The Nine-Step Scale of Alexander Tcherepnin: Its Conception, Its Properties, and Its Use

Dissertation

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Abstract

Like so many other composers of the twentieth-century, Russian-born Alexander Tcherepnin (1899-1977) adopted many compositional methods and styles over the span of his lifetime. One style that uniquely defines him is the one that incorporates a collection of nine pitches, affectionately known by many as the “Tcherepnin Scale.”

This collection, which is termed more generically the “nine-step scale,” is symmetrical in its construction. Having only four distinct transpositions, its structure can be viewed in a variety of ways. It is the combination of any three augmented triads. It is also the collection created by combining a hexatonic scale (the collection of pitches formed by alternating three semitones with one semitone) and its inversion $T_n I$ such that $n = \text{any even integer}$. In addition, it is a repeating sequence of two semitones and one whole tone. Finally, it is the complement of the augmented triad.

The presence of the scale in musical works is frequent and varied. In many cases, the scale somewhat defines the music; that is to say, the scale is used as a pre-compositional device for a piece of music. In other cases, composers are not even aware of the inclusion of the scale in their compositions. In these cases, the scale is a by-product of some other compositional goal. In this document, I look at examples of both types of music, in hopes of gaining an understanding of composers’ compositional goals.
The music of Alexander Tcherepnin, the composer with whom the scale is most often affiliated, is at the forefront of my musical analysis. Some musical examples are taken from before his theoretical formation of the scale. Most of the examples are drawn from his nine-step period of composition, roughly from 1922-1934. The final piece for which I provide an analysis was written much later in his life.

Select pieces by other composers, in which one or more nine-note collections appear, are also analyzed. These works are studied with the goal of understanding the composer’s compositional intention as it relates to the nine-step scale.
Dedication

Dedicated to Matt, Maria and Shawna
Acknowledgments

A work of this weight would never have come to fruition without the help, guidance, patience and encouragement from others. There are so many people I would like to thank.

First and foremost, I would like to thank my husband. He made great professional and personal sacrifices so that I could follow my dream. He has shown me tremendous support, particularly during the moments of greatest stress. He has been patient with me, he has encouraged me, he has shown me his love in countless ways. Thank you, Matt. I love you.

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I would also like to thank all my family and friends for their constant prayers, encouragement, and support.

Finally, I must give my utmost praise and thanks to God. It has been such a comfort to know that He is always by my side, guiding me through every moment of every day. He is my sole source of strength, and without him I can accomplish nothing.
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Introduction

Alexander Tcherepnin (1899-1977) has often been overlooked as a composer. Born in St. Petersburg, Tcherepnin grew up during one of the most tumultuous times in Russia’s history. Before his twenty-third birthday, Tcherepnin and his family had emigrated from Russia, never to return as permanent residents.

Throughout his compositional career, Tcherepnin used particular elements in his music that he adopted or developed during his earliest years. There are certainly rhythmic features that appear repeatedly in the works of Tcherepnin, as well as a chromatic-tonal harmonic structure that could be compared to that of Prokofiev. Tcherepnin was constantly aware of the musical innovations around him, and he used those elements that appealed to his fancy; the major-minor tetra/chord\(^1\) seemed to have had the most appeal. From the time Tcherepnin was first introduced to music, he heard the major-minor tetra/chord as the most stable harmony. In essence, this triad with a “split-third” would simultaneously encompass both the major and the minor tonic of any key. In “A Short Autobiography,” Tcherepnin describes his early affinity with the major-minor tetra/chord:

\(^1\) The use of the term “tetra/chord” reflects Tcherepnin’s method of distinguishing between four pitches that serve a harmonic function within the context of a musical composition (tetra/chord) and a general collection of four adjacent pitches in a scalar configuration (tetrachord).
It was my wish to see my family happy, in fact I wished for all humans to be happy. Even in my earliest childhood I realized that happiness and unhappiness, joy and sorrow [are always] coexisting and cannot be separated from each other. As childish as it seems, it brought me, in music, to the idea of the combination of major and minor. I had always felt that the major-minor triad of C, Eb, E, and G is a “fundamental,” “final” chord—therefore “consonant,” “stable,” and not in search of resolution like a dissonance in the classical concept of this term. I felt attracted to this major-minor chord. I heard it constantly sounding in me, and somehow, even in the early instinctive period of my composing, I used this chord as a final “consonant” chord.²

The use of the major-minor tetra/chord, as we will soon see, led to a very specific type of tonal organization that occupied Tcherepnin’s compositional palette almost exclusively for over a decade. He identified this tonal organization as the “nine-step scale,” a symmetrical collection of nine tones, which some eventually referred to as the “Tcherepnin scale.”³

Tcherepnin was by no means the only composer to have used this scale. The content of the nine-step scale is identical to the third of the “Modes of Limited Transposition” used by Olivier Messiaen. Some more recent composers have used the scale in their compositions because of the appeal of its symmetrical properties and its affinity with neo-Riemannian transformations. Still others appear to have unknowingly incorporated the nine-step scale into their music. Prior to Tcherepnin, other Russian composers, such as Rimsky-Korsakov and Glinka, used symmetrical constructions in their music that naturally resulted in the presence of the scale. The same types of

³ Nicolas Slonimsky mentions this in his article, “Alexander Tcherepnin Septuagenarian,” which was written in honor of Tcherepnin’s seventieth birthday. He states, “The scale has come to be known as the ‘Tcherepnin Scale,’ and is classified as such in the august Musik-Lexikon of Hugo Riemann.” (Korabelnikova, p. 169).
symmetrical constructions can be found in works of later composers as well. Tcherepnin himself was well aware of the natural accessibility and possible widespread use of “his” scale and some of his other compositional techniques. Reflecting on this truth, he states,

The ideas that were previously mentioned as being instinctively mine, then consciously used, might have been my own personal ideas as I saw them, but they were also the ideas of my generation. They were “in the air,” I felt them, faced them, and tried to use appropriate means to materialize them. I was not alone, and who of us was “first” is of no importance. The essential thing is to be honest and to compose music that one feels to be right. The ultimate value of a composition is the complete balance of the “what” with the “how.” The “how” can and should be analyzed. The “what” is extemporaneous and can only be felt, and escapes every cerebral investigation.

In January of 1962, Tcherepnin wrote a treatise about his compositional technique entitled, *Basic Elements of My Musical Language*. In the treatise, Tcherepnin addresses many subjects, but the most weight is given to the topic of his nine-step scale. The fact that Tcherepnin devoted such time and effort to record his theoretical thoughts motivates me to take a close look at what he has to say about this scale and to study how he used it in his own musical compositions. In an effort to form a theoretical basis for my own research, I will provide a critical analysis of Tcherepnin’s discourse.

Prior to a discussion of Tcherepnin’s treatise, however, I must first explore the scale as a modern music theorist and provide a method by which one can discuss such a scale. Resembling both the 7-tone diatonic universe and the 12-tone chromatic universe,

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4 Korabelnikova, pp. 187-188. This quotation also originates from “A Short Autobiography” in *Tempo* (no. 130, 1978).
5 *Basic Elements of My Musical Language*, by Alexander Tcherepnin, is published as an appendix to Ludmila Korabelnikova’s book, *Alexander Tcherepnin: The Saga of a Russian Emigré Composer*. The composer wrote the document in 1962, in English, and a photocopy of the manuscript is held at the Glinka State Central Museum of Musical Culture. The document also appears on The Tcherepnin Society’s website [http://www.tcherepnin.com/alex/basic_elm1.htm](http://www.tcherepnin.com/alex/basic_elm1.htm) and in Enrique Arias’s book, *Alexander Tcherepnin: A Bio-Bibliography*. All references to *Basic Elements* in my work will be directed toward Korabelnikova’s book, unless otherwise noted.
yet belonging wholly to neither, the nine-step scale provides some analytical challenges. These challenges are related to the identification of each distinct transposition of the scale, but are also related to the intervals contained within the scale.

After a basic understanding of the scale has been reached, I will look at the intrinsic properties of the scale. In looking at these properties, I will focus on two aspects: symmetry and tonality. There are other aspects of the scale that have yet to be explored in great detail, such as its position in the realm of neo-Riemannian theory. This is a realm that lies outside my expertise, and thus remains a topic for further study.

Finally, I will discuss the use of the nine-step scale in various musical compositions. My primary focus will be directed at Tcherepnin’s music, as I consider him the master of his art. My musical analysis will begin with some of Tcherepnin’s pre-nine-step compositions in an effort to trace the development of his tonal system. I will then look at two pieces written by Tcherepnin during his “Nine-Step Phase” and one piece written by him much later in his career.

I am, however, also interested in compositions by other composers in which the nine-step scale appears, so I will also conduct brief analyses of works by various other modern composers. In some cases, it will be clear that the composer knew of this collection of nine pitches and specifically chose to use it as the organizing force of their work. In other cases, the collection will appear as a result of some other compositional process.

The nine-step scale is a tonal system that has been neglected by musicians. In this paper, I will shed some light on its presence, its prominence, and its purpose.
Chapter 1: The Identity of the Nine-Step Scale

Before looking at the way Alexander Tcherepnin understood the nine-step scale, and before evaluating the use of the scale in musical compositions, I shall provide a brief overview of the basic construction and some of the properties of the scale.

The nine-step scale is perhaps better known to modern music theorists as Forte’s set class 9-12 or, in other words, the complement of the augmented triad. Although this identity is useful when dealing with this particular collection of pitch-classes within a mod-12 pitch-class universe, analysis becomes complicated when the music does not reflect a 12-tone system.

Similarly, one will run into difficulty when trying to explain nine-step music within the context of the 7-note diatonic scales. This is especially true when addressing distances between pitches within the nine-step scale.

Therefore, we must formulate a unique “language” through which the theoretical details of the nine-step scale can be discussed. Let us look at the basic construction of the nine-step scale. To start, I will use pitch-class integers in mod-12 pitch-class space because such a space is easily understood, and integer notation allows us to talk about specific notes. The nine-step scale in Figure 1-1 is arranged such that it begins on pitch-class 0 (C), and the scale is formed by the repeating semitone pattern of 1-2-1. As we will
see in Chapter 2, Alexander Tcherepnin refers to this particular arrangement of pitches as a “mode 1” scale. The mod-12 pitch-class integer for each note is labeled above the scale, and the distance from one scale-step to the next by quantity of semitones is labeled below the scale.

![Diagram of the Nine-Step Scale](image)

**Figure 1-1. Basic Construction of the Nine-Step Scale**

The specific scale in Figure 1-1 can be identified by the label NON\textsubscript{013}, where “NON” is an abbreviation for the descriptive term, “nonatonic,”\textsuperscript{6} and “013” designates the first three integers of the scale.

Sharing an identity with Messiaen’s third “Mode of Limited Transposition,” perhaps the nine-step scale’s most prominent feature is that of transpositional symmetry. The scale can only be transposed four times before it completely duplicates itself. Each transposition by four or eight semitones will yield an identical set of pitch-classes. Therefore, a scale identified as NON\textsubscript{457} is identical in pitch-class content to the scale NON\textsubscript{013}, and will be referred to exclusively as such.

\textsuperscript{6} The abbreviation “NON” for “nonatonic” follows the abbreviations of “OCT” and “HEX” for the octatonic and hexatonic scales, respectively, which are already in common usage.
Once again using “mode I” as the standard model, the four possible transpositions of the nine-step scale are presented in Figure 1-2. Identifying labels have been placed above each scale, while pitch-class integers reflecting the content of each scale have been placed underneath.

![Figure 1-2. The Four Transpositions of the Nine-Step Scale](image)

This particular labeling system will be used throughout this document. It allows for an effortless comparison between transpositions of the scale, which will be a useful tool in Chapter 3. It also provides a quick and easy way to identify the use of a particular scale within a musical composition, which will be invaluable in Chapters 4 through 6.

Although the construction of the scale appears straightforward and the labeling system introduced thus far is simple and logical, there are times when a deeper
understanding of the scale within a unique and, until now, relatively unexplored tonal system is necessary. It will be useful, at times, to discuss the nine-step scale and its use in music within a mod-9 pitch-class system. I take as my point of departure the work of John Clough and several of his collaborators, who have explored the diatonic set in mod-7 pitch-class space.\(^7\)

Let us begin with a familiar conceptual tool in set theory—the clockface. The circle in Figure 1-3 is surrounded by the integers 0, 1, 2 . . . 8, representing the nine steps of the nine-step scale. These integers, then, represent nine unique pitch-classes, to which I have assigned specific pitches, corresponding to a “mode I” scale beginning on C.

![Figure 1-3. Mod-9 Representation of the Nine-Step Scale](image)

The arrangement of the nine-step scale around a clockface helps us understand one very important feature of the scale—its interval content. Within the nine-step scale, there exist two types of intervals, which Clough and Myerson refer to as *generic* and *specific.* The distance between scale-steps, or between mod-9 pitch-classes, determines the size of a generic interval, while the distance between pitches, measured in semitones, determines the size of a specific interval. In the nine-step scale, each generic interval comes in either one or two sizes. The relationship between these intervals is shown in Table 1-1. Following the notational system of Clough, Engebretsen and Kochavi, generic intervals are placed inside angle brackets (e.g. \(<2>\)), and specific intervals are placed inside curly braces (e.g. \(\{2,3\}\)).

\[
\begin{array}{ll}
<1> & = \{1,2\} \\
<2> & = \{2,3\} \\
<3> & = \{4\} \\
<4> & = \{5,6\} \\
<5> & = \{6,7\} \\
<6> & = \{8\} \\
<7> & = \{9,10\} \\
<8> & = \{10,11\}
\end{array}
\]

Table 1-1. Generic and Specific Intervals Contained Within the Nine-Step Scale

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8 “Variety and Multiplicity in Diatonic Systems,” p. 249.
Generic intervals can also be understood as pitch-class intervals in a mod-9 system. These pitch-class intervals, as is the case in a mod-12 system, form complementary pairs—1-8, 2-7, 3-6, 4-5—and it follows that there exist four unique mod-9 interval classes (ic1, ic2, ic3 and ic4).

Understanding the scale as part of a mod-9 system will also be useful when comparing melodic segments. This will be especially true when looking at Alexander Tcherepnin’s work, *Symphonic Prayer*, and Gregory Proctor’s work, *Three Pieces for Violins*, which both incorporate serialism. To illustrate the importance of this understanding, let us look at two examples of pitch segments. Figure 1-4 shows an eight-note passage, beginning on C (labeled segment “A”), and a second eight-note passage beginning on E-flat (labeled segment “B”). The mod-12 pitch-class integers are given above the score, and the corresponding mod-9 pitch-class integers are provided underneath the score.
Looking at these two pitch segments through a mod-9 lens yields different results than those seen through a mod-12 lens. This is made clear by a simple comparison of ordered pitch-class intervals. Let us first compare the two melodies within a mod-12 system. Table 1-2 shows that the ordered pitch-class intervals of the two melodies are not the same, and therefore segment B is not a transposition of segment A. Table 1-3, on the other hand, shows the relationship between the two melodies using mod-9 arithmetic. Here, we see that, taken in the mod-9 system of the nine-step scale, segment B is the transposition by two scale-steps of segment A \( [B = T_2(A)] \). A mod-9 system can provide very important results in the analysis of nine-step music.
There are many other aspects of the nine-step scale as a mod-9 pitch-class system available for further investigation. For the purposes of this paper, however, the information presented thus far will suffice. Let us now explore the observations of Alexander Tcherepnin on the nine-step scale.
Chapter 2: The Nine-Step Scale as Explained by Alexander Tcherepnin

Background

Prior to the theoretical development of the nine-step scale, many of Tcherepnin’s compositions involved the use of both the major-minor tetra/chord and the major-minor hexachord. It is out of the pervasive use of these harmonic structures that the nine-step scale developed. An example of each of these constructs is given below:

![Diagram of Major-Minor Tetra/Chord and Major-Minor Hexachord](image)

**Figure 2-1. The Structure of Major-Minor Tetra/Chords and Hexachords**

The major-minor hexachord is formed by continuing the interval pattern of three semitones followed by one semitone that was established by the tetra/chord. As a result,

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10 Here, the use of both notational methods (-/chord and -/chord) is intentional. In Tcherepnin’s early works, the use of a tetra/chord as a harmonic simultaneity is common. Hexachords, on the other hand, appear almost exclusively as a melodic process.
two overlapping major-minor tetra/chords are contained within the hexachord. For this reason, it is no surprise that major-minor hexachords are quite prevalent in the early works of Tcherepnin—they create an ongoing pattern of major-minor tetra/chords.

These hexachords, furthermore, are directly related to the formation of the nine-step scale. In his treatise, Basic Elements of My Musical Language, Tcherepnin describes the scale as being the result of “the addition of two major-minor hexachords.” Specifically, as will be shown shortly within the context of his treatise, the combination is of one hexachord with its inversion. As the scale began to develop in his music, Tcherepnin included many major-minor hexachords, sometimes with their inversion, and sometimes without.

In an effort to better understand the concept and origins of the scale, as well as its properties and its possible uses, let us now turn to Tcherepnin’s Basic Elements of My Musical Language. In addition to presenting Tcherepnin’s own words and illustrations, I will provide a theoretical commentary on his work, clarifying some elements and adding some insights of my own.

A Critical Analysis of Tcherepnin’s “Basic Elements of My Musical Language”

On the first page of his treatise, Tcherepnin provides an explanation of his terminology for various types of “chords.” When speaking of a succession of adjacent pitches in a scalar arrangement, he uses one-word terms such as “tetra/chord,” “pentachord,” and “hexachord.” When speaking of notes that together form a musically harmonic structure, on the other hand, he uses compound words, separated by a slash.
mark, such as “tetra/chord,” “penta/chord” and “hexa/chord.” This system is certainly sufficient, and I will make use of it in this document as well.

In discussing the nine-step scale, Tcherepnin first explains how the scale is constructed:

**Major-minor tetrachords are constructed within the interval of a major third using two half-steps and one whole-step:**

![Figure 2-2. Excerpt 1 from Basic Elements of My Musical Language](image)

In general terms, these tetrachords are simply scalar constructions formed by every possible combination of two half-steps and one whole-step. The designation of each construction as a unique mode is necessary, as each mode ultimately leads to the construction of its own “mode” of the scale, just as Ionian, Dorian, Phrygian, and so forth are all different modes of the same diatonic set of pitches.

One may wonder why Tcherepnin chose to call the half-step/whole-step/half-step construction “mode I.” It is my assertion that Tcherepnin thought the symmetry of this construction to be the most stable. As stated previously, Tcherepnin was particularly

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11 Korabelnikova, p. 192.
12 Unfortunately, in Korabelnikova’s book, the use of compound words is eliminated and only the one-word terms are used. The Tcherepnin Society’s website (http://www.tcherepnin.com/alex/basic_elem1.htm) does, however, restore his notation of harmonic constructions with slash marks.
13 Korabelnikova, p. 193.
drawn to the major-minor tetra/chord. A simple glance at the construction of this chord, as shown in Figure 2-3, reveals its similarity in symmetrical construction to the mode I tetrachord. The four pitches of the tetra/chord are derived from the arrangement of one half-step (compared to the one whole-step of the tetrachord) surrounded by two one-and-a-half-steps (compared to the two half-steps of the tetrachord). Specifically, the “major-minor” construction places two equal-sized steps on the outside and a single unique step on the inside.

![Figure 2-3. Construction of the Major-Minor Tetra/Chord and the Mode I Tetrachord](image)

Additionally, just as the leading tone in a major scale creates a sense of stability in the tonic, and the lowered second scale degree performs similarly in the Phrygian mode, the half-step placement in the mode I tetrachord creates simultaneous stability in both its lowest and its highest pitches.

Tcherepnin continues:

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14 Throughout other portions of this document, I discuss most interval distances in terms of the number of semitones, rather than the number of whole- and half-steps. In my analysis of *Basic Elements*, however, I refer to whole- and half-steps to maintain consistency with Tcherepnin’s language.
Major-minor hexachords are constructed within the interval of a major seventh using an alternation of half-step and one-and-a-half-step intervals:

with one octave added to complete the row.\(^{15}\)

As shown in the example, mode II is the inversion of mode I.

Figure 2-4. Excerpt 2 from *Basic Elements of My Musical Language*\(^{16}\)

Once again, Tcherepnin identifies a unique mode for each construction. Here, it is likely that he chose to call the construction with a step-and-a-half as the lowest interval “mode I” because the four lowest pitches create a major-minor tetra/chord.

Tcherepnin’s major-minor hexachord is identical to the pitch-class collection Richard Cohn identifies as the “hexatonic system,” although Cohn’s collection is derived

\(^{15}\) The representations of *Basic Elements* by both Korabelnikova and the Tcherepnin Society website (and therefore most likely also the manuscript by Tcherepnin) present a colon after the word, “row.” I have made an editorial change, following “row” with a period, since the addition of the octave was already present in the preceding musical notation.

\(^{16}\) Korabelnikova, p. 193.
from chords rather than a scale.\textsuperscript{17} The major-minor hexachord is also one of six hexachords that Milton Babbitt describes as an “‘all-combinatorial’ source set.”\textsuperscript{18} It is this shared identity that creates an ambiguity of intent in the compositions of Tcherepnin and others in whose music the nine-step scale is present. One goal of this paper is to shed new light on some of this ambiguity.

One may be distracted by an inconsistency in how Tcherepnin presents the modes of each type of note group in his notational illustrations. In defining the three modes of major-minor tetrachords, Tcherepnin simply begins each tetrachord on the next “scale degree,” giving the modes a \textit{relative} relationship, much like the relationship shared by relative major and minor keys. With the major-minor hexachords, however, Tcherepnin presents each mode using C as the fundamental, or lowest, pitch. Although this may be seen as an inconsistency, it has no bearing on the basic theoretical characteristics of the scale. Basing both hexachords on C provides a logical setup to illustrate the formation of the nine-step scale.

Once Tcherepnin has explained the make-up of major-minor hexachords and their inversions, he illustrates how the union of a hexachord and its inversion forms the nine-step scale. It appears that Tcherepnin understands the inversion to be around the “tonic,” or the first “scale degree” of the hexachord. As it turns out, the inversion of any major-minor hexachord $T_n I$ such that $n = \text{any even integer}$ results in a nine-step scale. Observe Figure 2-5.

\begin{itemize}
\end{itemize}
Note that C, E, and Ab are common to both hexachords and that the major third interval between them is the same distance covered by the tetrachord of mode I.

The nine-step scale, which results from the addition of two major-minor hexachords is therefore based on three interlocking major-minor tetrachords and can have three modes:

Figure 2-5. Excerpt 3 from *Basic Elements of My Musical Language*¹⁹

With this illustration, Tcherepnin effectively arrives at his scale from two directions, yet the two directions are inherently associated. This is one reason why—as will be seen from my analyses of several musical compositions—so many composers use the nine-step scale, but only some are acutely aware of it.

¹⁹ Korabelnikova, pp. 193-194.
Once Tcherepnin has presented the basic structure of his scale, he provides a tremendous amount of detail about the various ways it can appear and its harmonic features. First, Tcherepnin details the possible configurations of the scale:

Because the interlocked tetrachords have identical intervals, the nine-step major-minor scale can have three points of departure (three tonics) in the same row, which are indicated by changes in notation (not by transposition).

As each nine-step row can have three modes and three tonics for each of the modes (differing by notation), this results in a total of 9 nine-step scales for each row: 3 fundamental and 6 derivative (through change of notation).

Figure 2-6. Excerpt 4 from Basic Elements of My Musical Language

Tcherepnin’s notational illustration provides the best explanation of the forms of the scale. It is useful to understand that a “row,” as Tcherepnin refers to it, is a fixed collection of nine pitch-classes. Any set of pitch-classes formed by the union of a major-

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20 ibid., p. 194.
minor hexachord and its inversion (as previously described) creates a row that can
generate nine possible nine-step scales, and all nine scales have the same pitch-class
content, regardless of their enharmonic spellings.

Tcherepnin refers to changes of notation when discussing the three tonics of a
nine-step scale, but he does not make it explicitly clear whether or not there is a standard
notation for the scales built on each tonic. For example, it would seem that in the tonic
G-sharp derivative scales, E-natural would be a more conventional spelling than D-
double-sharp. It is unclear why Tcherepnin uses the latter spelling.

Tcherepnin then theorizes about transpositions of the scale:
Each nine-step row can be transposed [only] three times in “fixed Doh” terms, [i.e., by a half-step, a whole step, or a step-and-a-half], since any further transposition merely yields one of the four fundamental rows (or scales), with the tonic (fundamental tone of the start) placed a major third above or below.

There can therefore be 4 fundamental nine-step scales, each with three modes (12 scales altogether) and 8 derivative nine-step scales (obtained by change of notation), each with 3 modes each (24 altogether).

This brings the number of nine-step-scale “tonalities” or modes to 36.

The notation will indicate the position of the point of departure (the tonic) and can be said to indicate the position of the tonal center.

Figure 2-7. Excerpt 5 from Basic Elements of My Musical Language

21 ibid., p. 194.
Tcherepnin’s terminology here warrants brief clarification. In Figure 2.6, Tcherepnin refers to the scales built on each tonic as “fundamental,” leading to the interpretation that each unique nine-step pitch-class collection includes three “fundamental” scales. In the case of Figure 2.6, then, the three “fundamental” scales are the mode I scales starting on the pitches C, E and G-sharp. Modes II and III, then, would be considered “derivative” scales. In Figure 2.7, on the other hand, Tcherepnin refers to each unique transposition of the nine-step collection as a “fundamental” scale. Here, the “derivative” scales refer to the scales built on the remaining two tonic pitches of each nine-step collection.

A total of 36 tonalities with overlapping pitch-class content inevitably creates a challenge when discussing the use of two or three of the tonalities. A row must first correspond to one of the fundamental scales—C, G, D or A—according to Tcherepnin’s model in ascending fifths. The row must then be identified with a specific tonic and mode. Breaking the nine-step scale down into rows, modes, and tonics makes absolute theoretical sense, but becomes cumbersome when used in music analysis.

One may find it useful to identify each scale simply by tonic pitch and mode, much as we do when we identify a key as “A major” or “D mixolydian.” Since there are only four unique rows and each row has three tonics, then there are exactly twelve possible tonic pitches, corresponding to the twelve pitch-classes of the dodecaphonic system. Therefore, one can identify a scale by first naming its tonic pitch. Then, to specify in which mode the scale appears, one could add the appropriate mode numeral: I, II or III. Mode I represents a scale whose pattern of whole-steps (W) and half-steps (H)
begins H-W-H. Mode II begins W-H-H, and mode III begins H-H-W. Figure 2-8 shows the three different modes of a C-tonic scale:

![C-I scale](image1)

![C-II scale](image2)

![C-III scale](image3)

**Figure 2-8. Tonic-Mode Labeling System for Nine-Step Scales**

This alternate labeling system, although straightforward and logical, is slightly discordant with Tcherepnin’s theory. The scale labeled C-I above does correspond to Tcherepnin’s mode I scale on the C-tonic of the C-row. The scale, C-II, however, corresponds to what Tcherepnin would describe as a mode II scale on the B-tonic of the G-row. Finally, C-III corresponds to a mode III scale built on the A-tonic of the A-row. Tcherepnin’s terms create a distinction between the “tonic” and the “tonal center,”

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22 The use of modes I, II and III in this labeling system remains consistent with Tcherepnin’s use of modes I, II and III.
suggesting that the two terms refer to two separate ideas. A labeling system where the
tonic pitch and the tonal center are one and the same seems to be a more sensible and
intuitive one, in this case, than Tcherepnin’s.

Labeling scales with tonic pitches and mode identifiers still allows for a total of
36 different tonalities. For this reason, the labeling system has the potential for being
extremely useful in the analysis of nine-step music, especially when a specific tonic pitch
is clearly presented. Many times, however, a tonal center is not apparent, and one cannot
easily identify a tonic or a mode, but can only identify one of the four transpositions of
the nine-step collection.

When a tonal center is not obvious, or when speaking of the four transpositions of
the nine-step scale in more generic terms, I would like to return to the labeling system
that was offered in Chapter 1. This system also operates in the dodecaphonic system,
using pitch-class integers to identify each transposition of the scale. These post-tonal
labels identify the nine-step scale as “NON” with pitch-class integer subscripts that
identify the specific transpositions. Recall that the subscript of three numbers represents
the lowest integers possible for the beginning of a mode I scale. While there are 36
different tonic-mode labels, there are only four post-tonal labels (NON_{013}, NON_{124},
NON_{235} and NON_{346}). Whereas the tonic-mode labels identify specific tonal centers, the
post-tonal labels do not. Table 2-1 illustrates the relationship between the tonic-mode
labeling system and the post-tonal labeling system. The music analyses in Chapters 4
through 6 will make exclusive use of the post-tonal labeling system.
<table>
<thead>
<tr>
<th>Tonic-Mode Label</th>
<th>Representation of Scale in Pitch-Class Integers</th>
<th>Post-Tonal Label</th>
</tr>
</thead>
<tbody>
<tr>
<td>C-I</td>
<td>0 1 3 4 5 7 8 9 e</td>
<td>NON\textsubscript{013}</td>
</tr>
<tr>
<td>C-II</td>
<td>0 2 3 4 6 7 8 t e</td>
<td>NON\textsubscript{346}</td>
</tr>
<tr>
<td>C-III</td>
<td>0 1 2 4 5 6 8 9 t</td>
<td>NON\textsubscript{124}</td>
</tr>
<tr>
<td>Db-I</td>
<td>1 2 4 5 6 8 9 t 0</td>
<td>NON\textsubscript{124}</td>
</tr>
<tr>
<td>Db-II</td>
<td>1 3 4 5 7 8 9 e 0</td>
<td>NON\textsubscript{013}</td>
</tr>
<tr>
<td>Db-III</td>
<td>1 2 3 5 6 7 9 t e</td>
<td>NON\textsubscript{235}</td>
</tr>
<tr>
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<td>2 3 5 6 7 9 t 1</td>
<td>NON\textsubscript{235}</td>
</tr>
<tr>
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<td>NON\textsubscript{124}</td>
</tr>
<tr>
<td>D-III</td>
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<td>NON\textsubscript{346}</td>
</tr>
<tr>
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<td>NON\textsubscript{346}</td>
</tr>
<tr>
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</tr>
<tr>
<td>Eb-III</td>
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<td>NON\textsubscript{013}</td>
</tr>
<tr>
<td>E-I</td>
<td>4 5 7 8 9 e 0 1 3</td>
<td>NON\textsubscript{013}</td>
</tr>
<tr>
<td>E-II</td>
<td>4 6 7 8 t e 0 2 3</td>
<td>NON\textsubscript{346}</td>
</tr>
<tr>
<td>E-III</td>
<td>4 5 6 8 9 t 0 1 2</td>
<td>NON\textsubscript{124}</td>
</tr>
<tr>
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<td>5 6 8 9 t 0 1 2 4</td>
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</tr>
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</tr>
<tr>
<td>F-III</td>
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<td>NON\textsubscript{235}</td>
</tr>
<tr>
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<tr>
<td>Gb-III</td>
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<tr>
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</tr>
<tr>
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</tr>
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<tr>
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<tr>
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<td>Bb-III</td>
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</tr>
<tr>
<td>B-I</td>
<td>e 0 2 3 4 6 7 8 t</td>
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<td>B-II</td>
<td>e 1 2 3 5 6 7 9 t</td>
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</tr>
<tr>
<td>B-III</td>
<td>e 0 1 3 4 5 7 8 9</td>
<td>NON\textsubscript{013}</td>
</tr>
</tbody>
</table>

Table 2-1. Two Alternate Labeling Systems for Nine-Step Scales
Next, Tcherepnin discusses the harmonic implications of the scales:

**If used in major thirds or minor sixths the scale will keep those intervals audibly intact (although some of the thirds will be written as diminished fourths, and some of the sixths will be written as augmented fifths):**

![Harmonic Implications Diagram](image1)

**If used in minor thirds or in major sixths, the intervals will audibly vary between minor thirds and major seconds (when in thirds) or between major sixths and minor sevenths (when in sixths):**

![Harmonic Implications Diagram](image2)

Figure 2-9. Excerpt 6 from *Basic Elements of My Musical Language*²³

At this point, it is important to make a distinction between traditional “tonal” harmony and Tcherepnin’s “nine-step” harmony. Tcherepnin’s theory makes use of both harmonic systems, and it is sometimes unclear to which system he is referring.

“Tonal” harmony reflects the conventional terminology commonly used to refer to intervals, chords, and functions within the major and minor diatonic systems. In the case of Tcherepnin’s illustration in Figure 2-9, a “minor third” is the span of three semitones, a “major third” is the span of four semitones, and so forth. With the exception

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²³ Korabelnikova, p. 195.
of the use of enharmonic spellings, tonal terminology is completely adequate when using
the nine-step scale in major thirds. When using the scale in minor thirds, on the other
hand, confusion arises.

To rationalize the use of minor thirds in the context of “nine-step” harmony, one
must understand the scale in terms of the mod-9 pitch-class system introduced in Chapter
1, where the scale and its resultant harmonies are best understood in a relationship
between scale steps. Figure 2-10 illustrates the nine scale degrees assigned to each of the
pitches of the NON\textsubscript{013} scale.

![Figure 2-10. Scale Degrees of the NON\textsubscript{013} Scale](image)

Looking once again at the combination of major thirds in the scale, one sees that
the tonal interval of a major third is formed between every three scale steps—between
scale degrees 1 and 4, 2 and 5, 3 and 6, and so on. In other words, the generic mod-9
interval <3> is present in the scale as one specific size: four semitones. A tonal minor
third, on the other hand, is formed between scale degrees 1 and 3 and between 2 and 4
(enharmonically), but a tonal major second is formed between scale degrees 3 and 5.
Here we see that the generic interval <2> comes in two specific sizes: two or three
semitones. Tcherepnin addresses this issue by calling the aural major second a “nine-step
In Tcherepnin’s terminology, the mod-9 interval <2> forms a “minor” third, whether it is tonal or not.

Tcherepnin continues, addressing other combinations of intervals:

**If used in fourths or fifths, the intervals will audibly vary between perfect and augmented fourths (when in fourths) or between perfect and diminished fifths (when in fifths).**

![Diagram of intervals in fourths and fifths](image)

**If used in minor seconds or major sevenths, the intervals will vary audibly between major and minor seconds (when in seconds) or major and minor sevenths (when in sevenths).**

![Diagram of intervals in minor seconds and major sevenths](image)

**Figure 2-11. Excerpt 7 from Basic Elements of My Musical Language**

The tonal intervals in the illustrations thus far have been named consistently in reference to a mode I scale, with the intervals of comparison being measured from scale degree 1 to the other steps of the scale. As a result, scale degree 1 to scale degree 2 forms

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24 ibid., p. 196.
a minor second, degrees 1 to 3 form a minor third, degrees 1 to 4 form a major third, degrees 1 to 5 form a perfect fourth (or simply, a fourth), degrees 1 to 6 form a perfect fifth (or simply, a fifth), degrees 1 to 7 form a minor sixth, degrees 1 to 8 form a major sixth, and degrees 1 to 9 form a major seventh. Notice that not all the possible tonal intervals are represented. It follows, then, that major seconds, tritones and minor sevenths theoretically do not exist in the nine-step system, except where they appear as unique “nine-step” intervals.

Table 2-2 adds Tcherepnin’s interval labels to the mod-9 generic and specific interval labels that were introduced in Chapter 1 (Table 1-1).

<table>
<thead>
<tr>
<th>m2</th>
<th>= &lt;1&gt; = {1,2}</th>
<th>5</th>
<th>= &lt;5&gt; = {6,7}</th>
</tr>
</thead>
<tbody>
<tr>
<td>m3</td>
<td>= &lt;2&gt; = {2,3}</td>
<td>m6</td>
<td>= &lt;6&gt; = {8}</td>
</tr>
<tr>
<td>M3</td>
<td>= &lt;3&gt; = {4}</td>
<td>M6</td>
<td>= &lt;7&gt; = {9,10}</td>
</tr>
<tr>
<td>4</td>
<td>= &lt;4&gt; = {5,6}</td>
<td>M7</td>
<td>= &lt;8&gt; = {10,11}</td>
</tr>
</tbody>
</table>

Table 2-2. Comparison of Tcherepnin’s Interval Labels to Generic and Specific Intervals Contained Within the Nine-Step Scale

Tcherepnin’s illustration of possible arpeggiations follows the same theoretical thought as the interval system previously addressed. Arpeggiations by “minor thirds” involve every other note of a mode I scale, beginning on its tonic (nine-step scale degrees
1-3-5-7-9-2-4-6-8-1). This produces a complete arpeggiation—the succession of unique pitches before returning to the tonic—which spans two octaves. Arpeggiations by “major thirds” involve every third note of the scale (scale degrees 1-4-8-1); its complete arpeggiation spans only one octave. In a similar fashion, arpeggiations by “fourths” use every fourth note of the scale, spanning four octaves; arpeggiations by “fifths” use every fifth note, spanning five octaves; arpeggiations by “minor sixths” use every sixth note, spanning two octaves; arpeggiations by “major sixths” use every seventh note, spanning seven octaves; and arpeggiations by “sevenths” use every eighth note, spanning eight octaves. Tcherepnin’s illustrates these arpeggios in Figure 2-12.
In arpeggiated form, the nine-step scale can be presented in the following ways:

by minor thirds:

by major thirds:

by fourths:

by fifths:

by minor sixths:

by major sixths:

by sevenths:

Figure 2-12. Excerpt 8 from Basic Elements of My Musical Language\textsuperscript{25}

\textsuperscript{25} ibid., p. 196.
The arpeggiation outlined above provide a theoretical aspect of the scale, albeit not a very practical one. Arpeggiation based on conventional tonality, such as triads and seventh chords, are generally confined to the octave. Although ninth, eleventh, and thirteenth chords are certainly common in tonal music, when one speaks of arpeggiation, they are typically referring to a broken triad or seventh chord. Quartal harmonies are also common in tonal music, but arpeggiation of quartal harmonies rarely exceed four or five chord members. Occurrences of full arpeggiation by minor thirds, fourths, fifths, major sixth or sevenths are rare in Tcherepnin’s music.

At this point, Tcherepnin moves away from purely intervallic constructions, introducing chords. Each chord is constructed from a fixed succession of scale degrees. For example, the nine-step triad is formed from scale degrees 1, 3 and 4, and the nine-step tetra/chord is comprised of scale degrees 1, 3, 4, and 6. Unlike with the intervals, Tcherepnin now includes the structure in each mode. Nine-step triads are outlined in Figure 2-13.
Tcherepnin also introduces the idea of an “inverse position” for the triads. In order to understand his process of inversion, one must employ set theory within a mod-9 pitch-class universe.\textsuperscript{27} The mode I triad illustrated in Figure 2-13 is made up of the pitches C, E-flat and E-natural, which correspond to the mod-9 pitch-classes (0, 2, 3). The mod-9 pitch-classes of its inverse position, as shown in Figure 2-13, are (0, 1, 3), arrived at through $T_3I(0, 2, 3)$. The mode II triad is represented by the mod-9 pitch-classes (1, 3, 4) and its inversion ($T_3I$) results in the mod-9 pitch-classes (1, 2, 4). Finally,

\textsuperscript{26} ibid., p. 197.
\textsuperscript{27} The mod-9 clockface presented in Figure 1-3 is a useful visual aid in understanding the inversions of nine-step triads.
the mode III triad is represented by the mod-9 pitch-classes (2, 4, 5) and its inversion (T:1) yields the mod-9 pitch-classes (2, 3, 5).\textsuperscript{28}

Tcherepnin then introduces the “fundamental perfect chord,” addressed in Figure 2-14. It is to this structure that the development of the nine-step scale is ultimately attributed.

The fundamental perfect chord of nine-step scale harmony is the major-minor tetra/chord.

\textbf{THE MAJOR-MINOR TETRA/CHORD AND ITS INVERSIONS:}

\textbf{Figure 2-14. Excerpt 10 from Basic Elements of My Musical Language}\textsuperscript{29}

Notice that the spelling of the mode II tetra/chord and the quality of the mode III tetra/chord are not consistent with what musicians commonly recognize as the major-

\textsuperscript{28} For the purpose of comparing the mod-9 pitch-class integers with the notated pitches of Figure 2-13, I provide the following key:

\begin{align*}
\text{Mode I:} & \quad \text{mod}-9 (0, 2, 3) = \text{mod}-12 (0, 3, 4) \quad \text{mod}-9 (0, 1, 3) = \text{mod}-12 (0, 1, 4) \\
\text{Mode II:} & \quad \text{mod} 9 (1, 3, 4) = \text{mod}-12 (1, 4, 5) \quad \text{mod}-9 (1, 2, 4) = \text{mod}-12 (1, 3, 5) \\
\text{Mode III:} & \quad \text{mod}-9 (2, 4, 5) = \text{mod}-12 (3, 5, 7) \quad \text{mod}-9 (2, 3, 5) = \text{mod}-12 (3, 4, 7)
\end{align*}

\textsuperscript{29} Korabelnikova, p. 197.
minor tetra/chord. Once again, Tcherepnin’s derivation of harmonic structure is dependent on the tones available from a given nine-step scale. Rather than defining the major-minor tetra/chord as a tonal triad with a split third, Tcherepnin’s chord is simply the union of scale degrees 1, 3, 4 and 6 in each of the three modes of the scale.

Due to the symmetrical nature of the nine-step scale, major-minor tetra/chords in this system have no distinct harmonic function. The quality of the “tonic” chord of the C-I scale (C-Eb-E-G) is audibly different from that of the “tonic” chord of the C-III scale (C-D-E-F#). Also, the frequent and regular occurrence of half-steps in the scale prevents any single pitch from acquiring the absolutely stable role of tonic. Tonic chords and tonic pitches in Tcherepnin’s world are entirely theoretical, and are used as an organizational tool rather than as an auditory compass. Tcherepnin continues:

**In five-part settings in which the fundamental tetra/chord is considered as stable (and final) any note of the fundamental tetra/chord can be doubled; the pentachord introduces the element of instability.**

![Diagram of minor and major penta/chords](image)

**Figure 2-15. Excerpt 11 from *Basic Elements of My Musical Language*[^30]**

One can interpret Tcherepnin’s penta/chords, in their simplest form, as a succession of every other pitch from a scale, remaining within an octave. In other words,

[^30]: ibid., p. 197.
it is a one-octave arpeggiation of “minor thirds.” In the example provided in Figure 2-15, the major penta/chord can be identified as scale degrees 1, 3, 5, 7 and 9 of the C-I scale. Similarly, the minor penta/chord can be identified as the same scale degrees, but of the G-III scale.\textsuperscript{31} If this is, in fact, the process by which Tcherepnin formed these penta/chords, then the designations of quality (major or minor) are related to the outside interval of the chord in “root position.” The penta/chord built on the tonic of the mode I scale has an outside tonal interval of a major seventh, while the penta/chord constructed from the mode III scale has an outside interval of a minor seventh.

Tcherepnin does not address the penta/chord built on the tonic of a mode II scale. Such a chord would result in an outside tonal interval of a major seventh, identifying it as another major penta/chord. The interior intervallic structure, however, differs from that of the mode I chord. Both chord constructions contain three minor thirds (or the enharmonically equivalent augmented second) and one major second. Once again, however, Tcherepnin’s theory results in four “equal” interior intervals of nine-step minor thirds. Compare the examples in Figure 2-16, which identify the interior intervals in terms of mod-9 generic and specific intervals.

\textsuperscript{31} Tcherepnin’s illustration of the minor penta/chord is shown in an inverted form. Viewing G as the “root” of the chord shows that this chord was formed from a G-III scale (with the D# enharmonically respelled as Eb).
But perhaps there is more to Tcherepnin’s omission of a penta/chord built on the tonic of a mode II scale. Looking back at the minor penta/chord, there may be a reason Tcherepnin uses an E-flat spelling rather than D-sharp. Perhaps the minor penta/chord is not simply a tertian harmony built on the tonic of a mode III scale, but is instead a “functional” harmony within the context of a mode I tonality. It is plausible that the minor penta/chord holds a “dominant” function in mode I. In this case, the minor penta/chord is not built on the tonic of the G-III scale, but rather on scale degree 6 of the C-I scale. This, in fact, makes greater theoretical sense, considering both the spelling of the chord and the omission of a third type of penta/chord.

As is explained at the end of his discourse on the nine-step scale, Tcherepnin views a “stable” harmony as being one in which the harmony includes one less pitch than
there are performing parts. If every part produces a unique pitch, the resulting harmony must resolve to a harmony that doubles one pitch. Such a resolution is specifically addressed in Figure 2-17.

**The minor penta/chord is tonal in its “resolution”:**

![Figure 2-17. Excerpt 12 from Basic Elements of My Musical Language](image)

The transformation of the penta/chord into a tetra/chord essentially involves the half-step resolution of all but one pitch. The pitch that does not resolve is common to both chords. Tcherepnin’s illustration does not make this clear. Figure 2-18 shows another way to view the voice-leading of the resolution of the same minor penta/chord.

![Figure 2-18. Parsimonious Voice-Leading in the Resolution of a Minor Penta/Chord](image)

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32 Korabelnikova, p. 198.
Two items remain unclear in Tcherepnin’s illustration in Figure 2-17. The first is the theoretical significance of his voice-leading. Why do some voices move by semi-tone while others move by a whole tone, when parsimonious voice-leading allows all voices to move by semi-tone while one pitch remains the same? The second item is the resolution of the penta/chord to a mode II fundamental tetra/chord. If the penta/chord does in fact serve a functional purpose, why does it not resolve to its own (mode I) fundamental tetra/chord? The answers to these questions, unfortunately, lie outside the scope of this study and will have to be determined at a later time.

Of course, one should notice that a resolution from this particular penta/chord to the C-I fundamental tetrachord would produce three common tones—not the type of resolution that creates a desirable effect of tension and release. Perhaps the true function of the minor penta/chord is that of a leading-tone harmony in mode II of the C tonic C row (Db-II), which is the relative mode II to C-I.

Overall, I see a deficiency in Tcherepnin’s explanation of penta/chord resolutions—a deficit that only increases when he explains the resolution of the major penta/chord, as shown in Figure 2-19.
The major penta/chord can effect modulation by nine-step resolutions that arrive at a new tonal center:

Figure 2-19. Excerpt 13 from *Basic Elements of My Musical Language*\(^{33}\)

In an effort to understand Tcherepnin’s theory of modulation, I considered every possible resolution that results in only one common tone and where the other voices move only by step (by tone or semi-tone). With these as the only criteria, there were fourteen different fundamental tetra/chords to which the major penta/chord could resolve. Two of these resolutions would be considered “tonal,” however, in that they resolve to tetra/chords of scales that are derived from Tcherepnin’s C scale (G-III and A-II). The fundamental tetra/chords to which the major penta/chord resolved are shown in Figure 2-20 (the resolution Tcherepnin highlights is enclosed with a box).

\(^{33}\) ibid, p. 198.
<table>
<thead>
<tr>
<th>FUNDAMENTAL SCALE:</th>
<th>C Scale</th>
<th>G Scale</th>
<th>D Scale</th>
<th>A Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCALES OF RESOLUTION:</td>
<td>G-III</td>
<td>Bb-III</td>
<td>D-I</td>
<td>A-I</td>
</tr>
<tr>
<td></td>
<td>A-II</td>
<td>Eb-I</td>
<td>Eb-II</td>
<td>Bb-II</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G-I</td>
<td>G-II</td>
<td>C-III</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bb-I</td>
<td>D-II</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>E-III</td>
</tr>
</tbody>
</table>

Table 2-3. The “Best” Resolutions of the Major Penta/Chord

Table 2-3 makes it clear that there are many more non-tonal resolutions available to the major penta/chord than tonal ones.

The particular example provided by Tcherepnin is unique among all the other resolutions in that three of the notes from the penta/chord move by tone and only one moves by semi-tone (the fifth pitch, of course, is the common tone). In every other resolution, at least two voices move by semi-tone. Perhaps this is the underlying circumstance that Tcherepnin intended to highlight.

Tcherepnin’s final comments on the theory of the nine-step scale, provided in Figure 2-20, address the relative stability and instability of chords in various musical settings.
In six-part settings the penta/chord (major or minor) is considered as stable, while the hexa/chord is unstable, finding resolution in a penta/chord, and so on, up to the point at which—in ten-part harmony—the entire nine-scale becomes a stable chord, with instability provided by an extra tonal appoggiatura (in ten-part settings), two extra appoggiaturas (eleven-part) or three appoggiaturas (twelve-part).

Figure 2-20. Excerpt 14 from Basic Elements of My Musical Language

An example of the entire nine-step scale being used as a stable harmony is present in Tcherepnin’s Symphonic Prayer, Op. 93, which I will focus on in Chapter Four of this work.

To end the nine-step scale portion of his treatise, Tcherepnin provides a list of pieces in which he used the scale, beginning with those pieces for which only the hexachord was the focus. I will now discuss several of these pieces, drawing my own conclusions about the relationship between the theory and practice of Tcherepnin’s nine-step scale.

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34 ibid., p. 198.
Chapter 3: Additional Properties of the Nine-Step Scale

Symmetrical Properties of the Scale

In addition to being transpositionally symmetrical, as discussed in Chapter 1, the nine-step scale is also inversionally symmetrical. In other words, it will map entirely onto itself under inversion. A set class that is inversionally symmetrical will have an index vector that contains an entry equal to the number of notes in the set. In the case of the nine-step scale, there are nine entries at index numbers 2, 6, and 10:

\[
\begin{align*}
T_0: & \ [ 0 \ 1 \ 2 \ 4 \ 5 \ 6 \ 8 \ 9 \ t ] \\
T_2I: & \ [ 4 \ 5 \ 6 \ 8 \ 9 \ t \ 0 \ 1 \ 2 ] \\
T_6I: & \ [ 8 \ 9 \ t \ 0 \ 1 \ 2 \ 4 \ 5 \ 6 ] \\
T_1I: & \ [ 0 \ 1 \ 2 \ 4 \ 5 \ 6 \ 8 \ 9 \ t ] \\
\end{align*}
\]

Table 3-1. Inversions of NON_{124} Showing Inversional Symmetry of the Nine-Step Scale

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35 All properties of the nine-step scale discussed in this chapter assume a mod-12 pitch-class space.
36 Korabelnikova, p. 85.
37 For $T_nI$ where “n” equals the index number, there will be “x” common tones between the two sets:

<table>
<thead>
<tr>
<th>n:</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>t</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td>x:</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>9</td>
<td>6</td>
</tr>
</tbody>
</table>
These basic symmetrical properties become apparent simply by looking at the scale as a single set class. More intricate properties emerge, however, when one looks at the subset structure of the scale. Figure 3-1 provides an inclusion lattice for the nine-step scale. Not only is the scale itself a symmetrical set, but many of its subsets are as well. Subsets that have inversional symmetrical properties\textsuperscript{38} are identified by blue (or boldface) print.

\textsuperscript{38} I only mention inversional symmetry here because transpositional symmetry does not occur in any set class, other than at $T_0$, without there also being inversional symmetry.
Figure 3-1. Inclusion Lattice for Forte Set Class 9-12
Table 3-2 presents some of the most familiar symmetrical subsets in the tonal and post-tonal theory of Western music. This list contains fewer than half the total symmetrical subsets of the nine-step scale, but it highlights some of the more common pitch collections.

<table>
<thead>
<tr>
<th>Set Class</th>
<th>Prime Form</th>
<th>Common Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-1</td>
<td>(012)</td>
<td>Chromatic steps</td>
</tr>
<tr>
<td>3-6</td>
<td>(024)</td>
<td>Whole steps; Opening of a major scale</td>
</tr>
<tr>
<td>3-9</td>
<td>(027)</td>
<td>Add 2 or suspended triad (D2, Fsus, etc.)</td>
</tr>
<tr>
<td>3-10</td>
<td>(036)</td>
<td>Diminished triad</td>
</tr>
<tr>
<td>3-12</td>
<td>(048)</td>
<td>Augmented triad</td>
</tr>
<tr>
<td>4-3</td>
<td>(0134)</td>
<td>Mode 1 tetrachord of a nine-step scale</td>
</tr>
<tr>
<td>4-7</td>
<td>(0145)</td>
<td>Opening of a hexatonic scale</td>
</tr>
<tr>
<td>4-17</td>
<td>(0347)</td>
<td>Major-minor tetra/chord</td>
</tr>
<tr>
<td>4-20</td>
<td>(0158)</td>
<td>Major-major seventh chord</td>
</tr>
<tr>
<td>4-21</td>
<td>(0246)</td>
<td>Whole steps; Opening of a Lydian scale</td>
</tr>
<tr>
<td>4-26</td>
<td>(0358)</td>
<td>Minor-minor seventh chord</td>
</tr>
<tr>
<td>6-20</td>
<td>(014589)</td>
<td>Complete hexatonic scale (Major-minor hexachord)</td>
</tr>
<tr>
<td>6-35</td>
<td>(02468t)</td>
<td>Complete whole tone scale</td>
</tr>
</tbody>
</table>

Table 3-2. Familiar Symmetrical Subsets of the Nine-Step Scale
These subsets serve various purposes when used as compositional material. Sequential use of many of these pitch-class collections has the potential to result in the appearance of the nine-step scale. The following are four arbitrary examples of sequences that, using some of the symmetrical trichords listed in Table 3-2, have the potential to produce the nine-step scale:

![Figure 3-2. Sequential Use of Symmetrical Trichords That Results in the Nine-Step Scale](image)

Composers may arrive at the nine-step scale by way of using a series of the less commonly known subsets as well. Figure 3-3 provides three examples of other
symmetrical pitch-class sets (not limited to trichords) that combine to form the nine-step scale when used in a sequential pattern.

Figure 3-3. Sequential Use of Other Symmetrical Pitch-Class Sets That Results in the Nine Step Scale

Some symmetrical subsets can *not* result in the nine-step scale unless they are combined with a different subset. The whole-tone scale and all its subsets illustrate this point: set classes 3-6 (024), 4-21 (0246), 5-33 (02468), and 6-35 (02468t) cannot be used in sequence to produce the nine-step scale.

From this list of set classes, the easiest to illustrate is that of 6-35, the complete whole-tone collection. Combining two transpositions of set class 6-35 will do one of two things: it will duplicate each of the six pitch-classes (at T0, T2, T4, T6, T8 and T10) or it will result in all twelve pitch-classes (at T1, T3, T5, T7, T9 and T11), as illustrated in Figure 3-4.
Tonal Properties of the Scale

Music composed using the nine-step scale has the potential to sound quite tonal. The inclusion lattice displayed in Figure 3-1 allows us to see exactly how many tonal harmonies are present within the scale.

The nine-step scale contains many triads and seventh chords that are typically understood within the context of major or minor scales. Each nine-step scale includes twelve occurrences of the major or minor triad (3-11) and three occurrences of the diminished triad (3-10). In addition, there are three unique occurrences of suspended triads (3-9). The scale also includes six major-major seventh chords (4-20), six major-minor or half-diminished seventh chords (4-27), and three minor-minor seventh chords (4-26).

In tonal music, major and minor triads create most of the tri/chord subsets of the diatonic collection. In the 7-pitch diatonic collection (the major scale or any modal reordering of the scale that contains the same pitch-class content), there are a total of six major or minor triads—three of each. The roots of the major triads fall on scale degrees 1,
4 and 5 of the major scale, while the roots of the minor triads fall on scale degrees 2, 3 and 6. Only scale degree 7 is deprived of its own major or minor harmony.

The nine-step scale, on the other hand, contains a total of twelve major or minor triads. In contrast to the major scale subsets, even though there are twice as many triads in the nine-step scale, the roots are only present on six of the nine scale degrees: 1, 2, 4, 5, 7 and 8. As such, each of these scale degrees is simultaneously the foundation of both a major and a minor triad, as shown in Figure 3-5.

![Figure 3-5. Major and Minor Triad Subsets of the Nine-Step Scale](image)

The placement of these triads in the context of the scale is quite different than in the more common major scale. As a result, the major and minor triads found in the nine-step scale will not hold the same harmonic function as the identical sonorities found in major or minor scales. Perhaps the most notable difference is that, in the above example, no major or minor triads can be built on G. In the key of C major, G is the dominant pitch, and the major triad built on G possesses a strong dominant function. In the nine-

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39 The scale degrees used here correspond to a mode I scale.
step scale, the pitch G has the potential to sound like the dominant, but a traditional major or minor triad cannot be built upon it. Although the nine-step scale possesses triadic subsets, harmonic function must operate in a completely different manner.

In the major scale, there exists only one diminished triad. Built on scale degree 7, this exclusive harmony holds a very distinct dominant function: its root is the leading-tone, which is almost always followed by the tonic pitch.

In the nine-step scale, as illustrated in Figure 3-6, diminished triads can be constructed on scale degrees 2, 5 and 8.

![Figure 3-6. Diminished Triad Subsets of the Nine-Step Scale](image)

Two observations emerge from this information. First, these scale degrees are shared as roots of some of the major and minor triads. This lessens the uniqueness of the diminished triad in nine-step music. Second, these diminished triads cannot function as leading-tone harmonies; their roots lie a whole step below the next scale degree. Nor can they function as a subdominant harmony, as the diminished supertonic does in minor keys. Again, the question of function arises.
Understanding the appearance and function of suspended triads is a little more complicated. Because of this chord’s inversionsal symmetry, a single sonority has two possible roots. For example, Csus4, with a root of C, is comprised of the pitches C, F and G. Meanwhile, Fsus2, with a root of F, contains the same three pitches. Hence, the major scale contains five occurrences of set-class 3-9, but ten different suspended triads can be identified if you take into account all the possible roots. These are displayed in Figure 3-7.

Figure 3-7. Forte Set Class 3-9 as a Subset of the Major Scale
If we transfer this scenario to the nine-step scale, we find three different occurrences of set class 3-9 with six possible roots, which lie on scale degrees 1, 2, 4, 5, 7 and 8, the same scale degrees upon which the major and minor triads were built. Observe Figure 3-8.

![Figure 3-8. Forte Set Class 3-9 as a Subset of the Nine-Step Scale](image)

Major-major seventh chords found within the nine-step scale are also built on scale degrees 1, 2, 4, 5, 7 and 8, as shown in Figure 3-9.

![Figure 3-9. Major-Major Sevenths as a Subset of the Nine-Step Scale](image)
Figure 3-10 illustrates that major-minor and half-diminished seventh chords are built on scale degrees 2, 5 and 8 of the nine-step scale. The presence of three of each of these sonorities eliminates the unique functions that the V\(^7\) and the vii\(^{o7}\) chords have in tonal music.

The minor-minor seventh chords of the nine-step scale, shown in Figure 3-11, are also built upon scale degrees 2, 5 and 8.

In summary, many tertian harmonies that have clear diatonic harmonic functions in tonal music are available without clear function in nine-step music. The one tonal
harmony that is absent from the nine-step scale is the fully diminished seventh chord. This absence, as well as the inability of other “dominant” sonorities to resolve to the nine-step’s tonic, prevent nine-step music from sounding traditionally tonal. In addition, none of the tertian tonal harmonies discussed thus far can be built upon scale degrees 3, 6 or 9. As a matter of fact, the only tertian harmony that can be built on those scale degrees is the augmented triad, which I will take a closer look at in the following pages.

In addition to the harmonic collections just discussed, there are some pitch-class collections that are not necessarily viewed as tonal—but are still a part of common tonal language—which are contained within the nine-step scale. These include not only the aforementioned augmented triad, but also the whole tone collection, the major-minor tetra/chord and the hexatonic collection. Altogether, there are three occurrences of the augmented triad (3-12), one occurrence of the whole tone collection (6-35), six occurrences of the major-minor tetra/chord (4-17), and two occurrences of the hexatonic collection (6-20) in each nine-step scale.

Let us first turn our attention to the augmented triad. In the simplest terms, one could view the nine-step scale as a collection of any three augmented triads. In Figure 3-12, the augmented triads are identified by the lowest pitch class member contained in each.
The augmented triad and the nine-step scale share a unique relationship—they are 12-tone complements. Because of the transpositionally and inversionally symmetrical properties of the augmented triad and the complementary relationship between the augmented triad and the nine-step scale, an augmented triad can be constructed on any
step of the scale. In Figure 3-13, we can see that once three unique triads have been formed, all other possibilities are transpositional duplicates.

![Figure 3-13. Transpositional and Inversional Symmetry in Both the Augmented Triad and the Nine-Step Scale](image)

The relationship between the augmented triad and the nine-step scale does not end there. Recall the sequential patterns that result in the formation of the scale from earlier in this chapter. Of the seven examples given in Figures 3-2 and 3-3 (which are by no means an exhaustive representation of possibilities), one example uses a sequence in which each stage ascends by half-step, and six examples use sequences that rise by major third. In the case of the latter examples, the initial pitches of each stage of the sequences outline augmented triads.

The whole tone collection is a close relative to the augmented triad, being a combination of two augmented triads placed a whole-step apart. The whole tone collection itself has very little tonal implication. In fact, its symmetrical properties, placing each pitch-class equidistant from its adjacencies, remove any tendency toward a tonal center.
The whole tone collection, which many identify as sounding impressionistic, dreamy and ethereal, is found most notably in the music of Claude Debussy. Debussy generally does not base an entire work on the scale, as Tcherepnin does with the nine-step scale, but rather, he uses the collection for passages of various lengths.

One piece in which Debussy makes extensive use of the whole tone scale is “Voiles,” one of this Preludes for solo piano (1910). In this piece, Debussy uses the scale containing pitch-class 0 in fifty-eight of its sixty-four measures; the remaining measures are governed by the G-flat major pentatonic scale, another of Debussy’s favorite collections. Not one note outside the whole tone collection is added to these measures. Interestingly, Debussy works against the atonal nature of the collection and provides a tonal center in the ever-present B-flat in the piano’s lowest register.

This type of tonal versus atonal exploration using the whole tone collection is also possible within the context of a nine-step piece, but in my score studies thus far, I have seen no examples of such exploration.

A fair amount of attention has already been given to the intrinsic relationship between the nine-step scale and the remaining vaguely tonal collections: major-minor tetra/chords and major-minor hexachords. At this point, I will delve deeper into those relationships.

One likely reason the nine-step scale appealed to Tcherepnin so much is the fact that the scale contains a total of six major-minor tetra/chords. Looking at NON_{013} in Figure 3-14, we can see each tetra/chord embedded in the scale.
The major-minor hexachord is also a major component of the nine-step scale. Theoretically, under 12-gamut constraints, only four unique major-minor hexachords and their inversions exist. A hexachord can be transposed up one, two, or three semitones; a transposition by four semitones yields the same pitch-class content as the original hexachord. In addition, since there are only four possible major-minor hexachords, each hexachord’s inversion yields a hexachord with the same pitch-class content as one of the original four transpositions. Figure 3-15 illustrates all the possible major-minor hexachords and their inversions.
For the purpose of comparison, I have provided a different alphabetical letter to identify each hexachord. Hexachord A is equal to the Forte prime form of the hexatonic collection (set class 6-20), which places the defining half-step between pitch-classes 0 and 1 (HEX_{01}). Hexachord B contains a half-step between pitch-classes 1 and 2 (HEX_{12}), and so forth.

Two major-minor hexachords can be combined to form the four possible transpositions of the nine-step scale, provided their “tonics” lie a half step apart. Using the alphabetical labels, we see that hexachords A and D combine to form NON_{013},
hexachords B and A combine to form NON$_{124}$, C and B combine to form NON$_{235}$, and D and C combine to form NON$_{346}$.

In order to see a structural relationship between these major-minor hexachords and the nine-step scales, let us also name each of the four transpositions of the scale by letter. The alphabetical names for the scale are provided in Figure 3-16.

![Figure 3-16. Construction of the Nine-Step Scale from Major-Minor Hexachords](image)

Focusing on hexachords as a “property” of the nine-step scale, we can make some additional observations. The first of these observations relates to the common tones that occur when two scales are used in combination. As a generalization, when two scales that lie a semitone apart are used in combination, the common tones, naturally, comprise the
hexachord common to both scales. Figure 3-17 shows that when NonA, which is formed by the combination of HexA and HexD, combines with NonD, which is formed by the combination of HexD and HexC, the common tones between the two scales are the six tones of HexD.

Figure 3-17. Common Tones in Nine-Step Scales That Lie a Semitone Apart

It follows, then, that the tones that are not common to these two scales, which I will refer to as unique tones, form the hexachord that is absent from both scales. In the case of NonA and NonD, the missing hexachord is HexB. Compiling the unique tones from these two scales illustrates this point, as seen in Figure 3-18.
The same hexachordal properties apply when scales B and A, C and B, and D and C combine: common tones will form the hexachord that is common to both scales, while unique tones will form the hexachord that belongs to neither scale.

The scale combinations described above illustrate what I consider “closely related” nine-step harmonies. Although common chords (specifically triads) certainly exist between each of these pairs, the determining factor of their “close” relationship is the shared hexachord. The use of closely related scales creates a smooth, sometimes undetectable, transition from one nine-step tonality to another.

Although two hexachords a semitone apart can combine to create a nine-step scale, two hexachords a whole tone apart can not, and instead forms the entire 12-tone collection. Two nine-step scales a whole tone apart combine in a similar manner. The two scales will still have six common tones and six unique tones; the resulting collection,
however, is not a major-minor hexachord. Rather, each collection results in a whole-tone scale. Using scales A and C as an example, Figure 3-19 illustrates their collections of common tones and unique tones.

![Diagram of common tones and unique tones](image)

**Figure 3-19. Common Tones and Unique Tones in Nine-Step Scales That Lie A Whole Tone Apart**

The hexachordal properties of the nine-step scale can be summarized in the following manner. Two major-minor hexachords of the same mode will combine to create a nine-step scale, provided the fundamental pitches of each hexachord are a semitone apart. Any two nine-step scales with fundamental pitches a semitone apart will combine in such a way that their common tones form the major-minor hexachord.
common to each scale, and their unique tones form the major-minor hexachord absent from each scale. Any two major-minor hexachords of the same mode whose fundamental pitches lie a whole tone apart will not combine to create a nine-step scale; instead, the two hexachords combine to create the 12-tone aggregate. Any two nine-step scales with fundamental pitches a whole tone apart will not have common tones or unique tones that combine to create major-minor hexachords; instead, their common tones will combine to create a whole-tone scale, and their unique tones combine to create the complementary whole-tone scale.

Tables 3-3 and 3-4 illustrate this summary, addressing all possible combinations of major-minor hexachords and nine-step scales and their resulting hexachordal properties:

<table>
<thead>
<tr>
<th>M-m Hexachord Combination</th>
<th>Scale Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>HexA and HexD</td>
<td>NonA</td>
</tr>
<tr>
<td>HexB and HexA</td>
<td>NonB</td>
</tr>
<tr>
<td>HexC and HexB</td>
<td>NonC</td>
</tr>
<tr>
<td>HexD and HexC</td>
<td>NonD</td>
</tr>
<tr>
<td>HexA and HexC</td>
<td>Chromatic</td>
</tr>
<tr>
<td>HexB and HexD</td>
<td>Chromatic</td>
</tr>
</tbody>
</table>

Table 3-3. Major-Minor Hexachord Combinations and Their Resultant Scales
<table>
<thead>
<tr>
<th>Scale Pairs</th>
<th>M-m Hex. Make-Up</th>
<th>Common Tones</th>
<th>Unique Tones</th>
</tr>
</thead>
<tbody>
<tr>
<td>NonA and NonD</td>
<td>HexA/HexD and HexD/HexC</td>
<td>HexD</td>
<td>HexB</td>
</tr>
<tr>
<td>NonB and NonA</td>
<td>HexB/HexA and HexA/HexD</td>
<td>HexA</td>
<td>HexC</td>
</tr>
<tr>
<td>NonC and NonB</td>
<td>HexC/HexB and HexB/HexA</td>
<td>HexB</td>
<td>HexD</td>
</tr>
<tr>
<td>NonD and NonC</td>
<td>HexD/HexC and HexC/HexB</td>
<td>HexC</td>
<td>HexA</td>
</tr>
<tr>
<td>NonA and NonC</td>
<td>HexA/HexD and HexC/HexB</td>
<td>Whole Tone 1</td>
<td>Whole Tone 0</td>
</tr>
<tr>
<td>NonB and NonD</td>
<td>HexB/HexA and HexD/HexC</td>
<td>Whole Tone 0</td>
<td>Whole Tone 1</td>
</tr>
</tbody>
</table>

Table 3-4. Hexachordal Properties of Nine-Step Scale Pairs
Chapter 4: The Early Stages of the Nine-Step Scale in the Music of Alexander Tcherepnin

As an illustration of the development of the nine-step scale in Tcherepnin’s music, I turn to his Inventions, Op.13, which were written in 1920-21. I shall focus on three of the Inventions in particular: numbers 2, 4, and 6. These short pieces are saturated with the intervals of three semitones (a minor third) and one semitone (a minor second). As we will soon see, these two intervals are not only the building blocks of the major-minor tetra/chord and the major-minor hexachord, but they are also essential to the formation of the nine-step scale.

The second Invention is constructed entirely out of the foundational intervals of one and three semitones and the major-minor tetra/chord. Quite often, Tcherepnin alternates between the intervals of three and one semitone, whether ascending or descending. I will term this feature the “3-1 Complex.” At other times, Tcherepnin uses only the interval of three semitones, which results in the frequent occurrence of fully-diminished seventh chords. Tcherepnin also includes a few straightforward major and minor triads, which help bind the piece together.

In Invention No. 2, the “3-1 Complex” is most visible in measures 1 and 3, as shown in Example 4-1. This opening musical passage returns in measures 27 and 29. Occasionally, the pattern breaks; in the case of this piece, the pattern breaks before it
begins, as it starts on the second pitch of measure 1. Out of the “3-1 Complex” naturally arises the formation of major-minor tetra/chords. One can find the tetra/chord buried in the midst of the sixteenth-note runs of measures 1 and 3:

![Example 4-1. The “3-1 Complex” and Major-Minor Tetra/Chords in Invention No. 2, mm. 1-4](image)

The prevailing use of the major-minor tetra/chord in this Invention is not of this buried, or hidden, type, however. Instead, Tcherepnin creates a very visible dichotomy between major and minor triads, sometimes with the use of the major-minor tetra/chord, and sometimes by other means.

Nowhere is this dichotomy more clear than in measures 9 through 26, where the interval of a minor second is highlighted in the left hand while a perfect fifth dances around it in the right hand. This form of interplay occurs a total of three times, each at a pitch level three semitones lower than the time before. This passage appears in Example 4-2.
Example 4-2. Dichotomy Between Major and Minor Triads in 
*Invention No. 2*, mm. 9-26

The major-minor tetra/chords formed by the combination of the right and left hand in these measures provide another illustration of the “3-1 Complex.” Each tetra/chord contains the interval pattern (in semitones) of 3-1-3:

![Diagram of tetra/chords](image)

Figure 4-1. The Major-Minor Tetra/Chord as a “3-1 Complex”
Another good illustration of the major-minor dichotomy occurs in the right hand in measures 2 and 4 (see Example 4-3). Here, Tcherepnin first expresses a G minor triad, only to “slide” the surrounding interval of a fifth down a half step to create an F-sharp major triad:

![Example 4-3. Major-Minor Dichotomy Created by SLIDE Function](image)

Despite the pervasive use of the major-minor tetra/chord, this piece exhibits only a trace of the nine-step scale. Figure 4-2 shows that the melody in measure 1, with the exception of the opening F-sharp, is obedient to NON$_{013}$.

![Figure 4-2. Early Stage of the Nine-Step Scale in Invention No. 2, m. 1](image)

David Lewin describes this transformation of a triad in the following manner: “We can define an operation SLIDE that preserves the third of a triad while changing its mode: (F,+)SLIDE = (F#,-); (F#,-)SLIDE = (F,+).” (Lewin, 178).
Much of the piece shows a strong link to the octatonic scale. In measures 5 through 8 (Example 4-4), Tcherepnin places an A-sharp fully-diminished seventh chord (right hand) above a D-sharp fully-diminished seventh chord (left hand). These two harmonies, when played together, present the eight tones of the (0,1) octatonic scale, as shown in Figure 4-3.

Example 4-4. Superimposed Fully-Diminished Seventh Chords in Invention No. 2, mm. 5-8

Figure 4-3. Superimposed Fully-Diminished Seventh Chords and the Resultant Octatonic Scale

In terms of the use of major-minor hexachords, the fourth Invention provides the most theoretical interest. Tcherepnin begins this piece with an incomplete ascending
hexachord (five of the six pitches of HexB are present), and follows it immediately with its descending inversion (HexA). In doing this, Tcherepnin achieves two interesting results. The first is that an uninterrupted “3-1 Complex” bridges a change of direction. In other words, for three continuous measures, Tcherepnin maintains the alternating pattern of one semitone followed by three semitones, regardless of direction. The second interesting result, which comes as no surprise given the earlier attention to its construction, is that measures 1 through 3 contain all but one pitch of a nine-step scale. Both features are illustrated in Example 4-5.

Example 4-5. A Nearly Complete Nine-Step Scale Formed by Two Inversionally-Related Major-Minor Hexachords in Invention No. 4, mm. 1-3

The one pitch that is absent from the nine-step scale in the right hand is A-sharp. A-sharp does appear in the left hand, however, completing the scale. The “extra” pitch, B, functions in this context as an upper neighbor and can be considered, in nine-step theory, a “non-harmonic” or “chromatic” tone.
A third major-minor hexachord (HexC) appears in the right hand in measures 8 and 9. Here, the ascending hexachord is not answered with its descending inversion. Instead, it is followed in measures 11 through 13 with a lengthened restatement of the opening hexachord, written one octave higher, as shown in Example 4-6.

Example 4-6. Additional Use of the Major-Minor Hexachord in *Invention No. 4*, mm. 8-13

As explained in Chapter 3, major-minor hexachords whose transpositions lie a semitone apart, or adjacent hexachords, can combine to create a nine-step scale. Referring once again to Table 3-3 (page 66), HexA can combine with HexD or HexB, HexB can combine with HexA or HexC, HexC can combine with HexB or HexD, and HexD can combine with HexC or HexA to form a nine-step scale. Hexachords that lie a whole tone apart, or opposing tetrachords (HexA and HexC, as well as HexB and HexD), are complementary, and simply combine to produce all twelve pitch-classes. In measures 1 through 3 of *Invention No. 4*, Tcherepnin combines hexachords B and A (which is equal to B and the inversion of B) to create NON_{124}.
Major-minor hexachords are a prominent feature of *Invention No. 6* as well. In contrast to *Invention No. 4*, however, Tcherepnin uses opposing hexachords, forming a twelve-tone aggregate rather than a nine-step scale. This *Invention* is a canon set at the tonal interval of a major ninth. As pointed out earlier, two major-minor hexachords that lie a whole tone apart are twelve-tone complements; this also means that the hexachord that lies a whole tone below a given hexachord is identical in content to the hexachord that lies a whole tone above the given hexachord. In the *dux* voice of *Invention No. 6*, Tcherepnin only makes use of hexachords B and D. The *commes* voice, entering at the interval of a major ninth below the lead voice, also only uses hexachords B and D, but in reverse order. This feature, as found in measures 1 through 3, is illustrated in Example 4-7.

![Example 4-7. Opposing Hexachords in Invention No. 6, mm. 1-3](image)

As with *Invention No. 2*, the intervals of three semitones and one semitone are essential to the construction of this piece. Tcherepnin once again uses the “3-1 Complex” quite often and quite freely. The minor third, in fact, becomes a motive for this piece,
appearing not only within the major-minor hexachords, but also in a chromatic ascent of broken thirds (mm. 10-12), accented third-pairs (mm. 14-15), “filled-in” thirds (mm. 15-17) and arpeggiated fully-diminished seventh chords (mm. 17-18). Illustrations of each variation of the motive are found in Example 4-8.

Example 4-8. Various Uses of the Minor Third Motive in Invention No. 6
The semitone is prominent in measures 5 through 8, serving as lower neighbors and chromatic passing tones for a single pitch in each measure (see Example 4-9).

Example 4-9. Various Uses of the Semitone in *Invention No. 6*

Another musical feature of this piece directly related to the intervals of three semitones and one semitone is the use of “3-1 Complex” fragments that hold the same arrangement of steps, whether ascending or descending, of 1-3-1. Some of these fragments are shown in Example 4-10.

Example 4-10. “3-1 Complex” Fragments in *Invention No. 6*, mm. 1-4
These fragments can be viewed as a type of inversion of the major-minor tetra/chord, where interval classes 1 and 3 map onto each other.

There are only two short passages in *Invention No. 6* that do not involve structures formed by one or three semitones. The first passage occurs in the right hand in measure 4, and the second is its transposition in the left hand in measure 5. In these two places, Tcherepnin provides a moment of tonality—G-sharp major in measure 4 and F-sharp major in measure 5 (see Example 4-11).

Example 4-11. Moments of Conventional Tonality in *Invention No. 6*, mm. 4-5

Although *Invention No. 6* is filled with major-minor hexachords and other building blocks of the nine-step scale, the scale is not present here. This is essentially due to the use of two opposing hexachords rather than adjacent ones. It is clear in this piece, however, that the major-minor hexachord and the “3-1 Complex” are at the forefront of Tcherepnin’s compositional mind. It is through this instinctive compositional activity that Tcherepnin’s music begins to head in a new direction.
In this chapter, I will analyze four of Alexander Tcherepnin’s compositions, beginning with an early work, and ending with one of his later works. In these analyses, I will return to the use of integer notation for major-minor hexachords and nine-step scales. While letter names worked well to compare relational properties between hexachord and scale structures, integers are more efficient in music analysis, since integers refer to specific pitches. With that said, let us take a closer look at the music of Alexander Tcherepnin.

_Sonatine romantique_

To understand the earliest stages of the use of the complete nine-step scale in Tcherepnin’s music, let us turn first to _Sonatine romantique_ (Op. 4). This four-movement piano solo is credited as being the first piece in which the nine-step scale appears.\(^{41}\) Completing the piece in 1918, Tcherepnin does not use the scale as its organizing compositional force, but rather as a subsidiary figure.

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\(^{41}\) Tcherepnin writes, “the first appearance of the nine-step scale is in the Romantic Sonatine, Op. 4, composed in 1918, while the conscious theorization of it came only in 1922 after the theorizing of hexachords that started around 1920,” in *Basic Elements of My Musical Language* (Korabelnikova, p. 207).
The first six pitches of the scale appear in eighth notes in measures 69, 71 and 72 of the first movement. Specifically, Tcherepnin uses a fragment of the $\text{NON}_{235}$ scale. These measures are provided in Example 5-1.

![Example 5-1. Sonatine romantique, First Movement, mm. 69-72]

A complete scale appears for the first time in measures 96 through 99 of the first movement (see Example 5-2). After several measures of right hand arpeggiations of three different major-minor hexachords, Tcherepnin writes a four-octave scalar descent through $\text{NON}_{235}$.

![Example 5-2. Sonatine romantique, First Movement, mm. 96-99]

This scale is the combination of $\text{HEX}_{12}$ and $\text{HEX}_{23}$, and its appearance is immediately preceded by two measures of $\text{HEX}_{23}$.
The scale does not appear at all in the second or third movements.

In the final movement, the scale appears in much the same context as in the first movement. Here, the scale is preceded by three measures of HEX\textsubscript{23}, and the final three eighth notes before the scale are in the hexachord’s inverted form, otherwise known as HEX\textsubscript{12}. This time the scale descends only two-and-a-half octaves, as shown in Example 5-3.

![Example 5-3. Sonatine romantique, Fourth Movement, mm. 99-101](image)

Although the nine-step scale is conspicuously present in Sonatine romantique, little information is given for my purpose. On the other hand, it confirms the theoretical relationship between major-minor hexachords and the development of the nine-step scale.

**Message**

*Message* (Op. 39, 1926) is a good example of Tcherepnin using the nine-step scale as the exclusive tonal basis of a piece. Tcherepnin uses all four transpositions of the scale, and presents several themes and motives in various combinations.

The piece has three primary themes, which are identifiable because of their initial prominence in the right hand and their melodic character that greatly contrasts the left-
hand accompaniment figures. Theme 1 begins with the anacrusis to measure 11, and it lasts four full measures, after which the melody is answered in the left hand. Theme 2 begins in measure 61 and lasts four full measures. The gypsy-like Theme 3 begins in measure 86 and also lasts four measures. The three themes are shown in Example 5-4.

Tcherepnin also uses one subordinate theme, which is introduced in measure 50 in the bass. I refer to it as a sub-theme because of its hidden nature. Its initial statement in measures 50 through 53 is overshadowed by virtuosic runs in the right-hand. Its second appearance is masked by intense motivic activity above and below it in measures 129 through 132. It does not take on a prominent role until the second half of the piece, in measures 181 through 188, where it is finally heard as a melodic line supported by a
subordinate accompaniment. The subordinate theme and all three of its occurrences are shown in Examples 5-5 through 5-8.

Example 5-5. Subordinate Theme, mm. 50-53

Example 5-6. Initial Statement of Subordinate Theme, mm. 50-53
Message is also littered with many smaller motivic ideas, some of which are fragments of the main themes, but others of which are entirely unique. The opening six measures present a rhythmically-driven motive reminiscent of Stravinsky’s *Rite of Spring*. This motive, \( k \), as I will refer to it, is one measure in length and consists of six
repeating eighth notes in a low register followed by an accented five- or six-voice chord.

Observe Example 5-9.

Example 5-9. Motive $k$, mm. 1-6

Motive $k$ appears again in measures 54 and 55 (Example 5-10), in a texturally inverted form, where chords are repeated as six eighth notes and a single accented pitch follows.

Example 5-10. Texturally Inverted Form of Motive $k$, mm. 54-55
The final two appearances of the $k$ motive return to the original texture, but the repeating low-register pitch is doubled at the octave. These appearances occur in measures 197 through 203 and measures 271 through 274 and are shown in Examples 5-11 and 5-12.

Example 5-11. Motive $k$, mm. 197-203

Example 5-12. Motive $k$, mm. 271-274
A contrasting motive, which I will refer to as motive $l$, is introduced immediately after the opening $k$ motive. This motive is characterized by percussive descending nine-step penta/chords in the right hand against an arpeggiation of nine-step minor thirds in the left hand. Each hand is offset rhythmically by an eighth note. Motive $l$ is shown in Example 5-13.

![Example 5-13. Motive $l$, mm. 7-9](image)

Motive $l$ returns in measures 79 and 80, but the left hand ascends through steps rather than through arpeggiated minor thirds (Example 5-14).

![Example 5-14. Variation of Motive $l$, mm. 79-81](image)

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42 A nine-step minor third is equal to the mod-9 pitch-class interval $<2>$. 

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The final appearance of motive \(l\) aligns the attacks of the right and left hand, as shown in Figure 5-15. Melodically, the motive is altered slightly. Measures 262 through 254 mirror measures 79 and 80, with the right hand chords descending by step and the left hand ascending by step rather than by arpeggiated minor thirds. In measures 265 through 268, the left hand increases from a single pitch to a dyad, and both hands ascend through major-minor hexachords:

![Example 5-15. Variation of Motive \(l\), mm. 262-268](image)

The final unique motive, which I will refer to as motive \(m\), essentially becomes the building block for the entire piece. The one-measure motive begins with four repetitions of a single pitch, followed by the outline of a major-minor tetra/chord. The first appearance is in measure 10 and is presented in Example 5-16.
There is certainly a developmental relationship between motive $m$ and Theme 1. The contour of the last four pitches of motive $m$ is the same as that of a similar figure found in the second full measure of Theme 1. In addition, both sets of four pitches outline a major-minor tetra/chord. The primary difference between these two statements is the arrangement of the split third of the chord. In motive $m$, the major third is introduced first, ascending by the interval of a minor third to the fifth of the chord; the minor third of the chord then descends to the root. In the Theme 1 fragment, the minor third of the chord begins the figure, creating the interval of a major third between it and the fifth of the chord, while the major third concludes the descent to the root. A comparison of these two melodic fragments is given in Example 5-17.
Example 5-17. A Comparison of the Structures of Motive \( m \) and Theme 1

Motive \( m \) reappears throughout the work, sometimes in its original state, but often with rhythmic alterations or slight melodic modifications. One of the most common melodic changes is reversing the order of the minor and major thirds, so that motive \( m \) appears more like the fragment of Theme 1 that was described above. A more thorough description of the development and use of motive \( m \) will enter into the analytical discussion that follows.

There is one more musical idea that appears in Message significantly enough to point out. Throughout the piece, most often in transitional moments, either between two scales or between two themes, Tcherepnin writes sequential passages based on nine-step minor thirds. The first of these appears in measures 46 through 48. In this passage,
presented in Example 5-18, the right hand plays blocked minor thirds that descend by step. The left hand plays broken thirds that first ascend by step, and then descend by a major third. This creates a sequence with an outer-voice linear intervallic pattern in mod-9 pitch-class intervals of 8-6 and is illustrated in Figure 5-1.

Example 5-18. Minor Third Motive Resulting in a Descending 8-6 Sequence, mm. 46-48

The next appearance of this minor third motive arrives in measure 171. Here, each hand plays blocked minor thirds, and each hand works through an arpeggiation of HEX\textsubscript{12}.

Figure 5-1. Outer-Voice Linear Intervallic Pattern of a Descending 8-6 Sequence, mm. 46-48

\[43\] I have borrowed this terminology from Jane Piper Clendinning and Elizabeth West Marvin’s music theory textbook, *The Musician’s Guide to Theory and Analysis*, p. 329 (discussion about harmonic sequences).
Each vertical simultaneity forms a major-minor tetra/chord, voiced with the major third of the chord in the bass. Observe the passage in Example 5-19.

Example 5-19. Minor Third Motive Creating Major-Minor Tetra/Chord Simultaneities

The final appearance of the minor third motive comes in measures 208 through 236. These measures appear in Example 5-20. The use of the motive here is quite altered from its first two appearances, and it also shows some resemblance to the \( l \) motive. In these measures, every vertical structure can be considered a tertian nine-step harmony, built from stacked nine-step minor thirds. Most contain four or five stacked thirds, but there are also some occurrences of six and even seven stacked thirds. About half the chords are complete, with the other half missing one chord member. There is no clear
harmonic “progression,” but beginning in measure 220, the left-hand pitches outline an arpeggiation of minor thirds.

Example 5-20. Minor Third Motive Creating Tertian Nine-Step Harmonies

This piano piece begins with NON$_{013}$, with an apparent tonic of B, although this is not made explicitly clear; such a claim is based solely on the marcato repetition of B in the first six measures and the returning prominence of B in the bass in measures 54 through 56. The piece’s first “key” change occurs in measure 58, where NON$_{346}$ becomes
the harmonic palette. These two scales, \( \text{NON}_{013} \) and \( \text{NON}_{346} \), are closely related, sharing the common major-minor hexachord, \( \text{HEX}_{34} \). The “modulation” takes place in measures 56 through 60, in a passage that serves as a transition between the first and second primary themes. Through the use of pitches common to both scales and the use of motive \( m \), Tcherepnin creates continuity through the passage. Here, the final descending triad of motive \( m \) is major, rather than minor, in all but one instance (measure 57); in some cases \( m \) is fragmented ([\( m \)]), and some of the fragments involve rhythmic alteration. In Example 5-21, the pivot area, comprised of common tones, is enclosed in a box.

![Example 5-21. Modulation From \( \text{NON}_{013} \) to \( \text{NON}_{346} \) Through the Use of Common Tones and Motive \( m \)](image)

The next change of scale occurs prior to the entrance of Theme 3 and is shown in Example 5-22. Measures 81 through 85 provide the modulation, and motive \( m \) is
involved in a very similar fashion to the first modulation. In the first three measures of this passage, motive $m$ appears three times. The first appearance is diatonic to only $\text{NON}_{346}$, the second is common to both $\text{NON}_{346}$ and $\text{NON}_{235}$, and the third is diatonic to only $\text{NON}_{235}$. Like the first two scales used in this piece, the second two are also closely related, sharing $\text{HEX}_{23}$.

Example 5-22. Modulation from $\text{NON}_{346}$ to $\text{NON}_{235}$

Thus far in *Message*, Tcherepnin has utilized three of the four possible transpositions of the nine-step scale. A new melodic theme is introduced with each new scale. Theme 1 was based on $\text{NON}_{013}$, Theme 2 on $\text{NON}_{346}$, and Theme 3 on $\text{NON}_{235}$. For the rest of the work, Tcherepnin develops these themes, as well as the smaller motives, as he continues to explore the various scale transpositions.

After Theme 3 concludes, Theme 2 returns, this time transposed up to the current scale, $\text{NON}_{235}$. Figure 5-2 compares the transposition in measures 102 through 105 to the original melody in measures 61 through 64 with an analysis of ordered pitch intervals,
using the semitone as the unit of measurement. Such a comparison shows a discrepancy in the transposition in the third measure of each melody.

This discrepancy arises because the melody is transposed within a mod-9 pitch-class system. In order to view the transposition appropriately, I shall provide two mod-9 clockfaces to serve as visual aids. The first clockface represents the \( \text{NON}_{346} \) scale (Figure 5-3), and the second represents \( \text{NON}_{235} \) (Figure 5-4). In each of these figures, the integers contained within the circle correspond to mod-12 pitch-classes, while the integers around the perimeter correspond to mod-9 pitch-classes.
Figure 5-3. Mod-9 Representation of $\text{NON}_{346}$

Figure 5-4. Mod-9 Representation of $\text{NON}_{235}$
With the help of Figures 5-3 and 5-4, we can now compare the melody in measures 61 through 64 with the one in measures 102 through 105 more appropriately. Figure 5-5 labels each melody with mod-9 pitch-class integers and ordered pitch-class intervals. The comparison shows that measures 102 through 105 are indeed a precise transposition of measures 61 through 64.

Before moving to the next scale, Theme 3 is heard at two different transposition levels. The first, in measures 110 through 113, takes place in the two highest voices, in canon. The second transposition is presents in measures 114 through 117 in an inner voice.

Motive \( m \) takes the forefront once again in measures 121 through 124, this time with significant rhythmic alteration. These measures serve as another transitional
passage between two different scales, but this time there is no pivot area; the modulation occurs suddenly at measure 125, bringing the tonal framework back to NON\textsubscript{235}. In measure 129, while the new rhythmic rendition of motive \textit{m} continues, the subordinate theme emerges in the middle voice, beginning on the same scale degree as it did the first time it appeared.

Tcherepnin remains with this scale for quite some time. Before the next modulation, motive \textit{m} is truncated and rhythmically altered once again, Theme 1 reappears with some melodic alteration, and Theme 2 enters on scale degree 6 of its respective scale, producing the same intervallic structure as that of measures 102 through 105.

Measures 171 through 178 present another transition between scales, but again, there is no pivotal area that includes common tones between the scales. This entire passage, comprised of the minor third motive, remains obedient to the NON\textsubscript{235} scale.

Measure 179 introduces the fourth and final transposition of the scale, NON\textsubscript{124}. In this, the largest section of the piece to use a single scale, Tcherepnin uses the subordinate theme and a rhythmically augmented version of Theme 2. This transposition of Theme 2 is an exact imitation of its original display. The conclusion of Theme 2 is elided with a transposed fragment, also of Theme 2. This large section of the NON\textsubscript{124} scale concludes with motive \textit{k}.

Once Tcherepnin has used all four transpositions of the nine-step scale in this piece, he cycles through them all once more, this time at a much quicker pace. In measure 204, he returns to the original scale, NON\textsubscript{013}. This is followed in the pick-ups to measure
234 by NON\textsubscript{346}. Shortly after, the music returns to NON\textsubscript{235}. One final statement of NON\textsubscript{124} follows in measure 249, which brings the piece back to NON\textsubscript{013} for its final conclusion.

Each move from one scale to the next is between two closely related scales. This, along with Tcherepnin’s use of several identifiable themes and motives, provides effortless transitions between “keys.” Table 5-1 displays a general diagram of the form of this piece.

<table>
<thead>
<tr>
<th>Measure</th>
<th>Duration</th>
<th>Scale</th>
<th>Thematic Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57</td>
<td>013</td>
<td>$k, l, m, 1, m3, 1A$</td>
</tr>
<tr>
<td>58</td>
<td>28</td>
<td>346</td>
<td>$m, 2, l$</td>
</tr>
<tr>
<td>86</td>
<td>32</td>
<td>235</td>
<td>3, 2</td>
</tr>
<tr>
<td>118</td>
<td>7</td>
<td>346</td>
<td>$m$</td>
</tr>
<tr>
<td>125</td>
<td>54</td>
<td>235</td>
<td>1A, m, 1, 2, m3</td>
</tr>
<tr>
<td>179</td>
<td>85</td>
<td>124</td>
<td>1A, 2, k</td>
</tr>
<tr>
<td>204</td>
<td>30</td>
<td>013</td>
<td>2, m3</td>
</tr>
<tr>
<td>234</td>
<td>3</td>
<td>346</td>
<td>m3</td>
</tr>
<tr>
<td>237</td>
<td>12</td>
<td>235</td>
<td>1, 3</td>
</tr>
<tr>
<td>249</td>
<td>6</td>
<td>124</td>
<td>1A</td>
</tr>
<tr>
<td>255</td>
<td>23</td>
<td>013</td>
<td>3, l, m, k</td>
</tr>
</tbody>
</table>

Table 5-1. Formal Diagram of *Message*
If one were to look only at the progression of “keys” in this piece, one might view this piece as some sort of binary form. Each half of the piece travels through the same sequence of scales: 013, 346, 235, 124, 013. The first half, however, includes a repeat of 346 and 235 before continuing on to the fourth scale. The two halves are elided in the middle section by the 013 scale. Figure 5-6 provides a visual representation of this binary structure.

---

**Part I**

(013) → ||: (346) → (235) :||→ (124) → (013) → (346) → (235) → (124) → (013)

**Part II**

---

**Figure 5-6. Binary Structure of *Message***

One could also view Tcherepnin’s path through the nine-step scales in *Message* as a transformational network, as in Figure 5-7. The path through the scales is mostly one-directional, except where measures 118 through 178 repeat the scalar succession of measures 58 through 117. The piece as a whole begins at the 013 node and moves through two complete counter-clockwise rotations of the network, ending where it began, at the 013 node.
Tcherepnin’s Message is a valuable example of relationships between nine-step scales with shared hexachords. His choice of which scales to use is entirely based on the common hexachordal properties between each pair of scales.

Tcherepnin’s use of NON\(_{013}\) to begin and end the piece, as well as to provide a divider for the piece in the middle tells us that this particular scale serves the function of a “tonic” key for the piece. In addition, of the four scales used in this piece, NON\(_{013}\) is contained in the most measures, confirming its role as the tonic.

**String Quartet No. 2**

Tcherepnin’s *String Quartet No. 2*, Op. 40, also written in 1926, provides another good example of an entire piece devoted to the nine-step scale. The piece comprises three movements, arranged in the classical succession of tempos: fast-slow-fast. The outside movements make use of all four transpositions of the scale, while the second movement only uses three transpositions.
The first movement is a simple binary form, using themes from the first half that are developed in the second half. Each main theme can be further divided into two sub-themes, which are often used as individual motivic ideas that are further developed after their initial exposition.

The first theme, which I well refer to as Theme A, is seen in measures 1 through 13 (Example 5-23). The first sub-theme, which I will label A₁, is made up of both a syncopated rhythmic pattern on a repeated pitch, most often played by the viola and cello, and a rhythmically jarring disjunct melody, most identifiable by its initial tonal interval of a fourth (either perfect or augmented). The second sub-theme, A₂, can be identified by its gypsy-like character, created by the distinctive melodic pattern of a minor second followed by an augmented second. Exotic as it may sound, A₂ is simply a melody based on the major-minor hexachord:
The second theme, or Theme B, begins with the anacrusis to measure 15. This theme also has two subordinate themes, the first of which (B₁) is a chromatically disjunct melody, with its repeated lower note interjected with three descending chromatic pitches. The second subordinate theme (B₂) consists of a scale-wise passage of dotted rhythms. Theme B is shown in Example 5-24.
As stated earlier, the first movement of the quartet visits all four transpositions of the nine-step scale. Tcherepnin begins and ends this movement with NON_{124}, with a possible tonic pitch of A. Such an inference is not arrived at, however, until the very end of the movement, when Tcherepnin conspicuously articulates an A major-minor tetra/chord at the onset of the last scale transposition (Example 5-25).
The assertion of A as tonic is further supported in the final measures, presented in Example 5-26, when the cello plays the upper third of an A major-minor tetra/chord and the three higher instruments end the movement with the complete tetra/chord.

Example 5-26. The A Major-Minor Tetra/Chord as the Final Harmony of the First Movement
A comparison between *String Quartet No. 2* and *Message* reveals more frequent changes of scales in the quartet. In addition, Tcherepnin remains on each scale for a very short period of time before moving on to the next one. Many of these quick scale changes are a result of chromatic sequences. Three such sequences occur in the first movement.

The first chromatic sequence is in measures 30 through 41, displayed in Example 5-27; each stage of the sequence is four measures long. The melodic material in the violins is a rhythmically augmented version of theme A₂. The two melodic lines are set apart by four semitones, resulting in parallel major thirds. At the end of the four-measure model of the sequence, the violins start the next stage nine semitones lower than the model, while the viola and cello begin three semitones higher. This move brings the music from NON₁₂₄ to the closely related NON₀₁₃. The pattern repeats four measures later, introducing the next scale, NON₃₄₆.
Example 5-27. Chromatic Sequence Resulting in Scale Modulation,  
First Movement, mm. 29-41

The next sequence has two-and-a-half stages and appears in Example 5-28. This takes place immediately following the codetta that concludes the first half of this binary piece. Measure 60 begins like a transposed return to measure 1, but the A₂ material does not arrive in measure 64 as expected. Instead, measures 60 through 63 are repeated twice at different pitch levels—pitch levels determined by the new scale. Incidentally, the first two scales used here, NON₀₁₃ and NON₂₃₅, are not closely related.
Example 5-28. A Second Modulatory Sequence, First Movement, mm. 60-70

The final chromatic sequence of this movement, illustrated in Example 5-29, is another three-stage sequence, but each stage is only two measures long. This sequence moves through the closely related scales NON_{235}, NON_{124}, and NON_{013}. 

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Each scale change in this movement appears to be quite abrupt, with non-common pitches appearing at the very end of one scale’s passage and at the very beginning of the next scale’s passage. One example of this type of direct modulation occurs in measures 44 and 45, given in Example 5-30. In this example, C, a diatonic tone of NON$_{346}$ but not of NON$_{235}$, is immediately followed by C-sharp, a diatonic tone of NON$_{235}$ but not of NON$_{346}$. A similar change happens for the viola, with an A-flat played at the end of measure 44, followed by an A-natural early in measure 45.
The succession of scales used in this piece differs from that of *Message* primarily in the fact that in the quartet, Tcherepnin occasionally moves between scales that are *not* closely related. We saw this once already, within the chromatic sequence from NON\textsubscript{013} to NON\textsubscript{235}. The only other example of this happening in this movement is slightly earlier in the piece, where, in measure 52, NON\textsubscript{235} moves to NON\textsubscript{013}. This passage of NON\textsubscript{013} is the same one from which the second chromatic sequence emerges, possibly leading one to view Tcherepnin’s inclusion of NON\textsubscript{013} at this point in the piece as a calculated “mis-step.”

There is one other analytical element I would like to point out here. The first movement of *String Quartet No. 2* appears to include some non-harmonic tones. One instance occurs in measure 51. Here, an A-flat appears in the first violin part, which is diatonic to the scale that follows (NON\textsubscript{013}), but not to the scale that precedes it (NON\textsubscript{235}). The other instruments play pitches that are common to both scales. One could argue that
the modulation occurs in measure 51 rather than 52. Theoretically, the vertical harmony formed in measure 51 could act as a secondary dominant would in tonal music, belonging in spelling and function to the new key. In this case, the A-flat played by the first violin provides a micro-tonicization to the A-natural (with octave displacement), while the D-flat played by the cello resolves by half-step to C. At the very least, the non-diatonic pitches in measure 51 anticipate the subsequent scale. This passage is illustrated in Example 5-31.

A different passage exists in which there are two non-harmonic tones in identical contexts. This happens within the third chromatic sequence. In measure 102, an E-natural appears among a NON_{235} scale, and in measure 104, a G-natural appears among a NON_{124} scale. These two pitches likely function as anticipations, melodically belonging to each sequential stage that immediately follows them. Observe Example 5-32.
The progression through nine-step scales in the first movement of *String Quartet No. 2* can be shown through the network in Figure 5-8. In this diagram, a move of “H” brings one scale to another that shares a common hexachord. A move of “W”, on the other hand, brings one scale to another that shares a common whole-tone collection.
The chronological path of the scales used is perhaps easier to follow by breaking the path into stages. Figures 5-9 through 5-13 show each step in the succession of scales used in the first movement of the *String Quartet*. The first stage involves a counter-clockwise move from NON$_{124}$ to NON$_{013}$, followed by a quick return to NON$_{124}$ (Figure 5-9). Then the path restarts its counter-clockwise path, moving again to NON$_{013}$, but then continuing on to NON$_{346}$ and NON$_{235}$ (Figure 5-10). The middle stage is unique because of its diagonal “W” move across the network, from NON$_{346}$ to NON$_{013}$ and back again (Figure 5-11). The final two stages are very similar to the first two. Figure 5-12 shows a counter-clockwise move from NON$_{235}$ to NON$_{124}$, followed by a quick return to NON$_{235}$. Figure 5-13 shows a restart of another counter-clockwise path, moving first from NON$_{235}$ to NON$_{124}$, and then to NON$_{013}$. To end the movement, Tcherepnin does not continue the counter-clockwise motion; instead, he returns to the home scale, NON$_{124}$.
Figure 5-9. The First Stage of a Network Representing the Passage Through Scales

Figure 5-10. The Second Stage of a Network Representing the Passage Through Scales
Figure 5-11. The Third Stage of a Network Representing the Passage Through Scales

Figure 5-12. The Fourth Stage of a Network Representing the Passage Through Scales
Table 5-2 combines the scale usage with the thematic material for the first movement of *String Quartet No. 2*. 

Figure 5-13. The Final Stage of a Network Representing the Passage Through Scales
### Table 5-2. Formal Diagram of *String Quartet No. 2*, First Movement

<table>
<thead>
<tr>
<th>Measure</th>
<th>Scale</th>
<th>Type of Modulation</th>
<th>Scale Duration</th>
<th>Thematic Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>124</td>
<td>H</td>
<td>26</td>
<td>A(_{1+2})</td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td>B(_{1+2})</td>
</tr>
<tr>
<td>27</td>
<td>013</td>
<td>H</td>
<td>2</td>
<td>B(_1)</td>
</tr>
<tr>
<td>29</td>
<td>124</td>
<td>H</td>
<td>5</td>
<td>A(_2)</td>
</tr>
<tr>
<td>30</td>
<td></td>
<td></td>
<td></td>
<td>A(_2)</td>
</tr>
<tr>
<td>34</td>
<td>013</td>
<td>H</td>
<td>4</td>
<td>A(_2)</td>
</tr>
<tr>
<td>38</td>
<td>346</td>
<td>H</td>
<td>7</td>
<td>A(_2)</td>
</tr>
<tr>
<td>43</td>
<td></td>
<td></td>
<td></td>
<td>B(_1)</td>
</tr>
<tr>
<td>45</td>
<td>235</td>
<td>W</td>
<td>7</td>
<td>B(_1) (varied)</td>
</tr>
<tr>
<td>52</td>
<td>013</td>
<td>W</td>
<td>12</td>
<td>B(_{1+2})</td>
</tr>
<tr>
<td>58</td>
<td></td>
<td></td>
<td></td>
<td>Codetta</td>
</tr>
<tr>
<td>60</td>
<td></td>
<td></td>
<td></td>
<td>A(_1)</td>
</tr>
<tr>
<td>64</td>
<td>235</td>
<td>H</td>
<td>4</td>
<td>A(_1)</td>
</tr>
<tr>
<td>68</td>
<td>124</td>
<td>H</td>
<td>8</td>
<td>A(_1) (fragments)</td>
</tr>
<tr>
<td>76</td>
<td>235</td>
<td>H</td>
<td>27</td>
<td>B(_2) (rhythm only)</td>
</tr>
<tr>
<td>80</td>
<td></td>
<td></td>
<td></td>
<td>B(_2)</td>
</tr>
<tr>
<td>88</td>
<td></td>
<td></td>
<td></td>
<td>A(_1), B(_1), B(_2) (rhythm)</td>
</tr>
<tr>
<td>94</td>
<td></td>
<td></td>
<td></td>
<td>B(_{1+2}, A(_2))</td>
</tr>
<tr>
<td>101</td>
<td></td>
<td></td>
<td></td>
<td>B(_1)</td>
</tr>
<tr>
<td>103</td>
<td>124</td>
<td>H</td>
<td>2</td>
<td>B(_1)</td>
</tr>
<tr>
<td>105</td>
<td>013</td>
<td>H</td>
<td>6</td>
<td>B(_1)</td>
</tr>
<tr>
<td>107</td>
<td></td>
<td></td>
<td></td>
<td>A(_2)</td>
</tr>
<tr>
<td>111</td>
<td>124</td>
<td></td>
<td>21</td>
<td>Coda</td>
</tr>
</tbody>
</table>
The second movement of *String Quartet No. 2* is quite brief and does not offer a lot of insight into the use of the nine-step scale. The piece reflects a ternary form, which is determined entirely by thematic content, and is not in any way punctuated by changes of scale. A primary melodic idea plus an accompanying rhythmic figuration identify the outer sections, the second of which begins in measure 23. The melody, as it appears in measures 1 through 5, is given in Example 5-33, and the rhythmic figuration is shown in Example 5-34.

![Example 5-33. Primary Melodic Idea in *String Quartet No. 2*, Second Movement, mm. 1-5](image)

The middle section’s main melodic interest, provided by the violins, is an elaboration of a descant they introduced in measure 6. This descant is supported by the lower instruments first with sustained block chords, and then by a counter-melody in the

![Example 5-34. Accompanying Rhythmic Figuration, Second Movement, mm. 10-13](image)
viola that draws from the primary melodic idea of the outer sections. Example 5-35 presents the beginning of the violin melody.

Example 5-35. Violin Melody, Second Movement, mm. 16-17

Tcherepnin only uses three of the four transpositions of the scale in this movement. Like other works we have seen thus far, this movement also begins and ends with the same scale. Here, the home scale is NON\textsubscript{235}, with an apparent tonic of B-flat. This time the notion of a tonic pitch is a little more credible, because the opening pitches of the viola outline the split-third and the fifth of a B-flat major-minor tetra/chord, while the cello supplies the root on beat four of the first measure. Measures 1-3 of the viola and cello parts are shown in Example 5-36.
The tonic of B-flat is confirmed at the end of the movement, as seen in Example 5-37. The cello and second violin, in measures 25 through 27, sustain the pitch F, simulating a dominant pedal function, which soon gives way to a B-flat major triad on beat three of measure 28. This is followed by a sequential pattern in the cello of a broken minor third followed by a major third at three different pitch levels, the first of which forms the B-flat tetra/chord’s split-third and fifth. The final four pitches of the cello outline an entire B-flat major triad, while the other three instruments conclude the movement on B-flat, D-flat, and D-natural.
Example 5-37. Confirmation of B-flat Tonic in Second Movement, mm. 25-31

After opening with the NON\textsubscript{235} scale, this movement moves first to the NON\textsubscript{124} scale. Before the end of the first section, however, the scale changes again, this time to NON\textsubscript{346}. Three measures into the middle section, the piece returns to NON\textsubscript{235}. This particular progression is significant for two reasons. First, it provides another example of movement between two scales that are not closely related. This “W” move is between the scales opposite the ones that were used in the first movement. The second significant feature of this progression is that the scale that is omitted happens to be the scale that
served as the “tonic” key in the first movement. Perhaps this is coincidence, or perhaps Tcherepnin chose these three scales with the intention of balancing tonalities between movements.

The path taken through the scales of the second movement of *String Quartet No. 2* can be illustrated by the network present in Figure 5-14.

![Network Structure]

**Figure 5-14. Network Representing the Passage Through Scales in *String Quartet No. 2*, Second Movement**

This movement, like the first, contains a few non-diatonic pitches. The “dissonance” in this movement, however, is more intentional. Three such instances occur. The first happens in measure 5 in the cello part. The commanding scale at this point is NON$_{235}$, which does not contain pitch-classes 0, 4 or 8, but the cello plays a C. The length of this pitch (a whole note) and its position as the lowest sounding pitch make it clear that there is some significance to its presence. The same pitch returns in exactly the same context in measure 23. These to occurrences are shown in Example 5-38.
Example 5-38. Non-Diatonic Pitches in Second Movement, mm. 5 and 23

The third non-diatonic pitch Tcherepnin uses is a passing tone, which is illustrated in Example 5-39. This also occurs while NON\textsubscript{235} is the governing scale. In this case, the viola plays an F-flat (enharmonic to E-natural, or pitch-class 4), which passes between E-flat and F-natural.

Example 5-39. Non-Diatonic Passing Tone in Second Movement, mm. 22-23

Although this movement is short, the use of non-harmonic tones and the choice of scales Tcherepnin uses significantly add to the tonal interest of the movement.

The third movement of the quartet, as in the second movement, brings little new information about the way Tcherepnin uses the nine-step scale. The movement as a whole
is quite typical of any classically organized three-movement work; it is a fast movement that begins and ends with NON124, the same scale that began and ended the first movement.

This work is an intricate display of contrapuntal writing that is based on three different themes. The themes have some motivic similarities, but can be distinctly identified by the contours of the entire melodies. Each theme is further divided into sub-themes, which are often used individually throughout the rest of the movement.

The first theme is introduced by the second violin, and I will refer to it as theme X. The divisions into sub-themes are labeled X1 and X2 in Example 5-40.

Of the three themes, X reappears the least throughout the rest of the movement. Its second appearance is in measures 14 through 17 and is shown in Example 5-41. Measure 14 begins exactly as measure 1 did, and X1 remains an identical repeat. When X2 enters in measure 16, however, the melodic intervals are altered. The perfect fourth of the first and third beats of measure 3 is replaced by a perfect fifth. Moreover, the
A semitone descent in beat two is replaced by a whole step descent, and the descending whole step in beat four is replaced by an ascending semitone. These changes are largely due to the position of the melody within its respective scale. The original melody is based on a mode II scale, while the melody in measures 16 and 17 is based on a mode III scale.

![Musical example](image)

**Example 5-41. Sub-Theme X₂ With Slight Melodic Alteration, Third Movement, mm. 16-17**

In measures 26 through 33, X₂ is used alone, passing alternately between the second and first violins. These representations alter the metric placement of the melody and eliminate the distinctive pitch alteration on beat four that measures 3 and 16 both displayed. In essence, the X₂ sub-theme in measures 26 through 33 is a repetition of only the first two beats of the sub-theme (Example 5-42).
Example 5-42. Sub-Theme X₂, Third Movement, mm. 26-33

The final appearance of theme X is in measures 65 through 68 (Example 5-43). The theme appears in its complete form in the second violin, while the first violin, viola and cello play truncated versions of theme X, each at a different pitch level, and each with a different metric placement.
The second theme of the third movement, theme Y, is introduced in measures 4 through 7. The opening of this theme is certainly related to the opening of theme X with its half step ascent and its continuous return to the highest pitch. The second half of the theme, however, departs and identifies Y as an independent theme. Theme Y is shown in Example 5-44.

Example 5-43. Final Appearance of Theme X, Third Movement, mm. 65-68
Example 5-44. Theme Y and Its Sub-Themes in *String Quartet No. 2*, Third Movement, mm. 4-7

The use of theme Y is much more innovative and varied compared to the use of theme X. Some presentations of the theme leave out the entire second measure (Example 5-45).

Example 5-45. Incomplete Statement of Theme Y, Third Movement, mm. 8-9

Some maintain only the rhythm of the theme (Example 5-46).

Example 5-46. Rhythmic Statement of Theme Y, Third Movement, mm. 14-18
Some use only fragments of $Y_1$ (Example 5-47).

Example 5-47. Fragmented Statement of Theme Y, Third Movement, mm. 19-21

Tcherepnin also augments the rhythm of this theme—one, two and three times (Examples 5-48, 5-49, and 5-50).

Example 5-48. Rhythmic Augmentation of Theme Y
The third and final theme of the third movement, theme Z, is the most distinct. It begins in measure 4 and is played by cello, as shown in Example 5-51.

In its later appearances, theme Z is either complete, or $Z_1$ appears by itself. There are no occasions when $Z_2$ is present by itself. Occasionally, the first group of four sixteenth notes appears in an inverted form, descending rather than ascending, as in Example 5-52.
Like theme Y, theme Z also appears with only its rhythmic identity (Example 5-53).

Theme Z also appears in fragmented configurations. The passage in Example 5-54 involves three different sections, each with fragments of decreasing length.
Near the end of the piece, Tcherepnin introduces one new melodic idea. In the cello and second violin parts in measures 70 through 79, he writes a “wedge” based on the nine-step scale he is using at the time (NON\textsubscript{124}). Each wedge, being eight pitches in length, omits one pitch-class (mod-12) from the scale. There are a total of four wedges represented, as shown in Figure 5-15.

The cello introduces the 5-4 wedge and repeats it several times before joining the violin with different transposed fragments. The second violin begins with the 9-8 wedge.
and then plays the 5-4 wedge. After repeating these two wedges, the violin continues on with 8-6 and 10-9 wedge fragments. These wedge formations can be observed in Example 5-55.

Example 5-55. Series of Wedge Formations, Third Movement, mm. 70-79
This movement, like the first, is centered around the NON_{012} scale, with a possible tonic pitch of A. This balances the quartet with outside movements that are in the same “key.” Although the pitch, D, is prominent through the first few measures, the other pitches—C, A, and C-sharp—suggest the A major-minor tetra/chord. A full A major-minor tetra/chord, held for two-and-a-half measures, ends the piece.

In many ways, Tcherepnin’s use of the scale in this movement is very similar to that of the first movement. He uses all four transpositions of the scale, and most moves from one scale to another are by way of common major-minor hexachords. Only one move, from NON_{346} to NON_{124} in measure 34, is by way of a common whole-tone scale. Most scales’ passages are quite short, with half of the passages being only two or four measures long.

There are a few unique traits in the use of the scale in this movement, however. The path through which the scales progress is planned out and nearly symmetrical. Again, we can use a network to follow the progression, but in order to see the symmetry in this piece, let us look at the progression in stages.

Beginning with NON_{124}, the first stage brings the music through the scales in a clockwise progression through the network, as shown in Figure 5-16. In measure 14, the scale changes to NON_{235}, in measure 16 to NON_{346}, and in measure 18 to NON_{013}. 

135
Then, in measures 18 through 34, rather than completing the path around the circle, Tcherepnin retracts to NON\textsubscript{346} (Figure 5-17). In a theoretical sense, this unexpected return to NON\textsubscript{346} could be viewed as an “interruption,” after which a whole-tone move brings the music back to NON\textsubscript{124}.

Figure 5-16. The First Stage of a Network Representing the Passage Through Scales in \textit{String Quartet No. 2}, Third Movement

Figure 5-17. The Second Stage of a Network Representing the Passage Through Scales in the Third Movement
This return to the original scale marks a “restart” for the progression of scales. This time, the scales progress in a counter-clockwise direction, completing an entire cycle, and ending where they began, with $\text{NON}_{124}$:

![Diagram showing scale progression](image)

**Figure 5-18. The Final Stage of a Network Representing the Passage Through Scales in the Third Movement**

Considering just the scale content, this movement is best described as having a simple binary form. The second half is articulated by the return to $\text{NON}_{124}$ in measure 34.

Like the first movement, the third movement includes a chromatic sequence. The length of each stage is determined by the length of theme Z, although only the rhythm of Z is used at this point. The sequence is contained in measures 42 through 53 and begins with the $\text{NON}_{124}$ scale. The two violins each play a repeated pitch in the rhythm of theme Z at the interval of two scale steps. The Z theme is accompanied by $Y_1$ fragments.

Table 5-3 articulates the scale changes and thematic development in the third movement of *String Quartet No. 2.*
### STRING QUARTET NO. 2, Op. 40, III

<table>
<thead>
<tr>
<th>Measure</th>
<th>Scale</th>
<th>Type of Modulation</th>
<th>Scale Duration</th>
<th>Thematic Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>124</td>
<td>↓ H</td>
<td>13</td>
<td>$X_{1+2}, Y_{1+2}, Z_{1+2}$</td>
</tr>
<tr>
<td>14</td>
<td>235</td>
<td>↓ H</td>
<td>2</td>
<td>$Z_1, X_1, Y_{1+2}$ (rhythm only)</td>
</tr>
<tr>
<td>16</td>
<td>346</td>
<td>↓ H</td>
<td>2</td>
<td>$Z_1, X_2, Y_2$ (rhythm only)</td>
</tr>
<tr>
<td>18</td>
<td>013</td>
<td>H</td>
<td>12</td>
<td>$Y_1$</td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td>$Z_1$</td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td>$Y_{1+2}, X_2$</td>
</tr>
<tr>
<td>30</td>
<td>346</td>
<td>↓ W</td>
<td>4</td>
<td>$Y_{1+2}, X_2, Z_1$</td>
</tr>
<tr>
<td>34</td>
<td>124</td>
<td>↓ H</td>
<td>12</td>
<td>$Y_{1+2}, Z_{1+2}$ (rhythm only)</td>
</tr>
<tr>
<td>46</td>
<td>013</td>
<td>↓ H</td>
<td>4</td>
<td>$Z_{1+2}$ (rhythm only), $Y_1$</td>
</tr>
<tr>
<td>50</td>
<td>346</td>
<td>H</td>
<td>11</td>
<td>$Z_{1+2}, Z_1, Y_1$</td>
</tr>
<tr>
<td>55</td>
<td></td>
<td></td>
<td></td>
<td>$Z_1$ (fragments)</td>
</tr>
<tr>
<td>61</td>
<td>235</td>
<td>H</td>
<td>4</td>
<td>$Z_1$ (fragments)</td>
</tr>
<tr>
<td>65</td>
<td>124</td>
<td>↓ H</td>
<td>26</td>
<td>$X_{1+2}, Z_1, Y_1$, “wedge”</td>
</tr>
</tbody>
</table>

**Table 5-3. Formal Diagram of String Quartet No. 2, Third Movement**

**Symphonic Prayer**

Tcherepnin’s music has often been categorized according to stylistic phases in his compositional style. His “nine-step scale” phase, according to Guy Wuellner, lasted
It is on music from this period that I have been focused on thus far. The culmination of Tcherepnin’s nine-step technique, however, arrives with a work written during his “synthesis” phase. This piece, *Symphonic Prayer*, Op. 93, written in 1959, will provide closure to my discussion of Alexander Tcherepnin’s nine-step music.

Tcherepnin’s use of the nine-step scale in *Symphonic Prayer* is quite different from what we have seen in any of his other pieces. There are two main differences that I will highlight here. The first is the frequency with which Tcherepnin changes nine-step collections. The second is the serial organization with which he molds his melodic material.

The opening nine measures of *Symphonic Prayer* possess a great deal of musical intensity. The very first sound of the work is the full orchestra in octaves on the pitch D. From that D, a series of pitches emerges—a tone row, if you will—building a sound pyramid as new pitches are added and previous ones are sustained, until finally nine different tones are sounded simultaneously. The harmonic tension is magnified by a gradual rhythmic crescendo; the first four pitches are each held for five beats, the next three pitches are sustained for only three beats each, and the final two pitches of the row are each sustained for only two beats.

The innovation of sounding all nine pitches of a scale at once must have excited Tcherepnin, for at this point, he goes one step further. Until this moment, only pitches from the NON\textsubscript{346} scale are used, and the nine-note harmony arrived at in measure 8 is

\footnotesize{44 Arias, p. 28.}
made up of all nine pitches from that particular scale. As the intensity of this passage increases, Tcherepnin continues the rhythmic crescendo by giving the last three chords a duration of only one beat. Harmonically, Tcherepnin maintains the fullness of nine-note harmonic structures, but he alters the collections from which each chord comes. Ultimately, in the ninth measure of Symphonic Prayer, all four transpositions of the nine-step scale are realized in four nine-note blocked chords, culminating with NON\textsubscript{013}.

The greatest tension of all arises with the last of the blocked chords. From the first sound of the orchestra, it is clear that D is the tonic, or at least a primary tonal center, of this piece. With the arrival of the NON\textsubscript{013} harmony, the importance of this pitch remains, as it is not diatonic to the NON\textsubscript{013} collection but is held fast in the flute, clarinet, bassoon, violin and bass parts. This nine-note harmony with an added tenth non-diatonic tone illustrates Tcherepnin’s point, “In six-part settings the penta/chord (major or minor) is considered as stable, while the hexa/chord is unstable, finding resolution in a penta/chord, and so on, up to the point at which—in ten-part harmony—the entire nine-scale becomes a stable chord, with instability provided by an extra tonal appoggiatura (in ten-part settings), two extra appoggiaturas (eleven-part) or three appoggiaturas (twelve-part)”\textsuperscript{45} (emphasis mine).

Measures 10 through 18 provide a “tonal answer” of sorts to the melodic subject that was introduced in the first nine measures. As the trumpets and trombones play this answer, the inversion of the subject is heard in the lower instruments. This nine-measure passage is governed by NON\textsubscript{124}.

\textsuperscript{45} Tcherepnin, Basic Elements of My Musical Language, from Korabelnikova, p. 198.
Aside from the four transpositions heard on four consecutive beats in measure 9, Tcherepnin’s use of the scale in these opening measures is similar to what we saw in some of his earlier pieces—complete musical ideas, bound by a single scale transposition. This pattern changes drastically in the measures that follow. Beginning in measure 19, Tcherepnin begins a journey through each nine-step “key,” visiting each one multiple times before settling into NON\textsubscript{013} in measures 37 through 43. Again, Tcherepnin uses graduated rhythmic durations to increase the dramatic intensity of sound.

We already encountered one serial property in this piece in measures 10 through 18, where the subject and its inversion are played simultaneously. The serialist aspect of these measures runs even deeper in the accompaniment figure of the upper strings. In Example 5-56, you will see four statements of a nine-tone series, each statement playing nine unique pitch classes, but all four statements belonging to the same nine-tone collection, NON\textsubscript{124}.

These statements are all ordered pitch-class segments related to each other by transposition, inversion, retrograde or retrograde inversion. The relationships, however,
are not apparent in a mod-12 context, but rather within the framework of a mod-9 system.

In the analysis of each series in Figure 5-19, the integers between pitches represent the mod-9 ordered pitch-class intervals used within the NON\textsubscript{124} scale.

![Figure 5-19. Serial Transformations in Symphonic Prayer](image)

Similar serial constructions are found throughout the subsequent measures, particularly in measures 30 through 36. Here, however, the transformations are not confined to a single scale transposition. Instead, each nine-tone series is part of a new nine-step scale. The first nine tones belong to NON\textsubscript{124}, the second nine belong to NON\textsubscript{235}, and the third and final group belongs to NON\textsubscript{346}.

After this serial passage, Tcherepnin proceeds to the remaining scale, NON\textsubscript{013}. Instead of continuing the serial transformations, however, he writes out the scale, in
contrary motion, for the higher instruments to play opposite the lower instruments, expanding out to an A in the bass against a G in the treble at extreme registers. This climax is intensified by the brass instruments filling in the other seven pitches of the NON₀₁₃ collection.

The forty-three measures we have looked at in *Symphonic Prayer* shed some welcome light on Alexander Tcherepnin as a composer. They show tremendous musical organization in the systematic movement through each of the nine-step tonalities. They show a synthesis of contemporary styles through passages of serial construction. They also show innovation in harmonic language with the use of a nine-tone chordal structure.
Chapter 6: The Nine-Step Scale in the Music of Other Modern Composers

The Music of Olivier Messiaen

Perhaps the most well-known composer to have used the nine-step scale as a compositional source is the French composer, Olivier Messiaen. Messiaen, in *The Technique of My Musical Language*, first published in 1944, includes the scale in his collection of seven “modes of limited transposition.” The importance of these modes in the music of Messiaen follows from his fascination with what he calls the “charm of impossibilities” in music. Referring to this “charm,” Messiaen states,

“This charm, at once voluptuous and contemplative, resides particularly in certain mathematical impossibilities of the modal and rhythmic domains. Modes which cannot be transposed beyond a certain number of transpositions, because one always falls again into the same notes; rhythms which cannot be used in retrograde, because in such a case one finds the same order of values again—these are two striking impossibilities.”

The modes themselves are formed “of several symmetrical groups, the last note of each group always being common with the first of the following group.” In other words, two- to six-note groups are formed by a specific pattern of semitones. Each of these note groups is then repeated, transposed to form a series of conjunct dyads,

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47 ibid., p. 58.
trichords, tetrachords, and so forth, that span one octave. Figure 6-1 displays the seven modes.

**Mode 1:** Divided into six groups of two notes each; transposable two times

![Diagram of Mode 1](image1)

Mode 2: Divided into four groups of three notes each; transposable three times

![Diagram of Mode 2](image2)

Mode 3: Divided into three groups of four notes each; transposable four times

![Diagram of Mode 3](image3)

Mode 4: Divided into two groups of five notes each; transposable six times

![Diagram of Mode 4](image4)

Figure 6-1. Messiaen’s Seven Modes of Limited Transposition
Figure 6-1 continued

Mode 5: Divided into two groups of four notes each; transposable six times

![Mode 5 Diagram]

Mode 6: Divided into two groups of five notes each; transposable six times

![Mode 6 Diagram]

Mode 7: Divided into two groups of six notes each; transposable six times

![Mode 7 Diagram]

With symmetrical structures being the primary criterion for the formation of these modes, it is surprising that Messiaen does not include the hexatonic (referred to earlier as the major-minor hexachord) scale in his collection. Using Messiaen’s method of organization, the hexatonic scale is divided into three groups of three notes each, and it is transposable four times, as shown in Figure 6-2.
Figure 6-2. An Eighth Mode of Limited Transposition

John Schuster-Craig points out this conspicuous omission in his 1990 journal article, “An Eighth Mode of Limited Transposition.” Schuster-Craig writes, “Messiaen stated in his treatise that ‘It is mathematically impossible to find others of them’, i.e. other Modes of Limited Transposition. Messiaen was wrong: there is one additional collection of pitches that is both symmetrical in its construction and ‘of limited transposition’. This mode is built around the alternation of the intervals of the semitone and minor 3\textsuperscript{rd} and has four transpositions.”\textsuperscript{48}

The absence of the hexatonic scale in Messiaen’s collection of modes is one indication that his music and his use of the nine-step scale are quite different from that of Tcherepnin. With Tcherepnin, the hexatonic scale is the entire basis for the formation of the nine-step scale. For Messiaen, on the other hand, the formation of mode 3 is based entirely on the scale’s symmetrical step-wise structure. In Tcherepnin’s music, there are clear references to the major-minor tetra/chords that occur so abundantly in the hexatonic scale; the designation of the major-minor tetra/chord as the “fundamental” chord is also testament to the importance of the hexatonic scale. Messiaen identifies a “typical chord” of the third mode as a six-note chord that includes a root, a major third and a minor third, a fifth, an augmented fourth, and a flat sixth.

Perhaps it is the close affinity between the nine-step and the hexatonic scales that warrants its exclusion from Messiaen’s modes. Perhaps Messiaen views them as one and the same structure. Such an argument is difficult to defend, however, when two of the other modes Messiaen uses are also so closely affiliated; mode 5 is a subset of mode 4, simply removing two pitches from the latter to create the former.

The aural effects of the harmonies used by Messiaen and Tcherepnin are quite different. Perhaps this is far too much of a generalization, but Tcherepnin’s harmonies tend to sound more percussive and dissonant, while Messiaen’s sound more aloof, impressionistic and, somehow, more consonant.

There is another aspect by which his use of the modes differs from Tcherepnin’s: modal duration, or the length of time or duration that is devoted to one mode or tonal system. As we have already seen in much of his music, Tcherepnin devotes entire compositional works to the nine-step scale. He modulates between the various transpositions of the scale, but the basic enneatonic collection is always present. Messiaen, on the other hand, uses all the modes interchangeably and modulates often. Among the musical examples given by Messiaen in *The Technique of My Musical Language*, Volume II, very few represent his third mode. Of the examples that do highlight the third mode, most only include a few measures of it before moving on to another of the modes. Messiaen tends to move effortlessly between modes, focusing primarily on modes 1, 2, and 3.

The method of using different tonal collections for different musical passages, as Messiaen did, became quite common in the late nineteenth century. I have already
commented briefly on this type of organization by Debussy, who writes multiple passages within the same composition using the whole-tone and pentatonic collections, often interspersed with diatonic sections. In the next section, I recognize works by two other composers who also focus on unique pitch collections for short passages in their musical compositions.

**The Music of Other Russian Composers**

The nine-step scale has been little explored in the area of music theory and analysis. Furthermore, a surprising number of musicians have never heard of Alexander Tcherepnin! One could say that Tcherepnin had a scattered existence, having spent his life moving from country to country after emigrating from Russia in 1921. Although he was considered by many to be a citizen of the entire world, having lived for long periods of time in France, China, the United States and England, Tcherepnin always considered himself, at the core, a Russian composer. It is for this reason that I have taken a special interest in the work of John Schuster-Craig.

John Schuster-Craig has studied extensively the music of Nicolai Rimsky-Korsakov and Mikhail Glinka, two Russian composers pre-dating Tcherepnin. Schuster-Craig is particularly interested in the music of these composers because of how their compositions make use of the “major-third system.” According to Schuster-Craig, the major-third system is made up of the whole-tone, hexatonic and enneatonic collections,
pointing out that “they are the only three pitch-class sets of cardinality 6 through 11 which maximize interval class 4.”

Schuster-Craig’s work is of particular interest here because he provides some compelling evidence of pre-20th century musicians who arrive at the enneatonic collection in their music as a by-product of other compositional means. Let us look at one work each from Glinka and Rimsky-Korsakov.

The first example comes from the Overture to Ruslan and Lyudmila by Glinka (Example 6-1). This particular passage is significant primarily because, according to Schuster-Craig, it is “generally cited as the earliest use of the whole-tone scale.”

Schuster-Craig, however, points out another feature of this passage that is particularly relevant to the topic at hand. In addition to the presence of the whole-tone scale, which is immediately recognizable as such, he points out that “the three triads in the upper register which accompany the scale—D major, B-flat major, and F# major—form a non-diatonic, chromatic mediant progression accountable to the hexatonic scale,” a scale with which we are now very familiar.

He continues, “The upper register’s hexatonic progression, in combination with the whole-tone scale in the bass, are accountable to a nine-note collection, Messiaen’s Third Mode of Limited Transposition, which has of late been described as an enneadic, or enneatonic scale.” He then cites some of the identifying features of the scale: it is the

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50 ibid., p. 1.
51 ibid., p. 1.
52 ibid., p. 2.
non-intersecting union of three augmented triads, interval-class 4 is maximally present, it is symmetrical in its construction, and it is transpositionally invariant.\textsuperscript{53}

Example 6-1. Reduction of an Excerpt from the Overture to \textit{Ruslan and Lyudmila} by Glinka

Although this combination does in fact form the enneatonic collection, I hesitate to call these measures a nine-step passage. In addition to the triads in the upper voices and the whole-tone scale in the lower voice, there is a tonal melody that emerges in the third measure of the excerpt. The three musical components combine in this measure to produce the dominant of B minor, a key that is clearly tonicized through its resolution in the fourth measure. The passage as a whole is convincingly tonal and in the key of D major. It is certainly a chromatic passage, and very distinct compared to rest of the diatonic overture. To summarize, the presence of the nine-step scale in \textit{Ruslan and Lyudmila} is worth noticing because of its early historic presence (it was written around 1840), but its function would be misunderstood if not viewed in a tonal context.

\textsuperscript{53} ibid., p. 2.
The second Russian work we will look at comes from Rimsky-Korsakov’s *The Tale of Tsar Saltan*, which was completed in 1900 (Example 6-2). Schuster-Craig states that this passage “shows Rimsky’s awareness of the enneatonic collection, as well as the inclusion relationships involving the hexatonic transpositions embedded within the enneatonic. The first eight bars at rehearsal 112 (the vocal part is the backstage voice of a swan) are wholly accountable to E₁. But the enneatonic collection is the result of the superimposition of two hexatonic collections: H₃ (the swan’s melodic line in the upper register) and H₀ (the triadic accompaniment in the lower register).”

![Example 6-2. Reduction of an Excerpt from *The Tale of Tsar Saltan* by Rimsky-Korsakov](image)

The significance of this passage is not so much *that* it was written, but rather, *who* wrote it. Rimsky-Korsakov was a very present figure in Alexander Tcherepnin’s life. Tcherepnin’s father’s position as a professor of composition at the St. Petersburg Conservatory and his involvement with both the Mariinsky Theater and Diaghilev’s “Russian Seasons” created an environment in which Alexander came into personal

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54 Only six bars are provided in the example.
55 John Schuster-Craig does not explain his labeling system for the enneatonic collection, but “E₁,” in this case, is the same collection as NON₀₁₃.
56 Schuster-Craig, p. 7.
contact with several of Russia’s most prominent musical figures. Rimsky-Korsakov was one of these figures, and Ludmila Korabelnikova writes that Tcherepnin often fell asleep at home at night “to the sounds of compositions by Rimsky-Korsakov, Borodin, or his father.” The musical context in which the enneatonic collection appears in The Tale of Tsar Saltan is so similar to the development of Tcherepnin’s nine-step scale that it is difficult to ignore the possible influence of Rimsky-Korsakov on Tcherepnin.

Schuster-Craig points out in one of his endnotes that there are several examples of composers superimposing one symmetrical collection onto another. Two fully-diminished seventh chords will form an octatonic collection when superimposed. Here we see the example of two hexachords superimposed to form the enneatonic collection. It is with this technique in mind that we begin an exploration of some more recent compositions.

**Recent Compositions**

Gregory Proctor’s compositions, *Pastorale* and *Three Pieces for Violins*, involve the nine-step scale in some manner. Each piece came to use the scale through a different process, which I will explore at this time.

*Pastorale* is a tonal, ternary piece whose middle section is divided into three contrasting sequential passages. The third of these passages is of particular interest, since it incorporates the nine-step scale. Observe the passage in Example 6-3.

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57 Korabelnikova, p. 3.
Example 6-3. *Pastorale* by Gregory Proctor, mm. 41-45

In these five measures, Proctor produces two different modes of the scale, utilizing two different pitch-class collections. This is different from how we would expect nine-step scales to appear in Tcherepnin’s music. In Tcherepnin’s music, one scale, or nine-step tonality, is used at a time. Transpositions of scales do not occur simultaneously, but rather in succession. In *Pastorale*, on the other hand, the oboe plays
one octave of the $\text{NON}_{346}$ scale, while the viola plays one octave of the $\text{NON}_{124}$ scale, as shown in Figure 6-3.

![Figure 6-3. Two Simultaneous Nine-Step Scales in *Pastorale*, mm. 41-44](image)

The two scales have six common tones, as do any two nine-step scales in combination. What makes this pairing unique, however, is that these two scales are set a whole tone apart. Again, this is something that has been seen only on occasion in Tcherepnin’s music. Tcherepnin tends to move from one scale to a closely related scale, one that shares a common major-minor hexachord with the scale that precedes it. There are only two possible pairings that lie a whole tone apart, and in such pairings, as discussed previously in this document, the common tones together form a whole tone scale.

The appearance of the nine-step scale in *Pastorale* is not intentional, but rather a by-product of other compositional intentions; specifically, this scale is the result of a melodic sequence. Each stage of the sequence is one measure long, and each stage begins a major third higher than the previous one. The oboe’s melodic pattern ascends by semitone, followed by a tone. The viola ascends in the opposite manner, moving first by
tone, and then by semitone. The organ supports these pitches with harmonies that could be analyzed locally as I-ii-V\(^7\)/IV. The secondary dominant is frustrated, however, and moves not the expected harmony, but to a harmony that shares one common tone—the third of the secondary dominant becomes the root of the subsequent harmony.

Furthermore, although the third chord is spelled like a dominant-seventh, its function is that of an augmented sixth chord. Here, the defining interval of an augmented sixth is represented as an enharmonic diminished third between the viola (e.g. D-flat in measure 41) and the right hand of the organ (e.g. E-flat in measure 41). Each voice then resolves inward by half-step in its respective octave, which is the fifth of the subsequent harmony.

Within a larger context, however, each enharmonically reinterpreted dominant-seventh chord is, in the simplest terms, a “dressed up” local tonic, resulting in a harmonic motion that recalls the opening of the piece—a tonic harmony embellished by a neighboring harmony.

The counterpoint formed by the oboe and viola in mm. 41-44 must also be noted here. In these measures, each instrument ascends one octave through the nine-step scale, creating harmonic intervals of major and minor sixths. The inversions of these intervals create major and minor thirds, providing one example of how thirds are an important component of this piece. Also, as noted before, within each measure of the sequence, one instrument ascends by a tone while the other does by a semitone, and *vice versa*. Once the instruments have reached the octave (at the downbeat of m. 44), the oboe continues its ascent, while the viola descends. At this point, contrary motion provides musical variety, but the pattern of movement by different sized steps continues. As the oboe
ascends with a H-W-H pattern in m. 44, the viola descends with a W-H-W pattern, as shown in Figure 6-4.

As shown earlier, the nine-step scale, as Tcherepnin derived it, is the combination of a major-minor hexachord and its inversion. The tonal harmonies formed on the downbeats of measures 41 through 44 together create a major-minor hexachord. On each downbeat, a major triad is formed, and each triad is separated by a major third. Between these three triads, there are only six distinct pitches—pitches that form a major-minor hexachord. These formations are articulated in Figure 6-5.
Major-minor hexachords are formed by two interlocking major-minor tetra/chords and by three major triads whose roots are a major third apart. Such hexachords are also formed by the combination of any two augmented triads that lie a semitone apart (two augmented triads that are a whole-tone apart produce the whole-tone scale), as in the example in Figure 6-6.

![Figure 6-6. Two Augmented Triads a Whole Tone Apart, Forming a Major-Minor Hexachord](image)

This feature, too, is very present in the score. One of the augmented triads is outlined by the pitches on the downbeats of both the organ’s pedal (Eb-G-B) and the oboe (G-B-D#). The other augmented triad is outlined by the downbeats of the viola (Bb-D-F#). In Example 6-4, these pitches are enclosed with boxes.
Example 6-4. Augmented Triads Outlined in *Pastorale*, mm. 41-43

Once again, the presence of the nine-step scale in *Pastorale* could be considered a by-product of a different compositional goal. Although the nine-step scale appears in *Pastorale*, it is only present in these five measures, and can in no way be considered an organizational force of the entire work.

A different piece by Gregory Proctor, *Three Pieces for Violin*, on the other hand, was created using the nine-step scale’s intrinsic features as its primary compositional tool.

The opening of the first piece instructs the three violins to “enter in order (I, II, III) at any interval of time. Violins I and II repeat as indicated until after Violin III has played his last note. Violins I and II then play up to asterisk (*), holding last note until they come together, then continue as below.” Examples 6-5, 6-6 and 6-7 display the introductory measures of *Three Pieces for Violins.*
Of all the pitches played by all three instruments, every pitch but one is accountable to the NON_{124} scale, but not every pitch of the collection is present in each part. In fact, each instrument plays only a seven-pitch segment of the scale, and each seven-pitch segment is unique. Violin I plays the segment from pitch-class 5 to 1 (5, 6, 8, 9, t, 0, 1), omitting pitch-classes 2 and 4; Violin II plays the segment from pitch-class 9 to
5 (9, t, 0, 1, 2, 4, 5), omitting pitch-classes 6 and 8; and Violin III plays the segment from pitch-class 1 to 9 (1, 2, 4, 5, 6, 8, 9), omitting pitch-classes 10 and 0. Let us take a closer look at these short passages, focusing on the theoretical items of interest.

The first item of interest involves an account of the pitches used, or rather, *not* used in all three violin parts. The six omitted pitch-classes (0, 2, 4, 6, 8, t) together form the WT₀ scale. This is significant because of the inclusion properties of the nine-step scale. Recall what John Schuster-Craig pointed out earlier—“any enneatonic transposition has one of the two possible whole-tone transpositions, and two of the four possible hexatonic transpositions, as subsets.”⁵⁹ In the case of NON₁₂₄, the whole-tone subset is WT₀ and the two hexatonic subsets are HEX₀₁ and HEX₁₂.

The fact that the omitted pitches of each instrument part form a subset of the larger enneatonic collection demonstrates that these pitches were not chosen at random. Whether or not the actual *intent* was to omit the pitch-classes of the whole tone subset or not is not clear, but there is certainly a symmetrical aspect at work here.

For example, the pitch-class segments chosen for each instrument are symmetrically constructed, and each is located equidistant from the others mod-9. The symmetrical construction of each segment reflects the first two conjunct tetrachords of a mode I nine-step scale. These pitch-class segments are presented in Figure 6-7.

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⁵⁸ The G-natural in the Violin I part.
Furthermore, the placement of each segment on a mod-12 clockface, as in Figure 6-8, shows that the intersections of the ends of each segment are located four semitones, or a major third, apart. These major thirds, creating an augmented triad, illustrate once again the internal symmetrical division of the scale.

Figure 6-7. Symmetrical Pitch-Class Segments in Each Violin Part

Figure 6-8. Clockface Representation of Pitch-Class Segments in Each Violin Part
Symmetry can also be found on a smaller scale within each instrumental part in this introduction. This is most obvious in the first and second violin parts. In the first two-and-a-half bars of the Violin I part, the violin plays a pitch wedge, expanding outward from the central pitch, A. The reduction in Figure 6-9 shows the symmetrical expansion away from A to the outer limits of the scale segment, C-sharp and F.

![Figure 6-9. Violin I Pitch Wedge in Three Pieces for Violins](image)

From A, there is first a half-step motion outward to B-flat and G-sharp. This is followed by a whole-step expansion to C-natural and F-sharp, which continues out by half-step to the goal pitches of C-sharp and F-natural. Horizontally, the intervallic succession displays symmetry by employing the semitone pattern of 1-2-1. Vertically, symmetry is apparent in the mirroring of this interval pattern between the upper and lower “voices.” The overall symmetry of this particular pitch wedge emphasizes the mode I tetrachord configuration of the nine-step scale.

The second violin also displays an element of symmetry in pitch space. In its fourth through sixth measures, pitches move symmetrically about C-sharp, but the
movement works through a different chronology than it did with the first violin. Whereas the first violin began with the central pitch and moved systematically outward from it, the second violin begins away from the central pitch, progresses toward the central pitch, and then moves away again. It does this through a retrograde-inversion of the first four pitches (of its fourth through sixth measures). Observe the ordered pitch interval pattern in Example 6-8.

Example 6-8. Pitch Symmetry in *Three Pieces for Violins*

The remainder of the first movement develops motives that were presented in the introduction, particularly those that were introduced by the first violin. Within that framework, however, Proctor does provide a “modulation” to a new collection of nine tones, recognizable as NON$_{346}$. Just prior to a return of the opening thematic material, there is a modulation back to NON$_{124}$. The significance of Proctor’s choice of scales is related once again (as was the choice of pitch-classes used in the movement’s introduction) to the whole-tone collection. The two “keys” chosen are two that share the common tones of pitch-class integers 0, 2, 4, 6, 8 and 10 (WT$_0$). Recall that this is the same whole tone collection that was represented by the omitted pitch classes of the introduction. This connection, whether it is intentional or coincidental, is one element that unifies this movement.
The third movement is also particularly interesting in its use of the nine-step scale. Similar to the first movement, this movement is entirely accountable to nine-step tonality, save one or two pitches. Like much of Tcherepnin’s music, Proctor uses in this movement various transpositions of the scale with pivot areas to connect each transposition. The primary “key” of this movement is NON\textsubscript{013}, but Proctor also utilizes NON\textsubscript{235} and NON\textsubscript{346} before returning to the original collection to finish the movement and the entire work.

Compositionally speaking, the most interesting aspect of the third movement is its serial construction. The movement is based on the five-note ordered pitch-class segment, [05e34]. Disregarding the order of the set for a moment, one finds the prime form of this set to be (01256), a subset of the enneatonic collection. It follows that transpositions up or down a major third will yield additional subsets of the same pitch-class collection. Similarly, inversion of the set and transpositions of the inversion up or down a major third will also produce subsets of the original collection. In other words, using P\textsubscript{0} = [05e34], which we know to be a subset of NON\textsubscript{013}, the serial transformations P\textsubscript{4}, P\textsubscript{8}, R\textsubscript{0}, R\textsubscript{4}, R\textsubscript{8}, I\textsubscript{0}, I\textsubscript{4}, I\textsubscript{8}, RI\textsubscript{0}, RI\textsubscript{4}, and RI\textsubscript{8} will all also produce subsets of NON\textsubscript{013}.

With this in mind, let us take a closer look at the third of Proctor’s *Three Pieces for Violins*. The movement opens with the following melodic gesture, spanning more than two octaves in its realization of the prime form of the ordered pitch-class set upon which the entire movement is based, presented in Example 6-9.
Example 6-9. Ordered Pitch-Class Segment in *Three Pieces for Violins*,
Third Movement

This opening phrase is repeated several times in the first half of the movement, not only maintaining the pitch-class order in each transformation, but also (with the exception of the final interval in P and R transformations and the initial interval in I and RI transformations) maintaining the ordered pitch interval relationships in seven of the eleven possible forms. These transformations are presented in Figure 6-10.

![Figure 6-10. Serial Transformations of [05c34]](image-url)
As in the first movement, the third movement modulates away from the “key” in which it begins and returns to the home “key” to close. NON\textsubscript{013} serves as the home collection, and the first modulation does not occur until well over half-way through the movement. In measure 29, the second and third violins move to NON\textsubscript{235}, with the first violin joining them three measures later. The next “key” change occurs in measures 38 and 39, where Proctor focuses on the NON\textsubscript{346} collection. The return to NON\textsubscript{013} happens only five measures before the end of the piece. Transitions between each scale are made smooth through the use of pivot areas, similar to the way Tcherepnin executed many of his modulations. Proctor was able to utilize transformations that were wholly accountable to the previous scale, with only the last half being wholly accountable to the new scale. For example, a pivot area between NON\textsubscript{235} and NON\textsubscript{346} occurs in measures 35 through 38. One of the transformations used in these measures is $R_{10}$ (Violin II). This ordered series, (2193t), is a subset of NON\textsubscript{235}. The last two pitch classes, 3 and 10, however, are also members of NON\textsubscript{346}, and immediately following the realization of these two pitch-classes in the score, the subsequent musical passage is entirely subject to NON\textsubscript{346}.

The path Proctor takes through different transpositions of the nine-step scale is reminiscent of the path Tcherepnin took in the second movement of his String Quartet No. 2. Like the quartet, Proctor’s third violin piece visits only three of the four scale transpositions. But unlike the quartet, Proctor’s first modulation occurs between two distantly related scales.

There are several other musical features to take note of in the third movement of Three Pieces for Violin. First of all, measure 4 introduces a motive that becomes a central
figure in this movement (Example 6-10). The motive is initially presented using the pitch-class segment P_4. Rhythmically, the motive is simply an evenly divided quintuplet.

Example 6-10. Motive in Third Movement of Three Pieces for Violins

Altogether, there are six statements of the motive, shown in Figure 6-11. The final statement implements rhythmic augmentation.

Figure 6-11. Additional Statements of the Motive

Another important musical feature is that, in addition to approximately thirty complete serial transformations of the initial series, Proctor includes some fragments of the series and some unordered collections of the series. Some of the fragments maintain order and can therefore be identified as P, R, I and RI transformations, but others display
an altered order, a chordal arrangement, or are missing interior pitch-class members rather than having truncated ends. Observe the examples of unordered sets and each type of fragment in Examples 6-11 through 6-14.

Example 6-11. Inversion with Altered Order

Example 6-12. Prime Form with Pitch-Class Omission

Example 6-13. Inversion with Pitch-Class Omission
Example 6-14. Retrograde in Stretto with Altered Order

Let us take a closer look at these alternate forms of the basic transformations. The first altered transformation occurs in measure 10. This representation of \( I_8 \), which should be \( (83954) \), places the last pitch class in the middle of the series, changing the order to \( (83495) \). Measure 15 contains the second alteration. Here, the prime form of the series is used, but pitch-class 3 is omitted, changing the series from \( (05e34) \) to \( (05e4) \). In the third alteration, found in measures 29 to 31, the first member of the \( I_8 \) series is missing and the next three pitch-classes occur simultaneously. The final examples included above are the most artistic and intricate representations of the original series. In measures 31 and 32, various retrograded segments appear, with slight alterations in pitch-class order, overlapping each other in a stretto-like fashion. These variations to the serial transformations illuminate the creativity of the composer, while he remains strict to the “tonal” nine-step surroundings of the moment.
Finally, not every pitch used in the third movement is bound to a serial structure. There are occasional vertical simultaneities that do not represent transpositions, inversions, or retrogrades of P₀. There are glissando-like runs that are formed by strict scalar passages. There are also brief melodic passages that do not conform to any of the serial transformations.

In the first movement of *Three Pieces for Violin*, we noticed that the whole-tone collection played an important role in the construction of the movement as a whole. It is no different in the third movement. The first involvement of the whole tone collection occurs in conjunction with the first modulation. The very choice of nine-step scales emphasizes the whole tone collection as the six common tones between the scales (NON₀₁₃ and NON₂₃₅). These common tones are very present in the score in measures 26, 29 and 30, where all six pitch-classes of WT₁ are played as a chord, providing harmonic support from the second and third violins. Figure 6-12 displays the chord.

![Figure 6-12. Whole Tone Collection in Three Pieces for Violins](image)

The second moment of importance for the whole-tone collection comes in measure 38, where two WT₀ scales are played, one by Violin I and the other by Violin II, articulated as the sixteenth-note quintuplet that was introduced by the motive we looked at earlier:
Much emphasis has been given in this paper to the distinction between using the nine-step scale as a compositional tool and arriving at its use through other compositional means. Gregory Proctor has provided us with examples of both scenarios. In *Pastorale*, we saw how a symmetrical structure, when used in sequence, resulted in the enneatonic collection. In *Three Pieces for Violins*, it is clear that various transpositions of the nine-step scale were pre-conceived and were central to the organization of each movement.

The most pointed illustration of this assertion arrives at the end of the third movement. In measures 44 and 45, Proctor replaces the whole-tone scale runs with a new scalar passage: the nine-step scale. First arranged in the motivic sixteenth-note quintuplet, and then in one beat of straight thirty-second notes, the NON\textsubscript{013} scale is played by the second violin in a repeating pattern of semitones: 2-1-1 (Example 6-16).
Example 6-16. Nine-Step Scalar Passages
Conclusion

Throughout this document, we have explored many aspects of the nine-step scale. We have brought to light several of the intrinsic properties of the scale. We have provided a critical overview of the scale as Alexander Tcherepnin, the primary architect of nine-step music, understood it, and we have followed the development and use of the scale in several of his musical compositions. Finally, we have discussed the appearance and use of the scale in the works of various other modern composers.

The nine-step scale is a symmetrical scale, like the whole tone, octatonic, and chromatic scales, but it has been largely overlooked by music theorists. As is the case with other symmetrical scales, one can arrive at the nine-step scale from a variety of directions. It is a repeating stepwise pattern of two half steps and one whole step. It is the combination of a major-minor hexachord and its inversion $T_n I$ such that $n = \text{any even integer in mod-12 pitch space}$. It is the combination of any three augmented triads, and is therefore also the complement of the augmented triad. The sum of all these attributes results in the appearance of the nine-step scale in numerous late Romantic to modern musical compositions.

The nine-step scale offers a tonal system that is theoretically sound, but few are aware of it. The most extensive theorization of the scale until now has been provided by
the Russian composer, Alexander Tcherepnin. He explained the construction of the scale as it developed from his instinctive use of two harmonic structures—the major-minor tetra/chord and the major-minor hexachord. He discussed a tonal system that involves tonic pitches, modal uniqueness, arpeggiations in a mod-9 pitch-class universe (although he did not have the language with which to fully explain it), and harmonic structures with tonal tendencies. Tcherepnin’s *Basic Elements of My Musical Language* provides evidence of his brilliance as a modern music theorist.

In addition to his theoretical writings, Alexander Tcherepnin has supplied the grandest musical examples of how the nine-step scale can be used in composition. Tcherepnin did not see in this scale a simple collection of notes. Rather, he created an entire tonal system around the scale. Very few composers have undertaken such a challenge. Tcherepnin’s music stands out from the music of other composers who merely incorporate the scale in passing; Tcherepnin bases entire works on the nine-step system. For over ten years, the nine-step scale was his tonality.

Because Tcherepnin based entire works on the nine-step scale, his works have displayed certain characteristics that are crucial to understanding why his use of the scale is so important. These characteristics are both melodic and harmonic. Tcherepnin’s melodies stand out because they are independent—not sequential. As we have seen, some composers arrive at the use of the nine-step scale in their music through sequential patterns. Tcherepnin’s harmonies stand out because they are, at the core, derived from common-practice tonality and are not *simply* a symmetrical collection of pitches. Derived from the major-minor triad, a harmony Tcherepnin viewed as the most stable of
harmonies, his nine-step music is based on a solid harmonic system—a system he
developed and used to its fullest extent.

Although the scale is not widely used or studied, its presence in a variety of
musical contexts cannot be ignored. In the case of Mikhail Glinka and Nicolai Rimsky-
Korsakov, the presence of the nine-step scale in some of their music was an unintentional
by-product of other current musical trends. Specifically, Glinka’s placement of three
major triads with roots a major third apart against a whole tone scale results in the nine-
step scale. Rimsky-Korsakov’s example, on the other hand, combines two major-minor
hexachords, which together form the nine-step scale. The more recent composer, Gregory
Proctor, unknowingly arrived at the use of the nine-step scale through a sequential pattern
that involved both the melody and its supporting harmonies.

There was one major composer other than Tcherepnin, however, that deliberately
wrote music based on the nine-step scale: Olivier Messiaen. But Messiaen’s interest did
not lie exclusively with the nine-step scale; this scale was one of seven “Modes of
Limited Transposition” that he incorporated into his compositions. As a result, one will
find the nine-step scale (Messiaen’s “Mode 3”) appearing primarily in short musical
passages rather than in entire works or movements.

Of all the pieces studied in this document, the one that most closely reflects the
same type of use of the nine-step scale as that of Tcherepnin is Three Pieces for Violins,
by Gregory Proctor. Like Tcherepnin, Proctor bases the entire work on the nine-step
scale, moving systematically from one transposition to another. The fundamental
difference between the music of the two composers is a harmonic one: Tcherepnin’s
pitches developed from the major-minor tetra/chord and hexachord, while Proctor’s
developed from a different harmonic structure.\textsuperscript{60}

It is my intention to conduct further studies on the nine-step scale. Specifically, I
have developed two research goals. First, I would like to gain a better understanding of
the harmonic aspects of the scale—a topic I barely acknowledge within this document.
Tcherepnin provides a point of departure for this task in his \textit{Basic Elements of My
Musical Language}, from which score studies of greater depth would be necessary.
Second, I would like to find a more exhaustive supply of musical examples from
composers other than Tcherepnin that incorporate the nine-step scale. It is my hope that,
in time, the nine-step scale will be recognized as an essential musical construction. In the
future, I would like to see this scale become a standard component of university-level
music theory curricula.

\textsuperscript{60} Gregory Proctor informs me that the nine-step scale appears in \textit{Three Pieces for Violins} as a result of the
transformational use of a “fourths” chord, or a quartal tri/chord (e.g. the pitches G, C and F). Specifically, it
comes about through parsimonious voice leading between three fourths chords and a single augmented
triad (each fourths chord has one pitch in common with the augmented triad), which is equivalent to
transposition of the tonic fourths chord by major thirds.
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Musical Scores


Unpublished Musical Scores


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Recordings


Appendix A: Inventions, Op. 13, Nos. 2, 4 and 6 by Alexander Tcherepnin
Appendix B: *String Quartet No. 2, Op.40* by Alexander Tcherepnin
III

Allegro moderato

f meno.
Appendix C: Symphonic Prayer, Op. 93 by Alexander Tcherepnin, Measures 1-43
Appendix D: *Three Pieces for Violins* by Gregory Proctor, Movements I and III