SIMULTANEOUS DIFFUSION OF HEAT AND MOISTURE IN
SPHERICAL POROUS BODIES

DISSERTATION
Presented in Partial Fulfillment of the Requirements for
the Degree Doctor of Philosophy in the Graduate
School of The Ohio State University

By


* * * * * *

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LIST OF SYMBOLS

D  Moisture Diffusion Coefficient \(D = \frac{R^2 K}{\pi^2}, \text{cm}^2 \text{hr}^{-1}\)

f  Void fraction of the porous solid

K  Drying constant \(K = \frac{\pi^2 D}{R^2}, \text{hr}^{-1}\)

K_t  Thermal conductivity of the sphere material

L  Latent heat of vaporization

\(L_{\text{em}}\)  Modified Lewis number

M(r)  Initial moisture concentration at \(r\)

\(m(r,t)\)  Moisture concentration at \(r\) and \(t\)

\(m_e\)  Average equilibrium moisture concentration

\(m_e(t)\)  Moisture concentration in equilibrium with changing air conditions

\(m_i\)  Moisture concentration at \(r_i\)

\(\bar{m}\)  Average moisture content

\(\bar{m}^*\)  Measured average moisture content

n  Integer number

P  Drying period in hours

R  Sphere radius

r  Position radius inside sphere

\(r_i\)  Mean radius of the \(i\)th shell

S  Specific heat of the sphere material

\(S_w\)  Specific heat of moisture

t  Time in hours
x,y,z  Rectilinear coordinates
\( \alpha \)  Thermal diffusion coefficient (\( \alpha = \frac{K}{S\rho}, \text{cm}^2 \text{hr}^{-1} \))
A,B  Temperature dependent constants
\( \theta(r,t) \)  Temperature at \( r \) and \( t \)
\( \theta_i \)  Temperature at \( r_i \)
\( \theta_e(t) \)  Temperature in equilibrium with changing air conditions
\( \bar{\theta}_e \)  Average equilibrium temperature
\( \bar{\theta} \)  Average sphere temperature
\( \bar{\theta}^* \)  Measured average sphere temperature
\( \theta(r) \)  Initial temperature at \( r \)
\( \theta_i \)  Initial temperature at \( r_i \)
\( \lambda \)  Constant (\( \lambda = \frac{L}{S}, ^\circ\text{F} \))
\( \rho \)  Density of the sphere material
\( \sigma_m \)  Standard deviation for moisture
\( \sigma_\theta \)  Standard deviation for temperature
\( \sigma_s \)  Sum of standard deviations
INTRODUCTION

The need for preserving and storing food grains over longer periods to meet the possible future food shortages demands the development of more effective and economical methods for preservation and storage. Moist grain stored at moderately high temperatures is subjected to the growth of harmful micro-organisms and insects, which damage and reduce its nutritive value. The best known preventive measure is to dry the grain below the moisture level at which it can be stored safely for prolonged periods.

Losses of food grains prior to storage may be reduced considerably by adopting improved harvesting practices. Harvesting the grain as soon as it is mature reduces field losses due to the ravages of rodents, birds, insects and weather. However some form of drying is necessary when these practices are adopted, because the grain, although mature, contains an excess of moisture which must be removed for safe storage.

Rational design of deep-bed drying systems requires an accurate description of the drying of individual kernels which involves simultaneous heat and moisture transfer.
Moisture movement can be described reasonably well with the diffusion equation in spherical coordinates using either a constant or a moisture dependent diffusion coefficient when the average kernel temperature does not change substantially during the process. This condition is realised in laboratory-controlled thin layer drying where the air temperature is constant and the kernel "quickly" attains this temperature. However under deep-bed drying conditions, kernels within the bed experience a substantial temperature rise during the drying process. The air temperature may be as much as 300°F higher than the initial grain temperature in such drying systems as parallel-flow drying. Since the moisture diffusion coefficient is directly related to the kernel temperature, heat transfer must be taken into consideration to describe moisture movement in deep-bed drying.

Heat transfer in grain kernels, like moisture transfer, can be appropriately described by the heat diffusion equation in spherical coordinates including a heat sink term to account for the latent heat of moisture vaporization within the kernel. The solution of the simultaneous heat and moisture diffusion equations in spherical coordinates using variable diffusion coefficients and under changing boundary conditions would be applicable to deep-bed drying.
REVIEW OF LITERATURE

Considerable literature has been published on the subject of grain drying over the past three decades. It falls into two categories, namely, thin layer or individual kernel drying, and deep-bed drying. A number of empirical and analytical models have been developed to describe both categories. However, most of the work has been with the former which in turn forms the basis for the latter.

Sherwood (19,20,21) and Newman (17) suggested that the drying of wet porous solids, when exposed to dry air, can be divided into the constant rate, and the falling rate periods. The former is similar to evaporation from a free water surface and continues until the water film around the material breaks and exposes its surface. When this happens the solid is said to have reached a critical moisture content after which the drying rate decreases and the falling rate period begins.

Sherwood (19,20,21), Newman (17) and Hall and Rodriguez-Arias (10) suggested that moisture moves through porous solids, including cereal grains, by the mechanism of diffusion during the falling rate period. Hougen et al. (13), suggested that other mechanisms such as capillarity, gravity, external pressure due to shrinkage,
convection, and a sequence of condensation and evaporation may exist and recommended that caution be exercised when characterizing moisture movement by the diffusion equation.

Crank (6) defines diffusion as "the process by which matter is transported from one point of a system to another as a result of random molecular motion." He derived the fundamental equation of diffusion in an isotropic medium from Fick's first law as follows:

\[
\frac{\partial m}{\partial t} = \frac{\partial}{\partial x} (D \frac{\partial m}{\partial x}) + \frac{\partial}{\partial y} (D \frac{\partial m}{\partial y}) + \frac{\partial}{\partial z} (D \frac{\partial m}{\partial z})
\]

This is sometimes referred to as Fick's second law of diffusion.

Much of the theory concerning the moisture movement in fully exposed porous bodies using the diffusion equation was developed by Sherwood (19,20,21) and Newman (17) who assumed that the moisture diffusion coefficient was constant and the potential causing the flow was the difference in moisture concentration. They described moisture movement as a function of time in porous bodies shaped as (1) an infinite slab, (2) a cylinder, and (3) a sphere.

Barre (3) found that vapor pressure gradient is the driving force in moisture movement and the amount of moisture transmitted was directly proportional to this force for relative humidity up to 75 percent of the drying air. Babbit (1) confirmed Barre's (3) findings by transferring
moisture in the direction of the vapor pressure gradient but against the moisture gradient. Van Arsdale (22) suggested that the potential could be either the concentration or the vapor pressure gradient, and both methods of expressing the diffusion equation are valid.

Becker and Sallans (2), Hustrulid and Flikke (15), Henderson and Perry (12), Whitaker et al. (24), Hamdy and Barre (7) and others describe moisture movement in grain kernels using the diffusion equation and assuming the kernel to be an isotropic sphere symmetrical with respect to its center.

\[
\frac{\partial m}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D r^2 \frac{\partial m}{\partial r} \right)
\]  

(1)

Crank (6) derived the solution to this equation for a constant D, constant boundary conditions \(m_e\) and uniform initial moisture concentration \(M\) as,

\[
\frac{m - M}{m_e - M} = 1 + 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n \pi r}{R} e^{-D \left( \frac{n \pi}{R} \right)^2 t}
\]

He also derived the average moisture content \(\bar{m}\) as,

\[
\bar{m} = m_e + (M - m_e) \sum_{n=1}^{\infty} \frac{1}{n^2} e^{-r^2kt}
\]  

(2)

Despite the restrictions on this solution Hustrulid and Flikke (15) used it to describe rather accurately the
drying rate of individual corn kernels. Chittenden and Hustrulid (4) observed that the diffusion coefficient which gives the best fit to measured drying rates varied with the initial kernel moisture and concluded that it must depend on moisture concentration at every point within the kernel [at any time]. They also found that a better prediction is obtained using equation (2) if a surface moisture slightly higher than the equilibrium moisture is assumed and that the surface concentration giving the best fit increases with higher initial moisture content. This led Hamdy and Barre (7) to conclude that a thin stagnant air film layer must have surrounded the kernel and obtained a better prediction to the drying data of Chittenden and Hustrulid (4) using equation (1) with a constant diffusion coefficient and assuming a thin stagnant film layer around the kernel.

Henderson and Pabis (11) applied the solution of the three dimensional equation of diffusion in a brick to a corn kernel. They assumed a constant diffusion coefficient D, a uniform initial moisture content $M$, and a constant environment.

$$\frac{m-m_e}{M-m_e} = \frac{512}{\pi^2} \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{(2j+1)^2(2m+1)^2(2n+1)^2} e^{-at}$$
where,

\[ \alpha = \frac{\pi^2 D}{4} \left( \frac{(2J+1)^2}{S^2} + \frac{(2m+1)^2}{W^2} + \frac{(2n+1)^2}{L^2} \right) \]

and \(2S = \text{Average kernel thickness}\)

\(2W = \text{Average kernel width}\)

\(2L = \text{Average kernel length}\)

\(D = \text{Moisture diffusion coefficient}\)

They concluded that better correlation between observed and predicted drying rates is achieved by considering the shape of the kernel to be a sphere rather than a brick.

Chu and Hustrulid (5) solved equation (1) on a digital computer using the finite difference technique and assuming constant initial and boundary conditions with the diffusion coefficient as a function of moisture \(D(m) = Ae^{\beta m}\), where \(\lambda\) and \(\beta\) are functions of drying air temperature. They reported close agreement between observed and predicted values of moisture content. Hamdy and Johnson (9) used the method of finite differences to solve the unidirectional diffusion equation in rectangular coordinates on an analog computer for a diffusion coefficient which is an arbitrary function of both position and concentration. Using the same technique, Whitaker et al. (24), solved equation (1) for a moisture diffusion coefficient which is an arbitrary function of both position and concentration. Their solution
can describe a non-homogenous kernel with a non-uniform initial concentration drying under changing air conditions. However, they found that the diffusion coefficient was more dependent on temperature than on moisture concentration and equation (1) is inadequate for describing the process completely. This agrees with Henderson and Pabis (11) who found that the drying constant of shelled corn, which is proportional to the diffusion coefficient, increased from 0.25 to 0.95 hr\(^{-1}\) when the temperature was raised from 100 to 200°F.

Wang and Hall (23) used the diffusion equation in spherical coordinates to characterize heat and moisture transfer within a corn kernel assuming constant boundary conditions. They suggested that for porous materials such as grains, the vapor pressure could be regarded as linearly dependent on temperature and moisture content between 15 and 25 percent moisture content dry basis and for temperature variations of about 10°F. They pointed out that the effect of temperature changes due to moisture vaporization within the kernel is important and has a profound influence on the rate of moisture diffusion.

Hamdy and Barre (8) described a mathematical model for deep-bed drying and obtained very good agreement with previous experimental data. Their kernel mathematical model comprises the diffusion equation in spherical coordinates for moisture movement with a temperature dependent diffusion coefficient, the logarithmic model for heat transfer
and a thin stagnant film layer around the kernel.

Young (25) described a mathematical model for a drying porous sphere using the diffusion equations for both moisture and heat transfer. Assuming that the moisture diffusion coefficient is linearly dependent on kernel moisture and temperature, and that thermal conductivity is a linear function of moisture, he obtained a solution to these equations on a digital computer for uniform initial conditions and constant boundary conditions. He studied the effects of various parameters on the solutions for a hypothetical sphere dried from 13.9 to 8.0 percent moisture content dry basis, and found that for low thermal conductivity $K_t$ and constant moisture diffusion coefficient $D$, $K_t$ has a significant effect on the drying rate. However, for $K_t$ values of the order expected for most materials, there is a negligible difference between the drying rate obtained by solving the heat and mass transfer equations simultaneously and the rate obtained by solving the mass transfer equation, assuming instantaneous thermal equilibrium. He defined a modified Lewis number $Le_m$ as,

$$Le_m = \frac{K_t (f+[1-f] \rho \beta)}{D (1-f) \rho (S+S_{wm}+L \gamma)}$$

and recommended that the mass transfer equation assuming instantaneous thermal equilibrium is sufficient if $Le_m$ is
greater than 60. If $Le_m$ is less than 60 the mass and heat transfer equations should be solved simultaneously.

The solutions of Wang and Hall (23) and Young (25), although more accurate than that of Hamdy and Barre (8) for single kernels, cannot be applied to deep-bed drying because they were derived for constant boundary conditions which do not exist in deep-bed drying. On the other hand, the solution of Hamdy and Barre (8) assumes that the kernel resistance to heat transfer is concentrated at the kernel surface. As such there exists a need to develop a method to solve the simultaneous diffusion equations for heat and mass transfer within the kernel under continuously changing boundary conditions encountered in deep-bed drying.
OBJECTIVES

The principal objective of this study was to develop a method for solving the simultaneous heat and moisture diffusion equations in homogeneous spherical porous bodies drying under varying conditions to make it directly applicable in deep-bed studies. The second objective was to develop a technique for evaluating these coefficients.
THEORETICAL ANALYSIS

The mathematical model characterizing radial diffusion of moisture and heat in a sphere, as described by Crank (6), is represented by the moisture diffusion equation,

$$\frac{\partial m}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( D \ r^2 \ \frac{\partial m}{\partial r} \right)$$

and the heat conduction equation

$$S \ \rho \ \frac{\partial \theta}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( K_r \ r^2 \ \frac{\partial \theta}{\partial r} \right) + L \ \rho \ \frac{\partial m}{\partial t} \tag{3}$$

The moisture diffusion coefficient $D$, the thermal conductivity $K_r$ of the material, the latent heat of vaporization $L$, the density $\rho$ and the specific heat $S$ of the material are, in general, functions of temperature and moisture concentration. No analytical solution to this complex and highly non-linear case has been found. Numerical or analog solutions would also be extremely difficult to obtain, especially when the nature of some of these functions is not known and assuming arbitrary functions would require unreasonably long computations to establish their nature and subsequently validate the model.
It was felt that considerable experience and information on the validity of this model could be gained at this stage by treating these parameters as constants. Therefore, this analysis assumes that D, K_t, L, ρ and S do not change substantially in the temperature and moisture ranges under consideration.

The solution to a differential equation under given initial and boundary conditions is valid only under those conditions. A proper choice of the initial and boundary conditions for equations (3) will determine the usefulness of its solution in deep-bed drying. The analytical solution of Wang and Hall (23) to an essentially similar model is not applicable to deep-bed drying because it is not valid for the changing boundary conditions encountered in deep-bed drying. This study alleviates this limitation by seeking a solution to the model under general initial and boundary conditions, defined as follows:

1. The initial temperature and moisture profiles in the sphere may vary along its radius.

2. The surface temperature and moisture concentration are in equilibrium with changing air temperature and humidity.

3. The temperature and moisture gradients at the center of the sphere would vanish because of the symmetry.
Mathematically these conditions can be expressed as follows:

\[
\begin{align*}
  m(r,0) &= M(r) \\
  m(R,t) &= m_e(t) \quad t > 0 \\
  \left. \frac{\partial m}{\partial r} \right|_{r=0} &= 0 \quad t > 0
\end{align*}
\]

and,

\[
\begin{align*}
  (r,0) &= \phi(r) \\
  (R,t) &= \theta_e(t) \quad t > 0 \\
  \left. \frac{\partial \theta}{\partial r} \right|_{r=0} &= 0 \quad t > 0
\end{align*}
\]

(4)

The solution to equation (3) with the initial and boundary conditions expressed in equation (4) can be used directly in deep-bed drying. The solution to the differential equations was not readily obtainable by classical methods under these general initial and boundary conditions. The possibility of using a digital or an analog computer was considered. The available analog computer was not used because of its limited capacity. Instead, an IBM 1130 digital computer using CSMP (continuous system modeling program) was used. The CSMP causes a digital computer to essentially simulate an analog computer. This computer was abandoned after some time because of its extremely slow speed in favor of an IBM 360 digital computer (model 75) utilizing the statement-oriented 360 CSMP.
Equations (3) and (4) have two independent variables, \( r \) and \( t \), and cannot be directly simulated by the CSMP which has only one independent variable, time. The method of Hamdy and Barre (7) was used to alleviate this difficulty by eliminating \( r \) as a variable. The method treats the kernel as a homogeneous sphere consisting of 10 concentric spherical shells of equal thickness and represents the moisture profile in each shell by a parabola. The coefficients of each parabola are calculated by matching the parabola to the moisture concentration at the mean radii of its shell and its adjacent shells. Furthermore, the coefficients of the innermost parabola are calculated such that the moisture gradient vanishes at the kernel center and those of the outermost parabola such that the moisture content at the surface equals \( m_e(t) \). Hamdy and Barre (7) gave the parabolas and their first and second derivatives and used them to transform the moisture diffusion equation into 10 simultaneous ordinary differential equations (one for each shell). Their method was also applied to heat transfer, thereby yielding a total of 20 equations as follows:

**Innermost shell** \((i = 0)\)

\[
\begin{align*}
    m_o &= M_o + \frac{100 D}{R^2} \int_0^t 3(m_1 - m_o) \, dt \\
    \theta_o &= \theta_o + \frac{100 \alpha}{R^2} \int_0^t 3(\theta_1 - \theta_o) \, dt + \lambda (m_o - M_o)
\end{align*}
\]
Intermediate shells \((i = 1, 2, ..., 8)\)

\[
m_i = M_i + \frac{100D}{R^2} \int_0^t \left( \frac{2i+3}{2i+1} m_{i+1} - 2m_i + \frac{2i-1}{2i+1} m_{i-1} \right) dt
\]

\[
\theta_i = \theta_i + \frac{100a}{R^2} \int_0^t \left( \frac{2i+3}{2i+1} \theta_{i+1} - 2\theta_i + \frac{2i-1}{2i+1} \theta_{i-1} \right) dt
\]

\[+ \lambda (m_i - M_i)\]

Outermost shell \((i = 9)\)

\[
m_9 = M_9 + \frac{100D}{R^2} \int_0^t \left( \frac{56}{19} m_e(t) - \frac{80}{19} m_9 + \frac{24}{19} m_8 \right) dt \tag{5}
\]

\[
\theta_9 = \theta_9 + \frac{100a}{R^2} \int_0^t \left( \frac{56}{19} \theta_e(t) - \frac{80}{19} \theta_9 + \frac{24}{19} \theta_8 \right) dt
\]

\[+ \lambda (m_9 - M_9)\]

where,

\[D = \frac{R^2K}{\pi^2} = \text{Diffusion coefficient, cm}^2 \text{ hr}^{-1}\]

\[a = \frac{K_t}{\rho} = \text{Thermal diffusivity, cm}^2 \text{ hr}^{-1}\]

\[\lambda = L/S = \text{Constant, °F}\]

It may be observed that equations (5) comprise 20 equations in 20 unknowns \(m_0, m_1, ..., m_9\) and \(\theta_0, \theta_1, ..., \theta_9\) and, therefore, have a unique solution.

Hamdy and Barre (7) derived an expression for
the mean moisture content by dividing the sum of the volume integrals of each parabola over its respective shell by the sphere volume. Using their expression for both the average moisture and temperature,

\[
\bar{m} = \frac{1}{1000} \left( 0.75 m_0 + 6.75 m_1 + 18.75 m_2 + 36.75 m_3 \right)
\]
\[
+ 60.75 m_4 + 90.75 m_5 + 126.75 m_6 +

\]
\[
168.75 m_7 + 207.825 m_8 + 282.175 m_9 \right)
\]

and,

\[
\bar{\theta} = \frac{1}{1000} \left( 0.75 \theta_0 + 6.75 \theta_1 + 18.75 \theta_2 + 36.75 \theta_3 \right)
\]
\[
+ 60.75 \theta_4 + 90.75 \theta_5 + 126.75 \theta_6 +

\]
\[
+ 168.75 \theta_7 + 207.825 \theta_8 + 282.175 \theta_9 \right)
\]

The mathematical model characterizing radial movement of heat and moisture in a homogeneous porous sphere drying under continuously changing air conditions described by equations (3) and (4) is transformed into equations (5) and (6), which can be simulated on either an analog or a digital computer using CSMP. The model characterizes heat and moisture transfer in a spherical porous body for

1. Constant thermal and moisture diffusion coefficients.
(2) Non-uniform initial temperature and moisture concentration,

(3) Continuously changing air temperature and humidity.
COMPUTER ANALYSIS

The computer circuit which solves equations (5) and (6) is shown in Figure 1. It may be noted that, unlike analog computer amplifiers, CSMP amplifiers cause no sign inversion and have unit-gain inputs. The continuously changing boundary conditions are implemented by using two function generators. The non-uniform initial temperature and moisture profiles are implemented by using the initial conditions at each shell on its temperature and moisture integrators.

The second objective of this study was to develop a method to determine the temperature and moisture diffusion coefficients giving the best fit of the model to observed data. The criterion for best fit was arbitrarily taken as the least sum of normalized standard deviations between computed and observed values for both moisture and temperature over a period of P hours.

\[
\sigma_s = \sigma_m + \sigma_\theta
\]

\[
\sigma_m^2 = \frac{1}{P} \int_0^P \frac{(\bar{m}^*-\bar{m})^2}{(\bar{M} - \bar{m}_e)^2} dt
\]

\[
\sigma_\theta^2 = \frac{1}{P} \int_0^P \frac{(*\theta - \bar{\theta})^2}{(\theta - \theta_e)^2} dt
\]

(7)
Figure 1. Computer Circuit for Equations (5) and (6).
where \( \bar{m} \) and \( \bar{\theta} \) are the average initial moisture and temperature of the sphere respectively, and \( \bar{m}_e \) and \( \bar{\theta}_e \) are its nominal average equilibrium moisture and temperature over the drying period.

The sum of normalized standard deviations was also calculated on the computer as shown in Figure 2, which is in fact a continuation of Figure 1.

The CSMP computer program for the IBM 360 is shown in Appendix 1. The observed average moisture and weighted mean temperature curves were generated by two non-linear function generators. Integrations were performed with respect to time \( t \) with an integration interval of 0.01 hour and the simulation was halted at \( t = P \) hours.

**Determination of \( D \) and \( \alpha \)**

Since \( \sigma_m \) is affected by \( D \) only and not by \( \alpha \), one computer run is made with several \( D \) values to obtain \( \sigma_m \) over a sufficient range. The calculation of \( \sigma_\theta \) is more involved because it depends on both \( D \) and \( \alpha \). A computer run is made for one value of \( D \) and several values of \( \alpha \) over a sufficient \( \alpha \) range including the minimum \( \sigma_\theta \) for that \( D \). The process is repeated many times until a sufficient \( D \) range is covered. Considerable time may be saved by the use of ENDJOB STACK cards in place of the ENDJOB card at the deck end (followed by a blank card), which makes it
Figure 2. Computer circuit for calculating standard deviations.
possible to use multiple decks at a time, limited only by the execution time allowable for this job class.
EXPERIMENTAL INVESTIGATION

Drying Apparatus

The experimental phase of this study involved the observations of the drying rate and of temperatures at selected points in a molded plaster sphere under controlled temperature, humidity and velocity of the drying air. The experimental apparatus shown in Figure 3, constructed primarily for experimental drying for grain, was adapted and used for this experiment.

Air Flow Control

An airfoil centrifugal blower capable of delivering 33.1 cu.m. of air per minute against a pressure of 22.9 cm. of water was used to force air through the system consisting of a conditioning tower, plenum, support chamber and tube. In the tower, the air passes through a sequence of humidifying, heating and mixing operations. The conditioned air then enters the support chamber through a perforated steel sheet with 101 holes, each 0.48 cm. in diameter. The air flow through the chamber and tube was determined by measuring the pressure drop across this sheet with an inclined water manometer to the nearest 0.125 cm. of water and with the aid of the calibration
data by Shedd (18). The total air flow into the system was controlled by two gates in the transition duct connecting the fan to the conditioning tower.

**Dry Bulb Temperature Control**

In controlling dry bulb temperature, the humidified air in the conditioning tower was heated with ten 3000 watt and eleven 1100 watt heaters before entering the mixing chamber. Each heater was fused independently and controlled by an on-off toggle switch. The temperature of the conditioned air was controlled automatically by two of the eleven 1100 watt heaters and a Honeywell Thermostat. The drying air temperature was measured in the tube (Figure 4) by a Leeds and Northrup copper-constantan 20-point temperature recorder (Figure 6). A system of baffles in the heating compartment of the tower insured thorough mixing of the heated and unheated air.

**Wet Bulb Temperature Control**

The wet bulb temperature of the drying air was controlled by a water spray supplied by a set of 36 nozzles mounted on the inside partition of the conditioning tower (Figure 3). It was measured in the plenum with a wick covered copper-constantan thermocouple. The wick was wetted by keeping its lower portion in a water bath, the level of which was maintained by adding make-up water from a supply outside of the plenum.
Figure 4. Support Chamber

Figure 5. Weighing Scale

Figure 6. Temperature Recorder
Support Chamber

A bottomless plywood box (45 x 45 x 43.8 cm.) placed over an opening in the top of the plenum served as a support for the aluminum tube. The tube and the perforated sheet were secured to the box for uniform air flow into the tube (Figure 3). The sphere was supported in the aluminum tube on a rotating tripod oscillating about its vertical axis, to insure uniform exposure of the sphere to the drying air. The tripod was connected to a 20 rpm electric motor through a gear and linkage arrangement such that the sphere oscillated, ten times a minute, through an angle of 210°. For removal of the sphere for weighing a hinged door was provided at the top of the tube. In addition a slotted opening was constructed in the door for removing the thermocouple leads connected to the sphere.

Experimental Procedure

Choice of Material

The material for the test sphere was selected to meet certain requirements, namely that: (a) moisture movement is by diffusion; (b) material is homogeneous and isotropic; (c) molds easily; (d) has sufficient strength to prevent breakage and flaking after molding. Sherwood (20) studied diffusion in a slab made of whiting (calcium carbonate and water). It was found to lack some of the above:
qualities. By experimenting with various modifications of whiting, Whitaker et al., (23) found that a mixture of two parts of whiting, one part of dental plaster (calcium sulfate), with an appropriate amount of water exhibits the necessary qualities. Accordingly the sphere was made of the same material as used by Whitaker.

Preparation of the Sphere

The selected material was molded and shaped into a sphere 12.5 cm. in diameter by slowly rotating it over a sharp bevelled edge steel tube 12 cm. in diameter. After it was dried, a hole 0.939 cm. in diameter was drilled along its radius and four thermocouples were placed in it at \( r = 0, \frac{R}{4}, \frac{R}{2}, \) and \( 3\frac{R}{4} \) (Figure 7) and secured with a thin paste of the molding material. A fifth thermocouple was placed in a shallow 0.3 cm. hole (Figure 7) and was secured such that its junction was positioned at the sphere surface to measure the surface temperature. After allowing a day for the paste to set, the sphere was partially submerged in water until it reached constant weight. It was dried in an air oven at 218°F to a moisture content of 25 percent dry basis and was then sealed in an airtight container and placed in an air oven at 130°F for about four days to allow the moisture to become uniformly distributed. The drying test was begun immediately after its removal from the sealed container.
Figure 7. Placement of thermocouples in the sphere.
Drying Procedure

The drying apparatus was brought to steady state conditions by operating the fan and the heaters for about six hours before starting to dry the sphere. The initial temperature and weight of the sphere were recorded immediately after it was removed from the sealed container and before it was placed in the tube (Figure 4). The temperature of each thermocouple in the sphere was recorded every four minutes and forty eight seconds by a Leeds and Northrup copper-constantan 20-point temperature recorder. Removal of the sphere from the tube and weighing it on a Toledo scale (Figure 5) required about thirty seconds. The time interval between two successive weighings was ten minutes during the initial stages of drying and was gradually increased to eight hours as the drying proceeded. Drying was terminated at the end of 48 hours.

The mean temperature of the sphere at any time during drying was determined by weighting the observed temperature in each shell with thickness R/4 (except the outer shell which has a thickness R/8 and the core with a radius of R/8 [Figure 7]) to the volumes of the respective shell. The observed temperature in each shell was assumed to be uniform throughout its volume. The outer shell was assumed to have a temperature equal to the surface temperature.
RESULTS AND DISCUSSIONS

Moisture

The observed mean moisture content of the test run is plotted against time in Figure 8 and tabulated in Table 1 (Appendix B). Table 1 shows that the sphere was still losing moisture after 48 hours of drying.

The drying air temperature deviated as much as 2°F from its nominal temperature; however, the deviations were of rather short duration. Therefore, the fluctuations in the humidity were small and judged to have very little effect on the equilibrium moisture. Consequently, the equilibrium moisture was assumed constant throughout the test run.

Taking the moisture content at $t = 48$ hours (9.6228 percent dry basis) as a first approximation for the equilibrium moisture, the measured average moisture ratio was plotted against time on a semi-log paper (Figure 9) and the straight portion was extended to give another moisture ratio at $t = 40$ hours which was used to obtain better estimate of the equilibrium moisture as 9.4935 percent dry basis.
Figure 8. Measured mean moisture content of the test sphere.
Figure 9. Semi-log plot of measured moisture ratios of sphere for $m_e = 9.6228$ and $9.4935$. 

\[ M_e = \frac{(m - m_e)}{(M - m_e)} \]
Temperature

The sphere temperature measured by the five thermo-couples at \((r = 0, R/4, R/2, 3R/4 \text{ and } R)\) and the air temperature are shown in Figure 10 and tabulated in Table 2 (Appendix B). The sphere weighted mean temperature is also listed in Table 2 (Appendix B). It may be noted that the sphere temperature dropped sharply as soon as drying started but later on it closely followed the fluctuations in air temperature.

Computer Results

The computer circuit (Figure 1) is capable of solving equations (5) and (6), which can take into account general initial and boundary conditions. The limitations of the experimental techniques, however, dictated that the initial temperature and moisture distributions within the sphere be considered uniform \((\phi_i = \bar{\phi} \text{ and } M_i = \bar{M} \text{ for } i = 2, \ldots, 9)\). The equilibrium moisture content of the sphere was assumed constant as described above; hence the surface moisture function generator was eliminated. Furthermore, the simulation time \(P\) was arbitrarily set at 45 hours.

Standard Deviations

The normalized moisture standard deviation, \(\sigma_m'\), was plotted against the moisture diffusion coefficient \(D\) (Figure 11). The normalized temperature standard deviation,
Figure 10. Measured temperatures at selected locations in the test sphere.
Figure 11. Normalized Standard Deviation for 45 hours drying time.
\(\sigma_0\), was plotted against thermal diffusion coefficient \(\alpha\) for numerous values of the moisture diffusion coefficient \(D\) (Figure 12).

The sum of normalized standard deviations, \(\sigma_s\), was plotted against thermal diffusion coefficient \(\alpha\) for numerous values of moisture diffusion coefficient \(D\) (Figure 13). The minimum \(\sigma_s\) (0.1189) occurred at \(D = 0.305\) and \(\alpha = 4.82\) cm.\(^2\) per hr.

It may be noted that the least standard deviations for moisture and temperature do not occur at the same value of the moisture diffusion coefficient \(D\). The least sum of normalized standard deviations, \(\sigma_s\), occurs at an intermediate value (Figure 13).

**Sphere Temperature and Moisture**

Observed and computed values of the mean moisture content for the least moisture standard deviation \(\sigma_m\) are shown in Figure 14. The weighted mean and computed average sphere temperatures for the least temperature standard deviation, \(\sigma_\theta\), are shown in Figure 15. The observed and computed mean moisture and temperature of the sphere for the least sum of standard deviations, \(\sigma\), are shown in Figure 16.
Figure 12. Normalized Temperature Standard Deviation for 45 Hours Drying Time.
Figure 13. Sum of Normalized Standard Deviation for 45 Hours Drying Time.
Figure 14. Moisture contents, observed and computed for best moisture fit.
Figure 15. Mean temperatures, observed and computed for best temperature fit.
Figure 16. Computed and observed mean moisture and temperature for best fit.
Discussion

The experimental results (Figure 10) show that the temperatures at the center of the sphere (r = 0) and at r = R/4 are very nearly the same throughout the drying period. This suggests, as assumed in the theoretical analysis, that there is no temperature gradient at the center of the sphere.

The computer results show that the standard deviation for mean moisture (Figure 11) has only one minimum, whereas the standard deviation for temperature (Figure 12) shows two minimums, probably because $\sigma_m$ depends only on one parameter, D, whereas $\sigma_\theta$ depends on two parameters, D and $\alpha$. They also show that the sum of normalized standard deviations for moisture and temperature has only one minimum (Figure 13) because the increase in $\sigma$ at the lower D values is roughly ten times greater than the corresponding decrease in $\sigma_\theta$. Thus, equally weighted, the sum of the standard deviations continues to increase.

Figure 14 shows that the moisture diffusion coefficient giving the best moisture fit, D = 0.335 is too high during the first 14 hours and too low thereafter. The close agreement between observed and computed temperatures for best temperature fit (Figure 15) indicates that the combination of $D = 0.18 \text{ cm}^2 \text{ hr}^{-1}$ and $\alpha = 3.70 \text{ cm}^2 \text{ hr}^{-1}$ gives a fairly good prediction of the temperature. However at this
value of \( D \) the corresponding moisture standard deviation is too large. Figure 16 suggests that \( D \) should be less than 0.305 at lower temperatures, and greater than 0.305 at higher temperatures. Since \( D \) normally increases with moisture content, and since the moisture content decreases in the experiment as drying progressed and the sphere temperature increased, it is likely that the change in \( D \) was even more significant than what Figure 16 reveals.

The criterion for the best fit of both moisture and temperature was arbitrarily taken as the least sum of the normalized standard deviations of moisture and temperature \( (\sigma_s = \sigma_m + \sigma_g) \). Other criteria such as their product, the sum of their squares, or their weighted sum, could have been used and would have given a different \( D \) and \( \alpha \). However, in the absence of any definitive information, the criterion for the best fit was arbitrarily taken as their sum.

The available similar works of Wang and Hall (23), Hamdy and Barre (7), Chu and Hustrulid (4) and Young (25) are not applicable to deep-bed studies because of the assumption of constant environmental conditions. Hamdy and Barre (8) modeled deep-bed drying by taking into account constantly changing drying air conditions and a temperature-dependent moisture diffusion coefficient but they used the logarithmic model for temperature variations. This study
improves on their model by using the more accurate heat conduction equation instead of the logarithmic model, but falls short by not accounting for the temperature dependence of the moisture diffusion coefficient. It can be extended though with minor modifications to take into consideration a temperature-dependent moisture diffusion coefficient once the nature of this dependence is known.

In short, the study provides a computer solution to the simultaneous partial differential equations (3) characterizing moisture and heat transfer through a porous sphere with constant thermal and moisture diffusion coefficients but under changing boundary and non-uniform initial conditions (equation 4). It also provides a method for evaluating the diffusion coefficients. The computer solution can be directly applied to the study of deep-bed grain drying.
SUMMARY AND CONCLUSIONS

Using the method of Hamdy and Barre (7), equations (3) characterizing the radial conduction of heat and diffusion of moisture in a porous sphere was reduced to a set of 20 simultaneous ordinary differential equations by dividing the sphere into ten concentric spherical shells of equal thickness. Moisture and temperature profiles in each shell were represented by one parabola each matched at the mean radii of the shell and its adjacent shells. The parabolas of the inner-most and outer-most shells satisfied the boundary conditions at the sphere center and surface, respectively. The analysis assumed the following conditions:

1. The moisture diffusion coefficient $D$ and thermal conductivity $K_t$ are constant.

2. As soon as drying starts, the surface moisture and temperature instantly assume the equilibrium values corresponding to the drying air conditions which may change with time.

3. The initial moisture and temperature distribution within the sphere may be non-uniform.

A test sphere was molded from a mixture of two
parts of finishing lime and one part of dental plaster. Five thermocouples were imbedded in the sphere at selected locations. The sphere was wetted, conditioned and dried to near equilibrium conditions in an experimental drying system. During drying, the sphere was weighed periodically and its temperature recorded.

Predictions of the moisture content and temperature were obtained by solving the equations on an IBM 360 digital computer using the CSMP. The standard deviation between the computed and the observed values of moisture were obtained for different values of D and α.

The best predictions for moisture and temperature occur at different values of the moisture diffusion coefficients, and the best fit for both moisture and temperature, taken together, occurs at some intermediate value. The best combination of moisture diffusion coefficient D and thermal diffusion coefficient α occurs when D = 0.305 cm²hr⁻¹ and α = 4.82 cm²hr⁻¹. However, a better fit may be obtained if both D and α are functions of moisture and temperature.

The study concluded that (1) the moisture diffusion coefficient depends more on temperature than on moisture; (2) and the method of least standard deviation can be successfully applied to evaluate the diffusion coefficients.
RECOMMENDATIONS FOR FUTURE STUDIES

The results of this study indicate a need for further research in several areas, namely:

a) the dependence of both the moisture and thermal diffusion coefficients upon the temperature and moisture of the material being dried;

b) an evaluation of the heat of vaporization and the specific heat during the drying process; and

c) an evaluation of the use of the film layer in describing the drying process.

d) The results of these studies can then be used to describe the deep-bed drying.
INITIAL
PARAMETER ALF=(4.60, 15.40, 0.02)
CONSTANTS AL=20.2700
AMC=24.9516
AME=9.4935
TNA=130.7826
TAO=111.2382
D=0.305
B=100.00/((2.5*2.54C)*2)
B1=B*D
B2=B*ALF
INCON C0=24.9516, C1=24.9516, C2=24.9516, C3=24.9516, C4=24.9516, ...
C5=24.9516, C6=24.9516, C7=24.9516, C8=24.9516, C9=24.9516, ...
I0=111.2382, T1=111.2382, T2=111.2382, T3=111.2382, T4=111.2382, ...
T5=111.2382, T6=111.2382, T7=111.2382, T8=111.2382, T9=111.2382, ...
DTIME0=0.0
FUNCTION CURVE1=(0.00, 24.9516), (0.00, 24.9516), (0.00, 24.9516), (0.00, 24.9516), (0.00, 24.9516), ...
(0.50, 24.9516), (0.50, 24.9516), (0.50, 24.9516), (0.50, 24.9516), (0.50, 24.9516), ...
11.666, 10.2311, (12.00, 19.8743), (12.25, 19.5358), (12.50, 19.1973), ...
(2.75, 18.8588), (2.00, 18.6170), (3.25, 18.3752), (3.50, 18.1335), ...
(3.75, 17.8917), (4.00, 17.6499), (4.333, 17.3598), (4.666, 17.0696), ...
(5.00, 16.8279), (5.333, 16.5089), (5.666, 16.3907), (6.00, 16.1492), ...
(6.333, 16.0058), (6.666, 15.8124), (7.00, 15.6190), (7.333, 15.2805), ...
(8.00, 14.9903), (8.50, 14.7485), (9.00, 14.4584), (9.50, 14.2166), ...
(10.00, 13.9749), (11.00, 13.5880), (12.00, 13.0212), (13.00, 12.3827), ...
(14.00, 11.7542), (15.00, 11.1857), (16.00, 11.0955), (17.00, 11.0504), ...
(18.00, 11.0417), (19.00, 11.0335), (20.00, 10.9982), (25.00, 10.2515), ...
(26.00, 10.1064), (28.00, 9.7913), (30.00, 9.3660), (33.00, 9.0769), (36.00, ...)
| 1.1666, 11.4960 | (1.3333, 112.8310) | (1.4833, 113.6445) | (1.65, ...) |
| 114.5478 | (1.80, 114.9570) | (1.9666, 115.6769) | (2.1333, 115.7460) |
| (2.2833, 116.0810) | (2.45, 117.1320) | (2.60, 117.1503) | (2.7666, ...) |
| 116.9902 | (3.00, 117.8166) | (3.25, 119.1025) | (3.4833, 119.4375) |
| (3.7333, 119.6502) | (3.9666, 119.9375) | (4.1333, 119.2724) | (4.2833, ...) |
| 119.0810 | (4.5333, 118.7724) | (4.7666, 117.9209) | (5.00, 119.5761) |
| (5.25, 119.6026) | (5.4833, 119.2261) | (5.7333, 118.5853) | (5.9666, ...) |
| 119.2246 | (6.00, 119.8154) | (6.45, 120.4801) | (6.7666, 121.1025) |
| (7.00, 122.0498) | (7.25, 122.2421) | (7.4833, 122.7412) | (7.9666, ...) |
| 122.8944 | (8.45, 123.1455) | (9.00, 122.7675) | (9.4833, 121.8896) |
| (9.9666, 122.6455) | (10.45, 121.6513) | (11.00, 122.9062) | (11.4833, ...) |
| 122.8369 | (11.9666, 123.1718) | (12.45, 123.3583) | (13.00, 123.4804) |
| (14.05, 124.1455) | (15.00, 124.7148) | (15.9666, 124.9804) | (17.00, ...) |
| 125.4541 | (18.05, 125.3501) | (19.00, 125.9531) | (20.05, 126.4277) |
| (21.00, 126.4277) | (22.05, 126.7880) | (23.00, 127.0498) | (24.05, ...) |
| 127.0499 | (25.00, 126.8583) | (26.05, 126.8837) | (27.00, 127.3584) |
| (29.00, 127.8388) | (31.00, 127.9199) | (33.00, 127.4306) | (39.00, ...) |
| 129.1914 | (37.00, 129.0019) | (39.00, 129.1923) | (41.00, 129.0009) |
| (43.00, 129.1923) | (45.00, 129.1660) | (47.00, 129.1523) |

**FUNCTION TTS=(0.00, 130.00), (0.0333, 130.00), (0.1166, 130.00), (0.20, ...)**

| 130.00 | (0.2833, 130.00) | (0.4333, 130.00) | (0.7666, 130.00) | (0.8333, ...) |
| 130.00 | (0.9166, 130.00) | (1.00, 130.00) | (1.1666, 130.00) | (1.3333, ...) |
| 130.00 | (1.4833, 130.00) | (1.65, 130.00) | (1.80, 130.00) | (1.9666, ...) |
| 130.00 | (2.1333, 130.00) | (2.2833, 130.00) | (2.45, 130.50) | (2.60, ...) |
| 130.50 | (2.7666, 130.00) | (3.00, 130.00) | (3.25, 131.00) | (3.4933, ...) |
| 131.00 | (3.7333, 131.00) | (3.9666, 130.00) | (4.1333, 129.00) | (4.2833, 128.50) |
| 132.00 | (4.5333, 128.50) | (4.7666, 127.50) | (5.00, 130.00) | (5.25, 130.00) |
| 132.00 | (5.4833, 128.50) | (5.7333, 128.00) | (5.9666, 128.50) | (6.00, 130.00) |
| 132.00 | (6.45, 130.50) | (6.7666, 131.00) | (7.00, 132.00) | (7.25, ...) |
| 132.00 | (7.4833, 132.00) | (7.9666, 132.00) | (8.45, 131.50) | (9.00, ...) |
| 131.00 | (9.4833, 131.50) | (9.9666, 131.00) | (10.45, 129.00) | (11.00, ...) |
| 131.00 | (11.4833, 132.00) | (11.9666, 130.00) | (12.45, 131.00) | (13.00, ...) |
| 129.50 | (14.05, 131.00) | (15.00, 132.00) | (15.9666, 131.00) | (17.00, ...) |
132.00, 18.05, 31.00, (19.00, 131.50, (20.05, 132.00, (21.00, 
132.00, (22.05, 132.00, (23.00, 132.50, (24.05, 131.50, (25.00, 
131.00, (26.05, 131.00, (27.00, 131.50, (29.05, 130.50, (31.00, 
130.00, (33.00, 131.50, (35.00, 131.50, (37.00, 131.00, (39.00, 
132.00, (41.00, 131.50, (43.00, 131.50, (45.00, 132.00, (47.00, 
131.50, (48.00, 131.50

DYNAMIC

xo=bl*(3.0*cc1-3.0*cc0)
x1=bl*(1.6667*cc2-2.0000*cc1+0.3333*cc0)
x2=bl*(1.4000*cc3-2.0000*cc2+0.6000*cc1)
x3=bl*(1.2857*cc4-2.0000*cc3+0.7143*cc2)
x4=bl*(1.2222*cc5+7.0000*cc4+0.7778*cc3)
x5=bl*(1.1818*cc6-2.0000*cc5+0.8182*cc4)
x6=bl*(1.1538*cc7-2.0000*cc6+0.8462*cc5)
x7=bl*(1.1333*cc8-2.0000*cc7+0.8667*cc6)
x8=bl*(1.1176*cc9-2.0000*cc8+0.8824*cc7)
x9=bl*(-4.2105*cc9+1.2632*cc8)

UTIME=INTEGRAL(UTIMEC,1.0)

YYT5=NLGEN(TT5,0,TIME)

CC0=INTEGRAL(C0,XC)
CC1=INTEGRAL(C1,X1)
CC2=INTEGRAL(C2,X2)
CC3=INTEGRAL(C3,X3)
CC4=INTEGRAL(C4,X4)
CC5=INTEGRAL(C5,X5)
CC6=INTEGRAL(C6,X6)
CC7=INTEGRAL(C7,X7)
CC8=INTEGRAL(C8,X8)
CC9=INTEGRAL(C9,X9)

YO=B2*(3.0*TT1-3.0*TT0)

Y1=B2*(-1.6667*TT2-2.0000*TT1+0.3333*TT0)
Y2=B2*(-1.4000*TT3-2.0000*TT2+0.6000*TT1)
Y3=B2*(-1.2857*TT4-2.0000*TT3+0.7143*TT2)
Y4=B2*(-1.2222*TT5-2.0000*TT4+0.7778*TT3)
Y5=B2*(-1.1818*TT6-2.0000*TT5+0.8182*TT4)
Y6 = 82 * (1.1538 * TT7 - 2.0000 * TT6 + 0.8462 * TT5)
Y7 = 82 * (1.1333 * TT8 - 2.0000 * TT7 + 0.8667 * TT6)
Y8 = 82 * (1.1716 * TT5 - 2.0000 * TT6 + 0.8824 * TT7)
Y9 = 82 * (2.9474 * YYTTS - 4.2105 * TT9 + 1.2632 * TT8)
TT0 = AL * (CC0 - CC1) * INTRL(T0, Y0)
TT1 = AL * (CC1 - C1) * INTRL(T1, Y1)
TT2 = AL * (CC2 - C2) * INTRL(T2, Y2)
TT3 = AL * (CC3 - C3) * INTRL(T3, Y3)
TT4 = AL * (CC4 - C4) * INTRL(T4, Y4)
TT5 = AL * (CC5 - C5) * INTRL(T5, Y5)
TT6 = AL * (CC6 - C6) * INTRL(T6, Y6)
TT7 = AL * (CC7 - C7) * INTRL(T7, Y7)
TT8 = AL * (CC8 - C8) * INTRL(T8, Y8)
TT9 = AL * (CC9 - C9) * INTRL(T9, Y9)
CAVER = (0.75 * CCC + 6.75 * CC1 + 18.75 * CC2 + 36.75 * CC3 + 60.75 * CC4 + 90.75 * CC5 + 126.75 * CC6 + 168.75 * CC7 + 207.825 * CC8 + 282.175 * CC9) / 1000.000
TTAVER = (0.75 * TTC + 6.75 * TT1 + 18.75 * TT2 + 36.75 * TT3 + 60.75 * TT4 + 90.75 * TT5 + 126.75 * TT6 + 168.75 * TT7 + 207.825 * TT8 + 282.175 * TT9) / 1000.000
YYC = NLFGEN(CURVE1, CTIME)
SC = (ARS (YYC - CAVER)) * #2.0
SSC = INTRL(T0, SC)
STD = ((SSC/45) * #C.50) / (AMQ - AME)
YYT = NLFGEN(CURVE2, CTIME)
ST = (ARS (YYT - TTAVER)) * #2
SST = INTRL(T0, ST)
STD = ((SST/45) * #0.50) / (TNA - TAD)
SGMSTD = STD + STCT
TERMINAL
FINISH CTIME = 45
TIMER DELT = 0.01, FINTIM = 45.0, PROEL = 5.0
PRINT CAVER, TTAVER, SSC, SST, STC, STD, SGMSTD
END
STCP
# APPENDIX B

Dry weight of the sphere = 1034 gms.

Flow rate of drying air = 2.15 cubic meters per minute.

Initial moisture content = 24.95 percent dry basis.

Equilibrium moisture content = 9.49 percent dry basis.

Initial sphere temperature = 111.24°F

Mean drying air dry bulb temperature = 130.78°F

Mean drying air wet bulb temperature = 77°F

## TABLE 1

<table>
<thead>
<tr>
<th>Time (Hrs.)</th>
<th>Sphere weight (grams)</th>
<th>Moisture Content (% dry basis)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>1292.0</td>
<td>24.95</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>1280.0</td>
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<td>20</td>
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<td>1266.0</td>
</tr>
<tr>
<td>0</td>
<td>40</td>
<td>1262.0</td>
</tr>
<tr>
<td>0</td>
<td>53</td>
<td>1257.5</td>
</tr>
<tr>
<td>1</td>
<td>02</td>
<td>1254.5</td>
</tr>
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<td>10</td>
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<tr>
<td>1</td>
<td>22</td>
<td>1249.0</td>
</tr>
<tr>
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24. Whitaker, T. B., Barre, H. J. and Hamdy, M. Y.  
"Theoretical and Experimental Studies of Diffusion in Spherical Bodies with a Variable Diffusion Coefficient." Transactions of the ASAE. 12 (5), 668-672, 1969.