THE EFFECTS OF LOCALIZED BLADE EN DWALL SUCTION ON SURFACE
HEAT TRANSFER

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ABSTRACT

Two methods of flow control were designed to mitigate the effects of the horseshoe vortex structure (HV) at an airfoil/endwall junction. An experimental study was conducted to quantify the effects of localized boundary layer removal on surface heat transfer in a low-speed wind tunnel. A transient infrared technique was used to measure the convective heat transfer values along the surrounding surface. Particle image velocimetry was used to collect the time-mean velocity vectors of the flow field adjacent to the endwall along three planes of interest. Boundary layer suction was applied through a thin slot, in the leading edge of the airfoil at two heights. The first height, referred to as Method 1, was immediately along the endwall, the second height, Method 2, was located at ~1/3 of the approaching boundary layer height. Five suction rates were tested, 0%, 6.5%, 11%, 15% and 20% of the boundary layer mass flow was removed at a constant rate. Both methods reduced the effects of the HV with increasing suction on the symmetry, 0.5-D and 1-D planes. Method 2 performed better at reducing the surface heat transfer but Method 1 outperformed Method 2 aerodynamically by completely removing the HV structure when the 11% suction rate was applied. This method however produced other adverse effects such as high surface shear stress and localized areas of high heat transfer near the slot edges at high suction rates.
Dedicated to my parents
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<td>Area</td>
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<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
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<tr>
<td>$D$</td>
<td>diameter of streamlined cylinder</td>
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<tr>
<td>$E$</td>
<td>energy</td>
</tr>
<tr>
<td>$Fo$</td>
<td>Fourier number</td>
</tr>
<tr>
<td>$H$</td>
<td>boundary layer shape factor ($\delta^*/\theta$)</td>
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<tr>
<td>$HV$</td>
<td>horseshoe vortex</td>
</tr>
<tr>
<td>$K$</td>
<td>Kelvin</td>
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<tr>
<td>$P$</td>
<td>pressure</td>
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<td>particle image velocimetry</td>
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<td>$St$</td>
<td>Stanton number ($h_c/\rho U_{\infty}c_p$)</td>
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<td>$SV$</td>
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<td>$m$</td>
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<tr>
<td>$q$</td>
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<td>$t$</td>
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SYMBOLS

\( \Gamma \) circulation
\( \Delta \) change in
\( \Phi \) dissipation function
\( \nabla \) del operator

\( \alpha \) thermal diffusivity \((k/\rho c_p)\)
\( \delta \) boundary layer thickness
\( \delta^* \) boundary layer displacement thickness
\( \theta \) boundary layer momentum thickness
\( \mu \) dynamic (absolute) viscosity
\( \nu \) kinematic viscosity \((\mu/\rho)\)
\( \rho \) fluid density
\( \tau \) shear stress OR dummy time integration variable
\( \varphi \) angle along x-y plane of coordinate system
\( \omega \) vorticity

SUBSCRIPTS

\( CV \) control volume
\( NS \) No Suction

\( cond \) conduction
\( conv \) convection
\( f \) fluid
\( gen \) generation
\( i \) index in X-direction
\( in \) into control volume
\( init \) initial
\( j \) index in Z-direction
\( out \) out of control volume
\( rad \) radiation
\( s \) surface
\( ss \) suction slot
\( st \) stored
\( t \) thermal
\( x \) component in the X-direction
\( y \) component in the Y-direction
\( z \) component in the Z-direction
\( \infty \) freestream
CHAPTER 1

INTRODUCTION

Gas turbine engines are constantly being pushed to the limit. Higher pressure ratios and hotter temperatures continue to increase the thermodynamic efficiencies of these complex systems. As time passes and efficiencies soar, smaller and smaller adjustments are being made to fine tune the engine, especially in its main components, the compressor, combustor and turbine. This study examines the effects of applying localized endwall flow control to mitigate the aerodynamic losses in a turbine blade passage.

1.1 BACKGROUND

Gas turbine engines are based off the standard thermodynamic Brayton cycle. The engine compresses an atmospheric intake, mixes it with a high-energy fuel, then burns the mixture and uses the combustive energy to generate power and thrust. Figure 1.1 below shows a cut-away of a typical axial-flow gas turbine engine; air flow through the engine is from left to right. As air enters the engine, it passes through a diffuser which slows the flow and prepares it for the compressor. Work, supplied by the turbine, is done on the air in the compressor to increase the pressure across a series of stages.
Axial-flow engines generally have multi-stage compressors, consisting of several rotor-stator pairs. The high-pressure air then enters the combustor where it mixes with fuel through a series of cans and ducts. The fuel-air mixture is then burned and passed immediately to the turbine where work is extracted from the flow and transmitted via the engine shaft to the compressor. This work can also be used to power multiple subsystems of an aircraft. Finally the air is exhausted through a throttling nozzle, accelerating the flow and creating thrust in the axial direction.

![Figure 1.1: Schematic of an axial-flow gas turbine engine; major components labeled.](image)

Work is extracted from the engine when the combustion products are expanded through the turbine section. The expanding gas passes over a series of airfoils that produce lift, spinning the engine’s central shaft and generating work. Ideally, this occurs without entropy generation; which is known to limit the engine’s ability to do useful work. However, no system is “ideal” and the gas-turbine engine is no exception. The turbine does not extract the ideal amount of work, as modeled by the Brayton cycle, due to increases in entropy known as “losses.”

A typical rotor passage is plagued with losses from a multitude of sources. Viscous effects, shock waves and even heat transfer across temperature differences, all
contribute to a reduction in the performance of the system. Endwall losses in the turbine are a major source of decreased efficiency; typically accounting for a third of the total loss in the turbine [9]. Sometimes called secondary losses, these describe entropy generation due to viscous effects along the annulus walls, surrounding a blade passage. A well known, but not well understood, contributor to secondary loss in a turbine blade passage is the infamous horseshoe vortex. This structure forms in the corner region between the endwall and a blade’s leading edge; and demonstrates high levels of randomness and complex three-dimensionality which makes it very difficult to understand. To date, much research has been conducted to discover the source, effects and methods to control this loss mechanism.

1.2 LITERATURE REVIEW

For years, researchers have studied the horseshoe vortex structure. This flow phenomena occurs adjacent to a surface when an obstruction is placed normally into a flow path. It has been seen at the intersection of aircraft wings with the fuselage, on submarines where equipment protrudes from the sides, and even at the base of bridge piers constructed in river beds. Only recently have investigators begun to understand the cause and behavior of the horseshoe vortex (HV).

Also known as a juncture vortex, the HV occurs when an impinging viscous boundary layer flow, traveling along an endwall, senses the pressure produced by an obstruction downstream. The strong adverse pressure gradient forces fluid to divert away from the obstruction’s leading edge. Consequently, higher momentum fluid will flow down the front face of the obstruction into the lower momentum fluid of the boundary
layer. But again, the endwall produces an adverse pressure gradient, turning the fluid back upstream into the approaching boundary layer. The boundary layer meets the reverse flow creating an unsteady roll-up of fluid known as a vortex. This structure is continuously fed by fluid downwash along the face of the obstruction, growing and strengthening the vortex tube in the juncture region. To the sides of the obstruction, fluid is able to freely travel downstream without the presence of an inhibiting pressure gradient. This fluid will carry the ends of the corner region vortex downstream, on either side of the obstruction, creating a “necklace” or “horseshoe” about its leading edge. This circumventing vortex structure is known as the horseshoe vortex [3, 14, 18].

A recent study by Sabatino et al. combines the work of previous researchers to summarize the flow features present in the corner region of a horseshoe vortex system. Figure 1.2 represents the time-mean streamline topologies of the HV structure along the streamwise plane located at the object’s leading edge; called the symmetry plane. Indicated in the figure is what Sabatino et al. call the “most important” flow structures of the system [20].

Figure 1.2 Typical time-mean, symmetry plane streamlines of a horseshoe vortex system [20].
Sabatino et al. indicate four separate vortices that develop at the endwall-bluff body juncture. The largest and most dominant structure of the system is the horseshoe vortex (HV). Not all fluid however is drawn into the HV, some is fed into two other vortex structures duly named, the secondary vortex (SV) and the tertiary vortex (TV). These occur upstream of the HV where the impinging boundary layer meets the reverse flow along the endwall. The TV rotates in the same direction as the HV, and the SV, which is found between the HV and TV, rotates in the opposing direction. The last, distinct vortex structure is a small, counter-rotating feature that fills the gap between the HV and the endwall juncture, called the corner vortex (CV). The size, position and rotation of these four vortices are designated in Figure 1.2. Also indicated is what is known as the saddle point of the system. This is the location where the boundary layer flow is lifted, or separated, from the surface by a downstream flow structure; for this reason it is also known as the separation point. This point usually occurs at an upstream distance equivalent to 1/2 the maximum thickness of a bluff body obstruction, as is indicated in the figure [18, 20].

The horseshoe vortex system is particularly detrimental in gas-turbine engines because of its use of blade rows. This allows the downstream leg of the HV from one blade to combine with the opposite leg of the HV of the adjacent blade. In a blade passage, the part of the HV that wraps around the pressure surface is referred to as the pressure side leg, and the part that wraps around the suction surface is known as the suction side leg. These vortices are counter-rotating when they meet in the blade passage.
The combining of opposing pieces of the HV is compounded by another loss mechanism known as the passage vortex (PV). Similar to the HV, the PV develops due to the inherent vortical motion of the endwall boundary layer, making it a secondary flow. As flow is turned through a curved passage the boundary layer vorticity reorients to the streamwise direction with the same sense as the pressure side leg of the HV. The PV combines with this leg of the HV, strengthening the vortex, and migrates across the passage from pressure surface to suction surface before interacting with the counter-rotating suction side leg of the adjacent blade. Velocity gradients then stretch and strengthen the vortex tubes as they travel toward the exit of the blade row [11, 15, 23]. This secondary flow interaction is demonstrated in the schematic of Figure 1.3; modeled by Langston et al.

![Figure 1.3: Three-dimensional separation of boundary layers through a turbine cascade as modeled by Langston et al. [15].](image)

The model by Langston et al. shows the suction side leg of the HV pushed to the suction surface by the PV near the entrance of the passage. The passage vortex then
holds the counter-rotating $HV$ in the corner region, as it follows along the suction surface to the exit of the passage. This is in contrast to a model by Sharma et al. who show the $HV$ wrapping around the passage vortex as it travels up the blade’s suction surface. With this model, presented in Figure 1.4, up to 50% of the aerodynamic losses in a stator row can be attributed to secondary flows in the endwall region [23].

![Figure 1.4: Three-dimensional separation of boundary layers through a turbine cascade as modeled by Sharma et al. [23].](image)

Several other models of this same flow field exist, but even with inconsistencies, it is evident that the complicated interactions of the horseshoe vortex system have a direct impact on the efficiency and attrition of turbomachinery. Numerous studies have shown turbulent boundary layers to have aperiodic, chaotic, large-scale horseshoe vortices that cause high turbulent intensities, surface pressure fluctuations, and heat transfer rates that scour the surface upstream of an obstacle [24]. For these reasons, many researchers have investigated the possibility of controlling the endwall boundary layer in order to attain more favorable results from these juncture flows.
Many types of passive techniques have been tested in an effort to reduce the HV and its effects through a blade passage. For example, a study by Mahmood et al. investigates the effects of leading edge modification and endwall contouring on the horseshoe and passage vortices in a linear cascade. Mahmood et al. tested two types of leading edge fillets and a variation in endwall height designed to retard the passage vortex flow by modifying the pressure gradient between blades. Results show all methods weaken the HV but do not remove it completely. Total pressure losses were mitigated in the near-wall region and surface heat transfer was lessened through the passage [16].

A 1997 study by Barberis et al. considered both passive and active methods of flow control. The passive technique, a fillet located at the wing-body junction, demonstrated no noticeable alteration to the horseshoe vortex structure. The active control technique however, had a significant effect. Barberis et al. pulled suction through a rectangular opening located along the endwall, upstream of an axisymmetric airfoil. Five rates, between 10% and 110% of the boundary layer mass flow, were removed from one of two locations centered about the airfoil’s leading edge. This method clearly decreased the vortex size and moved the separation point toward the airfoil’s leading edge; decreasing near wall vorticity. Barberis et al. also noted that of the two upstream positions tested, the location closest to the leading edge was most effective at reducing the size of the separation zone [1].

Many researchers have found boundary layer suction to be far more successful than previous passive techniques. Phillips et al. tested four removal rates, from 100% to 350% of the boundary layer flow, through a rectangular slot located just upstream of a
wing-fuselage juncture. This demonstrated a considerable effect on the size and significance of the HV because the source of spanwise vorticity from which the HV is generated had been removed [17]. Seal et al. found spatially limited surface suction to effectively weaken the surface interactions occurring in the vortex system and postulated that the technique could be used to mitigate multiple adverse effects of the system; such as noise, unsteady forces and buffeting resulting from the downstream extensions of the vortex. Seal et al. further noted that a reduction in associated heat transfer should be possible, thus eliminating problematic local “hot spots” in the turbine [22].

Many studies into the phenomena of juncture flows combine the analysis of aerodynamic and surface heat transfer measurements to fully identify the flow structures and their effects. For example, Kang et al. studied the effects of Reynolds number on the horseshoe vortex system as it traveled through a turbine vane passage using laser-Doppler velocimetry, infrared thermography and static pressure measurements. The authors were able to learn much about the size, strength, location and behavior of the HV by examining the flow field data in conjunction with the surface pressure and heat transfer measurements [14].

A recent study by Bloxham et al. combined the knowledge gained through these previous studies by locating boundary layer suction immediately at the endwall-airfoil intersection. Bloxham et al. applied very low removal rates through a 1-mm slot located at the leading edge of an axisymmetric airfoil. Total pressure measurements and partial image velocimetry (PIV) data identified a complete suppression of the HV structure when only 11% of the boundary layer mass flow was removed; reducing total pressure losses by approximately 30%. This active method of flow control has shown the most benefit
for the least cost, of any of the aforementioned studies. Bloxham et al. also suggest that
the mass flow removed from this location could be re-circulated back into the engine to
reenergize separating flows in other regions of the turbine [3]. The current study also
combines the ideas of previous work. Boundary layer removal is applied immediately at
the endwall-obstruction juncture and both the flow field and surface heat transfer data are
analyzed.

1.3 EXPERIMENTAL OBJECTIVES

The purpose of this study is to complement and further the research conducted by
Bloxham et al. Specifically, the study wishes to identify the effect of the horseshoe
vortex structure on the surface heat transfer along the endwall surrounding an airfoil and
the changes, if any, localized boundary layer removal produces. The study will
investigate five removal rates through a slot located immediately at the junction between
the endwall and airfoil’s leading edge. This technique has already been proven to
substantially reduce the aerodynamic losses created by the $HV$ system. By examining the
surface heat transfer, it can be determined whether this technique also produces a benefit
to the detrimental hot spots present in gas-turbine engine.

A second location for the removal of boundary layer fluid will also be tested and
compared at the same, suction rates. This method will again utilize a thin suction slot but
will be located above the endwall; at a location approximately 1/3 of the approaching
boundary layer’s height. This location wishes to remove the source of the horseshoe
vortex system. Recall, the $HV$ develops when higher momentum fluid strikes an
obstruction and washes down into the lower momentum fluid in the boundary layer. If
the downwash of fluid along the front face of the airfoil can be removed before it reaches the endwall, then no horseshoe vortex system should develop.

The deleterious consequences of the horseshoe vortex structure have well been documented. This study takes a slightly different approach by comparing the effects of the HV to a system that has already committed to the application of flow control. That is, the baseline case in this study is an airfoil with a slot cut into it for the purpose of boundary layer removal. It will be seen that this slightly alters the flow field from the standard junction flow.
CHAPTER 2

EXPERIMENTAL SET-UP

2.1 WIND TUNNEL & TEST SECTION

All tests were conducted in the test section of a low-speed, open-loop wind tunnel. The tunnel is powered by a centrifugal blower that pushes air through an electric heater, past a series of flow straighteners and into a square acrylic duct (0.15 m$^2$) upstream of the tunnel’s test section. The tunnel’s heater has the ability to vary the flow temperature from 20° to 60° Celsius, with a measured uniformity of ±1°C across the area of interest. The flow conditioners provide a two-dimensional velocity uniformity of ±2%. A 30-mm gap in the tunnel floor allows for removal of the developing boundary layer just upstream of the main test section. An 8-mm rod was placed on the tunnel floor, 90 mm downstream of the test section’s inlet to trip the boundary layer to turbulent. The boundary layer is then allowed 0.45 meters to develop before striking the leading edge of a faired cylinder. Measurements taken at the location of the leading edge, without the cylinder installed, yield a boundary layer thickness, $\delta$, of 38 mm and a momentum thickness, $\theta$, of 4.6 mm. The resulting shape factor, $H$, of the boundary layer is 1.5. Figure 2.1 shows a schematic of the tunnel test section.
A symmetric airfoil (faired cylinder) was placed in the center of the wind tunnel’s test section. The airfoil has a length five times its maximum width and spans the height of the tunnel. The maximum width is equivalent to the cylinder diameter, \( D = 76.2 \text{ mm} \). The fairing eliminates vortex shedding off the back of the cylinder which in turn, reduces upstream pressure fluctuations. The freestream velocity, \( U_\infty \), was set such that a Reynolds number of \( 1.8 \times 10^4 \) (based on the cylinder diameter, \( Re_D \)) was maintained for all tests. The Reynolds number, which is a ratio of the inertial to viscous forces in a flow, was selected for the development of a strong horseshoe vortex system.

Figure 2.2 is a schematic of the airfoil with both suction locations labeled. A 1.5-mm slot was cut into the leading edge of the cylinder, extending \( \pm 30^\circ \) from the airfoil’s centerline. Two locations were compared; Method 1 applied suction immediately against the endwall, at a slot centerline height of \( Z/D = 0.01 \) and Method 2 applied the same suction at a height of \( Z/D = 0.17 \). This allowed for a comparison between the two control locations.
Figure 2.2 also shows the right-handed Cartesian coordinate system employed with the origin located on the cylinder’s centerline, along the endwall. Therefore, the cylinder’s leading edge is located at $X/D=-0.5$ along the $Y=0$ plane, with flow in the positive $X$-direction. This system was chosen for ease of comparison between planes.

Recall, the horseshoe vortex develops at the cylinder’s leading edge due to the adverse pressure gradient generated by the presence of an obstruction. The legs of the vortex are then carried downstream by the bulk flow on either side of the airfoil. The present study wishes to examine both the development and the downstream effects of the $HV$ system. To do this, three planes are considered. The symmetry plane, located along the airfoil’s line of symmetry, shows the streamwise development of the horseshoe vortex. Next a plane located along the cylinder centerline, 0.5 diameters downstream from the cylinder’s leading edge is considered. Termed the 0.5-$D$ plane, this plane is rotated $90^\circ$ from the airfoil’s line of symmetry and shows the cross-stream movement of the vortex. Here, the cylinder’s edge is located at $Y/D=-0.5$. The last plane examined is located half a diameter downstream of the 0.5-$D$ plane, or one full diameter downstream.
of the cylinder’s leading edge. Referred to as the 1-D plane, it shows the cross-stream location of the vortex as it is carried downstream. In this case the taper of the fairing is evident, such that the edge of the airfoil is located at $Y/D=-0.45$. All planes are labeled in Figure 2.2.

2.2 FLOW CONTROL

Localized boundary layer suction was applied through the slot cut into the leading edge of the cylinder. The suction slot fed into the hollow cylinder which was attached to a rotometer by vinyl tubing. A vacuum pump was attached to the opposite end of the rotometer through a series of valves. This system, shown in Figure 2.3, could provide volumetric flow rates up to 2,000 cm$^3$/s. The amount of suction, referred to as suction rate, $SR$, is characterized as a percentage of the boundary layer mass flow. Five suction rates were tested for each slot location: ~0%, 6.5%, 11%, 15%, and 20%. Before testing, the system was set to the appropriate flow rate and held constant through the duration of each test.

![Figure 2.3: Schematic of suction system from suction slot to vacuum pump.](image)
2.3 VORTICITY MEASUREMENTS

In order to gain a better understanding of the mechanisms that cause surface heat transfer, knowledge of the flow field adjacent to the endwall was desired. Thus, a LaVision particle image velocimetry, \textit{PIV}, system was employed to attain flow field velocity data. The air was seeded with a lightweight olive oil injected near the blower. An Nd:YAG laser fired two consecutive sheets across one of the three test planes with a time separation of 100 µs. The time separation allowed the fluid particles to travel across the plane but not completely out of the 1-mm thick laser sheet. A high-speed camera, with view field of approximately $0.75D \times ID$, collected 1,000 image sets at a rate of 4.95 Hz. The images were relayed to the LaVision system where a two-dimensional autocorrelation was performed. Vector processing began with a 64 x 64 pixel interrogation window then refined to a 32 x 32 and a 16 x 16 pixel window. Finally, a second pass with a 16 x 16 pixel interrogation window was performed with a 50% overlap to determine the mean velocity vectors across each plane. The uncertainty in the seed particle displacement was approximately 0.2 pixels, which equates to a velocity variability of ~0.1 m/s.

From the time-mean velocity data, the vorticity, $\omega$, and circulation, $\Gamma$, of the flow could be calculated. Vorticity was numerically calculated using a central difference method. Final results are smoothed and filtered using a top-hat filter to remove large scale noise and a band-pass filter to mitigate small fluctuations about zero. Circulation data are the result of an area integral along the plane of consideration. An area was selected that contained the entire horseshoe vortex on each plane. The vorticity at every
point within the area was summed and compared for each suction rate. The MATLAB code used in these calculations can be found in Appendix C of the report.

2.4 HEAT TRANSFER MEASUREMENTS

A transient infrared (IR) measurement technique was employed to calculate the surface heat transfer coefficients. Based on the unsteady conduction equation (discussed in detail in section 3.2), this technique involves an instantaneously started hot flow over a cool test piece. In the present study, only one-dimensional conduction was considered. The tunnel’s baseplate was considered a semi-infinite solid exposed to a step-change in fluid temperature.

The tunnel floor is made of a 24-mm thick piece of acrylic with thermophysical properties listed in Table 2.1. These values are a result of a transient heat transfer technique conducted by the Thermal Properties Research Laboratory at Purdue University. The thermal diffusivity of the acrylic relates to the other properties by, \[ \alpha = \frac{k_s}{\rho_s c_p}, \] and is an important factor in the transient test. This is because the semi-infinite solid assumption is based on a Fourier number, \[ Fo = \frac{\alpha t}{b^2}, \] of less than 1/16, where \( b \) is the thickness of the test piece and \( t \) is time in seconds. Based on the properties of the baseplate, test durations were constrained to less than five minutes, otherwise the assumption is violated. The base of the cylinder was insulated from the tunnel floor by a 14-mm thick piece of foam to prevent heat conducting from the cylinder to the floor. Therefore, the contact of the warm freestream fluid with the cool baseplate was the only method of heat transfer in the system.
Table 2.1: Thermophysical properties of the test section baseplate at 25°C.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Heat</td>
<td>$c_p$</td>
<td>1345</td>
<td>J/kg K</td>
<td>±3%</td>
</tr>
<tr>
<td>Thermal Conductivity</td>
<td>$k_s$</td>
<td>0.231</td>
<td>W/m K</td>
<td>±6%</td>
</tr>
<tr>
<td>Plastic Density</td>
<td>$\rho_s$</td>
<td>1190</td>
<td>kg/m$^3$</td>
<td>±2%</td>
</tr>
<tr>
<td>Thermal Diffusivity ($k_s/\rho_s c_p$)</td>
<td>$\alpha_s$</td>
<td>1.45e-07</td>
<td>m$^2$/s</td>
<td>±5%</td>
</tr>
</tbody>
</table>

Prior to each test the airfoil and baseplate were allowed to cool to thermal equilibrium at ambient temperature. The heat exchanger near the tunnel’s inlet was used to raise the diverted freestream air temperature to approximately 45°C. Once data acquisition in the test section was started, the diverted hot gas flow was instantaneously routed over the airfoil, creating a step-change in the fluid temperature. For a period of four minutes a LabVIEW program collected temperatures and pressures across the test section at a sampling rate of 1 Hz. The freestream velocity was measured using a pitot-static pressure probe with a high-response thermocouple attached. Multiple thermocouples monitored both fluid and surface temperatures. A complete surface temperature mapping was collected by the precision Cedip Silver infrared camera mounted above the test section.

The critical measurement in the transient measurement technique is the time history of surface temperatures relative to some impulsively started flow. This measurement can be made optically through techniques such as liquid crystal application and infrared thermography or by standard contact methods such as thermocouple arrays. Although accurate, thermocouples are limited to points along a surface and cannot capture the finer details of a flow structure. The newer optical measurement techniques offer much higher spatial resolution while providing a non-intrusive, global mapping of
the surface temperature distribution. In this study, a highly precise infrared camera took a continuous surface temperature history that was then converted to convective heat transfer coefficients.

Infrared thermography is a non-contact technique for the measurement of surface temperature by the analysis of infrared radiation emitted by an object [25]. It provides a non-intrusive, non-destructive, means of collecting unsteady temperature data; ideal for the unsteady horseshoe vortex system. The camera in the experiments had a clear view of the tunnel floor and collected data at the same rate as the surface thermocouples, 1 Hz. Specifically, the Electrophysics Silver 420M produces a 320 x 256 snapshot readout at frames rates up to 100 Hz. The camera is equipped with an Indium Antimonide (InSb) detector that is cryogenically cooled to 70K allowing for a temperature sensitivity of 0.02°C. The detector responds to wavelengths in the infrared spectrum of 3.6 to 5.1 µm for this particular model. It was calibrated for a range of temperatures between 5° and 150°C, with an accuracy of ± 2% or ± 2°C [6].

The IR camera is equipped with a motorized 27-mm integrated lens that has a 20° x 16° field of view. Several tests were run with the camera mounted directly over the cylinder’s leading edge. From this location, each pixel of the Silver’s 320 x 256 readout equated to 1 mm² on the tunnel floor. The temperature mappings of these tests demonstrated a symmetric flow field about the airfoil. It was determined that only one side of the airfoil could be studied so long as the symmetry plane was in clear view. The camera was then shifted along the negative Y and Z-axes. This new camera orientation improved the spatial resolution to a pixel size of 0.44 mm² on the tunnel floor. This was desired as the finer
resolution aids in the identification of small scale structures of the horseshoe vortex system. All reported results were collected from the “fine resolution” position.

The computer software for the SILVER 420M has a built in calculation that uses several user inputs to determine the surface temperature from radiation emitted. These radiometry parameters include emissivity, atmospheric temperature, background temperature, transmission and distance. Because of the uncertainty in the emissivity of the baseplate and the transmissivity of the radiation path, an in situ calibration was performed using surface mounted thermocouples. This is common practice in many laboratory and industrial applications [25].

The convective heat transfer coefficient, $h_c$, was then calculated from the real-time surface heat flux, freestream fluid temperature and the surface temperature of the tunnel’s baseplate. The run-to-run variation was found to be ±2.5%, ±3%, and ±3.5% on the symmetry plane, 0.5-D plane and 1-D plane respectively. Results presented are the average of at least two, and no more than four, tests run for each case. The data analysis accounted for variability in the freestream temperature but not for radiative heat fluxes as they were less than 2% of the convective heat transfer to the baseplate and therefore, could be neglected [5].
CHAPTER 3

ANALYSIS

The study used a transient IR technique to investigate the surface heat transfer due to turbulent boundary layer impinging on a faired cylinder. This chapter develops the concepts and methods necessary for the understanding of the experimental results. First the basics of the flow field are discussed, followed by the analytical development of all data reported in the results section.

3.1 FUNDAMENTALS

3.1.1 Convection

If a fluid at temperature, $T_x$, flows over a surface at uniform temperature, $T_s$, such that $T_x \neq T_s$, convection heat transfer will occur. Convection is a term used to describe energy transfer between a surface and a fluid moving over it. Although the mechanism of diffusion (random motion of molecules) contributes to this transfer, the dominant contribution is made by the bulk or gross motion of the fluid particles. The local heat flux, $q$, is proportional to the difference between surface and fluid temperatures. The proportionality constant, $h_c$, is known as the convective heat transfer coefficient. This
relation is known as Newton cooling and is expressed in Equation 3.1; Newton’s law of cooling [13].

\[ q_s = h \left( T_\infty - T_s \right) \]  

Eqn. 3.1

3.1.2 The Velocity Boundary Layer

The velocity boundary layer is of fundamental importance in problems involving convection transport. Consider the flow over a flat plate as in Figure 3.1. When fluid particles make contact with the surface, they assume zero velocity, due to the “no slip” condition of viscous flows. These particles then act to retard the motion of the particles in the adjoining fluid layer, which then slow the motion of the particles in the next layer, and so on until at some distance, \( Z = \delta \), the effect becomes negligible. This retardation of motion is due to shear stress, \( \tau \), acting on the fluid parallel to its velocity. With increasing distance \( Z \) from the surface, the \( X \)-component of velocity, \( u_x \), must then increase until it reaches the free stream value \( U_\infty \). This region, where \( u_x \) varies from zero to \( U_\infty \), is called the boundary layer.

The entire fluid flow is thus characterized by two distinct regions: a thin layer (the boundary layer) where velocity gradients and shear stresses are large, and a region outside this layer in which velocity gradients and shear stresses are negligible. Boundary layer velocity profiles refer to the manner in which \( u_x \) varies with \( Z \) through the boundary layer. The thickness of a boundary layer, \( \delta \), is the distance, \( Z \), for where \( u = 0.99 U_\infty \).
When dealing with a Newtonian fluid, the near wall velocity gradient is used to evaluate the surface shear stress. This is described in Equation 3.2, where, $\mu$, is a fluid property known as the absolute, or dynamic, viscosity. Surface shear is directly related to other properties of external flows such as the coefficient of friction. Surface frictional drag, or drag force, on a body is found from the coefficient of friction.

$$\tau_s = \mu \frac{\partial u}{\partial Z}_{Z=0}$$

Eqn. 3.2

In this study the boundary layer is tripped turbulent. There are sharp differences between laminar and turbulent boundary layer flows. Fluid motion in the turbulent boundary layer is highly irregular and is characterized by random, three-dimensional motion of relatively large bundles of fluid. These random fluctuations enhance the transfer of momentum and energy, and hence increase surface friction and convective heat transfer rates [13].

### 3.1.3 The Thermal Boundary Layer

Just as a velocity boundary layer develops when there is fluid flow over a surface, a thermal boundary layer develops if the fluid’s freestream temperature differs from the
surface temperature. Consider flow over an isothermal flat plate as in Figure 3.1. At the leading edge the temperature profile is uniform, \( T(Z) = T_\infty \). However, fluid particles that come into contact with the plate will achieve thermal equilibrium with the plate’s surface temperature. In turn, these particles exchange energy with those in the adjoining fluid layer, and temperature gradients develop in the fluid. The region in which these temperature gradients exist is the thermal boundary layer. Like that of the velocity boundary layer, the thickness of the thermal boundary layer, \( \delta_t \), is defined as the height, \( Z \) for which the ratio, \( \frac{(T_s - T)}{(T_s - T_\infty)} = 0.99 \). As with the velocity boundary layer, with increasing distance from the leading edge the effects of heat transfer penetrate further into the freestream and the thermal boundary layer grows.

The thermal boundary layer directly relates to the heat transfer characteristics of a fluid flow. Equation 3.3 shows that at any distance from the leading edge, \( X \), the local surface heat flux is obtained by applying Fourier’s law to the fluid at the surface, \( Z = 0 \). In Equation 3.3, \( k_f \) is the thermal conductivity of the fluid.

\[
q_s = -k_f \frac{\partial T}{\partial Z} \bigg|_{Z=0} \quad \text{Eqn. 3.3}
\]

This expression refers to conductive heat transfer not convective heat transfer which is appropriate because of the “no-slip” condition discussed previously. If there is no fluid motion at the surface, energy transfer can only occur by conduction [13]. Recall that conduction is the transfer of energy from a highly energetic particle, to a less energetic particle due to normal particle interactions. The condition of the thermal boundary layer therefore, strongly influences the wall temperature gradient, which in turn determines the rate of heat transfer across the boundary layer.
3.2 VORTICITY CALCULATIONS

Velocity data from a LaVision particle image velocimetry system were used to calculate the normalized vorticity along each plane considered. Vorticity, \( \omega \), is found by taking the curl of the velocity vector, \( \vec{u} \), as is represented in Equation 3.4 [12].

\[
\omega = \nabla \times \vec{u}
\]  
\text{Eqn. 3.4}

The PIV data provided a two dimensional grid of time-mean velocity vectors. The curl of these vectors result in a one-dimensional vorticity oriented normal to the plane of view. For example, on the symmetry plane, Equation 3.4 results in Equation 3.5. Recall the orientation of the planes shown in Figure 2.2.

\[
\omega_j = \frac{\partial u_j}{\partial Z} - \frac{\partial u_j}{\partial X}
\]  
\text{Eqn. 3.5}

The resulting partial derivatives were numerically estimated using a central difference method derived from the Taylor series expansion. Equation 3.6 represents the Taylor series expansion in its most general form.

\[
\begin{align*}
&u_{i-1} = u_j - \Delta X \frac{du}{dX} + \frac{\Delta X^2}{2} \frac{d^2u}{dX^2} - \frac{\Delta X^3}{6} \frac{d^3u}{dX^3} + \cdots \quad \text{Eqn. 3.6}
&u_{j+1} = u_j + \Delta Z \frac{du}{dZ} + \frac{\Delta Z^2}{2} \frac{d^2u}{dZ^2} + \frac{\Delta Z^3}{6} \frac{d^3u}{dZ^3} + \cdots
\end{align*}
\]

The expansion can also be taken in the positive direction. This is shown in Equation 3.7 as the positive expansion in the Z-direction. Note the sign changes and the index \( j \), rather than \( i \).

\[
\begin{align*}
&u_{j+1} = u_j + \Delta Z \frac{du}{dZ} + \frac{\Delta Z^2}{2} \frac{d^2u}{dZ^2} + \frac{\Delta Z^3}{6} \frac{d^3u}{dZ^3} + \cdots \\
&\quad \text{Eqn. 3.7}
\end{align*}
\]

The central difference approximation is found by subtracting Equation 3.6 from Equation 3.7. Due to the magnitude of the gradients in this analysis, the higher order terms of the expansion become very small and are therefore neglected. Substituting the
results into Equation 3.5 yields the second-order accurate finite-difference solution shown in Equation 3.8. Again, the subscripts refer to the vorticity normal to the symmetry plane; the other planes are approximated using the same method, only the subscripts are changed.

\[
\omega_y = \frac{u_{x_j+i+1} - u_{x_j+i-1} - u_{z_j+j+1} - u_{z_j+j-1}}{2\Delta Z - 2\Delta X}
\]

Eqn. 3.8

A forward or backward difference approximation was used along the edges of the PIV grid as necessary.

The vorticity data were then smoothed and filtered. Two filters were applied, a top-hat filter to eliminate erroneous large scale fluctuations and a band-pass filter that removed small fluctuations about zero.

3.3 CIRCULATION CALCULATIONS

The vorticity contained within a fluid element is related to the circulation, \( \Gamma \), found around it. Specifically, circulation is defined as the integral of the velocity around a closed contour [8]. This is shown in Equation 3.9.

\[
\Gamma = \oint \vec{u} \cdot d\ell
\]

Eqn. 3.9

The relationship between circulation and vorticity is derived from Stoke’s Theorem. This theorem transforms the contour integral of Equation 3.9 into an area integral in which the area, \( A \), is defined by the contour around which circulation is calculated. The curl of the integrand must then be taken, resulting in Equation 3.10, where \( \vec{n} \) is the unit normal to the surface. Recall, Equation 3.4 allows the substitution of vorticity into the integral.
\[
\Gamma = \iiint_A \left( \nabla \times \mathbf{u} \right) \cdot \hat{n} dA = \iint_A \omega \cdot \hat{n} dA \quad \text{Eqn. 3.10}
\]

This integral can be approximated numerically using the two-dimensional data provided by the LaVision system. This is done by summing the vorticity calculated at each point within a pre-defined contour, approximately 0.43\(D\) wide and 0.16\(D\) tall. The result is then multiplied by the area of the \(PIV\) data grid; the product of the step size in each direction, \(dX\) or \(dY\) and \(dZ\). Again, the example of the symmetry plane is used in Equation 3.11.

\[
\Gamma_y = \sum_{i,j} \omega_{ij} dXdZ \quad \text{Eqn. 3.11}
\]

This type of approximation was developed by Philips et al. and has been used by researchers such as Seal et al. and Bloxham et al. [3, 17, 22].

### 3.4 THE TRANSIENT METHOD

The analytical solution to a transient conduction problem is found by satisfying the governing equation of heat transfer. This equation, known as the energy equation, is derived from the principle of energy conservation which states that energy can neither be created nor destroyed. During an interaction, energy can change from one form to another but the total amount will remain constant.

If the principle of energy conservation is applied to a body or system, a change in energy content can be described as the balance between energy into the system and energy output by the system; this is shown in Equation 3.12.
\[ \Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}} \quad \text{Eqn. 3.12} \]

Likewise, when applied to a fixed volume in space, known as a control volume, conservation of energy can again be used to describe the change in energy over a period of time, \( t \). Consider a control volume, bound by a control surface through which matter and energy may pass. If after some period in time, \( t \), the energy flux into the control volume is greater than the energy flux out of the volume, then energy is gained or stored within the control surface, \( E_{st} \). Also, some energy may be converted from one form (e.g. chemical energy) to another (e.g. thermal energy) within the control volume through a processes collectively referred to as energy generation, \( E_{\text{gen}} \). Equation 3.13 describes this by balancing the rates of energy transfer inside and through a control volume.

\[ \left( \frac{dE_{st}}{dt} \right)_{CV} \equiv \dot{E}_{st} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_{\text{gen}} \quad \text{Eqn. 3.13} \]

Internal energy is the sum of all microscopic forms of energy within a system and according to the first law of thermodynamics, is conserved. There are four types of internal energy; thermal energy, chemical energy, nuclear energy, and energy interactions. Energy interactions describe the forms of energy that cross a system’s boundary, or control surface, causing a gain or loss to internal energy.

There are several ways in which energy can be added to a system. The two main methods are through heat addition, \( Q \), and mechanical work, \( W \). If Equation 3.13 is altered to include these forms of energy, Equation 3.14 is generated. Note that \( \dot{Q} \), is the rate of heat transferred, and \( \dot{W} \), is the rate at which work is done by the control volume. Equation 3.14 is thus the sum of energy fluxes across the boundaries of a control volume.
\[ \left( \frac{dE_{st}}{dt} \right)_{CV} = \dot{Q}_{in} + \dot{Q}_{gen} + \dot{m}_{in} (e_{in}) - \dot{W} - \dot{m}_{out} (e_{out}) \]  
\text{Eqn. 3.14}

After converting Equation 3.14 into integral form and performing several manipulations, the energy equation is produced; shown as Equation 3.15. Notice energy, \( e \), is now being described as enthalpy, \( h \). The enthalpy of a system is defined by, \( h = e + PV \); the internal energy of the system plus the work done by the system to its surroundings. The term, \( \Phi \), in Equation 3.15 represents the dissipation of mechanical energy, commonly referred to as viscous dissipation or the dissipation function [12]. Each term in Equation 3.15 is integrated over the same control volume so the integrals are dropped.

\[ \frac{\partial}{\partial t} (\rho h) - \frac{\partial P}{\partial t} = -\nabla \tilde{q} - \nabla (\rho \tilde{u} h) + \dot{q}_{gen} - \dot{W} + \Phi \]  
\text{Eqn. 3.15}

In Equation 3.15, \( \rho \) is the fluid density and \( P \) is the pressure force.

Before an attempt at the solution is made, certain assumptions are utilized to reduce the equation. If the assumption that the fluid flow is an incompressible ideal gas, then Equation 3.15 becomes Equation 3.16. In this case the enthalpy is described using the specific heat at constant pressure, \( c_p \), and the density is considered constant.

\[ \rho c_p \frac{\partial T}{\partial t} + \rho \tilde{u} c_p \nabla T = \nabla (k_x \nabla T) - \nabla \tilde{q}_R + \dot{q}_{gen} - \dot{W} + \Phi \]  
\text{Eqn. 3.16}

The remaining assumptions made for this particular analysis are: no heat is generated, no work is done, there exist no losses due to viscous dissipation and radiative losses are negligible. This leads to the unsteady conduction equation shown in Equation 3.17.
Recall, this analysis only considers one-dimensional conduction into the tunnel floor. A schematic of the utilized control volume is shown in Figure 3.2.

Carrying out the Laplace operator of Equation 3.17 and noting that thermal diffusivity, \( \alpha \), is equal to \( k/\rho c_p \), yields Equation 3.18. This is the governing equation for one-dimensional unsteady conduction through a control volume.

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial Z^2} \tag{Eqn. 3.18}
\]

Equation 3.18 shows that temperature, \( T \), is a function of time, \( t \), and distance, \( Z \). The traditional analytical solution to this differential equation therefore necessitates an initial condition and a boundary condition. For this analysis, the tunnel floor was assumed to be a semi-infinite solid with uniform initial temperature. Equation 3.19 describes the initial condition of an isothermal surface and Equation 3.20 represents the semi-infinite solid boundary condition.
Equation 3.20 simply states that the test piece is thick enough that heat will not conduct completely through the tunnel floor during a single test. Therefore, for a finite length of time, the opposite end of the tunnel floor will remain at the initial temperature, $T_{\text{init}}$.

There are a variety of circumstances that will yield similar solutions to the governing equation with these boundary conditions. Considered here is one of the most common cases, a step change in the surface temperature, $T_s$. This physically means that at any time after the initial state, the temperature of the surface will be equal to the temperature measured by the IR camera. Expressed in equation form, that is $T(Z=0, t > 0) = T_s$, where, $T_s (t \leq 0) = T_{\text{init}}$. Using Duhamel’s theory of superposition, the general solution to the governing equation (Equation 3.18) is obtained; expressed in Equation 3.21. Note, this equation yields the unsteady temperature distribution along the surface, $Z=0$, where $\eta = \frac{Z}{2 \sqrt{\alpha t}}$.

$$\frac{T(Z,t) - T_{\text{init}}}{T_s - T_{\text{init}}} = \text{erfc} \eta$$

Eqn. 3.21

Out of convenience, Equation 3.21 makes use of the complementary error function, $\text{erfc}$. The complementary error function relates to the error function, $\text{erf}$, by, $\text{erfc} \beta = 1 - \text{erf} \beta$.

The surface temperature distribution can then be approximated as the summation of a series of steps. This is acceptable because the one-dimensional unsteady conduction equation is linear. Schultz et al. used this method to derive an expression for the unsteady surface heat flux, $q_s$ shown in Equation 3.22 [21]. This heat flux, although
estimated, is valid for any time after the initial state, \( t > 0 \). Note that here, \( \tau \), is a dummy time integration variable.

\[
q_{s,\text{cond}}(t) \equiv \frac{k}{\sqrt{\alpha \pi}} \sum_{i=1}^{n} \frac{T_i(\tau_i) - T_j(\tau_{j-1})}{\sqrt{t_i - \tau_i} - \sqrt{t_i - \tau_{i-1}}}
\]

Eqn. 3.22

An energy balance can then be evaluated at the surface, \( Z=0 \), to determine the total surface heat flux, \( q_s(t) \). The energy balance for the control volume in Figure 3.2 is represented by Equation 3.23a, which becomes Equation 3.23b after substitution of the appropriate methods of heat transfer.

\[
q_{s,\text{in}} = q_{s,\text{out}} \quad \text{Eqn. 3.23a}
\]

\[
q_{s,\text{conv}} = q_{\text{rad}} + q_{s,\text{cond}} \quad \text{Eqn. 3.23b}
\]

Heat loss due to radiation is neglected in this analysis, such that \( q_{s,\text{conv}} = q_{s,\text{cond}} \). With this, the convective heat transfer coefficient of a time-varying surface temperature distribution is found using Equation 3.24 [4] which is a form of Newton’s law of cooling (Equation 3.1).

\[
h_t = \frac{q_{s,\text{conv}}}{T_w(t) - T_i(t)}
\]

Eqn. 3.24

Neglecting radiative heat losses causes an under prediction of the \( h_t \) values in this analysis however, calculations considering these losses altered the convective heat transfer coefficient by less than 2%. Because these calculations are much more labor intensive and demonstrate little effect, radiation effects were neglected.
3.5 UNCERTAINTY

The experimental uncertainty was calculated for the variables of study. The uncertainty of the vorticity and convective heat transfer were found using the general uncertainty analysis as described by Coleman et al. [7]. A worst case scenario was considered for all calculations.

The uncertainty of the particle image velocimetry data was based on the calibration information provided by the LaVision system. Velocity data was taken across three planes for each suction method tested. The high-speed camera was focused and calibrated using the DaVis program’s pin-hole calibration at each plane. The greatest variance in the calibrations was seen along the symmetry plane of Method 1, yielding a standard deviation of .24 pixels. Based upon the sampling time separation of 100 µs, this corresponds to a velocity of 0.09 m/s. When compared to the freestream velocity, an uncertainty of approximately ±2.4% in the velocity measurements was established.

Once the uncertainty of the velocity data was found, a general uncertainty analysis of the numerically approximated vorticity data was performed. According to Coleman et al., the absolute uncertainty, U, of a quantity calculated with data reduction equation, r, is a combination of the uncertainties of all variables, X, used in its calculation. This is demonstrated in the general uncertainty equation; Equation 3.25 [7].

\[
U_r^2 = \left( \frac{\partial r}{\partial X_1} \right) U_{X_1}^2 + \left( \frac{\partial r}{\partial X_2} \right) U_{X_2}^2 + \left( \frac{\partial r}{\partial X_3} \right) U_{X_3}^2 + \ldots
\]

Eqn. 3.25

Based on Equation 3.8, vorticity is a function of two velocity vectors and the pixel size of the PIV system. The uncertainty of the velocity field was determined to be ±2.4% and the pixel size of the PIV system only varies about .02% due to machine error in the
calculations. Equation 3.25 was modified to contain the four variables of Equation 3.8, and their uncertainties were substituted in to yield a vorticity field uncertainty of 3.4%. A summary of the experimental uncertainties is shown in Table 3.1.

The convective heat transfer coefficient is calculated in two steps. First, the surface heat flux is approximated using Equation 3.22, then, it is divided by the temperature difference between the fluid and surface temperature, as is indicated in Equation 3.24. As was done with the velocity data, Equation 3.25 was modified to reflect the variables in these data reduction equations. The surface heat flux is a function of thermal conductivity, \( k \), thermal diffusivity, \( \alpha \), surface temperature, \( T_s \), and the recorded time, \( t \). The thermophysical properties of the tunnel’s baseplate were determined experimentally by Thermal Properties Research Laboratory at Purdue University and the accuracy of the measurements are listed in Table 2.1. The surface temperatures were measured using an infrared camera with accuracy of \( \pm 2^\circ C \). Based on a worst case surface temperature, the uncertainty of \( T_s \) is approximately 0.7%. The time recorded during testing was based on the computer’s internal clock and is considered to have negligible inaccuracies. Substituting all these values and solving for the uncertainty of the surface heat flux yields a value of 6.6%.

The convective heat transfer coefficient is a function of the test piece’s surface temperature, freestream fluid temperature and the surface heat flux. The freestream temperature was monitored using high-response thermocouples with an accuracy of \( \pm 1.1^\circ C \) according to manufacturer specifications. This, along with the uncertainties of the surface temperature and heat flux, were substituted into a modified version of
Equation 3.25 to yield an experimental uncertainty of 6.7% in the calculated convective heat transfer coefficient. These values are summarized in Table 3.1.

<table>
<thead>
<tr>
<th></th>
<th>$U_{\Delta x}$</th>
<th>$U_{ii}$</th>
<th>$U_{\omega}$</th>
<th>$U_{T_x}$</th>
<th>$U_{q_x}$</th>
<th>$U_{T_{\infty}}$</th>
<th>$U_{h_c}$</th>
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</thead>
<tbody>
<tr>
<td>Values</td>
<td>0.02%</td>
<td>2.4%</td>
<td>3.4%</td>
<td>0.7%</td>
<td>6.6%</td>
<td>0.4%</td>
<td>6.7%</td>
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CHAPTER 4

RESULTS

4.1 VALIDATION

To begin, results are compared with those found in previous studies to validate the methods and outcomes of the analysis. Because the applied method of flow control is unique, the characteristics of the baseline flow field, the “No Suction” case, are evaluated against experimentations with similar conditions. If the data here show comparable flow structures and trends then the remaining tests can be considered valid.

4.1.1 Velocity Data

Particle image velocimetry was used to find the time-mean velocity vectors on three planes surrounding the airfoil. From the velocity vectors, a time-averaged vorticity field was calculated. Figure 4.1 shows the normalized vorticity, $\omega_3 D/ U_\infty$, along the symmetry plane for the No Suction case of Method 2. The streamlines are overlaid to aid in visualization of fluid motion. The figure shows the corner region between the cylinder’s leading edge, $X/D=-0.5$, and the tunnel floor, $Y/D=0$. 
Upon examination, it is clear that Figure 4.1 shows indications of all of the “most important” flow structures of the HV system as indicated by Sabatino et al. in Figure 1.1. The obvious streamline roll-up within the concentration of strong positive vorticity located near $X/D = -0.68$ represents the horseshoe vortex, $HV$. The protuberance of negative vorticity very near the cylinder wall at $X/D = -0.52$ is due to the corner vortex, $CV$, which rotates in the opposite direction of the $HV$. There is also a pocket of negative vorticity, significantly smaller than the $HV$, located near the endwall at $X/D = -0.77$. This is the secondary vortex, $SV$, which is fed by the strong reverse flow under the $HV$. The fourth visible structure is that of the tertiary vortex, $TV$, indicated by the positive rotation just upstream of the $SV$, near $X/D = -0.85$. The saddle point of the flow is not obvious but can be estimated near $X/D = -1.0$, where the positive vorticity begins to migrate away from the endwall; congruent with the model from Sabatino et al.

Further, Figure 4.1 shows indications of the vortex’s temporal behavior. Notice the positive vorticity of the $HV$ extends between $X/D = -0.74$ and -0.58. This represents a
streamwise movement, where the vortex will switch between two primary positions along the symmetry plane. The streamlines indicate the dominant position, $X/D=-0.68$, is upstream and slightly elevated from the secondary position at $X/D=-0.65$. This bi-modal behavior has been found by many previous researchers; Seal et al., Praisner et al., Sabatino et al. and Simpson [19, 20, 22, 24]. Also, it has been found that quasi-periodically the SV will force packets of fluid away from the endwall, which is seen as turbulent “bursts” in the temporal data [18]. In the time-averaged results of Figure 4.1 this is signified by the confluence of streamlines immediately above the SV at $X/D=-0.77$; matching the flow field as seen by Praisner et al., and Sabatino et al., further confirming the PIV results along the symmetry plane [19, 20].

Along the airfoil’s line of symmetry, the vorticity data show the formation of the HV system as freestream fluid washes down the face of the cylinder and into the region of the lower momentum boundary layer. Away from the leading edge, the strong adverse pressure gradient due to the presence of the cylinder diminishes and the fluid can easily move around the airfoil. The horseshoe vortex that develops in the corner region along the endwall is carried downstream, on either side of the airfoil, creating “legs” about its leading edge. As the vortex travels around the cylinder the vorticity is turned from the cross-stream to the streamwise direction. The streamwise vorticity along the plane one half-diameter downstream of the cylinder’s leading edge is presented in Figure 4.2.
As in Figure 4.1, the non-dimensional vorticity data, $\omega_x D/U_\infty$, are presented; fluid streamlines in this case would be out of the figure. The surface of the cylinder is again located at -0.5 along the abscissa; which is now $Y/D$ rather than $X/D$. The HV is indicated by the area of positive vorticity near $Y/D=-0.76$ and again demonstrates a bi-modal behavior. It is clear that the strength of the vortex has been severely weakened from the symmetry plane. Also, the HV has grown in size while moving away from both the cylinder’s edge and the endwall. The corner vortex has grown in size, now extending out to nearly $Y/D=-0.58$, but is slightly stronger, most likely fed by boundary layer development along the surface of the cylinder. The secondary vortex has grown in size as well, now extending from $Y/D=-0.8$ to -0.95, and is noticeably weaker. Comparable to the HV, the SV has moved away from the endwall and cylinder’s edge. These findings are consistent with those of Seal et al., Praisner et al. and Bloxham et al. who found the horseshoe vortex system to distend and weaken as it progressed downstream. The
authors suggest viscous dissipation and the reorientation of the vortex axes away from the source, as the cause of loss in strength and coherence of the vortex structure [19, 22].

There are several different ways to quantify the size and strength of the horseshoe vortex system. Phillips et al. proposed using a net circulation as a single value that summarized the measurements across an entire plane. This was found by summing vorticity data in the area of interest across a measurement plane. Seal et al. and Bloxham et al. followed this method and present comparison of flow control against a baseline, “no control” case. Other researchers, such as Praisner et al., compare peak vorticity across a number of planes. Figure 4.3 shows the peak vorticity and non-dimensional circulation of the present study as compared to data collected by Praisner et al. and Seal et al. The value along each of the three planes of study is plotted against its percent difference from the value on symmetry plane. In the figure, results from the 0.5-D plane are plotted at $X/D=0.5$ and at $X/D=1.0$ for the 1-D plane.

![Figure 4.3: Percent difference in peak vorticity and net-circulation from the symmetry plane as compared to Praisner et al. and Seal et al. [19, 22]; Method 2, No Suction.](image)
Figure 4.3 demonstrates the effects of viscous dissipation as the HV system travels downstream. The figure shows the strength of the HV at the 0.5-D plane, has decreased over 50% from the symmetry plane; Bloxham et al. also saw an approximate 50% decrease along this plane. Then, by the time the HV reaches the 1-D plane, the vortex is nearly 75% weaker than it was initially along the symmetry plane. The net circulation data (solid lines) show the same trend, decreasing over 50% from the symmetry plane at the 0.5-D plane; then, an additional 25% decrease to the 1-D plane.

The data from Praisner et al. and Seal et al. confirm this roughly linear decrease in vorticity as the vortex travels downstream. Both plots show an approximate 50% decrease at the 0.5-D plane and a continued decrease to the 1-D plane. The maximum vorticity data of the present study matches very well with that of Praisner et al., differing by less than 1% along the 0.5-D plane and ~8% along the 1-D plane. The circulation data of Seal et al. however, does not match as well; differing by ~5% on the 0.5-D plane and ~18% at the 1-D plane. This difference is most likely due to the manner in which the net-circulation was calculated. Seat et al. integrated values within an iso-vorticity curve of the HV, summing only positive vorticity. The present study followed the method of Phillips et al. by summing the vorticity inside a 2-dimensional grid, extending from $X/D$ (or $Y/D$) = -0.95 through -0.525 and from $Z/D=0.02$ through 0.175, on all planes. With the 2-d grid, the negative vorticity of the secondary vortex moves away from the endwall and into the integration area on the downstream planes, greater reducing the net-circulation than if the HV alone were considered. Also, negative vorticity develops along the surface of the airfoil as the flow travels downstream due to boundary layer growth.
This again enters the integration area on the two downstream planes, reducing the calculated net-circulation.

4.1.2 Heat Transfer Data

The surface heat transfer during each method was studied through the use of a transient test using infrared imaging of the wind-tunnel floor. From the real-time temperature data, the convective heat transfer coefficient, $h_c$, was calculated. This is a direct measure of the energy passed from the hot fluid to the cool tunnel endwall. Figure 4.4 shows the surface mapping of $h_c$ values of Method 2 when no control is applied. The three planes of study, the symmetry plane, 0.5-D plane and the 1-D plane have been highlighted in blue, red and green respectively. The airfoil, with leading edge at $X/D=-0.50$ and $Y/D=0$, has been blacked out of the image.

![Figure 4.4: Convective heat transfer coefficient, $h_c$ (W/m²K) with planes of study highlighted; Method 2, No Suction.](image-url)
Figure 4.4 shows in the streamwise direction, that the heat transfer increases with proximity to the airfoil; with the maximum occurring just upstream of the cylinder’s leading edge. The values of $h_c$ then decrease around the sides of the airfoil. This is consistent with the findings of Goldstein, Blair, Giel, Mahmood, Praisner and Sabatino [2, 10, 11, 16, 19, 20]. The cross-stream direction also shows an increasing trend while approaching the airfoil, but again, the peak is not immediately at the airfoil’s surface.

To better observe the details of Figure 4.4, the convective heat transfer values along each plane were plotted against normalized distance. Figure 4.5 shows the $h_c$ values along the symmetry plane, 0.5-D plane and 1-D plane in blue, red and green respectively. First examining heat transfer along the symmetry plane in the streamwise direction, the values of $h_c$ remain fairly constant until a steep increase near $X/D=-1.0$. The heat transfer continues to increase steadily until it nearly levels at $X/D=-0.82$; forming an approximate plateau that extends to $X/D=-0.74$. Then, again a steep increase in heat transfer until peaking near $X/D=-0.54$. The final section shows a steep decrease, nearly 25%, to the leading edge of the cylinder at $X/D=-0.5$. 
If the heat transfer data are then compared to the flow field data; a clear connection can be made. Recall in the vorticity plot, Figure 4.1, the HV structure sits between \( X/D = -0.74 \) and \(-0.58\). Comparing this to the surface \( h_c \) values shows the peak heat transfer occurs in the region just downstream of the HV, before the CV. This is due to the rotation of the vortex drawing warm freestream fluid down to the cool endwall, significantly increasing the heat transfer. It is also noted that the initial increase in heat transfer is located at the saddle point of the flow field, near \( X/D = -1.0 \). These findings are consistent with the time-averaged results of Praisner et al. and Sabatino et al. [19, 20].

Upstream of the saddle point the surface heat transfer is equivalent to that of turbulent flow over a flat plate, equating to a Stanton number, \( St \), of approximately \( 2.5 \times 10^{-3} \). The peak in heat transfer along the symmetry plane is 340% greater than the upstream values. This is consistent with the time-mean findings of Prainer et al., Sabatino et al. and Goldstein et al. who found an increase of 350%, 340% and \( >325\% \) respectively [11, 19, 20].
Further comparison shows the plateau in $h_c$ occurs at the region directly beneath the $SV$. Praisner et al. and Sabatino et al. see a local increase, rather than a plateau at this location, that is between 65 and 70% of the magnitude of the global peak in heat transfer. In the present study, the downstream inflection point in surface heat transfer is approximately 60% the peak value. The streamwise distance of the plateau in Figure 4.5 is comparable to the span of the secondary peak seen by Praisner et al. and Sabatino et al. The disparity between the studies is most likely due to differences in the measurement technique. Praisner et al. and Sabatino et al. used liquid crystal thermography to measure heat transfer across a locally heated endwall. Localized heating can result in a relatively thin thermal boundary layer in the area surrounding the leading edge, which could exaggerate heat transfer effects due to the steep near-wall temperature gradients [19, 20]. The current study utilizes a transient test that subjects the test section to a step-change to fluid temperature. This causes a thermal boundary layer to develop much like a velocity boundary layer over the entire section. The height of the thermal boundary layer grows with downstream distance, making the temperature gradients more moderate than localized heating. Therefore, fluid must travel from far above the endwall to have a large impact on surface heat transfer.

It can be seen in Figure 4.5 that the trends present along the symmetry plane are mirrored along the downstream planes only less dramatically. The surface heat transfer is equivalent to that of flat plate flow very far from the cylinder wall. Then $h_c$ increases until the $SV$ creates an inflection point in the plot that leads to a pseudo-plateau before the peak in surface heat transfer; which is again located between the $HV$ and the $CV$. These effects spread and become less distinct with each downstream plane; matching the trends
of the flow field features. Clearly, the magnitude of heat transfer decreases as the flow travels to the downstream planes. This is consistent with the argument that the HV system weakens and moves away from the endwall as it is carried downstream [3, 19, 22].

Cross-stream vorticity data showed the horseshoe vortex system to move away from the surface of the airfoil as it traveled downstream. This is seen in Figure 4.5 as the primary peak shifting to the left on the downstream planes. The 1-D plane shows the primary heat transfer peak at a location very similar to that on the symmetry plane, $X/D=-0.54$, but if the extra distance to the airfoil is considered, it is clear that the structure is moving away from the cylinder’s surface. Also, the inflection points shift away from the airfoil’s surface, signifying a shift in the location of the SV on the downstream planes. The inflection point on the symmetry plane is found near $X/D=-0.72$, on the 0.5-D plane near $Y/D=-0.78$, and $Y/D=-0.84$ along the 1-D plane (although very slight). All these similarities solidify the notion that the surface heat transfer accurately tracks the movement of the horseshoe vortex system. Note, the time-mean vorticity data with respective surface heat transfer for all planes can be found in Appendices A and B of the report.

4.2 DIFFERENCE IN METHODS

Localized boundary layer suction was tested and compared when applied at two locations near the blade-endwall juncture. The first location, referred to as Method 1, applies suction immediately against the endwall, through a slot cut into the base of the leading edge of the cylinder. The second location, Method 2, utilizes the same leading
edge slot only shifted away from the endwall to a distance approximately 35% of the height of the approaching boundary layer. Theoretically, the difference in slot height should only be noticeable when the suction system is active and flow is being pulled into the cylinder. However, upon examination of the results, this is not the case. Some distinct differences are observed between the “No Suction” cases of Methods 1 and 2.

4.2.1 Velocity Data

The dissimilarity between the flows fields of Methods 1 and 2 is best illustrated along the symmetry plane. Figure 4.6 presents the symmetry plane normalized vorticity for Method 1’s No Suction case, as calculated from the time-mean velocity vectors. As in Figure 4.1, the fluid streamlines are overlaid to show the flow path.

![Figure 4.6: Non-dimensional vorticity, \( \omega_y D/U_\infty \), with streamlines along the symmetry plane; Method 1, No Suction.](image)

The suction slot in Figure 4.6 is located immediately against the endwall and it was hypothesized that fluid was able to enter the slot even though the suction was turned
off. This is supported by the total lack of a corner vortex in the region of the cylinder-endwall juncture. There is an absence of negative vorticity and no streamlines show the counter rotation indicative of the CV. Due to the loss of this structure, the HV structure has been shifted toward the cylinder wall. The peak normalized vorticity of the HV in this case is slightly larger than that of Method 2; Figure 4.1. This is most likely because more fluid is able to flow into the HV; which is supported by the streamlines in Figure 4.6. In contrast, it appears as though the SV and TV have been weakened. Examining the region between $X/D=-1.0$ and -0.7 it is noted that fewer streamlines are apparent and those present do not take the sharp turns of those in Figure 4.1. This could be due to a lessening in the interaction of the SV and TV with the HV. The figure shows a clear bimodal movement of the HV between $X/D=-0.74$ and $X/D=-0.55$, which spans a greater streamwise distance than in Method 2. Without the presence of the CV, the HV can freely travel downstream to the cylinder’s leading edge. If the HV is further downstream there will be less interaction with the secondary and tertiary systems since they remain at the same streamwise position. The separation point of the system also remains near $X/D=-1.0$. This means less fluid is exchanged and therefore less momentum is transferred between the systems, thus alleviating the strength of the SV and the TV.

It should be noted that the upstream band of paired positive and negative vorticity in Figure 4.6 (along $Z/D=0.09$) is due to an unfortunate glare that occurred during data acquisition and is in no way indicative of flow features at these locations. The same vorticity pairing of Figure 4.6 is present in all symmetry plane flow field data of Method 1; which is apparent upon examination of the figures in Appendix A. The PIV data are still considered accurate with the acknowledgement that these elements are artificial.
4.2.2 Heat Transfer Data

As with the velocity data, there are distinct differences present between the surface heat transfer of the two methods. Again, this is best demonstrated along the symmetry plane where effects are most dramatic. Figure 4.7 shows the calculated convective heat transfer coefficient, $h_c$, along the endwall of the symmetry plane for Method 1, in blue, and Method 2, in red.

![Figure 4.7: Convective heat transfer coefficient along the symmetry plane; Methods 1 & 2, No Suction.](image)

The figure corroborates the flow field variation previously discussed. Most noticeably, the peak heat transfer of Method 1 occurs further downstream, $X/D=-0.52$, than that of Method 2, $X/D=-0.54$; due to the shift in position of the $HV$. Also, the height of Method 1’s peak represents a stronger $HV$ nearer the wall, capable of drawing more freestream fluid down to the endwall. Lastly, this peak is less sharp than that of Method 2. This is most likely due to the absence of the $CV$ which acts as an obstruction to fluid approaching the endwall. The $SV$ and $TV$ affect the surface heat transfer by creating inflection points in the plot. In Figure 4.7, Method 2 shows a strong inflection point to a plateau in heat transfer near $X/D=-0.75$. The inflection of Method 1’s heat transfer plot is less abrupt than that of Method 2, indicating a weaker $SV$ and $TV$. 
These effects, though slight, are definite. Flow is lost through the suction slot during the “No Suction” case of Method 1. Later testing estimated the fluid loss to be ~0.1% of the boundary layer mass flow. In order to account for this when comparing different suction rates, each suction rate is normalized by its own No Suction case.

4.3 APPLICATION OF SUCTION

The effectiveness of localized endwall boundary layer removal on the mitigation of detrimental effects caused by the horseshoe vortex system was evaluated at five flow rates. The amount of fluid removal is characterized as a percentage of the approaching boundary layer mass flow. The suction rate, $SR$, defined in Equation 4.1, is a ratio of the mass flow through the suction slot, $\dot{m}_{st}$, to the boundary layer mass flow. Note that, $\delta^*$, in the equation represents the displacement thickness of the approaching velocity boundary layer at the cylinder’s leading edge (measured with the airfoil removed from the tunnel).

$$SR = \frac{\dot{m}_{st}}{\rho U_\infty D(\delta - \delta^*)} \times 100\%$$

Eqn. 4.1

Five suction rates, 0%, 6.5%, 11%, 15%, and 20%, were evaluated at two locations along the cylinder’s leading edge. The 0% suction case, also called the “No Suction” case, serves as the baseline for the other suction rates, as it represents a blade with no control. Recall the suction slot was not covered during the 0% suction cases and approximately 0.1% of the boundary layer mass flow was able to escape through the slot.
4.3.1 Method 2 Case Study: 11% Suction

To begin, the effects of suction at one flow rate are examined in detail. A suction rate of 11% was selected because it shows typical results for both methods of control. The horseshoe vortex system was shown to originate on the symmetry plane, and convect downstream as carried by the bulk fluid. Accordingly, the analysis begins on the symmetry plane and works its way to the downstream planes.

Figure 4.8 shows the normalized vorticity and streamlines along the symmetry plane for the 11% Suction case of Method 2. Recall that Method 2 applies suction at a height of $Z/D=0.17$. This is easily seen in the figure as multiple streamlines terminate at this location along the cylinder’s leading edge at $X/D=-0.5$. Examination of Figure 4.8 shows all the most important flow structures of the HV system are present. The positively rotating horseshoe vortex is located near $X/D=-0.6$, again showing a bi-modal movement between $X/D=-0.63$ and -0.55. A weak corner vortex is present immediately against the cylinder face, as is indicated by a concentration of negative vorticity and streamline roll-up against the endwall near $X/D=-0.52$. A secondary and tertiary vortex can be seen near $X/D=-0.64$ and -0.69 respectively. The saddle point of the flow field is estimated near $X/D=-0.90$ where the positive vortex structure begins to lift away from the endwall.
If Figure 4.8 is compared to the No Suction case of Method 2, presented in Figure 4.1, it is apparent that the application of suction has effectively reduced the size and strength of all features of the HV system. For example, the maximum vorticity of the HV has decreased by ~14% and the area in which it resides is markedly smaller than the baseline case. The same trend is seen by all vortices on the plane. This is because the suction is preventing 11% of the boundary layer mass flow from washing down the cylinder face into the HV system. Thus, the separation point of the system is shifted downstream, from $X/D=-1.0$ to -0.9, physically reducing the area in which the structures develop. On the other hand, the suction slot generates a new vortex pair near $Z/D=0.17$ where fluid is turned into the cylinder; which may show up on the downstream planes. A way to quantify the overall effectiveness of the suction is by examining the net-circulation, $\Gamma$. This was calculated from a summation of the vorticity in the region of the HV system. Upon comparison with the No Suction case, 11% boundary layer removal is shown to reduce the symmetry plane circulation by ~18%.
A reduction to the size and strength of the HV system will have a direct impact on the surface heat transfer about the airfoil. Figure 4.9 shows the convective heat transfer coefficient, $h_c$, when no control is applied, solid lines, and when 11% Suction is applied using control Method 2, dashed lines. The symmetry, 0.5-D and 1-D planes are represented by the blue, red and green plots respectively. Figure 4.9 clearly shows a reduction in surface heat transfer across all planes. This is most apparent on the symmetry plane where the peak $h_c$ value has dropped from 38.9 to 26.3 W/m²K; yielding a reduction of over 30% and making the symmetry plane heat transfer comparable to that of the 1-D plane without control. This effect is apparent on the downstream planes as well. The peak $h_c$ has decreased ~26% on the 0.5-D plane and ~24% on the 1-D plane. On all planes, this peak has moved closer to the cylinder’s edge. Recall, the vorticity data showed a downstream shift of the vortex structures. The heat transfer plots with control show the same correlations to the velocity field as was seen without control. This is demonstrated by the peak $h_c$ located downstream of the HV and inflection points near the locations of the SV and TV. However with suction, these characteristics are less distinct, meaning their effect has been mitigated.
Therefore, the application of 11% boundary layer removal through a slot located along an airfoil’s leading edge, above the endwall, significantly reduces the size and strength of the structures of the HV system. Both the aerodynamic losses and the surface heat transfer effects of the HV system are then mitigated on all planes of study. This confirms a positive impact from the flow control technique of Method 2. A complete data set from this technique can be found in Appendix B of this report.

4.3.2 Method 1 Case Study: 11% Suction

Figure 4.10 shows the normalized vorticity and streamlines along the symmetry plane when 11% boundary layer removal is applied utilizing control Method 1. Recall, Method 1 applies suction immediately against the endwall from a slot in the cylinder’s leading edge. In the figure, the slot acts as the termination point for many streamlines. Most significantly, Figure 4.10 indicates a total lack of the HV system. The faint yellow coloring throughout the corner region indicates there is a slight turning of the flow toward
the suction slot but minor in magnitude. As was seen with Method 2, the suction slot generates a vortex pair along its edges as streamlines turn to enter the cylinder. The effectiveness of the suction is quantified by examining the net-circulation in the region of the HV system. Compared to the No Suction case, 11% boundary layer removal reduces the net circulation by ~30% when applied at the cylinder-endwall juncture.

![Non-dimensional vorticity, \(\omega_y D/\nu\), with streamlines along the symmetry plane; Method 1, 11% Suction.](image)

Figure 4.10: Non-dimensional vorticity, \(\omega_y D/\nu\), with streamlines along the symmetry plane; Method 1, 11% Suction.

Figure 4.11 plots the convective heat transfer coefficient along the three planes of study for the no control case and the case with 11% boundary layer removal via Method 1. It can be seen on all planes that \(h_c\) is mitigated over the entire plane. The symmetry plane shows a decrease in the peak heat transfer of ~17%. Also, the shape of the plot along the symmetry plane has changed. The inflection points visible in the No Suction case have been expunged in the control case. Also, the surface heat transfer no longer demonstrates a peak upstream the cylinder’s leading edge. Instead, \(h_c\) increases sharply.
in proximity to the airfoil with its maximum at the suction slot. This is most likely caused by the suction pulling warmer fluid down toward the surface at this location.

![Convective heat transfer coefficient along 3 planes; Method 1, No Suction & 11% Suction.](image)

Figure 4.11: Convective heat transfer coefficient along 3 planes; Method 1, No Suction & 11% Suction.

The downstream planes in Figure 4.11 show a more traditional shape, with $h_c$ increasing to a peak before decreasing to the airfoil’s edge. Normally, this would suggest the presence of the horseshoe vortex, but Figure 4.10 clearly indicated the HV has been removed. Therefore, another feature is responsible for the trend in heat transfer along the two downstream planes. Even still, the peak $h_c$ has been decreased by $\sim 17\%$ on the 0.5-D plane and $\sim 13\%$ on the 1-D plane. In fact, $h_c$ is markedly less with control at every location along these planes except immediately at the airfoil surface, $Y/D=-0.5$, where it is comparable to the no control case. Finally, as on the symmetry plane, the downstream planes with control do not exhibit inflection points; suggesting the absence of a SV and TV structure.
The preceding figures indicated that the application of 11% boundary removal from a slot located at the cylinder-endwall juncture had a positive effect on both the aerodynamics and surface heat transfer in the region.

4.3.3 Boundary Layer Profiles

A useful technique in the visualization of flow patterns is a comparison of streamwise velocity profiles. Figures 4.12 and 4.13 show normalized velocity profiles of the boundary layer flow along the symmetry plane as calculated from the time-mean velocity vectors of suction Methods 2 and 1, respectively. Three suction rates are presented in the rows of each figure, No Suction, 11% Suction and 20% Suction; at five locations along the symmetry plane, $X/D=-0.99$, -0.88, -0.77, -0.66 and -0.55, representing the columns. Recall the cylinder’s leading edge is located at $X/D=-0.50$. It is important to note that a value of $U/U_\infty=1$, physically means the velocity component in the $X$-direction, $u_x$, is equivalent to the freestream velocity, $U_\infty$, and a negative value, signifies an upstream flow direction.
Figure 4.12: Normalized boundary layer profiles along the symmetry plane; Method 2, No Suction, 11% Suction & 20% Suction.

Figure 4.12 shows the normalized boundary profiles formed by control Method 2. Examining the top line, the No Suction case, the upstream position of $X/D=-0.99$ shows a typical flat plate boundary layer profile signifying the pressure gradient due to the downstream obstruction is not yet felt by the flow. Continuing downstream, the nearwall fluid begins to retard until a severe reverse flow is present at a distance of $X/D=-0.66$. 
The velocity near the endwall is nearly half the freestream velocity \( (U/U_\infty = -0.5) \). From the vorticity data, this location is known to be within the movement of the HV structure which feeds fluid to the upstream SV. Then by \( X/D = -0.55 \) the presence of the cylinder is felt by the entire flow field, slowing the entire boundary layer profile. At this point, the nearwall fluid only sees a small reverse flow.

The center profile of Figure 4.12 represents the 11% suction case of Method 2. Here, the strong reverse flow present in the No Suction case at \( X/D = -0.66 \) has clearly diminished. The reverse flow is now strongest at the \( X/D = -0.55 \) position, suggesting the HV still exists but has moved downstream; due to the application of boundary layer suction. Fluid flow into the suction slot located at \( Z/D = 0.17 \), is evident at the downstream position of \( X/D = -0.55 \). The velocity of the fluid is over half that of the freestream, \( U_\infty \), at this point. This effect is even greater in the 20% Suction case, the final row of Figure 4.12. Here the areas of reverse flow are again shifted toward the cylinder’s leading edge but not removed, again suggesting the presence of a HV, however weak.

Figure 4.13 shows the same locations and suction rates as Figure 4.12 but when suction Method 1 is utilized. In the first row, the baseline case, reverse flow is present at the three downstream locations, \( X/D = -0.55, -0.66 \) and -0.77, with the greatest reverse flow occurring under the HV near \( X/D = -0.66 \). The discontinuities near \( Z/D = 0.09 \) in the upstream profiles is due to an unfortunately glare present during data collection.
When 11% Suction is applied, all reverse flow is eliminated, thus, the HV has been removed. Recall, the vorticity plot of this case also indicated a complete absence of the HV structure. At the $X/D=-0.55$ location, the nearwall fluid is accelerated to more than half the freestream velocity. The upstream profiles do not feel the effect of suction.
until 20% boundary layer removal is applied. In this case the boundary layer fluid will enter the suction slot at a velocity equivalent to the uninhibited freestream velocity.

The large near-wall velocities of this method have an effect that has not yet been considered. Shear stress, as defined in Equation 3.2, is directly proportional to the velocity gradient at the endwall, $Z/D=0.0$. This viscous flow has a “no-slip” condition at the wall, meaning the velocity along the surface is zero. In some cases seen in Figure 4.13, the near wall fluid is accelerated very quickly into the suction slot, dragging the viscous fluid across the endwall. This viscous scraping could have detrimental effects such as scour or in some cases, high heat transfer rates. Praisner et al. found surface shear stress to correspond to certain flow structures of the $HV$ system; i.e. zero shear at the saddle point and large negative shear beneath the $HV$, but saw no clear relation to surface heat transfer [19].

4.3.4 Surface Heat Transfer

The convective heat transfer coefficient was calculated along the surface surrounding the leading edge of the airfoil. Figure 4.14 shows the $h_c$ mappings for selected suction rates of Methods 1 and 2. The left-hand column represents suction as applied using Method 1 and the right-hand column represents the same flow rates utilizing suction Method 2. The rows represent the cases of, No Suction, 11% Suction and 20% Suction respectively.
It was shown that surface heat transfer accurately traced the movement of the \( HV \) structure. This was due to the \( HV \) pulling warm fluid down into contact with the cool endwall, creating areas of high \( h_c \). The first row of Figure 4.14 shows the \( h_c \) mappings where the effects of the \( HV \) are most prominent; when no control is applied. In these plots the leading edge region demonstrates the greatest surface heat transfer which then decreases as the \( HV \) wraps around the leading edge. Also, the slight differences in Methods 1 and 2 can be seen as the values of \( h_c \) do not drop before the cylinder’s face in
Method 1. This was hypothesized to be due to the small amount of fluid entering the suction slot, ~0.1% of the boundary layer mass flow.

When suction is applied the values of surface heat transfer are dramatically decreased over the majority of the surface. However, when examining the 11% Suction case of Method 1, a local increase in $h_c$ develops near the downstream, or side, edge of the suction slot; $Y/D=-0.3$ and $X/D=-0.4$. This local bulge in heat transfer is not apparent when the same suction is applied via Method 2. Recall the vorticity plots, as presented in the case studies, showed a vortex pair that developed on either side of the suction slot as it pulled flow into the cylinder. Figure 4.14 shows a similar effect where the data field has been altered due to the pulling of fluid into the cylinder. Along the symmetry plane, 11% suction applied using Method 1 showed a decrease in peak heat transfer of ~17%. At the edge of the suction slot the reduction in peak $h_c$ is only ~12%.

This effect is magnified in the 20% Suction case of this Method 1. Along the symmetry plane, 20% suction applied using Method 1 decreases the peak $h_c$ by ~19%. The edge of the suction slot here demonstrates values of $h_c$ comparable the peak heat transfer when no control is applied. This means, that 20% boundary layer removal using Method 1 will merely shift the peak surface heat transfer from the leading edge of the cylinder, to the edge of the suction slot. Note, this flow field is symmetric; so this peak is also present on the opposing side of the airfoil. Downstream of this local heat transfer bulge, however, the values of $h_c$ are still significantly less than when no control is applied. This is true for 11% Suction case as well.

Method 2 does not demonstrate a localized increase as dramatically as Method 1. When 11% Suction is applied via Method 2, no bulge in surface heat transfer is present.
However, when 20% Suction is applied, there is a local increase to the $h_c$ in the region near the edge of the suction slot; $Y/D=-0.3$ and $X/D=-0.4$. This localized increase spans a much larger area than those indicated in Method 1, but is less intense. In this case, the symmetry plane demonstrates a 42% reduction in peak heat transfer. In the area of localized increase, this reduction is only 38%.

Heat transfer, or energy transfer, occurs across temperature gradients from the region of high energy (high temperature) to that of low energy (high temperature). This is the fundamental second law of thermodynamics. In the present study, the temperature gradient of the thermal boundary layer causes heat transfer down into the tunnel endwall. When the $HV$ is present, it draws warmer fluid down to the endwall, increasing the temperature gradient and thus increasing heat transfer for a fixed geometry.

Suction Method 1 applies suction immediately against the endwall. When large amounts of boundary flow are removed, the slot must draw fluid from outside the thermal boundary layer to achieve the proper mass flow. This is especially noticeable along the edges of the suction slot where fluid velocities are away from the suction slot. Warmer fluid is pulled from all directions to the surface and heats the areas surrounding the slot. For removal flow rates less than 20%, this localized increase in heat transfer is still less than the effects of an uncontrolled $HV$ system. However, when 20% Suction is applied via Method 1, this effect is locally comparable to no control case along the symmetry plane. However, the overall performance, i.e. along the three planes of study, is improved from the baseline. Method 2 does not demonstrate this effect to the magnitudes of Method 1. This is most likely because the slot is located above the surface, therefore drawing fluid away from the surface rather than towards it. The effect was present when
a suction rate of 20% was applied and it is postulated that this effect will worsen as suction rates increase.

4.3.5 Comprehensive Aerodynamic Effects

Both methods of control have been shown to reduce the time-averaged net-circulation along the symmetry plane when 11% of the boundary mass flow was removed. The overall effects of localized endwall suction are now presented. The results are best quantified by the net-circulation, $\Gamma$, data in the region of the $HV$. Vorticity data were used to calculate a single circulation value on each plane at every suction level. The percent difference from the baseline case for each method on each plane is calculated for all suction rates and plotted in Figure 4.15. The percent difference in net-circulation was found using Equation 4.2, such that a negative value represents a reduction.

$$\% \text{Difference} = \left( \frac{\Gamma - \Gamma_{NS}}{\Gamma_{NS}} \right) \times 100\%$$

Eqn. 4.2

Recall that the $HV$ is identified by a strong concentration of positive vorticity and is the most dominant feature in the $HV$ system.
Figure 4.15 shows suction has the greatest effect on the 0.5-D plane and the least effect on the 1-D plane. Recall, the 1-D plane was shown to have a very weak vortex due to viscous dissipation, so the effects of suction will most likely decay in the same way. Along the symmetry plane, the strong concentrations of negative vorticity of the $SV$ and $CV$ reduce the circulation value. So, if the strength of these vortices is reduced, the circulation value will increase. The $SV$ and $CV$ are not as prominent on the 0.5-D plane, and therefore a reduction to the positive vorticity of the $HV$ will have a substantial effect.

Figure 4.15 also shows a diminishing effect at high suction rates along the symmetry plane of Method 1. Net-circulation along the 0.5-D plane has a very linear slope, where the other planes seem to flatten out at suction rates greater than 11%. Method 2, on the other hand, seems to steadily reduce the circulation with increasing suction rates. As was shown in the case studies, Method 1 completely removed the $HV$ system with the application of 11% boundary layer removal. With the vortex system
eliminated, there is physically not much vorticity for the higher suction rates to remove. Method 2 reduced the size and strength of the vortex system but did not remove it completely. Therefore, removing a greater percentage of mass flow would further reduce the size and strength of the system. These trends were also seen by Phillips et al., Barberis et al., Seal et al. and Bloxham et al. [1, 3, 17, 22]. Apparently, this study did not remove the amount of mass flow necessary to completely eliminate the HV system with Method 2. Overall, Method 1 consistently outperforms Method 2 in the mitigation of aerodynamic losses. This suggests that the HV system, the primary source of vorticity in the junction region, is better removed by suction Method 1 than Method 2.

An inconsistency exists in Figure 4.15. The figure shows net-circulation decreases with increasing suction on every plane save the 1-D plane of Method 1. When 6.5% of the boundary mass flow is removed, the circulation is reduced by approximately 25%. When 11% suction is applied, only a ~23% reduction is seen. This is most likely due to the 2-dimensional grid used to calculate the net-circulation. When 6.5% suction is applied via Method 1 the HV system is not completely removed; meaning a TV and a counter-rotating SV also exist. Recall, when no control is applied, the negative vorticity of the SV moves away from the endwall with downstream distance. Along the 1-D plane, the SV is located well within the area considered when calculating circulation. When 11% suction is applied with Method 1 the HV system is removed, eliminating a large concentration of both positive vorticity, the HV, and negative vorticity, the SV. When 6.5% suction is applied, the existence of a SV, although weakened, helps to reduce the net-circulation value more than the removal of the HV alone. Thus the percent reduction is greater when 6.5% suction is applied than with 11% suction. This effect is not
apparent along the other planes because the majority of the \( SV \) is located outside the area of vorticity integration.

The technique of boundary layer removal at the immediate junction of the airfoil and endwall was first introduced by Bloxham et al. Similar suction rates were pulled through a 1-mm slot located at the intersection of a faired cylinder and endwall. A constant Reynolds number of \( Re_D = 7.2 \times 10^3 \) was maintained for all testing. Bloxham et al. characterized the effectiveness of the steady flow control along three planes, the symmetry plane and two downstream planes, using total pressure measurements and normalized circulation data. Figure 4.16 presents the circulation results of Bloxham et al. along the 1.5-D plane as compared to those of current study along the 0.5-D and 1-D planes.

![Figure 4.16: Percent difference in circulation from baseline along downstream planes as compared to Bloxham et al. [3]; Method 1, all suction rates.](image)

The trends present in all data sets of Figure 4.16 match very well. The data from Bloxham et al. show increasing suction to reduce the net-circulation along the 1.5-D
plane. When comparing this to the data of the present study, the effect of suction appears to dwindle as the same amount of suction reduces the circulation less and less with each downstream plane. This is understood since the HV system has been shown to dissipate with increasing downstream distance.

A comparison of circulation data with existing research confirms the validity of the present study. Overall, increasing suction reduces the net-circulation along each plane with the greatest effect being seen by the 0.5-D plane for both suction methods.

4.3.6 Comprehensive Heat Transfer Effects

Both methods of control have been shown to reduce the surface heat transfer along all planes, therefore reducing detrimental effects of the HV system. Heat transfer and suction effects are most prominent along the symmetry plane and decrease with increasing distance from the leading edge. Presented here is a direct comparison between methods along the symmetry plane, then, the downstream planes are discussed.

Figure 4.17 plots the convective heat transfer coefficient, $h_c$, along the symmetry plane for both Methods 1 and 2. There is a clear trend of decreasing heat transfer with increasing suction with both methods. Method 1 (left) shows a diminishing return with the suction at rates greater than 11% of the boundary layer flow. Method 2 (right) continues to reduce both the primary peak and plateau in heat transfer with increasing suction rates. Both methods also show a shift in the peak heat transfer. Method 1 moves the primary peak from $X/D=-0.52$ in the baseline, or No suction case, to the cylinder’s surface, $X/D=-0.50$, at a rate of 11%. Method 2 shifts the primary peak from the baseline location of $X/D=-0.54$ to $X/D=-0.52$ at the maximum SR of 20%. Method 2 never
completely removes this primary peak, nor does it completely remove the inflection points in the plot. Method 1 however, completely removes all these features at suction rates of 11% or more. This indicates that Method 1 is able to completely remove the $HV$ system where Method 2 is only able to lessen the system’s size and strength for suction rates up to 20% of the boundary layer mass flow. Nevertheless, Method 2 shows overall lower values of $h_c$ than Method 1. For example, when 20% Suction is applied utilizing Method 1, the value of $h_c$ at the location of the primary peak is reduced 36% from the baseline. When the same suction applied using Method 2, $h_c$ is reduced by 45% at this location; a clear advantage of Method 2.

![Figure 4.17: Convective heat transfer coefficient along the symmetry plane; Methods 1 (left) & 2 (right), all suction rates.](image)

In order to make a direct comparison between methods, the percent difference in convective heat transfer at every point along the symmetry plane was plotted in a single figure, Figure 4.18. Method 1 is represented by the dashed lines and Method 2 is represented by the solid lines. The local percent difference was calculated using
Equation 4.3; recall that each method was compared to the no suction case of that method.

\[
\%\text{Difference} = \left( \frac{h_{c,i} - h_{c,NS,i}}{h_{c,NS,i}} \right) \times 100\% \tag{Eqn. 4.3}
\]

Figure 4.18 clearly shows a “double peak” reduction in heat transfer for both methods. This is equivalent to the previous discussion that the peak value of \( h_c \) was reduced and the plateau, surrounded by inflection points, was removed. This is represented in Figure 4.18 as a large negative peak between \( X/D = -0.6 \) and -0.55, representing the weakening of the HV, and a second negative peak between \( X/D = -0.9 \) and -0.8, representing the weakening of the SV and TV. The figure also indicates the same decrease in \( h_c \) with increasing suction rates. Interestingly, the figure shows that although Method 1 has superior results for 6.5\% and 11\% suction, Method 2 displays a greater reduction in heat transfer at the larger suction rates. Also, it is noted that the end point of each plot, located at \( X/D = -0.5 \), shows that Method 1 does not reduce the heat transfer at this point by more than 15\%, where Method 2 shows reductions up to 30\% at this location. From this figure it can be said that Method 2 is more effective at reducing the surface heat transfer along the symmetry plane.
Finally, the surface heat transfer along all planes was examined. Similar to Figure 4.15 in the aerodynamic losses section, Figure 4.19 shows the percent difference in peak $h_c$ along each plane at each suction rate for both Methods 1 and 2.

It can be seen that the trends of the symmetry plane are apparent along the downstream planes as well. Surface heat transfer decreases with increasing suction on all planes with both methods. As on the symmetry plane, Method 2 shows a greater reduction in peak $h_c$ than Method 1, along the 0.5-D and 1-D planes. Method 1 shows a diminishing reduction in maximum $h_c$ for suction rates greater than 11% on the symmetry plane and on the 0.5-D plane; where the percent reductions are nearly identical. This is not as noticeable on the 1-D plane, where the trend is more of a linear decrease with increasing suction. Method 2 shows a slight diminishing effect on all planes, but not as great as that of Method 1. The reduction in peak surface heat transfer is clearly greatest along the symmetry of Method 2.
Figure 4.19: Percent difference in peak $h_c$ from baseline along 3 planes; Methods 1 & 2, all suction rates.

These observations are supported when examining the correlating vorticity plots of each case where the strong positive vorticity of the $HV$ becomes less and less apparent with increasing suction as applied using either method. A complete data set of the results for Method 1 and Method 2 can be found in Appendices A and B of the report respectively.
CHAPTER 5

SUMMARY AND CONCLUSIONS

The effects of localized endwall boundary layer removal were studied. Five removal rates were tested at two locations along the leading edge of an axisymmetric airfoil in cross-flow. Particle image velocimetry was used to collect time-mean velocity vectors of the flow field. The data were compared to surface heat transfer measurements made using a transient infrared thermographic technique.

Collected data agree with the findings of previous research considering an airfoil with no applied control. Vorticity data match the location, size and strength of the horseshoe vortex system, while circulation data identified downstream losses due to viscous dissipation. Also, the surface heat transfer was found to accurately track the size, strength and movement of the horseshoe vortex system as it convects around the airfoil.

Control Methods 1 and 2 were found to differ during the no control cases. It was shown that a small amount of fluid, approximately 0.1% of the boundary layer mass flow, was able to escape through the suction system when it was set to “off.” This was demonstrated best by a loss of a CV and the downstream shift of the HV in Method 1’s no control case. To account for this when comparing control, each suction rate was normalized by its own “No Suction” case.
Increased boundary layer removal was found to reduce the aerodynamic losses about the airfoil. Consistent with previous research, the net-circulation was reduced with higher removal rates. The greatest effects were seen along the cross-stream plane located at the cylinder’s centerline, the 0.5-D plane, but was followed closely by the symmetry plane for both methods. When boundary layer mass flow rates of 11% or more were pulled from a location immediately at the endwall-airfoil juncture, the HV system was removed completely. When suction was applied at a distance away from the endwall, the HV system was weakened but never completely removed for suction rates up to 20% of the approaching boundary layers mass flow.

These findings were confirmed by examination of select boundary layer velocity profiles. The HV is located above a strong reverse flow against the endwall that feeds into the SV. When suction was applied away from the endwall, the reverse flow was mitigated but never eliminated for the suction rates tested. However, when 11% suction was applied immediately at the endwall-airfoil juncture, all reverse flow was eliminated along the symmetry plane, where the HV system originates. The near-wall fluid was shown to accelerate into the suction slot at velocities near that of the freestream when 20% suction was applied. This effect was greatest with Method 1 which generated large velocity gradients at the surface and therefore, created high surface shear stresses.

Surface heat transfer was found to decrease with increased boundary layer removal. Without control, the greatest amount of heat transfer occurred near the leading edge of the airfoil, between the HV and the CV. This area showed a 350% increase over comparable flat plate heat transfer values, which agreed with the increase seen by previous researchers. The large increase was due to the ability of the HV structure to pull
warm freestream fluid down to the tunnel floor. When the HV was weakened from the application of boundary layer suction, the peaks of local surface heat transfer, and overall heat transfer, were also mitigated.

It was found that high suction rates created a vortex pair that developed on either side of the suction slot due to the turning of fluid into the leading edge of the cylinder. This effect showed up in the surface heat transfer data along the sides of the suction slot as a local bulge of increased $h_c$ values. These were prominent and distinct for the cases with suction applied along the endwall; at suction rates of 11% and greater. The effects were not as great when suction was applied away from the endwall via Method 2. The 20% suction rate saw a weak increase to surface heat transfer that spanned an area much larger than that of Method 1.

There are both similarities and differences between the comprehensive effects of the HV system. Both methods of control decreased the effects of the horseshoe vortex system as suction rate was increased. Suction along the endwall showed a decreasing trend but demonstrated diminishing returns at high suction rates. Suction applied above the endwall saw a slight diminishing return at high suction rates, but not as abrupt as the other method. The net-circulation data showed Method 1 consistently lessened the aerodynamic effects of the HV system better than Method 2. This is most likely due to the ability of Method 1 to completely remove the HV structure, the dominant source of vorticity on all planes, at suction rates of 11% and greater, where Method 2 only shifted and weakened the structure for the removal rates tested. In contrast, Method 2 outperformed Method 1 in the reduction of peak surface heat transfer on all planes, at all suction rates. The reduction in surface heat transfer was greatest along the symmetry
plane. The application of suction demonstrated a “double peak” reduction in surface heat transfer for all rates. Although at low suction rates control Method 1 showed better effectiveness, at suction rates greater than 11%, Method 2 displayed greater reductions to surface heat transfer. This may be explained by the drawing of warm fluid from outside the thermal boundary layer down to the endwall near the endwall-airfoil juncture during suction Method 1. This warmer fluid heats the surface thereby producing high values of $h_c$. Suction Method 2, on the other hand, draws fluid away from the endwall further reducing $h_c$ with increasing suction rates.
REFERENCES


APPENDIX A

METHOD 1 COMPLETE DATA SET
Figure A.1: Method 1, No Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer on 1-D plane.
Figure A.1 continued
Figure A.2: Method 1, 6.5% Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure A.2 continued
Figure A.3: Method 1, 11% Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure A.3 continued
Figure A.4: Method 1, 15% Suction; A) Surface $h_c$ map (W/m²K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure A.4 continued
Figure A.5: Method 1, 20% Suction; A) Surface $h_c$ map (W/m²K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure A.5 continued
Method 1, Symmetry Plane, SR = 0%

Method 1, Symmetry Plane, SR = 7%

Method 1, Symmetry Plane, SR = 11%

Method 1, Symmetry Plane, SR = 15%

Method 1, Symmetry Plane, SR = 20%

Figure A.6: Normalized boundary layer profiles along the symmetry plane; Method 1, No Suction, 6.5% Suction, 11% Suction, 15% Suction & 20% Suction.
APPENDIX B

METHOD 2 COMPLETE DATA SET
Figure B.1: Method 2, No Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure B.1 continued
Figure B.2: Method 2, 6.5% Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure B.2 continued
Figure B.3: Method 2, 11% Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure B.3 continued
Figure B.4: Method 2, 15% Suction; A) Surface $h_c$ map (W/m²K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure B.4 continued
Figure B.5: Method 2, 20% Suction; A) Surface $h_c$ map (W/m$^2$K), B) Normalized vorticity and streamlines with surface heat transfer on symmetry plane, C) Normalized vorticity with surface heat transfer on 0.5-D plane, D) Normalized vorticity with surface heat transfer.
Figure B.5 continued
Figure B.6: Normalized boundary layer profiles along the symmetry plane; Method 2, No Suction, 6.5% Suction, 11% Suction, 15% Suction & 20% Suction.
APPENDIX C

VORTICITY NUMERICAL CALCULATION
MATLAB PROGRAM FOR PIV CALCULATIONS

% Rebecca Hollis
% PIV Data Processing
% Master's Thesis Work
% March 2009

% close all
% clear all
% clear variables
clc
format long

% Enter Data Set Information
load Sym1_Ave_094.txt;
Data = Sym1_Ave_094;
Roto = 094 / 100;  % Applied Suction: 000, 030, 050, 070 or 094
Row = 3;
Plane = 1;  % Symmetry = 1, 90-degree = 2, 1-d = 3;
Method = 1;  % Suction Method: Matt's = 1, RMH = 2;
% Position = 1;  % NS= 1, 03= 2, 05= 3, 07= 4, and 094= 5

% Defining Limits
horz_lt = -0.95;
horz_rt = -0.5 - 0.025;
vert_top = 0.175;
vert_bot = 0.02;

% Raw Data
x = Data(:,1);  % Horizontal Distance (mm)
y = Data(:,2);  % Vertical Distance (mm)
u = Data(:,3);  % Horizontal Component of Velocity (m/s)
v = Data(:,4);  % Vertical Component of Velocity (m/s)

% Experimental Set-up
D = 76.2;  % Cylinder Diameter (mm)
Dia = D / 1000;  % Cylinder Diameter (m)
U_inf = 3.84;  % Freestream Velocity (m/s)
nu = 15.389 * 10^-6;  % Kinematic Viscosity of Air (m2/s)
Re = U_inf * Dia / nu;  % Reynolds Number
delta = 38.4 / 1000;  % Boundary Layer Thickness (m)
delta_star = 6.9 / 1000;  % Displacement Thickness (m)
SR = Roto/(5*U_inf*Dia*(delta-delta_star));  % Suction Rate (%)
SR_disp = round(SR);
Foam = 15;  % Foam Height (mm)
F = Foam / 1000;  % Foam Height (m)
f = F / Dia;  % Normalized Foam Height
Thick = 1.5;  % Slot Thickness (mm)
T = Thick / 1000;  % Slot Thickness (m)
t = T / Dia;  % Normalized Slot Thickness

% PIV Set-up
no = 1000;  % Number of PIV images collected
dt = 100 * 10^-6;  % Time Used to Find Velocities (s)

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % Plotting Axes %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if Plane == 1;  % Symmetry plane
  a = 2;
  ...
loc = -.5;
bottom = 'X';

if Method == 1;
% Matt's method 1
y_shift = -33.7333; % -34.9335; % -35.0085;
x_shift = 55.3097; % 55.6098;
else
% RMH's method
y_shift = -29.4641; % -29.5386; % -29.9111; % -30.0601;
x_shift = 52.3271; % 52.1036; % 52.4016; % 52.0291; % 51.9546; % 51.2840;
end

elseif Plane == 3;
% One-d plane
a = 1.5;
loc = -.5 + .05; % ??? Find distance away from outer diameter!!?? .1?
bottom = 'Y';

if Method == 1;
% Matt's method
y_shift = -39.8521; % -38.9219; % -38.7528; % -38.8374;
x_shift = 52.0449; % 52.3832;
else
% RMH's method
y_shift = -27.5530; % -27.8892;
x_shift = 52.0359; % 49.8468; % 49.7689
end

else
% Half-d plane
a = 1.75;
loc = -.5;
bottom = 'Y';

if Method == 1;
% Matt's method
y_shift = -39.1385; % -38.8114; % -38.8932; % -39.7928; % -39.9564; % -39.5666;
x_shift = 48.50; % 50.8311; % 50.9946; % 51.1582; % 51.6489; % 49.0318;
else
% RMH's method
y_shift = -28.7567; % -28.8205; % -28.6047;
x_shift = 52.0359; % 50; % 51.0481; % 49.5283; % 51.428; % 52.4159;
end
end

x_adj = x - x_shift;
y_adj = y - y_shift;

% Non-dimensionalizing
X_plot = (x_adj ./D) + loc;
Y_plot = y_adj ./D;

% Converting Data
dxx = diff(x);
d_x = find(dxx < 0);
dx = dxx(1)/1000;
Xplot = X_plot(1:d_x(1));

dyy = diff(y);
d_y = find(dyy < 0);
dy = abs(dyy(d_y(1)))/1000;
% dy = dyv(d_y(1))/1000;
Yplot = vertcat(Y_plot(d_y,:), Y_plot(end));
Yplot = flipud(Yplot);

for i = 1:length(Yplot);
% Rows
for j = 1:d_x(1);
% Columns
U(length(d_x)+2-i,d_x(1)+1-j) = u(i*d_x(1)-(j-1));
V(length(d_x)+2-i,d_x(1)+1-j) = v(i*d_x(1)-(j-1));
omega_x(length(Yplot)+1-i,d_x(1)+1-j) = (x(i*d_x(1)-(j-1)))/1000;
% omega_X(length(Yplot)+1-i,d_x(1)+1-j)=X(i *d_x(1)-(j-1))+x_shift+loc;
% omega_Y(length(Yplot)+1-i,d_x(1)+1-j)=Y(i *d_x(1)-(j-1))-y_shift;
omega_Y(length(Yplot)+1-i,d_x(1)+1-j)=Y_plot(i*d_x(1)-(j-1));
omega_Y(length(Yplot)+1-i,d_x(1)+1-j)=Y_plot(i*d_x(1)-(j-1));
% Calculating Vorticity
for i = 1:length(d_x)+1; % Rows (y)
  for j = 1:d_x(1); % Columns (x)
    if j == 1;
      dV_dx(i,j) = (V(i,j+1)-V(i,j))/dx; % Forward difference
    elseif i == 1;
      dU_dy(i,j) = (U(i+1,j)-U(i,j))/dy; % Forward difference
    elseif j == d_x(1);
      dV_dx(i,j) = (V(i,j)-V(i,j-1))/dx; % Backward difference
    elseif i == length(d_x)+1;
      dU_dy(i,j) = (U(i,j)-U(i-1,j))/dy; % Backward difference
    else
      dV_dx(i,j) = (V(i,j+1)-V(i,j-1))/(2*dx) ; % Center difference
      dU_dy(i,j) = (U(i+1,j)-U(i-1,j))/(2*dy) ; % Center difference
    end
  end
end
omega = dV_dx - dU_dy; % Forward difference
end

% Filtering and Smoothing
for i = 1:length(d_x)+1; % Rows (y)
  for j = 1:d_x(1); % Columns (x)
    if j == 1 && i == 1; % Top-Left Corner
      omega_new(i,j) = (2*omega(i,j)+ omega(i+1,j)+ omega(i,j+1))/4;
    elseif j == 1 && i ==length(d_x)+1; % Left Side
      omega_new(i,j) = (2*omega(i,j)+ omega(i+1,j)+ omega(i-1,j))... + omega(i,j+1))/5;
    elseif j == 1 && i == length(d_x)+1; % Bottom-Left Corner
      omega_new(i,j) = (2*omega(i,j)+ omega(i-1,j)+ omega(i,j+1))/4;
    elseif j == 1 && i == 1; % Bottom-Middle
      omega_new(i,j) = (2*omega(i,j)+ omega(i-1,j)+ omega(i,j+1))... + omega(i+1,j))/5;
    elseif j == d_x(1) && i == 1; % Top-Middle
      omega_new(i,j) = (2*omega(i,j)+ omega(i,j-1)+ omega(i,j+1))... + omega(i+1,j))/5;
    elseif j == d_x(1) && i == 1; % Bottom-Right Corner
      omega_new(i,j) = (2*omega(i,j)+ omega(i,j+1)+ omega(i,j-1))/5;
    elseif j == d_x(1) && i == length(d_x)+1; % Top-Right Corner
      omega_new(i,j) = (2*omega(i,j)+ omega(i,j+1)+ omega(i,j-1))/4;
    elseif j == 1 && i == length(d_x)+1; % Right Side
      omega_new(i,j) = (2*omega(i,j)+ omega(i,j+1)+ omega(i-1,j))... + omega(i,j-1))/5;
    elseif j == 1 && j ==d_x(1) && i ==1 && i ==length(d_x)+1; % Center
      omega_new(i,j) = (2*omega(i,j)+ omega(i+1,j)+ omega(i-1,j) + omega(i,j+1) + omega(i,j-1))/6;
  end
end
if omega_new(i,j)<= 30; % New Zero Filtering
  if omega_new(i,j) >= -30;
    omega_new(i,j) = 0;
  end
end
if omega_new(i,j) <= -1000; % Extreme Low Filtering
  omega_new(i,j) = -1000;
end
if omega_new(i,j) >= 1000; % Extreme High Filtering
  omega_new(i,j) = 1000;
end
end
omega_new = -1.*omega_new;

% Normalizing
omega_norm = (Dia.*omega_new)./U_inf;

% Defining Limits
Area_1x = find(omega_X(1,:) <= horz_lt,1,'last');
Area_2x = find(omega_X(1,:) <= horz_rt,1,'last');
Area_1y = find(omega_Y(:,1) >= vert_bot,1,'first');
Area_2y = find(omega_Y(:,1) >= vert_top,1,'first');

Vort_area_min = min(min(omega_new(Area_1y:Area_2y,Area_1x:Area_2x)));
Vort_area_max = max(max(omega_new(Area_1y:Area_2y,Area_1x:Area_2x)));
Vort_norm_area_min = min(min(omega_norm(Area_1y:Area_2y,Area_1x:Area_2x)));
Vort_norm_area_max = max(max(omega_norm(Area_1y:Area_2y,Area_1x:Area_2x)));

% Calculating Circulation
Circ_total = dx*dy*sum(sum(omega_new));
Circ_tot_norm = Circ_total * Dia / U_inf;
Circ_area = dx*dy*sum(sum(omega_new(Area_1y:Area_2y,Area_1x:Area_2x)));
Circ_area_norm = Circ_area * Dia / U_inf;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%
% setup_figure('figure')
% % % Normalized Vorticity Plot
set(figure,'Units','inches','Position',[8 5 9 4])
% subplot(2,1,1);
[B,l] = contour(omega_X, omega_Y,omega_norm,18);
clabel(B);
xlabel(bottom,'/D');
ylabel('Z/D');
colormap(jet(50))
colorbar('location','southoutside')
% grid on
axis([-1.25 0 .25])
% axis([min(omega_X(:))/a max(omega_X(:))-x_shift 0 max(omega_Y(:))/a])
caxis([-16 16])
title(['Normalized Vorticity Averaged Over ',num2str(no),' Images'];

elseif Plane == 2;
contourf(omega_X, omega_Y,omega_norm,26)
grid on
caxis([-16 16])

elseif Plane == 11;
contourf(omega_X, omega_Y,omega_norm,30)
% 'LineStyle','none')
grid on
caxis([-16 16])

% [sx, sy] = meshgrid(max(omega_X), 0.3);
% lines = streamline(stream2(omega_X, omega_Y, U, V, sx(:, sy(:, )));
% set(lines,'Color',[.6 .6 .6])
else
    contourf(omega_X, omega_Y, omega_norm, 30)', 'LineStyle', 'none')
    grid on
    caxis([-16 16])
    hold on
    plot(-.5, omega_Y(1:end, 1), 'k +', 'LineWidth', 2)
end
xlabel([bottom, '/D'], 'FontSize', 12, 'FontWeight', 'bold')
ylabel('Z/D', 'FontSize', 12, 'FontWeight', 'bold')
colormap(jet(50))
colorbar('location', 'eastoutside', 'FontSize', 12, 'FontWeight', 'bold')
grid minor
axis([-1.25 loc 0 .25])
axis([min(omega_X(:))/a max(omega_X(:)) 0 max(omega_Y(:))/a])
caxis([-20 20])
text(horiz+loc+1, vert-.05, ['Area Maximum Vorticity: ', ...
    num2str(Vort_norm_area_max), 'HorizontalAlignment', 'left', ...
    'BackgroundColor', [0 .9 .9 .9], 'FontSize', 12)
title([{'Normalized Vorticity Averaged Over ', num2str(no), ' Images'}; ...
    {'Method', num2str(Method), ' Plane', num2str(Plane), ...
    'SR = ', num2str(SR_disp), '%'}], 'FontSize', 12, 'FontWeight', 'bold');
hold on
plot(omega_X(:, Area_1x), omega_Y(:, 1), 'm--')
plot(omega_X(:, Area_2x), omega_Y(:, 1), 'm--')
plot(omega_X(1,:), omega_Y(Area_1y,:), 'm--')
plot(omega_X(1,:), omega_Y(Area_2y,:), 'm--')

% setup_figure('title', [{'PIV Data Analysis ', date, ' Rebecca Hollis'}; ''])
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% %%%%%%%%%%%%%%%%%%%%
% Defining Limits
bl_xmin = -.99;
bl_xmax = -.55;
bl_x = bl_xmin:0.11:bl_xmax;
bl_ymin = 0;
bl_ymax = 0.5;
% Finding Limits
BL_x = find(omega_X(1,:) <= bl_x(:,1), 'last');
BL_2x = find(omega_X(1,:) <= bl_xmax, 'last');
BL_1y = find(omega_Y(:,1) >= bl_ymin, 'first');
BL_2y = find(omega_Y(:,1) >= bl_ymax, 'first');
rows = 1:3;
color = ['c' 'r' 'g' 'b' 'k'];
set(figure(10), 'Units', 'inches', 'Position', [7 0 9 10])
for k = 1:length(bl_x)
    BL_x = find(omega_X(1,:), <= bl_x(k), 1, 'last');
    subplot(rows(end), length(bl_x), ((Row-1)*length(bl_x))+k);
    plot((U(BL_1y:BL_2y, BL_x)/max(max(U)), omega_Y(BL_1y:BL_2y, BL_x), 'c o')

Color - Here
hold on
plot(0, omega_Y(BL_1y:BL_2y, BL_x), 'k :')
axis([-5 1 0 .5])
if k == 1;
    ylabel('Z/D', 'FontSize', 12, 'FontWeight', 'bold');
    if Row == 1;
elseif k == 3;
    if Row == 1;
        title(['Method ',num2str(Method),', Symmetry Plane, SR = ',num2str(SR_disp),'

        fontsize',12,'weight','bold']);
        set(gca,'YTickLabel',[1 1 1]);
    elseif Row == rows(end);
        title(['Method ',num2str(Method),', Symmetry Plane, SR = ',num2str(SR_disp),'

        fontsize',12,'weight','bold']);
        xlabel('U/U_\infty',fontsize',12,'weight','bold');
        set(gca,'YTickLabel',[1 1 1]);
    else
        title(['Method ',num2str(Method),', Symmetry Plane, SR = ',num2str(SR_disp),'

        fontsize',12,'weight','bold']);
        set(gca,'YTickLabel',[1 1 1]);
    end
else
    if Row == 1;
        title(['X/D=',num2str(bl_x(k))],fontsize',12,'weight','bold');
    end
    set(gca,'YTickLabel',[1 1 1]);
end
hold on
end
APPENDIX D

HEAT TRANSFER NUMERICAL CALCULATION
MATLAB PROGRAM FOR CALCULATING HEAT TRANSFER COEFFICIENT

% modified from OSU_NoCool_HtXfr_jpb41.m
% by: J Bons, B. Barker and S. Lewis

close all
clear variables
clc

% Read in data from Excel spreadsheet, converting all Temps to Kelvin
dfile=[];
fprintf('The file should be closed if you wish to write to it \n');
dfile=input('Name of excel data sheet <fp_mar_11.xls> : ','s');
if isempty(dfile)
dfile='fp_mar_11.xls';
end
fprintf('
Loading data from %s ...
', dfile);
DATA1a = xlsread(dfile,1);
DATA2a = xlsread(dfile,2)+273.15;
DATA3a = xlsread(dfile,3);

pamb_HG = DATA1a(1,1);
% Ambient Pressure in "HG
pamb = 3377*pamb_HG;
% Ambient Pressure in Pa

T_IR_TR(1) = DATA1a(9,1);
% Starting time IR
T_LV_TR(1) = DATA1a(10,1);
% Starting time LV

% Aligns data with each other...

T_rate_IR = DATA1a(23,1);
% IR Data Acquisition rate

Pixel_Left = DATA1a(32,1);
% Zoom - Left boundary
Pixel_Right = DATA1a(33,1);
% Zoom - Right boundary
Pixel_Up = DATA1a(34,1);
% Zoom - Upper boundary
Pixel_Down = DATA1a(35,1);
% Zoom - Lower boundary

%*******************Transient LabView data**************
% Read in the Labview data
for i = 1 : length(DATA3a(:,2));
    T_inf_TR(i)= DATA3a(i,2)+273.15;
    % Freestream TC temperature upstream
    T_pitot_TR(i)= DATA3a(i,3)+273.15;
    % Freestream TC temperature at Pitot
    T_nfloor_TR(i)= DATA3a(i,4)+273.15;
    % Floor North TC temperature
    T_sfloor_TR(i)= DATA3a(i,5)+273.15;
    % Floor South TC temperature
    T_under_TR(i)= DATA3a(i,6)+273.15;
    % Underside TC temperature
    T_infds_TR(i)= DATA3a(i,9)+273.15;
    % Freestream TC temperature downstream
    Trans_voltage(i) = DATA3a(i,7);
    % Transducer Voltage
    Vel_TR(i)= DATA3a(i,8);
    % Freestream velocity from Pitot
    if i > 1
        t_LV_TR(i) = t_LV_TR(1) + DATA3a(i,1); %Time
    end
end
clear DATA3a
% Eliminate NaN's from Vel string
Veldum = zeros(size(Vel_TR));
Veldum(find(Vel_TR > 0.001)) = Vel_TR(find(Vel_TR > 0.001));
Vel_TR = Veldum;

%*******************Transient IR data**********************
fprintf('Interpolating IR data for different background Temps...\n');
lowTback = DATA2a(:,2);
highTback = DATA2a(:,5);
clear DATA2a
dum = lowTback;  %Initial guess
for k = 1:10 %10 iterations are probably unnecessary as this converges quickly
    for i = 1:length(lowTback)
        dum(i) = lowTback(i)-(lowTback(i)-highTback(i))*...
            (dum(i)-lowTback(1))/(highTback(end)-lowTback(1));
        if k == 1
            t_composite(i) = t_IR_TR(1)+(i-1) / t_rate_IR;
        end
    end
end
T_composite = dum;
dT_bkgnd = T_composite - lowTback;

figure(1)
plot(t_composite,T_composite,'k-');
hold on
plot(t_composite,lowTback,'r--');
plot(t_composite,highTback,'g-.');
plot(t_LV_TR,T_pitot_TR,'m:');
grid on
legend('T IR Composite','T IR using Low T Background','T IR using High T Background');
xlabel('Time in sec - NOT TRUE TIME');
ylabel('Temp in Kelvin');
title('Adjustment for Background Temperature Change');

figure(2)
plot(t_composite,T_composite,'k-');
hold on
plot(t_composite,lowTback,'r--');
plot(t_composite,highTback,'g-.');
grid on
legend('T IR Composite','T IR using Low T Background','T IR using High T Background');
xlabel('Time in sec - NOT TRUE TIME');
ylabel('Temp in Kelvin');
title('Adjustment for Background Temperature Change');

transfile = 'IR_fp_3-11.asc';
fprintf('%nThe transient IR data file to be loaded is %s 
', transfile);
newtransfile = input('Enter new filename or <accept>: ','s');
if isempty(newtransfile)
    transfile=newtransfile;
else
    transfile=newtransfile;
end
filid = fopen(transfile,'rt');

fprintf('%nThe total number of transient IR frames is %d.
', ...
    length(t_composite));
nframe=[];
if isempty(nframe);
    nframe=input('Enter number of transient frames to read <245>: ');  
    if isempty(nframe)  
        nframe = 245;
    end
end
fprintf('%nLoading transient IR data from %s',transfile);
tic
for i = 1:nframe;
    fprintf('Loading frame %d 
',i);
    for j = 1:460
        dum = fscanf(filid,'%s',1);
        if length(dum)==12 && dum(3)==':' && dum(6)==':'
            % Some frame headers are corrupted and only have the timestamp
            break
        end
    end
    time = dum; % From IR timestamp
    hr(i) = str2num(time(1:2));
    min(i) = str2num(time(4:5));
    sec(i) = str2num(time(7:8));
    % It appears that the camera does acquire data at the same fractional second
    T_IR_TR(:,:,i) = fscanf(filid,'%g',[320 256])'; % Temperature matrices
end

toc
% The IR maps must be adjusted by the dT_bkgnd so that they account for the
time-varying background temperature as well. Convert to Kelvin
for i=1:nframe
    T_IR_TR(:,:,i) = T_IR_TR(:,:,i) + dT_bkgnd(i) + 273.15;
    T_IR_mean(i) = mean(mean(T_IR_TR(:,:,i)));
end

% Create a true IR time data vector
trueIRtime = (hr*3600) + (min*60) + sec;
trueIRtime = trueIRtime - trueIRtime(1) + t_IR_TR(1);
adjtime=1;
adjtotal=0;
while adjtime~=0
    figure(3)
    plot(trueIRtime,T_IR_mean,'m+')
    hold on
    plot(t_composite,t_composite,'ko-',t_LV_TR,T_pitot_TR,'mx:');
    legend('True IR Time','T IR Composite','T pitot');
    xlabel('Time in seconds - NOT TRUE TIME FOR T IR Composite');
    ylabel('Temp in Kelvin');
    xlim([t_composite(1) t_composite(1)+50]);
    title('Composite adjustment for T Background');
    grid on
    adjtime=input('Enter time adjustment for IR data <0=no adjustment>: ');
    if isempty(adjtime)
        adjtime=0;
    end
    adjtotal=adjtotal+adjtime;
    t_composite=t_composite+adjtime;
end
fprintf('Total time shift for IR data is %g sec 
',adjtotal);
clear t_composite T_composite

fprintf('Interpolating Labview times into IR times...\n');
for i=1:length(trueIRtime)
    ind = max(find(t_LV_TR<trueIRtime(i)));
    fract = (trueIRtime(i)-t_LV_TR(ind))/(t_LV_TR(ind+1)-t_LV_TR(ind));
    T_inf_TR_interp(i) = T_inf_TR(ind)+fract*(T_inf_TR(ind+1)-T_inf_TR(ind));
T_pitot_TR_interp(i) = T_pitot_TR(ind) + fract * ...
   (T_pitot_TR(ind+1)-T_pitot_TR(ind));
T_nfloor_TR_interp(i) = T_nfloor_TR(ind) + fract * ...
   (T_nfloor_TR(ind+1)-T_nfloor_TR(ind));
T_sfloor_TR_interp(i) = T_sfloor_TR(ind) + fract * ...
   (T_sfloor_TR(ind+1)-T_sfloor_TR(ind));
T_under_TR_interp(i) = T_under_TR(ind) + fract * ...
   (T_under_TR(ind+1)-T_under_TR(ind));
Vel_TR_interp(i) = Vel_TR(ind) + fract * (Vel_TR(ind+1)-Vel_TR(ind));
end

%**********Time Synchronization*****************

fprintf('Identifying start of transient flow...\n');
for i = 1:length(Vel_TR_interp)
   if Vel_TR_interp(i+3)>Vel_TR_interp(i+2) && ...
      Vel_TR_interp(i+2)>Vel_TR_interp(i+1) && ...
      Vel_TR_interp(i+1)>Vel_TR_interp(i)
      if i<4
         Start_TR=1;
      else
         Start_TR = i-3;
      end
      break
   end
end
nullTemp = mean(T_IR_mean(Start_TR:Start_TR+3)) - ...
    mean(T_sfloor_TR_interp(Start_TR:Start_TR+3));
%Set starting time to 0 and truncate vectors and matrix accordingly
trueIRtime = trueIRtime(Start_TR:end)'-trueIRtime(Start_TR);
T_IR_TR = T_IR_TR(:,1,Start_TR:end);
T_IR_mean = T_IR_mean(Start_TR:end);
T_pitot_TR_interp = T_pitot_TR_interp(Start_TR:end)' + nullTemp;
T_nfloor_TR_interp = T_nfloor_TR_interp(Start_TR:end)' + nullTemp;
T_sfloor_TR_interp = T_sfloor_TR_interp(Start_TR:end)' + nullTemp;
T_under_TR_interp = T_under_TR_interp(Start_TR:end)' + nullTemp;
Vel_TR_interp = Vel_TR_interp(Start_TR:end)';
nframe = length(trueIRtime);

figure(4);
plot(trueIRtime,T_IR_mean,'ko')
hold on
plot(trueIRtime,T_sfloor_TR_interp,'rx')
plot(trueIRtime,T_pitot_TR_interp,'m:')
plot(trueIRtime,T_inf_TR_interp,'b-.');
grid on
legend('T IR Mean of entire image','T_s floor - surface temp', ...
       'T pitot - freestream','T inf - freestream');
xlabel('True Time in seconds');
ylabel('Temp in Kelvin');
title('Interpolated and Adjusted Temps');
grid on

% Calculate the St for the T_IR_mean temperature history and plot vs.
% empirical relation.
% Use the thermophysical properties that correspond to the Tinit of the
% stereolithography plastic (Somos 18420)
TinitC = T_IR_mean(1) - 273.15;
tcflow = T_pitot_TR_interp;
vel = Vel_TR_interp;
time = trueIRtime;
\[
\text{specheat} = 1093 + 10.6648 \times T_{\text{initC}} - 0.0232123 \times T_{\text{initC}}^2; \quad \text{\%J/kgK} \quad \text{---Dr. Bons}
\]

\[
\alpha = 3.102037e-12 \times T_{\text{initC}}^2 - 7.924196e-10 \times T_{\text{initC}} + 1.624137e-7; \quad \text{\%m}^2/\text{s} \quad \text{---Dr. Bons}
\]

\[
\text{density} = 1190; \quad \text{\%kg/m}^3
\]

\[
T_{\text{cond}} = \text{density} \times \alpha \times \text{specheat}; \quad \text{\%W/mK}
\]

\[
c_{10} = 2.0 \times \sqrt{\frac{T_{\text{cond}}^2}{\alpha \pi}};
\]

Nominal flow conditions are...

\[
\text{gasconst} = 287; \quad \text{\%J/kgK}
\]

\[
\text{cpair} = 1005; \quad \text{\%J/kgK}
\]

\[
Prt = 0.9; \quad \text{\%Turbulent Prandtl Number}
\]

% IR measurement location
\[
xts = 1.712; \quad \text{\%m}
\]

% Calculate the empirical smooth plate h value.
\[
p = 1;
\]

\[
\text{for} \quad m = 2: \text{length(time)}
\]

\[
\text{airdens} = \frac{pamb}{(\text{gasconst} \times \text{tcflow}(m))};
\]

\[
\text{visc} = 0.00001716 \times (\text{tcflow}(m)/273)^1.5 \times (273+111)/(\text{tcflow}(m)+111);\]

%Sutherland's
\[
T_{\text{condair}} = 0.0241 \times (\text{tcflow}(m)/273)^1.5 \times (273+194)/(\text{tcflow}(m)+194);\]

%Sutherland's
\[
\text{Pr} = \text{visc} \times \text{cpair} / T_{\text{condair}};
\]

\[
\text{if} \quad \text{vel}(m) > 3 \quad \text{\%6 m/s threshold is needed for some cases}
\]

\[
\text{if} \quad p = 1
\]

\[
\text{Reexp}(p) = \text{vel}(m) \times \text{airdens} \times xts/\text{visc};
\]

\[
\text{cfemp}(p) = 0.026/\text{Reexp}(p)^{(1/7)}; \quad \text{\%0.592/Reexp}(p)^{(1/5)};
\]

\[
\text{Stemp}(p) = 0.5 \times \text{cfemp}(p)/(\text{Pr} + \text{sqrt}(0.5 \times \text{cfemp}(p)) \times \ldots
\]

\[
(5 \times \text{Pr} + 5 \times \log(5 \times \text{Pr} + 1) - 14 \times \text{Prt}));
\]

\[
\text{hemp}(p) = \text{Stemp}(p) \times \text{airdens} \times \text{cpair} \times \text{vel}(m);
\]

\[
p = p + 1;
\]

\[
\text{end}
\]

\[
\text{end}
\]

% Perform 1-D Schulz-Jones on the T_IR_mean IR Temperature data
\[
\text{htc\_mean} = [];\]

\[
\text{htc\_mean}(1) = 0;
\]

\[
\text{q\_mean} = [];\]

\[
\text{q\_mean}(1) = 0;
\]

\[
p = 1;
\]

\[
\text{for} \quad m = 2: \text{length(time)};
\]

\[
\text{Tsum} = 0.0;
\]

\[
\text{for} \quad n = 2:m;
\]

\[
\text{Tsum} = \text{Tsum} + (\text{T\_IR\_mean}(n) - \text{T\_IR\_mean}(n-1)) / \ldots
\]

\[
(\text{sqrt}(\text{time}(n) - \text{time}(n-1)) + \text{sqrt}(\text{time}(m) - \text{time}(n-1)));\]

\[
\text{end}
\]

\[
\text{q}(m) = c_{10} \times \text{Tsum};
\]

\[
\text{htc}(m) = \frac{\text{q}(m)}{(\text{tcflow}(m) - \text{T\_IR\_mean}(m))};
\]

\[
\text{airdens} = \frac{\text{pamb}}{(\text{gasconst} \times \text{tcflow}(m))};
\]

\[
\text{if} \quad m >= \text{stind}
\]

\[
\text{Stexpir}(p) = \frac{\text{htc}(m)}{(\text{airdens} \times \text{cpair} \times \text{vel}(m))};
\]

\[
p = p + 1;
\]

\[
\text{end}
\]

% Plot the empirical St vs. the St from mean IR Temp history
\[
\text{figure}(5)
\]

\[
\text{plot(\text{time}(\text{stind}:\text{end}), \text{Stemp}, ['k-'])}
\]

\[
\text{hold on}
\]
plot(time(stind:end),Stexpir,'rx');
xlabel('Time [sec]');
ylabel('St');
ylim([0.001 0.004]);
title(['Data from file ',transfile,' at mean Rex = ', ...
      num2str(round(mean(Reexp)))]);
grid on
legend('Empirical','Mean Exp. -full IR field',4);

% Compute average Stexpir
avgst=[];
if isempty(avgst)
    avgst=100;
end
indst = find(time>avgst,1);
end
avgend = [];
if isempty(avgend)
    avgend=240;
end
indend = find(time > avgend,1);

fprintf('
Empirical St = %g',mean(Stemp(indst:indend)));
fprintf('
Experimental St = %g',mean(Stexpir(indst:indend)));
fprintf('
Error St = %2.1f',1-
(mean(Stemp(indst:indend))/mean(Stexpir(indst:indend)))
fprintf('
For Rex = %g
',mean(Reexp(indst:indend)));

ansa=[];
if isempty(ansa)
    ansa='n';
end
if ansa=='n'
    break
else

% Specify a region for spatially resolved St interrogation
adjbox=1;
while adjbox==0
    figure(6)
    contourf(T_IR_TR(:,:,nframe));
    colorbar
    title(['The Last IR Temp map loaded in from ',transfile]);
    xlabel('x-direction');
    ylabel('y-direction');
    hold on
    if Pixel_Left<1; Pixel_Left=1; end;
    if Pixel_Up<1; Pixel_Up=1; end;
    if Pixel_Right<1; Pixel_Right=1; end;
    if Pixel_Down<1; Pixel_Down=1; end;
    plot([Pixel_Left Pixel_Left Pixel_Right Pixel_Right Pixel_Left], ...
    [Pixel_Up Pixel_Down Pixel_Down Pixel_Up Pixel_Up],'k-');
    hold off
    fprintf('Coordinates of Interrogation Box:
');
    fprintf('Bottom Left x: %g \n',Pixel_Left);
    fprintf('Bottom Left y: %g \n',Pixel_Up);
    fprintf('Top Right x: %g \n',Pixel_Right);
    fprintf('Top Right y: %g \n',Pixel_Down);
    adjbox=[];
    adjbox=input('Select new box coordinates? 1=yes, 0=no <no> ');
    if isempty(adjbox)
adjbox = 0;
end
if adjbox ~= 0
    Pixel_Left = input('New Bottom Left x: ');
    Pixel_Up = input('New Bottom Left y: ');
    Pixel_Right = input('New Top Right x: ');
    Pixel_Down = input('New Top Right y: ');
end
figure(6)
end

% Calculate the mean Temp history on a limited domain downstream of the center 2 holes.
Tmap = T_IR_TR(Pixel_Up:Pixel_Down,Pixel_Left:Pixel_Right,:);
[nrow,ncol,nframe]=size(Tmap);
clear T_IR_TR
meanIRTemp_sub = mean(mean(Tmap));
meanIRTemp_sub = reshape(meanIRTemp_sub,nframe,1);
figure(7)
plot(trueIRtime,T_IR_mean';'gd',trueIRtime,meanIRTemp_sub,'ko');
hold on
plot(trueIRtime,T_pitot_TR_interp,'m:',trueIRtime,T_nfloor_TR_interp,'b+');
plot(trueIRtime,T_sfloor_TR_interp,'rx');
grid on
legend('Mean of entire IR field','Mean of selected IR region',...'
T_pitot','T_nfloor','T_sfloor');
xlabel('Time [sec]');
ylabel('Temp [K]');
grid on
hold off

% Use the thermophysical properties that correspond to the Tinit of the stereolithography plastic (Somos 18420)
TinitC = meanIRTemp_sub(1)-273.15;
specheat = 1093 + 10.6648 * TinitC - 0.0232123 * TinitC^2; %J/kgK
alpha = 3.102037e-12 * TinitC^2-7.924196e-10 * TinitC + 1.624137e-7; %m2/s
tcond = density * alpha * specheat; %W/mK
c10 = 2.0 * sqrt(tcond^2/alpha/pi);

% Perform 1-D Schulz-Jones on the mean IR Temp in this limited region
htc_mean=[];
htc_mean(1)=0;
qu_mean=[];
qu_mean(1)=0;
p=1;
for m=2:nframe
    Tsum=0.0;
    for n=2:m
        Tsum=Tsum+(meanIRTemp_sub(n)-meanIRTemp_sub(n-1))/(sqrt(time(m)-time(n))+sqrt(time(m)-time(n-1)));
    end
    q(m)=c10*Tsum;
    htc(m)=q(m)/(tcflow(m)-meanIRTemp_sub(m));
    airdens=pamb/(gasconst*tcflow(m));
    if m>=stind
        Stexpirsub(p)=htc(m)/(airdens*cpair*vel(m));
p=p+1;
    end
end

% Plot the empirical St vs. the St from mean IR Temp history
figure(8)
plot(trueIRtime(stind:end),Stemp,'k-',trueIRtime(stind:end),Stexpirsub,'rx');
xlabel('Time [sec]');
ylabel('St');
ylim([0.001 0.004]);
title(['Data from file ',transfile,' at mean Rex = ',num2str(round(mean(Reexp)))]);
grid on
legend('Empirical','Mean Experiment-IR',4);

% Schulz-Jones summation for all the pixels in the IR sub map.
% If this is the cooled case, the tcflow Temp must be replaced with a film
% temperature based on the local film effectiveness
fprintf('
Performing Schulz-Jones for Temp map subset...
');
tic
for j=1:nrow
    for k=1:ncol
        htc=[];
        htc(1)=0;
        q=[];
        q(1)=0;
        p=1;
        for m=2:nframe
            Tsum=0.0;
            for n=2:m
                Tsum = Tsum + (Tmap(j,k,n)-Tmap(j,k,n-1)) /
                ...((sqrt(time(m)-time(n)) + sqrt(time(m)-time(n-1))));
            end
            q(m) = c10 * Tsum;
            htc(m) = q(m)/(tcflow(m)-Tmap(j,k,m)); %W/m2K
            if htc(m) < 0; htc(m)=0.0001;
            end %Doesn't allow local values of
            hmap(j,k,m) = htc(m);
        end
    end
end
toc

% Average selected frames
avgst=[];
avgst=input('Enter start time for averaging <100>:');
if isempty(avgst)
avgst=100;
end
indst=find(time>avgst,1);
avgend=[];
avgend=input('Enter end time for averaging <240>:');
if isempty(avgend)
avgend=240;
end
indend=find(time>avgend,1);
navg=indend-indst+1;

hmap_avg=zeros(nrow,ncol);
for i=indst:indend
    hmap_avg=hmap_avg+hmap(:,:,i);
end
hmap_avg=hmap_avg/navg;

% Plot the hmap with coolant
figure(9)
% contourf(hmap_avg,[15:1:35])
contourf(hmap_avg,20)
% contourf(hmap_avg,20,'LineStyle','none')
% caxis([15 35]);
colorbar
grid on
axis equal
title(['h [W/m2K] for the data subset of ',transfile,...'
': ',num2str(navg),' frames'])
xlabel('x-direction');
ylabel('y-direction');

% Spatial average of hmap from x/d=0.5 and downstream
% h_spatavg = mean(mean(hmap_avg(:,1:126)));
% h_spatavg = mean(mean(hmap_avg(:,1:81)));
fprintf('Spatially averaged hmap from x/d=0.5 and downstream: %g 
\n',h_spatavg);
ansa=[];
ansa=input('Save the h data to excel sheet? <n>',s);
if isempty(ansa)
    ansa='n';
end

% Write the average hmap to the same Excel spreadsheet
if ansa=='y'
    ansb=input('Please specify sheet number? <5>');
    xlswrite(dfile,hmap_avg',ansb);
end
fprintf('\nWriting data to %s ...
', dfile);
end %for transient option