A simulation of the ANITA experiment with a focus on secondary interactions

DISSEPTION

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Abstract

The Antarctic Impulsive Transient Antenna (ANITA) experiment was designed to detect astrophysical neutrinos with energy greater than $10^{18}$ eV. It uses a cluster of antennas hanging from a balloon over Antarctica to look for radio signals from neutrino-induced particle showers in the ice. Neutrinos can also indirectly cause particle showers, one example being if the charged lepton produced in a charged current interaction experiences hard energy losses through bremsstrahlung and photonuclear interactions. One of the Monte Carlo simulations developed to determine the aperture of ANITA’s 2006-2007 flight focuses on these secondary showers. Inclusion of secondary showers increases the expected aperture by allowing a neutrino to be detected even if the primary shower is out of view. Because the spatial distribution of secondary showers depends on a particle’s cross section, the different arrival times of the radio pulses from the showers along a particle track can indicate the neutrino energy, neutrino flavor, and photonuclear cross section.
This dissertation is dedicated to my parents and stepparents.
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Chapter 1

Introduction

The universe is expected to be opaque to cosmic ray protons on a distance scale greater than tens of megaparsecs at energies greater than \( \sim 10^{20} \text{ eV} \) through what is known as the “GZK process,” in which the protons are attenuated by photopion interactions with the cosmic microwave background [1, 2]. If the drop in the cosmic ray spectrum above \( 10^{19.6} \text{ eV} \) [3] is caused by the GZK process instead of by a source cutoff, the charged pions created in the GZK process should decay to produce neutrinos.

\[
p + \gamma_{\text{CMB}} \rightarrow \Delta^+ \rightarrow n + \pi^+
\]

\[
n \rightarrow p + e^- + \bar{\nu}_e
\]

\[
\pi^+ \rightarrow \mu^+ + \nu_\mu
\]

\[
\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu
\]

These GZK neutrinos can travel almost entirely unattenuated from even the most distant known galaxies and provide a view of cosmic ray sources at distances and energies otherwise unavailable to cosmic ray detectors [4].

The small cross section that allows neutrinos to travel so far to reach the Earth also makes them difficult to detect when they arrive. The mean free path of a neutrino is hundreds of kilometers of water even at \( 10^{20} \text{ eV} \) [5]. A long neutrino mean free
path and a low expected flux of GZK neutrinos at ultrahigh energies ($E > 10^{18}$ eV) [6] cause the required exposure of a GZK neutrino detector to be $\sim 100$ km$^3$ sr yr or greater. Instead of trying to detect signals from the products of neutrino interactions, GZK neutrino detectors can achieve the required exposure by indirectly searching for neutrinos through a radio detection technique.

When an ultrahigh energy neutrino interacts in matter, the momentum transferred to a quark leads to a hadronic shower. The charged lepton coming out of a charged current (CC) interaction can also produce secondary particle showers such as an electron-induced electromagnetic (EM) shower or a shower induced by $\tau^{\pm}$ decay.

In early shower development, the high energy interactions of bremsstrahlung and pair production in an EM shower or in the electromagnetic component of a hadronic shower create a nearly equal number of electrons and positrons. Lower energy interactions with atomic electrons cause the number of electrons in a shower to exceed the number of positrons. Electron scattering interactions like Compton scattering add electrons to a shower and pair annihilation removes positrons from the shower [7, 8].

If a bunch of electrons travels through a transparent medium, the Cherenkov light will add coherently at wavelengths longer than the dimensions of the electron bunch as if the electrons were all a single point charge. The amplitude of Cherenkov light is proportional to the charge of the particle emitting the light, which makes the power of a Cherenkov pulse proportional to the square of the moving electric charge. The power of a coherent Cherenkov pulse is therefore proportional to the square of the number of electrons in the bunch. This method has been used to produce strong microwave Cherenkov pulses at particle accelerators [9].
In a dense ($\gtrsim 1 \text{ g cm}^{-3}$) radio transparent medium, the Molière radius of a shower is $\lesssim 10 \text{ cm}$ so the shower’s Cherenkov pulse is coherent up to frequencies greater than 1 GHz. This makes radio detection of neutrinos possible in targets such as ice, halite, and the lunar regolith [7]. The process by which particle showers emit coherent radio Cherenkov pulses is now known as the “Askaryan effect.”

Monte Carlo simulations of particle showers predicted a charge asymmetry of $(e^- - e^+)/ (e^- + e^+) = 15 - 25\%$ at the shower peak [10, 11]. Those and later simulations predicted the spectrum of the Cherenkov pulse for a wide range of pulse directions and shower energies [12, 13, 14].

The Askaryan effect has been confirmed experimentally. Bunches of 28.5 GeV electrons passed through a bremsstrahlung radiator to make photons that induced EM showers in a target of silica sand [15] or sodium chloride [16]. This resulted in the emission of radio pulses with the direction, polarization, and timing expected from Cherenkov pulses. A third experiment sent bunches of 28.5 GeV electrons into an ice target, resulting in radio Cherenkov pulses that had about 7 times the power that would have been expected from the initial electron bunches if there were no Askaryan effect [17].

Several experiments have been built to use the Askaryan effect for radio detection of neutrinos. The Goldstone Lunar Ultrahigh Energy (GLUE) neutrino experiment used large radio telescopes to search for Cherenkov pulses from the lunar regolith [18]. The Fast On-orbit Recording of Transient Events (FORTE) satellite searched for Cherenkov pulses from the Greenland ice sheet [19]. The Radio Ice Cherenkov Experiment (RICE) uses an array of antennas buried in ice near the south pole [20]. Proposed neutrino detectors include the Saltdome Shower Array (SalSA), which
uses antennas buried in a salt dome [21], and the Antarctic Ross Ice Shelf Antenna Neutrino Array (ARIANNA), which searches for reflections of Cherenkov pulses off the bottom of the Ross Ice Shelf [22].

The Antarctic Impulsive Transient Antenna (ANITA) experiment used a cluster of dual-polarization quad-ridged horn antennas hanging from a balloon over the Antarctic ice to search for Cherenkov pulses from the ice. The prototype detector (ANITA-lite) flew from December 2003 to January 2004 using two antennas and did not detect any signals with the properties that would be expected from particle showers in the ice [23]. Two complete detectors (ANITA-1 and ANITA-2) flew from December 2006 to January 2007 and from December 2008 to January 2009.

ANITA-1 used 32 antennas arranged in three rings. The bottom ring contained 16 antennas offset from each other by a 22.5° azimuthal angle. In this configuration, a pulse that hit an antenna straight on boresight would hit the adjacent antennas with a relative gain of about -3 dB. The antennas also faced a dip angle of 10° (4° below the Earth’s horizon at an altitude of 35 km) because early simulations showed this to be the antenna angle that gave ANITA the greatest sensitivity to neutrinos. The top two rings each contained 8 antennas offset from each other by a 45° azimuthal angle, with the second ring offset 22.5° from the top ring. Each antenna in the top two rings pointed in the same direction as an antenna in the bottom ring. The relative positions of the antennas allowed for the detector to be used as an interferometer to determine the direction of each radio pulse.

The first half of the signal chain included a low noise amplifier and a 200-1200 MHz bandpass filter. 1200 MHz corresponded to an in-ice wavelength of ~14 cm and was
less than the waveform sampler’s Nyquist frequency of 1300 MHz. The signal was then split into a trigger path and a digitizer path.

Along the trigger path, the two linearly polarized signals were converted to right and left circular polarization. They were then split into four non-overlapping frequency bands centered at \( \sim 265, 435, 650, \) and 990 MHz for a total of eight trigger channels per antenna. A tunnel diode system in each channel output a voltage roughly proportional to the power of the signal. Whenever the power in at least three of the eight channels was above those channels’ thresholds, the antenna triggered. Whenever several antenna triggers satisfied coincidence requirements to indicate passage of a plane wave through the payload, the detector recorded a possible neutrino event [24].

Along the digitizer path, the signals passed through 20 ns of cable and were recorded at a rate of \( 2.6 \times 10^9 \) voltage samples per second. When the trigger system indicated a possible neutrino signal, the digitizer recorded 100 samples for each polarization of each antenna, with the expected arrival time of the radio pulse near the center of the digital waveforms. Along with the waveforms, the GPS data and information about the health of the detector were also recorded [25].

The ANITA-2 payload was mostly the same as ANITA-1. An extra ring of 8 antennas was placed below the ring of 16 to improve ANITA’s ability to determine signal directions. Only the signal from an antenna’s mostly vertical polarization could be included in the trigger. The signal for a trigger was split into three non-overlapping frequency bands and a full band.

In order to compare the experimental results to theoretical neutrino fluxes, two independent Monte Carlo simulations of the ANITA experiment were built. The
simulation known as the “Hawaii Code” propagates neutrinos through the Earth to their interaction points. For each neutrino that interacts near the surface, the simulation calculates the electric field as a function of position in the sky relative to the position and direction of the neutrino interaction. Then it determines the payload’s response for different shower positions and directions relative to the payload [26].

The simulation known as “IceMC” first chooses an interaction point in the ice within or barely beyond the horizon of the payload. It traces a ray from the payload to the shower and assigns the shower a random direction. Then it calculates the electric field at the payload and decides the payload’s response. Each event is given an extra weight equal to the fraction of the neutrino flux that survives passage through the Earth to the interaction point [27].

One problem with the Hawaii Code and IceMC is how they handle showers caused by a $\mu^\pm$'s photonuclear energy losses, the decay of a $\tau^\pm$, and other secondary interactions. IceMC chooses the highest energy secondary shower from a probability distribution and places it at the position of the primary shower if it has more energy than the primary shower. Both simulations calculate the signal from the EM shower coming out of a $\nu_e$ CC interaction using parameters of single-peaked EM showers even up to energies where the cross section of an $e^\pm$ is too small to produce a single-peaked EM shower.

A third Monte Carlo simulation of ANITA-1 was built to determine ANITA’s response to secondary showers. It is known as the “Ohio simulation” and propagates neutrinos through an Earth model similarly to what is done in the Hawaii code, but propagation of neutrinos, charged leptons, or photons can continue after the first
Figure 1.1: a) If ANITA only detects the primary shower, there is no advantage to simulating secondary interactions. b) If ANITA does not detect the primary shower, it might still detect a secondary. c) Multiple bang event.

neutrino interaction until no particle has enough energy to produce another detectable shower. Ray tracing and the payload response are the same as in IceMC.

Figure 1.1 shows two advantages of simulating secondary interactions. If the primary shower is not detected, a secondary shower might still be detected, resulting in an increase in the aperture of ANITA-1. If the primary shower is detected and secondary showers are also detected, the secondaries do not affect the aperture, but they can provide clues about some of the physics involved in the interactions that produce secondary showers. A disadvantage of using this simulation is that much more computer time required to simulate a neutrino event, which results in fewer simulated events and a larger statistical error.
Chapter 2 describes the algorithm of the Ohio version of the simulation, including the parts that are shared with IceMC. Chapter 3 reports the exposure of ANITA-1 with respect to neutrino direction and energy, the calculation of statistical error, and new neutrino flux limits. Chapter 4 discusses the physics that could have been learned from “multiple bang” events, which are neutrino events that have more than one detected radio pulse. Chapter 5 describes how the systematic error is calculated and restates the exposure. Appendix A gives the cross sections for the electromagnetic interactions involved in some of the secondary particle propagation.
Chapter 2

Algorithm

2.1 Neutrino Propagation

The Earth model used for propagation of leptons is the Preliminary Standard Earth Model [28] surrounded by a shell of ice, as shown in Figure 2.1. The ice density can remain constant up to the surface or it can have a depth dependence based on a fit to the firn density of an ice core taken near the South Pole [29].

$$\frac{\rho}{\text{g cm}^3} = 0.36596 + 0.01472 \left( \frac{D}{\text{m}} \right) - 0.000344 \left( \frac{D}{\text{m}} \right)^2$$

$$+ 0.00000422 \left( \frac{D}{\text{m}} \right)^3 - (1.8 \times 10^{-8}) \left( \frac{D}{\text{m}} \right)^4 \quad (D < 85 \text{ m})$$

$$= 0.486979 + 0.0034915 \left( \frac{D}{\text{m}} \right) \quad (85 \text{ m} \leq D \leq 124.02 \text{ m})$$

$$= 0.917 \quad (D > 124.02 \text{ m}),$$

(2.1)

where $\rho$ is the ice density and $D$ is the distance below the surface. The nadir angle of the neutrino direction at the entrance point has a probability distribution of

$$\frac{dN}{d\theta_n} = 2 \sin(\theta_n) \cos(\theta_n),$$

(2.2)

which corresponds to an isotropic distribution of sources on the sky. The energy and flavor of a neutrino can be chosen by the user or from a theoretical flux density.
The distance to a source of GZK neutrinos is much larger than the oscillation length of neutrino mass-flavor mixing. The probability distribution of neutrino flavor is therefore the same as if the neutrinos were split into pure mass states. If a theoretical flux density is used, the flavor distribution is modified using the formulas

\[
\begin{align*}
P(\nu_e \rightarrow \nu_e) &= \cos^4(\theta_{1,2}) + \sin^4(\theta_{1,2}) \\
\cos^2(\theta_{1,2})
\end{align*}
\]

\[
\begin{align*}
P(\nu_e \rightarrow \nu_\mu) &= P(\nu_\mu \rightarrow \nu_e) = 2 \cos^2(\theta_{1,2}) \sin^2(\theta_{1,2}) \cos^2(\theta_{2,3}) \\
P(\nu_\mu \rightarrow \nu_\mu) &= [\cos^4(\theta_{1,2}) + \sin^4(\theta_{1,2})] \cos^4(\theta_{2,3}) + \sin^4(\theta_{2,3}) \\
P(\nu_e \rightarrow \nu_\tau) &= 1 - P(\nu_e \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) \\
P(\nu_\mu \rightarrow \nu_\tau) &= 1 - P(\nu_\mu \rightarrow \nu_e) - P(\nu_\mu \rightarrow \nu_\mu),
\end{align*}
\]

where \(\theta_{1,2} = 34^\circ\) [30] and \(\theta_{2,3} = 45^\circ\) [31]. The mixing angle \(\theta_{1,3}\) is assumed to be \(0^\circ\) [32].

The charged current (CC) and neutral current (NC) cross sections for a neutrino-nucleon interaction are given as

\[
\begin{align*}
\frac{d\sigma_{\text{CC}}}{dy} &= \frac{2G_F^2 M_N E_\nu}{\pi} \left( \frac{M_N^2}{Q^2 + M_N^2} \right)^2 \int_0^1 [xq(x, Q^2) + x\bar{q}(x, Q^2)(1 - y)^2]dx \\
\frac{d\sigma_{\text{NC}}}{dy} &= \frac{G_F^2 M_N E_\nu}{2\pi} \left( \frac{M_N^2}{Q^2 + M_N^2} \right)^2 \int_0^1 [xq^0(x, Q^2) + x\bar{q}^0(x, Q^2)(1 - y)^2]dx \\
Q^2 &= 2M_N x E_\nu y.
\end{align*}
\]

Figure 2.1: Earth model for lepton propagation (not to scale).
$M_N$ is the nucleon mass. $E_\nu$ is the neutrino energy. $Q^2$ is the square of the 4-momentum transfer. $M_W$ and $M_Z$ are the W boson and Z boson masses. $x$ is the fraction of nucleon mass held by a quark. $y = \Delta E / E$ is the fraction of the neutrino’s energy that is transferred to a quark. $q^0(x, Q^2)$ and $\bar{q}^0(x, Q^2)$ are the CTEQ6-DIS distribution functions for quarks and antiquarks. $q(x, Q^2)$ and $\bar{q}(x, Q^2)$ are the distribution functions for just the quarks and antiquarks that can participate in a CC interaction for the neutrino [5, 33].

As shown in Figure 2.2, the CC and NC cross sections can be well approximated as $AE^B$ lines with the same slope on a log-log scale in the energy range $10^{18}$ eV < $E_\nu$ < $10^{21}$ eV [33]. The column depth $X_\nu$ a neutrino travels before it interacts is calculated from the probability distribution

$$\frac{dN}{d(X_\nu/\lambda_\nu)} = e^{-X_\nu/\lambda_\nu}$$

$$\lambda_\nu = \frac{M_p}{\sigma_{\nu,p}} = \frac{1.66 \times 10^{-24} \text{ g}}{2.501 \times 10^{-35} (E_\nu / \text{GeV})^{0.3076} \text{ cm}^2},$$

where $\lambda_\nu$ is the mean free path of the neutrino, $M_p$ is the average nucleon mass for water, and $\sigma_{\nu,p}$ is the sum of the CC and NC cross sections. The probability of a charged current interaction is 0.6865. The energy transfer $E_\nu y$ is chosen from the probability distribution

$$\frac{dN}{dy} = \frac{e^{-\log_{2.5}(y)}}{y (1 - e^{-1}) \ln(2.5)}.$$  

The neutrino advances to the interaction point and loses $E_\nu y$ energy. If the interaction is NC, a new path length, interaction type, and energy transfer are chosen and neutrino propagation continues.
2.2 \( \mu^\pm \) and \( \tau^\pm \) Propagation

Propagation of \( \mu^\pm \) and \( \tau^\pm \) created in \( \nu_\mu \) and \( \nu_\tau \) CC interactions is handled by code mostly taken from Muons + Medium (MUM) version 1.6 [34, 35, 36]. The major sources of electromagnetic energy loss are photonuclear interactions, direct pair production, and bremsstrahlung. Electromagnetic interactions with relative energy transfer \( y < y_{\text{min}} \) are treated as a continuous (soft) energy loss given by

\[
\frac{dE_c}{dX} = \frac{E_c}{M_p} \int_0^{y_{\text{min}}} y \frac{d\sigma_{\text{tot}}}{dy} \, dy, \tag{2.8}
\]

where \( \sigma_{\text{tot}} \) is the sum of the electromagnetic cross sections, \( E_c \) is the energy of the \( \mu^\pm \) or \( \tau^\pm \), and \( dX \) is the increment in column depth. \( \frac{dX}{(\text{density})} = 1 \) m in the ice, which causes a \( \mu^\pm \) to lose about 0.01% of its energy per step. \( y_{\text{min}} = 10^{-3.6} \) in the ice in order to allow showers near the low end of ANITA’s energy range to be recorded from a \( \mu^\pm \) or \( \tau^\pm \) that has close to \( 10^{21} \) eV. \( y_{\text{min}} = 0.004 \) below the ice. The energy
loss rate from ionization and knock-on electron production is very small compared to
the other energy loss mechanisms in ANITA’s energy range, but these processes are
still included as a minor contribution to the soft energy loss. A $\mu^\pm$ or $\tau^\pm$ advances a
distance $\frac{dX}{(\text{density})}$ through the medium and loses $dE_c$ energy. Then a new value of $dE_c$
is calculated for the lower energy $E_c$.

Electromagnetic interactions with $y > y_{\text{min}}$ are treated as stochastic (hard) en-
ergy losses. A separate program calculates the mean free path as a function of energy,
while taking continuous energy losses into account. If a $\mu^\pm$ or $\tau^\pm$ reaches the inter-
action point, the interaction type (bremsstrahlung, pair production, or photonuclear
interaction) and $y$ are chosen. The $\mu^\pm$ or $\tau^\pm$ loses $E_c y$ energy and a new path length
is chosen for the next stochastic electromagnetic interaction. The differential cross
section formulas for bremsstrahlung, pair production, and photonuclear interaction
are shown in Appendix A.

In the rock, $\tau^\pm$ propagation is simplified to save time. The stochastic nature
of $\tau^\pm$ energy loss comes mostly from photonuclear interactions. For bremsstrahlung
and pair production, $y_{\text{min}} = 1$ in the rock, meaning they are treated as an entirely
continuous energy loss. $\frac{dX}{(\text{density})}$ is increased to 1.5 m for $\tau^\pm$ in rock less than 24.4 km
below the ice and 4 m for rock more than 24.4 km below the ice, with 24.4 km being
the approximate depth of the crust/mantle boundary.

$\mu^\pm$ and $\tau^\pm$ can also experience weak interactions. After every step of $dX$, a
path length is calculated for an NC or CC interaction with the same mean value as
Equation 2.6. If the path length is less than $dX$, the $\mu^\pm$ or $\tau^\pm$ loses $E_c y$ energy ,
with $y$ calculated from Equation 2.7. If the interaction is CC, the $\mu^\pm$ or $\tau^\pm$ turns
back into a $\nu_\mu$ or $\nu_\tau$. CC and NC interactions are more significant at higher energies ($E_c \gtrsim 10^{20}$ eV) because the mean free path is shorter.

At the start of $\tau^\pm$ propagation, a proper time is chosen from an exponential distribution for $\tau^\pm$ decay. After every step of $dX$, the remaining decay time is reduced according to time dilation and $dX$. If the decay time reaches zero, the $\tau^\pm$ decays through one of six channels ($e^\pm + \nu_e$, $\mu^\pm + \nu_\mu$, $\rho^\pm$, $\pi^\pm$, $a_1^\pm$, or a generic heavy meson) [37, 38, 39]. For the leptonic decay channels, propagation of the charged lepton and neutrino is simulated in addition to propagation of the new $\nu_\tau$. $\tau^\pm$ decay is more significant at lower energies, where it is less suppressed by time dilation.

A $10^{19}$ eV $\mu^\pm$ travels an average distance of 2.2 km in 2.6 g cm$^{-3}$ rock before dropping below $10^{17.4}$ eV. The CC cross section is insignificant below $10^{19}$ eV and time dilation prevents $\mu^\pm$ decay, so some $\mu^\pm$ in the rock have almost no chance to reach the ice. If a muon in the rock has less than $10^{19}$ eV and has to travel more than 5.82 km to reach the ice, it is assumed to be below the cutoff energy and propagation stops. For $\mu^\pm$ below $10^{18}$ eV, the maximum remaining distance in the rock is 2.8 km.

### 2.3 e\(^{\pm}\) Propagation

Electrons and positrons are created by $\nu_e$ CC interactions, pair production, or the $e^\pm + \nu_e$ channel of $\tau^\pm$ decay. Only bremsstrahlung, pair production, and photonuclear interactions are included in the energy loss of $e^\pm$ propagation. Because of the LPM effect, the bremsstrahlung cross section is suppressed for smaller energy transfers and the pair production cross section is suppressed for the new electron having about the same energy as the new positron (see Sections A.3 and A.4). Unlike the $\mu^\pm$ or $\tau^\pm$ total cross section for stochastic energy transfers that increases at higher energies,
the electron total cross section decreases at higher energies in ANITA’s energy range. Below about $10^{18}$ eV, the bremsstrahlung cross section for electrons is still high enough for an electron to produce a single-peaked purely electromagnetic (EM) shower [14].

For $E_c > 10^{18.1}$ eV, an electron is likely to produce a multiple-peaked shower or more than one shower. Electron propagation in this energy range in the ice is simulated in a way similar to $\mu^\pm$ and $\tau^\pm$ propagation. For continuous energy loss, $y_{min} = 0.00025$ and $dX = 1$ cm. The mean free path for a stochastic energy loss without considering continuous energy loss is calculated as

$$\lambda_e = \frac{M_p}{\int_{y_{min}}^{1} \frac{d\sigma_{tot}}{dy} dy}$$

(2.9)

If the chosen path length is less than 1 cm, the interaction type and $y$ are chosen, the electron loses $E_c y$ energy, and electron propagation continues.

2.4 $\gamma$ Propagation

Some bremsstrahlung interactions in the ice create photons with energy greater than $10^{17.4}$ eV. Photon propagation is handled by a Monte Carlo method similar to the simulation of neutrino propagation. A free path is calculated based on the total cross section and the photon advances to the interaction point. If the interaction point is in ice, the interaction type is chosen. The photon interaction type can be either pair production [40], which can require additional electron propagation, or a photonuclear interaction [41], which can result in a hadronic shower.

2.5 Attenuation in the Atmosphere

If the free path of a neutrino is less than the column depth of the atmosphere [37], neutrino attenuation in the atmosphere is calculated before the neutrino enters the
ice. If the interaction is NC, the neutrino just loses some energy. The air density is too low for an electron’s bremsstrahlung cross section to be suppressed by the LPM effect so electrons in the atmosphere are assumed to create air showers that can not be detected by ANITA.

To save time, the tracks used for $\mu^\pm$ and $\tau^\pm$ propagation have a constant air density of $0.00125 \text{ g cm}^{-3}$ for the remaining atmospheric column depth. If the $\mu^\pm$ or $\tau^\pm$ still has more than $10^{17.4} \text{ eV}$ when it reaches the ice, $\mu^\pm$ or $\tau^\pm$ propagation continues in the ice as if the neutrino had interacted at the surface. If $\tau^\pm$ decay occurs in the atmosphere, all decay products except electrons and mesons are propagated through the remaining air and then through the ice.

## 2.6 Shower Distribution

Depending on how well ANITA can detect signals close to the signal-to-noise threshold, the low energy cutoff for detectable showers can be somewhere between $10^{17.4} \text{ eV}$ and $10^{17.9} \text{ eV}$. Photonuclear interactions, weak interactions, and hadronic $\tau^\pm$ decays that transfer more than $10^{17.4} \text{ eV}$ in the ice are recorded as hadronic showers. Any electron with energy between $10^{17.4} \text{ eV}$ and $10^{18.1} \text{ eV}$ in ice is recorded as an EM shower. The list of showers is set aside to be tested later for a variety of payloads and flight paths.

A $\tau^\pm$ in ANITA’s energy range can travel an average of several tens of kilometers in ice. Because the Antarctic ice is only $\sim$1-3 km thick, most of the simulation’s time is spent propagating $\tau^\pm$ in rock to determine how much energy a $\tau^\pm$ or its decay products have when or if they enter the ice on an upward path. The average $\mu^\pm$ track length is on the order of 10 km in ice, so $\mu^\pm$ leave more showers in ice than $\tau^\pm$ do.
Figure 2.3: Blue: Distribution of the distances traveled by a single $e^\pm$. Black: Including the distance traveled by $e^\pm$ created later from pair production.

before entering the rock or the air. The average $e^\pm$ track length is in the hundreds of meters so it is more likely to be entirely contained in the ice. If the energy cutoff for $e^\pm$ is set to $10^{18.5}$ eV instead of $10^{18.1}$ eV, $e^\pm$ and $\mu^\pm$ will leave about the same number of showers in the ice. The mass dependence of cross sections and shower distributions is described in greater detail in sections 4.1 and 4.2.

The $e^\pm$ cross section is greater than the $\mu^\pm$ and $\tau^\pm$ cross sections and it decreases as the $e^\pm$ energy increases. For these reasons, an $e^\pm$ that loses more than 50% of
its energy in a bremsstrahlung or pair production interaction will usually run out of energy before the particles created from that interaction run out of energy. In most cases, the apparent track length of an electron is greater than the distance the electron actually travels in the ice. Figure 2.3 compares the distance traveled by just the $\nu_e$-induced $e^\pm$ to the combined distance traveled by the $\nu_e$-induced $e^\pm$ and all of the $e^\pm$ created later from pair production.

Below $10^{20}$ eV, the bremsstrahlung cross section for electrons is much greater than the pair production and photonuclear cross sections, and is also much greater than the total cross sections for $\mu^\pm$ and $\tau^\pm$. For photons, the pair production cross section is greater than the photonuclear cross section in this energy range so an electron track typically has a few hadronic showers and many EM showers that overlap each other. At $10^{21}$ eV, the bremsstrahlung cross section for electrons is LPM suppressed enough to be comparable in size to the direct pair production and photonuclear cross sections.

### 2.7 When Showers Overlap

There are two ways the simulation can handle interference between multiple radio pulses from the same particle track. As described in section 2.16, one method is to add the radio pulses in time domain to produce a long voltage waveform and then search that waveform for detectable pulses. This method is used for electron tracks because each shower is likely to overlap another shower.

The other method is to ignore the interference and decide whether the payload triggers off each radio pulse independently of the others. This method is used for all radio pulses when the simulation saves time by using a frequency domain trigger.
with a voltage threshold because the pulses can not easily be added semicoherently in frequency domain. This method is also used with the time domain trigger for a shower on a $\mu^\pm$ or $\tau^\pm$ track. At $10^{20}$ eV, the average track length in ice is 11 km for $\mu^\pm$ and 42 km for $\tau^\pm$. In most cases, a shower will be far enough away from the other showers on the track so that the radio signal it produces will not interfere with the radio signals produced by the other showers. When two showers do occur close to each other, producing radio pulses separated by $\sim 0.5-\sim 20$ ns, there is a risk that their interference will affect whether they can trigger the payload. The simulation might trigger off radio pulses whose destructive interference should prevent them from being detected or it might not trigger off radio pulses whose constructive interference should make them easier to detect.

This risk is reduced by combining some showers along the track. If two showers of the same type (EM or hadronic) are close enough to each other to satisfy the conditions described in Figure 2.4, they are combined into a single shower with energy equal to the sum of the two showers’ energies. Also, the cutoff energy for electron propagation becomes $10^{18.5}$ eV instead of $10^{18.1}$ eV. This is in the energy region where EM showers are double peaked but it reduces the number of showers that are artificially merged.

2.8 In-ice Showers as Seen by the Payload

After the showers have been recorded, the Preliminary Earth Model is replaced with a more realistic representation of the ice seen by ANITA. First, the Antarctic Bedmap [42] is plugged into Crust2.0 [43]. That roughly spherical model is then reshaped as a geoid [44].
Figure 2.4: a) Two showers that partially overlap. b) One shower as a combination of the two showers, made by summing their energies. c) If the peak of the combined shower is closer to the peak of the second shower than to the peak of the first shower, then the two showers are replaced with the combined shower.

The payload advances in steps of 15 minutes from GPS data taken during the flight [45]. The payload position used for a 15-minute block in the simulation is halfway between the first and last positions recorded during the 15-minute block. The orientation of the payload was averaged in 30-second blocks by a method similar to how the positions were averaged. Each shower is forced to occur under a point on the surface of the ice randomly chosen from within the payload’s horizon. One of the payload orientations in the 15-minute block is randomly chosen. If a shower is eventually detected, it is assigned an event weight according to the fraction of the Earth’s surface covered by ice within the payload’s horizon and according to the amount of live time recorded for the 15-minute block.

In order to calculate the time averaged sensitivity expected by a future flight, the payload can be placed at an altitude of 120,000 feet (∼36.6 km), a latitude of 80°S, and a random longitude. If this option is selected, only the area solid angle is calculated so the user can decide the expected live time.
Figure 2.5: a) A light path from the payload through the first guessed exit point passes a distance $\Delta x_1$ behind the shower maximum. The second guessed exit point is $\Delta x_1$ in front of the first point. b) A light path from the payload through the second guessed exit point passes a distance $\Delta x_2$ in front of the shower maximum. The third guessed exit point is $\Delta x_2$ behind the second point.

2.9 Cherenkov Light Path

The direction of the Cherenkov pulse is assumed to match a path that starts at the shower maximum and finishes at a point near the center of the payload. The local azimuthal component of the shower is chosen randomly from 0 to $2\pi$. The distance from the shower start to the shower maximum is calculated according to the shower energy and type (EM or hadronic) [46, 47].

The point where the Cherenkov pulse leaves the ice is first guessed to be directly above the shower maximum. Using Snell’s Law, a path is calculated from the payload through the first guessed exit point and into the ice. The displacement vector $\Delta \vec{x}_1$ from the shower maximum to the point where that path reaches the same altitude as the shower maximum is subtracted from the first guessed exit point to obtain a second guess for the exit point. See Figure 2.5a.

Again using Snell’s Law, a path is calculated from the payload through the second guessed exit point to a point in the ice at the same altitude as the shower maximum.
The displacement vector $\Delta \vec{x}_2$ from that point to the shower maximum is added to the second exit point to obtain the third guessed exit point, which is assumed to be the correct exit point for the Cherenkov pulse. See Figure 2.5b. Because the distance to the payload is much greater than the distances between the guessed exit points, $\Delta x_2$ will be much smaller than $\Delta x_1$. In rare cases, $\Delta x_2$ is more than 10 m and the Cherenkov pulse is declared to be undetectable.

In the ice shelves or at a depth greater than 150 m, the index of refraction in the ice is 1.783. The ice sheets over the continent include a firn with a depth approximated as 150 m. In the firn, the index of refraction is calculated as [26]

$$n = 1.325 + 0.463251 \left(1 - e^{-0.0140157 \left( \frac{\text{depth}}{m} \right)} \right).$$  \hfill (2.10)

If the shower is in an ice shelf, a path is calculated for a Cherenkov pulse that reflects off the bottom of the ice with 50\% attenuation and travels up through the top of the ice. The method used to find the exit point of the direct pulse is repeated for the reflected pulse. In rare cases, this allows ANITA to detect two radio pulses from the same shower. The possible paths from a shower to the payload in both types of ice are shown in Figure 2.6.

### 2.10 Radio Signals from Stations

Human activity at stations in Antarctica produces a background of radio signals that can be mistaken for the Cherenkov pulses that ANITA looks for. Abandoned stations can also produce neutrino-like signals from electric discharges in equipment. If ANITA detects a neutrino signal coming from the same direction as a station, the signal is assumed to be from the station instead of from a neutrino.
Figure 2.6: a) In the ice sheets over land, the upward path of a Cherenkov pulse is affected by the index of refraction in the firn. b) In the ice shelves over water, a downgoing Cherenkov pulse can reflect off the bottom of the ice and still be detected by ANITA.

In the data analysis, the direction of a radio signal could be found to within 4.4° in $\phi$ and 0.7° in $\theta$ [48]. In the simulation, any radio pulse coming from within that solid angle of a station is thrown out, which causes a decrease of about 3% in the aperture. Radio pulses coming from aircraft were also excluded in the data analysis but not in the simulation [49].

2.11 Spectrum of the Radio Pulse

The spectrum of the Cherenkov pulse before propagation losses is calculated as

$$|\tilde{E}(f, R, \theta)| = 2.53 \times 10^{-7} \left[ \frac{1}{R} \right] \left[ \frac{E_0}{\text{TeV}} \right] \left[ \frac{f}{1.15 \text{ GHz}} \right] \frac{1}{1 + (f/f_0)^{1.44}} \times$$

$$\left[ \frac{\sqrt{2}}{2} \right] \left[ \frac{\sin(\theta)}{\sin(\theta_c,d)} \right] e^{-(\theta-\theta_c)^2/(\Delta\theta)^2} \text{ V MHz}^{-1}$$

$$\Delta\theta = \left[ \frac{12.32^\circ}{\sin(\theta_c)} \right] \left[ \frac{0.5 \text{ GHz}}{f} \right] \left[ \frac{1 \text{ m}}{L_{EM}} \right] \left[ \frac{1}{\sqrt{\ln(2)}} \right]$$

(2.11) (2.12)
\[ \Delta \theta = \left[ \frac{1.473^\circ}{\sin(\theta_c)} \right] \left[ \frac{0.5 \text{ GHz}}{f} \right] \left[ 4.23 - 0.785\epsilon + 0.055\epsilon^2 \right] \left[ \frac{1}{\sqrt{\ln(2)}} \right] \]  

(hadronic showers with \( E_s < 10^{19} \text{ eV} \))  

\[ \Delta \theta = \left[ \frac{1.473^\circ}{\sin(\theta_c)} \right] \left[ \frac{0.5 \text{ GHz}}{f} \right] \left[ 4.23 - 0.785(7) + 0.055(7)^2 - 0.07(\epsilon - 7) \right] \left[ \frac{1}{\sqrt{\ln(2)}} \right] \]  

(hadronic showers with \( E_s > 10^{19} \text{ eV} \))  

\[ f_0 = 1.15 \left[ \frac{n_d \sin(\theta_{c,d})}{n \sin(\theta_c)} \right] \left[ \frac{\rho}{0.917 \text{ g cm}^{-3}} \right] \text{ GHz} \]  

\[ L_{EM} = 3.1 \left( 1 + 0.14 \left[ \frac{E_s}{2 \text{ PeV}} \right] \right)^{0.3} \text{ m} \]  

\[ \epsilon = \log_{10} \left( \frac{E_s}{\text{TeV}} \right) \]  

where \( f \) is the frequency, \( f_0 \) is the coherence frequency, \( R \) is the length of the path the radio pulse takes from the shower maximum to the payload, \( \rho \) is the ice density, \( \theta \) is the angle between the shower axis and the path of the radio pulse, \( E_s \) is the shower energy, \( n \) is the index of refraction, \( \theta_c \) is the Cherenkov angle at the ice depth of the shower maximum, and \( \theta_{c,d} \) is the Cherenkov angle in deep ice [12, 13, 14, 50].

Figure 2.7 demonstrates an advantage that hadronic showers have over EM showers in being detected. Because EM showers are longer, a deviation of the pulse direction from the Cherenkov angle produces more destructive interference in the Cherenkov pulses from different parts of the shower. A characteristic angular spread \( \Delta \theta \) is how far the signal must be off the Cherenkov cone for the field strength to be \( e^{-2} \approx 13.53\% \) of the field strength on the Cherenkov cone. If a signal is 1° off the Cherenkov cone of a \( 10^{18} \text{ eV} \) hadronic shower, it has \( e^{-2(1^\circ/1.28^\circ)} \approx 21\% \) the field strength of a signal on the Cherenkov cone at 700 MHz. For an EM shower with the same geometry, the signal has only \( e^{-2(1^\circ/0.643^\circ)} \approx 4.4\% \) the field strength of a signal on the Cherenkov cone at 700 MHz.
Figure 2.7: Angular spread at 700 MHz. The field strength decreases by a factor of $e^2 \approx 7.389$ for every $\Delta \theta$ the signal is away from the Cherenkov angle. Blue = EM, Red = hadronic.

Because of radio attenuation in the ice, showers are easier to detect if they are closer to the surface. However, the lower ice density in the firn has the opposite effect on showers near the surface. At shallower firn depths, the coherence frequency (Equation 2.15) is smaller, which suppresses the higher frequency component of the Cherenkov pulse. The Cherenkov angle is also smaller, which means the flux density of the Cherenkov pulse is emitted over a smaller solid angle. Figure 2.8 plots the field strength from Equation 2.11 at the Cherenkov angle for a shower in the firn relative to the field strength of an identical shower in deep ice at 1.15 GHz.

The radio attenuation length $\lambda_a$ in the ice is calculated according to the temperature [51, 52].

$$\lambda_a = 1250 \times 0.08886 e^{-0.048827 \times \frac{T}{\text{K}}}$$  (2.18)
The temperature $T$ in Equation 2.18 is replaced with a function of shower depth to give the attenuation length as a function of shower depth. The temperature can be like the colder ice of East Antarctica [53], the warmer ice of West Antarctica [54], or the ice shelves [55]. Because the ice thickness at the longitude and latitude of the shower is usually different from the ice thickness where the temperature was measured, the temperature profiles are converted to temperature as a function of (depth / ice thickness).

The average inverse attenuation length is calculated by integrating over the inverse attenuation length from the surface to the shower depth. For downgoing radio pulses, the path integral goes from the shower depth to the bottom of the ice shelf and back up to the surface.
In transmission from ice to air, the component of the electric field parallel to the surface is modified by the Fresnel equation

\[
\vec{E}_{\text{para,air}} = \vec{E}_{\text{para,ice}} \times \sqrt{1 - \left(\frac{\tan(\theta_i - \theta_a)}{\tan(\theta_i + \theta_a)}\right)^2},
\]  

(2.19)

where \(\theta_i\) and \(\theta_a\) are the hit angles in ice and air. The rest of the electric field is modified by the Fresnel equation

\[
|\vec{E}_{\text{perp,air}}| = |\vec{E}_{\text{perp,ice}}| \times \sqrt{1 - \left(\frac{\sin(\theta_i - \theta_a)}{\sin(\theta_i + \theta_a)}\right)^2}.
\]  

(2.20)

### 2.12 Diffraction

The lower ring antennas are the first part of the payload to be hit by a radio pulse from the ice so they can see the pulse as a simple plane wave. If the radio pulse is traveling at a local elevation of more than about 36°, it diffracts around the deck of the payload on the way to the upper ring antennas [56]. The radio pulse is not detectable if its elevation is more than about 68° because the interferometry used in the data analysis can not reconstruct the direction of such an upgoing radio pulse [24].

For elevation angles between 36° and 68°, the spectrum of the radio pulse at the upper ring antennas is modified according to Fresnel diffraction around a half plane as shown in Figure 2.9.

\[
\vec{E}(\nu) = \vec{E}(\nu)_{\text{plane wave}} \times \sqrt{\frac{1}{2} \left(\left(\frac{1}{2} + C\right)^2 + \left(\frac{1}{2} + S\right)^2\right)}
\]  

(2.21)

\[
C = \int_{0}^{w} \cos \left(\frac{\pi \tau^2}{2}\right) d\tau
\]

\[
S = \int_{0}^{w} \sin \left(\frac{\pi \tau^2}{2}\right) d\tau
\]  

(2.22)
Figure 2.9: How the deck affects the signal at the upper ring antennas: The point where the line of sight has the same altitude as the deck is a distance $x$ from the edge of the deck and a distance $s'$ from the phase center of the antenna.

$$w = \sqrt{\frac{2}{\lambda s'}} x \cos(\theta_z)$$ \hspace{1cm} (2.23)

This overestimates the decrease in field strength at upper ring antennas facing perpendicular to the direction of the radio pulse, but only the antennas facing toward the pulse are likely to trigger.

### 2.13 Antenna Height and Gain

Each antenna detects the radio pulse in two polarizations, labeled $V$ ($10^\circ$ away from vertical) and $H$ (horizontal). The frequency domain voltage for each polarization is calculated as

$$V(f) = \frac{c}{f} (\vec{E}(f) \cdot \vec{V}) \sqrt{\frac{\text{gain}_V(f, \theta_e, \theta_a) Z_r}{\frac{\pi}{Z_0}}}$$

$$H(f) = \frac{c}{f} (\vec{E}(f) \cdot \vec{H}) \sqrt{\frac{\text{gain}_H(f, \theta_e, \theta_a) Z_r}{\frac{\pi}{Z_0}}}$$ \hspace{1cm} (2.24)
where $\theta_e$ is the difference between the elevation angle of the radio pulse and the antenna’s elevation angle of 10°, $\theta_a$ is the difference between the azimuthal angle of the radio pulse and the antenna’s azimuthal angle, $Z_r$ is the antenna’s resistivity of about 50 Ω, $Z_0$ is the resistivity of free space, and $\vec{V}$ and $\vec{H}$ are the V and H polarizations.

The gain was measured for both polarizations on eight antennas at thirteen pulse directions and 131 frequencies [57]. In calculating the voltage, gain$_V$ and gain$_H$ are found by linear interpolation of the frequency and pulse direction, averaged across the eight test antennas.

### 2.14 Voltage (or Frequency Domain) Trigger

If run time is an issue, the simulation can use a faster but less realistic channel trigger in the frequency domain. The voltages in V and H polarizations are converted to left circular (L) and right circular (R) polarizations. Each of the new voltages is divided into frequency channels of 200-300 MHz (low band), 355-515 MHz (mid1 band), 525-775 MHz (mid2 band), and 795-1165 MHz (high band) for a total of eight channels on each antenna.

The root mean square of the noise voltage in a channel is calculated as

$$L_{\text{RMS}}(i) = R_{\text{RMS}}(i)$$

$$= \sqrt{Z_r \ k_B \ \text{band}_i \ T_{\text{avg}}},$$

where $Z_r$ is 50 Ω, $k_B$ is Boltzman’s constant, and band$_i$ is the bandwidth of the channel. $T_{\text{avg}}$ is the system temperature (200 K) plus the solid angle-averaged temperature of ice and sky seen by the antenna.

$$T_{\text{avg}} = \frac{\int_{-\pi}^{\theta_h-\theta_a} (T_{\text{sky}} + 200 \text{ K}) e^{-2(ln2)(\theta/50^\circ)^2} \ d\theta + \int_{\theta_h-\theta_a}^{\pi} (T_{\text{ice}} + 200 \text{ K}) e^{-2(ln2)(\theta/50^\circ)^2} \ d\theta}{\int_{-\pi}^{\pi} e^{-2(ln2)(\theta/50^\circ)^2} \ d\theta}$$

(2.26)
$\theta_a$ is the negative elevation angle of the antennas (10°) and $\theta_h$ is the negative elevation angle of the horizon (about 6°).

A random noise voltage from a Gaussian distribution with the same RMS as the noise can optionally be added to the signal voltage. If the signal voltage exceeds 2.3 of the noise RMS, the channel triggers.

### 2.15 Power (or Time Domain) Trigger

A slower but more realistic simulation of the ANITA channel trigger can be used instead of the frequency domain approximation. The voltage spectra are modified by the attenuation, amplification, and phase shifts measured for most of the equipment that carries the signal from the antennas to the trigger. Instead of using sharp frequency cutoffs for the frequency bands, the spectra are divided into low, mid1, mid2, and high bands using the measured frequency-dependent attenuation for each bandpass filter. $V_i(f, \phi(f))$ and $H_i(f, \phi(f))$ are then Fourier transformed to produce the voltage waveforms $V_i(t)$ and $H_i(t)$, where $\phi(f)$ is the frequency-dependent phase shift of the signal chain.

The first 100 samples (38.46 ns) of 19,759 recorded waveforms from the flight were Fourier transformed to produce 19,759 noise spectra. The average noise spectrum for the flight was calculated as the average of the measured noise spectra. The bandpass filters are applied to the average noise spectrum to get the average noise spectra of the frequency bands. For each simulated radio pulse, the average noise spectrum is Fourier transformed with random phases and added to $V_i(t)$ and $H_i(t)$. 
The linearly polarized V and H channels are converted to circularly polarized L and R channels using the formulas

\[
\Re(R_i(f)) = \frac{1}{\sqrt{2}} (\Re(H_i) + \Im(V_i)) \\
\Re(L_i(f)) = \frac{1}{\sqrt{2}} (\Re(H_i) - \Im(V_i)) \\
\Im(R_i(f)) = \frac{1}{\sqrt{2}} (\Im(H_i) - \Re(V_i)) \\
\Im(L_i(f)) = \frac{1}{\sqrt{2}} (\Im(H_i) + \Re(V_i)) .
\]

(2.27)

\(R_i(f, \phi(f))\) is 90° out of phase with the R channels on the actual payload, but that makes no difference in the power for those channels.

The power in each channel was measured as the voltage output of a tunnel diode and some supporting equipment. The output of the tunnel diode for each frequency channel was measured 100 times using a ∼1 ns input test pulse. The measured outputs for each channel were averaged and reversed in time to find the tunnel diode response function, which is approximated as

\[
S(t) = 0.8e^{-\frac{(t-15 \text{ ns})^2}{(4.6 \text{ ns})^2}} + 0.2e^{-\frac{(t-15 \text{ ns})^2}{(8 \text{ ns})^2}} - A_3 \left[ \frac{t-18 \text{ ns}}{1 \text{ ns}} \right]^2 e^{(t-18 \text{ ns})/(7 \text{ ns})}
\]

\[A_3 = 0.00275 \text{ (low), 0.00454 (mid1), 0.00964 (mid2 and high)}\]

(2.28)

The channel power for a simulated voltage is calculated as a convolution of the tunnel diode response function and the square of the voltage. A channel triggers when its power output exceeds a certain threshold.

The power threshold for each channel was found by simulating the trigger with an input of just noise and recording the average trigger rate. If the simulated noise trigger rate was close to its measured value of 2.65 MHz, the correct threshold was being used. The power thresholds required for a simulated noise trigger rate of 2.65 MHz are
shown in Table 2.1 as the instantaneous tunnel diode output divided by the average noise power.

### 2.16 Semicohherent Addition of Radio Pulses

The simulation can test each radio pulse for a trigger independently of the other pulses or it can add radio pulses to produce a long waveform and then test that waveform for a trigger. The frequency domain trigger tests one pulse at a time because there is no semicoherent addition of pulses in frequency domain.

In time domain, the radio pulses from a $\mu^+$ or $\tau^+$ track are also evaluated independently of each other. Because of the long track lengths, the ideal location and direction for the detectability of one shower usually requires other showers to be outside ANITA’s view or pointing in a bad direction. Even if several showers send strong radio signals to the payload, those signals are usually more than 20 ns apart and do not affect each others’ detectability.

The showers along an electron track are close enough to each other that their radio pulses add semicoherently. An electron track is also much shorter than a $\mu^\pm$ track, averaging about 250 m at $10^{20}$ eV, so it is unlikely to be at a distance where some of the showers are in ANITA’s range and some are not. All radio pulses from an electron track are calculated from shower parameters, attenuation, $90^\circ$ hybrid, etc., and then combined to produce one long waveform for each channel. Then the noise and tunnel

<table>
<thead>
<tr>
<th>channel</th>
<th>low</th>
<th>mid1</th>
<th>mid2</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>threshold ($P/ &lt; P &gt;$)</td>
<td>3.27</td>
<td>3.24</td>
<td>2.48</td>
<td>2.56</td>
</tr>
</tbody>
</table>

Table 2.1: Power thresholds.
diode response are simulated the same as they would be with a single radio pulse to determine the times that each channel is triggered by the entire electron track.

Random thermal noise triggers each channel at an average rate of $\sim 2.65$ MHz so there is some risk that the simulation will trigger off waveforms where the signal to noise ratio is too small for the waveform to have survived cuts that were used in the data analysis. Even without the risk of noise triggers, the simulation can waste time testing and then rejecting waveforms that are not even close to being detectable. To save time and to reduce the rate of false positives, a frequency domain signal is built parallel to the long waveform. The radio pulses are added with 100% coherence and no 90° hybrid. If the total voltage exceeds the RMS noise in fewer than 8 of the 256 channels, the simulation rejects the neutrino signal without checking the waveforms.

If the waveforms each require more than 524,288 voltage samples, which happens when the waveforms are longer than about 201,649 ns, they will use more than 2 gigabytes of RAM. This slows the simulation enough to prevent it from being used effectively on the computers where it was set up. If the waveforms do require more than 524,288 voltage samples, the neutrino event is rejected. The number of $\nu_e$ rejected for this reason is less than 0.1% of the number of $\nu_e$ that eventually pass the trigger simulation so this rejection is not a significant source of systematic error.

### 2.17 Coincidence Triggers

To prevent random noise triggers from overwhelming the detector system, three coincidence triggers were used to make the payload ignore any combination of channel triggers that did not match the timing expected for a plane wave passing through the payload.
The discriminator width of a channel is 11.19 ns after a trigger. The L1 (antenna) trigger requires a coincidence of three out of eight channel discriminator outputs on an antenna. In the voltage-based simulation, there is no definite time for a channel trigger to occur, so L1 requires three channels to be triggered by the same radio pulse.

The discriminator width of an antenna is also 11.19 ns after an L1 trigger. The L2 (phi sector) trigger occurs when an L1 triggers in coincidence with an L1 in at least one of the two adjacent antennas. Figure 2.10 illustrates the possible L1 coincidences for phi sector 4 in the top ring of antennas and the possible L1 coincidences for phi sector 4 in the bottom ring of antennas.

The L3 (payload) trigger occurs when the top ring of antennas and the bottom ring of antennas each experiences an L2 trigger in the same phi sector. For example, either of the L1 combinations in Figure 2.10a at about the same time as either of the L1 combinations in Figure 2.10b will result in an L3 trigger in phi sector 4. There was
no discriminator width for the L2 triggers. Instead, the L3 triggers used a coincidence window of 23.5 ns [58]. For the frequency domain trigger and for 0% coherence in the time domain trigger, the L2 and L3 triggers only require their L1s and L2s to be from the same radio pulse.

Every time an L3 trigger happened, the GPS data, housekeeping data, and 100 ns of H and V waveforms were recorded for the possible neutrino event. These waveforms were centered close to the probable arrival time of the radio pulse at the payload.

In order to reduce the number of false positive triggers in the simulation, the simulated L2 and L3 timing for semicoherent pulses is more restrictive than for the actual payload. The discriminator width of L1 triggers is 5 ns instead of 11.19 ns. 5 ns is enough time to cover the inverse frequency of the low band plus a small difference in arrival times. Depending on the direction of the radio pulse, the arrival time at the upper ring of antennas can be between 5.23 ns and 11.93 ns later than the arrival time at the lower ring. In the lower ring, the L2 triggers are shifted 5.23 ns later in time and are given a discriminator output of 6.7 ns. The L3 coincidence window is 5.5 ns in case one of the L2 triggers is delayed by a noise fluctuation.

2.18 Multiple Bang Events

If a shower is detected and if the event does not include semicoherent addition of pulses, all other showers from the same neutrino are tested for extra detectable pulses. Instead of being given a random position and a random direction, an extra shower is given the same direction as the detected shower and is placed at the previously recorded distance in front of or behind the detected shower. The shower parameters, radio propagation, trigger, etc. are simulated again for each extra shower.
Because a neutrino is already determined to have been detected when extra radio pulses are simulated, any extra detectable pulses have no effect on the sensitivity of the experiment. Instead of being counted as separate events, all of the detectable radio pulses from the same neutrino constitute a multiple bang event, named after the proposed “double bang” method of identifying $\nu_\tau$ [59].

2.19 Event Weights

In order to save time, a shower is forced to start directly below a point on the surface of the ice that is within or near the payload’s horizon. The lepton propagation part of the simulation uses a spherically symmetric Earth model and records showers at all latitudes and longitudes. In order to compensate for the improved shower positions, each event is weighted by a factor of

$$\text{position weight} = \frac{A_{BC}}{A_P},$$

(2.29)

where $A_{BC}$ is the area of ice seen by ANITA and $A_P$ is the surface area of the ice shell used for lepton propagation.

The column depth between the point where a neutrino enters the Earth and the point where a shower starts in the simulation of lepton propagation can be different from the column depth between the same two points in the BEDMAP and Crust2.0 Earth model. If a neutrino path in BEDMAP and Crust2.0 has a larger column depth than the path used in propagating the neutrino, an excess of showers will be recorded along that path. In order to compensate for the difference in column depth that a neutrino traverses in the two Earth models, each event is weighted by a factor of

$$\text{column depth weight} = e^{(D_P - D_{BC})/\lambda_{\nu}},$$

(2.30)
where \( D_{BC} \) is the column depth in the BEDMAP and Crust2.0 model, \( D_P \) is the column depth from propagating the leptons, and \( \lambda_\nu \) is the mean free path of the neutrino. The energy of the secondary neutrino is used to calculate \( \lambda_\nu \) if the path to a shower includes a secondary weak interaction like tau decay.

Different 15-minute blocks of the flight had different measured live times. Each event is weighted by a factor of

\[
\text{live time weight} = \frac{\text{measured live time}}{900 \text{ s}}.
\] (2.31)

This method slightly underweights the events because some seconds during the flight had no noise hits to cause a live time measurement to be made. If the expected aperture of a future flight is being calculated, only the effective area solid angle is calculated by the simulation. In this case, the live time weight is 1.

Most neutrinos pass through the ice without leaving a shower, so many neutrinos have to be simulated just to get a small number of showers. When a shower does happen in the ice, the Cherenkov pulse only reaches a small solid angle of the sky and sometimes does not even leave the ice, so each shower has to be tested for many positions, directions, and payload positions. If the aperture at a single energy is being calculated, each event is weighted by a factor of

\[
\text{sample weight} = \left[ \frac{1}{\text{neutrinos entered}} \right] \left[ \frac{1}{\text{number of times each shower is tested}} \right].
\] (2.32)

If the number of neutrinos detected from a theoretical flux density is being calculated, the number of neutrinos that are simulated depends on the amount of time that is multiplied by the flux density. In this case, each event is weighted by a factor of

\[
\text{sample weight} = \frac{900 \text{ s}}{\text{time of exposure}}.
\] (2.33)
There are two ways for the simulation to detect a double bang event. One is to detect a neutrino based on a radio pulse from the first shower and to identify the event as a double bang based on a pulse from the second shower. The other way is to detect a neutrino based on a radio pulse from the second shower and to identify the event as a double bang based on a pulse from the first shower. In order to compensate for double bang events having twice as many opportunities to be detected in the simulation, each event is weighted by a factor of

\[
\text{multiple bang weight} = \frac{1}{\text{number of radio pulses detected}}. \tag{2.34}
\]

The denominator should be the number of showers detected, but only a few percent of detected showers trigger with the direct pulse and the reflected pulse.

There is a risk that someone analyzing the data will decide the parameters of an analysis cut based in part on a preference that an L3 event be counted (or not be counted) as a neutrino event. In order to reduce this risk, the analysis cuts were tested on 10% of the events while the other 90% remained a blinded sample. This allowed the data analysis to be prepared without knowledge of how a change in a parameter would affect the number of neutrino events reported. Because only 90% of the events were available for the neutrino search, all events in a simulation of the ANITA-1 flight are multiplied by 0.9.

The analysis cuts to be used on the data were applied to a sample of simulated neutrino events. 19% of these simulated events were rejected by the analysis program. To account for the efficiency of the cuts, all events in a simulation of the ANITA-1 flight are multiplied by 0.81.
2.20 Effective Volume Solid Angle

Many subroutines are shared by IceMC and the Ohio simulation. One way to test whether these subroutines have been translated or copied correctly between the two simulations is to ignore the secondary showers and consider only the hadronic shower caused by a neutrino’s first interaction in the Earth. The two simulations should be equally sensitive to primary showers so any significant difference in the results indicates that at least one of them has a bug. The sensitivity should also be independent of neutrino flavor when only primaries are tested. The time averaged aperture from primaries for each flavor is multiplied by the neutrino mean free path to match the effective volume solid angles reported by IceMC.
Chapter 3

Results

3.1 Expected Exposure of ANITA-1

Table 3.1 lists the expected exposure and statistical error of ANITA-1 for several neutrino energies. The option of semicoherent addition of radio pulses was used. The average exposure assumes a 1:1:1 flavor ratio. Higher energy neutrinos can produce higher energy showers that can be detected farther from the payload. If the interaction is charged current, a higher energy charged lepton produces more extra showers that provide more possible viewing angles for the payload. For these reasons, ANITA is more sensitive to higher energy neutrinos.

At $10^{18.5}$ eV, the sensitivity to $\nu_e$ is more than twice the sensitivity to $\nu_\mu$ or $\nu_\tau$. This energy is close to the cutoff so a small difference in shower energy can have a large effect on the probability that the shower will be detected. A mean of 79% of a $\nu_e$'s energy becomes an EM shower in a CC interaction, which makes it much easier to detect than the smaller showers along a $\nu_\mu$ or $\nu_\mu$ track. For higher neutrino energies past $10^{20.5}$ eV, the result of a CC interaction for any neutrino flavor is a series of showers that are detectable but small compared to the initial neutrino energy. ANITA-1 is still more sensitive to $\nu_e$ at these energies but there is not as much of a flavor dependence.
Table 3.1: Sensitivity of ANITA-1. The statistical error is at 95.45% confidence level. The systematic error is not shown.

### 3.2 Primary Showers and Statistical Error

Because each recorded shower is tested for every 15-minute block of the payload position, a shower can be detected in the simulation several times with different event weights. This makes the calculation of statistical error more complicated than $2 \frac{\text{RMS}}{\sqrt{N}}$. Another method is to run the simulation many times and calculate the standard deviation of the mean exposure. The amount of time required to complete this calculation with currently available computers is too long.

The sanity check described in Section 2.20 enabled a method to estimate the statistical error in a reasonable amount of time. The simulation runs used in Section 5.1 and several fast simulation runs with only primaries produced several primary-only exposures that differed from each other only because of different random number sequences. For each energy, anywhere from 8 to 18 simulation runs were used. Because the standard deviation of many simulation runs is about equal to the standard error of a single simulation run, the statistical error can be calculated as 2 standard deviations divided by the mean from primary-only runs.
<table>
<thead>
<tr>
<th></th>
<th>default setting</th>
<th>higher PN cross section</th>
<th>lower PN cross section</th>
</tr>
</thead>
<tbody>
<tr>
<td>run 1 (km$^2$ sr yr)</td>
<td>0.00798</td>
<td>0.00971</td>
<td>0.00801</td>
</tr>
<tr>
<td>run 2 (km$^2$ sr yr)</td>
<td>0.00805</td>
<td>0.00899</td>
<td>0.00733</td>
</tr>
<tr>
<td>run 3 (km$^2$ sr yr)</td>
<td>0.00819</td>
<td>0.00899</td>
<td>0.00750</td>
</tr>
<tr>
<td>mean (km$^2$ sr yr)</td>
<td>0.00807</td>
<td>0.00923</td>
<td>0.00761</td>
</tr>
<tr>
<td>2 SD of mean</td>
<td>1.6%</td>
<td>5.2%</td>
<td>5.4%</td>
</tr>
</tbody>
</table>

Table 3.2: Exposures from running the simulation three times for each of three photonuclear cross sections. Each run used a different random number seed for a total of nine random number sequences.

There are two problems with reporting the fractional statistical error of one run of the full simulation as $2 \frac{\text{SD}}{\text{mean}}$ from primaries. One is that the sanity check in Section 2.20 produces three exposures, one for each neutrino flavor. The exposure of a full simulation is averaged across the three flavors and is the result of simulating 3 times as many neutrinos. In the full simulation, the statistical error for the mean exposure of $R$ runs can therefore be approximated as

$$2\sigma \text{ error} \approx \frac{2}{\sqrt{3R}} \left( \frac{\text{SD}}{\text{mean}} \right) \quad (3.1)$$

The other problem is that the relative statistical variance of ANITA’s sensitivity to primaries might not be equal to the variance of the sensitivity to all showers. This assumption could only be tested at $10^{19}$ eV, where the simulation had to be run 2 extra times for each photonuclear cross section to provide enough multiple bang events for the analyses in Chapter 4. Table 3.2 shows the exposures of the different runs done at $10^{19}$ eV. The errors in Table 3.2 have their own mean of 4.1% and their own relative error of 58% at the 1σ confidence level. This is consistent with the 3.7% statistical error obtained from Equation 3.1 at $10^{19}$ eV.
<table>
<thead>
<tr>
<th>energy</th>
<th>mean exposure from primaries (km² sr yr)</th>
<th>standard deviation (km² sr yr)</th>
<th>number of trial runs (R)</th>
<th>total aperture primaries aperture (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>$2.8 \pm 6%$</td>
<td>0.23</td>
<td>9</td>
<td>$2.0 \pm 11%$</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>$0.91 \pm 5%$</td>
<td>0.066</td>
<td>8</td>
<td>$1.9 \pm 10%$</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>$0.26 \pm 3%$</td>
<td>0.011</td>
<td>9</td>
<td>$2.1 \pm 6%$</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>$0.043 \pm 4%$</td>
<td>0.0025</td>
<td>8</td>
<td>$2.0 \pm 8%$</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>$0.0044 \pm 3%$</td>
<td>0.00025</td>
<td>18</td>
<td>$1.8 \pm 5%$</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>$0.000098 \pm 8%$</td>
<td>0.000012</td>
<td>8</td>
<td>$2.1 \pm 16%$</td>
</tr>
</tbody>
</table>

Table 3.3: Sensitivity of ANITA-1 when only primary showers are considered.

Table 3.3 lists the expected exposure of ANITA-1 using only primary showers. A comparison of the results in Table 3.1 to the means in Table 3.3 indicates that the detection scenario in Figure 1.1b contributes about half of the total aperture of ANITA-1.

### 3.3 Regenerated Neutrinos

In ANITA’s energy range, the neutrino-nucleon cross section is large enough for the Earth to be opaque to neutrinos. For example, a neutrino that hits the ground with a nadir angle of $75^\circ$ will have to pass through a column depth of about 10,000 km of water before it reaches the ice. Even at $10^{18}$ eV, the mean free path is about 1100 km of water so the neutrino will have to propagate through over 9 times its mean free path. However, a neutrino that interacts deep in the Earth can still be detected by a secondary interaction in the ice. One possibility, illustrated in Figure 3.1a, is for a $\nu_\tau$ to produce a $\tau^\pm$ that decays to produce one or two secondary neutrinos that can then reach the detector [60, 35]. For ANITA’s energy range, time dilation is expected
to suppress $\tau^{\pm}$ decay and prevent a significant number of secondary neutrinos above the cutoff energy from reaching the detector by this process.

A second possible regeneration mechanism is for a $\tau^{\pm}$ or $\mu^{\pm}$ to experience a CC interaction and become a secondary $\nu_\tau$ or $\nu_\mu$, as shown in Figure 3.1b. In a long track of 2.6 g cm$^{-3}$ standard rock, a $10^{21}$ eV $\tau^{\pm}$ has a $\sim 27\%$ chance to become a $\nu_\tau$ through a CC interaction before it can decay or drop below $10^{17.4}$ eV. The probability of a $\tau^{\pm}$ experiencing a secondary CC interaction is $\sim 16\%$ at $10^{20}$ eV, $\sim 7\%$ at $10^{19}$ eV, and $\sim 2\%$ at $10^{18}$ eV.

A neutrino can also experience one or more NC interactions before reaching the ice, like in Figure 3.1c. Because a neutrino loses an average of about 21% of its energy in a NC interaction, it can still have about $\frac{3}{5}$ of its original energy after two NC interactions and about half of its original energy after three NC interactions.

For each regeneration scenario, the simulation was run without the showers that occur before neutrino regeneration. In the first run, no showers could be recorded until neutrino propagation after a $\tau^{\pm}$ decay had occurred. In the second run, no
Table 3.4: ANITA’s sensitivity to regenerated neutrinos.

<table>
<thead>
<tr>
<th>energy</th>
<th>$10^{21}$ eV</th>
<th>$10^{20}$ eV</th>
<th>$10^{19}$ eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^{\pm}$ decay</td>
<td>0 km$^2$ sr yr</td>
<td>$0.0002 \approx 0$ km$^2$ sr yr</td>
<td>0 km$^2$ sr yr</td>
</tr>
<tr>
<td>$\tau^{\pm} \rightarrow \nu_{\tau}$</td>
<td>0.08 km$^2$ sr yr</td>
<td>0.002 km$^2$ sr yr</td>
<td>0.0000008 $\approx 0$ km$^2$ sr yr</td>
</tr>
<tr>
<td>$\mu^{\pm} \rightarrow \nu_{\mu}$</td>
<td>0 km$^2$ sr yr</td>
<td>0 km$^2$ sr yr</td>
<td>0 km$^2$ sr yr</td>
</tr>
<tr>
<td>$\nu_{x} \rightarrow \nu_{x}$</td>
<td>0.60 km$^2$ sr yr</td>
<td>0.066 km$^2$ sr yr</td>
<td>0.001 km$^2$ sr yr</td>
</tr>
</tbody>
</table>

Showers could be recorded until neutrino propagation after $\tau^{\pm} \rightarrow \nu_{\tau}$ had occurred.

In the third run, detectable showers required neutrino propagation after $\mu^{\pm} \rightarrow \nu_{\mu}$.

In the fourth run, detectable showers required neutrino propagation after $\nu_{x} \rightarrow \nu_{x}$, not including the antineutrinos created in some $\tau^{\pm}$ decays. The results for all three scenarios are listed in Table 3.4.

Because $\nu_{x} \rightarrow \nu_{x}$ does not involve any electromagnetic energy losses and requires only one weak interaction, it is the most viable mechanism to regenerate neutrinos that can be detected by ANITA. It is not known how often regenerated neutrinos in the simulation increase ANITA’s sensitivity compared to how often they provide secondary radio pulses for events that would have been detected anyway.

### 3.4 Declination of Detected Neutrinos

As shown by ANITA’s relatively small sensitivity to regenerated neutrinos, the Earth can still be considered to be opaque to neutrinos in ANITA’s energy range. Neutrinos on paths that enter the ice at large elevation angles are suppressed by attenuation so neutrino-induced showers in the ice should be mostly downgoing or almost horizontal.
If the axis of a particle shower in the ice points below the local horizontal direction, all parts of the Cherenkov cone point below the critical angle of total internal reflection. In such a geometry, only parts of the Cherenkov pulse beyond the Cherenkov angle can escape the ice. As neutrinos become more downgoing, less of their showers’ Cherenkov pulse energy is able to leave the ice. An exception shown in Figure 2.6b is the downgoing neutrinos in the ice shelves, where a very downgoing radio pulse can reflect off the bottom of the ice and then exit to the air.

With very upgoing neutrinos eliminated by attenuation and very downgoing neutrinos eliminated by total internal reflection, what remains are neutrinos that skim the Earth and are moving in a horizontal direction relative to the payload. Because of the high latitude of ANITA’s path, these skimming neutrinos come from the sky over the low latitudes. The distribution of declinations of detectable neutrinos shown in Figure 3.2 is consistent with these predictions. The largest effective area is at low latitudes. The reflected pulses produce a somewhat uniform tail of the distribution across the high southern latitudes, and there is almost no sensitivity to neutrinos from the high northern latitudes.

Table 3.5 lists the mean declinations of the sky maps. The energy dependence of the mean declination can be explained by the neutrino cross section. Upgoing neutrinos with less energy have a longer mean free path so they are less likely to attenuate before reaching the ice. For lower energies, this extra flux of upgoing neutrinos allows ANITA to be sensitive to sky with slightly larger declination.

Conversely, the energy dependence of the cross section favors higher energy neutrinos if they enter the ice on a downward path because they are more likely to interact
Table 3.5: Mean declination of ANITA-1’s aperture.

<table>
<thead>
<tr>
<th>energy</th>
<th>mean declination</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>-1.3°</td>
<td>12.2°</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>-0.5°</td>
<td>10.9°</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.4°</td>
<td>9.0°</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>1.1°</td>
<td>7.6°</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>1.9°</td>
<td>6.6°</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>3.1°</td>
<td>5.9°</td>
</tr>
</tbody>
</table>

before they have passed completely through the ice. In the ice shelves, where downgoing radio pulses can eventually be detected by ANITA, this increases the probability that a higher energy neutrino will leave a detectable shower.

The effect of the neutrino cross section can also be demonstrated by the fraction of detected neutrinos coming from the high southern latitudes. Table 3.6 shows the expected fraction of neutrinos detected from each of three sections of the sky for three neutrino cross section formulas. Because the cross section increases by a factor of about 2 when the energy increases by a factor of 10, it is seen to be the primary factor in determining the average declination of detected neutrinos. Another ANITA Monte Carlo simulation compared the neutrino event rate and average neutrino direction over the ice shelves to the event rate and average neutrino direction over the ice sheet. The difference between the two results can indicate the energy of the neutrinos. If the neutrino energy can already be measured in a different way, this comparison can instead be used to check the predicted neutrino-nucleon cross section [61].
Figure 3.2: Declination of ANITA-1’s sensitivity to neutrinos at $10^{21}$ eV.

<table>
<thead>
<tr>
<th>energy</th>
<th>declination $\leq -15^\circ$</th>
<th>$-15^\circ &lt; \text{declination} \leq 15^\circ$</th>
<th>declination $&gt; 15^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>6.9%</td>
<td>92.9%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>3.7%</td>
<td>96.1%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.9%</td>
<td>98.6%</td>
<td>0.5%</td>
</tr>
<tr>
<td>twice the expected neutrino cross section</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{21}$ eV</td>
<td>8.9%</td>
<td>90.9%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>4.6%</td>
<td>95.2%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>1.3%</td>
<td>98.5%</td>
<td>0.2%</td>
</tr>
<tr>
<td>half the expected neutrino cross section</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{21}$ eV</td>
<td>6.2%</td>
<td>93.6%</td>
<td>0.2%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>3.4%</td>
<td>96.2%</td>
<td>0.4%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.5%</td>
<td>98.0%</td>
<td>1.5%</td>
</tr>
</tbody>
</table>

Table 3.6: Percentage of detectable neutrinos expected to come from each of three sections of the sky.
3.5 Sky Maps

Figure 3.3 shows the complete directional dependence of ANITA-1’s aperture for six neutrino energies using the Mollweide projection. The surface integrals of the sky maps are the same as the total exposures listed in Table 3.1. The dependence on declination described in Section 3.4 is again seen by a bright band at the low latitudes and less sensitivity elsewhere. The dependence on right ascension is a combination of different ice properties near the payload and statistical fluctuations.

Because the simulation uses the average thermal noise of the sky instead of considering specific objects, the sensitivity dependence on the sun and galactic center is not shown in the sky maps. If this effect were included, the sky maps might show less sensitivity at around \(-90^\circ\) right ascension and more sensitivity elsewhere.

3.6 Response to Theoretical Fluxes

The number of neutrinos ANITA should detect from a theoretical flux of neutrinos can be calculated as

\[
N = \int_{10^{17.5} \text{eV}}^{10^{21} \text{eV}} \Phi(E)A(E)dE, \tag{3.2}
\]

where \(\Phi\) is the theoretical differential flux density and \(A\) is ANITA’s exposure. The results in Table 3.1 are multiplied by the ESS (\(\Lambda = 0.7\)) neutrino flux [6, 62] and the YK neutrino flux [63, 64] to get predictions of \(N = 0.097\) (ESS) and \(N = 0.12\) (YK) events.

The probability to get zero events when there are \(N\) predicted events is

\[
P_0 = e^{-N}. \tag{3.3}
\]
Figure 3.3: Sensitivity sky maps, using coordinates of right ascension and declination.
Figure 3.4: Black: The upper limits of monoenergetic neutrino fluxes at 90% confidence level. Red: The upper limit of an $E^{-2}$ flux is $E^2 \frac{dN}{dE} = 3.7 \times 10^{-7}$ GeV cm$^{-2}$ s$^{-1}$ sr$^{-1}$. These use the exposures from Table 3.1 without considering any margin of error, which will be considered in Section 5.6.

In a zero measurement experiment, the probability that the correct flux is less than a theoretical flux is

$$CL = 1 - P_0 = 1 - e^{-N}.$$  \hspace{1cm} (3.4)

In this case, ESS is ruled out with 9.2% confidence and YK is ruled out with 11% confidence. For a monoenergetic particle flux, $N = \Phi A$. The upper limits at 90% confidence level for monoenergetic fluxes and an $E^{-2}$ flux are plotted in Figure 3.4.

Because the simulation was run at only six constant energies and the aperture has a large dependence on energy, there is a risk of significant binning error. To get an idea of how the large bin size affects the number of predicted events, the simulation tested the neutrino spectra of ESS with bins of size 0.1 eV and $E^{-2}$ as a continuous spectrum. The result is 0.11 events from the ESS flux and 1 event from a flux of
Figure 3.5: Spectrum of neutrinos from the ESS flux that can be detected by ANITA-1.

\[ E^2 \left( \frac{dN}{dE} \right) = 4.3 \times 10^{10} \text{ GeV km}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}. \]  
In this case, ESS is ruled out with 11% confidence and YK is ruled out with 12% confidence. An \( E^{-2} \) flux has a 90% chance to be less than \( E^2 \left( \frac{dN}{dE} \right) = 3.1 \times 10^{-7} \text{ GeV cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}. \)

One concern about determining the response to a theoretical flux is that the result depends on the simulation’s energy range. If the upper limit of integration in Equation 3.2 increases from \(10^{21} \text{ eV}\) to \(10^{22} \text{ eV}\), more neutrinos are injected into the simulation so more neutrino detections can occur. ANITA’s response to the ESS flux in Figure 3.5 drops to zero near \(10^{17.5} \text{ eV}\) and near \(10^{21} \text{ eV}\). If the neutrino spectrum has the same shape as these GZK models, ANITA-1 was most likely to detect neutrinos with energies between \(10^{19} \text{ eV}\) and \(10^{20} \text{ eV}\). Beyond \(10^{21} \text{ eV}\), the incoming flux of neutrinos is too small to make a significant difference in the number of expected events. ANITA’s sensitivity below \(10^{17.5} \text{ eV}\) is too small for a change in the number of neutrinos at that energy to significantly change the number of expected events.
ANITA’s response to the $E^{-2}$ flux in Figure 3.6a shows no detected neutrinos near $10^{17.5}$ eV, but the detected event rate does not drop off before the energy reaches $10^{21}$ eV. The upper limit of the $E^{-2}$ flux in a zero event result can be decreased by extending the energy range of simulated neutrinos up to $10^{22}$ or $10^{23}$ eV. The simulation can not propagate charged leptons above $10^{21}$ eV so an extra set of neutrinos was run between $10^{21}$ eV and $10^{23}$ eV with just primary showers.
Figure 3.6b shows that the response to an $E^{-2}$ flux does get close to zero at $10^{23}$ eV when only primaries are used. ANITA’s total sensitivity below $10^{21}$ eV plus its sensitivity to primaries between $10^{21}$ eV and $10^{23}$ eV shows that a flux of

$$E^2 \left( \frac{dN}{dE} \right) = 3.4 \times 10^{10} \text{ GeV km}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}$$

produces a mean of 1 neutrino event. The 90% upper limit on the $E^{-2}$ flux is

$$E^2 \left( \frac{dN}{dE} \right) = 2.5 \times 10^{-7} \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}.$$ 

The effect of including secondaries in Table 3.3 does not show very much energy dependence. This allows for a tentative guess at what the upper limit would be if the simulation worked above $10^{21}$ eV. Assuming that the relative gain in aperture by including secondaries does not decrease significantly above $10^{21}$ eV, the $E^{-2}$ flux required for 1 neutrino event is

$$E^2 \left( \frac{dN}{dE} \right) = 3.1 \times 10^{10} \text{ GeV km}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}.$$ 

The 90% upper limit on an $E^{-2}$ spectrum would thus be

$$E^2 \left( \frac{dN}{dE} \right) = 2.0 \times 10^{-7} \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ s}^{-1}. $$

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Chapter 4

Multiple Bang Events

4.1 Mass Dependence of the Electromagnetic Cross Sections

The effect that a charged lepton’s mass has on the electromagnetic differential cross section is shown in Figure 4.1 for all three masses and all three interaction types at $10^{19}$ eV. In general, $e^\pm$ have the largest cross section and $\tau^\pm$ have the smallest cross section.

The photonuclear cross section has the least mass dependence and stays roughly proportional to $\frac{1}{y}$ if $y < 0.2$. For $\mu^\pm$ and especially for $\tau^\pm$, the photonuclear cross section is higher than the cross sections of the other interaction types for large energy transfers. Considering the wide angular spread of hadronic showers described in Section 2.11 and ANITA’s greater sensitivity to high energy showers, the photonuclear cross section can have a large effect on the spatial distribution of showers along a $\tau^\pm$ or $\mu^\pm$ track and on ANITA’s ability to detect them.

Direct pair production is the biggest source of continuous energy loss but does not produce very many hard energy transfers. The cross section for $\mu^\pm$ is 1 – 2 orders of magnitude higher than for $\tau^\pm$. The $e^\pm$ cross section is suppressed at lower energy
transfers through the LPM effect as shown in Section A.4.1. This results in $\mu^\pm$ having the largest continuous energy loss rate.

The effect of mass is largest for bremsstrahlung, where over two orders of magnitude separate the cross section of $e^\pm$ from $\mu^\pm$, and the cross section of $\mu^\pm$ from $\tau^\pm$ at $y$ close to 1. The LPM effect shown in Section A.3.1 suppresses the $e^\pm$ cross section for lower energy transfers, causing it to be the only electromagnetic cross section that decreases slower than $\frac{1}{y}$ as $y$ increases.

The differential cross sections are shown again in Figure 4.2 but at $10^{20}$ eV. The photonuclear cross sections are larger but still show the same relative differences with respect to mass. For $e^\pm$, the bremsstrahlung and pair production cross sections are
LPM suppressed further, though bremsstrahlung remains dominant where $y$ is close to 1.

Figure 4.3 shows the effects of mass and energy on the energy loss rate. Because the photonuclear cross section increases with energy, the $\mu^\pm$ and $\tau^\pm$ relative rate of energy loss also increases at higher energy. LPM suppression of bremsstrahlung and direct pair production for $e^\pm$ causes the $e^\pm$ relative rate of energy loss to decrease at higher energy, though it never is less than for $\mu^\pm$ and $\tau^\pm$ in ANITA’s energy range.

Figure 4.4 shows the effects of mass and energy on the mean free path for energy transfers greater than $10^{17.4}$ eV. In most cases, mass and energy have the opposite effect on the mean free path as they do on the energy loss rate. An exception is the mean free path for direct pair production from $\mu^\pm$ and $\tau^\pm$. As the particle energy
increases, $10^{17.4}$ eV matches a smaller value of $y$, which results in a large increase in the number of low energy EM showers along the particle track.

4.2 Shower Distributions

Examples of possible shower distributions for a $10^{20}$ eV neutrino in 1 km of ice are shown in Figure 4.5. For $\tau^{\pm}$, the photonuclear cross section contributes nearly all of the total cross section, so EM showers are rarely produced along the track. The exception is at $\tau^{\pm}$ energies close to $10^{21}$ eV, where more of the low $y$ pair production interactions can transfer energy exceeding $10^{17.4}$ eV. Because of time dilation, $\tau^{\pm}$ decay is mostly limited to where the $\tau^{\pm}$ energy is close to the cutoff and does not significantly shorten the average $\tau^{\pm}$ track length.
A $\mu^{\pm}$’s shorter mean free path makes it create showers that tend to be closer together than on a $\tau^{\pm}$ track. The three electromagnetic cross sections are comparable in size, and more than half of the showers are EM. ANITA-1 is still more likely to detect a hadronic shower from a $\mu^{\pm}$ track because the larger relative energy transfer of photonuclear interactions causes the hadronic showers to typically have more energy than the EM showers.

Because the relative rate of energy loss for electrons decreases as the electron energy increases, the distribution of showers on an electron track depends heavily on the first few stochastic energy losses. An electron that loses a large fraction of its energy in the first hard energy transfer will then experience a much greater rate of
energy loss per energy. An electron that travels the same distance before its first hard energy transfer but loses a small fraction of its energy will continue to travel with little attenuation. This effect should not apply to electron energies \( \gtrsim 10^{23} \text{ eV} \), where the LPM effect is expected to suppress bremsstrahlung and pair production so much that photonuclear interactions provide the most energy loss.

Figure 4.6 shows the distribution of track lengths for each charged lepton at three energies. The effect of \( \tau^\pm \rightarrow \nu_\tau \) is seen in the increased number of short \( \tau^\pm \) tracks at \( 10^{21} \text{ eV} \). The LPM effect is seen in the large energy dependence of the electron track lengths.

### 4.3 Timing Restrictions

After an L3 trigger, a copy of the H and V waveforms without the effect of the band pass filters was recorded for each antenna, covering a time span from \( \sim 50 \text{ ns} \) before the L3 trigger to \( \sim 50 \text{ ns} \) after the trigger. An L3 trigger could not happen again for another 120 ns. This created a dead time of \( \sim 70 \text{ ns} \) for each trigger.
The signal chain leading from the antennas to the waveform buffers created phase shifts that spread the pulses out by $\sim 20$ ns. This made a second pulse difficult to recognize if it arrived less than $\sim 10$ ns after the first pulse. With these restrictions, the simulation assumes a dead time from 0 to 10 ns and from 50 to 120 ns after a trigger when extra radio pulses are counted.
Figure 4.7: The fraction of neutrino events, excluding $\nu_e$, that have a certain number of radio pulses. A larger number of radio pulses indicates a higher neutrino energy.

4.4 Pulses Per Event

A higher energy $\mu^\pm$ or $\tau^\pm$ can leave more showers along its track. In addition to increasing the probability of at least one shower being detected, a larger number of showers can cause more radio pulses to be detected in a neutrino event. The number of radio pulses detected in a multiple bang event may be used as a clue regarding the energy of a neutrino.

Figure 4.7 shows the distribution of the number of recognizable radio pulses per neutrino event for an even mixture of $\nu_\mu$ and $\nu_\tau$. The probability that a neutrino event will be multiple bang ranges from about 35% at $10^{21}$ eV to less than 1% at $10^{19}$ eV. If the fraction of $\mu^\pm$ or $\tau^\pm$ events that are multiple bang is not close to 9%,
Figure 4.8: Two of the histograms from Figure 4.7 are split into $\nu_\mu$ and $\nu_\tau$. The flavor does not significantly affect the number of radio pulses seen.

This can indicate that the shape of the neutrino spectrum is not close to the spectrum predicted by ESS.

The effect that mass has on the number of radio pulses per event is not as straightforward. Although $\mu^\pm$ typically produce more showers than $\tau^\pm$ do, the extra radio pulses can arrive during the effective dead time. Figure 4.8 shows the distribution of the number of pulses for $\nu_\mu$ and $\nu_\tau$ at $10^{20}$ eV and the ESS spectrum. $\nu_\tau$ lead to slightly more detectable radio pulses per event, but it is not enough of a difference to be used as a method to identify neutrino flavor.
4.5 Flavor Identification

Any difference in the energy loss rate, shower energy, distance between showers, track length, and angular spread of Cherenkov pulses can affect the average amount of time separating the arrivals of radio pulses in a single neutrino event. As shown earlier, these properties of a $\mu^\pm$ or $\tau^\pm$ track are affected by the energy and mass of the particle. If many neutrinos are detected, the average time between the arrival times of the pulses in multiple bang events can hint at the energy and flavor of the neutrinos.

Figure 4.9 shows the time separating radio pulses in multiple bang events that contain exactly two recognizable pulses, with the results summarized in Table 4.1. The gap in each histogram is the result of the detector’s dead time between 50 ns and 120 ns after the first trigger. The $\tau^\pm$ energy has little effect on the geometric mean of the separation times, which increases by $\sim 50\%$ as the $\tau^\pm$ energy increases from $10^{19}$ eV to $10^{21}$ eV. The $\mu^\pm$ energy can be more easily extracted from the average separation time, which increases by $\sim 160\%$ as the energy increases from $10^{19}$ eV to $10^{21}$ eV.

Because the geometric mean of the separation times for $\mu^\pm$ is less than for $\tau^\pm$, a value significantly different from what is calculated for $\theta_{23} = 45^\circ$ can indicate that the flux of $\mu^\pm$ is not almost equal to the flux of $\tau^\pm$. This method of flavor identification does not work if large fractions of the detected neutrinos have energies near $10^{19}$ eV and $10^{21}$ eV because the average separation time for $\mu^\pm$ at higher energies will overlap the separation time for $\theta_{23} = 45^\circ$ at lower energies.

Figure 4.10 and Table 4.2 show the same results as Figure 4.9 and Table 4.1, except they include all multiple bang events involving $\nu_\mu$ or $\nu_\tau$. To calculate the
Figure 4.9: Distribution of times separating the radio pulses in events that contain just two radio pulses. There is dead time from 50 ns to 120 ns after a trigger.

<table>
<thead>
<tr>
<th>neutrino energy or model</th>
<th>( \nu_\mu )</th>
<th>RMS</th>
<th>( \nu_\tau )</th>
<th>RMS</th>
<th>( \theta_{23} = 45^\circ )</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{21} \text{ eV} )</td>
<td>2.22</td>
<td>0.68</td>
<td>2.57</td>
<td>0.64</td>
<td>2.39</td>
<td>0.68</td>
</tr>
<tr>
<td>( 10^{20} \text{ eV} )</td>
<td>2.03</td>
<td>0.64</td>
<td>2.55</td>
<td>0.62</td>
<td>2.30</td>
<td>0.68</td>
</tr>
<tr>
<td>( 10^{19.5} \text{ eV} )</td>
<td>1.96</td>
<td>0.58</td>
<td>2.49</td>
<td>0.45</td>
<td>2.25</td>
<td>0.61</td>
</tr>
<tr>
<td>( 10^{19} \text{ eV} )</td>
<td>1.79</td>
<td>0.57</td>
<td>2.39</td>
<td>0.59</td>
<td>2.05</td>
<td>0.64</td>
</tr>
<tr>
<td>ESS</td>
<td>2.12</td>
<td>0.60</td>
<td>2.44</td>
<td>0.62</td>
<td>2.28</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Table 4.1: \( \log_{10}(\text{time/ns}) \) separating the radio pulses in events that have two pulses.
Figure 4.10: Distribution of mean times separating the radio pulses in all multiple bang events. For events with more than two pulses, the mean separation time can have a value that is excluded by dead time in double bang events.

<table>
<thead>
<tr>
<th>energy or model</th>
<th>$\nu_\mu$</th>
<th>RMS</th>
<th>$\nu_\tau$</th>
<th>RMS</th>
<th>$\theta_{23} = 45^\circ$</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>2.23</td>
<td>0.58</td>
<td>2.55</td>
<td>0.52</td>
<td>2.40</td>
<td>0.58</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>2.05</td>
<td>0.61</td>
<td>2.56</td>
<td>0.57</td>
<td>2.34</td>
<td>0.64</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>1.95</td>
<td>0.56</td>
<td>2.50</td>
<td>0.58</td>
<td>2.25</td>
<td>0.63</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>1.79</td>
<td>0.58</td>
<td>2.39</td>
<td>0.45</td>
<td>2.05</td>
<td>0.61</td>
</tr>
<tr>
<td>ESS</td>
<td>2.16</td>
<td>0.59</td>
<td>2.45</td>
<td>0.57</td>
<td>2.31</td>
<td>0.60</td>
</tr>
</tbody>
</table>

Table 4.2: $\log_{10}(\text{mean(time/ns)})$ separating the radio pulses in all multiple bang events.
Figure 4.11: Photonuclear energy loss rates. Orange: $e^\pm$. Blue: $\mu^\pm$. Green: $\tau^\pm$. Solid: ALLM97 (see Figure 4.3). Dot-dashed: Increased by twice the stated error. Dotted: Decreased by twice the stated error.

separation time in this case, the arrival time of the first pulse is subtracted from the arrival time of the last pulse and then divided by one less than the number of detected radio pulses.

4.6 Proposed Measurement of the Photonuclear Cross Section

Because the difference in the average time separating radio pulses for $\nu_\mu$ and $\nu_\tau$ depends in part on their different photonuclear cross sections, an uncertainty in the photonuclear cross section can create an uncertainty in the expected pulse separation time. The simulation was run with the parameters of the ALLM97 [65] photonuclear cross section shifted by twice the margins of error stated by ALLM91 [66] in whichever
direction would increase $\frac{d\sigma}{dy}$. The same was done with the parameters shifted in whichever direction would decrease $\frac{d\sigma}{dy}$. Figure 4.11 shows the photonuclear energy loss rates as expected from ALLM97 (see Figure 4.3), increased by twice its stated error, and decreased by twice its stated error. Figure 4.12 shows the mean free paths as expected from ALLM97 (see Figure 4.4) and with the same two changes in cross section. In both examples, the uncertainty in the photonuclear cross section is greater than the effect of changing the particle mass.

A measurement of the flavor ratio by ANITA would also be redundant. $\theta_{23}$ has already been measured to within $\sin^2(2\theta_{23}) > 0.92$ [31]. If the $\nu_\mu$ and $\nu_\tau$ fluxes are assumed to be almost equal, the average time separating radio pulses in multiple bang events can instead be used to test the photonuclear cross section.

Table 4.3 lists the average logarithm of the time separating the pulses of double bang events if the photonuclear cross section is increased or decreased by twice its...
Table 4.3: Same as Table 4.1, but with $\theta_{23}$ fixed at 45° and the photonuclear cross section shifted by twice its stated error.

<table>
<thead>
<tr>
<th>Neutrino energy</th>
<th>Expected photonuclear cross section</th>
<th>RMS</th>
<th>lower</th>
<th>RMS</th>
<th>higher</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td></td>
<td>2.39</td>
<td>0.68</td>
<td>2.56</td>
<td>0.69</td>
<td>1.70</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td></td>
<td>2.30</td>
<td>0.68</td>
<td>2.43</td>
<td>0.69</td>
<td>1.75</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td></td>
<td>2.05</td>
<td>0.64</td>
<td>1.85</td>
<td>0.60</td>
<td>1.58</td>
</tr>
</tbody>
</table>

Figure 4.13: Top row: Distribution of times separating the radio pulses in events that contain just two radio pulses if the photonuclear cross section is decreased by twice its stated error. Bottom row: Same distribution but with the cross section increased by twice its stated error.
<table>
<thead>
<tr>
<th>neutrino energy</th>
<th>expected photonuclear cross section</th>
<th>RMS</th>
<th>lower</th>
<th>RMS</th>
<th>higher</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>2.40</td>
<td>0.58</td>
<td>2.56</td>
<td>0.60</td>
<td>1.74</td>
<td>0.59</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>2.34</td>
<td>0.64</td>
<td>2.44</td>
<td>0.66</td>
<td>1.78</td>
<td>0.55</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>2.05</td>
<td>0.61</td>
<td>1.85</td>
<td>0.59</td>
<td>1.58</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Table 4.4: Same as Table 4.2, but with $\theta_{23}$ fixed at 45° and the photonuclear cross section shifted by twice its stated error.

Figure 4.14: Same as Figure 4.13 but using the mean separation time in all multiple bang events.
stated error. The distribution of separation times is shown in Figure 4.13. A decrease in the cross section causes little change in the separation times at $10^{20}$ and $10^{21}$ eV and strongly suppresses double bang events at $10^{19}$ eV. An increase in the cross section causes the separation times to decrease by an amount similar to the root mean square (RMS), regardless of neutrino energy. If the mean logarithm of the separation time is less than 1.7, which corresponds to about 50 ns, this can indicate that the photonuclear cross section is at the high end of the range predicted by ALLM.

Table 4.4 and Figure 4.14 repeat Table 4.3 and Figure 4.13 using the mean separation times in all multiple bang events. The results are almost unchanged. Figure 4.15 shows the distribution of the number of radio pulses per event for the higher
and lower photonuclear cross sections. The results are close enough to the results in Figure 4.7 that a large uncertainty in the photonuclear cross section does not prevent ANITA from estimating the neutrino energy from the number of pulses per event.

4.7 Required Number of Events

The change in mean separation time for different photonuclear cross sections is similar to the RMS so several multiple bang events are required for the margin of error to be smaller than the effect of changing the cross section. Tens of multiple bang events are required to get a reliable measurement of the cross section. Even in the best case scenario where most of the detected neutrinos have energy near $10^{21}$ eV, there will be 2 or 3 single bang events for every multiple bang event, and ANITA will have to detect about 100 neutrinos in order to measure the photonuclear cross section. If the neutrino flux is closer to the GZK models, the typical neutrino energy will be lower and ANITA will have to detect several hundred neutrino events.

The flavor dependence of the pulse timing is even smaller, meaning thousands of single bang events might have to be detected in order for there to be enough multiple bang events to identify the ratio of $\nu_\mu$ to $\nu_\tau$. ANITA-1 did not detect any multiple bang events or any other neutrino events [67]. This method of identifying neutrino flavor and photonuclear cross section might still be used by other neutrino detectors in the future, but not by ANITA-1.
Chapter 5

Systematic Error

5.1 Uncertainty in the Photonuclear Cross Section

Because of the photonuclear cross section tends to dominate for high energy transfers and because a radio pulse from a hadronic shower has a wider angular spread than from an EM shower, a change in the photonuclear cross section can have a significant effect on the total sensitivity of ANITA. An increase in the cross section can cause an increase in the hadronic shower energy, which causes an increase in the distance a detectable shower can be from the payload. It can also shorten the particle track, which reduces the number of showers that occur in the rock or the air. An increase in the photonuclear energy loss rate causes a smaller fraction of the particle energy to be transferred to the mostly continuous energy loss of direct pair production.

It is also possible for an increase in the photonuclear cross section to decrease the total sensitivity. While the length of a very upgoing or downgoing track is likely to be limited by the distance to the rock or air, a particle moving horizontally can have over 100 km of ice in front of it. By shortening the track, an increase in the cross section can decrease the range of payload positions to be hit by a strong radio pulse.

Like in Section 4.6, the parameters from ALLM97 were shifted by twice the stated error in whichever direction would increase $\frac{d\sigma}{dy}$. To check for asymmetry in the error,
Table 5.1: The exposures of the ANITA-1 flight if charged leptons have different photonuclear cross sections.

<table>
<thead>
<tr>
<th>$10^{14}$ eV</th>
<th>ALLM97 cross section</th>
<th>higher PN cross section</th>
<th>change</th>
<th>lower PN cross section</th>
<th>change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.4 km$^2$ sr yr</td>
<td>5.5 km$^2$ sr yr</td>
<td>2%</td>
<td>4.7 km$^2$ sr yr</td>
<td>-13%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.54 km$^2$ sr yr</td>
<td>0.61 km$^2$ sr yr</td>
<td>13%</td>
<td>0.48 km$^2$ sr yr</td>
<td>-11%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.0081 km$^2$ sr yr</td>
<td>0.0092 km$^2$ sr yr</td>
<td>14%</td>
<td>0.0076 km$^2$ sr yr</td>
<td>-6%</td>
</tr>
</tbody>
</table>

The process was repeated with the parameters shifted in whichever direction would decrease $\frac{d\sigma_n}{dy}$. The photonuclear energy loss rates for both shifts are shown in Figure 4.11.

The simulation was then run at $10^{19}$, $10^{20}$, and $10^{21}$ eV to find how much sensitivity ANITA gained or lost from a change in the photonuclear cross section. Table 5.1 lists the results. The 2% increase in exposure at $10^{21}$ eV after increasing the cross section is within the simulation’s margin of statistical error. In the other 5 tests, the exposure increased slightly for a larger cross section and decreased slightly for a smaller cross section.

5.2 Uncertainty in the Neutrino-nucleon Cross Section

If the neutrino-nucleon cross section is small compared to its predicted value, too many neutrinos will pass through the ice without interacting and ANITA will have a smaller aperture. If the cross section is much larger than expected, more neutrinos will interact before they can enter the ice, which can also cause ANITA to have a smaller aperture. A small increase in the cross section can therefore increase the aperture by making a neutrino that enters the ice more likely to interact in the ice,
or it can decrease the aperture by suppressing the flux of upgoing neutrinos in the ice.

The neutrino-nucleon cross section is given a factor of 2 margin of error when the CTEQ4-DIS parton distribution function is used [5], and this analysis assumes the same fractional error for the more recent calculation [33]. The simulation was run at $10^{19}$, $10^{20}$, and $10^{21}$ eV to find the overall effect that a factor of 2 change in the neutrino cross section has on the exposure of ANITA-1. Table 5.2 lists the results. In all cases, the exposure increased for a larger cross section and decreased for a smaller cross section.

### Table 5.2: The exposures of the ANITA-1 flight if the neutrino-nucleon cross section is changed by a factor of 2.

<table>
<thead>
<tr>
<th>Energy</th>
<th>GQRS and Reno cross section $\times 2$</th>
<th>change</th>
<th>GQRS and Reno cross section $\div 2$</th>
<th>change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>5.4 km$^2$ sr yr</td>
<td>8.5 km$^2$ sr yr</td>
<td>57%</td>
<td>3.2 km$^2$ sr yr</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.54 km$^2$ sr yr</td>
<td>0.85 km$^2$ sr yr</td>
<td>58%</td>
<td>0.33 km$^2$ sr yr</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.0081 km$^2$ sr yr</td>
<td>0.0095 km$^2$ sr yr</td>
<td>18%</td>
<td>0.0069 km$^2$ sr yr</td>
</tr>
</tbody>
</table>

5.3 Uncertainty in Pulse Interference

The option to use semicoherent addition of radio pulses from electron tracks (Section 2.16) and the option to test the pulses one at a time each has its own correct and incorrect physics. Because semicoherence is only used for electron tracks, this difference in the trigger algorithms could be what causes the sensitivity to $\nu_e$ to be higher than the sensitivity to $\nu_\mu$ and $\nu_\tau$ in Table 3.1. To determine how this decision affects the estimate of ANITA’s sensitivity, the simulation was run without any coherent addition of radio pulses. Table 5.3 lists the results for a 1:1:1 flavor ratio and
for just $\nu_e$. The change in exposure at all energies except $10^{18.5}$ eV was within the margin of statistical error.

### 5.4 Uncertainty in Surface Roughness

The surface of the firn in the simulation follows a smooth slope from one ice bin to the next. On the path of a real radio pulse in Antarctica, the surface of the firn can have sastrugi and other roughness features. A rough surface can decrease the sensitivity by spreading out the radio pulses and decreasing their flux into the antennas. A rough surface can also increase the sensitivity by allowing pulses to exit the ice at angles that would otherwise result in total internal reflection.

One of the other ANITA simulations, the Hawaii code, estimates the strength of the signal from the average power transmitted through a rough surface [26], based on measurements taken with a small scaled model of the surface using sand and infrared irradiation.

Table 5.3: The exposure of the ANITA-1 flight if each radio pulse from an electron track is evaluated independently for a trigger.

<table>
<thead>
<tr>
<th></th>
<th>1:1:1 flavor ratio, semi-coherence (km$^2$ sr yr)</th>
<th>1:1:1 flavor ratio, no coherence (km$^2$ sr yr)</th>
<th>change</th>
<th>just $\nu_e$, semi-coherence (km$^2$ sr yr)</th>
<th>just $\nu_e$, no coherence (km$^2$ sr yr)</th>
<th>change</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>9.9</td>
<td>9.3</td>
<td>-6%</td>
<td>11.0</td>
<td>10.0</td>
<td>-9%</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>3.0</td>
<td>3.2</td>
<td>9%</td>
<td>3.7</td>
<td>4.1</td>
<td>11%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.85</td>
<td>0.81</td>
<td>-4%</td>
<td>1.1</td>
<td>1.0</td>
<td>-8%</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>0.13</td>
<td>0.13</td>
<td>-3%</td>
<td>0.19</td>
<td>0.17</td>
<td>-9%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.012</td>
<td>0.011</td>
<td>-7%</td>
<td>0.016</td>
<td>0.015</td>
<td>-10%</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>0.0003</td>
<td>0.0004</td>
<td>27%</td>
<td>0.0005</td>
<td>0.0007</td>
<td>51%</td>
</tr>
</tbody>
</table>
light [68]. At $10^{19}$ eV, inclusion of the surface roughness causes the sensitivity to increase by an average of 130%. This result had its own error of $+45\% -40\%$ [26].

### 5.5 Uncertainty from Noise Hits

The L0 trigger thresholds were set to allow thermal noise to trigger each channel at a rate of $\sim2.65$ MHz. This caused the payload to trigger at a rate of about 4-5 Hz in order to help monitor the health of the detector. For a noise trigger rate of 4-5 Hz, the payload’s full set of 100 ns noise waveforms has about a 0.000045% chance to get an L3 trigger. This probability is much greater when the simulation uses semicoherent pulses from electron tracks and has to search for triggers on waveforms much longer than 100 ns.

A simulated radio pulse that is too weak to trigger the payload on its own can be recorded as an event if the noise power is strong enough at that time to cause an L3 trigger. When the analysis cuts were applied to simulated events to determine the efficiency of the cuts, only a single radio pulse was used for each event. This kept the waveforms too short to determine how often a strong noise fluctuation occurred close to the arrival time of a weak radio pulse. The simulation uses more restrictive coincidence triggers to try to reduce the rate of false positives (see Section 2.17). It also includes some of IceMC’s cuts that prevent weak radio signals from being tested for a trigger.

In order to determine the rate of false positives, the simulation was run without the signal waveforms, meaning only the noise waveforms could cause triggers. For each energy and flavor, Table 5.5 gives the fraction of triggered events that would
have triggered anyway with just noise. This error is ~1% of the sensitivity to $\nu_e$ and is about 0% when only the 100 ns waveforms are used.

### 5.6 Total Systematic Error

Each systematic error was calculated by comparing 2 or 3 possible exposures, which are not enough to directly provide an error probability distribution function (EPDF) that can be used for error propagation. Several assumptions will therefore have to be made about the EPDFs in order to calculate an estimate of the total uncertainty in the exposure of ANITA-1.

The systematic errors calculated for ANITA represent relative changes in the aperture (± 10%) rather than absolute changes (± 3 km$^2$ sr yr). If the errors are small, for example 1% and 2%, they can be safely propagated on a linear scale because the result of adding [0.01 + 0.02] is nearly equal to the result of multiplying by [1.01 × 1.02]. If the error is large like the +58% -39% effect of changing the neutrino cross section at $10^{20}$ eV, error propagation on a linear scale tends to underestimate the total uncertainty regarding a possible increase in aperture and overestimate the uncertainty.

<table>
<thead>
<tr>
<th>$10^{21}$ eV</th>
<th>$10^{20.5}$ eV</th>
<th>$10^{20}$ eV</th>
<th>$10^{19.5}$ eV</th>
<th>$10^{19}$ eV</th>
<th>$10^{18.5}$ eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1:1</td>
<td>1:1:1</td>
<td>1:1:1</td>
<td>1:1:1</td>
<td>1:1:1</td>
<td>1:1:1</td>
</tr>
<tr>
<td>$\nu_e$</td>
<td>$\nu_{\mu}$</td>
<td>$\nu_{\tau}$</td>
<td>$\nu_e$</td>
<td>$\nu_{\mu}$</td>
<td>$\nu_{\tau}$</td>
</tr>
<tr>
<td>0.47%</td>
<td>1.2%</td>
<td>0.07%</td>
<td>0.06%</td>
<td>0.43%</td>
<td>0.04%</td>
</tr>
<tr>
<td>0.47%</td>
<td>0.06%</td>
<td>0.03%</td>
<td>0.03%</td>
<td>0.04%</td>
<td>0.03%</td>
</tr>
<tr>
<td>0.51%</td>
<td>1.1%</td>
<td>0.07%</td>
<td>0.03%</td>
<td>0.47%</td>
<td>0.06%</td>
</tr>
<tr>
<td>0.47%</td>
<td>0.93%</td>
<td>0.06%</td>
<td>0.05%</td>
<td>0.54%</td>
<td>0.99%</td>
</tr>
<tr>
<td>0.54%</td>
<td>0.99%</td>
<td>0.1%</td>
<td>0.08%</td>
<td>0.8%</td>
<td>1%</td>
</tr>
<tr>
<td>0.8%</td>
<td>1%</td>
<td>0%</td>
<td>0%</td>
<td>0.8%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Table 5.4: Percentage of reported sensitivity that was probably just a result of random noise in the simulation.
regarding a possible decrease in aperture. The EPDFs of ANITA’s systematic errors are on a logarithmic scale so \( \pm 58\% -39\% \) is instead \( \pm \ln(1.58) - \ln(1/(1 - 0.39)) \).

The errors described in Sections 5.3 and 5.5 are each a result of finding the exposure using a different Monte Carlo method. In this case, the EPDF is assumed to be a normalized Gauss function centered on the geometric mean of the two results. 68.27\% of the area is between the two reported exposures. If \( r' \) is the ratio of the two exposures \( \left( \frac{A_1}{A_0} \right) \), the EPDF is

\[
F\{\ln(r)\} = \frac{2}{\ln(r') \sqrt{2\pi}} e^{-\frac{[\ln(r) - \ln(r')/2]^2}{2[\ln(r')/2]^2}}.
\]

The errors described in Sections 5.1 and 5.2 involve an expected exposure and two other exposures that should bound the true exposure at a confidence level of about 2\( \sigma \) (95.45\%). These errors are asymmetric so their EPDFs can not be Gauss functions. Instead, the EPDF is assumed to be a normalized Beta distribution [69].

\[
F\{\ln(r), \alpha, \beta, C_0, C_1\} = \frac{(C_1 - C_0)\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \left[ \frac{\ln(r) - C_0}{C_1 - C_0} \right]^{\alpha-1} \left[ 1 - \frac{\ln(r) - C_0}{C_1 - C_0} \right]^{\beta-1} 
\int_{C_0}^{C_1} F\{\ln(r), \alpha, \beta, C_0, C_1\} \, d[\ln(r)] = 1,
\]

where \( r = A/A_0 \) is a relative change in exposure and \( \Gamma \) is calculated according to the approximation in [70]. The median of the EPDF is at the expected exposure, 2.275\% of its area is below the lower exposure, and 2.275\% of its area is above the upper exposure.

There is no unique solution to fitting four parameters to three exposures so the Beta distributions must be given one more property. Consider two Gauss functions, one with 2\( \sigma \) equal to the upper error and the other with 2\( \sigma \) equal to the lower error. The width of the Beta distribution at half the height of its median is equal to the
Figure 5.1: a) A nearly symmetric error probability distribution representing the $+58\% -39\%$ error from uncertainty in the neutrino cross section at $10^{20}$ eV. b) A more skewed error probability distribution representing the $+14\% -6\%$ error from uncertainty in the photonuclear cross section at $10^{19}$ eV. In both cases, the median is fixed at the point where there is no change in the exposure.

mean full width at half maximum of the Gauss functions that match the upper and lower margins of error. This causes the EPDF to be similar to a Gauss function when the errors are almost symmetric. In the case of a $+14\% -6\%$ error, the equations used to find the values of $\alpha$, $\beta$, $C_0$, and $C_1$ are
\begin{align*}
\int_{C_0}^{\ln(1-0.06)} F\{\ln(r), \alpha, \beta, C_0, C_1\} \, d[\ln(r)] &= 0.02275 \\
\int_{C_0}^{0} F\{\ln(r), \alpha, \beta, C_0, C_1\} \, d[\ln(r)] &= 0.5 \\
\int_{\ln(1+0.14)}^{C_1} F\{\ln(r), \alpha, \beta, C_0, C_1\} \, d[\ln(r)] &= 0.02275 \\
\text{full width at} \quad \frac{F\{0, \alpha, \beta, C_0, C_1\}}{2} &= \left[\ln(1 + 0.14) - \ln(1 - 0.06)\right] \sqrt{\frac{\ln(2)}{2}}. \quad (5.3)
\end{align*}

Figure 5.1 shows a Beta distribution that matches Equations 5.3, along with a less skewed Beta distribution used for an error of +58% -39%.

The errors from Sections 5.1 and 5.2 were not calculated at $10^{18.5}$, $10^{19.5}$, and $10^{20.5}$ eV. At $10^{19.5}$, and $10^{20.5}$ eV, the relative error is instead estimated by linear interpolation on the logarithmic energy scale. The relative error at $10^{18.5}$ eV is estimated to be equal to the relative error at $10^{19}$ eV.

The surface roughness error described in Section 5.4 has a +45% -40% systematic error of its own. First, an intermediate EPDF $I\{\ln(r')\}$ is calculated as a Beta distribution the same way the other asymmetric errors are calculated, but with the median at $\ln(2.3)$ instead of at $\ln(1)$. If some point $r'$ on the intermediate EPDF is assumed to be the result of simulating with a rough surface, a final EPDF can be calculated like in Equation 5.1. By integrating over all possible values of $I\{\ln(r')\}$, the final EPDF for the effect of surface roughness is calculated as

\[
F\{\ln(r)\} = \int I\{\ln(r')\} \left[ \frac{2}{\ln(r') \sqrt{2\pi}} \right] e^{-\frac{(\ln(r)-\ln(r'))^2}{2 \ln(r')^2}} \, d[\ln(r')]. \quad (5.4)
\]

Figure 5.2 shows the intermediate and final EPDFs from the surface roughness error.

To find the total error, a Monte Carlo program randomly chooses a value of $\ln(r)$ from each EPDF and records their sum. After 800,000 trials, the median value of $r$ is
Figure 5.2: a) Error probability distribution function $I\{\ln(r')\}$ representing the +45\% -40\% error of the Hawaii version of the ANITA simulation when surface roughness is used. Its median is 2.3 times the result with no surface roughness ($\ln(r) = \ln(A/A_0) = \ln(2.3)$). b) What the probability distribution due to surface roughness would be if the Hawaii version had no margin of error while using surface roughness. 68.27\% of the area is between $r = 1$ and $r = 2.3$. This would be $F\{\ln(r)\}$ if $I\{\ln(r')\}$ were a delta function at $r = 2.3$. c) The final EPDF due to surface roughness.
Figure 5.3: Blue, red, and green: The EPDFs at $10^{19}$ eV from Sections 5.3, 5.1, and 5.2. Black: The convolution of those EPDFs plus the negligible error from Section 5.5. 95.45% of the area is between the vertical bars.

recorded along with the values of $r$ that separate the top and bottom 2.275%. Figure 5.3 shows three of the EPDFs at $10^{19}$ eV along with the convolution of them.

The exposures in Table 3.1 are then multiplied by the different values of $r$ to find the expected range of exposures. Table 5.5 lists the results of combining the errors from photonuclear cross section, neutrino cross section, semicoherence, and thermal noise (Sections 5.3, 5.1, 5.2 and 5.5). Table 5.6 gives the same results as Table 5.5 but with the statistical error included. Table 5.7 also includes the error from surface roughness (Section 5.4). The numbers in Tables 5.6 and 5.7 are plotted in Figure 5.4.

Equation 3.3 with $N = \Phi A$ gives the probability to detect zero events from a monoenergetic particle flux when there is no margin of error. When there is an
<table>
<thead>
<tr>
<th>energy</th>
<th>average sensitivity (km$^2$ sr yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>5.1 +63% -44%</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>1.8 +66% -44%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.52 +67% -43%</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>0.088 +45% -34%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.0078 +28% -24%</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>0.00024 +44% -32%</td>
</tr>
</tbody>
</table>

Table 5.5: Sensitivity of ANITA-1, including the systematic errors from Sections 5.3, 5.1, 5.2 and 5.5, attempting a 95.45% confidence level.

<table>
<thead>
<tr>
<th>energy</th>
<th>average sensitivity (km$^2$ sr yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>7.5 +200% -62%</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>2.6 +200% -62%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.77 +200% -62%</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>0.13 +190% -59%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.011 +180% -56%</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>0.00035 +180% -59%</td>
</tr>
</tbody>
</table>

Table 5.6: Same as Table 5.5 but including statistical errors.

<table>
<thead>
<tr>
<th>energy</th>
<th>average sensitivity (km$^2$ sr yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{21}$ eV</td>
<td>7.5 +200% -62%</td>
</tr>
<tr>
<td>$10^{20.5}$ eV</td>
<td>2.6 +200% -62%</td>
</tr>
<tr>
<td>$10^{20}$ eV</td>
<td>0.77 +200% -62%</td>
</tr>
<tr>
<td>$10^{19.5}$ eV</td>
<td>0.13 +190% -59%</td>
</tr>
<tr>
<td>$10^{19}$ eV</td>
<td>0.011 +180% -56%</td>
</tr>
<tr>
<td>$10^{18.5}$ eV</td>
<td>0.00035 +180% -59%</td>
</tr>
</tbody>
</table>

Table 5.7: Same as Table 5.6 but including the systematic error from surface roughness.
uncertainty in the exposure, the probability to detect zero events is [71]

\[ P_0 = \int_0^{\infty} e^{-\Phi A} F\{A\} \, dA. \]  \hspace{1cm} (5.5)

Figure 5.5 shows the upper limits at 90% confidence level using the EPDFs that were used for Table 5.6. The results are almost unchanged from the upper limits calculated from the default simulation runs.
Figure 5.5: Black: Upper limits at 90% confidence level from Table 5.6. Purple: From Figure 3.4, the upper limits based on the default simulation runs.
A Monte Carlo simulation of ANITA-1 was used to study the effects of secondary particle showers. The expected aperture increased by 82 – 114%, though high fidelity in the simulation of secondaries comes at a cost of increased processing time per neutrino and thus a larger statistical error in the results. ANITA-1 was expected to detect roughly 0.11 neutrinos, assuming the ESS model of GZK neutrinos to be correct. The 90% upper limit on an $E^{-2}$ neutrino flux is $E^2 \left(\frac{dN}{dE}\right) = 2.5 \times 10^{-7}$ GeV cm$^{-2}$ sr$^{-1}$ s$^{-1}$.

ANITA-1 is mostly sensitive to neutrinos coming from declinations of between $-10^\circ$ and $+10^\circ$. The mean declination of the aperture per solid angle decreases slightly as neutrino energy increases. It also decreases slightly as the theoretical neutrino cross section increases.

If hundreds of neutrinos were detected, the distribution of the number of radio pulses detected per event could have indicated the energy of the neutrinos. Another proposed analysis of many neutrino events was to use the average time separating the radio pulses in multiple bang events to estimate the photonuclear cross section and to a lesser extent the neutrino flavor distribution. The possibility of using these methods to determine some of the properties of GZK neutrinos can not be used in ANITA-1 because no neutrino events were detected in the experiment.
Several sources of systematic error were analyzed and showed a total uncertainty in the exposure of roughly a factor of 1.5. The total uncertainty becomes a factor of 3 after including a surface roughness error analyzed in a different Monte Carlo.
Bibliography


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http://mahi.ucsd.edu/Gabi/rem.dir/crust/crust2.html.


http://nsidc.org/data/thermap/antarctic_10m_temps/.


Appendix A

Cross Section Formulas

A.1 Pair Production from Photons

The pair production cross section is from [40].

\[
\frac{d\sigma_{\gamma\rightarrow e^+ e^-}(E_{\gamma}, E_{e^+})}{dE_{e^+}} = \frac{4\alpha^2 e^2 \xi(\tilde{s})}{3E_{\gamma}} \left\{ G(\tilde{s}) + 2 \left[ \frac{E_{e^+}^2}{E_{\gamma}^2} + \left( 1 - \frac{E_{e^+}}{E_{\gamma}} \right)^2 \right] \phi(\tilde{s}) \right\}
\]

\[
G(\tilde{s}) = 3 - 3e^{-4\tilde{s}-8\tilde{s}^2/(1+3.96\tilde{s}+4.97\tilde{s}^2-0.05\tilde{s}^3+7.5\tilde{s}^4)} - 2\phi(\tilde{s})
\]

\[
\phi(\tilde{s}) = 1 - \frac{e^{-6\tilde{s}[1+(3-\pi)\tilde{s}]+\tilde{s}^3/(0.623+0.796\tilde{s}+0.658\tilde{s}^2)}}{1+(3-\pi)\tilde{s}+\tilde{s}^3/(0.623+0.796\tilde{s}+0.658\tilde{s}^2)}
\]

\[
\tilde{s} = \frac{E_{\text{LPM}} E_{\gamma}}{8E_{e^+} (E_{\gamma} - E_{e^+}) \xi(\tilde{s})} \approx \frac{E_{\text{LPM}} E_{\gamma}}{8E_{e^+} (E_{\gamma} - E_{e^+})}
\]

\[
\xi(\tilde{s}) = 2 \quad (\tilde{s} \leq \sqrt{2} s_1)
\]

\[
\xi(\tilde{s}) = 1 + h - \frac{0.08(1 - h)[1 - (1 - h)^2]}{\ln(\sqrt{2} s_1)} \quad (\sqrt{2} s_1 < \tilde{s} < 1)
\]

\[
\xi(\tilde{s}) = 1 \quad (\tilde{s} \geq 1)
\]

\[
h = \frac{\ln(\tilde{s})}{\ln(\sqrt{2} s_1)}, \quad s_1 = \frac{Z^{2/3}}{184^2}
\]

\[
E_{\text{LPM}} = 277406.21 \text{ GeV}
\]

A.2 Photonuclear Interaction from Photons

From Table 40.2 in [41].

\[
\frac{\sigma_{\gamma\rightarrow \text{mesons}}(E_{\gamma})}{ \text{mb}} = 0.00308 \left[ 35.45 + 0.308 \ln^2 \left( \frac{2E_{\gamma} M_p}{\left(5.38 \text{ GeV} \right)^2} \right) \right] + 0.032 \left( \frac{2E_{\gamma} M_p}{\text{GeV}^2} \right)^{-0.458}
\]

(A.2)
A.3 Bremsstrahlung

The bremsstrahlung cross section is calculated according to the ABB parameterization [72].

\[
\frac{d\sigma_b}{dy}(E_c, y) = \alpha \left( 2r_e Z \frac{M_e}{M_c} \right)^2 \frac{1}{y} \left[ (2 - 2y + y^2) \Psi_1(q_{\text{min}}, Z) - \frac{2}{3} (1 - y) \Psi_2(q_{\text{min}}, Z) \right]
\]

\[
\Psi_{1,2}(q_{\text{min}}, Z) = \Psi_{0,2}(q_{\text{min}}, Z) - \Delta_{1,2}(q_{\text{min}}, Z)
\]

\[
\Psi_0(q_{\text{min}}, Z) = \frac{1}{2} \left[ 1 + \ln \left( \frac{M_e^2 a_1^2}{1 + x_1^2} \right) - x_1 \arctan \left( \frac{1}{x_1} \right) \right]
\]

\[
+ \frac{1}{Z} \left\{ \frac{1}{2} \left[ 1 + \ln \left( \frac{M_e^2 a_2^2}{1 + x_2^2} \right) \right] - x_2 \arctan \left( \frac{1}{x_2} \right) \right\}
\]

\[
\Psi_2(q_{\text{min}}, Z) = \frac{1}{2} \left[ \frac{2}{3} + \ln \left( \frac{M_e^2 a_1^2}{1 + x_1^2} \right) \right] + 2x_1^2 \left[ 1 - x_1 \arctan \left( \frac{1}{x_1} \right) + \frac{3}{4} \ln \left( \frac{x_1^2}{1 + x_1^2} \right) \right]
\]

\[
+ \frac{1}{Z} \left\{ \frac{1}{2} \left[ \frac{2}{3} + \ln \left( \frac{M_e^2 a_2^2}{1 + x_2^2} \right) \right] \right\}
\]

\[
+ 2x_2^2 \left[ 1 - x_2 \arctan \left( \frac{1}{x_2} \right) + \frac{3}{4} \ln \left( \frac{x_2^2}{1 + x_2^2} \right) \right] \right\}
\]

\[
\Delta_1(Z \neq 1) = \ln \left( \frac{Z^{1/3}}{1.9} \right) + \frac{\zeta}{2} \ln \left( \frac{\zeta + 1}{\zeta - 1} \right)
\]

\[
\Delta_2(Z \neq 1) = \ln \left( \frac{Z^{1/3}}{1.9} \right) + \frac{\zeta}{4} (3 - \zeta^2) \ln \left( \frac{\zeta + 1}{\zeta - 1} \right) + \frac{2Z^{2/3}}{1.9^2}
\]

\[
\Delta_{1,2}(Z = 1) = 0
\]

\[
x_1 = a_1 q_{\text{min}}, \quad q_{\text{min}} = \frac{M_e^2 y}{2E_c(1 - y)}, \quad a_1 = \frac{111.7}{Z^{1/3} M_e}, \quad a_2 = \frac{724.2}{Z^{2/3} M_e}, \quad \zeta = \sqrt{1 + \frac{4Z^{2/3}}{1.9^2}}
\]  

(A.3)
A.3.1 LPM Effect in Bremsstrahlung

For $e^\pm$, the bremsstrahlung cross section is multiplied by an LPM correction that was modified from the one in Section A.1 by [73].

\[
\text{LPM correction} = \frac{\xi(s_p)}{3} \left[ \frac{y^2 G(\tilde{s})}{\Gamma^2} + 2(2 - 2y + y^2) \frac{\phi(\tilde{s})}{\Gamma} \right] \sqrt{\frac{4}{3} - \frac{4}{3}y + y^2}
\]

\[
G(\tilde{s}) = \frac{36\tilde{s}^2}{36\tilde{s}^2 + 1} \quad (0.710390 \leq \tilde{s} < 0.904912)
\]

\[
= 1 - \frac{0.022}{\tilde{s}^4} \quad (\tilde{s} \geq 0.904912)
\]

\[
\phi(\tilde{s}) = 1 - \frac{0.012}{\tilde{s}^4} \quad (\tilde{s} \geq 1.54954)
\]

\[
\tilde{s} = \frac{s_p\Gamma}{\sqrt{\xi(s_p)}}
\]

\[
\Gamma = 1 + \left[ \frac{0.917N_a(4\pi)}{2(1.00794) + 15.9994} \right] [2(1) + 8] \left[ \frac{r_e^3}{\alpha^2 y^2} \right]
\]

\[
s_p = \sqrt{\frac{E_{\text{LPM}}y}{8E_c(1 - y)}}
\]

\[
E_{\text{LPM}} = 297250.87 \text{ GeV}
\]

\[
s_1 = \frac{1.54(1.00794^{0.27})}{202.4} \quad \text{(hydrogen)}
\]

\[
= \frac{1.54(15.9994^{0.27})}{(0.5)173.4} \quad \text{(oxygen)}
\]
A.4 Pair Production from Charged Leptons

$\mu^\pm$ and $\tau^\pm$ cross sections are from [74, 75]

$$\frac{d\sigma_p}{dy}(E_c, y) = \alpha^2 \frac{2}{3\pi} r_e^2 Z [Z + \zeta(Z)] \frac{1-y}{y} \int_\rho [\Phi_e + (M_e/M_c)^2 \Phi_\mu] d\rho$$

$$\Phi_e(\xi \leq 0.001) = \left\{ [(2 + \rho^2)(1 + \beta) + \xi (3 + \rho^2)] \ln \left(1 + \frac{1}{\xi}\right) \right.$$  

$$+ \frac{1 - \rho^2 - \beta}{1 + \xi} - (3 + \rho^2) \right\} L_e$$

$$\Phi_e(\xi > 0.001) = \left[ \frac{3 - \rho^2 + 2\beta(1 + \rho^2)}{2\xi} \right] L_e$$

$$\Phi_\mu(\xi \geq 0.001) = \left\{ \left[ (1 + \rho^2) \left(1 + \frac{3}{2}\beta\right) - \frac{1}{\xi} (1 - \rho^2)(1 + 2\beta) \right] \ln(1 + \xi) \right.$$  

$$+ \frac{\xi(1 - \rho^2 - \beta)}{1 + \xi} + (1 - \rho^2)(1 + 2\beta) \right\} L_\mu$$

$$\Phi_\mu(\xi < 0.001) = [5 - \rho^2 + (3 + \rho^2)\beta](\xi/2)L_\mu$$

$$L_e(\tau^\pm) = \ln \left[ \frac{RZ^{-1/3} \sqrt{(1 + \xi)(1 + Y_e)}}{1 + [2M_e\sqrt{\rho} RZ^{-1/3} (1 + \xi)(1 + Y_e)]/\{E_e y (1 - \rho^2)\}} \right]$$

$$L_e(\mu^\pm) = L_e(\tau^\pm) - \frac{1}{2} \ln \left[ 1 + \left( \frac{3 M_e Z^{1/3}}{2 M_c} \right)^2 (1 + \xi)(1 + Y_e) \right]$$

$$L_\mu(\tau^\pm) = 0$$

$$L_\mu(\mu^\pm) = \ln \left[ \frac{2 M_c R / (3 M_e Z^{2/3})}{1 + [2 M_e \sqrt{\rho} R Z^{-1/3} (1 + \xi)(1 + Y_\mu)]/\{E_e y (1 - \rho^2)\}} \right]$$

$$Y_e = \frac{5 - \rho^2 + 4\beta(1 + \rho^2)}{2(1 + 3\beta) \ln(3 + 1/\xi) - \rho^2 - 2\beta(2 - \rho^2)}$$

$$Y_\mu = \frac{4 + \rho^2 + 3\beta(1 + \rho^2)}{(1 + \rho^2)(1.5 + 2\beta) \ln(3 + \xi) + 1 - 1.5\rho^2}$$

$$\beta = \frac{y^2}{2(1 - y)}, \quad \xi = \left( \frac{M_\mu y}{2 M_e} \right)^2 \left( \frac{1 - \rho^2}{(1 - y)} \right), \quad \rho = \frac{E_{e+} - E_{e-}}{E_{e+} + E_{e-}}$$

$$0 \leq |\rho| \leq \left( 1 - \frac{6 M_e^2}{E_c^2 (1 - y)} \right) \sqrt{1 - \frac{4 M_e}{E_c y}}$$

$$R = 202.4 (Z = 1), 176.6 (Z = 7), 173.4 (Z = 8), 165.8 (Z = 11), 174.3 (Z = 17)$$

(A.5)
Table A.1: Parameters for the ALLM97 photonuclear cross section. \( m_0^2, m_P^2, m_R^2, Q_0^2, \) and \( \Lambda^2 \) are in GeV^2.

<table>
<thead>
<tr>
<th>( m_0^2 )</th>
<th>( m_P^2 )</th>
<th>( m_R^2 )</th>
<th>( Q_0^2 )</th>
<th>( \Lambda^2 )</th>
<th>( c_{P1} )</th>
<th>( c_{P2} )</th>
<th>( c_{P3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.31985</td>
<td>49.457</td>
<td>0.15052</td>
<td>0.52544</td>
<td>0.06527</td>
<td>0.28067</td>
<td>0.22291</td>
<td>2.1979</td>
</tr>
<tr>
<td>( a_{P1} )</td>
<td>( a_{P2} )</td>
<td>( a_{P3} )</td>
<td>( b_{P1} )</td>
<td>( b_{P2} )</td>
<td>( b_{P3} )</td>
<td>( c_{R1} )</td>
<td>( c_{R2} )</td>
</tr>
<tr>
<td>-0.0808</td>
<td>-0.44812</td>
<td>1.1709</td>
<td>0.36292</td>
<td>1.8917</td>
<td>1.8439</td>
<td>0.80107</td>
<td>0.97307</td>
</tr>
<tr>
<td>( c_{R3} )</td>
<td>( a_{R1} )</td>
<td>( a_{R2} )</td>
<td>( a_{R3} )</td>
<td>( b_{R1} )</td>
<td>( b_{R2} )</td>
<td>( b_{R3} )</td>
<td></td>
</tr>
<tr>
<td>3.4942</td>
<td>0.58400</td>
<td>0.37888</td>
<td>2.6063</td>
<td>0.01147</td>
<td>3.7582</td>
<td>0.49338</td>
<td></td>
</tr>
</tbody>
</table>

### A.4.1 LPM Effect in Direct Pair Production

For \( e^\pm \), the pair production cross section has an LPM correction that was modified from [76] by [73].

\[
\Phi_e = [(1 + \beta)(A + B + \rho^2 B) + \beta(C + D + \rho^2 D) + E - \rho^2 E]L_{e}(\mu^\pm)
\]

\[
A = \frac{G}{2} (1 + 2Gx) \ln \left[ \frac{36s^2 (1 + x)^2 + 1}{36s^2 x^2} \right] - G
\]

\[
+ 6Gs \left[ 1 + \frac{(36s^2 - 1)x}{36s^2 + 1} \right] \left[ \arctan(6sx + 6s) - \frac{\pi}{2} \right]
\]

\[
B = \phi (1 + \phi x) \ln \left( \frac{6sx + 6s + 1}{6sx} \right) - \phi
\]

\[
C = -G^2 x \ln \left( \frac{36s^2 (x + 1)^2 + 1}{36s^2 x^2} \right) + G
\]

\[
- \left( \frac{36s^2 G^2 - G^2}{6s} \right) x \left[ \arctan(6sx - 6s) - \frac{\pi}{2} \right]
\]

\[
D = \phi - \phi^2 x \ln \left( \frac{6sx - 6s + 1}{6sx} \right)
\]

\[
E = -6s \left[ \arctan(6sx - 6s) - \frac{\pi}{2} \right]
\]

\[
\phi = \frac{6s}{6s + 1}, \quad G = \frac{36s^2}{36s^2 + 1}, \quad s = \frac{1}{4} \sqrt{\frac{E_{LPM}}{E_{c}\gamma(1 - \rho^2)}}, \quad E_{LPM} = 2354235.7 \text{ GeV}
\]

(A.6)
A.5 Photonuclear Interaction from Charged Leptons

The basic formula is from [77], the original parameterization is from [66], and this parameterization is from [65].

\[
\frac{d\sigma_n}{dy}(E_c, y) = \int_{Q^2_{\text{min}}}^{Q^2_{\text{max}}} \frac{F_2}{y} dQ^2
\left[ 1 - y - \frac{M_p xy}{2E_c} + \left( 1 - \frac{2M^2_p}{Q^2} \right) \frac{y^2(1 + \frac{4M^2_p x^2}{Q^2})}{2} \right] d(Q^2)
\]

\[x = \frac{Q^2}{(2M_p E_c y)}\]

\[F_2 = a[Z + (A - Z)P] \frac{Q^2}{Q^2 + m_0^2} (F^P_2 + F^R_2)\]

\[P = 1 - 1.85x + 2.45x^2 - 2.35x^3 + x^4\]

\[a = A^{-0.1} \quad (x < 0.0014)\]

\[a = A^{0.069 \log_{10}(x)+0.097} \quad (0.0014 \leq x < 0.04)\]

\[a = 1 \quad (0.04 \leq x)\]

\[F^P\text{,}_R = c_{P,R} a^{P\text{,}_R} (1 - x)^{b_{P,R}}\]

\[f = f_1 + f_2 t^{f_3} \quad (f = c_R, a_R, b_{P,R})\]

\[f = f_1 + (f_1 - f_2) \left( \frac{1}{1 + t^{f_3}} - 1 \right) \quad (f = c_P, a_P)\]

\[t = \ln \left( \frac{Q^2 + Q_0^2}{\Lambda^2} \right) \quad , \quad x_{P,R} = \frac{Q^2 + m^2_{P,R}}{m^2_{P,R} + 2M_p E_c y}\]

\[Q^2_{\text{min}} = M^2_c y^2 / (1 - y), \quad Q^2_{\text{max}} = 2M_p E_c y - [(M_p + M_{\pi})^2 - M^2_p]\]

(A.7)