TE SCATTERING FROM SHORT BUMPS

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by

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* * * * *

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Dedication

This thesis is dedicated to my family. I especially want to thank my parents, Charles and Deirdre Ryan, who have always supported me with their love and encouragement in all my endeavors.
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CHAPTER I

Introduction

A method of moments (MM) \cite{1} solution is applied to the problem of transverse electric (TE) scattering from a two-dimensional (2D) perfectly conducting bump of arbitrary cross section which is short in height and lies on an infinite and perfectly conducting ground plane. Image theory is used to reduce this problem to that of TE scattering by a 2D perfectly conducting cylinder of arbitrary cross section \cite{2}. The surface equivalence principle is used to replace the cylinder with free space and an unknown surface current density, $J_s$, on the outside surface of the cylinder \cite{2}. Rumsey's \cite{3} reaction concept is employed to formulate an integral equation for $J_s$, which is then solved by using Richmond's \cite{4} piecewise sinusoidal expansion for $J_s$ in the MM solution \cite{1}. Once the surface current density is determined, the scattered field from the cylinder can then be found. The total scattered field above the infinite and perfectly conducting ground plane is found by appropriately combining the scattered fields associated with the original incident plane wave and the scattered fields associated with the image plane wave.

The problem of 2D low frequency TE scattering by conducting cylinders has been addressed by several authors \cite{5}–\cite{12}. Mei and Van Bladel \cite{5} used the electric field integral equation (EFIE) and a point matching technique with rectangular pulses as basis functions in the expansion of the surface current density. Point matching is a numerical technique that is equivalent to using Dirac-delta
functions as the testing or weighting functions in a MM solution. Andreason [6] also used the point matching method, however, a piecewise parabolic expansion for the surface current density was used in his solution. Richmond [4], [7] used the reaction technique to formulate an integral equation for the problem of 2D TE radiation and scattering from a conducting cylinder of arbitrary cross section. In this solution, Richmond used piecewise sinusoidal functions in the expansion of the current density and also as the testing functions which resulted in a Galerkin [1] solution of the integral equation. Wang [8] extended this method for the case of TE radiation and scattering from a conducting cylinder of arbitrary cross section with a thin dielectric coating. Sarkar [13] has discussed the choice of weighting functions for a MM solution, and Sarkar, Siarkiewicz and Stratton have presented an overview of the method of solving large matrix equations resulting from MM solutions [14].

In addition to MM techniques, other numerical methods have been applied to 2D electromagnetic scattering problems. Marin [15] used a variational formulation and the finite element method to compute the scattering amplitudes for transverse magnetic (TM) polarized plane waves scattered from cylinders of arbitrary cross section geometries. The application of the boundary element method (BEM) to 2D electromagnetic field problems has been discussed by Kagami and Fukai [16]. Yashiro and Ohkawa [17] have applied this method to the problem of TE and TM scattering from cylinders of arbitrary cross section. Murthy, Hill and Thiele [18] have presented a hybrid iterative method (HIM) to solve the magnetic field integral equation (MFIE) for the induced currents on an arbitrary perfectly conducting scatterer. They present results for the current induced on smooth and edged 2D cylinder by a TE polarized incident plane wave. A paper of general interest on the choice of expansion functions in the numerical solution of operator equations has
been published by Sarkar, Djodjevic and Arvas [19].

Although, in this work, the main concern is with the 2D TE scattering problem, a recent solution of a three dimensional (3D) scattering problem is of interest. Arvas, Harrington, and Mautz [20] have presented a MM solution for the problem of radiation and scattering from electrically small conducting bodies of arbitrary shape close to and on an infinite ground plane. In particular, they treat the case of a conducting box on a ground plane. They make use of image theory to account for the presence of the ground plane, and then they approximate the time harmonic problem by two uncoupled problems, an electrostatic one and a magnetostatic one, each of which is solved separately by the MM. The basis for this approximation and a more detailed discussion of the MM solution is discussed further in an earlier work by the same authors [21], where they consider the radiation and scattering from electrically small conducting bodies of arbitrary shape. The static approximation used in [20] is valid only when all physical dimensions of the scattering body are much less than the wavelength of the incident wave, and therefore this solution would only apply to a limited number of the geometries considered in this study. Harrington and Mautz [22] present a new E field solution for the electric current and electric charge induced on a perfectly conducting surface which is specially formulated to deal with scattering in the low frequency portion of the Rayleigh region [23]. Their solution is valid for surfaces whose maximum dimension is as small as $10^{-15}$ wavelengths or as large as a few wavelengths.

This work deals with the time harmonic TE plane wave scattering from a 2D short perfectly conducting bump on an infinite and perfectly conducting ground plane. In all equations, the time dependence, $e^{j\omega t}$ is understood and suppressed, and the surrounding medium is free space. Chapter II describes the derivation of the integral equation and the MM solution. Numerical difficulties encountered in
the actual computer execution of the MM solution for some bumps are discussed. In Chapter III, the numerical results for the TE scattering from bumps of four different cross section geometries are presented. In particular, data is presented for an elliptic bump, a half cycle cosine bump, a full cycle cosine bump and a rectangular bump. The results are presented for a representative sample of physical dimensions of each bump configuration. For the case of the elliptic bump, the MM results are compared with results obtained by using an exact eigenfunction expansion solution [24], [25]. A conversion formula which is useful for obtaining the radar cross section (RCS) for a bump of finite length is discussed, and Chapter IV presents some general conclusions and a possible extension for our results.
CHAPTER II

The Integral Equation and Moment Method Solution

2.1 Introduction

This chapter will describe the integral equation and the method of moments (MM) solution to the problem of TE scattering by a short perfectly conducting 2D bump on an infinite and perfectly conducting ground plane. The basic problem geometry is shown in Figure 1a. We begin the solution by using image theory [2] to reduce the problem to that of TE scattering from a 2D perfectly conducting cylinder of arbitrary cross section in free space. In the equivalent problem, the perfectly conducting infinite ground plane is replaced by the image of the perfectly conducting bump and the image of the incident plane wave as shown in Figure 1b. Next, the surface equivalence theorem [2] is used to replace the cylinder with free space and an unknown surface current density, \( \mathbf{J}_s \), on the outside surface of the cylinder.

An integral equation for the unknown surface current density \( \mathbf{J}_s \) can then be formulated using the superposition principle and Rumsey's reaction technique [3]. Finally, the method of moments (MM) is used to solve for the unknown current density, \( \mathbf{J}_s \), from which the scattered fields from the cylinder can be found.
a) A short perfectly conducting 2D bump on an infinite perfectly conducting ground plane is excited by an incident TE polarized plane wave.

b) The infinite perfectly conducting ground plane is removed and replaced by the image of the perfectly conducting bump and the image of the incident TE polarized plane wave [2].

Figure 1: The Original Problem is simplified with the use of Image Theory
2.2 Derivation of the Integral Equation

This section describes the use of image theory, the surface equivalence theorem and Rumsey's reaction technique in the derivation of an integral equation for the problem of TE scattering by a perfectly conducting bump on an infinite and perfectly conducting ground plane [4]. Figure 1a shows a perfectly conducting 2D bump of arbitrary cross section on an infinite and perfectly conducting ground plane. The bump is infinite in the \( \hat{z} \) direction, and it is excited by an incident TE polarized plane wave. In the absence of the bump and the ground plane the incident TE plane wave electric field will be denoted \( \mathbf{E}_i^+ \). In the presence of the ground plane, but with the bump removed, this incident field produces the field \( \mathbf{E}_i \).

Image theory may be used to reduce the above problem to the more readily solvable problem of TE scattering by a 2D perfectly conducting cylinder. The equivalent problem, obtained from image theory, is shown in Figure 1b. Here the ground plane is removed and replaced by the image of the bump and of the incident field. The problems of Figure 1a and b are equivalent in that they produce the same fields in the half space above the ground plane. After the removal of the ground plane, the bump and its image form a perfectly conducting cylinder, enclosed by the surface \( S \), which is excited by a TE polarized incident wave, \( \mathbf{E}_i^+ \) and a TE polarized image wave, \( \mathbf{E}_i^- \). Henceforth, the bump plus its image will be referred to as simply the cylinder. Then the total field incident on this cylinder is related to the two incident fields in Figure 1b by

\[
\mathbf{E}_i = \mathbf{E}_i^+ + \mathbf{E}_i^-.
\]  

(2.1)

Following the methods of Richmond [4], let us now employ the reaction technique in arriving at an integral equation for the unknown current, \( \mathbf{J}_s \), flowing
on the surface of the cylinder. The surface equivalence principle [2] will be used to formulate an equivalent problem where the cylinder is replaced with an equivalent surface current density radiating in free space. Figure 2a shows the original incident wave and its image generating the unknown field \((\mathbf{E}, \mathbf{H})\) in the presence of the conducting cylinder. Note that in Figure 2a, the field inside the cylinder is zero, because this is a perfectly conducting cylinder [26]. On the outside surface of the cylinder, there will be an induced electric surface current density, \(\mathbf{J}_s\), given by Schelkunoff's [27] equivalent current formula such that

\[
\mathbf{J}_s = \hat{n} \times \mathbf{H} \quad \text{on} \ S \quad (2.2)
\]

where the unit normal vector, \(\hat{n}\), is directed outward from the surface \(S\) [2]. In Figure 2a, the field outside the cylinder will not be disturbed if the conducting cylinder is replaced by free space since the interior field in our equivalent problem is a null field [4]. The equivalent problem where the surface of the cylinder is replaced by free space is shown in Figure 2b. The free space fields of \(\mathbf{J}_s\) are denoted as \((\mathbf{E}_s, \mathbf{H}_s)\) or the scattered fields.

The total fields are the sum of the incident and the scattered fields; i.e.,

\[
\mathbf{E}_s = \mathbf{E} - \mathbf{E}_i = \mathbf{E} - (\mathbf{E}_i^+ + \mathbf{E}_i^-), \quad \text{and} \quad (2.3)
\]

\[
\mathbf{H}_s = \mathbf{H} - \mathbf{H}_i = \mathbf{H} - (\mathbf{H}_i^+ + \mathbf{H}_i^-). \quad (2.4)
\]

From Equations (2.2) and (2.4) the unknown surface current density, \(\mathbf{J}_s\), can be written as

\[
\mathbf{J}_s = \hat{n} \times (\mathbf{H}_i + \mathbf{H}_s). \quad (2.5)
\]

The reaction of the fields of the currents \((\mathbf{J}_a, \mathbf{M}_a)\) with the currents \((\mathbf{J}_b, \mathbf{M}_b)\) is given by Rumsey [3] and Harrington [2] as

\[
\langle a, b \rangle = \int \int \left( \mathbf{E}^a \cdot \mathbf{J}^b - \mathbf{H}^a \cdot \mathbf{M}^b \right) dv = \langle b, a \rangle \quad (2.6)
\]
a) The incident and image TE polarized plane waves generate the field \((E, H)\) in the presence of the perfectly conducting cylinder[6].

b) An externally equivalent problem is formulated by using the equivalent current density \(J_s\) and by replacing the surface of the cylinder with free space[6].

Figure 2: The Original and the Equivalent Exterior Scattering Problem
where $E^a$ and $H^a$ are the free space fields of sources $J^a$ and $M^a$ and the integration is carried out over the region occupied by $J^b$ and $M^b$. In Figure 2b, one can place an electric test source $J_t$ interior to the surface, $S$. Denoting $(E_t, H_t)$ as the free space fields of $J_t$, the reaction between the total electric field and the test source $J_t$ is

$$
\iint_S (E_s + E_i^+ + E_i^-) \cdot J_t dv
$$

where the integral is over the volume of the interior test source. Since

$$
E_s + E_i^+ + E_i^- = 0
$$

interior to $S$, Equation (2.7) becomes

$$
-\iint_S E_s \cdot J_t dv = \iint_S (E_i^+ + E_i^-) \cdot J_t dv.
$$

In practice, $J_t$ will be chosen as a surface current located an infinitesimal distance inside the surface $S$. In this case, Equation (2.9) becomes

$$
-\int J_t ds = \iint_S (E_i^+ + E_i^-) \cdot J_t ds \quad \text{on } S
$$

Finally, the reciprocity theorem, which is stated in terms of reactions in Equation (2.6), is used to express the left hand side of Equation (2.10) in terms of $J_s$ such that

$$
-\int J_s \cdot E_t ds = \iint_S (E_i^+ + E_i^-) \cdot J_t ds.
$$

### 2.3 Moment Method Solution of the Integral Equation

This section describes the moment method solution used to numerically solve the integral equation, Equation (2.11), derived in the previous section. Let us begin the MM solution by representing the unknown current, $J_s$, as a summation
of $N$ vector basis functions on the surface $S$ of the cylinder, so that we obtain
\[ J_s \approx \sum_{n=1}^{N} I_n J_n \]  
(2.12)

where the $I_n$'s are a sequence of $N$ complex constants to be determined in the MM solution. Substituting Equation (2.12) into (2.11) yields
\[ - \sum_{n=1}^{N} \int \int J_n J_n \cdot E_l ds \approx \int \int (E^+_l + E^-_l) \cdot J_l ds. \]  
(2.13)

Equation (2.13) is a single equation involving $N$ unknown coefficients. In order to obtain $N$ equations, so that we can solve for these coefficients, we enforce Equation (2.13) for $N$ linearly independent weighting functions by choosing the weighting functions identical to the expansion modes. In this case Equation (2.13) becomes
\[ - \sum_{n=1}^{N} I_n \sum_{m=1}^{N} \int \int J_n \cdot E_m ds = \sum_{m=1}^{N} \int \int (E^+_l + E^-_l) \cdot J_m ds \]  
(2.14)

for $m = 1, 2, \ldots, N$, and where $E_m$ and $H_m$ are the free space fields of $J_m$.

Equation (2.14) is usually written more compactly in matrix form as
\[ [Z] [I] = [V] \]  
(2.15)

where the $[Z]$ matrix is commonly referred to as the impedance matrix, and the $[V]$ column vector is known as the voltage vector or the right hand side vector. The elements of the impedance matrix and the right hand side vector are given by
\[ Z_{mn} = - \int \int J_n \cdot E_m ds \]  
(2.16)

and
\[ V_m = \int \int (E^+_l + E^-_l) \cdot J_m ds. \]  
(2.17)

Symbolically, the solution of the matrix equation can be written as
\[ [I] = [Z]^{-1} [V]. \]  
(2.18)
However, in practice the $N$ simultaneous linear equations are solved using linear algebra techniques [28].

Although the choice of basis and test functions is arbitrary, they are usually chosen to facilitate the numerical evaluation of the elements in the matrix equation and to yield a rapidly convergent solution. In his solution, Richmond [4] uses the piecewise-sinusoidal basis functions for the current expansion and testing functions. These modes are described below. This method of solution, where the test functions are chosen equal to the expansion functions, is a specialized case of the moment method known commonly as Galerkin's method. The use of Galerkin’s method in this MM solution reduces computational time by allowing some of the integrals given in Equations (2.16) and (2.17) to be evaluated in closed form [4].

2.4 Basis and Test Functions Used in the MM Solution

This section will describe the piecewise-sinusoid strip dipole which Richmond uses as basis and test functions in the MM solution of the integral equation. Let us begin by defining the electric strip monopole because an electric strip dipole is comprised of two electric strip monopoles, and the fields of a strip dipole are given by the sum of the field contributions from its two strip monopoles. Figure 3 shows an electric strip monopole of width, $h$, and infinite length radiating into free space. The vector potential, scalar potential, and fields of the electric strip monopole are:

\[
A = -\frac{j\mu_0}{4} \int_0^h J H_0^{(2)}(k\rho)dx'
\]  

\[
V = \frac{1}{4\omega\epsilon_0} \int_0^h \frac{dJ}{dx'} H_0^{(2)}(k\rho)dx'
\]  

\[
E_x = -\frac{k\eta_0}{8} \int_0^h J [H_0^{(2)}(k\rho) + H_2^{(2)}(k\rho)\cos(2\phi)]dx'
\]  

\[
E_y = -\frac{k\eta_0}{8} \int_0^h J H_2^{(2)}(k\rho)\sin(2\phi)dx', \text{ and}
\]  

12
Figure 3: An electric strip monopole radiating into free space[4].
\[ \mathbf{H} = -\frac{j k}{4} \int_0^h \mathbf{J} \times \hat{\rho} H_1^{(2)}(k \rho) dx' \]  

(2.23)

where the prime denotes the source coordinate and

\[ \rho = \sqrt{(x - x')^2 + y^2} \]  

(2.24)

\[ k = \omega \sqrt{\mu_0 \epsilon_0} = \omega / c, \text{ and} \]

\[ \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}. \]  

(2.25)

(2.26)

The electric field can be expressed in terms of vector and scalar potentials as

\[ \mathbf{E} = -j \omega \mathbf{A} - \nabla V. \]  

(2.27)

Substituting Equation (2.19) and Equation (2.20) into this formula yields the following equation for the electric field.

\[ \mathbf{E} = -\frac{k \eta_0}{4} \int_0^h J H_0^{(2)}(k \rho) dx' + \frac{\eta}{4} \int_0^h \hat{\rho} \frac{dJ}{dx'} H_1^{(2)}(k \rho) dx' \]  

(2.28)

Infinite-series expansions or numerical integration techniques must be used to evaluate the field integrals for most current functions; however, if a sinusoidal current distribution is used on the monopole, then \( E_x \) can be obtained rigorously in closed form. The equation for a sinusoidal monopole current distribution is given by

\[ \mathbf{J}(x) = \hat{x} \left[ I_1 \sin k (h - x) + I_2 \sin (k x) \right] / \sin (k h) \]  

(2.29)

where \( I_1 \) and \( I_2 \) are constants which represent \( \mathbf{J}(x) \) at \( x = 0 \) and \( x = h \) respectively \[4]. Whereas, the closed form expression for the \( x \) component of the field is

\[ E_x = \frac{\eta_0}{4 \sin (k h)} \left[ I_1 H_0^{(2)}(k \rho_1) \cos (k h) - I_1 H_0^{(2)}(k \rho_2) \right. \]

\[ + I_2 H_0^{(2)}(k \rho_2) \cos (k h) - I_2 H_0^{(2)}(k \rho_1) \]  

(2.30)
The sinusoidal monopole has line charges at its endpoints, however, the fields of these charges are omitted in Equation (2.30). The reason is that when two monopoles are combined to form a dipole the charges always cancel.

Now that the sinusoidal strip monopole has been defined, the sinusoidal strip dipole and its use in the MM solution can be examined. Figure 4 illustrates a planar strip dipole which lies in the xx plane and has infinite length in the \( \hat{z} \) direction. The surface current distribution shown on the strip dipole in Figure 4b is given by

\[
J = \hat{x} \frac{\sin k(x - x_1)}{\sin k(x_2 - x_1)} \quad x_1 < x < x_2, \text{ and} \tag{2.30}
\]

\[
J = \hat{x} \frac{\sin k(x_3 - x)}{\sin k(x_3 - x_2)} \quad x_2 < x < x_3. \tag{2.31}
\]

This current density vanishes at the ends of the strip dipole, is continuous across the terminals at \( x_2 \) and has a slope discontinuity at \( x_2 \).

A nonplanar strip dipole, also known as a strip V-dipole, is shown in Figure 5. The strip V-dipole is simply a generalized case of the planar strip dipole described above with edges at \( s_1 \) and \( t_1 \) and terminals at 0. If the angle \( \psi \) is 180 degrees, then the strip V-dipole will be reduced to a planar strip dipole. The sinusoidal surface current density along the strip V-dipole is

\[
J = \hat{s} \frac{\sin k(s_1 - s)}{\sin ks_1} \quad \text{on arm } s, \text{ and} \tag{2.32}
\]

\[
J = \hat{t} \frac{\sin k(t_1 - t)}{\sin kt_1} \quad \text{on arm } t \tag{2.33}
\]

where distances along the dipole arms, \( s \) and \( t \), is measured by the coordinates \( s \) and \( t \), and, the unit vectors, \( \hat{s} \) and \( \hat{t} \), are perpendicular to the \( z \) axis. As with the planar strip dipole, the current density vanishes at the edges of the source and has a value of unity at the terminals. The fields of a strip dipole are obtained by summing the field contributions from monopole \( s \) and monopole \( t \) which can
a) A planar strip dipole with edges at $x_1$ and $x_3$ and terminals at $x_2[4]$.  


Figure 4: A Planar Strip Dipole and its Current Distribution
be calculated for each monopole using Equations (2.19) through (2.23) along with appropriate coordinate transformations.

The current distribution along the strip V-dipole, given in Equations (2.32) and (2.33), is used as the basis function \( J_n \) in the series expansion of the current density, \( J_s \), on the outside surface of the conducting cylinder. Since a Galerkin solution has been used, the sinusoidal strip dipole is also used as the testing functions.

### 2.5 Computation of the Scattered Fields

After the expansion coefficients \( I_n \) are determined by solving the matrix equation, we refer back to Equation (2.23) to calculate the far zone fields. By superposition, the scattered field from the conducting cylinder is found by summing the contributions from all the modes defined on the cylinder in the series expansion for the surface current density, \( J_s \). Figure 6 shows a single planar strip extending from \((x_1, y_1)\) to \((x_2, y_2)\) where distance from the edge \((x_1, y_1)\) is measured by the
$t$ coordinate. For an electric current distribution $\mathbf{J} = \hat{t} J(t)$, the field at a distant point $(\rho, \phi)$ is

$$
\mathbf{H} = \frac{-z}{4\sqrt{2j}} \frac{k^2 \sin(\alpha - \phi)e^{-jk\rho}e^{j\psi_1}}{4\sqrt{k\pi\rho}} \int_0^h J(t)e^{jkct} dt
$$

(2.34)

where

$$
\psi_1 = k(x_1 \cos \phi + y_1 \sin \phi)
$$

(2.35)

$$
\psi_2 = k(x_2 \cos \phi + y_2 \sin \phi), \text{and}
$$

(2.36)

$$
c = \cos(\alpha - \phi)
$$

(2.37)

Equation (2.34) can be written in the form

$$
H_z = \frac{\sqrt{2j}}{4\sqrt{\pi k\rho}} \frac{k^2 \sin(\alpha - \phi)e^{-jk\rho}}{\sin k\rho} F
$$

(2.38)

where

$$
F = k \sin k\rho e^{j\psi_1} \int_0^h J(t)e^{jkct} dt.
$$

(2.39)

When the electric current on this segment has a sinusoidal distribution such that

$$
J(t) = \frac{I_1 \sin(kt - \alpha) + I_2 \sin kt}{\sin k\rho}
$$

(2.40)

Equation (2.39) can be integrated in closed form to yield

$$
F = \frac{I_1}{\sin^2(\alpha - \phi)}[e^{j\psi_2} - \cos k\rho + jc \sin k\rho e^{j\psi_1}]
$$

$$
+ \frac{I_2}{\sin^2(\alpha - \phi)}[e^{j\psi_2} - \cos k\rho - jc \sin k\rho e^{j\psi_2}].
$$

(2.41)

For the case when $(\alpha - \phi)$ is equal to 0 or $\pi$, Equation (2.39) is

$$
F = \frac{jcI_1}{2}[e^{j\psi_2} \sin k\rho - khe^{j\psi_2}]
$$

$$
- \frac{jcI_2}{2}[e^{j\psi_2} \sin k\rho - khe^{j\psi_1}].
$$

(2.42)
Figure 6: A single planar strip source extending from \((x_1, y_1)\) to \((x_2, y_2)\)
The field contribution from each mode on the cylinder is found using the above equations, and the total scattered field is obtained from summing the field contribution from each mode [4].

As mentioned previously, we are concerned with a perfectly conducting cylinder that is excited by a plane wave incident at angle $\phi_i = \psi$ and an image plane wave incident at angle $\phi_i = -\psi$. Since the total scattered field above the plane of the original ground plane is of interest, superposition is used to obtain the total scattered field generated by the incident plane wave and its image. Thus, the backscattered field at angle $\phi_s = \psi$ is obtained first from the current distribution which was found by solving the matrix equation using $E_i^+$ in the calculation of the right hand side vector. Then instead of calculating the bistatic field at angle $\phi_s = \psi$ resulting from $E_i^-$ incident at $\phi_i = 360 - \psi$, a reciprocity argument is used to get the equivalent contribution by calculating the bistatic field at angle $\phi_s = 360 - \psi$ resulting from $E_i^+$. The sum of the backscattered field and the bistatic scattered field gives us the total scattered field above the plane $x = 0, y = 0$; which by image theory, will be equivalent to the scattered field from the perfectly conducting bump on an infinite and perfectly conducting ground plane excited by a TE polarized plane wave incident at at angle $\phi_i = \psi$ [2].

For 2D plane wave scattering problems it is convenient to express the scattered field in terms of a quantity known as echo width. Harrington defines the echo width $A_e$ for two dimensional problems as the width of the incident wave which carries sufficient power to produce a cylindrically omnidirectional radiation with the same backscattered power density. In equation form the echo width is given by

$$A_e = \rho \lim_{\rho \to \infty} \left( \frac{2\pi \rho S^*}{S^1} \right)$$

(2.43)

where $S^*$ is the poynting vector of the scattered wave, and $S^1$ is the poynting vector.
for the incident wave. For our problem, this equation reduces to

$$A_e = \rho \lim_{\rho \to \infty} 2\pi \rho \left| \frac{\mathbf{H}^s}{\mathbf{H}^i} \right|^2 \text{meters}$$

(2.44)

In Equation (2.44), $\mathbf{H}^s$ denotes the magnetic field intensity of the scattered field, and $\mathbf{H}^i$ denotes the magnetic field intensity of the incident wave [2]. After the total scattered field is calculated, the echo width is found in terms of the wavelength of the incident and scattered field. The quantity echo width in wavelengths is then converted to decibels for presentation in graphical form by the conversion formula

$$A_e (\text{dB's relative to } \lambda) = 10 \log_{10} A_e (\lambda).$$

(2.45)

2.6 Numerical Problems

Our primary concern is with the TE scattering from bumps that are very short in height, less than 0.01$\lambda$ tall. In order to accurately define the geometry of each bump, at least twenty four dipole modes were used in the MM solution. This causes the physical length of the modes to be very small, which leads to some error in the computation of the elements of the impedance matrix and the right hand side vector, especially for very flat bumps. Newman [29] has dealt with this problem as it pertains to calculating the impedance matrix for a microstrip patch antenna. Since our problem is very similar, his explanation applies to our problem.

Consider the two current modes pictured in Figure 7. If we let the subscript $b$ refer to the single mode on the bottom half of the cylinder and the subscript $t$ refer to a single mode on the top half, then the MM matrix equation will have the form

$$I_b Z_{bb} + I_t Z_{bt} = V_b$$

(2.46)

$$I_b Z_{tb} + I_t Z_{tt} = V_t.$$ 

(2.47)
From the symmetry of our problem, $I_t = -I_b$, $V_t = -V_b$, $Z_{tt} = Z_{bb}$, and $Z_{tb} = Z_{bt}$. Therefore we can write

$$I_t = I_b = \frac{V_t}{Z_{tt} - Z_{tb}}. \quad (2.48)$$

Since for a very flat cylinder, the top and bottom modes are identical except for a small displacement, around a few thousandth's of a wavelength or less, $Z_{tb} \approx Z_{tt}$. When a full set of modes is used, this problem yields an ill conditioned matrix and erroneous results, and therefore is the main limitation on the size of a cylinder which can be treated accurately with the MM [29]. Thus, the self and mutual impedance between the top and bottom modes must be calculated very accurately in order to get a valid solution. To improve the accuracy of the originally calculated mutual impedance elements in the impedance matrix, we increased the minimum number of numerical integrations performed over a test mode.

In addition, we found that the computer subroutines used in computing the impedance matrix did not generate an entirely symmetric matrix for the cylinder geometries of interest. Since a Galerkin MM solution is being used, the impedance
matrix should be symmetric (i.e. $Z_{mn} = Z_{nm}$). In addition, geometrical symmetries should exist in the impedance matrix and the right hand side vector, because of the cylinder symmetry about the $z$ and $y$ axes. Figure 8 illustrates a typical perfectly conducting cylinder with sixteen modes in the expansion for $\mathbf{J}_s$, and Figure 9 lists the corresponding symmetry relationships that should exist between certain mutual and self mode impedances for this cylinder. However, we found that impedances that should be the same were not being calculated in precisely the same way and consequently were not coming out exactly equal as expected.

In an attempt to correct for the inconsistencies introduced by this problem, the impedance matrix elements are calculated by the code in the following manner. First, so that the impedance matrix would be symmetric, a new mutual impedance is defined as

$$Z'_{mn} = Z'_{nm} = \frac{Z_{mn} + Z_{nm}}{2} \quad (2.49)$$

where the prime indicates the elements of the new impedance matrix. Finally, the geometrical symmetries described earlier are enforced in the impedance matrix. This is done by simply setting all elements, which from geometric symmetries should be the same, to be exactly the same.

The proper symmetry relationships did exist in the right hand side vector. However, an accurate solution depended upon the proper cancellation of the fields generated by the incident wave and the image wave. This cancellation required that the elements of the right hand side vector be accurately computed. To improve the accuracy of our solution, the original subroutine used in calculating the right hand side vector uses a double precision routine.

With the above modifications to the original computer code, reasonable results for rectangular, elliptic and half cycle cosine bumps which have a $h/L$ ratio greater

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Figure 8: A perfectly conducting cylinder with sixteen modes.
Figure 9: Geometric Symmetries in the Impedance Matrix
than 0.006 are obtained. For some dimensions, accurate results for bumps with a $h/L$ ratio as small as 0.002 are obtained. Because of the small separation of modes on the right and left edges of the full cycle cosine cylinder, an accurate MM solution for some of the flatter bumps with a full cycle cosine configuration is not obtainable.
CHAPTER III

Numerical Results

3.1 Introduction

The results obtained for TE scattering from a short perfectly conducting bump on an infinite ground plane are discussed here. In this research, we study the TE scattering from perfectly conducting bumps of four different shapes. In particular, results for the scattering from an elliptic bump, a half cycle cosine bump, a full cycle cosine bump, and a rectangular bump are presented. For the case of an elliptic shaped bump, the MM results are compared with the results obtained using an exact eigenfunction solution. The modelling of the cylinders used in the MM computer code is discussed, and patterns are shown for various dimensions of each of the four bump configurations studied. The results are presented in graphical form so that the variation of the backscattered field with the dimensions and shape of the bump and the angle of incidence of the TE polarized plane wave can be seen. All results are shown in terms of the echo width described in the preceding chapter. Finally, a conversion formula for obtaining the radar cross section (RCS) for a bump with finite length $L_z$ in the $\hat{z}$ dimension from our 2D echo width results for a bump with infinite length in the $\hat{z}$ dimension is presented.
3.2 2D Results

Figure 10 shows the geometry of each of the bumps we studied. For purpose of illustration, each is \(0.5\lambda\) in length and \(0.005\lambda\) in height. As mentioned previously, the primary interest is in the scattering from bumps that are short in height (i.e. less than a few hundredths of a wavelength). Figure 10a shows an elliptic bump. Figure 10b and c are both bumps having the shape of a cosine function, however, the bump shown in Figure 10b is outlined by a half cycle cosine function, while, the bump shown in Figure 10c is the shape of a full cycle cosine function. The elliptic bump and the two cosine shaped bumps are of interest because they model common deformations that occur in practice. Some examples include a joint which joins two flat pieces of metal together and a curved deformation in a flat sheet resulting from a slightly imperfect fit. The rectangular bump, shown in Figure 10d is an exaggeration of any deformation that would realistically occur; however it is included in the study for general interest reasons.

Because the wave equation is separable in elliptic cylindrical coordinates, we were able to obtain exact results for the TE scattering by an elliptic bump with Richmond's exact eigenfunction expansion code for an elliptic cylinder [24], [25]. By comparing our MM solution for an elliptic shaped bump with this eigenfunction solution, we were able to assess the accuracy and limitations of the MM code. It was found that for especially flat elliptic cylinders that the segmentation of the ellipse used in Richmond's MM code influenced the accuracy of the solution. Three methods of setting up the ellipse geometry for use in the MM solution were examined, and the best results were obtained when the \(x\) and \(y\) coordinates defining the ellipse were generated using the following polar equations:

\[ x = a \cos \theta, \text{ and} \]  

\[ (3.1) \]
a) Elliptic shaped bump

b) Half cycle cosine shaped bump

c) Full cycle cosine shaped bump

d) Rectangular box shaped bump

Figure 10: Four different shaped bumps
\[ y = b \sin \theta \]  \hspace{1cm} (3.2)

where \( a \) is the half length of the major axis of the ellipse, and \( b \) is the half length of the minor axis of the ellipse. First, the parametric angle \( \theta \) is varied in equal increments from 0 through \( \frac{\pi}{2} \) to generate one quarter of the entire ellipse. Then, to insure geometrical symmetry, the rest of the points on the ellipse are determined by reflecting the points on the first quarter of the surface across the coordinate axes and the coordinate origin. An enlarged version of an ellipse generated by this method is shown in Figure 11. Note that the segment length gradually increases as \( \theta \) increases from 0 to \( \frac{\pi}{2} \). This method yields the shortest segments at the curved right and left edges of the ellipse and the longest segments along the flat top portion of the ellipse and generates an accurate polygon model of an ellipse.

Because this equal angle method gives the best MM results for the flat elliptic bumps, a similar approach is used to generate the cosine functions used in the MM solution of the half cycle cosine cylinder and the full cycle cosine cylinder. For both the cosine cylinders, the \( x \) coordinate is determined by

\[ x = \frac{L}{2} \cos \theta \]  \hspace{1cm} (3.3)

where \( L \) is the length of the bump, and \( \theta \) is the parametric angle which is again varied in equal increments from 0 through \( \frac{\pi}{2} \). For the half cycle cosine, the \( y \) coordinate is given by

\[ y = h \cos \frac{\pi x}{L} \]  \hspace{1cm} (3.4)

where \( h \) is the height of the bump, and \( L \) is the length. The \( y \) coordinate for the full cycle cosine is determined from

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\[ y = \frac{h}{2} \cos \frac{2\pi x}{L} + \frac{h}{2}. \] (3.5)

The reflection technique is used in generating both of the cosine cylinders as well as the rectangular cylinder.

Figures 12 through 14 show patterns calculated using the MM and the eigenfunction expansion technique, for three elliptic shaped bumps which have the same height 0.005\(\lambda\), but different lengths. In each figure, the solid curve shows the MM solution, and the dashed curve shows the eigenfunction solution. As discussed in Chapter II, the main limitation of the MM solution is the height to length ratio of the bump under consideration. If the height of the bump is held constant, the accuracy of the MM solution decreases as the length of the bump increases. Although the MM results agree fairly well with the eigenfunction results for the geometries shown, the accuracy does decrease as \(h/L\) decreases.

The figures that follow show the 2D, TE scattering patterns obtained using the MM for a representative sample of the four different shaped bumps. Patterns have been included for bumps which have the same physical dimensions so that the effect of bump shape on the scattering pattern can be seen. Figures 15 through 18 show the echo width in db's relative to a wavelength as a function of the incident angle \(\psi\) for bumps with \(L = 0.1\lambda\) and \(h = 0.005\lambda\). Twenty four modes were used in the MM solution for each of these bumps. The patterns for these bumps are very similar in shape, with the maximum echo width occurring at \(\psi = 0^\circ\) and the minimum occurring at \(\psi = 90^\circ\).

Figures 19 through 22 show the MM data obtained using twenty four modes for bumps with \(L = 0.5\lambda\) and \(h = 0.005\lambda\). As we the length of the bump is increased, we begin to see the influence of bump shape on the scattering pattern.
Segmentation used to model a 16 sided elliptic cylinder for a MM solution.

Figure 11: Segmentation of an elliptic cylinder.
Although the patterns of the half cycle cosine bump and the full cycle cosine bump are still fairly similar in shape, the patterns for the elliptic bump and the rectangular bump have taken on more distinctive shapes. Note in particular that a null occurs in the pattern of the rectangular bump at an incident angle of 10°. This null most likely occurs at the angle of incidence where the scattering from the front and back corner destructively interfere.

The number of segments used in the MM solution was determined by increasing the number of segments defining the cylinder geometry until a convergent solution was obtained. In a MM solution usually five segments per wavelength is sufficient for obtaining accurate results [4]; however, because of the similarities between the cylinder geometries of interest, more segments per wavelength were needed to accurately describe the geometry for each bump.

Figures 23 through 25 show the MM data obtained using forty modes for bumps having $L = 1\lambda$ and $h = .005\lambda$. The pattern for a full cycle cosine bump of $L = 1\lambda$ and $h = .005\lambda$ is not included, because reliable results for these dimensions are not obtainable. At this length, each of the bumps has a more uniquely shaped pattern. Although the patterns for the elliptic shaped bump and the half cycle cosine shaped bump appear to be somewhat similar, it is important to note that the echo width for the half cycle cosine shaped bump remains fairly constant as the angle of incidence varies from 0° to 30°; while, the echo width for the elliptic shaped bump increases steadily over this same range.

For low angles of incidence the total backscattered field is primarily composed of a diffracted field contribution from the front edge of the bump and another diffracted field contribution from the back edge of the bump. As seen in the patterns shown in Figures 15 through 25, the way in which these contributions combine is dependent on the shape of the bump as well as the dimensions of the
bump. It was found that when the length of each bump is fixed, the echo width versus incident angle characteristic does not change very much in shape as the height of the bump is varied. Figures 26 through 36 show some additional results for each bump in which the length of the bump is held constant while the echo width is plotted over the range $\psi = 0^\circ$ to $\psi = 90^\circ$ for different values of bump height, $h$. This behavior is noted for each bump with length $L = .1\lambda, .5\lambda$, however, only plots for each of the four shaped bumps having length $L = 1\lambda$ are shown.

So that the effect of bump height on the magnitude of the backscattered field can be seen, we have plotted the echo width as a function of bump height for a bump with constant length. We present this data at $\psi = 0^\circ, 30^\circ, 60^\circ$, and $90^\circ$ for each of the four different bumps with $L = .1\lambda, L = .5\lambda$, and $L = 1\lambda$ to show how the echo width varies as a function of incident angle $\psi$. Figures 37 through 48 show the MM result and the exact eigenfunction result at each different height for the elliptic bumps. Note that the eigenfunction expansion data for echo width as a function of bump height forms a smooth almost linearly ascending curve when plotted on a logarithmic scale as shown in the figures.

Because we are assuming that the eigenfunction expansion solution is the most accurate, the data points representing the MM solution which are not in agreement with the corresponding eigenfunction expansion data points are believed to be unreliable. We can see from Figures 37 through 48 that the unreliable MM data points are randomly located on the graphs of echo width versus height for the three different lengths of elliptic bumps. In most cases, this is also true for the unreliable points for the three remaining bump configurations. For this reason, we have not included unreliable data points in the figures showing the echo width as a function of height for the rectangular bump, and the two cosine shaped bumps. The first data point, at each angle, appearing on each graph in Figures 49 through
57 represents the first reliable data point obtained with the MM solution. For easy comparison, Figures 58 through 60 show the echo width as a function of height plotted for each bump configuration at lengths \( L = .1\lambda, .5\lambda \), and \( 1\lambda \) for the case of grazing incidence, \( \psi = 0^\circ \).

Figures 61 through 64 show the effect of bump length, \( L \), on the magnitude of the backscattered field at grazing incidence (\( \psi = 0^\circ \)) for the elliptic bump, the rectangular bump, the half cycle cosine bump and the full cycle cosine bump. In these plots, the bump height is held constant at \( h = .01\lambda \) while the length of the bump is varied. Data points are taken in \( .1\lambda \) increments, and they are connected using a spline interpolation. From the plots for the elliptic bump, the rectangular bump and the half cycle cosine bump, one can see that the echo width varies periodically with the bump length. Although the same behavior is expected for the case of the full cycle cosine bump, reliable data for this shape bump with length \( L > 1\lambda \) is not obtainable at this height.

In each of these plots, a line can be drawn connecting the peaks of the interference pattern to give a worst case scattering model. If this is done, it can be seen that the scattering from a rectangular bump is independent of the length \( L \) of the bump. For elliptic bumps with length \( L \leq 3\lambda \), the echo width has some dependence on the length of the bump, however, as the length of the bump is increased, the scattering becomes more independent of the bump length. This is because a long elliptic bump is very similar in shape to a long rectangular bump. For the case of the half cycle cosine bump, one can see that the scattering is more dependent on the bump length, \( L \).

From the results for the echo width of a bump with infinite length, the radar cross section, \( \sigma \), for a bump with finite length, \( L_z \) is given by the conversion
The formula [30] is given by:

$$\sigma = \frac{2A_e L_z^2}{\lambda}. \quad (3.6)$$

In Equation (3.6) $A_e$ is the echo width of the 2D bump, and as before, $\lambda$ is the free space wavelength of the incident TE polarized plane wave. Since the results have been presented in the units of dB's relative to a wavelength, it is convenient for us to write Equation (3.6) as

$$RCS_{db} = 10 \log_{10}(\frac{2L_z^2}{\lambda}) + 10 \log_{10} A_e. \quad (3.7)$$

In order for Equation (3.7) to be valid, the units of all quantities with dimension must be the same. For example, if $A_e$ is expressed in wavelengths, then $L$ must also be expressed in wavelengths, and the RCS will be in dB's relative to $\lambda^2$. Since we have computed the echo width $A_e$ in dB's relative to $\lambda$, it is most convenient for us to express $L$ in wavelengths to compute the radar cross section for a bump with finite length. The second term on the right hand side of Equation (3.7) is the quantity plotted in all of our graphs, echo width in dB relative to a wavelength. Therefore, the RCS for a bump of any one of the four cross sections studied could easily be obtained by shifting the appropriate 2D curve by the amount

$$10 \log_{10}(\frac{2L_z^2}{\lambda}). \quad (3.8)$$
MM and eigenfunction results for an elliptic bump with \( L = .1\lambda \), \( h = .005\lambda \) on an infinite ground plane.

Figure 12: Pattern for an elliptic bump; \( L = .1\lambda \) and \( h = .005\lambda \).
MM and eigenfunction results for an elliptic bump with $L = 0.5\lambda$, $h = 0.005\lambda$ on an infinite ground plane.

Figure 13: Pattern for an elliptic bump; $L = 0.5\lambda$ and $h = 0.005\lambda$. 

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MM and eigenfunction results for an elliptic bump with $L = 1\lambda$, $h = 0.005\lambda$ on an infinite ground plane.

Figure 14: Pattern for an elliptic bump; $L = 1\lambda$ and $h = 0.005\lambda$. 

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TE scattering from a short length short elliptic bump with $L = .1\lambda$ and $h = .005\lambda$.

Figure 15: Pattern for an elliptic bump; $L = .1\lambda$, $h = .005\lambda$. 
TE scattering from a short length half cycle cosine bump with $L = .1\lambda$ and $h = .005\lambda$.

Figure 16: Pattern for a half cycle cosine bump; $L = .1\lambda, h = .005\lambda$. 
TE scattering from a short length short full cycle cosine bump with \( L = .1\lambda \) and \( h = .005\lambda \).

Figure 17: Pattern for a full cycle cosine bump; \( L = .1\lambda, h = .005\lambda \).
TE scattering from a short length short rectangular bump with \( L = 0.1\lambda \) and \( h = 0.005\lambda \).

Figure 18: Pattern for a rectangular bump; \( L = 0.1\lambda, h = 0.005\lambda \).
TE scattering from a medium length short elliptic bump with $L = 0.5\lambda$ and $h = 0.005\lambda$.

Figure 19: Pattern for an elliptic bump; $L = 0.5\lambda$, $h = 0.005\lambda$. 

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TE scattering from a medium length short half cycle cosine bump with $L = .5\lambda$ and $h = .005\lambda$.

Figure 20: Pattern for a half cycle cosine bump; $L = .5\lambda$, $h = .005\lambda$. 
TE scattering from a medium length short full cycle cosine bump with $L = .5\lambda$ and $h = .005\lambda$.

Figure 21: Pattern for a full cycle cosine bump; $L = .5\lambda$, $h = .005\lambda$. 
TE scattering from a medium length short rectangular bump with $L = .5\lambda$ and $h = .005\lambda$.

Figure 22: Pattern for a rectangular bump; $L = .5\lambda$, $h = .005\lambda$. 
TE scattering from a long length short elliptic bump with $L = 1\lambda$ and $h = .005\lambda$.

Figure 23: Pattern for an elliptic bump; $L = 1\lambda$, $h = .005\lambda$. 
TE scattering from a long length short half cycle cosine bump with $L = 1\lambda$ and $h = .005\lambda$.

Figure 24: Pattern for a half cycle cosine bump; $L = 1\lambda$, $h = .005\lambda$. 
TE scattering from a long length short rectangular bump with $L = 1\lambda$ and $h = .005\lambda$.

Figure 25: Pattern for a rectangular bump; $L = 1\lambda$, $h = .005\lambda$. 
TE scattering from an elliptic bump with $L = 1\lambda$, and $h = .002\lambda$.

Figure 26: Pattern for an elliptic bump: $L = 1\lambda$, $h = .002\lambda$. 
TE scattering from an elliptic bump with $L = 1\lambda$, and $h = 0.008\lambda$.

Figure 27: Pattern for an elliptic bump; $L = 1\lambda$, $h = 0.008\lambda$. 
TE scattering from an elliptic bump with $L = 1\lambda$, and $h = .05\lambda$.

Figure 28: Pattern for an elliptic bump; $L = 1\lambda$, $h = .05\lambda$. 
TE scattering from a half cycle cosine bump with $L = 1\lambda$, and $h = .004\lambda$.

Figure 29: Pattern for a half cycle cosine bump; $L = 1\lambda$, $h = .004\lambda$. 
TE scattering from a half cycle cosine bump with $L = 1\lambda$, and $h = .008\lambda$.

Figure 30: Pattern for a half cycle cosine bump; $L = 1\lambda$, $h = .008\lambda$. 
TE scattering from a half cycle cosine bump with $L = 1\lambda$, and $h = 0.05\lambda$.

Figure 31: Pattern for a half cycle cosine bump; $L = 1\lambda, h = 0.05\lambda$. 

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TE scattering from a full cycle cosine bump with $L = 1\lambda$, and $h = .008\lambda$.

Figure 32: Pattern for a full cycle cosine bump; $L = 1\lambda$, $h = .008\lambda$. 
TE scattering from a full cycle cosine bump with $L = 1\lambda$, and $h = .05\lambda$.

Figure 33: Pattern for a full cycle cosine bump; $L = 1\lambda$, $h = .05\lambda$. 
TE scattering from a rectangular bump with $L = 1\lambda$, and $h = 0.002\lambda$.

Figure 34: Pattern for a rectangular bump; $L = 1\lambda$, $h = 0.002\lambda$. 

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TE scattering from a rectangular bump with $L = 1\lambda$, and $h = .008\lambda$.

Figure 35: Pattern for a rectangular bump; $L = 1\lambda$, $h = .008\lambda$. 

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TE scattering from a rectangular bump with $L = 1\lambda$, and $h = .05\lambda$.

Figure 36: Pattern for a rectangular bump; $L = 1\lambda$, $h = .05\lambda$. 
Echo width plotted as a function of bump height for a .1\(\lambda\) length elliptic bump for \(\psi = 0^\circ\).

Figure 37: Elliptic bump; \(L = .1\lambda, \psi = 0^\circ\).
Echo width plotted as a function of bump height for a \( .1 \lambda \) length elliptic bump for \( \psi = 30^\circ \).

Figure 38: Elliptic bump; \( L = .1 \lambda, \psi = 30^\circ \).
Echo width plotted as a function of bump height for a .1λ length elliptic bump for $\psi = 60^\circ$.

Figure 39: Elliptic bump; $L = .1\lambda, \psi = 60^\circ$. 
Echo width plotted as a function of bump height for a .1\(\lambda\) length elliptic bump for \(\psi = 90^\circ\).

Figure 40: Elliptic bump; \(L = .1\lambda, \psi = 90^\circ\).
Echo width plotted as a function of bump height for a \( 0.5\lambda \) length elliptic bump for \( \psi = 0^\circ \).

Figure 41: Elliptic bump; \( L = 0.5\lambda, \psi = 0^\circ \).
Echo width plotted as a function of bump height for a .5\(\lambda\) length elliptic bump for \(\psi = 30^\circ\).

Figure 42: Elliptic bump; \(L = .5\lambda, \psi = 30^\circ\).
Echo width plotted as a function of bump height for a \(0.5\lambda\) length elliptic bump for \(\psi = 60^\circ\).

Figure 43: Elliptic bump; \(L = 0.5\lambda, \psi = 60^\circ\).
Echo width plotted as a function of bump height for a .5λ length elliptic bump for $\psi = 90^\circ$.

Figure 44: Elliptic bump; $L = .5\lambda, \psi = 90^\circ$. 

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Echo width plotted as a function of bump height for a $1\lambda$ length elliptic bump for $\psi = 0^\circ$.

Figure 45: Elliptic bump; $L = 1\lambda, \psi = 0^\circ$. 
Echo width plotted as a function of bump height for a $1\lambda$ length elliptic bump for $\psi = 30^\circ$.

Figure 46: Elliptic bump; $L = 1\lambda, \psi = 30^\circ$. 
Echo width plotted as a function of bump height for a $1\lambda$ length elliptic bump for $\psi = 60^\circ$.

Figure 47: Elliptic bump; $L = 1\lambda, \psi = 60^\circ$. 
Echo width plotted as a function of bump height for a $1\lambda$ length elliptic bump for $\psi = 90^\circ$.

Figure 48: Elliptic bump; $L = 1\lambda, \psi = 90^\circ$. 
Echo width plotted as a function of bump height for half cycle cosine bump with $L = .1\lambda$.

Figure 49: Half cycle cosine bump; $L = .1\lambda$. 

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Echo width plotted as a function of bump height for a half cycle cosine bump with $L = .5\lambda$.

Figure 50: Half cycle cosine bump; $L = .5\lambda$.  

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Echo width plotted as a function of bump height for a half cycle cosine bump with $L = 1\lambda$.

Figure 51: Half cycle cosine bump; $L = 1\lambda$. 
Echo width plotted as a function of bump height for a full cycle cosine bump with $L = .1\lambda$.

Figure 52: Full cycle cosine bump; $L = .1\lambda$. 

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Echo width plotted as a function of bump height for a full cycle cosine bump with $L = 0.5\lambda$, length full cycle cosine bump.

Figure 53: Full cycle cosine bump; $L = 0.5\lambda$. 

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Echo width plotted as a function of bump height for a full cycle cosine bump with $L = 1\lambda$.

Figure 54: Full cycle cosine bump; $L = 1\lambda$. 
Echo width plotted as a function of bump height for a rectangular bump with $L = .1\lambda$.

Figure 55: Rectangular bump; $L = .1\lambda$. 
Echo width plotted as a function of bump height for a rectangular bump with \( L = 0.5\lambda \).

Figure 56: Rectangular bump; \( L = 0.5\lambda \).
Echo width plotted as a function of bump height for a rectangular bump with $L = 1\lambda$.

Figure 57: Rectangular bump; $L = 1\lambda$. 
A comparison of the Echo Widths of the $L = .1\lambda$ long bumps at $\psi = 0^\circ$.

Figure 58: Echo width as a function of bump shape and height; $L = .1\lambda$. 

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A comparison of the Echo Widths of the $L = .5\lambda$ long bumps at $\psi = 0^\circ$.

Figure 59: Echo width as a function of bump shape and height; $L = .5\lambda$. 
A comparison of the Echo Widths of the $L = 1\lambda$ long bumps at $\psi = 0^\circ$.

Figure 60: Echo width as a function of bump shape and height; $L = 1\lambda$. 

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Echo Width plotted as a function of bump length for an elliptic bump with $h = 0.01\lambda$ and $\psi = 0^\circ$.

Figure 61: Elliptic bump; $h = 0.01\lambda$, $\psi = 0^\circ$. 
Echo Width plotted as a function of bump length for a rectangular bump with $h = .01\lambda$ and $\psi = 0^\circ$.

Figure 62: Rectangular bump; $h = .01\lambda$, $\psi = 0^\circ$. 
Echo Width plotted as a function of bump length for a half cycle cosine bump with $h = .01\lambda$ and $\psi = 0^\circ$.

Figure 63: Half cycle cosine bump; $h = .01\lambda$, $\psi = 0^\circ$. 

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Echo width plotted as a function of bump length for a full cycle cosine bump with $h = 0.01\lambda$ and $\psi = 0^\circ$.

Figure 64: Full cycle cosine bump; $h = 0.01\lambda$, $\psi = 0^\circ$.
CHAPTER IV

Conclusions

A MM solution has been applied to the problem of TE scattering from a 2D perfectly conducting bump of arbitrary cross section which lies on an infinite and perfectly conducting ground plane. The main goal of this thesis was to generate a reliable body of data for the backscattered fields from small perturbations to a smooth surface. This has been achieved and presented in a series of echo width plots. We have found that the lower limit on the height to length \((h/L)\) ratio of a bump for which an accurate MM solution can be obtained is dependent on the shape and specific dimensions of the bump. The magnitude of the backscattered field from a bump is also dependent on the shape and specific dimensions of the bump as well as the angle of incidence of the plane wave. For low angles of incidence, the total backscattered field is composed of a diffracted field contribution from the front edge of the bump and another diffracted field contribution from the back edge of the bump, and the way in which these contributions combine depends on the shape and dimensions of the bump.

Although, no one shape was found to dominate the scattering properties of a short bump over the range of parameters investigated, some conclusions can be drawn from our results. For all bump geometries studied, we find that when the length of the bump is held constant, the echo width increases as the height of the bump increases. When the height of the bump is held constant, the backscattered
field is a periodic function of the the bump length. In addition, for a constant length bump, the 0° – 90° pattern remains basically the same shape as the height of the bump is varied. For bumps which are very short in length (i.e. \( L = .1\lambda \)), the shape of the bump does not have much influence on the shape of the pattern. The edge diffraction coefficients for each bump shape can be empirically derived from the MM results obtained in this research. We suggest this as a topic for further study.
References


[24] Richmond, J.H., Personal Communication


