Three Essays On The Interactions Between Regional Development And Natural Amenities

Dissertation

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ABSTRACT

The tension between economic development and ecological protection has become more intense in the United States in the past few decades as population is rapidly spreading into areas with high natural amenities, most notably the mountainous rural regions of the “New West” and exurban counties with abundant open space amenities. This dissertation addresses three different aspects related to the interactions between regional economic development and natural amenities. The first essay focuses on the question of how regional economic growth interacts with ecological processes and specifically, how the rate of economic growth relative to the rate of ecological change matters in terms the sustainability of the joint ecological-economic system. Within the theoretical framework of a coupled nonlinear ecological-economic system characterized by slow-varying migration and fast-varying ecological change, I find that fast regional economic growth can render the system more vulnerable to exogenous shocks that cause the system to collapse; however, faster economic growth can also increase the speed with which the system returns to a sustainable economic equilibrium whenever it survives a shock. We also find that economic and ecological interventions can enhance both social welfare and system resiliency, but to
degrees that depend on the underlying rate of regional economic growth. The common simplifying assumptions regarding the relative speeds of economic and ecological adjustment are assessed and the conditions under with such simplifying assumption can be applied are identified.

The second essay focuses on the optimal management of ecological thresholds when impacts from human activities not only trigger an ecological threshold effect, but also respond to subsequent ecological changes. Such two-way interactions are typical of many ecological-economic interactions. As an example, we study the management of a eutrophication threshold of a lake ecosystem when the run-off from developed land is affecting and at the same time responding to the lake water quality. I show that a naive policy that seeks to reduce run-off and improve lake water quality only by regulating the polluting source may not work due to the two-way interactions. In fact, such a policy can actually lead to more serious pollution in the long run and destabilize the ecological system. I also show that the effectiveness of a safe minimum standard hinges on two-way interactions as well. To manage a dynamically interacting ecological-economic system, it is necessary to adopt an integrated modeling approach that accounts for both human impacts on the ecology and ecological impacts on human behavior. Technically, through numerical simulation, I have solved a dynamic optimization problem with multiple equilibria and identified the coexistence of multiple solutions due to the non-convexity of the problem.
The third essay explores the mechanism underlying the pattern formation of exurban land uses given assumptions about household preferences for natural amenities and land heterogeneity in both the agricultural productivity and conversion costs. In particular, I contribute to the literature on land development by the provision of a micro-foundation for the capitalization process of non-marketable amenities. With the assistance of geographical information system, I am able to simulate my theoretical spatial economic model using actual data on land heterogeneity and the location of urban and natural amenities. The simulation result is then used to assess the theoretical predictions of the model in terms of the evolution of exurban land development patterns over a thirty year period. The model is unable to reproduce the observed land use pattern. This suggests that the spatial heterogeneity in agricultural productivity, conversion cost and natural amenities, open space amenities in particular, are insufficient to fully capture the complex spatial pattern of land development. Further research is needed to elaborate both the theoretical foundation for spatial land conversion decisions and to incorporate the impact of regulating policies on development and the provision of local public services such as sewer and water and educational services.
This is dedicated to my family.
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CHAPTER 1

RATE OF REGIONAL GROWTH, RESILIENCE AND OPTIMAL POLICY IN A COUPLED ECOLOGICAL-ECONOMIC MODEL
1.1 Introduction

The coupling of nonlinear economic and ecological models that characterize complex systems (Arrow et al., 1999, 2003; Levin, 2006; Carpenter et al., 1999) has presented new analytical challenges that were largely unforeseen in the traditional resource management and sustainable economic growth literatures, as reviewed in Cropper and Oates (1992), Hahn (2002) and Brock and Taylor (2004). In particular, nonlinear ecological models can generate multiple stable equilibria and ecological irreversibility. For example, Carpenter et al. (1999) examined a shallow lake system in which water quality remains in a “good” oligotrophic equilibrium so long as phosphorus loadings are low. When the phosphorus loadings increase, the lake shifts to a “bad” eutrophic equilibrium characterized by algal blooms, fish kills, and low water quality. Once the lake has become eutrophic, an equal amount of decrease in the phosphorus loadings cannot bring the lake back to the oligotrophic equilibrium. This phenomenon is referred to as irreversibility. Concerns about the shift between different stable equilibria, called regime shifts, and irreversible ecological changes lie in the core of the current debate of environmental protection and motivate regulation and policy prescriptions.

The integration of ecological and economic models has also given rise to analytical problems associated with coupling dynamic processes that change at significantly different rates. This is evident based on casual observation. Many biophysical processes evolve on the order of hours or days, whereas human systems often evolve
more slowly on the order of months, years or decades. For example, the population of a metropolitan area and the pollution that it generates will change significantly on the order of a decade or longer; yet, as a result of the associated increase in runoff, the adjacent lake can change from an oligotrophic state to an eutrophic state in the span of one year. The Cuyahoga River fire in Cleveland, Ohio that suddenly erupted in 1969 was a result of gradual build-up of industrialization and pollution; the subsequent effects of the fire on Cleveland’s image undermined migration to the area for decades.¹

Nonlinear dynamic systems with processes evolving at vastly different speeds have been studied extensively in the physical sciences (Ludwig et al., 1978; Chave and Levin, 2003; E and Engquist, 2003). As noted by Levin (2006): “dynamics on faster time scales, ..., are shaped by slow dynamics on larger times scales, which in turn arise from the aggregate of dynamics on faster and smaller scales.” This is also acknowledged in the general discussions in Carpenter et al. (2001), Dasgupta and Müller (2003), Chave and Levin (2003) and Folke (2006). In particular, Arrow et al. (1999) pointed out that ”the dynamics of slow variables hold the key to resilience and potential domain shifts”.

In this paper, we investigate the relationship between regional economic growth and environmental quality within a coupled ecological-economic system characterized by slow-varying migration and fast-varying ecological change. In contrast to the

¹The impaired image of Cleveland seems to last, despite the substantial improvements in water quality in the 1980s and 1990s.
literature of lake management and regional economics, in this paper, the economic activities (land use) of uncoordinated individuals pollute the ecology, which in return will affect their well-being and economic decisions. As a result, the policy maker has to balance local economic development and ecological protection. The optimal policy is not only about maintaining the lake quality to an optimal level, but also about the provision of the right incentives to coordinate individual behavior.

This paper differs from the existing ecological-economic models by the emphasis on the different change speeds between the interacting economic and ecological processes. Of special interest is how the different regional growth rate affects the system’s resiliency, that is, the system’s ability to return to a sustainable economic equilibrium after experiencing an exogenous ecological or economic shock. Resilience has typically been measured in terms of either the proximity to the undesirable domain or the time to recovery. We find that the rate of regional economic growth the economic and ecological processes has a profound impact on the resilience of the system. We show that faster regional growth renders the system more vulnerable to large exogenous shocks that cause the system to cross over into an undesirable domain in which regional development is unsustainable. However, faster regional growth will also increase the speed at which the system returns to a sustainable equilibrium when it survives the shock. These results reveal potential inconsistencies between these two commonly used measures for resilience.
This paper solves the dynamic optimization problem when the slow economic activities are responding to fast ecological changes in a general equilibrium framework. In contrast to the lake management literature (Carpenter et al., 1999; Brock and Starrett, 2003), we explicitly model the dynamics of economic activities and emphasize on the relative speed of change between the ecological and economic processes. This paper contributes to the literature by explicitly finding dynamic solutions for a system with both multiple stable equilibria and fast and slow-changing processes. We examine the potential benefits of policy interventions that control the rates of economic growth and ecological change. We find that such interventions can increase social welfare and the resilience of the system, but that the optimal policy depends on the underlying rate of economic growth. Specifically, our simulations indicate that, as the rate of regional economic growth increases, the intervention in the initial periods should become more aggressive. Our simulations also show that the relative net benefits of intervention are highly dependent on the underlying rate of economic growth. In particular, we find that the net welfare gain that can be achieved through intervention increases substantially with the rate of economic growth.

Finally, we examine the implications of simplifying assumptions commonly made in the literature regarding the relative speeds of economic and ecological adjustment. We find that assuming constant population (Carpenter et al., 1999) and instantaneous ecological adjustment (Mohtadi, 1996; Elbasha and Roe, 1996; Måler et al., 2003) can lead to profoundly misleading results, including the apparent existence
of false sustainable equilibria and highly inaccurate estimates of the magnitude of economic and ecological shock that the system can sustain without collapsing.

1.2 Literature Review

Since the pioneering works of Smith (1968), Clark (1971), Clark et al. (1979), Berck and Perloff (1984), dynamic modeling and economic decision making has flourished in the renewable resource literature. Subsequent research has extended these early dynamic analysis along three major fronts: (a) the introduction of more realistic biological/ecological processes, like multi-species management (Tu and Wilman, 1992; Mesterton-Gibbons, 1987) and management of bistable lakes (Carpenter et al., 1999, 2001); (b) the development of the spatially explicit models, like Sanchirico and Wilen (1999), Sanchirico and Wilen (2005), Holland and Schnier (2006), Costello and Polasky (2008); and (c) the introduction of stochasticity (Cropper, 1976; Reed, 1979, 1984; Clark and Kirkwood, 1986; Costello et al., 2001). Apart from these, Levhari and Mirman (1980) analyze the “fish war”, thus initiating a bulk of literature on differential games. While this paper differs conceptually from the renewable management literature with a focus on the interactions between regional development and lake ecology, it follows the tradition that tries to incorporate more realistic biophysical dynamics based on ecological studies. It extends the literature by explicitly finding the dynamic solutions for a system with fast-slow dynamics and bistability.

See While (2000) for a review.
A large and diffuse literature on ecological-economic interactions exists that includes a wide array of descriptive, empirical, conceptual and theoretical studies. The relevant literature can be classified into two categories based on the type of ecological-economic interactions. In the first category (Carpenter et al., 1999; Dasgupta and Mäler, 2003; Gude et al., 2006; Miller et al., 1998; Perrings and Walker, 1997a; Scotchmer, 2002), economic activities generate a pollution externality that degrades the ecosystem. However, the impact of the degraded ecology does not directly affect the polluting emitters. The ecological feedback is recognized only by the policymaker, who in turn influences polluting emitters through policy mechanisms. The other category (Anderies, 2003; Brander and Taylor, 1998; Janssen et al., 2004; Perrings and Walker, 2004) explicitly captures the ecological feedbacks that have a direct effect on the behavior of polluters. This paper falls into the second category, because the economic impact on ecology is modeled through the run-offs from total land use in the region. At the same time, changes in ecological quality directly affect individual utility and change their consumption behavior.

1.2.1 Fast-Slow Dynamics

Fast-slow dynamics (systems characterized by the interaction between fast-changing and slow-changing variables) have been studied in ecology, physics and sociology (Chave and Levin, 2003; E and Engquist, 2003; Ludwig et al., 1978) and references therein). In the environmental and resource economics literature, the difference in the change rates of the ecological and economic processes is most often ignored, which
results in an arbitrary treatment of the relative speed of change.3 Take the treatment of the environment as an example. On one extreme, environmental changes are regarded as a very slow process and assumed to be constant (Fujita et al., 1999; Fujita and Thisse, 2002). On the other extreme, they are regarded as fast process and assumed to respond to human activity instantaneously (Elbasha and Roe, 1996; Mäler et al., 2003; Mohtadi, 1996). In between, they are treated as processes that evolve at the same speed as human activity (Krautkraemer, 1985). For instance, in studying regional growth, Fujita et al. (1999) and Fujita and Thisse (2002) treat natural resources and natural amenities as constant, ignoring the pressure that regional growth exerts on the local environment. In fact, this is a common practice in regional growth models. In the discussion of natural amenities and urban spatial structure, Wu and Plantinga (2003) justifiably make the same assumption because of the static long-run equilibrium approach they pursue. On the other extreme, in models of economic growth with an endogenous environment, the environment is often assumed to instantaneously respond to human activity by specifying the ecological variable as an explicit function of capital stock (e.g., Elbasha and Roe (1996); Mohtadi (1996)) or assuming an instantaneous ecological adjustment (e.g., Mäler et al. (2003)). Between

3The resource management literature has a long tradition of modeling ecological-economic interaction, like the literature on the management of fishery and forestry. In this literature, human interventions (through harvesting) is typically a major factor determining the growth and stock of the resource and is typically coordinated with the changes in the stock. In this sense, human activity and the ecological process are usually changing at roughly the same speed. However, the interaction between the regional growth and local ecological conditions is different. While population migration is affected by the in situ ecological quality, its change is governed by uncoordinated individual behavior.
these two extremes, Krautkraemer (1985), treats the changes in natural environment as a process that evolves at the same speed as human activity.

On the other hand, when the focus of the research is on the ecological dynamics, similar simplifying assumptions regarding human activities are common in literature. For instance, in the lake management literature, Carpenter et al. (1999) essentially assume a zero regional growth, which corresponds to the fixed local population in this model.

A few studies of ecological-economic interactions have recognized the importance of the relative speed between different dynamic processes. Anderies (2003) develops a general equilibrium model of two sectors in which a renewable natural capital is used as an input factor in production, along with labor and human capital. He shows that the relative speed between the savings rate and the regeneration rate of natural capital is important because Hopf bifurcation may occur when the savings rate is faster than the regeneration rate of natural capital. Perrings and Walker (2004) investigates the optimal management of rangelands that can be the either regulated by fire or by the grazing pressure. They use the steady-state optimal level of harvest to simulate the management policy. They show that the relative rate of change between different ecological processes can affect the length of time that system is regulated by fire or by the grazing pressure. However, because of the way Anderies (2003) and Perrings and Walker (2004) set up their models, it is not clear whether the significance is due purely to something specific about the relative
speed or due to the simple fact that any parameter change may change the dynamic structure of the model. If it is the latter reason, the relative speed of change is just like other parameters that can change the location and stability of the steady-state, and thus affect welfare analysis. This paper shows that fast-slow dynamics have a more profound impact on system dynamics.

The impact of relative speed in the fast-slow dynamics is the focus of three recent papers in the resource economics and management literature (Grimsrud and Huffaker, 2006; Huffaker and Hotchkiss, 2006; Crepin, 2007). In these models, the ecological process itself has both a fast and a slow component. These authors propose the use of singular perturbation for models with fast-slow dynamics. In particular, Grimsrud and Huffaker (2006) explore the implication for pesticidal crops management when pest population evolve at a faster speed and the population’s genetic composition evolve at a much slower speed. Huffaker and Hotchkiss (2006) study the reservoir management by the introduction of a slow sedimentation process into the fast impounding water dynamics. Crepin (2007) suggests using singular perturbation theory as a general method to address the issue of fast-slow dynamics, highlighting the management of coral reefs with fishing pressure as an example. However, the application of the singular perturbation methods requires that the speeds of change are extremely divergent and it is limited in this sense as a general method in solving optimization problems with fast-slow dynamics. This is particularly true when modeling coupled ecological and economic systems since the speeds of change are
often not so vastly different. Moreover, because coupled ecological-economic models typically have more than one state variable, the dimensionality of the management problem remains a modeling challenge.\(^4\)

This paper does not assume the extreme divergence in the speeds of change as required by singular perturbation theory. The technique used here therefore has more general applications and can be used to address a larger variety of inter-temporal interactions. In addition, unlike these three papers, this paper considers multiple policy instruments, which act on processes changing at different speeds. This extension is important because it allows for the explicit consideration of the coordination of multiple policies that act on the fast and slow processes respectively.

1.2.2 Management of Systems subject to Multi-stability and Thresholds

Because bi-stability is a fundamental feature of the ecological-economic system considered in this paper, our work shares an important similarity with papers that address the optimal management of multi-stable systems. Carpenter et al. (1999) discusses the management of a lake with bistability to optimize the weighted welfare from phosphorus emitters and lake users. Brock and Starrett (2003) study the optimal management of an ecological process with nonlinearity. The focus of the paper is to prove the existence of multiple equilibria, instead of completely solve the management problem. In both papers, while the positive and negative effects

\(^4\)For a detailed discussion, please refer to Chen and Jayaprakash (2008).
of ecological changes can affect the objective function for policy makers, the human reaction is not explicitly modeled as a dynamic process. In contrast, this paper focuses on solving the dynamic optimization problem when human economic activities are responding to ecological changes in a general equilibrium framework. The dynamics of human activities are modeled explicitly.

The impact of ecological thresholds on the management of nonrenewable resources, eutrophying lakes and greenhouse gas emission has been considered in the environmental and resource management literature (Farzin, 1996; Nævdal, 2001, 2006; Keller et al., 2004). In these papers, the threshold is exogenously specified through a piecewise continuous function. In contrast, the threshold effect in this paper is endogenously determined by the dynamical structure of the coupled system. It arises naturally from the multiple steady states in the system. Most importantly, the threshold effect becomes endogenous to the interactions between the economic and ecological processes. As economic and ecological conditions change, the threshold will change accordingly.

1.3 A Coupled Ecological-Economic Model

In this section, we develop a coupled ecological-economic model characterized by slow-varying migration and fast-varying ecological change. We develop a simple general equilibrium model of regional economy in which people make migration decisions based on utility differentials between the region and the rest of the world. The

\footnote{The basic model is a modification of Chen et al. (2008).}
natural amenity services provided by an adjacent lake may attract or repel migrants, depending on water quality. As local population increases, local wages will increase due to increasing returns to scale in production. This, in turn, increases residential land consumption and run-off into the lake. The attendant increase in congestion and degradation in lake water quality reduce the utility of local residents.

1.3.1 The Economic System

In this subsection, we describe the micro-foundations for a regional growth model driven by agglomeration and congestion effects, when the lake ecological quality is fixed.

In the model economy, two goods are traded: a locally produced composite good and land. The composite good \( X \) is produced with local labor \( N \), which can be either consumed locally or exported to the rest of the world. Following Fujita and Thisse (2002), the composite good \( X \) is produced according to the production function:

\[
X = E(N)F(N)
\]  

(1.1)

where \( F(N) \) is a standard neoclassical production function with declining marginal product of labor. Agglomeration is captured by an externality function \( E(N) \), which represents scale effects that are external to individual firms.\(^6\) Under the assumption

\(^6\)Because the reason for agglomeration is not the focus of this paper, it is treated in the simplest way.
that regional output market is perfectly competitive, the local wage rate is determined by

\[ w = E(N)F'(N) \]  

(1.2)

Households that choose to stay outside of the region receive a constant utility \( \bar{U} \). Those that choose to live in the region accept the local wage rate and decide on their consumption of the composite good \( (x) \) and land \( (l) \). Their optimization problem is:

\[
U(N, P, w, r) = \max_{x, l} \left\{ x^a l^{1-a} + U_c(N) + U_e(P) \right\}
\]  

(1.3)

subject to

\[ x + r \cdot l = w \]

where \( x \) and \( l \) are household consumption of the composite good and land, respectively. \( a \) is the consumption elasticity of the numeraire good. The price of the composite good \( x \) has been normalized to 1; \( r \) is the land rent; \( U_c \) is the disutility from congestion; and \( U_e \) is the utility from enjoying the natural amenities in the region. \( P \) is an ecological variable that is inversely related to the quality of lake amenity service and which will be described in detail in the next subsection.

The optimal individual consumption for the numeraire good and residential land is:

\[
x = a w, \quad l = (1 - a) \frac{w}{r}.
\]  

(1.4)
Assuming that residents allocate one quarter of their expenditure on land/house payment, this gives a consumption elasticity of the numeraire good \( a = 0.75 \). Substitute the optimal consumption of \( x \) and \( l \) into the individual utility. This yields the indirect utility as a function of local population \( N \), ecological variable \( P \), local wage rate \( w \) and land rent \( r \).

\[
U(N, P, w, r) = a^a(1 - a)^{1-a}r^{a-1}w + U_c(N) + U_e(P) \quad (1.5)
\]

The incentive to migrate into the region is determined by the utility differentials between the region and the rest of the world:

\[
\dot{N} = U(N, P) - \bar{U}. \quad (1.6)
\]

Here, \( \dot{N} \) stands for the derivative with respect to time. Migration will continue until utility differentials disappear.

Finally, a land-market clearance condition closes the general equilibrium model. Rather than constraining the total amount of land in the region, we assume that, as local population grows, the urban area will expand into the surrounding rural areas where land is used solely for agriculture. Land will be bid away from agriculture for residential use so long as land rent for residential uses \( (r) \) is more than the agricultural rent \( (r_a) \). The total amount of developed land will be determined by the equalization of these two rents. The agricultural output is assumed to be produced with constant returns to scale technology and sold on a global market. This gives an exogenously determined output price and a constant agricultural land rent. As a
result, the equilibrium condition for the land market in this model is:

\[ r = r_a \] (1.7)

where \( r_a \) is set at value 1.2.\(^7\)

Equations (1.2), (1.3), (1.4) and (1.7), together with the full employment condition define a temporary equilibrium for a given \( N \) and \( P \). This gives the reduced form expression for migration rate in equation (1.6)

\[ \dot{N}(t) = \left( U(N(t), P(t)) - \bar{U} \right) \] (1.8)

In order to simplify the notation, the argument \( t \) in this migration equation is suppressed from here on.

For simplicity, we assume the neoclassical production function for \( X \) exhibits constant returns to scale: \( F(N) = N \); that the production externality is of the simple form: \( E(N) = bN \); that the congestion effect is \( U_c(N) = -N^2 \); and that the utility provided by the natural amenity service is of the form \( U_e(P) = -P \). This gives

\[ \dot{N} = A_0 N - A_1 N^2 - A_2 P - \bar{U} \] (1.9)

where \( N \geq 0 \) and \( A_0 = a^a(1-a)^{1-a}b^{1-a} \). \( A_1 \) and \( A_2 \) are the weighting coefficients for the congestion and the pollution respectively.

This dynamic equation is hump shaped in the local population, which incorporates a generic feature in regional development models: at the initial stages of

\(^7\)The price of the composite good is normalized to 1. We assume that \( r_a > 1 \) to reflect that land is scarcer than other in general. This is generally believed to be true in locations with high natural amenities.
development, the in-migration rate increases with population as a consequence of agglomeration. However, as population increases, the congestion effect will eventually dominate and the migration rate will slow down. The inverse relationship between the migration rate and the ecological variable $P$ reflects people’s preference for high quality natural amenities, exemplified by many of the fast growing areas of the Rocky mountainous and coastal areas in the U.S..

It is unlikely that, even with substantial growth, the attraction of environmental amenities alone will transform small rural areas into large metropolitan areas with large cities. Because the populations of moderately sized metropolitan areas in the U.S. are in the range of 50-100,000 people, the parameters of equation (1.9) are chosen so that the congestion effect dominates the agglomeration effect when local population is at around 90-100,000 people. Moreover, we believe that the ecological impact of economic activities is evident before the region fully develops. The most interesting case is when a region in its early stages of development faces the danger of severe ecological degradation. The coefficient of $P$ in equation (1.9) is set so that a population of around 60,000 is enough to trigger severe degradation in lake water quality.

Because one primary objective of this paper is to investigate how different rates of regional economic growth affect system’s resilience and policy, it is necessary to know the scale at which the growth rates vary in reality. In the U.S., on the regional level, annual population growth ranged from 0.52% in the Northeast to 1.48% in
the West between 1990-2003 (Markham and Steinzor, 2006, pp. 15-16). Because the variation in the regional growth rate is within a range of 4, the variations in the regional growth in this paper will be confined within the same range.

1.3.2 The Ecological System

Our lake ecological system is a stylized model for natural amenities, because it is tightly coupled with economic activities and because it evolves at a faster speed than migration. Run-off from developed land and urbanized areas is transported to lake through its tributaries. This process typically takes place over the time in a period of months. This run-off will contribute nutrients to the water column of the lake that over time, much of which will naturally fall to the bottom as sediment. Sedimentation also occurs over a time period of months. In spite of the sedimentation and out flow of nutrients, the nutrient can build up throughout the year and become sufficiently high in the late summer to spur nonlinear recycling. As a result, the nutrients bounded to sediments can be re-suspended back into the water column, potentially causing the lake to shift from an oligotrophic (good) to eutrophic (bad) state in late spring and summer seasons (Carpenter et al., 1999; Hein, 2006; Scheffer, 1998). An oligotrophic lake is characterized by a low nutrient level in the water column, relatively clear water and healthy ecosystem services. An eutrophic lake is characterized by a high nutrient level in the water column, high concentration of algae, toxicity, and turbidity and is prone to anoxic conditions and undesired events such as fish kills and noxious algal blooms. Hysteresis can occur in
nutrient dynamics (Scheffer, 1998; Carpenter et al., 1999). That is, once an increase in loadings causes the lake to shift from an oligotrophic to an eutrophic state, equal reductions in loadings cannot bring the lake back to the oligotrophic state. When a recovery does occur, it can take long time. Lake Erie is a good example. After the Cuyahoga River fire in 1969, substantial reductions in point sources failed to bring about meaningful improvements in water quality until ten years after the controls had been implemented.

This paper uses phosphorus concentration in lake water as an indicator of lake water quality because it is a major component of the nutrient loadings in lakes, and because its concentration is strongly inversely related to lake water quality. The ecological model in this paper is adapted from the simple model of Carpenter et al. (1999) which captures the basic dynamics of nutrient loadings and the possible shifts between oligotrophic and eutrophic states. It is assumed that the phosphorus loadings into the lake is proportional to the total land use in the region.

\[
\dot{P}(t) = L_0 + L_1 N(t) l(t) - s P(t) + \sigma_1 \frac{P(t)\sigma_2}{P(t)\sigma_2 + P_\sigma^2}.
\]  

(1.10)

Here, \( P(t) \) is the concentration of phosphorus in the lake water at time \( t \) and \( \dot{P} \) is its rate of change. The rate of nutrient input per unit of time is determined by the loading, outflow and recycling. The loading consists of \( L_0 \), which is a constant, and \( L_1 N(t) l(t) \), which is the total loadings from the residential land use in the region. \( L_1 \) captures the average impact of land use on lake water quality. The outflow of phosphorus is captured in \( s P(t) \), where \( s \) denotes the proportional rate of \( P \)-loss.
from the system. It accounts for both sedimentation and outflow, which remove phosphorus \((P)\) from the water column. The last term represents the recycling process of phosphorus. It captures the fact that, after sedimentation, phosphorus can be recycled back into lake water from the bottom. Recycling occurs only when the phosphorus concentration is sufficiently high. The maximum rate of recycling is \(\sigma_1\) and the steepness of the recycling curve is governed by \(\sigma_2\). \(P_c\) is the \(P\) value at which recycling reaches half its maximum rate; it is also a rough indicator of the ecological threshold where the recycling term becomes dominant in the phosphorus dynamics.

The recycling term that gives rise to threshold responses is a general feature of many ecosystems (Brock and Starrett, 2003). Therefore, with appropriate modifications, the analysis presented in this paper is also applicable to a more generic ecological-economic systems.

Significant changes in phosphorus concentration in the lake water system can occur in approximately one year (Carpenter et al. (1999) and references therein). The parameters are calibrated to conform with the existing ecological literature and to generate significant phosphorus changes in roughly one year if perturbed. This sets up the unit of time in the coupled ecological-economic model. In the numerical simulations, the immediate adjustment in phosphorus to shocks usually takes a time period of \(t = 10 \sim 20\), thus model time \(t = 10 \sim 20\) roughly corresponds to a year in real time.
Two examples of the evolution of the uncoupled system are presented in Figure 1.1. By "uncoupled", we mean that the interacting terms in equations (1.8) and (1.10) are neglected. In the example, the initial condition is assumed to be \((N_0, P_0) = (4, 1)\) in Figure 1.1(a) and \((N_0, P_0) = (6, 1)\) in Figure 1.1(b). The rates of change in \(N\) and \(P\) differ roughly by a factor of 10. This is evident from the initial rapid change in \(P\) as compared to that in \(N\).
1.3.3 Coupled System with Fast-Slow Dynamics

This section describes the dynamics of this coupled ecological-economic model with fast-varying lake water quality and slow-varying regional economy. The phosphorus dynamical equation (1.10) in this paper is adapted from ecological studies based on field experiments. While the nonlinear recycling term in equation (1.10) make it impossible to solve the dynamical system analytically, we consider it essential to retain the term in its nonlinear form, because it is the nonlinearity that gives rise to the most interesting phenomena, such as regime shifts and slowly reversible ecological changes. Concerns about possible regime shifts and slowly reversible changes lie in the core of the current debate of environmental protection and motivate regulation and policy prescriptions. The lack of an analytical solution makes comparative static analysis difficult, but not impossible. With an analytical solution, comparative static analysis often produces “general” results because the analytical formula holds for various parameter values. However, with numerical solutions, the implications of comparative static analysis are conditioned on the range of parameters chosen. Theoretically, we can try all the possible parameter values and the result will conform with the analytical solution. But this is impractical in reality. Fortunately, as a researcher, we are only interested in the realistic and/or theoretically more interesting cases rather than all the mathematically possible ones. These considerations restrain the relevant parameter space. Finally, there is a trade off between the analytical and
numerical solution. While analytical solutions (if they exist) are less restrictive in parameter specification than numerical solutions, they are more restrictive in function forms. For the analytical solution, in order to ensure the existence of solution, the function space is limited to extremely simple ones with nice mathematical properties. This implies that the analysis may inadvertently omit more complex dynamics, such as nonlinearities or threshold effects that are particularly important in ecological system dynamics. In contrast, numerical solutions do not suffer from this limitation and can allow more general and realistic function forms to study more complicated phenomenon. Because of the complicated nature of the coupled ecological-economic system and our interest in cases in which economic activities can cause discontinuous changes in the ecological system, numerical methods are used in this study.

**Baseline Results**

In this coupled system, the slow-varying population migration responds to ecological quality, which is determined by the fast-varying phosphorus concentration in the lake. At the same time, phosphorus concentration responds to the slow changes in land development that generates nutrient run-off. These coupled two-way interactions complete the ecological-economic model. The dynamics of this unregulated system are fully described by equations (1.8) and (1.10). Substituting for the residential land use \( l \), we have:

\[
\begin{align*}
\dot{N} &= A_0 N - A_1 N^2 - A_2 P - \bar{U} \\
\dot{P} &= L_0 + \frac{L_1 (1 - a) b}{r} N^2 - s P + \frac{P^{\sigma_2}}{P^{\sigma_2} + \bar{P}^{\sigma_2}}
\end{align*}
\]  (1.11)
A phase plot of the benchmark case is shown in Figure 1.2. Points along the broken line, called \( \dot{N} \)-nullcline, satisfy the condition \( \dot{N} = 0 \). In states below (above) the line, people will move into (out of) the region. The dotted line is called the \( P \)-nullcline. In states to the right (left) of the line, phosphorus concentration will increase (decrease). These two nullclines intersect at three fixed-points, at which the process will remain stationary. Point (A) is a non-trivial stable fixed point associated with positive population, point (B) is a trivial stable fixed-point associated with zero population, and point (C) is an unstable fixed point. Because point A is associated with a moderate population size and an oligotrophic lake, it is referred to as a ”sustainable” equilibrium state. At this sustainable state, regional development has pushed the lake to the edge of eutrophication under the current specification. As
shown in the phase plot, an ecological shock that increases phosphorus concentration (such as a variation in rain fall) or an economic shock that increases land development (such as government policy that promotes economic development) can shift the lake from the oligotrophic to eutrophic state. This sets up an interesting case for policy analysis in later sections. At fixed point B, because the local population is zero and the lake ecology is in its pristine state, it is called the “undeveloped” equilibrium state. Obviously, while this is not good from the perspective of regional development, it is not bad for the lake ecology.

The solid line, called the separatrix, partitions the state space into domains of attraction for the sustainable equilibrium (A) and undeveloped equilibrium (B). The separatrix is also the stable manifold of the unstable fixed point (C). That is, if the system starts on a point on this line, it will eventually approach the unstable fixed point. Starting from any state to the left of the separatrix, the system will eventually flow to the undeveloped equilibrium state (B). Starting from any state to the right, the system will eventually flow to the sustainable equilibrium state (A). Because of its association with sustainable equilibrium, we will refer to the area to the right of the separatrix as the "domain of sustainability".

Within this theoretical framework, we can now define system resilience. Resilience generally refers to the system’s ability to return to the sustainable equilibrium after an exogenous shock. There are two commonly used measures for resilience. One,
proposed by Holling (1973), uses the size of the domain of sustainability as a measure. The larger the domain, the greater the size of ecological or economic shock that the system can sustain and eventually return to the sustainable equilibrium. In particular, the vertical distance from the sustainable equilibrium (A) to the separatrix measures the maximum adverse ecological shock that the system can absorb and return to the sustainable state. Similarly, the horizontal distance from the sustainable equilibrium (A) to the separatrix measures the maximum economic shock the system can absorb and return to the sustainable state. A second measure, proposed by Pimm (1984), is based on the time required for the system to return to the sustainable equilibrium after sustaining an ecological or economic shock. The faster the recovery time, the more resilient is the system.

Given the lake ecological dynamics, the slow growth of the regional economy can surprisingly make the system less resilient as measured by the recovery time. This is shown in Figure 1.3. Suppose the system is at rest at the sustainable state, and then experiences a sudden influx of population, e.g., due to the improvement of local public services. The sudden influx causes the system to moved to the state \( N = 6.2 \) and \( P = 3.75 \), as shown in Figure 1.3(a). Because of the fast ecological change and slow economic adjustment, the recovery time is unexpectedly long. Moreover, due to the excess economic development, phosphorus loadings from residential land use exceed the carrying capacity of the lake. As a result, the phosphorus concentration surpasses the ecological threshold and the lake becomes eutrophic. This eutrophic
lake drives people away until the it recovers to the oligotrophic state. Only then will people begin to return, allowing the system to return to the sustainable equilibrium. The resilience as measured by recovery time is plotted in Figure 1.3(b).

**Impact of Regional Growth Rate on Resilience**

In this subsection, in order to investigate the impact of regional growth rate, we introduce a coefficient $\epsilon$ into the migration equation to control its growth.

\[
\begin{align*}
\dot{N} &= \epsilon(A_0 N - A_1 N^2 - A_2 P - \bar{U}) \\
\dot{P} &= L_0 + \frac{L_1(1-a)b}{r} N^2 - sP + \sigma_1 \frac{P^{\sigma_2}}{P^{\sigma_2} + P_c^{\sigma_2}} 
\end{align*}
\]

(1.12)

It is possible that changes in the regional growth rate can make the sustainable equilibrium unstable.\(^8\) As a result, the system will have zero resilience, because any perturbation will drive the system away permanently and the recovery time becomes infinite. In this case, the two measures of resilience (one by the domain of sustainability and one by the recovery time) are consistent with each other.

Except for this extreme case, simulation results show that the influence of the regional growth rate ($\epsilon \uparrow$) on the resilience of the system depends on the measure of resilience used. As shown in Figure 1.4(a), as the regional economic growth accelerates, the domain of sustainability shrinks. Thus an increase in the rate of regional growth decreases the resilience as measured by the domain of sustainability. This is due to the fact that the economic and ecological interactions are strengthened.

\(^8\)This occurs when the sustainable equilibrium lies “very” close to the critical point of the $P$-nullcline, the point at which the $P$-nullcline bends back in Figure 1.2. In the baseline model, such a change of stability does not occur.
Figure 1.3: An ecological degradation and slow recovery initiated by a sudden influx of population. The evolution of system is plotted in the state space (Figure a) and the time space (Figure b).

Figure 1.4: The impact on resilience when the regional growth rate doubles ($\epsilon = 2$). A decrease in the domain of sustainability is plotted in Figure (a). The recovery time to the sustainable equilibrium is plotted in Figure (b).
However, as also shown in Figure 1.4, the same increase in the regional economic growth rate can accelerate the speed at which the system returns to the sustainable equilibrium, provided that it does eventually return. Thus, an increase in the rate of regional economic growth can increase the resilience as measured by recovery time in some cases.

1.4 Optimal Policy with Variations in the Regional Growth Rate

This section calculates the optimal policy for this coupled model with fast-varying lake ecology and slow-varying regional development. In particular, it investigates how different economic growth rates may change the optimal type and time path of the policy. Since the unregulated system described above has two stable equilibria, the optimal policy should provide answers as to when and how policy should intervene to carry the system across the different domains of attraction. Faced with the danger of severe ecological degradation that can only recover slowly, the optimal policy should determine whether it is necessary and beneficial to prevent the possible severe ecological degradation or to intervene only to accelerate the recovery process. In addition, in a world of complex dynamics subject to adverse shocks, another objective of the policy maker is to identify the policy impact on the resilience of the system (Levin et al., 1998; Anderies, 2003). In this section, we first conduct an analytical analysis of the optimal management policy for this coupled ecological-economic system. Then,
numerical methods are applied to calculate the optimal policy and the results are discussed.

1.4.1 Analytical Analysis of the Optimal Policy

We first establish general policy framework for our problem and use an analytical approach to describe the impact of the regional economic growth rate on optimal policy. As in the baseline model, regional growth is driven by uncoordinated individual migration decisions, which affect and are affected by ecological changes in the region. The ecological system is governed by nutrient dynamics, which is affected by local population through land use. The policy maker cares about both regional economic development and environmental quality and is assumed to have two policy instruments: one helps to attract immigrants to the region \( v_1(t) \) and thus stimulates the local economy; the other helps to reduce pollution in the lake \( v_2(t) \). The adoption of either policy incurs a cost. The optimization problem is formulated as follows:

\[
\max_{\{v_1(t), v_2(t)\}} \int_0^\infty e^{-\rho t} \left\{ U_{pol}(N(t), P(t)) - C(v_1(t), v_2(t)) \right\} dt \tag{1.13a}
\]

subject to

\[
\dot{N}(t) = \epsilon F(N(t), P(t), v_1(t)), \tag{1.13b}
\]

\[
\dot{P}(t) = G(N(t), P(t), v_2(t)), \tag{1.13c}
\]

\[
N(t) \geq 0, \ P(t) \geq 0, \tag{1.13d}
\]

\[
N(0) = N_0, \ P(0) = P_0, \tag{1.13e}
\]
where $U_{pol}(N(t), P(t))$ is the instantaneous utility that the policy maker assigns to a given size of local economy and phosphorus concentration. $C(v_1(t), v_2(t))$ is the cost for the policy intervention with typical properties of a cost function: $C \geq 0$, $\nabla C > 0$ and $\Delta C > 0$. $v_1(t)$ is a policy that attracts in-migration, *ceteris paribus*, it increases the utility of local people and leads to more rapid immigration. Accordingly, $F(N(t), P(t), v_1(t))$ is assumed to be an increasing function of $v_1(t)$. Because $v_2(t)$ is an instrument that improves lake quality, $G(N(t), P(t), v_2(t))$ is assumed to be a decreasing function of $v_2(t)$. Under the assumption that the marginal impact of the policy intervention is non-increasing with intervention level, we have $\partial^2 F(N(t), P(t), v_1(t))/\partial v_1(t)^2 \leq 0$ and $\partial^2 G(N(t), P(t), v_2(t))/\partial v_1(t)^2 \leq 0$. For the sake of simplicity, the time arguments of $t$ are suppressed in the following discussions.

Assuming that optimal policy is an interior solution, the current value Hamiltonian is

$$H = [U_{pol}(N, P) - C(v_1, v_2)] + \lambda_1 \epsilon F(N, P, v_1) + \lambda_2 G(N, P, v_2) \tag{1.14a}$$

The necessary conditions are:

$$\frac{\partial H}{\partial v_1} = -\frac{\partial C}{\partial v_1} + \lambda_1 \epsilon \frac{\partial F}{\partial v_1} = 0 \tag{1.14b}$$

$$\frac{\partial H}{\partial v_2} = -\frac{\partial C}{\partial v_2} + \lambda_2 \frac{\partial G}{\partial v_2} = 0 \tag{1.14c}$$

$$\dot{\lambda}_1 = \rho \lambda_1 - \lambda_1 \epsilon \frac{\partial F}{\partial N} - \lambda_2 \frac{\partial G}{\partial N} - \frac{\partial U_{pol}}{\partial N} \tag{1.14d}$$

$$\dot{\lambda}_2 = \rho \lambda_2 - \lambda_1 \epsilon \frac{\partial F}{\partial P} - \lambda_2 \frac{\partial G}{\partial P} - \frac{\partial U_{pol}}{\partial P} \tag{1.14e}$$

$\nabla C$ is the Jacobian of $C(v_1(t), v_2(t))$ and $\Delta C$ is the Hessian.
When the ecological and economic subsystems are interacting with each other, any policy that neglects this interaction will be suboptimal, regardless of the regional economic growth rate, as shown in equations (1.14d) and (1.14e). These two equations govern the inter-temporal changes in the two costate variables ($\lambda_1$ and $\lambda_2$), which are the shadow price for local population ($N$) and phosphorus concentration ($P$) respectively. The changes over time are simply determined by the differences between the marginal benefits and costs of the two state variables, $N$ and $P$. Take $\lambda_1$ as an example. Under the current setting, local population pollutes the lake. The social net benefits of an increase in local population should account for this externality. This impact is embodied in the third term on the right hand side of equation (1.14d), which can become significant either because of a large value of $\partial G/\partial N$ or a large value of $\lambda_2$. Given this nonconvex ecological dynamics, a small change in the local population ($N$) can lead to dramatic changes in the ecological quality ($\partial G/\partial N$ is large). Nonlinearity may also arise from a threshold effect in $P$. That is, when phosphorus concentration $P$ exceeds some threshold level, the ecological system will undergo rapid degradation. This will result in rapid change in the shadow price for phosphorus ($\lambda_2$). In both cases, the shadow price for the local population ($\lambda_1$) will change substantially, and so will the optimal policy. These two equations also show that as the regional economic growth rate slows down ($\epsilon \to 0$), the impact of the regional growth on the shadow prices for state variables becomes negligible.
Equation (1.14b) determines the level of optimal policy for the slow process. It repeats the basic economic principle of economic optimality: the optimal policy is one that equates social marginal costs and social marginal benefits. The first term is the current value marginal cost of the policy regulating the slow process, and the second term is the marginal benefit of this policy. The economic growth rate, as controlled by the coefficient $\epsilon$, affects the marginal benefit: the slower the process, the smaller the marginal benefit. **Under the typical assumptions that marginal costs increase with the level of intervention** (that is, $\Delta C > 0$) **at a declining rate** (that is, $F$ is concave in $v_1$) **and assuming that system is in equilibrium state**, then the optimal intervention level in the slow process decreases as **the economic growth rate decreases** ($\epsilon \downarrow$). This confirms the typical practice that, **all else equal**, a policy instrument acting on the slower process should receive less emphasis. When the system is out of equilibrium, however, the direction of change in the optimal policy is unclear. From a policy perspective, the states out of the equilibrium are exactly the cases that policy makers are most concerned about and in which the policy intervention is most needed.

So far, the traditional analytical method of analysis has failed to provide any specific details for the optimal policy. These first order conditions only spell out the optimality conditions in a small neighborhood of the optimal path. When we have an explicit analytical solution, which serves as a reference point in the space of state variables and policy instruments, these first order conditions usefully specify
the optimal policy in case of small deviations. However, in this model, because we do not have an analytical solution for the optimal policy, we do not know the location of the optimal path, nor do we know the location of the small neighborhood of the optimal path in the whole space of the state variables and policy instruments. Without knowing the exact optimal policy, analysis of a small deviation from the optimal policy bears little information. More importantly, because this analysis only holds for marginal changes within a small neighborhood of the optimal path, we are unable to answer questions regarding the optimal policy when the system is close to an economic/ecological threshold or in danger of undergoing severe ecological degradation. Nor can this analysis provide adequate guidance for the optimal policy when the system is out of equilibrium, e.g., after the system is perturbed by a shock. The interactions between fast and slow processes exacerbate the problem because a small delay in policy response is enough to carry the system out of the neighborhood in which the first order conditions apply. In other words, we have exhausted the usefulness of analytics and are in a position in which numerical methods are required to fully answer these pressing policy questions.

1.4.2 Dynamic Solutions of the Optimal Policy

The application of numerical techniques allows for more realistic assumptions of the objective function.\textsuperscript{10} The functional form chosen here reflects the belief of some

\textsuperscript{10}Simpler functional forms can be more easily handled using the same algorithms.
key aspects of the policy maker’s objective function.\textsuperscript{11} First, we assume that the policy maker has a clear preference over a large local economy. In order to reflect this property, the objective function is assumed to be monotonically increasing in local population ($N$), which is a proxy for economic size of the region. The response to ecological conditions is more complicated however. In reality, policy makers are usually more responsive to ecological changes after severe ecological degradation has occurred or when they believe that such a danger is imminent. To reflect this reality, the objective function is assumed to be of a logistic form in the local ecological quality. When the lake is in an oligotrophic (eutrophic) state, the objective function from the ecological service is high (low). While the objective function is assumed to be an increasing function of the lake quality, it is not very responsive to changes in lake quality when water quality is high or low. On the other hand, when water quality is within an intermediate range and in the neighborhood of a threshold value, implying a transition from one state to the other is possible, the objective function is very responsive to small changes in water quality.

$$U_{pol}(N, P) = \alpha_1 N + \alpha_2 \frac{1}{P^2 + 1}, \quad (1.15a)$$

where $\alpha_1$ and $\alpha_2$ are the weighting coefficients for the economic development and ecological quality respectively.

\textsuperscript{11}The exact scale of net welfare gain in Table 1.1 depends on the specific functional forms, but the fact the economic growth rate has significant impact on net welfare gain is believed to hold in general.
The cost of the policy intervention is assumed to be quadratic in the level of intervention. This conforms with the typical convexity assumption of a cost function, reflecting the assumption of increasing marginal cost.

\[ C(v_1, v_2) = \alpha_3(v_1^2 + v_2^2), \]  

(1.15b)

where \( \alpha_3 \) controls the relative weighting of the cost as compared to the objective function.

Without additional data and further specification of specific policies, the precise relationship between the policy intervention and migration and phosphorus flows cannot be determined. As a first order approximation, the two policy instruments are assumed to have a linear impact on migration and phosphorus loadings respectively.

Adding these two linear controls to the equations (1.12), we have

\[ F(N, P, v_1) = A_0 N - A_1 N^2 - A_2 P - \bar{U} + \alpha_4 v_1, \]  

(1.15c)

\[ G(N, P, v_2) = L_0 + L_1 (1 - a)b N^2 - s P + \sigma_1 \frac{P^\sigma_2}{P^\sigma_2 + P_c^\sigma_2} - \alpha_5 v_2, \]  

(1.15d)

where \( \alpha_4 \) and \( \alpha_5 \) specify the effectiveness of the policy.

Substitute equations (1.15) into the optimization problem (1.13). In order to simplify the discussion, the notation for constant coefficients associated with the state variables are changed. All the coefficients in the \( \dot{N} \) equation are labeled as \( \theta \)'s
and those in the $\dot{P}$ equation are labeled as $\delta$’s. The Hamiltonian now becomes:

$$H = e^{-\rho t} \left[ \alpha_1 N + \alpha_2 \frac{1}{P^2 + 1} - \alpha_3 (v_1^2 + v_2^2) \right] + \lambda_1 \epsilon \left[ \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P + \alpha_4 v_1 \right] + \lambda_2 \left[ \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P_c^8} - \alpha_5 v_2 \right]$$  \hspace{1cm} (1.16)

The necessary conditions are:

$$\frac{\partial H}{\partial v_1} = -2\alpha_3 e^{-\rho t} v_1 + \alpha_4 \lambda_1 \epsilon = 0$$  \hspace{1cm} (1.19a)

$$\frac{\partial H}{\partial v_2} = -2\alpha_3 e^{-\rho t} v_2 - \alpha_5 \lambda_2 = 0$$  \hspace{1cm} (1.19b)

$$\dot{\lambda}_1 = -\lambda_1 \epsilon \left[ \theta_2 + 2\theta_3 N \right] - \lambda_2 \left[ \delta_2 + 2\delta_3 N \right] - \alpha_1 e^{-\rho t}$$  \hspace{1cm} (1.19c)

$$\dot{\lambda}_2 = -\lambda_1 \epsilon \theta_4 - \lambda_2 \left[ \delta_4 + \delta_5 \left( \frac{P^8}{P^8 + P_c^8} \right)' \right] - \alpha_2 e^{-\rho t} \left( \frac{1}{P^2 + 1} \right)'$$  \hspace{1cm} (1.19d)

With constraints

$$\dot{N} = \epsilon \left[ \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P + \alpha_4 \frac{1}{P^2 + 1} v_1 \right]$$  \hspace{1cm} (1.19e)

$$\dot{P} = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P_c^8} - \alpha_5 v_2$$  \hspace{1cm} (1.19f)

Initial conditions

$$N(t) \geq 0, \quad P(t) \geq 0$$  \hspace{1cm} (1.19g)

$$N(t_0) = N_0, \quad P(t_0) = P_0$$  \hspace{1cm} (1.19h)

Transversality conditions

$$0 = \lim_{T \to +\infty} \lambda_1(T) N(T)$$  \hspace{1cm} (1.19i)

$$0 = \lim_{T \to +\infty} \lambda_2(T) P(T)$$  \hspace{1cm} (1.19j)
The first order conditions along with the initial and transversality conditions formulate a boundary value problem, with initial conditions specified for state variables $N$ and $P$ and ending conditions for costate variables $\lambda_1$ and $\lambda_2$. The problem is solved with a boundary value problem solver (bvp4.m) in Matlab.

Before the discussion of results, some clarification on the relative weighting parameters ($\alpha$’s) is necessary. The sustainable equilibrium at point A is associated with a desired balance of population and ecological quality. This provides positive utility to the policy maker. When the system approaches this desirable equilibrium state, the instantaneous payoff $U_{pol}(N, P)$ will stay at a reasonable level. However, the instantaneous cost $C(v_1, v_2)$ will decrease gradually because the strength of intervention declines as the system moves closer to the sustainable equilibrium. Because the optimization horizon is $[0, +\infty]$, a small difference in the instantaneous cost and benefit $(U_{pol}(N, P) - C(v_1, v_2))$ can accumulate to a large value. In order to avoid overwhelming dominance of either the benefit or the cost of intervention, we assume that $\alpha_1 = 0.001$, $\alpha_2 = 0.003$ and $\alpha_3 = 0.5$.\textsuperscript{12} The results in this paper do not depend crucially on these parameter values. For simplicity, we assume that $\alpha_4 = 1$ and $\alpha_5 = 1$.

The results for the optimal policy response is summarized in Figure 1.5, which describes the optimal policy response for any given initial condition. The dotted black line is the $P$-nullcline and the broken black line is the $N$-nullcline. The solid black line

\textsuperscript{12}The optimization problem with $\alpha_1 = 0.01$ and $\alpha_2 = 0.03$ is also solved. No dramatic changes in the structure of solution space is observed.
is the separatrix that divides the domains of attraction of the "sustainable developed" equilibrium and the "undeveloped" equilibrium. The blue line\textsuperscript{13} demarcates the state space, it is beneficial to intervene to prevent the economic decline to the undeveloped equilibrium. In other words, it is beneficial to intervene and boost local economy to the sustainable equilibrium. For this reason, this region is called the “policy domain of sustainability” and this blue line is called the “policy line of sustainability”.\textsuperscript{14} As shown in Figure 1.5, the policy domain of sustainability is larger than that without policy. This increases the resilience as measured by the size of the basin of attraction. Simulations show that optimal policy also increases the resilience as measured by the recovery time. A special case is analyzed in Section 1.4.2.

The solid red line draws the Skiba points for this model, which demarcates a discrete transition of optimal policy.\textsuperscript{15} At each point on this line, the global optimum can be achieved by two different policies. Within a neighborhood on either side of

\textsuperscript{13}This line is plotted by trials and errors. Pick a point in the $N$-$P$ space as an initial condition. Find the solution that drives the system to the sustainable equilibrium state. Compare the consequent welfare with the welfare without any intervention. If the two generates the same welfare, the point is included in the blue line.

\textsuperscript{14}This blue line is not the separatrix for policy. However, this line is really what is relevant for policy makers. Mathematically, there should be a separatrices that divide the $N$-$P$ space in Figure 1.5 into two subareas. On one side, the optimal policy will drive the system close to the sustainable equilibrium. On the other side, the system will eventually go to the "undeveloped" equilibrium. In this subarea, there should be cases where policies try to linger at the unstable fixed point (C) as long as possible. While these are mathematically correct and may worth exploration for theoretical interest, we do not see its practical relevance, not to mention the numerical complications involved.

\textsuperscript{15}This line is derived in a similar way as the blue line. Different initial conditions in the neighborhood of this line are chosen and solved individually. The numerical algorithm finds two different policy solutions for each initial condition in this neighborhood, corresponding to the safety-first policy and slow-adjustment policy respectively. The welfare levels of the two policy solutions are compared. Each point on this red line identifies the initial condition that the two types of policies generate the same welfare level.
this line, one policy is preferred to the other. For this reason, we call it the “line of policy transition”. This line lies in the region where slowly reversible ecological degradation can occur, as explained in Figure 1.3. Based on intuitive reasoning, there are two types of policies that may apply in this region. The first type aims to prevent the undesirable switch to an eutrophic state and therefore requires strong intervention in the initial period. It is thus called the safety-first policy in this paper. However, such a policy may not always be the best solution because the cost of such intervention may be too high. The second type of policy does not prevent the ecological shift to an eutrophic state and focuses instead on the recovery process. Therefore the intervention will be less aggressive in the initial period and will change more gradually according to the state of the system. It is thus called the slow-adjustment policy.

In the neighborhood of this line of policy transition, we find that both policies are locally optimal and thus trial and error simulations are used to delineate the state space according to which policy corresponds to the global optimum. We find that if the initial condition lies within the neighborhood to the left of this line, the safety-first policy corresponds to the global optimum and thus is preferred. Conversely, if the initial condition lies within the neighborhood but to the right of this line, the slow-adjustment policy is preferred.

As we move upward along this line of policy transition, the difference between the two types of policies gradually diminishes. That is, the safety-first policy will
gradually allow for more ecological degradation while the slow-adjustment policy will be able to more rapidly accelerate the recovery process. If we look at the phase plot in $N-P$ space, the trajectories under these two policies will come closer and closer to each other. At the upmost point on the line of policy transition, the two solutions converge and the two local solutions converge to one.

Unlike the policy line of sustainability, across which the final state of the system is changed, on the two sides of this line of policy transition, the final state of the system is unchanged. While the system will finally evolve to the same sustainable developed equilibrium, the optimal policy on the two sides of the line of policy transition differs fundamentally.

With this optimal policy delineation of the state space in hand, we consider the potential welfare gains and the impact on resilience of three representative policy cases. Depending on their stage of regional development, regions with natural amenities can be classified into three categories, each faced with a different policy concern. First, there are regions that are growing reasonably well and are successful in preventing the threat of severe ecological degradation. Second, there are developed regions facing the problem of the potential overdevelopment and thus in danger of severe ecological degradation. Third, there are regions that are undeveloped and that compete for scarce economic resources in order to boost the local economy in the hope of becoming more developed. To consider the optimal policy for these three stages of regional development, we choose a representative case for each. Each representative
Figure 1.5: Optimal policy in phase plane. This figure adds a blue line and a red line onto the phase plot in Figure 1.2. The blue line addresses the multiple equilibria in the model. On the right side of the blue line, optimal policy will drive the system eventually to the desirable equilibrium state at point A. On the left, such a policy is suboptimal because of intervention cost. The red line is the line of policy transition which addresses the multiple local optimal solutions. It demarcates the preference over different local optimal solutions.

case corresponds to a different stage of regional development and is specified by a given initial condition and then simulated to examine the potential policy gains in terms of social welfare and resiliency. Based on the belief that all existing coupled systems with divergent time scales have existed for quite long time, it is unlikely for us to observe initial conditions far off the $P$-nullcline. Since in reality economic and/or ecological shocks may occur, small deviations from the slow manifold is not uncommon. As a result, all of the three cases use initial conditions that are close to the slow manifold.
Regions with Balanced Economic Growth and Environmental Protection

The first example represents a region that has already achieved some balance between economic development and environmental protection. We call this the "sustainable" case. The initial condition is chosen so that it lies deep in the domain of sustainability, but with a population level that is sufficiently low so that it is far from the lake eutrophication threshold. To illustrate this scenario, we choose a point close to the lower portion of the $P$-nullcline and on the right side of the separatrix. The initial condition is set at $(N_0, P_0) = (4, 2.32)$. The optimal path in the $N$-$P$ phase plane is plotted in Figure 1.6 as the red line. The welfare gain relative to no intervention is only 1.4%, as shown in Table 1.1. The optimal economic and ecological policy in the initial period are plotted as solid red lines on the first row in Figure 1.7. Both policies gradually decrease over time and eventually level off. The incentives to manipulate regional growth and ecological quality to achieve higher welfare is illustrated through a small oscillation in the ecological policy ($v_2$) at around $t = 200$ to 300. Under the optimal policy, instead of going directly to the sustainable equilibrium, the optimal policy allows a slight overshoot to get the additional welfare gain from regional growth at the cost of a small oscillation in the regional economy and lake quality. When the system is already close to the sustainable equilibrium, the possible gain from excessive economic development is limited, but the consequent cost is large. Therefore, with a reasonably specified discount rate, it is not beneficiary to sacrifice lake quality for the short term economic interest.
Developed Regions Facing the Overdevelopment Problem

The second case considers regions facing the potential of overdevelopment. The policy question here is when the ecology has been degraded due to overdevelopment, how should the optimal policy respond? We call it the “degraded ecology” case.

To illustrate this scenario, we choose a point close to the upper portion of the $P$-nullcline. The initial condition is set at $(N_0, P_0) = (6, 7.04)$. The optimal path in the $N$-$P$ phase plane is plotted in Figure 1.6 as the blue line. Note that when the system moves on the lower portion of the $P$-nullcline, the trajectory is indistinguishable from that of the sustainable case and thus coincides with the pink line. The relative welfare gain as compare with no intervention is 8.3%, as shown in Table 1.1. The optimal economic and ecological policy in the initial period are plotted as solid blue lines on the second row in Figure 1.7. Both the economic and ecological policy interventions are non-monotonic over time: they become more aggressive in the initial period and then diminish until they eventually level off. This non-monotonicity is due to the non-monotonic change of population and phosphorus concentration over time. Because population first decreases due to lake eutrophication and then increases due to improved water quality, it is beneficial to use economic policy to slow down the population loss in the initial period, which accelerates recovery later. The strength of the ecological intervention does not peak at the $t = 0$ where the lake quality is in the worst state along the optimal path. At this state, the incentive to flee is so strong, due to lake eutrophication, that stronger intervention will not have much
significant impact. In this example, the strongest intervention occurs shortly before the direction of migration is reversed. Similarly, the strongest ecological intervention occurs while the lake ecology is close to the ecological threshold. In summary, the major policy objective is to accelerate the speed at which the system recovers to the sustainable equilibrium. The strongest interventions occur when the policy impact is most significant.

**Undeveloped Regions with Growth Potential**

The third policy case corresponds to optimal policy for undeveloped regions with growth potential. We call it the "undeveloped" case. Without investment, these regions would eventually decline and die off, but with investment they could successfully attract regional growth and develop into a larger region. Because migrants are scarce and not all regions will attract the necessary population to fuel growth, the policy maker must make a decision regarding investment in any given region and whether the investment will pay-off in terms of pushing the region into the domain of sustainability and to the sustainable equilibrium. Thus the key policy question is under what conditions should investment in regional development be allocated or withheld from a region.

To illustrate this scenario, we choose a point close to the lower portion of the $P$-nullcline but are on the left side of the separatrix. The initial condition is set at $(N_0, P_0) = (2.8, 1.72)$. The optimal path in the $N$-$P$ phase plane is plotted in Figure 1.6 as the green line. We observe that this trajectory crosses the separatrix
and stays close to the $P$-nullcline. Just like the optimal path in the degraded ecology case, it tends to join the optimal path for the sustainable case. The relative welfare gain as compared with no intervention is 26.2%, as shown in Table 1.1. The optimal economic and ecological policy in the initial period are plotted as solid red lines on the third row in Figure 1.7. The policy intervention is the strongest at the very start and then declines over time. This is because the system is outside the domain of sustainability and thus, without policy intervention population will leave the region. Thus the sooner the intervention the better and the key task is to transport the system across the separatrix. In all three policy cases, we find that investment in the initial period is crucial. This is illustrated in Figure 1.6.

This is because the system is outside the domain of sustainability and thus without policy intervention people will leave the region. Thus the soonest and the key task is to transport the system across the separatrix. The sooner the intervention, the better. A small oscillation in the ecological policy at around $t = 400$ is also observed. The reason is the same as in the sustainable case.

For all three cases, the system evolves almost along the same route as it is approaching the sustainable equilibrium. This implies the optimal policy of the three cases are not much different from each other after the initial period. The fact that the trajectories are very close to the $P$-nullcline shows that the interventions are marginal. This phenomenon is due to the model characteristic of a fast-changing ecology and slow-changing economy. It also suggests that in such a coupled system,
policy design needs to focus on the initial period. It is in this initial period that the policy intervention achieves its major objectives: driving the system across certain thresholds (the unsustainable case), accelerating the slowly reversible changes (the degraded ecology case), or eliminating discrete ecological changes (safety-first policy to the left of line of policy transition). When these objectives are achieved, further intervention is marginal.

**Impact of the Regional Growth Rate on Optimal Policy**

The optimal policy results discussed so far have assumed constant regional growth and ecological change rates. In reality, regions grow at different speeds and thus it is of interest to consider how either slower or accelerated regional growth influences...
the optimal policy. We do this by simulating the model again using lower and higher values of $\epsilon$ in equation (1.12). The results show that changes in the rate of regional growth have fundamental impacts on the optimal policy.

In general, we find that changes in the rate of economic growth change the optimal policy (Figure 1.7). The strength of the policy intervention in the initial period increases as the economic growth rate increases ($\epsilon \uparrow$). This is observed in all three cases and applies to both the economic and ecological policy instruments ($v_1$ and $v_2$). More specifically, we find that optimal policies regarding investment in undeveloped regions and the management of overdeveloped regions both depend on the rate of growth in a region. For regions in which the local population is smaller than some minimal self-sustainable size (as determined by the separatrix), the major policy goal is to boost the local economy across the separatrix. With slower regional economic growth, however, the cost of intervention increases. As a result, the line of sustainability, which demarcates the initial conditions for which investment in lesser developed regions is socially optimal, moves closer to the corresponding separatrix. For example, when the regional economic growth rate decreases by 25% ($\epsilon$ decreases from 1 to 0.75), the horizontal distance between the blue and black lines changes by 14-43%.\textsuperscript{16}

\textsuperscript{16}This variation reflects the asymmetry in the fast and slow dynamic processes. In order to carry the system across the separatrix, the horizontal distance may not be the travelling distance under optimal policy. Because a pollution abatement policy can have the same effect. In addition, the separatrix will shift as well.
Because the rate of regional growth can shift the location of the line of policy transition, it determines whether a safety-first or slow-adjustment policy is preferred in case of overdeveloped regions that are in danger of lake eutrophication. Simulations show that different regional economic growth rates can lead to the preference of one policy over the other. For instance, in the baseline model, the slow-adjustment policy is preferred to the safety-first policy. When the regional economic growth rate doubles ($\epsilon = 2$), safety-first policy is preferred. The underlying reason is that the increase in the regional growth rate reduces the cost of intervention and increases the welfare gain from a policy that avoids large ecological degradation.

Finally, the magnitude of welfare gain from the optimal intervention is highly sensitive to the changes in the regional economic growth rate ($\epsilon$). Table 1.1 lists the relative welfare gains from the policy intervention as compared with no intervention. In the sustainable case, policy intervention generates an 1.4% increase in welfare in the baseline model. When the regional economic growth rate decreases by half ($\epsilon = 0.5$), the optimal policy improves the welfare by only 0.8%. The doubling of economic growth rate has no significant impact on relative welfare gain. The relative welfare gain is much more sensitive to the regional economic growth rate in the degraded ecology case and the undeveloped case. In the degraded ecology case, the optimal policy accelerates the ecological recovery process (under the slow-adjustment policy) which generates a relative gain of 8.3%. When the economic growth rate decreases by half, the welfare gain decreases to 2.2%. When the growth
rate doubles ($\epsilon = 2$), the net gain increases dramatically to 90.3%. This is because the increase in regional growth rate has shifted the separatrix so that the initial condition now lies between the new separatrix and the new line of sustainability. Finally, in the undeveloped case, the optimal policy in the baseline model generates a relative welfare gain of 26.2%. This is because the optimal policy shifts the system into the domain of sustainability. When the economic growth rate decreases by half ($\epsilon = 0.5$), the relative welfare gain decrease dramatically to 4.8%. When the growth rate doubles, the relative gain increases to 80%. Such sensitivity to the economic growth rate is due to the fact that the intervention cost and the speed of the system approaching the sustainable equilibrium are both a function to the economic growth rate.

In summary, we have numerically solved this nonconvex dynamic optimization model, in which a fast-changing ecology interacts with a slow-changing economy. We identify a “policy line of sustainability” which specifies the condition under which intervention should and should not drive the system to the sustainable developed

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$\epsilon = 0.5$</th>
<th>$\epsilon = 1$</th>
<th>$\epsilon = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sustainable Case</td>
<td>0.8%</td>
<td>1.4%</td>
<td>1.5%</td>
</tr>
<tr>
<td>Degraded Ecology Case</td>
<td>2.2%</td>
<td>8.3%</td>
<td>90.3%</td>
</tr>
<tr>
<td>Unsustainable Case</td>
<td>4.8%</td>
<td>26.2%</td>
<td>80.0%</td>
</tr>
</tbody>
</table>
equilibrium. We also identify a “line of policy transition” which specifies the condition under which the best strategy to approach the sustainable equilibrium is changed from the safety-first policy to the slow-adjustment policy. We analyze three representative cases corresponding to developing regions with the ecologically sustainable growth, developed regions with overdevelopment and degraded ecology and underdeveloped regions with growth potentials. We find that initial intervention is crucial for the long-run interest in this model. And optimal policy does not have to be most aggressive when the condition of the system is at its worst state. Variations in the regional growth rate change the resilience of the system. Faster regional growth leads to more aggressive optimal intervention in the initial period and increases the relative net welfare gain from the optimal intervention.

1.5 Simplification of the Fast-Slow Dynamics

In the existing economic literature, there are two common simplifications for models with fast-slow dynamics. One takes the slow process as constant and the other takes the fast process as instantaneous. This section investigates the consequences of these simplifying assumptions and the conditions for their applicability.

The first assumption arises when people are more interested in the fast process, such as the phosphorus dynamics in the lake ecosystem. This is an assumption commonly seen in the lake management literature like Carpenter et al. (1999). It is natural to ask whether we can treat the slow-changing regional growth as a constant, especially when we are dealing with already complicated models. In practice, such
an assumption can reduce the dimension of the original problem, because the slow dynamic process degenerates into a single value. Take the baseline model as an example. When we assume that the region does not grow, we are imposing the condition that the local population \( N \) is fixed. This degenerates the whole \( N-P \) plane into a vertical line, as in Figure 1.9(a). The simplified model predicts that the equilibrium is at the intersection of this vertical line and the \( P \)-nullcline. In general, this gives wrong predictions about the equilibrium unless the local population \( (N) \) is set exactly at the equilibrium level (the solid green line in the figure). Therefore, if we are interested in the steady state of the system, we should avoid such an assumption unless we have very good knowledge about the equilibrium level of \( N \).

But knowing the exact equilibrium population requires a thorough understanding of the interactions between the regional growth and the lake ecology. Thus it requires the same information as determining the exact equilibrium of the original system and the point of simplifying the original dynamic system is lost.

Even when the local population is miraculously fixed right at the equilibrium level, it can give false descriptions of the resilience and lead to both excessive and inadequate policy reactions, as shown in Figure 1.9(a). Firstly, the simplified model fails to capture the unsustainable equilibrium of the original coupled system at fixed point B. Therefore, any policy analysis based on this simplified model will definitely fail to address the possible economic decline and the related problems. Secondly, the simplified model creates a false economically sustainable but ecologically degraded
equilibrium at point E. It also creates a false unstable equilibrium at point D. This simplified model predicts that the sustainable equilibrium (A) can only resist a 20% adverse ecological shock. That is, the phosphorus concentration after shock should be less than $P_r$, the broken green line in the figure, or the lake will be driven to the eutrophic state at point E and stay there forever. But the analysis of the baseline model tells a different story. The distance of the sustainable equilibrium (A) to the separatrix shows that this equilibrium state can resist much larger ecological shocks. As a result, in this example, the simplified model underestimates the resilience of the true sustainable equilibrium state. Policy analysis based on this simplification will lead to policies that are too restrictive in their management of land use and therefore inefficiently limit human activities on the land.\textsuperscript{17} Thirdly, it ignores the possible economic decline due to large ecological shocks. When the ecological shock is so large that it shifts the system to the other side of separatrix in the baseline case, this will initiate economic decline and the system will eventually go to the unsustainable equilibrium at point B. The simplified model completely misses this possibility. Therefore, policy analysis based on this simplified model will underestimate the impact of large ecological shocks and ignore the necessity of the coordinating economic policies. In short, this simplification can lead to both excessive and inadequate responses to adverse ecological shocks.

\textsuperscript{17}Of course, when large ecological shocks occur, the lake will become eutrophic for some time, but this ecological change can be slowly reversed as discussed in the previous sections.
What if we are interested in the short-run behavior of the system? Intuitively, since the slow process is slow, the simplified system may provide an approximation for the original system in the short-run. But how good is this approximation? Take the dynamic system (1.20) as an example.

Assume that a dynamic system is governed by

\[
\begin{align*}
\dot{N}(t) &= f_1(N(t), P(t)) \\
\dot{P}(t) &= f_2(N(t), P(t))
\end{align*}
\]  

(1.20)

As shown in the appendix, an approximate simplification error (denoted as $\xi$) after time period $\Delta t$ can be written as:

\[
\xi \approx \frac{1}{2} \left[f_1 \frac{\partial f_2}{\partial N}\right]_{(N_0, P_0)} (\Delta t)^2
\]

It shows that the slower the regional growth ($|f_1|$ is small), the smaller the error after time period $\Delta t$. Also, when the regional growth has a weaker impact on the ecology ($|\partial f_2/\partial N|$ is small), the approximation error will be smaller. The impact of nonlinearity on the simplification error comes through the higher order terms, which can accumulate rather rapidly. Since the above formula has neglected all the higher order terms, the formula can only be regarded as an approximate lower bound. This formula can be re-arranged to show how long the simplified model can be used as an approximation.

In summary, treating the slow process as constant generates wrong predictions of the equilibria and the resilience of the system. The quality of the short-run approximation will depend on the rate of regional growth and the interaction between...
the regional economy and the ecology. Nonlinearity is important and may lead to a rapidly growing error. Policy responses based on this simplification can be both excessive and inadequate in the event of adverse ecological shocks.

The other simplifying assumption arises when people are interested in the slow process, like the regional economic growth in this model. It assumes that the fast process instantaneously responds to the changes in the slow variable. In the context of the baseline model, this assumes that, at any given population level, phosphorus concentration in the lake will instantaneously adjust onto the $P$-nullcline. That is, the system will move along the $P$-nullcline (the solid green line in Figure 1.9(b)) and any deviation from it will be reversed instantaneously. It is self-evident that the instantaneous adjustment is unable to capture the short-run changes. However, even though this simplification can correctly predict the location of the fixed points, it can generate wrong predictions about the long-run dynamics. This is demonstrated in the following example.

Suppose a dynamic system is governed by:

\[
\begin{align*}
\dot{x} &= -z \\
\epsilon \dot{z} &= -x + z
\end{align*}
\]

The assumption of instantaneous adjustment of the fast variable is equivalent to the condition of $\epsilon = 0$. This gives $z = x$. Substitute into the equation of $\dot{x}$, we will have

\[
\begin{align*}
\ddot{x} &= e^{-t}x_0^s \\
\ddot{z} &= e^{-t}x_0^s
\end{align*}
\]

which predicts that the system will converge to $(0, 0)$ regardless of the initial condition.
It is easy to verify that the system only has one fixed point at \((0, 0)\) and it is a saddle point. This means the system will only reach \((0, 0)\) if it starts exactly on the stable manifold for this saddle point. For an example, if the system starts from \(x_0 = 0\) and \(z_0 = 1\), the analytical solution is:

\[
\begin{align*}
  x(t) &= \frac{\epsilon}{\sqrt{4\epsilon + 1}} \left[ e^{\frac{(1-\sqrt{4\epsilon+1})t}{2\epsilon}} - e^{\frac{(1+\sqrt{4\epsilon+1})t}{2\epsilon}} \right] \\
  z(t) &= \frac{1}{2\sqrt{4\epsilon + 1}} \left[ e^{\frac{(1-\sqrt{4\epsilon+1})t}{2\epsilon}} (-1 + \sqrt{4\epsilon + 1}) + e^{\frac{(1+\sqrt{4\epsilon+1})t}{2\epsilon}} (1 + \sqrt{4\epsilon + 1}) \right]
\end{align*}
\]

It is obvious that the system will not approach \((0, 0)\), instead it explodes as \(t \to \infty\), even when \(\epsilon \to 0\). In short, unless the system starts right on the stable manifold of the fixed point, the simplified version gives incorrect prediction about the long-run dynamics.

This simplification also changes the domain of attraction for the fixed points, which is essentially the reason for the incorrect prediction of long-run behavior in the above example. This can be better explained in the baseline model. The real basin of attraction of the complete model is plotted in Figure 1.9(b). Under the simplifying assumption, the domains of attraction for stable fixed points A and B are separated by population level \(N_r\) in Figure 1.9(b), regardless of the \(P\) value. This is in effect saying that the separatrix is a vertical line in the \(N-P\) plane. For instance, starting from point F, the simplified model predicts that \(P\) will instantaneously adjust onto the \(P\)-nullcline and then evolve to point A. However, in the original system, because F is out of the domain of sustainability, the system will eventually go to the unsustainable equilibrium state at B.

For the resilience to the ecological shocks, measured by the vertical distance from the sustainable equilibrium to the separatrix, this simplifying assumption will
overestimate the resilience when phosphorus concentration is high. For instance, while the system is at point F, the simplified model predicts that the system is temporarily out of equilibrium. The phosphorus concentration will instantaneously decrease and the system will vertically jump onto the $P$-nullcline. Based on the observation that population level in the region is still relatively low, it is unlikely that the policy maker will intervene. But in the original model, point F is out of the domain of sustainability, implying that the ecological condition is poor enough to drive people out of the region. Since point F is very close to the separatrix, a small policy intervention, either through the improvement of ecological condition or the improvement in local public service, will be sufficient to prevent this eventual economic collapse. Therefore, the policy responses based on this simplification is inadequate.

Another complication arises due to the nonlinearity in the model. In the linear world, $\dot{P} = 0$ usually gives a unique solution of $P(N)$ \textsuperscript{18}. However, nonlinearity can generate multiple isolated $P(N)$ as solutions to $\dot{P} = 0$. For instance, in Figure 1.9(b), when $N = 5.5$, there are three real solutions of $P$. This will give rise to three possible instantaneous response function for $P(N)$. Then the selection among these branches will affect the dynamics of the simplified model and further information is required for the selection. Consider another example:

\textsuperscript{18}The infinite number of solutions or the nonexistence of solution is seldom seen in economic literature.
\[
\begin{align*}
\dot{x} &= -x + z \\
\epsilon \dot{z} &= z(z - 2x)
\end{align*}
\]

As \( \epsilon \to 0 \), we have \( z_1 = 0 \) and \( z_2 = 2x \). Substitute \( z_1 \) into \( \dot{x} \), we have an asymptotically stable solution

\[ x(t) = e^{-t}x_0 \]

Substitute \( z_2 \) into \( \dot{x} \), we have an asymptotically unstable solution

\[ x(t) = e^{t}x_0 \]

Given these two possibilities, which one gives a better approximation of the real model? How shall we determine the selection criteria?

In summary, if we are only interested in the immediate evolution of the system, we may ignore the slow process for a short period, but long-run prediction is invalid. If we are interested in the slower process, then treating the fast process as an instantaneous process can give us the right location of equilibrium but it may not give the right long-run dynamics of the system, nor can it be used to approximate the short-run behavior. In both cases, the predicted resilience of the system will be incorrect. The analysis in this section also points out that policy analysis based on these simplified models can be misleading. Policy analysis based on these simplifications cannot properly deal with the large variations in the model that shifts the system into and out of the domain of sustainability, nor can it effectively address the severe ecological degradations that recover slowly. The complete analysis of the coupled system with fast-slow dynamics is necessary.
1.6 Sensitivity Analysis

The results are robust to changes in parameter values so long as the topological structure of the model is not changed. In particular, this refers to the existence of two stable equilibria with one being sustainable.

In the dynamic equation governing the phosphorus dynamics, the non-interacting terms (namely, the constant loading term, the sedimentation term and the recycling term) are relatively well studied. The corresponding parameter values are constrained by field experiments. Because the nonlinear term is the most salient feature of this ecological dynamic equation, $\sigma_2$ is changed from 8 to 4, so that this recycling term becomes 'less' nonlinear. We find that population over-adjustment after severe ecological degradation become mitigated. The policy intervention in both tools are less intense in the initial period after ecological degradation. We find no qualitatively different result.

In the dynamic equation governing regional migration, the non-interacting terms (namely, the quadratic form in local population) are formulated to generate the humped shape in population and to capture the agglomeration and the congestion effects in regional growth literature. One key parameter is the agglomeration coefficient $b$ which controls the strength of agglomeration. When it is decreased by around 5%, we find only qualitative changes in results. Larger decreases, like a 10% decrease in $b$, will change the topological structure. Because the agglomeration effect is too weak to support the sustainable equilibrium state, the dynamic system has
only the unsustainable equilibrium. When the agglomeration effect is too strong, like a 10% increase in the coefficient $b$, the population and economic activities in the region will exceed the carrying capacity of the ecology and lead to severe ecological degradations. As a consequence, the sustainable equilibrium state in the baseline model becomes unstable.

We are particularly interested in the robustness with respect to the interacting terms: the individual disutility from the ecological degradation (the $P$ term in the $\dot{N}$ equation) and the runoffs from land use in the $\dot{P}$ equation. Again, we find that so long as the topological structure is not changed, changes in the parameter values do not qualitatively change the results. For instance, the topological structure is unchanged with a 10% increase in the coefficient $A_2$. With such a change, because pollution has a larger impact on individual utility, the sustainable equilibrium will have a smaller economic size but a better ecological quality. However, when people become very sensitive to ecological degradations, like a 25% increase in $A_2$, regional growth in the baseline model will not occur, because the ecological degradation will offset the agglomeration effect. On the other hand, if people become insensitive to ecological changes, local economy can expand beyond the carrying capacity of the local ecology, making the sustainable equilibrium unsustainable. This is found by a 10% decrease in $A_2$. By changing the ecological responsiveness to economic activities, we arrive at the same conclusion that parameter values will generate no qualitative
changes in results so long as the topological structure of the model is not changed. In short, the results in this paper are insensitive to parameter changes that do not change the topological structure of the model. Because this topological structure reflects our understanding of the key features of the coupled ecological-economic system in regions that experienced the amenity driven growth, we believe the result have general implications. For the specification of the function forms chosen in this paper, the topological structure is insensitive to a roughly 10% change in the parameter values. This number will change with different function forms.

1.7 Conclusion

In this paper, we have investigated the dynamic behavior of a coupled ecological-economic model in which a slow-varying economic growth interacts with fast-varying lake ecological processes. We have found that the relative speeds of economic and ecological change have profound implications regarding the resiliency of the system and the nature of optimal policy interventions. Our principal findings are:

Firstly, the relative speeds of economic and ecological adjustment are crucially important for the resilience of our coupled ecological-economic system. In our model, faster economic growth renders the system more vulnerable to large exoge-

uous shocks; however, faster economic growth can also decrease the time the system

19In fact, the consequences of the changes in ecological responses (changes in the coefficient $L_1$) are mirror images of those changes in human response to ecological quality (changes in $A_2$). This is an interesting phenomenon, and should be explored in future studies.

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needs to recover to return to the original sustainable state. In short, the coupled ecological-economic system becomes more resilient to small economic and ecological shocks, but less resilient to large economic and ecological shocks.

Secondly, this paper shows that policy intervention can increase the resilience of the system. In the presence of slow economic processes and fast ecological processes, the interventions in the periods immediately following an economic or ecological shock are crucial. Our simulations show that as the rate of economic growth increases, it is optimal to intervene more aggressively in the initial periods following a shock. We also find that as the rate of economic growth increases, the net welfare gains from intervention will increase substantially.

Thirdly, two common ways to simplify coupled ecological-economic models are to assume an unchanging population or an instantaneously changing ecological process. Such assumptions, however, can lead to incorrect assumptions regarding the resilience of the coupled system. If economic activity is treated as unchanging, the model will miss the true equilibrium states and draw misleading conclusions regarding the resiliency of the system. If ecological process is treated as instantaneously adjusting, the model will correctly capture the equilibrium states, but not the dynamic adjustment to equilibrium after an economic or ecological shock. Moreover optimal management prescriptions based on these assumptions will either be overly restrictive or inadequate.
We encourage future extensions of our work in two distinct directions. One direction is to find empirical evidence to support our theoretical analysis. Perhaps the single greatest obstacle to empirical work is that proper modeling of the slow economic process may require a longer series of observations than may currently be available. The second direction is the further exploration of models involving fast economic change and slow environmental change, as might be appropriate to understanding the impacts of global warming.
Figure 1.7: Impact of regional growth rate on optimal policy. The three rows plot the optimal policy over time in the sustainable, unsustainable and ecological degraded cases respectively. In each plot, the solid line depicts the optimal policy in the baseline model. The optimal policies when $\epsilon = 0.5$ and $\epsilon = 2$ are depicted as the thin and thick broken lines respectively.
Figure 1.8: Multiple local solutions. The green line depicts the phase plot of system without policy intervention. The blue line depicts the system evolution under the safety-first policy. The red line depicts the system under the slow-adjustment policy.

Figure 1.9: Simplifying assumptions on the relative speed in fast-slow dynamics. Figure (a) illustrates the case when the slow process is treated as constant over time. Figure (b) illustrates the case when the fast process is treated as an instantaneous change.
CHAPTER 2

THRESHOLD MANAGEMENT IN A COUPLED ECOLOGICAL ECONOMIC MODEL
2.1 Introduction

Threshold effects are commonly observed in ecological systems. Examples include the thresholds in biodiversity loss in forest, lake eutrophication, soil salinization, forest fire frequency, grazing pressure and global warming. The consequent large ecological damage after the breach of a threshold has caught the attention of both the ecologists and resource economists for decades. One commonly suggested policy is to identify the key contributor to the possible ecological disaster/hazard and use policy instruments to act in the opposite direction. For instance, to avoid lake eutrophication, the focus of a remedying policy would be to reduce the nutrient loadings and avoid over-enrichment. The common belief is that so long as a policy can reduce nutrient loadings contributing to the over-enrichment, we will reduce the risk of eutrophication and improve the ecological health of the lake. However, subsequent ecological improvements from such a policy can induce a human behavioral response, e.g., the region may become more attractive as a result of improved ecological services and induce a new round of household and migration into the region. If human activities are the major threats to ecological health, then policies aiming to improve the ecological services but failing to address these potential behavioral feedbacks are unlikely to be optimal and may in effect jeopardize the ecological service in question.

In this paper, we focus on complications due to this indirect effect of policy on human behavior. The objective is to assess threshold management policies in the context of a fully integrated human-ecological system and when the uncoordinated
individual economic decisions are dynamically affecting and responding to the ecological changes.

In reality, ecological thresholds can be triggered by two types of factors. The factors that are internal to the ecological system are called internal triggers. The others are called external triggers. The internal triggers are mainly studied in the ecological literature (Vogelmann and Ovitt, 1968; Wilson and Agnew, 1992; Ripple and Besca, 2004). The periodic outbreak of the spruce budworm is a good example because it is triggered by the interactions among forest, insectivorous birds and budworm (Ludwig et al., 1978). A policy that reduces the number of budworms can help to avoid or delay the outbreak of budworms. The indirect effect on human behavior is irrelevant. The external triggers and their impacts on ecological systems are of more interest to economists, especially when the triggers are related to human economic activities. This has been the focus of many resource and environmental studies (Ciriacy-Wantrup, 1952; Bishop, 1978; Clark, 1973; Henry, 1974; Cropper, 1976; Berck, 1979; Tsur and Zemel, 1995; Farzin, 1996). It is only recently that people have begun to explore the system dynamics of the integrated ecological-economic systems (Perrings and Walker, 1997b; Carpenter et al., 1999; Brock and Starrett, 2003). The policy analysis in these recent studies shares the same philosophy as the earlier studies. That is, they first isolate the ecological system from a more complicated system. Then the ecological changes are modeled as ecological dynamic processes linking to the human economic system through a few exogenous parameters.

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20 This distinction follows that of Resilience Alliance (http://www.resalliance.org).
The policy analysis in these studies is formulated as a way to manipulate these exogenous parameters so as to generate the most “desirable” ecological outcome. In the case of threshold management, the exogenous parameters are the triggers of the threshold effect, which respond to ecological changes only through policy.

Take Carpenter et al. (1999) as an example. Phosphorus loadings, the external trigger of the eutrophication threshold, respond to ecological changes because phosphorus loadings are fully controlled by the policy and the policy is responding to ecological changes. In other words, the external trigger responds to ecological changes only when such ecological changes have induced policy adjustments. These are reasonable and necessary assumptions to make when we first explore complex ecological dynamics, especially when the economic activities are not responding to ecological changes.

In reality, there are many cases in which human activities affecting the ecological system are also responding to ecological changes. In such cases, in which the ecological impact is determined by the cumulative effects of many autonomous and uncoordinated individuals that are responding in some way to the ecological change, it is unlikely that the ecological impact of these activities can be fully controlled by the policy makers. A fishery is a good example. The fishing activities are affecting and at the same time responding to the fish population. Controlling the fishing permits is easy but controlling the real ecological impact associated with the fishing activities is not (Tegner and Dayton, 1999; Blaber et al., 2000). This is true for
Figure 2.1: Decoupled v.s. integrated models of threshold management. In the decoupled model, external triggers respond to ecological changes only through the changes in policy as shown by the arrows with dashed lines. In an integrated model, external triggers are modeled as individual economic decisions that affect and respond to ecological changes. The management policy is interacting with both the economic and ecological systems, as shown by the arrows with solid lines.
many coupled ecological-economic systems, in which the above-mentioned external triggers of the ecological threshold are dynamical processes affecting and responding to ecological changes rather than a fixed parameter. In such cases, is it necessary that policy analysis be built upon an integrated ecological-economic dynamic system, that is, explicitly acknowledge the two-way interactions between the ecological and economic systems? Can we instead use the simplified model that separates the ecological system from its interacting counterparts? In this paper, we adapt the coupled ecological-economic system of Chen et al. (2009) to examine the implications of taking an integrated approach to optimal policy design versus the “naive” decoupled approach in which human behavioral responses are ignored.

The most closely related works lie in the lake management literature pioneered by Carpenter et al. (1999). In their seminar work, they analyze the lake management policy subject to eutrophication threshold that is triggered by excessive phosphorus loadings from economic activities. Because their focus is on the management of a dynamic ecological system, they carefully model the dynamics of water quality and necessarily omit the economic dynamics of phosphorus loadings. In particular, they take the cost-benefit analysis approach and posit an imaginary social planner to optimally manage the loading level, ignoring the reality that the nutrient loadings are difficult to monitor or regulate. More importantly, these nutrient loadings are endogenous to economic activities around the lake, like the nutrient run-off from agriculture and sedimentation from urbanization.
The follow-up studies of Brock and Starrett (2003); Crepin (2003); Dasgupta and Mäler (2003) make the same assumptions by separating the ecological system from its dynamically interacting counterparts. In contrast, in this paper we explicitly model the economic dynamics underlying the polluting activities and acknowledge the uncoordinated feature of these polluting activities. In this way, we establish two-way interactions between the economic activities and the ecological changes. That is, while people’s economic activities can affect the ecological system, the ecological changes can also affect economic activities. We also extend Brock and Starrett (2003)’s policy analysis on lake management to the policy analysis of a coupled ecological-economic system in which the lake ecosystem embodies an eutrophication threshold.

This paper uses a stylized lake ecological model with a regional model of land and labor markets as the basic framework for the coupled ecological-economic system. The regional economic development is modeled as a dynamic process driven by household relocation decisions, which in turn affects the land and labor markets. Economic development in the region will have negative ecological impacts through changes in land uses. Because of household preference for natural amenities (McGranahan, 1999; Hansen et al., 2002; Rappaport, 2007; Sutton and Day, 2004), the ecological conditions will also affect the relocation decision and thus affect the labor and land market outcomes. Because human activities are simultaneously affecting
and responding to ecological changes, the ecological threshold and its policy implications need to be considered from the perspective of a coupled ecological-economic system instead of the perspective of ecological management alone. This is particularly important when the policy maker cannot fully control the polluting activities. For instance, if excessive urban development is the trigger for ecological threshold effects, policies aiming to improve the ecological services but failed to fully constrain the polluting activities will likely make the region more attractive and stimulate more urban development.

The paper is organized as follows. Section 2.2 reviews the economic literature on threshold management. Section 2.3 describes a coupled ecological-economic model and the basic dynamics. Section 2.4 compares the policy effects on this coupled ecological-economic system with ecological threshold of three types of policies: the safe minimum standard, the optimal policy using a traditional decoupled approach and the optimal policy using the integrated approach. We show that in the context of this dynamically interacting ecological-economic model, neither the safe minimum standard nor the optimal policy using the decoupled approach will work effectively. In fact, these two types of policies may actually increase the instability of the system in face of the threshold effects. We propose the policy using the integrated approach when the ecological system under study exhibits strong two-way interactions between ecological-economic activities. In section 2.5, we investigate the technical challenges
associated with multiple equilibria and non-convexity. The solutions for various initial conditions are described. Section 2.6 concludes the paper.

2.2 Literature Review

Ecological thresholds triggered by internal factors have largely been studied in the ecological literature (Vogelmann and Ovitt, 1968; Ludwig et al., 1978; Wilson and Agnew, 1992; Ripple and Bescta, 2004). Thresholds triggered by external factors, especially human economic activities, and the related management issues have traditionally been of more interest to economists. Up to the end of 1990s, the economics literature on threshold management evolved roughly along two lines. One line of research posits that the functioning of ecosystem is fundamental to human society and thus the concern for a fundamentally damaging ecological threshold should dominate the concern for economic efficiency. The safe minimum standards is the most well-known policy recommendation (Ciriacy-Wantrup, 1952; Bishop, 1978; Woodward and Bishop, 1997; Farmer and Randall, 1998). The other focuses on the management efficiency and tries to find the optimal management policy with the presence of such threshold effects (Clark, 1973; Henry, 1974; Cropper, 1976; Berck, 1979; Tsur and Zemel, 1995; Farzin, 1996). Their conclusions vary from optimal depletion of natural resources to an immediate pollution abatement long before the breach of threshold. In these studies, the ecological resources are treated as another form of economic assets. The intrinsic features of the ecological dynamics are ignored.
More recently, economists and ecologists alike have argued for the need to better integrate economics and ecology (Arrow et al., 1999; Holling, 2001; Levin et al., 1998). Researchers have begun to incorporate complex ecological dynamics into the economic analysis, like strong nonlinearities, multiple stable states and potential discontinuous changes (Perrings and Walker, 1997b; Carpenter et al., 1999; Brock and Starrett, 2003; Crepin, 2003). However, the way that the management policies are posed follows the same perspective of the decoupled approach: isolating the ecological system from the human economic system and leaving only a few key exogenous parameters. The policy analysis is then formulated as a means to manipulate the exogenous parameters, like nutrient loadings in lake water management, so as to generate the most “desirable” ecological outcome. Take the seminar work of Carpenter et al. (1999) as an example. They analyze the lake management policy subject to eutrophication threshold that is triggered by excessive phosphorus loadings from economic activities. Because their focus is on the management of an dynamic ecological system, they carefully model the dynamics of water quality and necessarily omit the economic dynamics of phosphorus loadings. In particular, they take the cost-benefit analysis approach and posit an imaginary social planner to optimally manage the loading level, ignoring the reality that the nutrient loadings are difficult to monitor or regulate. More importantly, these nutrient loadings are endogenous to economic activities around the lake. In their model, the lake ecosystem is first
isolated from the surrounding economic development, except that phosphorus loadings from surrounding economic development serve as an exogenously-determined external trigger of the eutrophication threshold. In the policy analysis, phosphorus loadings, the external trigger of eutrophication threshold, are then decomposed into two parts: “fixed ($L_f$) and controllable ($L_c$) components”. In other words, phosphorus loadings will respond to ecological changes only when such ecological changes have induced policy adjustments. These are reasonable and necessary assumptions to make when we first explore the complex ecological dynamics, especially when the economic activities are not responding directly to ecological changes.

Brock and Starrett (2003) extend the analysis by exploring possible optimal behaviors associated with the framework of Carpenter et al. (1999). For a dynamic optimization problem with one state and one control variable, they identify some of the key ambiguities that are characteristic of non-convex problems, like the existence and location of Skiba points (the initial conditions that have more than one locally optimal solutions). Crepin (2003) investigates the management of a boreal forest with non-concave pine growth and reinforces the results from this lake management literature. Dasgupta and Mäler (2003) synthesize the literature by emphasizing the importance of threshold dynamics in economic analysis. All these papers make essentially the same assumptions by separating the ecological system from its dynamically interacting counterparts.
In contrast, this paper will explicitly model the economic dynamics behind the polluting activities and acknowledge the uncoordinated feature of these polluting activities. In this way, we establish two-way interactions between the economic activities and the ecological changes. That is, while people’s economic activities can affect the ecological system, the ecological changes can also affect economic activities. We also extend Carpenter et al. (1999)’s policy analysis on lake management to the policy analysis of a coupled ecological-economic system in which the lake ecosystem embodies an eutrophication threshold. With the assistance of numerical techniques, we are able to resolve the ambiguity about the existence and location of the Skiba points, as identified by Brock and Starrett (2003), in a dynamic optimization problem of two state variables and two controls.

2.3 A Coupled Ecological-Economic Model

The coupled ecological-economic model in this paper is based on the theoretical framework Chen et al. (2009) and Irwin et al. (2009). Lake water quality change is a dynamic ecological process incorporating an eutrophication threshold that can be triggered by economy-driven nutrient loadings. For simplicity, we assume that nutrient loadings increase with total land use in the region. We use the lake ecosystem as a stylized ecological system, because of the generality of its threshold effect and the functional form (Dasgupta and Mäler, 2003). Economic activities in the region around the lake will respond to ecological changes because households have a

21The derivation of complete model is described in Chen et al. (2009).
preference for high-quality ecological services. *Ceteris paribus*, good lake water quality attracts in-migration. This population growth will create jobs, generate scale economies and push up demand for land. As a result, more undeveloped land will be converted into developed land. This land conversion will in return increase the nutrient loadings into the lake and degrade the water quality, which then will feed back to household migration decisions and regional economy. Apart from the ecosystem service, people make migration decisions based on the best achievable consumption level and the crowdedness in the region.

### 2.3.1 Dynamics of Lake Water Quality and the Eutrophication Threshold

The ecological model is adapted from Carpenter et al. (1999). Our measure of lake water quality is reduced to a measure of total phosphorus concentration in the lake water under the assumption that phosphorus is the limiting factor in the lake eutrophication process. Phosphorus concentration increases with phosphorus loadings, decreases with outflow and sedimentation and is affected by phosphorus recycling.\(^{22}\) Because the phosphorus loadings are mainly determined by land uses around the lake, for simplicity we assume that phosphorus loadings are proportional to the total area of urban land use in the region. The recycling term is assumed to be S-shaped, which is a mathematical simplification for the complicated bio-physical and ecological dynamics underlies the observed eutrophication threshold. When the

\(^{22}\)For a detailed description, please refer to Carpenter et al. (1999).
phosphorus concentration level is low, this recycling term is insignificant. The phosphorus dynamics are determined mainly by loadings and outflows/sedimentation. In this state of the lake, the nutrient concentration level is relatively low in the water column, water is relatively clear and the ecosystem services are healthy. This state is often called “oligotrophic state”. When phosphorus concentration goes beyond the eutrophication threshold, the over-enrichment of phosphorus will trigger the nonlinear out-of-equilibrium bio-physical and ecological dynamics and will make this recycling term a major determinant in the phosphorus dynamics. In this case, water becomes turbid, algae concentration is high. Fish-kills and algal bloom may occur frequently. Such a state is often called “eutrophic state”. The over-enrichment in phosphorus due to excessive loadings will trigger the shift of lake water quality from the oligotrophic state to the eutrophic state. Such a transition is called “regime shift”. Once this regime shift has occurred, a reverse in loading cannot guarantee the reverse of the lake back into the oligotrophic state immediately, which is referred to as “hysteresis”. When a recovery does occur, it may take a long time. Lake Erie is a good example. After the Cuyahoga River fire in 1969, substantial reductions in point sources failed to bring about meaningful improvements in water quality until ten years after the controls had been implemented.

The dynamics of lake water quality is summarized in the following equation adapted from Carpenter et al. (1999):

\[
\dot{P}(t) = L_0 + L_1 N(t) l(t) - sP(t) + \sigma_1 \frac{P(t) \sigma_2}{P(t) \sigma_2 + P_c \sigma_2}.
\]  

(2.1)
Here, $P(t)$ is the concentration of phosphorus in the lake water at time $t$ and $\dot{P}$ is the change rate. The loading consists of $L_0$, which is a constant, and $L_1 N(t) l(t)$, which is the total loadings from the residential land use in the region. $L_1$ captures the average impact of land use on lake water quality. $N(t)$ is the total population in the region at time $t$ and $l(t)$ is the household demand for land. $N(t)l(t)$ gives the total demand of land in the region. The outflow of phosphorus is captured in $s P(t)$, where $s$ denotes the proportional rate of $P$-loss from the system. It accounts for both sedimentation and outflow, which remove phosphorus ($P$) from the water column. The last term represents the phosphorus recycling. In shallow lakes, phosphorus at the bottom of the lake can be re-suspended back into lake water. In stratified lakes, recycling depends on oxygen depletion in the hypolimnion during the stratified season (Carpenter et al., 1999). Recycling occurs only when the phosphorus concentration is sufficiently high. The maximum rate of recycling is $\sigma_1$ and the steepness of the recycling curve is governed by $\sigma_2$. $P_c$ is the $P$ value at which recycling reaches half its maximum rate. The recycling term that gives rise to threshold responses is a general feature of many ecosystems (Brock and Starrett, 2003). Therefore, with appropriate modifications, the analysis presented in this paper is also applicable to more generic ecological-economic systems.

### 2.3.2 Migration and Regional Economy

Land use pattern, which is the major determinant of nutrient loadings in this model, is mainly driven by regional development. To capture this, we build a simple
economic model based on the two fundamental forces of the regional economics: agglomeration and congestion. In addition, we also assume regional development will respond to ecological services. In this economic model, all these three forces—agglomeration, congestion and the impact of ecological changes—will act through the migration and consumption decisions of utility-maximizing households. If the maximized utility level in the region is higher than the reservation level from the rest of the world, households will migrate into the region, and vice versa. For simplicity, we assume that the regional migration rate is proportional to the utility differentials between the region and the rest of the world. This gives the following dynamic equation governing regional population. The derivation of this equation is included in the Section 1.3.

\[
\dot{N} = \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P
\]

Here, \(\dot{N}\) stands for the population change rate. Migration will continue until utility differentials disappear. \(\theta_1, \theta_2, \theta_3\) and \(\theta_4\) are parameters. The relation of these four parameters to the underlying parameters of household preference, production technology and the economic conditions of the rest of the world are described in the appendix.

This dynamic equation is hump shaped in the local population, which reflects a generic feature in regional development models: at the initial stages of development, the in-migration rate increases with population as a consequence of agglomeration. However, as population increases, the congestion effect will eventually dominate and
the migration rate will slow down. The inverse relationship between the migration rate and the ecological variable $P$ reflects people’s preference for high quality natural amenities, exemplified by many of the fast growing areas of the Rocky mountainous and coastal areas in the U.S. (Glaeser et al., 2001; McGranahan, 1999; Hansen et al., 2002; Rappaport and Sachs, 2003; Sutton and Day, 2004).

2.3.3 Dynamics of the Coupled Ecological-Economic System

The dynamics of this coupled ecological-economic system is governed by the two dynamic equations (2.1) and (2.2). To simplify the discussion, the notation for constant coefficients associated with the state variables are changed. All the coefficients in the $\dot{N}$ equation are labeled as $\theta$’s and those in the $\dot{P}$ equation are labeled as $\delta$’s.

\[ \dot{N} = \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P \]  
\[ \dot{P} = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P^c} \]

The nonlinear recycling term in equation (2.3b) makes the solution analytically untractable. But because it generates the threshold effects and because the concerns about ecological threshold, the consequent regime shifts and slowly reversible changes are major motivations for many regulating environmental policies, it is essential to the model. Without an analytical solution, the results from numerical comparative static analysis will be conditioned on the particular parameterizations of the model, thus losing the so-called “generalities” of the analytical solution. In fact, this is not a fair comparison between numerical and analytical solutions. As we know, any model
specification consists of the specification of functional forms and the specification of parameter values. While the form of analytical solution may not depend on particular values of parameters, the existence of the analytical solution is highly sensitive to the specification of the particular functional forms and is in most cases quite restrictive, especially in the models with two-way interactions between ecological and economic systems. Therefore, the analytical approach will inadvertently omit more complex dynamics, such as nonlinearities or threshold effects that are particularly important in ecological system dynamics. In addition, we cannot do much to test the sensitivity of analytical solutions on the specification of functional forms. In contrast, while a particular numerical solution can be sensitive to the parameter specification, it is quite flexible in the specification of functional forms. More importantly, the sensitivity of numerical solution to parameter values is testable but checking the sensitivity of analytical solutions to functional specification is not easy. In addition, for numerical solutions, we can in theory try all the possible parameter values on the numerical solutions and the result will conform with the analytical solution if it exists. Of course, this is practically unrealistic. Fortunately, as a researcher, we are only interested in the realistic and/or theoretically interesting cases rather than all the mathematically possible parameter specifications. These considerations often restrain the relevant parameter space. Because of the complicated nature of the coupled ecological-economic system and our interest in the threshold effects, this paper is mainly based on numerical methods.
Figure 2.2: Phase plot of the baseline model.

Because the coupled ecological-economic model is a two-dimensional dynamic system, we use a phase plot to describe the dynamics of the baseline case (Figure 2.2). Points on the $N$-nullcline satisfy the condition $\dot{N} = 0$. When the coupled system lies in the state as specified on these points, the utility differential between the region and the rest of the world is zero. Households have no incentive to migrate. The hump-shape of this $N$-nullcline reflects the trade-off between economic well-being and the preference for ecological services. When $N$ is small, the agglomerative forces dominate the congestion effect and the expansion of the regional population increases the incentive to migrate into the region. However, increasing population generates nutrient run-off due to urban land development, which degrades ecological services and reduces the attractiveness of the region. When population is high
enough so that the congestion effect dominates the agglomerative forces, further expansion of the regional population will increase the incentive to migrate out of the region, which has to be balanced with decreases in phosphorus concentration ($P$). In states below (above) this $N$-nullcline, households will move into (out of) the region, because of higher (lower) ecological services. The $P$-nullcline is S-shaped because of the nonlinear out-of-equilibrium dynamics captured in the recycling term. On the lower branch of this $P$-nullcline, phosphorus concentration in the lake water is relatively low; water is clear and algae concentration is low. This corresponds to the oligotrophic state. The upper branch has a higher phosphorus concentration and corresponds to the eutrophic state. The curvature in the middle reflects the out-of-equilibrium dynamics. In states to the right (left) of the $P$-nullcline, phosphorus concentration will increase (decrease) because of the excessive phosphorus loading from economic activities in the region. These two nullclines intersect at the three fixed-points, at which both the ecological and economic processes remain stationary. Point (A) is a non-trivial stable fixed point associated with positive population, point (B) is a trivial stable fixed-point associated with zero population, and point (C) is an unstable fixed point. Because point A is associated with a moderate population size and an oligotrophic lake, it is referred to as a balanced ecological-economic equilibrium state. At this state, regional development has pushed the lake to the edge of eutrophication under the current specification. An ecological shock that increases phosphorus concentration (such as a variation in rain fall) or an economic shock
that increases land development (such as government policy that promotes economic
development) may trigger the eutrophication threshold effect and shift the lake from
the oligotrophic to eutrophic state. This sets up an interesting case for policy analysis
in later discussions. At fixed point B, with insufficient economic build-up, the good
lake ecological service alone is unable to maintain local population. Because of the
ecological dominance, this steady state is called the “ecology-dominated” equilibrium
state.

The solid line, called the separatrix, partitions the state space into domains of
attraction for the balanced equilibrium (A) and ecology-dominated equilibrium (B).
The separatrix is also the stable manifold of the unstable fixed point (C). That is, if
the system starts on a point on this line, it will eventually approach the unstable fixed
point. Starting from any state in the area lying outside the separatrix, without policy
intervention, the system will eventually evolve to the ecology-dominated equilibrium
state (B). Starting from any state in the area lying inside this separatrix, the system
will eventually evolve to the balanced equilibrium state (A).

In contrast to the existing literature on ecological thresholds, the eutrophication
threshold in this paper is contingent on the economic conditions in the region. When
population level is low, consequent phosphorus loading into the lake is low. In this
case, even when the phosphorus concentration exceeds the ecological threshold level
so that the recycling term is active, the lake will not stay in the eutrophic state.
Instead, the phosphorus level will decrease until the lake becomes oligotrophic again.
When both the population level and the loadings are high, if the ecological threshold is breached, the lake may undergo eutrophication in the short run. So long as this is not an irreversible change, the lake quality will eventually recover in this simple model of coupled ecological-economic system. As lake water quality degrades, the region becomes less attractive, local residents will begin to migrate out. As population move out, the loadings into the lake will gradually decrease, which will eventually lead to ecological improvement. Of course, this recovery process can take very long time. In the baseline case, the recovery takes approximately 30-40 years.

2.4 Policy Discussion

In this section, we will discuss the policy implications when overdevelopment in the region may push the lake ecosystem across the ecological threshold. In particular, we will first discuss the implications of safe minimum standards as first proposed by Wantrup (1952) and Bishop (1978). Then we will discuss the implications of a “simple ecological management policy” as in Carpenter et al. (1999) and Brock and Starrett (2003). The policy analysis is based on separating the ecological system from the interacting economic counterpart. Finally, we will discuss the management policy based on the integrated ecological-economic model.

2.4.1 The Safe Minimum Standard

The safe minimum standard (SMS) stipulates that as the ecosystem is approaching the ecological threshold, the concerns for economic efficiency should be set aside
and policy intervention should be implemented to prevent the breach of the ecological threshold, unless the cost of policy becomes “intolerably too burdensome” (Bishop, 1978).

In this section, I will discuss the implications of SMS in the context of the baseline coupled ecological-economic system and show when the implementation of such a policy can endogenously make the cost of intervention “intolerably too burdensome”. The performance of the safe minimum standard hinges upon the indirect policy effect and people’s response to policy. For regions undergoing its economic expansion, this policy is likely to fail, because such a policy will likely make the region more attractive, thus attract more economic activities and create larger ecological pressure. Consequently, to deal with the increased economic activities and ecological pressure, the policy intervention has to become more aggressive and cost of intervention will also go up. This is illustrated in Figure 2.3(a).

Assume that the system originally sits at the balanced equilibrium. Denote the $P$ level triggering policy interventions as $P_{SMS}$. To make the policy meaningful, it should not exceed the ecological threshold, which in the figure is the $P$ level that the $P$-nullcline begins to bend back. Say that $P_{SMS}$ is set right at this level. When SMS is implemented, the ecological dynamics are transformed so that the phosphorus dynamics will follow the natural dynamics when its level is less than $P_{SMS}$. Whenever the natural evolution of the system tends to drive the phosphorus level above $P_{SMS}$, policy will intervene and hold phosphorus level at $P_{SMS}$. If before the intervention,
the system resides in the balanced equilibrium, the announcement of this SMS will stimulate additional in-migration. The new steady states will locate in $E_{1\text{SMS}}$ and $E_{2\text{SMS}}$. The system will not stay at $E_{1\text{SMS}}$ forever. At $E_{1\text{SMS}}$ the regional economic potential has not been fully exploited because agglomeration is still dominating the congestion. Therefore, a small perturbation due to in-migration will actually improve everyone’s economic welfare and induce more in-migration. This will continue until $E_{2\text{SMS}}$ is reached. However, as more people are migrating into the region, the phosphorus loadings will increase accordingly. To hold phosphorus level at $P_{\text{SMS}}$, policy intervention should become more and more aggressive and stabilize until $E_{2\text{SMS}}$ is reached. In any case, the implementation of a SMS will spur excessive development.
and produce more pollution into the lake in this coupled ecological-economic system. As a result, the policy makers have to fight a much more difficult battle because of the increased pollution and bear a much larger cost. Note the location of $E_{SMS}^2$. It lies outside the domain of the balanced equilibrium. This suggests that if by any chance, the policy makers fails to secure enough funding to maintain SMS for ever, the future ecological degradation will be far more severe than without SMS in the first place. Without proper policy intervention that brings the system back into the domain of the attraction of the balanced equilibrium, the coupled ecological-economic system will experience prolonged economic decline. As we make the SMS more stringent, that is decreasing $P_{SMS}$, the population level associated with $E_{SMS}^2$ will increase, leading to a more severe overdevelopment problem.

SMS may work when the regional economy has passed the phase of economic expansion, that is, the regional economy is at a stage in which congestion effect dominates the agglomeration effect. This is shown in Figure 2.3(b). At this stage, because the congestion effect is dominating, people have no incentive to further expand the local economy in the first place. Assume the system is resting at the balanced equilibrium and is hit by a sudden economic boost or a sudden influx of population. This will cause discrete ecological changes without a policy intervention. SMS will work in this case because it helps avoid the breaching of the ecological threshold. Because at this stage, congestion is dominating, the influx of people will
eventually leave, the system will eventually recover to the original equilibrium, in which case the SMS is no longer necessary.

In summary, the performance of SMS depends on the stage of regional economic development and hinges on the dynamic response of economic activities.

2.4.2 Ecological Management

In this section, we consider the management policy as discussed in many resource and environmental analysis (Carpenter et al., 1999; Brock and Starrett, 2003): while the ecological degradation may affect the policy makers, it does not feedback into the economic activities of autonomous economic agents. In the context of the baseline model, the policy makers are faced with the challenge that the overdevelopment in the region has imposed too much pressure on the ecology so that the ecological threshold is likely to be crossed. Assume that he has a policy instrument which can directly affect the level of phosphorus loadings in the lake with at some cost, such as the construction of a plant for phosphorus abatement. For simplicity, we assume the following objective function:

\[
\max_{\{v_2(t)\}} \int_0^{\infty} e^{-\rho t} \left\{ \alpha_2 \mu(P) - \alpha_3 v_2^2 \right\} dt \tag{2.4a}
\]

subject to

\[
\dot{P} = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P_s}{P_s + P_c} - v_2 \tag{2.4b}
\]

\[
\mu(P) = \frac{1}{P^2 + 1} \tag{2.4c}
\]
where \( v_2(t) \) is the policy instrument affecting phosphorus loadings with a quadratic cost function. The time arguments for these state and control variables are suppressed for simplification.

The current value Hamiltonian gives the following optimality condition:

\[
\begin{cases}
\dot{P}(t) = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P_c^8} + \frac{1}{2\alpha_3} \lambda_2 \\
\dot{\lambda}_2(t) = \rho \lambda_2 - \lambda_2 \left[ \delta_4 + \delta_5 \left( \frac{P^8}{P^8 + P_c^8} \right)' \right] - \alpha_2 \mu'(P)
\end{cases}
\]

(2.5)

The solution is plotted as the solid line in Figure 2.5.

By taking the current economic activities as given, which is a common assumption in many resource and environmental studies, the policy maker “believes” that the nutrient loadings from neighboring economic activities will not respond to ecological changes and thus remain constant over time. If the ecological threshold is triggered by ecological process internal to the ecosystem, this assumption may work well. However, in cases that the threshold effects are triggered by external factors, such an assumption can generate misleading policy recommendations.

Take the baseline model as an example. Let us consider the case in which overdevelopment puts too much pressure on the lake so that eutrophication will occur without policy intervention. If the policy maker takes the position that the regional economy is fixed, he will perceive the ecological change along the thin solid line passing through \( N_0 \) in Figure 2.4(a). As a result, he is afraid that the system will evolve along the path depicted in Figure 2.4(a) and end up in the eutrophic state at point \( A_{imag} \). Because the excessive phosphorus loadings are the direct trigger of
this eutrophication threshold, he will try to reduce the phosphorus loadings so as to avoid breaching the threshold. Under the optimal policy as specified in Equations 2.5, he thinks that the system will evolve along the path as specified in Figure 2.4(b) and ends up at equilibrium \( A_{Ecol} \).

Notice that while the eutrophication is avoided, at the perceived new equilibrium, the phosphorus level becomes higher. This is because policies that reduce the phosphorus loadings in effect enhance the carrying capacity of the lake so that it is able to accommodate more economic activities. As economic activity level is higher, the loadings into the lake will be higher at the equilibrium. Therefore, at the equilibrium \( A_{Ecol} \), the phosphorus level is closer to the eutrophication threshold as compared to the equilibrium level without policy intervention.

In fact, such a policy can destabilize the system. Because people will respond to ecological changes through their migration and consumption decisions, the system will not evolve as perceived in Figure 2.4(b). Under the above-mentioned optimal policy, the system will evolve along the path as plotted in Figure 2.4(c). Note that the starting point \((N_0, P_0)\) is below the \(N\)-nullcline, more people will migrate into the region even without any policy intervention. Policies that improve the water quality will also make the region more attractive and spurring further growth. Therefore, such ecological policies will worsen the overdevelopment problem and result in more severe ecological degradation. The greater ecological degradation will in return result in more severe and longer-lasting economic decline. The phase plot here omit one very
important piece of information. That is the speed of change along the trajectory. In fact, the transition from oligotrophic to eutrophic state takes only a couple of years. This corresponds to the initial increase in phosphorus in Figure 2.4(c). On the other hand, the reversal of the ecological condition takes much longer time, usually in decades.\(^{23}\)

### 2.4.3 Management Policy in An Integrated Framework

In order to manage an ecological system with externally triggered threshold effects, we propose the integrated modeling approach, especially when human activities serve as the external trigger and are responding to ecological changes. By integrated modeling approach, we mean that effective policy design should consider both the ecological and economic dynamics, especially when the two systems are closely linked. In this section, we will discuss the optimal management policy for a specific example. A optimal policy given any initial conditions and the technical challenges will be discussed in the next section.

Because of the co-evolution of the ecological process and economic activities and our interest in the balance between environmental protection and economic development, we assume that the policy maker for this integrated system cares about both the ecological and economic conditions so as to avoid the sacrifice of one for the other. The policy maker prefers more economic activities and better ecological service. He has two policy instruments: one acts on migration incentive and affects the local

\(^{23}\)For more detailed discussion on the speed of change, please refer to C Chen et al. (2008).
Figure 2.4: Policy implications when threshold is triggered by external dynamic factors. Figure (a) Perceived changes in ecological system when faced with overdevelopment pressure. Figure (b) Perceived changes when implementing the policy of the naive decoupled approach. Figure (c) Actual changes when the naive policy is implemented. Figure (d) Dynamic path under optimal policy of an integrated model.
economy; the other acts on phosphorus loadings into the lake and affects lake water quality. Policy intervention incurs a cost which increases with intervention intensity.

The necessity of the integrated approach in policy design for this externally triggered eutrophication threshold is illustrated by the comparison between Figure 2.4(c) and 2.4(d). By explicitly considering the dynamics in the human economic activities, the policy from the integrated approach can successfully lead the system to the targeted balanced equilibrium state and avoid undesirable boost and bust cycles. As shown in Figure 2.4(d), at the equilibrium $A_{Intg}$, the steady state ecological intervention is stronger than under the policy from ecological management, because the policy maker has considered the indirect impact of environmental policy on migration and land use, that is, ecological improvement attracts more in-migration and increases local economic activities. This is also shown in Figure 2.5. In addition, a complementary policy is used to mitigate migration incentives and helps to hold the system at this equilibrium state.

Figure 2.5 also shows that, from the perspective of optimal policy designs, policy interventions in the transient period and at the equilibrium state are equally important. Even though at the equilibrium state, the shift on the $N$-nullcline is insignificant in this example, this complementary policy is playing a crucial role in the transient period leading to the equilibrium as shown in Figure 2.5. In fact, in the initial period, the major part of the intervention cost comes from this complementary policy. The ecological intervention of this integrated approach is stronger than the
Figure 2.5: Optimal Policy. The solid line is the optimal policy under ecological management. The broken and dotted lines are the optimal ecological and economic policy of the integrated approach.

segregated counterpart, because the indirect impact on migration and consumption decisions are taken into consideration. As the ecological intervention improves lake water quality, it attracts more immigration and economic activities in the region. The phosphorus loadings will increase and putting greater pressure on the ecosystem. In order to avoid breaching the eutrophication threshold, the ecological policy needs to be more aggressive to counteract the increase in phosphorus loadings. As a result, in Figure 2.5, the optimal policy response are stronger than the segregated counterpart due to this indirect policy effect.

Finally, the ending of a project can be crucially important for the final outcome of the project. Because policy interventions change the system dynamics, the implementation of policies may shift the equilibrium into an otherwise unsustainable
state. That is, the new equilibrium under policy intervention lies outside the attraction basin of the balanced equilibrium in the unregulated system. Even though $A_{Intg}$ in this paper lie in the attraction basin of the balanced equilibrium, we actually find examples where it lies outside. We consider this important because in reality, it is not uncommon that the financial resources supporting a policy may run out, like the budget cuts due to economically hard times. The results in this paper suggest that we should not simply stop the funding but rather put aside some resources and implement policies that can bring the system from the new equilibrium under policy intervention back into the attraction basin of the balanced equilibrium without policy intervention. In short, a good end of a project is crucially important.

2.5 The Optimal Policy, A Technical Note

In this section, we discuss the technical challenge involved in the dynamic optimization of this integrated ecological-economic model with multiple equilibria and non-convexity. Because of the existence of multiple equilibria, the policy maker needs to decide when to drive the system to one equilibrium rather than the other.\textsuperscript{24} The presence of non-convexity can lead to the coexistence of multiple solutions.

As discussed in the previous section, the policy maker for this integrated system cares about both the ecological and economic conditions. He prefers more economic activities in the region and better ecological service. He has two policy instruments:

\textsuperscript{24}Such a decision may be simple and clear mathematically but in reality it can be quite subjective and subject to political pressures imposed by different interest groups.
one acts on migration incentive and affects the local economy, denoted as \( v_1(t) \); the other acts on phosphorus loadings into the lake and affects lake water quality, denoted as \( v_2(t) \). The cost of intervention increases with intervention intensity. The optimization problem is formulated as:

\[
\max_{\{v_1(t), v_2(t)\}} \int_0^\infty e^{-\rho t} \left\{ \alpha_1 N + \alpha_2 \mu(P) - \alpha_3 \left( v_1^2 + v_2^2 \right) \right\} dt \quad (2.6a)
\]

subject to

\[
\dot{N} = \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P - v_1, \quad (2.6b)
\]
\[
\dot{P} = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P^8_c} - v_2, \quad (2.6c)
\]
\[
N(t) \geq 0, \ P(t) \geq 0, \quad (2.6d)
\]
\[
N(0) = N_0, \ P(0) = P_0, \quad (2.6e)
\]

Using the current value Hamiltonian, the optimality conditions of interior solutions are:

\[
v_1 = \lambda_1 \quad (2.7a)
\]
\[
v_2 = -\lambda_2 \quad (2.7b)
\]
\[
\dot{N} = \theta_1 + \theta_2 N + \theta_3 N^2 + \theta_4 P + v_1 \quad (2.7c)
\]
\[
\dot{P} = \delta_1 + \delta_2 N + \delta_3 N^2 + \delta_4 P + \delta_5 \frac{P^8}{P^8 + P^8_c} - v_2 \quad (2.7d)
\]
\[
\dot{\lambda}_1 = \rho \lambda_1 - \lambda_1 \left[ \theta_2 + 2\theta_3 N \right] - \lambda_2 \left[ \delta_2 + 2\delta_3 N \right] - \alpha_1 \quad (2.7e)
\]
\[
\dot{\lambda}_2 = \rho \lambda_2 - \lambda_1 \theta_4 - \lambda_2 \left[ \delta_4 + \delta_5 \left( \frac{P^8_c}{P^8 + P^8_c} \right)' \right] - \alpha_2 \mu'(P) \quad (2.7f)
\]
along with the initial conditions (2.6e) and the transversality conditions:

\[ 0 = \lim_{T \to +\infty} e^{-\rho T} \lambda_1(T) N(T) \]  
\[ (2.7g) \]

\[ 0 = \lim_{T \to +\infty} e^{-\rho T} \lambda_2(T) P(T) \]  
\[ (2.7h) \]

The optimal policy response given any initial conditions is summarized in Figure 2.6. The line of equilibrium demarcation (the thick solid line) and line of policy transition (the thick broken line) are two important demarcations for discrete changes in the optimal policy. The former demarcates the conditions under which moving system to the balanced equilibrium \( A_{Intg} \) is desirable. This is due to the existence of multiple equilibria, the balanced equilibrium and the ecology-dominating equilibrium, in the unregulated system. The latter marks a discrete change in optimal policy leading to the balanced equilibrium. This is due to the non-convexity due to the ecological threshold. The coexistence of multiple solutions is confirmed in the region marked out by the thick solid line around the line of policy transition.

In order to discuss the policy implications, a short explanation of the notations in Figure 2.6 is necessary. The dotted lines are the \( P \)- and \( N \)-nullclines. The thin dotted lines are the nullclines without any policy intervention as shown in Figure 2.2. The thicker dotted lines are the nullclines under optimal policy at the equilibrium. The separatrix of the unregulated system, the thin solid line, and the steady states of the unregulated and regulated system are marked out in the figure. The line of
Figure 2.6: Optimal policy in phase plane. On the right side of the solid line of “equilibrium demarcation”, policy maker should utilize policy instruments to drive the system to the balanced equilibrium $A_{Intg}$. On the left, such a policy is suboptimal because of intervention cost. The thick broken line is called the Skiba set. It demarcates the discrete change in policy due to the preference over different local optimal solutions.

“equilibrium demarcation”\(^{25}\) lies on the upper left corner of the state space. It demarcates the state space into two parts. To its right, it is beneficial to intervene to prevent the economic decline to the ecology-dominating equilibrium. In other words, it is beneficial to intervene and boost local economy to the sustainable equilibrium.\(^{26}\)

As shown in Figure 2.6, many of the initial states that are outside the attraction

\(^{25}\)This line is plotted by trials and errors. Pick a point in the $N$-$P$ space as an initial condition. Find the solution that drives the system to the balanced equilibrium of the integrated system. Compare the welfare level with the welfare without any intervention. If the two generates the same welfare, the point is included in the blue line.

\(^{26}\)This line is not the separatrix for policy. However, this line is really what is relevant for policy makers. Mathematically, there should be separatrices that divide the $N$-$P$ space in Figure 2.6 into two subareas. On one side, the optimal policy will drive the system close to a balanced equilibrium, like $A_{Intg}$. On the other side, the system will eventually go to an ”ecology-dominating” equilibrium. And there should be an optimal policy that leads to the economic decline. While these
domain of balanced equilibrium $A$ are now to the right of the line of equilibrium demarcation. Without policy intervention, when the system starts from these initial states, the local economy will eventually undergo long-lasting decline and end up in an ecology-dominating equilibrium. For these initial states, the optimal policy will counteract this natural tendency and maintain the system at the balanced equilibrium at $A_{Intg}$. Notice that in this specific example, when the policy maker runs out of money and has to put an end to the intervention abruptly, because the equilibrium $A_{Intg}$ lies in the attraction domain of the unregulated balanced equilibrium $A$, the system will eventually approach equilibrium $A$. In this sense, the optimal policy is a sustainable policy because it “permanently” avoids the economic decline. In the short run, because at $A_{Intg}$, without policy intervention, the economic development will put too much phosphorus loadings into the lake and trigger the eutrophication process, an ecological degradation will occur if the policy is put to an end abruptly. The regional economy will suffer from economic decline due to the ecological degradation for quite a long time before it eventually recovers to the balanced equilibrium $A$. This suggests that additional resources put aside to artfully end the project and at the same time avoid the transient ecological and excessive economic decline is beneficial and necessary.

The broken line at the lower-right corner is line the “policy transition”. At each point on this line, the equilibrium $A_{Intg}$ can be optimally achieved by two different are mathematically correct and may worth exploration for theoretical interest, we do not see its practical relevance, not to mention the numerical complications involved.
policies. Unlike the policy line of sustainability, across which the final state of the system is changed, on the two sides of this line of policy transition, the final state of the system is unchanged. While the system will finally evolve to the same balanced equilibrium, the optimal policy on the two sides of the line of policy transition differs fundamentally.

This line lies in the region where slowly reversible ecological degradation can occur. Based on intuitive reasoning, there are two types of policies that may apply in this region. The first type aims to prevent the undesirable switch to an eutrophic state and therefore requires strong intervention in the initial period. It is thus called the safety-first policy in this paper. However, such a policy may not always be the best solution. Because the intervention cost is increasing with intervention intensity, the cost of such a policy can be too high. The second type of policy does not prevent the ecological shift to an eutrophic state and focuses instead on the recovery process. Therefore the intervention will be less aggressive in the initial period and will change more gradually according to the state of the system. It is thus called the slow-adjustment policy. To the left of this line of policy transition, the safety-first policy is the optimal policy leading to the balanced equilibrium at \( A_{Intg} \). To the right, the slow-adjustment is the optimal policy. At the bottom of this line of policy transition, because the phosphorus level is low, faced with the pressure of overdevelopment, the policy maker can manage to completely avoid the eutrophication by taking the safety-first policy. As we move up along this line, the phosphorus level becomes
higher and higher, completely avoiding the eutrophication becomes more and more difficult and the cost of the safety-first policy will increase gradually. To reflect such a cost increase, the policy maker will gradually become more tolerant for a short-run ecological degradation. On the other hand, because under the slow-adjustment policy, the policy maker is willing to tolerate the eutrophication and focuses on the recovery process, as we move up along the line of policy transition, more economic resources can be freed up from the initial period and used to accelerate recovery. Therefore, as we move up the line of policy transition, the safety-first policy becomes less and less intensive while the slow-adjustment policy becomes more and more aggressive. The difference between these two types of polices gradually diminishes. At the upper end of this line, the difference completely disappears and these two policies becomes one.

In the neighborhood of this line of policy transition, the area bounded by the solid line around this line of policy transition, we find the coexistence of both policies. In other words, both the safety-first policy and the slow-adjustment policy satisfy the optimality conditions (2.7) and are locally optimal. Because on this line, both locally optimal policies generate the same welfare, this line is also called Skiba set.\textsuperscript{27}

\textsuperscript{27}This line is derived in a similar way as the line of equilibrium demarcation. Different initial conditions in the neighborhood of this line are chosen and solved individually. The numerical algorithm finds two different policy solutions for each initial condition in the neighborhood, corresponding to the safety-first policy and slow-adjustment policy respectively. The welfare levels of the two policy solutions are compared. Each point on this red line identifies the initial condition that the two types of policies generate the same welfare level.


2.6 Conclusion

This study investigates threshold management in a dynamically interacting ecological-economic system. In order to manage such an integrated system, ecological policies need to consider the indirect policy effect on the related economic activities. For example, policies improving the ecological conditions in regions experiencing amenity-driven growth will in effect attract more economic growth and put more development pressure on the local ecological system. We show that the policies aiming to improve the ecological condition, but that ignore this indirect effect on the interacting economic activities, can actually result in long-run ecological degradation and instability. Take the safe-minimum standard as an example, the effect of such a policy hinges on the indirect policy effect on the economic activities. When the regional economic development reaches a stage that the congestion effect is dominating the agglomeration forces, such a policy can work as intended. However, when the regional economy is expanding and the agglomeration forces dominates the congestion, such a policy will have the opposite effect. That is it will attract more economic activities into the region and result in much greater ecological pressures. Similarly, when human polluting activities serve as the trigger of ecological threshold dynamics, policies that reduce the pollution but do not consider the indirect policy effect on the related economic activities can lead to more pollution and destabilize the ecological system. In order to fully incorporate the indirect effect of the threshold management policies, we need to take an integrated modeling approach that incorporates the dynamics
of both the ecological dynamics and the economic dynamics behind the trigger of threshold effects.

Such an integrated modeling approach is technically challenging due to the non-linearity and non-convexity in the problem. Because of these complexities, the analytical methods typically do not work. In such cases, numerically method are used to obtain the solutions. The solution of the dynamical problem needs to incorporate several key features. When there are multiple equilibrium states, optimal policy needs to provide the conditions under which one equilibrium is preferred to another. When there is non-convexity, the coexistence of multiple solutions should be explored and the conditions under which one local solution is preferred to the other needs to be identified.
CHAPTER 3

A SPATIALLY EXPLICIT SIMULATION OF LAND USE CHANGE
– LINKING MICRO LAND CONVERSION DECISIONS TO REGIONAL LANDSCAPE PATTERNS
3.1 Introduction

This paper explores how well the canonical economic theory of dynamic land conversion decisions predicts the observed spatial pattern dynamics of residential land use change in an exurban\textsuperscript{28} county in Maryland. In the last six decades, one of the most salient changes in the U.S. land use pattern is the rapid growth of low-density development, which has occurred largely in so-called exurban areas. According to Brown et al. (2005), from 1950 to 2000, the total area classified as exurban has increased by five times, while the corresponding population density has decreased. Such a rapid expansion of low density land development has raised serious concerns among ecologists, planners and others concerned with the environmental impacts of such growth. Hansen et al. (2005) find that such low density land development have reduced the survival and reproduction of the native species and increased those of the exotic or human-adapted species. Moreover, the effects of land development on bio-diversity sometimes exhibit strong threshold effects. Dale et al. (2005) identify four major pathways by which land use change can affect ecological processes: changes to species demography and diversity, changes in the habitat and land cover juxtaposition, changes in the disturbance regimes (e.g. fire, pest outbreaks) and biogeochemical cycles. Apart from these ecological concerns, such a scattered development has also fueled the decades-long debate among regional economists over the economic efficiency of different development patterns (Boyce, 1963; Heubeck, 2009;  

\textsuperscript{28}Areas with housing density between 1 house per 1 acre and 40 acres.
Lessinger, 1962; Ohls and Pines, 1975). Both environmental and economic concerns have motivated numerous policies at all levels of government trying to control the low density urbanization process. However, our understanding of the temporal and spatial evolution of land use pattern is still rudimentary. While much research focuses on the spatial modeling of the land use patterns, few theoretical models that are derived from microeconomic foundation have incorporated the spatial heterogeneity of key economic and topological factors that are observed in reality. While the assumption of homogeneous land eases the burden of theoretical analysis, it has prevented people from understanding how multiple sources of spatial heterogeneity enter the land conversion decision model and the extent to which these factors can explain the real observed patterns of land development.

To date, the complex landscapes, characterized by the coexistence of clustered land development near the urban centers and the scattered land development in more remote areas remains an unsolved puzzle. The focus of this paper is to explore mechanisms underlying the pattern formation and experiment on simulations of land use patterns. In order to generate the complex landscape, the underlying mechanism needs to include both agglomerative forces that generate the clustering and dispersive forces that generate the scattering. The former has been studied extensively in the economics literature. The latter is more of a recent development, especially when it concerns low density exurban development in rural areas. While more and more empirical evidences suggest that household preferences for natural amenities are a
key driver behind the low density land development, the microeconomic foundation that links such preferences with land conversion decisions has not be established. In this paper, we provide a microeconomic foundation that establishes the link between preferences for natural amenities and dynamic land conversion decisions. As a result, the theoretical model describing the pattern formation mechanism now has both agglomerative and dispersive forces. We focus explicitly on exurban areas, which implies that the primary land use change with which we are concerned is the conversion of rural land to a residential use. In Carroll County, for example, in the year 2000, 50,195 of the total 66,823 parcels are for residential use, 14,186 are undeveloped, and only 2210 are for the commercial and industrial use. The theoretical model is then applied to the real landscape of Carroll County with explicit consideration of spatial heterogeneities that affect the decisions of farmers, potential residents and land developers.

In the essence, the observed land development pattern in exurban areas of the United States can be characterized as land conversion decisions made by farmers (Capozza and Helsley, 1989, 1990; Capozza and Li, 1994) subject to the development pressures. The temporal ordering of individual conversion decisions will naturally generate a spatial land use pattern at each point of time. This is particularly true of fast growing areas in which, in the absence of land preservation policies, the relevant question is less about whether a parcel will be converted and more about when it will be converted. Therefore, it is important to start from the individual land conversion
decisions and look for factors that can affect the temporal ordering of individual land conversion decision. Conditions from both the demand and supply of residential land will affect the farmer’s land conversion decisions.

The demand for new residential land is driven by expansion of urban areas into the suburban and exurban areas, due to either the economic expansion or decline at the urban center. The outflow of population and economic activities from the urban centers puts development pressures on the owners of agricultural land and pushes up the land prices. According to the traditional wisdom (Alonso, 1826; Muth, 1969; Quigley, 1998), because of the travel cost and other agglomerative forces, the development pressure is first felt by the farmers at the urban fringe and gradually spreads into more distant areas. The pull of urban centers, which arises from scale economies is the key feature of urban economic models. This agglomerative force lead to clustered land development around one or more established employment districts, a pattern that has been confirmed by many empirical studies. In the last six decades, the significant improvement in the technology and the transportation infrastructure in the United States enables urban residents to seek more distant locations and is believed to have contributed to the land development in more distant areas. While homogenous land is an expedient assumption for theoretical modeling, the road network in reality imposes a topological structure on the rural landscape and has the potential to generate more complicated land use patterns than concluded in theoretical papers. Using Carroll County, Maryland as an example, this paper
explicitly considers the travel distance along the existing road network in the county and finds that it is a partial contributor to the sprawled land development.

On the supply side, converting the agricultural land into urban uses is an irreversible investment decision for farmers. The threshold rent to induce land conversion should incorporate agricultural rent, conversion cost and option value for the irreversibility in case of uncertainty (Capozza and Helsley, 1990). The agricultural rent is the opportunity cost of urban land conversion and needs to be compensated, because a rational farmer will only convert the land if the revenue after land conversion is greater than that from the agricultural production.

Preferences for natural amenities are another important source of sprawled exurban development (Carrion-Flores and Irwin, 2004; Irwin and Bockstael, 2002; McGranahan, 1999; Wu and Plantinga, 2003). Natural amenities, including open space amenities, are clearly more available in less urban areas and thus have the potential to pull people away from urban centers to the more rural areas. However, because natural amenities are non-market goods, preferences for natural amenities have not been incorporated into the microeconomic foundation of the theoretical models of land market, except for the two extreme cases: full and zero capitalization. In Wu and Plantinga (2003) and the simultaneous arrival game of Turner (2005), the Bertrand-type competition among potential migrantes gives land owners the market power to reap all the consumer surplus and preferences for natural amenities get
fully capitalized into the land value. In Caruso et al. (2007) and the sequential arrival game of Turner (2005), land owners participate in Bertrand-type competitions that give the market power to potential migrants. As a result, the land rent is the reservation value of land owners and no capitalization takes place. In all these cases, the value of natural amenities is taken as fixed and publicly known. As we know, in reality, there is no consensus on how to measure the value of non-market goods.

In contrast with these studies, the microeconomic foundation for land capitalization in this paper is more general in feature. The value of natural amenities is endogenous and varies across individuals. The capitalization not only reflects the individual valuation for natural amenities but also the market conditions. The above examples can be treated as the two extreme cases of the model here.

Based on this theoretical model on dynamic land conversion decisions, we adopt a bottom-up approach to simulate the landscape of Carroll County, Maryland. While the model is built on individual land conversion decisions, we are principally interested in the implications of these individual actions on the evolution of exurban land use patterns at a regional scale. While individual decisions are essentially governed by an optimal timing rule, by applying this rule of optimal timing to a spatially heterogeneous landscape, we hope to explore the spatial implications on land use pattern. We find that the economic theory on micro-level land conversion decision cannot fully explain the observed spatial pattern of land uses at the regional level.
3.2 Literature Review

The related economic literature on the formation of spatial land use patterns can be classified into four categories.

The first one has the longest history, which predicts the formation of clustered land use patterns. The pioneering works of Alonso (1826); Muth (1969); von Thünen (1826) take the transportation cost as a key driver behind the clustered development pattern around urban centers. These are followed by numerous studies providing alternative explanations of clustered development pattern. For instance, Buchanan (1965) introduces club theory, Mills (1967) assumes increasing return to scale in production.\textsuperscript{29} However, because these studies are focusing on the clustered development, they are insufficient to generate the right balance between the clustered and scattered development pattern observed in exurban areas.

The second type of literature studies the emergence of so-called “urban sprawl” (densely developed sub-centers around major cities), a more recent phenomenon that has appeared in the last several decades in the United States Anas et al. (1998). Some researches attribute sprawl to the falling transportation (Brueckner and Fansler, 1983; Glaeser and Kahn, 2003) and rising income (Margo, 1992) using the monocentric Alonso-Muth-Mills model. However, even though these models can predict the spatial extension of urban areas, they cannot explain the observed leap-frog development. Fujita and Ogawa (1982) develop the first model of polycentric city based on

\textsuperscript{29}Please refer to Quigley (1998) for a survey on alternative economic driving forces behind clustered development.
the key assumptions that benefits from firm interactions decay as a negative exponential function of the distance while the saving in commuting cost changes linearly with the distance. Henderson and Mitra (1996) generates polycentric cities by assuming a land market with imperfect competition. With the monopsony power, the large land developer in their model is able to strategically select the location of development to maximize his profit. Fujita and Krugman (1995) apply the new economic geography models to urban economics. The forward-linkage (more variety of consumption goods increase the real income of workers) and back-linkages (more consumers supports more specialized firms) are the backbone for the agglomerative forces. They show that when consumption goods are close substitutes and the transportation cost for consumption goods is high, polycentric cities may emerge. These extensions to monocentric models can explain the emergence of the dense development pattern in sub-centers around the major cities, but not the observed low-density suburban and exurban development.

In contrast to these theoretical models, in which the only source of spatial heterogeneity is due to transportation costs that differentiate locations with respect to their proximity to urban centers, the third type addresses the sprawl in a spatial explicit framework, emphasizing the importance of natural amenities. Wu and Plantinga (2003) show through a theoretical exercise that the co-existence of exogenously determined urban center and natural amenities can generate various development patterns. Turner (2005) and Caruso et al. (2007) show that the preference for open
space can generate leap-frog development pattern at the urban fringe. In addition, Caruso et al. (2007) shows through a spatial simulation model that the interactions between the preference for closeness to urban center and the preference for open space have the potential to generate both regular and “irregular” land use patterns. Filatova et al. (2009) show that the preference heterogeneity can contribute to the urban expansion. Apart from these theoretical explanations, many empirical studies suggest that households’ tastes for open space may have contributed to the household location decisions (Geoghegan et al., 1997). Irwin and Bockstael (2002) are the first that empirically identify the negative externalities among recently developed residential parcels. Carrion-Flores and Irwin (2004) find empirical evidence that low density areas close to existing urban development has a significant impact on the location of new residential development.

In contrast to the accumulating empirical and simulation-based evidences, the theoretical link between the preference for amenities and its capitalization into the land rent/price remains to be established. The major difficulty is the non-marketable nature of the natural amenities. This paper establishes the link by introducing auction into the literature on optimal land conversion decisions (Arnott and Lewis, 1979; Barilan and Strange, 1996; Capozza and Helsley, 1990). Arnott and Lewis (1979) show that agricultural land should be converted when the value of converted land exceeds the value of agricultural land plus the conversion cost. Capozza and Helsley (1990) emphasize on the fact that land conversion is irreversible and conclude that
the option value of this irreversible change should also be included in the threshold rent for land conversion. Barilan and Strange (1996) emphasize on the importance of the investment lag between the decision to convert the land and the completion of the land conversion. They show that investment lags make the land development less sensitive to uncertainty. In the presence of lags, uncertainty may even hasten the land development. This paper elaborates the work of Capozza and Helsley (1990) by incorporating residents’ preference for natural amenities, a non-market good, into the analysis of optimal dynamic decisions on land conversion.

More recently, people begin to study the spatial pattern in a temporally-and-spatially integrated framework (Boucekkine et al., 2009; Brock and Xepapadeas, 2008; Desmet and Rossi-Hansberg, 2009) consisting of forward-looking agents. Within a regular shaped domain, Brock and Xepapadeas (2008) finds that “the interaction among the discount rate, the curvature of the Hamiltonian and some spatial features of the problem can generate ‘diffusion-induced’ instability” and sustain an optimal spatial pattern. Boucekkine et al. (2009) try to extend the Ramsey model in a spatial explicit framework and show that the induced dynamic problem can give rise to ill-posed problems in general because the transversality condition cannot pin down the solution as in the aspatial models. An assumption of linear utility is used as a way to avoid such problems. Desmet and Rossi-Hansberg (2009) study the case in which technological shocks gets diffused spatially through factor mobility and generates non-uniform spatial patterns. One major limitation of these studies is the
counterfactual assumption of diffusion process. In reality, neither people nor capital relocate like particles in physics following diffusion processes. Instead, both people and capital can travel long distance without trespassing the nearby neighbors.

3.3 Microfoundations of a Model of Exurban Residential Land Use Patterns and Growth

In this section, we establish a structural land market model based on the interaction between farmers, potential residential migrants and land developers. The farmers own the land and make an irreversible decisions to convert the land from agricultural to residential use, as in Capozza and Helsley (1990). In the optimal period of land conversion, a developer is hired to convert the agricultural land to residential use. After the land conversion, the land is occupied by a household who pays the land owner the residential land rent. Households are potential migrants who make a decision whether to move into the region and locate on a particular parcel of residential land. If they decide to move into the region, they receive an income which will be allocated between the consumption of a numeraire good and the residential rent. They care about the location-specific amenities which are non-marketable, which can either be exogenous or endogenous to land conversion, like the weather, topography and open space in the neighborhood. Because households need to commute to the urban centers, if their residential location is far away the urban center, they need to pay higher transportation cost, which can be either the fuel cost.

\footnote{Following Wu and Plantinga (2003) and Caruso et al. (2007), we focus only on location choice. The housing choice is beyond the scope of this paper.}
and the opportunity cost of travel time. In this paper, we use the travel time. As a result, the residential rent they pay to the land owner includes the valuation for the amenities. This theoretical land market model is then applied to the real landscape of Carroll county, Maryland. The simulated land use pattern is compared with the pattern observed in the data.

Because the goal of this paper is to explore the mechanisms behind the pattern formation based on the land use change in Carroll County, Maryland, we follow the traditional urban economic model by incorporating transportation costs as a key driving force that attracts urban residents to locate within some proximity of urban centers. More generally, this transportation cost can be interpreted as the economic cost to enjoy the urban amenities, which increases with the distance to urban center. Acknowledging the importance of natural amenities for explaining exurban development, we also incorporate the preference for natural amenities into the urban resident utility in order to generate scattered development. But unlike Turner (2005) and Caruso et al. (2007), the model simulation is built upon a general equilibrium framework in which farmers optimally choose the timing of land conversion, residents optimally select and bid for land parcels, and land developers optimally choose the parcels to convert. More importantly, unlike the static models of Wu and Plantinga (2003); Turner (2005) and Caruso et al. (2007), the model in this paper is a dynamic one driven by the increasing income of the urban residents as in (Capozza
and Helsley, 1990).\textsuperscript{31} The farmers are forward looking when making the land conversion decisions. In contrast to Brock and Xepapadeas (2008), Boucekkine et al. (2009) and Desmet and Rossi-Hansberg (2009), because we are not interested in the analytical solution and we do not believe that an analytical function can generate the observed land use pattern, we relax the assumption on diffusion and regular domain. Because the goal of this paper is to explore the mechanisms behind the pattern formation based on the land use change in Carroll County, Maryland, we incorporate the transportation cost as a key driving force that attracts urban residents to the urban centers. More generally, this transportation cost can be interpreted as the economic cost to enjoy the urban amenities, which increases with the distance to urban center. Acknowledging the importance of the natural amenities, we incorporate the preference for natural amenities into the urban resident utility in order to generate scattered development. But unlike Caruso et al. (2007); Turner (2005), the model simulation is built upon a general equilibrium framework with farmers trying to optimally choose the timing of land conversion and residents trying to optimally select the residential site and maximize their utility. More importantly, unlike the static models of Caruso et al. (2007); Turner (2005); Wu and Plantinga (2003), the model in this paper is a dynamic one driven by the increasing income of the urban residents in (Capozza and Helsley, 1990).\textsuperscript{32} The farmers are forward looking when

\textsuperscript{31}Caruso et al. (2007) can be viewed as a special case of this model if the resident income is held constant over time, except for the relocation of local residents.

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making the land conversion decisions. In contrast to Boucekkine et al. (2009); Brock and Xepapadeas (2008); Desmet and Rossi-Hansberg (2009), because we are not interested in the analytical solution and we do not believe that an analytical function can generate the observed land use pattern, we relax the assumption on diffusion and regular domain. In addition, we include several sources of spatial heterogeneity in the analysis, such as the spatial heterogeneity generated by the road network, the land heterogeneity in the agricultural productivity and the cost of conversion.

3.3.1 Demand for Residential Land

In this model economy, a potential migrant chooses between migrating into the region and staying outside. If he chooses to enter the region, he receives an income \( Y(t) \) in period \( t \). He derives utility from a composite numeraire good \( (X) \), a parcel of land with size \( L \) and the location-specific amenities \( (A) \). If he chooses a parcel with a distance \( Z \) to the CBD, he needs to pay a unit distance cost of \( T \). The numeraire good is a market good, but the amenity is not. To successfully acquire the parcel, the potential migrant needs to participate in a first price sealed auction with \( N \) competitors including himself. The rest of his income is spent on the numeraire good. His valuation of the land with non-marketable amenity \( (A) \) is the maximum amount he would like to pay for it. Given that he has a reservation utility \( U_0 \), the value for the land \( (L) \) and amenity \( (A) \) can be expressed as:

\[
V = \sup \{ B \mid P_x X + B \leq Y(t) - T Z, \text{ and } U(X, A, L) = U_0 \}
\]
Assume that utility function is monotonic in $X$, and $X = U^{-1}(U_0, A, L)$, the maximum monetary value for the land with a specified amenity level of $A$ is the one that gives him the reservation level $U_0$. That is:

$$V(t, Z, A) = Y(t) - TZ - P_x U^{-1}(U_0, A, L)$$

To simplify the notation, the arguments in $V(t, Z, A)$ are suppressed.

Assume the income is growing at a rate of $g$ and subject to an i.i.d. random shock $\epsilon$ with cumulative distribution function $F$:

$$Y(t) = gt + \sigma \epsilon(t) \quad (3.1)$$

Each potential migrant knows his own income $Y_i$ but not that of others. As a result, each individual’s valuation for the land with amenity level $A$ is independent across potential migrants.

Denote the amount the potential migrant would like to bid for the land as $w$ and denote the bid function as $b : [0, V] \rightarrow \mathbb{R}_+$. In the first price sealed auction with $N$ participants, the bid should solve the following maximization problem:

$$\max_w (V - w) Pr(b(V_j) < w, \forall j \neq i) = \max_w (V - w) F^{N-1}(b^{-1}(w))$$

The optimal bid function $b(V)$ satisfies the condition:

$$F^{N-1}(b^{-1}(w)) = (V - w) \frac{d}{dV} F^{N-1}(b^{-1}(w)) \frac{db^{-1}(w)}{dw}$$
At the equilibrium of auction, we have \( b(V) = w \). The above optimality condition can be reformulated as:

\[
F^{N-1}(V) = (V - w) \frac{d}{dV} F^{N-1}(V) \frac{1}{b(V)}
\]

and solved:

\[
b(V) = \int_{-\infty}^{V} sdF^{N-1}(s) \frac{1}{F^{N-1}(V)}
\] (3.2)

It can be shown that as the number of bidders \( N \to \infty \), \( b(V) \to V \) regardless of the cumulative distribution function. For simplicity, from here on we assume that \( N = \infty \).\(^{33}\) In addition, we assume \( \epsilon(t) \) is a standard Brownian motion with drift 0 and variance 1.\(^{34}\) Because of the competition among the potential buyers of the land, the preference for natural amenities is fully capitalized. The residential rent at time \( t \) for a parcel with distance \( z \) to the CBD and amenity level \( A \) is:

\[
R(t, z, A) = b(V) = V = gt + \sigma \epsilon(t) - TZ - P_x U^{-1}(U_0, A, L)
\] (3.3)

\(^{33}\)This assumption also eliminates the possibility that bid price for land can reflect market conditions. For instance, as the number of potential buyers decreases, the competition for land is less intense and the migrant can potentially retain more of the private value for amenities. In the extreme case of no competitors, the migrant can in fact bid for the land at the reservation value of farmers and thus retain all the surplus. However, with finite \( N \), except for the special case that \( \epsilon(t) \) is an i.i.d. uniform, there is no analytical solution for the complete model. However, numerical methods can be applied to solve the problem.

\(^{34}\)If numerical methods are applied, this assumption can be relaxed as well.
3.3.2 Supply of Residential Land

Agricultural land that is available for residential development is owned by farmers who are risk neutral.\textsuperscript{35} The parcel size and the level of amenity on the parcel are exogenously determined. If the land is left in agricultural use, it will generate an agricultural return of $R_a$ that is not observed by the migrants. If a farmer decides to convert the land to residential use, he will hire a developer to complete the conversion. Land conversion is an irreversible change. That is, once the land is converted into residential use, it can never be used for agriculture. After the conversion, the farmer will rent the land to urban residents.\textsuperscript{36} We assume that farmers know the bid rent function $b(V)$ and its dynamics over time. Following the formulation of Capozza and Helsley (1990), the price for agricultural land that will be converted at time $t + s$ is:

$$P^a(t, s, z, A) = E\left\{ \left[ R_a e^{-r(t-s)} \int_t^{t+s} R(\tau, z, A) e^{-r(t-s)} d\tau \right] R(t, z, A) \right\}$$

$$= \frac{R_a}{r} + \frac{1}{r} E\left\{ \left[ R(t + s, z, A) + \frac{g}{r} - R_a \right] e^{-\tau s} R(t, z, A) \right\}$$

Let $R^*$ be the threshold rent for land conversion and reformulate the problem into a hitting time problem. The time of conversion, $t^*$, is the first hitting time for

\textsuperscript{35}The structure and derivations in this section follows closely to Capozza and Helsley (1990), except for the treatment of conversion cost. In this model, the conversion cost is absorbed by the land developer rather than the farmer.

\textsuperscript{36}This is not always the case in reality. In many cases, farmers may sell the land rather than rent it. Based on the economics theory, so long as the land market is at equilibrium and farmers are rational, the present value of cash flows from selling the land should equal that from renting. However, from the perspective of land modeling, renting is much easier to handle than sale, because the price of land includes the growth premium and the rent does not (Capozza and Helsley, 1990). With the assumption that farmer rent the land, we do not have to deal with the growth premium which hinges on farmer’s perception of the future evolution of urban boundary.
residential land rent $R(t + s, z)$ reaches the threshold rent $R^*$:

$$t^* = \min_s \{ t + s \geq t \mid R(t + s, z, A) \geq R^* \}$$

The expected price for agricultural land is:

$$E\{ P^a(t, t^*, z, A) \mid R(t, z, A), R^* \} = \frac{R_n}{r} + \frac{1}{r} \left[ R^* + \frac{g}{r} - R_n \right] E\{ e^{-r(t^*-t)} \mid R(t, z, A), R^* \}$$

Because $R(t, z, A)$ follows a Brownian motion, the distribution of the first hitting time $t^*$ is known. Using the moment generating function for $t^*$, we have

$$E\{ e^{-r(t^*-t)} \mid R(t, z, A), R^* \} = e^{-\alpha[R^*-R(t,z,A)]}$$

where

$$\alpha = \left( \frac{g^2 + 2\sigma^2 r}{\sigma^2} \right)^{1/2} - g$$

The expected price for agricultural land is:

$$E\{ P^a(t, t^*, z, A) \mid R(t, z, A), R^* \} = \frac{R_n}{r} + \frac{1}{r} \left[ R^* + \frac{g}{r} - R_n \right] e^{-\alpha[R^*-R(t,z,A)]}$$

The threshold land rent $R^*$ maximizes the above equation. Take the derivative of the above equation with respect to $R^*$, we have:

$$R^* = R_n + \frac{r - \alpha g}{\alpha r}$$

(3.5)
3.3.3 Land Conversion and Land Developer

A potential migrant will investigate the agricultural land and submit a bid for each parcel. If the bid price, or in other words, the residential rent $R(t, z, A)$, of a parcel exceeds the corresponding threshold rent $R^*$, a land developer will be hired to convert the land. In order to simplify the analysis, we assume that there is only one land developer in the economy and he observes both the current bid price and the threshold rent. In addition, he has the specialized knowledge about the cost of conversion. Using his market power, he is able to reap the current period consumer surplus, the difference between the bid price and threshold rent $(R(t, z, A) - R^*)$. He will not convert the land if the surplus is insufficient to cover the conversion cost $C(z)$ associated with the parcel of land. When the bid prices of a migrant exceed the threshold rents for multiple parcels, the land developer will choose the convert the parcel that maximizes his profit $R(t, z, A) - R^* - C(z)$.

3.4 Data Description

The above decision rules are applied to the real landscape of Carroll County, Maryland. Carroll County is an exurban county lying to the northwest of Baltimore City and north of Washington D.C.. According to the census of 2000, it has a

\[37\text{When there are multiple land developers, the competition among them may lead to strategic land conversion decisions. Intuitively, a land market with only a few big land developers may result in more clustered development because of the scale economy in land conversion. On the other hand, competition among numerous small land developers may result in a more scattered development. A market with a mix composition of big and small land developers will generate more complicated strategic interactions that can affect the land use patterns. These are interesting and important complications that worth further investigation. However, they are beyond the scope of this study.}\]
population of 150,897 and a total area of 452 square miles. Since the 1970s, the county has been faced with increasing sub-urbanization pressures as people in Baltimore are moving to the suburban and exurban areas. Because of this, the land use pattern in 1970 is used as the starting period of simulation. The City of Westminster is located at the center of Carroll County and serves as the County Seat. As of 2000, Westminster has a population of 16,731 and is the largest urban center in the county. Because many of the residents in Carroll County commute to Baltimore City and regions around, the Town of Sykesville is treated as another urban center in the analysis. The distance to Sykesville is used as a proxy to the distance to Baltimore City. In addition, because many residents in the southwest of the county are commuting to the Washington D.C., the Town of Mt Airy is treated as another urban center in order to proxy for the distance to the Washington D.C.

Land use change over time is derived from the MdProperty View data and the Tax Parcel Data from the Carroll County, Maryland. The two datasets are joined based on the common field: “ACCID” in order generate the vector dataset with parcel level information: the land use code (fieldname “LU”) and the year the building on the parcel is build (field name: “YRBLT”). The land use code (“LU”) can take on a number of values including agriculture (“A”), residential (“R”), commercial (“C”), industrial (“I”) and so on. As a first step in exploring the land use change pattern, we only distinguish between the developed and undeveloped land. All agricultural land is considered as undeveloped land. In addition, land for non-agricultural uses
Figure 3.1: Carroll County landscape. Figure (a) Land use pattern in 1970. Black cells are developed land. Figure (b) Land use pattern in 2000. Figure (c) Travel time to urban centers. The shorter the travel time, the darker the color. Figure (d) Conversion cost. The cheaper the cost, the darker the color.
are considered undeveloped land before the year the building is built. For instance, a residential parcel with “YRBLT”=1989 is considered to be undeveloped land before 1989 and developed land after 1989. The distinction among the residential, commercial, industrial land uses are ignored.\footnote{There can be interesting interactions among these different types of development land. However, they are beyond the scope of the current paper.} Starting with the landscape of 2000, we reconstruct the landscape in previous years based on the information of year built.\footnote{The generation of landscape data is done by Doug Wrenn as a part of Baltimore Ecosystem Study Project.} The land use pattern in 1970 and 2000 are plotted in Figure 3.1(a) and 3.1(b).

Three types of land heterogeneity are considered: the commuting cost, which affects the budget of migrants; the agricultural yield, which affects the threshold rent of the farmers; and the conversion cost, which affects the site choice of the land developer.

Road data of Carroll County is obtained from the TIGER/Line 2000 (U.S. Bureau of the Census). The commuting cost is calculated based on this dataset. Because the longest distance between any two parcels within the Carroll County is less than 33 miles (53 km), it is unlikely that the difference in gas price can be a key factor in determining the residents’ location decisions. However, the opportunity cost of travel time can still be substantial. Because many residents in Carroll County, especially those living in the southern part of the county, commute to the Baltimore city and the Washington D.C., in addition to the City of Westerminster (the County Seat), I also include the Town of Syksville (proxy for Baltimore city) and Mt Airy (proxy for...
Washington D.C.) as urban centers. The travel time from each parcel to each one of the urban centers is calculated using the Network Analysis Toolbox in ArcGIS. In the baseline case, the travel time for each parcel is assumed to be the sum of the travel time to the three urban centers and is plotted in Figure 3.1(c).

The agricultural yield is obtained from the Soil Data Mart website under the United States Department of Agriculture (USDA). Because Carroll County has an average annual precipitation of 42 inches, the report on the non-irrigated crop yield is used instead of irrigated yield. Of the three major crops: corn, soybean and wheat, the yield for corn is used because many of the soil types have no yield for the other two crops. The yield reflects the capability of land. According to the data description, it is the yield “that can be expected... under a high level of management”. It is based mainly on “the experience and records of farmers, conservationists, and extension agents” and can vary “because of variations in rainfall and other climatic factors”.

The conversion cost of each parcel is based on the rating of conversion difficulty generated by the Soil Data Mart website under the United States Department of Agriculture (USDA). For each soil type in the Soil Survey Area (SSA) data, a rating is generated to “indicate the extent to which the soils are limited by all of the soil features that affect building site development”. In particular, the Soil Data Mart website generates reports on the soil rating for three types of uses pertinent to this paper: “dwellings without basements”, “dwellings with basements” and “small

40 The description here are based on data description on the website of Soil Data Mart, Natural Resources Conservation Service, USDA.
commercial buildings”. For each use, the soil types in Carroll County are put into following categories.41

“Not limited” indicates that the soil has features that are very favorable for the specified use. Good performance and very low maintenance can be expected. “Somewhat limited” indicates that the soil has features that are moderately favorable for the specified use. The limitations can be overcome or minimized by special planning, design, or installation. Fair performance and moderate maintenance can be expected. “Very limited” indicates that the soil has one or more features that are unfavorable for the specified use. The limitations generally cannot be overcome without major soil reclamation, special design, or expensive installation procedures. Poor performance and high maintenance can be expected.

Two other soil types are classified as “Not Rated”. One is for “water”, which means not suitable for either dwelling or commercial use. The other is “manmade land”.42 Unable to find further information that links this rating to economic cost of land conversion, I assign numeric values for each rating in the manner as described in Table 3.1. The spatial distribution of the conversion cost is plotted in Figure 3.1(d):

41The description here are based on data description on the website of Soil Data Mart, Natural Resources Conservation Service, USDA.

4228 polygons fall into this category, all of them are small. Crosschecking with Google Earth reveals that most of them are either parts of roads or commercial centers. Depending on the relative timing between the land conversion decision and the time when the man made soil is created, this soil type can have different implications for the conversion cost. If it is a pre-existing condition at the time of land conversion decision, this soil type is favorable for land conversion. Otherwise it implies an unfavorable condition. In this preliminary exploration, I treat it as a preexisting condition and assigns the value of one. This has no significant impact on the result of this paper.
<table>
<thead>
<tr>
<th>Rating</th>
<th>Interpretation</th>
<th>Cost ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not limited</td>
<td>very favorable, low maintenance</td>
<td>1</td>
</tr>
<tr>
<td>Somewhat limited</td>
<td>moderately favorable, moderate maintenance</td>
<td>2</td>
</tr>
<tr>
<td>Very limited</td>
<td>unfavorable, high maintenance</td>
<td>3</td>
</tr>
<tr>
<td>Not rated water</td>
<td>not suitable</td>
<td>4</td>
</tr>
<tr>
<td>Not rated manmade</td>
<td>suitable if pre-existing</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 3.1: Soil ratings and interpretations.

3.5 Specifications and Implementation of the Simulation Model

Simulation is done in NetLogo, a commonly used modeling environment designed to model and simulate complex systems that evolve over time. Recently, a GIS extension is added to NetLogo which allows it to load GIS data. Because the “world” in Netlogo is comprised of patches, in order to facilitate comparison between simulation and real data, the above-mentioned vector datasets are transformed into raster data format using ArcGIS. The cell size is set at 1km x 1km to control the errors associated with the geographic data. When the data is projected into the coordinate system of “State Plane NAD83_Harn (Meter)”, the extent of Carroll County in the MDProperty View data lies 20 meters north of the extent of the soil data obtained from the SoilMart. This is due to the difference in the geographic coordinate systems of these two datasets. Setting the cell size at 1km x 1km ensures that the mismatched

43The rasterization is done carefully so that all files are in the geographic coordinate system of NAD83_Harn so that the Carroll County landscape is not tilted in the Netlogo world.
area in each cell is no more than 2% of the cell total. The extent of the data are fixed so that there are 43 rows and 45 columns in the landscape.\textsuperscript{44} Because the land use, travel time, agricultural yield and conversion cost are parcel level attributes, we need to transform them into attributes of the 1km x 1km cells. The dominant land use type in the cell is taken as the land use type for that cell. Agricultural yield and conversion cost is done similarly. For the travel time, because the road network is treated as missing value in the original data, using the same rule for aggregation will generate cells with missing values in the landscape. To avoid it, I use the average travel time of all the parcels in the cell as the travel time for that cell.

In the baseline model, residential migrants have a preference of the Cobb-Douglas form:

\[ U(X, A, L) = X^{a_1} (AL)^{a_2} \]

where location-specific amenity $A$ and the parcel size $L$ are non-separable. Assume $a_1 = 0.8$ and $a_2 = 0.2$.

The income growth of residents follows a Brownian motion with drift $g = 0.5$ and standard variation $\sigma = 0.5$.\textsuperscript{45} In addition, we set the price for the numeraire good $P_x = 1$ and the reservation utility level $U_0 = 2$. The initial income level is

\textsuperscript{44}If the cell size is fixed at 1000 meters, the rasterization of Carroll County in ArcGIS generates ASCII data with 42 rows and 45 columns. In Netlogo, the extent of the world always consists odd number of rows and columns. To minimize the error during the data importing procedure, I manually add a row of background value in all the ASCII data. The heading information in each ASCII data is changed so that the extent of the data are consistent across different data sources.

\textsuperscript{45}An annual income growth rate of 4% will generate roughly a 50% income growth at the end of 10th year. The $\sigma$ value implies that roughly 70% of the residents have an annual income growth in the range $[-4\%, 4\%]$
set at $Y(0) = 6$, which ensures that under the baseline specification the number of developed cells equals approximately the number of developed cells in the real landscape in year 2000. In Carroll County, the average travel time to urban centers is roughly 15 minutes, the fuel cost of such a short distance travel is minimal. However, the opportunity cost of travel time can still be significant in determining the choice of location. We assume that the unit travel cost $T = 2$, so that the commuting cost is roughly 5% of the income.

At the initial stage of the simulation, the raster data with cell size 1km x 1km of Carroll County are loaded into NetLogo. The values for the land use type, the commuting cost (as measured in minimum travel time to CBDs), the agricultural yield (as represented by the corn yield) and the conversion cost (as proxied by the suitability for dwelling and small business) are loaded as attributes for the patches in NetLogo. The location-specific amenity is calculated as the percentage of open space in the eight neighboring parcels. For instance, a parcel with four undeveloped neighbors has a value of 0.5.

Based on the stochastic process of resident income, the option value of the irreversible land conversion is calculated. The threshold rent for each parcel is then constructed combining this option value with the parcel-level agricultural yield, based on the Equation 3.5. In each period, 100 potential migrants are generated. Each one of them are assigned an income following the Equation 3.1 with $\epsilon(t) \sim N(0, \sigma)$. They investigate every undeveloped parcel and submit a bid price for each following

---

46 This number is unknown to the individual migrant.
Equation 3.3. If no parcel satisfies the condition $R(t, z, A) - R^* > 0$, the migrant will stay outside of the county. If there is only one parcel satisfies this condition, the land is converted. If there are multiple parcels satisfy this condition, the land developer will step in and choose the parcel that maximizes his profit $R(t, z, A) - R^* - C(z)$. At the end of the period, the value of open-space amenity is updated for each parcel. The model is run for three periods.

3.6 Simulation and Discussion

The results for 100 simulations are summarized in the Table 3.2. The ending time of the simulation is chosen so that the number of developed parcels roughly equals the total number of developed patches in Carroll County in 2000. If the location of a developed parcel in the simulation coincides with the location of a developed parcel in the real data, then it is considered as correct prediction of type A. The total number of these parcels are reported in the table as the “number of correct prediction A”. On average, 40% of the developed parcels in the 2000 data are developed in our simulation. This is a very stringent criterion for “correct” simulation. We can relax the criterion and consider a “correct” prediction if the simulated developed parcel misses the target slightly. For a developed parcel in 2000 data, if at least one parcel in its immediate neighboring parcels (including itself) gets developed in our simulation, it is considered as the correct prediction of type B. The “number of correct prediction
<table>
<thead>
<tr>
<th>Simulation result</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of developed parcels</td>
<td>142</td>
<td>126</td>
<td>153</td>
<td>5.2</td>
</tr>
<tr>
<td>Number of correct prediction A (%)</td>
<td>50</td>
<td>41</td>
<td>57</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>(29)</td>
<td>(24)</td>
<td>(33)</td>
<td></td>
</tr>
<tr>
<td>Number of correct prediction B (%)</td>
<td>137</td>
<td>128</td>
<td>144</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>(79)</td>
<td>(74)</td>
<td>(83)</td>
<td></td>
</tr>
<tr>
<td>Number of developed parcels in 2000:</td>
<td>174</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Prediction of location in 100 simulations.

B” is also listed in the table. On average, 80% of the developed parcels in 2000 data has at least one neighboring parcel gets developed in our simulation. In the case of random land conversion, the correct prediction of type A is approximately 17% and that for type B is approximately 60%.

The simulated landscape is shown in Figure 3.2(b). Compared with the real data, on the one hand, in areas around the urban centers, the simulated land use pattern is not as clustered as the real land use pattern; on the other hand, in the areas outside urban center, it is not as scattered.

In order to quantitatively compare the simulated land use pattern with that of the real data, several pattern metrics are generated using Fragstats, a software that

47 This second criterion suffers from the problem of double counting. In the worse scenario, one correct prediction of type A can be counted nine times under the criterion of the type B correct prediction.

48 There are 174 developed cells in the total of 1016 cell in Carroll County.
Figure 3.2: Carroll County landscape. Figure (a) Land use pattern in 2000. Black cells are developed land. Figure (b) Simulated land use pattern in baseline model. Figure (c) Simulated land use pattern with dominating conversion cost. Figure (d) Simulated land use pattern with alternative specification of travel time.
analyzes spatial patterns. To quantitatively measure land use pattern, we need measures that capture the characteristics of particular patches as well as measures that captures the spatial relation among different patches. In our case, we need measures that capture the characteristics of developed parcels as well as measures that captures the spatial relationship between the developed and undeveloped patches. Given a land use pattern, the mean area, the mean perimeter and the number of patches provide the most fundamental information of the land use pattern and are used in the calculation of many pattern metrics. The second key characteristic of a pattern is its shape. To measure the shape complexity, the perimeter-area ratio index and the shape index are chosen. The former is chosen for its simplicity. Because this index decreases with increases in the size of the patch, the shape index is included to control for this problem. In addition, the fractal dimension index is chosen because it measures cross-scale regularity in the spatial pattern. It gives a relationship between the perimeter and the area across different sizes of patches. To capture the spatial relationship between the developed and undeveloped patches, the edge contrast index is used because it measures the amount of contract along the patch perimeter.

The results are summarized in the following table. The first row specifies the pattern metrics used. The second row describes the value of pattern metrics for the real land use data in 2000. The third row gives the mean value for the 100 simulations. The fourth row shows the standard deviation across the simulation. The
simulated land use pattern differs significantly from the observed land use pattern in reality. Except for the shape index, all pattern metrics for the real land use pattern lie beyond two standard deviation from those of the simulated patterns, suggesting that the probability for the model to regenerate the same metrics as in the data is approximately less than 10%. The standard deviation of the fractal dimension is too small to be reliable. In general, the simulation model generates too many patches and the patch sizes on average are too small. This conforms with what we observe when comparing the simulated pattern in Figure 3.2(b) and the real land use pattern in Figure 3.2(a).

In short, it is unlikely that our simulation model can reproduce the observed land use pattern.

### 3.7 Sensitivity Analysis

The sensitivity is conducted over different specifications of travel time, conversion cost and open space amenities. For the sensitivity on conversion cost, I triple the conversion cost for each parcel, so that the conversion cost becomes the most important source of land heterogeneity. The resulted land use pattern is too scattered as compared to the real data, as shown in Figure 3.2(c). Comparing with the baseline case, the correct location prediction of type B is much better, this is mainly because the total number of developed parcels are about 15% more than the baseline case (see Table 3.4). The discrepancy in the pattern metrics are much larger as compared to the baseline case (see Table 3.5).
Table 3.3: Comparison between the simulated and real land use pattern.

The sensitivity analysis on the travel time is done through different specifications. Rather than defining the travel time for each cell as the sum of the time to the three urban centers (Westminster, Syksville and Mt Airy), I redefine it as the minimum of the three. This specification implies that the urban services provided by the three urban centers are complete substitutes so that one only needs to go to one of the centers. The simulated land use pattern is plotted in Figure 3.2(d). It looks quite similar to the baseline result. As shown in Tables 3.4 and 3.5, the location predictions and the pattern metrics under this different specification are very similar to the baseline result as well.
### Sensitivity to the specification of conversion cost

<table>
<thead>
<tr>
<th>Simulation result</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of developed parcels</td>
<td>167</td>
<td>152</td>
<td>184</td>
<td>7.01</td>
</tr>
<tr>
<td>Number of correct prediction A (%)</td>
<td>47 (26)</td>
<td>39 (22)</td>
<td>55 (32)</td>
<td>3.44</td>
</tr>
<tr>
<td>Number of correct prediction B (%)</td>
<td>148 (85)</td>
<td>136 (78)</td>
<td>156 (90)</td>
<td>4.88</td>
</tr>
</tbody>
</table>

### Sensitivity to the specification of travel time

<table>
<thead>
<tr>
<th>Simulation result</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of developed parcels</td>
<td>142</td>
<td>129</td>
<td>157</td>
<td>5.76</td>
</tr>
<tr>
<td>Number of correct prediction A (%)</td>
<td>49 (28)</td>
<td>42 (24)</td>
<td>58 (33)</td>
<td>3.30</td>
</tr>
<tr>
<td>Number of correct prediction B (%)</td>
<td>137 (78)</td>
<td>128 (74)</td>
<td>157 (83)</td>
<td>5.76</td>
</tr>
</tbody>
</table>

### Sensitivity to the specification of amenities

<table>
<thead>
<tr>
<th>Simulation result</th>
<th>Mean</th>
<th>Min</th>
<th>Max</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of developed parcels</td>
<td>160</td>
<td>144</td>
<td>173</td>
<td>6.2</td>
</tr>
<tr>
<td>Number of correct prediction A (%)</td>
<td>46 (26)</td>
<td>38 (21)</td>
<td>56 (32)</td>
<td>3.7</td>
</tr>
<tr>
<td>Number of correct prediction B (%)</td>
<td>142 (82)</td>
<td>130 (75)</td>
<td>154 (88)</td>
<td>5.76</td>
</tr>
</tbody>
</table>

Table 3.4: Sensitivity of location predictions to key sources of spatial heterogeneity.

In order to test the sensitivity on open space amenities, the weighting coefficient of the preference for open space amenities \(a_2\) is changed from 0.2 to 0.1. The results are similar to the baseline as shown in Tables 3.4 and 3.5.
### Pattern of the developed land use

<table>
<thead>
<tr>
<th></th>
<th>NP</th>
<th>Mean area</th>
<th>Total edges</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 2000</td>
<td>38</td>
<td>458</td>
<td>353000</td>
</tr>
<tr>
<td>Conversion cost</td>
<td>77</td>
<td>218</td>
<td>526890</td>
</tr>
<tr>
<td>(std)</td>
<td>(4.73)</td>
<td>(16)</td>
<td>(20364)</td>
</tr>
<tr>
<td>Travel time</td>
<td>46</td>
<td>312</td>
<td>424320</td>
</tr>
<tr>
<td>(std)</td>
<td>(3.91)</td>
<td>(31.43)</td>
<td>(15669)</td>
</tr>
<tr>
<td>Amenity</td>
<td>65</td>
<td>248</td>
<td>490530</td>
</tr>
<tr>
<td>(std)</td>
<td>(4.0)</td>
<td>(17.7)</td>
<td>(18375)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Fractal</th>
<th>PA ratio</th>
<th>Shape index</th>
<th>CE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data 2000</td>
<td>1.02</td>
<td>33.6</td>
<td>1.18</td>
<td>91.5</td>
</tr>
<tr>
<td>Conversion cost</td>
<td>1.02</td>
<td>36.6</td>
<td>1.14</td>
<td>95.1</td>
</tr>
<tr>
<td>(std)</td>
<td>(0.003)</td>
<td>(0.45)</td>
<td>(0.03)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>Travel time</td>
<td>1.02</td>
<td>35.8</td>
<td>1.24</td>
<td>96.7</td>
</tr>
<tr>
<td>(std)</td>
<td>(0.003)</td>
<td>(0.70)</td>
<td>(0.04)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>Amenity</td>
<td>1.02</td>
<td>36.4</td>
<td>1.17</td>
<td>95.2</td>
</tr>
<tr>
<td>(std)</td>
<td>(0.003)</td>
<td>(0.5)</td>
<td>(0.03)</td>
<td>(1.0)</td>
</tr>
</tbody>
</table>

**Note:**

NP = Number of patches  
Fractal = Fractal dimension  
PA ratio = Perimeter-area ratio  
CE = Contrast edges

Table 3.5: Sensitivity of simulated patterns to key sources of spatial heterogeneity.
3.8 Conclusion

This paper contributes to the literature in two aspects. First, we provide a theoretical framework that incorporates non-marketable amenities into the dynamic analysis of land conversion decisions. Through the introduction of a private value first-price sealed auction, it provides a micro-foundation for the observed capitalization of natural amenities in the land market. Second, with the assistance of geographical information system, we are able to link the economic theories on optimal land conversions to the observed suburbanization and exurbanization process and assess the theoretical predictions of pattern formation. While the model is constructed based on the individual-level land conversion decisions, the real focus is to assess the spatial land use change at the regional level. After the consideration of the spatially heterogeneity in commuting cost, land conversion cost and agricultural productivity, we are still unable to explain the observed landscape in Carroll County, Maryland. The spatial pattern of developed land use remains a puzzle to be solved.

3.9 Future Research

While the analysis in this paper is only exploratory, it sets up a framework that allow us to understand the micro-level economic decisions underlying the formation of regional land use patterns and test it on the real landscape. The spatial dynamic model proposed in this paper has the potential to improve its predictive ability of the spatial land use pattern. For the sake of analytical tractability of the land
conversion rules, we have assumed in this paper that potential resident migrant bid as if there are infinite competitors. With this assumption, migrants always set their bid price at the true private value of the land. As a result, their utility is always set at the reservation level. In the spatial equilibrium, they are indifferent among all the locations that satisfy the condition $R(t, z, A) - R^* > 0$. A parcel with higher natural amenities cannot generate more demand for the parcel. This issue could be addressed by relaxing the assumption on the number of bidders participating in the auction for land. With finite number of bidders $N$, except for the unrealistic case that income follows an i.i.d. uniform distribution, there is no analytical solution of the bid function. And the distribution of the first hitting time in the farmers problem becomes unknown. Both problems can be solved by the numerical methods. When the income does not follow the Brownian motion, the distribution of the first hitting time is analytically intractable, we do not need to reformulate the optimal timing problem as in Capozza and Helsley (1990). It is worthwhile to explore the direct solution of the optimal timing that maximizes agricultural land price, as specified in Equation 3.4.

While the parameter values in this paper are reasonable, it would be better if we could estimate their values using the structural model on household data. In this way, it can better reflect the demographic and economic characteristics of the Carroll County. Since we have the parcel-level land value in the most recent transaction, we

\[ \epsilon \sim \text{Uniform}[a, b]. \] This is unrealistic because income is not accumulative over time.
can construct the parcel-level land value of the “newly converted land”.\textsuperscript{50} In the short run, we can estimate the correlations between the land value of these newly converted land and the sources of observable spatial heterogeneity in this model, that is, the agricultural productivity, the transportation cost, open space amenities and the conversion cost. In the longer term, we can conduct the same estimation using the parcel-level sale price \textit{at} the time of conversion. This requires the time series of MDProperty View data, which we do not have right now. Since MDProperty View data starts in 1996, we can only go back to that year. Whether the spatial pattern metrics are sensitive enough to capture the landscape change is one concern for this approach. If we could have household level data on the income, neighborhood and household characteristics, we may directly estimate the parameters of the structural model on land conversion and use the estimates in the simulation model. This would be another interesting and important extension of the current model. So far, we have constrained our analysis to the equilibrium analysis. Given the characteristic of the low trading frequency in the land market and imperfect information, we have reasons to conjecture that the land market may be temporarily out of the equilibrium. With the household level data, we may be able to empirically estimate a structural model on both the supply and demand side of the land market and incorporate the out of equilibrium dynamics into the current framework.

\textsuperscript{50}If the most recent transaction date in the dataset is within one year of the YearBuilt of the parcel, we consider the parcel as newly converted.
In addition, the predicability of the model can be improved by incorporating other sources of spatial heterogeneities, such as the provision of the sewer and water services and land use regulation, including zoning and open space preservation policies. These will have significant impact on the land conversion decisions of the farmers, the location decision of the potential migrants and the decisions of land developers. Incorporating household heterogeneity may contribute to the predicative power of the model, because it may lead to sorting behavior. But unlike the exogenous heterogeneity in travel time, conversion cost and agricultural productivity, these factors are endogenous to development. Therefore, the analysis will be technically more challenging. We can also elaborate our representation of the behavior of land developers and potential migrants in the structural model. In reality, land developers play a crucial role in determining the land use patterns. Large developer may even have the market power in influencing the provision of local public goods. Finally, the interactions between different urban land uses can significantly affect the land use pattern. For instance, when a big company moves into the region, it will create employment opportunities and attract residential development. On the other hand, rapid increase in residential land use can spur the commercial land development because of the increasing market size.
APPENDIX A

GLOSSARY AND PARAMETER SPECIFICATION FOR ESSAY ONE
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Point</td>
<td>For a function, it is a point that is mapped onto itself. For a differential equation, it is a state that does not change with time.</td>
</tr>
<tr>
<td>Nullcline</td>
<td>Curves where either $\dot{N} = 0$ or $\dot{P} = 0$</td>
</tr>
<tr>
<td></td>
<td>Basin of Attraction Given an attracting fixed point $x^<em>$, it is the set of initial conditions $x_0$ such that $x(t) \to x^</em>$ as $t \to \infty$.</td>
</tr>
<tr>
<td>Separatrix</td>
<td>It partitions the phase space into regions of different long-run behaviors. In this paper, it corresponds to the stable manifolds of the unstable fixed point.</td>
</tr>
<tr>
<td>Resilience</td>
<td>It determines the persistence of relationships within a system and is a measure of the ability of these systems to absorb changes of state variables, driving variables, and parameters, and still persist. (Holling, 1973)</td>
</tr>
<tr>
<td>Stability</td>
<td>The ability of a system to return to an equilibrium state after a temporary disturbance.</td>
</tr>
<tr>
<td>Regime Shift</td>
<td>Shift of system from one attraction domain to another.</td>
</tr>
<tr>
<td>Skiba Point</td>
<td>In a one-variable model, it is a knife-edge point which the global optimum shifts from one local optimum to another.</td>
</tr>
<tr>
<td>Skiba Line</td>
<td>In a two-variable model, it is a line across which the global optimum shifts from one local optimum to another.</td>
</tr>
</tbody>
</table>

Table A.1: Glossary.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Consumption elasticity of the numeraire good</td>
<td>0.75</td>
</tr>
<tr>
<td>$b$</td>
<td>Coefficient of production externality</td>
<td>0.0536</td>
</tr>
<tr>
<td>$r_a$</td>
<td>Agricultural land rent</td>
<td>1.2</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>Reservation utility</td>
<td>0.036</td>
</tr>
<tr>
<td>$A_1$</td>
<td>Coefficient of disutility from congestion</td>
<td>0.0016</td>
</tr>
<tr>
<td>$A_2$</td>
<td>Marginal disutility from ecological degradation</td>
<td>0.0208</td>
</tr>
<tr>
<td>$L_0$</td>
<td>Agricultural phosphorus loadings to lake</td>
<td>0.14</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Average phosphorus loadings from residential land use</td>
<td>0.75</td>
</tr>
<tr>
<td>$s$</td>
<td>Average rate of phosphorus outflow</td>
<td>0.14</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>Maximum rate of recycling</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>Steepness of the recycling curve</td>
<td>8</td>
</tr>
<tr>
<td>$P_c$</td>
<td>Phosphorus level at half the maximum rate of recycling</td>
<td>5</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>Welfare weighting for the size of local population/economy</td>
<td>0.001</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Welfare weighting for the ecological quality</td>
<td>0.003</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>Welfare weighting for policy intervention cost</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>Average impact of economic policy on individual utility</td>
<td>1</td>
</tr>
<tr>
<td>$\alpha_5$</td>
<td>Average impact of ecological policy on individual utility</td>
<td>1</td>
</tr>
</tbody>
</table>

Table A.2: Parameter specification.
BIBLIOGRAPHY


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