CULTURAL DIVERSITY AND WHITE TEACHER SCAFFOLDING
OF STUDENT SELF-REGULATED LEARNING
IN ALGEBRA CLASSES

DISSERTATION

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ABSTRACT

The purpose of this descriptive qualitative study was to examine the ways in which teachers use classroom discourse for teaching and learning mathematics, developing self-regulated learning, and engaging culturally diverse students, including low SES students and students of color, in meaningful classroom interactions. Three classes participating in the Classroom Connectivity in Promoting Mathematics and Science Achievement (CCMS) research study were selected as cases for in-depth investigation. Each case included a White teacher and culturally diverse students. Data were gathered through videotaped classroom observations, observer notes, and demographic data reported by students. Post-observation and end-of-year teacher interviews provided additional data and opportunities for verification of analyses.

Data were analyzed to create descriptive narratives of classroom interactions for each case. Cross-case analysis was used to identify continuities and discontinuities for the purpose of understanding teachers’ and students’ uses of classroom discourse for learning mathematics with understanding and developing strategic learning skills in culturally diverse learning communities. Analyses revealed that several aspects of teacher-led classroom discourse have potential to support learning mathematics with understanding and developing self-regulated learning skills.
First, social and analytic scaffolding helped students know how to participate in discussion and to explore the mathematics more deeply when the relationships between classroom participation and learning were made explicit. The productive scaffolding observed involved pressing for students’ expressions of understanding and providing feedback. Furthermore, relating difficulty with problem solving to opportunities to learn with deeper understanding set norms for open discussion and created a safe atmosphere for taking risks, aspects of learning that are particularly important for students of color and students with fixed-entity theories of intelligence.

Explicit instruction in academic discourse supported communication in content-specific registers of language and may have increased engagement in dialogic discourse. In one case, student agency and the development of academic language were supported by highlighting students’ contributions to classroom discourse, which stimulated dialogic discourse. Additionally, students’ personal/cultural social discourse was described as the “lubricant that keeps the [mathematical] conversation going” in the class where students expressed the most mathematical reasoning. This has important implications for how engaged learning is defined in classrooms with culturally diverse learners.

Finally, the use of technology to support learning, along with the intention of addressing inequities, has potential to support dialogic discourse. A teacher’s philosophy and approach to teaching and learning may be more important than mere access to technology in addressing issues of equity with incorporation of technology. A teacher’s stance on what it means to teach and learn appears to work in concert with incorporating technology to create more equitable learning environments for culturally diverse students.
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CHAPTER 1

INTRODUCTION

Background of the Study

Since the time I began classroom teaching in the public schools, I have been interested in the ways in which young adolescents become more aware of themselves as students—in the ways that they develop identities as learners, become responsible for their education, and develop into self-motivated learners. Despite the many stories of successful learning that I can recall, I was often frustrated with my students’ apparent inability to monitor their own progress—to know how they are doing in their studies and to make decisions about what was important to do in order to obtain their goals. Students relied on me for reports of “how they were doing,” and their goals in school were often expressed in terms of “getting an A” rather than in terms of understanding. Their natural curiosity seemed to have been eclipsed by routines that placed them in passive roles of receptacles rather than in active roles of investigators.

Over time, influenced by collaborative work with critical friends connected to outside institutions, I was able to articulate my concerns about teaching and learning in language that was more in line with existing educational research and theory. I came to understand the methods that I had been developing over the years to encourage greater
student awareness of strengths and weaknesses in their approaches to learning as being the foundational self-knowledge of self-regulated learning (SRL). I also came to understand the issues related to working with racial, cultural, and socio-economic variability within my classroom as issues of equity and social justice. My experiences and subsequent realizations led to the present study of the development of student SRL while learning mathematics with understanding in culturally diverse classrooms.

*Self-regulated learning* (SRL) refers to learning that is guided by metacognition, strategic action (planning, monitoring, and evaluating personal progress against a standard), and motivation to learn (Butler & Winne, 1995; Zimmerman & Martinez-Pons, 1990). Self-regulated learners are aware of their academic strengths and weaknesses and are able to apply a variety of strategies to follow through on academic tasks. These learners tend to hold incremental views of intelligence and they attribute their successes or failures to factors within their control (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2000). Self-regulated learners believe that opportunities to take on challenging tasks, practice their learning, develop a deep understanding of subject matter, and exert effort will give rise to academic success (Perry, Phillips, & Hutchinson, 2006). *Cultural diversity* refers to a range of cultural characteristics of individuals within classes that include differences from the White middle-class majority in the United States. Classes exhibiting cultural diversity might include Hispanic/Latino/a, African American, Asian, and/or Native American students.

During my years teaching in the public schools, my students were predominantly African American, yet I was not prepared to think about cultural diversity as I made decisions for classroom instruction. I knew that all of my students needed to be more
aware of their roles as learners, but I also needed to know more about culturally relevant pedagogy and implications for scaffolding my students’ development as learners in a variety of ways. Professional development opportunities and collegial support were thus directed toward an effort to make sense of the task of teaching mathematics to early adolescent students who came from a variety of cultural groups in terms of race, ethnicity, and socio-economic status. The greatest support for making sense of my practice came through my involvement in a teacher-researcher project at The Ohio State University—work exploring the development of SRL with 7th-grade mathematics students that led to doctoral studies in education.

The theoretical foundation gained through my doctoral studies in mathematics education supports the use of reformed practices for teaching and learning mathematics and the use of student-centered practices. Sociocultural theory supports viewing relationships among culture, identity, and inequities in mathematics education as observed through classroom discourse to develop an understanding of what culturally relevant pedagogy means in mathematics education. Additionally, steps taken toward developing student SRL in mathematics classes have resulted in reflection upon both successes and limitations and thoughts about improved future work. In brief, recognition of the need for reformed practices in teaching and learning mathematics, sociocultural theory, and previous work with scaffolding the development of SRL have led me to the present investigation of teaching and learning mathematics with understanding and the developing SRL in culturally diverse classrooms.
Theoretical Context

NCTM presented a vision for reform in mathematics education with the publication of *Principles and Standards for School Mathematics* (NCTM, 2000). In this vision, all students have access to high quality, engaging mathematics instruction. Students are expected to learn mathematical concepts and procedures with understanding through the rich mathematical tasks provided by the teacher. Learning mathematics with understanding is an emergent, developmental process that focuses on mathematical thinking and transformational reasoning. Mathematical understanding comes from developing positive dispositions toward investigation, formulating good questions and solving mathematical problems, applying mathematical procedures strategically, communicating reasoning, making connections between mathematical concepts, representing mathematical ideas in a variety of forms, and recognizing that mathematics is a branch of human endeavor that is continually developing.

Within the NCTM vision, teachers are required to have deep professional knowledge of content and processes of teaching and learning mathematics. Teachers must provide rich mathematical tasks so that students engage effectively with the content and processes of the mathematics and work together with other members of the learning community. Scaffolding of engagement, motivation, and mathematical reasoning provides opportunities for students to take greater responsibility within processes of learning (Cobb, 1999; Turner, Meyer, Cox, Logan, DiCintio, & Thomas, 1998; Turner, Midgley, Meyer, Gheen, Anderman, & Kang, 2002). The culture of engagement and reasoning becomes one of inquiry, nurturing mathematical dispositions as well as thinking and learning dispositions.
There is general agreement among organizations concerned with teaching and learning mathematics that traditional approaches to mathematics instruction have been insufficient for meeting the needs of all American students (Herrera & Owens, 2001; National Council on Education and the Disciplines [NCED], 2001; Malloy, 2004). In addition, the changing needs in a world increasingly dependent upon skills in mathematics, science, and technology add urgency to the call for reform in mathematics education (NCTM, 2000; NCED, 2001; National Research Council [NCR], 2001). American mathematics curricula have been characterized as being shallow and wide (NCR, 2001), covering a great number of topics, but lacking depth of investigation that leads to understanding of concepts and useful applications.

It is also commonly agreed that the purpose of schooling is to help young people become well-adjusted, functional members of society (D'Ambrosio, 1994; Dowling, 2001; NCR, 2001). Being a functional member of a democracy involves being able to think in order to make informed decisions required to participate fully in the benefits of the democracy. Traditional approaches to mathematics teaching that emphasize procedures and memorization over understanding and application are not sufficient to produce citizens who are capable of functioning within a democracy increasingly dependent upon numerical data for reasoning about political ideas and social policies. Citizens need to have experienced more meaningful approaches to learning mathematics in order to be informed participants who are able to reason and justify the decisions that are made that affect all members in a democracy (D'Ambrosio, 1994; Dowling, 2001; NCR, 2001).
Sociocultural theory provides a foundation for theorizing about issues of equity and social justice in the context of schooling—as learning takes place through social processes, language is the medium for expressing, thinking, and conveying understanding. Building knowledge can then be defined as a dialogical process where language mediates between the outside world and the learner. It is through discourses that learners come to know about the world, come to know themselves as individuals within society, and learn how to function as part of society.

Sociocultural research in education includes analyses of the relative positioning of participants, or readers, in teaching and learning through texts. Typical mathematical pedagogical texts are either regulatory, which positions the reader as "a regulated object" where knowledge is implicit and strategies are localized, thus readers get more of what they already know, or transmission, which is the basis of "an apprenticing pedagogical action" that positions the reader as "a potential subject of mathematical practice—as a potential mathematician" (Dowling, 2001, p. 23). Within such an arrangement, a dominant polarizing effect results in apprenticing one group of students to academic mathematics (apprenticed subjects; the successful students) and regulating another group (regulated objects; the less successful students) in terms of what is intended to be practical and relevant mathematics. Neither side in this dichotomy is suited to the present change in the world economic structure or any future that students will encounter as adults (Dowling, 2001).

Instructional discourse and positioning of students are critical factors of reform in contemporary classrooms (Cobb, 1999; Dowling, 2001; Mukhopadhyay & Greer, 2001). Developing an instructional discourse where all participate to construct understanding of
mathematical concepts is essential in avoiding the polarizing and inequitable effects of "traditional" approaches to mathematics education. Students should be positioned as active participants in the construction of knowledge. Accordingly, students need to be aware of their roles as learners and to develop strategies that support active participation, both necessary components of SRL. Encouraging SRL in mathematics is important because it helps students plan, guide, and monitor their thinking when they encounter and solve challenging problems and it promotes mathematical understanding (De Corte, Verschaffel, & Op’t Eynde, 2000; Pape, 2005; Verschaffel & De Corte, 1997).

Previous Work

In previous work related to this proposal, two colleagues and I examined the development of mathematical thinking and student self-regulation in a seventh-grade classroom (Pape, Bell, & Yetkin, 2003). We worked to establish norms for mathematical behavior, provide rich mathematical tasks, and develop instructional discourse that supported and extended students' thinking about mathematics. At the same time, we encouraged students to develop greater awareness of themselves as learners and to reflect upon the strategies they used for studying. We developed the *Strategy Observation Tool* as one means of scaffolding the development of skills for self-regulated learning. The *Strategy Observation Tool*, which focused attention on observing the use of strategic behaviors for learning and attributing outcomes to student behaviors, was modified over the course of the year as students provided feedback and as we observed the results of its use.

Students grew in their ability to communicate mathematical reasoning over the course of the year. They were exposed to a wide variety of strategies and became better at
articulating the strategies they used. Many students were more aware of the decisions they made about studying and the effects of those decisions on grades and efficacy. A great number of students, however, generally did not choose to be strategic. Those who benefited most from the SRL instruction were those who were already inclined to have good study habits and who already valued doing well in school. Those who benefited least were those who already had difficulty following through on homework and had very few strategies for success in school learning. In other words, the actions we took toward encouraging the development of student SRL did not do enough to meet the needs of those students who we thought had the most to gain from SRL instruction, particularly non-white and economically disadvantaged underachieving students.

One way of addressing the failure to meet our goal for non-white and economically disadvantaged students might be to emphasize communal relationships as a means of support within the process of developing learning strategies. A communal orientation places priority on social bonds, the interconnectedness among people, and mutual responsibility (Hurley, et al., 2005). **Communal learning** refers to dialogic processes of acquiring knowledge and skills within a group (of students and teachers) that shares common purposes and goals (Jackson, Mackenzie, & Hobfall, 2000). This definition implies a caring and collaborative quality (interdependence) that may not be present within a more individualistic approach.

Despite the fact that feedback from external sources for development of self-regulation is acknowledged in the literature on self-regulation, cognitive psychological perspectives still convey the importance of the individual and the exercise of free will. This notion of SRL is not congruent with the learning styles of some non-mainstream
cultures, thus highlighting a need for further examination of communal aspects of self-regulation, critical issues of distributions of power in society, and access to social and economic resources necessary for independent control (Jackson, Mackenzie, & Hobfall, 2000). For non-mainstream populations, limited control over external factors such as economic, political, and social power may impede progress toward individual goals, therefore efforts to reach goals through personal control may be perceived by individuals as being ineffectual. Research that acknowledges and describes communal aspects of creating contexts for development of SRL would be an important contribution to the existing body of literature. Application of discourse analysis might help to gain understanding of the development of SRL and learning mathematics with students who possess communal orientations.

Research context: Classroom Connectivity in Promoting Mathematics and Science Achievement

My study of how classroom discourse is used to create contexts for the development of student SRL took place within the larger Classroom Connectivity in Promoting Mathematics and Science Achievement (CCMS) research study, which is an interdisciplinary, national, four-year, experimental study focused on teaching and learning of mathematics and science in grades 7 through 10. The main focus of CCMS is to examine the influence of connected classroom technology with interactive pedagogy and professional development on mathematics and science achievement. The goal of the CCMS study is to support the development of conceptual knowledge, increased achievement, effective use of self-regulated learning (SRL) strategic behaviors, and positive dispositions toward mathematics and science in connected classrooms (Pape et
al., 2006). The primary hypothesis is that classroom connectivity technology used with appropriate pedagogy increases student achievement and provides other desirable outcomes such as the development of self-regulated learning behaviors (e.g., strategic behaviors, metacognitively active stances toward learning, and problem-solving skills) and productive dispositions toward mathematics or science.

CCMS project participants come from 117 middle and high school teachers (96 Algebra I, 21 Physical Science) from a national sample (32 states, and 2 Canadian provinces), half in an experimental group and the other half in a control group. The project is based at The Ohio State University and includes the Better Education Foundation, the Center for Research on Evaluation, Standards, and Student Testing at the University of California Los Angeles, and the University of Florida as partners.

Intervention in the CCMS research project consists of (1) provision of connected classroom technology (TI-Navigator), (2) professional development consisting of a weeklong Summer Institute held at Ohio State University, (3) online web-based training as needed, (4) online discussion forum for the teacher community to exchange experiences, problems, and curricular materials, and (5) follow-up professional development at annual conferences (Owens, D., Abrahamson, L., Demana, F., Irving, K., Pape, S. J., & Herman J., 2004). The CCMS research is guided by the following main question: How do teachers’ professional development with appropriate pedagogy and the use of the connected classroom TI-Navigator system affect student achievement in Algebra 1 and Physical Science (Owens et al., 2004)?

The research design is a randomized crossover trial where the control group is exposed to intervention in the second year of participation. Quantitative data is used for
statistical analysis along with qualitative data for greater depth of analysis of classroom conditions. Measures include pre and post assessment of algebra and physical science achievement, teacher classroom practice, student dispositions toward mathematics or science, student motivation to learn mathematics or science as well as classroom observations, telephone interviews, and weekly technology use logs.

**My Roles within CCMS**

In July of 2005 I began as a participant within the CCMS study as a classroom algebra teacher. I participated in a weeklong summer workshop to learn how to use the TI-Navigator system for teaching and learning algebra. I carried out all of my obligations as a first-year participant. Because I left my teaching position, I was not directly involved in the CCMS study the following year (2006-2007). During the 2007-2008 school year, I worked in a variety of roles within the CCMS investigation as a graduate research associate. My duties included scoring end-of-year algebra tests for gauging inter-rater reliability, organizing data in preparation for analysis, conducting telephone interviews with teachers, assisting with videotaping of classroom sessions, and performing other tasks as needed. In the 2007-2008 school year, I was also a member of a group, with Dr. Stephen Pape (CCMS Co-PI) and two other graduate student associates, which coded first-year data of videotaped classroom sessions. I will continue working with the coding group during the 2008-2009 academic year.

**Summary**

The theoretical context of this study includes discussions around teaching and learning mathematics with understanding, awareness of creating culturally relevant learning experiences for students from a variety of cultural and socio-economic
backgrounds, and encouraging students to become more aware of the cognitive and strategic SRL processes they use while engaged in mathematical tasks. My views on all three of these areas of interest are shaped by critical theory. The present study is concerned with both the cognitive and the sociocultural aspects of teaching and learning mathematics. A qualitative, critical analysis of the discourse used to create contexts for the development of SRL in mathematics classrooms, with attention to both cognitive and sociocultural features, will be a unique and significant contribution to the body of knowledge gained through the larger CCMS experimental research.

Statement of the Problem

The problem addressed by this study is multi-layered. For both students and teachers, there is a need for awareness of strategies for learning and connecting to subject content. Related to long-recognized yet unaddressed patterns of underachievement for non-White, low SES students, there is a need for teacher understanding and awareness of culturally relevant pedagogies. Cultural relevance must be a central consideration in the development of lessons that address these inequities. Teachers must engage students in learning mathematics and encourage the development of strategic learning skills that are critical to both learning with understanding and progress toward self-regulation. Many teachers continue to struggle to meet the needs of non-mainstream learners, such as those with low SES and students of color. Addressing the problem involves identifying and understanding the ways that teachers support (scaffold) the development of learning strategies and skills for their particular students, who may or may not be classified as typical or mainstream, at the same time as fulfilling their obligations to their school districts as teachers of mathematics. An examination of classroom discourse, specifically
how teachers and students use language to support the development of SRL in mathematics classrooms that are representative of cultural diversity, may provide insight into ways of addressing inequities

Purpose of the Study

Much research has focused on the teacher’s role in using instructional discourse to encourage communication of mathematical reasoning while developing an appreciation for understanding over performance (Morrone, Harkness, D’Ambrosio, & Caulfield 2004; Rittenhouse, 1998; Williams & Baxter, 1996; Wood, 1999). There is also a great deal of literature available that describes models for teaching self-regulated learning (McCann & Turner, 2004; Schunk & Swartz, 1993; Schunk & Zimmerman, 1997; Turner, Midgley, Meyer, Gheen, Anderman, & Kang, 2002; Zimmerman, Bonner, & Kovach, 1996; Zimmerman & Martinez-Pons, 1990). However, there is a need to expand knowledge of the differences in how teachers use classroom discourse to scaffold student self-regulated learning and mathematical understanding within culturally diverse classrooms. The purpose of this study is to examine the ways in which teachers use classroom discourse for teaching and learning mathematics, developing SRL, and engaging non-mainstream and low SES students in meaningful classroom interactions. The present study will add a cultural perspective to the literature on SRL.

Guiding Questions

1. What do teaching mathematics for understanding and fostering SRL look like in culturally diverse algebra classrooms?

2. In what ways do students and teachers in these classrooms position themselves and each other through discourse?
Significance

Theory and research support the idea that instructional discourse can be used to support efforts in developing students' learning dispositions (Cobb, 1999; Jackson et al., 2000; Middleton & Midgley, 1997; Turner et al., 1998; Turner et al., 2002). There is a need to expand ideas about the use of instructional discourse to include the codevelopment (teacher and students) of self-regulation, including special attention to the variety of student needs in the development of learning strategies and goals. The goal for this study is to gain information through an examination of classroom discourse as participants create contexts for the development of SRL. A qualitative approach to understanding the development of SRL in a variety of cultural contexts, including both cognitive and sociocultural aspects of mathematics teaching and learning, will be a new contribution to the literature on SRL.
CHAPTER 2
LITERATURE REVIEW

This literature review is divided into three major sections, each of which contributes to the framework for this study. The first section, Teaching and learning mathematics with understanding, describes the call for reform in K-12 mathematics education and presents research related to classroom contexts that support learning with understanding. The second section, Culture, positioning, language, and power, addresses issues of cultural diversity, equity, and social justice with an emphasis on the use of language to negotiate power relations and the implications for self-theories. The third section, Self-regulated learning, outlines the major research on SRL, theories of intelligence, and the connections among SRL, sociocultural theory, and classroom discourse. Finally, the connections among major ideas from each of the sections, with an emphasis on the role of classroom discourse within them, are outlined in the Conclusion section.

Teaching and Learning Mathematics with Understanding

Teaching and learning mathematics with understanding is an emergent, developmental process of learning mathematical concepts deeply that focuses on mathematical thinking and transformational reasoning. Mathematical understanding
comes from practices that develop positive dispositions toward investigation, formulating
good questions and solving mathematical problems, applying mathematical procedures
strategically, communicating reasoning, making connections among mathematical
concepts, representing mathematical ideas in a variety of forms, and recognizing that
mathematics is a branch of human endeavor that is continually developing. The National
Research Council (Kilpatrick, Swafford, & Findell, 2001) and the National Council of
Teachers of Mathematics (NCTM, 2000) have called for changes in mathematics
education that support this conception of mathematical understanding as an emergent,
developmental process. In this first broad section of literature review, I address the call
for reform in school mathematics by briefly describing NRC’s conception of
mathematical proficiency and NCTM’s principles and standards, with particular attention
to the process standards. Discussion of these elements of the NRC and NCTM documents
are important to the foundation of the present study because they address what it means
to teach and learn mathematics with understanding. The research literature that follows
the discussion of NRC and NCTM documents relates to classroom contexts for learning
mathematics with understanding.

Mathematical Proficiency

In response to growing public concern over the quality of mathematics education
in the United States, the National Research Council engaged in a project to address the
need for guidance and leadership in answering questions about how to improve
mathematics learning for all students (Kilpatrick et al., 2001). The resulting publication,
*Adding It up: Helping Children Learn Mathematics*, outlines what students should know
and be able to do to be considered mathematically proficient. *Mathematical proficiency,*
the term used to describe successful learning of mathematics, is made up of five interwoven strands—conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions.

The first strand of mathematical proficiency, conceptual understanding, refers to an integrated and functional comprehension of mathematical concepts, operations, and relations. A student who has developed mathematical proficiency knows the meanings of symbols, diagrams, and procedures that are used in mathematical reasoning and is able to represent mathematical situations in a variety of ways (Kilpatrick et al., 2001). The second strand, procedural fluency, has been a primary focus in traditional approaches to mathematics education. The difference in procedural fluency as described by the NRC is that successful learners are not defined simply by their ability to carry out accurate algorithmic procedures, rather they are, over time, expected to carry out procedures flexibly, accurately, and efficiently. The third strand, strategic competence, “refers to the ability to formulate mathematical problems, represent them, and solve them” (Kilpatrick et al., 2001, pg. 124). This strand is similar to problem solving as it has been traditionally presented in mathematics education, the important difference being an emphasis on the ability to figure out what a problem actually is (problem formulation). The fourth strand, adaptive reasoning, “refers to the capacity to think logically about the relationships among concepts and situations” (Kilpatrick et al., 2001, pg. 129). Adaptive reasoning results in the ability to formulate arguments and justify decisions. Essentially, adaptive reasoning allows students to engage in mathematical processes with confidence because they do not have to rely on others for verification of the correctness of their work.

Finally, the fifth strand, productive dispositions, refers to the tendency to see
mathematics as useful and worthwhile, to see effort and perseverance as valuable characteristics of mathematical work, and to see oneself as competent learner and doer of mathematics. Each strand focuses on separate aspects of mathematical proficiency for the purpose of presentation, but the strands are interconnected aspects of successful learning of mathematics. The five strands are interwoven and work together as students develop mathematical proficiency.

NCTM (2000) produced *Principles and Standards for School Mathematics* (PSSM) based on the belief that all students should learn mathematics with understanding. PSSM outlines a developmentally appropriate mathematics education for students across grade levels (PreK-12) in the vision, principles, and content and process standards. The NCTM (2000) *Principles*, which are the philosophical foundation for the *Standards*, describe the characteristics of good quality mathematics education within six thematic categories: equity, curriculum, teaching, learning, assessment, and technology.

*Equity* refers to high expectations and strong support for all students. The *curriculum principle* addresses the need for a coherent and focused mathematics curriculum that is well articulated across grade levels. The *teaching principle* proposes that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (NCTM, 2000, pg. 16). The *learning principle* stresses that students should learn “mathematics with understanding, actively building new knowledge from experience and prior knowledge” (pg. 20). *Assessment* should support learning by providing feedback to both students and teachers about students’ development of mathematical understanding. Finally, *technology*
should be used to enhance students’ understanding of mathematics. All six categories of principles for school mathematics are inextricably linked with content and process standards.

The Process Standards include problem solving, reasoning and proof, communication, connections, and representation. For each of the five Process Standards, NCTM describes goals of what the standard should look like and what the teacher’s role should be in achieving the standard within grade-level bands (K-2, 3-5, 6-8, and 9-12). The following presentation of PSSM (NCTM, 2000) is limited to the Process Standards because the guiding questions for the present study focus on aspects of joint activity (among teachers and students) in the social construction of meaning as well as the development of learning environments. Each of these standards, problem solving, reasoning and proof, communication, connections, and representation, applies to all grade levels, but the emphasis placed on particular standards varies across the grade bands. The following descriptions are generalizations across all of the grade-level bands.

Problem solving involves a combination of cognitive and metacognitive processes. NCTM (2000) recommends that instruction in problem solving enable students to build new mathematical knowledge, solve problems that arise in mathematics and other areas, apply and adapt strategies to solve problems, and monitor and reflect on problem solving processes. Teachers must carefully choose contextual problems that provide opportunities to build on existing knowledge and pique student interest.

Because reasoning is a fundamental aspect of doing mathematics, reasoning and proof includes the processes that mathematicians use in “real life.” Students must learn to make and investigate mathematical conjectures, develop and evaluate mathematical
arguments and proofs, and select and use various types of reasoning and methods of proof (NCTM, 2000). The ability to reason, conjecture, and construct mathematical arguments develops through frequent classroom discussion among students and between the teacher and students. Listening is a critical component.

*Communication* is the process through which students are able to share the organization and consolidation of their mathematical thinking. Development of the ability to communicate mathematical thinking and strategy use coherently and clearly to other students and the teacher ensures that all members of the mathematics learning community will be able to evaluate and respond to each other’s thinking. It is through their precise expression that mathematical ideas can be made public and become objects of investigation. Language plays a crucial role in the negotiation of meaning in contexts that make use of rich tasks requiring mathematical thinking, problem solving, argumentation, and justification (Turner et al., 1998).

*Connections*, as a process standard, refers to the recognition and use of connections among mathematical ideas, comprehension of how mathematical ideas interconnect and build on one another to produce a coherent whole, and recognition and application of mathematical ideas in contexts outside of mathematics as a subject area (NCTM, 2000). Connections are formed not just among mathematical concepts but also between the learner and the mathematical concepts (facilitating the student’s identity as a doer of mathematics), between the concepts and real-world applications, and among learners and teachers (forming communities of learning).

*Representations* are signs or configurations of signs, characters, or objects. A representation can stand for something other than itself. “The term representation refers
both to process and to product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself” (NCTM, 2000, pg. 67).

*Internal representations* are the representations that are formulated in the mind of the learner, including mental images or imaginative systems of representation. They are not subject to direct observation. *External representations* are those which are shared outwardly—the spontaneous representations invented while making sense of problem situations and the conventional representations that have been developed over time. Representations are important in mathematics classes because they can be used as tools for understanding (Lehrer, Jacobson, Kemeny, & Strom, 1999).

Like the strands of mathematical proficiency, each process standard is complex and does not exist in isolation from the others. The process standards, as well as the components of mathematical proficiency presented above, provide descriptions of not only what students must know and be able to do but also of how they come to know the mathematical content in a vision of reformed mathematics. A thorough understanding of goals for students is critical to creating supportive classroom contexts where students take active roles in the construction of knowledge.

*Creating Contexts for Learning Mathematics with Understanding*

I have presented visions of reformed practice for mathematics education as described by two national organizations, NRC (2001) and NCTM (2000). Malloy (2004) asserts that reform in mathematics education can be thought of as consisting of three major components: curriculum that emphasizes access to valuable mathematics, pedagogy that invites students to become doers of mathematics, and assessment that guides the practice of teachers. In this section of literature review I examine the second
component of reform identified by Malloy, pedagogy that invites students to become doers of mathematics, through discussion of research on teachers’ and students’ roles in creating contexts for learning mathematics with understanding. Mathematics should make sense and be used as one way of understanding about how the world works (Carpenter & Lehrer, 1999). When mathematics becomes contextualized—used for the purpose of making sense—then it can become a language for thought that has relevance to the lives of students. Teachers must create norms for mathematical behavior, provide rich mathematical tasks, and develop instructional discourse to create contexts for student involvement with mathematics. To illustrate pedagogy that centers on making sense of mathematics, I present literature related to problem solving processes and mathematical tasks, developing discourse communities, and creating contexts for student involvement.

**Problem solving processes and mathematical tasks.** In a study of student problem solving, Artzt and Armour-Thomas (1992) present a framework for analysis of small-group problem solving that investigates three major variables—strategy selection, problem-solving processes, and social scaffolding or reciprocal teaching. The framework distinguishes between cognitive processes, or the cognitive actions and strategies used to make cognitive progress (the thinking/doing), and metacognitive processes, or the cognitive actions and strategies used to monitor cognitive progress (choosing and planning what to do, and monitoring progress). The authors identified eight types of problem-solving episodes, and each episode was categorized as cognitive, metacognitive, or both cognitive and metacognitive. *Reading* was categorized as cognitive, *understanding* as metacognitive, *analyzing* as metacognitive, *exploring* as either cognitive or metacognitive, *planning* as metacognitive, *implementation* as either
cognitive or metacognitive, *verifying* as either cognitive or metacognitive, and *watching-and-listening* was not assigned a cognitive level. Twenty-seven seventh-grade students, 11 girls and 16 boys, who attended an urban public middle school in New York City, participated in the study. The students were selected from average ability classrooms from three classes taught by two teachers.

Participating students were divided into six groups. The student group that displayed poor group functioning had the lowest occurrence of watch-and-listen behaviors and was not able to solve the problem. Another group’s poor functioning resulted in the domination of the problem solving process by one group member while all the others watched and listened. The other four groups were relatively functional. They demonstrated higher degrees of metacognitive episodes and more balance between watch-and-listen episodes and cognitive and metacognitive episodes.

The small-group format has the potential to encourage spontaneous verbalizations that allow students to externalize their ideas for critical examination (Artzt & Armour-Thomas, 1992). However, an appropriate or balanced interrelationship between metacognitive and cognitive processes is necessary for successful problem solving. The personalities and attitudes of the highest ability members of the group influenced the subsequent behaviors of other members of the group. High ability members can facilitate group processes by sharing and encouraging exploration of ideas, or they can stifle the group by dominating the process. It is important to note, then, that the monitoring and regulatory behaviors that follow externalization of ideas—questioning, elaboration, explanation, and feedback—account for the ability to solve the problem. If small group settings are used to provide situations for realistic mathematical problem solving and
activities, teachers will need to provide scaffolding to encourage functional group interactions, through modeling, monitoring, and feedback. They should choose problem-solving tasks strategically, keeping in mind the students’ prior experience with problem solving, their knowledge of problem-solving strategies, and their level of fluency with mathematical processes.

The small group setting also has potential to maximize the benefits of discourse (Artzt & Armour-Thomas, 1992). The patterns of behaviors displayed in the more functional of the problem solving groups were identified as similar to those of expert mathematicians. Individual students were able to contribute more often within the problem-solving time period. There were more opportunities for monitoring and regulating explorative behavior. Small group settings provided situations of realistic mathematical problem-solving processes and activities.

The problem solving processes identified by Artzt and Armour-Thomas (1992), reading, understanding, analyzing, exploring, planning, implementation, and verifying, can be likened to the processes involved in self-regulation (see Self-Regulated Learning below). Likewise, strategies that support the development of SRL can be likened to the self-management strategies that students use to solve problems. Self regulation, then, can be seen as “both an aptitude for and a potential outcome of schooling” (Randi & Corno, 2000, pg. 651), which has important implications for student problem solving in small groups and the ways in which teachers conduct classroom discourse for the purpose of making those connections explicit.

A teacher’s choice of mathematical tasks is important for encouraging meaningful student engagement with mathematical concepts. Rich dialogic discourse that encourages
students’ agentive roles in learning can only be expected as the result of students’ engagement with problem-solving tasks that require some degree of cognitive effort. Work done within the QUASAR (Quantitative Understanding: Amplifying Student Achievement and Reasoning) project provides a framework for choosing tasks to support mathematical reasoning (Smith & Stein, 1998). This framework specifies the criteria for categorizing mathematical tasks on the basis of the types of thinking and reasoning that students are required to use at four levels of cognitive demand:

Lower-level demands (memorization)

- Involve reproducing or memorizing previously learned facts, formulas, or definitions.
- Cannot be solved using procedures or time frame doesn’t allow for use of procedures.
- Do not contain ambiguity and involve reproduction of previously learned material.
- Have no connection to underlying concepts or meaning.

Lower-level demands (procedures without connections)

- Are algorithmic.
- Require limited cognitive demand for successful completion and little ambiguity.
- Do not connect to the concepts or meaning underlying the procedures.
- Focus on producing correct answers rather than mathematical understanding.
• Require no explanations or the explanations only describe procedures (how).

Higher-level demands (procedures with connections)
• Focus attention on procedures for developing deeper levels of understanding.
• Suggest pathways to follow that are broader than, but closely connect to, the underlying conceptual ideas.
• Are usually represented in multiple ways. Making connection among representations helps develop meaning.
• Require some degree of cognitive effort.

Higher-level demands (doing mathematics)
• Require complex and nonalgorithmic thinking.
• Require students to explore and understand the nature of mathematical concepts, processes, and relationships.
• Demand self-monitoring or self-regulation of one’s own cognitive processes.
• Require students to access and use relevant knowledge and experiences.
• Require students to analyze the task and actively examine restraints that might limit possible strategies and solutions.
• Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process. (Smith & Stein, 1998, p. 348)

These categories of cognitive demand were used for professional development sessions to guide teachers in understanding children’s thinking in relation to mathematical reasoning (Smith & Stein, 1998). Teachers often did not come to immediate agreement as they worked to sort the tasks into categories. Therefore, through the collegial professional activity, the teachers were not only sorting tasks, but they were also engaging in discourse about students’ mathematical reasoning. When the mathematical tasks are applied in their classrooms, the teachers will have the additional responsibility of developing their own listening skills in order to interpret the ideas that their students express and the relationships of those ideas to their mathematics agendas (Schifter & Riddle, 2004).

*Developing discourse communities.* Artzt and Armour-Thomas (1992) were specifically interested in student-to-student discourse during problem solving in the small-group setting. Rich mathematical discourse can only be expected as a consequence of engagement with rich mathematical tasks. Teachers can take the lead in encouraging both small-group and whole-class discourse related to problem-solving tasks for the purpose of promoting student understanding. One essential difference between discourses that promote understanding and those that do not is the degree to which teachers *press* students for articulation of reasoning and justifications of decisions made while solving problems (Kazemi, 1998).
Kazemi (1998) conducted an investigation of 23 upper-elementary classrooms to examine how teachers press for learning during whole-class and small-group problem solving. *Press for learning* was measured by the degree to which teachers “(1) emphasized students’ efforts, (2) focused on learning and understanding, (3) supported students’ autonomy, and (4) emphasized reasoning more than producing correct answers” (Kazemi, 1998, p. 410). Analysis revealed that the degree to which teachers pressed for understanding determined the amount of learning that occurred in the classroom or, in other words, the higher the press for reasoning and justification, the greater the evidence of mathematical reasoning that results in conceptual learning.

To better understand the differences between high-press and low-press classes, two classes were studied in greater depth. Both teachers valued problem solving and had established similar social norms in their classrooms. However, there were important differences in the quality of students’ engagement with mathematics. The characteristics of sociomathematical norms—norms specifically relating to normative aspects of mathematics discussion—were different in classrooms that led students to understand the mathematics deeply.

Four sociomathematical norms found to guide students’ mathematical activity in higher press classrooms related to explanations, errors, mathematical thinking, and collaboration. In the higher press classrooms, student explanations of their work consisted of mathematical arguments, not simply procedural steps. Students’ errors were used as opportunities to rethink problems and explore alternative solution paths.
Mathematical thinking involved understanding relationships among a variety of strategies, and all students were held accountable for collaborative work and reaching consensus through argumentation (Kazemi, 1998).

Through classroom discourse, students and teachers negotiated what counts as an acceptable explanation or justification of mathematical reasoning. Students came to understand what would count as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant. Additionally, within the classroom discourse, mathematical reasoning became an object of reflection, thus providing opportunity for development of deeper understanding (Kazemi, 1998; see also Cobb, Boufi, McClain, & Whitenack, 1997).

Attention to the implications of sociocultural learning theory has resulted in a trend toward acknowledging the importance of communities of practice for goal-directed learning (Forman, 2003). The role of the teacher in facilitating classroom discourse (as part of individual inquiry, small-group inquiry including explanations and justifications of ideas, and whole-class reflective discussion) may be the most important factor in creating a context for collaborative practice.

With the premise that the call for increased discourse among students in mathematics classrooms is clear, Mendez, Sherin, and Louis (2007) sought to examine the ways in which teachers develop mathematical discourse communities. They collected data over the period of one year in the form of videotaped classroom observations, field notes, and teacher observations. Participants included one teacher-researcher (Louis, third author) and two of his eighth-grade mathematics classrooms for a total of 50 students.
The authors explored students’ competencies with communicating mathematical ideas and changes in the ways that the teacher attended to students’ ideas to create a discourse community.

The authors present three perspectives on classroom discourse. The first is the teacher-researcher’s thoughts, which followed his participation in two case-writing workshops focused on discussion of a teacher-identified teaching dilemma (the case), narrative writing of the case, and feedback. The other two perspectives come from the remaining two authors’ thoughts as viewed through two research-based lenses—(1) robust mathematics discussion (RMD), which examines student moves in discussion in relation to mathematics content (substance) and discussion forum (process); and (2) professional vision for classroom discourse, which examines the way the teacher paid attention to and reflected on students’ ideas raised during discussion.

RMD and teacher professional vision provide lenses for teachers to examine, clarify, and work toward goals for creating a discourse community (Mendez et al., 2007). Mathematical dimensions of RMD provide foci for teacher awareness of how many representations students present on a given topic, whether or not discussion goes beyond specific and concrete topics toward generalization, and the level of reasoning that is evident in discussion. Discussion dimensions of RMD provide foci for awareness of students’ active involvement in discussion, the manner in which they enter discussions, and the ways in which they respond to and build upon comments made by others.

Teachers who are attempting to move toward more student-centered learning may have difficulty making shifts in their own practice to encourage dialogic discourse. In the case presented by Mendez et al. (2007), analysis over the year revealed a shift in the
teacher’s professional vision that included new ways of attending to student thinking. His focus on understanding the meaning of students’ ideas at a detailed level increased, he elicited follow up comments on students’ ideas, and he probed for further reasoning and additional ideas.

In one particular discussion that was identified as representing change in student communication, analysis of student involvement showed varied results. Students took more control of the discussion by making most of the comments, but that discussion included contributions from fewer than half of the students. Additionally, 30% of the comments were new ideas, 42% of student turns were coded as responses, meaning that they agreed or disagreed with a previous comment without further elaboration, and 28% were coded as elaboration on students’ ideas (the highest coding level). The authors concluded that the discussion dimension of RMD in this example illustrates the mixed nature of the discourse, which is complex rather than uniformly robust.

RMD as described above can take place in within small-group work and well as whole class discussion. During whole-class discussion, students may be exposed to a wider range of ideas and strategies for problem solving and learning. Such whole class discussion provides a forum for collective reflection, which may be considered a type of communal support for the development of skills and knowledge. Related to collective reflection, Cobb et al. (1997) conducted a yearlong study to explore the relationship between the practice of classroom discourse and the mathematical development of 18 first-grade students. Episodes of classroom discourse were chosen to illustrate reflective

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discourse and collective reflection. Their focus on reflective discourse and collective reflection highlight the shifts that must occur to transform previous actions into objects of reflection.

The researchers identified three main issues in student mathematical development through classroom discourse: the establishment of an inquiry microculture (in this case, mathematical enculturation and development of mathematical disposition), the sequence of activities, and the nature of the classroom discourse. Observations indicated that subsequent to mathematical activities, students’ ideas became objects of discussion during classroom discourse. Because the opportunity arose to participate in discussion, reflection was supported by the discourse. Additionally, students themselves contributed to the shifts that allowed ideas to become objects of discussion. Students reflected on and then reorganized their ideas from prior activities, therefore, support is bidirectional—discourse supports students’ reflection, and students’ reflection supports discourse and, consequently, students’ mathematical development.

Teachers must take proactive roles to initiate the shifts that result in development of mathematical dispositions (Cobb et al., 1997). Further, notational schemes, whether initiated by the teacher or students, facilitate the collective reflection that grows out of students’ activity. Mathematical disposition develops indirectly as a result of participation in collective reflection. Individual students contribute to the development of discourse that supports and sustains collective reflection. However, even as individual participation is a condition for the possibility of mathematical learning, it does not inevitably result in individual conceptual reorganization.
The actions that the teacher takes in supporting classroom discourse may be thought of as two distinct but related roles. In the first role, the teacher takes part in classroom discourse by stepping in, supporting students’ competence at engaging in discussion of the mathematics, listening to ideas, and asking questions to facilitate understanding (Rittenhouse, 1998). This role may include questioning and revoicing of student ideas to clarify mathematical ideas and terminology, thus making implicit ideas explicit and open to all members of the discourse community (Forman & Larreamendy-Joerns, 1998). This first role, which differs from traditional discourse patterns of asking questions in order to evaluate student responses, also may include contributions from the teacher that are based on his or her personal insight.

In the second role, the teacher steps out, becoming commentator on topics related to classroom rules and norms for classroom discourse (Rittenhouse, 1998). This may be thought of as a more didactic role where the teacher’s greater experience and expertise is used to lead students in becoming more competent participants in classroom discourse. In the role of commentator, the teacher is able to highlight particular ideas related to keeping classroom discourse productive, which, in mathematics classes, might be thought of as talking about talking about mathematics (Cobb et al., 1997). Talking about talking about mathematics helps set expectations for student talk, and may include such topics as what it means to give explanations, reasons, and justifications of ideas and how to participate in argumentation.

In another study of classroom discourse, Wood (1999) investigated teachers’ roles in creating contexts for discourse and collaboration with focus on engagement in the resolution of disagreements through argumentation. Use of the process of argumentation
can be seen as engagement in the work of real mathematicians. The central activity of mathematicians is argumentation within their communities of practice (Forman, 2003). Mathematicians must convince others of the validity of their ideas and be persuaded by the arguments of others. Therefore, teacher creation of contexts for student participation in mathematical argumentation can be seen as putting students in authentic roles of doing the work that “real” mathematicians do. *Argument* was defined as a discursive exchange among participants for the purpose of convincing others through the use of certain modes of thought (Wood, 1999). *Argumentation* was defined as an interactive process of knowing how and when to participate in the exchange. *Context* was defined as a situation that can be identified by distinct patterns of interaction and discourse.

To investigate interrelationships among context, student learning, and the role of teaching, Wood studied one second-grade teacher and the students in her classroom. The teacher was one of eighteen teachers who had previously participated in a professional development research study. Data included 50 lessons videotaped over the course of 18 months. Analysis of the data indicated a pattern of interaction and discourse that always involved a challenge: A student (explainer) provided an explanation of a solution; a listener issued a challenge, with or without a reason for disagreement; the explainer offered justification; the challenger either accepted the justification or continued to disagree by offering further explanation or rationale; the explainer offered further justification. Other listeners sometimes contributed during this process, and the exchange continued until members of the class were satisfied that the disagreement was resolved.

Listeners were expected to examine and evaluate the reasoning of others and to participate in the process of resolving disagreements. A distinction was made between
criticism that was personal and criticism of mathematical ideas. This issue was particularly important for students as they developed personal ownership of their ideas. Students had to learn how to disagree in order to develop shared meaning within the context of argumentation.

The explicit manner by which the teacher initiates and establishes expectations for participation creates the condition for private thought to become available for public examination (Wood, 1999). Although establishing norms for explaining are essential, the expectations established for listening are central to establishing a context for argumentation. *Showing and telling* isn’t enough in mathematics education. Teachers must guide students’ understanding by providing opportunities to construct meaning. The role of the teacher in establishing context is of particular significance and requires a proactive approach. While establishing the expectation for explanation is essential within the context, establishing expectations for listeners is equally important. When expected behaviors became routine, students were able to direct their mental activity to making sense of the mathematics rather than finding their place within the social setting.

In order to explore involvement in mathematics, Turner et al. (1998) examined 42 fifth- and sixth-grade students and their teachers. These students were randomly selected from seven classrooms at three elementary schools in a small, mostly White, middle-class town in rural Pennsylvania. The results of the study provide descriptions of the quality of students’ experiences during mathematics lessons as they are related to patterns of mathematical instruction.

Involvement was defined as a complex interaction of student cognition, motivation, and affect, and as a psychological state concerned with the quality of
experience during learning (Turner et al., 1998). The authors assumed that cognitive engagement is a prerequisite to involvement and they attempted to describe a momentary or situationally dependent quality of experience that participants seek to repeat. Involved learning was described as including focused concentration, attention, and deep comprehension. It could also be characterized as including positive affect, goal clarity, and intrinsic motivation.

Student rating of challenge and skill indicated higher involvement in three of the classrooms. Students in two of the three high-involvement classrooms tended to report significantly more experiences of flow. Flow is “a subjective state that people report when they are completely involved in something to the point of losing track of time and of being unaware of fatigue and of everything else but the activity itself” (Csikszentmihalyi, Rathunde, & Whalen, 1993, p. 14). Students who reported more experience of flow were also more likely to rate their experiences as involving and intrinsically interesting. Because of the high level of challenge in the third high-involvement classroom, students were more likely than others were to report feelings of anxiety. Students in low-involvement classrooms were more likely to report apathy or boredom.

In the three classrooms that were rated as more involving, teachers’ instructional approaches shared high rating of challenge matched with the students’ reported skill levels. Two of the three teachers demonstrated high frequencies across all three categories of scaffolding. In the lower involvement classrooms, teachers’ instructional approaches indicated patterns of higher frequency of non-scaffolding categories. Discourse could be characterized as frequent use of procedural talk and IRE sequences.
Whole-class teacher discourse did not indicate work toward student autonomy or taking responsibility for learning. These teachers used more extrinsic supports, and the portion of teacher talk within the scaffolding categories was low.

Scaffolding of classroom instruction through whole-class discussions helped to create a context for and support of student involvement. Teachers in high-involvement classrooms demonstrated a greater push for understanding, provided for autonomy, modeled successful strategies, attempted to relieve students’ frustration, and asked for explanation and justification of mathematical ideas. Errors were viewed constructively—as opportunities for learning. High-involvement teachers demonstrated respect for and interest in the mathematics, presenting mathematics as inherently interesting. They encouraged students to value and enjoy the process involved in learning and doing mathematics. The authors concluded that involvement was socially constructed through the development of mathematical dispositions as teachers provided motivational, emotional, and cognitive support.

Creating contexts for equitable teaching and learning. From an interactionist perspective, meaning is achieved through social interaction and interdependence, and therefore serves a communal function. Students must learn to use school discourse, often referred to as an academic register, in order to survive in school (Sierpinska, 1998). For Hispanic/Latino/a students who are English language learners, and whose culture may include strong communal values, the need to learn a new academic register may demand special attention to the nature of social interaction in the classroom.

In an article presenting implications for classroom practice gained from several studies on mathematics learning and Latino/a students, Khisty (2002) asserts the
importance of language in learning mathematics by stating that the majority of Latino/a students have some affiliation with Spanish regardless of their proficiency in the language. The Latino/a’s environment is bilingual; therefore teachers need to understand the linguistic strengths and experiences that children bring to school and use those strengths and experiences in positive ways.

Khisty observed that teachers who worked with Latino/a students and were effective in teaching mathematics firmly believed that their students were capable of doing advanced work in mathematics regardless of their past experience with mathematics and their home background. The teachers were also aware of two dimensions of language proficiency: the basic interpersonal language that is used in social conversations and the cognitive academic language that is used for school work. Proficiency in one does not necessarily indicate proficiency in the other. Students often needed support in building academic language even in cases where their basic interpersonal language may have appeared to be reasonably fluent. In describing one case, Khisty (2002) related that through the combination of bilingual and second-language acquisition theory with principles of reformed mathematics teaching, teachers successfully used mathematics as a context for teaching English at the same time as teaching it as a subject in itself.

To ensure that instruction in a second language is as comprehensible as possible, Khisty recommends that teachers minimize the need for learning strictly by listening, write words as they are spoken, contextualize instruction through the use of models, manipulatives, drawings, and other visual aids, and have students physically act out concepts or problems. Additionally, teachers should enunciate clearly and avoid
ambiguous pronouns. During instruction, a teacher’s speech serves two purposes (Khisty, 2002)—to guide students’ thinking and to provide a model of using the second language for mathematics.

In an effort to improve understanding of equitable and successful teaching of mathematics, Boaler and Staples (2008) conducted a study of 700 students in three California high schools, 1) Railside, an urban school with ethnically, linguistically, and economically diverse students, 2) Glendale, a coastal school with a primary White student body, and 3) Hilltop, a rural school with primarily White and Latino/a students. Railside School demonstrated the greatest gains in achievement. Railside School was described as having heterogeneous classes and reform-oriented teaching. The authors identify three sources of success at Railside: 1) the department, curriculum, and timetable, 2) group work and complex instruction, and 3) challenges and expectations. This discussion of contexts for learning mathematics with understanding will be limited to describing the three sources of success of Railside.

The first source of success was found to be the teachers’ curriculum planning. The teachers of the mathematics department worked very closely to develop curriculum over the years of the study. They followed a block schedule with 90-minute courses over a semester rather than a year. Because the algebra curriculum was designed around key concepts, they drew questions from a variety of published sources. The department only used problems that they considered to be groupworthy, which illustrated important mathematical concepts, suggested use of multiple representations, required the collective resources of a group in working on tasks, and had multiple solution paths.
The second source of success was a particular method of group work using the complex instruction approach, which was designed to respond to social and academic status differences within classroom. One aspect of complex instruction is multiple ability treatment, which contrasts with more hierarchical systems. The idea behind multiple ability treatment is that tasks will require a variety of abilities, and, while not everyone will be good at all of them, everyone should be good on at least one—when there are more ways to be successful, more students will be successful. One practice with the complex instruction approach was encouragement of justification. Through student justification of ideas, all students were exposed to mathematical ideas that might not otherwise come up in discussion. A second practice within complex instruction is the assignment of roles, such as facilitator, captain, recorder, and resource manager. The idea behind assigning roles was that each student would have an important part to play within the group without which the group could not function. Assigning competence, a third practice within complex instruction, refers to actions a teacher takes to raise the status of a student who might otherwise be of lower status within a group. Such actions might include praising something a student has done, in a group or to the whole class, because of intellectual value, or asking a particular student to present an idea with knowledge that the student is likely to be successful in doing so. Teachers also emphasized that students should be responsible for each others’ learning. Students learned to talk not just in terms of their own learning, but also in terms of benefits to their classmates.

The third source of success was challenges and expectations. Student groupings were heterogeneous, but no students were exempted from high expectations. Teachers challenged students through questioning and push for understanding and emphasized
effort over ability. They were explicit in discussing learning practices, particularly within groups. They stopped students at various points to highlight valuable ways in which they were working, to look back at the original problem, and to point out specific learning strategies.

The three sources of success identified by Boaler and Staples (2008) were enacted with the idea that the teachers and students in multicultural and multilingual classes have a responsibility to work together for all students to succeed. Mathematics lessons are often regarded as neutral in relation to culture and social awareness. The authors suggest that all students (from a variety of cultural backgrounds) have something to contribute to the promotion of equity (an aspect of social awareness) in mathematics classrooms. They concluded that the teaching practices at Railside School, in addition to supporting students’ gains in achievement, promoted respect and sensitivity among students.

Summary

Teaching mathematics with and for understanding involves moving away from traditional school practices that emphasize memorization and rote application of algorithmic procedures. Both the NRC and the NCTM have called for changes in classroom practices with the goal of helping students to develop mathematical proficiency. NRC (2001) identifies five interwoven strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive dispositions. A combination of NRC’s model of mathematical proficiency and the NCTM Process Standards, problem solving, reasoning and proof, communication, connections, and representation, provides guidance for how to help students develop positive dispositions toward investigation, formulate good questions and
solve mathematical problems, apply mathematical procedures strategically, communicate reasoning, make connections among mathematical concepts, and represent mathematical ideas in a variety of forms.

Research studies show that teachers can purposefully establish norms for mathematical behavior, provide rich mathematical tasks, and develop instructional discourses to create environments where students authentically engage in mathematical thinking and doing of mathematics. Instructional discourse plays an important role in reflection on mathematical activities when the activities and students’ mathematical contributions become objects of reflective discussion.

In addition to creating contexts that support learning mathematics with understanding, which is one concern of the present study, research indicates that instructional discourse plays a role in creating contexts that allow negotiation of students’ status, possibly through assigning competence, within the mathematics community (Boaler & Staples, 2008). Classroom discourse can be used to address issues of inequity and has the potential to promote respect and sensitivity among students. The nature of classroom discourse can be especially important for non-mainstream students groups that historically have not thrived in traditional school settings. To survive in school, all students, but especially those within cultures where historical inequities persist, “must notice the differences between discourses and put [their] efforts into becoming a practitioner of the new discourse” (Sierpinska, 1998, pg. 54).

Culture, Positioning, Language, and Power

In general, the educational system in our country has not done a good job of educating poor and minority students (Malloy, 2004; National Council on Education and
the Disciplines, 2001; Schoenfeld, 2002). Many studies show an achievement gap between White and minority students that has persisted for generations. Low achievement for non-White students often results in failure to thrive within the system. Much is known about meeting the needs of students from diverse backgrounds, but our schools typically function in the same way as they have for generations, retaining methods that have historically been sufficient for the relative success of middle-class White students. Considering rapidly changing world conditions, it is questionable whether or not maintaining the status quo even benefits students from the White middle class. Consequently, providing educational opportunities for diverse student populations includes doing more to identify the changing needs of all of our students.

Decisions made for instructional settings are equity issues because of the need to ensure provision of opportunities for all students to learn with understanding that results in their ability to use mathematics effectively in life, including in the workplace. Mathematics is a content area that many believe holds the key to future success. While the ultimate truth of that may be somewhat exaggerated, it is well known that students who are not successful in mathematics do not have as many opportunities open to them as those who are successful. Furthermore, we cannot assume that traditional approaches to mathematics education are meeting the needs of any of our students. Teachers must redefine what it means to be successful in mathematics classes, and then provide the opportunities, challenges, and responsive support for all students to feel some level of success, but especially to engage students who have been turned away by the mechanical modes of learning mathematics that prevail in classrooms in the United States.
Because of persistent patterns of underachievement for cultural groups that have traditionally been outside mainstream United States society, a mythology has built up around “problem” children, such as the poor and racial and ethnic minorities, essentially creating a deficit perspective on the issue of achievement (Khisty & Viego, 1999). The myth tells a story of lack of parental support, poor neighborhoods, language deficits, and school buildings that are falling apart. However, an alternate view recognizes public-policy and larger societal attitudes as the context of the achievement gap. The achievement gap tells us not about poor and minority children, but about a broader society “that condones high levels of child poverty, borne disproportionately by children of color, and significant segregation in its public schools” (Books, 2007, pg. 16).

Despite the widely accepted mythology surrounding lack of achievement for low SES and culturally diverse children, stories can be told of teachers who work successfully with African American students, Hispanic students, and other students of color. These teachers operate under the assumption that “poor social and economic status are political disadvantages, not educational deficits” (Khisty, 2002, pg. 33). Educators need to address issues of inequity in our schools, which may be thought of as arising from political disadvantages, and find ways to engage students so that learning becomes relevant in their lives. One approach to exploring culturally relevant pedagogy is to take a critical stance toward teaching and learning that deconstructs power relations as they now stand and opens up possibilities for reconstruction of self-theories. Sociohistorical models of learning acknowledge not only the historical aspects of socially constructed knowledge but also the present situatedness of each learning context and an envisioned future.
Therefore, a combination of critical theory and sociocultural/historical theory may support addressing issues related to making schooling relevant to historically disenfranchised groups of students.

To explore the issues involved in addressing low achievement and disenfranchisement of low SES and culturally diverse students, I present three sections of discussion. The first section, *Pedagogies of Liberation*, is a presentation of sociohistorical theory and pedagogy developed by Paolo Freire and discussions that have developed around his work. Freire’s *Pedagogy of the Oppressed* (2003) has inspired the development of alternative pedagogical practices in a variety of school contexts that use ideas of transforming experience through the recreation knowledge. I describe one such teaching experiment related to Freire’s work. In the second section, *Language, Knowledge, and Power*, I present the theoretical perspectives on power relationships of Bakhtin and Foucault that support the use of discourse for the disruption of power relations, and research on classroom discourse based on a Bakhtinian perspective. The third section, *Positioning and Possible Selves*, goes beyond themes introduced in the other two sections to look more specifically at development of individual and ethnic identity and ways that students might experiment with alternative roles and identities.

*Pedagogies of Liberation*

In their desire for guidance in transforming the educational experience of people who have been historically oppressed, some educators and researchers have turned to the work of Paolo Freire. In Freire’s work, the word—sign, text, or language—is essential to concepts of teaching and learning. It is through reading that learners are able to read a “previous reading of the world” (Freire, 1998, p. 18), or to understand their existence
within an historical context. Freire refers to the student as the subject of learning, and studying as an “uncovering” that requires risk taking in order to create and re-create. An understanding of place within the world, or the historical context of existence, is created through studying the word. In other words, through deconstructing the present, we are able to envision change and work toward future possibilities.

The concept of creating and recreating, especially with the student as subject, is of utmost importance in the idea of creating a pedagogy of freedom from oppression, inequity, and/or self-constraints. Teachers and students must be able to deconstruct before they reconstruct. They must understand who they are as in the social and political contexts of power and how they came to be who they are in those contexts, in order to re-create themselves as subjects. Freire’s position on human agency is that “men and women are human beings because they are historically constituted as beings of praxis, and in the process they are capable of transforming the world—of giving it meaning” (Freire, 1985, p. 151). The capacity to transform the world enables human beings to envision and create new meanings, perhaps more equitable meanings. A being that recognizes history is able to cultivate consciousness, particularly reflective consciousness. It is then through authentic dialogue, a subjective relationship between the self and others as knowing subjects, that education is able to serve a liberatory purpose.

Many questions have arisen about Freire’s pedagogy of the oppressed in regard to race, particularly as it might be applied to the oppression experienced by people of African descent. Race is historically an ill-defined construct in this country and continues to remain so as a categorical definition of human beings (Ladson-Billings, 1997). Ladson-Billings warns that we continue to use categories of “White” and “non-White”
without adequate analysis of what it means to be “White” in society. Historically, Whiteness has been constructed through physical markers such as skin color, hair texture, and facial features. Place in society, class or social hierarchy, was determined by degree of Whiteness. Even today, as many African American families are middle class in terms of economic stability, African American students continue to achieve at lower levels than their White counterparts (Ladson-Billings, 1997). Therefore, it is not strictly a matter of class that keeps African American students “in their place” within educational institutions.

Similar to Ladson-Billings’ argument, Haymes (2002) asserts that Freire’s pedagogy of the oppressed lacks a situated understanding of the nature of the problem of racial oppression. Freire’s model of education arose from political and social analysis of the living conditions of the working class people of Brazil. His analysis has its roots in the socialist tradition of working class education in nineteenth-century Spain that extended into Latin America (Haymes, 2002). Consequently, Freire’s model of liberatory education is considered a class-based pedagogy. Haymes argues that a class-based pedagogy cannot adequately address issues of race.

Freire’s theory emphasizes the need to transcend the individual experience in order to universalize it, but the experience of racism does not “universalize” in the same way as classism. In response to questions directed to him about race, Freire conceded that the problem of racism cannot be reduced to one of social class, but he maintained that racism cannot be fully understood without class analysis. Haymes (2002) suggests that Freire lacks an understanding of that particular relationship—the relationship between race and class. The reality of African Americans in an anti-black society is that class is
lived through race (Haymes, 2002); therefore, Freire’s theoretical framework for pedagogy can be seen to ignore the view that human beings are situated beings.

Black workers construct personal histories from their particular situations of existence as Black workers. Their history is based in conditions of slavery that situated them in machine-like roles that were presented as their “natural condition.” Their bodies were considered “thing-like,” so a question of self-actualization might be “What am I?” rather than “Who am I?” It is this relationship between racial bodily-consciousness and subjectivity that is left unaddressed in Freire’s pedagogy of liberation. To be relevant to people of African descent, a pedagogy of the oppressed has to engage the existential struggles against being reduced to merely physical bodies (Haymes, 2002). A body without thought is not capable of presenting an “I” self, and therefore doesn’t perceive agency, or capacity for becoming.

In a written response to Haymes’ propositions, Worsfold (2002) suggests that the key to actually carrying out a liberatory pedagogy that includes issues of race lies in the development of self-respect. A breakdown of beliefs about one’s behavior toward oneself initiates the realization of the need for an alternative self-concept. That realization must then be followed by intellectual imagination. Based on the knowledge gained through self-actualization, individuals are able to imagine possible selves.

In 1989, a group of legal scholars (Conference on Critical Race Theory) proposed a critical race theory to examine the reality of racial oppression (Delgado, 1990). This critical race theory includes themes of “naming one’s own reality,” realization of knowledge as power, critical examination of myths and stories used to justify racial subordination, and ways in which legal tools and thought structures impede law reform.
The application of critical race theory in education would include equivalent themes of naming one’s own reality, knowledge as power, and deconstruction of oppressive myths and structural determinism that impede reform. Unlike Freire’s pedagogy of the oppressed, critical race theory acknowledges the importance of voice in naming one’s own reality. Delgado (1990) justifies the idea of a non-White voice, even against arguments of the impossibility of a monolithic voice for any group of people, with the suggestion that people of color speak from a base of experience structured in racism. That structure gives their stories a commonality that warrants the term “voice.” Both Ladson-Billings and Delgado have called for discourses that are no longer “race neutral,” making race problematic and open to critique while giving voice to the reality of peoples’ lived experiences.

Drawing on the view of liberatory education as envisioned by Freire and “the historical struggles of African Americans for emancipator education” that predate Freire’s work, Gutstein worked to develop a pedagogy of liberation for his Latino/a students (Gutstein, 2007, pg. 423). The major goal was to create conditions for students’ development of agency where students would be able to sense themselves as subjects in the world. His two-year practitioner-research study with 26 middle school students took place in a Latino/a neighborhood school in Chicago. Gutstein identified himself as a White teacher attempting to learn about student agency by adopting the ethics of African American teachers that relate to the struggle for freedom.

During the two years of the study, Gutstein used a commercially produced mathematics curriculum based on the philosophy of teaching mathematics for understanding about 75 to 80% of class time. The curriculum, which follows the
guidelines of *PSSM* (NCTM, 2000), was designed to make use of realistic contexts and to connect mathematical content both across mathematical content strands and to the real world. The remaining class time was spent on projects that Gutstein developed to engage students in using mathematics to investigate racism and other injustices. The projects varied in length, from two days to two weeks. Over the span of the two year study, students worked on 17 such projects.

Based on his research, Gutstein suggests two possible starting points for developing student agency in mathematics. First, he suggests teaching students to use analytical tools, such as mathematics, to learn about social realities that include contradictions. A second approach is to start with students’ own lives and then link their mathematical knowledge production to their participation in larger social movements.

Even as their learning context was focused on social justice in combination with an NCTM-aligned curriculum, participating students achieved “conventional” academic success, as measured by standardized test and high school entrance tests. Gutstein attributes most of the students’ mathematical learning to the primary curriculum, but he does not dismiss the role of the real-world projects. Based on follow-up surveys and interviews conducted with students during their high school years, Gutstein makes the provisional claim that teaching mathematics for social justice in an urban public schools can make a difference in students’ lives outside the classroom.

Gutstein’s work has particular relevance to the present study because his attempt to realize liberatory principles took place in middle-school mathematics classrooms, which “are rarely the site of such attempts” (Gutstein, 2007, pg. 425). In addition, it draws together African American and Freirean theoretical perspectives on liberatory
pedagogy to examine a Latino/a population, a cultural group for which there is relatively little research literature available but whose presence in urban schools is increasing.

Gutstein gave prominence to the words of his students in carrying out and presenting the analysis of his data. A major goal of Gutstein's study was "to create conditions for students to develop agency, a sense of themselves as subject in the world" (Gutstein, 2007, pg. 420). The emphasis Freire places on the word in deconstructing in order to reconstruct ourselves as subjects suggests that teachers must attend to how students are developing agency while learning. In terms of research, discourse analysis might help to uncover ways that students develop agency while learning, including ways in which teachers create contexts that support the development of agency.

*Language, Knowledge, and Power*

Instructional discourse has been shown to be critical in creating classroom contexts for learning mathematics with understanding. In this section I present theoretical perspectives of Bakhtin and Foucault that support the use of discourse for disrupting power relations. The research presented is based on a Bakhtinian viewpoint on power relations within discourse. Understanding the use of language for development of agency in power relations will enhance understanding of possibilities for supporting student agency through classroom discourse in mathematics classes.

Bakhtin (1986) developed a pragmatic theory of language use—he believed that language is dialogic and learned through contextualized social interaction. Addressivity is important in Bakhtin’s conception of the speech utterance. When an utterance is formed, a response is presupposed—a presupposition that may influence the utterance in anticipation of approval or objection. The speech act, then, accounts for information such
as the addressee’s familiarity with a particular situation, specialized knowledge, biases or prejudices, and/or relational sympathies and hostilities. Inquiry into addressivity in traditional public education is important to understanding the relationship between student and teacher. Traditional forms of public education envisage learning in terms of a regulatory text, positioning the teacher as the holder of the knowledge and students as objects who do not yet know (Dowling, 2001). This conception of learning denies the dialogic and discursive character of communication and learning as well as the learner’s agency in the construction of knowledge.

The monoglossic language of a class or social position is potentially limiting as it constructs its own set of understandings that restrict imagination (Bakhtin, 1986). Monoglossia, then, can be seen as being dogmatic—one can think only what one's language allows one to think (Lye, 1998). Polyglossia, or the debate between languages, and heteroglossia, or the disagreement between voices and dialects within a language, provide arenas for debate between ways of thinking. All language use comes from a particular point of view, is used in a particular context, and is directed to a particular audience. From a Bakhtinian perspective, then, language use is ideological, contextual, and dialogic. The notion of ‘otherness’ is of critical importance because it emphasizes the role of difference in dialogue.

Given that self-other relationships and differences in understandings motivate dialogue, a Bakhtinian conception of the contextual and ideological nature of language and dialogue is useful in thinking about formulation of a pedagogy that addresses inequities in power relations in school settings. Knowledge and ideologies are socially
constructed; therefore the agentive role of language and dialogue in the formation of power relations, social identity, and patterns of beliefs must be better understood within the social context of schooling.

Foucault’s writings on knowledge and power provide another view on ways in which language is used to create disparities in the distribution of power throughout societies. Foucault proposes revisiting the character of discourse as an event in order to restore the “dangerous” elements that can lead to changes in the balance of power. The word *power* leads to misunderstandings with respect to its nature, form, and unity (Foucault, 1976/1999). Foucault describes the nature of power as “a multiplicity of force relations imminent in the sphere in which they operate” (p. 475). Its form is the process that transforms, strengthens, or reverses force relations. The question of unity in force relations refers to the support that these force relations find or to the disjunctions and contradictions that isolate them from one another.

Foucault believes that opportunity for the exercise of power does not exist solely in a central power. Force relations are not fixed, but rather they are built upon a supple foundation that allows for states of power that are local and unstable. Power is everywhere because it comes from everywhere (Foucault, 1976/1999); therefore, power can come from anywhere. Because it is unstable, power can be manipulated and used strategically. Complex strategic situations are present in every relationship and in every society by virtue of the impossibility of identical experience. Mechanisms of reproduction of power, which are inherent in knowledge relationships, are always at play throughout society as a whole, and, likewise, in our schools. Strategic situations (leading
to imbalance of power) that are explicit at a particular level become connected to one another, accepted as “the way things are,” and end up forming comprehensive systems of power relations.

Foucault (1976/1999) suggests that it is within discourse that power and knowledge are joined together. Knowledge is never pure because it is conditioned by beliefs, values, and practices of particular cultures and societal groups. Discourse is made up of discontinuous segments whose tactical function is neither uniform nor stable. Therefore, the complex and unstable quality of discourse can lead to instabilities in power relations. Awareness of discourse as an instrument of knowledge acquisition and as an instrument of positioning of participants in power relations can lead to conceptualization of discourse for specifically instrumental purposes—it can be an instrument and an effect of power; likewise, it can be a hindrance, a point of resistance, and a starting place for development of an opposing strategy (Foucault, 1976/1999).

Using a Bakhtinian conception of the nature of language as a theoretical background, Nystrand, Wu, Gamoran, Zeiser, and Long (2003) investigated the structure and dynamics of unfolding classrooms discourse in 16 Midwestern junior high and middle schools in eight Midwestern communities. The researchers analyzed discourse in English and Social Studies classrooms “in terms of the (a) antecedents and (b) consequences of discourse participant ‘moves’ as they (c) affect the inertia of the discourse and accordingly structure unfolding discourse processes” (Nystrand et al., 2003, p. 135). This quantitative study used event-history analysis to compute probabilities of the effects of discourse moves on subsequent patterns of discourse, and to measure and contrast conditions that lead to dialogic discourse.
Typical classroom instruction can be characterized as consisting mainly of recitation, where the teacher controls the content and patterns of classroom discourse (Nystrand et al., 2003; Nystrand, 1997). The purpose of such discourse is for students to demonstrate recall of information that had been previously assigned, or that which is expected to be already known. Dialogic interaction, which is far less common, occurs in the absence of recitation-type questions, when students are involved in non-scripted exchanges of ideas. Students have greater agentive roles within dialogic discourse.

Nystrand et al. (2003) found that specific types of teacher and student “moves” lead to dialogic discourse. Dialogic discourse occurred following “dialogic bids” from teachers in the form of authentic questions and uptake of students’ comments. Student questions, when taken up for discussion, were especially strong in initiating episodes of dialogic discourse. Dialogic episodes were characterized by engaged student questions and the absence of teacher recitation questions. The teacher’s role was to keep the discussion going while students made substantive contributions in the form of observations, conjectures, argumentation, and reasoning.

Analysis also revealed that authentic questions and uptake of student comments for discussion occurred with similar frequency in both high- and low-tracked classes (Nystrand et al., 2003). However, the pattern and clustering of authentic questions and uptake are less likely to occur as a sequence in low-track classrooms, and students in low-track classrooms did not ask questions that initiated dialogic episodes. This is an important finding in light of concern for issues of equity in mathematics education and the persistence of disparities in mathematics achievement and further educational opportunities for high- and low-track students.
Nystrand’s research and the theoretical perspectives of Bakhtin and Foucault support the notion that the use of dominant discourses as social practice within the institution of school must be problematized and politicized to reveal the hegemonic nature of traditional schooling practices and to form strategies that lead to development of agency, particularly for groups who have been historically marginalized or disenfranchised. Freire (2003) used discourse to deconstruct myths that justify oppression and to form resistance and opposing strategies with the Brazilian working class. Freire suggests that the belief systems of oppressed people often keep them in positions of oppression. This is much like Bakhtin’s conception of the dogmatic character of monoglossia—people’s imaginations can be limited by language. We must develop language that encourages imagination.

Deconstruction of myths through classroom discourse, specifically demystification of power forces, can lead to the transformation of power relations and to the envisioning of fresh possibilities for reconceptualization of self. Nystrand’s research in combination with the theoretical perspectives of Bakhtin and Foucault supports the notion that discourses, as social practice within the institution of school, might be used to form strategies that support the development of agency, particularly for groups who have been historically marginalized or disenfranchised. Analysis of classroom discourse in mathematics classes might provide insight into the ways that discourse actually unfolds during teaching and learning mathematics so that more can be known about the use of discourse for changing power relations in classrooms and supporting the development of student agency and reconceptualization of self.
Positioning and Possible Selves

The literature on teaching mathematics with understanding emphasizes pedagogy that puts students in the position of doers of mathematics, and suggests that classroom discourse can be used to create contexts that support students’ developing identities as doers of mathematics. Critical race theory suggests critically examining myths and stories that have been used to justify racial subordination, giving voice to one’s own reality, and engaging in self-actualization toward being able to image possible selves—one’s sense of reality cannot be changed if one cannot envision alternate possibilities. And theories of power as it relates to knowledge and language suggest that knowledge and ideologies are socially constructed; therefore language and dialogue have potential to change power relations and, consequently, social identity and patterns of beliefs. All three point to self-knowledge in relation to social contexts as fundamental to the development of agency. In this section I discuss concepts of self within social contexts and then look at research on how notions of self-concept are complicated by theories of ethnic identity.

In defining self during adolescence, peer groups take on greater importance than at any other time in a young person’s life. Early adolescents are starting to test their ability to step beyond the world as defined by their parents. Through exploration of possible selves, adolescents organize their interpretations of the world. They seek validation of their evolving identities, and their behaviors are typically in service of confirming those identities. “The more important the identity, the more it is in need of the validation” (Markus & Nurius, 1986, pg. 956). Validation or lack of validation from peers can influence the choices that an adolescent makes—which possible selves are worth further pursuit? However, self-concept is not a final goal or a resting point; it is not
a place where one ends up as an adult, in mid-life, or when finished growing. All people are continually in the process of modifying self-concept or identity, however subtle those changes may be. Adolescence is a time of particular urgency in developing a concept of self as adolescents are seeking to become increasingly independent of their families and to gain agency within the larger world.

Davies and Harré (1990) have written about theories of self in terms of positioning in the context of discourse. They define conversation as “joint action for the production of determinate speech acts” (p. 45), and discourse as “a multi-faceted process through which meanings are progressively and dynamically achieved” (p. 46). Positioning refers to the ability to take different stances within discourses. People can be characterized by both continuous personal identity and by discontinuous personal diversity, and they are able to make choices, or variably position themselves, within actual conversations. There is an aspect of self that is involved in a continuity of a multiplicity of selves (Davies & Harré, 1990). Analysis of this self to multiplicity of selves relationship may serve to guide teaching as the practice of freedom. Can an awareness of self-positioning help to understand the processes that signify people as powerless, or lacking agency, within dominant discourses? What practices within discourse might serve to reverse feelings of powerlessness and how might knowledge of those practices be used proactively meet the needs of people who have been historically marginalized?

Issues of education as the practice of freedom have been addressed above in discussion of the work of Freire. Theories of language in relation to knowledge and power can further strengthen understanding of the role of discourse in culturally relevant
and socially just pedagogical practices. Positioning within discourses can provide a means for playing out possible roles and experimenting with possible selves. Notions of self-efficacy, especially in mathematics, are extremely difficult to overcome. Self-beliefs about mathematical ability cannot be changed by lecturing about the importance of mathematics or by threats of negative consequences. Self-efficacy beliefs change in a positive direction when students experience success or believe in the possibility of success. To be able to experience success, they need opportunities to play various roles that are different from the ones that have contributed to the feeling of powerlessness resulting in low self-efficacy. Adolescents need opportunities to play with being producers and directors in their own drama, as well as taking various roles as audiences (Davies & Harré, 1990).

The notion of possible selves may suggest different approaches to teaching and learning depending upon students’ ethnicity, race, gender, and the social context. Specifically for African American youths, the context of urban living, along with a predominantly African American population, gives rise to questions about social context that may not arise for other specific populations (Oyserman, Gant, & Ager, 1995). While students who are relatively successful in school and who feel confident in their ability to be successful in the future may be able to comfortably experiment with a wide variety of possible selves, students who have been marginalized by mainstream society see fewer options for their future and may react by restricting possible selves. Because of racism, African American youths negotiating identity have a double task of assembling a positive sense of self while discrediting negative identities attributed to African Americans.
(Oyserman et al., 1995). This stereotype vulnerability is important in understanding identity construction as it forces a focus of self-protection for African Americans.

Balance of possible selves has been determined to be critical to working toward optimal achievement (Oyserman et al., 1995). Students need to create images of themselves as being successful and develop strategies for how to accomplish that vision of success. But students also have to be able to envision what they do not want to be and understand how to avoid becoming the undesired possible self. Oyserman et al. (1995) conducted a series of studies of predictors of school persistence in college-age and middle-school students. In one study (university students), individualism, Protestant work ethic, and balance in possible selves predicted generation of more achievement-related outcomes for white students. For African American students, collectivism, ethnic identity, and low endorsement of individualism were greater predictors of strategy generation. For middle school students, a gendered African American identity schema predicted performance for females, and balance in achievement-related possible selves predicted school achievement for African American males.

Proposing a communal model of contextualized African American identity, Oyserman et al. (1995) suggest a third component of identity, beyond envisioning positive possible selves and undesired possible selves, as essential to the development of successful possible selves—sense of self as part of kin and community. Giving and receiving support within a group is a valued and salient feature of many African American communities. African American identity does not necessarily include what can be characterized as a White Protestant individualized work ethic. If individual achievement contradicts connectedness to a group, and the implications of racism are
ignored, African American youths will find achievement to be problematic (Oyserman et al., 1995). If the desire for academic achievement is to be viewed in a positive light, achievement must be conceptualized as occurring within African American identity.

Another approach to understanding stereotype vulnerability is through examination of the development of ethnic identity. Social psychologists are interested in feelings of belonging to a group and the consequences of identification with a social group or with social groups in society, or the negotiation of social identity in the broader context of the value society has placed on a particular group membership (French, Seidman, Allen, & Aber, 2006). The process of developing ethnic identity involves redefining what it means to be a member of an ethnic group, and thus the individual no longer allows society to define that identity for them. With the premise that ethnic identity is a critical facet of adolescence, particularly for adolescents of color, French et al. (2006) examined the development of ethnic identity of African American adolescents (including African American and Caribbean American groups), Latino/a American adolescents (including Puerto Rican, Dominican, and “other Latino/a groups), and European American adolescents (including non-ethnic White, Italian, Greek, and “other” European ethnic groups) over three years. Two components of ethnic identity were assessed—group-esteem and exploration. Group-esteem refers to feeling good about being a member of a group and having “collective self-esteem” (French et al., 2006, pg. 2). Exploration refers to efforts directed toward finding out what it means to be a member of one’s racial or ethnic group.

The study included 420 student participants, 258 early adolescents with a mean age of 11.28 years and 162 middle adolescents with a mean age of 14.01 year at the time
of pretransition year assessment. *Pretransition* refers to the last year of elementary school before middle school and the last year of middle school before high school. Of the early adolescents, 102 were African American, 81 European American, and 75 Latino/a American. Of the middle adolescents, 45 were African American, 71 European American, and 46 Latino/a American. School demographics varied. Elementary and junior high schools tended to be relatively homogeneous. High schools were much more ethnically diverse.

A major goal of the study was to examine the developmental course of ethnic identity over two key transitional periods—from elementary to junior high school and from junior high to high school. Transition times were thought to be of particular importance because of the disruptive nature of school transitions and heightened awareness of self in relation to membership in groups that are historically devalued by the larger society. During the pretransition year, both early and middle adolescents attended predominantly homogeneous school, meaning that African American students attended schools that were predominantly African American, Latino/a American students attended schools that were predominantly Latino/a American, and European American students attended schools that were predominantly European American.

African American and Latino/a American students demonstrated the greatest amount of change in ethnic identity over the three years. European American adolescents also rose in group-esteem over the three years, but, because they started with relatively high group-esteem, they remained relatively stable over time. African American students reported low self-esteem at the time of pre-assessment. Over the following two years, however, group-esteem increased dramatically.
Throughout the two-year post transition, group-esteem was found to increase for both early and middle adolescent groups not separated by ethnicity. However, exploration rose only for the middle adolescent group. The authors question whether or not the rise in group-esteem for early adolescents really represents movement toward “development of an achieved identity” considering the lack of corresponding rise in exploration. It is noted, however, that the early adolescents transitioned from relatively homogeneous schools to similarly relatively homogeneous schools, and middle adolescents transitioned from relatively homogenous schools to extremely diverse schools. The disruption caused by change from a homogeneous setting to a diverse setting was missed by the early adolescents, therefore they did not experience heightened awareness of self in relation to group, and they did not have the opportunity to choose self-segregation. However, the middle adolescents did experience a change in ethnic composition in their transition from junior high school to high school, and thus were able to experience negative interaction with other ethnic groups. Negative interactions with other ethnic groups are more likely than not to provoke exploration of the meaning of being a member of one’s ethnic group. Exploration did increase during the transition year for the middle adolescent students, and group-esteem continued to rise in the subsequent year.

French et al.’s study (2006) was designed to look at the development of ethnic identity over time. However, the results have implications for the study of other facets of development in cultural diverse populations, such as the development of SRL and learning mathematics. As also suggested by Oyserman et al. (1995), the negotiation of ethnic or racial identity can complicate negotiation of identity in other areas. Diversity in ethnic composition of groups might factor into the types of interactions identified while
studying and analyzing classroom discourse for evidence of the development of understanding in mathematics and scaffolding of SRL. Awareness of identity formation during adolescence, especially while investigating culturally diverse populations, must include appreciation of not only the development of personal identity as discussed by Markus and Nurius (1986) and Davies and Harré (1990), but also the development of ethnic identity as reported by Oyserman et al. (1995) and French et al. (2006).

Summary

Teachers put themselves in the position of making hundreds of decisions a day, whether consciously or not, as they guide their students to develop knowledge that has historically been defined as essential to existence in the society and the larger world. As teachers make these decisions, they must become aware of the variety of needs of their students, especially as student populations increase in culturally diversity. In order to address issues related to providing equitable educational experiences for all students, teachers must work to understand what it means to teach and learn with understanding for all students while addressing historical patterns of underachievement for non-White and low SES students.

This second broad section of literature review—including pedagogies of liberation; language, knowledge, and power; and positioning and possible selves—has addressed issues of equity and power relationships and suggested ways that power relationships as they exist can be disrupted. Attending to issues of equity in schools, with the goal of providing high-quality mathematics instruction to all students, is becoming increasingly important as schools become more culturally diverse. Making necessary
changes toward more equitable school practices may require a shift in the way teachers and students think about what it means to teach and learn.

The discussion of pedagogies of liberation offered a way to conceptualize working toward equity in mathematics education, suggesting the use of critical analysis to reveal the hegemonic nature of a traditional transmission model of learning so that students demystify “the way things are” and envision possibilities. The discussion of language, knowledge and power suggested that the distribution of power is not stable, and that power structures as they exist can be disrupted through speech acts, or as applied to classroom teaching and learning, classroom discourse. And, finally, the discussion of positioning and possible selves highlighted the importance of the development of both individual and group identity during adolescence, and how cultural differences affects the ways in which cultural groups negotiate development of identity.

The common thread throughout the three sections of discussion is the role of discourse as a vehicle for change. Knowledge gained through analysis of classroom discourse might be a significant addition to existing literature on how teachers and students use discourse for students’ development of agency as learners, a process that may involve disruption of power structures. The development of agency in learning is crucial to both learning with understanding, which is a goal of reform school mathematics, and the development of SRL, which may be seen as both a goal for and a result of successful problem solving in mathematics.

Self-Regulated Learning (SRL)

In this third broad section of literature review, I present different perspectives on the development of SRL as well as discussion of critical elements of SRL. The first
section, *An Introduction to SRL*, is a presentation of SRL from a social cognitive perspective. The next three sections, *Self-Efficacy, Theories of Intelligence*, and, *Goal Theory*, provide more detailed discussion on concepts that are essential to the development of SRL. The final section, *Sociocultural Aspects of Self-Regulated Learning*, includes a presentation of SRL from a sociocultural perspective and research suggesting the need for acknowledgement of communal aspects of SRL as they relate to cultural diversity. SRL processes help students develop skills that lead to successfully achieving learning goals. Encouraging the development of SRL in mathematics classes can help students plan, guide, and monitor their thinking when they encounter and solve challenging problems, thus promoting mathematical understanding.

*An Introduction to SRL*

Self-regulated learning (SRL) processes involve conceptions of self in relation to successfully achieving learning goals. Defined from a social cognitive perspective, self-regulated students activate and sustain cognitions and behaviors systematically oriented toward attainment of learning goals (Schunk, 1990). SRL processes include attending to instruction, processing and integrating knowledge, rehearsing information to be remembered, and developing and maintaining positive beliefs about learning capabilities and anticipated outcomes of actions (Schunk, 1990). Self-regulation consists of three interacting subprocesses: 1) self-observation, which includes deliberate attention to aspects of one’s own behavior and may be assisted with self-recording; 2) self-judgment, defined as comparing current performance with a standard (absolute or normative); and
3) self-reaction, which includes evaluative responses to judgment of performance (Schunk & Zimmerman, 1997). Self-reaction can affect self-efficacy and motivation and is sometimes linked to tangible rewards.

Academic self-regulation is defined as “self-generated thoughts, feelings, and actions intended to obtain specific educational goals, such as analyzing a reading assignment, preparing to take a test, or writing a paper” (Zimmerman et al., 1996, p. 2). There are four basic phases to the self-regulatory cycle within the model for the systematic development of SRL proposed by Zimmerman et al. (1996):

1. Self-evaluation. This phase includes recalling prior knowledge of both the task and effectiveness of previous attempts to follow through on tasks or solve problems.

2. Analysis of the task. In this phase, students analyze tasks, set goals, and choose a strategy for accomplishing the goals.

3. Implementation of the strategy. While implementing the strategy, the learner manages and adjusts outcomes through self-monitoring and feedback from peers and/or adults.

4. Reflection and refinement. In this “final” phase, the learner expands monitoring based on analysis of the effectiveness of various strategies on the performance outcomes.

“Self-monitoring of strategic outcomes is essential for self-regulation because it produces corrective cognitive, emotional, and behavior reactive effects, such as strategy improvements following unfavorable results” (Zimmerman et al., 1996, p.13). Students who are unprepared for obstacles and/or negative learning outcomes may see themselves
as failures and, consequently, react unproductively (Dweck, 2000; Zimmerman et al., 1996). Understanding of the recursive nature of self-regulation—the cycle of analysis, planning, implementation, and reflection and refinement—can help students to recognize learning as an incremental process, thus setting the foundation for development or adoption of an incremental theory of intelligence (see Theories of Intelligence below).

Schunk and Zimmerman (1997) identify four levels of academic competence in the development of self-regulation: 1) observational, 2) imitative, 3) self-controlled, and 4) self-regulated. The process begins with development of academic skill through social sources and gradually shifts to self-source of control and regulation. Observational learning occurs through modeling, including explanations, demonstrations, verbalization of thoughts and reasons for performing actions. Adoption of cognitive models, which results in imitation, is determined by the perceived functional value of the model, such as whether it resulted in success or failure or was followed by reward or punishment. Observation of models and subsequent outcomes helps inexperienced learners to formulate outcome expectations. If the learner does not appreciate the functional value of the action, however, perceived similarity between the model and the observer takes on greater importance. At the self-controlled level, standards are internalized and the individual is able to self-reinforce desired behaviors. The definitive level, self-regulated, is characterized by strong, positive self-efficacy beliefs and consistent use of self-regulatory processes.

Self-Efficacy

Perceived self-efficacy refers to “people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect
their lives” (Bandura, 1994, pg. 71). Self-efficacy beliefs determine how people feel, think, motivate themselves and behave, and thus affect life choices, levels of motivation, quality of functioning, and resilience in difficult situations. Beliefs in efficacy are influenced by four main sources: mastery experiences (performance accomplishments), models of others managing tasks (observational experiences), social encouragement or persuasion, and assumptions developed in reaction to physiological and emotional states (Bandura, 1994; Schunk & Zimmerman, 1997).

An example of an effect on self-efficacy based on a reaction to performance accomplishment might be: “I was so embarrassed by the way I scrambled through that presentation that I never want to do that again.” A person with more positive self-efficacy might use the same performance to make corrections in preparation for the next presentation and do better the second time, thus maintaining or even improving positive feelings of efficacy. An observational reflection might be: “A friend was able to accomplish this task, so I should be able to do it also.” The more a student values a model, or perceives a similarity between the model and him/herself, the more likely a model will have a positive impact. Observational experiences can also be negative. If a person does not want to be associated with characteristics of the model, imitation of the model may be avoided.

Physiological reactions are obviously internal, but can also be positive or negative. I can feel nervous and become sick when anticipating a performance, or I can look forward to a feeling of elation. Self-efficacy, an important mediating factor in the individual’s adoption of self-control, is primarily social at the first two levels of academic competence (observational and imitative) with locus shifts to self in advanced stages of
SRL (self-controlled and self-regulated) (Schunk & Zimmerman, 1997). Accordingly, self-regulatory development can be conceptualized as “a series of inter- to intrapersonal shifts rather than a unidirectional age-related progression of mutually exclusive stages” (p. 200).

Because of its social origins, optimal levels of learning occur when the needed form of social instruction is tailored to the student’s level of regulatory skill on a learning task (Schunk & Zimmerman, 1997). Further development of SRL is most likely to occur when learning experiences socially convey the processes needed to regulate at the next phase level. But social learning interactions at developmental levels are influenced, and can be restricted, by value systems. Teachers need to become aware of the relationship between learners and their concepts of cognitive competence. It is often true that students come to mathematics classes with negative assumptions about what it means to learn mathematics. They may have already devalued learning mathematics because of prior experiences. Because perceived similarity to models is important for students who do not value a particular learning task (Schunk & Zimmerman, 1997), mastery models are not necessarily beneficial at the outset. Students need to see others encounter and overcome the obstacles that they will most likely come across while problem solving so that they will understand the true nature of problem solving; cognitive models they experience first should be those of encountering and overcoming difficulties.

Because self-efficacy in relation to mathematics is a context-specific assessment of competence to perform a specific task or judgment of capabilities to execute specific behaviors, it influences the choices one makes, the effort one expends, and how long one perseveres in the face of challenge in mathematics class. Pajares and Miller (1994) used
path analysis techniques to test the predictive and mediational role of self-efficacy in the area of mathematics. Participants in the study included 350 undergraduate students from a large public university in the South.

Results indicated that variables of gender, level of mathematics attained in high school, and the number of college credits all had a positive effect on math self-efficacy. Self-efficacy had a stronger direct affects on performance than did any of the other variables. Students’ judgments about their capability to solve math problems were more predictive of their ability to solve problems than were other variables found by previous research (Pajares & Miller, 1994). Additionally, self-efficacy mediated the effect of gender and prior experience on math self-concept, perceived usefulness of mathematics, and math problem-solving performance. Interestingly, regarding overestimating performance and overconfidence, it is suggested that some overconfidence is beneficial because it contributes to effort and persistence. Students who underestimate their performance ability and lack confidence are not as likely to persist in the face of difficulty.

These results suggest that researchers and school personnel should be looking to students’ beliefs about their capabilities as mediators as predictors of performance. Students who lack confidence in skills they possess are not likely to engage in tasks in which those skills are required, and they will exert less effort and persistence in the face of difficulty. It is important to know how students develop negative self-efficacy beliefs so that interventions can be developed to help prevent avoidance of novel or difficult learning tasks.
To examine how goals and progress feedback affect student self-efficacy and achievement, Schunk and Swartz (1993) conducted two related experiments. The experiments were also used to study the joint operation of learning goals and progress feedback. Sixty fifth-grade students of varying socio-economic backgrounds from three classes in two schools participated in the study. The research compared students who received strategy instruction along with 1) product goals, 2) process goals, 3) process goals and progress feedback, and 4) general goals as an instructional control.

Self-efficacy tends to be domain specific in academic contexts. Academic self-efficacy theory emphasizes students’ beliefs concerning their capabilities to effectively employ the skills and knowledge necessary to obtain learning outcomes. Attainment of the learning outcomes leads to increased feelings of agency, or positive self-efficacy, if the students themselves set realistic (challenging but obtainable) goals. Development of the ability to set realistic process goals may require explicit training or instruction (Schunk & Swartz, 1993). Goal setting conferences help children learn to realistically assess goal difficulty and present skills.

In the first of the two experiments, Schunk and Schwartz (1993) investigated how progress goals increase self-efficacy and skill and how process goals combined with progress feedback enhance the effect of process goals. The experiment consisted of a pretest of self-efficacy and writing skills and a posttest of progress in strategy learning, self-efficacy, and writing skills. Process goals were found to have raised students’ perceptions of progress in strategy learning, and progress feedback enhanced perceptions even further. The second experiment tested whether process goals lead to better transfer than product goals, and if process goals combined with progress feedback further
enhance transfer. The pretest was basically the same as the first experiment with addition of self-reporting on use of strategies and the posttest added strategy value, and self-reported strategy use. The ability to transfer learning to new situations based on perceived strategy usefulness and self-efficacy was not as clear in the second experiment.

The effect of goal setting depends on goal characteristics such as specificity, proximity, and difficulty level. In order for goals to have the greatest impact on performance for relatively inexperienced, developing self-regulated learners, there must be a specific, explicit relationship between the goal and performance standards. Goal setting must be proximal to the expected performance, as should subsequent cognitive reflection be to actual performance. Too great a lapse of time can blur recall of steps taken to reach the goal and make attributions for resulting outcomes more difficult. In addition, goals must be somewhat challenging so that students do not lose interest, but goals must also be reachable to prevent negative self-efficacy judgments.

Perhaps the most important implication based on these experiments relates to the value of progress feedback to help students learn about monitoring strategy use during self-regulation and to improve student self-efficacy. Children who received process goal feedback plus process feedback outperformed general goal students on posttest self-efficacy and skill, efficacy for improvement, and progress in strategy learning. Process goals were found to highlight strategy use as a means to improve performance, and progress feedback conveys the message that strategies are useful and that students are becoming skillful. Because of these findings, Schunk and Swartz conclude that progress feedback can be “a persuasive source of efficacy information” (p. 351).
Progress feedback can be conveyed in a variety of ways, ranging from very private to public. In classrooms where students work collaboratively to solve challenging problems, progress feedback will be relatively public, coming from both peers and the teacher; thus classroom discourse can be seen to be an important source of efficacy information as students and the teacher engage in discourse for reflection on learning strategies with special attention to highlighting effective strategies.

Theories of Intelligence

Development and maintenance of feelings of self-esteem, self-confidence, and self-efficacy is necessary for approaching challenging problems, overcoming obstacles, and adapting to and learning from successes and failures. Theories of intelligence also influence one’s ability to develop self-regulated learning. Dweck (2000) identifies two categories of theories of intelligence—an entity theory and an incremental theory. A person who holds an entity theory of intelligence believes that intelligence is fixed. That person believes that one receives a certain amount of intelligence, probably before birth, and there is not much that can be done to change. Because intelligence is conceptualized as a fixed entity, the student sees no point in setting learning goals. Looking smart, however, takes on greater importance because to appear otherwise confirms a lack of intelligence. Responses to failure may include helplessness, defensiveness, and, sometimes, complete shutdown.

A person who holds an incremental theory of intelligence believes that intelligence is malleable—difficulty is a natural part of being, and one can learn through unsatisfactory or unsuccessful experiences as well as satisfactory or successful ones. Setting learning goals, then, is an important part of responding to both failure and
success. An incremental theory, which includes an understanding of processes that lead to mastery, often results in a desire for challenging opportunities to learn or to add excitement to the process of learning. A person who holds an entity theory has difficulty in the adjustment to failure. It is often the case that underachievement or failure is attributed to an external source—“someone else failed to teach me, to show me, to tell me what to do...” Even when an individual who holds the entity theory recognizes ownership of a problem, the definition of the problem is what keeps that person from being able to respond or cope effectively. In such a case, the individual sees him/herself as the problem, thus internalizing the criticism (Dweck, 2000). The result is self-exoneration of personal responsibility because the individual claims no control over the fixed entity.

Students' theories of intelligence had an influence upon goal choices, causing students to focus on performance goals or learning goals (Dweck, 2000). Students given information on different theories of intelligence, whether they were originally identified as having an entity theory or an incremental theory, were more likely to choose learning goals over performance goals (Dweck, 2000). This finding has significant implications for classroom teachers. Through deliberate and explicit classroom discourse about multiple perspectives on intelligence, teachers can influence students’ choices of learning goals.

Goal Theory

Theories of intelligence address an important aspect of SRL: developing and working toward mastery goals. Goal theory provides additional ways of understanding goals in relation to SRL. Goal theory is a set of statements or principles devised to explain what motivates students to behave in the ways that lead to success or failure...
within the classroom. Based on previous school experiences, parental and teacher expectations, current classroom practices, and assessment outcomes, to name just a few possible factors, students pursue goals that are associated with certain behaviors and beliefs. Motivation has traditionally been described in terms of approach and avoidance tendencies (Middleton & Midgley, 1997). Goal theory can then be seen as an extension of motivation theory as it focuses on two approach goals—the demonstration of ability (sometimes referred to as performance-approach or ego) and developing ability (task, learning, or mastery).

Avoiding the demonstration of lack of ability is an aspect of goal theory that has not been explored as extensively as mastery and performance goals (Middleton & Midgley, 1997). With this in mind, Middleton and Midgley (1997) focused their research on avoiding the demonstration of lack of ability as an avoidance goal. Avoiding the demonstration of lack of ability differs from the goal of effort reduction in that the amount of effort expended by the student is not at issue. Low academic efficacy and the need to avoid appearing incompetent are the primary factors involved in avoiding the demonstration of lack of ability. Mastery goals are referred to as task goals, performance as performance-approach goals, and avoidance goals as performance-avoidance goals.

Participants included 703 sixth-grade students from four ethnically and economically diverse communities in southeastern Michigan. The Patterns of Adaptive Learning Survey (PALS) (Midgley et al., 1996) was used to collect data.

Middleton and Midgley (1997) reported that performance-approach goals were unrelated to academic efficacy and positively related to avoidance behaviors in the classroom and to test anxiety, confirming a basic principle of goal orientation theory—
the relative saliency of task and self. African American girls displayed a greater tendency to endorse task goals than did any other group. Students with lower GPAs were more likely than students with higher GPAs to endorse both dimensions of performance goals (hiding lack of ability and displaying superiority relative to others). Task goal orientation was the strongest predictor of academic self-efficacy and self-regulated learning; and the strongest predictor of avoiding seeking help in the classroom was orientation to performance avoid goals.

Teachers often attribute students' withdrawal of effort during early adolescence to student laziness, lack of value for school and/or education, and lack of parental support (Turner, Midgley, Meyer, Gheen, Anderman, & Kang, 2002). However, teachers play a large role in creating the environment for learning. Increased student involvement through a teacher's guidance and support may lead to change in attitudes about learning and intelligence and consequently greater student motivation and productive learning goals. (Dweck, 2000). In a study of teachers’ scaffolding of learning goals through classroom discourse, Turner et al. (2002) examined aspects of the learning environment related to students' reports of avoidance strategies in mathematics. They specifically wanted to look at the way in which achievement goals are emphasized through teachers' use of discourse and the relationship between students' perceptions of the classroom goal structure and the use of avoidance strategies. Avoidance strategies were categorized as self-handicapping, avoiding help seeking, or avoiding novelty.

Ten sixth-grade classrooms from nine ethnically diverse schools in Midwestern states were observed for evidence of scaffolding and nonscaffolding forms of instructional discourse. Students were subsequently surveyed on perceptions of classroom
goal structures and use of avoidance strategies. Findings indicated that supportive discourse patterns reflected scaffolding that emphasizes learning, improvement, and understanding; nonscaffolding discourse patterns were less about assisting learning and more about directing, assessing, and control; and whole-class discourse provided public messages about learning, performance, and expectations (Turner et al., 2002). The authors limited analysis to whole-class discourse based on the assumption that the public nature of whole-class discourse would relate most directly to students' reports of goal structure and avoidance strategies.

Classrooms in which students perceived an emphasis on learning, improving, and understanding had lower reports of self-handicapping, avoiding help seeking, and avoiding novelty. A mastery goal environment consists of both cognitive and motivational or affective components (Turner et al., 2002). In high-mastery goal classrooms, teachers devoted a large percentage of instructional discourse to instructional scaffolding (up to 50%) with an additional portion devoted to motivational support (20 to 25%). Motivational support and mastery messages were explicitly stated with advice to students not to feel inadequate or ashamed when they did not understand. A combination of instructional practices, rather than one salient feature, determined the differences in classroom context resulting in students' perceptions and reports of avoidance strategies. One implication is that students need a balance between scaffolding the transfer of responsibility and teacher cognitive and affective support.

Overuse of nonscaffolding discourse patterns (such as giving directions, answering questions about directions, commenting on off-task behavior, and asking questions with known answers) can be considered evaluative. When used in moderation
and in combination with scaffolding discourse, however, some instructional practices categorized as nonscaffolding (such as IRE patterns) can help students by directing attention to what is important, establishing prior knowledge, or giving the opportunity to demonstrate competence.

Using a different perspective related to goal theory, Lopez (1999) investigated the relationship between self-beliefs and academic achievement with a theoretical ground of action-control theory. Action-control theory is concerned with the relationships between an individual, a specified goal, and the behaviors that are necessary to achieve the goal (Lopez, 1999). Within this theoretical framework, three interrelated beliefs systems affect goal-directed behaviors. The first two belief-systems are means-end (“Ability is important for academic success”) and agency (“I am smart and able to figure this out”). The third set of beliefs has to do with control expectancy, which was defined as the general expectancy that the specified goal can be obtained. From an action-control perspective, one must be able to envision a goal in order to successfully reach it. Because an individual can envision successfully obtaining a goal without having to identify how the goal will be obtained, however, expectations for success can operate independently of one’s agency beliefs (Lopez, 1999).

Of the 120 children in grades 5 through 7 who participated in this study, about 30% were of South American, Cuban, Caribbean, Asian, or African American decent. For the purpose of analysis in regard to ethnicity, all ethnic minorities were combined into a single group. Participants completed a survey to measure action-control beliefs, academic goals, intrinsic motivation, and test anxiety. Performance information was collected from school records.
Evidence from the study (Lopez, 1999) suggests that White students had significantly higher agency for ability than non-White students. White students also reported significantly higher performance approach goals than non-White students. Action-control beliefs were strongly related to academic goals. Agency for effort was positively related to both mastery goals and performance-approach goals. Agency for ability was also positively related to mastery goals and negatively related to performance-avoidance goals.

As was expected, mastery goals were positively related to intrinsic motivation and performance-avoidance goals were negatively related to intrinsic motivation. Agency for ability was negatively related to test anxiety, and performance-avoidance goals were positively related to test anxiety. Academic goals mediate the relationship between action-control beliefs and achievement related outcomes. There were positive correlations between all action control beliefs and intrinsic motivation. Mastery goals were also positively correlated to intrinsic motivation. Agency beliefs were negatively correlated with approach-avoidance goals. Agency for ability and control expectancy beliefs were the strongest predictors of school grades. These results support the idea in other research related to self-efficacy that learning goals are translated into action through perceived self-efficacy (Lopez, 1999).

The results of analysis of variable correlations for minority students could be of critical importance for urban educators. Lopez (1999) reported that minority students’ differences in achievement were mediated by individual differences in action-control beliefs. This finding relates to the stereotype threat phenomenon that reveals minority students’ belief in academic tests as measures of ability. If a student perceives a test as a
threat to agency self-beliefs, the stereotype threat takes over and results in lack of ability to perform, a phenomenon which is sometimes referred to as self-fulfilled prophecy. Working to strengthen minority students’ agency for ability may result in improved minority student achievement—an issue of interest for the present study.

Goal theory provides a view for understanding the beliefs that students hold and insight into ways of scaffolding the development of students' beliefs about learning and intelligence. These beliefs ultimately affect students' learning and performance in the classroom. Each of the studies of goal theory suggest that teachers can have an effect on the goal orientation of their students through the kinds of tasks provided, the support of students' involvement in various levels of discourse, and the development of an inquiry learning environment.

*Sociocultural Aspects of Self-regulated Learning*

Sociocultural theory explains the development of self-control and self-regulation as it occurs through social interaction, which suggests strategic use of instructional discourse for the development of self-regulated learning in school. In terms of a child’s development within a social setting, self-control can be defined as the ability to follow through on a caregiver’s directions. An “internal voice” can be heard that “reminds” a child what is expected within the social setting. Self-regulation, on the other hand, indicates a more sophisticated level of development. Self-regulation can be defined as the ability to formulate a plan on one’s own and follow through on that plan. Self-regulation is developed through social interaction between a child and a caregiver (or the mediating culture), and, over time, the child internalizes social regulating interactions (Diaz, Neal, & Amaya-Williams, 1990).
The development of self-regulation can be described in terms of intersubjectivity within the structuring of responsibility in joint problem solving (Rogoff, 1990). Intersubjectivity refers to “the ability to show by coordinated acts that purposes are being consciously regulated” by adapting or fitting subjective control (the ability to exhibit to others at least the rudiments of individual consciousness and intentionality) to the subjectivity of others (Trevarthen, 2001, p. 5), or more simply, it refers to joint attention, sharing intentions and sharing affective states. Intersubjectivity provides grounds for communication and therefore bridges between what is already known and what is to be newly developed, acquired, or understood. Adults provide metacognitive support for the development of self-regulation by structuring activities that are just beyond a child’s skill level. They then must determine the problem to be solved, the goal, and the ways that the goal can be broken down into manageable pieces, or subgoals. Throughout the process, the adult must manage working for greater independence of the child, adjusting support as necessary, so that a transfer of responsibility occurs.

In Vygotskian terms, self-regulation is a higher psychological function. Higher psychological functions are “…(1) self-regulated rather than bound to the immediate stimulus field; (2) social or cultural rather than biological in origin; (3) the object of conscious awareness rather than automatic and unconscious; and (4) mediated through the use of cultural tools and symbols” (Wertsch, in Diaz et al., 1990, p. 128).

Vygotsky presents the development of self-regulation as characterized by four major milestones (Diaz et al., 1990):
1) The environment, based on the laws of stimulus-response, controls a child’s behavior, as a caregiver manipulates the environment for specific purposes.

2) The child, in the beginning stages of mediation, is able to use external signs to determine appropriate responses.

3) The child becomes aware of the usefulness of signs and is able to look for them. At this point, the child can create and manipulate signs in order to achieve a desired outcome. This stage is still limited by the availability of external signs.

4) The child is no longer dependent upon external mediating stimuli to carry out a plan of action. The relation between the child and the environment is now intrapsychological—a significant change to a new level of behavioral organization.

This progression toward self-regulation is a social process—signs are originally introduced in a social environment, the child begins to use signs for influence and action, words become the most useful tool in creating socially shared meaning, and thus transformation occurs primarily through the use of speech. In the context of the classroom, a sociocultural perspective on SRL might be seen to communal in character. In classroom settings, interactivity and joint problem solving are not private. Particularly in whole-class situations, but also in small-group discussion, all members of the learning community stand to benefit from discourse that supports the development of SRL. Responsibility to the community has been identified as highly valued within many non-mainstream cultures. Understanding of cultural values in relation to the development of
SRL might provide insight into the ways that scaffolding of SRL through classroom discourse might look different for students from varying cultural backgrounds.

Jackson, Mackenzie, and Hobfall (2000) argue for examining communal aspects of self-regulation in reaction to the individualistic approach implied in many other studies. Despite the fact that feedback from external sources for development of self-regulation is acknowledged in the literature on self-regulation, most perspectives still convey the importance of the individual and the exercise of free will. Jackson et al.’s argument also includes a discussion of issues of distributions of power in society. They dispute the assumption that all people have access to the social and economic resources necessary to control their lives independently. Economic resources, political power, and social power are unevenly distributed throughout society. Limited control over external factors such as economic, political, and social power impedes progress toward individual goals. Consequently efforts to reach goals through personal control may be perceived by the individual as being ineffectual. Additionally, there are cultural groups, typically outside the mainstream culture, that tend to operate in more socially mediated ways. “The causal social conditions that contribute to disempowerment must be confronted in order to understand self-regulatory behavior from a universal perspective” (Jackson et al., 2000, p. 286).

An alternative way of conceptualizing self-regulation, then, might be to recognize the development of individuals within the social context, or self-in-social-setting regulation (Jackson et al., 2000). Feedback loops and comparisons made for corrections in self-regulation happen within communities of interaction. Often comparisons for the purpose of eliminating discrepancies between the self and others are avoided in
communal setting, such as families, because comparisons cause stress and internal strife. On the other hand, in the case of families or familial-type relations, differences can be considered a source of diversity that is beneficial to the community. In communal settings, behavioral expectations are based on social norms and the roles that individuals take within the social setting. Members seek out support from others for guidance and confirmation of appropriate behaviors. Therefore, individual behavior can be understood to be a product of external, culturally based cues (Jackson, et al., 2000).

Classrooms are social settings where teachers often attempt to form communities of learning. Because the needs of the community are affected by individual behaviors, choices that are made and goals that are formed by individuals cannot be in conflict with community standards and goals. Individual development is a goal within the community, but not at the expense of other members of the community. But individual goals need not be void of individuality and uniqueness. The connectedness of members through interdependence is mutually supportive of the individual and the group. “We risk failure in our educational reforms by ignoring the significance of human connectedness in many communities of color” (Delpit, 1995, p. 95). It is within the context of a community of learners that teachers must strive to develop thinking and learning dispositions.

Summary

The literature on SRL, self-efficacy, theories of intelligence, and goal theory suggests that teachers can influence students’ development of self-regulated learning skills. Encouraging the development of SRL can help students plan, guide, and monitor their thinking when they encounter and solve challenging problems, thus promoting mathematical understanding at the same time. The teacher’s role in scaffolding SRL
includes awareness of students’ levels of regulatory skill on given tasks. Additionally, theories of intelligence influence goal choices, causing students to focus on either performance goals or learning goals. Contexts for the development of SRL can be created through supportive patterns of classroom discourse. Supportive discourse patterns offer scaffolding for learning, improvement, and understanding. Explicit classroom discourse about multiple perspectives on intelligence can influence students’ choices of learning goals, thus affecting the development of SRL. Sociocultural theory has implications for communal approaches to the development of SRL that might help to meet the learning needs of non-mainstream students.

Conclusion

The three sections of literature, *Teaching and Learning Mathematics with Understanding, Culture, Positioning, Language, and Power*, and *Self-Regulated Learning*, combine to support an argument for the examination of classroom discourse for the development of self-regulated learning with culturally diverse students in mathematics classrooms. The literature on *teaching and learning mathematics with understanding* contributes a stance on teaching and learning mathematics that requires the positioning of students as active participants in development of knowledge. Active participation involves communication of ideas and scaffolding through classroom discourse. Additionally, teacher scaffolding of students’ metacognitive behaviors during problem solving has potential to contribute to the development of SRL skills, supporting the argument that “self-regulation is both an aptitude for and a potential outcome of schooling” (Randi & Corno, 2000, p. 651). The literature on *culture, positioning, language, and power* addressed issues of cultural diversity, equity, and social justice with
an emphasis on the use of language to negotiate power relations and the implications for
the development of self-theories. The literature supports the notion that critical analysis
can be used to reveal the hegemonic nature of a traditional transmission model of
schooling, that distribution of power is not stable, and that power structures as they exist
can be disrupted through speech acts. Furthermore, the development of identity is
especially importance during adolescence, and cultural differences affect the ways in
which students negotiate development of identity. And, finally, the literature on self-
regulated learning supports the notion that development of SRL can be scaffolded
through activity and interaction (Pape, 2005). Students’ theories of intelligence influence
goal choices, causing students to focus on either performance goals or learning goals.
Explicit classroom discourse about multiple perspectives on intelligence can influence
students’ choices of learning goals, thus affecting the development of SRL.

The present study is concerned with examining the development of self-regulation
for culturally diverse students—a concern that recognizes the connectedness of group
members through interdependence for many students of color. Discourse has been
identified in the literature as a critical element in learning mathematics, negotiating
power relationships, and developing skills in SRL. Thus the examination of classroom
practice through discourse analysis may provide insight into the ways that teachers attend
to the development of SRL and support learning with understanding in mathematics
classes made up of students from diverse cultural backgrounds. It is through
discourse that power relations are challenged, negotiated, and restructured. That
possibility of restructuring holds the promise for equity and social justice in mathematics
education.
CHAPTER 3

METHODOLOGY

Restatement of the Purpose

There is a need to expand knowledge of the differences in how teachers use classroom discourse to scaffold student self-regulated learning and mathematical understanding within a variety of communities of learning, specifically culturally diverse communities. Learning communities, particularly those people who are responsible for teaching, might benefit from greater knowledge of how to support the development of student self-regulated learning. This qualitative study includes a critical stance toward the analysis of teacher practices, specifically the use of classroom discourse, for teaching and learning mathematics, developing SRL, and engaging non-White and low SES students in meaningful classroom interactions, and will add more to cultural perspectives in the literature on SRL.

Restatement of Guiding Questions

1. What do teaching mathematics for understanding and fostering SRL look like in culturally diverse algebra classrooms?

2. In what ways do students and teachers in these classrooms position themselves and each other through discourse?

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Participants

As described in the introductory chapter, CCMS project participants come from 117 middle and high school teachers (96 Algebra I, 21 Physical Science) from a national sample (32 states, and 2 Canadian provinces), half in an experimental group and the other half in a control group. First year participants included only teachers of Algebra I. Out of the first year participants, approximately 30 were observed while teaching participating classes of students. For this study the original group of 30 classroom observation cases was narrowed down to 8 cases based on 1) recommendations from CCMS observers of cases believed to illustrate cultural diversity and 2) demographic data obtained from the CCMS data base. The number of cases was further reduced to three based on the availability of sufficient teacher-student interaction to provide classroom discourse for analysis.

Case studies allow for the investigation of particular phenomena in real-life contexts (Yin, 1994) and can illustrate a particular problem (Stake, 2000). They assume …that ‘social reality’ is created through social interaction, albeit situated in particular contexts, and [seek] to identify and describe before trying to analyze and theorize. (Stark & Torrance, 2005, p. 33)

I was particularly interested in identifying and describing cases that would illustrate the nature of classroom discourse used to support learning in mathematics classes and how language is used to position students as learners and knowers within mathematics classrooms that are culturally diverse. The rationale for choices made for inclusion or exclusion of particular cases is provided below in Phase III of Data Analysis.
Data Sources

The following data sources for each teacher-participant/classroom case were used for analysis:

1. Videotaped sessions of teacher and students interacting during algebra classes (two for each case completed as part of the larger CCMS study),
2. Transcripts of the audio from the videotaped sessions described in #1 above,
3. Observer notes (that contain descriptions of physical characteristics of the classroom and details on lesson plans not evident on video, video transcripts, or audio),
4. Data on student race and ethnicity from the CCMS data base and as reported by students on pre and posttest exams from the second year of the study,
5. Audio taped post-observation interview with classroom teacher (conducted with the teacher in Case #2),
6. Audio taped end-of-year interviews with classroom teachers (conducted with all three teachers, as part of the CCMS study), and
7. Classroom artifacts and teacher lesson plans.

Videotaped classroom sessions and the corresponding transcripts of classroom dialogue were the main source of data for the present study. Observer notes helped to fill in minor details. Demographic data was used before coding to help identify classroom cases with cultural diversity. During the final write up, student self-identification of race/ethnicity was used to more accurately depict the makeup of each classroom. The audio taped post-observation interview provided an opportunity to collect information
about one teacher’s beliefs about her own teaching. The audio taped end-of-year interviews provided the opportunity to find out more about use of technology in the classroom, as required for CCMS, and also to ask about aspects of the classroom that I had observed during early phases of data analysis. Classroom artifacts and teacher lesson plans were used as part of providing a descriptive setting and identify learning and curricular objectives for each lesson within the cases.

Data Analysis

Erickson (1986) maintains that interpretive research on classroom teaching will play a significant role in educational research because of what it has to say about central substantive concerns such as

…(a) the nature of classrooms as socially and culturally organized environments for learning, (b) the nature of teaching as one, *but only one*, aspect of the reflexive learning environment, and (c) the nature (and content) of the meaning-perspectives of teacher and learner as intrinsic to the educational process. (p. 120)

For this study, I examined teacher practices within the socially and culturally organized environments of mathematics classrooms, and worked to understand how teachers’ verbal language might support the development of student SRL and learning mathematics with understanding. This work included examination of patterns of discourse that have the potential to affect student agency within the learning process.

Data sources were grouped by individual teacher-participant/classroom. The data for an individual teacher-participant/classroom are referred to as a case. I observed
teachers’ practices by watching videotaped sessions of algebra classes, listening to audiotapes of telephone interviews with teachers, and studying the corresponding transcripts and observer notes. During later stages of coding, I supplemented pre-existing CCMS data with a post-observation interview with one teacher and conducting end-of-year interviews with three teachers. The post-observation interview and telephone interviews were tasks assigned to me as part of the larger CCMS study.

**Phases of Data Analysis**

**Phase I.** Data analysis occurred in four major phases (see Table 3.1). The purpose of Phase I was to narrow the number of teacher cases from about 30 to eight. As described in the ‘Participants’ section above, decisions on which cases to include in this study were based on 1) recommendations from CCMS observers (cases believed to illustrate cultural diversity) and 2) demographic data from the CCMS data base.

I started work related to the present study by organizing CCMS classroom observation documents by individual teacher-participant into notebooks as part of my duties as a GRA working with CCMS. The notebooks contain teacher pre-observations forms, teacher lesson plans, observer notes, student questionnaires related to perceptions about use of the TI-Navigator system, and any other classroom artifacts gathered during the observations. A few of the observations included assessment of characteristics of instruction as identified in the *Classroom Observation Protocol* (Pape et al., 2006).
<table>
<thead>
<tr>
<th>Phase</th>
<th>Steps</th>
</tr>
</thead>
</table>
| I.    | 1. Organized data for notebooks and began working with Classroom Observation Protocol  
      2. Identified cases representing cultural diversity  
      3. Selected 8 cases to study in greater detail |
| II.   | 4. Coded data (total of 8 cases) two cases at a time  
      5. Wrote analytic memos in NVivo based on themes and categories that emerged during coding |
| III.  | 6. Selected 3 cases for detailed analysis and write up  
      7. Investigated corresponding teacher interviews for additional data and verification or negation of preliminary analyses  
      8. Revised codebook |
| IV.   | 9. Analyzed across cases  
      10. Revised codebook |

Table 3.1: Phases of data analysis

Later, I selected cases based on recommendations I solicited from research team members who conducted the classroom observations and on whole-school demographic data collected during teachers’ first year of participation in the CCMS study. Research team members were asked to recommend teachers and classes of students that would illustrate cultural diversity in student populations. Data from the CCMS data base was also used to identify potential cases. A total of 8 teachers/classes were identified as potential cases for the present study. Cases were grouped in pairs in order of participant number to create manageable portions for coding and annotating in Phases II. All
teachers who were recommended by CCMS observers were White; therefore differences in ethnic or cultural identification of the teachers did not become part of the study.

*Phase II.* Phase II began with coding of the transcribed texts of classroom observations using NVivo, which is data analysis software designed for qualitative research. Initial steps in coding and observation included watching the video to become familiar with the participants, the lesson, and the classroom environment. Transcripts were checked for accuracy, and changes were made as necessary. Next, nodes were created by carefully examining each video and corresponding transcript several times and naming portions of text—utterances, sentences, and exchanges—toward identify themes. Nodes are essentially organizational holders of data. Themes emerged from the data through naming portions of text, which are then organized into nodes. NVivo provides numerical reports of coded themes as references (number counts of occurrences) and as coverage of text (percentage). For example, in one case there were 14 references to *Humor* which can also be described as covering 4.87% of the lesson transcription. Percentages are reported in Chapter 4 to the nearest tenth of a percent.

Annotations, which facilitated identification and construction of connections and comparisons among teacher practices, were linked to data using NVivo. I annotated sections of text to highlight features of lessons that were of particular interest, to emphasize unusual or surprising elements, and/or to identify emerging patterns of discourse. Annotations were particularly helpful for writing about the individual cases in Phase III and contributed to cross-case analysis.
Following initial coding, I looked for patterns and disruptions of patterns within and among pairs of cases and wrote analytic memos based on the patterns and the annotations created during initial coding. As each additional pair of cases was coded, additional themes and sub-themes emerged. Previously coded cases were reexamined for evidence of the new themes and sub-themes as additional themes emerged. Initial grouping of themes into families was based upon judgments of relationships. Family groupings were modified in Phases III and IV. For example, *Facilitating classroom discourse* was included within a broad category of *Classroom discourse* during Phase II. During Phase III, *Classroom discourse* was separated into *Student contributions to classroom discourse* and *Teacher questioning and patterns of discourse*. During Phase IV, I realized that text coded as *Facilitating classroom discourse*, a theme within *Classroom discourse*, fit better with other themes that I grouped as a family under *Classroom norms*, which included *Press for classroom discourse*.

**Phase III.** Phase III began with further narrowing of the number of cases. I identified three cases that would provide rich and interesting data about how teachers and students create contexts for learning. Cases recommended by observers with extremely low numbers of non-White students were eliminated. In one such case, the regimented nature of classroom procedures resulted in little to analyze in terms of the few students of color. Cases that lacked sufficient discourse for analysis were also eliminated. For example, in one case, student small-group activity occurred in the hallway outside the classroom, therefore very little of the discourse was available for analysis.
Teachers in each of the three cases chosen for the study began participation in the CCMS study in Summer 2005, the onset of the project. A minimum of two classroom observations was available for each case. In cases where four or more classroom observation sessions were available for coding and analysis, I chose two of the available lessons. Only two classroom observations were available for Case #1, Ms. Brenner and her students. The observations in Ms. Brenner’s class occurred in December 2006 and April 2007, the second year of her participation in the CCMS study.

There were four available observations of Mrs. Blake and her students, Case #2. The observations from the first year did not include use of the TI-Navigator technology and classes were conducted in a different manner than in classes observed the following year. The changes in instructional practice, which occurred due to use of the TI-Navigator, were significant for Mrs. Blake and her students. Because of the importance of the transformation in Mrs. Blake’s role in supporting student learning from one year to the next, I included one lesson from April of 2006, the first year of the CCMS study, and one lesson from April of 2007, the second year of the CCMS study.

Five observations were available for Case #3, Mrs. Harmon and her students. I chose the first two lesson observations, which occurred in April of the 2006-2007 school year, the second year of the CCMS study. In Mrs. Harmon’s first lesson, students worked in small-groups for the entire class period. In the second, Mrs. Harmon led whole-class discussion the entire period. The nature of the small-group work and whole-class discourse illustrated in Mrs. Harmon’s two lessons provided an interesting view of discourse patterns in Mrs. Harmon’s class. After completing coding of the first lesson, I
was unsure about using the lesson because of the limited amount of student discourse for analysis as compared to other cases. The second lesson was conducted in a different style that was unexpected based on the first observation, which resulted in different styles of interaction between Mrs. Harmon and her students. Although many of the patterns of discourse between the teacher and her students were similar, I found the contrast between teacher and student interactions in the lessons to be intriguing.

The three cases reveal a variety of approaches to teacher and student interaction in mathematics classrooms. For each of the selected cases, I developed descriptive narratives of how classroom discourse is used to teach and learn mathematical processes and content and to encourage use of a variety of learning strategies in these particular situations. I looked to existing teacher interviews for data to support or refute the analysis from Phase II. I also conducted post-observation interviews and end-of-year interviews, which gave me the opportunity to clarify points of interest (‘member checks’ of my developing interpretations) and verify interpretations (providing additional lenses with which to view and inform). During the writing of descriptive narratives, I revised the codebook to eliminate unnecessary categories (those not related to cases not included in Phase III) and to refine families of themes, as described at the end of Phase II above.

Phase IV. Phase IV included cross-case analysis. The purpose of the cross-case analysis was to look for commonalities and distinctions among cases that might aid the comprehension of teacher practices in creating contexts for understanding mathematics and student self-regulated learning, and how those distinctions relate to culturally relevant pedagogy. Final decisions on family groupings of themes were made during
Phase IV, which resulted in nine families as shown in Table 3.2: Affect (including both teacher and student contributions), Classroom norms (as scaffolded by the teacher), Directions (from the teacher), Emotional scaffolding (directed to students by both teachers and students), Scaffolding mathematical thinking and understanding (teachers), Student contributions to discourse (students), Supporting SRL (teachers), Teacher questioning and patterns of discourse (teachers), and Technology (teacher and students).

Most of the data from classroom observations are in the form of language produced by the teacher. Although there is important student generated data, the unit of analysis for examining discourse is the classroom. I analyzed data across cases, revised descriptions of individual cases for consistency in form, and added sections of narrative describing physical features of the classrooms.

Citations and Coding

The codebook for the present study was created during coding of classroom observations using NVivo during Phases II, III, and IV, as described above. The codebook for the present study (see Table 3.2) shows themes with descriptions that are grouped into families of related themes. Themes were identified during coding through naming portions of text in NVivo. Themes were loosely grouped into families in Phase II. Fine-tuning of family groupings occurred in Phases III and IV.
<table>
<thead>
<tr>
<th>Family</th>
<th>Theme</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Affect</td>
<td>Appreciative comments</td>
<td>…comments that convey value for an idea or a thing in relation to another person</td>
</tr>
<tr>
<td></td>
<td>Enjoyment of activities</td>
<td>…comments that indicate positive affect in relation to classroom activities</td>
</tr>
<tr>
<td></td>
<td>Humor</td>
<td>…comments used to create amusing situations</td>
</tr>
<tr>
<td>Classroom norms</td>
<td>Facilitating classroom discourse</td>
<td>…comments made with the intent of initiating or sustaining classroom discourse</td>
</tr>
<tr>
<td></td>
<td>Press for classroom discourse</td>
<td>…comments or questions used to encourage students to participate in verbal exchanges</td>
</tr>
<tr>
<td></td>
<td>Press for elaboration</td>
<td>…comments or questions used to draw out additional information from a student in relation to previous comment</td>
</tr>
<tr>
<td></td>
<td>Press for inquiry</td>
<td>…comments or questions used to stimulate investigation of ideas</td>
</tr>
<tr>
<td></td>
<td>Press for justification</td>
<td>…comments or questions used to draw out additional reasoning for supporting a solution to a problem or a particular stance on an idea</td>
</tr>
<tr>
<td></td>
<td>Press for participation</td>
<td>…comments or questions used to stimulate engagement in classroom activity</td>
</tr>
<tr>
<td></td>
<td>Press for reasoning</td>
<td>…comments or questions used to draw out reasoning, such as explanations for the steps taken in problem solving</td>
</tr>
<tr>
<td></td>
<td>Setting expectations</td>
<td>…any of a variety of verbal messages used to convey values for social and sociomathematical behaviors</td>
</tr>
</tbody>
</table>

Table 3.2: Families of themes with descriptions
Table 3.2 continued

<table>
<thead>
<tr>
<th><strong>Giving directions</strong></th>
<th>…the initial statement of directions for expected behavior or instructional activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Introducing future plans</strong></td>
<td>…statements made concerning plans for learning that are not expected to occur within the current lesson</td>
</tr>
<tr>
<td><strong>Introducing next steps</strong></td>
<td>…instructions given subsequently to the initial directions, not repetition of initial directions</td>
</tr>
<tr>
<td><strong>Managing materials</strong></td>
<td>…directions related to distribution, use, and clean up of materials used during classroom learning activities</td>
</tr>
<tr>
<td><strong>Presentation of the problem</strong></td>
<td>…the initial presentation of a problem scenario for use during problem solving</td>
</tr>
<tr>
<td><strong>Restating directions</strong></td>
<td>…directions that are repeated without additional information</td>
</tr>
<tr>
<td><strong>Returning to the problem</strong></td>
<td>…repetition of the problem for the purpose of checking during problem solving</td>
</tr>
</tbody>
</table>

**Emotional scaffolding**

<table>
<thead>
<tr>
<th><strong>Emotional prep</strong></th>
<th>…statements made by the teacher for the purpose of preparing students for difficult work that would follow</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Emotional support</strong></td>
<td>…statements made by the teacher that were meant to help students overcome obstacles encountered while working on tasks</td>
</tr>
<tr>
<td><strong>Encouragement between and among students</strong></td>
<td>…statements made by a student to another student or group of students with the intention of providing positive affective support</td>
</tr>
<tr>
<td><strong>Encouragement from teacher</strong></td>
<td>…comments made to draw attention to good classroom participation and other forms of student work, such as student observations, questions, interactions, and products of student work</td>
</tr>
</tbody>
</table>

**Scaffolding mathematical thinking and understanding**

| **Challenging students to think** | …questions and statements used for the purpose of transferring responsibility for answering questions back to students |

Continued
Table 3.2 continued

| Clarifying mathematical terminology | …statements or questions that draw attention to meanings of mathematical words by such means as comparing, contrasting, highlighting, questioning, and emphasizing |
| Connecting ideas to representation | … statements or questions that draw attention to relationships between mathematical concepts and various representations |
| Connecting previous work to new work | … statements or questions that draw attention to relationships between previous classroom activities and the current activity |
| Elaborating | …statements or questions that add ideas to enhance mathematical understanding |
| Linking to prior knowledge | …statements or questions that draw attention to relationships between previously learned concepts and the current concept |
| Making sense of the mathematics | …discussion that focuses on the meaning of mathematical concepts [in contexts] rather than recall of procedures |
| Modeling mathematical thinking | …teacher think-aloud used for the purpose of demonstrating thought processes used during mathematical activity |
| Press for understanding | …comments or questions made for the purpose of directing attention to meaning |
| Review statement | …statements used to recall knowledge of a concept that was previously introduced |
| Review through questioning | …requests made for the purpose of recalling previously introduced concepts |
| Rewording | …clarification of an idea through restatement using different words |
| Soliciting multiple responses | …comments and questions used to elicit ideas from several students |
| Summarizing | …restatement of the substance of the lesson, or portion of the lesson, in an abbreviated form |

| Student contributions to discourse |
| Comment | …student contribution to mathematical discourse |
| Student-to-student question | …requests for information made by one student of another |
| Argumentation | …verbal challenging of an idea that includes alternatives and reasons for the challenge and the alternatives |

Continued
Table 3.2 continued

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conjecture</td>
<td>… proposal of a reasoned judgment that is uncertain</td>
</tr>
<tr>
<td>Explaining to another student</td>
<td>… one student assisting another student by talking about a mathematical concept</td>
</tr>
<tr>
<td>Explanation</td>
<td>… statements made about a mathematical concept without complete reasoning or justification</td>
</tr>
<tr>
<td>Finish sentence</td>
<td>… a student verbally completes an expressed thought initiated by the teacher before the teacher can finish</td>
</tr>
<tr>
<td>Justification</td>
<td>… reasoning provided by a student to support a claim, a solution to a problem, or a particular stance on an idea</td>
</tr>
<tr>
<td>Predicts question with answer</td>
<td>… a student voices an answer in anticipation of a question</td>
</tr>
<tr>
<td>Predicts what will be said</td>
<td>… a student voices an idea in anticipation of the direction of the discussion</td>
</tr>
<tr>
<td>Reasoning</td>
<td>… explanations for decisions made, usually in relation to steps taken in problem solving</td>
</tr>
<tr>
<td>Repetition</td>
<td>… exact restatement of an utterance</td>
</tr>
<tr>
<td>Rethinking</td>
<td>… the expression of changing thoughts in regard to a mathematical idea</td>
</tr>
<tr>
<td>Student-to-student discourse</td>
<td>… classroom conversation that does not include the teacher</td>
</tr>
<tr>
<td>Student thinking dispositions</td>
<td>… a student voices an idea that indicates thinking creatively about the mathematics</td>
</tr>
</tbody>
</table>

Supporting SRL

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strategy instruction</td>
<td>… statements made in relation to the development of skills for planning and carrying out plans for learning</td>
</tr>
<tr>
<td>Encouraging self-evaluation</td>
<td>… questions or statements that suggest reflection on behaviors related to learning</td>
</tr>
</tbody>
</table>

Teacher questioning & patterns of discourse

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dialogic episodes</td>
<td>… occurs when participants in classroom discourse exchange ideas in a nonprescripted way, expanding on or modifying the contributions of others</td>
</tr>
</tbody>
</table>

Continued
Table 3.2 continued

| IRE | …is a small-scale pattern of discourse that is usually initiated by the teacher who asks a question (I), students signal that they understand or can respond to the question, the teacher calls on a single student, the chosen student provides an answer to the question (R), and the teacher evaluates as either correct or incorrect (E) |
| IRIRE | …is an extended IRE pattern with the middle “I” representing the initiation of a modified question or information that provides a hint (an implied negative evaluation) |
| Uptake of student comments | …reference made to student comments or questions with extension; may be used to talk about correct and incorrect student statements, to explore or answer student questions, to clarify student comments, and/or to scaffold argumentation |
| Asking for clarification | …requesting more information from a student for the purpose of understanding a previous statement |
| Checking on progress | …assessment of movement toward a goal |
| Higher-order recitation question (HOrec) | …request made of student to think beyond what has been presented, but a specific answer is expected |
| Lower-order authentic question (LoAuth) | …request used for such purposes as finding out what steps a student took to solve a problem or how a student found information |
| Lower-order recitation question (LoRec) | …request that typically requires recall of previously known information, often used to involve students in providing “answers” |
| Open-ended question | …requests for information or ideas with few, if any, restrictions |
| Request for input | …asking a student to share an idea, solution, or information |
| Soliciting additional answers | …requests for more solutions |

Technology

| Students as experts | …includes discourse resulting from situations where students were given responsibility for the use or maintenance of classroom technologies |
Excerpts from observation transcriptions have citations that include a teacher code, the date, and the source of the excerpt. For example, the citation *(H8-08-06-16phint)* indicates participant H8, the date of June 16, 2008 (in the form yy-mm-dd), and phone interview as the source of the information. Blocks of direct quotes from transcribed texts of the observation include paragraphs numbers. Paragraphs are turns at speaking that were determined by a change in speaker. Direct quotes within other text, such as *(B5-06-12-14obs, 298)*, includes the paragraph number after the document source. Table 3.3 shows the abbreviations used in citations and their definitions.

<table>
<thead>
<tr>
<th>Abbreviation/ Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5</td>
<td>Ms. Brenner</td>
</tr>
<tr>
<td>B7</td>
<td>Mrs. Blake</td>
</tr>
<tr>
<td>H8</td>
<td>Mrs. Harmon</td>
</tr>
<tr>
<td>obs</td>
<td>Classroom observation</td>
</tr>
<tr>
<td>phint</td>
<td>End-of-year interview</td>
</tr>
<tr>
<td>postobs</td>
<td>Post observation interview</td>
</tr>
</tbody>
</table>

Table 3.3: Citation abbreviations
CHAPTER 4

DATA ANALYSIS

The analysis of the data is presented in two broad sections to address the questions guiding the present study. The first section is devoted to teacher/classroom cases, each consisting of two separate classroom sessions. The purpose of the first section is to describe the character of classroom practices in each of the three cases and examine the ways in which activities and interactions are structured for the development of student self-regulated learning and learning mathematics with understanding and the ways in which classroom practices might relate to cultural diversity. The teacher and the classroom are briefly introduced at the beginning of each case. The analysis is then organized in subsections of themes that emerged as lessons were coded. Because of differences among the three cases, major themes do not necessarily occur in every lesson or in every case. Salient themes, based upon either the coverage of transcribed text (by percent) or upon the importance as related to the questions guiding this study, are presented for each of the two lessons within a case.

The second section is devoted to cross-case analysis. The goal of the cross-case analysis was to highlight continuities and discontinuities across cases that might provide insight about the range of classroom practices that meet the needs of non-mainstream
students, in terms of cultural diversity (race, ethnicity, and SES), while learning mathematics with understanding and developing strategic learning behaviors that lead to SRL. Continuities and discontinuities are presented in family groups of themes across the three cases.

Case #1: Ms. Brenner and Students in New York

Ms. Brenner teaches algebra to 9th-grade students in a magnet school located in New York City. Ms. Brenner was in her fourth year of teaching mathematics at the time of the observations used in this study, which was her third year teaching algebra. Students’ self-identified racial and ethnic demographics indicated that the class included Asian (5%), African American (15%), Hispanic/Latino/a (60%), Hispanic/White/American Indian (10%) and Other (5%). One student did not self-identify in terms of race or ethnicity. Ms. Brenner is White. Ninety-three percent of the students in the school qualify for free lunch.

An agenda for starting activity—“Aim” and “Do now” (B5-06-12-14obs), information related to a health class, mathematics posters, a world map, and a variety of student-created posters were displayed on the classroom walls. Student-created posters included a class participation rubric with categories of class discussion, note taking, and listening (see Figure 4.1), a “thought bubble poster” with specific expectations for classroom participation, and another poster where “Math Lover” says to ask accountable questions and reminds that “there is more than one way to solve a problem.” (B5-06-12-14obs; B5-08-06-22phint)
Figure 4.1: Student-created rubric

*Ms. Brenner: The First Lesson*

The first of two lessons in Ms. Brenner’s algebra classroom took place in December of 2006. The objective of the lesson was for students to learn how to solve equations that involve absolute value. As a result of the lesson, students would be expected to be able to calculate the absolute value of a number, create tables of values for absolute value equations, predict the location of the vertex of an absolute value graph in a Cartesian plane, and identify various conditions that change the direction and the vertex of the graphed absolute value equation. The lesson plan began with “preparatory work,” which consisted of a story problem, or word problem of the day, and then moved on to steps for meeting a new objective. Salient features of this lesson, according to the percentage of transcribed text, are *making sense of the mathematics, uptake of student comments*, and *dialogic episodes*. *Student-to-student discourse* stood out not because of the percentage of text covered but because of the quality of the verbal interaction.
As Ms. Brenner began class, students were still settling into their classroom routine. Student chatter and shuffling of papers resonated in the room, which had high ceilings and large windows. Possibly due to the lack of sound absorbing materials in the room, the interactions of students seemed loud and chaotic. Ms. Brenner signaled that class would start by asking for a volunteer to read the posted problem aloud for everyone, but most students continued talking and organizing papers. Ms. Brenner set the expectation for student participation by asking the volunteer to wait for others to be ready to start rather than by asking others to be quiet. The class quieted down quickly, and the student read the problem to the class.

04 Denae: Create and solve an equation for the following problem: A toy company spends $1500.00 each day on plant costs, plus $8.00 per toy for labor and materials. The toys sell for $12.00 each. How many toys must the company sell in one day to equal its daily costs? (B5-06-12-14obs)

The students were instructed to set up and solve the equation that represents the given problem.

05 Ms. Brenner: Now, you may work together. I encourage you to work together at your tables to come up with an equation here. So we have to create the equation and we have to solve the equation and we’re going to ask you to discuss what you did and show your equations. (B5-06-12-14obs)

*Making sense of mathematics.* Ms. Brenner was very clear about expectations for processes the students were to use for making sense of the mathematics of the problem—they would work together, create equations, solve the equations (student small-group work), and then show and discuss the processes they used to create and solve the
equations (whole-class discussion). Talking about mathematics during whole-class discussion included encouraging students to express their understandings, exploring those understandings, and questioning ideas with the purpose of coming to agreement about meanings. Students’ engagement in mathematical argumentation can result in powerful mathematical discussions. The following student-led argument occurred in response to an equation that was written on the board by another student.

69 Denae: I wanted to disagree with something that’s on the board.

70 Ms. Brenner: Okay, Denae has a point. What is it you’d like to say?

71 Denae: You don’t divide 4T.

72 Ms. Brenner: That’s correct. It’s actually just divided by…

73 Students: 4.

74 Ms. Brenner: Right. Because if we divide by 4t then we get 1500 divided by 4t, or 375 divided by t and we get a 1 on this side. That’s a very good point. (B5-06-12-14obs)

Denae identified the error in the equation, but the opportunity for student elaboration on the reasoning used that resulted in discovery of the error was passed up in this example. Instead, Ms. Brenner provided the reasoning. The next excerpt, however, is an example of student analysis and explanation of an error. The student who was initially asked to share her work on the problem was reluctant to display what she thought might be incorrect. For the purpose of discussion, then, Ms. Brenner made up a likely student error. Making up a likely student error is an example of the teacher being aware of what students do during small-group work time and then directing attention toward important
mathematical issues. The made-up error corresponded to what another student, Cee-Cee, had done as she attempted to solve the problem.

74 Ms. Brenner: Zara, you want to put it up here? Okay, how about if I just… Now, I’m just making up this equation so we have something to talk about. Okay? What do you guys think? $8T + 12T = 1500$. Is that going to work? Cee-Cee.

75 Cee-Cee: That’s what I did in the beginning, but then I changed it.

76 Ms. Brenner: Why did you change it? Why won’t this work? Alana.

77 Alana: Because they’re not [spending] $8$ per toy because $12$ per toy equals what they spent so the total amount they spend—they’re saying that it’s $1500 + 8$ for the toy and then that would become $12$ per toy.

78 Ms. Brenner: Okay, that’s a good description. (B5-06-12-14obs)

Alana provided the reasoning for why the made-up equation would not work.

Reasoning is an integral part of doing mathematics (NCTM, 2000). Students’ explanations of reasoning can serve a variety of purposes in a mathematics classroom. Reasoning includes examining patterns and structures in search of regularities, formulating generalizations and conjectures about observed regularities, evaluating conjectures, and constructing and evaluating mathematical arguments (NCTM, 2000). Student explanations of work and descriptions of reasoning strengthen skills in reasoning and provide models for other students by making thinking processes explicit and available for public scrutiny. Those thinking processes can be examined for what works or doesn’t work in a particular context.
Ms. Brenner often encourages her students to make sense of what they are learning through *describing*. Students may describe the important parts of a problem, the meanings of mathematical terms, and/or the processes they used to solve a problem.

78 Ms. Brenner: This is a combination of, what is it, Lakina? A combination of…

79 Lakina: Profits.

80 Ms. Brenner: And… okay. We’ve got profit here, and what else do we have? This is profit…

81 Lakina: Yeah. And the other one is expenses.

82 Ms. Brenner: Expenses, so we’re combining our profits and expenses. And they’re asking us, “How many toys must they sell to equal the daily costs?” That’s a real important point. Very good, Alana. That’s a good description that we have a combination of these two. That [points to notation on the board] mixes us up, whereas here we have the expenses on one side and the profits on the other, okay? Good work. (B5-06-12-14obs)

Ms. Brenner was guiding students as they expressed their ideas, occasionally prompting, and, at the same time, providing cues for social behavior. She offered positive acclamation to encourage both norms for discourse and for mathematical thinking. She did not move on from the correct responses of “profits and expenses” without elaborating on the difference between the two versions of the equation, which is an illustration of her role of commentator after having pressed students to articulate knowledge of the important underlying concepts involved in the problem (Rittenhouse, 1998).

A significant amount of time was allowed for this preparatory work—the introduction, student work time, and discussion took about 20 minutes of the 90-minute
period. Students talked with one-another during work time, freely transitioning between personal social talk and ‘officially sanctioned’ classroom talk, which resulted in a seemingly chaotic atmosphere compared to classrooms where students sit quietly working individually or talk softly in their small work groups. Continued observation, however, revealed that students were engaged with solving the problem.

After the preparatory work time, at the beginning of the discussion of absolute value, students were again encouraged to describe a concept rather than formally defining it. By asking the students to describe the concept, Ms. Brenner was conveying importance of students’ emergent understanding. They were essentially being asking to talk about what they saw happening mathematically, and to develop their own way of expressing that understanding.

85   Marissa:    How do we solve equations involving absolute value?

86   Ms. Brenner: How do we solve equations involving absolute value? Now we’ve been talking a little bit about absolute value already, and is there anyone who would like to take a stab at describing absolute values? Someone else who hasn’t spoken yet.

87   Lakina:      Me?

88   Ms. Brenner: Let’s see. Lakina spoke a little bit earlier. Javier?

89   Javier:      [inaudible]

90   Ms. Brenner: Marissa.

91   Marissa.    Absolute value is the distance the number is from 0.

92   Ms. Brenner: The distance from 0, okay, very good description. … (B5-06-12-14obs)
This exchange took place at a time of transition in the lesson. Students knew that they were beginning a new topic, and a student read the “aim” of the lesson at the teacher’s request (paragraph 85). Ms. Brenner repeated it (paragraph 86), and her next question implied that she was going to build to the point of understanding how to solve this particular kind of equation. She started by exposing what the students already knew about absolute value, and then facilitated the building of understanding through the planned activities using the TI-Navigator.

Ms. Brenner clearly valued talking about mathematics as part of teaching and learning with understanding—making sense of the mathematics. In this case, talking about mathematics went beyond the typical classroom routine of the teacher posing questions to determine whether or not students know the correct answers. Often the episodes within the lesson that resulted in analytic discourse began with Ms. Brenner saying, "Let's talk…” Ms. Brenner was scaffolding student understanding by stepping in to focus student attention on the details that are critical to understanding the mathematics.

98  Ms. Brenner: [Coordinate points were submitted electronically to a Cartesian plane projected at the front of the room.] Hold on a second. Let’s talk about a few things. Now, someone moved a moment ago. If this point were on the line, would that point be correct or incorrect?

99  Student: Correct.

100 Ms. Brenner: Raise your hand. Why?

101 Student: It would be correct.

102 Ms. Brenner: Why?

103 Student: Because it’s on 3 on the x-axis.
Ms. Brenner: Okay, but it’s not on 0. Did I give instructions about y = 0?

Students: No.

Ms. Brenner: No, so this would actually have been correct if it had been on the line. (B5-06-12-14obs)

Ms. Brenner’s active role in facilitating mathematical discourse may be thought of as being comprised of dual roles—participant and commentator (Rittenhouse, 1998). Ms. Brenner did not simply accept correct answers to lower level questions. She stepped in, scaffolding analytic discourse by asking additional questions, to develop deeper mathematical understanding. After the initial question, she probes more deeply by asking such questions as “why?” and “but what about this?”

Successful learning of mathematics, or mathematical proficiency, includes adaptive reasoning (National Research Council, 2001). Adaptive reason refers to students’ capacity for logical thought, reflection, explanation, and justification. The following passage illustrates how Ms. Brenner pressed for reflection and justification of the student’s reasoning. The metacognitive activity that is required for justification of mathematical thinking can be likened to the metacognitive activity that is required in SRL. In order to justify an answer, a student must go back over the thought processes that led to an answer. Metacognitive reflection and justification of problem solutions give students practice for deeper understanding of mathematics and skills for the development of SRL.

Ms. Brenner: Somebody tell me, negative absolute value of negative 2 plus 2. What answer did you get? Cee-Cee. Sorry, Maria.

Maria: Zero.

Maria: Yes.

Ms. Brenner: Absolute value of negative 2 is positive. How did you end up with a negative 2, plus 2? [Other students try to tell Maria what to say.] Hold on, I’m asking Maria; she knows the answer.

Students: We’re helping.

Ms. Brenner: Well I know, but I want you to tell me how that happened. [Maria indicates that she is not ready to explain] Cee-Cee?

Cee-Cee: Because the absolute value is 2 and then there’s a negative sign outside of it, which means that it’s going to come out to be a negative 2.

Ms. Brenner: So it’s going to come out to be a negative 2, and when we add negative 2 and positive 2 that gives us 0. (B5-06-12-14obs)

Often teachers will only question a student or press for elaboration or justification when the student’s answer is judged to be incorrect. In the example above, as well as the previous one, justification of a correct response contributed a model of mathematical thinking. The classroom discourse was focused on conceptual understanding, which included investigation of reasoning that led to both correct and incorrect answers. Through exploration of both correct and incorrect answers, students were exposed to a wider variety of thinking and learning strategies—strategies that they might adopt for their own future use.
Uptake of student comments. Uptake can be defined as occurring when one participant in discourse, such as the teacher, asks a question about something that another participant, such as a student, said earlier in the conversation (Nystrand et al., 2003). Uptake of student comments and responses to questions was important, not only for examination of concepts or for the purpose of understanding mathematics (talking about mathematics), but also for modeling thought processes that are critical to the development of problem solving and self-regulated learning—the initial thinking, evaluating, rethinking, and testing or verifying ideas. Examining incorrect student responses was just as important as examining correct responses because of the opportunity to expose the thought processes that allow for testing and modifying conjectures.

199 Ms. Brenner: …What I’d like to discuss is this right here. I’m actually glad that someone answered this. Is this correct? Is the absolute value of 16 negative 16? And I need some justification. I want to hear what you’ve got to say. Lakina.

200 Lakina: No.

201 Ms. Brenner: Why?

202 Lakina: Because you know like the number is positive but it comes out negative.

203 Ms. Brenner: Hold on; let’s make sure everyone’s listening. Go ahead.

204 Lakina: I said when..., if a number is positive, I mean negative, it comes out as a positive.

205 Ms. Brenner: Why is that, though? Why is that? Romero.
206  Romero: Because absolute value is the number of the counted spaces from 0 to that number. So it should always be positive.

207  Lakina: Yeah, it always should.

208  Ms. Brenner: So even though it’s positive, it still has 16 spaces from 0. Good explanation. (B5-06-12-14obs)

When Lakina first tried to explain, she said the opposite of what she was probably trying to say, stating that the positive value would become negative. Ms. Brenner gave a reminder that all students should be listening. Lakina then reworded her previous assertion, this time making a correct mathematical statement. But, having made a correct statement was not enough to ensure student understanding of the underlying mathematical concept or the reasoning behind modification of the answer. Ms. Brenner pressed for justification of that correct answer. Another student, Romero, stepped in to help Lakina with the justification. Lakina then agreed with Romero’s comments. This exchange can be characterized as more communal in nature than what might be found in a traditional, individualistic school paradigm.

The concept of absolute value, when in the context of an equation that involves taking the opposite of the absolute value, can become confusing. In the following excerpt, a student had solved an absolute value equation correctly, but other students questioned her solution. Students had the authority to question ideas that were presented, whether correct or incorrect, in order to better understand a concept.

312  Student: The absolute value of $w$ is 7.

313  Ms. Brenner: The absolute value of $w$ equals 7. That is correct.

314  Students: No. / 7? / Negative 7, no.
Ms. Brenner: Ladies and gentlemen, let’s talk about something, okay? Now, shhh. I understand and we’re going to talk about this because it was a little bit unclear. It said—let’s take another look at the question—solve for \( w \), okay? Shhh. So in that case, \( w \) could have been what? Raise your hand. \( W \) could have been… Amelia?

Amelia: It could have been negative 7.

Ms. Brenner: It could have been negative 7 or positive 7, so “7” is absolutely correct. If we left the equation at absolute value of \( w \), that would have to be positive 7, and some of you, I know, have been trying to put negative and positive using this or this. Yes?

Student: But when you put in negative 7, after you come down to solve it, it will come down to a positive.

Ms. Brenner: Right. If you leave the absolute value symbol in there, that’s a good point, then it has to be positive. But inside the absolute value symbol it can be positive or negative. (B5-06-12-14obs)

In taking up the topic for discussion, Ms. Brenner acted as facilitator, providing the scaffolding for analytic discussion. She modeled going back to the original problem as part of rethinking the concept of absolute value. Ms. Brenner also provided analytic scaffolding by accepting the student’s description of what happens while solving the equation (paragraph 318) and rewording it so that the student’s mathematical point might be clearer for other students (paragraph 319). Simultaneously, she modeled standard mathematics terminology. By not explicitly pointing out the ambiguity of the student’s statement, she ensured that he made a successful contribution to the discussion, which is another example of non-competitive, communal approaches to learning. Looking back into the problem to verify the validity of a solution is an important part of the cycle of
problem solving. Rethinking in order to verify solutions is similar to the metacognitive activity in the self-regulated learning cycle. Modeling or scaffolding of either supports both.

**Dialogic episodes.** Dialogic episodes occur when participants in classroom discourse exchange ideas in a nonprescripted way, expanding on or modifying the contributions of others (Nystrand et al., 2003). In the following dialogic episode, Cee-Cee asked a question (paragraph 477) about whether or not a pattern she was seeing could be generalized. Ms. Brenner could have answered the question with a yes or no response. She did not do that, however, which allowed the shift to dialogic interaction. Ms. Brenner barely had the opportunity to take up the question as a topic of inquiry when Marco made a statement that other students challenged. The result was a discussion rich in mathematical content that led to deeper understanding of both absolute value and functions.

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477  Cee-Cee:  Can I say something? [...] If you continue like the chart, what, are there [always going to be] patterns?
481  
482  Ms. Brenner:  I don’t know. I’d like you guys to do something for me. Marco?
483  Marco:  Yes, because what if you do something like that, you also find a pattern. No pattern, it means you messed up.
484  Students:  Not always.
486  Alvia:  He said in an equation like that that if there’s no pattern it’s wrong, but I think when it hits 0 it
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stopped at its highest which was 2, and then repeated like a black hole.

487  Ms. Brenner:  Okay, let’s try something. I’d like you guys to all try something. Calculate… let’s add into our table here. I’d like to add negative 4 and positive 4. So we’ll have here x; negative absolute value of x, plus 2; and y. Wait a minute.

488  Student:  Negative 2.

489  Ms. Brenner:  So negative absolute value of negative 4 plus 2 gives you negative 2. What about for positive 4? Waylon.

490  Waylon:  I think it’s negative 2.

491  Ms. Brenner:  Negative 2, so, now we’re talking about patterns here. What pattern do you see? Emanuel?

492  Emanuel:  It’s negative and positive.

493  Ms. Brenner:  Okay, we have negative and then positive. What else do you see? Alvia?

494  Alvia:  I see it go from negative 2, negative 1 to 0, 1, 2, 1, 0, negative 1, negative… Oh, and that goes back to the flipping because if you flip 1, 1, 0, 0, negative 1, negative 1, negative 2, negative 2.

495  Ms. Brenner:  Okay, so we have a match here and here. Somebody give me another match. Marco.

496  Marco:  I have a question.

497  Ms. Brenner:  Yes.

498  Marco:  What I said wasn’t right?

499  Ms. Brenner:  Well, wait a second; I haven’t gotten that far yet. I haven’t gotten that far. Hold on a second. Let’s look at this. Lakina.

500  Lakina:  Negative 1, negative 1.
Ms. Brenner: Negative 1, negative 1. Anybody else see another match? Waylon.

Waylon: Positive 1 and positive 1.

Ms. Brenner: Positive 1 and positive 1. Any others?

Student: 0 and 0.

Ms. Brenner: 0 and 0. What do you attribute that to? How come we have matches on either side?

Student: Wouldn’t that be [inaudible]?

Ms. Brenner: How come we have matches on either side? Alana.

Alana: ‘Cuz like for negative 1, um… […] For negative 1 it says negative, bracket, negative 3, bracket, plus 2. It’s the same; since the bracket is there it’s going to still come out as +3 [inaudible] -1 on the bottom and then we multiply by negative so that comes out as a negative, and then just [inaudible].

Ms. Brenner: Okay, because we’re taking absolute value and there are two possible input values for the output value in absolute value you can have negative 1 and get a 1 or you can have positive 1 and get a 1. That’s why we’re having these duplicate answers. Let’s go back to what Marco said. He said, can you say it again for us? Do you remember exactly what you said?

Marco: When you find a pattern for y…,

Ms. Brenner: I’m trying to remember exactly the words that you said. Anyone remember exactly what Marco said? Lakina.

Lakina: He said that if you don’t get a pattern in the y column then you messed up.

Ms. Brenner: Okay, so let’s talk about that. What is a function? Have we talked about this? …. Somebody describe a function for me. Lakina.
Lakina: A function is when all the things of \( x \) the \( y \) value has one value to it.

Ms. Brenner: Good. Can anybody elaborate on what Lakina said? Denae?

Denae: A function is a relation in which every independent variable has only one dependent variable assigned to it.

Ms. Brenner: These are only being put in one place. We’re not trying to put them in two different places. So if in fact we went sort of all over the place would we be able to identify a pattern?

Students: No.

Ms. Brenner: Every independent variable has only one dependent variable assigned to it. So in that sense you will see a pattern in a function, okay? So even though when Marco said that it may have sounded like, “Oh, oh, wait, I don’t know if we can really prove that for all cases,” with a function, you probably will be able to see that pattern. (B5-06-12-14obs)

Eight identifiable students participated in this extended dialogic episode, which was prompted by Cee-Cee’s observation and question about the pattern formed by the graphical representation of the absolute value equation. Ms. Brenner guided students to focus on data that might be used as evidence to support or refute their conjectures. Her utterance in paragraph 515 could be interpreted as a shift away from the dialogic episode, back to a more “official” mode of classroom discourse; but Ms. Brenner was still focusing on student comments from the discussion to facilitate the construction of an answer to Cee-Cee’s original question.

Two dialogic episodes occurred during this lesson, covering 15.14% of the transcribed text. The dialogic episode started with uptake of a student’s question, but Ms.
Brenner continued to take up students’ comments throughout the dialogic episode. In paragraph 485, she took up Manny’s comments from 483. She refers to Manny’s words again in paragraphs 515 and 517. By taking up Manny’s comments, she recognized and highlighted the importance of his contribution.

*Student-to-student discourse.* Students were involved in conversation with each other throughout the class period. Much of their recorded conversation was related to the assigned mathematics activity. It was quite common, however, for students to intersperse non-mathematical conversation with discourse related to the mathematics. The non-mathematical discussion did not take away from the students’ discussion of the mathematics. Before the next excerpt of student discourse began, students had been asked to practice some absolute value equation problems.

271 Student 1: You shouldn't say it out loud.

272 Student 2: I don’t care.

273 Student 1: Oh my god! […]

274 Student 1: You shouldn’t.

275 Student 3: Shouldn’t what?

276 Student 1: His obsession.

277 Student 3: Oh. You shouldn’t.

278 Student 2: Exactly.

279 Student 1: Did you get [number] 3?

280 Student 4: Yes, I got negative 12.

281 Student 3: I didn’t get 12.

282 Student 4: Or it could be positive 12.
Student 1: Oh, multiply by the reciprocal, right?

Student 2: Duh.

Student 1: Yo! [Exaggerated reaction (pretend anger)]

Student 3: Why you got 8?

Student 1: Huh?

Student 3: Why you got 8?

Student 1: It will be 8 or negative 8.

Student 3: 32 times 4 is 8?

Student 1: Divided by…

Student 3: You have to divide?

Student 1: Yeah, because it’s multiplication and this is the opposite.

Student 3: I give up.

Student 2: I wonder why?

Student 1: Wait, so this side becomes what? […] Oh yeah, x equals 12.

Student 2: You gave up.

Student 3: I gave up a long time ago. I think it’s positive, though.

Student 2: A negative times a negative is positive.

Transitions between mathematical discussion and non-mathematical discussion occurred frequently. Teachers often assume that if students are not focused strictly on the assigned task during the allotted work time, they are not going to learn. Exchanges such as the example above support the idea that students are able to “multi-task” socially
during their small-group work time. The student who claimed to have given up still managed to get in a last word about the mathematics with “I think it’s positive, though” (B5-06-12-14obs, 298).

Summary. The focus of Ms. Brenner’s first lesson was on making sense of mathematics with her students (11.4% coverage of transcribed text). Additional support for making sense of mathematics was provided as she took up student comments for further discussion (17.0% coverage) and solicited multiple responses from students. Taking up students’ comments and soliciting multiple responses has the potential to add an element of authenticity to classroom discourse and support student autonomy. The freedom students felt to contribute to the discussion of ideas resulted in dialogic episodes that covered 15.1 percent of the transcribed text of the lesson. Student non-mathematical social discourse was often freely intermixed with discussion of mathematics during student work time. The “off topic” discourse did not give the impression of obscuring students’ ability to process the mathematics.

Ms. Brenner: The Second Lesson

Ms. Brenner and Ms. Cho team taught the second lesson, which took place in May of 2007, the same school year. The lesson had two distinct sections. The objective of the first section of the lesson was to have students work with quadratic equations \(y = ax^2 + bx + c\) to understand the effects of each of the coefficients, \(a\), \(b\), and \(c\). The second section of the lesson started with a presentation from Ms. Cho on using the distance formula to find the distance between two points on a coordinate grid, which was followed by guided practice. Ms. Brenner was the lead teacher in the first section, and Ms. Cho joined in during whole-class discussion of student work. Ms. Cho led at the
beginning of the second section. Ms. Brenner stepped in to scaffold the class discussion following the presentation of the lesson by Ms. Cho.

Salient features of the second lesson, according to percent of text coded, are 
\textit{uptake of student comments, student-to-student discourse, strategy instruction, and making sense of the mathematics.}

The first activity opened with use of the TI-Navigator technology to project a picture on the screen at the front of the room. In the picture, which had a coordinate grid overlay, a man is on a basketball court. He is crouched in position, ready to take a foul shot or free throw (see Figure 4.2). Students were instructed to work together to “come up with an equation that […] will match the parabola on the screen and get the basket[ball] in the hoop.” Work groups were made up of about four students, and each student was assigned a different role within the group: organizer, calculator manager, recorder, and reporter.

Figure 4.2: Ms. Brenner taking up a student solution for discussion
Each student group received a recording sheet that provided guidance for approaching the problem, including the hint to “look at the window settings on the screen [pictured on the worksheet] and set your calculator window to the same settings.”

Students were expected to experiment with finding an equation and then submit their equations to be viewed on the image at the front of the class. Then, in groups, they were to describe how a parabola changes as the coefficients of the equation are changed.

_Uptake of student comments._ Ms. Brenner took up student comments about the work they had done in their small groups so that the whole class could benefit from everyone’s experience and make sense of the mathematics together. As mentioned above in discussion of the first lesson, uptake of student responses places value on students’ ideas and validates their agentive roles in the discussion of mathematical ideas.

67 Ms. Brenner: Can we all talk for a second? I have a question for you, and I want you to raise your hand if you have an idea. This equation down here, what could the user do to make this equation a little closer up so that it’s shooting from here and landing in here? What Edgar?

68 Edgar: Make $b$ bigger.

69 Ms. Brenner: Okay, so Edgar’s suggesting that we make the $b$ bigger. So this equation, try making the $b$ bigger. What about this equation?

70 Student: It didn’t go in the hoop.

71 Ms. Brenner: What happened here to make this equation [inaudible]? Mike, do you have an idea what to change?

72 Mike: Maybe the $b$ has to be smaller.

73 Ms. Brenner: Maybe the $b$ has to be smaller. Dianna.
Dianna: The parabola has to be narrower.

Ms. Brenner: So how can they make the parabola more narrow?

Student: By putting a large number.

Ms. Brenner: By putting a larger number. Now, what is special about these parabolas? Which direction do they go?

Student: Upward.

Ms. Brenner: So that means that they’re what?

Student: Negative.

Ms. Brenner: They’re negative, so the *absolute value* of the A value is going to be bigger, correct? (B5-07-05-04obs)

The graphed equations (parabolas) submitted by groups of students were used as objects of whole-class discussion. This technique of scaffolding understanding through classroom discourse based on students’ work provided an element of authenticity for the students because it was *their* thinking and the products of *their* work that mattered. Ms. Brenner’s use of discourse also had the potential to support development of SRL as she took up student ideas about how to change coefficients of the equation to better fit the parabolic path. Throughout the analysis, Ms. Brenner did not evaluate the students’ comments. She accepted suggestions for making changes to the equations that would result in changing the shape of the parabola, but *the students* still had to process the meaning of the suggested changes and experiment with entering values for the coefficients of their equation into their calculators to create the parabolic path of the basketball.
Student-to-student discourse. Student-to-student discourse was observed throughout most of the second lesson in Ms. Brenner’s class. Additionally, because the video recording included the placement of an external microphone on a desk within one student work group, a significant amount of student-to-student discourse was captured on the video transcription (11.9% coverage of the transcribed text). These student exchanges might be characterized as “think-aloud” negotiations of mathematical activity embellished with occasional humorous comments, as illustrated in the following excerpt.

146 Student 1: Now we’re back to where we were.
147 Student 2: We have to put 9 point 5, look.
148 Student 1: You put 9 point 5.
149 Student 2: Oh, wait; 9 point 5 won’t work.
150 Student 3: (9 point 5)x plus B. No wait. Make that a little bit bigger.
152 Student 2: 9 point 6.
153 Student 3: There it is.
154 Student 2: We need a little bit more. It’s so close.
155 Student 1: What are we trying to get here?
156 Student 3: The math. He’s already got the basketball. (B5-07-05-04obs)

While there was non-mathematical discussion going on during the class time, the interjection of non-mathematically related comments during small-group work time occurred less frequently in the second lesson observation than in the first. As in the first lesson, the comments did not break the coherent flow of the mathematical conversation.
In this instance, the interjection added a humorous element in relation to the activity.

Students may also have become more familiar with one another and therefore might not have felt as much need for non-mathematical social interaction during small-group work. The social element involved in getting to know each other did not seem to be as urgent toward the end of the school year as it was in the beginning.

*Strategy instruction.* Ms. Brenner and Ms. Cho led students to reflect on their strategic thinking for problem solving during whole-class discussion. This explicit questioning about use of strategies had potential to support students’ development of SRL.

169 Ms. Cho: So what did you notice about when you were finding the equation for the parabola? What were some of your strategies? Vera.

170 Vera: We had to find the points of… We had to find out the [inaudible]. We had to go to 3 on X and then go up to see where the point is to find out how many [inaudible].

171 Ms. Cho: What were you finding exactly when you were looking for that point?

172 Vera: We were just trying to find the numbers to make the [inaudible].

173 Ms. Cho: Okay, Denae, what did you want to add to that?

174 Denae: We were finding coordinate points that are on the parabola and from there we could find the similarities between the numbers to find the equation. (B5-07-05-04obs)

This example of explicit strategy instruction preceded the excerpt below (paragraphs 179-211). While paragraphs 179-211 might be thought of as an extension of the explicit strategy instruction, paragraphs 179 through 211 were more about students
describing mathematical thinking rather than the strategies they used to solve the problem. Students disregarded the original question about what strategies they used to come up with the equation, even though it was repeated in paragraph 201.

*Making sense of the mathematics: Encouraging metacognition.* Metacognition, which is essentially thinking about thinking, can be defined as the conscious awareness of thinking processes used to solve problems and achieve goals (Artzt & Armour-Thomas, 1992). In reference to student self-regulated learning, it may include self-checking to determine if learning goals have been achieved and, if deemed necessary, selecting more appropriate strategies to achieve those goals. Ms. Brenner and Ms. Cho frequently asked students to think about their thinking about mathematics. As the teachers guided them, students were given the opportunity to solidify their awareness of the processes they used, reflect on the outcomes of their thinking, and use their new awareness of their own thinking and the thinking of others to judge the accuracy of their work or to make new conjectures.

179 Ms. Cho: So how did you come up with an equation that you submitted?

184 Student 1: We submitted the right one; well, we kept on working with the… Not the b, we left the b alone, but with the…

185 Student 2: No we didn’t, we changed the b.

186 Student 3: We changed it.

187 Student 1: We changed it?

188 Ms. Cho: What did you notice when you changed the… What did you start off with, and when you changed the b what happened?
Student 1: We started off with, it was negative $x$ to the 2nd power, plus 9$x$ plus 8.

Ms. Cho: Plus what?

Student 1: Plus 8. And we kept on adding to the 9, so we went to 9 point 5, 9 point 6, 9 point 7, and it kept on increasing, and then we stopped at 9 point 9.

Ms. Cho: Okay, so what happened as you increased the $b$ value?

Student 2: The equation rose.

Student 1: The equation rose. The parabola kept on widening.

Ms. Cho: It kept on widening? You said it rose…

Student 1: Increased.

Student 2: Going up.

Student 1: Like going up.

Ms. Cho: So it moved it up; what else? Lakina, I saw your hand.

Lakina: Yeah, um, what was the question again?

Ms. Cho: Well, first of all, for your table, what strategies did you use to come up with your equation? …what happens when you change the $c$ term? What does that affect in the graph?

Student: It goes up, the end, and how wide it is.

Ms. Cho: How wide it… well then what was the $a$ term?

Student: The $c$, it makes it be in the $y$ intercept, go a little higher, move it… it just makes the other part [inaudible].

Ms. Cho: Okay, good. What about the $a$ value?
209  Student: A makes the parabola wider.

210  Ms. Cho: So, you say width of the parabola? So what do we want to say about the $b$ value?

211  Student: It moves the parabola up and down. (B5-07-05-04obs)

Teacher support of students’ metacognitive behaviors, such as self-observation and self-evaluation during problem solving, has the potential to contribute to the development of SRL. The problem-solving situations examined by Ms. Cho, Ms. Brenner, and their students are ideal metaphors for the self-regulatory cycle: Students were asked to consider what they were thinking when they came up with equations, which corresponds to self-observing, evaluate the result of that thinking, which corresponds to self-evaluating, and determine what might create a desired change, which corresponds to goal setting. They were then given additional time to observe what happens after making the change, self-monitoring, and make additional changes as necessary, adjusting strategic methods.

Summary. All of the excerpts included in presentation of the second lesson with Ms. Brenner and Ms. Cho occurred during the first section of the lesson. The first section was focused on making sense of the mathematics (9.2% coverage of transcribed text) and helping students think about their roles in the learning process. Ms. Brenner and Ms. Cho took up students’ comments in order to explore mathematical concepts (14.8% coverage). By using students’ comments as objects of discussion, the teachers conveyed the value of students’ ideas and contributions to learning. The resulting ownership of the learning process by the students provided an element of authenticity that is often lacking in mathematics classes. Ms. Brenner and Ms. Cho encouraged metacognition by asking
students to reflect on the processes they used for problem solving. In addition, students were explicitly questioned about strategies they used for problem solving (6.4% coverage). The focus on making sense of the mathematics, student thinking, and strategy use potentially supported both learning mathematics with understanding and SRL.

The focus of the second part of the second lesson was more procedural—how to use the distance formula to find the length of a line segment on a coordinate grid. Instead of leading the students to generate the distance formula on their own, or perhaps leading them to see how the distance formula relates to what they already know about differences on a number line and the Pythagorean Theorem, Ms. Cho simply presented the formula and then students were guided in applying it to assigned problems. There were instances of challenging students to think, such as in understanding the necessity for keeping the order of the x and y values consistent in relation to the coordinate points when applying the distance formula, but the strong characteristics of the first half of the lesson, particularly uptake of student comments and making sense of the mathematics, were less prevalent in the second half of the lesson.

In both the first and second lessons, Ms. Brenner set norms for classroom discourse that included student verbal descriptions of mathematical situations, talking about and making sense of mathematics, taking up student ideas as objects of discussion, pressing for participation by all students, and encouraging metacognition. The emphasis on open discussion in this classroom produced dialogic episodes that were initiated by student observations and questions about the mathematics. Students were involved in describing mathematical situations, explaining processes they used to understand mathematics, and justifying solutions.
Case #2: Mrs. Blake and Students in South Carolina

Mrs. Blake teaches algebra to 8th-grade students at a magnet school in a South Carolina urban school district in a mid-sized city. The school houses grades Kindergarten through 8th grade. Mrs. Blake was in her 29th year of teaching at the time of the first observation. Her previous teaching experience included all levels of teaching, from Kindergarten through college. She had five years prior experience teaching algebra.

The student population in this school of about 400 students is predominantly White (61%). Other racial groups represented are African American (29%), Asian (8%), Native American (1%), and Hispanic (1%). Mrs. Blake is White. Seven percent of the students in the school qualified for free lunch.

The perimeter of Mrs. Blake’s classroom could be described as creatively cluttered (see Figure 4.3). Bookshelves were piled with books and papers that were tipping and appeared close to spilling over. Tubs of supplies on shelves and the floor had open lids and some had books and papers on top. Game boards were painted on small tables near Mrs. Blake’s desk at the back of the room, and boxes of games were on shelves. Walls and bulletin boards displayed two Einstein posters (“Do not worry about your difficulties in mathematics. I can assure you that mine are still greater.”), several student-created posters, and other student work. There were a few commercially produced “cultural” posters, such as “Math of Africa” and “Math of Arabia,” and a poster of a TI-83 calculator. A very long, horizontal poster spanning the length of the room showed the value of pi written out to the sixty-sixth decimal place value. The front of a T-shirt displayed on a side bulletin board read: “F.O.I.L.: Keepin’ Algebra Fresh.”
Mrs. Blake: The First Lesson

The first lesson observation in Mrs. Blake’s room took place on April 26, 2006. The classroom and adjacent hallway were set up with a total of four activity stations for working with the concept of quadratic functions. One of the most notable features of this lesson, particularly in contrast to the other two cases, was the emotional scaffolding that Mrs. Blake provided for her students. Other salient features of this lesson according to the percentage of transcribed text are IRE pattern of discourse, and strategy instruction.

As the observation began, students were working at all of the stations. Mrs. Blake was interacting with a group of students working from the textbook at one station. At other stations, students were examining weaving patterns in connection to functions, watching a DVD about throwing balls and quadratic functions, and using a CBR to collect data on dropping/bouncing a ball.
Emotional scaffolding. Mrs. Blake started the lesson with a question that acknowledges affect—“Which … do you like…” Students contributed their opinions quickly, along with concerns about what didn’t make sense to them.

02 Mrs. Blake: So, I want to know what you think now, about… You have to solve a quadratic equation. Which method do you like best?

03 Students: Quadratic formula. / I like graphing. [loudest] / Quadratic.

04 Mrs. Blake: You like graphing; which, what?

05 Student 1: [loudest, over other student comments:] It was weird because all of a sudden I got a negative inside the square root and then you can’t get a negative…

06 Mrs. Blake: No problem, no problem. You just wait until you see what happens next, because I haven’t told you the whole story.

07 Student 1: But I like the way it does, but it gets the numbers all messed up.

08 Mrs. Blake: Hang on. Hang on. [To another student:] That’s perfect, Cindy, I love that. [To another student:] So you still like graphing. You’re still pretty good on that. Anybody else; who likes… Um, what’s that?

09 Student: …factoring.

10 Mrs. Blake: You like factoring. I like factoring, too. But, the problem is, you can’t always use it. (B7-06-04-24obs)

Throughout her interactions with the textbook group, Mrs. Blake asked students to evaluate the methods that they were learning for solving quadratic equations and to compare them in terms of usefulness or benefits. She first elicited responses from students in order to generate a list of the possible solution methods. Students had a lot to express and many of their comments were inaudible as they talked over one-another.
Right from the start, Mrs. Blake provided emotional support for her students, assuring them that they would be able to move beyond their difficulty. As they went through the routine of checking answers to homework problems, however, it became clearer that some students were extremely frustrated with the numbers of errors they had made.

Later, Mrs. Blake pressed students to discover what was causing the difficulties they had with the mathematics (paragraph 66, below). Very soon after, however, she decided that whatever she had been pushing for in terms of understanding could wait until the students had more experience with the problems (in paragraph 76, “Okay, I think that’s a reason for right now.”). Mrs. Blake seemed to realize that development of understanding is emergent, not an all or nothing phenomenon. Students needed more experience working with the content.

66  Mrs. Blake:  Okay, okay, why do you think you might have gotten…? Hey. We’re going to get it, don’t worry. Why do you think you might have gotten one right but not the other? What’s that last step that you do there that could cause you some problems?

67  Student:  I only got the first one right. All the rest are wrong.

68  Mrs. Blake:  What happens at the very end?

69  Student:  I don’t like this. [a whiney tone]

70  Mrs. Blake:  You are going to… It’s going to be… First of all you have to…

71  Students:  Switch the numbers.

72  Mrs. Blake:  Switch it, right? You’ve got to either add or subtract to get, to solve for the variable. And then look what you’re doing. It’s either plus or minus, right? And I think that’s where you might get mixed up. [Student
interjection: Oh yeah.] And also, if it’s a decimal, sometimes don’t you – I think I kind of mess up sometimes on the decimals, do you?

73 Students: Yeah.

74 Mrs. Blake: Why?

75 Students: It’s hard. / … rounding…

76 Mrs. Blake: Yeah, rounding, but also I might not be thinking quite… It’s a little harder, don’t you think? Okay, I think that’s a reason for right now. You need to continue work on doing every step bit by bit. Okay. Let’s… What are we on?

77 Students: 16.

78 Mrs. Blake: 16. It’s negative four tenths.

79 Students: Right. / I got that one wrong.

80 Mrs. Blake: And, 4 and four tenths. Are you way off?

81 Students: If you were rounding… / I’m way off. / I’m nowhere close.

82 Mrs. Blake: Oh, okay.

83 Student: Way off.

84 Mrs. Blake: Okay. Here’s what I really need to do for those of you who are thinking "I’m not getting this." I’ve got to look at all of your work. I can’t just look at your answers. So you need to give me your papers with all the work on there so… And we’re going to need to go over it. (B7-06-04-24obs)

The process of going over homework was supposed to have been a quick review before the main assignment for that station in the rotation of activities. Students were becoming increasingly frustrated as they realized their poor performance on homework, and Mrs. Blake seemed to be having difficulty understanding the particular difficulties
students had with the homework. Because of the lack of flexibility with timing imposed by the small-group rotation format, Mrs. Blake decided against having students share their work for class examination; consequently students were not able to benefit from exploration of understandings and misunderstandings during this class period. The emotional scaffolding was evident, but exploration of the mathematical concept was deferred.

**IRE pattern of discourse.** IRE is a small-scale pattern of discourse that is usually initiated by the teacher who asks a question, students signal that they understand or can respond to the question, and the teacher calls on a single student. The chosen student then provides an answer to the question that the teacher evaluates as either correct or incorrect (Greeno, 2003). During the first lesson, Mrs. Blake spent most of her time supporting students at the homework/textbook station. Her interaction with students at that station started with checking homework and then introducing the new assignment. The resulting pattern of discourse was mostly IRE, with Mrs. Blake asking students to volunteer an answer to the homework problems, one at a time, a student responding with the solution, and then Mrs. Blake evaluating the answer. Double forward slash marks (///) indicate the boundaries between units of IRE.

30 Mrs. Blake: What is it, Conrad?
31 Conrad: 1, 2?
32 Mrs. Blake: 1, 2 is correct. // How about number 2?
33 Student: 4.
34 Mrs. Blake: 4 is correct. // Did anybody do number 3?
Student: I got 4 for number 3.

Mrs. Blake: You got 4 for number 3? It’s ¼. (B7-06-04-24obs)

This pattern of discourse continued throughout the lesson, but was interspersed with segments of less ritualistic-sounding discourse. In the next example of classroom discourse from the end of the lesson time with the second homework/textbook lesson group, students interjected comments in such a way that they appeared to be directing the conversation. They knew the direction of the reasoning, so they followed that path, essentially leading the teacher.

Mrs. Blake: So where have we gotten so far. Okay, simplify, // and what is the value of the discriminant?

Students: Negative 23.

Mrs. Blake: Negative 23. //

Student: It’s less than 0 so there's no roots.

Mrs. Blake: It’s less than 0 so…

Students: No roots. / Do you have to do anything?

Mrs. Blake: That’s it; you’re good to go. So, now, …

Student: We have another thing.

Mrs. Blake: Yeah, but what happens now?

Students: You don’t have to do it.

Mrs. Blake: Case is closed. (B7-06-04-24obs)

In paragraph 233, Mrs. Blake repeated the first part of a student’s comment, and the rest of the students finished the sentence. The remainder of the exchange shows the kind of “short-hand” speaking that occurred frequently in this class. The shifts between
Mrs. Blake and the students in leading the conversation broke the prevailing IRE pattern. Without knowledge of the context, a listener might have difficulty following along.

*Strategy instruction.* After the homework check routine and before the new lesson was discussed, Mrs. Blake talked with students about reading texts. Even though the exchange was not about correct and incorrect answers, the IRE pattern of discourse continued, which gives the impression that she was affirming the responses to the open-ended questions. Open-ended questions technically do not need affirming, except perhaps to encourage the students to feel good about having contributed to the discussion.

143 Mrs. Blake: When you read this what helped you or what hurt you? Any ideas? In the book or the page.

144 Student: Vocabulary helped.

145 Mrs. Blake: The vocabulary helped. // So the vocabulary helped. What vocabulary did you use?

146 Students: Discriminant.

147 Mrs. Blake: Discriminant, okay. //

148 Student: The charts helped.

149 Mrs. Blake: The charts helped; okay. // How many times did you read this?

150 Students: Twice.

151 Mrs. Blake: You read it twice? // When you read it the first time, what, what…

152 Student: I didn’t get it.

153 Mrs. Blake: You didn’t get it the first time? // What helped you the second time?
Students: (talk over each other)

(B7-06-04-24obs)

This exchange between Mrs. Blake and her students was about strategies for getting information from a textbook. However, even though multiple ideas are solicited, each student comment receives little if any further attention. Mrs. Blake did prompt more comments about reading a text more than once for understanding, which led to the student’s comment about discussing it and/or reading it again. Again, the discussion has a “short hand” character that either lacks depth or presumes communal understanding.

Throughout the lesson, Mrs. Blake cued students to be aware of a variety of strategies for learning, from how to read a page of text for understanding, as in the example above, to raising students’ awareness the “tricky” part of using the discriminant of a quadratic function to find the roots (see Table 4.1).

The reminders for strategic action were only one or two sentences long. They were not discussion of the strategies as much as reminders embedded within discussion of mathematics. Except for the discussion about how to use a text, Mrs. Blake did not try to draw out ideas about learning strategies from her students. Most of her discourse with students during the lesson took place during work with the group assigned to review of the homework and the textbook lesson. In the homework/textbook group, students tended to simply respond to Mrs. Blake’s questions and she did most of the talking.
So I want you to read this and see if you can figure out words within a word. (B7-06-04-24obs, 91)

Okay, let’s put this up here. If a discriminant is positive that means it has 2 roots. If it’s 0… (B7-06-04-24obs, 299)

Also, so that you write down that negative because sometimes, I do, I forget maybe that I need to put that negative in there. Be sure that you have that in there. (B7-06-04-24obs, 362)

Table 4.1: Examples of cues for awareness of learning strategies

Mrs. Blake encouraged students to participate in evaluation during lessons in a variety of ways. The encouragement of periodic assessment on a regular basis throughout the lesson is a form of strategy instruction that has potential to contribute to the development of self-regulated learning. Mrs. Blake also asked her students what they thought they needed, such as whether or not to work with more examples of a particular kind of mathematical problem or move on to something new. She encouraged the students to evaluate the utility of information or processes that they used to solve problems. This, too, supports development of metacognitive skills that may contribute to the development of SRL.
Mrs. Blake: Okay, $b$ squared minus $4ac$. So what good is this?

Students: It helps to find the root.

Mrs. Blake: Does it give you the root?

Students: No.

Mrs. Blake: What does it do?

Students: How many.

Mrs. Blake: Determines how many. Determines how many roots there are. So how does it do that?

Students: You put it into the equation.

Mrs. Blake: So all you’re doing is what? For $b$, $a$ and $c$?

Students: Substituting.

Mrs. Blake: Substituting your $a$, $b$ and $c$, and tell me what you might get. Does it give you the root?

Students: No. The number of roots. (B7-06-04-24obs)

In addition to identifying the purpose of using the formula ($b^2 - 4ac$), this exchange between Mrs. Blake and her students emphasized the opposite, what it was also not meant to find. The formula is used to find the number of roots, but not the roots themselves. This is an important distinction that can be quite confusing to students, especially when working with formulas and manipulation of symbols in algebra. The emphasis in this exchange was about using the formula for that specific purpose, not about the reason the formula works or why it leads to identification of the number of roots; consequently, the utility of the information was limited to formulaic application.

Summary. Emotional scaffolding was a prominent feature of Mrs. Blake’s first lesson (6.5% coverage of transcribe text). She was concerned about student efficacy—
making sure that her students knew that they were able to do mathematics even if they encountered difficulties along the way (B7-08-04-24postobs). Strategy instruction was included regularly within classroom discourse, but not as a separate topic of discussion (6.7% coverage). It was interwoven with regular classroom discourse about mathematics. Mrs. Blake encouraged students to evaluate class work, the utility of information they were working with, and their thinking processes. Her press for understanding, however, was most often related to the application of a formula rather than why something works or where it came from, with particular emphasis on using the discriminant to find specific information related to a quadratic equation. The predominant pattern of discourse consisted of segments of IRE (13.5% coverage).

Mrs. Blake: The Second Lesson

The second lesson observation of Mrs. Blake’s class took place in May of 2007. This lesson occurred just over one year after the first lesson observation, with a different class of students. The student population in this class was predominantly White (61%). Other racial groups represented were African American (28%), and Native American (6%). One student did not self-identify. Seven percent of the school student population qualified for free lunch.

The objective for the lesson was for students to understand graphing quadratic equations without using a table of x and y values, which implies that students would have to find the roots of the equation, the line of symmetry, and the vertex of the parabola. Salient features of this lesson, according to the percentage of transcribed text, are making sense of the mathematics, uptake of student comments, and challenging students to think.
Other topics of discussion included because of relevance to the guiding questions are *argumentation, assigning competence, and expression of enjoyment.*

The lesson began with an activity to review graphing quadratic equations on graph paper. The activity was initially presented to the students as a “pretend quiz,” but shortly thereafter students worked together in small groups. The goal for the second part of the class session was to reinforce what students knew about coefficients of quadratic equations through the generation of equations to match parabolic shapes within pictures, such as the spray of fountains and arches, using the TI-Navigator technology.

As students arrived, Mrs. Blake was organizing materials for the lesson. The classroom atmosphere was relaxed. Music was playing while students came in and settled into their routines. They were chatting with one another when Mrs. Blake turned down the volume of the music at the beginning of homeroom, but she did not turn the music off. There was no pressure to get started immediately. After several minutes, students were assigned the “quiz” problem. It was suggested that they work by themselves for a while. Within a few minutes there was a great deal of discussion, and Mrs. Blake went from one area of the room to another to support small-group work.

*Making sense of the mathematics and uptake of student comments.* Stepping in to scaffold student understanding as a technique to facilitate whole-class discussion was described above, in the case of Ms. Brenner and her students in New York City. In Mrs. Blake’s class, similar scaffolding was provided on an individual or small-group basis as she walked through the room and assessed student progress. She asked students guiding questions or made guiding comments to ensure that they would be able to progress.
Mrs. Blake: What do you have so far?

Student: Well the 2...

Mrs. Blake: Look at the vertex, Chantelle.

Chantelle: [inaudible]

Mrs. Blake: Solve for what?

Chantelle: [inaudible]

Mrs. Blake: Okay. (B7-07-05-03obs)

Even though the student’s comment and responses to questions were inaudible, it was clear that Mrs. Blake had directed Chantelle’s attention so that she was able to adjust her thinking to make sense of the mathematics. In the next example, Mrs. Blake used the same technique of questioning a student rather than telling him what had gone wrong with the work he had done.

Mrs. Blake: Ooh. Now, take a look at that. Your graph is going down. What's going to cause it to do that?

Student: [inaudible student response]

Mrs. Blake: The negative. Where should that negative be?

Student: I got it. (B7-07-05-03obs)

The student’s graph was constructed to correctly show the “upside down U” shape. Mrs. Blake, following a typical pattern of discourse, repeated what the student said in response to her question, “The negative” (paragraphs 39 through 40). But the coefficients of the variables in the equation written to represent the parabola were incorrect. This example illustrates the social construction of knowledge in this classroom.
as the teacher directed student thinking by questioning the student about observable representations of his thinking and then negotiating the meaning through additional questioning.

When whole-class discussion began, calling for student attention with ringing a small bell included the expectation that there would be discussion about the mathematics. In this particular case, discussion revolved around strategies of what to look for in an equation before graphing. Mrs. Blake guided students to understand the purpose of each of the terms of the equation so that, when graphing the equation, they would know what they could expect it to look like. Students would then have a way to assess their own work—to determine whether or not the graphs they produced were reasonable.

60  Mrs. Blake:  [Mrs. Blake rings a small bell] Everybody... Let's talk about this real quickly.... Okay. When you first see an equation up here, what is one thing that's going to be sort of your first thought, what stands out when you have a parabola, almost more than anything else? Andrew?

61  Andrew:  I would graph the line...

62  Mrs. Blake:  Okay, but...

63  Andrew:  ... axis is...

64  Mrs. Blake:  ... well that's a good plan. Meg?

65  Meg:  Oh, I was going to say that...

66  Mrs. Blake:  Say that? What?

67  Meg:  That x squared.

68  Mrs. Blake:  The x squared. Is that what you look at first? Why do you look at that first?

69  Meg:  'Cause it tells me it's a parabola.
Mrs. Blake: It tells you it's a parabola...

Patrice: And it tells you the negative... (B7-07-05-03obs)

Starting on paragraph 67 above, Meg contributed the idea about looking at $x$ squared first when examining a quadratic equation. Mrs. Blake took up Meg's idea and the conversation that follows built on it with Patrice adding that the $x$-squared term also let you know that the parabola was negative (paragraph 71), which was the way students described parabolas with a negative coefficient. Mrs. Blake took up students’ comments often, which can be seen in excerpts of classroom discourse below, such as paragraphs 88, 96, 98, 262, and 270. Uptake of student responses places value on students’ ideas and validates their agentive roles in the discussion of mathematical ideas. It also has the potential to facilitate negotiation of understanding and promote coherence by establishing intertextual links between speakers (Nystrand et al., 2003).

**IRE pattern of discourse.** IRE was not the dominant pattern of discourse in Mrs. Blake’s second lesson, but it stands out because of the difference in character between the two lessons. Unlike traditional use of IRE, where a teacher asks a question, a student responds, and the teacher evaluates it as either correct or incorrect, Mrs. Blake asked a question, a student answered, and Mrs. Blake acknowledged the answer; but then she often followed with a comment/question that suggested she wanted to hear more. This was not just different from traditional IRE, but different from the examples of IRE from the first observation lesson.

Mrs. Blake: Substitute. Substitute. What are you going to substitute? Shanelle, you're good at this.

133 Mrs. Blake: Substitute. Substitute. What are you going to substitute? Shanelle, you're good at this.
Shanelle: You’re going to substitute the …, the negative 1 for the x.

Mrs. Blake: Substitute negative 1? Where did you get negative 1?

Shanelle: [inaudible] from the [inaudible]

Mrs. Blake: Oh, positive 1. // Where do you get that 1 to substitute in?

Student: You do a table.

Mrs. Blake: Okay. You could do a table, couldn't you?

Student: Yeah.

Mrs. Blake: You could do a table. // Is that, um... But how else could you do it?

Student: From the line of symmetry.

Mrs. Blake: Oh. I see what you mean. Are you talking about... yeah, yeah. // But how else could you do that? Shanelle.

Shanelle: Figure out the y-intercept.

Mrs. Blake: Okay. // Then you do what? I'm still not quite sure what we're doing next.

Shanelle: Then you substitute all the x's, [inaudible] …

Mrs. Blake: Okay, // so Shanelle says that what we're trying to figure out now is a value for y when what?

Student: When x equals 1.

Mrs. Blake: When x equals 1. // (B7-07-05-03obs)

In the example above, Mrs. Blake was leading a particular student, Shanelle, to express her ideas accurately and more thoroughly. Shanelle did not articulate a correct
answer to the question of what to substitute initially. Mrs. Blake asked her to explain where she got her answer, which helped Shanelle to discover the error in her thinking.

The dialogue did not stop with the correct number answer of positive 1. Mrs. Blake continued to probe for voicing of student reasoning. The units of IRE built upon one another to scaffold students' mathematical thinking. By staying with the same student and questioning her response, Mrs. Blake conveyed important messages about learning; first, that any response to a question can be probed further, whether it is correct or incorrect, so that the thinking processes that students use become available for discussion, and second, that Mrs. Blake believes her students are all capable of thinking through the problems presented in class and that she is there to support them as they construct knowledge.

Just minutes before the exchange with Shanelle, another student had identified $x = 1$ as the line of symmetry for the parabola. It would follow, then, that the reason for using an $x$ value of 1 is to find the value of $y$, thus identifying the vertex of the parabola, in the form of coordinate point $(x, y)$, through substitution. The vertex is the point of the parabola that is on the line of symmetry. The $x$ value of 1 in this discussion came from the line of symmetry, $x = 1$. The students did not demonstrate a strong understanding of the relationship between the line of symmetry and the vertex of the parabola at that point in the discussion. The significance of the line of symmetry had been touched upon, but the knowledge was still fragile.

*Challenging students to think.* Mrs. Blake challenged students to think consistently through the second lesson. The subtle variety of wording from one instance to another was notable. In Table 4.2, several examples are displayed along with a brief
description of the context. These examples show that Mrs. Blake’s challenge encouraged the development of other behaviors related to understanding in mathematics and student self-regulated learning, (persistence, finding alternative solution paths, self-evaluation, evaluation of proposed ideas, articulation of ideas, metacognition, and reflection on learning).

Mrs. Blake’s question in paragraph 328 of Table 4.2, in response to students indicating that they "get it,” provided a context for the possibility of metacognition, which is an important part of the cycle of self-regulated learning. Students were encouraged to describe both what they didn't get, and to further process how to adjust or change, and what they did get, solidifying the knowledge and sharing their thought processes. In this instance, the student was asked to articulate what happened in the interim, between not knowing and knowing. By thinking about their own development of knowledge, students engaged in metacognitive processes, and through those processes, others might take on learning strategies for future independent use.

This type of encouragement of metacognition was also seen throughout this lesson as Mrs. Blake asked students to tell her something they learned that they didn’t know before. The knowledge, which may have been fragile during the initial stages of learning, became stronger through reflective discourse (Cobb et al., 1997). Additionally because “understanding also includes developing a stance toward what we know and how we come to know it,” reflection helped “students to develop a sense of the history of their learning and to build identities as mathematical thinkers” (Lehrer et al., 1999).
<table>
<thead>
<tr>
<th>Paragraph</th>
<th>Excerpt</th>
<th>Context</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>I don't know, guys. I don't know. I haven't done it yet.</td>
<td>Students were trying to get Mrs. Blake to verify their solution to a problem, but Mrs. Blake wanted them to keep processing it for themselves. (persistence)</td>
</tr>
<tr>
<td>91</td>
<td>So let's think about that again.</td>
<td>A proposal to find another way to approach the problem (finding alternative solution paths)</td>
</tr>
<tr>
<td>93</td>
<td>You're saying the negative 3 tells you whether the graph goes up or down.</td>
<td>Trying to get the students to evaluate their own ideas by presenting their own words back to them (self-evaluation, metacognition)</td>
</tr>
<tr>
<td>145</td>
<td>I'm still not quite sure what we're doing next.</td>
<td>An attempt to get a student to rethink and articulate her ideas more clearly (articulation of ideas, metacognition)</td>
</tr>
<tr>
<td>217</td>
<td>...wider than others. What do you think?</td>
<td>Asking students to evaluate her observation of their work (evaluation of proposed ideas)</td>
</tr>
<tr>
<td>328</td>
<td>What do you think, Alison? What is causing you to get it?</td>
<td>Asking a student to express a reason for her understanding. (metacognition)</td>
</tr>
<tr>
<td>337</td>
<td>Okay. Tell me what you know now that you didn't know a second ago.</td>
<td>This bid for student feedback was used frequently to get students to reflect on their learning. (reflection on learning, metacognition)</td>
</tr>
</tbody>
</table>

Table 4.2: Challenging students to think
Argumentation. Later in the lesson, while Brenan was graphing the vertex of the equation, another student (Student 2) challenged him inappropriately. The challenger interrupted the discussion of the usefulness of graphing the y-intercept. Students had already interrupted the flow of discussion by telling Brenan how to use the eraser on the interactive whiteboard. (The writing pen must be set down when using the eraser.) Here, the challenger did not identify what he thought to be incorrect nor did he explain his reasoning; but simply yelled out that the other student had “messed up.” Additionally, the comment could be interpreted as being directed at the student rather than at an idea.

151 Mrs. Blake: Brenan says negative 4. Is that what you got?
152 Students: Yes ma'am.
153 Mrs. Blake: You did? Okay. [To Brenan] You want to go ahead and put it up there? [To the rest of the class] So what good will that do?
154 Student 1: It makes an ordered pair, so that's where you'll find the vertex.
155 Mrs. Blake: Okay. And what...
156 Student 1: [To Brenan, who was at the interactive whiteboard] You've got to put the pen back down.
157 Mrs. Blake: [to Brenan] There you go. Go ahead and put that in. [back to the class] And then after your vertex, what are you going to do?
158 Student 1: Get the other point on the other sign [sic],
159 Student 2: Oh, wrong!
160 Student 1: ...on the other side, which is 2,
161 Student 2: Brenan messed up.
162 Student 1: ...negative 3.
Learning what was acceptable and not acceptable as argumentation was part of establishing classroom norms. Using very few words, Mrs. Blake let Student 2 know that his method of challenging was not acceptable. Mrs. Blake used a soft but distinctly different tone of voice, getting her point across with minimal disruption. She then proceeded with the lesson.

In the following passage, Mrs. Blake noticed that a student’s graph and the equation he had written did not match. Her question to him, which was meant to direct him to the coefficient of $x^2$ in the equation, elicited a response that was related back to the graph and not the equation. She then asked specifically about the equation, and he identified the incorrect term, negative 3, which was the constant. Mrs. Blake cued the student, Hayden, to express his idea as a whole sentence rather than just a phrase so that he might recognize his error. Another student made a comment about negative three being the $y$-intercept (paragraph 89), at which point Hayden recognized his error. Mrs. Blake then seemed to be cuing the whole class to finish the sentence, but Audra was still processing the original question. So Audra contributed an idea related to the question in paragraph 84, which was then challenged by another student (paragraphs 95 and 97).

84 Mrs. Blake: How do you know it’s going to go [shows upward with arms] or [shows downward with arms]? How do you know that, Hayden?
Hayden: The points, if it's negative, the points are going down.

Mrs. Blake: Well, yeah, but what if you just look at the equation, how are you going to tell?

Hayden: Oh, well, the negative 3.

Mrs. Blake: Negative 3 will tell you that...

Student 1: ...y-intercept...

Hayden: The y-intercept, so that’s… never mind.

Mrs. Blake: Okay. So let's think about that again. So negative 3... [waits for students to fill in]

Mrs. Blake: You're saying the negative 3... [no response from students]

Mrs. Blake: Jack, Carol, listen to this. See what you think. You're saying the negative 3 tells you whether the graph goes up or down.

Mrs. Blake: Say that, Audra.

Audra: If x is negative or positive.

Mrs. Blake: If x is negative or positive. That tells you.

Student 2: No; x squared.

Mrs. Blake: X squared! If x squared is negative or positive... everybody... eeverybody. If x squared is negative or positive. So what's x squared in this case?

Students: Positive.

Mrs. Blake: It's positive. How do you know?

Students: Because it doesn't have a negative sign.

Mrs. Blake: So what does that tell you?

Student 3: That it's going to be a positive parabola.
Mrs. Blake: And so which direction should this be?

Student 3: Up. (B7-07-05-03obs)

In this passage of text, Mrs. Blake did not evaluate the students’ ideas. Her repetition of words was neither verification nor contradiction of the correctness of students’ comments. She repeated what students had contributed so that other students might evaluate the comment, possibly question it, or make new conjecture—a effort to engage students in argumentation. However, her efforts did not produce the desired effect. This exchange lacked fluidity due to lack of discussion of student comments. The result is a disjointed set of student responses from paragraph 87 to paragraph 99. When students proposed different answers, they did not provide the reasoning, thus they never really made an argument. An argument for an alternative relies on the provision of reasoning. Additionally, it was never made clear that it is the coefficient of $x^2$ that determines the direction of the parabola on the graph. Mrs. Blake did, however, ask for justification of the assertion that the parabola would be positive [or negative], which was where the discussion started in response to Hayden’s graph that showed a parabola that opened downward.

In the next excerpt which occurred later in the lesson, Mrs. Blake consistently pressed for argumentation and she scaffolded students in the process by prompting them to respond to specific comments, giving suggestions for how to start rewording responses, and establishing the expectation for justification by asking, “Why?” Justification of mathematical processes used, including modeling and discussion of mathematical thinking, helps students develop a range of strategies from which to draw
when graphing quadratic equations. The excerpt started shortly after Mrs. Blake had asked students, “What do you know now that you didn't know four minutes ago?” The student comment that initiated the episode of argumentation was not necessarily wrong, but it was not acceptable within the established sociomathematical norms. The student could not make the generalization without clearly establishing what he meant by “the farther down you go.”

261 Student 1: Well, I kind of know this, but I expanded my knowledge with this, that the farther down you go with negative x squared, the fatter it goes. So like…

262 Mrs. Blake: What did we say about that?

263 Student 1: The more it goes...

264 Mrs. Blake: So is it the...

265 Student 1: …the greater the value...

266 Mrs. Blake: …the smaller the value… Is it really?

267 Student 1: Yes.

268 Mrs. Blake: The smaller the value. So if you have...

269 Student 2: No. Not necessarily.

(Andrew?)

270 Mrs. Blake: Not necessarily? Why?

271 Andrew: If you have negative 5, it's going to be a skinny negative.

272 Mrs. Blake: It's going to be. Did everybody hear what Andrew said?

273 Students: No. / Yes. / Can you repeat it? [a lot of student talking]
Mrs. Blake: Here's what I say. See if you agree with me. Listen to me guys. Listen to me guys. See if you agree with me. The smaller the number, the fatter the parabola. Andrew's disagreeing with me. You're disagreeing with me?!

Andrew: Yes.

Mrs. Blake: Why, Andrew?

Andrew: Because if you have negative 5, it's going to be a smaller negative parabola than if you have negative 1.

Mrs. Blake: So what are we going to say about that number?

Elliott: Absolute value.

Mrs. Blake: Say that louder. A little bit louder, now.

Elliott: Absolute value is... what... you...

Mrs. Blake: Okay. Can you say it? The smaller the... [hint]

Elliott: Absolute value, the wider the... [Student's voice fades out.]

Mrs. Blake: Okay. I don't think everybody heard that, but you're right, Elliott. Nice work, Andrew. (B7-07-05-03obs)

When Mrs. Blake took up Andrew’s argument (paragraphs 269 and 271), she put forward a generalization [italic in paragraph 274] that did not correspond to the example given by Andrew. She provided the generalization to give students an opportunity to think about how a negative coefficient of $x^2$ affects the appearance of a graphed quadratic equation. Andrew disagreed with her generalization, and was expected to explain why. But that still was not enough. A fourth student, Elliott, reworded the generalization so that it was a correct mathematical statement.
As in the other example of scaffolding student argument, Mrs. Blake had to press for involvement in argumentation; the argumentation was not spontaneous. But she also seemed to jump in too soon with additional questions in the process. Students were not given enough opportunity to challenge other students’ statements or to defend their own. In paragraph 274, Mrs. Blake put forward a new generalization instead of letting Andrew repeat what he had said. It was then Andrew who responded to the generalization, although he again used a specific example instead of a corrected generalization.

The terminology used throughout the excerpt lacks specificity, which may have caused some confusion. The first student comment included “negative x squared.” As the conversation continued, x squared was referred to as “the number,” which could mean any of the coefficients in the equation. It also may have been the case that students didn’t understand Mrs. Blake’s intention for the direction of the discussion. A more explicit question, such as, “What can we then say about all coefficients of $x^2$?” may have made it clearer to the other students that she was expecting to hear a generalization. The term coefficient was not used during the discussion.

**Assigning competence: Students as technological experts.** Teachers are not always as savvy with the latest technologies as they would like to be, or as their students are. Mrs. Blake had access to a lot of new technology for supporting teaching and learning in her classroom, but she was a novice user (B7-08-04-24postobs). The TI-Navigator system had been in her classroom for only a few months. However, students in her class were fairly comfortable with using technology, and some students took on specific roles
in providing technological support. As they moved to a new picture to use for matching parabolic shapes, the students’ roles in providing technological support with the TI-Navigator became more apparent.

300 Mrs. Blake: Okay, you want to put yours in? [To a student at the front with a jump drive]

301 Mrs. Blake: Yeah, it scanned. But, Andrew, I'm going to save it, can you save it in 'my pictures'. Because that's kind of cool, isn't it? Slide it down. It's pretty groovy. You think it's going to be hard?

302 Mrs. Blake: In my documents. Save it in there. Okay. And now get rid of it for right now. And, let's try it this way.

303 Mrs. Blake: You guys know, I don't know how to do this. I know Henry does. Do you know how to do this?

304 Henry: What?

305 Mrs. Blake: How are we going to do it because I've got to load a background image? I've got to go up here and, oh, yeah, I've got to load the background image. So where am I...? [Henry points.]

306 Mrs. Blake: Right here? [Henry gets up and does it for her.]

307 Mrs. Blake: There you go.

308 Students: [All students give advice at once]

309 Henry: There we go.

310 Students: Ohh! [Picture of two arches on the screen] (B7-07-05-03obs)

The students who took on the technical support roles were encouraged to do so by Mrs. Blake. In this particular class, the students who were observed in the roles of technical experts were African American males. As described in Case #1 where the teacher remarked on a student’s contribution in order to assign competence, Mrs. Blake
encouraged students who might otherwise not stand out as leading students of mathematics to take on technical support roles which highlighted their expertise. They might become more comfortable with contributing to discussion of mathematics as they came to be seen as competent contributors in other ways related to learning in the classroom. This was also evident within another excerpt from the lesson presented above where Mrs. Blake made a comment that had a dual role—to encourage the student to feel efficacious (assigning competence) and to draw her into the discussion: “What are you going to substitute? Shanelle, you're good at this” (B7-07-05-03obs, 133). Assigning competence raises the status of students who may be of lower status in a classroom by publicly praising something they have said or done that makes a positive contribution to the class (Boaler & Staples, 2008).

Enjoyment of activities. Mrs. Blake and her students often expressed appreciation for each other’s work. Students freely expressed their enjoyment while doing mathematics. In the following passage, Mrs. Blake conveyed her enthusiasm as she searched for a background picture that she thought the students would enjoy, and she did get a positive, appreciative response from the students. She then credited the student, Shalonda, who sent the picture. By praising her choice in front of the entire class, she was assigning competence to a student who might be overlooked in a more traditional activity setting (Boaler & Staples, 2008).

187 Mrs. Blake: Okay, so, um ... let's see what we can find, because look, a lot of people sent things to me. Oh, I want you to look at this one.

188 Students: Ooh! Aah. [Many simultaneous reactions]
That's pretty neat. Shalonda sent that one. [Enlarges picture] Can you see it?

Aah... We could go like on painting and drawing.

You could come up here and look at it. [Other student comments are voiced simultaneously. The teacher was responding to one that was not picked up by microphone of the video camera.] (B7-07-05-03obs)

The picture that Shalonda contributed for the class activity was aesthetically pleasing, and it was also an example of parabolic shapes as they occur in an urban setting. The photograph showed multiple arched sprays of water that spurted inward from both sides of a large, long rectangular fountain (see Figure 4.4). The assignment was to match parabolic shapes on the picture by graphing equations on the coordinate grid that was superimposed on the picture. Because of the number of arches and the variety of parabolic shapes, students had a great number of choices within the assignment.

Figure 4.4: Background picture from Shalonda
The previous night’s homework had been to find a picture that contained a parabolic shape that would be analyzed in the current lesson. Students found a wide variety of possibilities and were eager to share. The McDonald’s arches were mentioned early in the lesson while students were locating their picture files. The students reacted enthusiastically to the Golden Arches, a symbol recognized globally. Working with the Golden Arches would be the “ultimate quadratic experience” (See Figure 4.8).

Mrs. Blake: Okay, kids. Look. You're getting too good. So let's do one more. We've got about 7 minutes, so let's do one more.

Student: Let's do the McDonald's one.

Mrs. Blake: Yeah, let's do McDonald's. (B7-07-05-03obs)

Figure 4.5: Matching the golden arch shape

Mrs. Blake seemed to be looking forward to working with matching graphed parabolas to the McDonald’s arches as much as the students were. A bell rang very
shortly after the students began to work. Mrs. Blake told them that they could stay an extra seven minutes. Not a single student objected to staying longer or questioned the need to as they eagerly accepted the final challenge of the lesson.

**Summary.** Mrs. Blake strategically used questioning to support students in making sense of the mathematics in her classes (17.2% coverage of transcribed text). In the first lesson, she questioned students in order to get them to focus on specific aspects of the lesson, with questions such as “Okay, b squared minus 4ac. So what good is this?” (B7-06-04-24obs), to go over homework, and to get students to tell her what happens next in solving multiple-step problems. Her questioning during the second lesson was directed at challenging students to think (7.3% coverage). She often took up students’ comments to press students for discussion and argumentation (13.8% coverage), a process that did not appear to be comfortable for Mrs. Blake and her students.

Mrs. Blake made conscious efforts to engage all of her students in mathematics class. She *assigned competence* by giving important classroom roles to students, such as assistants with technology, who might not otherwise stand out in other classroom activity. Doing their jobs put these students in roles as experts, prevented disengagement, and allowed them to see themselves and to be seen by others as competent individuals within the community.

Mrs. Blake’s students seemed to enjoy the class and freely expressed their feelings. Mrs. Blake planned for a variety of activities, such as the rotation through stations in the first lesson, choice within homework assignments, such as when she asked students to find images that could be used to illustrate parabolic shapes, and connections to aspects of culture that are relevant to the students, such as the DVD of sports-related
applications of quadratic functions in the first lesson and student choice of images for the graphing activity in the second lesson. The students engaged enthusiastically perhaps in response to the motivating and relevant features of the lessons.

Case #3: Mrs. Harmon and Students in Texas

Mrs. Harmon teaches Algebra to 9th-grade students in a large urban school district in Texas. Mrs. Harmon was in her 14th year of teaching at the time of the observations—her 13th year teaching algebra. The participating students were in an Honors Algebra 1 class. The class was made up of high school freshmen who did not take Algebra 1 in eighth grade but wanted the credit for the higher level class, were recommended for the class by another teacher, or were pushed to take the class by their parents; “they were not necessarily interested in the mathematics” (H8-08-06-16phint).

Mrs. Harmon explained that in her district, when students are placed in Honors Algebra 1 as ninth graders, they are the “third level down” in the Honors program. The highest achieving students take the class as seventh graders, the next level down as eighth graders. This group of students would be earning the higher grades if they were evenly distributed into the other algebra classes in ninth grade (H8-08-06-16phint). This method of separating students essentially creates two additional levels in the system of tracking.

The student population of the class was predominantly Hispanic/Latino/a (42%). White students made up the second largest subgroup (23%). Other racial/ethnic demographic identifications included African American (19%), Asian (8%), and Other (4%). One student did not self-identify. Mrs. Harmon is White. Fifteen percent of the school population qualified for free lunch.
Mrs. Harmon’s classroom was well-lit with bright white walls and whiteboards across the front of the room. An interactive whiteboard was at the center of the wall. Supplies were neatly organized on black shelves. A few commercially-produced mathematics-related motivational posters were visible in various places around the room. One prominent poster, “We All Use Math Every Day,” is based on the show “NUMB3RS” and produced by Texas Instruments in conjunction with the Columbia Broadcasting System Corporation (see Figure 4.6). There were also a few colorful homemade posters. Framed pictures of family members were on the wall beside Mrs. Harmon’s desk. The Texas flag and a US flag were located at corners of the room.

Figure 4.6: Mathematics poster

Mrs. Harmon: The First Lesson

The first observation in Mrs. Harmon’s room took place in April of 2006. The objective of the first lesson was for students to work with a variety of rectangles of fixed
perimeter to create data sets of lengths and widths. The student-created data sets were to be used the following day for exploring quadratic functions. Salient features of this lesson, according to the percentage of transcribed text, are giving directions, managing materials, and IRE pattern of discourse.

**Giving directions and managing materials.** The introduction to the day’s lesson included extensive presentation of a problem. Small-group (4 students per group) work would follow the introduction of the problem. Mrs. Harmon wanted the students to understand the assignment, the directions for different parts of the assignment, and expectations for using class time. Presentation of the problem and initial directions for the assignment took approximately nine minutes. Mrs. Harmon walked around the room as she presented the problem, which was also on a handout and projected on the interactive whiteboard at the front of the class.

6 Mrs. Harmon: Okay, all throughout history, all throughout history people have traveled all over the world in an effort to get rich. In 1848, 1849, the gold miners, they discovered gold at Sutter's Mill in CA and the gold miners rushed to make their money.

7 Bryan: 1871.

8 Mrs. Harmon: 1871. You can tell me later on. In 1871 they discovered diamonds in South Africa and people rushed there to make their money in diamonds. From 1860 to 1900 they were giving away free land in the middle of the United States and so people rushed to the middle part of the United States, to the Midwest. Then in 1901, in Spindletop, Texas is where they discovered the big oil gushers and people rushed there. So all throughout history people have been trying to make it big, trying to make it rich. But these
prospectors and farmers, what they did was they staked a claim. They decided, "this land is mine, nobody else can have it..."

9 Bryan: (sings) This land is my land; this land is my land...

10 Mrs. Harmon: Yup.

11 Bryan: ...from California...

12 Mrs. Harmon: And they decided on which piece of land they wanted to work and then of course the ones who made smart claims made the money; the ones who didn't make such good claims didn't. And a funny thing, most of the people who made smart claims knew something about math.

13 Bryan: Are you saying, um, do we know math can make money?

14 Mrs. Harmon: I'm saying it gives you an advantage, Bryan. Now think about the math that you would use if you had some land that you're going to work. You'd have to survey to decide which kind of land, which land you want; surveying is just kind of looking and seeing where the straight lines are. Did everybody get a worksheet? (H8-06-04-24obs)

Students frequently repeated words or phrases from the teacher's narration. This was evident for the first time in paragraph 7 above, and happened several times during the two days of observation. Bryan voiced his thoughts/questions without regard to traditional classroom organizational routines, such as raising one's hand and waiting to be called on. He seemed unable to hold back commentary as he burst into song (to the tune of Woodie Guthrie’s *This Land is Your Land*) to echo the teacher's line, “...this land is
mine...,” with “This land is my land, this land is my land...,“ “...from California...” (paragraphs 9 and 11). This was also repetition, but with embellishment. Mrs. Harmon reacted with mild amusement and moved right on.

With Bryan's next interruption, however, he challenged the teacher’s assertion that smart claims that made money were related to knowing “something about math.” Mrs. Harmon responded with only a brief comment; as a result, an opportunity for a discussion of monetary success as related to possession of knowledge was missed. This issue, which is related to social justice, could have been tied to the problem context being used for this lesson. Mrs. Blake’s response (paragraph 14 above) essentially dismissed Bryan’s question. She reasserted what she “was saying,” implying that she was the one who controlled the knowledge and really shouldn’t be questioned about that particular topic at that time. As she continued reading directions, she improvised somewhat, adding details about “mathematical tools” to support her assertion.

17 Mrs. Harmon: Figuring out the amount of fencing, figuring out the amount of costs were all mathematical tools. Now, you and your group are a group of prospectors. You are headed to Mars, because it is the year two thousand one hundred, twenty-one hundred, and they have just discovered pershonium on Mars. Now pershonium is a very rare and valuable metal. You’re going to stake your claim; they're giving away free land. The only condition is each prospecting group can only claim a piece of land that can be surrounded by 20 meters of fencing; of laser fencing. Now of course you want to get the most land possible, because the more land you have the more metal is going to be on it. Now if we're talking about surrounding something by 20 meters of fencing, what are we talking about?

18 Students: Perimeter. It's area.
Mrs. Harmon:  Perimeter. The perimeter. Now the Mars Colony government puts one more restriction. It has to be a rectangular piece of land. They want these to fit together nicely, so it has to be a rectangular piece of land. [...] Now, the paper that you've gotten, that I've just handed out to you is your guide sheet for completing this activity. You're going to follow the instructions so you need to read it. I'm going to hit some highlights. Part number 1 says for you to sketch several rectangles. Each member of your group needs to sketch a different rectangle, so you're going to have four different rectangles that you will end up putting on your poster. The back of your worksheet has grid paper to make it easier for you to sketch your rectangle. But to be able to cut it out and put it on your poster you don't want to cut up this worksheet that you're going to need where?

Students: Keep the agenda in the folder.

Mrs. Harmon: Yeah, in your packets. (H8-06-04-24obs)

At the end of paragraph 17 above, Mrs. Harmon asked her students to identify what “surrounding something by 20 meters of fencing” was “talking about.” Student responses included both perimeter and area. In this occurrence of the initiation, response, evaluation pattern of discourse (IRE), Mrs. Harmon affirms the response of perimeter by repeating it. The other response was ignored. Mrs. Harmon again missed an opportunity to understand what her student was thinking because she focused only on the response she expected. Reexamination of her exact words reveals that the “something” she refers to in her question was area, not perimeter. The fencing would represent the perimeter or boundary of the “something,” which was area. Often that which is assumed to be a
student error, and then possibly not addressed, is in actuality an instance of
miscommunication. It may have been the ambiguity of the teacher’s question that elicited
both responses from students in paragraph 18.

The presentation of directions continued for another minute or two beyond what is
presented in the excerpt. There were no modifications to the assignment as printed on the
worksheet. Repetition of directions and elaboration on directions in various forms
continued throughout the class period (see Table 4.3), even though the worksheet
included everything that the teacher said.

<table>
<thead>
<tr>
<th>Category</th>
<th>% of text</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Presentation of the problem</td>
<td>17.1</td>
<td>The original presentation of the problem that students would be working on (6 occurrences as student remarks or responses to questions broke the presentation into segments)</td>
</tr>
<tr>
<td>Giving directions</td>
<td>12.1</td>
<td>The first time directions were given for the assignment or a task within the assignment</td>
</tr>
<tr>
<td>Introducing next steps</td>
<td>2.1</td>
<td>Teacher statements included phrases such as <em>Now we are going to work on...</em> or <em>Let’s get started on the table...</em></td>
</tr>
<tr>
<td>Elaboration on directions</td>
<td>3.4</td>
<td>A student asked a question and the teacher reworded or added to what was originally stated</td>
</tr>
<tr>
<td>Restating direction</td>
<td>5.5</td>
<td>Directions that were already stated and repeated in essentially the same form</td>
</tr>
</tbody>
</table>

Table 4.3: Directions
The norm for small-group work appeared to include a great deal of dependence upon the teacher. Student questions were mostly related to what was required within the assignment. Even though students had copies of the written directions, they still asked questions about them, and Mrs. Harmon gave additional unsolicited reminders. Part of the perception of dependence on the teacher may be due to the type of assignment that was given that day. Mrs. Harmon was looking for some very specific products of student work that would be used for whole-class discussion and analysis the next day. It also may have been habitual teacher behavior that set the norm for students’ expectation of having the teacher there to say what to do throughout the lesson.

Managing materials, which included statements for distribution and use of calculators, passing out papers, and turning in completed work, showed up as a prominent feature of the classroom in terms of percentage of text. The amount of time taken up in materials management, however, was not extraordinary. Lessons that involve working with poster board, scissors, glue, markers, etc., necessarily include attention to organization, and the processes involved in distribution and collection of materials in this class were quite efficient.

IRE pattern of discourse. The IRE pattern of discourse was another salient feature of this classroom. IRE patterns were extended to IRIRE, with the middle “I” representing the initiation of a modified question or information that provides a hint (an implied negative evaluation), when the student failed to provide the desired response. When she did evaluate student responses in traditional IRE, her evaluations were either explicit, with words like “correct,” or “right,” or implicit, with repetition of the student's response.
In the following excerpt, Mrs. Harmon made use of the traditional form of IRE but she also employed the extended form. She repeated the student's response if it was what she expected to hear, sometimes elaborating on the student's response; otherwise, the answer was evaluated as incorrect, or it was ignored. In either case, a new question followed. Double forward slash marks (//) indicate the boundaries between IRE and/or IRIRE units.

39  Mrs. Harmon:  Okay, let's look at the table that you need to make. Let's get started on the table you need to make. This table needs to include all possible rectangles, not just the four that your group came up with. // What are we going to title this first column?

40  Bryan:  Rectangle. Length.

41  Mrs. Harmon:  Length, Okay? It's the length of the side; that's what it tells you in problem two. // What's the other column?


43  Mrs. Harmon:  Look in section two; what does it say?

44  Student:  Area.

45  Mrs. Harmon:  Area. // What's the smallest length of side that anybody came up with?

46  Students:  1.

47  Mrs. Harmon:  1. // Okay, if you had an area, if you had a rectangle who had a side of length 1, what would that other side be?

48  Students:  9.

49  Mrs. Harmon:  9. // So what's the area?

50  Students:  9. / 10. 9.
Mrs. Harmon: How do you find area?

Student: Multiply length times width.

Mrs. Harmon: Length times width, so 9 times 1 is...

Student: 9.

Mrs. Harmon: 9. What would be the next one?

Students: 2. 8.

Mrs. Harmon: 2, and it could be 8. What would be your area?

Students: 8. I mean 16. / 16.

Mrs. Harmon: 16. Okay. (H8-06-04-24obs)

This passage consists of five instances of IRE and two of IRIRE. In the instances of IRIRE, Mrs. Harmon did not directly address the incorrect response, or multiple responses. She either asked another question, implying that the response was not acceptable, or she repeated the response she considered to be correct. The typical purpose of initiating questions in IRE and IRIRE is to elicit recall of information—that which is already known, such as “How do you find area?” in paragraph 51 above. Often in mathematics classes, instances of IRE are used for student numerical calculation, as in paragraphs 53 through 55 above.

In paragraph 40, Bryan called out an answer and then corrected himself. Mrs. Harmon acknowledged that his revised answer was correct by repeating it. She then moved on to the next question. Bryan called out again. Mrs. Harmon shook her head indicating “that's not it,” so he tried another response, which was also incorrect. During his modified response, Mrs. Harmon moved the exchange forward by telling students
where to find the response she expected to hear. She then initiated a new question and she affirmed the answer by repeating the student’s single word response (paragraph 45).

Mrs. Harmon attempted to support student learning by connecting mathematical ideas to representations (tables and graphs) and clarifying mathematical terminology. While guiding students through the first few steps of generating a table of values for the fixed perimeter problem might be interpreted as scaffolding student work, the actual work that students would need to accomplish beyond the introduction would not require anything more than continuation of the pattern already established in the excerpt above. Students did not have to do the work of figuring out how to organize the data to find a pattern and make sense of it.

Summary. Most of the discourse from Mrs. Harmon’s first lesson can be grouped into the categories of giving directions (40.2% coverage of transcribed text) and managing materials (6.7% coverage). The nature of the lesson—which was intended to be a sustained session of student small-group work that would result in production of a poster with very specific content—may account for the emphasis on the requirements of the assignment; but the amount of time expended in giving, repeating, and clarifying directions seemed excessive. The directions were already available on a worksheet for students. The use of lengthy introductions and frequent reminders may have been a norm that had already been set in this classroom. There were very few decisions that students had to make on their own. Frequent reminders did not scaffold student responsibility, and therefore did not encourage student autonomy.

The other salient feature of Mrs. Harmon’s first lesson, the IRE pattern of discourse (5.0% coverage), focused on correct student answers to lower-order questions.
This pattern of discourse does not encourage student agency as mathematical thinkers. Students were not asked to do any problem solving. They were only expected to remember and recall formulas used in earlier years, such as how to find the area of a rectangle.

Mrs. Harmon: The Second Lesson

The second observation in Mrs. Harmon’s class took place the following school day. The data sets generated by students during the first lesson were used in the second lesson for examining quadratic functions. The written objective of the second lesson was for students to recognize when a data set fits a quadratic relationship and to write an equation to represent the data set. Mrs. Harmon emphasized comparing tables and graphs as she led her students toward generalizing the data in the form of an equation. The products of students’ small-group work from the previous day were posted on and around the white boards at the front of the class. While there was some variety in the appearance of the group posters, they all looked remarkably similar, which may be explained by the step-by-step nature of the assignment that required specific elements (see Figure 5.7).

Salient features of this lesson, according to the percentage of transcribed text, are soliciting multiple responses, clarifying mathematical terminology, and IRE pattern of discourse. Other topics of discussion included because of relevance to the guiding questions are press for elaboration and justification and student thinking dispositions.
Picking up where the previous lesson left off, Mrs. Harmon started the second lesson by praising students for the work they had accomplished the day before. She reported that most of the students had completed everything that was assigned except the three questions on the study guide/worksheet. Mrs. Harmon asked students to write the descriptions asked for in “question 3” of the worksheet in their notes.

5 Mrs. Harmon: Okay, now, you did your scatter plot and it looked like that. Correct?

6 Students: Yeah.

7 Mrs. Harmon: Okay. Someone tell me some phrases to describe the graph, and listen [teacher emphasis, louder and more slowly]. If you did not write these things on your worksheet yesterday, your personal worksheet, if you just put them on the poster, jot them down on your paper. If you have lost that paper, get a plain sheet of paper and put it on there. (H8-06-04-25obs)
The emphasis placed on these instructions may have indicated the importance Mrs. Harmon placed on students’ descriptions of their understanding of the mathematics and note taking.

*Soliciting multiple responses, clarifying mathematical terminology, and IRE pattern of discourse.* Vocabulary and mathematics terminology was emphasized throughout the lesson as Mrs. Harmon solicited multiple responses from her students. Mrs. Harmon was aware of students’ emergent understanding, and she allowed for bridging between students’ use of their own words and standard mathematics terminology.

7 Mrs. Harmon: Give me some words to describe the graph, or some phrases.

8 Student 1: A “U” that's turned over.

9 Mrs. Harmon: Okay, upside down U. Okay, some other words to describe the graph.

10 Student 2: It's arched.

11 Mrs. Harmon: Okay, it's an arch; it's a curve.

12 Student 3: It’s an upside down U.

13 Mrs. Harmon: We got the upside down U, we got the arch...

14 Student 4: It's a parabola.

15 Mrs. Harmon: It's a parabola, good, good math word.

16 Student 5: Way to go.

17 Student 6: Couldn't you say it was a quadratic [sic] function?

18 Mrs. Harmon: Yes, you could say it's a quadratic function.
Student 7: Now we're trying to get math [inaudible], are we?

Mrs. Harmon: We've got to get some math words in here. Parabola, quadratic function, anything else to describe it?

Student 8: Looks like [inaudible].

Mrs. Harmon: Looks like what?

Student 8: Rainbow.

Mrs. Harmon: A rainbow, okay.

Students: Write that down. / Looks like a rainbow. Who said that? / She did.

Mrs. Harmon: Okay. You can use math words like parabola and quadratic function...

Student 9: Or kindergarten words.

Mrs. Harmon: ...as well as perfectly good English words that are equally as descriptive.

Student 9: Rainbow.

Mrs. Harmon: Because it just said describe the shape.

Student 4: Like in China do they call it parabola or something else?

Mrs. Harmon: Know what? That's a good question and I don't know the answer. You find out the answer and I'll give you extra credit.

Student 4: Okay.

.... [....]

Student 9: Rainbow. (H8-06-04-25obs)
Describing, or naming of ideas, was an important part of developing understanding. Some students were able to contribute standard mathematics terminology as well. This may be interpreted as an apprenticeship into mathematical language, which provides opportunities for students to contribute in increasingly substantial ways, as learning a *mathematics register* was intertwined with everyday classroom discussion (Forman & Larreamendy-Joerns, 1998; Lampert & Cobb, 2003). The solicitation of multiple responses also provided an opportunity for a kind of metadiscourse. In paragraphs 26 through 30, Mrs. Harmon talked with students, however briefly, about talking about mathematics, specifically about what kind of terminology is appropriate in relation to the given directions for the assignment. Mrs. Harmon encouraged her students to use words comfortably within their vocabularies in order to talk about the mathematical concepts of parabolas and parabolic functions. The clever retorts made in reaction to the description of “rainbow” did not come across as being derogatory as much as simply taking advantage of another opportunity to joke around. Even so, Mrs. Harmon stepped in to emphasize the importance of using any vocabulary to talk about the mathematics.

The focus on vocabulary was also evident in a student’s question about terminology in other countries (paragraph 31). The student who asked the question seemed genuinely curious. The words “extra credit” (paragraph 32) elicited an animated period of questions and comments. Mrs. Harmon’s suggestion to look it up seemed to be an acknowledgement of an interesting question and a light-hearted approach to
responding to it more than press for inquiry. The humorous tone of this first segment of classroom interaction set the tone for the remainder of the lesson, which was different from the tone of the previous day.

Later, the lighthearted mood manifested itself in another conversation about terminology. One student went beyond the usual concern over knowing the meaning of a term to actually expressing positive affect in relation to the word.

145  Student 1:  I like that word, discrete.
146  Mrs. Harmon:  Would this one be discrete? [in reference to the graphed parabola]
147  Students:  No. / Continuous. Oh yeah.
148  Mrs. Harmon:  It's continuous, good.
149  Student 2:  You like the word.
150  Student 1:  But it's a nice word.
151  Mrs. Harmon:  It is a nice word, but you can't use it for this one.
152  Student 1:  I like that word, too. [speaking of continuous] (H8-06-04-25obs)

Student demonstration of positive affect toward mathematics terminology was not found in any of the other classroom observations examined for this study.

Reasons for soliciting multiple answers from students during whole-class discussion include generating more than one idea or more than one solution path to explore with students and building connections between students’ lived experiences and the mathematics. In the next excerpt, Mrs. Harmon asks students if they have noticed anything about “greatest area rectangles,” in reference to the activity of finding the
possible areas of rectangles given a fixed perimeter. While not explicitly stated, the open-ended question implied that there were multiple features to notice.

269 Mrs. Harmon: What's the greatest area for this one?

270 Student: 36.

271 Mrs. Harmon: 36; and what would be the dimensions of that one?

272 Student: 6 and 6.

273 Mrs. Harmon: Have you noticed anything about the greatest area rectangles?

274 Students: They're always using the same numbers. / They're square.

275 Mrs. Harmon: They're square, okay. (H8-06-04-25obs)

Once answers were given, Mrs. Harmon stopped any further discussion by affirming one of them, essentially signaling that she heard what she wanted to hear. She neither explored the other student’s responses nor asked for elaboration. This tendency to prematurely evaluate resulted in dominance of the IRE pattern of discourse in the second lesson. If dialogue is stopped before multiple responses have been explored, students’ meanings and/or misconceptions are not available for public examination—to be used as objects of discussion. The connections between the dimensions of fixed-perimeter rectangles and the possible areas may not have been left entirely to chance, but the build up to a single question and response about “greatest area rectangles” being square would be easy to miss.

Later Mrs. Harmon did solicit multiple responses from her students, which may have been an attempt to provide students with knowledge of a variety of strategies for
approaching problems. This example is about using different representations of data to solve a problem, such as how to find the area that corresponds to a specific side of a rectangle, or to find the side length that will result in the greatest area.

Mrs. Harmon questioned students about how to find specific values within the representations being explored, but she appeared most interested in guiding them to use the graphing calculator for finding values in a table. Question C was, “If the length of a
side of the rectangle is 6 meters, what is the area?” In paragraph 379 above, Mrs. Harmon asked a student how he found the value “24.” The student’s response, “Multiply 6 times 4,” is one possible response if the student used the equation generated in the previous question on the worksheet: \( A = l \times (10 - l) \). With substitution, \( A = 6 \times (10 - 6) \), which is equal to \( 6 \times 4 \).

With her next question (paragraph 381), Mrs. Harmon was not explicit about finding the values in a different way. She had taken the worksheet problems out of order, which could account for some of the ambiguity. What she really wanted the students to know, however, was where the information could be found without using the equation. Some students followed the reasoning, but others didn’t.

Another possible interpretation of the confusion in this exchange is that Mrs. Harmon believes that she is supposed to solicit multiple responses from students, but she doesn’t really know what to do with those responses. An example is her question “Where did you look for the 6? [slight pause] In the \( x \)…”? A student immediately picked up on the fact that she was answering her own question with a leading question and said, “\( X \).” There really wasn’t any more that could be said about finding the 6.

*Press for elaboration and justification.* Earlier in the second lesson, as Mrs. Harmon attempted to scaffold student understanding, she continually maintained a margin of control of the knowledge. She asked Donella how she came up with an answer to a question on the worksheet (paragraph 66), but then quickly stepped in and took over Donella’s explanation. Mrs. Harmon may have stepped in so quickly because Donella spoke very softly—too softly for the whole class to hear.
Mrs. Harmon: You don't have to write this part down; just pay attention to this because this tricked some of you. Let's think about the rectangle. We know this side is 5 point 5 which means we know the other side. The opposite side is 5 point 5. Now we need to find the area so we need the width.

Donella: 4 point 5.

Mrs. Harmon: It would be 4 point 5; how did you come up with that, Donella?

Donella: I just multiplied 5 point 5 by 2 and then subtracted from 20...

Mrs. Harmon: And then divided. [overlapped with student finishing the sentence] She took 5.5 and multiplied it by 2, that's how much of the perimeter she's used, so she took 20 minus 11 and got 9, and then divided that by 2 to split it between the two sides. (H8-06-04-25obs)

Mrs. Harmon’s question in paragraph 66 gave the illusion of pressing for elaboration and justification, but, in this and similar instances, she responded very quickly to what became just another occurrence of looking for a correct answer without further discussion. There is no evidence here or elsewhere in the lesson of following through on pressing for elaboration and justification of students’ ideas. She specifically asked students to listen because they had been “tricked” when finding the width and area of rectangles with a non-whole number side length, but she neglected to check for understanding beyond Donella’s explanation of the steps she took to calculate the width and area of the rectangle.

When Mrs. Harmon talked with Donella in the example above, she drew a rectangle on the white board with the label “5.5” on one side (see Figure 4.8). Drawing the rectangle to help visualize the problem of finding the side lengths was a strategy of
providing a representation to help students understand the area problem. Representations, which are not simply visual displays of information, are important in mathematics classes because they can be used as tools for understanding (Lehrer, Jacobson, Kemeny, & Strom, 1999). They provide different ways to think about concepts, thus offering students additional ways to process ideas, possibly providing different entry points for understanding or presenting alternatives for connecting to what they already know. As Mrs. Harmon continued the lesson, she directed students’ attention from the data set represented as a table to the same data set represented as a graph, which became an additional tool used to support student understanding.

Figure 4.8: Rectangle with side measurement of 5.5 units

The table of values was projected on a screen using an overhead projector, and the graph was projected on the interactive whiteboard. Mrs. Harmon guided students to make the connection between the two representations to better understand how the “in
between” coordinate points make up the parabola. The data points by themselves will only be points. It isn’t until the points are “connected,” or a formula has been generalized and graphed, that the parabola, with an infinite number of solutions, is represented.

72 Mrs. Harmon: Now, let's look back over here on the graph. On this graph I need to go over 5 1/2. 1, 2, 3, 4, 5. I'm counting by ones, then 1/2.

73 Student: Can you draw a line?

74 Mrs. Harmon: And I want to put the point up here. I know this one's at 25 because these are counting by 5. 24 point 7, 5 is going to be right there. It's going to be down just a little bit. So did I go over my maximum area?

75 Student: No.

76 Mrs. Harmon: So let's think about it. As we add more fractions; as we put in... what's going to happen to the graph?

77 Student: It forms a line.

78 Mrs. Harmon: Okay, it's not going to form a line because it's not straight, it forms a...

79 Students: Scatter plot. / A curve?

80 Mrs. Harmon: A curve. Eventually what's going to happen if you keep taking more and more points...? It's going to do what?

81 Student: Connect.

82 Mrs. Harmon: It's going to connect. (H8-06-04-25obs)

Mrs. Harmon also modeled the process of graphing the points for her students (paragraphs 72 and 74), and she led them, or at least the one student who responded, to the realization that the points would “connect.” As she controlled the conversation,
however, she positioned herself as holder of the knowledge. She showed only one example, and then expected the students to draw a conclusion that would generalize the pattern to an infinite number of points—to understand that graphing the solution set of a quadratic equation would form a continuous curve rather than the discrete points that fit the equation when substituting whole numbers. Furthermore, with the question in paragraph 73, “Can you draw a line?” a student was anticipating the direction of the conversation, which was about moving from discrete data points to the continuous curve of the parabola. The recognition of that connection, possibly an extension of her prior experience with linear relationships, revealed the student’s positive disposition toward mathematical thinking. The student’s question was never acknowledged.

In the following excerpt, Mrs. Harmon was showing students a strategy for comparing representations in order to check for accuracy, or perhaps correctness. She used lower-level recitation questions to guide students through the process, however, which again resulted in a relatively ineffective message about connections between representations.

365 Mrs. Harmon: So does the table look like your table from yesterday?
366 Students: Yes.
367 Mrs. Harmon: Yes, it's exactly the same, so did we get the right equation?
368 Student: Yes, ma'am.
369 Mrs. Harmon: I've got my scatter plot already on here. You don't because you got rid of it; the calculator's been cleared.
370 Student: Yeah.
Mrs. Harmon: What should happen with my scatter plot if I graph?

Students: Upside down U. It's going to look the same. Oh yeah.

Mrs. Harmon: Did it work? Is it right?

Students: Yeah. No. (H8-06-04-25obs)

To press for mathematical thinking, Mrs. Harmon could have asked students to do the describing rather than having them confirm what she wanted to hear by asking LOrec questions. Or, assuming she felt pressured for time, she might have asked direct questions that would still put students in the position of doing the thinking, such as, "If the table looks the same, what does that mean in regard to the equation / the graph?" The methods Mrs. Harmon used were not strong examples of supporting student understanding, problem solving, or the skills necessary for SRL.

**Student thinking dispositions.** Productive dispositions for thinking and inquiry are important in learning mathematics with understanding and in the development of SRL. Productive thinking dispositions can be defined as the perception of the need for a particular thinking behavior and the inclination to act on the thinking behavior. Productive dispositions might include dispositions toward wondering, investigating, explaining, planning, evaluating, and rethinking. In the following interaction, one student in particular was being genuinely inquisitive about mathematical possibilities.

Mrs. Harmon: Any place your parabola crosses the X axis, any place your parabola crosses the X axis is going to be a root or zero.

Student 1: Can it go into negative numbers?
262 Mrs. Harmon: Yes, you probably can, but could it on this one? Well what are we talking about on this one? We're talking about the length of a side of a rectangle and the area. Can either one of those ever go negative?

263 Student 1: You could turn it inside out.

264 Mrs. Harmon: That would be in theoretical geometry. Not in the real world, which is what we're talking about.

265 Bryan: What, wha…, where and set up what?

266 Mrs. Harmon: She was playing "what if."

267 Bryan: Oh.

268 Student 1: I have questions and I need answers. (H8-06-04-25obs)

The student who proposed turning a rectangle inside out was stretching her mathematical thinking beyond what is typical in mathematics classrooms, particularly at the Algebra 1 level. The student took the lead as she pressed for more information, demonstrating a sense of agency and efficacy, also important components of SRL. Mrs. Harmon did not dismiss the student’s question, but it is interesting that she addressed it by making a distinction between theoretical geometry and the “real world” math of the problem. In the original problem, students were instructed to stake claims for areas of land to mine for “pershonium” on Mars. They may have considered the problem to be totally theoretical even though the teacher made reference to the gold rush in America in the 1840’s, a “real event” in American history, in the directions for the activity.

Summary. IRE, in its traditional form, was the dominant pattern of classroom discourse in both of Mrs. Harmon’s lessons. The use of the IRE pattern of discourse in the second lesson, as indicated by percentage coverage of transcribed text, increased
dramatically, from 5.0% in the first lesson to 18.4% in the second lesson. In the second lesson, Mrs. Harmon was aware of teaching mathematics vocabulary and helping her students make the transition from using their own language to using more standard mathematical terminology to describe mathematical situations (8.2% coverage). She also solicited multiple responses from students (7.8% coverage). However, solicitation of multiple responses did not include discussion in order to deepen understanding, which may explain the predominance of the IRE pattern of discourse. Mrs. Harmon gradually built up to the generalization of a quadratic formula based on student data, but, throughout the lesson, she controlled the information.

A few of Mrs. Harmon’s students demonstrated positive thinking dispositions in the comments they made in relation to mathematics and in their questions. Mrs. Harmon did not encourage students to think beyond what was presented in class aside from inconsequential acknowledgement of their questions. This lack of encouragement is another example of Mrs. Harmon’s weaknesses in supporting student agency and autonomy. Despite the weaknesses, her attempts to support development of mathematical terminology and to understand relationships between representations made the second lesson stand out as being more supportive of student learning than the first lesson.

Cross-Case Analysis

Data were analyzed across cases, examining within themes for continuities and discontinuities. The categories of themes were organized into family groupings during Phases II and III as described in the methods section of the present study. Families of themes that have proven to be most pertinent to the cross-case analysis are classroom norms and setting expectations, teacher questioning and patterns of discourse,
scaffolding emotions, scaffolding mathematical thinking and understanding, and student contributions to classroom discourse. Categories within the families of themes are identical to those used to present individual cases. Two categories, artifacts of setting classroom norms and thinking about equity and social justice, which were formed as a result of discussion with teachers during post-observation and end-of-year interviews, have been included to provide additional lenses for comparing cases.

Families of themes and the related headers used in the narrative descriptions of the cases are shown in Table 4.4. Headers are italicized in the columns beneath the teachers’ names. Similar to the interwoven strands of mathematical proficiency, the narratives written to describe these cases are complex and themes overlap. Consequently, many of the themes are embedded in descriptions under headings other than the primary family grouping. Embedded themes are indicated parenthetically, in a smaller font. To facilitate location of themes that are not identified by headers, I have color coded the families of themes to match with the corresponding embedded themes.

Comparisons among classroom interactions, teachers’ support for SRL and learning mathematics with understanding, and teachers’ awareness of and attention to issues of equity lead to greater understanding of the relationships between cultural difference and classroom discourse for these three cases. The stories of these teachers and their students may provide insight into the kinds of classroom practices that support meaningful engagement in classroom activities and contribute to development of students’ agency as learners.
<table>
<thead>
<tr>
<th>Family</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Brenner</td>
</tr>
<tr>
<td>Affect</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enjoyment of activities</td>
</tr>
<tr>
<td>Classroom norms</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Press for elaboration and justification</td>
</tr>
<tr>
<td></td>
<td>(Scaffolding mathematical thinking and understanding: Connecting idea to representation)</td>
</tr>
<tr>
<td></td>
<td>(Supporting SRL: Strategy instruction)</td>
</tr>
<tr>
<td>Directions</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Giving directions and managing materials</td>
</tr>
<tr>
<td></td>
<td>(Student contributions to discourse: Repetition)</td>
</tr>
<tr>
<td></td>
<td>(Teacher questioning and patterns of discourse: IRE)</td>
</tr>
<tr>
<td></td>
<td>(Classroom norms: Setting expectations)</td>
</tr>
<tr>
<td>Emotional scaffolding</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Emotional scaffolding</td>
</tr>
<tr>
<td></td>
<td>Emotional support</td>
</tr>
<tr>
<td></td>
<td>(Student contributions to discourse: Predicts what will be said)</td>
</tr>
<tr>
<td></td>
<td>(Supporting SRL: Encouraging self-evaluation)</td>
</tr>
</tbody>
</table>

Continued

Table 4.4: Families of themes and related headings for case descriptive narratives
<table>
<thead>
<tr>
<th>Family</th>
<th>Case</th>
<th>Harmon</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scaffolding mathematical thinking and understanding</strong></td>
<td><strong>Making sense of mathematics</strong></td>
<td><strong>Soliciting multiple responses</strong></td>
</tr>
<tr>
<td></td>
<td>(Classroom norms: Facilitating classroom discourse, Press for argumentation, Press for elaboration, Press for justification, Setting expectations)</td>
<td>(Affect: Humor)</td>
</tr>
<tr>
<td></td>
<td>(Student contributions to discourse: Argumentation, Reasoning)</td>
<td>Clarifying mathematical terminology</td>
</tr>
<tr>
<td></td>
<td>(Supporting SRL: Strategy instruction, Encouraging self-evaluation)</td>
<td></td>
</tr>
<tr>
<td><strong>Student contributions to discourse</strong></td>
<td><strong>Student-to-student discourse</strong></td>
<td><strong>Argumentation</strong></td>
</tr>
<tr>
<td></td>
<td>Student-to-student question</td>
<td>(Classroom norms: Facilitating classroom discourse, Press for argumentation, Setting expectations)</td>
</tr>
<tr>
<td></td>
<td>(Affect: Humor)</td>
<td></td>
</tr>
<tr>
<td><strong>Supporting SRL</strong></td>
<td><strong>Strategy instruction</strong></td>
<td><strong>Strategy instruction</strong></td>
</tr>
<tr>
<td></td>
<td>Encouraging self-evaluation</td>
<td>Encouraging self-evaluation</td>
</tr>
<tr>
<td><strong>Teacher questioning and patterns of discourse</strong></td>
<td><strong>Uptake of student comments</strong></td>
<td><strong>IRE pattern of discourse</strong></td>
</tr>
<tr>
<td></td>
<td>(Classroom norms: Press for justification)</td>
<td>(Classroom norms: Press for elaboration)</td>
</tr>
<tr>
<td></td>
<td><strong>Dialogic episodes</strong></td>
<td>Uptake of student comments</td>
</tr>
<tr>
<td></td>
<td>(Student contributions to discourse: Argumentation, Conjecture, Justification)</td>
<td></td>
</tr>
<tr>
<td><strong>Technology</strong></td>
<td><strong>Assigning competence</strong></td>
<td></td>
</tr>
</tbody>
</table>
Classroom Norms and Setting Expectations

Classroom norms refers to comments and questions made with the intent of facilitating mathematical discourse, including press for student elaboration, inquiry, justification, and reasoning. Setting expectations refers to verbal messages used to convey values for participation and social and mathematical behaviors. Teachers must provide the scaffolding to work toward discourse as the genuine sharing of ideas that results in greater mathematical understanding. Two kinds of teacher scaffolding are critical to discourse-oriented teaching (Williams & Baxter, 1996). Social scaffolding provides support for social behavioral norms and expectations regarding discourse. Analytic scaffolding has to do with scaffolding ideas that build understanding of mathematical concepts.

Both Ms. Brenner and Mrs. Blake provided social scaffolding by asking questions that would help students know how to participate in mathematical discussion. They also cued other students to listen when a student had something to say about the mathematics and directed students’ attention to particular aspects of concepts being discussed. This social scaffolding helped to communicate expectations and set norms for student participation in classroom discourse (see Table 4.5). Both Ms. Brenner and Mrs. Blake also pressed students to look further into the mathematics to bring out more detail for discussion (press for classroom discourse), which is both social and analytic scaffolding (see Tables 4.6 and 4.7). Ms. Brenner was more explicit, however, in facilitating and pressing for classroom discourse. She used specific language referring to dialogic interaction in all three of the examples from her classroom shown in Tables 4.4 and 4.5. Her explicit wording had potential to scaffold student engagement in dialogic discourse,
particularly English language learners (ELLs), who may not be acculturated to academic language, much less dialogic classroom discourse. Mrs. Blake’s language was not explicit as she pressed for classroom discourse.

<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenner</td>
<td>0.5</td>
<td>We’re going to talk about whether or not we agree or disagree with it. (B5-06-12-14obs, 64)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Okay, Denae has a point. What is it you’d like to say? (B5-06-12-14obs, 70)</td>
</tr>
<tr>
<td>Blake</td>
<td>0.2</td>
<td>How about this? (B7-06-04-24obs, 183)</td>
</tr>
<tr>
<td>Harmon</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5: Facilitating classroom discourse

Mrs. Harmon’s typical press for elaboration consisted of asking a student how he or she got a number answer, and then the student would reply with steps for calculation. Interestingly, a couple of Mrs. Harmon’s students did occasionally make conjectures about the mathematics. Mrs. Harmon neither encouraged nor discouraged this form of student input. It was treated as incidental.
<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenner</td>
<td>1.8</td>
<td>Can anybody elaborate on what Manny said? (B5-06-12-14obs, 457)</td>
</tr>
<tr>
<td>Blake</td>
<td>0.1</td>
<td>Negative 3 will tell you that... (B7-07-05-03obs, 88)</td>
</tr>
<tr>
<td>Harmon</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6: Press for classroom discourse

Discourse in Mrs. Harmon’s room was characterized by very traditional exchanges, often IRE, between teacher and students. Mrs. Harmon controlled the pace and direction of the lesson with questions about the subject material that students were expected to be able to answer. She did not press for discourse otherwise.

<table>
<thead>
<tr>
<th>Category</th>
<th>Brenner</th>
<th>Blake</th>
<th>Harmon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Press for elaboration</td>
<td>1.9</td>
<td>0.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Press for reasoning</td>
<td>0.4</td>
<td>0.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Press for justification</td>
<td>0.6</td>
<td>0.1</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.7: Teacher press (by % of coverage)
*Teacher Questions and Patterns of Classroom Discourse*

*Teacher questioning* includes the question categories of higher-order recitation (HOrec), lower-order authentic (LOauth), lower-order recitation (LOrec), open-ended, soliciting additional answers, asking for questions, checking on progress, and asking for clarification. The majority of the teachers’ questions in all three classrooms were lower-order recitation (LOrec). LOrec questions typically require recall of previously known information. Often in mathematics classes they are used to involve students in providing “answers” to math facts while working to solve multiple-step problems. Lower-order recitation questions usually initiate the IRE pattern of discourse. *Patterns of classroom discourse* observed in the present study include IRE, IRIRE, dialogic episodes, and uptake of student comments. As might be expected as a consequence of the predominant use the IRE pattern of discourse in Mrs. Harmon’s class, she asked LOrec questions at a higher rate within her classes (78.2% of the questions she asked were LOrec). Mrs. Blake’s rate was the lowest at 58.7%, and Ms. Brenner’s rate of LOrec questions fell in between, at 65.9%.

Lower-order authentic (LOauth) questions are used for eliciting unknown information, but do not require application of higher-level thinking. LOauth questions, which were relatively rare in all three cases, were used for such purposes as finding out what steps a student took to solve a problem or how a graph was used to find information. The only higher-order questions asked in the observed class sessions for all three cases were recitation (HOrec)—teachers asked students to think beyond what had been presented, but expected a specific answer (see Table 4.8, and Table 4.9 for examples).
<table>
<thead>
<tr>
<th>Category</th>
<th>Brenner</th>
<th>Blake</th>
<th>Harmon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower-order recitation (LOrec)</td>
<td>65.9</td>
<td>58.7</td>
<td>78.2</td>
</tr>
<tr>
<td>Lower-order authentic (LOauth)</td>
<td>3.2</td>
<td>3.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Higher-order recitation (HOrec)</td>
<td>10.3</td>
<td>12.8</td>
<td>7.9</td>
</tr>
<tr>
<td>Open ended</td>
<td>3.2</td>
<td>9.2</td>
<td>0.0</td>
</tr>
<tr>
<td>Soliciting additional answers</td>
<td>0.8</td>
<td>2.8</td>
<td>0.0</td>
</tr>
<tr>
<td>Asking for questions</td>
<td>0.8</td>
<td>1.8</td>
<td>1.0</td>
</tr>
<tr>
<td>Checking on progress</td>
<td>7.9</td>
<td>10.1</td>
<td>11.9</td>
</tr>
<tr>
<td>Asking for student clarification</td>
<td>7.9</td>
<td>0.9</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.8: Distribution of individual teacher’s questions (by % of total)

Many of the teachers’ questions had functions other than those coded as LOrec, LOauth, and HOrec for this study. Comparing for discontinuities in the types of questions asked by teachers between cases, the question categories open-ended and asking for student clarification stand out. Ms. Brenner and Mrs. Blake asked open-ended questions, and Ms. Brenner asked students to clarify comments or the responses they had given to questions. Mrs. Harmon did not ask open-ended questions; neither did she ask students to clarify meanings of their comments.

While Mrs. Blake did ask for clarification of a student response once, the relative lack of explicit questioning for clarification corresponds to the observed “short-hand” nature of many of the exchanges between Mrs. Blake and her students, as described in
Case #2 above. Her original question, “You got x equals 1?” (B07-07-05-03obs, 120) was interrupted by other questions and comments made by Mrs. Blake in response to ongoing comments made by the students. As a result, the purpose of the question and the student’s response, which he was writing on the board, were obscured. The exchange gave the impression of being left unfinished even though the student had drawn his graph to show how he got $x = 1$ as his answer.

<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenner</td>
<td>0.8</td>
<td>How come we have matches on either side? (B5-06-12-14obs, 479)</td>
</tr>
<tr>
<td>Blake</td>
<td>1.3</td>
<td>So what good will that do? (B7-07-05-03, 153)</td>
</tr>
<tr>
<td>Harmon</td>
<td>1.1</td>
<td>T: Suppose the dimensions were not restricted to whole numbers. In other words they could be fractions... S: Decimals? T: ...and decimals, yeah. How would that change? (H8-06-04-25obs, 50-52)</td>
</tr>
</tbody>
</table>

Table 4.9: Examples of teacher questions: Higher-order recitation

_Uptake of student comments._ When teachers take up students’ comments and questions, they are conveying the message that student contributions and discussion of student contributions are important for learning mathematics. Uptake serves a variety of purposes beyond conveying importance of students’ contributions, however. Uptake can be used to talk about both correct and incorrect student statements and the reasoning
behind them, but it can also be used to explore or answer student questions, to clarify student comments, and to scaffold argumentation.

Ms. Brenner took up correct answers five times, incorrect answers six times, student questions three times, and comments for the purpose of clarification three times during the two observed classes, for a total of 17 references. The example in Table 4.10 illustrates how Ms. Brenner took up an incorrect answer to bring out the reasoning that would help students understand the correct answer.

Mrs. Blake took up a student’s comment once for answering a student question, once to help students understand a correct answer, once for an incorrect answer, and once to clarify a student’s comment, for a total of four occurrences. The example for Mrs. Blake in Table 4.9 shows uptake of an incorrect answer. When Mrs. Blake questioned the student, he realized his mistake, but Mrs. Blake kept the conversation going by scaffolding argumentation, which the students did not engage in naturally. She prompted them more than once by repeating students’ comments and asking them to “listen to this. See what you think” (B07-07-05-03obs, 93).

Mrs. Harmon took up student comments five times, once to respond to a remark made about learning vocabulary, and four times to elaborate on correct answers. Her elaborations were not for the purpose of drawing out new student ideas, but rather to add her own idea. Other students participated in discussion, but their comments were simply in response to additional teacher questions.
Brenner 15.9

T: What I’d like to discuss is this right here. I’m actually glad that someone answered this. Is this correct? Is the absolute value of 16 [equal to] negative 16, and I need some justification. I want to hear what you’ve got to say. Latisha.

S1: No.

T: Why?

S1: …if a number is positive …[inaudible] comes out as a positive.

T: Why is that, though? Why is that? Roberto.

S2: Because absolute value is the number of the counted spaces from 0 to that number. So it should always be positive.

S3: Yeah, it always should.

T: So even though it’s positive, it still has 16 spaces from 0.

Blake 7.9

T: That’s a good question. Sidney said, “What happened to the –B plus or minus… [inaudible]” What happened to that?

S: You still do it, right?

T: You still do it. Why do you still do it?

S: It finds how many [inaudible].

T: This tells you how many, Simone says, and that tells you…

S: Where they are.

T: What they are. Okay?

Continued

Table 4.10: Uptake of student comments
IRE pattern of discourse. IRE was not a consistent pattern of discourse in Ms. Brenner’s first lesson. The two instances of back-to-back IRE units that occurred in her first lesson were used for the purpose of clarifying the idea of distance on a number line being a positive value, which is a fundamental concept for understanding absolute value equations.

112 T: Now, my question is, is this positive 3?

113 S: No, negative.

114 T: This is not positive 3. // Here, for those of you who did move to positive 3, how far on the number line did you move?

115 S: Y2. 3.

116 T: You moved 3 spaces. //

Other individual instances were used in Ms. Brenner’s classes for the purpose of eliciting information to highlight important concepts for building understanding of the mathematics, such as in the following example from the second lesson.
T: What happened here to make this equation [inaudible]? Mike, do you have an idea what to change?

S: Maybe the B has to be smaller.

T: Maybe the B has to be smaller. // […]

T: // So how can they make the parabola more narrow?

S: By putting a large number.

T: By putting a larger number. //

Mrs. Blake’s routine for going over homework consisted of very traditional IRE patterns of discourse (see Table 4.11). Other units of IRE occurred within less predictably structured discourse (see examples in Case #2 above).

<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Brenner| 6.3             | T: Jose, can you help her out?  
S: Subtract 5.  
T: Subtract 5, good. (B5-06-12-14obs, 244-246) |
| Blake  | 8.9             | T: What is it, Connor?  
S: 1, 2?  
T: 1, 2 is correct. (B7-06-04-26obs, 30-32) |
| Harmon | 11.7            | T: …so 40 plus 40 is... Is that what it was?  
Ss: 80. Yes.  
T: 80. (H8-06-04-25obs, 247-249) |

Table 4.11: IRE pattern of discourse
Mrs. Harmon used IRE throughout her second lesson. Continuous episodes (made up of back-to-back units) of IRE-patterned discourse were not necessarily lengthy because students often interrupted with observations, comments, and questions.

104 T:  // Does the graph go higher?

105 SS:  No.

106 T:  No, so... all that happens to the graph is that it becomes connected, or continuous, rather than individual points. // So that doesn't change the answer because what?

107 S:  Because 5, 25 [inaudible].

108 T:  The highest point is still at 5, 25. // [ Interruption] Yes, Ryan.

109 S:  [Inaudible]

110 T:  Several people said that. Okay. There were several people that missed that one. [Resume] Does the graph get wider?

111 Ss:  No.

112 T:  // Does it get narrower?

113 Ss:  No.

114 T:  No, all that happens is that it becomes continuous. //

115 S:  [ Interruption] I like that word.

116 T:  I want to go back now, to describing the graph. I want to refresh your memory about domain...

117 S:  And range?

118 T:  And range. [Resume] What does the domain refer to?

119 S:  The X values.

120 T:  The X values. // Okay, looking at our continuous graph now, what is the smallest X value?
Mrs. Harmon steered students back to IRE-patterned discourse very quickly, giving the impression of maintaining control of the direction of the discourse.

*Dialogic episodes.* Dialogic episodes occurred only in Ms. Brenner’s class (see Table 4.12). By pressing for student explanations, reasoning, and justifications, Ms. Brenner created an environment conducive to dialogic interaction in the classroom. The dialogic episodes consisted of sustained discourse about mathematical ideas.

<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenner</td>
<td>9.6</td>
</tr>
<tr>
<td>Blake</td>
<td>0.0</td>
</tr>
<tr>
<td>Harmon</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.12: Dialogic episodes

Both Mrs. Blake and Mrs. Harmon tended to maintain control of classroom discourse. Mrs. Blake recognized the importance of student contributions to classroom discourse and tried to solicit more than just correct answers from her students, but the control she imposed resulted in fewer student contributions than in Ms. Brenner’s classes and fewer opportunities for dialogic interaction. Mrs. Harmon did not encourage dialogic discourse. Student contributions were solicited when worksheet questions called for
description. Her lack of probing for students’ ideas and views took away the likelihood of negotiation of conceptual understanding.

Press for learning includes teachers’ positive emphasis on students’ efforts in the classroom, focus on learning with understanding, support of student autonomy, and emphasis on conveying reasoning over production of correct answers to questions (Kazemi, 1988). All of the aspects of press for learning as identified by Kazemi are included in the coding categories of emotional scaffolding and scaffolding student thinking and understanding.

Emotional scaffolding. Emotional scaffolding includes subcategories of encouragement from the teacher, emotional preparation, emotional support, and encouragement between and among students. The greatest number of episodes of encouragement from the teacher occurred in Ms. Brenner’s class, particularly in the first of the two observations (during the first half of the school year). These words of encouragement for students included setting classroom norms for student participation in learning about mathematics and calling attention to good work (see Table 4.13). By calling attention to good work, Ms. Brenner was also providing examples of what counts as good work in her class—a sociomathematical norm.

Words of encouragement from Mrs. Blake were in praise of students work or affirmation of their ability to do good work. Most of this type of encouragement occurred during the second observation, which was when Mrs. Blake was learning to use the TI-Navigator and essentially learning a new form of classroom interaction. Her words of encouragement might have been as much for herself as for her students. Mrs. Harmon did
not use the type of statements of encouragement that Ms. Brenner and Mrs. Blake did during the observations in her classes.

<table>
<thead>
<tr>
<th>Case</th>
<th>Avg. % Coverage</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brenner</td>
<td>1.5</td>
<td>Okay, so Vera does not have confidence in her answer, but at the same time it gives us a lot to talk about, right? So even though you have an awareness that it’s not the answer that you prefer, maybe if you put it up there it would just give us… Would you be willing to do that? (B5-06-12-14obs, 66)</td>
</tr>
<tr>
<td>Blake</td>
<td>1.1</td>
<td>Oh. Oh my gosh! What do you think? You did this yesterday. You keep getting these graphs. What up? Not too shabby. Good work. (B7-07-05-03obs, 290)</td>
</tr>
<tr>
<td>Harmon</td>
<td>0.0</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.13: Encouragement from the teacher

_Emeralol preparation_ refers to statements made by the teacher for the purpose of preparing students for work that would follow, assuring them that more information would be presented that would help them understand, warnings that a problem might be challenging, and suggestions for division of tasks to make them less overwhelming. Mrs. Blake is the only teacher who provided this kind of emotional preparation for students.

_Emeralol support_ refers to statements made by the teacher that were meant to help students overcome obstacles encountered while working on tasks. Mrs. Blake
provided the greatest number of statements of emotional support, with four occurrences in her first lesson and two in her second. Ms. Brenner provided such emotional support once to a student during her first lesson.

Encouragement between and among students occurred in Ms. Brenner’s and Mrs. Harmon’s classes, with such phrases as “It’s going to be fine” (B5-07-05-04obs, 422) and “Way to go” (H8-06-04-25, 292). The students in Mrs. Harmon’s class provided the encouraging words that they did not overtly hear from Mrs. Harmon. Use of such phrases between students resulted in the perception of a stronger sense of community between students in Ms. Brenner’s and Mrs. Harmon’s classes, particularly during whole-class discussion. Students didn’t interact with one another with words of encouragement during whole-class discussion in Mrs. Blake’s second lesson, and the only conversations audible during small-group work during the first lesson were led by Mrs. Blake.

Scaffolding student thinking and understanding. Scaffolding student thinking and understanding includes subcategories of challenging students to think, clarifying mathematical terminology, modeling mathematical thinking, making sense of the mathematics, pressing for understanding, reasoning, rewording of student comments, soliciting multiple responses, and summarizing. All three classes had similar numbers of episodes of clarifying mathematical terminology, connecting previous work to new work, elaborating on a student comment, rewording of student comments, and summarizing. Greater differences can be found in the categories of challenging students to think, making sense of the mathematics, modeling mathematical thinking, press for understanding, and soliciting multiple responses (see Table 4.14).
<table>
<thead>
<tr>
<th>Category</th>
<th>Brenner</th>
<th>Blake</th>
<th>Harmon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Challenging students to think</td>
<td>5.0</td>
<td>5.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Making sense of the mathematics</td>
<td>10.3</td>
<td>11.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Modeling mathematical thinking</td>
<td>1.3</td>
<td>0.0</td>
<td>1.3</td>
</tr>
<tr>
<td>Press for understanding</td>
<td>4.9</td>
<td>6.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Soliciting multiple responses</td>
<td>9.6</td>
<td>3.4</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 4.14: Scaffolding student thinking and understanding (by avg. % coverage; categories of greatest difference between cases)

The greatest number of instances of scaffolding student thinking and understanding occurred in Ms. Brenner’s and Mrs. Blake’s rooms in the categories of challenging students to think, making sense of the mathematics, and press for understanding. Both Ms. Brenner and Mrs. Harmon modeled mathematical thinking, but the type of thinking they modeled was different. Ms. Brenner’s modeling included describing what she was seeing on a graph and what it might mean, extending the description of thinking that was started by a student, and voicing the reasoning used to find the value obtained in an absolute value equation. Mrs. Harmon modeled her thinking about shortcuts for calculating the areas of rectangles. Ms. Brenner solicited multiple responses from students more often than both of the other teachers.

When all categories of scaffolding student thinking and understanding are included in the comparison, Ms. Brenner demonstrated the most scaffolding of student
thinking and understanding with an average of 40.3% of the two transcribed classroom texts. Mrs. Harmon demonstrated the least scaffolding of student thinking and understanding an average of 14.5% of the two transcribed texts. The corresponding number for Mrs. Blake’s class is 30.8%.

**Student Contributions to Classroom Discourse**

Analysis of classroom discourse thus far has been focused on the teachers’ roles in setting norms, asking questions, establishing patterns of classrooms discourse, scaffolding emotions, and scaffolding thinking and understanding. The ways in which teachers set norms, ask question, establish patterns of discourse, and scaffold student participation in mathematics class are critical to supporting students’ developing roles as mathematical thinkers and doers of mathematics. When teachers scaffold student explanation, justification, and argumentation, students develop skills for problem solving. This in turn has the potential to help students develop skills for SRL, as problem-solving skills have been shown to parallel the skills needed for self-regulated learning.

The essential difference between discourses that promote understanding and those that do not is the degree to which teachers press students for articulation of reasoning and justifications of decisions made while solving problems (Kazemi, 1998). It follows, then, that in classrooms where the teacher presses students for elaboration, reasoning, and justification, greater evidence may be found in the areas of student elaboration, reasoning, and justification. Furthermore, if there is also support of student autonomy, greater evidence of expression of mathematical thinking in the form of student
conjectures and argumentation should be found. Student contributions to classroom
discourse might substantiate the positive effects of the teaching practices that are meant
to lead to such student behaviors.

The apparent correspondence between teacher press and student contribution
holds true in the comparisons between Ms. Brenner’s and Mrs. Harmon’s classes (see
Table 4.15). Ms. Brenner’s students contributed more to classroom discourse than
students in the other two cases by explaining their thinking and processes used to solve
problems, justifying their solutions, and challenging ideas or proposing alternative ways
of thinking (argumentation) than did the students in either of the other two classrooms
(see Tables 5.15 and 5.16). Mrs. Harmon’s and Mrs. Blake’s students contributed very
little in terms of student conjectures, explanations, reasoning, and justifications.

This might be expected in Mrs. Harmon’s class given that less evidence of press
for student understanding was found in her class than in the others. Mrs. Blake’s class is
the anomaly in this comparison. Mrs. Blake did press for elaboration, reasoning, and
justification, but there was relatively little evidence of student contributions of
elaboration, reasoning, and justification to classroom discourse. This could be attributed
partially to Mrs. Blake’s lack of comfort with whole-class discussion. Mrs. Blake
indicated that she was uncomfortable with the lesson format, and she preferred to work
with students in small groups (B7-08-04-24postobs). Consequently, when her students
did not respond to her relatively new approach of pressing for further elaboration,
reasoning, and justification during whole-class discussion, Mrs. Blake would quickly
move to another question rather than trusting that the students could respond given more
time to process her questions. She was less skilled than Ms. Brenner at probing to engage
students at a more interpretive level. It might also be attributed to students’ lack of skills for engaging in mathematical conversation.

<table>
<thead>
<tr>
<th>Category</th>
<th>Brenner</th>
<th>Blake</th>
<th>Harmon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argumentation</td>
<td>1.4</td>
<td>2.8</td>
<td>0.1</td>
</tr>
<tr>
<td>Conjectures</td>
<td>0.9</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Explanations</td>
<td>1.1</td>
<td>0.0</td>
<td>0.1</td>
</tr>
<tr>
<td>Reasoning</td>
<td>0.5</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Justification</td>
<td>0.9</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 4.15: Student math talk: Argumentation, conjectures, reasoning, and justification (by avg. % coverage)

The richness of mathematical language and sophistication of student expression of ideas in Ms. Brenner’s class can be seen in the examples of student argumentation, conjecture, explanation, and justification in Table 4.16. A notable feature is the reasoning supplied by the student who made the statement of disagreement. Such occurrences were rare in Mrs. Blake’s and Mrs. Harmon’s classes. For example, one instance of argumentation occurred in Mrs. Blake’s room, after Mrs. Blake led the students to agree or disagree with her statement, “The smaller the number, the fatter the parabola” (B7-07-05-03obs, 274). The student’s comment was: [I disagree.] “Because if you have negative 5, it's going to be a smaller negative parabola than if you have negative 1” (B7-07-05-
Mrs. Blake did not press for further elaboration, despite the student’s terminology. Perhaps assumed what “smaller negative parabola” meant to the student who spoke, but to be consistent with her own terminology and not confuse it with her use of “smaller,” it would be necessary to clarify that the student had meant “narrower.”

<table>
<thead>
<tr>
<th>Category</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argumentation</td>
<td>I don’t think the negative 42 is the right answer because even if you’re moving to a negative side, the absolute value will be the number of spaces moved, not where to. (B5-06-12-14obs, 219)</td>
</tr>
<tr>
<td>Conjecture</td>
<td>Like when you submit it, maybe the place it was supposed to be a negative, they started adding more than necessary. See, it’s basically the opposite of the correct answer. You start seeing a trend. (B5-06-12-14obs, 401)</td>
</tr>
<tr>
<td>Explanation</td>
<td>It says that it’s the $1500 plus $8, so we wrote 1500 plus 8, and we put T because it’s for the toys, and the materials to make the toys, and then it says equals 12 because that’s how much they have to sell. I mean they sell for 12. Each toy is, 12; and then we put it here, subtract 8T from both sides and we get 4T = 1500 and divide by 14 and we get 375. (B5-06-12-14obs, 61)</td>
</tr>
<tr>
<td>Justification</td>
<td>Because the absolute value is 2 and then there’s a negative sign outside of it which means that it’s going to come out to be a negative 2. (B5-06-12-14obs, 411)</td>
</tr>
</tbody>
</table>

Table 4.16: Examples of student argumentation, conjecture, explanation, and justification (Ms. Brenner’s class)
One instance of argumentation occurred in Mrs. Harmon’s class as well, concerning the value of zero being included in the table of width and length values (0 and 10) for a rectangular perimeter of 20 units: “How can you have zero area? It's not even... It's not there” (H8-06-04-25obs, 129). Mrs. Harmon agreed that it was not really a rectangle and another student suggested that it would be just a line (H8-06-04-25obs, 130-132). The suggestion was evidence of mathematical thinking on the part of the second student involved in the exchange. This very short discussion of the student’s argument was typical for Mrs. Harmon’s class in that students’ interjections of any kind received little if any attention.

学生的对学生的讨论。学生的对学生的讨论事件在不同案例中在质量与数量上均存在差异。学生的对学生的讨论被观察到在Ms. Brenner的课堂上32次，占了两篇录音课本的平均8.2%（见表4.17）。在Ms. Brenner的课堂上，学生在学生的对学生的讨论中参与了更多的数学讨论。在表4.16的例子中，Ms. Brenner的学生们试图应用距离公式到一个给定的问题情境。这次对话集中在x和y值的减法顺序。随着一些教师的跟进，学生之间的交流使对x和y值的标记和在距离公式中使用这些值的澄清成为可能。

The observation recording done during Mrs. Blake’s first lesson did not pick up student-to-student discourse, so a comparison cannot be made for that lesson. Student-to-student discourse occurred during the second lesson seven times, which was 2.5% of the
single transcript text. The exchanges in Mrs. Blake’s class involved either one or two utterances between students or one student explaining a procedure to another student.

In Mrs. Harmon’s lessons, student-to-student discourse was observed four times, all of which occurred in the first lesson, making up 4.5% of the single transcript text. Student-to-student discourse consisted of working out the distribution of tasks for completing the assignment during small-group work time. These student discussions were characterized by declarations of intentions by the individual students, which were then questioned or remarked upon by other members of the small group. The example in Table 4.17 occurred as students were beginning to work on portions of the assignment that they each had claimed for themselves.
<table>
<thead>
<tr>
<th>Class</th>
<th>Avg. % Coverage</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Brenner | 8.2 | S1: Don’t you subtract the second X…  
S2: The one on top is the first one or the second one?  
S1: The first one.  
S2: Well all right then, it’s wrong. (B5-07-05-04obs, 294-297) |
| Blake | 1.2 | S1: David, that's way off.  
S2: I sent it again. |
| Harmon | 2.3 | S1: I can't do that, though.  
S2: I'm putting in there.  
S3: I'm going by 2s.  
S4: By 2s?  
S3: Yes, the universal number 2?  
S4: It's not the universal number.  
S4: What does that mean?  
S3: Hey you think I can't count.  
S4: Well obviously you can't. |

Table 4.17: Student-to-student discourse

Although more discussion of mathematical concepts took place among students in Ms. Brenner’s class, student-to-student discourse was often interspersed with other student non-mathematical conversation, such as in the following excerpt.

24 Student 1: I’m done.
25 Student 2: Alright, so talk to me about the problem.
26 Student 1: Okay, a toy store spends $1500 each day plus $8.
27 Student 3: She doesn’t get it. She isn’t serious.
Student 1: So 1500 is fixed so no matter what, you got to at least enter 1500. So then it’s 8 toys per day. So what I put was 8T, T for toys, because you don’t know how many toys you’re going to sell, less 1500. 1500, that’s what you’ve already spend. Equals, and they said 12 each, so I put 12T. So my equation was 8T plus 1500 equals 12T.

Student 3: It’s wrong.

Student 1: So I minus 8T from both sides to get 1500 equals 4T. I divided it by 4 and got 375 toys they must sell to get a profit.

Student 2: Okay, what you just said I did not even understand.

Student 1: Okay. Let's do it what you think.

Student 3: Let’s do it the opposite...

Student 4: You might get kicked out of school. [This comment was made in reference to the topic of pregnancy, which came up earlier during the observation.]

Student 2: I'm not getting kicked out of school.

Student 1: Yeah, let’s listen to you two.

Ms. Brenner: [Over the continued conversation of students] Once you have your equation, don’t forget, you’ve got to solve for the variables.

Student 3: I just want to be a mercenary.

Student 2: What?

Student 2: If that happens, I will choke you. (B5-06-12-14obs)

The whole-class discussion that followed small-group work time during this lesson was rich with mathematical discussion, which may not have been expected based on the student-to-student discourse in the example above. When questioned specifically
about these episodes of student-to-student discourse where students alternated between mathematical and non–mathematical discussion, Ms. Brenner stated that the students did have a remarkable ability to keep both lines of thought (non-school/social and school-focused/mathematical) going during student work time. She was bothered by the extra conversation when she was in her first years of teaching, but she now considers the non-mathematical student conversation to be “the lubricant that keeps the entire conversation going” (B5-08-06-22phint). Furthermore, she believed that if she were to discourage the more social student conversation, the mathematical discussion would suffer. She reported that when she discouraged “off-topic” conversation earlier in her teaching career, mathematical conversation also stopped. She found that it was important to allow the students to engage in conversation related to their social/cultural lives in order to teach them how to engage in a more formal school-based form of discourse, essentially facilitating the transition into an unfamiliar register.

Artifacts of Setting Classroom Norms

In Ms. Brenner’s class, student-made posters provided evidence of talking about how to participate in meaningful discourse in the classroom. The student-created rubric (Figure 4.1) included categories of classroom discussion, note taking, and listening. Each category was then described at four different assessable levels: impressive, expected, adequate, and minimal. The rubric made the expectations for classroom participation explicit, including how to assess one’s own participation in relation to the expectation.

When asked specifically about talking about metacognition, Ms. Brenner indicated that she and other teachers at the grade level probably did not use the term *metacognition* in their classroom discussion, rather they broke the concept down into
chunks the students could understand (B5-08-06-22phint). The emphasis was about *accountable talk*, which was also described in a student made poster, and what it means for individuals interacting in a learning community. Early in the school year the teachers described and modeled *accountable talk*, and then students made the posters as a process of filtering the information and putting it back together (B5-08-06-22phint). *Accountable talk* includes accountability to the learning community, accountability to accurate knowledge, and accountability to rigorous thinking. Ms. Brenner stressed working together, using mathematics vocabulary, using *accountable talk*, and asking thoughtful questions. She scaffolded accountability by teaching students to engage in processes of listening in class, echoing back questions, and explaining their own thinking. The next student would echo back what was already said and add to, and so forth (B5-08-06-22phint). This very explicit instruction may have led to more student input during classroom discourse, which in turn provided more evidence of student thinking.

Mrs. Blake’s room was filled with all sorts of posters and evidence of student work. Colorful student-created designs were on display. Posters and other creative student work indicated that students were given agency in carrying out assignments. While multiple pieces of student work were obviously from the same unit of study, the individuality of expression was clearly visible. Products of student work did not look too much alike.

Mrs. Blake also had a commercially made poster on the wall referring to metacognition. Throughout Mrs. Blake’s lessons, she asked students to think about specific things that were said or about other products of their work. In an interview, she indicated that she did talk about metacognition with her students:
We talk early on about metacognition. As often as possible, I ask them to think about what their thinking was when they heard the problem, if they thought it was hard, so I try to get them into that habit of mind. I think they start to assume that habit after a while. (B7-08-04-24postobs)

But even as she stressed thinking and thinking about thinking, as she spoke to her students during the two observed lessons, Mrs. Blake did not press for verbalization of their thoughts about their thinking. Students proposed different answers to a previous question rather than talking about their thinking. In addition, when questioned about their thinking in small groups, students indicated that they understood the error with comments such as “I got it,” and Mrs. Blake moved on. Evidence of students thinking about thinking was available only indirectly.

Mrs. Harmon’s classroom did not have student work up on the walls during the first of the two lessons described. She did have several mathematics posters, which were evenly distributed around the room. On the second day of observation, the student work from the previous day was on display (see Figure 4.7). No mention was made of the displayed posters, however, beyond declaration that the students had done good work. As described above in Case #3, the student posters were remarkably similar. Students’ individual creativity was not encouraged in Ms. Harmon’s two lessons. Furthermore, there was no evidence of talking about talking about mathematics and no evidence of talking about thinking about mathematics visible in the classroom. Mrs. Harmon did ask students to think about specific ideas that she proposed, but when she did, it was not a bid for student input.
All three teachers made it clear to their students that they wanted them to be successful and to be ready to go on to college. They all conveyed the importance of understanding the mathematics. What evidence from both the physical classroom and the classroom discourse convey, though, is that the three teachers went about providing scaffolding for student understanding and future success in very different ways. The different ways that teachers supported their students resulted in three very different pictures of student participation and input.

Thinking about Equity and Social Justice

Equity and social justice, as related to cultural diversity, are themes of interest within this study. Traditionally, little attention has been paid to meeting the needs of diverse learners. Because there were no obvious signs of teachers’ thoughts about equity and social justice during the classroom observations, I asked question about the classroom observations that might relate to efforts toward meeting the needs of culturally diverse students. I asked Ms. Brenner about the accountable talk posters and whether or not she thought about issues of social justice in connection to scaffolding students’ accountable talk. Mrs. Blake was asked about the specific extra roles that students have taken on in her classroom that might be efforts made toward assigning competence. With Mrs. Harmon, I asked about the students who seem to have positive dispositions for thinking about mathematics, and whether or not she has thought about issues of equity and social justice in planning to support that kind of thinking.

Ms. Brenner had not specifically thought about issues of equity and social justice when planning for supporting accountable talk and understanding mathematics. She did say, however, that she considers the work that she has done to expose students to as much
technology as possible to be part of a philosophy of social justice. She explained that brand new teachers often shun student use of calculators because they see the calculator as an answer box, not as a pedagogical tool. She believes that it is her duty to train students to use the technology, because they have just as much right to succeed on exams as the students who would be more likely to have personal and school access to such technologies. She believes in providing technological advantages to her students, who are students of color, many of whom live in poverty, as a way to “level the playing field” (B5-08-06-22phint).

Mrs. Blake also indicated that she did not specifically think about equity and social justice when planning for activities in her classroom. But she did acknowledge that she finds ways to engage particular students who she feels are not engaging enough in class activities:

If I know that there is somebody that may not be paying attention, I like giving them a job. It’s a real job. It needs to be done. But maybe it will engage them more in what’s going on. (B7-08-04-24postobs)

She went on to talk about specific students in the class who she had targeted for special duties. Mrs. Blake described one girl as really quite smart but not doing what she should to be successful in class and two boys as fooling around, not taking the situation seriously.

She likes feeling that she’s got something that she needs to do. I think I get a little more investment that way, if everybody has a job, even if it is the environmental protection agency. I had two boys that were kind of a mess, so I made them the EPA and they took it seriously. (B7-08-04-24postobs)
I also asked Mrs. Blake about a particular student, Henry, who I had noticed in the role of technological expert during the second observation. Henry and the girl mentioned above were African American students who were reluctant to share during class and directed their attention in ways that were considered “naughty.” Mrs. Blake’s first reaction to the question about her actions with students like Henry was that Henry was just plain good at technology. However, as she went on to talk about Henry it became clear that she was encouraging something that Henry liked to do as a way for him to be competent in the classroom—an action that teachers might take toward addressing equity and social justice.

Henry liked doing the tech stuff. The reason I chose Henry is because he’s a little bit of a naughty boy, or at least he was. I like picking those types out and giving them something to do, because it tends to make them calm down a little bit. (B7-08-04-24postobs)

Mrs. Blake’s believed that all of the described actions directed at individual students had to do with buy-in into the community of the classroom. It was important to her that everybody felt included in the community and everybody felt as though they could have some effect on the classroom (B7-08-04-24postobs). That buy-in might be equated with student agency within the community. She described it as part of engagement, emphasizing that “it all has to do with learning.”

I mean, that’s the bottom line for me. I just want everybody to achieve.

You don’t gain that [achievement] unless you have that kind of buy-in and participation. (B7-08-04-24postobs)
During an end-of-the-year interview, I asked Mrs. Harmon if she ever thought about social justice as she planned lessons or as she interacted with her students. She reacted very cautiously, asking me to tell her what I meant by social justice before responding to my question—to make sure that my definition was the same as what she had in mind. Following my description of what I thought could be included as components of teaching for social justice in mathematics class, she said that she did not specifically do any of the things I had mentioned, but she found it interesting. She did not talk about her philosophy of teaching or her thoughts about planning lessons in relation to traditionally underserved student populations.

All three teachers indicated that they had not specifically thought about equity and social justice during lesson planning, but Ms. Brenner and Mrs. Blake talked about philosophies they thought were related. As Ms. Brenner and Mrs. Blake implemented their philosophies, they were supporting their students in ways that had potential to promote equity and social justice. They wanted their students to feel as though they had agency within their learning communities.

Summary

In all three cases, teachers established positive working environments. There were friendly interactions between students and teachers and among students. Students seemed to enjoy the mathematics lessons.

Ms. Brenner and Mrs. Blake provided both social and analytic scaffolding in their classrooms through questioning. Social scaffolding helped students know how to participate in discussion. Analytic scaffolding helped them know how to explore the
mathematics more deeply. Ms. Brenner was the most explicit in her scaffolding of both social and analytic aspects of learning mathematics for understanding.

Teachers’ patterns of discourse also provided evidence of social and analytic scaffolding. Discourse in Mrs. Harmon’s class was made up segments of traditional IRE. All three of the teachers used IRE, and all three of the teachers used more LOrec questions than any other type of question. However, not all use of IRE was strictly the traditional pattern of the teacher asking the LOrec question for which there is a known correct answer, a student responding, and the teacher then evaluating the student response as correct or incorrect. Mrs. Blake’s use of IRE for checking homework was strictly traditional. At other times, she used IRE to explore mathematical concepts and student ideas. At times when Ms. Brenner used traditional IRE, the episodes functioned to draw student attention to very specific aspects of the lesson, such as the idea of distance on a number line as related to absolute value. Both Ms. Brenner and Mrs. Blake used open-ended questions, and Ms. Brenner asked students to clarify their statements. Mrs. Harmon did not ask open-ended question. Neither did she ask for clarification of students’ comments.

Uptake of students’ comments occurred in all three classrooms. The use of the uptake varied between classrooms, however. Ms. Brenner took up students’ comments and questions to examine both correct and incorrect answers to previous questions and other ideas proposed by students. Mrs. Blake also took up student comments to explore mathematical concepts, but she did so less frequently, four times as compared to 18 times for Ms. Brenner. Mrs. Harmon took up student comments five times, primarily to elaborate on a student’s answer to a question.

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Emotional scaffolding in Ms. Brenner’s class was used to clarify norms for student participation and to call attention to what counts as good work. It was a prominent feature of Mrs. Blake’s class as well, but was used more to affirm students’ ability to do good work. Mrs. Harmon did not make overt statements of encouragement during the observed lessons. Nonetheless, Mrs. Harmon’s students often encouraged each other, which gave the impression of a strong sense of community among students. Ms. Brenner’s students also routinely expressed encouragement to each other.

The greatest number of occurrences of scaffolding students’ thinking and understanding could be found in Ms. Brenner’s and Mrs. Blake’s classrooms. Ms. Brenner and Mrs. Blake challenged their students to think, supported making sense of the mathematics, and pressed for understanding. These teacher behaviors occurred most often in Ms. Brenner’s class and least often in Ms. Harmon’s class.

Ms. Brenner’s students contributed more to classroom discussion of mathematics than did students in the other two classrooms by making conjectures, explaining steps they took to solve problems, explaining their reasoning, justifying their ideas, and initiating argumentation. Student contributions to classroom discourse were relatively sophisticated in comparison to the other classes. Such student contributions were rare in both Mrs. Blake’s class and Mrs. Harmon’s class. While Mrs. Blake attempted to scaffold such student behaviors, her prompts were not explicit. She also did not question students multiple times to get them to explain their thinking more deeply. It was often the case that another student would contribute another idea, expressed in very few words, and she would then respond to that next idea, which eliminated the possibility of staying with the previous student’s idea.
Student-to-student discourse differed in quality between the three classroom cases. Ms. Brenner’s students engaged in more student-to-student discourse, and that discourse included deeper discussion of the mathematical topics than student-to-student discourse in the other classrooms. Ms. Brenner’s students were also able to sustain mathematical discourse even as they interjected comments and short exchanges that had nothing to do with the mathematics. Student-to-student discourse was relatively rare in Mrs. Blake’s and Mrs. Harmon’s classes.

The physical classrooms facilities also provided evidence of classroom norms. The walls of Ms. Brenner’s classroom were covered with student-created posters that had been made earlier in the year. The posters were a product of a lesson where teachers worked with students to understand what it means to contribute to classroom discourse (B5-08-06-22phint). In the lesson, teachers talked about what it means to be accountable to the learning community, to accurate knowledge, and to rigorous thinking. They modeled accountable talk, and had students practice it. Students worked together to process what accountable talk would mean for them, and they produced the posters to express their understanding. Mrs. Blake’s room was filled with evidence of creative student work. Mrs. Blake also had a poster that conveyed something about metacognition. Mrs. Blake talks with her students about metacognition, but there was no evidence of having pressed the students to understand what that would mean in terms of their contributions to classroom discourse. Mrs. Harmon’s room did not have any physical evidence of talking about talking about mathematics or talking about thinking about mathematics.
None of the teachers explicitly planned for addressing equity and social justice in their classrooms. Ms. Brenner talked about her teaching philosophy, however, that did address the specific needs and the success of students of color who have relatively little economic power within the larger community. Her concern for social justice and equitable access to technology were played out in the mathematics classroom, but not in exploring the mathematics of inequity and injustice. Ms. Brenner also worked to involve students to be successful during classroom discussion, sometimes assigning competence that might otherwise go unnoticed. As a result of our discussion during an end-of-year interview, Ms. Brenner expressed interest in literature that supports using mathematics to explore issues of equity and social justice.

Mrs. Blake consciously worked for equity in student classroom involvement. Even though her efforts at putting students in successful roles in the classroom were aimed at students of color, she was not conscious of issues of equity and social justice as related to race and ethnicity. Her goal was to create a strong, supportive classroom environment through “buy-in,” where all members were contributing members with a sense of agency with the community. Mrs. Harmon was uncomfortable talking about her classroom in terms of equity and social justice. Additionally, she did not volunteer any of her personal philosophy about teaching, equity and social justice, or building classroom community.

Overall, more evidence of teaching mathematics for understanding, scaffolding of SRL, and concern for equity and social justice for non-White, low SES learners was found in Ms. Brenner’s class. Mrs. Blake was not as effective at providing support for
understanding mathematics and developing SRL. She had a strong sense of providing support for all of her students, but she went about it in a colorblind fashion. Mrs. Harmon did the least to support student understanding of mathematics and the development of SRL. She was not able to talk about issues of equity and social justice. She did convey some interest in the topics, however, after I spoke with her about my own interests.
For the past three decades, a significant amount of research in mathematics education has focused on creating classroom contexts to support optimal student learning (Artzt & Armour-Thomas, 1992; Cobb, 1999; Mendez et al., 2007; Turner et al., 1998; Turner et al., 2002; Wood, 1999). The approach to creating classroom contexts as described by PSSM (NCTM, 2000) and as examined in related research has been referred to as reform mathematics. More recently, attention has been directed toward issues of equity for the purpose of understanding the ways in which traditional forms of mathematics education have not supported teaching and learning for understanding, particularly in relation to meeting the needs of students representing cultural diversity, such as non-mainstream, non-White and low SES students (Cobb, 1999; D'Ambrosio, 1994; Dowling, 2001; Herrera & Owens, 2001; NCED, 2001; Ladson-Billings, 1997; Malloy, 2004; Mukhopadhyay & Greer, 2001; NCTM, 2000; NCR, 2001; Oyserman et al., 1995). Questions still remain about ways in which reform mathematics practices support learning for culturally diverse learners and contribute to eliminating persistent gaps in mathematics achievement, and whether indeed reform practices, as they are presently understood, do help to reduce gaps in achievement.
In the literature on SRL, classroom context has been recognized as contributing to the development of self-regulated learning. While self-regulation implies an internal, individual cognitive development, research has provided evidence that students can be guided to use self-regulation strategies (Schunk & Swartz, 1993; Zimmerman et al., 1996) and to set learning or mastery goals (Dweck, 2000; Middleton & Midgley, 1997; Schunk & Swartz, 1993) through explicit instruction within the social setting of the classroom. Theory related to the relationships between language, knowledge, and power suggests that power relationships are unstable and inequities can be addressed through dialogic discourse. Bodies of literature on the development of SRL and on the relationships between language, knowledge, and power both recognize the importance of positive self-concepts in developing agency in the social context of schooling.

This study was motivated by possibilities for understanding the development of SRL for culturally diverse students (including non-White, non-mainstream student). In this quest for understanding, my first goal was to examine and describe classroom contexts (cases) that are representative of cultural diversity, and second, to analyze across cases for continuities and discontinuities that might provide evidence of classroom practices that support learning mathematics with understanding and the development of SRL for culturally diverse learners.

Coding of videotapes and transcripts of classroom observations provided insight into important classroom interactions. Coding resulted in identification of the following families of themes used for descriptions of cases and analysis across cases: affect, classroom norms, teacher questioning and patterns of discourse, scaffolding emotions, scaffolding mathematical thinking and understanding, and student contributions to
classroom discourse (see Table 3). Two additional themes, artifacts of setting classroom norms and thinking about equity and social justice are included because of their importance to my initial interest in issues of equity and social justice as they relate to cultural diversity. Analyses of themes from the three cases and relationships among them are discussed in sections below in terms of looking at what teaching mathematics for understanding and fostering SRL looks like when working with culturally diverse students.

Families of themes, grouped to facilitate discussion, are (1) affect and scaffolding emotions, which are critical factors in supporting students’ in developing agency as learners; (2) classroom norms, small-group problem solving, and development of SRL, which provide insight into ways that teachers support learning mathematics at the same time as scaffolding SRL; (3) patterns of classroom discourse as related to culture and equity, which addresses cultural diversity and positioning through dialogic interaction and the creation of opportunities for taking on subjective roles within dialogic discourse; (4) scaffolding mathematical thinking and understanding, and student contributions to classroom discourse which examines the related roles of teachers and students in creating contexts that include thinking and understanding and substantial student contributions to the discussion of mathematics; and (5) thinking about equity and social justice which addresses central issues in my quest to understand the relationships between cultural diversity and pedagogy.

Affect and Scaffolding Emotions

Five strands of mathematical proficiency are identified in Adding It Up: Helping Children Learn Mathematics (Kilpatrick et al., 2001). The fifth strand, productive
dispositions, refers to “the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics” (Kilpatrick et al., 2001, p. 131). The definition provided by NCR does not specifically use the words emotion and motivation to identify affective elements that shape learning, but emotion and motivation are integral to dispositions leading to engaged classroom participation and directly influence success in problem solving.

According to Turner et al. (2002), mastery goal learning environments consist of both cognitive and motivational components. Emotional reactions to negative performances can have different effects depending upon the confidence of the learner (McCann & Turner, 2004). In light of the influence of affective factors on motivation, teachers must attend to the affective components of learning mathematics so that students are able to see themselves as being competent and become more willing to take risks associated with involvement in classroom inquiry. Students may also find guidance on how to cope with the frustrations that are naturally part of problem solving and authentic inquiry.

In the three cases analyzed for this study, teachers attended to affective elements in varying degrees and for different purposes. Mrs. Blake’s use of encouragement stood out, and both Ms. Brenner and Mrs. Blake expressed concern for the quality of experience during learning. Of particular interest for this study, however, is the degree to which attention to affective elements explicitly supported meaningful engagement in classroom activities and students’ ability to cope with difficulties encountered during problem solving.
Mrs. Blake’s words of encouragement were most often a source of affirmation of her students’ ability to do good work. As she encouraged her students with comments such as, “Okay kids, look. You're getting too good. So let's do one more.” (B7-07-05-03obs, 334), she seemed to be cheering them on. The comments were made in an effort to make the students feel good about the work they had accomplished. At other times, Mrs. Blake reassured her students that they would be able to overcome difficulties with more practice, but analysis of the problem and further practice was not proximal—it did not occur during the observed lesson. Mrs. Blake neither drew her students into discussion for the purpose of analyzing difficulties they were having, nor guided them in developing their own goals for overcoming difficulties.

In contrast, Ms. Brenner worked to help students understand their feelings about difficulties with the mathematics by discussing them in relation to learning from errors and by conveying the value of discourse that includes analysis of both correct and incorrect student solutions to problems. Ms. Brenner’s explicit attention to relating the discussion of difficulty with solving mathematical problems to learning with deeper understanding helped to set the norm for open discussion and helped create an atmosphere of safety for taking risks, which can be important for students of color who may hesitate to participate in competition due to relatively communal orientations as well as students who may be developing fixed-entity theories of intelligence based on repeated failures within competitive environments. Students did not have to fear “not looking smart” in front of their peers. The message conveyed was that the smart thing to do when having difficulty is to discuss problems in order to understand what went wrong. In other
words, the emotional scaffolding provided by Ms. Brenner highlighted not knowing as a temporary state signaling a need for exploration of options.

“The importance of emotional support and social approval, achievement, self-efficacy, and positive coping (as opposed to reactive or avoidant coping) remain essential components for facilitating healthy identity formation” (Swanson et al., 1998, pg. 36).

By providing a safe atmosphere for discussion of problem solving, Ms. Brenner may have supported her students in developing an incremental theory of intelligence, toward understanding intelligence as being malleable (Dweck, 2000). She may have also supported the possibility of formation of identities as mathematical thinkers. Her actions reflect the notion that one can learn through unsatisfactory or unsuccessful experiences as well as satisfactory or successful ones. Additionally, attending to affective elements of learning helped to establish norms for participation in authentic mathematical thinking and problem solving. Her students were developing skills for working together to communicate ideas and solve problems—skills required for doing mathematics.

Attention to affect during mathematics classes can be directed toward more than assurance and praise. Feedback and discussion of affective elements can help establish norms for classroom discourse and scaffold development of skills related to problem solving and SRL. Perhaps not every instance of affective support would include additional scaffolding. Teachers can be aware of affective reactions from students, however, and realize the importance of attending to affective reactions as part of developing skills for problem solving, SRL, and productive dispositions. Attention to affective reactions might be especially important for students outside the cultural
mainstream of the United States who may already lack confidence in their ability to communicate and participate in classroom activities.

Classroom Norms, Small-group Problem Solving, and Development of SRL

Teachers convey both explicit and implicit messages about what is valued in mathematics class and what it means to learn mathematics. If the message is that mathematics should make sense and can be used as one way of understanding about how the world works, then mathematics can become a language for thought rather than merely a collection of procedures for getting right answers (Carpenter & Lehrer, 1999). The NCTM learning principle suggests that students learn “mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, pg. 20). This description of learning mathematics with understanding acknowledges the socio-historical nature of learning within a community, supporting the idea that students who have prior learning experiences (a history) must be put into active roles in defining their own goals and monitoring their own progress (a social present with attention toward a future). As they are involved in building knowledge by connecting prior knowledge and new experience, students must monitor their progress and set goals for further development.

Development of SRL is most likely to occur when learning experiences socially convey the processes needed to regulate at the next level of academic competence (Schunk & Zimmerman, 1997). Because perceived similarity to models is important for students who do not value a particular learning task, mastery models are not necessarily beneficial at the outset. Students need to see others encounter and overcome the obstacles
that they will likely come across while problem solving so that they understand the true nature of problem solving; cognitive models they experience first should be those of encountering and overcoming difficulties.

Small-group problem solving provides opportunities for putting students into active roles of defining their own goals and monitoring their own progress. In all three cases observed for this study, teachers incorporated small-group problem solving into their instruction. The ways that teachers attended to student progress varied. During small-group work time, Ms. Brenner supported students’ problem-solving attempts by providing social scaffolding. During both the small-group work time and during intermittent whole-class discussion, she questioned students about what they were observing mathematically and provided follow up questions to help students decide on next steps for problem solving. To provide models of encountering and overcoming difficulties for the benefit of the whole class, she asked students questions that required explanations of the thinking processes they used or descriptions of what they would try next. Ms. Brenner’s questions were specifically directed at eliciting descriptions of student thinking; she did not ask for, nor did she expect, a correct answer/solution to a problem.

When Mrs. Blake worked with small groups, she asked questions of individual students, but did not expect them to verbalize their thinking. Her focus appeared to be on making sure that individual students that she assessed as having difficulty understood how to do the particular part of the assignment they were working with at that moment. If students indicated that they understood their errors, Mrs. Blake moved on to check on
another group or individual. By moving on, Mrs. Blake missed opportunities to highlight student thinking and use it as an object of discussion, to set the norm for dialogic discourse during problem solving, and to help all students understand that problem solving is a cyclical process that includes making errors, rethinking processes used, creating new plans, and testing new methods for solving the problem.

In general, Mrs. Harmon did not support small-group work through questioning. Most of the discourse directed at students and in response to students’ questions during the small-group work time had to do with restating directions, reminders about directions, or clarification of directions. Overuse of nonscaffolding discourse patterns (such as giving directions, answering questions about directions, commenting on off-task behavior, and asking questions with known answers) can be considered evaluative, and can shut down genuine problem-solving processes (Turner et al., 2002). The students in Mrs. Harmon’s room were more concerned with requirements for the production of their posters than they were in understanding what the assignment had to do with mathematics. Mrs. Harmon’s introduction of the task resulted in student perception of a requirement for a rather mechanical set of procedures to follow rather than an invitation to explore the relationships between side lengths and areas of rectangles with a fixed perimeter measurement.

Small-group work formats have potential to encourage spontaneous verbalizations that allow students to externalize their ideas for critical examination. This type of discourse occurred during small-group problem solving in Ms. Brenner’s classes. Student discourse included asking each other questions, explaining processes used, questioning of explanations, and justifying answers. Students’ skills for engaging in problem solving can
be supported by scaffolding practice in problem-solving processes, which might include teacher modeling (student observation of problem-solving behaviors) and teacher scaffolding of student discourse. Equally important is the teacher’s choice and assignment of tasks that require authentic problem-solving behaviors. The ability to imagine and conjecture should be cultivated as competence and fluency are built up through experience with rich and challenging problem-solving tasks.

Teacher feedback during student small-group work has the potential to be “a persuasive source of efficacy information” (Schunk & Swartz, 1993, p. 351). Feedback during small-group problem solving can highlight strategy use as a means to improve performance and convey the message that students are becoming more skillful in the process. Both students and teachers can use feedback as formative assessment. When students participate in assessment of their own mathematical understanding, they are able to develop their own conceptions of standards of quality work and responsibility for learning.

In summary, small-group problem solving can be used as a vehicle for the establishment of norms for classroom problem-solving behaviors and to scaffold development of SRL. Ms. Brenner demonstrated the most support of students’ problem solving and development of SRL as she probed for student expressions of understanding and provided feedback. Her questions guided students to reflect on thinking processes and to voice plans for making changes, thus providing models of encountering and overcoming difficulties during problem solving for all of her students. As the students learned to apply these processes to problem-solving situations in mathematics, they were being supported in the development of SRL.
Patterns of Classroom Discourse as related to Culture and Equity

Traditional forms of public education envisage learning in terms of a regulatory text, positioning the teacher as the holder of the knowledge and students as objects who do not yet know (Dowling, 2001). This conception of learning denies the dialogic and discursive character of communication as well as the learner’s agency in the construction of knowledge. IRE patterns of discourse dominate traditional classrooms. IRE patterns of discourse, when used in their traditional form, do not scaffold student learning as envisioned by NCTM, nor do they support development of SRL. Nonscaffolding discourse patterns are less about assisting learning and more about directing, assessing, and controlling.

The language of traditional school mathematics might be categorized as monoglossic, which results in the construction of understandings that restrict imagination (Nystrand et al., 2003). Polyglossia, or the debate between languages, and heteroglossia, or the disagreement between voices and dialects within a language, provide arenas for debate between ways of thinking (Lye, 1998). A Bakhtinian stance toward language emphasizes self-other differences that motivate dialogue, thus differences in understanding can bring about productive dialogue and thought.

Given that self-other relationships and differences in understandings motivate dialogue, a Bakhtinian conception of the contextual and ideological nature of language and dialogue is useful in thinking about pedagogy that addresses inequities in power relations in school settings. Knowledge and ideologies are socially constructed; therefore the agentive role of language and dialogue in the formation of power relations, social identity, and patterns of beliefs must be better understood within the social context of
schooling. Especially as students from diverse cultures come together in classrooms, moves toward dialogism might help to provide more equitable learning environments.

Nystrand et al. (2003) found that specific types of teacher and student moves lead to dialogic discourse. In a study of hundreds of classroom sessions, dialogic discourse was found to occur following “dialogic bids” from teachers. Dialogic bids came in the form of authentic questions and uptake of students’ comments and questions. Student questions, when taken up for discussion, were especially strong in initiating episodes of dialogic discourse. Dialogic episodes were characterized by engaged student questions and the absence of recitation questions from the teacher. The teacher’s role was to keep the discussion going while the students make substantive contributions in the form of observations, conjectures, argumentation, and reasoning.

Dialogic discourse and discussion have been linked to greater student achievement (Nystrand, 1997). An additional benefit of supporting dialogic discourse is the resulting student subjectivity. Creating classroom contexts for students to take on subjective roles, and scaffolding students’ skills for taking on subjective roles within dialogic discourse is especially important for students who come from non-White, non-middle-class communities. Non-mainstream students often possess value systems that are not congruent with mainstream customs and values. When non-mainstream students take on subjective roles in the creation of knowledge, they are putting themselves into roles of creators of knowledge that they have determined to be pertinent to them rather than passively taking in what they are told within an unfamiliar culture.

Dialogic discourse occurred only in Ms. Brenner’s classes. These dialogic episodes were extended in form—the one noted in the first lesson lasted about six
minutes and took up about 10% of the transcribed text of the first observation. Although
these episodes may seem to occur spontaneously, specific types of teacher and student
utterances initiated dialogic episodes (Nystrand et al., 2003). Analysis of discourse in Ms.
Brenner’s classes confirms that dialogic episodes occur following “dialogic bids” in the
form of authentic questions, uptake of students’ comments, and uptake of student
questions. However, authentic questions, as defined by Nystrand, et al. (2003), did not
occur during observations in the three cases presented for this study. In instances where
dialogic discourse followed a question from Ms. Brenner, the question could be described
as having been strategically aimed at drawing students into discourse based on products
of student work, such as incorrect or correct answers, to deepen their understanding of the
thinking that led to such an answer. Accordingly, such occurrences were coded as uptake
of student comments.

Dialogic episodes did not occur in Mrs. Blake’s classroom. During the
observations of her second lesson, Mrs. Blake was experimenting with conducting her
classes differently than she had in the past (B7-08-04-24postobs). She was making a
transition to whole-class discussion focused on the products of student work supported by
the TI-Navigator at the time of the second observation. Mrs. Blake was aware of using
whole-class discourse as a tool for learning using products of student work as objects of
discussion. She knew that such discourse might provide deeper understanding of the
mathematical concepts being explored. She was not, however, comfortable with engaging
students in whole-class discourse toward meeting her objectives.

Near the end of the second lesson, when Mrs. Blake was less in control of the
direction of discourse, students noticed differences in the graphed equations and began to
discuss the differences based on what they knew about linear and quadratic equations. Students began exploring the mathematical concepts because Mrs. Blake’s usual form of directing the discourse through questioning was absent. While this was an interesting occurrence, it was not a deliberate move on her part. Mrs. Blake was not aware that she was stepping out of the way to allow students’ engagement in problem solving. She knew the students were actively processing the meaning of the graphs, but she did not necessarily comprehend the moves that led to the possibility of an authentic dialogic exchange.

The approaches taken by Mendez et al. (2007) to exploring classroom discourse might support developing classroom discourse in classes like Mrs. Blake’s. Their study included examining student moves during discussion in relation to mathematics content (substance) and discussion forum (process) and examining the ways the teacher paid attention to and reflected on students’ ideas raised during discussion. Mendez et al. (2007) conducted their study collaboratively with a teacher who was already involved in efforts to improve the quality of discourse in his classroom. Teachers who are attempting to move toward more student-centered learning must make shifts in their own practice to encourage dialogic discourse. The teacher’s initial, intentional awareness of the need to make shifts in practice combined with feedback based on the observation and analysis of teacher and student practices by the researchers supported the teacher in creating an environment that was conducive to dialogic interaction.

Dialogic episodes did not occur during Mrs. Harmon’s classes, where IRE was the typical pattern of discourse. Discourse in Mrs. Harmon’s classes could be characterized as frequent use of procedural talk and IRE sequences. Her use of whole-class discussion
did not indicate work toward student autonomy or development of SRL. Mrs. Harmon’s
description of her “honors” class indicated that her students were in a lower track. She
reported that while they were considered to be at the top of the ninth-grade algebra
classes, ninth-grade algebra classes were made up of students who did not take algebra in
seventh grade, the top tier, or eighth grade, the second tier. The practice of tracking of
students raises additional questions about efforts to encourage dialogic discourse in
mathematics classes. Nystrand et al. (2003) found that authentic teacher questions and
uptake of student comments for discussion occurred with similar frequency in both high-
and low-tracked classes. The patterns and clustering of authentic questions and uptake,
however, were less likely to occur as a sequence in low-track classrooms, and students in
low-track classrooms did not ask questions that initiated dialogic episodes.

This is an important finding in light of concern for issues of equity in mathematics
education and the persistence of disparities in mathematics achievement, and has
implications for further educational opportunities of students in low-track classes.
Nystrand et al.’s findings (2003) can be corroborated by analysis of data from Mrs.
Harmon’s classes. Mrs. Harmon did take up students’ comments, but uptake was not used
to engage students in dialogic discourse. Instead of probing for more evidence of student
thinking and greater student involvement, she would elaborate on a student’s comment
and then move on. Furthermore, when students did ask questions indicative of
thoughtfulness, the questions were essentially dismissed as though irrelevant to the
discussion. Her persistence in sticking with her agenda, which was not supportive of
students becoming active agents in their own learning or learning for understanding,
resulted in preservation of the hegemonic nature of traditional school practices.
Teachers should strive to engage students in dialogic discourse in order to create environments that are sensitive to cultural diversity so that teachers gain perspective on students’ views on the relationships between the content being studied and the students’ lives. Within such an environment, students would have opportunities to experiment with a variety of roles within the classroom discourse, enabling them to expand images of themselves as mathematical thinkers and doers of mathematics, and students would be provided opportunities for greater agency in the construction of knowledge.

Greater agency as individuals within the classroom need not come at the expense of group identity or group esteem. Environments that are sensitive to cultural diversity would meet the needs of students whose cultural or ethnic identities include a more communal quality than typical mainstream mathematics classes, as suggested by Oyserman et al. (1995) and French et al. (2006). The giving and receiving of support that is valued in many non-mainstream cultures does not necessarily conflict with the idea of dialogism, rather it may possibly be indicative of having processed expectations for members of the community in a more collective manner.

Scaffolding Mathematical Thinking and Understanding, and Student Contributions to Classroom Discourse

The explicit manner by which a teacher initiates and establishes expectations for participation in classroom discourse creates the condition for private thought to become available for public examination (Wood, 1999). In order to think about and respond to the thoughts of others, expectations must also include listening (Schifter & Riddle, 2004; Wood, 1999). The ability to reason, conjecture, and construct mathematical arguments develops through frequent classroom discussion among students and between the teacher
and students. Mathematicians must convince others of the validity of their ideas and also must be persuaded by the arguments of others. Therefore, teacher creation of contexts for student participation in mathematical argumentation can be seen as scaffolding of *mathematical thinking and understanding* by putting students in authentic roles of doing the work that “real” mathematicians do.

Classroom artifacts from observation of Ms. Brenner’s classes suggested that Ms. Brenner worked with her students to understand expectations for classroom discourse. Student-created posters indicating prior discussion of talking and listening were displayed around the room. Students had created a rubric earlier in the year that described varying levels of classroom participation and established what appropriate levels of participation would include for class discussion, note taking, and listening. Because it was a rubric, non-examples of appropriate behavior rated as *minimal* and *less than adequate* were also described, such as “making jokes about student or teacher,” “doesn’t care or doesn’t bother,” and “distracting other students.” The rubric provided evidence of students’ processing of expectations, and it showed that students had worked to understand classroom expectations as they might play out at different levels.

Other “thought bubble” type posters, indicating more focus on talking about mathematics, were presented as thoughts of sketched or implied characters. One such poster included the following:

- Think about your answer and comments.
- Learn how to listen to others.
- Be respectful to other peoples’ thoughts.
- “The solution meets the criteria because”
• “I don't understand. Can you explain it to me again?”

• “That solution is correct but another...” (B5-2006-12-18obs)

Another poster advised “Math Lover says: Ask accountable questions,” and “Remember: There is more than 1 way to solve a problem” (B5-2006-12-18obs). The two posters provided indirect evidence of talking about talking about mathematics.

Sociomathematical norms are social norms specifically relating to normative aspects of mathematics discussions (Yackel & Cobb, 1997). Talking about talking about mathematics, or the setting of sociomathematical norms, was not directly observed during classroom discourse in any of the three cases.

During an end-of-year interview, Ms. Brenner explained that all teachers of ninth-grade students at her school discussed accountable talk with their students at the beginning of the school year. They modeled accountable talk for students, and had students process what accountable talk means through role play, group discussion, presentations back to teachers, and creation of the posters. The accountable talk immersion provided both social and analytic scaffolding, giving students the opportunity to practice a register of language interaction that they may not have experienced before. Through classroom discourse, students and teachers negotiated what would count as an acceptable explanation or justification of mathematical reasoning, and they learned how to engage in mathematical argumentation. Additionally, the emphasis on making meaning of the mathematics through whole-class discussion supported a communal approach to learning. The participation rubric emphasized the cooperative and collaborative elements of classroom learning, but by extending instruction to include practice in how to engage

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in classroom discourse, the lessons in *accountable talk* went beyond mere mention of developing community to supporting authentic communal learning.

Coding and analysis of data from observations of Ms. Brenner’s classes showed that her students participated in mathematical discussion by making and testing conjectures, questioning each other about processes they were using to solve problems, explaining processes they used, challenging each other to justify assertions, and engaging in argumentation. Ms. Brenner also regularly pressed for student explanations, reasoning, and justification. These teacher and student behaviors occurred relatively rarely in the other two cases. Furthermore, there was neither discussion of how to engage in classroom discourse nor evidence of having previously discussed how to engage in classroom discourse in Mrs. Blake’s and Mrs. Harmon’s classrooms.

Talking about talking about mathematics is an important part of scaffolding students’ skills for participating in classroom discourse. Students can be encouraged to engage in processes that position them as doers of mathematics when provided with scaffolding in the processes that mathematicians use. As students participate by explaining their thinking or reasoning, making conjectures, justifying their ideas, and arguing for opposing ideas, they are learning to participate in a specialized register of language at the same time as learning with and for understanding.

Thinking about Equity and Social Justice

Explicit reference to issues of equity and social justice did not occur during the course of the observed classroom lessons, although a limited number of teachers’ actions implied attention to equity, such as when Ms. Brenner and Mrs. Blake took opportunities to assign competence to particular students during classroom discourse. Because of my
interest in exploring issues of equity and social justice as related to teaching and learning mathematics, however, I included the topic during post-observation and end-of-year interviews with the teachers. An approach I had envisioned to exploring culturally relevant pedagogy, as suggested by Bakhtin (1984), Foucault (1976/1999; 1983), Ladson-Billings (1995; 1997), and Delgado (1990), involves taking a critical stance toward teaching and learning, deconstructing power relations, and opening up possibilities for reconstruction of self-theories. In reaction to my questions about thinking about issues of equity and social justice when planning for teaching and learning mathematics, both Ms. Brenner and Mrs. Blake indicated that they did not think about equity and social justice when planning content for their classes. Mrs. Harmon was hesitant to respond to an unfamiliar topic, and she asked me to define what I meant by equity and social justice. Her caution gave the impression of discomfort with the topic.

Ms. Brenner talked about equity and social justice as part of her philosophy of teaching, even though she hadn’t thought of addressing the subjects of equity and social students directly with her students or exploring the issues mathematically as part of instruction. Ms. Brenner did not have any White students in her classes. Most of the students in the school, 93%, qualified for free lunch. She knew that her students did not have financial resources that would allow them to buy the latest in available technologies, such as computers and calculators; and other financial and social restrictions placed them at a disadvantage, especially as compared to their counterparts in wealthier districts. Consequently, she considers her work to expose students to as much technology as possible to be part of her philosophy of social justice. She believes that it is her duty to train students and other teachers in her school to use the technology, because her students
have just as much right to succeed on exams as students who are economically advantaged. She believes in providing technological advantages to her students, who are students of color, many of whom live in poverty, as a way to “level the playing field” (B5-08-06-22phint).

Positioning refers to the ability to take different stances within discourses. People can be characterized by both continuous personal identity and by discontinuous personal diversity, and they are able to make choices, or variably position themselves, within actual conversations (Davies & Harré, 1990). Ms. Brenner’s engagement of students in accountable talk and her work to provide technological advantages for the purpose of “leveling the playing field,” might be seen as positioning of students, or providing opportunities for them to position themselves to investigate possible selves, within the discourse of schooling. Ms. Brenner did not deconstruct the dominant discourses of schooling through critical discourse with her students, but she implemented other strategies to support the development of agency for her predominantly Hispanic/Latino/a and African American students—members of cultures that have been historically marginalized and/or disenfranchised.

In a study of the relationship between self-beliefs and academic achievement, Lopez (1999) reported that minority students’ differences in achievement were mediated by individual differences in action-control beliefs. This finding relates to minority students’ belief in academic tests as measures of ability, which is referred to as a stereotype threat phenomenon. If a student perceives a test as a threat to agency self-beliefs, the stereotype threat takes over and results in lack of ability to perform (as a self-fulfilled prophecy).
In relation to a stereotype threat phenomenon, it is important to note that Ms. Brenner’s class was the only one with no White students—Asian (5%), African American (15%), Hispanic/Latino/a (60%), Hispanic/White/American Indian (10%) and Other (5%). Both Mrs. Blake’s and Mrs. Harmon’s classes had White students, 61% and 23% respectively. The stereotype threat phenomenon may extend beyond testing, as described by Lopez (1999). A situation requiring participation in unfamiliar registers within a classroom that appears to favor a particular register, with a White teacher and anywhere from 6 to 12 White students, might be read as presenting the same kind of threat, especially if scaffolding in the uses of language within the dominant discourse has not occurred. Non-participation or limited participation of African American students in classroom discourse, such as was seen in Mrs. Blake’s class, could indicate student activation of avoidance goals (Middleton & Midgley, 1997). Differences between the student populations, such as in Ms. Brenner’s class (predominantly Hispanic/Latino/a and African American students) and the other two classes (up to 61% White students) as related to student engagement in unfamiliar language registers, require further investigation.

Because they had not been participating according to a standard expectation, a few students in Mrs. Blake’s class were given specific jobs to do. They were given jobs related to using technology in order to help them become more productive members of the community. These students happened to be African American. I initially interpreted Mrs. Blake’s actions as taking proactive measures to include her African American students, who seemed to avoid asking and answering questions. Mrs. Blake did see her actions as assigning competence, but she did not see her actions as being related to race.
This becomes problematic because, while Mrs. Blake was assigning competence, she was not yet engaging her African American students in the roles as competent members of the community in terms of mathematics and discourse about mathematical thinking, although their jobs might be thought of as parallel or supportive roles. Lopez (1999) wrote about working to strengthen minority students’ agency for ability in terms of improving minority student achievement, not in terms of keeping unruly children in line. The roles assigned to these students may or may not position them with the goal of improving academic achievement, but this potential conflict in conceptualization of classroom competence also warrants further investigation.

While Mrs. Harmon did not talk about issues of equity and social justice as Ms. Brenner and Mrs. Blake had, she did mention the role of technology use in her classroom during the end-of-year interview. Mrs. Harmon said that the introduction of the TI-Navigator technology within her classes made students’ classroom interactions seem friendlier (H7-2008-06-16phint), an observation I had previously noted during data analysis. Mrs. Harmon explained that her students were having fun, and often didn’t realize how much they were learning until later. Additionally, students who were normally very quiet and hesitant to participate in class started to engage more with others in activities that incorporated the technology. So although Mrs. Harmon did not plan for use of technology to engage her students more equitably, she did notice more equitable participation and increased positive affect resulting from the use of technology.

I underestimated the importance of technology in the larger CCMS study as a factor related to cultural diversity and striving for equity and social justice in teaching and learning mathematics with understanding. In all three classes the role of technology
surfaced, particularly during interviews with teachers. The relationships between the results of using technology, which I would identify as pedagogical (as seen in Ms. Brenner’s class), instrumental (as seen in Mrs. Blake’s class), and affective (as seen in Mrs. Harmon’s class), and creating equitable learning experiences may warrant further investigation. Based on the three cases studied, the strategic pedagogical use of the technology to develop student’s skills in using mathematics and communicating their understanding, as was observed in Ms. Brenner’s class with predominantly Hispanic/Latino/a and African American students, was the most effective in addressing issues of equity. Ms. Brenner’s use of technology in the classroom resulted in positioning her students as active participants in the creation of knowledge.

An unexpected finding related to culture and equity arose from analysis of student-to-student discourse during small-group problem solving. Throughout most of the first lesson observation, Ms. Brenner’s students kept two levels of conversation going, one related to the assigned mathematics problem and the other related to their personal lives. Sometimes the levels of conversations alternated, and sometimes they were woven together. The students’ facilities for keeping two conversations going at the same time were remarkable.

When asked her thoughts about my observation, Ms. Brenner agreed that her students had engaged in multiple levels of discourse quite successfully. Reflecting on her earliest years of teaching, Ms. Brenner told me that when she had stifled the “off topic” discussion, the discussion about mathematics also stopped. She now refers to what might have been considered “off topic” discourse as the “lubricant that keeps the [mathematical] conversation going.” This has important implications for how engaged
learning is defined in classrooms, particularly for culturally diverse learners that may include ELLs. Even as these students had support for learning a new register for classroom discourse, the latest in technologies to support learning mathematics, and a teacher who used reform pedagogical practices, discourse within their personal/social/cultural register remained important.

Revisiting the Guiding Questions

1. What do teaching mathematics for understanding and fostering SRL look like in culturally diverse, algebra classrooms?

2. In what ways do students and teachers in these classrooms position themselves and each other through discourse?

I have not found definitive answers for what teaching mathematics with understanding and developing student SRL skills with culturally diverse learners look like, nor would I expect to even if I were to have witnessed exclusively exemplary practice. What I have learned about, however, are situated instances of classroom practice that provide stories worth pondering. These stories of White teachers and their students in algebra classes provide mere glimpses into what teaching mathematics for understanding and fostering SRL might look like in classes with students representing a variety of cultures. Nevertheless, they have implications for classroom practice and future studies of classroom practice.

My analysis of the data for the three cases showed that there was considerable variety in the ways that teachers attended to teaching mathematics and scaffolding SRL. At the same time, there were differences in the demographic make-up of the student groups in each case. In other words, the cultural diversity of learners in each classroom
was not similar to the cultural diversity of learners in the other classes. Ms. Brenner’s class did not have any White students, whereas a little more than half of the students in Mrs. Blake’s were White. Mrs. Harmon’s class had less unevenness in terms of numbers of students within racial and ethnic groups. Therefore, assumptions cannot be made about the effectiveness of practices that seemed to support learning mathematics with understanding and the development of SRL in one classroom in relation to another classroom, nor whether practices in one case that did not seem to support learning mathematics with understanding and the development of SRL would necessarily shut down possibilities in another situation.

Nevertheless, I observed practices that hold potential to support learning with understanding and the development of SRL, and practices that seemed to have discouraged it. The knowledge gained from the stories provided by the three cases is potentially useful for understanding the application of theory relating to learning school mathematics and developing SRL in a variety of settings—for thinking about ways I might go about learning more about teaching and learning with culturally diverse students.

While causal claims cannot be made from a descriptive study, descriptions of situated practice have potential to inform teachers and researchers in their respective practices. In cases where students demonstrate mathematical thinking and strategic learning, knowledge of practices that supported those student behaviors may inform subsequent actions taken toward increasing student agency. Ms. Brenner’s students communicated to solve problems, argued for and against specific solutions, explained their thinking processes, and justified solutions to problems. Their approaches to problem
solving reflected the processes that mathematicians use during problem solving, and can be likened to the processes used in SRL. Although there were no students in the class who self-identified as exclusively White, there was still cultural diversity, and some students identified themselves as White along with Hispanic/Latino/a and African American. Conditions specific to this case, particularly the special attention to developing students’ ability to communicate effectively through *accountable* talk, suggest that explicit instruction in academic discourse holds promise for supporting communication in content-specific registers of language and may increase the possibility of engagement in dialogic discourse for students of color.

Likewise, knowledge of specific situations in which classroom interactions seems to disallow the possibility of greater student agency in learning with understanding and the development of SRL may help teachers to avoid such interactions in their own teaching and learning situations. Although it is not true that all instances of using IRE patterns of discourse discourage student agency, Mrs. Harmon’s use of IRE patterns of discourse might have limited opportunities for the development of agency for learning. The kind of teacher control observed in this case, possibly indicating teacher beliefs about who owns the knowledge and therefore holds the power, did not lead to dialogic interaction. Mrs. Harmon was positioning her students as relatively passive receivers of her knowledge. Additionally, overuse of IRE patterns of discourse did not allow for more communal approaches to learning that may have been valued within the students’ Hispanic/Latino/a and African American home communities.

Mrs. Blake’s attempts to engage students in classroom discourse were relatively ineffective, as compared to Ms. Brenner. She wanted students to engage in
communication processes as suggested by standards for reformed mathematics practices, but she struggled with her perception of losing control (B7-08-04-24postobs). She had been comfortable in her implementation of one method of classroom interaction and was uncertain in her approach to developing new routines. She also essentially took a colorblind viewpoint on addressing differences in student behavior, attributing lack of productive engagement to bad or naughty behavior (B7-08-04-24postobs). This case may epitomize the good intentions of White teachers who choose not to acknowledge implications of race and culture in the classroom. An important concern, however, is whether or not those good intentions may in reality do harm. The consequences of assigning competence through involvement in nonmathematical activities are not clear, but may be extremely important, especially if such actions result in the cultivation of greater inequities for African American and other culturally diverse students.

The present study also led me to take another look at the use of classroom technology. The use of technology was not one of my original concerns. However, it did prove to have some degree of significance in all three cases. The incorporation of TI-Navigator technology into Mrs. Blake’s classroom resulted in different approaches to learning mathematics and different interactions with students. Mrs. Harmon characterized her students’ interactions as being friendlier following incorporation of the TI-Navigator technology. I found Ms. Brenner’s views on incorporating technology to be the most intriguing, however. Ms. Brenner held strong beliefs about providing instruction that incorporated technology in order to “level the playing field” for her economically disadvantaged students (B5-08-06-22phint). The use of technology to support learning, along with the intention of addressing inequities, might have played a role in supporting
the classroom interactions that led to dialogic discourse. Common rhetoric around “the technology gap” identifies access to technologies as an equity issue. Based on analysis of the three cases in the present study, however, a teacher’s philosophy and approach to teaching and learning may be vital in addressing issues of equity with incorporation of technology. The teacher’s stance on what it means to teach and learn appears to work in concert with incorporating technology to create more equitable learning environments for students of color.

The aspect of the analysis of the data in terms of the guiding questions that I found most surprising, and perhaps of particular consequence for White teachers and culturally diverse learners, is the absence of attention to variation in terms of color and race. Ms. Brenner talked about SES when she discussed use of technology for “leveling the playing field,” which implies working to make changes within a culture of poverty. She mentioned that a large percentage of her students lived in poverty, and that they deserved as good an education as students in more affluent areas and schools. She also mentioned ELLs as another descriptor for some of her students, but she did not discuss ELLs in terms of culture. ELLs are usually immigrants who have come to the United States from another country or children born to recent immigrants. They are students who have cultural backgrounds that are different from White teachers and mainstream United States. Attending to academic and content-specific language is important in addressing issues of poverty and cultural diversity. Talking about talking in order to engage students in classroom discourse, however, may be critical to working toward academic success for ELLs, and Ms. Brenner did provide that support for her students as related to school-specific language.

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The present study is exploratory in that it provides glimpses into actual teaching and learning practice, including the ways that students participate in various classroom contexts. The stories I recount suggest a need for additional research into the creation of contexts where culturally diverse learners are supported in doing mathematics as defined in the NCTM (2000) vision and developing skills that lead to SRL. I would like to continue to study patterns of classroom discourse in relation to cultural diversity and teaching for understanding to inform my own practice teaching mathematics methods for both undergraduate and graduate students and to contribute more to research literature on classroom discourse and cultural diversity in mathematics classrooms. In the following two sections I present implications for theory and practice and recommendations for future research.

Implications for Theory and Practice

The descriptive findings of this study have important implications for both theory and practice. In relation to theoretical perspectives, descriptions of classroom practice that provided scaffolding for engagement in mathematical activities support the notion that reform in teacher practice is needed to achieve the vision of mathematics education as described by NCTM (2000). Teachers may need to shift the way they think about what it means to teach and learn. Although I cannot claim a causal link, students demonstrated more engagement in activities and reflective discourse when teacher practice included attention to the goals for mathematics education as outlined in the NCTM process standards and related research. Additionally, observations of scaffolding discourse toward dialogic interaction support conceptualizing classroom interactions as opportunities for experimentation with a variety of roles within the discourse. Scaffolding
of dialogic discourse has potential for engaging students in mathematical thinking with the goal of strengthening their identities as doers of mathematics.

Descriptions of actual discourse in mathematics classes illustrate ways in which teachers employ their understandings of reform mathematics practices and the ways in which they support the development of SRL. Descriptions of classroom practices in the three cases may guide teachers to understand the ways in which discourse can serve to position students within a vast range of possibilities as learners, a topic which inherently encompasses issues of equity and power relations. Analysis revealed variety in classroom practices among the three cases that provided different types and levels of scaffolding for students. The practices observed in Ms. Brenner classes appeared to provide more scaffolding for learning mathematics with understanding, for developing skills related to SRL, and for engaging in dialogic discourse. Based on analysis of data from Mrs. Brenner’s room I identify four areas of classroom activity that have practical implications for classroom teachers: 1) scaffolding of classroom discourse, 2) emotional scaffolding, 3) making student thinking public, and 4) engaging in dialogic discourse.

First, explicit scaffolding of classroom discourse may support student experimentation with unfamiliar language registers. Ms. Brenner provided explicit scaffolding of discourse. Although not observed during the two videotaped lessons, Ms. Brenner recounted the story of how she led students in negotiation of what it means to participate in class through accountable talk, and used accountable talk techniques to scaffold the process. Students’ posters documented their understandings and remained on the walls throughout the school year. Negotiation of understanding as part of scaffolding
an unfamiliar language register is also demonstration of a communal approach to learning, a possibly important way of addressing differences in learning styles for students of color and other non-mainstream learners.

Second, students may participate more effectively in mathematical discourse when words of encouragement include talking about difficulty in problem solving and errors in mathematical thinking as opportunities to learn with understanding. Ms. Brenner’s students were more willing than students in the other classes to take risks as they made their thinking available for public examination, which may have been related to the emotional scaffolding she provided through explicit discussion of coping and learning strategies.

Third, problem solving supported with intermittent teacher-led discourse for the purpose of highlighting strategic actions exposes students to a range of experiences and more examples of possibilities for strategic thinking and actions. Ms. Brenner provided scaffolding for student work during problem solving that included intermittent whole-class discussion. Requests for further explanations during whole-class discussion make explicit what may have been implicit in students’ explanations of mathematics content (Forman & Larreamendy-Joerns, 1998), but these requests potentially support learning about strategies used, as well. Whole-class discussion that makes student thinking public has the potential to support the learning of strategies that are important in problem solving and in the development of SRL. Because of the alternation of small-group work and whole-class discussion, Ms. Brenner’s press for explanations, reasoning, and justification had the potential to benefit individual students as well as the community as a whole.
And finally, when students are involved in dialogic discourse while discussing mathematics, they engage in the creation of knowledge, essentially taking agentive roles in the learning process. Dialogic episodes occurred when Ms. Brenner asked probing questions, her students made comments based on mathematical observations that were taken up for discussion, or students asked questions that were taken up for discussion. Ms. Brenner asked questions and made observations that functioned to position the student comments, conjectures, and argument at the forefront for the purpose of students’ construction of knowledge. This suggests that White teachers working with students of color might strategically support student agency and the development of academic and/or content-specific language registers by highlighting students’ contributions to classroom discourse for the purpose of stimulating dialogic discourse.

Recommendation for Future Research

Questions emerged during the present study in four areas: 1) the variety in the demographic makeup among the cases, 2) the boundaries used to differentiate between acceptable speech registers, 3) differential practice in low-track classes, and 4) the role of technology in working toward equitable learning experiences. These areas of concern suggest the need for further research.

First, the differences between cases in relation to student engagement in unfamiliar language registers warrant further investigation. Ms. Brenner appears to have provided effective scaffolding in negotiating language registers with her students, however Ms. Brenner did not have to negotiate the tensions that may exist between meeting the needs of White students at the same time as students of color, nor was there evidence of having to negotiate the tensions that may exist between cultural groups due to
racism and intolerance. Additionally, differences associated with SES complicate comparisons between cases. Variety of student demographics bring up questions about scaffolding practices in classrooms with wide discrepancies in SES. Such questions could be formulated to address how implementation of practices such as those in Ms. Brenner’s class and as described by Gutstein (2007) might look if those classes had included White students and/or wider variety in SES, and how greater polarity of demographics affects efforts to engage students and create “buy-in.”

Second, and closely related, are questions regarding use of a variety of registers while learning mainstream school registers. Even as Ms. Brenner’s students had support for learning a new register for classroom discourse, the latest in technologies to support learning mathematics, and were exposed to reform practices, discourse within the students’ personal/social/cultural register (off mathematics topic) remained important. There was less evidence of off-mathematics-topic talk, however, during the second observation, near the end of the school year. This leads to questions about the nature of personal/social/cultural talk that occurs in mainstream classrooms and how it compares to non-mainstream classrooms, as well as the role of social/cultural registers (off-mathematics-topic) in the development of fluency in academic registers.

Third, questions surfaced in relation to scaffolding dialogic discourse in low-track classes. Students in Mrs. Harmon’s class were at a relatively low level within the tracking system for that school district. Higher-tracked students had already taken algebra in seventh or eighth grade. Mrs. Harmon did not display the kinds of “moves” that have been found to initiate dialogic discourse. Nevertheless, in relation to findings from research, this case causes me to wonder about teachers who might attempt to scaffold
dialogic interaction in low-tracked classes. Nystrand et al. (2003) found that authentic teacher questions and uptake of student comments for discussion occurred with similar frequency in both high- and low-tracked social studies and language classes, but the pattern and clustering of authentic questions and uptake were less likely to occur as a sequence in low-track classrooms. Additionally, students in low-track classrooms did not ask questions that initiated dialogic episodes. The question of what it means for teachers and students to negotiate participation in low-track classrooms suggests a need to examine patterns of instruction and student self-images that are associated with reduced occurrence of effective dialogic bids, as well as the kinds of teacher scaffolding that might encourage dialogic interaction in low-track classrooms.

Finally, questions arose about the use of technology to address equity in mathematics class. Mrs. Blake was assigning competence to students by positioning them as experts in technology, yet she was not engaging them in roles as mathematical thinkers. The roles may have been considered relatively productive within the classroom community, and they may or may not lead to improved achievement, but assigning these parallel roles creates potential conflicts in conceptualizations of providing scaffolding for mathematical competence. Research might be designed to explore assigned roles and their relationship to acceptance of other roles as learners, specifically roles of academic competence, in the classroom.

Limitations

In an attempt to explore the development of SRL in mathematics classrooms for culturally diverse learners (classroom that include non-White, low SES students), I examined classroom practices, focusing on the use of discourse, of three algebra teachers
and their students. I provided descriptions of each case, comprised of two classroom sessions, and compared across cases to find patterns of practice that might help in understanding what it means to scaffold learning with understanding and the development of SRL with culturally diverse students. Because of the nature of case studies, understandings gained from these observations are limited in scope.

The three cases were part of a larger research study. Consequently, I did not have as much flexibility as would be ideal in determining participants for the study. I was limited to the teachers and students who were chosen for observation during the first year of the CCMS study. Additionally, a typical observation included videotaping two days of lessons and conducting a post observation interview. The prevailing standard of two-day observations limited the focus of the present study to relatively narrow descriptions of situated practice. I was not able to look for or describe the development of classroom contexts and conceptual knowledge over time.

In an argument for conceptualizing research on equity in mathematics differently, Gutiérrez (2002) suggests that proponents of reform directed toward issues of equity need to reframe their arguments.

Yet, even proponents of equity issues tend to frame their arguments in ways that suggest that benefits move from mathematics to persons and not the other way around. The assumption is that certain people will gain from having mathematics in their lives, as opposed to the field of mathematics will gain from having these people in their field. In other words, most
equity research currently assumes the deficit lies within the students who need mathematics as opposed to, or in addition to, lying in mathematics, which needs different people. (pg. 147)

I did touch on Gutiérrez’s concern in framing my own arguments for the present study with discussion of political disadvantages, which are systemic in both society and our schools, putting certain cultural groups in positions of relative powerlessness. I agree, however, that Gutiérrez’s observation applies to my own research in that I did not address what students of color might contribute to the field of mathematics. My concern was with the classroom context rather than the broader field of mathematics or mathematics education. Nevertheless, the broader picture does have implications for conceptions of what it means to engage in mathematical activities in classrooms.

More research about meeting the needs of culturally diverse students is necessary, particularly in relation to scaffolding the development of SRL and teaching mathematics for understanding. Future research might also address how variety in cultural perspectives might benefit school mathematics, as well as the field of mathematics as suggest by Gutiérrez (2002). Longitudinal studies, teaching experiments, descriptive studies that encompass a greater number of cases, and comparative studies could contribute additional perspectives on teaching and learning with culturally diverse students.
REFERENCES


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