MEASURING THE EFFECTS OF SATISFACTION: LINKING CUSTOMERS, EMPLOYEES, AND FIRM FINANCIAL PERFORMANCE

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

Jeffrey P. Dotson, B.S., M.B.A., M.STAT.

* * * * *

The Ohio State University

2009

Dissertation Committee:

Greg M. Allenby, Adviser
Robert E. Burnkrant
Rao H. Unnava

Approved by

__________________________

Adviser
Graduate Program in Business Administration
ABSTRACT

Firms are most successful when they are able to efficiently satisfy the wants and needs of their clientele. As such, customer satisfaction has emerged as one of the more ubiquitous and oft studied constructs in marketing. Central to the study of satisfaction is the desire to understand its antecedents and outcomes. Managers would ultimately like to know how their actions will impact the satisfaction of their consumer base and, by extension, the company’s financial performance. Through two essays, this dissertation develops quantitative models that allow for formal study of the relationship between customer satisfaction, employee satisfaction, and firm financial performance. The proposed models are designed to accommodate a variety of challenges often encountered in satisfaction studies including simultaneity, linkage of distributions, and the fusion of multiple data sets. The benefits of these models are demonstrated empirically using data from a national financial services firm.
To Holly, Henry, and Peter
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VITA

February 26, 1977 . . . . . . . . . . . . . . . . . . . . . . . . . . Born – Price, UT, USA

2002 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . B.S. Managerial Economics, Southern Utah University

2003 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . M.B.A., University of Utah

2005 . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . M.STAT. Business and Statistics, University of Utah

2005-Present . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . Graduate Teaching and Research Associate, The Ohio State University

PUBLICATIONS

Research Publications


FIELDS OF STUDY

Major Field: Business Administration
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CHAPTER 1

INTRODUCTION

The ability to demonstrate the impact of marketing action on firm financial performance is crucial for evaluating, justifying, and ultimately optimizing the expenditure of a firm’s marketing resources. This presents itself as a formidable task when one considers both the variety and potential influence of marketing activity. Although this applies to all types of firms, it is particularly true in the case of service organizations where transactional value often results from the qualitative interaction of customers and employees. In this context, linking action to outcome requires a thorough knowledge of the employee-customer relationship and its connection to key firm-level outcomes. The two essays in this dissertation develop the use of quantitative models in order to formally study the constructs of customer and employee satisfaction, their relationship to each other, and respective influence on behavioral and financial outcomes of the firm.

In Essay 1 (Chapter 2) the technique of linkage analysis is developed in order to study the relationship between employee and customer satisfaction. In many service organizations customers interact with many employees, and employees serve many customers, such that a one-to-one mapping between customers and employees is not possible. Analysis must proceed by relating, or linking, distribution percentiles among
variables. Such analysis is commonly encountered in marketing when data are from independently collected samples. The proposed model is demonstrated empirically in the context of retail banking, where drivers of customer and employee satisfaction are shown to be percentile-dependent. Simultaneous systems of equations with non-normal errors are also developed to allow for the potential for simultaneous causality in the customer-employee relationship.

Essay 2 (Chapter 3) proposes a Hierarchical Bayesian model in order to study the strategic influence of satisfaction on firm financial performance. Unit-level revenue production is modeled as a function of managerially controllable inputs, where latent levels of customer and employee satisfaction are allowed to exert an indirect influence on financial performance by altering the firm’s technology. Structure is imposed upon the parameters of the model through the estimation of a system of simultaneous supply and demand. The proposed model explicitly deals with the potential for endogeneity in the input variables, and produces managerially reasonable parameter estimates.

Empirically this model is applied to data provided by a national financial services firm, where data from three independently conducted studies are integrated in order to make inference. Customer and employee satisfaction are shown have both direct and indirect effects on branch-level revenue production. The proposed model also allows for the assessment of the relative benefits of engaging in short-term versus long-term marketing activities. This process is explored through the use of a marketing policy counterfactual scenario designed to determine when and under what cost structure it would become profitable for the bank to focus its efforts on increasing the latent
level of employee satisfaction as opposed to engaging in short-term sales incentive programs.

Collectively, essays 1 and 2 contribute to our understanding of customer satisfaction by studying its drivers, relationship to employee satisfaction, and ultimate influence on the financial performance of the firm. Empirically, this dissertation documents evidence of simultaneity in the connection between employee and customer satisfaction. Employee and customer satisfaction are also shown to have both direct and indirect influence on the firm’s financial performance. From a methodological perspective, models and estimation routines are developed in order to accommodate challenges commonly encountered in satisfaction studies. These include simultaneity, linkage of variables across distributions, and the fusion of multiple, independently collected data sets.
CHAPTER 2

NON-NORMAL SIMULTANEOUS REGRESSION MODELS FOR CUSTOMER LINKAGE ANALYSIS

2.1 Introduction

Customer linkage analysis investigates the relationship between the attitudes and behaviors of customers and employees. Linkage research is related to the service profit chain (Heskett et al., 1994; Heskett, Sasser, and Schlesinger, 1997), which connects managerial action to firm performance through a series of related effects. A critical link in this chain is the relationship between employees and customers, where it is believed that managers have the ability to positively influence their employees, who, in turn, can better serve customers. Satisfied customers and satisfied employees are thought to drive short-term and long-term profitability of the firm.

Although intuitively appealing, the Service Profit Chain has received mixed support within marketing literature. Rust and Chung (2006) conclude that the directional relationship between employees and customers has been demonstrated with only weak empirical support. Researchers investigating the relationship between employee and customer satisfaction have focused on the average influence of employees on customers (Kamakura, Mittal, DeRosa, and Mazzon, 2002). These studies are
typically conducted using regression methods and, as such, techniques have not been
developed for characterizing other quantiles of the relationship while allowing for the
possibility of simultaneous effects and non-normally distributed error terms. If dissat-
ished customers increase the likelihood of employee dissatisfaction, then analysis that
incorrectly assumes that employee satisfaction is independently determined will yield
inconsistent estimates of the relationship of these variables to their determinants, or
drivers. Furthermore, if customer and employee satisfaction data are asymmetrically
distributed, modeling approaches that rely on the assumption of normality may fail
to correctly estimate the true relationship between the same.

Mathematically, linkage analysis attempts to connect two datasets \((A\) and \(B\))
where the cardinality of the relationship is such that no single element in data set \(A\)
can be linked directly to an element in data set \(B\). Relationships that exhibit this
many-to-many mapping are prevalent in service organizations. They include natural
interactions between customers and employees, pupils and teachers, and patients and
nurses. Customers typically have multiple needs that are serviced over time by a team
of employees; pupils take courses from multiple instructors; and patients receive care
from many health-care professionals. It is often not possible to attribute successful
outcomes to any one team member. Organizations with many customer touch-points
must therefore rely on analysis at a more aggregate level to assess the effectiveness of
service delivery.

Linkage analysis is useful for analyzing data that were not originally collected from
the same units of analysis (e.g., same customers). Such analysis occurs, for example,
in brand tracking studies where the sample composition of respondents changes over
time. A distribution of responses is available for analysis at each time period, and the
goal of analysis is to measure the incremental value of a marketing initiative without being able to track the responses at the individual-level. Linkage analysis occurs when conducting analysis across data sets that are not, or cannot be, connected to the same individual. Examples of this involve linking customer satisfaction data with ad tracking and awareness data, linking scanner panel data with corporate ROI data such as sales and revenue, and relating satisfaction with multiple products that may not be bundled (e.g., DSL, wireless, long distance phone service) with an overall measure of satisfaction with the service provider. Such analysis is becoming more common in marketing as management looks to derive added value from existing data.

In this chapter we develop the use of non-normal simultaneous regression models to study the relationship between customer and employee satisfaction. We demonstrate our model in the context of retail banking, where customers are served by a variety of bank employees (tellers, loan officers, customer service managers, etc) who interact with a variety of customers. Cross-sectional surveys of both groups reveal only information about branch-level distributions of attitudinal and behavioral measures. We show that standard methods based upon linkage at the mean fail to fully characterize relationships that exist between the distributions of responses. Our method allows for estimation of functional relationships that exist for different portions of these outcome distributions.

The remainder of this chapter is organized as follows: in section 2.2 we describe the general form of our model and present three alternative error distributions that can flexibly accommodate non-normal data in a simultaneous equation framework. In section 2.3 we describe the data set used to illustrate the method developed in section 2.2. Results for this application are presented and discussed in sections 2.4
and 2.5. We conclude the chapter by discussing the implications of this research for linkage analysis and identify a number of potential areas for future work.

2.2 Simultaneity in Non-Normal Systems

In this section we develop a simultaneous equation model that allows for the possibility of non-normally distributed error terms. We do this in anticipation of customer linkage analysis where employees can affect customer satisfaction and customers can affect the satisfaction of employees. Allowing for the possibility of simultaneous effects enriches analysis by understanding the drivers of satisfaction for both sets of individuals. A natural question for Bayesian analysis is why one would want to assume the existence of asymmetric errors when specifying the likelihood for this model. We explore three answers to this question.

First, there has been much discussion in the marketing services literature regarding the existence of asymmetry in customer satisfaction and loyalty data (Anderson and Mittal 2000, Struikens and Ruyter 2004). Empirical work has documented evidence of negative asymmetry in a variety of applied settings (Mittal et al., 1998; Anderson and Sullivan, 1993). The existence of negative asymmetry in models of evaluative judgment (e.g. customer satisfaction) could suggest that respondents overweight negative experiences and underweight positive experiences. This notion is consistent with Kahneman and Tversky’s (1979) treatment of prospect theory. Consumers are inherently loss adverse, which causes losses to loom larger than gains. Recalling past service encounters from memory may therefore involve asymmetric errors.

A second justification for the use of asymmetric errors is the presence of scale ceiling effects. Responses at the extremes of a scale are susceptible to truncation,
giving rise to a distribution of skewed error terms. This is particularly problematic in services research where top-box scores are prevalent. The distribution of responses from surveys is often massed at the upper portion of a ratings scale, leading to an asymmetric distribution of responses with a thick left tail.

Finally, one could take a pragmatic view and test for the existence of asymmetric errors. We examine three error distributions that can flexibly accommodate both symmetric and asymmetric data. If the errors are, in fact, normally distributed, these distributions are capable of providing a reasonable approximation.

### 2.2.1 System of Equations

Our general model is of the form:

\[
y_A = \alpha_0 + \alpha_1 y_B + \sum_{j=2}^{J} \alpha_j x_j + \varepsilon_{yA} \quad \text{where} \quad \varepsilon_{yA} \sim f_A(\cdot) \tag{2.1}
\]

\[
y_B = \beta_0 + \beta_1 y_A + \sum_{k=2}^{K} \beta_k z_k + \varepsilon_{yB} \quad \text{where} \quad \varepsilon_{yB} \sim f_B(\cdot) \tag{2.2}
\]

where \(y_A\) and \(y_B\) are, respectively, employee and customer satisfaction, and \(x_j\) and \(z_k\) are covariates that affect \(y_A\) and \(y_B\), and are exogenous to the system. \(f_A(\cdot)\) and \(f_B(\cdot)\) are densities described below that can flexibly model non-normal errors.

Bayesian analysis proceeds by first specifying the likelihood for the model. Substituting for \(y_A\) and \(y_B\) and solving, equations (2.1) and (2.2) can be rewritten as:

\[
y_A = \frac{\left(\alpha_0 + \sum_{j=2}^{J} \alpha_j x_j + \varepsilon_{yA}\right) + \alpha_1 \left(\beta_0 + \sum_{k=2}^{K} \beta_k z_k + \varepsilon_{yB}\right)}{1 - \alpha_1 \beta_1} \tag{2.3}
\]

\[
y_B = \frac{\left(\beta_0 + \sum_{k=2}^{K} \beta_k z_k + \varepsilon_{yB}\right) + \beta_1 \left(\alpha_0 + \sum_{j=2}^{J} \alpha_j x_j + \varepsilon_{yA}\right)}{1 - \alpha_1 \beta_1} \tag{2.4}
\]
which demonstrates that the errors are non-linearly related to the observed data. The likelihood for the data, however, is easily computed using change of variable calculus. The likelihood for an observation pair \((y_A, y_B)\) can be written as:

\[
\pi(y_A, y_B) = \pi(\varepsilon_{y_A}, \varepsilon_{y_B}) \left| J_{(\varepsilon_{y_A}, \varepsilon_{y_B})\rightarrow(y_A, y_B)} \right| \tag{2.5}
\]

where:

\[
\varepsilon_{y_A} = y_A - \left( \alpha_0 + \alpha_1 y_B + \sum_{j=2}^{J} \alpha_j x_j \right) \sim f_A(\cdot) \tag{2.6}
\]

\[
\varepsilon_{y_B} = y_B - \left( \beta_0 + \beta_1 y_A + \sum_{k=2}^{K} \beta_k x_k \right) \sim f_B(\cdot) \tag{2.7}
\]

\[
J_{(\varepsilon_{y_A}, \varepsilon_{y_B})\rightarrow(y_A, y_B)} = \left| \frac{\partial \varepsilon}{\partial y'} \right| = 1 - \alpha_1 \beta_1 \tag{2.8}
\]

Given the selection of an error distribution, \(f(\cdot)\), Bayesian estimation proceeds by assigning prior distributions to all model parameters. Standard MCMC methods are then employed to sample from the posterior distribution of model parameters (Rossi, Allenby, and McCulloch 2005). Specific algorithms are provided in the appendix. Simulation experiments were conducted under a variety of settings to verify the validity of each of these estimation routines.

For comparative purposes, we investigate the performance of three error distributions that can flexibly accommodate the existence of asymmetry in the data: an asymmetric Laplace distribution (AL), a skewed \(t\) distribution (skewt), and a multivariate mixture of normals. The following is a discussion of the distributional assumptions for each of these models:

### 2.2.2 Asymmetric Laplace Distribution

Our first model assumes that \(\varepsilon_{y_A}\) and \(\varepsilon_{y_B}\) from equations (2.1) and (2.2) are independently distributed random variables that follow the 3 parameter Asymmetric
Laplace distribution of Yu and Zhang (2005).

\[ \varepsilon_{y_A} \sim AL(0, \sigma_{y_A}, p_{y_A}) \] (2.9)

\[ \varepsilon_{y_B} \sim AL(0, \sigma_{y_B}, p_{y_B}) \] (2.10)

where \( \sigma \) is a scale parameter and \( p \in (0, 1) \) is a scalar that governs the degree of asymmetry in the distribution. If \( p > 0.5 \) the distribution is left tailed skewed, if \( p < 0.5 \) the distribution is right tail skewed. The density function for the AL is presented in equation (2.11).

\[
f_p(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp \left\{ -\rho_p \left( \frac{y-\mu}{\sigma} \right) \right\}
\] (2.11)

where:

\[
\rho_p(y) = y (p - I(y < 0))
\] (2.12)

\( \rho_p \) is a loss function that applies a penalty \( p \) to positive residuals and a penalty \((p - 1)\) to negative residuals. Yu and Moyeed (2001) show that likelihood based inference that is conducted using independently distributed AL densities (where \( p \) is a priori specified) is directly related to the implementation of quantile regression (Koenker and Bassett 1978, Koenker 2005). Quantile regression is conducted by solving the mathematical programming problem presented in equation (2.13).

\[
\min_\beta \sum_t \rho_p(y_t - x_t'\beta)
\] (2.13)

where \( \rho_p \) is the same loss function presented in equation (2.12). Kottas and Krnjajic (2007) explore generalizations of the AL density for quantile regression using a Dirichlet process mixture model. In the context of this chapter, we do not exploit this duality between likelihood based inference with the AL and quantile regression.
Rather, we view the AL as a flexible family of densities that can be used to model asymmetrical error distributions. We treat $p$ as a free parameter in our model and use the data to estimate it.

Figure 2.1 compares the AL to the Standard Normal Distribution and illustrates how the skewness of the AL changes with differing values of $p$. The AL is linear in the exponent, in contrast to the normal distribution with a quadratic exponent. When $p = 0.5$, the AL is symmetrically distributed about its mean and assumes the form of the more common double exponential distribution. Relative to the Normal Distribution, the AL is characterized by a peaked mode with thick tails.

### 2.2.3 Skewed $t$ Distribution

Our second model assumes that $\varepsilon_{yA}$ and $\varepsilon_{yB}$ are independently distributed random variables that each follow the four parameter skewed $t$ distribution developed by Fernandez and Steel (1998).

\[
\varepsilon_{yA} \sim \text{skewt}(0, \gamma_{yA}, \sigma_{yA}, \nu_{yA}) \tag{2.14}
\]

\[
\varepsilon_{yB} \sim \text{skewt}(0, \gamma_{yB}, \sigma_{yB}, \nu_{yB}) \tag{2.15}
\]

Fernandez and Steel (1998) demonstrate that any symmetric, unimodal distribution can be transformed into a class of skewed distributions through the introduction of a parameter $\gamma$. The general form of this approach is presented in equation (2.16), where $\gamma \in \mathbb{R}_+$ is a scalar parameter that governs the direction and magnitude of asymmetry.

\[
f(\varepsilon|\gamma) = \frac{2}{\gamma + \gamma^{-1}} \left\{ f\left(\frac{\varepsilon}{\gamma}\right) I_{[0,\infty)}(\varepsilon) + f\left(\gamma \varepsilon\right) I_{(-\infty,0)}(\varepsilon) \right\} \tag{2.16}
\]
Moments of these skewed distributions can be computed according to equation (2.17), where $M_r$ indicates the $r^{th}$ moment of the original, symmetric distribution.

\[ E(\varepsilon^r | \gamma) = M_r \frac{\gamma^{r+1} + (-1)^r}{\gamma + \gamma^{-1}} \]  

(2.17)

This method is applied to a univariate student $t$ distribution in order to develop a modeling approach that can accommodate both asymmetry and thick tails. The density function for this skewed $t$ distribution is presented in equation (2.18).

\[
f(y_i | \mu, \sigma, \nu, \gamma) = \frac{2}{\gamma + \gamma^{-1}} \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\pi^\frac{\nu}{2}} \sigma^{-1} \times \left[ 1 + \frac{(y_i - \mu)^2}{\nu^2 \sigma^2} \left\{ \frac{1}{\gamma^2} I_{0, \infty}(y_i - \mu) + \gamma^2 I_{-\infty, 0}(y_i - \mu) \right\} \right]^{-\frac{\nu+1}{2}}
\]  

(2.18)

Where $\nu \in \mathbb{R}_+$ is the scalar degrees of freedom parameter that controls tail behavior, $\sigma$ is a scale parameter, and $\mu$ is a location parameter. Equation (2.18) reduces to a standard normal distribution as $\nu \to \infty$ for $\gamma = 1$, $\sigma = 1$, and $\mu = 0$. Figure 2.2 graphically depicts the shape of this distribution under various parameter setting. The left panel displays the skewed $t$ distribution for different values of $\nu$. The skewed $t$ exhibits thick tails for small values of $\nu$. As $\nu$ increases its tail behavior converges to that of a normal distribution. The right panel of Figure 2.2 depicts the skewed $t$ distribution for differing values of $\gamma$. The distribution is left tail skewed if $\gamma < 1$, right tail skewed if $\gamma > 1$, and symmetric if $\gamma = 1$.

2.2.4 Mixture of Multivariate Normals

A third approach to modeling asymmetric errors is to use a mixture of normals as described by Rossi, Allenby, and McCulloch (2005). That is, we assume that the $\varepsilon$’s are jointly distributed according to a mixture of $k$ bivariate normal distributions (equation 2.19).

\[
\begin{pmatrix} \varepsilon_{yA} \\ \varepsilon_{yB} \end{pmatrix} \sim \sum_k \phi_k N(\mu_k, \Sigma_k)
\]  

(2.19)
Where $\phi_k$ is the weight associated with the $k^{th}$ mixture component for $\sum_{k=1}^{K} \phi_k = 1$. The parameters $\mu_k$ and $\Sigma_k$ are the component specific mean vector and covariance matrix. The regression parameters $\alpha$ and $\beta$ from equations (2.1) and (2.2) are assumed to be common across all components.

The mixture distribution described in equation (2.19) has the potential to be the most flexible of all distributions discussed thus far. It can easily accommodate asymmetry and thick tails like the skewed $t$, in addition to multimodality and other deviations from normality. Additionally, the structure of the error distribution presented in equation (2.19) allows for correlation in the $\varepsilon$’s. Implementation of this model in the context of simultaneous equations requires a slight deviation from the standard algorithm presented in Rossi, Allenby, and McCulloch (2005). Specifically, the regression parameters, $\alpha$ and $\beta$, must be drawn using a Metropolis-Hastings step. This requires direct evaluation of the likelihood presented in equation (2.20).

$$
\prod_i \prod_k \varphi \left( \varepsilon_{yA,i,n_k}, \varepsilon_{yB,i,n_k} | \mu_k, \Sigma_k \right) \mid J_{\varepsilon \rightarrow y} \right) \tag{2.20}
$$

where $i$ indexes the respondent, $k$ indexes the mixture component, $\varphi (\cdot)$ is the multivariate normal density, and $J$ is the Jacobian defined in equation (2.8). Although this change can be easily implemented, it does substantially increase the computational burden of model estimation.

### 2.3 Empirical Application

Linkage analysis proceeds by first determining the quantile to use in studying the relationship between data set $A$ (e.g., employees) and data set $B$ (e.g., customers) within each unit of analysis. This involves selecting, or estimating, $p_A$ and $p_B$ such
that:

\[ y_{Ai} = F(A_i, p_A) \]  \hspace{1cm} (2.21)

\[ y_{Bi} = F(B_i, p_B) \]  \hspace{1cm} (2.22)

where \( i \) indexes the units of analysis. In our application, the index \( i \) corresponds to various branch offices. \( F(\cdot) \) is the cumulative distribution function, and \( y_{Ai} \) and \( y_{Bi} \) are the points of the distributions \( A \) and \( B \) where the percentage of lower valued observations are equal to \( p_A \) and \( p_B \), respectively. This process is applied to all variables, dependent and independent, included in the analysis. Thus, the data correspond to hypothetical agents described by the distributional quantiles.

The selection of quantiles \( p_A \) and \( p_B \) is often dictated by the business decision at hand. We may choose to select \( p_A \) and \( p_B \) to examine the relationship between the lower tails (e.g. the most dissatisfied employees and customers) or the upper tails (e.g. the most satisfied employees and customers) of the distribution of responses. Or, we may want to investigate how the most dissatisfied employees (lower tail) impact the most satisfied customers (upper tail). Alternatively, analysis could proceed by searching over all possible quantile combinations to find the best-fitting relationship.

### 2.3.1 Data

Data are provided by a national financial services firm, consisting of customer and employee survey responses for the firm’s consumer banking group. The data set is such that all respondents can be directly tied to one of the banks branches. Each consumer surveyed was asked to provide a holistic evaluation of the bank in addition to an assessment of the branch they frequent most often. In order to avoid confusion, the branch in question is explicitly defined in each consumer survey. Employee responses
are grouped according to their branch of employment. The data for both groups was collected during roughly the same time period.

A sample of 746 branches (employee and customer surveys) was obtained for model calibration. Descriptive statistics for the data sets are presented in Table 1. Included in this table are the respective customer and employee questions used as variables in the analysis. An average of 37 customer surveys were collected for each branch (minimum of 6, maximum of 87). In these surveys respondents were asked to rate their branch on a variety of service dimensions. Responses were recorded on a scale of 1 to 10, where 1 and 10 denote, respectively, “unacceptable” and “outstanding.” An average of 7 employee responses were recorded per branch (minimum of 5, maximum of 19). These responses were scaled from 1 to 5, where 1 and 5 indicate, respectively, “Very Dissatisfied” and “Very Satisfied.” In order to maintain consistency in the data and ease the interpretation of results, both customer and employee data were rescaled onto the [0, 1] interval, where 1 represents the maximum possible positive response (Outstanding or Very Satisfied).

The structure of this data is such that an overall measure of satisfaction is associated with various determinants. For each branch we have data on the distribution of the various measures included in the analysis. For example, on a branch-to-branch basis we have a sample approximation of the distribution of customer satisfaction. In order to estimate the model described in equations (2.1) and (2.2) we must first reduce these branch-level distributions to points that summarize the quantiles we wish to analyze. This is accomplished through the linking procedure described above, requiring the selection of within-unit linking quantiles \( p_A \) and \( p_B \) (see equations 2.21-2.22).
resulting data are then used to estimate the model for the second stage of the linking process.

Figure 2.3 presents plots of employee vs. customer satisfaction for data sets constructed at the quartiles of the data \((p_A = p_B = 0.25, 0.50, 0.75)\). The presence of scale effects is readily apparent in the data, with the distribution of scores truncated from above at 1.0, the maximum value. In addition, we find that the values associated with the first quartile (Q1) to have much greater dispersion than those associated with the third quartile (Q3). It is therefore likely that regression coefficient estimates may differ across these portions of the distributions.

### 2.3.2 Identification

In order to demonstrate that the system under study is identified, it is useful to re-express equations (2.1) and (2.2) in the following matrix form:

\[
\begin{bmatrix}
1 & -\alpha_1 \\
-\beta_1 & 1
\end{bmatrix}
\begin{bmatrix}
y_A \\
y_B
\end{bmatrix}
= 
\begin{bmatrix}
\alpha_0 & \alpha_2 & \cdots & \alpha_J & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & \beta_0 & \beta_2 & \cdots & \beta_K
\end{bmatrix}
\begin{bmatrix}
x_2 \\
\vdots \\
x_J \\
1 \\
z_2 \\
\vdots \\
z_K
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_{y_A} \\
\varepsilon_{y_B}
\end{bmatrix}
\tag{2.23}
\]

Equation (2.23) demonstrates that our model implicitly includes a number of exclusion restrictions. For example, we exclude employees’ feelings about how fairly they feel they are evaluated in their jobs as a potential driver of customer satisfaction. As a result of these restrictions, it is clear that our model meets both the rank and order conditions for identification in a system of equations (Greene 2003). While the validity of these restrictions is certainly debatable, we adhere to the model presented
above in order to maintain consistency with the existing service profit chain literature. That is, we model aggregate customer (employee) satisfaction as a function of customer (employee) specific covariates and an aggregate level of employee (customer) satisfaction. Customers and employees are linked only through their respective levels of satisfaction.

2.3.3 Models

We investigate the performance of six models fit to the data. The first three models are able to flexibly accommodate asymmetry, thick tails, and other deviations from the assumption of normality. The first model (M1) is the simultaneous regression model presented in equations (2.1) and (2.2) where the error terms are assumed to be independently distributed according the AL distribution described in equation (2.11). The second model (M2) utilizes independently distributed skewed t error terms presented in equation (2.18). The third model (M3) allows for correlated error terms using the mixture of bivariate normals described in equation (2.19).

The fourth model (M4) follows the same simultaneous equation specification as the first three where normally distributed errors are used in place of the AL, skewed t, or mixture distribution. Comparing the results from this model to those of the first three allows for assessment of the benefits of asymmetric errors. The fifth model (M5) uses the estimation technique of instrumental variables (i.e., 3SLS) to deal with the potential effects of simultaneity while ignoring the possibility of asymmetry. We estimate this model in a Bayesian framework using the Gibbs sampler outlined in Chapter 7 of Rossi, Allenby, and McCulloch (2005). We employ the determinants of overall satisfaction (e.g., friendliness of branch tellers, evaluation of waiting time)
as instruments for the overall measure. Thus, in this fifth model, analysis resembles traditional two-stage least squares. The sixth model (M6) is a standard regression model that ignores the possible presence of both asymmetry and simultaneity.

2.4 Results

Table 2.2 reports coefficient estimates for quartile 1 ($p_A = p_B = 0.25$) for the 4 simultaneous equation models described above (M1-M4). For illustrative purposes, we also report coefficients and model fit statistics for the two other alternative models (M5-M6). For the sake of brevity, we do not present posterior standard deviations with their corresponding parameter estimates. Rather, coefficients displayed in boldface type denote estimates (i.e., posterior means) with 95% of their posterior distribution mass away from zero. Although this hinders a direct comparison of statistics across models and linking quantiles, it substantially simplifies the presentation of results.

Estimates of $\gamma_A$, $\gamma_B$, $p_A$, and $p_B$ confirm the presence of asymmetry in both the customer and employee satisfaction data for this quartile. Not surprisingly, the models that are able to accommodate this asymmetry fit the data better than those that cannot. Model fit in this context is assessed using the Newton-Raftery (1994) approximation to the log marginal density. Coefficient estimates also differ for M1-M3 relative to the other competitive models. For example, M1-M3 identify a positive, directional link for customer satisfaction to employee satisfaction, whereas the other models do not. Table 2.3 reports coefficient estimates for Models 1-4 for different quartiles ($p_A = p_B = 0.25, 0.50, 0.75$) of the distributions of customer and employee satisfaction (see equations 2.21 and 2.22). We find that coefficient estimates for the
models that accommodate asymmetry (M1-M3) differ from the normal model (M4), and that the magnitude of difference is related to the estimated asymmetry in the error distribution. This is interesting in light of classical econometric results that suggest that all of these estimators are consistent. That is, as the sample size tends toward infinity Models 1-4 should produce identical results. The fact that we observe differences in parameter estimates illustrates the importance of correctly modeling the form of the error distribution when working with finite samples. An advantage of performing this analysis using Bayesian methods is that we are able to generate comparative statistics that allow us to select the model that best fits the data. This is true for both nested and non-nested models.

For example, consider the model coefficients reported in the lower right portion of the table corresponding to employee satisfaction data with $p_A = 0.75$ (i.e., $y_{Ai} = F(A_i, p_A = 0.75)$ corresponds to the 0.75 quantile (Q3) of the distribution of employee satisfaction at each branch). The estimated asymmetry parameters are $p_{yA} = 0.510$ and $\gamma_{yA} = 1.066$, indicating that the distribution of error terms are close to symmetric. Here, we find for that coefficient estimates for all four employee satisfaction models are similar. In contrast, the customer satisfaction estimates for $p_B = 0.75$ reported directly above these are not similar, with estimates for models M1-M3 differing from M4 which relies on symmetric errors. The estimated asymmetry parameters are $p_{yB} = 0.897$ and $\gamma_{yB} = 1.418$, indicating that customer satisfaction responses are best modeled by using error distributions that can flexibly approximate the asymmetric shape of the data. This is reasonable in light of Figure 2.3 where the marginal distribution of customer satisfaction scores for Q3 is seen to be severely skewed.
The log marginal density is reported at the bottom of the table, and indicates that the use of skewed t errors significantly improves the fit of the models in all cases. We find that the skewed t is better at modeling the data as the residual error becomes increasingly skewed. The severity of skewness is greater in the customer data, increasing as we move from using the 0.25 quantile (Q1) as a summary measure of the distribution of branch-level responses, to the 0.75 quantile ($p_{yB} = 0.736, 0.809, 0.897$). The skewed t is able to flexibly accommodate these changes by altering both its skewness parameter, $\gamma_{yB}$, and tail behavior $\nu_{yB}$. The AL is only able to model the distribution of the data through changes in skewness ($p_{yB}$).

We note that the variance of the error for the AL is a function of the model parameters:

$$\text{var}(\varepsilon) = \frac{\sigma^2 (1 - 2\rho + 2\rho^2)}{(1 - \rho)^2 \rho^2}$$

(2.24)

As a result, posterior estimates of for the AL model are not directly comparable to the other models. The same is also true for the scale parameter $\sigma$ reported for the skewed t distribution where the variance can be computed using equation (2.17).

We find that skewed t and AL estimates of the effect of customer satisfaction on employee satisfaction is positive and significant for all quartiles, whereas estimates for the normal model (M4) do not detect a relationship between the same. We also identify a positive, non-zero relationship from employee to customer satisfaction using the skewed t distribution, but only at the median (Q2).

Figures 2.4 and 2.5 provide a graphical summary of the posterior distribution of coefficients for deciles of the distribution of customer and employee satisfaction for models fit using the skewed t distribution. The first decile ($p_A = p_B = 0.10$) corresponds to the least satisfied portion of the distribution, and the ninth decile
corresponds to the most satisfied portion \( p_A = p_B = 0.90 \). The deciles are used to generate summary statistics from each branch using equations (2.21) and (2.22), with the resulting values used as data to fit the simultaneous set of equations (2.1) and (2.2). Figure 2.4 displays the posterior distribution of drivers of customer satisfaction, and Figure 2.5 displays the drivers of employee satisfaction.

In Figure 2.4, we see that customer satisfaction is associated with employee satisfaction only for the lower deciles (i.e., deciles 0.10 through 0.50) of the distribution of responses. For relatively satisfied customers, the overall level of employee satisfaction does not affect their service encounter. The factor that affects customer satisfaction most is teller friendliness. This aspect of the service encounter is consistently found to be positively associated with customer satisfaction across all levels of satisfaction - from least to most. It is found to be of greatest importance for deciles 0.60 through 0.90, and is of singular importance at decile 0.60 of the distribution of satisfaction. Finally, we find that satisfaction with wait time is important only for the lower deciles that correspond to relatively dissatisfied customers and employees (decile 0.10 through 0.50). Customers that are relatively satisfied are not influenced by either employee satisfaction or wait time.

Figure 2.5 displays posterior distributions of coefficients associated with employee satisfaction. Customer satisfaction affects employee satisfaction across a broad range of satisfaction. In addition, the pay-performance link is seen to be most important at low levels of satisfaction, while decision authority and fair evaluation exert the greatest influence on individuals that are moderately satisfied. Growth opportunities are more important for dissatisfied individuals, and rewards take on a “U” shaped
response, with higher importance for less-satisfied and more-satisfied individuals, and less importance for those of average satisfaction.

2.5 Discussion

The results have a number of interesting implications. The most important is that drivers of customer and employee satisfaction, and the relationship between them, are different for satisfied versus dissatisfied customers and employees. Simultaneity does exist, with employees and customers affecting each other’s satisfaction, but only at specific quantiles of the distribution of satisfaction. We find that customer satisfaction affects employees more often than employee satisfaction affects customers, and that the importance of determinants of overall satisfaction, for both employees and customers, is quantile dependent. Our results illustrate the richness of analysis available from investigating functional relationships across different quantiles of the distributions of response.

We find that customers are strongly influenced by the manner in which they are treated by frontline service representatives. Although the effect of teller friendliness is strong and positive across all quantiles, we find that satisfaction for customers (connected at deciles 0.60 and higher) is determined primarily by the friendliness of the branch’s tellers. Customers in the lower half \( (p_A = 0.1 \text{ to } 0.5) \) of the satisfaction distribution base their branch evaluations on a linear combination of how they feel they have been treated by the tellers, in addition to their assessment of how much time they spend waiting for service. This finding has obvious implications. If managers wish to improve the attitudes of their less-satisfied customers (lower half of the satisfaction distribution) they should concentrate on improving both the perceived
friendliness of their customer contact employees and the perception of time spent waiting in line. Conversely, if the firm were to a priori focus on improving teller friendliness, they would know which group of consumers those efforts would be most likely to affect.

More generally, it is important to ensure that employees are aware of the impact of their efforts. One way to accomplish this task is to simply disseminate results from customer satisfaction surveys to employees (frontline employees in particular). Although this seems like a simple task, a recent study of customer satisfaction information usage (CSIU) found that 40% of firms that collect customer satisfaction data do not routinely report it to their front line employees (Morgan, Anderson, and Mittal, 2005). This represents an opportunity to begin to help employees recognize and take credit (or responsibility) for the results of their actions.

We also find evidence that customer satisfaction is affected by the latent satisfaction of a branch’s employees, although this relationship only exits at certain quantiles of the distribution of customer satisfaction. This result is theoretically consistent with the service profit chain and offers evidence in favor of the same. It is interesting to note that we would not have identified evidence of this relationship if we had relied on standard models based upon linkage at the mean. Perhaps the weak empirical support for the employee-customer link documented Rust and Chung (2006) has resulted from model misspecification (e.g., normally distributed error terms, absence of simultaneous effects, etc.).

Finally, we find that there are many drivers of employee satisfaction, with customer satisfaction playing a role for most employees, irrespective of their level of satisfaction. The importance of other drivers is found to be quantile dependent, providing
management an opportunity to rely on different instruments to affect employees who are more versus less satisfied.

2.6 Conclusion

In this chapter we present a new approach to conducting customer linkage that allows for the estimation of relationships across seemingly disparate data sets. The specification of our model allows for the existence of both simultaneity and asymmetry in the linking variables. We compare the results of our procedure to other standard modeling approaches. Estimates of the log marginal density indicate that our proposed models provide superior in-sample fit, particularly in the presence of skewed data.

The method developed in this chapter provides a general approach for characterizing relationships between two distributions of data. Given the prevalence of this type of data in marketing, this approach should be of particular import to marketing research practitioners. In the case of employee-customer linkage analysis, it allows managers the ability to understand, for example, if and how their most disgruntled employees affect the attitudes of their least satisfied customers. In addition to the methodological developments, we discover a number of different empirical results that should be of interest to managers.

The development and application of non-normal regression models for linkage analysis raises several interesting questions that are worthy of future research. Consistent with the current intent of linkage analysis, an obvious extension of this work would be to tie attitudinal and behavioral outcomes to unit and firm profitability. Additionally, empirical results show the existence of asymmetry in the distributions
of both employee and customer satisfaction. Interestingly, the degree of asymmetry differs across groups. Distributions of customer satisfaction tend to be more skewed than those of employee satisfaction. Future work should focus on examining the types of behavioral processes that could give rise to asymmetrically distributed error terms. In particular, it would be interesting to determine if and how employees differ from customers when making holistic evaluations.

Methodologically, additional work is needed to more formally relate the distribution of responses among customers and employees. Our analysis conditions on specific distributional percentiles using equations (2.21) and (2.22), whereas a more formal analysis would account for the sample sizes used to obtain these point estimates, and develop a model structure that recognizes the nesting structure (e.g., respondents within branches) of the data.
Figure 2.1: Comparison of asymmetric Laplace and normal densities
Figure 2.2: Comparison of skewed $t$ densities for varying values of $\nu$ and $\gamma$
Figure 2.3: Joint distribution of employee and customer satisfaction quantiles
Figure 2.4: Posterior distributions of coefficients for customer satisfaction
Figure 2.5: Posterior distributions of coefficients for employee satisfaction
Table 2.1: Description of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>n</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Customer Measures (1 to 10 scale; 1 = Unacceptable, 10 = Outstanding)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall Branch Rating (proxy for customer satisfaction)</td>
<td>746</td>
<td>8.68</td>
<td>0.61</td>
</tr>
<tr>
<td>Rating of the courtesy and friendliness of branch tellers</td>
<td>746</td>
<td>8.92</td>
<td>0.55</td>
</tr>
<tr>
<td>Evaluation of time required to wait in line for service</td>
<td>746</td>
<td>7.42</td>
<td>1.37</td>
</tr>
<tr>
<td><strong>Employee Measures (1 to 5 scale; 1 = Very Dissatisfied, 5 = Very Satisfied)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall Job Satisfaction</td>
<td>746</td>
<td>3.56</td>
<td>0.64</td>
</tr>
<tr>
<td>Decision making authority required to do job effectively</td>
<td>746</td>
<td>3.77</td>
<td>0.53</td>
</tr>
<tr>
<td>Fair evaluation of job performance</td>
<td>746</td>
<td>3.55</td>
<td>0.63</td>
</tr>
<tr>
<td>Clear link between job performance and compensation</td>
<td>746</td>
<td>3.00</td>
<td>0.81</td>
</tr>
<tr>
<td>Satisfaction with rewards program (pay, bonus, 401k, etc)</td>
<td>746</td>
<td>3.05</td>
<td>0.63</td>
</tr>
<tr>
<td>Personal growth and development</td>
<td>746</td>
<td>3.71</td>
<td>0.53</td>
</tr>
</tbody>
</table>
### Table 2.2: Posterior mean of regression coefficients for quartile 1

<table>
<thead>
<tr>
<th>Variables</th>
<th>Skew t</th>
<th>ALD</th>
<th>Norm Mix</th>
<th>Normal</th>
<th>IV</th>
<th>OLS</th>
</tr>
</thead>
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<tr>
<td><strong>Customer Satisfaction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$ - Intercept</td>
<td>0.298</td>
<td>0.050</td>
<td>0.273</td>
<td>0.280</td>
<td>0.268</td>
<td>0.277</td>
</tr>
<tr>
<td>$\beta_1$ - Employee Satisfaction</td>
<td>0.005</td>
<td>0.000</td>
<td>0.027</td>
<td>0.003</td>
<td>0.026</td>
<td>0.009</td>
</tr>
<tr>
<td>$\beta_2$ - Friendliness of branch tellers</td>
<td>0.472</td>
<td>0.918</td>
<td>0.439</td>
<td>0.450</td>
<td>0.449</td>
<td>0.448</td>
</tr>
<tr>
<td>$\beta_3$ - Evaluation of waiting time</td>
<td>0.215</td>
<td>0.030</td>
<td>0.239</td>
<td>0.240</td>
<td>0.240</td>
<td>0.240</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.038</td>
<td>0.000</td>
<td>0.002</td>
<td>0.002</td>
<td>0.005</td>
<td>0.002</td>
</tr>
<tr>
<td>$\gamma_0$ (skew t) / $p_0$ (ALD)</td>
<td>1.425</td>
<td>0.736</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>df</td>
<td>83.03</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td><strong>Employee Satisfaction</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_0$ - Intercept</td>
<td>0.116</td>
<td>0.075</td>
<td>-0.025</td>
<td>0.029</td>
<td>0.009</td>
<td>0.020</td>
</tr>
<tr>
<td>$\alpha_1$ - Customer Satisfaction</td>
<td>0.054</td>
<td>0.071</td>
<td>0.114</td>
<td>0.051</td>
<td>0.071</td>
<td>0.057</td>
</tr>
<tr>
<td>$\alpha_2$ - Decision making authority</td>
<td>0.116</td>
<td>0.087</td>
<td>0.127</td>
<td>0.127</td>
<td>0.129</td>
<td>0.128</td>
</tr>
<tr>
<td>$\alpha_3$ - Fair evaluation of job performance</td>
<td>0.244</td>
<td>0.242</td>
<td>0.268</td>
<td>0.262</td>
<td>0.264</td>
<td>0.265</td>
</tr>
<tr>
<td>$\alpha_4$ - Link between job perform and comp.</td>
<td>0.159</td>
<td>0.148</td>
<td>0.155</td>
<td>0.164</td>
<td>0.160</td>
<td>0.161</td>
</tr>
<tr>
<td>$\alpha_5$ - Satisfaction with rewards</td>
<td>0.187</td>
<td>0.152</td>
<td>0.209</td>
<td>0.205</td>
<td>0.209</td>
<td>0.210</td>
</tr>
<tr>
<td>$\alpha_6$ - Personal growth</td>
<td>0.178</td>
<td>0.263</td>
<td>0.182</td>
<td>0.180</td>
<td>0.181</td>
<td>0.181</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.090</td>
<td>0.001</td>
<td>0.010</td>
<td>0.010</td>
<td>0.006</td>
<td>0.010</td>
</tr>
<tr>
<td>$\gamma_0$ (skew t) / $p_0$ (ALD)</td>
<td>1.428</td>
<td>0.623</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>df</td>
<td>78.98</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>k (number of mixture components)</td>
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<td>--</td>
<td>1</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>1992</td>
<td>1976</td>
<td>1944</td>
<td>1941</td>
<td>1699</td>
<td>1890</td>
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</table>

Quartile 1: 0.25
<table>
<thead>
<tr>
<th>Variables</th>
<th>Skew 1</th>
<th>ALD</th>
<th>Norm</th>
<th>Mix Normal</th>
<th>Skew 2</th>
<th>ALD</th>
<th>Norm</th>
<th>Mix Normal</th>
<th>Skew 3</th>
<th>ALD</th>
<th>Norm</th>
<th>Mix Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customer Satisfaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_0$ - Intercept</td>
<td>0.298</td>
<td>0.050</td>
<td>0.273</td>
<td>0.280</td>
<td>0.294</td>
<td>0.004</td>
<td>0.363</td>
<td>0.356</td>
<td>0.440</td>
<td>0.405</td>
<td>0.449</td>
<td>0.159</td>
</tr>
<tr>
<td>$\beta_1$ - Employee Satisfaction</td>
<td>0.005</td>
<td>0.000</td>
<td>0.027</td>
<td>0.003</td>
<td>0.037</td>
<td>0.000</td>
<td>0.035</td>
<td>0.039</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.002</td>
<td>0.007</td>
</tr>
<tr>
<td>$\beta_2$ - Friendliness of branch tellers</td>
<td>0.472</td>
<td>0.918</td>
<td>0.439</td>
<td>0.450</td>
<td>0.477</td>
<td>0.994</td>
<td>0.362</td>
<td>0.365</td>
<td>0.496</td>
<td>0.505</td>
<td>0.550</td>
<td>0.709</td>
</tr>
<tr>
<td>$\beta_3$ - Evaluation of waiting time</td>
<td>0.215</td>
<td>0.030</td>
<td>0.239</td>
<td>0.240</td>
<td>0.200</td>
<td>0.003</td>
<td>0.227</td>
<td>0.228</td>
<td>0.071</td>
<td>0.000</td>
<td>0.003</td>
<td>0.129</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.038</td>
<td>0.000</td>
<td>0.002</td>
<td>0.002</td>
<td>0.034</td>
<td>0.000</td>
<td>0.002</td>
<td>0.001</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>$y_4$ (skew t) / $p_4$ (ALD)</td>
<td>1.425</td>
<td>0.736</td>
<td></td>
<td></td>
<td>1.436</td>
<td>0.809</td>
<td></td>
<td></td>
<td>1.418</td>
<td>0.897</td>
<td></td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>83.03</td>
<td>0.000</td>
<td></td>
<td></td>
<td>84.41</td>
<td>0.000</td>
<td></td>
<td></td>
<td>0.44</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employee Satisfaction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha_0$ - Intercept</td>
<td>0.116</td>
<td>0.075</td>
<td>-0.025</td>
<td>0.029</td>
<td>0.101</td>
<td>0.172</td>
<td>0.103</td>
<td>0.062</td>
<td>0.079</td>
<td>0.100</td>
<td>0.094</td>
<td>0.055</td>
</tr>
<tr>
<td>$\alpha_1$ - Customer Satisfaction</td>
<td>0.054</td>
<td>0.071</td>
<td>0.114</td>
<td>0.051</td>
<td>0.038</td>
<td>0.039</td>
<td>0.007</td>
<td>0.016</td>
<td>0.030</td>
<td>0.039</td>
<td>0.007</td>
<td>0.072</td>
</tr>
<tr>
<td>$\alpha_2$ - Decision making authority</td>
<td>0.116</td>
<td>0.087</td>
<td>0.127</td>
<td>0.127</td>
<td>0.230</td>
<td>0.187</td>
<td>0.216</td>
<td>0.217</td>
<td>0.139</td>
<td>0.184</td>
<td>0.117</td>
<td>0.121</td>
</tr>
<tr>
<td>$\alpha_3$ - Fair evaluation of job performance</td>
<td>0.244</td>
<td>0.242</td>
<td>0.268</td>
<td>0.262</td>
<td>0.204</td>
<td>0.232</td>
<td>0.189</td>
<td>0.190</td>
<td>0.212</td>
<td>0.180</td>
<td>0.215</td>
<td>0.216</td>
</tr>
<tr>
<td>$\alpha_4$ - Link between job perform and comp.</td>
<td>0.159</td>
<td>0.148</td>
<td>0.155</td>
<td>0.164</td>
<td>0.123</td>
<td>0.059</td>
<td>0.139</td>
<td>0.137</td>
<td>0.142</td>
<td>0.144</td>
<td>0.138</td>
<td>0.148</td>
</tr>
<tr>
<td>$\alpha_5$ - Satisfaction with rewards</td>
<td>0.187</td>
<td>0.152</td>
<td>0.209</td>
<td>0.205</td>
<td>0.172</td>
<td>0.080</td>
<td>0.187</td>
<td>0.185</td>
<td>0.215</td>
<td>0.181</td>
<td>0.223</td>
<td>0.225</td>
</tr>
<tr>
<td>$\alpha_6$ - Personal growth</td>
<td>0.178</td>
<td>0.263</td>
<td>0.182</td>
<td>0.180</td>
<td>0.135</td>
<td>0.201</td>
<td>0.150</td>
<td>0.149</td>
<td>0.169</td>
<td>0.155</td>
<td>0.176</td>
<td>0.173</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.090</td>
<td>0.001</td>
<td>0.010</td>
<td>0.010</td>
<td>0.081</td>
<td>0.001</td>
<td>0.008</td>
<td>0.008</td>
<td>0.082</td>
<td>0.001</td>
<td>0.007</td>
<td>0.007</td>
</tr>
<tr>
<td>$y_4$ (skew t) / $p_4$ (ALD)</td>
<td>1.428</td>
<td>0.623</td>
<td></td>
<td></td>
<td>1.144</td>
<td>0.560</td>
<td></td>
<td></td>
<td>1.066</td>
<td>0.510</td>
<td></td>
<td></td>
</tr>
<tr>
<td>df</td>
<td>78.98</td>
<td>0.000</td>
<td></td>
<td></td>
<td>21.20</td>
<td>0.000</td>
<td></td>
<td></td>
<td>50.50</td>
<td>0.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>k (number of mixture components)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Marginal Density</td>
<td>1992</td>
<td>1976</td>
<td>1944</td>
<td>1941</td>
<td>2132</td>
<td>2111</td>
<td>2105</td>
<td>2105</td>
<td>4276</td>
<td>3662</td>
<td>3209</td>
<td>2627</td>
</tr>
</tbody>
</table>

Means and variances are reported for the largest of the mixture components.
CHAPTER 3

INVESTIGATING THE STRATEGIC INFLUENCE OF SATISFACTION OF FIRM FINANCIAL PERFORMANCE

3.1 Introduction

Marketing managers face increasing pressure to demonstrate the impact of their actions on firm financial performance. In practice, linking action to outcome poses a variety of methodological challenges. The influence of marketing intervention is often complex and can include multiple intervening, mediating, and moderating effects, whose results may be manifest on different time scales. Managers, for example, can directly influence sales through the use of short-term (i.e., tactical) activities like price and incentive promotions. Or, rather, they can indirectly influence sales by modifying consumer attitudes toward the firm through the use of long-term (i.e., strategic) actions like advertising, service climate improvements, or increasing customer satisfaction. In order to capture the effects of tactical and strategic actions, models are needed that can integrate data from a variety of sources.

Response models calibrated using market data must also account for the presence of endogenously determined covariates. If managers set the inputs of a marketing response model, $X$, with an expectation of how they will influence the outcome, $y$, 

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the inputs can no longer be treated as exogenous to the system of study. Endogeneity violates the assumptions of standard estimation approaches, thus leading to misestimation of the true relationship between $X$ and $y$. Resulting actions taken on the part of managers can lead to a misallocation of firm resources. Endogeneity in response models can be effectively addressed by modeling the joint distribution of both $X$ and $y$.

In this chapter we propose a Hierarchical Bayesian model that allows us study the strategic influence of satisfaction on firm financial performance. We model unit-level revenue production as a function of managerially controllable inputs, and we allow latent levels of customer and employee satisfaction to exert an indirect influence on financial performance by altering the firm’s technology. Structure is imposed upon the parameters of our model through the estimation of a model of simultaneous supply and demand. Our model explicitly deals with the potential for endogeneity in the input variables, and produces managerially reasonable parameter estimates.

We apply our model to data provided by a national financial services firm where we integrate data from three independently conducted studies. We show that customer and employee satisfaction have both direct and indirect effects on branch-level revenue production. Our model allows us to assess the relative benefits of engaging in short-term versus long-term marketing activities. We explore this process through the use of a marketing policy counterfactual where we identify when and under what cost structure it would become profitable for the bank to focus its efforts on increasing the latent level of employee satisfaction as opposed to engaging in short-term sales incentive programs.
The remainder of this chapter is organized as follows. Section 3.2 presents the
genral form of our model, the likelihood, and estimation strategy. In section 3.3 we
describe the data and setting used to empirically demonstrate our model. Alternative
models are outlined in section 3.4. Results are presented and discussed in section 3.5.
Section 3.6 illustrates the practical value of our proposed model by considering a
series of counterfactual scenarios. Final thoughts and areas for future research are
offered in section 3.7.

3.2 Model

The relationship between marking activity and financial performance has received
extensive attention in the marketing literature (Gupta and Zeithaml, 2006; Rust et
al., 2004). Existing models are often constructed using what are referred to as chain
links of effects. The service profit chain, for example, attempts to trace the influence of
managerial action to firm performance through its influence on employee and customer
satisfaction (Maxham III et al., 2008; Kamakura et al., 2002; Heskett et al., 1994).
While these models are useful in the sense that they provide directional evidence that
constructs like customer and employee satisfaction are, in fact, correlated with sales
production, they do not impose the sort of structure on the response process required
to optimize the firm’s utilization of resources.

In this section we develop a Hierarchical Bayesian model that integrates estima-
tion of the effects of multiple marketing activities through simultaneous analysis of
panel and cross-sectional data. Implicit in our approach is the notion that firms face
two fundamental types of decisions: short-term (i.e., tactical) and long-term (i.e.,
strategic). By relating these decisions to a scalar outcome we are able to formally
assess the tradeoffs associated with engaging in tactical versus strategic marketing activities.

Our general modeling approach consists of two major components:

### 3.2.1 Demand Model

We begin by specifying a response model that relates short-term marketing activities to a unit-level financial outcome. We express revenue, $y_{it}$, realized by unit $i$ in time $t$ as a multiplicative function of $k$ operational inputs, $\{x_{kit}\}$. Although we restrict our attention to a revenue generation process, $y_{it}$ in equation (3.1) could represent a variety of outcomes like unit sales or the number of new customers acquired.

$$y_{it} = \beta_0^i \left( \prod_{k=1}^{K} x_{kit}^{\beta_{ki}} \right) e^{\epsilon_{it}} \quad (3.1)$$

The functional form of this model has been used extensively in both economic and marketing applications (Lilien et. al, 1992). It enables us to capture diminishing returns to scale in the inputs and allows us to interpret the $\{\beta_{ki}\}$ as elasticities. We view the collection of $\{\beta_{ki}\}$ in equation (3.1) as a joint representation of the firm’s technology (Varian, 1992). $\{\beta_{ki}\}$ fully characterize the expected relationship between operational or tactical inputs, $\{x_{kit}\}$, and a realized outcome, $y_{it}$. We refer to marketing actions that alter the conditional distribution of $y_{it}\mid \{x_{kit}\}$ through the response coefficients, $\{\beta_{ki}\}$, as strategic.

We allow long-term, strategic actions to influence the revenue generation process in equation (3.1) by constructing a hierarchy on $\{\beta_{ki}\}$:

$$\beta_i = \Gamma' \mu^*_i + \eta_i \quad (3.2)$$

where $\mu^*_i$ is a vector of $S + 1$ variables that can include observable characteristics of the production process (e.g., product attributes, average advertising spend, etc.) or
unobservable constructs like customer satisfaction or brand equity.

\[
\mu_i^s = \begin{bmatrix} 1 & \mu_{1i}^s & \cdots & \mu_{Si}^s \end{bmatrix}^\prime
\]  

(3.3)

The former can be measured directly while the latter can be assessed through the use of survey data. Estimates of unobservable or latent constructs are related to observed data through equation (3.4):

\[
z_{ih}^s \sim N \left( \mu_i^s = \begin{bmatrix} \mu_{1i}^s \\ \vdots \\ \mu_{Si}^s \end{bmatrix}, \Sigma_i^s \right)
\]  

(3.4)

where \(z_{ih}^s\) is a vector of responses for individual \(h\) in unit \(i\) and \(\mu_{1i}^s\) is the estimate of interest included equation (3.3). This is done in anticipation of our empirical application where we include latent estimates of employee and customer satisfaction as covariates in equation (3.2). We allow for cross-unit heterogeneity by specifying distributions of random effects for both the location and covariance matrix of equation (3.4).

\[
\mu_i^s \sim N \left( \bar{\mu}^s, V^s_{\mu} \right)
\]  

(3.5)

\[
\Sigma_i^s \sim IW \left( \bar{\nu}^s, \Omega^s \right)
\]  

(3.6)

Collectively, equations (3.1-3.6) form the basis of a model of integrated decision making, where the influence of strategic action is manifest through the hyper-parameters of a hierarchical response model. We are informed about influence of tactical decisions on firm performance from within-unit variation across time (equation 3.1), while learning about the effects of strategic decisions occurs across units (equation 3.2). As illustrated in Section 3.6, this formal connection between tactical and strategic decisions allows us to compute the monetary benefit associated with altering as opposed operating within the firm’s extant technology.
3.2.2 Supply Model

Response models calibrated using market data must account for the possibility that the operational inputs are endogenous to the system of study (Yang et al., 2003). This occurs if managers set the inputs, $X$, with an expectation of how they will affect the outcome, $y$. The presence of endogenously determined covariates has been shown to yield parameter estimates that are both biased and inconsistent (Villas-Boas and Winer, 1999; Berry, 1994). We address this issue by constructing a model that reflects our belief about the managerial decision process that gives rise to observed input variables, $X$. Joint modeling of both the inputs, $X$, and output, $y$, of the response process has been shown to solve the issue of endogeneity, thus yielding consistent estimates of model parameters (Otter et al., 2008; Manchanda et al., 2004).

We implement this approach by specifying the supply-side model for $X$ defined by equation (3.7). In this model we assume that managers have at least an implicit knowledge of the response process defined by equation (3.1) and set levels of marketing inputs $\{x_{kit}\}$ in order to maximize firm profit over a finite time horizon, $T$, subject to a budget constraint. Managers identify optimal values of $\{x_{kit}\}$ by solving the constrained optimization problem presented equation (3.7):

$$
\max_{\{x_{kit}\}} \sum_i \sum_t \left( \beta_0 + \prod_{k=1}^{K} x_{kit}^{\beta_{ki}} - \sum_{k=1}^{K} p_{kit} x_{kit} \right) \quad \text{(3.7)}
$$

subject to

$$
\sum_i \sum_t p_{kit} x_{kit} \leq m_k
$$

where $p_k$ is the cost and $m_k$ denotes the budget constraint for input $k$.

The ideal solution to this allocation problem is obtained by first expressing the auxiliary function ($\mathcal{L}$) presented in equation (3.8) and then identifying the set of $\{x_{it}^{*}\}$
that jointly maximize this function.

\[
L = \sum_i \sum_t \left( \beta_{0i} \left( \prod_{k=1}^{K} x_{kit}^{\beta_{ki}} \right) - \sum_k p_{kit} x_{kit} \right) - \sum_k \lambda_k \left( \sum_i \sum_t p_{kit} x_{kit} - m_k \right) \tag{3.8}
\]

This is accomplished by solving the first-order conditions presented in equation (3.9) for each branch, input, and time period.

\[
\frac{\partial L}{\partial x_{kit}} = 0, \forall k, i, t \tag{3.9}
\]

By taking logs of the resulting system of equations we can solve for the profit maximizing values of all input variables, \( \{x_{*it}\} \), via equation (3.10). For a solution to exist, the response function defined in equation (3.1) must be a legitimate economic production function. That is, it must exhibit diminishing returns to scale for positively valued inputs, \( X \). This is accomplished if \( \beta_k > 0 \) for all \( k \), and \( \sum_{k=1}^{K} \beta_k \leq 1 \). If these conditions are met, equation (3.1) subsumes the properties of a Cobb-Douglas production function.

\[
\ln(x_{*it}) = \begin{bmatrix}
\ln(x_{*1it}) \\
\vdots \\
\ln(x_{*Kit})
\end{bmatrix} = \begin{bmatrix}
(\beta_{1i} - 1) & \cdots & \beta_Ki \\
\vdots & \ddots & \vdots \\
\beta_{1i} & \cdots & (\beta_Ki - 1)
\end{bmatrix}^{-1} \times \\
\ln(\lambda_1 + 1) - \ln(\beta_{0i}) - \ln(\beta_{1i}) + \ln(p_{1it}) \\
\vdots \\
\ln(\lambda_K + 1) - \ln(\beta_{0i}) - \ln(\beta_{Ki}) + \ln(p_{Kit})
\end{bmatrix} \tag{3.10}
\]

We allow observed realizations \( \{x_{it}\} \) to deviate from the optimal solution \( \{x_{*it}\} \) by introducing error \( \{\zeta_{kit}\} \) into the maximization problem defined in equation (3.7). Suboptimal allocation of marketing resources occurs as a result of uncertainty regarding the cost for each input, \( \{C_{kit}\} \). Management allocates across inputs, units and time periods using \( C_{kit}^* = C_{kit} e^{\zeta_{kit}} = (p_{kit} x_{kit}) e^{\zeta_{kit}} \), where input cost error can arise from a variety of sources, including uncertainty about input prices, \( \{p_{kit}\} \), unanticipated fixed and variable expenses, etc.
3.2.3 Likelihood and Estimation

We employ a full-information Bayesian approach in order to estimate our model, where the likelihood can be expressed as follows:

\[
\ell (\text{data}|\theta) = \prod_i \prod_t \pi (\varepsilon_{it}) \pi (\{\zeta_{kit}\}) \left| \frac{J_{it}}{\zeta \rightarrow x} \right| \tag{3.11}
\]

The quantities \( \varepsilon_{it} \) and \( \{\zeta_{kit}\} \) are defined in equations (3.12) and (3.13) and \( \left| \frac{J_{it}}{\zeta \rightarrow x} \right| \) is the Jacobian term that captures dependencies in the mapping of \( \zeta \rightarrow x \).

\[
\varepsilon_{it} = \ln (y_{it}) - \left( \ln (\beta_0) + \sum_{k=1}^K \beta_k \ln (x_{kit}) \right) \tag{3.12}
\]

\[
\begin{bmatrix}
\zeta_{1it} \\
\vdots \\
\zeta_{Kit}
\end{bmatrix} = \begin{bmatrix}
(\beta_{1i} - 1) & \cdots & \beta_{Ki} \\
\vdots & \ddots & \vdots \\
\beta_{1i} & \cdots & (\beta_{Ki} - 1)
\end{bmatrix} \ln (\mathbf{x}_{it}^*) - \begin{bmatrix}
\ln (\lambda_1 + 1) - \ln (\beta_{0i}) - \ln (\beta_{1i}) + \ln (p_{1it}) \\
\vdots \\
\ln (\lambda_K + 1) - \ln (\beta_{0i}) - \ln (\beta_{Ki}) + \ln (p_{Kit})
\end{bmatrix} \tag{3.13}
\]

Simultaneity present in the specification of equation (3.13) results in the non-trivial Jacobian defined by equation (3.14). Simultaneity in our model arises from the inclusion of the supply side model derived from equation (3.10). The optimal value of a given input, \( \{x_k\} \), is a function of all other inputs, \( \{x_{-k}\} \).

\[
\left| \frac{J}{\zeta \rightarrow x} \right| = \left| \sum_{k=1}^K \beta_k - 1 \right| \tag{3.14}
\]

It is important to note that equation (3.13) can only be evaluated for values of \( \beta_{ik} > 0 \). Furthermore, the Jacobian in equation (3.14) creates a ridge in the likelihood surface exactly equal to 0 when \( \sum_{k=1}^K \beta_k = 1 \). As such, equations (3.13-3.14) effectively bound the parameter space to include only reasonable values of \( \beta \), or values of \( \beta \) that would give rise to a solution to equation (3.10).
Bayesian estimation proceeds by recursively generating draws from the full con-
ditional distributions of all model parameters (Rossi, Allenby, and McCulloch 2005). The inclusion of the Jacobian term in equation (3.11) prevents us from utilizing stan-
dard conjugate results in order to implement an efficient Gibbs sampler for model estimation. Rather, we rely on a hybrid sampler where a subset of the param-
ters are drawn using the Metropolis-Hastings algorithm (Chib and Greenberg 1995). Although this is simple to implement, it does substantially increase the computa-
tional burden of the routine. The estimation algorithm for our proposed model of simultaneous supply and demand is provided in the appendix. Extensive simulation studies were conducted in order to assess both the efficacy and mixing properties of all estimation routines.

3.3 Data

Empirically, we study the strategic effect of satisfaction on firm performance in the context of retail banking. Data are provided by a national financial services firm and consist of three independently collected components: an employee satisfaction study, a customer satisfaction study, and a time series of unit-level financial statements. All data sets were collected during roughly the same time period.

3.3.1 Unit-Level Income Statements

The units of analysis in this study are retail banking branches. Income statements for approximately 13 months were made available for each of the firms 898 retail locations. Each income statement contains detailed information about branch-level expenses and revenues. Expenses include monthly outlays for base salary, incentive compensation, training, etc. Revenue in retail banking can be classified into two main
categories: production income and portfolio (or passive) income. Production income results from the accumulation of new business (e.g., new loans, deposit accounts, etc.). Passive income accrues as a result of existing loan and deposit balances.

Both sources of income are computed using the “value method” which assigns a fixed monetary value to new and existing business activities and consumer relationships. For example, a bank may assign a value of $2,000 for every $100,000 originated in new mortgages. The value method is used in a manner consistent with the premise of cost-based accounting. That is, to distribute aggregate revenue across the specific services provided by each branch. This facilitates a better understanding of the marginal contribution of various banking services to total profitability, and should therefore allow management to more easily identify and reward activities of greatest importance.

We focus our attention on three key short-term input variables: full-time equivalents (FTE), base salary, and incentive compensation. The dependent variable of interest in this study is total branch-level revenue (i.e., passive and production income). These are, respectively, the inputs and output of the production function presented in equation (3.1). FTE provides an aggregate measure of the number of full-time workers employed at a given branch. A part-time employee’s contribution to this measure is defined as the percentage of hours they are employed, where the basis is a 40-hour work week. Base compensation measures the total monthly unconditional compensation for all employees at a given branch. This includes both salaries for exempt employees and hourly wages for non-exempt employees. Incentive compensation consists of total monthly dollar expenditures in excess of base salary. Summary statistics of these key variables are presented in Table 3.1.
3.3.2 Customer and Employee Satisfaction Studies

Employee and customer satisfaction studies were conducted once during the time period in question. Each consumer surveyed was asked to provide a holistic evaluation of the bank in addition to an assessment of specific service aspects of the branch they frequent most often. In order to avoid confusion, the branch in question is explicitly defined in the survey instrument. Employee responses are grouped according to their branch of employment.

Descriptive statistics for these data sets are presented in Table 3.2. Included in this table are the respective customer and employee questions used as variables in the analysis. An average of 37 customer responses were collected for each branch (minimum of 6, maximum of 87). In the survey, respondents were asked to rate their branch on a variety of service dimensions. Responses were recorded on a scale of 1 to 10, where 1 and 10 denote, respectively, “unacceptable” and “outstanding.” An average of 7 employee responses were recorded per branch (minimum of 5, maximum of 19). These responses were also scaled from 1 to 10, where 1 and 10 indicate, respectively, “Very Dissatisfied” and “Very Satisfied.” In order to maintain consistency in the data and ease the interpretation of results, both customer and employee data were rescaled onto the [0, 1] interval, where 1 represents the maximum possible positive response.

Latent levels of aggregate customer and employee satisfaction are estimated using equation (3.4) and incorporated into the response model through equations (3.2) and (3.3). As presented in equation (3.4) responses to all survey questions are modeled as realizations from a heterogeneous multivariate normal distribution with branch-specific mean and covariance matrix. As illustrated in Section 3.6, the assumption
of multivariate normality will allow us to derive, for example, the conditional distribution of customer satisfaction given its determinants or drivers. This will allow us to trace the influence of specific changes in the service climate (e.g., customer wait time) through the response process to revenue generation.

### 3.3.3 Alternative Models

We explore the results of 7 alternative models. Model descriptions and characteristics are provided in Table 3.3. The first model ($M_1$) is a three input demand model defined by equation (3.15) without an informative supply side model for $\{x_{kit}\}$:

$$y_{it} = \beta_0 i x_1^{\beta_1} i x_2^{\beta_2} i x_3^{\beta_3} e_{it}$$

(3.15)

where $x_1^{it}$, $x_2^{it}$, and $x_3^{it}$ are respectively FTE, base salary in thousands of dollars, and incentive compensation in thousands of dollars.

In order to contrast $M_1$ with models of simultaneous supply and demand we must also estimate an implied model for the input variables, $X$. $M_1$ assumes that $\{x_{kit}\}$ are exogenous to the system of study. Realizations of the input variables $\{x_{it}\}$ are drawn from a multivariate normal distribution with a branch-specific mean and covariance matrix.

$$x_{it} \sim N(\bar{x}_i, \Sigma_{xi})$$

(3.16)

The second model considered ($M_2$) extends the first through the a priori imposition of constraints over the parameter space. Response models provide utility to managers only to the extent that parameter estimates or functions of those estimates are deemed reasonable. In this context the requirement for reasonability is that $\beta_k > 0$ for all $k$, and the $\sum_{k=1}^{3} \beta_k \leq 1$. In $M_2$ we impose these constraints upon the response process through the likelihood, but do not allow the supply-side model to further inform
estimation of $\beta$. This is accomplished by artificially inflating the variance of the supply-side shock to be large. This is similar in spirit to Allenby, Arora, Ginter (1995) and Boatwright, McCulloch, and Rossi (1999) who introduce parameter constraints through the prior. Unlike either of these papers, we do not have strong theoretical support to justify the imposition of constraints. As such, in $M_2$ we effectively utilize the likelihood as a computational device to achieve reasonable results instead of a reflection of our true belief about the data-generating process.

The third model studied ($M_3$) is a simultaneous supply and demand specification where $X$ is set with knowledge of the response parameters, $\beta$. This model, however, is not derived from the profit-maximizing behavior of managers. Rather, we model $X$ as a linear function of $\beta$. This is consistent in spirit with the descriptive supply side model introduced by Manchanda, Chintagunta, and Rossi (2004). We operationalize this by extending the model in equation (3.16) to include the hierarchal structure presented in equation (3.17).

$$\bar{x}_i = \Delta' \beta_i + \xi_i \quad (3.17)$$

Models $M_4$-$M_7$ are the simultaneous supply and demand models derived from the first-order conditions of the maximization problem defined in equation (3.7). They correspond to alternative assumptions regarding input budget constraints.

One advantage of using Bayesian estimation in this context is that it enables us to search over a wide variety of supply-side models in order to better understand the processes managers employee when making input-level decisions. We can compute Bayes factors for these alternatives in order to determine which model best corresponds to the observed data (Rossi et al., 2005; Kass and Raftery, 1995). This applies to both nested and non-nested model specifications.
Parameters of particular interest in our model are the set of \{\lambda_k\}, the Lagrange multipliers or “shadow prices” of inputs \{x_k\}. They correspond to the marginal increase in the objective function (e.g., profitability) resulting from a relaxation of the budget constraint, \{m_k\}:

$$\frac{\partial L}{\partial m_k} = \lambda_k$$  \hspace{1cm} (3.18)

Although typically defined in terms of dollars, budget constraints can be specified in a variety of units. In our application, we do not know the monetary cost of adding an additional unit of FTE to a branch and therefore define the budget constraint \(m_1\) as an upper bound on the total number of employees. Budget constraints \(m_2\) and \(m_3\) are bounds on total dollar expenditures for base and incentive compensation.

Estimates of \{\lambda_k\} inform us about the degree to which allocation decisions are coordinated across the bank. Given the existence of a budget constraint, optimal bank-level behavior would be achieved when (provided all inputs are measured in the same units):

$$\frac{\partial L}{\partial x_{kit}} = \lambda; \forall k, i, t$$  \hspace{1cm} (3.19)

That is, the marginal increase in profitability resulting from an increase in are balanced across all inputs, \(k\), units, \(i\), and time periods, \(t\). In this case, marketing resources are optimally allocated across the organization. A variety of deviations from optimal coordination are also possible. The following are alternative model specifications defined in terms of the budget constraint. As noted above, we do not observe prices for additional units of FTE and therefore investigate only the coordination of base salary and incentive compensation. Model 4 (\(M_4\)) presents a scenario where allocation decisions are made at the branch-level and separate budget constraints (and
corresponding Lagrange multipliers) are defined for each unit, $i$, and each input, $k$.

$$M_4 : \sum_t x_{kit} \leq m_{ki}$$

(3.20)

Allocation decisions in Model 5 ($M_5$) are still made at the unit level, but are coordinated across inputs. A single budget constraint is set for the sum of both base and incentive compensation:

$$M_5 : \sum_t \sum_k x_{kit} \leq m_i$$

(3.21)

Model 6 ($M_6$) defines a scenario where allocation decisions are coordinated across units, but not across inputs:

$$M_6 : \sum_t \sum_i x_{kit} \leq m_k$$

(3.22)

Model 7 ($M_7$) represents the optimal scenario described above. That is, allocative coordination across time, units, and inputs.

$$M_7 : \sum_k \sum_i \sum_t x_{kit} \leq m$$

(3.23)

### 3.4 Results

Table 3.3 presents descriptions and fit statistics for models $M_1$ through $M_7$. We compute Bayes factors for the respective models using the Newton-Raftery approximation to the log marginal density (Newton and Raftery 1995). Fit statistics are provided for the marginal distributions of both $y$ and $X$ implied by the model under investigation, in addition to the joint distribution of the same.

In terms of the joint distribution of both $X$ and $y$, we find that $M_5$ outperforms all other models, including the statistical model, $M_1$. This suggests that managers
optimally balance inputs within, but not across units. Within a given branch, the marginal increase in profitability resulting from a relaxation of the budget constraint is identical for both base-salary and incentive compensation. Results for the marginal distribution of $X$ indicate that the simultaneous supply and demand models allow us to better explain variation in the input variables relative to the model of exogeneity presented in equation (3.16). This supports our premise that managers set $X$ with an expectation of how it will influence $y$, or that $X$ is in fact endogenous.

A key object of interest in the MCMC output is the estimate of $\Gamma$, the coefficient matrix for the distribution of random effects for $\beta$ defined in equation (3.2). $\Gamma$ informs us about the relationship between customer and employee satisfaction and the firm’s technology (i.e., $\beta$). Posterior means for estimates of $\Gamma$ for $M_5$ are presented in Table 3.4. Parameter estimates with 95% of their mass above or below 0 are presented in bold face.

In expectation, both customer and employee satisfaction are positively correlated with the multiplicative intercept, $\beta_0$. Increasing average customer and employee satisfaction for a branch will yield a direct increase in its proclivity to produce revenue, conditional upon fixed values of the input variables, $X$.

As a general note, we observe that employee satisfaction appears to exert a greater influence on the firm’s technology than customer satisfaction. Specifically, employee satisfaction is significantly related to the multiplicative intercept and coefficients for base salary and incentive compensation, whereas customer satisfaction is only significantly related to the latter.
Employee satisfaction is negatively correlated with $\beta_2$, the response coefficient for base salary ($\gamma_{3,3} = -0.06$), and positively correlated with $\beta_3$, the coefficient for incentive pay ($\gamma_{4,3} = 0.09$). As the latent mean of satisfaction at a branch increases the efficacy of base salary as a driver of revenue decreases while the efficacy of incentive compensation increases. This suggests that, all else equal, branches whose employees are relatively more satisfied would make better use of their resources by designing employee compensation contracts that place greater emphasis on incentive relative to base pay. Our results also indicate that customer satisfaction is inversely correlated with the response coefficient for incentive compensation, $\beta_3$ ($\gamma_{4,2} = -0.07$). As customers become increasingly satisfied, incentive compensation becomes a less effective driver of branch profitability.

Figures 3.1 and 3.2 present a series of histograms of the mean of each branch’s posterior distribution of $\beta$. Figure 3.1 is constructed using MCMC results from $M_1$ while Figure 3.2 uses results from $M_5$. We observe considerable heterogeneity across branches in $\beta$ for both models. On average, the size of $\beta$ appears to be larger for base salary than for either FTE or incentive pay. In the case of the $M_1$, we observe average $\beta$’s for branches that are less than 0 and greater than 1. These results are counterintuitive and severely restrict $M_1$’s ability to provide guidance for future managerial decision making. A value of $\beta < 0$ implies an optimal expenditure of 0 dollars while $\beta > 1$ implies full allocation of all resources to that variable. These implied optimal values are inconsistent with current managerial action observed in our data.

Results of $M_5$, presented in Figure 3.2, are reasonable in the sense that all posterior mean estimates lie on the [0,1] interval. Closer examination of these results
demonstrates that our estimates of $\beta$ adhere to the restriction that $\sum_{k=1}^{3} \beta_k \leq 1$. As such, results from this model can be used to explore alternative allocation schemes and can provide useful managerial guidance. In the following section, we demonstrate how this can be accomplished using the results of our proposed model.

### 3.5 Optimal Resource Allocation

In this section of the chapter, we examine a hypothetical business scenario in order to demonstrate how our proposed framework can inform managerial decision making. Retail banks utilize short-term promotions in order to feature an existing service or introduce a new product. Although these promotions are supported by traditional marketing activities, their success or failure hinges upon interpersonal sales efforts conducted at the branch. Sales incentive programs are used to motivate employees to take an active part in these promotions.

Consider the case where one division of the bank (100 branches) is preparing to engage in a month-long promotion of an existing consumer product. Regional managers have access to $100,000 of discretionary funds to be used to incentivize branch employees to actively participate in the promotion. Management’s task is to determine how to best allocate these funds across the region in order to maximize the financial success of the campaign. That is, to set levels of in order to achieve the greatest region-wide contribution margin.

We describe incremental revenue less incremental cost as contribution margin (CM) as opposed to profit. Promotional revenue realized in excess of cost contributes to offset region fixed (e.g., advertising, signage, etc.) and administrative expenses. We compare results of the following 7 allocation scenarios:
1. Uniform allocation across branches: under this first scenario each branch in the region is granted an additional $1,000 dollars to be used for incentive compensation. Projections of the resulting contribution margin are made using the following equation: $\sum_i \beta_0 x_{1i} x_{2i} x_{3i} - x_{2i} - x_{3i}$, where the baseline levels for inputs $\{x_{1i}\}$, $\{x_{2i}\}$, and $\{x_{3i}\}$ are set to their corresponding averages observed in the data. We account for parameter uncertainty in these projections by integrating over the posterior distribution of $\beta$. This allows us to assess both the expected increase and variance in resulting CM.

2. FTE proportional allocation: greater funds are given to branches with a larger number of employees. We project CM using the method described above.

3. Allocation proportional to the posterior mean of $\beta_3$: greater resources are given to those branches whose employees are, on average, more sensitive to incentive compensation as a driver of revenue. This scenario follows Dorfman and Steiner’s (1954) optimal allocation rule for advertising expenditures.

4. Unconstrained optimal allocation (posterior mean): under this allocation scheme, $\{x_{3i}\}$ are set in order to solve the following mathematical programming problem: $\max_{\{x_{3i}\}} \sum_i \beta_0 x_{1i} x_{2i} x_{3i} - x_{2i} - x_{3i}$. Branch-specific point estimates equal to the posterior mean of the $\beta$’s are used to simplify the computations involved in obtaining a solution to this problem. As such, we are able to use standard, gradient-based optimization software. We do not impose any constraints on $\{x_{3i}\}$ and therefore admit the potential for corner solutions (i.e., $0$ allocations).

5. Unconstrained optimal allocation (full posterior distribution): in this scenario, we set $\{x_{3i}\}$ in order to maximize expected CM: $\max_{\{x_{3i}\}} E_{\beta|D} \left[ \sum_i \beta_0 x_{1i} x_{2i} x_{3i} - x_{2i} - x_{3i} \right]$. 

52
The expectation in this problem is taken with regard to the posterior distribution of $\beta$. In implementation, this is accomplished through the use of a simulation-based optimizer. Although this results in an increase in computational time, it allows us to formally assess variability in CM projections resulting from parameter uncertainty.

6. Constrained optimal allocation (posterior mean): with one exception, this is identical to scenario 4. In this case bounds are placed on the space of possible allocation solutions, $\{x_{3i}\}$. Specifically, we assert that all branches must receive at least $200 and no more that $2,000. Although sub-optimal relative to allocations 4 and 5, these constraints may constitute a more realistic representation of how managers would actually utilize our proposed model. That is, they would like to increase the efficacy of their promotional spend relative to scenarios 1-3, but are not willing to rely exclusively on the unconstrained optimized results of the model, or may wish to encourage some level of participation by all branches in the region.

7. Constrained optimal allocation (full posterior distribution): this is the constrained version of scenario 5, where the constraints are defined in scenario 6.

Means and standard deviations of expected incremental CM for all allocation schemes are presented in Table 3.5. We observe that expected CM is substantially improved for the model-based allocations (scenarios 3-7). Expected CM more than doubles as we move from the constant allocation scheme (scenario 1) to the unconstrained optimal allocation (scenario 5).
We also observe that the optimization-based allocations (scenarios 4-7) increase expected CM relative to the allocation based upon the size of the response coefficient, \( \beta_{3i} \) (scenario 3). For example, expected CM increases from $279K for scenario 3 to $471K for scenario 5. Although contrary to the Dorfman/Steiner (1954) result, this is consistent with our expectations given the functional form of our demand model. The optimal distribution of incremental funds will depend on both the size of \( \beta_{3i} \) as well as the extant allocation of \( \{x_{3i}\} \).

Our proposed model allows us to formally evaluate the costs and benefits associated with undertaking strategic as opposed to tactical actions. For example, under allocation scenario 5 we observe expected incremental CM for the region equal to $471K. In Table 3.4 we also observe a positive association between employee satisfaction and the efficacy of incentive compensation as a driver of revenue, as manifest through \( \beta_3 \). It may be of managerial interest to know how much employee satisfaction would have to increase in order to generate a per-period rise in CM of $471K. This can be easily computed using the chain rule for differentiation:

\[
\frac{\partial \Pi_t}{\partial \mu_{1t}} = \frac{\partial \Pi_t}{\partial \beta_3} \frac{\partial \beta_3}{\partial \mu_{1t}}
\]

(3.24)

As presented in Table 3.6, a one-unit increase in employee satisfaction yields an expected, per-period increase of $358.9K in CM. As such, employee satisfaction would have to increase by 1.3 points (on a 10 point scale) in order to generate a CM increase of $471K.

We can extend this analysis further by recognizing that our model also contains information about the relationship between aggregate employee satisfactions and the specific job characteristics that drive the same. As presented in equation (3.4), we
model employee responses to a battery of questions (including aggregate employee satisfaction) as realizations of a multivariate normal distribution with a branch-specific mean and covariance matrix. We can exploit the properties of the multivariate normal distribution in order to study the conditional distribution of aggregate satisfaction given its drivers. This is accomplished by partitioning the covariance matrix, $\Sigma^{e_i}$, as follows:

$$
\Sigma^{e_i} = \begin{bmatrix}
\Sigma^{e_{11}} & \Sigma^{e_{12}} \\
\Sigma^{e_{21}} & \Sigma^{e_{22}}
\end{bmatrix}
$$

where $\Sigma^{e_{12}}$ is a 1-by-5 matrix that reflects the covariance of aggregate employee satisfaction and its drivers and $\Sigma^{e_{22}}$ is a 5-by-5 matrix that contains the variance and covariance of the latter. We can compute the matrix of coefficients for the regression of employee satisfaction on its drivers through the expression $\Sigma^{e_{12}}\Sigma^{-1}_{22}$, as presented in Table 3.6.

The monetary impact of changes in employee satisfaction drivers can then be computed using an extension of the chain rule in equation (3.24). We observe that changes in employees’ satisfaction with opportunities for personal growth and development have the greatest impact on aggregate employee satisfaction and, by extension, incremental CM. A one point average increase in satisfaction with personal growth opportunities will yield a 0.314 increase in overall employee satisfaction and a corresponding increase in expected CM of $112.7K. Given estimated cost information for each of the drivers, this analysis allows us to assess specific actions that should be taken in order to most efficiently increase overall employee satisfaction.

Although our data set does not provide information about specific costs, we can utilize the results of our model to estimate an upper bound on the amount we would
be willing to spend to improve the various employee satisfaction metrics. In the scenario described above, management is presented with two alternatives: the firm can operate within the confines of its existing technology and increase the productivity of sales promotions through the use of incremental employee compensation or, rather, the firm can invest in systemic improvements designed to increase aggregate employee satisfaction and, by extension, the efficacy the current incentive compensation allocation. Given these options, the firm would like to know the maximum amount they should be willing to spend to increase employee satisfaction by 1.3 points and forgo the use of short-term sales incentives.

It is important to recognize that customer and employee satisfaction are constructs that evolve slowly across time. Satisfaction improvements will yield benefits not only in the period in which they are enacted, but throughout a finite future time horizon. The bank should view actions taken to improve satisfaction as capital investments (i.e., an investment in technology) and evaluate them accordingly. We know that, in expectation, increasing employee satisfaction by 1.3 points will yield an increase in region CM equal to a monthly outlay of $100K in short-term incentive compensation. Suppose the bank applies a $\delta = 12\%$ discount rate to capital investments of this type and believes that the proposed process changes will be sufficient to maintain the desired increase in employee satisfaction without further investment for a 3 year period. Given the discount rate and time horizon, we can compute the net present value of future CM realizations via equation (3.26):

$$\sum_{t=1}^{36} (1 - \delta)^{t/12} CM_t$$

(3.26)
where CMt is contribution margin realized in month t, in this case equal to $471K. Application of this formula results in a net present value of future CM of approximately $14 million.

We can also use this formula to compute the net present value of the future cash outlays associated with promotional expenditures (i.e., $100K per month), thus providing a present value equivalent of total promotional cost. This value, equal to approximately $2.97 million, is the maximum dollar amount that the firm should be willing to spend to engage in the employee satisfaction improvements described above. If the cost of improving average employee satisfaction by 1.3 points exceeds $2.97 million, the bank would be better off to continue to rely on short-term sales incentive promotions.

3.6 Conclusion

This chapter presents a new approach to relating tactical and strategic marketing initiatives. Specifically, we model revenue production in retail banking as a function of employee compensation, and allow customer and employee satisfaction to moderate the relationship between the same through a Hierarchical Bayesian model. We handle potential endogeneity in the input variables by jointly estimating a demand-side model (i.e., model for y) and supply-side model (i.e., model for X). Our supply-side model is formally derived from a constrained optimization problem where managers are assumed to maximize profitability subject to a budget constraint. The resulting likelihood imposes a variety on constraints on the parameter spaces of \( \beta \), thus yielding estimates consistent with the interior solutions observed in the data. The structure
imposed upon our model allows us to utilize its results to guide managers in the future allocation of resources.

Empirically, this work contributes to the literature on customer satisfaction. We find evidence that the influence of customer satisfaction is manifest through firm technology. That is, changes in customer satisfaction impact firm financial performance by altering the efficacy of the firm’s tactical inputs through the response coefficients, $\beta_{ki}$. In expectation, customer satisfaction is positively related to a firm’s baseline ability to generate revenue. However, increases in customer satisfaction also decrease the effectiveness of operational inputs like base salary and incentive compensation. These findings are congruent with recent calls for work exploring alternative influences of customer satisfaction (Luo and Homburg, 2007).

Our work raises a number of interesting questions that are worthy of future investigation. First, we define a strategic action to be any action that influences the technology of a firm. Technology in a regression-style response model includes both the location and scale of the conditional distribution of $y|X$. In this chapter, however, we examine only the influence of satisfaction on the mean of the conditional relationship of sales and compensation (e.g., the effect of satisfaction on $\beta$). It would also be interesting to explore the relationship between satisfaction and the variance $\sigma^2$. It is certainly possible that an inverse relationship could exist between the latent level of customer and employee satisfaction and the variability of revenue generation at a branch.

A second issue that should be explored is related to recent work by Dotson, Retzer, and Allenby (2008). Both the customer and employee studies used in this chapter provide sample information about the respective distributions of satisfaction. In
this chapter, we relate these distributions to financial performance through their latent mean. It would be interesting to see if other portions or percentiles of these distributions would yield different results than those observed in our current work. Furthermore, it would be useful to explore models that do not rely on the stringent assumption of normality in the distribution of consumer and employee responses.

Finally, the supply side models developed in this chapter were based upon an evaluation of presumed optimal behavior, conditional upon the structure of our proposed model. Supply-side models are needed that more accurately reflect the processes whereby managers actually make decisions. Rather than searching over a space of possible supply-side models defined by the researcher, it would be useful to elicit managerial input during model construction. This could be efficiently accomplished through closer collaboration between researchers and managers. We leave these issues to future research.
Figure 3.1: Distribution of posterior means of $\beta$ for $M_1$ - demand side only
Figure 3.2: Distribution of posterior means of $\beta$ for $M_3$ - simultaneous supply and demand
<table>
<thead>
<tr>
<th>Financial Variables</th>
<th>Mean</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weeks of Data</td>
<td>12.5</td>
<td>6</td>
<td>13</td>
</tr>
<tr>
<td>Total Income (000's)</td>
<td>$88.5</td>
<td>$1.4</td>
<td>$465.6</td>
</tr>
<tr>
<td>FTE</td>
<td>9.1</td>
<td>1.0</td>
<td>34.0</td>
</tr>
<tr>
<td>Base Salary Expense (000's)</td>
<td>$20.6</td>
<td>$2.7</td>
<td>$77.9</td>
</tr>
<tr>
<td>Incentive Compensation (000's)</td>
<td>$5.7</td>
<td>$0.0</td>
<td>$50.8</td>
</tr>
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</table>

Table 3.1: Descriptive statistics for branch-level income statements
<table>
<thead>
<tr>
<th>Variable</th>
<th>n</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Customer Measures (1 to 10 Scale; 1 = Unacceptable, 10 = Outstanding)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall branch rating (proxy for customer satisfaction)</td>
<td>898</td>
<td>8.9</td>
<td>1.6</td>
</tr>
<tr>
<td>Rating of the courtesy and friendliness of branch tellers</td>
<td>898</td>
<td>9.1</td>
<td>1.4</td>
</tr>
<tr>
<td>Evaluation of time required to wait in line for service</td>
<td>898</td>
<td>7.9</td>
<td>2.2</td>
</tr>
<tr>
<td><strong>Employee Measures (1 to 10 Scale; 1 = Very Dissatisfied, 5 = Very Satisfied)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall job satisfaction</td>
<td>898</td>
<td>7.7</td>
<td>1.9</td>
</tr>
<tr>
<td>Decision making authority required to do job effectively</td>
<td>898</td>
<td>8.1</td>
<td>1.8</td>
</tr>
<tr>
<td>Fair evaluation of job performance</td>
<td>898</td>
<td>7.7</td>
<td>2.0</td>
</tr>
<tr>
<td>Clear link between job performance and compensation</td>
<td>898</td>
<td>6.8</td>
<td>2.4</td>
</tr>
<tr>
<td>Satisfaction with rewards program (pay, bonus, 401k, etc.)</td>
<td>898</td>
<td>6.9</td>
<td>2.1</td>
</tr>
<tr>
<td>Opportunities for personal growth and development</td>
<td>898</td>
<td>7.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 3.2: Descriptive statistics for employee and customer satisfaction studies
<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
<th>Endogenous X</th>
<th>Constrained Parameters</th>
<th>LMD X</th>
<th>LMD Y</th>
<th>LMD TTL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_1$</td>
<td>Unconstrained demand-side model for $y$. Likelihood contribution of $X_i \sim N(\mu_i, \Sigma)$</td>
<td>--</td>
<td>--</td>
<td>14,709.88</td>
<td>1,645.71</td>
<td>16,355.59</td>
</tr>
<tr>
<td>$M_2$</td>
<td>Demand-side model with constrained parameter space for $\beta$. Likelihood contribution of $X_{it} \sim N(\mu_i, \Sigma)$</td>
<td>--</td>
<td>X</td>
<td>14,709.88</td>
<td>1,219.62</td>
<td>15,929.50</td>
</tr>
<tr>
<td>$M_3$</td>
<td>Simultaneous supply and demand model where supply-side is modeled as a linear function of the $\beta$s (MCR).</td>
<td>X</td>
<td>--</td>
<td>6,652.95</td>
<td>-732.18</td>
<td>5,920.78</td>
</tr>
<tr>
<td>$M_4$</td>
<td>Simultaneous supply and demand model with independent (unit-level) budget constraints over all inputs.</td>
<td>X</td>
<td>X</td>
<td>19,852.23</td>
<td>-2,943.99</td>
<td>16,908.24</td>
</tr>
<tr>
<td>$M_5$</td>
<td>Simultaneous supply and demand model with a single (unit-level) budget constraint over all inputs.</td>
<td>X</td>
<td>X</td>
<td>20,625.92</td>
<td>-2,889.55</td>
<td>17,736.38</td>
</tr>
<tr>
<td>$M_6$</td>
<td>Simultaneous supply and demand model with independent (bank-level) budget constraints over all inputs.</td>
<td>X</td>
<td>X</td>
<td>20,529.22</td>
<td>-4,650.15</td>
<td>15,879.07</td>
</tr>
<tr>
<td>$M_7$</td>
<td>Simultaneous supply and demand model with a single (bank-level) budget constraint over all inputs.</td>
<td>X</td>
<td>X</td>
<td>20,261.82</td>
<td>-5,247.83</td>
<td>15,013.99</td>
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</tbody>
</table>

Table 3.3: Fit statistics for alternative supply and demand side models
Table 3.4: Impact of satisfaction on response coefficients - $\Gamma$ matrix
<table>
<thead>
<tr>
<th>Scenario</th>
<th>Incremental Contribution Margin*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>mean</td>
</tr>
<tr>
<td>1. Uniform Allocation ($1,000 to each branch)</td>
<td>$220</td>
</tr>
<tr>
<td>2. FTE Proportional Allocation</td>
<td>$228</td>
</tr>
<tr>
<td>3. Allocation Proportional to size of $\beta_{si}$ (posterior mean)</td>
<td>$279</td>
</tr>
<tr>
<td>4. Unconstrained Optimal Allocation (posterior mean)</td>
<td>$465</td>
</tr>
<tr>
<td>5. Unconstrained Optimal Allocation (full posterior)</td>
<td>$471</td>
</tr>
<tr>
<td>6. Constrained Optimal Allocation $200 Min, $2000 Max (posterior mean)</td>
<td>$401</td>
</tr>
<tr>
<td>7. Constrained Optimal Allocation $200 Min, $2000 Max (full posterior)</td>
<td>$406</td>
</tr>
</tbody>
</table>

Table 3.5: Incremental contribution margin resulting from various allocation scenarios
<table>
<thead>
<tr>
<th>Employee Satisfaction Drivers</th>
<th>Expected impact of employee satisfaction on region CM*</th>
<th>Average impact of job characteristics on employee satisfaction</th>
<th>Expected impact of job characteristics on region CM*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{\partial \Pi}{\partial \mu}$</td>
<td>$\frac{\partial \mu}{\partial \mu}$</td>
<td>$\frac{\partial \Pi}{\partial \mu}$ $\frac{\partial \Pi}{\partial \mu}$ $\frac{\partial \mu}{\partial \mu}$</td>
</tr>
<tr>
<td>Personal growth and development</td>
<td>0.314</td>
<td>$112.7$</td>
<td></td>
</tr>
<tr>
<td>Satisfaction with rewards</td>
<td>0.204</td>
<td>$73.2$</td>
<td></td>
</tr>
<tr>
<td>Fair evaluation of job performance</td>
<td>0.197</td>
<td>$70.7$</td>
<td></td>
</tr>
<tr>
<td>Pay-performance link</td>
<td>0.158</td>
<td>$56.7$</td>
<td></td>
</tr>
<tr>
<td>Decision making authority</td>
<td>0.137</td>
<td>$49.2$</td>
<td></td>
</tr>
</tbody>
</table>

* Thousands of dollars

Table 3.6: Expected financial impact of changes in employee satisfaction and employee satisfaction drivers
APPENDIX A

ESTIMATION ALGORITHMS

A.1 Estimation algorithms for chapter 2

Estimation proceeds by recursively generating draws from the full conditional distribution of all model parameters. The likelihood of the data can be factored as:

\[ \pi(y_A, y_B | \alpha, \beta, \theta_A, \theta_B) = \pi(\varepsilon_{y_A}, \varepsilon_{y_B} | \alpha, \beta, \theta_A, \theta_B) \times \left| \frac{J(\varepsilon_{y_A}, \varepsilon_{y_B})}{\varepsilon_{y_A, y_B}} \right| \]

where the first factor corresponds to equation (2.1) and the second factor corresponds to equation (2.2). \( \theta_A \) and \( \theta_B \) represent other model parameters specific to the assumed distributions of the error terms. All model parameters are estimated using a random-walk Metropolis-Hastings algorithm where the likelihood contribution is of the form:

\[ [\alpha | \text{else}] \propto \prod_{j=1}^{J} \left( \varepsilon_{y_{A,j}, \varepsilon_{y_{B,j}} | \alpha, \beta, \theta_A, \theta_B} \right) \times \left| J(\varepsilon_{y_{A,j}, \varepsilon_{y_{B,j}}}) \rightarrow (y_{A,j}, y_{B,j}) \right| \]

where:

\[ \left| J(\varepsilon_{y_{A,j}, \varepsilon_{y_{B,j}}}) \rightarrow (y_{A,j}, y_{B,j}) \right| = \left| \frac{\partial \varepsilon_{j}}{\partial y_j} \right| = 1 - \alpha_1 \beta_1 \]

A.1.1 Model 1.1: Asymmetric Laplace

\[ \varepsilon_{y_A} \sim AL(0, \sigma_{y_A}, P_{y_A}) \]
\[ \varepsilon_{yB} \sim AL(0, \sigma_{yB}, p_{yB}) \]

\[ f_p(y|\mu, \sigma, p) = \frac{p(1-p)}{\sigma} \exp \left\{ -\rho_p \left( \frac{y - \mu}{\sigma} \right) \right\} \]

Prior distributions for all slope coefficients were specified as normal with mean zero and covariance equal to 100I. Inverted chi-square priors with 3 degrees of freedom and prior sum of squares equal to 0.01 were used for the scale parameters \((\sigma_{yA}, \sigma_{yB})\).

Priors for \(p_{yA}\) and \(p_{yB}\) were specified as Uniform(0,1).

**A.1.2 Model 1.2: Skewed t**

\[ \varepsilon_{yA} \sim skewt(0, \gamma_{yA}, \sigma_{yA}, \nu_{yA}) \]

\[ \varepsilon_{yB} \sim skewt(0, \gamma_{yB}, \sigma_{yB}, \nu_{yB}) \]

\[ f(y_i|\beta, \sigma, \nu, \gamma) = \frac{2}{\gamma+\gamma'} \frac{\Gamma \left( \frac{\nu+1}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right) \Gamma \left( \frac{\nu'}{2} \right)} \sigma^{-1} \]

\[ \times \left[ 1 + \frac{(y_i - x'_i \beta)^2}{\nu \sigma^2} \left\{ \frac{1}{\nu^2} I_{[0,\infty)} \left( y_i - x'_i \beta \right) + \frac{\gamma}{\nu} I_{(-\infty,0)} \left( y_i - x'_i \beta \right) \right\} \right]^{-\frac{(\nu+1)/2}{\nu}} \]

Prior distributions for all slope coefficients were specified as normal with mean zero and covariance equal to 100I. Inverted chi-square priors with 3 degrees of freedom and prior sum of squares equal to 0.01 were used for the scale parameters \((\sigma_{yA}, \sigma_{yB})\).

Priors for \(\gamma_{yA}\) and \(\gamma_{yB}\) were specified as Inverted chi-square with 3 degrees of freedom and prior sum of squares equal to 0.1. A uniform(0,150) prior was used for \(\nu_{yA}\) and \(\nu_{yB}\).
A.1.3 Model 1.3: Mixture of Multivariate Normals:

\[
\left( \begin{array}{c} \varepsilon_{yA} \\ \varepsilon_{yB} \end{array} \right) \sim \sum_k \phi_k N(\mu_k, \Sigma_k)
\]

1. Generate \( z|\mu_k, \Sigma_k, \phi \)

\[
z_i \sim \text{Multinomial} (\pi_i)
\]

\[
\pi_i = \phi_k \frac{\phi (\varepsilon_{yA}, \varepsilon_{yB}|\mu_k, \Sigma_k)}{\sum_k \phi (\varepsilon_{yA}, \varepsilon_{yB}|\mu_k, \Sigma_k)}
\]

where \( \phi \) is the multivariate normal density and \( z_i \) is a latent indicator variable that assigns each observation in the sample to one of the \( K \) mixture components.

2. Generate \( \phi|z \)

\[
\phi \sim \text{Dirichlet} (\tilde{\alpha})
\]

\[
\tilde{\alpha} = n_k + \alpha_k
\]

\[
n_k = \sum_{i=1}^{n} I(z_i = k)
\]

where \( \alpha_k \) is a prior value that is set equal to 1 for all \( k \).

3. Generate \( \mu_k, \Sigma_k|z, \alpha, \beta \)

Conditional on the latent indicator variable \( z_i \) each observation can be assigned to one of the \( K \) mixture components. Inference for \( \mu_k \) and \( \Sigma_k \) can then proceed through the use of a standard multivariate regression:

\[
\Sigma_k \sim \text{IW} (\nu_0 + n_k, V_0 + S_k)
\]

\[
\mu_k \sim N \left( \tilde{\mu}, \Sigma_k \otimes (X_k'X_k + A_\mu)^{-1} \right)
\]

where \( X_k = \iota_k \), the unit vector, with length equal to the number of observations assigned to component \( k \) and:

\[
\tilde{\mu}_k = \text{vec} \left( \tilde{M}_k \right)
\]
\[
\tilde{M}_k = (X'_kX_k + A_\mu)^{-1} \left( X'_kX_k\tilde{M}_k + A_\mu\tilde{M} \right)
\]

\[
S_k = \left( Y_k - X_k\tilde{M}_k \right)' \left( Y_k - X_k\tilde{M}_k \right) + \left( \tilde{M}_k - M \right)' A_\mu \left( \tilde{M}_k - M \right)
\]

\[
\hat{M} = (X'_kX_k)^{-1} (X'_kY_k)
\]

Prior parameters were specified as follow:

\[
\nu_0 = 3,
\]

\[
V_0 = \begin{bmatrix}
0.1 & 0 \\
0 & 0.1
\end{bmatrix}
\]

\[
A = 0.01
\]

\[
\tilde{M} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

4. **Generate** \(\alpha, \beta|\mu_k, \Sigma_k, z\)

\(\alpha\) and \(\beta\) can be drawn using a standard random walk Metropolis-Hastings algorithm where the likelihood for the model can be computed as follows:

\[
\prod_i \prod_k \phi \left( \varepsilon_{i,n_k}^{Y_A}, \varepsilon_{i,n_k}^{Y_B} | \mu_k, \Sigma_k \right) |J_{\varepsilon \rightarrow y}|
\]

Prior distributions for both \(\alpha\) and \(\beta\) were specified as multivariate normal with mean 0 and covariance equal to 100\(I\). To improve mixing of the Markov chain we found it useful to draw the regression coefficients in two distinct blocks.

**A.2 Estimation Algorithms for Chapter 3**

Bayesian estimation for the simultaneous supply and demand model proceeds by recursively generating draws from the full conditional distributions of all model parameters. The non-standard nature of our model prevents us from relying exclusively on conjugate results. As such, we implement a hybrid sampler and draw a subset
of the model parameters using the Metropolis-Hastings algorithm. We divide the MCMC sampler into six distinct blocks and alternate parameter draws within and across units. We define the following quantities in order to simplify exposition of the algorithm.

The full likelihood for the model can be expressed according to (A.1):

$$\ell (\text{data}|else) = \prod_i \prod_t \pi (\varepsilon_{it}) \pi (\zeta_{1it}, \zeta_{2it}, \zeta_{3it}) \left| J_{\zeta \rightarrow x} \right|$$  \hspace{1cm} \text{(A.1)}$$

where \( \pi (\cdot) \) denotes the multivariate normal density function and the quantities \( \varepsilon_{it} \) and \( \zeta_{kit} \) can be computed according to (A.2) and (A.3). The Jacobian term is defined in (A.4).

$$\varepsilon_{it} = \ln (y_{it}) - (\ln (\beta_{0i}) + \beta_{1i} \ln (x_{1it}) + \beta_{2i} \ln (x_{2it}) + \beta_{3i} \ln (x_{3it}))$$  \hspace{1cm} \text{(A.2)}$$

$$\begin{bmatrix}
\zeta_{1it} \\
\zeta_{2it} \\
\zeta_{3it}
\end{bmatrix} = \begin{bmatrix}
(\beta_{1i} - 1) & \beta_{2i} & \beta_{3i} \\
\beta_{1i} & (\beta_{2i} - 1) & \beta_{3i} \\
\beta_{1i} & \beta_{2i} & (\beta_{3i} - 1)
\end{bmatrix} \ln (x_{it}) - \begin{bmatrix}
\ln (\lambda_{1i}) - \ln (\beta_{0i}) - \ln (\beta_{1i}) \\
\ln (\lambda_{2i}) - \ln (\beta_{0i}) - \ln (\beta_{2i}) \\
\ln (\lambda_{3i}) - \ln (\beta_{0i}) - \ln (\beta_{3i})
\end{bmatrix}$$  \hspace{1cm} \text{(A.3)}$$

$$\left| J_{i} \right| = |\beta_{1i} + \beta_{2i} + \beta_{3i} - 1|$$  \hspace{1cm} \text{(A.4)}$$

The error terms for the supply and demand equations are assumed to be distributed as follow:

$$\varepsilon_{it} \sim N (0, \sigma_{i}^2)$$  \hspace{1cm} \text{(A.5)}$$

$$\begin{bmatrix}
\zeta_{1it} \\
\zeta_{2it} \\
\zeta_{3it}
\end{bmatrix} \sim N (0, \Sigma_{xi})$$  \hspace{1cm} \text{(A.6)}$$

Conditional on initial values the sampler proceeds as follows (repeating until convergence has been achieved):

**Block 1 - Within Units:** Iterate through each unit (i.e., branch) in the dataset drawing:

1. \( \{\beta_{ki}\} | else \propto \{y_{it}\} | \{x_{kit}\}, \{\beta_{ki}\}, \sigma_{yi}^2 \left[ \{x_{kit}\} | \{\beta_{ki}\}, \{\lambda_{ki}\}, \Sigma_{xi} \right] \{\beta_{ki}\} | \Gamma, \mu_{i}, \Sigma_{\beta} \)
Draw $\beta_i$ using a Metropolis-Hastings (M-H) step, where the contribution for the first two factors of the likelihood for unit $i$ is equal to:

$$\prod_t \pi(\varepsilon_{it}) \pi(\zeta_{1it}, \zeta_{2it}, \zeta_{3it}) \left| J_{it} \right|_{\zeta \rightarrow x}$$

where the Jacobian is defined in equation (A.4) and the hierarchical prior for beta is specified as:

$$\beta_i \sim N(\Gamma'_\mu^i, \Sigma_\beta)$$

Acceptance probabilities are computed using the standard algorithm (see Rossi, Allenby, and McCulloch 2005 - page 88)

2. $[\{\lambda_{ki}\}|else] \propto [\{x_{kit}\} | \{\beta_{ki}\}, \{\lambda_{ki}\}, \Sigma_{xi}] [\{\lambda_{ki}\} | \bar{\lambda}, \Sigma_{\lambda}]$

   $\{\lambda_{ki}\}$ are also draw using the M-H algorithm where the likelihood contribution of the first factor is equal to:

$$\prod_t \pi(\zeta_{1it}, \zeta_{2it}, \zeta_{3it}) \left| J_{it} \right|_{\zeta \rightarrow x}$$

with a corresponding hierarchical prior specified for $\{\lambda_{ki}\}$:

3. $[\sigma_i^2|else]$

   Conditional on a draw of $\beta_i$, $\sigma_i^2$ is a standard draw from an inverse chi-square distribution (see Rossi, Allenby and McCulloch 2005 - page 25)

**Block 2: Across Units**

4. $[\Gamma, \Sigma_\beta|else]$

   Conditional on realizations of $\beta$, $\mu^*$ inference for $\Gamma$ and $\Sigma_\beta$ proceeds through use of a standard multivariate regression:

$$\Sigma_\beta \sim IW(\nu_0 + N, V_0 + S_\beta)$$

$$\Gamma \sim N\left(\bar{\Gamma}, \Sigma_\beta \otimes \left(\mu^*\mu^* + A_\beta\right)^{-1}\right)$$
where:

\[ \tilde{\Gamma} = vec(\tilde{M}_\beta), \quad \tilde{M}_\beta = (\mu^*\mu^* + A_\beta)^{-1} \left( \mu^*\mu^*\tilde{M}_\beta + A_\beta\tilde{M} \right) \]

\[ S_\beta = \left( \beta - \mu^*\tilde{M}_\beta \right)' \left( \beta - \mu^*\tilde{M}_\beta \right) + \left( \tilde{M}_\beta - \tilde{M} \right)' A_\beta \left( \tilde{M}_\beta - \tilde{M} \right) \]

\[ \hat{M} = (\mu^*\mu^*)^{-1} (\mu^*\beta) \]

All priors were specified using standard, non-informative values.

5. \([\bar{\lambda}, \Sigma_\lambda] | else\]

Conditional on realizations of \(\{\lambda_{ki}\}\), \(\bar{\lambda}, \Sigma_\lambda\) can be estimated using a multivariate regression of \(\{\lambda_{ki}\}\) on the unit vector with length equal to the number of branches under study, \(\iota_N\). Full conditional distributions for the mean and covariance matrix follow those outlined in step (4).

**Block 3: Within Units**

6. \([\mu_i^c, \Sigma_i^c] | else\]

\(\mu_i^c\) and \(\Sigma_i^c\) can be drawn through the use of a multivariate regression of observed customer satisfaction survey responses on the unit vector, \(\iota_N\).

7. \([\mu_{i1}^c] | else\] \(\propto [\mu_i^c, V^e] [z_{i1}^c] [\mu_i^c, \Sigma_i^c] [\beta_i | \mu_i^*, \Gamma, \Sigma_\beta] \)

Inference for the latent level of aggregate customer satisfaction, \(\mu_{i1}^c\), proceeds by first recognizing that distribution of \(\mu_{i1}^c\) is proportional to the product of three multivariate normal distributions: \(A, B,\) and \(C\). We derive the posterior distribution for \(\mu_{i1}^c\) by re-expressing \(A, B,\) and \(C\) in terms of the univariate normal for \(\mu_{i1}^c\) and combining quadratic forms as described in Box and Tiao (1973):

\[ \mu_{i1}^c \sim N(\tilde{\mu}_{i1}^c, \Sigma_{\mu_{i1}^c}) \]

where:

\[ \tilde{\mu}_{i1}^c = (\Sigma_A^{-1} + \Sigma_B^{-1} + \Sigma_C^{-1})^{-1} (\Sigma_A^{-1}\mu_A + \Sigma_B^{-1}\mu_B + \Sigma_C^{-1}\mu_C) \]
\[ \Sigma_{\mu_i} = \left( \Sigma_A^{-1} + \Sigma_B^{-1} + \Sigma_C^{-1} \right)^{-1} \]

and the contribution for each factor, \(A\), \(B\), and \(C\), can be computed as:

Factor \(A\) (contribution from the prior):

\[ \mu_A = 0 \]
\[ \Sigma_A = 100I \]

Factor \(B\) (Contribution from the model for observed customer satisfaction responses):

\[ \mu_B = \mu_{1i}^c + \Sigma_{12}^c \Sigma_{22}^{-1} (a - \mu_{-1i}^c) \]
\[ \Sigma_B = \Sigma_{11}^c - \Sigma_{12}^c \Sigma_{22}^{-1} \Sigma_{21}^c \]

where:

\[ z_{ih}^c \sim N \left( \mu_i^c = \begin{bmatrix} \mu_{1i}^c \\ \vdots \\ \mu_{Ji}^c \end{bmatrix}, \Sigma_i^c \right) \]
\[ \Sigma_i^c = \begin{bmatrix} \Sigma_{ij_1}^c & \Sigma_{ij_2}^c \\ \Sigma_{21}^c & \Sigma_{22}^c \end{bmatrix} \]

Factor \(C\) (contribution from the model for the demand response coefficients, \(\beta\)):

\[ \beta_i = \Gamma' \mu_i^s + \eta_i \]

Begin by partitioning as follows:

\[ \Gamma'_{k \times 3} = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ k \times 2 & k \times 1 \end{bmatrix} \]

Compute:

\[ \beta_i - \Gamma_1 \begin{bmatrix} 1 \\ \mu_{1i}^c \end{bmatrix} = \Gamma_2 \mu_{1i}^c \]
\[ \mu_C = (\Gamma_2' \Gamma_2)^{-1} \Gamma_2' \left( \beta_i - \Gamma_1 \begin{bmatrix} 1 \\ \mu_{1i}^c \end{bmatrix} \right) \]
\[ \Sigma_C = \left( \Gamma'_2 \Gamma_2 \right)^{-1} \left( \Gamma'_2 \Sigma_\beta \Gamma_2 \right) \left( \Gamma'_2 \Gamma_2 \right)^{-1}, \]

**Block 4: Across Units**

8. \([\bar{\mu}^c, V^c_\mu | \text{else}]\)

Conditional on realizations of \( \mu^c_i \), estimation of \( \bar{\mu}^c \) and \( V^c_\mu \) proceeds through the use of a multivariate regression as defined in step 4.

9. \([\Omega^c | \text{else}]\)

We follow Jen, Chou, and Allenby (2007) when drawing parameters for the distribution of random effects specified for \( \Sigma^c_i \). The conditional posterior for \( \Omega^c \) is:

\[
IW \left( \nu^c_0 + N\bar{\nu}_c, \left( \Omega^c_0^{-1} + \sum_{i=1}^N \Sigma^c_i^{-1} \right) \right)
\]

10. \([\bar{\nu}^c | \text{else}]\)

The posterior distribution for \( \bar{\nu}^c \) does not have a closed form expression and must therefore be drawn using a M-H step, where the likelihood contribution for \( \bar{\nu}^c \) is equal to:

\[
\prod_{i=1}^N \left( 2^{\nu_c} \pi \frac{1}{2} \Gamma \left( \frac{\nu_c}{2} \right) \Gamma \left( \frac{\nu_c - 1}{2} \right) \right)^{-1} \left| \Omega^c_0 \right|^{\frac{\nu_c}{2}} \exp \left\{ -\frac{1}{2} tr \left( (\Sigma^c_i)^{-1} (\Omega^c)^{-1} \right) \right\}
\]

Steps 11 through 15 are the employee analogs of customer steps 6-10. Parameter estimation follows directly.

**Block 5: Within Units**

11. \([\mu^e_i, \Sigma^e_i | \text{else}]\)

12. \([\mu^e_i | \text{else}] \propto [\mu^e_i, \Sigma^e_i] [z_{ih} | \mu^e_i, \Sigma^e_i] [\beta_i | \mu^e_i, \Gamma, \Sigma_\beta] \)

**Block 6: Across Units**

13. \([\bar{\mu}^e, V^e_\mu | \text{else}]\)

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14. $[\bar{\nu}^e|else]$  
15. $[\Omega^e|else]$
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