ICE STREAM SHEAR MARGIN BASAL MELTING, WEST ANTARCTICA

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ABSTRACT

Basal water lubricates and enables the anomalous flow feature of ice streams in the West Antarctic Ice Sheet. As surface melt is insufficient to supply the base with the volume of water known to be there, basal melting must be the source of this water. How basal melt patterns vary spatial can be an insight into the dynamics of ice streams, which remain incompletely described by glaciological theory. Through a heuristic model extended from the work of Whillans and Van der Veen (2001) and Van der Veen et al. (2007) a spatial pattern of basal melt for the Whillans Ice Stream emerged that offer hypotheses for the onset of streaming flow, shear crevasse development and observed morphological changes of a slowing and widening ice stream.

The limitations and the uncertainties of this model make the determination of exact basal melt rates difficult, but the patterns of melt rate distribution are robust. This allows for a perspective to better understand current dynamics and how basal melt may play a role in the ice stream’s future development.
dedicated to my parents

for their continued support
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CHAPTER 1
INTRODUCTION

The future behavior of the terrestrial ice sheets is a great uncertainty, one with increasing significance as the vulnerability of human society to climate change and sea level rise becomes more fully recognized. Currently, small mountain glaciers and ice caps are contributing the majority to the global sea level rise (Meier et al., 2007). However, Greenland and Antarctica hold ~95% of possible future sea level rise not associated with thermal expansion (Alley et al., 2005 and Meier et al., 2007). Within the next fifty to one hundred years ice sheets are expected to become the major contributor to sea level rise, although it could be sooner given uncertainties in the response of ice sheets to a changing climate (Meier et al., 2007).

Understanding more fully what governs ice flow will allow for a more complete description for how ice sheets evolve though time. The Intergovernmental Panel on Climate Change (IPCC) Fourth Assessment Report has highlighted the deficiency in prognostic glacier models as it pertains to sea level and climate projections (IPCC, 2007). The deficiency is largely related to the effects of rapid, outlet glacier dynamical changes and large low probability events such as ice shelf disintegration and collapse of the West Antarctic Ice Sheet. Each of these events, should they occur, could lead to global sea level rise of multiple meters in a few hundred years or less (Oppenheimer, 1998).
IPCC projected a range of sea level rise by the decade 2090-2099 (relative to 1980-1999) of 18-59 cm but included the caveat that “larger values cannot be excluded, but understanding of these effects is too limited to assess their likelihood or provide a best estimate or an upper bound for sea level rise” (IPCC, 2007).

There are numerous recent observations which show how incompletely glacier behavior is captured in existing glaciological theory (for example: Rignot, 2006; Joughin et al., 1996; Scambos et al., 2004; Howat et al., 2007 and Zwally et al., 2002). These examples demonstrate that glaciers respond more quickly to climate forcings than was previously believed. It is now recognized that glacial dynamics can change on a temporal scale of a season or shorter when it was previously believed to take many years to centuries or longer.

Central to any discussion about rapid response time of glaciers is the effect of a thawed and lubricated base. This, and the effects of longitudinal stress gradients due to changes in terminal geometry (i.e. ice shelves and tongues) are the two boundary conditions which can allow for rapid change (Vaughan, 2007). It is necessary to understand the source of basal water and the effects of this water at changing volumes and pressures. These conditions must be more fully constrained and described if ice sheet models are to be improved.

Ice shelf collapse has become an ever increasing occurrence on the Antarctic Peninsula and elsewhere on the continent, with the complete collapse of the Larsen B ice shelf in 2002 (Scambos et al., 2004) and the partial collapse of the Wilkins ice shelf in early 2008. Particularly good documentation of glacier behavior before and after the Larsen B collapse has shown that glaciers behind an ice shelf which has collapsed
respond immediately by accelerating (Scambos et al., 2004). The southward migration of Antarctic ice shelf instability is closely linked with progressive atmospheric and oceanographic warming (Scambos et al., 2000). The break-up of ice shelves adjacent to the West Antarctic Ice Sheet (WAIS) is one process which also has been used to address the possible instability of the entire WAIS, which some researchers have suggested is prone to complete disintegration (Oppenhiemer, 1998 and Alley et al., 2001). Whether such a catastrophic event, capable of raising sea level by 5m, is possible is the focus of much debate. How the WAIS is to respond to the removal of a large ice shelf is linked to the present controls of its dynamical behavior. It is clear that understanding the current dynamic controls of the WAIS serve to better address the issue of ice sheet instability.

Understanding the controls of flow for the West Antarctic ice streams becomes especially significant when the specter of complete collapse is considered. More certainly, the behavior of the WAIS will continue to influence sea level. The ice streams are among the largest and most dynamic regions of the ice sheet and their behavior is inextricably linked to the future mass balance of the ice sheet. Variations in ice stream velocity structure could make the mass balance positive or increasingly negative, depending on the nature of future changes. Basal conditions and specifically the production and abundance of basal water are avenues to further the scientific understanding of this dynamic region and constrain uncertainties associated with glacial dynamical changes.

In the following chapters, the question of how basal melt rates vary beneath West Antarctic ice streams is addressed. After a general description of the location of the stuffy area and the remotely sensed data sets used, the theory as formulated by Whillans and Van der Veen (2001) and Van der Veen et al. (2007) is described, as are the simplifying
assumptions which allow for a solution to be found. The simplifications here are partially used to circumvent the need for the computationally heavy and sensitive full numerical force balance model. Chapter 4 describes a full numerical force balance model that was constructed and used to analyze the data sets of the ice streams. It was found given the sensitivities of a full numerical force balance model to the relatively noisy data sets and the high resolution needed to investigate processes below the shear margins that a stable solution was not possible. With this justification for a simpler model, analysis proceeded.

The sensitivity of the simpler model is assessed in Chapter 5, where it is shown, that although values for many variables can only be estimated, there is a robust solution. Following in Chapters 6 and 7 are the specific results of the model and the ramifications for the dynamics of the West Antarctic ice streams.
CHAPTER 2

OVERVIEW OF THE WEST ANTARCTIC ICE STREAMS

2.1 Introduction

There are five major ice streams along the Siple Coast Region of the WAIS which are 30-80 km wide and hundreds of kilometers long, each with numerous meandering tributaries. The Whillans Ice Stream (formally Ice Stream B) is the most extensively studied and will be studied here.

Figure 2.1: RADARSAT mosaic of Whillans Ice Stream with inset location of ice stream on West Antarctic Ice Sheet (Jezek, 2008). Names in quotations are regions of the Whillans Ice Stream referred to in text.
2.2 Ice Stream Location

The West Antarctic ice streams are located along the Siple Coast and drain into the Ross Ice Shelf. The upper tributaries of ice streams proven to be constrained by basal topography (Joughin et al, 1999). However the location of the main trunk of the ice stream is minimally controlled by basal topography (Shabtaie et al., 1987). The main truck of the ice stream is likely determined by some combination of basal sediment distribution, basal water abundance, geothermal flux patterns and internal dynamic feedbacks (Whillans et al., 2001).

2.3 Mass Balance

Due to the size and speed of the ice streams they are important to the mass balance of the WAIS. Siple Coast Ice Streams drain about 40% of the area of the WAIS (Price et al., 1998). Changes to the ice streams have the potential to significantly impact the mass balance of this region.

Rignot et al. (2008) calculated that the entire Siple Coast Ice Streams, in 2006, had a positive mass balance of 34 +/- 8 Gt yr\(^{-1}\) which is comparable to earlier estimates of 28 +/- 14.9 Gt yr\(^{-1}\) in 2002 (Joughin and Tulaczyk, 2002). Both of these vary markedly from an estimate in 1984 which showed a negative mass balance of – 17.4 +/- 13.7 Gt yr\(^{-1}\) (Shabtaie and Bentley, 1987). However, the Shabtaie and Bentley (1987) estimate relied on limited velocity measurements and maybe underestimated accumulation rates (Joughin and Tulaczyk, 2002). It is therefore unclear if there has been significant change since 1984 or if the earlier estimates were erroneous due to paucity of data.
The mass balance of the Whillans Ice Stream (WIS) in 1984 was estimated to be \(-11.5 +/\ - 5.4 \text{ km}^3 \text{ a}^{-1}\) (Shabtaie and Bentley, 1987). An estimate in 1988 found a thinning rate of 0.06 m yr\(^{-1}\) (Whillans et al., 2001) and is grossly equivalent to Shabtaie and Bentley’s (1987) estimate. A more recent analysis of measurements of the Whillans Ice Stream from 1988 and 1997 (Stearns et al., 2005), found a negative mass balance for the entire ice stream for the earlier date and a positive mass balance for the downstream section for the later year. This suggests that changes have been taking place over decadal time scales.

### 2.4 General Dynamics

The West Antarctic Ice Sheet is glaciologically unique because of the occurrence of ice streams which achieve fast velocities even as gravitational driving stress diminishes (Whillans et al., 2001). This makes them distinct from other fast glaciers that occur in other regions of West Antarctica, East Antarctica and Greenland.

The Siple Coast Ice Streams are regions of fast flowing ice, up to \(~800 \text{ m yr}^{-1}\), completely bounded by slower or stagnant ice ridges with surface velocities near zero to tens of meters per year. The WIS has a driving stress of \(<20 \text{ kPa}\), while other outlet glaciers often maintain a driving stress of over 200 kPa. Furthering the unusual conditions found in the ice streams, the driving stress tend to diminish down flow while the centerline velocity tends to increase. For most of the WIS this is the case, but about 100 km upstream from the grounding line the ice stream center line velocity begins to slow and there remainder of the ice stream there is direct relationship between the driving stress.
and centerline velocity. An inverse relation between driving stress and velocity does not fit within traditional glaciological theory and remains incompletely explained.

To achieve such fast speeds the base must be lubricated by highly pressurized basal pore water. It was shown by Kamb (2001) that basal pore water pressure is very near the ice over burden pressure. This creates a condition where the bed under the ice stream cannot supply much, if any, resistance to flow. Resistance to flow then must be supplied by the shear margins or longitudinal stress gradients (Whillans and Van der Veen, 1997).

2.5 Shear Margins

Rapid flowing ice streams being bounded by slower ice creates the presence of exceptional shear strain rates along the margins and development of large crevasse fields, a highly distinguishable morphological feature of the WAIS and clearly seen in the radar mosaic (see Fig. 2.1). Marginal shear is dominant in the controlling of the ice stream’s flow (Whillans and Van der Veen, 1997). Understanding how shear strain controls the ice stream is one goal of this study.

Resistance from lateral drag is close in magnitude to that of the gravitational driving stress. The lateral stresses, applied along vertical shear planes, must be transmitted and rotated to basal drag, applied along horizontal shear planes, below the interstream ridges where the ice is frozen to bedrock and capable of supporting greater basal drag (Whillans and Van der Veen, 2001).

Due to the high magnitude of shear stress characteristic crevasse fields are present in this location. From the interstream ridge into the ice stream there are typically two
types of crevasses found. First, a ~0.5 to 1 km wide band of arcuate crevasses, concave down flow, is encountered followed by a 2-4 km wide band of chaotic crevasses.

Figure 2.2: Close up of RADARSAT mosaic showing two types of shear crevassing present along shear margin of Whillans Ice Stream

2.6 Basal Conditions

Two proposed mechanisms to explain the ice streams’ rapid velocity are dilated till deformation and lubricated basal sliding (Bennett, 2003). It is possible that each mechanism contributes to some degree across or along an ice stream. Each mechanism also has a prerequisite for the presence of highly pressurized basal water. That prerequisite is satisfactorily fulfilled as shown by Kamb (2001) who used borehole data
to confirm that basal pore water pressure is near the ice overburden pressure for most of the ice stream.

The till has been characterized by Tulaczyk et al. (2000) as fine grained fairly homogeneous sediment with a small fraction (7%) of granules and pebbles. Basal sediment collected from boreholes was shown to behave plastically with a yield strength of only a few kPa (Tulaczyk et al., 2000). This supports the observation that ice stream flow cannot be resisted by the bed.

2.7 Morphological Changes

There have been observed changes in the velocity and size of the West Antarctic ice streams. The largest change has been the dramatic shut down of the Kamb Ice Stream 145 +/- 25 ybp (Retzlaff and Bentley, 1993). Current changes show that the Whillans Ice Stream is slowing and widening and the Bindschadler Ice Stream and the Echelmeyer Ice Stream are thickening and the MacAyeal Ice Stream is thinning (Joughin et al., 2002). The reasons for these changes have not been fully explained. Hypotheses include changes in the amount or presence of basal water and atmospheric changes which are affecting the accumulation rates.

2.8 Basal Melt

The source of the basal pore water is basal melt but where and at what rate melt water is produced is unclear. Prior to demonstrating that the ice stream was largely supported by the margins (Whillans and Van der Veen, 1997 and Whillans and Van der Veen, 2001), the basal drag on the interior ice stream was over estimated. As a result,
early estimates of basal melt in the interior ice stream were large (~2 cm a\(^{-1}\)) (Shabtaie et al., 1987). Since then numerous studies have been conducted to determine the nature of the basal energy regime. Raymond (2000 and 2001) reiterates the point from Whillans et al. (2001) that the interior of ice streams cannot support much basal melt, but explains the difficulty of generating heat in well lubricated areas. Parizek et al. (2003) and Joughin et al. (2003 and 2004) both conclude that basal melt primarily occurs in the tributary regions were basal drag and driving stress are both at relative highs. Localized areas of elevated melt are present throughout the ice stream and may be related to “sticky spots”, areas of elevated basal drag. Basal freeze-on is predicted in the interior of the main trunk of the ice stream and on the ice plain adjacent to the Ross Ice Shelf. Joughin et al. (2003 and 2004) predict a maximum melt rate of ~2 cm a\(^{-1}\) in the upper tributaries. Van der Veen et al. (2007) estimate that melt is concentrated across the shear margin. They compute the melt magnitude is ~4 cm a\(^{-1}\) and largely confined to areas beneath or just interior of shear margins.

Because the continued presence of water is necessary for streaming flow and a decrease in water availability is implicated in the shut down of the Kamb Ice Stream, understanding and quantifying water production is essential to understanding the present and future behavior of the ice streams.

2.9 Basal Hydrology

We are just now getting our first glimpse at the complexity and dynamic nature of the basal hydrological system. Subglacial lakes have been recognized since 1973 (Siegert et al., 1996) but their abundance is only now becoming clear. The speed and volumes of
basal water that move beneath the ice sheet are being quantified. Water volumes of 10 – 20 x10^6 m^3 have been inferred to have moved within a period of 24 days below the Kamb and Bindschadler Ice Stream (Gray et al., 2005) and a large subglacial lake below the Whillans Ice Stream is believed to have drained more than 2 km^3 over 3 years (Fricker et al., 2007).

It is also becoming clear that subglacial lakes may play a significant role in the onset of fast glacier flow and continental scale glacier dynamics (Siegert and Bamber, 2000). How water moves beneath the ice sheet is essential knowledge if we are to understand the ramification of basal melting. To what extent could melt in one region reach another is critical to understanding the effects of spatially varying melt rates. Fricker et al. (2007) suggest the presence of hydrologic potential ridges which isolate one region’s basal water from another. If this is true, melt from the margins may not influence the basal regime of the interior and could change the expected volume of basal water at those locations. Basal water may flow primarily in channels or as a thin spatially extensive layer. Knowing in what form basal water moves and how it evolves over time greatly affects the potential for glaciological changes (Bell, 2008). A diffuse spatial extensive layer of water can more easily induce large changes. With these uncertainties in mind, the true ramifications of basal melt rate can only be suggested.
CHAPTER 3
DATA

3.1 Introduction

Four primary data sets were used in this study: British Antarctic Survey’s BEDMAP for ice thickness (Lythe et al., 2000), the OSU DEM (Ohio State University Digital Elevation Model) relative to the geoid for ice surface slope (Liu et al., 1999), the RADARSAT Antarctic Mapping Project (RAMP) mosaic for ice stream dimensions (Jezek, 1999) and interferometric synthetic aperture radar (InSAR) surface velocities from RADARSAT-1 (Joughin et al., 1999). Other published data also used include temperature measurements from boreholes and calculations of geothermal flux.

3.2 Ice Thickness

BEDMAP is a British Antarctic Survey compilation of \( \sim 2 \times 10^6 \) observations of ice thickness collected since the mid 1950’s using a few ice core measurements, but mostly seismic reflection surveys, gravity surveys and ground and airborne radar echo sounding surveys. Spatial resolution and accuracy differs between surveys. The accuracy ranges from 10 m to over 180 m (Lythe et al., 2000). The data are presented as a gridded data set with 5 km grid spacing. Ice thickness was densely sampled over the Whillans and
Kamb Ice Streams giving rise to increased accuracy. Absolute accuracy of ice thickness is taken as 18 m (Shabtaie et al., 1987).

Figure 3.1: Ice Thickness of Antarctic Ice Sheet from BedMap (Lythe et al., 2000)

### 3.3 Surface Elevation

The OSU DEM is likewise a compilation of various surface elevation measurements collected from numerous sources each with differing spatial resolution and accuracy, including cartographical data, GPS and radar echo sounding sources. This interpolated data product is gridded at 200 m and has absolute mean errors ranging from
a few meters for GPS surveys to nearly 100 m for cartographical sources. The West Antarctic ice streams are primarily covered by cartographical data, from laser altimetry and airborne radar, at a scale of 1:1,000,000 with absolute errors near the higher end of the range. However, since surface slope is the measurement needed from this data set, only relative errors need to be considered. Relative errors over transects of 20 km (20 times ice thickness) can be taken as a few meters or less.

![Surface elevation of Whillans, Van der Veen, and Mercer Ice stream from the OSU DEM (Liu et al., 1999)](image)

3.4 Surface Features
Measurements of surface features are possible through the RADARSAT Antarctic Mapping Project (RAMP) mosaic (Jezek, 1999) (see Fig. 2.1). Over 5000 synthetic aperture radar frames were acquired using the RADARSAT-1 satellite over 18 days in 1997. These images were processed and calibrated at the Byrd Polar Research Center into a seamless mosaic of the entire continent that allows for an unobstructed look at surface and near surface features of the ice sheet. The shear margins of the West Antarctic ice streams are clearly visible due to their high surface roughness. The 100-200 m absolute geometric accuracy and minimal relative geometric distortion of this mosaic allows for direct measurement of relative ice stream width and length to within several tens of meters accuracy.

3.5 Surface Velocity

In conjunction with the RAMP product, interferometric image pairs were collected for select areas, including the West Antarctic ice streams. Processing of this data, completed by Joughin and others (1995 and 1999), allows for measurements of velocity. Control points from limited GPS velocity measurements were used to constrain the InSAR calculations. Through the use of speckle matching, both horizontal velocity components can be measured. This data set is presented in a 1000 m grid. Velocity error in the along track flow for InSAR is 4 m a\(^{-1}\). For speckle matching the error can be as great as 20 m a\(^{-1}\). A velocity error of 12 m a\(^{-1}\) is used for the calculations completed here.
3.6 Borehole Data

Published englacial temperature profiles (Kamb, 2001 and Engelhardt, 2004) were used to constrain the rate factor which is partially dependent on ice temperature and to estimate basal temperature gradient and geothermal flux.

3.7 Data Preparation

Each of the data sets were resampled to a uniform grid spacing of 400 m, the maximum distance necessary to resolve the processes of interest. Ice thickness and
surface velocity were cubically interpolated and the OSU DEM was degraded by averaging all the pixels to form the new larger pixel.
4.1 Introduction

Force balance is a technique that can be used to investigate basal processes through knowledge of surface velocity and glacier geometry, ice thickness and surface slope. All measurements are available from the remotely sensed data sets described in the preceding chapter. With the basic assumption that all the forces at a given point sum to zero, surface conditions can be used, given an understanding of how ice deforms under stress, to calculate how stresses and velocity vary with depth. In principle, this technique is useful to investigate how basal melt varies beneath the ice stream as it prevents the necessity for further simplifying assumptions. Its drawback is that it is computationally expensive and it is sensitive to small errors in initial conditions.

4.2 Force Balance Theory

A full numerical force budget model following from the scheme developed by Van der Veen and Whillans (1989) was constructed and summarized below.

Full stress horizontal force balance equations are used:
\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \quad (4.1)
\]
\[
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \quad (4.2)
\]

To solve these equations numerically it is convenient to normalize the vertical variable to the ice thickness. This is done by replacing the vertical coordinate, \( z \), with \( s \):

\[
s = \frac{(h - z)}{H} \quad (4.3)
\]

where \( h \) is ice surface elevation and \( H \) is ice thickness. Since horizontal gradients are in the \((x,y,z)\) coordinate system, it is necessary to relate them to the new \((x,y,s)\) system (see Van der Veen, 1999 pp. 45). This results in the force balance equations:

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\Delta_{xx} \, \frac{\partial \sigma_{xx}}{\partial s}}{H} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + \frac{1}{H} \, \frac{\partial \sigma_{sz}}{\partial s} = 0
\quad (4.4)
\]
\[
\frac{\partial \sigma_{xy}}{\partial x} + \frac{\Delta_{sx} \, \frac{\partial \sigma_{xy}}{\partial s}}{H} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{zy}}{\partial z} + \frac{1}{H} \, \frac{\partial \sigma_{ys}}{\partial s} = 0
\quad (4.5)
\]

The term \( \Delta_{si} \) is defined as:

\[
\Delta_{si} = \frac{\partial h}{\partial i} - s \frac{\partial H}{\partial i} \quad i = x,y
\quad (4.6)
\]
Next, the full stresses in equations (4.4) and (4.5) are partitioned into resistive, \( R_{ij} \), and lithostatic, \( L \), components, using equations (4.7) and (4.8), where \( \delta_{ij} \) is the Kronecker delta, \( \rho \) is the density of ice, \( g \) is the gravitational constant, \( h \) is surface elevation, \( H \) is ice thickness and \( s \) the dimensionless vertical coordinate defined above.

\[
\sigma_{ij} = R_{ij} + \delta_{ij} L \quad i,j = x,y,z \tag{4.7}
\]

\[
L = -\rho g H s \tag{4.8}
\]

The partitioned equations are integrated from some depth, \( s \), to the surface, \( s = 0 \). By reversing the order of differentiation and integration, per Liebnitz’s Rule, and applying an upper boundary condition that there must be a stress free upper surface:

\[
R_{ix}(0) \frac{\partial h}{\partial x} + R_{iy}(0) \frac{\partial h}{\partial y} - R_{iz}(0) = 0 \quad i = x,y \tag{4.9}
\]

equations (4.4) and (4.5) yield an equation for stress at depth:

\[
\frac{\partial}{\partial x} \int_s^0 HR_{xx} ds - \Delta_{xx} R_{xx}(s) + \frac{\partial}{\partial y} \int_s^0 HR_{xy} ds - \Delta_{xy} R_{xy}(s) + R_{xz}(s) - s \tau_{xs} = 0 \tag{4.10}
\]

\[
\frac{\partial}{\partial x} \int_s^0 HR_{yx} ds - \Delta_{yx} R_{yx}(s) + \frac{\partial}{\partial y} \int_s^0 HR_{yy} ds - \Delta_{yy} R_{yy}(s) + R_{yy}(s) - s \tau_{ys} = 0 \tag{4.11}
\]

Resistive stresses at the surface can be related to observable strain rates through the constitutive relationship. The constitutive relationship states:
\[ R_{ii} = B \dot{\varepsilon}_e^{(1/n-1)} (2\dot{\varepsilon}_{ii} + \dot{\varepsilon}_{jj}) + R_{zz} \quad i \neq j = x, y \]  
(4.12)

\[ R_{ij} = B \dot{\varepsilon}_e^{(1/n-1)} \dot{\varepsilon}_{ij} \quad i \neq j = x, y, z \]  
(4.13)

\[ \dot{\varepsilon}_e^2 = \dot{\varepsilon}_{xx}^2 + \dot{\varepsilon}_{yy}^2 + \dot{\varepsilon}_{zz}^2 + \frac{1}{2} \left( \dot{\varepsilon}_{xy}^2 + \dot{\varepsilon}_{xz}^2 + \dot{\varepsilon}_{yz}^2 \right) \]  
(4.14)

All strain rates can be calculated from surface velocity measurements except for \( \dot{\varepsilon}_{xz} \) and \( \dot{\varepsilon}_{yz} \). Rearranging these equations to solve for horizontal shear strain rate and making the simplifying assumption that \( R_{zz} \) is negligible results in an equation that can be used to solve for all stresses at any given depth:

\[ \dot{\varepsilon}_{xz} = \frac{-\partial}{\partial x} \int_s^0 H B \dot{\varepsilon}_e^{(1/n-1)} (2\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy}) ds + \Delta_{xs} B \dot{\varepsilon}_e^{(1/n-1)} (2\dot{\varepsilon}_{xx} + \dot{\varepsilon}_{yy}) \]  
(4.15)

\[ \dot{\varepsilon}_{yz} = \frac{-\partial}{\partial y} \int_s^0 H B \dot{\varepsilon}_e^{(1/n-1)} (2\dot{\varepsilon}_{xy} + \dot{\varepsilon}_{yz}) ds + \Delta_{ys} B \dot{\varepsilon}_e^{(1/n-1)} (2\dot{\varepsilon}_{xy} + \dot{\varepsilon}_{yz}) + s \tau_{dy} \]  
(4.16)

Although these equations have no analytical solution, a finite difference model can be used to estimate the values of \( \dot{\varepsilon}_{xz} \) and \( \dot{\varepsilon}_{yz} \) below any level \( s \) when velocities are known.
from previous calculations through iterative techniques. Vertical velocity gradients can be calculated from horizontal strain rates:

\[
\frac{\partial u_i}{\partial z} = 2\dot{\varepsilon}_i - \frac{\partial w}{\partial x_i} \quad i=x,y
\]  

(4.17)

After solving equations (4.15) and (4.16) for the surface, the velocity at the level below the surface is calculated through the use of equation (4.17). Stresses and velocity at each level can now be calculated given that stresses and velocity are known for all levels above. This process continues sequentially until the base \((s = 1 = b)\) is reached. All stresses at the basal layer are considered basal drag, defined as:

\[
\tau_{ii} = R_{iz}(b) - R_{ix}(b) \frac{\partial h}{\partial x} - R_{iy}(b) \frac{\partial h}{\partial y} \quad i=x,y
\]  

(4.18)

In addition to the resistive stress \(R_{zz}\) being assumed to be negligible, the magnitude of vertical velocity is considered zero. But to solve for effective stress, \(\dot{\varepsilon}_z\) it is necessary to know vertical strain rate, \(\dot{\varepsilon}_{zz}\). This can be accomplished by assuming ice incompressibility which yields:

\[
\dot{\varepsilon}_{zz} = -\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{yy}
\]  

(4.19)
4.3 Implementation

The force balance approach was initially applied to a test data set for the Nioghalvfjerdsfjorden Glacier (NFG). Encouraging results were found for this limited data set and these are presented in the following sections. As described later, many difficulties were encountered during the implementation of the force balance model for the gridded data sets of the West Antarctic ice streams and no stable solution was generated.

4.3.1 Nioghalvfjerdsfjorden Glacier

Figure 4.1 Synthetic Aperture Radar image of the NFG with inset location on Greenland Ice Sheet
The Nioghalvfjerdsfjorden Glacier is located along the 79th north parallel of northeast Greenland. An ice fall drains from the upper plateau where the surface velocity accelerates to ~1200 m a\(^{-1}\) near the grounding line. Surface velocities then slow to less than few hundred meters per year at the glacier terminus. The glacier is ~20 km wide at the grounding line and widens to ~30 km at the terminus and is ~80 km long. A majority of the study area includes an ice shelf, which rapidly thins away from the grounding line. The ice shelf is pinned at its terminus by five islands and perennial fast ice which prevents the development of icebergs. Mass loss is then primarily from basal melting below the ice shelf (Reeh et al., 1997). This glacier is a significant outlet glacier of the Greenland Ice Sheet with a large catchment area, ~8% of the ice sheet (Mayer et al., 2000) and fed by the Northwest Ice Stream (Joughin et al., 2001). The mass flux across the grounding line is ~15 km\(^3\) a\(^{-1}\) (Reeh et al., 1999 and Rignot et al., 2000).

4.3.1.1 Data

Glacier surface elevation, bed elevation and thickness measurements were taken during three days in mid July of 1998 by airborne laser altimeter and radar echo sounding. Five km spaced longitudinal and transverse profiles where collected over the ice shelf and 2.5 km spaced profiles over grounded ice. InSAR surface velocity measurements are available for the majority of the glacier. No measurements are available for the northern margin. The radar images were collected in the winter of 1995-1996 by ERS-1/2 satellites and processed by Eric Rignot at the Jet Propulsion Laboratory (Rignot et al., 2000). Thirteen repeat differential global positioning satellite centerline surface velocities were taken in the winter of 1996-1997 and they match well with InSAR velocities.
4.3.1.2 Results

Through the force balance equations the magnitude of the basal drag (see Fig. 4.2) and basal velocity (see Fig. 4.3) are calculated. With these conditions known the rate of basal ice melt, $\dot{M}$, can be calculated using the following equation, if basal ice is at the pressure melting point (Van der Veen et al., 2007):

$$\dot{M} = \frac{1}{L_i \rho} \left( G - k \frac{\partial T}{\partial z}(b) + \tau_b u(b) \right)$$  \hspace{1cm} (4.20)

where $L_i$ is latent heat of fusion, $\rho$ is the density of ice, $G$ is the geothermal heat flux, $k$ is thermal conductivity of ice, $\frac{\partial T}{\partial z}(b)$ is the basal thermal gradient, $\tau_b$ is basal drag and $u(b)$ is basal velocity.

The basal drag calculated by the numerical scheme predicts small amounts of basal drag beneath the ice shelf (see Fig. 4.3). This is physically impossible and represents an inconsistency in the model result. This could be the result of the boundary conditions applied along the northern margin. As the velocity data sets do not include the lateral shear present on the northern wall of the fjord the resistive stress of this region is poorly incorporated into the model. Also uncertainty in the rate factor could be under estimating resistive stresses. Calculated basal drag beneath the ice shelf results in the calculation of small non-zero basal melt magnitudes where there is known to be no frictional heating.
Figure 4.2: Basal velocity magnitude (m a\(^{-1}\)) for the NFG with the approximate grounding zone (red line).

Figure 4.3: Basal drag magnitude (kPa) for the NFG with the approximate grounding line (red line).
The amount of basal heat generated below the grounded section of the glacier would locally melt a maximum of 60 cm of ice a year (see Fig. 4.4). The average above the grounding line is ~19 cm. These values assume no geothermal heat flux and heat conduction into the ice, because there are no measurements of these values for the NFG. However, geothermal heat flux and conduction are small relative to frictional heat.

Assuming 1) a continental average geothermal flux of 56.7 mW m$^{-2}$ (Fahnestock et al., 2001) 2) a basal thermal gradient consistent with a cubically varying temperature with depth and 3) average ice thickness of 700 m for the grounded section, then the conduction and geothermal flux contribution to net basal heat budget is an order of magnitude smaller than frictional heat.
Despite high lateral strain rates, basal drag remains the dominant resistive stress for this glacier. This is due in part to basal topographic control of the shear margins and the limited ice thickness in this area. As a check on the accuracy of these results, a simple half width averaged force balance equation can be used where lateral drag and basal drag are the only forces which oppose the driving stress (Van der Veen, 1999, pp. 125):

\[ \bar{\tau}_b = \bar{\tau}_d - F_{\text{lat}} \]  \hspace{1cm} (4.21)

\[ F_{\text{lat}} = \frac{H\tau_s}{W} \]  \hspace{1cm} (4.22)

An estimate of lateral drag, \( F_{\text{lat}} \), for this glacier is computed to be \(~3.5\) kPa, where \( H \) is ice thickness at the margin, \( \tau_s \) is the maximum value of \( R_{xy} \) and \( W \) is the half width of the glacier. For the southern side of the NFG, \( H = 450 \) m, \( W = 10,000 \) m and \( \tau_s = 70 \) kPa. The average driving stress, \( \tau_d \), at this location is \(~130\) kPa. The difference results in a half width averaged basal drag, \( \tau_b \), of \(~126.5\) kPa. The half width averaged basal drag from the numerical force balance model equals \(~132\) kPa. The average disparity between the full numerical model and the half width average model for 20 different profiles up glacier from the grounding line is 7.9 kPa, always with the numerical model having the larger magnitude of width averaged basal drag.

The force balance model worked reasonably well, producing realistic values. However, improvements could be made. More exact integration approximations could be used as could more careful interpolation and smoothing techniques. Vertical resistive stress, \( R_{zz} \), could also be included, which could prove significant, especially in the steeper
ice fall region of this glacier. There are some issues with small, but non-zero, values of basal drag calculated for the floating ice shelf. This gives rise to the suspicion that calculated basal drag values may be exaggerated everywhere. Overall, the results do give perspective by which to understand the dynamics of this glacier. With reasonable results from the force balance model for the NFG and good agreement with an independent half width model, the numerics of the model are considered accurate.

4.3.2 West Antarctic Ice Streams

The same full numerical model was applied to the West Antarctic ice stream datasets described in chapter 3. It was unable to produce any stable results. Why the model was unstable for the Antarctic data sets is unclear but is partially linked to the inherent sensitivity to small errors in the initial data sets. Numerous techniques were employed to limit model instability, but none work to a sufficient degree. Various combinations of smoothing, horizontal grid spacing and distance between depth layers were tried, but never yielded stable results.

The variation of WAIS surface dynamics are very subtle, with exception of the shear margins. Velocity gradients, which are used to calculate strain rate through the constitutive relation, are very small and the signal to noise ratio in the InSAR velocity data set may simply be too low to employ a full numerical force balance model at the resolution necessary. Velocity gradients are high throughout the study area of the NFG both transverse to flow and longitudinally and although the NFG data sets may not be any more accurate the signal from glacier behaviour has a much greater magnitude.
The resolution of the NFG is also lower, 1 km verses 400 m. A resolution two or three times higher is necessary for investigation of processes below the ice stream shear margins. This necessity contributed to the model’s failure with the Antarctic data sets.

As a method to overcome the instabilities associated with the numerical force balance model was not discovered, this avenue of investigation was abandoned. Instead a heuristic, but analytical, approximation of basal conditions was employed to investigate how basal conditions vary beneath the ice streams and what impact this has on basal melt water generation and distribution.
CHAPTER 5
HUERISTIC BASAL DRAG MODEL

5.1 Theory

A model for estimating basal drag across the ice stream, first proposed by Whillans and Van der Veen (2001), is outlined here. Essentially, the model describes the transformation of shearing across horizontal planes, parallel to the bed, (basal drag) beneath the slow moving interstream ridge into shearing across vertical planes, parallel to the shear margin, (lateral drag) in the ice stream margins. The basic idea is that forces from basal drag give rise to lateral forces that support the flow of the interior ice stream over a near frictionless base. Establishing the form of that transformation yields an estimate of the magnitude of the basal drag across the ice stream margin. The basal drag, basal velocity and estimates of geothermal fluxes can then be used to compute the amount of melt beneath the ice stream margin. Our discussion highlights key equations and assumptions that are used in the subsequent analysis of basal melt water production.

5.1.1 Basal Drag

The gravitational forces on the ice stream are largely balanced by lateral drag within the margins (Jackson and Kamb, 1997; Whillans and Van der Veen, 1997; Harrison et al., 1998 and Van der Veen et al., 2007). For much of the ice stream
longitudinal resistive stress gradients are negligibly small (Whillans and Van der Veen, 2001). These conditions allow for assumptions that greatly simplify the force balance equations and allow for the development of a heuristic but analytical equation for basal drag when surface velocity, glacier geometry and sliding ratio are known.

An orthogonal coordinate system is used with a horizontal x-y plane and x being in the dominant flow direction. Following Whillans and Van der Veen (2001), the vertical shear stress, $R_{xz}$, is related to the vertical strain rate, $\dot{\varepsilon}_{xz}$, through the constitutive relation:

$$R_{xz} = B \dot{\varepsilon}^{1/\alpha-1} \dot{\varepsilon}_{xz},$$  \hspace{1cm} (5.1)

where $B$ is the rate factor, which is dependent primarily on ice temperature and ice fabric. $\dot{\varepsilon}$ is the effective strain rate (see Eq 4.14) and $\dot{\varepsilon}_{xz}$ is the vertical strain rate. The vertical strain rate is related to velocity gradients with $u$ being horizontal velocity in the $x$ direction and $w$ being vertical velocity in the $z$ direction:

$$\dot{\varepsilon}_{xz} = \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right).$$  \hspace{1cm} (5.2)

Vertical velocity is related to creep deformation, accumulation rate and basal melting. Both of the latter values are of the order of magnitude of cm a$^{-1}$ and have subtle spatial variability. The biggest source of horizontal variability in the vertical velocity component is basal melt rate. Through the results, presented in Chapter 6, we get a maximum spatial
variability in basal melt rates of 40 to 50 mm over 400 m. If there is only 1 m of horizontal shear in the entire 1000 m of ice thickness the average value for $\partial u/\partial z$ is an order of magnitude greater than $\partial w/\partial z$. It is likely that there is up to 30 m of horizontal shear in the ice column. As a result, the vertical velocity term in the vertical strain rate (Eq. 5.2) can be neglected.

A further simplification is accomplished by assuming completely parallel flow within the ice stream and the adjacent ridges. This makes $u$ the only component of velocity ($u = U$). Equation 5.1 can be rearranged to relate vertical strain rate to vertical shear using equation (5.2):

$$\frac{\partial U}{\partial z} = 2 \frac{R_{xz}^{n+1}R_{xz}}{B^n}. \quad (5.3)$$

Following Van der Veen (1999, pp. 35), basal drag is defined as:

$$\tau_{bx} = R_{xz}(b) - R_{xx} \frac{\partial b}{\partial x} - R_{xy} \frac{\partial b}{\partial y}. \quad (5.4)$$

With the simplifying assumptions that longitudinal resistive stress gradients are negligible and that basal elevation gradients are nearly zero (Retzlaff et al., 1993), the definition of basal drag becomes:

$$\tau_{bx} \equiv R_{xz}(b). \quad (5.5)$$
The integration of equation 5.3 from the base \((z = b)\) to the ice surface \((z = h)\) results in an equation which links the surface velocity to basal drag:

\[
\int_{u(b)}^{u(h)} \frac{\partial U}{\partial z} \, dz = \int_{h}^{b} 2 \frac{R_{x}^{n-1} R_{xz}}{B^n} \, dz. \tag{5.6}
\]

In order to evaluate the integrand on the right hand side the vertical variation of these parameters needs to be known. Due to changes in the strength of the ice, primarily in response to changing temperature, lateral shear stress is concentrated in the upper portion of the ice where it is stronger. With a stress free ice surface horizontal shearing is zero at the surface and concentrated at depth because the ice softens as the temperature approaches the pressure melting point at the base (Kamb, 2001). Based on this distribution of shear, Whillans and Van der Veen (2001) concluded that the effective shear stress, \(R_e\), then closely resembles the magnitude of \(R_{xy}\) at the surface and \(R_{xz}\) at the base. This condition allows for the use of a heuristic relationship which takes into account the vertical variations in shear stress and the rate factor:

\[
\left( \frac{R_{x}^{n-1}}{B^n} R_{xz} \right)^{1/n} = \left( \frac{h - z}{H} \right)^m \frac{\tau_{bx}}{B_b}. \tag{5.7}
\]

The ice thickness is represented by \(H\) and the basal rate factor by \(B_b\). The parameter \(m\) is related to the nature of shear stress depth variation. It can be considered as a shape factor similar to that previously adopted by Whillans and Van der Veen (2001). For a laminar
flow model there is a linear variation of vertical shearing from zero at the surface to some higher value at depth. This corresponds to the parameterization \( m = 1 \). For the ice stream, \( m \) is expected to vary by some value >1. An equation with an analytical solution which links surface velocity to basal drag is now available:

\[
\int_{U(b)}^{U(h)} \frac{\partial U}{\partial z} \, dz = \int_{b}^{h} \left( \frac{h - z}{H} \right)^{mn} \left( \frac{\tau_{bx}}{B_b} \right)^{n} \, dz
\]  

(5.8)

Whillans and Van der Veen (2001) solved equation (5.8) and rearranged the equation to solve for basal drag:

\[
\tau_{bx} = B_b \left( \frac{mn + 1}{2} \frac{U(h) - U(b)}{H} \right)^{1/n}
\]  

(5.9)

It is simpler to use a unitless sliding ratio which relates surface velocity, \( U(h) \), to basal velocity, \( U(b) \). The relationship

\[
S = \frac{U(b)}{U(h)}
\]  

(5.10)

creates a final equation which can be used to calculate basal drag for an ice stream given that the basal rate factor, surface velocity, ice thickness and the sliding ratio are known or estimated. This is equation 14 in Whillans and Van der Veen (2001):
\[ \tau_{mx} = B_b \left( \frac{mn+1}{2H} U(h)(1-S) \right)^{1/n}. \] (5.11)

### 5.1.2 Sliding Ratio

Variation in the sliding ratio determines how basal drag varies across the ice stream. With complete sliding, \( S = 1 \), there can be no basal drag as must be required to achieve high velocities in the interior of the ice stream. Analysis of tills beneath the ice stream show a yield strength of a few kPa (Tulaczyk et al., 2000), further suggesting that the sliding ratio must approach unity within the ice stream. Outboard of the ice stream and within the interstream ridge, there is virtually no sliding. Consequently, the sliding parameter must vary from near 0 to 1 from the interstream ridge, through the shear margin and into the interior ice stream. Exactly how the sliding ratio varies is not readily known, but through the use of force balance and the calculation of basal drag the location of complete sliding onset can be estimated.

The amount of driving stress supported by the lateral drag must be transferred to increased basal drag under the shear margin and interstream ridge (Whillans and Van der Veen, 2001). The magnitude of shear resistance in the shear margin, \( F_s \), can be estimated using this simple equation:

\[ F_s = H \tau_s. \] (5.12)
This is the equivalent of lateral drag (see Eq. 4.22) integrated over the half width of the ice stream. The term $\tau_s$ is the maximum value of the shear stress, $R_{xy}$, averaged over the ice thickness. The effective strain rate, $\dot{\epsilon}_e$, for the margin will closely approximate the value for $\dot{\epsilon}_{xy}$, because all other strain rates are much smaller then lateral shear when it is at its maximum. This allows:

$$R_{xy} = B \dot{\epsilon}_{xy}^{1/n}.$$  \hspace{1cm} (5.13)

Inherent in the assumption that $\dot{\epsilon}_s = \dot{\epsilon}_{xy}$ is that this location is dominated by pure sliding. An inspection of the sliding ratio magnitude predicted by the model at the location of maximum lateral shear stress shows incomplete sliding suggesting a non-zero $\dot{\epsilon}_{xz}$. If there is a non-zero $\dot{\epsilon}_{xz}$ contribution to effective stress it would have to have a depth averaged value of $\sim0.02 \text{ a}^{-1}$, given an lateral strain rate of 0.06 a$^{-1}$ to affect the outcome of these calculations. A lateral strain rate of 0.06 a$^{-1}$ is near the average lateral strain rate. This value for $\dot{\epsilon}_{xz}$ is equivalent to 40 m a$^{-1}$ of internal deformation, assuming no vertical component of velocity. This is perhaps the maximum expected amount of internal deformation. It is then likely that a realistic magnitude of horizontal strain rate, $\dot{\epsilon}_{xz}$, does not impact the result made with this assumption. It is then possible to make this assumption and have incomplete or no sliding at the location of maximum shear.

With the amount of resistance supplied by the shear margins estimated all that remains to be done is to find the location where there is enough basal resistance to balance shear resistance.
Following Van der Veen et al. (2007), a method for determining the location of complete sliding is used. This method begins by selecting a profile transverse to the surface velocity and using equation (5.11), while assuming no sliding ($S = 0$), to calculate basal drag. This is clearly not realistic, but gives a first order approximation, the maximum amount of basal drag possible given the surface condition. Next excess basal resistance for each point along the profile is calculated where excess basal resistance is the difference of basal drag and local driving stress. Any basal drag that exceeds the local driving stress can be used to balance the interior ice stream supported by lateral drag. Integrating the excess basal resistance from the interstream ridge into the ice stream gives the total excess basal resistance available, $F_{\text{bed}}$:

\[ F_{\text{bed}} = \int_{0}^{y} \left( \tau_b - \tau_d \right) dy . \]  

(5.14)

$\tau_d$ is the driving stress and is defined by the formula:

\[ \tau_d = \rho g H \alpha , \]  

(5.15)

where $\rho$ is ice density, $g$ is the gravitational constant, $H$ is ice thickness and $\alpha$ is surface slope. When $F_{\text{bed}} = F_s$ this is the location of complete sliding if there is an instantaneous jump from no to complete sliding. In this case it is here that there is enough basal drag to support local driving stress and the shear resistance offered by the shear margin.
It is more realistic to assume that the sliding ratio varies over some distance. The sliding ratio must vary from \( S = 0 \) in the interstream ridge, where the ice is known to be frozen to the basal substrate (Kamb, 2001), to \( S = 1 \) at the location just determined. For this investigation a surface velocity threshold of 30 m a\(^{-1}\) is defined as a location where there must be some component of basal sliding. The sliding ratio is assigned to vary linearly between these two locations.

The introduction of a sliding ratio that varies over some distance affects the form of the integrated excess basal resistance, \( F_{\text{bed}} \). A non-zero value of \( S \) diminishes the magnitude of basal drag calculated in that location and therefore shallows the slope of \( F_{\text{bed}} \). This moves the location where \( F_{\text{bed}} = F_s \) towards the interior of the ice stream. An adjusted location for the onset of complete sliding is calculated.

With the sliding ratio estimated, basal drag and basal velocity can be calculated. The equivalent basal melt can be calculated using the thermodynamic balance between geothermal flux, frictional heating and heat conducted into the ice assuming basal ice is at the pressure melting point:

\[
\dot{M} = \frac{1}{\rho L_i} \left( G - k \frac{\partial T}{\partial z}(b) + \tau_b U(b) \right) \tag{5.16}
\]

where \( \rho \) represents basal ice density, \( L_i \) the latent heat of fusion, \( G \) the geothermal flux, \( k \) the thermal conductivity of ice at the pressure melting point, \( \frac{\partial T}{\partial z}(b) \) the basal thermal gradient, \( \tau_b \) the basal drag and \( U(b) \) the sliding velocity. The sliding velocity (Eq. 5.10) is simply a product of the sliding ratio and surface velocity.
5.2 Analysis Considerations

The goal is to calculate basal melt rate through the basal thermodynamic balance equation (Eq. 5.16). The values $\rho = 910 \text{ kg m}^{-3}$ for basal ice, $L_i = 333.5 \text{ kJ kg}^{-1}$ for the latent heat of fusion and $k = 2.1 \text{ W (mºC)}^{-1}$ the thermal conductivity of ice near 0ºC are assigned. Selection of other variables is explained below.

5.2.1 Selecting Transverse Profiles

Calculating basal melt is best accomplished along transverse profiles orthogonal to the flow direction. Also, appropriately aligned profiles nearly eliminate transverse flow making that assumption in the theory valid.

The first consideration is to select the starting point of the profile on the interstream ridge. There is no a priori knowledge of the width of the ice stream margin and interstream ridge required to develop the forces that balance the interior ice stream through elevated basal drag, so it is not clear where integration of excess basal resistance should begin. As a first approximation, the location where the lateral shear strain rate first becomes non-zero is taken as the furthest location from the ice stream where elevated basal drag balances the interior ice stream. Functionally, this defines the width of the ice stream. The process of stress transmission from lateral drag to basal drag could extend far into the interstream ridge or be highly localized. Varying the start point of the profile affects the final melt calculations. A more complete error analysis is discussed in Chapter 5.3.1

Once the start point location is determined, the profile is aligned perpendicular to the dominant flow direction in the interior of the ice stream. There are variations in the
flow direction along each transverse profile which are especially prominent in the slower marginal regions. This may be partially due to errors in interferometric analysis, as interferometry has higher errors for slower moving regions, more than a 90º error in direction is possible, given the slow speeds and an uncertainty in velocity component magnitudes of +/- 12 m a⁻¹. However, localized variations in flow direction can be expected. GPS velocity data along transverse profiles of the Whillans Ice Stream (Van der Veen et al., 2007) show that the velocity vectors are closely parallel until the magnitude of the velocity vectors drops below 20 m a⁻¹. Below this velocity threshold a rotation of as much as 50º, relative to the interior ice stream velocity vectors, is observed. These variations in flow do not significantly impact the resulting calculations as they take place in slower regions where the transverse component of flow is minimal and more importantly so are velocity gradients. These normal stresses calculated from the GPS velocities are much smaller than lateral shear, a factor of five for the slowest region and one to two orders of magnitude once the velocity magnitude exceeds 20 m a⁻¹. The validity of the assumption that lateral horizontal shear dominates in this regime is retained.

5.2.2 Driving Stress

The driving stress (Eq. 5.15) is calculated from mean ice thickness measurements, surface slope measurements, ice density taken as 910 kg m⁻³ and the gravitational constant, 9.8 m s⁻². The driving stress is particularly sensitive to the magnitude of the slope. Three methodologies have been employed to obtain surface slope values from the OSU DEM. Orthogonal slope components are calculated for each grid point from the
linear regression of OSU DEM surface elevation over approximately 20 km. The resultant of these components is the maximum local slope. The directional derivative in the direction of flow was also taken at each grid point as a second estimate of surface slope. The third value for surface slope was taken from the mean of hand drawn longitudinal profiles. The error was taken as the standard deviation of a suite of measurements. There was variously good agreement and disparity between these differing methodologies (see Table 5.1).

An independent check on the minimum magnitudes of the driving stress is obtainable by determining the amount of basal drag needed to balance shear resistance. This magnitude offers the minimum driving stress for this region, because of the assumptions which limit resistive stress to lateral and basal drag. Error on these magnitudes is taken as maximum of 10% (see Chapter 5.3.5).

For some profiles calculated basal drag magnitudes did not exceed the minimum necessary value. The magnitude that driving stress exceeded lateral drag of near by profiles was used as a gauge for assigning driving stresses to profiles where calculated basal drag did not meet or exceed the necessary minimum. This could not be done for the four profiles just up stream of the Whillans and Mercer confluence, Whillans Ice Stream ‘B’ profiles (see Table 5.1). There is a topographical bump in the OSU DEM that is present just down stream from these profiles which is large enough to produce very near nil or negative surface slopes (sloping upstream). For this location the assumption was made that lateral drag must nearly balance the driving stress as these are closest to the location with the fastest centerline velocities.
Continued

Figure 5.1: Driving Stress calculated through various means (see Chapter 5.2.2) with error (+/-σ) (see Chapter 5.3.3). Balance driving stresses calculated using best estimates of parameters. Balance driving stress error is based on error estimates of lateral resistance (see Chapter 5.3.5). Chosen driving stress is the value used in melt calculations. WIS = Whillans Ice Stream, MIS = Mercer Ice Stream, VIS = Van der Veen Ice Stream. ‘B2’, ‘B’ and ‘Ice Plain’ are regions of the WIS (see Figure 2.1).
Table 5.1 (continued)

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<th>dot product +/- σ</th>
<th>longitudinal transect +/- σ</th>
<th>Driving Stress to balance lateral shear</th>
<th>Chosen Driving Stress</th>
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<td>10.8 +/- 10.9</td>
<td>14.6 +/- 1.5</td>
<td>15.8</td>
</tr>
</tbody>
</table>
It is unlikely the driving stress varies much over ~5 km, the approximate spacing of profiles. Using this assumption, these other driving stress estimations (Stearns et al., 2005 and Whillans and Van der Veen, 1997) the driving stress for each profile was chosen for the best consistency and applied uniformly over each profile.

### 5.2.3 Frictional Heating

Frictional heating is the product of the basal drag and the basal velocity. Each of these values can be calculated with knowledge of surface velocity and glacier geometry and the theory described in Chapter 5.1.

The basal drag, from equation (5.11), with $S = 0$ is calculated for all locations along the profile. The process is illustrated with a sample profile in figure 5.1. The velocity along the profile (see Fig. 5.1 b) determines the form of basal drag across the profile as the other variables are relatively constant. The basal rate factor is taken to be 120 kPa a$^{-1/3}$ which is appropriate for ice near 0°C. Following from convention, $n=3$. The value for $m$ is uncertain and is chosen to be 2. The total shear resistance is calculated from equation (5.12). Using published temperature values for the UpB borehole (Engelhardt, 2004) the depth averaged temperature is ~17°C. This corresponds with a rate factor of 500 kPa a$^{-1/3}$ which is used here as the depth averaged value. The point on the profile where $F_{bed} = F_s$ (see Fig. 5.1c) is taken to be the first approximation of onset of complete sliding ($S = 1$). The location where sliding first begins ($S<<1$) is defined by a surface velocity threshold calculated using a laminar flow model and constants appropriate for the West Antarctic ice stream. The assumption that basal drag supports all the driving stress, $\tau_b = \tau_d$, is made here. Using an ice thickness, $H$, of 1000m, a rate
factor, $A$, of $5.0 \times 10^{-8}$ kPa$^{-3}$ a$^{-1}$, appropriate for ice at -5°C, and driving stress, $\tau_d$, of 15 kPa, the laminar flow model from Paterson (1994, pp. 251):

$$U(h) - U(b) = \frac{2AH}{n+1} \tau_d^{n+1}$$  \hfill (5.17)

predicts the difference between the surface and basal velocities to be less than 1 m a$^{-1}$. But it is likely that strain heating and fabric alignment in the shear margin could increase the magnitude of surface velocity from internal deformation. As a result, a velocity threshold where there must be sliding, \~30 m a$^{-1}$, is assigned here. This threshold may be high, but it also tends to roughly coincide with the onset of shear crevassing. The sliding ratio is taken to vary linearly between the onset of sliding and the point where there is complete sliding (see Fig. 5.1d).

Through the introduction of a sliding ratio, the integrated excess basal resistance is slightly diminished. This requires a forward modeling approach to adjust the point where $S=1$. Typically the point where $S=1$ is moved 400 to 800 m further into the ice stream. Notice the difference in figure 5.1c where the two lines equal $F_s$. Once the variation of the sliding ratio is known basal drag can be calculated across the ice stream (see Fig. 5.1e).

Although the shear stress supports most of the driving stress for the ice stream, there may be a small component of basal drag across the width of the ice stream. Total
Figure 5.1: Determining basal drag magnitudes and sliding ratio along profile from interstream ridge into ice stream. a) location of example profile on Whillans Ice Stream; b) Surface velocity profile (blue) and basal velocity after determination of sliding ratio (black), when $S=1$, $U(h) = U(b)$; c) Integrated excess basal resistance before (blue) and after (black) the introduction of a sliding ratio, plotted with total shear resistance (dashed red line); d) sliding ratio; e) Basal drag across profile where $S=0$, (blue) and after (black) the introduction of the sliding ratio. Gray bands represent location of shear crevassing: narrow bands are arcuate crevasses and wide bands chaotic crevassing.
resistance is the integration of the driving stress across the half width of the ice stream, \( W \). Because the driving stress is applied uniformly across the entire profile the equation simplifies to:

\[
F_r = \tau_d W. \tag{5.18}
\]

The difference of total resistance and shear resistance is distributed evenly, as basal drag, over the width of the ice stream where \( S = 1 \). This establishes force balance. To allow for this residual basal drag, <2 kPa for most of the ice stream, in the interior \( S \neq 1 \). The sliding ratio for the interior ice stream is then recalculated to allow for this basal drag. This usually requires a value that is only a few thousandths smaller than unity.

There are certainly variations in the local magnitudes of basal drag across the profile associated with “sticky spots” (Alley, 1993), variations in basal conditions (substrate and topographic variability). The nature of these basal drag anomalies is still enigmatic and is not included in this analysis. Sticky spots may be identified by location of isolated crevassing within the ice stream (see localized bright spots in interior ice stream in Fig. 2.1). The low frequency of their occurrence argues against the importance of sticky spots as a control of flow.

Once the sliding ratio has been assigned basal drag and sliding velocity can now be calculated. From these, the frictional component of the basal thermodynamic balance equation is determined.
5.2.4 Geothermal Flux

There are limited measurements of geothermal flux beneath the ice sheet. Two ice cores in the region have been used to get values of geothermal flux. One at Siple Dome calculated ~70 mW m$^{-2}$ (Engelhardt, 2004) while the Byrd Station core temperature data enabled a calculation of ~60 mW m$^{-2}$ (Rose, 1979). From shallow core temperature data from the B/C interstream ridge, Alley and Bentley (1988) calculated a higher ~ 80 mW m$^{-2}$. Values as high as 100 mW m$^{-2}$ have been used in recent ice sheet models (Parizek et al., 2003).

Recent measurements of the magnetic field from the Antarctic continent have been used to estimate the thickness of the crust (Maule et al., 2005). With crustal thicknesses known, geothermal flux of the continent can be modeled if the lower boundary condition temperature is known and the amount of heat generated within the crust is assigned. With spatial resolution of hundreds of kilometers the continental variations of geothermal flux can be calculated. The continental average was determined to be 65 mW m$^{-2}$ with localized highs, >100 mW m$^{-2}$, near Siple Dome. For Whillans Ice Stream the values ranged from ~65 to ~90 mW m$^{-2}$. The error estimate of these figures is between 21-27 mW m$^{-2}$.

Based on the estimated age and thickness of the underlying continental crust of Antarctica and constraining borehole measurements, Llubes et al. (2006) made estimates of geothermal flux. In West Antarctica where the continental crust is believed to be of Mesozoic age and relatively thin, a geothermal flux of 64 mW m$^{-2}$ was determined. This study assumed constant geothermal flux for each major crustal type beneath the ice sheet.
although the crust is expected to be more complex resulting in varying geothermal flux magnitudes.

Given the large uncertainty of the magnetic field derived calculation and the values from direct temperature measurements, a value of \(~90\) mW/m\(^2\) is deemed too high and a more modest value of \(60\) mWm\(^2\) is assumed for the entire study area. Likely variations in geothermal flux are not incorporated into this study. This is probably a valid approach as frictional heat dominates in the thermodynamic equation and the effect of variation in geothermal flux is minor (see Chapter 5.3.7.1).
5.2.5 Basal Temperature Gradient

The determination of the basal temperature gradient also suffers from a scarcity of observations. Borehole temperature measurements do exist for a few locations on or near the ice stream. For the Siple Dome borehole, a basal temperature gradient of 0.032 °C m\(^{-1}\) was measured (Engelhardt, 2004). For boreholes drilled at Upstream B a basal temperature gradient of 0.038 °C m\(^{-1}\) was calculated from measured upward heat transport of 0.08 W m\(^{-2}\) and an assumed thermal conductivity of 2.1 W (m\(^{o}\)C\(^{-1}\)) (Kamb, 2001).

Assuming steady state and the bed is frozen the basal temperature gradient is defined as (Paterson, 1994, pp. 217):

\[
\left( \frac{\partial T}{\partial z} \right)_b = \frac{G}{k}
\]

where \(G\) is the geothermal flux and \(k\) is thermal conductivity. If \(G = 60\) mW m\(^{-2}\) and \(k = 2.1\) W (m\(^{o}\)C\(^{-1}\)), the basal temperature gradient equals 0.029 °C m\(^{-1}\). It is known, however, that long term climate fluctuations constantly affect the temperature distribution of deep ice and equilibrium is never achieved (Engelhardt, 2004). Furthermore the effects of basal friction and the concentration of strain heating in the basal layers will affect the magnitude of the basal temperature gradient and introduce spatial variability.

A temperature model was constructed by Joughin et al. (2004) which incorporated vertical heat conduction, horizontal advection and horizontal strain heating estimated from a laminar flow model. Their results indicate a widely variable basal temperature
gradient. Values of 0.03 to 0.07 °C m$^{-1}$ are modeled for the Whillans Ice Stream. Larger magnitude values are found in the down stream regions with a general trend of decreasing magnitude proceeding upstream.

The vertical temperature gradient is expected to diminish within the shear margin where strain heating is concentrated (Jacobson and Raymond, 1998). If this condition, termed “edge shield”, is true then the spatial variability of the basal temperature gradient would further concentrate the amount of basal melt below the shear margin by reducing the amount of heat conducted upwards.

For the calculations completed here a spatially uniform value of 0.04 °C m$^{-1}$ is assumed. This value is considered a reasonable average for the entire study area.

5.2.6 Basal Melt

With all the components of the basal thermodynamic balance equation (Eq 5.16) are known basal melt can be estimated. In figure 5.3, the variation of the basal melt rates across the ice stream half width is shown. The area of elevated melt is determined by the location where the sliding ratio varies between 0 and 1. If the sliding ratio is 0 then there is no basal velocity and no heat generation. Where S≈1, the amount of basal drag is low and limited heat is generated. Elevated basal melt rates neatly fall below the chaotic crevasse fields. Although not all profiles have such a good alignment there is a general pattern that elevated basal melt is observed in close proximity to the chaotic crevasse field.
Figure 5.3: Basal melt rate along example profile, see figure 5.1 for profile location. Gray bands represent location of shear crevassing: narrow bands are arcuate crevasses and wide bands chaotic crevassing.

5.3 Model Sensitivity

Even if all the assumptions needed to derive equation 5.11 are considered valid, there are still numerous parameters with varying sensitivities which need to be examined to determine their effect on the final basal melt rate calculation. There is error introduced through random error in measurements, uncertainty in the range of parameters and the algorithms used in these calculations. Below is a treatment of the numerous possible sources of error in the basal melt rate calculations.
5.3.1 Profile Length

The length of the profile determines the width over which elevated drag supports the interior ice stream. The end points of the profiles have previously been defined as the location where lateral strain rates, $\varepsilon_{xy}$, first become non-zero. However, the accuracy of this representation cannot be easily assessed. It is unlikely that profiles could begin any closer to the ice stream because the profile would not include the entire shear margin. Therefore, moving the endpoints up to 4 km away from the ice stream is considered to determine the effect of the placement of the profile end points on the final melt rate calculation. For most regions, the effect of moving the profile start point is consistent (see Fig 5.4). For each 2 km the start point is moved away from the ice stream the magnitude of the peak melt rate is diminished by up to 50%. For regions with small peak melt rates, the effect of start point placement is less important as a 50% decrease of a few millimeters is a small change in magnitude.

This sensitivity is significant. If the width over which elevated basal drag is large the melt rate peak will diminish. However, if there is confidence that defining the end points as the first deviation of lateral strain rate from zero is within 1-2 km of the actual width of the region which supports lateral drag then it is reasonable to define the width of the ice stream by this definition.

This condition can be explained by the dependence of basal drag on surface velocity. With the current parameter configuration a velocity of 1 m a\(^{-1}\) produces $\sim$18 kPa of basal drag. This value exceeds expected driving stress for most of the ice stream. As the surface velocity of the interstream ridge has velocities of up to $\sim$20 m a\(^{-1}\), the basal drag calculated in the interstream ridge can exceed the driving stress by 3 or more fold.
This creates the high sensitivity of profile length and suggests that perhaps the parameter choice exaggerates basal drag magnitudes. However, when variation in all parameters are considered there is a relatively consistent estimate basal melt rate (see Figs 5.6, 5.7 and 5.8).

Figure 5.4: Sensitivity of profile length on basal melt rate calculations. a) location of two example profiles (Profile 1 is the same as figures 5.1 and 5.3); b-c) Melt rate with the profile length as initially defined (Blue), 2km longer (red) and 4 km longer (black).

5.3.2 Profile alignment

The alignment of the profile relative to the dominant flow direction can influence the melt calculation by affecting the form of the velocity profile and thus variations in basal drag across the ice stream. By introducing a rotational error in the orientation of the
profile of +/- 5º the effect on the magnitude of calculated basal drag can been seen (see Fig. 5.5). Because of the effect of the sliding ratio the error is zero for most of the profile except where the sliding ratio varies. This creates an error of about +/- 2.0 mm a⁻¹ in the magnitude of the peak melt rate for the profile shown in Figure 5.4. This magnitude will vary from profile to profile depending on the magnitude of the basal velocity at the location of peak melt rate, greater velocity creating a large error and vice versa. This error may subtly affect the form of the basal melt rate profile but does not affect the integrated melt volume across the ice stream.

Figure 5.5: Variations in basal drag due to profile orientation. See figure 5.3 Profile 1 for profile location. a) Basal Drag with variable sliding ratio with no rotation (blue), with 5º rotation clockwise (black) and with 5º rotation counter-clockwise (red); b) percent change between no rotation and +/- 5º.

5.3.3 Driving Stress

The magnitude of driving stress has two fold importance in this basal melt model. The difference between driving stress and basal drag determines the form of integrated excess basal resistance and the difference between total resistance and shear resistance.
determines the magnitude of basal drag in the interior portions of the ice stream. Therefore, careful calculation of the driving stress is essential for accurate results.

The greatest sensitivity in the driving stress equation is surface slope, \( \alpha \), which can vary significantly depending on the methodology used (see Chapter 5.2.2).

Estimation of error in the driving stress, \( \tau_d \), is taken from standard propagation of random error equations (Taylor, 1996) and equation 5.15:

\[
(\delta F)^2 = \sum_i \left( \frac{\partial F}{\partial \alpha_i} \delta \alpha_i \right)^2
\]

\[
\delta \tau_d = \left( \rho g H \delta \alpha + \rho g \delta H \right)^{1/2}
\]

Error in the gravitational constant, \( g \), and ice density, \( \rho \), are ignored. The random error for the ice thickness, \( \delta H \), and surface slope, \( \delta \alpha \), are taken as the standard deviation of the suite of measured values. The error for each profile is presented in table 5.1 column 3 “longitudinal transect”. Estimated error in width averaged driving stress tends to be 4-6 kPa but ranges from less than 1 kPa to over 10 kPa (see Table 5.1).

A change in the magnitude of the width averaged driving stress produces an identical change in the interior basal drag. Because a change in the driving stress does not affect the magnitude of shear resistance, any additional or subtractive driving stress will need to be applied in the interior ice stream in order to retain force balance. In many locations, where interior basal drag is only a few kPa, a small change in width averaged driving stress can change the basal melt rates in these locations by a factor of 2 or more.
Take for instance the example profile used in figures 5.1, 5.3 and 5.4 (Whillans Ice Stream ‘B2’ Profile 3), the driving stress across this profile is ~15.6 kPa. It also has an estimated error of approximately +/- 6.5 kPa, taken as average of error estimates in Table 5.1. If the driving stress were 6.5 kPa greater additional resistive stresses are needed. Some of this additional resistance is taken up in the shear margin, however, anywhere \( S = \sim 1 \) the 6.5 kPa are directly applied as basal drag. This is the only way to achieve force balance across the width of the ice stream. With an average driving stress of 15.6 kPa, the interior basal drag is ~0.9 kPa. With an average driving stress of 22.1 kPa, interior basal drag is now ~7.4 kPa. This enacts a great change for the interior basal melt magnitudes changing a predicted freeze-on of a ~1 mm a\(^{-1}\) to melting >7 mm a\(^{-1}\). This is a very significant change in the total melt water production.

The physics of the ice stream preclude too large of a driving stress. It has been shown that the lateral drag nearly equals the driving stress (Whillans et al., 2001) and that basal sediment can only support a few kPa of basal drag (Tulaczyk et al., 2000). This does not speak to the relative accuracy of the driving stress calculation but does suggest a small reasonable range of driving stress that is within a few kPa of the given total shear resistance.

It is worth mentioning that it may be possible for a moderately sized sticky spot to support a few kPa of width averaged driving stress and decrease basal drag across the interior lowering melt rates and encouraging freeze on conditions. \( \tau_{ss} \) is the basal drag across a sticky spot of width, \( W_{ss} \), that will support the equivalent half width averaged basal drag, \( \bar{\tau}_b \), on an ice stream of half width, \( W \):

\[ \tau_{ss} = \bar{\tau}_b \]
\[
\frac{W\bar{\tau}_b}{W_{ss}} = \tau_{ss}. \tag{5.22}
\]

The example profile discussed above has a half width of 17 km. A sticky spot 500 m wide, either an individual spot or numerous sticky spots which are collectively this wide, can support 1 kPa of width average driving stress for every 34 kPa of local basal drag. If a 500 m wide sticky spot with 50 kPa of basal drag is present then it would support 1.5 kPa of half width averaged driving stress. This is equivalent to the amount of basal drag across the interior of most of the ice stream and would promote freeze-on conditions in the remainder of the profile.

The effect of the driving stress on the form of integrated basal resistance is addressed in chapter 5.3.5.

5.3.4 Shear Resistance

The accuracy of the shear resistance is mostly dependent on the assigned depth averaged rate factor necessary for calculating \( \tau_s \) (see Eq. 5.12). Given the uncertainty in the englacial variations in temperatures and the relative importance of differing depth layers in lateral shear, due to the temperature dependence of ice strength, the best value for the rate fact is not known. Previous studies (Whillans et al., 2001 and Van der Veen et al., 2007) have chosen a value which is appropriate for the average englacial temperature, 540 kPa a\(^{-1/3}\) and 510 kPa a\(^{-1/3}\) respectively. For this study a value of 500 kPa a\(^{-1/3}\) was chosen for the rate factor, \( B \). An error in the rate factor, \( \delta B \), of +/- 50 kPa a\(^{-1/3}\) is assigned. Using the standard equation for error propagation (Taylor, 1996) (Eq 5.20), a velocity
error, $\delta U$, of +/- 12 m a$^{-1}$, an ice thickness error, $\delta H$, of +/- 18 m and the equations for lateral shear (Eq 5.13 and 5.14):

$$
\delta F_s = \left[ B \left( \frac{1}{2 \frac{\partial u}{\partial y}} \right)^{1/n} \delta H \right]^2 + \left[ H \left( \frac{1}{2 \frac{\partial u}{\partial y}} \right)^{1/n} \delta B \right]^2 + \left[ H B \frac{1}{2 \Delta y n} \left( \frac{1}{2 \frac{\partial u}{\partial y}} \right)^{1/n-1} \delta U \right]^2 \right]^{1/2} \tag{5.23}
$$

Applying $B=500$ kPa a$^{-1/3}$, $H = 1000$ m, $\partial u/\partial y = .016$, $n=3$, $\Delta y=800$ m, equation 5.21 yields an error in lateral resistance, $\delta F_s$, of ~ 1x10$^4$ kPa m. The value of $F_s$ varies between profiles and ranges from 1x10$^5$ to 2 x10$^5$ kPa m. This creates an error of 5% to 10% depending on the profile location. In terms of melt rate, its effect varies from one location to another depending on the form of integrated excess basal resistance. Any additional errors in measurements are negligible. The rate factor dominates uncertainty.

### 5.3.5 Basal Drag

Excluding the sliding ratio, there are two measurements, ice thickness ($H$) and surface velocity ($U(h)$), and two parameters, basal rate factor ($B_b$) and depth variation parameter ($m$), which go into the calculation of basal drag. The expected error in ice thickness measurements, +/- 18 m, has a negligible effect on calculated basal melt rates. The error in the velocity measurements, +/- 12 m a$^{-1}$, becomes the most important if they are systematic. The effect of truly random errors would be minimized in terms of integrated excess basal resistance, which would minimize its effect on basal melt rates through the stabilization of the complete sliding onset location.
Given a systematic velocity error of +/- 12 m a\(^{-1}\), a basal rate factor of either 120 kPa a\(^{-1/3}\) or 200 kPa a\(^{-1/3}\), \(m = 1, 2, 3\), driving stress determined by each of the three techniques described above, a profile length which varies by 2 km and an error in \(F_s\) of 10\%, the range of possible melt rate profiles can be seen in Figure 5.6 – 5.8. Some parameter combinations can be thrown out as they place complete sliding outside of the shear crevasses and at unrealistically low velocities, such as <30 m a\(^{-1}\). Once the outliers are discarded, a majority of the melt rate peaks tend to group within a few kilometers, and have comparable melt rate magnitudes.

Figure 5.6: The effect of parameter variability on basal melt rate. Systematic velocity errors of +12 m a\(^{-1}\) (Red), -12 m a\(^{-1}\) (black) without introduced error (green) and with chosen best estimate parameters (thick blue line)
Figure 5.7: The effect of parameter variability on basal melt rate. Systematic velocity errors of $+12\ m\ a^{-1}$ (Red), $-12\ m\ a^{-1}$ (black) without introduced error (green) and with chosen best estimate parameters (thick blue line).

Figure 5.8: The effect of parameter variability on basal melt rate. Systematic velocity errors of $+12\ m\ a^{-1}$ (Red), $-12\ m\ a^{-1}$ (black) without introduced error (green) and with chosen best estimate parameters (thick blue line).
The variance in the melt rate profiles is primarily determined by the degree of lateral strain rate. For high strain rate areas, small changes in the position of complete sliding onset can create much greater melt rate peaks because the basal velocity increases markedly, this can be seen in the ice plain profiles (see Figure 5.8). The highest strain rates are located in this area. In lower shear areas, the magnitudes of melt rates are more closely comparable, as can be seen in the other profiles.

A full range of possible parameter values and measurement errors have been used and solution remains robust.

There are 324 combinations of parameters for each profile that, in some cases, extends beyond the full range of expected parameter values. Despite this there is a close grouping in location and magnitude of the basal melt rates found below the shear margin. It is possible to force the elimination of a melt rate peak with certain combinations of parameter values, however, nearly all produce a substantial melt rate peak. This creates confidence in the solution and that there is, indeed, elevated basal melt rates found beneath the shear margins, but the specific magnitudes may be uncertain.

5.3.6 Sliding Ratio

The distance over which sliding ratio varies affects the size of the region of elevated melt and can significantly increase the integrated melt beneath the shear margins. The assumption that the sliding ratio varies linearly between the location where surface velocity first exceeds 30 m a\(^{-1}\) and where \(F_s = F_{bed}\) is applied in this study. The form of the sliding ratio variation has minimal effect (see Fig. 5.6). The form of sliding ratio
variation influences the maximum melt rate and, more significantly, the integrated melt by broadening the area of elevated melt. Without any clear sense of the true form of sliding ratio variation, and the consistent results of the various forms, the linear approximation is considered a good first order approximation which would be difficult to improve on given the current understanding of these regions.

![Graph showing influence of sliding ratio form on basal melt rates](image)

**Figure 5.9**: Influence of the sliding ratio form on basal melt rates using an inverse cubic (yellow), inverse quadratic (black), linear (green), quadratic (blue) and cubic (red) variations in sliding ratio. Gray bands represent location of shear crevasse fields. See figure 5.6 for profile location.

### 5.3.7 Thermodynamic Equilibrium Melt Calculation

Given all the uncertainties in the magnitude of basal drag and the variation of the sliding ratio, the parameters used within the thermodynamic equilibrium equation (Eq. 5.16) induce a maximum error of +/- 2.3 mm a⁻¹.
5.3.7.1 Geothermal Flux

The chosen magnitude of the geothermal flux, 60 mW m$^{-2}$, may not be representative for all regions of the study area, but without more measurements the nature of its spatial variation cannot be assessed. Each 10 mW m$^{-2}$ of geothermal flux is equivalent to approximately 1 mm a$^{-1}$ of melt. A likely range of geothermal flux for this region is 55 – 85 mW m$^{-2}$. This creates a possible maximum error in melt rate of +/- 2.0 mm a$^{-1}$. In high melt rate regions this represents an error of only a few percent. However, in the interior ice plain region of the Whillans Ice Stream 2.0 mm a$^{-1}$ could be the difference between melting and freezing at the base of the ice sheet.

5.3.7.2 Basal Temperature Gradient

Perhaps a significant shortcoming of this investigation is a lack of a basal temperature model. A uniform basal temperature gradient, 0.04 ºC m$^{-1}$, is assigned to the entire study area. In a basal temperature gradient model used by Joughin et al. (2004) the basal temperature gradient approximately ranged from 0.02 to 0.07 ºC m$^{-1}$ for the Whillans Ice Stream. If this model accurately depicts the basal temperature gradient the error associated with the chosen uniform value, in terms of basal melt rate, would be approximately +/- 1.2 mm a$^{-1}$.

5.3.7.3 Thermal Conductivity of Ice

The thermal conductivity of ice is a function of temperature. Given the limited range of temperatures expected at the base of the ice sheet this does not impact the chosen uniform magnitude of thermal conductivity. The value 2.1 W (mºC)$^{-1}$ is
appropriate for slightly bubbly ice near 0°C. Given the range of temperature between ice stream and interstream ridge, -4 to near 0°C (Kamb, 2001), thermal conductivity might fluctuate by a few hundreds of a unit, resulting in a negligible impact of +/- 0.1 mm a⁻¹ of melt equivalent.

### 5.3.7.4 Basal Ice Density

The density of basal ice is taken to be 910 kg m⁻³. Only a small error of a few kg m⁻³ could be expected below 1000 m of overlaying ice. A variation of +/- 7 kg m⁻³ results in an error in melt rate of approximately +/- 0.05 mm a⁻¹, well below the expected accuracy of this investigation.

### 5.3.8 Summary

Summarizing, there are numerous sources of uncertainty in this study. Most importantly the length of the profile could change peak melt rates by tens of mm a⁻¹, and variation in the orientation of the profile could change the peak melt rate by a few mm a⁻¹. Super-imposed on this is the range of possible solutions given the uncertainty range of parameters used and an additional +/- 2.3 mm a⁻¹ for uncertainties in equation 5.16. With all these uncertainties working independently of one another the likely maximum range of outcomes is presented in figures 5.6-5.8. Although this range includes no melt to significant melt beneath the ice stream margins the results argue for elevated melt below the ice stream. It is difficult to state the exact magnitude of basal melt rate below the ice stream shear margin and the error for the peak melt rate is more than 10 mm a⁻¹. The important conclusion is that the pattern of elevated melt is robust. Even with the
uncertainty, more than half of the total melt across the ice stream can come from the short width beneath or adjacent to the ice stream. For the interior ice stream where melt rates are near zero mm a\(^{-1}\) the effect of uncertainty becomes sufficient to mask whether there is freezing or thawing at the base.
Chapter 6

Results and Discussion

6.1 Introduction
Over forty profiles were used to estimate the spatial variability of the basal melt rates below the Whillans (WIS), Van der Veen (VIS) and Mercer (MIS) Ice Streams. Lack of velocity data prevented calculation of basal melt rates for some portions of the ice stream. See figure 6.1 for velocity data set spatial coverage and location of profiles. The naming convention for each region is labeled. The profiles for each region are numbered sequentially beginning at the most downstream profile.

6.2 Spatial Patterns in Basal Melt Rates
There are two apparent patterns of melt rates across a transverse profile seen in the results (see Fig. 6.2, Fig. 6.3 and Fig. 6.4). One is where a high melt rate, 20 – 50 mm a\(^{-1}\), is concentrated beneath a narrow region below or adjacent to the shear margin and there is little to no melt or even freeze-on conditions across the interior of the ice stream. The other pattern is one where melt at intermediate levels, 3-7 mm a\(^{-1}\), is distributed across the entire width of the ice stream and there is a minor or non-existent melt rate peak beneath the shear margin. These will be referred to respectively as concentrated and distributed patterns.
Longitudinal patterns can be examined for the Whillans Ice Stream where there are profiles over a great distance of its length. The other ice streams have profiles over a shorter distance where the glaciological conditions do not change much. This prevents a specific examination of longitudinal patterns for the MIS and VIS. However, the pattern observed on the WIS may be extended to these other ice streams.

There is some variability and not all profiles fall neatly into just these two classifications. The Mercer Ice Stream, for instance, has characteristics of both classifications. Also there is asymmetry between the ends of the profiles. This variability in the melt rate is likely related both to the sensitivity of the model and to physical changes that take place between one region and another.
Figure 6.2 Modeled melt rate for portions of the Whillans Ice stream “B2” and Van der Veen Ice Stream
Figure 6.3: Modeled melt rate for portions of the ice plain and “B” of Whillans Ice Stream and Mercer Ice Stream.
Figure 6.4: Modeled melt across indicative profiles for each region of the ice stream.
6.2.1 Whillans Ice Stream

The most prevalent pattern of melt on the Whillans Ice Stream is the concentrated pattern. Progressing upstream from the ice plain every profile has concentrated melt below the shear margin. However, once the middle of the WIS B2 region is reached, there is a transition and the last seven profiles all have the distributed pattern of melt. This transition in melt rate pattern reflects a change in the relative importance of lateral resistance, the total volume of water that is produced and transition in the nature of shear crevassing.

The relative size of the peak melt rate under the shear margin changes upstream (see Fig 6.2 and Fig 6.3). The size of a local melt rate peak is primarily dependent on the magnitude of lateral resistance and absolute basal velocities. Regions of low lateral resistance, such as the ice plain, only require a small width of the shear margin to develop balancing basal drag. Consequently the sliding ratio varies across a region with slower basal velocities. This results in a smaller peak melt rate. Conversely, further upstream lateral resistance can be twice as large and a greater width of the shear margin is necessary to develop balancing basal drag. In these regions both basal drag and basal velocities are larger for the region where the sliding ratio varies resulting in greater melt rate peaks. These factors influence the relative magnitude of basal melt under the shear margin as you proceed upstream.

By considering the ratio, $R$, of the shear resistance, $F_s$ (see Eq. 5.12) to total resistance, $F_T$ (see Eq. 5.18), for each transverse profile we can determine if there are any longitudinal patterns in the relative importance of lateral resistance in controlling the flow of the ice stream (see Fig. 6.5). The ratio is defined by the simple expression:
\[ R = \frac{F_s}{F_T}. \]  
\[ \delta R = \left( \left( \frac{1}{F_T} \delta F_s \right)^2 + \left( \frac{F_s}{F_T^2} \delta F_T \right)^2 \right)^{1/2}. \]

The error in the ratio, \( \delta R \), is defined using standard propagation of error (Taylor, 1996) (see Eq. 5.20) and equation 6.1:

A 10% error in \( F_s \) (see Chapter 5.3.5) is used. The error assigned to \( F_T \) varies between regions of the Whillans Ice Stream. For ice plain profiles an error of 2 kPa width averaged driving stress was assigned. An error of 4 kPa width averaged driving stress was assigned for the remaining profiles, except the furthest up stream profile, WIS B2 Profile 17, where an error of 8 kPa was used. Error estimates are plotted as error bars in figure 6.5.

A distinct change is seen in the relative importance of lateral drag upstream of the shear crevassing. As shear crevassing begins, the relative importance of lateral resistance increases. This suggests that shear crevassing is indicative of nearly complete lateral control of the ice stream.
Figure 6.5: Ratio of lateral shear resistance to total resistance, with error bars, for each profile plotted along a flow line of the Whillans Ice Stream starting at the furthest upstream profile and extending to the furthest down stream profile of the ice plain (see Fig. 6.1). The vertical red line marks the location where lateral shear crevassing begins along the northern margin and the vertical black line is where lateral shear crevassing begins on the southern margin.

The ice plain profiles are calculated to have ~60% of the resistance from the lateral margins. This level of importance is reflective of the low driving stress of this region. Although there is only 1-2 kPa of basal drag across the interior of the ice stream, which is consistent with all the profiles with a concentrated melt pattern (see Fig. 6.6), it represents nearly half of the 4 to 5 kPa of driving stress in this region.
Figure 6.6: Basal drag in interior ice stream required to achieve force balance for each transverse profile plotted along flow line of Whillans Ice stream beginning at the furthest upstream profile and extending to the furthest down stream profile of the ice plain (see Fig. 6.1). The vertical red line marks the location where lateral shear crevassing begins along the northern margin and the vertical black line is where lateral shear crevassing begins on the southern margin.

Figure 6.6 shows the same transition in controlling dynamics as figure 6.5, but by a different metric. Indeed the longitudinal variation in basal drag more clearly shows the change in conditions between upstream and down stream of the northern margin onset of crevassing. Given what we know about the mechanical properties of the basal till, that it is very weak for most of the ice stream (Tulaczyk et al., 2000), it is possible that the appearance of the shear crevassing is an indication of a decrease in the strength of the basal substrate for this region. If a decrease in till strength does occur it could be accomplished by a change in lithology, thickness of the basal layer of till or the volume
or pressure of basal pore water. It seems likely that the latter is the case because of the
distributed pattern of melt which exist upstream from the onset of shear crevassing. Also
the transient nature of the location of shear margin argues against changes in lithology or
till thickness being the cause, as these properties may take a much longer time to change.
The distributed melt, due to its spatial extensiveness, could influence the strength of the
basal till across the entire width of the ice stream. The development of the shear
crevassing and thus near total lateral control of the ice stream could be related to a
threshold in basal water volume or basal pore water pressure which significantly weakens
the basal till and forces the development of increased lateral resistance.

6.2.2 Van der Veen Ice Stream

The few profiles of the Van der Veen Ice Stream are consistent with WIS B2
Profiles 1-10. Driving stress is nearly completely balanced by lateral resistance and the
interior ice stream has a melt rate very near zero. Likewise there is the presence of well
developed shear crevassing. Slight interior melt is calculated but the shear margins
dominate basal melting. Due to the complex nature of the flow in anastomosing
tributaries further upstream, profile placement is questionable. It is expected, however, as
driving stress increases and lateral resistance decreases upstream a transition from
concentrated melting below the shear margin will give way to melt distributed across the
width of the ice stream, perhaps just above the appearance of shear crevassing. If the
pattern is consistent with the WIS, an increase in total volume of melt is also expected
above this transition.
6.2.3 Mercer Ice Stream

The Mercer Ice Stream differs from the VIS and WIS because the driving stress is much greater. This creates conditions very favorable for interior ice stream basal melt. The driving stress is 50% to 100% greater than that on the WIS and has shear resistance of comparable magnitude. Basal drag must then be present in higher magnitudes across the ice stream. With basal velocities over 200 m a\(^{-1}\) basal melt rates are high. This could explain why the interior melt rates of 10 to 25 mm a\(^{-1}\) are simultaneously present with peak melt rates below of the shear margin of ~40 mm a\(^{-1}\).

For the northern shear margin there is, similarly, a sharp decline in melting below the shear margin upstream of the onset of shear crevassing. However, the southern margin does not follow this pattern. Diminishing but elevated melt is recorded below the southern shear margin for all MIS profiles. The southern margin of the MIS has a more rapidly varying surface velocity across the ice stream. This places the variation of the sliding ratio in a region with faster basal velocity sustaining elevated melt.

There is concern that basal topography could vary sufficiently to make the assumption of a level bed invalid. This could be contributing to the inconsistency of these melt rate patterns compared with other profiles.

6.3 Asymmetry in Transverse Melt Profiles

Asymmetry in the transverse melt rate profile is present in many of the profiles, including the WIS Ice Plain, WIS “B”, the MIS and upstream of southern shear margin crevassing of the WIS “B2” region. This asymmetry is likely related to model sensitivity,
asymmetry of the transverse velocity profile and physical changes between one end of the profile and the other.

There are no peak melt rates calculated below the southern shear margins of the ice plain. This could be the result of model sensitivity or represent an actual lack of melting beneath the shear margin. If it is model sensitivity it is related to the slower development of the maximum strain rate along southern margin compared to the northern margin of the WIS Ice Plain profiles. The sliding ratio approaches unity before the basal velocity magnitudes become sufficient to create significant heat. The velocity gradients on the southern margin are slightly lower and take a few more profile points to reach maximum than the northern (see Fig 6.7). As a result of the dependence of basal melt rates on basal velocity, the slower velocities for the southern margin create the difference in melt rates below the shear margins. It is possible, if slightly different parameter values were used that the sliding ratio could reach unity an additional 400-800 m further toward the center of the ice stream. If this is true, melt peaks below the southern shear margin would have magnitudes comparable to the northern margin, 5 - 10 mm a\(^{-1}\). This same pattern is true for the WIS ‘B’ region.

The lack of a melt peak below the shear margin of the most up-stream profiles of the WIS B2 is reasonable. As the magnitude of lateral resistive stress drops off as is evidenced by the lack of a southern shear, so does the magnitude of basal drag below the shear margin. Coupled with the slower absolute velocities in this region of the WIS basal melt rates below the shear margin rapidly decline. The opposite is true for the interior ice stream. As lateral resistance decreases interior basal drag increases, coupled with a general trend of increasing driving stress, to create higher melt rates (see Fig. 6.2 and Fig.
6.4). The asymmetry here is certainly related to variable glaciological conditions from one side of the ice stream to the other.

![Figure 6.7: Transverse velocity profile for WIS ‘B’ Profile 4 and WIS Ice Plain Profile 4](image)

**6.4 Melt Volumes**

We computed the volumes of water generated under each profile. First we take the profile to be 1 m wide in the along flow direction and calculate the total water volume. We also estimate the thickness of a water layer by assuming the melt volume is spread evenly over the entire width of the ice stream. This is essentially the average melt rate for the transverse profile. The results show a similar trend to that of relative importance of lateral shear and interior basal drag magnitudes. A transition takes place roughly at the appearance of shear crevassing.

Due to the uncertainties in basal melt rates (see Chapter 5.3), the volume of water produced reported here is uncertain. An uncertainty of 3 mm a\(^{-1}\) in the average melt rate across the ice stream is equivalent to 100-400 m\(^3\) of water per year, if the melt profile is 1 m wide in the along flow direction. This error, which is toward the lower end of error
estimates, is equal to or greater than the volume calculated for each profile. This creates difficulty to the analysis of these findings.

6.4.1 Whillans Ice Stream

Traveling upstream there is a jump from total melt volumes of near 0 - 100 m³ a⁻¹ to 100 to 250 m³ a⁻¹ where the shear crevassing on the southern margin of WIS B2 ceases (see Fig. 6.8). Not only is there a transition from concentrated shear margin melting to melting distributed across the ice stream but also an increase in total melt volume.

Figure 6.8: Total melt volume (a) and thickness of layer spread evenly (b) under each for each WIS profile plotted along flowline. Error bars equivalent to +/- 3 mm a⁻¹ in average melt rate. Dashed vertical lines represent appearance of shear crevassing along southern (red) and northern (black) shear margin.
A second increase upstream of the onset of northern margin shear crevassing is not seen because, although interior basal drag increases, basal velocities decrease lowering the net basal melt. Plus this profile is significantly shorter than the rest.

6.4.2 Van der Veen Ice Stream

As all the profiles are within a similar glaciological regime there is little variation in these profiles (see Chapter 6.2.2). They closely match the results for other profiles where lateral resistance nearly balances driving stress.

![Graph](image)

Figure 6.9: Total melt volume (a) and water layer thickness if distributed evenly under (b) each VIS profile plotted along flowline. Error bars equivalent to +/- 3 mm a^{-1} in average melt rate.
6.4.3 Mercer Ice Stream

Due to increased driving stress of this ice stream the volume of melt is much greater than calculated anywhere else. The drop in integrated melt values upstream from the crevassing onset is more closely linked to the decrease in driving stress (see Table 5.1) rather than to changes in the role of lateral resistance.

With two competing processes operating, a leveling of the ice surface and increasing magnitudes of lateral drag, changes in driving stress dominate the magnitude of basal drag across the interior ice stream. However, these results are consistent with

![Graphs showing integrated melt and melt water layer thickness](image)

Figure 6.10: Total melt volume(a) and water layer thickness if distributed evenly (b) under for each VIS profile. Error bars equivalent to +/- 3 mm a\(^{-1}\) in average melt rate. Dash vertical line is approximate location of onset of northern shear margin crevassing.
the notion that lateral resistance, which nearly balances driving stress, leads to near zero melt rate across the ice stream while areas with a diminished role of lateral resistance are characterized by higher average melt rates.

### 6.4.4 Melt Volumes for Each Ice Stream Region

Table 6.1 summarizes the amount of melt found below each region of the ice streams. These volumes were calculated by assigning each a width in the along flow direction which captures half the distance between each profile. Since not all profiles are parallel an increased estimation was required to account for the difference. Whillans Ice Stream B2 Profile 17, the furthest upstream, was not included in the calculation of values in Table 6.1. It was considered unrepresentative to apply that melt rate over a region longer the 10 km in the along flow direction. The disparity between the melt volumes between WIS B2 above and below crevassing is particularly significant and will be discussed in the following section. The area for each WIS B2 region is roughly equivalent.

<table>
<thead>
<tr>
<th>Profiles</th>
<th>Melt Volume (10⁶ m³ a⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whillans Ice Stream</td>
<td></td>
</tr>
<tr>
<td>Ice Plain</td>
<td>-2.8</td>
</tr>
<tr>
<td>B</td>
<td>-0.42</td>
</tr>
<tr>
<td>B2 total</td>
<td>6.2</td>
</tr>
<tr>
<td>Down-Stream of Northern Crevasses (Profiles 1-10)</td>
<td>1.3</td>
</tr>
<tr>
<td>Upstream of Northern Crevasses (Profiles 11-16)</td>
<td>4.9</td>
</tr>
<tr>
<td>Van der Veen Ice Stream</td>
<td>0.93</td>
</tr>
<tr>
<td>Mercer Ice Stream</td>
<td>13.1</td>
</tr>
</tbody>
</table>

Table 6.1: Total melt volume by region
6.5 Dynamical Ramifications

The nature of the spatial pattern of resistive stresses and basal melt rates suggest mechanisms to explain the onset of streaming flow, margin migration, bifurcation of ice streams and the possible significance of basal drag anomalies.

6.5.1 Onset of Streaming Flow

Streaming flow can be defined as highly lubricated basal sliding mostly controlled by lateral resistive stresses. Upstream from the appearance of shear crevassing, basal drag in the interior of the ice stream and thus basal melt rates are elevated (see Fig. 6.6) and support a large fraction of the flow. These regions where basal melt in the interior dominates also have the highest average melt rate (see Fig 6.8 b). This produces a greater volume of water per unit length of ice stream than further down stream. Even a melt rate of a few mm a\(^{-1}\) integrated over tens of kilometers produces a substantial volume of water. The fact that this water has a spatially diffuse source allows for a spatially enhanced effect. It could be assumed that water generated in the interior of the ice stream can more easily affect the basal condition of the interior of the ice stream. As opposed to further down stream, where melt is concentrated at the margins and would need to travel ten or more kilometers to reach the centerline. Because water is readily available in the interior it lubricates most of the width and limits the magnitude of basal drag. Once this water which is being produced across the entire width of the ice stream reaches a certain threshold in pressure or volume the till behaves plastically with yield strength of only a few kPa (Tulaczyk et al., 2000). This forces the development of alternative resistive
stresses, in the case of ice streams this is lateral resistive stress. Once lateral resistive stress becomes dominant, streaming flow is fully developed.

6.5.2 Margin Migration

A curious change has been taking place on the Whillans Ice Stream. The ice stream has been slowing and widening over time (Stearns et al., 2005). Based on the conclusion that lateral drag supports most of the driving stress, a model that links centerline velocity to fourth power of ice stream width can be made (Van der Veen, 1999, pp. 168). This creates the condition that a wider ice stream is also a faster one. As an ice stream accelerates, basal frictional heat increases and it is easier for the interstream ridge to become entrained in the ice stream. Conversely, if the ice stream is slowing less basal heat is generated and the interstream ridge can begin to reclaim the ice stream.

The findings presented here offer a new hypothesis for controls on changes in the width of the ice stream. For profiles where lateral resistance nearly balances driving stress the interior of the ice stream is not melting, while at the same time having significant basal melt, 5 to 40 mm a\(^{-1}\), concentrated under the shear margins. This creates conditions where heat and melt water at the margin can entrain the interstream ridge and the interior can slow due to increased friction due to basal freeze-on. These very conditions are predicted by this model to be occurring for the ice plain profiles, the WIS B profiles and some of the WIS B2 profiles. There is evidence that each of these areas is undergoing a widening and slowing. Due to the uncertainty in these calculations, areas with slightly positive melt rates could very well be experiencing basal freeze-on.
Profiles were the lateral resistance most completely balances driving stress, WIS ‘B’ Profiles and the down-stream half of WIS ‘B2’ Profiles (see Fig 6.3), have a net melt rate of nearly zero. This suggests a redistribution of basal water from the interior to the margins. As basal water is frozen to the base of the interior ice stream a similar volume of water is produced beneath the shear margins. The slow down is perhaps also changing the distribution of the water, but not the next flux and reinforcing the trend of centerline deceleration and margin outward migration.

6.5.3 Ice Stream Bifurcation

The conditions which would allow for a slowing and widening ice stream are conditions which could also serve to bifurcate the ice stream, a possible explanation for the formation of “the Unicorn”, the nearly stagnant region between the VIS and the upper portions of the WIS. If a portion of the interior can be sufficiently deprived of basal water, through basal freeze-on, and become securely anchored this region could essentially stagnate over time. The higher velocities near the margin would persist because of large volumes of basal water present near the margins. If this new frozen region is capable of supporting elevated basal drag $\approx 80$ kPa, new shear margins could develop because there would be sufficient basal strength to rotate the orientation of the dominant resistive stress from vertical (lateral drag) to horizontal (basal drag). Once new shear margins develop it would distribute the basal heat to reinforce the location of the newly bifurcated channel.
6.5.4 Sticky Spots

Variations in basal drag magnitude in the interior of the ice stream related to ‘sticky spots’ is a complication which has ramifications on the basal melt rate magnitudes. If a sticky spot exists that can support up to a few kPa of width averaged driving stress, it would significantly change the form of the basal melt rate curve. For many profiles, basal drag in the interior only balances a few kPa of width average driving stress. If a small region is sufficient to produce that amount of basal resistance the rest of the interior may supply no resistance, $\tau_b = 0$, and have basal freeze-on conditions despite that the model predicts some amount of melting. A further complication could develop if this sticky spot was sufficiently close to the margin to supply some of the basal drag necessary to develop lateral shearing. A large basal drag anomaly within or near the shear margin could change how the sliding ratio varies across the shear margin and either amplify or dampen the peak melt rates calculated through this theory. If sticky spots are isolated, as it is currently believed to be the case, this would not influence the nature of the analysis presented here. However, if sticky spots prove to be more pervasive they could completely reorganize spatial patterns of basal melt.

6.6 Limitations to Application of the Model

Limitations on the application of this model are imposed by the simplifying assumptions. The need for nearly parallel flow lines limits how far upstream profiles can be placed. Above the commencement of shear crevassing of the Whillans Ice Stream, the amount of turning flow increases appreciably. This increases the importance of normal resistive stresses and compromises the heuristic form of depth variation in resistive
stresses (Eq. 5.7). There is a possibility that curvilinear profiles could be used. However, whether this is a physically reasonable way to describe the transverse variations in resistive stress is questionable. If this technique was applied it would require a variable coordinate system and an adjustment to the calculation of spatial gradients.

The two regions of elevated flow within the Whillans Ice Stream may be separated by an island of decreased flow. However, these regions may only slow to 100 m a\(^{-1}\) and do not represent interstream ridges. It is possible for these regions to give rise to areas of high lateral strain rates suggesting partial support of the lateral resistive stress by elevated basal drag, but the question remains how basal drag and the sliding ratio vary below these regions and how long of a profile is necessary to achieve force balance. Without a region that is truly a ridge it is tenuous to assume that it can support all the lateral drag. Without a clear perspective, due to the increased complexity of the flow, the methodologies explained here are insufficient to address these regions.

There are areas where the ice stream widens as it progresses down flow, such near the confluence of the WIS and MIS. These areas have radially varying flow directions. Again, the magnitude of turning flow and normal resistive stress increases, making the assumption that lateral and basal drag dominate tenuous.

A general rule of thumb for the applicability of this methodology for calculating basal melt is the presence of sub parallel shear crevasse fields. If crevasses are present lateral resistance must be high but also likely supports a majority of the driving stress. If they are sub parallel turning flow is likely to have an insignificant effect.

Furthermore the assumption that the basal topography is near flat begins to break down above the location of the WIS B2 profiles (Retzlaff et al., 1993) and may not be
valid for the entirety of the MIS. This could be the source of some of the discrepancies observed in the MIS melt rate calculations.
CHAPTER 7

CONCLUSIONS

Basal water is the enabler of streaming flow in the West Antarctic Ice Sheet. As surface processes are insufficient to produce large volumes of water known to be at the base (Siegert et al., 1996 and Fricker et al., 2007), basal melting must be the source. Through the theoretical calculations reported here the spatial pattern of basal melt rates can be identified (see Fig 6.2, Fig 6.3). The melt rates range from freeze-on of \( \sim 3 \text{ mm a}^{-1} \) to melt in excess of \( 50 \text{ mm a}^{-1} \).

There are two prominent patterns of transverse variation of basal melt rates across the ice stream. One, which is present for most of the study area, has elevated melt rates localized beneath or directly adjacent to the shear margins and minimal melt, or even freeze-on conditions, across the interior. The second pattern, which is present in the furthest up stream profiles and likely persists until the upper most reaches of ice stream tributaries near the ice divide, has the melt distributed across the entire width of the ice stream. Melt volume for regions of the WIS range from \( 4.9 \times 10^6 \text{ m}^3 \text{ a}^{-1} \) to \( -2.8 \times 10^6 \text{ m}^3 \text{ a}^{-1} \). The seven most upstream profiles, the second melt pattern type, produce a total melt volume of \( \sim 4.9 \times 10^6 \text{ m}^3 \). The remainder of the WIS B2 profiles, the first melt pattern type, only produces \( 1.3 \times 10^6 \text{ m}^3 \) of melt despite being of a similar spatial extent.
Although the peak melt rate magnitudes of the distributed pattern are smaller the melt volume produced across an area of the ice stream is larger because melting occurs over a larger area.

A distributed pattern of melt creates a spatially extensive layer of basal water that is capable of influencing the basal drag over the entire width of the ice stream. Once the basal water layer becomes thick enough or pressurized enough and coupled with the basal water’s effect on the mechanical properties of the basal sediment it forces the further development of lateral resistive stresses as basal drag can no longer generate the necessary resistance. The location where this happens is closely linked with the appearance of shear crevasse fields. It is at or near these locations that the patterns of basal melt change from being distributed across the ice stream width to being localized across a few kilometers beneath the ice stream shear margin. It is also at these locations that full streaming flow commences. There is clearly a relationship between the onset of streaming flow and transition from one melt pattern to the other.

This transition from one melt pattern to another creates conditions that could possibly explain why the observed slow down of the WIS is paired with ice stream widening. Once lateral resistance supports most of the driving stress, basal drag across the interior is small, 1 to 2 kPa. Despite basal velocities of 500-700 m a⁻¹, the dynamic friction is insufficient to melt basal ice and may not be able to overcome the conduction of heat into the glacier. This is where basal freeze-on conditions occur. Basal freeze-on is a mechanism to explain centerline deceleration. Basal freeze-on could serve to strengthen subglacial sediments and physically attach the base of the ice stream to the basal substrate. However, simultaneously the margins adjacent to basal freeze-on are experiencing some
of the highest melt rate magnitudes found anywhere in this ice stream system. This excess heat could serve to thaw some of the interstream ridge and widen the area of basally lubricated flow.

This model strongly suggests the consistent occurrence of elevated melt rates beneath the shear margins when lateral stress nearly balances driving stress. However, once these conditions are not met the assumptions in the model become tenuous. Although the model cannot be used in more complex flow regimes it does provide a perspective to understand how basal drag may be distributed across the ice stream. Upstream of shear crevassing are the locations where interior basal drag starts to become important, supplying more than 40% of the total resistance to driving stress. It can be concluded that all areas upstream of the shear crevassing have larger values of basal drag distributed across the ice stream proportional to the driving stress of that region. This encourages the distributed pattern of basal melt.

Limitations to the application of this model could be overcome if highly accurate data sets were available. A full numerical force balance model can estimate the variations of basal drag and sliding ratio across the ice stream without making any simplifying assumptions. Until either an improved numerical scheme or nearly noiseless data sets are available the sensitivity of the numerical force balance model is too great to investigate the processes which are taking place over a few kilometers beneath the ice stream margins.

The pattern of spatial basal melt is clear but the specific magnitudes are subject to high uncertainty. However, the results of this model do indicate how these spatial
patterns are related to the dynamical processes which are currently occurring on the West Antarctic ice streams.
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