THEORETICAL AND EXPERIMENTAL STUDIES
OF ORIFICED, HOLLOW CATHODE OPERATION

DISSERTATION

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To my parents
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# TABLE OF CONTENTS

DEDICATION ................................................................. ii
ACKNOWLEDGEMENTS ....................................................... iii
VITA ................................................................. iv
LIST OF FIGURES ........................................................... ix
LIST OF TABLES .......................................................... xxiii
LIST OF SYMBOLS .......................................................... xxiv

## CHAPTER

### I INTRODUCTION

1.1 The hollow cathode ................................................. 2
1.2 Scope of the present work ......................................... 3

### II BACKGROUND

2.1 The Principle of the hollow cathode ............................... 6
2.2 Experimental and theoretical research ............................. 7
2.3 Cathode material and lifetime .................................... 17

### III A BASIC MODEL OF HOLLOW CATHODE ARC DISCHARGES

21

3.1 Introduction ......................................................... 21
3.2 Approach .......................................................... 21
3.3 Problem formulation ............................................... 24
3.3.1 Thermionic current density .................................. 24
3.3.2 Ion current density .......................................... 26
3.3.3 Plasma electron current density .............................. 27
3.3.4 Discharge current ........................................... 27
3.3.5 Cathode internal pressure ................................... 28
3.3.6 Mass flow rate ............................................... 29
3.3.7 Two-temperature Saha equation .............................. 30
3.3.8 Power balance ............................................... 31
3.3.9 Plasma electric conductivity ................................. 32
8.3 Recommendations ......................... 144

APPENDICES  
A  Calculation of the Species Composition  148

B  Calculation of electron number density  150

C  Evaluation of the Knudsen Number  153

BIBLIOGRAPHY  .............................................. 156

viii
LIST OF FIGURES

3.1 Description of hollow cathode physical processes .......................... 23
3.2 Effect of discharge current on cathode surface temperature (comparison between theory and experiment). ................................. 36
3.3 Effect of discharge current on electron temperature (comparison between theory and experiment). ................................. 37
3.4 Effect of discharge current on electron number density (comparison between theory and experiment). ................................. 38
3.5 Effect of discharge current on plasma potential (comparison between theory and experiment). ................................. 39
3.6 Effect of internal pressure on cathode surface temperature (comparison between theory and experiment). ................................. 40
3.7 Effect of internal pressure on electron temperature (comparison between theory and experiment). ................................. 41
3.8 Effect of internal pressure on electron number density (comparison between theory and experiment). ................................. 42
3.9  Effect of internal pressure on plasma potential (comparison between theory and experiment). ........................................ 43

4.1  Photograph of the NASA-Lewis Research Center bell jar 6 test facility 46

4.2  Schematic of hollow cathode test facility .................................................. 47

4.3  Schematic of bell jar 6 vacuum facility ........................................................ 49

4.4  Photograph of the hollow cathode setup inside bell jar 6 ......................... 50

4.5  Schematic of the main components of the gas feed system .................... 52

4.6  Photograph of the hollow cathode mounted inside bell jar 6 ................. 54

4.7  Schematic of the hollow cathode assembly .................................................. 55

4.8  Schematic of single Langmuir probe electric circuit .................................. 57

4.9  Photograph of electrostatic (top) and thermocouple (bottom) probes ..... 58

4.10 Schematic of the spectroscopic diagnostic apparatus ................................. 61

4.11 Schematic of pyrometry diagnostic instrumentation ..................................... 64

4.12 Photograph of pyrometric (top) and spectroscopic (bottom) probes ........ 65

5.1  The effect of discharge current on electron temperature along the cathode center line at different distances from the orifice (argon). .......................... 83

5.2  The effect of discharge current on electron temperature along the cathode center line at different distances from the orifice (xenon). ........................... 83

5.3  The effect of flow rate on electron temperature along the cathode center line at different distances from the orifice (argon). .............................. 84
5.4 The effect of flow rate on electron temperature along the cathode center line at different discharge currents (argon). .................. 84

5.5 Axial distribution of electron temperature along the cathode center line at different discharge currents (argon, \(d_o = 0.76\) mm). ........ 85

5.6 Axial distribution of electron temperature along the cathode center line at different discharge currents (argon, \(d_o = 1.21\) mm). .... 85

5.7 Axial distribution of electron temperature along the cathode center line at different discharge currents (xenon, \(d_o = 1.21\) mm). .... 86

5.8 Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, \(I = 3\) A). ............. 86

5.9 Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, \(I = 5\) A). ............. 87

5.10 Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, \(I = 9\) A). ............. 87

5.11 Axial distribution of electron temperature along the cathode center line in argon and xenon discharges (\(I = 5\) A). ............. 88

5.12 Axial distribution of electron temperature along the cathode center line in argon and xenon discharges (\(I = 9\) A). ............. 88

5.13 Axial distribution of electron temperature along the cathode center line in argon and xenon discharges (\(I = 15\) A). ............. 89

xi
5.14 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 3 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

5.15 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 7 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

5.16 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 10 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

5.17 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 3 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 1.27 \, mm \)).

5.18 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 7 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 1.27 \, mm \)).

5.19 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 10 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 1.27 \, mm \)).

5.20 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 15 \, A, \dot{m} = 0.93 \, A \, \text{eq.}, d_o = 1.27 \, mm \)).

5.21 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 7 \, A, \dot{m} = 0.5 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

5.22 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 10 \, A, \dot{m} = 0.5 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

5.23 Calculation of the electron temperature using relative line intensity
ratio (argon, \( I = 15 \, A, \dot{m} = 0.5 \, A \, \text{eq.}, d_o = 0.76 \, mm \)).

xii
5.24 Calculation of the electron temperature using relative line intensity ratio \( (\text{argon}, I = 7 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.25 Calculation of the electron temperature using relative line intensity ratio \( (\text{argon}, I = 10A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.26 Calculation of the electron temperature using relative line intensity ratio \( (\text{argon}, I = 15 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.27 Calculation of the electron temperature using relative line intensity ratio \( (\text{argon}, I = 20 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.28 Calculation of the electron temperature using relative line intensity ratio \( (\text{xenon}, I = 3 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.29 Calculation of the electron temperature using relative line intensity ratio \( (\text{xenon}, I = 10 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.30 Calculation of the electron temperature using relative line intensity ratio \( (\text{xenon}, I = 15 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.31 Calculation of the electron temperature using relative line intensity ratio \( (\text{xenon}, I = 20 A, \dot{m} = 0.5 A \text{ eq.}, d_o = 1.27 mm) \).  

5.32 Calculation of species composition for argon using two-temperature Saha equation \( (T_h = 0.15 \text{ eV}, p = 10 \text{ Torr}) \).  

5.33 Calculation of species composition for xenon using two-temperature Saha equation \( (T_h = 0.2 \text{ eV}, p = 10 \text{ Torr}) \).
5.34 Axial distribution of electron number density along the cathode center line (xenon, $\dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm, \ I = 5 \ and \ 9 \ A$). . . . 99

5.35 Axial distribution of electron number density along the cathode center line (xenon, $\dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm, \ I = 9 \ and \ 12 \ A$). . . 100

5.36 Axial distribution of electron number density along the cathode center line (xenon, $\dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm, \ I = 12 \ and \ 15 \ A$). . . 100

5.37 Axial distribution of electron number density along the cathode center line in argon and xenon discharges ($I = 5 \ A, \dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm$). ................................. 101

5.38 Axial distribution of electron number density along the cathode center line in argon and xenon discharges ($I = 9 \ A, \dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm$). ................................. 101

5.39 Axial distribution of electron number density along the cathode center line in argon and xenon discharges ($I = 15 \ A, \dot{m} = 0.5 \ A \ eq., \ d_o = 1.21 \ mm$). ................................. 102

5.40 Axial distribution of electron number density along the cathode center line for different orifice sizes. (argon, $I = 10 \ A, \dot{m} = 0.93 \ A \ eq., \ d_o = 1.21 \ mm$). ................................. 102

5.41 Axial distribution of electron number density along the cathode center line for different orifice sizes. (argon, $I = 13 \ A, \dot{m} = 0.93 \ A \ eq., \ d_o = 1.21 \ mm$). ................................. 103
5.42 Effect of discharge current on electron number density along the cathode center line at different distances from orifice (argon, $\dot{m} = 0.5 \ A eq., d_o = 1.21 \ mm$). ........................................... 103

5.43 Effect of discharge current on electron number density along the cathode center line at $1\ mm$ from orifice (xenon, $\dot{m} = 0.5 \ A eq., d_o = 1.21 \ mm$). ........................................... 104

5.44 Effect of flow rate on electron number density along the cathode center line at different distances from orifice (argon, $I = 10 \ A$, $d_o = 0.76 \ mm$). ........................................... 104

5.45 Effect of discharge current on plasma potential along the cathode center line at different distances from orifice (argon, $\dot{m} = 0.93 \ A eq., d_o = 0.76 \ mm$). ........................................... 105

5.46 Effect of discharge current on plasma potential along the cathode center line at different distances from orifice (xenon, $\dot{m} = 0.5 \ A eq., d_o = 1.21 \ mm$). ........................................... 105

5.47 Axial distribution of plasma potential along the cathode center line at different discharge currents. (argon, $\dot{m} = 0.93 \ A eq., d_o = 1.21 \ mm$). ........................................... 106

5.48 Axial distribution of plasma potential along the cathode center line at different discharge currents. (xenon, $\dot{m} = 0.5 \ A eq., d_o = 1.21 \ mm$). ........................................... 106
5.49 Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 5 \ A, \dot{m} = 0.5 \ A \ eq., d_o = 1.21 \ mm$). .......................... 107

5.50 Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 9 \ A, \dot{m} = 0.5 \ A \ eq., d_o = 1.21 \ mm$). .......................... 107

5.51 Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 12 \ A, \dot{m} = 0.5 \ A \ eq., d_o = 1.21 \ mm$). .......................... 108

5.52 Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 15 \ A, \dot{m} = 0.5 \ A \ eq., d_o = 1.21 \ mm$). .......................... 108

5.53 Effect of discharge current on plasma potential along the cathode center line for different orifice sizes (argon, $\dot{m} = 0.93 \ A \ eq., L = 4 \ mm$). .......................... 109

5.54 Axial distribution of plasma potential along the cathode center line for different orifice sizes (argon, $I = 10 \ A, \dot{m} = 0.93 \ A \ eq.$) .................. 109

5.55 Axial distribution of plasma potential along the cathode center line for different orifice sizes (argon, $I = 13 \ A, \dot{m} = 0.93 \ A \ eq., L = 4 \ mm$). ........................................ 110

xvi
5.56 Effect of discharge current on cathode internal pressure along the cathode center line at different flow rates (argon, \( d_o = 0.76 \ mm \)).  . 110

5.57 Effect of discharge current on cathode internal pressure along the cathode center line at different flow rates (argon, \( d_o = 1.21 \ mm \)).  . 111

5.58 Effect of discharge current on cathode internal pressure along the cathode center line in argon and xenon discharges (\( \dot{m} = 0.5 \ Aeq., d_o = 1.21 \ mm \)).  . 111

5.59 Effect of discharge current on cathode internal pressure along the cathode center line in argon and xenon discharges (\( \dot{m} = 0.93 \ Aeq., d_o = 1.21 \ mm \)).  . 112

5.60 Effect of Gas flow on cathode internal pressure along the cathode center line at different discharge currents (argon, \( I = 10 \ A, d_o = 0.76 \ mm \)).  . 112

5.61 Effect of discharge current on cathode internal pressure along the cathode center line for different orifice sizes (argon, \( \dot{m} = 0.93 \ A eq. \)). 113

5.62 Axial distribution of cathode external surface temperature at different discharge currents (argon, \( \dot{m} = 1.2 \ A eq., d_o = 0.76 \ mm \)).  . . 113

5.63 Axial distribution of cathode external surface temperature at different discharge currents (xenon, \( \dot{m} = 0.93 \ A eq., d_o = 1.21 \ mm \)).  . . 114
5.64 Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (argon, $\dot{m} = 1.24 \text{ Aeq.}, d_o = 0.76 \text{ mm}$).

5.65 Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (argon, $\dot{m} = 0.93 \text{ Aeq.}, d_o = 1.27 \text{ mm}$).

5.66 Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (xenon, $\dot{m} = 0.93 \text{ Aeq.}, d_o = 1.21 \text{ mm}$).

5.67 Effect of gas flow rate on cathode external surface temperature at different distances from the orifice plate (argon, $I = 7 \text{ A}, d_o = 1.27 \text{ mm}$).

5.68 Effect of discharge current on cathode external surface temperature in argon and xenon discharges ($\dot{m} = 0.5 \text{ Aeq.}, d_o = 1.21, L = 6 \text{ mm}$).

5.69 Effect of discharge current on cathode external surface temperature in argon and xenon discharges ($\dot{m} = 0.5 \text{ Aeq.}, d_o = 1.21 \text{ mm}, L = 19 \text{ mm}$).

5.70 Effect of discharge current on cathode external surface temperature for different orifice sizes (argon, $\dot{m} = 0.93 \text{ Aeq.}, L = 6 \text{ mm}$).

5.71 Comparison of axial distributions of insert and cathode external surface temperatures (xenon, $I = 4 \text{ A}, \dot{m} = 0.93 \text{ Aeq.}, d_o = 1.21 \text{ mm}$).
5.72 Comparison of axial distributions of insert and cathode external surface temperatures (xenon, $I = 9\ A$, $\dot{m} = 0.93\ A\ eq.$, $d_o = 1.21\ mm$). 118

5.73 Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 2\ A$, $\dot{m} = 1.24\ A\ eq.$, $d_o = 0.76\ mm$). 119

5.74 Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 2\ A$, $\dot{m} = 0.93\ A\ eq.$, $d_o = 1.21\ mm$). 119

5.75 Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 5\ A$, $\dot{m} = 1.24\ A\ eq.$, $d_o = 0.76\ mm$). 120

5.76 Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 5\ A$, $\dot{m} = 0.93\ A\ eq.$, $d_o = 1.21\ mm$). 120

5.77 Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 11\ A$, $\dot{m} = 1.24\ A\ eq.$, $d_o = 0.76\ mm$). 121

5.78 Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 10\ A$, $\dot{m} = 0.93\ A\ eq.$, $d_o = 1.21\ mm$). 121
6.1 Theoretical prediction of the radial variation of the heavy particle temperature within the cathode internal plasma column. ................. 128

6.2 Effect of discharge current on cathode surface temperature (comparison between theory and experiment, argon, $\dot{m} = 0.93 A eq., d_o = 1.27 \text{ mm}$). ........................................ 129

6.3 Effect of discharge current on cathode surface temperature (comparison between theory and experiment, xenon, $\dot{m} = 0.93 A eq., d_o = 1.21 \text{ mm}$). ........................................ 129

6.4 Effect of discharge current on cathode internal pressure (comparison between theory and experiment, argon, $\dot{m} = 0.93 A eq., d_o = 0.76 \text{ mm}$). ........................................ 130

6.5 Effect of discharge current on cathode internal pressure (comparison between theory and experiment, argon, $\dot{m} = 0.5 A eq., d_o = 0.76 \text{ mm}$). ........................................ 130

6.6 Effect of discharge current on cathode internal pressure (comparison between theory and experiment, xenon, $\dot{m} = 0.93 A eq., d_o = 1.21 \text{ mm}$). ........................................ 131

6.7 Effect of discharge current on cathode internal pressure (comparison between theory and experiment, xenon, $\dot{m} = 0.5 A eq., d_o = 1.21 \text{ mm}$). ........................................ 131
6.8 Effect of discharge current on electron number density (comparison between theory and experiment, argon, $\dot{m} = 0.5 \ A eq., \ d_0 = 1.21 \ mm$).

6.9 Effect of discharge current on electron number density (comparison between theory and experiment, xenon, $\dot{m} = 0.5 \ A eq., \ d_0 = 1.21 \ mm$).

6.10 Effect of discharge current on plasma potential (comparison between theory and experiment, argon, $\dot{m} = 0.5 \ A eq., \ d_0 = 1.21 \ mm$).

6.11 Effect of discharge current on plasma potential (comparison between theory and experiment, xenon, $\dot{m} = 0.5 \ A eq., \ d_0 = 1.21 \ mm$).

6.12 Effect of discharge current on electron temperature (comparison between theory and spectroscopy, argon, $\dot{m} = 0.93 \ A eq., \ d_0 = 0.76 \ mm$).

6.13 Effect of discharge current on electron temperature (comparison between theory and spectroscopy, xenon, $\dot{m} = 0.5 \ A eq., \ d_0 = 0.27 \ mm$).

7.1 Axial variation of electron temperature within the internal plasma column.

8.1 Theoretical prediction of the effect of cathode inner diameter on the maximum discharge current.
8.2 Theoretical prediction of the effect of orifice diameter on the maximum discharge current. ........................................ 145

8.3 Theoretical prediction of the effect of discharge current on the fall voltage for different orifice sizes. ........................... 146

8.4 Theoretical prediction of the effect of discharge current on the fall voltage for different insert diameters. .................... 146

8.5 Theoretical prediction of the effect of discharge current on cathode temperature. .................................................... 147

8.6 Theoretical prediction of the effect of flow rate on the maximum discharge current. .............................................. 147
LIST OF TABLES

B.1 Values of constants in equations B.4 and B.5. .......... 152

C.1 Debye length and charged particle mean free path for $T_e = 1 \text{ eV}, \ T_i =$ $.1 \text{ eV}$. ........................................ 154

C.2 Debye length and charged particle mean free path for $T_e = 2 \text{ eV}, \ T_i =$ $.2 \text{ eV}$. ........................................ 155
LIST OF SYMBOLS

$A$  area, $m^2$
$A_{pq}$ transition probability
$A_R$ Richardson’s constant, $1.2 \times 10^6 A/m^2.K^2$
$D$ cathode inner diameter, $m$
$d_o$ orifice diameter, $m$
$e$ electron charge, $1.602 \times 10^{-19}Coul$
$E$ electric field, $V/m$
$E_p$ excitation energy, $eV$
$f$ view factor
$h$ Planck’s constant, $6.626 \times 10^{-34} J - s$
$I$ current, $A$
$I_p$ spectral line intensity
$J$ current density, $A/m^2$
$k$ Boltzmann’s constant, $1.381 \times 10^{-23} J/K$
$l$ characteristic length, $m$
$L_e$ emission surface length, $m$
$m$ particle mass, $kg$
$M$ atomic mass, a.u.
$\dot{m}$ mass flow rate, $kg/s$
$n$ number density, $m^{-3}$
$p$ pressure, $N/m^2$
$Q$ cross section, $m^2$
$\dot{q}_r$ radiative heat flux, $W$
$r$ radial coordinate, $m$
$R$ Radius, $m$
$t$ characteristic time, $s$
$T$ temperature, $K$
$u$ flow velocity, $m/s$
$V$ potential, $V$
$x$ axial coordinate, $m$
\( Z \)  partition function or charge number  
\( \alpha \)  degree of ionization  
\( \varepsilon_0 \)  permittivity of free space, \( 8.85 \times 10^{-12} \)  
\( \varepsilon \)  normalized electric field  
\( \eta \)  plasma resistivity, \( \Omega - m \)  
\( \eta_c \)  normalized fall voltage  
\( \eta_e \)  normalized ion energy at the sheath edge  
\( \phi \)  material work function, \( eV \)  
\( \lambda \)  mean free path, \( m \)  
\( \lambda_D \)  Debye length, \( m \)  
\( \ln \Lambda \)  Coulomb logarithm  
\( \kappa \)  thermal conductivity, \( W/m - K \)  
\( \nu_i \)  ion to electron number density ratio at the sheath edge  
\( \nu_{ie} \)  electron-ion collision frequency, \( s^{-1} \)  
\( \rho \)  gas density, \( kg/m^3 \)  
\( \rho^c \)  charge density, \( coul/m^3 \)  
\( \sigma \)  electrical conductivity, \( (\Omega - m)^{-1} \)  
\( \theta \)  electron to heavy particle temperature ratio  
\( \xi \)  probe radius to Debye length ratio

Subscripts

\( a \)  atom  
\( b \)  emission beam  
\( c \)  cathode or characteristic  
\( D \)  discharge  
\( e \)  electron or emission  
\( eff \)  effective  
\( eq \)  equivalent or equipartition  
\( f \)  fall  
\( i \)  ion  
\( o \)  stagnation conditions  
\( or \)  orifice  
\( p \)  probe or plasma  
\( s \)  sheath edge or surface  
\( t \)  total  
\( th \)  thermionic
CHAPTER I
INTRODUCTION

Space missions demand technologically advanced propulsion systems which must meet severe requirements, exhibit longevity and demonstrate adequate reliability. Electric propulsion has appeared to be the most promising technology for space applications because of the substantially higher specific impulse developed by electrical thrusters compared to chemical or nuclear ones. Recently a great deal of attention has been focused on magnetoplasmodynamic (MPD) thrusters because they offer many attractive features such as: simplicity, high unit power, relatively high thrust..., but suffer from low efficiencies until now.

The improvement of this class of thrusters requires a detailed study of the different components of the thruster. The result of such a study would allow to acquire better understanding of the related physical phenomena, maximize component efficiency and optimize overall design. It is essential to know that the overall performance of the thruster depends on the successful design of each ele-
ment of the thruster. It has been established that the electrodes limit the lifetime of the MPD thruster because they are subject to erosion which can be severe under conditions required for space missions. The erosion in most cases is related to surface temperature, highly energetic ion bombardment, non-uniform temperature distribution, material properties, and a number of other processes that are not well understood at the present time. Therefore, it is the scope of the present work to focus on one of the main components of the MPD thruster which is the cathode.

1.1 The hollow cathode

The hollow cathode has been known as the best high-current plasma source. Under typical operating conditions, it produces strongly ionized plasmas with typical electron number densities of $10^{13}$ to $10^{15}$ cm$^{-3}$ and electron temperatures of 1 to 2 eV. As a plasma source, the hollow cathode has provided a valuable tool for research in atomic and molecular physics [7, 44]. This device was used as a high current density ion source to heat plasmas in controlled thermonuclear reaction experiments [42]. Moreover, the hollow cathode has been widely used in ion thruster technology since 1962 [43]. After the development of the hollow cathode in the SERT II neutralizer by Rawlin and Pawlik [36] and the main cathode of the SERT II by Kerslake et al. [19], all ion thrusters have adopted hollow cathodes. This technology has been applied successfully to a steady-state 100 KW class MPD
thrusters [28]. Recently, the hollow cathode has found other applications such as plasma contactors [49] where it provides a spacecraft with the ability to emit or collect charged particles from a surrounding plasma environment.

Despite the extensive research conducted on hollow cathode arc discharges, many aspects of this device remain unsolved. Some of these aspects are: the state of the plasma inside the cathode, the surface emission area, the interaction between plasma and solid boundary, the effect of cathode configuration, the effect of the propellant, the erosion rate, and the optimum operating conditions. Answers to these questions and good understanding of the physical processes governing hollow cathode arc discharges would allow the design of reliable, long lasting and efficient devices suitable for space applications.

1.2 Scope of the present work

The present work investigates the orificed hollow cathode discharge both theoretically and experimentally. This study is primarily aimed at the development of a purely theoretical model which would serve to predict plasma properties and other key parameters for a variety of configurations, propellants and operation modes. A number of investigations have been conducted to evaluate hollow cathode operations in the past. However, most of them were experimental in nature and did
not exhibit a thorough understanding of the different processes taking place. In addition, the few theoretical models included empirical correlations which limit their application and made them not attractive for preliminary design purposes.

The present experimental program was carried out at NASA-Lewis Research Center (Cleveland, Ohio) at the electric propulsion laboratory so as to acquire an extensive data base, determine general trends and parametric variations and establish the dependence of the plasma conditions on various operating modes. Emphasis was given to the implementation of a number of diagnostic techniques in order to minimize the randomness and uncertainties associated with data gathering. That is, due to the small size of the cathode (few millimeters in diameter), the experimental study of the internal plasma column presented some extraordinary laboratory challenges to the conventional diagnostic techniques. The basic goals of the experimental program were (1) to establish a basis for validation of the theoretical model, (2) to provide insight into the rigorous formulation of the problem, (3) to demonstrate new diagnostic concepts, and (4) to acquire a rich data base for the electric propulsion community.

Finally, it is the intent of the present work to establish a reliable theory which allows the definition of orificed, hollow cathode operation in the most efficient way and to set engineering guidelines for successful design of such devices. These cri-
teria encompass geometric consideration, material properties, suitable propellants and power levels.
CHAPTER II

BACKGROUND

2.1 The Principle of the hollow cathode

As early as 1934, v. Engel and Steenbeck [26] illustrated the principle of a hollow cathode by reducing the distance between two-parallel plate cathodes of a glow discharge. In this fashion, while the potential is kept constant, the current density rises substantially. This rise in current density has been attributed to the coalescence of the two separate negative glows. Thus, all the energy of the primary electrons is efficiently utilized to excite and ionize the heavy particles because the electrons are confined by the retarding fields of the cathodes. Moreover, the resulting highly ionized discharge has the potential to carry large current at low cathode falls.

In a study of glow discharges, Little and v. Engel [25], 1954, showed some particular interest to a special form of glow discharge that they refer to it as the hollow-cathode effect. In this case, the cathode is a hollow cylinder, spherical cav-
ity or a pair of parallel plates separated by a small gap. The cathode effect was described in terms of the high intensity increase of light (visible and ultra-violet) emitted by the glow compared to that of conventional cathodes. Furthermore, the gas temperature was reduced substantially due to the low cathode fall. These aspects of hollow cathode discharge have made it a good spectroscopic source. The ultimate goals of their study were to establish an elementary theory describing prominent processes in the negative region of the discharge that play an essential role in maintaining the glow discharge. Based on experimental results and theoretical analysis, the authors established that the intense glow in the hollow cathode could provide sufficient ultra-violet radiation to make photo-emission comparable to secondary electrons released by positive ions. The contribution of the photo-emission would lower substantially the cathode fall.

2.2 Experimental and theoretical research

In 1962, Lidsky [24] made measurements with a refractory metal hollow cathode discharge using a variety of gasses such as hydrogen, helium, argon and nitrogen under the current range of 2-300 A, various electrode configurations and a wide range of operating parameters. However, only external plasmas were studied. The hollow cathode discharge was described to be a highly ionized steady-state plasma where the electron temperature was estimated to be 1-2 eV and the charged par-
ticle densities were evaluated at $10^{13} - 10^{14}$ cm$^{-3}$, indicating a degree of ionization of 25-75%. Lidsky indicated that the condition $p_o d \approx 1$ cm x mm Hg has a direct control on the length of the active zone within the cathode. Moreover, based on temperature measurements, it was stated that the exact mechanism for high electron emission was still unknown since the currents exceeded those predicted on the basis of thermionic emission by large factors in many cases.

In 1968, Delcroix [8] attempted to establish a general rule that predicted the location of the active zone (maximum temperature region) within an open channel cathode as a function of the cathode diameter and operating conditions such as discharge current and volumetric flow rate. In this approach the gas temperature was assumed approximately equal to the external surface temperature. The thickness of the active zone was observed to increase when the flow rate is reduced or the diameter is increased. This observation led to suggest that the active zone location was characterized by a constant pressure whose value is independent of the cathode diameter and gas flow rate.

In 1969, Delcroix [9] studied a gas-fed multichannel hollow cathode and concluded that such a device was superior to the conventional one-channel cathode. The former operated with a lower voltage drop and higher maximum current density and had longer lifetime as well. In addition, the multichannel hollow cathode
could operate with extremely low gas flow rates. Delcroix adopted a simplified theoretical formulation to explain the principles of the operation of this type of cathode and presented some general rules for the cathode design. It was stated that the inner cathode diameter had to be large when high discharge currents are applied, in order to keep the current density below some critical values so that adequate lifetime is achieved. In this case, a high gas flow rate is required to prevent deep arc penetration inside the cathode which would cause large increase in voltage. The combination of low gas flow rate, reasonable operating voltage and long lifetime requirements have made multichannel hollow cathode very suitable for high discharge currents. This type of cathode was characterized by high efficiency due to thermal coupling between channels that could be enhanced by the use of thin walls.

In 1970, Fradkin [15] carried out an experimental investigation on a 25-kw lithium vapor MPD arcjet in which a hollow cathode was used instead of the conventional one. This work was not primarily directed to the study of the hollow cathode but instead to the external plasma. However, the performance achieved demonstrated the superiority of this device to the solid rod cathode in many respects. The use of a hollow cathode allowed a stable discharge, high-voltage mode operation, rapid and repeated extinguishing and restarting of the arc without melting of the cathode, a phenomenon that occurred frequently with the conventional
cathode. In addition, operation with no applied magnetic field was achieved without altering the arc structure.

In 1972, Lorente-Arcas, [27] attempted to investigate the up-stream limit of the internal plasma column in a hollow cathode. The author believed that this limit is a thin sheath (Adar sheath) created by ambipolar diffusion and recombination between electrons and ions. Of particular interest was the thickness of the sheath that was estimated in this work to be of the order of the cathode diameter from the active zone. Furthermore, similar to the active zone, the Adar sheath was characterized by a constant pressure. One of the implications of the existence of the ambipolar sheath is that the apparent thickness of the plasma column is greater than the emission length of the cathode surface.

The multichannel cathode was also studied by Babkin et al. in 1976 [2]. The experimental program carried out intended to investigate hollow cathodes in high power regimes (7 kW). Lithium was used as the working fluid because of its low ionization energy. The cathode was operated at discharge currents from 50 to 500 A at mass flow rates from 0.0015 to 0.018 g/sec. The cathode was made of 19 tungsten tubes 2.1 mm in diameter each. The results obtained showed that the electron temperature was much higher than the heavy particle temperature while small difference between the wall and heavy particle temperature was noted.
Moreover, the electron temperature was insensitive to the variation of the discharge current. Plasma densities in the range of $10^{15} - 5 \times 10^{15}$ cm$^{-3}$ and degree of ionization exceeding 75% were reported. It was also reported that the flow rate had little effect on the plasma properties. Even though the cathode was tested for a short period of time (few hours), the length of the emission region was estimated at 1 to 1.5 tube diameter. This result was based on the examination of the inside surface after completion of the experiment. This investigation had demonstrated several similarities to the single-channel cathode operating in a low power regime and provided valuable information for developing theoretical models.

In 1977, Krishnan [20] conducted an experimental study on open channel hollow cathodes with cavity diameters of 1.9 cm at discharge currents from 0.25 to 30 kA and argon mass flow rates from $10^{-3}$ to 16 g/sec. It was observed that the arc penetration depth was affected by both discharge current and mass flow rate. In all cases, the maximum current density peak penetration did not exceed one cavity diameter. It was stated that this behavior of migration of the upstream location of maximum surface current density inside the cathode, for certain combination of discharge current and gas flow rates, was related primarily to the energy-exchange mean free path. When the latter is either very small or very large compared to the cathode diameter the current density peak moved towards the cathode orifice, while, when the mean free path is of the same order as the cavity diameter, the
peak moves upstream by, at best, a distance on the order of the cathode diameter.

In 1978, Ferreira [11] attempted to establish a theory of an open channel hollow cathode arc which described the physical mechanisms of the discharge. In that study a simplified analysis of the gas flow inside the cathode was presented in which the gas pressure was estimated, using a semiempirical relation[10] between pressure, mass flow rate, tube conductances corresponding to the viscous and molecular regimen. To avoid solving the heat transport equation, the gas temperature was assumed to be equal to the wall temperature. In that approach, the high electron pressure was completely neglected. In addition, some efforts were devoted to the study of the formation of the internal plasma column by the ionization of the neutral gas by the primary electrons. The approach consisted of the thermalization of the primary electron through successive collisions with the heavy particles and the plasma electrons. In this fashion, the fast electrons are responsible of the ionization of the gas, while the thermalized, slow electron carry the discharge current. In this theory, called "cascade theory", only processes of collisional nature were taken into account and inelastic collisions were considered the most important mechanism of the electron energy loss. The study of both radial transport of ions and axial transport of the Maxwellian electrons and kinetic energy along the the internal plasma column showed large spatial variations in the plasma conditions such as number density, potential and electron temperature.
The importance of orificed, hollow cathodes to ion thruster technology has led to numerous studies to understand the physics of hollow cathode arc discharges and performance. These investigations intended primarily to develop, semi-empirical and ultimately theoretical, models for cathode operation. This goal has been pursued via a number of experimental efforts devoted to establish a data base for plasma conditions within the cathode, wall temperature distribution, internal pressure and current density profiles for typical operating conditions of an ion engine.

In 1980, Siegfried and Wilbur [37] proposed a so called "phenomenological model" to describe essential features inherent to orificed, hollow cathode. Their data base was collected through experiments on cathodes made of refractory metal tube (tantalum), housing a refractory insert coated or impregnated with barium compounds intended to lower material work function. The orificed, hollow cathode is obtained by covering one end by a thoriated tungsten plate. The choice of the orifice plate material is driven by the desire to assure current attachment on the inside of the cathode and not on the orifice plate. Typical hollow cathodes used in ion thrusters are rather small devices which have insert diameter of a few millimeters and orifice size of tenths of a millimeter. Similar sizes were adopted in these experiments. The propellant used in this case was mercury vapor because of its lower ionization potential compared to noble gases, and more importantly, its high
purity. Furthermore, it was noticed that mercury tended to reduce the contamination of the insert surface, which has serious implication in the cathode mechanical failure. For operating discharge currents of few Amperes, typical for ion thrusters, plasma densities of $10^{14}$ cm$^{-3}$ were measured inside the hollow cathode at plasma potential and electron temperature of approximately 8 Volts and 0.8 eV respectively. In that case, the insert operated at temperatures on the order of a 1000 °C which indicated the effect of the impregnate on the material work function.

In light of those results, the emission mechanism was modeled by field-enhanced, thermionic emission in which the strong electric field resulting from a potential drop across a very thin sheath was responsible for an effective work function lower than that of the insert material. Assuming a sheath thickness of one Debye length, the electric field was derived from Child's law. The electrons released from the surface and accelerated in the sheath ionized the injected gas through multiple collisions. The ion production region was defined by the volume bounded by the emitting region of the insert. From that idealized region, only ions were considered the major contributors to the heating of the emitting surface. The diffusion of those ions to the surface was assumed to satisfy the Bohm condition. Furthermore, the energy input by the ion flux to the surface was balanced by the surface thermal losses combined in the electron production, radiation and conduction. The total current density was expressed as the sum of the emission current and the ion flux to the cathode surface. The model, as described, lacked completeness since the
number of variables exceeds the number of equations. Therefore, it was necessary to make some assumptions regarding certain plasma properties to solve for the remaining variables. Subsequently, the model becomes useful only when some experimental data are available. In addition, this model lacks the capability of predicting spatial variation of both plasma properties and wall temperature.

In 1984 Siegfried and Wilbur [40] introduced an upgraded version of this phenomenological model. This version, also based on an idealized “ion production region”, defined as the volume delimited by the emitting portion of the insert, provided a qualitative description of the physical processes occurring within the hollow cathode. In that model, an assumed value of the electron temperature was based on related experiments, and the internal cathode pressure was evaluated using an empirical correlation with the discharge current, mass flow rate and cathode orifice diameter. In addition, the ion flux to the surface was formulated based on the Bohm condition. The surface and volume energy balances were used to solve for plasma density and potential in the ion production region. In addition to the prediction of plasma conditions, insert emission length was estimated based on the energy exchange mean free path of the primary electron. Finally, the insert surface temperature was predicted using field-enhanced thermionic emission as the primary surface electron emission. In this case, the electric field adjacent to the surface is determined from a double sheath approximation [34] instead of Child’s
law. The use of empirical correlations has imposed strict limitation on the application of this model to cathode arc discharges under different operating conditions.

In 1984, Siegfried [38] conducted experimental studies on orificed, hollow cathodes operating on inert gases such as argon and xenon. The main objectives of that work were to investigate the suitability of the phenomenological model developed for mercury cathodes and to determine the effects of argon and xenon on the physical processes identified in hollow cathodes. The experiments were conducted on hollow cathodes with a quartz outer tube intended for optical temperature measurements. In that case, the insert (emitting material) was a tantalum foil coated with a mixture of carbonate salts. In contrast with mercury cathodes, the author indicated great difficulties in collecting data. He attributed those problems to insert degradation which affected the material work function and subsequently the cathode operation. It was believed that those phenomena were related directly to the use of xenon and argon. The results obtained in that investigation showed that both plasma property and insert temperature axial distributions go through a peak upstream of the orifice. Furthermore, the ion production regions were 5 mm and 7 mm for xenon and argon respectively within a 3.8 mm diameter insert. Therefore, the scale size of the internal plasma column is of the order of the cathode diameter. It was also shown that argon cathodes operated at higher voltage that xenon cathodes. That behavior could be attributed to the ionization poten-
tial of the propellant. The major difference found between mercury and inert gas cathodes was the location of the electron emission region. In mercury cathodes the emission region was confined to the downstream of the insert (2 mm) while the xenon and argon cathodes that region tended to extend upstream on the insert. The analysis of the overall cathode operation showed that the basic physical processes governing cathode operation were independent of the propellant. However, different empirical equations were needed for the different propellants, and the corresponding measured electron temperature were used to apply the model developed for mercury cathodes. In addition, a semi-empirical correlation was adopted to evaluate the thermal power loss from the insert. In spite of the empirical correlations added to complete the theoretical model, agreement, between the experimental and theoretical results, was not obtained in all cases.

2.3 Cathode material and lifetime

Cathode lifetime has been among the essential design criteria that has a direct impact on the overall performance of electric thrusters. Since the cathode operates on the concept of thermionic emission, there has been a need to enhance the emission capability of the material at low temperatures. In this fashion, material failure under thermal stresses melting and evaporation would be prevented. In recent years, many advances have been made in manufacturing special materials
that combine both high melting temperature and low work function. Among the material used, refractory metals (e.g., tantalum) dip-coated with special chemicals such as carbonate salts and impregnated tungsten with barium compounds.

In an attempt to investigate the lifetime and failure modes of tungsten impregnated cathodes, an ongoing life test program has been carried out by NASA-Lewis Research Center since 1971 [13, 14]. In 1976, Mirrich [29] conducted an experimental study on hollow cathodes for 30-cm bombardment thrusters, where rolled tantalum foil inserts coated with low work function material and impregnated porous tungsten inserts were tested. The first kind of inserts had an emissive mix of barium carbonate and strontium carbonate mixture containing nitrocellulose binder and suspended in a mixture of organic solvents, while the second kind was impregnated with barium calcium aluminate. In either case, the lowering of the work function was attributed to the release of barium or barium oxide vapor from the chemical compounds when the insert was heated through chemical reactions. The barium is absorbed by cathode activated surfaces enhancing their emission capability. However, the absorbed barium is continuously lost from the surface by evaporation and ion bombardment. Therefore, the lifetime of the cathode depends on the rate of adequate replacement of the lost barium. This rate depends strongly on the cathode operating temperature. Based on the mechanism of barium depletion, attempts were made to define optimum operating conditions
corresponding to the desired lifetimes. Subsequently, at cathode temperature of 1000°C, a maximum lifetime of 100,000 hours was estimated. Higher temperature would lower substantially the cathode lifetime, for example, at 1100°C only 30,000 hours were expected. The relationship between barium depletion from the impregnated porous tungsten and the insert temperature was later presented in a model developed by Poeschel and Beattie in 1979 [32].

The cathode lifetime is not only affected by the cathode temperature but is also affected a great deal by the contamination of the cathode surface. This contamination is related to the gas feed system, integrity of the evacuated systems and the care taken in handling and assembling the hollow cathode. Cathode failures from orifice erosion [4], to cracking of the cathode tube [35], were attributed to oxidation and improper cathode activation procedures. The activation procedures were intended to eliminate poisoning agents such as oxygen, water vapors and carbon dioxide. This contamination problem has been addressed by a number of lifetest studies carried out by Verhey et al. [46, 47]. It was established that the presence of oxygen or water vapor could shorten the cathode lifetime by reacting with the insert material leading to material loss and formation of a number of oxides. The deposition of this material on the cathode internal surfaces lead to alter the normal cathode discharge, reduce the orifice size and possibly cause cathode failure. It was also found that by keeping the content of the residual gases at a volumet-
ric percentage of few parts per million, there was no evidence of the destructive symptoms of surface contamination.

Another aspect of the hollow cathode lifetime resides in the erosion of the orifice, which has serious implications on the cathode operation. A number of investigations were carried out to shed some light on the dependence of this phenomenon on the cathode operating conditions. An experimental research by Beattie [3] revealed that the cathode configuration played an essential role in orifice erosion. It was established that depending on the required power level there is a minimum orifice size that could be adopted to prevent destructive erosion. The correlation between orifice diameter and discharge current, for lifetime considerations, was first investigated in 1974 by Kaufmann [18]. It was found that the ratio of orifice diameter to discharge current had to be lower than 12 A/mm. Later, Rawlin [35] established a higher ratio criterion of 15 A/mm.

Summarizing the data of these publications it can be stated that hollow cathode lifetime could be improved considerably not only through the use of special materials but also through a proper combination of operating conditions and device configuration. Cathode geometry represents, in fact, a different arena of research to overcome the challenges imposed by this interesting device, the hollow cathode.
CHAPTER III

A BASIC MODEL OF HOLLOW CATHODE ARC DISCHARGES

3.1 Introduction

The present theoretical study is intended to identify and describe the major physical laws governing hollow cathode arc discharges based on first-principles. Therefore, it is the aim of this work to achieve adequate formulation of the problem with minimum complexity of the mathematical modeling. It is also the intention of this study to develop a purely theoretical model that could be applied to various regimes of hollow cathode operation. Thus, successful designs of the hollow cathode could be accomplished by simply varying the operating parameters and selecting the optimum conditions.

3.2 Approach

The cathode geometry considered for this study is a cylindrical cavity with a small orifice. In such configuration, the energy losses associated with radiation, positive
ions and excited atoms are substantially reduced. Therefore, the cathode fall will be reduced while large currents can still be carried by the discharge. This geometry also allows for reasonable boundary conditions vital to the solution of this problem.

The theoretical approach includes two-dimensional variation in plasma potential. The gas (i.e., heavy particle) is assumed in thermal equilibrium with the cathode wall [8, 39]. One aspect of the present formulation is the specification of an adiabatic wall. This condition reflects a special interest in high power density operation in which the principal cooling mechanism of the surface is electron emission and radiation. The present model takes into account the ohmic heating due to a resistive plasma. To a first approximation the conductivity of the plasma is considered to be that of a fully ionized gas. This assumption is reasonably justified due to the fact that hollow cathode can sustain highly ionized discharges [24, 2]. Finally, in line with the nearly closed geometry of the problem, plasma composition is determined by a two-temperature Saha equation derived based on the assumption of chemical equilibrium [33, 45]. The set of algebraic equations governing the power balance within the hollow cathode are solved to obtain the dependence of various characteristics on terminal parameters such as discharge current, mass flow rate, gas properties and geometry.
Figure 3.1: Description of hollow cathode physical processes
3.3 Problem formulation

The analytical formulation of the hollow cathode discharge is based on the physical processes described in figure 3.1.

3.3.1 Thermionic current density

Based on past experimental work conducted on low work function cathodes [38, 40], the most dominant surface electron emission mechanism appears to be thermionic emission (by comparison to other emission mechanisms such as photoemission). This phenomenon describes the ability of a metal to emit electrons when heated to a certain temperature. The electron flux emitted from the surface depends strongly on the work function of the material. The thermionic current density is expressed by Richardson equation:

\[ J_{th} = A_R T_e^2 \exp \left( -\frac{\phi_s}{kT_e} \right) \]  \hspace{1cm} (3.1)

In the case of a plasma surrounding a hot cathode (an emitting surface), a double sheath (a collisional, non-neutral region) is created adjacent to the cathode surface inducing high electric field. This electric field enhances the emitting potential of the surface by changing the material work function to a lower apparent (effective) work function. This effect, called Schottky effect, is expressed as:
\[ \phi_{\text{eff}} = \phi_s - \sqrt{\frac{|E_c|}{4\pi\varepsilon_0}} \] (3.2)

The analysis of the sheath was first presented by Langmuir in 1929 [23]. He showed that ions entering the sheath at negligible initial energies would limit the thermionic current by negative space charge. On the other hand, non-zero initial ion energy at the sheath edge would increase the space-charge limited current. This case was later treated by Crawford and Cannara in 1965 [6]. A more elaborate mathematical treatment of the double sheath associated with a hot cathode was presented, in 1975, by Prewett and Allen [34]. In that analysis, the electric field at the cathode was determined by solving Poisson's equation:

\[ \nabla^2 V = -\frac{\rho^c}{\varepsilon_0} \] (3.3)

in conjunction with the boundary conditions imposed by Andrews and Allen [1]. These boundary conditions are plasma quasi-neutrality and ion minimum energy criterion at the sheath edge. As a result of that model the electric field was given by:

\[ \varepsilon_c = \left\{ 4\nu_t\eta_o \left[ (1 + \eta_c/\eta_o)^{1/2} - 1 \right] - 2J_b(2\eta_c)^2 + 2\exp(-\eta_c) - 2 \right\}^{1/2} \] (3.4)

where the nondimensional parameters are defined as follows:
\[ \varepsilon_e = \frac{E_e e \lambda_D}{kT_e} \]  
(3.5)

\[ \eta_e = \frac{e V_F}{kT_e} \]  
(3.6)

\[ J_b = \frac{J_{th}}{\varepsilon_0 e \left( \frac{kT_e}{m_e} \right)^{\frac{1}{2}} (2\eta_e)^{\frac{3}{2}}} \]  
(3.7)

\[ \nu_i = 1 + 2\eta_e J_b \]  
(3.8)

\[ \eta_o = \frac{1 + 2\eta_e J_b}{2(1 - J_b)} \]  
(3.9)

The parameter \( \lambda_D \) is the Debye length defined by:

\[ \lambda_D = \left( \frac{\varepsilon_o kT_e}{n_e e^2} \right)^{\frac{1}{2}} \]  
(3.10)

In the case of zero beam (emission) current from the cathode surface, equation 3.9 shows that the predicted ion energy at the sheath edge corresponds to the one obtained by Bohn [16] for non-emitting solid boundary.

### 3.3.2 Ion current density

The electrons emitted from the cathode surface and accelerated through the sheath undergo a succession of collisions with the neutral gas. These collisions result in the
ionization of the gas and thermalization of the electrons. The surface temperature for electron emission is achieved, in part, through positive ion bombardment. For a singly ionized gas, the ion flux is assumed equal to the random thermal flux evaluated at the free-stream plasma density. Hence,

\[ J_i = e n_i \left( \frac{kT_i}{2\pi m_i} \right)^{1/2} \tag{3.11} \]

### 3.3.3 Plasma electron current density

Plasma electrons are assumed to have a Maxwellian distribution, and only electrons with energies higher than the fall voltage (tail of the distribution function) are able to reach the cathode surface. Therefore, the plasma electron flux to the cathode can be expressed by:

\[ J_e = e n_e \left( \frac{kT_e}{2\pi m_e} \right)^{1/2} \exp \left( -\frac{eV_f}{kT_e} \right) \tag{3.12} \]

### 3.3.4 Discharge current

Based on the contribution of all particle fluxes at the cathode surface, the discharge current can be expressed by:

\[ I_D = I_{th} + I_i - I_e \tag{3.13} \]

Under the assumption of operation of the orifice at plasma potential, it is useful to
\[ p_0 = (n_i + n_a)kT_i \left( 1 + \frac{T_e}{T_i} \right) \]  
(3.18)

### 3.3.6 Mass flow rate

The mass flow can be expressed in terms of the flow properties at the orifice as:

\[ m = \rho_{or} u_{or} A_{or} \]  
(3.19)

Assuming isothermal flow at the vicinity of the orifice, the integral of the one-dimensional momentum equation:

\[ \rho u \frac{du}{dx} = -\frac{dp}{dx} \]  
(3.20)

leads to a relationship between the density and flow velocity of the form:

\[ \frac{\rho}{\rho_0} = \exp \left( -\frac{u^2}{2 \frac{kT_i}{m_i} \left( 1 + \frac{T_e}{T_i} \right)} \right) \]  
(3.21)

The condition of shocked flow at the orifice requires that:

\[ u_{or} = \sqrt{\frac{kT_i}{m_i} \left( 1 + \frac{T_e}{T_i} \right)} \]  
(3.22)

Substituting equations 3.21 and 3.22 in equation 3.19 yields:

\[ m = \frac{p_o e^{-\frac{1}{2}}}{\sqrt{\frac{kT_i}{m_i} \left( 1 + \frac{T_e}{T_i} \right)}} A_{or} \]  
(3.23)
Equation 3.23 could be corrected for viscous effects by considering a parabolic velocity profile at the orifice. In this case, using an effective area equal to one half the orifice area, equation 3.23 becomes:

\[
\dot{m} = \frac{p_0 e^{-\frac{t}{\tau}}}{\sqrt{\frac{k\gamma}{m_i} \left(1 + \alpha \frac{T_e}{T_p}\right)^2}} \frac{A_{or}}{2}
\]  

(3.24)

3.3.7 Two-temperature Saha equation

Hollow cathodes are in general small devices, typically a few millimeters in diameter. This configuration tends to establish thermal equilibrium between the heavy particles and the cathode wall (due to the high rate of energy exchange). Measurements of particle temperatures carried out by Babkin [2] in multichannel cathode showed large differences between electron and heavy particle temperatures. On the other hand, maximum surface temperature differed from the heavy particle temperature by only 4%. Many investigators in hollow cathode discharges suggested that the gas temperature did not differ much from the surface temperature [8, 39]. This assumption implies that a two-temperature plasma would exist within the hollow cathode. Therefore, the Saha equation could no longer be valid to describe the plasma composition and degree of ionization. To calculate equilibrium ionization when electron temperature differs from that of the heavy particles, Potapov [33] derived a formula based on the condition of chemical equilibrium, without considering the interaction of plasma particles, given by:
\[ n_e \left( \frac{n_i}{n_a} \right)^{1/\theta} = \frac{2Z_i}{Z_a} \left( \frac{2\pi m_e kT_e}{\hbar^2} \right)^{3/2} \exp \left( -\frac{\epsilon_i}{kT_i} \right) \] (3.25)

where:

\[ \theta = \frac{T_e}{T_i} \] (3.26)

It can be noticed that equation 3.25 reduces to the Saha equation when \( \theta \) is unity.

The coulomb interactions of charged plasma particles were taken into account by Veis [45] in the form of a correction of the ionization energy.

### 3.3.8 Power balance

The cathode surface receives energy from ions accelerated through the fall, plasma electrons having energies exceeding the fall barrier and all other processes taking energy out of the plasma (e.g. excited atom flux and photons). In the limit of high power density, the cathode is cooled primarily by electron emission, so the power balance at the cathode surface is written as:

\[ \int J_{th} \left( \phi_{eff} + \frac{5kT_e}{2e} \right) dA_e = \int J_i \left( \epsilon_i + V_e - \phi_{eff} \right) dA_e \]

\[ + \int J_e \left( \phi_{eff} + \frac{5kT_e}{2e} \right) dA_e + f \varphi \] (3.27)
where the view factor \( f \) is expressed as:

\[
f = \frac{A_e}{A_s}
\]  

(3.28)

The integrals over the surface are needed because of the variation of the current density along the cathode surface. Electron emission provides energy to the plasma in the form of electrons accelerated by the cathode fall. This energy is utilized to ionize and excite the gas, and heat the plasma electrons. Energy is also added to the plasma by Ohmic heating with plasma electrical resistivity, \( \eta \). Energy is lost by particles flowing out of the hollow cathode in the form of enthalpy. This power balance could be expressed as:

\[
\int J_{ih} \left( V_c + \frac{5kT_e}{2e} \right) dA_e + \int \eta J^2 dV = \int J_i \left( e_i + \frac{5kT_i}{2e} \right) dA_s + \int J_e \left( \frac{5kT_e}{2e} \right) dA_e \\
+ (I_D + I_{eq}) \left( \frac{5kT_e}{2e} \right) + \frac{I_{eq}}{\alpha} \left( \frac{5kT_i}{2e} \right) + q_r
\]  

(3.29)

where the ionization/excitation energy per ion is lost to both the cathode surface and the orifice exit area so the ion flux \( J_i \) is integrated over the total surface area \( A_s \).

### 3.3.9 Plasma electric conductivity

In this analysis the plasma conductivity is approximated to that of a fully ionized gas since hollow cathodes are expected to have reasonable degree of ionization (\( \alpha \) much greater than \( 10^{-3} \)). Hence,
\[ \sigma(Spitzer) = \frac{1.53 \times 10^{-2}}{\log A} T_s^{9/2} \]  \hfill (3.30)

### 3.3.10 Plasma potential distribution

In this study an axisymmetric two-dimensional system is considered. The plasma potential distribution in the hollow cathode is obtained from the Laplace equation governing a neutral plasma:

\[ \nabla^2 V = 0 \]  \hfill (3.31)

with the following boundary conditions specified at the sheath edge \( r = R_c \), cathode inlet \( x = 0 \) and orifice \( x = L_e \):

\[ \frac{\partial V}{\partial r} = \eta J_s \text{ at } r = R_c \]

\[ \frac{\partial V}{\partial x} = \begin{cases} \eta J_{or} & \text{at } 0 \leq r \leq R_{or} \text{ and } x = L_e \\ 0 & \text{at } R_{or} \leq r \leq R_c \text{ and } x = L_e \end{cases} \]  \hfill (3.32)

\[ \frac{\partial V}{\partial z} = 0 \text{ at } 0 \leq r \leq R_c \text{ and } x = 0 \]

where:
\[ \int J_\sigma dA_\sigma = I_D \]  

(3.33)

\[ \int J_\sigma dA_\sigma = I_D \]  

(3.34)

The Laplace equation is solved numerically using finite difference technique. This solution is part of the overall solution which is obtained through an iterative process. Detailed description of the numerical solution is presented in appendix D. By solving the above set of equations, the plasma properties, sheath characteristics and wall temperature are obtained based on terminal conditions such as discharge current, mass flow rate, gas and wall properties and cathode geometry.

3.4 Comparison between theory and Siegfried’s experimental data

Experimental data published by Siegfried and Wilbur [38, 40] have provided a basis for comparison with the theoretical results obtained with the basic model. Plasma properties and wall temperature are plotted against discharge current and cathode internal pressure for mercury and xenon.

The variation of cathode surface temperature (figure 3.2) is governed by the Schottky-Richardson law which indicates that field-enhanced thermionic (vs. photo-) emission is the most important mechanism of cathode electron emission.
The difference between the calculated and measured temperatures does not exceed 10%. Figure 3.3 indicates that the predicted electron temperature is lower than the measured value about 0.1-0.2 eV, which is of the order of the experimental accuracy reported. The plasma electron number density (figure 3.4) varies linearly with the discharge current while the plasma potential (figure 3.5) appears to be insensitive to the discharge current for the specified operating conditions. Maximum discrepancies between theory and experiment of 35% and 36% are obtained for plasma potential and electron number density respectively. In general, the cathode pressure (figures 3.6 to 3.9) seems to have little effect on the overall cathode operation.

It is quite important to point out that Siegfried's experimental data were given as maximum and average values of the plasma characteristics without reference to spatial variation of these properties. This lack of information has prevented an overall description of the cathode operation. Therefore, a more detailed experimental investigation has become a necessity for further validation and improvement of the theoretical modeling of hollow cathode discharge. An extensive experimental work carried out as part of this study is presented in the subsequent chapters.
Figure 3.1: Effect of discharge current on cathode surface temperature (comparison between theory and experiment).
Figure 3.2: Effect of discharge current on electron temperature (comparison between theory and experiment).
Figure 3.3: Effect of discharge current on electron number density (comparison between theory and experiment).
Figure 3.4: Effect of discharge current on plasma potential (comparison between theory and experiment).
Figure 3.5: Effect of internal pressure on cathode surface temperature (comparison between theory and experiment).
Figure 3.6: Effect of internal pressure on electron temperature (comparison between theory and experiment).
Figure 3.7: Effect of internal pressure on electron number density (comparison between theory and experiment).
Figure 3.8: Effect of internal pressure on plasma potential (comparison between theory and experiment).
CHAPTER IV

EXPERIMENTAL APPARATUS

4.1 Introduction

In part of the present efforts, an extensive experimental program was carried out (by the author) at the NASA-Lewis Electric Propulsion Laboratory (EPL). This Laboratory is equipped with a number of test facilities which are designed to study a variety of electric propulsion devices such as MPD thrusters, ion engines and arcjets. These facilities are equipped with advanced instrumentation that allow both automatic and manual control, test monitoring, diverse diagnostics, fast data acquisition and safe operation.

Vacuum facilities are considered the essential elements of EPL, since they allow the duplication of space environment. Vacuum facilities can be classified into two categories based on their size: tanks and bell jars. Pressures as low as $10^{-7}$ Torr can be obtained in the vacuum tanks. These pressures are realized using turbo-molecular pumps.
4.2 Experimental facility

The present experimental work was conducted in the Bell Jar 6 (BJ6) test facility shown in figures 4.1 and 4.2. All tests were run in BJ6, a 0.53 m diameter, 0.36 m long stainless steel vacuum chamber. BJ6 was evacuated using a turbomolecular pump, in conjunction with a roughing pump (figure 4.3), which was capable of pumping 260 liters/sec of air at a range of 1 to $10^{-6}$ Pa ($7.5\times10^{-3}$ to $7.5\times10^{-9}$ Torr). The turbomolecular pump consisted of moving and stationary discs similar to turbine stages. The pumping action was realized through the relative velocities between gas molecules and moving discs. At rotor speeds of $1.6\times10^{4}$ rpm, the gas was driven to the exhaust port with compression ratios of $10^{6}$ to 1 for air.

BJ6 was equipped with a number of feedthrough flanges to accommodate power supply cables, gas feed tubbings, thermocouple connections and plasma diagnostic instrumentation (figure 4.4). In addition, it had Plexiglas windows which allowed visual monitoring of tests in progress and offered the capability to take photographs of the experiments, use pyrometers for temperature measurements and implement spectroscopic techniques. BJ6 was also equipped with a pressure control baffle mounted between the vacuum chamber and a turbomolecular pump to control the chamber pressure by varying the area of the exhaust port. Ionization gage and
Figure 4.1: Photograph of the NASA-Lewis Research Center bell jar 6 test facility
capacitance manometer were utilized to measure the chamber pressure to cover a range of pressures from $10^{-6}$ to $10^{-2}$ Torr.

### 4.3 Gas feed system

The gas feed system (figure 4.5) was comprised of two independent gas injection ports, one fed gas into the hollow cathode, while the other one fed gas into BJ6. In this fashion the pressure in the chamber could be varied independently of the cathode flow rate. The gas system consisted of a number of precision leak valves that allowed to eliminate fluctuations in flow rates. A Hastings linear mass flowmeter (Teledyne Hastings-Raydist, Hampton, Virginia) was used to establish desired flow rates. This flowmeter was designed to measure accurately mass flow rate without compensations for gas pressure and temperature. It operated on an electrical principle where a small tube is heated uniformly by a transformer. The gas flow through the capillary tube caused an asymmetrical temperature distribution. The differential output of two external thermocouples, mounted at both ends of the tube, was related to the mass flow rate and heat capacity of the gas. This flowmeter was calibrated using a bubble meter (Teledyne Hastings-Raydist Mini-Flo Calibrator, Hampton, Virginia) for flow rates ranging from 1 to 30 sccm of Argon.
Figure 4.4: Photograph of the hollow cathode setup inside bell jar 6
The gas feed system was also equipped with a capacitance manometer, rated for 100 Torr maximum pressure, that allowed to measure the pressure upstream of the hollow cathode. Ultra high purity grade (99.999 %) gases such as argon, helium, neon, xenon and nitrogen could be used as propellant in this facility. They were delivered from high pressure (250 psig) bottles.

4.4 Power Supply

A power supply capable of delivering 600 volts was used with a current regulator. This setup allowed operation at currents ranging from 1 to 30 A and voltages between 10 and 60 V depending on the anode size, flow rate and distance between the electrodes. The facility was also equipped with 1 kV ignitor required for arc ignition.

4.5 Hollow cathode assembly

The hollow cathode assembly (figures 4.6 and 4.7) consisted of an insert, cathode body, an orifice plate, a heater and a radiator shield. The insert was 3.81 mm inner diameter, 5.33 mm outer diameter and 25.4 mm long tube. It was made of low work function material (1.8-2.0 eV): impregnated tungsten with barium. The low work function allowed the operation of the cathode at low temperatures (1000°C) which had a direct impact on the cathode lifetime. The insert was housed in a molybdenum-rhenium cathode body: 5.59 mm inner diameter, 6.35 mm outer di-
Figure 4.5: Schematic of the main components of the gas feed system
ameter and 63.5 mm long tube. A 2% thoriated tungsten orifice plate was electron-beam welded at one end of the cathode body. It had 5.84 mm outer diameter, 1.24 mm thickness and came in different orifice sizes. The orifice sizes used in this research were 0.76, 1.21 and 1.27 mm diameters. The orifice was designed with a 45 degrees chamfer.

An 8-turn coil swaged heater is used to heat the cathode during activation and ignition procedures. The heater consists of a tantalum tube and wire insulated by magnesium oxide tubes. The heater is shielded by a 0.013 mm thick, 30.5 mm wide and 457 mm long tantalum foil wrapped around it.

To complete the electric circuit a 152 x 102 mm tantalum plate was used as anode. The size of the anode made it possible to operate at low discharge voltages (9 to 30 V).

4.6 Diagnostic Instrumentation

Two different plasma diagnostic techniques were used in the experimental investigation of the hollow cathode discharge: single Langmuir probes and spectroscopy.

4.6.1 Single Langmuir probe circuit

An all solid-state design bipolar power supply (Kepco BOP model 100-1M, Flushing, New York) was used (figure 4.8). It had two bipolar control channels (Voltage
Figure 4.6: Photograph of the hollow cathode mounted inside bell jar 6
Figure 4.7: Schematic of the hollow cathode assembly
or current mode). The bipolar power supply was driven by a 20 MHz function generator (Waveteck model 190) which was a source of a number of waveforms (sine, triangle and square) in addition to a dc voltage. All waveforms could be varied from 0.002 Hz to 20 MHz and externally modulated. Waveforms amplitude could be varied from 1.5 mV to 30 V peak-to-peak and the dc reference could be offset either positively or negatively.

The probes used were single Langmuir probes (figure 4.9) in which a thin tungsten wire 0.127-0.254 mm in diameter and 1-2 mm long was supported by a 99.8% alumina tube (Vesuvius-McDanel). The 99.8% Alumina tube could be used at operating temperatures of 1950°C in both oxidizing and reducing environments. These tubes were evacuated to $10^{-7}$ Torr at 1500°C during their manufacturing process. The probe circuit consisted of a three shunt resistors: 1, 10, and 100 Ohms. In this fashion, the measured voltage could be amplified at the desired level. Both input and output voltages were displayed on a digital oscilloscope (Nicolet model 310). Among the remarkable features of the Nicolet was its capability of storing full probe traces on floppy disks. The data were later reduced using 386 IBM-compatible computer.
Figure 4.8: Schematic of single Langmuir probe electric circuit
Figure 4.9: Photograph of electrostatic (top) and thermocouple (bottom) probes
4.6.2 Experimental setup for spectroscopy

Spectroscopy is an non-intrusive technique that allows the determination of plasma properties provided that adequate light intensity is generated by the radiation source. The experimental spectroscopic setup shown schematically in figure 4.10 consisted of a 500M Czerny-Turner spectrometer (SPEX Industries, Edison, New Jersey) characterized by a 0.5 meter focal length, linear drive in wave length, 0-1500 nm spectral coverage with 1.6 nm/mm dispersion, an accuracy of 0.1 nm and a drive step size of 0.00025 nm. The motor of the spectrometer is driven by a ministep driver (MSD). A compudrive (CD2A) handed total keyboard control over the spectrometer drive system. It allowed to repeat scanning of the spectral regions defined in advance and output the spectrometer position and scan status. The holographic grating with 1200 grooves/mm used in the spectrometer provided an average resolution of 0.4 A per pixel of the diode array used as a light detector.

An Optical Spectrometric Multichannel Analyzer (OSMA) was used for fast spectral data acquisition. The OSMA, a computer-controlled multichannel image detector, had the capability of detecting, measuring and manipulating spectra at high acquisition rates. The OSMA consisted of a computer console, a detector controller (ST-120) and an IRY detector head.

The detector head consisted of an optoelectronic image device with the neces-
sary electronics for its optimized manipulation. It was a self-scanning photodiode arrays with 1024 pixels arranged linearly on a 25.4 mm long single line. The detector was characterized by its wide spectral range, high dynamic range, geometric accuracy, thermal and temporal stability and the lack of readout lag problems.

The detector receives power, thermostating and timing signals from the ST-120 detector controller. The ST-120 coordinates data collection with experiment, digitizes and averages data, sets exposure time, stores data and transmits to the computer and provides a real-time display of the free running detector readout.

The spectrometer received light through a single mode optical fiber (quartz FC-2UV, Newport Corporation). This fiber had a core diameter of 200 μm, a cladding diameter of 250 μm, a jacket diameter of 1000 μm, a numerical aperture of 0.2 and an aperture half angle of 11.5 degrees. To improve fiber to spectrometer coupling an achromatic lens was positioned between the fiber and the entrance slit. In this fashion, the fiber beam was adjusted to the f# of the spectrometer. Furthermore, the alignment of the optical fiber with the spectrometer entrance slit was achieved by a 0.5 mW compact He-Ne laser.
4.6.3 Internal pyrometry

The internal surface temperature of the hollow cathode insert is an important parameter because it is related directly to the thermionic emission as well as the lifetime of the cathode. Therefore, it becomes essential to determine the temperature of the insert surface experimentally. In this case, pyrometry was implemented by coupling a quartz rod to a two-color photodiode.

The two-color photodiode (figure 4.11) is a silicon/germanium "sandwich" detector (J16Si-8A4-ROSM, EG&G Judson, Montgomeryville, PA). The high performance silicon detector mounted over the germanium detector responds to radiation from 400 nm to 1000 nm while the germanium responds to longer wavelength 1000 nm to 1800 nm that pass through the silicon photodiode. This kind of detector is suitable for two-color temperature measurements from 500°C to 2000°C.

A clear fused quartz rod (Suprasil2, Heraeus Amersil, Duluth, GA) was used to transmit radiation from cathode surface to the detector. The quartz rod was designed to withstand maximum working temperature of 950°C and 1200°C in a continuous and short-term operations respectively. It had a constant index of refraction for a wide wavelength band and exhibited a fairly constant transmission efficiency higher than 90% in the range of wavelengths between 200 nm and 2000 nm. The 1 mm diameter quartz rod used, was supported by a 99.8% alumina tube.
To reduce the contribution of the plasma radiation, the quartz rod was cleaved at 45 degrees and coated by a tantalum film few microns thick. A small window was left uncoated so that the incoming radiation through the window could be reflected by the 45 degrees surface and transmitted through the fiber to the detector. The pyrometric probe had to be (manually) moved rapidly in order to minimize the exposure time so that the optical properties of the material would not degrade, the coating would not heat up and start radiating and any physical damage to the material could be prevented. In this case, the high sampling rate required was achieved by automating the reading of both probe position and radiation intensity signals. This task was accomplished by the use of a multiple-channel A/D card. The use of a computer allowed fast data reading, processing and storing.

4.6.4 External temperature measurements

The external surface temperature of the cathode was measured using type R thermocouples (Platinum-Platinum 13% Rhodium, maximum working temperature 1600°C). In an attempt to determine axial temperature distribution on the cathode surface five thermocouples were embedded between the heater turns through tiny holes in the radiation shield (figure 4.7). Ceramic (99.8% alumina) tubes were used as insulation for its high working temperature (1800 °C). The presence of the heater coil and the radiation shield surrounding the cathode tube minimizes the thermal losses from the cathode surface. Thus, the uncertainty in temperature
Figure 4.11: Schematic of pyrometry diagnostic instrumentation
Figure 4.12: Photograph of pyrometric (top) and spectroscopic (bottom) probes
measurements due to thermocouple-surface contact, was reduced considerably.

4.7 Experimental procedures

4.7.1 Pumpdown

Every time the hollow cathode assembly was exposed to the atmosphere, BJ6 was evacuated, using turbomolecular pumps, to base pressures $10^{-5}$ to $10^{-6}$ Torr. In general, the pumpdown lasted twenty four hours before any operation. This procedure was of extreme importance because it assured a low base pressure and allowed adequate outgasing of the different equipment used. The system was usually evacuated up to the high pressure gas feed bottles. In this fashion, leaks in the gas feed system could be detected and fixed in order to guarantee the high purity of the propellant used. The knowledge of the exact properties of the working gas has a direct impact on the overall study of the hollow cathode discharge. That is, both operating conditions (e.g., mass flow rate) and physical processes depend strongly on the nature of the propellant.

4.7.2 Cathode activation

The impregnated tungsten insert is susceptible to absorb humidity, oxygen, carbon dioxide when exposed to the atmosphere. To prevent destructive reactions of the impregnate and tungsten with theses contaminants, the insert had to be subject to a conditioning procedure. The latter consisted of heating the cathode gradually, under vacuum, for several hours with a flow of an inert gas (e.g., argon or
xenon). This process consisted of three phases: low (100°C), medium (550°C) and high heat (1050°C) with a cool-down period of time between the different phases. The gradual heating was intended to allow the insert to release the residual gases slowly so that no chemical or physical damage could occur. In addition to this conditioning phase, an activation phase was necessary before every ignition of the cathode. The activation procedure consisted of maintaining the insert at 1050°C for a minimum period of 30 minutes in order to increase the thermionic emission potential of the insert surface. In this fashion, ignition was achieved quickly with low ignitor voltages. Furthermore, erosion of the cathode was substantially reduced if not prevented.

### 4.7.3 Internal temperature calibration

The internal pyrometry diagnostic required a careful calibration of the optical system adopted (quartz rod, photodiode, amplifier and A/D converter). It was necessary to manufacture a tungsten tube with identical dimensions to the insert except with a very thin wall. The tube was heated using a shielded heater coil. Since the tube was almost closed (only one end is open to receive the probe) and had a thin wall, the temperature of the inner surface would reach the outer surface temperature at thermal equilibrium. In this fashion, a set of thermocouples positioned on the shielded outer surface would allow the measurement of the inner surface temperature. By heating the tungsten tube to different temperatures and
measuring the amplified photodiode output, it was possible to calibrate the optical system. The A/D converter offered the ability of fast and frequent sampling which was essential to avoid heating of the quartz rod. Measurement were repeated to assure the quality and accuracy of the calibration.
CHAPTER V

EXPERIMENTAL RESULTS

Two diagnostic techniques were applied in order to acquire experimental data: single Langmuir probes and spectroscopy. The measurements were aimed primarily at the determination of the plasma properties such as electron temperature, electron number density and plasma potential within the hollow cathode. In this experimental study both argon and xenon were used as propellant in an attempt to establish the effect of the propellant on the hollow cathode characteristics. Furthermore, orifice diameters of 0.76 mm, 1.21 mm, and 1.27 mm were used to determine the geometry effects on cathode discharge. The Langmuir probe diagnosis was performed along the cathode center line. These measurements allowed to determine the axial distributions of plasma conditions. In addition to plasma properties, it was essential to determine the cathode external and internal surface temperatures to identify the different emission mechanisms and emission current density profile.
5.1 Plasma Properties

In the case of the single Langmuir probe, a sawtooth voltage pulse at frequencies of 1,000 to 10,000 Hz was applied to the probe which allowed to obtain the entire probe characteristic in tenths of a millisecond. In all measurements more than five probe traces were recorded, analyzed and averaged. Due to the high probability of contamination, special care was taken in cleaning the probe before each measurement by drawing saturation current to heat the probe tip until probe emission is observed. In addition, the probe was kept under a negative bias (-50 V) for one to five minutes between pulses.

5.1.1 Electron temperature

Electron temperature was measured using both diagnostic techniques for both argon and xenon. Electron temperatures from 0.6 to 2.2 eV for argon and 0.6 to 1.4 eV for xenon were measured along the cathode center line using single Langmuir probes. In all measurements, the probe traces exhibited Maxwellian distributions of the plasma electron. Figures 5.1 and 5.2 indicate that the electron temperature slightly decreased as the discharge current was increased. This behavior appears to be consistent at different distances from the cathode orifice. Moreover, the variation of electron temperature with current exhibits a linear relationship. On the other hand, the gas flow rate has very little effect on the electron temperature (fig-
ures 5.3 and 5.4). Figures 5.5 through 5.7 show a substantial variation in electron temperature along the cathode center line. Figures 5.8 through 5.10 illustrate the effect of the orifice size on the electron temperature. This latter tends to increase as the orifice diameter is increased. The effect of the propellant is illustrated in figures 5.11 through 5.13. The lower electron temperature obtained in the case of xenon is consistent with the lower ionization potential of xenon compared to argon.

In an attempt to shed some light on the state of the plasma within the hollow cathode, spectroscopic measurements were made under a wide range of operating conditions. These data could reveal whether or not the plasma is in local thermodynamic equilibrium (LTE) and subsequently the physical sense of the temperature determined using relative intensity measurements. In this case, one can define a parameter $T$ as given by [44]:

$$T = \frac{E_m - E_p}{k \ln(I_{pq} A_{mj} g_m \nu_{mj} / I_{mj} A_{pq} g_p \nu_{pq})}$$

(5.1)

The temperature $T$ would be the electron temperature only when the plasma is in LTE. This condition could be verified by constructing a Boltzmann plot where the quantity $\ln(I_{mj}/g_m A_{mj} \nu_{mj})$ for each spectral line is plotted versus the exci-
tation energy $E_m$ of the upper level of the transition. This test requires a wide spectral range in excitation energy which is, in general, not feasible experimentally. However, the relative error in $T$:

$$\frac{\Delta T}{T} = \frac{kT}{|E_m - E_p|} \left\{ \frac{\Delta(I_{m_j}/I_{p_q})}{I_{m_j}/I_{p_q}} + \frac{\Delta(A_{m_j}/A_{p_q})}{A_{m_j}/A_{p_q}} \right\}$$ (5.2)

could be reduced by making the ratio of $kT$ to the upper level excitation energy difference as small as possible.

Using ArII (singly ionized argon) spectral lines in the range 4000-5000 Å with excitation energies from 19 to 23 eV, and XeII spectral range 4000-7000 Å with excitation energies from 14 to 18.5 eV, Boltzmann plots ($ln(I_{m_j}A_{m_j}/A_{m_j}g_m) vs. E_m$) were constructed. In most cases, deviations from a straight line are less than 15%. However, obtaining a straight line within a limited spectral range is not sufficient to allow for a conclusive definition of the state of the plasma. Figures 5.14 to 5.31 illustrate the determination of the “electron” temperature using relative-intensity measurements. Temperatures from 0.91 to 1.3 eV for argon and 0.98 to 1.09 eV for xenon were calculated. The results obtained show a weak dependence of the “electron” temperature on the operating conditions. On the other hand, it coin-
cides with the average electron temperature obtained using Langmuir probe.

The analysis of both argon and xenon spectra, in the spectral range 3500-8500 Å, showed no evidence of the presence of doubly ionized species. Most of argon II spectrum lines were detected in the 3500-5000 Å, while argon I spectrum lines in the 7000-8500 Å region. In the case of xenon, the spectrum of xenon I was detected in the 7000-9000 Å while the spectrum of xenon II was detected in the 3500-7000 Å region. In both cases, the intensity of the neutral atom spectra varies from two times stronger to five times weaker than the strongest lines of singly ionized atom spectra depending on the discharge current applied.

In order to analyze the accuracy and consistency of the calculated electron temperature from intensity measurements, a computation of the species composition as a function of electron temperature, heavy particle temperature and pressure was performed. Using the equilibrium condition and the particle conservation equations, one can solve for the particle densities. The detailed solution of this problem is described in Appendix A. The results obtained are illustrated in figures 5.32 and 5.33 for some typical operating conditions. Based on the observation made earlier regarding the detected spectra, a most probable electron temperature would be in the 0.8-1.5 eV range for argon and 0.6 to 1.2 eV range for xenon. These results are consistent with the experimental data (spectroscopy and Langmuir probe di-
agnostics).

5.1.2 Electron Number Density

The electron number density was determined using cylindrical Langmuir probes. Due to the small size of the cathode insert, few millimeters in diameter, all measurements were performed along the cathode center line. In this case, the probe was inserted from the back of the cathode since the orifice size is a few tenths of a millimeter. The probe diagnostics are conducted using argon and xenon under a wide range of discharge currents, mass flow rates and orifice sizes.

In the regions where the electron density reached $10^{14}$ cm$^{-3}$ or higher, it became difficult to obtain the electron saturation current without severely perturbing the discharge. These perturbations usually showed up in the rapid increase of the cathode surface temperature and fluctuations in discharge current and voltage. In this case, the probe bias was negatively offset to avoid reaching the electron saturation current and consequently, the calculation of the plasma density was performed using the ion saturation current. In the latter, The plasma density was determined by applying Lafromboise's theory (details are given in Appendix B).

In general, plasma densities of $10^{11} - 10^{15}$ cm$^{-3}$ are calculated in the range of
operating conditions applied in these experiments. The electron number density reaches a maximum value in the vicinity of the orifice and decreases as we move away from the orifice to values of the order of $10^{12}$ cm$^{-3}$ at approximately two insert diameters. Farther away, the plasma density drops drastically to values difficult to measure by the probe setup. Figures 5.34 to 5.36 show clearly three different zones within the cathode. It is believed that the active zone where the ionization take place start at about one insert diameter from the insert, the second zone is a diffusion sheath [27] which extends also to about one insert diameter and the last zone describes the neutral gas. The xenon plasma region within the cathode is larger than that of argon (figures 5.37 to 5.39). This difference is consistent with the fact that xenon has a lower ionization potential than argon. The effect of the cathode orifice size is illustrated in figures 5.40 and 5.41. It is shown that a larger orifice diameter produces a higher plasma density. This effect appears to be related to the higher electron temperature measured in the case of the larger orifice size. On the other hand, this difference may be very small in reality due to the relative errors induced in the measurements and related to the limitation of the probe theory. While the plasma density is strongly dependent on the discharge current (figures 5.42 and 5.43), it appears weakly affected by the propellant flow rate (figure 5.44).

It is important to point out that the maximum values of the plasma densities
measured are of similar order to the one calculated using the method described earlier (figures 5.32 and 5.33) for typical operating parameters of the cathode.

5.1.3 Plasma Potential

Under the conditions presented here, the hollow cathode was operated at discharge voltages from 10 V to 40 V. The maximum plasma potentials measured within the hollow cathode did not exceed 35 V. In general, lower discharge voltages were obtained with the cathode when xenon was used. Also, an increase of the orifice size caused lower discharge voltages.

In the vicinity of the orifice where a full probe trace was not obtained for the reason mentioned in the previous section, the plasma potential was estimated by means of the following equation [17]:

\[ V_p = V_f - \frac{kT_e}{2e} \ln \left( \frac{\pi m_e}{2 m_i} \right) \]  

(5.3)

In all cases, the plasma potential for both argon and xenon decreased with the discharge current (figures 5.45 to 5.46). This behavior was related, in part, to an increase of the degree of ionization which lowered the plasma resistivity and
to the decrease in the internal plasma column thickness. The axial distribution of the plasma potential is illustrated in figures 5.47 and 5.48. The plasma potential decreased rapidly as we moved away from the cathode orifice. The plasma potential for argon was higher, in most case, than that of xenon but the difference in voltage decreased with the increase in discharge current (figures 5.49 to 5.52). In addition, this comparison shows clearly that, with xenon, the internal plasma column was much thicker than that of argon. The effect of the orifice size on the plasma potential is illustrated in figures 5.53 to 5.55. Even though the cathode with 1.21 mm orifice diameter operated at lower discharge voltages, the estimated plasma potential was higher than that of the cathode with 0.76 mm orifice size. This high potential was attributed mainly to the thickness of the internal plasma column.

5.1.4 Cathode Internal Pressure

The cathode internal pressure was measured upstream of the cathode using a highly accurate (0.1% accuracy) capacitance manometer rated for 100 torr. Cathode internal pressures from 5 to 30 torr were measured depending on flow rate, discharge current and orifice size.

In an attempt to determine the dependence of the cathode internal pressure on
the operating conditions, the pressure was plotted against the discharge current in figures 5.56 and 5.57 and flow rate in figure 5.60. The graphs show that the pressure increased linearly with the discharge current for a fixed gas flow rate. Furthermore, it is interesting to notice that the ratio of the slopes of these straight lines is equal to the ratio of the gas flow rates. On the other hand, the pressure is proportional to the mass flow rate which is consistent with the theoretical formulation of the mass flow rate of a choked flow.

The effect of the propellant on the pressure is illustrated in figures 5.58 and 5.59. By comparing xenon and argon, the pressure ratio is found to be equal to the parameter $\sqrt{\mathcal{M}_X/\mathcal{M}_A}$, which is also an indication of the validity of the mass flow rate equation for a choked flow. In this case, the dependence of the pressure on the propellant resides in the specific gas constant $R (\mathcal{R}/\mathcal{M})$. Finally, to establish the effect of the orifice size, the pressure was plotted against the discharge current for two different orifice sizes (figure 5.61). The latter shows that the pressure variation is substantially increased as the orifice diameter is decreased. It is important to indicate that at very low current the ratio of xenon to argon pressures approaches the area ratio of the cathode orifices.
5.2 Wall temperature

In an attempt to fully understand the hollow cathode operation, it is important to investigate the cathode wall temperature and its dependence on the operating conditions. Also, the knowledge of the surface temperature is vital to identify the important mechanisms of electron emission from the cathode surface. Furthermore, the distribution of the surface current density would lead to quantify the emission length. These different aspects of the cathode surface emission are tied with the lifetime of the cathode which represent a central question in the suitability of hollow cathodes for space applications.

In this study, there has been a special interest in determining the temperature of both external and internal surfaces of the hollow cathode. This would allow, in addition to the reasons indicated earlier, to evaluate the different heat transfer mechanisms through the cathode wall. In order to obtain the temperature profile on the external surface of the cathode, type R thermocouples were attached to the surface at several positions through the radiator shield and the heater of the hollow cathode. This setup allowed to construct figures 5.62 and 5.63 which describe the temperature distribution under various operating conditions. It is evident that the wall temperature reached a maximum in the vicinity of the orifice plate and then decreased rapidly as we moved farther upstream. In general, there was a difference
of 300 to 400 °C in temperature along the cathode surface. The cathode temperature depends strongly on the discharge current (figures 5.64 through 5.66) while it appears insensitive to the gas flow rate (figure 5.67). However, at very low mass flow rate, the cathode temperature increased substantially. This effect is related directly to the increase of the discharge voltage and subsequently the occurrence of plasma instabilities and voltage fluctuations. The effect of propellant and orifice size on the cathode temperature are apparently not significant (figures 5.68 through 5.70).

The insert temperature was also determined using the experimental technique described in chapter III. Knowing the insert temperature is extremely important to establish the surface plasma-wall interaction, surface cooling capability by electron emission, surface work function and the limit of the active zone. In figures 5.71 and 5.72, the insert temperature profile is presented and compared to the external surface temperature. It is very interesting to note that both profiles are similar and the temperature difference is quite small. This observation is consistent with the fact that the cathode walls are very thin and that the cathode is shielded on the outside. In this fashion, the radial heat loss through the wall is extremely low. Therefore, the outer temperature, which is easy to measure, could be used to predict the insert temperature and its axial distribution.
The analysis of the thermionic current density allowed to establish what part of the surface contributed mostly in the emission and compare that to the previous predictions of the scale size of the internal plasma column. Furthermore, this study would permit to estimate the surface work function and compare it to the properties of the material used.

The surface current density is determined using the measured wall temperature in conjunction with Richardson equation given by:

\[
j_s = A_R T_w^2 \exp \left( \frac{e\phi}{kT_w} \right)
\]

(5.4)

The work function is chosen so that the integral of the current density (equation 5.5), over the surface area of the insert, is equal to the corresponding discharge current neglecting the ion current, which is usually small compared to the emission current.

\[
\int j_s dA = I_D
\]

(5.5)
The current density profiles are illustrated in figures 5.73 through 5.78 for several operating parameters. It is evident that the current density falls drastically beyond one insert diameter from the orifice plate. It is evident that the effective length of the emission surface is scaled to one insert diameter. 65% to 80%, for argon and 50% to 70%, for xenon of the total current is emitted within one insert diameter from the orifice plate. The length of the emission surface is comparable to the length of the plasma internal column described in the previous sections. Also, the longer emission region for xenon compared to argon is indicated, once again, by the current density profile. Furthermore, the work function of the impregnated tungsten insert is known to be in the range of 1.8-2.0 eV [12]. The deduced work function from the above, rather simplified model of the surface emission, falls in this range. Consequently, this result provides a strong foundation to assume that thermionic emission could be the most important cooling mechanism for the cathode wall.
Figure 5.1: The effect of discharge current on electron temperature along the cathode center line at different distances from the orifice (argon).

Figure 5.2: The effect of discharge current on electron temperature along the cathode center line at different distances from the orifice (xenon).
Figure 5.3: The effect of flow rate on electron temperature along the cathode center line at different distances from the orifice (argon).

Figure 5.4: The effect of flow rate on electron temperature along the cathode center line at different discharge currents (argon).
Figure 5.5: Axial distribution of electron temperature along the cathode center line at different discharge currents (argon, $d_o = 0.76 \text{ mm}$).

Figure 5.6: Axial distribution of electron temperature along the cathode center line at different discharge currents (argon, $d_o = 1.21 \text{ mm}$).
Figure 5.7: Axial distribution of electron temperature along the cathode center line at different discharge currents (xenon, $d_o = 1.21\ mm$).

Figure 5.8: Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, $I = 3\ A$).
Figure 5.9: Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, $I = 5 \, A$).

Figure 5.10: Axial distribution of electron temperature along the cathode center line for different orifice sizes (argon, $I = 9 \, A$).
Figure 5.11: Axial distribution of electron temperature along the cathode center line in argon and xenon discharges ($I = 5 \, A$).

Figure 5.12: Axial distribution of electron temperature along the cathode center line in argon and xenon discharges ($I = 9 \, A$).
Figure 5.13: Axial distribution of electron temperature along the cathode center line in argon and xenon discharges ($I = 15 \, A$).

Figure 5.14: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 3 \, A, \dot{m} = 0.93 \, A_{eq.}, d_o = 0.76 \, mm$).
Figure 5.15: Calculation of the electron temperature using relative line intensity ratio (argon, \( I = 7 \, A, \dot{m} = 0.93 \, A \, eq., d_o = 0.76 \, mm \)).

Figure 5.16: Calculation of the electron temperature using relative line intensity ratio (argon, \( I = 10 \, A, \dot{m} = 0.93 \, A \, eq., d_o = 0.76 \, mm \)).
Figure 5.17: Calculation of the electron temperature using relative line intensity ratio \((\text{argon}, I = 3\, A, \dot{m} = 0.93\, \text{A eq.}, d_o = 1.27\, \text{mm})\).

Figure 5.18: Calculation of the electron temperature using relative line intensity ratio \((\text{argon}, I = 7\, A, \dot{m} = 0.93\, \text{A eq.}, d_o = 1.27\, \text{mm})\).
Figure 5.19: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 10 \, A$, $\dot{m} = 0.93 \, A \text{ eq.}, d_o = 1.27 \, mm$).

Figure 5.20: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 15 \, A$, $\dot{m} = 0.93 \, A \text{ eq.}, d_o = 1.27 \, mm$).
Figure 5.21: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 7 \, A$, $\dot{m} = 0.5 \, A \text{ eq.}$, $d_0 = 0.76 \, mm$).

Figure 5.22: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 10 \, A$, $\dot{m} = 0.5 \, A \text{ eq.}$, $d_0 = 0.76 \, mm$).
Figure 5.23: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 15\ A$, $\dot{m} = 0.5\ A\ eq.,\ d_o = 0.76\ mm$).

Figure 5.24: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 7\ A$, $\dot{m} = 0.5\ A\ eq.,\ d_o = 1.27\ mm$).
Figure 5.25: Calculation of the electron temperature using relative line intensity ratio (argon, \( I = 10 A, \dot{m} = 0.5 A \text{ eq.}, \, d_o = 1.27 \text{ mm} \)).

Figure 5.26: Calculation of the electron temperature using relative line intensity ratio (argon, \( I = 15 A, \dot{m} = 0.5 A \text{ eq.}, \, d_o = 1.27 \text{ mm} \)).
Figure 5.27: Calculation of the electron temperature using relative line intensity ratio (argon, $I = 20\,A, \dot{m} = 0.5\,A\,eq., d_o = 1.27\,mm$).

Figure 5.28: Calculation of the electron temperature using relative line intensity ratio (xenon, $I = 3\,A, \dot{m} = 0.5\,A\,eq., d_o = 1.27\,mm$).
Figure 5.29: Calculation of the electron temperature using relative line intensity ratio (xenon, $I = 10$ A, $\dot{m} = 0.5$ A eq., $d_o = 1.27$ mm).

Figure 5.30: Calculation of the electron temperature using relative line intensity ratio (xenon, $I = 15$ A, $\dot{m} = 0.5$ A eq., $d_o = 1.27$ mm).
Figure 5.31: Calculation of the electron temperature using relative line intensity ratio (xenon, $I = 20\, A$, $m = 0.5\, A\, eq.$, $d_o = 1.27\, mm$).

Figure 5.32: Calculation of species composition for argon using two-temperature Saha equation ($T_h = 0.15\, eV, p = 10\, Torr$).
Figure 5.33: Calculation of species composition for xenon using two-temperature Saha equation \((T_h = 0.2 \text{ eV}, p = 10 \text{ Torr})\).

Figure 5.34: Axial distribution of electron number density along the cathode center line (xenon, \(\dot{m} = 0.5 \text{ A eq.}, d_o = 1.21 \text{ mm}, I = 5 \text{ and } 9 \text{ A})\).
Figure 5.35: Axial distribution of electron number density along the cathode center line (xenon, $\bar{m} = 0.5$ A eq., $d_o = 1.21$ mm, $I = 9$ and 12 A).

Figure 5.36: Axial distribution of electron number density along the cathode center line (xenon, $\bar{m} = 0.5$ A eq., $d_o = 1.21$ mm, $I = 12$ and 15 A).
Figure 5.37: Axial distribution of electron number density along the cathode center line in argon and xenon discharges \((I = 5\, A, \dot{m} = 0.5\, A\, eq., d_o = 1.21\, mm)\).

Figure 5.38: Axial distribution of electron number density along the cathode center line in argon and xenon discharges \((I = 9\, A, \dot{m} = 0.5\, A\, eq., d_o = 1.21\, mm)\).
Figure 5.39: Axial distribution of electron number density along the cathode center line in argon and xenon discharges ($I = 15\ A, \dot{m} = 0.5\ A\ eq.,\ d_o = 1.21\ mm$).

Figure 5.40: Axial distribution of electron number density along the cathode center line for different orifice sizes. (argon, $I = 10\ A, \dot{m} = 0.93\ A\ eq.,\ d_o = 1.21\ mm$).
Figure 5.41: Axial distribution of electron number density along the cathode center line for different orifice sizes. (argon, $I = 13\, A$, $\dot{m} = 0.93\, A\, eq.$, $d_o = 1.21\, mm$).

Figure 5.42: Effect of discharge current on electron number density along the cathode center line at different distances from orifice (argon, $\dot{m} = 0.5\, A\, eq.$, $d_o = 1.21\, mm$).
Figure 5.43: Effect of discharge current on electron number density along the cathode center line at 1mm from orifice (xenon, $\dot{m} = 0.5$ A eq., $d_o = 1.21$ mm).

Figure 5.44: Effect of flow rate on electron number density along the cathode center line at different distances from orifice (argon, $I = 10$ A, $d_o = 0.76$ mm).
Figure 5.45: Effect of discharge current on plasma potential along the cathode center line at different distances from orifice (argon, $\dot{m} = 0.93\ A$ eq., $d_o = 0.76\ mm$).

Figure 5.46: Effect of discharge current on plasma potential along the cathode center line at different distances from orifice (xenon, $\dot{m} = 0.5\ A$ eq., $d_o = 1.21\ mm$).
Figure 5.47: Axial distribution of plasma potential along the cathode center line at different discharge currents. (argon, $\dot{m} = 0.93$ A eq., $d_o = 1.21$ mm).

Figure 5.48: Axial distribution of plasma potential along the cathode center line at different discharge currents. (xenon, $\dot{m} = 0.5$ A eq., $d_o = 1.21$ mm).
Figure 5.49: Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 5 \, A$, $\dot{m} = 0.5 \, A \,eq.$, $d_o = 1.21 \, mm$).

Figure 5.50: Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 9 \, A$, $\dot{m} = 0.5 \, A \,eq.$, $d_o = 1.21 \, mm$).
Figure 5.51: Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 12 \ A, \dot{m} = 0.5 \ A\ eq., d_o = 1.21 \ mm$).

Figure 5.52: Axial distribution of plasma potential along the cathode center line in argon and xenon discharges ($I = 15 \ A, \dot{m} = 0.5 \ A\ eq., d_o = 1.21 \ mm$).
Figure 5.53: Effect of discharge current on plasma potential along the cathode center line for different orifice sizes (argon, $\dot{m} = 0.93 \, A \, eq., \, L = 4 \, mm$).

Figure 5.54: Axial distribution of plasma potential along the cathode center line for different orifice sizes (argon, $I = 10 \, A, \dot{m} = 0.93 \, A \, eq.$).
Figure 5.55: Axial distribution of plasma potential along the cathode center line for different orifice sizes (argon, $I = 13\ A$, $\overline{\dot{m}} = 0.93\ A\ eq.$).

Figure 5.56: Effect of discharge current on cathode internal pressure along the cathode center line at different flow rates (argon, $d_o = 0.76\ mm$).
Figure 5.57: Effect of discharge current on cathode internal pressure along the cathode center line at different flow rates (argon, \(d_o = 1.21 \text{ mm}\)).

Figure 5.58: Effect of discharge current on cathode internal pressure along the cathode center line in argon and xenon discharges (\(\dot{m} = 0.5 \text{ A eq., } d_o = 1.21 \text{ mm}\)).
Figure 5.59: Effect of discharge current on cathode internal pressure along the cathode center line in argon and xenon discharges ($\dot{m} = 0.93$ A eq., $d_o = 1.21$ mm).

Figure 5.60: Effect of gas flow on cathode internal pressure along the cathode center line at different discharge currents (argon, $I = 10$ A, $d_o = 0.76$ mm).
Figure 5.61: Effect of discharge current on cathode internal pressure along the cathode center line for different orifice sizes (argon, $\dot{m} = 0.93 \, \text{A eq.}$).

Figure 5.62: Axial distribution of cathode external surface temperature at different discharge currents (argon, $\dot{m} = 1.2 \, \text{A eq.}, d_o = 0.76 \, \text{mm}$).
Figure 5.63: Axial distribution of cathode external surface temperature at different discharge currents (xenon, $\dot{m} = 0.93 \ \text{A eq.}, d_o = 1.21 \ \text{mm}$).

Figure 5.64: Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (argon, $\dot{m} = 1.24 \ \text{A eq.}, d_o = 0.76 \ \text{mm}$).
Figure 5.65: Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (argon, $\dot{m} = 0.93 \text{ A eq.}, d_o = 1.27 \text{ mm}$).

Figure 5.66: Effect of discharge current on cathode external surface temperature at different distances from the orifice plate (xenon, $\dot{m} = 0.93 \text{ A eq.}, d_o = 1.21 \text{ mm}$).
Figure 5.67: Effect of gas flow rate on cathode external surface temperature at different distances from the orifice plate (argon, $I = 7 \, A$, $d_o = 1.27 \, mm$).

Figure 5.68: Effect of discharge current on cathode external surface temperature in argon and xenon discharges ($\dot{m} = 0.5 \, A \, eq., \, d_o = 1.21, \, L = 6 \, mm$).
Figure 5.69: Effect of discharge current on cathode external surface temperature in argon and xenon discharges ($\dot{m} = 0.5 \text{ A eq.}, \; d_o = 1.21 \text{ mm}, \; L = 19 \text{ mm}$).

Figure 5.70: Effect of discharge current on cathode external surface temperature for different orifice sizes (argon, $\dot{m} = 0.93 \text{ A eq.}, \; L = 6 \text{ mm}$).
Figure 5.71: Comparison of axial distributions of insert and cathode external surface temperatures (xenon, $I = 4$ A, $\dot{m} = 0.93$ A eq., $d_o = 1.21$ mm).

Figure 5.72: Comparison of axial distributions of insert and cathode external surface temperatures (xenon, $I = 9$ A, $\dot{m} = 0.93$ A eq., $d_o = 1.21$ mm).
Figure 5.73: Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 2\, A$, $\dot{m} = 1.24\, A\, eq.,\, d_o = 0.76\, mm$).

Figure 5.74: Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 2\, A$, $\dot{m} = 0.93\, A\, eq.,\, d_o = 1.21\, mm$).
Figure 5.75: Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 5 \, A$, $\dot{m} = 1.24 \, A \text{ eq.}$, $d_o = 0.76 \, mm$).

Figure 5.76: Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 5 \, A$, $\dot{m} = 0.93 \, A \text{ eq.}$, $d_o = 1.21 \, mm$).
Argon
Flow Rate = 1.24 A eq.
Discharge Current = 11 A
Orifice Diameter = 0.76 mm
$\phi = 1.93$ eV

Figure 5.77: Calculation of thermionic emission current density based on measured cathode temperature (argon, $I = 11$ A, $\dot{m} = 1.24$ A eq., $d_o = 0.76$ mm).

Xenon
Flow Rate = 0.93 A eq.
Discharge Current = 10 A
Orifice Diameter = 1.21 mm
$\phi = 1.9$ eV

Figure 5.78: Calculation of thermionic emission current density based on measured cathode temperature (xenon, $I = 10$ A, $\dot{m} = 0.93$ A eq., $d_o = 1.21$ mm).
CHAPTER VI

IMPROVED THEORETICAL MODEL OF HOLLOW CATHODE ARC DISCHARGES

6.1 Heavy particle temperature

Experimental measurements of the gas temperature along the cathode center line indicate that the heavy particle temperature is higher than that of the wall. Consideration of the thermal conductivity for the conditions indicated by the basic model suggests that the electron temperature is rather uniform in the main body of the cathode plasma. The heavy particle thermal conductivity, however, is dominated by atom-atom collisions and allows a significant temperature gradient between the cathode wall and center line. The heavy particle temperature distribution can be estimated by balancing electron collisional heating [21] of ions with heavy particle heat conduction in the radial direction; It is assumed here that the ions immediately come to a thermal equilibrium with the neutral atoms. Hence,
\[
\frac{1}{r} \frac{d}{dr} \left( r \kappa \frac{dT_h}{dr} \right) = n_e \frac{T_e - T_h}{t_{eq}}
\]  

(6.1)

where \( t_{eq} \), the equipartition time, is given by:

\[
t_{eq} = 5.87 \times 10^6 \frac{M_e M_i}{n_e Z_e^2 Z_i^2 \ln \Lambda} \left( \frac{T_e}{M_e} + \frac{T_i}{M_i} \right)^{3/2}
\]  

(6.2)

Here, the parameter \( \Lambda \) is given by:

\[
\Lambda = 1.24 \times 10^{-7} \left( \frac{T_e}{n_e} \right)^{1/2}
\]  

(6.3)

An approximate formula for the thermal conductivity \( \kappa \) of the heavy particles, for partially ionized gas, could be derived based on the mean-free-path concept as [30]:

\[
\kappa = \frac{15}{8} k n_h \overline{C}_h \lambda_h
\]  

(6.4)

where \( \overline{C}_h \) is the heavy particle average thermal speed and \( \lambda_h \) represents the heavy particle mean free path defined by:
\[ \lambda = \frac{1}{\sqrt{2n_h Q_{hh}}} \]  

(6.5)

Here, \( Q_{hh} \) is the average heavy particle cross section including neutral-neutral, neutral-ions and ions-ions collisions. It is important to point out that for weakly ionized gas conduction is dominated by neutral-neutral collisions.

The heavy particle temperature is determined by solving equation 6.1 numerically in conjunction with the following boundary conditions:

\[ T_h = T_e \text{ at } r = R \]  

(6.6)

\[ \frac{dT_h}{dr} = 0 \text{ at } r = 0 \]  

(6.7)

The radial variation of the heavy particle temperature is illustrated in figure 6.1 for typical operating conditions.

### 6.2 Comparison between theory and experiment

In order to determine the validity of the theoretical model, it is necessary to make a comparison between the theoretical predictions and the experimental measurements of the plasma conditions and wall temperature under identical operating conditions. Therefore, the plasma properties and cathode surface temperature were plotted against the operating conditions (discharge current, mass flow rate,
propellant and cathode geometry).

6.2.1 Cathode temperature

It has been established in chapter 5 that small differences exist between the external and internal cathode surfaces. Therefore, the external surface temperature is compared to the calculated insert temperature. Since the insert work function has been reported by Forman [12] to vary in the range 1.8 - 2.0 eV, these two limits were used to predict the wall temperature. In figures 6.2 and 6.3, in addition to the measured tip temperature, the calculated temperature is plotted against the discharge current for two different values of the material work function. These figures indicate that the present model prediction of the cathode temperature coincide with the cathode tip temperature (maximum temperature). The agreement obtained in all cases, asserts that thermionic emission can be considered the most important surface emission mechanism.

6.2.2 Cathode internal pressure

The cathode internal pressure is one of the important parameters that describes the contributions of the operating conditions in establishing the final state of the internal plasma column. Figures 6.4 to 6.7 show that for a given flow rate, propellant and cathode geometry the internal pressure increases with the discharge
current. This behavior could be explained by the increase in the heavy particle
temperature with the discharge current. The good agreement obtained between
the computed and measured pressure provide confidence in the theoretical predic-
tion of the gas temperature.

6.2.3 Plasma conditions

Plasma density and potential

The internal plasma column is described to extend over two insert diameters.
Within this thickness the electron number density varies from $10^{12}$ to $10^{15} \text{cm}^{-3}$.
Agreements of the theoretical predictions are obtained in the upper limit (figures
6.8 and 6.9). This is related to the fact that the theory does not allow for density
variation within the plasma column inside the cathode. On the other hand, the
prediction of the maximum plasma density inside the hollow cathode is important
to define the state of the plasma before leaving the cathode.

In the case of plasma potential (figures 6.10 and 6.11), agreements are obtained
for discharge currents higher than 3 A. It is important to point out that low currents
as well as low mass flow rates tend to induce an instable discharge that manifests
itself in higher discharge voltages.
Electron temperature

The calculated electron temperature is compared with the measured value using electrostatic probe and spectroscopic diagnostics. Although agreement is satisfactory (figures 6.12 and 6.13), it is not clear whether the so called "configuration temperature" of ArII and XeII is identical with the electron temperature [44]. On the other hand, the substantial variation of the electron temperature (0.6-2.0 eV for argon and 0.6-1.6 eV for xenon) determined from the electrostatic probe measurements along the cathode center line prevents to establish a direct comparison. However, the theoretical predictions compare well with the measured electron temperature (Langmuir probe) on the cathode center line at distances beyond one insert diameter from the orifice.

The high value of the electron temperature measured in the vicinity of the orifice could be caused by several factors such as probe contamination, non uniformity of the plasma potential, the rapid increase in current density toward the small orifice and finally the size of the probe compared to the charged particle mean free paths (Knudsen number). The contamination of the probe is very likely since the vicinity of the orifice is characterized by a very dense plasma ($10^{14} - 10^{16} cm^{-3}$). Not only highly resistive coating could cause erroneous high values of the electron temperature but also a deposition of a low work function material (barium) which is known to flatten the probe current-voltage characteristic [48].
Figure 6.1: Theoretical prediction of the radial variation of the heavy particle temperature (argon, do = 0.76 mm, D = 3.8 mm).
Figure 6.2: Effect of discharge current on cathode surface temperature (comparison between theory and experiment, argon, $\dot{m} = 0.93 \text{ A eq.}, d_o = 1.27 \text{ mm}$).

Figure 6.3: Effect of discharge current on cathode surface temperature (comparison between theory and experiment, xenon, $\dot{m} = 0.93 \text{ A eq.}, d_o = 1.21 \text{ mm}$).
Figure 6.4: Effect of discharge current on cathode internal pressure (comparison between theory and experiment, argon, $\dot{m} = 0.93 \text{ A eq.}$, $d_o = 0.76 \text{ mm}$).

Figure 6.5: Effect of discharge current on cathode internal pressure (comparison between theory and experiment, argon, $\dot{m} = 0.5 \text{ A eq.}$, $d_o = 0.76 \text{ mm}$).
Figure 6.6: Effect of discharge current on cathode internal pressure (comparison between theory and experiment, xenon, $\dot{m} = 0.93 \text{ A eq.}$, $d_o = 1.21 \text{ mm}$).

Figure 6.7: Effect of discharge current on cathode internal pressure (comparison between theory and experiment, xenon, $\dot{m} = 0.5 \text{ A eq.}$, $d_o = 1.21 \text{ mm}$).
Figure 6.8: Effect of discharge current on electron number density (comparison between theory and experiment, argon, $\dot{m} = 0.5 \, A \, eq., \, d_o = 1.21 \, mm$).

Figure 6.9: Effect of discharge current on electron number density (comparison between theory and experiment, xenon, $\dot{m} = 0.5 \, A \, eq., \, d_o = 1.21 \, mm$).
Figure 6.10: Effect of discharge current on plasma potential (comparison between theory and experiment, argon, $\dot{m} = 0.5 \, \text{A eq.}, \, d_o = 1.21 \, \text{mm}$).

Figure 6.11: Effect of discharge current on plasma potential (comparison between theory and experiment, xenon, $\dot{m} = 0.5 \, \text{A eq.}, \, d_o = 1.21 \, \text{mm}$).
Figure 6.12: Effect of discharge current on electron temperature (comparison between theory and spectroscopy, argon, $\dot{m} = 0.93 \text{ A eq.}, d_o = 0.76 \text{ mm}$).

Figure 6.13: Effect of discharge current on electron temperature (comparison between theory and spectroscopy, xenon, $\dot{m} = 0.5 \text{ A eq.}, d_o = 0.27 \text{ mm}$).
CHAPTER VII

SPATIAL VARIATION OF ELECTRON TEMPERATURE

The variation of the electron temperature in the axial direction, obtained experimentally, may be explained by the Ohmic heating in the plasma. In addition, due to the rapid increase of the current density towards the orifice, substantial variation in electron temperature could be expected. It is believed that the plasma column inside the hollow cathode could be described by two different zones. One of these zones is a region in the vicinity of the orifice where high current density exists due to the small orifice cross section compared to the insert cross section. Therefore, from a very high spot at the orifice, heat conduction propagates upstream. The most appropriate coordinate system describing this propagation is the spherical coordinate system. It is expected that this zone will be scaled by the orifice radius. This analysis will be verified and quantified in the remaining of this chapter. The second zone, in the contrary, could be described by heat propagation in a cartesian coordinate system where the scale size of the conduction region is
expected to be of the order of the insert diameter along the cathode center line.

To verify the above analyses, the electron heat transfer equation, governing each zone in the plasma column, is integrated in conjunction with boundary conditions defined by experimental results. Since the scale size associated with the conduction region in the vicinity of the orifice is expected to be much smaller than the insert diameter, it is sufficient to determine this scale size without solving for the spatial variation of the electron temperature. This could be achieved through non-dimensionalization of the heat transfer equation. The balance between the heat conduction and the Ohmic heating may be expressed as:

\[
\frac{1}{r^2} \frac{d}{dr} \left( \kappa_e r^2 \frac{dT_e}{dr} \right) = -j^2 / \sigma_e \tag{7.1}
\]

Both thermal and electrical conductivities are evaluated based on classical formulation for fully ionized gases [21]. Hence,

\[
\kappa_e = 3.203 \frac{n_e k^2 T_e}{m_e \nu_{ei}} \tag{7.2}
\]

\[
\sigma_e = 1.975 \frac{n_e e^2}{m_e \nu_{ei}} \tag{7.3}
\]

where the collision frequency \( \nu_{ei} \) is given by:
\[ \nu_{ei} = 3.64 \times 10^{-8} n_i \ln \Lambda / T_e^{3/2} \]  
(7.4)

The combination of equations 7.2, 7.3 and 7.4 leads to:

\[ \kappa_e = K_1 T_e^{5/2} \]  
(7.5)

\[ \sigma_e = K_2 T_e^{3/2} \]  
(7.6)

where \( K_1 \) and \( K_2 \) are two constants that could be evaluated easily. Equation 7.1 could be non-dimensionalized by defining the following characteristic parameters:

\[ \alpha = r / r_c \]  
(7.7)

\[ \theta_e = T_e / T_c \]  
(7.8)

The current density, \( j \), may be expressed in terms of the total current as:

\[ j = \frac{I}{2\pi r^2} \]  
(7.9)

The substitution of \( r, T_e, \kappa_e, \sigma_e \) and \( j \) by their expressions in equation 7.1 yields:

\[ \frac{4\pi K_1 K_2 r_c^2 T_e^5}{I^2} \frac{1}{\alpha^2} \frac{d}{d\alpha} \left( \theta_e^{7/2} \alpha^2 \frac{d\theta_e}{d\alpha} \right) = -\theta_e^{-3/2} \]  
(7.10)

A nondimensional form of equation 7.10 is obtained when the characteristic radius \( r_c \) is equal to:
\[ r_c = \frac{I}{2\pi K_1^{1/2} K_2^{1/2} T_e^{5/2}} \]  

(7.11)

Electron temperatures measured in the vicinity of the orifice were in the range of 1.5 to 2.0 eV for operating conditions described in chapter V. For a typical value of discharge current of 10 A and an electron temperature of 1.5 eV, the conduction region scale size is evaluated at 0.26 mm. Comparison of \( r_c \) with a typical orifice radius of 0.38 mm shows that, indeed, the conduction region in the vicinity of the orifice is scaled by the orifice size. In addition, substantial variation in electron temperature would be expected within this small region.

The second region, further upstream from the orifice may be described by a similar expression to 7.1 assuming that only measurable variation in electron temperature occur in the axial direction (along the cathode center line). Hence,

\[ \frac{d}{dx} \left( \kappa_e \frac{dT_e}{dx} \right) = \frac{-j^2}{\sigma_e} \]  

(7.12)

In order to integrate equation 7.12, experimental data were used as boundary conditions as follows:
\[ T_e(x = 0) = \text{Measured value near orifice} \]

\[ \frac{dT_e}{dx}(x = 0) = 0 \]  \hspace{1cm} (7.13)

In spite of this rudimentary analysis, figure 7.1 shows that the theoretical prediction of the electron temperature profile is consistent with the measurements. In this case, the electron conduction region, evidently, scales to the insert diameter.

Figure 7.1: Theoretical prediction of the axial variation of electron temperature within the internal plasma column.
CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

8.1 General conclusions

The main objective of the present work has been achieved through the satisfactory agreements obtained between the theory and experiment in overall. The theoretical model includes enough physics, founded on first-principles, that adequate prediction of the cathode discharge is now possible under any desired operation. Optimization could be achieved by using the model to seek the best combination of the operating conditions. Among the many results obtained, both theory and experiment show that the discharge current, for a given cathode configuration, is the key operator that controls the cathode arc discharge and wall temperature. The mass flow rate, however, appears to have little effect on the overall plasma conditions and surface temperature. Because the wall cooling mechanism is realized by thermionic emission the material work function, for a given discharge current, is a critical parameter that dictates the cathode temperature and consequently its lifetime. It is of particular interest to note that both internal plasma column
(defined by one order of magnitude change in electron number density) and emission surface lengths are confined to one-insert diameter of the orifice plate. This finding is of considerable importance for the design of hollow cathodes suitable for high current regimes (with acceptable lifetime) because it indicates that the surface area for emission scales as the square of the insert diameter.

8.2 Hollow cathode scaling

The cathode lifetime depends strongly on the wall temperature. For impregnated porous tungsten inserts, a maximum lifetime of 100,000 hours was estimated at a temperature of 1000°C while, at 1100°C, only 30,000 hours were expected [29, 32]. Even though the existing hollow cathodes (for ion engines) operate at maximum wall temperatures less than 1200°C for discharge currents ranging from 1 to 20 Ampere, there has been a need to establish proper scaling of the hollow cathode for high current regimes. The present theoretical model could be used to determine the best cathode configuration that meets mission requirements (power level and lifetime). The capability of the model in conducting design analyses and predicting suitable operating conditions is illustrated in figures 8.1 through 8.6. It is important to note that the maximum discharge current is proportional to the cathode diameter rather than the expected dependence on the square of the diameter. This trend is consistent with the empirical results of Kauffman [18].
Physically, this behavior could be explained by analyzing the balance between the energy loss through the cathode orifice and the energy production in the internal plasma column. In this case, the energy loss is primarily proportional to the discharge current $I$ while the energy production within the hollow cathode is attributed in part to the energy of the emitted electrons accelerated through the fall and the resistive heating (which is proportional to $I^2$). The energy balance determines the magnitude of the fall voltage. Evidently, for a given discharge current, the smaller the orifice size the lower will be the fall voltage (figure 8.3). Furthermore, as the current increases, the fall voltages would reach extremely low values allowing for the following approximation:

$$\eta \left( \frac{I}{A} \right)^2 V_{ol} \simeq I \left( \frac{5kT_e}{2e} \right)$$

where $A$ and $V_{ol}$ are the insert cross section and volume of the internal plasma column respectively. Based on the fact that the length of the plasma column scales as one-insert diameter, equation 8.1 may be written as:

$$\frac{I}{D} \simeq \frac{5\pi kT_e}{8e\eta}$$

(8.2)
Because the electron temperature is a slow-varying function of the current (Chapters V and VI), the limiting discharge current appears to be proportional to the diameter:

$$ I \propto D $$ \hfill (8.3)  

Another interesting behavior of the theoretical solution regarding the relationship between the maximum discharge current and the propellant flow rate is described in figure 8.6. In this case, the current varies linearly with the flow rate up to a maximum value and then levels off. The previous analysis of the effects of the geometrical parameters on the maximum discharge current was conducted for a relatively high flow rate (1 A equivalent). This situation corresponds to the horizontal part of the curve in figure 8.6. In the linear region (low flow rate), the limiting current corresponds to a degree of ionization of one. (Attempts were made to include the second degree of ionization, but, no solution was obtained for currents higher than the limiting values corresponding to a singly ionized gas.) Therefore, as long as the mass flow rate is higher than the minimum value associated with the condition of a fully ionized gas, the orifice size is the sole parameter that limits the discharge current.
8.3 Recommendations

In spite of the acceptable success achieved in the theoretical modeling, further contributions could be made in upgrading the present theory. Among the possible improvements: (1) including a one-dimensional heat transfer equation to solve for the wall temperature profile, (2) solving a two-dimensional energy equation for the internal plasma to predict the spatial distributions of plasma properties, (3) deriving a more rigorous, theoretical formulation for non-equilibrium ionization, and (4) including applied magnetic field in order to predict the integration of hollow cathode in MPD thrusters.
Figure 8.1: Theoretical prediction of the effect of cathode inner diameter on the maximum discharge current.

Figure 8.2: Theoretical prediction of the effect of orifice to insert cross section area ratio on the maximum discharge current.
Figure 8.3: Theoretical prediction of the effect of discharge current on the fall voltage for different orifice sizes.

Figure 8.4: Theoretical prediction of the effect of discharge current on the fall voltage for different insert diameters.
Figure 8.5: Theoretical prediction of the effect of discharge current on cathode temperature.

Figure 8.6: Theoretical prediction of the effect of flow rate on the maximum discharge current.
Appendix A

Calculation of the Species Composition

Making the assumption that the plasma is in a state of equilibrium and using the particle conservation equations for a given plasma pressure, the species composition can be easily determined. For a two-temperature plasma \( \theta = \frac{T_e}{T_i} \neq 1 \), the equilibrium condition is expressed as [45]:

\[
n_e \left( \frac{n_i}{n_{i-1}} \right)^{1/\theta} = \frac{2Z_i}{Z_{i-1}} \left( \frac{2\pi m_e k T_e}{\hbar^2} \right)^{3/2} \exp \left( -\frac{\epsilon_i - \epsilon_{i-1}}{k T_e} \right) \quad (A.1)
\]

The particle conservation equations are given by:

\[
n_e = \sum_{i=1}^{N} i n_i \quad (A.2)
\]

and,

\[
n_t = \frac{P}{k T_e} = \sum_{i=0}^{N} (i + 1/\theta) n_i \quad (A.3)
\]

Equation A.1 can be written under the form:
\[ n_e \left( \frac{n_i}{n_{i-1}} \right)^{1/\theta} = K_i \]  \hspace{1cm} (A.4)

Therefore, equations A.2 and A.3 can be written as:

\[ n_e = n_0 \sum_{i=0}^{N} i \prod_{j=1}^{i} \left( \frac{K_j}{n_e} \right)^{\theta} \]  \hspace{1cm} (A.5)

and,

\[ n_i = n_0 \left[ \frac{1}{\theta} + \sum_{i=1}^{N} (i + 1/\theta) \prod_{j=1}^{i} \left( \frac{K_j}{n_j} \right)^{\theta} \right] \]  \hspace{1cm} (A.6)

By eliminating \( n_0 \) between equations A.5 and A.6 we obtain the following expression,

\[ n_i \sum_{i=1}^{N} i \prod_{j=1}^{i} \left( \frac{K_j}{n_e} \right) = n_e \left[ \frac{1}{\theta} + \sum_{i=1}^{N} (i + 1/\theta) \prod_{j=1}^{i} \left( \frac{K_j}{n_j} \right)^{\theta} \right] \]  \hspace{1cm} (A.7)

which is then solved for \( n_e \) by iteration. Once the electron density is known, the heavy particle densities are determined using equations A.4 and A.5.
Appendix B

Calculation of electron number density

An estimate of the electron density was, in most cases, derived from the ion saturation current. Lafromboise’s theory made it possible to calculate the electron number density based on the ion saturation current, the geometry of the probe and an estimate of the ion temperature. It is important to point out that the calculated density depends weakly on the ion temperature. Lafromboise [22] extended the method developed by Bernstein and Rabinowitz [5] to the case of a Maxwellian distribution of ions and conducted numerical computations of both ion and electron current collection for spherical and cylindrical probe geometry. This analysis is restricted to collisionless thin sheaths in which the charged particle mean free paths are much larger than the probe (a sphere or a long cylinder) radius.

Several approximate fits to Lafromboise’s results have been made. Peterson and Talbot [31] developed a curve fitting procedure for cylindrical probes which takes the form of both ion and electron current collection:
\[ j_{c,e}^* = (\beta + |\chi_p^*|)^\alpha \]  \hspace{1cm} (B.1)

where \( j^* \) is a normalized current collection defined by:

\[ j^* = (I/A_p)/(en\sqrt{kT_e/2\pi m}) \]  \hspace{1cm} (B.2)

The parameter \( \chi_p^* \) is the normalized probe potential measured with respect to the plasma potential defined as:

\[ \chi_p^* = eV_p/kT_e \]  \hspace{1cm} (B.3)

The remaining parameters in equation B.1 are given by:

\[ \alpha = a/(log\xi_p + b) + ce^m + d \]  \hspace{1cm} (B.4)

\[ \beta = e + \epsilon\{f + g(log\xi_p)^3 - l/\xi_p\} + l/log\xi_p \]  \hspace{1cm} (B.5)

where \( \xi_p \) is the ratio of the probe radius to the Debye length, epsilon is the ratio of the ion to electron temperature and the constants \( a, b, c, d, e, f, g, l \) and \( m \) have the values (assuming that \( \epsilon \leq 1 \)) given in table B.1.

Peterson-Talbot approximate formula agrees with Lafromboise’s numerical results within less than 3% error. This expression is not accurate in the orbital
motion limit \((R/\lambda_D \to 0)\) so it should be used only for \(\xi_p > 5\). According to Sonin [41] the expression:

\[
j^2_1\xi_p^2 = (R_p^2/\epsilon_e)(2\upi m_i/e)^{1/2}(e/kT_e)^{3/2}(I_i/A_p) \tag{B.6}
\]

depends only on \(\xi_p\) and could be solved for the electron number density by iteration.

Table B.1: Values of constants in equations B.4 and B.5.

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>l</th>
<th>m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion collection</td>
<td>2.90</td>
<td>2.30</td>
<td>0.07</td>
<td>-.34</td>
<td>1.50</td>
<td>0.85</td>
<td>0.135</td>
<td>0.00</td>
<td>0.75</td>
</tr>
<tr>
<td>Electron collection</td>
<td>2.90</td>
<td>2.30</td>
<td>0.11</td>
<td>-.38</td>
<td>-2.8</td>
<td>5.10</td>
<td>0.135</td>
<td>2.80</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Appendix C

Evaluation of the Knudsen Number

The Knudsen number (Kn) is defined as the ratio of the charged particle mean free path to the probe diameter. It is important to evaluate Kn to determine the extent of validity of Lafromboise’s collisionless theory when it is applied to calculate the plasma electron density from probe data.

According to Spitzer [21] the momentum-transfer cross section for charged particles for electron coulomb collision with charged particles is given by:

\[ Q_{ei} = Q_{ee} = 5.85 \times 10^{-10} \frac{ln\Lambda}{T_e^2} \]  \hspace{1cm} (C.1)

It follows that the mean free path for electron collision with charged particles is given by:

\[ \lambda_{ei} = \frac{1}{n_e Q_{ei}} \]  \hspace{1cm} (C.2)
Also, according to Spitzer [21] the ion-ion mean free path can be evaluated by the product of the root mean square velocity and the self-collision time. For a group of charged particles interacting with themselves, the self-collision time is given by:

\[ t_s = \frac{11.4 \mathcal{M}^{1/2} T^{3/2}}{n Z^4 \ln \Lambda} \]  

(C.3)

It follows that the ion-ion mean free path is given by:

\[ \lambda_{ii} = \left( \frac{3kT}{m} \right)^{1/2} t_s \]  

(C.4)

The comparison of the different mean free paths to the cylindrical probe radius is given in tables C.1 and C.2 for typical measured values of the electron temperature.

Table C.1: Debye length and charged particle mean free path for \( T_e = 1 \) eV, \( T_i = .1 \) eV.

<table>
<thead>
<tr>
<th>( n_e (cm^{-3}) )</th>
<th>( 5 \times 10^{13} )</th>
<th>( 1 \times 10^{14} )</th>
<th>( 5 \times 10^{14} )</th>
<th>( 1 \times 10^{15} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_D (\mu m) )</td>
<td>1.051</td>
<td>0.743</td>
<td>0.332</td>
<td>0.235</td>
</tr>
<tr>
<td>( \lambda_e (mm) )</td>
<td>0.598</td>
<td>0.313</td>
<td>0.070</td>
<td>0.037</td>
</tr>
<tr>
<td>( \lambda_{ii} (\mu m) )</td>
<td>13.33</td>
<td>7.238</td>
<td>1.809</td>
<td>1.013</td>
</tr>
<tr>
<td>( Kn_{ei} )</td>
<td>4.709</td>
<td>2.465</td>
<td>0.551</td>
<td>0.292</td>
</tr>
<tr>
<td>( Kn_{ii} )</td>
<td>0.053</td>
<td>0.029</td>
<td>0.007</td>
<td>0.004</td>
</tr>
</tbody>
</table>

It was shown by Sonin [41] that Lafromboise's collisionless theory, applied to ion collection current to a cylindrical probe, agreed with experiments for values of \( Kn_{ii} \) as low as 0.06 as long as \( Kn_{ei} \) is greater than unity. Based on the theoretical
prediction of the ion temperature (0.1 to 0.3 eV) the validity of Lafromboise's
theory would extend to an electron density of $5 \times 10^{14}$ cm$^{-3}$.

Table C.2: Debye length and charged particle mean free path for $T_e = 2$ eV, $T_i = 0.2$ eV.

<table>
<thead>
<tr>
<th>$n_e$ (cm$^{-3}$)</th>
<th>$5 \times 10^{13}$</th>
<th>$1 \times 10^{14}$</th>
<th>$5 \times 10^{14}$</th>
<th>$1 \times 10^{15}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_D$ (μm)</td>
<td>1.486</td>
<td>1.051</td>
<td>0.470</td>
<td>0.332</td>
</tr>
<tr>
<td>$\lambda_{ei}$ (mm)</td>
<td>2.107</td>
<td>1.097</td>
<td>0.243</td>
<td>0.127</td>
</tr>
<tr>
<td>$\lambda_{ii}$ (μm)</td>
<td>36.72</td>
<td>19.65</td>
<td>4.696</td>
<td>2.564</td>
</tr>
<tr>
<td>$Kn_{ei}$</td>
<td>16.59</td>
<td>8.640</td>
<td>1.914</td>
<td>1.000</td>
</tr>
<tr>
<td>$Kn_{ii}$</td>
<td>0.289</td>
<td>0.155</td>
<td>0.037</td>
<td>0.020</td>
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</table>
BIBLIOGRAPHY


