MEASUREMENT OF $CP$-OBSERVABLES WITH $B^- \to D^0 K^{*-}$ DECAYS

DISSERTATION

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By

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ABSTRACT

Using a sample of 379 million $\Upsilon(4S) \rightarrow B\bar{B}$ events collected with the $\text{BABAR}$ detector at the PEP-II $B$-factory, I study decays of $B^- \rightarrow D^0 K^{*-}$ where $K^{*-} \rightarrow K_s^0 \pi^-, K_s^0 \rightarrow \pi^+\pi^-$ and $D^0$ decays into $K^\pm\pi^\mp$ and $CP$-eigenstates. Both $CP^+$ $(K^+K^-, \pi^+\pi^-)$ and $CP^-$ final states ($K_s^0\pi^0, K_s^0\phi, K_s^0\omega$) are included. Using the Gronau-London-Wyler (GLW) and Atwood-Dunietz-Soni (ADS) methods, $CP$-observables which are sensitive to the CKM angle $\gamma$ are measured.
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CHAPTER 1

INTRODUCTION

Charge-Parity, or $CP$, violation has been an exciting yet elusive subject in the field of elementary particle physics for more than forty years. The laws of physics were thought to be unchanged under a $CP$-transformation which changes a particle into its antimatter partner. However, $CP$-symmetry was observed to be violated in the neutral kaon system in 1964. Since then, enormous experimental and theoretical efforts have been put into the exploration of this fascinating phenomenon. In 1967, Andrei Sakharov [1] showed that $CP$ violation was one of the three conditions necessary for the emergence of a matter-antimatter asymmetry in the universe from an initial symmetric state created from the Big Bang, thereby linking physics in the smallest to the astronomical scale. About seven years later, Kobayashi and Maskawa generalized the quark mixing matrix which was first introduced by Cabibbo, in order to explain $CP$ violation in weak interactions. The so-called CKM matrix has been very successful, using a model with three families of quarks. It would be another 20-plus years before another experimental discovery was made: $CP$ violation in the neutral $B$-meson system.

Today, $CP$ violation is firmly established in the Standard Model through the aforementioned CKM matrix, and experimental results have shown great consistency
with the Standard Model predictions in this area. However, people anticipated that the degree of \( CP \) violation in the \( K \)-system and \( B \)-system (which was going to be measured in \( B \)-meson factories) would be far too small to account for the baryon-antibaryon asymmetry from astronomical observations, and thus unable to explain the dominance of matter over antimatter in the universe. This inevitably points to potential new sources of \( CP \) violation beyond the Standard Model. The search for this “new physics”, along with precision measurements of \( CP \) violation parameters in the Standard Model, has also been performed at the \( B \)-meson factories, \( BABAR \) in the United States and Belle in Japan.

This thesis measures physical observables relating to a \( CP \) violation parameter, the CKM angle \( \gamma \), in the Standard Model, through the decays of \( B^- \rightarrow D^0 K^{*-} \). My measurements, when combined with the results of other similar experiments, will greatly improve the precision of \( \gamma \), which in turn will provide a powerful constraint to the CKM model of \( CP \) violation. The measurements are made on data samples provided by the \( BABAR \) experiment, which contain hundreds of millions of \( B \)-meson events.

The structure of this thesis is as follows: In Chapter 2, I provide an overview of the theoretical background on \( CP \) violation in the Standard Model, as well as the methods which allow the measurement of \( \gamma \): the GLW and ADS methods. In Chapter 3, a description of the experimental environment, the PEP-II accelerator and the \( BABAR \) detector, is presented. The analysis procedures of the two \( \gamma \)-extracting techniques are described in details in Chapters 4 and 5. Finally, the combined results and conclusion are presented in Chapter 6 and 7, respectively.
CHAPTER 2

THEORY

2.1 Discrete Symmetries

In contrast to continuous transformations such as translation and rotation, discrete transformations are ones which cannot be reduced to infinite series of infinitesimal steps. There are three such types of transformations we are interested in: parity reversal \( P \), charge conjugation \( C \) and time reversal \( T \).

Parity symmetry refers to the invariance of physics under the parity transformation. \( P \) changes the sign of the spatial coordinates \((t, \mathbf{x}) \rightarrow (t, -\mathbf{x})\). This is equivalent to a mirror reflection plus a 180\(^\circ\) rotation about the axis perpendicular to the plane of the mirror. Parity transformation of a particle reverses also its momentum, but its spin remains unchanged.

Charge conjugation transforms a particle into its anti-particle. The anti-particle has the same mass, spin and momentum of the particle, but opposite internal quantum numbers like electric charge, baryon and lepton number, and so on. The laws of physics assume that anti-particles behave in exactly the same way as their corresponding particles.
Time transformation flips the sign of the time component of a state \((t, \mathbf{x}) \rightarrow (-t, \mathbf{x})\). Momentum and spin are also reversed.

It was believed that all elementary processes respect the three symmetries individually. This is true for strong and electromagnetic interactions in the Standard Model. However, Lee and Yang \([2]\) suggested that parity violation is present in weak interactions; and was later confirmed by experiments. Subsequently the combined \(CP\) violation was also observed in weak decays of \(K\) and \(B\) mesons.

### 2.2 \(C\) and \(P\) Violation

Wu et al. \([3]\) discovered parity violation in \(\beta\) emission of polarized Cobalt-60 nuclei: \(^{60}\text{Co} \rightarrow ^{60}\text{Ni}^{++} \, e^- + \bar{\nu}_e\). It was found in the experiment that electrons were emitted preferentially in the direction opposite to that of the spin of the Cobalt-60 nuclei. Subsequent experiments that examined the electron momentum spectra in muon decays showed that electrons are preferably left-handed and positrons preferably right-handed \([4]\). A left-handed electron means that its spin is opposite to the direction of the \(z\)-component of its momentum.

The discovery of exclusively left-handed neutrinos and right-handed anti-neutrinos would come not long after \([5]\) and further demonstrate that parity is (maximally) violated in weak interactions. \(C\) is also violated in weak interactions. Both parity and charge symmetry were observed to be violated individually in \(\pi^+ \rightarrow \mu^+ \nu_\mu\) decays \([6]\). The muon neutrino \(\nu_\mu\) from the \(\pi^+\) decay is left-handed. The \(P\)-conjugate process in which the \(\nu_\mu\) is right-handed and the \(C\)-conjugate process in which the the \(\bar{\nu}_\mu\) is left-handed, never occur. Thus parity is violated because of the definite handedness
of the neutrinos, and $C$ is violated because the neutrinos in the observed decays have opposite handedness (the decays are not $C$-conjugate of each other).

### 2.3 CP Violation in the $K^0$ System

After $C$ and $P$ violations were observed, it was a considerable relief that $CP$ was still conserved, for $CP$ invariance was considered to be the replacement of the separate $C$ and $P$ invariance of weak interactions [7]. However, the discovery of $CP$ violation in neutral $K$ mesons changed all that.

A $K^0$ is a bound state of a quark and an anti-quark ($K^0 = \bar{s}d$, $\bar{K}^0 = s\bar{d}$). $K^0$ and $\bar{K}^0$ differ in one quantum number, strangeness $S$, in which $S = +1$ and $-1$ for $K^0$ and $\bar{K}^0$ respectively. The $K^0$'s and $\bar{K}^0$'s are produced by strong interactions and decay via weak interactions. $K^0$ and $\bar{K}^0$ themselves are not $CP$ eigenstates. Instead, $CP$ eigenstates are constructed out of a linear combination of $K^0$ and $\bar{K}^0$, as proposed by Gell-Mann and Pais [8]:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \quad CP|K_1\rangle = +|K_1\rangle,$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \quad CP|K_2\rangle = -|K_2\rangle.$$

(2.1) (2.2)

The dominant decays of neutral kaons are 2 or 3 pions. The two-pion system has $P = +1$ and $C = +1$ while the three-pion system has $P = -1$ and $C = +1$. Therefore if $CP$ is to be conserved then only the following decays are allowed:

$$K_1 \rightarrow \pi\pi, \quad CP \text{ even},$$

$$K_2 \rightarrow \pi\pi\pi, \quad CP \text{ odd}.$$

Gell-Mann and Pais pointed out that $K_1$ should have a much shorter lifetime than $K_2$, since there is more decay phase space (energy) available for $K_1$ ($m_{K_1} - 2m_{\pi} \approx$
219 MeV, \( m_{K_2} - 3m_\pi \approx 80 \) MeV). Hence we also identify \( K_1 \) as \( K_S \) (short) and \( K_2 \)
as \( K_L \) (long). In 1964, Christenson et al. [9] discovered that \( K_L \) also decays to \( \pi\pi \) with a branching ratio of \( \sim 2 \times 10^{-3} \). Thus \( CP \) is violated in weak interactions\(^1\).

### 2.4 CP Violation in The Standard Model

Cabibbo [10] first attempted to explain generation-changing charged-current processes observed in experiments by the concept of quark mixing, and it was later extended to three generations of quarks by Kobayashi and Maskawa [11] through the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in the Standard Model. In essence, the CKM mechanism suggests that the weak flavor eigenstates in the Standard Model are not the same as the mass eigenstates of the Hamiltonian. \( CP \) violation is also explained by the CKM model, with the source being a complex phase in the CKM matrix. I now proceed to discuss how \( CP \) violation is incorporated in the Standard Model.

#### 2.4.1 V-A Theory and the GIM Mechanism

The experimental observation of left-handed neutrinos and right-handed anti-neutrinos implies that the Lagrangian of weak interactions should be composed of not only vectors (V) but also axial-vectors (A). This led to a generalization of Fermi’s theory of \( \beta \) decay [12] by Feynman and Gell-Mann [13]. They proposed that the Lagrangian that describes weak interactions should be:

\[
L(x) = -\frac{G_F}{2\sqrt{2}} J_\lambda^\dagger(x) J^\lambda(x),
\]

\(^1\)CPT invariance is still obeyed by any local gauge quantum field theory that is Lorentz-invariant.
where the weak current \( J_\lambda(x) \) consists of a leptonic and a hadronic part:

\[
J_\lambda(x) = J_{\lambda L}(x) + J_{\lambda h}(x).
\]  

(2.4)

The leptonic current can be written as (after the discovery of the tau lepton):

\[
J_{\lambda L}(x) = \overline{e} \gamma_\lambda (1 - \gamma_5) e + \overline{\mu} \gamma_\lambda (1 - \gamma_5) \mu + \overline{\tau} \gamma_\lambda (1 - \gamma_5) \tau.
\]  

(2.5)

\((1 - \gamma_5)\) is the helicity projection operator. Therefore, only left-handed fermions are present in weak currents.

Neutron \( \beta \) decays proceed through the quark transformation of \( d \rightarrow u \) (neutron is composed of \( udd \) and proton \( uud \)), therefore the hadronic weak current should be:

\[
J_{\lambda h}(x) = \overline{u} \gamma_\lambda (1 - \gamma_5) d.
\]  

(2.6)

However, strangeness-changing and quark transformation \( u \rightarrow s \) decays (e.g. \( K^+ \rightarrow \mu^+ \nu_\mu, A^0 \rightarrow p \pi^- \)) discovered in the 1950s indicate that the above expression is not enough to describe weak interactions. To account for both strangeness-conserving (\( \Delta S = 0 \)) and strangeness-changing (\( \Delta S = 1 \)) decays, Cabibbo [10] showed that the hadronic weak current should instead be:

\[
J_{\lambda h}(x) = \overline{u} \gamma_\lambda (1 - \gamma_5) d_c
\]  

(2.7)

where

\[
d_c = d \cos \theta_C + s \sin \theta_C
\]  

(2.8)

and \( \theta_C \sim 13^\circ \) is the Cabibbo angle, an empirical parameter. The Cabibbo model provides a full description of two-generation quark mixing.

In 1970, in order to explain the highly suppressed rate of \( K_L^0 \rightarrow \mu^+ \mu^- \), a first-order flavor-changing neutral weak interactions, Glashow, Iliopoulos, and Maiani [14]
postulated the existence of charm, the fourth quark (at a time when only three quarks: up, down and strange, were believed to exist) which couples with

\[ s_c = -d \sin \theta_C + s \cos \theta_C. \]  

The hadronic weak current is now written as

\[ J_{th} (x) = \bar{u} \gamma_\lambda (1 - \gamma_5) d_c + \bar{c} \gamma_\lambda (1 - \gamma_5) s_c. \]  

Note that although Cabibbo’s model and the GIM mechanism fully describe two-generation quark mixing (until the discovery of the fifth quark, the bottom quark, in 1977 at Fermilab), neither of them could incorporate \( CP \) violation in weak interactions.

2.4.2 The Standard Model

The Standard Model is a theory of elementary particles and their interactions (the strong and electroweak interactions). Together with the theory of quantum chromodynamics (QCD), it has thus far explained all experimental results in elementary particle physics, with the exception of the generation of neutrino masses and the unobserved Higgs boson.

In the Standard Model, all matter is made out of fermions and bosons. Fermions are subdivided into leptons and quarks. Leptons carry integer charge while quarks carry one-third or two-third integer charge. Leptons are subjected to the electroweak force and quarks the strong force in addition to the electroweak force. Quarks also have color charges with each quark coming in three possible colors: red, blue and green. Each fermion has a corresponding anti-particle (anti-fermion), which has the same mass as the particle but with opposite electric charge (some particles are their
own anti-particles, e.g. photon). Anti-quarks have opposite color charge to their corresponding quarks as well. Figure 2.1 provides an illustration of the constituents of the Standard Model.

Figure 2.1: The constituents of the Standard Model of particle interactions.

The fundamental interactions of nature are mediated by the exchange of vector bosons. Photons and three massive intermediate vector bosons, $W^\pm$ and $Z^0$, are responsible for the electroweak interaction, while the strong force is propagated by gluons. When two quarks are close to one another, they exchange gluons and create a very strong color force field that binds the quarks together. They bind either as mesons, a quark and anti-quark pair, or as baryons, combinations of three quarks and/or anti-quarks.

## 2.4.3 The CKM Matrix

The Standard Model is based on the $SU(3) \times SU(2) \times U(1)$ gauge group, where the $SU(3)$ subgroup characterizes the strong interaction and the $SU(2) \times U(1)$ describes
the electroweak interaction as formulated in the Glashow-Weinberg-Salam (GWS) model [15]. The quarks come in families of left-handed doublets and right-handed singlets:

\[ Q_L = \begin{pmatrix} U \\ D \end{pmatrix}_L, \quad U_R, \quad D_R, \quad (2.11) \]

where \( Q_L \) is the left-handed quark field, and \( U_R \) and \( D_R \) are the right-handed up-type quarks and down-type quarks respectively. The three generations of quarks (a total of six “flavors”) are:

\[ \begin{pmatrix} u \\ d \\ c \\ s \\ t \\ b \end{pmatrix}. \quad (2.12) \]

Leptons are described similarly by families of doublets and singlets. There are also three generations of leptons:

\[ \begin{pmatrix} \nu_e \\ e \\ \nu_\mu \\ \mu \\ \nu_\tau \\ \tau \end{pmatrix}. \quad (2.13) \]

The up and down components of each doublet differ by unit electric charge. Note that the above doublets are in flavor eigenstates.

The \( SU(2) \) component in the GWS model is composed of a triplet of vector bosons, the carriers of electroweak interaction:

\[ W_\mu = \begin{pmatrix} W_{\mu 1} \\ W_{\mu 2} \\ W_{\mu 2} \end{pmatrix}, \quad (2.14) \]

The interactions of quarks with the \( W \)-bosons are given by

\[ L_W = \frac{g}{2\sqrt{2}} \sum_{i,j} U_{L,i} \gamma^\mu D_{L,j} W_\mu^+ + h.c. \quad (2.15) \]

The indexes \( i, j = 1, 2, 3 \), denote the number of families of quarks. The SM contains a single \( SU(2) \) doublet of Higgs fields. The Yukawa interactions (interactions between
scalar and Dirac fields) of Higgs fields $\phi$ with the quarks are given by

$$L_Y = \sum_{i,j} (G_U)_{ij} \overline{Q}_{L,i} \left( \begin{array}{c} \phi^0 \\ \phi^- \end{array} \right) U_{R,j}$$

$$+ \sum_{i,j} (G_D)_{ij} \overline{Q}_{L,i} \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) D_{R,j} + h.c.$$ \hfill (2.16)

$G_U$ and $G_D$ are the Yukawa couplings (complex $3 \times 3$ matrices). Fermion masses arise once the neutral Higgs field acquires a vacuum expectation value $\langle \phi \rangle = (0, v/\sqrt{2})$.

The mass terms for the up-type and down-type quarks are

$$M_U = \frac{v G_U}{\sqrt{2}}, \quad M_D = \frac{v G_D}{\sqrt{2}},$$ \hfill (2.17)

which are given by

$$L_M = \frac{v G_U}{\sqrt{2}} \overline{U}_{L,i} U_{R,i} + \frac{v G_D}{\sqrt{2}} \overline{D}_{L,i} D_{R,i} + h.c.$$ \hfill (2.18)

To transform the mass matrices from the bases of flavor eigenstates to the bases of mass eigenstates, we diagonalize them by defining four unitary matrices such that

$$T_{U,L} M_U T_{U,R}^\dagger = M_U^{diag}, \quad T_{D,L} M_D T_{D,R}^\dagger = M_D^{diag}.$$ \hfill (2.19)

The left-handed and right-handed quark fields are then transformed to their mass eigenstates:

$$U^m_{L} = T_{U,L} U_{L}, \quad U^m_{R} = T_{U,R} U_{R},$$ \hfill (2.20)

$$D^m_{L} = T_{U,L} D_{L}, \quad D^m_{R} = T_{U,R} D_{R}.$$ \hfill (2.21)

The charged interactions in the mass eigenbasis is then, from Equation 2.15,

$$L_W = \frac{g}{2 \sqrt{2}} \sum_{i,j} \overline{U}_{L,i} \gamma^\mu D_{L,j} W^\mu + h.c.$$ \hfill (2.22)

$$= \frac{g}{2 \sqrt{2}} \sum_{i,j} \overline{U}^m_{L,i} T_{U,L,i,j} \gamma^\mu T_{D,L,j,i}^\dagger D^m_{L,j} W^\mu + h.c.$$ \hfill (2.23)

$$= \frac{g}{2 \sqrt{2}} \sum_{i,j} (V_{ij} \overline{U}^m_{L,i} \gamma^\mu D^m_{L,j} W^\mu + V_{ij}^\dagger \overline{D}^m_{L,j} \gamma^\mu U^m_{L,i} W^-) ,$$
where

\[ V = T_{U,L} T_{D,L}^\dagger, \quad (2.24) \]

a 3×3 unitary matrix, is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It can be written as

\[
V = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}. \quad (2.25)
\]

The CKM matrix contains \(2n^2 = 18\) free parameters, where \(n\) is the number of columns (generations) of the matrix. Unitarity of the matrix imposes \(n\) real normalization constraints and \(n(n-1)\) complex orthogonality constraints on its columns. In addition, the freedom to arbitrarily choose the global phase of a quark field allows \(2n - 1\) complex phases to be removed. Thus we are left with \((n - 1)^2 = 4\) independent parameters, which are three rotation angles and one irreducible complex phase. Kobayashi and Maskawa argued that at least three generations of quarks must exist to have enough physical degrees of freedom to allow for a non-zero and non-trivial phase. This phase is the origin of \(CP\) violation.

The 90\% Confidence Level of the absolute values of the CKM matrix elements are [16]:

\[
\begin{pmatrix}
0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.00010} & (3.96^{+0.09}_{-0.09}) \times 10^{-3} \\
0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80}) \times 10^{-3} \\
(8.14^{+0.32}_{-0.64}) \times 10^{-3} & (41.61^{+0.12}_{-0.78}) \times 10^{-3} & 0.999100^{+0.000034}_{-0.000004}
\end{pmatrix}. \quad (2.26)
\]

The CKM matrix is almost diagonal, which means that the weak eigenstates are almost the same as the mass eigenstates. Hence the coupling between quarks from different generations is strongly suppressed.

The CKM matrix can be parameterized in a variety of ways. The “standard parameterization” [17] utilizes the mixing angles \((\theta_{12}, \theta_{23}, \theta_{13})\) between the three generations of quarks and an overall phase \(\delta\). This phase can not be removed by
redefining the quark phases. It is responsible for CP violation in weak interactions.

This parameterization is obtained by the product of three complex rotation matrices:

\[
V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13}e^{-i\delta} \\
0 & 1 & 0 \\
-s_{13}e^{-i\delta} & 0 & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}
\]

(2.27)

\[
V = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}s_{13}
\end{pmatrix}
\]

(2.28)

where \(c_{ij} = \cos \theta_{ij}\) and \(s_{ij} = \sin \theta_{ij}\) for \(i < j = 1, 2, 3\).

Wolfenstein [18] proposed an expansion of the CKM matrix in terms of four variables, \(\lambda, A, \rho\) and \(\eta\). Using experimental facts that \(|V_{cb}| \gg |V_{ub}|, |V_{cb}| \sim |V_{us}|^2\) and \(|V_{us}| \ll 1\), he expanded the CKM elements in powers of \(\lambda\), with \(\lambda\) defined as \(s_{12}\), the sine of the Cabibbo angle \((|V_{us}| \approx s_{12} \approx \lambda, \text{since } \theta_{13} \approx 0)\). Inserting the other two definitions, \(s_{23} \equiv A\lambda^2\) and \(s_{13}e^{-i\delta} \equiv A\lambda^3(\rho - i\eta)\), into Equation 2.27, we have the “Wolfenstein parameterization” to order \(\lambda^3\):

\[
V = \begin{pmatrix}
1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4).
\]

(2.29)

The parameters \(A, \rho\) and \(\eta\) are of order 1. CP violation exists because of the imaginary parts of the matrix elements \(V_{ub}\) and \(V_{td}\).

### 2.4.4 The Unitarity Triangle

Unitarity of the CKM matrix implies that

\[
V^\dagger V = VV^\dagger = I,
\]

(2.30)

which results in six orthogonality equations:

\[
\sum_i V_{ij}V_{ik}^* = 0,
\]

(2.31)
with $j, k = 1, 2, 3, j \neq k$. Each equation represents a triangle in the complex plane. Four of the triangles are nearly degenerate and only two of them have all three sides with same order of magnitude in lengths. The most useful relation for understanding the Standard Model predictions for $CP$ violation is the orthogonality condition between the first and third columns of $V$:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (2.32)$$

$V_{ub}$ and $V_{td}$ are complex and can be represented in polar form:

$$V_{td} = |V_{td}| e^{-i\beta} \quad (2.33)$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}. \quad (2.34)$$

Equation 2.32 then becomes

$$|V_{ud}V_{ub}^*| e^{i\gamma} + |V_{cd}V_{cb}^*| + |V_{td}V_{tb}^*| e^{-i\beta} = 0. \quad (2.35)$$

Each of the three terms in the equation can be represented as a vector in the complex plane. The requirement that they sum to zero forces them to form a triangle. This triangle is referred to as the “Unitarity Triangle” (Figure 2.2).

In $B$ physics, the Unitarity Triangle is usually presented with the Wolfenstein parameterization in the complex $(\rho, \eta)$ plane. It is convenient to divide Equation 2.35 by $|V_{cd}V_{cb}^*|$. The resulting triangle is shown in Figure 2.3. It has fixed vertices at $(0,0)$ and $(1,0)$ and coordinates of the remaining vertices depend on the two Wolfenstein parameters, $\rho$ and $\eta$. The three angles of the re-scaled Unitarity Triangle are defined as:

$$\alpha \equiv \arg \left[ \frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right], \quad \beta \equiv \arg \left[ \frac{-V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right], \quad \gamma \equiv \arg \left[ \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]; \quad (2.36)$$
and the lengths of the sides are:

\[ R_u \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{\rho^2 + \eta^2}, \quad R_t \equiv \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right| = \sqrt{(1 - \rho)^2 + \eta^2}. \] 

(2.37)
We can see that the existence of \( CP \) violation keeps the Unitarity Triangle from collapsing to a line because of a non-zero \( \eta \). These quantities are measured redundantly from \( CP \) asymmetries in \( B \) decays to verify and provide a stringent test of the CKM model of \( CP \) violation in the Standard model.

2.5 \( CP \) Violation in \( B \) Decays

The \( B \) meson is a bound state of a \( b \) quark and a light \((u, d, s, c)\) anti-quark with a spin of 0 and parity of -1. \( B \) decays via weak interactions with a relatively long lifetime of approximately 1.5 ps. \( CP \)-violating effects are expected to be large in \( B \) decays, therefore the \( B \) system provides an excellent probe of \( CP \) violation in the Standard Model.

\( CP \) violation can be classified into three categories: \( CP \) violation in decay, in mixing and in the interference between decay and mixing. All these types of \( CP \) violation involve interference between different amplitudes that lead to the same final state with different phases.

2.5.1 \( CP \) Violation in Decay

\( CP \) violation in decay is also known as “direct” \( CP \) violation (Figure 2.4). It occurs when \( B \to f \) and \( \bar{B} \to \bar{f} \), for some final state \( f \) and its charge-conjugate \( \bar{f} \), do not proceed at the same rate, i.e. \( |\bar{A}_f/A_f| \neq 1 \), where \( A_f \) and \( \bar{A}_f \) are the total \( B \to f \) and \( \bar{B} \to \bar{f} \) decay amplitudes respectively. It is usually expressed as a branching fraction asymmetry:

\[
A_{CP} = \frac{B(\bar{B} \to \bar{f}) - B(B \to f)}{B(\bar{B} \to \bar{f}) + B(B \to f)}
\]

\[
= \frac{|\bar{A}_f|^2 - |A_f|^2}{|\bar{A}_f|^2 + |A_f|^2}. \tag{2.39}
\]
Figure 2.4: Direct CP violation is due to the interference between two amplitudes \( A_1 \) and \( A_2 \) with a relative CP-violating phase \( \phi \) and a CP conserving phase \( \delta \) for the transition between an initial state \( i \) and a final state \( f \).

There are two types of complex phases that can enter the amplitudes \( A_f \) and \( \overline{A_f} \). We write

\[
A_f = \sum_i A_i e^{i(\delta_i + \phi_i)} \quad \text{and} \quad \overline{A_f} = \sum_i A_i e^{i(\delta_i - \phi_i)},
\]

where \( \delta_i \) is called the strong phase and \( \phi_i \) the weak phase. Weak phases come from complex parameters in the Lagrangian and enter the transition amplitude for a decay process and the amplitude for its complex conjugate process. It has opposite signs in the two amplitudes. Strong phases arise from intermediate on-shell states re-scattering into the final state. It has the same sign in both \( A_f \) and \( \overline{A_f} \). Strong phases do not change under CP. CP violation in decay requires at least two interfering channels with different strong and weak phases. Therefore,

\[
A_f = A_1 e^{i(\delta_1 + \phi_1)} + A_2 e^{i(\delta_2 + \phi_2)}, \quad (2.40)
\]

\[
\overline{A_f} = A_1 e^{i(\delta_1 - \phi_1)} + A_2 e^{i(\delta_2 - \phi_2)}, \quad (2.41)
\]

so that the numerator of the asymmetry

\[
|A_f|^2 - |\overline{A_f}|^2 = -4 |A_1| |A_2| \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2) \quad (2.42)
\]

is proportional to the interference between the amplitudes.
Direct $CP$ violation has been observed in the kaon system at the level of a few parts in a million [19], and in the decays $B^0 \rightarrow K^+\pi^-$ and $B^0 \rightarrow \pi^+\pi^-$ at the 10% and 50% levels respectively [20]. My analysis of $B^- \rightarrow D^0 K^{*-}$ is also a search for direct $CP$ violation.

2.5.2 $CP$ Violation in Mixing

$CP$ violation in mixing is also called “indirect” $CP$ violation. It could be present in $B^0$ and $\bar{B}^0$ mixing through second-order weak processes as the $B$ mesons propagate through space. Mixing is a process in which a particle turns into its anti-particle. Consider the $B^0 - \bar{B}^0$ system. Figure 2.5 shows the Feynman diagrams responsible for $B^0 - \bar{B}^0$ mixing. The Schrodinger equation gives

$$i\hbar \frac{\partial}{\partial t} \Psi(t) = H\Psi(t), \quad \Psi(t) = \begin{pmatrix} B^0 \\ \bar{B}^0 \end{pmatrix}. \quad (2.43)$$

The mass matrix in the $B^0 - \bar{B}^0$ basis is given by

$$H = M - \frac{i}{2} \Gamma = \begin{pmatrix} M_{11} - \frac{i}{2} \Gamma_{11} & M_{12} - \frac{i}{2} \Gamma_{12} \\ M_{21} - \frac{i}{2} \Gamma_{21} & M_{22} - \frac{i}{2} \Gamma_{22} \end{pmatrix}, \quad (2.44)$$

where $M_{ij} = M_{ji}^*$ and $\Gamma_{ij} = \Gamma_{ji}^*$ are obtained by summing over intermediate states using second-order perturbation theory. Constraints from $CP$ and $CPT$ invariance give us $M_{11} = M_{22} \equiv m$, $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$ and $ImM_{12} = 0 = Im\Gamma_{12}$. The mass eigenstates and eigenvalues are

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle, \quad \text{eigenvalue} = m - \frac{i}{2} \Gamma + \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}), \quad (2.45)$$

$$|B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle, \quad \text{eigenvalue} = m - \frac{i}{2} \Gamma - \frac{q}{p} (M_{12} - \frac{i}{2} \Gamma_{12}), \quad (2.46)$$

with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}}, \quad (2.47)$$
and $|p|^2 + |q|^2 = 1$. $L$ and $H$ refer to the lighter and heavier mass eigenstates.

Now consider the time evolution of the system. We derive,

$$
|B^0(t)\rangle = f_+(t)|B^0\rangle + \frac{q}{p} f_- (t)|\overline{B}^0\rangle, \quad (2.48)
$$

$$
|\overline{B}^0(t)\rangle = f_+(t)|B^0\rangle + \frac{p}{q} f_- (t)|\overline{B}^0\rangle, \quad (2.49)
$$

where

$$
f_{\pm}(t) = \frac{1}{2} [e^{(\pm i m_L - \frac{i}{2} \Gamma_L) t} \pm e^{(\pm i m_H - \frac{i}{2} \Gamma_H) t}]. \quad (2.50)
$$

From the above equations we can see that $B^0$ can oscillate to $\overline{B}^0$ and vice versa, since

$$
|\langle \overline{B}^0(t)|B^0(t)\rangle|^2 = \frac{1}{4} \frac{q}{p} [e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-\frac{1}{2}(\Gamma_L+\Gamma_H) t}]
\times \cos[(m_H - m_L)t], \quad (2.51)
$$

$$
|\langle B^0(t)|\overline{B}^0(t)\rangle|^2 = \frac{1}{4} \frac{p}{q} [e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-\frac{1}{2}(\Gamma_L+\Gamma_H) t}]
\times \cos[(m_H - m_L)t]. \quad (2.52)
$$

$CP$ conservation requires the rates of $B^0 \to \overline{B}^0$ and $\overline{B}^0 \to B^0$ to be equal. This is true if and only if $\left|\frac{q}{p}\right| = 1$. $\left|\frac{q}{p}\right| \neq 1$ indicates $CP$ violation in mixing, which results from the mass eigenstates being different from the flavor eigenstates. This type of $CP$ violation is negligibly small in the $B$ system with the most current best
measurements (performed using both inclusive dilepton events [21, 22] or with $B$ mesons fully reconstructed into flavor or $CP$ eigenstates [23]) averaging [24]

$$\left| \frac{q}{p} \right| = 1.0024 \pm 0.0023,$$

(2.54)

which is consistent with unity.

### 2.5.3 $CP$ violation in the Interference between Decay and Mixing

The third type of $CP$ violation occurs when $B^0$ and $\overline{B^0}$ decay to the same final state (Figure 2.6). It is due to the interference between the direct decay of the $B^0$ into the final state and the alternate path of first mixing into $\overline{B^0}$ and then decay into the final state. Both decay ($A_f$ and $\overline{A_f}$) and mixing amplitudes ($p$ and $q$) are involved and we define:

$$\lambda_{CP} \equiv \eta_{CP} \frac{q \overline{A_{CP}}}{p A_{CP}},$$

(2.55)

where $\eta_{CP}$ is the $CP$ eigenvalue of the final state.

![Diagram](image.png)

Figure 2.6: The third type of $CP$ violation results from the interference between decay ($A_f$ and $\overline{A_f}$) and mixing ($p$ and $q$) amplitudes.
Consider the $CP$-violating asymmetry in the time-dependent decay rates between $B^0$ and $\bar{B}^0$:

$$A_{CP}(t) = \frac{\Gamma(B^0 \to f)(t) - \Gamma(\bar{B}^0 \to f)(t)}{\Gamma(B^0 \to f)(t) + \Gamma(\bar{B}^0 \to f)(t)} \quad (2.56)$$

$$= \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m t) - \frac{2Im(\lambda_f)}{1 + |\lambda_f|^2} \sin(\Delta m t) \quad (2.57)$$

$$= C \cos(\Delta m t) - S \sin(\Delta m t), \quad (2.58)$$

where $\Delta m$ is the difference in mass between the heavy and light $B$-meson eigenstates and

$$C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}, \quad S = \frac{2Im\lambda}{1 + |\lambda|^2}. \quad (2.59)$$

In the absence of $CP$ violation, both $C$ and $S$ must be zero, since they occur only when weak phases do not cancel. $C$ is non-zero only when the ratio of the decay amplitudes differs from unity, which is the signature of direct $CP$ violation. $S$ is a measure of $CP$ violation in the interference between decay and mixing. When $Im\lambda \neq 0$, i.e. $S \neq 0$, even if there is no $CP$ violation in decay or mixing ($|q/p| = 1$ and $|\lambda| = 1$), the $CP$ asymmetry in Equation 2.56 is non-zero. This is the third type of $CP$ violation and results from a phase in mixing that is not cancelled by the decay.

The first observation of $CP$ violation in $B$ decays and also the first in such interference is the so-called “Golden Mode” $B^0 \to J/\psi K^0_S$ [25]. This mode is a $b \to c$ transition and proceeds primarily through the tree diagram. Penguin diagrams or virtual $D^0$ exchange involving long distance re-scattering contribute as well. The tree diagram has the same phase of $2\beta$ as the penguin diagrams. The decay amplitudes have the same weak phase and thus there is no direct $CP$ violation:

$$S_{J/\psi K^0_S} = \sin 2\beta, \quad (2.60)$$

$$C_{J/\psi K^0_S} = 0. \quad (2.61)$$
This is considered the cleanest mode to measure $\sin 2\beta$, with a non-zero observed value of $\beta$ and thus a non-zero area of the Unitarity Triangle. Several other charmonium modes have been measured by BABAR and Belle for $\sin 2\beta$, including $J/\psi K_S^0$, $\psi(2S)K_S^0$, $\chi_{c1}K_S^0$, and $\eta_c K_S^0$.

2.6 Magnitudes of CKM Elements

The latest values of the magnitudes of CKM matrix elements are summarized in Equation 2.26. This section briefly describes the methods used to measure each of the elements. More information can be found in the website of Particle Data Group [26].

2.6.1 $|V_{ud}|$

The most precise determination comes from the study of nuclear $\beta$ decays which are pure vector transitions. Another precise measurement is also obtained from the measurement of neutron lifetime.

2.6.2 $|V_{us}|$

$|V_{us}|$ is traditionally extracted from semileptonic kaon decays. Other determinations include leptonic kaon, hyperon and $\tau$ decays.

2.6.3 $|V_{cd}|$

The most precise measurement is by studying neutrino and antineutrino production of charm off valence $d$ quarks.

2.6.4 $|V_{cs}|$

Direct determination is possible from semileptonic $D$ or leptonic $D_S$ decays.
2.6.5 \( |V_{cb}| \)

This matrix element can be measured from exclusive and inclusive semileptonic \( B \) decays to charm.

2.6.6 \( |V_{ub}| \)

The determination of \( |V_{ub}| \) has been obtained by combining measurements from inclusive and exclusive \( B \to X_u l \nu \) decays.

2.6.7 \( |V_{td}| \) and \( |V_{ts}| \)

These two CKM elements cannot be measured from tree-level decays of the top quark, so one has to rely on determinations from \( B \to B \) oscillations mediated by box diagrams or loop-mediated rare \( K \) and \( B \) decays. However, theoretical uncertainties in hadronic effects limit the accuracy of current determinations. These can be reduced by taking ratios of processes that are equal in the flavor \( SU(3) \) limit to determine \( |V_{td}/V_{ts}| \).

2.6.8 \( |V_{tb}| \)

The direct determination of \( |V_{tb}| \) from top decays uses the ratio of branching fractions of \( t \to Wb \) to \( t \to Wq \) where \( q = b, s, d \).

2.7 Unitarity Triangle Angles

\( CP \) violation measurements in \( B \) meson decays provide direct information on the angles of the Unitarity Triangle. These over-constraining measurements serve to improve the determination of the CKM elements or to reveal effects from new physics beyond the Standard Model.
2.7.1 \( \beta \)

As mentioned previously, the \( b \to cs \) decays to \( CP \) eigenstates \((B^0 \to \text{charmonium } K_{S,L}^0)\) are the theoretically cleanest modes. The \( CP \) parameters from the interference between decay and mixing are \( S \) and \( C \) defined in Equation 2.59. \( S \) is \( -\eta_f \sin 2\beta \) and \( C \) is zero in these decays. \( 2\beta \) is the phase difference between the \( B^0 \to f \) and \( B^0 \to \bar{B}^0 \to f \) decay paths and \( \eta_f \) is the \( CP \) eigenvalue of \( f \).

The \( b \to s \) penguin dominated decays such as \( B^0 \to \phi K^0 \) and \( \eta' K^0 \) also provide \( \sin 2\beta \) measurements. They have the same CKM phase as the \( b \to c \) tree level decays, up to corrections suppressed by \( \lambda^2 \) (defined in Equation 2.55), since \( V_{tb}^* V_{ts} = -V_{cb}^* V_{cs}[1 + O(\lambda^2)] \). These modes are also used to search for new physics. \( S \) could be different than \( -\eta_f \sin 2\beta \) and \( C \) different than 0 if new physics contributes to the \( b \to s \) loop diagrams with different weak phases.

The \( b \to c\bar{c}d \) decays, such as \( B^0 \to J/\psi \pi^0 \) and \( B^0 \to D^{(*)} + D^{(*)-} \), also measure \( \sin 2\beta \) approximately. However, the effect of penguins could be large, since the dominant component of \( b \to d \) penguin amplitude has a different CKM phase \((V_{tb}^* V_{td})\) than the tree amplitude \((V_{cb}^* V_{cd})\) and their magnitudes are of the same order in \( \lambda \). This could result in \( S \neq -\eta_f \sin 2\beta \) and \( C \neq 0 \). The most updated world-averaged results of \( S \) and \( C \) are consistent with those from \( B^0 \to \text{charmonium } K_{S,L}^0 \) decays, although with sizable uncertainties. The world average of \( \sin 2\beta \) is [27]:

\[
\sin 2\beta = 0.681 \pm 0.025, \tag{2.62}
\]

which corresponds to \( \beta = (42.92 \pm 0.72)° \).
2.7.2 \( \alpha \)

\( \alpha \) is measured with charmless \( B \) decays. Since it is the angle between \( V_{tb}^* V_{td} \) and \( V_{ub}^* V_{ud} \), only time-dependent \( CP \) asymmetries in \( b \rightarrow u \bar{u}d \) dominated tree modes can directly measure \( \sin 2\alpha \). The penguin contribution can be sizable because \( b \rightarrow d \) penguin amplitudes have a different CKM phase than \( b \rightarrow u \bar{u}d \) tree amplitudes, and their magnitudes are of the same order in \( \lambda \). This complication makes the extraction of \( \alpha \) very difficult. \( \alpha \) has been measured in \( B \rightarrow \pi \pi, \, \rho \pi \) and \( \rho \rho \) decay modes, with \( B \rightarrow \rho \rho \) giving the best precision. Estimating the penguin contribution (or pollution) with respect to tree contribution requires isospin analysis [28] for \( \pi \pi \) and \( \rho \rho \), and Dalitz-plot analysis [29] for \( \rho \pi \). Combining these three decay modes, \( \alpha \) is constrained as

\[
\alpha = (99^{+13}_{-8})^\circ. \tag{2.63}
\]

2.7.3 \( \gamma \)

\( \gamma \) does not depend on CKM elements involving the top quark, so it can be measured in tree level \( B \) decays. As a result direct measurements of \( \gamma \) are unlikely to be affected by physics beyond the Standard Model. The methods to determine \( \gamma \) utilize the measurement of direct \( CP \) violation in \( B^- \rightarrow D^{(*)0}K^{(*)-} \) decays [30, 31, 32, 33], where the neutral \( D \) meson can be \( D^0 \) or \( \bar{D}^0 \). The final states of the \( D \) meson are mostly reconstructed in either \( CP \) eigenstates, hadronic modes, or self-conjugate three-body final state \( K_S^0 \pi^+ \pi^- \) [34]. The last method can be optimized by performing a Dalitz plot analysis. All variations are sensitive to the same \( B \) decay parameters and therefore can be treated in a combined fit to extract \( \gamma \).
This thesis focuses on the study of $B^- \rightarrow D^0 K^{*-}$, where $D^0$ decays to $CP$ ($K^+ K^-$, $\pi^+ \pi^-$, $K^0_S \pi^0$, $K^0_S \phi$, $K^0_S \omega$) and non-$CP$ modes ($K^{+} \pi^{\pm}$). The details of the analysis techniques and formalism will be discussed after next section.

### 2.8 The Global CKM Fit

Using the independently measured CKM elements mentioned in the previous sections, the unitarity of the CKM matrix can be checked. The following numbers are obtained [16]:

\begin{align}
|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 &= 0.9992 \pm 0.0011 \quad \text{(1st row)} \\
|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 &= 1.003 \pm 0.027 \quad \text{(2nd row)} \\
|V_{ud}|^2 + |V_{cd}|^2 + |V_{td}|^2 &= 1.001 \pm 0.005 \quad \text{(1st column)}.
\end{align}

These results are consistent with the unitarity of the CKM matrix.

The sum of the three angles of the Unitarity Triangle is also consistent with the Standard Model expectation [16]:

\[ \alpha + \beta + \gamma = (184^{+20}_{-15})^\circ. \]

If we recall the Wolfenstein representation of the CKM matrix, the parameters $\lambda$, $A$, $\rho$ and $\eta$ parameterize the weak interactions of quarks and $CP$ violation in the Standard Model. Measurements of semileptonic decays of strange and beauty particles are the main sources of information on $\lambda$ and $A$ respectively. $\rho$ and $\eta$ can be most precisely constrained by a global fit that uses all available measurements and imposes the SM constraints (i.e., three generation unitarity). The fit consists of maximizing a likelihood built upon relevant experimental measurements and their Standard Model predictions. There are several approaches to combine all the experimental
data. CKMfitter [35] uses frequentist statistics with different presentations of the theoretical errors, while UTfit [36] uses a Bayesian approach [37]. These approaches provide similar results. Viola Sordini [36] of the UTfit group has kindly created plots of $\gamma$ and $r_B$ from the newest measurements of $CP$ observables in this thesis, which will be presented in the Combined GLW and ADS Results Chapter. Only UTfit will be discussed in this thesis.

UTfit uses the values of $|\epsilon_K|$, $|V_{ub}/V_{cb}|$, $\Delta m_d$, $\Delta m_d/\Delta m_s$, $\alpha$, $\beta$ and $\gamma$ to constrain $\varphi$ and $\eta$, where $\varphi = \rho(1 - \frac{\lambda^2}{2})$ and $\eta = \eta(1 - \frac{\lambda^2}{2})$. $\Delta m_d$ and $\Delta m_s$ are the mass differences between the light and heavy mass eigenstates of the $B_d^0 - \bar{B_d^0}$ and $B_s^0 - \bar{B_s^0}$ system respectively, while $|\epsilon_K|$ parameterizes $CP$ violation in the kaon system. These constraints are obtained by comparing the most recent experimental measurements with theoretical calculations, while taking into account different sources of uncertainties. Statistical errors and systematic effects in experiments, as well as theoretical uncertainties are combined to deduce a global uncertainty for $\varphi$ and $\eta$. Details of the method is beyond the scope of this thesis and readers can refer to [38]. The Wolfenstein parameters determined by UTfit are:

$$
A = 0.815 \pm 0.013 \quad \lambda = 0.2258 \pm 0.0014 \\
\varphi = 0.197 \pm 0.031 \quad \eta = 0.351 \pm 0.020.
$$

(2.68)

The allowed regions for $\varphi$ and $\eta$ using the constraints above at 68% and 95% probability are shown in Figure 2.7.

### 2.9 $B^- \to D^0 K^{*-}$ Decays

The search for direct $CP$ violation in $B^- \to D^0 K^{*-}$ decays, where $CP$ asymmetries have clean theoretical interpretations in terms of $\gamma$, is performed in this thesis. The $D^0$ corresponds to the leading $b \to c$ transition, whereas the $\bar{D}^0$ is produced
Figure 2.7: Confidence levels in the $(\overline{\rho}, \overline{\eta})$ plane for the global CKM fit performed by UTfit are determined by constraints from measurements of $|\epsilon_K|$, $|V_{ub}/V_{cb}|$, $\Delta m_d$, $\Delta m_d/\Delta m_s$, $\alpha$, $\beta$ and $\gamma$. Closed contours at 68% and 95% probability are shown.

by a color-suppressed $b \to u$ transition. The interference of $B^- \to D^0 K^{*-}$ and $B^- \to \overline{D}^0 K^{*-}$ transitions are studied in final states accessible in both $D^0$ and $\overline{D}^0$ decays. The two interfering amplitudes result in observables that are sensitive to $\gamma$, the relative weak phase between the two decays. We will use two different methods which have different sets of $D^0$ final states to extract $\gamma$. 

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2.9.1  The GLW Method

The first method is called the GLW method. Gronau, London and Wyler (GLW) [39, 40] proposed a theoretically clean way to measure the CKM angle $\gamma$ with $B^- \rightarrow D^0 K^{*-}$ decays, where the $D$-meson decays into $CP$-eigenstates ($K^+ K^-, \pi^+ \pi^-, K^0_S \pi^0, K^0_S \phi, K^0_S \omega$) and non-$CP$ final state ($K^- \pi^+$). At quark-level, the amplitudes for $B^- \rightarrow D^0 K^{*-}$ and $B^- \rightarrow \bar{D}^0 K^{*-}$ are color-favored $b \rightarrow c\bar{s}s$ and color-suppressed $b \rightarrow u\bar{c}s$ respectively. When the $D^0$ and $\bar{D}^0$ decay into the same final state, these two $B$-decays are indistinguishable and the quantum interference gives rise to a $CP$ asymmetry proportional to $\sin \gamma$. Figure 2.8 shows the Feynman diagrams for the decays.

![Feynman Diagrams](image)

Figure 2.8: Feynman diagrams for the color-favored decay of $B^- \rightarrow D^0 K^{*-}$ (left) and the color-suppressed decay of $B^- \rightarrow \bar{D}^0 K^{*-}$ (right).

The amplitudes of the two decays can be expressed as:

$$A(B^- \rightarrow D^0 K^{*-}) = a$$  \hspace{1cm} (2.69)

$$A(B^- \rightarrow \bar{D}^0 K^{*-}) = ar e^{i\delta} e^{-i\gamma}$$  \hspace{1cm} (2.70)
where $\delta$ is the strong phase difference between the two decays that does not violate $CP$; and $\gamma$ is the weak phase difference that does. $r_B$ is the ratio of the amplitudes of the color-suppressed to the color-favored process,

$$r_B = \frac{|A(B^-(\rightarrow \bar{D}^0 K^{*-} ))|}{|A(B^-(\rightarrow D^0 K^{*-} ))|}. \quad (2.71)$$

$r_B$ has not been measured precisely and it is one of the parameters we want to extract from this analysis$^2$.

When the $D^0$ and $\bar{D}^0$ decay to the same $CP$-eigenstate, $D^0$ and $\bar{D}^0$ are indistinguishable and the two processes in Equation 2.69 interfere. Since

$$|D_{CP^+}^0\rangle = \frac{1}{\sqrt{2}}(|D^0\rangle \pm |\bar{D}^0\rangle), \quad (2.72)$$

we can re-write Equation 2.69 as:

$$A(B^- \rightarrow D_{CP^+}^0 K^{*-}) = \frac{1}{\sqrt{2}}(a \pm ar_B e^{i\delta} e^{-i\gamma}) \quad (2.73)$$

$$A(B^+ \rightarrow D_{CP^+}^0 K^{*-}) = \frac{1}{\sqrt{2}}(a \pm ar_B e^{i\delta} e^{i\gamma}). \quad (2.74)$$

Now we add (subtract) the squares of the above two equations, we have

$$|A(B^- \rightarrow D_{CP^+}^0 K^{*-})|^2 + |A(B^+ \rightarrow D_{CP^+}^0 K^{*-})|^2 = a^2(1 + r_B^2 \pm 2r_B \cos \delta \cos \gamma) \quad (2.75)$$

$$|A(B^- \rightarrow D_{CP^+}^0 K^{*-})|^2 - |A(B^+ \rightarrow D_{CP^+}^0 K^{*-})|^2 = \pm 2a^2 r_B \sin \delta \sin \gamma. \quad (2.76)$$

$^2$We are actually measuring $r_s$ due to the large $K^*$ natural width. However for purpose of this thesis, we will assume that $r_s$ is equivalent to $r_B$. Readers can refer to Appendix C for a detailed discussion.
The $CP$-asymmetry $A_{CP}$ and another independent observable $R_{CP}$ are defined as:

$$A_{CP \pm} = \frac{\Gamma(B^+ \rightarrow D_{CP \pm} K^{*-}) - \Gamma(B^- \rightarrow D_{CP \pm} K^{*-})}{\Gamma(B^+ \rightarrow D_{CP \pm} K^{*-}) + \Gamma(B^- \rightarrow D_{CP \pm} K^{*-})} = \frac{\pm 2r_B \sin \delta \sin \gamma}{1 \pm 2r_B \cos \delta \cos \gamma + r^2_B}$$

(2.77)

$$R_{CP \pm} = \frac{\Gamma(B^+ \rightarrow D_{CP \pm} K^{*-}) + \Gamma(B^- \rightarrow D_{CP \pm} K^{*-})}{\Gamma(B^- \rightarrow D^0_{NON-CP} K^{*-})} = 1 \pm 2r_B \cos \delta \cos \gamma + r^2_B$$

(2.78)

These observables can be measured experimentally,

$$A_{CP +} = \frac{N(B^-, CP+) - N(B^+, CP+)}{N(B^-, CP+) + N(B^+, CP+)}$$

(2.79)

$$A_{CP -} = \frac{N(B^-, CP-) - N(B^+, CP-)}{N(B^-, CP-) + N(B^+, CP-)}$$

(2.80)

and

$$R_{CP +} = \frac{N(B^-, CP+) + N(B^+, CP+)}{N(Non - CP)} \times \frac{\epsilon_{Non-CP}}{\epsilon_{CP +}}$$

(2.81)

$$R_{CP -} = \frac{N(B^-, CP-) + N(B^+, CP-)}{N(Non - CP)} \times \frac{\epsilon_{Non-CP}}{\epsilon_{CP -}}$$

(2.82)

where $N(B^+, CP \pm)$ and $N(B^-, CP \pm)$ represent the number of $B^+ \rightarrow D^0_{CP \pm} K^{*+}$ and $B^- \rightarrow D^0_{CP \pm} K^{*-}$ decays respectively. $D^0_{CP +}$ includes events where the $D^0$ is reconstructed in either $K^+ K^-$ or $\pi^+ \pi^-$ final states. $D^0_{CP -}$ includes events where the $D^0$ is reconstructed in either $K^0_s \pi^0$, $K^0_s \phi$ or $K^0_s \omega$ final states. $N(Non - CP)$ refers to the number of charged-$B$ decaying to $D^0 K^*$ and where the $D^0$ is reconstructed in $K^- \pi^+$ final state.

In the measurements of $R_{CP \pm}$, the raw MC efficiency should be corrected so that the differences between the MC and real data will be taken into account. The number of events of each type of decays are corrected by their respective relative reconstruction efficiency $\epsilon$:

$$\epsilon_{rel}(K^+ K^-) = BR(D^0 \rightarrow K^+ K^-) \times \epsilon(K^+ K^-) \times \text{efficiency correction},$$

(2.83)

31
The efficiency correction include tracking, particle identification, $\pi^0$ and $K_S^0$ corrections, with most of the correction recipes provided by corresponding BABAR special working groups. The details and calculations of the corrections will be discussed in Appendix A.

Another set of observables $(x^\pm, y^\pm)$, called the “cartesian coordinates”, are useful for combining the GLW analysis with the Dalitz $B^- \to [K_S^0\pi^+\pi^-]D^0K^+\pi^-$ analysis. They are defined as:

$$x^\pm \equiv r_B \cos(\delta \pm \gamma)$$

$$y^\pm \equiv r_B \sin(\delta \pm \gamma),$$

Since $r_B$ has a physics boundary (it must be positive), it is found that its fit values have a biased distribution and the bias is larger for smaller values of $r_B$ and smaller data sample size [41]. In addition, $(r_B, \delta, \gamma)$ are significantly correlated while the cartesian coordinates are not. Therefore, they have the advantage over $r_B$ of being Gaussian distributed, uncorrelated and unbiased. The cartesian coordinates are related to $A_{CP\pm}$ and $R_{CP\pm}$ through

$$x^\pm = \frac{R_{CP+}(1 \mp A_{CP+}) - R_{CP-}(1 \mp A_{CP-})}{2},$$

and

$$r_B^2 = (x^\pm)^2 + (y^\pm)^2 = \frac{R_{CP+} + R_{CP-} - 2}{2}.$$
Both $x^\pm$ and $y^\pm$ are directly measured in the Dalitz analysis. However only $x^\pm$ can be extracted from the GLW analysis due to the way $A_{CP}^\pm$ and $R_{CP}^\pm$ are constructed ($y^\pm = r_B (\sin \delta \cos \gamma \pm \cos \delta \sin \gamma)$, but neither $A$ nor $R$ has a $(\sin \times \cos)$ term).

2.9.2 The ADS Method

In the second part of my thesis, I turn the focus to another method suggested by Atwood, Dunietz and Soni [42]. The ADS methodology aims to extract the weak phase $\gamma$ from the interference of $B^- \to D^0K^{*-}$ and $B^- \to \bar{D}^0K^{*-}$ decays where the $D^0$ and $\bar{D}^0$ decay to $K^+\pi^-$ final state. This final state may occur by two paths: either by a color-suppressed $B$ decay, $B^- \to \bar{D}^0K^{*-}$, followed by a Cabibbo-favored $D^0$ decay, $\bar{D}^0 \to K^+\pi^-$ (Figure 2.9); or by a color-favored $B$ decay, $B^- \to D^0K^{*-}$, followed by a Doubly-Cabibbo-suppressed $D^0$ decay, $D^0 \to K^+\pi^-$ (also called the Wrong-Sign (WS) decay) (Figure 2.10). The two processes have similar magnitude and so one expects a large associated $CP$ asymmetry.

![Feynman Diagram](image)

Figure 2.9: Feynman diagrams for the color-suppressed decay of $B^- \to \bar{D}^0K^{*-}$ (left) followed by the Cabibbo-favored decay of $\bar{D}^0 \to K^+\pi^-$ (right).
Figure 2.10: Feynman diagrams for the color-favored decay of $B^- \rightarrow D^0 K^{*-}$ (left) followed by the Doubly-Cabibbo-suppressed decay of $D^0 \rightarrow K^+ \pi^-$ (right).

The amplitudes of the two $B$ decays can be expressed as:

\[
A(B^- \rightarrow D^0 K^{*-}) = a \tag{2.90}
\]

\[
A(B^- \rightarrow \overline{D}^0 K^{*-}) = ar_B e^{i\delta_B} e^{-i\gamma} \tag{2.91}
\]

where $\delta_B$ is the strong phase difference between the favored and suppressed $B$ decay.

$\gamma$ is the weak phase difference. $r_B$ is the ratio of the amplitudes of the color-suppressed to the color-favored process,

\[
r_B = \frac{|A(B^- \rightarrow \overline{D}^0 K^{*-})|}{|A(B^- \rightarrow D^0 K^{*-})|}. \tag{2.92}
\]

The amplitudes of the two $D^0$ decays are:

\[
A(\overline{D}^0 \rightarrow K^+ \pi^-) = b \tag{2.93}
\]

\[
A(D^0 \rightarrow K^+ \pi^-) = br_D e^{i\delta_D} \tag{2.94}
\]

where $\delta_D$ is the strong phase difference between the favored and suppressed $D^0$ decay and $r_D$ is the ratio of the amplitudes of the Doubly-Cabibbo-suppressed $D^0 \rightarrow K^+ \pi^-$ to the Cabibbo-favored $D^0 \rightarrow K^- \pi^+$ decay:

\[
r_D = \frac{|A(D^0 \rightarrow K^+ \pi^-)|}{|A(D^0 \rightarrow K^- \pi^+)|}. \tag{2.95}
\]
The PDG [43] has \( r_D^2 = 0.00362 \pm 0.00029 \).

When the amplitudes interfere, they become:

\[
A(B^- \rightarrow D_{ADS} K^{*-}) = A(B^- \rightarrow [K^+\pi^-]_{D^0} K^{*-}) + A(B^- \rightarrow [K^+\pi^-]_{\bar{D}^0} K^{*-}) \quad (2.96)
\]
\[
= abr_D e^{-i\delta_D} + abr_B e^{i(\delta_B - \gamma)} \quad (2.97)
\]

\[
A(B^+ \rightarrow D_{ADS} K^{*+}) = A(B^+ \rightarrow [K^-\pi^+]_{D^0} K^{*+}) + A(B^+ \rightarrow [K^-\pi^+]_{\bar{D}^0} K^{*+}) \quad (2.98)
\]
\[
= abr_B e^{i(\delta_B + \gamma)} + abr_D e^{-i\delta_D} \quad (2.99)
\]

Adding and subtracting the squares of the amplitudes gives us:

\[
|A(B^- \rightarrow D_{ADS} K^{*-})|^2 + |A(B^+ \rightarrow D_{ADS} K^{*+})|^2 = 2a^2b^2(r_B^2 + r_D^2 + 2r_Br_D \cos \delta \cos \gamma) \quad (2.100)
\]

\[
|A(B^- \rightarrow D_{ADS} K^{*-})|^2 - |A(B^+ \rightarrow D_{ADS} K^{*+})|^2 = 4a^2b^2r_Br_D \sin \delta \sin \gamma, \quad (2.101)
\]

where \( \delta \) equals to \( \delta_D + \delta_B \). Finally we have our observables, \( R_{ADS} \) and \( A_{ADS} \). \( R_{ADS} \) is the ratio of wrong-sign to right-sign (RS) \( (D^0 \rightarrow K^-\pi^+ \text{ or } \bar{D}^0 \rightarrow K^+\pi^-) \) decays.

\( A_{ADS} \) is the asymmetry in the wrong-sign events. They are defined as:

\[
R_{ADS} \equiv \frac{\Gamma(B^- \rightarrow D_{ADS} K^{*-}) + \Gamma(B^+ \rightarrow D_{ADS} K^{*+})}{\Gamma(B^- \rightarrow [K^-\pi^+]_{D^0} K^{*-}) + \Gamma(B^+ \rightarrow [K^-\pi^+]_{\bar{D}^0} K^{*+})} \quad (2.102)
\]
\[
= r_B^2 + r_D^2 + 2r_Br_D \cos \delta \cos \gamma \quad (2.103)
\]

\[
A_{ADS} \equiv \frac{\Gamma(B^- \rightarrow D_{ADS} K^{*-}) - \Gamma(B^+ \rightarrow D_{ADS} K^{*+})}{\Gamma(B^- \rightarrow D_{ADS} K^{*-}) + \Gamma(B^+ \rightarrow D_{ADS} K^{*+})} \quad (2.104)
\]
\[
= \frac{2r_Br_D \sin \delta \sin \gamma}{R_{ADS}}. \quad (2.105)
\]

The observables can also be measured experimentally,

\[
R_{ADS} = \frac{N(B^-, WS) + N(B^+, WS)}{N(B^-, RS) + N(B^+, RS)}, \quad (2.106)
\]

and

\[
A_{ADS} = \frac{N(B^-, WS) - N(B^+, WS)}{N(B^-, WS) + N(B^+, WS)}, \quad (2.107)
\]

where \( N(B^-, WS) \) is the number of \( B^- \rightarrow [WS]_{D^0} K^{*-} \) events measured and so on.
CHAPTER 3

THE EXPERIMENTAL ENVIRONMENT

3.1 Introduction

$B$ mesons are an ideal particle laboratory for $CP$ violation studies within and beyond the Standard Model. These studies require copious amounts of $B$ mesons, precise measurement of the $B$ time of flight and flavor, and reasonably low background in the event sample. The $B$ factory at the Stanford Linear Accelerator Center (SLAC) in Menlo Park, CA, comprising the PEP-II accelerator complex [44] and the BABAR detector [45], is designed and optimized with these goals in mind.

The SLAC $B$ factory studies $e^+e^-$ collisions at a center-of-mass (CM) energy of 10.58 GeV. This energy corresponds to the $\Upsilon(4S)$ resonance, which provides a very clean environment for $B$ reconstruction. At this energy, about 17% of the hadronic $e^+e^-$ cross-section is $b\bar{b}$ production (Table 3.1). The $\Upsilon(4S)$ resonance is a spin-1 bound state of a $b$ quark and a $b$ anti-quark. The $\Upsilon(4S)$ mass is just above the $B\bar{B}$ production threshold, and it decays almost exclusively to $B\bar{B}$ pairs (equal number of $B^+B^-$ and $B^0\bar{B}^0$) through the strong interaction.

A $B\bar{B}$ pair produced by $\Upsilon(4S)$ decay is in a coherent $L=1$ state (P-wave). The two mesons evolve in phase, therefore they have opposite flavor before one of them
Table 3.1: Production cross-sections at $\sqrt{s} = 10.58$ GeV.

<table>
<thead>
<tr>
<th>$e^+e^- \rightarrow$</th>
<th>Cross-Section (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$bb$</td>
<td>1.10</td>
</tr>
<tr>
<td>$cc$</td>
<td>1.30</td>
</tr>
<tr>
<td>$u\bar{u}$</td>
<td>1.39</td>
</tr>
<tr>
<td>$d\bar{d}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$s\bar{s}$</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau^+\tau^-$</td>
<td>0.94</td>
</tr>
<tr>
<td>$\mu^+\mu^-$</td>
<td>1.16</td>
</tr>
<tr>
<td>$e^+e^-$</td>
<td>$\approx 40$</td>
</tr>
</tbody>
</table>

decays. In the measurement of time-dependent $CP$ asymmetries, a key ingredient is to determine whether the meson decaying to a $CP$ final state at $\Delta t = 0$ is a $B$ or $\bar{B}$. $\Delta t$ is the difference in decay times of the two $B$’s. This can be achieved by the flavor-tagging technique, which is to infer the flavor from the flavor of the other $B$ in the event through charge correlations of its daughters. After the flavor of the $B$ is tagged, $\Delta t$ is calculated from the distance between the decay vertices of the two $B$ mesons. However, in $\Upsilon(4S)$ decays the $B$’s are produced almost at rest in the CM frame, resulting in a vertex separation of only about 30 $\mu$m, on average, by the time they decay. Such distance would be too small to be measured by any vertex tracker, since a typical silicon-vertex detector would only have a spatial vertex resolution of about 50 $\mu$m. The PEP-II B factory solved this problem by colliding asymmetric $e^+$ and $e^-$ beams. Head-on collisions between a 9.0 GeV electron beam and a 3.1 GeV positron beam provide a Lorentz boost of $\beta\gamma = 0.56$ in the laboratory frame to the $B$ meson pair. The $B$ particles are carried downstream in the direction of the higher energy beam and this forward boost makes it possible to separate the decay vertices of the two $B$ mesons. This allows us to observe the distances between their points.
of decay (the resulting Lorentz time dilation of the $B$-meson lifetime elongates the average decay-vertex separation in the lab frame to an average of 270 µm).

I now provide a detailed description of the PEP-II accelerator and the $\text{BABAR}$ detector.

### 3.2 The PEP-II Asymmetric Collider

A schematic representation of the acceleration and storage system at PEP-II is shown in Figure 3.1. Electrons and positrons are produced by the electron-gun positioned near the beginning of the two-mile long linear accelerator (LINAC). The gun is a thermally heated cathode filament held under high voltage. The electrons are accelerated into copper waveguides by an electric field. They are then collected into bunches and ejected out of the gun into the LINAC. The electron beam in the LINAC is accelerated by Klystron tubes which generate high power microwaves in radio-frequency (RF) cavities.

After the electron beam is accelerated to approximately 1 GeV of energy, it is directed into a damping ring to be stored for some period of time. The purpose is to reduce the transverse momentum of the electrons in the beam. As the beam circulates in the ring, it loses energy through synchrotron radiation and is continuously re-accelerated by RF cavities. After this process, the electron beam is re-directed back to the LINAC and accelerated to 8.9 GeV.

To generate positrons a portion of the electron beam is separated, accelerated to approximately 30 GeV and collided onto a tungsten target to produce electromagnetic showers that contain large numbers of electron-positron pairs. The positrons are separated, collected, accelerated and returned to the LINAC. The positron beam in
the LINAC will then be accelerated and directed to its own damping ring like the electron beam. Finally, it is dumped back into the LINAC and accelerated to its target energy of 3.1 GeV.

After reaching their respective desired energies at the end of the LINAC, the electron and positron beams are injected into the PEP-II storage rings. The electron beam is injected into the High Energy Ring and travel clockwise around the ring while the positrons are injected into the Low Energy Ring and travel counter-clockwise. As they circulate, magnets and RF cavities around the storage rings focus the beams and replace the energy lost due to synchrotron radiation. They are then brought into collision at an interaction point (IP). During data taking, each ring contains about 1600
circulating bunches colliding every 5 ns. The \textbf{BABAR} detector is constructed around the IP to detect and analyze the decay products of the resulting $e^+e^-$ collisions.

Note that about 10\% of the time the beams are collided at an energy 40 MeV below the peak of the $\Upsilon(4S)$ resonance to allow studies of non-resonant background (continuum, $e^+e^- \rightarrow q\bar{q}$, where $q = u,d,s,c$) in data. No $B$ mesons are produced since this energy is below the $B\bar{B}$ threshold.

The PEP-II collider was designed to operate at an instantaneous luminosity of $3 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$. The accelerator system has since significantly exceeded the original expectation and reached a highest value of $1.2 \times 10^{34} \text{ cm}^{-2}\text{s}^{-1}$. The increased luminosity is achieved by using higher beam currents and by improvements in the RF cavities, beam-shaping cavities and magnet systems. The original goal of an integrated luminosity of 3.3 fb$^{-1}$ per month has also been exceeded with a record month of 18.84 fb$^{-1}$ from 07/18/2006 to 08/17/2006. PEP-II has delivered a total integrated luminosity of 550.45 fb$^{-1}$ to date, while \textbf{BABAR} has recorded 529.06 fb$^{-1}$, a 96\% efficiency. Figure 3.2 shows the delivered and recorded integrated luminosity since operations first began in June 1999 and Figure 3.3 shows the luminosity per day. With a $B\bar{B}$ production cross-section of 1.1 nb, this corresponds to 605 million $B\bar{B}$ pairs. It is important to note that the data has been collected in seven different periods (so-called runs), so the actual dataset corresponds to Run 1 to Run 7 (Run 7 was recorded on $\Upsilon(3S)$ and $\Upsilon(2S)$ resonances). The PEP-II $B$ factory and \textbf{BABAR} detector stopped running in April after the end of Run 7.
3.3 The BABAR Detector

3.3.1 Overview

The BABAR detector is designed to satisfy the following requirements in order to measure the $CP$-asymmetry in the $B$-system due to the very small branching ratios of decays of $B$ mesons to $CP$-eigenstates. The need for full reconstruction of final states...
with two or more charged particles and several $\pi^0$'s adds to the stringent requirements [45]:

- Large acceptance at small polar angles relative to the boost direction.
- Excellent reconstruction efficiency for low energy charged particles and photons ($< 100$ MeV).
- Excellent momentum resolution.
• Excellent energy and angular resolution for photon detection.

• Excellent vertex resolution, in directions transverse and parallel to the beam.

• Efficient electron and muon identification and accurate hadron identification.

• Detector subsystems that can withstand and operate reliably under large doses of radiation, especially for the inner-most subsystems.

Figures 3.4 and 3.5 show schematic drawings of the side and front view of the detector. The $\text{Babar}$ detector consists of two end-caps and a cylindrical barrel surrounding the beam-pipe along the $z$-direction and roughly symmetric in the azimuth $\phi$. The detector has five major subsystems, the Silicon Vertex Tracker (SVT), the Drift Chamber (DCH), the Detector of Internally Reflected Cherenkov Light (DIRC), the Electromagnetic Calorimeter (EMC), and the Instrumented Flux Return (IFR), in the order of increasing distance from the beam-pipe. The SVT, the innermost detector, measures trajectories of charged particles very close to the interaction point and is the main source of information on their polar angles. The Drift Chamber is $\text{Babar}$’s main tracking device. It determines the momentum of charged particles by measuring the curvature of tracks in a 1.5 T magnetic field created by a superconducting solenoid located between the EMC and IFR. The DCH also provides a measurement of ionization energy loss ($dE/dx$) for particle identification. The DIRC is a charged particle identification device and has a primary task of distinguishing charged pions and charged kaons at high momentum. The EMC is built from CsI(TI) crystals and is used to determine the position, energy and identity of electrons, photons and neutral pions. The IFR is $\text{Babar}$’s outermost subsystem. It is designed to detect muons and long-lived neutral hadrons.
A trigger system uses information from the DCH, EMC and IFR to select useful data from a particular event to be stored for later analysis.

The BABAR coordinate system is defined as a right-handed system:

- The $+z$ axis is parallel to the magnetic field of the solenoid and in the direction of the electron beam.
- The $+y$ axis points vertically upward.
- The $+x$ axis points horizontally, away from the center of the PEP-II ring.
Figure 3.5: Transverse cross-section of the barrel of the BABAR detector.

- The coordinate system origin is the IP, which is offset in the $-z$ direction from the geometrical center of the detector to increase coverage of the boosted $\Upsilon(4S)$ decays.

The relevant BABAR subsystems will be described in more details in the following sections. Original design of the BABAR detector can be found in [45] and more technical details in [46].

### 3.4 The Silicon Vertex Tracker

The SVT [47] was designed to measure the positions and decay vertices of $B$ mesons and other charged particles near the IP precisely. Good vertexing is particularly critical to $CP$ violation studies, in particular measurement of the distance
between the decay positions of two $B$ mesons in an event. It is necessary for the detector to have a $\Delta z$ resolution and single vertex resolution better than 130 $\mu$m and 80 $\mu$m respectively, for full reconstruction of $B$ decays. In addition, since many of the $B$ decay products have a low transverse momentum ($p_T$), the SVT must also provide track reconstruction for particles with $p_T < 120$ MeV, as the DCH cannot provide reliable tracking information for these low-momentum tracks.

3.4.1 Design

The SVT consists of five layers of silicon double-sided with conductive strip sensors. The $z$-side of a layer is oriented perpendicularly to the beam ($z$) direction on the inner side of the detector to measure $z$-coordinates; while the $\phi$-side on the outer side of the detector runs parallel to the beam-pipe for phi-coordinate measurements. Figure 3.6 shows the $r - z$ view of the upper half and Figure 3.7 the layout of the detector in the $x - y$ plane. The SVT covers 90% of the solid angle in the CM frame. The first three layers of silicon strips are located as close as possible to the beam-pipe to reduce the effect of multiple scattering within the detector. The effect is more profound as the particle trajectories are farther away from the IP. The outer two layers are closer to the DCH to facilitate matching of SVT tracks with DCH ones.

The first three layers of the SVT are divided into sextants. They are to provide precision measurements of the azimuthal angle ($\phi$), polar angle ($\theta$) and impact parameters of a track. The outer layers are required for pattern recognition and stand-alone tracking. There are 6 modules of silicon wafers in the first 3 layers. Each module in the first two layers has 4 wafers and 6 each in the third layer. Layers 4 and 5 have 16 and 18 modules with 7 and 8 wafers each, respectively. Each module is divided into
Figure 3.6: Longitudinal ($r$-$z$) cross-section of the SVT.

Figure 3.7: Transverse ($x$-$y$) cross-section of the SVT.
forward and backward half. The design was chosen to reduce the amount of silica required to cover the solid angle while maintaining maximum efficiency.

3.4.2 Theory of Operations

The sensors in the strips are 300-\(\mu\)m-thick high-resistivity silicon wafers. When a high-energy charged particle traverses through the sensor it displaces orbital electrons and creates positive holes. A bias voltage attracts the electrons and holes in opposite directions, thus creating a current. The resulting electrical signal is read out from the strips, amplified, and discriminated by front-end electronics.

As the SVT is the closest subsystem to the IP, it is the most vulnerable to radiation damage. The silicon sensors are designed to have a high threshold for radiation damage to tolerate an anticipated life-time accumulated doses of 2 Mrad. Nevertheless, they are easily damaged by high instantaneous or accumulated doses. Therefore, a radiation monitoring system with twelve silicon PIN diodes, located at a radius of 3 cm from the beam-pipe, was installed to constantly monitor the radiation doses. The beams are automatically aborted if radiation levels are above the 1 Rad/s threshold. However, the PIN diodes have degraded since initial installation and during the 2002 Summer detector shutdown, a system of two Chemical Vapor Deposition (CVD) diamond sensors was installed inside the SVT [48]. The diamond sensors were designed, tested and assembled by the Ohio State University. They have virtually no leakage current and are much more radiation hard than the silicon PIN diodes. The SVT is well below the operational limit of 4 Mrad integrated dose and the monitoring system has prevented any significant damage from occurring to date.
3.4.3 Performance

The SVT has been operating efficiently since its installation, with an average track reconstruction efficiency of over 97% (Figure 3.8) [49]. The average $z$- and $\phi$-resolutions for tracks originating from the IP are 20-40 $\mu$m. It has satisfied the original goal for vertex resolution and low transverse momentum hit resolution.

![Figure 3.8: SVT reconstruction efficiency in the $z$ (left) and $\phi$ view (right) as measured in $e^+e^- \rightarrow \mu^+\mu^-$ events.](image)

3.5 The Drift Chamber

The Drift Chamber [50] is the primary tracking device of BABAR for obtaining trajectory information and the best possible momentum resolution for charged tracks. The trajectory of a charged particle in a uniform magnetic field is a helix. The radius of curvature and pitch angle of this helix are related to the magnetic field strength and the transverse momentum of the particle. The DCH is placed in uniform...
magnetic fields parallel to the beam direction to allow measurement of the transverse momentum. The DCH is also used for particle identification by measuring track ionization loss as a function of position ($dE/dx$), particularly for tracks with momenta less than 700 MeV/c. The total charge collected by a sense wire for a given track is proportional to the $dE/dx$ for that track. The particle velocity ($\beta\gamma$) is then calculated using the $dE/dx$ information. The momentum and velocity are used to determine the mass of the particle.

### 3.5.1 Theory of Operations

The DCH is a gas-filled chamber traversed by a series of wires. A high-energy particle passing through the chamber ionizes the gas along its path of travel. If an electric potential is maintained between the wires, the ionized electrons are accelerated toward the positively charged (sense) wires and away from the negatively charged (field) wires. A “hit” is defined when an avalanche of second ionization caused by the accelerating electrons amplifies the original signal and produces a pulse in the current on the sense wire.

The ingredients of the gas mixture has to be chosen carefully to maintain a balance between ionization energy and radiation length. A low ionization energy is desirable in order to generate large number of electrons with a minimum reduction in the energy of the particle passing through the gas. Meanwhile a long radiation length is crucial in minimizing the reduction in the particle energy when it passes through the chamber.

Another consideration to be taken into account is the potential damage of the chamber due to accumulated charge. In addition, excited atoms within the DCH emit photons that have sufficient energy to eject electrons from the field wires. New
avalanches started by these electrons will disrupt the operation of the chamber. Therefore a quenching gas capable of absorbing and thermalizing a wide range of photon energies must be added in the gas mixture.

### 3.5.2 Design

To facilitate track-matching between the SVT and DCH, the DCH is placed close to the SVT outer wall and its own walls are made as thin as possible. Figure 3.9 shows a schematic of the Drift Chamber. The chamber is 2.8 m long and contains 40 cylindrical layers of 1.2 cm by 1.9 cm hexagonal cells, each consisting of six gold-coated aluminum field wires at the corners and one gold-coated tungsten-rhenium sense wire at the center. The cells are filled with a mixture of 80% helium and 20% isobutane (as quenching gas). There are a total of 7104 sense wires and 21664 field wires. The layers are grouped by four into ten superlayers, with the wires in each superlayer oriented as either axial (directly parallel to the $z$-axis) or stereo (at a small angle in phi with respect to the $z$-axis, in order to obtain longitudinal position info). Six of the ten superlayers are stereo, and the other four are axial. Figure 3.10 shows the layout of the four innermost layers. The DCH is asymmetric in $z$ about the IP, to accommodate the forward boost of the CM of physics events.

The field wires are 120 $\mu$m in diameter and the sense wires are 20 $\mu$m in diameter. The field wires are grounded, while the sense wire is held at a nominal operating voltage at 1930 V. The grounded wires produce a uniform electric field in the cell with evenly distributed isochrones (contours) of equal drift time, as shown in Figure 3.11. The gain relative to the charge of the primary ionization is about $5 \times 10^4$. The chamber has a typical position resolution of 140 $\mu$m.
3.5.3 Performance

The DCH has demonstrated excellent performance throughout its lifetime. The transverse momentum resolution is represented by

$$\sigma_{p_T} / p_T = (0.13 \pm 0.01)\% \times p_T + (0.45 \pm 0.03)\%,$$

where $p_T$ is in units of GeV/c. The $dE/dx$ measurement resolution has been measured to be around 7%. Tracking efficiency is a function of transverse momentum and polar angle. Figure 3.12 shows the measured efficiencies for operating voltages at 1900 V and 1960 V. The efficiency is measured based on multi-hadron events as the fraction of tracks detected in the SVT that are also detected in the DCH. The efficiencies for tracks with $p_T > 200$ MeV/c and polar angle $\theta > 500$ mrad at 1960 V and 1900 V are well above the 95% and 90% level respectively.

The estimated error in the measurement of the difference between the decay vertices of the two neutral $B$ mesons along the $z$ axis is shown in Figure 3.13. The rms
Figure 3.10: A schematic of the arrangements of the wires in the hexagonal cells in the four innermost layers of the DCH.

width of 190 $\mu$m is dominated by the reconstruction of the partially reconstructed $B^0$ (170 $\mu$m) while the r.m.s. of the resolution for the fully reconstructed $B^0$ is 70 $\mu$m.
Figure 3.11: Isochrones in a typical DCH cell at 1.5 T of magnetic field.

3.6 The Detector of Internally Reflected Cherenkov Light

The DIRC [51] is the main particle identification device of BABAR, which uses the Cherenkov angle of a charged track to determine the track velocity. Combined with momentum measurements from the SVT and DCH, the velocity is used to determine the mass and thus the ID of the particle. Additionally, it is particularly crucial for the DIRC to meet BABAR’s stringent requirements for $\pi$-$K$ discrimination (2.5 $\sigma$ or more) over a large momentum range (700 MeV/$c$ - 4.2 GeV/$c$). Good particle ID is essential for measurements that depend on efficient $\pi$-$K$ separation or the need to identify the $B$ meson flavor.
Figure 3.12: The track reconstruction efficiency in the DCH at operating voltages of 1900 V and 1960 V as a function of transverse momentum (top) and polar angle (bottom). The measurement at the DCH voltage of 1900 V (open circle) and 1960 V (solid circle) are shown.

3.6.1 Theory of Operations

The DIRC is a ring-imaging Cherenkov detector which uses total internal reflection to transfer Cherenkov radiation generated by charged particles traversing the detector to photo-multiplier tubes (PMTs). A particle moving at a velocity greater than the speed of light in certain medium emits Cherenkov radiation at an angle $\theta_C$ relative
Figure 3.13: Estimated error in the difference $\Delta z$ between the $B^0$ meson decay vertices for a sample of events in which one $B^0$ is fully reconstructed.

to the particle’s direction. The angle is related to the particle’s velocity ($\beta$) by

$$\cos \theta_C = \frac{1}{n\beta},$$

(3.2)

where $n$ is the index of refraction of the medium. Through total internal reflections, the Cherenkov light is carried to a large water tank and is focused onto an array of PMTs mounted at the outside of the tank. The Cherenkov angle is preserved by total internal reflections and can be reconstructed from the PMT signals, timing information, and track momentum vectors obtained by matching the signal with tracks from the DCH and SVT.

3.6.2 Design

Figure 3.14 illustrates the principles of light production, transportation and imaging in the DIRC. The DIRC uses thin, long rectangular bars made of synthetic fused
silica (quartz) with a refractive index of $n = 1.47$. It consists of 144 bars 17 mm thick, 35 mm wide and 4.9 m long running along the $z$ direction. Each set of 12 bars are housed in a bar box filled with nitrogen to prevent moisture from condensing on the bars. The silica serves as the Cherenkov radiator and as a waveguide. A mirror with $\approx 92\%$ reflectivity is placed on one end of the bar. A 9 mm quartz window at the other end separates the bar from a water tank filled with 6000 liters of purified water, called the Standoff Box (SOB). The water has a similar refractive index ($n = 1.35$) to the quartz to minimize refraction at the silica-water boundary. The rear surface of the SOB is instrumented with 12 sectors of 896 PMTs each. Each PMT has a diameter of 29 mm. The PMTs collect the photons, convert them to electrons and amplify the signal.

Figure 3.14: A schematic of the DIRC fused silica radiator bar and imaging region.
3.6.3 Performance

The DIRC has performed well throughout BABAR’s operational lifetime. The single-photon Cherenkov angle resolution is about 9.6 mrad, while the resolution for a track is \( \sigma_\gamma / \sqrt{N_\gamma} \), where \( \sigma_\gamma \) is the resolution of a single photon and \( N_\gamma \) the number of detected photons (about 30 for a normal incidence track). For a dimuon event, the Cherenkov angle resolution is 2.5 mrad, resulting in a \( \pi-K \) separation of 4.2 \( \sigma \) at 3.3 GeV/c of momentum (Figure 3.15).

![Figure 3.15: DIRC \( \pi-K \) separation versus track momentum measured in \( D^0 \to K^-\pi^+ \) decays in units of standard deviations.](image)

A control sample of \( D^0 \to K^-\pi^+ \) was used to measure the efficiency for correctly identifying a charged kaon and the probability of wrongly identifying a \( \pi \) as a \( K \). Kaon selection efficiency and pion mis-identification probabilities as a function of momentum are shown in Figure 3.16. The \( K \) efficiency is above 90% for the full
momentum range from 0.6 to 3.4 GeV/c. Pion mis-identification is roughly constant around 2% for momenta less than 2.4 GeV/c and increases to a little over 10% for tracks with momenta $\approx 3.2$ GeV/c.

![Graph showing kaon efficiency and pion mis-identification as a function of track momentum.]

Figure 3.16: The DIRC kaon identification efficiency versus track momentum is shown on the top plot. The bottom plot shows the probability of a pion being mis-identified as a kaon as a function of track momentum.

### 3.7 The Electromagnetic Calorimeter

The main purpose of BABAR’s electromagnetic calorimeter is to determine the position, energy and identity of electrons and photons with excellent efficiency, energy and angular resolution over the energy range of 20 MeV to 9 GeV. The EMC is designed to measure the energy in electromagnetic showers. This capability allows reconstruction of $\pi^0$ and $\eta^0$ mesons that decay to two photons, as well as identification of high-energy photons from rare radiative $B$ decays. Electron ID is necessary for
$J/\psi$ reconstruction, flavor-tagging of the non-signal $B$ in semileptonic decays, and reconstruction of semileptonic and rare $B$ decays.

### 3.7.1 Theory of Operations

In an electromagnetic calorimeter, particles are distinguished based on how much they are absorbed and their different shower shapes. Absorption of the particle generates showers of new particles, and the energy and momentum of the original particle is distributed among the shower particles. Electrons and photons are fully absorbed and have short and narrow showers. Hadrons, on the other hand, are only partially absorbed and generate wide and scattered showers. Muons are not absorbed and do not shower, despite being electromagnetic particles.

### 3.7.2 Design

The EMC is composed of 6580 Thallium-doped Cesium iodide (CsI(Tl)) scintillating crystals, separated into a cylindrical barrel and a conical forward end-cap. Figure 3.17 shows a schematic diagram of a typical crystal including the readout electronics. The barrel portion has 5760 crystals arranged in 48 polar angle rows with 120 crystals each. The forward end-cap contains 820 crystals grouped into 20 modules of 41 crystals each. The EMC covers a CM solid angle of $-0.916 \leq \cos \theta \leq 0.895$, with the backward-forward asymmetry reflecting the boost of the collision in the lab frame. Figure 3.18 is the schematic of the EMC in longitudinal view.

The crystals have a trapezoidal shape with dimensions of $47 \times 47 \text{ mm}^2$ at the front face and $60 \times 60 \text{ mm}^2$ in the forward and the end-cap regions respectively. They have radiation lengths of 1.85 cm. The crystals serve as radiators for the traversing electrons and photons. They scintillate under the influence of electromagnetic showers,
and the light is then passed to the outer face of the crystal through total internal reflection, where it is read out by silicon PIN diodes. As the electromagnetic showers spread through several crystals, a reconstruction algorithm is used to associate the activated crystals into clusters and either to identify them as photon candidates or to match individual maxima of the deposited energy to the extrapolated tracks from the DCH-SVT tracker. Additional PID is obtained from the spatial shape of the shower.

Figure 3.17: Schematic of a typical CsI(Tl) crystal (not to scale).
3.7.3 Performance

The energy resolution at low energy is calibrated with a radioactive source. At high energy the resolution is the r.m.s. error in the energy measurement divided by the energy \((\sigma_E/E)\) and is measured from Bhabha scattering events. It is parameterized by [52]:

\[
\frac{\sigma_E}{E} = \frac{(2.32 \pm 0.30)\%}{\sqrt{E \text{ (GeV)}}} \oplus (1.85 \pm 0.12)\% ,
\]

where \(E\) is the photon energy in GeV and \(\oplus\) represents a sum in quadrature. Figure 3.19 shows a plot of the energy resolution of the EMC for various processes. At lower energies it is dominated by fluctuations in photon statistics and beam generated backgrounds; whereas at higher energies (> 1 GeV) the non-uniformity in light collection from leakage or absorption in the material in front of or between the crystals dominates in the energy resolution.
The angular resolution is determined from analyses of \( \pi^0 \) and \( \eta^0 \) decays to two photons of roughly equal energy. The angular resolution \( (\sigma_\theta) \) can be parameterized by [52]:

\[
\sigma_\theta = \sigma_\phi = \frac{(3.87 \pm 0.07)\text{mrad}}{\sqrt{E(\text{GeV})}} \oplus (0.00 \pm 0.04)\text{mrad}.
\] (3.4)

The angular resolution is approximately 12 mrad at low energies and 3 mrad at high energies. Figure 3.20 shows the angular resolution of the EMC for photon energies between 0 and 3 GeV.

The reconstructed \( \pi^0 \) mass has a width of 6.9 MeV/\( c^2 \) and is shown in Figure 3.21. Energy resolution dominates the mass resolution at lower energies (\(< 2 \text{ GeV}\)), while the angular resolution dominates at higher energies.
Figure 3.20: Angular resolution in the EMC as a function of photon energy.

3.8 The Instrumented Flux Return

The Instrumented Flux Return (IFR) is designed for detection of muons and long-lived neutral hadrons, primarily $K^0_L$ and neutrons. It also serves as the flux return for the solenoid magnet. The principal requirements for IFR are large solid angle coverage, good efficiency, and high background rejection for muons down to momenta under 1 GeV/$c$. For neutral hadrons, good angular resolution and high efficiency are most important. These capabilities allows identification of kaons with high efficiency and good purity and detection of neutral hadrons over a wide range of momenta and angles. Muons are important for flavor tagging of neutral $B$ mesons via semileptonic decays, for the reconstruction of $J/\psi$'s, and for the study of semileptonic decays and rare decays involving leptons of $B$ and $D$ mesons and $\tau$ leptons. $K^0_L$ detection allows the study of exclusive $B$ decays, in particular $CP$ eigenstates.
Figure 3.21: Invariant mass of two photons in $B\overline{B}$ events. The energies of the photons and the $\pi^0$ are required to be between 30 MeV and 300 MeV. The solid line is the fit to data.

The IFR is divided into a hexagonal barrel, which covers 50% of the solid-angle in the CM frame, and forward and backward end-caps (Figure 3.22). Originally, it was designed with only Resistive Plate Chambers (RPCs) [53], sandwiched between layers of steel plates in the barrel and both the two end-caps. Figure 3.23 shows the cross-section of a RPC. There were 19 and 18 layers of RPCs in the barrel and each end-cap respectively. The steel serves as a flux return for the solenoidal magnet as well as a hadron absorber, limiting pion contamination in the muon ID.

3.8.1 Theory of Operations

The RPCs in $BABAR$ detect high-energy particles through gas-avalanche formation in a high electric field. They operate in the so-called streamer mode. Charged particles passing through a gas ionize the gas along their paths. An applied electric
Figure 3.22: Drawings of the barrel sectors and forward and backward end doors in the IFR.

Figure 3.23: Cross-section of a BABAR Resistive Plate Chamber (RPC).
field causes the ionized electrons to accelerate toward the anode creating additional
ionized electrons and photons through a variety of electromagnetic processes. The
“avalanche” of electrons and photons grows into a streamer, which is a mild, con-
trolled form of electrical discharge in the gas. The streamer charge is then read out in
both the \(z\) and \(\phi\) directions by aluminum strips located outside of the RPCs which are
capacitively coupled to the chambers. One potentially serious problem is that high-
energy photons produced by a streamer of ionized electrons can photo-ionize other
electrons and create secondary avalanches or streamers, which will lead to break-
down of a RPC. Therefore, a quenching gas of freon and isobutane is used, with the
isobutane absorbing high-energy photons and the freon absorbing excess electrons.

The RPCs consist of 2 mm-thin bakelite sheets kept 2 mm apart by an array of
spacers located every 10 cm. The space in between is filled with a non-flammable gas
mixture of 56.7% argon, 38.8% freon, and 4.5% isobutane, while the sheets are held
at a potential of 8000 V. The inside surface of the bakelite is covered with a linseed-oil
coating to ensure the uniformity of the electric field in order to prevent discharges
in the gas and large dark currents. The RPCs performed well in the beginning. It
had over 90% efficiency as expected geometrically from inactive space in the detector,
resulting in a muon detection efficiency of 90% for a pion mis-identification rate of 6
- 8% in the momentum range of \(1.5 < p < 3.0 \text{ GeV/c}\).

3.8.2 Limited Streamer Tubes

In 1999, shortly after the beginning of \textit{BaBar} data-taking, the performance of the
RPCs started to deteriorate rapidly. Many chambers began drawing dark currents and
developing large areas of low efficiency. The overall efficiency of the RPCs started to
drop and the number of non-functional chambers, defined as chambers with efficiencies less than 10%, rose dramatically. Figure 3.24 shows the average RPC efficiency since 1999. It is obvious that the deterioration of efficiency, and thus muon ID, had become a serious problem. After investigations, it was traced to insufficient curing and R&D of the linseed-oil coating and to the high temperature at which the RPCs were operated. Uncured oil droplets would form columns under the action of the strong electric field and the high temperature, up to $37^\circ C$, bridging the bakelite gap and resulting in large currents and dead space. Various remediation measures were attempted, but none solved the problem. Extrapolating the efficiency trend showed a clear path towards losing muon ID capability at BABAR within a couple of years of operations. Therefore replacement of RPCs, or at least an upgrade of the IFR detector with new and more reliable RPCs, was deemed essential by the collaboration.

The collaboration decided to upgrade the IFR with Limited Streamer Tubes (LST) in the barrel region. In Fall 2004, the RPCs in the top and bottom sextant were removed and replaced by 12 layers of LSTs and 6 layers of brass to improve hadron absorption. The installation of LSTs in the remaining four sextants was completed on November 13, 2006. (The forward end-cap was retrofitted with new and improved RPCs in 2002. The new RPCs were screened much more stringently with Quality Control (QC) tests and had a much thinner linseed-oil coating. The backward end-cap was not retrofitted.)

**Design**

The LSTs consist of a PVC comb of eight 15 mm × 17 mm cells about 3.5 m in length, encased in a PVC sleeve, with a 100 µm gold-plated beryllium-copper wire running down the center of each cell. Figure 3.25 shows photos of a LST. The cells
in the comb are covered with graphite, which is grounded, while the wires are held at typically 5500 V and held in place by wire holders located every 50 cm. The gas mixture consists of 3.5% argon, 8% isobutane, and 88.5% carbon dioxide. Like the RPCs and as their name implies, the LSTs are operated in streamer mode. The signal is read off directly from the wires through AC-coupled electronics (granularity of two wires per channel in the \( \phi \) direction) and from strips running perpendicular to the tubes and capacitively coupled to the wires (35 mm pitch in the \( z \) direction).

The LSTs were constructed at PolHiTech, an Italian company. The construction and QC procedures were conducted under the supervision of BABAR personnel. During construction, the mechanical quality of the graphite surface was inspected and the resistivity tested. The chambers were strung with wires tested for thickness and
tested for gas leaks after sealing. The tubes were then conditioned under progressively higher applied voltages to burn off any dirt accumulated during construction. Only tubes that could hold the operational voltage without drawing excessive currents were accepted.

Figure 3.25: Photos of a BABAR limited streamer tube.
Quality Control

After construction, the LSTs were shipped to the Ohio State University and Princeton University to undergo another round of rigorous QC tests and high-voltage conditioning. An important test was the “singles rate” test. As the streamer signals are effectively digital, given a constant incident flux of particles, the chamber should show a counting-rate plateau over a range of applied voltage where the charge of every streamer is above the read-out threshold. The plateau provides operational tolerance of the applied HV, allowing operations of the LSTs at the middle of the plateau to safeguard against fluctuations in efficiency due to changes in the gas gain from pressure or voltage fluctuations. A good LST should have a plateau wider than 300 V in all of its cells. A short plateau is an indication of poor aging behavior. Defects in the surface of the graphite or dirt accumulation on the wire can result in large discharges in the tube that raise the singles rate and spoil the plateau. Examples of a good and bad plateau are shown in Figure 3.26. Counting rates were recorded every 100 V from 5000 V to 6000 V. Note that the plateau eventually fails at above 5900 V, due to multiple streamers formed from electrons photoelectrically ejected from the graphite by UV photons radiated by the original streamer. At high voltages, enough UV photons are produced to overwhelm any signal dead-time imposed by the electronics, thus raising the singles rate.

Another powerful QC procedure was the scan test, in which the LSTs were scanned with a localized, focused radioactive source, subjecting small regions of the tube to intense radiation rates. Although the incident flux is then much higher than what the tube would experience in the experiment, the stress reveals weak points in the tube, where the source initiates a self-sustaining discharge of high current that continues
even when the source is removed while the high-voltage is applied. This happens when a conductive channel is formed in the gas around a mechanical defect. Only tubes that do not exhibit this behavior are accepted for installation. A 7 μC Cs$^{137}$ source was used in OSU for this purpose. A robot arm, controlled by user via a networking software, carried the source and scanned each wire of a LST. A tube that displayed a self-sustaining discharge in any of its wire would be put under HV conditioning for a few days before a second scan. The tube would not be used for installation in the IFR if it failed the scan test twice. Figure 3.27 shows a bad tube with a self-sustaining discharge in one of its wires.

After all QC tests, the tubes were held under high voltage for a month to ensure no premature aging behavior occurred. Thereafter, they were assembled into modules of two or three tubes at The Ohio State University and Princeton University and then shipped to SLAC for yet another round of QC tests and HV conditioning before being installed in the IFR.

Performance

The project involved the manufacture of 1500 LSTs, with more than 1200 installed in the detector. It also necessitated the design and fabrication of custom read-out electronics (done by INFN Ferrara in Italy), HV power supplies (The Ohio State University), and gas system (SLAC). The project was completed successfully and ahead of schedule. The LSTs have performed extremely well since installation in all sextants, with failure rates below 0.5% for both the tubes and z-strips. The efficiencies of all layers are around 92%, which are consistent with expectation (under 100% due to geometrical limitations). Regular tests of singles rates with cosmic rays have verified continuing excellent behavior with long singles rate plateaus (Figure
Figure 3.26: Singles rate plateau versus applied voltage for two LSTs. The LST on the top showed a very good plateau; while the one on the bottom had bad (no) plateau and was due further HV conditioning before being tested again.

3.28). Figure 3.29 and Figure 3.30 show the efficiency maps of a sample layer and the improved muon ID of the new and fully functional muon system.

3.9 The Trigger and Data Acquisition System

The trigger system [45] for BABAR is designed to collect and store data relevant for $B$ physics with a high and stable efficiency. It is implemented as a two-tier hierarchy, the Level 1 (L1) trigger followed by the Level 3 (L3) trigger. The trigger system
must function efficiently under extreme background situations. It should contribute no more than 1% dead time.

The L1 trigger, a hardware-based trigger system, analyzes data from the front-end electrons (FEEs) of the DCH, EMC and IFR to make the trigger decision. The L1 trigger must accept or reject an event within a time window of 12.9 $\mu$s. The selections are optimized to maintain nearly perfect $B\bar{B}$ efficiency while rejecting most of the beam-induced background events in order to prevent overloading of the downstream event processing system. The L1 is configured to have an output rate of 1 kHz. It is $> 99.9\%$ efficient for $B\bar{B}$ events.

The L3 trigger is a software-based trigger running on a farm of commercial PCs which performs a full event reconstruction and classification. After an event is accepted by the L1 trigger (L1 accept), the L1 output is passed on to L3. The trigger at this level is performed in three stages. First, events are classified according to
Figure 3.28: Singles rate plateaus from a sample layer of installed LSTs in the IFR.

the trigger information from the Fast Control and Timing System (FCTS). Then the events are applied with \textit{BABAR} event reconstruction algorithms to find quantities of interest and filters to test if these quantities satisfy imposed selection criteria. Lastly, L3 output information, a set of classifications for the events, are generated. The related data is stored on tapes in collections which will be retrieved later for high-level analysis by individual users. The L3 trigger maintains the $B\bar{B}$ selection efficiency at more than 99% while reducing the data rate to about 200 Hz.
Figure 3.29: Color-coded efficiency maps for six sample layers in the barrel. The scale goes from 0% (red) to 100% (green). All other layers of LSTs display similarly exceptional efficiencies.
Figure 3.30: Muon ID performance plot of the LSTs in the barrel for muons with momenta between 2.0 and 4.0 GeV/c, represented by pion rejection versus muon efficiency. Note that the LST performance bested the initial performance of the RPCs.
CHAPTER 4

ANALYSIS OF $B^- \rightarrow D^0 K^{*-}$ DECAYS USING THE GLW METHOD

4.1 Overview

The most important step of this analysis is the separation of signal events from the backgrounds to achieve a high signal-to-background ratio. This is essential as the branching fractions of the decay channels in this analysis are of the order of $10^{-6}$ or $10^{-7}$ (Table 4.1). The main technique is to exploit the kinematic and topological information of an event. The dominant background in this analysis is continuum $q\bar{q}$ events ($c\bar{c}$, $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$). The event shape of continuum background is distinctively different from that of a $B$ decay. In the $\Upsilon(4S)$ rest frame, the two $B$ mesons are produced nearly at rest without a preferred direction because the $\Upsilon(4S)$ is just above the $B\bar{B}$ invariant mass. Therefore, $B$ decays are roughly isotropic and spherical in nature. In contrast, continuum events are more “jet-like”, as shown in Figure 4.1. The two jets are roughly collinear in the CM frame, conserving momentum in the two-body process. Thus, event-shape variables which display different distributions for signal and continuum events can be used to suppress this background. In addition, kinematic variables which describe the reconstructed $B$ meson kinematically can further separate signal from background events.
This analysis proceeds in several steps. First, events in the BABAR dataset which are reconstructed as the decay chains that we are interested in are selected. Then rejection of background and poorly reconstructed signal events is accomplished through a series of selections on topological and kinematic variables. Only a small sample of candidate events remains after the selection process. Finally, an extended maximum likelihood fit to the distributions of discriminating variables of the candidate decays is performed to extract signal yields and subsequently the \(CP\) observables.

Figure 4.1: A graphical representation of a \(B\) decay event (left) and a continuum background event (right). A \(B\) decay event has a more spherical structure while a continuum event is jet-like. The “signal \(B\)” in the \(B\) decay event represents the \(B\) candidate of interest in the event.
<table>
<thead>
<tr>
<th>$D^0$ Mode</th>
<th>Total Branching Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+K^-$</td>
<td>$4.68 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$1.67 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K_S^0\pi^0$</td>
<td>$9.46 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K_S^0\phi$</td>
<td>$1.76 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K_S^0\omega$</td>
<td>$8.23 \times 10^{-7}$</td>
</tr>
<tr>
<td>$K^-\pi^+$</td>
<td>$4.65 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

Branching fractions used in the calculations:

- $B^- \rightarrow D^0K^-$: $5.3 \times 10^{-4}$
- $K^{*-} \rightarrow [\pi^+\pi^-]K_S^0\pi^-$: $2.3 \times 10^{-1}$
- $\pi^0 \rightarrow \gamma\gamma$: $9.9 \times 10^{-1}$
- $D^0 \rightarrow K_S^0[K^+K^-]_\phi$: $2.1 \times 10^{-3}$
- $D^0 \rightarrow K_S^0[\pi^+\pi^-\pi^0]_\omega$: $9.8 \times 10^{-3}$

Table 4.1: Total branching fractions of the decay modes studied in the GLW analysis. All numbers are provided by the PDG [16].

4.2 Data and Monte Carlo Samples

4.2.1 Data Samples

This analysis uses the Run 1 - 5 dataset collected by BABAR from 1999 to the summer of 2006. The total integrated luminosity at the $\Upsilon(4S)$ is $344.7$ fb$^{-1}$ (Table 4.2), corresponding to 379 million $B\bar{B}$ pairs.

4.2.2 Signal Monte Carlo Samples

Table 4.3 shows the statistics of signal Monte Carlo samples used in this analysis.

4.2.3 Background Monte Carlo Samples

Table 4.4 shows the statistics of the background Monte Carlo samples used in this analysis.
Table 4.2: Luminosities of the Run 1 - 5 data samples used in this analysis. The equivalent luminosity ($\mathcal{L}$) is calculated based on the cross-section of $b\bar{b}$ production (1.1 nb) and according to the formula $\mathcal{L} = N/\sigma$ where $\sigma$ is the cross-section and $N$ is the number of generated events.

<table>
<thead>
<tr>
<th>Data (Run Period)</th>
<th># $B\bar{B}$ Pairs</th>
<th>Luminosity $\mathcal{L}$ (fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>22082143</td>
<td>20.07</td>
</tr>
<tr>
<td>Run 2</td>
<td>65968708</td>
<td>59.97</td>
</tr>
<tr>
<td>Run 3</td>
<td>35088049</td>
<td>31.90</td>
</tr>
<tr>
<td>Run 4</td>
<td>109718240</td>
<td>99.74</td>
</tr>
<tr>
<td>Run 5</td>
<td>146326920</td>
<td>133.02</td>
</tr>
<tr>
<td>Total</td>
<td>379184060</td>
<td>344.70</td>
</tr>
</tbody>
</table>

Table 4.3: Statistics of signal Monte Carlo samples used in this analysis. $K^{*-}$ from $B^- \rightarrow D^0 K^{*-}$ decays with $K^{*-} \rightarrow K_s^0 \pi^-$ and $K_s^0 \rightarrow \pi^+\pi^-$ and various decay modes of $D^0$.

<table>
<thead>
<tr>
<th>Signal MC (Mode)</th>
<th># events</th>
<th>Luminosity (fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^- \rightarrow [K^+K^-]_{D^0} K^{*-}$</td>
<td>704000</td>
<td>1364700</td>
</tr>
<tr>
<td>$B^- \rightarrow [\pi^+\pi^-]_{D^0} K^{*-}$</td>
<td>704000</td>
<td>3832200</td>
</tr>
<tr>
<td>$B^- \rightarrow [K_s^0\pi^0]_{D^0} K^{*-}$</td>
<td>704000</td>
<td>676560</td>
</tr>
<tr>
<td>$B^- \rightarrow [K_s^0\phi]_{D^0} K^{*-}$</td>
<td>704000</td>
<td>3645900</td>
</tr>
<tr>
<td>$B^- \rightarrow [K_s^0\omega]_{D^0} K^{*-}$</td>
<td>704000</td>
<td>777550</td>
</tr>
<tr>
<td>$B^- \rightarrow [K^-\pi^+]_{D^0} K^{*-}$</td>
<td>175000</td>
<td>34190</td>
</tr>
</tbody>
</table>

4.3 Reconstruction and Selection

In this section I describe how an event is reconstructed into one of the decay modes we are interested in and the selection requirements of the related particles. Charged $B$ mesons are reconstructed in the $D^0K^{*-\pm}$ mode, with the $K^{*-\pm}$'s reconstructed in $K_s^0\pi^{\pm}$ and $K_s^0$ in $\pi^+\pi^-$ final states. The $D^0$'s are reconstructed in the final states $K^+K^-$, $\pi^+\pi^-$, $K_s^0\pi^0$, $K_s^0\phi$, $K_s^0\omega$ and $K^-\pi^+$. Different particles are reconstructed/selected
<table>
<thead>
<tr>
<th>Background MC</th>
<th># events or $B\bar{B}$ pairs</th>
<th>Luminosity $\mathcal{L}$ (fb$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y(4S) \rightarrow B^+B^-$</td>
<td>612640000</td>
<td>1113.89</td>
</tr>
<tr>
<td>$Y(4S) \rightarrow B^0\bar{B}^0$</td>
<td>606700000</td>
<td>1103.09</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>765288000</td>
<td>588.68</td>
</tr>
<tr>
<td>$u\bar{u}/d\bar{d}/s\bar{s}$</td>
<td>791406000</td>
<td>378.66</td>
</tr>
<tr>
<td>Off Peak Data</td>
<td>-</td>
<td>35.09</td>
</tr>
</tbody>
</table>

Table 4.4: Statistics of Run 1 - 5 background Monte Carlo samples used in the analysis. The equivalent luminosity is calculated based on the cross-section of the related process at the resonance: 1.1 nb, 1.30 nb and 2.09 nb for $b\bar{b}$, $c\bar{c}$ and $u\bar{u}/d\bar{d}/s\bar{s}$ production respectively.

with different mass constraints and/or kinematic requirements. This section details how $\pi^\pm$ and $K^\pm$ candidates are selected, and how $K^0_s$, $K^{*\pm}$, $\pi^0$, $\phi$, $D^0$ and $B^\pm$ candidates are reconstructed. Note that additional selection cuts will be applied to choose events which are more likely to be a signal event than a background. Those final selection criteria will be discussed in the next section.

4.3.1 Reconstruction of $K^0_s$

$K^0_s$ candidates are reconstructed by combining two charged tracks which have zero net charge ($\pi^\pm$’s) with a vertex fit. The $\pi^+\pi^-$ invariant mass is required to be within $\pm 25$ MeV/$c^2$ of the $K^0_s$ mass (497.6 MeV/$c^2$) provided by the Particle Data Group (PDG) [16].

4.3.2 Reconstruction of $K^{*\pm}$

A $K^0_s$ and a charged track (charged pion) are combined with a vertex fit to form a $K^{*\pm}$ candidate. The $K^{*\pm}$ mass must be within $\pm 125$ MeV/$c^2$ of the PDG mass (891.66 MeV/$c^2$).
4.3.3 Selection of $K^\pm$

The $K^\pm$ in $D^0 \rightarrow K^+ K^-$, $D^0 \rightarrow K^- \pi^+$ and $\phi \rightarrow K^+ K^-$ modes are selected from candidates belonging to the “GoodTracksVeryLoose” list. It is a list of charged tracks which satisfy the following requirements:

- A maximum momentum of 10 GeV/c,
- Distance of the track in the $x$-$y$ plane to the $z$-axis defined by distance of closest approach (DOCA) has a minimum of -10 cm and maximum of 10 cm.
- Distance of the track in the $z$ direction to the origin of the coordinate system defined by DOCA to be less than 1.5 cm.

In addition, the $K^\pm$ candidate must pass a likelihood-based selector called “KLH-NotPion”. The selector uses information of the track from the SVT, DCH and DIRC to determine if the information is more consistent with the candidate coming from a kaon than from a pion. More details can be found in [54].

4.3.4 Reconstruction of $\pi^0$

$\pi^0$ candidates are reconstructed as $\pi^0 \rightarrow \gamma \gamma$. All local maxima of the deposited calorimeter energy not matched with any track are contained in a neutral cluster list. The individual $\gamma$ selected from the list must have a minimum energy of 30 MeV and the sum of energies of the two $\gamma$'s a minimum of 200 MeV. The invariant mass of the two photons is required to be in the range of $115 < m(\gamma \gamma) < 150$ MeV/c$^2$. Finally, the photon pair should have a lateral shower shape (LAT) consistent with the expected energy deposit pattern for an electromagnetic shower, as determined by a cut of LAT < 0.8. The LAT variable is described in detail elsewhere [55].

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4.3.5 Reconstruction of $\phi$

$\phi$ candidates are reconstructed in the $\phi \to K^+K^-$ mode. The charged kaon candidates are selected with the requirements listed earlier and combined with a vertex fit. The mass of a $\phi$ candidate must be within $\pm 30$ MeV/c$^2$ of its PDG mass (1019.46 MeV/c$^2$).

4.3.6 Reconstruction of $\omega$

$\omega$ candidates are reconstructed in the $\omega \to \pi^+\pi^-\pi^0$ mode. The two charged pions are selected from the “GoodTracksVeryLoose” list while the neutral pion has to meet the requirements listed earlier. The $\omega$’s are required to be within $\pm 50$ MeV/c$^2$ of the PDG mass (788.65 MeV/c$^2$).

4.3.7 Reconstruction of $D^0$

The $D^0$ candidates are reconstructed in the final states mentioned in the beginning of this section. In the $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+\pi^-$ modes, the $\pi^\pm$’s are selected from the “GoodTracksVeryLoose” list. Other daughter particles are reconstructed/selected according to the requirements already mentioned. In addition, the mass of a $D^0$ candidate must be within $\pm 70$ MeV/c$^2$ of its PDG mass (1864.5 MeV/c$^2$). The mass constraint is applied after the $D^0$ is reconstructed from its daughter particles and before it is combined with a charged $K^*$ to make a $B^\pm$. Note that an additional constraint is applied in $D^0 \to K^0_S\pi^0$. This constraint forces the $D^0$ to originate from the beamspot in the transverse direction. The size of the beamspot in $y$ is increased by 30 microns, which corresponds to the average transverse flight length of the $B$ [56]. This extra constraint is used due to the fact that $D^0 \to K^0_S\pi^0$ is the only decay
we study where there is not enough geometrical information to reconstruct the $D^0$ decay point, as neither the production vertex of the $K_s^0$ nor the $\pi^0$ is well-known.

### 4.3.8 Reconstruction of $B^\pm$

A charged $B$ meson is reconstructed by combining a $D^0$ candidate and a $K^{*\pm}$ candidate with a vertex fit.

### 4.4 Discriminating Variables

In this section I describe the discriminating variables used to separate signal from background events. Both topological and kinematical variables are considered. Topological variables describe the spatial structure of the events and furnish separation between $B\bar{B}$ events and continuum backgrounds. Kinematic variables, on the other hand, discriminate signal from non-continuum background, while also assist in additional continuum rejection.

#### 4.4.1 Topological Variables

Here we introduce three variables for the fight against continuum backgrounds. More variables will be introduced in the next chapter when we discuss the use of a Neural Network.

$$\cos \theta_{\text{Helicity}}(K^*-)$$

The $K^*$ helicity angle is the angle between the direction of the $K^*$ measured in the parent $B$ meson rest frame and the momentum vector of the daughter pion measured in the $K^*$ rest frame. The $K^*$ is a spin-1 particle, therefore the angular distribution is a function of the helicity angle. For signal events, $dN/d(\cos \theta_{\text{Helicity}})$ follows a $\cos^2$ distribution. It is flat for background events.
\( \cos \theta_{\text{Helicity}}(\omega) \) and \( \cos \theta_{\text{Dalitz}}(\omega) \)

The \( \omega \) normal helicity angle is the angle between the normal to the plane where the three \( \pi \)'s decay from the mother \( \omega \) in the \( \omega \) rest frame and the direction of the \( \omega \) in the mother \( D^0 \) rest frame. Since it is a spin-0 particle (\( D^0 \)) decaying into a spin-1 particle (\( \omega \)), the \( \omega \) normal helicity angle has a \( \cos^2 \) distribution for signal events; it is flat for background events.

The \( \omega \) Dalitz angle is the angle between the direction of one of the three daughter \( \pi \)'s in the \( \omega \) rest frame and the direction of one of the other two \( \pi \)'s in the rest frame of those two \( \pi \)'s. For signal events, it follows a \( \sin^2 \) distribution (spin-1 particle decays into three spin-0 particles). It is flat for background events.

### 4.4.2 Kinematic Variables

The signal \( B \) candidates are characterized by two kinematic variables: \( m_{ES} \), the beam-energy-substituted mass, and \( \Delta E \), the energy difference. In the CM frame, by four-momentum conservation, each \( B \) meson has the same energy as the beam \((E_B^* = E_{\text{beam}}^* \text{, the asterisk denotes a variable in the CM frame})\). This property will be exploited in the definitions of the kinematic variables. \( m_{ES} \) is especially important as the maximum-likelihood fit for signal extraction will be applied on this very distribution of signal candidates.

**Beam-Energy-Substituted Mass**

\( m_{ES} \) is defined as:

\[
m_{ES} = \sqrt{E_{\text{beam}}^* - p_B^*} \quad \text{(CM frame)}
\]

\[
= \sqrt{(\frac{s}{2} + p_0 \cdot p_B)^2 / E_0^2 - p_B^2} \quad \text{(Lab frame),}
\]

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where the subscripts 0 and $B$ refer to the $e^+e^-$ system and $B$-meson candidate respectively. $\sqrt{s}$ is the $e^+e^-$ center-of-mass energy ($= 2E^*_{\text{beam}}$) and $(E_0, p_0)$ is the four-momentum of the $e^+e^-$ system in the lab frame. In practice, $m_{ES}$ is computed in the lab frame. This is because as the $B$ momentum is measured in the lab frame, transforming it to the CM frame would require knowledge of the masses of the $B$ daughters, which would in turn require PID assumptions. The $m_{ES}$ distribution is described by a Gaussian centered at 5.28 GeV/$c^2$ (near the $B$ mass) for signal $B$ events, and an Argus function for continuum backgrounds. An Argus function is defined as [57, 58]:

$$A(m_{ES}, \xi, E^*_{\text{max}}) = A_0 m_{m_{ES}} \sqrt{1 - \frac{m_{ES}^2}{E^2_{\text{max}}}} \cdot \exp(-\xi(1 - \frac{m_{ES}^2}{E^2_{\text{max}}})),$$

where $A_0$ is the normalization constant. $\xi$ is called the shape parameter and $E^*_{\text{max}}$ defines the machine’s threshold and equals to $\sqrt{s}/2$. $E^*_{\text{max}}$ is fixed at the beam energy, 5.2910 GeV/$c^2$, which corresponds to $p^*_B = 0$.

**Energy Difference**

The second kinematic variable, the energy difference, is defined as:

$$\Delta E = E^*_B - \frac{\sqrt{s}}{2} = E^*_B - E^*_{\text{beam}}.$$  

The $\Delta E$ distribution is likewise described by a Gaussian (centered at 0) for signal events and a linear polynomial for continuum events.
4.4.3 Other Variables

$D^0$ Invariant Mass

A mass cut will be put on the $D^0$ invariant mass, $|m(D^0) - M(D^0)_{PDG}|$. The resolutions of the signal Monte Carlo invariant mass distributions are listed in the next section in Table 4.7.

$K^*$ Invariant Mass

The full width of $K^*$ mass from signal Monte Carlo is $50.8 \text{ MeV}/c^2$ and a cut will be put on $|m(K^*) - M(K^*)_{PDG}|$.

$K^0_S$ Invariant Mass

The $K^0_S$ candidates which come from $K^{*+}$’s have a mass resolution of $2.4 \pm 0.1 \text{ MeV}/c^2$ measured from signal Monte Carlo (with a double Gaussian fit). A mass cut $|m(K^0_S) - M(K^0_S)_{PDG}|$ will be imposed. We also have the same mass cut for $K^0_S$’s which are daughters of $D^0$ in the three $CP$- modes. These $K^0_S$’s have the same mass resolution as the ones from $K^*$.

$\phi$ and $\omega$ Invariant Mass

The mass resolutions of $\phi$ and $\omega$ are fitted with a convoluted Gaussian and Breit-Wigner (Voigtian) and a double Gaussian, respectively. The mass resolution of $\phi$ (the Breit-Wigner width) is $4.66 \pm 0.17 \text{ MeV}/c^2$, which includes the $\phi$’s intrinsic width. The mean and sigma of the Gaussian are $0.37 \pm 0.01 \text{ MeV}/c^2$ and $0.98 \pm 0.09 \text{ MeV}/c^2$ respectively. The core sigma of the $\omega$ mass resolution is $8.44 \pm 0.13 \text{ MeV}/c^2$, also including its intrinsic width. The tail sigma and the fraction of the core Gaussian in
the fit are 46.3 ± 1.3 and (83.8 ± 9.2)% respectively. A cut on \(|m(\phi) - M(\phi)_{PDG}|\) and \(|m(\omega) - M(\omega)_{PDG}|\) will be applied.

\section*{K^0_s Decay Length}

The distance of flight before decay (DOF) is one of the variables used to select \(K^0_s\) candidates. The variable is called “signed, 2D, DOF pull” \cite{59} and is defined as:

\[
\text{signed, 2D, DOF pull} = \frac{\bar{p} \cdot \bar{v}}{|\bar{p} \cdot \bar{v}|} \cdot \frac{||\bar{v}|^{2D}|}{\sigma(\bar{v})^{2D}},
\]

where \(\bar{p}\) is the momentum vector of the \(K^0_s\) candidate and \(\bar{v}\) is the distance vector from the \(K^*\) decay to the \(K^0_s\) decay. \(||\bar{v}|^{2D}|\) is the projection of this distance vector on the \(x-y\) plane perpendicular to the beam. The “signed” part in the equation refers to \(\frac{\bar{p} \cdot \bar{v}}{|\bar{p} \cdot \bar{v}|}\).

\section*{PID Selectors}

Standard BABAR PID selectors are used for selecting \(K\)’s and \(\pi\)’s from \(D^0\) in \(CP^+\) mode, and \(\pi\)’s from \(\omega\)’s in \(CP^-\) mode. They are likelihood-based selectors with certain kinematic requirements, similar to the ones described earlier in Section 4.3. The selectors for charged kaons and pions are called “KLHVeryLoose” and “LHPionLoose” respectively. More details can be found in \cite{54}.

\section*{4.5 Neural Network}

A Neural Network (NN) is a very powerful technique to discriminate between signal and background events. It makes use of the different topologies exhibited by signals and continuum backgrounds. It is extremely useful for this analysis since majority of the backgrounds in the decay channels studied are of the continuum type. Moreover, since the number of signal events is small due to small branching ratios of
the decay modes, background suppression is most essential and the Neural Network technique is an extremely efficient and powerful tool for this purpose.

A Neural Network is a computer model which attempts to mimic the learning capability and fault-tolerance of the human brain. The brain is composed of a large number of neurons which are interconnected via synapses (there are an average of several hundred thousand of synapses per neuron). When a neuron receives an input, or is activated, it fires an electrochemical signal to other neurons through the synapses, which may in turn fire. A neuron only fires if the total signal received exceeds its own threshold.

An artificial neuron receives inputs either from original data or outputs from other neurons in the NN. Each input comes via a connection that has a weight. The weighted sum of all the inputs is calculated and the neuron’s threshold is subtracted. The number is then passed through a transfer function to produce the output of the neuron. In biological systems, the transfer function is a step function, which gives an output of 1 if the input is greater than or equal to 0 and 0 if the input is less than 0. We will not exactly use a step function in the NNs in this analysis, but the above provides a brief description of an artificial NN.

4.5.1 Multi-Layer Perception

The NN model we use is called the Multi-Layer Perception (MLP). The MLP network is a simple feed-forward network which has an input layer, one or more hidden layer(s) and an output layer. All connections within the network are unidirectional. The input layer serves to introduce the values of the input variables. The
neurons in the hidden and output layers are each connected to all of the neurons in
the preceding layer.

The software used for creating and training NNs for this analysis is called *MLPFit*
[60]. In *MLPFit*, when the network is activated, the input neurons forward the
inputs they receive to the hidden layer. The neuron $j$ of the hidden or output layer
computes a linear combination of the outputs from the neurons of the previous layer
$y_i$. Referring to Figure 4.2 [60], the output of a neuron in the hidden layer is computed
as:

$$u_j = A(w_{0j} + \sum_i w_{ij}x_i),$$

(4.7)

where is $w_{0j}$ is a bias and $w_{ij}$ are the weights. $A$ is the transfer function and is a
Sigmoid function in our NNs:

$$A(x) = \frac{1}{1 + e^{-x}}.$$  

(4.8)

The output in the output layer neuron is calculated as:

$$y_k = w_{0k} + \sum_j w_{jk}u_j,$$

(4.9)

where again $w_{0k}$ and $w_{jk}$ are bias and weights respectively.

### 4.5.2 Neural Network Training

The most important step in setting up a successful NN is to find a set of weights
such that the prediction error made by the network is minimized. This process is
called “training” and a training sample of data is needed. In this analysis, this
sample contains 30000 events each from signal and continuum Monte Carlos (20000
events for $K^0_S\pi^0$ and $K^-\pi^+$ samples). A signal event is denoted 1 and continuum 0.
Training is equivalent to fitting the NN model to the training data. Each case is run through the network, and the actual output generated from the NN is compared with the desired output. The differences from all training cases are combined together to give the network error. The weights in the network are then adjusted in order to minimize this error.

In *MLPFit*, the weights are initially set to random numbers between -0.5 and 0.5. The NN outputs $o_p$ are then compared with the desired outputs $t_p$ in the training examples. The error is given by:

$$E = \sum_p \frac{1}{2} (o_p - t_p)^2.$$  

(4.10)

As mentioned above, *MLPFit* changes the values of the weights in the NN to minimize this error using a certain training algorithm. There are several algorithms available in *MLPFit* and the user has to specify which one to use.

Beside the training sample, the user is also required to provide another “validation sample” to *MLPFit* for testing the NN after the weights are adjusted after each
training cycle. We gather 30000 events each from signal and cuds Monte Carlos for this validation sample (20000 events for $K_S^0\pi^0$ and $K^-\pi^+$ samples).

### 4.5.3 Input Topological Variables

Each topological variable described here displays very different distributions between signal and continuum events, and is thus ideal for use as input variables of the NNs.

$R_2$

$R_2$ is called the second order Fox-Wolfram moment [61]. It is a ratio of two Fox-Wolfram moments and is defined as:

$$R_2 = \frac{H_2}{H_0},$$

where

$$H_l = \sum_{i,j} |\vec{p}_i||\vec{p}_j|P_l(\cos \phi_{i,j}),$$

with $i, j$ running over the particles produced in the event, $P_l$ being the $l$-th order Legendre polynomials, and $\phi_{i,j}$ being the angle between the momentum vectors of particles $i$ and $j$. The zeroth moment serves as a normalization in the ratio and equals to 1 by energy and momentum conservation, assuming perfect reconstruction. For two-jet events, the even moments peak at 1 while the odd moments are approximately 0. Meanwhile $R_2$ tends toward 0 for spherical (signal) events. Therefore an upper cut can be put on $R_2$ to suppress continuum events.

**Thrust Angle**

The thrust axis, $\hat{T}$, of a collection of particles is the direction in which their combined longitudinal momentum is maximized in the center-of-mass frame. Thrust,
$T$, is related to this direction [62, 63] by

$$T = \frac{\sum_i |\hat{T} \cdot P_i|}{\sum_i |P_i|}. \quad (4.13)$$

The thrust angle $\cos \theta_T$ is defined as the angle between the thrust axis of the reconstructed $B$ candidate and the thrust axis of particles constituting the rest of the event (particles not belonging to the reconstructed $B$ candidate). The $|\cos \theta_T|$ variable has a nearly flat distribution for $B\bar{B}$ events while it is sharply peaked at 1 for continuum background events. This variable gives a strong discrimination power between signal and background events.

**Monomials**

$L^0$ and $L^2$ are the zeroth- and second-order monomial, respectively. A monomial is a set of momentum-weighted sums of the charged and neutral tracks in the rest of the event after removal of the tracks and clusters making up the $B$ candidate,

$$L^j = \sum_{i=1}^{ROE} p_i^* |\cos \theta_i^*|^j \quad (4.14)$$

where $j = 0, 1$ or 2 and $\cos \theta_i^*$ is the angle of the $i$th track with respect to the thrust of the reconstructed $B$ in the CM frame.

$\cos \theta_{Mom}$

$\theta_{Mom}$ is the angle between the momentum vector of the reconstructed $B$ and the beam axis in the $Y(4S)$ CM frame. The signal distribution follows a $\sin^2 \theta$ form whereas the distribution for $q\bar{q}$ is flat.

$\cos \theta_{Helicity}(D^0)$

The helicity angle of the $D^0$ decay (angle between a $D^0$ daughter momentum vector in the $D^0$ rest frame and the direction of the $D^0$ momentum in the $B$ rest frame) is
flat for two-body decays of true $D^0$ mesons (which are pseudo-scalar particles); while it peaks toward 1 for fake $D^0$ candidates reconstructed from random combinations of tracks and clusters in $q\bar{q}$ background.

4.5.4 NN Settings

One training and one validation sample of each decay modes are prepared from signal and continuum Monte Carlo samples. The samples are then run by MLPFit for testing of different configurations. After performing studies on signal and background efficiencies, the following settings for the NNs are chosen:

- Transfer function: Sigmoid function
- Training method: Hybrid [64]
- Number of hidden layer = 1
- Number of neurons in hidden layer = 8
- Number of epochs (training cycles) = 500

4.5.5 NN Output ($O_{NN}$)

After setting the NN configurations and training the NNs, another set of data is put through the NNs to produce visual NN outputs ($O_{NN}$) for each decay mode. The contents of the dataset are similar to that of the training and validation samples (produced from signal Monte Carlo events and $cuds$ continuum backgrounds). In addition, we run the NNs on off-peak data to check if the NN outputs are consistent with those of the corresponding continuum samples. Table 4.5 lists the size of various samples used in producing the NN outputs. The $O_{NN}$’s are shown in Figures 4.3
(CP+ modes) and 4.4 (CP- and Non-CP modes). The continuum and off-peak data are in very good agreement with each other.

<table>
<thead>
<tr>
<th>Sample Size (# Events)</th>
<th>Signal, cuds MC</th>
<th>Off-Peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \to K^+K^-$</td>
<td>30000</td>
<td>5000</td>
</tr>
<tr>
<td>$D^0 \to \pi^+\pi^-$</td>
<td>30000</td>
<td>20000</td>
</tr>
<tr>
<td>$D^0 \to K_s^0\pi^0$</td>
<td>30000</td>
<td>20000</td>
</tr>
<tr>
<td>$D^0 \to K_s^0\phi$</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>$D^0 \to K_s^0\omega$</td>
<td>30000</td>
<td>20000</td>
</tr>
<tr>
<td>$D^0 \to K^-\pi^+$</td>
<td>20000</td>
<td>20000</td>
</tr>
</tbody>
</table>

Table 4.5: Sizes of the samples used to produce Neural Network outputs.

Figure 4.3: Neural Network outputs ($O_{NN}$) of $D^0 \to K^+K^-$ (left) and $D^0 \to \pi^+\pi^-$ (right) from signal MC, cuds continuum and off-peak data samples. Red solid line represents signal and blue represents continuum. The dots are off-peak data.

4.6 $D^0 \to K_s^0\pi^+\pi^-$ Veto

Besides the aforementioned selection variables and a Neural Network, another selection cut for the $B^- \to (\pi^+\pi^-)D^0(K_s^0\pi^-)_{K-}$ mode is needed. The main source of
background in this channel is $B^– \rightarrow (K^0_S \pi^+ \pi^-)D^0 \pi^-$, which has a 500-times larger branching ratio than the signal $\pi^+ \pi^-$ mode. Therefore the suppression of this background is of great importance. The main weapon is the $D^0 \rightarrow K^0_S \pi^+ \pi^-$ veto cut.

In the veto, we first choose any two of the three pions in the candidate event (excluding the two daughter pions from $K^0_S$) and combine them with the $K^0_S$ to calculate the invariant mass of the three particles. Then a candidate is vetoed if the $K^0_S \pi^+ \pi^-$ invariant mass is within a certain MeV/$c^2$ of the candidate $D^0$ mass. Details of the choice of this cut window is explained in Section 4.7.3.
4.7 Optimization of Selection Cuts

All selection cuts (except $\Delta E$, $|m(D^0) - M(D^0)_{PDG}|$ and $D^0 \rightarrow K_s^0\pi^+\pi^-$ veto) on the discriminating variables are optimized individually by maximizing the significance, which is defined as:

$$\text{significance} = \frac{S}{\sqrt{S + B_B + B_C}}.$$  \hspace{1cm} (4.15)

$S$, $B_B$ and $B_C$ are the number of signal Monte Carlo, $B\bar{B}$ and $q\bar{q}$ background events that pass all the applied selection cuts respectively.

4.7.1 $\Delta E$ Cut

The choice of the width of a $\Delta E$ cut window is based on the width of the signal peak measured in signal Monte Carlo. We have $|\Delta E| < 25$ MeV for all modes except $D^0 \rightarrow K_s^0\pi^0$, for which the window is set at 50 MeV. The worse resolution in $K_s^0\pi^0$ is due to poor $\pi^0$ reconstruction in the detector. Table 4.6 records the resolutions of $\Delta E$ distributions in signal Monte Carlos.

<table>
<thead>
<tr>
<th>$D^0$ Mode</th>
<th>Fit Type</th>
<th>Core $\sigma$ (MeV)</th>
<th>Tail $\sigma$ (MeV)</th>
<th>Fraction (%)</th>
<th>Cut (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+K^-$</td>
<td>Double Gaus.</td>
<td>11.4 ± 0.1</td>
<td>76.2 ± 0.7</td>
<td>60.0 ± 1.7</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>Double Gaus.</td>
<td>11.5 ± 0.1</td>
<td>77.4 ± 0.6</td>
<td>58.2 ± 1.6</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>$K_s^0\pi^0$</td>
<td>Double Gaus.</td>
<td>15.5 ± 0.2</td>
<td>81.1 ± 1.0</td>
<td>50.5 ± 3.6</td>
<td>&lt; 50</td>
</tr>
<tr>
<td>$K^0\phi$</td>
<td>Double Gaus.</td>
<td>11.9 ± 0.1</td>
<td>79.3 ± 0.8</td>
<td>52.6 ± 2.2</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>$K_s^0\omega$</td>
<td>Double Gaus.</td>
<td>12.2 ± 0.3</td>
<td>66.5 ± 3.3</td>
<td>79.4 ± 9.0</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>$K^-\pi^+$</td>
<td>Double Gaus.</td>
<td>11.4 ± 0.1</td>
<td>71.9 ± 1.0</td>
<td>54.0 ± 3.3</td>
<td>&lt; 25</td>
</tr>
</tbody>
</table>

Table 4.6: List of $\Delta E$ cut windows for various modes. The column “Fraction” refers to the fraction of the fit in the core Gaussian. The last column is the $\Delta E$ cut value that will be used in this analysis.
4.7.2 \( |m(D^0) - M(D^0)_{PDG}| \) Cut

The choice of this cut window also depends on the resolution of the \( D^0 \) invariant mass. The resolutions of the invariant mass in signal Monte Carlos and the cut window widths of various modes are summarized in Table 4.7.

<table>
<thead>
<tr>
<th>( D^0 ) Mode</th>
<th>Fit Type</th>
<th>( \sigma ) (MeV/c^2)</th>
<th>Cut (MeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^+K^- )</td>
<td>Double Gaus.</td>
<td>5.9 ± 0.1</td>
<td>&lt; 12</td>
</tr>
<tr>
<td>( \pi^+\pi^- )</td>
<td>Double Gaus.</td>
<td>7.3 ± 0.2</td>
<td>&lt; 12</td>
</tr>
<tr>
<td>( K_s^0\pi^0 )</td>
<td>Asym. Gaus. + Gaus.</td>
<td>23.6 ± 0.2 (L)</td>
<td>&lt; 30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.4 ± 0.2 (R)</td>
<td></td>
</tr>
<tr>
<td>( K^0\phi )</td>
<td>Double Gaus.</td>
<td>3.4 ± 0.1</td>
<td>&lt; 12</td>
</tr>
<tr>
<td>( K_s^0\omega )</td>
<td>Double Asym. Gaus.</td>
<td>10.6 ± 0.2 (L)</td>
<td>&lt; 20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.9 ± 0.2 (R)</td>
<td></td>
</tr>
<tr>
<td>( K^-\pi^+ )</td>
<td>Double Gaus.</td>
<td>6.7 ± 0.1</td>
<td>&lt; 12</td>
</tr>
</tbody>
</table>

Table 4.7: The fitted mass resolutions of all reconstructed \( D^0 \) candidates in signal MC. The last column is the value of the \( D^0 \) invariant mass cut that will be used in this analysis.

4.7.3 \( D^0 \rightarrow K_s^0\pi^+\pi^- \) Veto

Optimization of this cut is a little bit different than others, in that we will resort to a signal Monte Carlo sample of 878000 \( B^- \rightarrow D^0\pi^- \), \( D^0 \rightarrow K_s^0\pi^+\pi^- \) events. Figure 4.5 is a histogram of the difference between the mass of a \( D^0 \) candidate and the \( D^0 \) PDG mass in the Monte Carlo sample. The slight cut-off at about ±40 MeV/c^2 is the result of a selection cut at the production stage of the sample (more details in [65]). Table 4.8 lists the signal efficiencies of different \( D^0 \) mass cuts. As the cut window on this sample widens, the signal efficiency increases because more signal \( D^0 \rightarrow K_s^0\pi^+\pi^- \) events are included. At first glance, a veto window of 80 MeV/c^2 seems to be the
logical choice as all the $D^0 \rightarrow K^0_s \pi^+ \pi^-$ events (the background events we want to reject in our $D^0 \rightarrow \pi^+ \pi^-$ mode) will be discarded. Another check is made on a 1113.9 fb$^{-1}$ $B^+B^-$ sample to see how many $D^0 \rightarrow \pi^+ \pi^-$ signal events (events that we want) will be thrown away while we increase the cut window. It shows that we do not lose any $D^0 \rightarrow \pi^+ \pi^-$ signal events even though the veto goes from 25 to 80 MeV/$c^2$. However, the final veto cut window is chosen at 60 MeV/$c^2$ since there is no evidence that at 80 MeV/$c^2$ more backgrounds will be cut. The risk of unnecessary loss of signal events is also lessened with a smaller cut window.

![Figure 4.5: Mass difference between a $D^0$ candidate and its PDG mass in a signal MC sample of 878000 $B^- \rightarrow D^0 \pi^-, D^0 \rightarrow K^0_s \pi^+ \pi^-$ events.](image)

4.8 Multiple Candidates Selection

In the cases when more than one candidate is found passing all selection cuts in a single event, a decision is needed to be made on which candidate is the best one to
use. The decision is based on choosing the candidate with the lowest $\chi^2$ determined from the differences between the measured and PDG values of $M_{D^0}$ and $M_{K^{*}-}$. The $\chi^2$ is defined as:

$$\chi^2 = \chi^2_{m(D^0)} + \chi^2_{m(K^{*-})} = \frac{(m(D^0) - M(D^0)_{PDG})^2}{\sigma^2_{m(D^0)}} + \frac{(m(K^{*-}) - M(K^{*-})_{PDG})^2}{\sigma^2_{m(K^{*-})} + \Gamma(K^{*-})^2},$$

(4.16)

where $m(D^0)$ and $m(K^{*-})$ are the masses of the candidates in the event, $\sigma_{m(D^0)}$ and $\sigma_{m(K^{*-})}$ are their corresponding errors, and $\Gamma(K^{*-})$ is the natural width of $K^{*-}$ (50.8 MeV/c$^2$ from PDG). Table 4.9 displays the multiplicity rates of the $CP+$, $CP-$ and Non-$CP$ modes. We count the number of events in signal Monte Carlo samples which have more than one candidate that pass all the selection cuts. The table also shows the success rate of choosing the correct candidates.

### 4.9 Summary of Analysis Cuts

Tables 4.10 - 4.12 list all the analysis cuts used in the GLW analysis. Here we want to introduce the term “$m_{ES}$ Signal Region.” An event that successfully passes all the selection cuts and has a $m_{ES}$ value between 5.2 and 5.3 GeV/c$^2$ is said to be
<table>
<thead>
<tr>
<th>Mode (Signal MC)</th>
<th>Multi. Cand. Event Rate</th>
<th>Correct %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow K^+ K^-$</td>
<td>3.6%</td>
<td>86%</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^+ \pi^-$</td>
<td>3.6%</td>
<td>86%</td>
</tr>
<tr>
<td>$D^0 \rightarrow K_S^0 \pi^0$</td>
<td>7.8%</td>
<td>86%</td>
</tr>
<tr>
<td>$D^0 \rightarrow K_0^0 \phi$</td>
<td>4.3%</td>
<td>89%</td>
</tr>
<tr>
<td>$D^0 \rightarrow K_S^0 \omega$</td>
<td>7.4%</td>
<td>91%</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^- \pi^+$</td>
<td>4.3%</td>
<td>88%</td>
</tr>
</tbody>
</table>

Table 4.9: Percentage of events with multiple candidates that pass all selection cuts in signal MC samples. Truth-matching is used to determine the percentage of correctly-chosen candidates.

in the $m_{ES}$ signal region. All events in this region are signal candidates and will be analyzed (with maximum-likelihood fit) for measuring the signal yield of a particular decay mode.

<table>
<thead>
<tr>
<th>Selection cut</th>
<th>$K^+ K^-$</th>
<th>$\pi^+ \pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>&gt; 0.65</td>
<td>&gt; 0.82</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(K^*)</td>
<td>&gt; 0.35$</td>
</tr>
<tr>
<td>signed, 2D, DOF pull ($K_S^0$)</td>
<td>&gt; 3$\sigma$</td>
<td>&gt; 3$\sigma$</td>
</tr>
<tr>
<td>$</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
</tr>
<tr>
<td>$(K^{<em>-}) - M(K^{</em>-})_{PDG}$ (MeV/$c^2$)</td>
<td>&lt; 75</td>
<td>&lt; 75</td>
</tr>
<tr>
<td>$</td>
<td>m(K_S^0) - M(K_S^0)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
</tr>
<tr>
<td>$</td>
<td>m(K_S^0 \pi^+ \pi^-)_{D^0}</td>
<td>$ veto (MeV/$c^2$)</td>
</tr>
<tr>
<td>PID on dau. 1 from $D^0$</td>
<td>KLHVeryLoose</td>
<td>LHPionLoose</td>
</tr>
<tr>
<td>PID on dau. 2 from $D^0$</td>
<td>KLHVeryLoose</td>
<td>LHPionLoose</td>
</tr>
<tr>
<td>$</td>
<td>\Delta E</td>
<td>$ (MeV)</td>
</tr>
</tbody>
</table>

Table 4.10: Summary of selection criteria of $CP^+$ modes.
<table>
<thead>
<tr>
<th>Selection cut</th>
<th>$K^0\pi^0$</th>
<th>$K^0\phi$</th>
<th>$K^0\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>&gt; 0.91</td>
<td>&gt; 0.56</td>
<td>&gt; 0.80</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(K^*)</td>
<td>$</td>
<td>&gt; 0.35</td>
</tr>
<tr>
<td>signed, 2D, DOF pull ($K^0_S$)</td>
<td>&gt; 3$\sigma$</td>
<td>&gt; 3$\sigma$</td>
<td>&gt; 3$\sigma$</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(\omega)</td>
<td>$</td>
<td>-</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Dalitz}(\omega)</td>
<td>$</td>
<td>-</td>
</tr>
<tr>
<td>$</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>&lt; 30</td>
</tr>
<tr>
<td>$</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>&lt; 75</td>
</tr>
<tr>
<td>$</td>
<td>m(K^0_S) - M(K^0_S)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>&lt; 13</td>
</tr>
<tr>
<td>$</td>
<td>m(K^0_S(D^0)) - M(K^0_S)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>&lt; 6</td>
</tr>
<tr>
<td>$</td>
<td>m(\phi) - M(\phi)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>-</td>
</tr>
<tr>
<td>$</td>
<td>m(\omega) - M(\omega)_{PDG}</td>
<td>$ (MeV/$c^2$)</td>
<td>-</td>
</tr>
<tr>
<td>PID on dau. 1 from $\phi/\omega$</td>
<td>-</td>
<td>KLVVeryLoose</td>
<td>LHPionLoose</td>
</tr>
<tr>
<td>PID on dau. 2 from $\phi/\omega$</td>
<td>-</td>
<td>KLVVeryLoose</td>
<td>LHPionLoose</td>
</tr>
<tr>
<td>$</td>
<td>\Delta E</td>
<td>$ (MeV)</td>
<td>&lt; 50</td>
</tr>
</tbody>
</table>

Table 4.11: Summary of selection criteria of $CP$- modes.

<table>
<thead>
<tr>
<th>Selection cut</th>
<th>$K^-\pi^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>&gt; 0.73</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(K^*)</td>
</tr>
<tr>
<td>signed, 2D, DOF pull ($K^0_S$)</td>
<td>&gt; 3$\sigma$</td>
</tr>
<tr>
<td>$</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
</tr>
<tr>
<td>$</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
</tr>
<tr>
<td>$</td>
<td>m(K^0_S) - M(K^0_S)_{PDG}</td>
</tr>
<tr>
<td>PID on dau. 1 from $D^0$</td>
<td>KLVVeryLoose</td>
</tr>
<tr>
<td>$</td>
<td>\Delta E</td>
</tr>
</tbody>
</table>

Table 4.12: Summary of selection criteria of Non-$CP$ modes.

103
4.10 Signal Efficiencies

Efficiencies of selection cuts on signal Monte Carlo samples of each decay mode are calculated. In each signal Monte Carlo sample, candidates that satisfy all the cuts listed in last few sections are selected. Then an extended maximum likelihood fit to the $m_{ES}$ distributions of the selected candidates (details of the fit will be described in Section 4.11) is performed. The number of signal events is then extracted from the fit and used to calculate the efficiency. The efficiencies are calculated run-by-run and then averaged with a weighted-sum according to the luminosity of each run. Table 4.13 shows the Run 1 - 5 signal efficiencies of $CP^+$, $CP^-$ and Non-$CP$ modes. Individual cut efficiencies are also calculated (Tables 4.14 - 4.16). For a specific cut efficiency, we measure the number of signal events passing all cuts except the one being studied.

<table>
<thead>
<tr>
<th>$D^0$ Mode</th>
<th>Signal $\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+K^-$</td>
<td>12.78 ± 0.05%</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>12.34 ± 0.05%</td>
</tr>
<tr>
<td>$K^0\pi^0$</td>
<td>5.59 ± 0.03%</td>
</tr>
<tr>
<td>$K^0\phi$</td>
<td>8.90 ± 0.04%</td>
</tr>
<tr>
<td>$K^0\omega$</td>
<td>2.35 ± 0.02%</td>
</tr>
<tr>
<td>$K^-\pi^+$</td>
<td>12.76 ± 0.09%</td>
</tr>
</tbody>
</table>

Table 4.13: Signal efficiencies of the six decay modes studied in this analysis.
### Table 4.14: Summary of individual cut efficiencies of $CP^+$ modes.

<table>
<thead>
<tr>
<th>Selection Cut</th>
<th>$K^+ \bar{K}^-$</th>
<th>$\pi^+ \pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>70.2%</td>
<td>65.9%</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(K^*)</td>
<td>$</td>
</tr>
<tr>
<td>signed, 2D, DOF pull ($K_s^0$)</td>
<td>96.2%</td>
<td>96.2%</td>
</tr>
<tr>
<td>$</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
<td>$</td>
</tr>
<tr>
<td>$(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
<td>$</td>
<td>91.1%</td>
</tr>
<tr>
<td>$</td>
<td>m(K_s^0) - M(K_s^0)_{PDG}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>m(K_s^0\pi^+\pi^-)_{D^0}</td>
<td>$ veto</td>
</tr>
<tr>
<td>PID on dau. 1 from $D^0$</td>
<td>95.7%</td>
<td>96.4%</td>
</tr>
<tr>
<td>PID on dau. 2 from $D^0$</td>
<td>95.5%</td>
<td>96.4%</td>
</tr>
<tr>
<td>$</td>
<td>\Delta E</td>
<td>$</td>
</tr>
</tbody>
</table>

4.11 Extended Maximum Likelihood Fit

4.11.1 The Formalism

The maximum likelihood method is a technique to estimate the parameter value that makes the observed data most likely. In other words, it is used to calculate the best way of fitting a model to some data sample. Consider a random variable $x$ (or a multidimensional random vector $\hat{x} = (x_1, ..., x_n)$) distributed with a distribution function $f(x; \theta)$. Suppose the expression $f(x; \theta)$ is well known, but at least one of the parameters, called $\theta$ (or parameters $\hat{\theta} = (\theta_1, ..., \theta_n)$) is unknown. After a normalization to 1, the expression $f(x; \theta)$ represents the hypothesized probability function (PDF) for the variable $x$. Then, suppose an experiment is performed where a measurement has been repeated $N$ times, with values of $x_1, ..., x_N$. The total PDF is:

$$P(\theta) = \prod_{i=1}^{N} f(x_i; \theta). \quad (4.17)$$
<table>
<thead>
<tr>
<th>Selection Cut</th>
<th>( K^0\pi^0 )</th>
<th>( K^0\phi )</th>
<th>( K^0\omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>54.8%</td>
<td>77.5%</td>
<td>58.1%</td>
</tr>
<tr>
<td>(</td>
<td>\cos \theta_{Helicity}(K^*)</td>
<td>)</td>
<td>95.1%</td>
</tr>
<tr>
<td>signed, 2D, DOF pull (( K^0_s ))</td>
<td>96.2%</td>
<td>96.6%</td>
<td>96.7%</td>
</tr>
<tr>
<td>(</td>
<td>\cos \theta_{Helicity}(\omega)</td>
<td>)</td>
<td>-</td>
</tr>
<tr>
<td>(</td>
<td>\cos \theta_{Dalitz}(\omega)</td>
<td>)</td>
<td>-</td>
</tr>
<tr>
<td>(</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
<td>)</td>
<td>78.7%</td>
</tr>
<tr>
<td>(</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
<td>)</td>
<td>91.3%</td>
</tr>
<tr>
<td>(</td>
<td>m(K^0_s) - M(K^0_s)_{PDG}</td>
<td>)</td>
<td>98.1%</td>
</tr>
<tr>
<td>(</td>
<td>m(K^0_s(D^0)) - M(K^0_s)_{PDG}</td>
<td>)</td>
<td>90.2%</td>
</tr>
<tr>
<td>(</td>
<td>m(\phi) - M(\phi)_{PDG}</td>
<td>)</td>
<td>-</td>
</tr>
<tr>
<td>(</td>
<td>m(\omega) - M(\omega)_{PDG}</td>
<td>)</td>
<td>-</td>
</tr>
<tr>
<td>PID on dau. 1 from ( \phi/\omega )</td>
<td>-</td>
<td>94.7%</td>
<td>99.6%</td>
</tr>
<tr>
<td>PID on dau. 2 from ( \phi/\omega )</td>
<td>-</td>
<td>94.5%</td>
<td>99.5%</td>
</tr>
<tr>
<td>(</td>
<td>\Delta E</td>
<td>)</td>
<td>99.0%</td>
</tr>
</tbody>
</table>

Table 4.15: Summary of individual cut efficiencies of \( CP \)- modes.

<table>
<thead>
<tr>
<th>Selection Cut</th>
<th>( K^-\pi^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>65.3%</td>
</tr>
<tr>
<td>(</td>
<td>\cos \theta_{Helicity}(K^*)</td>
</tr>
<tr>
<td>signed, 2D, DOF pull (( K^0_s ))</td>
<td>96.3%</td>
</tr>
<tr>
<td>(</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
</tr>
<tr>
<td>(</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
</tr>
<tr>
<td>(</td>
<td>m(K^0_s) - M(K^0_s)_{PDG}</td>
</tr>
<tr>
<td>PID on dau. 1 from ( D^0 )</td>
<td>95.8%</td>
</tr>
<tr>
<td>(</td>
<td>\Delta E</td>
</tr>
</tbody>
</table>

Table 4.16: Summary of individual cut efficiencies of Non-\( CP \) modes.
This is also called the likelihood function, $\mathcal{L}$. We want to seek the value of the parameter that maximizes the likelihood function (i.e. maximum likelihood). If $x_i$ are measured and $f(x; \theta)$ is well-known, then $\mathcal{L}$ only depends on the parameters we want to fit.

We may also want to measure the total number of observed events $N$ in an experiment. If the total expected number of events is $n$, the probability to have $N$ observed events is given by the Poisson probability:

$$e^{-n} \frac{n^N}{N!}.$$  

The likelihood function is then given by the product of this Poisson probability and the function in Equation 4.17 for $N$ values of $x$,

$$\mathcal{L}(n, \theta) = e^{-n} \frac{n^N}{N!} \prod_{i=1}^{N} n f(x_i; \theta).$$  

This is called the extended likelihood function. Now we will see how the extended maximum likelihood technique allows us to measure the number of signal and background events in a data sample where every event has $h$ observable quantities $\hat{x} = (x_1, ..., x_h)$. Suppose that the parameters to be evaluated are the number of events $n_1, ..., n_s$, each one corresponding to a particular categories of events (e.g. signal, continuum background, non-continuum background). To distinguish between events of different categories, we determine the PDFs that present a high discriminant power for each observable quantity (e.g. Gaussian, Argus). We fit these distributions with their corresponding PDFs, indexed with $f^1_j, ..., f^h_j$, with $j = 1, ..., s$. As a result, we have $h$ PDFs for each category and $(h \times s)$ PDFs in total. If the observable quantities are independent (if not, correlation terms are to be considered), we can define the
total PDF for event $i$ with observable quantities $\hat{x}^i = (x^i_1, ..., x^i_n)$ and category $j$ as

$$P^i_j = \prod_{l=1}^{h} f^i_j(x^i_l).$$

(4.20)

The extended likelihood function is thus:

$$L = \frac{e^{-(\sum_{j=1}^{s} n_j)}}{N!} \prod_{i=1}^{N} \sum_{j=1}^{s} n_j P^i_j.$$  

(4.21)

The extended maximum likelihood fit consists of maximizing this function with respect to the event yields and the floating parameters in $P^i_j$.

### 4.11.2 Overview

The number of signal events (as well as signal-like and non-peaking background) of all the decay modes studied in this analysis are obtained by performing a simultaneous extended maximum likelihood fit to the $m_{ES}$ signal regions, and $\Delta E$ and $\Delta m(D^0)$ sidebands (the sidebands will be discussed below). The signal candidates and background events of $K^+K^-$ and $\pi^+\pi^-$ modes will be combined into one $CP+$ dataset, and likewise for $K^0_S\pi^0, K^0_S\phi$ and $K^0_S\omega$, where they will be merged into a single $CP-$ sample. Therefore we have a total of three ($CP+$, $CP-$ and Non-$CP$) data samples.

In the $m_{ES}$ signal region, the probability density function (PDF) is a Gaussian for signal candidates and an Argus threshold function for backgrounds. The $\Delta E$ sideband is modeled with an Argus function only. A Gaussian for fake $D^0$ backgrounds and an Argus for non-peaking backgrounds are used in the $m(D^0)$ sideband (to be explained later in the “Fake $D^0$ Backgrounds in Signal Region” section). In addition to signal yields, the $CP$ observables $A_{CP\pm}, R_{CP\pm}$ and $x^\pm$ will be extracted from the fit using Equations 2.79, 2.81 and 2.88.
4.11.3 ΔE and Δm(D⁰) Sidebands

Two “sidebands” are defined to help us better define the background shape in the m_{ES} signal regions. They are the ΔE and |m(D⁰) - M(D⁰)_{PDG}| (or Δm(D⁰)) sidebands. Their definitions are summarized in Table 4.17.

<table>
<thead>
<tr>
<th>Sideband (Mode)</th>
<th>ΔE (MeV)</th>
<th>Δm(D⁰) (MeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>D⁰ → K^-π⁺</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>20 &lt;</td>
</tr>
<tr>
<td>D⁰ → K⁺K⁻</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>20 &lt;</td>
</tr>
<tr>
<td>D⁰ → π⁺π⁻</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>20 &lt;</td>
</tr>
<tr>
<td>D⁰ → K⁰_sπ⁰</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>60 &lt;</td>
</tr>
<tr>
<td>D⁰ → K⁰_sφ</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>20 &lt;</td>
</tr>
<tr>
<td>D⁰ → K⁰_sω</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>30 &lt;</td>
</tr>
<tr>
<td>D⁰ → K^-π⁺</td>
<td>60 &lt; ΔE &lt; 200</td>
<td>20 &lt;</td>
</tr>
</tbody>
</table>

Table 4.17: Sideband definitions for various modes in this analysis.

4.11.4 Fake D⁰ Backgrounds in Signal Region

The most dominant source of background comes from events with fake D⁰ candidates. These events have wrongly-reconstructed D⁰ candidates which are combined with a true K^{*}. Potentially significant peaks may be present in the region away from the signal Δm(D⁰) region. Thus it is necessary to account for these fake candidates so that signal yields will not be overestimated.

A Gaussian is used to describe the fake D⁰ backgrounds in the Δm(D⁰) sideband. To extrapolate the number of these background events in the m_{ES} signal region, we multiply the number of peaking background events in the Δm(D⁰) sideband obtained from the Argus+Gaussian fit, by the ratio between the width of the Δm(D⁰) signal and sideband window. A summary of the window widths is shown in Table 4.18. A
Gaussian will be used to model the fake $D^0$ backgrounds in the $m_{ES}$ signal region and now we have a total of two Gaussians in the region. The Gaussian which represents the $D^0$ peaking backgrounds has the same parameters as the one representing $B^\pm \rightarrow D^0 K^{*\pm}$ signal events. The number of $D^0$ peaking backgrounds in the $m_{ES}$ signal region is fixed in the final simultaneous fit.

<table>
<thead>
<tr>
<th>$D^0$ Mode</th>
<th>Signal Region (MeV/$c^2$)</th>
<th>Sideband (MeV/$c^2$)</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^0 \rightarrow K^+K^-$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 12$</td>
</tr>
<tr>
<td>$D^0 \rightarrow \pi^+\pi^-$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 12$</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0_\pi\pi^0$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 30$</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0_\phi$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 12$</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^0_\omega$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 20$</td>
</tr>
<tr>
<td>$D^0 \rightarrow K^-\pi^+$</td>
<td>$</td>
<td>\Delta m</td>
<td>&lt; 12$</td>
</tr>
</tbody>
</table>

Table 4.18: A summary of the $|\Delta m(D^0)|$ signal region and sideband definitions, and the ratio of the $|\Delta m(D^0)|$ signal window to sideband window.

### 4.11.5 Fit Strategy

We will fit simultaneously the $m_{ES}$ distributions of signal candidate events in $CP^+$ ($D^0 \rightarrow K^+K^-, \pi^+\pi^-$), $CP^-$ ($D^0 \rightarrow K^0_\pi\pi^0, K^0_\phi, K^0_\omega$) and Non-$CP$ ($D^0 \rightarrow K^-\pi^+$) mode. Each of these three modes ($CP^+, CP^-, Non-CP$) has a $m_{ES}$ signal region, a $\Delta E$ sideband and a $|\Delta m(D^0)|$ sideband. In order to measure the physical $CP$ quantities, $A_{CP\pm}, R_{CP\pm}$ and $x^\pm$, from the fit, we need to split the $CP^+$ and $CP^-$ samples in the $m_{ES}$ signal region into $B^+$ and $B^-$ sub-samples. The sideband samples will not be split, as we assume that the fake $D^0$ backgrounds do not violate $CP$. The peaking background estimation will be divided equally between the $B^+$ and $B^-$ sub-samples. The systematic error associated with this assumption will be discussed in

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a later chapter. In summary, we have eleven \((3\times3+2)\) \(m_{ES}\) datasets contributing to the extended maximum likelihood fit.

There are a few advantages for using a single simultaneous fit across all regions. First, the high-statistics Non-\(CP\) sample helps constrain the PDF shape of signal peaks of the low-statistics \(CP\)-modes. Second, the \(\Delta E\) sideband helps define the background Argus shape in the \(m_{ES}\) signal region. Third, the \(\Delta m(D^0)\) sideband provides information on the fake \(D^0\) background which are also present in the signal region.

A few more assumptions are made with the simultaneous fit. To simplify the fit, we assume that a single Gaussian is sufficient to describe the signal peaks for all decay modes. This assumption is valid because in the six modes studied in this analysis, they differ only by the content in the \(D^0\) decay. It can be verified by measuring the means and widths of the individual Gaussians of the different modes using data or Monte Carlo samples. We use Monte Carlo cocktails composed of signal Monte Carlos, generic backgrounds and continuum backgrounds for \(CP^+\) and \(CP^-\) modes. Monte Carlos are more fitting for these two modes (instead of real data) because their corresponding signal peaks are not as significant as the Non-\(CP\) one. The results show that the Gaussian shapes are consistent among Non-\(CP\), \(CP^+\) and \(CP^-\) modes.

Similarly, we only use a single Argus function to describe the backgrounds in different modes. To validate, we follow the same procedure as above as we allow the Argus shape parameter \(\xi\) to vary in all nine data subsets. The floating \(\xi\)'s in different modes are all consistent with each other. Therefore it is safe to use only one single background Argus function in the fit. All in all, there are three floating (mean and
sigma if the signal is Gaussian and $\xi$ of the background Argus) and one fixed (the Argus end-point $E_{Max}$) shape parameters in the fit.

Another assumption to be made is related to fake $D^0$ backgrounds in $m_{ES}$ signal regions. We assume the fake $D^0$'s have the same shape as the signal Gaussian. The number of that background is extrapolated from the number of signal events measured in the $\Delta m(D^0)$ sideband multiplied by the ratio of the widths of the $\Delta m(D^0)$ signal to sideband window.

To summarize, we have a total of three types of PDF and three floating parameters (Gaussian mean and $\sigma$, and $\xi$) for the signal region and two sidebands in the final fit:

- $m_{ES}$ signal region: Argus+Gaussian+Gaussian. The first Gaussian represents the $B^\pm \rightarrow D^0 K^{*\pm}$ signal and the second one represents the fake $D^0$ backgrounds.
- $\Delta E$ sideband: Argus.
- $\Delta m(D^0)$ sideband: Argus+Gaussian. The Gaussian models fake $D^0$ backgrounds.

Likelihood Functions

In the final fit, the natural log of the likelihood function ($L$) of each region is maximized. The $L$ of the $m_{ES}$ signal region is:

$$
L_{SR} = e^{-(N_{Sig} + N_{D^0 Bkg} + N_{Bkg})} \prod_N (N_{Sig} \cdot G + N_{D^0 Bkg} \cdot G + N_{Bkg} \cdot A).
$$

(4.22)

$N_{Sig}$ is the number of signal yields, $N_{D^0 Bkg}$ and $N_{Bkg}$ are the number of fake $D^0$ backgrounds and Argus backgrounds respectively. The $L$'s of the $\Delta E$ and $\Delta m(D^0)$
sidebands are:

\[ \mathcal{L}_{\Delta E} = e^{-(N_{Bkg})} \prod_{N} (N_{Bkg} \cdot \mathcal{A}) \]  

(4.23)

\[ \mathcal{L}_{\Delta m(D^0)} = e^{-(N_{D^0_{Bkg}} + N_{Bkg})} \prod_{N} (N_{D^0_{Bkg}} \cdot \mathcal{G} + N_{Bkg} \cdot \mathcal{A}). \]  

(4.24)

4.11.6 Fit Result

The results from the simultaneous fit on 344.7 fb\(^{-1}\) (379 million \(B\overline{B}\) pairs) of data are shown in Table 4.19. Figure 4.6 shows the plots of \(m_{ES}\) signal regions of \(CP^+\), \(CP^-\) and Non-\(CP\) data samples. The ones in Figure 4.7 are the \(\Delta E\) and \(\Delta m(D^0)\) sidebands. Figures 4.8 - 4.13 are the \(m_{ES}\) distributions of individual modes. Note that in the fit to single decay modes, the values of the mean and sigma of the signal Gaussian and the shape parameter of the background Argus are fixed and taken from the fit result of the \(D^0 \rightarrow K^- \pi^+\) mode. The fixed parameters from the high-statistics Non-\(CP\) mode help the fits in modes without significant signal peaks (\(CP^-\) modes).

For the purpose of checking for asymmetry, the Non-\(CP\) sample is split by \(B\) charge. The result gives 126.8 ± 12.3 and 98.7 ± 11.3 signals for \(B^+\) and \(B^-\) respectively and it corresponds to an asymmetry of \((12.5 \pm 7.5)\%\). Using a signal Monte Carlo sample of 175000 events, the signal yield for \(B^+\) is 11333 ± 108 and 10994 ± 107 for \(B^-\) (an asymmetry of \((1.5 \pm 0.7)\%\)). Because the same fitting procedure is used on both the data and signal Monte Carlo samples, the apparent asymmetry in data is not due to any mistakes in the fit (the signal Monte Carlo samples are created with no asymmetry).

From Table 4.19, we also see an asymmetry in the number of backgrounds in the \(m_{ES}\) signal region of the \(CP^-\) data sample. A Monte Carlo cocktail of \(CP^-\) signal, generic and continuum background normalized to the Run 1 - 5 data luminosity is fit
to extract the number of background events in the \( m_{ES} \) signal region. The result is \( 90.8 \pm 10.0 \) for \( B^+ \) and \( 93.3 \pm 10.1 \) for \( B^- \). Again, the asymmetry does not indicate a problem in the fitting procedure.

<table>
<thead>
<tr>
<th>Asymmetry Fit</th>
<th>( CP^+ )</th>
<th>( CP^- )</th>
<th>Non-CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{Shape Parameters} )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argus end-point (MeV/( c^2 ))</td>
<td></td>
<td>5291.00 (FIXED)</td>
<td></td>
</tr>
<tr>
<td>Argus shape parameter ( \xi )</td>
<td>-19.41 ( \pm ) 2.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian mean (MeV/( c^2 ))</td>
<td>5278.70 ( \pm ) 0.16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian sigma (MeV/( c^2 ))</td>
<td>2.40 ( \pm ) 0.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yield (Signal Region)</th>
<th>( CP^+ )</th>
<th>( CP^- )</th>
<th>Non-CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian (Signal)</td>
<td>31.15 ( \pm ) 6.20 ( (B^+) )</td>
<td>22.97 ( \pm ) 4.77 ( (B^+) )</td>
<td>230.69 ( \pm ) 16.80</td>
</tr>
<tr>
<td>Gaussian (Fake ( D^0 ) Bkg)</td>
<td>37.43 ( \pm ) 6.77 ( (B^-) )</td>
<td>15.50 ( \pm ) 5.16 ( (B^-) )</td>
<td>348.31 ( \pm ) 19.80</td>
</tr>
<tr>
<td>Argus</td>
<td>0.34</td>
<td>0</td>
<td>5.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yield (( \Delta E ) Sideband)</th>
<th>( CP^+ )</th>
<th>( CP^- )</th>
<th>Non-CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argus</td>
<td>334.00 ( \pm ) 18.30</td>
<td>353.99 ( \pm ) 18.80</td>
<td>888.98 ( \pm ) 29.80</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Yield (( \Delta m(D^0) ) Sideband)</th>
<th>( CP^+ )</th>
<th>( CP^- )</th>
<th>Non-CP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian (Fake ( D^0 ) Bkg)</td>
<td>3.96 ( \pm ) 3.23</td>
<td>0.00 ( \pm ) 3.04</td>
<td>20.96 ( \pm ) 7.72</td>
</tr>
<tr>
<td>Argus</td>
<td>437.08 ( \pm ) 21.20</td>
<td>177.01 ( \pm ) 13.30</td>
<td>579.03 ( \pm ) 25.00</td>
</tr>
</tbody>
</table>

\[ A_{CP^+} = 0.092 \pm 0.134, \quad A_{CP^-} = -0.194 \pm 0.205 \]
\[ R_{CP^+} = 2.172 \pm 0.337, \quad R_{CP^-} = 1.170 \pm 0.266 \]
\[ x^+ = 0.144 \pm 0.136, \quad x^- = 0.357 \pm 0.138 \]

Table 4.19: \( CP \)-asymmetry fit results on 344.7 fb\(^{-1} \) of Run 1 - 5 data.

### 4.12 Systematic Errors of \( A_{CP} \) and \( R_{CP} \)

In this section, I discuss the systematic errors involved in the calculation of \( A_{CP} \) and \( R_{CP} \). Systematic errors of \( x^\pm \) can be found in Appendix B.
4.12.1 Systematics from Same-Final-State Backgrounds

We need to address the non-resonant background for $D^0 \rightarrow K^0_s \phi$ and $D^0 \rightarrow K^0_s \omega$ in $CP$- modes. $K^0_s \phi$ suffers from any background that has the $K^0_s K^+ K^-$ final state. Similarly for $K^0_s \omega$, there is contamination from decays that have the $K^0_s \pi^+ \pi^- \pi^0$ same final state. In Appendix E of [66], a detailed method to correct $A_{CP}$ and $R_{CP}$ due to these background was deduced.

When we integrate over the full width of the resonating state, i.e. $\phi/\omega$, we can ignore quantum interference effects and consider only the number of decays in the resonance ($N_{Res}$) and the number of same-final-state decays that do not belong to the resonance ($N_{Non}$). The total asymmetry, which includes both events from resonant and non-resonant decays, is

$$A = \frac{N_{Res}^- + N_{Non}^- - (N_{Res}^+ + N_{Non}^+)}{(N_{Res}^- + N_{Non}^-) + (N_{Res}^+ + N_{Non}^+)}.$$

(4.25)

The true asymmetry is then, according to [66],

$$A_{Res} = (1 + \epsilon)A - \epsilon A_{Non}.$$  

(4.26)

$\epsilon$ is defined as $N_{Non}/N_{Res}$ and we assume that to be much smaller than 1.

There is no error if the background is of the same $CP$ eigenstate and exhibits similar $CP$ asymmetry. If the background is of opposite $CP$ eigenstate, $A_{CP-}$ is corrected according to the following equation:

$$A_{CP-} = (1 + \epsilon)A_{CP-}^{Meas} - \epsilon A_{CP+},$$

(4.27)

where $A_{CP-}^{Meas}$ would be the $A_{CP-}$ we obtain from our simultaneous fit. Similarly, we adjust $R_{CP-}$ by:

$$R_{CP-} = \frac{R_{CP-}^{Meas}}{1 + \epsilon}.$$  

(4.28)
For $K^0_s\phi$ mode, background events mainly come from $K^0_s a_0(980)$ decays [67]. In [67], which is a Dalitz plot analysis of $D^0 \rightarrow K^0_s K^+ K^-$, the ratio of the number of non-resonant $K^0_s K^+ K^-$ decays to the number of resonant $K^0_s \phi$ decays is measured to be $(24.7 \pm 1.3)\%$. For $K^0_s \omega$ mode, no information for $K^0_s \pi^+ \pi^- \pi^0$ decays is available so we assume a $(30 \pm 30)\%$ for the background to signal ratio. To calculate $\epsilon$, we need the expected (calculated from PDG branching fractions and signal efficiencies) number of total $CP$- signal yield, and the expected number of $K^0_s \phi$ and $K^0_s \omega$ signals. With 344.7 fb$^{-1}$ of data, the numbers are 39.7, 7.1 and 8.7 respectively. Hence we have:

$$\epsilon = \frac{(24.7 \pm 1.3)\% \times 7.1 + (30 \pm 30)\% \times 8.7}{39.7}$$

$$= 0.110 \pm 0.069$$ (4.30)

The measured values of $A_{CP-}$ and $R_{CP-}$ are thus adjusted as follows:

$$A_{CP-} = (1 + \epsilon)A_{CP-}^{Meas} - \epsilon A_{CP+}$$

$$= (1.110 \pm 0.069)(-0.305 \pm 0.269) - (0.110 \pm 0.069)(-0.017 \pm 0.168)$$

$$= (-0.337 \pm 0.299) \pm 0.020$$ (4.33)

$$R_{CP-} = \frac{R_{CP-}^{Meas}}{1 + \epsilon}$$

$$= (1.06 \pm 0.314)/(1.110 \pm 0.069)$$

$$= (0.996 \pm 0.289) \pm 0.062$$ (4.37)

where the second error is the additional systematic error introduced by the uncertainty on the magnitude of $\epsilon$.  

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4.12.2 Systematics from the Interference with $B \rightarrow D^0 K_s^0 \pi$

Another non-resonant background in the $K^{*-\pi}$ region is the $b \rightarrow c$ process, $B^- \rightarrow D^0 K_s^0 \pi^-$. In Chapter 6 of [66] a detailed model of the $B^- \rightarrow D^0[K^{*-} + K_s^0 \pi^-]$ system was built and analyzed using Mathematica, where two related variables, $\rho_c$ and $\Delta_c$, were measured. $\rho_c$ is the ratio of the amplitudes of the three-body background and the signal for $b \rightarrow c$ processes:

$$\rho_c = \frac{|A(B^- \rightarrow D^0 K_s^0 \pi^-)|}{|A(B^- \rightarrow D^0 K^{*-})|};$$  \hspace{1cm} (4.38)

while $\Delta_c$ is the phase between the three-body background and the signal for $b \rightarrow c$ processes. The measured values with 211 fb$^{-1}$ of data are:

$$\rho_c = 0.185^{+0.045}_{-0.025} \hspace{1cm} (4.39)$$

$$\Delta_c = (-90 \pm 37)^\circ. \hspace{1cm} (4.40)$$

In [68], using the above measurements, the variation of the observables $A_{CP}$ and $R_{CP}$ were calculated. We will use those calculated variations (Table 4.20) as the systematic errors for the non-resonant $B \rightarrow D^0 K_s^0 \pi$ background.

<table>
<thead>
<tr>
<th></th>
<th>$A_{CP\pm}$</th>
<th>$R_{CP\pm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Upward Variation</td>
<td>0.051</td>
<td>0.035</td>
</tr>
<tr>
<td>Max. Downward Variation</td>
<td>-0.051</td>
<td>-0.024</td>
</tr>
<tr>
<td>Systematic Error Used</td>
<td>0.051</td>
<td>0.035</td>
</tr>
</tbody>
</table>

Table 4.20: Results of the $B^- \rightarrow D^0[K^{*-} + K_s^0 \pi^-]$ system modeling presented in BAD1141.
4.12.3 Other Systematics of $A_{CP}$

- Charge bias in the analysis: The charge asymmetry of the $B\bar{B}$ detector is the potential charge bias in tracking efficiency or PID. We adopt results from the study carried out in [31]. The authors measured the asymmetry in a number of control samples from data and Monte Carlo. They measured the charge bias on data for the decays of $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow K^{-}\pi^{+}]$, $B^{-} \rightarrow D^{0}K^{-}[D^{0} \rightarrow K^{-}\pi^{+}]$, $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow K^{-}K^{+}]$, $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow \pi^{+}\pi^{-}]$, $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow K_{S}^{0}\pi^{0}]$, $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow K_{S}^{0}\phi]$, $B^{-} \rightarrow D^{0}\pi^{-}[D^{0} \rightarrow K_{S}^{0}\omega]$. The same decay processes were measured on signal Monte Carlo. No evidence of charge asymmetry was found. The average asymmetry in data control samples is $-(1.6 \pm 0.6)\%$ and $-(0.4 \pm 1.0)\%$ in Monte Carlo samples. We will take $(1.6 + 0.6)\% = 2.2\%$ as the systematic error.

- Assumption of charge symmetry in the fake $D^{0}$ background: The systematic error introduced by this assumption in the $\Delta m(D^{0})$ sideband is related to the number of signal and peaking background events in the $m_{ES}$ signal region and the hypothetical $CP$ asymmetry in the background by:

$$\sigma = A_{Bkg} \times \frac{N_{Bkg}}{N_{Sig}}. \quad (4.41)$$

I assume the hypothetical $CP$ asymmetry in the background to be 50%. The number of signal and background events will be taken from the results in the Fit Result section. In the $\Delta m(D^{0})$ sideband of $CP$- mode zero fake $D^{0}$ background events $(0.00 \pm 3.04)$ is measured. To be conservative, I use 3.04 as the number of peaking background events in the $m_{ES}$ signal region. The resulting systematic errors for $A_{CP+}$ and $A_{CP-}$ are 0.25% and 3.95% respectively.
4.12.4 Summary of Systematic Errors of $A_{CP\pm}$

Table 4.21 summarizes the systematic error calculation for $A_{CP\pm}$. The total systematic error is calculated by adding the individual systematic error sources in quadrature. The final numbers are $\pm 0.058$ for $A_{CP+}$ and $\pm 0.089$ for $A_{CP-}$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$A_{CP+}$</th>
<th>$A_{CP-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of Same-Final-State Bkg</td>
<td>-</td>
<td>0.020</td>
</tr>
<tr>
<td>Effect of $B \rightarrow D^0 K_s^0 \pi$ Bkg</td>
<td>0.051</td>
<td>0.051</td>
</tr>
<tr>
<td>Inherent Asymmetry</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Asymmetry in fake $D^0$ Bkg</td>
<td>0.003</td>
<td>0.040</td>
</tr>
<tr>
<td>Total Systematic Error</td>
<td>0.051</td>
<td>0.068</td>
</tr>
</tbody>
</table>

Table 4.21: Systematic errors of $A_{CP\pm}$.

4.12.5 Summary of Systematic Errors of $R_{CP\pm}$

Table 4.22 summarizes the systematic error calculation for $R_{CP\pm}$. The systematics from efficiency corrections are obtained by multiplying the systematic errors of $\epsilon_{CP\pm}/\epsilon_{NCP}$ to the corrected central values of $R_{CP\pm}$. The final numbers are $\pm 0.083$ and $\pm 0.113$ for $R_{CP+}$ and $R_{CP-}$ respectively.

<table>
<thead>
<tr>
<th>Source</th>
<th>$R_{CP+}$</th>
<th>$R_{CP-}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic Error in $\epsilon_{CP\pm}/\epsilon_{NCP}$</td>
<td>0.075</td>
<td>0.088</td>
</tr>
<tr>
<td>Effect of Same-Final-State Bkg</td>
<td>-</td>
<td>0.062</td>
</tr>
<tr>
<td>Effect of $B \rightarrow D^0 K_s^0 \pi$ Bkg</td>
<td>0.035</td>
<td>0.035</td>
</tr>
<tr>
<td>Total Systematic Error</td>
<td>0.083</td>
<td>0.113</td>
</tr>
</tbody>
</table>

Table 4.22: Systematic errors of $R_{CP\pm}$. 


4.13 Summary of the GLW Analysis Results

From the study of $B^- \to D^0 K^{*-}$ decays with 379 million $B\overline{B}$ events (Run 1 - 5) the following results of the GLW analysis are presented:

\begin{align*}
A_{CP^+} &= 0.092 \pm 0.133 (\text{stat.}) \pm 0.051 (\text{syst.}) \\
A_{CP^-} &= -0.199 \pm 0.206 (\text{stat.}) \pm 0.068 (\text{syst.}) \\
R_{CP^+} &= 2.174 \pm 0.352 (\text{stat.}) \pm 0.083 (\text{syst.}) \\
R_{CP^-} &= 1.142 \pm 0.273 (\text{stat.}) \pm 0.113 (\text{syst.}) \\
\end{align*}

\begin{align*}
x^+ &= 0.144 \pm 0.136 (\text{stat.}) \pm 0.046 (\text{syst.}) \\
x^- &= 0.357 \pm 0.138 (\text{stat.}) \pm 0.051 (\text{syst.}) \\
\end{align*}
Figure 4.6: Simultaneous fit to $m_{ES}$ distributions in signal regions on 344.7 fb$^{-1}$ of data. First row: CP+ modes ($K^+K^-$, $\pi^+\pi^-$), left($B^+$), right($B^-$). Second row: CP-modes ($K^0\pi^0$, $K^0\phi$, $K^0\omega$), left($B^+$), right($B^-$). Third row: Non-CP mode ($K^-\pi^+$), $B^\pm$. 
Figure 4.7: Simultaneous fit to $m_{ES}$ distributions in sidebands on 344.7 fb$^{-1}$ of data. Left column: ($\Delta E$ sideband), right column: ($\Delta m(D^0)$ sideband). First row: CP+ modes ($K^+K^-$, $\pi^+\pi^-$), $B^\pm$. Second row: CP- modes ($K^0\pi^0$, $K^0\phi$, $K^0\omega$), $B^\pm$. Third row: Non-CP mode ($K^-\pi^+$), $B^\pm$. 
Figure 4.8: Simultaneous fit to $m_{ES}$ distributions in signal regions and sidebands on 344.7 fb$^{-1}$ of data for $D^0 \rightarrow K^+K^-$ ($CP+$) mode. First row: Signal region. Second row: $\Delta E$ sideband. Third row: $\Delta m(D^0)$ sideband.
Figure 4.9: Simultaneous fit to $m_{ES}$ distributions in signal regions and sidebands on 344.7 fb$^{-1}$ of data for $D^0 \rightarrow \pi^+\pi^- (CP^+)$ mode. First row: Signal region. Second row: $\Delta E$ sideband. Third row: $\Delta m(D^0)$ sideband.
Figure 4.10: Simultaneous fit to $m_{ES}$ distributions in signal regions and sidebands on 344.7 fb$^{-1}$ of data for $D^0 \rightarrow K_s^0 \pi^0$ ($CP$-) mode. First row: Signal region. Second row: $\Delta E$ sideband. Third row: $\Delta m(D^0)$ sideband.
Figure 4.11: Simultaneous fit to \( m_{ES} \) distributions in signal regions and sidebands on 344.7 fb\(^{-1}\) of data for \( D^0 \rightarrow K^0_s \phi \) (\( CP \)-) mode. First row: Signal region. Second row: \( \Delta E \) sideband. Third row: \( \Delta m(D^0) \) sideband.
Figure 4.12: Simultaneous fit to $m_{ES}$ distributions in signal regions and sidebands on 344.7 fb$^{-1}$ of data for $D^0 \to K^0\omega$ ($CP$-) mode. First row: Signal region. Second row: $\Delta E$ sideband. Third row: $\Delta m(D^0)$ sideband.
Figure 4.13: Simultaneous fit to $m_{ES}$ distributions in signal regions and sidebands on 344.7 fb$^{-1}$ of data for $D^0 \rightarrow K^- \pi^+$ (Non-CP) mode. Top: Signal region ($B^\pm$). Middle: $\Delta E$ sideband ($B^\pm$). Bottom: $\Delta m(D^0)$ sideband ($B^\pm$).
CHAPTER 5

ANALYSIS OF \( B^- \to D^0 K^{*-} \) DECAYS USING THE ADS METHOD

5.1 Overview

As already discussed in the Theory section, the ADS method which studies the \( D^0 \to K\pi \) final states is another clean way to measure the CKM angle \( \gamma \). It is complementary to the GLW analysis so the results from the two studies will be combined in Chapter 6 for a more precise determination of \( r_B \) and \( \gamma \).

The ADS method analyzes both the Wrong-Sign (\( B^- \to D^0 K^{*-}, D^0 \to K^+\pi^- \)) and Right-Sign (\( B^- \to \bar{D}^0 K^{*-}, \bar{D}^0 \to K^+\pi^- \)) \( K\pi \) decays. Since we have the same final states, there is quantum interference between these two decays and its size is sensitive to \( \gamma \).

The analysis technique is in many ways similar to that of the GLW analysis. Discriminating variables are chosen and the corresponding cuts are optimized to suppress continuum background. Extended maximum likelihood method is used to fit the \( m_{ES} \) distributions of both the Wrong-Sign (WS) and Right-Sign (RS) modes in the signal region to extract the signal yields. The \( CP \) observables \( R_{ADS} \) and \( A_{ADS} \) will also be measured.
5.2 Discriminating Variables and Optimizations

The discriminating variables in the ADS analysis are the same as the ones used in the GLW $K^-\pi^+$ analysis. They are $\Delta E$, invariant masses of $D^0$, $K^*$ and $K^0_s$, $K^0_s$ decay length, $\cos\theta_{\text{Helicity}}(K^{*+})$, a Neural Network and PID selector for $K^*$’s.

All of the selection cuts are optimized by maximizing the significance defined in Equation 4.15, except for $\Delta E$ and $D^0$ invariant mass cuts, where the cut windows are chosen according to their resolutions in signal MC. Table 5.1 records the resolutions and Figure 5.1 has the corresponding plots.

The multiple candidate selection criteria is also the same as the GLW one, defined in Equation 4.16.

<table>
<thead>
<tr>
<th>$D^0 \rightarrow K^+\pi^-$</th>
<th>Fit Type</th>
<th>Core $\sigma$</th>
<th>Tail $\sigma$</th>
<th>Fraction (%)</th>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta E$ (MeV)</td>
<td>Double Gaus.</td>
<td>10.9 ± 0.1</td>
<td>50.2 ± 0.7</td>
<td>58.0 ± 6.3</td>
<td>&lt; 25</td>
</tr>
<tr>
<td>$\Delta m(D^0)$ (MeV/c^2)</td>
<td>Double Gaus.</td>
<td>6.8 ± 0.1</td>
<td>34.3 ± 0.6</td>
<td>75.4 ± 3.0</td>
<td>&lt; 18</td>
</tr>
</tbody>
</table>

Table 5.1: The choices of $\Delta E$ and $\Delta m(D^0)$ cut windows are based on the widths of their signal peaks in signal MC. The column “Fraction” refers to the fraction of the fit in the core Gaussian. The last column is the $\Delta E$ and $D^0$ invariant mass cut values that will be used in this analysis.

5.3 Neural Network

The Neural Network technique will be utilized to suppress continuum backgrounds in both Wrong-Sign and Right-Sign $K\pi$ modes. The input topological variables are the same as in GLW: $R_2$, the thrust angle, the zeroth- and second-order Legendre monomials, $\cos\theta_{\text{Mom}}$ and $\cos\theta_{\text{Helicity}}(D^0)$. The Neural Network settings are:
Figure 5.1: $\Delta E$ of signal $B$ candidates (left) and $D^0$ invariant mass difference (right) distributions of $D^0 \rightarrow K^+\pi^-$ signal MC with fits. Both distributions are fitted with double Gaussians.

- Transfer function: Sigmoid function
- Training method: Hybrid
- Number of hidden layer = 1
- Number of neurons in hidden layer = 8
- Number of epochs (training cycles) = 500

After setting the NN configurations and training the NN, another set of data is put through the NN to produce visual NN output ($O_{NN}$). The contents of the dataset are similar to that of the training and validation samples in that it has 20000 signal events and 20000 cuds continuum backgrounds. In addition, we run the NN on off-peak data to check if the output is consistent with that of the continuum. The $O_{NN}$ of $D^0 \rightarrow K^+\pi^-$ is shown in Figure 5.2. The continuum and off-peak data agree with each other well.
Figure 5.2: Neural Network output \((O_{NN})\) of \(D^0 \to K^+\pi^-\) from samples of signal, \textit{cuds} and off-peak data with 20000 events each. Red solid line represents signal and blue represents continuum. The dots are off-peak data.

5.4 Summary of Analysis Cuts

Table 5.2 lists the selection cuts that will be applied on data for the WS \(K^+\pi^-\) mode (as well as the RS \(K^-\pi^+\)) in the ADS analysis.

<table>
<thead>
<tr>
<th>Selection cut</th>
<th>(K^+\pi^-)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>&gt; 0.85</td>
</tr>
<tr>
<td>(</td>
<td>\cos\theta_{Helicity}(K^*)</td>
</tr>
<tr>
<td>(</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
</tr>
<tr>
<td>(</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
</tr>
<tr>
<td>(</td>
<td>m(K_s^0) - M(K_s^0)_{PDG}</td>
</tr>
<tr>
<td>PID on dau. 1 from (D^0)</td>
<td>KLHVeryLoose</td>
</tr>
<tr>
<td>(</td>
<td>\Delta E</td>
</tr>
</tbody>
</table>

Table 5.2: Selection criteria of WS \(D^0 \to K^+\pi^-\) and RS \(D^0 \to K^-\pi^+\).
5.5 Signal Efficiencies

Table 5.3 shows the total Run 1 - 5 signal efficiencies of the WS and RS modes. Individual cut efficiencies are shown in Table 5.4.

<table>
<thead>
<tr>
<th>$D^0$ Mode</th>
<th>Signal $\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+\pi^-$</td>
<td>$9.59 \pm 0.08%$</td>
</tr>
<tr>
<td>$K^-\pi^+$</td>
<td>$9.45 \pm 0.08%$</td>
</tr>
</tbody>
</table>

Table 5.3: Signal efficiencies of WS $D^0 \to K^+\pi^-$ and RS $D^0 \to K^-\pi^+$.

<table>
<thead>
<tr>
<th>Selection Cut</th>
<th>$K^+\pi^-$</th>
<th>$K^-\pi^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neural Network</td>
<td>49.2%</td>
<td>48.7%</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_{Helicity}(K^*)</td>
<td>)$ signed, 2D, DOF pull $(K^0_s)$</td>
</tr>
<tr>
<td>$</td>
<td>m(D^0) - M(D^0)_{PDG}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>m(K^{<em>-}) - M(K^{</em>-})_{PDG}</td>
<td>$</td>
</tr>
<tr>
<td>$</td>
<td>m(K^0_s) - M(K^0_s)_{PDG}</td>
<td>$</td>
</tr>
<tr>
<td>PID on dau. 1 from $D^0$</td>
<td>95.5%</td>
<td>95.6%</td>
</tr>
<tr>
<td>$</td>
<td>\Delta E</td>
<td>$</td>
</tr>
</tbody>
</table>

Table 5.4: Summary of individual cut efficiencies of the two ADS modes.

5.6 Peaking Backgrounds

In this section the main source of peaking background in the WS $m_{ES}$ signal region (Right-Sign pollution) is studied. Possible fake $D^0$ (from $\Delta m(D^0)$ sideband)
and peaking background (from $\Delta E$ sideband) contaminations are also investigated.

First, the $\Delta E$ sideband is checked and no evidence of peaking background is found (Figure 5.3). Next the background in which a fake $D^0$ is combined with a true $K^{*-}$ is checked. The $m_{ES}$ distribution of the $\Delta m(D^0)$ sideband of Run 1 - 5 Data (344.7 fb$^{-1}$) is fit with a PDF of Argus+Gaussian. The plot of the distribution along with the fit is shown in Figure 5.3. No peak is found and thus fake $D^0$ background is not a source of background in this analysis.

Figure 5.3: $m_{ES}$ distributions in $\Delta E$ (left) and $\Delta m(D^0)$ (right) sideband. The Argus+Gaussian fit is also shown. The sample is $B^- \to (K^+\pi^-)_{D^0}K^{*-}$ with Run 1 - 5 data.

### 5.6.1 Right-Sign Pollution

The ratio of the signal efficiency of the ADS selection cuts on WS events ($\epsilon_{WS}$) to RS events ($\epsilon_{RS}$) is used to estimate the number of right-sign (Cabibbo-favored $D^0 \to K^-\pi^+$ decay) pollution in the wrong-sign (doubly-Cabibbo-suppressed $D^0 \to K^+\pi^-$ decay) $m_{ES}$ signal region. The RS contamination is defined as:

$$\text{RS contamination} = \frac{\epsilon_{WS}}{\epsilon_{RS}} \times \text{RS data signal yield}.$$  \hfill (5.1)
Selection cuts are applied on a signal MC sample of 175000 RS $B^- \rightarrow (K^-\pi^+)_{D0}K^{*-}$ events and the number of RS events that are mis-reconstructed into WS events and survive the cuts is measured to calculate $\epsilon_{WS}$. The RS efficiency ($\epsilon_{RS}$) is taken from last section. Table 5.5 summarizes $\epsilon_{RS}$ and $\epsilon_{WS}$ for Run 1 - 5 $K^+\pi^-$ signal MC. This contribution will be subtracted from the wrong-sign peak in the simultaneous fit. The measured RS contamination will be reported in the next section when the simultaneous fit for signal extraction is performed.

<table>
<thead>
<tr>
<th>RS-Pollution</th>
<th>Run 1 - 5 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{RS}$</td>
<td>9.45 ± 0.08</td>
</tr>
<tr>
<td>$\epsilon_{WS}$</td>
<td>0.13 ± 0.01</td>
</tr>
</tbody>
</table>

Table 5.5: $\epsilon_{RS}$ and $\epsilon_{WS}$ for Run 1 - 5 $K^+\pi^-$ signal MC for calculation of RS pollution.

### 5.7 Extended Maximum Likelihood Fit

#### 5.7.1 Fit Strategy

We perform an extended maximum likelihood fit to the $m_{ES}$ distributions in the WS and RS modes simultaneously. A Gaussian is used for modeling both the WS and RS signals and an Argus function is used for describing the background. The likelihood function of the fit is:

$$L^{SR} = e^{-(N_{\text{Sig}} + N_{\text{Bkg}})} \times \prod_{N} (N_{\text{Sig}} \cdot \mathcal{G} + N_{\text{Bkg}} \cdot \mathcal{A}).$$  \hspace{1cm} (5.2)

The number of signal events in the WS $m_{ES}$ distribution, $N_{\text{Sig}}$, contains two parts. The first is the real WS signal and the second is the RS contamination explained in last section.
To simplify the fit, only a single Argus function is used to describe the background shapes in the WS and RS modes. To validate this, the Argus shape parameters are allowed to float in the fits to these two \( m_{ES} \) distributions. The \( \xi \) for \( D^0 \rightarrow K^+\pi^- \) is \(-29.0 \pm 20.5\) and the \( \xi \) for \( D^0 \rightarrow K^-\pi^+ \) is \(-28.0 \pm 18.7\). The two numbers are statistically consistent and the assumption is justified. Hence, there are a total of three floating (mean and sigma for the signal Gaussian and \( \xi \) for the Argus background) and one fixed (the Argus end-point \( E_{Max} \)) shape parameters in the simultaneous fit. The WS sample is split by the charge of \( B \) candidates to measure \( A_{ADS} \).

### 5.7.2 Right-Sign Pollution

As described in Section 5.6, we need to account for the presence of the RS events in the WS \( m_{ES} \) signal region. We first run the fit to measure the RS signal yield. Note that the number of RS pollution does not affect the shapes of the Gaussian and Argus function, it only reduces the number of \( K^+\pi^- \) signal events. The fit yields 172.8 ± 14.2 \( K^-\pi^+ \) right-sign events on Run 1 - 5 data. Therefore, according to Equation 5.1, the RS pollution is 2.38 ± 0.27 events. We will take 0.27 to be the associated systematic error.

The signal yields from fits of \( B^+ \rightarrow [K^+\pi^-]_{D^0}K^{*+} \) and \( B^- \rightarrow [K^-\pi^+]_{D^0}K^{-} \) are 102.3 ± 10.8 and 69.9 ± 9.3 events respectively. Therefore the right-sign contaminations are 1.41 ± 0.18 for \( B^+ \) and 0.96 ± 0.15 for \( B^- \).

### 5.7.3 Fit Results

The results from the simultaneous fit on 344.7 fb\(^{-1}\) (379 million \( B\bar{B} \) pairs) of data are shown in Tables 5.6 and 5.7. Figure 5.4 shows the plots of the fits to the WS and RS samples (for calculation of \( R_{ADS} \)) and the two WS samples that are separated by
$B$ charge (for calculation of $A_{ADS}$). An asymmetry of $(18.8 \pm 8.4)$% seen in the RS $K^{-\pi^+}$ sample when RS contamination was calculated in last section warrants a check on the signal MC. A 175000-event $K^{-\pi^+}$ signal MC sample reads an asymmetry of $(1.6 \pm 0.8)$%. Again, the asymmetry observed in data is not due to a mistake in the fitting procedure.

<table>
<thead>
<tr>
<th>Asymmetry Fit</th>
<th>Wrong-Sign</th>
<th>Right-Sign</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Shape Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Argus end-point (MeV/$c^2$)</td>
<td>5291.00 (FIXED)</td>
<td></td>
</tr>
<tr>
<td>Argus shape parameter $\xi$</td>
<td>-30.09 ± 8.60</td>
<td></td>
</tr>
<tr>
<td>Gaussian mean (MeV/$c^2$)</td>
<td>5278.50 ± 0.20</td>
<td></td>
</tr>
<tr>
<td>Gaussian sigma (MeV/$c^2$)</td>
<td>2.38 ± 0.16</td>
<td></td>
</tr>
<tr>
<td><strong>Yield (Signal Region)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gaussian (Signal)</td>
<td>11.47 ± 4.98</td>
<td>172.79 ± 14.20</td>
</tr>
<tr>
<td>RS-Contamination</td>
<td>2.38 ± 0.27</td>
<td>-</td>
</tr>
<tr>
<td>Argus</td>
<td>74.15 ± 9.23</td>
<td>154.13 ± 13.50</td>
</tr>
<tr>
<td>$R_{ADS} = 0.066 \pm 0.029$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.6: Fit results for $R_{ADS}$ on 344.7 fb$^{-1}$ of Run 1 - 5 data.

### 5.8 Systematic Errors of $R_{ADS}$ and $A_{ADS}$

#### 5.8.1 Asymmetry in the Detection Efficiency

For $A_{ADS}$, we use the same number quoted in [31] for the systematic error due to detector asymmetry. In the $B^- \rightarrow D^0K$ analysis, the authors used several control samples from data and Monte Carlo to measure the charge asymmetry. The asymmetry in Monte Carlo is $-(0.4 \pm 1.0)$% and it is $-(1.6 \pm 0.6)$% in the data samples. We will take $(1.6 + 0.6)$% = 2.2% as the systematic error.
\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
\textbf{Asymmetry Fit} & \textbf{Wrong-Sign} & \textbf{Right-Sign} \\
\hline
\textit{Shape Parameters} & & \\
Argus end-point (MeV/c^2) & 5291.00 (FIXED) & \\
Argus shape parameter $\xi$ & -30.09 $\pm$ 8.60 & \\
Gaussian mean (MeV/c^2) & 5278.50 $\pm$ 0.20 & \\
Gaussian sigma (MeV/c^2) & 2.38 $\pm$ 0.16 & \\
\hline
\textit{Yield (Signal Region)} & & \\
Gaussian (Signal) & 7.73 $\pm$ 3.84 ($B^+$) & 172.79 $\pm$ 14.20 \\
& 3.77 $\pm$ 3.05 ($B^-$) & - \\
RS-Contamination & 1.41 $\pm$ 0.18 ($B^+$) & - \\
& 0.96 $\pm$ 0.15 ($B^-$) & - \\
Argus & 38.86 $\pm$ 6.67 ($B^+$) & 154.13 $\pm$ 13.50 \\
& 35.27 $\pm$ 6.31 ($B^-$) & - \\
\hline
\end{tabular}
\caption{Fit results for $A_{ADS}$ on 344.7 fb$^{-1}$ of Run 1 - 5 data.}
\end{table}

For $R_{ADS}$, the derivation of the detector asymmetry is as follows. Let $W^\pm$ and $R^\pm$ be the measured wrong-sign and right-sign signal yields. $\epsilon_{D^0}$ ($\epsilon_{\bar{D}^0}$) is the efficiency to reconstruct a $K^+\pi^-$ (or $K^-\pi^+$) at the $D^0$ ($\bar{D}^0$) mass. $\epsilon_{K^{*\pm}}$ is the efficiency to reconstruct a $K^{*\pm}$ which decays to $K^0_S\pi^{\pm}$. We also define the detector asymmetry of $D^0$ and $K^*$ with the variables $A_D = \frac{\epsilon_{D^0} - \epsilon_{\bar{D}^0}}{\epsilon_{D^0} + \epsilon_{\bar{D}^0}}$ and $A_K = \frac{\epsilon_{K^{*+}} - \epsilon_{K^{*-}}}{\epsilon_{K^{*+}} + \epsilon_{K^{*-}}}$. We rewrite $R_{ADS}$ in first order as:

$$R_{ADS} = \frac{W^+\epsilon_{D^0}\epsilon_{K^{*-}} + W^-\epsilon_{\bar{D}^0}\epsilon_{K^{*-}}}{R^-\epsilon_{D^0}\epsilon_{K^{*-}} + R^+\epsilon_{\bar{D}^0}\epsilon_{K^{*-}}}$$

(5.3)

$$= \frac{W^-(1 + A_D)(1 + A_K) + W^+(1 - A_D)(1 - A_K)}{R^-(1 - A_D)(1 + A_K) + R^+(1 + A_D)(1 - A_K)}$$

(5.4)

$$= \frac{W^- + W^+ + (W^- - W^+)(A_D + A_K)}{R^- + R^+ - (R^- - R^+)(A_K - A_D)}$$

(5.5)

$$= \frac{W^- + W^+}{R^- + R^+} \times \frac{1 + \frac{W^- - W^+}{W^- + W^+}(A_D + A_K)}{1 - \frac{R^- - R^+}{R^- + R^+}(A_K - A_D)}$$

(5.6)

$$= R_{ADS}^{Meas}(1 + A_D^{Meas}(A_D + A_K) + \frac{R^- - R^+}{R^- + R^+}(A_K - A_D)).$$

(5.7)
Figure 5.4: Simultaneous fit to $m_{ES}$ distributions in signal regions on Run 1 - 5 data. First row: Left: Wrong-sign $D^0 \rightarrow K^+\pi^-$ (the dashed-peak represents the RS-contamination). Right: Right-sign $D^0 \rightarrow K^-\pi^+$. Second row: Left: Wrong-sign $B^+$. Right: Wrong-sign $B^-$. The last term is negligible because of the small asymmetry in right-sign yields. The middle term is dominated by the charged pion detection efficiency imbalance $A_K$. Hence,

$$R_{ADS} = R_{ADS}^{Meas} (1 + A_{ADS}^{Meas} \times A_K) \quad (5.8)$$

$$= 0.076(1 + 0.222 \times 0.027) \quad (5.9)$$

$$= 0.076 + 0.00046. \quad (5.10)$$

Therefore, the systematic error due to detector asymmetry for $R_{ADS}$ is $\pm 0.00046$. 

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5.8.2 Right-Sign Peaking Background Subtraction

In Section 5.7.2 the associated systematic uncertainty due to right-sign events being identified as signal wrong-sign events is 0.27. From the fit we have 172.8 right-sign and 11.5 wrong-sign events. Therefore the systematic errors for $R_{ADS}$ and $A_{ADS}$ are 0.0016 ($=0.27/172.8$) and 0.0235 ($=0.27/11.5$) respectively.

5.8.3 Systematics from the Interference with $B \to D^0K^0_S\pi^-$

This source of systematic error will be quantified similarly to that in the GLW analysis: the results presented in [68] will be used. The biggest variations of $R_{ADS}$ and $A_{ADS}$ are quoted as the systematic errors due to the interference with $B \to D^0K^0_S\pi^-$ (Table 5.8).

<table>
<thead>
<tr>
<th></th>
<th>$R_{ADS}$</th>
<th>$A_{ADS}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. Upward Variation</td>
<td>0.0073</td>
<td>0.126</td>
</tr>
<tr>
<td>Max. Downward Variation</td>
<td>-0.0023</td>
<td>-0.126</td>
</tr>
<tr>
<td>Systematic Error Used</td>
<td>0.0073</td>
<td>0.126</td>
</tr>
</tbody>
</table>

Table 5.8: Results of the $B^- \to D^0[K^{*-} + K^0_S\pi^-]$ system modeling presented in [68].

5.8.4 Summary of Systematic Errors of $R_{ADS}$

Table 5.9 summarizes the systematic error calculation for $R_{ADS}$. The final number is ±0.0075.
5.9 Summary of the ADS Analysis Results

From the study of $B^- \rightarrow D^0K^{*-}$ decays with 379 million $\bar{B}B$ events (Run 1 - 5) the following results of the ADS analysis are presented:

\[
R_{ADS} = 0.066 \pm 0.029(stat.) \pm 0.008(syst.)
\]

\[
A_{ADS} = -0.344 \pm 0.451(stat.) \pm 0.130(syst.).
\]
Combination of results from the GLW and ADS analyses provide a determination on $r_B$. In addition, together with other experiments sensitive to $\gamma$, which include $B^- \to D^{(*)0}K^{(*)-}$ (GLW+ADS) and $B^- \to [K_\pi^0\pi^+\pi^-]_{D^{(*)0}K^*}$ (Dalitz), we can constrain the values of $\gamma$. The values of $r_s$ and $\gamma$ are extracted by the UTfit group [36] with our combined GLW and ADS results. Figure 6.1 shows the probability density function for $r_B^3$ at the 68 and 95% probability intervals. The value at 68% C.L. and probability range at 95% are:

$$r_B = 0.268 \pm 0.07 \text{ (68\% C.L.)}, \ [0.112, 0.397] \text{ (95\% C.L.).} \quad (6.1)$$

This is the first significant non-zero measurement of the ratio of the color-suppressed $B$ decay to the color-favored decay among all the GLW and ADS analyses. This result will produce a meaningful constraint on $\gamma$ when combined with other analyses (a zero $r_B$ gives no information on $\gamma$, according to Equations 2.77 (GLW) and 2.104 (ADS)).

Figure 6.2 is the probability density function for $\gamma$. It corresponds to a value of $\gamma$ of $(0 \pm 55)^\circ$ or $(180 \pm 49)^\circ$ at the 68% C.L. The lack of constraint from the

$^3$The UTfit uses $r_s$ but we assume that $r_s$ equals to $r_B$ in this thesis. More details can be found in Appendix C.
GLW and ADS measurements is expected due to the ambiguities introduced by the strong and weak phases ($0 \rightarrow \pi$). Therefore, our results should be combined with other $\gamma$-sensitive experiments if we want a better constraint on $\gamma$. Figure 6.3 is the determination of $\gamma$ using the latest published results from all GLW, ADS and Dalitz experiments [36]. The extracted value is:

$$
\gamma = (66.7 \pm 6.4)^\circ \ (68\% \ C.L.) .
$$

(6.2)

All the measurements of the $CP$ observables in this thesis are greatly limited by statistics, as evident from the statistical errors which are as many as four times larger than the systematics. The extra $\text{BABAR}$ data sample with approximately 95 million $B\bar{B}$ pairs, which will be ready in a couple months, will help the continuing effort of improving the statistical limits of the GLW and ADS modes. For example, with the additional data, we are expected to see much cleaner signal peaks in the $CP$- (GLW) and ADS modes. For the past several years, constant updates of precision measurements of CKM Unitarity Triangle parameters have greatly improved the constraints on $CP$-relating parameters (e.g. Figure 6.4 shows the evolution of the allowed region in the $(\rho, \eta)$ plane during the past 15 years). With the potential future Super-$B$ factory targeted at producing about 50 $ab^{-1}$ of $e^+e^-$ collision data, the $CP$ violation picture in the Standard Model will become very clear.
Figure 6.1: The PDF for $r_s$. The 68 and 95% C.L. are shown. They are extracted from UTfit [36] using the combined GLW and ADS results in this thesis.
Figure 6.2: The PDF for $\gamma$. The 68 and 95% C.L. are shown. They are extracted from UTfit [36] using the combined GLW and ADS results in this thesis.
Figure 6.3: The PDF for $\gamma$ extracted by UTfit [36] with the most updated results from all $\gamma$-sensitive experiments. The 68 and 95% C.L. are shown. (Note: results from this thesis were not used.)
Figure 6.4: Evolution during the last 15 years of the allowed region in the $(\bar{\rho}, \bar{\eta})$ plane from theoretical and experimental constraints [36].
CHAPTER 7

CONCLUSION

This thesis analyzes $B^- \rightarrow D^0 K^{*-}$ decay using the GLW and ADS methods. The measurement of $CP$ observables with 379 million $B\bar{B}$ events produced by the BABAR $B$-factory are summarized in Table 7.1. From the combined GLW and ADS results, $r_B$ is constrained to be $0.268 \pm 0.07$ at 68\% C.L. and $\gamma$ is constrained to be $(0 \pm 55)^\circ$ or $(180 \pm 49)^\circ$ at the 68\% C.L. Our $r_B$ measurement is the first non-zero measurement and will greatly the determination of $\gamma$ in future experiments.

<table>
<thead>
<tr>
<th>$CP$ Observables</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>GLW:</td>
<td></td>
</tr>
<tr>
<td>$A_{CP^+}$</td>
<td>$0.092 \pm 0.133(stat.) \pm 0.051(syst.)$</td>
</tr>
<tr>
<td>$A_{CP^-}$</td>
<td>$-0.199 \pm 0.206(stat.) \pm 0.068(syst.)$</td>
</tr>
<tr>
<td>$R_{CP^+}$</td>
<td>$2.174 \pm 0.352(stat.) \pm 0.083(syst.)$</td>
</tr>
<tr>
<td>$R_{CP^-}$</td>
<td>$1.142 \pm 0.273(stat.) \pm 0.113(syst.)$</td>
</tr>
<tr>
<td>$x^+$</td>
<td>$0.144 \pm 0.136(stat.) \pm 0.046(syst.)$</td>
</tr>
<tr>
<td>$x^-$</td>
<td>$0.357 \pm 0.138(stat.) \pm 0.050(syst.)$</td>
</tr>
<tr>
<td>ADS:</td>
<td></td>
</tr>
<tr>
<td>$R_{ADS}$</td>
<td>$0.066 \pm 0.029(stat.) \pm 0.008(syst.)$</td>
</tr>
<tr>
<td>$A_{ADS}$</td>
<td>$-0.344 \pm 0.451(stat.) \pm 0.130(syst.)$</td>
</tr>
</tbody>
</table>

Table 7.1: The measurement of $CP$ observables with $B^- \rightarrow D^0 K^{*-}$ decay using the GLW and ADS methods.
APPENDIX A

EFFICIENCY CORRECTIONS TO $R_{CP\pm}$

To account for the differences between the data and Monte Carlo samples, the raw Monte Carlo efficiencies are corrected to that expected in the data.

A.1 Efficiency Correction to $A_{CP}$

By construction, $A_{CP}$ (Equation 2.79) does not require efficiency corrections.

A.2 Efficiency Correction to $R_{CP}$

As we see from the definition of $R_{CP}$ (Equation 2.81) in Section 2.9.1, corrections that belong to decays common to both $CP$ and Non-$CP$ modes (e.g. $K^{*-} \rightarrow K_{S}^{0}\pi^{-}$) will cancel in the ratio and therefore can be disregarded. These include the efficiency corrections on the $K^{*}$, $\Delta E$ and continuum suppression cuts. We will only discuss systematics that remain after the top-and-bottom cancellation.

We use two methods to correct for the MC and data differences. For tracking efficiency, particle identification, $K_{S}^{0}$ identification efficiency and $\pi^{0}$ detection efficiency, we use standard recipes developed by the corresponding $BABAR$ task forces.

For analysis cuts, since we do not have a large sample of $B^{-} \rightarrow D^{0}\pi^{-}$, we are going to assume the correction to be unity and assign systematic error due to the specific
cut. To determine the error, we relax the cut by 10\% in signal MC and calculate according to this definition:

\[
\sigma(\text{cut}) = \frac{N_{\text{MC}}^{\text{Relaxed Cut}} - N_{\text{MC}}^{\text{Original Cut}}}{N_{\text{MC}}^{\text{Original Cut}}}. \tag{A.1}
\]

### A.3 Tracking Corrections

The Tracking Task Force recommends no tracking efficiency correction [69]. Both $K^\pm$’s and $\pi^\pm$’s from a $D^0$, $K^*$ or $\omega$ are chosen from the $\text{GoodTracksVeryLoose}$ list and the suggested systematic error is 0.22\% per track. $\pi^\pm$’s coming from a $K_s^0$ are chosen from the $\text{ChargedTracks}$ list and has a 0.38\% systematic error per track. The Task Force uses the so-called “Tau3-1 Tracking Efficiency Method”, which utilizes $\tau$ pair events with a 1 vs 3 topology to determine the tracking efficiency and the associated correction factor. Readers can refer to [69] for more details. Note that the total systematic of a system of tracks is the sum of the errors of its individual tracks. For example, the systematic of two $\text{GoodTracksVeryLoose}$ tracks is $(0.22 + 0.22) = 0.44\%$.

### A.4 PID Corrections

The correction due to PID is calculated using PID weighting. The recipe is provided by the Particle ID AWG [70]. We use the systematic errors cited in [66]. [66] compared with several similar analyses which used either $\text{Loose}$ or $\text{VeryLoose}$ PID criteria and assigned a systematic error of 2\% per use of PID. A systematic error of 4\% is taken where PID is used twice.
A.5 $\pi^0$ Corrections

The Neutral Reconstruction Analysis Working Group [71] recommends a flat correction. The correction depends on the list we take our $\pi^0$ candidates from. We use the $pi0AllDefault$ list and therefore a correction of 0.968311 will be applied. The systematic error is 3% per $\pi^0$.

A.6 $K^0_s$ Corrections

The $K^0_s$ correction recipe is also provided by the Tracking Task Force [72]. For $K^0_s$ candidates which come from $K^*$'s we choose the correction class “3DSign3DAlpha” with the distance of flight and momentum information. For $K^0_s$'s from $D^0$ in $CP$- modes, the correction class of “noSignnoAlpha” is used since we do not have a cut on the distance of flight. The results are shown in Tables A.1 and A.2.

All the above corrections and their systematics for $CP+$, $CP$- and Non-$CP$ modes are summarized in Table A.3.

A.7 $CP-$ Analysis Cuts

We use the second method mentioned earlier in this chapter for studying corrections in $CP$- modes, i.e. we assume no correction to the efficiency from the $K^0_s\pi^0$, $K^0_s\phi$ and $K^0_s\omega$ cuts and estimate their corresponding errors by relaxing the cuts by 10%. Those cuts include $\phi$ and $\omega$ invariant masses, and the cosine of the helicity and Dalitz angle of the $\omega$ candidates. In addition, since the $D^0$ invariant mass cuts in $K^0_s\pi^0$ and $K^0_s\omega$ are different from the Non-$CP$ one, we need to re-calculate the systematic errors due to this cut in these two modes.
Table A.1: Run-by-run and luminosity-weighted average efficiency corrections due to $K_s^0$ (from $K^*$) identification and associated systematic errors for $CP^+$, $CP^-$ and Non-$CP$ modes.

The results of the calculation are recorded in Table A.4. The systematic of the $m(D^0)$ cut is added since the cut windows are different in $K_s^0\pi^0$ and $K_s^0\omega$ than in $K^-\pi^+$. 

A.8 Summary

A summary of the efficiency corrections required in the calculations of $R_{CP}$ is shown in Table A.5. Since relative efficiency is defined as a ratio between $CP$ and Non-$CP$ modes, only corrections that survive the cancellation are listed in the table.
Table A.2: Run-by-run and luminosity-weighted average efficiency corrections due to $K^0_s$ (from $D^0$) identification and associated systematic errors for $CP$- modes.

<table>
<thead>
<tr>
<th>Run</th>
<th>$K^0_s\pi^0$</th>
<th>$K^0_s\phi$</th>
<th>$K^0_s\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>0.987 2.6%</td>
<td>0.998 2.0%</td>
<td>0.992 3.5%</td>
</tr>
<tr>
<td>Run 2</td>
<td>1.000 2.0%</td>
<td>1.006 1.6%</td>
<td>0.998 2.2%</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.998 2.5%</td>
<td>0.999 1.8%</td>
<td>0.994 2.8%</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.956 3.5%</td>
<td>0.966 1.1%</td>
<td>0.954 2.0%</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.953 1.9%</td>
<td>0.981 1.1%</td>
<td>0.957 1.8%</td>
</tr>
<tr>
<td>$\mathcal{L}$-weighted average</td>
<td>0.968 2.0%</td>
<td>0.984 1.3%</td>
<td>0.969 2.1%</td>
</tr>
</tbody>
</table>

The same reasoning is applied in the case of the absence of corrections in $K^-\pi^+$ mode, as all sources of corrections are cancelled out top-and-bottom.

Table A.3: A summary of efficiency corrections due to tracking, PID, $\pi^0$ reconstruction and $K^0_s$ identification and associated systematic errors for $CP+$, $CP$- and Non-$CP$ modes.

<table>
<thead>
<tr>
<th>$CP+$</th>
<th>$K^+K^-$</th>
<th>$\pi^+\pi^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tracking</td>
<td>1.0000 ± 0.0142</td>
<td>1.4%</td>
</tr>
<tr>
<td>PID</td>
<td>0.9946 ± 0.0400</td>
<td>4.0%</td>
</tr>
<tr>
<td>$K^0_s$</td>
<td>0.9902 ± 0.0142</td>
<td>1.4%</td>
</tr>
</tbody>
</table>

The same reasoning is applied in the case of the absence of corrections in $K^-\pi^+$ mode, as all sources of corrections are cancelled out top-and-bottom.
Table A.4: Estimation of systematic errors which will be used in the $R_{CP-}$ calculation.

<table>
<thead>
<tr>
<th></th>
<th>$K_s^0\pi^0$</th>
<th>$K_s^0\phi$</th>
<th>$K_s^0\omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m(\phi)$</td>
<td>-</td>
<td>1.2%</td>
<td>-</td>
</tr>
<tr>
<td>$m(\omega)$</td>
<td>-</td>
<td>-</td>
<td>2.7%</td>
</tr>
<tr>
<td>$\cos \theta_{\text{Helicity}}$</td>
<td>-</td>
<td>-</td>
<td>1.2%</td>
</tr>
<tr>
<td>$\cos \theta_{\text{Dalitz}}$</td>
<td>-</td>
<td>-</td>
<td>3.8%</td>
</tr>
<tr>
<td>$m(D^0)$</td>
<td>4.1%</td>
<td>-</td>
<td>2.6%</td>
</tr>
<tr>
<td>$R_{CP-}$ Systematic</td>
<td>4.1%</td>
<td>1.2%</td>
<td>5.5%</td>
</tr>
</tbody>
</table>
Table A.5: Corrections to the relative efficiencies, $\epsilon_{\text{Rel}}$, which are required to calculate the observables $R_{CP}$. The raw signal efficiencies are taken from Section 4.10. Note that for $K^0_S$, only the corrections to $K^0_S$ from $D^0$ are recorded. Corrections to $K^0_S$ from $K^+$ in CP and Non-CP modes cancel each other out. Tracking and PID corrections in CP modes listed are after dividing by Non-CP numbers as well.
APPENDIX B

SYSTEMATIC ERRORS OF $x^\pm$

B.1 Systematics from Same-Final-State Backgrounds

As we have seen in the discussion of systematics of $A_{CP}$ and $R_{CP}$, $K^0_s\phi$ and $K^0_s\omega$ modes suffer contaminations from backgrounds that share the same final states of $K^0_s K^+ K^-$ and $K^0_s \pi^+ \pi^- \pi^0$ respectively. Following [66], we calculate the biases due to the non-resonant backgrounds. The errors on the biases are assigned as the systematic errors of $x^\pm$.

We note that since $N(B^+ \to D_{CP+}^0 K^{s+})$ as the sum of signal $S_{CP+}$ and background $W_{B^\pm}$, we can rewrite $x^\pm_{Meas}$ as:

$$x^\pm_{Meas} = \frac{N_{B^+}^{B^\pm}/\epsilon_{CP+} - N_{B^-}^{B^\pm}/\epsilon_{CP-}}{2 \cdot N_{NC}/\epsilon_{NC}}$$

(B.1)

$$= x^\pm - \frac{\epsilon_{NC}}{\epsilon_{CP-} \cdot 2/\epsilon_{NC}} \cdot W_{B^\pm}$$

(B.2)

The bias can then be written as:

$$\delta x^\pm = x^\pm - x^\pm_{Meas} = \frac{\epsilon_{NC}}{\epsilon_{CP-} \cdot 2 \cdot N_{NC}} \cdot W_{B^\pm}$$

(B.3)

Since

$$W_{B^\pm} = \frac{(W_{B^+} + W_{B^-})}{2} (1 \mp A_{CP-}) = (\epsilon \cdot N_{CP-}/(1 + \epsilon)) \cdot \frac{1}{2} A_{CP-},$$

(B.4)
where $\epsilon = N_{Non}/N_{Res}$ as defined in Section 4.12, we have:

$$
\delta x^{\pm} = \frac{\epsilon_{CP}}{\epsilon_{CP-}} \cdot \frac{1}{4 \cdot N_{NCP}} \cdot \frac{\epsilon \cdot N_{CP-} \cdot (1 \mp A_{CP-})}{1 + \epsilon}.
$$

(B.5)

Plug in the appropriate numbers and we get,

$$
\delta x^+ = 0.035 \pm 0.024 \quad \text{(B.6)}
$$

$$
\delta x^- = 0.023 \pm 0.017. \quad \text{(B.7)}
$$

The systematic errors due to same-final-state backgrounds for $x^+$ and $x^-$ are thus $\pm 0.024$ and $\pm 0.017$ and respectively.

**B.2 Systematics from the Interference with $B \rightarrow D^0 K_s^0 \pi$**

We will use the numbers in [66]. Using their model for the non-resonant component to calculate the maximum and minimum of $(x^{\pm}_{NR} - x^{\pm})$, the systematic error for both $x^+$ and $x^-$ due to $B^- \rightarrow D^0 K_s^0 \pi^0$ background is $\pm 0.0277$.

**B.3 Systematics from Fake $D^0$ Background**

Again, we need to estimate the systematic error due to our assumption that there is no charge asymmetry in the fake $D^0$ background measured in the $m(D^0)$ background. We define $P_{CP}$, the number of peaking backgrounds in the signal region,

$$
P_{CP} = \frac{1}{2}(P_{CP}^{B^+} + P_{CP}^{B^-})
$$

(B.8)

$$
A_{CP}^p P_{CP} = \frac{1}{2}(P_{CP}^{B^+} - P_{CP}^{B^-}).
$$

(B.9)

$A_{CP}^p$ is the hypothetical $CP$ asymmetry in the fake $D^0$ backgrounds and we assume it to be 50% and -50% for $A_{CP}^p$ and $A_{CP}^p$ respectively. We can rewrite the equation of $x^{\pm}$ as:

$$
x^{\pm} = \frac{(N_{CP}^{B^+} + P_{CP}^{B^+})/\epsilon_{CP} - (N_{CP}^{B^-} + P_{CP}^{B^-})/\epsilon_{CP}}{2 \cdot N_{NCP}/\epsilon_{NCP}}.
$$

(B.10)
The systematic error is then:

\[
\Delta x^\pm = x^\pm (A_{CP\pm}^p = \pm 50\%) - x^\pm (0\%)
\]
\[
= \frac{P_{CP+} A_{CP+}^p / \epsilon_{CP+} - P_{CP-} A_{CP-}^p / \epsilon_{CP-}}{2 \cdot N_{NCP} / \epsilon_{NCP}}
\]

(B.11)

(B.12)

The systematic from fake $D^0$ background is $\pm 0.026$ for both $x^+$ and $x^-$.

**B.4 Other Systematics of $x^\pm$**

- Charge bias in the analysis: This is the same error as described in Section 4.12. We quote the result from the study carried out in [31]. The authors measured the asymmetry in a number of control samples from data and Monte Carlo. A 2.2% systematic error due to inherent asymmetry will be used.

- Systematic from efficiency corrections: This error is determined by multiplying the systematic errors of $\epsilon_{CP\pm} / \epsilon_{NCP}$ to the central values of $x^\pm$. The final result is $\pm 0.006$ and $\pm 0.028$ for $x^+$ and $x^-$ respectively.

**B.5 Summary of Systematic Errors of $x^\pm$**

Table B.1 summarizes the systematic error calculation for $x^\pm$. The final result is $\pm 0.046$ and $\pm 0.051$ for $x^+$ and $x^-$ respectively.
Table B.1: Summary of systematic errors of $x^\pm$.

<table>
<thead>
<tr>
<th>Source</th>
<th>$x^+$</th>
<th>$x^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effect of Same-Final-State Bkg</td>
<td>0.024</td>
<td>0.017</td>
</tr>
<tr>
<td>Effect of $B \to D^0 K^0 \pi$ Bkg</td>
<td>0.028</td>
<td>0.028</td>
</tr>
<tr>
<td>Inherent Asymmetry</td>
<td>0.002</td>
<td>0.002</td>
</tr>
<tr>
<td>Asymmetry in fake $D^0$ Bkg</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>Systematic Error in $\epsilon_{CP}/\epsilon_{NCP}$</td>
<td>0.006</td>
<td>0.028</td>
</tr>
<tr>
<td>Total Systematic Error</td>
<td>0.046</td>
<td>0.050</td>
</tr>
</tbody>
</table>
APPENDIX C

$B^- \rightarrow D^0 X_s^-$ DECAYS

In the study of $B^- \rightarrow D^0 K^{*-}$ decays, one must consider the effect of the large $K^{*-}$ natural width (50.8 MeV/$c^2$) on the relationship between the CKM angle $\gamma$ and the experimental observables. In this chapter, we will detail the difference between $r_S$ and $r_B$ and the introduction of the observables $x^\pm$. A brief description of three-body $B^- \rightarrow D^0 X_s^-$ decay rate is also included.

We will write the amplitudes of the $B^- \rightarrow (D^0 X_s^-)_p$ and $B^- \rightarrow (\overline{D}^0 X_s^-)_p$ processes, where $p$ indicates a point in the phase space of the final state and $X_s^-$ is a state with strangeness, as:

\begin{align}
A(B^- \rightarrow (D^0 X_s^-)_p) &= A_{cp}e^{i\delta_p} \\
A(B^- \rightarrow (\overline{D}^0 X_s^-)_p) &= A_{up}e^{i\delta_p}e^{-i\gamma} \\
A(D^0 \rightarrow f) &= A_f e^{i\delta_f} \\
A(D^0 \rightarrow \overline{f}) &= A_{\overline{f}} e^{i\delta_{\overline{f}}},
\end{align}

where the index $p$ indicates the position in the $D^0 X_s^-$ phase space, that is, $A_c$, $A_u$, $\delta_c$ and $\delta_u$ generally vary as a function of $p$. The subscript $c$ and $u$ refer to the $b \rightarrow c$ and $b \rightarrow u$ transitions respectively. $A_{cp}$, $A_{up}$, $A_f$ and $A_{\overline{f}}$ are real and positive. The amplitudes $A_{cp}e^{i\delta_p}$ and $A_{up}e^{i\delta_p}e^{-i\gamma}$ generally include both the resonant $B^- \rightarrow$
$D^0/\overline{D}^0K^{-}$ and the non-resonant contributions. The non-resonant part will not be discussed here.

The partial rates $\Gamma(B^{-} \to D^0X_s^-)$ and $\Gamma(B^+ \to \overline{D}^0X_s^-)$ are:

\[ \Gamma(B^{-} \to D^0X_s^-) = \int dp A_{cp}^2 \]  
\[ \Gamma(B^+ \to \overline{D}^0X_s^-) = \int dp A_{up}^2, \]  
\[ (C.5) \]
\[ (C.6) \]

The quantities $r_s$, $k$ and $\delta_s$ are introduced as:

\[ r_s^2 = \frac{\Gamma(B^+ \to \overline{D}^0X_s^-)}{\Gamma(B^+ \to D^0X_s^-)} = \frac{\int dp A_{up}^2}{\int dp A_{cp}^2}, \]
\[ k e^{i\delta_s} = \frac{\int dp A_{cp} A_{up} e^{i\delta}}{\sqrt{\int dp A_{cp}^2 \int dp A_{up}^2}}, \]
\[ (C.7) \]
\[ (C.8) \]

where $0 \leq k \leq 1$ and $\delta_s$ is between 0 and $2\pi$. $k$ is introduced to include the non-resonant $B^+ \to D^0[K\pi]_{non-K^*}$ contributions and the fact that the $b \to c$ and $b \to u$ amplitudes may vary over the $K^*$ phase space (width). In the limit of a $B \to 2$-body decay, in which the $X_s^-$ has a small natural width, such as $B^{-} \to D^0K^-$, we have:

\[ r_s \to r_B \equiv \frac{|A(B^{-} \to \overline{D}^0K^-)|}{|A(B^+ \to D^0K^-)|}, \]
\[ \delta_s \to \delta_B, \]
\[ k \to 1. \]
\[ (C.9) \]
\[ (C.10) \]
\[ (C.11) \]

In this thesis, we assume that the color-favored $B^+ \to D^0K^{*-}$ and color-suppressed $B^+ \to \overline{D}^0K^{*-}$ amplitudes are independent of the phase space in the $K^*$ mass region, i.e. $r_s = r_B$. This is a reasonable assumption as [73] has shown that the distribution of $k$ is narrow and its values are no more than 10% smaller than 1.

The amplitude of $B^+ \to D^0X_s^-$ when the $D^0$ decays to two bodies can be written as:

\[ A(B^+ \to D^0X_s^-) = A_{cp} e^{i\delta_{cp}} + A_{up} e^{i\delta_{up} + \gamma}, \]
\[ (C.12) \]
and the rate is
\[ \Gamma(B^+ \to D^0 X^-) \propto 1 + r_s^2 \pm 2k r_s \cos \delta_s \cos \gamma. \]  \hspace{1cm} (C.13)

A set of “cartesian coordinates” is defined as:
\[ x_s^\pm = Re[k r_s e^{i(\delta_s \pm \gamma)}], \]  \hspace{1cm} (C.14)
\[ y_s^\pm = Im[k r_s e^{i(\delta_s \pm \gamma)}]. \]  \hspace{1cm} (C.15)

In the case of \( r_s \to r_B \) and \( k \to 1 \), we have:
\[ \Gamma(B^+ \to D^0 X_s^-) \propto 1 + r_B^2 \pm 2r_B \cos \delta_B \cos \gamma \]  \hspace{1cm} (C.16)
\[ x^\pm = r_B \cos(\delta_B \pm \gamma) \]  \hspace{1cm} (C.17)
\[ y^\pm = r_B \sin(\delta_B \pm \gamma). \]  \hspace{1cm} (C.18)
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