ESSAYS ON STRATEGIC COMPETITION IN E-COMMERCE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

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2008

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ABSTRACT

This dissertation consists of two essays on retail market change in the context of e-Commerce and one essay on keyword auction at the search engines.

In the first essay, we extend Balasubramanian’s analytical model (1998) on competition between the traditional retailer and the direct marketer. We examine how possible entry to the online channel by a conventional brick-mortar retailer changes the result of Balasubramanian (1998). We characterize the equilibriums that arise in that context. Interestingly, the addition of online store to brick-mortar store does not always increase profits. The analysis suggests that successfully entering the e-commerce market can be determined, in part, by considering pros and cons of the new competition dual-channel retailers will face. Internet market sequential entry is examined next. Interestingly, web-only retailer always enters the market, no matter their level of service disadvantage.

In the second essay, we expand the notion of consumers to revisit the price competition in e-commerce. Previously, retail consumers are only geographically differentiated. A new type of consumer - virtual consumers emerge in the environment of e-commerce. The proportion of this virtual consumer turns out to have a direct effect on the characteristics of retail pricing equilibrium.
In the third essay, we look at the bidding practice of selling keyword at the search engines. In complete information setting where the assumption of bidder heterogeneity does not hold, we show that the pay-for-placement approach yields suboptimal performance for media owner. The result sheds some light on the management of pay per click.
ACKNOWLEDGEMENTS

I am grateful to my advisor Professor Waleed Muhanna for his guidance and encouragement on this research. I am also grateful to my committee member, Professors Richard Young and John Fellingham, for their time spent conversing about this work. I benefit from Eric Spires for his valuable suggestion. I want to express my appreciation to Lixin Ye for his feedback on this work. Also I am indebted to conference participants at INFORMs Marketing Science Conference in Pittsburgh and Atlanta. Thanks to the Fisher College of Business and the Department of Accounting & MIS for financial support. My greatest thanks go to my family for their support.
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A STRATEGIC ANALYSIS OF ONLINE CHANNEL ENTRY OF
CONVENTIONAL BRICK-MORTAR RETAILERS

ABSTRACT

Internet technology has changed many aspects of the modern retail industry. One specific change is the birth of web-only retailer. From the start of e-commerce, the web-only retailer intended to compete with conventional brick-mortar retailers. Conventional brick-mortar retailers’ response to web-only retailer has significant implications for market-place performance. The business literature, however, has been dominated by a focus on web-only retailer versus conventional brick-mortar retailers. This chapter strategically models conventional brick-mortar retailers’ adoption of e-commerce, for the purposes of this chapter called dual-channel retailers, as a way to challenge web-only retailer.

Balasubramanian (1998), in particular, analyzes the competition found by conventional brick-mortar retailers due to the emergence of web-only retailers. In his setting, conventional brick-mortar stores lose remote consumers to a web-only store because remote consumers find online shopping more convenient than traveling to
physical store. That study did not consider the strategic adoption of e-commerce by conventional brick-mortar retailers, however. This study examines the response of conventional brick-mortar retailers to web-only retailer through launching their own online stores.

This study begins by modifying Balasubramanian’s basic model (1998) and the new model accommodates the following retailer characteristics for dual-channel retailers: the existing physical store supplements the online store; the online store leverages the retail service advantages of the physical store to differentiate from web-only store. These characteristics underpin the analysis of competition in the retail market.

First, the model analyzes a setting where the retail service advantage allows for the co-existence of web-only retailer and dual-channel retailer. In the resulting equilibrium, dual-channel retailers squeeze the market share and profit of web-only retailer. Interestingly, the addition of online store to brick-mortar store does not always increase profits. The analysis suggests that successfully entering the e-commerce market can be determined, in part, by considering pros and cons of the new competition dual-channel retailers will face.

Next, a setting is introduced where the retail service advantages drive web-only retailer out of business. The value of having a physical store for dual-channel retailer is so strong that it eliminates the service benefits of web-only retailer. But this domination comes at a considerable cost, from the conventional brick-mortar retailers’ perspective, because the exit of web-only retailer from the market results more intense competition
between dual-channel retailers. This outcome has implications for whether or not conventional brick-mortar retailers enter the Internet market.

Equipped with results from the previous scenarios, Internet market sequential entry is examined next. The analysis draws on business anecdotes such as web-only retailer leveraging the web first, followed by conventional brick-mortar retailers. Multiple entry equilibriums are derived and justified with associated retailer characteristics. Interestingly, web-only retailer always enters the market, no matter their level of service disadvantage. This outcome has implications for the perseverance of web-only retailers, as the economic viability of these virtual stores has been questioned since their inception.

When considering online market competition, the dual-channel retailer’s ability to compete with web-only retailer is not affected by the prices charged at physical store. The value of differential pricing across two channels is analyzed. Although separate pricing moderates internal consumer choice, it does not affect marginal consumers at equilibrium. Therefore, the resulting equilibrium will be the same as uniform pricing. Spinning off the online operation as a separate entity for a dual-channel retailer provides another strategic variable to compete with web-only retailer. In the situation of spin-off, more severe competitions occur and profits drop across the board. This finding is consistent with Barnes and Noble’s infamous decision to reverse previous move on separating the physical and online store into two distinct businesses.

The chapter concludes with a discussion of issues for future research in the context of the modern retail market.
1. Introduction

The use of Internet as a channel for commerce has catalyzed interest into how the new channel impacts retail competition. E-commerce is a fast growing segment of the 21st century retailing industry. According to a report by the U.S. Census Bureau, projected online retail sales will reach $184 billion in 2008 (U.S. Census, 2007). Because of its low cost and powerful communication capacity, online has bypassed previous sales channels such as catalogs and television, becoming the retail industry’s most popular tool to reach mass consumers. This trend presents a threat to conventional brick-mortar retailers. It fuels tremendous growth of web-only retailers, who operate virtual stores and sell directly to mass markets. This chapter is an effort to analyze such market changes from a strategic viewpoint and explore retail market issues of managerial interest in the Internet era.

A glimpse of the retail market today reveals that dual-channel retailers and web-only retailer increasingly compete for online sales (Bughin & Zeisser, 2001). For example, consumers can order a computer online either from web-only retailer, such as Amazon.com, or from online store of dual-channel retailer, such as BestBuy.com. The Marketing literature (Alba et. al., 1997; Balasubramanian, 1998) blazed the research trail on the rising roles of web-only retailer in the world of conventional brick-mortar retailers. However, competitive interface of web-only retailer versus dual-channel retailer has received limited attention in the existing literature. To address this gap, this study extends Balasubramanian’s (1998) framework to conceptualize competition between web-only and dual-channel retailers, and analyzes issues of research and managerial interest.
Web-only retailers are emerging as a powerful distribution force. Meanwhile, it is important to recognize that the same technology is within the reach of conventional brick-mortar retailers. Many traditional retailers now have expanded to operate across two channels. How can the competition between web-only retailer and dual-channel retailer be conceptualized? What are the variables of interest and how they can be accommodated in the model? What are the implications to Internet market entry? This chapter addresses these issues.

The popular business literature often discusses the importance of direct channel marketing, mainly in terms of web-only retailers. Among many business analysts, Bughin (2001) predicts that web-only retailers will have a lower average cost due to their virtual existence. Rent, utilities and staffing costs will disappear. Other analysts such as Kroll (2000) recognize the high fixed costs of developing state-of-the-art order-fulfillment and logistics systems. These cost factors certainly contribute to the discussion on web-only versus conventional brick-mortar stores. However, when the point of analysis is dual-channel retailers rather than conventional brick-mortar retailers, the cost difference vanishes because the same technology is used. Consumer preferences, rather than differences in retailers’ cost structure, start to play a stronger role in determining e-commerce competition.

Among online shoppers, consumers living close to physical store favor shopping at online store of dual-channel retailers because they can access traditional retail service from physical stores. However, consumers who live far away from physical stores find web-only retailer more attractive. Therefore, dual-channel retailers chip away some of the
online market share described in Balasubramanian (1998). This paper investigates how an expansion into an Internet channel changes industry competition and results in profit shifts for participating retailers.

The paper is organized as follows: section 2 reviews related literature, while the core of the model is introduced in section 3. The post-entry Bertrand-Nash equilibrium in prices is analyzed in section 4, and section 5 explores pre-entry incentives and strategic issues related to post-entry competition. Section 6 concludes with the discussion.

2. Related Literature

Information System literature in digital marketing has been dominated by a focus on the e-retailer sector (e.g., Brynjolfsson & Smith, 2000; Clemons, Hann, & Hitt 2002; Chen & Hitt, 2000). Among many important issues, the competitiveness of digital markets and consumer online switch costs received early attention. In music, movie, and book markets, the price dispersion is not lower in e-commerce markets, as compared to conventional markets (Bailey 1998a, 1998b; Brynjolfsson & Smith, 2000). In the airline industry, the prices for airline tickets can differ by as much as 20% across online travel agents, even after controlling for observable product heterogeneity (Clemons, Hann, & Hitt, 2002). Researchers have collected substantial empirical data that show the imperfect competitiveness of digital market. The self-enforcing collusion by firms on the price of commodity products is studied by Campbell, Ray, and Muhanna (2005). Balasubramanian (1998) examines how direct marketer establishes herself as a serious
competitor in the traditional retailing market. Cheng & Nault (2007) analyzes how retail market coverage and entry cost affect the Internet channel entry game. They assume that retail market is not fully served and retailers’ Internet channel entry costs are not the same. We consider a fully served retail market and the cost of entry into Internet channel is the same across the board. Although factors such as market immaturity, retailer segmentation strategies, price discrimination and collusion, and heterogeneity in retailer attributes can all contribute to the development of digital market, the literature lacks a consideration of dual-channel retailing. This analysis addresses that gap by highlighting two different kinds of retailing forces in the digital market: web-only store and online store of dual-channel retailer. The recognition of two forces in the digital market establishes a context in which the performance of the market is analyzed on a broader base of participants.

Economic literature in multimarket operation has focused on cross-market competition and cross-market collusion (Bulow, Geanakoplos, & Klemperer, 1985; Berheim & Whinston, 1990). While these models offer tools to understand multimarket performance, they do not consider the transition from a one-market operation to multimarket operations. This paper examines the issue of market transitions, from a conventional brick-mortar retailer’s perspective. Our approach closely parallels the situation that traditional retailers face in the Internet era.

3. Basic Model

Following Salop (1979), we consider a circular spatial model in which consumers are located evenly on a circle of unit circumference (Figure 1.1). Each consumer
purchases a single unit of a product in each period. Consumer demand is assumed to be inelastic. This assumption is equivalent to postulating a high reservation price for each consumer, compared to their shopping cost. Consumer’s utility function is to maximize the gain minus cost of shopping. Sellers procure the product at a marginal cost $c$ which is normalized to zero for simplicity.

![Circular spatial model](image)

Figure 1.1 Circular spatial model

$N$ dual-channel retailers (R in figure 1) are equally distanced from each other on the circumference. The consumers incur traveling costs at a linear rate of $t$ per unit distance when visiting a brick-mortar store. Traveling costs can include the actual cost of travel, an opportunity cost of shopping time, or inconvenience associated with travel. It serves to differentiate brick-mortar retailers, with respect to ease of shopping. As $t$ increases, the market power of brick-mortar retailers over their proximate consumers improves.
Following Balasubramanian (1998), a web-only retailer also serves the market, with no physical location on the circumference; it is useful to locate the web-based retailer at the center of circle (D in figure 1). The web-only store run virtual trading floor. Consumers desiring to purchase items via the Internet contact the web-only store, which sell the items at a quoted price. A third-party carrier delivers the products to consumers, charging a fixed freight per unit regardless of each consumer’s location. As pointed out in Balasubramanian (1998), convenience is a major reason consumers buy online. When buying online, consumers give up benefits from purchasing the good at a brick-mortar retailers in exchange of convenience. The brick-mortar retailers offer instant order fulfillment for physical goods, product inspection which avoids problems such as adverse selection found on online trading, and easy return and exchange services. These benefits can mean a little for the trade of branded electronics or a lot for the trade of jewelry. Product parameter $\mu$ captures disutility of shopping online. Customers have complete information on product availability and price.

Like the direct marketer in Balasubramanian (1998), online store of dual-channel retailer does not provide instant order fulfillment and product inspection. However, the physical store can provide exchange and return service if online purchase is not satisfactory. Physical stores also can participate in online order fulfillment enabling consumers to buy online and pick up at retail store. Therefore, online store of dual-channel retailer has advantage over web-only store. Since these services are provided by existing physical store, it is reasonable to assume that the advantage over web-only store diminishes as the consumer’s distance to the physical store increases. The advantage of
purchasing from online store of dual-channel retailer over web-only store is assumed to start from utility $S$ for consumer collocating with brick-mortar store. The combination of $S$ and $k$ captures the service advantage the online store of dual-channel retailer has over the online store of a web-only retailer. If product sold at online store is expected to incur after-sale service like exchange or return with high possibility, for example personal clothing, then the advantage of dual-channel retailer over web-only retailer is large. In this case, $k$ will be low and $S$ is high. Low $k$ means the advantage of buying from online store of dual-channel retailer over web-only store diminishes slowly.

The prospect of e-commerce expansion by conventional brick-mortar retailers lies critically on the value of $k$ and $S$, which capture the relative advantage of buying from online store of dual-channel retailer instead of buying from web-only store. $k$ and $S$ vary with the product or purchase context. For example, when the need for product inspection is low and quality of product is known constant, the product is well suited for e-commerce expansion and $k$ would be high and $S$ would be low (e.g., branded music CDs). $\mu$ is also low in this context. When quality is uncertain and exchange or return is likely, $S$ would be high and $k$ would be low. An example is clothing market. In Balasubramanian (1998), $\mu$ captures the relative advantage of buying offline. Similarly, $k$ and $S$ in our analysis capture the relative advantage of buying from online store of dual-channel retailer. A clothing retailer providing in-store exchange or return offers a higher service value compared to web-only clothing retailer, as modeled in our analysis.

A high $k$ implies a fast dissipation of service benefit of dual-channel retailer. If that benefit decreases faster than the transportation cost (i.e. $k > t$), visiting a brick-mortar
store would cost less than shopping at online store. Therefore, online expansion adds no value to conventional retailer when \( k > t \). On the other hand, when online consumers order from online store of dual-channel retailer, they need to travel to the physical store for exchange or return service only if the delivered product is not satisfactory. In this way, consumers incur travel cost with less than probability 1, compared to shopping from a brick-mortar store. Hence \( k \) is less than \( t \). It is assumed that \( k < t \) in the following retailer competition analysis because of the reasons offered above.

For customers who locate far away from physical stores, there is evidence supporting that shopping at an online store of dual-channel retailer could cost more in shipping than shopping at a web-only store. For instance, Target.com imposes a special shipping fee depending on the zip code destination; this is in addition to the standard shipping charge. Wal-Mart.com calculates shipping costs based on an item’s characteristics, shipping method, and delivery address. In both cases, the shipping charges increase as distance from the physical store increases. On the contrary, web-only store Amazon.com’s shipping fee is flat across the country. In the model, this is captured by the continuity of \( k \) reflecting the steady dual-channel retailer’s advantage loss over web-only retailer by distance. The assumption of discontinuity of \( k \) does not change equilibrium result. See discussion in appendix A.
4. Post-entry equilibrium in the price setting game

The equilibrium is studied for a simple situation, where \( n \) conventional brick-mortar retailers turned dual-channel retailers compete with web-only retailer. The results assume that both types of retailers make positive profits in equilibrium. The case where the web-only retailer is driven out of business, and the competition among dual-channel retailers, is considered later.

Customers are modeled as living on a circular city (perimeter is scaled to unit length). There are \( n \) conventional brick-mortar retailers, evenly located on the circle. A web-only retailer locates at the center of circle implying “equal distance” to all customers. All shopping options are depicted in the Figure 1.2. Consumers can shop from three sources: physical store of dual-channel retailer (B henceforth), web-only store (D henceforth) or the online store of dual-channel retailer (C henceforth). Assume a uniform price across B and C. If consumers shop at B, total costs include store charge \( P_{bc} \) and transportation cost \( t^*x \). If they shop at D, expenditures include price \( P_d \) and online shopping disadvantage cost \( \mu \). If they go to C, they pay the price \( P_{bc} \), incur online disutility of buying through the online channel \( \mu \) and benefit from additional utility of buying from the online store of dual-channel retailer \( S - k^*x \).
The line (I) represents shopping cost at C.

The line (II) represents shopping cost at B.

The line (III) represents shopping cost at D.

Line (I): \( P_{bc} + \mu + k \cdot x - S \).  
\( S \): maximum relative advantage of buying from online store of dual-channel retailer rather than web-only store (consumers collocating with physical store). \( S - k \cdot x \): advantage for consumers with distance \( x \) from physical store.

\( k \): service premium decreases at rate \( k \) per unit distance.

\( \mu \): online shopping disutility.
Line (II): $P_{bc} + t*x$. $t*x$ is transportation cost with unit travel costing $t$.

Line (III): $P_d + \mu$.

D and B-C (dual-channel retailer) set up prices simultaneously to optimize profits. The detail of formulation is as follows: There are two key locations $x_1$ and $x_2$ segmenting customers into three groups. Customers at location $x_1$ are indifferent between shopping at store B and C; i.e.,

$$P_{bc} + t*x_1 = P_{bc} + \mu + k*x_1 - S,$$

$$x_1 = (\mu - S) / (t - k).$$

Customers at location $x_2$ are indifferent between shopping at C or D; i.e.,

$$P_{bc} + \mu + k*x_2 - S = P_d + \mu,$$

$$x_2 = (P_d - P_{bc} + S) / k.$$

In the competition between dual-channel retailer and web-only retailer, the outcome hinges on the critical parameter $S$ and $k$. In order for dual-channel retailer to gain new customers, $x_2$ must be greater than $x'$ which is the location point for customers who are indifferent between shopping at B or D. If this does not hold, opening an online store does not generate extra customers.

We consider the case which opening store C helps dual-channel retailer gain online customers. Then customers can be segmented into three groups based on their geographical location: $0 < x < x_1$, $x_1 < x < x_2$ and $x_2 < x < 1/2n$. Group 1 shop at B. Group 2 shops at C. Group 3 shops at D.
Dual-channel and web-only retailer’s profit maximization functions are specified as:

\[
\text{Max } \pi_{bc}(P_{bc}, P_d) = 2P_{bc} \times x_2, \\
\text{Max } \pi_d(P_{bc}, P_d) = 2n \times P_d \times (1/2n - x_2).
\]

**Proposition 1.1**

Consider a circular market with a web-only retailer \( D \) and \( n \) evenly spaced dual-channel retailers, each having an online outlet. If a unique pure strategy Nash equilibrium in prices exists where the web-only retailer and dual-channel retailers make positive profits, the following conditions must be met:

1. \( x_2^* > x \) \[ i.e. (k/2n + S)/k > ((k/2n – 2S) + 3 \mu)/t \]
2. \( x_2^* < 1/2n \) \[ i.e. S/k < 1/n \]

Equilibrium prices are given by:

\( P_{bc}^* = (k/2n + S)/3 \) and \( P_d^* = (k/n - S)/3. \)

When condition 1 does not hold, competition degenerates to Balasubramanian (1998).

Proof: See Appendix A.

**Corollary 1.1**

Compared with Balasubramanian (1998) model, the dual-channel retailers’ market share increases and the web-only retailer’s market share decreases. The expansion to online market increases conventional brick-mortar retailers’ profit if condition \( A \) holds:

\[
4 \times n^2 \times (S^2/k - \mu^2/t) + 4 \times n \times (S - \mu) + k - t > 0. \tag{A}
\]

The results contrasting Balasubramanian’s model (1998) are displayed in Table 1.1. If \( (k/2n + S)/k > ((k/2n – 2S) + 3 \mu)/t \) and \( S/k < 1/n \), internet expansion increases the market share of brick-mortar retailers while leaving room for web-only retailer.
### Table 1.1 Comparison Summary

<table>
<thead>
<tr>
<th></th>
<th>Total industry profit</th>
<th>Traditional store profit</th>
<th>Direct marketer profit</th>
<th>Sales online/offline</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Decrease</td>
<td>Depends on condition A; increase if condition A holds</td>
<td>Decrease</td>
<td>Increase/decrease</td>
</tr>
</tbody>
</table>

### Table 1.2 Comparison Detail

<table>
<thead>
<tr>
<th>Balasubramanian’s setting vs. the setting in Proposition 1.1</th>
<th>Price</th>
<th>Market share</th>
<th>Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web-only retailer</td>
<td>$P_d^* = c - \mu/3 + t/3n$</td>
<td>$S_d^* = 2/3 - 2\mu n/3t$</td>
<td>$\pi_d^* = 2(t-\mu n)^2/9nt$</td>
</tr>
<tr>
<td></td>
<td>$P_d^* = (k/n - S)/3$</td>
<td>$S_d^* = 2(1-Sn/k)/3$</td>
<td>$\pi_d^* = 2n(1/n-S/k)^2/9k$</td>
</tr>
<tr>
<td>Retailer with physical store</td>
<td>$P_r^* = c + \mu/3 + t/6n$</td>
<td>$S_r^* = 1/3n + 2\mu/3t$</td>
<td>$\pi_r^* = (t+2\mu n)^2/18n^2t$</td>
</tr>
<tr>
<td></td>
<td>$P_{bc}^* = (k/2n+S)/3$</td>
<td>$S_{bc}^* = (1/n+2S/k)/3$</td>
<td>$\pi_{bc}^* = 2(k/2n+S)^2/9k$</td>
</tr>
</tbody>
</table>

Both $P_{bc}^*$ and $P_d^*$ are increasing in $k$. When the relative advantage of buying from online store of dual-channel retailer is small, competition between web-only store and online store becomes less severe. This benefits both web-only retailer and dual-channel
retailer by allowing high prices in equilibrium. However, a higher $k$ adds to greater utility loss for consumers buying from online store of dual-channel retailer and reduces retailer market share. The comparative statics of $S$ in equilibrium are different. An increase in $S$ leads to an increase of market share for dual-channel retailer and equilibrium price, while depressing price and market share of the web-only retailer. Finally, large $n$ lowers equilibrium prices and market shares across the board, leading to a lower profitability for all retailers. The model shows if $S/k > 1/n$; web-only retailer cannot co-exist in the market. $S/k$ is dual-channel retailer’s competitive advantage index.

Note that in the resulting equilibrium, dual-channel retailers do not necessarily receive higher profits than brick-mortar retailer in Balasubramanian (1998). The expansion to the Internet has two effects on the bottom line. It captures some consumers who would originally buy from a web-only retailer, and it depresses equilibrium prices through online competition. The ultimate impact of expansion into e-commerce is determined by these two factors. The condition A is the direct layout of the profit difference between conventional brick-mortar retailer in Balasubramanian’s model and dual-channel retailer in this model. It is expressed as follows.

$$4 * n^2 *(S^2/k - \mu^2/t) + 4 * n *(S - \mu) + k - t > 0$$

(A)

**Summary**

Balasubramanian (1998) maintains a clear separation of direct marketer and traditional retailer throughout the analysis. Results presented above reveal the strategic
effects of Internet entry by traditional retailers. Under certain conditions the entry puts pressure on web-only retailer but web-only retailer still makes positive profit. Here, the advantage for consumers to buy from online store of dual-channel retailer is moderate, allowing the co-existence of dual-channel retailer and web-only retailer. While the analysis captures key competitive aspects of retail market, other conditions of $S$ and $k$ are also worth examination. The assumption is relaxed in the next section. It is clear that the possibility of expanding into e-commerce by conventional brick-mortar retailers can alter the strategic valuation of market entry by direct marketer in Balasubramanian (1998). It is important to analyze market entry in a sequential setting, where the direct marketer (web-only retailer) makes strategic entry decision anticipating the conventional brick-mortar retailer’s following move. The next section also analyzes separate pricing between physical store and online store of dual-channel retailer and scrutinizes the impact of splitting dual-channel retailer into two separate, independent retail operations.

5. Entry decision and post-entry competitive strategies

Extension 1: Relaxation of condition 2 in proposition 1.1

The benefit of buying from online store of dual-channel retailer indicated by $k$ translates into the critical leverage of online competition for dual-channel retailers. If $S$ is relatively high and $k$ relatively low, dual-channel retailer can serve all consumers with a lower cost than web-only retailer, so web-only store is eliminated (Figure 1.3). In this
case, neighboring dual-channel retailers start to compete with each other. This scenario features more severe competition and results in lower profits.

The same basic circular spatial model structure is used, with \( n \) dual-channel players, placed evenly on the circumference and a web-only retailer in the center. The competitiveness of the online store of dual-channel retailer is described by shopping cost (line I). If \( S \) and \( k \) are such that line I undercuts line III (shopping cost at the web-only store), the web-only store is driven out of competition. (Figure 1.3) The resulting competition is similar to Salop (1979) with parameter modification.

**Proposition 1.2**

*In a circular model with a web-only retailer and \( n \) dual-channel retailers evenly spaced, the web-only retailer is driven out of the market if online outlet service premium \( S \) is large and the rate of loss \( k \) is small (\( S > k/n \)). A symmetric pure strategy Nash equilibrium in prices exists. The equilibrium price is given by: \( P^* = k/n \).*

Proof: See appendix A.
Lower equilibrium prices are observed compared to the Balasubramanian’s model (1998). This is not surprising. If the product provided is well suited to dual-channel retailing in terms of high service value $S$ and low rate of service loss $k$, dual-channel retailer provides a better option for consumers than web-only retailer. With the online channel lowering the cost of serving consumers, competition becomes more intense for retailers.

**Extension 2: Entry decisions of web-only entrepreneur and conventional brick-mortar retailers**

The online retail channel started to emerge in the mid-1990s, following the technology breakthrough of the Internet. The idea of selling goods via web interface and bypassing conventional retailers caught on very early. Many entrepreneurs in the business-to-consumer world followed this strategy. Examples include Amazon.com, Buy.com and Crutchfield.com, among many others. The conventional brick-mortar retailer did not prepare for Internet commerce coming at first. Entrepreneurs built online business ahead of conventional retailers. A sequential model of Internet market entry captures the market opportunity presented to each type of player. The analysis contributes to the understanding of the formation of a new market. Earlier results on competition from proposition 1.1 and 1.2 build the foundation to analyze the sequential market entry model shown in Figure 1.4.
A game tree is constructed in Figure 1.5, where the first mover entrepreneur (E) decides whether to launch web-only store, and the second mover conventional retailer (T) decides whether to follow.

E and T’s choice to enter online or not (NE, for no entry) determines four competitive arrangements as the possible outcomes of this sequential game. Payoffs are
not displayed explicitly in the game tree because they are implicitly suggested by those competitive arrangements.

As usual, the analysis is carried out backwards. First, consider time 1. If expansion to e-commerce by traditional retailer adds consumers but does not have enough service advantage to price out the web-only retailer (as described in proposition 1.1) and conditional A holds, launching online store will increase profits of the conventional retailer. When condition A doesn’t hold, launching the online store decreases the profit. Therefore, the decision of a brick-and-mortar retailer to launch an online store at time 1 depends on key indicators $k$ and $S$ of the proposed outlet.

Next consider time 0. An entrepreneur decides whether to invest in the web-only store, given the opportunity. In the situations of proposition 1.1, the entrepreneur finds an online market niche where sustainable and positive profit can be made, in anticipation of competition from dual-channel retailers. Hence, the entrepreneur enters at time 0. Two scenarios of sequential market entry are possible: (entry, entry) and (entry, NE).

If dual-channel retailer can eliminate the web-only retailer as described in proposition 1.2, launching the online store further intensifies market competition and generates a lower profit for dual-channel retailers. Therefore, in time 1, conventional brick-and-mortar retailers choose not to launch online stores. Next, consider time 0. The entrepreneur foresees the tradeoff presented to conventional retailers and enters into the market. Only one scenario of sequential market entry is possible: (entry, NE). It is interesting to note that conventional retailers restrain their market power, and in the process give web-only store a monopoly on the Internet channel. The resulting strategy in
equilibrium is the same as Balasubramanian’s static model (1998). Proposition 1.3 summarizes above analysis and Table 1.3 lists conditions and examples for these possible arrangements.

**Proposition 1.3**

*In a sequential online market entry model where a web-only store investor moves first, there are two equilibriums of market entry depending on the characters of dual-channel store (or the characters of goods distributed).*

1. **If** \((k/2n + S)/k > ((k/2n - 2S) + 3\mu)/t\) and **S/k < 1/n**, dual-channel store coexist with web-only store. When condition A holds, conventional brick-and-mortar retailer enters into online market. A \((entry, entry)\) equilibrium is obtained. When condition A does not hold, the conventional retailer does not enter into the online market. A \((entry, NE)\) equilibrium is obtained.

2. **If** \(S > k/n\), dual-channel retailer can force web-only store exit. But it will lead to stronger competition and lower profit. Therefore conventional brick-mortar retailers choose to stay out. A \((entry, NE)\) equilibrium is obtained.

<table>
<thead>
<tr>
<th>Sequential Entry Equilibrium</th>
<th>Condition</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposition 1</td>
<td>Condition A, 1, 2 of proposition 1.1 hold</td>
<td>Travel agencies</td>
</tr>
<tr>
<td>Balasubramanian (1998)</td>
<td>Condition 1, 2 of proposition 1.1 hold, but A does not hold or condition 2 does not hold</td>
<td>Casinos</td>
</tr>
<tr>
<td>Proposition 2</td>
<td>(\mu &gt; t/N) and condition 1 of proposition 1.1 hold</td>
<td>Automobiles</td>
</tr>
<tr>
<td>Salop (1979)</td>
<td>(\mu &gt; t/N) and condition 1 of proposition 1.1 not hold</td>
<td>Groceries</td>
</tr>
</tbody>
</table>

Table 1.3 Sequential Entry Equilibrium Examples
Table 1.3 summarizes proposition 1.3 and suggests product categories which can be the example of sequential online market entry equilibrium. It also makes use of Balasubramanian (1998) analysis and gives conditions (for example, $\mu > t/N$) that web-only retailer can not sustain in the competition.

**Extension 3**

If conventional brick-mortar retailers expand into e-commerce, several strategic options are worth consideration. The rest of this section discusses these choices. The first strategic option is in regards to uniform pricing across physical and online store of dual-channel retailers. Relaxing this assumption allows dual-channel retailer to have two price variables to play in competition. The new equilibrium involving three price variables is introduced and compared with the earlier equilibrium with uniform pricing across two stores. The finding is summarized in the next proposition. Table 1.3 summarizes

**Proposition 1.4:**

*The profit of a web-only store does not depend on the pricing scheme of a dual-channel player. Separate pricing over channels increases the profit for dual-channel retailer from uniform pricing but does not affect web-only store.*

Proof: See appendix A.

Equilibrium prices under separate pricing scheme are matched against their counterparts under uniform pricing scheme. The equilibrium price for the web-only retailer does not change, and the market share stays the same. Hence, the profit in this
scenario remains the same for web-only retailer. On the other hand, the dual-channel retailer improves its profit by adopting separate pricing schemes. The prices at physical store and online store only affect internal consumer choice over these two stores. A separate pricing scheme does not have an effect on online consumer. This scheme also can be understood in a sequential context. A dual-channel retailer considers its online competition with a web-only retailer first. The competitive interface determines equilibrium prices at the web-only store and the online store of dual-channel retailer. Then the dual-channel retailer sets the physical store’s price to optimize total profit.

Proposition 1.4 has an obvious, though useful corollary.

**Corollary 1.2:**

*Price differentiation over physical store and online store does not expand dual-channel retailer’s online reach at equilibrium; neither does it change its online store price. But price differentiation increases profit compared with uniform pricing. A dual-channel retailer sets the physical store price higher than its online store price, sending more customers to its online store than in the uniform price setting.*

**Extension 4**

A separation of dual-channel retailer’s physical store and online store is another strategic variable. The physical store and online store of dual-channel retailer are treated as independent business units. In the resulting setup, three types of players involve in a simultaneous price setting game in the market. A web-only retailer, a brick-mortar retailer, and an online retailer each operate with unique characteristics. The online retailer
competes with the web-only retailer at one end of the consumer spectrum and with the brick-mortar retailer at the other end. Whether this move increases the dual-channel retailer’s overall profit is the main interest. Management literature (Gulati, Garino 2000) voices strong support for the integration of physical and online stores. Integration nurtures brand recognition, cross-selling, purchasing leverage and customer benefit. Our analysis provides an economic diagnosis on this issue. The result is consistent with the proposition in management literature.

**Proposition 1.5**

*Consider a circular market with a web-only retailer D and n evenly spaced retailers B, each spinning off an online outlet C. A unique pure strategy Nash equilibrium in prices exists. Equilibrium prices are given by:*

\[
\begin{align*}
p_b^* &= k(\mu - S)/3t + (k/4n + 3\mu/2 - S) * (t - k)/3t, \\
p_c^* &= (2k - 3t) (\mu - S)/3t + 2(k/4n + 3\mu/2 - S) * (t - k)/3t, \\
p_d^* &= k(\mu - S)/3t + (k/4n + 3\mu/2 - S) * (t - k)/3t + k/4n - \mu.
\end{align*}
\]

*Proof: See Appendix A.*

Equilibrium prices in the three-player setting are below counterpart prices in the earlier two-player setting. Separation of the physical store from the online store depresses the dual-channel retailer’s profit. The market share of each player in the three-player setting is contingent on parameters. Barnes & Noble once singled out its online operation, arguing that online retailing asks for a totally different set of skills. Our analysis suggests that operating a physical store and online store separately does not contribute to the retailer’s bottom line. Separation intensifies market competition.
6. Conclusions, Limitations, and Future Research

The growth of online marketing in two forms, web-only retailer and dual-channel retailer, is changing the competition in the consumer market. However, the strategic incentive of Internet market entry has not been explored sufficiently in the earlier literature (Alba, et. al., 1997; Balasubramanian, 1998). The analysis of entry and market structure in this work provides such an amendment.

We explore the strategic implications of conventional brick-mortar retailer’s entry into e-commerce. We assume that dual-channel retailers receive a favorable appreciation from consumers, compared to web-only retailer, in terms of service. This advantage dwindles over distance. We also assume the same production cost across all retailers. It is shown that launching online store by conventional retailer does not necessarily destroy web-only retailer. Conventional brick-mortar retailers do not always enter the online market.

The analysis has its limitations. It assumes complete information to gain insight. Real-life markets are more complicated. For example, the cross marketing and synergy effect may contribute to the incentive of launching an e-commerce outlet. Online retailing is so new that firms definitely learn by doing. The analysis of retail market does not address wide price dispersion observed in market-place. Next chapter studies this important issue.
Future research looks at multiple web-only players in the model, the incentive of vertical integration of the retailing industry, and the study of alternative online retailing mechanisms such as auctions.

References:


CHAPTER 2

EFFECT OF THE PROPORTION OF VIRTUAL CONSUMERS ON THE RETAIL PRICE OF HOMOGENOUS PRODUCTS

ABSTRACT

The contemporary retail market is encountering more virtual consumers, whose access to the channel is starkly different from conventional consumers; these virtual consumers have uncommonly high cost of accessing the traditional retail channel. For example, it is difficult for a traveler at an airport to find a retail store carrying the merchandise, or some senior citizens find shopping trips extremely cumbersome. However, instead, they can access online stores with ease. These virtual consumers exist in parallel with conventional consumers who are not restricted to either traditional channel or online channel. The literature in retailing has been dominated by the sector focused on conventional consumers. This chapter is an effort to model, from a strategic viewpoint, retailing markets that include both virtual and conventional consumers.

Bailey (1998) and Brynjolfsson and Smith (2000) found that price dispersion is surprisingly not lower in online markets as compared to conventional markets. The
authors attribute their unexpected findings to several factors, including market immaturity and heterogeneity in online retailer attributes such as trust and awareness. This chapter suggests that in a mature market, where retailers remain constant and consumers are fully aware of online offerings, price dispersion also could exist. The analysis finds a relationship between the ratio of two consumer types -- conventional and virtual -- and the characteristic of price competition in this retail market. I find that the proportion of virtual consumers plays a pivotal role in the market equilibrium. A pure strategy price equilibrium exists when the proportion of virtual consumers is small, while a mixed price equilibrium exists when the proportion of virtual consumers is large. In contrast to extant literature, my analysis incorporates two types of retailers, pure web based retailer and land-based retailer with online outlet, and two types of consumers, virtual and conventional. It also offers an analysis of industry performance by capturing key participants in contemporary retail markets.

1. Introduction

“Retailing in the 21st century will no doubt be very different from retailing in the 20th century, just as retailing in the 20th century was very different from retailing in the 19th century.” - Robert A. Peterson and Sridhar Balasubramanian (2002)

The e-commerce, or online, market channel is the status quo for the modern retail industry. According to a report from Forrester Research Inc. (2008), online retail sales
reached $175 billion in 2007. This is after six consecutive years of high annual growth rate of at least 20 percent. As the perceived risk of online shopping continues to drop and improved technology better facilitates e-commerce transactions, more and more consumers feel comfortable shopping online. Among these shoppers, some consumers forgo completely any traditional in-store shopping and become solely “virtual consumers”. These virtual consumers represent a significant change for the industry from the demand side. This chapter is an effort to examine the effect of virtual consumers on pricing in a modern retail market with multiple retail channels.

A casual observation of retail markets today reveals that more and more consumers shop online. However, many consumers still prefer to shop in land-based channel. Among online consumers, some use the online channel to check for merchandise information, then subsequently shop in a land-based store. Some, though, shop online exclusively. The implication of this more diverse shopping behavior has largely been unexplored in the literature. To address this gap, this chapter provides an equilibrium analysis that accommodates e-commerce consumer diversity and applies the analysis to specific problems of research and managerial interest, yielding an immediate implication for price competition.

The online shopping channel is becoming a powerful marketing and distribution medium. It is important to recognize that the traditional channel, a land-based store, provides services that an online channel cannot match, such as product inspection and instant fulfillment. Traditional retailers now adapt to market change and operate both web-based and land-based stores, called brick-and-click stores for the purposes of this
chapter. The coexistence of online and traditional shopping becomes an enduring reality for the retailing industry.

Previous research has focused on the competitiveness of the online channel (Bailey, 1998; Brynjolfsson & Smith, 2000). The wide price dispersion for a homogeneous product in the online channel raises a question: Why does the electronic market not reflect anything close to perfect competition given the low cost of search and comparison? There have been some efforts to understand consumers’ cost in switching to another online retailer (Chen & Hitt, 2000). If the switching cost is not negligible online, then store can charge a high price without losing customer. The effect of brick-and-click formats on the level of price dispersion in the market has been studied empirically (Xing & Tang, 2004). However, a systematic analysis of price dispersion in electronic markets is lacking.

The literature in the context of web-only stores versus brick-and-click retailer competition is relatively new. The following seminal question has not been fully answered:

“Many traditional retailers will find themselves in multiple channels—maintaining their bricks-and-mortar operations while also creating an electronic presence. What are the economics of such dual systems?” (Alba, et al., 1997)

Earlier literature (Balasubramanian, 1998) studies the strategic competition between direct marketers and conventional retailers. In that research stream, brick-and-click stores are not considered, however. Our early model (the first chapter) fills that gap
by extending the presence of land-based retailers into multiple channels. A limitation, though, is that it cannot explain price dispersion.

One underlying contention of the essay is that virtual consumers are different from conventional consumers usually featured in the literature. Store location is not a salient factor for virtual consumers. Every store is just a click away. The volume of purchases from virtual consumers can substantially change the dynamics of the retailing market. There are many situations that online shopping is preferred. For example, the following are all examples of people who may depend exclusively on online shopping: college students without transportation, people who prefer to shop at night when most stores are closed to the general public, elderly who find trips to busy stores overwhelming, employees who cannot leave their office, and consumers of digital goods. These various situations, combined sometimes with a dislike of the conventional shopping experience, can shift the person’s preference from conventional shopping to virtual shopping. Their purchases truly rely on the convenience of a virtual marketplace.

The popular business literature often has discussed the impact of virtual consumers. An article in The Economist (April 2005) applauds the arrival of consumers power, which force firms into hyper competition. While such descriptions remain current, a formal analysis on hyper competition is lacking in the literature. This paper provides an early understanding on the issue.

The next section introduces a model of retail market competition, considering virtual and conventional consumers. In Section 3, the static equilibrium analysis is provided. Section 4 introduces the strategic implications of virtual consumers and the
resulting mixed strategy equilibrium. The paper concludes in Section 5 with a summary of equilibrium conditions and a discussion on the managerial lessons derived from the analysis.

2. Model

Consumers are in the market for a single unit of standardized product in each period. The market is modeled as a residential circle (Salop 1979). Retailers procure the product at a marginal cost $c$, and it is normalized to zero for simplicity. Consumer demand is assumed to be inelastic. This assumption is equivalent to postulating a high reservation price for all consumers, compared with shopping cost. Consumer’s objective is assumed to minimize the cost of shopping.

There are two types of retailers. There is a web-only store with no physical location in the market and $n$ conventional retailers who are evenly located within the market. Those conventional retailers also set up online stores.

There are two types of consumers, conventional consumers who are uniformly distributed over the market and consider travelling cost to conventional retailers by distance and virtual consumers who bypass the travel required by the conventional method and exclusively purchase online.

The web-only store sells to any consumer who desires to make an online purchase and pays the listed price. Additional to the listed price, purchasing from the web-only store incurs cost $\mu$, a disutility capturing reduced retail services compared to purchasing from conventional stores where product inspection and immediate fulfillment are
included. This disutility also applies to purchase from online stores of dual-channel retailers.

The conventional retailers sell to conventional consumers with linearly increasing travelling cost at rate of \( t \) per unit distance. Online stores of dual-channel retailers sell to conventional consumers with the same cost \( \mu \) due to online format, but the “brick” portion of the brick-and-click stores makes returns and exchanges easy for conventional consumers. It is assumed that this associated benefit decreases over distance. It starts from a value of \( S \) for conventional consumers collocating with land stores and dissipates with a linear rate of \( k \) per unit distance. Online stores of conventional retailers sell to virtual consumers too.

The total demand is normalized to 1. Let the proportion of conventional and virtual consumers be \( \alpha \) and \((1 - \alpha)\) accordingly. Virtual consumers only are interested in shopping via an online format. They exclusively compare online offers from \( n \) online stores of dual-channel retailers and the web-only store, purchasing from the store with the lowest price. The consumers’ reservation price is assumed to be the same across the two groups.

3. Static Equilibrium in the Price-Setting Game

Consider first a simultaneous price-setting game where \( n \) brick-and-click stores and a web-only retailer compete for two groups of consumers. The setting implicitly
assumes that the web-only retailer survives the competition. Let the price listed by the web-only retailer and the brick-and-click retailer be $p_d$ and $p_{bc}$ respectively.

**Proposition 2.1:**

*A unique, pure strategy Nash equilibrium in prices exists where both web-only retailer and brick-and-click retailer make profits, equilibrium prices are given by:

\[
p_{bc}^* = \frac{(n + 1 + an)/2an + S/k}{2n + 1} \frac{k}{k},
\]

\[
p_{d}^* = \frac{(n + 1 + n(n + 1 - an))/2an - nS/k}{2n + 1} \frac{k}{k},
\]

when $\alpha > 1/(1 + 2S/k)$ and $p_{d}^*$ is lower than $p_{bc}^*$.

Proof: See Appendix B.

Figure 2.1 provides the shopping cost of the conventional consumers in a market under equilibrium conditions. (Details are in the Appendix B.)

![Figure 2.1 Shopping Cost](image-url)
In pure strategy equilibrium specified in proposition 1, all virtual consumers are captured by the web-only store since \( p_d^* \) is lower than \( p_{bc}^* \). The web-only store has to charge a lower price to compensate the shortage of conventional retail service.

Looking at comparative static, the web-only store and brick-and-click store’s prices in equilibrium \( p_d^* \) and \( p_{bc}^* \) are increasing in \( k \). That means the less competitive the market is, the higher equilibrium prices for both. Increasing \( S \) adds to the value of shopping at the brick-and-click retailer’s online store; therefore, it boosts \( p_{bc}^* \) while pushing down the price of web-only store \( p_d^* \). Increased retail competition, in the form of bigger \( n \) in the model, lowers equilibrium prices across the board. A higher proportion of conventional consumers in terms of larger \( \alpha \) lowers equilibrium prices for both stores. More conventional consumer means less virtual consumers for the web-only store. Reduced demand for the web-only store forces it to charge a lower price. In a price-setting game, a lower price of one participant drags down the price of the other participant in equilibrium.

4. Strategic Implication of Virtual Consumers: Proportional Effect in Multi-channel Markets

The proportion of conventional consumers plays an important role in the static equilibrium of the previous section. If the fraction of conventional consumer \( \alpha \) is less than a threshold value, the pure strategy equilibrium does not exist. This is because the
decreasing value in $\alpha$ creates an inconsistency for fixed price sellers in the market. There are three possible ways of setting price in a pure strategy game. The seller can set its price higher, lower or equal to its rival. None of them can prevail in this market, however, if $\alpha$ is below the threshold value. For example, in a low $\alpha$ value market, if a seller decides to set price lower than rival, he will capture $1-\alpha$ fraction representing virtual consumer, considerably large since $\alpha$ is low. Facing a large increase in demand, his low price strategy no longer maximizes the profit opportunity across conventional and virtual consumers. He then would like to raise the price, but that move would foil price strategy. If he decides to match a rival’s price, so to split the market share of virtual consumer, the uneven market powers prescribe heterogeneous equilibrium prices which will not support market share split. That setup defies the feasibility of equal price at equilibrium. Lastly, to set price higher than rival mirrors the case of its rival setting lower price, therefore, that strategy will not achieve stable pure strategy price equilibrium either. In summary, when $\alpha$ value is below a threshold, sellers find impossible to achieve pure strategy equilibrium in prices in this market. A different solution is needed to address this situation.

A mixed pricing strategy, where retailers simultaneously pick a price from a set of values, represents a strategy that hides price information and thus avoids the dilemma in pure strategy competition. A stable market outcome can be produced.
Proposition 2.2:

When $\alpha < 1 / (1 + 2S/k)$ a unique pure strategy Nash equilibrium in prices does not exist. However, a mixed pricing strategy for both the web-only retailer and brick-and-click retailer exists and the distribution functions are given by:

$F_{bc}(p) = \frac{[2pn\alpha / (n+1)k - \nu ] / (1 - \alpha)(a+bp)^{(n-1)(n-1)} / (n-2)}{(1 - \alpha)(a+bp)^{(n-1)(n-1)} / (n-2)}$ and

$F_d(p) = \frac{(a + b p)^{n-1/n-2}}{(a + b p)^{n-1/n-2}}$

where $a = -\alpha(1 - 2n(S - p')/k) / (1- \alpha)(n+1)$, $b = 4an(n-2) / (1- \alpha)(n+1)(n^2-2)k$,

$p'$ is the mean of brick-and-click retailer’s price and $\nu$ is a constant.

Proof: See Appendix B.

From the distribution functions of Proposition 2.2, probability density functions can be derived. The density functions are skewed to high end of distributions. A quick examination on prices of books or music CDs at pricegrabber.com is consistent with this pattern. Most of the prices, for a single item, cluster around the high end, while few prices at the low end. An increasing number of traditional retailers reduces the degree of upper skewness in the distribution of random pricing and leads to a lower average price in the market.

Utilizing the boundary condition of the distribution function $F_{bc}(\max(p_{bc})) = 0$ and $F_{bc}(\min(p_{bc})) = 1$, the upper and lower bounds of the price can be derived as functions of $S$, $\alpha$, $n$ and $k$.

$\max(p_{bc})$ is the solution of equation $2n\alpha / (1 - \alpha)(n+1)k - \nu / (1 - \alpha)(a+bp)^{(n-1)(n-1)} / (n-2)$.

$\min(p_{bc}) = (n + 1)*k* \nu / 2na.$

$\max(p_d) = [n + 1 - na - 2na(S - p')/k] * (n^2 - 2)k / (n-2)4na.$

$\min(p_d) = [1 - 2n(S - p')/k] * (n^2 - 2)k / (n-2)4n.$

$p'$ is the mean of the brick-and-click retailer’s price.
If $\alpha$ increases, $\max(p_d)$ decreases and $\min(p_d)$ remains unchanged. The increase of $\alpha$ also leads to decreasing $\min(p_{bc})$. Therefore, the web-only retailer lowers its top price when $\alpha$ increases. In this light, brick-and-click retailers find the increasing proportion of conventional consumers beneficial, while the web-only retailer finds that harmful.

5. Summary of equilibrium and discussion

The equilibrium in the online market with two groups of consumers can be summarized in Figure 2.2.

Figure 2.2 Mixed and Pure Strategy
Below the curve $\alpha = 1/(1+2S/k)$ (area I), retailers play a mixed strategy in equilibrium. Above the curve (area II), retailers set a fixed price in equilibrium. In the fixed-price strategy equilibrium, the web-only retailer sets a lower price. This result is consistent with the assumption that the brick-and-click retailer holds a service advantage over the web-only retailer. The analysis suggests a useful metric $S/k$. It can be interpreted as a brick-and-click retailer comparative advantage index. When $S$ is large and $k$ is small, (i.e. the advantage of online store of dual-channel retailer over web-only store starts high and dissipates slowly), there is a big advantage for a brick-and-click retailer over a web-only retailer. In this situation, the brick-and-click retailer is content losing the price competition at equilibrium. The comparative advantage of $S/k$ allows the brick-and-click retailer profit-making from conventional consumers. A lower percentage of conventional consumers are needed to maintain pure strategy equilibrium. On the other hand, if $S/k$ is small, i.e. the advantage of a brick-and-click retailer over the web-only retailer is less significant, the market condition is less favorable for the brick-and-click retailer. Then a high percentage of conventional consumers are needed to maintain pure strategy equilibrium. The smaller the comparative advantage brick-and-click retailers stores hold, the smaller the threshold of virtual consumers that demands mixed strategy by the brick-and-click retailers.

Results also suggest that the development of retail market competition can hinge on the ratio of virtual versus conventional consumers. When the digital market starts with a small number of virtual consumers, pure strategy equilibrium is still possible. Brick-
and-click retailers do not need to serve the virtual consumers, leaving them completely to the web-only retailers. When the e-commerce gains mainstream awareness by the increasing ratio of virtual versus conventional consumers, it becomes more difficult to ignore virtual consumers. In this situation, if the comparative advantage of brick-and-click retailers $S/k$ is too weak to compete with web-only stores, retailers institute mixed pricing strategy in equilibrium.

Contribution of this chapter to the literature is twofold. First, most digital market literature focuses solely on e-commerce market behavior, paying little attention to contributions from the traditional retail sector. Although web-only stores have gained solid ground in modern retailing industry, traditional services are still relevant to many consumers. The incorporation of two retailing worlds – online and traditional - is the foundation of this analysis. Second, there are strong empirical evidences that support the hypothesis of efficiency gain in digital markets. Both Bailey (1998) and Brynjolfsson and Smith (2000) find that price dispersion is not lower in e-commerce markets as compared to conventional markets. The authors attribute their findings to several factors, including market immaturity and heterogeneity in retailer attributes such as trust and awareness. As e-commerce enters its 14th year in 2008, it shows no clear signs of price merging online. The analysis in this paper suggests that the price dispersion could exist in a mature digital market, which is signaled by a high fraction of virtual shoppers. To date, this perspective has not been emphasized in the literature.
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CHAPTER 3

STRATEGY OF MANAGING PAY PER CLICK: THE CASE OF SYMMETRIC ADVERTISERS

ABSTRACT

To display advertisements in the “sponsored links” location on search engine and content web sites, advertisers must compete for the premium ad placements through a bidding process. Advertisers have to judge the value generated from consumers’ actions, which in most cases is measured as the number of “clicks” on the advertisement that connect to more information. The bids that advertisers submit signal their commitment to winning this advertising space among potential competitors. Media owners of these search sites usually charge for advertisements as a “pay-per-click” rate; advertisers bid on the amount they will pay for each “click.” Therefore, the strategy of managing these pay-per-click bids has significant implications for media owners.
The practice in business and in marketing and economics literature has been dominated by a focus on “pay-for-placement,” in which advertisers bid on their placement, or rank, of their advertisement within the collection of sponsored links. This mechanism recognizes that advertisers have different threshold amounts that they are willing to pay. By merging the ad’s location rank with the bidding process, media owners can utilize the scarce ad space most efficiently. However, this chapter examines bids, as a management strategy, if the willingness of advertisers to pay is similar.

At search engines or content networks, advertisers bid so that their ad appears based on the keywords entered into search query; the keywords can be broad or narrow. The more narrow the keyword, the fewer the advertisers who are interested in bidding on that term. When potential advertisers want their information to appear based on the same keyword, the bidding for the ad’s rank does not reflect any difference among them. Instead, the bidding becomes a strategic move to obtain an ad location rank near close competitors, since an ad rank has the same value to all advertisers. A framework of bidding on common-value items consequently yields a modification in the strategy of managing pay-per-click. The analysis demonstrates that media owners can improve their advertising revenue, when the advertiser heterogeneity is absent, by switching from a pay-for-placement bid process to a bid-for-inclusion format.
1. Introduction

Pay per click is an Internet advertising model used on search engines, advertising networks, and content websites, such as blogs, where advertisers only pay when a user actually clicks on an advertisement (ad) to visit the advertisers' website.

-Wikipedia.com

The pay per click model has generated billions of dollars of revenue for search engines and advertising networks. Google’s advertising revenue from Google websites in 2007 reached $10.624 billion. Yahoo!’s total revenue in 2007 equaled $6.969 billion. It is believed that more than 50% of Yahoo!’s revenue is derived from sales via pay per click.

Media owners find the bidding process, or auction, naturally appealing, as managing the problem of asymmetric information from advertisers can be handled by pay per click. Google AdWords, Yahoo! Search Marketing, and Ask Sponsored Listings all use advertisers’ bids to determine their ad rank. However, the advertiser heterogeneity assumption has not been closely scrutinized in the context of Internet advertising. Particularly, situations where the advertisers are close competitors have received little attention in the existing literature. This paper is an effort to analyze asymmetry-proof situations, from the media owner’s perspective, and explores issues of managerial interest in this Internet advertising model.

A casual look at search entries reveals that they can be either very broad or very narrow terms. For example, a consumer can search by the keyword “allergy,” or he can
search for a very specific allergy product, such as the medication Zyrtec. The broad search term, allergy, will attract a variety of advertisers with differing levels of interest, while the narrow search term, Zyrtec, is more selective. In the extreme, when the term is most selective, potential advertisers’ backgrounds become very closely matched and their willingness to pay is leveled. This paper focuses on using narrow search terms; it addresses a gap in literature by providing analysis of pay per click management in this new situation.

Research into Internet advertising has evolved from early banner ads to recent search ads. Chatterjee et al. (2003) collected consumer click stream data to analyze consumer responses to banner ads. They identify the effects of ad exposure time, location, and iteration on the likelihood of the advertisement being clicked by the consumer. They also identify consumer heterogeneity in clicking patterns. Mangani (2003) discusses the pricing situation of banner ads, in which a web publisher compares pay-per-view pricing, pay-per-click pricing, and mixed pricing for banner ads. The general findings are that pay-per-click is better suited when consumers are looking for information, and pay-per-view is better suited when consumers are more impressed by their perception of the banner image.

In the research stream of search ads, Feng and Bhargava et al. (2003) focus on controlling of advertising content. The media owner favors advertisements that suit the web page theme; meanwhile the advertiser’s goal may not align well with media owner’s goal. For example, a repulsive advertisement can deteriorate the reputation of the media
owner’s site. It is in the interest of the media owner to put controls on the advertising contents accepted at his site. Feng and Bhargava, et al. (2003), in particular, simulate the revenue impact of various ranking methods on pay-per-click in search ads, including the method that considers past advertisement click-through rate. Edelman et al. (2007) closely models the Yahoo! pay-per-click practice. All the research builds on the auction literature, in which bidders’ willingness to pay is diverse and assumed unknown by the media owner.

The recognition of narrow keywords in search entries suggests a new area of study, and the analysis leads to a different strategy on pay-per-click management when bidder heterogeneity is absent. For example, if the keyword entry is narrow enough that very few advertisers are interested, and these turn out to be close competitors in the same industry, the assumption of asymmetric advertisers is lost. Allocating ad location rank among close bidders becomes the focus of pay per click management. In a setting where oligopoly advertisers bid for rank at search ads sites, this analysis shows that current private-auction driven policy generates sub-optimal revenue for the media owner. The content relevancy (Feng & Bhargava, et al. 2003) is still an important area of work for media owners. However, in the long run, irrelevant advertising will not attract adequate clicks, and, thus, it will dissipate. Editorial filtering also can provide additional content guards. This paper selects two genuine-fit advertisers and examines the ad rank choices (the top or bottom of the page) through bidding at a search ads’ website.
Intuitively, the auction of ad ranks at a search engine’s web page will force advertisers to compete. Because advertisers hold same willingness-to-pay for these ad rankings, the auction fails to separate advertisers, and the competition increases the cost to obtain these positions. In contrast, a randomizing strategy where the media owner gives advertisers equal chances to obtain positions through a random allocation process decreases the cost of obtaining the preferred rank that turns into a higher value of the listing service for advertisers that media owners can extract.

The popular business literature often has discussed the importance of search ads. Jupiter Media Metrix conducted an online advertising effectiveness study in 2001 among several ways to advertise on the Internet: banner, e-mail, paid inclusion, and search ads at the search engine. The search ads came out at the top by ROI comparison.

A study of Jupiter Research (Stain, 2005) yielded the following finding:

*Online advertising will continue the steady growth it has seen since 2002, with total revenue growing to $18.9 billion in 2010. Paid search advertising will be critical to this burgeoning growth and will eventually comprise a larger portion of total spent than display advertising.*

Given the importance of search ads to the industry, a thorough study of its practice is warranted. This chapter complements current on-going literature on this subject.
The remainder of this chapter is organized as follows. The model is introduced in Section 2, while Section 3 establishes the decreasing value of advertising positions at search ads. In Section 4, the media owner’s auction policy is analyzed and compared to a new policy that shows an improvement of the media owner’s revenue. Section 5 relates this study to other economics literature, and Section 6 concludes with a discussion of the findings.

2. Model

The setting involves two advertisers and two ad locations with different ranks at a search advertisement service. Both advertisers are interested in being associated with a certain search keyword. However, each advertiser can only purchase one space associated with that entry, and one space is listed ahead of the other.

The value of the space for an advertisement is assumed to link directly with the number and the quality of clicks generated from that ranking. It is assumed that the more sales generated from an advertisement location, the more valuable that space is for the advertiser. Therefore, the value of the ad location is driven by consumer activity rather than an advertiser’s idiosyncratic taste, as held in auction literature.

This new interpretation of the value of an ad location consequently changes the meaning of pay per click to advertisers. In the existing literature, pay per click reflects advertiser’s valuation of the clicks, such as the amount advertisers would pay to have
their ad clicked and viewed. Now, pay per click is the up-front cost to reach consumers and subsequently generates sales.

Two advertisers procure the same product with marginal cost $c$, assumed to be zero without loss of generality. Two advertisers want to obtain an ad location associated with a certain search entry. The media owner sets up a policy to sell space. One policy is to accept bids from two advertisers and award the top space to the winning bid. It is a common knowledge that the space located at the top of the Web page generates more sales; hence, that space is worth more money. To bid high or low is no longer connected with an advertiser’s desire to place the ad in a specific location, but the strategic willingness to win or concede in bidding war. The analysis examines the efficiency of this auction mechanism to a media owner under this new perspective. A valuable insight is uncovered through a modeling exercise.

The model assumes that advertisers can predict future sales from each ad space prior to submitting bids. Given the uncertain nature of Internet buyers, this may seem too strong and unrealistic at first glance. But in the real-time interactive environment of search ads advertising, where advertisers can monitor sales and adjust bids in a very short period, advertisers may be not too far from learning the real demand information. After the learning period in Internet advertising, the bid is likely to be in-line with the actual effect of click advertising on sales.

It is important to recognize that the search ads are different from a multi-unit auction, where all units are identical and bidders’ valuations are unknown. In the search
ads, the locations are not identical: a low-ranked ad space attracts fewer clicks than a high-ranked space, in general. This is because the ranks are subject to self-selection by customers. Only shoppers tend to reach low-ranked advertisements. Therefore, lower ranked ad spaces face fewer and tougher demands. This observation suggests that multi-unit auction framework does not fit the search ads advertising.

3. Common knowledge: ranking value of ad location at search ads service

The value of the ranking of search ads is studied in this section, as a means to build the first step of analysis. First, it is important to recognize that the common knowledge that the top-ranked space generates more sales is not because of more clicks on the advertisement. An issue with search ads is the sale conversion rate. Advertisers pay for actual clicks. But real sales generated depend on sale conversion rate. That rate can be very tricky. For example, a high-ranked position can get lots of clicks, but few purchases. A low-ranked position can get few clicks, but percentage-wise, yields a higher purchase ratio. The analysis in this paper does not incorporate sale conversion rate. The values of ad spaces are established by competitive force between two advertisers, as will be shown.

The analysis implicitly assumes that consumers visiting the online media are heterogeneous in the amount of time willing to spend in a search. Similar to search literature (Salop & Stiglitz, 1977; Varian, 1980; Stahl, 1989), there are high search cost consumers and low search cost consumers in the market. Low search cost consumers click on both spaces, while high search cost consumers limit their exposure to the top ad.
space only. Therefore, a low rank position will only receive visits from low search cost consumers, while a top rank position reaches both types of consumers. Because low search cost consumers check both spots, advertisers have to compete based on that type of consumer.

The demand curves of high search cost consumers and low search cost consumers are assumed independent to each other and both linear in price. The top ranked advertiser has a monopoly over high search cost consumers. The bottom ranked advertiser needs to maintain a price advantage to obtain low search cost consumers because they compare prices. The two advertisers engage in price competition over high and low search cost consumers through the interface of search ads.

**Proposition 3.1**: Consider a market with two firms ranked at the top and bottom of the sponsored links ad location, selling the same product and two sets of consumers. The top-ranked ad has monopoly control over one set of consumers. The other set of consumers are shared by both firms. A unique Nash equilibrium in prices exists where

1. The top-ranked firm receives higher revenue than the bottom-ranked firm at equilibrium. Top firm sets price at monopoly level to consumers.
2. The equilibrium price at top-ranked firm is higher than bottom-ranked firm.

Proof: See Appendix C.

The firm of the top-ranked advertisement has a favorable advantage at the competitive interface at search ads because it captures high search cost consumers, and its
price can affect both types of consumers in the market. The top firm, thus, maintains a higher revenue and price at equilibrium. Note that the portions of the two types of consumers are implicitly assumed equal in the proof. The next analysis shows the result in proposition 1 is not affected by that simplification.

The proportion of two types of consumers

Assign proportion $\rho$ to high search cost consumers and proportion $(1 - \rho)$ to low search cost consumers ($\rho$ is strictly between 0 and 1). The variation of $\rho$ is perfectly correlated to a comparative position shift of demand curve $D_h$ and $D_l$. The top-ranked position’s revenue from high search cost consumers at monopoly level is $p^*_h \cdot D_h (p^*_h)$.

If the firm of the top-ranked ad undercuts the bottom firm’s price $p_l$ to get both high search cost and low search cost consumers, then its revenue is

$$p_l \cdot D_h (p_l) + p_l \cdot D_l (p_l).$$

In order to defend its market, the firm of the bottom-ranked ad sets its price to $p^*_l$ making the top-ranked position indifferent to these two choices:

$$p^*_h \cdot D_h (p^*_h) = p^*_l \cdot D_h (p^*_l) + p^*_l \cdot D_l (p^*_l). \quad (1)$$

The top-ranked firm will not undercut the bottom-ranked price at this level, because it receives lower revenue than it can obtain from high search cost consumers alone. At equilibrium, the top-ranked ad’s revenue is $p^*_h \cdot D_h (p^*_h)$ and bottom position has $p^*_l \cdot D_l (p^*_l)$. From the equation 1, the top firm’s revenue is higher than that of bottom
firm’s: \( p_h^* * D_h(p_h^*) > p_l^* * D_l(p_l^*) \). There also is a situation in which price undercutting is not a threat to the bottom-ranked firm, however. This happens if low search cost consumers demand is too weak to be a target of price competition. The firm in the top rank simply ignores the bottom firm’s position and acts on its own monopolized demand (high search cost consumers). Therefore, top- and bottom-ranked firms do not interact with each other. Because the demand for the top rank is stronger than the demand for the bottom rank, the top-ranked firm ends up with higher revenue. In conclusion, the proportions of the two types of consumers do not change the revenue difference previously observed.

4. Advertisers’ bidding game under auction and new policy improving media owner’s revenue

The game involves two advertisers bidding for two spaces at search ads service. The winning bid advertiser is awarded the top-ranked ad space by the media owner. After the spaces are sold, advertisers generate sales from consumer clicks. The top space can generate more sales than the bottom space. A two-period game between advertisers looks like Figure 1, with advertisers bidding to acquire the sale position, with future sales in mind.
Assume the value of the top-ranked location and bottom-ranked location \( V_A \) and \( V_B \) respectively.

**Lemma 3.1:** The range of bidding is from 0 to \( V_A \) (the revenue value of top-ranked location).

**Lemma 3.2:** There is no pure strategy Nash equilibrium for this bidding game at period 0.

Proof: When one bids between 0 and \( V_B \), the other can outbid slightly to win the better position. When one bids between \( V_B \) and \( V_A \) and it is outbid by the other, it pays too much to get position B. So each player will continually outbid the other. When one bids for \( V_A \), there is no gain by outbidding, so the other player’s best move is to get \( V_B \) as cheap as possible, by bidding 0. This triggers the bidding process again. The result is that any bid between 0 and \( V_A \) is subject to be outbid. The advertiser’s bidding incentives are summarized in Figure 3.2.
Proposition 3.2: A mixed-strategy equilibrium for this bidding game is to bid between 0 and $V_A - V_B$, with equal chances. In equilibrium, each bidder’s expected payoff is $V_B$ and the average bidding revenue for the media owner is $V_A - V_B$.

Proof: See Appendix C.

Bid for Inclusion Policy

It is shown next that the media owner can randomize the allocation process of ad spaces among advertisers, so that each advertiser gets same chance of being listed at the top. The media owner then sets a bidding game for advertisers to be included in the random allocation process. The media owner can establish a reservation price in the game. Since random process makes advertisers indifferent to top and bottom rankings prior to the lottery draw, their expected value of inclusion is the expected revenue from search ads site $(V_A + V_B)/2$. Advertisers can bid up to that value in the inclusion bid game. Then, the media owner, with this policy, can set a reservation price up to $(V_A + V_B)/2$. 
Proposition 3.3: *The media owner can increase the revenue up to \((V_A + V_B)\) by adopting a bid for inclusion policy instead of a bid for position policy.*

5. Related literature and discussion of weakness in modeling

The deviation of this study from the existing search literature is the ranked nature of positions. The top-ranked position is more valuable than the bottom-ranked position because it attracts both types of consumers. Advertisers are able to differentiate through bidding on positions. In the search literature, however, all firms are symmetric to consumer searches. The ranked nature of position generates resulting pure strategy equilibrium, which sets it apart from the mixed strategy commonly found in consumer search literature. Search literature has shown at theoretical level that price dispersion can be equilibrium outcome (Salop & Stiglitz, 1977; Varian, 1980; Stahl, 1989). Empirical evidence shows that the average cost of products purchased for those consumers who visit more stores is lower (Carlson, 1983).

It is recognized that, for consumers, a search ad is one of many sources for information. Newspapers, word-of-mouth, television, etc. also can lead to a purchase. Consumers are assumed to be acting alone in this paper; it simplifies the problem. In order to model a complete consumer behavior across all channels, more assumptions on the consumer would need to be included, which is beyond the scope of this analysis. The assumption on demand curves used in the analysis is a strong one. It takes away difficult issues in modeling search ads such as click through rate, sale conversion rate and
dynamics of uncertain demand. There is unfortunately little consensus information available to model the pattern of these market variables.

In the real world, multidimensional challenges face the media owner managing pay per click bids. Content control and asymmetric information of the bidder’s willingness to pay are parts of an enduring reality. This paper is relevant to a particular situation in which the issue of heterogeneous bidders is resolved by highly selective search entry. This model can help media owners adjust their advertising policy when potential bidders hold similar demand to advertising positions.

6. Conclusion

This paper compares bidding for position versus bidding for inclusion at search ads services, from the media owner’s perspective. The pay-for-position policy is shown to generate suboptimal revenue for the media owner if the problem of bidder information asymmetry is minimized. The competition for positions increases the cost perceived by advertisers. The media owner should consider instead bidding for inclusion policy.

References:


APPENDIX A

NOTES OF CHAPTER 1

Discussion of discontinuity of $k$

If we assume discontinuity of $k$ beyond $x_2$, consumers regard the shopping cost the same between online store of dual-channel retailer and web-only store. Because there is no cost difference among these stores, any store charges little less than the peers will gain the whole sector. The web-only retailer has strong incentive to undercut the price because this is its sole source of profit. By contrast, online stores of dual-channel retailers do not want to engage in price-cutting because it affects the profit from its physical stores. Therefore, dual-channel retailers set the price at the same level in the continuity of $k$ case and web-only retailer sets the price slightly lower than the level in the continuity of $k$ case. The result is not much different from the case of continuity of $k$. 
Proof for proposition 1.1:

To find equilibrium, simultaneously optimize these profit functions:

\[ \frac{\partial \pi_{bc}(P_{bc}, P_d)}{\partial P_{bc}} = 0, \]  
\[ \frac{\partial \pi_d(P_{bc}, P_d)}{\partial P_d} = 0. \]

That is

\[ \frac{\partial (2 P_{bc} (P_d - P_{bc} + S) / k)}{\partial P_{bc}} = 0, \]
\[ \frac{\partial (2n P_d (1/2n - (P_d - P_{bc} + S) / k))}{\partial P_d} = 0. \]
That reduces to

\[ P_d - 2P_{bc} + S = 0, \]

\[ 1/2n = (2P_d - P_{bc} + S)/k. \]

Optimal value \( P_{bc}^* \) and \( P_d^* \) are obtained by solving two equations above.

Equilibrium prices are:

\[ P_{bc}^* = (k/2n + S)/3, \]

\[ P_d^* = (k/n - S)/3. \]

Given these equilibrium prices, conditions 1 and 2 are explicitly stated as:

\[ (P_d^* - P_{bc}^* + S) / k > (P_d^* - P_{bc}^* + \mu) / t \quad \text{and} \]

\[ (P_d^* - P_{bc}^* + S) / k < 1/2n \]

i.e. \( (k/2n + S)/k > ((k/2n - 2S) + 3 \mu)/t \quad \text{and} \]

\[ S/k<1/n. \]

Given these equilibrium prices, market shares for dual-channel and web-only retailer \( (S_{bc} \) and \( S_d \)) are\( 2x_2^* \) and \( 2n^*(1/2n - x_2^*) \) respectively where

\[ 2x_2^* = 2(P_d^* - P_{bc}^* + S) / k. \]

\[ S_{bc} = (1/n + 2S/k)/3. \]

\[ S_d = 2(1 - Sn/k)/3. \]

Profits at equilibrium according to profit maximization functions are as follows.

\[ \pi_{bc}(P_{bc}, P_d) = 2(k/2n + S)^2 / 9k. \]

\[ \pi_d(P_{bc}, P_d) = 2n(1/n - S/k)^2 / 9k. \]

The traditional retailer makes a profit of \( (t+2\mu n)^2/18tn^2 \) in Balasubramanian’s model of a static competition with web-only retailer. In order to justify the e-commerce
expansion, the dual-channel retailer’s profit must exceed the traditional retailer’s profit in Balasubramanian (1998), i.e. $\pi_{bc}(P_{bc}, P_d)^*$ must be greater than $(t+2\mu n)^2/18t n^2$. 2$(k/2n + S)^2 / 9k > (t+2\mu n)^2/18t n^2$. After simple algebraic operation, this condition is stated as below named condition A.

$$\begin{align*}
4* n^2*(S^2/k – \mu^2/t) + 4 * n *(S – \mu) + k – t > 0 \\
\end{align*}$$

(A)

Proof for proposition 1.2

Let the price of a dual-channel retailer A be $P_a$, the price of neighbor dual-channel retailer B be $P_b$, online shopping disutility $\mu$, rate of service loss $k$, maximum service premium $S$. Dual-channel retailers now use the online store as a less expensive delivery channel than web-only store. The web-only store is priced out of business. Customers far away from physical stores now choose between neighboring online stores. Due to the symmetric setup of our model, we only need to consider the interaction of a pair of retailers. A customer incurs a cost of $P_a + \mu + kx – S$ when buying from store A. A customer incurs a cost of $P_b + \mu + k(1/n – x) – S$ when buying from store B. If the cost at store A is lower than the cost at store B, customer orders from store A. Therefore, all the customers buying from store A meet this cost condition: $P_a + \mu + kx – S < P_b + \mu + k(1/n – x) – S$. Simplifying the expression, we have $x < (P_b – P_a + k/n)/2k$. Because two stores are symmetrically positioned at each side of store A, the demand of store A $D_a$ is defined as follows.

$$D_a = (P_b – P_a + k/n)/k.$$ 

The profit function $\pi_a$ is

$$\pi_a = P_a * D_a = P_a * (P_b – P_a + k/n)/k,$$
assuming zero marginal cost. Store A maximizes its profit $\pi_a$ by setting $P_a$.

$$\frac{\partial \pi_a}{\partial P_a} = 0.$$ 

By symmetry assumption, equilibrium price $P_a^*$ is equal to $P_b^*$.

$$P_a^* = P_b^* = k/n.$$ 

**Proof for proposition 1.4**

If dual-channel retailer considers using two price variables for the competition, (physical store charging $P_b$ and online store charging $P_c$) the equilibrium takes the form of these simultaneous profit optimizations.

Max $2 \left( P_b^* X_1 + P_c^* (X_2 - X_1) \right)$ \hspace{1cm} (a)

Max $2n^* P_d^* \left( \frac{1}{2n} - X_2 \right)$ \hspace{1cm} (b)

$P_b$: price charged at physical store of dual-channel retailer

$P_c$: price charged at online store of dual-channel retailer

$P_d$: price charged at web-only store

$X_1$: location of customer indifferent between physical store and online store, $X_1 = \frac{(P_c - P_b + \mu - S)}{(t - k)}$.

$X_2$: location of customer indifferent between online store of dual-channel retailer and web-only store, $X_2 = \frac{(P_d - P_c + S)}{k}$.

Since (a) is a two-variable function, Hessian matrix’s definitiveness is tested. $H(a) =$

$$
\begin{vmatrix}
\frac{\partial^2 a}{\partial P_b^2} & \frac{\partial^2 a}{\partial P_b \partial P_c} \\
\frac{\partial^2 a}{\partial P_c \partial P_b} & \frac{\partial^2 a}{\partial P_c^2}
\end{vmatrix}
$$

where function (a) is expressed as
\[ 2\{P_b^*(P_c - P_b + \mu - S)/(t - k) + P_c^*((P_d - P_c + S)/k - (P_c - P_b + \mu - S)/(t - k))\}. \]

\[ H(a) = \begin{bmatrix} 2/k-t & -2/k-t \\ -2/k-t & -2/k + 2/k-t \end{bmatrix}. \]

Given any a pair of variable \(<x y>\), calculate Hessian matrix’s definitiveness.

\[ <x y> \cdot H \cdot <x y>' = <x y> \cdot 2*1/[(k-t)k] \begin{bmatrix} k & -k \\ -k & t \end{bmatrix} \cdot <x y>' \]

\[ = 2 * [k(x-y)^2 + y^2(t-k)] / [(k-t) * k]. \]

If \( k < t \), this Hessian matrix is negative definite. Thus, it ensures a pair of variables \(<x y>\) supportive of a strict local maximum of (a).

Dual-channel and web-only retailer profit functions are specified as:

Max \( \pi(P_b, P_c) = 2 \{P_b * X_1 + P_c * (X_2 - X_1)\}, \) \hspace{1cm} (a)

Max \( \pi(P_d, P_c) = 2n * P_d * (1/2n - X_2). \) \hspace{1cm} (b)

By the examination of the Hessian matrix of (a), a strict local maximum of function (a) exists at \((P_b^*, P_c^*)\). Therefore, to find \((P_b, P_c, P_d)\) simultaneously optimize dual-channel retailer and web-only retailer’s profit functions to solve following equations:

\[ \partial \pi(P_b, P_c) / \partial P_b = 0, \] \hspace{1cm} (1)

\[ \partial \pi(P_b, P_c) / \partial P_c = 0, \] \hspace{1cm} (2)

\[ \partial \pi(P_d, P_c) / \partial P_d = 0. \] \hspace{1cm} (3)

The following equations result from (1)-(3):
Equilibrium prices are derived by solving equations (4)-(6) simultaneously.

\[
P_d^* = \frac{k}{3n} - \frac{S}{3}.
\]

\[
P_c^* = \frac{k}{6n} + \frac{S}{3}.
\]

\[
P_b^* = \frac{k}{6n} - \frac{S}{6} + \frac{\mu}{2}.
\]

**Proof for proposition 1.5:**

We derive the price decisions of three players C, B and D in this game. The location of a customer who is indifferent between shopping store B and store C is \(x_1\), determined by:

\[
P_b + t \cdot x_1 = P_c + \mu + k \cdot x_1 - S.
\]

\[
x_1 = \frac{(P_c - P_b + \mu - S)}{(t - k)}.
\]

The location of a customer who is indifferent between shopping at store C and store D is determined by:

\[
P_c + \mu + k \cdot x_2 - S = P_d + \mu.
\]

\[
x_2 = \frac{(P_d - P_c + S)}{k}.
\]

New customer segmentation can be shown on a horizontal line:

\[
0 \quad x_1 \quad x_2 \quad \frac{1}{2n}
\]
B land store sells to customers up to $X_1$, C sells to customers between $X_1$ and $X_2$.

Store D is the choice of customers beyond $X_2$.

Each store sets prices that maximize their own profit:

$P_b * 2X_1,$

$P_c * 2(X_2 - X_1),$  

$P_d * 2(1/2n - X_2).$

i.e.

B store profit function:  $\pi(P_b) = P_b * (P_c - P_b + \mu - S)/ (t - k) * 2,$

C store profit function:  $\pi(P_c) = P_c * [(P_d - P_c + S)/k - (P_c - P_b + \mu - S)/(t - k)] * 2,$

D store profit function:  $\pi(P_d) = P_d * [1/2n - (P_d - P_c + S)/k] * 2n.$

A Nash equilibrium in price exists by simultaneously solving profit maximization functions.

$\frac{\partial \pi(P_b)}{\partial P_b} = 0. \quad P_c - 2P_b + \mu - S = 0.$

$\frac{\partial \pi(P_c)}{\partial P_c} = 0. \quad (P_d - 2P_c + S)/k = (2P_c - P_b + \mu - S)/(t-k).$

$\frac{\partial \pi(P_d)}{\partial P_d} = 0. \quad 1/2n = (2P_d - P_c + S)/k.$

$P_b^* = k(\mu - S)/3t + (k/4n + 3\mu/2 - S) * (t - k)/3t.$

$P_c^* = (2k - 3t) (\mu - S)/3t + 2(k/4n + 3\mu/2 - S) * (t - k)/3t.$

$P_d^* = k(\mu - S)/3t + (k/4n + 3\mu/2 - S) * (t - k)/3t + k/4n - \mu.$
APPENDIX B

NOTES OF CHAPTER 2

Proof of Proposition 2.1

Virtual consumers search and shop online exclusively. They buy from the lowest offer among \( n \) brick-and-click retailers and web-only retailer. Depending on which type of retailer offers the lower price at equilibrium, there are two cases, logically. In case one, the brick-and-click store charges a lower price at equilibrium, thus capturing business of virtual consumers. In case two, the web-only retailer sets a lower price at equilibrium and only sells to virtual consumers.

Case One

When equilibrium \( p_{bc}^* < p_d^* \), the virtual consumers are shared by \( n \) brick-and-click stores on the circumference because of symmetry setup. In a static price competition, the equilibrium prices of brick-and-click stores and web-only retailer can be derived as follows.

Conventional customers compare location advantage and online shopping costs. Online shopping suffers a disutility cost \( \mu \) associated with disadvantages such as product delivery wait times and the inability to physically inspect the product. However, conventional consumer shopping from the click store of a brick-and-click retailer can benefit from the traditional retailer’s presence; the land-based store can participate in
fulfilling online orders and offer after-sale services such as exchanges and returns. This benefit decreases as the consumer’s distance to the store increases; conventional consumers receive less value from shopping with brick-and-click stores when their geographical proximity to the store diminishes. At the extreme, the brick-and-click store fails to offer more benefits than the web-only store; thus they become equally valuable to conventional consumers.

Conventional customers’ shopping cost from the web-only retailer is $p_d + \mu$.

$p_d$: price charged by web-only retailer.

$\mu$: online shopping disutility cost.

Conventional consumers’ shopping cost from the online outlet of a brick-and-click store is $p_{bc} + \mu - (S - k*x)$.

$p_{bc}$: the price charged by the brick-and-click store.

$\mu$: online shopping disutility cost.

$S$: maximum benefit from brick-and-click store for conventional customers shopping at online store.

$k$: the rate at which the advantage of online store of dual-channel retailer decreases.

$x$: conventional consumer’s distance to land-based store on the circle.

If $k$ is small, consumers living far away still perceive a high premium. If $k$ is large, consumers living far away drop the premium value sharply. $k$ measures how fast the land-based store’s benefit decreases with distance.

Therefore, conventional consumers who value brick-and-click stores more than web-only store are defined as follows.

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\[ p_{bc} + \mu - (S - k*x) < p_d + \mu. \]
\[ x < (p_d - p_{bc} + S) / k. \]

Because of the symmetry of the circular model, the size of conventional consumers strictly favoring a brick-and-click store is 2x. For conventional consumers who are not attracted to brick-and-click stores, i.e. \( x > (p_d - p_{bc} + S) / k \), they randomly choose one store among \( n \) online stores and the web-only retailer. The size of those conventional consumers is defined by: \( 1/n - 2x = 1/n - 2(p_d - p_{bc} + S) / k \). Virtual consumers only notice the prices of the web-only retailer and \( n \) online stores of brick-and-click stores; they purchase from whichever store posts the lowest price.

Assuming the web-only retailer’s equilibrium price \( p_d^* \) and other \( n-1 \) brick-and-click store’s equilibrium price \( p^* \) are given, a brick-and-click store’s demand is affected by its price level \( p_{bc} \) through three ways. The first is local conventional consumers, who strictly favoring this store 2x, where \( x = (p_d^* - p_{bc} + S) / k \) is the marginal conventional consumer indifferent from this brick-and-click store and the web-only store. The second is conventional consumers around each store who favor online shopping \( (n-1)[1/n - 2(p_d^* + S - p^*)/k] + 1/n - 2x \). The third is virtual consumers segment \((1- \alpha)\). The first part of demand belongs to one store and the remaining stores share the balance of the demand. The brick-and-click store sets its price \( p_{bc} \) to maximize its profit

\[ p_{bc} \{ \alpha \cdot [2x + (1/(n+1))] \cdot [(n-1)(1/n - 2(p_d^* + S - p^*)/k) + 1/n - 2x] \} + 1/n * (1- \alpha). \]

(1)
Given \( n \) brick-and-click stores’ equilibrium price \( p^* \), the web-only retailer’s demand consists of only shared conventional consumers who are indifferent towards online shopping and the web-only retailer \( n/(n+1) \* [1/n – 2(p_d + S – p^*)/k] \). The web-only retailer sets its price \( p_d \) to maximize its profit

\[
p_d^* \alpha \* n/(n+1) \* [1/n – 2(p_d + S – p^*)/k].
\] (2)

Take first order conditions of (1) and (2) relative to \( p_{bc} \) and \( p_d \), then let optimal \( p_{bc} \) equal \( p^* \) (symmetry). This creates two equations with two unknowns \( p^* \) and \( p_d^* \). Solutions have the form as follows.

\[
p_{bc}^* = [(2n+2- \alpha)*k/2n\alpha + S]/(2n+1).
\]

\[
p_d^* = [(n+1+ \alpha n)*k/2n\alpha – nS]/(2n+1).
\]

**Case Two**

In case two, when the equilibrium price \( p_{bc}^* \) and \( p_d^* \) have the opposite ordinal relationship as \( p_{bc}^* > p_d^* \), virtual consumers shop at the web-only retailer. In a static price competition setting, equilibrium prices of a brick-and-click store and a web-only retailer can be derived as follows.

Given the web-only retailer’s equilibrium price \( p_d^* \) and \( n-1 \) brick-and-click store’s equilibrium price \( p^* \), the demand of a brick-and-click store consists of a conventional consumer strictly favoring local store \( 2x \) and shared conventional consumers who are indifferent towards the click portion of the brick-and-click store and the web-only retailer \( (n-1)*[1/n – 2(p_d^* + S – p^*)/k] + 1/n – 2x \). The brick-and-click store sets the price \( p_{bc} \) to maximize its profits

\[
p_{bc}^* \alpha \* {2x + (1/n+1) * (n-1)(1/n – 2(p_d^* + S – p^*)/k) + 1/n – 2x}\}
\] (3)
where \( x = \left( p^*_d + S - p^*_bc \right) / k. \)

Given \( n \) brick-and-click stores’ equilibrium price \( p^* \), the web-only retailer’s demand consists of shared conventional consumers \( n/(n+1) \ast [1/n - 2(p^*_d + S - p^*/k)] \) and virtual consumers \((1-\alpha)\). The web-only retailer sets its price \( p^*_d \) to maximize its profit

\[ p^*_d \ast \left[ \alpha \ast n/(n+1) \ast (1/n - 2(p^*_d + S - p^*/k)) + (1-\alpha) \right]. \]  

Similarly we obtain two equations with two unknowns \( p^* \) and \( p^*_d \). The solutions have the form as follows.

\[ p^*_bc = \left[ (n + 1 + \alpha n)/2\alpha n + S/k \right] * k/(2n + 1), \]

\[ p^*_d = \left[ (n + 1 + n(n + 1 - \alpha n))/2\alpha n - nS/k \right] * k/(2n + 1). \]

Because the established price equilibrium in case two implies \( p^*_d < p^*_bc \), a restriction on \( \alpha \) follows:

\[ \alpha > 1 / (1 + 2S/k). \]

Similarly in case one, the established price equilibrium requires \( p^*_d > p^*_bc \). A restriction on \( \alpha \) follows:

\[ \alpha > 1 / (1 - 2nS/k). \]

As defined, \( \alpha \) is the proportion of conventional consumers in this retailing model. It shows in both cases that when the proportion of conventional consumers is beyond a certain point, a pure strategy price equilibrium exists. Immediately, case one is ruled out because \( \alpha \) is bigger than 1 under the assumption both \( S \) and \( k \) are positive from condition (6).
Proof of Proposition 2.2

In this proposition, there are $n$ traditional retailers with web capacity (brick-and-click) and one web-only retailer. Each retailer picks prices randomly from a continuous distribution. Let the cumulative probability function of the price of the brick-and-click retailer be $F_{bc}(p)$, and the cumulative probability function of the price of the web-only retailer be $F_d(p)$. Each retailer simultaneously picks a price without knowing the price at the rival retailers. Each retailer’s randomizing strategy is the best response to its rival’s randomizing strategy. Each store maximizes its expected profit.

By the definition of cumulative probability function, the brick-and-click retailer’s price $p_{bc}$ is lower than the web-only retailer’s price $p_d$ with $F_{bc}(p_d)$ probability. Similarly, the web-only retailer’s price $p_d$ is lower than the brick-and-click retailer’s price $p_{bc}$ with $F_d(p_{bc})$ probability.

Given that other $n$ brick-and-click retailers pick price $p_i$ out of the distribution $F_{bc}(p_i)$, at any price level $p_d$, the expected profit for the web-only retailer is:

$$\pi_d(P, p_d) = p_d \ast (1 - \alpha) [1 - F_{bc}(p_d)]^n + p_d \ast \alpha \frac{\int \ldots \int (1 - 2(n(p_d + S) - \sum p_i)/(n + 1) f(p_i, p_n) \, dp_1 dp_2 \ldots dp_n)}{n}.$$  

The profit maximization condition says

$$\frac{\partial (\pi_d(p_d))}{\partial p_d} = 0.$$  

(7)

The close form solution of $F_{bc}(p_{bc})$ comes from the solution of the differential equation derived from (7) which is

F.O.C. of $p_d \ast (1 - \alpha) [1 - F_{bc}(p_d)]^n + p_d \ast \alpha *[1 - 2n(p_d + S - p')/k]/(n+1)$.
The cumulative probability function of $F_{bc}(p_{bc})$ in turn is:

$$1 - F_{bc}(p) = (a + b \ p)^{n^{-1/2}}$$

(8)

where $a = -\alpha(1 - 2n(S - p')/k) / (1-\alpha)(n+1)$,

$b = 4\alpha n(n-2) / (1-\alpha)(n+1)(n^2-2)k$.

Because $1 - F_{bc}(p)$ is equal to $F_d(p)$ by definition, we have

$$F_d(p) = (a + b \ p)^{n^{-1/2}}.$$

Given other $n-1$ brick-and-click retailers price $p_i$ drawn out of distribution function $F_{bc}(p_i)$ and web-only retailers price pick $p_d$ out of distribution function $F_d(p_d)$, at any price level $p_{bc}$, the expected profit for a brick-and-click retailer is:

$$\pi_{bc}(P, p_d) = p_{bc} \ (1-\alpha) * [1 - F_{pi}(p_{bc})]^{n-1} \ *[1 - F_d(p_{bc})] + p_{bc} \ * \alpha * \int \ ... \ \int [2(p_d + S - p_{bc})/k + (1/n - 2(p_d + S - p_{bc})/k)/(n+1) + \sum [(1/n - 2(p_d + S - p_i)/k)/(n+1) \ f(p_i, p_d) \ dp_i ..dp_d].$$

The profit maximization condition says

$$\partial(\pi_{bc}(p_{bc})) / \partial p_{bc} = 0.$$  \hfill (9)

The close form solution of $F_d(p_d)$ comes from the solution of the differential equation derived from (9) which is:

F.O.C. of

$$p_i \ (1-\alpha) * (1-F_{bc}(p_d))^{n-1} (1-F_d(p_d)) + p_i \ * \alpha * [2(p' + u')/k - 4np_i / (n+1)k + v].$$

The cumulative probability function of $F_d(p_d)$ turns out to be:

$$F_d(p) = 1 - [2p\alpha / (n+1)k - v] / (1 - \alpha)(a+bp)^{(n-1)(n-1) / (n-2)}. \hfill (10)$$

Because $1 - F_{bc}(p)$ equals $F_d(p)$ by definition, we have

$$F_{bc}(p) = [2p\alpha / (n+1)k - v] / (1 - \alpha)(a+bp)^{(n-1)(n-1) / (n-2)}.$$
where \( v \) is a constant.

Math note on the derivation of \( F_d(p) \) and \( F_{bc}(p) \)

A general solution for a non-homogeneous linear differential equation of order 1 such as \( y' + A(x)y = B(x) \) has a normal form : 

\[
y = e^{\int A(x)dx} * (\int B(x)e^{\int A(x)dx} dx + D)
\]

where \( D \) is a constant. By inserting special instances for differential equation (9), we can solve (9) and derive the close form of \( F_d(p) \). Because \( F_d(p_d) \) is a mathematical function describing a cumulative probability which ranges from 0 to 1, the constant term \( D \) in solution form must equal zero.

The close form of \( F_d(p) = 1 - [2pn\alpha / (n+1)k - v] / (1 - a)(a+bp)^{(n-1)(n-1) / (n-2)} \).

A linear differential equation such as \( y' + A(x)y = B(x)y^n \) can be transferred to a simply form by letting \( y = z^{1/(1-n)} \). A new differential equation involving \( z \) can be expressed as \( z' + (1 - n)A(x)z = (1 - n)B(x) \). Utilizing the general solution above for new equation generates a normal form solution : 

\[
z = e^{\int (1-n)A(x)dx} * (\int (1-n)B(x)e^{\int (1-n)A(x)dx} dx + D)
\]

where \( D \) is a constant. Then the original differential equation \( y' + A(x)y = B(x)y^n \) has a normal form solution as : 

\[
y^{1-n} = e^{\int (1-n)A(x)dx} * (\int (1-n)B(x)e^{\int (1-n)A(x)dx} dx + D).
\]

This result is used to make transition from (7) to (8).
APPENDIX C

NOTES OF CHAPTER 3

Proof for proposition 3.1

Since the setting involves two independent sets of consumers, it is required to analyze pricing game under all situations. Consider identical demand curves first, then heterogeneous demand curves.

a. The homogeneous demand curves

In this case, low search cost consumers and high search cost consumers pose the same demand curve. A solid line representing demand of high search cost consumer $D_h$ is identical to a dashed line the demand of low search cost consumers $D_l$ in the graph below.

![Figure C.1 HIGH - LOW SEARCH COST CONSUMER DEMANDS EQUAL](image-url)
The low search cost consumers compare the bottom-ranked firm’s price with top-ranked firm. Therefore, bottom-ranked firm should set price lower than top firm in order to obtain consumers. The firm with the top-ranked ad has an incentive to undercut price to capture both sets of consumers. Depending on the bottom-ranked position’s price level, the top-ranked position will not cut prices when it makes lower revenue from \( D_h \) and \( D_l \) combined than it can make with \( D_h \) alone. At such a low price level, posted by the bottom-ranked firm, the top-ranked firm has no incentive to undercut the price. This is the equilibrium point. The bottom-ranked position protects its market of low search cost consumers by setting a low and unattractive price. The top position focuses on high search cost consumers only and achieves monopoly revenue from \( D_h \). Two observations can be made from this equilibrium: First, the top-ranked position receives more revenue than the bottom-ranked position at equilibrium. The top-ranked position receives monopoly revenue from high search cost consumers. Second, the equilibrium price at the top-ranked ad position is higher than bottom-ranked position.

**B. The heterogeneous demand curves**

Two possibilities stand out.

1. **The low search cost consumer is more likely to make a purchase decision than the high search cost consumer at a given price.**

   This could be the case when the lack of market knowledge discourages high search cost consumers to make a purchase decision. They are more likely to look
somewhere else, such as price comparison websites or recommendations from peers. On the other hand, low search cost consumers are more likely to seize the ‘deal’ when the price is right.

![Figure C.2 LOW SEARCH COST CONSUMER DEMAND HIGHER](image)

Figure C.2 LOW SEARCH COST CONSUMER DEMAND HIGHER

It is a same price setting game except demand curves are apart.

2. **The low search cost consumer is less likely to make a purchase decision than the high search cost consumer at a given price.**

Another possibility to consider is the opposite. High search cost consumers have limited time in price search, or their reservation price is high. Low search cost buyers always look for bargain or their reservation price is low.
There are two situations here. In one case, undercutting is feasible from the top-ranked ad position. In order to protect its market, the bottom-ranked position has to lower its prices so that the top position has no incentive to undercut and capture the low search cost consumers. The top-ranked position is better off selling to its own high search cost consumers instead of undercutting. In another case, because of the weak demand for the bottom-ranked ad position, undercutting is not desirable at all for the top position. In this case, there is no strategic interaction between the two positions. The top-ranked position’s prices reach monopoly revenue from its own demand (high search cost consumers). Bottom-ranked positions set their prices to reach the monopoly revenue from low search cost consumers. But they share the same feature of the strategic interaction case. Either situation generates the same equilibrium revenue and price pattern.
Summary observation:

Regardless of homogenous or heterogeneous demand assumptions made about low search cost consumers and high search cost consumers, the same ordinal relationship of optimal price and revenue is observed.

Proof for proposition 3.2

A mixed-strategy equilibrium of this bidding game is a symmetric play game. Assume each bidder bids between 0 and $V_A$, with the cumulative probability distribution function of $P_1(x)$ and $P_2(x)$. Mixed strategy equilibrium exists when given the other player’s probability bid; a player cannot do better by changing weights on probability.

For any bid $x$, the expected payoff given the other’s cumulative probability distribution function $P(x)$ is

$$P(x) (V_A - x) + [1 - P(x)] (V_B - x).$$

To maximize expected payoff, take first order condition

$$d\{ P(x) (V_A - x) + [1 - P(x)] (V_B - x) \} / dx = 0.$$

The solution to the above differential equation is $P'(x) = 1 / (V_A - V_B)$. The normative form of $P(x)$ is thus
\[ P(x) = \left[ \frac{1}{V_A - V_B} \right] x + C. \] Since the probability distribution function of \( p(x) \) has two end points that \( P(0) = 0 \) and \( P(V_A) = 1 \), the normative form of the solution can be further determined. It is shown that \( C = 0 \).

Therefore the cumulative probability distribution function of the bidding price is \( P(x) = \left[ \frac{1}{V_A - V_B} \right] x \), where \( x \) is between 0 and \( (V_A - V_B) \). With \( P(x) \) solved above, if we plug into the bidder’s expected payoff function under mixed strategy, \( P(x) (V_A - x) + [1 - P(x)] (V_B - x) \), the expected payoff is calculated as \( V_B \). Each bidder randomly bids between 0 and \( V_A - V_B \) with equal chance. Therefore, the average bid value is \( (V_A - V_B)/2 \) per bidder. The web publisher’s bidding revenue on average is \( V_A - V_B \).
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