THE EFFECTS OF A COMPUTER MICROWORLD ON MIDDLE SCHOOLS STUDENTS' USE AND UNDERSTANDING OF INTEGERS

DISSERTATION

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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1995

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ACKNOWLEDGMENTS

I express sincere appreciation and heartfelt thanks to Dr. Patricia Brosnan for her insight and guidance in this research and throughout my doctoral program. Special thanks go to the other members of my advisory committee, Dr. Sigrid Wagner and Dr. Thomas Ralley, for their time, consideration, and suggestions. Gratitude is expressed to the personnel of the Columbus Public Schools for their never-ending support of my work.

I am grateful to my parents, Holly and Doris Smith, for the sacrifices they have endured providing for my education and for being an inspiration whenever the task seemed unbearable. To my friends, I thank you for continually reminding me—sometimes in the oddest ways—that the journey has only begun.
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CHAPTER I

PROBLEM DEFINITION

1.0 Introduction

For over a decade, the importance of implementing technology in the mathematics classroom has been underscored. From *The Agenda for Action* (National Council of Teachers of Mathematics, 1980) to *Measuring Up* (Mathematical Sciences Education Board, 1993), every major document which paints a vision of the future for mathematics education includes a description of technologically-enhanced instruction. In fact, powered by the educational reform movement, a tenet for the classroom of tomorrow purports a computer workstation for every child (see National Education Association, 1989). And while significant transitions are taking place in children's learning environments, computer software continues to evolve as well, augmenting the teaching tools that promote active construction of mathematical knowledge.
1.1 Nature of the Problem

In the past, instructional software for the K-12 mathematics classroom could fundamentally be classified according to one of four categories: computer games, "business" programs adapted to mathematics needs, Computer Assisted Instruction (CAI) tutorials, or simulation modules. Computer games have been used to teach content – for example, equations and graphing in *Green Globs* (Dugdale, 1982) – and to teach heuristics and problem-solving strategies – as in *The King's Rule* (O'Brien, 1985). Business application programs (primarily spreadsheet and charting utilities) are frequently cited as a malleable tool in assisting data-analysis activities. CAI tutorials are commonly referred to as drill-and-practice programs, serving as "electronic flashcards." Finally, computer simulations, such as *Interactive Physics* (Knowledge Revolution, 1990) and *Geometer's Sketchpad* (Klotz, 1991), model either real-world events or theoretical possibilities in an attempt to provide students with an environment for experimentation – one that encourages the formation, testing and refinement of hypotheses.
And although the preponderance of existing computer packages for mathematics education remains CAI tutorials (Kaput, 1992), the production and use of microcomputer simulations is enjoying heightened appeal. Especially in secondary classrooms, where the need for a semi-concrete bridge to highly symbolic mathematics may arise, software is being created that acts in lieu of students' physical manipulation of traditional tools for mathematics. For example, The Geometric Supposer (Schwartz & Yerushalmi, 1985) was developed for use as an "electronic compass and protractor," bisecting line segments and measuring angles with a few simple menu commands. Even the work of graphing utilities can be considered, in a sense, a microcomputer simulation, replacing paper-and-pencil plotting with a computer-generated sketch.

In younger grades, the development of non-CAI programs has been comparably slower (Kaput, 1992) – although computer "microworlds" are being invented now which simulate the use of primary, hands-on manipulatives. In Blocks World (Thompson, 1991), for example, children work with computer-generated Diene's blocks to explore place value and arithmetic operations. If asked to add 17 and 8, students
manipulate cubes on the screen while the computer simultaneously translates any onscreen activity into a symbolic representation (see Figure 1).

![Diagram of Base Ten Blocks World Microworld Screen]

**Figure 1.** *Blocks World* Microworld Screen

Of course, students could perform the same operation with hands-on manipulatives; however, individuals would be responsible for making the correct exchanges and translating their task into symbolics. Thus, a microworld can highlight the process of "natural abstraction"—converting real objects and physical actions into the language of
mathematics. In the preceding example, transforming the bundles of blocks and translating to the sum 25 is a "natural abstraction."

In addition to "natural abstraction" – abstracting from substantive items – microworlds offer the power of "hypernatural abstraction." A "hypernatural abstraction" can convey meaning from concepts that have no tangible, "real" counterpart. From a mathematics education standpoint, a child's first encounter with a "hypernatural abstract" might be the concept of integer. Until the notion of zero and negative numbers, which have no true physical referent, students can sort, count, and operate on concrete manipulatives, providing underlying meaning for the mathematics encountered. At best, integer instruction from a hands-on approach must impose arbitrary conditions or rules: "the red colored chips are for negatives" or "when you subtract a negative number, move opposite the negative direction."

With a microworld, however, a student enters an alternate reality, a land with new undeniable laws – where the hypernatural can become natural. Using concrete manipulatives, a child modeling 5 – (-2), not knowing what else to do, may simply remove two positive chips and unwittingly break "external" rules. Through proper programming, a
microworld can disallow such a move, with the child not yielding to
teacher mandates but perceiving this precept as "just the way it is."
From a learning theorist's perspective, the use of a microworld appears
meritorious with its immediate feedback and endless patience; from a
researcher's perspective, however, a problem surfaces— it is currently
unknown whether the use of a computer microworld will assist in
students' construction of integer concepts.

1.2 Problem Statement

Conceptual development, as defined by the networking of
representations, may be expedited via the hybrid of computer
simulations. Through the marriage of manipulatives and cybernetics,
students are afforded the opportunity to use technology as a dynamic
tool for building a personal, ontogenic framework of concepts. The
objective of this study is to investigate the effects of a computer
microworld, entitled *The Cy-Bee Chips*, on middle school students' use
and understanding of integers. Related to this objective are several
research questions:
1. Will students who are exposed to The Cy-Bee Chips be more successful in understanding and using integers than students who are exposed to instruction with concrete manipulatives?

2. Will students exposed to both The Cy-Bee Chips and concrete manipulatives be more successful in understanding and using integers than students who are exposed to instruction that is mutually exclusive?

3. Which of the three teaching methods will be the most effective for students who have had previous instruction on integers?

4. Which of the three teaching methods will be the most effective for students who have had no previous instruction on integers?

1.3 Definition of Terms

The following list is a compilation of frequently-used terms in this study which require local definitions:

*achievement:* a quantitative measure of knowledge gained (for example, scores on pre- and post-tests).

*dynamic medium:* a system that is capable of changing over time (for example, a videotape which, when played, demonstrates the joining of two sets).
**static medium:** a system that is incapable of changing over time (for example, an overhead transparency which displays the union of two sets).

**interactive medium:** a system that when acted upon responds with new information (for example, a calculator which produces the sum of two numbers after the addends are entered and the equal sign is pressed).

**inert medium:** a system that when acted upon responds only with a display of the user's input (for example, paper-and-pencil which can record the data and solution for an addition problem).

**computer microworld:** a software environment that is a dynamic and interactive medium.

**cybernetic manipulative:** an object that is controlled through communication between person and computer.

**constraint-support structure:** the network of programmed rules that govern the user within the microworld; the hyphenated term "constraint-support" is coined to reflect the idea that disallowing certain actions may act as a support for other actions.

**user-directed agents:** the network of automatic mechanisms that perform actions for the user.
natural abstraction: the translation of real objects and physical actions into mathematical symbols.

hypernatural abstraction: the translation of physically-intangible concepts and constructs into mathematical symbols.

previous instruction: formal instruction in the schools, regarding either adding or subtracting integers

1.4 Hypotheses

The following primary hypotheses will be tested:

H1. There will be no significant difference in post-test scores between the three treatment groups (concrete manipulatives, computer microworld, and both concrete manipulatives and computer microworld).

H2. There will be no significant difference in post-test scores between students with previous instruction on integers and students without previous instruction on integers.

H3. There will be no significant interaction between treatment levels and previous instruction levels.

Then, the following secondary hypotheses will be tested:
$H4$. There will be no significant difference in post-test scores between students with previous instruction across the three treatment groups.

$H5$. There will be no significant difference in post-test scores between students without previous instruction across the three treatment groups.

1.5 Significance of the Study

For many students, it appears positive (+) and negative (-) signs have only operational meanings (to combine or to "take away"). Furthermore, without a clear understanding about the nature of integers, students experience interference when attempting to compute with signed numbers. Teaching via "rules" – which are seemingly contradictory (for example, "negative + negative = negative"; "negative x negative = positive") – muddies conceptualization and compounds student frustration. Thus, being an underpinning to algebra, teaching methods and learning difficulties regarding the construction of integer knowledge must become a focus of research in mathematics education (Kieran & Chalouh, 1993).
To teach effectively, constructivists would argue for inquiry-based, experiential learning (Wood, Cobb, Yackel, & Dillon, 1993), allowing students to create their own knowledge in an exploratory setting. With respect to integers, this might mean hands-on teaching with a number line or using colored counters for the "paired-zero" approach. Both models of instruction, however, have shortcomings. The number line [directional] method relies upon convoluted movement edicts, which may be as confusing as "simply teaching the rules." On the other hand, the colored counter [attribute] method can be easily derailed without constant teacher-imposed screening. To overcome these externally-oriented issues, the use of a computer microworld seems appropriate.

Computer microworlds provide user-directed agents and constraint-support (CS) structures that empower students to independently create and develop constructs (Kaput, 1992). User-directed agents, for example, can be programmed to link representations so that, when one notational system is altered, parallel translations occur in another notational system. By monitoring actions on objects in the microworld, constraint-support structures disallow student errors when operating on integers. Hence, computer microworlds may cultivate pseudo-
experiential learning in an environment that sanctions understanding from a dynamic, interactive semi-concrete vantage.

1.6 Limitations

As with all research, limitations occur which may be related to the content being investigated or the design of the study. With respect to content, this project introduces positive and negative signs as an attribute of quantity – a state of being. The software, therefore, delimits the conceptualization of integers in terms of an existential characteristic, purposefully omitting the commonly-used "vector" interpretation (magnitude and direction). Furthermore, only addition and subtraction are considered in this study; whether an effective multiplication/division microworld can be built which extends the essence of an existential model for integers remains in question.

As for the design of the study, the researcher created the software and the instructional worksessions to be used in this study. With the help of classroom teachers, the researcher wrote the worksheets and the pre- and post-tests. Unfortunately, because of this inextricable involvement, researcher bias may be of concern. It is imperative,
therefore, that control measures, such as inter-rater reliability checks for the scoring of tests, be employed. Furthermore, to promote as naturalistic a study as possible, the classroom teachers will provide the instruction for the duration of the study.

Finally, because of the technology required to conduct this research, only classes and schools that have access to Macintosh computer labs will be considered. This may significantly hinder the generalizability of results.
CHAPTER II

REVIEW OF LITERATURE

2.0 Introduction

In reviewing the use of microworlds during mathematics instruction, the following resources were used: the ERIC CD-ROM, the Dissertation Abstracts International CD-ROM, the Education Index, the Journal for Research in Mathematics Education, the Research Agenda for Mathematics Education Series (NCTM), the Research Ideas for the Classroom Series (NCTM), and the Handbook of Research on Teaching and Learning Mathematics (NCTM). Initially, for computer-based searches, the descriptors selected were mathematics, mathematics education, computer microworlds, computer assisted instruction, integers, signed numbers, negative numbers, and research. Because few articles subsumed using computer microworlds in mathematics instruction, the search arena was broadened to include manipulatives, multiple representations, constructivism, and experiential learning. Similar key words were used to hand-search the Education Index (1980-1993) and
the *Journal for Research in Mathematics Education* (July issues, 1975-1993), as well as the aforementioned research documents. After this comprehensive scan of research from 1975 to 1993, findings related to the teaching and learning of mathematics via multiple representations were categorized according to (a) *effects of manipulatives in mathematics instruction*, (b) *effects of software in mathematics instruction*, and (c) *necessary conditions for conducting multi-modal research in mathematics*.

### 2.1 Effects of Manipulatives in Mathematics Instruction

Using manipulative materials to teach mathematics is certainly not a novel idea. Since the nineteenth century, educators such as Pestalozzi advocated their use, and concrete representations were an integral part of the activity curricula of the 1930s (Hartshorn & Boren, 1990). Furthermore, following the emergence of Cuisenaire rods during the mid-1960s, another period of emphasis began on using objects in elementary mathematics classrooms. In fact, today, most of the nation's primary schools report availability of manipulatives, and several states (including Ohio) encourage their use (Pfeiffer, 1992). In the Model Curriculum (1990) supplied by the Department of Education
for the State of Ohio, for example, manipulatives are frequently used in scenarios describing appropriate and effective mathematics instruction.

With regard to elementary school mathematics, Suydam and Higgins (1977), released a comprehensive review of studies on activity-based learning in mathematics instruction, reporting that using concrete materials produced greater achievement than not using them. In particular, the use of counters and base-ten blocks aid the learning of the four arithmetic operations (addition, subtraction, multiplication, and division), as well as increasing the understanding of place value and number sense (Kennedy, 1986; Morrow, 1990; Rawl & O'Tuel, 1983). Moreover, students at the elementary level develop better proportional reasoning skills when instructed with concrete materials (Tourniere, 1986; Hiebert, 1991). Thus, at all grade and age levels in elementary school, the use of "hands-on" materials appears to be effective with children.

Although the "blanket" hypothesis that the use of manipulatives increases mathematical understanding is alluring, research supporting increased student achievement due to concrete representations in
secondary mathematics topics has been scarce. First of all, in the past, secondary mathematics textbooks (in particular, middle grades textbooks) tended to be a recycling of basic operations instruction with minimal new content (Flanders, 1987). Therefore, few manipulatives – with the exception of geometric solids – were designed to embody abstract concepts in secondary mathematics (Hartshorn & Boren, 1990).

Secondly, middle school students (and middle school teachers) may not be inclined to use manipulatives because of their perceived "primary mathematics" stature. In fact, throughout the intermediate grades (grades 3 through 6), students in mathematics classes are typically weaned from enactive and iconic representations – including "counting on fingers" – in favor of symbolic notation (National Council of Teachers of Mathematics, 1989). Abstraction is stressed to the point that students claim "juggling numbers, variables, and operations is what algebra is all about" (Pfeiffer, 1992). Hence, middle school students often react negatively when asked to use manipulatives during math class, feeling that to do so would be demeaning as well as unnecessary (Baroody, 1987; Howden, 1986; Morrow, 1990).
With respect to algebraic concepts in secondary mathematics, Antosz (1989) reports that seventh grade students who were exposed to manipulatives during instruction on solving linear equations scored significantly higher than students who were taught using standard symbolic notation on a test of linear algebra skills. On the other hand, Wohlgehagen (1989) found no evidence to tout the effectiveness of a concrete approach in teaching algebraic concepts to ninth-and tenth-grade students. Analysis of his data revealed no statistical difference in the mean scores of students instructed with or without manipulatives when the test was administered immediately after three weeks of instruction. Nor was there any statistical difference in the mean scores when the test was administered two months after instruction.

Thus, although recent meta-analyses (Lenoir, 1989; Sowell, 1989) indicate that mathematics achievement – in grades K-16 – is increased through the long-term use of concrete instructional materials, the disproportionate number of studies from the "elementary school concepts" raises questions regarding the accuracy of such claims for secondary topics. Scant research on the effects of manipulatives on the
teaching and learning of integer operations is especially problematic (Kieran & Chalouh, 1993). In addition, even when using concrete manipulatives, the threshold age of integer concept acquisition may be questionable. Some claim to have successfully introduced negative numbers as early as the third grade, using the "zero-pairs" approach with two-color counters (Duncan & Saunders, 1980; Thompson & Dreyfus, 1988). Others have gleaned poor results with seventh-grade students, who received integer instruction utilizing a thermometer model (Human & Murray, 1987; Kohn, 1978). Hence, when combined with disparate accounts concerning the usefulness of manipulatives, teaching younger children integer concepts from a multi-modal perspective may become a cornerstone of constructive mathematics education.

2.2 Effects of Software in Mathematics Instruction

Of all the influences that are currently affecting mathematics education, technology in the classroom may stand out as being the greatest catalyst for reform. As software becomes more sophisticated, researchers are examining the effects of computer exploration.
environments on mathematics conceptualization. The phrase "computer as a tool" is now synonymous with software that can graph equations, manipulate symbols, or create geometric figures. In fact, at the high school level, studies have shown increases in students' mathematical understanding after using computer graphers (Heid & Kunkle, 1988) and software such as the Geometric Supposers (Bobango, 1987; Yerushalmy, Chazan, & Gordon, 1988).

In attempting to answer – "How well will computer 'toolkits' assist elementary school students in constructing a rich understanding of mathematics?" – the response has been less resounding. First of all, most microworlds for elementary-aged students either assist in building primary arithmetic skills (Ippel, 1992; Thompson, 1991) or teach transformational geometry in Logo-derived environments (diSessa, 1982; Edwards, 1992). In both cases, research reveals that students are fully capable of performing in the microworld and of solving problems within the context of the immediate study situation. However, evidence of transfer of knowledge – internalization and generalization – has been equivocal.

Thompson (1992), for example, had ten fourth-graders use the Blocks
World program to model the base-10 number system. The students were able to navigate the software and, for the most part, mimicked whole-number addition and subtraction methods which were already automatized. Then, the microworld was used to introduce decimal notation and conventions. Unfortunately, at the end of the study, when asked to explain decimal notation, the children "were largely in a state of disequilibrium" (p. 123) and unable to construct "decimal numeration as a numerical system" (p. 145).

In addition to Blocks World, Thompson helped to create a microworld entitled Integers. Integers was designed to present integers as unary translation operators acting on position on a number line (Dreyfus & Thompson, 1985). By entering a movement command, a "turtle" icon is displaced according to the value of the integer (see Figure 2). Thompson and Dreyfus (1988) reported that, after eleven 40-minute sessions using Integers, two sixth-grade students were able to construct mental operations for negating arbitrary integers and determining the sign and magnitude of a sum.
While this evidence is encouraging in that it supports the use of a microworld to teach about integers, the number line model may be ineffectual for students who will most likely interpret addition or subtraction symbols as binary operators. In fact, using a thermometer and an "integers-as-transformations" approach, Human and Murray (1987) found that students erroneously responded to written integer arithmetic exercises on the basis of whole numbers. Students declared, for example, "-6 – 4 is 2 and you add the negative sign to the answer because of the minus in front of the 6" (p. 441). Thus, the representational system of the number line does not "inherently
embody" the written form of integer arithmetic which is an extension of operating with sets. To this end, inventing a microworld which acts as a conceptual amplifier for "integers-as-sets" might prove to be more beneficial.

2.3 Necessary Conditions for Conducting Multi-Modal Research in Mathematics

Whenever a new representational system is introduced, time and care must be taken to instruct students on the use of the system, translating between representational domains and transforming within representational domains. Without an experiential background, as suggested by Bruner (1966), facilitation of learning may be impeded by manipulative models. As Stephenson (1978) found, children who are not formally taught how to use interactive models concentrate on the manipulatives themselves rather than the conceptual connections to mathematics. Furthermore, selection of manipulatives must guarantee that the objects embody the mathematical construct. Fennema (1975), for example, had students use Cuisenaire rods (a length representation) to model discrete quantities. As a result, subjects reported difficulty in transferring the extension of continuous
rods to whole number operations. Hence, mathematics achievement or understanding may be hampered through the interference of inappropriate or underdeveloped manipulative use.

2.4 Summary

Summarizing the implications from this review of literature, the use of multiple representations (including the use of manipulatives and technology) appears to be advantageous to the learner. Unfortunately, multi-modal research concerning elementary students' acquisition of "secondary" knowledge has been scant. Even fewer studies have been conducted using microworlds as a vehicle for instruction. With the available documentation and the resultant shortcomings in mind, this research is conducted under the assumption that a well-designed and properly-used microworld can enhance conceptual and procedural knowledge in mathematics.
CHAPTER III
THEORETICAL RATIONALE

3.0 Introduction

While developing the software for the study and designing the research, learning theories were of critical importance. Not only did the study need to be firmly grounded in a theoretical perspective to help clarify results, the software had to accurately reflect a psychological position as well. Two components to educational psychology became the underlying foundation upon which the study was built: cognitive constructivism and representations.

3.1 Cognitive Constructivism and Representations

As a psychological view, constructivism has received much attention within the past few decades, including interpretations for the mathematics education community by such prominent authorities as Confrey (1990), Steffe (1983), and von Glasersfeld (1984, 1987). It was even the subject of a special monograph, Constructivist Views on the Teaching and Learning of Mathematics (Davis, Maher, and Noddings,
1990), under the *Journal for Research in Mathematics Education* banner. As this position continues to gain momentum, manifold beliefs regarding how individuals actually learn in a meaningful way have led to the fissuring of constructivism into branches, including theories deemed cognitive constructivism, pedagogical constructivism, social constructivism, and radical constructivism.

Although there are conceptual differences between the branches, in general, the constructivist position suggests that all knowledge is constructed and learners create knowledge for themselves by acting on the world (Piaget, 1980). The following five tenets, as described by Wood (1993), have been identified as the basis for the constructivist perspective:

- Mathematical knowledge is constructed by the individual.
- Individuals create new mathematical knowledge by reflecting on their physical and mental actions.
- Constructs undergo a process of continual revision.
- Individuals create their own personal interpretations of mathematics.
- Opportunities for learning occur during social interaction as individuals resolve conflicting points of view.
In particular, cognitive constructivism focuses on each learner building a mathematical way of knowing by reorganizing his/her experiences in an attempt to resolve problems which block a goal. It calls into question the transmission-of-knowledge model that is found in many "traditional" classrooms. In opposition to the notion of "being taught," cognitive constructivism asserts that meaningful learning occurs only when individuals reflect on their activity and reorganize their experiential framework.

Thus, if students are to learn about integers, individuals must tackle problem situations, which can include the construction of operational algorithms (Cobb, Yackel, Wood, 1993) and actively interpret their actions and experiences with positive and negative numbers. The manner in which students experience mathematics is also important. They must be given the opportunity to independently explore how the integer number system works. Rather than inundating students with rules and symbolic examples, cognitive constructivists would argue that students need to investigate the world of integers by manipulating representations of positive and negative numbers, running into conflicts, and resolving those conflicts in search of winning
the "rational game" (von Glasersfeld, 1990).

Mental representations, therefore, are of prime importance to cognitive constructivism. They can be "replayed, shelved, or discarded according to their usefulness and applicability in experiential contexts" (von Glasersfeld, 1987, p. 219). In a denotative sense, mental representations suggests what cognitive constructivists have heralded as a cornerstone to their position: To re-present an idea, event, or action, is to form an interpreted construct within the mind and to let that construct resurface in a personally-adapted mold. At some point, however, the mental representation must have held some form of physical presence. Moreover, some theorists would contend that the psychological term of representation is dichotomous in nature — is it an external embodiment of an internal conceptualization or vice versa?

Regardless of this contrived argument, translations among and transformations within representations convey the very essence of what is meant by "actively constructing knowledge." To that end, finding appropriate and powerful representations in mathematics can be a catalyst for such learning (Lesh, Post, and Behr, 1987). According to Piaget (1952), manipulative materials are significant aids in the
maturation of the learner, especially during the move from concrete operations to formal operations. Recognizing that students' mental images and abstract ideas are based on experiences, teachers who encourage the manipulation of a variety of objects enhance mathematics conceptualization.

Learners who are at the concrete operational stage, for example, may benefit from the manipulation of cybernetic two-color chips, representing the process of adding integers. After using the manipulatives, when asked to perform with mathematical symbols, students can recreate the understood microworld process via mental images. Those mental representations are then translated into appropriate abstract concepts and functions.

Furthermore, Bruner's Tri-Modal Theory (1966) and Dienes' Multiple Embodiments Theory (1973) claim that translations between and transformations within modes of representations (enactive, iconic, and symbolic) make mathematics meaningful for children. If a student can "touch" a negative number, as well as see its pictorial representation and symbolization, the notion of an integer has been enriched. By providing multiple sources for encoding information,
different learning styles can be accommodated. Thus, students who are exposed to multiple embodiments are capable of forming a stronger ontogenic framework for mathematical constructs.

3.2 Description of the Software

To reflect the theoretical foundation adopted for the study, the researcher created a computer microworld which promoted active construction of knowledge and provided multiple representations for the integers. To build a computerized "mathematical world" that embodied in some tangible sense the abstract nature of integers, the researcher linked multiple representations using a constraint-support structure and user-directed agents. Figure 3 illustrates how using a computer microworld, such as The Cy-Bee Chips, can influence the user's conceptualization of integers. The learner enters the microworld with some degree of knowledge regarding what an integer is, what addition and subtraction mean, and how to perform arithmetic operations. Once the learner interacts with the computer, he/she builds an initial, working model of "integers" by reflecting on the microworld's responses to user actions. As the microworld provides feedback (or non-
feedback in the case of computer programmed constraints), the user must interpret the screen output and can revise his/her integer-related constructs.

**Smith's Model for Microworld Interaction**

**Global Factors of Instruction:**
- Interpersonal Communication
- Hardcopy Materials

![Diagram of Smith's Model for Microworld Interaction](image)

**Figure 3.** Smith's Model for Microworld Interaction
According to Edwards (1991), it is important to note that microworld feedback comes about as a natural consequence of interacting with the computer, often requiring the learner to reconcile or re-link an understanding of the symbolic with the visual representation. Continuing the interactive cycle, therefore, should result in an expert refinement of integer constructs. Ideally, the intermediate contemplation of microworld behavior should become routine, allowing users to concentrate on directly linking microworld feedback to integer schemata.

Software Specifications

Using HyperCard (version 2.1 for the Apple Macintosh), the researcher developed and created a computer microworld entitled The Cy-Bee Chips. Consisting of two modules, The Cy-Bee Chips provides students an opportunity to use cybernetic manipulatives to model adding and subtracting integers. Students can place two-color counters on a screen workmat, manipulate the counter configuration, add or remove "zero-pairs", suggest solutions to problems posed, and receive feedback on correctness. Figure 4 shows an example of the first
interactive screen for the addition module:

![Interactive Screen](image)

**Figure 4 Cy-Bee Chips Addition Module Screen 1**

In Figure 4, the user has dragged three positive chips and two negative chips onto the addend-partitioned workmat. Whenever a new chip is dragged onto the mat, the integer indicator near the bottom of the workmat is appropriately incremented. At this point, the user can:

- Place more chips on the workmat;
- Use the "Snap" Agent button to combine the addends and snap the chips together into an "easy-to-read" configuration;
· Use the Problem/Solution button to enter his/her proposed answer to the problem currently represented on the workmat;

· Use the Reset button to clear the mat and begin a new problem; or

· Use the Quit button to exit the microworld program.

If the user clicks on the "Snap" Agent button, the screen will transform into Figure 5. The User Action Script box will display "Combining +3 and -2" and the "Snap" Agent button is replaced with a Remove Zeroes button. If the user highlights "zero-pairs" of chips – by clicking on positive and negative chips – the Remove Zeroes button can be used to return the chips to their respective containers.
The subtraction module works in a similar manner. If, for example, the problem to be addressed is \((+3) - (-2)\), the user might assemble the screen shown in Figure 6. Because the workmat contains no negative chips to remove, the Add Zeros button may be pressed and "zero-pairs" will be pulled onto the mat from the chip containers.

![Cy-Bee Chips Subtraction Module Screen 1](image)

**Figure 6 Cy-Bee Chips Subtraction Module Screen 1**

Once enough "zero-pairs" have been added to the workmat, the user can highlight the (-2) chips and drag them away from the (+3) chips (see Figure 7).
Although *The Cy-Bee Chips* emulates the use of concrete manipulatives, the software is designed to go beyond mimicking what could easily be done with hands-on materials. Once a problem has been locked in place by snapping the chips together, the constraint-support (CS) structure does not allow the user to change the value on the workmat. The user can only add or remove "zero-pairs", for example, if a single chip is highlighted for removal, the Remove Zeroes button is disabled and nothing will happen if it is pressed. Furthermore, user-directed agents immediately translate all user actions into a verbal script, generating an artifact of the temporal
manipulations. This allows the user to retrace procedures and provides a secondary representation for interpreting the meaning behind a symbolic operation. By employing the principles of constraint-support structures and user-directed agents, the infrastructure of The Cy-Bee Chips provides new insight into cognitive constructivism.

3.3 Applied Theory

During the early 1960s, Cuisenaire, Dienes, Montessori, and Piaget led cognitive constructivism to mainstream educational psychology. Arguing that instruction must be based on activities that use concrete materials, each became a proponent of the "mathematics laboratory" approach to education. Among the "mathematics laboratory" theorists, however, Dienes was the most specific regarding recommendations for teaching and learning mathematics (Lesh, Post, & Behr, 1985). As described in his book Building Up Mathematics (1960), Dienes' perspective is based on the following four principles:

1. The Constructive Principle

Dienes asserted that "hands-on" mathematical labs provide a learning environment rich with opportunities to promote active construction of knowledge.
Mathematics, however, is not "inherent" in cubes, blocks, sticks, or chips. In order for students to begin building knowledge, the representing materials must be organized and patterned after mathematical concepts and procedures. Lesh, Post, and Behr (1985) write: "When individual activities cease to be treated as isolated actions and start to be treated as part of a systematic pattern of activities, the student begins to shift from playing with blocks to playing with mathematical structures" (p. 649). Thus, according to Dienes, students create mathematical knowledge through the abstraction of "relational/operational/organizational systems" imposed on sets of objects.

In The Cy-Bee Chips, for example, if students are assembling the number "positive eight" on the workmat, they may do so in an infinite number of ways. Students can pull up eight gray (positive) chips, or nine grays and one red (negative), or ten grays and two reds, etc. Because the microworld has been organized and programmed to recognize pairs of opposite colors as being zero, all of these collections represent the integer (+8). The mathematics comes not from the fact that two-color counters are used to represent positive and negative
numbers – rather, the nature of integers is revealed when students abstract the relationship imposed on the chips by the "zero-pairs" constraint-support structure.

2. The Multiple Embodiment Principle

Dienes claimed that mathematical abstractions occur when students recognize structural similarities shared by several related models.

Just as mathematics can not be abstracted by isolated actions, construction of knowledge may not occur if students are using single isolated models to "embody" mathematical patterns. Multiple representations are a necessary component to mathematics teaching and learning, allowing the same concept or the same mathematical structure to be experienced in a variety of ways (Bednarz & Belanger, 1983). When presented to the learner, it is hoped that the student will be able to grasp the common properties of the diverse embodiments and ultimately abstract the underpinning structure.

Lesh, Post, and Behr (1987) have identified five distinct types of representational systems: (1) "real scripts" - knowledge organized around real world events, (2) "manipulative models," (3) "static pictures," (4) "spoken language," and (5) "written symbols." Of the five, "manipulative models" and "written symbols" are clearly evident in
The Cy-Bee Chips microworld. Controlled onscreen by the computer's mouse interface, the two-color counters can be considered a "manipulative model"; while the integer indicators (located in the center of the workmat) and the mathematical sentence (shown after pressing the Problem/Solution button) are both examples of written symbols.

An argument can be made that the microworld invokes "spoken language" as well. The User Action Script box automatically translates into written form all user activities, enabling the individual to examine his/her entire solution path. The phrases shown in the User Action Script box were purposefully written in terms of natural, spoken language. Although the words are not digitally sounded by the computer, reading the script does provide some sense of an "inner voice." Certainly the representation offered by the User Action Script box is different from the typical symbolic solution path. It should also be noted that digital voicing technology is available, at the expense of computer memory. To produce software that is capable of being run on "modest" computers, such as those in the schools, it was the decision by the researcher to sacrifice the voicing option.
3. *The Dynamic Principle*

*Dienes believed learners should be exposed to models capable of demonstrating transformations within the representation.*

Mathematics can not be constructed from "static patterns." In fact, according to Dienes, students must not only investigate transformations within a single model, they must also investigate corresponding transformations within other models. A primary goal for teachers, therefore, is to help students make sense of how relationships in one representation compare to relationships in another representation. Moving from one mode of representation to another (for example, from a collection of chips to an integer numeral) is often referred to as a psychological "translation." Both transformations and translations are critical in facilitating the abstraction process.

Students using *The Cy-Bee Chips* are afforded the opportunity of experiencing instantaneous translations between representations. Suppose a student was trying to solve the problem \((+4) - (-2)\). After pulling up four red (positive) chips, the student may recognize that there are no negatives to "take away." In an attempt to place negatives on the workmat without changing the initial value of the problem, the
student then clicks on the Add Zeroes button. For each click of the Add Zeroes button, several things happen at once: a "zero-pair" of chips is dragged onto the workmat, the User Action Script box records the action in written form, and the mathematical sentence invoked by the Problem/Solution button reads "+(4) – (0) = ." As described by this episode, the manipulative model changed – although the microworld retains the value (+4), the chip configuration does in fact look different. While the cybernetic manipulatives are being modified, user-directed agents translate the action and apply homomorphic transformations to the language and symbolic modes of representation.

4. The Perceptual Variability Principle

*Dienes thought students needed to work with mathematical models under varied conditions, to discover the "pure" properties of the construct.*

Regardless of the representing form, every model clarifies some aspects of the represented mathematical structure and obscures others. Frequently models confound the abstraction process because of "perceptual distractors." Perceptual distractors are those characteristics present in a representation, but are irrelevant in terms of mathematical content. Therefore, it is important to orchestrate experiences with multiple embodiments in a manner designed to
eradicate perceptual distractors.

When working with two-color counters on addition problems, for example, students may feel as if they must always place the chips representing the first addend on the left and the chips for the second addend on the right. In this case, virtual positioning of the counters is a perceptual distractor. *The Cy-Bee Chips* attempts to eliminate this misconception by randomly organizing addends in rows once the Snap Agent button is pressed. Sometimes the positives are placed in the top row; sometimes the negatives are placed in the top row. The positioning of the counters is irrelevant and the random placement is made without regard to the first or second term of the symbolic problem.

Furthermore, the software exhibits no preferences nor any control over methods of solution. *The Cy-Bee Chips* was founded on the tenet that the user should exercise high control over the microworld environment and that user-directed agents should act as a support in an open-ended investigation. For example, suppose that students are trying to simplify the problem (+6) + (-4) by removing "zero-pairs" of chips. In the computer microworld, they can highlight and remove each
pair one at a time, or they may highlight and remove two or more pairs all at once. Taken independently, the individual numbers of reds and grays being removed is irrelevant, but the value of the collection \( 0 \) is critical. By allowing the student the opportunity to experience a variety of solution paths, the software muffles perceptual distractors and emphasizes the mathematical structure of zero.

As the arguments have shown, all of Dienes' learning principles are found in the representational system of computer microworlds. Along with the "Age of Technology" and innovative learning tools though, come new ideas regarding how teachers can help students construct mathematical knowledge. Note that Dienes' assertions all relate to the objects used in the modeling of mathematical concepts; they all describe ways to select objects in a representing world that will facilitate abstraction. Nothing is mentioned, however, about imposing structure upon actions when working in the representing world.

Even if Dienes' four principles are present, certainly it is not guaranteed that children will use the objects in a way that parallels "true" mathematical behavior. If children approach the addition and subtraction of integers using concrete two-color counters, for example,
there are many opportunities for "experiential" errors to occur – everything from reversing color meanings to recklessly displacing chips. The teacher, in these instances, is usually the sole source for guidance and when mishaps transpire, he/she emerges as the correction officer after the erratum has been practiced. On the other hand, when using The Cy-Bee Chips, users are disallowed making errors that change the value of their problems.

For example, suppose students are adding (+3) + (-2). Once the problem is locked into place on the workmat, students may want to simplify the screen by "removing zeroes." To do this, they must press the Remove Zeroes button along the bottom control panel. However, the button will do nothing unless pairs of opposite colors are highlighted – in other words, students must alert the software that zeroes are in fact on the workmat. If users establish a one-to-one correspondence between highlighted red and gray chips, and then press the Remove Zeroes button, the computer will drag those chips off the workmat and return them to their original containers. This proactive prohibition of wrongdoing conveys the underlying message that, in the realm of integers, certain actions are simply impossible (one can
"neither create nor destroy matter"). If students attempt to "rip the fabric" of what has been woven as integers, the computer microworld sends immediate feedback – in fact, it is *instantaneous feedback* – to redirect their actions.

Immediacy of feedback has long been an ideal basic to good instruction. Computer microworlds are able to employ this ideal in a way that most instructional methods and tools, even computer tutorials, are incapable of matching in speed or delivery. Rather than stamping an *answer* as "correct" or "incorrect", the *Cy-Bee Chips* analyzes the appropriateness of students' *actions*. Furthermore, it provides instantaneous feedback in an inconspicuous manner, forcing students to interpret the computers' response (or non-response). In turn, students then may reflect on what it means to operate with positive and negative numbers. Thus, via the instantaneous feedback supplied by the constraint-support structure, the *Cy-Bee Chips* organizes and maintains the "zero-pairs" relational system in an effort to help children understand the nature of integers.

The researcher contends, therefore, that computer microworlds enkindle the emergence of a fifth learning principle:
5. The Action Feedback Principle

*Students should explore mathematics using representational systems capable of providing instantaneous feedback, feedback that supports actions corresponding to "true" mathematical behavior and blocks actions inconsistent with mathematics.*

In conclusion, as an interactive, cybernetic tool, *The Cy-Bee Chips* strives to maintain all of the eminent features established by hands-on materials, while functioning in a hypernatural world. The software was designed to bridge the gap between concrete manipulatives and the abstract nature of integers. As a tool for learning, its intent is to provide students with a powerful representational system for integers that encourages the active construction of knowledge.
CHAPTER IV
METHODOLOGY

4.0 Introduction

The purpose of this research was to determine the effects of a computer microworld, The Cy-Bee Chips, on middle school students' use and understanding of integers. Prior to the study, the software and worksession tasks were field tested with four students from the fourth grade. After debugging the software, the worksessions were revised and refined to highlight the salient aspects of operating with integers. This chapter describes the experimental methodology that will be employed to investigate the impact of using the latest version of The Cy-Bee Chips. Sections contained in the chapter include selection of the sample, design of the study, instrumentation, procedures, and statistical analyses.
4.1 Selecting the Sample

The research was conducted during the 1994-1995 school year in three middle schools of an urban school district in central Ohio. The schools were selected because of their proximity to The Ohio State University, the availability of Macintosh computer labs for classroom use, and the willingness of three teachers (referred to as Teacher A, Teacher B, and Teacher C) to participate in the study.

With the target population being all students learning about integers, six classes were required for the research: three sixth grade classes and three eighth grade classes. According to The Ohio Model for Mathematics Instruction (Ohio Department of Education, 1990), the seventh grade has been recommended as the introductory year for integer operations. Prior to participating in the study, the students were polled to determine whether or not they had received any formal instruction on operating with integers. Students in the sixth grade subsample indicated (according to their recollection) that they indeed had not received any formal instruction on operating with integers. On the other hand, eighth grade students had received instruction on integers prior to Winter Break and were using this study's
workshops as a review for the final exam at the end of the school year. Thus, participants provided data from a pre-formal instruction and a post-formal instruction standpoint.

Each class was randomly assigned to one of three treatments: the microworld treatment, the concrete manipulatives treatment, or the combination (microworld and concrete manipulatives) treatment. An illustration of the class assignments is given in Figure 8.

<table>
<thead>
<tr>
<th></th>
<th>Micro</th>
<th>Concrete</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>6th Grade</td>
<td>n = 20</td>
<td>n = 21</td>
<td>n = 23</td>
</tr>
<tr>
<td>8th Grade</td>
<td>n = 22</td>
<td>n = 24</td>
<td>n = 18</td>
</tr>
</tbody>
</table>

Figure 8 Treatment Groups Configuration

The sample of 128 subjects was retained throughout the study, with no loss of participants, due to the short treatment period (seven days).

Note that the nature of the research, which requires schools to have a Macintosh computer lab, necessitates a sampling of convenience. Thus, in order to procure the accessible population, a letter was sent to the principals of the nine middle schools in the Columbus area which
have Macintosh computer labs (see Appendix A). Although the use of voluntary, intact classrooms can challenge generalizability of results, efforts were made to strengthen the sampling plan. Teacher C, for example, taught all three treatment groups for the eighth grade; these participants were the students from his first, fifth, and sixth period math classes. Furthermore, the study’s worksessions were implemented at varied times in the school year, when the course of study deemed it appropriate.

It was important to make the students feel the teaching being done was as natural as possible. The teachers were critical components to this end and, thus, need to be described.

Teacher A

Teacher A taught the concrete manipulatives treatment group and the microworld treatment group for the sixth grade subsample. She has been an elementary-certified teacher for 16 years (9 years in the current district). She has experienced a wide range of classrooms (Grades 1, 2, 4, and 6) and was trained to use Mathematics Their Way (Baratta-Lorton, 1976). For this study, Teacher A used her own sixth grade classroom for the concrete manipulatives treatment and "guest-
taught" in another sixth grade teacher's classroom – a teacher with whom she has team-taught on several occasions – for the microworld treatment.

Teacher A openly acknowledges her belief that manipulatives are important tools to use in mathematics education. At the beginning of the school year, for example, she used Base-Ten Blocks to review concepts of whole-numbers and to introduce the notion of decimal. Just prior to implementing the study treatments, Teacher A had been using fraction bar manipulatives to teach about adding and subtracting rational numbers. The researcher observed Teacher A for three days during the treatment period and taped the remaining two (non-testing) days. It was noted that Teacher A was able to follow the provided worksession scripts closely (see Appendix C), with little variation of extraneous factors between the two treatment groups.

Teacher B

Teacher B taught the combination treatment group for the sixth grade subsample. For the past year, she was appointed the position of mathematics resource teacher for her building, which means she provides support for teachers by ordering materials, planning and
conducting mathematics education workshops, and guest-/team-
teaching in classrooms. She has taught for five years in this district
(two years at the current school). Prior to becoming a resource teacher,
she received a teaching award at her school, an award based on student
nomination. Teacher B is well-known and admired by the entire school
community.

For this study, Teacher B guest-taught in a sixth grade classroom
which had received little exposure to manipulatives and no experience
with computers. This class was selected because of the willingness of
the "home" teacher to allow her students to participate and the
scheduled availability of the school's computer lab. The researcher
observed Teacher B for the duration of the seven-day program, and it
can be noted that her conduct was consistent with the other
participating teachers.

Teacher C

Teacher C taught all three treatment groups of eighth grade students.
He was a first year teacher, having recently received his Master's
Degree in Mathematics Education. His classroom contained 16
Macintosh computers that he used frequently for teaching and
remediating skills, practicing operations with fractions, decimals, and percents. Teacher C's computer program library was limited to "drill software." One day a week, he would have the class use the computers to "brush up on their computation." Teacher C also invited students to use the computers whenever they had free time, during math class, study periods, or before school.

With Teacher C being a first-year teacher, students were used to a variety of individuals coming in to observe their teacher. The researcher, therefore, was able to take field notes during three days of the treatment program. From the field observations, it was evident that Teacher C felt comfortable with all aspects of the treatment methods. Furthermore, outside the realm of issues specifically related to particular treatments, Teacher C used the same questions, examples and explanations for each class.

4.2 Design of the Study

By using a non-equivalent control group design (see Figure 9), the majority of threats to internal and external validity were checked. Regarding internal validity, the greatest threat is posed by the
interaction of selection and maturity, which was controlled for by the short treatment period (seven days). Unfortunately, regarding external validity, the target population of "all students learning about integers" could not readily encompassed, by virtue of the characteristic differences between suburban, urban, and rural school systems. In order to support "global" generalizations, replications of the study are recommended in other district settings.

![Schema of Quasi-Experimental Design]

Figure 9 Schemata of Quasi-Experimental Design

4.3 Treatments

Materials

The treatment materials consisted of computer labs, classroom sets of two-color counters, classroom sets of software, and a unit entitled Working with Integers (written by the investigator; see Appendix C). Calculators were not allowed during the study. Some of the problems which appeared in the materials were adapted from Bennet, Maier,
and Nelson (1988). The problems were typical of integer exercises used to develop vocabulary, concepts, and skills when operating with positive and negative numbers.

**Groups**

All groups received treatments for five sessions over a consecutive seven-day period. The first and seventh days were used for testing. The treatments consisted of presenting the students with discussions and supportive instructional materials designed to teach about integers and operating with integers. Each group received the same amount of instructional time and the same problems on the same days. The microworld treatment group used only the computer microworld to explore adding and subtracting integers. The concrete manipulatives group used only the two-color counters to explore adding and subtracting integers. The combination treatment group worked with two-color counters on the second, third, and fifth sessions of the seven-day program and used the computer microworld on the fourth and sixth sessions.
4.4 Measuring Instruments

Designed for use in this study, a pre-test and a post-test (Appendix B) was administered to assess each participant's knowledge of integers and integer operations (addition and subtraction). To strengthen the validity of the measuring instruments, the tests and grading standards were designed with the help of five classroom teachers from the central Ohio area and represent the common body of knowledge expected when assessing students on integer knowledge. Two types of problems appear on the instruments. On each test, twenty items have been classified as Type I problems (skill-based exercises) and six items have been classified as Type II problems (exercises that address issues of conceptualization and application). Although items on the pre- and the post-test are not identical, only superficial changes have been made. The tests do not allow use of manipulatives, calculators, or computer software.

Free response to all items prevents cueing and guessing-by-elimination, threats associated with multiple-choice tests in mathematics. Answers to Type I problems will be scored 0 (incorrect magnitude, incorrect sign), 1 (correct magnitude, incorrect sign), or 2
(correct magnitude, correct sign). Type II problems are similar to higher order worksession tasks. Designed to ascertain depth of integer knowledge, responses to these open-ended questions will be evaluated using item-specific rubrics based on the following prototype:

| Level 4 Response (Highest Rating) | Contains a complete response with clear, coherent, unambiguous, and elegant explanation. 
Includes clear and simple diagram. Communicates effectively to an identified audience. Shows understanding of the question's mathematical ideas and processes. Identifies all the important elements of the question. Includes, if appropriate, examples and/or counter-examples Gives strong supporting arguments. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 3 Response</td>
<td>Contains a good response with some of the characteristics of Level 4, but not all. Explains less elegantly, less completely.</td>
</tr>
<tr>
<td>Level 2 Response</td>
<td>Contains a complete response, but the explanation is unclear. Presents incomplete arguments. Includes inappropriate or unclear diagrams. Indicates understanding of mathematical ideas, but not expressed clearly.</td>
</tr>
<tr>
<td>Level 1 Response</td>
<td>Omits significant parts of the questions and/or response. Has major errors. Uses inappropriate strategies.</td>
</tr>
<tr>
<td>Level 0 Response</td>
<td>No response provided.</td>
</tr>
</tbody>
</table>

Adapted from *Mathematics Assessment: Myths, Models, Good Questions and Practical Suggestions* (Stenmark, 1991)

Using Cronbach's alpha, the reliability of each instrument was established for the study. The results of this analysis and similar correlational analysis of item types can be found in Table 1.
Table 1

<table>
<thead>
<tr>
<th>Reliability of Measuring Instruments</th>
<th>Pre-test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I Items</td>
<td>.80</td>
<td>.84</td>
</tr>
<tr>
<td>Type II Items</td>
<td>.85</td>
<td>.86</td>
</tr>
<tr>
<td>Entire Test</td>
<td>.77</td>
<td>.82</td>
</tr>
</tbody>
</table>

In addition to the analysis using Cronbach's alpha, inter-rater reliability was examined by having two graders independently score each item on the pre- and post-tests. The inter-rater percentage of agreement is reported in Chapter V.

4.5 Procedures

Prior to the treatment, the three classroom instructors participating in the study were trained in the use of manipulatives and in the use of the microworld to teach integers. For two evenings, a total of three hours was spent modeling the addition and subtraction process using the color counters and the computer software. At the end of the training sessions, the instructors were asked to teach three 10-minute videotaped lessons to a panel of volunteers. The tapes were analyzed
to insure that each teacher was using either the manipulatives strategy or the microworld strategy (or the appropriate mix of both), without any additional instructional methods.

Classes were randomly assigned to either the concrete manipulatives, the microworld, or the combination treatment. Students took the integer pre-test immediately prior to the five day treatment period. The treatment worksessions were held on consecutive school days, following the normal class schedule. Each worksession lasted approximately 45 minutes. Previous work with fourth-graders using the software indicated that attention waned after 40 minutes and two to three days were necessary for the instruction on both addition and subtraction of integers. On the day immediately following the treatment period, the post-test was administered. The course of events for participant activities was as follows:

Day 1: Pre-test
Day 2: Introduction to the manipulatives or the microworld (& free exploration)
Day 3-4: Addition of Integers Module
Day 5-6: Subtraction of Integers Module
Day 7: Post-test
Scripts for each workshopsession (including lesson outlines) can be found in Appendix C. In general, the sessions were similar for all groups. During each workshopsession, the teacher checked attendance and then gave a brief explanation of the lesson for that day. Students were then handed activity sheets to be completed, working in pairs (using the appropriate treatment materials). The teacher provided individual assistance to those students who needed or requested it and periodically checked work to assure that students stayed "on-task." At the end of each session, students were to continue working at home on any of the problems not completed in class; activity sheets were then collected on the day following the instruction. Most students were able to complete all activity sheets by the end of each workshopsession.

Students who missed a day during the treatment phase were asked to make up the instructional time during a study period. If a student missed the pre-test, he/she was required to take the test on the first day returning to class; he/she then made up any missing treatment sessions during study periods. Students who missed the post-test were required to take the test on the day following completion of the five
treatment workshops. Thus, although the treatment period was short, the actual collection of data for each group required weeks, due to spotty attendance and chronic absences.

4.6 Statistical Analyses

The statistical procedures for this study were categorized into three parts: (1) analysis of pre-test data, (2) analysis of post-test data, and (3) regression analysis of critical factors.

Pre-test Analysis

Although pre-test scores were used as a covariate for analyzing post-test data, a one-way analysis of variance (ANOVA) was performed to determine if significant differences were indeed evident between groups, prior to treatment sessions. To ascertain whether or not the treatment groups were similar in abilities, with respect to using and understanding integers, the following hypothesis was formed: There will be no significant difference between all three treatment groups on the pre-test. This assertion was tested at the .05 level of significance (alpha level).
Post-test Analysis

A 3x2 factorial design was employed to study the main effect of instructional method, the main effect of grade level, and the interaction between those two independent variables. An analysis of covariance (ANCOVA), using the General Linear Model, was used to determine, at the 0.05 significance (alpha) level, whether to reject each of the primary hypotheses investigated:

H1. There will be no significant difference in post-test scores between the three treatment groups.

H2. There will be no significant difference in post-test scores between students with previous instruction on integers and students without previous instruction on integers.

H3. There will be no significant interaction between treatment levels and previous instruction levels.

Pre-test scores were used as a covariate to strengthen the power of the statistical analysis, by reducing within-group (error) variance.

After separating the data according to previous instruction level, an analysis of variance was used to test the secondary hypotheses:
**H4.** There will be no significant difference in post-test scores between students with previous instruction across the three treatment groups.

**H5.** There will be no significant difference in post-test scores between students without previous instruction across the three treatment groups.

Pre-test scores were used as a covariate to strengthen the power of the statistical analysis, by reducing within-group (error) variance. Furthermore, scores from Type I and Type II problems were separated to perform additional factorial analyses on skill-based and higher order response items, testing these same hypotheses.

*Regression Analysis*

As a final note, the factors of pre-test scores, gender, teacher, class, and treatment were used in a stepwise multiple regression analysis, to determine which variable(s) may be the best predictor(s) of student achievement on the integer post-test.
CHAPTER V
STATISTICAL ANALYSES AND RESULTS

5.0 Introduction

Two testing instruments were used in this study. The internal consistency reliabilities for both the pre- and the post-test were reported for the sample in this study in Chapter IV. Since the items on the tests were scored using rubrics on an interval scale, the researcher and each respective classroom teacher graded both tests and inter-rater reliability was established. Every item score for each student was examined to determine the percentage of agreement between raters (see Table 2).

Table 2
Percentage of Agreement Between Raters

<table>
<thead>
<tr>
<th>Raters</th>
<th>No. of Items</th>
<th>Pre-Test</th>
<th>Post-test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teacher A/Researcher</td>
<td>1066</td>
<td>96%</td>
<td>94%</td>
</tr>
<tr>
<td>Teacher B/Researcher</td>
<td>598</td>
<td>89%</td>
<td>92%</td>
</tr>
<tr>
<td>Teacher C/Researcher</td>
<td>1644</td>
<td>97%</td>
<td>98%</td>
</tr>
</tbody>
</table>
5.1 Analysis of Pre-Test Data

The pre-test was used to determine students' initial understanding of integers. Used as a covariate for post-test data analyses, the pre-test scores help to statistically control for differences between the intact classes participating in the study. It is assumed that there must be some differences among these six groups – some have had previous instruction on integers and some have not. However, whether or not the three treatment groups were different prior to participating in the study is important to discern. The descriptive statistics of pre-test data, organized by treatment group, can be found in Table 3.

Table 3

Descriptive Statistics of Pre-test Data

<table>
<thead>
<tr>
<th></th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>Range</td>
<td>0.21</td>
<td>0.20</td>
<td>0.21</td>
</tr>
<tr>
<td>$M$</td>
<td>8.2</td>
<td>8.81</td>
<td>8.561</td>
</tr>
<tr>
<td>Median</td>
<td>6.0</td>
<td>7.0</td>
<td>6.0</td>
</tr>
<tr>
<td>$SD$</td>
<td>5.199</td>
<td>5.052</td>
<td>5.491</td>
</tr>
</tbody>
</table>
A one-way analysis of variance (ANOVA) was performed on the pre-test data for the 128 subjects participating in the study to determine if initial differences existed among the treatment groups. The following hypothesis was tested: There will be no significant differences between all three treatment groups on the pre-test. Table 4 presents the results of the one-way ANOVA on pre-test data.

Table 4  
ANOVA for Pre-test Data

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>2</td>
<td>8.19</td>
<td>8.19</td>
<td>4.10</td>
<td>0.15</td>
<td>0.862</td>
</tr>
<tr>
<td>Error</td>
<td>125</td>
<td>3441.77</td>
<td>3441.77</td>
<td>27.53</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>3449.97</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Results indicated no significant differences existed between the treatment group means. The hypothesis can not be rejected at the .05 level of significance. Therefore, since the cell means did not differ significantly, the treatment groups were considered similar in abilities, prior to participation in the worksessions.
5.2 Analyses of Post-test Data for the Complete Sample

The pre-test was given immediately before the treatment period.

The post-test was given immediately after the treatment period.

Possible scores on each test were from 0 to 64. Actual scores ranged from 0 to 21 on the pre-test and from 17 to 59 on the post-test. Table 5 contains a summary of the descriptive statistics for the three treatment groups of subjects.

Table 5
Means and Standard Deviations for Treatment Groups on Pre- and Post-tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>( M )</td>
<td>8.2</td>
<td>8.81</td>
<td>8.561</td>
</tr>
<tr>
<td>( SD )</td>
<td>5.199</td>
<td>5.052</td>
<td>5.491</td>
</tr>
<tr>
<td>Dependent Variable (Post-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n )</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>( M )</td>
<td>32.42</td>
<td>34.86</td>
<td>32.10</td>
</tr>
<tr>
<td>( SD )</td>
<td>9.59</td>
<td>7.35</td>
<td>8.15</td>
</tr>
<tr>
<td>( Adj M )</td>
<td>32.77</td>
<td>34.54</td>
<td>31.64</td>
</tr>
</tbody>
</table>
There were pre- and post-test scores for 45 subjects in the concrete manipulatives only treatment group, 42 subjects in the microworld only treatment group, and 41 subjects who were taught with both representational treatments. The pre-test mean for the concrete group was 8.2 \((SD = 5.199)\), compared to the microworld group mean of 8.81 \((SD = 5.052)\) and the concrete-micro group mean of 8.561 \((SD = 5.491)\). The post-test means for all three groups were an increase from the pre-test, with the microworld group showing the greatest increase. On the post-test, the mean for the concrete group was 32.42 \((SD = 9.59)\), 34.86 \((SD = 7.35)\) for the microworld group, and 32.10 \((SD = 8.15)\) for the concrete-micro group. When adjusted for the covariate of pre-test scores, the means for the concrete, microworld, and concrete-micro groups were 32.77, 34.54, and 31.64 respectively. The microworld group had the highest average scores on both the pre- and the post-test. Furthermore, it maintained the highest mean when adjusting for pre-test scores.

Descriptive statistics were also compiled according to whether or not students had received previous formal instruction on integers. The results of this analysis can be found in Table 6.
Table 6

Means and Standard Deviations According to Previous Instruction

Classification

<table>
<thead>
<tr>
<th>Test</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>10.484</td>
<td>6.547</td>
</tr>
<tr>
<td>$SD$</td>
<td>5.342</td>
<td>4.283</td>
</tr>
<tr>
<td>Dependent Variable (Post-test)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>34.53</td>
<td>31.703</td>
</tr>
<tr>
<td>$SD$</td>
<td>8.87</td>
<td>7.861</td>
</tr>
<tr>
<td>$Adj\ M$</td>
<td>31.79</td>
<td>34.18</td>
</tr>
</tbody>
</table>

There were pre- and post-test scores for 64 subjects in the previous instruction category and 64 subjects who had no previous formal instruction on integers. The pre-test mean for the previous instruction category was 10.484 ($SD = 5.342$), compared to the no previous instruction mean of 6.547 ($SD = 4.283$). The post-test means for both groups were an increase from the pre-test, with subjects who have received no previous formal instruction on integers showing the
greatest increase. On the post-test, the mean for the previous
instruction group was 34.53 ($SD = 8.87$), and 31.703 ($SD = 7.861$) for
those with no previous formal instruction. When adjusted for the
covariate of pre-test scores, the means for the groups with and without
previous formal instruction were 31.79, and 34.18 respectively. Even
though the previous instruction group had the highest average score on
both the pre-test and the post-test, the group without previous formal
instruction on integers yielded the highest mean after adjusting for pre-
test scores.

Adjusted means for all configurations of factors (3 levels of
treatment and 2 levels of previous instruction) were calculated. The
source table of adjusted means for these configurations is presented in
Table 7.
Table 7

*Adjusted Means According to Type of Treatment and Previous Instruction*

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Only</td>
<td>33.51</td>
<td>32.03</td>
</tr>
<tr>
<td>Microworld Only</td>
<td>33.47</td>
<td>35.60</td>
</tr>
<tr>
<td>Both</td>
<td>28.38</td>
<td>34.90</td>
</tr>
</tbody>
</table>

The adjusted mean scores for the post-test range from a low of 28.38
(subjects with previous instruction who were treated with both concrete
and microworld methods) to a high of 35.60 (subjects without previous
instruction who were treated with the microworld method only).

An analysis of covariance was used to determine whether or not to
reject each of the following null hypotheses:

*H1.* There will be no significant difference in post-test scores between
the three treatment groups.

*H2.* There will be no significant difference in post-test scores between
students with previous instruction on integers and students
without previous instruction on integers.

*H3.* There will be no significant interaction between treatment levels
and previous instruction levels.
The results from this analysis of covariance are shown in Table 8.

**Table 8**

*ANCOVA Comparison of Treatment, Previous Instruction, and Interaction*

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>5079.70</td>
<td>4541.12</td>
<td>4541.12</td>
<td>161.3</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>132.21</td>
<td>175.92</td>
<td>87.89</td>
<td>3.12</td>
<td>0.048</td>
</tr>
<tr>
<td>Previous</td>
<td>1</td>
<td>157.01</td>
<td>154.62</td>
<td>154.62</td>
<td>5.49</td>
<td>0.021</td>
</tr>
<tr>
<td>Treat*Prev</td>
<td>2</td>
<td>336.37</td>
<td>336.37</td>
<td>168.19</td>
<td>5.98</td>
<td>0.003</td>
</tr>
<tr>
<td>Error</td>
<td>121</td>
<td>3405.94</td>
<td>3405.94</td>
<td>28.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>9111.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pre-test accounted for a significant amount of the variation in the post-test scores (p < 0.001). Scores prior to the treatment period were significantly related to scores immediately following the treatment period. The null hypotheses were tested at α = 0.05. Therefore, each of the three null hypotheses can be rejected. There is a significant difference between the post-test mean scores among the treatment groups. There is a significant difference between the post-test mean scores among subjects with and without previous formal
instruction on integers. Furthermore, there is a significant interaction between levels of treatment and levels of previous instruction (see Figure 10 for a graphical representation).

**Interaction: Treatment*Prev Instruction**

![Graph of Interaction: Treatment*Previous Instruction](image)

**Figure 10**
Graph of Interaction: Treatment*Previous Instruction
Although the statistics indicate that significant differences were evident among the three treatment groups, it was not known – besides glancing at descriptive statistic means – which treatment was "outstanding." To determine significant differences between pairs of treatments, the Scheffe test was used for multiple comparisons of post-test means. Results of the Scheffe test appear in Table 9.

**Table 9**

*Scheffe Test for Pairwise Comparisons of Treatments*

<table>
<thead>
<tr>
<th></th>
<th>Micro</th>
<th>Concrete</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>34.86</td>
<td>Micro</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>32.42</td>
<td>Concrete</td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>32.10</td>
<td>Both</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(n = 128; \( \alpha = 0.05 \))

Thus, there were significant differences in the post-test scores, favoring the microworld over the concrete manipulatives treatment and the microworld over the combination treatment.
5.3 Analyses Within Previous Instruction Categories

In order to determine the effects of the treatments for students who have or have not experienced previous formal instruction on integers, data was analyzed according to groups partitioned by previous instruction categories.

Subjects with Previous Formal Instruction on Integers

The following null hypothesis was proposed and tested: There will be no significant difference in post-test scores between students with previous instruction in the three treatment groups (Hypothesis H4, from Chapter I). 64 subjects who had previous formal instruction on integers participated in the study. Table 10 contains a summary of the descriptive statistics for the three treatment groups of subjects with previous instruction.
Table 10

Means and Standard Deviations for Treatment Groups of Subjects with Previous Instruction

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>10.96</td>
<td>10.09</td>
<td>10.33</td>
</tr>
<tr>
<td>$SD$</td>
<td>5.03</td>
<td>4.95</td>
<td>6.39</td>
</tr>
<tr>
<td>Dependent Variable (Post-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>36.58</td>
<td>35.45</td>
<td>30.67</td>
</tr>
<tr>
<td>$SD$</td>
<td>9.39</td>
<td>6.75</td>
<td>9.67</td>
</tr>
<tr>
<td>$Adj\ M$</td>
<td>36.04</td>
<td>35.90</td>
<td>30.84</td>
</tr>
</tbody>
</table>

Within the category of previous formal instruction, there were pre- and post-test scores for 24 subjects in the concrete manipulatives only treatment group, 22 subjects in the microworld only treatment group, and 18 subjects who were taught with both representational treatments. The pre-test mean for the concrete group was $10.96 (SD = 5.03)$, compared to the microworld group mean of $10.09 (SD = 4.95)$ and the concrete-micro group mean of $10.33 (SD = 6.39)$. The post-test means for all three groups were an increase from the pre-test, with the
concrete group showing the greatest increase. On the post-test, the mean for the concrete group was 36.58 (SD = 9.39), 35.45 (SD = 6.75) for the microworld group, and 30.67 (SD = 9.67) for the concrete-micro group. When adjusted for the covariate of pre-test scores, the means for the concrete, microworld, and concrete-micro groups were 36.04, 35.90, and 30.84 respectively. The concrete group had the highest average scores on both the pre- and the post-test. Furthermore, it maintained the highest mean when adjusting for pre-test scores.

An analysis of covariance was used to determine if there was any significant difference between the mean scores among the three treatment groups for subjects with previous instruction. The source table for this analysis of covariance is presented in Table 11.
Table 11

ANCOVA Comparison of Treatment for Subjects with Previous Instruction

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>2356.83</td>
<td>2309.83</td>
<td>2309.83</td>
<td>61.23</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>341.64</td>
<td>341.64</td>
<td>170.82</td>
<td>4.53</td>
<td>0.015</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>2263.46</td>
<td>2263.46</td>
<td>37.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>4961.94</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pre-test accounted for a significant amount of the variation in the post-test scores (p < 0.001). Scores prior to the treatment period were significantly related to scores immediately following the treatment period. The null hypothesis (tested at α = 0.05) can be rejected. There is a significant difference between the post-test mean scores among the treatment groups for subjects with previous formal instruction.

Subjects without Previous Formal Instruction on Integers

The following null hypothesis was proposed and tested: There will be no significant difference in post-test scores between students without previous instruction in the three treatment groups (Hypothesis H5, from Chapter I). 64 subjects who had no previous formal
instruction on integers participated in the study. Table 12 contains a summary of the descriptive statistics for the three treatment groups of subjects without previous instruction.

**Table 12**

*Means and Standard Deviations for Treatment Groups of Subjects without Previous Instruction*

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>M</td>
<td>5.048</td>
<td>7.40</td>
<td>7.174</td>
</tr>
<tr>
<td>SD</td>
<td>3.294</td>
<td>4.90</td>
<td>4.324</td>
</tr>
<tr>
<td>Dependent Variable (Post-test)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>M</td>
<td>27.67</td>
<td>34.20</td>
<td>33.22</td>
</tr>
<tr>
<td>SD</td>
<td>7.51</td>
<td>8.08</td>
<td>6.76</td>
</tr>
<tr>
<td>Adj M</td>
<td>29.85</td>
<td>32.96</td>
<td>32.30</td>
</tr>
</tbody>
</table>

Within the category of participants with no previous formal instruction, there were pre- and post-test scores for 21 subjects in the concrete manipulatives only treatment group, 20 subjects in the microworld only treatment group, and 23 subjects who were taught
with both representational treatments. The pre-test mean for the concrete group was 5.048 ($SD = 3.294$), compared to the microworld group mean of 7.40 ($SD = 4.90$) and the concrete-micro group mean of 7.174 ($SD = 4.324$). The post-test means for all three groups were an increase from the pre-test, with the microworld group showing the greatest increase. On the post-test, the mean for the concrete group was 27.67 ($SD = 7.51$), 34.20 ($SD = 8.08$) for the microworld group, and 33.22 ($SD = 6.76$) for the concrete-micro group. When adjusted for the covariate of pre-test scores, the means for the concrete, microworld, and concrete-micro groups were 29.85, 32.96, and 32.30 respectively. The microworld group had the highest average scores on both the pre- and the post-test. Furthermore, it maintained the highest mean when adjusting for pre-test scores.

An analysis of covariance was used to determine if there was any significant difference between the mean scores among the three treatment groups for subjects without previous instruction. The source table for this analysis of covariance is presented in Table 13.
Table 13
ANCOVA Comparison of Treatment for Subjects without Previous Instruction

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>2714.71</td>
<td>2300.31</td>
<td>2300.31</td>
<td>128.57</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>105.18</td>
<td>105.18</td>
<td>52.59</td>
<td>2.94</td>
<td>0.061</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>1073.47</td>
<td>1073.47</td>
<td>17.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>3893.36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pre-test accounted for a significant amount of the variation in the post-test scores (p < 0.001). Scores prior to the treatment period were significantly related to scores immediately following the treatment period. The null hypothesis (tested at $\alpha = 0.05$) can not be rejected. There is no significant difference between the post-test mean scores among the treatment groups for subjects without previous formal instruction.

5.4 Analyses for Test Item Categories

Two types of items were included in both the pre- and post-tests. Type I items were skill-based exercises which involved arithmetic computation. Type II items were higher-order questions, requiring
written conjectures and justifications. Each of the three primary hypotheses presented in Chapter I were adapted and analyzed after partitioning the pre- and post-tests into subsets of Type I or Type II items.

*Analyses of Type I Items*

Possible scores for Type I items were from 0 to 40. Actual scores for Type I items ranged from 0 to 17 on the pre-test and from 17 to 40 on the post-test. Tables 14, 15, and 16 contain summaries of descriptive statistics with respect to Type I items.

**Table 14**

*Means and Standard Deviations for Treatment Groups on Type I Items*

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Covariate (Pre-test Type I Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>$M$</td>
<td>7.067</td>
<td>7.595</td>
<td>7.463</td>
</tr>
<tr>
<td>$SD$</td>
<td>4.303</td>
<td>4.085</td>
<td>4.489</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type I Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>$M$</td>
<td>29.38</td>
<td>31.667</td>
<td>29.54</td>
</tr>
<tr>
<td>$SD$</td>
<td>7.30</td>
<td>5.771</td>
<td>6.92</td>
</tr>
<tr>
<td>$Adj , M$</td>
<td>29.65</td>
<td>31.43</td>
<td>29.07</td>
</tr>
</tbody>
</table>
The microworld group had the highest average scores on both the pre- and the post-test for Type I items. Furthermore, it maintained the highest mean when adjusting for pre-test scores.

Table 15

*Means and Standard Deviations on Type I Items According to Previous Instruction Classification*

<table>
<thead>
<tr>
<th>Test</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type I Items)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>9.016</td>
<td>5.719</td>
</tr>
<tr>
<td>$SD$</td>
<td>4.297</td>
<td>3.566</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type I Items)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>31.453</td>
<td>28.906</td>
</tr>
<tr>
<td>$SD$</td>
<td>7.266</td>
<td>6.381</td>
</tr>
<tr>
<td>$Adj\ M$</td>
<td>29.270</td>
<td>30.830</td>
</tr>
</tbody>
</table>

Even though the previous instruction group had the highest average score on both the pre- and post-test Type I items, the group without previous formal instruction on integers yielded the highest mean after adjusting for pre-test scores.
Table 16

Adjusted Means on Type I Items According to Type of Treatment and Previous Instruction

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Only</td>
<td>30.80</td>
<td>28.51</td>
</tr>
<tr>
<td>Microworld Only</td>
<td>30.81</td>
<td>32.05</td>
</tr>
<tr>
<td>Both</td>
<td>26.21</td>
<td>31.94</td>
</tr>
</tbody>
</table>

The adjusted mean scores for post-test Type I items range from a low of 26.21 (subjects with previous instruction who were treated with both concrete and microworld methods) to a high of 32.05 (subjects without previous instruction who were treated with the microworld method only).

An analysis of covariance was used to determine whether or not to reject each of the following null hypotheses:

H1a. There will be no significant difference in post-test scores on Type I items the three treatment groups.

H2a. There will be no significant difference in post-test scores on Type I items between students with previous instruction on integers and students without previous instruction on integers.
H3a. There will be no significant interaction between treatment levels and previous instruction levels, with respect to Type I items.

The results from this analysis of covariance are shown in Table 17.

**Table 17**

*ANCOVA Comparison of Treatment, Previous Instruction, and Interaction on Type I Items*

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>3117.49</td>
<td>2640.21</td>
<td>2640.21</td>
<td>128.80</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>95.42</td>
<td>124.98</td>
<td>62.49</td>
<td>3.05</td>
<td>0.051</td>
</tr>
<tr>
<td>Previous</td>
<td>1</td>
<td>67.97</td>
<td>65.12</td>
<td>65.12</td>
<td>3.18</td>
<td>0.077</td>
</tr>
<tr>
<td>Treat*Prev</td>
<td>2</td>
<td>337.61</td>
<td>337.61</td>
<td>168.80</td>
<td>8.23</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Error</td>
<td>121</td>
<td>2480.38</td>
<td>2480.38</td>
<td>20.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>6098.87</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pre-test Type I items accounted for a significant amount of the variation in the post-test scores for Type I items (p < 0.001). The null hypotheses were tested at $\alpha = 0.05$. There is no significant difference between the post-test mean scores on Type I items among the treatment groups. There is no significant difference between the post-test mean scores on Type I items among subjects with and without
previous formal instruction on integers. However, the null hypothesis of no significant interaction between treatment and previous instruction can be rejected (see Figure 11 for a graphical representation).

**Interaction: Treatment*Prev Instruction for Type I Items**

![Graph of Interaction: Treatment*Previous Instruction (Type I Items)](image)

**Figure 11**
Graph of Interaction: Treatment*Previous Instruction (Type I Items)
Analyses of Type II Items

Possible scores for Type II items were from 0 to 24. Actual scores for Type II items ranged from 0 to 4 on the pre-test and from 0 to 19 on the post-test. Tables 18, 19, and 20 contain summaries of descriptive statistics with respect to Type II items.

Table 18

Means and Standard Deviations for Treatment Groups on Type II Items

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>( M )</td>
<td>1.133</td>
<td>1.214</td>
<td>1.098</td>
</tr>
<tr>
<td>SD</td>
<td>1.057</td>
<td>1.138</td>
<td>1.281</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>45</td>
<td>42</td>
<td>41</td>
</tr>
<tr>
<td>( M )</td>
<td>2.978</td>
<td>3.190</td>
<td>2.561</td>
</tr>
<tr>
<td>SD</td>
<td>3.064</td>
<td>2.361</td>
<td>1.582</td>
</tr>
<tr>
<td>Adj ( M )</td>
<td>3.004</td>
<td>3.127</td>
<td>2.589</td>
</tr>
</tbody>
</table>

The microworld group had the highest average scores on both the pre- and the post-test for Type II items. Furthermore, it maintained the highest mean when adjusting for pre-test scores.
Table 19
Means and Standard Deviations on Type II Items According to Previous Instruction Classification

<table>
<thead>
<tr>
<th>Test</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type II Items)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>1.469</td>
<td>0.828</td>
</tr>
<tr>
<td>$SD$</td>
<td>1.272</td>
<td>0.918</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type II Items)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>64</td>
<td>64</td>
</tr>
<tr>
<td>$M$</td>
<td>3.078</td>
<td>2.750</td>
</tr>
<tr>
<td>$SD$</td>
<td>2.559</td>
<td>2.303</td>
</tr>
<tr>
<td>$Adj\ M$</td>
<td>2.659</td>
<td>3.154</td>
</tr>
</tbody>
</table>

Even though the previous instruction group had the highest average score on both the pre-and post-test Type II items, the group without previous formal instruction on integers yielded the highest mean after adjusting for pre-test scores.
Table 20

Adjusted Means on Type II Items According to Type of Treatment and Previous Instruction

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Previous</th>
<th>No Previous</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concrete Only</td>
<td>2.887</td>
<td>3.121</td>
</tr>
<tr>
<td>Microworld Only</td>
<td>2.783</td>
<td>3.470</td>
</tr>
<tr>
<td>Both</td>
<td>2.306</td>
<td>2.871</td>
</tr>
</tbody>
</table>

The adjusted mean scores for post-test Type II items range from a low of 2.306 (subjects with previous instruction who were treated with both concrete and microworld methods) to a high of 3.470 (subjects without previous instruction who were treated with the microworld method only).

An analysis of covariance was used to determine whether or not to reject each of the following null hypotheses:

H1b. There will be no significant difference in post-test scores on Type II items between the three treatment groups.

H2b. There will be no significant difference in post-test scores on Type II items between students with previous instruction on integers and students without previous instruction on integers.
H3b. There will be no significant interaction between treatment levels and previous instruction levels, with respect to Type II items.

The results from this analysis of covariance are shown in Table 21.

Table 21

ANCOVA Comparison of Treatment, Previous Instruction, and Interaction on Type II Items

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>232.244</td>
<td>226.362</td>
<td>226.362</td>
<td>54.35</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>5.497</td>
<td>6.534</td>
<td>3.267</td>
<td>0.78</td>
<td>0.459</td>
</tr>
<tr>
<td>Previous</td>
<td>1</td>
<td>7.183</td>
<td>7.179</td>
<td>7.179</td>
<td>1.72</td>
<td>0.192</td>
</tr>
<tr>
<td>Treat*Prev</td>
<td>2</td>
<td>1.178</td>
<td>1.178</td>
<td>0.589</td>
<td>0.14</td>
<td>0.868</td>
</tr>
<tr>
<td>Error</td>
<td>121</td>
<td>503.953</td>
<td>503.953</td>
<td>4.165</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>750.055</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The pre-test Type II items accounted for a significant amount of the variation in the post-test scores for Type I items (p < 0.001). The null hypotheses were tested at α = 0.05. All of the null hypotheses must be accepted. There is no significant difference between the post-test mean scores on Type II items among the treatment groups. There is no significant difference between the post-test mean scores on Type II
items among subjects with and without previous formal instruction on integers. There is no significant interaction between treatment and previous instruction, with respect to Type II items (see Figure 12 for a graphical representation).

Interaction: Treatment*Prev Instruction for Type II Items

![Graph of Interaction: Treatment*Previous Instruction (Type II Items)](image)

Figure 12
Graph of Interaction: Treatment*Previous Instruction (Type II Items)
5.5 Analyses for Test Item Types Within Previous Instruction Categories

Type I and Type II Item data were further organized according to the partitioning of previous instruction categories.

*Subjects with Previous Formal Instruction and Type I Items*

The following null hypotheses, adapted from the secondary hypotheses in Chapter I were proposed and tested:

*H4a.* There will be no significant difference in post-test scores on Type I items between students with previous instruction across the three treatment groups.

*H4b.* There will be no significant difference in post-test scores on Type II items between students with previous instruction across the three treatment groups.

Tables 22 and 23 contain summaries of the descriptive statistics for the three treatment groups of subjects with previous instruction, according to item type.
Table 22

Means and Standard Deviations of Type I Items for Treatment Groups of Subjects with Previous Instruction

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type I Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>9.375</td>
<td>8.727</td>
<td>8.89</td>
</tr>
<tr>
<td>$SD$</td>
<td>4.158</td>
<td>3.966</td>
<td>5.03</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type I Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>33.17</td>
<td>32.41</td>
<td>28.00</td>
</tr>
<tr>
<td>$SD$</td>
<td>7.31</td>
<td>5.75</td>
<td>8.04</td>
</tr>
<tr>
<td>$Adj M$</td>
<td>32.77</td>
<td>32.72</td>
<td>28.14</td>
</tr>
</tbody>
</table>
Table 23

Means and Standard Deviations of Type II Items for Treatment Groups of Subjects with Previous Instruction

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>1.583</td>
<td>1.364</td>
<td>1.444</td>
</tr>
<tr>
<td>$SD$</td>
<td>1.100</td>
<td>1.177</td>
<td>1.617</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>24</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>$M$</td>
<td>3.417</td>
<td>3.045</td>
<td>2.667</td>
</tr>
<tr>
<td>$SD$</td>
<td>3.635</td>
<td>1.430</td>
<td>1.879</td>
</tr>
<tr>
<td>$Adj , M$</td>
<td>3.296</td>
<td>3.156</td>
<td>2.692</td>
</tr>
</tbody>
</table>

For both Type I and Type II items, the concrete manipulatives group had the highest average scores on the pre- and post-tests. Furthermore, it maintained the highest means on Type I and Type II items when adjusting for pre-test scores.

With respect to each type of test item, an analysis of covariance was used to determine if there was any significant difference between the mean scores among the three treatment groups for subjects with previous instruction. The results from these analyses of covariance are
presented in Table 24 and Table 25.

Table 24
ANCOVA Comparison of Treatment for Subjects with Previous Instruction, Type I Items

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>1409.99</td>
<td>1379.88</td>
<td>1379.88</td>
<td>50.46</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>275.10</td>
<td>275.10</td>
<td>137.55</td>
<td>5.03</td>
<td>0.010</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>1640.77</td>
<td>1640.77</td>
<td>27.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>3325.86</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 25
ANCOVA Comparison of Treatment for Subjects with Previous Instruction, Type II Items

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>113.696</td>
<td>111.827</td>
<td>111.827</td>
<td>22.75</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>3.953</td>
<td>3.953</td>
<td>1.977</td>
<td>0.40</td>
<td>0.671</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>294.960</td>
<td>294.960</td>
<td>4.916</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>412.609</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For both Type I and Type II Items, the pre-test accounted for a significant amount of the variation in the post-test scores (p < 0.001).
Scores prior to the treatment period were significantly related to scores immediately following the treatment period. However, only the $H4a$ null hypothesis (tested at $\alpha = 0.05$) can be rejected; there is a significant difference between the post-test mean scores of Type I items among the treatment groups for subjects with previous formal instruction. There is no significant difference between the post-test mean scores of Type II items for this category of subjects.

**Subjects without Previous Formal Instruction and Type I Items**

The following null hypotheses, adapted from the secondary hypotheses in Chapter I were proposed and tested:

$H5a$. There will be no significant difference in post-test scores on Type I items between students with no previous instruction across the three treatment groups.

$H5b$. There will be no significant difference in post-test scores on Type II items between students with no previous instruction across the three treatment groups.

Tables 26 and 27 contain summaries of the descriptive statistics for the three treatment groups of subjects without previous instruction,
according to item type.

**Table 26**

*Means and Standard Deviations of Type I Items for Treatment Groups of Subjects without Previous Instruction*

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Covariate (Pre-test Type I Items)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>$M$</td>
<td>4.429</td>
<td>6.350</td>
<td>6.348</td>
</tr>
<tr>
<td>$SD$</td>
<td>2.675</td>
<td>3.937</td>
<td>3.761</td>
</tr>
<tr>
<td><strong>Dependent Variable (Post-test Type I Items)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>$M$</td>
<td>25.05</td>
<td>30.85</td>
<td>30.74</td>
</tr>
<tr>
<td>$SD$</td>
<td>5.95</td>
<td>5.83</td>
<td>5.82</td>
</tr>
<tr>
<td>$\text{Adj } M$</td>
<td>26.74</td>
<td>30.02</td>
<td>29.92</td>
</tr>
</tbody>
</table>
Table 27

 Means and Standard Deviations of Type II Items for Treatment Groups of Subjects without Previous Instruction

<table>
<thead>
<tr>
<th>Test</th>
<th>Concrete</th>
<th>Micro</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>Covariate (Pre-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>$M$</td>
<td>0.619</td>
<td>1.050</td>
<td>0.826</td>
</tr>
<tr>
<td>$SD$</td>
<td>0.740</td>
<td>1.099</td>
<td>0.887</td>
</tr>
<tr>
<td>Dependent Variable (Post-test Type II Items)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>21</td>
<td>20</td>
<td>23</td>
</tr>
<tr>
<td>$M$</td>
<td>2.476</td>
<td>3.350</td>
<td>2.478</td>
</tr>
<tr>
<td>$SD$</td>
<td>2.228</td>
<td>3.117</td>
<td>1.344</td>
</tr>
<tr>
<td>$Adj M$</td>
<td>2.800</td>
<td>3.006</td>
<td>2.481</td>
</tr>
</tbody>
</table>

For both Type I and Type II items, the microworld treatment group had the highest average scores on the pre- and post-tests. Furthermore, it maintained the highest means for Type I and Type II items when adjusting for pre-test scores.

With respect to each type of test items, an analysis of covariance was used to determine if there was any significant difference between the mean scores among the three treatment groups for subjects with no previous instruction. The results from these analyses of covariance are
presented in Table 28 and Table 29.

**Table 28**

*ANCOVA Comparison of Treatment for Subjects without Previous Instruction, Type I Items*

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>1609.28</td>
<td>1281.63</td>
<td>1281.63</td>
<td>93.97</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>137.85</td>
<td>137.85</td>
<td>68.93</td>
<td>5.05</td>
<td>0.009</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>818.30</td>
<td>818.30</td>
<td>13.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>2565.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 29**

*ANCOVA Comparison of Treatment for Subjects without Previous Instruction, Type II Items*

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>130.496</td>
<td>123.031</td>
<td>123.031</td>
<td>36.82</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Treatment</td>
<td>2</td>
<td>3.008</td>
<td>3.008</td>
<td>1.504</td>
<td>0.45</td>
<td>0.640</td>
</tr>
<tr>
<td>Error</td>
<td>60</td>
<td>200.496</td>
<td>200.496</td>
<td>3.342</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>63</td>
<td>334.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For both Type I and Type II Items, the pre-test accounted for a significant amount of the variation in the post-test scores (p < 0.001).
Scores prior to the treatment period were significantly related to scores immediately following the treatment period. However, only the $H5a$ null hypothesis (tested at $\alpha = 0.05$) can be rejected; there is a significant difference between the post-test mean scores of Type I items among the treatment groups for subjects with no previous formal instruction. There is no significant difference between the post-test mean scores of Type II items for this category of subjects.

5.6 Regression Analyses of Critical Predictors

In addition to the ANCOVA tests to determine the significance of treatments, previous instruction, and interactions, other factors were identified throughout the study that must be examined for impact on achievement. To determine critical predictors of post-test achievement, the following six factors were coded to perform regression analyses: pre-test score (PreS), treatment (T), previous instruction (PI), gender (G), teacher (Te; one of three instructors), and class (C; one of six groups of students). Data for the complete sample ($n = 128$) were used.
**BREG Analysis**

BREG does best subsets regression, using the maximum R-squared criterion. BREG first looks at all one-predictor regression models and selects the model giving the largest R-squared. Information on this model and the two next best models having one predictor is organized before moving on to the two-predictor models. Table 30 presents the BREG analysis of the top three models, whenever possible, in each category of one-predictor to six-predictors.

### Table 30

**BREG Analysis for Best Subsets Regression of Post-test**

<table>
<thead>
<tr>
<th>Vars</th>
<th>R-sq</th>
<th>Adj. R-sq</th>
<th>C-p</th>
<th>Pre</th>
<th>T</th>
<th>PI</th>
<th>G</th>
<th>Te</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>55.8</td>
<td>55.4</td>
<td>16.7</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>7.5</td>
<td>6.8</td>
<td>169.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>1</td>
<td>3.7</td>
<td>2.9</td>
<td>182.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>2</td>
<td>57.3</td>
<td>56.6</td>
<td>13.7</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>56.7</td>
<td>56.0</td>
<td>15.7</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>55.9</td>
<td>55.2</td>
<td>18.3</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>60.7</td>
<td>59.7</td>
<td>5.0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>60.0</td>
<td>59.0</td>
<td>7.2</td>
<td>X</td>
<td></td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>3</td>
<td>57.5</td>
<td>56.5</td>
<td>15.0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>61.5</td>
<td>60.2</td>
<td>4.4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>61.1</td>
<td>59.8</td>
<td>5.6</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>4</td>
<td>60.9</td>
<td>59.7</td>
<td>6.2</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>61.8</td>
<td>60.3</td>
<td>5.4</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>61.6</td>
<td>60.3</td>
<td>6.0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>5</td>
<td>61.5</td>
<td>59.9</td>
<td>6.5</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
<tr>
<td>6</td>
<td>61.9</td>
<td>60.0</td>
<td>7.0</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td>X</td>
</tr>
</tbody>
</table>
Note that the pre-test was included in all models with more than one predictor. Class, teacher, and previous instruction are the next frequent predictors, respectively, according to simple tallying. When comparing models with different numbers of predictors, choosing the model with the highest adjusted R-squared is equivalent to choosing the model with the smallest mean square error (MSE). Furthermore, in general, models where the C-p statistic is small and is also close to the number of parameters in the model has small variance and is relatively precise in estimating the true regression coefficients and predicting future responses. Therefore, according to BREG analysis, the following four models appear to be the best at predicting post-test scores, from among the six possible factors:

- 3-variable model of pre-test, previous instruction and class (adj. R-sq = 59.7; C-p = 5.0)

- 4-variable model of pre-test, previous instruction, teacher, and class (adj. R-sq = 60.2; C-p = 4.4)

- 5-variable model of pre-test, previous instruction, gender, teacher, and class (adj. R-sq = 60.3; C-p = 5.4)

- 6-variable model with all coded factors (adj. R-sq = 60.0; C-p = 7.0)
Stepwise Regression Analysis

To further analyze the six coded factors, a stepwise regression of analysis was performed. The F-statistic for entrance and removal was 2.29 (n = 128, k = 6, and α = 0.05). Table 31 shows the results of the stepwise regression analysis.

Table 31
Stepwise Regression of Post-test

<table>
<thead>
<tr>
<th>Step Constant</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1.213</td>
<td>1.297</td>
<td>1.258</td>
<td>1.257</td>
</tr>
<tr>
<td>T-ratio</td>
<td>12.60</td>
<td>12.63</td>
<td>12.63</td>
<td>12.70</td>
</tr>
<tr>
<td>Prev Instruction</td>
<td>-2.3</td>
<td>-8.0</td>
<td>-5.5</td>
<td></td>
</tr>
<tr>
<td>T-ratio</td>
<td>-2.14</td>
<td>-3.93</td>
<td>-2.20</td>
<td></td>
</tr>
<tr>
<td>Class</td>
<td>1.91</td>
<td>2.67</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-ratio</td>
<td>3.26</td>
<td>3.57</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Teacher</td>
<td></td>
<td>-2.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>T-ratio</td>
<td></td>
<td>-1.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>5.66</td>
<td>5.58</td>
<td>5.38</td>
<td>5.34</td>
</tr>
<tr>
<td>R-sq</td>
<td>55.75</td>
<td>57.31</td>
<td>60.68</td>
<td>61.50</td>
</tr>
</tbody>
</table>

The stepwise regression analysis yields the regression equation:

\[
\text{Post-test} = 21.8 + 1.26 \text{ (Pre-test)} - 5.53 \text{ (Prev Instruction)} - 2.90 \text{ (Teacher)} + 2.67 \text{ (Class)}
\]
Results of an analysis of variance on this regression equation can be found in Table 32.

Table 32

Analysis of Variance for Regression Equation

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>SS</th>
<th>MS</th>
<th>F</th>
<th>p</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regression</td>
<td>4</td>
<td>5603.1</td>
<td>1400.8</td>
<td>49.11</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Error</td>
<td>123</td>
<td>3508.1</td>
<td>28.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>127</td>
<td>9111.2</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>1</td>
<td>5079.7</td>
</tr>
<tr>
<td>Prev Instr</td>
<td>1</td>
<td>142.1</td>
</tr>
<tr>
<td>Teacher</td>
<td>1</td>
<td>17.7</td>
</tr>
<tr>
<td>Class</td>
<td>1</td>
<td>363.6</td>
</tr>
</tbody>
</table>

Once again, the pre-test appears to be the most critical predictor for post-test scores (this relationship is underscored by the Pearson product moment correlation between pre- and post-test means, 0.747).

Furthermore, statistically speaking, the stepwise regression analysis and subsequent analysis of variance support the claim that the 4-variable configuration of pre-test, previous instruction, teacher, and class is an adequate model and fits the data well.
CHAPTER VI

SUMMARY AND CONCLUSIONS

6.0 Summary

Successfully developing the concept of integer is a cornerstone to understanding algebra. The review of literature indicated that the little research which has been devoted to teaching about integers is inconclusive. A preferred model of integers has not been found, nor a particularly effective method of instruction. Students have misconceptions about positive and negative numbers (as well as zero) and inappropriately apply operating rules. So that students might overcome these difficulties, there is a need to strengthen or provide instructional strategies that promote making meaning of integers.

The purpose of this study was to investigate approaches for developing the concept of integer. The objective was to determine the effects of a computer microworld on middle school students' use and understanding of integers. Previous exposure and formal instruction on
Therefore, two primary and two secondary research questions were examined:

1) Will students who are exposed to *The Cy-Bee Chips* microworld be more successful in understanding and using integers than students who are exposed to instruction with concrete manipulatives?

2) Will students exposed to both *The Cy-Bee Chips* and concrete manipulatives be more successful in understanding and using integers than students who are exposed to instruction that is mutually exclusive?

3) Which of the three teaching methods will be the most effective for students who have had previous instruction on integers?

4) Which of the three teaching methods will be the most effective for students who have had no previous instruction on integers?

This study took place at two urban middle schools and involved three sixth grade classes and three eighth grade classes. The treatments of computer microworld, concrete manipulatives, or both computer microworld and concrete manipulatives were randomly assigned to the sixth and to eighth grade classes. All 128 subjects were retained throughout the study (64 sixth grade students and 64 eighth
grade students). Existing classroom teachers were trained and implemented the testing and the teaching modules. The execution of the seven-day program took place at various times during the school year, when each of the three teachers believed the course of study and "natural" flow deemed it appropriate.

Seven 45-minute worksessions were used for each group. The sessions were on consecutive days, with testing on the first and seventh day and treatments on the second through sixth days. Prior to treatments, students' understanding and use of integers were assessed by a pre-test, which was used as a covariate for analyses. After the treatments, a post-test that paralleled the pre-test was administered. The participating classroom teachers assisted in creating both testing instruments, which contained two types of items: computational exercises and higher-order questions.

The experimental design was a non-equivalent control group design with three levels of treatment. The primary independent variables were treatment group membership and level of previous formal instruction. The dependent variable was the post-test score. Statistical analysis for this study was divided into three major parts:
(a) factorial analysis of covariance for the entire sample, (b) analysis of variance for subsets of the sample, and (c) regression analysis for the entire sample.

The following section provides a list of the hypotheses that were tested and the outcomes.

H1: There will be no significant difference in post-test scores between the three treatment groups.

Outcome: H1 was rejected at the .05 level of significance.

H2: There will be no significant difference in post-test scores between students with previous instruction on integers and students without previous instruction on integers.

Outcome: H2 was rejected at the .05 level of significance.

H3: There will be no significant interaction between treatment levels and previous instruction levels.

Outcome: H3 was rejected at the .05 level of significance.

H4: There will be no significant difference in post-test scores between students with previous instruction across the three treatment groups.

Outcome: H4 was rejected at the .05 level of significance.

H5: There will be no significant difference in post-test scores between students without previous instruction across the three treatment groups.

Outcome: H5 could not be rejected at the .05 level of significance.
H1a: There will be no significant difference in post-test scores on Type I items between the three treatment groups.

Outcome: H1a could not be rejected at the .05 level of significance.

H2a: There will be no significant difference in post-test scores on Type I items between students with previous instruction on integers and students without previous instruction on integers.

Outcome: H2a could not be rejected at the .05 level of significance.

H3a: There will be no significant interaction between treatment levels and previous instruction, with respect to Type I items.

Outcome: H3a was rejected at the .05 level of significance.

H1b: There will be no significant difference in post-test scores on Type II items between the three treatment groups.

Outcome: H1b could not be rejected at the .05 level of significance.

H2b: There will be no significant difference in post-test scores on Type II items between students with previous instruction on integers and students without previous instruction on integers.

Outcome: H2b could not be rejected at the .05 level of significance.

H3b: There will be no significant interaction between treatment levels and previous instruction, with respect to Type II items.

Outcome: H3b could not be rejected at the .05 level of significance.

H4a: There will be no significant difference in post-test scores on Type I items between students with previous instruction across the three treatment groups.

Outcome: H4a was rejected at the .05 level of significance.
H4b: There will be no significant difference in post-test scores on Type II items between students with previous instruction across the three treatment groups.

Outcome: H4b could not be rejected at the .05 level of significance.

H5a: There will be no significant difference in post-test scores on Type I items between students without previous instruction across the three treatment groups.

Outcome: H5a was rejected at the .05 level of significance.

H5b: There will be no significant difference in post-test scores on Type II items between students without previous instruction across the three treatment groups.

Outcome: H5b could not be rejected at the .05 level of significance.

Regression Analysis: Six factors (pre-test score, treatment, previous instruction, gender, teacher, and class) were coded to determine which were critical factors in predicting post-test achievement.

Outcome: Pre-test, previous instruction, class, and teacher appear as the critical factors in the best model for prediction.

6.1 Discussion

The purpose of the investigation was to determine the effects of a computer microworld on students' ability to understand and use integers. This method was compared to a concrete manipulatives approach and an approach that used both the computer microworld and concrete manipulatives. From the rejection of Hypothesis 1 and the
results of the Scheffe test for multiple comparisons. In general, the microworld appears to be the favorable method of teaching (adjusted means for the concrete, microworld, and combination being 32.77, 34.54, and 31.64, respectively). Data indicate that previous instruction is also statistically significant in analyzing post-test scores; students without previous instruction actually had higher adjusted mean scores than students with previous instruction (34.18 to 31.79, respectively). Furthermore, previous instruction interacts with the teaching method, as indicated by the rejection of Hypothesis 3. In fact, students with previous instruction had a highest adjusted mean score for the concrete manipulatives group (33.51), whereas students without previous instruction had a highest adjusted mean score for the microworld group (35.60).

However, analyses within previous instruction categories indicate that only the students with previous instruction differ significantly in post-test scores between the three treatment groups. Ranking the three treatment groups from highest to lowest on adjusted mean scores reveals that concrete manipulatives is the most effective teaching strategy, with the microworld treatment second, and the combination
treatment third. In particular, there was a significant difference in post-test scores for Type I (computational) items for students with previous instruction in the three treatment groups. No significant difference existed for Type II items (higher order questions) under any conditions or categories (acceptance of Hypotheses H1b, H2b, H3b, H4b, and H5b).

Other analyses for Type I items indicate that, there was no significant difference between all students in the three treatment groups and no significant difference between students with previous instruction and without. However, a significant interaction between treatment levels and previous instruction does exist. On Type I items, adjusted mean post-test scores for students with previous instruction were 30.81 for the microworld treatment, 30.80 for the concrete manipulatives treatment, and 26.21 for the combination treatment. For students without previous instruction, adjusted mean post-test scores on Type I items were 32.05 for the microworld treatment, 31.94 for the combination treatment, and 28.51 for the concrete manipulatives treatment. In terms of procedural/computational knowledge, the most effective teaching technique may depend upon
whether or not a student has had previous instruction on integers.

With respect to the regression analyses, it appears that factors other than treatment may be critical in predicting students' post-test scores. After the BREG and stepwise regression analysis, a four variable model can be applied to accurately fit the data. In order of highest to lowest statistical significance, those four variables are (1) pre-test scores, (2) previous instruction, (3) class, and (4) teacher. Gender and treatment were not well-represented in the BREG analysis, nor did they load into the stepwise regression analysis. Throughout the ANCOVA and regression analyses, the pre-test score appears to be the most critical and statistically significant factor that can be used to explain variance in post-test scores.

6.2 Conclusions

On the basis of the statistical analyses of the quantitative data, five conclusions can be drawn:

- *In general, the microworld treatment appears to be the most effective teaching method.*

Referring to the entire sample (n = 128), there is a statistically significant difference in post-test scores between students in the three
treatment groups. The Scheffe test of multiple comparisons indicated that the microworld treatment is favored over both the concrete manipulatives treatment and the combination treatment, with statistically significant differences in post-test means. Although the microworld treatment group had the highest pre-test mean score entering the program, they retained the highest post-test mean score even after adjusting for the covariate of the pre-test. Looking at subsamples, students who had previous instruction on integers (n = 64) did have the highest adjusted mean post-test score for the concrete manipulatives treatment (36.04); however, their adjusted mean post-test score for the microworld treatment group was 35.90. The concrete manipulatives score represents an increase of less than one-half of one percent from the microworld score. Furthermore, for students without previous instruction on integers (n = 64), the microworld adjusted mean post-test score is highest once again, clearly surpassing the concrete manipulatives treatment (32.96 compared to 29.85).
- Previous instruction may interfere with students' acquisition of knowledge under "new" methods.

In all of the analyses comparing students with previous instruction to students without previous instruction on integers, students with previous instruction had the lower adjusted mean post-test scores. With respect to the entire sample (n = 128), students with previous instruction had an adjusted mean post-test score of 31.79, as opposed to 34.18 for students without previous instruction. Subjects who had received no formal instruction on integers showed the greatest increase, with statistical significance. Although statistical significance was not determined according to analyses involving Type I and Type II items, in both categories, students without previous instruction outscored students with previous instruction, after adjusting for the covariate of the pre-test. Thus, the evidence suggests that some form of proactive inhibition (Houston, 1986) detracted students with previous knowledge and possibly interfered with their learning the "new" material.

- The effectiveness of different strategies may depend upon whether or not students have experienced previous instruction on integers.

Analyzing the entire sample (n = 128), there was a statistically significant interaction between treatment levels and the levels of
previous instruction. For students with previous instruction, the strategies may be ranked from high to low, according to adjust mean post-test scores: (1) concrete manipulatives treatment, (2) microworld treatment, (3) combination treatment. For students without previous instruction, the high-to-low order is (1) microworld treatment, (2) combination treatment, (3) concrete manipulatives treatment. It should be noted, however, that these results may partially reflect a novelty effect. From observations and short interviews in the field, the investigator noticed that the eighth graders enjoyed using the concrete manipulatives more than the sixth graders, whereas the sixth graders responded most favorably to the computer microworld. Students in the eighth grade classes had routinely used the computers throughout the school year; they had not, however, used any manipulatives. In the sixth grade, the opposite circumstance was evident – students had not yet used computers in their math class, but were quite experienced with Cuisenaire rods, fraction bars, and geoboards.
• *The treatments seem to impact procedural knowledge more than conceptual understanding.*

Of the five analyses which specifically involved Type I items, three yielded results with statistical significance. None of the analyses involving Type II items generated statistically significant results. The lack of statistical significance for the Type II analyses, however, may be the direct result of mean and adjusted mean scores being extremely low, with standard deviations nearly matching – and at times surpassing – those means. Significant variance is impossible, for example, when descriptive statistics for the entire sample (n = 128) include adjusted mean scores of 3.004 (SD = 3.064), 3.127 (SD = 2.361), and 2.589 (SD = 1.582), for the concrete manipulatives treatment, the microworld treatment, and the combination treatment respectively. In general – during worksessions and testing – students hesitated or were reluctant to write conjectures and explanations when attempting to answer higher-order questions.

• *Factors other than the type of treatment may provide a better gauge for predicting student outcomes.*

Although final scores vary with statistical significance according to treatment, results of regression analyses indicate that a student’s pre-
test score is the best predictor of post-test achievement. Other statistically significant factors which help to predict outcomes are previous instruction, class, and teacher variables. Using stepwise regression analysis on data for the entire sample (n = 128), the cumulative effect of these four factors accounts for 61.5% of the variance in post-test scores. Thus, several attribute variables appear to influence students’ use and understanding of integers.

6.3 Implications

The conclusions suggest that students exposed to the computer microworld instruction perform significantly better on the integers post-test than those exposed to the other two treatments. In particular, the computer microworld groups were more successful in completing computational exercises that involved adding and subtracting integers. The conclusions also assert that different techniques are appropriate for different students. Implications of these conclusions can be classified under two headings: implications for practice and theoretical implications.
Implications for Practice

A teacher need not be reluctant to use a computer microworld as a primary source of instruction. According to analyses for the entire sample and subsamples, the microworld treatment groups did as well as or better than the other treatment groups. Especially in the area of learning or retaining computational skills, the microworld treatment seems quite effective. Teachers often fear procedural knowledge will erode (or never be acquired) if students use technology. This study may help some teachers to overcome their "techno-phobia."

Teachers must continue, however, to determine on a case-by-case basis which methodology will truly benefit their students most. For students who have some working knowledge of integers, using new techniques may not be as productive as continuing along the same instructional vein. If new techniques are used, teachers may find that helping these students see connections between the methods can possibly remove some retroactive interference/proactive inhibition and even deepen understanding. Above all, it should be noted that this computer microworld is not offered as a panacea – it is merely an option which appears to be effective in some circumstances.
Theoretical Implications

In some sense, this study supports the notion that students do actively construct personal and meaningful knowledge. In the microworld treatment, students were never told about rules for adding and subtracting integers. Through experimentation, they learned how to interact in the microworld and then had to transfer that knowledge into mathematical understanding. In essence, the students were forced to discover how integers work. Furthermore, the action feedback principle appears to be a legitimate extension of Dienes' theory.

Students working in the microworld – a representational system with a constraint-support structure imposed upon actions – outperformed students in the concrete manipulatives groups (students whose errors were corrected rather than disallowed). Students in the concrete manipulatives group had to be explicitly told and consistently reminded of how to use the chip model by the teacher.

The linked representations offered by the microworld may also have helped students bridge the gap between the semi-concrete and the symbolic worlds. The automatic translation of chip action to scripted text was a unique feature for students exposed to the microworld
system. By being relieved of this translating duty, students may have been able to examine the underlying operating structure of the representing world, as opposed to worrying about the details of how to use the appropriate and conventional mathematical terms. Students in the concrete manipulatives group, on the other hand, had to concentrate on changing "actions into words;" this additional step may have opened yet another opportunity for misconception.

For theorists who support the benefits of multiple representations, it may be surprising to note that students fared worst with the combination treatment. Perhaps students simply became tired of working in both the semi-concrete and concrete representing worlds. With the similarity of their structures, fatigue – and annoyance – from vacillating back and forth may have played a key role in the downfall of this method with older students in particular. These are students familiar with both manipulatives and technology, and they may have already had clear preferences. On several occasions, the eighth graders asked the teacher, "Can't we just stick to either the chips or the computer?"

Furthermore, quickly shifting between several representing worlds
can be self-defeating. Students need time to develop concepts; understanding how to operate in a representing world is sometimes a slow process. Thus, before the students may have been able to "get a handle" on using the concrete chips, they were pulled into the arena of cyberspace. Then, just as swiftly, the students were forced to use the hands-on approach again. With this rapid movement, making connections between the two representing worlds may have been impossible. Ultimately, the combination treatment should have provided a catalyst for students' examination of the constraint-support structure of the microworld: "Why won't the computer let us do this? Is this similar to what the teacher was saying when we were playing with the chips?" Unfortunately, with relatively little "inclusive" time for concept development in either representing world, as Dufour-Janvier, Bednarz, and Belanger (1987) note, a multitude of representations may be detrimental:

The child comes to the point where he no longer sees what is important in what is presented to him. Either he starts thinking that everything presented is pertinent, or at the other extreme, that all is accessory and, in a sense, nothing is relevant. Either case is equally bad. (p. 115)
6.4 Recommendations

On the basis of the conclusions from this study, it is apparent that more research is needed in the area of students' use and understanding of integers. Some factors affecting the acquisition of conceptual and procedural knowledge of integers have been explored in this study and many others have been identified which need further investigation. The following areas are recommended for future research:

1. Design a study incorporating "traditional" methods of teaching about integers. If previous knowledge is somehow "blocking" the effectiveness of new techniques, perhaps a return to tradition is what is needed to help some students excel. Furthermore, at this point, it is not known how the "rule-based" approach compares to the treatments used in this study.

2. Replicate the study, with adaptations for other school districts and grade levels. To strengthen the ability to generalize results, the key elements of this study must be used in other places with other people. Some interesting questions to consider include: At how early a grade level can significant results be obtained? Looking at various classroom cultures, is a novelty effect evident when using the treatments from this study? Will regression analyses of the pooled sample (including the subjects from this study) reveal more attribute variables which significantly effect students'
achievement on the integers post-test?

3. Modify the study to include longer durations of treatments. Most teachers do not teach addition and subtraction of integers in only one week. Although the pilot study indicated that students could learn about integers in a short time span, perhaps more impressive gains would have been evident if students had been given more time with the materials. Especially for students in the combination treatment, time may have been a key element of downfall in processing information. Students in the combination treatment were quickly pulled to and fro in the representing worlds and may not have been able to deeply or adequately reflect on either. As a corollary to the longer duration suggestion, the combination treatment could be re-designed. Instead of alternating days between the concrete manipulatives and the computer microworld, allow students to work several days in one representing world and then switch to the other. By chunking the time frame of the design, students in the combination treatment may show an improvement in scores, benefiting from the "inclusive" (one-world-at-a-time) development of concepts.

4. Design a study using alternative assessments, as opposed to paper-and-pencil tests. If students are unwilling to write about mathematics or feel uncomfortable when asked to do so, other avenues must be pursued to determine what they truly understand. In this case, performance task assessments or
interviews may be an appropriate method to use. The reliability and validity of these types of instruments, of course, becomes a critical factor. It may become important to cross-analyze results of a study using these alternative assessments with a study using a paper-and-pencil test. Furthermore, using classrooms where writing in mathematics has become a staple can introduce another attribute variable worth considering.

5. Design a qualitative research study to investigate how students are building their knowledge of integers when interacting with the computer microworld. Some critics may argue that the microworld treatment merely substitutes one "rule" for another – instead of having students memorize "when you subtract, you add the opposite," they simply learn that "you can only add or remove pairs of opposite chips." While it is true that (hopefully) students leave the worksessions with a sense of the "paired-zero rule," the legitimate question of concern should be: "How do students come to recognize and form those 'rules'?") Does the microworld underscore the difference between a seemingly arbitrary rule and a "law of nature?" Is there something fundamentally different that students do and learn when working with the computer microworld? These are questions which can only be answered through qualitative research.
APPENDIX A

LETTERS OF PERMISSION
Date

Principal
School
Street Address
Columbus, Ohio  ZIP

Dear Principal Lastname:

Recently I spoke with Judy Silbaugh, the district's mathematics specialist, on the subject of enhancing mathematics learning with technology. In particular, my dissertation research deals with teaching integer arithmetic using computer cybernetics. Judy recommended that I inquire as to whether your school would be willing to participate in this study.

To participate, I would need either one sixth grade teacher who teaches two classes of mathematics or one eighth grade teacher who teaches two classes of mathematics. Students would then engage in an seven day program of learning about integers and integer addition & subtraction.

All program materials would be provided, including daily lessons, "homework" assignments, and tests. For participating, teachers will be compensated $50.00 for three hours of training time and the school will be able to keep the classroom set of manipulatives and educational software.

If you believe a teacher in your building may be willing to participate in this venture, please pass this information along to him/her. Enclosed you will find a response postcard, indicating whether or not your school would like to be involved. Once I have received the postcards, detailed information will be provided for those interested.

If you have any questions or concerns, please feel free to contact me anytime (office telephone: 292-1257; home telephone: 878-3251). The opportunity to forge a link between the university and your school is an exciting prospect — thank you for your time and consideration.

Sincerely,

Jeffrey P. Smith
The Ohio State University
Dear Parents & Guardians:

In an effort to describe how students' process and come to understand mathematics, The Ohio State University will be providing materials specially designed for a "hands-on" teaching approach. To help us determine the effectiveness of these materials, we are asking that students participate in a dissertation research project during our unit on integers. The information and test scores gathered from the students during the project will remain confidential; the teaching content and methods will not in any way detract from the required course of study. The specific content is identified as "adding and subtracting positive and negative numbers."

This letter serves as notification of the research project and returning the bottom portion with your signature indicates your acknowledgment of permission to participate.

If you have any questions or concerns, please feel free to contact either of us at the numbers listed below. With the goal being greater understanding, our work together may be a vital key to opening a window on mathematics. Thank you for your time and consideration.

Sincerely,

TeacherName
SchoolName
Telephone: SchoolTele

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Jeffrey P. Smith
The Ohio State University
Telephone: 292-1257

__________________________ has my permission to participate in the dissertation research project Working with Integers. I understand that all information and test scores will remain confidential and that participation is purely voluntary.

__________________________
Parent/Guardian Signature

__________________________
Student Signature
APPENDIX B

PRE- AND POST-TESTS
Integers Pre-Test

Part A

1. Determine the following sums:
   a. (-13) + (-9)  
   b. (+6) + (-11)  
   c. (-7) + (+7)  
   d. (+23) + (-10)  
   e. (-8) + (+12) + (-3)  
   f. (+34) + (+42)  
   g. (+27) + (-58)  
   h. (-82) + (+79)  
   i. (-53) + (-96)  
   j. (-23) + (+54) + (-61)  

2. Fill in numbers to make this sentence true:
   \[ \bigcirc + \bigcirc + \bigcirc + \bigcirc = -5 \]

3. Is it possible to add a positive integer and get a final answer that is negative? Explain your answer.

4. Can you predict what the sign of the final answer is when you add two integers? Explain your answer.
Part B

1. Determine the following:
   a. \((-8) - (-5)\)  
   b. \((+25) - (-11)\)  
   c. \((+2) - (+7)\)  
   d. \((-6) - (+6)\)  
   e. \((-4) - (-6) + (+18)\)  
   f. \((+27) - (+35)\)  
   g. \((+18) - (-41)\)  
   h. \((-58) - (+77)\)  
   i. \((-49) - (-86)\)  
   j. \((+21) - (-37) - (+24)\)

2. Fill in numbers to make this sentence true:
   \[\Box - \Box - \Box = +6\]

3. Is it possible to subtract a positive integer and get a final answer that is negative? Explain your answer.

4. Can you think of any rules or shortcuts to use when you subtract integers? Explain your answer.
Integers Post-Test

Part A
1. Is it possible to add a negative integer and get a final answer that is positive? Explain your answer.

2. Can you predict the sign of the final answer when two integers are added together? Explain your answer.

3. Fill in numbers to make this sentence true:

\[ \square + \square + \square + \square = +6 \]

4. Determine the following sums:
   
   a. \((-15) + (-6)\)  
   b. \((+5) + (-10)\)  
   c. \((-8) + (+8)\)  
   d. \((+30) + (-17)\)  
   e. \((-4) + (+6) + (-18)\)  
   f. \((+32) + (+49)\)  
   g. \((+28) + (-57)\)  
   h. \((-84) + (-72)\)  
   i. \((-51) + (-96)\)  
   j. \((-24) + (+55) + (-63)\)
Part B

1. Is it possible to subtract a negative integer and get a final answer that is positive? Explain your answer.

2. Can you think of any rules or shortcuts to use when you subtract integers? Explain your answer.

3. Fill in numbers to make this sentence true:

\[ \_{\text{-}} \_{\text{-}} \_{\text{+}} = +4 \]

1. Determine the following:
   a. \((-9) - (-6)\)   f. \((+26) - (+34)\)
   b. \((+35) - (-10)\)   g. \((+16) - (-45)\)
   c. \((+3) - (+8)\)   h. \((-62) - (-88)\)
   d. \((-9) - (-9)\)   i. \((-53) - (-79)\)
   e. \((-2) - (+8) + (+12)\)    j. \((+23) - (-36) - (+29)\)
APPENDIX C

WORKSESSION SCRIPTS
Counting Chip Collections

Overview
This portion of the unit is used to introduce students to signed numbers. Using two-colored chips, a model for the integers is developed.

Prerequisites
None

Materials
Gray and red counting chips or Cy-Bee Chips Intro Module Software
Activity Sheet 1
Overhead colored chips

Teaching Notes
The teacher should avoid "giving" answers to questions. Many of the tasks have multiple solutions and flexibility in thinking is a desired goal. In particular, during this portion of the unit, finding several different chip collections to represent the same integer is of primary concern. Encourage the use of the manipulatives.
Activity
1. Draw a chart like the one shown below on the chalkboard. Drop a small handful of overhead chips on the overhead. Record the information about this collection on the first line of the chart.

<table>
<thead>
<tr>
<th>Collection #</th>
<th>Total Pieces</th>
<th># of Gray</th>
<th># of Red</th>
<th>Net Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>2 gray</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Examples
This collection contains 8 total pieces, 5 gray and 3 red. Its net value is 2 gray.

This collection contains 12 total pieces, 8 gray and 6 red. Its net value is 0.

Notes
1. Counters are red on one side and gray on the other.

Gray and red chips are said to be of opposite color. The net value of a collection of counting chips is the number of gray or red chips in the collection that can not be matched with a chip of the opposite color.

A collection in which all chips can be matched has a net value of 0.

2. Ask a student to take a small collection of overhead chips (about a dozen) from a container of red and gray mixed. Have the student put the chips on the overhead. Record the information about this collection on the chart. Repeat this action with different students until there are several entries on the chart.

3. Discuss the information contained in the chart. In particular, draw out the students’ observations concerning net values.

2. You may want to have the student record the information on the chart as you move around the room with the container. You can have a student report the number of chips in their collection and then ask the class for the net value of the collection.

3. For a net value of 3 red, for example, good discussion questions include:
What are some other collections that have a net value of 3 red?
What is the collection containing the fewest number of chips that has a net value of 3 red?
If a collection has a net value of 3 red and contains 10 gray chips, how many red chips are in the collection?

Note the following:

* Many different collections may result in the same net value.

* Adding or removing an equal number of red and gray chips from a collection does not change its net value.

* For a given non-zero net value, the collection with the fewest chips that has that net value contains all red pieces or all gray chips — the number and color matches the net value. For example, the collection with the fewest chips that has a net value of 3 red is a collection of 3 red chips.

* The collection with the fewest chips that has net value zero is the empty collection — the collection containing no chips.

4. A plus sign will indicate a gray net value and a minus sign will indicate a red net value.

For example, a net value of 3 gray will be written +3 (read "positive three"); a net value of 2 red will be written -2 (read "negative two").

Numbers to which a plus or minus sign are attached will be called signed numbers. You may want to return to the chart developed earlier and write the appropriate signed number alongside the net values.

<table>
<thead>
<tr>
<th>Collection #</th>
<th>Total Pieces</th>
<th># of Gray</th>
<th># of Red</th>
<th>Net Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>2 gray +2</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Drop a small handful of counting chips onto the overhead. Ask the students for the net value of the resulting collection. Then, ask the students for the net value of the collection that would be obtained if all of the chips were reversed. Repeat this action for two or three other collections.

5. By the end of this activity, students should recognize that reversing all chips in a collection changes the sign of its net value.

Thus, if all chips in a collection whose net value is +2 are reversed, the resulting collection will have a value -2. See the example below.

Example

This collection has a net value of +2.

This collection has a net value of -2.

Use the results of #5 to explain opposite collections and opposite net values to the students.

Two collections are called opposites of each other if one can be obtained from the other by reversing all of the chips.

The collections above are opposite collections. Their net values, +2 and -2, are opposite net values. That is, +2 is the opposite of -2; and, -2 is the opposite of +2.

Note that a collection which has the same number of gray and red chips is its own opposite. Thus, the opposite of 0 is 0.

6. Distribute Activity Sheet 1 to each student. Distribute materials to pairs. “Work in pairs, but fill out your own worksheet.” Discuss with them the methods they used to arrive at their answers.

6. You may need to remind students that a minus sign indicates a red net value and a plus sign indicates a gray net value. Some students may try to arrive at answers without using counting chips.

Encourage students who are having difficulty to use them.

7. Explain to students that net values are a model of the integers.

7. The collection of gray net values -1, +2, +3, ... represents the positive integers. The collection of red net values -1, -2, -3, ... represents the negative integers. A 0 net value represents the zero integer. Perhaps reinforce with a number line.
Activity Sheet 1

Part A
Fill in the missing numbers:

<table>
<thead>
<tr>
<th>Collection</th>
<th>Total # of Pieces</th>
<th># of Gray Pieces</th>
<th># of Red Pieces</th>
<th>Net Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>6</td>
<td></td>
<td>+3</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>2</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>12</td>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>4</td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>E</td>
<td>10</td>
<td></td>
<td></td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>7</td>
<td></td>
<td></td>
<td>+7</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td></td>
<td>8</td>
<td>+3</td>
</tr>
<tr>
<td>H</td>
<td>13</td>
<td></td>
<td></td>
<td>-5</td>
</tr>
</tbody>
</table>

Part B
1. Draw three different collections that have a net value of +8.

2. Why do you think the integer 0 is never given a sign?
Adding Integers

Overview
The colored chip model for integers is used to find the sums of signed numbers.

Prerequisites
Counting Chip Collections activity

Materials
Gray and red counting chips or Cy-Bee Chips Addition software
Workmats (for Manipulatives Group)
Computer Reference Cards (for the Computer-Based Group)
Activity Sheet 2
Overhead workmat
Overhead computer screens for addition
Overhead colored chips

Teaching Notes
For the Computer-Based Group, it is important that the teacher demonstrate using the overhead computer screens. Showing the pressing of the buttons and the subsequent result is a necessary component to the instruction. Before the students are "sent out on their own," they should have an understanding of what each button does and how to enter a solution to a problem. Also, draw students' attention to the script screen in the lower left-hand corner - this recounts in words the actions that the students have performed.
Activity

1. Write \((+3) + (-7)\) on the chalkboard. Ask the students to suggest ways in which the two-colored chips can be used to compute the value of this expression.

Notes

1. The students may have a variety of suggestions. One way to use the pieces to determine the sum of \((+3) + (-7)\) is to combine a collection whose net value is \(+3\) with a collection whose net value is \(-7\) -- then, find the net value of the combined collection.

Example

Step 1: Placing the addends on the workmat.

Step Two: Combining the addends. Ask students to interpret the net value.

Step Three: Remove zeroes, if students deem it appropriate. The combined collection has a net value of \(-4\).
2. If necessary, do a second example: \((-6) + (+8)\)

Example

Step 1: Placing the addends on the workmat.

Step Two: Combining the addends. Ask students to interpret the net value.

Step Three: Removing zeroes, if students deem it appropriate. The combined collection has a net value of +2.

3. Distribute a copy of Activity Sheet 2 to each student. Distribute materials to pairs. "Work in pairs, but fill out your own worksheet."

3. Some students may try to arrive at answers without using the tools. Encourage those who are having difficulty to use them.

2. To be consistent, put the 6 red chips in the left-hand partition and the 8 gray chips on the right-hand partition. Also, when combining addends, put the gray chips in the top row and the red chips in the bottom row.
Activity Sheet 2

Part A
Use the colored chips to determine the following sums:

a. (+ 5) + (+ 8)  
   e. (-4) + (+ 4)

b. (+ 7) + (-5)  
   f. (-7) + (+ 11)

c. (-4) + (-5)  
   g. (+ 11) + (-7)

d. (+ 2) + (-6)  
   h. (+ 8) + (-2) + (-14)

Part B
1. Write five different number sentences that show two integers adding to -7.

   $\bigcirc + \bigcirc = -7$  
   $\bigcirc + \bigcirc = -7$  
   $\bigcirc + \bigcirc = -7$

   $\bigcirc + \bigcirc = -7$  
   $\bigcirc + \bigcirc = -7$

2. Have you thought of any shortcuts - or rules - to help you add integers? Describe your rules and give examples of how they work.

Part C
First, predict the sign. Then, try to determine the following sums:

a. (-25) + (-40)  
   b. (-60) + (+52)  
   c. (-35) + (+50)

   +   -   +   -   +   -
Subtracting Integers

Overview
The colored chip model for integers is used to find the differences of signed numbers.

Prerequisites
Adding Integers activity

Materials
Gray and red counting chips or Cy-Bee Chips Subtraction software
Workmats (for Manipulatives Group)
Computer Reference Cards (for the Computer-Based Group)
Activity Sheet 2
Overhead workmat
Overhead computer screens for subtraction
Overhead colored chips

Teaching Notes
For the Computer-Based Group, it is important that the teacher demonstrate using the overhead computer screens. Showing the pressing of the buttons and the subsequent result is a necessary component to the instruction. Before the students are "sent out on their own," they should have an understanding of what each button does and how to enter a solution to a problem. If students encounter difficulty in getting correct answers, alert students as to where to "find" the solution to the problem - the partitioning of the workmat may be interfering with their notion of subtraction (the instructional purpose of the partition is to demonstrate the inverse relationship between addition and subtraction). Also, draw students' attention to the script screen in the lower left-hand corner - this recounts in words the actions that the students have performed.
Activity
1. Write \((-5) - (-3)\) on the chalkboard. Ask the students to suggest ways in which the two-colored chips can be used to compute the value of this expression.

Notes
1. One way is to start with a collection of 5 red chips. Then, take away 3 red chips. Using the partitioned workmat reinforces the idea that subtraction is the inverse of addition.

Example
Step 1: Start with \(-5\) on the workmat.

Step Two: Remove two red chips; place them on the right-hand side of the partition. Ask for the interpretation of the net value. The net value is \(-3\).

2. Write \((+3) - (-2)\) on the chalkboard. Ask the students to suggest ways in which the colored chips can be used to compute the value of this expression.

2. One way is to take away a collection whose net value is \(-2\) from a collection whose net value is \(+3\). In order to do this, one must build an appropriate collection for \(+3\).

The collection with the fewest pieces for \(+3\) is 3 gray chips. Unfortunately, there are no red chips to remove. However, adding 2 reds & 2 grays to the \(+3\) collection would not change the net value.

The collection now has 5 grays and 2 reds, with a net value of \(+3\). At this point, we are able to remove \(-2\) (i.e. 2 red chips), leaving 5 grays.
Example

Step 1: Start with +3 on the workmat.

Step Two: Add 2 reds and 2 grays (this has the effect of adding zero). Ask students to interpret the net value of the collection. (The net value is still +3.)

Step 3: Remove -2 from the collection. This leaves a net value for the collection of +5.

3. If necessary, do a third example: (+5) – (+8).

4. Distribute a copy of Activity Sheet 3 to each student. Distribute materials to pairs. “Work in pairs, but fill out your own worksheet.”

3. If students have problems determining the final answer, it might be a result of not understanding “where” to find the answer on the workmat (on the left-hand side of the partition).

4. Some students may try to arrive at answers without using the tools. Encourage their use.
Activity Sheet 3

Part A
Use the colored chips to determine the following:

a. \(+8\) \(-\) \(+3\)

b. \(-7\) \(-\) \(-2\)

c. \(-4\) \(-\) \(+7\)

d. \(+2\) \(-\) \(-5\)

e. \(+2\) \(-\) \(+5\)

f. \(-4\) \(-\) \(-4\)

g. \(0\) \(-\) \(-5\)

h. \(+8\) \(-\) \(-2\) \(-\) \(+14\)

Part B
1. Solve each of these number sentences:

\[
\begin{align*}
0 - \bigcirc &= +8 \\
0 - \bigcirc &= -3 \\
+2 - \bigcirc &= +6 \\
0 + \bigcirc &= +8 \\
0 + \bigcirc &= -3 \\
+2 + \bigcirc &= +6 \\
\end{align*}
\]

2. How are subtraction problems related to addition problems?

Give two examples to demonstrate your ideas.

\[
\begin{align*}
\bigcirc - \bigcirc &= \bigcirc \\
\bigcirc + \bigcirc &= \bigcirc \\
\bigcirc - \bigcirc &= \bigcirc \\
\bigcirc + \bigcirc &= \bigcirc \\
\end{align*}
\]

Part C
1. Give three examples of subtraction problems that have a result of zero.

\[
\begin{align*}
\bigcirc - \bigcirc &= 0 \\
\bigcirc - \bigcirc &= 0 \\
\bigcirc - \bigcirc &= 0 \\
\end{align*}
\]

2. How can you tell if a subtraction problem will have a result of zero?
Activity Sheet 3 (continued)

Part D
First, predict the sign. Then, try to determine the following:

a. \((-25) - (-50)\) \(+\) \(-\)

b. \((+80) - (+73)\) \(+\) \(-\)

c. \((+70) - (-35)\) \(+\) \(-\)

d. \((-45) - (-40)\) \(+\) \(-\)
APPENDIX D

SOFTWARE REFERENCE CARD
Cy-Bee Chips Addition Reference Card

To move a chip onto the workmat:

- Put the mouse pointer over the chip. Click the mouse button and hold it down. Drag the chip onto the workmat.

The following buttons are onscreen at all times:

- Use the Question Button when you are ready to present a solution to your problem.
- Use the Reset Button when you want to clear the workmat and begin a new problem.
- Use the Quit Button when you are ready to clear the workmat and quit the program.

The following buttons perform special functions:

- Use the Snap Button when you are ready to combine two collections of chips.
- After clicking on pairs of gray & red chips, use the Remove Zeros Button to remove collections of “0” from the workmat.

Cy-Bee Chips Subtraction Reference Card

To move a chip onto the workmat:

- Put the mouse pointer over the chip. Click the mouse button and hold it down. Drag the chip onto the workmat.

The following buttons are onscreen at all times:

- Use the Question Button when you are ready to present a solution to your problem.
- Use the Reset Button when you want to clear the workmat and begin a new problem.
- Use the Quit Button when you are ready to clear the workmat and quit the program.

The following buttons perform special functions:

- Use the Snap Button to prepare the initial collection for the subtraction process.
- If necessary, use the Add Zeros Button to add collections of “0” (pairs of gray & red chips) to the workmat.

To “take away” a chip from the initial collection:

- Move the mouse pointer over the chip. Click the mouse and the chip will be marked (with an X). Hold the mouse down and drag the marked chip to the right-hand side of the workmat.
BIBLIOGRAPHY


