THE EFFECTS ON STUDENT ACHIEVEMENT
AND ATTITUDES OF INCORPORATING
A COMPUTER ALGEBRA SYSTEM INTO
A REMEDIAL COLLEGE MATHEMATICS COURSE

DISSERTATION

Presented in Partial Fulfillment of the Requirements for
the Degree of Doctor of Philosophy in the Graduate
School of The Ohio State University

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ABSTRACT

The current study examined the effects of using a computer algebra system (CAS) and graphing utility on the achievement and attitudes of students in remedial college mathematics. The participants were enrolled in night sections of a basic algebra course and were identified for remedial work by a placement examination. Students came from various mathematical backgrounds, but all had experienced a cycle of mathematical failure that needed to be broken.

Both adult learning theory and social constructivist theory informed the design of the research and influenced the instruction. The use of technology was designed to further support and better facilitate success for all students in the treatment sections. A CAS offered students a fresh look at mathematics previously studied, facilitated an inductive approach to content, and provided immediate feedback for the students. The CAS use was primarily focused on the rules of exponents, solving linear equations in one variable, and graphing linear equations in two variables. However, the Texas Instruments' TI-92s were available at all times in the classroom for treatment students, except during assessments.
The effect of CAS on achievement was examined by comparing scores from the three researcher-developed tests and the departmental Final Examination. These assessments focused primarily on pencil-and-paper procedures and were designed to reflect the curricular goals of the department. Independent samples $t$-tests were used to compare the means of the treatment (n=25) and control (n=25) groups on each assessment and CAS-specific questions. Multiple regression analysis was used to factor in the effect of gender and age on both the Final Examination and CAS-specific questions. No statistically significant difference in achievement was found on the assessments.

Affective factors were examined quantitatively using a Likert-type instrument. Paired sample $t$-tests were conducted in order to identify significant changes in attitudes, and multiple regression analysis was employed to determine whether gender or age were significant factors in attitude results. The end-of-term questionnaire and classroom observations provided qualitative data for investigation of the affective domain. Significant difference in attitudes was found and themes indicating differences in classroom culture emerged.
Dedicated in Memory of
Elijah H. Greene, my father

Because he always believed I could accomplish anything.

&

Dedicated in Honor of
Kelly M. Costner, Lynda S. Greene,
Loran, Kristy, Luke, Andy, & Jake Greene,
Paul, Ellen, Carley, & Stephanie Moore

Because they supported me as I tried.
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FIELD OF STUDY

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CHAPTER 1

STATEMENT OF THE PROBLEM

Changing the way that mathematics is taught has been a focus of reports and policy statements by such groups as the National Council of Teachers of Mathematics (NCTM), Mathematical Sciences Education Board (MSEB), Mathematics Association of America (MAA), the American Mathematics Association of Two-Year Colleges (AMATYC), and the National Research Council (NRC). Advocates of change suggest new curriculum and theory-based instructional design will help students grasp mathematical concepts. Students need an education different from what they have received in the past; this education needs to help them think through significant mathematical problems (Thompson & Zeuli, 1999).

Today, teachers must structure the classroom so that students concentrate on learning how and why mathematics works (Sowder, 1989). Students and teachers should expect and demand justification for work, and consequently, should question and explore the work’s appropriateness. Through the process of exploration and justification, students begin to appreciate the current mathematical culture.
Classrooms that use inquiry in order to allow students to discover the nature of mathematics help students understand mathematics as more than a set of rules.

Mathematics is more than sets of procedures to be mastered or rules to be memorized. It includes, among other things, the investigation of patterns, the formulation of hunches about what generalization might fit a pattern, the communication of arguments for or against various conjectures, and the understanding of fundamental concepts well enough to judge when to use a particular skill (Silver, Kilpatrick, & Schlesinger, 1990, p. 6).

A classroom that reflects the ideas expressed above is quite different from many of the classrooms that students and teachers experience today.

Inquiry in Mathematics Classrooms

 Calls for change are targeted at various levels and have varying degrees of specifics. NCTM, AMATYC, and MAA have suggestions for specific grade level groups, while groups such as MSEB and NRC have provided arguments that address mathematics education at all levels. Mathematics educators have answered these policy statements by conducting specific research that examines the implementation of these recommendations in the classroom.

The Vision of Reform

Activity, communication, and exploration are important for all students (Jarrett, 1997). Students at various levels of mathematical experience and expertise deserve the opportunity to see mathematics in a meaningful way so that they can construct their own mathematical understanding. According to the MSEB (1990), research indicates that students learn mathematics only when they construct their own mathematical
understanding. By including inquiry approaches in the curriculum, teachers can provide students with the necessary avenues to construct meaningful mathematical understanding.

*Classroom Level Goals*

Inquiry classrooms should be filled with opportunities for students and teachers to question one another (Jarrett, 1997; Silver, Kilpatrick, & Schlesinger, 1990). Students should be encouraged to make conjectures and then justify them in convincing ways to colleagues. Mathematics can no longer be portrayed as a neat discipline; instead, students must be involved in the uncertain and changing face of mathematical culture (Nickerson, 1995; also see NCTM, 2000). According to Silver et al., a mathematics classroom should be open-ended, be full of conjecture making, encourage pattern discovery, delineate clear expectations for justification, and encourage communication of ideas. Jarrett also challenges teachers to design a classroom where students pose their own questions and then find different ways to reach answers, making communication central to the learning process.

Inquiry activities allow students to be involved both physically and mentally and foster the use of authentic materials (including technology) to explore problems. Further, inquiry-oriented teaching encourages communication among all members of the classroom (Waxman & Walberg, 1991). Hands-on experience also increases students’ positive attitudes. According to Lawton (1997), three out of five 10- to 17-year-olds said they would be more interested in science if they could do more experiments themselves and use computers to communicate with peers and experts about their discoveries.
Similarly in mathematics, teachers need to use exploration, conjecturing, reasoning, and validation to help students develop understanding while modeling mathematics as a discipline.

*Teacher Level Goals*

Teachers at all levels should take the risks necessary to allow students to explore mathematics. Risks may be aversive to both students and teachers, but changing one’s practice at a gradual pace can reduce the degree of risk. Jarrett (1997), with input from a science curriculum coordinator, provides an outline for gradual change in practice. She suggests a progression that starts with activities that focus on the text, teacher demonstrations, and explorations with no opportunity for student choice. Then, teachers can begin to use activities that lead to student discovery or are initiated by a teacher-generated open-ended question. Finally, courses should be filled with experiments that are student-initiated and student run. The end result is a more thorough learning experience for students. To achieve this goal, change in the classroom must be gradual, but focused.

Changing practice is not an easy task, yet today many teachers at all levels are in the process of changing from traditional approaches to inquiry-based ones. Kokol-Voljc (1999) suggests that classroom change comes in three stages. Teachers first change their instructional style, then their course goals. Finally, the curriculum must be adjusted to better reflect the changes that have taken place in the teacher.
Society Level Goals

The NRC (1991) extends the call for change to undergraduate mathematics courses so that the way in which future teachers are taught mathematics in college reflects the most recent research on learning. According to a study conducted by Waits and Demana (1988) of the students testing into the remedial mathematics courses at a major university, the majority who graduated majored in either education or social work. It follows that if mathematics educators can find ways to teach students in remedial mathematics courses they will make an impact in the way mathematics is viewed and taught for generations to follow. Future teachers are not the only ones who will be affected by potential changes in undergraduate mathematics courses, especially for those courses in the first two years of college. A cross-section of society moves through these courses and will thus be able to experience the benefits of reform-based teaching firsthand. This experience could be a vital step to their own understanding of school change, leading them to support reform at all levels of mathematics education—both for themselves and their children.

The NCTM Vision of Reform

NCTM’s Principles and Standards (2000) stress the importance of helping students see the beauty of mathematics and encourage teachers to engage students in meaningful activity that stimulates the construction of mathematical knowledge. The Learning Principle states that: “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (p. 19). This same principle for learning is applicable for all students, so by examining what is said to
the K-12 mathematics educators, ideas can be gleaned for college educators as well. The standards reflect the notion that mathematics as a discipline is not a set of manipulations, theorems, and definitions to be mastered (NCTM, 2000; 1989). Instead, school mathematics must prepare students for an unpredictable future, providing the tools they need to solve unfamiliar problems that occur over a lifetime. This implies that the school curriculum and teacher expectations should be reexamined regularly to incorporate the ever-growing field of mathematics. The Principles and Standards encourage teachers to provide all students with opportunities to use mathematics in activities related to a variety of situations and disciplines.

Classroom Level Goals

The search for meaningful mathematical understanding for all students began with NCTM’s original standards document. The Curriculum and Evaluation Standards (NCTM, 1989) were written, “(1) to ensure quality, (2) to indicate goals, and (3) to promote change” (p. 2), with emphasis on reform.

As society changes, so has NCTM’s vision of the mathematics classroom. Using expectations for school mathematics similar to those in previous documents, the Principles and Standards (2000) reflect changes that have occurred over the past decade in the field of mathematics as well as in mathematics teaching and learning, but maintain five overarching goals. The first of these goals encourages the use of opportunities for students to explore, develop, and refine problem-solving techniques with the goal of mathematical literacy. Second, mathematics classrooms must foster and support the desire and ability for life-long mathematical exploration and discovery. The third goal
challenges educators to put mathematical knowledge and mathematical success within all students' reach, stressing that all students should have the opportunity and the tools to explore and grow in mathematics. The fourth goal stresses that students need to communicate mathematically through problem-solving tasks that encourage interaction around mathematical concepts. Finally, the fifth goal encourages teachers to help students develop reasoning skills by expecting students to question situations, make hypotheses, test their conjectures, and support their conclusions mathematically (NCTM, 2000; 1989). All of these goals lead to a comprehensive learning experience for mathematics students, and could effectively eliminate misconceptions and problems previously experienced.

Teacher Level Goals

In 1991, NCTM published a companion to the *Curriculum Standards*, the *Professional Standards for Teaching Mathematics (Professional Standards)*. These standards provided guidelines for teacher practice in the implementation of mathematics education reform. The philosophy of the *Professional Standards* and the *Principles and Standards* advocates that students need to construct their own knowledge, and that teachers must be prepared to offer activities and environments that encourage this construction (NCTM 2000; 1991). Mathematical knowledge is not an accumulation of facts, and to teach mathematics effectively, reform-oriented classrooms must help students develop tools for solving unfamiliar problems (Resnick, 1987). This student-inquiry focus should be extended to college level classrooms.
The MAA and AMATYC Vision of Reform

Just as NCTM calls for change in grades K-12, college mathematics has also been targeted for reform. In an MAA policy document, Steen (1989) challenges university mathematics departments to examine entry-level courses. He sees these courses as an opportunity to encourage future mathematical study, provide students with a clearer understanding of the nature of mathematics, and ensure appropriate preparation for all students. Steen suggests that time should also be devoted to the remedial curriculum. Remedial courses should not simply be modeled after traditional courses that have proved less beneficial for these students.

Crossroads in Mathematics: Standards for Introductory College Mathematics

Before Calculus (Crossroads) was written in order to fill the gap between the NCTM Standards documents and the suggestions for calculus reform (Cohen, 1995). The AMATYC standards are currently in the process of being updated to reflect the growing research on appropriate teaching and learning, but contain suggestions that are still applicable for today’s classrooms and students. The Standards for Intellectual Development call for a focus on (a) problem solving, (b) realistic world connections, (c) reasoning skills, (d) connections, (e) communication, (f) appropriate technology use, and (e) rich experiences for all students in lower division and remedial college mathematics.

The teaching strategies recommended in Crossroads mirror those provided by Bonk and Kim (1998) for adult learners and NCTM (2000) for younger learners. Faculty teaching entry level mathematics courses must (a) incorporate technology in appropriate ways, (b) foster interaction using various forms of communication, (c) choose activities
and problems that allow students to use previous knowledge and experiences, (d) model and value multiple appropriate strategies, and (e) provide an active learning environment that requires independent thinking and sustained effort. Even though learners at different developmental stages have unique needs, many teaching strategies overlap the age bands. By offering the students in remedial mathematics courses an open and active learning environment, instructors will be able to meet the needs of the students. As students are asked to communicate their needs, expectations, and understanding through various modes, instructors will be able to adapt to their needs, while drawing upon the vast experiences and combined knowledge of all the students. Finally, by presenting mathematics as more than a set of rules to be memorized, students will be given the opportunity to develop their own reasoning skills and prepare for future mathematical development and exploration.

Technology Use Supporting Reform

The Principles and Standards clearly state that “electronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics” (NCTM, 2000, p. 24). NCTM emphasizes technology’s ability to provide multiple representations, to support investigation, and to allow students to “focus on decision making, reflection, reasoning, and problem solving” (p. 24). The emphasis placed on technology in the Principles and Standards makes clear NCTM’s dedication to helping teachers and students use technology to promote mathematical understanding.

Examination of technology use is occurring on college campuses as well. With a call for calculus reform, technology has been examined as a possible way to improve the
understanding of students on college campuses large and small (e.g. Heid, 1988; Herman, 1995; Judson, 1990; Porzio, 1995; Quesada, 1995). The use of technology in algebra courses is now seen as a possible pathway for reform in lower division and remedial courses (Hillel, Lee, Laborde, & Linchevski, 1992; Hollar & Norwood, 1999; O’Callaghan, 1998; Shore, 1999; Tynan & Asp, 1998).

Effect on Classrooms

Use of technology is supported by the final principle of the Principles and Standards, the Technology Principle. “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 24). Classroom goals can be strengthened when new opportunities, such as expanding technological tools, are applied to further learning. Just as change is inevitable as the availability of technology increases, technology will greatly change the classroom. Technology will change what is taught and how it is assessed. For example, extensive exercises in unsituated pencil-and-paper computation or symbol manipulation may no longer be appropriate when students use technology (Kutzler, 2000; Waits & Demana, 2000a). Affordable and accessible handheld technologies perform the repeated operations with numbers and symbols that once dominated courses in mathematics. As the curriculum moves away from skill practice, assessment must also look at more than skills; students must be expected to demonstrate their understanding of mathematical ideas (NCTM, 1989, 1991, 2000; Resnick, 1987; Thompson & Zeuli, 1999).
The vision for technology use supported in the *Principles and Standards* is one in which technology is not "a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics" (NCTM, 2000, p. 24). The mathematics of today has been redefined and expanded by technological advances. Computers have offered new areas of mathematical study, while providing tools that can be used to explore information and ideas in ways previously unavailable. The *Principles and Standards* emphasize the need for all students to experience the use of technology as an agent in mathematical discovery. However, in order to truly incorporate technology much research is needed.

**Effect on Students**

Technology is an important part of the reform movements at all levels of mathematics education and for all students. The Equity Principle, included as part of the *Principles and Standards* (NCTM, 2000), stresses the need to provide all students with opportunities to engage in meaningful mathematical learning. By using technology as a tool for inquiry, students who have trouble with even basic manipulations will have an opportunity to explore the patterns and algorithms that can help them improve their mathematical understanding. Research must begin to examine how technology can be used appropriately with specific populations, including remedial students.
Effect on Teachers

Simply purchasing technology for the classroom is not the answer at any level. Technology must be used to improve student understanding. To realize this goal, teachers must experience for themselves ways to effectively incorporate technology into the curriculum to increase student understanding (Salomon, 1998). A teacher's most difficult role in a reform-based classroom is helping students bridge the connection between the activity, manipulative, technology, or application and the mathematical concept being explored. Technology must be more than a hook to get students' attention; its use must challenge student thinking and help foster deeper examination of the material.

Mathematics classrooms need to reflect the changing face of the discipline. Change in teaching mathematics has not kept pace with the fact that mathematics itself has been greatly affected by advances in technology (Silver, Kilpatrick, & Schlesinger, 1990). With the addition of a technology standard in the Principles and Standards, NCTM (2000) calls for more effective use of technology as a tool to help students understand mathematics and prepare for their futures in a technological world. Discussion and questioning will play an important role in this integration. Teachers must expect students to become more involved in the learning process. Students must be expected to do more than give the solution to a problem or answer only recitation type questions; instead students will need to justify their solutions and question the work of peers (Wood, 1999). As technology becomes a more essential element of the classroom culture, teachers need to find ways to successfully implement its use into the learning process.
Using Technology in Developmental Mathematics Classrooms

Can technology help students who have been introduced to symbolic manipulations expected for college level mathematics, and who are still struggling to apply these skills? The purpose of the present study was to examine how the introduction of technology for exploring algebraic manipulation in a remedial college mathematics course might change student outcomes in terms of achievement on exams and attitudes towards mathematics. Calls for technology use increase and, concomitantly, research indicates that students benefit from such exposure. Thus it is important to find effective ways to introduce its use into established courses at all levels. In order to continue the introduction of technology into additional mathematics classrooms, we need further examination of different technologies to effectively change the curriculum, instruction, and assessment. In time, technology can become an integral part of learning and understanding mathematics for all students.

Breaking the Cycle of Failure

Finding strategies to help students in developmental mathematics courses at the college level is essential to break the cycle of failure in mathematics. Many of these remedial classes remain traditionally organized, mirroring the type of classroom instruction in mathematics that has already proven to be unsuccessful for the students (Steen, 1989). Effective use of technology requires that the culture and expectations in a classroom change (Dunham, 2000; Waits & Demana, 2000b). Students who have failed in traditional classrooms may find the new technology-based approach a way to break the cycle of failure and lead to understanding in the discipline.
Inductive Approach

Technology works as a scaffold on which students can explore the processes of algebra even though they are still learning basic skills (Kutzler, 2000). In the present study, students used the Computer Algebra System (CAS) included with the Texas Instruments’ TI-92 to discover the patterns and rules that constituted the focus of the remedial course, instead of first being told the abstract rules, followed by a series of examples that used the rules. As the students became more comfortable with the procedures and meaning, they were encouraged to move away from calculator usage (Kutzler, 2000; Waits & Demana, 2000a). Heid (1988) used this reversal of concept and skill with calculus students in order to focus students’ attention on the broader ideas of calculus, and then allowed the students to strengthen their skills in a situation where they could see the application of the skills. Adult learning theory suggests that an inductive format, in which application precedes skill development, be used to give adult students a situation in which applying the skill is understood to be necessary (Birkenholz, 1999).

Challenging Misconceptions

This format also recognizes that many adult students have had at least some exposure to algebraic concepts in the past, but developed misconceptions or did not fully understand the concepts presented. By allowing students to examine problems they feel they already understand and highlighting the process, many opportunities to examine misconceptions will arise. For example, when solving the equation \(5 - 2x = 13\), a common misconception held by many students is that they should add five to both sides of the equation, rewriting the equation as \(2x = 18\). The CAS allows the students to try this
operation and then check the result. Once the calculator gives them $10 - 2x = 18$, there is room for discussion and a modification of their understanding. When technology is used in this way, students have the opportunity to build upon what they know and identify areas of misunderstanding. Finally, the use of a CAS in a remedial college course is appropriate because it can provide the immediate feedback that is often absent in an environment where the teacher or peers provide all the feedback (Kutzler, 2000; Pierce, 1999; Tyran & Asps, 1998).

**Problem Statement and Research Questions**

Change needs to occur in the structure of remedial courses at the college level (Hillel et al., 1992; Steen, 1989). Remedial courses should encourage exploration of mathematics while maintaining accessibility to all. Technology can play a major role in that process. Students who have experienced years of failure in mathematics need an opportunity to be successful. The curriculum must be altered to emphasize the role of algebraic manipulation in realistic and meaningful contexts before focusing on abstract rules and processes. Finally, course design must respect and recognize that many students in remedial courses have developed misconceptions about mathematics that must be dismantled before future mathematical understanding is realized.

The purpose of this study was to examine the use of the CAS and graphing utilities included with the TI-92 as a tool for mathematical exploration in a college remedial mathematics course. Students were encouraged to develop a deeper understanding of mathematics, to explore many algebraic manipulations, and to see why
they were learning a manipulation skill. These remedial students were given the opportunity to break the cycle of mathematical failure in which they found themselves. By providing them with occasions to develop abstract rules for themselves, by challenging mathematical misconceptions, and by providing multiple entry points into the learning environment, these students would gain knowledge and confidence in mathematical competencies. In connection with introducing technology into the remedial classroom, this study addressed the following questions:

1. What is the effect of the use of a CAS and graphing utility on remedial mathematics students' achievement?

2. What is the effect of the use of a CAS and graphing utility on remedial college mathematics students' attitudes toward, beliefs about, and confidence in mathematics?

Conceptual Framework

Ideas from constructivist theory as well as ideas from adult learning theory formed the foundation for the research questions in this study. The teaching strategies incorporated into both control and treatment classrooms were also in line with key ideas from these theories. Students in the remedial college classroom have diverse experiences with mathematics education. Mathematics backgrounds range from students who have completed a calculus course to students who may have had only minimal exposure to algebra. Students include freshmen who have been assigned to this course due to placement scores on the college entrance examination, as well as transfer students who have not taken courses sufficient to place them into college level mathematics. Upperclassmen who have avoided mathematics find themselves in these remedial courses.
in hopes of gaining the confidence and skills to succeed in the mathematics courses required for graduation. Many nontraditional students returning to college after a long break from school begin their study of mathematics in the remedial classroom. Finally, some students are repeating the course after several previous attempts so that they can move on to the mathematics course required for their major. The developmental mathematics sections used in this study were taught at night, and were therefore filled with many students who were either nontraditional by age and background, or nontraditional due to their work status even though they were of traditional college age.

_Constructivist Theory_

Constructivist theory takes the position that all knowledge is the result of individual construction based on interaction with the world in which the individual acts (Piaget, 1970; 1980). In order to explain and expand Piaget’s statement, Wood (1993) identified five principles that are primary to this learning perspective. These include:

(a) The child actively constructs mathematical knowledge; (b) children create new mathematical knowledge by reflecting on their physical and mental actions; (c) children’s constructions undergo a process of continual revision; (d) children create their own individual interpretations of mathematics; and (e) opportunities for learning occur during social interaction as children resolve conflicting points of view (p. 7).

As Wood describes, children develop a life-long relationship with mathematics through these pivotal activities. Thus, it is important that learners be given opportunities to interact with mathematical situations in order to begin to construct an understanding of the discipline. Much of the writing of constructivists such as von Glasersfeld (1996) and Piaget has concentrated on the individual construction of knowledge and failed to highlight the effects of the social situation in which learning takes place. Cobb, Yackel,
and Wood (1993) highlight their own change in perspective as they examined the learning of children in a second grade classroom. It became evident to the researchers as they examined student knowledge construction that classroom interactions and social norms played a primary role in the content and processes of knowledge construction. Theorists such as von Glasersfeld and Piaget place primary emphasis on activity as the road to individual knowledge construction, yet they concede that if the student’s language is mature enough, language provides cues that allow others to form a model of the student’s thinking (Sierpńska, 1998). However, language is not enough to account for the influence that social interaction has on the learning process.

Constructivist learning theory also includes the interactionist and emergent perspectives. For the interactionist, language and communication provide for shared meaning within the culture, and context greatly affects the meaning of what is said (Sierpńska, 1998). Finally, the emergent perspective views communication as working with activity to help students construct understanding. Through communication the teacher can monitor this understanding and help students relate their discoveries to those of the accepted mathematical culture (Cobb, Jaworski, & Presmeg, 1996). These theories place greater importance on the role of language, but fail to emphasize the effects of existing social norms on the construction of knowledge.

*Social Constructivist Theory*

Like Cobb, Yackel, and Wood (1993), I find that social constructivist theory offers the best explanation of the role of both individual construction of knowledge and the effects of social norms on the construction. The social constructivist perspective,
typified by much of the writing of Vygotsky, emphasizes the use of language and social interaction in the learning process. From this perspective it is vital that the study of technology in the mathematics classroom take into account the use of discourse. Vygotsky speaks of the zone of proximal development (ZPD) as the stage in which a more knowledgeable teacher or peer can help the novice solve problems that would be out of his or her grasp if attempted without assistance (Oers, 1996). Technology can serve as the expert, the bridge between what students can understand alone and what they can understand with help from technology (Pierce, 1999; Waits & Demana, 2000a). Communication will be essential for the teacher to monitor students’ growth and understanding while making decisions on when to begin to help students break away from a dependence on technology. The teachers must help students gain insight into the manipulations the technology can perform without prompting inappropriate dependence on the technology itself.

Technology as a scaffold. As learners begin to explore a mathematical concept at the bottom edge of their ZPDs, much assistance is needed to help guide them through the concept. For example, as remedial mathematics students begin to examine their misconceptions about solving simple equations, technology can be relied upon to perform the basic manipulations of terms and thus eliminate any misconceptions that can be formed or reinforced by performing an operation incorrectly. If the goal of a course is to develop students’ paper-and-pencil manipulation skills, reliance on technology can be
gradually reduced. Concurrently, students will develop their own strategies for choosing operations, and can begin to concentrate on how to carry out those operations without the aid of the CAS.

The idea of using technology as a scaffold is found in the work of Kutzler (2000). In a discussion on solving equations, as in the previous example, Kutzler distinguishes between the ability to perform an algebraic manipulation and the decision of which algebraic skill to use. Kutzler contends that the latter should be the focus of the mathematics classroom, and that tools such as a CAS can provide a means by which students can examine the process without interruption. In light of Vygotsky’s work with the ZPD, technology can provide expertise as students begin to explore new concepts and make connections between various mathematical representations.

*Classroom Discourse.* Social constructivist theory supports the role of interactions between students and experts as knowledge is constructed, but social constructivist theory also highlights the importance of a classroom culture that allows students to explore mathematics independently of the instructor (Cobb, Yackel, & Wood, 1993). Simon (1993) highlights some areas of concern when the use of class discussion and presentation of solutions is incorporated into a classroom of older students. First, the research base at more advanced levels is not as complete as the research that backed up the work of Cobb, Yackel, and Wood in the second grade classroom. Second, the curriculum at the second grade level should be accessible to all, but unfortunately it seems that more advanced topics such as fractions, ratio, and function have eluded a large segment of society. This can be a direct result of Simon’s third point. That is, older
students have significant gaps in their mathematical understanding, which in turn makes further mathematical understanding more difficult. Fourth, Simon addresses the fact that students in higher grades have more socialization in traditional mathematics classrooms, where the social norm dictated only one correct way to pursue an answer and did not require an explanation. In addition, incorrect answers in the traditional classroom were viewed as failure rather than an opportunity to learn more about the mathematics. Finally, due to the increased complexity of mathematics in the middle and high school years, teachers may be less prepared to address multiple ways of approaching problems. Yet, the more complex ideas studied in the courses at these grades and beyond offer many opportunities to involve students in realistic mathematical exploration.

In the present study, social constructivist theory helped guide the ways the researcher and instructors approached the course. Discussion and questioning was given emphasis over other instructional strategies, and students' ideas were encouraged and validated. In light of the reluctance and inexperience of some learners to participate in full class discussions, small groups and individual writing assignments were used on a regular basis to provide additional opportunities for discourse in the classroom. When technology was to be used for exploration, the instructor could use student discovery to introduce or formalize mathematical concepts. Technology also acted as the expert for many students as they were exploring a concept, and challenged students to recognize mistakes in a private setting, which increased the safety of the learning environment. Students were encouraged to develop mathematical processes and to understand mathematical language so that they could solve problems. Development of the
mathematical power necessary to succeed in future work was put in light of students' own exploration and discovery when possible. This focus on both the individual and the social was the basis for the overall design of both the control and treatment courses. However, the use of technology ultimately affected the classroom cultures that were created and either strengthened or detracted from this focus.

**Adult Learning**

Technology appears to be a means through which many adult learning needs can be met, and thus it offers promise as a means for facilitating learning in this student population. Exploration involving technology respects and reinforces the knowledge that adult learners bring to the classroom. Adult learners have different characteristics and thus different needs from students in grades K-12 (Brookfield, 1991; Deshler & Hagan, 1989; Knowles, 1989). A technology-enhanced environment provides the opportunity for adult learners to conceptualize the subject and construct their own understanding, which has proven to be especially important when working with adults (Brookfield, 1991). Simply telling adult learners how to manipulate the algebraic symbols will not foster full participation in the learning environment; instead, these learners need to see how this manipulation relates to them and facilitates success outside the classroom.

There are many definitions of adult education, and each highlights a different avenue through which adults increase their understanding and knowledge about the world in which they live. Formal adult education has many times been associated with remediation for individuals who were not adequately served by the K-12 school system (Riley, 1993; Wagner, 1993). Adult education has also been linked to the need to help
adults keep current in their chosen field of employment or to increase their abilities in order to improve their positions within their fields (Cohen, 1995; NCES, 1994). Studies by the NCES (1994) and the National Household Education Survey (Korb, Chandlers, & Wert, 1991) show that a major factor in whether adults choose to further their education is the amount of education they already have. Birkenholz (1999) attributed this to the fact that, as adults increase their knowledge of the world, they begin to see that there is much they do not know and will intrinsically want to learn more.

_Epistemology_

Adults have not only developed a concept of their own personal mathematical ability that is difficult to change, it is also difficult for them to change long held beliefs about how to carry out a mathematical procedure. Adults are many times willing to spend long periods of time studying content presented in the classroom, but at the same time will have great difficulty letting go of long-held misconceptions about that content (Schommer, 1998). Unless their misconceptions are challenged, incentive to relearn is provided, and tools for this level of mathematical thinking are developed, adults will be stifled in their ability to break a cycle of failure in the mathematics classroom. The use of technology to expose and re-examine these procedures can provide all students with the fresh perspective on the content that is necessary to foster mathematical growth.

There is a great debate in the field of adult education between _development_ and _learning_. Granott (1998) offers an alternative, _developing learning_ that links the two as one, providing suggestions for adult learning situations. When developing learning, adults show progress toward more advanced knowledge levels, reorganize existing
knowledge structures to incorporate new experiences, and continue to learn outside the structured learning situation. Granott stresses the need to encourage developing learning among adults because it "prevent[s] learning from being shallow, superficial, reversible, and rote memorizing" (p. 18). These undesirable outcomes characterize non-developing learning. In order to enable developing learning, (a) adequate time should be allowed to construct an intuitive understanding, (b) learners should see merit in the effort to increase their knowledge, (c) individuals need choice in approach, strategy, and activity, and (d) the situation should challenge learners while supporting their needs.

The beliefs individuals hold about the nature of knowledge, their epistemology, greatly affects their approach to learning (Schommer, 1998). Learners' beliefs about whether or not knowledge potential is permanently set or can increase will affect their willingness and tendency to continue their formal education beyond mandatory levels. Also, their stance on whether knowledge is acquired quickly or is a slow process will help determine their study habits. Research has shown that some of these beliefs are tied to age and others to education level (Schommer, 1992; Schommer & Hunter, 1995; Schommer & Walker, 1997). This research also indicates that as people age, they believe that knowledge capacity is not set, and that through persistent study they can increase their current level of understanding. Adults are less likely to recognize the variability of knowledge without continued formal education. Continued formal education, however, is often dependent on successful completion of mathematics courses that appear early in a higher education program. This fact makes Schommer's research pertinent to mathematics education.
Understanding the way adults learn and how their beliefs play a role in their approach to learning is important as one plans the instructional environment. Theory on adult learning can provide insights for educators. Adult learning theory has many similar attributes to theory for younger students, but there are differences that must be considered when a classroom includes adults.

Theory

What factors enhance learning for college students? As more and more adults choose to continue education beyond mandatory levels, educators must be sensitive to their needs. Merriam and Caffarella (1991) describe four learning theory orientations that they feel are prevalent in adult educational settings.

1. The behaviorist orientation places an emphasis on learning as a relationship between a stimulus and a response. Educators who teach from this orientation believe that all behaviors are learned and can therefore be affected by educational techniques.

2. The cognitive orientation is a second belief system that many adult educators aspire to as they design programs. This position can be compared to the work of Piaget and other constructivists. Learning is believed to be a function of individual mental actions. These educators are looking at the acquisition, transformation, and application of new ideas into new settings.

3. The humanist orientation posits that individuals have an intrinsic desire to move beyond their current state once their needs at that level have been achieved. Educators who align their courses with this philosophy attempt to determine the current level of need of adult students, are and try to find means to help them fulfill their needs so that they can continue along a path to more knowledge and satisfaction.

4. The social learning theory orientation holds that adults learn through observation and reflection. Adults will modify their own understanding based on the interactions they have with others and the environment.
Regardless of the educational philosophy of the adult educator, the learning environment must respect the experiences and needs of adult learners, giving them as much control over the learning situation as possible (Brookfield, 1993; Cranton, 1996; Jarvis, 1995; Tennant & Pogson, 1995). The use of socioculturally based learning theory in the classroom increases the opportunity for adults to interact, use previous experiences, and construct knowledge together. Even though this provides great opportunities for adult learners, it is also in opposition to the individualized, teacher-centered learning that many have experienced in previous learning environments (Bonk & Kim, 1998). It is not enough to design a course that is in line with the tenets of sociocultural theory; rather, courses must meet the needs of the increasing population of adult learners. Educators must recognize the specific needs of adults in the classroom and help them adapt to the new learning environment described by sociocultural theory.

*Characteristics of adult learners.* In the past, researchers believed that adults reached their peak learning capability at age 25 and steadily lost their capacity to learn as they grew older (Miles & Miles, 1932). Today, however, researchers recognize the effects of deterioration of some sensory organs in adults’ learning processes, but believe that adults maintain a capacity for learning that is higher than many adolescents (Birkenholz, 1999). Clearly, adults learn differently and have different needs than adolescents. Adults come to the classroom with different responsibilities, usually participate on a voluntary basis, and have a higher level of self-motivation than their adolescent counterparts (Birkenholz, 1999). In addition, adult educators must appreciate the fact that age, educational activities, and experiences tend to solidify an individual’s
concept of his or her abilities and of the environment in which he or she lives (Birkenholz, 1999). These characteristics of adult learners do not suggest that the strategies available to help younger students learn are not applicable to this level, but they indicate that the adult educator should consider the characteristics of adults mentioned above in designing a program. For example, the educator will not need to spend as much time on motivational techniques to convince the adult it is important to learn, but will need to help adults channel their own desires to learn into productive activities. In addition, when working with adults, the educator will need to be aware of the fixed nature of adults’ perception of their own ability and try to encourage them to challenge these abilities.

*Differentiating adult learning from that of younger students.* Adult educators must recognize the differences found in adult learning patterns (Brookfield, 1991; Deshler & Hagan, 1989; Knowles, 1980). Pascual-Leone and Irwin (1998) concluded that adults are more self-motivated and reflective than younger learners are. In addition, they pointed out that adults are better able to recognize various perspectives in a single situation and have accumulated more life experiences that play a role in what they wish to learn and how they approach this learning. As with younger learners, Knowles (1980) suggested that adult educators should plan programs that allow adults to be more involved personally by using real life situations for the basis of learning. Brookfield (1991) agreed that adult programs should be more practice oriented, but argued that they must also incorporate opportunities for learners to reflect, conceptualize the subject, and construct
individual understandings. Again this suggestion is supported by literature on the
importance of meaningful discourse to encourage learning in younger children (Cobb,

Birkenholz (1999) offers eight principles of adult learning in his book. Some of
these are more applicable to the remedial mathematics classroom containing adult
learners than others. Three of these principles are (a) adults learn by doing, (b) learning
should focus on realistic problems, and (c) experience affects the adult learner. Adults
learn best, just as younger children do, when they take an active role in the process.
Educators will need to encourage this participation among adults, however, because they
are more reluctant to get involved for fear of failure, because they believe that they lack
the skills, and because they have low self-esteem (Birkenholz, 1999). Adults demand to
see the benefit of the knowledge they are to acquire. One approach used by some adult
educators, the inductive approach, is to start with examples of the principle to be studied
and allow adults to generalize the principle from the examples provided (Birkenholz,
1999). Technology has been identified as an aid in this process (Dunham, 2000; Kutzler,
2000). Not only will adults come into the classroom with more practical experience to
apply to real-life problems, but they will also come to the classroom with many
misconceptions of the content. Therefore, when necessary, the adult educator must be
able to help adults unlearn what they have held as fact in order to understand the complex
situation at hand.
The complexities of examining adult learners challenged the researcher to design a course in which the needs of adults were respected. Activities offered both control and treatment students multiple levels of entry, thus providing autonomy in the learning environment while respecting the knowledge brought to the activity. With the focus of full class discussions on students’ discovery, every student was responsible for engagement with the mathematics. Finally, as much as possible, students in both sections were provided with applications outside the classroom for concepts approached in the course. When this was not possible, concepts were introduced as stepping-stones toward more applicable procedures. The use of technology offered the treatment group additional opportunities. The technology provided immediate feedback on many procedures that forced participants to acknowledge disconnections between what they expected and what was produced. Further, technology enhanced the activities by promoting more student-initiated discovery and discussions centered on those discoveries.

Designing the Technology-Rich Remedial Mathematics Course

As explained earlier, students in a remedial mathematics course come from diverse populations, and the classroom culture must work to meet the needs of all. The ideas from social constructivist theory are obviously applicable to the youngest students in the classroom, and adult learning theory can be applied to nontraditional students. But what about the others—is there compatibility between the theories for all types of students?
**Instructional Strategies**

Social constructivist theory emphasizes learning as a social endeavor. Society determines what is to be both learned and accepted as legitimate knowledge. Learning itself occurs as individuals interact around a topic. In addition to the four divisions explained by Merriam and Caffarella (1991), there is considerable promise in applying the tenets of sociocultural learning theory to adult education. Bonk and Kim (1998) offer adult educators ten teaching techniques in line with the sociocultural perspective on learning that can be applied to adult learning situations.

1. Modeling can show expectations and highlight processes used to solve problems.

2. Teachers should observe and supervise students in order to help move them toward needed experiences for understanding.

3. Adult educators are encouraged to scaffold students’ learning by providing assistance with tasks with which learners are unfamiliar, and then gradually reduce involvement until the students have internalized the process and understanding themselves.

4. Educators are also encouraged to use questioning to gather information on the understanding and needs of the learners.

5. The instructor must reflect his or her own thinking orally.

6. Students must also be expected to verbalize their own understanding.

7. Adult educators wanting to establish learning environments that support the sociocultural view of learning should also provide students with ample exploration and opportunities for reflection.

8. Educators must be aware of students’ levels of understanding in order to structure appropriate activities to encourage understanding and internalization of new content.
9. The adult educator should provide students with feedback and encouragement.

10. Use of direct instruction should be included to provide clarity or additional information. (p. 72)

These ten strategies are similar to those that are associated with classrooms for younger children as well. For example, as instructors model their own thinking and encourage their students to do the same, they are supporting the idea that knowledge is a social construction. Accepted knowledge is based on the norms of the society and new ideas must be tested against what is already known. In addition, the interplay between students and teachers as they learn is very similar to the ZPD. Bonk and Kim (1998) encourage teachers to scaffold students' learning and gradually remove assistance as students are able to understand more individually. Adult educators are also encouraged to honor what the students already know and then help them move forward from there, again an important part of sociocultural theory for younger children.

Bonk and Kim (1998) suggest teaching strategies for adults that mirror teaching strategies successful in working with younger students. But could Birkenholz’s (1999) suggestions for adult learners help the traditional student as well? Regardless of age, many students enrolled in remedial mathematics courses think that if knowledge does not come quickly it is unattainable (Schommer, 1992). Therefore all learners need to have their misconceptions challenged throughout the educational experience.
Discussion of Theoretical Model and Teaching Strategies

Figure 1.2 illustrates the connection between the main tenets of sociocultural theory and teaching strategies for adult learners. The three tenets of sociocultural theory included are (a) ZPD, (b) social norms, and (c) discourse. The ZPD is the stage in which a more knowledgeable teacher or peer can help novices solve problems that would be beyond their grasp if attempted without assistance (Oers, 1996). The idea of social norms to the importance of establishing a classroom environment that reflects the true nature of mathematics. Discourse, an essential element in a classroom, includes various forms of communication between student and teacher, and between student and student.

Many strategies from the work of Bonk and Kim (1998), as well as suggestions from Birkenholz (1999), can be paired with these three basic tenets of sociocultural theory. Through scaffolding the students’ learning and giving timely feedback, an adult educator is able to assist students as they move within their ZPD. Technology is one tool that can be used to assist this process; however, small group and instructor-student interaction can also assist in supporting students as they explore new ideas. Instructors can also establish a classroom culture that encourages students to explore mathematics and reflect on what they see, so that they can better understand the concepts. This culture can be established through instructor modeling and classroom expectations. Finally, discourse offers a means through which the instructor can better understand the thinking of students. Interaction can also offer opportunities to explore and validate alternative solution methods that can honor an adult’s past experience.
Sociocultural theory offers a basis for establishing a classroom that benefits both traditional and adult students. The characteristics of, and suggestions for, strategies to reach adult students become a natural part of a classroom that recognizes the importance of discourse, the effects of social norms, and a student’s ZPD. The introduction of technology is the final piece in this relationship that can be used to support and foster suggestions provided. Even though many of the elements of the interplay between sociocultural and adult learning theory can be incorporated into a classroom without the use of technology, it is the researcher’s hope that technology will strengthen the connection and therefore help these remedial students break the cycle of mathematical failure. The present study examined a CAS used within an environment honoring the tenets of constructivist learning theory and what is known of adult learning, and tracked its effect on student achievement and attitudes toward mathematics.
Figure 1.1. Use of Social Constructivist Theory and Adult Learning Theory in Course Design.
CHAPTER 2

REVIEW OF RELATED LITERATURE

Research on Technology in the Mathematics Classroom

The appropriate use of technology will cause much change in mathematics education and may result in fear among many teachers, parents, administrators, and students. The literature is full of calls for the use of technology, yet many teachers and schools do not incorporate it into the curriculum. The way that many teachers view the nature of mathematics makes them hesitant to incorporate technology into their classroom (Fleener, 1995; Schmidt & Callaghan, 1992; Simmit, 1997). Simonsen and Dick (1992) found that, with teacher education and experience, many of the negative views on technology usage could be altered. Nevertheless, negative views, such as the fear that students will lose computational skills or that technology will be used when it is not appropriate, prevent full application of technology into the mathematics curriculum (Mayes, 1995; Simonsen & Dick, 1992; Zand & Crowe, 1997).

Change Facilitated by Technology Use

Education of teachers, administrators, and parents will be essential in realizing the power of technology in the mathematics curriculum (Waits & Demana, 2000a). Pollack (1984) has highlighted three changes that are inevitable when technology is introduced
into the mathematics curriculum. These include a de-emphasis of some topics that traditionally been part of the curriculum, for example, many paper-and-pencil arithmetic and symbolic manipulation techniques. As these topics and skills are de-emphasized, others will gain emphasis. That is, more time can be devoted to topics such as discrete mathematics, data analysis, parametric representations, and non-linear mathematics. Finally, there will also be opportunities to introduce topics that have not been included in traditional, non-technology-rich curricula, such as fractal geometry (see also NCTM, 2000).

Kutzler (2000) examined studies in Austria's general high schools to conclude that the use of technology leads to more efficient mathematics curricula, more opportunities for students' individual activity, more creativity, and an increased emphasis on the role of the teacher. Teachers must help students by choosing appropriate activities for knowledge construction, and guide students as they learn to use technology in appropriate ways.

The use of technology can help students see the true nature and value of mathematics (NCTM, 2000; Waits & Demana, 2000a). In addition, research indicates that the use of technology does not negatively affect basic skills and does strengthen problem solving ability and attitudes (Hembree & Dessart, 1992; Tynan & Asps, 1998). The fact that students and even teachers and parents fail to see the true nature of mathematics cripples many and prevents the level of understanding they need to be successful (van Oers, 1996). It seems that the combination of these two facts would encourage the use of technology, but more education of those involved in school decision
making is necessary. This education must not only address the years of modeling mathematics as a set of manipulative skills that adults have endured, but must also present the necessary research to show the power of technology.

*Means of Incorporating Technology*

Technology can add to the mathematical experience of students. Waits and Demana (2000a) offer three ways to incorporate technology into the curriculum. First, students can solve problems using paper-and-pencil methods and then rely on technology to check their solutions. Conversely, problems can be solved using technology and the solution can be confirmed using paper-and-pencil techniques. Finally, students can be encouraged to choose whatever methods and tools are best for the situation at hand. All three ways can enhance learning for students.

I am interested in ways that a computer algebra system (CAS) can be incorporated into the classroom. With the use of a CAS in a beginning algebra course, questions arise concerning the extent to which students must demonstrate the ability to perform algebraic manipulations by hand (Heid, 1995; Herget, Heugl, Kutzler, & Lehman, 2000). In order to address these questions, it may be helpful to divide algebraic processes into two areas—deciding what operation to use and applying the operation chosen. Kutzler (2000) views the choosing of the operation as requiring a higher level of thinking, but sees this process repeatedly interrupted by the need to apply the operation. A CAS can be used to highlight appropriate choices by performing the operation chosen and thus eliminating the interruption of algebraic manipulations. Students can then immediately see the results of their choice, decide on its appropriateness, and then choose the next step. The use of
technology in this manner can serve as a scaffolding tool to help low-ability students advance farther than such students have in the past (Kutzler, 2000; Owens, 1995; Shoaf-Grubbs, 1994; Zand & Crowe, 1997).

The use of Computer Algebra Systems in the Mathematics Classroom

"The power of technology is needed to capture the emerging mathematics" (Stone, 1995, p. 6). The Principles and Standards (NCTM, 2000) echo this call for the use of technology in mathematics classrooms. Heid, Sheets, and Matras (1990) highlight the use of symbolic manipulators as a way to address many of the issues raised by the Curriculum and Evaluation Standards (NCTM, 1989). CAS were not originally designed for the classroom; therefore, ever since they have appeared in that domain there has been discussion of how they should be used and how that use will change the mathematics that is emphasized (Heid, 1995; Herget et al, 2000; Hillel et al, 1992).

With the availability of CAS for student use, the definition of what it means to understand an algebraic procedure changes. According to Tynan and Asp (1998), this new definition includes: (a) knowing what the procedure is used for; (b) recognizing needed input; (c) choosing conditions under which the procedure should be used; (d) analyzing the meaning of the answer given by the technology; and (e) detecting errors in answers given by technology. When using a CAS, one must decide what operations are the responsibility of the technology and assure that the student is performing the cognitive steps of the problem. Just as in the use of calculators for arithmetic, using technology means little if students are not thinking about the meaning of the problems.
Reasons to Incorporate CAS Use

Hillel et al. (1992) recognize three aspects of a CAS that make them an appropriate tool for the classroom. These are:

1) CAS make it possible to change the type and complexity of problems which students typically are asked to solve.

2) CAS allow for a shift in emphasis in instruction, away from learning techniques, toward trying to build better conceptual understanding.

3) With CAS, students have the opportunity to try many examples and to receive immediate feedback. (p. 124)

Hillel et al. also provide three reasons for using CAS in remedial courses:

1) A CAS allows students to develop mathematical ideas that are not rigidly structured in terms of a hierarchy of skills, and particularly not in terms of prerequisite algebraic manipulation skills.

2) The students’ benefit from a mathematical learning experience that is substantially different from their previous experiences.

3) A single system can be used in a unified way to deal with mathematics covered in all the courses in a program. Those students who take further mathematics courses would be able to make repeated use of such a system. (pp. 126-127)

Small, Hosack, and Lane (1986) add five possible changes that can result in the classroom due to CAS use.

1) Students’ attitudes will change.

2) Students will become more actively involved with mathematics.

3) Students will spend more time organizing their thoughts.

4) Students will be exposed to more realistic examples.

5) There will be broad advantages to the mathematics curriculum, [including the fact that] numerical approximations [can] be used in conjunction with symbolic methods. (p. 424)
Factors to consider. Through their work with grade nine girls, Tynan and Asp (1998) identified nine factors to consider when using CAS in early courses on algebra. These are: (a) amount of access; (b) teacher beliefs and style; (c) student skills and learning styles; (d) student motivation to succeed; (e) time pressures; (f) level of communication in classroom; (g) support for teachers; (h) level of interaction in task design; and (i) balance of open-ended and routine tasks. The use of a CAS in this setting did not seem to reduce the students’ by-hand algebra skills and seemed to positively influence their ability to choose appropriate manipulations. The use of CAS also impacted the students’ methods for solving problems.

Changes in instructional emphasis. Advances in technology change what can be included in the mathematics curriculum and the way in which mathematics is taught. Teachers have the opportunity to design lessons to highlight conceptual understanding as opposed to mastering paper-and-pencil manipulations (Kutzler, 2000; Waits & Demana, 2000a). This can allow lessons to focus on the desired skills. By removing the need to interrupt the learning process with repetitive manipulations, students are able to go far beyond what was once possible (Kutzler, 2000). For example, as a student begins to learn what it means to find a solution to a linear equation, the teacher can use a CAS to highlight the process, while removing the need to perform operations of combining like terms. The student can examine the equation $2x + 3 = -15$ and decide to rewrite the equation by subtracting 3 from both sides. The students can then perform this operation on the calculator without having to stop and think about what $-15 - 3$ would be (see Figure 2.1).
Figure 2.1. TI-92 screenshot example of removing difficulty of arithmetic computations.

The use of a CAS allows teachers to highlight specific skills and provides a means for immediate feedback to students. Using CAS, students can experiment to find patterns and discover misconceptions for themselves (Kutzler, 2000). As in the previous example, Kutzler suggests using CAS to support students as they solve equations by eliminating the need to perform algebraic manipulations and giving immediate and accurate feedback based on the choices made by the students. This allows students to learn the process of solving equations even if they still have difficulty with the algebraic manipulation skills needed. At the same time, it emphasizes areas of process misunderstanding that are many times reinforced when paper-and-pencil exercises are used without immediate and accurate feedback.

Improvements in the classroom. Stone (1995) looked at how the use of Derive affected curriculum, teaching practice, and assessment in 3 secondary and 16 undergraduate mathematics classrooms. Through observations, interviews, and document analysis, he found three specific ways in which technology can improve mathematics classrooms. First, it can provide opportunities for empirical investigation, by allowing
students to concentrate on the data as opposed to time-consuming manipulations. Second, visual representations can be used to form generalizations that can then lead to conjectures for exploration. And third, tools, such as symbolic manipulators, can be used to verify abstract conjectures. When used properly, technology has promise for mathematics classrooms. Stone highlighted additional ways in which the classroom culture was different when technology, specifically the computer-based symbolic manipulator Derive, was used. These included less student dependence on the teacher, increased student-student conversation, and more opportunities for one-on-one attention from the teacher. Teachers were also affected by the use of Derive. They enjoyed increased discussion with colleagues that led to more frequent and diverse use of technology, a positive outcome for both students and instructors.

*Examples of CAS in the Classroom*

*The Algebra Classroom.* Hunter et al. (1995, as cited in Tynan & Asp, 1998) examined the use of CAS with 14-15-year-olds and found that it was a benefit for those students who had a basic understanding of the mathematical concepts studied. Tynan and Asp (1998) examined the impact of CAS use with Grade 9 girls. Two groups of students were compared, one using the TI-92 (includes a CAS and graphing utility) and one using the TI-82 (traditional graphing calculator). Both groups were completing a unit on linear equations, and through a pre- and posttest, the effects of CAS on paper-and-pencil symbol manipulations, expression and equation formulations, and problem solving were examined. Students also kept journals that allowed the researcher to explore their impressions of the effect of CAS on learning.
In this experiment, the CAS was used as a tool to check student work and give immediate feedback on the choice of manipulation steps. When completing the posttest without the aid of the CAS, students in the treatment group still used algebraic manipulations to approach problems, while more reliance on arithmetic and trial and error was observed in the control group. The majority of errors in solving equations, poor order-of-operation skills, and difficulty with negative number operations, were similar between both groups. These errors disappeared in the treatment group when they were able to use the CAS to complete a problem. During the exam in which students had access to the CAS, they were asked to describe the steps used to solve a problem. Those choosing to use the CAS were more successful at this task than those who did not use the CAS. This finding led Tyuan and Asp (1998) to believe that the instruction with the use of CAS led to an increased understanding of how algebraic manipulations affect equation solving and the ability to formalize the process used.

Examining the concept of function. O'Callaghan (1998) designed an instrument to examine the understanding of function among students who were able to use a computer graphing tool in a lab setting as they explored functions. Hollar and Norwood (1999) used this same instrument in a follow-up study to compare the understanding of two groups, one receiving traditional instruction in which paper-and-pencil skills were stressed and another group receiving instruction based on explorations with a graphing calculator. In addition to pre- and posttest scores on the paper-and-pencil instrument used by O'Callaghan, attitude measures were also taken for students in both groups.
The content in both groups was the same, although the graphing calculator did allow for exploration of real world situations that were too time consuming for the control group. Hollar and Norwood (1999) were interested in (a) students' modeling ability, (b) interpretation of functions, (c) use of multiple representations, and (d) the move from operational understanding to that of structural understanding. In both Hollar and Norwood's study and O'Callaghan's (1998) study, the treatment group did significantly better in the first three areas. Difference in the final category, being the most difficult, was not found to be significant in O'Callaghan's research but was in Hollar and Norwood's work with graphing calculators. The researchers attributed this to the fact that students were able to use the graphing calculators whenever they chose to, but the computer program used in O'Callaghan's study was available only in a lab. The attitude scales did not show significant differences between the two groups, nor were significant differences seen on the standardized skill-based departmental test given as a final. These results indicate that the use of graphing calculators did not get in the way of skill building, but did add to the structural understanding of algebra. Further research that corroborates the idea that basic paper-and-pencil skills are not hampered by the use of technology includes the work of Heid (1997), Hillel et al. (1992), Liu (1994), Mayes (1995), Tynan and Asp (1998), Shore (1999), and Wilkins (1995).

Understanding multiple representations in calculus. The power of technology also allows students to link various representations of data together in order to make a more complete connection between representations and to use this information to solve new problems (Waits & Demana, 2000a). Pierce (1999) examined student uses and
conceptions of a CAS in a first-year calculus course. The CAS provided an environment in which students could quickly switch between symbolic, graphic, and numerical representations. The students could also use the CAS as an expert and receive immediate feedback absent in paper-and-pencil examinations. Pierce found that the computer offered a context for student discussion and exploration in groups, allowing for multiple viewpoints when solving a problem. She also highlighted the importance of modeling various uses of technology throughout the problem-solving process. Through observations and questionnaires she found that, even though the students were free to use any approach they thought best, the methods chosen were similar to those modeled by the teacher. For example, students rarely relied on the table function when solving problems. Pierce attributed this to the fact that the table feature was used almost entirely at developmental stages and the instructor did not model its use in problem solving for the students.

Even though students used the CAS extensively and responded positively to the notion that it was a tool for exploration in the Pierce (1999) study, there was strong belief among students that paper-and-pencil work led to a better understanding of the subject. They saw speed and confidence as the main reasons for using the CAS, but learning primarily came from pencil-and-paper exploration. Again, this reinforces the importance of one’s background and how that experience has formed impressions about learning. It will be interesting to see whether teachers are better able to utilize the power of
technology at all levels and whether students will change their notion of the role of technology. In order to fully grasp technology’s role in learning mathematics, both students and teachers must examine its usefulness.

Porzio (1995) conducted another example of research investigating the potential power of technology. In this study three different approaches to a first course in calculus at the college level were examined. The author explored the connection between instruction and students’ abilities to use multiple representations (numerical, graphical, and symbolic) in order to solve problems. Previous research had explored the impact of technology on students’ conceptual and procedural knowledge; Porzio’s research concentrated on examining various representations of problems and the use of these representations in problem solving. For this study, classroom observations were used to classify instruction and activities in which multiple representations were used. Students from three different classes (n=100) were chosen to take a pre- and posttest that included problems that any first-year calculus student should be able to address, regardless of instructional method. Then 12 students from each section were interviewed. The tests and interviews were designed to examine the students’ use and understanding of different representations for calculus concepts.

Porzio (1995) found that students in the classroom where symbolic representations were the primary means for exploring calculus concepts had the most difficulty using graphical representations and making connections between representations. A second course was taught from a graphical perspective, using graphing calculators to explore symbolic representations. These students were able to use graphical
representations to solve problems, but lacked skills in understanding symbolic representations and had difficulty making connections between the two, even though this was stressed in the instruction. Porzio felt that even though connections are explained in instruction, in order for students to understand and use these connections, they need opportunities to explore these connections for themselves. The final version of the calculus course was one in which Mathematica, a computer-based CAS, was used to emphasize the use of and connections between symbolic, numeric, and graphic representations through problems that reinforce these connections and the concepts and procedures pertinent to explore calculus. Students in this course were most successful at making connections between representations in the posttest, leading the researcher to stress that more than just the use of multiple representations, discussion of connections, and incorporation of technology is required to help students grasp connections in calculus concepts.

Exploring patterns in college algebra. Mayes (1995) used a CAS with college algebra students. The CAS was used to produce data and explore a series of related manipulations in order to find patterns. Much of the data collection and exploration was carried out in the context of application problems. CAS use by the students occurred during lab sessions that were added to the traditional course structure and by instructors for modeling during lecture. Results from the study showed that students in the group using technology scored significantly better on the final achievement measures. Mayes
suggests that even though positive results were recorded, that in order to gain the benefit of a CAS, it must be an essential part of the course and not simply an add-on component.

Calls for Further Study

Even though the power of technology has been demonstrated in many studies, there are limitations to its use, and further research is required. The various technologies can lead to misconceptions because of design limitations; therefore, teachers must be trained to expect these troubles and learn to help their students recognize these weaknesses. This demonstrates that students still need to be exposed to the algorithms that dominate the traditional curriculum so that they can judge the meaning and reasonableness of the technology output (NCTM, 2000). Dunham (2000) suggests that further research not only concentrate on questions of whether technology works, but also who uses calculators and how often, when and what kinds of technology are used, whether student and teacher characteristics affect the way the technology is used, and how technology affects different groups. These types of questions allow for comparisons of pertinent factors in order to discover why, when, and how technology provides benefits for the student.

Waits and Demana (2000b) have suggested that further research on the use of CAS focus on finding appropriate teaching techniques, on determining what paper-and-pencil algebraic manipulations are needed to help students develop symbol sense, and on identifying which algebraic manipulations are obsolete. When students are asked to spend large amounts of time on manipulations of algebraic or arithmetic expressions, the
student has the opportunity to use only low level thinking skills (Kutzler, 2000). Increasing evidence indicates that the practice of performing repeated algorithmic manipulations does not help the student better understand the mathematical concepts they are using (Waits & Demana, 2000b). However, a balanced mix of technology and paper-and-pencil exercises can provide the necessary time to explore mathematical concepts important for true mathematical understanding.

Remedial Mathematics Literature

When designing a remedial course at the college level involving individuals of various ages, it is important to take into consideration the various needs of the students in the classroom. This requires recognizing the past experiences and belief structures that these students have formed. Many of these beliefs are formed on the basis of either years of failure in formal mathematics classes, misunderstandings of the nature of mathematics and learning, misconceptions about the content, and unwillingness or lack of opportunity to get fully involved in the learning process.

Students in remedial classes need to view mathematics differently (Robinson, 1995). Crossroads in Mathematics: Standards for Introductory College Mathematics Before Calculus (Crossroads) includes standards set forth by AMATYC for lower division and remedial mathematics courses at the college level and indicates a commitment to helping students see the nature of mathematics as more than a set of rules (Cohen, 1995). These standards also encourage teachers in these courses to move away from traditional lecture formats in order to help their students gain a better understanding
of the mathematical concepts. The primary goals of these standards are to teach mathematics for all students, give students opportunities to engage in rich, meaningful problem solving activities and experiments, highlight connections between mathematics and other disciplines, and link mathematical thinking with lifelong learning (Hector, 1993). These are lofty goals, but the standards do not stop here. They also discuss instructional strategies that can aid in building confidence in and enjoyment for mathematics. The use of technology plays a prominent role in these standards. Hector explains that technology can allow students to see multiple representations quickly and explore mathematical concepts with more confidence. The standards also encourage the use of cooperative learning, writing, real-life problem solving, and connections with other subjects. Finally, the standards for instruction stress that instructors in remedial courses should model multiple solution strategies and reassure students that their individual ideas will be valued in the classroom.

Teaching Strategies Used in Remedial Courses

Nesbit (1995) examined three adult basic education courses in mathematics that were taught by different instructors at the same community college. The researcher explored the teaching practices used in these mathematics courses for adults. Because mathematics achievement has been associated with helping to determine the level and status of one’s job opportunities (NCTM, 2000; NCES, 1994), Nesbit thought it fitting to examine the ways in which students were taught. Many of the students enrolled in these courses had not been successful in mathematics courses in the K-12 arena, but still needed to complete a certain number of mathematics credits to be eligible for degrees in
areas in which they had interest. Nesbit was alarmed to find that the portrayal of mathematics as a set of skills and rules to be memorized prevailed in these remedial college courses. The teacher and textbook remained the authority as in the traditional classroom, and students were given little opportunity to use their experiences outside of the classroom to help them understand the material. For these individuals, mathematics continued to be a set of rules and procedures that had little connection to their lives outside of the school setting. In addition, the classroom was structured around individual learning, and students were not encouraged to participate. In light of the principles for adult learning presented by Birkenholz (1999), these classrooms did not meet the needs of the students, even though the teachers expressed the intent to do so. Remedial mathematics classes at the college level must veer away from the traditional classroom that fails to help these developmental students construct lasting mathematical understanding.

*Technology's Role in Changing Teaching Strategies*

Robinson (1995) suggests that appropriate use of technology can allow remedial students to explore mathematics in a more meaningful way. The speed and accuracy of technology allows students to concentrate on concepts as opposed to a set of individual skills. Robinson contends that a “focus on rote memorization of formulas, algorithms, and rules is a hindrance to students at the developmental level” (p. 3). Technology can assist remedial students in making further insights into mathematical processes.
This belief in the need to help students understand the nature of mathematics is also supported by the MAA. Steen (1989), in a position paper for the MAA, encourages university mathematics departments to change mathematics courses that are taken during the first two years of college in order to encourage future mathematical study and appropriate preparation for all students. One suggestion for change is the use of technology and especially CAS in these entry-level courses. The MAA also chooses to highlight the role of mathematics courses in the first years of college as a time to help students learn about the nature of mathematics and what it means to learn mathematics. Finally, Steen warns that colleges need to make sure that remedial courses are not simply a rehash or a speedier rehash of traditional school courses that proved not to be beneficial for these students. The remedial course should be new and refreshing for the students—helping them to gain a better understanding and appreciation for mathematics. There is room to explore mathematics in remedial courses. There is plenty of need and opportunity to make mathematics accessible to all, and I believe that technology can play a major role in that process. Technology can level the playing field for students in marginalized populations (Dunham, 2000). The use of technology can serve as a scaffolding tool to help low-ability students and nontraditional college students advance farther than in the past (Kutzler, 2000; Owens, 1985; Shoaf-Grubbs, 1994; Zand & Crowe, 1997). To meet the standards dictated by the NCTM and AMATYC, changes must take place in contemporary curriculum. Technology is a vital player in this new vision for the classroom.
CAS use in Remedial Mathematics Classrooms

Research on the use of CAS in developmental mathematics is sparse and is focused on functions and multiple representations. These research projects are similar to many of those described in the previous section. Hillel et al. (1992) conducted such research with remedial students in a functions course at Concordia University of Montreal, Canada. The authors recognized both the difficulty of and necessity for change in remedial mathematics courses. Maple was used during one of two class meetings per week. The self-guided labs encouraged students to use multiple representations to solve problems and confront misconceptions about functions. Final evaluations for treatment and control groups were different in light of the focus taken by each course. The assessment for the treatment group was more conceptual in nature and viewed by the mathematicians as more difficult. A few procedural level questions were included on both tests for comparison. The group using a CAS to explore functions outsored the traditional group on the overall assessment as well as the specific comparison questions.

Hillel et al. (1992) set out to find ways to help improve the mathematical experience of remedial students. Instituting change can be difficult, so they set two goals for this first attempt at change. First, they hoped to create a more active environment, and second, they wished to make the content more accessible to students who have difficulty with basic mathematics. The study indicated that if both goals were met, more change would be necessary to reap the full potential that CAS hold for remedial settings.
Shore (1999) studied the use of a CAS with elementary and intermediate algebra students. Functions and their multiple representations again served as the focus of the study and course. Students participated in a pre- and post-measure of their procedural and conceptual understandings. This measure was in addition to the regular course exams that were more traditional in nature. Students were allowed to use the CAS not just on Shore’s tests, but also on regular course exams. Results indicated that students in the technology-integrated course did improve significantly on both procedural and conceptual understanding. In addition, this change was significantly greater than the small change that was measured among students in the traditional course. Yet there were not significantly different success rates in the courses themselves, indicating that the differences in achievement were linked to whether the CAS was available for assessment.

Working within the constraints of the departmental design of the remedial course used in the present study, the role of exploration on the CAS was examined. One advantage over the Hillel et al. (1992) and Shore (1999) studies lies in the fact that, since CAS are now readily available on handheld calculators, students could have access to technology throughout the course. However, in line with departmental policy and course design, the CAS was not allowed on the tests, and these formal assessments continued to focus on procedural skills. As expressed in previous sections, change takes time, is difficult, and must be gradual. By introducing the CAS into the traditional curriculum, some of the possible benefits will not be realized, yet it is a first step towards realigning the goals and practices to better meet the needs of students.
Attitudes and Beliefs

The literature on the role of affective variables in the mathematics classroom is quite complex. Various researchers have targeted issues relating to attitudes, beliefs, emotions, demographics, learning styles, and motivation. Many of these studies simply examine or develop inventories that can be used to measure various components and factors affecting attitude (e.g., Hagedoran, Siadat, Fogel, Pascarella, & Nora, 1999; Tapia & Marsh, 2000). Attitudes and beliefs have been closely linked with motivation and therefore are important in the learning process (Carre, 2000). This section will briefly examine some of the current work on attitudes and concentrate on those that directly examine ways that changing attitudes can help students be more successful and less fearful in their mathematical studies.

Classroom Culture

One interesting line of research examines the connection between teaching strategies and student satisfaction. Brown and Uhde (2000) examined the effect of appropriate teaching strategies with adult learners. Case studies were conducted using artifacts and observations from three graduate mathematics methods courses. The students enrolled in the courses were identified as individuals who were uncomfortable with mathematics and felt inadequate in the discipline. Teaching strategies were pulled from literature on adult learning. These are: a) variety in format, b) assist in search for learning, c) set affective and cognitive goals, d) provide connections to past and future learning, and e) attend to the needs and goals of the learners. (p. 56). During the graduate course, Brown and Uhde grouped positive student outcomes into five major categories:
1) Removal of fear
2) Increase in mathematical competence
3) Deeper understanding of mathematical processes
4) Development of ability to foster learning in self and students
5) Increased confidence in ability to do and teach mathematics. (p. 58)

In addition, the students agreed that the adult educators’ techniques had a positive impact on student learning. As the students’ mathematical fears lessened, they were able to achieve more and consequently demonstrate greater depths of understanding.

Self-handicapping. The link between the classroom environment and self-handicapping was examined among high school students by Dorman, Adams, and Ferguson (2001). The researchers defined self-handicapping as “a form of proactive avoidant behavior which is designed to manipulate other people’s perceptions of performance outcomes so that the student appears worthy to other people in the school” (p. 2). These behaviors interfere with student achievement and understanding. Dorman et al. found that a more student-centered approach lowered the negative levels of self-handicapping among the participants, indicating that classroom climate can have an impact on students’ performance by affecting attitude.

Motivation. Motivation plays an important role in student achievement and can negatively or positively affect learning (Givvin, Stipek, Salmon, & MacGyver, 2001). Givvin et al. compared students’ perceptions of their own motivation with that of the teacher’s perception. The study found that when teachers went on their own impressions as opposed to talking with the student, they were highly likely to incorrectly identify
motivations for a negative behavior. This misunderstanding could then lead to interventions that either did nothing to address the problem, or in some cases, made the problem worse.

Transition times also provide interesting settings to study motivation and attitude. Walmsey (2000) examined students transitioning from high school to college. Poor attitude and motivation to study seemed to carry over from high school to college for many students. However, Walmsey was able to identify three interventions that could help improve attitudes:

1) Provide extra support in terms of computers or tutors.
2) Use student centered approaches in the classroom.
3) Incorporate applications when teaching to emphasize relevance. (p. 49)

Anderson, Goulding, Hatch, Love, Morgan, Rodd, and Shiu (2000) examined the impressions that graduate students preparing to be secondary mathematics teachers have of mathematics. The participants had completed mathematics related degrees before entering the teacher preparation program and had, for the most part, been successful mathematically in school. Yet, many of these students expressed feelings of fear and dislike for their mathematical experiences in traditionally taught college level courses. If these students, who have been successful mathematically, were turned off by the traditional approach, how much more fear must a remedial student with little prior success feel in mathematics classes?
Miller (2000) examined literature on attitudes and teaching in order to identify a set of teacher characteristics that could be used to combat the negative attitudes of remedial students. Fear was identified as the greatest stumbling block that must be overcome. The teacher characteristics that were identified in the study as offering promise in breaking the cycle of poor mathematical attitudes and low motivation were: a) enthusiasm, b) use of motivating techniques and encouragement, c) relevant content, and d) genuine concern. Therefore, improvement in remedial mathematics education relies not only on student attitude, but also the teacher attitudes and behaviors.

Lesson context. Leonard and Derry (2001) examined the relationship between the context of a lesson and student outcomes in achievement and attitude. After identifying situations that were considered either masculine or feminine, students were presented lessons that either contained a context that matched their gender or did not. Although the context did not seem to affect the achievement of students on the material covered, it did affect the attitudes and motivation to become involved which could, over time, lead to achievement concerns.

The Role of Fear

Tobias (1991) has studied the effects of mathematics anxiety in relation to its effect on success in mathematics and has found that fear of mathematics can many times outweigh personal intellect. Students who believes they cannot do mathematics will find it almost impossible to perform to their full potential in a mathematics course. This finding is relevant to all mathematics students. The Fennema-Sherman Mathematics Attitude Scales (1976a) have been used to show that there is a link between low
confidence and low achievement. Many students enrolled in remedial mathematics courses are fighting their own feelings of inadequacy and, therefore, often second-guess or ignore intuition that has originated outside of the classroom (Bernstein, Coti-Bonanno, Reilly, Carver, & Doremus, 1995). Students must be encouraged to deconstruct their fears within the context of mathematics courses. This goal requires teachers to use methods specifically geared toward this process.

The end goal for students in remedial mathematics courses is more than just absorption of content. Due to the well-documented emotional concerns students bring to the classroom, teachers must do more than present processes. The cycle of failure must be broken. Because attitude and achievement are linked, opportunities to improve understanding must go hand-in-hand with interventions that work to improve student attitude. The present study addressed the attitudes and beliefs of participants by using research-based techniques for instruction, incorporating the use of technology, choosing instructors who believed in supporting student learning, and using writing assignments as opportunities for students to express and even confront feelings.
CHAPTER 3

DESIGN OF THE STUDY

The present study examined the use of a computer algebra system (CAS) with remedial college students. The research design focused on achievement and affective factors, while incorporating what is known about how mathematics is learned by students of various ages. Adult learning and constructivist learning theories played a key role in choices about the research design and instructional practices implemented.

Setting

The students in the present study were enrolled in four sections of a remedial mathematics course. The classes took place over two academic quarters at a large state university in the Midwest. Students were placed in the remedial course as a result of scores on a departmentally designed placement test administered on-line. No college credit is awarded for the remedial course, but students are expected to successfully complete the course and possibly a corresponding follow-up course before enrolling in college level mathematics for credit.
Two night sections each quarter were used for the present study; one was designated as the control and the other treatment before the term began. Students in both sections accomplished the same tasks. They completed the same writing assignments, covered material in the same exact order, and took the same exams. The instructors for both the treatment and control sections remained the same for each quarter that the study covered. As a result of the consistency in instructors, activities, and content, the experience for all control students (n=25) and all treatment students (n=26) were the same regardless of whether they were enrolled in the Winter or Spring terms. Therefore, all students from both terms were used together for the analysis in the current study, and will be referred to as one group throughout the discussion of the study.

The primary difference between the instruction provided in the treatment section and the control section was the use in the treatment section of a CAS to introduce topics, serve as a means for students to discover algorithms, and identify misconceptions they may have held about them. Activities designed to facilitate students' exploration with the CAS in the treatment group were redesigned for the control group to address the same content and ideas without the use of technology.

Enrollment in the two sections was based on students' self-selection due to scheduling and personal preference. Because these sections were at night, the population of students included several nontraditional as well as traditional college students. Graduate students from the college of education taught these sections and were selected based on their willingness to participate and their past experience with the course. These instructors had used many inquiry-based techniques in their teaching prior to this study
and were encouraged to continue to do so throughout the study. Inquiry-based activities were designed by the researcher for students in the treatment group to use with the technology to explore the algebraic manipulations. After this exploration, the instructor facilitated a whole class discussion in order to formalize student observations and correlate their observations with more abstract mathematical processes. In the control group the instructor first introduced the abstract mathematical processes and then used non-technology activities to reinforce these processes.

The structure of the course remained similar to that of the traditional daytime course taught during the same terms. This means, for example, that students in both the treatment and control groups were tested entirely without the aid of the CAS. In addition, the text chosen by the department was traditional in nature and did not incorporate technology. A writing component was added to all sections involved in the study so that students took the time to reflect on what they had experienced either with technology or without. Further, it was important to highlight the true nature of mathematics in order to clarify for the students that mathematics requires more than memorization of steps and rules. When possible, students in both the treatment and control groups were asked to explore mathematical processes in order to understand why they were used and how they worked, thus emphasizing that mathematics is a discipline of reason, not a set of magical procedures.
Participants

The students enrolled in this remedial course had a variety of mathematical backgrounds. Table 3.1 shows that some students had taken at least two courses in algebra at the high school level, while others had not been formally exposed to algebraic skills prior to their enrollment in the remedial course. Both traditional and nontraditional students were in these sections, which contributes to vast differences in their exposure to mathematics.

For this study traditional students were defined as those students between the ages of 17 and 22, while students aged 23 or older were classified as nontraditional (see Table 3.2). Students enrolled in the remedial course ranged from freshman to graduate rank or came into the course as continuing education students not yet accepted into degree programs. In addition to attending classes, 89% of these students had full- or part-time employment. Many of the students enrolled in the course had family responsibilities adding to their time constraints and schedules. Finally, 38% of the students in these sections were previously unsuccessful either in this specific course (19%) or in another university mathematics course (19%). All students needed to complete this course successfully in order to continue to progress at the university.

Even though the students enrolled in the remedial class came from diverse backgrounds, a common characteristic was that the majority of the students had experienced difficulty with mathematics in the past and entered this remedial course with many beliefs about the subject, some of which were inaccurate. This was not only
observed by the researcher through past experiences in this and other remedial course, but also apparent in the mathematics autobiographies written by these students during the first week of the course.

<table>
<thead>
<tr>
<th>Control Group n=25</th>
<th>Basic Math</th>
<th>Basic Algebra</th>
<th>Algebra II or College Algebra</th>
<th>Calculus or Pre Calculus</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (12%)</td>
<td>11 (44%)</td>
<td>5 (20%)</td>
<td>2 (8%)</td>
<td></td>
</tr>
<tr>
<td>Treatment Group n=26</td>
<td>2 (8%)</td>
<td>9 (35%)</td>
<td>9 (35%)</td>
<td>3 (12%)</td>
</tr>
</tbody>
</table>

Table 3.1. Most Recent Mathematics Course Taken by Participants

<table>
<thead>
<tr>
<th>Section</th>
<th>Gender</th>
<th>Age</th>
<th>Employment</th>
<th>Class Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M</td>
<td>F</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group n=25</td>
<td>11 (44%)</td>
<td>14 (56%)</td>
<td>11 (44%)</td>
<td>i6 (64%)</td>
</tr>
<tr>
<td>Treatment Group n=26</td>
<td>7 (27%)</td>
<td>19 (73%)</td>
<td>15 (58%)</td>
<td>12 (46%)</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>N</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>13 (52%)</td>
<td>11 (44%)</td>
<td>13 (52%)</td>
<td>8 (32%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Full Time</td>
<td>Part Time</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Full Time</td>
<td>Part Time</td>
</tr>
</tbody>
</table>

$T$ = Traditional (17 to 22 years old) and $N$ = Nontraditional (23 years and older)
One control group student did not provide an age.

Table 3.2. Demographic Summary of Participants

Limitations

The low number of participants for whom complete data sets were available limits the application of this study to the entire population of remedial students. It is important to note that the dropout rate among students in this remedial course is traditionally high.
and thus limited an already small population. In addition, some of these students were frequently absent and/or failed to complete out-of-class assignments. With data collected from greater numbers of students and a broader range of classrooms, could better indicate the role technology use might play in helping remedial students gain deeper mathematical understanding. A further limitation in the present study was the lack of random assignment to treatment or matched (with respect to composition) samples. In addition allowing instructors to teach both control and treatment sections would help eliminate any variability caused by the instructors themselves and thus strengthen application to diverse settings. I recognize the difficulty this causes in establishing statistically valid results, but do believe that much can still be gleaned from this work in learning more about the use of technology in remedial courses and how this technology facilitates work with adult students.

Instruments

Affective factors have been shown to impact the amount of effort students are willing to invest when studying, and these factors influence what courses they elect to take (Galbraith & Haines, 1998; Fennema & Sherman, 1976b). Therefore, in examining the use of CAS, it was important to document its effect on both achievement and affective measures in order to get a better picture of how students will approach future mathematical endeavors. By studying the effect technology had on achievement and attitudes, further insight will be gained into ways to improve the learning of mathematics (Fennema & Sherman, 1976b).
Achievement Measures

Students in both the treatment and control groups completed a pretest containing problems similar to those found on past departmental Final Exams. This instrument provided a baseline measure for the students' basic skills in mathematics. The students then completed three tests throughout the quarter that were designed by the researcher using past tests in an attempt to confirm that these students acquired the skills expected by the department. The division of material on the first test changed slightly from Winter to Spring quarters due to university scheduling. In order to standardize analysis, the questions involving negative exponents in the Winter quarter were reassigneed to reflect the design of the first test of Spring quarter (see Appendix B). Even though there was a slightly different arrangement of topics on the exams from Winter to Spring quarters the material was presented in the same way and order both terms. Finally, these students took the same departmental Final Exam as all other students enrolled in this first level remedial course. The questions on the Winter and Spring term versions of the Final Exam were identical in structure, and covered topics previously assessed on regular tests as well as a few concepts addressed after the third test of each quarter (see Appendix B).

These assessments yielded important indicators of skill level, indicated the level of achievement in learning the course content, and provided insights into student understanding of the processes used to solve problems. The researcher-designed tasks included as part of the end-of-term interviews were intended to enrich the examination of
student achievement and provide a more detailed picture of student understanding. To ascertain differences in understanding, students completed skill level problems and then explained their understanding (see Appendix B).

Affective Measures

Likert-Type Scales

In order to obtain an indication of the students’ attitudes toward how mathematics achievement will affect them, the Attitude Toward Success in Mathematics Scale was used (Fennema & Sherman, 1976b). This scale was designed to measure “the degree to which students anticipate positive or negative consequences as a result of success in mathematics” (p. 325). Students indicate that they have a fear of negative consequences by indicating that success in mathematics is the result of luck or that they will lose acceptance as a result of success.

The Confidence in Learning Mathematics Scale was used to gain insight into students’ confidence in their “ability to learn and to perform well on mathematical tasks” (Fennema & Sherman, 1976b, p. 326). This scale rates the students’ confidence levels from “distinct lack of” confidence to “definite” confidence. This scale was used to judge whether students increase their belief in their own ability over the term.

The final scale used was the Mathematics Usefulness Scale. This scale was designed to “measure students’ beliefs about the usefulness of mathematics currently and in relationship to their future education, vocation, or other activities” (Fennema & Sherman, 1976b, p. 326). The results of this instrument indicate the range of how useful students feel mathematics is to them outside the classroom setting.
Revised versions. In the present study Round's (1998) revised versions of the Confidence in Learning Mathematics and Usefulness of Mathematics scales were used. In order to shorten the length of the instruments presented by Fennema and Sherman (1976a), Round eliminated questions that were repeated in each of the subscales. The original scales included positive and negative statements that paralleled each other. Round's shortened scales eliminated these duplicates, while maintaining the balance of positive and negative items. The shortened version of the Confidence in Learning Mathematics and Usefulness of Mathematics scales had reliabilities of .82 and .78, respectively, on Round's test-retest measure of reliability. These reliabilities were somewhat lower than those calculated for the entire subscales (.93 and .88, respectively [Fennema & Sherman, 1976a]), but were still in the acceptable range.

In piloting the shortened version of the scales, Round (1998) found, by comparing his own test-retest reliabilities with the split-half reliabilities from Fennema and Sherman (1976a), that the Attitude Toward Success in Mathematics Scale showed the most difference in reliability. Fennema and Sherman found a reliability of .87, while Round found a reliability of .56. He attributed this difference primarily to two students whose scores changed drastically over the 11 days between the test and retest. Removing these two scores raised the reliability to .73. This would place all the reliability scores above Henerson's (1987) benchmark of .70 for respectable reliabilities. In order to keep each of the scales closer to the .80 benchmark for reliability set by Mueller (1986) for a well-constructed scale, the entire Attitude Toward Success in Mathematics Scale was used in the present study.
A list of the questions in the Attitude Toward Success in Mathematics scale, with indications of whether each statement is positive or negative is given in Appendix A. The questions from the Confidence in Learning Mathematics and Usefulness of Mathematics scales included in Round's shortened versions can also be found in Appendix A. Questions tested by Round in the Confidence in Learning Mathematics and Usefulness of Mathematics scales were incorporated with the Attitude Toward Success in Mathematics Scale to create one affective instrument used with the current study.

The Fennema-Sherman (1976a) inventory was originally designed for use with high school students. However, many researchers have used these scales in university and college settings (e.g., Austin-Martin, Waddell, & Kincaid, 1980; Benton, 1979; Betz, 1977; Bretscher, Dwinell, Heyl, & Higbee, 1989; Eckard, 1995; Goolsby, Dwinell, Higbee, & Bretscher, 1987; Hackett, 1985; Hackett & Betz, 1989; Thompson, Melancon, & Becnel, 1993). Therefore these scales were used with slight changes in the wording of a few questions to reflect modern slang terms and the age of the students. For example, the word grind in the Attitude Toward Success in Mathematics Scale was replaced with geek. In addition there were several questions in the Mathematics Usefulness Scale that were phrased in the future tense about mathematics use in adulthood that were rewritten in present tense for the participants.

End-of-Term Questionnaire and Semistructured Interview

In addition to the affective instrument, a researcher-designed end-of-term questionnaire and semistructured interview were used with both the treatment and control groups (see Appendix A). The end-of-term questionnaire was used with a previous
remedial course taught by the researcher and was modified slightly to reflect the technology differences between the two groups in the current study. This questionnaire asked students in both sections to comment on (a) what aspects of the course that they felt were most beneficial in helping build their understanding, (b) whether the writing assignments specifically helped them understand the content, and (c) how the present course compared with mathematics courses they had previously taken. The treatment group was additionally asked whether using technology to introduce new concepts had aided or interrupted their understanding. The semi-structured interview format allowed the researcher to explore emerging themes during each interview. However discussion starters were prepared in advance and included: (a) past experiences with mathematics, (b) what it meant to be good at mathematics, (c) learning strategies that work for the student across disciplines and in mathematics specifically, and (d) strategies used by the present instructor that had facilitated their understanding. In addition students from the treatment group were asked to discuss their experiences with and impressions of technology in relation to the remedial course they were taking. Students were also given three problems to complete during the interview, and were asked to explain their thinking on each. These problems included one from each of the content areas in which planned CAS activities were designed – exponent rules, linear equations, and graphing (see Appendix A).

All students involved in the study were expected to complete the questionnaire as a writing assignment, but volunteers were solicited for interviews. Private, semi-structured interviews were conducted by the researcher outside of class time. The response rate on
the questionnaires was sufficient to allow for analysis; however, because the number of students willing to sit through the interview was very small, these data could not be used to explore the general attitude of the groups.

Writing Assignments

In addition to the instruments above, the researcher in collaboration with the instructors designed writing assignments that provided additional qualitative data for continued examination of the attitudes and beliefs held by the participants. These assignments included questions relating to their personal beliefs about the nature of mathematics, past experiences they had in mathematics that have affected their beliefs about their abilities, what it takes to do well in mathematics, and their reflections on class work and exams. For the purposes of this report, these assignments were only partially examined in order to gain better understanding of the students enrolled in the sections and their past experiences with mathematics.

Procedures

Recruiting Instructors

Recruitment of instructors for the control and treatment groups was important for a number of reasons. The instructors needed to be not only familiar with the course as designed by the department, but also willing to participate in the added duties of collecting the homework and writing assignments necessary for this research. Because the sections participating were offered at night, the instructors’ schedules needed to be flexible enough to accommodate nighttime teaching for two consecutive quarters.
Mathematics education graduate students were sought, because of their familiarity with the constructivist philosophy and because they would be more likely to incorporate questioning and student centered approaches. Additionally, the treatment instructor needed to be familiar with the technology and preferably to have used technology in the classroom previously. The treatment instructor was identified first. She was a third year doctoral student with five years of high school teaching experience, as well as previous experience with this remedial course. She was familiar with the TI-92 and had incorporated technology into courses she had taught in the past. The instructor for the control group was in his second year as a mathematics education doctoral student, and even though his past education training was not as extensive as that of the other instructor, he was familiar with the reform movement in mathematics education and had completed coursework specifically addressing the constructivist learning philosophy. I had previously observed this instructor and was impressed by his use of questioning and a student centered approach even before he had participated in considerable education coursework. After discussing my goals with both instructors, they agreed to participate.

Recruiting Student Participants

During the first week of each term the researcher visited both the control and treatment groups and explained the project to the students. At that time students were asked to participate by completing a consent form, a demographic information sheet, and the affective instrument. By agreeing to participate, students were also giving permission to the researcher to examine their course materials and grades. This request was explained thoroughly before consent was solicited. Of those students providing
permission for the researcher to examine course grades and assignments, only those who completed all tests and the Final Exam were considered in the analysis. This reduced the number of subjects from an already small population to an even smaller subset.

Approximately two weeks before the end of the term, the researcher returned to the classroom to solicit volunteers for the semistructured interview that was to be conducted outside of class time. A list of possible interview times was provided to the students, and the researcher also agreed to meet volunteers at their convenience. The number of students willing to schedule an interview was very low, and even then some of these students did not keep their appointments. Due to the small number of students completing the interview (n = 5), the data collected were not considered representative, and therefore were not used for analysis.

Classroom Visits

Periodically throughout both terms the researcher visited both classrooms, especially when those key concepts involving the use of technology were addressed. Due to the lack of student familiarity with technology, the researcher acted as an assistant to the instructor in the treatment classroom, moving between groups to facilitate use of the CAS. Opportunities also arose in the control classroom for the researcher to interact with the students by answering questions or talking informally during breaks. Even though this involvement was not initially planned, it proved to be beneficial in helping the treatment students use the technology more efficiently and become more comfortable with the researcher. It also provided insight into student thinking and difficulties with technology use.
Course Assignments

Homework problems, writing assignments, review sheets, and quizzes, some designed by the researcher, were supplied to the instructors throughout the term. After assignments were completed and tests were graded, the researcher made copies of these so that originals could be returned to students. As needed and requested, changes were made in assignments to better meet the needs of participants. Upon the suggestion of the treatment instructor and a request by students, solutions including a possible process for completing homework problems were made available to students in both groups. The researcher formulated grading guides with input from the instructors for all tests and quizzes, helping to standardize the grading process. The department provided a similar grading guide for the Final Exam.

Vignettes of Classroom Observations

Through classroom visits and discussion with the instructors, the researcher explored the presentation of material and creation of classroom culture. Special attention was given to those concepts for which the CAS was incorporated deliberately. It is important to note that the TI-92s were available to students in the treatment group during every class period except for test days. Some students used the calculators in ways beyond those prescribed by the instructor, such as for computation or checking answers.
Monomials

In the present study the primary difference in instruction for the control and treatment groups was the use of a TI-92 calculator as a means for inquiry. However, by introducing calculator use for instruction, the treatment group experienced learning conditions identified by adult learning theory and social constructivist learning theory as desirable. The first lesson designed to use the CAS covered operations with monomials. In both classes students were expected to multiply, divide, and raise monomials to a specified power without the use of technology, and both instructors explained why the rules for exponents work. However, differences arose in the instructors’ approaches.

Control Group

In this section the instructor began by providing the rules for operations on monomials. For example, $a^m \cdot a^n = a^{m+n}$ is the rule for multiplying monomials. After providing students with this abstract rule, the instructor demonstrated several examples that incorporated the rule, such as $(x^2 y)(x^3 y^5)$ and $(3x^4 y^2 z)(4xz^5)$. This was then followed by a discussion of why the rule works, using an example such as $x^3 \cdot x^2$. The instructor broke each term apart by writing $(x \cdot x \cdot x) \cdot (x \cdot x)$, and then made the connection between multiplying the term five times and obtaining the solution from the rule, $x^5$.

The same process was then repeated for the other two rules. Throughout the lesson presentation the instructor asked for questions from the students and asked them to help him complete the examples when appropriate. Some examples that involved more than one of the rules were examined, and students then practiced several exercises of
each type. While mathematically sound, this approach ignores the facts that many of these students have been exposed to these rules in the past, and that adult learners need to first see the reason for a skill before they are motivated to learn it. These students were asked to comment on the rules for exponents using the X-potent X-ploration [Control Group] assignment (see Appendix C). Most of these students completed this assignment outside of class.

**Treatment Group**

In the treatment section, the students began with the opportunity to derive the rules for monomials after exploring problems with the CAS included in the TI-92. The students worked cooperatively to complete the X-potent X-ploration [Treatment Group] during class time (see Appendix C). Included in this exploration were groups of items and follow-up questions that encouraged the students to find patterns in similar situations. For example, one set of exercises ask the students to explore negative bases by comparing exercises such as $-2^3$ and $(-2)^3$ with $-2^4$ and $(-2)^4$ using the CAS to compute the values (see Figure 3.1). Another set involved computing several problems requiring the multiplication of monomials with the CAS (see Figure 3.2). Students were asked to write in their own words any patterns or differences that they observed. This activity was designed to allow students to explore problems with the CAS and then generalize patterns. The instructor was able to use these student-generated patterns to tie into the more abstract rules in the text. After discussing the rules, she had the opportunity to explore with the students why these generalizations work.
Figure 3.1. TI-92 Screenshot of Exponent Example.

Figure 3.2. TI-92 Screenshot of Multiplication Rule Example.

There are several aspects of the X-ponent X-ploration [Treatment Group] activity that reflect the suggestions provided in the literature for teaching mathematics in a meaningful way to all students. First, this activity respects the fact that adult learners prefer to see a reason for an abstract skill before spending time learning it. Because the students had the opportunity to work with several examples and began to form their own understanding of a pattern before discussing the rule as a class, they could find importance in the activity. This approach did not take away from the fact that many of the
students had been exposed to the abstract rules in the past, and some may have retained them. On the contrary, it provided an opportunity for students who did recall the abstract rule to confirm their understanding. An environment was established in which students who had long since forgotten the rule, who had developed misconceptions about the rule, or who had not been exposed to the rule could examine the situation from a new perspective. The idea of putting the application before the skill is promoted in the work of Granott (1998) and Birkenholz (1999) for adult learners and Heid (1988) for younger students.

A second component of adult learning theory and social constructivist theory found in this lesson was the scaffolding that occurred within an individual’s Zone of Proximal Development (ZPD). The technology acted as the expert when the students began to work problems. However, they were encouraged to watch for a pattern and when they felt they had a conjecture, to try working a problem by hand and checking the conjecture on the calculator. Here, the technology filled the skill gap until the students were able to branch out alone. In addition, the design of the worksheet also required more from the students as they moved through the various sets. As the students progressed through the various sets of problems, fewer and fewer examples were provided. This activity was designed to promote student autonomy.

The third aspect of adult learning theory and social constructivist theory addressed by this activity was that students used knowledge they brought with them, while exploring and testing their ideas with those of other students. The instructor modeled what it means to mathematically justify a conjecture and then asked the students
to do the same. As a class the students explored why the rule for multiplication is
\[a^m \cdot a^n = a^{m+n}\], and then worked in smaller groups to discuss the other rules. First a
pattern was searched for, then a conjecture was made and tested, followed by appropriate
modifications, until finally a proof of why the conjecture works was developed. This
process experienced by the students provided a picture of the way that many
mathematical discoveries are made.

Solving Equations

The CAS was also used extensively as students solved linear equations. Again the
students were encouraged to form their own understandings of the mathematics based on
the activities and on the skills they brought with them into the classroom. The instructors’
goal was to help them justify this understanding and match it to the established
mathematical culture.

Control Group

As the students explored solving simple linear equations without technology, the
instructor took a traditional approach. He provided the students with techniques they
could use in specific instances, and provided opportunities for them to look at several
examples. As before, the students were encouraged to help solve examples the instructor
provided and to ask questions as needed. In addition, alternative paths to solutions were
discussed as questions arose.

The instructional plan used with the control group was mathematically sound and
similar to that used in many algebra classrooms in both university and K-12 settings. One
problem with this approach is that it ignores the fact that most of these students have
solved equations in the past by applying these same techniques, but have developed misconceptions or only memorized the rules on a short-term basis. In the case of students who have resilient misconceptions, this strategy did not provide the needed incentive to change or move beyond their current levels of understanding. Most of these students spend long periods of time practicing these techniques, but at the same time have great difficulty letting go of long-held misconceptions (Schommer, 1998). These students, trying simply to memorize a set of steps, have once again missed the opportunity to understand the mathematical concept that allows them to apply the techniques in new circumstances and future course work.

Treatment Group

In the treatment group, the use of the CAS started at the beginning of the unit. After a brief review of equations, what solving means, and an overview of the CAS procedures, students were given a set of equations to solve using the scaffolding feature provided by the calculator (see Appendix C). Students were given a table on which to keep track of the steps they tried in order to solve each equation, as well as whether each attempt resulted in the desired outcome (see Appendix C).

Some of the students had an idea of how to solve the problems and simply used the calculator to verify their work, while others needed more assistance from the instructor or classmates in deciding what steps to take in solving the equation. The technology did cause some confusion by showing the results of certain steps in a different format from what the students had recorded. However, this difference provided an opportunity to discuss alternative solution methods and equivalent expressions. Students
who had worked with equations before could test their processes with immediate feedback, while other students who did not have knowledge of the techniques could experiment and find some that worked. Using this instructional plan, the instructor was able to provide students with different entry points into the lesson.

The items provided in the activity included several different misconceptions that the researcher had observed in her past work with remedial students. For example, when given an equation such as \[ \frac{2}{3} - w = \frac{5}{8} \], many students remember that, when fractions are present, they often multiply by the reciprocal. Therefore, in this case, they might multiply by \( \frac{3}{2} \), predicting a result of \( -w = \frac{15}{16} \). With the CAS, students could ask the system to multiply by \( \frac{3}{2} \), but the CAS would give them the result of \( 1 - \frac{3}{2} = \frac{15}{16} \) (see Figure 3.3). The students must then try to decide what happened and adjust their process. The immediate feedback from the CAS does not allow students to repeat a step based on a misconception without an indication of something being wrong.

![Figure 3.3. TI-92 Screenshot of Scaffolding Procedure.](image)

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Students were encouraged to keep track of incorrect attempts, thus creating a running log of mistakes. The instructor and students could then track mistakes for each individual, and this provided opportunities for further discussion around specific need areas. For example, if a student repeatedly made an error when combining like terms, the instructor or other students could help address this specific issue. For some students, the technology also effectively removed arithmetic processes and difficulties from the exercise, thus placing emphasis on choosing the algebraic tool to apply. Here the technology focused the learner on choosing the operation, an act that has been identified as a higher order thinking skill (Kutzler, 2000). Still other students simply used the calculator to check their work, reaffirming their own abilities and pinpointing any areas that needed additional attention. Again, the technology-based activity offered students multiple entry points into the lesson.

*Graphing Linear Equations in Two Variables*

The course material in this unit included determining slope; using T-tables, slope-intercept form of an equation, or x- and y-intercepts to draw a graph; and matching graphical and symbolic representations for a linear function.

*Control Group*

Throughout this unit the instructor provided students with information and techniques for working with linear equations in a specific form. At one point, students worked in groups to begin the *Graphing Conjectures [Control Group]* worksheet (see Appendix C). Groups were assigned a specific set of equations to explore. Students found solutions and plotted these to draw the graphs of the lines on a transparency to
share with the class. They also calculated the slopes and intercepts. The results were then shared with the whole class in order for students to make conjectures. Examples of such conjectures included relating the numerical value of the slope with the appearance of line when graphed, or establishing a relationship between the slopes of two parallel lines. Students were then encouraged to use the concepts discussed as they graphed other equations.

As a result of the class work and instructor’s lectures, students in the control group were provided with numerous specific techniques for obtaining answers to problems in the unit. These students even had specific techniques for looking at the equation in standard form \((ax + by = c)\) and immediately determining needed components. For example, the slope was discussed as merely a transformation of components of the standard form \(\left(\frac{-a}{b}\right)\), and little focus was placed on conceptual understanding. Thus, many students spent more time and effort determining which technique to apply in a given exercise than trying to understand the problem.

*Treatment Group*

The unit in the treatment class also began with an introduction to many of the concepts that would be used while graphing. However, the technology afforded the instructor the opportunity to neatly and efficiently offer graphical representations and examples of many of the concepts addressed. For example, when discussing the role of slope, the instructor could graph several lines and show how changing the value of the
slope even slightly affected the appearance of the line. In the case of the difference of a negative and positive slope, the instructor might graph both $y = 2x + 4$ and $y = -2x + 4$ and allow the students to discuss the difference they observed (see Figure 3.4).

![TI-92 Screenshot of Slope Example.](image)

The technology not only played a role in the way the instructor could present the material, but also acted as an aid when the students were working on the Graphing Conjectures [Treatment Group] activity (see Appendix C). The CAS offered a quick and easy way to check solutions or carry out the steps of solving for $y$ in a particular equation. The graphing feature of the calculators also allowed students to examine the sets of equations in a neat and efficient manner. Equations were grouped in order to highlight some aspect of graphing. For example one set may contain four equations in which all slopes were equal and then students could make a conjecture that if the slopes are equal then the lines are parallel. Another set of equations contained two pairs of equations with equal slopes, but the two slopes in the set of four were opposite reciprocals to highlight the relationship between the slopes of perpendicular lines. Other concepts addressed in
the sets of equations included comparing intercepts and the given equation. Groups were still asked to present one set to the class, but the technology allowed groups to quickly look at neat representations of several of the sets for themselves. As a result of past work with students in remedial courses, the researcher and instructors were familiar with the fact that inaccuracy and messiness in graphing had many times perpetuated misconceptions. For example, having neat and accurate representations eliminates some of the misconceptions that are possible when students look at hand-produced images to determine the relationship between two lines that may be parallel. The precision of technology-produced graphs eliminated many of the factors that may lead to misconceptions when hand-drawn sketches are the primary mode of representation.

Classroom Culture

In observing the classes, the researcher paid close attention to the culture created in the classroom noting ways in which sociocultural and adult learning theories were reflected. It was evident that the cultures created in each classroom had similarities, but definite differences arose primarily in the interaction and validation patterns. Although it is not possible to say that technology was the cause of all of these differences, it certainly played a role in the culture that was created.

Control Group

The primary mode of teaching in this classroom was lecture, but the teacher did make specific efforts to involve the students. By specifically asking for questions, providing ample time for students to pose questions, and creating a relaxed and open environment, the instructor created the opportunity for students to become part of the
instruction. Many times the instructor used suggestions from students as he worked examples at the board, asking questions that could help them develop effective suggestions. He also used group assignments, some designed by the researcher and others that he had used in the past, to allow students to work together. At times class time was allowed for students to start practicing skills, and the instructor encouraged them to work together. On the majority of the occasions when the researcher observed students working together in small groups, the students were primarily comparing answers to exercises. Rarely did they go on to compare techniques used to obtain the results. The students were demanding of the instructor’s time, constantly seeking his validation for their work. Students placed an emphasis on knowing and using instructor- or text-provided strategies for solving problems. Student-to-student interaction increased somewhat as the term progressed. During both terms, students in the control classrooms sat at tables, but rarely repositioned their seats to encourage interaction without specific directions from the instructor.

_Treatment Group_

The treatment classroom was also open for students’ participation in lectures, and questions were encouraged in ways similar to the control group. However, more time was devoted to small group discussion, even when technology was not being used. Students consulted each other and the calculator for validation of work as much as, if not more than, consulting the instructor. Students in this group also asked questions about answers, but seemed more willing than those in the control group to discuss processes, both when working in small groups and participating in full class discussions. Many lectures were
based on explorations and discoveries made by the students themselves, and efforts to show connections between methods were made when possible. Students questioned the instructor’s choice of methods by providing alternatives that worked for them. Students were working together from the beginning of the term and would many times quietly ask each other about what the instructor was suggesting during lectures. The classrooms in which the treatment groups met were filled with single student desks, but these were frequently rearranged by the students—without direction from the instructor—into small clusters around the room.
CHAPTER 4

ANALYSIS

The analysis in this study focuses on two research questions. First, the effect of the use of a CAS and graphing utility on remedial mathematics students' achievement is examined. Data from course assessments will be used to explore possible differences between achievement in the control and treatment groups. Second, the effect of the use of CAS and graphing technology on remedial college mathematics students' attitude toward, beliefs about, and confidence in mathematics is investigated. Likert-type attitude scales and an open-ended questionnaire were used to learn more about the students' attitudes, beliefs, and confidence.

The following statistical analysis was conducted using SPSS version 5.0 for Windows. t-tests were used throughout the examination of data and it should be noted that three criteria must be met in order to validate the results of a t-test: The samples must be normal, have equal variance, and be independent sets. According to Hopkins, Hopkins, & Glass (1996), recent research has indicated that the t-test can still be considered valid even if the data is not normally distributed. Hopkins et al. went on to say that the homogeneity of variances could also be disregarded if the sample size was equal in both the control and treatment groups as is true in the current study. A 2-tailed test was
used throughout due to the fact that the mean of the treatment group could have fallen either above or below the mean of the control group and the use of a 2-tailed significance test allows for both possibilities. At the outset of the research it was not known whether the effect of technology would be positive or negative; however, in future examinations of CAS use with remedial students, the current study will offer support for the use of 1-tailed significance testing, thus strengthening the power of the examination.

**Effect on Achievement**

*Research Question 1. What is the effect of the use of a CAS and graphing utility on remedial mathematics students' achievement?* The achievement data were primarily student scores on class assessments. Throughout the term, students in the control and treatment groups participated in a pretest, three tests, and the departmental Final Exam. In examining the scores on the tests and finals, one student emerged as an outlier. This student, in the treatment group, was considered an outlier because she fell more than three standard deviations below the mean on all three tests and the Final Exam. Due to the small number of participants with complete assessment data, this one score had a measurable effect on the results. Therefore, data collected from this participant was removed before analysis was conducted. Only one other score fell more than three standard deviations below the mean. A student in the control group was an outlier on Test Three; however, her scores fell within the accepted range on all other assessments, so her scores were not removed. Thus, there were 25 complete assessment data sets for both the control group and the treatment group included in the following analyses.
**Pretest Analysis**

Even though students are placed in mathematics courses based on a departmental placement test, an additional pretest similar to an old departmental final for the course was given on the first day of class to all students in the control and treatment groups. Scores were converted to percentages that ranged from 0% to 65%. In order to compare the achievement level of students in both groups on course-related content, an independent samples t-test was run on the pretest scores, and it was determined that there was not a significant difference in entering achievement at the .05 level (see Table 4.1).

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Treatment Group</strong></td>
<td>23</td>
<td>15.57</td>
<td>15.051</td>
<td>0.190</td>
<td>0.851</td>
</tr>
<tr>
<td><strong>Control Group</strong></td>
<td>23</td>
<td>14.78</td>
<td>12.873</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note that the outlier was removed for this analysis and 4 students were absent on the first day of class.

Table 4.1. Comparison of Entering Achievement for Treatment and Control Groups Using an Independent Samples t-Test on Pretest Scores.

**Regular Tests**

The material covered in the remedial course was grouped into four main sections. The first section—basic computation of real numbers—and the second—basic computation of algebraic expressions—were both included on the first test. Due to scheduling needs, problems involving negative exponents were placed on the second test even though these problems better fit the material covered on the first test. The second
test was primarily focused on the third section—solving equations—including both linear and quadratic equations. Therefore, factoring was included as part of the third section. The fourth section—graphing linear equations in two variables—constituted the third test. Simple percents and basic inequalities were covered after the third test but before the Final Exam. This deviated from the sequence of topics covered by the text and daytime course as determined by the department. The concepts were reordered for both the control and treatment groups in such a way that students could see the connections between concepts.

*Test One*

Based on classroom observations, discussion with the instructor, and preparation of activities, the CAS was used before the first test primarily when students explored the rules of exponents. Students in the treatment group were given the opportunity to work several examples of each rule using the symbolic manipulator and were asked to make a conjecture about what they saw. After working in groups and discussing as a class, the abstract rules the students had developed were summarized in symbolic notation. Students in the control group, however, were first presented with the abstract rule and then worked several examples illustrating that particular rule. The last step in both the treatment and control groups was to examine why the rules worked. A detailed explanation of the CAS activity exploring exponent rules is included in Chapter 3 and the activity sheet can be found in Appendix C.
An independent samples \( t \)-test was used to compare the mean total scores on Test One for the control and treatment groups. As in the case of the pretest, the criteria necessary to run this statistical test were satisfied. Statistically the means on Test One were not significantly different and were, in fact, less than one percentage point apart (see Table 4.2). The null hypothesis that there was no significant difference between the mean score for the control group and the mean score for the treatment group could not be rejected.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>( t )</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>25</td>
<td>69.36</td>
<td>18.497</td>
<td>-.091</td>
<td>.928</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>69.80</td>
<td>15.395</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.2. Comparison of Mean Scores for Treatment and Control Groups on Test One Using an Independent Samples \( t \)-Test.

**Test Two**

Before Test Two the students in the treatment group used the scaffolding feature of the CAS as a class in order to solve linear equations in one variable. Students were asked to solve a set of equations, recording the steps they would carry out and predicting the result. The same steps were then completed with the CAS, and if the results were not the same, students were encouraged to think about why. A detailed explanation of this CAS activity is included in Chapter 3 and the activity sheet can be found in Appendix C.
Even though this was the only time between Test One and Test Two that a specific activity was designed to incorporate the CAS, several students also used the CAS to check homework or in class activities that involved linear equations and factoring.

Again an independent samples $t$-test was used to compare the mean scores from the treatment and control groups. The null hypothesis that the mean scores were the same could not be rejected (see Table 4.3). However, the difference between the means, seven percentage points, was the greater on this test than any of the other achievement measures.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$t$</th>
<th>Significance (2 –tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td>25</td>
<td>66.20</td>
<td>19.343</td>
<td>-1.328</td>
<td>.190</td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td>25</td>
<td>73.20</td>
<td>17.898</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3. Comparison of Mean Scores for Treatment and Control Groups on Test Two Using an Independent Samples $t$-Test.

It seems that the use of a CAS c as a check for work was important and helpful to several students. On the end-of-term questionnaire the treatment group was asked if the technology helped them better understand the material, and several comments indicated that the greatest benefit in using the technology was the ability to check their work. Both checking equations and checking factoring were mentioned specifically. Some sample responses are included in Table 4.4. Although not all responses were positive with regards to technology use, several students indicated that having immediate feedback did
help them discover areas that needed more attention. Even though these statements primarily reflect the students’ attitudes, the statements are mentioned here because they indicate that the students linked checking answers with learning the content will be discussed in more detail later in this chapter.
<table>
<thead>
<tr>
<th>Students in Treatment Group</th>
<th>Did using technology to introduce new concepts help you understand the course materials? Explain how you feel the technology encouraged understanding and/or interrupted your understanding of the mathematics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female Nontraditional (01)</td>
<td>Learning to use TI-92 calculators to check work was great, but nothing but a tease because they are not allowed on exams.</td>
</tr>
<tr>
<td>Male Nontraditional (02)</td>
<td>I feel that the technology introduced was very helpful. I used the TI-92s to check homework and various other assignments after they were graded to understand minor errors I had made.</td>
</tr>
<tr>
<td>Male Traditional (03)</td>
<td>Using the TI-92 helped in graphing and solving any problem. The thing encouraging was figuring out the problem then using the calculator to check the answers.</td>
</tr>
<tr>
<td>Female Nontraditional (04)</td>
<td>The use of technology was helpful in the fact that you could check your work and play around with the different equations and/or solutions. However, it was not helpful to spend so much time learning about the new calculator and it’s features because we couldn’t use them on the test and therefore I only wanted to know the basics of the calculator for problem checking. It’s good to have them for the folks who want them, but I didn’t need one.</td>
</tr>
<tr>
<td>Female Traditional (05)</td>
<td>Technology was not only a good way to check your work, but was a good tool to help me to understand how the problem was solved and how it got to that stage.</td>
</tr>
<tr>
<td>Female Nontraditional (06)</td>
<td>Using the TI-92s reinforced the concepts in most cases. I was able to test my knowledge and check my answers.</td>
</tr>
<tr>
<td>Female Nontraditional (07)</td>
<td>Yes it helped. I could never figure out how to use scientific calc, but I learned and it helped in checking problems and using them to solve.</td>
</tr>
<tr>
<td>Female Traditional (08)</td>
<td>I think it increased my understanding by giving me another means of working/checking a problem I had worked out in my head previously.</td>
</tr>
</tbody>
</table>

Responses are typed as written by the student with no corrections.

Table 4.4. Sample Responses from Treatment Group Students’ End-of-Term Questionnaire Indicating Benefit of Technology for Checking Work.
Test Three

The instruction between the second and third tests was most heavily influenced by use of technology. A less powerful graphing calculator could have been employed for these topics, but the treatment group continued to use the TI-92s. Students were therefore able to use the CAS to assist with algebraic manipulation of equations. Use of the CAS began early in the unit and continued throughout many lessons. At times the instructor was using the graphing feature with the overhead to demonstrate concepts, and at other times the students worked in groups or individually with the calculator. The goal of the calculator use was again to allow the students to generate conjectures about the connection between the symbolic form of the equation and the graphic representation. The hope was that students would be able to see examples in a neat and more efficient manner. The focus of the text was on the process of graphing and finding slopes and intercepts, without emphasis on the application. In an attempt to make the skill more meaningful, situated examples were used in both groups. For instance, students were asked to apply their knowledge of linear equation to an economics application in one of the writing assignments.

The independent samples $t$-test used to compare results of this test did not show a statistically significant difference (see Table 4.5). Again the null hypothesis that the mean of the treatment group was equal to the mean of the control group could not be rejected. The mean difference in scores between the two groups for this test was approximately six
percentage points. The mean scores of both groups on this exam represented the highest exam scores in the course for both groups, with the mean in the treatment group being at the B level and the mean in the control group being at the C level.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2−tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>25</td>
<td>77.48</td>
<td>18.360</td>
<td>-1.220</td>
<td>.229</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>83.12</td>
<td>14.048</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.5. Comparison of Mean Scores for Treatment and Control Groups on Test Three Using an Independent Samples t-Test.

**Final Exam**

The Final Exam was created by the department for all sections of the course and consisted of material covered on the previous tests as well as simple percentage problems and basic linear inequalities. This exam was worth 200 points, while the tests were worth 100 points each. Using the results from the independent samples t-test on the Final Exam data, the null hypothesis that the means were the same again could not be rejected (see Table 4.6). However, both groups recorded means in the C range, indicating that as a whole, they had successfully learned the material expected by the department. The use of writing, rearranging the material, and decreased time spent on presenting rules in the treatment group had not negatively affected the achievement of the students.
<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td>25</td>
<td>145.80 (72.9%)</td>
<td>29.971</td>
<td>-.671</td>
<td>.505</td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td>25</td>
<td>151.76 (75.88%)</td>
<td>32.734</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.6. Comparison of Mean Scores for Treatment and Control Groups on Final Exam Using an Independent Samples $t$-Test.

*CAS Specific Assessment Questions*

In order to get a better picture of how the use of the CAS may have affected achievement on the specific focus areas for this research, those assessment questions directly related to the subjects taught using CAS—namely, rules of exponents, solving linear equations in one variable, and graphing linear equations in two variables (see Appendix B)—were grouped to get a total number of points in each area (see Table 4.7). The mean results on these questions were compared using an independent samples $t$-test in order to statistically test the following null hypotheses:

1. There was no statistically significant difference between the control group and the treatment group in scores on the exponent rules items.

2. There was no statistically significant difference between the control group and the treatment group in scores on the linear equations items.

3. There was no statistically significant difference between the control group and the treatment group in scores on the graphing linear equations items.
4. There was no statistically significant difference between the control group and the treatment group in scores on all CAS-specific items combined. In each case the treatment group scored higher than the control group, but not enough higher that the null hypotheses could be rejected.

The mean score earned on the Final Exam for the treatment group indicates that using a CAS did not interfere with the students' ability to perform paper-and-pencil tasks at an acceptable level. The mean number of points earned by the treatment group represents a minimum 70% accuracy rate in all cases, except the CAS-specific questions for exponent rules. These questions came from the first test. The mean scores on Test One were the lowest assessment means in both the treatment and control groups.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2 –tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total for Exponent assessment items</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>25.04</td>
<td>9.922</td>
<td>-.866</td>
<td>.391</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>27.28</td>
<td>8.289</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total for Equation assessment items</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>25.88</td>
<td>7.833</td>
<td>-.981</td>
<td>.331</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>28.00</td>
<td>7.438</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total for Graphing assessment items</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>64.04</td>
<td>17.838</td>
<td>-.567</td>
<td>.573</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>66.84</td>
<td>17.072</td>
<td></td>
<td></td>
</tr>
<tr>
<td>All CAS assessment items combined</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Control Group</td>
<td>25</td>
<td>114.96</td>
<td>28.365</td>
<td>-.951</td>
<td>.346</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>25</td>
<td>122.12</td>
<td>24.727</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.7. Comparison of Mean Scores for Treatment and Control Groups on CAS-Specific Items Using an Independent Samples t-Test.
Analysis by Age and Gender

In order to examine the relationship of more than one variable on the achievement of students, a multiple regression analysis was performed. Multiple regression analysis is defined by Kerlinger (1973) as a “method of analyzing the contributions of two or more independent variables to one dependent variable” (p. 150). For this analysis I looked at the total scores on Final Exam and the total on the CAS-specific concepts individually in relation to group placement, gender, and age category. Throughout the design of the research, the fact of whether the student was traditional or nontraditional was seen as an important factor to consider. Whether the treatment or traditional style of instruction was more effective for the two age groups was of considerable interest to the researcher. Gender was also factored in to see if the effect on males versus females was different. The Final Exam was chosen as the achievement measure because it was the achievement measure most independent of the design of the study. The CAS total was used in order to get an overall picture of the CAS effect.

A univariate analysis of variance was used to statistically test the hypothesis that there was no difference in the achievement of students when age and gender of students were considered. The univariate analysis showed no statistically significant results, though there were some results to highlight (see Tables 4.8 & 4.9). Note that one student in the control group did not provide her age, so she was not identified as either traditional (17 to 22 years old) or nontraditional (23 years or old) and was therefore removed from this analysis.
Table 4.8. Univariate Analysis Examining Effects of Gender, Age, and Group on Final Exam Scores.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENDER</td>
<td>813.845</td>
<td>.834</td>
<td>.366</td>
</tr>
<tr>
<td>AGECAT</td>
<td>2165.179</td>
<td>2.219</td>
<td>.143</td>
</tr>
<tr>
<td>GROUP</td>
<td>377.111</td>
<td>.386</td>
<td>.537</td>
</tr>
</tbody>
</table>

R Squared = .076 (Adjusted R Squared = .014)

Table 4.9. Univariate Analysis Examining Effects of Gender, Age, and Group on CAS-Specific Item Total.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENDER</td>
<td>337.393</td>
<td>.463</td>
<td>.500</td>
</tr>
<tr>
<td>AGECAT</td>
<td>750.487</td>
<td>1.031</td>
<td>.315</td>
</tr>
<tr>
<td>GROUP</td>
<td>673.786</td>
<td>.926</td>
<td>.341</td>
</tr>
</tbody>
</table>

R Squared = .054 (Adjusted R Squared = -.009)

When the totals were compared by age category, the means were not significantly different, yet age was the variable that resulted in the lowest \( p \)-value in both statistical tests. This was noteworthy to the researcher because the way in which different generations judge the role of technology in the classroom has been affected by their experience. As technology is incorporated into more mathematics classrooms in ways that help students understand the material, the acceptance and ability to use technology as a tool will change.

In order to explore the age factor further, an independent samples \( t \)-test was conducted on the treatment group only for the Final Exam. The null hypothesis statistically tested was: There was no statistically significant difference between the nontraditional students’ and the traditional students’ scores on the Final Exam. The \( t \)-test
results indicated that the hypothesis could not be rejected (see Table 4.10). However, exam scores showed that the nontraditional students performed somewhat better than did the traditional students on the Final Exam. It was also true that in the control group the mean score for the nontraditional students on the final exam was higher, but not significantly so (see Table 4.11). So even though use of technology and course structure did not significantly help one age group more than another, it is important to note that even though most of the older students had not had experience with powerful technology during their K-12 education, the way many of the younger students had, the use of the new technology did not interfere with their achievement in this study.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nontraditional</td>
<td>14</td>
<td>156.71</td>
<td>30.454</td>
<td>.832</td>
<td>.416</td>
</tr>
<tr>
<td>Traditional</td>
<td>11</td>
<td>145.45</td>
<td>35.887</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equal variances were not assumed.

Table 4.10. Comparison of Mean Scores for Traditional and Nontraditional Students in the Treatment Group on the Final Exam Using an Independent Samples t-Test.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nontraditional</td>
<td>11</td>
<td>155.55</td>
<td>26.576</td>
<td>1.425</td>
<td>.168</td>
</tr>
<tr>
<td>Traditional</td>
<td>13</td>
<td>138.38</td>
<td>32.428</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.11. Comparison of Mean Scores for Traditional and Nontraditional Students in the Control Group on the Final Exam Using an Independent Samples t-Test.
Controlling for Pretest Measure

The control and treatment groups were also compared using univariate analysis by taking into account the pretest scores when comparing means for the Final Exam and CAS-specific items. Results indicated that neither the pretest nor the group were significantly related to the students' score on the Final Exam (see Tables 4.12 and 4.13). Scores on the pretest were very low in both treatment and control groups (16% and 15%, respectively) which can account for the lack of correlation between pre- and post-measures for achievement.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRETEST</td>
<td>621.077</td>
<td>.902</td>
<td>.348</td>
</tr>
<tr>
<td>GROUP</td>
<td>313.681</td>
<td>.455</td>
<td>.503</td>
</tr>
</tbody>
</table>

R Squared = .031 (Adjusted R Squared = -.014)

Table 4.12. Univariate Analysis Using Pretest Scores as a Covariant and Fixing Group on Final Exam Total.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRETEST</td>
<td>614.447</td>
<td>1.006</td>
<td>.321</td>
</tr>
<tr>
<td>GROUP</td>
<td>529.947</td>
<td>.868</td>
<td>.357</td>
</tr>
</tbody>
</table>

R Squared = .043 (Adjusted R Squared = -.002)

Table 4.13. Univariate Analysis Using Pretest Scores as a Covariant and Fixing Group on CAS-Specific Item Total.
Effect on Attitude

Research Question 2. What is the effect of the use of CAS and graphing technology on remedial college mathematics students' attitudes toward, beliefs about, and confidence in mathematics? In order to examine the attitudes of students toward mathematics, the Fennema-Sherman (1976a) scale for Attitude Toward Success in Mathematics and revised Round (1998) scales for Confidence in Learning Mathematics and Mathematics Usefulness were given at the beginning and end of both terms to all participants. Complete descriptions of these scales can be found in Chapter 3.

The data collected from the Fennema-Sherman scale (1976a) and revised Round scales (1998) were re-coded in order to account for the wording of some questions as positive and others as negative. Responses ranged from strongly agree, which was given a score of 1, to strongly disagree, which was scored as a 5. A drop in participants’ scores would therefore indicate a more positive attitude on the scale.

An end-of-term questionnaire adapted for technology usage was also administered as a writing assignment to participants in the control and treatment groups at the end of the each term (see Appendix A). Even though an unstructured interview was planned, the data collected were not used in this analysis due to the small number of students willing to participate.

Attitude scores were taken only from the 50 participants used in the achievement analysis. In order to compare pretest and posttest measures, students needed to be present
and complete both measures. Due to absences, 39 of the 50 students used in the achievement analysis could be included in the attitude analysis. Refer to Table 4.14 to see the breakdown of students included in the attitude analysis.

<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Gender</th>
<th>Age Category</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>M</td>
<td>F</td>
</tr>
<tr>
<td>Control Group</td>
<td>21</td>
<td>9 (43%)</td>
<td>12 (57%)</td>
</tr>
<tr>
<td>Treatment Group</td>
<td>18</td>
<td>5 (28%)</td>
<td>13 (72%)</td>
</tr>
</tbody>
</table>

T = Traditional, N= Nontraditional


Attitude Scales

Attitude Toward Success in Mathematics

The full version of the Attitude Toward Success in Mathematics developed by Fennema and Sherman (1976a) was used in this study. On each of the 12 statements, students indicated their attitude from strongly agree to strongly disagree. Scores could range from 12 to 60, where 12 resulted from strong agreement on all items, and 60 resulted from strong disagreement on all items. A neutral response for all statements would result in a score of 36 for this scale.

An independent samples t-test showed no significant difference in attitude toward success between the control and treatment groups on the pretest administration of the
scale (see Table 4.15). The treatment group showed a slightly more positive attitude toward success in mathematics. However both groups’ means indicated a generally positive feeling, since the mean values both lie below a completely neutral mark of 36.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>21</td>
<td>29.333</td>
<td>5.379</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment Group</td>
<td>18</td>
<td>26.500</td>
<td>7.278</td>
<td>1.363</td>
<td>.183</td>
</tr>
</tbody>
</table>

Equal variances were not assumed.

Table 4.15. Comparison of Mean Scores for Treatment and Control Groups on Pretest Attitude Toward Success Scale Using an Independent Samples t-Test.

A paired samples t-test was used to compare the pretest scores with the posttest scores. This statistical test matches participants’ scores on the pre- and posttests and compares whether the change over the entire sample is significant. For both the treatment and control groups the change in scores for the Attitudes Toward Success in Mathematics scale were significant at the .01 level and both groups changed their attitudes in the positive direction (see Table 4.16). This indicated that over the semester the students began to anticipate that success in mathematics would result in positive consequences.
<table>
<thead>
<tr>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Paired Differences</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Group</td>
<td>Pre</td>
<td>21</td>
<td>29.333</td>
<td>5.378</td>
<td>7.095</td>
<td>5.319</td>
<td>6.113</td>
<td>.000</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>21</td>
<td>22.238</td>
<td>6.387</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Treatment Group</td>
<td>Pre</td>
<td>18</td>
<td>26.500</td>
<td>7.278</td>
<td>6.139</td>
<td>7.851</td>
<td>3.317</td>
<td>.004</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>18</td>
<td>20.361</td>
<td>7.336</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.16. Comparison of Pre- and Posttest Scores for Attitudes Toward Success in Mathematics Scores for the Treatment and Control Groups Using a Paired Samples t-Test.

Since both groups had a significant change in attitude it appears something happened during the semester to make the students feel more positive; however, technology cannot be identified as the reason for change. As explained in previous chapters the structure of the course was changed for both groups using the suggestions from Bonk and Kim (1998), NCTM (2000), and AMATYC (Cohen, 1995). The combination of these changes may have played a role in the change in attitude.

**Confidence in Learning Mathematics Scale**

This scale was used to judge whether students increase their belief in their own ability over the term. If students strongly agree (have high confidence), with each statement they would accumulate a score of 6, while a set of all negative responses, strongly disagree (have low confidence), would result in a score of 30. Students who responded as neutral to all questions would receive a score of 18.
Again an independent samples $t$-test was used to determine whether the groups began the treatment with feelings of confidence that were statistically the same. This statistical test failed to reject the null hypothesis that the means were equal (see Table 4.17).

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$t$</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td>21</td>
<td>22.071</td>
<td>5.459</td>
<td>.910</td>
<td>.370</td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td>18</td>
<td>20.222</td>
<td>6.984</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equal variances were not assumed.

Table 4.17. Comparison of Mean Scores for Treatment and Control Groups on Confidence in Learning Mathematics Scale Using an Independent Samples $t$-Test.

A paired samples $t$-test was used to examine whether the control students and the treatment students changed in their confidence level about their own ability to learn and perform the mathematics expected of them. The results from the $t$-test indicated that there was not a significant change in confidence in either group (see Table 4.18). At the beginning of the study, both groups’ means fell above the neutral score of 18 indicating a lack of confidence in their own ability. Both means dropped from pre to post, indicating a slight gain in confidence, with the treatment group moving within a point of that neutral score.
<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>t</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre</td>
<td>21</td>
<td>22.071</td>
<td>5.459</td>
<td>.405</td>
<td>2.782</td>
<td>.667</td>
<td>.513</td>
</tr>
<tr>
<td>Post</td>
<td>21</td>
<td>21.667</td>
<td>6.061</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pre</td>
<td>18</td>
<td>20.222</td>
<td>6.984</td>
<td>1.444</td>
<td>3.666</td>
<td>1.672</td>
<td>.113</td>
</tr>
<tr>
<td>Post</td>
<td>18</td>
<td>18.778</td>
<td>5.663</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.18. Comparison of Pre- and Posttest Scores for Confidence in learning Mathematics Scores for the Treatment and Control Groups Using a Paired Samples \( t \)-Test.

Mathematics Usefulness Scale

Round’s (1998) shortened version of the original Fennema-Sherman (1976a) Mathematics Usefulness Scale was administered (see Chapter 3). Like the Confidence Scale, six statements were included such that if students strongly agreed with each statement (math is useful) they would accumulate a score of 6, while a set of all negative responses, strongly disagree (math is not useful), would result in a score of 30. Students who responded as neutral to all questions would receive a score of 18.

Scores on the pre-test were compared using an independent samples \( t \)-test to determine whether there was a statistically significant difference in views of usefulness at the beginning of the treatment (see Table 4.19). The null hypothesis that the mean scores were equal was not rejected. Both group means were well below the neutral score of 18 indicating that the students did see mathematics as useful to their present and future endeavors.
<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$t$</th>
<th>Significance (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td>21</td>
<td>14.143</td>
<td>2.555</td>
<td>1.550</td>
<td>.134</td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td>18</td>
<td>12.167</td>
<td>4.866</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equal variances were not assumed.

Table 4.19. Comparison of Mean Scores for Treatment and Control Groups on Pretest Mathematics Usefulness Scale Using an Independent Samples $t$-Test.

Using a paired samples $t$-test, the mean differences from pre- to post measures were examined. The statistical test indicated that there was not a significant change in views of the usefulness of mathematics (see Table 4.20). This was the only measure that indicated any change in the negative direction. The mean treatment score did rise less than a point for this measure.

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>$t$</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Control Group</strong></td>
<td>Pre</td>
<td>21</td>
<td>14.143</td>
<td>.238</td>
<td>2.555</td>
<td>.422</td>
<td>.678</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>21</td>
<td>13.905</td>
<td></td>
<td>3.700</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Treatment Group</strong></td>
<td>Pre</td>
<td>18</td>
<td>12.167</td>
<td>-.500</td>
<td>4.866</td>
<td>-.686</td>
<td>.502</td>
</tr>
<tr>
<td></td>
<td>Post</td>
<td>18</td>
<td>12.667</td>
<td></td>
<td>4.614</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4.20. Comparison of Pre- and Posttest Scores for Mathematics Usefulness Scale for the Treatment and Control Groups Using a Paired Samples $t$-Test.
Analysis by Age and Gender

As with achievement, the researcher used a multiple regression analysis to explore the effects of age and gender on the attitudes in both groups. Only posttest attitude scores were used in this analysis. By using the additional variables of age and gender, the multiple regression indicated that the attitudes of students did seem to cluster by age, gender, and/or exposure to technology in the Confidence and Usefulness scales. On the Attitudes Toward Success in Mathematics scale, the univariate analysis failed to indicate that the gender, age, or group could be shown as significant variables in determining the score (see Table 4.21). The scores on the confidence subscale did seem to cluster by both age and group; however, the p-values were .03 and .02 respectively (see Table 4.22). Even though these results should indicate that the grouping most likely did not occur by chance, one must be careful when too many statistical tests are run on a set of data, because the chance of Type II errors increases (Hopkins, Hopkins, & Glass, 1996). Likewise, gender appeared as a significant factor on the Confidence posttest (see Table 4.23). Here the p-value was much stronger, indicating there is more than a 99% certainty that the result was not due to chance.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENDER</td>
<td>30.359</td>
<td>.619</td>
<td>.437</td>
</tr>
<tr>
<td>AGECAT</td>
<td>25.385</td>
<td>.518</td>
<td>.477</td>
</tr>
<tr>
<td>GROUP</td>
<td>58.696</td>
<td>1.197</td>
<td>.282</td>
</tr>
</tbody>
</table>

R Squared = .053 (Adjusted R Squared = .031)

Table 4.21. Univariate Analysis Examining Effects of Gender, Age, and Group on the Attitude Toward Success in Mathematics Subscale Results.
<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENDER</td>
<td>64.046</td>
<td>2.266</td>
<td>.141</td>
</tr>
<tr>
<td>AGECAT</td>
<td>144.510</td>
<td>5.114</td>
<td>.030</td>
</tr>
<tr>
<td>GROUP</td>
<td>169.784</td>
<td>6.008</td>
<td>.020</td>
</tr>
</tbody>
</table>

R Squared = .244 (Adjusted R Squared = .177)

Table 4.22. Univariate Analysis Examining Effects of Gender, Age, and Group on the Confidence in Learning Mathematics Subscale Results.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>GENDER</td>
<td>109.928</td>
<td>7.857</td>
<td>.008</td>
</tr>
<tr>
<td>AGECAT</td>
<td>.820</td>
<td>.059</td>
<td>.810</td>
</tr>
<tr>
<td>GROUP</td>
<td>45.146</td>
<td>3.227</td>
<td>.081</td>
</tr>
</tbody>
</table>

R Squared = .220 (Adjusted R Squared = .151)

Table 4.23. Univariate Analysis Examining Effects of Gender, Age, and Group on the Mathematics Usefulness Subscale Results.

Controlling for Pretest Measure on Each Subscale

The control and treatment groups were also compared using univariate analysis by taking into account the pretest scores when comparing means for the posttest for each subscale. Results indicated that the pretest was a significant factor in the posttest score at the .01 level. However, the group assignment was not significantly related to the students' score on the posttest even when the scores on the pretest were controlled (see Tables 4.24, 4.25 and 4.26).
<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRETEST</td>
<td>430.605</td>
<td>11.923</td>
<td>.001</td>
</tr>
<tr>
<td>GROUP</td>
<td>1.117</td>
<td>.031</td>
<td>.861</td>
</tr>
</tbody>
</table>

R Squared = .263 (Adjusted R Squared = .222)

Table 4.24. Univariate Analysis Using Pretest Scores as a Covariant and Fixing Group on Attitudes Toward Success in Mathematics Subscale Posttest.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRETEST</td>
<td>945.560</td>
<td>101.850</td>
<td>.000</td>
</tr>
<tr>
<td>GROUP</td>
<td>18.109</td>
<td>1.951</td>
<td>.171</td>
</tr>
</tbody>
</table>

R Squared = .754 (Adjusted R Squared = .741)

Table 4.25. Univariate Analysis Using Pretest Scores as a Covariant and Fixing Group on Confidence in Learning Mathematics Subscale Posttest.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>F</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRETEST</td>
<td>357.073</td>
<td>46.117</td>
<td>.000</td>
</tr>
<tr>
<td>GROUP</td>
<td>1.302</td>
<td>.168</td>
<td>.684</td>
</tr>
</tbody>
</table>

R Squared = .572 (Adjusted R Squared = .548)

Table 4.26. Univariate Analysis Using Pretest Scores as a Covariant and Fixing Group on Mathematics Usefulness Subscale Posttest.

**End of Term Questionnaire**

At the end of each term all students in both sections were asked to complete an end-of-term questionnaire as one of their writing assignments. By using the questionnaire as a graded assignment, more students were likely to complete the questions. This required that they place their names on the responses, possibly preventing them from being as critical as they might have been on an anonymous response.
Design of Course

When the researcher asked for participants from both the control and treatment groups each term, she explained that she was interested in looking at how different instruction helped or hindered students in their ability to better understand the content. In relation to this, on the end-of-term questionnaire students were asked to compare their experience in this remedial course with past experiences. After weeding out the students who responded that this was their first college course or that they could not compare this to another course, some interesting responses emerged in both the treatment and control groups.

Control Group. Many students referred to the instructor and the open atmosphere as positive elements that helped them learn and feel more confident. However, few saw their own effort as a real factor. One student said:

I believe the various approaches used were effective, but my lackadaisical, let’s get it over with attitude affected my achievement, my attitude, and my confidence (Female Nontraditional Student in Control Group 09).

The students seemed very appreciative of the effort put forth by the instructor to create a comfortable learning environment:

I liked how the teacher was open to comments, which gave a laid-back atmosphere. It was easy to learn and re-learn a lot of things I had forgotten how to do (Female Traditional Student in Control Group 10).

The culture created in class and many of the activities and resources were also cited as beneficial:

I did think the approach i.e. present, practice, homework, review, was good and I always felt that the atmosphere in the classroom encouraged participation and questions (Female Nontraditional Student in Control Group 11).
Not only did these comments support the decision to choose an instructor who was using a student-centered approach, but they also highlighted the fact that the control section was more traditional in structure. The overall sense of answers about the class was positive, but this may have been affected by the fact that students put their names on responses. A few negative comments were also included. A couple of negative responses referred directly to the fact that the order of material had changed and did not mirror the text.

I thought it was rather confusing the way we jumped around from section to section, and how we really never followed the book (Female Traditional Student in Control Group 12).

These comments reinforced my feelings that the control students were exposed to many of the teaching strategies that are recognized for adult learners, but the sense that mathematics is a subject to be practiced and memorized remained in many of the responses, even when alluding to more reform-based practices such as questioning and group use.

There was more opportunity for practice, with the opportunity for assistance by the instructor. This is also the first time I have had opportunity to work with other students in class to compare how and if our functions differ (Female in Control Group with Unknown Age 13).

Treatment Group. Again in the treatment group much of the credit for a successful course was attributed to the instructor and the atmosphere in the classroom, with very little attributed to their own effort. This is exemplified by the comment:

I feel the teacher has a lot to do with what is taught. If they are very knowledgeable in their field they have many ways of explaining it (Male Nontraditional Student in Treatment Group 14).
Within these comments about the instructor, however, there was more of a sense that the students played a prominent role in the direction of the instruction. For example, one student mentioned the instructor’s practice of providing alternative solution methods, but then went on to relate that to herself:

You showed us more than one way to do a problem and showed us our way (if we had one) and why or why not it was a good way to solve a problem (Female Nontraditional Student in Treatment Group 04).

There were a few comments that mentioned the benefits of working with others as well.

As with the control group, these students had primarily positive comments to share about the class, but I think that was skewed because they were asked to put their names on the assignment. The only negative comment dealt with the fact that a student did not feel that the assignments helped him prepare, and he was unhappy with the way that assessments were arranged and used to determine his grade. Just as in the control group one student did mention, within a response to a separate question, that the order of topics was a problem for her.

We jumped around from chapter to chapter. There didn’t seem to [be] much of a flow of consistent lessons (Female Nontraditional Student in Treatment Group 01).

**Class Activities**

Both groups were also asked about the activities that helped them the most in the course. In the treatment group, the majority of students focused on nontechnology assignments because a separate question had already specifically asked them to talk about how technology had affected their understanding. Both groups described similar
experiences. The focus in the control group was slanted to traditional practice and the benefits that it provided in learning, but the treatment group seemed more focused on the discussion around these activities.

*Control Group.* The most frequently referenced aids to success mentioned were the homework and review assignments. Homework came primarily from the text while the review sheets were both practice tests created by the researcher and commercially published puzzles (Marcy & Marty, 1996a; 1996b). The commercially produced puzzles were self-checking and consisted of drill and practice only, whereas the researcher included some open-ended thought questions on practice tests. Answers and possible solution paths were provided to students for the homework questions and reviews. This was the practice of the treatment instructor in past classes and was also made available to the control group. Several students thought this practice was beneficial to their learning.

I’m a visual person, so just simply going over concepts in class helps me. Or being able to see the answers to homework also helps as a good review for future exams (Female Traditional Student in Control Group 15).

Other comments about homework and reviews focus solely on the repetition and practice.

Homework assignments were the most beneficial. They forced me to understand the concepts presented in class and the book. Working repeated problems helped reinforce the material (Male Nontraditional Student in Control Group 16).

Another student commented:

The homework and worksheets handed out [were most helpful]. The repetition of the worksheets helped to indelibly etch concepts and processes in my head (Male Nontraditional Student in Control Group 17).
In reference to the review sheets one student commented:

The review sheets with answers so I could practice myself before the test (Female Traditional Student in Control Group 18).

A few students also mentioned working in groups, in-class activities, and the tutor room as other areas that benefited them.

In class practice. Because I was able to discuss the subject with my classmates and my tutor and be guided in the right direction (Female in Control Group with Unknown Age 13).

Each time students in the treatment group completed an activity using the CAS, a similar activity was used in the control group. One student referenced these in-class activities.

The most beneficial were the in-class math participation activities. While homework is extremely important, the in-class better helped me in understanding the concepts of algebra. Because the in-class activities had the advantage of the teacher being there (Female Nontraditional Student in Control Group 09).

Treatment Group. The responses provided in the treatment group seemed to fall primarily in two categories – working in groups and discussions during class. The use of technology forced the use of groups from the very beginning of the term and the instructor and students continued to rely on groups even when technology was not being used as a way to help students understand and get different perspectives.

We were allowed to work in groups during class time. Even some students exchanged phone numbers. We even talked about our class work/homework before class started. Each person has their own method of solving problems. And it is great when we all come up with the same answer. (Female Traditional Student in Treatment Group 19)
Discussion and questions were also a major part of the instruction in the treatment classroom.

Class discussions and activities not involving technology did help me the most. I think group activities and being able to discuss the subject (homework) with classmates was beneficial. Sometimes just going over the material in class without discussion didn’t make things sink in. Sometimes it would hit me what I was doing wrong when involved with others (Female Traditional Student in Treatment Group 20).

This focus on discussion and sharing shifted the focus from drill and practice to more understanding. This can also been seen in a response about the open-ended questions that were used.

I also think the worksheets that made us explain why the non-technological way was different than the calculator exercises improved our understanding as a class in whole because it made us analyze and understand the basic components of math (Female Nontraditional Student in Treatment Group 21).

Even though many students included comments that went beyond the learning of processes, references were still made to outlines provided for notes, homework, and reviews as a way to help them learn the needed processes.

The practice materials (pink sheets) as they were good examples to see if you knew what you were doing and if you didn’t, you could make sure you asked before test time (Female Traditional Student in Treatment Group 05).

Writing Assignments

Writing has been used in mathematics courses by the researcher as a way to help students focus more on meaning than memorizing. When using technology, writing can also be a tool to force the students to think about what they are doing with the calculators. In this study, writing was introduced into both groups to remove the difference in communication and discourse that these assignments would add. As a result, I was
interested in seeing how the students perceived these writing assignments and if they felt they had gained from them. Writing assignments ranged from explaining a strategy to reflecting on one's own practice and experience. Writing assignments were included in the final grade, but accounted for only about 50 (or 8%) out of the 600 points possible for the course.

Control Group. In general the students in the control group received less feedback on writing assignments than did those in the treatment group, and that may have had an effect on the overall attitude toward them. However, more than twice as many students had positive things to say about writing in mathematics than did those who felt they were not beneficial at all. Some students identified the more personally oriented assignments as most beneficial.

The assignments provided an opportunity for me to be self-critical regarding my performance (Male Nontraditional Student in Control Group 17).

Others had a change of heart as the term progressed.

Initially, I thought they had no foundation, but as time progressed, I realized how each built on the other and formed a pattern. Especially when I did not do well in my quizzes, they made me realize that I needed a new approach and extra help (Female in Control Group with Unknown Age 13).

Positive comments also included those from individuals who saw writing as an opportunity to put into words what they were thinking.

They forced me to explain the concepts in my own words, reinforcing those concepts (Male Nontraditional Student in Control Group 16).
Treatment Group. The student responses regarding writing in the treatment group were overwhelmingly positive and contained many of the same ideas that students in the control group had expressed. Some mentioned the understanding they had gained.

Those assignments really did help me understand the material better because it made me actually think about what I was doing. By writing it down it helped the material sink in (Female Traditional Student in Treatment Group 20).

Others found benefit in reflecting upon their own practice and effort.

I loved it! Writing about/voicing my feelings and understanding about math helped me to admit failings and strengths and take responsibility for them (Female Nontraditional Student in Treatment Group 06).

The writing assignments were great because it made me exam myself and what I was learning. It also helped me realize my mistakes, identify what I might need to work on. I feel this gave me some control and better understanding of how I learn (Female Traditional Student in Treatment Group 08).

Those students who did not see benefit in the assignments for the most part either did not explain or said that practicing problems was the best way of studying mathematics. One student who did not see personal benefit in the assignments did acknowledge that they provided more information to the instructor.

I don’t believe that I benefited from this area but I think it is a good way for a teacher to gauge student progress (Male Nontraditional Student in Treatment Group 02).

Technology Usage

Students in the treatment group were asked one additional question: whether using technology to explore new material had helped or hindered their understanding. Students also used this as an opportunity to express their feelings toward the use of
technology in the classroom. Overwhelmingly, the positive comments centered on being able to check work and get immediate feedback.

I feel that the technology introduced was very helpful. I used the TI-92s to check homework and various other assignments after they were graded to understand minor errors I had made (Male Nontraditional Student in Treatment Group 02).

The technology used in class to introduce concepts did help me to understand more. For example, the use of the TI-92 calculators helped me to see how and why certain math problems are solved the way they are solved. Sometimes you had to input the problem differently than it was on paper to get the correct result, which required a good sense of order of operations. I think it encourages understanding (Female Nontraditional Student in Treatment Group 21).

The use of technology was helpful in the fact that you could check your work and play around with the different equations and/or solutions (Female Nontraditional Student in Treatment Group 04).

Two students also praised the calculator for helping them see the mathematics differently.

Even though several students had positive comments, a large number of students were frustrated that the calculators took up time and could not be used on assessments. Even students who found benefit in the calculator use like Female Nontraditional Student in Treatment Group 04 from above did not like the fact that the calculators could not be used for the exam. Another student explained:

Learning the TI-92 calculators to check work was great, but nothing but a tease because they are not allowed on [assessments] (Female Nontraditional Student in Treatment Group 01).

The students attributed very little of their new mathematical understanding to the technology activities, and saw its primary benefit as the immediate feedback provided. Some even feared that they could have done better on the procedures had there been more lecturing instead of exploration. Even though the students did not see the role of technology as completely as it was intended, that is understandable given the fact that
they would be assessed on procedures in order to measure their understanding. However, achievement data indicated that the time spent with the calculators, as opposed to paper-and-pencil practice, did not put them behind the control group.
CHAPTER 5

CONCLUSIONS

The current study examined the effects of using a computer algebra system (CAS) on the achievement and attitudes of students in remedial college mathematics. The participants in the study were enrolled in night sections of a basic algebra course offered at a large research university in the Midwest. These students had been identified for remedial work by the departmental placement exam, and did not receive college mathematics credit for the course. Successful completion of this course was required before they could enroll in credit-earning mathematics coursework. The students came from various mathematical backgrounds, but they all had experienced a cycle of mathematical failure that needed to be broken.

Both adult learning theory and social constructivist theory informed the design of the research project, and influenced the instruction planned for the sections involved. Even though both of these theories were used in the overall design, the use of technology with the treatment group further supported and better facilitated success for both traditional and nontraditional students. A CAS was used to offer students a fresh look at mathematics previously studied, to allow for exploration of a procedure before the abstract rule was presented, and to provide immediate feedback for the students. The use
of CAS was primarily focused on the rules of exponents, solving linear equations in one variable, and graphing linear equations in two variables. Despite the focus on these areas, the technology was available at all times in the classroom for students in the treatment group, except during assessments.

The effect of CAS on achievement was examined by comparing scores from the three tests developed by the researcher and the departmental Final Exam. These assessments focused primarily on pencil-and-paper procedures and were designed to reflect the curricular goals of the department. Independent samples $t$-tests were used to compare the means of the treatment ($n=25$) and control ($n=25$) groups on each assessment as well as on CAS-specific questions. Multiple regression analysis was used to factor in the effect of gender and age on both the total Final Exam scores and the scores on CAS-specific questions involving rules for exponents, solving linear equations, and graphing linear functions.

Affective factors were examined quantitatively using the Attitude Toward Success in Mathematics Scale (Fennema & Sherman, 1976b), and Round's (1998) revised Confidence in Learning Mathematics Scale and Mathematics Usefulness Scale. Paired sample $t$-tests were conducted in order to identify any significant changes in attitudes, and multiple regression analysis was employed to determine whether gender and/or age were significant factors in attitude results. The end-of-term questionnaire designed by the researcher provided qualitative data for additional investigation of student attitudes and beliefs.
Major Findings

Important information can be gleaned from these results that address the needs of remedial mathematics students at the university level. The following list delineates significant results in the areas of achievement and attitudes:

1. Exam Scores in Relation to Time Spent With Technology
   
a. Some students expressed concern over the time spent with technology, and felt that since the CAS was not available for assessments it should not be used in class. However, the increased time spent using the CAS to perform algebraic and numeric manipulations did not negatively affect students in the treatment group on the assessments, which primarily involved paper-and-pencil manipulations.

b. On both Tests Two and Three, the mean for the treatment group was a letter grade higher than that of the control group, though their scores on the achievement pretest did not differ significantly.

2. Differences in Attitudes
   
a. Gender was a significant factor in student scores on the Confidence in Learning Mathematics Scale.

b. There was a significant positive change from pretest to posttest in the mean scores for both treatment and control groups on the Attitudes Toward Success in Mathematics.
3. Differences in Classroom Culture

a. The students in the treatment group seemed more willing to work together, to share strategies in small and large group discussions, to suggest alternative solution methods, and to seek validation from sources other than the instructor (such as each other and the technology itself).

b. Students in both treatment and control groups attributed success more to the teacher than to their individual efforts. However, comments from the treatment group indicated that they saw themselves as an important component in the learning environment, and that their ideas played a role in how material was approached in the classroom.

c. The control group students identified homework and practice as the aspects of the course most likely to lead to understanding; whereas, the treatment group students indicated that working with classmates and class discussions were primary factors in their understanding.

4. Positive Instructional Aspects Identified by Students

a. The treatment students identified the immediate feedback provided by the CAS when checking answers as a means for increasing their understanding of mathematical procedures and improving their abilities to perform them.

b. Students from both control and treatment groups indicated that writing, working together, and being able to ask questions were all positive aspects of the course design.
Achievement Conclusions

Data sources used to examine the achievement of students in the remedial course were three regular tests, the Final Exam, and CAS-specific questions from these assessments. Even though CAS technology was used with the treatment students, both control and treatment groups completed the same assessments. As a result of the analysis conclusions were drawn as to whether the use of CAS technology affected the achievement of students enrolled in the treatment sections.

*Exam Scores in Relation to Time Spent With Technology*

Independent samples *t*-tests were used to compare mean scores of the treatment and control groups on all three tests, the Final Exam, and CAS-specific questions from these assessments. No statistically significant differences were found between any of the means. The CAS did not appear to positively affect the pencil-and-paper performance of the treatment group; however, the time spent exploring patterns and performing manipulations with the aid of the CAS did not prevent these students from performing at a level at least equivalent to that of the control students. On each achievement measure used, the treatment group mean was higher than that of the control group, although not significantly so. On the second and third tests, students in the treatment group had means in the C and B ranges, respectively, while the control group means fell within the D and C ranges, respectively. Multiple regression analysis was also used to determine if gender and age categories were significantly linked to achievement on the Final Exam and on the CAS-specific questions. In both cases, the tests failed to identify any significant links between these factors and achievement.
The departmental goal of this remedial course is to provide students with the opportunity to review algebraic processes that are necessary for success in college-level courses. As a result, assessments are focused on exercises that require students to demonstrate their abilities to perform these procedures using paper-and-pencil methods. The use of a CAS directs students’ attention to choosing the operations to perform (a higher-order thinking skill) as opposed to the manual process of the manipulation (Kutzler, 2000; Waits & Demana, 2000a). Whether or not CAS use affected students’ abilities to choose appropriate strategies cannot be ascertained from the assessment data. By observing and questioning student thinking as they worked problems posed during the interview, the researcher might have gained insight into whether there was a difference in the abilities of students in the two groups to choose appropriate strategies in a setting where their facility in carrying out these operations by hand did not cloud the picture. However, due to the small number of participants (n=5) willing to complete an interview, these data were not sufficient for analysis.

Hillel et al. (1992) discussed the difficulty encountered as the mathematics department at Concordia University in Montreal, Canada attempted to restructure the remedial mathematics program. One part of the plan at this university was to eventually incorporate the use of CAS into each of their courses in the remedial track, thus providing students with a tool that could allow them to further explore mathematics, even though they would need to rely on technology for what many mathematicians would consider simple manipulations. Had the CAS been available for testing in the course used in the current study, the outcome might have been different.
When considering the use of CAS on exams, several factors arise. First, if the primary goal were pencil-and-paper manipulation skills, then access to CAS on exams would be contrary to this goal. The structure of the course in the present study did not support CAS use on assessment due to the focus on paper-and-pencil manipulation skills. Second, the availability of any type of technology requires assessment to change, and the use of CAS would imply the need to shift away from an emphasis on simple manipulation exercises to tasks of conceptual understanding and application. Third, whether the CAS can be used as a tool in subsequent courses must also be considered, especially when remedial courses are most likely preparing students for the required mathematics courses in their degree programs.

**Implications for Teaching**

In light of the current study and the results on assessments, there are several implications for using a CAS with remedial students. First, access to technology during each class meeting was a positive aspect of the design. Using TI-82s in the classroom was a more streamlined way of incorporating technology than had previously been used in other experiments in which a CAS was added as a computer lab component (e.g., Mayes, 1995; O’Callaghan, 1998). The availability of the CAS facilitated scaffolding of student learning, timely feedback, and use of alternative paths to problem solutions all of which are identified as instructional methods that support learning for all students in the theoretical model in Chapter 1.
A second implication is that even though change in practice is a slow process, the addition of technology to an existing course will be more meaningful if assessment is altered in a way that parallels the use of technology in everyday activities. Several students in the treatment group expressed concern over the availability of a CAS during classes, but not during assessments. This sentiment surfaced both during the term (especially the first times technology activities were completed) and then again on the end-of-term questionnaire. The mean for the treatment group was higher than that of the control group on all assessments, although not significantly, indicating that the use of technology did not negatively affect student performance on paper-and-pencil manipulation exercises. On the contrary, by incorporating technology use and exploration involving the CAS, classroom norms could be established that supported the exploration of mathematical meaning.

A third implication to consider when technologies that are unfamiliar and possibly intimidating to many of the students is to make a concentrated effort to match students who are more comfortable with technology with those who are less experienced. These pairings may eliminate the confusion and frustration felt by some who were overwhelmed by the simple mechanics of operating a graphing calculator. The goals of university mathematics courses are many and diverse, but for instructors who consistently seek opportunities to deconstruct student misconceptions developed in prior mathematics experiences, CAS may be a tool to reach this goal.
Implications for Research

In light of the disconnect between the classroom use of CAS and the paper-and-pencil manipulation skills emphasized on assessments, further research to explore CAS in a remedial environment completely designed for CAS use would be beneficial. One possible course for future research is to determine whether changing the class structure might result in different outcomes. For example, the university in which the current study was conducted is on an academic quarter system, allowing for less than ten weeks of exposure to the CAS. Would more sustained use of the technology on a semester calendar allow for greater benefits as the students became more familiar with this type of learning? In addition, if a CAS was consistently offered in relevant and consecutive mathematics courses, would students become even more comfortable with its use throughout their mathematics education?

Affective Conclusions

Attitude was examined in addition to achievement. Scores from the affective instrument, a researcher constructed open-ended questionnaire, and classroom observations were used to inform conclusions concerning student attitudes and beliefs. A detailed discussion of these instruments can be found in Chapter 3.

Differences in Attitudes

Paired sample t-tests were used to test for significant change between pretest and posttest measures on Fennema-Sherman’s (1976a) Attitude Toward Success in Mathematics Scale and Round’s (1998) revised Confidence in Learning Mathematics and
Usefulness of Mathematics scales. A paired sample t-test indicated significant change 
($p < .01$) on the Attitude Toward Success measure in both the control ($p \leq .000$) and 
treatment ($p \leq .004$) groups. Further analysis using multiple regression indicated that 
there was a significant link ($p < .01$) between gender and the results on the Mathematics 
Usefulness Scale ($p \leq .008$). Multiple regression analysis also indicated that age ($p \leq .03$) 
and group ($p \leq .02$) were significantly linked ($p < .05$) to results on the Confidence in 
Learning Mathematics Scale. However, this evidence is less conclusive than the data for 
the Mathematics Usefulness Scale, given the number of statistical tests run on such a 
small amount of data. Hopkins, Hopkins, and Glass (1996) warn that the chances of Type 
II errors increase when multiple statistical tests are conducted on the same set of data.

Even though these results do not indicate any significant differences between the 
control and treatment groups, there is some indication that the design of the courses 
affected the attitudes of students. In all cases, with the exception of the Mathematics 
Usefulness Scale for the treatment group, the mean scores changed in a positive direction, 
indicating that the students felt that success in mathematics would benefit them, and that 
they were more confident in their abilities to do mathematics. The Usefulness measures 
were already positive on the pretest for both groups, which did not leave much room for 
improvement.

Differences in Classroom Culture

After visiting the classrooms of both groups throughout the terms, it was apparent 
to the researcher that the culture created in the treatment section was different from that 
in the control section. In Chapter 3, a description of the courses is provided for the reader.
to compare. From these observations and by reading students’ responses on the end-of-term questionnaire, themes of difference became evident. It is impossible to attribute the differences strictly to the use of technology, but the two courses had differing qualities that emerged through the process of data analysis.

*Discourse.* Group work was included in both sections. However, the treatment students seemed to use sharing with other students at a much deeper level. After interpreting responses on the questionnaire and visiting the classroom, it became evident that students in this section not only consulted classmates on results, but also worked together on solution strategies. This implied that validation was sought not only from the instructor, but also from fellow classmates. In the control group, instructor validation seemed to be valued above student interaction, and some students even lamented the fact they could not have more one-on-one time in class with the instructor. From data obtained during classroom observations of the control group, the researcher concluded that students were more likely to merely compare results with each other rather than question one another on the processes themselves. In the treatment classroom, students in both small group and large group discussions often suggested alternative approaches to problems. The treatment instructor was praised for her ability to use “student methods” when teaching.

*Attribution of Success.* In both groups, students gave the instructors credit for student understanding with little mention of their own efforts. However, when discussing what helped them understand the material, differences can be seen. The control group used language indicating that learning mathematics entailed practicing
procedures provided in lectures. The treatment group, however, indicated that discussion of the various approaches and problems, along with the practice afforded by homework, provided the most productive means for increasing their understanding. The implication is that these students are beginning to see mathematics as more than a set of magical processes to be memorized.

*Teaching Strategies.* The outcomes described above lead the researcher to conclude that constructivist theory provides insights into the difficulties of teaching adult learners. The students responded positively to some of the teaching strategies suggested within the conceptual framework of the current study (see Chapter 1 for a detailed explanation). Specifically, the use of discourse allowed the treatment students to become more involved in the learning process. Bonk and Kim (1998) encouraged adult educators (a) to use questioning and discussion to highlight areas of understanding and misunderstanding, (b) to expect students to justify their solutions and share discoveries or relevant experiences, and (c) to respect and encourage the use of alternative forms of solving problems, especially with skills they use outside the classroom. Many students in the treatment group identified positively with these instructional strategies as evidenced by their actions and comments. Students were also affected by social norms that are not present in many remedial classrooms. Bonk and Kim identified important norms for adult classrooms including (a) encouraging exploration and reflection as a means for constructing knowledge, (b) providing students with models for exploring mathematics, and (c) modeling thinking processes for students and then expecting them to do the same.
when discussing problems. As the current study demonstrates, these aspects of the classroom can be incorporated without technology, but they are definitely facilitated by its use.

Implications for Teaching

The fact that the students responded to many of the specific teaching strategies planned for the research is further evidence that when teachers are cognizant of the learners' needs in their particular environment, students respond. The present study addressed the attitudes and beliefs of participants by using research-based techniques for instruction, incorporating the use of technology, choosing instructors who believed in supporting student learning, and using writing assignments as opportunities for students to express and even confront feelings. Hence, adult learning strategies need to become a more significant component of the professional development of university mathematics instructors. Finding ways to integrate both the vast research on appropriate teaching strategies for adolescents and the growing literature on adult learning with research on teaching mathematics in general will enable instructors to meet the needs of the increasingly wide range of students found on college campuses.

Implications for Research

It follows from the teaching implications that researchers must continue to explore adult learning in general, develop more complete learning theories for this population, begin to identify specific strategies that are particularly helpful for remedial students, and focus research in adult learning on content area issues. AMATYC is in the beginning stages of revising Crossroads (Cohen, 1995) in order to better reflect the
increasing knowledge in the fields of teaching and learning. Scholars in adult learning must also look at the suggestions from AMATYC and NCTM (2000) and begin to help instructors modify the best practices already known for younger learners for use in the college mathematics classroom.

Other Conclusions

The research questions focused the current research on achievement and attitudes. However, the analysis led the researcher to make additional conclusions concerning the instruction planned in relation to adult learning theory and social constructivist theory. Refer to Chapter 1 for a more complete discussion of these theories. Even though different instructors were used with the control and treatment groups, these instructors were already implementing many of the same teaching strategies in their teaching, including several that are directly relevant to a course taught from a social constructivist perspective. Both instructors were providing their students with a learning environment that support the course design as discussed in Chapter One and therefore teacher difference was not considered a variable in the design.

Positive Instructional Aspects Identified by Students

A review of the research on learning was a major factor in planning the instructional activities for this project. Information gleaned from this review affected the way in which the researcher observed the classroom. Through analysis of the quantitative data, it became clear that the students responded to many of the specific plans and
strategies that were intended to support their learning. This finding leads the researcher to conclude that classroom activities and their effects on the development of classroom culture increase positive outcomes for college mathematics students.

Adults are often more willing than younger students to spend long periods of time studying content presented in the classroom, but at the same time, they will have great difficulty letting go of long-held misconceptions about that content (Schommer, 1998). The students in the current study identified the immediate feedback available through CAS technology as a benefit. When students used technology to perform manipulations or when they were checking their paper-and-pencil work, they were provided with immediate feedback on their attempts. They could not continue to practice a process incorrectly without some indication of a procedural problem, which limits the time many students might spend trying a process that does not work.

Brookfield (1991) suggested that environments providing opportunities for adult learners to conceptualize the subject matter and construct their own understandings are especially important. This sense of student autonomy was also highlighted in the work of Granott (1998), who espoused the theory of “developing learning.” Granott suggested that adult students (a) need to be given adequate time to construct an intuitive understanding, (b) should see merit in the effort to increase their knowledge, (c) need to have choice in approach, strategy, and activity, and (d) require that their needs be supported during challenging exercises. The students in the present study recognized some of these aspects within the instruction in both the treatment and control groups. Writing, which was positively viewed in both groups, seemed extremely important for the
treatment group to keep track of what that they were learning and to examine their own efforts and practice in the classroom. Autonomy was provided through the activity design, but students also found empowerment through the openness of the classroom culture. The ability to freely ask questions was mentioned by many students in both groups. By working in groups, students, especially those in the treatment group, found support and increased understanding. The students in the treatment group also received support through the ability of the technology to act as a scaffold in their explorations.

An area that needs more attention in the design of this course is that of helping students see relevance outside of the classroom for the subject matter they are studying. Even though meaning and connections were emphasized in the classroom by the instructors and discussed by the students, these did not constitute a large portion of the assessment, and therefore did not significantly impact student learning.

*Implications for Teaching*

The support and autonomy available when a CAS is incorporated into the classroom offer much promise for learning. Students can enter a learning task at various levels and progress according to their abilities. The present study has strengthened my resolve that when technology is added to the classroom, much change is required, and that time is necessary to improve the integration process. The perceived lack of connection between the CAS exploration and the assessment process hindered the ability of many students to gain full benefit from the activities. The focus on doing the mathematics—which for the students meant completing the process on paper—interrupted the ability of many students to focus on why the exploration was important. I feel this is
where time, the refinement of activities, and a change in assessment focus are key to increasing student learning and understanding in college mathematics courses. The two terms in which the present study was conducted provided many ideas for the researcher about ways to improve both the activities that were used and the overall integration of the CAS.

Implications for Research

As Dunham (2000) suggested, research with technology must become more specific. Further research is needed to pinpoint the role of writing, which was not a focus of the current study, in the integration of technology. Ways to provide adult and remedial students with the support they need through technology also offers a vast field for additional exploration.

Final Thoughts

Even the small number of results gained from the current study encourages me to find additional opportunities to explore CAS use. This technology was used with students who had already been exposed to many of the mathematical ideas, but a CAS also offers promise for students who are first learning the concepts. Finding ways to use technology to encourage deeper understanding is the responsibility of all mathematics educators. As all students are expected to solve mathematics problems at increasing levels of difficulty, technology can be incorporated as a viable tool to help students.
The positive response to the writing assignments also prompts me to look deeper into the student responses to discover ways to better incorporate these open-ended assignments into future mathematics courses. The students’ concern over time spent on technology reinforces the need to make its use and purpose clear. Modifying assessment to incorporate technology is one way to address this concern, but the way in which the instructor presents technology-based activities also plays a major role in the student perception.

The disconnect between the classroom exploration and assessment focus challenges me to explore effects on achievement and attitudes in a larger scale study. Using technology with a larger number of students and in more diverse settings could better uncover the role that technology can play in student understanding. Changing assessment to examine mathematical understanding by including more authentic problem solving, application of mathematical processes in unfamiliar situations, and discussion of the mathematical reasoning and relationships within processes could open the door to a better understanding of technology’s role in classrooms for all students.

Mathematics is a dynamic field of study, and teachers at all levels have the important responsibility to help their students construct meaning. Mathematics is used as a gatekeeper at many levels, thus understanding must be the goal of all teachers. Remedial mathematics students deserve instruction that can help them break the cycle of failure in which they find themselves. Technology can play an important role in offering these students the tools and opportunities they need to be successful.
Fennema-Sherman Attitudes Toward Success in Mathematics Scale (Fennema & Sherman, 1976a)

<table>
<thead>
<tr>
<th>Statement</th>
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<tbody>
<tr>
<td>It would make me happy to be recognized as an excellent student in math.</td>
<td>P</td>
</tr>
<tr>
<td>I’d be proud to be the outstanding student in math.</td>
<td>P</td>
</tr>
<tr>
<td>I’d be happy to get top grades in mathematics.</td>
<td>P</td>
</tr>
<tr>
<td>It would be really great to win a prize in mathematics.</td>
<td>P</td>
</tr>
<tr>
<td>Being first in a mathematics competition would make me pleased.</td>
<td>P</td>
</tr>
<tr>
<td>Being regarded as smart in mathematics would be regarded as a great thing.</td>
<td>P</td>
</tr>
<tr>
<td>Winning a prize in mathematics would make me feel unpleasantly conspicuous.</td>
<td>N</td>
</tr>
<tr>
<td>People would think I was some kind of grind if I got A’s in math.</td>
<td>N</td>
</tr>
<tr>
<td>If I had good grades in math, I would try to hide it.</td>
<td>N</td>
</tr>
<tr>
<td>If I got the highest grade in math I’d prefer that no one knew.</td>
<td>N</td>
</tr>
<tr>
<td>It would make people like me less if I were a really good math student.</td>
<td>N</td>
</tr>
<tr>
<td>I don’t like people to think I’m smart in math.</td>
<td>N</td>
</tr>
</tbody>
</table>

P = Positive Statement and N = Negative Statement

Note the word grind in question 8 was replaced with geek after consulting several mathematics and science education students about meaning.
### Fennema-Sherman Confidence in Learning Mathematics Scale adapted by Round (1998)

<table>
<thead>
<tr>
<th>Statement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Generally I have felt secure about attempting mathematics.</td>
<td>P</td>
</tr>
<tr>
<td>I am sure I can do advanced work in mathematics.</td>
<td>P</td>
</tr>
<tr>
<td>I have a lot of self-confidence when it comes to mathematics.</td>
<td>P</td>
</tr>
<tr>
<td>I’m not the type to do well in mathematics.</td>
<td>N</td>
</tr>
<tr>
<td>For some reason even though I study, mathematics seems unusually hard for me.</td>
<td>N</td>
</tr>
<tr>
<td>Most subjects I can handle okay, but I have a knack for flubbing up math.</td>
<td>N</td>
</tr>
</tbody>
</table>

P=Positive Statement, N=Negative Statement

### Fennema-Sherman Mathematics Usefulness Scale adapted by Round (1998)

<table>
<thead>
<tr>
<th>Statement</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I study mathematics because I know how useful it will be.</td>
<td>P</td>
</tr>
<tr>
<td>I’ll need a firm mastery of mathematics for my future work.</td>
<td>P</td>
</tr>
<tr>
<td>I will use mathematics in many ways as an adult.</td>
<td>P</td>
</tr>
<tr>
<td>Mathematics will not be important to me in my life’s work.</td>
<td>N</td>
</tr>
<tr>
<td>Taking mathematics is a waste of time.</td>
<td>N</td>
</tr>
<tr>
<td>In terms of my adult life it is not important for me to do well in mathematics in high school.</td>
<td>N</td>
</tr>
</tbody>
</table>

P=Positive Statement, N=Negative Statement
End of Term Questionnaire [Treatment Group]
We have used technology and inquiry-based teaching activities throughout Math 050. Each of the following questions asks you to reflect on your experiences participating in activities, completing assignments, and using technology during this quarter.

1. Did using technology to introduce new concepts help you understand the course materials? Explain how you feel the technology encouraged understanding and/or interrupted your understanding of the mathematics.

2. Did class discussions and activities not involving technology prove to be most beneficial in helping to build your understanding? Explain what non-technology activities you felt were most beneficial and what activities you felt interrupted your understanding of the mathematics.

3. Throughout the quarter you were asked to write about your understanding of concepts presented during class time. Explain how these assignments affected your understanding of the material.

4. Briefly compare and contrast your impressions of this course with other mathematics courses you have been enrolled in. Please be specific as to how various approaches to teaching affected your achievement, attitude, and confidence.
End of Term Questionnaire [Control Group]
We have used technology and inquiry-based teaching activities throughout Math 050. Each of the following questions asks you to reflect on your experiences participating in activities and completing assignments this quarter.

1. Did class discussions and activities prove to be most beneficial in helping to build your understanding? Explain what activities you felt were most beneficial and what activities you felt interrupted your understanding of the mathematics.

2. Throughout the quarter you were asked to write about your understanding of concepts presented during class time. Explain how these assignments affected your understanding of the material.

3. Briefly compare and contrast your impressions of this course with other mathematics courses you have been enrolled in. Please be specific as to how various approaches to teaching affected your achievement, attitude, and confidence.
Semi-Structured Interview Outline

The following outline will be used to start conversation with a small number of students in both the control and treatment groups. The interviewer will use this outline to begin a conversation about mathematics, but allow the interviewee to tell about his/her experience.

Discussion starters:

1. Tell me about your experiences with mathematics. [Encourage the student to give examples of mathematics courses they enjoyed as well as those that they struggled in.]

2. Describe what it means to you for someone to be good at mathematics.

3. Describe the way you learn best. [Encourage discussions about mathematics as well as other disciplines.]

4. If you could tell your mathematics instructors both now and in the future how to best help you learn mathematics, what would you say?

5. What has been the most useful part of the instruction in your math 050 course so far?

6. Describe your experiences this year with technology. Did it help you understand the mathematics better? How?

7. If you could change the way the technology was used in the Math 050 course what would you change?
Semi-Structured Problems

What are the x and y intercepts of the equation:

\[ y = 3x - 4 \]

Solve the equation: \( 5 - 4x = 18 \)

Simplify: \( 2x^3 \cdot 3x^6 \)
APPENDIX B

ASSESSMENT TOPICS
<table>
<thead>
<tr>
<th>Question Numbers</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Order of operations</td>
</tr>
<tr>
<td>2</td>
<td>Addition of rational numbers</td>
</tr>
<tr>
<td>3</td>
<td>Least Common Multiple (Numeric)</td>
</tr>
<tr>
<td>4</td>
<td>Addition and subtraction of rational numbers</td>
</tr>
<tr>
<td>5</td>
<td>Evaluate algebraic expressions</td>
</tr>
<tr>
<td>6</td>
<td>Order of operations</td>
</tr>
<tr>
<td>7</td>
<td>Order of operations</td>
</tr>
<tr>
<td>8</td>
<td>Distributive property</td>
</tr>
<tr>
<td>9</td>
<td>Additive inverse</td>
</tr>
<tr>
<td>10</td>
<td>Simplifying algebraic expressions</td>
</tr>
<tr>
<td>11</td>
<td>Perimeter of a rectangle (algebraic)</td>
</tr>
<tr>
<td>12</td>
<td>Order of operations</td>
</tr>
<tr>
<td>13</td>
<td>Order of operations</td>
</tr>
<tr>
<td>14</td>
<td>Exponent Rules (Power of a power)</td>
</tr>
<tr>
<td>15</td>
<td>Exponent Rules (Power of a power)</td>
</tr>
<tr>
<td>16</td>
<td>Zero exponent</td>
</tr>
<tr>
<td>17</td>
<td>Addition of polynomials</td>
</tr>
<tr>
<td>18</td>
<td>Subtraction of polynomials</td>
</tr>
<tr>
<td>19</td>
<td>Multiplication of polynomials by a monomial</td>
</tr>
<tr>
<td>20</td>
<td>Product of binomials</td>
</tr>
<tr>
<td>21</td>
<td>Square of a binomial</td>
</tr>
<tr>
<td>22</td>
<td>Converting to scientific notation</td>
</tr>
<tr>
<td>23</td>
<td>Multiplying numbers expressed in scientific notation</td>
</tr>
<tr>
<td>24</td>
<td>Addition of polynomials</td>
</tr>
<tr>
<td>25</td>
<td>Area of a rectangle (algebraic)</td>
</tr>
</tbody>
</table>
Topics Evaluated by Test Two

<table>
<thead>
<tr>
<th>Question Numbers</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Simplify algebraic expression with negative exponents *</td>
</tr>
<tr>
<td>2</td>
<td>Greatest common factor (algebraic)</td>
</tr>
<tr>
<td>3</td>
<td>Factor (common term)</td>
</tr>
<tr>
<td>4</td>
<td>Factor (common term; polynomial)</td>
</tr>
<tr>
<td>5</td>
<td>Factor (grouping)</td>
</tr>
<tr>
<td>6</td>
<td>Factor (trinomial)</td>
</tr>
<tr>
<td>7</td>
<td>Factor (trinomial)</td>
</tr>
<tr>
<td>8</td>
<td>Division (polynomial by monomial)</td>
</tr>
<tr>
<td>9</td>
<td>Creating a factorable trinomial</td>
</tr>
<tr>
<td>10</td>
<td>Factor (trinomial)</td>
</tr>
<tr>
<td>11</td>
<td>Factor (trinomial; perfect square)</td>
</tr>
<tr>
<td>12</td>
<td>Factor (difference of squares)</td>
</tr>
<tr>
<td>13</td>
<td>Simplify algebraic expression with negative exponents *</td>
</tr>
<tr>
<td>14</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>15</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>16</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>17</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>18</td>
<td>Consecutive integer problem</td>
</tr>
<tr>
<td>19</td>
<td>Mixture problem</td>
</tr>
<tr>
<td>20</td>
<td>Solve quadratic equation (in factored form)</td>
</tr>
<tr>
<td>21</td>
<td>Solve quadratic equation</td>
</tr>
<tr>
<td>22</td>
<td>Solve formula, given values for all but one variable</td>
</tr>
<tr>
<td>23</td>
<td>Solve quadratic equation</td>
</tr>
<tr>
<td>24</td>
<td>Write and solve quadratic equation from word problem</td>
</tr>
<tr>
<td>25</td>
<td>Distance problem</td>
</tr>
</tbody>
</table>

* Negative exponents appeared on Test One in the Winter term.
## Topics Evaluated by Test Three

<table>
<thead>
<tr>
<th>Question Numbers</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Identify location of a point from description</td>
</tr>
<tr>
<td>2</td>
<td>Choose point on graph that matches given coordinates</td>
</tr>
<tr>
<td>3</td>
<td>Identify a solution of a linear equation (2 variables) from list</td>
</tr>
<tr>
<td>4</td>
<td>Draw graph (slope intercept form)</td>
</tr>
<tr>
<td>5</td>
<td>Find intercepts (standard form)</td>
</tr>
<tr>
<td>6</td>
<td>Draw graph (standard form)</td>
</tr>
<tr>
<td>7</td>
<td>Find 3 solutions and draw graph (slope intercept form)</td>
</tr>
<tr>
<td>8</td>
<td>Find slope from picture of the graph of a line</td>
</tr>
<tr>
<td>9</td>
<td>Identify slope of a line parallel to a line given the equation</td>
</tr>
<tr>
<td>10</td>
<td>Identify slope of a line perpendicular to a line given two points</td>
</tr>
<tr>
<td>11</td>
<td>Identify location of a point from description</td>
</tr>
<tr>
<td>12</td>
<td>Match graph of a line to its equation (slope-intercept form)</td>
</tr>
<tr>
<td>13</td>
<td>Find slope given two points</td>
</tr>
<tr>
<td>14</td>
<td>Find slope given two points</td>
</tr>
<tr>
<td>15</td>
<td>Find slope given equation (horizontal/vertical line)</td>
</tr>
<tr>
<td>16</td>
<td>Identify slope of a line parallel to a line given two points</td>
</tr>
<tr>
<td>17</td>
<td>Find intercepts ((x = Ay + b)\ form)</td>
</tr>
<tr>
<td>18</td>
<td>Match graph of a line to its equation (various forms)</td>
</tr>
<tr>
<td>19</td>
<td>Find 3 solutions and draw graph ((x = Ay + b)\ form)</td>
</tr>
<tr>
<td>20</td>
<td>Draw a line with a positive/negative slope</td>
</tr>
<tr>
<td>Question Numbers</td>
<td>Topic</td>
</tr>
<tr>
<td>------------------</td>
<td>-------</td>
</tr>
<tr>
<td>1</td>
<td>Relationship between absolute value of integers</td>
</tr>
<tr>
<td>2</td>
<td>Multiplication of rational numbers</td>
</tr>
<tr>
<td>3</td>
<td>Order of operations</td>
</tr>
<tr>
<td>4</td>
<td>Evaluate algebraic expressions</td>
</tr>
<tr>
<td>5</td>
<td>Addition and subtraction of rational numbers</td>
</tr>
<tr>
<td>6</td>
<td>Order of operations</td>
</tr>
<tr>
<td>7</td>
<td>Distributive property</td>
</tr>
<tr>
<td>8</td>
<td>Simplifying algebraic expressions</td>
</tr>
<tr>
<td>9</td>
<td>Subtraction of polynomials</td>
</tr>
<tr>
<td>10</td>
<td>Write an algebraic expression given a word phrase and simplify</td>
</tr>
<tr>
<td>11</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>12</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>13</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>14</td>
<td>Find solution to a mathematical word sentence with one unknown</td>
</tr>
<tr>
<td>15</td>
<td>Simple Percent Problem</td>
</tr>
<tr>
<td>16</td>
<td>Non-Routine word problem (Spring – Distance and Winter – Mixture)</td>
</tr>
<tr>
<td>17</td>
<td>Solve formula for indicated variable</td>
</tr>
<tr>
<td>18</td>
<td>Solve and graph inequality (one variable)</td>
</tr>
<tr>
<td>19</td>
<td>Complete a solution of a linear equation (2 variables) given either x or y</td>
</tr>
<tr>
<td>20</td>
<td>Find 2 solutions and draw graph (slope intercept form)</td>
</tr>
<tr>
<td>21</td>
<td>Draw graph given one point and slope</td>
</tr>
<tr>
<td>22</td>
<td>Find intercepts (slope-intercept form)</td>
</tr>
<tr>
<td>23</td>
<td>Find slope given two points</td>
</tr>
<tr>
<td>24</td>
<td>Find slope given equation (standard form)</td>
</tr>
<tr>
<td>25</td>
<td>Identify slope of a line perpendicular to a line given equation (slope-intercept form)</td>
</tr>
<tr>
<td>26</td>
<td>Simplify numeric expression with negative exponents</td>
</tr>
<tr>
<td>27</td>
<td>Subtract trinomials</td>
</tr>
<tr>
<td>28</td>
<td>Multiply binomials</td>
</tr>
<tr>
<td>29</td>
<td>Simplify algebraic fraction with negative exponents</td>
</tr>
<tr>
<td>30</td>
<td>Find greatest common factor of monomials</td>
</tr>
<tr>
<td>31</td>
<td>Factor (trinomial)</td>
</tr>
<tr>
<td>32</td>
<td>Factor (trinomial)</td>
</tr>
<tr>
<td>33</td>
<td>Solve quadratic equation (in factored form)</td>
</tr>
<tr>
<td>34</td>
<td>Solve quadratic equation</td>
</tr>
<tr>
<td>35</td>
<td>Find dimensions of a rectangle given area and relationship between dimensions</td>
</tr>
</tbody>
</table>
### CAS-specific questions Identified for Separate Analysis from Tests and the Final Exam

<table>
<thead>
<tr>
<th>Question</th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Exponent Rules - Midterm</strong></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Power of a power</td>
</tr>
<tr>
<td>15</td>
<td>Power of a power</td>
</tr>
<tr>
<td>16</td>
<td>Zero Power</td>
</tr>
<tr>
<td>19</td>
<td>Multiplication (monomial by polynomial)</td>
</tr>
<tr>
<td>21</td>
<td>Multiplication (product of binomials)</td>
</tr>
<tr>
<td>26</td>
<td>Simplify algebraic expression with negative exponents</td>
</tr>
<tr>
<td>33</td>
<td>Division (polynomial by monomial)</td>
</tr>
<tr>
<td>38</td>
<td>Simplify algebraic expression with negative exponents</td>
</tr>
<tr>
<td><strong>Exponent Rules – Final Exam</strong></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Simplify numeric expression with negative exponents</td>
</tr>
<tr>
<td>29</td>
<td>Simplify algebraic fraction with negative exponents</td>
</tr>
<tr>
<td><strong>Solving Linear Equations - Midterm</strong></td>
<td></td>
</tr>
<tr>
<td>39</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>40</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>41</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>42</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td><strong>Solving Linear Equations – Final Exam</strong></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>12</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td>13</td>
<td>Solve linear equation (one variable)</td>
</tr>
<tr>
<td><strong>Graphing Linear Equations – Midterm</strong></td>
<td></td>
</tr>
<tr>
<td>54</td>
<td>Draw graph (slope intercept form)</td>
</tr>
<tr>
<td>56</td>
<td>Draw graph (standard form)</td>
</tr>
<tr>
<td>57</td>
<td>Find 3 solutions and draw graph (slope intercept form)</td>
</tr>
<tr>
<td>60</td>
<td>Identify slope of a line parallel to a line given the equation</td>
</tr>
<tr>
<td>61</td>
<td>Identify slope of a line perpendicular to a line given two points</td>
</tr>
<tr>
<td>63</td>
<td>Match graph of a line to its equation (slope-intercept form)</td>
</tr>
<tr>
<td>66</td>
<td>Find slope given equation (horizontal/vertical line)</td>
</tr>
<tr>
<td>67</td>
<td>Identify slope of a line parallel to a line given two points</td>
</tr>
<tr>
<td>69</td>
<td>Match graph of a line to its equation (various forms)</td>
</tr>
<tr>
<td>70</td>
<td>Find 3 solutions and draw graph (x = ay + b form)</td>
</tr>
<tr>
<td>71</td>
<td>Draw a line with a positive/negative slope</td>
</tr>
<tr>
<td><strong>Graphing Linear Equations – Final Exam</strong></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Find 2 solutions and draw graph (slope intercept form)</td>
</tr>
<tr>
<td>21</td>
<td>Draw graph given one point and slope</td>
</tr>
<tr>
<td>24</td>
<td>Find slope given equation (standard form)</td>
</tr>
<tr>
<td>25</td>
<td>Identify slope of a line perpendicular to a line given equation (slope-intercept form)</td>
</tr>
</tbody>
</table>
APPENDIX C

EXAMPLE ASSIGNMENTS

Please note the example assignments have been reformatted slightly to better fit the pagination of this document.
X-ponent X-ploration [Control Group]

Negative Bases: How does the sign of the base affect the answer?
\[-2^3 = \underline{_______}\]
\[-(-2)^3 = \underline{_______}\]

Question: Is there a difference when you use parentheses when a negative sign is involved? Explain why or why not?

Product Rule: What happens when we multiply two powers that have the same base?
\[2^1 \cdot 2^1 = \underline{_______}\]
\[x^5 \cdot x^{10} = \underline{_______}\]

Question: The general form of the Product Rule is \(a^m \cdot a^n = \underline{_______}\)? Explain, using the “definition” of exponent, why this rule is true.

Power of a Power Rule: What happens when we raise a power to another power?
\[(x^1)^2 = \underline{_______}\]
\[(a^{-3})^5 = \underline{_______}\]

Question: The general form of the Power of a Power Rule is \((a^m)^n = \underline{_______}\)? Explain why this is true.

Power of a Product Rule: What happens when we raise a product to a power.
1. \((2x)^2 = \underline{_______}\)
2. \((3z)^4 = \underline{_______}\)
3. \((-2a^2bc^4)^4 = \underline{_______}\)
4. \((x^3y^2z^3)^6 = \underline{_______}\)

Question: The general form for the Power of a Product Rule is \((ab)^m = \underline{_______}\)? Why? What was different about 1 and 2 compared to 3 and 4?

Power of a Quotient Rule: What happens when we raise a quotient to a power
\[
\left(\frac{2}{x}\right)^2 = \underline{_______}
\]
\[
\left(\frac{3x}{4z}\right)^4 = \underline{_______}
\]

Question: The general form of the Power of a Quotient Rule is \(\left(\frac{a}{b}\right)^m = \underline{_______}\)? What happens when \(b = 0\)? Does this rule make sense in terms of the other rules such as the power of a product rule? Why does this rule work?
**Quotient Rule:** Just as the power of a quotient rule was the division counterpart to the power of a product rule, the quotient rule is similar to the product rule. So, what happens when we divide powers that have the same bases?

\[
\frac{12x^3y^5}{15x^5y^2}
\]

Question: The general form of the Quotient Rule is \( \frac{a_m}{a^n} = \) _______. Explain why this rule is true. How do you know whether the remaining terms are in the numerator or denominator?

**Zero Exponents:** What happens when the exponent is 0? When you explore this, make sure to use lots of different kinds of bases (for some hints/examples, look at the first section of this worksheet) Record the examples you tried below. Make sure you try positive and negative examples!

\[
2x^0
\]
\[
(2x)^0
\]

Question: What can we say about the answer when we raise a base to the zero power?
**X-ponent X-ploration [Experimental Group]**

This worksheet will help guide you towards discovering some of the exponent rules that you will use to simplify expressions and equations. It starts off giving you specific examples to work out, but by the end, you are encouraged to explore on your own to discover the rules. Just make sure you have explored enough to answer the questions.

**Powers, Bases, Exponents:** For each of the following powers, just identify the base and the exponent. You do not need to simplify or evaluate the powers in this section. You also will not need the calculator for this part.

1) \(2^3\) Base = ________ Exponent = ________
2) \(x^5\) Base = ________ Exponent = ________
3) \((2 + x)^7\) Base = ________ Exponent = ________
4) \(y^{-z}\) Base = ________ Exponent = ________
5) \(-a^{2+x}\) Base = ________ Exponent = ________
6) \((-3z)^{p-q}\) Base = ________ Exponent = ________

**Negative Bases:** How does the sign of the base affect the answer?

5. \(-2^3 = \) ________
6. \((-2)^3 = \) ________
7. \((-2)^4 = \) ________
8. \(-2^4 = \) ________
9. \(-x^5 = \) ________
10. \((-x)^5 = \) ________
11. \((-x)^10 = \) ________
12. \(-x^{10} = \) ________

Question: Is there a difference when you use parentheses when a negative sign is involved? Can you explain why or why not?

**Product Rule:** What happens when we multiply two powers that have the same base?

13. \(2^1 \cdot 2^1 = \) ________
14. \(3^2 \cdot 3^1 = \) ________
15. \((-4)^3 \cdot (-4)^4 = \) ________
16. \(x^5 \cdot x^{10} = \) ________
17. \((a + b)^{12} \cdot (a + b)^8 = \) ________
18. \(y^m \cdot y^3 = \) ________
19. \(z^n \cdot z^{-5} = \) ________
20. \((2x + 5)^{3x-2} \cdot (2x + 5)^{2x+7} = \) ________

Question: The general form of the Product Rule is \(a^m \cdot a^n = \) ________? Explain, using the “definition” of exponent, why this rule is true.

**Power of a Power Rule:** What happens when we raise a power to another power?

21. \((x^2)^2 = \) ________
22. \((x^2)^2 = \) ________
23. \((x^3)^3 = \) ________
24. \((x^5)^{10} = \) ________
25. \((a^3)^5 = \) ________
26. \((z^2)^7 = \) ________

Question: The general form of the Power of a Power Rule is \((a^m)^n = \) ________? Explain why this is true.
**Power of a Product Rule:** What happens when we raise a product to a power? When putting these into your calculator, make sure you type a multiplication sign between variables. For example, to type in \(xy\) (meaning \(x\) times \(y\)), type \(x*y\). Otherwise, the calculator will treat \(xy\) as one variable whose name is “\(xy\)”.

27. \((2x)^2 = \) _______
28. \((3z)^4 = \) _______
29. \((-2a^2bc^5)^4 = \) _______
30. \((x^5y^2z^3)^6 = \) _______

Question: The general form for the Power of a Product Rule is \((ab)^m = \) ______? Why?
What was different about 27 and 28 compared to 39 and 30?

**Power of a Quotient Rule:** What happens when we raise a quotient to a power?
Remember, the fraction bar acts like a grouping symbol that groups the numerator and the denominator separately, so be careful when you put these into your calculator. (You will need more examples.)

31. \(\left(\frac{2}{x}\right)^2 = \) _______
32. \(\left(\frac{3x}{4z}\right)^4 = \) _______

Question: The general form of the Power of a Quotient Rule is \(\left(\frac{a}{b}\right)^m = \) ______? What happens when \(b = 0\)? Does this rule make sense in terms of the other rules such as the power of a product rule? Why does this rule work?

**Quotient Rule:** Just as the power of a quotient rule was the division counterpart to the power of a product rule, the quotient rule is similar to the product rule. So, what happens when we divide powers that have the same bases? Explore this one on your own, and report your results below.

33. \(\frac{12x^3y^5}{15x^5y^2}\)

Question: The general form of the Quotient Rule is \(\frac{a^m}{a^n} = \) ______? Explain why this rule is true. How do you know whether the remaining terms are in the numerator or denominator?

**Zero Exponents:** What happens when the exponent is 0? When you explore this, make sure to use lots of different kinds of bases (for some hints/examples, look at the first section of this worksheet!) Record the examples you tried below. Make sure you try positive and negative examples!

34. \(2x^0\)
35. \((2x)^0\)

Question: What can we say about the answer when we raise a base to the zero power?
Solving Linear Equations [Experimental Group]

Record important calculator functions you might need to complete the activity below.

For each of the following equations please record the following information:
1. Record what you would do to begin finding the solution to the given equation.
2. Predict what the equation will look like after performing this step.
3. Perform the step with CAS and record the result you receive.
4. If your prediction and the CAS result do not match, explain why the mismatch occurred.
5. Choose the results from step 2 or 3 as correct or begin again with the equation in step 1 and record it in the Equation column.
6. Repeat steps 1-4 until you have a solution.

Please follow the steps above for the following equations:

1) \( 5 - 2x = 18 \)  
6) \( 2(3b - 5) = 10 \)

2) \( 2x - 3 = 10 \)  
7) \( \frac{2}{3} - w = \frac{5}{8} \)

3) \( 2a + 10 = 5 - 4a \)  
8) \( 3x + 8 = 2x - 6 \)

4) \( 5x + 6 - 2x = 18 \)  
9) \( \frac{2}{5} x - \frac{1}{2} = \frac{4}{5} \)

5) \( \frac{2}{3}(2x - 4) = 21 \)  
10) \( 8 - 2x = 10x - 5 \)
Solving Equations Chart

<table>
<thead>
<tr>
<th>Equation</th>
<th>Suggested Operation</th>
<th>Prediction</th>
<th>CAS Result</th>
<th>Explanation of Mismatches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<table>
<thead>
<tr>
<th>Equation</th>
<th>Suggested Operation</th>
<th>Prediction</th>
<th>CAS Result</th>
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</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>
**Graphing Conjectures** [Control and Experimental Groups]
[Students in Experimental Group were encouraged to also look at sets using CAS.]

In class:
1. After being assigned a set of equations produce a T-table with at least three solutions for each graph in your set.
2. Use the solutions you have found to graph the set of lines on a single set of axes.
3. Also find the slope and x- and y-intercepts for each equation in your set.
4. Look at the graphs and the calculated slope and intercepts to come up with as many observations as you can about the graphs, the equations, how the graphs relate to the equations, how the equations relate to each other, etc.

<table>
<thead>
<tr>
<th>Set #1</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = \frac{3}{5} x + 2 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{3}{5} x )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{3}{5} x + 4 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = \frac{3}{5} x - 1 )</td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Set #2</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = 2x + 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 2x + 3 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 2x + (-2) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y = 2x - 5 )</td>
<td></td>
<td></td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Set #3</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-5x + 10y = -15)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-10x - 5y = -20)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-5x + 10y = 5)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(10x + 5y = 5)</td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Set #4</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7x + 6y = -18)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-7x - 6y = -24)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6x + 7y = -7)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6x + 7y = 14)</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Set #5</td>
<td>Slope</td>
<td>y-intercept</td>
<td>x-intercept</td>
</tr>
<tr>
<td>-------</td>
<td>--------</td>
<td>-------------</td>
<td>-------------</td>
</tr>
<tr>
<td>x = -3y + 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = -3y + 7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = -3y - 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = -3y</td>
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<table>
<thead>
<tr>
<th>Set #6</th>
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<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 2y + 2</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>x = -2y + 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = \frac{3}{2}y + 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = \frac{1}{4}y + 2</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Set #7</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 4x + 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = -\frac{1}{4}x + 5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = 4x - 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = -\frac{1}{4}x - 2</td>
<td></td>
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<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Set #8</th>
<th>Slope</th>
<th>y-intercept</th>
<th>x-intercept</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = -\frac{3}{2}x - \frac{3}{2}</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = 2x + (-\frac{3}{2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = -5x + (-\frac{3}{2})</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = \frac{7}{8}x - \frac{3}{2}</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Homework:
- Complete the chart for the graphs you did not work on in class.
  What observation can be made about each set of graphs?
  Attach your work on a separate paper.
- Make a list of things to know about graphing based on your own observations and those of your classmates.
LIST OF REFERENCES


Granott, N. (1998). We learn therefore we develop: Learning versus development - or developing learning? In M. C. Smith & T. Pourohot (Eds.), Adult learning and development: Perspectives from educational psychology (pp. 15-34). Mahwah, NJ: Lawrence Erlbaum.


Kutzler, B. (2000). The algebraic calculator as a pedagogical tool for teaching mathematics. In E. D. Laughbaum (Ed.), *Hand-held technology in mathematics and science education: A collection of papers* (pp. 98-116). Columbus, OH: Teacher Teaching with Technology College Short Course Programs at The Ohio State University.


Waits, B. K., & Demana, F. (2000b). A new breed of calculators: They will change the way and what you teach! In E. D. Laughbaum (Eds.), Hand-held technology in mathematics and science education: A collection of papers (pp. 81-84). Columbus, OH: Teacher Teaching with Technology College Short Course Programs at The Ohio State University.


