PREDICTION OF FORCES, STRESSES, TEMPERATURES AND TOOL WEAR IN METAL CUTTING

DISSERTATION

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By

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In the present work, an analytical computer program (OXCUT) is developed to predict cutting variables. The temperatures and forces, predicted by OXCUT, have been found to be in reasonable agreement with published experimental results.

Moreover, flow stress data, at the high strains, strain rates and temperatures, encountered in metal cutting operations, has been determined using OXCUT in conjunction with 2-D orthogonal slot milling experiments and multidimensional minimization using the Downhill Simplex Method. The OXCUT and FEM predicted cutting variables, using the obtained flow stress data, have been found to be in reasonable agreement with the measured results. The obtained flow stress data has been used, in conjunction with the FEM code DEFORM, to study the influence of edge preparation of cutting tools on tool stresses, cutting temperatures and forces. It has been concluded that the hone-radius edge with hone radius of 0.1 mm may have the least probability of chipping and the chamfered edge (20°X0.1 mm) may have the minimum flank and crater wears for the conditions used in the present study.
Furthermore, OXCUT and published measured crater and flank wear results have been used to estimate the empirical parameters of a characteristic wear equation to predict crater and flank wear of HSS, Carbide P20 and Carboloy 370 inserts when cutting low carbon steels. The estimated parameters of the characteristic wear equation have been used, in conjunction with OXCUT, to approximate the flank and crater wear rate distributions along the twist drill lip when drilling low carbon steels. A parametric study has been conducted to investigate the influence of spindle speed, feed, workpiece material, and drill helix angle on the crater and flank wear rates along the lip of twist drills made of Carboloy 370 and Carbide P20. The important observations of this parametric study are: (1) increasing the drill helix angle reduces crater and flank wear, (2) Carboloy 370 is more resistant to drill crater wear than Carbide P20 at low feeds and the opposite is true at high feeds, and (3) Carboloy 370 is less resistant to drill flank wear than Carbide P20.
Dedicated to my parents and my wife
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<td>(MPa)</td>
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<td>$r_n$</td>
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<td>$r_l$</td>
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<td>$\Delta s_2$</td>
<td>(mm)</td>
<td>Deformation zone thickness</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>(°K)</td>
<td>Temperature rise</td>
</tr>
<tr>
<td>$\Delta T_C$</td>
<td>(°K)</td>
<td>Average temperature rise in chip</td>
</tr>
<tr>
<td>$\Delta T_M$</td>
<td>(°K)</td>
<td>Maximum temperature rise in chip</td>
</tr>
<tr>
<td>$\Delta T_{SZ}$</td>
<td>(°K)</td>
<td>Temperature rise in Chip formation zone</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>(-)</td>
<td>Effective strain</td>
</tr>
<tr>
<td>$\dot{\varepsilon}$</td>
<td>(s$^{-1}$)</td>
<td>Effective strain rate</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_o$</td>
<td>(s$^{-1}$)</td>
<td>Strain rate constant</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_o^\ast$</td>
<td>(s$^{-1}$)</td>
<td>Ratio of test strain rate to reference strain rate</td>
</tr>
<tr>
<td>$\varepsilon_{AB}$</td>
<td>(-)</td>
<td>Effective strain along shear plane AB</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{AB}$</td>
<td>(s$^{-1}$)</td>
<td>Effective strain rate at shear plane AB</td>
</tr>
<tr>
<td>$\varepsilon_i$</td>
<td>(-)</td>
<td>Input pulse in the Hopkinson's bar apparatus</td>
</tr>
<tr>
<td>$\dot{\varepsilon}_{int}$</td>
<td>(s$^{-1}$)</td>
<td>Effective strain rate at the tool-chip interface</td>
</tr>
<tr>
<td>$\varepsilon_r$</td>
<td>(-)</td>
<td>Reflected pulse in the Hopkinson's bar apparatus</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>(-)</td>
<td>Output pulse in the Hopkinson's bar apparatus</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(°)</td>
<td>Clearance angle</td>
</tr>
<tr>
<td>$\gamma'$</td>
<td>(s$^{-1}$)</td>
<td>Shear Strain Rate</td>
</tr>
<tr>
<td>$\gamma_s$</td>
<td>(-)</td>
<td>Shear Strain</td>
</tr>
<tr>
<td>$\eta$</td>
<td>(-)</td>
<td>Chip formation zone temperature factor</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>(°)</td>
<td>Angle between the chip flow direction and the normal to the side cutting edge in 3-D cutting</td>
</tr>
<tr>
<td>$\eta^\ast_c$</td>
<td>(°)</td>
<td>Projection of $\eta$ in the cutting face plane for oblique nose radius tools</td>
</tr>
<tr>
<td>$\eta_a$</td>
<td>(°)</td>
<td>Drill chisel edge angle</td>
</tr>
<tr>
<td>$\phi$</td>
<td>(°)</td>
<td>Shear angle</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>(°)</td>
<td>Pitch angle of the ball end mill cutter</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>(°)</td>
<td>Angle in vertical plane between a point on the flute and the Z-axis of the ball end mill</td>
</tr>
</tbody>
</table>
\( \lambda \)  \( (\degree) \)
Mean friction angle at the tool-chip interface \( (\lambda = \tan^{-1} \mu) \)

\( \lambda_s \)  \(-\)
Guess of the problem's characteristic length scale for the
Downhill Simplex Method

\( \mu \)  \(-\)
Coulomb friction coefficient

\( v \)  \(-\)
Constant in the velocity modified temperature equation

\( \theta \)  \( (\degree) \)
Angle made by the resultant cutting force \( R \) and the
shear plane direction \( AB \) in orthogonal cutting
Angle made by the undeformed chip element with the
positive \( X \) axis for nose radius tools

\( \theta_1, \theta_2, \theta_3 \)  \( (\degree) \)
Limits of integration used to obtain the equivalent cutting
edge for nose radius tools

\( \theta_e \)  \( (\degree) \)
Tool end relief angle in 3-D turning

\( \theta_t \)  \( (\degree) \)
Tool rotation angle in Milling

\( \theta_b \)  \( (\degree) \)
Tool side relief angle in 3-D turning

\( \rho \)  \( (\text{kg/m}^3) \)
Density of the workpiece material

\( \sigma \)  \( (\text{MPa}) \)
Stress

\( \bar{\sigma} \)  \( (\text{MPa}) \)
Effective Stress

\( \sigma_0, \sigma_1, \sigma_{11} \)  \( (\text{MPa}) \)
Constants in the stress-strain equation

\( \sigma_{0,AB} \)  \( (\text{MPa}) \)
Flow stress constant evaluated at the strain rate and
temperature of the shear plane \( AB \)

\( \sigma_{1b} \)  \( (\text{MPa}) \)
Value of the flow stress constant for a workpiece of
carbon content of 0.16%

\( \bar{\sigma}_{AB} \)  \( (\text{MPa}) \)
Flow stress at the shear plane \( AB \)

\( \bar{\sigma}_{\text{flank}} \)  \( (\text{MPa}) \)
Effective Stress on the flank face

\( \sigma_{N,B}, \sigma_{N,B}' \)  \( (\text{MPa}) \)
Normal stress acting at \( B \)

\( \bar{\sigma}_{\text{rake}} \)  \( (\text{MPa}) \)
Effective Stress on the rake face

\( \sigma_r \)  \( (\text{MPa}) \)
Normal stress

\( \sigma_{\text{rake}} \)  \( (\text{MPa}) \)
Normal stress on the rake face

\( \tau \)  \( (\text{MPa}) \)
Shear stress

\( \tau_{\text{int}} \)  \( (\text{MPa}) \)
Shear stress at the tool-chip interface

\( \Omega \)  \( (\degree) \)
Local chip flow angle measured relative to tool axis for
an uncut chip element (nose radius tools)

\( \bar{\Omega} \)  \( (\degree) \)
Chip flow angle measured relative to tool axis in 3-D
cutting

xxvi
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>(-)</td>
<td>Tool-chip interface temperature factor</td>
</tr>
<tr>
<td>$\psi_0(z)$</td>
<td>(°)</td>
<td>Lag angle between the tip of the flute at $z = 0$ and a point on the helical flute at height $z$ in ball end milling</td>
</tr>
<tr>
<td>$\Psi_j(z)$</td>
<td>(°)</td>
<td>Lag angle in global coordinates, measured from the $+Y$ axis clockwise, between the tip of flute $j$ at $z = 0$ and a point at an axial location $z$ on flute $j$</td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>(°)</td>
<td>Stipline field angle</td>
</tr>
<tr>
<td>$\zeta_\infty$</td>
<td>(°)</td>
<td>Reference angle for an element $e$ on the drill lip</td>
</tr>
</tbody>
</table>
CHAPTER 1

INTRODUCTION

Metal cutting processes such as turning, milling and drilling are used extensively in all industrial applications, usually, to machine castings and forgings to assembly-ready dimensions.

1.1 Prediction of Cutting Forces and Tool Wear

In today’s practice, to avoid excessive tool wear and to make sure that deflections due to high cutting forces will not affect the geometric tolerances of the machined product, CNC programmers tend to develop their programs based on conservative cutting conditions. They specify cutting speeds, feeds and depths of cut that are usually far below the optimal values, which reduces productivity drastically. Thus, prediction of cutting forces and rate of wear of the cutting tools are two vital tasks that can enhance the design of metal cutting processes.
1.2 High Speed Machining

High speed machining (HSM) has many advantages such as higher productivity due to the high material removal rate and better product quality in terms of surface finish and surface integrity because the fast chip removal minimizes heat transfer from the deformation zone to the machined surface [Rigby; 1993]. Moreover, HSM is characterized by lower cutting forces due to the thermal softening of the workpiece material at the high temperatures generated in the deformation zone and at the chip-tool interface [Altintas; 1993].

1.3 Hard Machining

Nowadays, using the modern technology of machine tools and cutting inserts, it is possible to machine materials, in the hardened state, to their final shapes and tolerances with no need of further heat treatment and little need of grinding operations [Borchers; 1997]. Thus, high speed machining of hard materials helps to increase productivity and improve product quality. However, to use this technology effectively, the process must be well designed to avoid its disadvantages. The higher the hardness of the workpiece material, the higher the cutting forces. Moreover, high speed machining of hard materials results in high temperatures, stresses and relative velocities at the tool-workpiece interface which cause accelerated tool wear.

Thus, to expand the application of HSM of hard materials, it is necessary to understand the fundamental relationships between workpiece and tool material properties, tool geometry and cutting conditions, on one side, and a) cutting forces, b) the developed stresses, temperatures and relative velocities, and c) tool wear, on the other side.
1.4 Methods to Analyze Metal Cutting Operations

The purpose of analyzing metal cutting operations is to relate process variables such as stresses, temperatures and cutting forces to tool geometrical parameters (rake angle, clearance angle and honing radius), cutting conditions (feed, cutting speed and depth of cut) and workpiece and tool material properties. Towards this goal, experimental, Finite Element Method (FEM), mechanistic and analytical techniques are used.

1.4.1 Experimental Techniques

Experimental techniques are reliable but they are also very expensive and time consuming. It is relatively easy to use experimental techniques to measure cutting forces and cut chip thickness, however, it is difficult and sometimes impossible to measure the stresses, temperatures and relative velocities, induced at the tool-chip interface during the cutting process, and relate them to tool wear.

1.4.2 Finite Element Method (FEM)

FEM has been used extensively in the literature to analyze metal cutting operations. Most researchers tend to write their own FEM codes which can only analyze a specific theoretical problem [Obikawa; 1996] [Shinozuka; 1996] [Shih; 1996]. Moreover, because of the difficulty of obtaining material property data, the validity of the results obtained from the FEM models, published in the literature, is questionable. Even researchers who used commercial codes such as NIKE2D [Strenkowski; 1985], ABAQUS [Mason; 1995], DEFORM [Cerretti; 1996] could not analyze most practical 3-D cutting operations because those commercial codes are not yet capable to solve such problems with reasonable effort.
1.4.3 Mechanistic Modeling

Mechanistic modeling has been used by many investigators to predict cutting forces in practical 3-D metal cutting operations such as drilling [Wiriyacoso; 1976] and ball end milling [Lee; 1996]. In this approach, the cutting forces, measured from orthogonal turning experiments, were used to predict cutting forces in 3-D cutting operations by geometrically relating the oblique cutting edges to the orthogonal cutting edges. Mechanistic modeling does not have the potential to predict temperatures and stresses at the tool-workpiece interface required for tool wear prediction.

1.4.4 Analytical Modeling

Analytical modeling of metal cutting processes has been developed and used to predict cutting forces from flow stress data and thermal properties of the workpiece material, tool geometry and cutting conditions [Oxley; 1963, 1966, 1976, 1977]. This technique has the potential to predict average stresses and temperatures at the tool-workpiece interface. Analytical modeling has advantages over FEM because it needs relatively short calculation time and it requires no geometric modeling of the tool or workpiece which is very cumbersome, especially in 3-D machining processes such as drilling and ball end milling. Analytical modeling was used to predict forces in metal cutting processes such as orthogonal and 3-D turning, simplified 3-D face milling operations and drilling [Lin; 1982] [Hu; 1986] [Young; 1987, 1994] [Arsecularante; 1995] [Elhachimi; 1999a, 1999b]. Analytical modeling was also used to predict temperatures at the tool-chip interface and tool wear in orthogonal turning [Mathew; 1989].
1.5 Rationale and Motivation of the Present Work

The following observations explain the rationale for the present research work:

1) The technology of high speed machining (HSM) of hard materials, characterized by high temperatures, stresses and relative velocities at the tool-workpiece interface and high cutting forces, is being applied gradually in the metal cutting industry.

2) The prediction of cutting forces is essential to the design of jigs and fixtures such that the deflections do not affect the geometric tolerances of the final product.

3) Tool parameters (rake and clearance angles, edge preparation and coating) and cutting conditions (cutting speed, feed and depth of cut) need to be selected such that product quality, productivity and tool life are maximized for a certain workpiece material.

4) The prediction of stresses, temperatures and relative velocities at the tool-workpiece interface is essential to the estimation of tool wear and tool life.

5) Analytical-based computer modeling provides short calculation time and can be used to predict cutting forces, average temperatures and stresses. Thus, this modeling technique can be used to optimize HSM of hard materials.

6) The geometric models of 3-D cutting processes such as drilling and ball end milling, found in the literature and developed for mechanistic modeling, can be applied to analytical modeling as well.
7) Analytical-based computer modeling can only analyze cutting with sharp tools. Thus, to be able to analyze cutting with radius or chamfered tools, FEM should be used.

8) Both FEM and analytical modeling of metal cutting operations require flow stress data of the workpiece material as function of strain, strain rate and temperature. Conventional tension, compression or torsion tests can not be used to develop flow stress data for practical high speed metal cutting operations where the strains can be higher than 2, the strain rates can be higher than \(10^5\) s\(^{-1}\) and temperatures can be higher than 1000 °C. Thus, other techniques such as the high speed compression test and the high speed impact compression test (Hopkinson’s technique) are used to obtain flow stress data for metal cutting operations. However, those techniques are expensive and can not predict flow stress data for strain rates higher than 2000 s\(^{-1}\) [Maekawa; 1983] [Lee; 1998].

The above listed observations indicate that it is necessary to:

1) Develop a methodology to estimate flow stress data for metal cutting using analytical-based computer modeling in conjunction with orthogonal cutting experiments. This methodology should be inexpensive and automated in order to generate a database of flow stresses in the practical range of strains, strain rates and temperatures encountered in metal cutting.

2) Investigate the relations between tool wear (crater and flank wear) and the stresses, temperatures and relative velocities for different tool materials.
1.6 Research Objectives

The overall objective of the present study is to analyze the High Speed Machining (HSM) process of hard materials, using FEM and analytical techniques, to predict cutting forces, stresses, temperatures and relative velocities at the tool-workpiece interface and relate them to tool wear. This will facilitate the design of jigs and fixtures, the specification of the power requirements of the machine tools, and the estimation of tool life. The specific research objectives are to:

1) Develop an analytical-based computer code that can predict cutting forces, average stresses and temperatures, using flow stress data and thermal properties of the workpiece material, tool geometry and cutting conditions, for 2-D orthogonal cutting operations and extend the analysis to 3-D practical cutting operations.

2) Develop an automated technique to obtain flow stress data, as function of the practical range of values of strains, strain rates and temperatures encountered in HSM. This technique will use the developed analytical-based computer code in conjunction with orthogonal cutting experiments.

3) Establish a methodology to estimate wear of cutting tools and inserts. For this purpose, the present study aims at using empirical equations that can relate the wear rate, measured from orthogonal cutting experiments, to the stresses, temperatures and relative velocities, predicted at the tool-workpiece interface using FEM and/or analytical-based computer modeling. Those empirical relations will be used to estimate tool wear and tool life in practical 3-D cutting operations such as drilling.
4) Establish a methodology to design the tool geometrical parameters, select the type of tool material and select the cutting conditions to maximize product quality, productivity and tool life.

1.7 Results and Contributions

In addition to the fundamental understanding of the HSM process of hard materials, the present study will contribute to:

1) Creating a database of flow stresses for various workpiece materials of various hardness levels used in the metal cutting industry.

2) Developing a user-friendly computer program, based on a well-established analytical model to predict cutting forces and average stresses and temperatures at the tool-workpiece interface for practical 2-D and 3-D metal cutting operations. This program will have a built-in database of flow stresses and material properties for selected materials used in industry.

3) Predicting tool failure, increasing tool life, and reducing damage to the machined surface in some selected machining operations.

1.8 Organization of Dissertation

The following is a brief description of each chapter of the present work:

Chapter 1: INTRODUCTION. In this chapter, the technology of high speed machining of hard materials is discussed briefly and the different techniques to analyze the metal cutting process are introduced.
The rationale and motivation of the present work, research objectives and contributions are also discussed.

Chapter 2: BACKGROUND IN CUTTING MECHANICS. In this chapter, the different analytical and mechanistic models to analyze the metal cutting process are described. The techniques used to obtain flow stress data in metal cutting are discussed. Moreover, famous models of friction at the tool-workpiece interface are described. Furthermore, the mechanics of wear in metal cutting are discussed briefly.

Chapter 3: UTILIZATION AND VALIDATION OF ANALYTICAL – BASED COMPUTER MODELING TO PREDICT PROCESS VARIABLES IN METAL CUTTING. In this chapter, the temperature and force results, predicted by the developed computer code OXCUT, are compared to the published experimental results in 2-D turning and slot milling, and 3-D turning, face milling, ball end milling and drilling. Moreover, a method of approximating the stresses and temperatures on the flank face, using OXCUT, is suggested from investigating the FEM results.

Chapter 4: DETERMINATION OF FLOW STRESS DATA. In this chapter, 2-D orthogonal slot milling experiments in conjunction with OXCUT and a multidimensional minimization Algorithm (the Downhill Simplex Method) are used to predict flow stress data. In this chapter flow stress data of P20 mold steel (30 HRC), H13 tool steel (46 HRC) and the aluminum alloy Al2007 (100 HB) are obtained.

Chapter 5: VALIDATION AND APPLICATION OF THE DETERMINED FLOW STRESS DATA. In this chapter, the flow stress data, obtained in
Chapter 4, is further validated by using it to predict process variables and compare the predicted results with the published measured results and results of experiments conducted in the present work. Moreover, the obtained flow stress data of H13 tool steel (46 HRC) is used to investigate the influence of edge preparation of cutting inserts on forces, stresses and temperatures.

Chapter 6: PREDICTION OF TOOL WEAR IN METAL CUTTING. In this chapter, the empirical parameters of a characteristic wear equation, developed in the literature based on diffusion and adhesion, are determined by fitting the OXCUT stress, temperature and relative velocity results to the published measured crater and flank wear results. The empirical parameters are determined to predict crater and flank wear of Carbide P20, Carboloy 370 and MTM 41 (HSS) inserts when cutting low carbon steels. The wear equations are used to investigate the effect of feed, spindle speed, drill material, workpiece material and drill helix angle on crater and flank wear along the twist drill lip.

Chapter 7: CONCLUSIONS AND FUTURE WORK. In this chapter, the important conclusions of the present work and suggestions for future work are stated.
CHAPTER 2

GENERAL BACKGROUND IN CUTTING MECHANICS

2.1 Mechanics of Orthogonal Cutting

In orthogonal cutting, the tool has a single cutting edge perpendicular to the cutting velocity direction, as shown in Figure 2.1, whereas the cutting edge in oblique cutting is not perpendicular to the cutting velocity. Plane strain conditions can be approximated in orthogonal cutting if the width of cut \((W)\) is more than or equal to 10 times the uncut chip thickness \((t_1)\) [Oxley; 1989]. In this case, there will be no flow in the direction parallel to the cutting edge and the cutting process can be modeled as two-dimensional as illustrated by the schematic diagram of Figure 2.2 (the direction of the width of cut \(W\) is normal to the plane of the page). Orthogonal cutting experiments can be done by turning a disk of thickness (width of cut) at least 10 times the feed (uncut chip thickness) with the tool cutting edge parallel to the axis of rotation of the disk. Another way to do orthogonal cutting experiments is by turning a tube to shorten its length with the wall thickness of the tube (width of cut) at least 10 times the feed (uncut chip thickness). The diameter of the tube should be large enough, compared to its wall thickness, to avoid variation of the cutting velocity along the width of cut [Oxley; 1989].

Merchant analyzed the orthogonal plane strain metal cutting process based on the following assumptions [Merchant; 1945]:
Figure 2.1: (a) orthogonal cutting (b) oblique cutting [Boothroyd, 1966]

Figure 2.2: Schematic diagram used in Merchant’s model of 2-D orthogonal cutting
1) The shear plane AB does not have a thickness.
2) The tool tip is sharp and no rubbing or ploughing occurs between the tool and the workpiece.
3) The stresses on the shear plane are uniformly distributed.
4) The resultant cutting force $R$ applied at the shear plane is equal, opposite and collinear to the force $R$ applied at the tool-chip interface as shown in Figure 2.2.

Merchant used a force balance to obtain expressions for the cutting force $F_C$ and the feed force $F_T$ as functions of the shear stress of the material, the shear angle ($\phi$), Coulomb friction at the tool-chip interface ($\mu = \tan \lambda$), uncut chip thickness ($t_1$), width of cut ($W$) and tool rake angle ($\alpha$) shown in Figure 2.2. Merchant obtained an expression, relating the shear angle ($\phi$) to the tool rake angle ($\alpha$) and the friction angle ($\lambda$), by expressing the cutting force $F_C$ in terms of the shear flow stress (assumed constant) along the shear plane AB, $W$, $t_1$, $\alpha$, $\lambda$ and $\phi$ and then selecting $\phi$ that makes the work done by $F_C$ a minimum.

While it explains, approximately, the mechanics of orthogonal cutting, Merchant’s model does not predict forces with reasonable accuracy [Armarego; 1969]. Other researchers attempted to improve the expression of the shear angle $\phi$ [Lee; 1951][Shaw; 1953], however, all the developed expressions were proven not to provide accurate results [Oxley; 1989].

### 2.2 Application of Mechanistic Modeling to Predict Cutting Forces in Machining

A mechanistic approach was used to predict cutting forces in end milling processes [Tlusty; 1975][Kline; 1983][Yellowley; 1985][Altintas; 1991]. This
mechanistic approach depends on milling force components coefficients that must be determined experimentally for each cutter geometry.

A “unified mechanics of cutting” approach was developed and is being extended by ongoing research [Armarego; 1969, 1983, 1985, 1990, 1993]. This approach mathematically relates the cutting analysis of practical metal cutting operations such as oblique turning or oblique milling to the basic cutting quantities (shear angle, friction coefficient and shear stress) determined from orthogonal cutting tests. Thus, the “unified mechanics of cutting” approach reduces the laborious effort that was required in the old mechanistic approach that needed experimentally determined coefficients for each cutter geometry.

The “unified mechanics of cutting” approach was used to predict torques and thrust forces produced in drilling [Wiriyacosol; 1979]. Moreover, that approach was used by many researchers to predict cutting forces in practical milling operations [Yang; 1991][Budak; 1996][Lee; 1996][Yucesan; 1996]. In these studies, the tool geometry was conceptually divided into differential oblique cutting edge segments. The researchers calculated the cutting forces for each segment using an orthogonal cutting model such as the Merchant’s model described above. The fundamental cutting parameters (shear angle, friction coefficient and shear stress) required in Merchant’s model, were determined using orthogonal cutting experiments. The predicted cutting forces were found to be in good agreement with the measured results.

The major disadvantage of the mechanistic modeling of metal cutting operations is that it can not predict the stresses or temperatures produced at the tool-workpiece interface. Thus, tool wear and tool life can not be estimated using that approach.
2.3 Application of Analytical Modelling to Predict Cutting Forces and Average Stresses and Temperatures in Orthogonal Machining

Oxley [Oxley; 1966] developed a predictive machining theory that can predict cutting forces, average temperatures and stresses, in orthogonal cutting, from thermal properties (thermal conductivity and specific heat) and flow stress data as function of strain, strain rate and temperature of the workpiece material, tool geometry and cutting conditions. The theory is based on a chip formation model derived from a slipline field analysis and a strain rate analysis of experimental flow fields [Oxley; 1989]. Oxley’s theory is developed based on the assumptions of plane strain, steady state conditions and a sharp tool. Schematic diagrams showing the model of chip formation, used in the analysis, are shown in Figure 2.3. A diagram from an FEM simulation showing the parameters of Oxley’s theory is shown in Figure 2.4.

In Oxley’s theory the shear plane is considered as a thick plane extending on both sides of the shear plane center AB which was considered as the thin shear plane in the previous theory of Merchant. In the slip line filed analysis, used to develop Oxley’s theory, line AB is considered as a straight slipline near the center of the slipline fields of the chip formation zone [Oxley; 1989]. The basis of the theory is to analyze the stress distributions along line AB which is the center of the primary deformation zone and along the tool-chip interface (both are assumed to be directions of maximum shear stress and maximum shear strain rate). That stress analysis is conducted in terms of the shear angle $\phi$, flow stress data and thermal properties of the workpiece material, and friction angle $\lambda$. Once the value of $\phi$ is determined, the uncut chip thickness $t_2$, average temperatures, average stresses and the various components of forces can be calculated.

At the early stages of developing the theory, two experimentally determined constants were required which are C and $\delta$. C is a constant that
Figure 2.3: Schematic diagram showing the parameters used in the Algorithm of Oxley’s Theory
Figure 2.4: Picture from a real FEM simulation showing the parameters of Oxley's Theory
relates the shear strain rate at plane AB to the quantity \( \frac{V_s}{L} \) where \( V_s \) is the velocity of the workpiece material along plane AB and L is the length of AB as shown in Figure 2.3. The other constant \( \delta \) is the ratio of the tool-chip interface plastic zone thickness to the cut chip thickness. As shown in Figures 2.3 and 2.4, \( \delta t_2 \) is the plastic zone thickness at the tool-chip interface and \( t_2 \) is the cut chip thickness.

Oxley and Hastings [Oxley; 1977] used the stress boundary condition at B to determine the value of the constant C as part of the solution. This stress boundary condition is that the normal stress at point B on the tool rake face, calculated assuming uniform normal stress distribution at the tool-chip interface, should be equal to the normal stress at point B on the shear plane AB calculated from a slip line field equation.

Regarding the determination of the value of \( \delta \), Oxley and Hastings [Oxley; 1976] proposed that it should satisfy the minimum work condition. That is the value of \( \delta \) can be determined, as part of the solution, as the value that causes the cutting force to be a minimum. They showed that \( \delta \) predicted in this way agreed well with the measured results.

The Algorithm of Oxley's predictive machining theory is illustrated in Appendix A. Analytical derivations of some of the important equations in Oxley's theory are illustrated in Appendix D. For orthogonal cutting, the values of the cut chip thickness and the values of forces, predicted by Oxley's theory, were found to be in good agreement with the measured results [Oxley; 1989].

The Algorithm of Oxley's predictive machining theory is summarized in the flow chart of Figure 2.5. In the present work, this Algorithm is used to develop a computer program called OXCUT [Shatla; 1998]. OXCUT can predict cutting forces, average stresses and temperatures at the tool-chip interface in 2-D.
Figure 2.5: Simplified flow chart of Oxley's theory and the Computer code OXCUT developed in the present work
orthogonal cutting operations such as orthogonal turning and orthogonal slot milling operations. The program has been further developed, as will be discussed later, to analyze 3-D practical cutting operations such as 3-D face milling, ball end milling and drilling.

The advantages of Oxley’s predictive machining theory are:

1) It takes into account material flow stress variation with strain, strain rate and temperature.
2) It takes into account the thermal properties of the workpiece material as function of the workpiece temperature.
3) It considers the variation of the shear angle \( \phi \) with the friction angle \( \lambda \) at the tool-chip interface and the shear stress at the primary and secondary deformation zones.
4) It performs the analysis in two deformation zones, primary and secondary, as shown in Figure 2.4 which is close to reality.
5) Its predictive nature because it requires, like FEM, only material property data, tool geometry and cutting conditions to calculate forces and average stresses and temperatures.
6) Its fast calculation time compared to FEM.

The disadvantages of Oxley’s predictive machining theory are:

1) It is developed based on the assumption of a perfectly sharp tool.
2) The normal and shear stresses and temperatures at the tool-chip interface are assumed to be uniform.
3) It calculates average values of stresses and temperatures not distributions like FEM.
2.4 Application of Analytical Modeling (Oxley’s theory) to Analyze Oblique and 3-D Cutting Operations

Lin et al. [Lin; 1982] utilized Oxley’s theory to predict cutting forces for oblique machining conditions. The work of Lin et al. is described in detail in section B.1 in Appendix B. Hu et al. [Hu; 1986] used Oxley’s theory and the concept of the equivalent cutting edge to predict cutting forces in bar turning using oblique sharp nose tools where cutting occurs at two cutting edges (side or main cutting edge and end cutting edge).

Young et al. [Young; 1987] developed a model for predicting the chip flow direction and cutting forces for bar turning using non-oblique nose radius tools. The chip is treated as a series of elements of infinitesimal width where the thickness and orientation of the uncut chip section corresponding to each chip element vary. A detailed description of the work of Young et al. [Young; 1987] is provided in section B.2.2 in Appendix B.

Arsecularante et al. [Arsecularante; 1995] utilized Oxley’s theory to predict chip flow direction and cutting forces in bar turning using oblique nose radius tools. Basically, they obtained the equivalent cutting edge for a corresponding non-oblique nose radius tool using the technique developed by Young et al [Young; 1987]. Then, they projected this equivalent cutting edge into the cutting face plane of the oblique nose radius tool using three dimensional geometric analysis. A more detailed description of the work done by Arsecularante et al. [Arsecularante; 1995] is provided in section B.2.3 in Appendix B.

Young et al. [Young; 1994] utilized Oxley’s theory to predict cutting forces in face milling using a non-oblique nose radius tool with a side cutting edge angle of 45°. They divided the uncut chip conceptually into elements each of which corresponds to a certain tool rotation angle. They calculated the current uncut
chip thickness for each element and used it to predict cutting forces in a similar manner to that of turning with a nose radius tool.

Elhachimi et al. [Elhachimi; 1999a, 1999b] used Oxley's theory to predict thrust forces and torques in twist drilling where they divided the drill geometry into oblique cutting elements similar to the work of Wiriyacosol and Armarego [Wiriyacosol; 1979] [Armarego; 1969]. They used Oxley's theory to conduct the analysis for each element and they summed the calculated forces and torques over all the elements. They found that the predicted thrust forces and torques are in good agreement with their measured results.

2.5 Application of the Finite Element Method (FEM) to Analyze the Machining Process

Up till now, FEM has not been proven to be fully capable of analyzing practical machining processes. One reason for that is the tendency of many researchers to write their own FEM codes that can handle a specific case which is, mostly, orthogonal cutting [Strenkowski; 1990][Sasahara; 1994][Shih; 1995, 1996][Obikawa; 1996][Shinozuka; 1996] [Kim; 1999]. Even researchers who used commercial FEM codes could not analyze the practical 3-D cutting operations such as ball end milling and drilling because those codes still need further development to handle such complicated operations [Strenkowski; 1985] [Komvopoulos; 1991][Zhang; 1994][Rakotomalala; 1993][Marusich; 1995][Ceretti; 1996][Kumar; 1997][Fourment; 1997][Özel; 1998] [Lei; 1999][Zhang; 1999][Hong; 1999].

Another reason for the incapability of FEM to handle practical metal cutting operations is the lack of material property data (flow stress) at the high strains, strain rates and temperatures encountered in metal cutting. Many
researchers used flow stress data that made their FEM results questionable. Stevenson et al. [Stevenson; 1983] modeled the flow stress in the primary deformation zone as a function of strain rate at room temperature. Iwata et al. [Iwata; 1984] assumed the flow stress to be a function of strain alone in their metal cutting FEM simulations. Joshi et al. [Joshi; 1993] assumed a constant flow stress obtained at the average strain rate and temperature in the deformation zone. Ceretti et al. [Ceretti; 1996] assumed the flow stress to be constant for values higher than the test data and as a function of strain, strain rate and temperature for values in the range of the test data. Ng et al. [Ng; 1999] ignored the effect of strain rate on flow stress in their FEM analysis.

2.6 Development of Flow Stress Data for Practical Cutting Operations

Both FEM and analytical modeling require flow stress data of the workpiece material, as function of strain, strain rate and temperature, to analyze the metal cutting process ($\sigma = f(\varepsilon, \dot{\varepsilon}, T)$). Thus, it is necessary to have flow stress data at the high strains (1 and over), strain rates ($10^3$ to $10^5$ s$^{-1}$ or more) and temperatures (200 to 1000 °C or more) encountered in metal cutting operations.

2.6.1 High Speed Compression Tests

Some researchers used high speed compression tests to obtain flow stress data of carbon steels at the high strains, strain rates and temperatures encountered in the metal cutting process [Oyane; 1967]. The disadvantage of that technique is that the flow stress data are obtained at strain rates below 450 s$^{-1}$ and strains below 1. The equations that describe flow stress data for carbon steels, developed using high speed compression tests, are shown in detail in section C.1 in Appendix C.
2.6.2 High Speed Impact Compression Test (Hopkinson's Bar Technique)

The Hopkinson's bar technique is used to obtain flow stress data for various materials at the high strain rates and temperatures encountered in metal cutting [Usui; 1982][Maekawa; 1983, 1996][Obikawa; 1996][Childs; 1997][Lee; 1998]. A description of the Hopkinson's bar technique and a list of some of the flow stress equations, developed using that technique, are given in section C.2 in Appendix C. The disadvantages of the Hopkinson's bar technique are:

1) The results may not be accurate due to the metallurgical phase transformation of the specimen during the test [Shirakashi; 1983].
2) The flow stress data obtained using the test is limited to strains less than 1, strain rates less than $10^3$ s$^{-1}$, whereas the strains in machining can be higher than 2 and the strain rates can reach $10^5$ s$^{-1}$.
3) It is a complicated and relatively expensive technique.

2.6.3 Obtaining Flow Stress Data for Metal Cutting Using the Metal Cutting Process

Oxley was one of the first researchers suggesting to use the machining process to obtain flow stress data at the high strains, strain rates and temperatures encountered in machining [Oxley; 1989]. That idea was applied earlier with limited success [Stevenson; 1970-1971]. Later, this technique was implemented by using the results of orthogonal turning experiments in conjunction with the finite element method [Kumar; 1997]. The investigators tried to obtain the flow stress equation for AISI 1045, suitable for metal cutting, by tuning the flow stress data, given to the finite element code DEFORM, until the predicted cutting forces matched the experimental results. This approach required the calculation of the average strain rate and temperature values in the primary deformation zone using a separate code that can process the FEM
results. Then, the flow stress data, given to DEFORM, was tuned based on those calculated average strain rates and temperatures. The disadvantages of the methodology, developed by Kumar et al. [Kumar; 1997], are:

1) It needs extensive time and effort because the flow stress data was tuned using 10 cutting velocities each of which required 4 iterations of FEM simulations to let the predicted and measured cutting forces match (average 5 hours of CPU time per iteration).

2) It considers the friction at the tool-workpiece interface to have a shear friction factor of 0.5 which may not always agree with most friction models of metal cutting found in the literature.

Özel [Özel; 1998] tried to improve the methodology of flow stress determination, developed by Kumar et al. [Kumar; 1997], by improving the friction model at the tool-workpiece interface using Zorev’s friction model [Zorev; 1963] shown in Figure 2.6. He tried to modify both of the flow stress data and the parameters of the friction model (the length of the sticking region \( l_p \) and the Coulomb friction coefficient of the sliding region \( \mu \) as shown in Figure 2.6) until the machining forces, predicted by FEM (DEFORM), matched those measured experimentally. He had to use the same code, used by Kumar et al., to calculate the average strain, strain rate and temperature in the deformation zone from the FEM results to be able to tune the flow stress data corresponding to those values. The disadvantages of the technique, used by Özel [Özel; 1998], are:

1) It needs more iterations than the technique, developed by Kumar et al. [Kumar; 1997], because it requires modifying both of the flow stress data and friction parameters to let the predicted and measured forces match.

2) It may give false results because there are an infinite number of combinations of friction and flow stresses that can give the same predicted machining
forces (by increasing friction and decreasing flow stress or vice versa one can get the same cutting forces).

3) It requires a separate code to calculate the average strains, strain rates and temperatures in the deformation zone from the FEM results.

Thus, it is still necessary to develop an "automated" technique to obtain flow stress data at the practical range of strains, strain rates and temperatures used in the High Speed Machining applications. This technique, in the present work, will be based on the analytical modeling of the metal cutting process (Oxley’s theory) in conjunction with orthogonal slot milling experiments as described in Chapter 4.

![Diagram](image)

Figure 2.6: Curves representing normal ($\sigma_t$) and frictional ($\tau$) stress distributions on the tool rake face [Zorev; 1963].

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2.7 Friction at the Tool-Workpiece Interface in Metal Cutting

Many researchers tried to establish models that can describe the frictional and normal stress distributions at the tool-workpiece interface in metal cutting. As shown in Figure 2.6, Zorev [Zorev; 1963] divided the tool-chip interface length into a sticking region where the frictional stress equals the shear flow stress of the material and a sliding region where the frictional stress follows the Coulomb friction model. The normal stress in Zorev's model is assumed to follow an exponential curve throughout the tool-chip interface length. Barrow and Lee [Barrow; 1982] [Lee; 1995] did minor improvements to Zorev's model based on experimental evidence.

Figures 2.7 and 2.8 show the normal and shear stress distributions measured at the tool-chip interface in metal cutting using the split tool technique [Childs; 1989][Li; 1997]. As shown in those figures, the contribution of the sliding region to the shear and normal forces is small compared to the contribution of the sticking region. Thus, the assumption made by Oxley that the normal stress is constant along the tool-chip interface and that the frictional stress at the tool-ship interface is also constant and equal to the shear flow stress of the material may not be very far from reality.

2.8 Tool Wear in Metal Cutting

Due to the high temperatures, stresses and relative velocities that occur at the tool-workpiece interface, excessive wear might occur on both of the rake and flank sides of the tool as shown in Figure 2.9. There are different wear mechanisms in metal cutting. Therefore, tool wear can not be attributed to a unique wear mechanism but to a combination of several of them. In view of the complexity of the various wear mechanisms and theories, it is very difficult and
Figure 2.7: Experimental results of normal and shear stress distributions measured along the tool-chip interface using the split tool technique [Childs; 1989]
Figure 2.8: Experimental results of normal and shear stress distributions measured along the tool-chip interface using the split tool technique [Li; 1997]
sometimes impossible to implement a wear model that takes into account all wear mechanisms that are present in machining. A simple but practical way to evaluate tool wear is to consider only the dominant mechanism. Different tool wear mechanisms are illustrated qualitatively in Figure 2.10.

Tool wear mechanisms in metal cutting can be classified as sliding and non-sliding. The most important “sliding wear mechanisms” are:

**Abrasive wear:** This type of wear is caused by the contact of a tool body with a harder particle that slides over the tool body removing material during its movement. Abrasive wear causes material loss in both of the flank and rake faces of the cutting tool [Yamaguchi; 1990][Painter; 1995][Vardan; 1987][Ravinowicz; 1984].

**Adhesive wear:** Basically adhesive wear is the result of formation and breakage of interfacial bonds between the tool and the workpiece. Adhesive wear depends on surface roughness, crystal structure, atomic configuration and hardness [Suh; 1986][Painter; 1995][Gahr; 1987] [Jain; 1979][Rhee; 1970].

The important “non-sliding wear mechanisms” are:

**Solution and diffusion wear:** When two materials come in contact, they have a tendency to dissolve in each other to form a solution. The solution rate increases with the interface temperature. Once the solution has formed, it is carried away by diffusion. If the rate of dissolution is higher than the rate of diffusion, the wear is termed solution wear, otherwise, it is called diffusion wear [Loladze; 1981][Suh; 1986][Strenkowsky; 1991] [Kramer; 1985].
Figure 2.9:  Typical tool wear locations in metal cutting [Holmberg; 1994]

Figure 2.10:  Different tool wear mechanisms [König; 1984]
Oxidation wear: In the chip formation zone, the generated high temperatures encourage the tool material to react with oxygen forming thin oxide layers on the cutting tool surface. Those oxide layers are carried away by the relative movement between the tool and the workpiece. This is called oxidation wear which is the material loss from the tool body in the form of oxides [Lim; 1987].

Intuitively speaking, wear between two surfaces sliding against each other should be influenced by the stresses, temperatures and relative velocities at the interface between the two surfaces. Many researchers tried to use fundamental tool wear equations derived by considering the possible types of wear mechanisms on the rake and flank faces of cutting tools.

Mathew [Mathew; 1989] used a wear equation, derived from a diffusion wear model, in the form:

\[ \dot{w} = Ae^{-\frac{B}{T}} \]  

(2.1)

where

\( \dot{w} \): wear rate.

A, B : empirical constants to be determined.

T : interface temperature.

Mathew [Mathew; 1989] obtained the constants A and B of equation 2.1, for Carboloy 370 when cutting 0.2% and 0.38% carbon steel, by curve fitting his measured flank wear results to the predicted temperature results using Oxley's theory.

Takeyama and Murata [Takeyama; 1963] used an empirical equation derived, based on abrasion and diffusion, in the form:

\[ \dot{w} = A^* V + Be^{\frac{-E}{KT}} \]  

(2.2)
where
\[ \dot{\omega} \quad : \text{wear rate.} \]
A,B,K : empirical constants to be determined.
\[ E \quad : \text{activation energy of the process} \]
\[ T \quad : \text{flank temperature.} \]
\[ V \quad : \text{velocity at the tool-workpiece interface.} \]

It can be noticed that the second term in equation 2.2 is similar to that in equation 2.1 (basically it is the exponential of a constant divided by the temperature). Takeyama and Murata obtained the constants A, B and K, for carbide P10 and K20 when cutting cast iron and the heat resistant steel alloy G18B, by curve fitting their measured flank wear results to their measured flank temperature results. They used the cutting velocity \((u)\) as the velocity at the tool-workpiece interface on the flank side, required in equation 2.2.

Usui et al. [Usui; 1978] investigated crater wear of Carbide P20 using an empirical equation, derived from a wear model based on diffusion and adhesion, in the form:
\[ \dot{\omega} = A * \sigma_r * V * e \left( -\frac{B}{T} \right) \]

(2.3)
where
\[ \dot{\omega} \quad : \text{wear rate.} \]
\[ \sigma_r \quad : \text{normal stress at the tool-workpiece interface.} \]
\[ V \quad : \text{relative velocity between tool and workpiece at the interface.} \]
\[ T \quad : \text{temperature at the interface.} \]
A, B : constants to be determined.

They obtained the constants A and B by relating their crater wear results measured during turning of low carbon steel (0.15% C, 0.25% C, 0.35% C and 0.45% C) using carbide P20 to their temperature, normal stress and relative velocity results at the tool-chip interface. Usui et al. argued that equation 2.3 with their predicted constants can be used to predict crater wear of Carbide P20 when cutting any low carbon steel. This is because the carbon content should affect
the stresses, temperatures and relative velocities, induced at the tool-workpiece interface, however, the wear mechanics should be the same for low carbon steels with different carbon contents.

Later, Kitagawa and Maekawa [Kitagawa; 1988][Maekawa; 1989] argued that equation 2.3, with the constants A and B estimated for crater wear, can be used to predict flank wear by relating the flank wear geometry to the crater wear geometry. This will be explained in detail in Chapter 6.

A more detailed and comprehensive literature review about tool wear models and experiments in metal cutting can be found elsewhere [Castellanos; 1999].
CHAPTER 3

UTILIZATION AND VALIDATION OF ANALYTICAL – BASED COMPUTER MODELING TO PREDICT PROCESS VARIABLES IN METAL CUTTING

3.1 Introduction

Oxley’s theory [Oxley; 1989] is used in the present work to develop a computer code, called OXCUT, that can predict cutting forces and average temperatures and stresses at the tool-workpiece interface in orthogonal and 3-D cutting operations. Results of orthogonal turning and orthogonal slot milling, obtained from OXCUT, are compared to the published experimental results [Childs; 1997] [Maekawa; 1989] and to results obtained from experiments conducted at the ERC/NSM [Özel; 1998] and Fraunhofer Institute [Sterr; 1998]. Moreover, 3-D cutting results of turning and face milling, using oblique nose radius inserts with two cutting edges, ball end milling and drilling, obtained from OXCUT, are compared to the experimental results, published in the literature [Arsecularante; 1995][Lee; 1996][Wiriyacosol; 1979][DeVries; 1968][Agapiou; 1990]. Furthermore, the temperature and stress results predicted by OXCUT are compared to the results of the Finite Element method (FEM) using DEFORM 2D. The FEM results are utilized to induce a method to use OXCUT to predict temperature and effective stress on the flank side of the tool.
3.2 Applying Oxley’s Analytical Model Using General Forms of Flow Stress Equations

Oxley’s analytical model of cutting, described in Appendix A, was developed based on a flow stress equation in the form:
\[
\bar{\sigma} = \sigma_1 \varepsilon^n
\]  
(3.1)

where \(\bar{\sigma}\) is the flow stress of the material and \(\varepsilon\) is the strain. \(\sigma_1\) and \(n\), both of which vary with the strain rate (\(\dot{\varepsilon}\)) and temperature (\(T\)), are parameters of the flow stress equation. The strain hardening exponent \(n\) of equation 3.1 is required in equations A.14, A.23 and A.34 in the Algorithm of Oxley’s theory described in Appendix A. To be able to use more general forms of flow stress equations such as those described in Appendix C (e.g. equation C.15), two techniques are suggested in the present work. The first technique, explained in detail in Appendix E, is to calculate the equivalent \(\sigma_1\) and \(n\) that make the flow stress equation 3.1 curve fit the given general flow stress equation that is assumed to be in the form
\[
\bar{\sigma} = \text{function (\(\varepsilon\), \(\dot{\varepsilon}\), \(T\))}
\]  
(3.2)

The other technique is to modify Oxley’s analytical model to account for more general forms of flow stress equations. Flow stress equations similar to equation C.15 can be handled by modifying Oxley’s analytical model as follows:

At a certain strain rate and temperature, equation C.15, which is used extensively in the present work will take the form:
\[
\bar{\sigma} = (A + B \varepsilon^n)C_1C_2
\]  
(3.3)

where \(C_1\) and \(C_2\) are constants that represent the values of the temperature and strain rate terms at a certain strain rate and temperature. Equation 3.3 can be written in the form
\[
\bar{\sigma} = (\sigma_0 + \sigma_1 \varepsilon^n)
\]  
(3.4)
where $\sigma_0$ and $\sigma_{11}$ are constants that depend on strain rate and temperature. Performing analytical derivations similar to those of Appendix D, Equations D.18, D.22 and D.35 are modified, in the present work as shown in Appendix E, to equations 3.5, 3.6 and 3.7, respectively, for the more general flow stress model described by equation 3.4.

$$\tan \theta = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - 3Cn + \frac{3\sigma_{0,AB}}{\bar{\sigma}_{AB}}$$

$$\sigma_{n,AB}^* = k_{AB} \left[ 1 + \frac{\pi}{2} - 2 \alpha - 6Cn \right] + \frac{6Cn \sigma_{0,AB}}{\sqrt{3}}$$

$$h = \frac{t_c}{\cos \lambda} \cos \theta \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) - 2Cn - 2Cn \frac{\sigma_{0,AB}}{\bar{\sigma}_{AB}} \right]$$

where $\sigma_{0,AB}$ is the flow stress constant of equation 3.4 evaluated at the strain rate and temperature of the shear plane AB and $\bar{\sigma}_{AB}$ is the flow stress at plane AB (Figure 2.3). Equations 3.5, 3.6 and 3.7 were used instead of equations A.14, A.23 and A.34, respectively, in OXCUT. It was found that the solution of OXCUT became less robust when those modified equations were used. Thus, in the present work, it is decided to use the first technique to apply Oxley's theory for general forms of flow stress equations.

3.3 2-D Orthogonal Turning

In this work, the cutting forces and temperatures developed at the tool-workpiece interface and the shear zone angle, predicted by OXCUT, are compared to the experimental results published by Childs et al. [Childs; 1997] and Maekawa et al. [Maekawa; 1989] for the workpiece material, tool geometry and cutting conditions described in Tables 3.1 and 3.2, respectively.
<table>
<thead>
<tr>
<th>Workpiece</th>
<th>Low Carbon Free Cutting Steel (LCFCS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>equation C.14 in Appendix C</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>$\sigma = 62 - 0.044T, \quad T \text{ in } ^\circ \text{C}$ (3.8)</td>
</tr>
<tr>
<td>Thermal conductivity $K$ (W/m°C)</td>
<td>$K = 0.0166 + (0.0035 + 0.0016)$, $T \text{ in } ^\circ \text{C}$ (3.10.a)</td>
</tr>
<tr>
<td>Specific heat $S$ (J/kg °C)</td>
<td>$S = 450 + 0.38T, \quad T \text{ in } ^\circ \text{C}$ (3.9)</td>
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<tr>
<th>Cutting Conditions</th>
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<tr>
<td>Cutting speed ($u$)</td>
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<tr>
<td>Uncut chip thickness ($t_u$)</td>
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<td>Width of cut ($W$)</td>
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<td>Friction</td>
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<tr>
<th>Tool Material and Geometry</th>
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<tr>
<td>Tool rake angle ($\alpha$)</td>
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<tr>
<td>Hone radius ($r_h$)</td>
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<td>Tool material</td>
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Table 3.1: Data for orthogonal cutting of LCFCS [Childs; 1997]

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<tr>
<th>Workpiece</th>
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<tbody>
<tr>
<td>Material</td>
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<tr>
<td>Chemical composition for 0.45% Carbon steel</td>
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<tr>
<td>Chemical Composition for 0.25% Carbon steel</td>
</tr>
<tr>
<td>Flow stress data</td>
</tr>
<tr>
<td>Thermal conductivity $K$ (W/m°C)</td>
</tr>
<tr>
<td>Specific heat $S$ (J/kg °C)</td>
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<td>Hone radius ($r_h$)</td>
</tr>
<tr>
<td>Tool material</td>
</tr>
</tbody>
</table>

Table 3.2: Data for orthogonal cutting of 0.46% and 0.25% Carbon steels [Maekawa; 1989][Kitagawa; 1988] [Oxley; 1989]
3.3.1 Orthogonal Turning of Low Carbon Free Cutting Steel (LCFCS)

As shown in Figure 3.1, the average temperature on the tool rake face, calculated by OXCUT for the conditions in Table 3.1, is in reasonable agreement with the temperature distribution on the tool rake face measured experimentally [Childs; 1997]. Figures 3.2 and 3.3 illustrate the cutting force ($F_C$) and feed force ($F_T$), respectively, vs cutting velocity, obtained experimentally and from OXCUT, for the conditions in Table 3.1. The cutting force ($F_C$) is the force in the direction of the cutting velocity (i.e. tool motion) and the feed force ($F_T$) is the force in the feed direction (perpendicular to $F_C$). Good agreement is found between the cutting force results, obtained experimentally and from OXCUT, for different cutting velocities, as shown in Figure 3.2. However, as shown in Figure 3.3, the agreement between the predicted and measured feed forces vs cutting velocity is not as good.

The measured feed forces, shown in Figure 3.3, can be observed to increase with cutting velocity for cutting velocities higher than 150 m/min which contradicts most of the experimental data found in the literature [Oxley; 1989]. The cutting and feed forces in machining should decrease with increasing the cutting velocity due to the thermal softening of the workpiece material. As the cutting velocity increases, both the temperature and strain rate of the workpiece material increase. The flow stress of the workpiece material increases with increasing the strain rate and decreases with increasing the temperature. However, for most metals, the influence of the temperature on flow stress is more significant than the influence of strain rate. Thus, at higher cutting velocities, both the cutting and feed forces should decrease.

The increase of the measured feed force results at higher cutting velocities (higher than 150 m/min), shown in Figure 3.3, may be due to tool wear that increases at higher cutting velocities and causes the cutting edge to be more
Figure 3.1: Comparison between the predicted and measured temperatures on the tool rake face for orthogonal turning of LCFCS (cutting data is given in Table 3.1)
Figure 3.2: Comparison between the predicted and measured cutting forces vs cutting velocity for orthogonal turning of LCFCS (cutting data is given in Table 3.1)
Figure 3.3: Comparison between the predicted and measured feed forces vs cutting velocity for orthogonal turning of LCFCS (cutting data is given in Table 3.1)
round and more dull (larger hone radius). Increasing the hone radius of the cutting edge causes both the cutting and feed forces to increase but it has a larger influence on the feed force than on the cutting force. OXCUT performs the calculations based on the assumption of a sharp tool which may be the reason why the calculated feed force results do not agree very well with the measured results at higher cutting velocities where tool wear is expected to be higher.

Figure 3.4 shows the variation of the shear zone angle (\( \phi \)), obtained experimentally and from OXCUT, with the cutting velocity. The shear zone angle (\( \phi \)) can be obtained experimentally by measuring the cut chip thickness (\( t_2 \)) and knowing the uncut chip thickness (\( t_1 \)) and the tool rake angle (\( \alpha \)) from the relation [Oxley; 1989]:

\[
\phi = \tan^{-1} \frac{(t_1/t_2) \cos \alpha}{1 - (t_1/t_2) \sin \alpha}
\]  

(3.12)

3.3.2 Orthogonal Turning of 0.46% and 0.25% Carbon Steel

The cutting results of orthogonal turning of 0.46% and 0.25% carbon steel, predicted by OXCUT, are compared to the experimental results published in the literature [Maekawa; 1989], for the conditions in Table 3.2. The temperature, the cutting force, the feed force and the shear zone angle results are shown in Figures 3.5, 3.6, 3.7 and 3.8, respectively. The discussion of the results of orthogonal turning of LCFCS, provided in section 3.3.1, applies to the results of this section. Figure 3.5 shows the predicted and measured temperatures on the tool rake face. Figures 3.6 and 3.7 show that the agreement between the cutting and feed force results for 0.25% C steel is better than that for 0.46% C steel. This may be due to dulling of the cutting edge because of tool wear that is expected to increase with the carbon content of the steel workpiece.
Figure 3.4: Comparison between the predicted and measured shear zone angle vs cutting velocity for orthogonal turning of LCFCS (cutting data is given in Table 3.1)
Figure 3.5: Comparison between the predicted and measured temperatures on the tool rake face for orthogonal turning of 0.46% C steel (cutting data is given in Table 3.2)
Figure 3.6: Comparison between the predicted and measured cutting forces vs cutting velocity for orthogonal turning of 0.46% and 0.25% C steel (cutting data is given in Table 3.2)
Figure 3.7: Comparison between the predicted and measured feed forces vs cutting velocity for orthogonal turning of 0.46% and 0.25% C steel (cutting data is given in Table 3.2)
Figure 3.8: Comparison between the predicted and measured shear zone angle vs cutting velocity for orthogonal turning of 0.46% and 0.25% C steel (cutting data is given in Table 3.2)
3.4 2-D Orthogonal Slot Milling

The slotting operation of a thin plate, shown in Figure 3.9, can be modeled as 2-D orthogonal cutting because cutting occurs along one cutting edge that is parallel to the axis of rotation of the tool. In this case, the uncut chip thickness or depth of cut \( t_1 \) varies with the tool rotation angle \( \theta_r \) according to the relation [Young; 1994]:

\[
t_1 = f \sin \theta_r
\]  
(3.13)

Where \( f \) is the feed in mm/tooth. This relation is established based on the assumption of a circular tool path of the milling cutter. This circular path is nearly the same as the cycloidal path generated due to the combined rotation and translation of the milling cutter relative to the workpiece.

To analyze the process using OXCUT, the circular tool path, between the tool inlet and exit angles shown in Figure 3.10, is digitized at tool rotation angles in steps of 5° similar to the work found in the literature [Young; 1994]. At each increment, the current depth of cut \( t_1 \) at a certain rotation angle of the tool \( \theta_r \) is calculated from equation 3.13. Then, the cutting and feed forces are calculated using OXCUT for each depth of cut at each angular increment of tool rotation.

To compare with experimental results, the cutting and feed forces, \( F_C \) and \( F_T \), are resolved at each tool rotation angle in the global X and Y directions shown in Figure 3.9 using the relations [Young; 1994]:

\[
F_X = F_C \cos \theta_r + F_T \sin \theta_r \\
F_Y = F_C \sin \theta_r - F_T \cos \theta_r
\]  
(3.14.a) (3.14.b)

Figure 3.11 shows the variation of the X and Y forces \( F_X \) and \( F_Y \) and the cutting and feed forces \( F_C \) and \( F_T \) with tool rotation angle \( \theta_r \) in orthogonal slot milling. At a tool rotation angle of 90°, the uncut chip thickness reaches its maximum value.
Figure 3.9: The 2-D orthogonal slot milling operation [Özel; 1998]

Figure 3.10: Tool inlet and exit angles in milling
Figure 3.11: Variation of the X and Y forces $F_X$ and $F_Y$ and the cutting and feed forces $F_C$ and $F_T$ with tool rotation angle in 2-D orthogonal slot milling.
Thus, the cutting and feed forces reach their maximum values \( F_C - 90 \) and \( F_T - 90 \) at a tool rotation angle of 90° as shown in Figure 3.11.

3.4.1 Orthogonal Slot Milling of P20 Mold Steel (30 HRC)

For orthogonal slot milling of P20 mold steel (30 HRC), the cutting forces in the X and Y directions, predicted by OXCUT, are compared to the measured results published in the literature [Özel; 1998]. The workpiece material properties [Oxley; 1989], tool geometry and cutting conditions are illustrated in Table 3.3. The workpiece material properties of P20 mold steel (30 HRC) are assumed to be the same as those of a carbon steel of the same chemical composition.

<table>
<thead>
<tr>
<th>Workpiece Material</th>
<th>P20 mold steel (30 HRC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical composition</td>
<td>0.3% C, 0.85% Mn, 0.015% P, 0.003% S, 0.3% Si, 1.1% Cr, 0.08% V, and 0.55% Mo</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>equations C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity ( K (W/m°C) )</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat ( S (J/kg °C) )</td>
<td>Equation (3.11) in Table 3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cutting Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed ( (u) )</td>
<td>200 m/min</td>
</tr>
<tr>
<td>Feed ( (f) )</td>
<td>0.1 and 0.155 mm/tooth</td>
</tr>
<tr>
<td>Width of cut ( (W) )</td>
<td>1 mm</td>
</tr>
<tr>
<td>Tool inlet angle (Figure 3.10)</td>
<td>0°</td>
</tr>
<tr>
<td>Tool exit angle (Figure 3.10)</td>
<td>180°</td>
</tr>
<tr>
<td>Friction</td>
<td>Shear flow stress of the material</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool Material and Geometry</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool rake angle ( (\alpha) )</td>
<td>-11.4°</td>
</tr>
<tr>
<td>Hone radius ( (r_h) )</td>
<td>OXCUT assumes sharp tool</td>
</tr>
<tr>
<td>Number of cutting edges ( (N_c) )</td>
<td>1</td>
</tr>
<tr>
<td>Tool material</td>
<td>uncoated tungsten carbide</td>
</tr>
</tbody>
</table>

Table 3.3: Data for orthogonal slot milling of P20 mold steel (30 HRC) [Özel; 1998]
Figures 3.12 and 3.13 show the cutting forces, in the global X and Y directions, $F_x$ and $F_y$, vs tool rotation angle, obtained experimentally [Özel; 1998] and from OXCUT, for feeds of 0.1 mm/tooth and 0.155 mm/tooth, respectively, and the cutting data of Table 3.3. In both figures the solid circles represent the average measured forces of 20 milling cycles (revolutions). Similarly, the light lines indicate the range of these average forces. The heavy line represents the results of OXCUT. It can be observed from Figures 3.12 and 3.13 that the forces obtained from OXCUT are in reasonable agreement with those measured experimentally.

3.4.2 Orthogonal Slot Milling of the Titanium Alloy Ti6Al4V

The workpiece material properties, tool material properties and geometry, and cutting conditions are shown in Table 3.4. The force results, predicted by OXCUT, are compared to these, obtained from experiments performed at the Fraunhofer Institute [Sterr; 1998] and from FEM (DEFORM) simulations as shown in Figure 3.14.

The temperature at the tool-workpiece interface, predicted by OXCUT, is compared to that predicted by FEM (DEFORM) as shown in Figure 3.15. The FEM temperature results are displayed by DEFORM as contour lines. Therefore, the temperature predicted by DEFORM at the tool-workpiece interface, shown in Figure 3.15, is given in terms of the maximum value, the minimum value and the average value. The temperature contour lines at a tool rotation angle of 55°, obtained from DEFORM, are shown in Figure 3.16.
Figure 3.12: Forces vs tool rotation angle obtained from OXCUT and experiments during orthogonal slot milling of P20 mold steel (30 HRC) using a feed of 0.1 mm/tooth (cutting data is given in Table 3.3)
Figure 3.13: Forces vs tool rotation angle obtained from OXCUT and experiments during orthogonal slot milling of P20 mold steel (30 HRC) using a feed of 0.155 mm/tooth (cutting data is given in Table 3.3)
<table>
<thead>
<tr>
<th>Workpiece</th>
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</thead>
<tbody>
<tr>
<td>Material</td>
<td>Ti6Al4V</td>
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</tr>
<tr>
<td>Flow stress data</td>
<td>equation C.15 in Appendix C</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity K (W/m °K)</td>
<td>6.7</td>
<td></td>
</tr>
<tr>
<td>Specific heat S (J/kg °K)</td>
<td>542</td>
<td></td>
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<table>
<thead>
<tr>
<th>Cutting Conditions</th>
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<th></th>
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</thead>
<tbody>
<tr>
<td>Cutting speed (u)</td>
<td>74.81 m/min</td>
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<tr>
<td>Feed (f)</td>
<td>0.25 mm/tooth</td>
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</tr>
<tr>
<td>Width of cut (W)</td>
<td>3 mm</td>
<td></td>
</tr>
<tr>
<td>Tool inlet angle (Figure 3.10)</td>
<td>0°</td>
<td></td>
</tr>
<tr>
<td>Tool exit angle (Figure 3.10)</td>
<td>180°</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Shear flow stress of the material for OXCUT calculations</td>
<td></td>
<td></td>
</tr>
<tr>
<td>• Frictional stress-vs-normal stress data, obtained by OXCUT at the tool-chip interface and input to FEM (DEFORM), for FEM simulations</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool Geometry</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Tool rake angle (α)</td>
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<td></td>
</tr>
<tr>
<td>Tool Clearance Angle (γ)</td>
<td>15°</td>
<td></td>
</tr>
<tr>
<td>Rake radius (r)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OXCUT assumes sharp tool, 0.01 mm for FEM</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of cutting edges (N_e)</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool Material Properties</th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Tool material</td>
<td>K10 uncoated carbide</td>
<td></td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>600 GPa</td>
<td></td>
</tr>
<tr>
<td>Poisson's ratio</td>
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<td></td>
</tr>
<tr>
<td>Thermal Conductivity (W/m °K)</td>
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<td></td>
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<tr>
<td>Heat capacity N/mm²/°C</td>
<td>3.12</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.4: Data for orthogonal slot milling of Ti6Al4V [Lee; 1998][Obikawa; 1996][Sterr; 1998][ASM; 1997]
Figure 3.14: Forces vs tool rotation angle obtained from OXCUT, FEM (DEFORM) and experiments during orthogonal slot milling of Ti6Al4V (cutting data is given in Table 3.4)
Figure 3.15: Temperature at the tool-workpiece interface vs tool rotation angle obtained by OXCUbT and FEM (DEFORM) for orthogonal slot milling of Ti6Al4V (cutting data is given in Table 3.4)

Figure 3.16: Temperature contours (°C) for orthogonal slot milling of Ti6Al4V
3.5 3-D Oblique Turning

Practical three dimensional metal cutting processes such as turning and face milling using oblique nose radius tools are characterized by cutting along two cutting edges that define an oblique plane instead of the one straight cutting edge of orthogonal cutting. Schematic diagrams of the oblique nose radius tool with two cutting edges, showing the American Standards Association nomenclature and the nomenclature used by Oxley and his co-workers, are shown in Figures 3.17 and 3.18, respectively. To use OXCUT to predict forces in 3-D turning, the concept of the equivalent cutting edge, developed by Oxley and his co-workers [Lin; 1982][Hu; 1986][Young; 1987][Arsecularante; 1995], as discussed in section B.2 in appendix B, is utilized in the present work.

The tool angles, required to perform the analysis are the side cutting edge angle ($C_s$), the normal rake angle ($\alpha_n$) and the inclination angle ($i$) shown in Figure 3.18. The normal rake angle ($\alpha_n$) is defined as the angle measured, in a plane normal to the side cutting edge, between the cutting face plane (dark plane in Figure 3.18) and the horizontal YZ plane as shown in Figure 3.18 [Hu; 1986]. The inclination angle ($i$) is the angle measured in a vertical plane passing through the side cutting edge, between the side cutting edge and the horizontal YZ plane as shown in Figure 3.18 [Hu; 1986]. The side cutting edge angle ($C_s$) is defined as the angle measured, in the horizontal YZ plane, between a vertical plane passing through the side cutting edge and a vertical plane passing through the Z-axis [Hu; 1986] as shown in Figures 3.17 and 3.18.

The majority of cutting tool manufacturers use the nomenclature of the American Standards Association to define the geometry of their turning tools and inserts. Only the side cutting edge angle ($C_s$) is the same in both of the nomenclatures used by the American Standards Association and that used by Oxley and his co-workers as shown in Figures 3.17 and 3.18. The inclination
Figure 3.17: Schematic diagram of an oblique nose radius turning tool with two cutting edges (side cutting edge and end cutting edge) showing American Standards Association nomenclature [Shaw; 1984].
X-X : Cutting Force Direction, Vertical Axis
Y-Y : Feed Force Direction
Z-Z : Radial Force Direction, Depth of Cut

Figure 3.18: Schematic diagram of an oblique nose radius turning tool with two cutting edges (side cutting edge and end cutting edge) showing the nomenclature used by Oxley and his co-workers
angle (i) and the normal rake angle ($\alpha_n$), shown in Figure 3.18, can be calculated from the back rake angle ($\alpha_b$), the side rake angle ($\alpha_s$) and the side cutting edge angle ($C_s$) using the relations [Shaw; 1984][Stephenson; 1997]:

$$i = \tan^{-1}(\tan \alpha_b \cos C_s - \tan \alpha_s \sin C_s)$$

(3.15)

$$\alpha_n = \tan^{-1}[\cos i(\tan \alpha_s \cos C_s + \tan \alpha_b \sin C_s)]$$

(3.16)

As shown in Figures 3.19, 3.20 and 3.21, the cutting forces, predicted by OXCUT, are compared to the published results of 3-D turning using oblique nose radius inserts for the workpiece material, tool geometry and cutting conditions stated in Table 3.5 [Arsecularante; 1995].

<table>
<thead>
<tr>
<th>Workpiece</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>0.19% C steel</td>
</tr>
<tr>
<td>Chemical composition</td>
<td>0.19% C, 0.88% Mn, 0.27% Si, 0.085% Cr, 0.021% P, 0.02% S, 0.02% Ni, 0.02% Cu, 0.02% Ti, and 0.005% Mo</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>equations C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity $K$ (W/m$^2$C)</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat $S$ (J/kg °C)</td>
<td>Equation (3.11) in Table 3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cutting Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed ($u$)</td>
<td>200 m/min</td>
</tr>
<tr>
<td>Feed ($f$)</td>
<td>0.224 mm/rev</td>
</tr>
<tr>
<td>Depth of cut ($d$)</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>Friction</td>
<td>Shear flow stress of the material</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool Geometry</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclination angle ($i$)</td>
<td>10°</td>
</tr>
<tr>
<td>Side cutting edge angle ($C_s$)</td>
<td>10°</td>
</tr>
<tr>
<td>Normal rake angle ($\alpha_n$)</td>
<td>-10°, 0°, 10°</td>
</tr>
<tr>
<td>Nose radius ($r_n$)</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>Hone radius ($r_h$)</td>
<td>OXCUT assumes sharp tool</td>
</tr>
</tbody>
</table>

Table 3.5: Data for 3-D turning of 0.19% C steel [Arsecularante; 1995]
Figure 3.19: Comparison between the cutting force ($P_1$) predicted by OXCUT and measured experimentally during 3-D oblique turning of 0.19% C steel (cutting data is given in Table 3.5)
Figure 3.20: Comparison between the feed force ($P_2$) predicted by OXCUT and measured experimentally during 3-D oblique turning of 0.19% C steel (cutting data is given in Table 3.5)
Figure 3.21: Comparison between the radial force \( P_3 \) predicted by OXCUT and measured experimentally during 3-D oblique turning of 0.19% C steel (cutting data is given in Table 3.5)
3.6 3-D Face Milling Using Oblique Nose Radius Inserts

Figure 3.22 illustrates a schematic diagram of a practical face milling cutter with four inserts adapted from a catalogue prepared by Ingersoll Cutting Tools Co. [Ingersoll; 1999]. The inserts are usually located in the tool in such a way that they have a certain axial rake angle ($\alpha_a$), radial rake angle ($\alpha_r$) and lead angle ($C_s$) as shown in Figure 3.22. The inserts also have a nose radius ($r_n$). For the practical 3-D face milling cutter, shown in Figure 3.22, the axial rake, the radial rake and the lead angles correspond to the back rake, the side rake and the side cutting edge angles, respectively, of the 3-D turning tool shown in Figure 3.17. Thus, the analysis of the practical 3-D face milling operation can be performed similar to the analysis of the 3-D turning operation.

Young et al. [Young; 1994] analyzed a special case of the face milling operation in which the normal rake angle ($\alpha_n$) and the inclination (i) of the face milling cutter were both zeros. In the present work, the face milling operation is analyzed using the general case of non-zero axial rake, radial rake and lead angles and with non-zero nose radius inserts. The normal rake angle ($\alpha_n$) and the inclination (i) of the 3-D face milling insert, shown in Figure 3.22, can be calculated using equations 3.15 and 3.16. Then, a three dimensional analysis, similar to that used by Arsecularante et al. [Arsecularante; 1995] for oblique 3-D turning, is used to obtain the equivalent cutting edge for the oblique face milling inserts with nose radius. Then, the forces were obtained vs tool path using a technique similar to that used by Young et al. [Young; 1994]. Forces in the global X and Y directions are calculated using equations 3.14 and the force in the Z direction which corresponds to the force in the radial direction in 3-D oblique turning is calculated using equation B.2 in Appendix B.

Comparisons between forces, in the X, Y and Z directions, predicted by OXCUT and measured experimentally [Özel; 1998] during 3-D face milling of P20
Figure 3.22: Schematic diagram of the 3-D face milling operation [Ingersoll; 1999]
mold steel (30 HRC), for the workpiece material, tool geometry and cutting conditions illustrated in Table 3.6, are shown in Figures 3.23 through 3.25. The agreement between the calculated and measured forces in the Y and Z direction is better than the agreement in the X direction as shown in Figures 3.23 through 3.25. This might be due to inaccuracies in flow stress data. The high fluctuations in the measured Z forces, shown in Figure 3.25, may be due to chatter and vibration during cutting.

<table>
<thead>
<tr>
<th>Workpiece</th>
<th>P20 mold steel (30 HRC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical composition</td>
<td>0.3% C, 0.85% Mn, 0.015% P, 0.003% S, 0.3% Si, 1.1% Cr, 0.08% V, and 0.55% Mo</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>equation C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat S (J/kg °C)</td>
<td>Equation (3.11) in Table 3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cutting Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed (u)</td>
<td>200 m/min</td>
</tr>
<tr>
<td>Feed (f)</td>
<td>0.1 mm/tooth</td>
</tr>
<tr>
<td>Depth of cut (d)</td>
<td>1 and 2 mm</td>
</tr>
<tr>
<td>Tool inlet angle (Figure 3.10)</td>
<td>0°</td>
</tr>
<tr>
<td>Tool exit angle (Figure 3.10)</td>
<td>180°</td>
</tr>
<tr>
<td>Friction</td>
<td>Shear flow stress of the material</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tool Material and Geometry</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Radial rake angle (αₚ)</td>
<td>-11.4°</td>
</tr>
<tr>
<td>Axial rake angle (αₚ)</td>
<td>0°</td>
</tr>
<tr>
<td>Lead angle (Cₚ)</td>
<td>0°</td>
</tr>
<tr>
<td>Nose radius (rₚ)</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Hone radius (rₚ)</td>
<td>OXCUT assumes sharp tool</td>
</tr>
<tr>
<td>Number of inserts (Nₑ)</td>
<td>1</td>
</tr>
<tr>
<td>Tool material</td>
<td>uncoated tungsten carbide</td>
</tr>
</tbody>
</table>

Table 3.6: Data for 3-D face milling of P20 mold steel (30 HRC) [Özel; 1998]
Figure 3.23: Forces in X direction for 3-D face milling of P20 mold steel hardened to 30 HRC (cutting data is given in Table 3.6)
Figure 3.24: Forces in Y direction for 3-D face milling of P20 mold steel hardened to 30 HRC (cutting data is given in Table 3.6)
Figure 3.25: Forces in Z direction for 3-D face milling of P20 mold steel hardened to 30 HRC (cutting data is given in Table 3.6)
3.7 Drilling

The geometry of the twist drill is shown in Figure 3.26. To analyze the process, the drill lip is divided into orthogonal cutting elements. The rake angle, the uncut chip thickness, the cutting velocity and the width of cut are calculated for each element, using equations from the literature [Armarego; 1969][Wiriyacosol; 1979], as explained in section B.4 in Appendix B. The cutting temperatures in the primary deformation zone (shear zone) and in the secondary deformation zone (rake face) are calculated for each element on the drill lip using Oxley’s theory. The FEM results, discussed at the end of the chapter, reveal that the temperature on the flank face of an orthogonal cutting tool can be approximated as the average of the temperatures in the shear zone and on the rake face. Thus, the temperature distribution on the flank face of the twist drill lip can be approximated using OXCUT.

The temperature distribution on the flank face of the twist drill lip, predicted by OXCUT, is in reasonable agreement with the published experimental results [DeVries; 1968] as shown in Figures 3.27, 3.28 and 3.29. The workpiece material, tool geometry and cutting conditions, used in this study, are illustrated in Table 3.7.
Figure 3.26: Geometry of the twist drill [Armarego; 1969]
<table>
<thead>
<tr>
<th><strong>Workpiece</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>0.45% C steel</td>
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<tr>
<td>Chemical composition</td>
<td>0.45% C, 0.1% Si, 0.77% Mn, 0.015% P</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>equation C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity (K (\text{W/m}^\circ\text{C}))</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat (S (\text{J/kg} ^\circ\text{C}))</td>
<td>Equation (3.11) in Table 3.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Cutting Conditions</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spindle speed (N)</td>
<td>45, 61, and 80 rpm</td>
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<tr>
<td>Feed (f)</td>
<td>0.28 and 0.43 mm/rev</td>
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<tr>
<td>Friction</td>
<td>Shear flow stress of the material</td>
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</table>

<table>
<thead>
<tr>
<th><strong>Drill material and geometry</strong></th>
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</tr>
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<tbody>
<tr>
<td>Drill diameter (D)</td>
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<td>Drill web thickness (2W_b)</td>
<td>5.84 mm</td>
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<tr>
<td>Drill helix angle (\delta_0)</td>
<td>32°</td>
</tr>
<tr>
<td>Drill chisel edge angle (\gamma_0)</td>
<td>135°</td>
</tr>
<tr>
<td>Drill point angle (2p_0)</td>
<td>118°</td>
</tr>
<tr>
<td>Drill material</td>
<td>High Speed Steel (HSS)</td>
</tr>
</tbody>
</table>

Table 3.7: Data for drilling of 0.45% C steel [DeVries; 1968][Agapiou; 1990]
Figure 3.27: Temperature distribution on the flank face of the twist drill lip using a spindle speed of 45 rpm (cutting data is given in Table 3.7)
Figure 3.28: Temperature distribution on the flank face of the twist drill lip using a spindle speed of 61 rpm (cutting data is given in Table 3.7)
Figure 3.29: Temperature distribution on the flank face of the twist drill lip using a spindle speed of 80 rpm (cutting data is given in Table 3.7)
3.8 Ball end milling

The ball end mill, shown in Figure 3.30, has 3 flutes each of which lies on the surface of a hemisphere. The geometry of a ball end milling cutter is usually defined by the ball radius \( R_0 \), the normal rake angle of the cutting edges of the flutes \( \alpha_n \), the helix angle at the flute-shank meeting point \( i_0 \) and the number of flutes \( N_f \). Similar to the twist drilling process, Oxley’s theory can be used to analyze the ball end milling process by dividing the complex 3-D geometry of the ball end milling cutter, shown in Figure 3.30, into infinitesimal oblique cutting elements. The cutting parameters for each element (the uncut chip thickness, the width of cut, the cutting velocity and the inclination angle) can be calculated using the equations described in section B.3 in Appendix B [Lee; 1996]. The temperatures, stresses and forces can be calculated for each element on each flute using Oxley’s theory. The differential cutting forces for each element are summed in the global X, Y and Z directions to get the global cutting forces vs tool rotation angle.

The ball end milling forces, predicted by OXCUT, are compared to the published results [Lee; 1996] for the tool geometry and cutting conditions in Table 3.8. The workpiece material is the titanium alloy Ti6Al4V (workpiece material properties are given in Table 3.4).

<table>
<thead>
<tr>
<th>Sim</th>
<th>Figure</th>
<th>( R_0 ) (mm)</th>
<th>( \alpha_n )</th>
<th>( f ) (mm/flute)</th>
<th>( a ) (mm)</th>
<th>( N_f ) (flutes)</th>
<th>( N ) (rpm)</th>
<th>( i_0 )</th>
<th>Inlet angle</th>
<th>Exit angle</th>
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<tr>
<td>1</td>
<td>3.31</td>
<td>9.525</td>
<td>0</td>
<td>0.0508</td>
<td>6.35</td>
<td>1</td>
<td>269</td>
<td>30</td>
<td>0°</td>
<td>180°</td>
</tr>
<tr>
<td>2</td>
<td>3.32</td>
<td>9.525</td>
<td>5</td>
<td>0.1016</td>
<td>3.81</td>
<td>1</td>
<td>269</td>
<td>30</td>
<td>0°</td>
<td>180°</td>
</tr>
<tr>
<td>3</td>
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<td>10</td>
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<td>1</td>
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<td>1</td>
<td>269</td>
<td>30</td>
<td>0°</td>
<td>90°</td>
</tr>
</tbody>
</table>

Table 3.8: Ball end mill geometry and cutting conditions [Lee; 1996]
Figure 3.30: Geometry of a ball end milling cutter with three flutes [Lee; 1996]
Figures 3.31 through 3.35 show that the cutting forces, obtained in the present work, using flow stress data and thermal properties of the workpiece material, are in good agreement with those obtained mechanistically, using results of orthogonal turning tests, and experimentally, as published in the literature [Lee; 1996]. The cutting forces, shown in Figures 3.31 through 3.33, are produced during slot cutting with full radial immersion using ball end mills (tool inlet angle = 0 and tool exit angle = $\pi$). The cutting forces, shown in Figures 3.34 and 3.35, are produced during half radial immersion cutting using ball end mills (tool inlet angle = 0 and tool exit angle = $\pi/2$). The tool inlet and exit angles were defined before as shown in Figure 3.10.

The predicted average temperature at the tool-chip interface during the ball end milling of Ti6Al4V as a function of axial distance $z$ from the tool tip is shown in Figures 3.36. The predictions are for elements, on the flute, at height $z$ from the tip of the ball end mill. The tool geometry and cutting conditions are from Table 3.8. As shown in Figure 3.36, the higher the value of $z$ (axial distance from the tool tip to the element on the flute), the higher the value of the temperature at the tool-chip interface. If needed, normal and shear stresses can also be displayed as done for temperatures.
Figure 3.31: Ball end milling forces (a) OXCUT (b) measured [Lee; 1996]
Figure 3.32: Ball end milling forces (a) OXCUT (b) measured [Lee; 1996]
Figure 3.33: Ball end milling forces (a) OXCUT (b) measured [Lee; 1996]

(a) Predicted by OXCUT

(b) Measured [Lee; 1996]
Figure 3.34: Ball end milling forces (a) OXCUT (b) measured [Lee; 1996]
Figure 3.35: Ball end milling forces (a) OXCUT (b) measured [Lee; 1996]
Figure 3.36: Predicted temperature at the tool-chip interface for some of the ball end milling conditions of Table 3.8
3.9 Utilization of OXCUT to Predict Stresses and Temperatures on the Flank Face

Because OXCUT performs the analysis in two zones which are the primary deformation zone (shear zone) and the secondary deformation zone (tool-chip interface or tool rake face), it can not predict temperatures and stresses on the tool flank face required for flank wear prediction. In this section, the FEM results will be utilized to investigate the possibility of using OXCUT to approximate temperatures and stresses on the flank face. This will help to predict tool flank wear as well as crater wear in 3-D cutting processes such as drilling.

FEM simulations of orthogonal turning of LCFCS using the cutting data, stated in Table 3.1, were conducted. The tool properties, input to FEM, are Young's modulus, thermal conductivity and heat capacity of 558 GPa, 60 W/m °K and 2.79 N/mm²/°C, respectively [ASM; 1997]. The hone radius, used in the FEM simulations, is 0.02 mm. The friction model, input to FEM (DEFORM), is in the form of frictional stress vs normal stress measured by Childs et al. [Childs; 1997] at the tool-chip interface as shown in Table 3.9.

<table>
<thead>
<tr>
<th>Cutting Velocity = 150 m/min</th>
<th>Cutting Velocity = 250 m/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal Stress (MPa)</td>
<td>Frictional Stress (MPa)</td>
</tr>
<tr>
<td>1100</td>
<td>370</td>
</tr>
<tr>
<td>650</td>
<td>350</td>
</tr>
<tr>
<td>250</td>
<td>210</td>
</tr>
<tr>
<td>90</td>
<td>80</td>
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<td>40</td>
<td>40</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 3.9: Measured normal and frictional stress at the tool-chip interface [Childs; 1997] used to define the friction model in FEM (DEFORM)
The temperature (°C) and effective stress (MPa) distributions, predicted by FEM, for orthogonal turning of LCFCS at a cutting speed of 150 m/min are shown in Figures 3.37 and 3.38, respectively. It can be observed in Figure 3.37 that the temperature contour lines (contour lines C, D and E) in the shear zone (primary deformation zone) are nearly parallel to the shear zone direction which implies that the temperature does not vary much along the shear zone as assumed by Oxley [Oxley; 1989]. The same observation can be made at the tool-chip interface where the temperature contour lines H and I, shown in Figure 3.37, are nearly parallel to the tool rake face which implies that the temperature does not vary much along the tool rake face as assumed by Oxley [Oxley; 1989].

The temperature contour lines C, D and E of the shear zone and the temperature contour lines H and I of the tool rake face meet in the area close to the tool flank face as shown in Figure 3.37. Thus, one way to approximate the temperature on the flank face is to calculate it as the average of the temperatures in the shear zone (primary deformation zone) and the temperature on the rake face (secondary deformation zone). Applying the same reasoning to Figure 3.38, the effective stress on the tool flank face can be approximated as the average of the effective stresses in the shear zone and on the rake face. The same observations were made from the FEM results of orthogonal turning of LCFCS using a cutting velocity of 250 m/min.

To further investigate the validity of the conclusions made from the FEM results of orthogonal turning of LCFCS, another set of simulations of orthogonal turning of 0.2% C steel were conducted using the workpiece material properties, tool geometry and cutting conditions of Table 3.10. The friction was modeled according to Zorev’s friction model shown in Figure 2.6. This was done in DEFORM using two friction windows as shown in Figure 3.39. The lower window specifies the sticking friction region at the tool-workpiece interface and the upper window specifies the sliding friction region. Throughout the present work, the
Figure 3.37: Temperature results (°C) predicted from the FEM modeling of orthogonal turning of LCFCS using \( u = 150 \) m/min (cutting data is given in Table 3.1)
Figure 3.38: Effective stress results (MPa) predicted from the FEM modeling of orthogonal turning of LCFCS using $u = 150$ m/min (cutting data is given in Table 3.1)
### Workpiece

<table>
<thead>
<tr>
<th>Material</th>
<th>0.2% C steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical composition</td>
<td>0.2% C, 0.15% Si, 0.015% S, 0.72% Mn, 0.015% Al</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>equation C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat S (J/kg °C)</td>
<td>Equation (3.11) in Table 3.2</td>
</tr>
</tbody>
</table>

### Cutting Conditions

<table>
<thead>
<tr>
<th></th>
<th>Cutting speed, u (m/min)</th>
<th>Tool rake angle, α°</th>
<th>Uncut chip thickness, t₁ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation 1</td>
<td>200</td>
<td>3.5</td>
<td>0.125</td>
</tr>
<tr>
<td>Simulation 2</td>
<td>400</td>
<td>3.5</td>
<td>0.125</td>
</tr>
<tr>
<td>Simulation 3</td>
<td>100</td>
<td>3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 4</td>
<td>200</td>
<td>3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 5</td>
<td>400</td>
<td>3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 6</td>
<td>200</td>
<td>-3.5</td>
<td>0.125</td>
</tr>
<tr>
<td>Simulation 7</td>
<td>400</td>
<td>-3.5</td>
<td>0.125</td>
</tr>
<tr>
<td>Simulation 8</td>
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<td>-3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 9</td>
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<td>-3.5</td>
<td>0.25</td>
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<tr>
<td>Simulation 10</td>
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<td>-3.5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 11</td>
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</tr>
<tr>
<td>Simulation 12</td>
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<td>0.125</td>
</tr>
<tr>
<td>Simulation 13</td>
<td>400</td>
<td>5</td>
<td>0.125</td>
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<tr>
<td>Simulation 14</td>
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<td>-5</td>
<td>0.125</td>
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<tr>
<td>Simulation 15</td>
<td>200</td>
<td>5</td>
<td>0.25</td>
</tr>
<tr>
<td>Simulation 16</td>
<td>400</td>
<td>-5</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**Friction**: Zorev’s model of Figure 2.6 (Length of sticking region = two times the uncut chip thickness, Coulomb friction coefficient of the sliding region = 0.3)

### Tool Geometry

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Clearance angle (γ)</td>
<td></td>
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<tr>
<td>Hone radius (rₜ)</td>
<td>0.02 mm</td>
</tr>
</tbody>
</table>

Table 3.10: Data for FEM modeling of orthogonal turning of 0.2% C steel [Oxley; 1989][Mathew; 1989]
Figure 3.39: Applying Zorev’s friction model to FEM modeling by DEFORM

sticking friction region length $l_p$ is taken as two times the uncut chip thickness $t_1$ and the Coulomb friction coefficient of the sliding region is taken as 0.3.

Similar results of temperature and effective stress to those of LCFCS were obtained as shown in Figures 3.40 and 3.41 for the conditions of simulation 11 described in Table 3.10. The temperature and effective stress results of the 16 simulations, listed in Table 3.10, are summarized in the bar charts of Figures 3.42 through 3.44 where the predictions of FEM (DEFORM) and OXCUT are compared.

As shown in Figure 3.42, the average temperature in the primary zone, predicted by FEM (DEFORM) is in good agreement with that predicted by OXCUT. On the other hand, the average temperature on the tool-chip interface (secondary deformation zone), predicted by OXCUT, is about 30% larger than that predicted by FEM for the majority of the simulations. It was proven in Figures 3.1 and 3.5 that the temperature at the tool-chip interface, predicted by OXCUT, is in reasonable agreement with the measured results published by Childs et al.
Figure 3.40: Temperature results (°C) predicted from the FEM modeling of orthogonal turning of 0.2% C steel for Simulation 11 in Table 3.10
Figure 3.41: Effective stress results (MPa) predicted from the FEM modeling of orthogonal turning of 0.2% C steel for Simulation 11 in Table 3.10
Figure 3.42: Summary of the temperature results predicted by FEM and OXCUT using the cutting data of Table 3.10.
Figure 3.43: Summary of the effective stress results predicted by FEM and OXCUT using the cutting data of Table 3.10.
Figure 3.44: Summary of the temperature and effective stress results on the flank face predicted by FEM and OXCUT using the cutting data of Table 3.10.
[Childs; 1997] and Maekawa et al. [Maekawa; 1989], respectively. Therefore, there is a high probability that FEM tend to underestimate the value of the temperature at the tool-chip interface for the conditions used in this study.

Figure 3.43 shows that the effective stress in the shear zone (primary deformation zone) and at the tool-chip interface (secondary deformation zone), predicted by OXCUT, is in reasonable agreement with that predicted by FEM for the majority of the cutting conditions used in this study. It can be observed in Figure 3.43 that the agreement in the primary deformation zone is better than that in the secondary deformation zone.

Figure 3.44 compares the temperature and effective stress: (1) obtained directly on the flank face from the FEM (DEFORM) contour plots, (2) calculated by OXCUT as the average of the primary and secondary deformation zone values, and (3) calculated as the average of the primary and secondary deformation zone values obtained from FEM (DEFORM). As shown in Figure 3.44, the white and the black bars are in reasonable agreement with each other, for the majority of the cutting conditions used in the present study. Thus, the temperature and effective stress on the flank face can be approximated as the average of the values in the primary and secondary deformation zone. The gray bars in the effective stress chart are in reasonable agreement with the black bars, for the majority of the cutting conditions, which implies that the effective stress on the flank face, predicted by OXCUT, is in reasonable agreement with that predicted by FEM. On the other hand, the temperature on the flank face predicted by OXCUT (gray bars) is always higher than that predicted by FEM because the temperature at the tool-chip interface, predicted by OXCUT, is higher than that predicted by FEM as was explained in Figure 3.42.

The purpose of this FEM study is to check whether the temperature and effective stress on the flank side can be approximated as the average of the
values in the shear zone and on the rake face not to check the validity of the OXCUT predictions which was validated in Figures 3.1 and 3.5. However, if we assume that the FEM temperature results of the present work are more accurate than the OXCUT results, the OXCUT temperature results on the flank face are still about 30% higher than the FEM results. Thus, the predictions, made by OXCUT, should be on the safe side and should not be very much exaggerated because the temperature predicted by OXCUT is only about 30% higher than that predicted by FEM.

3.10 Summary

In this chapter the results of the computer code OXCUT, developed in the present work based on Oxley's approach, are compared to the published experimental results in orthogonal turning, orthogonal slot milling, 3-D turning, 3-D face milling, ball end milling and drilling. It has been found that the temperature at the tool-chip interface in orthogonal turning, predicted by OXCUT, is in good agreement with the measured results published in the literature [Maekawa; 1989][Childs; 1997]. Moreover, the orthogonal turning forces in the cutting and feed directions, predicted by OXCUT, have been found to be in reasonable agreement with the published measured results [Maekawa; 1989][Childs; 1997]. The agreement between the predicted and measured cutting force results has been found to be better than the agreement between forces in the feed direction, especially, at higher cutting velocities. This is attributed to the cutting tool edge wear at higher cutting velocities which has more influence on forces in the feed direction than in the cutting direction.

The orthogonal slot milling forces in the global X and Y directions, vs tool rotation angle, predicted by OXCUT, are in reasonable agreement with the measured results [Özel; 1998][Sterr; 1998]. The same kind of agreement has
been found between the predicted and measured 3-D turning forces, 3-D ball end milling forces and 3-D face milling forces in the Y and Z directions [Arsecularante; 1995] [Özel; 1998]. The predicted 3-D face milling forces in the X direction, on the other hand, have not been found to be in good agreement with the measured results. This might be attributed to inaccuracies in the flow stress data, input to OXCUT, due to assuming that the flow stress data of P20 mold steel (30 HRC) is the same as that of low carbon steel of the same carbon content. This assumption was found to have less influence on the accuracy of the predicted force results in orthogonal slot milling of P20 mold steel (30 HRC).

In the present study, it is observed in the contour lines of temperature and effective stress distributions in the workpiece, predicted by FEM (DEFORM), that the contour lines in the primary and secondary deformation zones meet close to the flank face. Thus, it was concluded that the temperature and effective stress on the flank face may be approximated as the average values in the primary (shear) and secondary (rake face) deformation zones. Using that conclusion, reasonable agreement has been found between the temperature on the flank face of the twist drill lip, predicted by OXCUT as the average of the values in the primary and secondary deformation zones, and that measured experimentally [DeVries; 1968].

Because OXCUT can use flow stress data to predict cutting forces with reasonable accuracy, there is a high chance that the opposite is true. In other words, if OXCUT is given the orthogonal cutting and feed forces for different cutting conditions, it should be able to use them to predict flow stress data of the workpiece material. This is the subject of the next chapter.
CHAPTER 4

DETERMINATION OF FLOW STRESS DATA

4.1 Methodology to Use OXCUT and 2-D Orthogonal Slot Milling Experiments to Obtain Flow Stress Data for Machining Conditions

As seen in equation 3.13, the uncut chip thickness \( t_1 \) varies with tool rotation angle in the 2-D orthogonal slot milling operation shown in Figure 3.9. Thus, in one cycle of tool rotation, there is a large number of cutting conditions corresponding to the variation of the uncut chip thickness with the tool rotation angle. This is illustrated in Figures 4.1 and 4.2, where the average strain rate and temperature, calculated by OXCUT, in both of the primary and secondary deformation zones vary with tool rotation angle.

The methodology, used in the present study, starts by conducting orthogonal slotting experiments using the set up shown in Figure 3.9. During those experiments, the variation of cutting forces in the global X and Y directions with tool rotation angle is measured. The measured cutting force data vs tool rotation angle are, then, input to OXCUT which tunes the parameters of a flow stress equation until the predicted cutting forces match the experimental results. When this matching occurs, the parameters of the flow stress equation are the output. The predicted flow stress equation will, then, be verified by using it to predict cutting forces for other cutting conditions and compare the results with the
Figure 4.1: Variation of strain rate in the primary and secondary deformation zones, calculated by OXCUT with tool rotation angle
Figure 4.2: Variation of temperature in the primary and secondary deformation zones, calculated by OXCUT with tool rotation angle.
experimental data. If the flow stress equation can predict cutting forces with reasonable accuracy for those cutting conditions, then the flow stress equation should be correct and can be considered as a representation of the flow stress behavior of the material under cutting conditions.

In applying the method, described above, several experimental conditions were satisfied, as follows:

1. A sharp insert was selected to conduct the experiments because OXCUIT is based on the assumption of a sharp tool. In reality, there is nothing called a perfectly sharp tool because any tool should have a very small hone radius. The hone radius of the sharp tool, used to conduct the experiments for this work, is less than 0.02 mm [Kennametal; 1998]. The error due to this small hone radius should be insignificant. For each experiment, a new part of the cutting edge was used to assure a reasonable sharpness of the cutting edge.

2. The cutting velocity was selected high enough to avoid the formation of a built-up edge which is not taken into account in Oxley’s theory and may affect the accuracy of the results.

3. The plane strain assumption was satisfied by making the width of cut which is the plate thickness (W), shown in Figure 3.9, more than 10 times the maximum uncut chip thickness in the cycle which is the feed in mm/tooth. The workpiece plate thickness (W), used in the present work, is 3 mm.

4. To avoid high fluctuations in the measured cutting forces due to chatter and vibrations that may affect the accuracy of the measured forces, only one side of the insert, shown in Figure 3.9, is used to perform the cutting and the other side of the insert was ground.
5. To minimize experimental scatter of results, the measured forces vs tool rotation angle, used to obtain the flow stress data, are the average of 20 milling cycles. The maximum and minimum forces vs tool rotation angle for those 20 milling cycles are also recorded to make sure that variation in the results due to chatter and vibration, discussed under condition 4, is minimum.

6. The shapes of the chips were examined under the microscope to make sure that the conditions, used to obtain the flow stress data, are based on continuous chip formation as assumed by OXCUT.

7. OXCUT considers the normal stress and the frictional shear stress at the tool-chip interface to be constant. The frictional stress is assumed to be constant and equal to the shear flow stress of the material at the tool-chip interface. As discussed in Chapter 3, this assumption is close to reality.

4.2 Experimental Set-Up

In the present work, 2-D orthogonal slot milling experiments, described schematically by the setup of Figure 3.9, were conducted on a Makino A55 Delta high-speed 4-axis machining center located at the Net Shape Manufacturing Laboratory of the ERC/NSM. This machine tool is equipped with a horizontal 14,000 rpm-18.5 KW spindle.

The geometry of the insert, used in the present work, is illustrated schematically in Figure 3.9. The characteristics of the various inserts of the same geometry, used in the experiments of the present work, are summarized in Table 4.1. The rake surface of some of the uncoated inserts was ground to increase the surface roughness from $Ra = 0.06 \mu m$ to $Ra = 0.5 - 0.7 \mu m$. This is
to verify friction conditions at the tool-workpiece interface in cutting as will be
discussed later.

The cutting forces during the milling process were measured with a three-
component piezoelectric force dynamometer (Kistler type 9257). The charge
amplifier (Kistler type 5001) of the force platform was connected to a PC-Pentium
data acquisition system (National Instruments, Labview V 3.1.1) [Kistler; 1999].
The sampling rate was set to 180 samples per tool revolution. The attenuation of
the amplifying signal was 180 kHz.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool Type</td>
<td></td>
<td>Inserted flat bottom mill</td>
</tr>
<tr>
<td>Tool Diameter</td>
<td>D</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>No. of Cutting Edges</td>
<td>(N_e)</td>
<td>1 (the other cutting edge was ground)</td>
</tr>
<tr>
<td>Insert Material</td>
<td></td>
<td>tungsten carbide (WC), ANSI C2-C4 (ISO M20-M35)</td>
</tr>
<tr>
<td>Insert Type and Coating</td>
<td></td>
<td>no coating: FBR 1000N-R1/32-F</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TiN-coated: FBR 1000N-R1/32-FPT</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TiAlO(_3)N-coated: FBR 1000N-R1/32-FPI</td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
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</tr>
<tr>
<td>Rake Angle</td>
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<td>Clearance Angle</td>
<td>(\gamma)</td>
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<tr>
<td>Nose Radius</td>
<td>(r_n)</td>
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</tr>
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<td>Hone Radius</td>
<td>(r_h)</td>
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</tr>
<tr>
<td>Manufacturer</td>
<td></td>
<td>DAPRA Corp., Bloomfield CT, USA</td>
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</tbody>
</table>

Table 4.1: Description of the inserts used in the 2-D orthogonal slot milling
experiments (Figure 3.9)
For determining the surface properties of the inserts and measuring the roughness of the cutting edges after cutting, the Federal Surface Analyzer System 4000 was used. The surface analyzer provides total profile as well as roughness and multi-parameter measurement capability. Microscopic photographs of the chips and cutting edges of the inserts were taken using a Nikon A4-B microscope, equipped with an XY table. More details about the experimental set-up, used in the present work, can be found elsewhere [Kerk; 1999].

4.3 Experimental Results and Evaluation

To check the validity of the condition 7, given above in section 4.1, inserts with different types of coating and surface roughness were tried. As shown in Figure 4.3, varying the insert type or roughness has an insignificant influence on the measured forces in slot milling of H13 tool steel hardened to 46 HRC using a cutting velocity of 200 m/min and a feed rate of 0.1 mm/tooth. The same results were found for three other cutting conditions which are 200 m/min and 0.2 mm/tooth, 300 m/min and 0.1 mm/tooth, and 300 m/min and 0.2 mm/tooth. The same experiments were conducted using AISI P20 mold steel hardened to 30 HRC and the same observations were made. A summary of all those results is shown in Figures 4.4 through 4.13. In these Figures, the forces \( F_{C-90} \) and \( F_{T-90} \), described in Figure 3.11, are used for comparison and to show all the conditions in one chart.

As shown in Figure 4.4, during slot milling of AISI P20 mold steel hardened to 30 HRC, the maximum measured cutting forces, at tool rotation angle 90° (\( F_{C-90} \)), change insignificantly with varying the type of coating or the rake face surface roughness for all the cutting conditions displayed in the Figure.
Figure 4.3: Effect of the type of coating and surface roughness of the insert on the measured forces in orthogonal slot milling of AISI H13 tool steel (46 HRC)
Figure 4.4: Maximum cutting and feed forces in a milling cycle for different types of coating and rake surface roughness during orthogonal slot milling of AISI P20 mold steel (30 HRC)
Figure 4.5: Shape of the cutting edge of the uncoated inserts after slot milling of AISI P20 mold steel (30 HRC)
Figure 4.6: Shape of the cutting edge of the uncoated inserts with roughed rake face after slot milling of AISI P20 mold steel (30 HRC)
Figure 4.7: Shape of the cutting edge of the TiN-coated inserts after orthogonal slot milling of AISI P20 mold steel (30 HRC)
Figure 4.8: Shape of the cutting edge of the TiAlO$_3$N-coated inserts after orthogonal slot milling of AlSi P20 mold steel (30 HRC)
Figure 4.9: Maximum cutting and feed forces in a milling cycle for different types of coating and rake surface roughness during orthogonal slot milling of AISI H13 tool steel (46 HRC)
Figure 4.10: Shape of the cutting edge of the uncoated inserts after orthogonal slot milling of AISI H13 tool steel (46 HRC)
Figure 4.11: Shape of the cutting edge of the uncoated inserts with roughed rake face after orthogonal slot milling of AISI H13 tool steel (46 HRC)
Figure 4.12: Shape of the cutting edge of the TiN-coated inserts after orthogonal slot milling of AISI H13 tool steel (46 HRC)
Figure 4.13: Shape of the cutting edge of the TiAlO$_3$N-coated inserts after orthogonal slot milling of AISI H13 tool steel (46 HRC)
The maximum measured feed forces, at tool rotation angle 90° (F_T-90), on the other hand, are higher for the inserts with roughed rake face than the other three insert types. This is because roughing the rake face was done manually on the grinding machine which caused the cutting edge to be more round and more dull (higher hone radius) than the cutting edges of the new inserts as shown in Figures 4.5 through 4.8. The higher hone radius of the insert resulted in the higher feed forces.

Moreover, Figure 4.4 shows that the F_T-90 forces of the uncoated inserts are usually higher than those of the coated inserts, especially, at the feed rate of 0.2 mm/tooth. This is because the uncoated inserts have less resistance to wear and micro chipping than the coated inserts. That wear and micro chipping caused the cutting edge of the uncoated inserts to be more round and more dull (higher hone radius) than the cutting edges of the coated inserts. Therefore, F_T-90 forces are observed to be higher for the uncoated inserts than the coated inserts and this is more clear at the feed rate of 0.2 mm/tooth, where the wear and micro chipping are larger than at the feed rate of 0.1 mm/tooth.

The same discussion applies to Figures 4.9 through 4.13 which display results similar to those of Figures 4.4 through 4.8, for AISI H13 tool steel hardened at 46 HRC. As expected, wear and micro chipping of the cutting edge, during cutting of AISI H13 (46 HRC), are higher than those observed during cutting of AISI P20 mold steel (30 HRC). The insert that is found to be most resistant to wear and micro chipping during cutting of AISI H13 (46 HRC) is the TiAlO_3N-coated insert as reflected in the results of Figures 4.9 through 4.13. This is very clear at the most severe cutting conditions (u = 300 m/min and f = 0.2 mm/tooth). In this case wear and micro chipping, resulting in more dulling of the cutting edge (higher hone radius), are high for the uncoated and TiN-coated inserts causing the F_C-90 and F_T-90 to be higher for those inserts than for the TiAlO_3N-coated insert.
Thus, from the experimental results given in Figures 4.3 through 4.13, one may conclude that the measured forces during cutting, using inserts with sharp cutting edges, do not change significantly by varying the rake face surface texture or by varying the type of coating or surface roughness. This supports the assumption that the frictional stress at the major portion of the tool-chip interface length is equal to the shear flow stress of the material at the interface. If this assumption is incorrect (i.e. the friction at the major portion of the interface is governed by sliding conditions that obey Coulomb’s friction model for example), then varying the rake face texture, by varying the type of coating or surface roughness, should have an influence on the Coulomb friction coefficient at the interface and, consequently, on the measured forces. Thus, the assumption that the friction at the tool-chip interface is nearly constant and equal to the shear flow stress of the workpiece material (made by Oxley when he proposed his predictive machining theory) is not very far from reality. Consequently, the use of OXCUT together with the experimental data is justified in determining flow stress data in cutting.

4.4 Implementation of the Methodology for Flow Stress Determination

In the present work, the flow stress equation which will be tuned by OXCUT is:

\[ \sigma^* = (A + B e^\gamma)(1 + C \ln \varepsilon)(D - E T^* m) \]  
(4.1.a)

where, \[ T^* = \frac{T - T_{room}}{T_{melting} - T_{room}} \]  
(4.1.b)

This equation is similar to the Johnson-Cook’s model of flow stress [Lee; 1998] with some modifications to make it more flexible. \( T^* \) is the homologous temperature as defined by equation 4.1.b, \( T \) is the workpiece temperature. A, B,
n, C, D, E and m are material constants that need to be determined by the tuning process as illustrated in the Flow Chart in Figure 4.14.

The matching process between the calculated and measured forces is based on least mean square fit. As long as the sum of deviations between the measured and calculated forces, at each tool rotation angle, does not reach a selected minimum value, the “fine tuning” of the flow stress equation 4.1 is continued by varying the parameters A, B, C, D, E, n and m. This iteration procedure can be considered a task of minimization of a multi dimensional function as described by equation 4.2.

\[
\text{MIN} \left[ \sqrt{\sum_{\theta_r} \left( \left( F_{c,\text{measured}} (\theta_r) - F_{c,\text{OXCUT}} (\theta_r) \right)^2 \right)} \right] = \text{function (A, B, C, D, E, n, m)}
\]

(4.2)

The Downhill Simplex Algorithm [Press; 1992], described in Appendix F, provides an easy and effective way to find the minimum of multi dimensional functions. The Downhill Simplex Algorithm and OXCUT are combined in one computer program. Because one calculation with OXCUT takes about thirty seconds, many iteration steps are possible in a short time. Figures 4.15 through 4.18 illustrate the iteration of the parameters A, B, C, D, E, m and n, to obtain flow stress data for AISI P20 mold steel (30 HRC), until the deviation between the measured and calculated forces reaches the required minimum value. It can be observed, in Figures 4.15 through 4.18, that the variation of the parameters A, B, C, D, E, n and m is large initially and reaches steady state after about 2 hours of computer calculation time.

Not every combination of the parameters A, B, C, D, E, n and m describes the flow stress of a material with metallic characteristics. Unrealistic combinations
Figure 4.14: Flow chart of flow stress determination
Figure 4.15: Change of parameters A and B during flow stress determination

Figure 4.16: Change of parameters C and n during flow stress determination
Figure 4.17: Change of parameters D, E and m during flow stress determination

Figure 4.18: Deviation of forces during flow stress determination
of these parameters result in calculation errors in OXCUT which are output as high peaks as shown in Figure 4.18.

4.5 Determination of Flow Stress Data

The density and thermal properties of the workpiece materials, used in the present work, are shown in Table 4.2. These properties are required for the calculations by OXCUT.

The methodology, developed in the present study, is used to determine the flow stress data for AISI P20 mold steel (30 HRC), AISI H13 tool steel (46 HRC) and Al2007 (100 HB). The results of the determination of the flow stress data for AISI P20 mold steel (30 HRC) are shown in Figures 4.19 and 4.20 using the cutting conditions \( u = 200 \text{ m/min} \) and \( f = 0.1 \text{ mm/tooth} \). The Downhill method and OXCUT iterated the parameters of the flow stress equation 4.1 until the agreement between the measured and calculated forces is as shown in Figure 4.19. As shown in Figure 4.20, the chip is nearly continuous at the cutting conditions \( u = 200 \text{ m/min} \) and \( f = 0.1 \text{ mm/tooth} \), used to determine the flow stress data for AISI P20 mold steel.

To validate the obtained flow stress equation of AISI P20 mold steel (30 HRC), the equation is used in OXCUT to calculate the cutting forces vs tool rotation angle for three other cutting conditions. The calculated force results are compared to the measured results for the cutting conditions \( u = 200 \text{ m/min} \) and \( f = 0.2 \text{ mm/tooth} \), \( u = 300 \text{ m/min} \) and \( f = 0.1 \text{ mm/tooth} \) and \( u = 300 \text{ m/min} \) and \( f = 0.2 \text{ mm/tooth} \) as shown in Figures 4.21, 4.23 and 4.25, respectively. Pictures of the chips and the tool cutting edges are shown in Figures 4.22, 4.24 and 4.26 for the cutting conditions \( u = 200 \text{ m/min} \) and \( f = 0.2 \text{ mm/tooth} \), \( u = 300 \text{ m/min} \) and \( f = 0.1 \text{ mm/tooth} \) and \( u = 300 \text{ m/min} \) and \( f = 0.2 \text{ mm/tooth} \), respectively.
<table>
<thead>
<tr>
<th></th>
<th>AISI P20 (30 HRC)</th>
<th>AISI H13 (46 HRC)</th>
<th>Al2007 (100 HB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DIN</td>
<td>1.2330, 35CrMo4</td>
<td>1.2344, X40CrMoV51</td>
<td>1725 T1: AlCuMgPb, 3.1645</td>
</tr>
<tr>
<td>Composition</td>
<td>0.3% C, 1.1% Cr, 0.3% Si, 0.55% Mo, 0.85% Mn, 0.08% V, and 0.015% P</td>
<td>0.32-0.45% C, 5.32% Cr, 1.25% Mo and 0.92% V</td>
<td>0.8% Si, 0.8% Fe, 3.3-4.6% Cu, 0.5-1% Mn, 0.4-1.8% Mg, 0.1% Cr, 0.8% Zn, 0.2% Ti, 0.2% Ni, 0.2% Bi, 0.8-1.5% Pb and 0.2% Sn</td>
</tr>
<tr>
<td>( T_{\text{melting}}, \degree K )</td>
<td>1753</td>
<td>1760</td>
<td>780</td>
</tr>
<tr>
<td>Density ( \frac{\text{kg}}{\text{m}^3} )</td>
<td>7850</td>
<td>7760</td>
<td>2850</td>
</tr>
<tr>
<td>Specific Heat ( \frac{\text{J}}{\text{Kg \degree K}} )</td>
<td>( 420 + 0.504 T, ) ( T ) in \degree C</td>
<td>( 420 + 0.504 T, ) ( T ) in \degree C</td>
<td>879</td>
</tr>
<tr>
<td>Thermal Conductivity ( \frac{\text{W}}{\text{m} \degree K} )</td>
<td>( 418.68 \left( 0.065 + \right) \left( 0.10241-0.065 \right) \left( 1.0033-11.095E-4 \right) \left( T-273 \right), T ) in \degree C</td>
<td>28.4 ( \left( 350 \degree C \right) ), 28.4 ( \left( 475 \degree C \right) ), 28.7 ( \left( 605 \degree C \right) )</td>
<td>130-160 [Kaiser; 1999], 204 [Ziebeil; 1995]</td>
</tr>
<tr>
<td>Hardness</td>
<td>30 (\pm 2.5) HRC</td>
<td>46 (\pm 2) HRC</td>
<td>Cold hardened to 100 HB</td>
</tr>
</tbody>
</table>

Table 4.2: Properties of the workpiece materials used in the present study [ASM; 1980] [ASM; 1997][Oxley, 1989][Ziebeil; 1995][Kaiser; 1999].
Figure 4.19: Comparison between calculated and measured forces at the cutting conditions ($u = 200$ m/min and $f = 0.1$ mm/tooth) used for flow stress determination of AISI P20 mold steel (30 HRC)
Figure 4.20: Shape of the chip and tool cutting edge at the cutting conditions 
\((u = 200 \text{ m/min}, f = 0.1 \text{ mm/tooth})\) used for flow stress 
determination of AISI P20 mold steel (30 HRC)
Figure 4.21: Comparison between calculated and measured forces at the cutting conditions \( (u = 200 \text{ m/min} \text{ and } f = 0.2 \text{ mm/tooth}) \) used for flow stress validation of AISI P20 mold steel (30 HRC)
Figure 4.22: Shape of the chip and tool cutting edge at the cutting conditions (\( u = 200 \text{ m/min} \), \( f = 0.2 \text{ mm/tooth} \)) used for flow stress validation of AISI P20 mold steel (30 HRC)
Figure 4.23: Comparison between calculated and measured forces at the cutting conditions \(u = 300\ \text{m/min}\) and \(f = 0.1\ \text{mm/tooth}\) used for flow stress validation of AISI P20 mold steel (30 HRC)
Figure 4.24: Shape of the chip and tool cutting edge at the cutting conditions 
(u = 300 m/min, f = 0.1 mm/tooth) used for flow stress validation of 
AISI P20 mold steel (30 HRC)
Figure 4.25: Comparison between calculated and measured forces at the cutting conditions \((u = 300 \text{ m/min} \text{ and } f = 0.2 \text{ mm/tooth})\) used for flow stress validation of AISI P20 mold steel (30 HRC)
Figure 4.26: Shape of the chip and tool cutting edge at the cutting conditions 
(u = 300 m/min, f = 0.2 mm/tooth) used for flow stress validation of 
AISi P20 mold steel (30 HRC)
As shown in Figures 4.22, 4.24 and 4.26, although the chip is serrated for the three cutting conditions used to validate the obtained flow stress equation, the agreement between the predicted and measured forces is still good. The pictures of the tool cutting edge and its profile, measured by the surface analyzer, for the cutting conditions used in the flow stress determination and validation of AISI P20 mold steel (30 HRC), shown in Figures 4.20, 4.22, 4.24 and 4.26, suggest that there is no major damage or wear of the cutting edge that might have affected the validity of the results.

The cutting conditions, used for flow stress determination of AISI H13 tool steel (46 HRC) are \( u = 50 \, \text{m/min} \) and \( f = 0.1 \, \text{mm/tooth} \). Although, the cutting velocity is low and might cause some built-up edge formation, it had to be used in the flow stress determination to avoid serrated chip formation that is observed to occur at relatively low cutting velocities for AISI H13 (46 HRC) compared to AISI P20 (30 HRC). For this cutting velocity (50 m/min), chatter and vibration were found to be high during cutting AISI H13 (46 HRC). The effect of chatter and vibration is reflected on the charts in Figure 4.27 as a larger difference between the maximum and minimum cutting forces in the X and Y directions of 20 milling cycles as compared to the other results. As shown in Figure 4.28, the chip is continuous and there is no wear or damage of the tool cutting edge for the cutting conditions \( (u = 50 \, \text{m/min} \) and \( f = 0.1 \, \text{mm/tooth} \) used for the flow stress determination of AISI H13 tool steel (46 HRC).

Similar to what was done for AISI P20 mold steel (30 HRC), to validate the obtained flow stress equation of AISI H13 tool steel (46 HRC), the equation is used in OXCUT to calculate the cutting forces vs tool rotation angle for three other cutting conditions as shown in Figures 4.29 through 4.36. The calculated force results are compared to the measured results for the cutting conditions \( (u = 200 \, \text{m/min} \) and \( f = 0.1 \, \text{mm/tooth} \), \( (u = 200 \, \text{m/min} \) and \( f = 0.2 \, \text{mm/tooth} \)
Figure 4.27: Comparison between calculated and measured forces at the cutting conditions (u = 50 m/min and f = 0.1 mm/tooth) used for flow stress determination of AISI H13 tool steel (46 HRC)
Figure 4.28: Shape of the chip and tool cutting edge at the cutting conditions (u = 50 m/min, f = 0.1 mm/tooth) used for flow stress determination of AISI H13 tool steel (46 HRC)
Figure 4.29: Comparison between calculated and measured forces at the cutting conditions ($u = 200$ m/min and $f = 0.1$ mm/tooth) used for flow stress validation of AISI H13 tool steel (46 HRC)
Figure 4.30: Shape of the chip and tool cutting edge at the cutting conditions (u = 200 m/min, f = 0.1 mm/tooth) used for flow stress validation of AISI H13 tool steel (46 HRC)
Figure 4.31: Comparison between calculated and measured forces at the cutting conditions \((u = 200 \text{ m/min} \text{ and } f = 0.2 \text{ mm/ tooth})\) used for flow stress validation of AlSi H13 tool steel (46 HRC)
Figure 4.32: Shape of the chip and tool cutting edge at the cutting conditions ($u = 200$ m/min, $f = 0.2$ mm/tooth) used for flow stress validation of AISI H13 tool steel (46 HRC)
Figure 4.33: Comparison between calculated and measured forces at the cutting conditions ($u = 300$ m/min and $f = 0.1$ mm/tooth) used for flow stress validation of AISI H13 tool steel (46 HRC)
Figure 4.34: Shape of the chip and tool cutting edge at the cutting conditions (u = 300 m/min, f = 0.1 mm/tooth) used for flow stress validation of AISI H13 tool steel (46 HRC)
and \((u = 300 \text{ m/min and } f = 0.1 \text{ mm/tooth})\) as shown in Figures 4.29, 4.31 and 4.33, respectively. Pictures of the chips and the tool cutting edges are shown in Figures 4.30, 4.32 and 4.34 for the cutting conditions \((u = 200 \text{ m/min and } f = 0.1 \text{ mm/tooth})\), \((u = 200 \text{ m/min and } f = 0.2 \text{ mm/tooth})\) and \((u = 300 \text{ m/min and } f = 0.1 \text{ mm/tooth})\), respectively.

Similarly, the flow stress data for Al2007 (100 HB) was obtained using the cutting conditions \((u = 200 \text{ m/min and } f = 0.2 \text{ mm/tooth})\). No microscopic pictures of the chips were taken because Al2007 (100 HB) is a relatively soft material and the chip is expected to be continuous. The microscopic pictures of the cutting edge showed that the edge damage and wear due to cutting were minor. Figure 4.35 illustrates a comparison between the cutting forces calculated by OXCUT and measured experimentally for the conditions used to determine the flow stress data. Three other cutting conditions, namely \((u = 100 \text{ m/min and } f = 0.2 \text{ mm/tooth})\), \((u = 100 \text{ m/min and } f = 0.4 \text{ mm/tooth})\) and \((u = 200 \text{ m/min and } f = 0.4 \text{ mm/tooth})\), were used to validate the obtained flow stress equation as shown in Figures 4.36, 4.37 and 4.38, respectively.

The shapes of the flow stress curves for AISI P20 mold steel (30 HRC) and AISI H13 tool steel (46 HRC) are shown in Figure 4.39. The parameters of the flow stress equation 4.1 for AISI P20 mold steel (30 HRC), AISI H13 tool steel (46 HRC) and Al2007 (100 HB) are shown in Table 4.3. In the same table, the ranges of strains, strain rates and temperatures, used to determine the flow stress data based on the selected cutting conditions, for the three materials are also indicated.
Figure 4.35: Comparison between calculated and measured forces at the cutting conditions (u = 200 m/min and f = 0.2 mm/tooth) used for flow stress determination of aluminum Al2007 (100 HB)
Figure 4.36: Comparison between calculated and measured forces at the cutting conditions ($u = 100$ m/min and $f = 0.2$ mm/tooth) used for flow stress validation of aluminum Al2007 (100 HB)
Figure 4.37: Comparison between calculated and measured forces at the cutting conditions \((u = 200 \text{ m/min} \text{ and } f = 0.4 \text{ mm/tooth})\) used for flow stress validation of aluminum Al2007 (100 HB)
Figure 4.38: Comparison between calculated and measured forces at the cutting conditions (\( u = 400 \text{ m/min} \) and \( f = 0.2 \text{ mm/tooth} \)) used for flow stress validation of aluminum Al2007 (100 HB)
Figure 4.39: Determined flow stress curves for AISI P20 mold steel (30 HRC) and AISI H13 tool steel (46 HRC)
<table>
<thead>
<tr>
<th></th>
<th>AISI P20 (30 HRC)</th>
<th>AISI H13 (46 HRC)</th>
<th>Al2007 (100 HB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>145 MPa</td>
<td>674.8 MPa</td>
<td>320.9 MPa</td>
</tr>
<tr>
<td>B</td>
<td>565.6 MPa</td>
<td>239.2 MPa</td>
<td>316.0 MPa</td>
</tr>
<tr>
<td>C</td>
<td>0.03</td>
<td>0.027</td>
<td>0.0228</td>
</tr>
<tr>
<td>D</td>
<td>1.26</td>
<td>1.16</td>
<td>1.05</td>
</tr>
<tr>
<td>E</td>
<td>1.07</td>
<td>0.88</td>
<td>1.18</td>
</tr>
<tr>
<td>n</td>
<td>0.154</td>
<td>0.28</td>
<td>0.209</td>
</tr>
<tr>
<td>m</td>
<td>1.8</td>
<td>1.3</td>
<td>0.9</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>$0.9 \leq \varepsilon \leq 1.5$</td>
<td>$0.9 \leq \varepsilon \leq 1.7$</td>
<td>$0.75 \leq \varepsilon \leq 1.1$</td>
</tr>
<tr>
<td>$\dot{\varepsilon}$ (s$^{-1}$)</td>
<td>$2E4 \leq \dot{\varepsilon} \leq 8E5$</td>
<td>$6E3 \leq \dot{\varepsilon} \leq 9E5$</td>
<td>$14E3 \leq \dot{\varepsilon} \leq 325E3$</td>
</tr>
<tr>
<td>T (°C)</td>
<td>$600 \leq T \leq 1200$</td>
<td>$300 \leq T \leq 1200$</td>
<td>$100 \leq T \leq 400$</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters of the flow stress equation 4.1 for the three materials used in the present study

4.6 Summary

An automated technique is developed to determine the flow stress data at the high strains, strain rates and temperatures, encountered in metal cutting. This technique uses 2-D orthogonal slot milling experiments in conjunction with a computer code (OXCUT) that is based on Oxley’s predictive machining theory. The parameters of a selected flow stress equation are iterated and tuned using the Downhill Simplex Algorithm, added to OXCUT, until the calculated force results vs tool rotation angle match the experimental results measured during 2-D orthogonal slot milling experiments. In the present work, flow stress data were determined for 3 materials: AISI P20 mold steel (30 HRC), AISI H13 tool steel (46 HRC) and Al2007 (100 HB). These data were verified by (a) conducting more 2-D orthogonal slot milling experiments and (b) comparing the measured force results vs tool rotation angle with predictions made by using the determined flow stress data.
CHAPTER 5

VALIDATION AND APPLICATION OF THE DETERMINED FLOW STRESS DATA

In this chapter, the flow stress data determined in the previous chapter (Chapter 4) is validated by using it as input to OXCUT and DEFORM to predict process variables such as forces, cut chip thickness and tool chip contact length. The predicted results are compared to published experimental results. Moreover, the flow stress data of H13 tool steel (46 HRC) is used, in this chapter, to study the effect of the tool edge preparation on tool stresses and cutting forces and temperatures.

5.1 3-D Face Milling of P20 mold steel (30 HRC) and H13 tool steel (46 HRC)

The 3-D face milling force results, predicted by OXCUT vs tool rotation angle, are compared to the published measured results during 3-D face milling of P20 mold steel (30 HRC) [Özel; 1998] and to the results of 3-D face milling experiments of H13 tool steel (46 HRC) conducted in the present work. The 3-D face milling analysis, conducted in this chapter, is similar to that of section 3.6 in Chapter 3. In section 3.6, it was found that the forces vs tool rotation angle, predicted by OXCUT using the data in Table 3.6, were not in good agreement with those measured experimentally during 3-D face milling of P20 mold steel (30
HRC) [Özel; 1998] especially in the X direction as shown in Figure 3.23. This was attributed to inaccuracies in the flow stress data due to the assumption that the flow stress data of P20 mold steel (30 HRC) is the same as that of a low carbon steel of the same chemical composition.

In this section, the OXCut simulations of 3-D face milling of P20 mold steel (30 HRC) for the conditions in Table 3.6 are repeated using the flow stress data, determined in Chapter 4 and shown in Table 4.3. The cutting forces in the X, Y and Z directions, predicted by OXCut, using the flow stress data of Table 4.3, are compared to the measured results [Özel; 1998] as shown in Figures 5.1 through 5.3. As shown in Figures 5.1 and 5.3 the agreement between the predicted and measured force results in the X and Z directions has become better than the agreement in Figures 3.23 and 3.25.

Using the same cutting conditions and tool geometry as those shown in Table 3.6, except for a different radial rake angle ($\alpha_s$) of 9°, experiments of 3-D face milling of H13 tool steel (46 HRC) are conducted in the present work. The cutting forces in the X, Y and Z directions, predicted by OXCut using the flow stress data of Table 4.3 and the thermal property data of Table 4.2, are compared to the measured results of 3-D face milling of H13 tool steel (46 HRC) as shown in Figures 5.4, 5.5 and 5.6, respectively. As shown in Figures 5.4 and 5.5, the agreement between the predicted and measured forces in the X and Y directions is good. The agreement for forces in the Z direction is not as good especially at the higher depth of cut (2 mm) as shown in Figure 5.6. This might be attributed to chatter and vibration that increase with the depth of cut and have a larger influence on forces in the Z direction than forces in the X and Y directions.
Figure 5.1: Forces in X direction for 3-D face milling of P20 mold steel (30 HRC) using the flow stress data of Table 4.3 (cutting data is given in Table 3.6)
Figure 5.2: Forces in Y direction for 3-D face milling of P20 mold steel (30 HRC) using the flow stress data of Table 4.3 (cutting data is given in Table 3.6)
Figure 5.3: Forces in Z direction for 3-D face milling of P20 mold steel (30 HRC) using the flow stress data of Table 4.3 (cutting data is given in Table 3.6)
Figure 5.4: Forces in X direction for 3-D face milling of H13 tool steel (46 HRC) using the material data of Tables 4.2 and 4.3 (cutting data, except for a different radial rake angle ($\gamma_r$) of 9°, is given in Table 3.6)
Figure 5.5: Forces in Y direction for 3-D face milling of H13 tool steel (46 HRC) using the material data of Tables 4.2 and 4.3 (cutting data, except for a different radial rake angle ($\alpha_r$) of 9°, is given in Table 3.6)
Figure 5.6: Forces in Z direction for 3-D face milling of H13 tool steel (46 HRC) using the material data of Tables 4.2 and 4.3 (cutting data, except for a different radial rake angle ($\alpha_r$) of 9°, is given in Table 3.6)
5.2 Orthogonal Turning of H13 Tool Steel (46 HRC), P20 Mold Steel (30 HRC) and the Aluminum Alloy Al2007 (100 HB)

5.2.1 Orthogonal Turning of H13 Tool Steel (46 HRC)

The cutting and feed forces, predicted by FEM (DEFORM), using the determined flow stress data of AISI H13 tool steel (46 HRC) of Table 4.3, are compared to the published experimental results of orthogonal turning of AISI H13 tool steel (52 HRC) [Ng; 1999] as shown in Figure 5.7. The difference in hardness between the H13 steel, used in the flow stress determination of the present work (46 HRC), and the H13, used in the published experimental work (52 HRC), is small and should not have a major influence on the results. The workpiece material properties, cutting conditions and tool material and geometry, used in the FEM simulations, are shown in Table 5.1. The friction model, used in the FEM simulations is the same as that of Figure 3.39 (Zorev's model [Zorev; 1963]).

The FEM predicted temperature distributions in the workpiece for the cutting velocities of 75 and 200 m/min (Table 5.1) are shown in Figure 5.8. The FEM temperature predictions of the present work are about 17% higher than the published FEM temperature results [Ng; 1999]. This may be due to the difference in flow stress data. It is worth mentioning that Ng et al. [Ng; 1999] used a flow stress equation that neglects the effect of strain rate on flow stress.

5.2.2 Orthogonal Turning of P20 Mold Steel (30 HRC)

Both FEM and OXCUIT simulations, using the determined flow stress data of AISI P20 mold steel (30 HRC), shown in Table 4.3, are conducted in the present work using the cutting data of Table 5.2. The friction model, used in the FEM simulations, is Zorev's model [Zorev; 1963] as described in Figure 3.39.
Figure 5.7: Cutting and feed forces for orthogonal turning of H13 tool steel hardened to 46 HRC (cutting data is given in Table 5.1)
<table>
<thead>
<tr>
<th>Workpiece</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>AlSi H13 tool steel (46 HRC)</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>Table 4.3</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>Table 4.2</td>
</tr>
<tr>
<td>Specific heat</td>
<td>Table 4.2</td>
</tr>
<tr>
<td><strong>Cutting Conditions</strong></td>
<td></td>
</tr>
<tr>
<td>Cutting speed (u)</td>
<td>75, 150 and 200 m/min</td>
</tr>
<tr>
<td>Uncut chip thickness (t₁)</td>
<td>0.25 mm</td>
</tr>
<tr>
<td>Width of cut (W)</td>
<td>2 mm</td>
</tr>
<tr>
<td>Friction</td>
<td>Zorev's model (Figure 3.39)</td>
</tr>
<tr>
<td><strong>Tool Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>Tool rake angle (α)</td>
<td>-5°</td>
</tr>
<tr>
<td>Tool clearance angle (γ)</td>
<td>5°</td>
</tr>
<tr>
<td>Edge preparation</td>
<td>20°X0.2 mm T-land</td>
</tr>
<tr>
<td><strong>Tool material</strong></td>
<td>PCBN</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>852</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.128</td>
</tr>
<tr>
<td>Density (Kg/m³)</td>
<td>3399.5</td>
</tr>
<tr>
<td>Thermal conductivity (W/m °K)</td>
<td>100</td>
</tr>
<tr>
<td>Specific heat (J/Kg °K)</td>
<td>960</td>
</tr>
</tbody>
</table>

Table 5.1: Data for orthogonal turning of H13 tool steel (46 HRC) [Ng; 1999]
Figure 5.8: Cutting Temperatures for orthogonal turning of H13 tool steel hardened to 46 HRC (cutting data is given in Table 5.1)
Table 5.2: Data for orthogonal turning of P20 mold steel (30 HRC) [Özel; 1998][ASM; 1997]

As shown in Figure 5.9, the cutting and feed force results, predicted by OXCUT using the data of Table 5.2, are in good agreement with the published results measured during orthogonal turning of AISI P20 mold steel (30 HRC) [Özel; 1998]. FEM (DEFORM), on the other hand, slightly over predicts the cutting forces and under predicts the feed forces for many of the cutting conditions used in the present study. The error in the FEM cutting and feed force results, however, is still within less than 20% as compared to the measured results for the majority of the cutting conditions used in the present work.
Figure 5.9: Cutting and feed forces for orthogonal turning of P20 mold steel hardened to 30 HRC (cutting data is given in Table 5.2)
Good agreement can be observed between the FEM and OXCUT predicted tool-chip contact length (h) and predicted cut chip thickness (t₂) as illustrated in Figure 5.10.

5.2.3 Orthogonal Turning of Al2007 (100 HB)

Both FEM and OXCUT simulations, using the determined flow stress data of the aluminum alloy Al2007, shown in Table 4.3, are conducted in the present work using the cutting data of Table 5.3. The friction model, used in the FEM simulations, is, again, Zorev's model [Zorev; 1963] as described in Figure 3.39.

The cutting force results, predicted by OXCUT and FEM (DEFORM) using the data in Table 5.3, are in good agreement with the cutting force results measured during orthogonal turning of Al2007 (100 HB) as shown in Figure 5.11. The feed force results predicted by OXCUT, on the other hand, are observed to be in poor agreement with the measured results. The agreement between the FEM and measured feed force results is better because a hone radius of 0.04 mm is used in the FEM modeling of the tool. No information is mentioned about the edge preparation of the inserts used in the experimental study [Ziebeil; 1995].

The chip–tool contact lengths, predicted by OXCUT and FEM, are found to be in reasonable agreement with the measured results [Ziebeil; 1995] as shown in Figure 5.12. As illustrated in Figure 5.13, the normal and shear stress distributions, predicted by FEM at the tool-chip interface, are found to be in good agreement with the measured results [Ziebeil; 1995].

165
Figure 5.10: Cut chip thickness and tool-chip contact length for orthogonal turning of P20 mold steel hardened to 30 HRC (cutting data is given in Table 5.2)
<table>
<thead>
<tr>
<th>Workpiece</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>Aluminum alloy Al2007 (100 HB)</td>
</tr>
<tr>
<td>Flow stress data</td>
<td>Table 4.3</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>Table 4.2</td>
</tr>
<tr>
<td>Specific heat</td>
<td>Table 4.2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cutting Conditions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting speed (u)</td>
<td>200 m/min</td>
</tr>
<tr>
<td>Uncut chip thickness (t&lt;sub&gt;c&lt;/sub&gt;)</td>
<td>0.05 to 0.7 mm</td>
</tr>
<tr>
<td>Width of cut (W)</td>
<td>2 mm</td>
</tr>
</tbody>
</table>
| Friction | - Zorev's model (Figure 3.39) for FEM (DEFORM)  
- Shear flow stress of the material for OXCUT |

<table>
<thead>
<tr>
<th>Tool Geometry</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Tool rake angle (α)</td>
<td>-6°</td>
</tr>
<tr>
<td>Tool clearance angle (γ)</td>
<td>6°</td>
</tr>
</tbody>
</table>
| Edge preparation | - 0.04 mm hone radius for FEM simulations  
- Sharp edge for OXCUT calculations |

<table>
<thead>
<tr>
<th>Tool material</th>
</tr>
</thead>
</table>
| **Al<sub>2</sub>O<sub>3</sub>-ZrO<sub>2</sub>**  
used in the experiments to measure the chip tool-contact length and cutting forces |
| **Tungsten Carbide P25**  
used in the experiments to measure the stresses at the tool-chip interface |

<table>
<thead>
<tr>
<th>Property</th>
<th>Al&lt;sub&gt;2&lt;/sub&gt;O&lt;sub&gt;3&lt;/sub&gt;-ZrO&lt;sub&gt;2&lt;/sub&gt;</th>
<th>Tungsten Carbide P25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus (GPa)</td>
<td>303</td>
<td>558</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>Thermal conductivity (W/m °K)</td>
<td>24</td>
<td>60</td>
</tr>
<tr>
<td>Heat capacity (N/mm² °C)</td>
<td>3.5</td>
<td>2.79</td>
</tr>
</tbody>
</table>

Table 5.3: Data for orthogonal turning of Al2007 (100 HB) [Ziebel; 1995] [ASM; 1997]
Figure 5.11: Cutting and feed forces for orthogonal turning of Al2007 hardened to 100 HB (cutting data is given in Table 5.3)
Figure 5.12: Chip-tool contact length for orthogonal turning of Al2007 hardened to 100 HB (cutting data is given in Table 5.3)
Figure 5.13: Normal and shear stresses for orthogonal turning of Al2007 hardened to 100 HB (cutting data is given in Table 5.3)
5.3 FEM study of the Effect of Edge Preparation on Process Variables

As shown in Figure 5.14, there are different types of edge preparations of the cutting inserts [Stephenson; 1997]. If there is no hone or chamfer, the insert has an up-sharp edge. Practically speaking, there is nothing called perfectly sharp, therefore, the up-sharp edge has a hone radius value within the vicinity of 0.01 mm [Kennametal; 1998]. The up-sharp edge is not suitable for inserts made of brittle materials, like PCBN, or inserts used in interrupted cutting operations, such as face milling, because the cutting edge will fail easily by breaking or micro-chipping. Thus, edge preparation such as a hone radius, chamfer, negative T-land or combinations of these are added to the cutting inserts to strengthen them and prolong their life. The size of the hone radius ranges from 0.025 mm (light hone) to 0.18 mm (heavy hone) according to the size of the insert. However, the hone radius should always be less than the uncut chip thickness [Stephenson; 1997].

The chamfered edge, shown in Figure 5.14, has a chamfer angle ($\alpha_c$) that usually varies from 20° to 45°. The width of the chamfer ($W_c$) can be smaller, equal or larger than the uncut chip thickness. Preferably it should not exceed about 70% of the uncut chip thickness [Stephenson; 1997]. The negative T-land edge is similar to the chamfered edge but with smaller angle ($\alpha_c$) that usually ranges from 5° to 20°. A hone radius is, sometimes, added to the chamfer or negative T-land edges to prevent micro-chipping at the intersection of the chamfer or negative T-land with the rake or flank faces of the insert [Stephenson; 1997]. The hone radius, added to the chamfer or T-land (Figure 5.14) at the intersections with the rake and flank faces, usually ranges from 0.025 mm to 0.05 mm. More details about edge preparations of cutting inserts can be found elsewhere [Stephenson; 1997].
The workpiece and tool material properties and cutting conditions (only one cutting velocity \( u = 200 \text{ m/min} \)) used in this study are given in Table 5.1. The different edge preparations with the symbols used to differentiate between them, investigated in this study, are listed in Table 5.4.

Figure 5.15 summarizes the cutting and feed force results of orthogonal turning of AISI H13 tool steel (46 HRC) using the cutting data in Tables 5.1 and 5.4 (\( u = 200 \text{ m/min} \)). As shown in the Figure, the shape of the cutting edge has more influence on the feed forces than on the cutting forces. This supports the argument, mentioned above, that the disagreement between the predicted and measured feed forces, assuming a sharp cutting edge, may be due to the change of shape of the cutting edge because of wear and micro chipping.

![Figure 5.14: Different types of edge preparations of cutting inserts](image)

[Stephenson; 1997]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>c111</td>
<td>chamfered edge ($\alpha_c = 10^\circ$, $W_c = 0.1$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c115</td>
<td>chamfered edge ($\alpha_c = 10^\circ$, $W_c = 0.1$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c121</td>
<td>chamfered edge ($\alpha_c = 10^\circ$, $W_c = 0.2$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c125</td>
<td>chamfered edge ($\alpha_c = 10^\circ$, $W_c = 0.2$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c211</td>
<td>chamfered edge ($\alpha_c = 20^\circ$, $W_c = 0.1$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c215</td>
<td>chamfered edge ($\alpha_c = 20^\circ$, $W_c = 0.1$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c221</td>
<td>chamfered edge ($\alpha_c = 20^\circ$, $W_c = 0.2$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c225</td>
<td>chamfered edge ($\alpha_c = 20^\circ$, $W_c = 0.2$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c311</td>
<td>chamfered edge ($\alpha_c = 30^\circ$, $W_c = 0.1$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c315</td>
<td>chamfered edge ($\alpha_c = 30^\circ$, $W_c = 0.1$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c321</td>
<td>chamfered edge ($\alpha_c = 30^\circ$, $W_c = 0.2$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c325</td>
<td>chamfered edge ($\alpha_c = 30^\circ$, $W_c = 0.2$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c411</td>
<td>chamfered edge ($\alpha_c = 40^\circ$, $W_c = 0.1$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c415</td>
<td>chamfered edge ($\alpha_c = 40^\circ$, $W_c = 0.1$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>c421</td>
<td>chamfered edge ($\alpha_c = 40^\circ$, $W_c = 0.2$ mm, $r_n = 0.01$ mm)</td>
</tr>
<tr>
<td>c425</td>
<td>chamfered edge ($\alpha_c = 40^\circ$, $W_c = 0.2$ mm, $r_n = 0.05$ mm)</td>
</tr>
<tr>
<td>h01</td>
<td>0.01 mm hone radius ($r_h$)</td>
</tr>
<tr>
<td>h05</td>
<td>0.05 mm hone radius ($r_h$)</td>
</tr>
<tr>
<td>h1</td>
<td>0.1 mm hone radius ($r_h$)</td>
</tr>
<tr>
<td>h2</td>
<td>0.2 mm hone radius ($r_h$)</td>
</tr>
</tbody>
</table>

Table 5.4: Different types of edge preparations used in the present study
Figure 5.15: FEM predicted cutting and feed forces of orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
As shown in Figure 5.15, for the chamfer or T-land, the feed force generally increases with increasing the chamfer angle ($\alpha_c$), the width of the chamfer ($W_c$) and the hone radius added to the chamfer. For the hone radius edges h01, h05, h1 and h2, the value of the feed force, as expected, increases with increasing the hone radius. The values of the cutting and feed forces for the T-land edges, c111 and c115, are close to those of the hone-radius edges h01 and h05, respectively. This may be because the chamfer angle of the T-land edges c111 and c115 is small (10°) which makes the shape of these cutting edges very similar to the hone-radius cutting edges h01 and h05 which do not have chamfers. The maximum cutting and feed forces are obtained using the chamfered edge c425 and the hone-radius edge h2. Low cutting and feed forces are obtained with chamfers of chamfer angle $\alpha_c = 10°$ which are c111, c115, c121, and c125, however, as mentioned above, these edges are very similar to the edges without chamfer and may not provide enough strength. As shown in Figure 5.15, the chamfered edge c211 provides relatively low cutting and feed forces. Another edge preparation that provides low cutting and feed forces is the hone-radius edge h1 ($r_h = 0.1$ mm).

The temperature distributions in the workpiece for the cutting data in Tables 5.1 and 5.4 (cutting velocity $u = 200$ m/min) are shown in Figures 5.16 through 5.20. A summary of this temperature data is given in the bar charts of Figure 5.21. The lower the temperature, the lower the wear rate of cutting inserts. The lowest rake face temperature corresponds to the hone radius edge h01 (hone radius $r_h = 0.01$ mm) which is nearly sharp and is not preferred for PCBN. The next lowest rake face temperature corresponds to the chamfered edge c111 which is similar in shape to the h01 and not preferred for PCBN. The next lowest rake face temperature corresponds to the chamfered edge c211. This edge, c211, also has one of the lowest flank face temperatures as shown in Figure 5.21. Other edges that have low flank face temperatures are the c111 and c121.
Figure 5.16: FEM predicted temperature distribution (°C) in the workpiece during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 10^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.17: FEM predicted temperature distribution (°C) in the workpiece during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 20^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.18: FEM predicted temperature distribution (°C) in the workpiece during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 30^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.19: FEM predicted temperature distribution (°C) in the workpiece during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 40^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.20: FEM predicted temperature distribution (°C) in the workpiece during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (different hone radii) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.21: Summary of the temperature results in Figures 5.16 through 5.20 during orthogonal turning of H13 tool steel (46 HRC) using the data in Tables 5.1 and 5.4 (cutting velocity $u = 200$ m/min)
which are not preferred for PCDN because they look similar to an up sharp edge. Another good option for the shape of a cutting edge with low flank face temperature is the hone-radius edge h1.

As expected, a tool with an Up-sharp edge (low hone radius close to 0.01 mm) does the cutting easier and more efficient than a tool with a higher hone radius. Thus, the stresses in the tool should decrease with decreasing the hone radius. However, this is not the case because a sharp tool, as expected, should have more stress concentration, especially in the vicinity of the cutting edge, than a less sharp tool. This stress concentration is usually the cause of edge chipping and breakage and it is the reason why edge preparation is used to relief the stress concentration and strengthen the cutting edge. Thus, to minimize edge breakage and chipping, the shape of the cutting edge should be optimized based on the trade off between the higher stress concentration and the easier cutting (less induced stresses) of the sharp tool and the lower stress concentration and the more difficult cutting (higher induced stresses) of the dull tool.

The effective stress distributions in the tool, for orthogonal turning of H13 tool steel (46 HRC), using the cutting data in Tables 5.1 and 5.4 (cutting velocity \( u = 200 \text{ m/min} \)), are shown in Figures 5.22 through 5.26. In those figures, it can be observed that the stress concentration at the tool cutting edge is relieved by adding a hone radius of 0.05 mm between the chamfer and the flank face of the tool. Moreover, it can be observed in Figures 5.22 through 5.26 that the edge with the least stress concentration and the least maximum value of effective stress in the tool is the hone-radius edge h1 (\( r_h = 0.1 \text{ mm} \)). For the chamfered edge c211 where the flank and rake temperatures were observed to be the least, the predicted maximum effective stress in the tool is 2800 MPa, as shown in Figure 5.23, which is relatively not very high. Usually, chamfered edges are preferred more than hone radius edges because they are easier to manufacture.
Figure 5.22: FEM predicted effective stress distribution (MPa) in the tool during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 10^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1).
Figure 5.23: FEM predicted effective stress distribution (MPa) in the tool during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 20^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.24: FEM predicted effective stress distribution (MPa) in the tool during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 30^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1).
Figure 5.25: FEM predicted effective stress distribution (MPa) in the tool during orthogonal turning of H13 tool steel (46 HRC) using the different edge preparations listed in Table 5.4 (chamfer angle $\alpha_c = 40^\circ$) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
Figure 5.26: FEM predicted effective stress distribution (MPa) in the tool during orthogonal turning of H13 tool steel (45 HRC) using the different edge preparations listed in Table 5.4 (different hone radii) and a cutting velocity of 200 m/min (workpiece and tool material properties and cutting conditions are given in Table 5.1)
As shown in Figure 5.26, the stress concentration is relatively high for the sharp edge ($r_h = 0.01$ mm) where the cutting is the easiest but the shape of the cutting edge causes highly concentrated stresses on the tool edge. When the hone radius value is increased from 0.01 mm to 0.05 mm, some relief of the stresses occurs and the maximum effective stress in the tool drops from 3948 MPa to 2689 MPa as shown in Figure 5.26. The further increase of the hone radius to 0.1 mm causes more relief of the stress concentration in the tool and results in the decrease of the maximum effective stress to 1974 MPa. Increasing the hone radius to 0.2 mm makes the cutting more "difficult". This effect is more significant than the stress relief obtained with the higher hone radius. Thus, as shown in Figure 5.26, the maximum effective stress in the tool increased to 3800 when the hone radius is increased to 0.2 mm. For the conditions used in the present study, it can be concluded that the hone radius edge ($r_h = 0.1$ mm) may have the least probability of chipping.

5.4 Summary

In this chapter, the flow stress data of AISI H13 tool steel (46 HRC), P20 mold steel (30 HRC) and the aluminum alloy Al2007 (100 HB), developed in the previous chapter (Chapter 4), are validated. The results of OXCU T and FEM simulations of 3-D face milling and orthogonal turning, respectively, using the predicted flow stress data, have been found to be in reasonable agreement with the published measured results and results of experiments conducted in the present work. Moreover, the flow stress data of AISI H13 tool steel (46 HRC) is applied to study the effect of edge preparation of the cutting insert on insert wear and breakage using FEM. It has been concluded that the hone-radius edge, with a hone radius of 0.1 mm, may have the lowest probability of chipping and the chamfered edge ($20^\circ$X0.1 mm) may have the least crater and flank wears for the conditions used in the present study.
CHAPTER 6

PREDICTION OF TOOL WEAR IN METAL CUTTING

6.1 Introduction

The following wear equation [Usui; 1978] will be used in this study:

\[ \dot{w} = AV \sigma_i e^{\frac{-B}{T}} \]  

\( \dot{w} \) : wear rate (mm/min)
\( A \) : constant (l/MPa)
\( V \) : relative velocity at the tool-workpiece interface (m/min)
\( \sigma_i \) : normal stress at the interface between the two surfaces (MPa)
\( B \) : constant (K)
\( T \) : temperature at the interface between the two surfaces (K)

This study is based on curve fitting the wear data, from the orthogonal turning experimental results published in the literature [Mathew; 1989][Lau; 1980][Usui; 1978][Maekawa; 1989][Kitagawa; 1988], to the temperatures, stresses and relative velocities at the tool-workpiece interface, predicted by OXCUT, in order to obtain the constants A and B of the wear equation (6.1) for crater or flank wear. OXCUT can predict the effective stress, normal stress and temperature on the rake face (secondary deformation zone). It was shown in Chapter 3 that the temperature, predicted by OXCUT on the rake face, agrees
well with the measured published results (Figures 3.1 and 3.5). OXCUT can, as well, approximate the temperature and effective stress values on the flank face as the average of the values in the primary and secondary deformation zones as was argued at the end of Chapter 3 (Figures 3.42 through 3.44).

The advantage of using OXCUT instead of FEM, in the present study, is that the study can be extended to tool wear prediction in 3-D cutting processes such as drilling where 3-D FEM is very difficult to apply. As discussed in Chapter 3, a 3-D cutting tool can be analyzed by dividing it into infinitesimal orthogonal cutting elements and using OXCUT to analyze each element. It will require so much time and effort to use 2-D FEM to analyze each infinitesimal element of the 3-D cutting tool. Moreover, it was shown in Chapter 3 that the OXCUT predicted temperature distribution, on the flank face of the twist drill lip (3-D cutting), reasonably agrees with the measured results (Figures 3.27 through 3.29).

Some investigators who focused on flank wear used orthogonal turning experiments to measure the time rate of change of the flank wear length (fl) shown in Figure 6.1 [Mathew; 1989][Lau; 1980]. Others who focused on crater wear measured the time rate of change of the average crater wear depth (cr), shown in Figure 6.1, which is the wear volume per unit area of the worn surface [Usui; 1978].

Equation 6.1 was developed, initially, as a characteristic equation to predict crater wear [Usui; 1978]. However, it was proven that flank wear can be predicted using the same characteristic equation [Kitagawa; 1988]. Moreover, if the constants of the wear equation 6.1 are obtained using crater wear measurements, the equation with those constants can be used to predict flank wear as well [Kitagawa; 1988]. This can be done by using equation 6.1, with
Figure 6.1: Crater and flank wear of cutting tools
constants A and B developed to predict the time rate of change of the average crater wear depth (cr) of Figure 6.1, to calculate the time rate of change of the average flank wear depth (fd) shown in Figure 6.1 (fd is the average flank wear depth that corresponds to the average crater wear depth cr). Then, the geometry of Figure 6.1 can be used to relate the flank wear length (fl) to the average flank wear depth (fd). Thus, the required time rate of change of the flank wear length (fl) can be calculated [Maekawa; 1989]. Maekawa et al. [Maekawa; 1989] developed an equation which relates the flank wear length (fl) to the average flank wear depth (fd). That equation is suitable for the special case of the zero-rake angle tool, used in their study. In the present work the relationship between (fl) and (fd) is re-established for the general case of the non-zero rake angle tool of Figure 6.1. From the geometry of Figure 6.1:

\[
\frac{fl}{\sin(90 - (\alpha + \gamma))} = \frac{x}{\sin(180 - (\gamma + 90 - (\alpha + \gamma)))} \quad (6.2.a)
\]

then

\[
\frac{fl}{\cos(\alpha + \gamma)} = \frac{x}{\sin(\gamma) \cos(\alpha)} \quad (6.2.b)
\]

\[
flank\_wear\_volume = \frac{1}{2} fl \times W, \quad W = \text{width of cut} \quad (6.2.c)
\]

\[
flank\_wear\_area = fl \times W \quad (6.2.d)
\]

\[
\text{average\_flank\_wear\_depth(fd)} = \frac{\text{flank\_wear\_volume}}{\text{flank\_wear\_area}} \quad (6.2.e)
\]

\[
= \frac{1}{2} \frac{fl \times W}{fl \times W} = \frac{1}{2} x
\]

if we substitute \( x = 2fd \) from equation (6.2.e) into equation (6.2.b)

\[
\frac{fl}{\cos(\alpha + \gamma)} = \frac{2fd}{\sin(\gamma) \cos(\alpha)} \quad (6.2.f)
\]

then

\[
fl = \frac{2fd \cos(\alpha + \gamma)}{\sin(\gamma) \cos(\alpha)} \quad (6.2.g)
\]
and

\[ fd = \frac{f_l \sin(\gamma) \cos(\alpha)}{2 \cos(\alpha + \gamma)} \]  \hspace{1cm} (6.2.h)

This study is focused on crater and flank wear of cutting tools and inserts during cutting of low carbon steel. The constants A and B of the wear equation 6.1, estimated in the present study, will be suitable to predict flank and crater wear of tools and inserts, made of Carboloy 370, MTM 41 High Speed Steel (HSS) and Carbide P20, when cutting low carbon steels with different carbon contents. Usui et al. [Usui; 1978] argued that low carbon steels with different carbon contents induce different stresses and temperatures at the tool-workpiece interface on the rake and flank faces, however, they can be considered the same from the point of view of wear characteristics.

6.2 Tool Wear of Carboloy 370 Inserts When Cutting Low Carbon Steels

Mathew [Mathew; 1989] measured the time rate of change (\( \dot{W}_{fl} \)) of the flank wear length (\( f_l \)), shown in Figure 6.1, of an insert, made of Carboloy 370, during orthogonal turning of 0.2% and 0.38% carbon steels using the conditions listed in Table 6.1. In the present work, to test the validity of the tool wear prediction methodology, some of the conditions, shown in bold in Table 6.1, are used to estimate the constants A and B of the wear equation 6.1. This is done by curve fitting the measured time rate of change of the flank wear length (\( \dot{W}_{fl} \)) to the relative velocity on the flank face (relative velocity on the flank face can be approximated as the cutting velocity \( u \) [Maekawa; 1989]) and the OXCUT predicted effective stress (\( \sigma_{flank} \)) and temperature (\( T_{flank} \)) as shown in Figure 6.2. Equation 6.1 uses the normal stress not the effective stress, however, the effective stress is used in this study because OXCUT can not predict the normal
<table>
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<tr>
<th>Workpiece</th>
<th>0.2% and 0.38% Carbon steels</th>
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<td>Chemical composition for 0.2% Carbon steel</td>
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<td>Flow stress data</td>
<td>equations C.1 to C.8 in Appendix C</td>
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<tr>
<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
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<td>Specific heat S (J/kg °C)</td>
<td>equation (3.11) in Table 3.2</td>
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### Tool geometry and material and cutting Conditions

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<tr>
<th>Cutting speed, ( u ) (m/min)</th>
<th>Depth of cut, ( t_1 ) (mm)</th>
<th>Rake angle, ( \alpha ) (°)</th>
<th>Cutting speed, ( u ) (m/min)</th>
<th>Depth of cut, ( t_1 ) (mm)</th>
<th>Rake angle, ( \alpha ) (°)</th>
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<td>Width of cut (W)</td>
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<td>Friction</td>
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<tr>
<td>Tool material</td>
<td>Carboloy 370 (P30)</td>
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</table>

Table 6.1: Data for orthogonal turning wear experiments using tools made of Carboloy 370 [Mathew: 1989]
Figure 6.2: Estimating the constants A and B of the wear equation (6.1) for an insert made of Carboloy 370 (P30) during orthogonal turning of 0.2% and 0.38% carbon steel (cutting data are shown in bold in Table 6.1)
stress on the flank face. The other conditions in Table 6.1, not shown in bold, are utilized to validate the results by comparing the flank wear rate \( \dot{W}_{fl} \), predicted using OXCUT results and equation 6.1 with the estimated constants A and B, to the published measured values corresponding to those conditions [Mathew; 1989] as shown in Figure 6.3. The predicted and measured wear results reasonably agree for the majority of the conditions shown in the Figure. The end of tool life criterion, used in Figure 6.3, is the flank wear length (fl) of 0.18 mm [Lau; 1980].

Now, after the tool wear prediction methodology is validated, all the conditions in Table 6.1 are used to estimate the constants A and B of the wear equation 6.1 to make it more reliable to predict the time rate of change of the flank wear length fl \( \dot{W}_{fl} \) as shown in Figure 6.4. To be able to utilize Mathew’s data to predict crater wear, the average flank wear depth fd (Figure 6.1) is calculated from the measured flank wear length fl (Figure 6.1) using equation 6.2.h. Then, the constants A and B of the wear equation 6.1 are estimated, for crater wear prediction, by curve fitting the cutting velocity (u) and the predicted effective stress \( \overline{\sigma}_{flank} \) and temperature \( T_{flank} \) on the flank face to the time rate of change of the flank wear depth \( \dot{W}_{fl} \) as shown in Figure 6.5. This is based on the analogy that the average flank wear depth fd (Figure 6.1) corresponds to the average crater wear depth cr (Figure 6.1) [Maekawa; 1989]. Thus, the equation for flank wear prediction of a tool or an insert made of Carboloy 370 when cutting low carbon steel is:

\[
\dot{W}_{fl} = 1.038 E - 5 * \overline{\sigma}_{flank} * u * e^{-\frac{9703}{T_{flank}}} \tag{6.3.a}
\]

and the equation for crater wear prediction is:

\[
\dot{W}_{cr} = 5.4328 E - 7 * \overline{\sigma}_{rake} * V_{ch} * e^{-\frac{9700}{T_{rake}}} \tag{6.3.b}
\]
Figure 6.3: Predicted and measured flank wear rate during orthogonal turning of 0.2\% and 0.38\% carbon steel using Carboloy 370 (cutting data are shown as non-bold in Table 6.1)
Figure 6.4: Estimating the constants A and B of the wear equation 6.1, using all the data of Table 6.1, for flank wear prediction of an insert or tool made of Carboloy 370 when cutting low carbon steel.

\[
y = 1.0380 \times 10^{-5} e^{-9.7026 \times 10^{-3} x}
\]

\[
y = \frac{\dot{W}_f}{\bar{\sigma}_{\text{flank}} \cdot u} \quad x = \frac{1}{T_{\text{flank}}}
\]

- Tool material: Carboloy 370
- Work materials: 0.20% & 0.38% C steel
- Rake angles = 3.5° & -3.5°
- Clearance angle = 6°
- Cutting speeds = 100, 200 & 400 m/min
- Uncut chip thickness = 0.125, 0.25 & 0.5 mm
- Width of cut = 1.27 mm
Figure 6.5: Estimating the constants A and B of the wear equation 6.1, using all the data of Table 6.1, for crater wear prediction of an insert or tool made of Carboloy 370 when cutting low carbon steel.
6.3 Tool Wear of MTM 41 (HSS) Inserts When Cutting Low Carbon Steels

Lau et al. [Lau; 1980] measured the flank wear rate (time rate of change of the flank wear length $f_l$ in Figure 6.1) of MTM 41 High Speed Steel (HSS) tools during orthogonal turning of 0.2% carbon steel using the data of Table 6.2. Using the same technique as that of the previous section (Section 6.2), the constants $A$ and $B$ of the wear equation 6.1 are determined for flank and crater wear prediction as shown in Figures 6.6 and 6.7, respectively. The equation to predict flank wear of MTM 41 HSS tools when cutting low carbon steel is:

$$\dot{w}_{fl} = 2.1271E - 4*\overline{\sigma}_{flank} * u * e^{T_{flank}}$$  \hspace{1cm} (6.4.a)

and the equation for crater wear prediction is:

$$\dot{w}_{cr} = 8.1037E - 6*\overline{\sigma}_{rake} * V_{ch} * e^{T_{rake}}$$  \hspace{1cm} (6.4.b)

6.4 Tool Wear of Carbide P20 Inserts When Cutting Low Carbon Steels

Usui et al. [Usui; 1978] measured the time rate of change of the average crater wear depth (cr) of Figure 6.1 (they called it the wear volume per unit area of worn surface) of carbide P20 when cutting low carbon steels (0.15%, 0.25%, 0.35% and 0.45% C). They estimated the constants $A$ and $B$ of the wear equation 6.1 for crater wear prediction by curve fitting their normal stress, temperature and velocity results on the rake face to the measured crater wear rate results to obtain the following equation:

$$\dot{w}_{cr} = 0.01198*\overline{\sigma}_{rake} * V_{ch} * e^{T_{rake}}$$  \hspace{1cm} (6.5.a)

Kitagawa et al. [Kitagawa; 1988] argued that this crater wear equation can be used to predict flank wear. However, it was found that this equation is not
<table>
<thead>
<tr>
<th>Workpiece</th>
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<td>Chemical composition</td>
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</tr>
<tr>
<td>Flow stress data</td>
<td>equations C.1 to C.8 in Appendix C</td>
</tr>
<tr>
<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
</tr>
<tr>
<td>Specific heat S (J/kg °C)</td>
<td>equation (3.11) in Table 3.2</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Tool geometry and material and cutting Conditions</th>
</tr>
</thead>
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<tr>
<td>Cutting velocity, u (m/min)</td>
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<table>
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<tr>
<th>Width of cut (W)</th>
<th>Uncut chip thickness (t_c)</th>
<th>Friction</th>
<th>Hone radius (r_h)</th>
<th>Tool material</th>
</tr>
</thead>
<tbody>
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<td>2 mm</td>
<td>0.1016 mm</td>
<td>Shear flow stress of the material</td>
<td>OXCUT assumes sharp tool</td>
<td>MTM 41 (HSS)</td>
</tr>
</tbody>
</table>

**Table 6.2:** Data for orthogonal turning wear experiments using tools made of MTM 41 (HSS) [Lau; 1980]
Figure 6.6: Estimating the constants A and B of the wear equation 6.1, using the data of Table 6.2, for flank wear prediction of an insert or tool made of MTM 41 (HSS) when cutting low carbon steel.
Figure 6.7: Estimating the constants A and B of the wear equation 6.1, using the data of Table 6.2, for crater wear prediction of an insert or tool made of MTM 41 (HSS) when cutting low carbon steel.
suitable for wear prediction (crater or flank) if the interface temperature is below a critical point (1150 K) [Kitagawa; 1988][Maekawa; 1989]. Therefore, they conducted another set of experiments to cover the low temperature range and came up with different A and B constants of the wear equation 6.1 to predict wear rate for temperatures lower than 1150 K as follows:

\[
\dot{W}_{cr} = 7.8E - 9*\sigma_{t,rake} * V_{ch} * e^{-\frac{5302}{T_{rake}}} \tag{6.5.b}
\]

Crater wear rate can be predicted using equations 6.5 and the normal stress, temperature and relative velocity results at the tool-chip interface calculated by OXCUT. In order to be able to predict the flank wear rate (time rate of change of the flank wear length f of Figure 6.1), equations 6.5 need to be modified to be functions of the effective stress instead of the normal stress. To achieve this task:

1) A set of OXCUT simulations were conducted to predict the effective stress, normal stress, chip velocity and temperature at the tool-chip interface, using the conditions of Table 6.3 which are similar to the conditions used in the published experimental work [Usui; 1978][Kitagawa; 1988][Maekawa; 1989].

2) The OXCUT predicted normal stress, temperature and relative velocity results at the tool-chip interface were used in equations 6.5 to calculate the crater wear rates for the conditions of Table 6.3.

3) The constants A and B of the wear equation 6.1, suitable for making equations 6.5 functions of the effective stress instead of the normal stress, were obtained by curve fitting the calculated wear rate from step (2) to the predicted effective stress, temperature and relative velocity results from step (1) as shown in Figure 6.8.

Thus, the following equations to predict crater wear of carbide P20 inserts when cutting low carbon steels were obtained:

\[
\dot{W}_{cr} = 1.2717E - 1*\bar{\sigma}_{rake} * V_{ch} * e^{-\frac{24807}{T_{rake}}} \text{, } T_{rake} \geq 1150 \text{ K} \tag{6.6.a}
\]
<table>
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<tr>
<th>Workpiece Material</th>
<th>0.15%, 0.25%, 0.35% and 0.45% Carbon steels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemical composition for 0.15% Carbon steel</td>
<td>0.15% C, 0.2% Si, 0.72% Mn, 0.02% P, 0.02% Ni, 0.02% Cr, 0.01% Cu</td>
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<td>Flow stress data</td>
<td>equations C.1 to C.8 in Appendix C</td>
</tr>
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<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
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<tr>
<td>Specific heat S (J/kg °C)</td>
<td>equation 3.11 in Table 3.2</td>
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<td>Tool rake angle, α(°)</td>
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<tr>
<td>Tool clearance angle, γ(°)</td>
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<td>Hone radius (r)</td>
<td>OXCUIT assumes sharp tool</td>
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<td>Tool material</td>
<td>P20 carbide</td>
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Table 6.3: Data for orthogonal turning of low carbon steel using P20 Carbide to estimate the constants A and B of the wear equation 6.1 to make it a function of the effective stress instead of the normal stress in order to be able to predict flank wear as well as crater wear [Maekawa; 1989][Kitagawa; 1988] [Usui; 1978]
Figure 6.8: Estimating the constants A and B of the wear equation 6.1 to make it a function of the effective stress instead of the normal stress, using the data of Table 6.3, in order to be able to predict the flank wear as well as the crater wear of an insert or tool made of Carbide P20 when cutting low carbon steels.
\[ \dot{W}_{cr} = 6.0781E - 9 \cdot \sigma_{rake}^{-\frac{5259}{e^{T_{rake}}}} \cdot V_{cr} \cdot e^{T_{rake}}, \ T_{rake} \geq 1150 \ K \]  

(6.6.b)

Now equations 6.6 can predict crater wear using effective stress instead of the normal stress. Thus, equations 6.6 can be used in conjunction with OXCUT to predict flank wear by:

1) Using the OXCUT predicted temperature, effective stress and relative velocity results on the tool flank face to calculate the time rate of change of the flank wear depth \( f_d \) (Figure 6.1) which corresponds to the average crater wear depth \( (cr) \) as was argued in section 6.1 [Kitagawa; 1988][Maekawa; 1989].

2) Using equation 6.2.g to calculate the required rate of change of the flank wear length \( f_l \) (Figure 6.1) from the rate of change of the average flank wear depth \( f_d \) (Figure 6.1) calculated in step (1).

6.5 Prediction of Crater and Flank Wear Rate Distributions on the Twist Drill Lip

Stephenson and Agapiou [Stephenson; 1997] mentioned that the common types of drill wear are chisel edge wear, lip wear, margin wear and crater wear. The lip wear is called flank wear and it occurs along the relief face of the drill cutting lip as shown in Figure 6.9. Drill flank wear causes the thrust force, the power consumption and the lip temperature to increase. Moreover, drill flank wear leads to a larger size of the burr produced when drilling through holes. Excessive drill flank wear causes the drill to be unable to cut and leads to its failure by chatter or breakage [Stephenson; 1997].

Stephenson and Agapiou [Stephenson; 1997] defined the drill crater wear as that occurring on the flute surface along the drill cutting lip as shown in Figure 6.9. They argued that if the crater wear is not high, it is of minor importance.
However, excessive crater wear weakens the cutting edge of the drill and may cause edge deformation, chipping or breakage.

The present study of wear in twist drilling is focused on the drill lip flank and crater wears. The wear rate equations 6.3, 6.4, 6.5 and 6.6 are utilized to estimate, approximately, the crater and flank wear rate distributions in the twist drill lip region. By dividing the twist drill into orthogonal cutting elements, as was done in Chapter 3, OXCUT is utilized to predict the stresses, temperatures and relative velocities on the rake and flank faces for each element. Then, using the proper tool wear equation, according to the drill material, crater and flank wear for each lip element can be estimated.

DeVries et al. [DeVries; 1968] used a HSS twist drill that cut AISI 1045 steel to conduct their experimental work on drilling as indicated in Table 3.7. The temperatures, stresses and relative velocities on both of the rake and flank faces,
predicted by OXCUT using the data of Table 3.7, were plugged in equations 6.4 to predict the distributions of the crater and flank wear rates on the HSS twist drill lip as shown in Figure 6.10. The maximum crater and flank wear rates which are around 0.00034 and 0.00054, respectively, are relatively low for such cutting conditions. Using higher values of spindle speed, both the crater and flank wear rate increase as shown in Figure 6.11. Both the crater and flank rates decrease as the cutting element approach the center line except for the flank wear corresponding to a spindle speed of 300 rpm as observed in Figure 6.11. The reason might be that as the element approaches the center line of the drill, both the cutting velocity and the rake angle of the element decrease until the rake angle of the element, closest to the drill center line, reaches high negative values (sometimes less than -30°) [Armarego; 1969]. The cutting temperatures on both of the rake and flank faces increase with decreasing the rake angle and decrease with decreasing the spindle speed. Those opposing effects may cause the existence of a point on the drill lip where the influence of the decreasing rake angle on increasing the temperature is more than the influence of the decreasing cutting speed on decreasing the temperature. At this point the temperature and consequently the wear rate will start rising as the cutting element approaches the drill center line. The stresses also play a role because the stresses get lower as the temperatures get higher. Moreover, the relative velocity is also an input to the wear equation. Thus, the mechanics is complicated and the predicted wear rate distribution is expected, sometimes, not to have the trend of a decreasing wear rate as the element approaches the center line of the twist drill lip.

Similarly, the crater and flank wear rate distributions on the lip of twist drills, made of Carboloy 370 and Carbide P20 when cutting low carbon steels, using the conditions in Table 6.4 and equations 6.3, 6.5 and 6.6, are obtained as shown in Figures 6.12 through 6.19. In these figures, parametric studies of the effect of spindle speed, feed rate, carbon content of the workpiece material and drill helix angle ($\delta_0$), shown in Figure 3.26, on the crater and flank wear rate
Figure 6.10: Crater and flank wear rate distributions on a HSS twist drill lip (cutting data are given in Table 3.7)
Figure 6.11: Crater and flank wear rate distributions on a HSS twist drill lip (cutting data are given in Table 3.7, spindle speed = 100, 200 and 300 rpm)
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<td>equations C.1 to C.8 in Appendix C</td>
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<td>Thermal conductivity K (W/m°C)</td>
<td>equation (3.10) in Table 3.2</td>
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<tr>
<td>Specific heat S (J/kg °C)</td>
<td>Equation (3.11) in Table 3.2</td>
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<td>Shear flow stress of the material</td>
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</tr>
<tr>
<td>Drill helix angle (δ_o)</td>
<td>15°, 20°, 25°, 30°, 35°</td>
</tr>
<tr>
<td>Drill chisel edge angle (η_o)</td>
<td>135°</td>
</tr>
<tr>
<td>Drill point angle (2p_o)</td>
<td>110°, 120°, 130°, 140°</td>
</tr>
<tr>
<td>Drill material</td>
<td>Carboloy 370 (P30), Carbide P20</td>
</tr>
</tbody>
</table>

Table 6.4: Data used for the parametric study of wear in drilling [Elhachimi; 1999a, 1999b]
Figure 6.12: Effect of spindle speed on the crater and flank wear rate distributions on a Carbide P20 twist drill lip (cutting data are given in Table 6.4)
Figure 6.13: Effect of spindle speed on the crater and flank wear rate distributions on a Carboloy 370 twist drill lip (cutting data are given in Table 6.4)
Figure 6.14: Effect of feed on the crater and flank wear rate distributions on a Carbide P20 twist drill lip (cutting data are given in Table 6.4)
Figure 6.15: Effect of feed on the crater and flank wear rate distributions on a Carboloy 370 twist drill lip (cutting data are given in Table 6.4)
Figure 6.16: Effect of carbon content of the workpiece material on the crater and flank wear rate distributions on a Carbide P20 twist drill lip (cutting data are given in Table 6.4)
Figure 6.17: Effect of carbon content of the workpiece material on the crater and flank wear rate distributions on a Carboloy 370 twist drill lip (cutting data are given in Table 6.4)
Figure 6.18: Effect of the drill helix angle ($\delta_0$) on the crater and flank wear rate distributions on a Carbide P20 twist drill lip (cutting data are given in Table 6.4)
Figure 6.19: Effect of the drill helix angle ($\delta_0$) on the crater and flank wear rate distributions on a Carboloy 370 twist drill lip (cutting data are given in Table 6.4)
distributions on the twist drill lip, are presented. The drills, used in this parametric study, are made of Carboloy 370 and Carbide P20.

As shown in Figures 6.12 and 6.13, both of the crater and flank wear rates increase with increasing the spindle speed. It can be observed in these figures that the crater and flank wear rates, generally, decrease with approaching the drill center line except for the flank wear rate of Carboloy 370 which reaches its maximum values near the center line. Moreover, Figures 6.12 and 6.13, show that, for the feed used, the maximum crater wear rates of Carboloy 370, for all the spindle speeds shown in the Figures, are less than 60% of the corresponding values for Carbide P20. This suggests that, for the used feed rate of 0.12 mm/rev, Carboloy 370 drills are more resistant to lip crater wear than Carbide P20. The opposite is true for lip flank wear rate as shown in Figures 6.12 and 6.13.

As shown in Figures 6.14 and 6.15, at low feeds, Carboloy 370 is more resistant to drill lip crater wear than Carbide P20, as was shown in Figures 6.12 and 6.13 for different spindle speeds, however, as the feed rate increases to 0.24 mm/rev both Carboloy 370 and Carbide P20 tend to have the same resistance. As the feed rate increases to 0.3 mm/rev, Carbide P20 which is less resistant to lip crater wear at low feeds than Carboloy 370 starts to be more resistant than Carboloy 370. For all the feeds, used in Figures 6.14 and 6.15, Carbide P20 has less drill lip flank wear rates than the corresponding values for Carboloy 370. Thus, Carbide P20 is more resistant to drill lip flank wear than Carboloy 370 which is the same conclusion derived from Figures 6.12 and 6.13.

In Figures 6.16 and 6.17 increasing the carbon content of the workpiece material causes both of the drill lip crater and flank wears to increase as expected. Moreover, Figures 6.16 and 6.17 show that the validity of the conclusions, derived from Figures 6.12 and 6.13 (Carboloy 370 is more resistant
to drill lip crater wear and less resistant to drill lip flank wear than Carbide P20), are not affected by changing the carbon content of the workpiece material.

In Figures 6.18 and 6.19, it can be observed that increasing the drill helix angle ($\delta_0$) (Figure 3.26) causes both of the drill lip crater and flank wear rates to decrease for Carbide P20 and Carboloy 370. Figures 6.18 and 6.19 also show that the largest decrease, in both of the drill crater and flank wear rates, occurs when the drill helix angle increases from 15° to 20°. Thus, in industrial applications, it is recommended, from the observations of Figures 6.18 and 6.19, that a value of 20° or more should be used for the drill helix angle in order to minimize both drill crater and flank wear rates. Sometimes it is difficult to increase the drill helix angle beyond a certain limit due to geometric and manufacturing limitations.

6.6 Summary

In this chapter, the constants of a characteristic wear equation are estimated by curve fitting the OXCUT predicted stresses, temperatures and relative velocities to the published measured flank and crater wear results. The constants are estimated to predict both crater and flank wear rates of Carbide P20, Carboloy 370 and HSS when cutting low carbon steels.

The wear equations are then used to predict crater and flank wear distributions in the drill lip. A parametric study is conducted to investigate the effect of spindle speed, feed rate, carbon content of the workpiece material, and helix angle on the drill lip crater and flank wears for twist drills made of Carboloy 370 and Carbide P20. Considering the assumptions made in OXCUT and the approximate nature of the analysis, the observations are made, as much as
possible, from a qualitative rather than a quantitative point of view to derive conclusions that can be used as suggestions for industrial applications.

It has been concluded from that parametric study that at a low feed and different spindle speeds, Carboloy 370 is more resistant to drill lip crater wear and less resistant to drill lip flank wear than Carbide P20. At higher feeds, Carbide P20 is more resistant to drill lip crater wear than Carboloy 370. The validity of these conclusions is not affected by changing the carbon content of the workpiece material or the spindle speed.

Moreover, it has been concluded from a parametric study of the effect of drill helix angle on drill lip crater and flank wears that increasing the helix angle causes both of the drill lip crater and flank wear rates to decrease. Considering the range of values of helix angles used in this study (15° to 35°), it has been observed that the largest decrease in drill lip crater and flank wears occurs when the value of the drill helix angle is increased from 15° to 20°. Considering the geometric and manufacturing limitations imposed on the value of the drill helix angle, it is recommended, from the observations made in the present study, that its value should be at least 20°.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1 Conclusions

1) The temperature at the tool workpiece interface and forces, predicted by the developed analytical computer code OXCUT (based on Oxley's theory [Oxley; 1989]), are in reasonable agreement with the published experimental results in orthogonal turning.

2) The cutting force results, predicted by OXCUT, in 3-D turning, face milling and ball end milling are in good agreement with the published experimental results.

3) An FEM study using DEFORM 2D revealed that the temperature and effective stress on the flank side of the tool can be approximated as the average of the values in the shear zone and on the rake face.

4) The temperature distribution on the flank face of the twist drill lip, predicted by OXCUT using the conclusion in (3), is in reasonable agreement with the published experimental results.

5) The technique, developed in the present work to obtain flow stress data for metal cutting using 2-D orthogonal slot milling experiments in conjunction with OXCUT and a multidimensional minimization Algorithm (the Downhill Simplex Method), has been found to be easier and less
expensive than the other techniques such as the Hopkinson's bar technique found in the literature.

6) The predicted process variables, using OXCUIT and FEM (DEFORM) in conjunction with the obtained flow stress data, have been found to be in reasonable agreement with the published measured results and the results of experiments performed in the present work.

7) The FEM study of the effect of edge preparation on tool stresses and cutting temperatures revealed that the hone radius edge (0.1 mm hone) and the chamfered edge (20° X 0.1 mm) may have the least amounts of chipping and wear, respectively, for the conditions used in the present study.

8) The curve fitting between the OXCUIT temperature, stresses and relative velocity results and the published measured flank and crater results from orthogonal turning experiments could lead to obtain the empirical parameters of a characteristic wear equation (developed in the literature based on adhesion and diffusion wear).

9) Tool crater and flank wears along the twist drill lip can be approximated using a characteristic wear equation with its empirical constants evaluated as mentioned in (8).

10) The parametric study of the effect of spindle speed, feed, carbon content of the workpiece material, drill helix angle has lead to many observations, the most important of which are:

   a) twist drills, made of Carboloy 370, are more resistant to crater wear than those made of Carbide P20

   b) at high feeds, the opposite of the observation in (a) occurs

   c) twist drills, made of Carbide P20 are more resistant to flank wear than those made of Carboloy 370

   d) the observations in (a) and (c) do not change with varying the spindle speed or the carbon content of the workpiece material

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e) increasing the drill helix angle reduces both crater and flank wear along the twist drill lip.

7.2 Future Work

1) Using the technique, developed in the present work, to establish a data base of flow stresses for workpiece materials at different hardness values used in the metal cutting industry.

2) Conduct experiments to investigate the influence of edge preparation of cutting inserts on insert life and cutting forces in order to validate the theoretical results of the present study.

3) Conduct experiments to measure twist drill flank and crater wears along the lip to validate the methodology used and the results obtained in the present study.

4) Obtain the empirical parameters of the characteristic wear equations for more workpiece material-tool material combinations.
REFERENCES


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<table>
<thead>
<tr>
<th>Reference</th>
<th>Details</th>
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[Sterr; 1998] Sterr, M., 1998, Private Communication with Fraunhofer Institute, Boston, USA.


APPENDIX A

ALGORITHM OF OXLEY’S THEORY

The geometric symbols, used in this Algorithm, are illustrated in Figure 2.3. The Algorithm is as follows:

0 - Input material properties, Tool geometry and cutting conditions which are:
Flow Stress of Workpiece = function (strain, strain rate, temperature)

\[ = \text{func}(\varepsilon, \dot{\varepsilon}, T) \]

Thermal Conductivity of Workpiece
Specific Heat of Workpiece
\( \alpha \): tool rake angle.
\( u \): cutting velocity.
\( t_1 \): depth of cut (uncut chip thickness).
\( W \): width of cut.
\( T_W \): initial temperature of the workpiece material.

1- Start of loop to calculate \( \delta \) ( \( \delta \) is the ratio of tool-chip interface plastic zone thickness to cut chip thickness):
Assign values for \( \delta \).
II- Start of loop to calculate C (C is the slope of the curve of the shear strain rate of the material versus \( V_s/L \) where \( V_s \) is the workpiece material velocity parallel to AB and L is the length of AB):
Assign values of C.

III- Start of loop to calculate the shear zone angle \( \phi \):
Assign values of \( \phi \).

1- Calculate the following:

\[
L = \frac{t_i}{\sin \phi} \quad (A.1)
\]

Where L is the length of AB.

\[
V_s = \frac{u \cos \alpha}{\cos(\phi - \alpha)} \quad (A.2)
\]

Where \( V_s \) is the workpiece material velocity parallel to AB.

\[
\varepsilon_{AB} = \frac{1}{\sqrt{3}} \left[ C \frac{V_s}{L} \right] \quad (A.3)
\]

Where \( \varepsilon_{AB} \) is the strain rate at plane AB.

\[
\varepsilon_{AB} = \frac{1}{\sqrt{3}} \left[ \frac{\cos \alpha}{2 \sin \phi \cos(\phi - \alpha)} \right] \quad (A.4)
\]

Where \( \varepsilon_{AB} \) is the strain at plane AB.

2- Loop to calculate the temperature at plane AB:
Assume \( T_{AB} = T_W \), where \( T_{AB} \) is the temperature along AB

\[
k_{AB} = func\left( \varepsilon_{AB}, \varepsilon_{AB}', T_{AB} \right) \quad (A.5)
\]

Where \( k_{AB} \) is the shear flow stress of the material at plane AB.

\[
n = func\left( \varepsilon_{AB}, \varepsilon_{AB}', T_{AB} \right) \quad (A.6)
\]

Where \( n \) is the strain hardening exponent of the workpiece material at plane AB.

\[
S_{AB} = func(T_{AB}) \quad (A.7)
\]

Where \( S_{AB} \) is the specific heat of the workpiece at plane AB.

\[
K_{AB} = func(T_{AB}, \text{material content}) \quad (A.8)
\]
Where \( K_{AB} \) is the thermal conductivity of the workpiece at plane AB.

\[
F_s = k_{AB} \times L \times W \tag{A.9}
\]

Where \( F_s \) is the shear force at plane AB.

\[
\tau_{1,AB} = \frac{\rho S_{AB} \mu t_i}{K_{AB}} \tag{A.10}
\]

Where \( \tau_{1,AB} \) is a non-dimensional thermal number required to calculate the proportion of heat conducted into the workpiece at the primary deformation zone.

\[
\beta = \text{function (} \tau_i \text{)} \tag{A.11}
\]

Where \( \beta \) is the proportion of heat conducted into the workpiece.

\[
\Delta T_{SZ} = \frac{(1 - \beta) F_s \cos \alpha}{\rho S_{AB} t_i W \cos(\phi - \alpha)} \tag{A.12}
\]

Where \( \Delta T_{SZ} \) is the temperature rise in the chip formation zone.

Calculate new \( T_{AB} \) from the following relation:

\[
T_{AB} = T_W + \eta \Delta T_{SZ} \tag{A.13}
\]

Where \( 0 < \eta \leq 1 \), \( \eta \) is a factor which allows for the fact that not all of the plastic work of the chip formation has occurred at plane AB (\( \eta = 0.7 \) [Oxley; 1989]).

Repeat step 2 until \( T_{AB} \) converges.

3- Calculate the following:

\[
\theta = \tan^{-1} [1 + 2 \left( \frac{\pi}{4} - \phi \right) - Cn] \tag{A.14}
\]

Where \( \theta \) is the angle made by the resultant cutting force \( R \) with the shear plane AB.

\[
\lambda = \theta + \alpha - \phi \tag{A.15}
\]

Where \( \lambda \) is the mean friction angle at the interface.

\[
R = \frac{F_s}{\cos \theta} \tag{A.16}
\]

Where \( R \) is the resultant cutting force.
\[ F = R \sin \lambda \]  \hspace{1cm} (A.17)

Where \( F \) is the friction force at the interface.

\[ N = R \cos \lambda \]  \hspace{1cm} (A.18)

Where \( N \) is the normal force at the interface.

\[ F_c = R \cos (\lambda - \alpha) \]  \hspace{1cm} (A.19)

Where \( F_c \) is the cutting force (force in the cutting direction).

\[ F_T = R \sin (\lambda - \alpha) \]  \hspace{1cm} (A.20)

Where \( F_T \) is the feed force (force in the feed direction).

\[ t_2 = \frac{t_1 \cos(\phi - \alpha)}{\sin \phi} \]  \hspace{1cm} (A.21)

Where \( t_2 \) is the cut chip thickness.

\[ V_{ch} = \frac{u \sin \phi}{\cos(\phi - \alpha)} \]  \hspace{1cm} (A.22)

Where \( V_{ch} \) is the rigid chip velocity.

\[ h = \frac{t_1 \sin \theta}{\cos \lambda \sin \phi} \left\{ 1 + \frac{Cn}{3[1 + 2(\frac{\pi}{4} - \phi) - Cn]} \right\} \]  \hspace{1cm} (A.23)

Where \( h \) is the tool-chip interface length.

\[ \tau_{int} = \frac{F}{hW} \]  \hspace{1cm} (A.24)

Where \( \tau_{int} \) is the shear stress at the interface.

\[ \varepsilon_{int} = \frac{1}{\sqrt{3}} \left[ \frac{V_{ch}}{\delta t_2} \right] \]  \hspace{1cm} (A.25)

Where \( \varepsilon_{int} \) is the strain rate at the interface.

**4- Loop to calculate the mean chip temperature:**

assume \( T_C = T_W + \Delta T_{SZ} \), where \( T_C \) is the mean chip temperature

\[ S_C = \text{func} (T_C) \]  \hspace{1cm} (A.26)

Where \( S_C \) is the specific heat of the chip.

\[ K_C = \text{func} (T_C, \text{material content}) \]  \hspace{1cm} (A.27)
Where $K_C$ is the thermal conductivity of the chip.

$$\Delta T_C = \frac{F \sin \phi}{\rho S_C t_1 W \cos(\phi - \alpha)}$$ \hspace{2cm} (A.28)

Where $\Delta T_C$ is the average temperature rise in the chip.

Calculate new $T_C = T_W + \Delta T_{SZ} + \Delta T_C$ \hspace{2cm} (A.29)

Repeat step 4 until $T_C$ converges.

5- Calculate the following for the tool-chip interface:

$$r_{t, \text{chip}} = \frac{\rho \ S_C \ u t_{\text{f}}}{K_C}$$ \hspace{2cm} (A.30)

Where $r_{t, \text{chip}}$ is a non-dimensional thermal number required to calculate $\Delta T_M$ which is the maximum temperature rise in the chip.

Calculate the maximum temperature rise in the chip $\Delta T_M$ using the following equation:

$$\lg \left( \frac{\Delta T_M}{\Delta T_C} \right) = 0.06 - 0.195 \delta \left( \frac{r_{t, \text{chip}} t_2}{h} \right)^{1/2} + 0.5 \lg \left( \frac{r_{t, \text{chip}} t_2}{h} \right)$$ \hspace{2cm} (A.31)

Calculate the average temperature at the tool-chip interface from the following equation:

$$T_{\text{int}} = T_W + \Delta T_{SZ} + \psi \Delta T_M$$ \hspace{2cm} (A.32)

$0 < \psi \leq 1$, where $\psi$ is a factor which allows for possible variation of temperature along the interface ($\psi = 0.7$ [Oxley; 1989]).

$$k_{\text{chip}} = \text{func}(\varepsilon_{\text{int}}, \dot{e}_{\text{int}}, T_{\text{int}})$$ \hspace{2cm} (A.33)

Where $k_{\text{chip}}$ is the shear flow stress of the workpiece material at the interface.

Compare $k_{\text{chip}}$ and $T_{\text{int}}$ for the assumed value of $\phi$.

repeat $\phi$-loop until $\phi$ converges when $k_{\text{chip}} = T_{\text{int}}$
Find the converged value of C at which the normal stress at B calculated from the following two equations is the same.

\[ \sigma'_{N,B} = k_{AB} \left[ 1 + \frac{\pi}{2} - 2 \alpha - 2 Cn \right] \quad (A.34) \]

Where \( \sigma'_{N,B} \) is the normal stress at point B calculated from the slipline field analysis.

\[ \sigma_{N,B} = \frac{N}{hW} \quad (A.35) \]

Where \( \sigma_{N,B} \) is the normal stress at point B assuming uniform normal stress at the interface.

Repeat C-loop until C converges when \( \sigma'_{N,B} = \sigma_{N,B} \)

Repeat \( \delta \)-loop until \( \delta \) corresponding to the minimum cutting force \( F_c \) is obtained.

End.
APPENDIX B

APPLICATION OF OXLEY'S THEORY TO OBLIQUE AND 3-D CUTTING OPERATIONS

B.1 Oblique Cutting

In oblique cutting, the tool cutting edge is not perpendicular to the direction of the cutting velocity as shown in Figure B.1. The angle between the tool cutting edge and the normal to the velocity direction is called the inclination angle (i). The tool rake angle in oblique cutting, called the normal rake angle (\(\alpha_n\)), is the angle measured in a plane normal to the oblique cutting edge between a plane normal to the cutting velocity and the rake face plane. According to Stabler's flow rule [Stabler; 1951]

\[
\eta_r = i
\]  \hspace{1cm} (B.1)

where \(\eta_r\) is the chip flow angle measured in the rake face plane between the chip flow direction and the normal to the cutting edge as shown in Figure B.1.

Lin et al. [Lin; 1982] utilized Oxley's theory to predict cutting forces for oblique machining conditions. Based on their experimental observation that the cutting and feed forces are not influenced by the inclination angle, they used the normal rake angle (\(\alpha_n\)) to predict cutting and feed forces (\(F_C\), \(F_f\)). Then, using Stabler's flow rule, in conjunction with the fact that the resultant cutting force must lie in a plane normal to the tool cutting face and containing the resultant frictional force acting in the chip flow direction, they obtained an expression,
Figure B.1: Schematic diagram showing oblique chip formation model [Oxley; 1989]
using 3-D vector analysis, for the third component of cutting forces \( F_R \) in the radial direction shown in Figure B.1 as:

\[
F_R = \frac{F_C (\sin i - \cos i \sin \alpha_n \tan \eta_c) - F_T \cos \alpha_n \tan \eta_c}{\sin i \sin \alpha_n \tan \eta_c + \cos i} \tag{B.2}
\]

Lin et al. [Lin; 1982] conducted oblique turning experiments of tubes, shown in Figure B.2, with the side cutting edge of the tool \( (C_s) \) equal zero and not equal zero. In the case of oblique turning with a zero side cutting edge, shown in Figure B.2.a, the cutting and feed forces \( F_C \) and \( F_T \) are acting in the cutting direction (normal to the plane of the page) and feed direction (parallel to the axis of rotation of the tube), respectively. The radial force \( F_R \) is acting in the radial direction of the tube normal to the plane containing the cutting and feed forces. In the case of Figure B.2.b, where oblique cutting is performed using a tool with a non-zero side cutting edge, the cutting, feed and radial forces in this case are not acting in the cutting, feed and radial directions. Moreover, the uncut chip thickness does not equal to the feed and the width of cut does not equal to the thickness of the tube. To obtain the cutting forces, the uncut chip thickness \( (t_1) \) and the width of cut \( (W) \) should be calculated as follows [Lin; 1982]:

\[
t_1 = f \cos (C_s) \tag{B.3}
\]

\[
W = t / \cos (C_s) \tag{B.4}
\]

where \( f \) is the feed (mm/rev) and \( t \) is the tube wall thickness. The values of \( t_1 \) and \( W \), calculated from equations B.3 and B.4, in conjunction with the normal rake angle of the tool \( (\alpha_n) \) and the cutting velocity should be used in Oxley’s theory, summarized in Appendix A, to predict the cutting and feed forces \( F_C \) and \( F_T \). Then, \( F_C \) and \( F_T \), \( \alpha_n \) and the inclination angle \( (i) \) should be used in equations B.1 and B.2 to calculate the radial force \( F_R \). Then, \( F_C, F_T, F_R \) and the side cutting edge angle \( (C_s) \) should be used in the following equations to calculate the forces \( P_1, P_2 \) and \( P_3 \) in the cutting, feed and radial directions respectively [Lin; 1982]:

\[
P_1 = F_C \tag{B.5.a}
\]

\[
P_2 = F_T \cos C_s + F_R \sin C_s \tag{B.5.b}
\]
Figure B.2: Oblique turning of a tube
\[ P_3 = F_T \sin C_s - F_R \cos C_s \]  \hfill (B.5.c)

Lin et al [Lin; 1982] found that the predicted cutting forces match their experimental results.

## B.2 Application of Oxley’s Theory in 3-D Turning Operations

Schematic diagrams of the 3-D turning process using an oblique nose radius tool with two cutting edges are shown in Figures 3.17 and 3.18. The X axis is the axis parallel to the cutting direction and it is called, in the present work, the vertical axis. The Y axis is the axis of the workpiece and it is parallel to the feed direction. The Z axis is the axis of the tool and it is parallel to the direction of the depth of cut. The plane defined by the Y and Z axes is called the horizontal plane. The cutting face plane is the plane defined by the side and end cutting edges. The radius between the side and end cutting edges, measured in the cutting face plane, is called the nose radius.

There are many angles that describe the geometry of the tool for a 3-D turning process as shown in Figure 3.18. The inclination angle \((i)\) is the angle, measured in a vertical plane passing through the side cutting edge, between the side cutting edge and the horizontal YZ plane. The angle \(C_s\) which is called the side cutting edge angle is defined as the angle, measured in the horizontal YZ plane, between a vertical plane passing through the side cutting edge and a vertical plane passing through the Z axis (Z plane). The end cutting edge angle \(C_e\) is the angle, measured in the horizontal plane (YZ plane), between the vertical plane passing through the Y axis (Y plane) and a vertical plane passing through the side cutting edge. The normal rake angle \(\alpha_n\) is the angle, measured in a plane perpendicular to the side cutting edge, between the cutting face plane and the horizontal YZ plane.
B.2.1 Bar Turning Using Oblique Zero-Nose radius Tool

Hu et al. [Hu; 1986] applied Oxley’s theory to predict cutting forces in 3-D bar turning using oblique sharp nose tools (nose radius = 0) where cutting occurs at two cutting edges (side or main cutting edge and end cutting edge). They did that via the concept of the equivalent cutting edge which combines both the side and end cutting edges. They utilized 3-D vector analysis to obtain the equivalent oblique cutting edge that can be used in the oblique analysis of Lin et al. [Lin; 1982] to obtain the cutting forces. They did their analysis for a tool with a zero nose radius which is not practical, however, their work was further developed as will be discussed later.

B.2.2 Bar Turning Using Non-Oblique Nose Radius Tools

Young et al. [Young; 1987] developed a model for predicting the chip flow direction and cutting forces for bar turning using non-oblique nose radius tools. As shown in Figure B.3, the chip is treated as a series of elements of infinitesimal width [Young; 1987]. The thickness and orientation of the undeformed chip section, corresponding to each chip element, vary. Therefore, the frictional force components change in magnitude as well as in direction. Those frictional force components are summed in order to find the resultant and it is assumed that this resultant coincides with the chip flow direction. The chip flow angle due to nose radius effect, $\overline{\Omega}$, measured from the positive Y axis can be determined from the relation:

$$\overline{\Omega} = \tan^{-1} \left( \frac{\int \sin \Omega \, dA}{\int \cos \Omega \, dA} \right)$$

(B.6)

where dA is the area of the uncut chip element shown in Figure B.3. It is assumed that the magnitude of the elemental friction force varies linearly with the local uncut chip thickness. This assumption is based on the experimental orthogonal cutting results published by Armarego and Brown [Armarego; 1964].
Figure B.3: Resolution of differential friction force at nose radius edge into components in a set of reference directions at right angles (the X and Y axes lie in the cutting face plane) [Oxley; 1989].
By integrating the numerator and denominator of equation B.6 over the entire area of uncut chip section, the chip flow angle $\Omega$ can be determined as follows [Young; 1987]:

1) Case 1 (Figure B.4):

If $r_n^*(1-\sin C_3) \geq d$ then:

$$NUM = [-r_n \sin \theta \theta_{01}^0 + \frac{1}{2} \sin \theta (r_n^2 - f^2 \sin^2 \theta)^{1/2}$$

$$+ \frac{r_n^2}{f} \sin^{-1}(\frac{f}{r_n} \sin \theta)]_{\theta_0}^0_0 + f [\frac{\sin 2\theta}{4} + \frac{\theta}{2}]_{\theta_0}^0_0 +$$

$$[(r_n - d) \ln(\sin \theta)]_{\theta_0}^0_0$$

(B.7)

$$DEN = [-r_n \cos \theta \theta_{01}^0 + \frac{1}{2} \cos \theta (r_n^2 - f^2 \sin^2 \theta)^{1/2} +$$

$$\frac{r_n^2 - f^2}{f} \ln\{f \cos \theta + (r_n^2 - f^2 \sin^2 \theta)^{1/2}\}_{\theta_0}^0_0 +$$

$$\frac{f}{4} [\cos(2\theta)]_{\theta_0}^0_0 + [-(r_n - d)\theta]_{\theta_0}^0_0$$

(B.8)

where the limits of integration are:

$$\theta_1 = \cos^{-1}(\frac{f}{2r_n})$$

(B.9)

$$\theta_2 = \pi - \tan^{-1}\left\{\frac{r_n - d}{(2r_n d - d^2)^{1/2} - f}\right\}$$

(B.10)

$$\theta_3 = \pi - \sin^{-1}(\frac{r_n - d}{r_n})$$

(B.11)
Figure B.4: Geometry of the chip flow model for nose radius tools when \( r_n(1-\sin C_s) \geq d \) [Oxley; 1989].
2) Case 2 (Figure B.5):

If \( d > r_n^*(1 - \sin C_s) \) then:

\[
NUM = \left[ -r_n^2 \sin \theta \right]_{\theta_1}^{\theta_2} + \frac{r_n^2}{2} \left[ \sin \theta \left( r_n^2 - f^2 \sin^2 \theta \right)^{1/2} + \frac{r_n^2 \sin^{-1}\left( \frac{f}{r_n} \sin \theta \right)}{2} \right]_{\theta_1}^{\theta_2} + r_n f \left[ \frac{\sin 2\theta}{4} + \frac{\theta}{2} \right]_{\theta_1}^{\theta_2} + f \left[ d - r_n (1 - \sin C_s) \right] - \frac{f^2}{4} \sin(2C_s) \cos C_s
\]  

(B.12)

\[
DEN = \left[ -r_n^2 \cos \theta \right]_{\theta_1}^{\theta_2} + \frac{r_n^2}{2} \left[ \cos \theta \left( r_n^2 - f^2 \sin^2 \theta \right)^{1/2} + \frac{r_n^2}{f} \ln\left\{ \left( \frac{f}{r_n} \cos \theta \right) + \left( r_n^2 - f^2 \sin^2 \theta \right)^{1/2} \right\} \right]_{\theta_1}^{\theta_2} + \frac{r_n f}{4} \left[ \cos(2\theta) \right]_{\theta_1}^{\theta_2} + f \left[ d - r_n (1 - \sin C_s) \right] - \frac{f^2}{4} \sin(2C_s) \sin C_s
\]  

(B.13)

where the limits of integration are

\[
\theta_1 = \cos^{-1}\left( \frac{f}{2r_n} \right)
\]  

(B.14)

\[
\theta_2 = \pi - C_s
\]  

(B.15)

The chip flow angle due to the effect of the nose radius can be expressed as

\[
\Omega = \tan^{-1}\left( \frac{NUM}{DEN} \right)
\]  

(B.16)

As shown in Figure B.5, \( \Omega \) is the angle between the chip flow direction and the Y-axis. The angle \( \eta_c \) between the chip flow direction and the normal to the side cutting edge can be found from the relation

\[
\eta_c = \frac{\pi}{2} - C_s - \Omega
\]  

(B.17)
Figure B.5: Geometry of the chip flow model for nose radius tools when $r_n^*(1-\sin C_s) < d$ [Oxley; 1989].
B.2.3 Bar Turning Using Oblique Nose-Radius Tools

Arsenio et al. [Arsenio et al.; 1995] used Oxley's theory to predict chip flow direction and cutting forces in oblique turning with nose radius tools (Figure B.6). Basically, they obtained the equivalent cutting edge for a corresponding non-oblique nose radius tool using the technique developed by Young et al. [Young; 1987]. Then they projected this equivalent cutting edge into the cutting face plane of the oblique nose radius tool using three dimensional geometric analysis.

To obtain the equivalent cutting edge of an oblique nose radius tool, the chip flow angle due to nose radius ($\eta_c$) should be obtained using equation B.17 [Arsenio et al.; 1995]. Then, this angle is projected into the tool rake face plane of the oblique nose radius tool as shown in Figure B.6. The projection of the chip flow angle in the tool rake face plane ($\eta^*_c$) can be found from the relation obtained using 3-D vector analysis [Arsenio et al.; 1995]:

$$\eta_c^* = \cos^{-1} \left[ \frac{\sec i - \tan i \tan \eta_c \tan \alpha_n}{\left( \tan i - \tan \eta_c \tan \alpha_n \sec i \right)^2 + \sec^2 \eta_c} \right]$$  \hspace{1cm} (B.18)

where $\eta^*_c$ is defined as the angle between the normal to the equivalent cutting edge and the chip flow direction in the rake face plane. Again, using three dimensional geometric analysis, the equations of the rake angle, inclination angle and side cutting edge angle for the equivalent cutting edge are, respectively [Arsenio et al.; 1995]:

$$i^* = \sin^{-1} (\cos \eta_c^* \sin i - \sin \eta_c^* \sin \alpha_n \cos i)$$  \hspace{1cm} (B.19)

$$\alpha_n^* = \sin^{-1} \left( \frac{\sec \eta_c^* \sin i - \sin i^*}{\tan \eta_c \cos i} \right)$$  \hspace{1cm} (B.20)

$$C^* = C_s + \eta_c$$  \hspace{1cm} (B.21)
These angles are shown in Figure B.6. Thus, a tool with a nose radius \( r_n \) and side cutting edge angle \( C_s \), inclination angle \( i \) and rake angle \( \alpha_n \) can be replaced by a tool having a single straight cutting edge, that is the equivalent

![Diagram of equivalent cutting edge and tool angles](image)

Figure B.6: Schematic diagram of the equivalent cutting edge and tool angles for oblique nose radius tools [Arsecularante; 1995].
cutting edge, with a side cutting edge angle $C_s^*$, inclination angle $i^*$, and a rake angle $\alpha_n^*$. The cutting forces of the oblique cutting edge, defined by those angles, can be predicted using the oblique cutting analysis of Lin et al. [Lin; 1982] described above.

B.3 Analysis of the Ball End Milling Process Using Oxley’s Theory

The idea is to divide the 3-D geometry of the ball end mill into oblique cutting elements and perform the analysis for each oblique element using the technique of Lin et al. [Lin; 1982] explained in section B.1 of Appendix B. If the ball end mill is divided into elements, then for each element at an axial location $z$, Figure 3.30, the lag angle in global coordinates, $\Psi_j(z)$, measured from the $+Y$ axis clockwise, between the point representing the element on flute $j$ and the tip of the mill at $z = 0$, is calculated from the relation [Lee; 1996]:

$$\Psi_j(z) = \theta_r + (j - 1)\phi_p - \frac{z}{R_0}\tan(i_o) \quad (B.22)$$

where $\phi_p = 2\pi/N_f$ is the pitch angle of the cutter. $\Psi_j(z)$ is used to determine whether this part of the flute is in cut or out of cut. If inlet angle of the cutter $< \Psi_j(z) <$ exit angle of the cutter, then this part of the flute is in cut, otherwise, it is out of cut. The inlet and exit angles of the cutter, shown in Figure 3.10, are determined based on the type of operation. For example, in slot cutting, the inlet angle is zero and the exit angle is $\pi$.

If the part of the flute is in cut, then the required cutting parameters are calculated as follows [Lee; 1996]:

$$\Psi_b(z) = \frac{z\tan(i_o)}{R_0} \quad (B.23)$$

$$R_b(\psi_b) = R_0\sqrt{1 - (\psi_b\cot i_o - 1)^2} \quad (B.24)$$
\[ \kappa(\psi_b) = \sin^{-1} \frac{R_b(\psi_b)}{R_0} \]  
(B.25)

\[ W(\psi_b) = \frac{dz}{\sin(\kappa)} \]  
(B.26)

\[ t_i(\psi_b) = f \sin(\Psi) \sin(\kappa) \]  
(B.27)

\[ i(\psi_b) = \frac{R_b(\psi_b) \tan(i_0)}{R_0} \]  
(B.28)

\[ u(\psi_b) = \frac{2\pi N}{60} R_b(\psi_b) \]  
(B.29)

For each element, the width of cut \( W \), the uncut chip thickness \( t_i \), the inclination angle \( i \), and the cutting velocity \( u \), calculated from equations B.26, B.27, B.28, and B.29, respectively, in addition to the given normal rake angle \( \alpha_n \) are the parameters required to perform the oblique cutting analysis. Using Oxley's theory and the methodology to analyze oblique cutting, developed by Lin et al. [Lin; 1982], the differential cutting forces in the cutting \( df_c \), radial \( df_r \), and feed \( df_f \) directions for each element, shown in Figure 3.30, can be calculated. Average temperatures and stresses for each element along the mill can also be calculated. The predicted cutting forces in local coordinates for each element are transformed in global coordinates using the relations [Lee; 1996]:

\[ dF_x = -\sin(\kappa) \sin(\Psi) df_r - \cos(\Psi) df_c - \cos(\kappa) \sin(\Psi) df_f, \]  
(B.30)

\[ dF_y = -\sin(\kappa) \cos(\Psi) df_r + \sin(\Psi) df_c - \cos(\kappa) \cos(\Psi) df_f, \]  
(B.31)

\[ dF_z = \cos(\kappa) df_r + \sin(\kappa) df_f, \]  
(B.32)

The total cutting forces in the global \( X \), \( Y \) and \( Z \) directions are calculated by summing the differential cutting forces for each element \( (dF_x, dF_Y, dF_Z) \).
B.4 Analysis of the Twist Drilling Process Using Oxley's Theory

The idea is to divide the 3-D geometry of the twist drill, shown in Figure 3.26, into oblique cutting elements and perform the analysis for each element using Oxley's theory and the technique of Lin et al. [Lin; 1982], similar to the analysis of ball end milling. In the present work, the analysis is limited to the lip region because convergence problems occurred when Oxley's theory (OXCUT) was applied to conduct the analysis for the elements in the chisel edge region. This might be due to the high negative rake angles and the low cutting velocities at the chisel edge elements [Armarego; 1969]. The shear angle $\phi$, shown in Figure 2.3 and iterated in the Algorithm of Oxley's theory of Appendix A, decreases with decreasing the cutting velocity and with decreasing the rake angle. Thus, the shear angle gets very small for elements on the drill lip near the chisel edge. As the element approaches the chisel edge more and more, OXCUT can not find the shear angle that causes the solution to converge.

The geometry of the twist drill, shown in Figure 3.26, can be described as follows [Wiriyacosol; 1979][Armarego; 1969]:

$$\omega_0 = \sin^{-1}\left(\frac{2W_b}{D}\right)$$  \hspace{1cm} (B.33)

where $\omega_0$ is the drill web angle at the outer surface of the drill, $D$ is the nominal drill diameter, and $W_b$ is the half web thickness at the drill point.

$$\omega' = \pi - \eta_d$$  \hspace{1cm} (B.34)

where $\omega'$ is the drill web angle at the chisel edge corner and $\eta_d$ is the drill specified chisel edge angle.

$$D' = \frac{2W_b}{\sin(\pi - \eta_d)}$$  \hspace{1cm} (B.35)

where $D'$ is the chisel edge diameter.

$$W = \frac{D \cos \omega_0 - D' \cos \omega'}{2MI \cdot \sin p_a}$$  \hspace{1cm} (B.36)
where \( W \) is the width of cut of all elements on the lip, \( M_l \) is the selected number of elements on the lip, and \( \rho_a \) is the specified drill half point angle.

\[
r_e = \sqrt{\frac{D \cos w_e}{2} - (e - 0.5)W^2 + W^2}
\]

(B.37)

where \( r_e \) is the distance from the drill center line to the midpoint of the cutting edge of an element \( e \) where \( e = 1 \) to \( M_l \)

Then, the equations of the geometrical quantities of the \( e^{th} \) element on the lip, \( (e = 1 \) to \( M_l) \), required for OXCUT calculations are as follows:

\[
w_e = \sin^{-1} \left( \frac{W_{b_e}}{r_e} \right)
\]

(B.38)

where \( w_e \) is the local web angle for an element \( e \) on the drill lip

\[
\delta_e = \tan^{-1} \left( \frac{2r_e \tan(\delta_0)}{D} \right)
\]

(B.39)

where \( \delta_e \) is the helix angle for an element \( e \) on the lip and \( \delta_0 \) is the helix angle at the outer surface of the drill.

\[
\zeta_e = \tan^{-1} (\tan w_e \cos \rho_a)
\]

(B.40)

where \( \zeta_e \) is the reference angle for an element \( e \) on the lip.

\[
i_e = \sin^{-1} (\sin \rho_a \sin w_e)
\]

(B.41)

where \( i_e \) is the inclination angle for an element \( e \) on the lip.

\[
\alpha_{ref,e} = \tan^{-1} \left( \frac{\tan \delta_e \cos w_e}{\sin \rho_a - \cos \rho_a \sin w_e \tan \delta_e} \right)
\]

(B.42)

where \( \alpha_{ref,e} \) is the reference rake angle for an element \( e \) on the drill lip

\[
\alpha_{n,e} = \alpha_{ref,e} - \zeta_e
\]

(B.43)

where \( \alpha_{n,e} \) is the normal rake angle for an element \( e \) on the drill lip

\[
t_{1e} = \frac{f \sin \rho_a \cos \zeta_e}{2}
\]

(B.44)

where \( t_{1e} \) is the uncut chip thickness for element \( e \) on the drill lip
\[ u_c = \frac{2\pi N}{60} r_e \]  \hspace{1cm} (B.45)

where \( u_c \) is the cutting velocity for an element \( e \) on the drill lip.

\[ \tan Cl_{nc} = \frac{\cos p_a [\cos w_0 \tan w_c - (\tan w_0 - \tan Cl_0 \tan p_a)]}{\cos w_0 + \tan w_c \cos^2 p_a [\tan w_0 - \tan Cl_0 \tan p_a]} \]  \hspace{1cm} (B.46)

where \( Cl_{nc} \) is the normal clearance angle of an element \( e \) on the twist drill lip and \( Cl_0 \) is the nominal clearance angle at the periphery [Armarego; 1969].

For each element \( e \) on the twist drill lip, \( e = 1 \) to \( M \), the width of cut \( W \) (equation B.36), the normal rake angle \( \alpha_{n,e} \) (equation B.43), the uncut chip thickness \( t_{1e} \) (equation B.44) and the cutting velocity \( u_c \) (equation B.45), are used to calculate the temperature and effective stress on the rake and flank faces of the drill lip. Similar analysis could be performed for the chisel edge, however, at the chisel edge, convergence problems occurred due to the high negative rake angles as mentioned above. Temperatures are expected to be higher for elements on the drill lip than on the drill chisel edge. This is because the elements on the drill lip are at larger distances from the center line of the drill than elements on the chisel edge causing the cutting velocities and consequently the temperatures to be higher on the drill lip than the chisel edge.
APPENDIX C

DETERMINATION OF FLOW STRESS DATA FOR PRACTICAL CUTTING OPERATIONS

Both FEM and analytical modeling require flow stress data, as function of strain, strain rate and temperature of the workpiece material, to analyze the metal cutting process \( (\bar{\sigma} = f(\varepsilon, \dot{\varepsilon}, T )) \). Thus, many researchers tried to obtain flow stress data at the high strains (1 and over), strain rates \( (10^3 \text{ to } 10^5 \text{ s}^{-1} \) or more) and temperatures \((200 \text{ to } 1000 \text{ °C} \) or more) encountered in metal cutting operations.

C.1 High Speed Compression Tests

Some researchers used high speed compression tests to obtain flow stress data of carbon steels at the high strains, strain rates and temperatures encountered in metal cutting processes [Oyane; 1967]. The disadvantage of that technique is that the flow stress data are obtained at strain rates below \(450 \text{ s}^{-1} \) and strains below 1. The obtained flow stress equation was expressed as

\[
\bar{\sigma} = \sigma_1 \varepsilon^n
\]

(C.1)

where \(\bar{\sigma}\) is the flow stress of the material and \(\varepsilon\) is the strain. \(\sigma_1\) and \(n\) are parameters that vary with the strain rate and temperature. The strain rate \((\dot{\varepsilon})\) and
temperature \(T\) were combined in a single parameter called the velocity modified temperature \(T_{\text{mod}}\), defined as

\[
T_{\text{mod}} = T \left(1 - v \log \frac{\dot{e}}{\dot{e}_0} \right)
\]

(C.2)

where \(v\) and \(\dot{e}_0\) are constants which have the values of 0.09 and 1 \(\text{s}^{-1}\), respectively. The variations of \(\sigma_1\) and \(n\) with \(T_{\text{mod}}\) for 0.2% and 0.38% carbon steels, obtained using high speed compression tests, are shown in Figure C.1. The values of \(\sigma_1\) and \(n\) can be calculated for 0.16% carbon steel using the following equations [Oxley; 1989]:

**Calculations for \(\sigma_1\) (0.16% carbon steel):**

For \(458 \geq T_{\text{mod}}\)

\[
\sigma_1 = 1126.62 - 0.98421 \ T_{\text{mod}}
\]

For \(748 \geq T_{\text{mod}} > 458\)

\[
\sigma_1 = 19914.15 + 135.07 \ T_{\text{mod}} - 0.20137 \ T_{\text{mod}}^2 - 3.1090E-4 \ T_{\text{mod}}^3 + 7.2551E-7 \ T_{\text{mod}}^4 + 7.3255E-10 \ T_{\text{mod}}^5 - 2.2977E-12 \ T_{\text{mod}}^6 + 1.2673E-15 \ T_{\text{mod}}^7
\]

For \(1200 \geq T_{\text{mod}} > 748\)

\[
\sigma_1 = 17756.97 - 97.198 \ T_{\text{mod}} + 0.23022 \ T_{\text{mod}}^2 - 2.4637E-4 \ T_{\text{mod}}^3 + 2.8921E-8 \ T_{\text{mod}}^4 + 1.8495E-10 \ T_{\text{mod}}^5 - 1.6072E-13 \ T_{\text{mod}}^6 + 4.2722E-17 \ T_{\text{mod}}^7
\]

For \(T_{\text{mod}} > 1200\)

\[
\sigma_1 = 172.42
\]

(C.3)

**Calculations for \(n\) (0.16% carbon steel):**

For \(73 \geq T_{\text{mod}}\)

\[n = 0.04768\]

For \(396 \geq T_{\text{mod}} > 73\)

\[
\begin{align*}
n &= 0.04937 + 3.5861E-4 \ T_{\text{mod}} - 1.4026E-5 \ T_{\text{mod}}^2 + 1.7680E-7 \ T_{\text{mod}}^3 - 9.4992E-10 \ T_{\text{mod}}^4 + 2.7341E-12 \ T_{\text{mod}}^5 - 4.1361E-15 \ T_{\text{mod}}^6 + 2.5569E-18 \ T_{\text{mod}}^7 \\
&\quad + \frac{1}{264}
\end{align*}
\]
Figure C.1: Flow stress and strain-hardening index obtained for 0.2% (solid line) and 0.38% (dashed line) carbon steels from high speed compression tests (strain rate = 450 sec⁻¹) [Oxley; 1989]
For $T_{mod} > 396$

\[
n = 0.19109
\]

For $T_{mod} > 528$

\[
n = -145.26 + 0.81927 \cdot T_{mod} - 0.88538 \cdot 10^{-3} \cdot T_{mod}^2 - 2.5350 \cdot 10^{-6} \cdot T_{mod}^3 + 5.0364 \cdot 10^{-9} \cdot T_{mod}^4 + 2.4501 \cdot 10^{-12} \cdot T_{mod}^5 - 1.04279 \cdot 10^{-14} \cdot T_{mod}^6 + 5.8410 \cdot 10^{-18} \cdot T_{mod}^7
\]

For $T_{mod} > 693$

\[
n = -21.227 + 0.08507 \cdot T_{mod} - 4.4837 \cdot 10^{-5} \cdot T_{mod}^2 - 1.3310 \cdot 10^{-7} \cdot T_{mod}^3 - 3.5910 \cdot 10^{-11} \cdot T_{mod}^4 + 5.1253 \cdot 10^{-13} \cdot T_{mod}^5 - 5.1724 \cdot 10^{-16} \cdot T_{mod}^6 + 1.5471 \cdot 10^{-19} \cdot T_{mod}^7
\]

For $T_{mod} > 827$

\[
n = -65.632 + 0.30193 \cdot T_{mod} - 0.49548 \cdot 10^{-3} \cdot T_{mod}^2 + 2.7300 \cdot 10^{-7} \cdot T_{mod}^3 + 9.1267 \cdot 10^{-11} \cdot T_{mod}^4 - 1.0362 \cdot 10^{-13} \cdot T_{mod}^5 - 3.1959 \cdot 10^{-17} \cdot T_{mod}^6 + 3.0674 \cdot 10^{-20} \cdot T_{mod}^7
\]

For $T_{mod} > 974$

\[
n = 0.18388 \quad \text{(C.4)}
\]

To obtain the values of $\sigma_1$ and $n$ for a carbon steel of a carbon content other than 0.16%, the following procedure can be used [Oxley; 1989]:

To calculate $n$ for carbon steels of carbon content other than 0.16%:

\[
n = n_b \cdot \text{NFAC} \quad \text{(C.5)}
\]

Where $n_b$ is the value of $n$ for carbon steel of 0.16% carbon content and NFAC is a rescaling factor that can be multiplied by $n_b$ to give the value of $n$ at the required carbon content. NFAC is calculated from the relation:

\[
\text{NFAC} = (0.244 - 0.3396 \cdot C)/0.189 \quad \text{(C.6)}
\]

Where $C$ is the percentage of carbon content which is different from 0.16%.
To calculate $\sigma_1$ for steels of carbon content other than 0.16%:

For $700 \geq T_{\text{mod}}$

$$\sigma_1 = \sigma_{1b} \left[1+(\text{SIGFAC}-1)(1400 - T_{\text{mod}})/900\right]$$

For $T_{\text{mod}} \geq 1100$

$$\sigma_1 = \sigma_{1b} \left[1+C^*(T_{\text{mod}} - 700) / T_{\text{mod}}\right]$$  \hspace{1cm} (C.7)

Where $\sigma_{1b}$ is the value of $\sigma_1$ for a carbon content of 0.16% and C is the carbon content of the steel that is different from 0.16%. SIGFAC is a rescaling factor that can be calculated from the relation:

$$\text{SIGFAC} = \frac{(531.31 + 753.17 \times C)}{651.72}$$  \hspace{1cm} (C.8)

C.2 High Speed Impact Compression Test (Hopkinson’s Bar Technique):

Because of the limited value of the maximum strain rate that can be attained using the high speed compression tests, many researchers used the Hopkinson’s bar technique to obtain flow stress data at the high strains, strain rates and temperatures encountered in metal cutting. The Hopkinson’s bar apparatus, shown schematically in Figure C.2, consists of one input bar and one output bar between which the tiny specimen (about 6 mm in diameter and 10 mm in length) is located [Shirakashi; 1983][Lee; 1998]. The specimen is quickly heated by a high frequency induction coil and pressed between the input and output bars. An impact or striker bar, propelled by an air pressure system acting as a gun to fire the striker bar, is projected to hit the input bar and cause the specimen to, plastically, deform. The amount of strain is controlled by a stopping ring of a certain length that is located between the input and output bars.

When the striker bar impacts the input bar, a constant-amplitude input pulse is generated and propagates through the input bar to the specimen causing
Figure C.2: The Hopkinson's High Speed Impact Compression Test [Shirakashi; 1983]
the specimen to deform. The input pulse is modified by the specimen response and propagates through the specimen as an output pulse to the output bar while the input pulse is reflected back as a reflected pulse through the input bar. The passage of the pulses is detected by 2 electric-resistance strain gages one of which is mounted on the input bar and the other one is mounted on the output bar. The strain gage on the input bar records the input \( \varepsilon_i \) and reflected \( \varepsilon_r \) pulses that pass through the input bar and the strain gage on the output bar record the output pulse \( \varepsilon_o \) that passes through the output bar. Based on one-dimensional elastic wave propagation theory and on the records of the input, reflected and output pulses, the average compression strain \( \varepsilon \), strain rate \( \dot{\varepsilon} \) and stress \( \bar{\sigma} \) in the specimen can be obtained from [Lee; 1998]:

\[
\varepsilon = (2C_0/L_0) \int \varepsilon_r \, dt
\]

\[
\dot{\varepsilon} = 2C_0 \varepsilon_r / L_0 \\
\bar{\sigma} = E(A/A_0) \varepsilon_i
\]  

(C.9)

where \( C_0 \) is the longitudinal wave velocity in the split bar; \( L_0 \) is the effective gage length of the specimen; \( E \) is the Young's modulus of the split bar; and \( A \) and \( A_0 \) are the cross-sectional areas of the split bar and the specimen, respectively.

The temperature of the specimen during the test is measured, as shown in Figure C.2, using thermocouples that are spot welded at different locations in the specimen. When the specimen is pressed between the input and output bars before the impact, heat conduction takes place between the specimen and the bars. The impact bar is not projected until a quasi-stable temperature distribution in the specimen is achieved [Shirakashi; 1983].

Many researchers utilized the Hopkinson's bar technique to develop expressions of flow stress data for various materials as function of strain, strain rate and temperature as follows [Özel; 1998]:
• For low carbon (0.15%C) steel [Usui; 1982]:

\[
\sigma = \left[ 103.0 \exp(-1.42 \times 10^{-3} T) + 34.6 \exp\left(-2.39 \times 10^{-4} T - 1.84 \times 10^{-5} \left(T - 7670.0 + 23.5 \ln \left(\frac{\varepsilon}{1000}\right)\right)\right) \right] \\
\times \left(\frac{\varepsilon}{1000}\right)^{0.028} \left\{0.0079 + \int \exp(0.001147) \left(\frac{\varepsilon}{1000}\right)^{-0.0157} d\varepsilon\right\}^{0.21}
\]

(C.10)

• For 0.18% C mild steel [Maekawa; 1983]:

\[
\sigma = A(T)\left(\frac{\varepsilon}{1000}\right)^{0.0195} e^{0.21} \\
A(T) = 1394 \exp(-0.001187T) + 339 \exp\left[-0.0000184 \left(T - \left(943 + 23.5 \ln \left(\frac{\varepsilon}{1000}\right)\right)^2\right)\right]
\]

(C.11)

• For cold work mold steel (Cr-Mo steel) [Maekawa; 1996]:

\[
\sigma = A \left(10^{-3} \varepsilon\right)^{M} e^{k_{1}T} \left(10^{-3} \varepsilon\right)^{N_{1}} [ \int_{T_{1} = \varepsilon^{\alpha_{1} \gamma_{1}^{\beta_{1}}} \left(10^{-3} \varepsilon\right)^{-m_{1}^{N_{1}}} d\varepsilon ]^{N_{1}}
\]

where

\[
A(T) = 1.46 \exp(-0.00137) + 0.196 \exp\{-0.000015(T-400)^2\} - 0.0392 \exp\{-0.01(T-100)^2\}
\]

\[
N_{1}(T) = 0.162 \exp(-0.0017) + 0.092 \exp\{-0.0003(T-380)^2\}
\]

\[
M(T) = 0.047, k_{1} = 0.000065, m_{1} = 0.0039
\]

• For the titanium alloy Ti6Al4V [Obikawa; 1996]:

\[
\sigma = 2.28 \left(\frac{\varepsilon}{1000}\right)^{0.028} \exp(-0.001557T) \\
\times \left[\left(\frac{\varepsilon}{1000}\right)^{0.029} \exp(0.001757T) \times \int \left(\frac{\varepsilon}{1000}\right)^{0.029} \times \exp(-0.001757T)d\varepsilon + 0.12\right]^{0.5} + 0.239
\]

(C.13)
• For low carbon free cutting steel (LCFCS) [Childs; 1997]:

\[
\bar{\sigma} = A\left[\frac{\dot{\varepsilon}}{1000}\right]^M \varepsilon^{n_i}
\]

\[
A = 910e^{-0.0011T} + 120e^{-0.30004(T-280)^2} + 50e^{-0.90001(T-600)^2}
\]

\[
M = 0.018 + 0.000038T
\]

\[
n_i = 0.16e^{-0.0017T} + 0.09e^{-0.00003(T-370)^2}
\]

(C.14)

• For the titanium alloy Ti6Al4V [Lee; 1998]:

\[
\bar{\sigma} = (A + B \varepsilon^{n_i})(1 + C \ln \dot{\varepsilon}^\ast)(1 - T^\ast m)
\]

(C.15.a)

where \(\bar{\sigma}\) is the flow stress of the workpiece in MPa and \(\varepsilon\) is the equivalent plastic strain. \(\dot{\varepsilon}^\ast = \frac{\dot{\varepsilon}}{\varepsilon_0}\) is the ratio of the test strain-rate to the reference strain rate \((\varepsilon_0)\), taken as \(10^{-5}\) s\(^{-1}\).

\[
T^\ast = \frac{T - T_{room}}{T_{melting} - T_{room}}
\]

(C.15.b)

where \(T^\ast\) is the homologous temperature, \(T\) is the workpiece temperature, \(T_{room} = 25^\circ\)C and \(T_{melting} = 1676.67^\circ\)C. A, B, n\(_i\), C, and m are material constants determined experimentally and have the values 782.7 MPa, 498.4 MPa, 0.28, 0.028 and 1.0, respectively.
APPENDIX D

OXLEY'S PARALLEL-SIDED SHEAR ZONE THEORY

Oxley's theory was developed based on a flow stress equation in the form of equations C.1 and C.2. In the present work, more general forms of flow stress equations, such as the form of equation C.15, are required to be used. There are two options: (1) to modify the constitutive equations of the theory or (2) to curve fit the equations in the form of equation C.15 to equation C1. The second option will be discussed in Appendix E. To be able to try the first option, some of the equations of Appendix A are required to be derived. This is the subject of this Appendix.

The stress equilibrium equations referred to the sliplines, shown in Figure D.1, when the x and y directions coincide with the sliplines directions ($\psi_s = 0$), are [Oxley; 1989]:

\[
\frac{\partial p}{\partial s_1} + 2k \frac{\partial \psi_s}{\partial s_1} - \frac{\partial k}{\partial s_2} = 0 \quad \text{along a I line} \quad (D.1.a)
\]

\[
\frac{\partial p}{\partial s_2} - 2k \frac{\partial \psi_s}{\partial s_2} - \frac{\partial k}{\partial s_1} = 0 \quad \text{along a II line} \quad (D.1.b)
\]

If the shear plane AB, shown in Figure D.2, is considered as a I slipline and the normal to it as a II slipline, then the variation of the hydrostatic stress (p) along AB can be expressed from equation D.1.a as:
Figure D.1: Curvilinear element bounded by sliplines showing stresses acting on sliplines [Oxley; 1989]
Figure D.2: Schematic diagram showing Oxley's thick shear zone model of orthogonal cutting
\[ dp = \frac{dk}{ds_2} ds_1 \]  

(D.2)

because \( \frac{\partial y}{\partial s_1} = 0 \) since AB is assumed to be a straight line. In equation D.2, the differential \( \frac{dk}{ds_2} \) can be replaced by the finite difference \( \frac{\Delta k}{\Delta s_2} \), where \( \Delta k = k_{EF} - k_{CD} \) is the difference between the shear stress along lines CD and EF that form the boundary of the shear zone. Thus applying equation D.2 along slipline AB [Oxley; 1989]:

\[ p_A - p_B = \frac{\Delta k}{\Delta s_2} L \]  

(D.3)

where \( p_A \) and \( p_B \) are the hydrostatic (mean compressive) stresses at A and B and \( L \) is the length of AB. The variation of \( p \) along AB is assumed to be linear with \( p_B < p_A \) and \( \frac{\Delta k}{\Delta s_2} > 0 \) as shown in Figure D.3 [Oxley; 1989]. The normal and shear forces acting on AB are given by:

\[ F_N = \frac{p_A + p_B}{2} LW \]  

(D.4)

\[ F_S = k_{AB} LW \]  

(D.5)

where \( W \) is the width of cut. Then, the angle \( \theta \) made between the directions of the resultant cutting force \( R \) and the shear plane AB, shown in Figure D.2, can be expressed, using equations D.3, D.4, and D.5 as:

\[ \tan \theta = \frac{F_N}{F_S} = \frac{p_A + p_B}{2k_{AB}} = \frac{p_A}{k_{AB}} - \frac{\Delta k}{2k_{AB} \Delta s_2} L \]  

(D.6)

From the slipline field analysis, Oxley and Welsh [Oxley; 1963] determined an expression for \( p_A \) as:

\[ \frac{p_A}{k_{AB}} = 1 + 2\left(\frac{\pi}{4} - \phi\right) \]  

(D.7)

Substituting \( \frac{p_A}{k_{AB}} \) from equation D.7 into equation D.6
Figure D.3: Schematic diagram showing the pressure distribution on the shear plane AB
\[
\tan \theta = 1 + 2\left(\frac{\pi}{4} - \phi\right) - \frac{\Delta k}{2k_{AB}} \frac{L}{\Delta s_2} \quad \text{(D.8)}
\]

The shear flow stress is usually a function of the shear strain \((\gamma_s)\), shear strain rate \((\dot{\gamma})\) and temperature \((T)\). Thus, an expression of \(\frac{d k}{d s_2}\) can be expressed as

[Oxley; 1989]:

\[
\frac{\partial k}{\partial s_2} = \frac{\partial k}{\partial \gamma_s} \frac{\partial \gamma_s}{\partial s_2} + \frac{\partial k}{\partial \dot{\gamma}} \frac{\partial \dot{\gamma}}{\partial s_2} + \frac{\partial k}{\partial T} \frac{\partial T}{\partial s_2} \quad \text{(D.9)}
\]

To determine \(\frac{d k}{d s_2}\) at plane AB from equation D.9, the second and third terms disappear because of the assumptions that the strain rate reaches a maximum at plane AB and the change in temperature with respect to time at plane AB is very small at steady state [Oxley, 1989]. Thus, an expression of \(\frac{d k}{d s_2}\) at plane AB is

[Oxley; 1989]:

\[
\frac{d k}{d s_2} = \frac{d k}{d \gamma_s} \frac{d \gamma_s}{d s_2} \quad \text{(D.10)}
\]

Using an expression of flow stress in the form \(\bar{\sigma} = \sigma_{,e}\), Oxley [Oxley; 1989] obtained an expression of \(\frac{d k}{d \gamma_s}\) as:

\[
\frac{d k}{d \gamma_s} = \frac{n k_{AB}}{\gamma_s} \quad \text{(D.11)}
\]

where \(k_{AB}\) and \(\gamma_{sAB}\) are the shear flow stress and strain, respectively, at plane AB. An expression of \(\gamma_{sAB}\) is as follows [Oxley; 1989]:

\[
\gamma_{sAB} = \frac{\cos \alpha}{2 \sin \phi \cos (\phi - \alpha)} \quad \text{(D.12)}
\]

\(\frac{d \gamma_s}{d s_2}\) can be expressed as: 277
\[
\frac{dy_s}{ds_2} = \frac{dy_s}{dt} \frac{dt}{ds_2}
\]  \hspace{1cm} (D.13)

where

\[
\frac{dy_s}{dt} = \dot{y} = C \frac{V_s}{L}
\]  \hspace{1cm} (D.14)

C is a constant, used to be determined experimentally and now numerically as was described in Appendix A, and \( V_s \) is the velocity parallel to the shear plane AB, expressed as [Oxley; 1989]:

\[
V_s = \frac{u \cos \alpha}{\cos(\phi - \alpha)}
\]  \hspace{1cm} (D.15)

where \( u \) is the cutting velocity, \( \alpha \) is the tool rake angle, and \( \phi \) is the shear angle.

\[
\frac{dt}{ds_2}
\] is the reciprocal of the material velocity normal to the shear plane AB and can be expressed as [Oxley; 1989]:

\[
\frac{dt}{ds_2} = \frac{1}{u \sin \phi}
\]  \hspace{1cm} (D.16)

Using equations D.10 through D.16, Oxley obtained an expression for \( \frac{dk}{ds_2} \) as

\[
\frac{dk}{ds_2} = \frac{2Cn k_{AB}}{L} = \frac{\Delta k}{\Delta s_2}
\]  \hspace{1cm} (D.17)

Substituting expression of \( \frac{\Delta k}{\Delta s_2} \) from equation D.17 into equation D.8, Oxley obtained the following relation for the angle \( \theta \), shown in Figure D.2:

\[
\theta = \tan^{-1}[1 + 2(\frac{\pi}{4} - \phi) - Cn]
\]  \hspace{1cm} (D.18)

To obtain an expression of the normal stress at the tool-chip interface at point B (Figure D.2), the following equation of the hydrostatic pressure at B (\( p_B \)), derived from equations D.3 and D.7, can be used:

\[
p_B = k_{AB}[1 + 2(\frac{\pi}{4} - \phi)] - \frac{\Delta k}{\Delta s_2} L
\]  \hspace{1cm} (D.19)
Oxley and Hastings [Oxley; 1977] used slipline field analysis to obtain an expression of the normal stress at the tool-chip interface at point B ($\sigma'_{n,B}$) as:

$$\sigma'_{n,B} = p_g + 2k_{AB} (\phi - \alpha)$$  \hspace{1cm} (D.20)

Then, using equations D.19 and D.20

$$\sigma'_{n,B} = k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right] - \frac{\Delta k}{\Delta s_2} L + 2k_{AB} (\phi - \alpha)$$

$$= k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) + 2(\phi - \alpha) \right] - \frac{\Delta k}{\Delta s_2} L$$ \hspace{1cm} (D.21)

Substituting $\frac{\Delta k}{\Delta s_2}$ from equation D.17, into equation D.21, the following expression of the normal stress at the tool-chip interface was obtained [Oxley; 1989]:

$$\sigma'_{N,B} = k_{AB} \left[ 1 + \frac{\pi}{2} - 2\alpha - 2Cn \right]$$ \hspace{1cm} (D.22)

An expression of the tool-chip interface length ($h$), can be obtained by taking moments about B of the normal stresses on the shear plane AB to find the position of the resultant force ($R$) on AB as shown in Figure D.4 [Oxley; 1989]. Then, the position of the resultant force at AB can be related to the position of the resultant force at the interface through geometric analysis. Then, the tool-chip interface length can be obtained because the resultant force ($R$) is located at the middle of the interface length due to the assumption of a uniform normal stress distribution at the interface [Oxley; 1989]. Thus, the expression of the tool-chip interface length is derived in the present work as follows, using Figure D.4:

$$\sum M_A = 0 \quad \text{(moments about B of the normal forces along AB)}$$ \hspace{1cm} (D.23)
Figure D.4: Distance on the shear plane between the resultant cutting force $R$ and the cutting edge ($x_1$) and its relation to the tool-chip interface length ($h$)
\[ R \sin \theta \cdot x_1 - p_b \cdot L \cdot \frac{L}{2} - \frac{1}{2} \left( \frac{\Delta k}{\Delta s_2} L \right)^2 = 0 \quad \text{(D.24)} \]

\[ R \sin \theta \cdot x_1 = \frac{p_b \cdot L}{2} + \frac{\Delta k \cdot L}{\Delta s_2} \frac{L}{3} = 0 \]

since \( R \sin \theta = \frac{L \cdot (p_A + p_B)}{2} \quad \text{(D.25)} \)

Then by substituting (D.25) in (D.24)

\[ L \left( \frac{p_A + p_B}{2} \right) x_1 = \left( \frac{p_b}{2} + \frac{\Delta k \cdot L}{\Delta s_2} \frac{L}{3} \right) L^2 \Rightarrow x_1 = \frac{2 \left( \frac{p_b}{2} + \frac{\Delta k \cdot L}{\Delta s_2} \frac{L}{3} \right) L^2}{(p_a + p_b)L} \quad \text{(D.26)} \]

Using equation D.3 in D.26

\[ x_1 = \frac{p_A - \frac{\Delta k}{\Delta s_2} L + \frac{\Delta k}{\Delta s_2} \frac{2L}{3} L}{p_A + p_B} \quad \text{(D.27)} \]

From equation D.6

\[ p_a + p_b = 2k_{AB} \tan \theta \quad \text{(D.28)} \]

Substituting (D.28) in (D.27)

\[ x_1 = \frac{p_A - \frac{\Delta k}{\Delta s_2} L + \frac{\Delta k}{\Delta s_2} \frac{2L}{3} L}{2k_{AB} \tan \theta} \quad \text{(D.29)} \]

Using equation D.7 to substitute for \( p_A \) in D.29

\[ x_1 = \frac{k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right] + \frac{\Delta k}{\Delta s_2} L \left( \frac{2}{3} - 1 \right) L}{2k_{AB} \tan \theta} = \frac{L \{ k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right] - \frac{1}{3} L \frac{\Delta k}{\Delta s_2} \}}{2k_{AB} \tan \theta} \quad \text{(D.30)} \]

\[ = L \{ - \frac{k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right]}{2k_{AB} \tan \theta} + \frac{1}{3} \frac{L \frac{\Delta k}{\Delta s_2}}{2k_{AB} \tan \theta} \} \]

Substituting \( \frac{\Delta k}{\Delta s_2} \) from equation D.17, into equation D.30
\[ x_1 = L \left\{ \frac{k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) \right]}{2k_{AB} \tan \theta} - \frac{1}{3} \frac{L}{L} \right\} = L \left\{ \frac{1 + 2 \left( \frac{\pi}{4} - \phi \right)}{2 \tan \theta} - \frac{Cn}{3 \tan \theta} \right\} \]  

(D.31)

\[ = L \left\{ \frac{\tan \theta + Cn}{2 \tan \theta} - \frac{Cn}{3 \tan \theta} \right\} = L \left( \frac{1}{2} + \frac{Cn}{6 \tan \theta} \right) \]

From Figure D.4:

\[ L = \frac{t_1}{\sin \phi} \]  

(D.32)

Also, from Figure D.4:

\[ \frac{x_1}{\sin(90 - \lambda)} = \frac{\sin \theta}{2} \]  

(D.33)

then

\[ h = \frac{2x_1 \sin \theta}{\cos \lambda} \]  

(D.34)

then by substituting \( L, x_1 \) and \( \tan \theta \) from equations D.32, D.31, and D.18, respectively, into equation D.34, the following expression of \( h \) can be obtained [Oxley; 1989]:

\[ h = \frac{t_1 \sin \theta}{\cos \lambda \sin \phi} \left\{ 1 + \frac{Cn}{3 \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) - Cn \right]} \right\} \]  

(D.35)
APPENDIX E

MODIFICATION OF OXLEY'S ANALYTICAL MODEL TO ACCOMMODATE GENERAL FORMS OF FLOW STRESS EQUATIONS

Oxley's predictive machining theory, described in Appendix A, was developed based on a flow stress equation in the form:

\[ \bar{\sigma} = \sigma_1 \varepsilon^n \]  \hspace{1cm} (E.1)

where \( \bar{\sigma} \) is the flow stress of the material and \( \varepsilon \) is the strain. \( \sigma_1 \) and \( n \), both of which vary with the strain rate (\( \dot{\varepsilon} \)) and temperature (\( T \)), are parameters of the flow stress equation. The strain hardening exponent \( n \) of equation E.1 is required in equations A.14, A.23 and A.34 in the Algorithm of Oxley's theory described in Appendix A. To be able to use general forms of flow stress equations such as those described in Appendix C (equation C.15), two techniques are suggested in the present work. The first technique is to calculate the equivalent \( \sigma_1 \) and \( n \) that make the flow stress equation E.1 coincide with a given flow stress equation in the general form

\[ \bar{\sigma} = \text{function}(\varepsilon, \dot{\varepsilon}, T) \]  \hspace{1cm} (E.2)

This can be done by the following technique:

1) Determine the strain (\( \varepsilon \)), strain rate (\( \dot{\varepsilon} \)) and temperature (\( T \)) from the analytical model

2) Find \( \bar{\sigma} \), using the given flow stress equation E.2, at the determined strain (\( \varepsilon \)), strain rate (\( \dot{\varepsilon} \)) and temperature (\( T \))
3) Find $\sigma_1 = \sigma^*$, using equation E.2, at the determined strain rate ($\dot{\varepsilon}$) and temperature (T) and a strain ($\varepsilon$) = 1

$$\ln \frac{\sigma^*}{\sigma_1} \quad \text{(E.3)}$$

4) Calculate $n = \frac{\sigma_1}{\ln \varepsilon}$

The determined $\sigma_1$ and $n$ are the parameters of the flow stress equation $\sigma^* = \sigma_1 \varepsilon^*$ that coincides with the given flow stress equation E.2.

The validity of this technique is checked by implementing it in OXCUT to analyze the 2-D orthogonal slot milling of the Titanium alloy Ti6Al4V described in Chapter 3. The given flow stress equation of this alloy, stated as equation C.15 in Appendix C and repeated here for convenience, is as follows [Lee; 1998]:

$$\sigma^* = (A + B\varepsilon^\alpha)(1 + C\ln \varepsilon^*) (1 - T^{*m}) \quad \text{(E.4.a)}$$

where $\sigma^*$ is the flow stress of the workpiece in MPa and $\varepsilon$ is the equivalent plastic strain. $\varepsilon^* = \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}$ is the ratio of the test strain-rate to the reference strain rate ($\dot{\varepsilon}_0$), taken as $10^{-6}$ s$^{-1}$.

$$T^* = \frac{T - T_{room}}{T_{melting} - T_{room}} \quad \text{(E.4.b)}$$

where $T^*$ is the homologous temperature, $T$ is the workpiece temperature, $T_{room} = 25^\circ C$ and $T_{melting} = 1676.67^\circ C$. A, B, $n_1$, C, and m are material constants determined experimentally and have the values 782.7 MPa, 498.4 MPa, 0.28, 0.028, and 1.0, respectively.

The average strain ($\varepsilon_{AB}$), strain rate ($\dot{\varepsilon}_{AB}$) and temperature ($T_{AB}$), in the primary deformation zone, are calculated using equations A.4, A.3, and A.13, respectively, as stated in Appendix A. Then, the steps (1) through (4), mentioned above, are used to obtain $\sigma_1$ and $n$ of the flow stress equation E.1 such that it coincides with the given flow stress equation E.4. The stress-strain curves,
described by the obtained flow stress equation in the form of E.1 and the given flow stress equation E.4, are compared in Figure E.1. As shown in Figure E.1, the curve, describing the obtained flow stress equation E.1 (filled circles), fits the points representing the given flow stress equation E.4 (hollow squares), especially, within the vicinity of the strain value equal to 0.684 where the parameters \( \sigma_1 \) and \( n \) of equation E.1 are obtained. Thus, the calculated \( n \) can be used in equations A.14, A.23 and A.34 in the Algorithm of Oxley's theory described in Appendix A.

The other technique is to modify Oxley's analytical model to account for other forms of flow stress equations. Flow stress equations such as equations C.10, C.12 and C.13 which include an integral that accounts for the strain history cannot be handled in Oxley's theory because the strain, strain rate and temperature are calculated at steady state and the strain history cannot be obtained. Flow stress equations, such as equation C.14, do not need any modifications because at a certain strain rate and temperature equation C.14 will be exactly the same as E.1. Flow stress equations similar to equation C.15, repeated in this Appendix as equation E.4, can be handled by modifying Oxley's analytical model to account for such equation as follows:

At a certain strain rate and temperature, equation E.4 will take the form:

\[
\bar{\sigma} = (A + B\varepsilon^n)C_1C_2 \tag{E.5}
\]

where \( C_1 \) and \( C_2 \) are constants that represent the values of the temperature and strain rate terms at a certain strain rate and temperature. Equation E.5 can be written in the form

\[
\bar{\sigma} = (\sigma_0 + \sigma_{11}\varepsilon^n) \tag{E.6}
\]

where \( \sigma_0 \) and \( \sigma_{11} \) are constants that depend on strain rate and temperature. Differentiating equation E.6 with respect to strain:
Figure E.1: Comparison between flow stress equations E.1 and E.4
\[
\frac{d\sigma}{de} = 0 + n_1 \sigma_1 e^{n_1 - 1} = n_1 \sigma_1 \varepsilon^{n_1 - 1} \varepsilon^n = \frac{n_1 \sigma_1 \varepsilon^{n_1}}{\varepsilon} = \frac{n_1 (\sigma - \sigma_0)}{\varepsilon}
\]
\[
\frac{n_1 (\sigma - \sigma_0)}{\sqrt{3}} = \frac{\varepsilon}{\sqrt{3}}
\]

Since \( k = \frac{\sigma}{\sqrt{3}} \), \( \gamma_s = \varepsilon \sqrt{3} \), and \( \dot{\gamma} = \varepsilon \sqrt{3} \), then
\[
\frac{dk}{d\gamma_s} = \frac{n_1 (k - \frac{\sigma_0}{\sqrt{3}})}{\varepsilon \sqrt{3}} = \frac{n_1 (k - \frac{\sigma_0}{\sqrt{3}})}{\gamma_s} = \frac{3n_1 (k - \frac{\sigma_0}{\sqrt{3}})}{\gamma_s}
\]

(E.7)

(E.8)

Then at plane AB
\[
\begin{bmatrix}
\frac{dk}{d\gamma_s} \\
\frac{d\theta}{d\gamma_s}
\end{bmatrix}_{AB} = \frac{3n_1 (k_{AB} - \frac{\sigma_0}{\sqrt{3}})}{\gamma_{sAB}}
\]

(E.9)

Using equation E.9 instead of equation D.11 of Appendix D, the new expression for \( \frac{\Delta k}{\Delta s_2} \) is
\[
\frac{\Delta k}{\Delta s_2} = \frac{3n_1 (k_{AB} - \frac{\sigma_0}{\sqrt{3}})}{\cos \alpha} \frac{u \cos \alpha}{\cos(\phi - \alpha) L} \frac{1}{u^* \sin \phi} = \frac{6Cn_1 (k_{AB} - \frac{\sigma_0}{\sqrt{3}})}{L}
\]

(E.10)

Substituting \( \frac{\Delta k}{\Delta s_2} \) from equation E.10 into equation D.8 of appendix D, the new expression for the angle \( \theta \) is
\[
\tan \theta = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - \frac{\Delta k}{2k_{AB} \Delta s_2} L = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - \frac{L}{k_{AB}} \frac{6Cn_1 (k_{AB} - \frac{\sigma_0}{\sqrt{3}})}{L}
\]
\[
= 1 + 2 \left( \frac{\pi}{4} - \phi \right) - 6Cn + \frac{\sigma_{0,AB}}{k_{AB} \sqrt{3}} = 1 + 2 \left( \frac{\pi}{4} - \phi \right) - 3Cn + \frac{3 \sigma_{0,AB}}{\sigma_{AB}}
\]

(E.11)

where \( \sigma_{0,AB} \) is the flow stress constant of equation E.6 evaluated at the strain rate and temperature of the shear plane AB; \( k_{AB} \) and \( \sigma_{AB} \) are the shear flow stress.
and the flow stress at plane AB, respectively. Thus, equation E.11 should be used instead of equation D.18 of appendix D to calculate the angle \( \theta \) in Oxley’s theory for a flow stress equation similar to E.4.

In the present work, to modify the expression of the normal stress at the tool-chip interface at point B, given by equation D.22 of appendix D, substitute \( \frac{\Delta k}{\Delta s_1} \) from equation E.10, into equation D.21 of appendix D

\[
\sigma_{n,B} = k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) + 2(\phi - \alpha) \right] - \frac{6Cn(k_{AB} - \sigma_{0,AB})}{\sqrt{3}L} L
\]

\[
= k_{AB} \left[ 1 + 2 \left( \frac{\pi}{4} - \phi \right) + 2(\phi - \alpha) \right] - 6Cnk_{AB} + \frac{6Cn\sigma_{0,AB}}{\sqrt{3}} \tag{E.12}
\]

To modify the expression of the tool-chip interface length, given by equation D.35 of appendix D, substitute \( \frac{\Delta k}{\Delta s_2} \) from equation E.10, into equation D.30 of Appendix D as follows:
\[
x_i = L\left\{k_{AB} \left[1 + 2\left(\frac{\pi}{4} - \phi\right)\right] - \frac{1}{3} \frac{L}{2k_{AB} \tan \theta} \right\} - \frac{6Cn(k_{AB} - \frac{\sigma_{0,AB}}{\sqrt{3}})}{2k_{AB} \tan \theta}
\]

\[
= L\left[1 + 2\left(\frac{\pi}{4} - \phi\right)\right] Cn(k_{AB} - \frac{\sigma_{0,AB}}{\sqrt{3}}) - \frac{1}{2k_{AB} \tan \theta} \right\}
\]

\[
= L\left[1 + 2\left(\frac{\pi}{4} - \phi\right)\right] Cn\frac{\sigma_{0,AB}}{\sqrt{3}} + \frac{Cn \sigma_{0,AB}}{k_{AB} \tan \theta \sqrt{3}}
\]

\[
= \frac{L}{2\tan \theta} \left[1 + 2\left(\frac{\pi}{4} - \phi\right) - 2Cn + \frac{2Cn \sigma_{0,AB}}{\sigma_{AB}}\right]
\]

(E.13)

Then by substituting \(x_i\), from equation E.13, and L, from equation D.32 of appendix D, into equation D.34 of Appendix D, the following expression for the tool-chip interface length can be obtained:

\[
h = \frac{t_i \cos \theta}{\sin \phi \cos \lambda} \left[1 + 2\left(\frac{\pi}{4} - \phi\right) - 2Cn + \frac{2Cn \sigma_{0,AB}}{\sigma_{AB}}\right]
\]

(E.14)

Equations E.11, E.12 and E.14 were used instead of equations D.18, D.22 and D.35, in Oxley's theory, for the flow stress equation in the form of E.4. It was found that many convergence problems occurred due to applying those modified equations. Thus, in the present work, it is decided to use the first technique by curve fitting equation E.1 to equation E.4, to apply Oxley's theory for general forms of flow stress equations.
The Downhill Simplex Method

One of the techniques that can be used to perform multidimensional minimization is the Downhill Simplex Method [Press; 1992]. The method requires evaluations of functions not derivatives. The Downhill Simplex Method has a geometrical naturalness which makes it descriptive. A “simplex” is the geometrical figure that consists, in N dimensions, of N+1 points or vertices and their interconnecting line segments, polygonal faces, ... etc. In two dimensions, a simplex is a triangle. In three dimensions, a simplex is a tetrahedron which is not necessarily the regular tetrahedron. The simplex, used in the Downhill Simplex Method, is nondegenerate which means that it encloses a finite inner N-dimensional volume. If any point of a nondegenerate simplex is considered the origin, then the N remaining points define vector directions that span the N dimensional vector space.

To find the minimum of a N-dimensional function, the Downhill Simplex Method must be started with an initial guess consisting of N+1 points that define the vertices of a Simplex. The Algorithm is supposed to use that initial guess to find its way downhill through the N-dimensional space until it locates at least one local minimum. If one of the N+1 initial guess points (it does not matter which) is taken as the starting point $P_0$, then the N other points can be defined as:

$$P_i = P_0 + \lambda e_i \quad (i = 1 \text{ to } N) \quad (F.1)$$
where the $e_i$'s are $N$ unit vectors and $\lambda_s$ is a constant which is a guess of the problem's characteristic length scale (different $\lambda_s$'s can also be assigned to the unit vectors $e_i$'s).

The Downhill Simplex Method performs multidimensional minimization using the basic moves illustrated in Figure F.1. Those basic moves are based on maintaining the volume of the simplex. Most of the steps, done by the Algorithm, are reflections (Figure F.1) during which the Algorithm moves the point where the function has its maximum value across the opposite face of the simplex to a point where the function has a lower value. If the Algorithm can achieve a reflection, it tries to take larger steps by expanding the simplex in one or another direction as shown in Figure F.1. It is required, sometimes, to perform single or multiple contractions of the simplex to continue the solution. The Downhill Simplex Algorithm is explained in more details by Press et al. [Press; 1992].
Figure F.1: Basic moves of the Downhill Simplex Method to achieve multidimensional minimization [Press; 1992]