TIME REVERSAL BASED SIGNAL PROCESSING
TECHNIQUES FOR ULTRAWIDEBAND
ELECTROMAGNETIC SENSING IN RANDOM MEDIA

DISSERTATION

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ABSTRACT

This dissertation focuses on the application and development of time reversal (TR) based signal processing techniques for ultrawideband (UWB) electromagnetic waves in random media. TR techniques exploit the time reversal invariance of the wave equation to take advantage of the retransmitted (time-reversed) fields for improved imaging and focusing. This is because retransmitted signals propagate backwards through the same medium and undergo similar reflection, refraction and multiple scattering that they underwent during the forward propagation, resulting in focusing around the initial source locations. If multiple scattering occurs in the intervening media, refocusing resolution can overcome the classical diffraction limit (i.e. when no multiple scattering is present) that characterizes superresolution, a somewhat counterintuitive result. With the success of initial TR experiments in acoustics, there has been a strong interest in the application of TR methods using radio frequency electromagnetic (EM) waves. It is also the motivation of this dissertation to develop TR techniques for UWB electromagnetic waves. We start by investigating the superresolution effects of time-reversed UWB EM waves under continuous random background media and examine their dependency on the first- and second-order statistics. In contrast to the acoustic case, polarimetric TR exploiting the polarization of the EM waves is also analyzed to observe the depolarization effects on the refocusing of time-reversed EM waves. We continue with the application of TR in lossy and dispersive
media (such as soil or biological tissues) where the TR invariance is broken and yields a performance degradation. We introduce a physical compensation technique based upon both space and frequency dependent inverse filters to improve the performance of TR techniques for homogeneous and random dispersive media. We also develop a physical full time-domain selective focusing method on the desired scatterers in the presence of others. The method is applied both in homogeneous and random media.

Both narrowband and UWB TR-based imaging techniques for detection and localization of distinct scatterers in inhomogeneous background media are investigated under several perturbations. Specifically, we investigate the effects due to clutter, noise, dispersion and losses. Moving TRAs and restrictions on array elements on these TR-based imaging methods are also considered. Finally, we introduce a new TR-based imaging functional based on the simultaneous utilization of spatial and UWB frequency data to obtain a novel method for UWB imaging of embedded scatterers in homogeneous and random media.

As can be understood, although we consider typical subsurface sensing scenarios in presence of inhomogeneous and dispersive soil models, the same algorithms can be adapted to different applications, as is the case with microwave breast cancer detection or nondestructive testing.
To my dear wife Seda and my baby boy Ömer Kerim The Burrito
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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Active and passive remote sensing systems in the microwave and millimeter wave (mm-wave) range provide unique capabilities for detection and imaging of obscured objects and people. For example, ground penetrating radar (GPR) can detect landmines, locate underground pipes, and identify subsurface defects in runways. Microwaves can also penetrate through foliage to detect obscured vehicles and personnel in forested environments. Millimeter-waves can penetrate through non-conductive walls and clothing, and many packaging. Not limited to metal targets, mm-wave based systems allow detection of drugs, contraband, plastic explosives, and ceramic weapons, while providing good image identification and resolution. X-rays can also provide an effective search of luggage and other items, but their perceived health risks from ionizing effects make them unacceptable for searching concealed weapons and explosives. Metal detectors are limited to detecting metal objects and provide little on the nature or location of the detected items. Because microwaves and mm-waves can better penetrate through dust, fog, and smoke (an advantage over optical and infrared-based systems) remote sensing at these frequencies is particularly attractive for battlefield scenarios, terrorist attack responses, and fire rescue operations. Even
for scenarios where there is no clear advantage among various sensing technologies, microwave and mm-wave remote sensing systems will likely play an increasingly important role because of their ease of integration into multi-sensor/distributed-sensor systems (sensor fusion).

However, there are still challenges for the full exploitation of mm-waves for remote sensing systems, such as vulnerability to certain atmospheric and meteorological phenomena and increased attenuation inside lossy materials. In addition, intervening media in remote sensing scenarios are often disordered, i.e., may exhibit complex constitutive properties or include many secondary scatterers (clutter) about which precise information on properties and/or location is not available. As a result, signals from target object(s) are often weak and/or distorted by clutter and multipath (multiple scattering), which confounds detection, causes erratic tracking and makes it difficult to extract relevant information for imaging and classification purposes. For example, in the case of digital communications, the multipath components can cause inter-symbol interference resulting in errors while making decision as to whether the signal is a zero or one. Additionally, performance of traditional inverse scattering techniques, which are limited by the classical diffraction limit (when no multiple scattering is present) [1], are shown to degrade with increasing multiple scattering. However, contrary to the conventional thoughts, recent findings have shown that multipath components can actually be utilized to create independent paths between the transmitter and receiver in a remote sensing and/or communication system [2, 3, 4] which converts the traditionally hostile multipaths to performance-boosting elements in cluttered environments. One such novel technique is the time-reversal (TR) method.
introduced by Fink et al. [5] which utilizes the multipath components in the intervening media to achieve superresolution, i.e. resolution that beats the classical diffraction limit. TR involves the physical or synthetic retransmission of signals acquired by a set of transceivers in a time-reversed fashion, i.e. like playing a movie backwards in time. Retransmitted time-reversed signals propagate backwards through the same medium and experience similar multiple scattering, reflections and refraction that they underwent during the forward propagation, resulting in energy focusing around the initial source locations. Relying on the TR invariance of the wave equation in stationary (time-independent) and lossless media, TR utilizes ultrawideband (UWB) signals which offer a number of attractive properties to overcome the aforementioned challenges. First, UWB systems can exploit the advantages of simultaneous operation at both low (more penetration into lossy materials) and high frequencies (better resolution). Second, they are more immune to atmospheric effects and multipath interference and can exploit multipaths in improving the imaging capabilities and third, UWB enables the development of imaging techniques that depend only on the statistical properties of the random medium providing statistical stability in random media.

Fink’s successful TR experiments using low frequency acoustic waves and ultrasound have recently created an interest in the application of TR methods using radio frequency (RF) electromagnetic (EM) waves. The research carried out in this dissertation is among the first studies of TR techniques as applied to UWB EM waves in random media. It is devoted to introduce TR techniques using the EM waves and develop new extensions for its possible application in complex inhomogeneous environments. Within this general objective, we can divide the dissertation into three
main parts. The first part investigates the superresolution effects of time-reversed UWB EM waves under continuous random media. The second part deals with the development of compensation methods for the application of TR in dispersive and conductive media where an inverse filtering approach is introduced. Finally, the third part is devoted to the development and improvement of several TR-based imaging methods for application in continuous random media. But, before discussing the details of the contributions, let us provide a literature review on TR and its applications.

1.2 Literature Review

In the frequency domain, time-reversal corresponds to phase conjugation which has found applications in optics [6] as well as in electromagnetics where it is used to cancel distortions in the medium and provide beam steering action by using self-phasing or retrodirective array antennas [7, 8, 9, 10, 11]. The main difference between the phase conjugation and TR is that phase conjugation applies to monochromatic waves whereas TR utilizes pulsed time-domain signals (UWB). The first successful time-domain experiments to show time-reversal focusing were by Fink and his group [12] in the early 1990s. They have developed the concept of TR cavity and TR mirrors and built several devices to illustrate the feasibility of this concept [13, 14, 15]. Several physical TR experiments have been conducted by using acoustics [16, 15, 17, 18, 19, 20] and ultrasonic waves [21, 14, 22, 23, 24, 25, 26]. Underlying principles of TR have been investigated through theoretical analysis [27, 28, 29, 30] and numerical simulations [31, 32, 33, 34, 35]. Time-reversal has found many applications in various different fields. For example, in medicine, it has been used for the destruction of kidney stones [36, 37], ultrasonic focusing through the skull [38], hyperthermia [39]
and microwave breast cancer detection [40]. Similarly, nondestructive testing applications have utilized it for detecting defects in materials and structures [26, 41, 42, 43]. Applications in geophysics and geoscience have employed TR for finding the center of an earthquake or finding objects buried in the ground [44, 45]. Underwater applications include sonar and acoustic communication [19, 46, 47], intruder detection and echo-to-reverberation enhancement [48, 49]. Apart from these, imaging using synthetic (computational) time-reversal is also possible [50, 51, 52]. For most of the imaging applications, analysis of the TR operator (TRO) is fundamental. Eigenspace analysis of the TRO provides information about the scattering scenario under study. Specifically, in the case of multiple scatterers, selective focusing on the desired scatterers is possible via the decomposition of the TRO method (DORT under its French language acronym) [22, 25, 53, 26, 54, 45, 55, 56, 57]. Similarly, TR-MUSIC (Multiple signal Classification) method developed in [50, 58, 59, 60, 61] utilizes TRO to obtain TR-based imaging for the localization of target(s).

Currently, there is an increasing research effort for the application of TR techniques using the EM waves. One of the earliest application of TR using EM waves at RF frequencies is the demonstration of TR focusing using the phase-conjugation approach in the frequency domain [62] where it was experimentally shown that by using a single frequency phase conjugation approach, energy focusing around two targets is possible. Recently, Fink’s group also performed a TR focusing experiment using the EM waves [63, 64] where they experimentally demonstrated that a 1 MHz wide band pulse can be focused in a cavity environment. Various target detection and interference cancelation algorithms accompanied by experiments have been demonstrated
in [65, 66, 67, 68, 69]. Recently, microwave breast cancer detection using ultrawideband EM waves has been of interest in the EM community [70]. TR also found application in this field by its natural ability to focus energy on the scattering malignant tissues. Algorithms employing TR concept were introduced in [40, 71, 72, 73] and a TR pulse design method was developed in [74]. Apart from these, EM imaging and target detection in discretely cluttered environments have been experimentally demonstrated in [75, 76, 77]. DORT method has been used for sensing of the buried objects in [78, 55, 45]. Additionally, the TR operator for electromagnetic waves in homogeneous medium was recently analyzed in [79]. TR focusing in a forest environment has been numerically shown in [80]. In [81], a TR synthetic aperture radar imaging was developed and applied in an environment filled with trees. Telecommunications and wireless communications applications include the development of time-reversal based spatio-temporal matched filters to reduce channel dispersion and inter-symbol interference thereby increasing the capacity of the channel [82, 83, 84, 85, 86, 87, 68]. Last but not least, a practical implementation of time-reversal of broadband microwave signals is demonstrated in [88].

1.3 Contributions

In addition to the aforementioned studies, this dissertation work has also produced promising results for the application of TR using EM waves. Here, we summarize its various contributions and state where it stands in regards to the already published literature as follows:
• Time-reversal using EM waves: TR technique has been applied to UWB EM waves to investigate the superresolution effects obtained under continuous random background media. Additionally, for the first time in the literature, polarimetric TR has been investigated to observe the effects of depolarization on the refocusing resolution of the time-reversed fields [89].

• Frequency Dispersion Compensation for TR techniques: A novel space and frequency dependent inverse-filter based compensation method utilizing the Short-Time Fourier Transform (STFT) has been developed. This method compensates for the degradation in the TR performance induced by the breaking of the TR invariance in lossy and dispersive media (such as soil, breast tissue etc.) [90].

• Full time-domain DORT and Selective Focusing: In media with multiple scatterers, it is possible to focus selectively only on the desired one(s) by using TR techniques. A time-domain fully polarimetric selective focusing method has been developed that can be applied both in homogeneous and random media [91].

• Sensitivity of TR methods to Perturbations: The performance and sensitivity of TR methods to external perturbations such as noise, clutter, loss and model perturbations are studied. [92].

• Space-Frequency UWB TR imaging: A new UWB imaging method based on the simultaneous utilization of spatial and UWB frequency data has been developed [93]. Its performance is compared with that of the time-domain DORT method.
1.4 Outline of the Dissertation

The organization of the dissertation is as follows. In Chapter 2, we introduce the time-reversal process and apply it using the UWB EM waves in random inhomogeneous media. Then, Chapter 3 discusses the development of the full time-domain polarimetric DORT method. This is followed, in Chapter 4, by the introduction of the frequency dispersion compensation method. Chapter 5 demonstrates the sensitivity of synthetic TR imaging methods to several perturbations. Space-frequency TR imaging is introduced in Chapter 6 and finally, conclusions are provided in Chapter 7 along with the suggestions for future works. It should also be noted that, for our studies, we have considered configurations that match those of typical subsurface sensing scenarios with random medium models based on inhomogeneous soil models having spatially fluctuating random dielectric permittivities. We have utilized two or three dimensional finite-difference time-domain (FDTD) [94] simulations for the solution of EM wave equation.
CHAPTER 2

TIME-REVERSAL OF ELECTROMAGNETIC WAVES AND SUPERRESOLUTION

Time-reversal concept exploits the invariance of the wave equation under time-reversal, i.e. for the following wave equation in lossless media

\[ \nabla^2 \vec{E}(\vec{r}, t) - \mu(\vec{r}) \epsilon(\vec{r}) \frac{\partial^2}{\partial t^2} \vec{E}(\vec{r}, t) = 0 \]  \hspace{1cm} (2.1)

\( \vec{E}(\vec{r}, -t) \) is also a solution to the same wave equation where \( \vec{E}(\vec{r}, t) \) is the vector electric field in space and time, \( \vec{r} = \hat{x}x + \hat{y}y + \hat{z}z \) is the spatial position, \( \mu \) and \( \epsilon \) are the permeability and permittivity of the medium, respectively. This governing wave equation guarantees that for every wave diverging away from the source, there exist a reversed wave that would precisely retrace the path of the original wave back to the source. This holds true even if the propagation medium consists of scattering objects and permittivity/permeability variations causing the wave to reflect, scatter and refract. The reversed waves would undergo all these intricate paths and converge \textit{coherently} at the original source location as if time were going backwards. In other words, if it was possible to record the video of forward propagation \( \vec{E}(\vec{r}, t) \), watching it backwards would correspond to \( \vec{E}(\vec{r}, -t) \). This concept has been exploited to re-create the wave and send it back to its source as if time had been reversed, thereby yielding
in applications like destruction of kidney stones, detecting defects and communicating with submarines. In the ideal situation, wave fields propagating in all possible directions should be taken into account so that the whole wave could be generated. However, in practical scenarios, it is often impossible to entirely surround the source and record all the wave fields diverging from it. Therefore, TR is generally performed with a limited number of antennas which is called as the time-reversal array (TRA). Since this causes some information loss, the focal spot size gets larger as the array aperture size becomes smaller. However, recovering some of the lost information is possible through the utilization of the multipaths where possible. In this chapter, we will focus our attention to multipaths created by the continuous random media and investigate its effects on the refocusing resolution of the TR fields. However, before going into more details, let us start with an overview of the time-reversal experiment.

2.1 Time-Reversal Experiment

Suppose a point source located at $\vec{r}_0$ transmits a short UWB pulse $s(t)$ as shown in Fig. 2.1(a). The transmitted signal propagates through the medium and is received by an antenna array. The signal received at the $i^{th}$ antenna is

$$f_i(t) = s(t) *_t h_{\vec{r}_0\vec{r}_i}(t)$$  \hspace{1cm} (2.2)

where $*_t$ denotes convolution in time and $h_{\vec{r}_0\vec{r}_i}(t)$ is the impulse response between the antennas at $\vec{r}_0$ and $\vec{r}_i$. Reciprocity allows us to write $h_{\vec{r}_0\vec{r}_i}(t) = h_{\vec{r}_i\vec{r}_0}(t)$. Received signals at each array element are recorded, reversed in time and transmitted back to the same medium (Fig. 2.1(b)). The time-reversed signal received at the original
Figure 2.1: Time reversal experiment using a limited aspect array

source point $\vec{r}_0$ due to $i^{th}$ antenna is then given by

$$p_i(t) = s(-t) \ast_{\infty} h_{\vec{r}_0\vec{r}_i}(-t) \ast_{\infty} h_{\vec{r}_i\vec{r}_0}(t)$$

(2.3)

where the last two terms $(h_{\vec{r}_0\vec{r}_i}(-t) \ast h_{\vec{r}_i\vec{r}_0}(t))$ represent a correlation filter (time-correlator). This correlation function has a maximum at $t = 0$ which corresponds to the energy of $h_{\vec{r}_0\vec{r}_i}(t)$, i.e. $\int |h_{\vec{r}_0\vec{r}_i}(t)|^2 dt$. With multiple antennas, TR system
performance improves since each antenna will have a maximum at the original source location and they will constructively interfere to improve the TR peak signal. For an $N$ element TR array (TRA), the received signal at the original source location becomes

$$p(\bar{r}_0, t) = \sum_{i=1}^{N} s(-t) * t h_{\bar{r}_0\bar{r}_i}(-t) * t h_{\bar{r}_i\bar{r}_0}(t) \quad (2.4)$$

In addition to being a time-correlator, TR also acts as a space-correlator. In the above analysis, the TR waveform is exactly matched to the original source point $\bar{r}_0$. However, at any other point $\bar{r}$ in the domain, this signal becomes

$$p(\bar{r}, t) = \sum_{i=1}^{N} s(-t) * t h_{\bar{r}_0\bar{r}_i}(-t) * t h_{\bar{r}_i\bar{r}}(t) \quad (2.5)$$

As the probe antenna location $\bar{r}$ gets further away from the original source location $\bar{r}_0$, then, similar to time-correlation analysis, uncorrelated terms tend to cancel each other. For media with rich multipath components, correlation peak gets sharper and a better (sharper) focusing spot both in time and space can be achieved. These steps can be best illustrated by the visualization of the time-domain progress of the forward and backward propagations as in Fig. 2.2 and Fig. 2.3. In these figures, the snapshots of the electric field propagation in an homogeneous medium taken at different times are shown for both forward and backward propagations. It is observed that although only a limited number of antennas is used to receive and record the propagating signals, it is enough for the backpropagated signals to focus around the original source location and the focusing spot size is dictated by the classical diffraction limit [1] which states that in an homogeneous media, cross-range ($d_c$) resolution is given by

$$d_c = \lambda \frac{L}{a} \quad (2.6)$$
Figure 2.2: Snapshots of the forward electric field component \( (E_z) \) propagation in an homogeneous medium when a short pulse is transmitted from a source point (shown by the + sign). Signals received by the TRA (shown by the x signs) are recorded and time-reversed for backpropagation.

Figure 2.3: Snapshots of the backward electric field component \( (E_z) \) propagation when the time-reversed fields recorded at the array antennas are transmitted back into the same homogeneous medium. At \( t=0 \), focusing around the original source location occurs.
where $\lambda$ is the wavelength, $a$ is the antenna aperture length and $L$ is the distance between the array and the source (Fig. 2.1). Now, let us see what would happen in a similar scenario but with multipath components this time. For this end, the same short pulse is transmitted from the same location to a similar medium whose only difference from the homogeneous case is the two embedded perfectly electric conductor (PEC) walls. The snapshots of the forward and backward propagations recorded in this case are shown in Fig. 2.4 and Fig. 2.5. The PEC walls are located in such a way that they guide the waves toward the TRA antennas. In this particular configuration, the waves diverging away from the TRA are redirected towards the TRA antennas thereby creating the multipath components. In this case, the focusing spot size seems to be much narrower as compared to the homogeneous case which can be considered as a counter intuitive result at first. However, since the TRA acts as a mirror to refocus the time-reversed signals back on the source, the multipath tends to interfere constructively only at the focusing point. This can also be understood as an increase in the effective aperture ($a_e$) of the TRA due to multipathing [5] yielding the following cross-range resolution

$$d_c = \lambda \frac{L}{a_e} \quad (2.7)$$

Therefore, in inhomogeneous media, a better focusing resolution than the homogeneous medium case, i.e. superresolution, can be achieved as long as the inhomogeneities in the medium redirect the diverging waves toward the TRA thereby effectively increasing the aperture length ($a_e > a$) [5]. This is illustrated in Fig. 2.6. Note that while Fig. 2.6(b) correspond to the multipaths caused by a waveguide-like structure, Fig. 2.6(c) illustrates the multipathing due to discrete scatterers in the medium. Similarly, continuously varying inhomogeneities also act as sources of multipath and
Figure 2.4: Snapshots of the forward electric field component ($E_z$) propagation when a short pulse is transmitted from a source point. The medium has two PEC walls (shown by the white-bordered boxes) creating multipaths in the medium. The received signals by the TRA are recorded and time-reversed for backpropagation.

Figure 2.5: Snapshots of the backward electric field component ($E_z$) propagation when the time-reversed fields recorded at the array antennas are transmitted back into the same homogeneous medium with PEC walls. At $t=0$, focusing around the original source location occurs.
in this chapter we will focus our attention on their effects on the TR refocused fields. Thus, the next section introduces the random medium model used throughout this work.

2.2 Physics-Based Ultrawideband Clutter Models

Natural media such as snow, vegetation, rocks, soils and biological tissues are often inhomogeneous and cannot be described in a deterministic manner. Therefore, statistical models (random medium models) should be employed instead [95]. A random or disordered media can be classified either as (i) discrete random media, characterized as a discrete set of scatterers (e.g. trees, buildings) at random positions and/or random constitutive properties in a background medium, or as (ii) continuous random media, characterized by pointwise fluctuations on its properties (e.g. biological media, soils, smoke) described in terms of a stochastic process with appropriate correlation functions.
With subsurface sensing applications in mind, we will focus our attention on the characterization of continuous random media where the parameters are approximately based on particular soil models. Naturally, soil contains fluctuations in density, material and moisture that may affect the scattering results. Additionally, since for most applications, the aim is to detect man-made buried objects, the soil between the target and the ground surface has usually been previously excavated and therefore it is not expected to have a homogeneous distribution. In the absence of experimental data to support a specific choice of random medium model, continuous random medium models with Gaussian distributions are preferred for their generality and mathematical properties. This is due to the fact that Gaussian distributions for the constitutive properties require only the second order statistics. Such media are also characterized by a spatial correlation function. In our models, random media correlation lengths are on the order of a wavelength (resonant regime) so that fluctuations are directly mapped into the computational domain.

The relative permittivity of the medium is defined as

$$\epsilon(\bar{r}) = \epsilon_m + \epsilon_f(\bar{r}) \quad (2.8)$$

where $\bar{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the spatial position, $\epsilon_m$ is the average value of the relative permittivity and $\epsilon_f(\bar{r})$ is a function of position characterizing the random fluctuation on $\epsilon(\bar{r})$ with $\langle \epsilon_f(\bar{r}) \rangle = 0$. At every point in space, the fluctuating permittivity is a Gaussian random variable with zero mean and probability density function given by:

$$P_{\epsilon_f}(\zeta) = \frac{1}{\sqrt{2\pi}\delta} \exp \left( -\frac{\zeta^2}{2\delta} \right) \quad (2.9)$$

The random medium is characterized by transverse and vertical correlation lengths ($l_s$ and $l_z$) and variance ($\delta$). The correlation function between the permittivities at
two points is also given by a Gaussian function as follows:

\[ C(\vec{r}_1 - \vec{r}_2) = \langle \epsilon_f(\vec{r}_1)\epsilon_f^*(\vec{r}_2) \rangle \]
\[ = \delta \exp \left( -\frac{|x_1 - x_2|^2 + |y_1 - y_2|^2}{l_s^2} - \frac{|z_1 - z_2|^2}{l_z^2} \right) \]  
\[ (2.10) \]

Note that this correlation function has previously been used in various studies for the characterization of geophysical data [96, 97]. However, it should be noted that any general correlation function can also be incorporated as well, e.g. exponential function [98, 99].

Now that the parameters are defined, it is time to generate a random medium realization. For this purpose, we follow the procedure discussed in [100] which is also explained in Appendix A. The results are shown in Fig. 2.7 where sample homogeneous and random media with different variance and correlation lengths are demonstrated. It is observed that for a fixed \( l_s \), increasing \( \delta \) yields fluctuations with higher amplitudes. Similarly, increasing \( l_s \) for a fixed \( \delta \), results in relatively slower variations along both directions. Since, EM wave propagation in the continuous random media is of interest throughout this dissertation, it is worth to demonstrate the time-domain progress of a short pulse in such a media as shown in Fig. 2.8. The multiple scattering due to random media can clearly be seen during the forward propagation. Additionally, Fig. 2.9 show the snapshots of the backward propagation of the corresponding time-reversed signals in the same random medium. A better focusing around the original source location is observed.

### 2.3 Computational Setup

In the recent years, numerous studies concentrating on the modeling of subsurface problems have appeared in the literature, e.g. [101, 102, 103]. While the initial
(a) (Deterministic) Homogeneous medium ($\delta = 0$ and $l_s = \infty$)

(b) Continuous random medium with $\delta = 0.025\epsilon_m$ and $l_s = 5\Delta_s$

(c) Continuous random medium with $\delta = 0.0875\epsilon_m$ and $l_s = 5\Delta_s$

(d) Continuous random medium with $\delta = 0.0875\epsilon_m$ and $l_s = 10\Delta_s$

Figure 2.7: Relative dielectric permittivity distribution ($\epsilon(\bar{r})$) of two dimensional homogeneous and random media with different variance ($\delta$) and correlation lengths ($l_s$) ($\epsilon_m = 5.5$)
Figure 2.8: Snapshots of the forward electric field component \((E_z)\) propagation when a short pulse is transmitted from a source point in a continuously random media.

Figure 2.9: Snapshots of the backward electric field component \((E_z)\) propagation when the time-reversed fields recorded at the array antennas are transmitted back into the same continuous random media. At \(t=0\), focusing around the original source location occurs.
studies gave attention to approximate analytical techniques [104], recent studies focused on numerical methods such as the method of moments [105] and the FDTD method [102, 106, 107]. FDTD has gained popularity due to several advantages it offers such as the ability to model arbitrary complex geometries and material structures with relative ease. Additionally, it provides total field solution with the direct implementation of the Maxwells equation and can be applied to narrowband, broadband and harmonic time-domain problems [94]. Therefore, FDTD is also the method adopted throughout this dissertation. We employ either 2-D or 3-D FDTD simulations to produce data from scattering by objects in continuous random media. To model wave propagation in unbounded media, the FDTD domains are terminated by perfectly matched layers (PML) [108] with stretched coordinates [109, 110] matching the interior random medium to provide reflectionless truncation of the computational domains.

As for this chapter, we employ a two-dimensional FDTD computational domain with \( N_x \times N_y = 200 \times 240 \) grid points with uniform space discretization size of \( \Delta_x = \Delta_y = \Delta_s = 0.0137 \) m. A linear TRA of \( N = 7 \) electric dipole antennas with even distribution along the array apertures are located just above a lossless random medium with spatially fluctuating permittivity as explained in the previous section. The array lies parallel to the \( x \)-direction and the dipoles are separated by \( \lambda_c/2 \) where \( \lambda_c \) corresponds to the wavelength at the central frequency for the mean permittivity value. The location of the central antenna is accepted as the origin, i.e. \( \bar{R}_4 = (0,0)\Delta_s \) where \( \bar{R}_i \) is the location of the \( i^{th} \) TRA antenna. An electric current dipole located at \( \bar{r}_s = (x_s, y_s) = (0,155)\Delta_s = (0,2.12)\text{m} \) is initially fed by the current source \( \bar{J}(\bar{r}_s,t) = \hat{e} s(t)\delta_d(\bar{r}_s) \) where \( \hat{e} \) (\( \hat{x} \), \( \hat{y} \) or \( \hat{z} \)) is the unit vector.
representing the polarization of the dipole, $\delta_d(\vec{r})$ is the Dirac delta function and $s(t)$ is the ultrawideband time-domain excitation taken as the first derivative of the Blackmann-Harris (BH) pulse [111] that vanishes after a time period of $T = 1.55/\tilde{f}_c$ (Fig 2.10) where $\tilde{f}_c = 400$ MHz is the central frequency. Since the incident electric field on the scatterers is related to the derivative of the current by

$$\bar{E}^{inc}(\vec{r}, t) = \mu_0 \bar{G}(\vec{r}, \vec{r}', t) * t \frac{\partial}{\partial t} \vec{J}_i(\vec{r}', t)$$  \hspace{1cm} (2.11)$$

where $\bar{G}$ is the dyadic Green’s function, the incident field appears as the second derivative of the BH pulse and the central frequency is shifted towards a higher frequency (Fig. 2.10(b)). Note that for the FDTD grid, chosen $\Delta_s$ corresponds to $\lambda_c/30 - \lambda_c/40$ for the mean permittivity of the random media. To isolate volumetric scattering effects, surface roughness and mutual interactions among array elements are not included. Results presented depend on the size of the computational box used for the random medium, which is essentially limited by the computational resources at our disposal. These truncation effect decrease for larger computational boxes or smaller permittivity variances. In the setups considered, this dependency is weak because the boundary reflections affect the received signals at very later times and with relatively very small amplitudes.

### 2.4 Numerical Results

In this section, we investigate the (retro)focusing resolution of UWB TR electromagnetic (EM) pulses in continuous random media with respect to statistical parameters, i.e. variance ($\delta$) and correlation length ($l_s$). Their effects are studied for both $TM_z$ and $TE_z$ waves. For $TM_z$ case, the isolated dipole source is $z$-directed ($J_z$) and the linear TR array consists of $z$-directed dipoles whereas for the $TE_z$ case, the
Figure 2.10: (a)-(d) Time and frequency domain representations of several ultrawideband Blackmann-Harris pulses, (e)-(f) Comparison of the first derivatives of the BH and Gaussian pulses. Note that while both have similar frequency domain characteristics, BH pulse is shorter in time domain (more compact).
isolated dipole source is $x$-directed ($J_x$) and three different linear TR array configurations are used to receive the incoming signals: (a) $x$-oriented dipoles (co-pol) (b) $y$-oriented dipoles (cross-pol) and (c) both $x$ and $y$-oriented dipoles (fully polarimetric). We verify that multiple scattering in the intervening random medium produces super-resolution, i.e. a better focusing resolution of the retrofocused EM waves than is achievable according to the classical diffraction limit (in homogeneous media). The effects of (de-)polarization on the retrofocusing resolution of time reversed EM waves are also considered with no analogy in the acoustic case.

### 2.4.1 First-order medium-statistics effects

We first investigate the effect of the variance of the random medium on the retrofocusing properties of the time reversed signals in the $TM_z$ case. The variance ($\delta$) is changed from $0.025\epsilon_m$ to $0.125\epsilon_m$ while the correlation length is fixed at $l_s = 8\Delta_s$. The snapshots of the $z$-component of the electric field ($E_z$) distribution at the time of refocusing are plotted in Fig. 2.11. It can be observed from the plots that as the variance increases, the amplitude of the focused field increases and the spot size of the focused field is reduced, characterizing super-resolution. This seems at first a counterintuitive result, but it can be explained from the fact that as the variance increases more multipath is produced. For the time reversed signals, the multipath tends to interfere constructively only at the focusing point where coherent perfect phase conjugation exists. This can also be understood as an increase on the effective aperture of the TRA due to multipathing [5, 89]. Fig. 2.12(a) shows the spatial distribution of the field components of Fig. 2.11 at the time of refocusing at the source plane ($y = y_s$) with respect to the (transverse) $x$-coordinate (cross-range).
Figure 2.11: Spatial distribution of the time-reversed $E_z$ field component of the electric field at the time of refocusing for increasing $\delta$ and fixed $l_s$ of $l_s = 8\Delta_s$. 

(a) $\delta = 0$ (Homog.) 
(b) $\delta = 0.025\epsilon_m$ 
(c) $\delta = 0.0875\epsilon_m$ 
(d) $\delta = 0.125\epsilon_m$
2.4.2 Second-order medium-statistics effects

Similar simulations are performed to assess the effect of different correlation lengths on the resolution for the $TM_z$ case. The correlation length is changed from $l_s = 8\Delta_s$ to $l_s = 20\Delta_s$ for fixed variance of $\delta = 0.04\epsilon_m$. Results are shown in Fig. 2.13 and Fig. 2.12(b), where it is seen that increasing the correlation length degrades the focusing properties. This can be explained from the fact that, for larger correlation lengths, the random medium starts to behave closer to an homogeneous medium and multiple scattering effects are diminished. Note that this observation applies only for the range of correlation values and problem size considered here. If the correlation length becomes sufficiently small against the wavelength, then a homogenization
would apply and the random medium results would again approach those of a homogeneous medium. In general, the effect of changes on the correlation length upon the focusing resolution is larger when the correlation length is comparable to the wavelength.

2.4.3 Effects of (de-)Polarization

In this section, the refocusing resolution for the $TE_z$ case is examined where the isolated dipole source is $x$-directed. Three different setups are considered for the TRA. These are: Arrays composed of (1) only $x$-directed (co-pol) dipoles, (2) only $y$-directed dipoles (cross-pol) and (3) both $x$- and $y$-directed dipoles (fully polarimetric). These arrays are used to receive the incoming signals and back-propagate the time-reversed versions of them and for all cases the focusing of the $E_x$ field component is observed at the source location. Note that for the fully polarimetric case, the focused $E_x$ component is obviously the sum of the co-pol and cross-pol cases. These are: (1) An array composed of only $x$-directed (co-pol) dipoles is used to receive the incoming signals and backpropagate the time-reversed versions and observe the focusing of the $E_x$ field component at the source location. (2) An array composed of only $y$-directed (cross-pol) dipoles is used for both reception and backpropagation and again the focusing of the $E_x$ component (not the $E_y$ component) is observed around the source location. (3) Finally, a full polarimetric array consisting of both co- and cross-pol dipoles is employed. In this case, the focused $E_x$ component which is obviously the sum of the previous two setups is observed. In Fig. 2.14, the refocused $E_x$ components for the co-pol and cross-pol setups in both an homogeneous and a random medium defined with $\delta = 0.0875\epsilon_m$ and $l_s = 8\Delta_s$ are shown. Additionally, Fig. 2.15 plots the
Figure 2.13: Spatial distribution of the time-reversed $E_z$ field component of the electric field at the time of refocusing for increasing $\delta$ and fixed $l_s$ of $l_s = 8\Delta_s$. 
Figure 2.14: Spatial distribution of the time reversed $E_x$ field components at the time of refocusing due to: co-pol ($x$-directed) and cross-pol ($y$-directed) TRAs in a homogeneous medium and a random medium (a)-(d) source located under the central antenna (e)-(f) for an off-central source location
$E_x$ components along the cross-range at the source location for all three array setups mentioned above for the same homogeneous and random medium. For both of these cases, it is observed that although the $E_x$ component due to cross-pol TRA does not focus well at the source location, it helps to improve the refocusing resolution when it is superimposed to the $E_x$ component due to the co-pol TRA (fully polarimetric case) as compared to the co-pol only case. Variations in the multiple scattering by changing the medium-statistics have the same effects on the focusing resolution as in the $TM_z$ case. However, it is observed that increased multiple scattering tends to increase depolarization more, hence the improvement on the refocusing resolution between the fully polarimetric and the co-pol TRA cases may become more pronounced in certain cases.

Figure 2.15: The directivity pattern of the focused $E_x$ field component for all the three array setups at positive maxima for (left) homogeneous and (right) random medium (for the source point located below the central antenna).
2.5 Summary and Conclusions

The effects of different statistical parameters of the random medium on the focusing resolution of UWB EM waves generated by TRAs in continuous random media have been studied computationally. It has been shown that an increase in the multiple scattering effects (multipaths) results to better focusing resolution of the time-reversed signals. Although only the inhomogeneous background medium is considered here, any means increasing the multiple scattering in the medium such as discrete scatterers would improve the focusing resolution of the focused time-reversed fields (as long as the TRA can receive those multipaths). Additionally, the UWB nature of the signal augments the super-resolution effects since the TR retrofocusing produces a coherent sum (for all frequency components of the signal) only at the original source position. At points in the domain away from the original source position, the different components of the signal add only in a incoherent fashion \cite{5}. It is also shown that, in the $TE_z$ case, better (super-)resolution can be obtained with fully-polarimetric TRA over that achieved with co-pol TRA only.
CHAPTER 3

PHYSICAL TIME-DOMAIN DORT METHOD AND SELECTIVE FOCUSING

In media containing multiple discrete scatterers, physical backpropagation of TR scattered fields results in generation of focal spots on all scatterers simultaneously, and more strongly on the dominant scatterer (standard TR). As the standard TR process is iterated, the wavefront becomes increasingly localized on the dominant scatterer [23]. Hence, standard TR iteration does not allow focusing on other (weaker) scatterers unless time-gating is applied. However, time-gating can distinguish only temporally well-resolved scatterers and its performance strongly depends on the usable bandwidth. Decomposition of the time-reversal operator (DORT under its French language acronym) method overcomes this problem by isolating and classifying different scattering centers (in the presence of others) without the need for any time gating or iterative process [22, 25] (Fig. 3.1). To achieve this, DORT utilizes the time-reversal operator (TRO) which is obtained using the multistatic data matrix (MDM) of a time-reversal array. The eigenspace structure of the TRO (i.e. eigenvalues and corresponding eigenvectors) contains valuable information about the scattering scenario under study. Particularly, it was shown in [22, 25] that for well-resolved point-like isotropic scatterers, each eigenvalue and the corresponding eigenvector of
Figure 3.1: DORT operation: (a) MDMs are first obtained by launching a short pulse from each TRA antenna and recording the scattered field signals at all the antennas. (b) Selective focusing is achieved by (back-)propagation of the associated eigenvectors from the TRA. (c) Standard TR yields focusing around all the scatterers.

The TRO are associated with a distinct scatterer in the probed domain. Therefore, by performing an eigenvalue decomposition (EVD) of the TRO and using the TR array to transmit the signals produced by particular eigenvectors, selective focusing of point-like scatterers becomes possible. This strategy forms the basis of the so-called DORT method [22, 25]. DORT was first applied for EM waves in [78] using time-harmonic waves with the scatterer completely surrounded by the TRA (full aspect configuration). Extensions of this analysis to linear arrays (limited aspect configuration) [55], crosswell boreholes [44], and ultrawideband (UWB) signals [45] have further demonstrated the potential of DORT for application in EM remote sensing problems. Recently, the TRO for isolated spherical EM scatterers has been derived by considering TRAs with dipole antenna elements in [79] and extended targets have been considered in [112].
Most prior works on DORT have assumed homogeneous background media and
time-harmonic fields. Although robustness to noise was demonstrated by considering
random phase shifts on scattered fields [78] or by using discrete random media (RM)
composed of many point-like scatterers [45], the effect of clutter from background
continuous (inhomogeneous) random media (where volumetric scattering effects are
important) on the performance of DORT for UWB signals are not considered before.
Under such conditions, distributed source effect of clutter can be translated to the
signal or noise subspace (or both) of the TRO which may mislead the number of
scatterers or selective focusing. Since DORT differs from standard TR with such a
preprocessing step, it is important to analyze the response of the DORT method to
these conditions. Therefore, the work carried out in this chapter complements the
previous works on DORT by:

- Developing a full time-domain (TD)-DORT algorithm [91] with phase smoothing,
  instead of the previously employed central-frequency DORT,

- Addressing the application of the TD-DORT using different polarizations of the
  EM signal,

- Discussing the effects of random medium statistics on the eigenvalue (or singular
  value)/eigenvector distribution of the TRO and selective focusing performance
  of DORT.

Note that TD-DORT allows the generation of time-domain eigenvectors for back-
propagation which will be explained in details in the next section. But before going
into that, let us mention several advantages DORT offers. First of all, by analyzing
the eigenvalues and eigenvectors, it is possible to determine the number of scatterers in the media provided that they can be separated at the frequency of operation. Additionally, since it can isolate distinct scatterers in the domain, it can be used as a pre-processing step to improve the efficiency of the inverse scattering algorithms. This is possible through restricting the inverse problems to smaller domains (domain reduction) by including the localization data provided by the DORT into the inversion process, as well as dramatically reducing the number of required iteration steps (faster convergence). Finally, focusing the wave around a selected scatterer increases the signal to noise ratio (SNR), thereby improving the quality of the extracted target signatures at poor SNR conditions.

3.1 Full Time-Domain DORT

In the DORT method, an active TR array of \( N \) transceivers produces an \( N \times N \) symmetric (due to reciprocity) MDM denoted as

\[
K(t) = \begin{pmatrix}
k_{11}(t) & \cdots & k_{1N}(t) \\
\vdots & \ddots & \vdots \\
k_{N1}(t) & \cdots & k_{NN}(t)
\end{pmatrix}
\] (3.1)

where \( k_{ij}(t) \) corresponds to the signal received at the \( i^{th} \) antenna when a short pulse \( s(t) \) is transmitted from the \( j^{th} \) antenna as the sole transmitter (Fig. 3.1(a)). The Fourier transform of \( K(t) \) yields \( K(\omega) (= [k_{ij}(\omega)]) \) for \( \omega \in \Omega_s \) where \( \Omega_s \) is the bandwidth of operation. For point-like and well-resolved scatterers, \( k_{ij}(\omega) \) can be written as

\[
k_{ij}(\omega) = \sum_{m=1}^{M} G(\bar{r}_i, \bar{x}_m, \omega) \tau_m(\omega)G(\bar{x}_m, \bar{r}_j, \omega)s(\omega)
\] (3.2)

where \( M \) is the total number of scatterers, \( G(\bar{r}_i, \bar{x}_m, \omega) \) is the reciprocal medium Green’s function between the \( i^{th} \) antenna location \( \bar{r}_i \) (for \( i = 1, \ldots, N \)) and the \( m^{th} \)
scatterer location $\bar{x}_m$ (for $m = 1, .., M$), $\tau_m$ is the scattering coefficient of the $m^{th}$ scatterer and $s(\omega)$ is the frequency domain representation of the input pulse (Fig. 3.2(a)). Note that the well-resolvedness criterion implies that the multiple scattering among the scatterers can be neglected. Therefore, only the direct scattering from the embedded scatterer(s) in the domain is included in Eq. 3.2. Using this expression, $K(\omega)$ can be written as

$$K(\omega) = s(\omega) \sum_{m=1}^{M} \tau_m(\omega) g_s(\bar{x}_m, \omega) g_s^T(\bar{x}_m, \omega)$$

(3.3)

where

$$g_s(\bar{x}_m, \omega) = [G(\bar{r}_1, \bar{x}_m, \omega), ..., G(\bar{r}_N, \bar{x}_m, \omega)]^T$$

(3.4)

is the $N \times 1$ background Green’s function vector (steering vector) that connects the $m^{th}$ scatterer location ($\bar{x}_m$) to the array antennas as illustrated in Fig. 3.2(b). This representation of the MDM is helpful for the analysis of the eigenstructure of
the TRO, as will be shown shortly. But before that, let us define the TRO as the following self-adjoint matrix

\[ T(\omega) = K^\dagger(\omega)K(\omega) \]  

(3.5)

where \( K^\dagger(\omega) \) is the Hermitian conjugate of \( K(\omega) \) and \( \dagger \) stands for the complex transpose operation. This comes from the fact that time-reversal corresponds to a phase conjugation in the frequency domain, and therefore, Fourier transform of \( K(-t) \) can be written as \( K^\dagger(\omega) \). Thanks to the TRO definition, its eigenstructure can be found by utilizing the singular value and singular vectors of the MDM. For this end, singular value decomposition (SVD) is applied to \( K(\omega) \) which yields

\[ K(\omega) = U(\omega)\Lambda(\omega)V^\dagger(\omega) \]  

(3.6)

where the real diagonal matrix of singular values \( \Lambda(\omega) \) and the unitary matrices \( U(\omega) \) and \( V(\omega) \) which are formed by the left and right singular vectors, respectively, are given as follows

\[
\Lambda(\omega) = \begin{pmatrix}
\lambda_1(\omega) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \lambda_N(\omega)
\end{pmatrix}
\]  

(3.7)

\[
U(\omega) = [u_1(\omega), \cdots, u_N(\omega)] = \begin{pmatrix}
u_{11}(\omega) & \cdots & u_{1N}(\omega) \\
\vdots & \ddots & \vdots \\
u_{N1}(\omega) & \cdots & u_{NN}(\omega)
\end{pmatrix}
\]  

(3.8)

\[
V(\omega) = [v_1(\omega), \cdots, v_N(\omega)] = \begin{pmatrix}
v_{11}(\omega) & \cdots & v_{1N}(\omega) \\
\vdots & \ddots & \vdots \\
v_{N1}(\omega) & \cdots & v_{NN}(\omega)
\end{pmatrix}
\]  

(3.9)
Using the SVD of the MDM, the eigenvalue decomposition of the TR operator can be written as

\[ T(\omega) = V(\omega)\Lambda(\omega)U^{\dagger}(\omega) \]

\[ = V(\omega)\Lambda(\omega)\Lambda^{\dagger}(\omega)V^{\dagger}(\omega) \]

\[ = V(\omega)S(\omega)V^{\dagger}(\omega) \quad (3.10) \]

where \( I \) is the identity matrix and \( S(\omega) \) is the real diagonal matrix of eigenvalues as shown below

\[ S(\omega) = \Lambda(\omega)\Lambda^{\dagger}(\omega) = \begin{pmatrix} \mu_1(\omega) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mu_N(\omega) \end{pmatrix} \quad (3.11) \]

Note that eigenvalues of the TRO correspond to the square of the singular values of the MDM, i.e., for \( i = 1, \ldots, N \)

\[ \mu_i(\omega) = \lambda_i^2(\omega) \quad (3.12) \]

Hence, both terms are used interchangeably throughout this dissertation. Similarly, eigenvectors of the TR operator correspond to the columns of the unitary matrix \( V(\omega) \) \( (v_i(\omega), i = 1, \ldots, N) \), i.e., the right singular vectors of \( K(\omega) \), which are normalized and orthogonal. As a result, it is observed that finding the singular value and the singular vectors of \( K(\omega) \) is enough and necessary to obtain the eigenstructure of the TRO. Therefore, we first recall the following singular system

\[ K(\omega)v_i(\omega) = \lambda_i(\omega)u_i(\omega) \quad (3.13) \]

\[ K^{\dagger}(\omega)u_i(\omega) = \lambda_i^*(\omega)v_i(\omega) \quad (3.14) \]

where * denotes the phase conjugation. Substituting Eq. 3.3 into Eq. 3.13 yields

\[ s(\omega)\sum_{m=1}^{M} \tau_m(\omega) g_s(\bar{x}_m, \omega) g_s^T(\bar{x}_m, \omega)v_i(\omega) = \lambda_i(\omega)u_i(\omega) \quad (3.15) \]
where \( g_s^T(\bar{x}_m, \omega) v_i(\omega) \) is a scalar identity allowing us to obtain the following expression for the left singular vector \( u_i(\omega) \)

\[
 u_i(\omega) = \frac{\sum_{m=1}^{M} s(\omega) \tau_m(\omega) \left< v_i(\omega), g_s^*(\bar{x}_m, \omega) \right>}{\lambda_i(\omega)} g_s(\bar{x}_m, \omega) 
\]  

(3.16)

where \( \left< x, y \right> = y^\dagger x = \sum_{i=1}^{N} y_i^* x_i \) is the standard inner product. Note that Eq. 3.16 states that left singular vector \( u_i(\omega) \) is a linear combination of the steering vectors \((g_s(\bar{x}_m, \omega) \text{ for } m = 1, .., M)\) connecting the scatterers to the TRA. A similar expression for the right singular vector is also given below

\[
 v_i(\omega) = \frac{\sum_{m=1}^{M} s^*(\omega) \tau_m^*(\omega) \left< u_i(\omega), g_s(\bar{x}_m, \omega) \right>}{\lambda_i^*(\omega)} g_s^*(\bar{x}_m, \omega) 
\]  

(3.17)

Note that, as observed from Eqs. 3.16 and 3.17, left and right singular vectors satisfy the following property

\[
u_i(\omega) = v_i^*(\omega) \text{ for } i = 1, .., N
\]  

(3.18)

which is due to the fact that \( K(\omega) \) is symmetric. Now that, if a specific nonzero \( u_i(\omega) \) can be found such that the inner product \( \left< u_i(\omega), g_s(\bar{x}_m, \omega) \right> \) becomes zero only when \( u_i(\omega) \neq g_s(\bar{x}_m, \omega) \), then \( v_i(\omega) \) can be written in terms of only a single steering vector. Actually, this condition is satisfied for well-resolved targets for which

\[
\left< g_s(\bar{x}_p, \omega), g_s(\bar{x}_m, \omega) \right> = \begin{cases} 
\|g_s(\bar{x}_m, \omega)\|^2 & \text{when } \bar{x}_p = \bar{x}_m \\
0 & \text{when } \bar{x}_p \neq \bar{x}_m 
\end{cases} 
\]  

(3.19)

where \( \|x\| = \sqrt{x^\dagger x} \) denotes the norm of the vector \( x \). Thus, the following choice of left and right singular vectors satisfies the singular system of Eqs. 3.13 and 3.14

\[
u_i(\omega) = \frac{g_s(\bar{x}_i, \omega)}{\|g_s(\bar{x}_i, \omega)\|} \text{ for } i = 1, .., M 
\]  

(3.20)

\[
v_i(\omega) = \frac{g_s^*(\bar{x}_i, \omega)}{\|g_s(\bar{x}_i, \omega)\|} \text{ for } i = 1, .., M 
\]  

(3.21)
where the corresponding singular value becomes

\[ \lambda_i(\omega) = s(\omega)\tau_i(\omega)\|g_s(\bar{x}_i, \omega)\|^2 \quad \text{for} \quad i = 1, \ldots, M \] (3.22)

Therefore, it is observed that for well-resolved, point-like scatterers in the probed media, each significant eigenvalue and the corresponding eigenvector of the TRO are associated with a single scatterer (for isotropic scattered fields) as shown in Fig. 3.3. Subsequent backpropagation of the corresponding eigenvector yields wavefront focus-

![Diagram of eigenvectors and scatterers](image)

**Figure 3.3:** For well-resolved, point-like scatterers each significant eigenvalue and corresponding eigenvector of the TRO are associated with a single scatter in the domain. Specifically, each eigenvector is proportional to the steering vector connecting the single scatterer to the TRA antennas.

In prior works, eigenvalue decomposition of the TRO was carried out only at the central frequency of operation \( (\omega_c) \). This approach is denoted here as *central*
frequency (CF)-DORT. In this case, the backpropagated fields radiated from the TRA for focusing on the \( p \)th scatterer are produced by the associated eigenvector \( \mathbf{v}_p(\omega_c) \) and singular value \( \lambda_p(\omega_c) \) which satisfy

\[
\mathbf{T}(\omega_c)\mathbf{v}_p(\omega_c) = \lambda_p^2(\omega_c)\mathbf{v}_p(\omega_c).
\]  

(3.23)

The normalized vector \( \mathbf{v}_p(\omega_c) \) is the \( p \)th column of \( \mathbf{V}(\omega_c) \). The components of the \( N \times 1 \) column vector \( \mathbf{e}_p(\omega_c) \) give the excitation amplitudes for \( N \) element TRA via

\[
\mathbf{e}_p(\omega_c) = \mathbf{K}^\dagger(\omega_c)\mathbf{u}_p(\omega_c)
\]

\[
= \lambda_p^*(\omega_c)\mathbf{v}_p(\omega_c) = \lambda_p^*(\omega_c)\frac{\mathbf{g}_s^*(\bar{x}_p, \omega_c)}{\|\mathbf{g}_s(\bar{x}_p, \omega_c)\|}
\]  

(3.24)

In central frequency DORT, the same amplitude distribution \( \mathbf{e}_p(\omega_c) \) is used along the entire bandwidth. However, for UWB signals a similar decomposition should be applied over the entire bandwidth to yield an amplitude distribution (excitation) that is truly frequency dependent. This is particularly important to improve the focusing resolution (super-resolution) of the TR UWB signals by taking advantage of frequency decorrelation due to multiple scattering [89, 33].

When different eigenvalues are associated with the same well-resolved scatterers throughout the entire bandwidth, the corresponding (frequency dependent) eigenvectors can be used to obtain a set of UWB pulses to excite each array element, by means of an inverse Fourier transformation. In particular, the UWB pulses to be fed to the array elements for selective focusing of the (well resolved) \( p \)th scatterer are given by

\[
\mathbf{e}_p(t) = \mathcal{F}^{-1}\left(\mathbf{K}^\dagger(\omega)\mathbf{u}_p(\omega)\right)
\]

\[
= \mathcal{F}^{-1}\left(\lambda_p^*(\omega)\mathbf{v}_p(\omega)\right)
\]  

(3.25)
where $\mathcal{F}^{-1}$ stands for inverse Fourier transform. Substituting Eqs. 3.21 and 3.22 into Eq. 3.25 yields

$$
e_p(t) = \mathcal{F}^{-1}(s^*(\omega)\tau_p^*(\omega)\|g_s(\bar{x}_p, \omega)\| g_s^*(\bar{x}_p, \omega)) \quad (3.26)$$

Defining $g_s(\bar{x}, t) = \mathcal{F}^{-1}(g_s(\bar{x}_p, \omega))$ as the time-domain steering vector, Eq. 3.25 can be written as

$$
e_p(t) = [e_{p1}(t), \ldots, e_{pN}(t)]^T \quad (3.27)$$

$$
e_p(t) = s(-t) *_t \tau_p(-t) *_t \alpha_p(t) *_t g_s(\bar{x}_p, -t) \quad (3.28)$$

$$
e_p(t) = s(-t) *_t \tau_p(-t) *_t \alpha_p(t) *_t [G(\bar{x}_p, \bar{r}_1, -t), \ldots, G(\bar{x}_p, \bar{r}_N, -t)]^T \quad (3.29)$$

where $*_t$ is the time-convolution, $G(\bar{x}_s, \bar{r}_i, t)$ is the time-domain background Green’s function between the point $\bar{x}_p$ and the $i^{th}$ TRA antenna location $\bar{r}_i$ and $\alpha_p(t) = \mathcal{F}^{-1}(\|g_s(\bar{x}_p, \omega)\|)$ is a scalar identity with no phase information. Once $e_p(t)$ is transmitted back to the background medium, the field at any arbitrary search point $\bar{x}_s$ in the probed domain can be found through the following expression

$$
h_p(\bar{x}_s, t) = \sum_{i=1}^{N} e_{pi}(t) *_t G(\bar{x}_s, \bar{r}_i, t)$$

$$
h_p(\bar{x}_s, t) = s(-t) *_t \tau_p(-t) *_t \alpha_p(t) *_t \sum_{i=1}^{N} G(\bar{x}_p, \bar{r}_i, -t) *_t G(\bar{x}_s, \bar{r}_i, t) \quad (3.30)$$

where the last two terms represent a spatio-temporal filter which is maximum when $t = 0$ and $\bar{x}_s = \bar{x}_p$. While a coherent summation occurs around $\bar{x}_s = \bar{x}_p$, incoherent summation takes place at other points in the domain. Therefore, focusing occurs around only the $p^{th}$ scatterer in the presence of other scatterers. This characterizes the full time-domain (TD)-DORT employed in this dissertation. As in the case of standard TR, TD-DORT can also take advantage of the frequency decorrelation to
achieve superresolution. This is possible in cases where the multiple scattering increases the effective aperture of the array and when actual physical backpropagation of the generated time-domain excitation signals into the original medium is carried out (so that an exact phase cancelation occurs around the selected scatterer location). It is also required that the SVD operation yields singular vectors proportional to the Green’s function vectors connecting the scatterers to the TRA antennas. This is achieved when the scatterers are well-resolved and point-like.

Up to this point in this section, we have discussed the mathematical development of the TD-DORT method. However, during the code implementation of the algorithm, an unaccounted numerical problem arises. This problem is addressed next.

3.1.1 Phase Smoothing Algorithm

The mathematical development of the TD-DORT method has been carried out using the continuous time and/or frequency domain functions. However, numerically, we have to work with their discrete counterparts, as it is impossible to represent continuous functions using computers. Therefore, when we use the term inverse Fourier transform of a function $F(\omega)$, we implicitly refer to the discrete inverse Fourier transform of the discrete function $F[k]$ which consists of the uniformly taken samples of the continuous function $F(\omega)$, i.e. $F[k] = F(\omega_k)$ for $k = 1, \ldots, K$ where $\omega_K - \omega_1 = \Omega_s$ is the bandwidth of operation. Similarly, we apply the SVD operation to the MDMs only at the corresponding sampling points. Actually, this is where the numerical problem arises which basically comes from the way SVD algorithm works. Since the SVD is applied to the MDM at each single frequency one-by-one and without having any information of the SVD results obtained at other frequencies, the singular vectors
at each frequency carry an arbitrary and frequency dependent phase $\phi_{\text{svd}}(\omega)$ [32]. As a result, Eq. 3.21 should be updated as follows to include the numerical error as well:

$$v_i(\omega) = e^{j\phi_{\text{svd}}(\omega)} \frac{g^*_s(\bar{x}_i, \omega)}{\|g_s(\bar{x}_i, \omega)\|} \text{ for } i = 1, \ldots, M$$  (3.31)

Once this is substituted in Eq. 3.25, the corresponding time-domain signals become incoherent as illustrated in Fig. 3.4(a). Therefore, a preprocessing step is necessary to obtain coherent time-domain excitation signals. Several methods such as the power SVD or projecting the incoherent eigenvectors onto the columns of MDM are possible remedies [31]. However, here, we have utilized a smoothing algorithm based on phase difference tracking between the adjacent frequency eigenvectors. The algorithm starts by analyzing the phase distribution of the eigenvectors along the bandwidth. Using

![Figure 3.4](image-url)

Figure 3.4: (a) Comparison of incoherent and coherent time-domain signals. Note that although (b) the magnitudes of the frequency domain representation of both signals are same, (c) phases are not. Incoherency comes from the oscillations in the phase which comes from the SVD algorithm.
Eq. 3.31, we can write the total phase of the \( i \)th eigenvector \( \angle \mathbf{v}_i(\omega) \) as follows

\[
\begin{pmatrix}
\angle v_{i1}(\omega) \\
\vdots \\
\angle v_{iN}(\omega)
\end{pmatrix}
= \phi_{\text{svd}}(\omega) + \begin{pmatrix}
\angle G^*(\bar{x}_i, \bar{r}_1, \omega) \\
\vdots \\
\angle G^*(\bar{x}_i, \bar{r}_N, \omega)
\end{pmatrix}
\]

(3.32)

where \( \angle g^*_i(\bar{x}_i, \omega) \) is the phase of the Green’s function vector connecting the \( i \)th scatterer to the TRA. For well-resolved and point-like scatterers embedded in homogeneous media, the medium Green’s function has a linear phase with respect to the frequency \( \omega \), specifically,

\[
\angle G^*(\bar{x}_i, \bar{r}_l, \omega) = \phi_{l,l}^G(\omega) = \omega \sqrt{\mu \epsilon |\bar{x}_i - \bar{r}_l|} \text{ for } l = 1, \ldots, N
\]

(3.33)

Note that for this case, the first derivative of \( \phi_{l,l}^G(\omega) \) with respect to \( \omega \) is a constant and therefore the second derivative is zero. On the other hand, \( \phi_{\text{svd}}(\omega) \) is oscillatory in nature, since at each frequency, it is randomly selected by the SVD engine\(^1\). Actually, we have realized that both MATLAB and LAPACK chooses this frequency dependent term as either \( \pm \pi \) or \( \pm \pi/2 \). Thus, the total phase distribution has a continuously (slow) varying envelope and a (fast) oscillating part that comes from the numerical SVD as illustrated in Fig. 3.4(c). Therefore, if we can extract the envelope or get rid of the oscillating part, then we can use it as the new phase distribution along the frequency domain. For this end, we can use the derivative information where we know that \( \frac{d}{d\omega} \phi_{l,l}^G(\omega) \) should be a nonzero constant. Therefore, by looking at the phase difference obtained at each adjacent frequency samples, we can shift each related phase by either \( \pm \pi \) or \( \pm \pi/2 \) so that its derivative yields a constant. As long as the randomness is not very strong, this phase tracking and shifting algorithm works well for our purposes as shown in Fig. 3.5. However, for increased randomness this

\(^1\)Throughout the dissertation we employ both MATLAB© 7.3.0 and LAPACK© SVD engines.
algorithm fails to work and alternative techniques should be employed instead (One such method is introduced in Chapter 6).

![Figure 3.5: Comparison of incoherent and coherent time-domain signals obtained at different random media. (top row) $\delta = 0.5$, (bottom row) $\delta = 0.1$](image)

### 3.2 Computational Setup

Since the aim of this chapter is to apply the full time-domain DORT under 3-D scenarios, we employ 3-D FDTD simulations to produce the synthetic data from scattering 3-D objects in random media. For this end, an FDTD grid with $N_x \times N_y \times N_z = 220 \times 200 \times 250$ cells is used (Fig. 3.6). The FDTD grid domains are terminated by PML to provide reflectionless truncation of the computational domains. The random medium has spatially fluctuating permittivity $\epsilon(\bar{r}) = \epsilon_m + \epsilon_f(\bar{r})$
Figure 3.6: (Left) 3-D simulation setup used. The linear array is composed of $x$, $y$ and/or $z$ polarized infinitesimal electric dipoles, (right) slices from a sample 3-D continuous random medium

where $\epsilon_m$ is the average relative permittivity and equal to 5.5 (dry soil). The fluctuating permittivity $\epsilon_f(\bar{r})$ is a zero mean Gaussian random variable which has a Gaussian correlation function $C(\bar{r}_1 - \bar{r}_2)$ with variance $\delta$ and transverse (horizontal) and longitudinal (vertical) correlation lengths denoted as $l_s$ and $l_z$, respectively (refer to Section 2.2). The linear TRA consists of $N = 7$ dipole transceivers. Each dipole is initially fed by current source $\bar{J}_i(x, y, x, t) = \hat{e}_i s_i(t)$ where $\hat{e}_i$ ($\hat{x}$, $\hat{y}$ or $\hat{z}$) is the unit vector representing the polarization of the $i^{th}$ dipole and $s_i(t)$ is the UWB time-domain excitation taken as a Blackmann-Harris (BH) pulse derivative $[111]$ centered at $f_c = 400$ MHz. Since the incident electric field on the scatterers is related to the derivative of the current by $\bar{E}^{inc}(\bar{r}, t) = \mu_0 \bar{G} * (\partial \bar{J}_i / \partial t)$ where $\bar{G}$ is the Green’s function and $*$ is convolution in time, the incident field appears as the second derivative of the BH pulse and the central frequency is shifted approximately to 500 MHz. A uniform discretization cell size of $\Delta_x = \Delta_y = \Delta_z (= \Delta_s) = 0.7995$ cm is chosen
for the FDTD grid. This corresponds to $\lambda/93$ in free space and $\lambda/40$ for the mean permittivity of the random media at the central frequency (physical dimensions of the grid are $1.75 \times 1.59 \times 1.99$ m$^3$). The TRA lies parallel to the $x$-direction and the dipoles are distributed evenly along an aperture length of $a_{3D} = 120\Delta_s$, with the first dipole at $(x_{t1}, y_{t1}, z_{t1}) = (50, 100, 40)\Delta_s$ (Fig. 3.6(b)). To isolate volumetric scattering effects, surface roughness and mutual interactions among array elements are not included. We should point out that the results presented depend on the size of the computational box used for the random medium, which is essentially limited by the computational resources at our disposal. These truncation effect decrease for larger computational boxes or smaller permittivity variances. In the setup considered, this dependency is weak because the boundary reflections affect the received signals at very later times and with relatively very small amplitudes.

### 3.3 Results and Discussions

Among the attractive properties of a TRA is that multiple scattering in the intervening medium *increases* the effective aperture [33, 113], or equivalently, the refocusing resolution. Here, we consider similar effects due to both full polarimetric operation and random medium statistics on the full TD-DORT. We initially consider the effects of both first- and second- order statistics on singular value (eigenvalue) distribution of the MDM (TRO) obtained in 3-D lossless random media.

#### 3.3.1 Discrete Scatterer in Homogeneous Media

For reference, we first analyze the TRO of a single spherical PEC scatterer with radius $r = 4.79$ cm centered at $(x_{s1}, y_{s1}, z_{s1}) = (110, 100, 200)\Delta_s$ in an homogeneous medium. Linear TRAs are employed under three orthogonal linear polarizations.
Both scatterer and the TRA lie on the same \( y \)-plane. For each \( j \)-polarized incident field \((j = x, y, z)\), three scattered field components are produced. Therefore, the full polarimetric TRA response consists of a total of nine MDMs, \( K_{ij} \), \( i, j = x, y, z \). However, by reciprocity only six of them are independent as observed in Fig. 3.7 where the first two significant singular values of all MDMs are plotted over frequency. The remaining ones are not shown here since they are insignificant compared to the first two. It is observed that, for this TRA and scatterer configuration, \( K_{yy} \) allows for a more isotropic scattering compared with other polarization combinations. This case is similar to the acoustic or a 2-D EM case with perpendicular polarization (\( TM_z \) or s-polarization) \([55]\). This can also be noticed from the ratios between the first and second significant eigenvalues as shown in Fig. 3.7(c), with the maximum ratio obtained for \( K_{yy} \). Therefore, we can conclude that only one significant singular value is present for \( K_{yy} \), but two significant singular values are observed for \( K_{xx} \), \( K_{xz} \), \( K_{zx} \) and \( K_{zz} \). Having more than one significant singular value for a single scatterer is simply a consequence of non-isotropic scattering for a particular polarization combination \([79, 78]\). The remaining TRA polarization combinations do not provide comparably significant data due to weak scattered signals. This can also be concluded by observing the ratio between the second and third singular values in this configuration, as shown in Fig. 3.7(d), which is considerably larger than that between the first and second ones over the entire frequency band.

### 3.3.2 Discrete Scatterer in Random Media

Next, a single spherical PEC scatterer is considered in Gaussian random media having different first- and second-order statistics.
Figure 3.7: Significant singular values (and their ratios) of the MDMs $\mathbf{K}_{ij}$ in a homogeneous medium. A single spherical PEC scatterer is considered. For this particular configuration, $K_{yy}$ allows the most isotropic scattering.
Effect of First-Order Statistics

The effect of random medium variance on the singular values is first investigated. MDMs corresponding to different polarized TRAs are obtained in random media with fixed correlation lengths \( l_s = l_z = 8 \Delta_s = 6.33 \text{ cm} \) and variance \( \delta \) changing between 0.1, 0.5 and 1.0 which represent approximately 30%, 54% and 81% of \( \epsilon_m(= 5.5) \), respectively. The first three singular values for \( K_{xx}, K_{yy}, K_{zz} \) and \( K_{zx} \) are plotted in Fig. 3.8. As it can be observed, for larger variances, the magnitudes of the singular values increase. Moreover, other significant singular values may also appear. These can often be distinguished as clutter as long as they are much smaller than the significant ones corresponding to (primary) discrete scatterers. Once the time-domain signals produced using TD-DORT applied to \( T_{yy} = K_{yy}^\dagger K_{yy} \) are transmitted from the TRA, the resulting wave fields converge and interfere constructively around the intended scatterer location. We show snapshots of \( E_y \) at the focal time for homogeneous and random media cases in Fig. 3.9 In addition, a cross-range field pattern defined by

\[
D_{E_y}(x) = \frac{1}{N_2 - N_1} \sum_{i=N_1}^{N_2} E_y^2(x, y_s = 100 \Delta_s, z = i \Delta_s) \tag{3.34}
\]

is shown for homogeneous and random media for various variances and fixed correlation length. Here, \( N_1 = 160 \) and \( N_2 = 220 \), so that whole focusing spot is included.

For the range of variances considered, the field produces a larger peak in the focal spot for increasing variances, as shown in Fig. 3.9. This can be attributed to an increase in the frequency decorrelation among the different frequency components of the UWB field due to multiple scattering and to the phase conjugation enforced by TR. This produces a coherent summation (over the entire bandwidth) of different frequency components of the field only nearby the focal point, and an incoherent
Figure 3.8: Three most significant singular values of the MDMs $K_{ij}$. The singular values are obtained in lossless random media with increasing variance and fixed correlation lengths. A single PEC scatterer is embedded within the media.
Figure 3.9: Spatial distribution of the focused $y$-component of the electric field ($E_y$) at $xz$-plane ($y = 100\Delta_s$) and cross-range patterns $D_{E_y}(x)$ at the time of first minimum focusing. The TRA antennas are fed with the time-domain signals generated by the highest eigenvalue and corresponding eigenvector of the TRO obtained using $K_{yy}$. The scatterer location is denoted by a small circle.
summation elsewhere [89, 33, 113]. Moreover, when they are compared in a normalized fashion, the focal spot becomes slightly narrower for larger variances, as also shown in Fig. 3.9. This latter phenomena is caused by spatial decorrelation due to increasing multiple scattering. Equivalently, this produces an increase on the effective aperture of the TRA, characterizing superresolution [33]. This directivity gain (over the homogeneous case) for the pattern defined in Eq. 3.34 is equal to 1.21 for \( \delta = 1.0 \).

**Effect of Second-Order Statistics**

Additional simulations are performed to evaluate the effect of different correlation lengths on the singular value distribution and subsequent focusing. Here, the correlation length of the random medium with fixed variance of \( \delta = 0.5 \) is progressively increased from \( l_s = 3.2 \text{ cm (}4\Delta_s\text{)} \) to \( l_s = 11.2 \text{ cm (}14\Delta_s\text{)} \) where \( l_s = l_z \). Significant singular values for different TRA combinations are shown in Fig. 3.10. Generally speaking, it is observed that larger correlation length yield smaller singular values. This can be explained from the fact that media with larger correlation lengths produce less multiple scattering (in this range of correlation lengths).

### 3.3.3 Multiple Well-Resolved Scatterers in Random Media

In this case, two PEC spheres with radii \( r_1 = 3.99 \text{ cm} \) and \( r_2 = 3.18 \text{ cm} \) are centered at \((x_{s_1}, y_{s_1}, z_{s_1}) = (50, 100, 160)\Delta_s\) (first scatterer) and \((x_{s_2}, y_{s_2}, z_{s_2}) = (150, 100, 180)\Delta_s\) (second scatterer), respectively. This can be considered as a well-resolved configuration where the contribution from multiple scattering *between* the scatterers is much lower than first-order scattered signals. If multiple scattering between the scatterers were not negligible, then the significant eigenvalues would not
Figure 3.10: Significant singular values of $K_{ij}$ obtained in various random media with different correlation lengths ($l_s = l_z$) and fixed variance. A single spherical PEC scatterer is present.

correspond to individual scatterers but to some linear combination. A possible remedy to reduce multiple scattering between the discrete embedded scatterers in that case would be to form the MDMs after applying iterative time-reversal a certain number of times. As mentioned elsewhere [23], iterative TR works as a power SVD method and therefore, the highest eigenvalue dominates as the number of iterations
goes to infinity. We also note that techniques that can naturally incorporate multiple scattering between targets (in the time-harmonic regime) have been presented in \[50, 59, 114\] and references therein.

The TRO singular values for different TRA polarization combinations are shown in Fig. 3.11, for both homogeneous and random media. It can be observed that in

![Graphs showing TRO singular values for different media](image)

(a) Homogeneous medium

(b) Random media with $\delta = 0.5, l_s = l_z = 8\Delta_s$

Figure 3.11: First four singular values of $K_{ij}(\omega)$ obtained in homogeneous and random media with two embedded spherical PEC scatterers.

both media two significant singular values can be distinguished for $K_{yy}$. For the
other cases, there are no such clear amplitude separation of the first two singular values with respect to the others, and all four singular values are within a smaller degree of (successive) amplitude separation. Analysis of the phase distributions of the eigenvectors at the whole bandwidth shows that, for the $K_{yy}$ case, the first and second eigenvalues are associated with the first and second scatterer, respectively. On the other hand, two significant singular values exist for each of the individual scatterer in the other cases. Using the first two dominant eigenvalues and corresponding eigenvectors of $T_{yy}$, the time-domain signals to be transmitted by the TRA antennas are shown in left column of Fig. 3.12. Once these signals are transmitted by the TRA, they propagate and selectively focus around the associated original scatterer locations, as shown on the right column of Fig. 3.12. The standard TR result is also shown on the last row, where the rightmost antenna (7th from left) is used for initial excitation. In this latter case, received signals at the TRA are simply time-reversed and backpropagated without any additional pre-processing. Since the signatures of both scatterers are included in these backpropagated signals, focusing occurs on both scatterers (and more strongly on the dominant scatterer). As noted before, standard TR can achieve selective focusing by time-gating. However, this is limited to scenarios where the signatures of each scatterer are resolved in time. Additionally, it should be pointed out that time-gating suppresses some multipath contributions and thus can reduce super-resolution. Finally, it is observed that changes on the variance or correlation length produce effects similar to the single scatterer case as in Sec. 3.3.2, and are not shown here.
Figure 3.12: Left column: Time-domain signals used in the backpropagation step from the TRA. Right Column: Corresponding spatial distribution of the $y$-component of the electric field ($E_y$) at $xz$-plane ($y = 100\Delta s$) at the time of focusing. Time-domain signals are obtained using $K_{yy}$ for homogeneous medium and shown here normalized to unity. The scatterer locations are indicated by small circles.
3.3.4 Fully Polarimetric DORT

Polarimetric DORT operation can exploit decorrelation between the different polarizations to extract more information than available with a single polarization. Here, we investigate the cross-range patterns of the transverse electric field $D_{E_x}(x)$ and $D_{E_y}(x)$, for a single PEC sphere with radius $r = 3.99$ cm and centered off the TRA plane at $(x_s, y_s, z_s) = (110, 30, 110)\Delta_s$ (in such offset case, depolarization effects are more pronounced than in the prior symmetric setup). First, we employ a $y$-directed TRA to backpropagate the time-domain signals generated by the first eigenvector associated with $K_{yy}$, and observe the resulting cross-range patterns, $D_{E_x}(x)$ and $D_{E_y}(x)$ around the scatterer location on $xz$-plane at $y = 30\Delta_s$. We then repeat this procedure for $K_{yx}$. Similarly, we use $x$-directed TRA to backpropagate signals generated using $K_{xx}$ and $K_{xy}$, separately. The focused $E_y$ field components due to $K_{yy}$ and $K_{yx}$ are shown at the focal time for the homogeneous medium case in Fig. 3.13. It is observed that while (peak) signal focusing occurs at the scatterer location for $K_{yy}$, null-focusing is observed for $K_{yx}$. The resulting cross-range patterns for different TRA configurations are shown in Fig. 3.14 for homogeneous and a random medium realization, where $D_{E_t} = D_{E_x} + D_{E_y}$. As it can be seen, the various MDMs and TRA polarizations can create distinct focusing behavior, which can be exploited in target discrimination via polarimetric DORT. Likewise, the resulting DORT focusing can also be tailored by the use of co-pol and cross-pol components together. These observation are valid for both homogeneous and random media as long as each distinct TRA combination provides partially uncorrelated data. However, since multiple scattering increases depolarization, the use of full polarimetric DORT is expected to become relatively more effective in random (inhomogeneous) media.
\( (a) E_y(x, y = 30\Delta s, z) \) due to \( K_{yy} \)

\( (b) E_y(x, y = 30\Delta s, z) \) due to \( K_{yx} \)

Figure 3.13: Spatial distribution of \( E_y \) field component at the \( xz \)-plane at \( y = 30\Delta s \). For this configuration, \( K_{yx} \) generates null focusing around the scatterer location. The scatterer location is denoted by a circle.

3.4 Summary and Conclusions

The application of a full time-domain and polarimetric DORT for UWB EM waves has been discussed and investigated in 3-D continuous random media. Linear dipole antenna arrays with different polarizations are used to obtain (limited aspect) multi-static data matrices and converted into the TRO. The number of well-resolved scatterers and their locations can be determined, and selective focusing can be achieved via backpropagation of the fields generated by TRO eigenvectors associated with significant singular values. Time-domain probing signals are generated from TRO eigenvectors by employing the full bandwidth available, instead of only central frequency data.

Effects of first- and second-order (Gaussian) random medium statistics on the TRO eigenvalues and corresponding eigenvectors were considered for MDM associated
Figure 3.14: Cross-range patterns (dB scale) $D_{Ex}$, $D_{Ey}$, and $D_{Et}$ around scatterer location at $xz$-plane and $y = 30\Delta_s$ obtained using different TRA combinations.

with different polarization combinations that provide full polarimetric data. It has been found that decorrelation between polarizations can be exploited in continuous random (inhomogeneous) media. Moreover, multiple scattering effects in the intervening continuous random media increases the effective aperture of the TRA through spatial decorrelation. Increase in multiple scattering can result from an increase on the variance of the background permittivity, or from a decrease on the correlation
length. Increase in multiple scattering also allows for enhanced frequency decorrelation among the different frequencies of the UWB probing signal. This produces enhanced focusing on the target location because of the phase conjugation enforced by TR. At the scatterer location, this phase conjugation leads to a coherent summation over the entire bandwidth, whereas at other locations, the field summation is only incoherent. The sensitivity of the singular values against the additive noise at the receiver has also been briefly discussed and suggested as a possible criterion to distinguish background (volumetric scattering) clutter.

We have restricted ourselves to lossless background media. Compensation methods recently proposed in [44] and [90] can be applied for either standard TR and DORT in lossy and dispersive media (where TR invariance is broken). The incorporation of such compensation techniques into the full time-domain DORT will be the subject of the next Chapter. For well-resolved point-like scatterers, fields created at the TRA elements can be considered as the Green’s functions connecting scatterers to the TRA, and also depend on the geometry of setup and scatterers. Therefore, parameters such as number of antennas, spacing and frequency of operation play an important role in the overall performance. Depending on the particular application in mind, these parameters can be optimized to further increase the signal to noise ratio (SNR) and improve selective focusing capabilities of TRA in inhomogeneous, random media.
The invariance of the wave equation under time-reversal (TR) in lossless and stationary media enables optimal refocusing of the time-reversed signals. Applications employing this concept have already been mentioned in the first Chapter of this dissertation. One common property of the previous studies is that most of them have assumed propagation in nondispersive and lossless media. However, dispersive and lossy (conductive) media are often encountered in the nature. For example, soils, rocks, ice and biological tissues exhibit dispersive properties. In such cases, the time domain Maxwell’s equations can be written as

\begin{align}
\nabla \times \vec{E}(\vec{r}, t) &= -\frac{\partial}{\partial t} \vec{B}(\vec{r}, t) \quad (4.1) \\
\nabla \times \vec{H}(\vec{r}, t) &= \frac{\partial}{\partial t} \vec{D}(\vec{r}, t) + \sigma(\vec{r}) \vec{E}(\vec{r}, t) \quad (4.2)
\end{align}

where

\begin{align}
\vec{B}(\vec{r}, t) &= \mu(\vec{r}) \vec{H}(\vec{r}, t) \quad (4.3) \\
\vec{D}(\vec{r}, t) &= \epsilon(\vec{r}, t) *_t \vec{E}(\vec{r}, t) \quad (4.4)
\end{align}
In these equations, $\vec{E}$ is the electric field intensity (V/m), $\vec{H}$ is the magnetic field intensity (A/m), and $\vec{D}$ and $\vec{B}$ are the electric and magnetic flux densities, respectively. Additionally, $\sigma(\vec{r})$ is the static medium conductivity, $\mu(\vec{r})$ is the medium permeability (assumed constant over frequency) and $\epsilon(\vec{r}, t) = \mathcal{F}^{-1}(\epsilon(\vec{r}, \omega))$ is the medium permittivity which varies as a function of frequency. Using these expressions yields the following wave equation for dispersive and conductive media

$$\nabla^2 \vec{E}(\vec{r}, t) - \frac{\mu(\vec{r})}{\epsilon(\vec{r}, t)} \frac{\partial^2}{\partial t^2} (\epsilon(\vec{r}, t) \ast_t \vec{E}(\vec{r}, t)) - \frac{\mu(\vec{r}) \sigma(\vec{r})}{\epsilon(\vec{r}, t)} \frac{\partial}{\partial t} \vec{E}(\vec{r}, t) = 0 \quad (4.5)$$

As compared to Eq. 2.1, the TR invariance is broken due to the last term containing the first derivative of the electric field. Therefore, conventional TR operation cannot be directly applied. In the literature, several compensation methods have already been discussed. For example, in [38], a simple amplitude compensation technique for TR signals in a nondispersive but lossy medium has been proposed for the attenuated acoustic signals through the skull. In that work, absorption was assumed uniform over the entire bandwidth and therefore, compensation was performed at once for the entire received signal. Since the attenuation is supposed to affect the signal in a frequency independent fashion (which is a prior approximation for wideband signals), this method does not take the (dispersive) frequency dependent attenuation into account. Later, this approach has been extended to multipath propagation where the absorption from the ocean bottom is considered [115]. Since the ocean bottom is modeled as a waveguide, the arrival from each bottom reflection is treated separately by assuming that each of them can be separated in time and that the absorption for each reflection is uniform over the bandwidth. Therefore, compensation is again applied at once for each of these reflections, hence, it does not fully account the frequency dependent nature of the attenuation. In addition to these works, a
method for compensating losses for the DORT method has also been proposed in [44] where the eigenvectors obtained in lossy media are compensated. This method considers compensation performed only at a single frequency and again does not take the frequency dependent attenuation into account. Another very recent proposal for the compensation of (only) conductive losses is considered in [40] where the authors propose a numerical algorithm by changing the sign of conductivity parameter in the FDTD simulations and thereby creating a nonphysical active-like medium. All these approaches provide partial compensation for the conductive losses (frequency independent) in the medium. However, to our knowledge, compensation of dispersive effects for the time-reversal of UWB signals has not been considered before. Since, in a dispersive medium, the attenuation undergone at each frequency is different and depends on the medium characteristics, a compensation incorporating all these differences should be considered. For example, as shown by the experimental data [116], the effective permittivity and conductivity of wet soils exhibit frequency dependent characteristics at the operating frequency range (50-1000 MHz) of ground penetrating radar (GPR) [117]. In such a dispersive medium, the broadband electromagnetic waves propagates and attenuates in a frequency-dependent manner. Therefore, to have a realistic model for compensation, each frequency component of the received signal should be treated differently. In this Chapter, we propose such a method for the compensation of dispersive effects in order to improve the refocusing of time-reversed EM waves in linear dispersive media where at least the statistical dispersive characteristics are assumed known. Compensation is achieved here by a spatial and frequency dependent filtering approach. We demonstrate that using such an approach,
the strength and resolution of the refocused TR signals can be increased. Additionally, we extend the numerical compensation algorithm of [40] to general dispersive media as well.

4.1 Dispersive Media Models

The causality requirement of the Kramers-Kronig relations [118] dictates that when the medium permittivity varies as a function of frequency, the conductivity also varies with frequency. Two casual models that satisfy the Kramers-Kronig relations are the Lorentz and Debye relaxation models where the permittivity value is complex, i.e., having both a frequency dependent real and imaginary part [119, 120]. The imaginary part can be considered as the frequency dependent loss or conductivity in addition to the static medium conductivity. In our simulations we employ an $M_s$-species Lorentz model for the complex permittivity function which is given by

$$
\epsilon(\bar{r}, \omega) = \epsilon_0 \left[ \epsilon_\infty(\bar{r}) + \chi(\bar{r}, \omega) \right] - \frac{\sigma}{j\omega} = \epsilon_0 \epsilon_\infty(\bar{r}) + \epsilon_0 \left[ \epsilon_s(\bar{r}) - \epsilon_\infty(\bar{r}) \right] \left( \sum_{p=1}^{N} \frac{G_p \omega_p^2}{\omega_p^2 - i2\omega\alpha_p - \omega^2} \right) - \frac{\sigma}{j\omega} \quad (4.6)
$$

where $\chi(\bar{r}, \omega)$ is the medium susceptibility, $\omega_p (= 2\pi f_p)$ is the resonant frequency for the $p^{th}$ species, $\alpha_p$ is the corresponding damping factor, $G_p$’s are constants satisfying $\sum_{p=1}^{M_s} G_p = 1$, $\sigma$ is the static conductivity, $\epsilon_s(\bar{r})$, $\epsilon_\infty(\bar{r})$ are the static and infinite frequency permittivities, respectively. Note that $\epsilon_\infty(\bar{r})$ is a misnomer, and simply represents the limit of $\epsilon(\bar{r}, \omega)$ for larger $\omega$ in the finite range of frequencies considered by the model. For inhomogeneous random dispersive medium, $\epsilon_\infty(\bar{r}) = \epsilon_m + \epsilon_f(\bar{r})$ where $\epsilon_m$ is the average relative infinite frequency permittivity, $\epsilon_f(\bar{r})$ is a zero mean Gaussian random variable and $\epsilon_s(\bar{r})$ has the same distribution as $\epsilon_\infty(\bar{r})$ except having
Table 4.1: Lorentz-Model Parameters of the Puerto Rico Type Clay Loams. (Up to 1 GHz range).

<table>
<thead>
<tr>
<th>Moisture</th>
<th>$\langle \epsilon_s(\bar{r}) \rangle$</th>
<th>$\langle \epsilon_\infty(\bar{r}) \rangle$</th>
<th>$\sigma(mS/m)$</th>
<th>$f_1$ (MHz)</th>
<th>$f_2$ (MHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5%</td>
<td>4.25</td>
<td>3.54</td>
<td>0.397</td>
<td>130.39</td>
<td>330.58</td>
</tr>
<tr>
<td>5.0%</td>
<td>6.55</td>
<td>4.76</td>
<td>1.110</td>
<td>95.39</td>
<td>336.58</td>
</tr>
<tr>
<td>10.0%</td>
<td>9.50</td>
<td>6.67</td>
<td>2.000</td>
<td>90.39</td>
<td>416.58</td>
</tr>
</tbody>
</table>

a different mean $\langle \epsilon_s(\bar{r}) \rangle = \epsilon_m + \Delta \epsilon$ where $\Delta \epsilon > 0$ to guarantee that the medium is passive. Note also that when the homogeneous dispersive medium term is used, it corresponds to constant $\epsilon_s(\bar{r})$ and $\epsilon_\infty(\bar{r})$ over the domain, i.e. $\epsilon_s(\bar{r}) = \epsilon_s$ and $\epsilon_\infty(\bar{r}) = \epsilon_\infty$.

For our simulations purposes, the parameters of the dispersive medium are obtained by curve fitting a two-species Lorentz model to the experimental data reported in [116] and a Debye model as reported in [121] for Puerto Rico type of clay loams. These realistic soil parameters used for the Lorentz model are given in Table 4.1. Additionally, Fig. 4.1 shows the corresponding frequency domain distribution of the complex permittivities obtained at different moisture levels. The damping factors are $\alpha_1 = \sqrt{2}\omega_1$, $\alpha_2 = \sqrt{2}\omega_2$ and $G_1 = 0.75$, $G_2 = 0.25$. Note also that by an appropriate choice of parameters, the Lorentz model reduces to the Debye model. Since the former model is more general [121], it is the one employed in this study.

### 4.2 Forward Simulation Setup

To evaluate the frequency dispersion compensation technique, we simulate EM wave propagation in a dispersive medium using the FDTD method [94] summarized in Appendix B. The FDTD algorithm includes a piecewise-linear recursive
Figure 4.1: Relative dielectric permittivity of Puerto Rico type of clay loams obtained both experimentally and with curve fitting with a two species Lorentz model in different moisture levels.

convolution (PLRC) [122] and perfectly matched layers (PML) [108] to model wave propagation in unbounded and dispersive media [121]. Two different sets of simulations are carried out. The first one is for the application of the compensation technique for *homogeneous* dispersive media. For this case, a two-dimensional domain of $N_x \times N_y = 200 \times 200$ grid points with a uniform space discretization of $\Delta_x = 0.998$ cm and a time step of $\Delta t = 44.19$ ps is employed. A TRA (composed of equally spaced 13 ideal $z$-directed electric dipoles) with an aperture size of $a = 96\Delta_s$, parallel to the $x$-axis is located in this homogeneous dispersive medium along with a cylindrical PEC scatterer of radius $r = 3\Delta_s$ centered at $(x_c, y_c) = (100, 70)\Delta_s$ and two point PEC scatterers at $(x_1, y_1) = (36, 50)\Delta_s$ and $(x_2, y_2) = (164, 60)\Delta_s$. These scatterers act as sources for the signals received by the TRA and also mimic the effect of multipathing in the medium. Note that since these scatterers are PEC,
their reflectivities are independent of frequency. The central TRA antenna located at \((x_s, y_s) = (100, 142)\Delta_s\) transmits a UWB BH pulse centered at \(f_c = 400\) MHz into the medium defined by a two species Lorentz medium with resonant frequencies of \(f_1 = 17.39\) MHz and \(f_2 = 436.58\) MHz. Note that these parameters approximately correspond to a moisture level between 5\% to 10\%. As for the second set, we employ random media distribution with dispersive and conductive properties corresponding to moisture levels ranging from 2.5\% to 10\%. Linear TRA utilizes \(N = 11\) dipoles with \(\Delta_s = 1.0\) cm and a physical domain of \(3.6 \times 3.6\) m\(^2\). The dipoles are evenly distributed along an aperture length of \(a = 160\Delta_s\) with the first one positioned at \((x_{t1}, y_{t1}) = (100, 35)\Delta_s\). Note that both sets considers only the \(TM_z\) case to guarantee isotropic scattering from point-like scatterers.

After transmission of the UWB pulse from the source dipole(s), the signal propagates through the dispersive medium, is scattered, and then received back by the TRA. The received signals on each array element are then recorded, time reversed, and transmitted back to the medium, where they tend to automatically focus on the scatterer position(s). The same setups are also used for the simulation in a non-dispersive medium to provide a reference solution for comparison where the (real) relative permittivity \((\epsilon_r)\) is chosen equal to the \(\epsilon_{\infty}\) of the corresponding dispersive medium. Note also that numerical dispersion is also present in the FDTD simulations. However, for the parameters considered here, medium dispersion is more dominant compared to numerical dispersion. Hence, we ignore the latter in what follows.
4.3 Compensation of Dispersive Attenuation

In general terms, a dispersive medium acts as a filter for signals propagating in it. For such a medium, the electric field at a single frequency is given as follows

\[
\tilde{E}(\bar{r}, \omega) = \tilde{F}(\bar{r}) exp \left( -j\omega \sqrt{\mu \epsilon(\omega)} \bar{r} \right)
\]

(4.7)

where \( \tilde{F}(\bar{r}) \) is the frequency independent amplitude coefficient and permittivity function \( \epsilon(\omega) \) is assumed to be invariant over the spatial domain \( (\epsilon(\bar{r}, \omega) = \epsilon(\omega)) \). The non-zero imaginary part of \( \epsilon(\omega) \) yields additional attenuation on the received signals by the TRA relative to the non-dispersive case as shown in Fig. 4.2. Moreover, the real part of the permittivity function causes an additional phase shift (delay) as well. Since the TR signals are phase conjugated coherently along all the bandwidth, any additional phase shift induced from dispersion during the forward propagation is exactly compensated by the TR process during back-propagation. In a dispersive medium however, the signal is attenuated during both forward and back-propagation. As a

Figure 4.2: Magnitude of the exponential term \( exp(-j\omega \sqrt{\mu \epsilon(\omega)} \bar{r}) \) plotted with respect to frequency and spatial distance for complex \( \epsilon(\omega) \) corresponding to the Puerto Rico soil with moisture level of %2.5. Space and frequency dependent attenuation is observed.
result, refocusing of the time-reversed signals can be significantly degraded relative to the non-dispersive (lossless) case. To overcome this degradation, a compensation technique should be applied to act as an inverse filter with respect to the attenuation. Additionally, dispersive effects are *cumulative* in time, i.e., the more the signal travels in the medium, the more it is attenuated. In other words, dispersive medium acts as a frequency-dependent, cumulative-in-time filter to the propagating signals. Therefore, the inverse filters associated with signal components corresponding to different effective propagation distances (or, equivalently, propagation delays) should be different. This requires the use of *space and frequency dependent* filters. Here, we propose such a space and frequency dependent compensation method for the application of TR techniques in dispersive media. The implementation of the compensation method does not change the basic TR experiment, i.e., forward and backpropagation steps stay the same. But, received and recorded signals after the forward propagation are modified (compensated) and then retransmitted. During the forward propagation of the transmitted short pulse in the dispersive medium, different multipaths go through different dispersive attenuation depending on the total distance (or time) traveled in the medium. In other words, received signals at later times (equivalently at further distances) need stronger compensation than those obtained at earlier times. Ideally, different filters should be applied for each particular instant, but this is impractical. Instead, it is more convenient to time-window the received signals and apply different frequency-dependent filters to each windowed signal. Note that each time-window utilized here corresponds to the signals received from a certain range of spatial distances (spatial window). As the time-window is shifted to cover later parts of the time-domain signal, the spatial window also shifts to a further distance. The process
of time-windowing and filtering can be best achieved by applying the Short-Time Fourier transformation (STFT) [123] to the received signals at the TRA. Here, the mathematical development of this process is carried out using the discrete signal notation since we utilize discrete signals in our simulations. However, its extension to continuous time and frequency domain signals is straightforward and not carried here for the sake of brevity.

The process starts by taking the discrete STFT, $X[k, i]$, of the (sampled) received signal at each TRA antenna $x[n]$ which is given as

$$X[k, i] = \sum_n x[n] w_i[n] e^{-j(2\pi/L)nk} \text{ for } i = 1, \ldots, M$$  \hspace{1cm} (4.8)

where $w_i[n]$ is the $i^{th}$ window, $L$ is the FFT length, and $M$ is the total number of windows. Note that $M$ depends on the window length and overlapping factor (between windows). The product $x[n] w_i[n] = s_i[n]$ is referred to as the $i^{th}$ windowed signal. Each windowed signal has traveled different amount of distance to reach the receiver and each of its frequency components has gone through different attenuation.

Therefore, each windowed signal has to be filtered with a different space and frequency dependent filter $H[k, i]$ obtained for the $i^{th}$ window. In the Fourier domain, the filtering is carried out by:

$$Y[k, i] = X[k, i] H[k, i] \text{ for } i = 1, \ldots, M$$  \hspace{1cm} (4.9)

The final, dispersion-compensated signal $x_c[n]$ is then obtained by an inverse STFT:

$$x_c[n] = \sum_{i=1}^{M} \frac{1}{w_i[n]} \sum_k Y[k, i] e^{j(2\pi/L)nk}$$  \hspace{1cm} (4.10)

A block diagram of the procedure is given in Fig. 4.3. Note that in practice (spurious) additional phase shifts can be introduced due to the finite length of the filters.
Figure 4.3: The block diagram of the dispersion compensation method.

However, these shifts do not affect the focusing, because the same filters are applied to all received signals.

Filters play an important role here and to obtain them, the dispersive characteristics of the media should be known (as assumed in this study). One way to obtain these filters is to compare the solution of the wave equation in a homogeneous test medium having the exact dispersive model of interest against a nondispersive test medium, where the relative dielectric permittivity ($\varepsilon_r$) used is equal to the infinite frequency permittivity ($\varepsilon_\infty$) of the dispersive medium. For this end, distance versus time (DvT) plots can be used which are obtained by transmitting a short UWB pulse from a source point and recording it at increasing distances (see Fig. 4.4). At each distance, a broadband time-domain signal is received for both dispersive and nondispersive media. By comparing these received UWB signals at specific distances, it is possible to obtain frequency-dependent filters that correspond to the selected window(s).
Figure 4.4: Distance vs. time plot (DvT) of the propagating pulse in the dispersive and non-dispersive homogeneous test media. Space and frequency dependent attenuation and phase shift in the dispersive case can clearly be observed.
4.3.1 Example

Now, let us illustrate the compensation procedure by means of an example where
the received signal at one of the TRA elements is considered in more detail. The
received signal is windowed using five Hamming windows \([123]\) of length \(256\Delta t\) each
and an overlapping factor of 0.5 as shown in Fig. 4.5. A Hamming window is chosen
due to its nonzero end points to avoid singularities in the inverse STFT operation
(Eq. 4.10). For each windowed signal, a filter is designed as follows: First, for each
central point of window \(w_i[n]\), a corresponding effective distance is found for the
specific dispersive characteristics. The ratio between the frequency domain represen-
tations of the signals received in the dispersive and non-dispersive test media at this
effective distance gives the frequency-dependent attenuation due to dispersion to be
compensated by \(H[k,i]\). This is illustrated in Fig. 4.6. Note that, to avoid noise
contamination, the amplification factors can be smoothly set to unity for frequencies where the spectral density are below acceptable levels (high frequency ends) depending on the particular application. Then, these filters are applied to corresponding windowed signals yielding the filtered windowed signals shown in Fig. 4.7. These filtered windowed signals are then inverse Short-Time Fourier transformed to obtain the final compensated signal which is shown in Fig. 4.8 along with the original signal received in the dispersive medium and a reference signal that would have been received in the non-dispersive case. As seen in these plots, apart from the phase shift (which is automatically compensated by the TR operation), amplitude of the compensated signal is much closer to the reference signal amplitude than that of the original signal. Any discrepancy between the compensated and the reference signal is due to the finite window length used. A better agreement can be obtained with a larger number of
Figure 4.7: Time and Frequency domain representations of the windowed signals and their compensated counterparts after space and frequency dependent filtering.
Figure 4.8: Signal received by one of the TRA antennas in dispersive medium, corresponding compensated signal and the reference signal that would be received in non-dispersive medium in (left) time and (right) frequency domains.

windows, however, trade-off is the increased computational cost. This procedure is applied to all the signals received by each TRA antenna and then, these compensated signals are time-reversed and backpropagated into the dispersive medium. Note that, since backpropagation occurs in the same dispersive medium, additional attenuation is produced on the signals. Therefore, for each windowed signal, two different sets of compensation filters should be applied for each windowed signal. The first one is for the propagation from the source (TRA) to scatterers and from scatterers back to the TRA, and the second one for the propagation from TRA to scatterers. The compensation filters used for backpropagation should be different from those for the initial propagation, since the effective propagation duration for the backpropagation is approximately half of the initial propagation. The second set of filters for each windowed signal can be obtained by using half of the effective distances used for the first set of compensation filters. Once these compensated and time-reversed signals
are backpropagated into the same medium, the refocused fields around the scatterers have closer amplitudes to those obtained in the nondispersive case as will be shown in the next section. However, it should also be mentioned that the compensation filters would also amplify noise inherent in the system and, hence, would not be of use in scenarios where the medium dispersion reduces the signal to noise ratio below acceptable levels.

4.4 DORT in Random, Dispersive Media with Conductive Losses

The previous section has introduced the compensation method that could be used for the TR applications in dispersive media where it was assumed that the static and infinite frequency permittivity distributions ($\epsilon_s$ and $\epsilon_\infty$, respectively) of the dispersive media were homogeneous. In this section, we extend the compensation method for its application in the inhomogeneous random media with dispersive and lossy characteristics. For this end, rather than the standard TR process, we apply the compensation method to the time-domain signals obtained in the TD-DORT method under dispersive and conductive (DPC) media. Although the basic implementation of the DORT method in DPC media is somewhat similar to the nondispersive and lossless (NDL) case, the effects of dispersion and losses have to be considered in order to adequately interpret the eigenvalues and eigenvectors and later correctly use them in the DORT method. We study those effects and apply the compensation technique to improve the DORT method performance in such conditions.
4.4.1 TRO Behavior in DPC and NDL Homogeneous Media

We first compare the TRO obtained for a single cylindrical PEC scatterer with radius \( r = 4.0\Delta_s \) centered at \((x_{s1}, y_{s1}) = (180, 170)\Delta_s\) for both DPC media with moisture level of 2.5% (Table 4.1) and NDL homogeneous \((\delta = 0)\) media. In both cases, we obtain \( K_{zz} \) whose first two singular value distributions are shown in Fig. 4.9 (others are much smaller than these, hence not shown). Dispersion and losses (attenuation) decrease the singular values. As expected, this effect becomes more pronounced at higher frequencies. The ratios of the first and second singular values remain almost the same. This large ratio implies that there is only one significant singular value corresponding to a single scatterer in the medium [25]. Once the eigenvectors at different frequencies along the bandwidth are analyzed, it is observed that despite the decrease on the singular value magnitudes, relative phase distribution among the array elements is preserved. In Fig. 4.10, phase and magnitude distributions of the first
eigenvector versus TRA antennas are shown at several frequencies along the bandwidth (Similar behavior is also observed at other frequencies but not shown here). The scatterer location along the cross-range can be estimated from these distributions [45, 55]. These singular values and eigenvectors are employed in the TD-DORT to generate the signals to be retransmitted from the TRA. The resulting spatial distribution of the $E_z$ field components at the time of focusing are shown in Fig. 4.11 for both DPC and NDL media. In both cases, selective focusing around the original scatterer is observed. However, focusing performance is degraded in the DPC case, as clear from the cross-range profile (note, in particular, the decrease in cross-range resolution). Therefore, compensation techniques become of interest to improve the focusing performance.

Figure 4.10: Phase and magnitude distribution of the first eigenvector for each TRA antenna at several frequencies along the bandwidth. (Solid line: NDL case. Dash-dotted line: DPC case)
Figure 4.11: Spatial distribution of the $E_z$ field component in the $xy$-plane and at cross-range ($y_s = 170\Delta_s$) due to time-domain signals produced by the largest eigenvalue and corresponding eigenvector in both DPC and NDL homogeneous media. The scatterer location is indicated by a small circle.

4.4.2 Dispersion and Loss Compensation for DORT in Dispersive, Inhomogeneous Media with Conductive Losses

Effects of the random medium statistics on the TRO were considered for the NDL case in Chapter 3. It was verified that factors increasing multiple scattering improve the focusing resolution (superresolution). Similar behavior is also observed in DPC case as shown in Fig. 4.12. In this figure, the largest three singular value distributions for particular realizations of two different random media with variances $\delta = 0.005$ and $\delta = 0.01$ and fixed correlation lengths $l_s = 3\Delta_s$ are compared with those of homogeneous medium for the single cylindrical PEC of the previous subsection.

As mentioned in previous sections, in a DPC medium, UWB signals transmitted from each TRA antenna undergo attenuation. When the background medium is homogeneous, this attenuation can be partially compensated using the approach described in Section 4.3. However, in the case of inhomogeneous background media
Figure 4.12: First three singular value distributions of $\mathbf{K}_{zz}$ obtained for a single PEC scatterer in DPC and NDL random backgrounds (Solid line: in NDL media, dashed line: in DPC media)

(as encountered in practice), the attenuation depends on position in general. Unless the characteristics of the medium are known pointwise (which is not a realistic assumption), it is not possible to find exact filters to compensate the losses in this case. The weaker assumption here is that only first- and second- order statistics of the medium are known. In this case, approximate compensation filters can be found as described next.
To simulate the attenuation undergone in the original DPC random medium, multiple realizations of the random medium are employed to obtain an average DvT plot as in the case of homogeneous distribution. For this end, an UWB signal is transmitted from a point source in the test random media and a set of received signals are recorded at increasing distances. For each realization of the random test media, the total electric field at a specific distance is written as

\[
\vec{E}(\vec{r}) = \vec{E}_{\text{inc}}(\vec{r}) + \int d\vec{r}' \vec{G}(\vec{r}, \vec{r}') \cdot o(\vec{r}') \vec{E}(\vec{r}')
\] (4.11)

where \(\vec{r}\) is the distance between the source and observation points, \(\vec{E}_{\text{inc}}(\vec{r})\) is the incident field (invariant for all realizations), \(\vec{G}\) is the Green’s function for an homogeneous medium (which may include a static conductivity) having relative dielectric permittivity \(\epsilon_m\), and \(o(\vec{r}) = \omega^2 \mu_0 (\epsilon(\vec{r}) - \epsilon_m)\) is the contrast function, which depends on the random fluctuations (function of each particular realization). An average DvT plot can be obtained by taking the (ensemble) average of the DvT plots of the different realizations. On the other hand, the ensemble averaged field can be written as

\[
\langle \vec{E}(\vec{r}) \rangle_{N_e} = \vec{E}_{\text{inc}}(\vec{r}) + \int d\vec{r}' \vec{G}(\vec{r}, \vec{r}') \cdot \langle o(\vec{r}') \vec{E}(\vec{r}') \rangle_{N_e}
\] (4.12)

where \(\langle \cdot \rangle_{N_e}\) denotes an ensemble average over \(N_e\) realizations of a random variable, i.e. \(\langle \xi \rangle_{N_e} = N_e^{-1} \sum_{i=1}^{N_e} \xi_i\). For weak fluctuations, we can apply a Born approximation [118] to linearize the dependency of \(\langle \vec{E}(\vec{r}) \rangle_{N_e}\) on \(\langle o(\vec{r}) \rangle_{N_e}\), and write the ensemble averaged electric field as

\[
\langle \vec{E}(\vec{r}) \rangle_{N_e} \approx \vec{E}_{\text{inc}}(\vec{r}) + \int d\vec{r}' \vec{G}(\vec{r}, \vec{r}') \cdot \langle o(\vec{r}') \rangle_{N_e} \vec{E}_{\text{inc}}(\vec{r}')
\] (4.13)

Under this assumption, the ensemble average of the random media properties can be used to approximate the average electric field response. For a sufficiently large
number of realizations, the average response in this approximation approaches that of an homogeneous medium with \( \epsilon_{\text{ens}} = \langle \epsilon_s(\bar{r}) \rangle_{N_e} \) and \( \epsilon_{\infty\text{ens}} = \langle \epsilon_{\infty}(\bar{r}) \rangle_{N_e} \). Therefore, DvT plot of an homogeneous medium with \( \epsilon_{\text{ens}} \) and \( \epsilon_{\infty\text{ens}} \) can be used to obtain approximate time-dependent filters for weak fluctuations. Note that this Born approximation is assumed only in the process of obtaining \textit{approximate} compensation filters, and not for obtaining the TRO itself or for actual backpropagation.

Associated with DvT plots, compensation filters can be constructed as described in Section 4.3 to partially compensate for the losses. This is shown in Fig. 4.13, where the compensation is applied in one of the TRA received signals in a random medium. As shown in Fig. 4.13(a), the received signal from the PEC scatterer is centered at around 20 ns. In the DPC case, the signal amplitude is much weaker than the NDL case. By applying the (time-dependent) compensation, the scatterer signal is

![Figure 4.13: Comparison of the (i.) received signal at one of the TRA antennas (\( k_{(1,4)}(t) \)) in DPC random medium without compensation), (ii.) its compensated version, and (iii.) a reference signal that would have been received if the medium were NDL. Both time and frequency domain representations are shown. The random medium has \( \delta = 0.026 \) and \( l_s = 3\Delta s \).](image)

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considerably amplified. Note that a comparable amplification is not produced on the clutter signal present at earlier and later times. The focusing results are shown in Fig. 4.14 where the spatial distributions at the focal time of the $E_z$ field components in DPC and NDL random media ($\delta = 0.026$ and $l_s = 3.0\Delta_s$) are given for the cases with and without compensation. Similarly, Fig. 4.15(a) shows the corresponding cross-range resolutions. As observed from these figures, the performance can vary for each realization, but compensation yields a better performance than the case without compensation (in terms of closeness to NDL case). A performance study is discussed next.

4.4.3 Compensation Performance

It is important to point out that the approximate compensation filters are not specific to a particular realization, but depend only on the statistics. However, the actual performance of the dispersion compensation depends on several factors, such as the amount of losses (e.g., from the moisture level in the soil), the scatterer location (e.g., depth), and the amount of fluctuations on the complex permittivity (which impacts the validity of Born approximation).

For cases where the propagation distance $d$ is much larger than the correlation length $l_s$, the overall attenuation undergone in random media with different correlation lengths having the same variance approaches to each other. This is due to the fact that for larger distances signals effectively see the same attenuation and this attenuation converges to that observed in a homogeneous medium whose dispersion and conductivity values are equivalent to the corresponding mean values of the considered random media. In other words, for $d/l_s \gg 1$, the attenuation produced by
Figure 4.14: Snapshots of the spatial distribution of $E_z$ field component in the $xy$-plane and at cross-range ($y_s = 170\Delta_s$) in both DPC (with and without compensation) and NDL random media with $\delta = 0.026$ and $l_s = 3.0\Delta_s$. Additionally, result obtained in a modified medium using numerical compensation is also shown (refer to Section 4.5). The scatterer location is indicated by a small circle.
each realization depends only on the first-order statistics of the conductivity distribution. This is illustrated in Fig. 4.16, where the total energy of a UWB pulse as it propagates in media with random conductivity (and constant permittivity) having different $l_{s\sigma}$ and $\delta_{\sigma}$ is computed versus the propagation distance. In this case, $\sigma(\vec{r})$ is a truncated (positive at all points) Gaussian random variable with Gaussian correlation function $C_{\sigma}(\vec{r}_1 - \vec{r}_2) = \delta_{\sigma}\exp(-|\vec{r}_1 - \vec{r}_2|/l_{s\sigma})^2).$ Hence, it has a nonzero mean conductivity ($\langle \sigma \rangle$) varying with $\delta_{\sigma}$. Note that random conductivity characteristic of this homogeneous medium (i.e. constant $\epsilon$) can be considered as a simple model for the random attenuation undergone in an inhomogeneous medium (i.e. random $\epsilon(\vec{r})$) with dispersion and conductivity. For reference, the decay rate in the lossless case ($\sigma(\vec{r}) = 0$) and uniform lossy ($\sigma(\vec{r}) = \langle \sigma \rangle$) cases are also shown in the figure. As observed, the decay rate with respect to the mean conductivity (or $\delta_{\sigma}$) is much more pronounced than that with respect to different $l_{s\sigma}$ (for fixed $\delta_{\sigma}$). Actually,
Figure 4.16: UWB pulse energy in media with homogeneous permittivity ($\epsilon_r = 5.5$) and random conductivity, for various correlation lengths $l_s$ and variances $\delta_\sigma$ (a single realization result is plotted). Two different sets of mean/variance are considered (the means are as indicated). The same inset legend is used for both sets of mean/variance values. The lossless ($\sigma(\bar{r}) = 0$) case is also shown as reference.

this dominant attenuation is the factor considered during the design process of the compensation filters. Therefore, these results suggest that although negligible local variations may exist, compensation filter performance in randomly conductive media approaches that of the homogeneous case with uniform conductivity for propagation distances sufficiently larger than the correlation length.

However, the use of compensation filters in this case poses additional challenge in terms of the validity of the Born approximation (in addition to the overall degree of attenuation). For Born approximation to hold, the second term of right hand side of Eq. 4.12 should be much smaller than the first term. Since UWB signals are used, the higher frequency components should be considered vis-a-vis the condition,

$$k_H L \langle \Delta \epsilon \rangle_{Ne}/\epsilon_m \ll 1 \quad \text{for} \quad k_H L \gg 1$$

(4.14)
where $L$ is the maximum propagation distance considered and $k_H = \omega_H^2 \epsilon_m \mu_0$ is the largest wavenumber of interest [118]. This imposes a stringent condition on the level of fluctuations amenable for compensation if a single realization is used to obtain the filters. Thanks to the ensemble averages, $\langle \Delta_{\epsilon,\sigma} \rangle_{N_e} \to 0$ for $N_e \to \infty$, rendering the Born approximation still adequate for obtaining the filters. However, the filter performance will degrade for increasing fluctuation levels (i.e., increased variance $\delta$, having other parameters fixed). This is shown in Fig. 4.17(a) where the normalized error distribution of each MDM element is plotted for homogeneous and random media with increasing variance. This error measure is defined here as

$$e_I = \frac{\int_\Omega |s_{\text{ref}}(\omega) - s_{\text{comp}}(\omega)| d\omega}{\int_\Omega |s_{\text{ref}}(\omega)| d\omega}$$

where $s_{\text{ref}}$ represents the signal received in the reference NDL media, $s_{\text{comp}}$ is the compensated one in DPC media and $\Omega$ is the available bandwidth. Since the random media used differ only in the variance values and share the same ensemble averaged medium, the same set of filters is used for compensation. As the fluctuations increase, the filters become relatively less effective. Note that, since additive noise present in the system is amplified by the compensation filters, the error between compensated signals and reference signals is naturally expected to increase with increasing signal attenuation. This is illustrated in Fig. 4.17(b) where $e_I$ at different distances and moisture levels (both affecting the overall signal attenuation) is plotted for a random media with $\delta = 0.026$ and $l_s = 3\Delta_s$.

### 4.5 Numerical Compensation

An alternative algorithm that compensates for medium losses has been recently discussed in [40]. In this approach, the compensation is performed using synthetic
Figure 4.17: Normalized error between compensated signals in DPC random media and reference signals in NDL random media for different fluctuations, moisture levels, and scatterer distances.

data from FDTD simulations where the sign of the conductivity term is reversed (active-like medium). This approach is unstable and is limited for small conductivity values because of the ill-posedness of the resulting initial-value problem, and requires point-wise knowledge of the medium conductivity at all spatial points. Despite the disadvantages, this compensation technique can still be extended to Lorentz or Debye dispersive media by modifying the medium susceptibility function $\chi(\omega)$ (Eq. 4.6) accordingly. In the Lorentz case, this corresponds to reversing the sign of the damping factors $\alpha_p$. In Fig. 4.18, the frequency dependence of the relative dielectric permittivity of homogeneous Puerto Rico type soil at moisture level of 2.5% is shown with and without modification.

As long as the original attenuation is small and required backpropagation distance is not large, this method can compensate for the dispersive and conductive effects. However, instability and high sensitivity to numerical noise (ill-posedness)
Figure 4.18: Frequency dependence of the relative dielectric permittivity function $\epsilon(\omega)/\epsilon_0$ for modified and unmodified DPC media.

again exist since the entire media amplifies the signal. We apply this technique for the same dispersive random medium of the previous subsection. The time-domain signals (without compensation) are transmitted into the modified medium and the spatial distribution of the focused $E_z$ field component is given in Fig. 4.14(d). The resulting cross-range distribution is compared with the other results in Fig. 4.15(b). It is observed that modified medium amplifies the signals much more than the reference case which is not realistic. Also, long time instability exists occurs for this simulation.

Finally note that, this compensation can only be applied numerically, i.e. it cannot be implemented physically in practice since it is impossible to change the medium characteristics. On the other hand, the compensation via filters is suitable for practical implementation since only the received signals are modified and the same actual physical medium is used for backpropagation.
4.6 Summary and Conclusions

Time-reversal invariance of Maxwell’s equations is broken in a (lossy) dispersive medium. Although (additional) phase shift produced by dispersion is automatically compensated by the time-reversal operation via phase conjugation, attenuation during both forward and backpropagation occurs. A method based on Short-Time Fourier transform has been proposed here to partially compensate losses due to dispersive attenuation under some assumptions. The method was first developed for homogeneous dispersive medium (except the existence of secondary impenetrable scatterers), where the dispersive characteristics of the medium are assumed known. On the other hand, for cases where background medium is both inhomogeneous and lossy, and only statistical information is available on the intervening medium, it is not possible to exactly compensate for losses. However, partial compensation can be achieved for relatively low fluctuations and losses by extending the current method to employ inverse filters constructed using ensemble averages of the random medium under consideration. A performance study of this method is carried out for different varying parameters. Additionally, we have considered the effects of dispersion and conductive losses on TD-DORT performance using realistic soil types. It has been observed that, although losses result in degradation on the time-reversal operator (TRO) eigenvalue magnitudes, scatterer localization can still be extracted from the phase information that the eigenvectors possess. Using the existing phase information along with attenuation compensation results in a focusing performance closer to that achieved in the nondispersive and lossless case. Finally, an alternative numerical compensation method has been extended to dispersive case as well. Here, only impenetrable, well-resolved point-like scatterers have been considered. Parameters such as number of
antennas, spacing, and frequency of operation also play an important role in the overall performance. Depending on the particular application in mind, these parameters can be optimized to further improve selective focusing capabilities of ultrawideband DORT in inhomogeneous, random media.
CHAPTER 5

SENSITIVITY OF SYNTHETIC TIME-REVERSAL TECHNIQUES TO PARAMETER AND MODEL PERTURBATIONS

As discussed in the previous chapters, TR techniques involve physical or synthetic retransmission of signals acquired by a set of transceivers in a time-reversed fashion, and can be used for a host of applications. For most of the TR-based applications, the analysis of the TR operator (TRO) is fundamental. Particularly, the eigenvalue decomposition (EVD) of the TRO forms the basis for both the so-called DORT [22, 25] (Chapter 3) and TR-based MUSIC (MUltiple SIgnal Classification) [124, 50] methods. These two approaches allow for TR-based imaging via the use of complementary subspaces of the TRO. Specifically, DORT employs the signal subspace (SS) whereas TR-based MUSIC employs the null subspace (NS).

As shown in Chapter 3, for well-resolved point-like scatterers, information on scatterer strength(s) and location(s) are partially encoded by the eigenvalues and associated eigenvectors of the TRO signal subspace [25]. Backpropagation of these eigenvectors (in the same background media) yields target images. However, DORT performance degrades if the well-resolvedness criterion is not met. In this case, the SS eigenvectors become linear combinations of the Green’s function vectors connecting scatterers to the TRA [50]. Backpropagation of such SS eigenvectors creates
overlapped image fields which hamper target imaging and localization. On the other hand, regardless of the well-resolvedness criteria, the TRO null subspace is orthogonal to the TRO signal subspace, i.e. projection of any vector formed by the linear combination of signal subspace eigenvectors onto the null subspace is (ideally) zero. This property is the basis of TR-based MUSIC method which provides better detection and localization properties than DORT even for poorly-resolved scatterers (assuming homogeneous media). TR-based MUSIC was first considered in [50, 58] assuming Born approximation and coincident arrays, and later extended to more general array configurations [51]. Multiple scattering effects have been considered in [60, 59, 61] and ionospheric applications in [114].

In both DORT and TR-based MUSIC for applications involving physical TR (i.e., where actual retransmission of signals occurs in the same physical medium in which the forward propagation was carried out) [5, 36, 62, 89, 91], knowledge of the background Green’s function is not required to obtain the TRO. However, knowledge of the Green’s function is fundamental for imaging purposes (synthetic TR) where retransmission occurs numerically in a synthetic medium [60, 59, 61, 114, 32, 31]. In applications involving inhomogeneous media, the Green’s function is often not known in a deterministic fashion. The drawback is noisy and unreliable images [32, 31].

In Chapter 3, we have analyzed the performance of time-domain DORT method for physical TR of UWB signals in random media. Two recent papers [76, 77] have experimentally studied the effects of changing media on TR source localization. In this chapter, we investigate the performance of some narrowband (NB) and UWB synthetic TR-based imaging methods under different types of perturbations (non-ideal conditions), and we divide the discussions into four main parts. In the first
part, we investigate the robustness of TR-based imaging techniques using DORT and MUSIC algorithms under clutter and (additive) noise assuming that the (inhomogeneous) background Green’s functions is not known in a deterministic fashion. In the second part, we examine the effect of TR-invariance breaking caused by losses in the intervening media on the performance of DORT and TR-based MUSIC. In the third part, the effect of translational perturbations on the location of TRA elements is investigated as a class of model perturbations. This perturbation is then exploited, in a controlled fashion, as a means to estimate second order spatial statistics (such as correlation length) of the probed media using physical TR. Finally, in the fourth part, we consider the effects of restrictions on the array response elements.

The scenarios considered here consist of half-space problems where a linear TRA (limited aspect sensor array) located at the interface is used to probe inhomogeneous ground below it. Ground media are modeled as a continuous random media having spatially fluctuating permittivities with prescribed averages and correlation functions. Perfectly electrical conducting (PEC) objects embedded in the ground are considered as primary scatterers.

5.1 DORT and TR-based MUSIC

The procedure followed for the DORT method has already been discussed in Section 3.1. It was shown that for each frequency $\omega$, an active TRA with $N$ transceivers produces an $N \times N$ symmetric (due to reciprocity) MDM denoted as $K(\omega)$. Then, the TRO is defined as the self-adjoint matrix $T(\omega) = K^\dagger(\omega)K(\omega)$ [91]. A singular value decomposition (SVD) of the MDM gives $K(\omega) = U(\omega)\Lambda(\omega)V^\dagger(\omega)$, where $U(\omega)$ and $V(\omega)$ are unitary matrices and $\Lambda(\omega)$ is the real diagonal matrix of singular
values $\lambda_1(\omega),..,\lambda_N(\omega)$. Similarly, EVD of the TRO yields $T(\omega) = V(\omega)S(\omega)V^\dagger(\omega)$, where $S(\omega) = \Lambda^\dagger(\omega)\Lambda(\omega)$ is the real diagonal matrix of eigenvalues $\mu_1(\omega),..,\mu_N(\omega)$ satisfying $\mu_i(\omega) = \lambda_i^2(\omega)$.

Columns of the unitary matrix $V(\omega)$ ($v_i(\omega), i = 1,..,N$) correspond to normalized and orthogonal eigenvectors of the TRO. Up to this point, the procedure for the TR-MUSIC is exactly the same. The difference comes from the employed eigenvectors of the TRO. Specifically, DORT method utilizes the eigenvectors of the TRO signal subspace which is formed by the eigenvectors with non-zero eigenvalues, i.e.,

$$SS(\omega) = \{v_1(\omega),...,v_{M_s}(\omega)\} \quad \text{with} \quad \lambda_1^2(\omega) > .. > \lambda_{M_s}^2(\omega) > 0 \quad (5.1)$$

where $M_s$ is the number of significant eigenvalues. On the other hand, TR-MUSIC utilizes those of the null subspace which is formed by the remaining eigenvectors having almost zero eigenvalues, i.e.,

$$NS(\omega) = \{v_{M_s+1}(\omega),...,v_N(\omega)\} \quad \text{with} \quad \lambda_{M_s+1}^2(\omega) \approx .. \approx \lambda_N^2(\omega) \approx 0 \quad (5.2)$$

It can be shown [25] that for point-like scatterers and isotropic scattering in homogeneous media, the number of targets $M$ in the probed medium is equal to the number of significant eigenvalues $M_s$ ($M = M_s$) provided that $M \leq N$. This may not hold for non-isotropic scattering where more than one eigenvalue may be associated with a single scatterer [78, 55, 79, 112]. Similarly, with increasing clutter and/or noise, additional TRO eigenvalues may become significant which makes it increasingly difficult to distinguish the clutter/noise contribution from that of discrete scatterers. In this case, a threshold criteria can be set to determine the SS and NS. Note that this threshold may depend on the frequency and clutter/noise level.
Forming the image of the scatterers requires knowledge of the exact background Green’s function vector \( g(\bar{x}_s, \omega) \) \((steering vector)\) at each search point \( \bar{x}_s \) in the probed domain \([114, 32]\). Its conjugate provides the necessary phase and amplitude distribution for the array excitation to focus on the desired point. The steering vector is defined by

\[
g(\bar{x}_s, \omega) = [G(\bar{x}_s, \bar{r}_1, \omega), ..., G(\bar{x}_s, \bar{r}_N, \omega)]^T
\]

(5.3)

where \( G(\bar{r}, \bar{r}', \omega) = |G(\bar{r}, \bar{r}', \omega)|e^{j\phi(\bar{r}, \bar{r}', \omega)} \) is the Green’s function of the problem and \( \bar{r}_i, i = 1, \ldots, N \), represents the \( i^{th} \) antenna location of the TRA. In inhomogeneous media, \( g(\bar{x}_s, \omega) \) is often not known in an deterministic (exact) fashion. As a result, approximate steering vectors should be used instead of \( g(\bar{x}_s, \omega) \). In this work, we assume the average constitutive parameters of the medium to be known and utilize

\[
g_0(\bar{x}_s, \omega) = [G_0(\bar{x}_s, \bar{r}_1, \omega), ..., G_0(\bar{x}_s, \bar{r}_N, \omega)]^T
\]

for backpropagation, where \( G_0 \) is the Green’s function of an homogeneous medium with same mean constitutive parameters of the underlying inhomogeneous (random) medium that produces \( K(\omega) \).

**5.1.1 Central Frequency (CF)-DORT**

The DORT method provides both selective and collective imaging by employing either single or multiple signal subspace eigenvectors, respectively \([78, 55, 44]\). For well-resolved point-like scatterer cases, the signal subspace eigenvectors are related to complex conjugates of the Green’s function vectors connecting the scatterer locations to the TRA via \( \mathbf{v}_p(\omega) = g^*(\bar{x}_p, \omega)/\|g(\bar{x}_p, \omega)\| \), where \( \bar{x}_p \) is the location of the \( p^{th} \) scatterer as illustrated in Fig. 5.1(a). Using these eigenvectors, imaging functionals
Figure 5.1: Illustration of well-resolved and non well-resolved cases for a scenario having $M_s = 2$ scatterers and probed with $N = 3$ antennas. While the signal subspace is formed by the plane $(P)$ formed by the first two eigenvectors ($v_1$ and $v_2$), null subspace is formed by the eigenvector ($v_3$) orthogonal to $P$.

Peaked at $\bar{x}_p$ can be constructed as

$$D_p(\bar{x}_s, \omega) = \langle g_0(\bar{x}_s, \omega), v_p^*(\omega) \rangle = v_p^T(\omega)g_0(\bar{x}_s, \omega)$$

$$= \frac{1}{\|g(\bar{x}_p, \omega)\|} \sum_{n=1}^{N} G^*(\bar{x}_p, \bar{r}_n, \omega)G_0(\bar{x}_s, \bar{r}_n, \omega)$$

$$= \frac{1}{\|g(\bar{x}_p, \omega)\|} \sum_{n=1}^{N} |G^*(\bar{x}_p, \bar{r}_n, \omega)||G_0(\bar{x}_s, \bar{r}_n, \omega)| e^{j\phi_0(\bar{x}_s, \bar{r}_n, \omega) - j\phi(\bar{x}_p, \bar{r}_n, \omega)}$$

for $p = 1, .., M_s$, where $\langle , \rangle$ represents the standard inner product. This functional is the point spread function (PSF) of the TRA [58] that produces selective imaging of the $p^{th}$ scatterer. Most prior works on DORT have employed the central frequency of operation ($\omega_c$) to obtain $D_p$, denoted here as central frequency (CF)-DORT.

### 5.1.2 TD-DORT

When the UWB signals are considered (as discussed in Chapter 3), the time-domain excitation of each array element can be constructed by combining SS eigenvectors in a coherent fashion over the entire bandwidth of operation via $r_p(t) =$...
\( \mathcal{F}^{-1} \left( K^*(\omega) u_p(\omega) \right) = \mathcal{F}^{-1} \left( \lambda_p(\omega) v_p(\omega) \right) \), where \( \mathcal{F}^{-1} \) stands for the inverse Fourier transform. Once \( r_p(t) \) is (synthetically) transmitted back to the (approximate) background medium, the following imaging functional is obtained:

\[
D_p^\Omega(\bar{x}_s) = \langle g_0(\bar{x}_s, t), r_p(t) \rangle \big|_{t=0} = \sum_{i=1}^{N} r_p^i(t) * g_0(\bar{x}_s, \bar{r}_i, t) \big|_{t=0}
= \int_{\Omega} \lambda_p(\omega) v_p^T(\omega) g_0(\bar{x}_s, \omega) d\omega \quad (5.5)
\]

where \( r_p^i \) and \( v_p^i \) are the i-th element of \( r_p \) and \( v_p \), respectively, \( \Omega \) is the bandwidth of operation. Note also that the above formulation assumes that no numerical phase problem exists. This is denoted as the full time-domain (TD)-DORT \([91]\) for imaging.

### 5.1.3 CF-MUSIC

In homogeneous media, the lateral size of the main lobe of the PSF at a single frequency is related to the classical diffraction limit \([1]\), which is proportional to the wavelength and propagation distance and inversely proportional to the aperture size of the TRA (Eq. 2.6). In inhomogeneous media, physical TR can beat this limit due to an increase on the effective aperture length caused by multipath \([35]\). However, as the well-resolvedness criterion for the scatterers is weakened, signal subspace eigenvectors become linear combinations of the Green’s function vectors connecting the scatterers to the TRA (Fig. 5.1(b)). As a result, imaging using these eigenvectors creates wavefield interferences that degrades the scatterer location estimates. However, even for closely-spaced scatterers, null subspace is still orthogonal to the signal subspace. Therefore, inner products of the steering vectors with the null subspace eigenvectors would vanish only at the scatterer location(s) which can provide a good imaging functional. This forms the basis for the TR-based MUSIC \([50, 58, 114, 60]\), where the

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The following imaging functional (pseudospectrum) is employed:

\[
M(\bar{x}_s, \omega) = \left( \sum_{i=M_s+1}^{N} | \langle g_0(\bar{x}_s, \omega), v^*_i(\omega) \rangle | \right)^{-1}
\]  

(5.6)

The resulting TR imaging scheme using a single (central) frequency (CF) \( \omega_c \) as above is referred to as CF-TR-MUSIC, or simply CF-MUSIC in what follows. As shown previously, TR-based MUSIC resolution is superior than DORT in homogeneous media \([50, 58]\). However, TR-based MUSIC requires a non-trivial NS, i.e., \( M \leq N \).

### 5.1.4 UWB-MUSIC

As mentioned before, when it is not possible to obtain the exact background steering vectors in inhomogeneous media, approximate steering vector \( g_0 \) is employed instead of \( g \) in (5.4) and (5.6). However, \( g_0 \) does not cancel out exactly the phase of the SS eigenvectors \( v_p (= g^*/\|g\|) \) around the scatterer locations as \( g \) would (see the phase factor in (5.4)). Similarly, the inner product of \( g_0 \) with the NS eigenvectors may not necessarily produce a minimum (ideally zero) at the original scatterer locations. In addition, since \( g \) has phase factors that depend on the (random medium) realization, using \( g_0 \) does not yield statistical stability. For that, one can take advantage of frequency decorrelation. This is achieved by wideband operation and combining images obtained at different frequencies via the following frequency averaging \([50]\)

\[
M_\Omega(\bar{x}_s) = \left( \int_{\Omega} \sum_{i=M_s(\omega)+1}^{N} | \langle g_0(\bar{x}_s, \omega), v^*_i(\omega) \rangle | d\omega \right)^{-1}
\]  

(5.7)

where \( \Omega \) is the bandwidth of operation. We might call this strategy as UWB-TR-MUSIC, but since the TR context is clear here, we will denote it simply as UWB-MUSIC in what follows.
5.2 Clutter and Additive Noise

In this section, we investigate the aforementioned TR-imaging methods for imaging of a PEC cylinder in (in)homogeneous media. For this end, we employ 2D FDTD simulations with a grid of \( N_x \times N_y = 200 \times 220 \) cells. We assume random media with spatially fluctuating and statistically homogeneous permittivity \( \epsilon(\vec{r}) = \epsilon_m + \epsilon_f(\vec{r}) \) where \( \epsilon_m = 2.908 \) (dry sand) [100], unless otherwise noted. To better isolate the effects of perturbations, we consider only 2D scenarios involving point-like scatterers and restrict to isotropic scattering by utilizing TM\(_z\) polarization. This purposefully excludes finite scatterer size effects, as well as non-isotropic scattering and depolarization effects that might occur in 3D. The linear TRA consists of \( N = 7 \) dipole transceivers which are initially fed by a Blackmann-Harris pulse derivative, centered at \( f_c = 400 \) MHz. A uniform discretization cell size of \( \Delta_{x,y,z} = \Delta_s = 1.37 \) cm is chosen for the FDTD grid. This corresponds to about \( \lambda_c/32 \) for the mean permittivity of the random media. The TRA lies parallel to the \( x\)-direction and it is located just above the random media at \( y = 0 \) m. Each dipole is separated by \( \lambda_c/2 \) to reduce mutual coupling, which are not included. The central dipole located at the origin, i.e. \( \vec{r}_4 = (0,0)\Delta_s \). Throughout this work, we assume \( N > M = M_s \).

The buried PEC cylinder has a radius of \( r = 4.1 \) cm and it is embedded at 1.71 m below the central antenna in random media with different variance (\( \delta \)) and correlation lengths (\( l_s \)). For the range of frequencies considered, this is a point-like scatterer, as it can also be seen from Fig. 5.2, where only a single dominant eigenvalue is associated with the scatterer. Further reducing the scatterer radius would only affect the eigenvalue magnitude but not the relative phase differences among the eigenvector.
components. As a result, imaging performance would not be affected, except for an increased signal-to-noise ratio (SNR) and signal-to-clutter ratio (SCR).

Clutter is produced by the background medium inhomogeneities (e.g., permittivity changes caused varying water content and/or different material formations). Compared to the additive system noise, clutter is multiplicative in nature and can have signatures similar to those of the targets depending on the spatial spectrum of
the background medium inhomogeneities. The SCR (or alternatively, the clutter-to-signal ratio CSR) is used to quantify the amount of clutter received by all MDM elements and defined here as

$$\text{SCR} = \frac{1}{\text{CSR}} = \left( \frac{\sum_{l,m=1}^{N} \int_{\Omega} |E_{lm}^R(\omega)|^2 d\omega}{\sum_{l,m=1}^{N} \int_{\Omega} |E_{lm}^H(\omega) - E_{lm}^R(\omega)|^2 d\omega} \right)^{1/2}, \quad (5.8)$$

where $E_{lm}^R(\omega)$ and $E_{lm}^H(\omega)$ are the scattered fields (elements of $K(\omega)$ for $l,m = 1,\ldots,N$) received by the $l$-th element due to initial excitation applied at the $m$-th element in random (RM) and homogeneous (HM, no clutter) media cases, respectively. Note that SCR depends on the scatterer dimension and distance from the TRA elements as well as amount of fluctuations of the background media defined by $\delta$ and $l_s$. It was shown in [89] that for the considered range of $\delta$ and $l_s$, increasing $\delta$ at fixed $l_s$ or decreasing $l_s$ at fixed $\delta$ tend to increase SCR (Fig. 5.2). The above definition for SCR (note the square root) is adapted from [45]. The signal-to-noise ratio (SNR) is defined in a standard fashion as

$$\text{SNR} = \frac{\sum_{l,m=1}^{N} \int_{\Omega} |E_{lm}(\omega)|^2 d\omega}{\sum_{l,m=1}^{N} \int_{\Omega} |\eta_{lm}(\omega)|^2 d\omega}, \quad (5.9)$$

where $E_{lm}(\omega)$ is the received scattered field in either homogeneous or random media, $\eta_{lm}(\omega)$ is the (filtered) additive white Gaussian noise (AWGN) forming the matrix $N_{SNR}$, and $\Omega$ is the -30 dB bandwidth of the received signals. For the clutter-only case ($SNR = \infty$), SVD is applied to the MDM obtained either in HM or RM. For noisy cases, the MDM is modified as $\tilde{K} = K + N_{SNR}$.

The effects of both clutter and noise on the singular value distribution along the bandwidth are shown in Fig. 5.2, where all seven singular values and a threshold value (solid line) corresponding to 10% of the maximum singular value are plotted. For the noise-free HM case, there is only one singular value above the threshold
(along the entire bandwidth). However, with increasing clutter due to changes on $\delta$ and/or $l_s$, additional singular values exceed the threshold. This affects the estimate of the number of discrete scatterers (especially at higher frequencies). The difference between the effects of clutter and noise is reflected on the frequency distribution of the singular values. As expected, singular values associated with additive noise assume a more uniform (white) distribution over frequency, while singular values associated with clutter have a frequency behavior akin to the dominant singular value (associated with the discrete target). Since both SS and NS depend on the choice of threshold, it is prudent to analyze each individual eigenvector behavior before threshold selection. For the remainder of this section, results are presented for a threshold of 10% of the maximum singular value at each frequency.

The images and cross-range distributions obtained from each imaging method applied for noise-free data from HM and RM media are shown in Fig. 5.3 and Fig. 5.4, respectively. In HM, both TR-based MUSIC methods provide good range and cross-range resolutions. TD-DORT provides better range and slightly better cross-range resolutions compared to CF-DORT, but both these resolutions underperform those of TR-based MUSIC. In the RM case, the clutter affects the resulting images as follows: For CF-MUSIC, multiple spurious image peaks appear, which can be misinterpreted as discrete scatterer locations. Moreover, the images vary with RM realization (with same statistical properties) since the imaging functionals utilize $g_0$ and not $g$. Therefore, CF-MUSIC does not provide reliable or statistically stable images. UWB-MUSIC, on the other hand, combines images obtained at different frequencies to yield a statistically stable image. This is produced at the expense of poorer range and cross-range resolutions (blurring) compared to the homogeneous
Figure 5.3: Images obtained in noise-free \((SNR = \infty)\) homogeneous and random media (RM: \(\delta = 0.013\) and \(l_s = 3\Delta s\), SCR=1.12). (All plots are in dB scale and normalized to unity)
Figure 5.4: (a-d) Cross-range resolutions of all the methods for various SCR ratios and -3 dB resolutions for increasing (e) CSR and (f) SNR values. (I) represents the points where TD-DORT fails to work due to inability to form appropriate time-domain signals to focus only on the scatterer. In this case, there are multiple spurious peaks in the TD-DORT images. Similarly, (II) and (III) represent the points where the MUSIC methods fail due to either nonexistent null space or spurious peaks.
case. Note that for both TR-based MUSIC methods, increased clutter (i.e. decreased SCR) yields wider cross-range resolutions and higher noise floors (Fig. 5.4(d)). Also, both TR-based MUSIC fail to work in cases where NS is zero, e.g. when all clutter eigenvalues are above the threshold. On the other hand, both DORT images have similar range and cross-range information even for increasing clutter (Fig. 5.4). In other words, DORT is more robust against clutter and provides more stable images. Sidelobe levels for TD-DORT are below those of CF-DORT in general, but with decreasing SCR, sidelobe levels increase for both methods. With increasing clutter and/or noise, each SS eigenvector (at different frequencies) can be associated with different scattering centers in the domain. Therefore, time-domain signals obtained using such eigenvectors can yield multiple peaks in the image which limits the use of TD-DORT method under strong clutter/noise conditions. Another point to consider is the resolution ability. If the scatterers are spaced closer than the cross-range resolution, individual scatterers cannot be resolved. Therefore, one may conclude that for weak clutter/noise, TR-based MUSIC has stronger resolution capability than DORT. However, as mentioned above, DORT is more robust under increasing clutter and/or noise.

Processing time is another metric to be considered in evaluating the imaging techniques. The total processing time depends on several factors such as the number of search points in the probed domain, number of SS or NS eigenvectors, and number of frequency samples. Assuming the same forward processing for all techniques, the time $t_{PT}$ to obtain the images can roughly be written as

$$t_{PT} = t_{EV} N_F + t_{IP} t_{SV} N_{EV} N_{SV}$$  \hspace{0.5cm} (5.10)
where $t_{EV}$ is the time it takes to calculate the EVD at each frequency, $t_{IP}$ is the time to calculate each inner product, and $t_{SV}$ is the time to calculate the background steering vector at each search point and frequency. Additionally, $N_F$ is the number of frequency samples, $N_{EV}$ is the number of eigenvectors, and $N_{SV} = N_xN_yN_F$ is the number of background steering vectors. The parameters $t_{EV}$, $t_{IP}$ and $t_{SV}$ depend on the processor used and not on the particular imaging technique. Relative differences among the processing times depend on $N_F$ and $N_{EV}$ values which are listed in Table 5.2. The variable $\Omega_s$ in that table denotes the number of frequency samples to represent the available bandwidth. As expected, NB methods take less time as compared to their UWB counterparts. Additionally, as long as the number of SS eigenvectors is less than the number of NS eigenvectors (i.e. $M_s < N - M_s$), the DORT method is faster than the MUSIC method.

### 5.3 Background Media Dispersion and Loss

In many remote sensing scenarios, losses cannot be neglected and TR invariance is broken. This constitutes another source of perturbation to the ideal TR scenario. In this section, we study loss and frequency-dispersion effects on the aforementioned TR-imaging methods. For this purpose, consider a 2D imaging scenario in an heterogeneous

<table>
<thead>
<tr>
<th>$N_F$</th>
<th>CF-DORT</th>
<th>TD-DORT</th>
<th>CF-MUSIC</th>
<th>UWB-MUSIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Omega_s$</td>
<td>$\sum_{i=1}^{\Omega_s} M_s(\omega_i)$</td>
<td>$N - M_s(\omega_c)$</td>
<td>$\sum_{i=1}^{\Omega_s} (N - M_s(\omega_i))$</td>
</tr>
</tbody>
</table>

Table 5.1: Number of frequency samples ($N_F$) and eigenvectors ($N_{EV}$)
medium with uniform conductivity $\sigma(\omega)$. In such a medium, the Green’s function between two points $\bar{x}_s$ and $\bar{r}_n$ sufficiently apart has the following approximate behavior

$$G_0(r = |\bar{x}_s - \bar{r}_n|, \omega) \approx \frac{e^{(-jkr - \sigma(\omega)\eta^2 r)}}{\sqrt{r}}$$ (5.11)

where $k = \omega\sqrt{\mu\varepsilon}$ and $\eta = \sqrt{\mu/\varepsilon}$. Once this behavior is incorporated into the components of the signal subspace eigenvectors and background steering vectors (assuming deterministic knowledge of $\sigma$) and substituted into (5.4), we get

$$D_p(\bar{x}_s, \omega) = \sum_{n=1}^{N} \frac{G_0^*(\bar{x}_p, \bar{r}_n, \omega)G_0(\bar{x}_s, \bar{r}_n, \omega)}{\Vert g_0(\bar{x}_p, \omega) \Vert} \approx \frac{1}{\sqrt{\sum_{i=1}^{N} e^{-\sigma(\omega)|\bar{x}_p - \bar{r}_n|}}} \sum_{n=1}^{N} e^{-j\left(|\bar{x}_s - \bar{r}_n| - |\bar{x}_p - \bar{r}_n|\right)} e^{-\sigma(\omega)\eta^2 \left(|\bar{x}_s - \bar{r}_n| + |\bar{x}_p - \bar{r}_n|\right)}$$ (5.12)

When the search point is close to the original scatterer location, i.e. $\bar{x}_s \approx \bar{x}_p$, the above equation reduces to

$$D_p(\bar{x}_p, \omega) \approx \sqrt{\sum_{n=1}^{N} e^{-\sigma(\omega)\eta|\bar{x}_p - \bar{r}_n|}}$$ (5.13)

which shows the dependence of the DORT image on $\sigma$. Assuming that other parameters such as frequency of operation, scatterer depth, and TRA characteristics are kept constant, the general effect of increasing $\sigma$ on the CF-DORT is a degradation on both the (focusing) amplitude and cross-range resolution, as the received signals at the TRA as well as background steering vectors undergo more attenuation. This is illustrated in Fig. 5.5(a), where cross-range resolution of a cylindrical object in homogeneous media for various $\sigma$ is shown.

In order to carry out a similar analysis for the CF-MUSIC method, we employ the projection of the background steering vector onto the TRO null subspace ($P_Ng_0$). When the projection yields null vector, i.e. $P_Ng_0 = 0$, the MUSIC functional peaks.
Figure 5.5: Cross-range resolutions obtained for a cylindrical PEC scatterer (radius 4.12 cm) buried 75 cm below the central TRA antenna of 7 (=N) elements. (a) CF-DORT and (b) CF-MUSIC obtained in HM (with $\epsilon_m = 3.54$) for increasing $\sigma$ values. (c) TD-DORT and (d) UWB-MUSIC in HM with increasing moisture levels creating more dispersion. NDL stands for the ideal nondispersive and lossless case. All plots are given in magnitude. DORT plots are normalized to the maximum value obtained in the lossless cases.

This projection can be written by using either NS or SS eigenvectors as follows

$$P_N g_0(\bar{x}_s) = \sum_{i=M_s+1}^{N} (g_0(\bar{x}_s), v_i^*) v_i^* = g_0(\bar{x}_s) - \sum_{i=1}^{M_s} (g_0(\bar{x}_s), v_i^*) v_i^* \quad (5.14)$$

where the $\omega$ dependence is suppressed for brevity. The SS eigenvectors can be related to the background Green’s function vectors ($v_i = g_0/\|g_0\|$) as long as the $\sigma$ value is not too large so as not to degrade SNR and impede an adequate eigenvector...
decomposition. In such case, (5.14) becomes

\[ P_N g_0(\bar{x}_s) = g_0(\bar{x}_s) - \sum_{i=1}^{M_s} \frac{g_0^\dagger(\bar{x}_i) g_0(\bar{x}_s)}{\|g_0(\bar{x}_i)\|^2} g_0(\bar{x}_i) \]  

(5.15)

where \( \zeta_{\bar{x}_i,\bar{x}_s} \) is given by

\[ \zeta_{\bar{x}_i,\bar{x}_s} = \left( \sum_{n=1}^{N} e^{-jK(|\bar{x}_s-\bar{r}_n|-|\bar{x}_i-\bar{r}_n|)} e^{-\sigma \frac{4}{4}(|\bar{x}_i-\bar{r}_n|+|\bar{x}_s-\bar{r}_n|)} \right) \left( \sum_{n=1}^{N} e^{-\sigma |\bar{x}_i-\bar{r}_n|} \right) \]  

(5.16)

Around the original scatterer locations \( \bar{x}_s \approx \bar{x}_p \) for \( p = 1, \ldots, M_s \), \( \zeta_{\bar{x}_i,\bar{x}_p} \approx 1 \) for \( i = p \) and \( \zeta_{\bar{x}_i,\bar{x}_p} \approx 0 \) for \( i \neq p \) (assuming well-resolved scatterers). Since the dependence of \( \zeta_{\bar{x}_i,\bar{x}_s} \) on \( \sigma \) is weakened around the scatterer location, the projection yields null vector for any \( \sigma \) value. As a result, CF-MUSIC performance appears more stable against increasing \( \sigma \). Again, this is the case as long as the SNR is large enough to allow adequate eigenvector decomposition. If the received signals are below the noise level, the evaluated NS may cease to be orthogonal to the scatterers’ Green’s function vectors. This can be observed when cross-ranges obtained for different \( \sigma \) are compared in Fig. 5.5(b). In this particular example, CF-MUSIC cross-range patterns and noise floors are very similar to each other whereas CF-DORT shows a gradual degradation for conductivity values below \( \sigma = 0.04 \) S/m. However, for \( \sigma > 0.04 \) S/m, CF-MUSIC shows a more abrupt transition to higher noise floors and poorer cross-range resolutions.

These observations can be generalized for TD-DORT and UWB-MUSIC, both of which employ wideband operation. In those cases, it is important to incorporate the fact that the imaginary part of the complex permittivity (generalized conductivity or loss factor) is frequency-dependent in general. For this end, we employ the dispersive media parameters given in Section 4.1 corresponding to different moisture levels. The
general effect of increasing moisture is to increase both the real and imaginary parts of
the complex permittivity. The effects of changes on the real part of the permittivity
are automatically compensated by the TR operation (complex conjugation in the
frequency domain). This is not the case, however, for the imaginary part of the
permittivity. The resulting cross-range resolutions for different moisture levels are
shown in Fig. 5.5(c) and 5.5(d). For increased moisture levels (losses) in the frequency
interval considered, the TD-DORT cross-range degrades whereas the UWB-MUSIC
cross-range remains nearly constant. This suggests that UWB-MUSIC is more robust
than TD-DORT under increasing loss. However, for cases where received signals
from scatterers are not distinguishable due to attenuation, neither method provides
accurate images.

5.4 Translational Perturbations

This section considers the effects of horizontal displacement of the TRA on the
performance of TR techniques. Specifically, we consider time-reversed signals received
at a certain TRA location and backpropagated from a different TRA location. In such
cases, perfect phase cancelation around the original scatterer locations is not observed
since backpropagated signals do not travel back along the original paths. When
imaging is considered in homogeneous (or horizontally layered) background media,
the only resulting effect is a translation of the images. However, in inhomogeneous
background media, spatial translation of the TRA introduces random phase shifts on
the background Green’s function resulting in blurring of the image.

When TRA shifting is performed in a controlled fashion for physical TR in inho-
mogeneous random media, it is possible to extract statistical information from the
media (assuming statistically homogeneous parameters). Consider the scenario shown in Fig. 5.6 where the antenna at \( \bar{r}_i \) is used as source. The signal received at \( \bar{r}_s \) is time-reversed and retransmitted from a shifted position \( \bar{r}_s + \vec{d} \). At a single frequency \( \omega \), the signal received by the antenna at \( \bar{r}_s \) (due to the source at \( \bar{r}_i \)) is given by [118]

\[
\tilde{E}_{\bar{r}_i}(\bar{r}_s) = \tilde{E}_{\bar{r}_i}^{inc}(\bar{r}_s) + \omega^2 \mu \int_{\mathcal{V}_1} d\bar{r}' G_0(\bar{r}_s, \bar{r}') \epsilon_f(\bar{r}') \tilde{E}_{\bar{r}_i}(\bar{r}'),
\]

(5.17)

where \( \mathcal{V}_1 \) is the random medium volume, \( \epsilon_f \) is the permittivity fluctuation as defined in Section III, \( G_0 \) is the (scalar) Green’s function of the homogeneous medium with mean dielectric permittivity \( \epsilon_m \) (under TM\(_z\) excitation), and \( \tilde{E}_{\bar{r}_i}^{inc}(\bar{r}_s) \) is the incident field. Assuming small \( \delta \) (weak fluctuations), we can employ Born approximation and rewrite the above as

\[
\tilde{E}_{\bar{r}_i}(\bar{r}_s) \sim \tilde{E}_{\bar{r}_i}^{inc}(\bar{r}_s) + \omega^2 \mu \int_{\mathcal{V}_1} d\bar{r}' G_0(\bar{r}_s, \bar{r}') \epsilon_f(\bar{r}') \tilde{E}_{\bar{r}_i}(\bar{r}'),
\]

(5.18)

where the second term is the scattering contribution due to the random medium only, \( \tilde{E}_{\bar{r}_i}(\bar{r}_s) \). Since \( \tilde{E}_{\bar{r}_i}^{inc}(\bar{r}') \sim \hat{z} G_0(\bar{r}_i, \bar{r}') \) for a point (line) source in 2D (3D) located at \( \bar{r}_i \), the second term of (5.18) can be written as

\[
\tilde{E}_{\bar{r}_i}(\bar{r}_s) \sim \hat{z} \omega^2 \mu \int_{\mathcal{V}_1} d\bar{r}' G_0(\bar{r}_s, \bar{r}') \epsilon_f(\bar{r}') G_0(\bar{r}_i, \bar{r}').
\]

(5.19)
Similarly, when an antenna at $\vec{r}_s + \vec{d}$ (shifted position) is used as the source, the received scattered field at $\vec{r}_i$ due to random medium fluctuations can be written as

$$E^s_{\vec{r}_s + \vec{d}}(\vec{r}_i) \sim \hat{z} \omega^2 \mu \int_{V_1} d\vec{r}'' G_0(\vec{r}_i, \vec{r}'') \epsilon_f(\vec{r}'') G_0(\vec{r}_s + \vec{d}, \vec{r}'').$$

(5.20)

Then, the received signal at $\vec{r}_i$ due to backpropagation of the time-reversed (i.e., phase conjugated) signal received at $\vec{r}_s$ and transmitted from a shifted position can be written as

$$\hat{E}^s_{\vec{r}_s}^* (\vec{r}_s) \cdot \hat{E}^s_{\vec{r}_s + \vec{d}} (\vec{r}_i) \sim (\omega^2 \mu)^2 \int_{V_1} \int_{V_1} d\vec{r}' d\vec{r}''. G^*_0(\vec{r}_s, \vec{r}') G^*_0(\vec{r}_i, \vec{r}') G_0(\vec{r}_i, \vec{r}'') \epsilon_f(\vec{r}') \epsilon_f(\vec{r}'') G_0(\vec{r}_s + \vec{d}, \vec{r}'') \cdot \delta \exp \left(-\frac{|\vec{r}' - \vec{r}''|^2}{l^2_s}\right).$$

(5.21)

Note that $G_0$ is a deterministic function, while $\epsilon_f(\vec{r})$ is a random variable. We next consider the following statistical average

$$\langle \hat{E}^s_{\vec{r}_s}^* (\vec{r}_s) \cdot \hat{E}^s_{\vec{r}_s + \vec{d}} (\vec{r}_i) \rangle_{N_r} \sim (\omega^2 \mu)^2 \int_{V_1} \int_{V_1} d\vec{r}' d\vec{r}''. G^*_0(\vec{r}_s, \vec{r}') G_0(\vec{r}_s + \vec{d}, \vec{r}'') G^*_0(\vec{r}_i, \vec{r}') G_0(\vec{r}_i, \vec{r}'') \langle \epsilon_f^*(\vec{r}') \epsilon_f(\vec{r}'') \rangle_{N_r},$$

(5.22)

where $N_r$ stands for the number of realizations. For $N_r \to \infty$, $\langle \epsilon_f^*(\vec{r}') \epsilon_f(\vec{r}'') \rangle_{N_r}$ approximates the correlation function of the dielectric permittivity distribution. For a Gaussian correlation, we have

$$\langle \hat{E}^s_{\vec{r}_s}^* (\vec{r}_s) \cdot \hat{E}^s_{\vec{r}_s + \vec{d}} (\vec{r}_i) \rangle_{N_r \to \infty} \sim (\omega^2 \mu)^2 \int_{V_1} \int_{V_1} d\vec{r}' d\vec{r}''. G^*_0(\vec{r}_s, \vec{r}') G_0(\vec{r}_s + \vec{d}, \vec{r}'') G^*_0(\vec{r}_i, \vec{r}') G_0(\vec{r}_i, \vec{r}'') \cdot \delta \exp \left(-\frac{|\vec{r}' - \vec{r}''|^2}{l^2_s}\right).$$

(5.23)

This expression is valid only for low fluctuations regimes where Born approximation (5.18) holds. A similar analysis can be done for 3D problems. The main difference is that depolarization factors would be present in 3D.
In practice, the average in (5.22) can be calculated by a frequency averaging assuming a UWB signal that spans a frequency range many time larger than the decorrelation frequency. For this end, the following procedure is carried out (Fig. 5.7): First, a short pulse is transmitted from a single TRA antenna located at $\vec{R}_p$, $1 \leq p \leq N$. The scattered signals are recorded by all TRA elements, and denoted as $h_{\vec{R}_p}\vec{R}_q(t)$ for $q = 1, ..., N$. These signals are time-reversed and backpropagated from $\vec{R}_q + \vec{d}$. After backpropagation, the received signal at the original source antenna becomes $S_{\vec{R}_p}(t, \vec{d}) = \sum_{i=1}^{N} h_{\vec{R}_p\vec{R}_q}(-t) \ast t h_{(\vec{R}_q+\vec{d})\vec{R}_p}(t)$. If $\vec{d} = 0$, the resulting signal is the standard TR process, with peak at $t = 0$. By observing the correlation at $t = 0$ for different $\vec{d}$, it is possible to obtain a decorrelation rate as follows

$$\Gamma_{\vec{R}_p}(\vec{d}) = \frac{S_{\vec{R}_p}(t = 0, \vec{d})}{S_{\vec{R}_p}(t = 0, 0)} = \frac{\int_{\Omega} S_{\vec{R}_p}(\omega, \vec{d}) d\omega}{\int_{\Omega} S_{\vec{R}_p}(\omega, 0) d\omega}$$

(5.24)

The same can be repeated for all $p$ ($p = 1, ..., N$). Depending on the medium and/or scatterer characteristics, $\Gamma_{\vec{R}_p}(\vec{d})$ can yield information about second order statistics of the probed domain. As an example, Fig. 5.8(a) shows decorrelation rates obtained
from random media with increasing correlation lengths and fixed variances where shifting is carried out in the $x$-direction, i.e. $\hat{d} = \hat{x}d_x$. For small $l_s$, the decorrelation rate is faster because of the increase in the *effective* aperture length of the TRA due to increased multiple scattering. The decorrelation rate is also affected by the nature of correlation function. This is illustrated in Fig. 5.9 where the decorrelation rates obtained in random media with either exponential $C(\vec{r}_1 - \vec{r}_2) = \delta \exp(-|\vec{r}_1 - \vec{r}_2|/l_s)$ or Gaussian correlation functions are shown (the same correlation length $l_s = 8\Delta_s$ is used).

When a discrete scatterer is embedded in the probed medium, the decorrelation rate depends both on the background medium properties and on the discrete scatterer properties. As an example, Fig. 5.8(b) shows the decorrelation rate obtained for a cylindrical PEC scatterer with radius $r = 4.12$ cm buried in homogeneous medium at increasing depths from the TRA. Targets closer to TRA yield faster decorrelation.

Figure 5.8: Decorrelation rates for two different scenarios.
(Figure 5.9: (a) Comparison of Gaussian and Exponential correlation function used during the random medium generation with $l_s = 8\Delta_s$, (b) Decorrelation rate obtained using these random media)

because the TRA cross-range resolution is proportional to the distance between the scatterer and the TRA. For a given shift distance, the field at larger depths have larger cross-range resolutions, and therefore, the relative effect of shifting the TRA is diminished, resulting in a slower decorrelation.

Next, consider the same scatterer buried in random media at a fixed position 90 cm below the central antenna. The decorrelation rates obtained for various $\delta$ and $l_s$ of the random media are shown in Fig. 5.10. For the range of $\delta$ considered, it is observed that increasing $\delta$ results in faster decorrelation. Similarly, faster decorrelations are also obtained by decreasing $l_s$ for fixed $\delta$, as expected. However, as $l_s$ is further increased (e.g. $l_s = 12\Delta_s$), the decorrelation rate converges to that produce by the embedded scatterer in homogeneous media because the $l_s$ scale becomes much larger than the scatterer size.
5.5 Restrictions on Array Response Elements

The performance of the physical TR experiment and the TR-based imaging methods depends on how much information the TRA (or equivalently MDM) possesses about the scattering scenario under study. Therefore, interelement responses between the array antennas play a crucial role for the imaging performance using these methods. However, in many applications, a full MDM is not available. In this section, we will consider two kinds of restrictions: (i.) Only the main diagonal $K_{i,i}(t)$ is available (e.g., in synthetic aperture imaging (SAI)), or (ii.) both the main and one lower diagonal terms (subdiagonal) $K_{i,i}(t)$ and $K_{i,i-1}(t)$ are available (e.g., in a simple interferometric imaging).

5.5.1 Effects on Physical TR experiment

First, we start with the physical TR experiment. Consider that $p^{th}$ antenna of the array located at $\vec{r}_p$ transmits a short UWB pulse $s(t)$ into an unknown medium with $M$ point-like scatterers each located at $\vec{x}_m$ for $m = 1, .., M$. We assume that
scatterers are well-resolved (i.e. no multiple scattering among them) and \( M \leq N \). Transmitted signal propagates through the medium and is reflected by the scatterer(s) and received by the antenna array. Signal received by the antenna at \( \bar{r}_i \) due to \( m^{th} \) scatterer can be written as \( \phi^{m}_{t,p}(t) = \tau_m s(t) \ast_t h_{\bar{x}_m \bar{r}_p}(t) \ast_t h_{\bar{r}_i \bar{x}_m}(t) \) where \( \tau_m \) is the scattering coefficient of the \( m^{th} \) scatterer and \( h_{\bar{y} \bar{x}}(t) \) is the impulse response between the two points \( \bar{x} \) and \( \bar{y} \) (due to reciprocity, \( h_{\bar{y} \bar{x}}(t) = h_{\bar{x} \bar{y}}(t) \)). This process is repeated for each transmitter antenna, \( p = 1, \ldots, N \), and the resulting received signals at each receiver antenna are summed. Then, the time reversed version of these received signals are backpropagated into the same medium. During backpropagation, signals undergo through the same reflection, refraction and scattering that they have undergone during the forward propagation. As a result, focusing around the scatterer location(s) occurs. Mathematically, the resulting focused signal around the original \( m^{th} \) scatterer location can then be given by

\[
F^{m}_{TR-PHY}(t) = \sum_{i=1}^{N} h_{\bar{x}_m \bar{r}_i}(t) \ast_t \sum_{p=1}^{N} \phi^{m}_{i,p}(-t)_{\text{backpropagation}} \sum_{p=1}^{N} \phi^{m}_{i,p}(-t)_{\text{time-reversal}} = \tau_m s(-t) \ast_t \sum_{i=1}^{N} h_{\bar{x}_m \bar{r}_i}(t) \ast_t h_{\bar{r}_i \bar{x}_m}(-t) \ast_t \sum_{p=1}^{N} h_{\bar{x}_m \bar{r}_p}(-t) \quad (5.25)
\]

where \( \beta_{\bar{r}_i \bar{x}_m}(t) \) represents a correlation filter (time-correlator) which has a maximum at \( t = 0 \). This is the result obtained when the full MDM matrix is available. When only the main diagonal of the MDM is accessible, Eq. 5.25 reduces to

\[
F^{m}_{TR-SAR}(t) = \sum_{i=1}^{N} h_{\bar{x}_m \bar{r}_i}(t) \ast_t \sum_{p=i}^{i} \phi^{m}_{i,p}(-t)_{\text{backpropagation}} \sum_{p=i}^{i} \phi^{m}_{i,p}(-t)_{\text{time-reversal}} = \tau_m s(-t) \ast_t \sum_{i=1}^{N} h_{\bar{x}_m \bar{r}_i}(t) \ast_t h_{\bar{r}_i \bar{x}_m}(-t) \ast_t h_{\bar{x}_m \bar{r}_i}(-t) \quad (5.26)
\]
which is called the TR-SAR procedure. Similarly, when both the main diagonal and a lower diagonal of the MDM are available, we obtain the TR-Interferometry and Eq. 5.25 becomes

\[
F_{TR-INT}^m(t) = h_{\bar{x}_m\bar{r}_1}(t) * t \phi_{1,1}^m(-t) + \sum_{i=2}^{N} h_{\bar{x}_m\bar{r}_i}(t) * t \sum_{p=1}^{i} \phi_{i,p}^m(-t) \\
= \tau_m s(-t) * t h_{\bar{x}_m\bar{r}_1}(t) * t h_{\bar{r}_1\bar{x}_m}(-t) * t h_{\bar{x}_m\bar{r}_1}(-t) \\
+ \tau_m s(-t) * t \sum_{i=2}^{N} h_{\bar{x}_m\bar{r}_i}(t) * t h_{\bar{r}_i\bar{x}_m}(-t) * t (h_{\bar{x}_m\bar{r}_i}(-t) + h_{\bar{x}_m\bar{r}_{i-1}}(-t))
\]

Note that both \(F_{TR-SAR}^m(t)\) and \(F_{TR-INT}^m(t)\) have the same correlation filter term \(\beta_{i,\bar{x}_m}(t)\) as in Eq. 5.25. However, while the focused field \(F_{TR-PHY}^m(t)\) exploits all the interelement responses, only the main diagonal terms in \(F_{TR-SAR}^m(t)\) and only the main diagonal and one lower diagonal terms contribute to the focused fields in \(F_{TR-INT}^m(t)\).

In other words, all three methods perform \(N\) simultaneous TR experiments, but while \(F_{TR-PHY}^m(t)\) employs all of the array antennas for each experiment, \(F_{TR-SAR}^m(t)\) uses only a single antenna (different at each experiment) and \(F_{TR-INT}^m(t)\) utilizes only two adjacent antennas of the array (a different couple at each experiment). Finally, TR common source (CS) configuration where a movable receiving antenna is used at different locations to record the incoming signals originating from a fixed source antenna. Assuming the first antenna is used as the source, TR-CS signal around the scatterer location can be written as follows:

\[
F_{TR-CS}^m(t) = \sum_{i=2}^{N} h_{\bar{x}_m\bar{r}_i}(t) * t \phi_{i,1}^m(-t)
\]

Because of the different configuration of antennas, each scheme has a different focusing, range and cross-range data. We demonstrate this difference by applying all these methods for a single cylindrical PEC scatterer with radius \(r = 4.12\) cm and located 1.3 m below the central antenna of an array with \(N = 7\) elements. Fig. 5.11
shows the focused electric field components at the time of focusing for all the methods both in homogeneous and a random medium. Also, a comparison of the cross-range directivity pattern defined by $D_{E_z}(x) = \frac{1}{y_2 - y_1} \int_{y_1}^{y_2} E_z(x, y) dy$ is provided for $y_1 = 0.89$ m and $y_2 = 1.44$ m (roughly the full focusing spot in the range direction). It should be noted that TR-SAR, TR-INT and TR-CS employ synthetic backpropagation to obtain the images shown in the first two rows of the figure. This is due to the fact that physically, these methods use either single or double antennas to transmit and receive whose backpropagation result the images shown in the third row. Since the synthetic backpropagation is employed, the exact background Green’s function is not employed in the images shown. As it can be observed, full MDM case provides the smallest focusing spot and best cross-range information. On the other hand, TR-SAR has the worst performance since it exploits the least amount of spatial decorrelation available. As expected, TR-INT has a performance in between these two methods which can clearly be seen in the amplitudes of the focused fields and directivity patterns (Figs. 5.11(m), 5.11(n)). Note also that the non-symmetry in the TR-INT directivity pattern comes from use of the lower diagonal. One way to overcome this is to take advantage of the reciprocity by using the upper diagonal of the MDM which will be carried in the next section. It is also observed that TR-CS results are better than those of the TR-INT and TR-SAR. This is because of the spatial decorrelation utilized by the TR-CS method as it carries out a single TR experiment with an array of $N - 1$ elements whereas TR-SAR combines $N$ TR experiments utilizing arrays of 1 antenna.
Figure 5.11: Spatial distribution of the focused $E_z$ field and cross-range comparisons in homogeneous (HM) and random media (RM). First two rows use synthetic backpropagation whereas the third row can be achieved physically. For the inhomogeneous case, approximate background Green’s function is employed.
5.5.2 Effects on TR-based Imaging Methods

Now, we continue to investigate the effects of interelement response restrictions on the TR based imaging methods DORT and TR-MUSIC. Both methods employ the eigenvalue and eigenvectors of the MDM obtained at each frequency of the available bandwidth. Therefore, the following mathematical discussion is carried out using a single frequency, but its extension to time-domain is can be found in [91].

First, let us recall Eq. 3.3 where the full MDM $\mathbf{K}(\omega) = [K_{i,j}]$ is written in terms of the steering vectors $\mathbf{g}_s(\vec{r}_m)$ (Fig. 3.2(b)). Here, another $M \times 1$ vector $\mathbf{g}_a(\vec{r}_p, \omega) =$

![Figure 5.12: Illustration of array and sub-steering vectors](image)

$[G(\vec{x}_1, \vec{r}_p, \omega), ..., G(\vec{x}_M, \vec{r}_p, \omega)]^T$ is introduced to represent the connection between the $p^{th}$ array antenna location ($\vec{r}_p$) to all the scatterer locations (Fig. 5.12(a)). Note that for steering vectors to be orthogonal, they should form an orthogonal set or equivalently, the inner product $\langle \mathbf{g}_s(\vec{x}_m), \mathbf{g}_s(\vec{x}_{m'}) \rangle$ should be zero when $m \neq m'$. This is achieved when the scatterers are well-resolved, i.e. when the distance between the scatterers is large enough so that there is no multiple scattering among them.
Similarly, for the array vectors to be orthogonal, antennas should also be well resolved. Therefore, it should be pointed out that when the array elements are separated by only a half wavelength distance (which is the case employed here), orthogonality between the array vectors may not be satisfied.

In the TR-SAR scheme, only the diagonal terms of the full MDM are available. Therefore, MDM for TR-SAR is given as follows:

\[
K_{SAR}(\omega) = \text{diag}\{K(\omega)\}
\]

\[
= \sum_{m=1}^{M} \tau_m(\omega) \begin{pmatrix}
G(\bar{r}_1, \bar{x}_m) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & G(\bar{r}_N, \bar{x}_m)
\end{pmatrix}
\begin{pmatrix}
G(\bar{r}_1, \bar{x}_m) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & G(\bar{r}_N, \bar{x}_m)
\end{pmatrix}
\]

\[
= \begin{pmatrix}
g^T_a(\bar{r}_1, \omega) \varpi(\omega) & g^T_a(\bar{r}_1, \omega) & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & g^T_a(\bar{r}_N, \omega) \varpi(\omega) & g^T_a(\bar{r}_N, \omega)
\end{pmatrix}
\]

(5.29)

where \(\varpi\) is an \(M \times M\) diagonal matrix whose entries are the scattering coefficients, i.e. \([\varpi]_{i,i} = \tau_i\). Since all the diagonal elements of this matrix are nonzero, the matrix is nonsingular and has a full rank. Therefore, it does not have a null space and its eigenvectors are the unit vectors \(e_i = [\delta_{i,1}, \ldots, \delta_{i,i}, \ldots, \delta_{i,N}]^T\) where \(\delta_{i,j}\) is the Kronecker delta and its corresponding eigenvalues are equal to \(g^T_a(\bar{r}_i, \omega) \varpi(\omega) g_a(\bar{r}_i, \omega)\) for \(i = 1, \ldots, N\). Note that each eigenvalue has contributions from all of the \(M\) scatterer(s), in other words, eigenvalue decomposition applied to this matrix cannot resolve the scatterers, i.e. selective focusing is not possible. When these eigenvectors are backpropagated as in the case of the DORT method, only a single antenna would be excited with a signal proportional to the summation of all the backscattered fields from the scatterers. However, the resulting image would not yield any cross-range information since the spatial decorrelation cannot be exploited (Fig. 5.13(b)). However, by employing all the eigenvectors and the corresponding eigenvalues simultaneously, we can exploit
somewhat better spatial decorrelation since all the antennas are excited at the same time. This can be achieved by backpropagating the following vector which is obtained by vectorizing the $K_{SAR}(\omega)$ matrix as follows

$$k_{SAR}(\omega) = \begin{pmatrix} K_{1,1}^*(\omega) \\ \vdots \\ K_{N,N}^*(\omega) \end{pmatrix} = \sum_{i=m}^{M} \tau_m \begin{pmatrix} G^*(\bar{r}_1, \bar{x}_m) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & G^*(\bar{r}_N, \bar{x}_m) \end{pmatrix} \begin{pmatrix} g_s^*(\bar{x}_m, \omega) \\ \vdots \\ \vdots \end{pmatrix}$$

Note that, since all the individual eigenvectors are added together to obtain the $N \times 1$ vector $k_{SAR}(\omega)$, it has contributions simultaneously from all the scatterers. Therefore, its backpropagation would yield focusing on all the scatterer locations and thus selective focusing is not possible. The results obtained in time domain using this vector and one of the unit eigenvectors are compared with that of the full MDM case in Fig 5.13 for two PEC scatterers symmetrically located below the central antenna of the array. As it can be observed, single unit vector does not provide any cross-range information and cannot resolve the scatterers. On the other hand, when $k_{SAR}(\omega)$ is

![Figure 5.13: Spatial distribution of the focused $E_z$ field obtained by backpropagating a single eigenvector of $K_{SAR}$ and $k_{SAR}(t)$ compared with full MDM case.](image-url)
used, somewhat better cross range is obtained, however, it is still not optimal as in
the full MDM case.

Another point to note is the application of TR-MUSIC method. Since there is no
null space in this case to be employed, TR-MUSIC cannot be applied as it is. One
can think that if an orthogonal vector $k_{SAR}^\perp(\omega)$ to $k_{SAR}(\omega)$ can be found, TR-MUSIC
can also be applied by exploiting this orthogonal vector instead of $v_i(\omega)$. Such a
vector can be found by the TR nulling algorithm [65]. However, being orthogonal to
$k_{SAR}(\omega)$ does not guarantee orthogonality to the individual steering vectors $g_s(\bar{x}_s, \omega)$
of the scatterer locations. Therefore, TR-MUSIC pseudospectrum of Eq. 5.6 would
not have peaks around the original scatterer locations.

Similarly, when the TR-Interferometry is considered, the available MDM is a
tridiagonal matrix given as:

$$K_{INT}(\omega) = \text{diag}\{K(\omega)\} + \text{lowerdiag}\{K(\omega)\} + \text{upperdiag}\{K(\omega)\}$$

$$= \sum_{m=1}^{M} \tau_m(\omega) \begin{pmatrix}
G(\bar{r}_1, \bar{x}_m) & G(\bar{r}_1, \bar{x}_m) & 0 & \cdots & 0 \\
G(\bar{r}_2, \bar{x}_m) & G(\bar{r}_2, \bar{x}_m) & G(\bar{r}_2, \bar{x}_m) & \ddots & \vdots \\
0 & G(\bar{r}_3, \bar{x}_m) & G(\bar{r}_3, \bar{x}_m) & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & G(\bar{r}_{N-1}, \bar{x}_m) & G(\bar{r}_{N-1}, \bar{x}_m)
\end{pmatrix}
\times
\begin{pmatrix}
G(\bar{x}_m, \bar{r}_1) & 0 & \cdots & 0 \\
0 & G(\bar{x}_m, \bar{r}_2) & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & G(\bar{x}_m, \bar{r}_N)
\end{pmatrix}
\times
\begin{pmatrix}
g_a^T(\bar{r}_1) & g_a(\bar{r}_1) & g_a^T(\bar{r}_1) & g_a(\bar{r}_1) & \cdots & 0 \\
g_a^T(\bar{r}_2) & g_a(\bar{r}_2) & g_a^T(\bar{r}_2) & g_a(\bar{r}_2) & \cdots & 0 \\
0 & g_a^T(\bar{r}_3) & g_a(\bar{r}_3) & g_a^T(\bar{r}_3) & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & g_a^T(\bar{r}_{N-1}) & g_a(\bar{r}_{N-1}) & g_a^T(\bar{r}_{N-1}) & g_a(\bar{r}_{N-1})
\end{pmatrix}
$$

(5.31)
Note that, upper diagonal of $\mathbf{K}(\omega)$ is also included in $\mathbf{K}_{INT}(\omega)$ since $K_{i,j-1}(\omega) = K_{i-1,i}(\omega)$ thanks to the reciprocity (which makes $\mathbf{K}_{INT}(\omega)$ symmetric). Since all the columns of this MDM are linearly independent from each other, time-reversal operator $\mathbf{K}_{INT}^H \mathbf{K}_{INT}$ has a full rank, i.e. all eigenvalues are significant. Using these column vectors, one can obtain an orthogonal set of eigenvectors (e.g. by the Gramm-Schmidt orthogonalization method). However, obtained eigenvectors are not necessarily orthogonal to the steering vectors $\mathbf{g}_s(\bar{x}_s, \omega)$ connecting the scatterers to the array. Therefore, using such eigenvectors in the DORT or TR-MUSIC scheme does not guarantee maxima or minima focusing around the original scatterer locations, but at other locations determined by the chosen vectors and their resulting orthogonalization. Additionally, it is rather difficult to interpret these eigenvectors since they are not proportional to the steering vectors $\mathbf{g}_s(\bar{x}_s, \omega)$ (or to their sub-vectors) of the scatterer locations.

However, submatrices of $\mathbf{K}_{INT}(\omega)$ allows the use of easy-to-interpret eigenvectors and eigenvalues. Recalling the special case that for an array with $N = 2$ elements, $\mathbf{K}_{INT}(\omega) = \mathbf{K}(\omega)$, we can form $2 \times 2$ submatrices $\mathbf{K}_{sub}(\omega)$ from the main, sub- and super-diagonals of $\mathbf{K}_{INT}(\omega)$ as follows:

$$
\mathbf{K}_{sub}^i(\omega) = \begin{pmatrix}
K_{i,i}(\omega) & K_{i,i+1}(\omega)
\end{pmatrix}
\begin{pmatrix}
K_{i,i}(\omega) & K_{i,i+1}(\omega)
\end{pmatrix} 
\text{ for } i = 1, \ldots, N-1 \quad (5.32)
$$

Assuming $M \leq 2$, each submatrix yields eigenvector(s) that connects the scatterer location(s) to the related antenna locations as shown in Fig. 5.12(b). By combining each sub-eigenvector (or sub-steering vector), full steering vectors for the scatterers connecting them to the whole array antennas can be obtained (Fig. 5.12(c)). By employing these eigenvectors in the DORT and TR-MUSIC methods and adding the images obtained for each case, we can have focusing around the scatterer location.
This is shown in Fig. 5.14 where both the DORT and TR-MUSIC images obtained using each of these submatrices are shown for a single scatterer located below the central antenna of the array. The resulting DORT and TR-MUSIC images obtained from the summation of the submatrix images are also compared with those of the full MDM case in Fig. 5.15. Note that these images correspond to those obtained in the central frequency of operation. For this specific example, there is only one scatterer in the domain. Therefore, while one of the eigenvectors of each submatrix belongs to the signal space of that submatrix, the other one is in its null space. This allows us to have both the DORT and TR-MUSIC images using $K_{INT}(\omega)$. However, when there are more than one well-resolved scatterers in the domain, TR-MUSIC images using the submatrices cannot be obtained since there is no null space left in the submatrices.
We can still have DORT images, but when $M > 2$, we cannot selectively focus on the scatterers. In this case, each submatrix can have different associations with the scatterers in the domain. Depending on their distances or the strength of signals they received from specific scatterers, the eigenvectors of each submatrix may correspond to different scatterers. Therefore, it is important to have the image of each submatrix before combining them to get the final DORT images.

The submatrix concept can easily be extended to contain other diagonals of the $K(\omega)$ matrix. Assuming that we have $S$ of the diagonals (i.e. $K_{i,i+s}$, $i = 1, \ldots, N - s$...
for each \( s = 0, \ldots, S - 1 \text{ and } S < N \), we can form \( S \times S \) submatrices and utilize them for imaging. Note that, as the number of diagonals \( (S) \) increases, the number of utilized submatrices decreases.

5.6 Summary and Conclusions

We have analyzed the sensitivity of some TR-based imaging methods to noise and clutter. DORT and TR-based MUSIC were considered under both NB and UWB operation and under different (volumetric scattering) clutter and (additive) noise levels. For homogeneous background media, it was found that CF-MUSIC and UWB-MUSIC outperform CF-DORT and TD-DORT, respectively, in terms of both range and cross-range resolutions. On the other hand, CF-DORT and TD-DORT are more robust under increased clutter and/or noise levels compared to CF-MUSIC and UWB-MUSIC, respectively. Under further increase on clutter and/or noise, however, CF-MUSIC images may exhibit spurious relative maxima and suffer from statistical instability. UWB-MUSIC overcomes these drawbacks because it naturally combines images over different frequencies, resulting in statistically stable process. TD-DORT also provides statistically stable images by exploiting frequency decorrelation. Furthermore, it provides better range resolution than CF-DORT due to the increased bandwidth. Since MUSIC requires a non trivial null space of the TRO, TR-based MUSIC operation is prevented in highly cluttered environments, whereas DORT can still be employed. For low noise and low clutter environments, TR-based MUSIC is a better option due to its superior resolution. Another source of perturbation considered was from TR invariance breaking due to losses. As a general rule, increasing losses result in focal amplitude attenuation and resolution decrease for DORT images.
As long as the signal subspace received from the scatterer is preserved, TR-based MU-SIC results yield more stable images. The decorrelation rate of signals emanated from TRAs with (systematic) translational perturbations was studied. When performed in a controlled manner, such class of perturbations allows the extraction of second order statistical information of the probed media. Finally, the effects of array restrictions have been investigated.
CHAPTER 6

SPACE-FREQUENCY ULTRAWIDEBAND TIME-REVERSAL IMAGING

As discussed in the previous chapters, TR-based imaging methods, specifically the DORT and TR-MUSIC, employ the eigenstructure of the TR operator. Traditionally, the eigenvalue decomposition of the TRO (or equivalently the singular value decomposition (SVD) of the MDM) is carried out at a single frequency by utilizing the space-space MDMs one-by-one. UWB extensions of these methods, viz., time-domain (TD)-DORT [91] and UWB-MUSIC [92] utilize the same space-space MDMs and combine them at multiple frequencies to obtain statistically stable images. However, at each frequency, the eigenvectors carry an arbitrary and frequency-dependent phase from the eigenvalue decomposition. As a result, eigenvectors in the time domain become incoherent and a preprocessing step is necessary to obtain coherent time-domain eigenvectors. This is especially important in inhomogeneous background media with strong multiple scattering, where multipath components need to be combined coherently over the entire bandwidth at the target(s) location(s).

Such preprocessing can be done by projecting the incoherent eigenvectors onto columns of the MDM [31]. An alternative way to overcome this problem is through
the application of SVD directly to the space-frequency MDM (SF-MDM), as practiced in this Chapter. Instead of utilizing a MDM at a specific frequency, we combine MDMs obtained at different frequencies to yield a new MDM whose columns and rows correspond to space and frequency components of the received signals, respectively. SF-MDM was first proposed in [125] in the context of the conventional MUSIC algorithm [124] to determine three-dimensional (3-D) positions and electrical parameters of focal lesions for biomedical applications. The proposed application here of applying SVD to this new MDM yields a set of singular values and corresponding left and right singular vectors. The left singular vectors encode target location in the form of relative phase shifts among elements. The right singular vectors encode the frequency dependence of the received signals (target signatures) which can be directly employed to provide an UWB imaging method without any pre-processing step. In this Chapter, we introduce new TR-based imaging approaches using the singular vectors of the SF-MDM for backpropagation from the TR array antennas. We analyze the singular values and vectors obtained for various scattering scenarios involving continuous random media. A limited-aspect linear TR array is used to probe the (random) media with UWB signals for the imaging of embedded discrete scatterers.

6.1 Space-Frequency TR-imaging

Since SF-imaging utilizes the whole available bandwidth, it can be considered as an UWB imaging scheme. Additionally, as it employs the signal subspace of the SF-MDM, its performance will be compared against that of the TD-DORT method. For this end, it is worth to recall the TD-DORT functional which is given as follows.
\[ D_p^\Omega(\bar{x}_s) = \langle g_0(\bar{x}_s, t), r_p(t) \rangle \big|_{t=0} = \sum_{i=1}^{N} r_p^i(t) \star G_0(\bar{x}_s, \bar{r}_i, t) \big|_{t=0} = \int_{\Omega} \lambda_p(\omega) v_p^T(\omega) g_0(\bar{x}_s, \omega) d\omega \]  

(6.1)

As observed, TD-DORT method utilizes the same space-space MDMs employed by CF-DORT, and further combines them along the bandwidth. However, the eigenvalue decomposition (or SVD) operation produces eigenvectors with arbitrary frequency dependent phases \( \phi_{svd}(\omega) \) as given below.

\[ v_p(\omega) = e^{j\phi_{svd}(\omega)} g^*(\bar{x}_p, \omega) / \| g(\bar{x}_p, \omega) \| \]  

(6.2)

where \( \bar{x}_p \) is the location of the \( p^{th} \) target. As a result, the direct combination of these eigenvectors does not produce coherent time-domain signals. Although this is not a problem for UWB-TR-MUSIC (since only the magnitudes of the inner-products are combined along the bandwidth), TD-DORT excitation signals are severely affected.

As mentioned, preprocessing steps such as projecting the incoherent eigenvectors onto the columns of the MDM [31], or tracking the phase difference between adjacent frequency eigenvectors by a phase smoothing algorithm [91] are required. In this section, we introduce an alternative strategy based on the application of SVD to two new types of MDMs which incorporate simultaneous sensor location and UWB frequency data. This is denoted here as space-frequency (SF)-MDM.

### 6.1.1 SF-Imaging Using Individual-Based SF-MDMs

Individual-based SF-MDMs are obtained by transmitting a short pulse from the \( n^{th} \) antenna and recording the received signals from all TR array antennas, which
yields a $N \times M_f$ matrix given by

\[
\mathbf{K}^n_{SF} = \begin{pmatrix}
  k_{1n}(\omega_1) & \cdots & k_{1n}(\omega_{M_f}) \\
  \vdots & \ddots & \vdots \\
  k_{Nn}(\omega_1) & \cdots & k_{Nn}(\omega_{M_f})
\end{pmatrix}
\]  

(6.3)

where the $i^{th}$ row consists of uniformly distributed frequency samples of the Fourier transform of the time-domain signal at the corresponding receiver: $k_{in}(\omega)$ for $\omega_1 \leq \omega \leq \omega_{M_f}$ where $\omega_{M_f} - \omega_1 = \Omega$ with $M_f$ being the number of samples. The number $M_f$ can be chosen in such a way that the error between $k_{in}(\omega)$ and a linear interpolation function of the samples $\tilde{k}_{in}(\omega)$ is below a threshold criterion $\delta_e$, i.e.

\[
\int_{\omega_1}^{\omega_{M_f}} |k_{in}(\omega) - \tilde{k}_{in}(\omega)| d\omega < \delta_e.
\]

Once this is repeated for each TR array antenna ($n = 1, \ldots, N$), $N$ of these individual SF-MDMs are obtained. SVD applied to each $\mathbf{K}^n_{SF}$ yields $\mathbf{K}^n_{SF} = \mathbf{U}^n_{SF} \Lambda^n_{SF} (\mathbf{V}^n_{SF})^\dagger$, where $\mathbf{U}^n_{SF}$ is the $N \times N$ matrix of left singular vectors, $\mathbf{V}^n_{SF}$ is the $M_f \times M_f$ matrix of right singular vectors, and $\Lambda^n_{SF}$ is the $N \times M_f$ matrix of singular values. Note that $\mathbf{K}^n_{SF}$ can be viewed as a mapping from the frequency to receiver spaces via $\mathbf{K}^n_{SF} \mathbf{v}^n_{SF_i} = \lambda^n_{SF_i} \mathbf{u}^n_{SF_i}$, where $\lambda^n_{SF_i}$ is the $i^{th}$ singular value, $\mathbf{v}^n_{SF_i}$ is the $i^{th}$ $M_f \times 1$ right singular vector that represents the frequency content of the received signals, and $\mathbf{u}^n_{SF_i}$ is the $i^{th}$ $N \times 1$ left singular vector containing spatial (position) information. The left singular vectors $\mathbf{u}^n_{SF_i}$, $i = 1, \ldots, N$, form an orthonormal set spanning the sensor location space. Similarly, the right singular vectors $\mathbf{v}^n_{SF_i}$, $i = 1, \ldots, M_f$, form an orthonormal set spanning the frequency space. Once the elements of $\mathbf{v}^n_{SF_i}$ are similarly treated as samples of the continuous function $v^n_{SF_i}(\omega)$ (for $\omega_1 \leq \omega \leq \omega_{M_f}$) obtained by linear interpolation, time-domain signals can be simply obtained by an inverse Fourier transformation, i.e.

$v^n_{SF_i}(t) = \mathcal{F}^{-1}(v^n_{SF_i}(\omega))$.

These time-domain signals are coherent in the sense that SVD applied to SF-MDM
does not create an arbitrary frequency dependent phase factor as in Eq. 6.2. Therefore, they can be employed to approximate the TR array excitation signals $s^n(t)$, $n = 1, \ldots, N$, for use in backpropagation as follows:

$$s^n(t) = \sum_{k=1}^{P} \lambda^n_{SF_k} v^n_{SF_k}(t)$$

(6.4)

where $P$ is the total number of time-domain signals being included. The number $P$ is chosen by examining the singular values and the associated singular vectors. Time-domain signals corresponding to small singular values and erratic behavior are not included. The resulting time-domain excitation signals still cannot provide the necessary relative phase shifts (or time delays) among the TR array antennas (spatial information) in order for the backpropagated wave to focus on the embedded scatterer(s). For this end, we need to create a new $N \times 1$ time-domain vector whose elements employ a time-domain function with different amplitude weights and time shifts. Therefore, we define the following vector functional

$$f(a, z(t)) = \mathcal{F}^{-1}\{[A_0 e^{j\phi_0} z(\omega), \ldots, A_N e^{j\phi_N} z(\omega)]^T\}$$

(6.5)

where $z(t)$ is the employed time-domain function and $a = [A_0 e^{j\phi_0}, \ldots, A_N e^{j\phi_N}]^T$ is an $N \times 1$ vector determining the amount of relative time-delays and amplitudes. Using the left singular vectors and the approximated time-domain signal as inputs to this vector functional yields the required $N \times 1$ time-domain vector as

$$r^n_{SF_i}(t) = f(u^n_{SF_i}, s^n(t))$$

(6.6)
Once this vector is transmitted back to the approximate background medium, we get the following pseudospectrum for each search point $\bar{X}_s$ in the probed domain

$$I_{SF_i}^n(\bar{X}_s) = \langle g_0(\bar{X}_s, t), r_{SF_i}^n(t) \rangle \mid_{t=0}$$

$$= \sum_{k=1}^N r_{SF_i}^n(-t)^* G_0(\bar{X}_s, \bar{R}_k, t) \mid_{t=0}$$

$$= \int_\Omega (r_{SF_i}^n(\omega))^\dagger g_0(\bar{X}_s, \omega) d\omega$$

$$= \int_\Omega (s^n(\omega))^* (u_{SF_i}^n)^\dagger g_0(\bar{X}_s, \omega) d\omega \quad (6.7)$$

This procedure is repeated for all $n = 1, \ldots, N$ and the resulting images are averaged via $I_{SF_i}(\bar{X}_s) = 1/N \sum_{n=1}^N I_{SF_i}^n(\bar{X}_s)$. The final image obtained using this approach is denoted as $SF$-image. Alternatively, instead of the left singular vector(s), eigenvectors obtained from the CF-DORT method can also be combined with the time-domain singular vectors as follows:

$$r_{SF_i}^n(t) = f(v_i^*(\omega_c), s^n(t)) \quad (6.8)$$

Once Eq. 6.8 is used in Eq. 6.7, we get

$$I_{SF_i}^n(\bar{X}_s) = \int_\Omega (s^n(\omega))^* v_i^T(\omega_c) g_0(\bar{X}_s, \omega) d\omega \quad (6.9)$$

In the case of well-resolved and point-like scatterers, CF-DORT eigenvectors correspond to individual scatterers, and the use of Eq. 6.8 in Eq. 6.7 allows for selective focusing on individual scatterers in the presence of others. The next subsection discusses some of the underlying assumptions for SF-imaging and examine the impact of some geometrical and frequency parameters on the optimal range of applicability.

### 6.1.2 Assumptions and Applicability Range

The two foremost assumptions are that the scatterers are point-like and well-resolved, which are also the conditions assumed for conventional DORT. Given these
assumptions, one can substitute Eq. 6.2 into the TD-DOR T spectrum in Eq. 6.1 to write

\[
D^\Omega_p(\bar{X}_s) = \int_{\Omega} \lambda_p(\omega) e^{j\phi_{svd}(\omega)} \frac{g^\dagger(\bar{X}_p, \omega) g_0(\bar{X}_s, \omega)}{\|g^\dagger(\bar{X}_p, \omega)\|} d\omega \tag{6.10}
\]

The TD-DOR T functional in this form takes the inner product \(\langle g_0, g \rangle = g^\dagger g_0\), where both \(g_0\) and \(g\) are evaluated at the same frequency \(\omega\), and multiplies it by the product of the associated singular value \(\lambda_p(\omega)\) and the arbitrary phase factor \(e^{j\phi_{svd}(\omega)}\). This product is integrated along the bandwidth and the process is repeated for all search points \(\bar{X}_s\) in the probed domain. The main drawback in this procedure is the phase incoherency introduced by the arbitrary frequency-dependent phase \(\phi_{svd}(\omega)\). The remedy proposed here is the use of SF-MDMs and their possible combination with space-space MDM eigenvectors (more specifically, CF-DOR T eigenvectors). Let us analyze the SF-imaging functional when Eq. 6.2 is substituted into Eq. 6.9. This yields

\[
I^n_{SF_p}(\bar{X}_s) = \int_{\Omega} (s^n(\omega))^* \frac{g^\dagger(\bar{X}_p, \omega_c) g_0(\bar{X}_s, \omega) d\omega}{\|g^\dagger(\bar{X}_p, \omega_c)\|} \tag{6.11}
\]

In this case, the issue arising from the arbitrary frequency dependent factor is not present anymore because \(s^n(t)\) is coherent and has a continuously varying phase (thanks to the SVD applied to the SF-MDM). However, while the background steering vector \(g_0\) present in the inner product factor is evaluated along all the frequencies of the bandwidth, \(g\) is evaluated only at the central frequency of operation \(\omega_c\). To examine the impact of this dichotomy, we write \(g_0(\bar{X}_s, \omega)\) in terms of \(g_0(\bar{X}_s, \omega_c)\). In a homogeneous medium, the scalar Green’s function between two points \(X_s = \bar{r}\) and \(\bar{R}_i = \bar{r}'\) sufficiently apart can be written as \(G_0(\bar{r}, \bar{r}', \omega) = G_0(\bar{r}, \bar{r}', \omega_c) e^{-j(k-k_c)|\bar{r}-\bar{r}'|}\) where \(k = \omega \sqrt{\mu \epsilon}\) and \(k_c = \omega_c \sqrt{\mu \epsilon}\). Defining \(|\bar{X}_s - \bar{R}_i| = d_{si} = d_{si}^{aw} + \Delta d_{si}\), where
\[ d_{av} = 1/N \sum_{i=1}^{N} d_{si} \] is the average distance between the search point and the TR array elements and \( \Delta d_{si} \) is the deviation of each element from \( d_{av} \) allows us to write

\[
G_0(X_s, \bar{R}_i, \omega) = G_0(X_s, \bar{R}_1, \omega_c) e^{-j\Delta k d_{av}^w} e^{-j\Delta k \Delta d_{si}}
\]  

(6.12)

where \( \Delta k = k - k_c \). Once the above is substituted into \( g_0(\bar{X}_s, \omega) \), we get

\[
g_0(\bar{X}_s, \omega) = e^{-j\Delta k d_{av}^w} \left( G_0(\bar{X}_s, \bar{R}_1, \omega_c) e^{-j\Delta k \Delta d_{s1}} \right) \text{;} \ldots \left( G_0(\bar{X}_s, \bar{R}_N, \omega_c) e^{-j\Delta k \Delta d_{sN}} \right)
\]

(6.13)

If \( \Delta k \Delta d_{si} \ll 1 \), then \( g_0(\bar{X}_s, \omega) \) becomes

\[
g_0(\bar{X}_s, \omega) \approx e^{-j\Delta k d_{av}^w} g_0(\bar{X}_s, \omega_c)
\]

(6.14)

Substituting the above in Eq. 6.11 yields

\[
\mathbf{I}_{SFp}^s(\bar{X}_s) = \int_{\Omega} (s^n(\omega))^* e^{-j\Delta k d_{av}^w} \frac{g_0^\dagger(\bar{X}_s, \omega_c) g_0(\bar{X}_s, \omega_c)}{\| g_0^\dagger(\bar{X}_p, \omega_c) \|} d\omega
\]

(6.15)

which is peaked at \( \bar{X}_s \simeq \bar{X}_p \) if \( \Delta k \Delta d_{si} \ll 1 \). If \( d_{si}/a \gg 1 \), where \( a \) is the effective aperture length of the TR array, i.e. for target(s) sufficiently far from the TR array, then \( \Delta d_{si}/a \propto a/d_{si} \ll 1 \) and \( \Delta k \Delta d_{si} \ll 1 \) may hold even under wideband operation.

In this case, since the inner product factor taken at the central frequency is weighted and integrated afterwards, the resulting cross-range is close to that of the CF-DORT method whereas a better range resolution is achieved because of the integration along the bandwidth. Another possibility is to employ narrowband operation where \( \Delta k \to 0 \). In this case, both range and cross-range resolutions are close to those of CF-DORT.

Even if \( \Delta k \Delta d_{si} \ll 1 \) does not hold, Eq. 6.11 can still be used as an alternative to Eq. 6.10. However, a trade-off is present between the arbitrary phase factor and...
the inner-product taken at different frequencies. Note that this occurs in addition to, of course, the degradation on performance expected in both Eq. 6.10 and Eq. 6.11 caused by any background medium inhomogeneities (clutter) since the inner product factor combine both the exact and approximate background Green’s function vectors.

6.1.3 SF-Imaging Using Full SF-MDM

The second kind of SF-MDM is obtained by combining all the $K^n_S F, n = 1, \ldots, N$ of the previous subsection into a single $N^2 \times M$ matrix

$$K_{SF}^{full} = \begin{pmatrix} K^1_{SF} \\ \vdots \\ K^N_{SF} \end{pmatrix} = \begin{pmatrix} k_{11}(\omega_1) & \cdots & k_{1M}(\omega_1) \\ k_{11}(\omega_2) & \cdots & k_{1M}(\omega_2) \\ \vdots & \ddots & \vdots \\ k_{N1}(\omega_1) & \cdots & k_{NM}(\omega_1) \end{pmatrix}$$ (6.16)

denoted as full SF-MDM. The SVD of the above gives $$K_{SF}^{full} = U_{SF}^{full} \Lambda_{SF}^{full} (V_{SF}^{full})^\dagger,$$ where $U_{SF}^{full}$ is the $N^2 \times N^2$ matrix of left singular vectors, $V_{SF}^{full}$ is the $M \times M$ matrix of right singular vectors, and $\Lambda_{SF}^{full}$ is the $N^2 \times M$ matrix of singular values. The matrix $V_{SF}^{full}$ is similar to $V^n_{SF}$ and therefore time-domain signals $s_{SF}^{full}(t)$ can be calculated similarly to Eq. 6.4. As for $U_{SF}^{full}$, each of its columns $u_{SF_i}^{full}, i = 1, \ldots, N$ contains $N$ sub-vectors $u_{SF_i,sub_k}^{full}, k = 1, \ldots, N$ with $N$ elements each encoding the relative phase information of each TR array antennas for backpropagation, i.e., $u_{SF_i}^{full} = [(u_{SF_i,sub_1}^{full})^T, \ldots, (u_{SF_i,sub_N}^{full})^T]^T$. As shown in the simulations below, for well-resolved and point-like scatterers, each sub-vector of a specific left singular vector corresponds to the same specific scatterer. By combining these sub-vectors and using the time-domain singular vectors, the following $N \times 1$ time-domain vector can be constructed

$$r_{SF_i}^{full}(t) = f \left( \sum_{k=1}^{N} u_{SF_i,sub_k}^{full}, s_{SF}^{full}(t) \right)$$ (6.17)
Once this vector is convolved with \( g_0(\vec{X}_s, t) \), we obtain the pseudospectrum of the full SF-image via

\[
I_{SF}^{full}(\vec{X}_s) = \langle g_0(\vec{X}_s, t), r_{SF}^{full}(t) \rangle
\] (6.18)

As long as the left singular vectors of the full SF-MDM can be associated with specific scatterers, selective focusing is possible with the full SF-MDM without the need for CF-DORT eigenvectors. For this end, phase and magnitude distributions of the left singular vectors should be examined and consistent ones used for spatial information.

### 6.2 Results and Discussion

We employ FDTD simulations with a grid of \( N_x \times N_y = 200 \times 200 \) cells to synthesize TR operators both in homogeneous and random media [94]. The random medium has a spatially fluctuating permittivity \( \epsilon(\vec{r}) = \epsilon_m + \epsilon_f(\vec{r}) \) where \( \vec{r} = x\hat{x} + y\hat{y} \) denotes spatial position and \( \epsilon_m = 2.908 \) (dry sand) [100] is the average relative permittivity. The fluctuating term \( \epsilon_f(\vec{r}) \) is a zero-mean Gaussian random variable with Gaussian correlation function as outlined in Section 2.2. Only TM\(_z\) case is considered here to provide isotropic scattering. The linear TR array consists of \( N = 7 \) dipole transceivers. Each dipole is initially fed by a current source \( \vec{J}_i(x, y, t) = \vec{z}r_i(t) \) where \( r_i(t) \) is the derivative of the Blackmann-Harris (BH) pulse, centered at \( f_c = 400 \) MHz (Fig. 6.2(m)). A spatial cell size \( \Delta_{x,y} = \Delta_s = 1.37 \) cm is chosen, corresponding to about \( \lambda_c/32 \) for the mean permittivity \( \epsilon_m \). The TR array lies parallel to the \( x\)-direction. The dipoles are separated by \( \lambda_c/2 \) to reduce mutual coupling, which is neglected. The central dipole is located at the origin, \( \vec{R}_4 = (0, 0)\Delta_s \).
6.2.1 Single Embedded Target

We first investigate individual and full SF-MDMs obtained for a single PEC target (embedded discrete scatterer) with radius $r = 4.12$ cm located at $(-2, 100)\Delta_s$ (i.e., 1.37 m below the TR array and slightly off the central antenna) in homogeneous ($\delta = 0$ and $l_s = \infty$) or inhomogeneous random (or simply, random) ($\delta = 0.02$ and $l_s = 8\Delta_s$) media. Fig. 6.1 shows the resulting singular value distributions.

![Figure 6.1](image)

Figure 6.1: Singular values of (a) individual-based ($K_{SF}^{j=4}$), and (b) full SF-MDMs in homogeneous (HM) and random media (RM) for a single scatterer.

As observed, the first dominant singular values obtained in homogeneous and random media are close to each other for both individual and full SF-MDM cases. Additionally, as shown in the first row of Fig. 6.2, the first right singular vectors exhibit similar time-domain behavior (except for the fluctuations due to clutter in the random medium case). On the other hand, among the remaining (non-dominant) singular values, those obtained in the random medium are considerably larger than those in the homogeneous medium. In addition, the corresponding time-domain singular vectors behave quite differently in homogeneous and random media, as seen in...
Figure 6.2: First six (dominant) time-domain right singular vectors for a single scatterer embedded in (first two rows) homogeneous and (third and fourth rows) random media. Both individual-based and full SF-MDM cases are shown. Insets show the corresponding spectra. (m) Original excitation signal (first derivative of BH pulse) and its derivative. Two of the received signals by the TRA (n) $k_{14}(t)$, (o) $k_{17}(t)$
Fig. 6.2, because the background inhomogeneities act as distributed scattering centers (distinct spatial spectra from the discrete target).

Once the time-domain excitation signals are obtained, they can be used to excite the TR array antennas with relative phase shifts construed by the left singular vectors $u_{SF_i}^j$ or $u_{SF_i}^{full}$ that contain spatial information. Fig. 6.3 shows the phase of the first three left singular vectors obtained in homogeneous and random medium for the individual SF-MDM case, along with the TR array and scatterer configuration.

![Figure 6.3](image)

Figure 6.3: (a)-(c) Phase distribution (radians) of first three left singular vectors, in sequence. The relative phase distribution encodes information about scatterer location(s). These results corresponds to a single scatterer in either HOM or RM, as sketched in (d), for the individual-based SF-MDM case.

Only the first left singular vectors provide the necessary phase information for focusing of the backpropagated wavefield around the target, as seen in Fig. 6.3(d). Similarly, Fig. 6.4 shows the phase distribution of the first two left singular vectors of the full SF-MDM case. Note that corresponding phase distribution of the sub-vectors are also shown. As observed, each sub-vector of the first left singular vector corresponds to the embedded scatterer. Thus, backpropagating the approximated signals using these sub-vectors would yield the necessary focusing around the embedded
Figure 6.4: Phase distribution of the (a) first (top row results) and (d) second (bottom row results) left singular vectors and corresponding sub-vectors for a single scatterer obtained both in (b), (e) homogeneous and (c), (f) random media, for the full SF-MDM case.

scatterer. The remaining left singular vectors are not shown since they correspond to smaller singular values and therefore do not provide useful phase distribution.

Fig. 6.5 shows images using the left and right singular vectors of both SF-MDMs and using TD-DORT, as well as a comparison of their respective cross-range resolutions. As observed, both individual and full SF-MDMs yield similar images. Although the cross-ranges obtained in the homogeneous medium are close to each other, in random media TD-DORT cross-range is slightly better than those from SF-MDM. However, a phase smoothing algorithm is used for TD-DORT and with increasing background random media fluctuations the resulting time-domain excitation signals become more incoherent hindering the application of TD-DORT (see next section).
Figure 6.5: Images obtained from a single scatterer in homogeneous (first row results) and random (second row results) media. (a),(d): TD-DORT images. (b),(e): individual-based SF-MDM images. (c),(f): full SF-MDM images. The associated cross-range field intensity distributions are shown in (g) and (h) for homogeneous and random media, respectively. All plots are normalized.
However, by using SF-MDMs, it is always possible to obtain coherent time-domain excitation signals to be backpropagated from the TR array, regardless of background media fluctuations.

### 6.2.2 Multiple Embedded Targets

Next, we investigate the individual and full SF-MDMs when multiple targets are present, both in homogeneous and random medium (with stronger fluctuation: $\delta = 0.04$ and $l_s = 8\Delta_s$ at which generation of TD-DORT excitation signals is not possible). Specifically, we analyze the singular values and singular vectors in the case of two identical discrete scatterers with radii $r = 4.12$ cm and buried at $(32, 70)\Delta_s$ (first scatterer) and $(-48, 100)\Delta_s$ (second scatterer).

The corresponding singular values are shown in Fig. 6.6. Compared to the single scatterer case in Fig. 6.1, the major difference occurs in the homogeneous medium case, where the small singular values are relatively more significant in Fig. 6.6. This is caused by multiple scattering between the two scatterers that increases the associated

![Figure 6.6: Singular values of (a) individual-based ($K_{SF}^{j=4}$) and (b) full SF-MDMs in homogeneous and random media from two embedded scatterers.](image-url)
singular values. This is also verified by examining the time-domain singular vectors. Fig. 6.7 shows the first four significant right singular vectors. Additionally, two MDM elements, \( k_{11}(t) \) and \( k_{14}(t) \), are also shown where the scattered signals from the two scatterers and the multiple scattering can be distinguished. In the case of well-resolved scatterers and weak multiple scattering, SVD applied to the SF-MDM can extract the contribution from each scatterer individually. However, when multiple scattering is stronger, the extracted signal by the SVD is masked by the multiple scattering contribution. The remaining time-domain singular vectors correspond to multiple scattering contributions and scattering from background inhomogeneities.

The time-domain excitation signals can again be approximated by considering the time-domain behavior of the right singular vectors. Similarly to the single scatterer case, the time-domain singular vectors obtained with full SF-MDM have less ripples as compared to those obtained with individual SF-MDM.

Once the excitation signals are generated, they should be backpropagated with appropriate phase shifts. However, as shown in Fig. 6.8, the left singular vectors of the individual SF-MDM case cannot provide the necessary phase shifts since none of them corresponds to a specific embedded scatterer, as opposed the single scatterer case. As a result, selective focusing on the desired scatterers using these left singular vectors is not feasible. One solution is to combine the spatial information encoded by the CF-DORT eigenvectors with the time-domain singular vectors of SF-MDMs. For this end, the first two CF-DORT eigenvectors are shown in Fig. 6.8(d)-6.8(e) where it is observed that they provide the necessary phase shifts required to focus selectively on each of the targets in both homogeneous and random media.
Figure 6.7: First six (dominant) time-domain right singular vectors for two scatterers embedded in (first two rows) homogeneous and (third and fourth rows) random media. Both individual-based and full SF-MDM cases are shown. Insets show the corresponding spectra. Three of the received signals by the TRA (m) $k_{11}(t)$, (n) $k_{14}(t)$, (o) $k_{44}(t)$
Figure 6.8: (a)-(c) Phase distribution of the first three left singular vectors of individual SF-MDM obtained from two scatterers embedded in both homogeneous and random media. (d)-(e) First two eigenvectors of CF-DORT and the TR array and (f) scatterer configuration.

Alternatively, left singular vectors of full SF-MDM can also be employed for selective focusing. As shown in Fig. 6.9, the sub-vectors corresponding to the first left singular vector have the necessary phase shifts for focusing on the first scatterer in both homogeneous and random media. In contrast, sub-vectors of the second left singular vector are associated with the second scatterer only in the homogeneous case. Note that in this case, the second scatterer is further away than the first scatterer and that the background fluctuations $\delta$ are stronger than in the single scatterer case ($\delta = 0.04$ versus $\delta = 0.02$). As a result, the second left singular vector does not adequately discriminate the scattering contribution of the second scatterer from the background clutter. It should be pointed out that in this highly scattering case,
TD-DORT cannot yield the necessary time-domain signals to focus on the second scatterer as well even if the phase smoothing algorithm is applied. However, coherent time-domain signals can still be approximated with the SF-MDMs. Fig. 6.10 shows images obtained using the first two left and right singular vectors of the individual and full SF-MDM cases, and the first two eigenvectors of CF-DORT in homogeneous media. The SF-images using left singular vectors of the individual SF-MDM case show spurious secondary peaks. On the other hand, SF-images using sub-vectors of the full SF-MDM case and CF-DORT eigenvectors are better localized around the selected scatterer. This illustrates that selective focusing around the desired scatterers in the presence of others is possible with the use of the full SF-MDM and CF-DORT
Figure 6.10: Images obtained for two scatterers embedded in a homogeneous background medium. SF-image using (a) the first and (d) the second left singular vector of the individual-based SF-MDM, using (b) the first and (e) the second left singular vector of the full SF-MDM, and using (c) the first and (f) the second eigenvector of CF-DORT.

eigenvectors. In the context of TR imaging using DORT, selective focusing is possible for well-resolved scatterers with negligible multiple scattering. This condition is also necessary for the SF-imaging to provide satisfactory selective focusing since both the left and right singular vectors are adversely affected by the multiple scattering between targets.
6.3 Conclusions

In this Chapter, we considered TR imaging techniques that employ two unconventional MDMs with simultaneous spatial and UWB frequency information on the scattered field. Such MDMs are denoted as space-frequency MDM to distinguish them from those of the conventional TR-imaging methods which normally encode spatial information only (space-space MDM). Application of SVD to SF-MDMs provides new sets of singular values and singular vectors that can be employed to generate signals for UWB imaging of targets in inhomogeneous random media. The required spatial information for target focusing is provided by either the left singular vectors or by eigenvectors of the space-space MDM.

SF-imaging yields TR-based imaging functionals with a performance comparable to TD-DORT. In scenarios with strong multiple scattering (clutter) where TD-DORT cannot provide coherent time-domain signals, SF-imaging can adequately extract target signatures to generate time-domain excitation signals for backpropagation. In principle, target signatures from SF-imaging could be employed for classification purposes as well, but that analysis is left as a future work.
CHAPTER 7

CONCLUSIONS AND FUTURE WORK

Time-reversal techniques involve the physical or synthetic backpropagation of signals received by a sensor array in a time-reversed fashion (first-in last-out sequence) and they can achieve superresolution by utilizing traditionally hostile multipath components. Fink’s successful TR experiments using low frequency acoustic and ultrasound waves have showed great potential in remote sensing applications and have also created an interest in the application of TR methods using radio frequency electromagnetic waves. In this work, we have applied and developed TR techniques using ultrawideband electromagnetic waves in homogeneous and random media. First, we have investigated the influence of different statistical parameters of continuously random medium on the refocusing resolution of EM waves. Similar to the acoustic case, we have observed that an increase in multiple scattering effects leads to better focusing resolution of the time-reversed EM signals. Additionally, the effects of depolarization is also studied where a better superresolution can be obtained with the fully polarimetric TRA over that achieved with co-pol TRA only. Second, we have considered the application of TR in dispersive media where the TR invariance is broken. To compensate for the losses due to dispersive attenuation, we have developed an inverse filtering approach based on the STFT of the received signals. It
was shown that by utilizing these filters, a partial compensation taking into spatial and frequency dependent attenuation has been achieved. As a third step, we have developed a time-domain selective focusing method that can be applied both in homogeneous and random media. This method allows focusing on the selected scatterer in the presence of others. Additionally, we have analyzed the sensitivity of both narrowband and UWB TR-based imaging methods to several perturbations. Specifically, we have studied the effects of clutter, additive noise, dispersion and array restrictions on the focusing resolution of the EM waves. Finally, we have developed new UWB TR imaging functional utilizing unconventional MDMs with simultaneous spatial and frequency data.

For the most part, we have restricted ourselves to typical subsurface sensing scenarios where the random medium models are based on inhomogeneous soil models. However, the techniques developed here can easily be transferred to other applications such as microwave detection of breast cancer or nondestructive testing.

7.1 Future Work

Application of TR using the electromagnetic waves is a relatively new and open topic. Therefore, many different applications regarding target detection problems can be explored using the TR techniques. Some of these applications are as follows:

7.1.1 Classification of Subsurface Objects via TR techniques

For many subsurface sensing problems, it is important to know what target features are characteristic and are most easily detectable. For example, in the case of metallic targets in free space, edges provide strong scattering centers and are often used for scattering signature analysis. However, for the case of non-metallic objects,
edges do not provide as strong signatures. Moreover, the frequency of operation also affects the signature from targets. In the low frequency regime, shape details of targets may not be recovered. The TROs of scatterers with different shapes in UWB regime can be investigated by performing an eigenspace analysis to study the effects of different scattering centers. TR-imaging using eigenvectors can also be applied to determine the shape of the scatterers in general.

7.1.2 DORT-enhanced TR Adaptive Interference Cancelation for Change Detection

One possible application of TR is the nulling of clutter in highly scattering environments as discussed in [126]. This can be achieved by obtaining the response of clutter-only case at first. Then instead of applying traditional TR where the back-propagated signals focus on the clutter, the time-reversed signals are modified in such a way that backpropagated signals avoid the clutter. Therefore, when a target appears in the same medium, more echo with a higher SNR can be received from the target. This modification algorithm is named as TRAIC, i.e. TR Adaptive Interference Cancelation. It was shown that this method provides better results in terms of detection of targets as compared to conventional change detection algorithm [127, 65, 66]. This method can be improved with the use of the DORT method which will also provide the localization information in addition to the detection. Since the clutter only channel response is needed, instead of subsurface sensing applications, scenarios where targets enter the probed domain will be considered (e.g. through-wall imaging).
7.1.3 Study of Time-Reversal Arrays for GPR applications

Antennas used in ground penetrating radars (GPR) generally have wide illumination and limited directivity properties. Phased array transmitters can overcome this problem by directing the electromagnetic (EM) energy in the desired directions \cite{128}. This is achieved by feeding array elements with relative phase differences and amplitudes so that effective radiation pattern is reinforced in the desired direction and suppressed in undesired directions. Similarly, TR array (TRA) antennas are fed by time-reversed (phase conjugated in frequency domain) versions of the original received signals so that EM energy focusing occurs around the original scatterer location(s). This, in turn, provides better signal-to-noise ratio (SNR) and resolution capability as compared to single transmitter - single receiver systems. Moving TR arrays and fixed receiving antennas (common-receiver) can be utilized for GPR subsurface imaging. It is anticipated that a better SNR, higher amplitude and deeper penetration can be obtained by using TRAs as compared to single antenna receiver and transmitter systems.

7.1.4 TR-based Microwave Breast Cancer Detection

As recently shown in \cite{70}, malignant tumors possess dielectric permittivity values greater than the rest of the surrounding breast tissue which can then be detected using the UWB EM waves via methods adapted from the GPR concept. The complex nature of the breast tissue and its surrounding environment (e.g. the skin or chest wall) allow the utilization of TR technique to achieve super-resolution imaging. Additionally, the dispersive and lossy characteristics of the tissues require the
compensation methods similar to discussed in this dissertation. Thus, as future research projects, we plan to adapt, transfer and deploy the developed algorithms for utilization in microwave based early breast cancer detection application.
APPENDIX A

Spectral Domain Random Medium Generation

In this appendix, we provide the mathematical and numerical details of the continuous random medium generation. The fluctuating part of the medium dielectric permittivity is represented by $\epsilon_f(\vec{r})$ where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the spatial position. Each spatial position fluctuates with a correlation function defined as $C(\vec{r}_1 - \vec{r}_2) = \langle \epsilon_f(\vec{r}_1)\epsilon_f^*(\vec{r}_2) \rangle$. Here, we consider three types of correlation functions; namely, Gaussian ($C_g$), Exponential ($C_e$) and Rayleigh ($C_r$) [129] correlation functions which are given below (Fig. A.2(b)):

\begin{align*}
C_g(\vec{r}_1 - \vec{r}_2) &= \delta_g \exp \left( -\frac{|\vec{r}_1 - \vec{r}_2|^2}{\ell_s^2} \right) \quad (A.1) \\
C_e(\vec{r}_1 - \vec{r}_2) &= \delta_e \exp \left( -\frac{|\vec{r}_1 - \vec{r}_2|}{\ell_s} \right) \quad (A.2) \\
C_r(\vec{r}_1 - \vec{r}_2) &= \delta_r |\vec{r}_1 - \vec{r}_2| \exp \left( -\frac{|\vec{r}_1 - \vec{r}_2|^2}{\ell_s^2} \right) \quad (A.3)
\end{align*}

Once the correlation function is chosen, random medium can be generated in the spectral domain by passing an array of random numbers through a digital filter $W(\vec{k})$ [100]. Actually, this digital filter is the spectral density function of the dielectric fluctuation, hence, it is the Fourier Transform of the correlation function, i.e. $W(\vec{k}) = \mathcal{F}(C(\vec{r}_1 - \vec{r}_2))$. Mathematically, this filtering process can be carried out by
the following formulation

$$\epsilon_f(\vec{r}) = \int_{-\infty}^{\infty} (a(\vec{k}) + jb(\vec{k})) \sqrt{W(\vec{k})} e^{-j\vec{k} \cdot \vec{r}} d\vec{k} \quad (A.4)$$

where $a(\vec{k})$ and $b(\vec{k})$ are independent random arrays satisfying

$$a(\vec{k}) = a(-\vec{k}) \quad (A.5)$$

$$b(\vec{k}) = -b(-\vec{k}) \quad (A.6)$$

$$W(\vec{k}) = W(-\vec{k}) \quad (A.7)$$

These conditions enforce the conjugate symmetry which ensures that the generated dielectric permittivity is real. In practice, $a(\vec{k})$ and $b(\vec{k})$ are generally chosen to have Gaussian distribution (independent from the correlation function distribution).

Fig. A.1 shows sample random media in 2D using the same random arrays but with different correlation functions. The principle difference among these random media is that Gaussian medium has a stronger low frequency content as compared

![Figure A.1: Random media with different correlation functions with same correlation length](image)

(a) Gaussian  (b) Exponential  (c) Rayleigh

Figure A.1: Random media with different correlation functions with same correlation length
to others, hence, its distribution is smoother. On the other hand, exponential and Rayleigh distributions have more contributions from the higher frequencies resulting in faster fluctuations as can be seen in Fig. A.2

\[ x (\times \Delta s) \]

(a) Cross section of the generated random media at \( y = 100\Delta_x \)

(b) Correlation functions

Figure A.2: Cross section of the generated random media and the correlation functions employed

It should also be mentioned that since we employ finite domains to generate these random media, there are some inherent statistical errors. For example, the spatial average permittivity of the finite domain realization is not equal to the theoretical average \( \epsilon_m \). This statistical error decreases as the fluctuations of the random media decrease (which is possible through increasing correlation length or decreasing variance values). However, as far as the media generated in this dissertation is considered, these errors are very small. Additionally, in 3D problems, vertical and horizontal correlation lengths can be chosen to be different.
APPENDIX B

FDTD for Dispersive and Lossy Medium

In this Appendix, we briefly summarize the finite-difference time-domain (FDTD) method utilized in this dissertation. The soil dispersion is modeled by Lorentz and Debye models and incorporated into the FDTD scheme via the use of piecewise-linear recursive convolution technique [121]. We use the complex coordinate stretching PML formulation to obtain the following modified Maxwell’s equations ($e^{-j\omega t}$ convention)

$$\nabla_s \times \vec{E} = j\omega \vec{B} \quad (B.1)$$

$$\nabla_s \times \vec{H} = -j\omega \vec{D} + \sigma \vec{E} \quad (B.2)$$

where $\sigma$ is the medium conductivity and $\nabla_s = \hat{x} \frac{1}{s_x} \frac{\partial}{\partial x} + \hat{y} \frac{1}{s_y} \frac{\partial}{\partial y} + \hat{z} \frac{1}{s_z} \frac{\partial}{\partial z}$ with $s_i = a_i + j\Omega_i/\omega$ (for $i = x, y, z$) being the frequency-dependent complex stretching variables (Note that $a_i$ and $\Omega_i$ are frequency-independent). Eqs. B.1 and B.2 are split as follows:

$$\frac{1}{s_x} \frac{\partial}{\partial x} \hat{x} \times \vec{E} = j\omega \vec{B}_{sx} \quad (B.3)$$

$$\frac{1}{s_x} \frac{\partial}{\partial x} \hat{x} \times \vec{H} = -j\omega \vec{D}_{sx} + \sigma \vec{E}_{sx} \quad (B.4)$$

which can then be written as:

$$\frac{\partial}{\partial x} \hat{x} \times \vec{E} = j\omega a_x \vec{B}_{sx} - \Omega_x \vec{B}_{sx} \quad (B.5)$$

$$\frac{\partial}{\partial x} \hat{x} \times \vec{H} = -j\omega a_x \vec{D}_{sx} + \Omega_x \vec{D}_{sx} + a_x \sigma \vec{E}_{sx} + j\Omega_x \sigma \vec{E}_{sx} \quad (B.6)$$
In the time domain, these equations are written as follows:

\[
\frac{\partial}{\partial x} \hat{x} \times \vec{E} = -a_x \frac{\partial}{\partial t} \vec{B}_{sx} - \Omega_x \vec{B}_{sx}
\]  
(B.7)

\[
\frac{\partial}{\partial x} \hat{x} \times \vec{H} = a_x \frac{\partial}{\partial t} \vec{D}_{sx} + \Omega_x \vec{D}_{sx} + a_x \sigma E_{sx} + \Omega_x \sigma \int_0^t E_{sx}(\tau) d\tau
\]  
(B.8)

These equations are to be discretized both in the time and space. However, before that, we have to introduce the dispersive characteristics into these equations. This is carried out through the use of complex permittivity function which results in the following magnetic and electric flux equations:

\[
\vec{B}_{sx} = \mu \vec{H}_{sx}
\]  
(B.9)

\[
\vec{D}_{sx} = \epsilon(t) \star t \vec{E}_{sx}
\]  
(B.10)

where \(\epsilon(t) = \mathcal{F}^{-1}(\epsilon(\omega))\) is the \(N\)-species Lorentzian dispersive medium given by the following frequency-dependent relative permittivity function

\[
\epsilon(\omega) = \epsilon_0 [\epsilon_\infty + \chi(\omega)]
= \epsilon_0 \epsilon_\infty + \epsilon_0 (\epsilon_s - \epsilon_\infty) \sum_{m=1}^{N} \frac{G_m \omega_m^2}{\omega_m^2 - j2\omega \alpha_m - \omega^2}
\]  
(B.11)

where \(\chi(\omega)\) is the medium susceptibility, \(\omega_m\) is the resonant frequency for the \(m^{th}\) species, \(\alpha_m\) is the correspondent damping factor and \(\epsilon_0\) and \(\epsilon_\infty\) are the static and infinite frequency permittivities, respectively. A corresponding time-domain susceptibility function can be defined as [120]

\[
\hat{\chi}(t) = \sum_{m=1}^{N} \hat{\chi}_m(t) = \sum_{m=1}^{N} j\gamma_m e^{-(\alpha_m + j\beta_m)t} u(t)
\]  
(B.13)

where \(\beta_m = \sqrt{\omega_m^2 - \alpha_m^2}\), \(\gamma_m = (\epsilon_s - \epsilon_\infty)\omega_m^2 G_m / \beta_m\) and \(\sum_{m=1}^{N} G_m = 1\). Note that \(\chi(t) = \mathcal{F}^{-1}(\chi(\omega)) = \mathcal{R} \epsilon(\hat{\chi}(t))\). Substituting Eqs. B.11 and B.13 into Eq. B.10 yields
the following electric flux definition

\[ \vec{D}(t) = \epsilon_0\epsilon_\infty \vec{E}(t) + \epsilon_0 \chi(t) *_t \vec{E}(t) \quad (B.14) \]

\[ \vec{D}(t) = \epsilon_0\epsilon_\infty \vec{E}(t) + \epsilon_0 \sum_{m=1}^{N} \Re\{\chi(t) *_t \vec{E}(t)\} \quad (B.15) \]

Electric field at \( t = l\Delta t \) using the piecewise-linear approximation for the time discretization can be written as

\[ \vec{E}(t) = \vec{E}^t + \frac{t - l\Delta t}{\Delta t}(\vec{E}^{t+1} - \vec{E}^t) \quad (B.16) \]

Once substituted into Eq. B.15, we get

\[ \vec{D}^t = \epsilon_0\epsilon_\infty \vec{E}^t + \epsilon_0 \sum_{m=1}^{N} \Re\{\tilde{Q}^l_m\} \quad (B.17) \]

where

\[ \tilde{Q}^l_m = \sum_{p=0}^{l-1} \left[ (\hat{\chi}_m^0 - \hat{\zeta}_m^0) E^{l+p} + \hat{\zeta}_m^0 E^{t-p-1} \right] e^{-(\alpha_m + j\beta_m) p\Delta t} \quad (B.18) \]

with the constants given as follows:

\[ \hat{\chi}_m^0 = \int_0^{\Delta t} \hat{\chi}_m(t)dt = \frac{j\gamma_m}{\alpha_m + j\beta_m} \{1 - e^{-(\alpha_m + j\beta_m) \Delta t}\} \quad (B.19) \]

\[ \hat{\zeta}_m^0 = \int_0^{\Delta t} t\hat{\chi}_m(t)dt = \frac{j\gamma_m}{\Delta t(\alpha_m + j\beta_m)} \{1 - [(\alpha_m + j\beta_m) \Delta t + 1] e^{-(\alpha_m + j\beta_m) \Delta t}\} \quad (B.20) \]

The following recursive calculation can be carried out for \( \tilde{Q}_m^l \)

\[ \tilde{Q}_m^l = \begin{cases} 0 & l = 0 \\ (\hat{\chi}_m^0 - \hat{\zeta}_m^0) E^l + \hat{\zeta}_m^0 E^{l-1} + \tilde{Q}_m^{l-1} e^{-(\alpha_m + j\beta_m) \Delta t} & l \geq 0 \end{cases} \quad (B.21) \]

Once this is substituted in Eq. B.17, we obtain the following update equation for the electric flux

\[ \vec{D}^t = \epsilon_0 \left( \epsilon_\infty + \sum_{m=1}^{N} \Re(\hat{\chi}_m^0 - \hat{\zeta}_m^0) \right) \vec{E}^t + \epsilon_0 \sum_{m=1}^{N} \Re(\hat{\zeta}_m^0) E^{t-1} + \epsilon_0 \sum_{m=1}^{N} \Re(\tilde{Q}_m^{l-1} e^{-(\alpha_m + j\beta_m) \Delta t}) \]

\[ \quad = \epsilon_0 (\lambda_0 \vec{E}^t + \lambda_1 \vec{E}^{t-1} + \vec{D}^{l-1}) \quad (B.22) \]
Note that in this equation, $\bar{P}^{l-1}$ depends only on $\bar{Q}^{l-1}_m$.

At this point, we need to apply both time-stepping and space discretization schemes for Eqs. B.7 and B.8. The space discretization follows the Yee staggered grid with central difference scheme and therefore the time discretization for them becomes

$$\frac{\partial}{\partial x} \hat{x} \times \bar{E}^l = -a_x \Delta t^{-1} \left( \bar{B}^{l+1/2}_{sx} - \bar{B}^{l-1/2}_{sx} \right) - \Omega_x \bar{B}^{l+1/2}_{sx} \quad (B.23)$$

$$\frac{\partial}{\partial x} \hat{x} \times \bar{H}^{l+1/2} = a_x \Delta t^{-1} \left( \bar{D}^{l+1}_{sx} - \bar{D}^l_{sx} \right) + \Omega_x \bar{D}^{l+1}_{sx} + a_x \sigma \bar{E}^{l+1}_{sx} + \sigma \Omega_x \bar{F}^l_{sx} \quad (B.24)$$

where $\bar{F}^l_{sx} = \bar{F}_{sx}(l \Delta t) = \int_0^{l \Delta t} \bar{E}(\tau) d\tau$. These equations can be arranged to give the time-stepping scheme as follows

$$\bar{B}^{l+1/2}_{sx} = -(a_x + \Omega_x \Delta t)^{-1} \left( \Delta t \frac{\partial}{\partial x} \hat{x} \times \bar{E}^l - a_x \bar{B}^{l-1/2}_{sx} \right) \quad (B.25)$$

$$(a_x + \Omega_x \Delta t) \bar{D}^{l+1}_{sx} + a_x \sigma \Delta t \bar{E}^{l+1}_{sx} = \Delta t \left( \frac{\partial}{\partial x} \hat{x} \times \bar{H}^{l+1/2} \right) + a_x \bar{D}^l_{sx} - \sigma \Omega_x \Delta t \bar{F}^l_{sx} \quad (B.26)$$

Since the left hand side of Eq. B.27 depends both on $\bar{D}^{l+1}_{sx}$ and $\bar{E}^{l+1}_{sx}$, it is unsuitable for time stepping in its current format. However, substituting Eq. B.22 into Eq. B.27, we have

$$\left( (a_x + \Omega_x \Delta t) \lambda \epsilon_0 + a_x \sigma \Delta t \right) \bar{E}^{l+1}_{sx} = \Delta t \left( \frac{\partial}{\partial x} \hat{x} \times \bar{H}^{l+1/2} \right) + a_x \bar{D}^l_{sx} - \sigma \Omega_x \Delta t \bar{F}^l_{sx}$$

$$- (a_x + \Omega_x \Delta t) \epsilon_0 (\lambda \bar{E}^l_{sx} + \bar{F}^l_{sx}) \quad (B.27)$$

In this format, $\bar{E}^{l+1}_{sx}$ is easily updated for time stepping. This is also used for updating of $\bar{B}^{l+1/2}_{sx}$ and $\bar{H}^{l+1/2}_{sx}$. The other quantities are updated as follows:
\[ D_{sx}^l = \epsilon_0 (\lambda_0 \bar{E}_{sx} + \lambda_1 \bar{E}_{sx}^{l-1} + \bar{P}_{sx}^{l-1}) \] (B.28)

\[ \bar{F}_{sx}^l = \bar{F}_{sx}^{l-1} + 0.5 \Delta_t (\bar{E}_{sx}^l + \bar{E}_{sx}^{l-1}) \] (B.29)

\[ \bar{Q}_{m,sx}^l = (\chi^0_m - \zeta^0_m) \bar{E}_{sx}^l + \zeta^0_m \bar{E}_{sx}^{l-1} + \bar{Q}_{m,sx}^{l-1} e^{-(\alpha_m + j\beta_m)\Delta_t} \] (B.30)

\[ \bar{P}_{sx}^l = \sum_{m=1}^{N} \Re \left( \bar{Q}_{m,sx}^l e^{-(\alpha_m + j\beta_m)\Delta_t} \right) \] (B.31)

This scheme is then repeated for \( y \) and \( z \) by replacing them with \( x \). This constitutes the complete FDTD algorithm used for the dispersive and lossy medium used throughout this dissertation. More details on this scheme and general FDTD algorithms can be found in [121] and [94].


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