A Qualitative Study of Interactions, Concept Development and Problem Solving in a Calculus Class Immersed in the Computer Algebra System Mathematica™

Dissertation

Presented in Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy in the Graduate School of The Ohio State University

By

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Approved by

Advisor
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To My Mother and
in Memory of My Father
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CHAPTER I
INTRODUCTION

The mathematics education reform movement spans all levels. Appropriate methods for teaching and the content of calculus are primary targets of reform. This study investigated the classroom interactions and problem solving skills developed by students participating in one attempt at such reform. The entire course was taught using the computer algebra system Mathematica™ in a computer laboratory setting. The data were collected via classroom observations, student interviews over a period of two quarters, and a questionnaire on the backgrounds and attitudes of students about the experience. Changes in problem solving skill development, understanding of calculus concepts, and appropriate use of technology were documented.

Review of Literature

Theoretical Base for Learning Calculus

The teaching and learning of mathematics is affected by many factors. Primary among these factors are classroom environment and interactions or discourse. Some attention has been devoted to the categorization of interactions in the mathematics classroom and the duration of these interactions (Farrell, 1990; Fraser, Malone, & Neale, 1989; Wandzilak & Mortensen, 1988). Information has been gathered, mostly via interviews, (Browning, 1989; Dunham, 1990; Heid, 1988)
indicating that the nature of discourse in the classroom is significantly related to the
construction of knowledge and should be investigated.

The National Council of Teachers of Mathematics (NCTM) in its Professional
Standards for Teaching Mathematics (Commission on Professional Teaching
Standards, 1991) states:

The discourse of a classroom—the ways of representing, thinking, talking,
agreeing and disagree— is central to what students learn about mathematics .
. . . The discourse is shaped by the tasks in which students engage and the
nature of the learning environment. (p. 34)

Investigation of the types of interactions in the learning environment are essential to
implementing changes in mathematics teaching and learning. The National Council
of Teachers of Mathematics (Commission on Standards for School Mathematics,
1989) also states:

Students' learning of mathematics is enhanced in a learning environment that is
built as a community of people collaborating to make sense of mathematical
ideas. (p. 58)

This statement supports the idea that current mathematics education research should
be directed toward investigation of the learning environment.

The current study involved the environment of a computer laboratory and the
use of the computer algebra system Mathematica™ to teach calculus. Prior to this
study some research (Hawker, 1987; Heid, 1988; Judson, 1989; Palmiter, 1986;
Schrock, 1990) had been carried out with calculus classes using computer algebra
systems. These studies incorporated computer algebra systems into traditional
classroom settings. This was usually done in a limited way and accomplished by
adding a lab day or days. The researchers compared test performance of students using the computer with students not using the computer and explored the effects of resequencing topics on performance and development of concepts. The studies share the common result that the use of a computer algebra system can enhance the learning of calculus concepts prior to the development of computational skills. Conceptual understanding was better in the groups using computer algebra systems (Hawker, 1987; Palmiter, 1986; and Schrock, 1990).

The unique classroom environment and materials separate this study from all of the above. The focus of this study was not a comparison of groups nor an interest in skill development, but rather the reactions of the students to the learning environment, their interactions within the environment, the development of their understanding of the concepts of calculus, and the development of their problem solving strategies.

Interest in the use of technology in mathematics combined with resounding calls for reform involving methods of teaching and content of calculus, increase the timeliness and interest in research such as that reported here. Douglas (1986) presents the essence of the issue concerning the difficulty with current methodology for teaching calculus by stating,

The principal problem with calculus today is the way it is taught... The demoralizing effect this has on both faculty and students cannot be overstated.

Calculus cannot be learned passively (p. 5).

A strong case was made for promoting and researching students' understanding of calculus by Steen (1987):
in order to teach students what we want them to learn, we have to understand the interaction that goes on when students construct their own images of mathematics which are quite likely different than the ones we have in our minds or that we are trying to convey to them. (p. 13)

The understanding of what occurs when students construct their own mathematics can only come through new methods and techniques of research. In order to ascertain what and understand how students construct mathematics, researchers must employ qualitative techniques. Careful observation and in-depth interviews with students are primary techniques for obtaining information. The information on what and how students construct mathematics will lead to new methods of instruction. One strong message concerning methods of instruction in calculus is to incorporate technology as a tool. Tucker (1987) voiced this call for incorporation of technology with his hope that calculus "uses calculators and computers, not for demonstrations but as tools, tools that raise as many questions as they answer" (p. 16). This study involved the use of technology in teaching calculus and employed qualitative methods to research its use.

**Theoretical Base for Constructivism**

A brief, but complete, description of the theoretical position of the researcher will help with interpretations of the findings of the study and promote an understanding of the directions taken as the research design emerged. Mathematics education research has traditionally focused on achievement in various content areas. A shift is occurring toward investigations of learning and the construction of knowledge by the learner. Von Glasersfeld (1983) states that education and
educational research will change, by the adoption of a constructivist perspective, in the following ways:

(a) There will be a radical separation between educational procedures that aim at generating *Understanding* (teaching) and those that merely aim at the repetition of *behaviors* (training).

(b) The researcher's and to some extent also the educator's interest will be focused on what can be inferred to be going on inside the student's head rather than on overt responses.

(c) The teacher will realize that knowledge cannot be transferred to the student by linguistic communication but that language can be used as a tool in a process of guiding the student's construction.

(d) The teacher will try to maintain the view that students are attempting to make sense in their experiential worlds. Hence he or she will be interested in students' errors and, indeed, in every instance where students deviate from the teacher's expected path because it is these deviations that throw light on how the students, at that point in their development, are organizing their experiential world. (p. 3)

The term constructivism has become popular in the last several years, but the ideas have been around for hundreds of years. The term is often used in ways that are different from the original intent. Von Glasersfeld (1989c) points out that constructivist ideas are pre-Socratic. This is when people started to question whether knowledge of the real world is certain or derived from experience.

The idea of cognitive construction was first discussed in a Latin treatise on epistemology, by the Italian philosopher Vico in 1710 (Von Glasersfeld, 1988b).
The question of how knowledge is acquired moved through several philosophical views during the 17th and 18th centuries (Fabricius, 1983). The three major views were (a) rationalism, (b) empiricism, and (c) romanticism. The theories of rationalism and empiricism place the structure of knowledge in objects. The theory of romanticism placed the structure of knowledge within the cognizing subject.

Rationalism was based on the idea that the entire world was rational and patterned after mathematical systems (Fabricius, 1983). Certain accepted premises enabled people to deduce conclusions that were considered facts about the world. The entire world was considered a mathematical model.

Empiricists did not use deductive reasoning in their view of the world, but rather inductive reasoning (Fabricius, 1983). They tried to generalize and predict based only on what was observed or experienced. Knowledge was based on perceptions of sight, sound, feel, and other senses.

Hume revealed the problems of empiricism and rationalism (Fabricius, 1983) in their inability to explain the structure of knowledge. If knowledge had to be deduced or induced, its structure could not be explained. Von Glasersfeld (1989c) explained this paradox by the statement that if true knowledge is representative of the world, then true knowledge cannot be tested. He writes:

If experience is the only contact a knower can have with the world, there is no way of comparing the products of experience with the reality from which whatever messages we receive are supposed to emanate. . . . The paradox then is this: to assess the truth of your knowledge you would have to know what you come to know before you know it. (cited in Narode, 1987, p. 22)
Rationalism and empiricism both placed the structure of knowledge in objects. Rousseau (Fabricius, 1983) felt that the structure of knowledge might not be so far from the subject. He felt these structures could be found from within. Knowledge was within the subject, not the object. This theory was romanticism.

In the late 18th century, Kant looked at a combination of subject and object (Fabricius, 1983), in particular, how objects appeared to the subject. He argued that the structure of knowledge was in neither subject nor object, but in the experience created by their interaction. This was a major break through in what has become known as constructivism.

Narode (1987) said constructivism is the process of individuals defining their own, constructed worlds. Blais (1988) says "knowledge is something learners must construct for and by themselves. . . . Discovery, reinvention, or active reconstruction is necessary" (p. 627). He claimed constructivists distinguish information, that can be told or given to someone, from knowledge, that requires gaining expertise. Piaget (1973) stated a similar idea:

to understand is to discover . . . The goal of intellectual education is not to know how to repeat or retain ready-made truths. It is in learning to master the truth by oneself. (p. 106)

Kamii (1985) saw constructivism as the theory that children build their own knowledge and it comes from inside. This means that knowledge is not based on any pre-existing truths. Knowledge is constructed as a result of the experiences of the learner. Brooks (1987) thought constructivism explained how people "come to know their world" (p. 67). Perhaps this would be better stated as how people understand their world. This knowing or understanding takes place as a result of
the interactions people make and the perceptions they develop as a result of these interactions.

Orton (1988) explained that constructivists consider the processes learners go through to create representations or relationships between ideas and that process involves reflection. Von Glasersfeld (1988a) brought to attention the following:

Constructivism is a way of thinking which, at this stage in the development of our ideas, seems the most adequate; like all we call knowledge, it is not, and cannot be the description of an ontological reality and therefore it may change as our ways of experiencing and our purposes change. (p. 5)

Perhaps most widely known is Piaget's theory, considered by many to be constructivist in orientation. Piaget states that experience is assimilated and accommodated in order to create knowledge (Piaget, 1954). As the subject seeks a state of equilibrium, knowledge is created. These ideas underlie most of the description given above for constructivism. Von Glasersfeld (1989a) stated Piaget's theory the following way:

1. a) Knowledge is not passively received either through the senses or by way of communication;
   b) knowledge is actively built up by the cognizing subject.

2. a) The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability;
   b) cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (p. 5)

Von Glasersfeld (1989b) explains Piaget's stance with the following:
Knowledge for Piaget is never (and can never be) a representation of the real world. Instead it is a collection of conceptual structures that turn out to be adapted or viable within the knowing subject’s range of experience. (p. 125)

This range of experience was the focus of this study. Specifically, the target of this study was the types of classroom interactions that contributed to the construction of knowledge and the problem solving strategies used by the students as a result of their experiences with Mathematica™. The beliefs of the researcher concerning constructivism as a learning theory contributed not only to the directions the study took, but also to the interpretations made of the data collected.

As illustrated in Figure 1, expanded from Confrey (1990b), the cycle of construction of knowledge is affected by many factors. Among them are the classroom environment, interactions (discourse), and the nature of the interactions.
Figure 1. Cycle of construction of knowledge

A student encounters a problem situation. As explained by Confrey (1990a), the problem is not the same as the problematic in that the following holds:

A problematic is defined only in relation to the solver and only becomes a problematic to the extent to which and in the manner in which it feels problematic to the solver. When defined this way, as a roadblock to where a student wants to be, the problematic is not given an independent status. The problematic acts as a perturbation, i.e. a call to action. (p. 12)

This begins a cycle of action and reflection. The actions and reflections are affected by the prior and current experiences of the student as movement is made toward solutions. At the point of reflection, the student examines their solutions
and attempts to decide if equilibrium has been achieved. For example, in a mathematics problem solving situation, the student is presented with a problem. Some action is taken to clarify or manipulate the information in the problem. A solution, partial or complete is arrived at and reflection on the reasonableness or accuracy of the solution based on prior experiences may lead to repetition of the process. More specifically, given a function in a calculus class to explore and sketch a graph of, students may investigate the first derivative and make decisions concerning intervals where the function increases and intervals where it decreases. After these decisions are made, maximum and minimum points may be identified. Further investigation, possibly involving the second derivative, may lead the student to reflect and determine points of inflection or possibly reconsider intervals where the function is increasing or decreasing.
CHAPTER II
ENVIRONMENT AND PARTICIPANTS

The environment at The Ohio State University is a factor in this research. Many faculty in mathematics support and are active in the use of technology in the teaching and learning of mathematics. Due to this, the environment at this university may be more conducive to projects involving experimentation with new technology and methods in the classroom than in other locations. This is evident by the number of funded projects involving technology over the years, such as the use of Maple in calculus survey courses and the Computer and Calculator Pre-Calculus (C^2PC) Project. The Department of Mathematics has hosted an international conference on technology in collegiate mathematics on campus for three consecutive years. For these reasons, the atmosphere in the department and the institution can be viewed as supportive.

Environment

The study was carried out in a beginning calculus course at The Ohio State University in the Fall and Winter quarters of 1990-1991. Information was gathered during the first two quarters of a five quarter sequence designed primarily for mathematics, science, and engineering students. The course was taught using Mathematica™ notebooks written by Brown, Porta, and Uhl (1991) at the University of Illinois, but revised and adapted for use in the calculus sequence at The Ohio State University by Davis. Mathematica™ notebooks are electronic text.
They provide an interactive environment where students are able to execute sections of the notebook and see graphics created, problems solved, and experiment with the mathematics. The ability to experiment by changing numbers, functions, and various parameters is one of the more powerful aspects of the notebooks. No written text allows such student experimentation and flexibility to test what occurs and, therefore, aid the student in learning. A notebook corresponds, in some ways, to a chapter or collection of sections in a traditional calculus textbook. The notebooks served as the only text for the course at the time this study was carried out. Each notebook consisted of five sections. They were: (a) A "Guide" that gave a somewhat brief explanation of the lesson contained in the notebook and served as an introduction to a new unit of material, (b) a section called "Basics" that presented example problems on the new topics along with new calculus tools as well as new Mathematica™ tools to be used, (c) a "Tutorial" containing more in-depth problems to serve as examples and show the students what the new topics involved, (d) a section called "Give it a Try" containing problem assignments, and (e) a "Literacy Sheet" consisting of things students should be able to do without the aid of the computer.

The interest of Brown, Porta, Uhl, and Davis in calculus reform can be seen in the design of the notebooks as well as the approach to calculus. As stated by Brown, Porta, and Uhl (1990),

Calculus is nothing more or less than a course on how to use the tools of differentiation, integration and approximations to make precise measurements.

... If a topic contributes to measurement, then the topic is not tossed out; if a topic is purely mathematical and may serve to whet the students' interest in
mathematics, then it is not tossed out. Otherwise the topic is probably a goner.

All calculus cottage industries like L'Hopital's rule, convergence at endpoints,
etc. are de-emphasized or missing. (p. 104)

This attitude toward the content of calculus is rooted in the reform movement for a
"lean and lively" calculus. Some of the changes involving content that Brown,
Porta, and Uhl (1990) have incorporated into the course are:

Feeling for limits and convergence is set up through plots showing the curves
(f[x + h] - f[x])/h crawl onto f[x] as hhl gets small.

The chain rule forms the keystone of our treatment of differentiation. We get at
the logarithms by using the chain rule to differentiate the functional equation of
the logarithm. The power rule and product rules are obtained by logarithmic
differentiation.

Continuity and limits do not sit at the front of the course but emerge in a natural
way throughout the course. Students work on continuity by being able to
report how many accurate decimals of x are needed to calculate, say, eight
accurate decimals of f[x].

The mean value theorem is studied as a consequence of something we call the
Race Track Principle. This principle says that if f(a) = g(a) and f'(x) ≥ g'(x)
for x ≥ a, then f(x) ≥ g(x) for x ≥ a. Another version of the principle says that
if f(a) = g(a) and f'(x) is close to g'(x) for x near a, then f(x) is close to g(x)
for x near a. The Mean Value Theorem is a corollary of the Race Track
Principle.
Early in the course we confront the problem of plotting $f[x]$ given only $f'[x]$ and one value of $f[x]$. Differential equations appear liberally in this course, but maybe not so much as in some other calculus revision projects. Our course is not in the business of defining slopes, areas, arc lengths and volumes. We are in the business of measuring these quantities. Newton never heard of a Riemann sum, but Newton did teach us via the fundamental theorem . . . that we can measure any quantity once we calculate its derivative. Finding the derivative of area, volume, arc length and the like is good geometric mathematics. (pp. 104-105)

The involvement of the notebook authors and professor for this course in the calculus reform efforts resulted in an environment for learning calculus that was different in approach as well as content.

The physical setting for the class meetings was a computer laboratory equipped with Apple Macintosh computers networked with a file server. Once each week, a recitation meeting was scheduled with the main purposes of discussion of problems and administering a weekly quiz. The quizzes were all paper–and–pencil. Some questions were answered during this session, but formal lectures were not presented. Occasionally, small groups of students gathered around the professor in the lab to clarify a common problem. Some short presentations on particularly difficult topics were presented by the instructor in similar sessions. No time for a traditional lecture was scheduled for the class.

It was observed by the researcher that the role of the professor and teaching assistants was changed drastically from the typical classroom role in beginning calculus courses at the college level. The primary changes were: (a) facilitating
learning activities rather than presenting information, (b) stimulating discussion and
group work, (c) raising questions rather than providing answers, and
(d) troubleshooting problems with the materials as well as the equipment. This
changing, multi-dimensional role of the teacher is similar to the one presented by
the Mathematical Sciences Education Board (1990) as follows:

- A role model who demonstrates not just the right way, but also the false
  starts and higher-order thinking skills that lead to the solution of problems;
- A consultant who helps individuals, small groups, or the whole class to
decide if their work is keeping to the subject and making reasonable
  progress;
- A moderator who poses questions to consider, but leaves much of the
decision making to the class;
- An interlocutor who supports students during class presentation,
  encouraging them to reflect on their activities and to explore mathematics on
  their own;
- A questioner who challenges students to make sure that what they are doing
  is reasonable and purposeful, and ensures that students can defend their
  conclusions. (p. 40)

The professor and teaching assistants worked as a support mechanism for the
learning taking place and not as the providers of knowledge. The relaxed and open
atmosphere of the classroom promoted students working together. The professor
and teaching assistants for the course worked as participants with the students. The
students were relaxed and open to discuss the mathematics with each other as well
as the professor and teaching assistants.
Students enrolled in the course, though registered for a particular meeting time, came to the lab at whatever time was convenient, based on their schedule. The lab was reserved for the calculus courses on weekdays from 8 am until 12 noon and during the evenings for 2 to 3 hours. At other times, the lab was open to the public as well as the calculus students. It was also available on weekends. Students in the class tended to take advantage of the morning lab time for at least an hour daily. The other times were popular, but help during the morning hours was always available from the professor or teaching assistants. Some night hours were staffed by teaching assistants as well. The lab arrangement of the class, the flexibility allowed regarding attendance, and the nature of the materials used all contributed to the increased time involvement on the part of the professor for this course. It was not unusual for the professor to spend the majority of every morning in the lab and large amounts of outside time at the computer revising and writing notebooks. The teaching assistants also contributed a large amount of time due, in part, to the fact that the homework they graded had to be graded electronically and returned electronically. It should be noted that the time investment is significant for all involved, the students, the teaching assistants, and the professor.

Two sections of first quarter calculus were offered in the Fall quarter of 1990. Two graduate students and one student from fourth quarter calculus were assigned to work as teaching assistants with one professor in these courses. One section of fourth quarter calculus was offered during this quarter by this same professor and the fourth quarter students were often in the computer lab along with the first quarter students. Some interaction between the fourth and first quarter students occurred. While some of this was social, the fourth quarter students did answer
some questions for the first quarter students. This did not occur on a regular basis and was not a major component of the learning environment. It most often occurred in the evening hours when the lab was available and a teaching assistant or professor was not there.

Participants

The two sections of first quarter calculus used for the study were the only two Mathematica™ sections offered during Fall 1990. A total of 38 students registered for the two sections. The researcher attended the two recitation classes during the fifth week of the quarter to explain briefly the purposes of the study and ask for volunteers to participate in a series of four interviews. The recitations were used to recruit volunteers since attendance at these meetings was high due to the weekly literacy quizzes. This resulted in a total of 27 volunteers out of 36 students attending recitation on the days visited. They provided addresses and phone numbers for contact to be made.

A purposive sampling process was used in that maximum variation in perceived ability of the students was used as the criteria for narrowing the group of volunteers to 12. Patton (1980) describes purposive sampling versus random sampling for use in qualitative research. Maximum variation sampling is one type of purposive sampling. Lincoln and Guba (1989) stated that,

In naturalistic investigations . . . the purpose of sampling will most often be to include as much information as possible, in all of its various ramifications and constructions; hence, maximum variation sampling will usually be the sampling mode of choice. the object of the game is not to focus on the
similarities that can be developed into generalization, but to detail the many specifics that give the context its unique flavor. (p. 201)

The two teaching assistants working with the classes were asked to assist in the process of selecting subjects since they had the most contact with the students to this point. They had worked individually with students daily and had written, administered, and graded the weekly quizzes. Each teaching assistant was asked to identify three students they perceived as performing at one of three levels, high, medium, and low. Their selection criteria were discussions involving calculus they had participated in and the performance on written quizzes to this point in the quarter. Two students in each category were contacted for participation in the interviews and the third was identified as an alternate if a student did not wish to participate or was not planning to remain in the Mathematica™ course during second quarter calculus.

The initial 12 contacts resulted in 2 alternates being contacted due to 2 of the original students contacted planning not to continue the Mathematica™ sequence into the second quarter. The first interview with each participant was conducted near the end of the first quarter course. At the beginning of the second quarter course, contact was made to schedule the remaining interviews. It was discovered that 2 of the 12 participants had moved to a traditional second quarter class and 1 had left the university. These 3 were dropped from the sample and the study was carried out with 9 participants taking part in a total of 36 interviews. The researcher chose not to replace the 3 with alternates since it would not be possible to obtain the first interview under the same conditions, particularly at the same point in the course, as it was obtained from the initial participants.
Parts of a questionnaire given to the participants at the end of the last interview provided information on the make-up of the group and their background. A copy of the questionnaire and a tally of the complete results can be found in Appendix A. The group of nine participants consisted of six males and three females. Eight had been enrolled at the university for two quarters when the questionnaire was administered. This indicated that they started the calculus sequence with Mathematica™ as their beginning mathematics course during the first quarter of their freshman year. The remaining participant had been enrolled for six quarters and indicated completing two quarters of precalculus mathematics at the university before entering the calculus sequence. The precalculus courses required the use of graphing calculator technology. The intended majors cited by the participants were mechanical engineering (2), botany, international studies, business, microbiology/pre-med, accounting, environmental engineering, and one undecided.

Five of the participants had completed a calculus course in high school. Among the four who did not, three indicated that the content of their pre-calculus course in high school included limits and derivatives. Seven participants took some form of computer course prior to enrolling in the calculus sequence. These courses varied from programming in BASIC, Fortran, or Pascal to introductory courses with little or no programming. None of the participants had taken a mathematics course that required use of a computer prior to this, but one indicated that a calculus course in high school made use of a computer for a few days to picture graphs of functions. Only three participants indicated that they owned a computer and only one of these was the same type used in the laboratory.
CHAPTER III
ORGANIZATION AND METHODS OF DATA COLLECTION

Naturalistic or qualitative research requires gaining entry into the site to be studied. In the case of educational research that often means getting the approval of a teacher to enter the classroom and interact with the students. Boundaries and limitations of that interaction must be agreed upon. The design of the study begins to emerge. A research plan is established and often revised as the research progresses.

Gaining Entry

In this study, entry was gained in the Mathematica™ classroom during the Spring quarter of 1990 when a pilot study or prior ethnography was conducted involving students from the third quarter of a calculus sequence using Mathematica™. Corsaro (cited in Lincoln & Guba, 1985) recommends a prior ethnography described as, "becoming a participant observer in a situation for a lengthy period of time before the study is actually undertaken" (p. 251). Lincoln and Guba (1985) describe this as a time to lessen the obtrusiveness of the researcher, become oriented in the environment, and to become aware in order to be more effective during the formal study.

Initially, it was agreed that the researcher would only observe in the classroom and would avoid more direct interaction with the students. At the end of the quarter a written questionnaire was administered to the students to obtain information about
their background and attitudes toward the course. Due to the restrictions, the researcher devised a plan for recording the types of interactions that took place in the classroom. This was done with the intention of developing a thorough understanding of the learning environment. The interactions of interest were categorized as follows:

\[ S \leftrightarrow S \] (Student with Student Interactions)
\[ S \leftrightarrow M \] (Student with Mathematica™ Interactions)
\[ S \leftrightarrow T \] (Student with Teacher Interactions)

The categories of interaction are illustrated in Figure 2.

![Diagram of interaction categories]

**Figure 2.** Categories of interaction recorded during the study

The data collected from classroom observations was combined to arrive at percentages in each category of observation. These percentages are presented in Table 1 and Figure 3.
Table 1

Percentages in Group Observation Categories Over Seven Weeks

<table>
<thead>
<tr>
<th>Week</th>
<th>S&lt;--&gt;S</th>
<th>S&lt;--&gt;M</th>
<th>S&lt;--&gt;T</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.76%</td>
<td>39.02%</td>
<td>21.95%</td>
<td>29.27%</td>
</tr>
<tr>
<td>2</td>
<td>15.79%</td>
<td>56.14%</td>
<td>27.19%</td>
<td>10.53%</td>
</tr>
<tr>
<td>3</td>
<td>16.46%</td>
<td>56.96%</td>
<td>6.33%</td>
<td>20.25%</td>
</tr>
<tr>
<td>4</td>
<td>15.74%</td>
<td>64.81%</td>
<td>2.78%</td>
<td>16.67%</td>
</tr>
<tr>
<td>5</td>
<td>28.69%</td>
<td>50.82%</td>
<td>10.66%</td>
<td>13.11%</td>
</tr>
<tr>
<td>6</td>
<td>21.53%</td>
<td>62.50%</td>
<td>9.72%</td>
<td>6.25%</td>
</tr>
<tr>
<td>7</td>
<td>22.92%</td>
<td>56.25%</td>
<td>6.25%</td>
<td>14.58%</td>
</tr>
</tbody>
</table>
Figure 3. Bar graph of group observations over seven weeks

Kendall's Tau was used to determine if there was significant increase over time in student-student interactions and student-technology interactions. Strong evidence exists that student-student interactions increased over time (p-value .035). This was not found with student-Mathematica™ interactions (p-value .281).

As a result of the pilot study, agreement was reached to conduct a study involving observations, a questionnaire, and individual interviews with students. The focus of the planned study was selected to be the development of the problem solving skills of the students taught using Mathematica™.

Chronology of Events

The formal study took place during the Fall and Winter quarters of the academic year 1990–1991. The study was conducted with classroom observation, interviews, examination of student work on paper and at the computer, and a student questionnaire involving background and attitudes. Triangulation of
methods, through the use of observations, examination of written work, and interviews was used to establish the trustworthiness of the results. The basis for the design of the study was taken from the Teaching Experiment devised by Steffe (cited in Skemp, 1987). The purpose of the Teaching Experiment is to make and test hypotheses about the nature of a student's thinking at particular times and about how this thinking develops. Steffe summarizes the Teaching Experiment by:

1. daily teaching of small groups of students
2. intensive observation of individual students as they engage in mathematical behavior
3. prolonged involvement with the same students over periods ranging from about six weeks to the academic year
4. clinical interviews with students
5. detailed records of observations through audio–video taping and written work (p. 138)

Modifications of this design were required due to limitations in this study. During this study, emphasis was on the types of the interactions, development of the concepts of calculus, and the problem solving strategies employed by the students.

During any academic year, the first quarter is from September until December, second quarter from January through March, and third quarter from March through May. A timetable of data collection activities is presented in Table 2.
Table 2

**Time Line of Data Collection**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pilot study</td>
<td>March–May (1990)</td>
</tr>
<tr>
<td>Observation in the classroom and recording of observations</td>
<td>September–November (1990)</td>
</tr>
<tr>
<td>Request for volunteers to be interviewed</td>
<td>October (1990)</td>
</tr>
<tr>
<td>Interview 1</td>
<td>November (1990)</td>
</tr>
<tr>
<td>Interview 2</td>
<td>January (1991)</td>
</tr>
<tr>
<td>Interview 3</td>
<td>January–February (1991)</td>
</tr>
<tr>
<td>Interview 4 and questionnaire</td>
<td>February–March (1991)</td>
</tr>
</tbody>
</table>

The first interview was scheduled near the end of the first quarter of the calculus sequence. At the beginning of the second quarter, a schedule for the remaining interviews was established. They were placed at intervals of ten days to two weeks apart. The interviews were to focus on solving problems and getting the participants to verbalize what they were thinking as they approached each problem situation. Initially, the researcher planned to alternate between interviews using paper–and–pencil approaches and interviews with the computer algebra system Mathematica™ available. This plan was adjusted after the first interview, which was done only with paper–and–pencil, and the subsequent interviews were
conducted with Mathematica™ as well as paper-and-pencil available. The researcher was able to observe when a student elected to use each in problem solving situations.

Interview Questions

In all of the interviews, students were presented with problems and asked to solve and discuss the problems. The first interview was planned to allow each participant a chance to talk about the course and the approach as well as some problems. The purpose of the conversation was twofold. It allowed the students to become comfortable with the interviewer/researcher and enabled information about attitudes to be collected before the interviews as well as after for comparison. All of the subsequent interviews were centered around problems and solutions.

Since most of the first interview was discussion of attitudes toward the course, the problems used were selected from an earlier quiz in the course. They were:

1. Suppose \( f(x) = x - \sin x \)
   
   What is \( f'(x) \)?
   
   Where is \( f(x) \) decreasing?

2. Given \( f(x) = xe^{-x} \), examine the derivative to determine the following.

   The maximum value that \( f(x) \) can have
   
   The intervals in which \( f(x) \) is increasing
   
   A sketch of the graph (include global behavior)

Students were asked to answer the above questions without the use of Mathematica™ in the interview.
The second interview consisted of the following questions:

1. Differentiate \( g(x) = \log(x) \cos(x) \). What can you say about the graph of this function?

2. Differentiate \( f(x) = xe^{-x} \). What can you say about the graph of this function? (The use of the same function here as in interview 1 was intentional to observe how the students approached the problem when Mathematica™ was available.)

3. \( f(x) \) is negative for \( 3 \leq x \leq 5 \), which is larger, \( f(3) \) or \( f(5) \)?

Mathematica™, paper–and–pencil, and graph paper were available at this interview and all subsequent interviews. Each was used at the discretion of the student.

The third interview consisted of the following question and problems:

1. What is an integral?

2. Find the area of the region bounded by the curve \( y = x^2 \), the \( x \)-axis, and the line \( x = 3 \).

3. A particle is moving on a straight line; \( s \) is the number of feet in the directed distance of the particle from the origin at \( t \) seconds of time, \( v \) is the number of \( \text{ft/sec} \) in the velocity of the particle at \( t \) seconds, and \( a \) is the number of \( \text{ft/sec}^2 \) in the acceleration of the particle at \( t \) seconds. If \( a = 2t - 1 \) and \( v = 3 \), and \( s = 4 \) when \( t = 1 \), express \( v \) and \( s \) as functions of \( t \).

4. A particle is moving along the \( x \)-axis under the action of a force of \( f(x) \) pounds when the particle is \( x \) feet from the origin. If \( f(x) = x^2 + 4 \), find the work done as the particle moves from the point where \( x = 2 \) to the point where \( x = 4 \).
5. A trough having a trapezoidal cross section is full of water. If the trapezoid
is 3 ft wide at the top, 2 ft wide at the bottom, and 2 ft deep, find the total
force owing to water pressure on one end of the trough.

The fourth interview was comprised of the following questions:

1. What is a derivative?

2. Evaluate \[ \int_{-1}^{1} \frac{1}{x^2} \, dx \]

3. Estimate the value of \[ \int_{1}^{\infty} \frac{1}{x^3 + x + 1} \, dx \]

4. A spring has a natural length of 14 in. If a force of 5 lb is required to keep
the spring stretched 2 in, how much work is done in stretching the spring
from its natural length to a length of 18 in?

5. Suppose a gas is pumped into a spherical balloon at a constant rate of 50
cubic centimeters per second. Assume that the gas pressure remains
constant and that the balloon always has a spherical shape. How fast is the
radius of the balloon increasing when the radius is 5 centimeters?

6. Find values of a and b so that the line \(2x + 3y = a\) is tangent to the graph of
\(f(x) = bx^2\) at the point where \(x = 3\).
7. The figure below shows the graphs of two functions, one of which is the derivative of the other. The vertical scales may be different, but the horizontal scales are the same. Which is which? Explain how you know.

![Graph of two functions](image)

8. Sketch the graph of a function \( f \) such that \( f' = g \), where the graph of \( g \) is shown below.

(a)
At the end of the fourth interview, a written questionnaire, shown in Appendix 1, was administered to obtain information similar to that recorded verbally in the first interview involving attitude toward the course. Additional background information was also obtained from this questionnaire.
CHAPTER IV
SUMMARY AND COMPARISON OF INTERVIEWS

This chapter contains summaries of each set of interviews, comparisons across interviews, results from a written questionnaire administered after the last interview, and data on classroom interaction collected through observation. Within interview summaries are presented first and changes demonstrated across interviews are described second. Statements made about courses numbered 151, 152, 153, and 254 are in reference to a traditional lecture sequence in beginning calculus while statements about 151M, 152M, 153M, and 254M pertain to the calculus with Mathematica™ sequence. Complete transcriptions of all interviews are in Appendix B. Excerpts from the interviews are referenced with their page numbers from the appendix.

Summaries Within Sets of Interviews

Interview One

The first interview consisted primarily of discussion about the course. Students were given an opportunity to respond openly on their views of the class, notebooks, and Mathematica™. The researcher prompted the participants with questions to ensure responses concerning both positive and negative aspects of their experience. This interview was scheduled during the ninth and tenth weeks of first quarter calculus. The only mathematics questions presented during the first interview involved the computation of a derivative and the use of the derivative to
sketch the graph of the function. The two functions used for these questions were 
\[ f(x) = x - \sin x \] and 
\[ g(x) = xe^{-x}. \] A computer was not made available for this interview.

All of the students expressed or exhibited discomfort with the use of paper and 
pencil alone. This often led to explanations of how they would approach the 
problem, rather than the actual computations being done. Only two of the nine 
participants correctly evaluated the derivative with pencil and paper. Three 
incorrectly treated or attempted to treat a problem involving the product and chain 
rules as a sum and compute derivatives on each part. Two correctly identified the 
problems as product and chain rule, but were unable to calculate the derivative 
correctly. The others neither identified their approach nor calculated accurately.

When questioned about the use of the derivative to sketch the graph of the 
function, responses again centered around what they would do if they had the aid of 
the computer rather than actual pencil and paper work. Seven of the nine, at 
varying levels of explanation and understanding, accurately associated maxima and 
minima with points where the derivative was zero and correctly described methods 
for determining where the function was increasing and decreasing using the first 
derivative. Excerpts from those responses are given below and illustrate the varied 
levels of understanding and expertise:

Participant H:

\[ S_H: \] The first derivative is the slope so if the slope is negative then you know 
that the function F of X is going down and if it's positive then you know 
it's increasing. (p. 150)

The above description was rather brief, but accurate. The participant was making 
appropriate connections between the derivative and the graphs of functions.
Recalling that this interview was near the end of first quarter calculus lends credibilidad to the worth of this response.

Participant A:

SA: So what we can do is we can take the derivative assuming that it's right or try set it equal to zero. Zero we could find out... it could be a maximum and a minimum. It could be either one.
I: How would you decide?
SA: If I'm on the computer it takes care of that for me. As far as paper I'm not really sure. I'd probably go through and try and get a rough outline of the function if I could and see where it falls into on that line.
I: What about that next part? Intervals where that function might be increasing?
SA: I vaguely remember doing this. It had to do with dividing it up into - with the x value. As you're approaching - on second thought I wonder if we could take the second derivative and that would tell us where it's increasing but I'm not exactly sure when and where. I know if you take the second derivative you can look at it that way.
I: Can you do that at all with the first derivative?
SA: Yeah. You have to - it's like kind of a coordinate system - kind of like this. Here is x represented by this line. And I believe you (inaudible) the first derivative and by going through and figuring out - I'm not sure if it's the limit or whatever you can figure out what it's doing and by dividing it this way you know this is where the point - whether your max or your min is you know that it is increasing on this side and decreasing on this side you've found the max. (pp. 111-112)

Participant A gave more detail about the techniques used for curve sketching. The mention of maximum and minimum points as well as the second derivative should be noted. This participant verbalized some discomfort working without the computer.

Participant B:

I: All right. What else can you find by using the first derivative besides places where the graph is decreasing.
SB: Places where the graph it hits maximum and minimum points so the function would hit it's maximum at - the first derivative shows where the original function will hit it's maximum and minimum.
I: What about the first derivative tells you those places? What parts of the first derivative would tell you where those places lie?
SB: I can't remember. I just know they do. (pp. 117-118)
Participant B demonstrated a knowledge of some of the aspects of derivatives associated with graphing, but was unable to discuss how to carry out the processes involved. The ideas were not as well formed as with some of the other participants.

Participant C:

SC: When you find that first derivative of any function and you set to zero usually you can find the maximum or your minimum and then what we learned in the computer is when you find the intervals it can be where it's like increasing then you know the graph has to follow that and also you can find the first derivative usually as your range. You know that the graph has to go from there to there somewhat. We also learned second derivatives too which can determine concavity. It is going to go up or down and if the second derivative is positive usually that means F of X would be increasing. And if the second derivative is negative then F of X is decreasing so you can figure out from that too.

I: You said that when you take the first derivative and set that equal to zero you can find maximums and minimums. How do you know what you have is a maximum or a minimum?

SC: Hm - because I was taught usually if it's like less than zero - like if you get one thing usually it's a maximum but if you get two things you always look at the last one as a minimum and the greater one as a maximum. That's how I was taught. (p. 122)

The ideas of finding maximum and minimum points on a graph are known to Participant C, but specific details concerning them are missing. The "greater one is the maximum" comment may have been an attempt to make sense of a concept that is not yet clear.

Participant D:

SD: A graph of the derivative. Wherever it equals zero will tell you like maximum and minimums and wherever it's positive it will tell you where it's increasing; wherever it's negative it will tell you where it's decreasing.

I: If you don't have the graph can you do that algebraically?

SD: I think you can but I don't remember how to do it.

I: Okay. What else could you tell me? Is there anything else you could tell me about the graph of the original function with the derivatives?

SD: Just that whenever the derivative is negative then it is decreasing and whenever it's positive it is increasing and zero is always the maximum.

I: How do you decide if it's a maximum or if it's a minimum?
SD: Usually just look at the graph and just like if it looks like this you know that this would be the minimum and this would be the maximum.
I: Look at the graph of the function?
SD: Right. By looking at the derivatives you probably get something like this. This is all positive. You'd get the minimum. It would hit zero right here at the minimum and then this is also positive so it's still keeping the same. Somehow it is going to hit zero again.
I: Okay.
SD: And it starts to decrease. That's where it is. And then you know that these two are where it's maximum and by looking at this you can tell which is which. (p. 127)

The concept of derivative and its implications for graphing are stronger for

Participant D. Still, a lack of knowledge on how to proceed with the process

algebraically is missing.

Participant E:

I: If you have the derivative then how would that help you with that other part? How can you tell where the function that you started with is decreasing if you've got the derivative?
SE: I'd plug in things for x. If you can see where the - if you have the derivative. If it's cos x then I know that it's increasing when it's positive; decreasing when it's negative. And we have a max and mins here.
I: Okay. But you kind of knew what that graph - you knew what the graph of cosine looked like. So you're saying that where it's increasing and decreasing is where the function is going to be increasing and decreasing?
SE: Yes. I'm saying where the derivative is positive then it's increasing.
I: The function.
SE: The function is increasing. F of X is increasing and where it's negative F of X is decreasing.
I: Okay. So that would help you be able to sketch this graph x minus sin x.
SE: Kind of sketch it, yeah. So I know the kind of - I'm guessing it would be out here somewhere.
I: Increasing and then -
SE: Decreasing from this point and then increasing from this point.
I: You're making little faces that look like maxes and mins. Can you go a little further than this session asks you to? What would you do if you wanted to find the maximum points or minimum points of this function x minus sine x?
SE: Find out where the derivative has zeros. I know that it's going to be a max or a min and then I would just plug things in for the function. Plug things in for x and find out where it's actually going up and down so I know if it's going to be a max or a min. (pp. 132-133)
The understanding demonstrated by Participant E is one of the better ones verbalized during this first interview. The connections between the derivative and the graph of a function are stated clearly as well as techniques for working problems.

Participant G:

I: So just in general to sort of summarize with the first derivative what kinds of things can you tell about the graph?
SG: If the slope is increasing or decreasing like with wings. Like before this point it's increasing because it's positive and after this point the slope is negative so it's decreasing.
I: So you can tell if the function is increasing or decreasing using the first derivative.
SG: Then you can also find the zero points of the first derivative when you solve for it. You can find one where the function crosses the zero.
(p. 145)

Participant G also appears to have a partial picture of the concept of derivative as it relates to graphs. The idea that the zero points of the derivative are also the zero points of the function was an indication of a less than accurate concept at this time.

It appeared at this point in the calculus course that the students were developing a sense of the concept of derivative, but were not developing pencil and paper computational skills. The concept of derivative and its relationship to graphs of functions was forming for all participants, at varying levels of accuracy in understanding.

A major part of the first interview involved discussion of the pros and cons of the course. Similarities among the responses from the participants were surprising. Many of the comments involve suggestions for improving the course. The amount of time spent on the course, in the lab, and the lack of a strong feeling of how much they were learning, whether it was similar to a more traditional class, were two of
the most often mentioned negative aspects of the course. Comments included the following:

Participant H:

I think I'm spending too much time and not learning a lot. . . . if I was taking calculus for the first time, I wouldn't know how much I'm learning. . . . you don't have time to go back and look at your homework and see what you did wrong. . . . See, I don't really know how much material we're covering. We may be covering a lot. But using Mathematica™, it's hard to tell. (p. 152)

Participant F:

I spend twice as much time in Mathematica™ as I spend in all my other classes put together. . . . how can you have a life and still take Mathematica™? (p. 141)

Participant C:

I was afraid if I switched to regular 152 I'd be totally lost. . . . I wouldn't be used to doing graphing, everything by hand. (p. 125)

Participant G:

I look at people that are in regular 151 and they're a lot further along. . . . We don't know if we did it [homework] right or not. (pp. 146-147)

Participant B:

I don't realize homework, the grading and stuff. . . . I really don't catch what they're doing grade policy-wise. . . . I don't think I'm getting too bad a grade. . . . they say eventually you do catch up to where everyone [in 151] else is. (p. 120)

Participant A:

I wish I was doing better. I'm not really sure what I'm getting in the class. (p. 115)

The participants were all uneasy about their status as far as grades were concerned. Some of this uneasiness could have been due to the lack of written feedback since all homework was submitted and graded electronically. The expected
paper-and-pencil reinforcement of correct problem solutions to measure progress through the course was absent. None of these students had been in a completely electronic learning situation before.

The most often mentioned positive aspects of the course included the small class size, ability to work with other students in class, individual attention of the teaching assistants and professor, relaxed learning environment, and flexibility in choosing work time. These are illustrated by the following excerpts from the interviews:

Participant F:

... we kind of share information and insights into our problems. I think that's a definite plus too. (p. 143)

Participant C:

... the course is more close between students because like in a regular 151 class you have like 300 people in your lecture and like 30 people and you never talk to these people and we are like so close (p. 123)

Participant D:

... when you see everybody else is struggling just as hard and then you get working together and you get motivated like come on, let's get this done. Let's figure out how to do it. It's better to work with a group. (p. 129)

Participant A:

It was a smaller class and Dr. Davis is very friendly and the TA's are very friendly. It just seemed more like people cared for you than the regular 151 class. ... I'm impressed with the TA's and Dr. Davis. They have a lot of patience. (p. 113)

Participant E:

... there's about 5 or 6 of us that work together... It helps a lot because when you're just one person your mind may be going on one track and you can't get out of it even though you know you're wrong, but another person might enlighten you and then together you can push it out. (pp. 134-135)
Participant B:

You get to work at a nice, not slow, pace, but you're learning a lot but it's still you can work on your problems any time you want to. You're not forced to go over there [the lab] every morning at 9 in the morning. You can go over any time you want and work... you get personalized help because it's such a small group of people. (p. 119)

The lab setting of this course forced a small class size. The professor and teaching assistants encouraged working in groups and discussion of problems. A stringent time schedule for the class was not enforced and students could decide when to spend time in the lab to do their assignments.

Several suggestions were made for improving the class. The suggestions mentioned by more than one participant included: (a) Use more teaching assistants or make help available more hours per week, particularly in the evening; (b) make answers to problems available in written or electronic form and allow time to reflect on problems missed and discuss the correct approaches; and (c) incorporate at least one day per week in a lecture format to introduce new material and discuss problems.

Interview Two

The second interview was scheduled during the first and second weeks of the second quarter of the calculus sequence. The questions for the second interview were focused on differentiation even though the participants had started the study of integral calculus. The two reasons for this were to observe students attempting similar problems as in the first interview, but with the aid of the computer, and to attempt to discover whether a better understanding of the concept of derivative had developed.
Participants were asked to explore the function \( g(x) = \log(x)\cos(x) \) using the computer, derivative, or any other tools they chose. All but one of the nine students graphed the function immediately. All then had Mathematica™ compute and graph the derivative. They proceeded to discuss, accurately, the connections between the graph of the function and the graph of the derivative. The graphing capabilities of Mathematica™ led the students to graph everything they came in contact with in a problem situation. With the power of the Mathematica™ tool, students do not attempt to predict the graph of a function using the derivative, but, instead, to confirm that the graph and derivative are related in the ways they had learned.

An interesting aspect of this problem was the choice of a range for graphing the function. Mathematica™ would ignore negative values requested for the range to a certain point and graph the positive values along with a warning message about the ignored values. The researcher started to take note of how many students seemed to arbitrarily choose a range versus how many thought through the problem ahead of time and selected a more appropriate range. Six of the nine participants reasoned through first and selected positive values for the range. When asked about this, they explained that it was due to the \( \log(x) \). The remaining three seemed to rely on the computer without question and would change the range when an error message appeared, but the changes were arbitrary even then. More than one arbitrary change was made by these three to adjust to an appropriate range for the graph and no reason for the difficulty was offered. The process was trial and error.

For the second question in this interview, participants were asked to explore the function \( f(x) = xe^{-x} \) in a similar fashion. The researcher asked that they look at
the derivative first and predict the graph of the function. Only one student was unable to do this. In that one case, the student stated that the function behaved exactly as the derivative. The following is an excerpt from that interview:

Participant C:

I: Let me have you look at another one. Let’s sort of take a different attack since we’ve talked about some of these things. Let’s look at \( f(x) = xe^{-x} \).

SC: Okay. Oh negative X. Okay.

I: Instead of looking at the graph of the function right at the start -

SC: Do you want me to get the derivative?

I: Let’s look at the graph of the derivative and see if you can make any predictions about what the graph of the function might look like....

SC: If (inaudible) negative X. There is the derivative.

I: Now if you had just that graph of the derivative and you wanted to make some predictions about what the graph of the function would look like what would you predict?

SC: Right here where it crosses I’d say - 5 - I’d say that would be a (inaudible) and minimum and right here where it’s like (inaudible) I think the real function will cross at 2. Here it’s decreasing so the function would probably be decreasing or negative and here it is increasing so I think the function would be positive. And it’s almost touching E so it would probably be the max number 8, the real function.

I: So you think there will be -

SC: Minimum at one point (inaudible). It will cross at 2. Here it’s decreasing, increasing and then maybe a maximum of 8 because it’s not really crossing there. Should I plot it?

I: Yeah. Check and see.

SC: I said it would be decreasing but it’s not. It’s increasing.

I: Okay.

SC: At 1.5 I said maximum is a minimum. At .8 still the same thing. It’s almost the opposite. When this is increasing - no, when this is negative then this would be decreasing. So like this 1.5 to 8 the real function would be decreasing. (p. 168)

Participant C seemed to be at the lowest point of understanding of the group at this stage. It appeared as though some of the ideas were forming, but appropriate connections had not been made between concepts. The ability to work problems was demonstrated, but the understanding was not present. The problem \( f(x) = xe^{-x} \) was one of the problems in the first interview. During the first interview, students
were asked to explore this problem by hand. All expressed an inability to actually carry out the computations even though some of them could verbalize the steps that needed to be taken.

The last question of the second interview was intended to be more conceptual. Participants were asked the following:

\[ f'(x) \text{ is negative for } 3 \leq x \leq 5, \] which is larger, \[ f(3) \] or \[ f(5)? \]

All nine of the participants correctly answered the question. Eight of the nine gave an accurate explanation for their answer. One participant needed to look at an example of the situation on the computer before answering and stated that it would depend on the given situation.

**Interview 3**

Interview three was scheduled during the third and fourth weeks of the second quarter of the calculus sequence. The first task for each participant in the third interview was to describe an integral. The participants were approximately four weeks into the study of integral calculus at this point. The explanations included the following:

Participant H:

It's the sum of area. It's a way to find area of a given function. If you're given a function and you want to find the area and if it's not just simple, easy you can integrate and find the area under the curve or under whatever, or a volume, anything really. (p. 224)

Participant F:

An integral is the area under a curve. . . . Uh - well the area under a curve. I guess between the X axis and the curve I should really say. (p. 213)
Participant G:

An integral? You want to know what integration is? ... It's like the area between - like an integral between like A and B. So it's like the area between A and B on the function. ... Integration is the opposite of derivative. (p. 218)

Participant J:

That's funny that you asked. Just last night I was discussing the term integral with a 254 student and my definition of an integral is simply a sum of differentially small areas so I guess you could either say it's the area under a curve or like I said I like to think of it as the sum of the differentially small areas because it is a sum and that's the way we think of it in physics, as a sum. Not just any area. It's a sum of very small particles. ... Yeah. I was saying in my opinion - I look at it as a sum of the very small areas under the curve and he agreed with me but he wanted to stress the fact that it wasn't a summation. And I said no, no I didn't mean that it was simply a summation. It's just you add up all of the small areas. That would be what I would call an integral. (p. 228)

Participant E:

An integral. From what I understand of it now it's the space underneath the curve. (p. 209)

Participant D:

It's - I don't know the exact definition of it. I can tell you what you'd do in it. When you evaluate an integral it would give you - it's usually a derivative of a function and they want you to integrate it from a certain point to another point which means you have to integrate it up to the original function and then plug in those two numbers and subtract and the answer is the difference. ... Area of a function from a certain point to another point. (p. 205)

Participant C:

An integral is a derivative and you have to find the function that the derivative came from. And they give you like a certain range and stuff to plug into the derivative in order to find the function that it came from. (p. 201)

Participant B:

An integral - okay - it's something that has a function inside of it and it measures the area under the function from a point A to B. ... It measures from A to B. The area. (p. 198)
Participant A:

An integral means to me that you're going to measure the area under a function, between the function and the X axis so we have F of X running somewhere and we take - the integral is going to measure the area between F of X and the X axis and it could be positive or negative or it could be both and cancel each other out, whatever the total area is. (p. 195)

Eight of the above descriptions included mention of area. It should be noted that memorizing formal definitions was not part of the course. Three of the participants above attempted to describe integral as somehow involving intervals. Some understandings of the concept of integral are developing, but seemed to be neither complete nor totally accurate at this point. This may explain the lack of success on some subsequent problems in this interview.

The first problem was a strictly computational problem to find the area bounded by the curve $y = x^2$, the x-axis, and the line $x = 3$. Eight of nine participants chose to graph the problem before attempting to find the area. The attempts to graph led to an interesting result. None of the participants understood why the plotting command on Mathematica™, which is a function plotter, would not plot the vertical line $x = 3$. Even when questioned by the interviewer, they still did not realize that this was not a function. Various attempts led to graphing the horizontal line $y = 3$ or deciding, sometimes on the suggestion of the interviewer, to proceed without the graph of the vertical line since they knew where it would have been. There is a way to draw vertical lines using Mathematica™, but none seemed to be aware of this and none could explain why the plotting command would not work. Eight of the nine eventually arrived at a correct answer and five of them checked their work by using a combination of the computer and paper and pencil. When asked how the computer found its answer, they mentioned adding up the
areas of either triangles, rectangles, or trapezoids. Three mentioned the Fundamental Theorem of Calculus in their explanations.

On a question involving the distance, velocity, and acceleration of a particle, none arrived at a complete, correct solution. The interviewer questioned them about any relationship they might know between expressions for distance, velocity, and acceleration. Four could correctly identify the relationships in terms of derivatives or integrals, two gave partially correct answers, and three could give no relationship. Some attempts were made to use the algebraic relationship of \( d = rt \).

The interviewer recognized growing discomfort on the part of the participants during this interview. Perhaps it was due to the approaching midterm. For this reason, not all participants were asked to complete the last two questions involving force on a particle and force of water pressure. Three participants were on the verge of leaving the interview, so only six attempted the water pressure problem. Four of them manipulated the quantities given algebraically and ended up with no answer, while the other two were able to arrive at partial responses. These two were unable to complete the problem due to what they considered a lack of information. By this point, almost all of the participants seemed overly anxious and disturbed. The interviewer, due to similarities in the problems, chose not to pressure students to answer the problem involving force on a particle. Only one correct response was given and one partial response.

After the last of this round of interviews, the researcher felt, due to student comments, that the nature of the questions was more difficult than anticipated and the pressure of an approaching midterm was on the mind of each participant. Several unfortunate incidents involving missed problems and grading took place in
the class around this time and the students were alarmed by the situation. The participants felt that problems included on weekly quizzes were unfair and different from the work expected in class. Due to these events and feeling, the unexpected problems in the interview that were not identical to problems from their homework brought these recent negative events back to mind. On reflection, the researcher felt that the decision not to pressure participants to respond was the correct one. One of the premises for this study was to be as unobtrusive as possible. To arouse the students over past incidents would not fit into this premise.

Interview 4

The fourth interview was the most lengthy. It was scheduled during the fifth and sixth weeks of the second quarter of the calculus sequence. An attempt was made to question the participants on both of the tools, the derivative and the integral, of calculus they had available. A short explanation by the interviewer at the beginning of the interview informed the participants that both differentiation and integration could be considered as tools for solving the problems. The first question in this interview was to describe what a derivative is. The responses included:

Participant H:

A derivative is the slope of the tangent line to the curve. Or its the velocity. If you have the distance you can find the velocity. Acceleration - you can take the integral you can find the velocity. (p. 263)

Participant F:

A derivative is a - hm - a derivative is a way to find the instantaneous growth rate of a function or a point of a line or it's the slope of a line at a certain point. (p. 252)
Participant G:

Opposite of integral. . . . First derivative defines the slope of the equation at whatever point. It's the opposite of an integral. (p. 258)

Participant J:

What is a derivative? Well I guess the first thing that comes to mind is it's the slope function meaning that if you have a function F of X and you take the derivative using the power rule, chain rule, product rule and so forth you end up with the function at F prime of X from which if you plug in a value of x you get the slope of the original function at that point. (p. 268)

Participant E:

A derivative is - if you're going from - if you're going from A to B let's say it would be your function at F of (A + B) minus F of A all over B as B is going to zero. . . . It's the average slope. (p. 248)

Participant D:

It's the slope of a function. (p. 244)

Participant C:

A derivative is - I forget. You take the function and - I forget what a derivative. I know what it is but it's hard to explain. You take the function and you just take the derivative of it. What is it? I don't know. I can't think - I just know what it is. (p. 241)

Participant B:

The derivative of a function is when you take the function whatever it is - say if it was x^2. Take x^2 and take the exponent and move it down to the coefficient and you subtract 1 to it so it would be 2x. And you would use that to determine zero points of the function. So wherever the function crosses zero usually that's maximum and minimum point. (p. 236)

Participant A:

A measure of the slope of a function. (p. 232)

Five of the above explanations involved the slope. That was stated as the "slope of the function" in all but one case. This may have been an inability to formally
verbalize the slope of the tangent or secant. The above explanations involve, in
almost all cases, some application of the derivative. No one gave a textbook
definition of derivative based on limits. This was expected since the limit definition
is not included and stressed as in more traditional calculus courses. The goal of this
course was to teach calculus as a set of tools for solving problems so the
explanation of derivative, a tool, would, of necessity, be expected to include
references to applications.

The next two problems were given because of the inability of Mathematica\textsuperscript{TM} to
produce results. In the case of the problem \[ \int_{-1}^{1} \frac{1}{x^2} \, dx \], Mathematica\textsuperscript{TM} will give
the answer -2 when asked to find a numerical solution and will indicate it fails to
reach accuracy. The answer -2 is typical from computer algebra systems due to
their inability to identify singularities of the integrand. Students will often obtain a
similar answer by hand when they simply substitute into the antiderivative. This
problem was an opportunity, in some cases, to observe whether the participants
ever questioned the accuracy of Mathematica\textsuperscript{TM}.

The responses were varied. Two participants obtained the correct answer
without question. One did not use the computer at all and the other used the
computer as a check and correctly explained the problem the computer was having
around zero. The other responses included agreeing with the computer on -2
without question as stated by one participant, the computer is the authority;
reasoning through a correct answer, but changing to -2 when the computer gave
that value: using the computer, failing to reach accuracy, and computing -2 by hand; and arriving at -2 both by hand and on the computer.

The second question proved to be troublesome on Mathematica™. Most students resorted to their own reasoning when they failed to get the computer to evaluate \( \int_{1}^{\infty} \frac{1}{x^3 + x + 1} \) dx. Most had difficulty even using large values in the place of infinity. Two participants were incorrect in their computer generated estimates. Five estimated the answer as close to zero since the function \( \frac{1}{x^3 + x + 1} \) approaches zero as \( x \) approaches infinity. Two estimated between one and two as the answer. One reasoned through the approximate antiderivative and the other viewed the graph on the computer.

A question involving force on a spring resulted in four incorrect responses that all involved attempts to set up ratios and proportions. Three participants were convinced that this was a physics problem that only required the appropriate physics formula to substitute the numbers into. They felt it did not involve calculus. Two participants gave partial solutions that were not entirely correct. Force problems had been encountered in class, but none like the spring problem. This could have been part of the difficulty.

Related rates problems as a group were not singled out and studied in the course, but the ideas were mixed in among other problems. The related rate problem involving the spherical balloon resulted in one response of "I don't know" and eight partial, but incomplete answers. One of the partial answers was considered, by the researcher, as weak since the participant stated that the problem
might involve calculus since there was a change in rate. The other seven responses
involved more detail and all mentioned the use of derivatives since there was a
change in radius occurring. No participants gave complete answers to the problem
even when the expression for the volume of a sphere was given upon request.

The next problem was used by Selden, Mason, and Selden (1989) in a study
of average calculus students ability to solve nonroutine problems. The participants
in the present study had been exposed to calculus in an environment where
nonroutine problems were the norm. Yet, four of the participants could give no
solution to the problem. Five gave partial solutions and two of those were carried
out after discussion with the interviewer. Unlike the results from Selden, Mason,
and Selden, the partial solutions all included some attempt to use calculus.

The last questions in the fourth interview were from presentations made at a
calculus poster session in Louisville, Kentucky at the January 1990 joint meetings
of AMS–MAA. They involve graphical representations and are taken from
presentations of projects involving calculus reform that attempt to create better
understanding of the concepts of calculus. Given two graphs and asked to identify
which was the function and which was the derivative proved trivial for all of the
participants in this study. All nine of the participants correctly identified the graphs
and gave accurate explanations, based on the concepts of calculus, of how they
determined which was which. The other exercise presented the participants with
graphs of derivatives of functions and asked them to sketch a graph that might
represent the function. While the participants were not 100% successful, the results
were impressive. The periodic function was answered correctly by all nine
participants. The middle graph, which should have led to a parabola, was
answered correctly by eight of the nine. The most difficulty was encountered on the first graph, that should have resulted in a linear graph with positive slope. Seven students accurately answered that question. It would seem, based on the graphing responses from the first interview, that somewhere between the fourth and eighth weeks of the second quarter, during the study of integral calculus, the concept of derivative had been solidified in the minds of all the participants.

Comparisons Across Interviews

The comparison of data across interviews could encompass any number of areas of interest. The aspects chosen for discussion by the researcher for this study were:

1. Development of the concept of derivative
2. Choice of computer or paper-and-pencil for solving problems
3. Skills and strategies employed for solving problems
4. Attitude concerning reliability of computer answers

Each of these will be discussed separately below. A participant representing each of the perceived levels of performance and understanding will be used to trace each aspect of interest.

Concept of Derivative

Questions and problems involving the derivative were included in all of the interviews. Questions the researcher considered conceptual, in particular, were found in interviews one, two, and four. Those questions will be the focus of this discussion. They include:

1. Explore the graph of \( f(x) = xe^{-x} \) using the derivative.
2. \( f'(x) \) is negative for \( 3 \leq x \leq 5 \), which is larger, \( f(3) \) or \( f(5) \)?
3. What is a derivative?

4. The fourth interview questions involving graphs.

The first question above was used in the first and second interviews since the participants were restricted to paper-and-pencil during the first interview. This allowed the opportunity to observe the handling of the problem with the computer and without.

Participant D, perceived to be high in performance and understanding, was able to correctly compute the first derivative of $f(x)=xe^{-x}$ by hand as well as describing the process as the multiplication rule. The process of finding the points where the first derivative was zero to locate maximums and minimums and then determine where the function was increasing and decreasing was explained, but when asked to do this algebraically, on paper, the participant responded, "I think you can, but I don't remember how to do it" (p. 127). When asked to look at the same problem using Mathematica™, Participant D graphed the function first, which was typical of all participants, and then the derivative. A discussion of the connections between the graph of the derivative and the graph of the function consisted of:

The pink one is regular function. The blue one is it's derivative. And the blue one is decreasing. The function is negative. It decreases all the way down to about 2.5 and - if the derivative is positive like it's above the X axis it's right here. That means the function is increasing and so it is from here to here so at that same point it is increasing. Then the derivative goes under the X axis it's negative and that means that the function will be decreasing and it is. And then it decreases all the way down to here and it starts going back up and that's exactly where the derivative is positive. If you follow it along the way every time that the derivative is above the X axis then the function is going to be increasing. When the derivative is below the X axis the function is going to be decreasing. Every time that the derivative crosses the X axis there is a slope of zero or maximum or minimum. It crosses right here and this is one of the
maximums. Crosses right here and it is one of the minimums every time it crosses. (p. 170)

Participant D appears to have a good grasp of the relationship. The connection of the idea of value of the derivative with slope is apparent. This same participant’s description of derivative was simply, "It's the slope of a function" (p. 244). While all nine of the participants in the interviews accurately answered question two from above, the discussion accompanying their responses varied. This particular participant gave an example of a quadratic function that fit the conditions of the problem and indicated that the function was figured out by integration. All nine of the participants performed well on the graphical items in interview four. This participant had no difficulty with the questions and even identified the graphs drawn, with the exception of the trigonometric graph. The trigonometric one was described as possibly from a third power equation with some uncertainty voiced.

Participant C, perceived to be performing and understanding at a middle level, correctly identified the product rule as a way to compute the derivative of \( f(x) = x e^{-x} \), but was inaccurate with the actual manipulation by hand. When questioned about finding maximums, this participant did set the derivative equal to zero and find a solution by hand. Since the derivative was not accurate, this did not result in an accurate solution, but the procedure was correct. The indication seemed that the places where the first derivative was zero were maximums. No mention of minimums was made at this time. Later, this summary was offered:

When you find that first derivative of any function and you set to zero usually you can find the maximum or your minimum and then what we learned in the computer is when you find the integrals it can be where it's like increasing then you know the graph has to follow that and also you can find the first derivative usually as your range. You know that the graph has to go from there to there somewhat. We also learned second derivatives too which can determine concavity. It is going to go up or down and if the second derivative is positive
usually that means f(x) would be increasing. And if the second derivative is negative then f(x) is decreasing so you can figure out from that too. (p. 122)

It can be seen that the ideas involving maximums and minimums both seem accurate. Participant C also mentioned the second derivative and concavity. With the computer as a tool, the student made predictions about the graph of the function by first graphing the derivative. The interviewer asked that the derivative be examined first. The predictions concerning the graph of the function were exactly opposite what was observed when the function was graphed. This realization made the student quite uncomfortable as evidenced by the comment about going that was made below:

At 1.5 I said maximum is a minimum. At .8 still the same thing. It's almost the opposite. When this is increasing - no, when this is negative then this would be decreasing. So like this 1.5 to 8 the real function would be decreasing. I need to go. (p. 168)

Participant C's explanation of derivative was:

A derivative is - I forget. You take the function and - I forget what a derivative. I know what it is but it's hard to explain. You take the function and you just take the derivative of it. What is it? I don't know. I can't think - I just know what it is. (p. 241)

The above comments might indicate that derivative is not a very clear concept even though the participant can often respond to questions in a seemingly correct fashion. The lack of ability to verbalize a definition of derivative may indicate a process, perhaps Piaget's assimilation, of construction of knowledge is taking place or a case of not knowing. This student correctly answered the second question from above and produced the sketch of a linear function for illustration. The graphical questions in the fourth interview presented a slight problem for this participant. A sketch of a function with constant positive slope was inaccurate and there was confusion attempting to name, but not sketch, the other graphs.
Perceived as low in performance and understanding, Participant F, when asked to approach the problem \( f(x) = xe^{-x} \) by hand, indicated that the chain rule should be used but was unable to calculate the derivative at all. An explanation of what should be done with the derivative to determine maximums was:

I'd set the derivative equal to zero and then find negative \( x \). If there is more than one we'd have to find out if it was a maximum or minimum once we got the derivatives. . . . I would have tried values on each side of the \( x \) to see what the curve was doing. At this point I'd probably take the second derivative of the function. And see if it was positive or negative to see if the curve was concave or convex. (p. 138)

This participant also mentioned the use of the second derivative. When asked about determining where the function was increasing and decreasing the response was:

If the derivative was both positive and negative. It was positive for increasing. . . . I'd probably check on both sides of maximum. (p. 138)

This does not appear to be a solid understanding of the concepts involved. This participant's definition of derivative was:

A derivative is a - hm - a derivative is a way to find the instantaneous growth rate of a function or a point of a line or it's the slope of a line at a certain point. (p. 252)

This description relies heavily on applications of the derivative. This should be expected since the use of the tools of calculus for solving problems was stressed in this course. Participant F was taking physics simultaneously with calculus and starting to see the connections between the two courses. The correct answer was given to problem two from above and the explanation of the answer was, "Well if the slope is negative, Yeah. As \( x \) increases the function decreases" (p. 178). This was offered without hesitation. Participant F offered correct answers to all of the graphical problems in interview four. No effort was made to name the functions sketched.
By tracing the development of the concept of derivative with one participant from each perceived level of performance and understanding, it is possible to discuss some similarities and differences. The participants presented above were representative of the entire group. It was interesting that the middle and low participants mentioned the second derivative when discussing graphing and the high category participant did not. Difficulty with the graphical representations of derivatives was experienced by the middle level participant only, while the perceived lower level participant was able to answer all of those questions. The middle level participant seemed to have the most difficulty of all with expressing a definition of derivative and the connections between the derivative and the graph of a function. This may indicate that the stronger and weaker students in the group gained larger benefit from the use of Mathematica™. The middle students may have been caught in a struggle between the software and the understanding of the mathematical concepts.

**Choice of Computer or Paper-and-pencil**

The first interview was planned to make use of only paper-and-pencil. The decision to make the computer and paper-and-pencil available at the remainder of the interviews was made after this. The discomfort observed and verbalized with only paper-and-pencil has been mentioned. That could have been due to the timing of the first interview. Even though it was scheduled near the end of the first quarter course, most of the students stated that they had started to feel comfortable using Mathematica™ after four and sometimes five weeks. Comfortable was a combination of knowing the commands, which was achieved sooner, and having
some feeling of what they were doing in terms of the mathematics. Soon after this, the interviewer asked them to put the computer aside and use paper—and—pencil.

The last three of the interviews were held in a room with a computer. Paper—and—pencil were always beside the computer and participants could choose to use either. It is interesting to note that two participants reached a point, during the third interview, where they asked the interviewer whether the problems should be done on the computer or with pencil—and—paper. The interviewer always informed them that it was their choice.

During the second interview, no students chose to use paper—and—pencil. Questions that did not lend themselves to working on the computer were verbalized. The third interview brought change in this behavior. All of the participants used pencil—and—paper to some degree during the interview. Some simply sketched out the parameters of the problems and turned to the computer, others attempted to work problems completely by hand and sometimes check with the computer. Some of this may have been due to their uncertainty with some of the problems. It seemed that exploring on paper was still more comfortable than exploring on the computer when the direction to pursue was not evident. By the fourth interview, the use of pencil—and—paper was regular. Participants even commented that it helped them solve problems if they outlined the steps on paper first. One student even insisted on working the problems with paper—and—pencil with only an occasional check on the computer. This was by choice and involved a belief that the paper—and—pencil work somehow guaranteed better understanding. It is remarkable that the student who chose to do this was also one of the students that expressed the most discomfort during the first interview when asked to use
pencil-and-paper. During that first interview, the following comments were
offered by the student:

If you give me this problem on the computer I don't have a problem with it. If
you give it to me on paper I've got a big problem with it. I don't think it's
because I'm stupid. I think it's just because I can't make the translation.
Because I look at a problem and I can come up with strategies to attack it even
though I'm not sure exactly what I'm doing all the time. I don't think that's
cheating. You're letting the computer do it for you simply because a lot of 151
students can sit there. They're given a formula. They can plug and chug it but
they still don't know the reasoning behind why they used that particular
format. It's the same type of thing. (p. 113)

Perhaps this feeling early in the course prompted this participant to strive to be able
to make the translations mentioned above. This participant was convinced that the
proof of understanding was shown on paper. This had not changed by the fourth
interview.

**Skills and Strategies for Solving Problems**

The questions used for comparison of problem solving skills and strategies
were as follows:

1. Interview one and two questions to explore graphs of functions with
   paper-and-pencil only and then with Mathematica™

2. Interview three question:

   A particle is moving on a straight line; s is the number of feet in the directed
distance of the particle from the origin at t seconds of time, v is the number
of ft/sec in the velocity of the particle at t seconds, and a is the number of
ft/sec² in the acceleration of the particle at t seconds. If a = 2t - 1 and v = 3,
and s = 4 when t = 1, express v and s as functions of t.
3. Interview four questions:

A spring has a natural length of 14 in. If a force of 5 lb is required to keep the spring stretched 2 in, how much work is done in stretching the spring from its natural length to a length of 18 in?

Suppose a gas is pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second. Assume that the gas pressure remains constant and that the balloon always has a spherical shape. How fast is the radius of the balloon increasing when the radius is 5 centimeters?

Find values of a and b so that the line $2x + 3y = a$ is tangent to the graph of $f(x) = bx^2$ at the point where $x = 3$.

A discussion of the specific skills and strategies employed by three participants representing each of the three levels of performance and understanding follows.

Participant A, from the highest level group, verbalized a definite lack of confidence in ability when asked to explore a graph with pencil-and-paper as illustrated by the comments, "Just forgive me if I'm not very good at this. I'll probably get it wrong. Math is not my favorite subject" (p. 111). When asked to verbalize what and why choosing to do the steps, the participant commented, "I'd like to hear myself, what I think when I do these" (p. 111). This may indicate an attempt by the participant to fit problems to an algorithm even though that was not promoted in the course. This seems to also indicate a lack of reflection on the problems. After the derivative of a function was incorrectly computed by hand, the participant said, "This is not what I was expecting" (p. 111). An apparent mental computation did not seem to match the procedure carried out on paper.
The same problem was asked in the next interview and the computer was available. Participant A used Mathematica™ to plot the function at the very start and commented that the graph was, "About what you expect" (p. 156). The subsequent computation and graph of the derivative did not produce much comment from the participant. The participant repeatedly mentioned that practical applications of the derivative were more sensible than the ones typically presented in class. This was unusual since most of the problems from the course would be considered practical applications. Specifically, Participant A referred to business applications of the derivative involving cost and demand functions. When asking Mathematica to graph a function involving a logarithm, the participant did not anticipate any problem with the graphing range and attempted a plot of negative values, the resulting Mathematica™ messages forced an adjustment of the range selected to plot. The student indicated that the problem was probably due to the logarithm, but this was a case of reacting rather than acting ahead of the difficulty.

During the third interview when asked the problem involving acceleration, velocity, and displacement, the participant proceeded to write all the given information on paper in an attempt to outline the problem. Several questions regarding the information followed and the participant seemed unsure of the question. When the interviewer asked about knowledge of the relationship between acceleration, velocity, and displacement, a response of "I don't know" (p. 196) resulted. After a hint, the simple integration of the given function was performed with no attention given to the possible use of a constant to ensure that the conditions given in the problem were met. Questions from the interviewer about this resulted in the following exchange:
I: Would that one have value 3 when t is 1?
SA: One way to find out. What did you say t was?
I: 1.
SA: What’s that equal to? 3?
I: Yes, and you want that to come out 3 when t is 1.
SA: All right. It didn’t.
I: Can you do something to it so that it will come out equal to 3? If you simplify that what you really have there is \(x^2 - t\).
SA: \(2t^2\) divided by 2 is 1.
I: Oh \(t^2 - 2\). Okay, so what can you do so that the answer would come out with 3? How can you modify that so that the answer would come out to be 3?
SA: I don’t know. (p. 197)

Participant A refused to engage in different strategies even when questioned. If the first response was not absolutely correct, no other attempt was made and not other strategy used.

The last two problems in the fourth interview followed the same pattern. The student would attempt to outline the information in the problem on paper and a description of what might be attempted followed. The problems were not worked. In both cases, an equation, that the participant could not come up with, was needed to work the problem and no attempt was made. This again points to an attempt on the part of the participant to force an algorithmic process. The comment "I try to do less on the computer nowadays" (p. 233) was made near the middle of the interview. The lack of willingness to use the computer seemed to detract from the experimentation to attack a problem. One attempt was made and, if unsuccessful, no other strategies were used.

Participant H was from the middle group in understanding and performance. When asked to explore the graph of a functions using pencil–and–paper only, this participant correctly computed the derivative, explained the process, and proceeded
to identify one possible maximum or minimum point by setting the derivative equal to zero. This participant also discussed information that could be obtained from the second derivative and correctly computed the second derivative on paper. The willingness and skill to perform computations with pencil—and—paper was demonstrated. A similar exploration using Mathematica™ was approached in an equally competent fashion. The participant looked at the graph of the derivative and predicted the graph of the function with accuracy. The participant also computed and graphed the second derivative to predict concavity. After graphing the first and second derivatives, the following explanation was offered:

Between - when x is between 1 and 2 or just after 2 the derivative is negative so the - between 1 and 2 the first derivative is positive indicating that it's increasing, the function is increasing. Then at just before 2 the first derivative is negative up 'til maybe just before 3 or 4 so it's decreasing. At 2 there would be a point of inflection or just after 2. Then when it crosses the axis again right before 4 the first derivative is 0 again. That means that since the first derivative is 0 it's a critical point here. That means it would be a maximum. It would switch then. It would go back - the function would then start going up since the first derivative is positive and then it would come just after 6 and it would cross again at 0 so it would be another point of inflection and the second derivative would be negative. It means it would be - the second derivative negative means concave down. It would be a maximum. I think it would be a maximum at just past 6 and then the derivative is negative again so it's going down. Yeah, that would be right. It's concave down again until it gets to - it's still going down. It decreases all the way to just before 10 but at 8 it switches concavity and the concavity is positive again. It means it is going up. It's concave up again. (p. 186)

The accuracy of the above comments was confirmed by graphing the function using Mathematica™. This participant demonstrated a willingness to explore more than the higher level participant. The second derivative was used as an additional tool to further explain or confirm the findings from the first derivative.

Participant H also acted rather than reacted regarding a function containing a logarithm. This participant recognized the difficulty with using negative values in
the function without waiting messages from Mathematica™. The explanation offered before plotting the derivative of the function was, "I'm plotting g'(x) from I'll say 1 because the derivative of zero of a log is going to go crazy" (p. 185). While this may not be eloquent, the point made is accurate and the participant considered the domain of the function prior to approaching the problem with the computer.

Participant H approached the questions of interviews three and four with a combination of paper-and-pencil and computer. In the case of the problem involving acceleration, velocity, and displacement, the participant knew the relationships between the quantities and chose to use paper to compute integrals for the following reason:

I'm integrating - I forget how to write that. It's an indefinite integral because I don't know t. What if I just write it on a paper and tell you? . . . I forget how to write it on the computer. I'm going to take the integral of the acceleration which is velocity and that's 2t - 1. (p. 225)

The participant had been focusing on the evaluation of definite integrals in class and could not recall how to use Mathematica™ to assist with indefinite integrals. No difficulty was encountered while working the problem with paper and the entire problem was completed. The appropriate constants were computed to fit the constraints in the problem. Part of the conversation leading to and involving the computation of the constant was:

My acceleration is wrong. There has to be a constant. I know there is a constant out here. This has to be plus C but this constant must be - let me see. This has to equal 3 so if T is 1, 1 minus zero. It would have to equal 3. The constant would have to be 3. (p. 225)

The participant realized that the acceleration function would not yield the value stated in the problem and had no difficulty recognizing the omission of a constant.
A definite difference between the performance of this participant and participant A was observed. Participant H was more willing to experiment and more likely to anticipate difficulties with a problem and work through those difficulties.

During the fourth interview, participant H employed proportions to attempt to solve the spring problem. When questioned about connections with calculus, the participant offered that there was some connection since, "I think the work is the integral of the force" (p. 264). The answer found using proportions was discarded and another attempt was made with the integral. An inappropriate function was used for the computation and a wrong answer resulted. Yet, this participant was thinking and still exploring. No evidence of refusal to explore or approach the problem with a different strategy was observed. Similar difficulty, involving an inappropriate formula was encountered with the balloon problem. Even though a correct solution was not achieved, the following illustrates the strategies the participant employed:

Pumping gas in at 50 centimeters cubed/second is going to increase the radius at some rate. When the gas is pouring in at this amount so if I could find - I want to find the rate of change of the radius since having the rate of 50 centimeters cubed/second so - I don't know how to do it. I know that it's logical that the radius is going to get bigger as the gas is pumped in since the pressure inside has to remain the same so if the pressure inside remains the same the only thing the balloon can do is expand so if it expands - if you're adding more to this volume so every second you're adding 50 centimeters cubed more of gas to this. So when the radius is 5 - that's the thing I don't know how to figure out how quick it changes. That's what you want to find though, \( \frac{dR}{dt} \), and then you could plug in - the radius is 5 and then you find out what the size of the balloon is there and then you can figure out what constant - it wouldn't be 50 centimeters cubed/second. Do you need to use circumference or volume or anything in there to figure it out? I don't know. I don't know the formulas to use. (p. 265)

The definite association between the derivative and rate of change is verbalized here and an understanding of the strategy that should be employed to solve the problem.
The tangent line problem was handled with paper by participant H and once again, even though no absolute solution was presented, strategies were explained that were insightful. Some of that discussion was:

I wanted to solve for either a or b. Since you know the slopes. You know one is a negative reciprocal of the other you can solve for y and then you know the slope of the first line. \( 2x + 3y = a \). The slope of that is negative \( \frac{2}{3} \). Slope y–intercept form or whatever. You get y equals \( -\frac{2}{3}x + \frac{a}{3} \) so you know the slope of that line is \( -\frac{2}{3} \). If it's tangent to the other curve, \( y = bx^2 \) then the slope of that one would be \( \frac{3}{2} \) and I don't know if b would be the slope of that or not. All I know is that is the formula for a parabola. It's \( bx^2 + 0x + 0 \). b times quantity x plus 0 squared plus zero so it's centered at \( (0, 0) \). The slope at 3 would be \( \frac{3}{2} \). It should. So x is \( \frac{3}{2} \). So - if I put in negative 2 - if I put in 3 into that... I'd just find the slopes and then try to set them equal to each other at the point equals 3 and then find out what a and b are from there but I know c is 3. I don't know y. I should be able to just solve for y and then tell you what s and b are. (p. 266)

Participant H recognized a relationship between the slopes even though it was not completely accurate. Again, the willingness to engage in discussion of strategies was evident and that was missing with participant A.

Participant E represents the low performing and understanding group for this discussion. Approaching the graphing problem with pencil–and–paper led to the participant's statement of how to decide what "rule" to apply when computing derivative. The participant came up with the following:

**SE:** I'm terrible at doing this. I don't know this - I know how to do it kind of but I'm really terrible at like the (inaudible) rule and the chain rule and all that stuff. I don't know when to apply them so basically I just look and see, \( \sin(x) \) - let's see.

**I:** When you look and you try to decide which one of those, what do you think about when you're trying to decide which one of those things to apply?

**SE:** Can I say of? Is it x of \( \sin(x) \)? Like if this is sin of x. And if it's of then I usually do the chain rule. But if it's not then I just attack it like it's a regular like \( x^2, ax^2 - bx, \) or + bx+c. (p. 132)
Participant E stated that this was a self-devised approach. This is an attempt to make sense of the different types of differentiation problems and illustrates some of the thinking employed by a student when extracting what is perceived, by the student to be pertinent information. When approaching a graphing problem with the computer, the student was able to predict using the first derivative. It is interesting that participant E also employed the second derivative to further explain the graph of the function. Participant E acted rather than reacted on the logarithm problem as well. The problem with negative values for the domain of the function was anticipated prior to entering the problem into the computer. The explanation offered was "Well you can't take the log of the negative so, and it was zero, so you'll get an infinity so I started with .1" (p. 173).

The acceleration, velocity, and displacement problem of interview three was approached with paper initially and checked with the computer. The units given in the problem caused some experimentation to recall the relationships among the quantities. Algebraic methods were used to determine the units. Since some evidence of knowledge of the relationships was found, confirmation from the interviewer allowed the participant to follow through and work part of the problem. Here, experimentation and an attempt to employ multiple methods was demonstrated.

Participant E also used proportions to approach the spring problem of interview four. Hints from the interviewer after questioning the connections to calculus did not lead to any further attempts on the problem. The sphere problem led to more algebraic manipulation in attempts to discover relationships between the quantities given. The attempts included looking at a cross section of the sphere, a
circle. This would seem to be the result of recent classwork on volumes of solids. More experimentation led to the decision that such attempts would not lead to how fast the rate was increasing and the statement, "Rate of increase would be the derivative" (p. 250). Similar experimentation with the quantities in the tangent line problem was performed on paper. Still, the recent work interfered as illustrated by this excerpt:

If I can separate my variables and integrate left and right and then solve left equals right I should be able to get values for y and x. If I get values for y and x then I can get values for a and b. So it will integrate this from whatever to whatever. From 2b = x - 2x when x equals 3. If x equals 3. Tangent to the graph. Or I could set up a parametric can't I? Or can I? Set up a parametric. Let's see, bx², No. I give. I don't know. I don't even know which direction to go. Tangent lines. Tangent lines. I don't know. (p. 250)

The experimentation and multiple strategies employed were of interest here rather than complete solutions.

As demonstrated by the above cases, the perceived upper performing and understanding participants were less likely to experiment and employ multiple strategies on problem. A solution was either evident and achievable with one strategy, or no solution was attempted. The middle and lower level participants were more willing to experiment and more likely to employ multiple strategies on problems.

This observation could be attributed to several factors. The confidence of the upper level participants may have been such that further experimentation was deemed unnecessary. Attempts to create algorithmic processes for categories of problems were seen as the result of years of schooling where such actions led to success in mathematics. Even though algorithms and categories of problems were
not emphasized in this course, such habits had been developed in previous courses. The middle and lower level participants made better use of a combination of pencil- and paper and computer. A more natural flow of thinking and planning problem strategies resulted. The upper level participants fell into extreme categories that used only paper-and-pencil or only computer. Discomfort with switching between the two often interfered with problem solution.

**Attitude Toward Computer Solutions**

This section focuses on participant's trust, often dependence, in the accuracy of the answers generated by Mathematica™. Two questions in the fourth interview are used to gauge this. They are:

1. Evaluate \[ \int_{-1}^{1} \frac{1}{x^2} \, dx \]

2. Estimate the value of \[ \int_{1}^{\infty} \frac{1}{x^3 + x + 1} \, dx \]

The first question results in an answer of \(-2\) when Mathematica™ is asked to find a numerical approximation to the integral and messages regarding an inability to converge on an answer otherwise. The second question requires some experimentation on the part of the students and led to demonstration of their trust, or lack of it, in Mathematica™.

The responses to the first question above were obtained in three ways. Two participants worked the problem only with pencil-and-paper, two only with the computer, and five used both. One of the two working the problem with paper-and-pencil arrived at an incorrect answer of \(-2\) by attempting to apply the power
rule to the problem and substituting. The other participant indicated the use of logarithms and arrived at an answer of 0. Both were confident in their solutions and chose not to attempt the problem with Mathematica™. The two participants using the computer only arrived at an answer of \(-2\) by asking for a numerical approximation of the answer. One indicated that some thought was given to the accuracy of the answer after the computer was finished, but saw no problem with the answer \(-2\). The conversation between the other participant using the computer and the interviewer was:

\begin{verbatim}
I: Okay. Does that sound like a reasonable answer to you?
S: Yeah. It sounds fine. The computer says it's right.
I: All right. Is it the authority here?
S: It is in this class. (p. 258)
\end{verbatim}

This was the most obvious referral to the computer as an authority that could not be wrong or questioned. The participants who chose to use both the computer and pencil--and--paper were divided in their reactions. Two participants arrived at the answer \(-2\) by hand and elected to go with that result. One of these two tried to integrate with the computer and failed to reach accuracy resulting in another attempt with the computer to find a numerical approximation that resulted in the answer \(-2\). Since this was in agreement with the paper--and--pencil computations, the answer was accepted and the failure to reach accuracy warning ignored. The other participant could not get an answer from the computer and was confused by the failure to reach accuracy. No attention was given to the problem or problems that might be causing the accuracy error on the computer. No explanation of the message was offered. One student using written and computer computations, pointed out the need to use logarithms, but indicated that \(-2\) would probably result
from that computation after getting −2 as an answer from the computer. The uncertainty of dealing with logarithms caused concessions to be made to the computer. Two of the participants using both means of computation decided on an answer of zero before using the computer. One of the explanations was, "Right off the bat I can tell you that the integral is going to equal zero because it's going from negative 1 to 1 and that's a closed interval and it's 1 on each side so the two areas the integral is going to cancel each other out so the integral is going to equal zero" (p. 236). This participant believed that any function would have integral zero if integrated from negative a to positive a where a was any number. The other explanation was valid and stated as, "I know already that it's going to be zero because it's an even function" (p. 252). This participant also indicated that the function "blows up" at zero and that explained the failure to reach accuracy on the computer.

The responses to this first question indicated a range of trust concerning the accuracy of computer generated answers. Some participants agreed with the computer no matter what the result; others questioned, but conceded to the computer if their answers did not agree with the computer generated ones; and some were at the point where they questioned and even checked the accuracy of the computer generated answer with some success at identifying problems.

The second question involving trust in the computer answers yielded quite different result. None of the students were successful in getting numerical approximations for the problem when trying to use larger and larger values in place of infinity. Eight of the nine attempted the problem with this approach though. One participant simply stated a loss of memory on how to use the computer when
dealing with infinity, while the other chose not to use the computer to calculate successively larger numbers, but to plot the function and observe from the graph that the estimate would seem to be between zero and one. No further estimation was used. Of the seven participants who attempted to use the computer for successive approximations, three estimated zero by reasoning that the fraction was going to zero and therefore the integral was approaching zero. Another participant estimated that the answer was one with a similar argument, and the remainder of the participants gave no estimate when the computer failed to offer an answer. As with the first question, the trust or dependence on computer generated answers ranged from a total trust or dependence to a questioning and reasoning through answers.

Observation Results

Observations of interactions between students and other students (S<->S), students and Mathematica™ (S<->M), and students and the teacher (S<->T) were recorded during the study as in the pilot or prior ethnography discussed in Chapter III. Over a period of six weeks during the middle and later part of the first quarter of the calculus experience, these observations were recorded during thirty minute intervals. It was anticipated that results similar to those from the pilot study, increased student–student and student–computer interactions would occur while decreased student–teacher interactions would be found. The percentage of each type of observation appear in Table 3 and Figure 4.
Table 3

Percentages in Group Observation Categories Over Six Weeks

<table>
<thead>
<tr>
<th>Week</th>
<th>S&lt;--&gt;S</th>
<th>S&lt;--&gt;M</th>
<th>S&lt;--&gt;T</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.24%</td>
<td>64.37%</td>
<td>9.31%</td>
<td>6.07%</td>
</tr>
<tr>
<td>2</td>
<td>5.14%</td>
<td>82.29%</td>
<td>9.71%</td>
<td>2.86%</td>
</tr>
<tr>
<td>3</td>
<td>8.79%</td>
<td>62.64%</td>
<td>20.88%</td>
<td>7.69%</td>
</tr>
<tr>
<td>4</td>
<td>6.49%</td>
<td>75.97%</td>
<td>9.74%</td>
<td>7.79%</td>
</tr>
<tr>
<td>5</td>
<td>19.77%</td>
<td>60.45%</td>
<td>16.38%</td>
<td>3.39%</td>
</tr>
<tr>
<td>6</td>
<td>6.20%</td>
<td>70.54%</td>
<td>10.08%</td>
<td>13.18%</td>
</tr>
</tbody>
</table>
Figure 4. Bar graph of group observations over six weeks

Kendall's Tau was used to determine if there was significant increase over time in student–student interactions and student–technology interactions. The findings were not statistically significant. This differs from the pilot results. The explanation may be due to the fact that the observation for the study occurred during first quarter calculus while the pilot observations took place during third quarter calculus. Further investigation into when such changes in interactions occur should be carried out. It seems as though a large part of the first quarter is spent becoming comfortable with the technology and the laboratory setting. This could prevent the increase in these interactions. The large percentage of student–technology interaction that was recorded during this first quarter, while expected, would seem to indicate attempts to adjust to the computer and the laboratory environment.
Results from Questionnaires

Information from the written questionnaires administered to participants at the end of the interview has been presented in previous chapters when it concerned the background and experiences of the group. Results from other questions, involving attitudes toward the course, are revealing and helpful in understanding how the students react. Two questions in particular are of interest in this area. They are:

What do you like best about these courses? (List at least three aspects.)

What do you like least about these courses? (List at least three aspects.)

The responses to these questions are tallied in Appendix A. The best liked aspects of the course are categorized and presented in Table 4 with the total number of responses in each category as a summary of the most prevalent responses.

Table 4

<table>
<thead>
<tr>
<th>Best Liked Aspects of the Course</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Best Liked Responses</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Response</td>
</tr>
<tr>
<td>Working problems</td>
</tr>
<tr>
<td>Working with other students</td>
</tr>
<tr>
<td>Using the computer</td>
</tr>
<tr>
<td>Seeing problems and graphing</td>
</tr>
<tr>
<td>Self-teaching</td>
</tr>
<tr>
<td>Flexible scheduling and work hours</td>
</tr>
</tbody>
</table>
It seems that working with applications, group activity, and the computer are the most popular aspects of the course. The graphing capabilities of the computer were singled out for mention separately from the general use of the computer almost as often. The students interviewed and observed, without exception, tended to use graphs to solve problems as often as possible.

The least liked aspects of the course are categorized similarly in Table 5 and presented as a summary of the responses.

Table 5

Least Liked Aspects of the Course

<table>
<thead>
<tr>
<th>Least Liked Responses</th>
<th>Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Response</td>
<td></td>
</tr>
<tr>
<td>Amount of time required for course</td>
<td>2</td>
</tr>
<tr>
<td>Lack of class lectures</td>
<td>6</td>
</tr>
<tr>
<td>Classwork on computer and quizzes on paper</td>
<td>4</td>
</tr>
<tr>
<td>Unsure of what I'm doing sometimes</td>
<td>2</td>
</tr>
<tr>
<td>Unsure of amount being learned</td>
<td>2</td>
</tr>
</tbody>
</table>

The time investment on the part of the students was estimated to be about 15 hours per week. While some seemed to believe this was equitable with the time they
would spend in a more traditional course, a few mentioned it as a negative aspect. In discussions, it seemed the students resented that the time had to be spent in the computer lab rather than the actual amount of time. The dislike of a completely laboratory class was mentioned by six of the nine participants. Some of this may be due to their background of lecture oriented mathematics classes. The most often mentioned suggestion for improving the course was to add at least one day a week of lecture where new material was introduced and problems were discussed with paper-and-pencil techniques emphasized. The perception of a large part of the group was that it was unfair to be taught and encouraged to use the computer with all homework and to be tested using paper-and-pencil. There was constant discussion of the problems associated with trying to "learn" the paper-and-pencil skills prior to weekly quizzes just for the sake of taking the quiz and never having the opportunity to be graded on performance using the computer. The feelings of uncertainty about what they were doing when typing commands into the computer was expressed, particularly at the beginning of new lessons when new computer commands were presented. Participants commented that they often just used the cut-and-paste option on the computer without really knowing why they were doing what they were doing. The feelings of uncertainty about their progress in the course and the amount of calculus they were learning decreased by the end of the interviews. Initially, these feelings were mentioned often in the interviews, but on the questionnaire, only two participants expressed this as a negative aspect.

On the written questionnaire, five participants indicated that they spent five days a week in the lab while all the others mentioned between four and six days. No one mentioned less. Seven students indicated spending between twelve and
fourteen hours per week in the lab. This is indicative of the amount of time invested on the part of the students.

The participant's views of their calculus experience were, for the most part, positive. Eight of the participants indicated they would recommend the course to someone else. The only student who hesitated on this stated that the amount of time and self-teaching involved would determine the recommendation.

Mixed results came from a question concerning whether Mathematica™ helped their understanding of calculus. Four participants replied yes, three yes and no, one maybe, and one no. The mixed responses involved uncertainty about how much was learned and what was known without Mathematica™. Of the affirmative responses, three specifically mentioned the graphing capabilities of Mathematica™ as helpful. Three also mentioned a lack of understanding the answers or how Mathematica™ arrived at the answers as points that proved not to be helpful.

When questioned about working with other students in the class, which was promoted by the instructor and teaching assistants, all nine participants responded that they viewed this as helpful in understanding calculus. The positive response to cooperative learning was not surprising. Though no structured working groups were assigned by the instructor, several groups did evolve and remained in tact throughout the study.

Five participants indicated they would be comfortable, to some degree, doing calculus without the aid of Mathematica™. Only three responded no to this question. One student failed to answer this question. It appeared that some of the early fears concerning paper-and-pencil work had diminished. The degree of comfort with calculus without Mathematica™ varied and some participants indicated
they would understand how to work a problem, but might not be able to perform
necessary computations.
CHAPTER V
SUMMARY AND SUGGESTIONS FOR FURTHER STUDY

Summary of Findings

The intent of this study was to examine the interactions, concept development, and problem solving skills of students learning calculus in a laboratory environment using the computer algebra system Mathematica™. The following findings are supported by the data collected during this study.

1. Nature of interactions:
   
a. Unanimous positive response to group or cooperative learning, which was encouraged but not structured by the teacher in this course, is consistent with the calls for change in methodology issued by the National Council of Teachers of Mathematics (Commission on Standards for School Mathematics, 1989; Commission on Teaching Standards for School Mathematics, 1991)

   Students' learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas. (Commission on Standards for School Mathematics, p. 58)

   We need to shift—toward classrooms as mathematical communities—away from classrooms as simply a collection of individuals. (Commission on Teaching Standards for School Mathematics, P. 3)
b. Students viewed the course as primarily self-taught. They viewed the notebooks and the laboratory experiences as teaching themselves instead of learning via the more traditional lecture method. While they perceived this as a negative, the researcher looked at it as positive in that the students took responsibility for their own learning. This led to a development of confidence in their willingness to approach and ability to work problems.

c. Even though statistical significance was not found regarding the increase of student–student (S<->S) and student–Mathematica™ (S<->M) interactions or the decrease in student–teacher (S<->T) interactions, valuable insight was gained regarding the quantity of time spent engaged in each type of activity. Information gathered from the pilot study for this research indicated that statistically significant change over time could be found among students after three quarters of exposure to this type of learning environment. The interaction observations for this study were made during a first quarter calculus experience.

d. Observations of class sessions and comments during interviews demonstrated the large amount of conversation concerning mathematics prompted by this approach to instruction. Students discussed mathematics in attempts to make sense of what they encountered.

e. The choice of use of paper–and–pencil or computer changed over the period of the interviews. Initially, students were uncomfortable and unable to perform with paper–and–pencil. By the end of the study, all participants chose to use paper–and–pencil, to some extent, when working problems.
2. Concept development:

a. Data from this study documents the improvement in the understanding of the concept of derivative over the course of the first two quarters of calculus. The focus was not on the building of skills or paper-and-pencil computational ability. All of the participants demonstrated a strong understanding of the concept by the middle of the second quarter of the course.

b. Students had the ability to approach and solve problems much sooner than they were able to verbalize, accurately, the meaning of derivative. This would indicate that simply testing skills could lead to the incorrect assumption that understanding is present.

c. Even though formal definitions of derivative were not stressed during the course, participants developed a strong connection between the concept of derivative and slope. This was a result of the numerous and varied applications they encountered.

d. The concept of integral was beginning to form by the end of the interviews, the end of the second quarter of calculus. The development of this concept was not complete just as the development of the concept of derivative was not complete at the end of the first quarter of experience.

e. As with the derivative, formal definitions of integral were not stressed. Yet, participants clearly developed the sense of integral as associated with area.
3. Problem solving skills and strategies:

a. No hierarchy of problem solving was exhibited by the data collected during this study. Rather, four categories of response type were demonstrated. These did not fall into a hierarchy as students were not observed progressing through them as stages. All of the categories were demonstrated throughout the study. The four categories observed were: (a) non-response, failure to engage in problem solving activity; (b) single-strategy, or attempts to solve problems using only one approach with no attempt to try alternate methods whether success or failure was met; (c) multiple-strategy without success, attempts to solve problems using multiple strategies from calculus as well as algebra that resulted in various possibilities but no partial or complete solutions; and (d) multiple-strategy with success, attempts to solve problems with multiple strategies using calculus and algebra that led to partial or complete solutions.

These categories correspond to some of the levels in a cognitive network developed by Collis, Romberg, and Jurdak (1986), but did not appear to be hierarchical. The basis of their hypothesized cognitive network was closely related to Piaget's stages of cognitive development. The work of Collis, Romberg, and Jurdak involved young children and the group studied here consisted of young adults. This could account for the lack of an observable hierarchy in this study. Further investigation would be necessary to support not only the existence of the categories of response detailed above, but whether or not they would follow a hierarchical pattern. Within the network, a description of individual
responses given in problematic situations is broken down within another structure. The components of this response structure are:

(a) prestructural, refusal to become engaged in the problem;
(b) unistructural, a response making use of only one relationship or fact;
(c) multistructural, a response making use of multiple relationships or facts; (d) relational, responses based on the relationships of elements in the situation; and (e) extended abstract, responses involving use of given data, related constructs, and abstract principles.

b. Students fell into two distinct categories by the end of the interviews regarding their approach to solving problems using Mathematica™. The researcher has referred to these groups earlier as those who act versus those who react. Some students, the ones who "acted", planned problem solving strategies before engaging the computer. This often led, as with the problem \[ \int_{-1}^{1} \frac{1}{x^2} \, dx, \] to discovering that the computer was not always correct in its responses. Others simply plunged in with the computer and accepted the solutions offered or became confused if solutions were not obtained. Some of these, students who "reacted" rather than acted, offered explanations for the computer when difficulties arose.

c. Problem solving skills changed to the extent that participants became willing to make multiple attempts to solve problems. The increase in attempts led to more successful results. The findings support the claim
that better problem solvers are created by exposing students to more problems.

d. Students perceived to be middle or low in ability at the beginning of the course were observed to be more likely to experiment and try more approaches to problems. Contrary to popular belief, the lower ranked students were not left further behind when technology was used.

e. The higher level students exhibited the most difficulty in problem solving situations. They failed to attempt problems, failed to use multiple strategies, and became easily discouraged if a first attempt resulted in failure. Some of this behavior could be attributed to the attempts, due to past mathematics training, to force problems into categories and use an algorithmic approach. Perhaps students at this level were confident in their abilities and saw no need for exploration. Further study may yield some answers to this observed behavior.

f. Students were moderately successful at the end of the study in determining when problem solving situations required the use of derivatives and when they required the use of integrals. Mistaken choices involved the misuse of integrals and could most likely be attributed to the fact that the concept of integral was still undergoing development.

Through observation and a questionnaire, additional findings were acquired by concerning teaching methodology and student attitudes. It should be noted that this was the second year a course of this type had been offered at this university. The approach, using Mathematica™ notebooks in a laboratory setting, was new and methods of approaching such instruction were still evolving as this data was
collected. The professor and teaching assistants were continually making adjustments in their approaches to the class as the study progressed. This makes it difficult to discuss a method or style of teaching. As mentioned earlier, the role of the professor and the teaching assistants was observed to be significantly different from similar roles in more traditional classes. They (a) facilitated learning activities rather than presenting information through lecture, (b) stimulated discussion and group work, (c) raised questions rather than provided answers, and (d) helped troubleshoot problems with the materials as well as the equipment.

The professor and teaching assistants were observed making a large commitment of time to the course. They each spent at least fifteen hours per week in the laboratory as resources for the students. The amount of time spent outside of the laboratory exceeded this. The professor was continually developing new notebooks or improving the quality of already existing ones. The teaching assistants held recitations for answering questions and administering weekly written quizzes and graded quizzes and homework. The use of technology in the classroom requires careful planning and a large commitment of time.

The students also spent large amounts of time in the laboratory. All of their work had to be done at the computer. The majority estimated spending five days per week and fifteen hours per week in the laboratory. This appeared to be an accurate estimate. While some students viewed this as no more time than they would spend on any other course, the inconvenience of spending it in the laboratory was seen as negative.

The attitudes of the students concerning the course overall were positive. They felt that Mathematica™ and the ability to work with other students in the class
enabled them to understand calculus better than they might have otherwise. The learning environment was perceived to be friendly and they viewed the teaching assistants and professor as approachable, caring, and willing to listen. As an indication of their positive attitudes, eight of the nine indicated they would recommend the course to others. The only one who answered maybe indicated that such a recommendation would be contingent on the amount of time and effort a person would be willing to commit.

The findings presented above and supported by the data from this study are the aspects that were of interest to the researcher. In the process of the study, other questions were raised for future study.

Suggestions for Further Study

One of the dominant aspects of qualitative research is the questions raised. As the design evolves or emerges, many aspects of the situation under investigation are probed and many questions are raised for future investigation. This was the case with this study. Primary among the suggestions for further study is the use of qualitative methods to investigate learning at all grade levels. These methods have been employed predominantly at the elementary level. This study should serve to illustrate that valuable information can be obtained at higher grade levels employing such methods. Investigations of this type could lead to changes in our theories of instruction and learning.

Certain questions specific to the learning of calculus and use of technology arose. They are presented below:
Do the types of interactions in a technologically rich classroom change over time and eventually evolve into a new and stable pattern?

Conflicting results were found during the pilot and the study reported here. The types of interactions recorded changed significantly over time during the gathering of the pilot data in a third quarter calculus experience. No such significance was found during this study when similar data was collected during a first quarter calculus experience. Studies of groups over extended periods of time should be made to determine whether a pattern of interaction change takes place and stabilizes.

Does the concept of integral develop further in subsequent quarters much the same way the concept of derivative was observed to develop in this study?

The development of the concept of derivative was solidified during the second quarter calculus experience after beginning integration topics. The concept of integral was starting to form during the last interview at the end of the second quarter of the calculus experience. Studies through the entire calculus sequence should be carried out to see if concept development follows the same pattern as observed with the concept of derivative. Reasons for this solidification of concepts could then be conjectured and tested. Studies such as this, the development of concepts, should be done with more traditional classes of calculus as well as those incorporating technology. Time-series analysis of quantitative data could offer some explanations.

How do students exposed to this type of environment in a beginning calculus sequence perform in subsequent courses?
The true proof of the success of this approach to teaching calculus as with any approach is the level of success experienced by students who proceed into subsequent courses with a calculus prerequisite. As sufficient numbers of students complete entire sequences of calculus using this approach, studies of their performance in later courses should be done. These studies would involve comparisons of the computer algebra group to students who had experienced a more traditional approach to calculus.

Do paper–and–pencil skills continue to develop? Is this an issue with the availability of technology?

The issues of paper–and–pencil computational skills, though deemed unimportant by some researchers, should still be tested by research. The question of whether these skills develop over periods of time should be put to rest.

Do the categories of problem solving observed evolve into a hierarchy?

The four categories of problem solving observed during this study: (a) non-response, failure to engage in problem solving activity; (b) single–strategy, or attempts to solve problems using only one approach with no attempt to try alternate methods whether success or failure was met; (c) multiple–strategy without success, attempts to solve problems using multiple strategies from calculus as well as algebra that resulted in various possibilities but no partial or complete solutions; and (d) multiple–strategy with success, attempts to solve problems with multiple strategies using calculus and algebra that led to partial or complete solutions did not
appear to be hierarchical in nature. Longitudinal studies of this phenomena might reveal otherwise and would also lend credibility to the existence of the categories.

**Do students transfer their knowledge into other areas, such as business and science, better after learning calculus with this approach?**

Since the applications of the tools of calculus, the derivative and integral, are the focal point of the course studied here, the possibility exists that students will transfer the ideas of calculus more readily after this experience. This would again involve studies of students in comparison with more traditionally taught groups in courses following calculus.

**How do students with background in mathematics involving graphical approaches perform in a calculus course with a computer algebra system versus students with no graphing background?**

Curriculum and instruction that focuses on graphical representations has become popular at the pre–calculus as well as calculus level. The power of graphical representation in the teaching of mathematics should be examined. One participant commented:

> I never knew what the plot of a log of x looked like until I came to college. I didn’t even know what x squared looked like even though that’s simple. x squared is an obvious plot and x cubed and so forth. I didn’t know what they looked like until I came to college. Started taking this class and started looking at them. (p. 191)
Only one student in the group of participants for this study had experienced a pre-calculus course that required the use of a graphing calculator. This does not provide enough information for inferences to be made about the differences in the way such students approach a computer algebra calculus. While this student was seen to make large gains in ability to approach problems, no attempt was made during this study to compare this with students with no experience with graphing utilities. Since one of the most liked aspects of the computer algebra system was the graphing capabilities, investigations of student performance when graphing utility experience should be examined.

Do students with no previous calculus experience perform as well in a course such as the one studied here?

All but one of the students in this study had a high school calculus course or a pre-calculus course that included limits and derivatives. The question of whether some calculus knowledge prior to entering an experience such as the one studied here seems necessary. Could students with no knowledge of calculus succeed to the extent that the participants in this study did?

Most of the questions that emerge as a result of this study involve longitudinal studies of groups and individuals. They must also involve qualitative techniques to ascertain what the students are thinking and how concepts are formed. Longitudinal studies employing qualitative techniques should be developed in the near future to answer the many questions that arise when technology such as the computer algebra system is introduced into the classroom in such a drastic manner. Little to no research has been done in this area.
Is this type of approach for teaching calculus cost effective for general implementation?

The question of the cost effectiveness of this approach to teaching should be considered. Perhaps of primary concern is the cost incurred due to the large amount of time required from faculty members and graduate teaching assistants to organize and maintain a class taught using the computer laboratory methods studied here. Additionally, providing computer laboratories such as the one used for this class is expensive due to the space, equipment, and staff needed.
List of References


Confrey, J. (1990b, January). *The design of student–centered software for teaching mathematics: An example*. Presentation at The Ohio State University, Columbus, OH.


Bibliography


Questionnaire
Background Information

1. Check one: __________Male __________Female

2. I have been enrolled at OSU for ___________ quarters.

3. My intended major is ________________________________ .

4. Did you take calculus in high school? YES NO

5. Did you take a computer course or courses prior to enrollment in 151M?
   If so, please list.

6. Had you taken a mathematics course that required you to use the computer before 151M?
   If so, please list and explain the computer use.

7. How did you find out about the 151M–153M calculus sequence?

8. What do you like best about these courses? (List at least three aspects.)

9. What do you like least about these courses? (List at least three aspects.)
10. In your opinion, has using Mathematica™ helped your understanding of calculus?

   If yes, list at least one way you have found it helpful.

   If no, explain why not.

11. In your opinion, has working with other students in class helped your understanding of calculus?

   If yes, list at least one way you have found it helpful.

   If no, explain why not.

12. Would you feel comfortable doing calculus without Mathematica™ as a tool?

   Why or why not?

13. Estimate the number of days you spend working in the lab each week. __________

14. Estimate the number of hours you spend working in the lab each week. __________

15. Do you own a computer? YES  NO

   If yes, what type of computer?
16. Would you recommend calculus with Mathematica™ to others? YES  NO

Why or why not?
Tally of Results of Questionnaire
Background Information

Numbers appearing in parentheses ( ) indicate the number of responses of each particular type.

1. Check one: 6 Male 3 Female

2. I have been enrolled at OSU for 2 (8); 6 (1) quarters.

3. My intended major is
   mechanical engineering (2)
   botany (1)
   business (1)
   international studies (1)
   accounting (1)
   microbiology/pre-med (1)
   environmental engineering (1)
   unknown (1)

4. Did you take calculus in high school?
   Yes (5) No (4)

5. Did you take a computer course or courses prior to enrollment in 151M?
   Yes (7) No (2)
   If so, please list.

FORTRAN
Apple BASIC
PC applications, BASIC programming, and Pascal
use of Apple IIe, IBM, and Macintosh and some programming
introduction to computer programming and advanced computer
programming in BASIC
independent study course with IBM's senior year using dBase,
Plus3, and Lotus 1-2-3
TRS 80 in 6th and 7th grade

6. Had you taken a mathematics course that required you to use the computer
   before 151M?
   Yes (0) No (9)
   If so, please list and explain the computer use.
I took 148 and 150 with the graphing calculator, if calculators are still considered computers. We graphed functions and discussed relations of graphs.

No, but for my calculus class in high school we did spend a few days on the computer to look at various graphs of functions.

7. How did you find out about the 151M-153M calculus sequence?

My 150 TA is now a TA for the series and told me about it.
Handout during orientation (4)
Found out from TA in recitation of regular 151
Knew someone in the course from the previous year (2)
My OSU UVC teacher (she's also my counselor) told me about it.
Student who was in it along with my advisors recommendation to take it

8. What do you like best about these courses? (List at least three aspects.)

The problem solving techniques (seeing how calculus actually applies).
The interaction of students.
The interaction of students/TA/professor.
Math is funner for me as a non-math major because theorems are derived, not memorized.
computer familiarity
real life problems
seeing the results
working with students
the computer experience
the instructor
the interaction among students & instructor
work on Macintoshes
personalized contact with the professor
no textbook
The class size is small, you get to work together and experiment different ways of doing one problem on the computer, and just the fact that you get to use the computer instead of writing out everything.
being able to look at functions without having to do them by hand
learning more about math than I would if I too regular math since then I'd only be memorizing formulas
being free to experiment with problems (not limited to only a certain way)
the ability to see what's going on
avoiding all the tedious calculations by using the computer
more convenient to do homework
They are very challenging.
hard work
self-teaching (very independent)
graphing capabilities
flexible scheduling and work hours
the challenge of solving difficult problem without a formal
discussion or laid out formula to plug into

9. What do you like least about these courses? (List at least three aspects.)

My handwork and simple algebra is going down the tubes
because I have the computer to do it.
Sometimes I forget past subjects because we move so fast i.e.
tangent lines!
The amount of time needed in front of the computer.
little teacher/student teaching
degree of difficulty
the lab is too far away
I can only do homework there, because other computers are too slow.
Sometimes the questions seem useless.
Correlation between computer homework and quizzes is poor.
having to know two "kinds" of calculus—computer calculus and
paper calculus
The tutorial and basics sections aren't usually written real well.
It does not have a book to supplement what is going on in the
computer.
Outside the lab there is not too many people that can help you if
you have a problem.
There is no lecture class to explain what is happening on the
computer.
hardly any lectures
sometimes don't know what I'm really doing, because I just type
in commands telling the computer to do it for me
quizzes are written and it's hard to go from homework on the
computer to a written quiz
You don't really learn as much as you would in regular math
class.
having to take quizzes and tests on paper after learning
fundamentals on the computer
not enough teaching going on—students are expected to teach
themselves
takes too much time
not sure if you are learning math, or if just are finishing the
homework
not many lectures
not drilled with equations that are fundamental to learning math
no discussion or lecture about new topics
little information retained after lessons
too many variables in problems, not enough numbers to relate to
I hate parametric equations!

10. In your opinion, has using Mathematica™ helped your understanding of
    calculus?
    Yes (4)
    No (1)
    Yes and No (3)
    Maybe (1)

    If yes, list at least one way you have found it helpful.

    I never knew calculus, so it had to help. It helped me realize that
    it is a functioning part of life, not some archaic formula-ridden
    monstrosity that we had to slug through to get our diplomas.

    The plotting is great for seeing what I'm trying to find.

    In some ways yes. It has showed me simple ways to produce
    answers out of something that looks like a big mess.

    learned about how function play a part in calculus not just a
    formula

    It does graphs and problems much quicker.

    visualizing complex graphs

    If no, explain why not.

    It's hard to tell. I can't exactly explain what calculus is. I know
    parts of it (derivatives, integrals, etc) but the total concept
    eludes. Mathematica is a good program, but just like a regular
    calculus textbook, you have to learn from it. The key to this
    class is the professor and the TA's.

    Because you learn something for one lesson then forget it
    because Mathematica doesn't make use techniques over and over
    again.

    Sometimes I did not know a lot of what I was doing because I
    wasn't sure why I was using a certain command, except just to
    get the answer.

    I'd already learned everything in high school to some extent.
    Since teaching wasn't adequate, didn't learn any more in depth.
You do not know how Mathematica got the answers, not given step-by-step so when doing by hand, you get confused.

11. In your opinion, has working with other students in class helped your understanding of calculus?

Yes (9)    No (0)

If yes, list at least one way you have found it helpful.

They have the math background and I have the problem solving sense at times, so together we help each other out through sticky problems. You always work best when studying with others. Everyone has a different approach to things.

I learn more thoroughly when we brainstorm and discuss problems together.

Exchanging ideas with different people usually leads to new ideas and strategies for solving a problem.

Because different people have different ways of looking at things and sometimes they are easier.

It is really impossible to do the work on your own and be sure you're right. With others you can get a variety of approaches to a problem and in the end compare your answers to see if you did it correctly. Everyone really helps each other out.

Group discussion was the only way to understand anything as the instructors failed to adequately explain problems.

Because it is interesting to see how other people solve problems and it is easier to work together.

Working with others is a way of looking at different problems different ways. Sometimes you just get more confused.

If no, explain why not.

12. Would you feel comfortable doing calculus without Mathematica™ as a tool?

Why or why not?

Not really. I understand concepts and theorems, but my algebra is lacking and I'd have to work a few problems before I was completely comfortable.
Somewhat, it's nice to see the results on the screen. Exp graph

Yes, because Dr. Davis makes us do a lot of our work by hand as well as our quizzes and exams.

No. Without the computer, I probably never would have managed to get through regular 151. A textbook and paper and pencil are just too plan and boring.

In some aspects yes, like taking derivatives, in others I'm very lost without Mathematica.

Not really, because I'd be limited. I wouldn't be able to look at functions as easily and I would not know what I was doing except memorizing formulas.

Yes, as long as I was able to understand the lesson content. If so, it could be quite helpful.

Maybe, because aside from my high school education—I really have not learned anything new.

13. Estimate the number of days you spend working in the lab each week.

   4 days (1)
   5 days (5)
   6 days (2)
   5–6 days (1)

14. Estimate the number of hours you spend working in the lab each week.

   between 12 and 15 hours (7)
   between 8 and 10 hours (1)
   25 hours (1)

15. Do you own a computer?

   Yes (3)   No (6)

   If yes, what type of computer?

   IBM (1); Macintosh LC (1); Commodore 64 (1)

16. Would you recommend calculus with Mathematica™ to others?

   Yes (8)   No (0)   Maybe (1)
Why or why not?

I like computers and think that they make the class more interesting. Also, Dr. Davis is a great teacher in my opinion.

The class atmosphere makes calculus somewhat interesting. Solving real world problems with a computer is much more enjoyable than memorizing formulas out of a book.

I would recommend it to people who like computers and are willing to put in the extra time working with the computers. Once you understand the system it makes calculus a breeze.

It's more interesting than just plugging in numbers into formulas and you learn more.

On one condition, that they were able to teach themselves. If not, no way.

If they can work by themselves or if they have a lot of time on their hands, if math is not their major.
Appendix B
The participants in the study are coded by the letters A through H and I. In all of the following interview transcriptions, the letter I refers to the interviewer's comments and the letter S refers to the student's comments. References made to courses numbered 151, 152, 153, and 254 are regarding the traditional four quarters of beginning calculus and courses numbered 151M, 152M, 153M, and 254M are in reference to the beginning calculus sequence using Mathematica™.
Student A - Interview 1

I: This problem was off of an earlier quiz and the way I understand it, this week you've been reviewing. In recitation you reviewed a little bit for the final and I think you looked back at this problem again. A couple of people I've interviewed since Thursday said that you had another look at this problem in recitation. \( f(x) = e^{-x} \)

S: Uh huh.

I: Find the derivative first.

S: I vaguely remember doing this yesterday. Go ahead and work on through here?

I: Sure. And try to talk about what you're doing and when you decide what to do, how you decide it. When you're asked to take the derivative of \( f(x) = e^{-x} \), what are you going to use to take the derivative?

S: I get the two of them confused. I believe this one is (inaudible). I get the names confused but I think I know the strategies behind them but I get the names interchanged.

I: Okay. Then just do what you would do.

S: Just - forgive me if I'm not very good at this. I'll probably get it wrong. Math is not my favorite subject.

I: I'm not interested in the accuracy necessarily. I'm interested in what you think about when you decide what to do.

S: I'd like to hear myself, what I think when I do these. Should I go ahead and go through all the steps?

I: Right. So you took what? The derivative was - x -

S: The derivative of x and then just (inaudible) just straight to negative x and then took the derivative of the negative x just times x.

I: All right. What you're supposed to do with the derivative then is use it to try to figure out where the maximum values or value whichever the case may be of the function.

S: Right.

I: Why? How can you use the derivative to figure that out?

S: As I recall whenever the derivative is equal to zero then you're going to have a place in the \( f(x) \) graph where there isn't going to be a change so you would be at max right in there. We don't have a specific interval on this so we're not going to have a max or a mid at any point.

I: Okay.

S: So what we can do is we can take the derivative assuming that it's right or try set it equal to zero. Zero we could find out. And just going to try and see if I can - this looks plausible. You have to forgive me. This is not what I was expecting.

I: That's fine, that's fine. I didn't expect you to study for the interview.

S: Calculus is one of those things where I've just gotten progressively worse in it.

I: It's okay. If you don't want to follow through with that that's okay. Let's suppose that you did follow through and you could find a value of x where this derivative equaled to it. Then what would you do with it? Whether you actually find it or not.

S: For me if you knew what the x value was - if you would know half the coordinate then you could plug it into the \( f(x) \) and you could get the y coordinate. You'd have on your graph wherever that maximum may be.

I: So when you set the derivative equal to zero and you solve, finding x that makes that true then that x that you get is going to be the x coordinate of the maximum.

S: Well it could be a maximum and a minimum. It could be either one.

I: How would you decide?

S: If I'm on the computer it takes care of that for me. As far as paper I'm not really sure. I'd probably go through and try and get a rough outline of the function if I could and see where it falls into on that line.

I: What about that next part? Intervals where that function might be increased?
S: I vaguely remember doing this. It had to do with dividing it up into - with the x value. As you're approaching - on second thought I wonder if we could take the second derivative and that would tell us where it's increasing but I'm not exactly sure when and where. I know if you take the second derivative you can look at it that way.

I: Can you do that at all with the first derivative?

S: Yeah. You have to - it's like kind of a coordinate system - kind of like this. Here is x represented by this line. And I believe you (inaudible) the first derivative and by going through and figuring out - I'm not sure if it's the limit or whatever you can figure out what it's doing and by dividing it this way you know this is where the point - whether your max or your min is you know that it is increasing on this side and decreasing on this side you've found the max.

I: Okay.

S: And then as far as graphing it I'd probably first find the limits as it approaches a negative infinity and infinity so you have the global behavior and then look at the points (inaudible) and probably between negative 2 and 2 or depending on where the max was and trying to see kind of like the local action is rather than the origin. It works out like that.

I: All right. That sounds good. What other kinds of things can you tell about the graph or anything else, if you can think of anything else, once you have the derivative of a function or does this kind of include everything here?

S: Since we have been told that one of the fundamentals of calculus is the derivatives and all this and more than just finding the max and mins is where it's changing. I'm sure there's more but I'm not thinking of them right this second.

I: Can you call besides the graphing problems trying to get sketches of graphs other types of things you've used the derivative for this quarter?

S: We used it to find tangent lines.

I: Tangents to -

S: Tangents to points on the graph. Things that had to do with Newton's method and all those tangent lines. Derivatives came into play when we used the range track principal and when we came and did on proportions what we had had saying that f(x) is proportional to it's derivative.

I: Okay.

S: Those are the ones that stick right out of my head.

I: Uhhuh.

S: Then of course there is the stuff we just did with the higher derivatives and order of contact and things of that nature. Seeing where things are similar and (inaudible) and things of that nature.

I: Did you have a calculus course in high school?

S: I had a very bad pre-calculus course. We had 6 or 8 people in it and the class we didn't really get along with the teacher and I don't think she was a very talented teacher. Kind of like in high school we all moved just as kind of a group through math classes and we all did well in geometry and algebra II and analytic trig and then we got to our senior year and pretty much all of us cracked out. I've talked to some of those other people who were going here to Ohio State and other schools and none of them except really one is doing well in calculus and his dad is a professor of aeronautical engineering here at Ohio State so he has some help.

I: In that pre-calculus class that you had whether it was good or bad did you look at limits and derivatives or did you get that far?

S: I remember looking at limits and I remember looking at - she didn't call them derivatives but I remember looking at f prime but it was like we just got to that at the end of the year. We didn't do a whole lot with it. I do remember doing that. I remember doing it with just simple plots and stuff of that nature. We were doing polar coordinates. I'm trying to remember what else. We really didn't get much into derivatives. I do remember doing limits.
I: Okay. How did you find out about the calculus with Mathematica™?
S: I had signed up for regular 151 and I went to the lecture on the Wednesday, the first day of the quarter and the first recitation on Thursday and I found that I was in there with 40 other students and this huge calculus book. It was very intimidating. I didn't think it was a very friendly atmosphere and right at the end of the lecture, recitation or whatever he said they need more people to try this 151 on the computer. So I went over that day. Talked to Dr. Davis. Sounded like a great guy. I like Macintosh. I used them all through high school. I'm very familiar with them so I felt very comfortable going into class.
I: And it was a smaller class too.
S: It was a smaller class and Dr. Davis is very friendly and the TA's are very friendly. It just seemed more like people cared for you than the regular 151 class.
I: What are your feelings at this point now that you've got one quarter behind you sort of about the class?
S: I like the class. I'm going to finish it up next quarter because I'm taking 152 because unlike most of the other people in there I'm not a science major or anything. Really I'm interested in International studies. I don't even need to take calculus classes. Just screwy things happen. I ended up taking this class and I want to finish the series through 152. I like it a lot. There are some things that need to be improved but overall things went pretty well.
I: What kinds of things would you suggest changing for those improvements you mentioned?
S: I think we need more recitation time than just once a week. Because what happens is we get in there. We talk about a couple of problems. We have to take a quiz so we don't get enough time to work. Especially with the way my classes are scheduled I'm not in the lab the same time every day so I don't see the same people so it's harder to work in groups. So I think we just need two recitations a week. One you could have a quiz and once you could just talk and let problems out in the air. What is going on in the class.
I: Would that be the only change you'd recommend?
S: I think they need another TA because when we get to the peak times in the lab if they're right around between 10 and 12 are real peak and you've got 254 students in there. 151 students. You've got two TA's and Dr. Davis which is not enough to answer all the questions that are going on because you're faced with two kinds of problems with the students. Either your reasoning and everything can be off and solving the problem or you could just simply have a mistyped command in the computer which could be throwing it off. And in either case you're screwed. There is not a whole lot you can do so you need someone to look over your shoulder and say you just put a space in the wrong place or whatever or you're just using the wrong type of approach. I think that's the thing. I'd be interested in because I've seen that they have the overhead projector and I'd be really interested if they did use that to show how to solve a particular problem, actually how you key it into the computer. I think that would be a great aid - if it works. And then I think just a little more translation between doing it on the computer and doing it by hand because -
I: Do you think some of that should take place actually in the lab or -
S: I'm not sure how but there's just - if you give me this problem on the computer I don't have a problem with it. If you give it to me on paper I've got a big problem with it. I don't think it's because I'm stupid. I think it's just because I can't make the translation. Because I look at a problem and I can come up with strategies to attack it even though I'm not sure exactly what I'm doing all the time. I don't think that's cheating. You're letting the computer do it for you simply because a lot of 151 students can sit there. They're given a formula. They can plug and chug it but they still don't know the reasoning behind why they used that particular format. It's the same type of thing.
I: Okay. Can you estimate about how many hours a week you spend working on this class in the lab?
S: Between 10 and 15. A lot of it just depends what day the homework is due and when I can get there. I'm a computer so I'm only going to be here during the day. I'm very rarely here
in the evening to work. So again that kind of hurts me because I know they have a 254 class in there from 7 to 9 at night and when 151 is supposed to be over at 2 and ask questions of the TA but if you're living off campus you're not going to be in there at that time.

I: How do you feel about putting that much time in on the course?
S: Well if I wasn't doing it in the lab I'd be doing it at home with books so -
I: It's an even trade.
S: I figure it's an even trade. I'd much rather work on a computer than a paper and pencil.
I: Do you print the notebooks in there?
S: I print the literacy sheets. I've printed a couple of the notebooks but generally I don't do that because each of those notebooks can be 40-50 pages long. It takes forever to print out. I don't know how I'm going to study for my finals. I've got a couple of them printed out so I may come down one day on Saturday and look on the computer. But it's just too much to print out. I think it would be nice if they had a companion book. Like the regular students they can go home and study the strand book. If we had some kind of a book with formulas or commands or something so - it's nice to have when you're working on a computer a copy here you can look at instead of having to flip back and forth on the screen. Because if you have 2 or 3 windows on there then your space is really limited. But if you have a thing here with all the commands, no problem (inaudible) and what the formulas were. That would be nice.
I: Do you ever look things up in a calculus book? Have you at all this quarter gotten hold of a calculus book and looked anything up?
S: No. I bought the calculus book and took it back. I might buy a used one next quarter but brand new books are $65 - they're big, they're heavy and way Dr. Davis described it to me I wasn't going to really need it and I'm not so sure if I looked at a regular calculus book I'd find what I'm looking for. I might but - personally I think math books are notoriously awful. You cannot understand them. I never have run into a book that was written well.
I: When you mean written well and so forth - even though you don't print out the notebooks do you read the tutorials?
S: Yeah, usually. We usually at the beginning - the first day when I signed up and I opened the basic tutorial. I look for the main things and scan the detail, whatever needs to be done. Then I usually go home and the next day I actually start my lesson. Give it time to sink in and by just scanning you know where you have to take a look to get the information when you approach a problem because they're pretty good about it - it's going to be in the tutorial. It's going to be somewhere in the problems. I like that. I like that they don't just put the stuff there that's of no use. It's just garbage to throw you off.
I: Are they fairly easy to read? The explanations that are there as compared to -
S: Some are, some aren't.
I: A typical book.
S: As a typical calculus book or other math books I've read in the past actually yes, I think they're fairly well written. Because by example right away for you and the great thing about the computer is the graphs. That's probably the greatest thing about it.
I: You use that probably more than -
S: If you know how you can be looking for something. If you're looking for derivatives, functions, whatever and you have a general idea then you can plug that in and get the graph out and then you're ahead. You don't have to sit there plugging in points, numbers and whatever is (inaudible). I hate it. You've got something to look at right away. I think that makes a difference. Because while I think it's good that you know how to do it by hand, the theory behind it, the chance of you ever doing it compared to time so much time it takes less to do it on the computer if you're going to do it on the computer in the real world.
I: You said you printed the literacy sheets. Do you then sit down and try to do some things off of those by hand or -
S: I try to do them by hand. I wish they would give you the answers on the literacy sheets. That would be nice. It would be nice if they had literacy and then a separate thing, a separate answer sheet you could take out so you could do the problems and check them. Because that's probably one really bad thing with the classes. You know most books give the odd problems in the back you get the answers to them. In this case even though the TA and everything will help you out to get the answer it's still nice to have an answer so you don't need to ask all the time.

I: You said that because of the times that you are usually in there working there are not always the same people around. Do you work very much with the other people in the class?

S: I have more towards the end of the quarter. At the beginning of the quarter I didn't.

I: That seems to be something that is promoted in the class by -

S: yeah, they want you to work together and personally I'm a loner person but I'm getting more used to working with people and that doesn't bother me at all. It's just a matter of finding (inaudible) which you like and which you can work with at the time that you are there.

I: But that's not something typically you've done in the past?

S: I wouldn't say I did it that much the first five weeks of the quarter. The last couple of weeks I've done it a lot more, now that I know people. But I'm sure I'll do it a lot more. I'll probably actually do it less next quarter because of the time I have to take the class. The thing is if I get the schedule I want and I think that I will because of the way I sent it to the classes is I won't be able to get into the lab on most days until 11 o'clock. They're supposed to have a second 151 class in there from 12-2 or 11-1. So I'll be with 151 classes as opposed to 152 class which will still be in the morning so I'll have less people to work with really.

I: You said you were going to stay with the 152M. I've asked a couple of people who have come in if a 152M wasn't available how would you feel about going into a regular section of 152 next quarter.

S: I don't think I'd go to 152. I'd probably - personally I don't think it's because - if I went to a 152 class I think I'd feel lost. Because I have a hard enough time (inaudible) right now but because of what I'm interested in I'd probably go to 132 and take business calculus if I couldn't take it on the computer. Just because of where my interests are because I don't have the time or the patience to do all the math, like physics or engineering. I'm just not that persistent.

I: Okay. Do you have any other general comments you are willing to make or anything you would like to say about the course in general? How you think it's going or how you feel about how you're doing or -

S: I wish I was doing better. I'm not really sure what I'm getting in the class. But I don't have any real complaints. I think it's in a period of just growth. Of growing pains and I think like maybe 3 or 4 quarters down the line that they'll really have it be like clockwork. It will be going really smooth and maybe by then they'll have some separate books for it and more help in the lab and things like that.

I: There is a book that goes along with this course that's due out the first of the year. The same people who originally developed these notebooks, Dr. Davis has done some major revisions to use them here but Porter, Uhl, and Brown actually have a book that's coming out the first of the year.

S: That would be good because like 151 all the way through or -

I: I think eventually it will be for all the way through. I'm not sure if in the beginning they're going to have it all available. So it may be the first quarter or two of course work that they'll come out with it at the beginning.

S: That would be nice. I think that's definitely in the right direction.

I: Those things have been in process. I think some of it is kind of a catch-22 because in a way you want to try this out with students and see sort of what's needed and then put it together but then in that interim period those students sort of have a hard time of it.
S: I think probably last year's class, this year's class - it probably won't be right until next year that those are - if you want to say competent calculus students or whatever that will get the maximum out of it.

I: Do the 254 people ever answer questions for you?

S: Oh yeah they do. When they're in the lab and you're in there. They're more than happy to. Everyone is real friendly and nice and that's kind of refreshing because I don't know if it's a stereotype or whatever but I do find that a lot of math people are just not friendly in general. I don't know why they're that way. They just are. And I'm impressed with the TA's and Dr. Davis. They have a lot of patience because a lot of people do not have patience, math teachers and if you don't understand it screw you, whatever. I think that's why most students don't like math. People don't have the patience like a history teacher or an English teacher or something else.

I: I understand that.
Student B - Interview 1

I: This is the problem that was on the quiz that was earlier in the quarter. You might remember it if you flip back through anything studying for exams and that kind of thing. I have problems like that and I want you to kind of work through them and talk about what you do and why you do it. Or if you can't remember what to do or how to attack the problem just take a piece of paper if you need it.

S: Just work it.

I: Yeah, but on this one you were given the function x - sin(x). The first thing is to find the derivatives. So, -cos? Now how did you decide what to do?

S: First of all, the derivative of x because it's an addition problem so you take each by itself is 1. Actually it should be 1 minus (inaudible) and the derivative of sin(x) is cos(x) and it's negative and so it would be a -cos(x).

I: All right. Now decide if the function is decreasing. How can you decide that? What's something you can use or -

S: You use the first derivative and you should plot it.

I: Plot the first derivative.

S: Plot the first derivative and wherever it touches like zero and it is going down and the function is decreasing at that interval between those two intervals. Like say it was zero and π. I'm not sure if that's where it would be. x - 0 - π - x 1 - so it would be π minus. Yeah, that's what it would be. It would be like from zero to π. And then the function would go back up and it would not decrease until like 2π. And you can figure it out by first derivative and plotting it.

I: That's what I wanted to be sure of. Make sure I'm understanding. You would look at a graph of the first derivative and where the first derivative was what?

S: Zero.

I: You'd look at those places and then you would decide between those places?

S: Yeah, what was decreasing.

I: Okay. And you would decide it was decreasing by what between those places?

S: Like how much?

I: Uh huh.

S: It only goes between 1 and negative 1.

I: Right, but I'm just trying to get you to clarify for me. Say that does touch zero at zero and at π, between zero and π how do you make your decision if it's decreasing or not?

S: Well if the function is going down to a certain point it's decreasing so it decreasing -

I: If the derivative is going down or if the function is going down? I keep losing track of whether you're talking about the graph of the function or the graph of the derivative.

S: This is the graph -

I: Of the derivative.

S: Of the derivative and at this point where it touches zero it decreases to here and this increases. So -

I: So if the first derivative decreases and increases then the function also decreases and increases there. Is that what you're saying?

S: Yeah.

I: That's what I was trying to figure out.

S: Yeah.

I: All right. What else can you find by using the first derivative besides places where the graph is decreasing.

S: Places where the graph it hits maximum and minimum points so the function would hit it's maximum at - the first derivative shows where the original function will hit it's maximum and minimum.

I: What about the first derivative tells you those places? What parts of the first derivative would tell you where those places lie?
S: I can't remember. I just know they do.
I: That's fine, that's fine. What else? Is there anything else that you could find out about graphing or graphs with the first derivative?
S: Not that I can -
I: Besides using that first derivative to sketch graphs to get an idea of what they look like what are some other things that you used it for during the quarter that -
S: Find tangent lines. The slopes of tangent lines to the function. You take the first derivative and then that gives you the slope and then you plug it into the formula and get the slopes of the tangent lines. I think that's the only other thing we've used it for.
I: Okay. Have there been other things that don't involve graphing at all that you've used the derivative for that you can recall?
S: No, not that I can recall. That's one thing - we mostly plot everything like we would find the first derivative and plot it.
I: On Mathematica™?
S: On Mathematica™.
I: Do you usually graph the derivative or the derivative and the function or -
S: Both usually. On the same graph so it can show you where (inaudible) between the two what it's doing.
I: Some of the people I've talked to in your class had taken calculus before in high school. Did you do that?
S: I took a year of calculus but it wasn't really - the teacher said either you kept up with him - the pace he was going or you were lost and they finished on lesson 102 and I was on 52 still trying to figure out what I was doing so I had it but it wasn't really - and I think if I wouldn't have had it I would be lost in this class because some of the things you need even a little bit of calculus background to figure what the derivatives and stuff are.
I: So the course you had, you did find derivatives and things like that.
S: Yes.
I: But mostly by hand.
S: By hand, yeah. Didn't -
I: Didn't do much with graphing and things like that.
S: No, nothing like that. I forget a lot of the functions (inaudible).
I: What about the course? Do you have any comments you want to share about the course itself?
S: I think that if they had a book or something that explained what Mathematica™ is doing with the problems it would be more helpful because I find myself going over there and reading through the tutorials and then trying to do the homework. Then I'm always constantly referring back to the tutorials because there is nothing to go home and study and I think maybe that's one big hangup I have with class. I noticed that if I realized what was going on I got better grades on my quizzes but as I got lost my grades went out the door. And I'm not retaining anything. Some of the things we learned earlier in the course I'm not keeping and they're like well don't you remember and I'm like no because you don't get a chance to take it home and actually study. Dr. Davis I guess said there was a book coming out next quarter that he is going to tell everybody to buy so maybe that will help.
I: That might make a difference.
S: Might make a difference.
I: Do you ever print those notebooks?
S: Yeah. I print em all out and I told Dr. Davis I was having a problem and he told me why don't you print em out and I just now started to do them and I've been reading through them and that's been helping me so - another thing is I don't understand how the literacy sheets help you. We never have classes to go over and this was so and so and did you get this right? And stuff like that.
I: So looking at them and somebody saying this is what you need to know (inaudible) is not necessarily enough.
S: Yeah. I don't think it's necessarily enough. I think maybe like a lecture class or something along with going to the computer lab and working would help. Sometimes if you do problems on the computer but then you don't realize what you're getting out of it.
I: So if you were recommending to somebody how they ought to set this up and organize it then what would you recommend? A day a week lecture or two.
S: Even like Monday, Wednesday and Friday in the lab and then Tuesdays and Thursdays - like on Thursdays we go over some of the problems and but it's really not enough time to go over a lot and then we take a quiz. I think just another day or even like go to lab and like one day in the lab set up one day specifically you go over problems in the lab and they explain them to you as you're doing a problem on the computer what you are supposed to be getting out of it.
I: Doing it even at the computer too.
S: Yeah.
I: That's interesting.
S: That way you could be seeing what you're doing on the computer and then putting what you're supposed to be getting out of it together with what you are getting out of it.
I: What's the best thing that comes to mind about this course and the way it's set up in your opinion? What's good about it?
S: You get to work at a nice, not slow pace but you're learning a lot but it's still you can work on your problems any time you want to. You're not forced to go over there every morning at 9 in the morning. You can go over any time you want and work. I like that about it. You're free to do - you can ask a lot of questions and stuff and since it's new they're not afraid to say - you get personalized help because it's such a small group of people. I think I like that better because I was talking to one of my friends and she's in 151 and she says there is like 30-40 people and she says it's too hard. You don't get the attention you need. I like it in that aspect. That it is small and you get personalized attention.
I: What's not so good about it?
S: I think just that sometimes they just expect you to get the things out, off the computer and you're not getting them. Sometimes you don't catch like certain things. They don't point out make sure you know the arc sign or make sure you know the arc tangent. That has come up on a couple of quizzes and I never caught it when I was going through the lessons. I think that would be helpful if they like pointed out certain things that you should know. You should know this but other than that it's really not too much wrong with it.
I: Can you estimate about how much time you spend a week working on these assignments?
S: I would say probably about 15-20 hours depending on how long the lesson is.
I: That's all in lab time.
S: That's all in lab time. It works out to about 4 hours a day. I really don't put like four hours a day in. Not all lab time because sometimes I'm printing out the lessons. I've been taking them home and working on them so that 15-20 hours would be with -
I: Including -
S: Including taking it home, seven days a week because you can work in the lab Saturday and Sunday.
I: Do you work a lot in the evenings and on weekends?
S: Yeah, a lot at night. I work two days a week in the mornings and my schedule doesn't allow me to work (inaudible) because it is so far out. That's another thing bad about it. It's out of the way. They can move it closer to campus but I guess that's just where they had to put it.
I: Where it was available. What about working with other people in class? Do you work with a group of people in class?
S: Usually no I don't. But if I have like a problem or something I'll ask somebody what did you get for this or how did you go about it but I don't do that too often because usually when I'm over there no one else is over there.
I: I know that's encouraged I think.
S: Yeah it is.
I: Okay. Do you have any other comments just on the course in general and how it is going or
S: That's another thing. I don't realize homework, the grading and stuff. I don't - to where I am
fitting in the grade-wise. I really don't catch what they're doing grade policy-wise so - I guess
I'll see at the end of the quarter.
I: It will be interesting then.
S: I don't think I'm getting too bad a grade. I haven't caught up on the grading policy.
Everyone's grading policy is so different that - it is hectic every once in a while trying to
figure out -
I: If they only had a 151M course and they didn't have a 152M and you had to go into a regular
152 class.
S: I don't think I'd be ready to go in - I really. Some of the people that I've been talking to
they're like yeah, we're running all this stuff and I'm like we haven't even touched on that stuff
yet. And it's in the lessons that we're going to touch on next quarter and I think that's - we go
at a slower pace but yet they say as it goes along once you get familiarized with the system
they start speeding you up. So they say eventually you do catch up to where everyone else is.
But I talked to someone that jumped from the 151M into 152 and they had to retake 152
because they didn't have enough of a background.
I: Somebody from last year?
S: It was somebody from last year.
I: How did you find out about the class?
S: Somebody that was in the class last year.
I: That's still in or not?
S: Yeah, he's still in. He's still taking the class. He was talking about it and then there were
sheets on it at orientation and they said if you were interested to come on out but I had talked
to him about the class. He said you should get in this class. You like computers and that's
another thing - if you don't like computers the class isn't for you. If you don't get along with
computers.
I: Is that the kind of area you want to major in?
S: No. It's just that I've always been, I've always worked on computers since I was like real
little. It was one of the big things in our school was making sure everyone was computer
literate and they just made working with computers fun and stuff. We had Macintosh at our
school. I used to fiddle around with them all the time so -
I: You felt pretty comfortable with them before you went in?
S: Yeah.
I: So you got some advice from someone else in the class. Did those people in 254 ever answer
questions and things for you?
S: Yeah. In fact Brian that's the kid that's in 254 is one of my main resources. Brian, I'm
having problems. Can you explain it to me? He sits down and he shows me how to work
things out when I bring things back because he lives in the same dorm I do and so I just run
upstairs and ask questions. Go back downstairs and figure out the problem.
I: Do you do very much on paper?
S: No, not really. This lesson I did because I knew I wasn't going to have the time. I had two
finals this week in the mornings and I knew I wasn't going to get to go over. Lucky I had
class today because I thought the lesson was due Friday. It's due tonight.
I: So does that mean you're going to be in the lab this evening?
S: Yep. As soon as I get done here that's where I'm headed. I'll probably be over there for 2 or 3
hours because I have everything worked out. I just have to take them in. I guess that's why
(inaudible).
I: What about Mathematica™ itself, the software?
S: I think it's great. I think the software is great. If they could incorporate it with 151 like a regular class with a lecture class I think that it would really help people understand because I think graphing in a (inaudible) class is a waste of time. And the computer just spits it out for you. You can see it instantly. I think that was one of the hardest things in high school to figure out in trig and stuff was why does it go up and then it went down and you're like oh, what did I do? It's just the graphs there you (inaudible) calculate. I wish I had those in high school. They made everything so easier.

I: So the graphing is something you think is really valuable off Mathematica®?

S: Yeah.

I: What about the fact that it does all those computations and things for you? Manipulates the symbols for you.

S: When you think about it you do the same thing with a calculator so - other than that - I think you lose some but not much. Not much that would make a difference.

I: Okay. This first one was a kind of let's get acquainted and look at a simple kind of problem and talk a little bit about the course and see how things are going. Unless you have some last comments you want to make - some things you want to say.

S: No.

I: About the course or about derivatives -

S: I hope that's where it goes. I can't quite remember. I always have to look back in derivatives. I even keep a sheet in my book that I keep how to take derivatives and stuff because I can never - there are so many different rules for derivatives. They always get confusing.

I: Do you ever look back at the book you used in high school to look things up?

S: No, not the book I used. I have my brother's college book and I use it, I reference it quite frequently when I'm trying to go through the literacy pieces.

I: Has it been helpful?

S: Some, but calculus is so - the words in the textbook are so - unless you know math really well they are hard to understand and I get lost sometimes trying to figure them out. That's another reason I think a lecture class explains what you're supposed to learn and be helpful.

I: Okay.
Student C - Interview 1

I: The problem I picked for you to look at in this first interview is from one of the earlier quizzes this quarter now. I understand that you guys have started reviewing a little bit for the exam. I believe you might have looked back at this problem in your review this week so the first people I interviewed sort of hit this cold and you might have the advantage if you were in recitation this week. Did you look at that one in review?

S: Yeah.

I: This is the function f(x) = xe^{-x} and what you were supposed to do here is find the derivative and then use it to answer some questions. So I wanted you to try that problem out and talk to me about what you’re doing and how you decided what to do.

S: Your first question is to find the derivative. You look at it and you know you need the product (inaudible) to solve for it. So - the derivative - I'll just put x here and derivative of e to the negative x is negative e to the negative x plus the derivative of x which is just 1 plus the same thing which is e to the negative x. So if I simplify it I could take the e to the negative x out and that would just be 1 - no, it would be x minus 1. Okay. And then the next question is the maximum value that f(x) can have. I could solve for maybe the zero to find the maximum value of f(x).

I: Solve the derivative?

S: The derivative for zero. So make these e to the negative x - x - 1, zero. Let’s see. x - x can equal I think 1. Yes, because x can equal 1 because e to the negative x if you put 1 in for that that would equal zero and then 1 - 1 equals 0 so I'd say the maximum value that x can have is a 1.

I: When you find the places where the derivative equals zero those are the places you're going to have a maximum?

S: Uhhuh, yes. The intervals in which f(x) is increasing - well since I solved for zero in the first one the next (inaudible) is 1. You’d have to increase between zero and the 1. Between zero and 1. A sketch of the graph. So much easier using the computer. First of all since I have an e to the x or e to the negative x you know it's got to have like a behavior approaching the x-axis. First of all you know that it's going to be increasing between 1 and zero so it's going to be increasing there. You know the maximum is 1. So far I have something like this and the bottom part - let us. A sketch of the graph (inaudible). Well it has to go down because it can only increase between these little interval - between zero and one. When it starts to decrease it will go like that but it should just do something like that.

I: Can you just kind of summarize just in general how the derivative works to help you figure out maximum and minimum, increase and decrease?

S: When you find that first derivative of any function and you set to zero usually you can find the maximum or your minimum and then what we learned in the computer is when you find the integrals it can be where it's like increasing then you know the graph has to follow that and also you can find the first derivative usually as your range. You know that the graph has to go from there to there somewhat. We also learned second derivatives too which can determine concavity. It is going to go up or down and if the second derivative is positive usually that means f(x) would be increasing. And if the second derivative is negative then f(x) is decreasing so you can figure out from that too.

I: You said that when you take the first derivative and set that equal to zero you can find maximums and minimums. How do you know what you have is a maximum or a minimum?

S: Hm - because I was taught usually if it's like less than zero - like if you get one thing usually it's a maximum but if you get two things you always look at the last one as a minimum and the greater one as a maximum. That's how I was taught.

I: Okay.

S: Plus you asked for maximum -
I: Well on this one I asked for maximum.
S: But if you don't know - yeah. Or you can just like find the second derivative and see if it is increasing in that little integral and which one is the maximum and minimum. Like the graph helps too. Like if they just - if you can graph it on the computer and then you can figure out which one is the maximum.
I: Graph the derivative or the function?
S: The derivative definitely.
I: Did you do any other things with derivatives in class this quarter? Other uses besides deciding how the graph might look?
S: We used higher derivatives to find out which the order of contact and that means simply that if you have two functions you can find out when they're like equal to each other. They share the same point. So that's good to know because like when you're graphing something and you have two functions to graph you can at least get a general idea where they're going to look similar. When they're graphed they're going to touch each other at the same point. Let's see - we didn't go into integrals yet - we just went to derivatives and it's just about graphing things.
I: Did you have calculus in high school?
S: Yeah, I had honors pre-cal so we got into integrals. We got into a lot of stuff so it's basically this quarter was a review for me which is good but if someone is coming in cold it was probably going to be very hard for them to be quite honest. You had to have a background in it to really understand what we're doing.
I: How did you find out about the Mathematica™ course?
S: When I went to orientation they - we had to fill out our schedules and they gave us a piece of paper that was an application for Mathematica™. So I was looking at it and thought well the computer you know that would be good because I'm okay with computers so I filled it out and I thought for sure I wasn't going to get in it because my schedule came back to me and it didn't have it in it but then he wrote me a letter and so that's how I got into it.
I: So you just found out about it at orientation and filled out the form.
S: Yeah, uhhuh.
I: Do you have any general feelings just about the course after one quarter in there? You already said I think that if people didn't have background they might have some problems.
S: It's basically a self-taught course. That's how I feel about it. You read the lesson and you do the homework. They're always there to help though which is good if you get stuck. I think we should have more than one recitation a week because when you take the quiz it takes away some of the time you're in there and Dr. Davis I think he should maybe give like a lecture or something and teach us. Maybe before every lesson what we're going to learn. Because a lot of people I talked to they're like you're (inaudible) the computer. You don't know what it's doing. It's like I had this and so I could somewhat guess what it's going to do but for people that didn't - they're like I don't know why the computer did that but it's right because it did it so I'm just going to go onto the next problem. The quizzes help a lot because she picks problems that you should know how to do and then she goes over them like (inaudible) TA and the course is challenging which is good because you spend a lot of hours in the lab doing homework and the course is more close between students because like in a regular 15! class you have like 300 people in your lecture and like 30 people and you never talk to these people and we are like so close. We share homework; we help each other. You do everything. I think that helps a lot because when you're in this we need all the help we can get. I took 152 next quarter. That's the highest math I have to take for a grade so - I like the course. It's challenging so you have to think like that.
I: You said that it was really a self-taught course. Did you expect that when you signed up for it?
S: No I didn't. Like the first week we just like walked through the computer, getting used to it and he's like do this lesson for the homework and you have to go to the lesson and think for
yourself. Look at the lesson yourself and I didn’t expect that. I expected him to teach us and so it was like sort of we had to be really smart to think about it.

I: You mentioned the time that you spent in the lab. Estimate for me just on the average how much time a week say you spend in there.

S: A week - I'd say like a homework that is due I usually spend like 3 hours a day on it. So close to 15 hours I spend in there. But the fact that I like it though because if you have math homework at home you might not do it. You'll get distracted or something but when you walk to the lab you know you have to get it done. There's nothing to distract you. You have a computer and you get it done and you leave which is something I like because you don't have to take your homework home but you do spend lots of hours in the lab. I suppose it's good to a point because you learn a little bit more and you're more responsible because you know the homework is due. You have to get it done because usually in 151 regular they don't collect their homework so you could just like slack off but here you can't. It's due. You get graded on it. That's the end of it. So they're very lenient too. Like if we have a problem and we know the homework is going to take longer he'll like postpone it for a couple of days which I feel is really nice because a lot of teachers don't do that. But since this isn't a course he might think well I'll give them a break or something. I think this is like the second year they've had it.

I: That's right.

S: We're still new at it. I think it's going pretty good so far.

I: You said that the working together with people in the class you were closer and that kind of thing.

S: Yeah.

I: Do you work with a specific group in class on a regular basis or is it just whoever is in the lab when you're in the lab?

S: Yeah. It's whoever is in the lab, exactly. There's like - I have friends in there but usually if they're working next to someone somebody happens to sit down you'll start working so I think I know everyone in that class just from working with them. And if one person has a problem - they have solved it or something - I'll (inaudible) explain it so - you find out everybody - you go to different people for different things. So it's good, though.

I: Do you ever get any questions answered by the people who are in 254 who are in the lab?

S: Yeah I do. One time I was by myself doing it. It was like late at night and there was a couple of 254 students and they were trying to help me and you learn a lot from them because they remember what they went through and they helped a lot so you do get a variety of things instead of just 151 students so it's good.

I: If there was one thing that you could absolutely have changed about the course - just one - what would you suggest changing?

S: I think more teaching. Where Dr. Davis teaches really one day a week where we get away from the computer. We actually sit in the classroom. Because like when I read the application it was like computer enhanced so I thought oh, once in a while we'll go to the computer. I think that would help a lot because sometimes he's not there. It's like where's Dr. Davis? I want to talk to him but maybe just one day a week we just sit there in a regular classroom and talk about what we're learning, if we have any problems so he could directly answer them. I think that would help a lot. Maybe two recitations a week say with him teaching one of them and then the TA having the other one and you having a quiz in that one. That would help a lot.

I: What about the software Mathematica™? Had you seen that before?

S: No, I never saw it before.

I: What did you think of it?

S: I like it. It's - you get like three problems compared to one problem. Do you understand what I'm saying? Like in regular math you might get one problem because we do it so quickly we can get like 10. I like it. Sometimes they're not too clear in explaining why
they got that. Usually they give good definitions and I like the fact that it's really fast. You can sketch a graph in like no time at all. So I like that part because you don't have to be bothersome with doing simple calculations and if you get those wrong you get the whole thing wrong. So it's good software. I could see there is a lot of work into it and it's good. It teaches you a lot.

I: What do you think is the best feature of that software that you might not have available in regular class? What's one thing that you do with that that you think is most helpful maybe?

S: That's most helpful - well when you're doing your homework you could just put the lesson on top of the screen. You can see how they do it like in a regular book you just look on pages and stuff and what's most helpful usually they give direct examples. And they explain usually each step and you just like plug in the numbers and you can usually get it because usually when we do our homework it's like cut and paste. You cut from the tutorial and put in there but I would say maybe because they don't expect a lot. Because they try to explain everything they're doing and everything in the homework is usually found in the tutorial, the basics which is good.

I: Do you ever print the tutorial and the basic facts on paper?

S: One time I did because it was like on a lesson that I did not understand at all so I printed the tutorial but usually you print the (inaudible) issue and you work them out and like when you go to recitation you can ask her about it which is good but usually when you're doing the homework you get a feel for what's in the tutorial. But one time I did print it out and it took like 12 pages but I had to do it because we have to really sit down and learn it which is good.

I: Do you use the literacy sheets with (inaudible)?

S: Yeah. I have like all of them to all lessons and it's a great review. Because usually they have easier questions on the homework but you need them to study for the quizzes and for your test because you don't know like where they're coming from or anything and you have to know what's in each lesson and you print them out. They're a big help there.

I: Do you do any problems or any work by hand?

S: In the literacy you have to. In the sheet you have to because you could do it on the computer but I usually do it by hand because you know you have to do your quizzes and your final by hand so you do it by hand. That's another thing - I think they should emphasize - Dr. Davis is like don't bring a pencil or anything like that. Sometimes you have to because a lot of people when they write it and when they do it it helps. I think maybe we should get away from the computer maybe again once a week so that you sit down and do it - maybe what regular classes are doing because I was afraid if I switched to regular 152 I'd be totally lost. I wouldn't know what to expect at all. So I stuck with Mathematica™ and it's not that I don't like the course. I do but I would probably be lost if I were in the other class. Not the material but I wouldn't be used to doing graphing, everything by hand. I had that last year in senior high school and I was pretty good at it but after a while you forget how to do it. You're like - you're so used to the computer just doing it. The course is good though. It is. I'm glad I've taken it to tell you the truth because it is hard but yet it's easy. You don't have to bother with all this stuff like regular classes to do. You have just quizzes. You have no mid-terms and you have one final. So the homework is hard but that's the only part I think is really hard in the whole course.

I: Do you think you're learning the same amount as, more than or less than those people who are in the other class?

S: I asked people. They're learning about the same. This one kid said he started integrals. We didn't start that yet. This quarter was like a review for most people but I think they learned about the same. I said do you learn - we're learning derivatives. He's like yeah, I know all that stuff so about the same I thought.

I: But you think if you went to a regular 152 the reason you would kind of feel lost is more that you'd be doing things by hand and not really that -
S: Yeah. Not really the subject matter because you can read a book and it's just like the lessons you know. You're learning calculus here but it's just the things by hand you know. You may not be sure. We do a lot of derivatives by hand, though. She made sure we knew how to do those but graphing I'd probably be lost. But that's about it. I'd be scared to go into that class. I wouldn't know what to expect. You go into a big lecture with 300 people and that would be a great shock.

I: Different.

S: Yeah.

I: Do you ever look things up in a calculus book or anything?

S: No, I didn't buy one. I have my old pre-calculus notebook but I just brought that here like last week. I haven't looked up anything really. It's usually self-explanatory. You don't need to go any further. Some people said they do but I don't so - I don't know. If you get the same answer I guess you can look it up and make sure you're doing it right but (inaudible).

I: Anything else you want to - any other comments you want to make?

S: No, not really.
Student D - Interview 1

I: This is a problem that was from a quiz last quarter and what you were asked to do with that problem $y = x - \sin(x)$ is first of all just find the derivatives. I'll give you a pencil and let you attack it and see what you come up with. Something you might not be used to doing is sort of talking about what you're doing and how you decide to do it so I might ask you to do that.

S: Want me to do that now?

I: Sure.

S: We just - I learned this in high school so you just use the - I forget - some kind of multiplication rule and you take the derivative. You take the first part times the derivative of the second. The first part is $x$ and the derivative of the second is cosine of $x$ and then you add that to the derivative of the first which is just $1$ so you don't have to write that down and times the second one which stays the same. So it's $x$ goes to $\sin(x)$.

I: Okay. When you have the derivative of that function what can you tell me about the original function? Can you tell me where it's decreasing?

S: Not by just looking at it. With the graph you can. I can't. If I saw a graph of it yeah. Wherever it's -

I: If you saw a graph of the function or a graph of the derivative?

S: A graph of the derivative. Wherever it equals zero will tell you like maximum and minimums and wherever it's positive it will tell you where it's increasing; wherever it's negative it will tell you where it's decreasing.

I: If you don't have the graph can you do that algebraically?

S: I think you can but I don't remember how to do it.

I: Okay. What else could you tell me? Is there anything else you could tell me about the graph of the original function with the derivatives?

S: Just that whenever the derivative is negative then it is decreasing and whenever it's positive it is increasing and zero is always the maximum.

I: How do you decide if it's a maximum or if it's a minimum?

S: Usually just look at the graph and just like if it looks like this you know that this would be the minimum and this would be the maximum.

I: Look at the graph of the function?

S: Right. By looking at the derivatives you probably get something like this. This is all positive. You'd get the minimum. It would hit zero right here at the minimum and then this is also positive so it's still keeping the same. Somehow it is going to hit zero again.

I: Okay.

S: And it starts to decrease. That's where it is. And then you know that these two are where it's maximum and by looking at this you can which is which.

I: All right. Last quarter you worked with derivatives just about all quarter. Can you recall some other things you used the derivative for other than looking at a function and trying to tell what the graph would look like using the derivative? What other way did you use derivatives?

S: To find the slope. The first derivative is the slope and it's like with the function. To get the slope of the function you just took the first derivative.

I: Okay.

S: And we just - I don't know - we used it throughout the entire quarter. Like almost every problem we used to had to find the first derivative for some reason or other. Off hand I don't remember.

I: What types of problems did you use it for, do you remember?

S: Any graphing and stuff like - every time we had to do a graph you almost always took the first derivative. We even took second derivative and stuff.

I: What did you do with the second derivative?
S: That tells you the point where it changes like from - that right there it will tell you where this point is, critical point, no, critical points are these. I forget what that's called. I don't remember but it tells you like where it changes.

I: Okay. Did you say you took calculus in high school?

S: Uhuh. Well it's called math 5 and it was like calculus.

I: Did you spend a whole year in that course or a semester?

S: A whole year.

I: Did you do derivatives -

S: Yeah. We spent the whole second semester on it. The first semester we did like logs and other basic principles and then the second semester we did first derivatives, second derivatives and applying it to calculus and stuff. That was all by hand. It was mostly just memorization and you knew when to use, you memorized the formula and when to use it and then they'd give you another one and you'd do that. We did graphs by hand and usually we got out of doing them because they were too difficult to do by hand. So that's why I think it's a benefit of having the computer first so you can see the graphs. Before it just looked, I couldn't picture it.

I: Did you do anything beyond the derivative that second semester or did you stop with the derivatives the first and second derivatives in second quarter, can you recall?

S: No. We just - I think all we did was derivatives and then we learned different ways to apply them. That was over a year ago.

I: That's okay. We forget.

S: But we're doing parametric equations right now and Mathematica™ and we never did that in high school so - I don't know if we skipped it or didn't get that far or what.

I: All right. That sounds good. Now you finished a quarter with Mathematica™.

S: Uhuh.

I: If you look back over that first quarter are there any just general comments about the course because it is quite different from other math courses you've taken I suspect. Are there any comments you'd like to make on good points, bad points? Anything you'd like to see done differently? Something that's just super fantastic about the course or -

S: First of all the only computer experience I had had was in the computer programming course that lasted one year. At our school the first semester was introduction and then you could take the second semester which was advanced programming and that was all I had and that was in basic and we didn't use the Macintosh. I got here and took this class. First of all I took it because it's on the computer and everything is in computers and the technology is expanding and I'm going to be in business and I want to know how to operate everything. So learning math and computer I think helped me a lot because I had never used the Macintosh. Went there the first day; took the tour. Go in and everything and I thought that was really neat and then I learned how to use it and everything and I think that it has it's pros and cons taking Mathematica™ on the computer because it's good that you get to see the actual graphs and know what you're doing with these problems. Not just memorizing a formula and not knowing what it does. But then you don't have to use this formula. You don't have to memorize them. You're just telling the computer to do it so that's a con. So if there could be some kind of a combined course it would be really good. It was fun because you get to work with people and see how they do it and then you could think oh, that's better. Maybe I should try doing it that way. Or the opposite - somebody will see you doing it that way and they'll take over and do it that way, change the way they do things and you've learned from other people. What I didn't like about it - it wasn't really structured that well and they've made changes to accommodate that now this quarter. People were lost and they didn't know - they wanted to lecture and it's not a lecture class. You're not supposed to have any lecture but they've done some things to accommodate that now. You're supposed to go five days a week like last quarter. Not everybody went every single day but now you're supposed to go and each day they're going to make sure. Somebody was always available but they're going
to make sure there is somebody there for a longer period of time and plus I think once a week we have an actual lecture - you can suggest - figure out what part of the lesson most people are having problems with and then they'll discuss that. That's what we did today and it helped. And then we have quizzes once a week still. Those are written so that kind of makes it weird switching over from the computer and taking a written quiz. That was always hard because before the quiz we had to tell her could she explain how to do this by hand. Yeah, we understand on the computer but we have no idea how to do it by hand. So she had to review for half the period and then we'd take the quiz. It's hard to switch back and forth from paper to computer. That's about it. They're going to help us with that now more.

I: With the paper work versus the computer work?
S: Uhhuh.
I: Okay. Did you work a lot or a little with the other people in the class? You mentioned working with other people.
S: I worked with them probably a lot because some people sat there and just worked by themselves and I mostly was working with other people. They encouraged that. Dr. Davis said for this quarter he said don't work by yourself because you're not going to get done. It's going to be difficult and everything so I was like well that's fine with me because I did that last quarter and it's easier that way and it makes the class more fun. If you're there by yourself you feel lost and what am I doing. That's where I was the first two weeks. I wanted to drop it. It was really confusing but then when you see everybody else is struggling just as hard and then you get working together and you get motivated like come on, let's get this done. Let's figure out how to do it. It's better to work with a group.

I: Is this the first time you've been in a math class where you worked together like that or did you do that in high school?
S: We did that in my high school like on several occasions you'd get a lecture and then you'd get some problems and work together in groups and then go over them and you did your homework of course separately but now we're doing our homework together but in high school we did our homework separately. We just did examples together. So I've been used to working in groups.

I: Estimate about how many hours you spend working in the lab each week - say in a week. Just on the average.
S: Okay. I'm there usually from 11 til probably 2:30 every day. About 3 1/2 hours times 5. Seventeen and a half hours a week.

I: Do you think that's normal for a calculus class or a lot?
S: That's a lot. I thought that was a lot because I wasn't accustomed to that. In high school I would do my math homework and it would maybe take me an hour each day and I was in math - like class would be done at 12 and I would hang out for 2 1/2 more hours after class was out doing work and I thought gosh this is a lot. Even though during class time you get to work. Still you need more time than just class time. You could never get it done in 5 hours a week. Impossible. Unless you're Dr. Davis.

I: Do you read the tutorials and the basics in each notebook or do you jump into the problems or do you do a little of both?
S: I try to just go and do the problems. Give it a try but you usually can't - you need some sort of introduction. Some kind of review and see how they go about doing it on the computer because you may know how to do it by hand but you've got to learn how to do it on the computer. So I go back usually to the tutorial. I don't go through basics because it's really basic. I just go to the tutorial and try to find a problem that looks like the one that we have and give it a try and run through the whole thing and see how they did it and then go through and try to do mine. And then rather than going through the rest of the tutorial for every other single problem I skip back to, give it a try and see what kind of a problem there is and then look for that one in the tutorial and keep jumping back and forth.

I: Did you use the literacy sheet from the notebook?
S: I always run them off before we had a quiz because that's what the quizzes came off of. And I looked over and I glanced at them. They didn't really help that much because they didn't have the answers on them. They just had the problem and you can put down an answer but you don't know if you would get it right or wrong and then some of them are just really difficult and you didn't know how to do them and not having an answer you can't even try and figure out the way they derived that answer. So they really didn't help me that much. I had them but I didn't even study them for the final. I just studied all my quizzes and my notes.

I: Uhhuh. If you look back at that first quarter and some people have a basis to kind of answer a question like this and some don't - it just depends I guess on your friends and what kinds of classes they're taking and what you're familiar with but thinking about that first quarter of calculus in your opinion do you think you got a strong first quarter calculus course. An average one or a weak one? Because it was with Mathematica™?

S: In relation to the other ones that are offered here?

I: Uhhuh.

S: I think it was strong just because you got - you already had the written background and now you were presented to a computer and how it's carried on out there and that kind of makes you more well-rounded in knowing how things occur in calculus. Whereas if I would have taken a written course I still would have been deprived of knowing how to do it on the computer and knowing how it works. I'd just be memorizing formulas and not knowing what I'm doing.

I: Do you know any people - do you have any friends who are taking the lecture 151, not the Mathematica™?

S: Uhhuh. Most of my friends that's what they're taking. I had actually 3 people from my high school were in my Mathematica™ class so we compared you know. We're on different levels. Like it was kind of easy for me and one of the girls really struggled with it even though we had been through all the same classes. So I don't know. I think that was because she worked more on her own. She didn't really get into the groups and stuff but my friends say that they have to do lots of work also. Sit there and write everything out by hand but I don't think that they do as much work as we do.

I: You think you're doing similar kinds of work to the people who are in the other classes?

S: I haven't compared the material or anything. Because I hardly see them since I commute. I just hear about it. (inaudible) this calculus takes so long. It's like I can't imagine it taking that long, as much time as we spend on it.

I: You said you go usually from 11 - or you did last quarter 11 to 2:30. Did you ever work in the lab in the evening?

S: Once we had to get my homework done because I hadn't understood the lesson and I didn't have time to go there during the week that much so I went on the night that it was due because they're due at 10 or midnight, something like that and everybody went there like around 7 o'clock and the whole lab was full. I had been there the whole day. I was at 11 and I stayed there until 5 and then I went and I had dinner and then I came back at 6 and I stayed until 9 so that was a long day. Spending 9 hours in the lab. But I finished everything but then I said I'm never going to put it off until the last day. I had like maybe 1/3 of it done. Still saving that much for the last day was not a good idea. I didn't do it purposely.

I: But you've stuck by that and not done that again, huh?

S: No. But still you finish on the day of usually. There was only one day I finished the day before. Because you usually have questions and you have to skip those if there is not a person there to help you you go onto other ones that you can do by yourself and then you wait for when the professor is there or when a TA is there to help you or when your friends are there and you can figure it out together and select what they did if they got it more. You still go to the deadline. Most people do.

I: Okay. Any other comments you would like to make just about the course in general?
S: It's just really difficult to grasp at first, the concept and everything of how it's run and how you do everything because I was really discouraged the first two weeks. I wanted to drop it. I went to my counselor. Do I really need this? It's an engineering math. I'm a business major. Business majors are instructed to take the 30's, the 130's. Do I really need this? She said well you placed into it You placed in 151.. Try and stick with it and see what happens and I had spoken to Dr. Davis and he was like everybody is lost the first two weeks and I really didn't realize that because the first two weeks you don't know anybody in the class and you don't know if everybody else is struggling or not and then I started talking to people - oh my gosh this is hard so I realized I was in the same boat as everybody else. I wasn't behind or anything. And I stuck with it and now I'm glad I did because as we went along it got a lot easier and from seeing what we have this quarter I think last quarter was real easy now. To get an A in it I was in shock because - I had a lot of potential but I thought that - I can't do this, I can't do this. I want to but I don't know how and then I picked up on how to do it and learned how to do it and I guess I achieved what I set out to do.

I: It took those first couple of weeks -

S: Definitely. Maybe even at the third week I was still confused but after that I think like the fourth week it started like okay.

I: What happened about that time that sort of made it turn around?

S: Just that you had like different problems and they all related to each other. Like at first you were like what does this have to do with anything. You were doing certain commands and functions and stuff and then as you went along they had other ones and you're like now you use what you did before. Like we were given the basics at first and then you started applying them and then it was like the same thing over and over. Then that made it easier because you're like okay, I've done this before. And then they would introduce new things and you would catch on to those. It was just dealing with the computer and knowing how it works and what everything does. What the commands do.

I: So it was more of that adjustment to the computer during that time than anything else.

S: Right, yeah. I think that was the main thing. I don't know how to do this on the computer because you're trying to figure out what you're doing. You're like show me by hand. Show me what I'm doing but they're like it's written right here. They tell you what you're doing. You don't need to know how to do it by hand because it's on the computer. So it's just the change from written to the computer. I think that was probably my biggest problem.

I: Now you've gone the other way because I was showing you one on paper and you were saying if I had the computer then -

S: Then I could show you on the graph. I'm glad I took it and this is my last quarter of math before I start taking statistics. I'm not going to advance any further (inaudible). I'm glad I took it and I'm sure this quarter will be fine now too. This is the first week again. Everybody is panicking. How do you do this? Because we're learning more new concepts and stuff but I'm sure we'll all catch on.

I: You're a little bit more comfortable this time than you were last time.

S: Yeah. See now the problem is not the computer. We don't know what parameters are. Now he's doing parametric equations and we're like what's a parameter? We never had this in calculus. Now that's why we had our little lecture today and now I understand it more.
Student E - Interview 1

I: I thought today that we would start by looking at a problem that was on one of your quizzes this quarter. And you remember I'm interested in what you think about when you look at the problems. And I don't mind - I have a pencil and paper and I don't mind if you write things down to sort of think through what you're doing or what you would do but you don't need to completely finish problems. I'm not going to ask you to come and sit and work a bunch of problems for me. I want to know what you're thinking about and why you're thinking it and that kind of thing. You probably remember this one. I think it was from maybe your third quiz.

S: I probably don't.
I: You're given a function, $x - \sin(x)$ and asked some questions about first just finding the derivative and then how to tell, using the derivative if the function is decreasing. That all ties in with how to get a picture of the graph using the derivative.

S: I'm terrible at doing this. I don't know this - I know how to do it kind of but I'm really terrible at like the (inaudible) rule and the chain rule and all that stuff. I don't know when to apply them so basically I just look and see, $\sin(x)$ - let's see.
I: When you look and you try to decide which one of those what do you think about when you're trying to decide which one of those things to apply?

S: Can I say of? Is it x of $\sin(x)$? Like if this is sin of x. And if it's of then I usually do the chain rule. But if it's not then I just attack it like it's a regular like $x^2$, $ax^2 - bx$, or $+ bx + c$.
I: Did you come up with that scheme there? If you can say of? Or did somebody -
S: Kind of - yeah. Cause if it's of I've noticed it's always a product because it's always f of g of x.
I: Right.
S: And all that so - I had Dick last quarter and he doesn't stress getting the answer correct as much as he stresses you doing the problem correctly.
I: Using the right methods.
S: Yeah. As long as you have the method down. Let's see - my guess here is cos so it would just be $\cos(x)$.
I: Now you used what? You treated that as a product there?
S: Yes.
I: All right.
S: And it's 1 - that's how I treated it. Put the x and negative $\sin(x)$ so it would be 1 times $\cos(x)$.
I: If you have the derivative then how would that help you with that other part? How can you tell where the function that you started with is decreasing if you've got the derivative?
S: I'd plug in things for x. If you can see where the - if you have the derivative. If it's $\cos(x)$ then I know that it's increasing when it's positive; decreasing when it's negative. And we have a max and mins here.
I: Okay. But you kind of knew what that graph - you knew what the graph of cosine looked like. So you're saying that where it's increasing and decreasing is where the function is going to be increasing and decreasing?
S: No. I'm saying where the derivative is positive then it's increasing.
I: The function.
S: The function is increasing, f(x) is increasing and where it's negative f(x) is decreasing.
I: Okay. So that would help you be able to sketch this graph x minus $\sin(x)$.
S: Kind of sketch it, yeah. So I know the kind of - I'm guessing it would be out here somewhere.
I: Increasing and then -
S: Decreasing from this point and then increasing from this point.
I: You're making little faces that look like maxes and mins. Can you go a little further than this session asks you to? What would you do if you wanted to find the maximum points or minimum points of this function $x - \sin(x)$?

S: Find out where the derivative has zeros. I know that it's going to be a max or a min and then I would just plug things in for the function. Plug things in for $x$ and find out where it's actually going up and down so I know if it's going to be a max or a min.

I: When you had this problem to work on a written quiz on a Thursday in class if you were sitting at the computer instead what would you do with this problem?

S: If I was sitting at the computer?
I: Uhhuh.
S: I usually type out - I go through this myself and type it out underneath here so that I've got something to look at and then I'll go down to the questions and try and apply what I've typed in and go from there because Mathematica™ is nice and everything but I'm not real good with computers so I don't know the commands as well and I'm not real comfortable with commands as everybody else is. So I like to do a lot of pencil work first if I can. If I can't do the pencil work then I'll use Mathematica™.

I: Would you initially or after a while actually ask Mathematica™ to find the derivative for you?
S: Yeah.
I: You would try to do it by hand first.
S: I'd try and do it by hand. Then I'd go back and do the clear $x$ and $f(x)$ equals - it's more to find out some (inaudible).

I: To check and see if what you did by hand is the same thing that comes up.
S: Usually.
I: Okay. And then to tell where the function is increasing or decreasing you could look at the derivative and do the kinds of things you're talking about doing on Mathematica™. Is that what you would do on Mathematica™?
S: Whenever they ask me like where $f(x)$ is decreasing, everything I'll find the derivative first. Then I'll just plot both of them and that way I've got both of them on the screen.

I: Like the function and the derivative.
S: Right. He doesn't like that as much but it gives me - I can visualize it better when it's on the computer. It is right there in front of me and if he doesn't like it then I'll take the derivative off or something but it gives me an idea to see if I'm correct.

I: In your thinking.
S: In my thinking.
I: So that helps reinforce that idea of what the derivative has to do with the graph to see both of them up there together?
S: Right.
I: Okay. Not necessarily just with this problem but just in general would you make some comments or statements about derivatives. Things that you've learned this quarter about derivatives? What are they and when are they useful? Here they're useful for sketching a graph or something.

S: I never had derivatives before so I guess I learned is new. But it really helps to understand where there is going to be a maximum and minimum. If you need to know where those maximums are going to be - and like when we used them to do boxes. You have a certain cubic foot or something those become helpful. Now I'm finding that they're useful in tangent line. Determining tangent lines. You take double derivatives and all that. How it's useful. I'm not quite certain how it is but I see that calculus seems to be based on derivatives. Everything we're doing is derivative of some sort.

I: Do you know what the connection is yet with the tangent lines? You mentioned tangent lines.
S: No. I have a sketchy idea that somehow the double derivative is tangent to the curve so how that applies I have no idea.
I: Does the first derivative have anything to do with tangents?
S: I'm not sure but from what I'm guessing is like a double derivative is perpendicular to the
curve and then you take the double derivative to the Cotangent. I don't know.
I: Well you're still in that discovery stage on that.
S: Yeah.
I: How did you manage to sign up for the Mathematica™ course to start with?
S: (inaudible) and he said that he was going to be doing this and he mentioned it to us and I just
need to go to 152. I just need to go to next quarter. I might even go further because I'm
really enjoying math. It's not such drudgery and he mentioned that a computer course was
coming and how they give you, that you make up your own theory. They give you the basis
and then you work up a theory from it and that interested me.
I: That's kind of what you're doing here with figuring out the first derivatives and the
connections.
S: Right. See how they apply.
I: You're still on that theory building process. What course were you in with him? 150?
S: 150.
I: Was that with the graphing calculators?
S: Yes. I didn't like those at all. Graphing calculators you just push buttons and it pops up
whereas with Mathematica™ you usually have to work through and understand what buttons
you're going to have to push to do it. The 148 and 150 are more like monikied series and I
lost so much of my math basis because before then I was doing it all by hand and you could
understand what you're doing. Or at least you would quasi understand it and then the
calculator you'd lose it all because it's right there in front of you. You just push it. Now
I'm starting to work back up to that with 151 with Mathematica™ because you have to
understand it before you can apply it which helps.
I: You used the Casio in that 150 class didn't you?
S: Yes.
I: Have you seen the new TI graphing calculators? They're a little different from the Casio.
S: I've seen them but I haven't really used them.
I: Is the graphing capability of Mathematica™ helpful to you?
S: Yeah.
I: But in a different way from the calculator.
S: The calculator had a limited graph ability and it's a lot sharper with Mathematica™ and to
make a graph by hand it seems like purgatory or something. Having to actually go through
and plot points and everything so you can actually do it and see what your product is. I like
it a lot better.
I: You said you needed 151 and 152 but you might do more because you actually are enjoying
math now.
S: Yeah.
I: Why now? I know Mathematica™ plays a role there but -
S: Mathematica™ plays a big role and because it's more discovery process than input in, output
out. They present you with a problem and you have to tackle it however you would and it's
more real life and everything and I enjoy that discovery process more than learning theories,
applying theories, learning theories, applying theories and that just seems like it's monkey
see, monkey do. This is more - and all the people here - before you never really interacted
with everybody. It's a big class of math. You may have 1,2 people who study together but
here you have everybody working together, all the minds working.
I: Do you work with a lot of people in your class on homework problems and things like that?
S: I work over in the corner and there's about 5 or 6 of us that work together. Which I think is
helpful. It helps a lot because when you're just one person your mind may be going on one
track and you can't get out of it even though you know you're wrong but another person might enlighten you and then together you can push it out.

I: Is it more the Mathematica™ or more that working together in that class that's making you like math more than you did before?

S: If I had a choice I'd say I would do Mathematica™ even if I was in a cell by myself because I really enjoy that discovery process but the people are a help. They are a lot of fun. Even though I'm older than half of them.

I: That's okay. So the course - are there any drawbacks to doing calculus this way that you can see?

S: Yeah because some problems are just so hard that you don't understand them because the basics in the tutorial that they have sometimes it is so sketchy and you read it and you really don't understand it. You need the input from the teacher. Somebody who will put it up on the board and kind of hit you over the head with it. We need that and they said they were going to do and they did it last week but it's already dropping off again and I could really tell this time because I didn't finish three problems because they were just too hard. We didn't have the bull sessions that we were having for a little while. I think we need that. It's nice to have the computer in front of you but the basics are lacking in some parts.

I: So you think that puts you at a disadvantage compared to students who are in the regular calculus sequence or do you have an advantage because of Mathematica™?

S: I think we have an advantage because for them to work out a problem it takes 15 minutes by hand and if it is wrong then they have to go back through and do it again but with Mathematica™ we've got it right there and we can just go up and retype in something and go in a different direction. It's a lot faster for us so even though we may not have the theory that they have we've got the means to overcome that part and keep going.

I: So if you needed to take another math course beyond calculus, even beyond the whole sequence if you are starting the whole sequence then you feel like you would probably be an advantage over students who went through the other course when you got there maybe?

S: Uh - I have an advantage at tackling a problem because problem solving is what mathematics is all about but they would have an advantage over with the theory I would think, the basics. Because we do so much computer work and problem solving that we kind of overlook some of the basics sometimes.

I: When you say basics do you mean things that you would do by hand -

S: Like the theories that come because they have the theories in the basics and they show you how to do them but how many of us actually go through and do all that by hand and everything. I guess it is the handwork that we're missing. The basic handwork.

I: How do those literacy sheets fit into all of this, fit into all the network?

S: I guess they're kind of - I don't look at the literacy sheet that much because I found basically everything we've done in the lessons so as long as I understand the lesson I don't have to understand the literacy sheet and sometimes I overlook the literacy sheet because we do much and it takes so much time that to do another hour or two hours or home it's just too much so I just never look at them.

I: Because they always preface those with this is the stuff you should be able to do away from the computer.

S: Yeah. They always say that but they don't give us any nudging or any help. Like I said the basics are lacking in a lot of lessons. You'd have to have an outside source in a lot of them.

I: Those little quizzes that you take every week like the one this problem came from did they nudge you to work on some of that away from Mathematica™ or?

S: Yeah. It nudged me to do derivatives a lot.

I: Did it?

S: Yeah, it really did because when you're sitting in Mathematica™ you throw up a derivative and it's no problem but with quizzes it tells me that derivatives are important and I should be able to do them and it does.
I: Okay, so if you could change something about the course would you keep those Thursday recitation of ideas? Would you add a day of class? Would you make more lab hours? What would you do to make things better because you had mentioned having those bull sessions and that helping.

S: I think I'd definitely add a bull session.

I: Once a week maybe.

S: Yeah. And I'd have the bull session more with the theory and not with applying Mathematica™ to the problem because when we have it he just shows us what commands to type in for this and what commands to type in for that. He mentions the theory but we're so intent on the commands that we lose the theory and I think we need to do bull sessions on theory. That's my - I know that's what mathematics is about - to build up a theory but we need to do some type of work on it.

I: As often as every week or -?

S: As often as every lesson. Do - like the tangent line. We're doing the tangent lines with Mathematica™ is no problem but when we have to apply the tangent lines it became a little bit harder because I don't know - I don't understand how to. So -

I: You mentioned something a little bit ago about time. How much time do you spend on this class?

S: About 10-15 hours/week. I realize that if you do your homework at home that another hour or two hours out of class and everything but when you're sitting in front of a computer you start blacking out and everything. You're staring at a screen and it's not like I can't this stuff home. I have to go to the computer lab and everything. I guess I spend more time than that. I guess I spend 20 hours. I spend about 3 hours/day, 4 hours/day. Then when the deadline comes I'm usually here more.

I: How do you feel about spending that much time?

S: This quarter is kind of hard because I have this killer chemistry but I don't mind spending the time, not really. As long as there is more people in there but everybody else kind of blows off at the regular times and they come in at 7 at night and stuff. I'm never there at 7 so - I don't have the interaction as much but I like going in now because there's hardly anybody there, Cheryl and Dick and Bill are all there. And a lot easier to get to.

I: In the mornings.

S: Yeah. And now that everybody knows each other it's a lot more comfortable and they'll come over and talk to you and before if you didn't understand something they were the basic math people who - silly, it's this way. What's the problem? That's the problem. I don't know it but now they're a lot better. They sit down and talk which is a lot better in Mathematica™ because you don't have the professor interaction or the TA interaction like you do in Mathematica™ or other classes.

I: What about those students who are there in 254? Do you talk much with them about how things are going?

S: No, because they're usually so busy with their own things. It's hard to go over and actually talk to them. Some of them talk to us but they have their own problems so I don't really say anything.

I: So if they were to add a new section of 151 and there were some people just starting out in there next quarter with you guys do you think you would be real busy with your own work or do you think there would be more interaction? Those two groups would be a little closer together than what (inaudible).

S: I assume there would be more interaction. For one thing (inaudible) to answer somebody else's problems you can get up away from your own problems which sometimes you need that. Sometimes your thought processes are going the wrong way so you could get up and talk. And the age group would - it would be (inaudible) but we've had one section after another would come it would make this a lot more bureaucratic and there would be more people involved and more things involved.
I: So do you think that Mathematica™ ought to have a limited (inaudible)?
S: I think with the scale that we're looking at they ought to limit it the way they are with just taking one class through a year but if they were to have their own computer room like something this big then they could do three classes in here. That would be okay.
I: Do you think they'll ever teach all the calculus courses that way?
S: I don't think so -
I: A lot of equipment.
S: That would be a lot of equipment but who would have thought they would do this 20 years ago.
I: So if you were going to talk to a friend who was thinking about taking calculus would you recommend this way or the other way?
S: I already have. I talked to a lot of people. I recommend it. They gear it towards engineering students but I think - I'm a forest biology student and I think it's just as helpful for me if not more because a lot of these people already know whereas for me it's all fresh. These people have already had calculus in high school.
I: A lot of the people in your class have?
S: Yeah, most of them. Because they all placed into this class or (inaudible).
I: Any parting comments on derivatives and what you can do with them?
S: I'm still learning.
I: You're still learning.
S: Yeah. I'm still learning.
I: Do you think you're going to switch over and learn about something else besides the derivative here before long or are you going to focus on that for a while? Do you know what's coming up?
S: No. I have no idea.
I: It's a surprise, notebook to notebook.
S: Yeah. I don't go ahead because I spend so much time with what we're doing and I never look ahead so it will surprise me. I don't know. Whatever we do will probably involve derivatives. I need to know these cold before I go on. Thanksgiving I plan on doing this, over the break.
I: When you say know them cold do you mean know how to do them by hand?
S: Yeah. At least know the basic, how to get the basic derivatives. If you can get the basic derivatives then you can go on.
Student F - Interview 1

I: Today I picked a problem that was on the third quiz you had this quarter to start with and I don’t know if you remember that one or not, or if it looks familiar to you.

S: That looks like what we’ve been doing I guess.

I: Okay. I’m interested in talking to you about this function f(x) you’re given. First it says that you should find the derivative, right?

S: Uh huh.

I: And then use that derivative to make some comments about some aspects of the graph. Can you talk to me first about what you would do in order to find the derivative of that function?

S: I’d probably use the (inaudible). It might be wrong.

I: That’s okay if it is. I want to know what you would do. Do you want to write something down - I may try to get you to say things as you’re writing too because ultimately what I’m interested in is what you think about when you’re doing these problems.

S: I just try to remember what I learned in practice.

I: You try to fit them into a pattern of some kind or -

S: Uh huh.

I: Okay.

S: Oh -

I: That’s all right if you can’t think right off hand how to do it. It’s not a test you know.

S: I guess I’d use the chain rule.

I: You write that as \( \frac{1}{e^x} \) because -

S: Because e has a negative x.

I: Negative x. Okay. Is that the kind of thing you would do then to find the derivative?

S: Yeah.

I: Once you find the derivative this question is asking you to find the maximum value that f(x) can have and whether you actually do that or not how might you find the maximum value of the function using the derivative?

S: I’d set the derivative equal to zero and then find negative x. If there is more than one we’d have to find out if it was a maximum or minimum once we got the derivatives.

I: How do you do that?

S: At the time of this, this was the third (inaudible). I would have tried values on each side of the x to see what the curve was doing. At this point I’d probably take the second derivative of the function. And see if it was positive or negative to see if the curve was concave or convex.

I: Okay.

S: That would be easier.

I: What about the intervals where the function is increasing using the derivative? How would you decide where it was increasing?

S: If the derivative was both positive and negative. It was positive for increasing.

I: You just check random points to do that or -

S: I’d probably check on both sides of maximum.

I: Check on both sides to see what it’s doing.

S: Uh huh.

I: This asks for a sketch of the graph. And it said include global behavior. What’s that asking for?

S: Once x gets really large than (inaudible) shape can go somewhere - negative infinity, infinity or zero. It could be any number actually but I’d look and find the dominant terms which in this case I guess would be x. When x got large I guess it would put it there. So I’d just check to make (inaudible) it was a graph like that.
I: When you said check to make sure it was graphed that way you mean check your graph or check by putting some points in to see if that is in fact what it does?
S: I could do that. I guess I could find the maximum and then plug them in and find out where my limits were and then kind of draw it. I could put in points to check it but if it was a quiz or something I'd probably skip that one. It depends on how specific I was supposed to be. Most of our graphs on the quizzes are pretty general and so you can just sort of put the graph where it's supposed to be without going through the exact points.
I: So a sketch really means a sketch in other words.
S: Right.
I: Okay. Can you make some comments just in general based on what you've learned this quarter about the derivatives and what a derivative is and here you are asked to use it to help you look at the behavior of a graph. Are there some general statements you can make about things you can always do no matter what your function is? Things you can always do with the derivative in general to help you figure out the graph -
S: I think it's pretty interesting actually. I think you can use lots of different things from engineering to economics to find out where instantaneous growth rate (inaudible) for x at a certain point and to find a slope of a graph at a certain point we can always take the derivative. Of course in this stuff we're doing now about the reflections of parabolas and stuff like that you need to have a slope and I guess that has kind of opened the doors to what a derivative is used for.
I: So instantaneous growth and slopes of curves at different points.
S: Uhhuh. And it always tells us like where the graph is increasing and decreasing if you don't have a graph to help. Take the derivative at a certain point. It can tell us that,
I: Is it helpful to know that if you're going to use Mathematica™ to do the graphing?
S: Yes.
I: Why?
S: Well Mathematica™ isn't always going to be able to help me really.
I: Okay. How about the course work? Did you take any calculus before this?
S: Well I guess my situation is kind of unique because I've been out of school for two years. I took - I went to school out west and I took a calculus course is what they called it and it was quite difficult. It was difficult but when I came here and tried to take the calculus course in the midwest it was a lot harder and so I had to drop down in pre-calculus which was just about a step above the calculus course I took in the west. So I took two years of pre-calc as I see it and then I was out of school for three years, 2 1/2 years and so -
I: Was that two years of pre-calc here at OSU or some place else?
S: It was in high school.
I: Okay. It was also in high school that you took what they call a calculus course.
S: They called it calculus but I don't think it was a pre-calc. It was one of those years that I consider pre-calc.
I: Did you do things like derivatives in that course?
S: Yeah. I remember taking the derivatives of numbers, not so much e's and x's and logs and that sort of thing.
I: Polynomials and -
S: yeah and I don't remember ever graphing it and understanding that it was the slope of a function at a certain point or anything like that. I guess I didn't understand that until I saw it on the screen with Mathematica™ and I could see what it was doing and the questions applied where from lots of different fields and so you could see how it works in the world.
I: So then do you think the Mathematica™ is a good way to learn calculus?
S: I think it's excellent.
I: Based on your experiences from before.
S: Yes, definitely.
I: Do you think you will have the same calculus skills, the same understanding of calculus that students who take it in a regular lecture course will have?

S: Well it's hard to say what - I don't know exactly what they learn or how they learn but as I understand it they don't do a lot of graphing and it seems to me like the graphing is what helped me understand what the derivative really meant and how it could be used. So I would say that - I'm not sure if their understanding of other things is greater but I would say that Mathematica™ adds something to the understanding of derivatives in that way.

I: Because of the graphing?

S: Because of the graphing.

I: You think they could do something like that in the other courses using something like these graphing calculators?

S: I never used one of those so I don't know how it works.

I: Well this one is strictly - this is a Casio and it will just do graphs but there are some that do a little bit of symbol manipulation. Some that will actually evaluate that derivative in other words for you. Similar to what Mathematica™ does. But you can have this calculator graph that function x times - I'll let you see it after I get it in there. I made the (inaudible) probably not too helpful to you. e raised to (inaudible) minus x power. If you just take the graph key, can you see that? I can do a little bit of contrast here. Just put in there. Well you can see the one that it graphed is this one right here. I think I had a couple of sine curves there already I didn't know I did. Let me get out of here and clear that out. I cleared that out. Let's try this without that there. You can get the alphabet by using the alpha key times - let's try it again. e to the negative x. Graph it by itself then. You hold it so you can see it. So that you could graph that curve. Maybe you would if you're using this type of calculator evaluate derivative by hand the way you would have on the quiz and make your predictions about the increase and decreasing maximum, minimum values. But you can get the graph of that up there just the way you could on Mathematica™. Then you can have it graph the derivative right along with this. And see how the two compare.

S: I think that's good. I think it doesn't compare to Mathematica™. First of all I don't know how much these cost. It could be an investment for each student. (inaudible) take calculus (inaudible) do that. And if they have to buy one of those - Mathematica™ you can - it does a lot of work for you too. You can put a point on there. Then you close it up (inaudible). I don't know exactly what you can do with that but you can put a point on there. You can evaluate S and X at any point. The person writes their answers right there on the screen. Actually the reason that I took Mathematica™ was first of all I thought computers are (inaudible) pretty convenient. You wouldn't have to write out a whole lot of things when you're doing stuff and I guess another thing was that I needed help in my typing skills.

I: You took Mathematica™ to practice typing?

S: Well it was just a secondary benefit. I don't think that really compares with Mathematica™.

I: Not as far as the power and what you can do with it. But I was wondering if since the graphing seemed to be one of the important features of Mathematica™ if you could replace at least that much of Mathematica™ and all the other (inaudible).

S: Can you expand a graph on this?

I: Uh huh. You can zoom in and out. I can zoom in or out. That zoomed in a little. I can zoom back out and out further. Of course screen display is not as large as it would be on Mathematica™. So I can get a bigger picture if I was going on or I can go in, zoom in on a small part of the graph. I can actually place my cursor at a point on the graph and zoom in on that point. And you can on this do something like read points off. There is a trace feature. If you press trace then you can move over. These are cursor keys just like the arrow keys on the keyboard and you can move over. There is (inaudible) if you hold it so you can see it and you can read the x value of point and you can flip back and read the y value too. So you do have some of those capabilities. I agree - not as much as you have on Mathematica™.
S: I think that would help. I think it would help other people - first of all graphing - I talked with one of the professors of one of the 151 courses and he said that they don't do much graphing and just because it takes so long to find each value for (inaudible) and draw the line. I think this would help definitely. You could (inaudible). I think seeing the graph is important.

I: Okay. How did you find out about the Mathematica™ course? You said you took it for several reasons but how did you find out about it?

S: Freshman orientation was kind of confusing. They gave us the placement test and told us - actually I was given the wrong placement test so I had to take the right one and like the first week of the course I barely got into Mathematica™ so - I was allowed to. He gave us a little printout of the different courses that were offered and talking about a 151M, working with computers and I thought it sounded interesting. I figured that the class would be probably the most popular of all the calculus courses. I was surprised when I learned that there were few that actually signed up for it. So I went to Dr. Davis and said do you have room for one more? And he did. I got in okay. They gave us a printout -

I: With it listed on there. Okay. But not as many people are signed up for it as you would have expected.

S: No.

I: You thought they'd be jumping in it.

S: I thought they would.

I: Maybe they don't know exactly what it involves. Maybe they're a little computer shy still or something like that.

S: I don't know. I always took computer class in high school. They didn't have a whole lot of computer classes offered but I took them all. (inaudible) and it was kind of different because every computer is different.

I: Did you know about Mathematica™ before you took the class?

S: No.

I: You knew it was with the computer but you didn't know about Mathematica™ itself.

S: No. I'd seen a little bit of the Macintosh but never worked on one or anything like that.

I: How much time do you spend doing your homework? How does that compare with -

S: With my other classes?

I: Your other classes.

S: I spend twice as much time in Mathematica™ as I spend in all my other classes put together.

I: Really.

S: That includes class time and homework.

I: How do you feel about that?

S: To tell you the truth even though I like Mathematica™ a lot because I had been out of school for a long time I felt like I was way behind everybody else. That I wasn't grasping it. In fact I considered dropping down. That's why I talked to some other professors and found out what they were doing. They kind of said it's not compatible because we do things in a different (inaudible) and stuff so I said okay, I'm just going to try to stick out. I found out that I'm not the only one that felt frustrated and like everybody else understood everything and they didn't. But I was (inaudible). But last night as I was in there until 5:30 trying to get that done some of those thoughts come again - how can you have a life and still take Mathematica™?

I: Okay.

S: Last week - everybody has some personal things they have to take care of and I had something and I just didn't have enough time to be in there for three hours those days and you just get behind. That's why I was in there all day yesterday. It's tough that way for me. Just the time that you - I was working, before I was working on a job being a waiter and I was only doing that a few nights a week but the combination of having a family and having school that takes this much time I had to find something else.
I: Are you going to stay in the Mathematica™ sequence?
S: That's one thing I've thought about and since I haven't been working as much it has been a lot better. I think it would be easy for someone that lives on campus here, like at the dorms and stuff. I think it would be a lot easier but when you have a family and you have to work and stuff it's tough to find enough time to do it but I decided anyway just take (inaudible) because I felt like I was learning. I was learning more in there. That's one setback I would say with the Mathematica™ course. The only other one is that the time with TA's is very limited. So when you - luckily Dick was there last night doing some homework or else I would have had to turn in my (inaudible) incomplete. That happens quite a lot with us. I had classes until 11 and Cheryl and Dick are there from 8 until 12. So that leaves me one hour with them and that's just - a lot of these ideas of just basic calculus I don't remember that I need refreshing on plus Mathematica™ plus the new material which is a lot of the actual instruction that I feel like at least I need. It is hard to get.
I: And then that hour that you have in there that they are are there are lots of other people.
S: I figure if you ask an in depth question in that hour - I came in here Sunday night and I worked Sunday night and luckily Cheryl was in there so I could ask her a couple of questions but she's really overrun too with all the homework. Not only does it take us a long time to do it it takes her a long time to correct it and when she does go in there to correct it there is all those students asking questions and stuff. So - two things that I would like to see happen with the course is one that there is more time with the TA's. I think the TA's here are great but I think they need some more of them and there are (inaudible) that if maybe we could have a little classroom instruction as to what we're going to be doing in Mathematica™. I don't see that we need to know the answers. That we don't need to figure things out on our own. I just feel like we need some basics and sometimes it's hard to understand the basics in tutorials. If you do have time to read them. I usually print them out and take them home and read them but still they're often difficult to understand completely.
I: So would you suggest something like maybe meeting in a classroom setting, more typical classroom setting once a week or something like that?
S: Yes. I think that every student has to spend on the average of 2 hours in the computer room and I think that that student probably needs 3 hours of instruction to be able to really know what we're doing in the course and they say that it's common that if I do (inaudible) 254 felt like we did. That they are frustrated and didn't know what was going on and they're doing okay now and I think that's fine but I think that still (inaudible).
I: Have any of you said anything like that to -
S: Yes.
I: Because I know last year they did make some adjustments along the way in what they were doing to benefit the students needs and it helped some.
S: Well Mathematica™ is still in it's growing period.
I: Right.
S: And I think it's getting more and more popular and there is going to be a need for more and more TA's in the course.
I: Right.
S: I asked Cheryl if maybe we could on our quiz day if we could have a little bit of review and maybe take the quiz and then maybe a little preview of what we will be doing. I think she liked the idea but then there is that factor of time. We only have an hour in that classroom and that's just not time to do everything. I talked to Bill Davis too and asked him about it. I think that they hear those things that they're kind of shorthanded and I feel like sometimes the TA's are as frustrated as we are so I think we're going to (inaudible) together on that. Going together.
I: Right. How do you like that feeling - working with the people in your class, of kind of learning together.
S: That's really interesting because our class as far as the class goes I think that we know - this might sound kind of funny but I think we have sort of a close class. We're kind of - I think we talk more than other students do usually in class. That we kind of know each other more because we're all going through the same trials sort of. Like this party we're having. That kind of shows that we're kind of - it's interesting. It's a good setting. I enjoy it. And I like the feeling that - I like knowing the TA's. They have tough times in it too and that (inaudible) together because like I said when I felt like I was - I was the only one that didn't understand it because it was tough. When you get into other people who are just about ready to give up and you all help each other - I like that.

I: Do you work on problems with other people in class a lot? I know they kind of promote that.

S: Well sometimes I do. I do more than I used to. This last (inaudible) I didn't because I was here at odd hours. I had to be. But usually yes and we kind of share information and insights into our problems. I think that's a definite plus too. I think that helps. That's a problem (inaudible).

I: Well I wanted to use the first interview just to look at a small problem -

S: I think you'll find that most people (inaudible) even two weeks ago if you had talked to people they would have been more expressing their concerns and then as it goes later it's mostly just the problems anyway.

I: But you went through a kind of a panic stage as a group?

S: Yeah. I'm not saying that it's completely over. There are students that won't be enrolling in it next quarter.

I: Do you think it would be all right to stop Mathematica™ this quarter and go into a regular lecture section next quarter? You said you had talked to a professor earlier this quarter and considered dropping back into one of those but they said you didn't cover things in the same order.

S: I'm not sure if each 151 course has to cover certain material but I know that within that 151 course they're not covering the same thing at the same time so maybe we'll cover at the end what they covered at the beginning or vice versa. I'm not sure how it goes from one course to the other but I'm really not willing to take a chance.

I: I think you're probably right that they cover an amount of material and you cover an amount and maybe at the end you end up -

S: I'm not sure if it's in the end of the quarter or if it's at the end of the sequence that we would have caught up to each other. Like I said, I'm not really willing to take a chance. If I felt like Dr. Davis and the TA's weren't really caring or understanding of our complaints I probably would have dropped it but I think they are and that they're doing their best to help us.
Student G - Interview 1

I: What I’m going to do in all of the interviews is look at some problems that you’ve either seen maybe in homework, maybe on a quiz. Sometime later in the interviews we’ll probably look at a problem or two that you’ve not seen before. What I’m interested in is how you approach the problems. What you do to them. How you decide what to do and that kind of thing so I’m going to ask you to try to do problems with me during the interview times. I’m going to ask you to talk about what you’re thinking about when you decide to use in a problem and that kind of thing. The first interview I got a problem off one of your quizzes from this quarter that I thought we’d look at to just kind of get the feel for how that would work and then spending some time on the first interview just getting general comments from people on how they feel about the course after one quarter since it is so different from other types of courses. Other types of math classes that you might have had. And this interview is fairly short. It’s just to sort of get started and see how it’s going to work and that kind of thing and get a little bit of feedback from you. The interviews next quarter I hope that at least one of them we can do with a Mac and let you do some stuff on Mathematica™ with a problem so that you can also talk about how you decide what you’re going to use in Mathematica™ when you do a problem. Let’s start with the problem first and then we’ll get some comments from you.

S: We did that problem today in class.
I: Did you.
S: Uh huh.
I: I didn’t know you did this today. This isn’t going to be fun then. You probably know everything there is to know about it. Why did you go back over this problem today in class?
S: Because we’re having an exam on Monday.
I: Oh really. In recitation -
S: I can tell you all the answers in Mathematica™.
I: Can you really? I picked a bad problem here didn’t I? People who have interviewed before you up until now have had a time with it.
S: I had those problems in math. See I took calculus in high school and we did this but it was so long ago.
I: Really. Well you can give me a review of what you decided you were supposed to do to this - xe^-x.
S: You just want the first derivative?
I: Yeah, the first derivative. You might have looked at a little more than that today.
S: On this you have to do the product. So you just take the derivative of the first times the second times the regular second plus the first times the derivative of the second, which is negative x. I think that’s right. You factor this out and (inaudible). It would be 1 minus x.
I: When you had that on the quiz you didn’t necessarily do that. Now that you’ve gone back over it it’s -
S: On the quiz I think I bombed this problem. I had no idea - because see she wanted us to graph it and everything. I can usually graph it but not e’s and stuff because I’ve always used the calculator and computer. The maximum value f(x) in half. Increasing (inaudible) the graph. So there is no way I can sketch it without a calculator. Maximum value f(x) can have. As x gets larger - is that what you mean? (inaudible).
I: The maximum value - the largest value that whole function f(x) can take on.
S: Hmm. I hate this kind of problem. This is engineering math and I should be in business math. As this gets larger e is going to get smaller. This goes through - this goes -
I: You’re looking at the function and deciding what happens when you put large values in for x.
S: Yeah.
I: Are those the places that the function itself is necessarily going to have it’s maximum value?
S: No. Maximum value is going to be when you solve this. Maximums and minimums when you solve the first derivative which is 1.
I: Okay. You solve for the first derivative how?
S: This is the first derivative and you solve for it which would be x equals 1 so 1 it's either max or minimum and you determine from - maximum value. Is this what you want to know?
I: Uhhuh.
S: Like 1 - so that's it.
I: Is it a maximum or a minimum? How do you decide?
S: You take it from negative infinity and 1 - this is from 1 to positive and then you plug it in. You plug in a zero into this.
I: Into the first derivative.
S: Right and you get plug 0. This would be 1 times whatever times 0 which is 1 and this is 1. It's positive and plug in a 2 from positive infinity you get - this would be negative 1. You get negative 2. This would be a negative number so it's increasing and decreasing. So it's a max at 1. And intervals -
I: Where it's increasing.
S: I just did that so it's increasing and to negative infinity and it's decreasing to positive infinity and to the graph because you know at 1 there is a max and then you want to know - it goes through - see I'm remembering it from - because I know it goes through (0,0). I forget how she got that. I know if you take the second derivative you would get - you could leave it like that. Negative x times minus x times the derivative of this. Negative 1 plus x times x minus x. So this would be negative. A negative x plus the negative x minus - I don't want to do that. Yeah, why not? Minus x times (inaudible) and carry these two. So you have a negative. That's going to be the second derivative.
I: What can you use that second derivative to find?
S: The second derivative refers to the concavity. The concave up or concave down. So at zero this is going to be - this is going to be a zero. That's not right. This isn't right. It's like a 2e^2. I know that this is concave down. It's going to go like this.
I: Uhhuh.
S: Because you know when you take the limit of this it's going to zero. Because the e never goes - e is never negative. So that's what's going (inaudible).
I: So just in general to sort of summarize with the first derivative what kinds of things can you tell about the graph?
S: If the slope is increasing or decreasing like with wings. Like before this point it's increasing because it's positive and after this point the slope is negative so it's decreasing.
I: So you can tell if the function is increasing or decreasing using the first derivative.
S: Then you can also find the zero points of the first derivative when you solve for it. You can find one where the function crosses the zero.
I: By setting the first derivative equal to zero?
S: Uhhuh. (inaudible) do that.
I: You said that equals zero there.
S: I was trying not to make that problem (inaudible) maximum. Yeah you find the - I can't remember - the second derivative is the concave up. She doesn't like to teach us that because we're not supposed to know that yet, supposedly.
I: The second derivative.
S: Right. Like today when she explained this she didn't use the second derivative but it's a lot easier because we went into it last year and then this part with the answer (inaudible) because it's increasing before and after.
I: Now what other kinds of things have you done with the first derivative this quarter? What other uses could there be for the first derivative rather than figuring out some things about the sketch of the graph.
S: You can use the - find the tangent line. Find the tangent to a point on the graph you need to know the slope so you'd have to use the first derivative.
I: So the first derivative gives you the slope of -
S: The slope of a function of a particular point which you need to know so you can get the tangent line. You have to use it for the like the order of contact that we had to do. I had never done that before. Like an order of contact of 2, the function has to be equal. The first derivative has to be equal for two functions and then the second derivative has to be equal so you don't use first derivative for that. Basic thing is just the slope. That's what it's mainly used for. Plus you need to have the first derivative to get to the second derivative. That's obvious. If you skip the first derivative you're never going to get to the second derivative like I did down here.
I: Or not get it.
S: Yeah, or not get it.
I: Yeah.
S: See these - I don't like the e's at all. I can (inaudible) a lot better in other derivatives.
I: Is it useful then the first derivative?
S: Oh yeah.
I: For the kinds of things you're doing with it?
S: It probably is - this quarter - first quarter was probably the most important thing, definitely because I mean last year in calculus in high school we did a lot of first derivative. This year we did it but it was more we got it all in one unit. Like last year we like spread it out. Like we'd do just like the first derivative of polynomials and then like cosine and sine and tangent. Like broke it up. This year it's just all in one. That's what I like. A lot of people this year I don't think really took it real well. That's why a lot of people aren't taking this class next quarter.
I: Really? Because it was all together and not -
S: It was too - because they had to learn how to use the computer and to learn the math at the same time and they couldn't handle it.
I: Do you think that's mostly people who hadn't had calculus before?
S: Uhhuh, or hadn't done that well in calculus. I know a lot of people aren't going to take it next year because of next quarter. If I hadn't taken calculus I'd probably be lost in it.
I: Really?
S: Yeah - because it wasn't really - because I look at people that are in regular 151 and they're a lot further along. But since this type of math isn't that important to my major I don't mind.
I: So they're doing things like way ahead of where you are then?
S: Uhhuh, yeah.
I: What are they doing? Do you know?
S: Not really. I'm just like I can just tell people I'm doing this and they'll say oh we did that weeks ago. Like my girlfriend at Miami she's taking 151 which is basically the same thing and she did this stuff a long time ago because I sat in on one of her classes and they were doing a lot harder stuff.
I: What about the kinds of problems that you did? Are they easier or harder?
S: Like in the homework -
I: Than what they're doing -
S: They're harder I think because they're more like word problems. They're not like just the cosine and find the first derivative like we have to do. I think they're harder. They're probably more useful. Because they apply to like every day life.
I: We're kind of getting on that subject of the course. Do you have any comments just to make about the course in general after one quarter of experience behind you or almost behind you?
S: I don't know. I like it because I like computers. I think it could be taught a lot better. Because Dr. Davis is never in there. I have not seen the man for three weeks. I saw him
yesterday and Dick and Cheryl are never there. When we have questions no one can answer them.

I: So you've kind of been on your own for a while now.

S: Yeah. It's been like that a lot of the quarter. Even when we do ask them they'll explain it but I always don't understand it and the tutorials I don't know (inaudible) the tutorial. I don't understand the tutorials at all.

I: Do you usually print those out and read them?

S: No. I don't have enough time. I think they're worthless. I mean I look at the examples - what they do. I don't like read - like beforehand I look at the examples and how they did it and I can understand it better from that. (inaudible) the course is - I think it's worthwhile but it could be taught a lot better I think. It needs to be changed.

I: How? How would you like to see it done?

S: I would like to see it like when we do the homeworks that they don't explain the homework at all. We don't know if we did it right or not. That's what happened last year in calculus. I hated it. The teacher would assign us the homework. We'd do it and then the next day he'd start on a unit and we had no idea of what we did and if we were lost in the unit before because of math you build - if you were lost in the unit before you're lost for the rest of the year and I think that after they had the homework, after they have graded it they should sit down and explain to everyone now this is what you did wrong and this is why it's wrong because I've gotten bad grades on some homework and I have no idea why. Because they don't explain it. They go on and they assign something the next day and you have to start on that. You don't have time. I think they should cut out like a couple of units - you have to understand what you're doing. It's like the race track principle. I don't understand what that is. I never did understand it. It showed in my homework too and no one since has explained it to me and they're not going to because we're in a new unit. So I think that they should have to explain it afterwards.

I: That's pretty frustrating.

S: yeah. I don't like that at all because my teacher did that last year and it showed. I got a C in calculus last year and when I took the calculus placement test here at Ohio State I got placed in the highest level and I could have taken 161 if I wanted to but I didn't because I didn't want to be so advanced my first quarter. I wanted to stay with something I already knew. But they still really don't - I think they should still teach it a lot better. It's only the second year so -

I: How would you suggest they do that? Have more recitation or lecture class or -

S: I think they should have maybe like 2 instead because we only meet once a week. I think we should do it like maybe twice a week. Like the one we have a quiz. That takes out because we have to take a quiz at the end. We can't talk about the homework the entire time because we have to take a quiz. I think we should have like another one where we talk about the homework. About the problems and how to do it like if we don't understand it. I think that would be a lot more helpful. Then we could go on to the next unit and we'd know what we did in the last one.

I: have you made that suggestion to anybody?

S: We had to fill out a thing today and I told her today. I said this is what is wrong a lot of people really - because they're supposed to be - I think she said 17 people in our class and 4 haven't even shown up the past month. They just gave up on it. They knew they weren't going to pass it so they didn't show up any more. I'm not going to do that. I understand it but the only reason I can understand it is because I took it last year. If I didn't I really think I'd be lost.

I: What other feelings about the course in general?

S: I don't know. I like it because we get to use the computer because I want to know how to use computers. I want to go into them in my career. The math itself like I said the word problems, I don't like word problems but it's more helpful because word problems are more pertaining to like regular life. But I think it's a good idea but it's only the second year. It's
going through it's problems. So I mean just have to accept it I guess. It is not going to change by next quarter. (inaudible)

I: There might be some changes. Maybe from the suggestions that you guys make.

S: They have to realize that they can't do that. They just can't - they can't just grade your homework, give you an F and then give you a new homework assignment. What do you learn? Obviously you didn't know what to do and then they just assign you another homework the next day and you can't do that. You go to college to learn. You don't go to college to do work and just get pass or fail. You're supposed to learn what you're doing.

I: You don't see any place that that's happening in the way it's set up now?

S: No.

I: Okay. What about the time you spend on that course? About how much time do you think you spend on that class a week say?

S: In college they say you're supposed to spend like 2 hours for every hour you're in class. So I'm supposed to spend 10 hours a week in it and I spend easily over that. I spend probably 14 or 15 - maybe more. It depends on the homework assignment in a way. Granted, that's probably, granted that's not all doing the math. People don't do that in class either. People don't go home and study for two hours straight. Just sit there and talk or whatever. I probably spend maybe up to 16, 17 hours a week in there just working on math. And that's not that bad. I don't mind that. Like some of my other classes - you've got a book and you have to write out all this stuff. I'd rather do it with the computer. That's why I'm taking English 110C next quarter. It's on the computer. And 152M. Next quarter I think there are about 4, 5 kids aren't going to take it. The class will be plenty small which is probably better.

I: I called a couple of people to set up interviews who said that they weren't going to take it next quarter so -

S: Yeah, they're not. (inaudible) isn't. The one girl - I don't even know her name. She hates it. She's not going to take it. A lot of people aren't because they're just intimidated by they have to use the computer and learn the math at the same time or like I said they have no idea what they're doing because no one explains it to them.

I: How did you find out about this class?

S: It was my academic advisor. After I took my placement test we had to go in and schedule classes. She gave us a paper and she said or you can take this class if you place in the L level which I did. And I didn't really think that much about it. One of my friends, he has an IBM, he really loves computers and he saw it and he's like why don't you do it? And I still didn't want to do it. But then I talked to Java - he took it last year in the first year. His brother was taking it and his dad is a professor here at Ohio State and he's like yeah, it's a really good class. Why don't you take it? So I did because basically Java said it was really good but even Java is in 254. He's not taking it next quarter. He's going back to the regular because he said next year is really hard and I'm not going that high so I'm not going to take it. I'm taking 152 and that's it.

I: That's just two quarters of calculus is what you need.

S: That's all I need, yeah. That's all I'm taking. I'll take statistics and stuff. I took statistics in high school and I liked that. So I don't need any more calculus. That's all I want.

I: If you decided now that you didn't want to stay in the Mathematica™ class do you think it would be easy, hard or not make much difference to move into a regular 152 now?

S: I think it would be harder. It would be a lot harder because you're not used to getting out the book, doing 20 problems on a piece of paper and then having to turn them in the next day. So we're used to having an entire week to do it. Granted like the first day we might not do anything. Do more at the end but everyone does that. But I think it would be a lot harder. I think the people that are going back to the regular 152 have more problems and so if they stayed with it and got used to it now.
I: What do you think about Mathematica™ itself? Not just the class and the way it's set up and operates but the actual software Mathematica™?

S: I think it's a good idea. This world is going more and more into computers. Kids nowadays they have graphing calculators that they use. They had to buy a graphing one in 148. It was required that they had to have this graphing calculator.

I: Did you use those in high school?

S: No. I just had a regular scientific - my friends did though. They had - and I'm using them - they would graph them and you wouldn't have to like do this. You'd just plug it in and it would graph it for you.

I: Right.

S: But I think the computer is a lot better because you don't have to go from learning how to use the calculator and then learn how to use the computer. Just go straight to the computer - what you're going to have to use, whatever. I mean computers are everywhere. You might as learn how to do them now.

I: What's the most useful thing that Mathematica™ will do for you and that you have used at least so far in calculus? It will do a lot of things. It will graph. It will do the computations for you and simplify.

S: Just the computations. Because I'm just lazy. I mean I could do these by myself but I mean they're a lot slower and definitely more prone to error because computers don't make errors. I think more though it's not what the calculus - it's with the computer and using it which is a lot more important. Because I know people that are just hate computers. They won't even touch them. They think they're going to blow up if you touch them so - you have to learn how to use it or you go in your first day say I want to be an accountant you go in and you have to enter all this stuff in the computer and they're going to be lost. They're just afraid to touch the key. Come on. Nothing is going to happen to you.

I: You do or don't have your own computer?

S: I don't. I used to have like - but I never had anything like this. One of my friends did but I just use the ones down here at the lab. I don't need to buy my own. Maybe in the future I'll probably get one definitely. I can do my work in my own house. I won't have to go to work in the morning.
Student H - Interview 1

I: The first interview I'll pick a problem off of one of the quizzes you had this quarter to start with. Just to see what you would do with it here and to talk a little bit about why you do what you do and that kind of thing. You were just given this function \( f(x) = x - \sin(x) \).

S: You want me to do this?

I: Yeah. Okay. You wrote that the derivative was \( 1 - \cos(x) \). What did you think about when you looked at that problem to decide how to work it?

S: Well I had to take the derivative of x and the derivative of \( \sin(x) \) so each one separately. So it's \( 1 - \cos(x) \).

I: What kinds of tools, for example, I guess would be a good word for that did you sort of sort through in your mind in deciding to do the derivative of x and then the derivative of \( \sin(x) \)?

S: You have to know the rules of derivatives and that since you have two quantities x and \( \sin(x) \) you have to take the derivative of both of them.

I: The next part of this question on the quiz was to use that then to decide where that function is decreasing. What would you do to -

S: Set the first derivative equal to zero and find the critical points and then take the second derivative and if it's positive it's evaluated at the critical points it's a maximum and then you know if it's a max it's positive. You know \( f(x) \) is decreasing and beyond that point it's a minimum. You know that it's increasing over the interval for that point.

I: Could you at least start some of that and let's see kind of where it goes?

S: If you take the first derivative - equal to zero - x equals 1 so cosine is 1, zero and 2 pi. So if I know the first derivative is 1 then I evaluate - I could do this second derivative which is \( \sin(x) \) and \( \sin(x) \) is already at 1.

I: If you evaluated the sine of 1 then you would use that then to decide -

S: If it's increasing or decreasing because if you evaluate the second derivative at the critical points then you know if it's positive then it's functionally increasing.

I: Is there any way to decide if it's decreasing and only use the first derivative?

S: If you find the slope and it comes out negative. The first derivative is the slope so if the slope is negative then you know that the function \( f(x) \) is going down and if it's positive then you know it's increasing.

I: What about on this one?

S: The cosine is 1 so it's not decreasing. If you would evaluate the sine of 1 then it would tell you - even if you didn't pick it up here if you went through and solved for the second derivative and you got zero or something or you couldn't do it then you would know that the cosine would never be decreasing.

I: So do you think from what you just told me that this function then is never decreasing?

S: I don't think it would decrease. No. I don't think so.

I: Okay. Now with this problem it looks like we could tell some things about increasing and decreasing. You mentioned that. You mentioned something about slope with the first derivative. Explain to me a little bit more about what you mean. What's the connection between slope and the first derivative?

S: Well instantaneous slope - if you take the first derivative finding the slope at a point, you evaluate the first derivative at a point and finding the slope at a point.

I: The slope of what?

S: The slope of the \( f(x) \), the graph that you're trying to find. If you have a graph and they give you a point at that point where \( f(x) \) is over the interval then you can find the instantaneous slope at the point.

I: Now you've done several things this quarter with derivatives. You used derivatives to solve several kinds of problems. What are some other things besides using the derivative to get an idea of what the sketch of a graph would look like for a function? What are some other uses for the derivatives that you have encountered so far?
S: Finding (inaudible). We used it in functions in like max and mins of boxes, areas of boxes, volumes. And once we had graphs of polynomials that you didn't really know for a fact that they were like $x^2$ is kind of easy to graph but when you get into $x^3$, $x^4$, $x^5$, stuff like that they're kind of difficult to solve. And just stuff like stuff that wasn't really as easy as the stuff we'd encountered before.

I: Right. Did you take calculus before in high school?

S: Yeah I did but see it's hard for me because we used - like when I do a problem like this I have my notes organized where it's not so much that I have them memorized where I can look back and find them. Like if I take the first derivative and then set it equal to zero I know I have critical points but then you can find concavity by taking the second derivative and evaluating it at the critical points. Point of inflection by evaluating the second derivative at equal to zero and then you find concavity. If it's concave up or concave down and if it's increasing or decreasing. I just get those two mixed up. Concavity and increasing or decreasing so I just look back at my notes every time.

I: Your notes from high school?

S: Right. We went through more of a regular calc class. He'd explain it and then we'd do the problems and that's why it makes this class a little easier but still it takes a ton of time. This class does.

I: So you've done a lot of paper and pencil type work like this before in high school.

S: Right. I haven't done it for a year so it's been - we did really a lot of calculus in high school but then coming back and doing it again the second time through it's a lot clearer but it's hard to pick it up again because I remember seeing it but it's hard to remember exactly what I did so then I have to go back to my notes.

I: Have you used those notes from high school a good bit this quarter?

S: Yeah I have. It's a lot easier for me studying for the test rather than using a literacy sheet. I use a literacy sheet and find out what's going on but then I go back to my notes because I get stuck and bring out my notes.

I: What do those people who don't have notes like yours do? What do they have to fall back on?

S: A lot of them print out the literacy sheet all the time. They print out basics or tutorial or something like that and then try to work that but the way I took my notes in high school and I rewrote them and stuff like that. That book is everything to me. If I wouldn't have it - I'd be in trouble.

I: You don't want to lose it.

S: Yeah.

I: Do you ever print out those tutorials -?

S: I do. I print out the literacy sheets and look at them but I don't print out the basics or tutorial because usually they're real long so I go through them and get an idea what's going on. Then I go and try the homework.

I: Do you try to read all of those or do you sort of jump in the problems and refer back to it or - what do you do?

S: At the first couple of homework assignments we had I went and I went through the basics completely, tutorial completely and I went to give it a try. I printed out the literacy sheet for our quiz. But now you don't have time to go through that any more like that.

I: What has happened? Have they shortened the times or are the problems just harder?

S: The problems take a lot of time to do and when you were going through it before you could remember everything in the proof. The first early lessons you remember everything and then go through and zip through the homework without having to look back but now there are so many steps that you almost have to look back. It's like your notebook looking back. Into the tutorial or basics. And then going back to the problem so you kind of flip back and forth so it's just the time is shortened down and the problems are lengthy.

I: How much time do you spend a week, about, working on homework for that class?
S: At least 15 hours. I'm in that lab at least 15 hours a week because I go - Monday and Wednesday I spend from 9 til noon in there and Friday if our homework is not due until Friday I can spend from 9 until noon in there. And Monday night, Tuesday, Wednesday, Thursday night usually I'm in there from 7 - 9 every night. So at least 15 hours a week depending -

I: How do you feel about that?

S: I think I'm spending too much time and not learning a lot. I can't really say that but it is giving me a lot of experience with like story problems and practical problems but I don't really know if I'm gaining enough - like if I was taking calc for the first time I wouldn't know how much I'm learning. It just seems like the regular calc class - I know it's a lot different situation. It's typical give them problems, do the assignment, whatever but I couldn't really tell you. I'm just going to stick it out. He says we're learning a lot so I'll stick it out.

I: But you don't feel like you really are getting any feedback as to how much you're learning or - how could you do that?

S: I don't really know how much I'm learning. It just doesn't seem like - like in high school you knew exactly how you were doing because he'd give you problems. You'd go; you'd do the problems. If you couldn't do them then you know you didn't grasp it. Well you don't really know if you do or not. The thing that bothers me is you don't have time to go back and look at your homework and see what you did wrong. Because there is so much time you spend putting your homework together and turning it in, getting everything done because it is hard to get it done on time. To get it done on time you don't have time to go back and look if you did anything wrong. So what you learn doing your homework is what stays with you. You don't go back and correct anything. I don't really know - see we don't go over our homework at all so - I don't know if that's good.

I: But you get that graded and given back to you?

S: Yeah you do get it graded but what I do is I just look back in my homework file and open it up and see what kind of grade I got and I don't know. I'm doing all right right now but who knows - I'll have to stick it out and find out if I'm doing - I'll know pretty soon. The end of the quarter if I'm doing all right.

I: Time it sounds like is a big factor in what you're talking about.

S: Yeah it is.

I: More than anything. How much of it is involving the fact that you're doing your homework on the computer rather than on paper?

S: I don't do any homework on paper. I write a few things down - I figure out a few things and then plug it into the computer but as far as working the problem on paper before I do it in the computer I don't do that. I don't think anybody does that to tell you the truth. It's hard to get done.

I: Do you need to do more stuff on paper to feel like you're learning something?

S: Maybe it's just because it's a different way of teaching because really there isn't any really lectures. It's just pretty much self-learning so maybe it's just different. Maybe it's something that I haven't done before. I like the class and I don't mind going to it and working hard but I don't know if I'm learning a whole lot. I know I'm getting the problems done and I have to be learning something but I don't know if I'm learning as much as if I would spend 15 hours in a regular calc class.

I: What would have to happen in this course? What would have to change before you could decide whether you thought you were learning?

S: I don't know. I'd like to keep like a notebook just for the fact that I like to look - me, myself I don't learn. I don't have a memory where I can go back and remember that like first derivative, second derivative, whatever those are kind of easy but then you get some stuff where there are so many rules and you can look back, check on those rules, refresh yourself or something instead of remembering everything like the mean value theorem. Everything is
explained in letters. It's easier if you have the problem and then you can go back and flip
back to the mean value theorem and look at it and say oh yeah, I can do it. That way rather
than trying to remember because if you screw up one little thing then the whole thing is
wrong. I like to keep a notebook but I guess I really don't need one right now.

I: Could you keep a Mathematica™ notebook? Sort of take notes from the Mathematica™
notebooks or something?

S: You could, you could if you wanted to but it would take -

I: Time.

S: I don't have that much time because I have a heavy load this fall, I think heavy load this
quarter and my other classes are taking up as much time as I can allot. I'm probably at my
maximum right now for time I can allot for math class. Because I have a physics class too
and English and some UVC right now and each one of them is taking too much time.

I: How did you find out about this course?

S: Jason Sterns, a kid that I played football with in my junior year in high school. Introduced
me to it my senior year and asked me to take the class. Asked me if I wanted to and I said
sure, I'll take it. It sounded like a really nice class and he's in 254 so -

I: Does he answer questions for you when you have them sometimes?

S: Every once in a while. Not really. I don't really see him that much. I see him in there but
he's usually busy doing whatever, his own assignments.

I: So what is the best thing so far in your opinion about this class?

S: Just probably experience with the type of problems that he has given us. Really there hasn't
really been - hasn't really had any problems where you see where the derivative fits in, like
practical things. Finding maximum and mins, it's kind of practical - the best cost functions
given a certain amount of material. I can find out the dimensions of a box or whatever which
is kind of nice because something that I'm kind of a visual person. I like to see the graphs
on the computer. The graphs that you haven't seen before. Like weird ones with large
powers and stuff like that you can see those. That's nice seeing those and also doing these
problems that seem like they're real life.

I: So you use the graphing capabilities in Mathematica™ a good bit then?

S: Yeah. I like that a lot. That's a major advantage I think in this course, especially for
someone like me.

I: They really help you figure out what's going on in the problem?

S: Yeah. If you can see it it's a lot easier to visualize what you're doing and why you're doing
it.

I: What's the worst part of it?

S: Oh probably that it is about a mile from where I live and I spend a lot of time in there each
week. So -

I: I figured from what you said before time would be one of those.

S: Time is a major factor for me. I was going to talk about the TA's. It's hard to get - really
on some of the homework problems it's hard to - there's like a trick in them and without
somebody in there unless you know the trick or give you a hint there's no way you can do a
problem. And just that little handle - from nothing without the hint to you can finish the
problem with the hint. It's almost impossible to do it without it or unless you have that
little spark there. So it's kind of difficult getting the problems done if there is nobody in
there to help you and that's the only complaint I have - sometimes there is no one in there
especially when the homework is getting down towards the due date there is still no TA's in
there or -

I: In the evenings or -

S: Usually in the evenings. That's when I go in and a lot of people go in to do their homework
in the evening time because a lot of people don't wake up in the morning so -

I: So maybe there should be some more help available in the evening hours.
S: Yes. Especially towards test time because in the early part of the lesson there is always something else you can go onto and keep busy but towards the end when you'd like to complete those and turn your homework in and there's no one in there it's kind of hard to get done. You just sit in there and get frustrated.

I: Do you work with other people in the class a good deal? Or do you do it sort of on your own.

S: I work with people in the class. We talk back and forth but I don't really work with anybody. You're kind of on your own. The way my hours in there are weird. Because I have Monday, Wednesday, Friday go to noon and Tuesday I can't go at all. And except at night and Thursday go in the mornings.

I: Because I think they kind of promote that working together on problems too.

S: Yeah.

I: I've seen a lot of people who do that.

S: Right. We give hints back and forth. That's what's nice but sometimes like the way I'm in there I go in usually at night. There's usually the same people in there at night and I kind of work with them a lot more than I do anybody else really. So - I work with John a lot.

I: You're going to stay with Mathematica™

S: I'm going to stick it out next quarter.

I: And see how it goes.

S: Yeah. I think right now.

I: Then what? Decide whether to stay with it or go to a regular lecture class or -

S: Probably - I like the class but I don't know. I think maybe I should take this class and 151 at the same time. But that's kind of ridiculous so - one or the other will be fine.

I: With the number of hours you're putting in on this one at night.

S: Yeah. I don't have time for the other one, that's for sure. Because I think I'd be getting a better grade. With the pre-calc or calc class I had in high school I think I'd be getting a real good grade in calc 151 because my roommate is taking it and I help him on his homework and his quizzes - net his quizzes before he takes them home you know and it just seems that I could be doing A work in there and I don't know if I'm doing A work in Mathematica™ 151 so - which is - it's not all grades but it would help to keep my grade point up.

I: Sure. Is that the case they're doing easier problems than you're doing or - are they covering as much material as you're covering?

S: See I don't really know how much material we're covering. We may be covering a lot. But using Mathematica™ it's hard to tell.

I: Can you tell where you are compared to where he is? If you're doing the same kinds of things or have done the same kinds of things?

S: We're pretty much - most calc is in derivatives and stuff like that so we're doing more or less - we haven't done related rates or anything yet but he's somewhere around where we are I think but I don't know if has graphs or not. He doesn't put nearly the time that I put into it so I don't know. It's that time factor.

I: I was going to say you keep coming back to that time thing. That might be the deciding factor for you in the end.

S: It depends how it goes with physics. That is a class that does not come easy so it depends if I do decent next quarter in physics we'll see how next quarter goes. If it's really hard I'll probably end up getting out of Mathematica™ and back into 152 or 153.

I: If you go all the way through the next quarter in Mathematica™ do you think you'll have to go into a 152 lecture in order to understand what's going on or do you think you can jump right into 153?

S: I don't know. I'll have to be told when I get in there. I'll probably enroll into a 153. If I go through next quarter in 152M I'll go and physics is just too much then I'll have to jump into 153 regular. I don't know how it will go. He says we're kind of keeping up with the regular class so it shouldn't be a problem.
I: Well if you're helping your roommate with some of his homework with 151 so it sounds like you're -
S: Yeah, I'm kind of keeping up both ways. I think I'll be all right.
Student A – Interview 2

I: What I'm going to have you do is explore some things on Mathematica with me. Now I know you're working with integration already this quarter but since the first interview I had you take a look at just a small problem involving derivatives with pencil and paper I want before we move onto integration with the interviews to give you a chance to do something with derivatives using Mathematica. Use recall here. The first thing I want you to do is I have a function $g(x) = \log(x) \cos(x)$. What I'd like for you to do is use Mathematica any way you choose. Explore the graph of this function and what the derivative tells you about the graph. How the derivative might help you understand the graph. Now you can graph anything you want to graph. You can have Mathematica evaluating anything you want to evaluate or any kind of computation. Those are all your choices. But think of it as just sort of an open-ended exploration of this function $g(x) = \log(x) \cos(x)$.

S: Get started. Do you want me to sit there and tell you what commands I'm using?

I: Well tell me why you're doing what you're doing more than what the commands are specifically.

S: First let's clear everything. That's how we start. And just try a plot here.

I: Plot of the function there.

S: Right. Plot $g(x)$. Let's plot it here just to see what we get. Still haven't quite gotten keyboarding down yet. Let's see what we have. Good. Okay. About what you expect.

I: All right. How does the derivative work with the function? Is it useful? What can you tell me about the connections between the two? If you were exploring this graph would you use the derivative at all?

S: Oh you could. A lot of it depends on what you want to do. For me the derivative is telling instantaneous growth rate and slopes at different points on the graph and the you know if the function is constant. You can tell if the function is constantly increasing, decreasing constant or whatever. To me from last quarter as far as pure mathematics goes the derivative doesn't say a whole lot. Now that I'm in economics this quarter and I can see how you use derivatives to figure out economics and things it makes a little more sense.

I: You see an application of it in economics?

S: Yeah.

I: That means more to you?

S: Yeah. I guess I'm just a practical person but if I don't see an application right away it's hard for me to correlate it but I've seen a lot where you can use - depending on the kind of curves we're doing. There are a certain type of curve that when you took the derivative you could find demand and different functions which would be what you'd want.

I: Okay.

S: Just for fun - if you take the derivative and then we could also plot the derivative just for the fun of it. Then again it's a negative. Any negative numbers - (inaudible) and ranges.

I: Why doesn't that evaluate the negatives?

S: I would guess the log because I know in practice experience cosine is going to be like this and it can be negative and the log - I think you can't have log of negative numbers.

I: Okay.

S: There we can see the correlation.

I: Okay. Talk about some of those correlations for me.

S: Let's see if I can remember. Whenever the derivative is zero we should have a max or a minimum I believe. So we've got somewhere around 1 and then around 5. Let's see if that makes a liar out of me now. No, didn't make too much of a liar. There we have the max. So that's how we can get maximums. That's real helpful in economics when you're trying to maximizing profits or minimizing costs, things like that. Those type of things.

I: Good thing to do.

S: Yeah.
I: Anything else? Correlations you see between the two?
S: Not that I can see really. You could get the second derivative.
I: What would that do for you?
S: The second derivative tells you whether or not the first derivative would be increasing or not.
I: Does it tell you anything about the function itself?
S: It does but I can't remember off the top of my head.
I: That's fine.
S: They're all - derivatives and functions - they're all integrated but I can't remember the relationship off the top of my head.
I: Between the function and the second derivative.
S: Yeah.
I: Do you ever take a close look at them to see if you think they're what they're supposed to be? Or do you trust MathematicaTM?
S: I generally trust the computer. I think the big problem with the computer though sometimes it will give you - like some problems like this quarter you work them out by hand if you've got different derivatives and intervals. If you work it out by hand you get it in one form and then you do it on the computer and it gives you the same answers in a different form. That especially is concerned with the logs. And you think oh boy, I got the wrong answer here and you're looking around and then you realize that it's the same answer. It's in a different form.
I: So telling the equivalent forms and trying to test some things.
S: Yeah. When you're not prepared for it you take three pages of notebook paper and figure out something and then you're like what's this? Where did this come from?
I: Now in a more traditional class, different from the kind of calculus class that you're getting students would I think depend more heavily on the first derivative to make some predictions about the graphs of functions.
S: Probably.
I: Just to see what happens with you since your experience has been different let's look at another function. Let's look at \( f(x) = xe^{-x} \).
S: Clear everything out. I guess a good point Dr. Davis made to me - you're supposed to use this more of a tool than as something - a crutch which I guess I probably didn't do. I probably used it more as a crutch which has come back to haunt me on several tests.
I: Let's sort of take the approach you might take in a more traditional class. Let's have a look at the first derivative without looking at the function first.
S: First derivative on the computer?
I: Yeah. You can look at it and look at it's graph if you'd like. I don't know how much you can tell by just looking at the symbolic form. So if you want to look at the graph you can.
S: Doesn't say a whole lot to me right at this second.
I: And the algebraic form?
S: It doesn't say a whole lot to me. I guess I'm more a geometric person. I have to see it to believe it.
I: There are a lot of people like that.
S: I think that's what I like about the computer. Just the fact that it generates graphs like that. You've got them there to look at. Make sense out of it.
I: Now if you just had that first derivative to look at and there you have the graph and you know there are some correlations between that and the graph of the functions, what kind of predictions can you make about what the graph of that function is going to look at just by looking at the first derivative or can you make any?
S: Let's see. It looks like it is eventually going to approach some kind of constant level off on the positive side of the axis. And then it's going to have it's greatest positive slope somewhere maybe just like 7. This great big positive slope. Kind of level off and down. I'm almost sure that's going to be a constant. Got no change in the graph. It's equal to zero.
I: Okay.
S: Just make sure (inaudible). Yes it is going to be a constant. And have a huge increase in the slope.
I: Is there going to be a maximum anywhere or anything like that or can you tell?
S: My guess is that it's product would be maximum unless you want to consider the whole - I would guess that the graph is going to start down here; have a huge positive increase and just level off and if you consider that level off a maximum then -
I: Okay. I got you. Well let's test it and see. Let's look at that function.
S: Let's see if I know what I'm talking about. What?
I: Where is your confidence?
S: I don't have confidence in math any more. All my confidence went out when I got to calculus.
I: Did you have it before?
S: I had confidence before but I don't have confidence now. One week I understand something. The next you get lost. So I'm just looking forward to being done with this quarter so I'll be done with math. Oh, I didn't lie. Good.
I: Do you feel better? That helps your confidence.
S: When you get one right it helps your confidence.
I: All right. That looks pretty good. We probably don't need to deal with the second derivative because we didn't get too much from that before, right?
S: Not really. We didn't really do a whole lot with the second derivative.
I: It was kind of at the end of the quarter?
S: It was at the end of the quarter. We did some things with it but we really didn't - I'm trying to remember. We didn't do a whole lot. I remember one week we worked with it and we only worked with it as a homework session. We didn't really get that deep into it.
I: I have another question here I'm going to ask you. This is kind of tied in with the things that we've been talking about. Suppose you have some function f - you know about function f, right? If you know that the derivative is negative in this closed interval from 3-5 then which do you think is larger? The function value at 3 or the function value at 5? And why.
S: That sounds so familiar. I had a problem almost exactly like this and I remember I got it wrong.
I: See, you get a second chance at it.
S: Get a second chance at it. You are saying that the derivative is constantly negative?
I: It's negative. Now it may not have a constant value. It's always negative but it could be any negative value.
S: I'll tell you what I'm thinking. You're on 3-5 and you know that this derivative is negative between 3 and 5. If the derivative is negative then that tells you the function is going to be going down. So I would guess that f(3) is larger. Because if it's going to be constantly, if it's always going to be negative between 3 and 5 then that means the function has got to be going down because it would have to hit zero. The derivative would have to hit zero and that means you'd have a max or a min and it never does so I would guess that the 3 is larger.
I: It sounds like you (inaudible). Okay.
S: Test it out on this or -
I: If you want to make up an example to test that out, yeah. Or if you've got something in mind that would work.
S: I can't think of anything off the top of my head. I was thinking of drawing it.
I: Look back up at something like that first function. Look at the graph of it's derivative.
S: The cosine model.
I: Yeah. The graph of it's derivative - is that the graph of the derivative? Oh it is. How far did you go out on this one? Maybe it was this one. No. What about something like - what would be negative between 3 and 5?
S: We need a function that - I don't think the sine graph would. (inaudible) derivative might.
I: You could do a simple polynomial function.
S: Just invent something.
I: What about something like just minus x squared? It's going to be negative everywhere isn't it?
S: Yeah, any - Yeah that actually should work because it will just be a reverse. Parabola.
I: Well that's the derivative. It is definitely negative from 3 to 5.
S: Right. It's negative at a constant rate.
I: That parabola between 3 and 5, particularly at 3 and 5 would the value at 3 be bigger than the value at 5?
S: Uhhuh. Plot it anyhow.
I: A simple little polynomial might be a way to illustrate it.
S: It should be. And there we go.
I: The value at 3 -
S: Is greater. All right. What have you got next?
I: Those are the only problems I had today and you just got warmed up. Your confidence is there.
S: Confidence is there.
I: Those are the only 3 problems I had for today. Two specific ones to look at and this one that is more generalization about a specific function. Now what I'd like to do for the next interview is get more on target with what you're doing in the class right now on integration. Give you some problems to look at involving integration and then our last time I would like to give you a problem - I'll probably stick with the integration because you'll still be working on that. I want to finish this up this month. I want them about a week apart.
S: This month being February.
I: Right. I'd like to give you a problem situation and see what you would do with a problem on paper and try the same problem instead of different ones on the computer and then I'm going to get everybody to do a little written questionnaire. In talking with everybody, especially in the first interview I've got a pretty good feel for and I have it recorded the kind of background everybody is coming from. And how you found out about the course and got involved in it and that kind of thing but it seems like in going through the interviews that I missed some of those comments from one or two people so just to insure that I do have that information about everybody I'm going to have you do a written questionnaire. It will sort of rehash some of the things we talked about but maybe it will insure that I have some uniform questions answered from everybody. Just about your background which will be a little bit helpful if I think when I start looking at how you respond to questions and things. So I'm not giving you any real tough problems. I'm not interested in how difficult a problem you're able to handle but rather in the way you approach problems in general and I think you look at the problems in your class that are on a different level of difficulty than a traditional calculus student might look at them. And so I'm not really trying to see if you can handle harder problems because I think you handle them daily. But rather if some of the ideas that are hidden in some of those problems you're dealing with are coming out enough so that you've tucked them away in memory. You attack just simple problem solving situations in maybe a different fashion because of it. I'm not going to ask you to try to do the impossible.
S: How many people are doing this now?
I: I started out with 12 people with the thought that if I needed to I could use a couple of them just as guinea pigs and actually use data from 10 and as it turns out you had a lot of people drop from 151 to 152. They were not taking 152 but were in 151 and I guess I got kind of lucky in my interviewee because out of the 12 I started with I'm sure that 9 of you are still in 152. One person I still have not been able to contact by telephone. I suspect she may not be in there still so it may be that I ended up with 9 instead of 10 people which won't be a bad
thing. But I can't get in touch with her. I've only been to the lab once or twice this quarter and I've seen a few people there but she has not been one of them. I suspect she's probably not in class. I think it worked out pretty well since a good many people didn't take the 152 I think if I am able to follow 9 people through both then that's probably good. What's the size of the 152 class? Do you have any idea right now? I've got the idea that there's only around 15 or so. Both sections of 152.

S: It's really hard for me to say. I would guess 15-25 but I could - I don't go to recitation so I don't know how many are actually in my section because I have a conflict. It's on Thursdays at 11 and on Thursdays from 10-11 I have Botany recitation labs. Most of the time what they do in recitation so far has been they ask questions and do some simple problems and take a quiz. The quiz being the main thing so I just come in right at 12 when I can. I just go to the lab and one of the TA's, Amy brings a quiz from Dick. I just take it then. So I don't know how much I'm missing out on. It's difficult to say but -

I: Is there any way for you to go to the one that Dr. Davis is doing in that section?

S: No, because he has it at 9. My classes are 8,9,10. I have classes straight through in the morning. In the afternoon I'm free to do the computer. I'm going to get through this quarter and get math wrapped up and move onto some other subject material.

I: But you are seeing some usefulness in your -

S: yeah with economics. I can see some usefulness.

I: Maybe you want to go ahead and take another quarter?

S: Economics. Math is not so bad but it takes up a lot of time.

I: And it's really destroying your confidence level here.

S: In math yes. In math I have very little confidence but in other subjects I'm -

I: I hate to hear that. Especially if you had it once but you don't now.

S: I don't have a great deal of confidence in my answers. I just do em. That just goes with the territory I guess. I don't know how other calculus students handle it or whatever. Like algebra and geometry that stuff really - most of trig clicked but then I got to calculus and it just doesn't make a whole lot of sense. Functions make sense. And now I can see something with the derivatives. I can see how you can use integrals but a lot of the symbolic stuff doesn't make any sense. I kind of still have a lot of problem taking derivatives by hand. I have a lot of problem doing logs and I don't know any of the stuff - like cosine or sine off the top of my head. Like as far as being pi over 4. Any of that I really don't know. The symbolism part of math I'm really vague in that area and that really hurts on quizzes because I come up to things that should be off the top of my head symbolic stuff that I just really don't know. I've never really learned it.

I: Maybe on down the road some of those things you might get a little better at if you see some use for them, like in the economics course or some other business course that you take. And you may not. It may strictly be a case of your seeing applications for certain of the tools you study. Just using those for those applications.

S: Yeah. A lot of it just depends on whatever.

I: Sometimes everything just seems to click with you on a subject or something and then it all falls into place quickly and sometimes it doesn't.

S: Math isn't quick like it used to. Things like history and stuff really click well with me. Those are my best courses. But math -

I: Do you like the business courses you're taking?

S: Outside of economics I'm not really taking any business courses.

I: When will you get to take them?

S: I'm not really interested in business. I'm more interested in international relations and international studies and some of the requirements are economics classes so that's why I'm taking economics. I'm not really into - I don't know if I'm really into business or not. I'm not really sure what I'm into. I really like computers and I think computer science would be interesting but again it's the math. I know I could do it and I could stick with the math but
the hard decision to make right now is I'm really split between two things and if I want to stick with Russian and stuff then you've got to keep taking the language every quarter and if you want to stick with math, computer science this means stick with math every quarter and then you get really squeezed.

I: So you're going to get out of the math and try the other route and see what happens?
S: Yeah, I'll try it the language route. I've enjoyed the language. That stuff really goes with me. What I'll ever do with it I'm not really sure. Computer science is a direct correlation to jobs out there. They're just waiting but I don't - I really don't know at this point in life. I have no idea.

I: You've got some time to decide.
S: That's another thing I really don't like about computer science. It just takes so much math to keep going through it.
Student B – Interview 2

I: The first function I want you to look at is \( g(x) = \log(x) \cos(x) \). Now I realize that you’re already working with integration in class.

S: That’s okay. I’ve had enough of integration for the moment.

I: Since our first interview was involving derivatives with paper and pencil I would like to let you have a chance to do something with derivatives using Mathematica\textsuperscript{TM}. So I would like for you to take this function and some of the things we had talked about before and you talked about in class are how you can use the first derivative to tell you information about the graph of the function. So what I would like to set this up as is just an exploratory problem. Given this function \( \log(x) \cos(x) \) explore, using Mathematica\textsuperscript{TM} and making use of the first derivative what the graph of this function looks like. Do what you would do normally if you were given a kind of an open-ended question.

S: Should I go through - as I’m typing should I go through the procedures?

I: Yes. You can talk about what you’re doing as you’re typing.

S: First of all you’ve got to clear all the variables you’re going to use which is \( g \) and \( x \). And then you plug in the function which is \( g(x) = \log(x) \cos(x) \). And then if you hit enter it will give you what \( g(x) \) is supposed to be and then if you type in \( g’(x) \) Mathematica\textsuperscript{TM} will tell you what \( g’(x) \) is and then if you go to the bottom and type in plot \( g(x) \) and if you want to plot \( g’(x) \) the same thing you type in \( g’(x) \) and you say from where to where. You want to go from \( 0 \) to \( 2\pi \) or 0 to \( 2\pi \) would be better. And since we’re typing in 2 we want to see two different colors so we type in plot style. And type in color and you can have red, green, blue. Different colors. And colors can only range from 0 and 1 but any decimal in between so it should change color. See if this does - and then you enter it. Where’s it at? Right here. Now I do this. \( g(x) \) does not evaluate at negatives. Pi, changing zero to pi. The computer is telling me that it won’t do it from end to the other and the function is here, red. We have no green but we have blue. Make it purple - red and blue so that’s this line. That’s \( g(x) \) and the other line is blue and that’s this. So \( g’(x) \) when it hit zero shows that yes it is a zero point there. When it crosses zero this is one of the (inaudible). Make sure it (inaudible) here. It’s a zero point and this when it crosses here is telling you that there’s a maximum point here.

I: Now which one is the function and which one is the derivative again? The blue one or the red?

S: \( g(x) \). It goes red, green, blue. So red and blue - that’s purple so this is \( g(x) \) here.

I: So the purple one is \( g(x) \).

S: I’m not sure. I think. We’ll change the color here and find out. He tells us - the experiment and change the color you find out which is which. That changed the color. Hm. That looked nothing like the graph we just had. Let’s see black. This is the second one so this is \( g’(x) \). If it doesn’t change on me again. Make sure this is right. So change (inaudible). Don’t have the (inaudible) this long for any other. Then this is \( g’(x) \). Because it’s almost black but it has some color to it and this is \( g(x) \). This is the function \( g(x) \).

I: So go back over for me. You look at this place where -

S: Right here it crosses zero. This is the minimum -

I: The g’ does.

S: It crosses. This is the minimum point. We don’t see - the maximum point should be like wherever this crosses is zero but we don’t see that because we’re not going far enough but if we put negative \( 2\pi \) to \( 2\pi \) we should get where it crosses. Go away - I hate (inaudible). It’s still evaluating so we may be (inaudible). This is a graph (inaudible). Maximum point on this way and then if it crosses here there’s another point. And crosses probably right around here there’s another point here. And it goes - this line going up to infinity shows that this line will never cross here. It constantly goes down so it goes to infinity.
I: So summarize just in general for me the things you are pointing out that you can tell with the first derivative.

S: This is g '(x). It's going down. Therefore, the function is going down at that point, g(x). As it crosses here it is going out. As it crosses at like 1 it's going down which function then is going down. As it crosses here that is another point. It starts to go up and the function does start again to go up. As it crosses here it should go down again. The function probably turned up here and goes down. Since we can't seem to get it out any further than that we can't tell.

I: You looked a little bit at the second derivatives toward the end of the quarter. Does the second derivative give you any information that might be helpful with the graph?

S: I can't remember what we did. You can ask Mathematica™ what the second derivative is and you can ask it to plot the second derivative. From the same intervals negative 2pi to 2pi. I'm getting statements, probably because it doesn't evaluate certain numbers.

I: This one might be a little smaller than the (inaudible). That's the second derivative.

S: That's the second derivative and if you wanted to plot them against each other you can go up and call this plot a function, say 1 and reenter it. Then you can go down here. Call the second plot two and then reenter it. And then this one - the second one. Then you can take - show 1,2 and it should show and make that other one tell you a little bit of body language. These must be really small k's.

I: Smaller than the ones you're using in the lab.

S: If you enter the show command it will put all the plots together. So this is the g(x). This is the g ''(x). So the function is going down here and around.

I: So what kinds of things does that second derivative graph indicate on the graph of the function?

S: I'm trying to think. I can't remember. I can't what it - because we didn't do that much. We just like - they like popped up. It has something to do with zeros. I remember that but other than that -

I: That's fine. Now in a more traditional calculus class students would probably look at that first derivative and maybe even the second derivative and try to predict what a sketch of the function looked like but with Mathematica™ you are able to guess the sketch of that function just right off. So is the first derivative as useful? Or -

S: I think it is because really I think in a regular calculus class that sketching graphs is a waste of time because you spend so much time worrying about what the graph looks like. You're not really exploring what the functions are doing for you which in Mathematica™ that's what you're exploring more. You're exploring what g '(x) is telling you about the function, not is it zero here and do I have a high, low point here and things like that. Just takes that all away and then you can look at the thing and go well, g(x) is zero there so it must be 0 point there and 0 again here. There must be a max or min. When it's zero again that's maximum and minimum. You don't have to worry about how low does it go and how high does it go. You just know that it's zero there so I can plug that in and say at that point the function touches the graph.

I: Let me have you look at another function here. f(x) = xe^-x.

S: I'm clearing the functions again because Mathematica™ sometimes thinks you're calling some functions and saying I'm typing in the function.

I: Now what I'd like for you to do with that one is look at the derivative first.

S: You're just asking Mathematica™ what the derivative is and it tells me that it's the log of x to the e over x to the e to the x.

I: You can look at the graph of that derivative. I'd like for you to see if you can predict for me what the graph of the function is going to look like -

S: From the graph of the derivative.

I: From the graph of the derivative.
S: See what that does. Sometimes you have to look at it bigger. Doesn't evaluate negatives so it tells me. Got a misnomer on that because it won't evaluate. You have to ask it to go from certain other points. It doesn't evaluate zero. Let's see if we can plot - let's try 1. Sometimes it works. It doesn't evaluate at 1. Hmm. I wonder what it evaluates at? So I'm going to ask Mathematica\textsuperscript{TM} solve f'(x) for x. And see what values it gives me. Hmm - nothing. Sometimes that works; sometimes it doesn't. Just curious about why none of these seem to work. They should. Ten maybe? 20? Can't seem to find an X that Mathematica\textsuperscript{TM} likes. So I wonder if I'm getting the correct - proofs. Come on down. That wasn't what we got before. Ah ha. I made a mistake typing in the x and e to the x. Sometimes if you don't leave a space they think it's the same. That you're naming the variable 2, x and e instead of 1 variable by itself which would be just plain old x. You have to watch how you type things in in Mathematica\textsuperscript{TM}. It likes certain things. To graph Mathematica\textsuperscript{TM} we have, - I'd say it would be around 1. It's either going to be a maximum, minimum point and then there's one at 8 where the graph is going to touch zero. It's not going to cross but it's going to touch. I'm just going to check to see if Mathematica\textsuperscript{TM}, if it does anything on the other side of the axis by taking it from negative 10 to 10. It may loop around and be positive. Didn't like that. Sometimes you use - even 10 is too big. You have to get a smaller and see what the function is doing and it's still a little bit - five is still a little bit too big so I'm going to go from negative 2 to 2. This should bring the function down to size. The bigger you plot, the bigger scale you get. Okay, this is good. I go from negative 1 to 1 because the function is not doing much here but it's doing a lot in the area around 1. So if you leave it at 2 - After it's going up and will continue going up and it doesn't look like it comes back down so this part of the graph is going to infinity.

I: Positive or negative?

S: It's going to negative infinity. It would be going this way. Here it crosses zero. And it keeps going to infinity here, this way. And I can check by asking it to plot f(x). Negative 1 to 2. So it should come in from this side and cross and then go back down. So it's coming in from negative infinity, crossing and then crosses here at 1, 0. Then here is the max point so when it crosses it either tells us that it's a maximum point. That seems to be the maximum that it curved before it drops back down and then this tells us it's dropping down. It's going down. So it was dropping down but I don't know whether it goes - whether it crosses (inaudible).

I: So are you going to increase the range?

S: I'm going to increase the range just to see - it comes down and then it rides along the x-axis out to infinity and it comes up from positive and there is your maximum point. I want my pointer in there.

I: All right. With the things that you've been telling me, here is a problem that I wanted to ask you about. If you have the derivative for some function f and you know that derivative is negative when x is between 3 and 5 inclusive which would be larger? The value of the function at 3 or at 5?

S: It's negative. The value would be larger at f(3) because if it's negative the values are coming in this (inaudible) here. The values of f. That's the y. So if they're coming in from 3 - 5 and they're negative they would be coming this way which when you graph f(x) it would be coming this way so the one on the lower end of it would be higher. Like here - this would be like f(5) would be where it's crossing here.

I: I'm saying if the value of the derivative is negative now - your picture is not exactly going to fit up there - the value of the derivative is negative so the y value of the derivative is negative when x is between 3 and 5.

S: It's coming in this way. It would still be - f(3) would still be larger. For the same reasons because it's like it would be - it will be - I can't think of words. Negative - I guess what I'm trying to say is the flip - when it goes negative the flip side will be positive and f(3) will just be larger. I can't think of the words right now that describe why it would be
larger. Just because it's - when you reverse a negative - negative 3 is larger than negative 5. When you flip them around 3 would be on the up side and 5 would be on the down side.

I: Are you thinking about flipping the graph? Is that what you were thinking about?

S: No. The derivative tells you - like the max and min points and if it was - the function between 3 and 5 was negative you would have the section would be going - the 3 - 5 would be going negatively, right? So it's going down. When you flip the graph, when you change it back to f(x) that would take the graph up to infinity. It would take it to negative infinity instead of positive infinity which would be on the graph so when you're going negatively 3 is larger than the 5. The value for f at 3 would be larger than the value for f at 5. Because it would be going negative instead of positive. If it was going positive it would be larger than the negative.

I: Okay.

S: Because that's just like the log(x), the function would be 1/X is the derivative and if you have 1 - 3 that's larger than 1 - 5. And when you plug it back in it's still - negative log is still - if you had like negative log - the negative log of it would be down below the graph but 3 would be larger than 5. I was trying to think of an example that I could explain that with and that would be an example to explain that with.

I: Well those are the three problems I wanted you to look at to let you use Mathematica™.

S: More and more as we're getting into integral we're starting to use paper and pencil more because they also want us to - if we get into physics we're going to need to know how to integrate in order to - so I think

I: You're using more paper and pencil.

S: More paper and pencil and starting to understand more. Last quarter I kept saying the help wasn't there. This quarter they broke it up and changed a few things and now I'm getting more individual help and the classes are smaller. There is only 6 left in our class.

I: It sounds like things are working out pretty well.
Student C - Interview 2

I: I've picked out a couple problems but I want you to approach them as sort of explorations. The first one is $g(x) = \log(x)\cos(x)$. Now you know that there are some things that you could predict or tell about the graph of this using the derivative so I would like for you to just explore the graph of this function and explore the derivative and talk to me about some of the connections between the two. You can use Mathematica™ to do anything you like. If you want to do computations, if you wanted to do graphs, whatever. Just whatever you would do if you were left with this problem to explore the graph and what you can do with the derivative.

S: I'll just make it the function $f(x)$. Let's see.

I: What about - I don't think you have the right function in there.

S: I was (inaudible).

I: Now do you need to just put the x's in parentheses?

S: Hm -

I: Just going to leave a space in there?

S: (inaudible) space.

I: I'm thinking right between here and here. Do you need to take the space out or something?

S: Surely taking (inaudible).

I: Use parenthesis for those arguments or braces or brackets or something.

S: Sometimes you use parenthesis but for the $\log(x)$ it should just take - let's see. There you go.

I: Okay.

S: There is that. Is there any specific range?

I: No. You get to decide. Talk about what you're doing and why you're doing it.

S: I'm just going to plot it and I choose negative 5 to 5 to see what they would give me.

(inaudible) means that - I think it would just automatically plot the points to what it thinks it should be. Okay. So it's trying to (inaudible). It should give me something. All right.

I: So it wouldn't plot negative 5.

S: No. So I'll just make it zero. This one times this. So it would be 1 now it's like (inaudible). Okay. Let's try zero, maybe 2pi. It doesn't seem to be evaluating. Hm. Let's see - it won't take 1 (inaudible) pi. I don't know why it's not taking this (inaudible). Did you try this one? Did it work out? I don't know what range I'm going to do now though.

I: I'm sort of curious as to whether you really have the appropriate function. Let's try something different with that. Why don't you try putting the brackets -

S: Around the (inaudible)?

I: Just like you did the cosine and taking the spaces out between and see if

S: Like that?

I: Now try to do a plot.

S: Okay. (inaudible).

I: You can change it and see what you can get it to do now.

S: I don't know if it will take a negative now.

I: It just won't plot anything.

S: Yeah it goes right and it closes off.

I: How does it do that?

S: Because you don't - they're saying you don't get a real number so you probably get an 'i', imaginary number. And you can't really plot those.

I: What part of that function, $\log(x)\cos(x)$ causes that problem?

S: Oh. By the $\log(x)$ because the cosine just keeps going on negative.

I: So what you have now is the graph that that function is. Right?

S: Uhhuh.

I: What about the derivative?
S: I could probably just ask it to take the derivative.
I: How is the derivative helpful in explaining that graph, what the graph does or (inaudible)?
S: Usually the derivative like will tell you like - when it's equal to zero that means at those places it is crossing the x-axis.
I: The function is crossing the axis.
S: Yeah, the real function. Not the derivative. It also helps like getting a good range.
I: So if you were given this problem would you (inaudible) the derivative along? If you're interested in the graph of this or would you just -
S: Oh yeah. You couldn't just plot the graph.
I: But if you could just plot the graph you would just plot the graph.
S: Yeah. You could use the derivative.
I: Go ahead and find the derivative.
S: That's the derivative.
I: Can you graph the derivative?
S: Sure, I'll just use the same range. What had happened when I graphed there - Okay so it's negative.
I: Now what parts of the derivative would indicate something about the graph of the function? You said the function is where it's zero.
S: Also like where it's decreasing. You know that usually the function - well see it's decreasing so the function you know would be negative at that point. Here it's like increasing a little bit so the function would be like positive and like from here to here the real function would be positive but from here to here it would be negative.
I: So from just past 2 to just past 4 the function would be positive.
S: Yeah, like 5, yeah.
I: Let's go look at it just past 2 to just past 4.
S: So you have - well (inaudible) the same thing. Because it's starting to go up to the positive range. Like here it's negative. And here it's decreasing and now it is negative.
I: Okay.
S: Seven here it looks like it's crossing at 6.2. And at 6.2 it looks like you have like a maximum. It's usually the maximum (inaudible). Which also causes the minimum - like 3 (inaudible). Right here (inaudible) so that also - because it will probably cross like the derivative will cross right here the X axis and here is your minimum. You could tell that too.
I: Okay.
S: That's crossing the (inaudible).
I: Did you do anything with second derivatives?
S: The second derivative I think you just - I'm not sure if it was the range. You could tell the range of the first derivative. I'm not sure. I could try it.
I: Can you tell anything about the graph of the function from the second derivative?
S: The second derivative I think that gives you maybe where it is equal to zero.
I: Where the function is equal to zero?
S: Yeah. I could see. This is the second derivative.
I: What part of that derivative in that second derivative is causing this (inaudible) to zero? What's causing the difficulty?
S: Probably this part where the x is - if x equals zero you end with a fraction. Sometimes it doesn't really take the (inaudible) ratio. So that probably (inaudible). Okay. See it crosses here.
I: Just past here?
S: Yeah. It almost looks like the same - I was going to say a little bit different because you have this little increase in it. The same one is increasing. See what it is increasing to. It also crosses at 8 and I think that's really the first one to cross (inaudible). I think it should cross to 8 too. I think these functions are usually similar - the second derivative is usually
similar to that (inaudible) function. The first one usually tells you when your function is
going to be positive or negative or stuff like that.

I: Let me have you look at another one. Let's sort of take a different attack since we've talked
about some of these things. Let's look at \( f(x) = xe^{-x} \).

S: Okay. Oh negative x. Okay.

I: Instead of looking at the graph of the function right at the start -

S: Do you want me to get the derivative?

I: Let's look at the graph of the derivative and see if you can make any predictions about what
the graph of the function might look like. In a more traditional calculus class that's what
students would tend to do but they don't have this nice program to help them graph the
function so let's see what may happen if we try that.

S: If (inaudible) negative x. There is the derivative.

I: Now if you had just that graph of the derivative and you wanted to make some predictions
about what the graph of the function would look like what would you predict?

S: Right here where it crosses I'd say - 5 - I'd say that would be a (inaudible) and minimum
right here where it's like (inaudible) I think the real function will cross at 2. Here it's
decreasing so the function would probably be decreasing or negative and here it is increasing
so I think the function would be positive. And it's almost touching E so it would probably
be the max number 8, the real function.

I: So you think there will be -

S: Minimum at one point (inaudible). It will cross at 2. Here it's decreasing, increasing and
then maybe a maximum of 8 because it's not really crossing there. Should I plot it?

I: Yeah. Check and see.

S: I said it would be decreasing but it's not. It's increasing.

I: Okay.

S: At 1.5 I said maximum is a minimum. At .8 still the same thing. It's almost the opposite.
When this is increasing - no, when this is negative then this would be decreasing. So like
this 1.5 to 8 the real function would be decreasing. I need to go.

I: I have another question I want to ask you about. I don't know that Mathematica™ will
necessarily help you with this but it's kind of the thing we've been talking about. If there is
some function \( f \) and you don't know what the function is but you know that it's derivative is
negative and this closed interval from 3-5 then which would be larger? The value of the
function at 3 or the value of the function at 5? And why do you think that? If you want to
try something with Mathematica™ you can. Or if you want to just think about it -

S: You're saying that from here to 5 it's negative which is almost like this. Then you know the
- I'd say \( f(3) \) is larger because when this is negative, the derivative you know that like right
here this would be decreasing. The real function would be decreasing so whatever value you
had at 3 would be greater than the value you had at 5. I would say \( f(3) \) would be greater than
\( f(5) \).

I: That sounds good. I just have those 3 problems I wanted you to tackle for me today on the
computer.

S: I had some of those in calculus in high school so they're not bad but - it's okay. It's going
pretty good. It is very tiring this quarter. I keep thinking 5 more weeks.

I: What I want to do is finish up the two interviews before the end of the month. The next one
probably won't be lengthwise, time wise much different than this. That last one will
probably take longer than any of them have because what I think I'll do for the next one is
have you take a look at some problems involving integration using the computer and then
the last one I think I'll still stick with the integration because you'll still be working on that
topic in class. I might have you look at something with pencil and paper and then after look
at the computer to see the difference between how you might try to do something with pencil
and paper. The same problem. And then I have a little questionnaire I'm going to have all of
you do that I've interviewed. Some of it is to give me some background information so that
I can tell what you came into the course with. In most cases I've talked to people about that but I want to have it on paper. Just some general information just about where you came from and things like that. A lot of things that we talked about especially in the first interview and I just want you to write it down so I have a written record of it, that kind of thing.

S: I don't know why I don't like this class so much. It's just all the work.
I: Did you like math before in high school?
S: I liked it because I knew what was going to be on the tests and stuff and here they don't really go over it that much. I may talk to Dick, Dr. Davis and tell him to give us a recitation that you just go over everything and then have a quiz. Normal classes do that. But I don't know. I just keep thinking five more weeks. Maybe I can hold out for that long. I want to know if we're having a mid-term or not too. This quarter is flying by though. I'll probably want it to drag on.
Student D - Interview 2

I: I still want to look at derivatives a little bit and I know you guys are going on looking at integrals now so we're going to still be lagging a little behind. By next time we might be actually asking some questions on what you're doing. But since the first interview we did look at one problem involving derivatives that we looked at with pencil and paper I thought we'd look at some things involving derivatives and let you use Mathematica™. So I have this function. \( g(x) = \log(x)\cos(x) \). Now one of the kinds of things you have done is look at the derivatives of functions and figure out things about the functions. I want you sort of an open-ended type question to take this function. Use it's derivative in whatever way you choose and tell me anything you can tell me about this function. It might involve increasing, decreasing. It might involve maximums, minimums, points of inflection. The kinds of things you can remember that you could do with the help of Mathematica™ and the derivative. How much can you tell me about what the graph of this function is going to look like? Aside from just having Mathematica™ graph the function.

S: Want me to tell you what I'm doing while I do it?

I: Yeah, that would be good.

S: First you have to clear all the variables you're going to use just in case they've already been entered on the computer. Then you enter in the function. Then I know what \( g(x) \) is and then I say \( g'(x) \). It will give me the derivative. I can say -

I: Now you can do anything with that derivative you want. And tell me as much as you can tell me about the graph.

S: Okay.

I: Because you're just exploring the graph of this function.

S: I'm going to go ahead and plot just the function. And get a good range so then \( x \) goes from -20 to 20. All my graphs will be pretty so I can color them. My real number with the negative. I assume it is going to be from zero up. So I'll just change my range from zero to 20. I'll make it really, really close to zero. Just at zero point, zero, zero point. And then I'll get any of that red stuff. And then I'll plot it's derivative. Within range.

I: Now why are you deciding to plot functions and then plot it's derivative? How did you decide to do that?

S: I first wanted to see what the actual function looked like and now I've got it's derivative and then I'm eventually going to put them one on top of the other to see how they coincide.

I: What do you have there?

S: The pink one is regular function. The blue one is it's derivatives. And the blue one is decreasing. The function is negative. It decreases all the way down to about 2.5 and - if the derivative is positive like it's above the X axis it's right here. That means the function is increasing and so it is from here to here so at that same point it is increasing. Then the derivative goes under the X axis it's negative and that means that the function will be decreasing and it is. And then it decreases all the way down to here and it starts going back up and that's exactly where the derivative is positive. If you follow it along the way every time that the derivative is above the X axis then the function is going to be increasing. When the derivative is below the X axis the function is going to be decreasing. Every time that the derivative crosses the X axis there is a slope of zero or maximum or minimum. It crosses right here and this is one of the maximums. Crosses right here and it is one of the maximums every time it crosses.

I: Okay. Are there different kinds of maximums and minimums then?

S: There is one maximum. They're all slopes of zero and they're all maximum but actually they're all (inaudible). But there is just one maximum and that's where it reaches the highest value and I assume if I made this graph even bigger that the maximum would keep going up and up and up. I'm going to put it 50 - 100. Because each lump is getting bigger and
bigger. And the minimum also gets bigger. And it really seems like - it's really neat because you can never do this by hand. I never could anyway. Forget that.

I: Can you use what you find out about the derivative to get a pretty good idea of what the graph is supposed to look like or do you usually always look at the graph and the derivative to get (inaudible) in here?

S: Can I do what? Just look at the derivative to figure out the function?

I: Uhuh.

S: You can.

I: Get a good idea.

S: Uhuh. But it's just easiest to go ahead and look at the function and then the derivative and put them together and see how the one makes the other one go where you want it.

I: Okay.

S: Is that the derivative controls the function? So I just expanded the range more so I would see if as x increased then the maximum would increase and it did. And so did the minimum.

I: Now you looked some at second derivatives too and how (inaudible). Does the second derivative help you with that at all?

S: I think it's best to show you where the graph changes directions. If I say show me g''(x). I'm not sure what this (inaudible).

I: Have you plotted the second derivatives before in class?

S: I think we have. Only on a couple of occasions.

I: Making it work (inaudible).

S: All I did was name it. Ah - whoa. Okay. The second derivative is exactly the opposite of the function once it starts doing the lumps but not as far as concerned in here. It just seemed to be shifted a little bit. I don't know why (inaudible). But it's opposite. So I guess it is kind of an indicator of direction. I'm not sure if that's always the case or not. Dead end.

I: Take a look at one more for me.

S: Uhuh.

I: f(x) = xe^-x

S: (inaudible). So the actual function is going to (inaudible). It's (inaudible). If I really want to see a function (inaudible) I just decrease the scale. It looks like it's becoming (inaudible) up around negative 10. I'll just go to negative 10. This is all one color. (inaudible). It's just going in that quadrant across, along the x-axis. The derivative is doing the same thing. The derivative is positive and of course increasing and then it's doing along the x-axis it's neither positive nor negative in either of the functions. Just going along with (inaudible).

S: It's not increasing or decreasing.

I: So is it straight up there?

S: Yeah, because it - that would be my guess.

I: Does it ever touch the x-axis? Does that function ever touch the x-axis do you think?

S: Here.

I: Does the function itself?

S: Yeah, I think so right along here. It goes from negative 5 to - get a clearer picture of it because sometimes it may look like it's on the X axis but it's not quite there. It's just approaching the limit. See it looks like it goes just exactly over the x-axis. And the derivative is under it.

I: So can you decide for sure? Can you somehow find out if there is a place where it actually touches the x-axis?

S: I can keep reducing the range.

I: And see what it looks like.

S: Yeah. There probably is a way to find it. But (inaudible) right now I don't know.

I: That's all right. So - go back over for me again. If the derivative is what then that means that the graph of the function is what?
S: If the derivative is positive above the x-axis the function is increasing. If the derivative is below the x-axis, negative then the function is decreasing and wherever the derivative crosses the x-axis it's a critical point on the function, either a maximum point or a minimum point. It looks like it crosses right there. But it flipped so it looks like it is a maximum right there.
I: Okay. All right. Tell me something - this is a question - the kind that you've seen once before. Just sort of a general question. If you have a derivative that you know is negative for x between 3 and 5 inclusive then which is larger? The function value at 3 or the function value at 5?
S: It's negative for 3 and 5?
I: The derivative.
S: I'd say 3 is larger just because your slope is negative.
I: Now explain why you're saying that f(3) is larger. Based on what you read there.
S: Okay. If the derivative is negative between 3 and 5 - right here - then - oops, I went - this is negative. The larger - a negative 3 and negative 5.
I: You can try to make up an example and try something out on Mathematica™ if you want to. It might fit this.
S: If you had the function - 1/2 x squared and you took it's derivative, it's derivative is just going to be negative x. And if you plug in 3 in there you're going to negative 3. If you plug in 5 you're going to get negative 5. Obviously the larger one is negative 3 so the 3 would you give you the larger one.
I: Okay.
S: I figured that out by integration which we're working on now. If I just said negative x would be the derivative and then I found the function by integrating.
I: That's pretty good. So you do make that connection between integration and you're working on derivatives. That's good. All right. I want to save your file here.
Student E – Interview 2

I: It's going to be kind of odd because you're not accustomed to typing at the computer and talking about what you're doing while you're typing. So I might have to say say what you're doing. The first interview we looked at some problems involving the derivatives with paper and pencil. I know you are working on integration, started integration but I would still like for you to have a chance to do something with derivatives using Mathematica™. So we want to do this time and then get another interview right away in which you do some stuff with integration. You're a pro at derivatives now. That's why these answers will be so easy for you.

S: Command-wise with the computer doing it.

I: I'm going to ask you to - now first thing I want you to go is look at this function \( g(x) = \log(x) \cos(x) \). Now you've studied the derivative and you know that there are some things that you can tell about the graph of a function using the derivative and so forth so I would like to leave this problem as just an exploratory type problem for you using Mathematica™ to do what you would like using the derivative and this function and see what you can come up with and what you can tell me about the graph of this function. Do what you would do if you were just left with this and said you can use the derivative to help you.

S: To do what?

I: To look at the graph of this function and tell me what it will tell you about the graph and that type of thing.

S: Start off here - okay. \( g(x) = \log(x) \cos(x) \). Like in equation first. So it's (inaudible). Then find the derivative and let's name the derivative so you can use it and apply - uh, uh okay. Then plot it - \( g(x) \) - oops. I'm not real good at typing so this takes a lot -

I: No penalty.

S: \( x \), from - let's see. This goes from .1 to N.

I: How did you decide to do that?

S: Well you can't take the log of the negative so and it was zero so you'll get an infinity so I started with .1.

I: Okay.

S: Oops. Let's go back here. And color these so we can tell them apart. Style - oops.

I: Now you're graphing the function and the derivative.

S: Right. That way because if I'm left to see if they give me full reign I like to do both of them so that I can see how the derivative and the function interact.

I: So when you look at that graph then what do you - what can you tell from it about how they interact?

S: Well the red is our function \( g(x) \). And you can see the derivative when it crosses the x-axis. It has a min or a max and you can see that in each case here so where it touches we have a max touch and we have a min. They'll just keep going that way and you can see when it's negative. When - let's see - how did that go again? When it's less than zero then \( g(x) \) is declining, it's going down and when it's greater than zero it's going back up. That's what I look for.

I: All right. You looked a little bit at second derivatives at the end of the quarter. Did they tell you anything about the graph?

S: If I remember second derivative tells if the function has an upward bowl or a downward bowl. If it's positive it has an upper; if it's negative it has a downward I believe. Sounds right.

I: Test out what you just said for me.

S: Okay. So if we said x equals x squared and we said -

I: Test it with this we had.

S: That one up there?

I: Yeah.
S: Okay. So g"(x) - oops. Uh - let's see. That says that it is negative. I want to make sure it is still doing the same commands that I'm doing. It says it's negative. Yeah, because it's downward so -

I: Now where does that say it's negative? Are you saying it's saying that the second derivative is negative all the time?

S: Well not all the time. You can see that by these steps here. But it will, depending on what x is. Second derivative is still (inaudible) which way it is going.

I: Can you graph the second derivative?

S: I don't know.

I: Have you ever done that?

S: I don't know if we have or not. Well, in a sense we did tangent once. So I can't remember exactly how to do it. Let's go x equals - cut and paste. I never knew about it until Dick told me.

I: About cut and paste?

S: Yeah. I always just went back in and did everything. Oops.

I: Okay now. What was your theory again?

S: That when the double derivative is negative this will go down. Or the bowl, whatever you want to call it, the bowl goes down and when it's positive it goes up. It kind of looks like it's doing that here.

I: So your theory must have been right.

S: Must have been. Hot dog.

I: Usually in a traditional calculus class students will look at those second derivatives and the first derivative and try to predict what the sketch of the curve is going to look like. Now using Mathematica™ you just start off looking at all of them together.

S: Unless he tells me otherwise.

I: So is the derivative as useful in graphing for you do you think as it would be without using Mathematica™?

S: Without Mathematica™ I'm sure it would be.

I: But with Mathematica™ -

S: But with Mathematica™ it can be at times I guess. Because you can put in whatever your function is and then put in the derivative. You can plot your function and you can see in your function what your derivative is going to do so either way. You have it right there.

I: Okay. I want you to look at another function. What I would like for you to do with this one f(x) = xe⁻ˣ is not look at the graph of the function. Instead do whatever you want with the derivative. Either the algebraic form of the derivative, the graph, whatever you want to look at and see if you can predict for me what you think that graph is going to do. And then we can check it again to see if the prediction was on target.

S: All right. That tells me nothing. The derivative tells me nothing so I graph it to predict what xe⁻ˣ looks like, f(x). So plot it. x can't be zero. Well it can be zero. Couldn't it? I want to see. Huh. Okay. Let's see. It would be positive up to this point where it would do some type of - it would be rising, the function. The function would be rising here until it got to 1 where it would have some type of max or min. Probably a max. And then it would decrease again and get closer to zero so it would look like that. Go up and then down. Go up towards zero.

I: Okay. So you think the function itself is going to be increasing?

S: Increasing until -

I: Until it gets to 1 and you think it's going to have a maximum?

S: I think it will have a maximum at 1.

I: And then you think it's going to -

S: Decrease to zero.

I: Go down towards the x-axis.

S: Right. For a long time. At least until (inaudible).
I: And you think that because this derivative is why?
S: I think that because why?
I: Why do you think that?
S: By looking at the derivative. The derivative is positive until it hits 1. It crosses the x-axis at 1 and then it's negative thereafter and it is going towards the x-axis. It goes down and then back up towards the x-axis.
I: Check out your predictions.
S: All right. So we've got - ha -
I: I like the ha. All right. Pretty good prediction I would say. Does that function ever touch the x-axis?
S: Will it ever touch the x-axis? No, because e is a limiting factor and since e is a limiting factor I don't think x will go to zero and this goes to infinity or whatever. I don't know if it limits that but I know this is a limiting factor so it will never touch the (inaudible) I think.
I: Okay.
S: And of course we could do it because if this was 10 this would be a billion so 10 over a billion would be a very small number so it would never actually touch. All right?
I: Sounds good to me. This will look real familiar too probably or something like it. You've got a function f, the derivative is negative in the interval from 3 to 5 inclusive. Which would be larger? f of 3 or f of 5?
S: Is negative for the interval between 3-5.
I: The derivative.
S: The derivative is negative between 3-5. Which is larger?
I: The function value at 3 or 5.
S: Uh -
I: Well you can think it over. You can try to make something up that fits it with Mathematica™ if you'd like to. Probably (inaudible) way to do it. Think that over.
S: Hm. I would forget the commands. I'm trying to figure this out in my head.
I: Okay.
S: Which would be larger? Well - negative between 3-5. Uh - let's see. (inaudible) the function. Goes like this. So it's negative in here. This would be -
I: The derivative is negative.
S: Right. I would say it would be greater after (inaudible).
I: Why?
S: Because if it's negative at 3 and 5 - actually it depended on which way because if it's going down this way or if it's going up this way. If it's going down this way - if it's going this way then - it would depend on which way the slope is going.
I: The slope of what?
S: Of f'(x). Because we could either have a max or a min at 3 or a max or a min at 5 depending on which way the slope is going.
I: I'm not saying the slope.
S: Okay, but it would be either trailing off of a max or going to a max or trailing off a min or going to a min, either one, depending on which way it was going.
I: Can you go back through that again one time for me?
S: If I can.
I: The derivative is negative.
S: The derivative is negative.
I: Between 3 and 5, including 3 and 5.
S: Including 3 and 5 so we actually don't know where the derivative is crossing the x-axis. So if it's crossing the x-axis after 5 or - if it's after 5 so we're either going to a max with our function. We're either going towards a maximum or going towards a minimum but if it crosses over here at 3 it's either going towards a max or going towards a min, the function is.
That's why I'm saying it depends on which way, what type of slope we have or if we have any humps in here somewhere.

I: All right. This graph you had been working on kind of fits the picture.
S: Yeah.
I: Between 3 and 5, the derivative - is that the derivative?
S: This is the derivative here.
I: Is negative.
S: Right.
I: Okay. And the function value at 3 and the function value at 5.
S: Right. Three is higher than five.
I: So if the derivative here had been doing what - how did that change those function values?
S: If the derivative was - if the derivative was inversed here. So if it was going from right to left instead of left to right the function would be going this way also. So it would be greater at 5.
I: So how is it that the function. Somehow you're connecting the slope and the graph of the function to the slope and the graph of the derivative it looks like.
S: No. I'm saying if it's a positive slope for the derivative in the negative, when it's a negative then it's probably going to - it all depends on the function.
I: Okay.
S: Because we could be going all types of directions.
I: Right.
S: So I would have to see a picture of it. For this one 3 would be greater but if I saw another picture of a derivative and it had a totally different shape than it could be depend.
I: So this might not be enough information for you to settle on where (inaudible).
S: Right.
I: So it would depend on the particular f.
S: Right.
I: So how is the start with integration going?
S: I'm understanding it kind of slowly. I need to organize my notes because when I'm doing it on computer I just grab anything and write it down in any old corner and I've got all these - I don't know if you'd call them identities or not but all these equations and functions and everything going through my head and the swing on this paper and I know what it is but sometimes it's hard to see which one goes to what and which one applies to where. I think the integrals are okay.
Student F – Interview 2

I: I’m going to give you some problems to look at. I know you’ve already started working with integration but since our first interview last quarter involved the derivative and it was with pencil and paper I did want to do something with the derivative on the computer and then the next couple of interviews we’ll look at some problems involving integration. But this time I want to step back I guess and look at derivatives one last time. If you are given the function \( g(x) = \log(x) \cos(x) \) you know that you can use the derivative to figure out some things about the graph. We talked about that with pencil and paper before. What I’d like for you to do is with Mathematica\textsuperscript{TM} do what you would do if you were given this problem and just asked to explore the graph and what the derivative tells you about the graph on Mathematica\textsuperscript{TM}. You can have it do computations for you, graphing for you. Whatever you choose.

S: Okay. Start out - clear x And my function. Set up the function. Okay. Now we can take the derivative and then usually plot (inaudible). Here is the plot.

I: Of the function.

S: Of the function. And I could come out with (inaudible) and the derivative too. Okay. So where the derivative crosses zero that is where my function up here is going to be flat. That’s right. That’s where it will be flat.

I: Anything else? Any other connections between the derivative and the function?

S: Well, where this one down here is - some of them have this little thing. You can screw it up and down and see what (inaudible). Where this is - let’s see. Where my derivative graph is positive above the \( x \)-axis that means that my graph of my function up here will be increasing which it is. Like right there for example. And all of this will be increasing on another graph and this will be decreasing underneath the X axis.

I: You looked a little bit at second derivatives before the end of last quarter. Right?

S: Yeah.

I: Does the second derivative tell you anything about the graph of functions? Any connection between -

S: Well - let’s see - yes it does. It tells me like this curve right here it will tell me if it is opening up or opening down.

I: That is your first derivative. What is your second derivative? Does the second derivative tell you when the first derivative opens up? Is it a function of (inaudible).

S: From the function.

I: Okay.

S: I think the (inaudible) is.

I: Now in a more traditional calculus class -

S: (inaudible).

I: Not exactly like you’re doing it, no. What students would do is use the first derivative and the second derivative. Maybe there are graphs. Not necessarily. Sometimes just do algebraic (inaudible) to try to predict what the graph of the function is going to look like using the same things that you’re talking about. You’re comparing the 3 in showing how they sort of work together, graph. But then we’d use the first derivative and then probably the second to just sketch that original function. Now I want you to look at another problem and at the very beginning instead of going straight and looking at the graph of the function I want you to do whatever you want to with the first derivative. Compute it and/or look at the graph of it, whatever and try to predict using the things you were telling me about. what the graph of the function is going to look like. The function \( f(x) = xe^{-x} \)

S: I’ll clear my work up here. Okay. \( xe^{-x} \). It’s kind of fanny I’m learning this in programming. In instrument graphics I have to think (inaudible). I have to use commands like this.

I: Tell me what you’re doing and why you’re doing it.
S: First I just typed in the function and the derivative of the function. Then I'm solving the derivative from (inaudible) equal to zero.

I: Why do you want to do that?

S: Because that will tell me where my peaks and (inaudible) are. My height and low, my max and mins. Then this tells me that there is no one at 1. And so if I

I: Where you look at the value of the second derivative it's 1?

S: Yeah.

I: And at 1.5.

S: On both sides of it. Okay. That turned out to be a peak. On both sides of where the derivative is equal to zero it will tell me what my graph is doing there and it told me down so it looks something like a (inaudible). Like a parabola. Up and down.

I: Anything else you'd like to investigate with the derivatives before you check out this graph of this function and see how accurate you are in your prediction?

S: Not with the derivatives. It could be - it might be a parabola and just be going up from say a limit somewhere which is possible.

I: All right. Check out that theory then and see if it works, if you can predict. Go ahead and look at the graph. They're good, aren't they?

S: Yeah. Actually I could have found before I did the graph it (inaudible) max-wise by plugging in the answer I got for x back in the original function. Or f(x), (inaudible). I got x equal to 1 so I knew there was an (inaudible) and I could just plug that 1 into f(x) down here. It would tell the point that (inaudible). It's .36, .37 at the max. So that kind of makes it (inaudible).

I: I think that kind of (inaudible). It sort of fits that description of (inaudible).

S: Uhhuh but that might change when it gets bigger but this will never increase again. It's (inaudible)

I: But that maximum stays (inaudible) because that concavity enters into it. Now students who don't have Mathematica™ fall back on the kind of things you were doing. First they do them by hand and then with the aid of a computer. And they try to predict what the graph will look like, how it will be sketched. And we sort of forced you into that. Does that mean that the first derivative and the second derivative are not as useful to you because you have Mathematica™ or -

S: Well - if I look at the graph right here I can tell. I can tell what it looks like and the reason that you take derivatives is like you said for them to know what it is going to look like and I can tell that there is a max here and of course it's always good to - we use them a lot on the computer also. Just to make sure you know. Because looking at this right here you wouldn't know if this curve doesn't come up here again for example so it's useful and I think it is essential for them but it's useful for us.

I: Now this little problem is one that you've seen before. There is some function f. You know that the derivative of this function is negative in that closed interval from 3-5. Which would be larger? The value of the function at 3 or the value of the function at 5 and why?

S: Okay. The slope is negative. f at 3 would be greater.

I: Is that your explanation of why? Since the slope is negative?

S: Well if the slope is negative, yeah. As x increases the function decreases. (inaudible).

I: Those are the only three questions I wanted you to look at.

S: Did I tell you everything you wanted to know? It's getting easier. It's more interesting doing math. I think it's easier. It's easier to keep up with the work in school.

I: Why do you think that's the case? What happened to make it easier?

S: Maybe we're getting more interested in it. Maybe we're getting a little better with figuring out what needs to be done. Actually the way it is this quarter for me is that I don't feel like I need as much - I feel like I'm just a little farther ahead than I was last quarter. I felt like I was a little bit behind last quarter and I'm moving ahead this quarter. And (inaudible) essential and stuff for some reason. I don't know if it's just the (inaudible) we had.
I: Maybe you're kind of getting the hang of using Mathematica™ and the different way of thinking (inaudible).
S: That's probably true. And I felt like I was a little behind just because I (inaudible) school. So (inaudible).
I: Closed that gap a little bit too, huh.
S: yeah.
I: That's good to know. There are some things that have changed this quarter. The way they've got things organized and (inaudible) getting maybe a little more of lecture type thing once in a while.
S: Yeah a little bit. I have noticed a change in what they're doing. There's more - I think there is a little more help available and a little less help needed. I think it's more exploratory than (inaudible). I did just have a quiz that I flunked. I couldn't do it. I was kind of frustrated because I felt something like what we were tested on wasn't exactly what we had been working on. And I know that the problems in the lesson are made there to prove a point to us but just because we can do the problems doesn't always mean that we (inaudible) or the principle behind it. Which I didn't obviously.
I: And the test was on that point, the principle that was behind underlying -
S: On those few. What it was we were working on integrals and they discussed a little bit in the lesson what integrals were - they said even ones do this and odd ones do this. They never said that odd integrals are the ones that have all odd exponents. The even ones have all even exponents. That was never pointed out and they never asked why are these odd and why are these even and I guess I maybe should have paid more attention but a lot of these guys have had calculus before. They already knew that so it was okay on the quiz and stuff like that. I couldn't figure out how to - and I didn't know how to compute integrals by hand.
I: Can you do that now?
S: Yeah I think so. I learned it - he went over on the board on the day of the test and taught us how to (inaudible). I got a zero on the (inaudible).
I: Are you in Dick's section or (inaudible)?
S: In Dicks. Actually as it turns out we didn't have to know how to compute integrals on the test. Sometimes you can go through a lesson and the times that you're really learning is when the TA stands over your shoulder and helps you out with the concept. Cheryl is the one that told me the difference between even and odd functions when I asked her about it. That's really the only way I think I could have learned it. She told me how it goes and so I think that there is like a (inaudible).
I: it sounds like you're getting better at dealing with the (inaudible) without as much of that personal interaction but you still see a need for some of it.
S: Uhhuh.
I: I don't think the course was ever intended to be a teach yourself kind of course but with whatever reason I think there is not as much of that interaction with you and the faculty member or the TA's as might have been intended at the beginning. It's kind of easy I think to have (inaudible) on the computer and students just get turned loose in them and sort of go at their own pace and kind of turns it into teach yourself kind of situation. I'm not sure that was what it was meant to be at the beginning.
S: I think it's kind of surprising that they expect us to understand all of this (inaudible). Even though they're supposed to be (inaudible) there as the teacher, principal. It's hard to get a lot out of it. It is amazing that (inaudible) there. Even though you don't always understand (inaudible).
I: Somebody was telling me - I knew the books were supposed to be ready in January but they're not ready yet. Are you guys going to get the book that goes along with this?
S: I will.
I: Any word on when you'll be able to get them?
S: I don't know. I'm not sure. I think there have been a couple people that have started in Mathematica™ in the 152 series after they had regular 151 and for people like that it has got to be essential. Because they don't know any of the commands or anything.
I: I didn't realize that there were a couple that had done that.
S: I don't know if they're still in there but - half the learning of this is learning the computer and they're a whole quarter behind when it comes to learning it.
Student G - Interview 2

I: This is the first function I want you to look at and I want to leave it as kind of an open-ended question. Given the function $g(x) = \log(x)\cos(x)$. I want you to kind of think about everything you learned that might involve the graph of this function and derivatives. If you were given this function and said examine it's graph using the first derivative and you had Mathematica® to aid you. What would you do first? Talk about what you’re going to do too. You would put the function in?

S: Put the function in, $g(x) = \log(x)\cos(x)$. Clear - enter (inaudible). Three and zero and (inaudible). Does not plot between there. Try 1, $\log(x)\cos(x)$?

I: Uhhuh.

S: Well it doesn't plot. Ah - I can't even get it to plot.

I: The function?

S: Yeah, plot it.

I: Okay. Do you change your range in some other way?

S: (inaudible). Zero, zero - I can't plot it anywhere.

I: I think it might -

S: Maybe clear.

I: You think it hit your clear until it hit your function?

S: Yeah. There you go. Now what do you want me to do?

I: How can you use the first derivative to tell me something about this graph? Why is the first derivative useful when you talk about graph of the function or is it useful?

S: Yeah it tells you the slope. It tells you about the slope and the function you're doing. So it's a negative -

I: So if you wanted to explore this function using the first derivative what would you do?

S: Using the first derivative plot it. Plot zero. Plot 10. Smaller - (inaudible) smaller. There. The first derivative over (inaudible). It looks like you'd have to (inaudible). They tell you it's going to be (inaudible) that it's a negative - no, that's not right. It should be positive. No, like (inaudible) is right here.

I: Right.

S: So it tells you it’s going to be three and then like (inaudible).

I: So the value of the first derivative is negative at 3?

S: Uhhuh. And the value at 4 is positive.

I: Now that's the graph of the derivative.

S: No, that's the graph of the function. Yeah, that's the graph of the derivative. See you went into positive.

I: So how is that useful concerning the graph of the function?

S: When this is positive it tells you the function, no, when it's negative it tells you the function is going down because it's a negative slope.

I: Is that true? Does that fit with what you have up here as a feature of your function?

S: Well here this is between zero and (inaudible).

I: All right. Okay. So -

S: See at 2 - that's where it intersects and it sits at zero because there is no slope. Because that would be a min here. A min here. So you have three. See it's negative. It gets below it's negative and then at 3 here.

I: Let me see if I hear what you're telling me. Are you telling me that when the first derivative is negative the function is also negative?

S: No. It doesn't mean it's negative. It means that the slope is negative. See like right here - these would be all zero - we're at zero. That's where it is min and max. Look at this 6 here. See here's your max here because your slope is zero so that means it's going up and coming back down. Back here it crosses at A, see down here. Look right here - if we put this out farther this would probably be the max or something. It changes (inaudible). You can plot
it all the way out. See - (inaudible) or the 10. See over here is a min. That's where the slope is zero so it would give you the minimum and maximum. It will tell you what it's doing.

I: Is there more than one kind of maximum and minimum?
S: This is a relative min. See this goes to negative infinity so it's not the minimum because there is no minimum. This is a relative min and it's probably a relative max too. The function (inaudible). It might not be because when you put it out to a hundred this might be the max. You can't tell unless you're going to put it out a lot farther.

I: Now you did some work with second derivatives before the end of last quarter. What kind of information will the second derivative give you?
S: The second derivative gives you -
I: So predict for me.
S: Predict it?
I: Uhhuh.
S: What it's going to be or what it's going to say?
I: What it's going to be. You already have the graph of your function up above there, right?
S: Right.
I: This is the derivative. The function is up above.
S: It's going to say - the second derivative is going to be positive and this is concave up so this is concave up and this is concave down so here should be negative. It will be positive until it gets like in this area. It will be negative like out to 2. It will be positive after 4 because see this is positive so here it is negative, positive.

I: Let's see if that's (inaudible).
S: Doesn't like zero. (inaudible).
I: Uhhuh.
S: It doesn't like zero. Blows up here when you (inaudible).
I: That second derivative looks like it's negative until you get -
S: See it's negative so you (inaudible). Positive - oh, see this would be concave up because it's going upwards and it's always positive here. This is positive. Like at 14 see they all where they cross is the same. It's the same because there's nothing there. Like at 10 so this is concave down here. See it's going down to this - because this is going down. So it shows the general concavity of the function at certain points.

I: I have a question for you. The first thing you did is plot the function. Then the derivative and then the second derivative.
S: Right.
I: If the derivative and the second derivative give you clues about how the graph of the function looks are they still useful? If you just use Mathematica™ and graph the function what you're interested in is the graph. How useful are the first and second derivative to you?
S: In using Mathematica™?
I: Uhhuh.
S: Not much.
I: Would you typically graph a function and it's derivatives and look at it's derivatives -
S: No, because you can tell just by looking at the function what it's doing.
I: That's the graph of the function.
S: Right. It's useful like when you have to find the min and max and stuff.
I: The derivative is useful?
S: Right.
I: You can't find those from the graph of the function?
S: Well you can but I mean this could just be a little plane and little graph and like it's plummet over here, you don't know.
I: But you can figure that out from the derivative?
S: Uhhuh. I can solve the derivative. (inaudible).
I: Now what are you looking for when you do that?
S: (inaudible). Well -
I: What are you trying to look for when you do this?
S: You're supposed to look for - solve for the first derivative you look for the - it should give mean means of access. Obviously it's not going to -
I: Does that make sense when you look at this function?
S: I don't know. It's between - this can be (inaudible). It might be in there. Let's see if x was 3.37 - find the derivative - it says find - I don't know if that command is right.
I: Scroll it down so that you can see your function at about that location. That's the second derivative.
S: Yeah, I got rid of the function. This is the second derivative. This is the (inaudible) function.
I: Right.
S: That's probably why this is (inaudible). x is zero. This computer takes a while. There it is. Set up 3.37 (inaudible) zero. 6.3 is here where it crosses. These zeros of the function where it crosses the x-axis. To solve this doesn't work if you're (inaudible) for some reason. (inaudible).
I: That's the zeros of the derivatives. Now what does that tell you about the function?
S: That's (inaudible). So you look here where there is 4 which is the relative min and here it is 6 which is the relative max. And between 10 and 12 here we have about what's left. (inaudible).
I: Let's explore one more function in a similar kind of way. If f(x) = xe^-x.
S: I hate these.
I: But you have Mathematica™ to help you.
S: It doesn't matter. I still hate them. Do you want to talk about a derivative with this one?
I: Well you can. I have a question. Does that function touch the x-axis?
S: Uh - I don't think so.
I: How would you decide?
S: Solve g.
I: Ah -
S: You could do the (inaudible) and make the g solved. g' won't work so if (inaudible) equals zero or x.
I: Oh.
S: I don't think it liked that.
I: Stretch it's memory a lot.
S: Erased my graph. Oh well it's (inaudible) stuff.
I: Why don't you close out a lot of this stuff. I expect it might be eating up some stuff too. I don't want you to get rid of all of yours because I want you to save that file.
S: (inaudible) file. Just close it.
I: See if that helps a little. It made a terrible noise. There is the graph. Still not - How about if you - why don't you go ahead and save yours and then done.
S: This doesn't have the work space does it.
I: No. It's not on (inaudible). I want to have you dump it on the hard drive so save yours with your name/my last name. Just have you drop it on the hard drive right now. I'm going to move it afterward. There is a lot of other stuff on the hard drive. Now if you want to clear some stuff out of there you can to help you. That would help you decide if it touched the axis or not.
S: Or you just make your range smaller.
I: And try to tell by looking?
S: Just look here between it looks like 2, 7, 10.
I: I think we've got some stuff eating up memory now. It doesn't want to talk to you any more. I don't think the space is used up on the hard drive.
S: (inaudible). It hasn't loaded the Mathematical™ stuff either. It touches at zero. It goes out towards infinity and never touches.
I: All right. Since we've been talking about this kind of thing this is the kind of thing you've seen too. If you know that the derivative of your function is negative what value is $x$ between 3 and 5 inclusive? Which is larger? $f(3)$ or $f(5)$ and why?
S: Why? If the function is negative it means it has a negative slope.
I: Now the derivative is negative.
S: If the derivative is negative it's going to have a (inaudible) slope so it's going to be like this and so $f(3)$ would be this one. $f(5)$ would be out here so 3 would be bigger because $f(3)$ would be the first and $f(5)$ would be the second.
I: All right.
Student H – Interview 2

I: The first thing that I want to do is have you look at a function. I know you're studying integration already but since in the first interview I had you look at some things involving the derivative with pencil and paper I did want you to have a shot at doing something with the derivative using Mathematica™. So this is still going to be concerning derivatives and then we'll move on the next interview probably and get up to the integration and make you feel like we're doing exactly what you're doing kind of thing. I have a function - g(x) = log(x)cos(x). What I'd like for you to do is use this as kind of an open-ended exploration type question. You know that there are some things about the graph of the function that you can tell from the first derivative. Explore in any way you would like using the graphs, using the evaluation capabilities of Mathematica™. Make use of the first derivative however you would just naturally to explore the graph of this function.

S: So you want me to plot it?

I: If you want to.

S: And explain what it's doing? Is that the idea? Do you want me to tell you what that graph looks like?

I: Sure and how the first derivative might help you or how you could use the first derivative.

S: First what I'm going to do is label g(x) as log x cosine x. And enter it. Whoa.

I: It will catch up. This machine may not have the memory that you're accustomed to either so if it is a little slower than the response time you're used to don't -

S: Okay. Now what I'm going to do is ask it for the first derivative. See what that gives me. Okay. Now I'll just - do you want me to explain g(x) or the derivative?

I: I want you to explain g(x) but I want you to explain it by using things you know about it from the first derivative. So maybe sort of explain how they're related might be a good way to put it.

S: Okay.

I: You're going to plot the derivative, right?

S: Right. I'm plotting g'(x) from I'll say 1 because the derivative of zero of a log is going to go crazy. To maybe 10. Okay. The plot of derivative shows where the function is increasing. Where it is decreasing. And max and mins.

I: What part of that plot of the derivative tells you what about increasing, decreasing max and mins?

S: Well you know the plot of g'(x) gives the slope so between 2 and maybe 5 the function is increasing between maybe 5 and 8 it's decreasing and then it starts increasing again between 8 and maybe 10 or so.

I: Let me make sure I'm understanding you. This is the graph of the derivative and you're looking at what it's doing and saying that the function is going to do that too.

S: Well no. See where the derivative is positive, where it's above the axis is where it's increasing. Functionally increasing. Where it's below the axis it's decreasing. This is just a plot of the points. Now the derivative is positive between maybe 3 1/2 and a little over 6 - 6 1/4 so it's positive any time between those points. g(x) is positive.

I: Now you said the derivative is positive between here and here?

S: Right. See because the graph goes up and then comes back down so everything above the x-axis that's a plot of g'(x) so that means the slope is positive over that range.

I: What about back here between - just before 2 and just before 4?

S: It's negative. That means it's going down. The function is going down.

I: What else? Anything else?

S: Well you know the derivative is - at these points the derivative is zero right there where it crosses the axis so if the derivative is zero it's a horizontal line at just before 2, just before 4, just after 6 and just after 1.

I: So what's that going to mean in the graph of the function?
S: It's going to be a horizontal line there.
I: Okay. So what does that mean?
S: It's not increasing or it's decreasing.
I: So what's it doing?
S: It's going straight across. The function is. At the - minimums would occur at - let's see - g '
' (x) - that means it's negative. I'm trying to determine - I know the slope is positive
between just before 4 and just after 6 and the hump in the graph would indicate that - it is
where the function is increasing the most I think. I think that's it.
I: So what about your maxs and mins you were mentioning?
S: Maxs and mins. The maximum - let's see - if the first derivative is equal to zero that's the
critical points so that's where it crosses the axis so that's where it is horizontal. Where the
first derivative is positive it's increasing. That's presumed just before 4, just after 6. And
where the second derivative - I took the derivative there and it's positive. That would mean
it's concave up at this point so that would be a minimum. At 4 and maybe 5 it would be a
minimum I think. And at 2 it would be a maximum because of the second derivative. Can I
take the second derivative?
I: Sure.
S: Okay. I could type that all in but - do you want me to type all this in? All I know is when
the second derivative is positive it's a maximum. Now you look at this graph - let me copy
this real quick. Just going to copy this so I can show the difference. Okay. At this point at
just before 4 the second derivative is 0 so you know these graphs are like reciprocals of one
another. At the first derivative when it's equal to zero it's at a maximum. When the first
derviative is positive at 5 the second derivative is 0 meaning that the second derivative is
equal to zero it's point of inflection in the graph I think. And when the second derivative is
at a minimum you know the first derivative is zero so if the first derivative is 0 means it is a
horizontal line so this is going to be a minimum. And the second derivative is equal to zero
when the first derivative is at this peak. The first derivative is negative. That means that it's
another point of inflection so if the function increases - the point of inflection is like when
it increases and then switches and goes concavity and it will be concave up and then it
switches and goes concave down. It may be still increasing but it's just going concave down
and then the point of inflection is right at that point where it switches - like at the S - the
middle.
I: Now looking at this first derivative and second derivative can you kind of summarize for me
what you think the functions graph is going to look like?
S: Between - when x is between 1 and 2 or just after 2 the derivative is negative so the -
between 1 and 2 the first derivative is positive indicating that it's increasing, the function is
increasing. Then at just before 2 the first derivative is negative up til maybe just before 3 or
4 so it's decreasing. At 2 there would be a point of inflection or just after 2. Then when it
crosses the axis again right before 4 the first derivative is 0 again. That means that since the
first derivative is 0 it's a critical point here. That means it would be a maximum. It would
switch then. It would go back - the function would then start going up since the first
derviative is positive and then it would come just after 6 and it would cross again at 0 so it
would be another point of inflection and the second derivative would be negative. It means it
would be - the second derivative is negative means concave. It would be a maximum. I think
it would be a maximum at just past 6 and then the derivative is negative again so it's going
down. Yeah, that would be right. It's concave down again until it gets to - it's still going
down. It decreases all the way to just before 10 but at 8 it switches concavity and the
concavity is positive again. It means it is going up. It's concave up again.
I: All right. Check out your theory.
S: I got a min right here just before 4 where the first derivative is equal to zero point of
inflection. Then at - you get a maximum just after 6 and you have a minimum at just before
10. So the function from 1 - just before 2 - it looks like it's increasing. Then it's a
maximum of 2 it’s concave down until it gets to maybe just before 4. Then it turns. It’s a minimum - just before 4 it turns back up, concave up until it hits just after 4, crosses the x-axis. Then it goes concave down, up until just after 6 where it’s a maximum and then it stays concave down til it crosses the x-axis at 8 or just before 8 and then it turns concave up but still decreasing til just before 10 and it stays concave up, increasing after 10.

I: All right.

S: That’s the first derivative there so we know - you can’t see it real good right there but right at 2 the first derivative should be negative from just before 2 to just before 4. It’s going down. It’s decreasing. Then at just before 4 til just after 6 it should be increasing and it is. And then just after 6 to just about 8 it should be decreasing. And it is. Until - oh, just before 10. I’m sorry, just before 10. And it is all the way just before 10. It changes concavity at 8 and it goes from concave down to concave up and then it starts increasing again.

I: Okay. All right. That’s pretty good. Can you do a similar type of exploration except look just at the first derivative. And see what you can predict for me about the graph of this function. \( f(x) = xe^{-x} \)

S: So you just want to look at the first derivative.

I: Uhhuh. And tell me what you can tell me about what the graph of the function is going to look like.

S: Do you want me to plot it or just explain from this?

I: You can plot it if you want to plot it. If you want to explain from that you can. Can you explain from that?

S: \( xe^{-x} \). You know e to the minus x is 1 over e to the x so if you take a derivative you have to use a product rule and you get derivative to the bottom times the top which is e to the x times x minus -

I: You’re explaining how to get this from this.

S: Right. So it would be the derivative of the bottom times the top so it would be e to the x, derivative of e to the x which is e to the x times x minus the derivative of the top which is 1 times e to the x which is e to the x over e to the x squared. If that made any sense. So now if you plug in x values you get - that should be e to the x squared.

I: Maybe if it would simplify out or something.

S: Yeah it could. I could simplify that. Okay. Now I know that if I plug in x values to the derivative I can find out where the derivative is positive so - I know the first derivative is zero at x equals 1. Because if I plug in 1 negative 1 plus 1 is zero so it would be zero over who cares. So you know the first derivative would be zero at X equals 1 so it wouldn’t be increasing or decreasing. It would be a horizontal line. Then as X increases if I plug in 2 I get 1 over - okay. The first derivative would be positive but the values would be getting smaller so that means the slopes of the lines wouldn’t be as steep. So it would be gradually - so the function would go something like it would look something like the log of x I believe but it’s negative. It just means a negative slope so it would just be going the other way. Let me see if I’m right. I don’t know.

I: That’s a good idea.

S: I think I’m going to have to make a negative because of the first derivative. I’m not sure. Okay. As x gets bigger and bigger negative the value of the derivative went up so that means the function is increasing. As x gets bigger the function decreases. So if you know the value of the function decreases as x gets bigger. The slope is getting smaller. So f(x) would - the slope is getting smaller. So f(x) would be leveling off and at 1 it would be - the slope of it would be zero. I think. You know the derivative is positive so it is increasing. It’s going like that I think. The value of the function should be like that.

I: Rising up from negative (inaudible).

S: Right. As x gets - I think it should be as x gets more and more negative close to negative 1 the value will be positive and it will remain positive as x gets bigger and bigger negative but it won’t be, it will increase rapidly at first and then it will level off I think so it should go
something like that. However you explain that, I think. Because - if this is increasing rapidly at negative numbers. I think that's right. I'm not sure.

I: Take a look.

S: It went the other way. So if you go from (inaudible) zero - that's right. It's increasing. The graph looks like a parabola of the derivative. So you know that it's increasing - as x moves toward zero the value of the function gets larger. So it's increasing and if I set the value at negative 1 it would be zero and then I said f(x) would be looking - oh man. Okay. So from the graph you can see that if you took the slope of the line at negative 10 it would be positive so the negative number would produce a positive slope and as it gets closer and closer towards zero the value of the slope gets less and less so it's decreasing. The function is increasing but the value of the slope is decreasing to where it levels off at negative 1.

I: Okay. All right. Can you kind of just in general pull together those ideas you're working with about what you find from the first derivative? What you find from the second derivative for me?

S: The first derivative where it's equal to zero you know that the function is horizontal so it's not increasing. I found that out to be at negative 1 because when we first took the derivative so then I plugged in - in order to find out what the derivative looks like you just plug in a few points and find out where their values are. So then when we plotted it we found out that the derivative is like connected dots. I found the point at negative 1 and found it was horizontal. Then as it got less and less negative I guess you should say then the value of the slope is increasing so at like the number negative 10 the slope was almost undefined because it looks it was pretty much vertical so if it was pretty much vertical then that means the slope is increasing so the function has to be increasing near negative 10 and it is. It's in the third quadrant. So it's increasing and then it has to level off at up towards negative 1 so if you know, if the value of the derivative is a parabola you know that near negative 10 the slope is really big and it's positive and you know that near 1 it's horizontal and if you plugged in maybe a point at negative 4, negative 6 or so you could plot the function. You know it's increasing and levels off at negative 1. So as the slope gets less and less the function has to becoming more and more horizontal.

I: All right. In a more traditional calculus class students would tend to do the kind of thing you're doing. Of course they'd be doing it with pencil and paper. They would look at the derivative and try to predict what the graph of the function is going to look like. With the use of Mathematica™ typically is that what you do? Or do you just have Mathematica™ plot the function for you?

S: Well see I had - the way I first learned how to do calculus was pencil and paper. So that's why I try to plug in points but it's hard without the pencil and paper here to plug in the points you use Mathematica™. So that's the way I do it. So then instead of me having to connect the dots Mathematica™ just shows you a picture and then to know if you're right you just look at the picture and decide. Instead of having to say it's increasing here, it's decreasing here you can just look at it and you know Mathematica™ is right when it plots it if you were right typing in the command so then you just look at your graph and if your graph makes sense then you should have done it right and then if you plot a few points, like if you know at negative 1 the slope is zero so if you look on the graph at negative 1 the slope looks like it's zero and then as it gets at negative 10 you know the slope is positive and if you go down here at negative 10 and f(x) the slope is positive. The value of the function is negative but the slope is positive.

I: Do you think with Mathematica™ that the first derivative is as useful a thing to know when you're interested in the graph of a function as it would have been without Mathematica™?

S: Without Mathematica™ you have to have the first derivative but with Mathematica™ it's almost - I don't know if I'd say it's a substitute but it's not as necessary to know because Mathematica™ will do it for you.

I: Just graph the function for you.
S: Right. If you can graph the function then you can look at it and say what the slopes are going to be. You can predict the graph of the derivative. What you're supposed to be able to do from the derivative after the function but I guess I can't do it all.

I: You did a pretty good job of it. Let me ask you one general kind of question here. I don't know that you can use Mathematica™ for this but if you think you can though you can. Again, along the same lines just to kind of summarize it. You say something like this before probably. There is some function f and you know that it's derivative is negative and this closed interval from 3-5. Which is larger? The value of the function at 3 or the value of the function at 5?

S: You know the derivative is negative so the value at 3 would be larger. f at 3 would be larger.

I: Why?

S: Because you know the graph of the function somehow is going down so as x gets bigger the value of y is less and less so at 3 it's going to be higher up on the graph than at 5.

I: That's what I wanted to know.
Student J - Interview 2

I: The first problem I wanted you to have a look at is \( g(x) = \log(x)\cos(x) \). Now you've done a lot of things with the derivatives involving the graphs of (inaudible). I want to leave this as a little bit of an open-ended question and let you explore the graph of this function. It's derivative, what you can tell about the graph of the function with the derivative and that kind of thing. I'm not going to have you do something specific. I want you just to decide what you would like to do on Mathematica™ with it. And try to talk about what you're going to do and why as you go along.

S: Obviously I would first tell the computer what the function is and find the derivative to be as such which I would assume using the product since it's negative sine \( x \) - why it didn't take the derivative of - oh, okay. I over \( x \) times the first one. Exactly what I would expect it to be. That's the derivative. I would go ahead and plot \( g(x) \) where \( x \) goes from - if you have log of \( x \) I would probably say from (inaudible) point 1 to let's say 2\( x \) and I would choose that. I may be incorrect but with it being well the log of \( x \). I mean take the derivative of log of \( x \). \( x \) ends up being on here so you obviously wouldn't want a zero as your range. You would mathematically have a fit with that. And then the 2\( x \) because of the trig functions. Well, I typed it wrong.

I: Little syntax error.

S: Yes. They happen quite often. That's the part that I don't run into whereas other people - it seems they do sometimes with the fact that they aren't quite as familiar with computers. I would say from this you can definitely see some of the sine in it and then the log being 1 over \( x \) and the log ends up negative from the derivative. It looks interesting. I probably should have named that as just graph and then I would come down and derivative and then another graph just to look at it. And always use the copy command whenever possible. That's interesting.

I: Okay. Now, what are you doing here?

S: I'm showing both graphs on the same coordinate system. With this one being the derivative and -

I: And what do you see from doing that?

S: Well, what I would expect is wherever the graph of the function is at perhaps in maximum, a minimum, another maximum here. The derivative is always zero. I could use a find route or so forth to find what the function is at that point. The minimums and the maximums. As this function is increasing here - that's the derivative going down. This is zero and then when it changes concavity - I'm trying to remember the difference on a graph between increasing and decreasing and so forth. I'm trying to follow which graph is - see here they coincide. You know what I mean? They literally coincide. Here it would seem that the log has perhaps - the start of what looks like a sine and cosine function is skewed and I would assume it would be due to the log as far as what it is actually doing. Would there be something else that you would want me to find?

I: Well what is the connection between the function and when it's increasing and decreasing and the derivative since you have the graph up there? What aspect of the derivative indicates to you whether you could see the function or not when the function is increasing and when it is decreasing?

S: Well I'm trying to remember. I'm pretty sure, when the derivative here is negative the function would be decreasing which it is here. And when the - when the derivative is positive in both these places here it is increasing. Although here it's positive, the derivative is always positive it does change direction and that would be due to the fact that here it shows that the function is increasing but it's also negative and then here the function is still positive so it's still increasing but since the derivative shows up as going down the function is above here.
I: All right. You looked a little bit at second derivatives toward the end of the quarter. What kind of information could you get from the second derivative or the graph of the second derivative to help you see what's going to happen with the graph of that function when you actually had the graph.

S: So before I type this up - the second derivative would show me - wherever there is an inflection point in the graph the second derivative would be zero. So for example here on the function here and here and up here the second derivative would be zero versus the first derivative which shows the minimums and maximums as zero and it would also show where the function is concave up and where it is concave down by whether it's positive or negative.

I: Well you've got that plot (inaudible). Go ahead and plot that one and let's see.

S: Whoops.

I: You didn't compute the second derivative yet.

S: Yeah. I was thinking it would do it all at the same time. That's the derivative. Now plot it.

Hm - plot - cosine x, log x, minus 2 sine x. I can't seem to figure out why that won't plot.

I: Let's try this. I name it. I unfortunately can't seem to notice what's wrong with it.

S: I don't see any kind of (inaudible).

I: Unless I got the plot command typed properly. No I didn't. I didn't use the capital P. That will do it. Here we go.

S: Okay.

I: Now I'm going to show the second derivative as well as the original draft. Okay. This is a zero on the second derivative so it obviously is an inflection point even though it's a difficult one to notice and I'm trying to see here. It would seem to me that there would have to be another one. Show one here.

S: Now what is an inflection point? A place where the graph does what?

I: Excuse me - it changes concavity, right. Here is one and here is another and originally it's concave down from this zero all the way to this zero it's concave up and then at this zero it changes back to concave down. It doesn't necessarily here. That's not where it changes concavity. That's where it changes to increases versus decreases and that's where this is zero on the first derivative. I just looked at it improperly. And then if I wanted to find what those values are or which ever you could do that with different commands and so forth but as far as what can I tell from the actual graph that's what I can tell from the graph. The graph of the first derivative and the plot of the second derivative. Now as I, you have probably gathered, my weakness comes in at looking just at the plot of the original graph. In determining why it looks as such. I could plug in numbers to the function log x cosine x and I could perhaps figure out from there but as far as why is it way down here in the negative range I honestly don't know for a fact. I'm curious as to what a plot of just log of x looks like over the same interval. That's what I would do. The computer gives me the power to do that if like I said if I'm not real familiar with what it looks like I'll just call it up. I did know that but I guess that's one downfall is because of the computer I don't memorize what it looks like. I can just dig it out of the computer.

S: But you often do that when you have functions that are more complicated that are built as several functions.

I: Right, which you can't remember.

S: You'll look at the parts -

I: Right, you can't remember what it - see, I never knew what the plot of a log of x looked like until I came to college. I didn't even know what x squared looked like even though that's simple. x squared is an obvious plot. And x cubed and so forth. I didn't know what they looked like until I came to college. Started taking this class and started looking at them.

S: But after doing this and looking at those on Mathematica\textsuperscript{TM} could you now identify those on a piece of paper do you think or - ?

I: Yes I could. Now log of x that was one that I had to refresh my memory. Yeah, now I see. I remember it looks as such. But as far as x squared and so forth those I could figure out.
relatively easily. One over x squared or x squared plus one. Simple polynomials such as that. Now after taking this class last quarter I can recognize them. But I learned that last quarter I did not know that previously. And so yeah I am memorizing them as I see them. Trying to remember - we just took a quiz yesterday and it dealt with integrals but when we don’t really know the formula for finding integrals and so forth and so what we had to do was draw the graph and compute the area within that graph. Well most often the area was zero because it was even/odd function or whichever so we didn’t actually have to do the integral but the hardest part was simply drawing the graph. I had to - it took me a while to draw those different graphs and so forth and that’s what this has helped me with. If I want to see a graph I can. I’ve just got to instruct myself to memorize some more of them.

I: Now usually in a calculus class students use the first derivative and the second derivative if they’re doing it with pencil and paper to try to get a sketch of the graph of the function but now you because you’re using Mathematica™ can just tell Mathematica™ to start out by graphing the function so how useful really are the first and second derivative to you since if you want to see what the function looks like you simply graph the function.

S: Well I would say the majority of the time we use the derivatives to find actual values. We would put them into story problems and find the minimum and maximum depth and height, the volume and so forth. Somewhere in the depth of my brain I do know how to take a function, find the second derivative, find the minimums and maximums. Take the first derivative, take the second derivative then and find the inflection points and sketch the graph. We did do that. Do I remember exactly the process? No. That is one weakness since you don’t repeat it over and over again you only - you learn it when you know you’ve got a test coming up and on the final we had to do just that and I did get it right so I do somewhat like I said have the ability.

I: So you did acquire those skills through the work on Mathematica™?

S: Right.

I: By the end of the quarter. I have another question. It’s along the lines of the things we’ve been talking about. You’ve seen one like this before I know but if you have a first derivative. You know it’s negative on this closed interval from 3-5. Then which is larger? The function value at 3 or the function value at 5?

S: So the derivative is negative from 3 - 5. That would mean that the function is decreasing so therefore it’s decreasing from 3 - 5 because it’s negative so therefore f of 3 would be larger than f of 5.

I: Okay, that’s right.

S: Okay, good.

I: That’s what I wanted you to look at today.

S: I’m curious now. I would probably be stricken with fear if I saw log x, cosine x and had to do like we mentioned, sketch a graph. Although that’s dumb because I know what log of x looks like and I know what cosine of x looks like. But could I put them together? I’m not real sure but I’m curious if you would give me a polynomial I would hope - like I said I did on the final - I would hope that I could take it and sketch the graph. Like I said I’ve done it before. I did it last quarter. As a matter of fact I did do it in high school so I would hope that I could do it.

I: If you did it for the final I would say you probably could again.

S: It’s just a matter of recalling the process.

I: And I think given enough time you maybe could tackle the log x, cosine x because when you had the computer compute the derivative for you back here you looked and checked to see if it was what you thought it was supposed to be. Now granted it might be harder for you by hand to try to find those critical points but given enough time I think you might even be able to think through that. Figure out when it would be zero. And put some values in even by hand, maybe with the use of the calculator. And which you would do if you were taking
calculus without Mathematica™. And figure out where that first derivative is positive and where it is negative.

S: Yeah, that's -
I: Because you had a feel for whether this is the answer you would expect for the derivative or not.
S: That's true. That is exactly true.
I: Now if you had just let it compute that and not bothered to even comment on whether you thought it was correct or incorrect or say what you said about it then I might be more worried that you wouldn't be able to handle something like that by hand because -
S: Find the derivative in itself.
I: You wouldn't know - if you can't get to that point then you're really stuck.
S: I like that feeling. When I entered g'(x) and it showed up and I went wow, exactly what I expected. It's exactly how I would do it by hand and come up with. So I was glad that I was right in my head.
I: It's interesting to me because some people will look and sort of like they're double checking the computer's computations the way you did. Is that what I think it is supposed to be? Some people when I talk to them just sort of take it for granted that whatever pops out of there is correct. They don't fall back and look and think now is that what I thought it would be?
S: I can't allow myself to fall into that group because I need to know how to take a derivative by hand. I want to know how to take a derivative by hand.
I: Now are you going to take the whole calculus sequence?
S: Yes.
I: That's what I thought. There are a few people who are taking the first couple quarters because that's all they need.
S: That's all they need. I need all the way up to 415 so yes, I plan to stay in it. Even right now in my physics course we're taking integrals and unfortunately in my calculus class I haven't learned that the derivative of x cubed is x to the 4th divided by 4. But I learned it in physics so I'm just like 3 days behind in calculus. So I need to know how to do this by hand. I need to know how to integrate things. I need to know how to differentiate things. Otherwise I'm going to have a list of formulas 2 miles long to memorize. I've got to be able to derive the formulas and as a matter of fact I plan to go to Dr. Davis in my free time if there is such and show him a physics problem that we went over yesterday in recitation and just ask for a little bit of explanation. I feel comfortable doing that. I've heard him mention to the people that just finished 254 that are in 415 he says come to me. I will help you. So that's reassuring. This problem we did in physics it took in an entire notebook page and a half to do the entire problem. Everything was integration and I followed it but I couldn't do it myself. So I want to get some help on it and so forth.
I: I know he's willing to do that and sometimes it might just be a matter of helping you connect what you're doing in physics with what you've been seeing on Mathematica™. It might be just a small little gap there.
S: It's so nice when you see something and you're like wow, that correlates exactly with what I'm doing in math. It just makes perfect sense. Because it's supposed to. But sometimes it doesn't always. I guess in math you're just doing it with random variable and random things and so forth whereas physics you're taking the derivative of dr/t and it's all those variables mean something and you've got to know exactly what's happening and when you're integrating from zero to r1 and then the next time you integrate from r1 to r2 and then from r2 to infinity and so forth you've got to be able to catch all that and right now it's a little overwhelming.
I: Sounds like you're getting there though and I think the gap is going to close between what you're doing in your calculus class and what you're doing in your physics class fairly quickly
and that will help you pull it all together. The fact that you're seeing the connections between them and you can tell you're real close to having that gap closed is good.

S: I'm really looking forward to the day when everything clicks and it's like wow because I know people that are in 162 for example and they just see that integration stuff and they just think oh, of course. I'm like - wait a minute. How did you integrate that one yet? Could you do just that one integration and show me how that one works? So I'm learning it in physics. I'm just ready to catch up.

I: If it makes you feel any better I know students who take calculus with or without Mathematica™. Even the ones who are taking it without I think still feel some of the same things that you're feeling because they're not as far as the physics is by the time they get to that point in physics usually.

S: Most of the people I'm referring to are in the 162 which is accelerated and they have been integrating since they were seniors in high school. I know that the 152 classes are having just as much trouble as the 152M classes. I do know - I try to keep up on it - I know that the 152 classes like I said maybe a couple of days ahead. They're just a little bit further than we are but I feel that it takes more time to cover - well, I don't want to say that - I just think that the Mathematica™ course we have to work at the pace that we're at. I really don't want to go any faster in that class. I don't feel like I'm losing out on anything. I feel like we're keeping up. It's important since I am taking 132 physics - 152 is a concurrent. Sometimes I think it should be a pre-req but it's a concurrent and so I'm doing it the way they say.

I: That's good and it's fairly typical. You're not in any worst situation than you would have been if you were in 152 without Mathematica™.

S: Certainly, certainly. Anyone that's in 152 and 132 is feeling the same thing. But I have a cousin who is a chem engineering major and because of the fact that she was a chemical engineer she took chemistry her freshman year and saved physics for her sophomore year. Well she was in 254 when she started taking 131. It was simple to her and so - sometimes I wonder. Well maybe the University is wrong. Maybe you should start your calculus series and then start your physics instead of taking them together but I'm getting by. Working a little harder this quarter than I did last quarter in physics. That was the one class that I kind of let slide. No more. I didn't do it because I hated the class or because of boredom or so forth. There was just not the time and I won't even blame the lack of time on Mathematica™ which a lot of people who dropped the class, dropped Mathematica™ said it took too much time. I didn't feel that way at all. Mathematica™ takes time but I don't think it takes much more than the regular math class would. It's inconvenient sometimes because when I want to work in the computer lab I want to do it all in one setting and I want to do it in the morning when Dr. Davis or a TA is there so that's sort of inconvenient with your scheduling but back to what I was saying is that as far as the time that was not the factor.
Student A – Interview 3

I: What's an integral?
S: We just did this today, too. An integral means to me that you're going to measure the area under a function, between the function and the x-axis so we have f(x) running somewhere and we take - the integral is going to measure the area between f(x) and the x-axis and it could be positive or negative or it could be both and cancel each other out, whatever the total area is.
I: So you could have negative area?
S: In the real world you can't but mathematically you can.
I: Does it measure, does it have to be area between a curve and the x-axis?
S: What do you mean?
I: I mean could it be area that's cut off and marked by something besides -
S: You could set whatever parameters you want. If you say you've got function f(x) and you're only interested between A and B then you could just do A and B and integrate from A to B.
I: Does one part of the boundary of that thing have to be the x-axis? If you have two curves?
S: Yeah. You'd have to - let me make sure - you have two curves and you're interested in the area between the two curves?
I: Can you do that?
S: It's just a matter of subtracting one from the other - manipulation.
I: You said a curve and x-axis and I wondered if -
S: That's for one function but you can manipulate it any way you want.
I: Okay. Let's let you do one of those areas.
S: By hand or by computer?
I: Whichever way you prefer. Find the area of the region bounded by the curve y = x^2. The x-axis and the line x = 3.
S: Let me just change some variables. You're only interested in the - you've got x-axis. Here is your curve. x equals 3. You want the area between it and y-axis or - it's going to bottom out anyhow but it's going to come back up over here.
I: Right, but I want the area that's bounded by the curve, the axis and the line equals 3.
S: So in other words that area in there.
I: Right.
S: I can't remember if that is 3 or not. I don't think so.
I: Did you look at the graph to get an idea of how to do the integration? Is that why you were able to see that or -
S: A little bit. Since it bottoms out at zero, zero anyhow I'm just going to integrate from 3, f(x). Is that what you want?
I: Uhhuh. So the area is 9?
S: Looks like it to me.
I: Do you know what Mathematica™ is doing when it computes this integral for you? Do you have any idea -
S: Yeah. The other way we did - f(x) = x^2 so we just could have taken f of 3 when it's f of zero. It's a fundamental theorem in calculus.
I: f of 3 and f of 0 where f(x) is x^2?
S: Yeah.
I: So if I take 3 squared minus 0 squared?
S: (inaudible).
I: So if it's just the value of the function at 3 minus the value of the function at zero where's this thing called an integral fit in? Is that the integral?
S: I hope so.
I: If it were another function f(x) = x^3 -
S: That also depends because the integral would be different. Say you're only in step 1. It wouldn't be so clean. That's pretty clean I think.
I: So would the area between 1 and 3 be f of 3 minus f of 1?
S: Yeah.
i: What is f of 3 minus f of 1 in this case?
S: 8.
I: Have it compute the integral for me from 1 to 3 and let’s see if that’s where it is. I’m just wondering if there is another thing called an integral or if it is just using the function and putting values in.
S: I made an error.
I: So is it using the function and putting the values in or is it using something else?
S: I’m not really sure.
I: Well even if it were using the function how is that going to be the area underneath that curve? How is the value rating at that point and the value rating at this point going to be the area under the curve?
S: It has to do with the fundamental theorem which states - I really don’t know. I’m not with it today. I can’t remember if it’s f’(x) or f(x). But I think this should be the fundamental theorem. I should have done the derivative. No, because we already knew the function. So we really should have integrated the derivative. There you go. Just should have integrated the derivative.
I: So if you integrate the derivative, what do you get?
S: Hopefully the answer.
I: As far as the function goes? Not the numerical evaluation.
S: What do you mean? I don’t understand what you’re asking.
I: You have a function f, right? f(x) = x^2.
S: So f of x is 2x. I mean f’ of x is 2x.
I: All right. And if you integrate 2x what do you get?
S: You got back 8 in this case.
I: That’s a numerical answer. Give me a formula. What is the integral? This is a definite integral. This is looking at the curve and finding the area beneath it.
S: Right.
I: So that’s a definite integral.
S: You mean as far as f of B minus f of A?
I: No, not f of B minus f of A.
S: Well then I don’t know it. I just don’t.
I: That’s all right. Let’s look at another problem. This one you’ll have to read a little bit and see. A particle moving in a straight line.
S: Write everything down and see what we’re getting.
I: Some people have been confused about how this is worded but what I intended here is the value of this function of V is 3 and the value of this function at S is 4 at time 1 second. So those are the values when T is 1.
S: Okay. Isn’t A going to be 1?
I: At time one second it would be 1, yes.
S: Isn’t that what it’s saying?
I: Uh huh.
S: All three of these are when T is 1.
I: This is when T is anything because this is just a general function for Acceleration.
S: These two aren’t general?
I: No. You’re trying to find those general functions for these two
S: General functions.
I: Do you know any relationship between acceleration, velocity and distance?
S: Not really but I can figure it out. Measured in feet for T. T squared. This is distance. I don’t know.
I: Okay.
S: I give.
I: Well - what if I tell you something? I'll tell you something and see if you can do one part of it. The other part would work similarly. If I tell you that acceleration just in general is the derivative of velocity can you then find a function for V in terms of T? If I give you that help? Acceleration is the derivative of velocity.
S: Get that. It's not necessarily right but I can get it.
I: What did you do to get that?
S: Well you just said that acceleration - so I said that was F of T is equal to 2T minus 1 and then I just thought of a function being velocity whose derivative would be 2T minus 1.
I: Hope that it's right.
S: Would that one have value 3 when T is 1?
I: One way to find out. What did you say T was?
S: 1.
I: What's that equal to? 3?
S: Yeah, and you want that to come out 3 when T is 1.
I: All right. It didn't.
S: Can you do something to it so that it will come out (inaudible)? If you simplify that what you really have there is 1. x squared minus T.
I: 2T squared divided by 2 is 1.
S: Oh T squared minus 2. Okay. so what can you do so that the answer would come out with (inaudible)? How can you modify that so that the answer would come out to be 3?
I: I don't know.
S: I don't know.
I: Okay. That's fine.
S: What's the next one?
I: Those are the only two I have for you to work on today.
S: Sorry to disappoint you there.
I: I'm not disappointed. I have some other problems but I think they might be a little ahead of the kinds of things you're doing right now in class so I'm going to hold onto them for the next time.
S: If I probably can't do these then I can't do the other ones.
I: That's all right. Let's save it.
S: Not much to save.
Student B - Interview 3

I: ... ask you what is an integral?

S: An integral - okay - it's something that has a function inside of it and it measures the area under the function from a point A to B.

I: Okay.

S: It measures from A to B. The area.

I: All right. Let me give you one of those. I want you to try it. Find the area of the region bounded by the curve $y = x^2$ and the line $x = 3$.

S: Find $x = 3$, okay. First of all - clear $f$, $x$, $y$ and type in what $f$ of $x$ is. Call $f$ of $x$ equal to $x^2$ which was $y$ so you can also say that $y$ equals $f$ of $x$. $y = x^2$ and then - okay it's bounded by the $x$-axis and $x = 3$. Then you want to integrate $f$ of $x$ from $x$ - from zero to $3$. It gives you $9$ so evidently that should be $9$. But that's not right. Because it should come out to be - maybe it is right.

I: Why did you say it wasn't right?

S: Well I was looking - when you integrate $f$ of $x$, $f$ of $x$ equals the prime of the outside so the integral would equal the inside of the integral $f$ of $x$ is equal to something on the outside and the integral of that would be like $g$ of $x$. $g$ of $x$ would be $x$ to the third over $3$. Okay, it is right. I was doing stuff in my head and it wasn't coming out right but it would be $27/3$ which is $9$ so that would be correct.

I: Now what is Mathematica doing when you tell it integrate $f$ of $x$ and you give it $x$ from zero to $3$?

S: it's going from the point zero to the point $3$ and it's taking that region and it's measuring all the area from $0$ to $3$.

I: How does it measure all that area?

S: it does what it calls trapezoids and it's like it gets them down so small. You can it yourself but it just keeps like subdividing trapezoids until it gets an approximate. Nine is just an approximate answer of the area under that curve.

I: Can you get a more exact answer?

S: No.

I: Let me give you another problem here to tackle. A particle is moving on a straight line. $f$ is the number of feet in the directed distance from the origin at $T$ seconds. $V$ is the number of feet/second and the velocity at $T$ seconds. $A$ is the number of feet/square second in the acceleration at $T$ seconds. So if $A$ is $2T$ minus $1$ and $V$ has to be $3$ and $S$ has to be $4$ at time equal to $1$. Can you write $V$ and $S$ as functions of $T$?

S: So it's going from $S$ to $T$. $S$ is the number of feet in the directed distance. From the origin at $T$ seconds and time $V$ is the number of feet seconds and the velocity. So it's going $S$ far and $A$ is the number of feet per second in the acceleration. $T$ seconds. It will take me a while here to get it. We're integrating from zero to $S$. $S$ is the direct distance and it's taking $T$ seconds to do it in and let me write this down. Pictures always help, I find anyway. From zero to $S$. At $T$ seconds of time. $V$ is the number of feet. Feet/second. $T$ equals time. The velocity of the particle at $T$ seconds. So this is $T$ seconds equals that and $A$ is the number of feet/second. $T$ over second squared. $T$ seconds. Okay. $T$ equals $1$ so we can get rid of that (inaudible). Since $T$ equals $1$ it's going to really not matter where (inaudible) because you wanted $T$ equal to $1$, right? These are the ones we did this time and we like skipped them. We didn't do any acceleration so I'm trying to figure in my mind. $T$ times $1$. So $A$ is going to equal $1$ because you're telling me that $T$ is $1$. So it's going to be $1$ feet/second squared and the velocity is also going to be - what was $S$ again? $S$ is the number of feet so this is four. So from zero to $4$. So what you're going to do is you're going to name the variables - going to clear the variables first. $S, A, T, V$ and $f$ and $x$ always. $f$ of $X$ equal to $2T$ (inaudible). $2T$ minus $1$. And then you're (inaudible) and you're going to say $T$ equals $1$. $S$ equals $4$ and $V$ equals $3$. We're going to eat (inaudible). Just telling the computer what all the variables...
stand for so when I go plugging them in it won't go what are you talking about. T.S and I know there was another one. V. What I did the first time (inaudible) give me the last variable which I put in which I need to know. We're going to integrate A.T.V. f of x. x -from the starting point where the velocity would equal 1 - I mean it will equal 3 because that's where it was telling me it started out at and then it's going to go to 4 so from 3-4 and then we get 1. So that would say that A equals 1 which plug it in T equal to 1 would give us 1. It is striking me as not right but - I didn't even look at this part of the lesson because we didn't have anything to do with it so -

I: You don't have (inaudible).
S: No. We did the density ones.
I: Do you know any relationship between acceleration, velocity and distance?
S: One is like f of x equals f ' of x which that would give you the acceleration and then f " of x equals the velocity. So I guess I would just ask it - that would be smart of me to do. What f of x is. f ' of x is zero. And I don't know why. It should be 2. I don't know why it gave me zero. It should have been 2. I did everything right there. It should give me 2. The computer is not working. Let's try it again here. For the sake of being stupid I'm just to rename T'H' since the computer doesn't seem to want to give me - 2H. I would say it wouldn't be zero. It would be 2.

I: Aside from the fact that you keep getting these zeros now, tell me what you're doing. You're putting in the acceleration function and you're taking the derivative and that would give you what?
S: That should give me the velocity but it doesn't seem to want to give me the velocity so it would be 2 times T and T is 1 so the velocity would be 1 and the acceleration is 1. And then it's moving - for every 1 second, 1 per 3.

I: Maybe you need to tell it with what variable, with respect to what variable you want it to take the derivative or something like that.
S: I know what it is. I'll try that. That was what it was - 2. Because we weren't taking the derivative of x. We were taking the derivative of H. I do that all the time.

I: Use two different variables.
S: Yeah. You can't use two different. You have to use the same. it doesn't matter what.
I: So say this for me again so I understand what you're saying. Acceleration, velocity and distance are related. You took the function for acceleration and you took the derivative.
S: Right.
I: And that gave you velocity. So then where's the distance figure in there?
S: Actually the distance is like the first function you use. And then the derivative of the distance will give you acceleration. That's wrong. It goes DVA. It's the distance, velocity and then the acceleration so the derivative of the velocity gives you the acceleration so in order for me to get the velocity here I'm going to have to integrate g at H. For H. H to zero - 4. It is 12. That's the growth. So it would be the constant 12 so you would add 12 to this. This would be -1H plus 2H squared over 2 so it would be H squared plus 12.
I: So that would be a function then for -
S: For velocity. And then in order - if you would take that function and call it another thing and then integrate that you would get the distance of 1.
I: Did you (inaudible) H from zero to 4 because - why did you type that?
S: Well S is the distance between 0 and 4 so the line is definitely only going to go between zero and 4. Because S equals the 4. That would keep everything constant if you told it always to integrate from 0-4.
I: Let me have you look at something else. Now you don't necessarily have to write or think about this in terms of all numbers but just give me an idea of how you might attack a problem like this. A (inaudible) having a trapezoid cross section is full of water. The trapezoid is 3 feet wide at the top, 2 feet wide at the bottom and 2 feet deep. Find the total force owing to water pressure on one end of the trough.
S: Well you would figure out the trapezoid which is based on its height base 1 plus base 2 divided by 2 multiplied by a half. Add up the bases - its base 1 plus base 2 so the top is 2 feet and then you would add the bottom which is 3 feet wide. 3 and 2 which is 5 divided by 2 which is 2 and then it's 2 feet deep so you would times it by 2. Because it's the depth -

I: Then that gives you what?

S: The volume of the trough. That would give you the volume of the trough and then you would have to find out the pressure somehow. That would be the same way with finding the density where we find what P sub S is which would be the weight of the water and stuff which would be the same as the volume and then that's equal to the prime of the pressure. Whatever the function of the pressure would be and then you would take the derivative of it and that would equal the volume of the water.

I: Okay. Well those are my problems for the day. You are going to have plenty of time here.

S: Were those from the beginning of this week's lesson?

I: Yeah I think so. I went over to the lab yesterday and looked a little bit at some of the things you have.

S: The only thing he did was the weight density problems with the metal rods and stuff. That took me a week to do that. I seem to spend so much time over there. It's getting better though. I'm starting to understand it. The class is getting smaller so I think that helps.
Student C - Interview 3

I: The first thing I want to do is just get you to tell me what is an integral?
S: An integral is a derivative and you have to find the function that the derivative came from. And they give you like a certain range and stuff to plug into the derivative in order to find the function that it came from.
I: Okay. You've been looking at finding areas using integrals. So this is the first problem I want you to consider. Find the area of the region bounded by the curve y equals x squared, the x-axis and the line x equals 3. Any of these - if you want to use any paper any time - if not for this problem for others that's available and what not. That's the first problem I want you to consider.
S: I'll just do the curve, y equals x squared. And here we get a graph.
I: You're going to graph y equals x squared.
S: The curve and the line. So you want the area of this, right?
I: Uhhuh.
S: You could take maybe the integral of function from negative 2 to 2.
I: Okay. I think maybe we have something a little different in our graph there. Where do you expect the line x equals 3 to be? Do you expect that to be a horizontal line or vertical line?
S: A vertical line. (inaudible). Here, I'm going to put - above here a g of x. I'll just put equals - because you would want a vertical line.
I: Can you approach the problem then without having it draw the line?
S: Without the line?
I: Without having it plot the line.
S: Oh yeah. There's the curve and you want the area -
I: Bounded by the curve. The x-axis and the line x equals 3.
S: So it would be this area in here, right?
I: Yeah.
S: So this would be from zero to 3. It would be the integral. The range. The integral. Of f of x as x is from 0 to 3. I forgot to put an integral - what integral there? Oh integrate. All right. So it's saying that the integral from 0 to 3 which is from here to here of the curve is 3. So I would say just that the area from here to here would be 9. Because you're going - you're going 3 and going up to about (inaudible) you're hitting 9. Right here is the (inaudible). Here is about 3 so this line - I would say it intersects it like maybe 9. So then that area would be 9 in between there. That's how you use like the integral function.
I: Do you have an idea of what Mathematica™ is doing when it calculates -
S: Oh it's like breaking it down into a triangle so then it's 1/2 the base times the height and then that would give you the area. See it's sort of hard like with the curve. It makes it easier - because of the curve you have to like -
I: So how does it make use of these values you gave it? Zero and three?
S: Well it like isolates the curve only from that function. Like if I put from - even if I put from 0 to -3 it should give me the same value.
I: Why would that be?
S: Because it's symmetrical to the y's of the area. It should be the same. I'll show (inaudible) - 9.
I: So does that mean the area is negative?
S: No. Area can't really be negative but you just sort of know that. Area has to be positive.
I: So why didn't that give you a positive answer?
S: Because it's taking negative 3 as like the base - let's see. The area is 1/2 base times height and I put in the base for negative 3 so it just does the calculations without realizing that. Meaning the area. It seemed to like ignore the negative signs usually. The area isn't exactly the same on both sides.
I: You said that it was doing a triangle. Or something like a triangle but since that is curved - can you get it to plot a function so that that actually would be a triangle over there? In other words can you make a linear graph over there so that it would be triangular?

S: A line (inaudible) there?

I: Uhhuh.

S: I don't know what kind of graph it would be. Let's see - x.

I: What line would be the diagonal with those (inaudible)?

S: It could be just y equals x.

I: Okay. That one down below somewhere.

S: Like graph it differently?

I: Yeah, on a different axis. Okay.

S: That will take it but you might want to shift the line a little bit. It's sort of - maybe x - 1 or something and see I want to shift it up.

I: If you left it y equals x which would have larger area? The area under the curve you're computing or would the triangle that was cut off it?

S: The curve. This line doesn't justify where the curve meets the other one. See you have to like plug in different values to shift in (inaudible) so it's almost like tangent to it, almost.

I: And what kinds of values might shift it out?

S: Let's see. If you shift it out maybe a plus 1. Let's see. x equals 3. That just moved a little too much.

I: Uhhuh.

S: I'm going to use x plus 1. I don't know if that will - hm.

I: Can you get that line to go up without moving it from the origin?

S: I don't know. Let's try 2x. It's sort of getting there. Might want to shift it down 1. 2x maybe minus 1 and see where that leads to. That's better. Maybe if I put 3x then that would do it. There - that's pretty close.

I: Okay.

S: So it's taking that line which is (inaudible) to the x minus 1. It would be plus 1, minus 1. Then 3 and the base is 3. That looks fine already. When the base is 3. That's probably what it's doing. It's probably (inaudible) drawing the line. Like this little area is sort of little. It's not that much to worry about. But it probably did an exact triangle though.

I: If it's a curve rather than x squared then the triangle wouldn't really fit underneath the curve, does it still use triangles to approximate?

S: I don't know if it would use a triangle. It might use like trapezoids or something else that would fit the shape of the curve. It depends on the curve.

I: All right. I have another one here. This one might be something you're just getting into and you might have to think this over a little. A particle is moving on a straight line. S is the number of feet in the directed distance of the particle from the origin in T seconds of time. V is the number of feet/second and the velocity of the particle at T seconds. A is the number of feet/second squared and the acceleration at T seconds. If A is 2T minus 1 and V is 3 and S is 4 when T is 1 can you write a function for V and S in terms of T?

S: Oh - let's see. All right. So we're given that A equals 2T minus (inaudible).

I: Uhhuh.


I: As functions of T. That's kind of a breaking integral.

S: Okay.

I: Functions of T.

S: You're saying that T has to be 1, right?

I: When T is 1 you know that S is 4 and V is 3.
S: Okay. So if T is 1 I could always put it in there and get what A is. So that would be 2 - 1. So A would have to equal 1, too. So if A equals 1 and A is the number of feet/second squared.
I: Acceleration.
S: So you're going 1 foot/second. That's the first thing squared in one second.
I: Yes
S: And you're moving at 3 feet/second. It doesn't make sense. Moving at 3 feet/second and in one second you only go one foot - you're accelerating 1 foot/second squared. So in 1 second you're moving 3 feet because it says 3 feet/second, right? So you're moving 3 feet and A is saying that in those 3 feet you're accelerating 1 foot/second squared. So just when (inaudible) wouldn't it just be 3 then?
I: Well, one of the conditions is that S is 4 from T.
S: When A is 1 - Hm - (inaudible) A has to equal 1. So it would be 0-
I: Do you know how acceleration and velocity in distance are related?
S: Isn't it V times A equals D?
I: Okay.
S: I think it's V times A equals - it would equal S then. So I could put V - hm. V times A would have to equal 4. I don't know what to do. (inaudible). A equals 2T minus 1.
I: So what you're looking for is a way to write V in terms of T and a way to write S-
S: In terms of T. I know that T is 1. Hm - I don't know what you're going to do with this.
I: That's all right. I wanted to see if you can attach it and see where you get with it. I have one I was going to ask you to approximate something with a polynomial but I think you've not done that yet so let's see if you have some idea here. The trough with a trapezoidal cross section is full of water. If the trapezoid is 3 feet wide at the top and 2 feet wide at the bottom and 2 feet deep can you find the total force in one (inaudible)?
S: Trough having a trapezoidal (inaudible). It would be like -
I: You can draw that on paper if you want. I have paper and here is a pencil.
S: Let's see. 3 feet wide at the top. So this is 3. 2 feet at the bottom. It is 2 feet deep. Okay. Find the total force (inaudible) to water pressure on one end of the trough. Do they mean like I have 2 ends here?
I: Yeah.
S: I'm not a very good artist but that's 2 ends. And this is 2 feet deep. So wouldn't the total be 6 up here? It's a trapezoid. No, it would just be 3.
I: 3 at a given end.
S: But I'm doing it with 2 ends aren't I?
I: The trough has 2 ends. We're interested in what the force would be exerted on one end.
S: I have 3 feet. Oh - Force equals pressure - maybe I could find the area of the trapezoid on the side which has the area for the trapezoid. Uh - hm. That 2 feet deep has to give me something.
I: Yes
S: Two feet deep. What is the total force - given the pressure - total force and force is related to the - it's not related to the area. Hm. I have to do something with the area but I can't figure out what.
I: Okay.
S: So the area of the trapezoid - if I draw a line diagonal. This is 2. This is 2 feet deep. Okay. Maybe I could do that. A right triangle. And this is 2. This one is hard. Because if I make a right triangle here I don't know where the base would be (inaudible) that. And up here that maybe just be the (inaudible). That's one too. It's probably really a simple solution. I can't think of it. Did the other people get this or not?
I: I've just started these interviews. You're close to the beginning of this round of interviews.
S: Because (inaudible) class and I'm trying to remember from last year. Hm.
I: if you could figure out the area of the trapezoid where would you go from there?
S: I don't know. Force equals the area times something. See I don't do anything else in that class.
I: Really?
S: (inaudible). So maybe just the force equal the area. But I don't think so. I don't know where you would go with that one.
I: Okay.
S: Did you figure it out?
I: I had a little bit of an advantage on this one. Can you come up with a formula even for force?
S: Oh force.
I: If you can't recall that, that's all right.
S: I know it has something to do with area but or maybe even the (inaudible). Would it be the area? I don't know. Force times the area. Force equals area times - or maybe force equals area squared. Hm.
I: That's all right. You're having those kind of problems -
S: Now it's going to bother me.
I: A long way back.
S: Now I'm trying to think. Area times - I keep thinking about pressure but they don't give you any pressure. They give you water.
I: You could use that as a variable maybe.
S: Area times P? Well then you have to solve for two variables. Unless you could find the pressure of water which from chemistry I should know. I guess you could do area times pressure. I'm not sure. Because I need to find out how now. I think so. I'm not sure.
I: All right. We'll go with that.
Student D - Interview 3

I: What I want to do is ask you a question. What’s an integral?
S: It’s - I don’t know the exact definition of it. I can tell you what you’d do in it. When you evaluate an integral it would give you - it’s usually a derivative of a function and they want you to integrate it from a certain point to another point which means you have to integrate it up to the original function and then plug in those two numbers and subtract and the answer is the difference.
I: So what do you have then when you plug in those two numbers?
S: Area.
I: Area of what?
S: Area of a function from a certain point to another point.
I: The area of the function?
S: Yeah.
I: Let’s let you do one. You knew that was coming. Here is one of those typical kinds of problems. Find the area of the region bounded by the curve $y$ equals $x$ squared, the $x$-axis and the line $x$ equals 3.
S: Okay.
I: That’s the graph of $x$?
S: Yes. And (inaudible).
I: So you’re going to try to plot the line too?
S: Yeah. I guess not. I am definitely guessing here. I don’t know how to plot this line.
I: Well, can you handle it without plotting the line?
S: Yeah I suppose I could. So it (inaudible) and let you know. It’s not from the (inaudible).
I: Where would that line $x$ equals 3 be?
S: Over here. I guess I should extend its range.
I: It would be vertical?
S: Uh huh.
I: So what area -
S: This is easy. What am I doing that’s so hard? Okay, wait. Got a function of $x$ equals $x$ squared, right? I want the area between the $x$-axis and where $x$ equals 3 line, right?
I: Uh huh.
S: So all we do - say integrate this function from $x$ equals 0 to 3.
I: You are going to have to integrate from 0 to 3?
S: Uh huh. To find the area between from here to here. All this area going down to the $x$-axis. So I was trying to make it too hard at the beginning. You just put the area underneath and really what it’s doing is integrating this function and then plugging in the value of 3 for it and getting an answer and then plugging in the value of zero into it and then subtracting the answer you got from zero from the answer you got from 3 to give you the area between, the triangle.
I: Why does that give you the area? Why does putting 3 into that function you get for the integral and putting zero in and subtracting them give you the area?
S: I have no idea. I don’t know. Not today. I’m sorry.
I: Do you know what it’s doing when it computes it? I think you probably just told me that. You said it was calculating the answer when you’re putting 3 and putting 0 in.
S: Yeah. If you do it by hand then all you do - it’s written like this. Derivative of 3. Integrate $f$ of $x$ which is $x$ squared. From 0-3 so what you do is you raise the power like to give you the anti derivative. You raise the power and then divide the coefficient by whatever power you get.
I: That’s (inaudible).
S: Yeah. And then I would plug in 3 (inaudible) and then plug in zero which is zero and the answer is 9. Some theory that we learned but I don’t really understand the principle behind it.
I'm sure I do but I don't know it right now off the top of my head. But that's what it is doing. This is just as easy to do by hand as it is on the computer for a simple function like this.

I: Would you sketch the graph if you were doing it by hand probably or would you just compute it?

S: I usually don't draw them but they told us to show this one. If it's more complicated I do but I know that $x$ squared looks like this so I can just draw a picture of it anyway. If it was like $e$ to the something, I'm not familiar with those functions then I usually try and plot them on the computer because I can't compute those in my head because $e$ completely loses me. Even if it was like $x$ squared plus $4x$ plus $3$ or something I'd try and plot that to see what it would look like. And something like this.

I: Let me give you another one. I know you've been taught about these. Let's see what you can do with this. A particle is moving on a straight line. $S$ is the number of feet in the directed distance of the particle from the origin at $T$ seconds of time. $V$ is the number of feet/second in velocity as a particle of $T$ seconds. $A$ is the number of feet/square second and the acceleration at $T$ seconds. If $A$ is $2T$ minus $1$ and $V$ is $3$ and $S$ is $4$ when $T$ is $1$, would you write functions for $V$ and $S$ in terms of $T$ for me?

S: on the computer or by hand?

I: Whatever way you want.

S: Okay.

I: It looks like you've got what?

S: I've got $T$ at the origin and then I've got like a certain point and making this distance $S$. Number of feet. $A$ equals $2T$ and $V$ equals $3$. $T$ equals $1$. As functions of $T$.

I: $S$ is $4$ when $T$ is $1$.

S: $S$ is (inaudible) number of feet. So you want a function that will combine all these together?

I: Two functions. One function for $V$ and one function for $S$. A function of $V$ in terms of $T$ and a function of $S$ in terms of $T$.

S: Did we do this last quarter?

I: I think you're just starting to look at some things like this now?

S: Maybe this is the problem that we're (inaudible).

I: Do you know how acceleration and velocity and distance are all related?

S: Yeah.

I: How are they related?

S: Distance equals rate times time.

I: And how is that related to velocity and acceleration?

S: Acceleration is - velocity - I don't know how to do this problem. Hopefully we'll get some rules or something in the math book. I'm really blanking out today.

I: That's okay. We can leave this one.

S: I wouldn't even do this on the computer because this looks like a hard problem to me. If we're going to work on this on the computer I'm sure we'll know afterwards but never having seen this before.

I: I think you could do this one fairly easily by hand.

S: Let me look at it.

I: Has this stuff got anything to do with calculus? With the things you've been studying in calculus?

S: This looks more like physics to me. What I did last year in high school.

I: Is physics connected to calculus?

S: Yeah. Physics uses calculus to figure out problems and geometry and algebra, uses everything.

I: So how does it use calculus in a problem like this?

S: I wish I could tell you. I don't know how to do this one.
I: Okay. Let me give you another one. You might attack me over this one too. A trough has a trapezoidal cross section and it’s full of water. If the trapezoid is 3 feet wide at the top; 2 feet wide at the bottom and 2 feet deep can you find the total force owing to or attributed to water pressure on one end of the trough?

S: Hm. These are physics problems. What are you doing to me? Did you really get these from our homework and stuff?

I: These are things similar to some things you’re going to look at.

S: Oh, that we’re going to look at.

I: That you should have started looking at. The other one I think you might have already started looking at. You might not have put those things together yet.

S: Right now we’re working on arc lengths. We haven’t even done velocity, force, pressure, any of that stuff.

I: All right.

S: I’m sorry.

I: Don’t be sorry. Let me see if I can find something else here for you. I’ve got some other problems written down. Let me stay away from things new then. Tell me what you’ve been doing with arcs.

S: Finding the lengths.

I: Finding the lengths of arcs?

S: Uhhuh.

I: I’ve got a problem for you. Something like you have been doing. You’re looking at function and you’re looking at arcs. What are you using those arc lengths for?

S: They just give us like an arc like this and they tell you to find it’s length and we use this formula that’s the square root of 1 plus the derivative of a function squared. And -

I: Is the length of the arc?

S: That would give you the length.

I: Why would you want to have the length of the arc?

S: Good question. We’re not applying it to anything. They’re just telling us find this length and they gave us the formula and plug in the numbers and stuff. I think this is it but I don’t know if there is anything else - 1 plus the derivative squared and then take the square root of that whole thing. It’s like right now we’re just learning things and it seems like we’re going to apply them later because right now it seems like we’re just learning these concepts and then dumping them and go onto new ones and forgetting those and learning a bunch of different ones and not doing anything with them.

I: Okay. I’m not going to ask you another question. I don’t have any more (inaudible) know about today.

S: I wish I could have told you about those other ones.

I: That’s all right. You might get a chance that last interview to tell me something. Maybe you’ll know something about those things by that time. Maybe you’ll take all these ideas you’ve been looking at and throwing away and start applying them by then.

S: Now you’re going to make me think about these.

I: That’s good. If you think about it then when you do start applying it then you’ll have it.

S: It’s got to be (inaudible). That’s not right, though.

I: What if I did this? What if I told you how acceleration and velocity and distance are related mathematically? What if I told you that acceleration is the derivative of velocity and velocity is the derivative of the distance function?

S: Velocity is the derivative of distance?

I: Then could you answer that question? If you are given the acceleration function can you with these conditions come up with a function of V in terms of T and a function for S in terms of T?

S: You want S and you want V?

I: Uhhuh, functions for them with variable T.
S: Does that work?
I: 2T minus 1 is V, the derivative of V. What's V? T squared minus T?
S: Uhhuh.
I: It has to be a function so that when T is 1 V comes out to be 3. Would that work?
S: No.
I: What do you need?
S: Constant. Have to integrate it. So I would have 2T minus 1 as a derivative of V so I integrated it to T squared minus T plus 3 to get just V. Then when T is 1 V is 3.
I: Now can you write a function for S in terms of T?
S: I remember doing this at the very beginning of last year. Double checking. It had better equal or I'll be mad. Okay. So I just went through and integrated it and then I had to be plus a constant and then given the conditions that S equals 4 and T equals 1 I plugged in the 1 and said it equal to 4 and solved for the constant which came out to be 7/6. So S is 1/3 T cubed minus 1/2 T squared plus 3T plus 7/6 and V was T squared minus T plus 3. You had to give me these conditions so I would know what I was doing. I didn't know these. This was the only one that I knew.
I: Does it make sense that the derivative of the velocity, with acceleration, with velocity? (inaudible).
S: I don't know. I faintly remember this one from physics. The derivative of velocity equals acceleration but not this one.
I: Okay.
S: Got further than I was about 5 minutes ago.
I: That's good. Those are the ones I wanted to ask you about.
Student E, Interview 3

I: The first thing I'm going to do is just ask you a question. What is an integral?
S: An integral. From what I understand of it now it's the space underneath the curve.
I: In that case I have a problem for you. Start you off with one I know you can do here. Find the area of the region bounded by the curve y equals x squared, the x-axis and the line x equals 3.
S: Okay. On the computer?
I: If you want.
S: Can I draw a picture first just for me to play with?
I: Uhhuh.
S: All right. How do I do that? Okay. So we're taking - okay. I'll try it.
I: You're graphing the function x squared?
S: Right, and also graph x equals 3. The line x equals 3. g of x - see if it works. I'm not real sure if it will work. Are we going anywhere with this?
I: Are we going anywhere?
S: Yeah.
I: I think the problem tells you where you're going if you're careful.
S: All right. The area of the region bounded - the x-axis - Okay. I was going to say (inaudible). I just want to see a picture.
I: So you're going to graph it from 0 to 10?
S: Yeah. Just to see a picture. Okay. You want the area bounded by the curve x equals 3 and the x-axis. By bounded are you talking about this region or this region? I'm not sure what you mean.
I: Whichever one is contained by those formulas that's marked off by those things into something that you can actually find the area of.
S: Okay.
I: Bounded by means that those are lines in the curves provide the boundaries of the region you're trying to find the area of.
S: Okay. I'll try that one. Square root.
I: Tell me what you're trying to do.
S: All right. First of all I'm going to make this shorter so that I can see it. Get a better representation here.
I: So you cut down the (inaudible).
S: Right. I'm going to go for this area here because this area here I'd have not a chance of so I'm going to find out where x equals 3 and y equals x squared meet and of course it will be through the - x - about 1.4. So - I'll need to do that if I can just do it.
I: So you're going to have Mathematica™ find the route?
I: Do you know what Mathematica™ is doing when you reach that find route command?
S: It's using Newton's Law. It was a big long thing that instead of memorizing that I just memorized the find route. But I know it's Newton's. Now I'll integrate. Maybe I won't integrate. Uh - 2x minus f of x. I'm not sure if this will work. It's 7.3205.
I: Explain what you were doing before you integrated.
S: All right. What I'm doing is having it take the area of this, of the x equals 3 minus y equals x squared and taking it from zero to 1.3205. I hope. Yes. So the area is 3.464.
I: Do you know what Mathematica™ is doing when you put that in integrate command there?
S: Roughly. Uh - let's see if I can think of it here. I know it's taking the area and it's doing it by - let's see. In this case it would be doing it by taking this rectangle and then it would be subtracting this area underneath the y equals x squared curve and it does that by taking the trapezoids. And it takes many different trapezoids.
I: Okay. Let me give you another problem. See what you can do with it. Cars are moving on a straight line. S is the number of feet in the directed distance. A particle is (inaudible) at T seconds of time. V is the number of feet/second in velocity of a particle at T seconds and A is the number of feet/square second in acceleration at T seconds. If you know that A is 2T-1, and you know that when T is 1 V has to be 3 and S has to be 4 can you write functions for V and S in terms of T?
S: Can I?
I: Uhuh.
S: I don't know. I did arc length today. A particle is moving in a straight line. S is the number of feet in the directed distance that the particle is from the origin T seconds. So - S is the number of feet in the directed distance. If T equals S - T is the number of feet/second and velocity of the particle is T (inaudible). T is the number of feet/second. And the velocity - T seconds. A is the number of feet/second squared. The acceleration. S equals T obviously. Anywhere along here would be a P. So if it is 3 seconds then it would be (inaudible). I haven't got - T is the number of feet/second so it is S/T. So velocity equals feet/time. A is the number of feet/second squared so A equals V squared at T seconds. If T (inaudible). S/T. So if A equals 2T minus 1. (inaudible). S equals 4. T equals 1. Oh.
I: Do you know any relationships between acceleration and velocity and distance?
S: No. I know from what Dr. Davis has said just off hand that to get the velocity if you take the integral of acceleration - would that work? What did he say? Or if you know velocity you take the integral - I don't know. I'm stuck. I don't know.
I: Okay. When did he mention that to you?
S: Today. We were talking about arc 1. So it just kind of stuck.
I: So was he telling you you are going past that arc length towards something like this?
S: Yeah. I don't know if it's in this homework because - we did arc 1. More than arcs. We did something else. I had to do that Tuesday and I can't remember what it was. But I don't know the relationships.
I: That's all right. Did he tell you about that because somebody asked the question or did he just tell you that?
S: No, because we were learning today that the arc length is S - if S is the arc line S of x equals the square root of 1 plus f'(x), f'(x) being the function squared. Then define the arc length. You take the integral from A to x and then just put in the function. The square root of 1 plus f of t squared. That would be S of x.
I: And while everybody is in the middle of doing this he's talking about -
S: Well how did he bring that up? We were talking about - he was showing us an example of his famous function. Always looks like that. And he did the mean value theorem to prove this function.
I: Prove that that was arc 1?
S: Right. Did the mean value theorem to do that and then he mentioned with taking the integral of the derivative - dv/dt is important in finding velocities of other things. And he said that finding the integral of the derivative will always be important and it has many, many functions in calculus he says. The basic theorem, the fundamental theorem is very important in everything that we do from here on out.
I: What is the fundamental theorem?
S: The fundamental theorem is f of t equals, dx - f of t would equal g of t minus g of a. I don't know if this is correct or if it's f' of t. And basically f' of t is equal to g' of t.
I: What does that mean? That looks like an attempt to make a statement that you might see in the tape but what does that really mean?
S: What does that really mean? That -
I: What does it mean to you maybe?
S: There you go. That whatever function we're trying to find is equal to the derivative of the integral from wherever you're going and then you can prove that by taking the function
whose derivative would look like your function and subtracting - let's see, where you're going to from where you've been. (inaudible). It's just - what does it mean to me? I don't know yet. I'm working on it. I'm finding relations with - like if you are on a curve. You start off at f of x and go to f of x plus h. That the derivative of h goes to zero as your area? Is that right? Goes back up to here. Something like that.

I: What is that you write up there? f of x plus h?
S: f of x plus h minus f of x divided by h.
I: What's that?
S: That's what goes back to the fundamental and I'm trying to find out what the relationship is because if you take the limit as h goes to zero you get your f'. You get your derivative.
And cause if you go from - let's see. If we were going to go from A to T then add h that - let's see. That would be your - what do you call that? Your slope? Not your slope. What do you call that? Oh no. I'll call it slope or rise for a better word and that is how it helps, how it works on this one. If you can find that then you can find the function. It's all tieing in but I'm still trying to figure out how.

I: Okay. Well since he said what he said to you in class what if I told you that acceleration is the derivative of velocity?
S: Acceleration is the derivative of velocity.
I: And rather than trying to attack this whole problem if I just said that acceleration is the derivative of velocity could you do the - at least tell me or show me how to get a function for velocity in terms of T with what's given in this problem? Would that help?
S: Yeah, I think so. Because -
I: Why don't you use another piece of paper.
S: So acceleration is equal to the derivative of velocity.
I: With what you know here ignoring the S as the distance for right now could you write a function for V in terms of T? If you know that?
S: Yeah I think so. T minus 1. Came out with these two. Okay. So if I take (inaudible) plus - see prime of T would equal 1 plus - let's see. Prime of T squared. That would find area.
I: Wouldn't it? f' of T. Well yeah. You just take - you do the rule.
S: The rule?
I: The rule. How did he do that? There's a fundamental - by applying the fundamental theorem you can come up with what this is. V equals 3 and that equals that.

I: When T equals what?
S: When T equals 1.
I: This was V is 3 and S is 4 when T is 1. So you can ignore the comments about S for the moment because -
S: Then T would be 3 plus this. 2T minus 1. Call this x. So it would be x.
I: Tell me what you did there now and why you did that.
S: All right. I know that the derivative of V of T is 2T minus 1. So, and I also know that when T equals 1 the velocity equals 3 so V of 1 equals 3. So if we take that V of 1 and add to it the integral from 1-T of this one, 2T minus 1. I put X in there since I put it into the integral so I don't confuse all of this. If I add to it that integral then I'll get my velocity.
I: What does that come out to be?
S: Let's see.
I: Can this help you? Can Mathematica™ help you?
S: Sure can. I can make it do anything.
I: You can do it by hand if you want. It's your choice.
S: How about if I do it by hand and check myself on the computer?
I: Okay.
S: So it would 3 plus - this is the derivative. You've got to find the integral which would be y. It would be 2. If you have the negative that's - I know that N to the A plus 1 - A plus 1 but if it's a negative - so it would be 2x squared over 2. Then it's (inaudible). Let's see. So it
would be - I can’t drop that, can I? Let’s try it. 2x squared over 2. If you put in T. So it would be 2T squared over T squared minus 1. So it would be 3 plus T squared minus 1 which would be 2 plus T squared. I don’t know. We’ll see.

I: Two plus T squared is what you’ve got?

S: Yeah. We’ll see. There we go. And let’s see - we’ll call it V of x. V of x equals 3 plus - is that the command? I don’t know. We’ll find out. Integral 2 plus 1. From 1 - T.

I: Now what are you doing? Checking your -

S: I’m checking - it’s this one. V of T equals 3 plus the integral from 1-T, 2x minus 1, I think. No, it doesn’t like that. Ah, integrate. 1 plus T plus T squared. I got the T squared.

I: You’ve got 2x plus 1 up here too.

S: Which is the A. I’m taking the integral.

I: But you’ve got 2x plus 1 here and you’ve 2x minus 1 here.

S: Oh! 3 minus T plus T squared. 3 minus T so that 1 was a T. Oh. This should have gone to a T so it should have gone 2x squared over 2 minus x. So 2T squared over 2 minus T. So it would be T squared minus T. If you do one. I think I understand.

I: Okay. What happened?

S: I think what happened is when I was at 2x minus 1, when I took my function that should have - that this should be the derivative of, it should have been 2x squared over 2 minus x. And that would have given me - because these would cancel so it would be 2x minus 1. That’s (inaudible).

I: Is this T squared plus T minus 3 must be what? A function of V in terms of T?

S: Right.

I: Okay. I think that’s all I’m going to ask you about that.
Student F – Interview 3

I: The first thing I want to do today is just have you tell me what is an integral.
S: An integral is the area under a curve.
I: Okay. Anything else you want to add?
S: Uh - well the area under a curve. I guess between the x-axis and the curve I should really say.
I: Okay.
S: And it - oh there's lots of different stuff about it. I don't know if you -
I: Just wanted to hear your definition.
S: Is that right?
i: That sounds like a pretty good place to begin. Since you mentioned area under a curve let's let you see if you can find that. Can you find the area of a region that's bounded by the curve y equals x squared? Looking for the region down by the (inaudible) y equals x squared, the x-axis and the line x equals (inaudible).
S: Can I move out here? I just want to see -
I: Sure. You can do anything you want. So you're going to plot that function x squared first?
S: Yes. That's what I'm plotting.
I: Now how did you decide to choose your range?
S: You said between -
i: I just want you to say what you decided.
S: You said the line x equals 3 so I went out to 3. If it had of gone to 0 I would have to choose a negative range too but in this case it's 0. 0 - 3. Line squared unit.
I: Okay. Now -
S: What's that? For the (inaudible).
i: How is that computing? Do you have any idea?
S: Well -
i: Because if you look at this area that you're computing it's not a regular (inaudible) or anything.
S: Actually I think it finds the - let's say that H of x is equal to this integral right here. Then - H of x, the derivative of H of x would be equal to the integral of (inaudible).
I: Okay.
S: So that it just might find the, might go backwards instead of finding the derivative. It might find the function where the derivative came from and plug the values in, 0 and 3 or subtract the value from 3. Put in the value of 3 and subtract it from the function. Then the value is 0. Do you understand that?
i: Uhhuh.
S: In this case the function would be x to the third over 3.
i: If you take x to the third over 3 and you put in 3 and 0 and subtract like you said you would get it.
S: You should get 9.
i: True. Okay. Why would putting 3 into that function and 0 into that function of H be the area under the curve?
S: Why would it be the area under the curve?
i: Uhhuh.
S: Oh -
i: And it looks like it is.
S: Uhhuh. We were just working on something along those lines. In this case - I have a hard time explaining it on paper too, actually. Area consists of two different dimensions and so there is a - like length times (inaudible) length times the height is going to equal - there is a point in here that if you take the height of that, whatever it is times the length of the whole curve on the x-axis will equal the same area as is under the curve right here.
I: Why is that?
S: Well I don't why that is.
I: Okay. Is that point going to be - you sort of made it look like it's about half way. Is it going to be half way?
S: Well no. If it was a right triangle it would be. I mean if it was a 45 degree triangle. What do you mean? Half way along the x or half way along the (inaudible)?
I: Half way along the x is about where you - well you were a little past half way actually.
S: I guess it wouldn't be if it was - (inaudible).
I: Use a function that would make it look like a right triangle. Can you graph a function that would make it look like a right triangle?
S: Sure.
I: (inaudible). Do it below. I didn't want you to delete.
S: You know how your brain kind of goes weird sometimes? That's the way I feel. Not just in here but in a few of my classes.
I: Maybe a lot of ideas are about to come together for you.
S: Maybe that's it.
I: You graphed g of x equals x. (inaudible). All right. That looks like a right triangle.
S: What's half way out on the x-axis?
I: 1 1/2.
S: So -
I: But that's not right.
S: I looked at the area, at the value of the function. It's at 1 1/2. You decided now it wouldn't be the same as the area.
I: 1 1/2 times 3 if you take the whole area it would be yes. I was thinking that it would be 1 1/2 times (inaudible) and that wouldn't work. I was thinking I would take half from this area over here but if you take this point, the height of this point times the length of the whole one (inaudible). And so when you manipulate these ones up here, they turn out to where - I can't quite remember how we did it. But they're not the same length.
I: Would the area under that line y equals x? Is it going to be more than the area of y equals x squared or less?
S: Uh -
I: You have that same problem with function y equals x instead of y equals x squared.
S: Do you want me to just - this would be -
I: You can computer it but tell me what you think it is going to be first?
S: I think it will be - this would be actually less.
I: This area will be less?
S: yeah.
I: Why would you think that?
S: This area should be about (inaudible). It's easier with right triangles. Because you can just use trigonometry to find out the area. 6.
I: Would you think that by looking at those two graphs?
S: Since this one goes all the way up to 9 up here -
I: Oh.
S: It would (inaudible). This one below here goes up to about 3, 3 1/2. That makes sense.
I: All right. I don't know if you've looked at anything like this or not. This might be something you had (inaudible) but so far in using integration have you looked at particles moving?
S: We're just starting on that.
I: Oh you did.
S: Actually we're doing that in physics too.
I: You have a double dose. If a particle is moving along a straight line, S is the number of feet of directed distance for the particle from the origin in T seconds. V is the number of
feet/second. Velocity is the particle at T seconds. A is the number of feet/second squared. Direction of the particle in T seconds. If A is 2T minus 1 and V is 3 and S is 4 when T is 1 express V and S as functions of T.

S: Oh.
I: I don't know how useful Mathematica™ is going to be to you here.
S: Really?
I: Maybe. I think it can do some of the work for you but I think you've got to think a little bit.
S: We did this. It was tough.
I: You can do it on Mathematica™. You can do it with Mathematica™ and paper or whatever. How would you attack this problem?
S: Well - A equals 2T minus 1? Is that for all the T's? And V equals 3 of where?
I: Well - V equals 3 when T equals 1 and S equals 4 when T is 1.
S: Okay. Is it starting from a stand still?
I: Keep them on the same line.
S: Okay. I'll probably do some thinking on paper first.
I: I can provide the paper.
S: Okay. A equals 2 T minus 1. V equals 3. So we're looking for -
I: Functions.
S: Functions of V and functions for T.
I: Yeah. A function for V and a function for S.
S: In terms of T. Okay. So velocity - this should be more than that. If I can just think of how to get it.
I: Can you tell me how this -
S: I did this wrong.
I: This A and V and S are related?
S: Sure. Velocity is the derivative of distance. Acceleration is the derivative of time. Did I say that right?
I: You said velocity is the derivative of distance and acceleration is the derivative of time.
S: Velocity is the derivative of distance. Acceleration is the derivative of velocity.
I: Velocity is the derivative of distance. Acceleration is the derivative of velocity.
S: Right. So it would be - A equals - is that right? Would A - Oh - bad day. Acceleration is the slope of velocity. That's right. Does that make sense?
I: Uhhuh.
S: Acceleration is the derivative of velocity.
I: Uhhuh.
S: isn't that right?
I: Yes.
S: That means that - A of T minus 6 is the same as the integral of (inaudible).
I: Okay.
S: yeah. I was trying to think if I did this right. Zero. That would be zero. I must have done something wrong.
I: These are the relationships you know (inaudible).
S: yeah.
I: Remember, you're going to look (inaudible) for the function itself too.
S: I figured they'd give it to me in terms of T. I'm really having troubles today. I have a quiz today too on this.
I: What are you asking the computer to do when you put these values out here? See you're telling it to integrate when you're giving it this acceleration function 2T minus 1.
S: Right.
I: And then what are you telling it to do when you put the values out there -
S: Actually with a function (inaudible) I'd probably - I can't remember how to do it. I'd have to go look at it. I'd probably have to do it by hand. I think probably I should have done that from the start.
I: Okay. so you're saying V of T is T squared minus x.
S: Uhuh.
I: And S of T is T cubed over 3 minus x squared over 2?
S: Yeah.
I: Will that fit the conditions that were given in this problem for V being 3 and S being 4 when T is 1?
S: I don't know. Let's see.
I: Well - if T is 1 -
S: This should be (inaudible). If V is 1 -
I: If T is 1 -
S: 1-1 is zero. Nope. They sure don't.
I: What do you need there?
S: Need to add a 3 to it.
I: Okay. When T is 1, S is 4.
S: This is 4. 1/3 minus 1/2- what is S supposed to equal?
I: 4.
S: Negative (inaudible) plus 4 and a 6. Okay. Ah - my brain (inaudible). The strain is too hard I think. There is a way to do it this way but I just couldn't remember.
I: Can you get Mathematica™ to give you those functions?
S: Yeah you sure can.
I: How do you do it?
S: Well - have (inaudible) disc.
I: Is that the same disc that you can't get things off of?
S: Well - it is working now. If I had to do it I'd probably - I'd probably have to look at - I can't remember. I know that AT - it has something to do with the points. You need the points and you have the points there. You have everything you need with what you gave me for Mathematica™ to figure it out.
I: Will Mathematica™ integrate without you giving it these values over here?
S: It should do it (inaudible). 2T minus 1. (inaudible).
I: Do you know what a definite integral is?
S: That's where the values are assigned. Where it goes from one point to another. It may be (inaudible) but I just don't know how to do it.
I: All right.
S: A definite integral is when you have your x values defined and your indefinite is when you don't.
I: Okay.
S: I'm sorry I can't remember that.
I: That's all right.
S: I know it has to do with the fundamental theorem of calculus.
I: You're not to the point yet - the fundamental theorem of calculus. What's the fundamental theorem of calculus?
S: (inaudible) telling me about the wrong theorem.
I: I'm not saying you got it wrong. I was just curious since you brought it up. Tell me something about the fundamental theorem of calculus.
S: I thought it was an integral call it f ' (x) from A to B equals f of B minus f of A.
I: Okay.
S: Is that the right one?
I: Now what were you saying about it?
S: I said that's how you figure these integrals out. And it's really simple. It's really simple to plug them in and get them out.
I: Have you gotten to the point where you're using polynomials to approximate numbers yet?
S: I don't think so.
I: We've gone on and on about these kinds of problems.
S: I'm sorry.
I: That's okay. I expected that to be the case because I think you're probably just getting a good idea about these particle moving type problems.
S: I know you figure it out using - plug it in in a way that you have to add - f of D minus f of A equals (inaudible) If you know the point here, f of A. If you do zero. You add f of A to both sides.
I: You say your quiz today is on this kind of thing?
S: I think so.
I: If that's the case, just so you won't leave here with any bad ideas you should really figure out if that constant has to be a 3 before you go on and compute this function S of T.
S: That's true.
I: Because this is going to mean that this function -
S: It's wrong.
I: Yeah. That's for the sake of your quiz.
S: That's right.
I: But you had these all put together in the right way. So you've got the ideas right.
S: I can't quite remember how to do this problem. Maybe by the next time I will. We just started getting into this and this was something we barely touched on our last lesson.
I: Did you know how to do this integration that you did? I know it was simple by hand as a result of being in this class or did you do that before you came into the class or is that a result of physics?
S: As I mentioned before my calculus experience was 3 years ago before I came into this class and that was pre-calculus and we might have done stuff like this but I cannot remember any of it.
I: Had you done something like this in physics before you hit it in here?
S: No. Actually we have gone just past that in physics and they never required that you go from one step to the other. It was just understanding the relationship of something traveling at this velocity or this distance and so the slope of that would be velocity for example. They never make it equations that you have to figure out. So I learned everything new. At least I learned this from Mathematica\textsuperscript{TM} or from doing the problems. Figure it out by hand and then go check it with Mathematica\textsuperscript{TM} so I did that.
I: Okay. That sounds good. Sometimes if you're doing a problem for me and you do make a mistake I'll let you go with the mistake and see where you go. But since we're (inaudible) and you are going to have a quiz I didn't want you to leave thinking that problem was absolutely correct since you were having a quiz so you could deal with it.
Student G – Interview 3

I: First thing I want to do is just ask you a question. What's an integral?
S: An integral? You want to know what integration is?
I: What is an integral?
S: It's like the area between - like an integral between like A and B. So it's like the area between A and B on the function.
I: The area of a function?
S: Uhuh. Between -
I: Of a function?
S: Just of any function. Like between A and B. Like between 1 and 4 of like f of x or whatever.
I: What if I had said what's integrate or integration?
S: Integration is the opposite of derivative.
I: All right. Since you brought up this area, let's do one of those first. We'll do an easy one. Find the area of the region bounded by the curve y equals x squared. The x-axis and the line x equals 3.
S: Oh joy. Plot - x squared - x is the line. x equals 3. Plot the line x equals 3. (inaudible). That did do it.
I: Is that the line x equals 3?
S: Hm -
I: What is that?
S: That's the line y equals 3.
I: Oh.
S: I don't know how to do it. They never taught us how to do that. I have no idea how to make that line.
I: Can you do this problem without graphing that line?
S: Yeah, because it would just be from here to here. It would be the top function.
I: Let's get rid of that y and just do this problem. Just do this problem without it.
S: Okay. This goes to zero.
I: Now explain to me where that line is and what it is you're hearing now that you're typing in the (inaudible).
S: The line is right here.
I: Okay.
S: Then this function goes to zero so you want to know the area between here and the 3. So you would integrate. x and squared. x times 0 and 3 so that's where the (inaudible) is. And it's 9.
I: 9 what?
S: I don't know. It's just 9.
I: Okay. What is Mathematica™ doing when you tell it to integrate? How does it compute the integral?
S: Like in high school you would use the trapezoids. If you use the trapezoids and then as they get smaller that's what - I mean you can use the trapezoidal method but it's not going to be exact integration doing that.
I: Is that what Mathematica™ does do you think?
S: I have no idea. I just type it in.
I: What does it do with the zero and the (inaudible)?
S: That's telling it to (inaudible) from zero to 3 because if I changed it, if I put it to say 4 then it's going to be bigger because now it's sticking up here. That's the unit from 0-3.
I: Okay. If you were doing that by hand, what would you do?
S: If I was doing it by hand? I would take the integral, X squared plotting at 3, and the 3 and subtract it.
I: What would you (inaudible)?
S: Subtract the integral from (inaudible). Integration of - was it x squared? x squared and 3-0. The integration of that is - this is x to the third and it's over 3 so you plug this in. 3 cubed is 27. 27 divided by 3 is 9. Take that minus and you plug the zero in and you're just going to get zero so it's 9.
I: So you'd plug in 3 and you'd plug in the 0 and you'd subtract.
S: Yep and then it's 9.
I: What does that give you the area of?
S: I have no idea. I really don't. In high school we were taught to do like the trapezoids. That's self-explanatory because the area of the trapezoid is underneath the curve and no one ever told me -
I: What does that function that's the integral there in that x cubed over the 3 what does it look like?
S: What does it look like?
I: Yeah. Can you graph it up here and get this x squared curve? Somebody told me you would never use colors?
S: Who told you that? Did Lola say that? Lola doesn't know what she's doing. Hey - this should be colored. There. What's wrong with this?
I: What is wrong with this?
S: I have no idea. It should be right. Color, style. Oh come on. There's nothing wrong with that. Wait, wait. The green one is x squared and the blue one is X cubed over 3.
I: So when you take that x cubed over 3 and you find the value of that function at 3 and the value of that function at zero and you subtract them and you have the area underneath x squared (inaudible). Hmm.
S: Why I don't know. I never learned that. We did this in high school and I liked integrals but he never told us why. He just told us - we had to do it for like a trapezoid and I did the first - there was some of them (inaudible). It was f of B minus f of A is equal to the integration of B to A of f of x. I think you can use that one too and it gives you a more accurate but it's not as - integration exact.
I: Is that the one you're using when you did what you did on paper and put the 3 in there and put the zero in there.
S: Yeah, that's just the integration.
I: Is that (inaudible) in calculus?
S: Kind of, not really. Well - no. Because the fundamental theorem is saying that f of 3 minus f of 0. See this is just the (inaudible). Would be equal to f' of squared. Which is (inaudible) minus. If you plug that in it's going to equal 6 and that's not going to work so - it isn't like the fundamental theorem. Oh well. That's why I didn't do good on that quiz this morning.
I: I'm going to give you a problem now.
S: I hate to do problems.
I: See if you can deal with this.
S: (inaudible).
I: Acceleration is 2T minus 1 and if the V is 3 and S is 4, T is 1. Can you write functions for V and S in terms of T?
S: Velocity is the derivative and acceleration.
I: Is it?
S: I think it is. I can't remember. I did this in physics. Acceleration - is acceleration integral (inaudible)? It's one of the two.
I: Now what did you say? You said either it's -
S: If you integrate acceleration you get velocity or if you take the derivative of velocity you can acceleration or it's the opposite. I don't know.
I: Choose one and try it. I want you to end up with two new functions. One for V and one for S.
S: So it goes to SV? From the origin and it goes - S equals 4 when T equals 1. Oh man. I have no idea to tell you the truth.
I: You have some suspicion about a relationship between acceleration and velocity.
S: I know there is. I just don't know what it is though.
I: That will get you somewhere.
S: It would be T - x equals 4. So (inaudible) equals 4.
I: You're taking the integral of 2T minus 4 from 0 - 4?
S: Yeah, which obviously is not right. Take the derivative - it's not (inaudible). It's equal to 2.
I: Now how are you making your decision if it's right or not?
S: Because there is no 12 written on that page.
I: Oh,
S: So it can't be.
I: There is a 2 written on that page. It's in that A equals 2T minus (inaudible). Does that count?
S: No. That's just part of the equation. That 2 has nothing to do with that I'm afraid. If T takes 1 second, velocity is 3. I can't remember. I took physics my junior year. I never did any good with these problems. Six up there.
I: My goodness. When you integrate 3 from 0-4 you also get 12. Isn't that amazing.
S: You can figure out another one. Distance equals 1/2 AT squared. I remember that. And so it goes 4. 4 equals 1/2 AT squared. Acceleration of 2, T-1. AT squared. Solve that. 4. T. Distance equals 1/2 AT squared. This isn't right. 1/2 A which is acceleration with T squared. 1/2 T - what is T? Equals 1. This is - T equals 4.5. That's what T equals.
I: When?
S: When the acceleration - when it's the point - when it's at 4. It's going 4.5 in whatever units/second.
I: Oh. Feet/second.
S: 4.5 feet/second. It has to travel 4 units. So it would probably be -
I: When you have Mathematica™ do things like integrate or find the derivative it gives you (inaudible) and what we're really looking for here is the function.
S: That is a function.
I: Well -
S: You don't like that function?
I: No.
S: It works.
I: But it's not a function for V in terms of T or a function for S.
S: Velocity equals distance divided by time so like if velocity is 3 it would be equal to (inaudible) variables than that. Distance is equal to velocity over time I think. I can't remember. No, it's distance divided by time. No, that's not right. You only have 3 and we have 4 divided by 1 and that's not true. I can't remember if you integrate (inaudible) remember about these stupid equations.
I: But would - even if that's right - would having Mathematica™ evaluate that integral from 0-4 give you what you're asked for in this problem? Which is a function of V in terms of T? That is giving you a numerical value of something, isn't it?
S: I have no idea. That's basically what it comes down to. Every problem I've ever had like that I've missed.
I: Suppose I give you a helpful hint and I tell you you are on the right track. I tell you that acceleration is the derivative of velocity so that when you tried to integrate that you were going the right way, all right? And I'll even go a step further and I'll tell you that velocity is the derivative of this distance function S.
S: Velocity is the derivative of the distance.
I: Yeah. So in other words if you can find that velocity function you could integrate it and you could get this function. But now you need a formula for that. Mark a numerical value and some integral. Don't delete all that good stuff. Leave that good stuff up there.
S: It was worthless.
I: It's not worthless.
S: That was my physics. That's why I got a B in physics.
I: It makes me happy to see it though.
S: It's gone forever. Now you can't see how stupid I am.
I: I'll remember. Now can you, knowing that - I told you you're doing the right kind of thing but I need a function now and not a numerical value.
S: Oh a function of that that would be for T?
I: In terms of T.
S: That would be - T squared minus T. Like that.
I: Would Mathematica™ do that? Is there a way you can get Mathematica™ to give you that function T squared minus T?
S: I don't know. There you go.
I: Oh yeah.
S: T squared minus T.
I: Okay. Now let's work with this. Is T squared minus T then actually the function for V? Does it satisfy the condition that when T is 1 V is 3?
S: Uhhuh. No.
I: Uhhuh, no.
S: You can't tell. There's no V in that equation. There is just T's.
I: I thought that equation was an equation for V.
S: If it is then it's not true because it's 1-1 which is 0. So it is 3.
I: So what do you need? When T is 1 V has got to be 3.
S: You need a constant.
I: Okay. What does that need to be?
S: How about a constant T?
I: So if I had T squared minus T plus 2 then I would put 1 in for T, I'd get 3 for V?
S: Yep. Because when you integrate it and (inaudible) there is always a constant.
I: I want you to show me that. I don't believe it.
S: What do you mean you want me to show you that?
I: I want you to show me that T squared minus T plus 2, evaluated at T equals 1 gives me 3.
S: First you'd say T equals 1. Then you just put it in.
I: Oh, T, -
I: With what I told you can you take the T squared minus T plus 3 now and give me a function of V?
S: Yeah. Just integrate that. One is a number which cannot be used as a variable. What's wrong with that? I don't have to integrate it. I can stay - it would be -
I: Write it down for me.
S: It would be T cubed over 3 minus T squared over 2 plus 3T.
I: Okay. When T is 1 you want S to be 4. So that's S. Is that right?
S: If T is 1?
I: Uhhuh.
S: It's going to be 2/6 - no, that's not right so you have to have another constant. Wait a second. 2-3. 17/6 or something. Why isn't this 1? Integrate 3. I got - I can't (inaudible) T plus 1.
I: (inaudible) that T plus 1 back up there where you had it.
S: See now I can't.
I: I know. Come in here before to integrate. Can't you clear it out at that point?
S: There. That's what I had. And now - not again. T squared over 2. 17/6. So it doesn't work. You want it to equal 4?
I: Uhhuh.
S: Add another constant. Equal 4 - T -
I: Oh that's ugly.
S: (inaudible). No wait, that's not going to work.
I: When T is 1 I want that to equal 4. I don't want to know in general when that's equal to 4.
S: That's right. Why don't I (inaudible) T? It's been a long day. That will solve it. Ah! Oh come on. I don't like those (inaudible). There is the constant 7/6. You want me to leave this here?
I: Yeah, please.
S: There is the constant, 7/6.
I: Okay.
S: It's a bad problem. I hate those problems.
I: You shouldn't hate those problems.
S: I do because I'm never going to have to do them ever again.
I: You're getting near the end of the quarter.
S: I have 27 days left. No more calculus ever again.
I: You have counted the days. What is your major?
S: Accounting. I'm in engineering calculus in the business school. My academic advisor told me to take it.
I: Is there any business course that has anything to do with calculus you're taking?
S: No. Well like interest formulas. Those are just like powers.
S: There is (inaudible). It's all straight forward. There's not calculus like this. You don't have to find the areas and stuff. It's not accounting. Why? Do you disagree?
I: I think there might be some connection between those cost functions and some of this calculus.
S: It's not as in depth though.
I: Let's see what you do with this problem.
S: Trapezoidal cross sections (inaudible) find the force (inaudible) water pressure - see force equals mass times acceleration.
I: Okay. You might end up with something that has variables in it as your answer.
S: Trapezoidal cross section.
I: The top of the trapezoidal cross section. The top is 3 feet wide. It is 2 feet wide at the bottom and 2 feet deep.
S: Three feet at the top, 2 feet at the bottom. And it is 2 feet tall. Find the total force owing to water pressure on one end? You want to knew the pressure at the end of it?
I: It's a trough.
S: Yeah but it sits going down.
I: Right.
S: You don't want to find the pressure at the bottom of it?
I: No, at the end.
S: I can't do that.
I: That's good. That's all right. That's good.
S: If you could tell me how much the water weighed I could figure out the force at the bottom because gravity is 9.8 meters/second and then the force of the water times 9.8 would be the force at the bottom. On the end I have no idea.
I: We'll leave that one.
S: You don't want me to find that one?
I: No. I just wanted to see what you'd do with it.
S: That's what I'd do but there's not enough information. There's not just some golden formula.
I: All right.
Student H - Interview 3

I: The question is what is an integral.
S: It's the sum of area. It's a way to find area of a given function. If you're given a function and you want to find the area and if it's not just simple, easy you can integrate and find the area under the curve or under whatever, or a volume, anything really.
I: I'll let you do one then to demonstrate. Find the area of the region bounded by the curve $y$ equals $x$ squared. The $x$-axis and the line $x$ equals 3.
S: Okay. Do you want me to just find the area or do you want me to plot it or what?
I: Whatever you would prefer.
S: Okay. I'll plot it first. I like to see it. $x$ squared.
I: Now tell me how you decided what kind of range that (inaudible).
S: Do you want the area of the curve $y$ equals $x$ squared bounded by the $x$-axis and the line $x$ equals 3. So it's from 0 - is that the $x$-axis or $y$-axis?
I: $x$-axis.
S: Three to what? I'm assuming it's zero. Is it zero?
I: The line $x$ equals 3 is -
S: That's one $x$ value but I have to have another $x$ value to take the integral of it. I'm assuming it's zero. Because it intersects at zero.
I: Okay.
S: Because it's a function of symmetry.
I: Bounded by these three things pins that down for you doesn't it?
S: Well it intersects at zero but it has symmetry with respect to the $y$-axis.
I: What area would be bounded by those three things? That is determined by the fact that it has to be an area bounded by those three. Right?
S: Right, but what I'm saying is if you have the line $x$ equals 3 and it has symmetry there is an area at negative 3 to 3. Then that area would be zero. If you keep going, if it's negative infinity to 3 -
I: But would that area be bounded by the curve, the $x$-axis and $x$ equals 3?
S: That's why I'm assuming it's zero because (inaudible) starts at zero.
I: Okay.
S: That's why I choose that. There is the curve. So I want to integrate, oop. $x$ squared -
I: Okay. Now what?
S: Whatever the units are for the $y$ equals $x$ squared.
I: Okay. Do you know what Mathematica™ does when you ask it to integrate with that command?
S: It's approximating the area by saying finding an infinite small number of straight line segments and then finding an area that each little trapezoid and then (inaudible) supposedly.
I: Okay. What does it do to decide how many little trapezoids to use? How does that (inaudible)?
S: It's an approximation. It takes as many - it takes a lot more than we can. If we were to divide it up into a specified number, maybe 6, over the interval from 0-3 if we made 6 then you could get a rough estimate of what it is as you go littler and littler. The difference between your number and what Mathematica™ does is just a little bit because it just divides it a lot more than you did. It's just a small area difference.
I: I've got a problem for you to look at. A particle is moving on a straight line. $S$ is the number of feet in the directed distance for the particle from the origin in 2 seconds of time. $V$ is the number of feet/second in velocity of the particle at $T$ seconds. $A$ is the number of feet/square second near acceleration of 2 seconds. If $A$ is 2$T$ minus 1 and if $V$ is 3 and $S$ is 4 when $T$ is 1 can you write functions for $V$ and $S$ in terms of $T$?
S: If you know the acceleration you can integrate the acceleration that will find velocity. So you just want - you want me to plug in $T$ as 1?
I: Well I want the functions that you come up with for V and S to give these values when T is 1.
S: Okay, I'm integrating - I forget how to write that. It's an indefinite integral because I don't know T. What if I just write it on a paper and tell you?
I: That's okay.
S: I forget how to write it on the computer. I'm going to take the integral of the acceleration which is velocity and that's 2T minus 1. Okay. The velocity would T squared minus T?
I: Does that satisfy the condition?
S: And then I have to say that this equals 3. So I -
I: When T equals (inaudible), you want that to equal 3.
S: No it doesn't. It's either T equals 3 or T equals 4. If I add 1, T equals 3 seconds or T equals 4 seconds at Velocity of 3. By the integral of acceleration.
I: Now what did you do to do that?
S: I took the integral of -
I: No, this part down here. I'm with you to here. Now you've put -
S: Okay T times T - 1 equals 3 so either T equals 3 or you solve for T. Then you say T equals 3 or T equals 4, right? Isn't that right?
I: Okay.
S: I hope so. I hope my algebra is right.
I: I just wanted to see what you were doing.
S: I think it's T equals 3 and T equals 4. I think that would be right.
I: When T is 1 you want the value of that function you wrote for velocity to be 3 so what -
S: So if I plug in 1 here I get 1 equals 3. That's not right. No I don't. I get zero equals 3.
I: So what can you do?
S: My acceleration is wrong. There has to be a constant. I know there is a constant out here. This has to be plus C but this constant must be - let me see. This has to equal 3 so if T is 1, 1 minus zero. It would have to equal 3. The constant would have to be 3.
I: So what is the function then for V?
S: T squared minus T plus 3. That should be the function for V since you know it is a constant. It should be the velocity and if you take the integral of the velocity then which is T squared minus the angle of T plus integral of 3 that's T cubed over 3 minus T squared over 2 plus 3T plus some constant, K. Uhhuh.
I: Okay.
S: And this constant here - that has to equal distance of 4 and 4 has to equal T cubed over 3 minus T squared over 2 plus 3T plus this new constant K so if I plug in 1 to T I get 1/3 minus 1/2 plus 3 plus K equals 4. So if I solve for K I get 4 minus 1/3 plus 1/2 minus 3. Four minus 3 is 1. 1 plus 1/2 is 3/2. 3/2 minus 1/3 - so K equals 7/6 of distance and would equal - this is a new constant so 4 equals 1/3 minus 1/2 plus 3 plus 7/6K and that has to equal 4. V and S is a function of T so S equals this integral. S equals T cubed over 3 minus T squared over 2 plus 3T plus 7/6.
I: Now summarize for me all of these relationships between velocity and acceleration.
S: What you have is the integral is defined as sum f' of let's say T dx. So if A is f' of T, take the integral of the acceleration and that will be the velocity. It's what we learned. The integral of acceleration of velocity and then if you take the integral of velocity you get the distance. So when I found the velocity, when you take the integral of 2T minus 1 you get T squared minus T but you have to add that constant on so then you solve for the constant and you know that's 3. Then you take the integral of the velocity which you just found and you solve for the new constant, K and then you plug that back into what you found for the integral and that should be the formula for the distance I think.
I: You said that that is something you just learned.
S: I forgot the constant here. That's why zero didn't equal 3. I forgot that constant on there.
I: Since you said what you said then my question is going to be a why question. Why is acceleration the derivative of velocity and why is velocity the derivative of (inaudible)?

S: Okay because if you take the distance, if you work backwards and you go from the distance to the velocity a distance graph, if you take the derivative of a distance graph that is the velocity because it's the slope of the lines. And the integral is just the anti derivative. So you're just working the opposite way. So if you know the distance you can find the velocity and if you know the velocity you take the slope of that graph and that's acceleration.

I: What is the slope of that graph acceleration?

S: Because you have the velocity is in meters/second and time is in seconds so if you take the rise of a run it's meters/second/second. So it's meters/second squared. That's just meters/second squared unit for acceleration but it works the same way with feet/second or whatever. Feet/second squared.

I: let me ask you about another one here. That you may end up using variables in but let me just get your feel for what you might do with this problem whether you actually do it or not. You have a trough that's trapezoidal full of water. And the trapezoid is 3 feet wide at the top, 2 feet wide at the bottom and 2 feet deep. How would you find the total force owing to water pressure on one end of the trough? What kind of thing would you try to do? Whether you know all of the numbers you might use or not.

S: First I'd draw a picture to find out what the trough looks like because you know it's 3 feet at the top. Three feet across at the top. It's 3 feet wide. It's 2 feet wide at the bottom. This is 2. This is 3. And 2 feet deep so you know if you have a perpendicular right here. Find the total force owing to the water pressure on one end. Okay. Well what you could do - let me think here.

I: You don't have to actually try to work it out. Just explain the process here or something like that.

S: If you know water is 2 feet deep I think the force on the sides is going to be the same as the force on the bottom. I don't know if that's true or not because it seems to me if you fill it up with 2 feet full of water at the trough that the water would be pushing on it with the same force on the sides that it would on the bottom so if - let me see. You could take - I don't know if this would be right but since the trapezoid looks something like this from the end this angle right here, since you know this is 2 and this is 3 - this angle will be the same as this angle. I don't know if you'd have to use like the - I'm trying to think if you have to multiply the force of gravity times some angle to get the force on the side here. But the force on the side is not what we're looking for.

I: Is there any calculus involved in working this problem that you can see at this point?

S: I don't know. I think it would be - I'm sure there is because it's almost like a related rate problem. Water falling in; water falling out kind of thing. You could find the force, the water pressure on one end so if you knew the total pressure on the box, the force on one end would be 1/4 of that I think. I don't know if you could find the volume of water in the trough and then find the volume of water in the box and find the weight of that and relate the weight to the pressure. You could find the force on the one corner of the box maybe by dividing that by 4. I'm not really sure.

I: Okay. I didn't want you to actually work it out. I wanted to get your feeling on what direction that might go into.

S: I'm sure there is calculus but it's more or less a physics problem I think.

I: Okay. Anything else you want to say about that?

S: No.

I: Another particle moving along the x-axis under the action of a force. f of x pounds. When the particle is x feet from the origin if f of x equals x squared plus 4 find the work done as the particle moves from the point where x equals 2 to the point where x equals 4.
S: The work done. So the force is x squared plus 4 and you want to find the work done at the point where x equals 2 or x equals 4. Work done is I plus - I think it’s the squared of I plus f’ of x squared is the work field of force. Is that from the lesson that we just did?
I: It’s like one you just turned in today.
S: I think that’s what it is. I think it would be the square root of I plus -
I: Did you turn that lesson in?
S: I got it done last night and I turned it in this morning but I think that’s the formula for it. Let’s see here. x squared plus 4.
I: You’re going to plot it and look at it, right?
S: Uhhuh. I always do. That’s practically the reason why I took this course. From 2 to 4
I: So you could look at the graphs?
S: That’s the greatest part of this course. So you can see what you’re looking at I think. So we want to find the area from 2 - 4. Plot from 0 - 4. Okay. So we want to find work done as (inaudible).
I: Does that graph look right to you?
S: x squared plus 4. It intersects at 4. This should be at 4. There it is. So it is 0 4 right there. This is just - the x-axis is just - I don’t know why it’s up here.
I: That’s what I was wondering.
S: There is one function you can plot it in true scale. I forget what the word is. It’s not (inaudible). It’s integral 1. Try to plot (inaudible). Yes. Aspect ratio automatic. Is this (inaudible) problem?
I: We know how the graph goes.
S: It’s supposed to intersect at 4. So it’s supposed to be - yeah the intercept should be 0 4. It should be a parabola. I don’t know, whatever. Anyway, it is supposed to be intersect at 0 4 and if I’m trying to find the work done I’d take, I’d integrate the force - I didn’t tell it what f’ of x is. I’ll have to go back.
I: Is this just a formula you memorized or why does this work?
S: Oh man. Okay. The work done - the work done is - no, that is not it at all. That is arc 1. Gosh, work done. Oh, wait a minute. Work done. Role of the force I think. I think it’s just the integral. I don’t think it’s - I would have to look in my notes. I think it’s just the integral of X squared plus 4 from 2-4. The area under a force graph - if that’s work that’s the answer. If the area under a force graph is work then that is the answer. I don’t know if the area under force - I think that’s right. I think that the integral of a force is work. But I wouldn’t bet money on it. I’m not sure. I forget that. I’d have to look at my sheet.
I: That looks pretty good.
S: I think arc length is square of 1 plus f’ of x squared. That’s what I was thinking of. If work is the area under the force curve then you’ve got it but otherwise we’re totally wrong.
I: Otherwise we’ve got something else.
S: Yes we do.
Student J - Interview 3

I: I want to just ask you a question. What's an integral?
S: That's funny that you asked. Just last night I was discussing the term integral with a 254 student and my definition of an integral is simply a sum of differentially small areas so I guess you could either say it's the area under a curve or like I said I like to think of it as the sum of the differentially small areas because it is a sum and that's the way we think of it in physics, as a sum. Not just any area. It's a sum of very small particles.
I: Was that the result of the conversation you mentioned?
S: Yeah. I was saying in my opinion - I look at it as a sum of the very small areas under the curve and he agreed with me but he wanted to stress the fact that it wasn't a summation. And I said no, no I didn't mean that it was simply a summation. It's just you add up all of the small areas. That would be what I would call an integral.
I: Since you said you may be able to talk about area under a curve let me give you something along that line for your first problem. Find the area of the region bounded by the curve \( y = x^2 \). The x-axis and the line x equals 3.
S: All right. In terms of numbers - ?
I: Yes.
S: On the computer you use the \texttt{N integrate} command whereas if I were evaluating it by hand I would do it differently obviously. I would integrate \( x^2 \) from 0 to 3 because x goes - I assume you mean from 0 to the x-axis up to the curve but you also mean from zero to three, right? So I'm going to integrate from zero to 3. The computer is going to tell me 9 which if I would plug in, if I would differentiate \( x^2 \) squared that would be - I don't do that very well off the top of my head. It's \( x^2 \). In a function you have \( x^3 \) cubed over 3.
I: Now you said differentiate it -
S: No, I didn't mean - to get that from something - anti-differentiate is what I'm trying to say. Because that would be \( 3x^2 \) squared over 3. See I have to go backwards and then go forwards to check myself. So if I were to plug 3 into this I would have \( 7/3 \) minus - plug in 0 which is 9 minus 0. 9.
I: Why does plugging 3 into that formula and plugging zero into that formula and subtracting - why does that give you the area?
S: Fundamental theorem of calculus.
I: What's the fundamental theorem of calculus?
S: You want it in letters? I guess it would be \( f' \) of x dx is equal to - from A to B is equal to \( f \) of B minus \( f \) of A.
I: What does that mean? Why is that?
S: Excuse me?
I: Why is that?
S: Why is that true?
I: Yes
S: You mean not simply because I'm plugging in - okay.
I: Yes
S: I'm sure we did prove that and I do remember doing it. Why is it exactly true? Because if you have a curve and this is B and this is A and the curve is specified by this function - let's see. Good question. I don't know if I can actually say it in an actual definition, words. I don't know. It's a formula I memorized.
I: That's good enough.
S: I remember doing it but if I had to go backwards, like how would I go backwards, I don't remember if I could or not. We would probably deal with estimations and so forth.
I: Okay. When you gave me your definition of integral you said you liked to think of it as summing up infinitesimal little areas? What kinds of little areas?
S: Like when we started this quarter we would draw lines like this and create trapezoids and find the area of them and then add them up. And then first we would draw ten blocks and then twenty five, then fifty and keep getting bigger and bigger numbers until we were getting closer and closer to what the computer told us with the integrate command so that's what I mean. Until eventually you're going like this and you're just adding them up and they're so small.

I: Adding up these smaller and smaller trapezoids.

S: Yeah. We just worked with work in terms of x and that's equal to the integral of f of x dx and that's just like - it's the force needed to move from here to here plus force needed to move from here to here plus force needed to move from here to here and so forth. Just adding up those differentially small values.

I: All right. Let me give you a problem then to look at. One of these particle moving problems. The particle is moving on the straight line. If f is the feet in directed distance from the origin in T seconds; V is the feet/seconds in the velocity in T seconds and A is the feet/square second in acceleration in T seconds. You know that A is equal to 2T minus 1. And V has got to be 3 and S has to be 4 at T for 1. Can you write functions for V and S in terms of T?

S: V of 1 equals 3? Is that what you mean? Because V is a function of T so V of 1 equals 3?

I: Right.

S: Since acceleration is the derivative of velocity with respect to time so if I integrate the formula for acceleration I have a formula for velocity in terms of time and if T is 1 this equals 3 because you said that there would be equals 3.

I: If negative T plus T squared is your function then for velocity, when T equals 1 does that equal 3? Does negative T plus T squared equal 3?

S: No, unfortunately. Oh, I forgot to add the constant on there and then that would be - plus some constant. Does that work on the computer? Yeah. Because then you'd differentiate the T out of there and you're just going to have - that's the acceleration formula. It doesn't have a - so we'd do that and then say velocity plus some constant C equals 3 and then we'd solve that. I always forget to put the constant in there. Solve velocity is equal to 3. For C when T is 1. I forgot to put in the of T. So that tells me that C is 3, obviously because if you plug in 1 you're going to get 0 so the constant is going to have to be 3 so then if I would integrate that formula 3 in terms of T that would give me the formula for S, displacement and then if I didn't name it - Then there is going to be another constant on that too. Plus another constant. What did you say? Time equals 4?

I: Uhhuh.

S: I'm just going to solve the displacement formula plus the constant equals 4 for the constant when T equals 1. And there the constant is 7/5 so what do you want then? Express V and S as functions of T?

I: Right.

S: Well it would just be - that would be velocity and displacement of T plus this constant. That would be displacement.

I: All right. Good. This relationship between acceleration and velocity and displacement.

S: I can picture it backwards. Displacement would be F of T. Velocity would be f' of T. And acceleration is f" or the second derivative. And since you told me acceleration I just worked backwards and integrated them.

I: That looks good. Now -

S: That's cool because that makes sense. I went through the whole first quarter of physics and I didn't understand it really but now I can go from one to the other and back and forth -

I: And it makes sense. All right. Let's have you look at this one. See what you can come up with here. A trough has trapezoidal cross section and it holds water. If the trapezoid is 3 feet wide at the top, 2 feet wide at the bottom and 2 feet deep can you find the total force owing to water pressure on one end of the trough?
S: It's 3 feet wide at where? At the top. And 2 feet wide on the bottom.
I: And 2 feet deep.
S: Meaning this way?
I: Uhhuh.
S: Find the total force water pressure on the end of the trough.
I: Any ideas on that? I don't know if you've looked at any problems involving force or not.
S: Well we just had a problem that involved work which we were just moving an object from here to there and it required varying forces instead of a constant force. You are wanting to know the pressure? It obviously has no - it doesn't matter how long this trough is.
I: You're looking for an expression maybe would be a better way to say it for the total force due to the water pressure.
S: Force due to the water pressure. Hm - I guess it would be - let's see the cross of the area. I don't even know the formula for area of the trapezoid. No, I really don't know where to begin that would be the area or the sum of the force on the surface. Each unit of the surface area. I wouldn't know where to find what the pressure is on the small units.
I: Okay.
S: It probably would be the integral I guess of all the little - well - if this were a - if it were a rectangular thing or a square better yet it would be the sum of - well it would all be even - or would it? It would just be the total of the area pushing on this square by the water but I wouldn't - even without calculus or anything I'm not sure where I would begin on that. Total force.
I: I think you've got the right idea though. You're looking at the problem.
S: it would be the sum. If it's pushing on this thing - I would assume you would have to use the integral because - well, no I'm stumped.
I: I think you said some good things there. I wanted to see what you could come up with. I didn't give you numbers for everything you might possibly need to completely work that problem out. But you could make up variables for those places.
S: Water pressure is - how do you determine water pressure?
I: Right.
S: Is pressure a function of something? I don't know.
I: Okay.
S: Got me. I flunked.
I: No you didn't. There is no such thing as flunking. Those are the problems I wanted you to look at today.
S: The work - like I said we just did that and that's not too tough at all and that was interesting because we didn't just - they didn't just tell us work equals the integral of the force. We actually went through and derived it and they asked why is the derivative of the work equal to the force? It was easy to see that if you went backwards, if the work equals the integral of the force if you differentiate both of those things you're going to have the work first equals the anti derivative of the integral so it was easy to go backwards but we had to go forward.
We had to start from an inequality - F of X is less than equal to the work at X naught plus H minus the work at X naught is less than or equal to F of X naught plus H and then times H. And then if you divide through by H then it all made sense. I can do that one. And this one wasn't too tough because I knew - I didn't necessarily know from math - I knew from calculus that the acceleration was the derivative of velocity which was the derivative of displacement. That's how I knew how to do that one. The mathematical process I learned in calculus. The concept that acceleration being the second derivative of time I knew from physics.
I: Are you in a physics class now?
S: Right.
I: What class are you in now?
S: 132. We're dealing with magnetism. We just took a midterm on Thursday and we're starting magnetism today.
I: So you were in 131 last quarter?
S: Yes.
I: That's the physics course with the computer labs, right?
S: 131 we have computer labs, yeah. It's all done on Macs.
I: Are those helpful?
S: The labs? No, I didn't think because most of the process dealt with - a lot of it was velocity - well it was all pretty much displacement and then velocity and acceleration, graphs and stuff. Like I told you earlier I didn't really grasp the concept, especially of looking at a displacement graph and then realizing that the velocity graph is going to be the derivative of that so that wherever the displacement is at zero you're going to have a max on the velocity and then you're going to have an inflection point on the acceleration. I didn't catch that until about the middle of the quarter.
I: Because your math caught up with your physics class at that point?
S: Yeah. That's when I personally understood graphs by about that time. Before that I would look at a graph and think oh well, this is x. This is y. If I plug in x and y I get this point up here. Other than that - that was about my total concept of graphs. So then I caught on but the biggest problem with the physics lab was that we would have a force probe or a motion probe and say we had a motion probe sitting here, you had to walk away from it at a constant velocity, sure. Your graphs ended up looking like this because you couldn't - it was physically impossible to not have - and I just thought it was kind of a waste. I didn't like it myself. It was nice. The one thing that was nice about it in comparison to this quarter is that when we had to graph something you just said hey computer, plot this versus this. Now we sit in lab for 20 minutes and do it ourselves and I don't know. I'm a firm believer that if you have a resource that will do it for you use it. It's just busy work. that's what computers are for is to take care of the busy work and so forth. I'd rather plug in values into a computer and let the computer plot them for me.
I: Did you feel as strongly that way before taking calculus with Mathematica™?
S: That I would rather have a computer do the work? Yeah. I've always thought if a computer can do it for me let it.
I: Have you been in situations where you were able to let it before?
S: Yeah, as a matter of fact. In my anatomy class in high school we did standard deviations and while everyone else was doing it on hand and plotting things by hand, plotting the standard deviation, not me. I threw it into a spreadsheet and used the graph function on the spreadsheet and it was right there. I knew how to do it by hand but why should I do it by hand if the computer can do it for me? I would always cut corners. That's kind of funny that that was my problem, the variables because just the other day in math class Dr. Davis we took a quiz and everyone eventually got everything right but everyone seemed to be stumbling over variable names and after the quiz Dr. Davis said I wish I could remember this happening to me. He said I don't remember it happening to me so therefore I don't remember how I got over it but he said it seems like every student I've ever taught has this difficulty understanding that a variable is a variable is a variable and he said if I could remember how I got over it I could teach you guys that it doesn't matter what the variable is. He said if you've got f of T and you want to change it to f of x, just go through and change all the t's to x's which seems simple in itself but when you're working with integrals and you have the integrands and like if you have the integral from 1 to x of f of t and you want to change the x's it just seems like everybody in the class, myself included gets confused in the variables. You get lost. Wait a minute. Where was this x from and stuff. Then you've got x and x naught. It's like wait a minute. Like in physics we use r, R, R'. R' is just a variable that we use in the integrals and it's just like well you get confused in all the variables. You just get lost.
Student A – Interview 4

I: I'm going to start you out with a question like I did before. What's a derivative?
S: A measure of the slope of a function.
I: I have some questions for you today. Since this is our last interview I've sort of got a combination of questions. So consider it fair game to use integration or differentiation on the questions. The problems - it's up to you. You can use whatever you like. Whatever you think applies. If there is a problem that you can't work then just your ideas on what you think would be usable. But sort of consider anything you've done fair game to use on the problems. Some will be obviously one or the other. The first thing I'm going to have you look at is just an integration problem. Find the integral from negative 1 to 1 of 1 over x squared dx.
S: Okay. On the computer?
I: If you want. Either way you want to do it. You've got paper over there if you want to use paper for anything. Do it on the computer.
S: So I look at that and I see - I'm going to say that f prime of x is equal to 1 over x squared. The fundamental theorem of calculus here and I'm going to guess that f of x then is log of x squared. And then to measure the integral take f of 1 minus f of negative 1. And just plug those into the various logs. So you'd have log of 1 minus the log of 1 which would be zero. Minus itself is zero.
I: All right. That sounds reasonable enough. Let me get you to do another integral type problem here. Could you estimate the value of - you can't find exact - the integral from 1 to infinity of 1 over x cubed plus x plus 1 dx?
S: Okay. Try kind of the same type of strategy. f prime of x equals x cubed plus x. So therefore trying to estimate it - this is going to be a very small number I would guess. The log of anything is usually pretty small and even though you're going from 1 to infinity taking this minus 1 log of 1 is zero. So I was just going to subtract out of - so you're just going to be left with log of infinity and I would guess that would be a very small number. Somewhere close to zero.
I: Okay. Can you get Mathematica™ to give you a pretty close estimate of that in any way?
S: Not without infinity. I can't remember how you do - there is a series of commands to find the limits and you could probably do that but I can't remember how you do it because I only did it once.
I: Now let me show you a couple of problems here and see how you might try to attack those. If you can solve them solve them but if not just give me sort of a plan of attack. We've got a spring. The natural length is 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches how much work is done in stretching the spring from it's natural length to a length of 18 inches?
S: We had work problems last week. So length of the spring is 14. If you use 5 pounds to get 2. So (inaudible) function. Call it Y. So it is 2Y. How much work is done stretching the spring from it's natural length to 18. So that means we want to go from 14 to 18.
I: Uhhuh.
S: So if we - given a point and if we can come up with an equation we can use the - can integrate I believe and come out with something. The trick is to find the equation.
I: That's good. You would find an equation and then you would integrate.
S: Yeah.
I: An equation to express what? What are you looking for an equation to express? What aspect of this problem?
S: Probably something related to work and length. I'm trying to remember work is a product of density. We had this set of problems - density, something and work. I'm trying - I was think P of X was equal to W prime of X but that's a density problem. Work relates to function of whatever the weight of the object is times how much distance it has moved.
How much effort it is going to take so we'd probably like to come up with a function of work. But I'm not really sure from what we're given how we'd do that.

I: In the cases where you might be able to come up with a specific equation if you just give me the kind of response you gave me - if I could come up with an equation then this is what I would do with it. That's good. Let's look at another one. Suppose you have a gas and it's pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second and that pressure remains constant. And the balloon is always spherical. Ideal conditions in other words. How fast is the radius of the balloon increasing when the radius is 5 centimeters?

S: The initial radius is 5 centimeters?

I: It's not necessarily the initial but at the time when the radius is 5 centimeters how fast is the radius increasing?

S: Okay. So let's see - a sphere - I think in terms of a circle - that constitutes to area so it would be pi R squared. We've got a sphere so we want the volume of the sphere and I can't remember what the volume of a sphere is off the top of my head.

I: We'll consider that one of those equations then we were talking about before. I think this is the stuff we just started yesterday. It looks like it if it has got to do with volume. What I'm thinking is a guess to try and solve this. I'm not sure now. It's not really the same type of problem. See what I was thinking initially if you took your sphere and you mounted it on the X and Y axis and you took a cross section of it and you could find that area which would just be the circle basically and then once you had a function or an equation for that area then plug it in. You'd want to integrate that area because the derivative of volume is equal to area and you could take that and integrate it from something to something. I'm not exactly sure what and that would allow you to use your fundamental theorem of calculus and you could say the volume of say B minus volume of A equals the volume you want.

I: Okay.

S: And you would know that when you were doing this - I guess another way to look at it is if you want to know how much the radius is increasing at 5 centimeters if you could somehow come up with a function of radius in terms of function and then you can measure it's derivative at 5. You could see how much it is increasing.

I: So measuring the derivative at 5 could show you that.

S: Yeah. That's the only way I can think to measure something. Because you'd want to know basically the slope. How much it is increasing.

I: Let me ask you some other here. They are typed a little differently here. Can you find the values of A and B so that the line 2X plus 3Y equals A? Is tangent to the graph of F of X equals DX squared at the point where X is (inaudible)?

S: I can try.

I: If you're going to use that paper you can write on that.

S: I try to do less on the computer nowadays.

I: That's just something that you've decided to do yourself?

S: Makes it - it makes it easier to understand what you're doing if you can sit down and can understand the basic concepts and then use the computer to do the advanced problems. That works better for me because then like before - like all last quarter and the majority of this quarter I had been trying to read the tutorial and go in and do the problems and I really didn't have the working knowledge of what was going on and that wasn't too bad last quarter. You could get away with that. When we got out of the fundamental calculus and we started doing all this manipulation with it then you could really shoot yourself in the foot so I kind of changed that for me. Made it a little bit easier. If you've got a function and you want the two to be tangent but at that point where X equals 3 they're both going to have the same slope. so their derivatives at that point are both going to have to be equal.

I: The derivative of VX squared function and the 2X plus 3Y equals A?

S: Yeah. Their growth rate is going to be the same. You'd have to make them into a function. Basically you want their slopes to be - so that would be the main thing.
I: Any way you could figure out what values of A and B you would need to use there to insure that?
S: Let's take our function say at F3. That's the point we're going to be working on. This is 3. That means we're going to have 9B. And let's see. Use the other one (inaudible). Six plus 3Y equals A. Then if - There's another (inaudible). I'm not really sure.
I: But you think that - their slopes are what you'd be interested in?
S: Yeah.
I: What were you going to say? Obviously what?
S: It would be easier if you could work that into a function but I don't see an obvious way to do it.
I: All right. Let's go down to this one. People have enjoyed the fact that they have some pictures to look at on these questions. These two graphs are supposed to be the graphs of two functions. One is the derivative of the other one. Now there is no vertical scale on the graph because the vertical scales could be different from one to the other but the horizontal scales are the same. So which one of these is the function and which one is the derivative and how do you know?
S: Okay. Well let's see. We know the derivative is equal to zero. Then you're going to have a max or a min. I would guess that this is probably the derivative because every time it is equal to zero at the max or the min of a function.
I: So you think the second one is the derivative.
S: Yeah. Because when you go up here you - it's kind of hard to tell.
I: You can draw lines on there if you want to try the lines (inaudible) or anything like that.
S: I'll just look at it. This is the derivative. That means it's showing it's greatest slope there. And it's greatest negative slope there which does kind of correspond. This kind of worries me.
I: What kind of worries you?
S: No, I guess it corresponds because this would mean that the function should be increasing rapidly which it would be doing. My key thing I think would be seeing that the derivative is zero when it functions (inaudible). Of course it does kind of go - it looks like it goes both ways in a way because there that is zero and that's a min and there it's zero. That looks slightly off. Of course that one does too so that could just be drawing.
I: Mark those on there for me so I - I'll put this paper away with your other written work. Which one is the function and which one is the derivative. Let's look at these. In this one let's mark the - I'm not interested in satisfying this condition. Now what I want you to do down here is just sketch and it can be a rough sketch the graph of some function F so that this would be it's derivative. So what I'm giving you here are the graphs of the derivatives in other words. Can you give me the function?
S: Okay. So these are all F primes.
I: Uhhuh.
S: I would guess that this is something like a parabola because it's - so if it is decreasing it hits at a max or min if it increases. I don't think it would be like this because that would mean it's increasing and then decreasing. This means that you have a constant slope. So I'd guess it is just a straight line. And I'd guess you'd have something more or less kind of like the opposite of that I guess because the slope should be increasing. Then you should have a max and then it's going to be going down, down, down. And then it's going to - there's your max. Oh yeah. It is still continuing to go down. Should you come up, hit the max starts coming down, down, down. Then you come back up a little bit and you hit the opposite of it there. Do you understand what I'm trying to say here?
I: Yeah.
S: So kind of flips in a way.
I: Okay. Can you give me any indication of what type equations or formulas you might have to go with these functions you've got here? You've indicated a straight line, a parabola but what type of -
S: Like a parabola is X squared?
I: Yeah.
S: So like Y equals X squared. I know what this is going to be. This is guessing here but this is the derivative. This is AA, sine graph so this is derivative of cosine graph. Cosine graph. Because - let me think. Derivative - must be the tops. The derivative of a cosine is a negative sine. Derivative of sine is cosine. So this is the original function. I know that's the regular sine graph and that's got to be negative cosine.
I: Okay.
S: And this would be Y equals X.
I: All right. That sounds good. Those are the problems I wanted you to look at today.
Student B - Interview 4

I: The first thing I'm going to do is ask you a question like the last interview up front. This interview is going to be anything goes sort of as far as working problems. The problems might involve derivatives or they might involve integrals and you get to make your choice. Last time I asked you to tell me what an integral was so this time I thought I'd start by getting you to tell me what a derivative is.

S: The derivative of a function is when you take the function whatever it is - say if it was X square. Take X square and take the exponent and move it down to the coefficient and you subtract 1 to it so it would be 2X. And you would use that to determine zero points of the function. So wherever the function crosses zero usually that's maximum and minimum point.

I: All right. Now I have several problems here and we'll go through as many of them as we have time to go through basically is what it amounts to. Have you done any work yet in class with series, to approximate things?

S: No, I think that's next.

I: Let's start with a problem. Now again this could be a combination. Some of them will be real obvious and some of them you'll have to make the decision. The first one - find the integral from negative 1 to 1 of 1 over X squared DX.

S: Right off the bat I can tell you that the integral is going to equal zero because it's going from negative 1 to 1 and that's a closed interval and it's 1 on each side so the two areas the integral is going to cancel each other out so the integral is going to equal zero.

I: Why does that do that? Would any function work that way?

S: Any function as long as it was like negative 1 to 1 or negative 3 to 3. It should work that way.

I: What does the graph of that 1 over X squared look like?

S: It would be 1 over negative X so it would be a line going from the upper left hand corner to the lower right. A straight line. Do you want me to check?

I: Okay.

S: I'm going to check. What I'm just doing is I'm typing in the (inaudible) and the function is 1 over X into the computer and letting Mathematica™ -

I: It's 1 over X squared.

S: Sorry. I didn't mean to say that and I'm going from X - from negative 1 to 1. And the computer is telling me it's negative 2. I'm taking X squared from the bottom to the top which would make it X to the negative 2 and adding one and then dividing by negative 2 plus 1. Seeing what that gives me.

I: Now why are you doing that?

S: That's the way you take an integral out of - the integral defined - what the function of it is. I'm reversing - reversing what a derivative does. I'm taking the anti-derivative of the integral which would give what the integral is and it's saying it's negative 1 plus X to the negative 2 which is negative 1 plus 1 over X squared. So F at X equals negative 1. Then negative 2. Make sure - if you want to plot that we can plot F at X. X - negative 1 to 1. And I was wrong about the function. It should be - the way the graph looks it's two lines that are approaching the zero point but it will never get there because if it gets to zero it isn't a function because you can't divide by zero in a fraction.

I: That would make sense for that integral to be negative 2?

S: Actually it doesn't. But I guess what they're doing is they're approximating what it would be. Mathematica™ is saying it's about negative 2. We can't tell you for sure that it is but it's about negative 2.

I: Okay.

S: Sometimes Mathematica™ does that. It gives you an approximate answer of what it thinks.
I: Let's try another one. You can use the computer or not use it. It's up to you. Estimate the value of the integral from 1 to infinity if 1 over X cubed plus X plus 1.

S: I'm going to use the computer and I'm just typing in - I'd better clear X first because it might think - I'm integrating the function. It's 1 divided by X cubed, X plus 1. Plus explanation point. X from 1 to infinity. And I don't know whether that will work. No. What I'm going to have to do is I'm going to say 1 - T. From 1 - T that tells the computer that yes we're going from 1 and T could be anywhere out there. And it's still not going to give me anything. Did I spell integrated wrong? I do that quite frequently. Evidently Mathematica™ won't integrate it either.

I: Can you figure out a way that you might at least be able to estimate it with Mathematica™?

S: You can ask Mathematica™ to n integrate which it's telling it to give it the best you can. And even that it's not doing it so what you can do is you can take and integrate it by parts. And you can say - I'm trying to figure out which part. 1 plus X to negative 1 plus X wouldn't be 1 plus - it would just be X to the negative third. All I'm doing is I'm taking everything from the bottom up to the top. And seeing if Mathematica™ will evaluate that as an integral and it's giving me negative 1/T minus 1/T squared plus T plus log of T and what that's telling me is T is infinite, depending on what value put in T. You'll get an estimation. So if you make T very large you would get - if you want to make T like 100,000, that's quite large and see what I get for the function and it comes out to be - if I asked it to n integrate it it will give me a more exact number. It comes out to be about 1000. About 100,000 so - and you could probably make this as big as you want and it would just keep coming out times (inaudible). It's going to be about whatever T is. So you can't - let's see if it gives me something different in there. You get an approximation.

I: That sounds good. Now let me have you look at one of these problems. A spring has a natural length of 14 inches. A force of 5 pounds is required to keep the spring stretched 2 inches. How much work is done in stretching this spring from it's natural length to a length of 18 inches?

S: Want to stretch it 4 extra inches so it's going to require 10 pounds because for each - see how much work is done. Five pounds - so every 2 inches you use 5 pounds and you're stretching 18 and the natural length is 14 so you're stretching an extra 4 inches which would be 10 extra pounds so work is force times something length so it would be 10 times 4 or 40 working units, whatever the units would be. Pounds squared. Or inches squared. Pounds per inches squared. So I think it's distance squared. So it would 4 inches times the poundage which is 5 so it would be 20 pounds per inches squared.

I: Does that problem have anything to do with calculus?

S: No. I don't think so. I suppose they could probably fit it in there somewhere.

I: Let's try this one. A gas is pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second. If the gas pressure remains constant and the balloon always has a spherical shape - we will make it an ideal situation here how fast is the radius of the balloon increasing when the radius is 5 centimeters?

S: You mean how fast is the radius of the balloon increasing when it starts at 5 centimeters?

I: At the time when it is 5 centimeters how fast does the radius increase?

S: If it is 50 cubic centimeters/second and the radius - it would be 25 centimeters in diameter. Let's see. This is a velocity which is - you would take 50 cubic centimeters per second which is the velocity of something and then if you took the anti derivative of that you would get an acceleration and if you would take the anti derivative of that you would get a function of the rate. Rate, acceleration. Rate of something is velocity. The rate and velocity are equal so we would want to set up a function and take it's derivative to get it's acceleration and take the derivative of that function and then you would get the velocity or the rate of change of something. It's related to the derivatives. You would have DY of the changing of the radius over the changing of the rate so it would be DR. DR would be the derivative of the
radius - how much it is changing over the derivative of how much the rate is changing of the balloon so something over 50 cubic centimeters per second, whatever it would be.

I: Is there any way for you to work that out with what you have?

S: Let me think. Let DR stand for the derivative of the rate. I don't want the radius. I want the rate. I'll say DT was the rate. So as the rate is growing so is the radius. So you would say - F at S equals rate of changes. Which we decided were DR divided by DT. And RT - cancel them. Okay. I left a space and I shouldn't have. DR/DT. Then you say it's changing at a rate of 50 per second so you can say that - you can ask it when S is 5 what would that be? It's telling me that it's going to be a negative 24 with 5 and you have to times that by 50. And 48 so it is changing at 48 centimeters per second. Because you squared the centimeters.

I: All right. Now I've got some actualy typed because there are some pictures involved down here. Look at this part first. Can you find values of A and B so that the line 2X plus 3Y equals A is tangent to the graph of F of X equals BX squared at the point where X equals 3?

S: X equals 3. Yes. I'm clearing the variables. X, A, B and the other one is - first of all what I'm going to do is I'm going to set the F of X equal to VX squared. Then it is saying at the point where X equals 3 so if we set the - X equals - we're going to tell the computer that X equals 3. We're going to say X equals 3. And we also have another function that's telling us that Y which we can also say is equal to G of X equals A minus 2X divided by 3. And the way I got that I just rearranged the other equation which gave us the tangent line - it's a graph. So -

I: So for what values of A and B would that line be tangent to this graph?

S: Well if we ask it what G prime of X is that's the slope and it says the slope is zero which I don't think is right. I know what happened. I named that value too soon. Okay. We ask what G prime of X is. It gives you the slope and the slope of the line is going to be negative 2/3 so therefore for Y you take Y equals MX. M which is the slope so it would be negative 2/3. X which in this case they say is 3. And you ask it what B is. And it's negative 2 plus B so Y equals negative 2 plus B. I used the wrong variable. If we say Y I'm going to think it's the same as (inaudible). It's the equation (inaudible). So I'm - Y is not a point. Hmm. I know you can do this. This is the tangent along there. So I'm tangent to the graph. Must have a slope -. It is already given me that the equation of Y is 2/3X plus A which is - and then if I fill in - get rid of this - and this side give it the value of X equals the 3 because that's the point where it's going to be ask it - G at 3 and D. That's going to give you the point at which to meet so at this point A and B will be equal. So therefore we have 9B. We're going to solve F of X equal to G of X or - it would be - say the A is equal to B. And you have to ask it to solve G of X and equal to B of X. B equals negative 3/13. So to find the values of A and B - so - that can't be right. Oops. My head - a lot of thinking going on. Sometimes you try things and you notice well that didn't work because that would be impossible for it to work. I'm going to plot F of X, G of X from X. From zero -

I: F of X would be equal to zero to plot?

S: Yeah.

I: Then you're going to solve for G of X with A -

S: Zero to five and to see if - already has a value. I see what they do. See up here we didn't say what - we're going to have a coefficient times X squared so whatever the coefficient is the first line is going to be a straight line. Wait a minute. I see something. Plot X squared because I have a feeling I'm thinking plot of X squared is totally different than what it - whoops. Now the computer isn't giving me anything.

I: You're just trying to get a picture of what the graph looks like?

S: Yeah. I'm just trying to see what the graph - I thought that X squared was a straight line and I was wrong, it's not and all that a coefficient will do in front of it if I'm not mistaken is move it either left to right or up or down. What's the difference between those? I didn't see any difference.

I: Between X squared and 2X squared?
S: The graph - I put a coefficient in front of it to see what the difference between because all B is going to do is add a coefficient in front of it which is just going to make it wider across from point to point. It's going to fill the same area but it's going to be going up to a further point. Like if it was 2 it would be 2 squared and since it's 3 I think we should do 3. It already told us X was 3. Okay. I'm going to do the same thing with - makes it 3. I'm going to go back up here and find out what the slope of F of X is by asking it's derivative and it's 2. The slope is 2. F prime of X. I'm going to solve F prime of X. (inaudible) G of X. Ask it what G prime of X is. I'm going to ask it to solve F prime of X equals G prime of X for B. And hopefully it knows X equals 3. Make it solve that. That should give us B equals 1/9. So we have B equal to - we ask it what F of X is again and it's 9B because we told it X was equal to 3 and if you put in for B the negative 1/9 you'll get F at X equal to 1. And B equal to a negative 1/9. So when X is 3 - that doesn't make any sense. I'm lost. Trying to - you take the derivatives and you find where the slopes are and at that point 3 of the slopes should - it has got a slope of negative 2/3. B has got (inaudible) the X axis. Okay, okay. The reason this graph isn't making any sense is because I have X squared. It's definitely X squared because we figured out if B was equal to 1/9 so it's going to be negative 1 divided by 9X squared. I'm asking it to plot that. Where will the line - negative 6, where will it touch? Where will AB? And it's going to touch at 3. It's going to be a negative number. Down here. And it looks to be that whole expression is going to have to equal negative 1. So that would be plus 9. A would be 9 and B is negative 1/9.

I: So when A is 9 B is negative 1/9.

S: Let me check the - so that would be - F at X. We'll call this G. Clear. G, X. Equal to A which is 9. The equation of the line is (inaudible) and F of X equals negative (inaudible) X squared. F of X, G of X, X - I'm going to go from zero to five. We'll see the point 3 then. Forgot to put brackets. It doesn't come anywhere close. I did something wrong. Try something else. I'm going to change 9 to 2 because looking at the graph it looks like somewhere near 2. What was that? It keeps telling me that F of X is not evaluating at that number. I don't know why it wouldn't be. If it was 1 it would be a negative 1/9. Hm. And it's now the point is going through - I've got the line going through 3. Now negative 1/9 sure isn't the right one. Let me try - X squared to see what that gives. Evidently it's a fraction because when I just let it be 1 instead of a negative number it came out above the tangent line which is not - you want it to be tangent at that point so it's got to be a fraction of some type. I'm thinking since it was 2 it would be 1/2.

I: So this point is the best - keep trying different numbers?

S: There is a formula that you can use but right at the moment it's not coming to the top of my head which if I had Mathematica™, all the programs in it that are over there all you have to do is look back in the lessons and find it. It would be the same as if you had a book.

I: Look back in the notebook?

S: Yeah. Look back in a notebook or something to find it. It's going to keep getting - sooner or later it will get to a point and it will be tangent to the here it's tangent but it's at the wrong point. That makes me believe that this shouldn't be 2. Y keeps giving me these messages that it doesn't evaluate because it should. And at 1 it's not tangent. It crosses so evidently the slope of the line it's not reading so I could sit here and play with it all day but - but you'd rather go on.

I: All right. Let's look at something down here. Here is a figure. I have the graph of two functions. One of them is the derivative of the other one. The vertical scales might be different on the two but the horizontal scales are the same. Which is which? Which is the function and which is the derivative and tell me how you decided. You can look at that and do whatever.

S: Now these scales - do they line up this way?

I: Yeah. The horizontal scales are the same.
S: Okay. The one on the bottom it crosses zero at a point and that's telling you maximum or minimum point and if you look up here that is where the minimum point is so this would be the function. The top one is your function and the bottom one is the derivative because wherever it's crossing the X axis it's hitting a minimum or maximum point on the upper graph so the top one would be your function and the bottom one would be the derivative of the function.

I: Can you sketch a graph of some function so that the derivative of your function is equal to G where the graph of G is here? In other words give me a function that this would be the derivative of.

S: Okay. The derivative of X is 1. If X equals - that would give you a straight line. the derivative of X squared would be 2X and that would give you a line vertical.

I: Let's write down the ones you are saying here. You're saying F of X - let's put them under the pictures. Let's write which ones you are saying are which here. F of X would be what for this one?

S: F of X would equal X and F prime of X would equal 1. F of X equal 2X - now you're saying approximate where these are?

I: Uh huh.

S: You're not saying that - and F prime would be 2X. And this one would be - I always get these confused. I'm just asking it to plot the sine of X because I always get the sine and the cosine goofed up, which one starts where. I really should remember those but I don't. So this would be - let's see. F of X equals - would equal the negative cosine of X and F prime of X would equal the sine of X. You could always check these with Mathematica™. Which would be easy. F of X equal to X and F prime of X is 1. So F prime of X - and the line is - I'm not going to touch that. For the sake of just naming things different I'm going to name this one G of R equals X squared. G prime of R - ask it what it is. Where did that come from? I don't know. I thought it was 2/3 and I know it's not. X - I'll just name it G of X and clear em.to make everything much more simpler. G prime of X hopefully it's going to be 2X. There we go. And we're going to plot G prime of X and X from zero and clear them. Clear X again. And we'll say H of X - whoops - equal negative cosine and the reason I did the negative cosine of X was because when you take the derivative of cosine of X it gives you the negative sine of X and the function we wanted was sine of X. So we have to use the negative.
Student C - Interview 4

I: The first thing I'm going to do this time is kind of what I did before which is to start by asking a question. I asked you before what's an integral so this time I'm going to ask you what's a derivative?

S: A derivative is - I forget. You take the function and - I forget what a derivative. I know what it is but it's hard to explain. You take the function and you just take the derivative of it. What is it? I don't know. I can't think - I just know what it is.

I: Sometimes it's hard to find the words. Now the problems that I'm going to have you look at today could be a mixture. Some of them I'll just ask you to do some integration but there may be situations where you may want to think back to the things you know about derivatives and there may be others where you want to use what you've learned about integration so we'll say that either of those that you want to use is fine. You get to decide if it's a problem situation. The first one is to find an integral. The integral from negative 1 to 1 is 1 over X squared DX.

S: By looking at it I'd say it's with logs because usually the integral of 1 over X is the natural log of X so it would probably be like the natural log squared. Do you want me to do it on the computer too?

I: If you want.

S: Okay. Hm - 1 divided by X squared. Negative 1 -

I: So you're just telling it to integrate -

S: Yeah the function.

I: Okay.

S: Negative 2.

I: Does that fit in with the kind of thing you were saying you suspected?

S: Uhhuh because like when you take the natural log. It would be natural log of X squared and then you'd take the X squared and you want to integrate that so it would be 2X. So then you just like plug in - of the 2X you plug in the values negative 1 and 1 so for - it would be 2X times the one. Plug in the one for the X and you get 2 and then for the negative 1 2X you get negative 2. So you plug it in and that's what I came up with.

I: Okay. So you felt pretty satisfied with that answer.

S: Yeah I did.

I: Can you estimate the value of this integral for me? The integral from 1 to infinity with 1 over X cubed plus 1?

S: It would probably just be zero because if you have 1 on top then no matter what number it will keep getting bigger and bigger so you'll have a smaller fraction so as X goes to infinity the integral will probably be zero. Do you want me to put it in there?

I: Can you put that in there (inaudible)?

S: I don't know - because infinity I have to like have a value. I don't think you can.

I: Do you know of any way you could estimate this using Mathematica™?

S: You could pick a very big number.

I: Well try that and see -

S: Okay. I'll make X (inaudible). I'll put 1 - 1000. I'll put 100,000. (inaudible). No it's not. Hm. Let's try - huh.

I: Anything with the spacing? Did you get the space in here?

S: I don't see why. I spelled integrate right. Sometimes it won't come out if you (inaudible). Maybe the number is too big. Sometimes it has a hard time with a (inaudible) function. Huh. 1 divided by all of this. Put this space. No. It's not even doing that one. See this might be giving it problems.

I: X cubed plus X (inaudible).

S: Yeah. Huh. I estimated it as zero but it's not like doing anything.
I: Okay. Let's (inaudible) and see what else we can come up with here. Problems (inaudible). Here is one. The spring has a natural length of 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches how much work is done in stretching the spring from it's natural length to a length of 18 inches?

S: The spring has a natural length of 14 inches.

I: Uhhuh.

S: How much work equals force times pressure. How much work is required to keep the spring stretched two inches? Hm. Force (inaudible). It's like if I added 10 pounds to the force would that move it 4 inches then? Required to get the spring stretched 4 inches?

I: I don't know.

S: So maybe you can just keep - force equal (inaudible).

I: Is that what you think there?

S: Yeah.

I: Do you need calculus to do that problem?

S: I don't think so. It's more like a physics problem than a calculus problem.

I: All right. So if it works the way you said that it worked then to stretch it from it's natural length out to 18 what force would do it?

S: Ah - I think about 40 pounds.

I: And how are you figuring that?

S: If force is (inaudible). Because you have this spring and it's like coiled up and you have 5 pounds and it stretches it 2 inches. So for every 2 inches there is like an additional 5 pounds so if you want to do 18 there is like 8 intervals of 2. So it would be 8 x 5 - 40 pounds.

I: Okay. Let's take a look at this one. Suppose a gas is pumped into a spherical balloon at a constant rate, 50 cubic centimeters per second. Now if you assume that the gas pressure is constant and the balloon is going to always have that spherical shape how fast is the radius of the balloon increasing when the radius is 5 cm?

S: So it has 5 cm as a radius and there's 50 cubic - is that -

I: Well the gas is pumped in at a constant rate of 50 cubic centimeters. At the particular instant when the radius of that sphere is 5 cm -

S: They want to know how fast?

I: How fast is the radius changing at that particular instant?

S: I'm not (inaudible) as far as physics. Constant - 50 cubic centimeters and it's 5 centimeters. For every 50 cubic centimeters it's increasing (inaudible) and it would be 1 centimeter.

I: Did you have any thoughts on this problem while I was -

S: I fooled around. Everything I got wrong. Oh -

I: At this point what are your thoughts?

S: Okay. You know that there is a constant rate of 50 cubic centimeters per second being pumped into this balloon. So you know that the rate is always supposed to increase so at that particular point if it's 5 centimeters the rate of increase probably will be the 5 centimeters divided by the 50 cubic centimeters per second. That would make the (inaudible) - because it is probably increasing probably maybe one centimeter for every 50 cubic centimeters that it (inaudible) the balloon.

I: Let me ask you what I asked the other ones - do you think this problem involves calculus?

S: Calculus - maybe with the rate. Finding the rate of increasing but it's sort of like a physics problem.

I: Now I had these others typed for you because there are some pictures that involve some of those things but not with that one. This first one - could you find values of A and B so that the line 2X plus 3Y equals A would be tangent to the graph of the function F of X equals VX squared at a point where X equals 3?

S: Okay. X is 3 - so if you know X is 3 then you could plug it in here because I guess that would be 6. Six (inaudible) 3Y equals A and then you can put 3Y equals A minus 6. Then you have to find the - I think maybe Y would be -
I: You can write on this if you'd like instead of trying to remember what you're telling me.
S: (inaudible) 2X. Is tangent to the graph. Any X squared function usually gives you a
parabola and B is just like the coefficient outside of it so it either raises it or lowers it. So
(inaudible). They want to know - find values of A and B so that the line 2X plus (inaudible)
tangent to the graph at the point where X is 3. So they want to know - so you know the
point is 3-something because you know X is 3. But you don't know what Y is. Since
(inaudible) Y equals B minus 6. Values of A and B. Maybe for this X you can (inaudible)
there. Hm. You don't know what Y is. Hm. (inaudible) be. A - (inaudible) 3. Hm. You
want B from this line. Y equals (inaudible). I don't know where you would go from here.
I: What do you have information-wise with what you've got there? You've got Y equals A over
3-2.
S: Uhhuh.
I: Can you see a direction to go?
S: No. Because you have to find A and B. And also Y because (inaudible). Hm. I don't really
know where to go from here.
I: Okay. All right. Let's move on down the line here and see what we can come up with. You
have two figures here. Graphs of two functions and one of them is the derivative of the other
one. Which one is the function and which one is the derivative and explain how you know
which is which. Notice in the directions the vertical scales could be different on these so
there is no vertical scale but the horizontal scales are the same.
S: Okay. (inaudible) be the derivative.
I: You think the second one is the derivative. And (inaudible) with the function.
S: Because this one is above the X axis so this one is increasing. And then at that point and
that point when it is below this whole (inaudible) is below this is decreasing and this from
here to here this is above the X axis so it would be increasing and then you would get here
it's below so that's where it starts to decrease. So that would be the derivative and that would
be the function.
I: Label those for me. Let's look at these. Can you sketch a graph for me of a function F so
that the derivative of F equals G where this is the graph of G? So in other words this is -
S: That's the derivative, okay.
I: Yeah.
S: The derivative -
I: Each one of those.
S: Hmm. This would be (inaudible). Okay. (inaudible). (inaudible). This will be like that.
I: For the first one here?
S: Yeah because that's just above the X axis and it will probably increase and go to infinity.
But it can't go like over here because then it would be (inaudible). This will (inaudible).
Well I'm pretty sure this will be a parabola and that one will be -
I: Okay.
S: Because that's below, decreasing, increasing. Then that one will be (inaudible). But the
derivative it's just like, will still be equal to zero so I was thinking at first it could be maybe
a straight line. But - let me see the third one. I'm not sure. First of all this is above so the
function has to be increasing. This crosses the X axis. That would probably be the
maximum and then this is all below and it crosses here so I put - it is all decreasing and then
what crosses that's like the minimum and it is all above here so mine is gradually increasing
and it crosses here again so that's just the maximum and then it's decreasing because it's
(inaudible).
I: What kind of function does that look like? You said this looked like a parabola.
S: It's either a quadratic or a (inaudible). Probably a quadratic.
I: Okay. That's good.
Student D - Interview 4

I: I asked you a question first thing last time. I'm going to do the same thing this time. this time I'm going to ask you what's a derivative?
S: It's the slope of a function.
I: Now in the interview today I have several questions and problems and some of them will obviously be integrals to find or something like that. But some of the problems you have a choice. You can choose to use integration in solving the problem or you can choose to use derivatives to solve the problem so there could be some mixture. So feel free to use either techniques, whatever seems appropriate in the problem. First I want you to evaluate this integral for me. The integral from negative 1 to 1 of 1 over X squared, dX.
S: What happened? Something is wrong. I could try this by hand.
I: Okay. It says it failed to reach specified accuracy after (inaudible) subdivisions and it gives you (inaudible).
S: I'll just do it by hand. I always check it. I over there. Okay. It could be negative 2. Do you know what the answer is?
I: Uhhuh.
S: probably doing some stupid mistake.
I: So you're thinking the answer should be negative 2 and it's giving you some messages -
S: A huge number.
I: What do you think is happening there?
S: Maybe it's because - I don't know. Increase power - I think it should be less. I don't know what that is.
I: So you think it should be negative 2.
S: Yeah.
I: We'll go with that.
S: Is that the right one? Probably picked the wrong one.
I: Can you estimate the value of the integral from 1 to infinity of 1 over X cubed plus X plus 1?
S: You mean like off top of my head or something?
I: If you can do that off the top of your head it's okay. If you want to use Mathematica™. If you want to use paper, whatever.
S: It already didn't work for me on the first one.
I: Integrate the function from 1 to whatever you've got out there.
S: Oh yeah.
I: A million? Why did you choose that?
S: A big number, close to infinity.
I: Okay.
S: it won't do it. Zero.
I: Does that make any kind of sense?
S: Oh yuk. You know, this Mathematica™ is getting to me. Yes that makes sense because when you integrate this - the bottom is going to have more X variables on it and the small power and it's infinity dominates over everything else so the denominator is going to get close to infinity and the numerator is going to be like nothing compared to it so it's going to be a zero eventually.
I: A spring problem.
S: You know we didn't do those troughs and stuff that you were talking about last time?
I: Did you do those acceleration and velocity?
S: No.
I: Have you started using theories to approximate (inaudible) to?
S: No. We're measuring volume now.
I: Okay.
S: We're getting to that.
I: Have you done springs?
S: No.
I: Let's see if we can make any sense out of this. A spring has a natural length of 14 inches. If a force - done any force problems?
S: One maybe.
I: If a force of 5 pounds is required to keep the spring stretched 2 inches how much work is done in stretching the spring from its natural length to a length of 18 inches?
S: Oh my gosh. Spring - 14 inches - force - required to get the spring stretched - you mean - you mean pushing it together to be 2 inches? Is that what you mean?
I: Stretched 2 inches further.
S: Oh 2 inches more. So it's 16 - this force. How much work it does. 18. I'm going to do basic algebra for this one. Maybe it will work. It takes 5 pounds of force to make it stretch to 16 and then it will take how many pounds to get it to 18? About 6 pounds of force maybe.
I: Okay. And you did that with ratios?
S: Proportions, yeah. Five pounds to get 16 and how many to get 18. About 6.
I: All right. Let's try another one. This time we have a balloon. Now gas is being pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second. Assume the gas pressure remains constant. The balloon is always spherical. How fast is the radius of the balloon increasing when the radius is 5 centimeters?
S: Constant rate - I don't know. How fast per second?
I: Uhhuh.
S: I don't know how to do this. The only thing I could think of would be distance equals rate times time. So if it's going to be the rate is 50 - we have 5 but we also have (inaudible). I don't know how to do this problem.
I: Okay. That's all right. I just wanted to see what you'd do.
S: Nothing.
I: It's all right. Let's try this one. Can you find values of A and B so that the line 2X plus 3Y equals A is tangent to the graph of F of X equals VX squared if X is 3? Can you find A and B?
S: Constant times X squared. X plus 3 Y equals (inaudible). X equals 3.
I: You can talk about what you're doing. Okay. What did you come up with?
S: I'm thinking if somehow if you take this line 2X plus 3Y equals A and set it equal to the function VX squared when X is 3 then if you solve them you put them equal to each other and solve them then you can find the values.
I: Does that (inaudible) calculus?
S: Yeah. We're doing that. We're taking like certain things and setting them equal and solving for the variable. If I knew A and B it would be real easy to find Y. This is X, the line and this? X is 3? So it is -
I: You put 3 in the place of X on the line (inaudible).
S: Yeah and the same thing - Line would be - so you say 6 plus 3 Y minus A, equals 9B.
I: Now why say 6 plus 3Y minus A equals 9B? Why did you decide to do that?
S: Setting them equal to find the solution. But it's going to be a problem because I have three different variables I have to find here. A and B are constants, right?
I: Uhhuh.
S: There is probably a real easy way to do this. I'm going to graph this.
I: You're going to plot X squared in the (inaudible)?
S: Uhhuh. Okay. Then X is 3. Y is about 9. If X is 3, Y is 9. Try and plot this line. It doesn't work. I don't know. This was right. We can try - say B is 1. And A is -
I: You just arbitrarily try A's and B's?
S: Yeah. But I don't know if that's right. I don't know if you can do that. I'm not sure. I'm lost.
I: Okay. Let's look at this one. Here we have two figures. They're graphs of functions. One of them is the derivative of the other one. There are no vertical scales on these graphs because the vertical scales could be different but the horizontal scales you see are the same. Which is the function and which is it's derivative and explain how you know.
S: Are they in between the same points?
I: Uhhuh. The horizontal scales are the same.
S: This is zero and this is one. This is zero and this is one also. This is the derivative. The second one is the derivative because when we look at the derivative wherever it is positive the function is increasing and wherever it is negative the function is decreasing so if you look at it from here it's positive to this point and the function is increasing to that point. Then it's negative to this point and the function is decreasing from here to here. It is positive again until this point and the function is increasing and it's negative again till there where it is decreasing.
I: Mark those on there for me. Function, derivative. So I can remember. All right. Let's look at this.
S: I like that one.
I: Here we have 3 graphs. I want you to try to sketch a graph of some function S so that the derivative of F is what you see here. So this graph is -
S: The derivative.
I: Show me what the function would look like. Ignore this condition here. We’re not going to do this.
S: This is the derivative.
I: Uhhuh. It can be a rough sketch.
S: A line with this. Keeps always increasing forever because this is positive and this will always increase.
I: Why did you choose the line instead of a coefficient? Any reason?
S: No, no particular reason. I think I was inclined to because this was a straight line. To show no variation because I think if this was a curve then I would probably make this a curve even though this would be positive for that still it would be negative for this.
I: Okay.
S: Like I said, this is going right through the X axis. Then I'd say it would be a parabola because this is negative so it is going to be always increasing from here. (inaudible) that this goes down like that and this one keeps going up and the parabola will keep going up too. This is the derivative.
I: You are starting way back and so now you have to go - what direction do you go?
S: I didn't like what was going to end up.
I: Oh.
S: Have the maximum there and max and min there. This is positive. Increasing. Increases to here so it's going to be going (inaudible). Decreasing till there. Negative. There is (inaudible) I can do this. To show more of an effect of having a minimum at that point. Where the derivative crosses the X axis. Here, to here. Then it is increasing again to here and it's going (inaudible) and it's decreasing. So we've got maximums and minimums every time it crosses the X axis.
I: What kind of function might that be? This is a parabola. What might that be?
S: I think this was to the third power. I could be wrong. Because we never learned what each kind of graph looked like. You probably know don't you what it looks like.
I: Am I interviewing you or are you interviewing me?
S: Nope. I think -
I: Keep (inaudible) to see what happens?
S: Uhhuh. It's got to be 5. The three went like this. No. Gosh - Probably has plus at X or something. Probably nothing basic. What am I doing? I don't know.
I: Good guess. That's it. Those are all the problems for today.
Student E - Interview 4

I: Similar to the first question last time except this time instead of what's an interval what's a derivative?
S: What's a derivative?
I: Uhhuh.
S: A derivative is - if you're going from - if you're going from A to B let's say it would be your function at F of A plus B minus F of A all over B as B is going to zero.
I: What does that mean?
S: It's the average slope.
I: Okay. All right. Now for the other questions for this interview you can consider integration or differentiation either one as fair tools to use. Some of the problems it will be obvious. Others it will be up to you which one you want to use. The first problem I want you to look at is this integral. Integral from minus 1 to 1 of 1 over X squared DX.
S: All right.
I: Can you evaluate that?
S: Well - by hand or with the machine?
I: Either way you want.
S: It would be 1 over - oh - it would be X to the minus 2 plus 1 all over (inaudible) so it would be - that's not it. X - negative 1 to 1. I don't know. Let's see if I can it by machine. Clear - integrate. X squared - X goes to negative 1. Minus 2.
I: Does that seem like a reasonable answer?
S: Does it seem like a reasonable one?
I: Uhhuh.
S: Sure.
I: Okay. Let me have you have a look at another one here. Can you somehow estimate the value of the integral from 1 to infinity of 1 over X cubed plus X squared?
S: From 1 to infinity. I could try.
I: Okay.
S: First I want to look at a plot because I like pictures if that is okay. Plot - might as well look at 100 to get an idea.
I: So you will be graphing 1 - 100.
S: Yeah I just want to. Okay. That's what I thought. It will be super tiny.
I: Okay. And how are you basing that? Are you looking at the graph?
S: I'm looking at the graph how it reacts at 1 and then going down and there's not much between the X axis and the curve so - even though it will probably go out for infinity to the zero these numbers are going to be so small that it doesn't really matter. This thing is out to get me. X cubed - I'll just copy this. It won't do it for me. But my estimate is that it is going to be very small.
I: Is that smaller than 1 or 2? What's your guess?
S: It will be between - let's see. It will be - it will be X. I think it will be between zero and 1 or 1 and 2 but it's hard to tell. It would be below 2 I think.
I: All right. Let me have you look at a problem for me. A spring has a natural length of 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches, how much work is done in stretching the spring from it's natural length to a length of 18 inches?
S: All right. Let's see - natural length equals 14 inches. 5 pounds applied. In order to keep the spring stretched 2 inches. How much work is done in stretching the spring to 18 inches? So 5 pounds - 14 inches - all right. Zero at work equals 14 inches. X pounds equals 18 inches. My guess would be 10 but let's see.
I: How did you get your guess of 10?
S: By just setting up a ratio but I don't know if work is set up in a ratio like that. Let's see, - stretch to 14. Let's see. Work equals - I don't know what the relationship for work is, Work equals force - no. Chemistry it's energy times mass. I'll make a guess and say 10.

I: From ratios. Do you think that problem has anything to do with calculus? Would it seem like?

S: It would if I knew the relationship of work because if I knew the relationship of work then I could set it up I think.

I: What if I tell you that work is force times displacement?

S: Force times displacement so it would be 5 times 2 inches. No. Equals force times displacement. X - I don't know if it would or not. Force times displacement. So I'm sure I could but I don't see it. Not at the moment.

I: Let's have you look at this one. Gas is pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second. The gas pressure is constant and the balloon is always spherical. How fast is the radius of the balloon increasing when the radius is 5 centimeters?

S: How fast is it?

I: Increasing.

S: Increasing when the radius is 5 centimeters. Oh - okay. So you have centimeters down here. This would be radius. This would be increase. No, this would be gas. And it's 50 XT. Spherical shape - how fast is the radius of the balloon increasing? Radius 50 T equals (inaudible). I don't know. Let's see. All right. Clear. See how bad we can screw this up. Empty. Let's see. It's going into here, increasing 50/second. So it would be 50 times time. Time being seconds. So it (inaudible). That is a perfect circle. It would be 2 and so it is a sphere. I'm going about a thousand directions at once. So you want to see rate of change which would be (inaudible). Because this is going to go with this and this one here so if you (inaudible). I've got to set something up don't I. Let's see. Sphere. Let's see. A circle cross section. I'm sure it's much easier than this is but - let's see. Be a circle here and a circle - wherever it cuts - it intersects. (inaudible) would be here. From zero to - I have no idea.

I: Give me the ideas that you're tossing around. You say you're going in a thousand directions. Give me some of those stops.

S: I know that the fastest rate of increase will be at double prime of X.

I: How do you know that?

S: Because I'm assuming it's going to be a positive type of - it's going to be some type of graph that looks positive this way because we're pumping in 50 cubic centimeters per second and so the derivative is going to look like this if this were to go off because the balloon has gone so big that pumping 50 in in a second won't really matter for it because the volume will be so huge it will be a drop in the bucket for it but it will keep going to infinity so the derivative will look like that and since we can't really find the root here of where it's increasing the fastest we need the double derivative to find whether the derivative from this is enough. Even if it's at 5 we need to know where it's going to be max out so we need that point right there.

I: And then what would you do with that point?

S: If this was - well I guess we'd make the (inaudible) equal 5. And then wherever it equalled 5, even if it's here this is the balloon increasing when the radius is 5 centimeters. Oh. If this was T - this is R - R equals 5 here. And if the radius is going up here like this we would need to know what time it was. Oh. We wouldn't want to know that. This would be - if I wanted to know the fastest rate of increase. That's when I'd bring out the double derivative but now 5 equals this. I'd just - whatever my equation was just set it equal to 5 and solve for T. That's all I'd do.

I: And that would tell you how the fast the rate is it's increasing when the radius is 5?

S: Would tell how fast is the rate - oh how fast. Okay. It has nothing to do with that. Or would it? How fast is the radius of the balloon increasing when the radius is 5 centimeters?
I was looking at that wrong. Suppose the gas pumped into the spherical balloon at the rate of 50 per second - (inaudible) you know it's positive. Spherical shape. Increasing the radius (inaudible). Do you want - you want rate of increase. You'd want the derivative at some point wouldn't you? At 5 seconds. That's 5 second. Rate of increase would be the derivative, or are we looking for a (inaudible)? I don't know. I don't even know how to set that up. What to do. How fast is the radius of the balloon increasing (inaudible). The radius is (inaudible). How fast is the radius increasing when the radius is (inaudible)? I guess it would be derivative. My guess is you would be looking for the derivative.

I: Okay.
S: Went all around this page.
I: Let's look at another one.
S: Oh good.
I: You might want to start a new page though. You're running out of room there. Can you find values of A and B so that the line 2X plus 3Y equals A is tangent to the graph of F of X equals VX squared at the point where X is 3?
S: A line 2X plus 3Y equals A - 2X plus 3Y equals A - Oh, it is tangent. Tangent to so it would be X squared and X equals 3.
I: Uhhuh.
S: Find the value of A and B. 2X plus 3Y equals A. All right. Okay. The tangent line like this and X equals 3. Tangent equal the derivative? Isn't that right? Or is it (inaudible)? You have V X squared - how can I fit this in? X goes to 3. Wait a minute. 2X plus 3 equals A. X goes to 3 so if we had a 6 plus 3Y equals A - I'd like to explain this out loud if I knew what I was doing. Let's see - the slope would be - change (inaudible). Which equals (inaudible). I don't know what I'm doing. Let's see. 2X equals A. Tangent to F of X equals B squared at X equals 3. So (inaudible) I don't know what I'm doing. (inaudible) X equals 2VX. Which would equal 2X plus 3Y. Or equals (inaudible). So find values of A. Try to integrate everything. I don't if that's wise.
I: What made you decide to try to integrate everything?
S: Because if I can separate my variables and integrate left and right and then solve left equals right I should be able to get values for Y and X. If I get values for Y and X then I can get values for A and B. So it will integrate this from whatever to whatever. From 2B - X minus 2X when X equals 3. If X equals 3. Tangent to the graph. Or I could set up a parametric can't I? Or can I? Set up a parametric. Let's see. B X squared. No. I give. I don't know. I don't even know which direction to go. Tangent lines. Tangent lines. I don't know.
I: All right. Let me give you one with pictures here.
S: Oh good. I can deal with pictures.
I: Here you go. You can write on that one if you want to. This figure shows the graph of two functions. One of them is the derivative of the other one. There are no vertical scales here because the vertical scales could be different but the horizontal scales are the same. Which one is which and tell me how you know? First label which one is which for me. Which is the function and which is it's derivative and then explain how you know.
S: Okay. All right. (inaudible) equals zero and the maximum (inaudible). So what is it equals zero when we have maximum (inaudible)? Okay. If the derivative is positive it should be increasing. If the derivative is negative it should be decreasing so it's positive. It's positive. This is increasing. So this is F of X. This is F prime.
I: So the first one you're saying is the derivative. Explain how you figured that out.
S: First I want to see where there are maximums and where the other one was crossing zero. This one is crossing zero and it has a minimum. Crosses zero, has a maximum, crosses zero. Also I looked at when the derivative is positive the function should be increasing and here the derivative is positive and it's increasing. If the derivative is negative and it is decreasing the entire line. If it is positive it is increasing all the way. The derivative equals
zero here where it's at a maximum. The derivative is negative so it's starting to decrease. That's how I know.

I: When you were looking at those places where - this second graph crossed zero and you were looking for max and mins did you do the same thing with the other one to see where it crosses zero - you had maximums and minimums?

S: Yeah. I did that was kind of what threw me off and then I went to the positive and negative -

I: I noticed the funny reaction to -

S: Right.

I: So do you think that just looking at those critical points where one of them equals zero to see if the other one has max and mins is that enough to determine or does it take both?

S: I think it takes both.

I: I've got another one with pictures.

S: Oh boy.

I: I want you to sketch for me and you can do it right here on the paper graphs of function so that the derivative of your function would be the G where the graph of G is going - so I'm giving you the graph of the derivative. Draw for me a sketch of what the function might look like. that's the derivative. What might the function look like? If that's the derivative what might the function look like? We're not going to do the condition. I didn't change my mind just because of you. I changed it before I did it with anybody.

S: Let's see - it's going to be decreasing and increasing - this is positive which would be - and oh my gosh. Here, here and here. It's going to be increasing. Okay. It's increasing. Okay. Increasing and increasing. Decreasing. So it's going to be around here somewhere. Up -

I: Now you've drawn a picture for this first graph that looks like the first graph.

S: Right.

I: So are you indicating to me then that the derivative of this function is itself?

S: Well - let's see. Yeah, because this would be nX equals something. Equals 3 or the line would be F of X equals X. The derivative would be 1 so I'm just assuming it would be the same thing.

I: On this one what type of graph would that be? If you had to tie a function with it?

S: Square.

I: Can you tie any kind of function with that last one?

S: Function with it -

I: Like you did with the X squared - a function here.

S: Right. Let's see. It would be like sine or cosine. It would be some type of repeating graph. It would be some type of - like a odd - odd or even. It would have to be odd because this side would equal this side and it would have to be odd.

I: All right. That sounds good.
I: The first thing I want to do this time is kind of like I did last time. I want to ask you what's a derivative.

S: A derivative is a - hm - a derivative is a way to find the instantaneous growth rate of a function or a point of a line or it's the slope of a line at a certain point.

I: I wanted to ask you that one because the last time I did ask you about integrals so I wanted to get your definition. Now -

S: Is that right?

I: It sounds pretty good to me. The problems today could be mixed. If you think you need to use what you know about derivatives to solve a problem then go ahead. If you think you need to use what you've learned about integrations to solve the problem go ahead but I want you to know that anything that you've done in first or second quarter could be a possible tool for you on these problems and some will be real obvious and others you'll have to make a decision about what to do with them. The first one may be one of the obvious ones. I want you to evaluate the integral from negative 1 to 1 over 1X squared DX.

S: Okay. I can use this if I want to?

I: Uhhuh.

S: Okay.

I: Got computer, paper, pencil, whatever you want. Chalkboard even.

S: I could solve this without it I'm sure but this is probably the easiest way to do it so - I know already that it's going to be zero because it's an even function.

I: So you're saying it's going to be zero because the function is even without doing anything. Right?

S: I think so. I know if it's an even function it's going to be the same on each side of the Y axis. And since it's even there I can just take it from zero to one and then take - go up here and say 2 - oh my goodness.

I: What does that mean?

S: I don't know.

I: Has anything like that ever happened? What does that say?

S: Complex (inaudible).

I: What does that mean? Has that happened in another problem you worked before?

S: I hope not. It probably has but - let's see if I can -

I: So now you're going to try negative 1 to 1?

S: That's because - when you use Mathematica™ it will skip over the points that don't work but in actuality this won't work because as we say the function blows up. It's zero. That's why.

I: So is the integral really negative 2 or - did what you find out before indicate what the integral route is?

S: Yeah it did. It (inaudible) that's why. Because it blows up at zero. I didn't look at that.

I: So can you evaluate this integral?

S: Not really. The way Mathematica™ does it it kind of hops over the point where it doesn't work and just figures it as if it was a continuous line without that one point in there where it didn't work -

I: What's happening at that point where it doesn't work? I think you told me - what you keep referring to as the point where it doesn't work.

S: That's where the denominator is close to zero and so it doesn't - undefinable area. Because you don't know where it ends.

I: All right. Can you estimate the value of the integral from 1 to infinity of 1 over X cubed plus X equals 1?

S: Yeah. Again there's - we could either do this by hand or we could do it on the computer. Actually we just started a lesson these with where we integrate by parts and then we could - but I'm not sure (inaudible) yet. We just were introduced to it yesterday but -
I: Just to get an estimate of it what could you do?
S: Well we could take this as being F of X, F prime of X and then find the function and then let - solve the function from there as X goes to infinity. Take the limit of the function from infinity to 1. But Mathematica™ will do it too. Integrate. No, it won't do it. It's too much.
I: If you were going to make an estimate could you make an estimate without using Mathematica™? Mathematically can you predict approximately what that value would be?
S: Sure. It would be zero.
I: Why is it zero?
S: Because if you let X go to infinity the function goes to zero.
I: Why?
S: Because the dominant terms are on the bottom and as the bottom gets larger and larger then the function is zero.
I: All right. Lets give you some problems to look at and see what you do with them. A spring has a natural length of 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches, how much work is done in stretching the spring from it's natural length to a length of 18 inches?
S: Well actually I don't know if - I can't think of anything in Mathematica™ - the way I would solve this I would use physics.
I: Okay. What would you do with this? Do you want paper?
S: Yeah. That's interesting that you asked this actually.
I: Why is that?
S: It makes me wonder if I missed something in our lessons that I should have known.
I: Not necessarily.
S: I'm sure you can relate this to derivatives but I wouldn't do that. I would use physics.
I: What would you do?
S: Let's see -
I: What does that say? Is that an F?
S: Yeah that's an F. I was trying to think if it was F or W actually. And you could use this to figure out the spring constant and I think force of 5 pounds - you'd have to know the spring constant. A force of 5 pounds. I'd have to figure it to Newton's and (inaudible) for it to be (inaudible).
I: Let's suppose you did do that. You don't have to actually do that but suppose you did convert.
S: And say it's like -
I: What would you do after that? Just - are you moving toward plugging numbers into a formula or what are you doing? What would come after that even if you did the conversion? Would you be done?
S: Well with this I could find the spring constant. Then I'd just plug it back into the formula and I'd look for force. X equals 4. That would tell me how much work was done.
I: That would tell you how much work was done?
S: That would tell me how much force is needed. Force times - that should be - be able to work.
I: Force times work.
S: For that distance. Yeah. That's the way I'd solve that.
I: You've got gas being pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second. The balloon is always spherical. One of those perfect balloons. How fast is the radius of the balloon increasing when the radius is 5 centimeters?
S: For this one I'd use actually our very last lesson.
I: Which was?
S: Which was with -
I: it doesn't have to be the title but what kind of things were you doing there?
S: It talked about the rate - if you doubled the dimensions of something and how much the volume increases. From this you're kind of going backwards from that. If you double the dimensions - or double the volume what is going to happen to the dimensions I guess in this case.
I: So what kind of mathematics would you use on that problem?
S: Well you'd have to know - I guess you'd just have to know the formula.
I: So you could find the formula and then substitute what? That constant rate the gas is being pumped in or the radius or what?
S: I'd use 50 cubic centimeters to find out what - to find out what - you'd probably also use derivatives.
I: How could you use derivatives?
S: Because it wants the - because it wants the rate and when you're talking about rate then you could take - when you're talking about rate then you're talking about the derivative of something.
I: Any way to use derivatives with what you're given there or do you still feel like you need some kind of formula?
S: 50 centimeters - no. You'd have to find the formula.
I: A formula involving what?
S: Involving that relationship between the volume and the radius.
I: Okay. Do you know (inaudible) with the sphere is?
S: Sure.
I: That's a good start.
S: That's (inaudible).
I: Is that the formula you need?
S: Yes.
I: Can you solve that problem?
S: Maybe. Let's see. The radius - the radius is 5 centimeters.
I: Tell me what you did there.
S: I just plugged in a radius.
I: Did you plug in 5?
S: Uhhuh. And this tells me the volume. And then I would find the (inaudible). From this I could find out the time.
I: Now you divided that volume by 50.
S: That's how much time it took to get to the radius of 5.
I: Got your function in there again and you're taking the derivative?
S: Well I was wondering what the rate of this growth would be in terms of (inaudible).
I: You took the derivative and evaluated it at 5.
S: Uhhuh. So the volume increases - this is the volume at any radius. So the rate that that volume is changing is - is that.
I: 4pi R squared.
S: Yes. And then plugging the 5 into that at the - plugging the 5 in at that point it's (inaudible).
I: Now what does that give you? The derivative to 4pi R squared you said is the rate that the volume is changing.
S: Right.
I: So you put the number in and what do you have?
S: And then I have the rate at that R. That's the rate the volume is changing.
I: Okay.
S: The rate the radius is changing. That wasn't right.
I: Are the two connected in any way?
S: I'm sure they're proportional but - hm. Radius is increasing. (inaudible).
I: I see that you're trying to solve the equation for R there.
S: Right.
I: Trying to solve the V equals 4 piR cubed for R.
S: Uhuh. I get R at any volume. And then well - this didn't work out.
I: Okay.
S: I tried to find an equation for R and then take the derivative of that equation and plug in 5.
I: Okay.
S: That's what I'd do.
I: All right.
S: I don't know how - I'm not sure how that - I'm not sure how to plug in 50 cubic centimeters/second. But I have to put it in there in order to get the right equation. I'm not sure exactly how to do it.
I: Suppose you want to find values of A and B so that the line 2X plus 3Y equals A, is tangent to the graph F of X equals V X squared at the point where X is 3. Can you find A and B so that would be true?
S: Sure. We can do that. We did do that. We did something similar to that. Like point of order and stuff like that. That was last quarter.
I: Do you remember anything about it?
S: Hm - if (inaudible). Well I can give it a try. See if I can remember.
I: Have you gotten stuck like this and had to break out before?
S: Yeah, but usually it just push (inaudible).
I: Hold it down for a minute.
S: I've never been stuck with an arrow.
I: This is not good.
S: it seems pretty stuck.
I: There you go. Let's keep all this nice stuff you've got here from the other problems.
S: Let's see - G of X. It's equal to 2X plus 3Y plus A. And F of X - DX squared.
I: You put 2X plus 3Y plus A in as a function? Is that what you did?
S: Yep.
I: And DX squared in as a function. Take the derivatives. What did it take the derivative of when you said find G prime of X and it came out with 2?
S: Just took the variable of this. I need to (inaudible). Let's see.
I: There's 2X over 3 plus A.
S: 2X over 3 plus A.
I: Now what did you do?
S: I changed my - I wasn't thinking right. I made this equation right here a function of X and this is already a function of X. So let's try that. Now I can make A be numbers?
I: Uhhuh.
S: I can make them anything.
I: You want to find A and B numbers so that this line over to (inaudible). You set the two derivatives equal and then you set the two derivatives as (inaudible) three. Take out this equation.
S: Uhhuh.
I: 6V to negative 2/3.
S: Okay.
I: So that makes V as negative (inaudible)?
S: Yeah.
I: What's A?
S: A is - A - it doesn't matter what A is. A can be a thousand.
I: So if I take the line 2X plus 3Y equals 1000 then it should be tangent to the graph of F of X equals what? Negative 1/9? Right. X squared - where X is 3.
S: I think so, I hope so. This is how much I can remember.
I: Write for me that value you ended up with B and what you decided on for A here. Just write on this piece of paper for me so I'll have a record of it.

S: Oh, I did something wrong.

I: What?

S: I forgot something. I left out something.

I: F of 3 equals GF3.

S: Yeah. It is starting to come back. This might not be right though.

I: F prime of 3 equals G prime of 3.

S: I think that's right. The functions have to be equal (inaudible) and the derivatives have to be equal so that they're tangent of each other. Hm. I wonder where B is? B must be zero.

I: 3 minus (inaudible). Now what are you doing?

S: I was just seeing if this statement is true. That part is true. Okay.

I: So you've got A is 1 and B is negative 1/9?

S: That's it.

I: Put that on this paper. Just what you found for A and B there.

S: Check these I can plug them in.

I: (inaudible) that constant (inaudible). Okay.

S: That's true.

I: So you have (inaudible).

S: The derivatives are down (inaudible).

I: So the derivatives are right.

S: Oh good.

I: Let's look at one with pictures. In that picture you have two graphs. That is two functions. One of them is the derivative of the other one. There are no vertical scales on the graph because the vertical scales can be different but the horizontal scales are the same. Which one is the derivative and which one is the function and how do you know?

S: Uh, this is the derivative.

I: The second one is the derivative and the first one is the function.

S: Uhhuh.

I: How do you know that?

S: The reason you know is the slope of the function is zero at these points and that's where the derivative is because the slope is zero and since the derivative is actually the slope it's zero at these points. It's not the same with these. This is true. But these aren't true.

I: All right. Mark them function and derivative. Now I have three other pictures. When you sketch graphs, just rough sketches will be fine of some function F so that the derivative of F would be this function of G. So in other words this is the derivative. Can you tell me what the function looks like? Or show me what the function looks like?

S: This makes this convenient.

I: Okay. The axis could be anywhere but it's -

S: Yeah. This one - the slope is increasing

S: This is negative -

I: So then it looks like a parabola?

S: I guess so.

I: Okay.

S: Here the slope is negative but it's increasing. So the slope is negative here but it's getting less and less steep in a negative direction and it hits zero here. That's what tells you there is a max or min - min is increasing. Slope is increasing. This one - okay. This is the derivative. This is where - the derivative is zero. This is (inaudible). The slope is positive here so that's (inaudible). The slope is increasing all this time. Slope is increasing. The slope is negative here so the F of X is negative here so it doesn't get as steep but here there's another - the slope is zero at that point. So it is zero at that point.

I: Okay. All right. That's it. I've got -
S: You kind of stumped me on that first one.
I: That's all right.
Student G - Interview 4

I: I want to start with a question like I did before. Last time it was what is an integral. This time it's what is a derivative?
S: Opposite of integral.
I: Well, what does that mean?
S: You already asked me this before.
I: I asked what was an integral.
S: First derivative defines the slope of the equation at whatever point (inaudible). It's the opposite of an integral.
I: Now in the questions I'm going to ask you today you might need to use what you know about derivatives. You might need to use what you know about integrals. It will be up to you. You can use either one. So consider anything to be fair game to apply to any problems. Some will be obvious. It will be if you can find integrals and things but others it will be your choice. Now the first one is find an integral for me. Go from negative 1 to 1 with 1 over X squared DX.
S: You want to know the integral.
I: Uhhuh.
S: I think I can handle that hopefully. It is negative 2.
I: Okay. Does that sound like a reasonable answer to you?
S: Yeah. It sounds find. The computer says it's right.
I: All right. Is it the authority here?
S: It is this class.
I: What about this one - can you estimate the value of the integral from 1 to infinity of 1 over X cubed plus X plus 1?
S: 1 to infinity - it would go to zero.
I: Why do you say that?
S: Because when you plug infinity into this 1 divided by a huge number is going to go to zero.
I: Can you test your theory out in any way?
S: Sure. I can plug it into here. Big number - you can put infinity in here somehow but it would probably blow up if I did.
I: Is it having problems with big numbers here?
S: Yeah. It doesn't like this at all. Integrate. Hm.
I: Well if you can't get it to do anything for you we'll go with what you were saying.
S: It's going to zero.
I: Okay.
S: It won't even integrate it.
I: That's interesting. Now let's get over here to the problem. A spring has a natural length of 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches, how much work is done in stretching the spring from its natural length to a length of 18 inches?
S: I need the spring formula. In physics.
I: The spring formula?
S: Yeah. There is a formula that has - I think they're not proportional. I don't remember. We did it in physics class.
I: Is it a formula for spring particularly or is it a formula for force or for work?
S: It's a formula just for like spring when they are stretched because there is a point if you stretch it too far.
I: So what do you do with this formula if you had it? Just plug the numbers into it?
S: Yeah. Just plug the numbers in.
I: So do you think that problem has got anything to do with calculus?
S: No.
I: Just one of those physics problems?
S: You just plug it in.
I: Plug in a formula, okay. Do another one.
S: Suppose a gas.
I: Spherical balloon -
S: Constant 50 cubic centimeters. Assuming the gas pressure remains constant (inaudible).
How fast is the radius of the balloon increasing when the radius is 5 centimeters? We'd have to use the first derivative or something like that to figure it out.
I: Can you?
S: Can I?
I: First, why did you say use the first derivative?
S: Because the balloon is increasing. Because how fast is the radius and you want to know that's the rate and that would be the first derivative, how fast it is increasing.
I: Can you figure out a way to use the first derivative to do it?
S: Constant rate of 50 cubic centimeters per second. Spherical. Always in a spherical shape.
How fast is the radius of the balloon increasing? That's another formula.
I: Formula for what?
S: For the volume of the sphere which is - that's a cone. That's another volume of the sphere and you can just plug it in.
I: If you knew the volume of the sphere formula you'd do what with it?
S: You'd plug those numbers in.
I: Plug 50 into it or 5 or both?
S: You would plug in 50 and you'd take the first derivative and you could plug in 5.
I: Into the first derivative?
S: Yeah. And then the radius because that would tell you how fast it's going when the radius is 5. How much it is increasing. Is that what you want to know?
I: I might have a formula over here for the volume of a sphere.
S: How convenient.
I: I could possibly have that somewhere. I think it is $4/3 \pi R$ cubed.
S: Isn't that a cone?
I: No, I think that's a sphere.
S: Is it? $4/3 \pi R$ cubed?
I: Uh huh.
S: Pumped into a spherical balloon at a constant rate of 50 cubic centimeters per second.
Radius is 5. (inaudible) into 50. Hm. $\pi R^2$ - Hm - now - (inaudible) that. Zero. That's not (inaudible).
I: What happened?
S: Well the constants will drop out. $\pi R$ cubed shouldn't. This machine is rigged.
I: This machine is rigged. This particular one?
S: This computer is messed up. It's not zero.
I: How about if I help you over this hurdle.
S: That's not right.
I: Let me give you a hand here. Let me give you a hint. You told this machine that's not rigged that you're putting a function in in terms of X and then you put a function in in terms of R and then you've asked it to take the derivative in terms of X.
S: I'll put the R here. I knew there was something -
I: Let's try it one more time. Now what's the difficulty?
S: I don't know. I'm getting mad. It doesn't like R for some reason. There you go.
I: Now are you happier now with this $4\pi$ times X squared. Now you're evaluating that at 5?
S: Uh huh. The radius is 5 so it would be - it wouldn't be increasing that much. 314 cubic centimeters/second. I don't know.
I: What seems unreasonable?
S: it doesn't sound right. This is volume. This is rate. It should be right I do believe.
I: maybe this is a hot air balloon. Instead of a balloon balloon.
S: Maybe and maybe I'm just stupid.
I: Give you another one. Are you ready for another one?
S: Have one off the midterm.
I: I don't think I have any off the midterm. Can you find values of A and B so that the line 2X
plus 3Y equals the tangent to the graph if F of X equals VX squared at the point where X is 3?
S: The line is tangent to the graph of that where the point where X equals 3? That sounds too
complicated. Then tangent to it. X equals 3. Have to admit I've never seen a problem like
this in my life.
I: We had to give you some that you haven't seen one like to see what you'll do with them.
S: Crash and burn. If A and B -
I: Can you use what you know about calculus in any way?
S: Let's change this.
I: You are going to solve that for Y?
S: Yeah, you want the point slope. Or not from the point slope. From the regular point. This
is A over 3. Now you want it where X equals 3. So it would be where it intersects. Hm.
So I would put A.
I: So you solved it for Y and now you're solving it for A?
S: Uhhuh. Oh I didn't solve for Y. I just changed it. Changed the formula.
I: But you did in the beginning solve it for Y.
S: Yeah, but that was just changing the formula. I didn't care what Y was. This is A. Okay.
It looks okay.
I: So you've got A in terms of B?
S: Uhhuh. X is equal to 3. I don't know how you'd find B.
I: I want to ask you another question.
S: That would be good.
I: Here is one with pictures. Two graphs - one is the function and one is it's derivative. Now
there are no vertical scales on the graph because they could be different but the horizontal
scales are the same. So which is which? Which is the function? Which is it's derivative and
explain how you know it.
S: I always love these. Which is the function and which is the derivative. This would be the
derivative and this would be the function because here -
I: Mark those on there for me if you will and -
S: Derivative and this would be the function.
I: Want to explain to me how you know that?
S: Cause when the first derivative is equal to zero right here and here, here we have a min and
here we have a max and here it's going to another min. And like right here this is increasing
like - it's positive here because it's increasing and here it's negative because - no, here it's
negative because it's decreasing so I had them backwards. Right here - since it's negative it's
negative up here.
I: So it's negative up here which means what now?
S: That this - no, no. This (inaudible).
I: So -
S: This is backward.
I: So the first one is the function and the second one is the derivative?
S: That's not right.
I: What about what you told me about them?
S: Well the slopes don't correspond to this function. This is (inaudible).
I: Now why?
S: Because here - right here we have where the first derivative is zero and we have a max. We
have a max. Where it is zero here we have a min. Where it is zero here we have a max and
here because the slope is negative - so the slope is negative - since the slope is negative up here this is the first derivative is negative and now it is positive here. The derivative is positive.

I: Okay. Is that because you looked just at those critical points or -
S: Yeah, I was just looking at these at first but then the slopes didn’t correspond with it.
I: More pictures. You’re not going to look at the condition on this one but - suppose that these are three graphs, three functions and these are the derivatives. Can you sketch, just a rough sketch of the function that this would be the derivative of?
S: These are the derivatives?
I: Uhhuh. So what would a function look like if this is what it’s derivative looked like?
S: I’ll do the easy part.
I: Draw the axis.
S: Okay. Here - derivative there is no change in the slope so it’s rise would be 1. And so it would just be this. It would just be like Y equals 4 or whatever.
I: So that one is it’s own derivative?
S: This has a zero slope which doesn’t surprise me. So that’s (inaudible). Take the derivative of constant would be a zero. I’m not used to easy stuff. I’m used to disgusting calculus. I’ll do this one. I like this one better. Here the slope is positive. Here we’re going to have max. the slope is negative. And here we have min. It’s too small. It’s decreasing so it’s going to be positive. It’s going to get to the max and it’s going to be decreasing - I’m getting this backwards. These are the derivatives, right?
I: Uhhuh.
S: This is zero. This is the max or min. Since this is (inaudible) this is negative. It’s going to be a max. Here - now it’s going to be negative. Over here it’s going up again and going to be a min. It’s still positive. Drawing the same thing and it isn’t working out.
I: What do you mean the same thing?
S: I’m drawing the function. I’m drawing the derivative. It’s the same thing.
I: That graph doesn’t look like that graph.
S: It’s close. And then this is - it’s increasing after that. Take it to there which is decreasing so that’s going to be a max right here like that.
I: You kind of gave this one a name like Y equals 4. What kind of function is that?
S: This is a cosine isn’t it. Since the derivative of cosine is sine and this is the sine curve.
I: This is the cosine. Now this is cosine. This is constantly increasing that same amount. It wouldn’t be a straight line. Hm. Slope at any point. Here it is negative. Here it is positive. Hm - (inaudible) cosine. Increasing all the way. Get zero (inaudible). Just leave this. Hm. This would be XY. That would be the X squared curve. So if it’s decreasing in here it’s a min. Now it’s increasing and it’s positive. Back to this. This constant is positive. It is not changing. (inaudible). It’s positive. It’s not (inaudible). It couldn’t be that same line. Because this is Y equals 4. The derivative of that is zero. I don’t know. That bugs me. I should know that. Derivative of X - that can’t be. Maybe it is. It might just be this. It doesn’t have to go through there. Just (inaudible) like this. Because it is always positive.
I: Well what do equations of lines like those look like?
S: Y equals X or Y equals (inaudible). The derivative of that would be Y equals 1. It would have to be like that one. Something like Y equals 4X which would be 1. I could plot it. 400 - negative 4. Take a look at it. Something like Y equals 4. Just the derivative you get 4,
I: Okay. That’s all the questions I had unless in the process of thinking about anything you want to add anything to any of your other answers. Don’t look at me like that.
S: I have no idea how to do that.
I: How to do that line tangent at that point.
S: Line tangent thing. I've done it before and I always had A's and B's in there. I don't know how to do it.
I: All right.
Student H - Interview 4

I: ...kind of like we did last time. Last time I asked you what's an integral so this time I'm going back up and say what's a derivative and let you tell me.

S: A derivative is the slope of the tangent line to the curve. Or its the velocity. If you have the distance you can find the velocity. Acceleration - you can take the integral you can find the velocity.

I: Now I have several questions for today and one thing I like for everybody to be aware of for this last interview is that anything that you've done first or second quarter in calculus is fair game as a tool to use on these problems. So if you need to back up and use the derivative you can. If you need to use integration you can. Whatever seems to apply. And some of them will be fairly straightforward and others you'll have to find what it is The first one hopefully is very straightforward. I want you to define the integral for negative 1 to 1 of 1 over X squared DX.

S: I can use the computer?

I: Sure. Use whatever you like.

S: Negative 2.

I: Okay. Does it seem reasonable?

S: I'd say...

I: When you do an integral on the computer like that do you ever think about whether it seems like a reasonable answer?

S: Once I get an answer then I think about what integrals - or if you just an integral that it's just the other end of the curve - negative 1 to 1. So it seems like it would be pretty good.

I: All right. Now look at this one. Can you estimate the value of the integral from 1 to infinity if 1 over X cubed plus X plus 1 (inaudible)?

S: Okay. It would be roughly 1.

I: Now how do you come up with that?

S: Well if you - just looking at it - as X gets larger from 1 to infinity the denominator - X cubed is going to get really small. So X cubed is going to be real little. X is going to be real little as X gets to infinity. The only thing left is 1 over 1 -

I: X is going to be real little as X goes to infinity?

S: As X goes to infinity 1 over X is going to be real little. So you're adding 1 over 1 plus a little and a little bit more. So it's roughly 1 plus a little bit or a tiny bit.

I: So really close to 1.

S: Uhuh.

I: Any way to check out your estimate in any way to see? With paper and pencil or the computer, whatever.

S: I'll check it. It takes a second. 1 over X cubed plus X. Just go to 1000 so the computer doesn't take all day. Oops.

I: Just spit it back out at you. Has that ever happened?

S: 1 over X cubed plus X plus 1. Is there something I should know?

I: I don't know. Is there something you should know?

S: I don't know. Maybe I spelled something wrong.

I: I don't think you spelled anything wrong.

S: Maybe it's just like impossible to do because - oh I don't know to tell you the truth. Maybe - you can do this by partial fractions I think. I think that's what Dr. Davis told us and it's the idea of separating the denominators like 1 over X cubed plus 1 over X plus 1 over 1. But you have to get some kind of factor on the top. You have to multiply by some constant factor, whatever.

I: So it maybe can't deal with these.

S: Maybe it doesn't know what the constant factor is but it's going to be close to 1.

I: Are you taking physics too?
S: Uhhuh.
I: Are you in 131 or 132?
S: 132. I just came from it.
I: Since this is fresh on your mind everybody looks at this and says this looks like a physics problem so maybe you'll - a spring has a natural length of 14 inches. If a force of 5 pounds is required to keep the spring stretched 2 inches how much work is done in stretching this spring from it's natural length to a length of 18 inches? All these non-physics people say it's a physics problem and they kind of cop out on me. Even if you think it is a physics problem then since you're in physics you ought to be able to deal with this. Right?
S: Well -
I: It is a physics problem you say?
S: Yes it is. It's a force problem. That's all it is. If the spring has a natural length of 14 inches and 5 pounds stretches it 2 inches how much work is done to stretch the spring from it's natural length to 18? So you've got to stretch it 6 inches? So if 5 pounds -
I: How did you get that?
S: The natural length is 14 inches. Oh I'm sorry. It is 4 inches. Then the other length - from it's natural length to a length of 18 inches so the difference in length is 4 inches so if it takes 5 pounds to stretch it 2 inches then it should be - I think it was negative KX - that means the force - as you stretch this spring the force grows linearly I think so I'd say it would be 10 pounds but since it takes 5 to stretch it 2 inches, 4 inches would hopefully stretch it, or require a force of 10 pounds?
I: Does that involve calculus?
S: It does because - force is - it's related to calculus in that it's the integral of something.
I: So that 10 pounds that you're telling me, you're saying is the force then that it would take?
S: I think the work is the integral of the force. So - I'd say it would be 10 pounds but I don't know if that's right or not.
I: Ten pounds of work or force?
S: Ten pounds of force but I know force is - force is the integral - the integral of force is work - let's see. Well if force is kilogram meter/second squared and work is kilogram meter squared/second squared if you multiply by 4 inches because that's the difference in length times 5 pounds - 20 - wait a minute. I changed that to foot pounds. So wait a minute. I'm going to see here. We've got to stretch 12 - or 14. Stretch 14 and we want to stretch it 18, right?
I: Uhhuh.
S: And it takes 5 pounds to stretch it 2 inches. Work is the integral of force and the force is 5 pounds. Stretch it 2 inches. Work done would be - if you convert inches to feet to get foot pounds I think then - I'd say it would be 5/6 - wait a minute. No it wouldn't. 3 1/3 whatever the unit is. It's foot pounds.
I: And that would be the work?
S: I think. All I know I think work is integral of force and force times distance is work. If a force of 5 pounds - the work to stretch it 2 inches would be 10 but that's in inches so if you convert that to feet, divide by 12 it would be 5/6 so since it is 5 pounds and you divide 2 by 12 - that's divided by 6 - so 5/6. That would be the work to stretch it 2 inches so if it - what force is required to stretch it 4 inches would have to be - I'd say the force would be 10 pounds. I don't know if that's right. I think it is right since F equals negative KX. So it would be 10 pounds times 1/3 since 4 inches is 1/3 of a foot. So I'd say it would be 3 1/3. Foot pounds.
I: Okay. I've got another problem for you now and I didn't plan this but we have gas pumped into a spherical balloon at a constant rate of 50 cubic centimeters/second. Assume that the gas pressure remains constant and that the balloon always has a spherical shape. How fast is the radius of the balloon increasing when the radius is 5 centimeters? Don't let the fact that there is a spherical balloon in this problem deter you from the - it wasn't planned this way.
S: I've had enough of spheres forever. Gas is pumped into a balloon. So gas is pumped in at a rate of 5 cubic centimeters/second. So 50 - that's the rate. Assume that the gas pressure remains constant and that the balloon always has a spherical shape. How fast is the radius of the balloon increasing when the radius is 5? I don't know the rate of change of the radius. RDT would be the rate of change. The radius with time - how fast is the radius increasing when the radius is 5? When the balloon is deflated at time equals zero the balloon is - there is nothing in it. When time equals 5 - the radius equals 5 and the gas has been - at the rate of 50 cubic centimeters/second so that would be - so it would take 1/10 of a second I think? That can't be right. Or the balloon with a radius of 5, I don't know what I'm doing.

I: That's okay. But rehash what you've said here.

S: You want to know how fast the radius of the balloon is increasing when the radius is 5-

I: Uhuh.

S: Pumping gas in at 50 centimeters cubed/second is going to increase the radius at some rate. When the gas is pouring in at this amount so if I could find - I want to find the rate of change of the radius since having the rate of 50 centimeters cubed/second so - I don't know how to do it. I know that it's logical that the radius is going to get bigger as the gas is pumped in since the pressure inside has to remain the same so if the pressure inside remains the same the only thing the balloon can do is expand so if it expands - if you're adding more to this volume so every second you're adding 50 centimeters cubed more of gas to this. So when the radius is 5 - that's the thing I don't know how to figure out how quick it changes. That's what you want to find though. DRTD and then you could plug in - the radius is 5 and then you find out what the size of the balloon is there and then you can figure out what constant - it wouldn't be 50 centimeters cubed/second. Do you need to use circumference or volume or anything in there to figure it out? I don't know. I don't know the formulas to use.

I: That's all right. Let me ask you another question. Here is another situation. Can you find the values of A and B. It says if a line 2X plus 3Y equals A is tangent to the graph of F of X equals B X squared where X is 3?

S: Find the values of A and B. So the line - 2X plus 3Y equals A. It's tangent to the graph of F of X equals BX squared? If it is tangent then the slope of line A is the negative reciprocal of line B. The point where X equals 3.

I: Now why is that?

S: So it's tangent - because the slope is - well see - just for instance the slope of a line, of slope 1. A tangent to the graph. If it only touches once, - if it only intersects the line with slope 1 once then the other slope would have to be negative 1 because it would have to intersect at right angles where it would only touch once. So if you want to find - if you have a line 2X plus 3Y equals A and you know tangent BX squared then the slope F 3 - F line 3 - both of these graphs intersect so if I set them equal to each other and let X equal 3 I could find what Y is equal to. So two times three plus 3Y equals (inaudible) times 3 squared at the point 3 so I have 3 Y equals 9B minus 6 so Y equals - would be 3B minus 2. That would be - let me see now. At the point X equals 3 this would be the Y value so it would be 3. Let me see - 3. 2X plus (inaudible). If I know F equals 3 to 2 - (inaudible) X plus A/3. I know the slope of the first line is negative 2/3. So if it's tangent at 3 then I know that the slope of the other graph would have to be 3/2. The slope of this one is negative 2/3. This one is perpendicular so it would have to be 3/2. So this is F of X is the same - B times X plus 0. So there is 0. So F of X is a parabola. Just because with BX squared multiply this by B. It would have to be slope (inaudible) you know what B was. You know the slope is 3/2 so it is positive. This is negative slope. Okay. They tell you at the point X equals 3 you know that the slope of the parabola would have to be positive since the slope of the 2X plus 3Y equals A. The A line is negative 2/3 so at this slope it's positive so you know that F of X value is a parabola. It's increasing here so the slope of that line would have to be positive because (inaudible). Now define values of A and B. Of the line at X equals 3 the Y values
would be equal; the X values would be equal. So if this is Y then I could plug that in for Y equals BX squared and Y equals this. So if those are equal I'd still have A and B there. Let me see here. Got 2/3 X plus A/3 (inaudible). X is 3. A/3 is (inaudible). Negative 2 plus negative 3. So A equals 9B plus 2 times 3. Put that back in and solve for B. What a mess. I know what X equals 3 - but why shouldn't I - should just solve for X here. I don't know what I'm doing. We know at three they're equal so if I solve for Y/X - Y equals - (inaudible). (long pause). I guess it's just retarded. I know that A is the intercept and A equals 3Y. Since X is zero the Y intercept would be A. So 3Y equals negative 2X plus A. A equals - okay. So if that is the intercept. Hm - that's a zero?

I: What kinds of things are you chasing around over there on that piece of paper?

S: I wanted to solve for either A or B. Since you know the slopes. You know one is a negative reciprocal of the other you can solve for Y and then you know the slope of the first line. 2X plus 3Y equals A. The slope of that is negative 2/3. Slope Y intercept form or whatever. You get Y equals negative 2/3X plus A/3 so you know the slope of that line is negative 2/3. If it's tangent to the other curve, Y equals BX squared then the slope of that one would be 3/2 and I don't know if B would be the slope of that or not. All I know is that is the formula for a parabola. It's B times X plus 0 squared plus 0. B times quantity X plus 0 squared plus zero so it's centered at 0 0. The slope at 3 would be 3/2. It should. So X is 3/2. So - if I put in negative 2 - if I put in 3 into that -

I: Okay, you've given me an idea of what direction to go in there.

S: I'd just find the slopes and then try to set them equal to each other at the point equals 3 and then find out what A and B are from there but I know X is 3. I don't know Y. I should be able to just solve for Y and then tell you what A and B are but -

I: It's not happening.

S: Right.

I: I'm going to give you one with pictures. Have two functions graphed here. One of them is the derivative of the other one. Now there are no vertical scales on the graphs because the vertical scales could be different but the horizontal scales are the same. Now which one is the derivative and which one is the function and how do you know?

S: Okay. This one is the derivative of that one.

I: The first one is the function. The second one is the derivative. Go ahead and label them. Just write that on there somehow for me and I can follow what you answered.

S: I'll put this as derivative and this is the function.

I: Now how do you know that?

S: I just picked the peaks. If this is the function the derivative at the peak would be - the slope would be zero so a slope here if you look straight down the slope is zero. The slope would be changing the most at the point of inflection right here and that would be the derivative down here at the bottom. The slope of this one at the bottom would have to cross the axis again where it's zero. This peak right up here would have to cross again at zero. And the slope here would be positive and that's above the X axis slope all the way until this point. Wait a minute. The slope would be positive here - that's right, to here. Then all of a sudden from the peak down to the bottom would be negative so from here to here would have to be negative. You'd trace this down from here to here as negative and from here to this peak would have to be positive again. Positive from here to here and then from here down it would have to be negative and have a negative slope. It's negative down here. It's the derivative.

I: Got some more pictures. Not going to have you deal with these conditions though because that wouldn't be necessary. I want you to sketch the graph of some function S so that the derivative of S would be G where G is what is given to you here. So in other words this is the derivative. What would the function look like? This is the derivative. What would it look like?

S: If these are graphs of the derivative -
I: These are graphs of the derivative.
S: Then G would be - let me see here.
I: You can just sketch.
S: Okay. You know the slope of this one - if the derivative is 2 - or whatever. I'm just saying it's a value of 2 then the slope is 2 here so for the slope to be 2 it would have to be greater than 45 degrees. I would just say it is something like that. I would say that would be G of X. And if the slope is - it's okay. well here we go. I'd say this derivative keeps increasing as X gets larger. So if somehow the slope of the line would have to be increasing as X gets larger so as X gets larger I would say it would be something like a parabola but I don't know if that goes exactly through zero or not but it's close but then I'd say towards X being negative the slope would have to be more and more negative which would be a parabola going down because the slope just keeps getting greater and greater negative. So if you add those up I think it should be a straight line for the slope. If you were to take the derivative of this I hope it gives you that. Now this last one - the slope is negative right here. From here to here with the graph somewhere down here and somewhere up here. So if the (long pause). The slope is negative for this integral. From here to here would be negative. What I've got - if this is a peak, if this slope here - the slope is zero. That means it's either a maximum or minimum so at this point it's going to be A. The slope would be zero. Okay, the slope is positive. Then backwards again. The slope is positive (inaudible). I've got it but I've got it reversed. This slope over this equals - the slope has to be positive up to this point. So that is positive. Now the slope has to go negative so it goes down to zero. Now the slope has to be positive again and so it goes back up to this point and then also the slope turns negative so it goes down something like this. You know the slope is positive here so this line is going up. Then at this point the slope is negative - wait a minute. Right. The slope is negative from this point to this point so it would have to be going down with this integral and it's positive again so it would have to be going back up til this point and then the slope is negative again and it would have to be going down. You know where the slope is zero it would have to be at either maximum or minimum so it's critical point. So that's critical point, that's critical point. This right here - that would be one. So it should be 1,2 - right here. Wrong one. This would have to be a maximum or minimum. This would have to be a maximum or minimum and this would have to be a maximum or minimum from there. So I hope that's right.
I: Okay.
S: Clean that up a little bit. You can tell me I failed and to go home.
I: It's not a test. That's good. Those are all the questions for today.
Student J - Interview 4

I: What's a derivative?
S: What is a derivative? Well I guess the first thing that comes to mind is it's the slope function meaning that if you have a function F of X and you take the derivative using the power rule, chain rule, product rule and so forth you end up with the function at prime of X from which if you plug in a value of X you get the slope of the original function at that point.
I: Now in the questions that I'm going to give you today you should consider the things that you've done with derivatives as well as the things you've done with integrals to be fairly tools so if it's a word problem for example you can choose whichever you would like to use to apply to that particular problem. Some problems will be fairly straight forward; others it will be your choice and part of what I want to do is see what you decide to use. So some will be straight forward and there will be no questions. Others consider anything fair game that you have covered in the two quarters that you have been in calculus. The first one is probably one of the more obvious ones. Evaluate the integral from negative 1 to 1 of 1/X squared DX.
S: Okay. I like to rewrite my stuff. 1/X squared is one that I have memorized. I think it is negative 1/X evaluated from negative 1 to 1. But I'll check that. Negative 1 over X. That's negative X to the minus 1. I'm reversing the process. So it is - I was right and then it's just negative - (inaudible) I minus negative - wait a minute. There's a lot of negatives there. Minus from the formula and then negative 1 over negative 1 is just 1 over 1 so I've got minus 1 over 1. Or I've got minus 2. Can I use the computer?
I: Sure. Since you did it on paper now you're going to check it with the computer?
S: Yes. If I could spell integrated. Ah ha.
I: What did it say to you?
S: Failed to reach -
I: Failed to reach accuracy after 7 subdivisions.
S: Yeah. I don't know if this will do the trick.
I: What's it doing? Minus 2 instead of 1 over -
S: Oops. Huh.
I: So what do you think is going on there?
S: I don't know. That's very - subdivisions near (inaudible).
I: What does n integrate do versus integrate?
S: I wanted it to give me a numerical value but perhaps that was a mistake. You know I really don't - to me that seemed like a situation to use n integrate because I wanted a numerical value whereas integrate I figured would probably although I can see now that since I have X in it's specific interval and so forth that it would get rid of the value but normally when you integrate a function you use integrate if you want it terms of variables and so forth and the n integrate uses Newton's method to guesstimate.
I: So why didn't the n integrate work here? Does that make you feel confident in this answer negative 2 or less confident?
S: Well that's what I got myself so I'm pretty confident in it.
I: You feel good about it because it matched yours.
S: Right. The n integrate - like I said the computer was - instead of directly calculating it as it did with integrate it was trying to estimate it and obviously didn't quite make it. That's weird. I don't remember ever having that happen where one worked and one didn't. Do you know what I mean? Usually if you want a number you use - or at least that's a rule I follow. If you want a numerical answer you use n integrate.
I: Okay. Can you estimate then, whether you can find exactly or not, estimate the value of the integrate from 1 to infinity? If 1 over X cubed plus X plus (inaudible)?
S: Hm. Well - for that instead of messing around by hand I would do this.
I: That might not be messing around.
S: Well yeah. One over the denominator. I've got to go back and put some parenthesis in here. Figure out why I used capital X. And then I'll just integrate this in terms of X or just (inaudible). Just like that. That's just an empty integral. Thank you very much. You've got me real - just where I wanted to go. Maybe it's like this. Oh. Integrate. Spelled that right. It's obviously not doing something and I don't know why. I thought that -
I: I thought it was too. I wonder if there is some spacing problems or something.
S: 1 over X plus X.
I: Do you need a space after that comma, before that X or something?
S: No.
I: (inaudible).
S: I don't know why it's not reading that. Integrate I spelled right and then it's - that's how I remember doing it but all I was trying to do was just find out what this is. If this is F prime of X - I was just trying to find out what F of X was so I could evaluate it in terms of infinity. Shoot. But I guess if I would estimate that I would just - if X is going to go to infinity I would just consider the 1/X cubed DX which would be X to the 4th over 4. So X to the 4th over 4. Evaluated from 1 to infinity would be - this is going to be infinity over 4 minus 1/4 so - I mean it's going to go to infinity.
I: So it's going to go to a very large number.
S: Yes. I would want to estimate and I'm at a loss as to why this won't do it.
I: Get the machine to cooperate.
S: I have all my terms right here? That's an X. That's an X. Why won't that go? Maybe - hm. Wait a minute.
I: (inaudible)
S: You never know. Didn't help. I am confused. I don't know why its not working. I'm sure this isn't helping you as far as what you want to hear but it's just not - that shouldn't make any difference but -
I: It's all right. Other than just double checking your spacing I thought about variables but that didn't seem to help any.
S: Unless, hm.
I: It should be able to integrate that.
S: Yeah, that's what I thought.
I: Does real well without the (inaudible).
S: Now can I just invert everything there? Probably not. Hm. That's dumb. It's just not going to cut that is it. I was just going to try - oops. Is that right?
I: Trying to slip it by.
S: That is equivalent - 1 over this, 1 over this and 1 over X cubed.
I: Yeah.
S: That's not what I would have expected. It's still going to go to infinity -
I: That's not - is that equivalent? Is 1 divided by the quantity 1 plus X plus X cubed equivalent to 1 divided by 1 and 1 divided by X and 1 divided by X cubed?
S: 1 over - that's not right. I'm just trying to come up with some -
I: Something you can slide by the computer.
S: Yeah. Trick it into believing me but no luck. Has anyone else tried this?
I: Uhuh, yeah.
S: And they got it?
I: They could get the computer to cooperate a little better than you can. I can't see a syntax error or anything right off hand though so I'm not being very helpful.
S: Can I look?
I: No.
S: They did just like this? They integrated that?
I: I don't know if anybody did integrate it this way but some of them did get it to do something for them.
S: Did they use some like 1 - 10?
I: Some people tried some things like that.
S: And then it's just going to get bigger and bigger and bigger.
I: You're not doing the correct problem there.
S: Is that what you meant?
I: Yeah. Don't worry about it. The computer has a bad day once in a while too you know. Just leave those. It's all right.
S: I don't know what's going on.
I: Maybe after we're done we'll go back and try to fiddle around with it.
S: I'll ask Dr. Davis.
I: Let's look at another problem here. Suppose we have a spring. It has a natural length of 14 inches. A force of 5 pounds is required to keep the spring stretched 2 inches. How much work is done in stretching the spring from it's natural length to a length of 18 inches? I thought you liked physics.
S: I hate physics. Whatever gave you that idea? You need 5 pounds hanging on it. That's a force. Per pounds like or something?
I: Uhhuh.
S: Okay. Excuse me.
I: Actually I said it in pounds of inches.
S: Oh yeah. Required to keep the spring stretched 2 inches. Like 5 pounds stretches it 2 inches. X - how much work is done? Now we're talking work and force and we're going 4 inches. So - I can't do this problem without knowing the formula for - unless I'm wrong - as far as I'm concerned if you use a spring you have to know the spring constant.
I: Suppose you did know that. What would you do to this problem?
S: What is the work done by the spring? That's a formula.
I: Don't try to think of the formula. If you had the formula then what would you do with it?
S: If I had the formula for the work done by a spring I could find out how much work was done by 5 pounds of force.
I: Will it help you any with this problem if I tell you that force - that work is force times displacement?
S: Yeah, force times - FD - isn't it force - to me it's force dotted into displacement. Meaning they're both vectors. It's a dot product. It's not just a (inaudible).
I: But do you have enough information in this problem to deal with that?
S: Just ignore the fact that they're - so force times distance is the work.
I: What I hear you saying is if I give you a formula that you can just put these numbers in then you can tell me how much work is done. Of course if I give you a formula and you just have to put the numbers in you can obviously do that. Right?
S: Yeah.
I: So with what you're given here is there another way to get at this without being handed a nice pretty formula that you can just substitute a couple numbers in and boom, out pops work.
S: The work is force times the distance. Then you've got the work done by the 5 pound weight is 10. And then if you want to do that's just algebra. I don't see - work equals - is it how much work?
I: Uhhuh.
S: In stretching the spring from - so if the distance is 18 but do we know the force? How much work is done in stretching the spring from it's natural length to a length of 18 inches by the same force? Or by -
I: You know the force that's required to stretch it 2 inches.
S: Okay. I don't know what to do with it.
I: We'll fall back on given a formula that you can substitute numbers in - answer not given. Let's look at this one and see what you think of this one. You might have similar feelings. I don't know. Gas is pumped into a spherical balloon at a constant rate of 50 cubic centimeters/second. If the gas pressure remains constant and the balloon is always spherical in shape how fast is the radius of the balloon increasing when the radius is (inaudible)?
S: 50 cubic centimeters/second. The rate that the radius is increasing is going to be - if you have some function F at 5 centimeters equals - I'm not sure what that would equal. If you have some function at 5 centimeters that tells you how much gas is in the balloon at 50 cubic centimeters/second, I don't know how to get it from there but then F prime at 5 would be the rate, would be how fast the radius of the balloon is increasing when the radius is at 5. It would be the derivative of that function at 5. But as far as what would be the function of the amount of pressure in there it's going to be 50 cubic centimeters with respect to time or for something like that.
I: Okay. How about this one? Can you find values of A and B with the line 2X plus 3Y equal to A as tangent to the graph of F of X equals BX squared at the point where X is 3?
S: Is that polar equations?
I: it's up to you.
S: I just wondered -
I: Why don't you started on a new sheet of paper so you'll have plenty of room there.
S: 2X plus 3Y equals A. X equals 3. (inaudible) X squared. Plug in this - you've got X equals 3 and Y is going to equal B times X squared 9. That's going to equal A. If you plug that into the formula of the line.
I: What did you do there?
S: I just took the formula of the line and plugged in the given X and the given Y.
I: Okay.
S: Just because and - what does that tell me? 6 plus - (inaudible) equals A. So - oh, so that line is tangent to the curve.
I: Uhhuh.
S: I read it - curve B times X squared. So times some constant B. X equals 3 (inaudible) tangent line of X. The line tangent to that would be would just be F prime of X. So that's - I guess that would be line B. Take the derivative of that. Maybe I should do it in terms of what is given first. So B X squared - I guess it would be - I don't know what that tells me. If I want to find this point here - the point tangent to that point it's going to be the derivative at that point. But now I want to find, now I want to make 2X plus 3Y equals A. I want to make that this line. Sounds like fun. I'm not going to be able to do it. Not just sitting here.
I: What would you need in order to be able to do it?
S: A hint.
I: Okay.
S: Some more insight into exactly where to head, maybe.
I: What if you know you're going in the right direction working with that derivative? Does that give you enough?
S: Then I know 2BX is -
I: What is that formula that you've just found there? 2BX?
S: I guess that would actually be that line. Well that's the derivative of this function so if I - I: And in general then what is that formula?]
S: Excuse me?
I: In general what is that formula? That is the derivative of that function.
S: The slope.
I: The slope of what?
S: Of the curve. Is that what you mean?
I: The slope of the curve so -
S: At any point.
I: Okay.
S: So if I want - if I go and throw in the given X at 6B equals the slope of that point and I also have 2X plus 3Y equals A where 3Y equals minus 2X plus A. In that form where - let's see. Where negative 2/3 is the slope so B must be - divide this - sorry. Minus 2/3 divide by 6. Three minus 1/3. Excuse me? That's not right. I did my algebra wrong I guess.
I: I don't think you can multiply 6 times any number and get zero unless that other number is zero.
S: Right.
I: Or divide.
S: I'm just trying to do this stupid algebra. I'm just trying to get rid of - this is not my week that I can do algebra. Divide both sides by 6 so that's multiply by - that was a minus. Multiply, that's what I'm trying to do so I've got 1/3 times - so I've got negative 1/3 equals B. And how does that relate to A? Did you see how I got my B?
I: Uhhuh.
S: And then -
I: You took this like you found out the Y and let it equal to the derivative.
S: Right, at that point and I got a B. And now A is going to come from - A is going to be A/3. So function is - negative 1/9 X squared is equal to Y and that's where I intercept - Y squared to zero but I want to get - equal to A/3.
I: And now you set the function that is a parabola equal to A/3?
S: The function F of X given with the Y thrown in there is minus 1 over 9 times X squared. And I want that to equal F of X and I want to know where that is when that is the Y intercept because the Y intercept is given as A over 3.
I: Why should the function of the parabola equal A/3? That's basically what you have written down there,
S: Right. I want F of zero - no. I want F of some point is going to be zero. It's going to be the slope - not the slope but the Y intercept so F of something is going to equal A over 3.
I: What F are you talking about here? Is this the F? F of X equals VX squared?
S: Yeah. The function at some point, the function is going to equal - it's going to cross the Y axis and it's going to be -
I: So are you saying that where this crosses the Y axis this is the same place that this line crosses the Y axis?
S: That's the function of the line. I see what you mean. Yeah. I see what you mean. No, that's not true. The slope crosses the line at A/3; the curve doesn't. So let's see what I was doing wrong. I'm not sure how we'd go to get A then without -
I: Let me show you some of the pictures. Everybody breaths a sigh of relief when they have pictures. Here are two pictures. Graphs of two functions. One is the derivative of the other. Now there are no vertical scales on either of these graphs because the vertical scales could be different for the graphs.
S: Uhhuh.
I: But the horizontal scales are the same. Should be considered the same. So which one is the function? Which one is it's derivative? And how do you know?
S: Where the function is at a maximum or minimum. Where the function is at a maximum or minimum the slope at that point is going to be zero. So that means the graph of the derivative is going to have zeroes at that point. And that right there is the most negative slope is right here and the most positive slope is right in here so that's the derivative.
I: Okay, label them. Which is which. And now go back and tell me how you figured it out.
S: The first thing I looked at was if you have a function, wherever it has a maximum or a minimum the derivative is going to be zero and since you said it has the same horizontal scales that looks like zeroes where the maxes and mins are.
I: Would that work the other way around? Where this has a maximum or a minimum has this one got a zero?
S: Where this one has a maximum or minimum?
I: On this picture.
S: It's pretty close isn't it. That brings me to another question. This has got a slope -
I: But that's not the only thing you based your decision on.
S: Right. I also looked at the fact that right here this is a negative, it's the most negative slope and the derivative is at it's lowest negative value and I looked at it here - that's almost straight up and down. That's the most positive slope and this is where it's at it's highest positive value. That was the other criteria. I was looking at this. This is a positive slope and it's at a positive value. But I still wouldn't change my answers.
I: But your first criteria wouldn't by itself distinguish the two maybe?
S: Obviously not. If I looked back at it it didn't but I didn't base my conclusion on that.
I: Here are three graphs and we're going to ignore the condition I have right here about finding a particular function but suppose that these are the graphs of the function S. So that the derivative of S equals some function G. What's the G? In other words, let me translate for you. The directions don't make sense. This is the graph of the derivative of some function.
S: This one here?
I: What does the function look like?
S: Oh all of these are -
I: This is the graph, right.
S: All of these are derivatives.
I: Right. So sketch the graph of the function.
S: I've got to sketch the function myself.
I: Just a rough sketch.
S: If the derivative is constant that means that the function itself has a constant slope so it could look for example - a slope of 2 I guess you could say. Something like that with a constant slope since the derivative is a straight line that means that the slope is constant and you can see how I did that with the two. But regardless, it's just going to be a constant slope and it's going to be a positive slope because it's above the X axis. This one has a increasing - it's going from negative slope to a positive slope. Trying to figure out what that means right there. It's going to - I guess right here it is going to be a zero or a minimum. It doesn't tell you where it will be - I don't think it does. Like if this is a minimum it has got a negative slope here and then it's got a positive slope like such. This is just negative but increasing. Increasing negatively. No. What am I saying? Increasing - negative. It's a contradiction of terms. I'd leave it like that. And then this one - that's easy. That's sine.
I: That's sine? Okay.
S: It looks like sine to me. I'm going to draw this one like - well let's see. This is - that's pi and this is 3/2. That's zero. This is pi. And then this one goes the same amount. So this is - that's half way. That's pi. And that's (inaudible). That's 2 pi. But then it's not (inaudible). To see if this is - wait a minute. Sine is the derivative. Sine is F pi. This should be - derivative of sine is cosine. The derivative of cosine is negative sine. So I've got this backwards. It should be negative cosine. So I would just - like that would be because that's a zero here and a zero here. Either way I've still got at the zeros on the derivative I've still got my minimum, I've got a minimum and a maximum but here it's increasing and then it - and then here it's increasing. Increasing positive, negative slope. Can you tell I've been increasing and decreasing from this one or can you only look at F double prime to get increasing and decreasing?
I: You can tell it.
S: You can. If it's got a positive slope it's increasing. It can't be increasing and have a negative slope.
I: Right.
S: If it's got a negative slope here it's going to be like so and then it's going to get to the zero and come right down (inaudible). So this one is (inaudible).
I: Okay.
S: I can see it from the graph itself but it's much easier to know that this is sine and this is going to be negative cosine if you go in the right direction instead of the wrong way which I did first.
I: Looks good.