specification. As discussed in chapter two of this dissertation, Friedman (1959) suggested a measure of permanent income would be appropriate due his view of this money demand function as the total demand for money in the macroeconomy (Laidler (1982)). This also implies then that broad measures of money are most appropriate as measures of the money stock. This is in contrast to the approach that considers money to be a means by which to conduct transactions. Therefore, real income, in the sense of real GDP

\[^{13}\text{In the MQT regressions, we used } R_t \text{ and } \ln R_t \text{ to test simultaneously which had more explanatory power. Unfortunately, not much was learned from this test because both parameter estimates were not significant in any of the MQT regressions. The likely reason for this is the high degree of correlation between the two opportunity cost variables. For the January 1959 to May 1999 data period, the correlation between } R_t \text{ and } \ln R_t \text{ is 0.9617.}\]
or real personal income, explains the demand for money. In line with this view, narrower measures of money are more appropriate.\footnote{As noted in Chapter 2, money demand instability is not resolved by the choice of permanent income (or wealth) as the scale variable or some measure of current income as the scale variable. So, siding with one view of the use for money or the other probably will not determine the success of this research effort. In the case of U.S. money demand at a monthly frequency, we choose real personal income (in natural logs) as the scale variable because it appears to be the best proxy for U.S. income at the monthly frequency.}

Adoption of one of these two views of money, the portfolio approach or the transactions approach, then specifies the appropriate opportunity cost of money to use. The portfolio approach views money as a component in the consumers' portfolio so that the appropriate opportunity cost is the return on assets that make up this portfolio. The transactions approach specifies that the opportunity cost of money as the return on assets that are most closely substitutable for money.

The view that this paper adopts is slanted more toward the transactions view of money demand, but not because there is necessarily a large body of evidence in support of that view over the other. In fact, this paper's focus is not on money demand function itself; but, instead, it focuses on the adjustment process when the money market is in disequilibrium. But, the money demand literature has focused predominantly on narrow definitions of money, like M1 and M2. This line of investigation moves away from Friedman's ideas of the total demand for money in the macroeconomy. The rationale for this appears to be the fact that the US Federal Reserve Bank had, over the years, adopted activist monetary policy that use narrower monetary aggregates. It is in light of the operational policies that the Fed had used to coordinate monetary policy that precipitates our choice of one view over the other. Also, this paper will present evidence for which of the monetary aggregates seems to best explain the adjustment process postulated by the MQT. This empirical investigation would have less logical appeal if a Friedman-like portfolio approach were the underlying theory behind money demand.

As a last note toward making the MQT empirically testable, the final term in the theory, \(c \frac{dz}{dt}\), which is the stochastic differential that represents microeconomic shocks, is expected to have mean zero, conditioned on prior information. While it will dominate instantaneous inflation because \(\frac{dz}{dt}\) has a scale
which approach infinity as \( dt \) approaches zero for \( \alpha > 1 \), it will not effect the mathematical expectation of inflation. This treatment of the stochastic shock term is justified in the sense that empirically it will be impossible to specify actual microeconomic shocks for any real-world economy with any degree of accuracy. In other words, the empirically testable model of the MQT will not include a term meant to represent the stochastic shock term specified by the theory.

The regressions used to test the MQT here focus on monthly frequencies. Much of the money demand research focuses on quarterly horizons. However, the Federal Open Market Committee meets far more regularly than once every quarter. For models of money demand to be useful in these instances, monthly frequencies must also be investigated. Moreover, the MQT postulates that forecasts of pertinent variables in the MQT specification will be more accurate the shorter the lag length between periods. It would certainly be possible for consumers to gather information on output and prices at lag length of shorter than one quarter. In this sense, the more uncertainty consumers have about these pertinent variables in the current time period, the more likely these consumers are to try to gather the most recent information about these variables.

The scale variable for the money demand function (equation (4.2)) is US real personal income (see Figure 4.1) for the monthly frequency. That variable is deflated by the US CPI-UX\(^ {15} \) (see Figure 4.2) to make it a real quantity and placed on a natural logarithmic scale. The opportunity cost variables are the US 3-month T-bill rate (see Figure 4.3), the Moody’s Aaa rated corporate bond (see Figure 4.10), and the Moody’s Baa rated corporate bond (see Figure 4.11). The US CPI-UX is the price index used to calculate US inflation. The various monetary aggregates are also transformed into real quantities by deflating by the CPI-UX and are placed on a logarithmic scale as well. Besides examining the conventional monetary aggregates of M1, M2, and M3 (see Figures 4.4-4.6), we will also examine the various other aggregates that have been created. These other aggregates are in response to criticisms that conventional monetary aggregates contain components that are not close substitutes to other components within the aggregate. Specifically, these aggregates are M1plus, MzM, and M2minus (see Figures 4.7-

\(^{15}\) See chapter 3, ‘Inflation Expectations’ for an explanation of the price index, CPI-UX.
4.9. \(^{16}\) All data is obtained from the FRED database located at and maintained by the St. Louis Federal Reserve Bank. The monthly observations are from 1959:1 to 1999:5. \(^{17}\)

Rewriting the theoretical derivation of the MQT to incorporate the transformations to make it empirically testable, we have

\[
\pi_t = \gamma (\ln m_{t-1} - (\alpha_1 + \alpha_2 \ln y_{t-1} + \alpha_3 R_{t-1})) + \pi_{t-1}^e + \epsilon_t
\]  \hspace{1cm} (4.3)

which is a combination of (4.1), (4.2), and the comments made above on the stochastic shock term.

Equation (4.3) implies that next period's inflation is a linear function of the excess supply (or excess demand) of money and last period's expectation of this period's inflation rate plus an error term.

Assuming there is no systematic error in inflation forecasts, \(\pi_{t-1}^e\), the coefficient on this term is one.

This is imposed by rewriting (4.3) as

\[
UAI_t = \pi_t - \pi_{t-1}^e = -\alpha_1 - \gamma + \gamma \ln m_{t-1} - \alpha_2 \gamma \ln y_{t-1} - \alpha_3 \gamma R_{t-1} + \epsilon_t
\]  \hspace{1cm} (4.4)

where \(UAI_t\) is unanticipated inflation. This equation allows for estimation and identification of the parameters of interest; the adjustment parameter, \(\gamma\); the elasticity of money demand with respect to income, \(\alpha_2\); and the semi-elasticity of money demand with respect to the interest rate, \(\alpha_3\).

2. Unit Root Tests

Estimation of (4.4) is straightforward at this point. However, following Granger and Newbold's (1974) warning about the possibility of spurious regressions when using time-series data, first we check the variables, \(UAI_t\), \(\ln m_{t-1}\), \(\ln y_{t-1}\), and \(R_{t-1}\), for their order of integration. Table 4.1 presents tests for a unit root for each of the variables for the monthly series. For most of the series investigated in Table 4.1, the null hypothesis of a unit root cannot be rejected at the 95% level of significance. The notable exception to this is the UAI series.

In terms of equation (4.1) this means that a variable integrated of order 0 (I(0)) is being regressed on a linear combination of variables that are each integrated of order 1 (I(1)). Recall the results

\(^{16}\) Definitions of these monetary aggregates, as well as the conventional ones, appear in Appendix A.
from Chapter 2 in which the presence of a cointegrating relationship was tested on the set of variables, U.S. M1 (or M2), U.S. personal income, and the U.S. three-month T-bill rate using Johansen’s (1988) trace test for the existence of at least one cointegrating vector. The existence of a cointegrating vector was shown, although invoking Engle-Granger’s methodology with the monetary aggregate as the normalizing variable rejected that cointegrating vector. This is an important result for the remainder of this work. Since the left hand side of (4.4) is I(0) while each of the variables on the right hand side is I(1), it is necessary that a cointegrating relationship exist between the variables on the right hand side of (4.4). Otherwise, the unbalanced regression in (4.4) will be misspecified. Without a cointegrating relationship between $m_t$, $y_t$, and $R_t$, the right hand side of (4.4) will drift infinitely far away from the left hand side of (4.4) as $T \to \infty$. The trace tests for the existence of a cointegrating vector in Chapter 2 give strong evidence for a cointegrating vector between the right hand side variables.

3. Monte Carlo Simulations

To test whether a regression of an I(0) variable on a combination of I(1) variables might lead to spurious significance of the parameter estimates, a Monte Carlo simulation was run. The size of each sample is 500 observations. Ten thousand samples were created with a regression run on each set of data. As a basis of comparison, we first calculated a regression of an I(1) series on a constant and two other I(1) series where each I(1) series was generated by a random walk process. This is the type of spurious regression to which Granger and Newbold refer (1974). It often has the characteristic that the adjusted coefficient of determination is greater than the absolute value of the Durbin-Watson statistic.

Using the standard normal critical value of 1.645 for a 90% confidence level, 1.96 for a 95% confidence level, and 2.576 for a 99% confidence level, the Monte Carlo simulation is displayed in Table 4.2, Panel A. We should expect, for example, that roughly 5% of the time the t-statistic for a slope coefficient different from zero should be greater than the critical value of 1.96. Looking at the entries under the 95% level of confidence, the parameter estimate on the first regressor is significant 87.89% of

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17 MzM observations do not begin until 1974:1 due to the non-existence of certain components of this aggregate.
the time. The parameter estimate on the second regressor is significant 87.38% of the time. And the F-statistic testing the joint significance of the slope parameter estimates is significant 98.61% of the time.\textsuperscript{18} This clearly illustrates that spurious correlation occurs, even in completely unrelated series, when unit roots are not taken into account.

The same Monte Carlo experiment was done for the regression of an I(0) series on a constant and two I(1) series. Here, the I(1) series are still generated by a random walk process. A white noise process generates the I(0) series. In this case, Table 4.2, Panel B shows, for the 95% level of confidence, that 4.74% of the time the first slope parameter estimate is significant. For the second slope parameter, 5.06% of the time the second slope parameter estimate is significant. And, the F-statistic testing the joint significance of the two slope parameters is significant 4.65% of the time. It appears then that the fact that we have an I(0) regressand regressed on I(1) regressors will not cause a problem in terms of spurious correlations. The assumption of a standard normal distribution for the parameter estimates appears to hold even in this case.

\textbf{4. Moderate Quantity Theory Regressions}

Turning toward the results of the UAI regressions (equation 4.4), we present Table 4.3. Each regression uses a different monetary aggregate as an independent variable. The monetary aggregate used in a regression is represented in the leftmost column with the parameter estimates for that regression on the same row as its respective monetary aggregate. Estimation of (4.4) by ordinary least squares (OLS) produced residuals that were uncorrelated, according to a Ljung-Box Q statistic at up to 121 (1/4 of the observations) lags.

in every case except for M3 in the monthly data, the adjustment parameter, $\gamma^*$, is at least significant at a 90% level of confidence. It also has the correct sign meaning that an excess supply of money has a positive effect on unanticipated inflation. The dependent variable, unanticipated inflation, is a decimal indicating an annualized rate. The excess supply of money is in natural logarithms. A one

\textsuperscript{18} The significance values for the F-statistic are 2.303, 2.996, and 4.605 for the 90%, 95%, and 99% levels of confidence, respectively.
percent excess supply of money is a logarithmic excess supply of money of 0.01. As an example, the γ coefficient using the M1 monetary aggregate is 0.0579. A sustained excess supply of money of one percent, according to the γ estimate, creates a 0.0579% per year inflation in excess of the public's expectations. Figure 4.12 shows the adjustment of prices to a monetary disequilibrium using the M1 monetary aggregate's γ parameter under the assumption that the public's expectations of next period's inflation are constant over time. In other words, we focus on the speed of adjustment due only to the monetary disequilibrium. After approximately thirteen years, prices have adjusted to cut the excess supply of money in half. After approximately twenty-five years, the excess supply of money is effectively zero. This illustrates a relatively slow speed of adjustment of prices to a monetary disequilibrium. But, it is important to note that this assumes, very unrealistically, that the public's expectations of next period's inflation rate are constant over time. Incorporating those expectations into the price adjustment will dissipate the monetary disequilibrium much more quickly.

Moreover, the parameter estimates for the coefficient on the scale variable, except for the M3 aggregate, are all statistically significant. For the parameter estimates of the opportunity cost variable, we see that statistical significance occurs only for the M1, M1plus, and M2minus aggregates. This is not necessarily an indictment of the MQT, however. Instead, it is more an indication that using the 3-month T-bill rate as a proxy for the opportunity cost of holding money is not appropriate for money aggregates that are more broadly defined.

To investigate this hypothesis, we consider other opportunity cost variables in the MQT regressions. Specifically, we include a six month and twelve month U.S. T-bill rate (secondary market) and return on Moody's rated AAA and BAA bonds. Since the components of M3, for example, include instruments like jumbo CD's and large RP agreements, it is certainly plausible that the three month T-bill rate is not a good proxy for the opportunity cost of holding M3.

Tables 4.4-4.9 present the MQT regressions with various opportunity cost variables. None of the T-bill rates were nested together in any of the MQT regressions because there was a correlation of the three-month T-bill with the six-month T-bill of 0.9970, a correlation of the three-month T-bill with the
twelve-month T-bill of 0.9887, and a correlation of the six-month T-bill with the twelve-month T-bill of 0.9965. Since these series are so highly correlated, multicollinearity is an issue with any regression that contains more than one of these series.

It is apparent from Table 4.4-4.6 that the best proxy for opportunity cost is short-term interest rates on T-bills when the monetary aggregate is M1, M1Plus, or M2Minus. At best in MQT regressions with these monetary aggregates, the corporate bond rate is marginally significant. In most cases, the AAA or BAA rated corporate bond rates do not have statistically significant parameter estimates. Also, the parameter estimates on each of the explanatory variables is fairly robust to the proxies chosen to represent the monetary aggregate, the scale variable, and the opportunity cost variable.

In Tables 4.8 and 4.9, it appears that a short-term interest rate on a fairly liquid asset is not the appropriate opportunity cost variable, at least not by itself. With both M2 and M3 as the monetary aggregates, the opportunity cost variable is best represented by a relatively short-term interest rate and a corporate bond rate. In fact, once an appropriate set of opportunity cost variables is used in the M3 MQTI regression, the parameter estimates on the speed of adjustment variable and the scale variable become statistically significant. This result is not surprising because M2 and M3 are more broadly defined and therefore encompass more financial assets. These assets have more alternatives that have higher rates of return and less liquidity.

5. Excess Supply of Money

Figures 4.13 to 4.18 show the excess supply of money, in percentage terms, for M1, M1Plus, M2Minus, MzM, M2, and M3\(^{20}\) from January 1959 to April 1999. In each case, except for MzM, which did not exist prior to 1974, there is a large excess supply of money throughout the 1960's. From February 1961 to December 1969, this excess liquidity translated into the second longest peacetime expansion in

\(^{19}\) All data is monthly and from FRED (Federal Reserve Economic Data) established and maintained by the St. Louis Federal Reserve Bank.

\(^{20}\) The variables in the Cagan form of money demand for M1, M1plus, M2Minus, and MzM were a constant, monthly U.S. personal income, and the U.S. monthly three-month T-bill rate. The variables for M2 and M3 money demand were a constant, monthly U.S. personal income, the U.S. monthly three-month T-bill rate, and the rate on Moody’s BAA rated corporate bonds.
U.S. history.\textsuperscript{21} However, this excess supply of money did not begin to translate into higher levels of inflation, presumably in part because consumers’ expectations of permanent higher inflation levels did not materialize and because Nixon’s price controls from 1971-4 did not allow the monetary disequilibrium to adjust on its own, until the mid to late 1970’s.

Once the price controls were lifted, circa 1974, the excess supply of money began to nosedive hovering right around 0\% until 1980. The U.S. inflation rate, however, remained well above 5\% from 1977 and on. While the money supply in the U.S. economy appeared to be in line with consumers’ money demand, the inflation that had made itself apparent earlier in the decade had led to inflationary expectations that did not dissipate even in the face of less expansionary monetary policy. Toward the end of the 1970’s inflation started to turn up in response to a loosening of monetary policy reins and consumers acted quickly to incorporate price level increases into their inflation expectations. This spurred the U.S. Federal Reserve Bank not only to bring the excess supply of money toward zero, but also to try to realign consumers’ expectations of future inflation.

In May 1980, there is a precipitous drop in the excess supply of money in which the demand for money far outstrips the supply. The U.S. Federal Reserve Bank, chaired by Paul Volcker, had decided on October 6, 1979 to announce publicly a new set of operating procedures. Instead of focusing on the behavior of short-term interest rates, the Federal Reserve would now emphasize a measure of bank reserves, non-borrowed reserves. The Fed began to target specific bounds on money supply growth. This money supply targeting allowed interest rates to vary markedly (cf. Figure 2.4, circa January 1980-January 1983), so much so that in March 1980, December 1980, January 1981, and August 1982, the U.S. three-month T-bill rate spiked to over 15\% per annum. Consequently, and soon thereafter, the excess supply of money dipped to its lowest levels in response to this tightening of the money supply (cf. Figures 4.4-4.16). Tight monetary policy reigned in response to unacceptably high inflation rates and a large excess demand for money developed.\textsuperscript{22}

\textsuperscript{21} This is according to NBER Business Cycle Data.
\textsuperscript{22} This period of time also saw the implementation of credit controls that did not allow banks to issue all the loans they would have otherwise. This had the effect of sharply reducing the excess supply of money as well.
Unfortunately, this decision to cut back severely the money supply was almost perfectly synchronized with the beginning of the U.S. recession that began in July 1980. This led to an unusually severe recession in which the supply of money dropped to as much as 80% of money demand. This recession and the monetary policy that accompanied did have the effect of reducing inflation quite dramatically. By the mid 1980’s, inflation was consistently below 5% and the excess supply of money fluctuated between quite narrow bands around 0%. In fact, it is remarkable in the graphs how well behaved, relative to its earlier history; the excess supply of money has been since the early 1990’s. It is no wonder that the U.S. has enjoyed its longest peacetime expansion coupled with low level of price inflation over the 1990’s period.

6: Further Evidence for the Existence of a Cointegrating Relationship

While the Johansen trace test above showed that a cointegrating relationship exists between $m_t$, $y_t$, and $R_t$, further tests, using the residual from the MQT regressions, can be done to solidify that fact. The literature has referred to regressions of the MQT type as unbalanced regressions (Banerjee, Dolado, Galbraith, and Hendry (1993)). Pagan and Wickens (1989) and Banerjee, Dolado, Galbraith, and Hendry (1993) note that unbalanced regressions are not necessarily a problem. For an unbalanced regression to be a statistically sound regression, we require as a necessary condition that there be two or more regressors integrated of the same order. Baffes (1997) notes that a logical way to check that an unbalanced regression is statistically valid is to examine the properties of the error term from the regression. If the error term from an unbalanced regression is I(0), then it may be the case that the regression is statistically valid. However, if the parameter estimates on the I(1) regressors are statistically insignificant from zero, the properties of the error term will be directly attributable to the regressand and not to a cointegrating relationship between the regressors. Baffes (1997) suggests that not only should the error term in the MQT regressions be I(0), but the parameter estimates must be statistically significant as well.

Table 4.10 examines the properties of the error term from the MQT regressions as well as how many of the regressors from table 4.3 had significant parameter estimates. We see in every case that the null hypothesis of a unit root process governing the error term is rejected at better than 99% confidence.
This provides more evidence that a cointegrating relationship exists between the variables on the right hand side of (4.4), \( m_t \), \( y_t \), and \( R_t \), and that MQT regressions are statistically valid unbalanced regression equations.

7. Elasticity Estimates

Table 4.11 presents the income elasticity and interest rate semi-elasticity for each of the regressions. The MQT has the feature that the adjustment parameter in the adjustment equation (4.4) is estimated simultaneously with the parameters of the money demand equation. For the most part, the income elasticity of money demand increases as the monetary aggregate becomes more broadly defined. This is understandable because the various components in a monetary aggregate fulfill more objectives that consumers may have for their asset holdings as the definition of the monetary aggregate becomes broader. In other words the broader the monetary aggregate is defined, the more a one-percent increase in income will spur an increase in consumers' desire to hold more broadly defined money balances.

The interest rate semi-elasticity is similar for the more broadly defined monetary aggregates. It does differ markedly between M1 and the broader aggregates, though. The opportunity cost, as measured by the three month T-bill rate, is greater for monetary aggregates that include savings deposits and money market mutual funds (MMMF). Presumably, savings deposits and MMMFs pay higher rates of interest than do demand and other checkable deposits, so the effect of the opportunity cost of holding M1 balances, as opposed to broader aggregate balances, should be higher. This result, however, may be due to the fact that the opportunity cost used here is a gross rate of return that does not take in to account the aggregate's respective own rate of return.

8. The Best Monetary Aggregate

It is well known that monetary theory has very little to say theoretically on the appropriate measures of money, prices, income, or opportunity costs to be used in money demand models. While theory (either demand theory or inventory theoretic money demand theory) clearly specifies what sort of variables should be important in the determination of consumers' money demand, it does not say how
these variables should be measured. Ultimately, the question of the appropriate proxies for the variables in money demand equations is an empirical one.

To this end, the scale variable has been proposed to be best represented by permanent income (weighted GNP), measured income (GNP), direct measures of wealth, or measures of transactions. For opportunity costs, researchers have used short-term rates (yields on government securities, commercial paper, or savings deposits), long-term rates (yields on long-term government bonds or equities), or the entire term structure of interest rates. While the literature has spent time investigating these issues, little has been said on which monetary aggregate best explains money demand (and the corresponding equilibrium adjustment process). The MQT allows us to examine just this question in a context of nested regression models.

Table 4.12 presents the comparisons of various monetary aggregates in equation (4.4). The table presents the estimated coefficients for the two monetary aggregates being compared. $\beta_2$ represents the coefficient on the monetary aggregate listed first in column one of the row of interest. $\beta_3$ represents the coefficient on the monetary aggregate listed second in column one of the row of interest. For instance, in the first row, M1 is being compared to M1plus. The coefficient, $\beta_2$, represents the estimate of the parameter for M1 while $\beta_3$ represents the estimate of the parameter for M1plus.

As Table 4.12 shows, the monetary aggregate that dominates in terms of explanatory power in every case is M1plus. M1plus is significant (and of the correct sign) for every regression at the 95% level of confidence or better while the monetary aggregate to which it is compared is insignificant in the same regression. This implies that, relative to the other monetary aggregates investigated here, M1plus is the best aggregate to use to explain inflation and the underlying adjustment process.

9. Money Demand Stability

Prior to the mid 1970's, money demand forecasts, based on a functional form of money demand similar to
\[
\ln m_t = \alpha + \beta_1 \ln y_t + \beta_2 R_t + \beta_3 \ln m_{t-1} + \beta_4 \ln \left( \frac{P_t}{P_{t-1}} \right) + \varepsilon_t
\]  

(4.5)

did a good job in predicting the amount of money held in the U.S. Goldfeld's (1976) thorough investigation of money demand first exposed a problem with the conventional forecasts when data from the period of 1974-1976 was examined. During this period, Goldfeld documented that conventional money demand forecasts overpredicted the amount of money that the U.S. would be holding. The explanations for this phenomenon were varied (see Judd and Scadding (1982), Goldfeld and Sichel (1990), or Chapter 2 of this dissertation), but the message was clear. Simply examining the goodness of fit of a given model was not good enough. Since ultimately a model of the demand function for money is to facilitate the implementation of an activist monetary policy, not knowing the position of the money demand function at any given time hinders the effectiveness of monetary policy. If the parameters of the money demand specification changed over time and these parameter shifts were not modeled, then the usefulness of the particular money demand specification would be questionable. This led economists to examine not only the goodness of fit of particular models, but also to examine their out-of-sample fit and the stability of the parameter estimates.

Many economists subsequently found forecasts of money demand in later periods to be inexact. Gordon (1984); Roley (1985); Rose (1985); Hetzel and Mehra (1989); Miller (1989); and Baba, Hendry, and Starr (1992) all note a velocity decline occurs in money demand starting in late 1981 or early 1982 and continuing through much of 1983. Through that period, termed the great velocity decline, conventional money demand specifications underpredict U.S. money holdings. Miller (1989) and Baba, Hendry, and Starr (1992) also mention that the period from early 1985 through 1986 saw an overforecast of money holdings. Lastly, Mehra (1997) finds that the period from early 1990 through 1994 again saw an overprediction of money holdings.

In light of these findings by other economists, we examined the parameter stability of MQT with respect to these different time periods. We used Chow's (1960) test of the equality between sets of coefficients in two linear regressions. To implement Chow's test, the data was partitioned into two samples with the break point being the first month where the money demand forecasts began to deviate from actual money holdings. Separate regressions were run for each of the two samples (the unrestricted
regressions). And, one regression was calculated for the overall sample (the restricted regression). Table 4.13 presents the results of the Chow tests for each money aggregate for monthly data.

Encouragingly, the MQT specification holds up well when examining the stability of the parameter estimates over the various sub-periods. Only for the "missing money" episode of Goldfeld (1976) is there a statistically significant difference between the parameter estimates for the two sub-periods. This suggests that the MQT specification is robust to many of the structural changes that the aforementioned authors noted. It is also apparent that it matters which monetary aggregate is used. MQT regressions where M1 is the monetary aggregate appear to suffer from parameter instability across all of the sub-periods investigated here.

To investigate these periods of instability, we can represent shifts in the money demand function as shifts in the intercept term. Specifically, if the intercept term shifts randomly, then the MQT specification can be rewritten as a random walk plus noise model and can be estimated using a Kalman (1960) filter. From equation 4.4, let $z_t$ be

$$z_t = \pi_t - \pi_{t-1} - y \ln m_{t-1} + \alpha_2 y \ln y_{t-1} + \alpha_3 y R_{t-1} = -a_{1t} y + \varepsilon_t.$$  \hspace{1cm} (4.6)

In a Kalman filter representation, (4.6) is the measurement equation in which the state vector, $a_{1t}$, can vary over time. Specifically, $a_{1t}$, behavior is governed by a transition equation of the form

$$a_{1t} = a_{1t-1} + \eta_t,$$  \hspace{1cm} (4.7)

which is simply a random walk model plus noise for the intercept term. In this model, the

$$\text{Var}(\varepsilon_t) = \sigma^2 \varepsilon,$$  \hspace{1cm} (4.8)

and

$$\text{Var}(\eta_t) = \sigma^2 \eta.$$  \hspace{1cm} (4.9)

The model can be parameterized in terms of $\sigma^2 \varepsilon$ and the signal-to-noise ratio, $q$, by rewriting (4.8) and (4.9) as

$$\text{Var}(\varepsilon_t) = \sigma^2 \varepsilon$$ and $\text{Var}(\eta_t) = \sigma^2 \varepsilon q$.  \hspace{1cm} (4.10)
Equations 4.6-4.7 and 4.10 can be estimated using a Kalman filter. In the case of the random walk plus noise model the prediction equations are (Harvey, 1989):

\[ a_{t+1} = (1 - k_t) a_{t-1} + k_t z_t \]  

(4.11)

where the gain is

\[ k_t = \frac{p_{t|t-1}}{p_{t|t-1} + 1} \]  

(4.12)

and the covariance of the estimation error is

\[ p_{t+1|t} = p_{t|t-1} - \left[ \frac{p_{t|t-1}^2}{1 + p_{t|t-1}} \right] + q \]  

(4.13)

where \( a_t \) and \( z_t \) are defined above.

If the disturbances and the initial state vector, \( \alpha_0 \), are normally distributed, the distribution of \( z_t \) conditional on \( z_{t-1} \) is normal as well. Furthermore, the mean and covariance matrix of the conditional probability density function, \( p(z_t \mid z_{t-1}) \), can be estimated with a Kalman filter or equivalently by means of maximum likelihood estimation. Maximum likelihood estimation allows us to estimate the signal-to-noise ratio directly. If \( q \) is zero, this implies that \( Var(\eta_t) = 0 \) and the intercept term, \( a_{1t} \), does not vary randomly over time. This method of estimation by maximum likelihood is known as the prediction error decomposition form of the likelihood.

Table 4.14 presents the results of this estimation. In every case, the signal-to-noise ratio that maximizes the log-likelihood function is very close to zero. This means that allowing the money demand intercept to drift randomly adds almost no likelihood to the data. It also implies that the noise, \( Var(\varepsilon_t) \), dwarfs the signal, \( Var(\eta_t) \). If a time-varying money demand intercept can be interpreted as the term that soaks up changes in transaction technology, then this result is not hard to accept. This result implies that changes to the transaction technology, or some other plausible explanation to money demand functions' instability, are not random.
To investigate further the instability exhibited for all the monetary aggregates around the "Missing Money Episode" we first run expanding window Chow tests on the monthly data from January 1970 to December 1997.\textsuperscript{23} Figures 4.19 to 4.24 display the results of these expanding window Chow tests. The parameter estimates for the MQT regression with M1Plus and M2 as the monetary aggregate are very stable across the periods considered. Except for the Missing Money Episode, both MQT regressions indicate very stable relationships between the variables in money demand. MQT regressions with M2Minus, MzM, and M3 are also relatively stable after the early to mid 1980's. It is only when one looks at a MQT regression with M1 that there is an indication that a serious money demand instability problem exists. This further emphasizes the importance of the monetary aggregate and the definition of money when trying to explain money demand.

10: Endogeneity Tests

Traditional single equation money demand specifications of the general form,

$$m_t = f(y_t, R_t, X_t)$$

(4.14)

where $X_t$ is a set of variables that are also thought to explain money demand (cf. Baba et al., 1992), can suffer from endogeneity problems if one believes that explanatory variables, $y_t$, $R_t$, and $X_t$ are functions of the money stock, $m_t$. If the U.S. Federal Reserve chooses the interest rate to influence the behavior of the money stock, then past and current values of the money stock would certainly seem to influence interest rates. If this is true then estimated coefficients on the explanatory variables would be inconsistent. Cooley and Leroy (1981) go so far as to argue "...that there is no obvious way to formulate models of equilibrium in financial markets in which the demand for money is identified." Hetzel (1984)

\textsuperscript{23} The expanding window Chow tests utilize all the data beginning from January 1959. However, Chow tests require estimating regression equations for the two sub periods determined by the on which you split the data. Starting at January 1970 and stopping at December 1997 as split points allows a sufficient number of observations for accurate estimation of all sub periods specified by the expanding window.
and Roley (1985) take a less pessimistic view to the problem, but they still note that it is a serious
problem with conventional money demand specifications.\footnote{With cointegrated money demand variables in an Engle-Granger cointegrating equation there is no
care for any endogeneity problems because the estimated coefficients in these types of regressions are
superconsistent (assuming that a cointegrating relationship exists).}

To illustrate this problem, we examine the causality/exogeneity of the various money demand
variables in a conventional money demand specification as in (4.14). Granger (1969) put forth the idea of
causality with the notion that the future cannot cause the present or the past. If an event $X$ occurs after
event $Y$, then $X$ cannot cause $Y$. However, Granger causality is not causality as thought of in the usual
sense. Granger causality does not test for a causal connection between two variables. Instead Granger
causality examines whether a given variable $X$ precedes a variable $Y$ (Leamer, 1985). Specifically, a
time series $x_t$ fails to Granger cause a time series $y_t$ if in a regression of $y_t$ on $\sum_{i=1}^{k} y_{t-i}$ and on
\[
\sum_{j=1}^{l} x_{t-j},
\]
the coefficients on $\sum_{j=1}^{l} x_{t-j}$ are zero. In other words, if $\beta_j = 0 \ \forall j = 1, 2, \ldots, l$ in
\[
y_t = \sum_{i=1}^{k} \alpha_i y_{t-i} + \sum_{j=1}^{l} \beta_j x_{t-j} + \varepsilon_t,
\]
then $x_t$ does not Granger cause $y_t$.

Another causality test proposed by Sims (1972) says that a time series $x_t$ fails to cause a time
series $y_t$ if a regression of $x_t$ on $\sum_{i=-k}^{l} y_{t-i}$ results in coefficients on the future values of $y_t$ to equal
zero. In other words, if $\alpha_i = 0 \ \forall i = -k, -k+1, \ldots, 1$ in
\[
x_t = \sum_{i=-k}^{l} \alpha_i y_{t-i} + \varepsilon_t,
\]
then $x_t$ does not Sims cause $y_t$. Chamberlain (1982) shows that both notions of causality, Granger and
Sims, are functionally equivalent.
Table 4.15 presents the results of Sims causality tests for the conventional money demand specification. It is important to note that Geweke, Meese, and Dent (1982) examined several forms of causality tests and found the Sims test was sensitive to the failure to correct for serial correlated errors. They proposed an alternative test that added to the two-sided distributed lag terms of Sims a set of lagged dependent variables. The form of their alternative is

\[ x_t = \sum_{i=-k}^{l} \alpha_i y_{t-i} + \sum_{j=1}^{m} \beta_j x_{t-j} + \epsilon_t. \]  

(4.17)

The alternative Sims causality tests were run with real U.S. Personal Income (in natural logs), the three-month T-bill rate, and the Moody’s rated Aaa and Baa bonds each as dependent variables. The regressors were a constant, a monetary aggregate (in leads and lags), and the lags of the dependent variable. Table 4.15 presents the F-statistics for the joint significance of the first two leads of the monetary aggregate regressed on each respective variable. For example, row 1 of Table 4.15 shows that the first two leads of M1 have no joint explanatory power for U.S. Personal Income. But, the first two leads of M1 have a very statistically significant influence on the three-month T-bill rate, the Aaa bond, and the Baa bond. In the Granger-Sims sense, M1 Granger-Sims causes the three-month T-bill rate, the Aaa bond rate, and the Baa bond rate. This is evidence that there is an endogeneity problem with the conventional money demand specification for the M1 aggregate.

Examining the remainder of Table 4.15, it is apparent that this endogeneity problem is prevalent regardless of the monetary aggregate considered. It is not surprising, given the discussion above, that there is some evidence interest rates in general, as represented by the short-term T-bill rate and the corporate bond rates, depend on the real money stock. It is another indication that partial adjustment models of money demand have serious econometric problems that make their usefulness suspect.

Moderate Quantity Theory of Money (MQT) regressions will not suffer from the same problem. Since unanticipated inflation is the dependent variable instead of the money stock, there is no a priori reason to believe that the real money stock (in natural logs), real U.S. Personal Income (in natural logs), and a short-term T-bill rate or corporate bond rate should be a function of unanticipated inflation. It is
much harder to paint a realistic picture of the U.S. Federal Reserve examining unanticipated inflation before it sets interest rate policy. Nevertheless, it may still, in fact, be the case that Granger-Sims tests for causality indicate some level of reverse relationship.

Table 4.16 provides the results using alternative Sims tests for causality. In this case, the dependent variable is a real monetary aggregate (in natural logs), real U.S. Personal Income (in natural logs), the three-month T-bill rate, the Aaa-rated bond rate, or the Baa-rated bond rate. The regressors are a constant and unanticipated inflation.\textsuperscript{25} Examining Table 4.16, there is very little evidence of an endogeneity problem as the only variable that is statistically significant is real U.S. personal income and that is only marginally significant. Thus, the MQT regressions do not suffer from the same problems that standard PAM regressions do. We can feel reasonably confident that the parameter estimates in the MQT regressions are not inconsistent due to endogeneity.

11: Summary

This chapter provided evidence for the Moderate Quantity Theory (MQT) as a means by which to estimate money demand functions. A conventional Cagan (1956) form of money demand was used and appropriate proxies for the monetary aggregate, the scale variable, and the opportunity cost variables were established.

Following Granger and Newbold’s (1974) warning of the possibility spurious regressions when using time series data, unit root tests examined the time series properties of the series. It was noted that the MQT regressions have an I(0) regressand and I(1) regressors. Econometrically, these types of regressions are what are known as unbalanced regressions. As such, it is important to establish that the regressors in these types of regressions are cointegrated. Support for this hypothesis was given in Chapter Two and further supported in this chapter. We also showed via Monte Carlo simulation that

\textsuperscript{25} Eight lags of the dependent variable were used to try to whiten the residuals in the Sims regressions. A LM test for higher order serial correlation indicated in each regression that the residuals were sufficiently whitened so that serial correlation was not an issue.

\textsuperscript{26} The number of leads of unanticipated inflation is two. In each alternative Sims test the regression residuals were whitened by including lagged terms of the dependent variable. LM tests for higher order serial correlation found in every case that serial correlation was not present to any significant degree.
there is no fear of underestimation of the standard errors of parameter estimates from unbalanced regressions.

The MQT regressions, using various monetary aggregates, all had significant parameter estimates on the adjustment, scale, and opportunity cost variables that were of the right sign. Income elasticity estimates ranged from approximately 0.5 for the narrowest definitions of money to approximately 1.0 for the broadest definitions of money. This range of estimates is consistent with previous models of money demand. This range of estimates also makes good intuitive sense as more general definitions of money encompass more monetary assets and, therefore, provide more instruments for agents to transfer personal income into. The range of values for the interest rate semi-elasticity was from -0.026 to -0.044.

Another advantage of the MQT specification for money demand is the ability to test various monetary aggregates against each other in a nested MQT regression. Doing this leads us to the conclusion that M1Plus, which is M1 plus savings deposits, has the best explanatory power when examined against other monetary aggregates. This compares tellingly to most previous money demand studies that focus on either M1 or M2 as the monetary aggregate and suggests M1Plus may be the aggregate that should be considered.

Lastly, we examine the Moderate Quantity Theory in light of well-documented problems with previous money demand research, namely the instability of estimated money demand functions and the possible endogeneity problems with single equation money demand specifications. MQT regressions with M1Plus and M2 experience little parameter instability over the U.S. post War monetary history. The only instability in the parsimonious MQT specification is over the Goldfeld (1976) Missing Money Episode. MQT regressions with M2Minus, M2M, and M3 exhibit instability through the early 1980’s, but do not have the same instability problems that other research have found with conventional money demand specifications post 1982. These stable MQT parameter estimates are all the more remarkable given the parsimonious money demand specification used in this research.

The possibility of U.S. personal income and interest rates being a function of the level of the real money stock is not a problem often investigated in conventional money demand research, but the Granger-Sims causality analysis done in this chapter suggests it should be. Of course, it is well known
that inconsistent parameter estimates arise in regressions where endogeneity is an issue. However, the same analysis applied to the MQT specification does not lead to the same conclusion of an endogeneity problem. In the Granger-Sims sense, unanticipated inflation does not temporally precede the regressor variables used in the MQT regressions.

Thus, this chapter can conclude that the Moderate Quantity Theory is a promising and fruitful way to investigate U.S. money demand. The next chapter will examine the Moderate Quantity Theory with respect to the international data of the G7 countries.
CHAPTER 5

Empirical Tests of MQT Using International Data

1. Previous Work on International Money Demand

As with the literature on U.S. money demand, previous examinations of international money demand were primarily in the context of the Partial Adjustment Model (PAM) paradigm (see Chapter 2 for more details on this type of money demand model). Boughion (1981) and Fair (1987) were among the first to examine international money demand on a comparative scale, that is, examining industrialized countries together.

Fair (1987) examined twenty-seven different countries in a PAM framework.\(^\text{27}\) He found similarity in the money demand functions across OECD countries. Specifically, the signs on the parameter estimates for both the income elasticity and the interest semi-elasticity were, for the most part, consistent with theory. Countries like Germany, the Netherlands, Sweden, and Finland had per capita income elasticities in the 0.34-0.64 range while countries like Canada, Japan, France, Italy, the U.K., and the U.S. had much lower (0.05 to 0.11) income elasticities. The interest rate semi-elasticities were of the same magnitude across countries, however.

As discussed in Chapter 2, PAM specifications of money demand suffer from a number of methodological and econometric shortcomings that are insurmountable. Recognizing this fact, the international money demand literature moved toward cointegration theory as a way of overcoming the problems inherent in the PAM methodology.

\(^{27}\) Besides doing a comparative examination of money demand across countries, Fair was also interested in whether the adjustment of actual to desired money holdings is in nominal or real terms.
Most of the research that followed in the cointegration theory vein found evidence for cointegrating relationships between a monetary aggregate, a scale variable, an opportunity cost variable, and, sometimes, a price level across the various countries considered here. In cases where the evidence was less than conclusive, researchers would “…cautiously proceed under the assumption of cointegration” (Hendry and Ericsson, 1991). Engle and Granger’s (1987) two-step estimation technique, Johansen’s (1988) ML estimation technique, and Stock and Watson’s (1993) dynamic OLS were most often the methods used to reach the conclusion that a money demand cointegrating relationship existed. Table 5.1 gives an incomplete list of the studies by country that investigated money demand using a cointegration framework. Each of the studies listed there found some evidence of a money demand cointegrating relationship.

2. Unit Root Tests

To examine evidence for the Moderate Quantity Theory (MQT) of money in an international setting, we will examine data from the major industrialized G7 countries, Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States (see Figures 5.1 through 5.28). Unfortunately, the common data frequency that is available for these countries is quarterly. As the Moderate Quantity Theory implies, consumers’ information about current period values for prices and output becomes better as the informational lag length decreases. Plus, it is more reasonable to assume consumers’ inflation expectations adjust more quickly than quarterly. But, available proxies for the scale variable in these international MQT regressions, like industrial production, do not correspond well to the traditional idea of an income or wealth variable in money demand specifications. So, the choice of examining international money demand in the MQT framework is based more on availability of the data than anything else.

The data used for the analyses in this chapter all come from the OECD Main Economic Indicators Historical Statistics 1960-1997 CD-ROM. For each of the countries examined, a monetary aggregate was chosen along with a short-term interest rate. In some cases, the only short-term rate that was available was the discount rate. Rather than use that rate, a long rate with the shortest time to
maturity was chosen. For each country the monetary aggregate and the scale variable, a country's GDP, were deflated by that country's consumer price index (all items).\textsuperscript{28} So, the monetary aggregate and GDP are in real terms. For Germany, GDP data provided by the OECD did not exist prior to Germany's reunification in 1990. In that case, the GDP data for Germany was obtained from the DRI-McGraw-Hill International Data CD-ROM for West Germany up to 1995. This data series was spliced together with the OECD data through 1998. A description of the data and its sources can be found in Appendix C.

To investigate the existence of at least one cointegrating vector in an international setting, unit root statistics are calculated. Both augmented Dickey-Fuller (1981) and Phillips-Perron (1988) tests are done. The differenced lag structure of the augmented Dickey-Fuller test is determined by Campbell and Perron (1991) method of including twenty lag-differenced terms and testing the significance of the parameter on the longest lag. The lag length for these differenced lags is determined by including the longest lagged difference term for which its respective parameter estimate is significant. To check for robustness of conclusions with regard to the presence of a unit root, the differenced lag lengths were also determined by appealing to Akaike and Bayes-Schwarz information criteria, a LM test for serial correlation in the ADF specification, and a Ljung-Box Q test of the residuals from the ADF regression as well. Table 5.2 presents the results of the ADF unit root tests by country.

Phillips-Perron tests for unit roots allow for a possible MA(q) structure in the DF residuals. If the residuals are generated by a MA(q) process, then the unit root statistic estimated from an ADF unit root regression will be inconsistent regardless of the lag length in that regression. This is because an MA process can only be represented by an infinite order AR process. To allow for this possibility, Table 5.3 presents the Phillips-Perron unit root test statistics.

In every case, the series, unanticipated inflation (UAI), rejects the null hypothesis of a unit root regardless of the test used. In most of the cases with respect to the monetary aggregate, the scale variable, and the interest rate variable, the null hypothesis of a unit root cannot be rejected. Both results taken together suggest that each country's MQT specification is an unbalanced regression in which an I(0) series is regressed on a set of I(1) regressors. As in Chapters 2 and 4, this is not problematic if the set

\textsuperscript{28} The lone exception to this was in the case of the U.S. The price index used there was the CPI-UX and its construction is detailed in Chapter 3.
of I(1) regressors move together so that a linear combination of those series is I(0). In other words, there must exist a cointegrating relationship between the set of regressors for a given country for an MQT regression to make sense econometrically.

3. Tests for the Existence of a Cointegrating Vector

As discussed in Chapter 2, Johansen (1988) and Johansen and Juselius (1990) proposed a procedure for estimating a cointegrating relationship that allows one to test for the existence of a cointegrating vector among a set of I(1) variables. Specifically, Johansen's method uses maximum likelihood estimation in a pth order VAR where if \( x_t \) is a vector of economic variables thought to be cointegrated, then \( x_t \) can be set up such that

\[
    x_t = \sum_{i=1}^{p} A_i x_{t-i} + \varepsilon_t .
\]

Reparameterizing (5.1) as

\[
    \Delta x_t = \sum_{i=1}^{r+p} \Pi_i \Delta x_{t-i} - \Pi x_{t-p} + \varepsilon_t
\]

(5.2)

where \( \Pi = (I - \sum_{i=1}^{p} A_i) \) allows for the estimation of the rank of \( \Pi \). \( \Pi \) is the matrix of eigenvalues such that its rank determines the number of cointegrating vectors for the system. If the rank of \( \Pi \) is zero, then equation 5.2 corresponds to a traditional, differenced VAR. If the rank of \( \Pi \) is non-zero and the system has rank, \( r \), then there exists \( r \) independent cointegrating vectors.

\( \Pi \) can be estimated and the trace of \( \Pi \) can be computed. The trace test can be used (Johansen, 1988 and Johansen and Juselius, 1990) for determining the existence of at least cointegrating vector. The trace test is:

\[
    \hat{\lambda}_{trace}(r) = -T \sum_{i=r+1}^{n} \ln(1 - \hat{\lambda}_i) \quad \text{(5.3)}
\]
where $\lambda_i$ is the $i$th ordered characteristic root of $\Pi$. If the variables in $x_t$ are not cointegrated, the rank of $\Pi$ will be zero and all the characteristic roots of $\Pi$ will be one. Therefore, for each $i$, $\ln(1 - \lambda_i)$ will be zero and the trace statistic will be zero.

Tables 5.4 to 5.10 presents the results from Johansen’s trace test for the existence of a cointegrating vector for each of the G7 countries. The third column presents the trace test for a cointegration space of the monetary aggregate, a scale variable, an opportunity cost variable, and the inflation rate. The fourth column presents the trace test for a cointegration space of the monetary aggregate, a scale variable, and an opportunity cost variable. The null hypothesis for the trace test is no cointegrating effect vs. the alternative of the existence of at least one cointegrating vector. Since the Johansen trace test can be biased in the presence of serial correlation in the residuals, a Ljung-Box Q test for higher order serial correlation is computed. Column five has the results of this test. The first number in of the pair in column five is the number of error processes (out of four) in the VAR that exhibit serial correlation with 95% confidence. This corresponds to the Johansen VAR run in column three. The second number of the pair in column five is the number of error processes (out of three) in the column four VAR that exhibit serial correlation with 95% confidence.

As can be seen from the table, the null of no cointegrating vector can be rejected at better than a 99% confidence level for Canada, the United Kingdom, Italy, and France. For Japan and Germany, the null of no cointegrating vector can be rejected with at least 95% confidence. For the United States, the null of no cointegrating vector for the cointegration space of the monetary aggregate, real GDP, a short-term interest rate, and price inflation can be rejected with a 99% degree of confidence. There is also evidence that the cointegration space of the monetary aggregate, real GDP, and a short-term interest rate rejects the null of no cointegrating vector with better than 95% confidence. In each case, the trace statistic is used from the table where there is no evidence of serial correlation in any of the residuals from the VAR. These lag length show that the error processes in the VAR have been whitened satisfactorily.
4. MQT Regressions Using International Data

As stated in Chapters 2 and 4, the Moderate Quantity Theory (MQT) posits a stable relationship between unanticipated inflation and the disequilibrium between money supply and money demand.

\[ UAI_t \equiv \pi_t - \pi_{t-1}^a = -\alpha_1 \gamma + \gamma \ln m_{t-1} - \alpha_2 \gamma \ln y_{t-1} - \alpha_3 \gamma R_{t-1} + \epsilon_t \] (5.4)

From (5.4), a speed of adjustment parameter, \( \gamma \), plus an income elasticity, \( \alpha_2 \), and an interest rate semi-elasticity, \( \alpha_3 \), can be estimated.

In the MQT specification in (5.4), \( \pi_{t-1}^a \), agents’ expectations of inflation at time \( t \) made at \( t-1 \) are estimated using the Fuller (1996) median unbiased estimator of the coefficient on the first lag of the dependent variable in an ADF weighted symmetric OLS regression. As in Chapter 3, the lag length for ADF specification is chosen so that a general LM test for higher order serial correlation shows no serial correlation in the ADF residuals. Once this median unbiased estimate is obtained, the inflation series is differenced by \( \bar{\beta} y_{t-1} \), where \( \bar{\beta} \) is the median unbiased estimate for the coefficient on the first lag of the dependent variable and \( y \) is the inflation series. This pseudo differencing makes unanticipated inflation an I(0) time series allowing us to use Wold’s Decomposition Theorem to find a parsimonious, finite order ARMA representation of this pseudo differenced inflation series. Once the data generating process for the pseudo differenced inflation time series is chosen based on a Bayes-Schwartz information criterion across alternative ARMA models, next period’s inflation expectation is generated based on this model.

At time \( t \), an econometrician would have only past and current inflation rates with which to generate next period’s expectation of inflation. Of course, this means when using inflation data for a given country, any inflation rates after time \( t \) that the econometrician may have cannot be used. In the context of this research, this means that for an inflation series from time 1 to time \( T \), when generating an expectation of inflation for time \( t + 1 \), only inflation observations up to time \( t \) were used. So, for
example, when generating agents’ inflation expectation for January 1975, only inflation data through December 1974 was used. However, the complete estimation process, the median unbiased WSLS and the ARMA estimation, was redone each time a new data point was added. This expanding window method of estimation allowed for complete flexibility when modeling agents’ expectations of inflation as new information (new inflation data) was made available.

Figures 5.29-5.35 show the median unbiased estimate for the coefficient on the first lag of the dependent variable for each G7 country. For the most part, every G7 country has a unit root or near unit root process governing its inflation series. The notable exception over the period 1970 to 1998 is Germany in which that country’s inflation rate appears to be more like an I(0) process. Given Germany’s past experience with hyperinflations (cf. Cagan, 1956) and its deep-rooted fear of price inflation, it is not hard to understand why Germany’s inflation series behaves differently than the remainder of the G7 countries.

Table 5.11 presents the parameter estimates for the MQT regressions by G7 country. For four of the countries, Germany, Italy, Japan, and the United States, the parameter estimates for the independent variables are statistically significant and the right sign. To examine the effect of the data splicing for Germany, we use a dummy variable to represent a reunified Germany starting in the first quarter of 1995. The dummy variable has an insignificant parameter estimate in the German MQT regression indicating that there is no need to account for the pre-unified West Germany and the post-unified Germany separately. For the remainder of the G7 countries, Canada, France, and the United Kingdom, most of the parameter estimates are statistically insignificant.

The international evidence for the MQT presented here is encouraging especially when one considers that the availability of data sometimes forced the use of a proxy for a money demand variable that was less than ideal. (The criterion for the selection of a money aggregate for each country was how well it explained unanticipated inflation, i.e., how well it did in the MQT regression). Table 5.12 displays the estimated income elasticity and interest rate semi-elasticity for the G7 countries in which statistically significant relationships were found.
5. Unit Root Tests on the MQT Residuals

As further evidence for a cointegrating relationship between the independent variables in the MQT regressions, unit root tests are run on those residuals. Of course, if the independent variables have no explanatory power in a MQT regression then the residuals will be I(0) because the regressand is I(0). So, we perform unit root tests only for the MQT regressions of Germany, Italy, Japan, and the United States. If the variables have explanatory power and they are all integrated of order I(1), yet the MQT regression residuals are I(0), then we can be reasonable sure that there exists a cointegrating relationship between the explanatory variables. If not, than the unbalanced MQT regressions would not make sense econometrically.

Table 5.13 presents the unit root tests for the MQT regressions in which the parameter estimates for the MQT regressions were statistically significant. In other words, unit root tests were performed on the residuals from the MQT regressions for Germany, Italy, Japan, and the United States. Both augmented Dickey-Fuller unit root tests and Phillips-Perron unit root tests were computed. For the augmented Dickey-Fuller tests, the lag length was determined by the Campbell-Perron (1991) method of including a large number of lag lengths and decreasing the lag lengths until the longest lag length has a significant parameter estimate. An examination of Table 5.13 reveals strong evidence for a cointegrating relationship between the monetary aggregate, the income variable, and the interest rate variable. In almost every case, the null of a unit root process governing the residual from each MQT regression is rejected at better than 95% confidence regardless of which unit root test is used. Therefore, the unbalanced regressions conducted in Table 5.11 make sense econometrically.

6. Money Demand Stability

As with U.S. money demand, the mid to late 1970's were a period of money demand instability across the world. So, we investigated this issue in the context of the Moderate Quantity Theory. Unfortunately, data for Italy are available here only since January 1975. However, Germany, Japan, and the United States can be investigated throughout the documented period of money demand instability.
To investigate the instability exhibited across the data period, we first run expanding window Chow tests on the quarterly data from January 1973 to December 1996. The break point for the Italian data started in January 1978. Figures 5.36 to 5.39 display the results of these expanding window Chow tests. The stability for the parameter estimates from the U.S. and German MQT regressions are especially encouraging as there is little indication of instability across the data period except for a couple of months in the 1970's. (For Germany, there is no indication of parameter instability).

The Japanese MQT regressions also exhibit stability for most of the data period. There are some serious instabilities exhibited, however, during the mid-1970's. The Italian expanding window Chow tests are the least persuasive of the four sets of data. There is significant instability throughout the late 1970's and early 1980's. However, this instability is no longer an issue after that period as there is no indication of parameter instability after the early 1980's. Since the proposed money demand function is the Cagan form that has historically shown more parameter instability across many more periods than we have found here, the results of this exercise are encouraging both for estimation and forecasting money demand.

7: Summary

In this chapter, we examined the empirical evidence for the Moderate Quantity Theory using international data from the G7 industrialized countries. The data period extended quarterly from January 1970 to December 1998. Unfortunately, the quality of data available does not rival the U.S. monthly data that was used in the previous three chapters. Often, the OECD database used here did not have every data series extend back to January 1970. Moreover, the data series available at times did not include the ideal proxies for interest rates. Nonetheless, a strong case can be made for the Moderate Quantity Theory holding across countries.

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29 The expanding window Chow tests utilize all the data beginning from January 1970. However, Chow tests require estimating regression equations for the two sub periods determined by the on which you split the data. Starting at January 1973 and stopping at December 1996 as split points allows a sufficient number of observations for accurate estimation of all sub periods specified by the expanding window.
As a first step to ensure the econometric soundness of the MQT regressions, unit root tests on all the appropriate time series were conducted. As was the case in Chapter 4, the regressand in each MQT regression, unanticipated inflation, was I(0) and each of the regressors, the monetary aggregate, the scale variable, and the interest rate variable, were I(1). This means that each MQT regression is what is called an unbalanced regression (Banerjee, Dolado, Galbraith, and Hendry, 1993) in which an I(0) time series is regressed on a set of I(1) variables.

For an unbalanced regression to be sound econometrically, it must be shown that the set of I(1) variables are cointegrated of order 1. In that way, the regressors do not move infinitely far from the regressand in an infinite period of time. Johansen's (1988) maximum likelihood estimation of cointegration matrix and the Johansen trace test statistic give strong evidence for the existence of at least one cointegrating vector for each country's cointegration space. Further evidence for the existence of a cointegrating vector is the unit root tests of the residual from a MQT regression. In each case, the residuals from a given MQT regression is I(0) while at the same time the parameter estimates from that regression are statistically significant.

The MQT regressions are estimated for each of the G7 countries. We find that a statistically significant relationship, as specified by the MQT, exists for Germany, Italy, Japan, and the United States at a quarterly frequency for the time period January 1970 to December 1998. While a MQT relationship does not exist for the data examined from Canada, France, and the United Kingdom, the concern is not with the theory, but instead with the availability and quality of the data.

A last issue investigated here is a problem that has plagued conventional money demand specifications every since Goldfeld's (1976) famous "Missing Money Episode." We examine the stability of the parameter estimates from the MQT regressions, where statistically significant estimates were computed, using expanding window Chow tests over the majority of the time period. In all cases, the MQT parameter estimates were remarkably stable throughout the mid to late 1980's and 1990's. For Germany, there was no parameter instability exhibited across the entire time period considered. In the

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30 The exception is Italy in which the data period starts January 1975.
case of the United States, instability existed for only a couple of quarters during the mid 1970's. For Japan, there was more marked instability during the period, but it came almost as infrequently as in the U.S. case. Only Italy had consistent instability during the late 1970's and early 1980's. But, as already has been noted, other studies have found more prominent and more numerous instabilities over the time period examined here. Therefore, the MQT is an encouraging alternative to estimate and forecast money demand.
CHAPTER 6
Conclusions

1. Summary

This dissertation presents the Moderate Quantity Theory (MQT) of Money as an adjustment process for the periods of disequilibrium between money supply and money demand. The theory is similar to what is normally called the Quantity Theory of Money in that there is long-run money neutrality. But, unlike the Quantity Theory, the MQT does not have prices adjusting instantaneously to monetary disequilibria. Also, the MQT allows for inflation even during periods where there is monetary equilibrium.

We test the MQT by first examining conventional money demand models and contrasting them to the Moderate Quantity Theory. Examined were Partial Adjustment Models, Cointegration/Error Correction Models, and Buffer Stock Models. Chapter II detailed each of those models and pointed out the drawbacks to each of them in the context of the MQT. Importantly, it was shown in Chapter II that there exists at least one cointegrating relationship between a monetary aggregate, a scale variable, and an opportunity cost variable for U.S. monthly data from January 1959 to May 1999. This result validates the MQT regressions in Chapter IV as being sound econometrically. These types of regressions are called unbalanced regressions in the literature and require the regressors to be cointegrated of order 0.

A method for estimating expectations of next period’s inflation rate was presented in Chapter III. There it was noted that the monthly U.S. inflation series certainly appears to exhibit mixed orders of integration across the January 1959 to May 1999 time period. This necessitates a different approach to modeling this time series, namely, estimating the order of integration instead of imposing one order of integration across the whole time series.
Using median unbiased estimators, this chapter focused on estimating the order of integration rather than simply using unit root tests to discern whether a unit root process was present. Once the order of integration was estimated for a sub period, the inflation series was pseudo differenced by this estimate to return a covariance stationary series. It was this series that was estimated using ARMA models. Wold’s Decomposition Theorem states that an ARMA representation is fundamental for any covariance stationary series. Of course, this could mean that an infinite order ARMA process might best represent this pseudo differenced inflation series. But, it certainly is reasonable to assume that a more parsimonious, finite-order ARMA representation would represent the pseudo differenced series as well.

All nested ARMA models up to order (10, 10) were searched over using the Schwartz-Bayes Criterion to determine the model of best fit. The Schwartz-Bayes Criterion penalizes for spurious overfitting, so its use here is especially appropriate. Once the model was chosen, next period’s forecast for inflation was generated. This forecast was used as the proxy for agents’ inflation expectations in the tests of MQT.

To simulate the incorporation of new inflation information into agents’ expectations, each “new” data point for the inflation series caused a reestimation of the order of integration for the series as well as the appropriate ARMA model for the subsequent pseudo differenced series. This expanding window approach, while computationally intensive, allowed for the realistic reassessment of agents’ inflation expectations as new inflation information became available. In the context of the MQT, it was more this readjustment of inflation expectations than a monetary disequilibrium that accounted for aggregate price changes neutralizing a money shock.

Chapter IV details the MQT (as does Appendix B). It is shown that all the variables in a MQT regression are $I(1)$ except for unanticipated inflation which is $I(0)$. This means that the MQT regressions are unbalanced regressions and require the regressors to be cointegrated of order 0. Evidence is provided in Chapter II for the cointegration of the MQT regressors. Moreover, Chapter IV shows via Monte Carlo simulation that the standard errors for the parameter estimates in an unbalanced regression will have the same critical values as OLS regression standard errors.

From an examination of the parameter estimates for the MQT regressions, it is apparent that each parameter estimate is significant and of the right sign as stipulated by the MQT. This provides
strong evidence in favor of the Moderate Quantity Theory. Also, an examination of the residuals from
the MQT regressions shows them to be \( f(0) \) as well. This is further evidence that the regressors in the
MQT regressions are cointegrated of order 0.

Estimates of the income elasticities and interest rate semi-elasticities for the MQT regressions
using different monetary aggregates are presented. Our elasticity estimates suggest that for less broadly
defined monetary aggregates, the income elasticity is significantly less than one. The income elasticity
tends to increase as the definition of the monetary aggregate becomes broader. This makes intuitive sense
as a one percent increase in income leads to larger percentage increases in the demand for the monetary
aggregate as the monetary aggregate becomes more broadly defined, i.e., more broadly defined monetary
aggregates encompass more of the services that agents might demand. The interest semi-elasticity is
significantly different than zero, but is small (approximately -0.03) in absolute terms.

Unlike conventional money demand models that specify the monetary aggregate as the
explanatory variable, begging the question of endogeneity problems, the MQT allows for the explicit
testing of each monetary aggregates explanatory power in the context of aggregate money demand. We
test the explanatory power of the monetary aggregates by making pairwise comparisons of the monetary
aggregates side-by-side in the MQT regressions. From this comparison, we find that M1plus is the
monetary aggregate that is most appropriate, in terms of explanatory power. This result suggests that
closer examination of M1plus, as a monetary aggregate in money demand models, might be fruitful.

Two last issues that have become major stumbling blocks in the money demand literature are
instability of the parameter estimates and endogeneity across the economic variables. Chapter IV
examines both of these problems in the context of the MQT. The Moderate Quantity Theory favorably
addresses the issue of parameter stability. Chow tests of parameter stability around periods documented
by other researchers to be periods of instability show parameter stability for the MQT parameters. The
notable exception is the “Missing Money Episode” of the mid-1970’s. While this finding is less than
ideal, it must be noted that the MQT does not specify the money demand model to use in the MQT
specification. We use the Cagan form of money demand, but there is no assurance that this is the
appropriate money demand specification. Specifically, that money demand model suffers from the
assumption that transactions costs remain constant over the entire estimation period. Unfortunately, a
money demand model that incorporates transactions costs, while not hard to derive theoretically, is extremely difficult to implement because of the lack of data on transactions costs.\footnote{We tried some previously used proxies of transactions costs like ratchet variables based on previous peak income and/or interest rate levels (c.f. Goldfeld, 1976; Goldfeld and Sichel, 1990) in the MQT regressions. But, these proxies where at best imperfect measures of changes in transactions costs and did not resolve the parameter instability problem over the "Missing Money Episode". So, the results of these attempts are not presented in this work.}

It is remarkable nonetheless that the MQT specification can use a money demand specification that has been noted by other researchers to generate unstable parameter estimates over certain period of U.S. monetary history and find no instability over the same sub period. This is an indication that the out-of-equilibrium adjustment process specified by the MQT has power to explain previous periods of money demand instability.

Conventional money demand specifications also suffer from an endogeneity problem. In a conventional PAM where the monetary aggregate is the dependent variable, it is difficult to tell a story that can convincingly argue that aggregate income or the interest rate is not a function of the monetary aggregate. In fact, Chapter IV found that using causality tests to examine this hypothesis, in a PAM framework, resoundingly rejected the idea that aggregate income temporally precedes the monetary aggregate. However, endogeneity problems were also shown in Chapter IV not to exist for the MQT specification. This is very encouraging news in terms of the parameter estimates from the MQT regressions being identified and well defined in the single equation system.

Chapter V presented evidence for the Moderate Quantity Theory using international data from the G7 countries. Unfortunately, an adequate proxy for aggregate income at a monthly frequency does not exist for the G7 countries, except, obviously for the U.S. We used quarterly data, then, to test MQT at the international level. Favorable, but mixed evidence, was found for the theory. In four out of the seven countries, significant parameter estimates were found on each of the explanatory variables that were the right sign. In the remainder of the cases, the parameter estimates on the explanatory variables were statistically insignificant.
These findings are supportive of the MQT as an explanation of money demand for international countries, but they are less than conclusive. A couple of reasons might explain why this was so. First, as explained in Chapter V, the data used for some of the countries was less than ideal. In some cases, a long term interest rate would be the opportunity cost variable coupled with a narrowly defined monetary aggregate. Second, for at least one country, the data existed for only approximately twenty years. This meant only a little over eighty total data points with which to investigate the MQT. In fact, the data period began after the mid-1970's making the examination of a period of worldwide money demand instability impossible to investigate for some of the countries. Overall, better quality data might mean more conclusive evidence for the Moderate Quantity Theory. At the very least, it would allow for a thoroughly convincing test of the MQT in an international framework.

In conclusion, a plethora of evidence for the Moderate Quantity Theory was presented here. The theory itself merits significant attention because of the microfoundations that underlie the specification of the adjustment process of a monetary disequilibrium. Its biggest advantage is that it the monetary adjustment process is not ad hoc as in competing models. Moreover, this expectations-augmented price adjustment equation is, in large part, supported by the data. Considering its use in monetary policy formulation as a way of estimating stable money demand parameters would seem to be worthy of consideration.
Appendix A

Definitions of the various monetary aggregates:

$M1 = \text{Currency outside the US Treasury, Federal Reserve Banks, and the vaults of depository institutions }$
$\text{ + Travelers checks of non-bank issuers + Demand deposits at all commercial banks other than those due to depository institutions, the US government, and foreign banks and official institutions + other checkable deposits consisting of negotiable orders of withdrawal (NOW) and automatic transfer service (ATS) accounts at depository institutions, credit union share draft accounts and demand deposits at thrift institutions }$
$\text{ - cash items in the process of collection and Federal Reserve float.}$

$M1_{\text{plus}} = M1 + \text{Savings deposits.}$

$M2M (\text{zero-maturity money}) = M1 + \text{Savings deposits (including money market deposit accounts) + Balances in retail money market mutual funds (money funds with minimum initial investments of less than $50,000). Excludes individual retirement accounts (IRA) and Keogh balances at depository institutions and money market funds.}$

$M2_{\text{minus}} = M2 - \text{Small time deposits.}$

$M2 = M2M + \text{Small-denomination time deposits (time deposits including retail RPs in amounts of less than$100,000).}$
M3 = M2 + Large-denomination time deposits (in amounts of $100,000 or more) + Balances in institutional money funds (money funds with minimum initial investments of $50,000 or more) + RP liabilities (overnight and term) issued by all depository institutions + Eurodollars (overnight and term) held by US residents at foreign branches of US banks worldwide and at all banking offices in the United Kingdom and Canada. Excludes amounts held by depository institutions, the US government, money funds, and foreign banks and official institutions.
Appendix B

1: Derivation of MQT

In establishing the adjustment model considered here, a number of simplifying assumptions are required. The first is that there is no growth in this assumed economy. It will be apparent why this assumption is important, but a word or two about it can be said here at the outset. With a no-growth economy, the expected marginal utility will remain constant for each good. With a constant expected marginal utility, the riskless rate of return for all goods will be the same. And this riskless rate of return will be equal to the consumers' rate of time preference. This will facilitate the solution of the overall model. Of course, the no-growth assumption is a strong one, but it may be an innocuous one as well if the monetary adjustment is rapid.

The second assumption is that the supply curve for each good is perfectly inelastic. This assumption, in essence, obtains for us a money neutrality result because we are not allowing output (or interest rates) to accommodate money shocks. However, we will allow the production of any given good to be governed by a stochastic process that will introduce some uncertainty into output. In this sense then consumers will have an incomplete information problem as they will not know the exact level of output for any given good. This will enable some stickiness in prices as consumers are unable to adjust immediately to a monetary shock because of their uncertainty regarding the level of output. As consumers gain information regarding output levels, though, prices adjust to their long-run levels nominalizing a monetary shock.
The third assumption is that relative prices are constant. This assumption that price movements are assumed to be proportional to some fixed base price scalar is the same thing as saying that the utility function is Hicksian separable. This assumption is made to make aggregation over our $n$ goods in this economy possible. While the existence of a base price scalar (price index) is ultimately an empirical question, much of the money demand literature has assumed Hicksian separability without proof of a base price scalar. We use this assumption to make aggregating across goods easier.

The last assumption is the specific form of the average consumer’s utility function. Here we assume a CRRA utility function. The closed form solution of the adjustment equation will have the adjustment parameter being a function of, among other things, the coefficient of relative risk aversion. This assumption, in effect, implies that the level of risk aversion for the average consumer does not vary with the level of consumption. This could be a problematic assumption for large changes in consumption. But, by the previous assumptions, aggregate consumption will not change by much due to no-growth and inelastic supply assumptions. So, small changes in consumption are more likely, making the assumption of CRRA less problematic.

We specify the economy by allowing for $n$ goods, the production of each represented by a time-dependent production function, $g_{it}$. The output from any good’s production function at time $t$ is a per capita flow consistent with a representative agent paradigm. Each of those goods is produced and sold competitively by firms who then pass their earnings on to consumers. Consumers own the firms by means of well-diversified mutual funds that distribute the proceeds via dividend checks that are equal in value to current production.

The utility of any given consumer is represented by a twice differentiable, additively separable, lifetime utility function of the form:

$$U = \sum_{i=1}^{n} \int_{0}^{\infty} u(g_{it}) e^{-\delta(t-t)} d\tau$$

(B1)

where $g_{it}$ is good $i$’s production technology at time $\tau$ and the parameter, $\delta$, is the consumer’s rate of time preference. $u(g)$ specification is a utility function of the constant relative risk aversion type that is commonly written as:
\[ u(g) = \frac{g^{1-\eta} - 1}{1 - \eta} \]  \quad (B2)

where \( \eta \) is the coefficient of relative risk aversion.

The production function, \( g_{i\tau} \), for any good \( i \) is not a function of any arguments by which agents in the economy can control. Therefore, output for any good in this economy is specified by a stochastic process with the following properties:

1) the probability that actual output of good \( i \) at time \( \tau \) is what agents expected it to be at time \( t \) is zero for all \( i \) and \( \tau \). Mathematically, this can be represented by

\[ \Pr(g_{i\tau} = \bar{E}_tg_{i\tau}) = 0 \quad \forall \tau > t, \forall i. \]  \quad (B3)

This means that there will always be at least some unpredictability to \( g_{i\tau} \) that cannot be forecasted. This, of course, translates into some unpredictability for the output of any given good. Consequently, consumption streams and prices will also be subject to uncertainty. Property 2 specifies just how bad forecasts of \( g_{i\tau} \) are:

2) as the difference between the time period the forecasts are made and the time period forecasted becomes smaller and smaller, the probability that the current period's value of output for good \( i \), \( g_{i\tau} \), will be different from the future period's value of output for good \( i \), \( g_{i\tau} \), is zero.

Mathematically,

\[ \Pr(\lim_{\tau \rightarrow t} g_{i\tau} = g_{i\tau}) = 1. \]  \quad (B4)

In other words, output for any good \( i \) evolves continuously; there are no discontinuous jumps in output. This implies that forecasts made \( \tau - t \) periods ahead will become more and more accurate as \( \tau - t \rightarrow 0 \), but will only be completely accurate in the limit as \( \tau \) approaches \( t \) from the right.

In essence, what properties 1 and 2 of \( g_{i\tau} \) imply is that a consumer in this economy will never know exactly what the production of any given good is. Microeconomic events will cause unforecastable deviations in production. But these microeconomic events will never be so large as to cause a discontinuity in output from previous output levels. Nor will there be any underlying structure to these
events that would make them related. But the difference between recent past output and current output will be small. So, consumers do not need to know exact current output of every good in this economy in order to compare the marginal utility of any good and its price with money and its price. The consumers can rely on past levels of consumption and prices to give them fairly accurate estimates to make comparisons. Yet, due to the stochastic process governing production of any given good, consumers cannot be certain about equilibrium price levels. This also admits uncertainty into marginal utility comparisons because consumers' consumption streams will also be stochastic. The uncertainty in these variables on the part of consumers in this economy makes adjustment to monetary shocks less than instantaneous (and, therefore, non-neutral in the short-run).

The real value of aggregate output for any time, \( t \), can be calculated as

\[
y_t = \frac{1}{P_t} \sum_{i=1}^{n} p_{it} g_{it}
\]

(B5)

where \( p_{it} \) is the price of good \( i \) at time \( t \), \( g_{it} \) is the quantity produced of good \( i \) at time \( t \) and \( P_t \) is a price index. And the value of lifetime real income, discounted back to the present, is

\[
w^T_t = \frac{y_t}{r} \quad .33
\]

(B6)

Allow \( m^d \) to represent the per capita level of real cash balances that the average consumer desires to hold given the economic variables that determine this desired level of cash balances. And \( M_t \) is the level of the nominal, per capita money stock.

Given these definitions, total wealth for the consumer is the sum of his/her lifetime, discounted real income, in per capita terms, and the level of real money balances, in per capita terms, that he/she wants to hold. But consumers have the desire to hold the amount of \( m^d \) in real cash balances. The motivation to hold \( m^d \) could stem from a transactions, precautionary, and/or speculative motive in the sense that Keynes (1936) first specified. The motivation to hold \( m^d \) here is not important. It is that \( m^d \) is held that must be accounted for in total wealth. As above, total wealth is the sum of lifetime,
discounted, real income and money holdings. This is to be differentiated from total consumable wealth that is total wealth net money balances desired. In other words, total consumable wealth is

$$w_i^C = w_i^F + \frac{M_i}{P_t} - m_i^d.$$  \hspace{1cm} (B7)

This is the budget constraint that finishes the consumer choice problem posed in (B1). Specifically, (B7) can be further written out as

$$w_i^C = \sum_{i=1}^{n} p_n g_{it} + \frac{M_i}{P_t} - m_i^d$$ \hspace{1cm} (B8)

which completely specifies the consumers’ maximization problem.

At any point in time, assuming the proper regularity conditions for the utility function of the average consumer, consumers will want to consume exactly what is produced if the economy is in a state of monetary equilibrium. However, if the economy is not in a state of monetary equilibrium, then consumers will either want to consume more than what is produced (the aggregate real money stock, $\frac{M_t}{P_t}$, is greater than the aggregate money demanded, $m_t^d$) or consumers will want to consume less of what is produced ($\frac{M_t}{P_t}$ is less than $m_t^d$). This, of course, assumes that aggregate prices are not fully flexible so that they cannot adjust instantaneously to equate output supplied and output demanded. When the economy is in a state of monetary disequilibrium, the average consumer will either want to consume more or less of the current output depending on whether there exists an excess money supply or an excess money demand.\(^3^{34}\) Due to the homotheticity of CRRA utility function and due to constant relative prices,\(^3^{35}\) the average consumer will want to consume a fixed proportion more (or less) of each good

\(^3^{33}\) Under the no-growth assumption, the rate of time preference, $\delta$, is equal to the riskless rate of return, $r$.

\(^3^{34}\) This is Mises’ idea that consumers experience the inequality between the price ratio of money and a given good with the marginal utility of money and the marginal utility of the given good. This causes consumers to bid up the appropriate prices to return the two ratios to equality depending on whether there is an excess supply or an excess demand for money.

\(^3^{35}\) Again the assumption of relative prices remaining constant comes from our assumption that individual goods prices can be aggregated together, or that Hicksian separability applies.
produced. That fixed proportion is the amount of consumable wealth that the average consumer has over the amount of total wealth they have. In other words, the proportion is \( \frac{w_t^C}{w_t^T} \).

The optimal level of consumption of good \( i \) for consumers is

\[
\dot{g}_i^* = g_{it} \cdot \frac{w_t^C}{w_t^T},
\]  

(B9)

where \( g_i^* (t) \) is the optimal level of consumption of good \( i \). Of course, only \( g_{it} \) of good \( i \) was produced at time \( t \). So, for \( \frac{w_t^C}{w_t^T} \neq 1 \), prices will ultimately have to adjust for the market for good \( i \) to clear. (\( \frac{w_t^C}{w_t^T} > 1 \) implies an excess supply of money currently exists, while \( \frac{w_t^C}{w_t^T} < 1 \) implies an excess demand for money exists).

Given our assumptions of a no-growth economy and an inelastic supply for each good, the expected lifetime utility from consumable wealth, \( w_t^C \), is

\[
U^* = \frac{1}{r} \sum_{i=1}^{n} u_i (g_{it}^*).
\]  

(B10)

In other words, the sum of each of the additively separable utility functions, discounted at the riskless rate of return, gives expected lifetime utility for the average consumer. (B10) is an indirect utility function that represents the maximum level of lifetime utility given prices and income.

To derive the marginal utility of money, we take the derivative of (13) with respect to \( M \)

\[
\frac{dU^*}{dM} = \frac{1}{r} \sum_{i=1}^{n} \frac{du_i (g_{it}^*)}{dg_{it}} \cdot \frac{dg_{it}^*}{dM},
\]  

(B11)

Using (B11), along with (B9) and (B7) gives

\[
\frac{dU^*}{dM} = \frac{1}{r} \sum_{i=1}^{n} \frac{du_i (g_{it}^*)}{dg_{it}} \cdot g_{it} \cdot \frac{1}{w_t^T} \cdot \frac{1}{P_t},
\]  

(B12)

Simplifying (B12) with the help of (B6) has
\[
\frac{dU^*}{dM} = \frac{1}{P_{Y_i}} \sum_{i} g_{iu} \cdot \frac{du_i(g_{iu}^*)}{dg_i}.
\]

Taking a first order Taylor Series expansion of \(\frac{du_i}{dg_i}\) around the point \(g_{iu}\) gives

\[
\frac{du_i(g_{iu}^*)}{dg_i} = \frac{du_i(g_{iu})}{dg_i} + \frac{d^2 u_i(g_{iu})}{dg_i^2} (g_{iu}^* - g_{iu}) + R(2)
\]

where \(R(2)\) is the second order remainder term of this expansion.

To simplify (B14), first recall (B9) where \(\frac{g_{iu}^*}{g_{iu}} = \frac{w_i^c}{w_i^t}\). Subtracting one from both sides gives

\[
\frac{g_{iu}^* - g_{iu}}{g_{iu}} = \frac{w_i^c - w_i^t}{w_i^t}.
\]

Then using the definition of \(w^*\) from (B7) and substituting it in (B15) leaves

\[
\frac{w_i^c - w_i^t}{w_i^t} = \frac{M_i / P_i - m_i^d}{w_i^t}.
\]

This makes (B14) appear (ignoring the remainder term) as

\[
\frac{du_i}{dg_i} = \frac{du_i(g_{iu})}{dg_i} + \frac{d^2 u_i(g_{iu})}{dg_i^2} \left[ \frac{M_i / P_i - m_i^d}{w_i^t} \right] \cdot g_{iu}.
\]

Further simplification requires the use of our assumption that the average consumer's utility function is well represented by the CRRA type. If this is true then the relative risk aversion coefficient is equal to

\[
\eta = \frac{\frac{d^2 u_i(g_{iu}^*)}{dg_i^2} \cdot (g_{iu}^*)}{\frac{du_i(g_{iu})}{dg_i}}.
\]

Equation (B17) can then be rewritten, using (B18) as
\[
\frac{du_i}{dg_i} \equiv \frac{du_i(g_{ii})}{dg_i} \left[ 1 - \eta \left( \frac{M_t}{P_t} - \frac{m_i^d}{w_i} \right) \right].
\]  

(B19)

Factoring out \( \frac{M_t}{P_t} \) from the final parenthesized terms in (B19) and noticing that \( \ln(x) \equiv (1 - x) \) for small \( x \) gives\(^{36}\)

\[
\frac{du_i(g_{ii})}{dg_i} \equiv \frac{du_i(g_{ii})}{dg_i} \left[ 1 - \frac{\eta m_t}{w_i} \left( \ln m_t - \ln m_i^d \right) \right].
\]  

(B20)

Combining the simplification of \( \frac{du_i(g_{ii})}{dg_i} \) with (B13) gives us a simplified version of the marginal utility of money

\[
\frac{dU^*}{dM} = \frac{1}{P_t} \sum_{i=1}^{n} g_i(t) \cdot \frac{du_i(g_{ii})}{dg_i} \left[ 1 - \frac{\eta m_t}{w_i} \left( \ln m_t - \ln m_i^d \right) \right].
\]  

(B21)

Having calculated the expected marginal utility of money given the average consumer’s optimal level of lifetime consumption of any given good, we can focus on the information mechanism that consumers have to ascertain price levels. Recall that prices (and output) are stochastic in this framework. Consumers’ expectations of output and, consequently prices and consumption, are reasonable approximations to what the actual levels turn out to be. But, the approximations are never exactly accurate, given the uncertainty in output that is not forecastable. Consumers incorporate what past information they do have on price levels. And they use this information to make decisions on consumption expenditures this period. The problem again is that consumers do not know today’s prices. In this case, they have an incomplete information problem with the information on today’s prices coming to them only at a lag.

\(^{36}\) To make the approximation that \( \ln(x) \equiv (1 - x) \) requires that the excess supply or demand for money not be too large. This makes the use of this model to describe hyperinflationary economies somewhat inappropriate.
We can specify this lag in consumers' price information as $\lambda$. Not only do consumers have past information on price levels; but, of course, given this information on past price levels, consumers also have information on past price changes. Consumers can use the past levels of prices as well as their changes to form expectations of what price levels are in the current period in which they must conduct transactions.

Under the assumption that relative prices are constant, all goods prices will move by the same rate. So what consumers believe the current level of prices to be can be represented by a price index that grows at the non-constant rate, $\pi^a$, where $\pi^a$ is the average consumer's expectation of the inflation rate. In other words, if $P_t$ is the current (as yet unknown) level of prices then consumers expect in period $t - \lambda$ for prices in period $t$ to be $P_{t-\lambda}e^{\lambda \pi_{t-\lambda}^a}$. So,

$$E_{t-\lambda}(P_t) = P_{t-\lambda}e^{\lambda \pi_{t-\lambda}^a} = P_{t-\lambda}(1 + \lambda \pi_{t-\lambda}^a).$$  

(B22)

It is this expectation of what current prices are that consumers use to evaluate the marginal utility of money.

The marginal utility of money in period $t$ (B21), using consumers' expectations of what the price level is in period $t$ (B22), is

$$\frac{dU^*}{dM} = \frac{1 - \eta m_{t-\lambda}^d (\ln m_{t-\lambda} - \ln m_{t-\lambda}^d)}{P_{t-\lambda}y_{t-\lambda} (1 + \lambda \pi_{t-\lambda}^a)} \sum_{i=1}^{n} g_{it-\lambda} \frac{du_i(g_{it-\lambda})}{dg_{ii}}.$$  

(B23)

Given this expression for the marginal utility of money, we can now use the general conditions for the consumer optimization problem (equation (3)) to derive a time path for the price level. If we normalize the price of money to be one, then the first order conditions become

$$P_t(t) = \frac{du_i(g_{it})}{dU^*/dM}.$$  

(B24)

---

37 This assumes that the expected inflation rates prevalent in the economy are sufficiently small so that $(1 + \lambda \pi_{t-\lambda}^a)$ is a good approximation to $e^{\lambda \pi_{t-\lambda}^a}$. 

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This is the optimization rule that describes the consumers' choices in this economy. However, \( \frac{dU_i(g_{it}^*)}{dg_{it}} \) is unknown to consumers at time \( t \) because \( g_{it}^* \) is unknown at that time. Consumers can only approximate \( \frac{dU_i(g_{it}^*)}{dg_{it}} \) at time \( t \). That approximation can be represented by

\[
\frac{dU_i(g_{it}^*)}{dg_{it}} = \frac{dU_i(g_{it-\lambda}^*)}{dg_{it-\lambda}} + \varepsilon_i
\]

(B25)

where \( \varepsilon_i \) is a martingale process. In other words, the forecasting error committed by consumers when they try to estimate the marginal utility of a given good is a random, mean zero process.

To fully specify equation (B24) and make it amenable to empirical testing, a method of aggregating individual goods prices must be used to replace \( P_{it} \). We can represent the price index, \( P_t \), as an index where individual goods current period prices are weighted by the contribution that each makes to total real output. That is, the price index is a Laspeyres index

\[
P_t = \frac{1}{Y_{t-\lambda}} \sum_{i=1}^{n} g_{it-\lambda} P_{it}.
\]

(B26)

Using the expressions for \( \frac{dU^*}{dM} \) (B23), \( P_{it} \) (B24), \( \frac{dU_i(g_{it}^*)}{dg_{it}} \) (B25), and \( P_t \) (B26), we can combine them to obtain

\[
\frac{P_t}{P_{t-\lambda}} = \left[ 1 + \frac{\sum_{i=1}^{n} g_{it-\lambda} \varepsilon_i}{\sum_{i=1}^{n} g_{it-\lambda} \frac{du_i(g_{it-\lambda}^*)}{dg_{it-\lambda}}} \right] \cdot \left[ 1 + \frac{\sum_{i=1}^{n} g_{it-\lambda} \varepsilon_i}{\sum_{i=1}^{n} g_{it-\lambda} \frac{du_i(g_{it-\lambda}^*)}{dg_{it-\lambda}}} \right].
\]

(B27)

Rewriting (B27) by subtracting one from both sides, we find an expression for the rate of change of prices in this economy from time period \( t - \lambda \) to time period \( t \)
\[
\begin{align*}
\frac{P_t - P_{t-\lambda}}{P_{t-\lambda}} &= \frac{\eta m_{t-\lambda}^* (\ln m_{t-\lambda} - \ln m_{t-\lambda}^e) + \lambda \pi_t^a}{1 - \frac{\eta m_{t-\lambda}^*}{w_{t-\lambda}^*} (\ln m_{t-\lambda} - \ln m_{t-\lambda}^e)} + \kappa^* \\
\text{where } \kappa^* &= \left[\frac{1 + \lambda \pi_t^a}{1 - \frac{\eta m_{t-\lambda}^*}{w_{t-\lambda}^*} (\ln m_{t-\lambda} - \ln m_{t-\lambda}^e)} \right] \left[ \frac{\sum_{i=1}^{n} g_{it-\lambda} \epsilon_i}{\sum_{i=1}^{n} g_{it-\lambda} \frac{du_i(g_{it-\lambda})}{dg_{it-\lambda}}} \right].
\end{align*}
\]

If the difference between the quantity supplied of money and the quantity demanded of money is small in percentage terms then the denominator of equation (B28) will be close to one. This gives the final form of MQT:

\[
\lambda \pi_t = \gamma (\ln m_{t-\lambda} - \ln m_{t-\lambda}^e) + \lambda \pi_t^a + c \frac{dz}{dt} \quad (B29)
\]

\footnote{Once again, this model will not hold well in a hyperinflationary economy due to the assumptions made that the excess supply of money does not deviate too much from the excess demand for money at any point in time.}
Appendix C

International Data:

The international data used in Chapter 5 was taken from the OECD Main Economic Indicators Historical Statistics 1960-1997 CD-ROM. The following lists the macroeconomic variables used for each country:

Canada:

**Monetary aggregate:** M1, data source Bank of Canada, Canadian $ (billion), SA.

**Scale variable:** GDP from National Accounts, data source Statistics Canada in Canadian $ (billion), SA.

**Price index:** data source Statistics Canada, 1990=100, SA.

France:

**Monetary aggregate:** M3, data source Bank of France, FF (billion), SA.

**Scale variable:** GDP from National Accounts, data source Statistical Office of France (INSEE) FF (billion), SA.

**Price index:** data source Statistical Office of France (INSEE), 1990=100, SA.
Germany:

**Monetary aggregate:** M2, data source Federal Bank of Germany, DM (billion), SA.

**Scale variable:** GDP from National Accounts, data source Federal Bank of Germany DM (billion), SA for a unified Germany 1995-8. Data from 1970 through 1994 is from the DRI-McGraw Hill International Data CD-ROM for West Germany. The two data series were spliced together to obtain a quarterly GDP data series from 1970 through 1998.

**Price index:** data source Federal Statistical Office of Germany, 1991=100, SA.

Italy:

**Monetary aggregate:** M1, data source Bank of Italy, Lira (trillion), SA.

**Scale variable:** GDP from National Accounts, data source National Institute of Statistics (ISTAT) Lira (trillion), SA.

**Price index:** data source National Institute of Statistics (ISTAT), 1990=100, SA.

Japan:

**Monetary aggregate:** M2, data source Bank of Japan, Yen (trillion), SA.

**Scale variable:** GDP from National Accounts, data source Economic Planning Agency (EPA) Yen (trillion), SA.

**Price index:** data source Statistics Bureau, 1990=100, SA.

United Kingdom:

**Monetary aggregate:** M4, data source Bank of England, Pd (billion), SA.

**Scale variable:** GDP from National Accounts, data source Office for National Statistics Pd (billion), SA.
Price index: data source Office for National Statistics, 1990=100, SA.

United States:

Monetary aggregate: M2, St. Louis Federal Reserve Bank FRED database, $ (billion), SA.

Scale variable: GDP from National Accounts, data source U.S. Bureau of Economic Analysis (BEA) $ (billion), SA.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>constant</th>
<th>$\gamma$</th>
<th>$R$</th>
<th>$m_{t-1}$</th>
<th>$P_t / P_{t-1}$</th>
<th>$\rho$</th>
<th>$\bar{R}^2$</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>-0.0023</td>
<td>0.0141</td>
<td>-0.0009</td>
<td>0.9700</td>
<td>-0.0656</td>
<td>-</td>
<td>99.92%</td>
<td>1.01</td>
</tr>
<tr>
<td>(AR1 correction)</td>
<td>-0.0041</td>
<td>0.0204</td>
<td>-0.0013</td>
<td>0.9564</td>
<td>-0.0768</td>
<td>0.5332</td>
<td>99.94%</td>
<td>2.13</td>
</tr>
<tr>
<td>M2 (AR1 correction)</td>
<td>0.3099</td>
<td>0.0371</td>
<td>-0.0013</td>
<td>0.8250</td>
<td>-0.0750</td>
<td>0.9941</td>
<td>99.99%</td>
<td>2.16</td>
</tr>
</tbody>
</table>

*t-statistics are in parentheses. The Beach and MacKinnon (1978) maximum-likelihood procedure was used to estimate $\rho$. All variables are in natural logs except the short term interest rate.

**Table 2.1: Conventional Money Demand Model**
<table>
<thead>
<tr>
<th>Researcher(s)</th>
<th>M1 as monetary aggregate</th>
<th>M2 as monetary aggregate</th>
<th>Data frequency</th>
<th>Data period</th>
<th>Cointegration test(s) used</th>
</tr>
</thead>
<tbody>
<tr>
<td>King, Plosser, Stock, and Watson (1991)</td>
<td>-</td>
<td>CI relationship</td>
<td>SA quarterly</td>
<td>1953-88</td>
<td>Johansen</td>
</tr>
<tr>
<td>Hafer and Jansen (1991)</td>
<td>CI relationship</td>
<td>No CI relationship</td>
<td>SA quarterly</td>
<td>1915-88</td>
<td>Johansen</td>
</tr>
<tr>
<td>Hoffman and Rasche (1991)</td>
<td>CI relationship</td>
<td>-</td>
<td>SA monthly</td>
<td>1953-88</td>
<td>Johansen</td>
</tr>
<tr>
<td>Miller (1991)</td>
<td>No CI relationship</td>
<td>CI relationship</td>
<td>SA quarterly</td>
<td>1959-87</td>
<td>Augmented Dickey-Fuller</td>
</tr>
<tr>
<td>MacDonald and Taylor (1992)</td>
<td>CI relationship</td>
<td>-</td>
<td>SA annually</td>
<td>1871-1975</td>
<td>Johansen</td>
</tr>
<tr>
<td>Lee (1992)</td>
<td>Mixed results</td>
<td>-</td>
<td>SA quarterly</td>
<td>1953-91</td>
<td>ADF, JOH, SW, PO</td>
</tr>
<tr>
<td>Miyao (1996)</td>
<td>-</td>
<td>No CI relationship</td>
<td>SA quarterly</td>
<td>1959-93</td>
<td>ADF, JOH, SW</td>
</tr>
<tr>
<td>Ball (1998)</td>
<td>CI relationship</td>
<td>-</td>
<td>SA annually</td>
<td>1900-96</td>
<td>ADF, JOH</td>
</tr>
</tbody>
</table>


Table 2.2: Previous evidence on money demand cointegration*
<table>
<thead>
<tr>
<th></th>
<th>ADF (AIC)</th>
<th>ADF (BIC)</th>
<th>ADF (Ljung-Box)</th>
<th>ADF (Lagrange Multiplier)</th>
<th>ADF (Campbell-Perron (1991) reduction)</th>
<th>PP</th>
<th>3% critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>-1.59 (9)</td>
<td>-0.92 (3)</td>
<td>-1.44 (8)</td>
<td>-1.44 (8)</td>
<td>-1.44 (8)</td>
<td>-0.67 (4)</td>
<td>-2.87</td>
</tr>
<tr>
<td>M2</td>
<td>-2.14 (8)</td>
<td>-2.67 (1)</td>
<td>-2.14 (8)</td>
<td>-2.49 (2)</td>
<td>-2.14 (8)</td>
<td>-3.40 (4)</td>
<td>-2.87</td>
</tr>
<tr>
<td>Personal Income</td>
<td>-2.34 (16)</td>
<td>-3.08 (1)</td>
<td>-2.00 (13)</td>
<td>-1.83 (12)</td>
<td>-2.34 (16)</td>
<td>-3.03 (4)</td>
<td>-2.87</td>
</tr>
<tr>
<td>3 month t-bill rate</td>
<td>-2.43 (20)</td>
<td>-1.91 (6)</td>
<td>-2.65 (16)</td>
<td>-2.43 (14)</td>
<td>-2.43 (20)</td>
<td>-2.48 (4)</td>
<td>-2.87</td>
</tr>
<tr>
<td>Inflation</td>
<td>-2.40 (11)</td>
<td>-2.99 (5)</td>
<td>-2.40 (11)</td>
<td>-2.25 (8)</td>
<td>-2.40 (8)</td>
<td>-9.47 (4)</td>
<td>-2.87</td>
</tr>
</tbody>
</table>

*T-test scores are provided in the table along with the lag length (in parentheses) used in the Augmented Dickey-Fuller t-tests. For the Phillips and Perron unit root test, a lag length of 4 in the spectral estimation window was used.

**Table 2.3: Unit root tests of the money demand variables**
<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>constant</th>
<th>$y$</th>
<th>$R$</th>
<th>$P_t / P_{t-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 (equation 2.7)</td>
<td>-0.1428 (-9.27)</td>
<td>0.4417 (60.82)</td>
<td>-0.0265 (-21.60)</td>
<td>0.0316 (0.32)</td>
</tr>
<tr>
<td>M1 (equation 2.8)</td>
<td>-0.1426 (-9.27)</td>
<td>0.4416 (60.96)</td>
<td>-0.0263 (-26.11)</td>
<td>-</td>
</tr>
<tr>
<td>M2 (equation 2.7)</td>
<td>0.0821 (5.86)</td>
<td>0.7992 (121.04)</td>
<td>-0.0002 (-0.14)</td>
<td>0.3621 (4.06)</td>
</tr>
<tr>
<td>M2 (equation 2.8)</td>
<td>0.0852 (5.99)</td>
<td>0.7977 (119.11)</td>
<td>0.0024 (2.62)</td>
<td>-</td>
</tr>
</tbody>
</table>

*t-statistics are in parentheses. All variables are in natural logs except the short-term interest rate.

Table 2.4: Money demand cointegrating equations*
<table>
<thead>
<tr>
<th></th>
<th>ADF (AIC)</th>
<th>ADF (BIC)</th>
<th>ADF (Ljung-Box)</th>
<th>ADF (Lagrange Multiplier)</th>
<th>ADF (Campbell-Perron reduction)</th>
<th>PP</th>
<th>5% critical value**</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 residuals</td>
<td>-3.29</td>
<td>-2.93</td>
<td>-3.70</td>
<td>-3.70</td>
<td>-4.15</td>
<td>-3.00</td>
<td>-4.22</td>
</tr>
<tr>
<td>(equation 2.7)</td>
<td>(20)</td>
<td>(2)</td>
<td>(16)</td>
<td>(16)</td>
<td>(18)</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>M1 residuals</td>
<td>-3.31</td>
<td>-2.91</td>
<td>-3.72</td>
<td>-3.15</td>
<td>-4.15</td>
<td>-3.00</td>
<td>-3.93</td>
</tr>
<tr>
<td>(equation 2.8)</td>
<td>(20)</td>
<td>(2)</td>
<td>(16)</td>
<td>(15)</td>
<td>(18)</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>(equation 2.7)</td>
<td>(11)</td>
<td>(1)</td>
<td>(10)</td>
<td>(6)</td>
<td>(10)</td>
<td>(4)</td>
<td></td>
</tr>
<tr>
<td>M2 residuals</td>
<td>-2.99</td>
<td>-2.22</td>
<td>-2.40</td>
<td>-1.86</td>
<td>-3.25</td>
<td>-1.95</td>
<td>-3.93</td>
</tr>
<tr>
<td>(equation 2.8)</td>
<td>(13)</td>
<td>(2)</td>
<td>(4)</td>
<td>(1)</td>
<td>(12)</td>
<td>(4)</td>
<td></td>
</tr>
</tbody>
</table>

*T-test scores are provided in the table along with the lag length (in parentheses) used in the Augmented Dickey-Fuller t-tests. For the Phillips and Perron unit root test, a spectral estimation window of lag length 4 was used. **The critical values for the tests are taken from Engle and Yoo (1987).

Table 2.5: Unit root tests of the residuals from the cointegrating regressions*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m1, y, $\pi$, and R</th>
<th>$\lambda_{trace}$ for m2, y, $\pi$, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>1</td>
<td>310.31</td>
<td>406.17</td>
<td>(4, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>127.39</td>
<td>119.49</td>
<td>(4, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>110.27</td>
<td>96.75</td>
<td>(4, 4)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>80.75</td>
<td>70.23</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>77.91</td>
<td>66.78</td>
<td>(3, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>57.98</td>
<td>57.48</td>
<td>(3, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>63.51</td>
<td>60.50</td>
<td>(0, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>59.27</td>
<td>46.99</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 95% and 99% critical values are 47.21 and 54.46, respectively. The last column in the table shows the number of error processes (out of 4) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes with m1 as the monetary aggregate. The second number in the pair signifies the number of autocorrelated error processes with m2 as the monetary aggregate.

Table 2.6: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and inflation*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m1, y, and R</th>
<th>$\lambda_{trace}$ for m2, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r &gt; 0</td>
<td>1</td>
<td>104.37</td>
<td>85.20</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>44.43</td>
<td>30.54</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>36.80</td>
<td>26.88</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>33.25</td>
<td>28.26</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>35.56</td>
<td>27.21</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>28.43</td>
<td>28.82</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>30.45</td>
<td>26.85</td>
<td>(1, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>30.52</td>
<td>24.63</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values are 26.79, 29.68, and 35.65, respectively. The last column shows the number of error processes (out of 3) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes with m1 as the monetary aggregate. The second number in the pair signifies the number of autocorrelated error processes with m2 as the monetary aggregate.

Table 2.7: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, and an opportunity cost variable *
<table>
<thead>
<tr>
<th>Model Description</th>
<th>Forecast Root Mean Square Error (FRMSE)</th>
<th>Theil's (1961) U statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hallman et al., (1991) Price-gap model (M2 velocity constant)</td>
<td>0.5193</td>
<td>0.4757</td>
</tr>
<tr>
<td>Hallman et al., (1991) Price-gap model (M2 velocity varying)</td>
<td>0.4966</td>
<td>0.4550</td>
</tr>
<tr>
<td>Christiano (1989) interest rate model</td>
<td>0.5216</td>
<td>0.4778</td>
</tr>
<tr>
<td>ARIMA(0,1,1)</td>
<td>0.5019</td>
<td>0.4598</td>
</tr>
<tr>
<td>Fama-Gibbons (1984) Real Interest Rate Model</td>
<td>0.4949</td>
<td>0.4534</td>
</tr>
<tr>
<td>Fuller-ARMA expanding window model</td>
<td>0.4411</td>
<td>0.4041</td>
</tr>
</tbody>
</table>

Table 3.1: Measuring the Forecasting Accuracy of the Fuller Expanding Window Method vs. Various Other Models (01/70-04/99)
<table>
<thead>
<tr>
<th>Model</th>
<th>Forecast Root Mean Square Error (FRMSE)</th>
<th>Theil's (1961) U statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hallman et al., (1991) Price-gap model (M2 velocity constant)</td>
<td>0.3855</td>
<td>0.4676</td>
</tr>
<tr>
<td>Hallman et al., (1991) Price-gap model (M2 velocity varying)</td>
<td>0.3764</td>
<td>0.4565</td>
</tr>
<tr>
<td>Christiano (1989) interest rate model</td>
<td>0.3883</td>
<td>0.4710</td>
</tr>
<tr>
<td>ARIMA(0,1,1)</td>
<td>0.3746</td>
<td>0.4544</td>
</tr>
<tr>
<td>Fama-Gibbons (1984) Real Interest Rate Model</td>
<td>0.4090</td>
<td>0.4960</td>
</tr>
<tr>
<td>U. of Mich Consumers' Inflation Expectations</td>
<td>0.4765</td>
<td>0.5302</td>
</tr>
<tr>
<td>Fuller-ARMA expanding window model</td>
<td>0.3226</td>
<td>0.3913</td>
</tr>
</tbody>
</table>

Table 3.2: Measuring the Forecasting Accuracy of the Fuller Expanding Window Method vs. Various Other Models (01/79-04/99)
<table>
<thead>
<tr>
<th></th>
<th>t-statistic (monthly series)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln(M1,)$</td>
<td>-1.47</td>
</tr>
<tr>
<td>$\ln(M1\ plus,)$</td>
<td>-2.01</td>
</tr>
<tr>
<td>$\ln(MzM,)$</td>
<td>-0.38</td>
</tr>
<tr>
<td>$\ln(M2\ min\ us,)$</td>
<td>-0.67</td>
</tr>
<tr>
<td>$\ln(M2,)$</td>
<td>-2.23</td>
</tr>
<tr>
<td>$\ln(M3,)$</td>
<td>-1.99</td>
</tr>
<tr>
<td>$\ln(Y,)$</td>
<td>-2.23</td>
</tr>
<tr>
<td>$R_t$</td>
<td>-2.46</td>
</tr>
<tr>
<td>$UAI_t \equiv \pi_t - \pi_{t-1}^\circ$</td>
<td>-5.76**</td>
</tr>
</tbody>
</table>

The form of the augmented Dickey-Fuller (1981) test is

$$\Delta y_t = \mu + (\gamma - 1)y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t$$

where the test consists of testing the null $H_0$: $\gamma = 0$ (unit root). And the lag length, $p$, for the differenced lags of the series in question is determined by the method suggested by Campbell and Perron (1991). For each monthly series, there are in excess of 300 observations with all series but one having far more. The augmented Dickey-Fuller critical value for the 95% confidence level of these series is -2.87.

**Table 4.1: Augmented Dickey-Fuller Test for Unit Roots**
### Panel A: Regressing a Random Walk on Two Other Random Walks

<table>
<thead>
<tr>
<th>Level of Confidence that a given statistic is significant</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW1(t-statistic)</td>
<td>89.65%</td>
<td>87.89%</td>
<td>84.86%</td>
</tr>
<tr>
<td>RW2(t-statistic)</td>
<td>89.27%</td>
<td>87.38%</td>
<td>83.62%</td>
</tr>
<tr>
<td>F-statistic</td>
<td>98.86%</td>
<td>98.61%</td>
<td>97.94%</td>
</tr>
</tbody>
</table>

### Panel B: Regressing White Noise on Two Random Walks

<table>
<thead>
<tr>
<th>Level of Confidence that a given statistic is significant</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>RW1(t-statistic)</td>
<td>9.70%</td>
<td>4.74%</td>
<td>0.91%</td>
</tr>
<tr>
<td>RW2(t-statistic)</td>
<td>10.24%</td>
<td>5.06%</td>
<td>1.02%</td>
</tr>
<tr>
<td>F-statistic</td>
<td>9.48%</td>
<td>4.65%</td>
<td>0.99%</td>
</tr>
</tbody>
</table>

*The panels above depict the percentage of time that a given statistic was significant at a certain level of confidence. The number of simulations was 10,000 and the sample size for each was 500. RW represents a random walk series as a regressor.

**Table 4.2: Monte Carlo Simulations***
<table>
<thead>
<tr>
<th></th>
<th>$\alpha_1\gamma$</th>
<th>$\gamma$</th>
<th>$\alpha_2\gamma$</th>
<th>$\alpha_3\gamma$</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.0248** (0.0066)</td>
<td>0.0579** (0.0186)</td>
<td>-0.0316** (0.0085)</td>
<td>0.0015* (0.0006)</td>
<td>0.026</td>
</tr>
<tr>
<td>M1plus</td>
<td>-0.0278* (0.0122)</td>
<td>0.0574** (0.0133)</td>
<td>-0.0302** (0.0061)</td>
<td>0.0021** (0.0006)</td>
<td>0.044</td>
</tr>
<tr>
<td>MzM</td>
<td>0.0468* (0.0231)</td>
<td>0.0838** (0.0266)</td>
<td>-0.0842** (0.0265)</td>
<td>0.0027** (0.0010)</td>
<td>0.023</td>
</tr>
<tr>
<td>M2minus</td>
<td>-0.0063 (0.0097)</td>
<td>0.0525** (0.0162)</td>
<td>-0.0381** (0.0101)</td>
<td>0.0020** (0.0008)</td>
<td>0.028</td>
</tr>
<tr>
<td>M2</td>
<td>0.0117† (0.0067)</td>
<td>0.0510* (0.0203)</td>
<td>-0.0465** (0.0161)</td>
<td>-0.0002 (0.0004)</td>
<td>0.019</td>
</tr>
<tr>
<td>M3</td>
<td>0.0220** (0.0078)</td>
<td>0.0170 (0.0185)</td>
<td>-0.0237 (0.0186)</td>
<td>-0.0002 (0.0005)</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.3: Monthly UAI Regressions
<table>
<thead>
<tr>
<th>$\alpha_{1,y}$ (constant)</th>
<th>$\gamma$ (M1)</th>
<th>$\alpha_{2,y}$ (Personal Income)</th>
<th>$\alpha_{3,y}$ (3 mo T-bill)</th>
<th>$\alpha_{3,y}$ (6 mo T-bill)</th>
<th>$\alpha_{4,y}$ (12 mo T-bill)</th>
<th>$\alpha_{5,y}$ (AAA bond)</th>
<th>$\alpha_{5,y}$ (BAA bond)</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0248** (0.0066)</td>
<td>0.0579** (0.0186)</td>
<td>-0.0316** (0.0085)</td>
<td>0.0015* (0.0006)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0241** (0.0066)</td>
<td>0.0581** (0.0188)</td>
<td>-0.0316** (0.0086)</td>
<td>-</td>
<td>0.0015* (0.0007)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0228** (0.0068)</td>
<td>0.0557** (0.0192)</td>
<td>-0.0303** (0.0088)</td>
<td>-</td>
<td>-</td>
<td>0.0015* (0.0007)</td>
<td>-</td>
<td>0.022</td>
<td></td>
</tr>
<tr>
<td>0.0230** (0.0071)</td>
<td>0.0349† (0.0197)</td>
<td>-0.0217* (0.0103)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0005 (0.0008)</td>
<td>-</td>
<td>0.016</td>
</tr>
<tr>
<td>0.0215** (0.0070)</td>
<td>0.0275 (0.0201)</td>
<td>-0.0174† (0.0102)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0001 (0.0007)</td>
<td>0.015</td>
</tr>
<tr>
<td>0.0220** (0.0071)</td>
<td>0.0484* (0.0203)</td>
<td>-0.0249* (0.0103)</td>
<td>0.0021* (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0012 (0.0010)</td>
<td>-</td>
<td>0.027</td>
</tr>
<tr>
<td>0.0217** (0.0069)</td>
<td>0.0437* (0.0208)</td>
<td>-0.0226* (0.0103)</td>
<td>0.0022** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0013 (0.0009)</td>
<td>0.029</td>
</tr>
<tr>
<td>0.0206** (0.0071)</td>
<td>0.0482* (0.0203)</td>
<td>-0.0242* (0.0103)</td>
<td>-</td>
<td>0.0023* (0.0009)</td>
<td>-</td>
<td>-0.0014 (0.0011)</td>
<td>-</td>
<td>0.027</td>
</tr>
<tr>
<td>0.0204** (0.0069)</td>
<td>0.0435* (0.0207)</td>
<td>-0.0219* (0.0103)</td>
<td>-</td>
<td>0.0024** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0015† (0.0009)</td>
<td>0.029</td>
</tr>
<tr>
<td>0.0186* (0.0074)</td>
<td>0.0461* (0.0203)</td>
<td>-0.0227* (0.0104)</td>
<td>-</td>
<td>-</td>
<td>0.0026* (0.0011)</td>
<td>-0.0016 (0.0012)</td>
<td>-</td>
<td>0.024</td>
</tr>
<tr>
<td>0.0184* (0.0072)</td>
<td>0.0412* (0.0208)</td>
<td>-0.0205* (0.0104)</td>
<td>-</td>
<td>-</td>
<td>0.0027** (0.0010)</td>
<td>-</td>
<td>-0.0017† (0.0009)</td>
<td>0.026</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.4: Monthly UAI Regressions with Different Opportunity Cost Variables (M1 Monetary Aggregate)*

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<table>
<thead>
<tr>
<th>$\alpha_1 \gamma$ (constant)</th>
<th>$\gamma$ (M1Plus)</th>
<th>$\alpha_2 \gamma$ (Personal Income)</th>
<th>$\alpha_3 \gamma$ (3mo T-bill)</th>
<th>$\alpha_4 \gamma$ (6mo T-bill)</th>
<th>$\alpha_5 \gamma$ (12mo T-bill)</th>
<th>$\alpha_6 \gamma$ (AAA bond)</th>
<th>$\alpha_7 \gamma$ (BAA bond)</th>
<th>Adj. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0278* (0.0122)</td>
<td>0.0574** (0.0133)</td>
<td>-0.0302** (0.0061)</td>
<td>0.0021** (0.0006)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.044</td>
</tr>
<tr>
<td>-0.0295* (0.0125)</td>
<td>0.0583** (0.0135)</td>
<td>-0.0304** (0.0062)</td>
<td>0.0021** (0.0007)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.044</td>
</tr>
<tr>
<td>-0.0348** (0.0133)</td>
<td>0.0614** (0.0139)</td>
<td>-0.0308** (0.0063)</td>
<td>-</td>
<td>-</td>
<td>0.0024** (0.0007)</td>
<td>-</td>
<td>-</td>
<td>0.044</td>
</tr>
<tr>
<td>-0.0162 (0.0124)</td>
<td>0.0457** (0.0147)</td>
<td>-0.0273** (0.0081)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0015† (0.0008)</td>
<td>-</td>
<td>0.029</td>
</tr>
<tr>
<td>-0.0136 (0.0130)</td>
<td>0.0413** (0.0152)</td>
<td>-0.0242** (0.0081)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0010 (0.0007)</td>
<td>0.026</td>
</tr>
<tr>
<td>-0.0263* (0.0129)</td>
<td>0.0549** (0.0150)</td>
<td>-0.0283** (0.0080)</td>
<td>0.0022** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0004 (0.0011)</td>
<td>-</td>
<td>0.042</td>
</tr>
<tr>
<td>-0.0243† (0.0134)</td>
<td>0.0523** (0.0156)</td>
<td>-0.0268** (0.0081)</td>
<td>0.0023** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0006 (0.0009)</td>
<td>-</td>
<td>0.042</td>
</tr>
<tr>
<td>-0.0275* (0.0130)</td>
<td>0.0546** (0.0150)</td>
<td>-0.0275** (0.0080)</td>
<td>-</td>
<td>0.0025** (0.0009)</td>
<td>-</td>
<td>-0.0006 (0.0011)</td>
<td>-</td>
<td>0.042</td>
</tr>
<tr>
<td>-0.0254† (0.0135)</td>
<td>0.0520** (0.0155)</td>
<td>-0.0261** (0.0081)</td>
<td>-</td>
<td>0.0026** (0.0008)</td>
<td>-</td>
<td>-0.0008 (0.0009)</td>
<td>-</td>
<td>0.043</td>
</tr>
<tr>
<td>-0.0331* (0.0136)</td>
<td>0.0579** (0.0153)</td>
<td>-0.0281** (0.0081)</td>
<td>-</td>
<td>-</td>
<td>0.0028** (0.0010)</td>
<td>-0.0006 (0.0012)</td>
<td>-</td>
<td>0.042</td>
</tr>
<tr>
<td>-0.0310* (0.0141)</td>
<td>0.0554** (0.0158)</td>
<td>-0.0268** (0.0081)</td>
<td>-</td>
<td>-</td>
<td>0.0029** (0.0009)</td>
<td>-0.0008 (0.0010)</td>
<td>-</td>
<td>0.043</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.5: Monthly UAI Regressions with Different Opportunity Cost Variables (M1Plus Monetary Aggregate)*
<table>
<thead>
<tr>
<th>$\alpha_{1, y}$ (constant) (M2Minus)</th>
<th>$\gamma$ (M2Minus)</th>
<th>$\alpha_{2, y}$ (Personal Income)</th>
<th>$\alpha_{3, y}$ (3mo T-bill)</th>
<th>$\alpha_{4, y}$ (6mo T-bill)</th>
<th>$\alpha_{5, y}$ (12mo T-bill)</th>
<th>$\alpha_{6, y}$ (AAA bond)</th>
<th>$\alpha_{7, y}$ (BAA bond)</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0063 (0.0097)</td>
<td>0.0525** (0.0162)</td>
<td>-0.0381** (0.0101)</td>
<td>0.0020** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.028</td>
</tr>
<tr>
<td>-0.0081 (0.0101)</td>
<td>0.0544** (0.0166)</td>
<td>-0.0391** (0.0103)</td>
<td>-</td>
<td>0.0021** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.025</td>
</tr>
<tr>
<td>-0.0106 (0.0106)</td>
<td>0.0552** (0.0171)</td>
<td>-0.0390** (0.0106)</td>
<td>-</td>
<td>-</td>
<td>0.0023** (0.0009)</td>
<td>-</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0061 (0.0095)</td>
<td>0.0283 (0.0182)</td>
<td>-0.0241† (0.0131)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0007 (0.0010)</td>
<td>-</td>
<td>0.014</td>
</tr>
<tr>
<td>0.0097 (0.0100)</td>
<td>0.0186 (0.0188)</td>
<td>-0.0167 (0.0131)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.0061 (0.0009)</td>
<td>0.013</td>
</tr>
<tr>
<td>-0.0042 (0.0102)</td>
<td>0.0458* (0.0193)</td>
<td>-0.0325* (0.0134)</td>
<td>0.0023** (0.0009)</td>
<td>-</td>
<td>-</td>
<td>-0.0007 (0.0011)</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>-0.0017 (0.0106)</td>
<td>0.0403* (0.0201)</td>
<td>-0.0286* (0.0137)</td>
<td>0.0023** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0010 (0.0009)</td>
<td>0.028</td>
</tr>
<tr>
<td>-0.0059 (0.0104)</td>
<td>0.0463* (0.0193)</td>
<td>-0.0321* (0.0133)</td>
<td>-</td>
<td>0.0025** (0.0009)</td>
<td>-</td>
<td>-0.0010 (0.0012)</td>
<td>-</td>
<td>0.037</td>
</tr>
<tr>
<td>-0.0034 (0.0108)</td>
<td>0.0409* (0.0201)</td>
<td>-0.0284* (0.0136)</td>
<td>-</td>
<td>0.0026** (0.0009)</td>
<td>-</td>
<td>-</td>
<td>-0.0012 (0.0010)</td>
<td>0.029</td>
</tr>
<tr>
<td>-0.0087 (0.0109)</td>
<td>0.0470* (0.0194)</td>
<td>-0.0318* (0.0133)</td>
<td>-</td>
<td>-</td>
<td>0.0029** (0.0011)</td>
<td>-0.0011 (0.0012)</td>
<td>-</td>
<td>0.025</td>
</tr>
<tr>
<td>-0.0060 (0.0112)</td>
<td>0.0416* (0.0202)</td>
<td>-0.0282* (0.0136)</td>
<td>-</td>
<td>-</td>
<td>0.0029** (0.0010)</td>
<td>-</td>
<td>-0.0013 (0.0010)</td>
<td>0.027</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 99% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.6: Monthly UAI Regressions with Different Opportunity Cost Variables (M2Minus Monetary Aggregate)*
<table>
<thead>
<tr>
<th>$\alpha_1\gamma$ (constant)</th>
<th>$\gamma$ (MzM)</th>
<th>$\alpha_2\gamma$ (Personal Income)</th>
<th>$\alpha_3\gamma$ (3mo T-bill)</th>
<th>$\alpha_4\gamma$ (6mo T-bill)</th>
<th>$\alpha_5\gamma$ (12mo T-bill)</th>
<th>$\alpha_6\gamma$ (AAA bond)</th>
<th>$\alpha_7\gamma$ (BAA bond)</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0468** (0.0231)</td>
<td>0.0838** (0.0266)</td>
<td>-0.0842** (0.0265)</td>
<td>0.0027** (0.0010)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.023</td>
</tr>
<tr>
<td>0.0429† (0.0229)</td>
<td>0.0814** (0.0272)</td>
<td>-0.0810** (0.0267)</td>
<td>-</td>
<td>0.0027* (0.0011)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.020</td>
</tr>
<tr>
<td>0.0389† (0.0229)</td>
<td>0.0717** (0.0269)</td>
<td>-0.0720** (0.0262)</td>
<td>-</td>
<td>-</td>
<td>0.0025* (0.0012)</td>
<td>-</td>
<td>-</td>
<td>0.015</td>
</tr>
<tr>
<td>0.0354 (0.0232)</td>
<td>0.0211 (0.0215)</td>
<td>-0.0285 (0.0222)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0001 (0.0011)</td>
<td>-</td>
<td>0.000</td>
</tr>
<tr>
<td>0.0381 (0.0233)</td>
<td>0.0196 (0.0213)</td>
<td>-0.0200 (0.0218)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0007 (0.0009)</td>
<td>0.002</td>
</tr>
<tr>
<td>0.0540* (0.0237)</td>
<td>0.0738** (0.0276)</td>
<td>-0.0760** (0.0271)</td>
<td>0.0034** (0.0011)</td>
<td>-</td>
<td>-</td>
<td>-0.0016 (0.0012)</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0574* (0.0238)</td>
<td>0.0680* (0.0280)</td>
<td>-0.0722** (0.0272)</td>
<td>0.0034** (0.0011)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0017† (0.0010)</td>
<td>0.030</td>
</tr>
<tr>
<td>0.0508* (0.0235)</td>
<td>0.0731** (0.0277)</td>
<td>-0.0742** (0.0270)</td>
<td>-</td>
<td>0.0037** (0.0013)</td>
<td>-</td>
<td>-0.0019 (0.0012)</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0542* (0.0236)</td>
<td>0.0675* (0.0281)</td>
<td>-0.0705** (0.0271)</td>
<td>-</td>
<td>0.0037** (0.0012)</td>
<td>-</td>
<td>-</td>
<td>-0.0019† (0.0010)</td>
<td>0.029</td>
</tr>
<tr>
<td>0.0467* (0.0233)</td>
<td>0.0658* (0.0270)</td>
<td>-0.0670* (0.0263)</td>
<td>-</td>
<td>-</td>
<td>0.0039** (0.0014)</td>
<td>-0.0022† (0.0013)</td>
<td>-</td>
<td>0.021</td>
</tr>
<tr>
<td>0.0505* (0.0234)</td>
<td>0.0603* (0.0273)</td>
<td>-0.0636* (0.0263)</td>
<td>-</td>
<td>-</td>
<td>0.0039** (0.0013)</td>
<td>-0.0022* (0.0011)</td>
<td>-</td>
<td>0.025</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.7: Monthly UAI Regressions with Different Opportunity Cost Variables (MzM Monetary Aggregate)
<table>
<thead>
<tr>
<th>$\alpha_1 \gamma$ (constant)</th>
<th>$\gamma$ (M2)</th>
<th>$\alpha_2 \gamma$ (Personal Income)</th>
<th>$\alpha_3 \gamma$ (3mo T-bill)</th>
<th>$\alpha_3 \gamma$ (5mo T-bill)</th>
<th>$\alpha_4 \gamma$ (12mo T-bill)</th>
<th>$\alpha_5 \gamma$ (AAA bond)</th>
<th>$\alpha_6 \gamma$ (BAA bond)</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0117‡ (0.0067)</td>
<td>0.0510* (0.0203)</td>
<td>-0.0465** (0.0161)</td>
<td>-0.0002 (0.0004)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.019</td>
</tr>
<tr>
<td>0.0117‡ (0.0067)</td>
<td>0.0513* (0.0203)</td>
<td>-0.0466** (0.0161)</td>
<td>-</td>
<td>-0.0002 (0.0004)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.019</td>
</tr>
<tr>
<td>0.0092 (0.0070)</td>
<td>0.0571** (0.0208)</td>
<td>-0.0502** (0.0163)</td>
<td>-</td>
<td>-</td>
<td>-0.0003 (0.0005)</td>
<td>-</td>
<td>-</td>
<td>0.020</td>
</tr>
<tr>
<td>0.0096 (0.0068)</td>
<td>0.0600** (0.0207)</td>
<td>-0.0501** (0.0161)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0010† (0.0005)</td>
<td>-</td>
<td>0.026</td>
</tr>
<tr>
<td>0.0098 (0.0067)</td>
<td>0.0616** (0.0207)</td>
<td>-0.0510** (0.0161)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0010* (0.0004)</td>
<td>0.029</td>
</tr>
<tr>
<td>0.0056 (0.0069)</td>
<td>0.0680** (0.0209)</td>
<td>-0.0529** (0.0161)</td>
<td>0.0020* (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0030** (0.0010)</td>
<td>-</td>
<td>0.036</td>
</tr>
<tr>
<td>0.0067 (0.0068)</td>
<td>0.0689** (0.0207)</td>
<td>-0.0542** (0.0160)</td>
<td>0.0021** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0027** (0.0008)</td>
<td>0.042</td>
</tr>
<tr>
<td>0.0042 (0.0071)</td>
<td>0.0684** (0.0209)</td>
<td>-0.0526** (0.0160)</td>
<td>-</td>
<td>0.0022* (0.0009)</td>
<td>-</td>
<td>-0.0033** (0.0010)</td>
<td>-</td>
<td>0.037</td>
</tr>
<tr>
<td>0.0055 (0.0069)</td>
<td>0.0691** (0.0207)</td>
<td>-0.0538** (0.0160)</td>
<td>-</td>
<td>0.0023** (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0029** (0.0008)</td>
<td>0.043</td>
</tr>
<tr>
<td>0.0005 (0.0075)</td>
<td>0.0738** (0.0213)</td>
<td>-0.0556** (0.0163)</td>
<td>-</td>
<td>-</td>
<td>0.0026* (0.0010)</td>
<td>-0.0035** (0.0011)</td>
<td>-</td>
<td>0.037</td>
</tr>
<tr>
<td>0.0019 (0.0072)</td>
<td>0.0744** (0.0211)</td>
<td>-0.0569** (0.0162)</td>
<td>-</td>
<td>-</td>
<td>0.0026** (0.0009)</td>
<td>-</td>
<td>-0.0031** (0.0009)</td>
<td>0.043</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.8: Monthly UAI Regressions with Different Opportunity Cost Variables (M2 Monetary Aggregate)*

125
<table>
<thead>
<tr>
<th>$\alpha_1\gamma$ (constant)</th>
<th>$\gamma$ (M3)</th>
<th>$\alpha_2\gamma$ (Personal Income)</th>
<th>$\alpha_3\gamma$ (3mo T-bill)</th>
<th>$\alpha_3\gamma$ (6mo T-bill)</th>
<th>$\alpha_3\gamma$ (12mo T-bill)</th>
<th>$\alpha_3\gamma$ (AAA bond)</th>
<th>$\alpha_3\gamma$ (BAA bond)</th>
<th>Adj R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0220** (0.0078)</td>
<td>0.0170 (0.0185)</td>
<td>-0.0237 (0.0186)</td>
<td>-0.0002 (0.0005)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.008</td>
</tr>
<tr>
<td>0.0222** (0.0079)</td>
<td>0.0175 (0.0185)</td>
<td>-0.0242 (0.0187)</td>
<td>-</td>
<td>-0.0002 (0.0005)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.008</td>
</tr>
<tr>
<td>0.0218** (0.0080)</td>
<td>0.0208 (0.0189)</td>
<td>-0.0268 (0.0189)</td>
<td>-</td>
<td>-</td>
<td>-0.0003 (0.0005)</td>
<td>-</td>
<td>-</td>
<td>0.006</td>
</tr>
<tr>
<td>0.0248** (0.0078)</td>
<td>0.0332† (0.0197)</td>
<td>-0.0363† (0.0192)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0011† (0.0006)</td>
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<td>0.015</td>
</tr>
<tr>
<td>0.0262** (0.0078)</td>
<td>0.0373† (0.0197)</td>
<td>-0.0401* (0.0192)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0011* (0.0005)</td>
<td>0.018</td>
</tr>
<tr>
<td>0.0231** (0.0078)</td>
<td>0.0371† (0.0197)</td>
<td>-0.0372† (0.0191)</td>
<td>0.0017* (0.0008)</td>
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<td>-</td>
<td>-0.0029** (0.0010)</td>
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<tr>
<td>0.0251** (0.0078)</td>
<td>0.0407* (0.0196)</td>
<td>-0.0412* (0.0192)</td>
<td>0.0018* (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.0027** (0.0008)</td>
<td>0.028</td>
</tr>
<tr>
<td>0.0220** (0.0078)</td>
<td>0.0375† (0.0197)</td>
<td>-0.0370† (0.0191)</td>
<td>-</td>
<td>0.0020* (0.0009)</td>
<td>-</td>
<td>-0.0031** (0.0011)</td>
<td>-</td>
<td>0.023</td>
</tr>
<tr>
<td>0.0241** (0.0078)</td>
<td>0.0410* (0.0196)</td>
<td>-0.0410* (0.0192)</td>
<td>-</td>
<td>0.0021* (0.0008)</td>
<td>-</td>
<td>-</td>
<td>-0.0029** (0.0009)</td>
<td>0.029</td>
</tr>
<tr>
<td>0.0203* (0.0080)</td>
<td>0.0392* (0.0199)</td>
<td>-0.0380* (0.0192)</td>
<td>-</td>
<td>-</td>
<td>0.0022* (0.0010)</td>
<td>-0.0033** (0.0012)</td>
<td>-</td>
<td>0.021</td>
</tr>
<tr>
<td>0.0223** (0.0080)</td>
<td>0.0428* (0.0198)</td>
<td>-0.0420* (0.0193)</td>
<td>-</td>
<td>-</td>
<td>0.0023* (0.0009)</td>
<td>-</td>
<td>-0.0031** (0.0009)</td>
<td>0.028</td>
</tr>
</tbody>
</table>

* Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 4.9: Monthly UAI Regressions with Different Opportunity Cost Variables (M3 Monetary Aggregate)*
<table>
<thead>
<tr>
<th></th>
<th>ADF test</th>
<th>PP test</th>
<th>Number of significant parameter estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>-6.415**</td>
<td>-21.269**</td>
<td>4</td>
</tr>
<tr>
<td>M1plus</td>
<td>-7.149**</td>
<td>-21.676**</td>
<td>4</td>
</tr>
<tr>
<td>MzM</td>
<td>-6.187**</td>
<td>-14.888**</td>
<td>4</td>
</tr>
<tr>
<td>M2minus</td>
<td>-6.450**</td>
<td>-21.322**</td>
<td>3</td>
</tr>
<tr>
<td>M2*</td>
<td>-6.886**</td>
<td>-21.680**</td>
<td>4</td>
</tr>
<tr>
<td>M3*</td>
<td>-6.499**</td>
<td>-21.384**</td>
<td>5</td>
</tr>
</tbody>
</table>

The form of the augmented Dickey-Fuller (1981) test is

$$\Delta y_t = \mu + (\gamma - 1)y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t$$

where the test consists of testing the null $H_0$: $\gamma = 0$ (unit root). And the lag length, $j$, for the differenced lags of the series in question is determined by the method suggested by Campbell and Perron (1991). For each monthly series, there are in excess of 300 observations with all series but one having far more. The augmented Dickey-Fuller critical value for the 95% confidence level of these series is -2.87. The Engle-Yoo (1987) critical value for the null of a unit root is -4.22. The Engle-Yoo critical value takes into account the residuals used here to test for a unit root are estimated residuals and not the actual residuals. *Note: the estimated money demand model includes both the three-month U.S. T-bill rate and the Moody’s BAA rated U.S. corporate bond rate.

**Table 4.10: Whether the error term from the MQT regressions has a unit root**
<table>
<thead>
<tr>
<th></th>
<th>Income elasticity</th>
<th>Interest rate semi-elasticity**</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.547**</td>
<td>-0.026*</td>
</tr>
<tr>
<td>M1plus</td>
<td>0.525**</td>
<td>-0.036**</td>
</tr>
<tr>
<td>MzM</td>
<td>1.005**</td>
<td>-0.039**</td>
</tr>
<tr>
<td>M2minus</td>
<td>0.726**</td>
<td>-0.033**</td>
</tr>
<tr>
<td>M2*</td>
<td>0.787**</td>
<td>-0.031**</td>
</tr>
<tr>
<td>M3*</td>
<td>1.012**</td>
<td>-0.044**</td>
</tr>
</tbody>
</table>

Note: † indicates significance at the 90% level of confidence, * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence. *Note: the estimated money demand model includes both the three-month U.S. T-bill rate and the Moody’s BAA rated U.S. corporate bond rate. **The interest rate semi-elasticity is estimated from the coefficient on the three-month U.S. T-bill rate in all the above MQT regressions, even if the Moody’s BAA rated corporate bond rate was used as an explanatory variable.

Table 4.11: Elasticity estimates for the monthly data
<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>M1 vs. M1plus</strong></td>
<td>-0.017</td>
<td>0.028</td>
<td>0.049**</td>
<td>-0.039**</td>
<td>0.003**</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.021)</td>
<td>(0.015)</td>
<td>(0.009)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td><strong>M1plus vs. MzM</strong></td>
<td>-0.048</td>
<td>0.069*</td>
<td>-0.005</td>
<td>-0.028</td>
<td>0.003*</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.033)</td>
<td>(0.050)</td>
<td>(0.038)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td><strong>M1plus vs. M2minus</strong></td>
<td>-0.030*</td>
<td>0.070**</td>
<td>-0.019</td>
<td>-0.024*</td>
<td>0.002*</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.024)</td>
<td>(0.029)</td>
<td>(0.011)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td><strong>M1plus vs. M2</strong></td>
<td>-0.026*</td>
<td>0.054**</td>
<td>0.012</td>
<td>-0.038*</td>
<td>0.002**</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.023)</td>
<td>(0.016)</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td><strong>M1plus vs. M3</strong></td>
<td>-0.029*</td>
<td>0.058**</td>
<td>-0.004</td>
<td>-0.027</td>
<td>0.002**</td>
<td>0.042</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.019)</td>
<td>(0.018)</td>
<td>(0.001)</td>
<td></td>
</tr>
</tbody>
</table>

The parameter estimates in this table are derived from

$$UAI_t = \beta_1 + \beta_2 \ln m_{t-1,i} + \beta_3 \ln m_{t-1,j} + \beta_4 \ln y_{t-1} + \beta_5 R_{t-1} + \varepsilon_t.$$ 

$m_{t-1,i}$ corresponds to the first monetary aggregate listed in the first column of the row of interest. $m_{t-1,j}$ corresponds to the second monetary aggregate listed in the first column of the row of interest. * indicates significance at the 90% level of confidence. ** indicates significance at the 95% level of confidence, and *** indicates significance at the 99% level of confidence. Standard errors are in parentheses. *Note: All of these comparisons presented are using only the U.S. three-month T-bill rate as the opportunity cost variable. However, the same set of comparisons was done with both the U.S. three-month T-bill rate and the Moody’s BAA rated corporate bond rate as the opportunity cost variables with similar results.

**Table 4.12: Comparison of Monetary Aggregates for Monthly Data***
## Monthly Data

<table>
<thead>
<tr>
<th>Time Period</th>
<th>M1</th>
<th>M1Plus</th>
<th>M2Minus</th>
<th>M2</th>
<th>MzM</th>
<th>M3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959:1 - 1974:3 vs. 1974:4 - 1999:5</td>
<td>4.826  (0.0025)</td>
<td>3.889  (0.0091)</td>
<td>5.742  (0.0007)</td>
<td>3.470  (0.0083)</td>
<td>-</td>
<td>3.736  (0.0053)</td>
</tr>
<tr>
<td>1959:1 - 1982:1 vs. 1982:2 - 1999:5</td>
<td>5.336  (0.0013)</td>
<td>0.257  (0.8562)</td>
<td>2.252  (0.0816)</td>
<td>0.863  (0.4861)</td>
<td>3.416  (0.8178)</td>
<td>3.159  (0.0140)</td>
</tr>
<tr>
<td>1959:1 - 1985:1 vs. 1985:2 - 1999:5</td>
<td>5.485  (0.6010)</td>
<td>0.395  (0.7568)</td>
<td>1.713  (0.1634)</td>
<td>0.527  (0.7160)</td>
<td>0.201  (0.8954)</td>
<td>2.301  (0.0578)</td>
</tr>
<tr>
<td>1959:1 - 1990:1 vs. 1990:2 - 1999:5</td>
<td>3.141  (0.0251)</td>
<td>0.070  (0.9758)</td>
<td>0.800  (0.4941)</td>
<td>0.4048  (0.8053)</td>
<td>0.391  (0.7599)</td>
<td>1.720  (0.1442)</td>
</tr>
</tbody>
</table>

**P-values are in parentheses.** Shaded cells indicate that the null hypothesis of parameter stability is rejected at the 95% level of confidence or better.

### Table 4.13: Chow Tests for Parameter Stability
<table>
<thead>
<tr>
<th></th>
<th>Signal-to-Noise Ratio</th>
<th>Likelihood Ratio Test Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>2.33499E-05</td>
<td>2.56</td>
</tr>
<tr>
<td>M1Plus</td>
<td>2.47869E-05</td>
<td>2.46</td>
</tr>
<tr>
<td>M2Minus</td>
<td>2.2303E-05</td>
<td>2.39</td>
</tr>
<tr>
<td>MzM</td>
<td>4.24727E-05</td>
<td>1.71</td>
</tr>
<tr>
<td>M2</td>
<td>2.40689E-05</td>
<td>0.30</td>
</tr>
<tr>
<td>M3</td>
<td>2.20515E-05</td>
<td>1.15</td>
</tr>
</tbody>
</table>

The model is a random walk intercept term in the money demand specification. The signal-to-noise ratio is the ratio of the variance of the error for the random walk state equation (4.7) to the variance of the observation equation (4.6). The likelihood ratio test examines the null hypothesis of $q = 0$ vs. the alternative of $q \neq 0$. The LR test statistic is distributed $\chi^2(1)$ with 10%, 5%, and 1% critical values of 2.71, 3.84, and 6.63, respectively.

**Table 4.14: Signal-to-Noise Ratio for Random Walk Intercept**
<table>
<thead>
<tr>
<th>Monetary Aggregate</th>
<th>Real U.S. Personal Income</th>
<th>Three-month T-bill rate</th>
<th>Aaa Bond</th>
<th>Baa Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.968</td>
<td>37.372**</td>
<td>35.922**</td>
<td>29.592**</td>
</tr>
<tr>
<td>M1Plus</td>
<td>1.763</td>
<td>34.521**</td>
<td>40.249**</td>
<td>31.147**</td>
</tr>
<tr>
<td>M2Minus</td>
<td>2.628†</td>
<td>53.876**</td>
<td>55.063**</td>
<td>44.347**</td>
</tr>
<tr>
<td>MzM</td>
<td>2.369†</td>
<td>37.088**</td>
<td>31.487**</td>
<td>24.801**</td>
</tr>
<tr>
<td>M2</td>
<td>3.420*</td>
<td>36.203**</td>
<td>34.565**</td>
<td>30.058**</td>
</tr>
<tr>
<td>M3</td>
<td>5.406**</td>
<td>20.607**</td>
<td>19.790**</td>
<td>19.426**</td>
</tr>
</tbody>
</table>

† indicates significance at the 90% level of confidence. ** indicates significance at the 95% level of confidence, and *** indicates significance at the 99% level of confidence. All value are F-statistics for the joint significance of the parameter estimates on the first two leads of the monetary aggregate. Each regression used eight lags of the dependent variable to whiten the residuals. In each case a LM test for higher order serial showed that the residuals did not exhibit statistically significant autocorrelation.

Table 4.15: Alternative Sims Tests for Causality (Conventional Money Demand Specification)
<table>
<thead>
<tr>
<th></th>
<th>Unanticipated Inflation</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>1.951</td>
</tr>
<tr>
<td>M1Plus</td>
<td>0.685</td>
</tr>
<tr>
<td>M2Minus</td>
<td>0.796</td>
</tr>
<tr>
<td>MzM</td>
<td>1.246</td>
</tr>
<tr>
<td>M2</td>
<td>2.118</td>
</tr>
<tr>
<td>M3</td>
<td>2.122</td>
</tr>
<tr>
<td>U.S. Personal Income (ln)</td>
<td>2.662†</td>
</tr>
<tr>
<td>Three-month T-bill rate</td>
<td>0.375</td>
</tr>
<tr>
<td>Aaa Bond</td>
<td>0.690</td>
</tr>
<tr>
<td>Baa Bond</td>
<td>0.260</td>
</tr>
</tbody>
</table>

† indicates significance at the 90% level of confidence. ** indicates significance at the 95% level of confidence, and *** indicates significance at the 99% level of confidence. All value are F-statistics for the joint significance of the parameter estimates on the first two leads of unanticipated inflation. Each regression used eight lags of the dependent variable to whiten the residuals. In each case a LM test for higher order serial showed that the residuals did not exhibit statistically significant autocorrelation.

**Table 4.16: Alternative Sims Tests for Causality (MQT Money Demand Specification)**
<table>
<thead>
<tr>
<th>Country</th>
<th>Author(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>Bose and Rahman (1996)</td>
</tr>
<tr>
<td>Germany</td>
<td>Beyer (1998)</td>
</tr>
<tr>
<td></td>
<td>Lutkepohl and Wolters (1998)</td>
</tr>
<tr>
<td></td>
<td>Hubrich (1999)</td>
</tr>
<tr>
<td></td>
<td>Bahmani-Oskooee and Bohl (2000)</td>
</tr>
<tr>
<td></td>
<td>Bohl (2000)</td>
</tr>
<tr>
<td>Italy</td>
<td>Muscatelli and Papi (1990)</td>
</tr>
<tr>
<td></td>
<td>Bagliano (1996)</td>
</tr>
<tr>
<td>Japan</td>
<td>Bahmani-Oskooee and Shabsigh (1996)</td>
</tr>
<tr>
<td></td>
<td>Morimune and Zhao (1997)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Hendry and Ericsson (1991)</td>
</tr>
<tr>
<td></td>
<td>Choudry (1992)</td>
</tr>
<tr>
<td></td>
<td>Engsted and Haldrup (1997)</td>
</tr>
<tr>
<td></td>
<td>Howells and Hussein (1997)</td>
</tr>
<tr>
<td>European Union</td>
<td>Wesche (1997)</td>
</tr>
</tbody>
</table>

Table 5.1: International Studies that Have Found a Money Demand Cointegrating Relationship
<table>
<thead>
<tr>
<th></th>
<th>Gross Domestic Product</th>
<th>Monetary Aggregate</th>
<th>Interest Rate</th>
<th>UAI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-2.98*</td>
<td>-0.27</td>
<td>-1.92</td>
<td>-4.25**</td>
</tr>
<tr>
<td>France</td>
<td>-2.34</td>
<td>-0.93</td>
<td>-0.68</td>
<td>-3.24*</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.51</td>
<td>-0.49</td>
<td>-5.26**</td>
<td>-10.17**</td>
</tr>
<tr>
<td>Italy</td>
<td>-2.68†</td>
<td>-1.79</td>
<td>-1.07</td>
<td>-4.18**</td>
</tr>
<tr>
<td>Japan</td>
<td>-1.74</td>
<td>-0.88</td>
<td>-0.07</td>
<td>-5.28**</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.73</td>
<td>-0.06</td>
<td>-0.75</td>
<td>-10.39**</td>
</tr>
<tr>
<td>United States</td>
<td>-0.84</td>
<td>-0.83</td>
<td>-2.37</td>
<td>-9.89**</td>
</tr>
</tbody>
</table>

The form of the augmented Dickey-Fuller (1981) test is

$$\Delta y_t = \mu + (\gamma - 1)y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t$$

where the test consists of testing the null $H_0: \gamma = 0$ (unit root). And the lag length, $j$, for the differenced lags of the series in question is determined by the method suggested by Campbell and Perron (1990). The augmented Dickey-Fuller critical value for the 95% confidence level of these series is -2.87.

**Table 5.2: Augmented Dickey-Fuller Test for Unit Roots**
<table>
<thead>
<tr>
<th></th>
<th>Gross Domestic Product</th>
<th>Monetary Aggregate</th>
<th>Interest Rate</th>
<th>UAI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>-3.05</td>
<td>-0.53</td>
<td>-2.11</td>
<td>-9.37**</td>
</tr>
<tr>
<td>France</td>
<td>-2.76†</td>
<td>-0.90</td>
<td>-0.73</td>
<td>-9.09**</td>
</tr>
<tr>
<td>Germany</td>
<td>-0.99</td>
<td>-0.73</td>
<td>-2.65†</td>
<td>-10.28**</td>
</tr>
<tr>
<td>Italy</td>
<td>-1.99</td>
<td>-3.53**</td>
<td>-1.00</td>
<td>-9.67**</td>
</tr>
<tr>
<td>Japan</td>
<td>-2.42</td>
<td>-1.97</td>
<td>-0.29</td>
<td>-9.53**</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>-0.80</td>
<td>-0.04</td>
<td>-1.10</td>
<td>-10.39**</td>
</tr>
<tr>
<td>United States</td>
<td>-0.06</td>
<td>-2.01</td>
<td>-2.46</td>
<td>-9.10**</td>
</tr>
</tbody>
</table>

The critical value for the 95% confidence level of these series is -2.87.

**Table 5.3: Phillips-Perron Test for Unit Roots**
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m2, y, $\pi$, and R</th>
<th>$\lambda_{trace}$ for m2, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r &gt; 0</td>
<td>1</td>
<td>144.43</td>
<td>33.20</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>63.84</td>
<td>22.92</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>54.73</td>
<td>23.21</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>47.11</td>
<td>25.13</td>
<td>(1, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>43.41</td>
<td>30.03</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>34.00</td>
<td>20.94</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>37.08</td>
<td>24.73</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>29.08</td>
<td>23.97</td>
<td>(1, 1)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 46.71, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autoregressed error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autoregressed error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.\n
**Table 5.4: Johansen’s (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for the United States**
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for ( m_1, y, \pi, \text{, and } R )</th>
<th>$\lambda_{trace}$ for ( m_1, y, \text{ and } R )</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 0 )</td>
<td>( r &gt; 0 )</td>
<td>1</td>
<td>135.83</td>
<td>98.31</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>105.91</td>
<td>79.85</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>100.69</td>
<td>68.23</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>88.08</td>
<td>57.29</td>
<td>(0, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>88.65</td>
<td>58.11</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>72.60</td>
<td>45.54</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>65.43</td>
<td>33.68</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>70.64</td>
<td>28.18</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autoregressed error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.

**Table 5.5: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for Canada**
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{\text{trace}}$ for m4, y, $\pi$, and R</th>
<th>$\lambda_{\text{trace}}$ for m4, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>1</td>
<td>135.94</td>
<td>38.97</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>55.14</td>
<td>23.67</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>42.15</td>
<td>18.52</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>48.13</td>
<td>26.76</td>
<td>(4, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>52.90</td>
<td>30.20</td>
<td>(0, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>61.66</td>
<td>33.61</td>
<td>(1, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>56.71</td>
<td>35.86</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>58.46</td>
<td>32.73</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{\text{trace}}$ test used here allows a drift term. This corresponds to Johansen and Juselius’ (1990) Table A1 Monte Carlo simulation. Österwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.

Table 5.6: Johansen’s (1988) $\lambda_{\text{trace}}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for the United Kingdom*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m2, y, $\pi$, and R</th>
<th>$\lambda_{trace}$ for m2, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = 0$</td>
<td>$\tau &gt; 0$</td>
<td>1</td>
<td>240.65</td>
<td>63.33</td>
<td>(3, 3)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>60.81</td>
<td>25.11</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>55.07</td>
<td>30.03</td>
<td>(2, 2)</td>
</tr>
<tr>
<td></td>
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<td>4</td>
<td>56.90</td>
<td>34.96</td>
<td>(0, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>48.58</td>
<td>28.74</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>33.46</td>
<td>17.16</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>32.90</td>
<td>16.33</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>33.43</td>
<td>16.38</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.

Table 5.7: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for Japan*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m1, y, π, and R</th>
<th>$\lambda_{trace}$ for m1, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r &gt; 0</td>
<td>1</td>
<td>111.10</td>
<td>52.40</td>
<td>(3, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>67.13</td>
<td>40.92</td>
<td>(1, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>58.79</td>
<td>30.40</td>
<td>(1, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>59.91</td>
<td>28.87</td>
<td>(1, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>56.45</td>
<td>31.20</td>
<td>(1, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>89.68</td>
<td>41.63</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>66.58</td>
<td>40.64</td>
<td>(0, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>59.79</td>
<td>40.62</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.

Table 5.8: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for Italy*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m3, y, $\pi$, and R</th>
<th>$\lambda_{trace}$ for m3, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = 0$</td>
<td>$r &gt; 0$</td>
<td>1</td>
<td>67.12</td>
<td>16.48</td>
<td>(3, 2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>42.74</td>
<td>16.95</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>40.35</td>
<td>18.47</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>38.47</td>
<td>19.92</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5</td>
<td>50.34</td>
<td>18.69</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
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<td>55.97</td>
<td>22.86</td>
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<td></td>
<td></td>
<td>7</td>
<td>61.94</td>
<td>30.24</td>
<td>(0, 0)</td>
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<td></td>
<td></td>
<td>8</td>
<td>62.06</td>
<td>32.74</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius’ (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.*

Table 5.9: Johansen’s (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for France*
<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>Alternative Hypothesis</th>
<th>Lags</th>
<th>$\lambda_{trace}$ for m2, y, $\pi$, and R</th>
<th>$\lambda_{trace}$ for m2, y, and R</th>
<th>Number of error processes with significant autocorrelation</th>
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</thead>
<tbody>
<tr>
<td>r = 0</td>
<td>r &gt; 0</td>
<td>1</td>
<td>99.99</td>
<td>36.69</td>
<td>(3, 2)</td>
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<td>84.77</td>
<td>42.26</td>
<td>(2, 2)</td>
</tr>
<tr>
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<td>3</td>
<td>72.99</td>
<td>36.05</td>
<td>(2, 1)</td>
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<tr>
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<td>4</td>
<td>63.64</td>
<td>27.58</td>
<td>(2, 1)</td>
</tr>
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<td></td>
<td></td>
<td>5</td>
<td>59.54</td>
<td>31.70</td>
<td>(2, 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6</td>
<td>60.78</td>
<td>37.03</td>
<td>(1, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7</td>
<td>55.13</td>
<td>35.43</td>
<td>(1, 0)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>8</td>
<td>48.66</td>
<td>30.02</td>
<td>(0, 0)</td>
</tr>
</tbody>
</table>

*The Johansen $\lambda_{trace}$ test used here allows a drift term. This corresponds to Johansen and Juselius' (1990) Table A1 Monte Carlo simulation. Osterwald-Lenum (1992) refines the critical values in Johansen and Juselius (1990) and his are the critical values used here. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for four explanatory variables are 43.95, 47.21, and 54.46, respectively. For at least one cointegrating vector, the 90%, 95%, and 99% critical values for three explanatory variables are 26.79, 29.68, and 32.56, respectively. The last column in the table shows the number of error processes (out of 4 and 3, respectively) in the VAR that show significant autocorrelation according to a Ljung-Box Q-test. The first number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, the opportunity cost variable and inflation. The second number in the pair signifies the number of autocorrelated error processes in a system of the monetary aggregate, the scale variable, and the opportunity cost variable.*

Table 5.10: Johansen's (1988) $\lambda_{trace}$ statistic for the existence of at least one cointegrating vector between a monetary aggregate, a scale variable, an opportunity cost variable, and with and without inflation for Germany*
<table>
<thead>
<tr>
<th>Country</th>
<th>$\alpha_1 \gamma$</th>
<th>$\gamma$</th>
<th>$\alpha_2 \gamma$</th>
<th>$\alpha_3 \gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada</td>
<td>0.0763† (0.0410)</td>
<td>0.0165 (0.0183)</td>
<td>-0.0246† (0.0125)</td>
<td>-0.0002 (0.0007)</td>
</tr>
<tr>
<td>France</td>
<td>-0.0677 (0.1808)</td>
<td>0.0359 (0.0669)</td>
<td>-0.0097 (0.0296)</td>
<td>0.0008 (0.0011)</td>
</tr>
<tr>
<td>Germany</td>
<td>1.7599** (0.5361)</td>
<td>0.0727** (0.0250)</td>
<td>-0.1563** (0.0477)</td>
<td>0.0019* (0.0008)</td>
</tr>
<tr>
<td>Italy</td>
<td>-0.1500† (0.0757)</td>
<td>0.3038** (0.0948)</td>
<td>-0.1234* (0.0542)</td>
<td>0.0022† (0.0012)</td>
</tr>
<tr>
<td>Japan</td>
<td>0.5233* (0.2297)</td>
<td>0.2157* (0.0987)</td>
<td>-0.3612* (0.1598)</td>
<td>-0.0002 (0.0025)</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>0.0985 (0.0898)</td>
<td>0.0216 (0.0417)</td>
<td>-0.0826 (0.0870)</td>
<td>0.0004 (0.0022)</td>
</tr>
<tr>
<td>United States</td>
<td>0.3568** (0.1229)</td>
<td>0.0844** (0.0265)</td>
<td>-0.0580** (0.0199)</td>
<td>0.0033** (0.0013)</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

Table 5.11: Quarterly UAI Regressions
<table>
<thead>
<tr>
<th>Country</th>
<th>Income elasticity</th>
<th>Interest rate semi-elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>2.1504**</td>
<td>-0.0261*</td>
</tr>
<tr>
<td>Italy</td>
<td>0.4062**</td>
<td>-0.0072*</td>
</tr>
<tr>
<td>Japan</td>
<td>1.6746*</td>
<td>-0.8009†</td>
</tr>
<tr>
<td>United States</td>
<td>0.6872**</td>
<td>-0.0391**</td>
</tr>
</tbody>
</table>

Standard errors are in parentheses. † indicates significance at the 90% level of confidence. * indicates significance at the 95% level of confidence, and ** indicates significance at the 99% level of confidence.

**Table 5.12: Estimates of International Income Elasticities and Interest Rate Semi-Elasticities**
<table>
<thead>
<tr>
<th></th>
<th>ADF test</th>
<th>PP test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>-3.176</td>
<td>-12.126**</td>
</tr>
<tr>
<td>Italy</td>
<td>-6.064**</td>
<td>-11.602**</td>
</tr>
<tr>
<td>Japan</td>
<td>-3.614†</td>
<td>-9.694**</td>
</tr>
<tr>
<td>United States</td>
<td>-12.223**</td>
<td>-13.715**</td>
</tr>
</tbody>
</table>

The form of the augmented Dickey-Fuller (1981) test is \( \Delta y_t = \mu + (\gamma - 1)y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t \), where the test consists of testing the null \( H_0: \gamma = 0 \) (unit root). And the lag length, \( j \), for the differenced lags of the series in question is determined by the method suggested by Campbell and Perron (1991). The augmented Dickey-Fuller critical value for the 95% confidence level of these series is -2.87. The Engle-Yoo (1987) critical value for the null of a unit root is -4.22. The Engel-Yoo critical value takes into account the residuals used here to test for a unit root are estimated residuals and not the actual residuals.

**Table 5.13: Whether the error term from the MQT regressions has a unit root**
Figure 2.1: Real U.S. Monthly M1
Figure 2.2: Real U.S. Monthly M2
Figure 2.3: Real U.S. Monthly Personal Income
Figure 2.4: Nominal U.S. Monthly 3 month T-bill Rate
Figure 2.5: U.S. Monthly CPI-UX
Figure 3.1: Monthly Inflation (CPI-UX) Annualized
Figure 3.2: Median Unbiased Estimate of the AR(1) Coefficient for Monthly Annualized U.S. Inflation
Figure 3.3: Time Series of the Difference between Actual Inflation at Time $t$ and Forecasted Inflation at Time $t$
Figure 3.4: U.S. M2 Velocity at a Monthly Frequency
Figure 3.5: Actual vs. Forecasted (Fuller-ARMA) Monthly U.S. Annualized Inflation
Figure 3.6: Adjusted R-Squared from the Fuller-ARMA Inflation Models
Figure 4.1: Real U.S. Monthly Personal Income
Figure 4.2: U.S. Monthly CPI-UX
Figure 4.3: Nominal U.S. Monthly 3 month T-bill Rate
Figure 4.4: Real U.S. Monthly M1
Figure 4.5: Real U.S. Monthly M2
Figure 4.6: Real U.S. Monthly M3
Figure 4.7: Real U.S. Monthly M1Plus
Figure 4.8: Real U.S. Monthly M2Minus
Figure 4.9: Real U.S. Monthly MzM
Figure 4.10: Moody's Aaa rated Corporate Bond
Figure 4.11: Moody's Baa rated Corporate Bond
Figure 4.12: Adjustment Period for Prices to a Sustained 1% Nominal Money Market Disequilibrium
Figure 4.13: Excess Supply of Money (M1)
Figure 4.14: Excess Supply of Money (M1Plus)
Figure 4.15: Excess Supply of Money (M2M)
Figure 4.16: Excess Supply of Money (M2Minus)
Figure 4.17: Excess Supply of Money (M2)
Figure 4.18: Excess Supply of Money (M3)
Figure 4.19: Expanding Window Chow Test for Parameter Stability (M1)
Figure 4.20: Expanding Window Chow Test for Parameter Stability (M1Plus)
Figure 4.21: Expanding Window Chow Test for Parameter Stability (MzM)
Figure 4.22: Expanding Window Chow Test for Parameter Stability (M2Minus)
Figure 4.23: Expanding Window Chow Test for Parameter Stability (M2)
Figure 4.24: Expanding Window Chow Test for Parameter Stability (M3)
Figure 5.1: Canadian Real GDP
Figure 5.2: Canadian Real M1 Money Stock
Figure 5.3: Canadian Nominal Short Term Interest Rate (SA)
Figure 5.4: Canadian Aggregate Prices (SA)
Figure 5.5: French Real GDP
Figure 5.6: French Real M3 Money Stock
Figure 5.7: French Nominal Long Interest Rate
Figure 5.8: French Aggregate Price Level
Figure 5.9: Real German GDP
Figure 5.10: Real German M2
Figure 5.11: German Nominal Short Interest Rate
Figure 5.12: German Aggregate Price Level
Figure 5.13: Italian Real GDP
Figure 5.14: Italian Real M1 Money Stock
Figure 5.15: Italian Nominal Long Interest Rate
Figure 5.16: Italian Aggregate Price Level
Figure 5.17: Japanese Real GDP
Figure 5.18: Japanese Real M2 Money Stock
Figure 5.19: Japanese Nominal Long Interest Rate
Figure 5.20: Japanese Aggregate Price Level
Figure 5.21: United Kingdom Real GDP
Figure 5.22: United Kingdom Real M4 Money Stock
Figure 5.23: United Kingdom Nominal Long Interest Rate
Figure 5.24: United Kingdom Aggregate Price Level
Figure 5.25: U.S. Real GDP
Figure 5.26: U.S. Real M2
Figure 5.27: U.S. Nominal Short Interest Rate
Figure 5.28: U.S. Aggregate Price Level
Figure 5.29: Estimate of Canadian Inflation Order of Integration
Figure 5.30: Estimate of French Inflation Order of Integration
Figure 5.31: Estimate of German Inflation Order of Integration
Figure 5.32: Estimate of Italian Inflation Order of Integration
Figure 5.33: Estimate of Japanese Inflation Order of Integration
Figure 5.34: Estimate of United Kingdom Inflation Order of Integration
Figure 5.35: Estimate of U.S. Inflation Order of Integration
Figure 5.36: Expanding Window Chow Test for Parameter Stability
(M2 Italian Quarterly data)
Figure 5.37: Expanding Window Chow Test for Parameter Stability (M2 Japan Quarterly data)
Figure 5.38: Expanding Window Chow Test for Parameter Stability (M2 U.S. Quarterly data)
Figure 5.39: Expanding Window Chow Test for Parameter Stability
(M1 German Quarterly data)
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