AN APPROXIMATE UTD DEVELOPMENT FOR THE RADIATION BY ANTENNAS NEAR OR ON THIN MATERIAL COATED METALLIC WEDGES

DISSERTATION

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By

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ABSTRACT

Approximate and relatively simple but closed form Uniform Geometrical Theory of Diffraction (UTD) solutions are obtained for describing the radiated, scattered and surface wave fields, respectively, excited by sources near or on a canonical geometry consisting of a thin, planar, three-dimensional (3-D) double positive/double negative (DPS/DNG) material coated metallic wedge with an arbitrary wedge angle. Thus, unlike previous works that consider primarily plane wave scattering by such DPS structures via the Wiener-Hopf (W-H) or Maliuzhinets (MZ) methods, the present development can also treat radiation and coupling problems of antennas near or on finite material coated metallic surfaces. The latter is made possible mainly through the introduction of higher order UTD slope diffraction terms added to first order UTD. It is noted that all previous solutions based on rigorous W-H and MZ formulations employ approximate boundary conditions; in contrast, the present solutions, which are developed via a heuristic spectral synthesis approach, recover the proper local plane wave Fresnel reflection and transmission coefficients and surface wave constants of the DPS/DNG material. They also include the presence of backward surface waves in DNG media. Besides being asymptotic solutions of the wave equation, the present UTD diffracted fields satisfy reciprocity to first order UTD, the radiation condition, boundary conditions on the conductor, and the Karp-Karal lemma which dictates that the first order UTD space waves vanish on a material interface.
To my parents, Nongnuch, Narong, and my family.
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CHAPTER 1

INTRODUCTION

In this work, new, approximate, uniform geometrical theory of diffraction (UTD) [1, 2] based ray solutions are developed for describing the high frequency electromagnetic (EM) wave radiation/coupling mechanisms for antennas on or near material coated metallic wedges. This work is useful for analyzing the radiation and scattering from edges on electrically large complex platforms such as those shown in Figs. 1.1 and 1.2. Platforms illustrated in the latter figures involving modern aircraft or naval ships often contain material treatments over their otherwise metallic surfaces to control their scattering. Also modern antenna platforms may be built from composite materials. Furthermore, in many of these airborne or naval applications, there could be several antennas mounted on the same platform for multi-functional communication systems; thus it is important not only to be able to predict the effects of the platform on the performance of antennas placed thereon, but it is also important to predict the effects of mutual coupling between such antennas on a common platform. It is possible to control antenna mutual coupling by inserting material treatments around antennas in order to decrease the coupling. It is therefore clear that efficient and reliable computational tools for analyzing and accurately predicting the performance of such antennas which operate in the presence of material coated complex metallic
platforms are crucial to the design and the development of modern antenna systems for airborne, naval, and other applications.

The present UTD solutions provide relatively simple and accurate closed form expressions for the ray fields excited by antennas in the presence of edges in material coated metallic planar as well as wedge shaped surfaces. In these cases, the edges can be formed by material and/or geometrical discontinuities which are used to model the appropriate situations that may be encountered in practice. In contrast to most previous works which deal mostly with plane wave scattering problems and surface impedance approximations to coated metallic surfaces, the present work retains the Fresnel reflection coefficients and surface wave propagation constants of the actual coating, and allows for antennas to be not only far or near, but also directly on such surfaces.

It is noted that material coatings can be classified as double positive/double negative (DPS/DNG). DPS materials are those which exhibit positive values of electrical permittivity and permeability while DNG materials are supposed to exhibit negative values for these quantities [3–6]. One can also have materials with one of their electrical parameters positive with the other being negative; all of these types of materials are included in the UTD solutions developed in this work. While the existence and usefulness of DNG metamaterials is currently being investigated by others, and is somewhat controversial at this point, it is nevertheless included in this work as simply a hypothetical material for the sake of the completeness of analysis. It may be remarked in the passing that the DNG materials have generated considerable interest for their novel use in a variety of applications, e.g., miniaturization of microwave components, flat lens designs with enhanced focus, and antennas with special
Figure 1.1: Examples of antennas on modern aircraft with associated rays which describe the radiation mechanisms.
Figure 1.2: Examples of antenna arrays on a complex platforms in which there can be interactions or coupling between different antennas/arrays on the same platform.
performance [3,6]. In regard to the present study, which emphasizes the radiation characteristics of practical, and hence finite, material antenna structures, it is noted that most previous studies deal with reflection and transmission at DNG planar material slabs of only infinite extent. Thus, the important effects arising from wave diffraction at edges formed by the truncation of the DNG materials due to the finite nature of practical structures is totally ignored in most previous work. Furthermore, most previous work models DNG materials by periodic structure elements embedded in an infinite planar host slab of finite thickness, and their analysis is typically based on a Finite Difference Time Domain (FDTD) numerical approach which is generally inefficient and lacks physical insight. It is shown in the present work that one can clearly identify and analytically quantify all the dominant radiation/scattering mechanisms in terms of UTD rays including the backward surface (BSW) launching coefficients for the BSW rays, which in turn may lead to a useful physical description for DNG flat lens focusing. The latter BSW flat lens DNG focusing mechanism is quite different from the bulk refractive based DNG focusing mechanism of Pendry’s lens [6].

With the availability of large and fast computers, numerical approaches such as the method of moments (MoM), the finite element method (FEM), the finite element boundary integral (FE-BI) method, or the finite different time domain (FDTD) method, etc., are able to deal with EM radiation/scattering problems at low to moderately high frequencies for which the radiation/scattering platforms are relatively small to moderately large in terms of the EM wavelength, respectively. However, all of these numerical techniques become very poorly convergent, inefficient, and sometimes even intractable at high frequencies because they must satisfy EM field
self-consistency requirements in a global sense. For example, it is still a great challenge to solve very large problems such as for the fields of antennas on an aircraft or a naval ship platform at X-band frequencies using conventional numerical methods because of the exorbitantly large number of unknowns that need to be solved in these electrically large problems. One also notes more importantly that conventional numerical methods of solution generally provide no physical insights into the EM wave radiation/scattering mechanisms. An attractive alternative way to solve these types of electrically large problems is via the high frequency techniques such as the geometrical theory of diffraction (GTD). The GTD was introduced in the 1950s by Keller and his associates [7]. This GTD provided, for the first time, a very powerful and physically appealing simple ray theory for describing the phenomena of wave diffraction in a systematic fashion. It is noted that diffracted rays exist in addition to the conventional incident and reflected/transmitted ray fields of geometrical optics (GO). One notes that GO when used alone exhibits failure within the shadow region where it predicts zero fields because it contains no diffraction effects whatsoever. On the other hand, the diffracted rays penetrate into both, the shadow and lit zones of structures illuminated by an external incident wave field or by an antenna, thereby overcoming the failure of the GO ray method in the shadow zone. In its original form, the GTD fails at and near ray shadow boundaries and caustics where it exhibits singularities. In order to overcome the latter failure, a uniform version of the GTD, namely the UTD [1], was introduced to patch the GTD singularities across the ray shadow boundaries (SBs). This UTD employs the same GTD rays, and it not only remains valid within the SB transition regions where the GTD fails, but it also automatically reduces to the conventional GTD ray picture outside these ray shadow
boundary transition regions. This UTD therefore complements numerical solution methods which are tractable for treating radiating/scattering objects that are small to moderately large in terms of the wavelength. The UTD approach has been widely used with great success to analyze electrically large problems for over decades. One notes that complex and highly inhomogeneous electrically small antenna configurations can not be handled by the UTD and therefore require numerical methods for their accurate local modeling. Thus, numerical methods can be used “locally” for accurately modeling complex large or small antennas placed on large platforms in which the latter in turn can be modeled for more efficiently via UTD, thereby leading to a tractable hybrid numerical-UTD approach for calculating antenna-platform wave interactions [8].

Due to the local properties of high frequency fields, the diffracted rays propagate from specific points of diffraction (satisfying Keller’s extension of Fermat’s principle) on the radiating/scattering object, in a manner similar to points of reflection pertaining to GO reflected rays. Also, as in the GO description where the reflected GO ray fields are characterized by a Fresnel type surface reflection coefficient at the point of reflection, the diffracted ray fields are characterized by diffraction coefficients at the point of diffraction on the surface (or by launching/attachment coefficients at the source/receiver antenna from where diffracted rays can also be initiated/terminated). The local properties of high frequency wave phenomena allow these reflection, diffraction and launching/attachment coefficients to be found from the asymptotic solutions to simpler canonical problems which only need to model the local geometrical and electrical properties of the surface at points of reflection, diffraction, and launching/attachment, respectively. Thus, in contrast to the globally based conventional
purely numerical methods of analysis, the UTD analysis of the radiation by antennas on large complex platforms can be performed by locally identifying all the various dominant ray contributions that arrive at an observer from the antenna directly and also via the platform, and then by superposing the fields of these rays as seen in Fig. 1.1. Clearly, one must therefore not only find the relevant UTD (i.e., GO plus diffracted) ray paths which satisfy Keller’s generalization of Fermat’s principle, but one must also know all the relevant UTD (i.e., reflection and diffraction) coefficients accurately to arrive at an overall prediction of the antenna radiation and also antenna coupling (between different antennas) in the presence of the platform. The applicability of the UTD for analyzing the problems in Figs. 1.1 and 1.2 depends on the availability of all the relevant UTD coefficients necessary to model the platform effects accurately. Thus it is necessary to know all the UTD coefficients which are required for predicting the fields of antennas on material coated metallic platforms, that are modeled by approximately generalizing (by invoking the local properties of high frequency fields) the solutions to canonical material coated perfect electric conductor (PEC) configurations. In this work, the focus is primarily on the UTD analysis of EM diffraction by edges in material coated PEC planar and wedge configurations.

The specific canonical configurations of interest in the present work are shown in Figs. 1.3. In particular, Fig. 1.3(a) illustrates a two dimensional (2-D) junction between two thin, planar DPS/DNG grounded material slabs of different thickness and different electrical properties. Fig. 1.3(b) illustrates a 2-D thin, DPS/DNG material half plane. Also Fig. 1.3(d) shows a 2-D material coated PEC wedge. The three dimensional (3-D) skew incidence cases where the source can be a uniform plane wave, a uniform line source, or a current moment (a point source) as shown in Figs. 1.3(c)
and 1.3(e) are also of interest here. All of configurations of interest are surrounded by free space. The material thickness is assumed to be a small fraction (e.g., one tenth) of the free space wavelength. The material is assumed to be an isotropic and homogeneous DPS/DNG material.

Unfortunately, exact analytical solutions to the above problems of wave diffraction by discontinuities in material coated metallic wedge configurations are not currently available in a form suitable and tractable for engineering applications. Also as mentioned earlier, conventional numerical approaches for solving problems illustrated in Fig. 1.3 become highly inefficient and lack physical insight. In contrast, the present work which is based on a heuristic spectral synthesis method [9, 10], provides relatively simple and reasonably accurate closed form solutions that describe, in a physically appealing manner, the fields associated with the various UTD ray contributions arising in these configurations, namely, the GO, surface wave, and diffraction effects, respectively. It is noted that while the source may be near, or directly on, or far from the surface, it must remain at a distance which is somewhat larger than a free space wavelength from the edge or discontinuity. When the source is sufficiently far from the discontinuity and from the material surface, and when this source produces an incident field with a rapid spatial variation in which its radiation pattern exhibits a null either in the direction toward the edge (or the discontinuity) or close to it, then the UTD slope diffraction phenomena dominates over the first order UTD based diffraction effects. One notes that the first order UTD diffracted field is proportional to the value of the incident field at the discontinuity, while the slope diffraction is proportional to the derivative (slope) of the angular variation of the incident field at the discontinuity; hence, the slope effect alone remains dominant when the first
Figure 1.3: Canonical problems of interest
order diffraction either vanishes or becomes small in comparison. Furthermore, it is important to note that the first order UTD ray field directly incident from a source, which is placed either in or directly on a DPS/DNG material also vanishes along the material interface in accordance with the Karp-Karal lemma [11], irrespective of the source orientation, but again the slope of the field incident from the source in this case does not. Thus, in the latter case, which arises for antennas placed on a DPS/DNG material, the space wave excited directly by the source also undergoes slope diffraction at the discontinuity, in addition to surface wave diffraction which exists if that source is also able to directly excite a surface wave (SW) in the material. Clearly, slope diffraction effects are therefore very important in most practical antenna and scattering problems involving material discontinuities. It is noted that DPS materials typically excite forward surface waves (FSWs), while DNG materials can support backward surface waves (BSWs) [12]. BSWs are also included in the UTD solution developed below.

Most previous works in the literature dealing with the analytical solutions to diffraction by canonical material discontinuities [13–17] generally replace the original coated metallic surfaces or material slabs by approximate impedance or transmissive (for the geometry of Fig. 1.3(b)) boundary conditions, respectively. The latter approximation allows one to arrive at a rigorous analytical solution to the resulting approximate problem configuration. These previous works primarily address the scattering problem in which the illumination is a uniform plane wave that is incident on the thin material discontinuity. In contrast, the present work is expected to be very useful not only to the analysis of scattering situations but also to antenna problems which are equally importance from a practical standpoint. Moreover, slope diffraction
effects were not treated in almost all previous works on the problems of scattering by material discontinuities, since the illumination used therein was typically a uniform plane wave. The present work incorporates plane, cylindrical, spherical and surface wave illumination as well as sources which can be placed directly on the material coated metallic wedges. Among related previous works, which is described in [14] provides a solution to the problem of the plane wave diffraction by a two dimensional (2-D) impedance wedge. The later work in [15] allows a cylindrical wave illumination; also another paper [16] analyzes surface wave diffraction by the same geometry. All of these solutions [14–16] are based on the Maliuzhinets (MZ) method [13]. Some initial, useful, related work is discussed in [18] where the earlier work in [1] for a perfectly conducting wedge is generalized heuristically for constructing a UTD solution to describe the diffraction by a DPS material half plane; however, the resulting diffracted field is non reciprocal and does not satisfy the Karp-Karal lemma on the material half plane; also, the solution does not contain surface wave (SW) effects. In [19–22], the work of [18] was directly extended to study the approximate UTD scattering by a wedge with impedance boundary conditions; these solutions also suffer from the same limitations as those in [18]. An asymptotic UTD like 3-D solution based on a rigorous formulation has been obtained in [23] for the problem of EM wave diffraction by an impedance wedge with an interior wedge angle of $(2 - n)\pi$ (in which $n$ may not be necessarily be an integer) when it is illuminated by an obliquely incident plane wave. The formulation in [23] employs the MZ approach; however, it is shown in [23] that this rigorous formulation for the exterior problem is applicable only for three special wedge angles, namely a half plane ($n = 2$), an entire plane ($n = 1$) and a right angle wedge ($n = 1.5$). For the interior wedge problem, the work in [23] is
applicable to an interior angle which is $\frac{\pi}{2}$ (or $n = 0.5$). Other 3-D solutions based on the MZ method include the diffraction of an obliquely incident plane wave on a half plane with different face impedances [24], and the solution for the diffraction of an obliquely incident plane wave on a planar two part impedance surface [25, 26], and a solution for an obliquely incident plane wave diffracting from a right angle wedge with one face PEC and the other characterized by the impedance boundary condition [26–28]. The work in [25] does not explicitly provide a ray solution e.g. as in the UTD, and related formats, via an asymptotic high frequency approximation of the resulting complex integral representations for the fields arising in such diffraction problems. On the other hand, the works in [23, 24, 26–28] provide a high frequency ray solution via asymptotic reduction of the governing integral representation for the total field. Approximate MZ based solutions for 3-D wedge diffraction of an obliquely incident plane wave is given in [29, 30]; however, they are restricted primarily to small interior wedge angles. Recently another solution has been obtained in [31] for the problem of the diffraction of a plane wave at skew incidence on an impedance wedge; that solution is based on the MZ method, but it in turn involves the solution of a complicated Fredholm integral equation of the second kind thereby making it unwieldy for engineering applications. In addition, the solution in [31] is not a closed form solution because the numerical quadrature method is used to solve the Fredholm integral. At the time of this writing, it has come to the author’s attention that a new solution for the diffraction of a skew incident plane wave by a wedge with anisotropic surface impedance has very recently been obtained in [32]; this work in [32] also reduces the MZ problem to a Fredholm integral which is solved numerically. In contrast to previous works, the present UTD solutions can deal with a metallic wedge containing a
thin material coating which is not only different on the two faces, but without the use of an impedance boundary condition, and with plane, cylindrical as well as spherical wave illumination. In addition, the present work also includes slope diffraction effects which become important for antennas placed directly on metallic wedges with coatings.

Some additional solutions based on the W-H method, rather than MZ method are available for a plane wave diffraction by a thin DPS material half plane modeled using an approximate transmissive boundary condition in [33], and for a plane wave diffraction by a DPS half plane, and also by a junction formed by a thin, planar two-part DPS material backed by an infinite ground plane which was modeled approximately by higher order impedance boundary conditions on either side of the junction in [34], respectively. Also a W-H based analysis of the EM diffraction by an impedance discontinuity in a planar surface and by an impedance half-plane with an obliquely incident plane wave is developed in [17]. A W-H solution for an impenetrable wedge for a plane wave at skew incidence has been recently developed in [35] based on the generalized W-H equations. It is not possible to find a closed-form W-H factorization for a general wedge problem. The work in [35] employed the Fredholm integral equation of the second kind to approximate the W-H factorization problem. Again this leads to a solution in [35] which involves a numerical evaluation of the complicated Fredholm equation thus making the solution unwieldy for engineering applications. Unlike the W-H and MZ type solutions based on the impedance type approximation, the solutions developed in this work recover the proper, local plane wave Fresnel reflection and transmission coefficients (FRTCs), and surface wave constants, respectively, for the actual material, and they also allow for arbitrary wedge
angles and include important slope diffraction effects which allow for finite sources on or near such structures. The solutions developed in the present work appear to be versatile, accurate and useful for engineering applications, because they are essentially in closed form and are expressed in terms of ray fields which allow a physical picture for the radiation/scattering from such configurations.

In this dissertation, the approximate high frequency ray solutions to the 2D problems in Figs. 1.3(a) and 1.3(b) are formulated initially in terms of a cylindrical wave spectral (CWS) integral for the scattered field (satisfying the wave equation). First the spectral weight function in the CWS integral for the problem in Fig. 2.1 is synthesized heuristically based on an ansatz provided by the W-H solution to a special canonical problem of the plane wave diffraction by a two part impedance surface [34]. A bisection method, also described in [34], can be employed to directly synthesize a CWS integral for the problem in Fig. 1.3(b) in terms of the CWS integral for the problem in Fig. 1.3(a). The solutions to corresponding 3D problems in Figs 1.3(a) and 1.3(b) can be obtained by extending the 2D solution via an approach similar to that in [17]. In particular, the plane wave spectral (PWS) integral for the diffraction of an obliquely incident plane wave by a two part grounded material slab is first constructed from the ansatz provided by the W-H solution [17]. The Fourier transformation, also described in [36], can be used to synthesize a spherical wave spectral (SWS) integral in terms of the PWS integral so a point source illumination can be accommodated. It is important to note that the expressions for configurations in Figs. 1.3(a) and 1.3(b) are appropriately approximated via physical reasoning so that they can be made free of the complicated integral forms of the W-H split (or factorization) functions. Unfortunately, for the more complicated wedge configuration,
namely that of a material coated PEC wedge with arbitrary wedge angle illuminated
by obliquely incident plane waves and spherical waves, it has not been possible in this
present work to find an ansatz which allows one to have a closed-form solution that
are free of the MZ functions. Thus it was found necessary in this work to keep the
MZ functions in the first order diffraction solution for the coated PEC wedge case.
It is noted that since a direct numerical evaluation of the MZ functions can be diffi-
cult, approximate analytical expressions have been studied elsewhere [37–40]. Each
of these expressions in [37–40] have their own pros and cons. In the present work, it
is decided to employ each of the different expressions in [37–40] in separate regions,
i.e., over different parameters ranges of the MZ functions for which each remains
most accurate and efficient thereby allowing one to obtain an accurate and efficient
computer subroutine to calculate the required MZ functions. One can first obtain the
SWS integral for a material coated PEC wedge, with a spherical wave illumination,
from the CWS integral developed in [14] via inverse Fourier transformation [36]. The
resulting SWS integral for the 3-D EM case needs some additional modifications to
include the coupling of the $TE_z$ and $TM_z$ waves for the coated wedge with planar
faces where the edge is along the $\hat{z}$ direction, and to also satisfy other physical re-
quirements. Unlike the available MZ based solutions which utilize an approximate
impedance boundary condition on the wedge faces, the present closed-form solution
is allowed to retain the integrity of the actual thin material coating on the metallic
wedge as indicated before.

This dissertation is organized as follows. Chapter 2 summarizes the development
of approximate UTD ray solutions for the DPS/DNG material discontinuity problems
in Figs. 1.3(a) and 1.3(b), respectively, including the slope diffraction terms. The
UTD for the launching and diffraction of SWs is also discussed later in that Chapter. Furthermore, the UTD solution for the problem in Fig. 1.3(a) with a perfect magnetic conductor (PMC) (instead of PEC) ground plane is also included in that Chapter. In Chapter 3, the solutions in Chapter 2 are extended to treat the corresponding more general 3-D configuration to include point source (spherical wave) illumination. Chapter 4 provides details of the development of an asymptotic UTD solution for a material coated PEC wedge of Fig. 1.3(e) with an arbitrary wedge angle when it is illuminated by a spherical wave. Numerical results for the radiation and scattering from all the canonical problems of interest are calculated in Chapter 5 using the corresponding UTD solutions obtained in the earlier Chapters, and are shown to compare very well with the modified and extended W-H and MZ based reference solutions obtained from [17, 23, 33, 34] for those special cases in which the reference solutions are applicable; the latter had to be modified or extended because they were developed originally to deal with very thin DPS materials (via approximate boundary conditions) so that they could now also deal with DNG materials that are included in this work, as well as to allow line source excitation rather than just plane wave illumination in these reference solutions. Furthermore, the present UTD solution agrees well with an independent numerical method (Finite Element Boundary Integral (FE-BI)) for the case of an antenna in the presence of a thin finite material strip. Other numerical results are presented, for example, for the wedge with an arbitrary chosen interior wedge angle, but no comparisons have been included in these remaining cases because of unavailability of sufficiently general reference solutions. Finally, some conclusions and future work are given in Chapter 6. It is noted that
all the fields in this work are assumed to have an $e^{i\omega t}$ time dependence which is suppressed throughout this dissertation.
In this chapter, it is of interest to develop a UTD solution for the 2-D problems of wave diffraction by thin planar positive/negative material discontinuities with and without a ground plane backing. The ground plane can be either PEC or PMC. Plane wave and cylindrical wave illuminations are considered here; the plane wave is incident normal to the line (or edge) defining the discontinuity. The materials are assumed to be homogeneous and isotropic. An approximate UTD solution for the diffraction by a junction between two different planar material slabs on a PEC ground plane is developed first by using an ansatz based on the W-H solution to a related simpler problem. The corresponding solution for the diffraction by a material junction discontinuity on a PMC ground plane is then developed by using the duality theorem. By employing the bisection method, one can combine the aforementioned dual solutions (for the PEC and the PMC cases, respectively), to also obtain an approximate UTD solution for a thin material half plane without any ground plane.
2.1 Diffraction by a junction between two different planar material slabs on a PEC ground plane

The geometrical configuration of the problem of interest is shown in Fig. 2.1(a). An approximate UTD solution to the problem in Fig. 2.1(a) is obtained first in Section 2.1.1 for the case of plane wave incidence. This solution is based on a useful ansatz which is provided by a heuristic simplification of the W-H solution to a related special canonical two-part impedance diffraction problem as discussed in Section 2.1.1. The simplification in question involves the replacement of a term involving the W-H split functions with a constant term (unity). This replacement occurs within the steepest descent path (SDP) integral in W-H solution, and it greatly simplifies the solution. Next, in Section 2.1.2, the ansatz of Section 2.1.1 is extended to treat the case of line-source illumination.

2.1.1 Ansatz for the plane wave illumination case

For the sake of simplicity, and with no loss of generality, consider a special case of the geometry of Fig. 2.1(a) in which the $o$-face is defined to be a uniform surface impedance (or admittance) whose value is $Z_o^s$ (or $Y_o^s$) for the transverse magnetic (TM) (or transverse electric (TE)) case, respectively, while the $n$-face is just a PEC. The TE (or TM) terminology pertains to the case where the electric (or magnetic) field is transverse to the $x-y$ (or $z=0$) plane. Here, the $o$-face exists for $x > 0$ and $y = 0$, while the $n$-face exists for $x < 0$ and $y = 0$. When this special two-part geometry is illuminated by an incident, unit amplitude, plane wave field, $u_{pw}^i$, where $u_{pw}^i$ is the incident electric (or magnetic) field $\hat{z}E_z^i$ (or $\hat{z}H_z^i$) for the TE (or TM) case,
then the total field, \( u_{pw} \) for \( y > 0 \) (free space) may be expressed as

\[
\begin{align*}
u_{pw} &= u_{pw}^i + u_{pw}^s \\
\text{(2.1)}
\end{align*}
\]

where \( u_{pw} \) represents the total electric field \( \hat{z}E_z \) for the TE case (or the total magnetic field \( \hat{z}H_z \) for the TM case). The \( u_{pw}^s \) is the corresponding scattered field component corresponding to \( \hat{z}E_z^s \) (or \( \hat{z}H_z^s \)) for the TE (or TM) case. It is noted that

\[
\begin{align*}
u_{pw}^i &= e^{jk\rho \cos(\phi - \phi')} \\
\text{(2.2)}
\end{align*}
\]

From the W-H solution for the canonical two part problem in [34], the \( u_{pw}^s \) at \( P(\rho, \phi) \) is

\[
\begin{align*}
u_{pw}^s &= R^p_{e,h}(\phi')e^{jk\rho \cos(\phi + \phi')} + u_{pw}^s(\rho, \phi) \\
\text{(2.3)}
\end{align*}
\]
where \( k \) is free space wave number, and \( R_{e,h}^o \) is the \( o \)-face reflection coefficient, namely

\[
R_{e,h}^o(\phi') = \frac{\sin \phi' - \delta_{e,h}^o}{\sin \phi' + \delta_{e,h}^o}.
\] (2.4)

It is noted that the special case of plane wave illumination of the discontinuity results when the line source at \((\rho', \phi')\) as in Fig. 2.1(a) is allowed to recede to infinity (i.e. \( \rho' \to \infty \)). The first term on the right hand side (RHS) of (2.3) is chosen here to correspond to the field reflected from an “unperturbed” surface which is assumed to be an entire (infinite) plane at \( y = 0 \) characterized by the impedance \( Z_s^o \) (or admittance \( Y_s^o \)) for the TM (or TE) case. The \( \delta_{e,h}^o \) in (2.4) are defined by \( \delta_e^o = Y_s^o/Y_o \) and \( \delta_h^o = Z_s^o/Z_o \), respectively, where \( Z_o (= Y_o^{-1}) \) is the free space impedance. Thus, the second term, \( u_{pw}^p \), on the RHS of (2.3) constitutes a “perturbation” to the first term; it arises from the fact that the special geometry being considered is actually a two-part problem (of which one part, namely that for \( x < 0 \) and \( y = 0 \) is PEC) rather than just an “unperturbed” entire impedance (admittance) surface at \( y = 0 \).

From [34], one obtains

\[
 u_{pw}^p = -\frac{1}{2\pi j} \int_{C_\alpha} \left[ R_{e,h}^o(\phi') - R_{e,h}^o(\phi') \right] \Lambda_{e,h}(\alpha, \phi') e^{-jk\rho\cos(\alpha-\phi)} d\alpha
\] (2.5)

where

\[
\Lambda_e = \left\{ \frac{\sin \frac{\phi'}{2}}{\cos \frac{\phi'}{2}} G_e^c(k \cos \alpha) \right\} \left( +\frac{1}{2} \right) \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) + \sec \left( \frac{\alpha + \phi'}{2} \right) \right]
\] (2.6)

and

\[
\Lambda_h = \left\{ \frac{\cos \frac{\phi'}{2}}{\sin \frac{\phi'}{2}} G_h^c(k \cos \alpha) \right\} \left( -\frac{1}{2} \right) \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) - \sec \left( \frac{\alpha + \phi'}{2} \right) \right].
\] (2.7)

The contour \( C_\alpha \) in the complex angular spectral \( \alpha \) plane is shown in Fig. 2.2; it is chosen to satisfy the radiation condition. In the above, the reflection coefficient
\( R^n_e(\phi') = -1 \) and \( R^n_n(\phi') = 1 \) for the \( n \)-face because it is PEC in this canonical problem. The \( G_{e,h}^+ \) and \( G_{e,h}^- \) constitute the W-H factors (or split functions) of the functions,

\[
G^e(\alpha) = \frac{1}{k(\sin \alpha + Y_s/Y_o)}
\]

and

\[
G^h(\alpha) = \frac{\sin \alpha}{\sin \alpha + Z_s/Z_o}
\]

, respectively. The W-H factorization of \( G^{e,h}(\alpha) \) into \( G_{e,h}^+(\alpha)G_{e,h}^-(\alpha) \) leads to explicit expressions for the split functions in terms of an integral for each as given in [34]. The spectral integral in (2.5) may be evaluated asymptotically for large \( k\rho \) via the method of steepest descent. Thus, deforming \( C_\alpha \) into the steepest descent path (SDP) through the saddle point at \( \alpha \equiv \alpha_s (= \phi) \), as in Fig. 2.2, allows one to obtain

\[
u^p_{pw} = - \left[ R^o_{e,h}(\phi') - R^n_{e,h}(\phi') \right] \cdot e^{jk\rho \cos(\phi' + \phi')} U(\phi - \pi + \phi') - u^sw_{pw} U_{sw}
\]

\[- \frac{1}{2\pi j} \int_{SDP} \left[ R^o_{e,h}(\phi') - R^n_{e,h}(\phi') \right] \Lambda_{e,h}(\alpha, \phi') \cdot e^{-jk\rho \cos(\alpha - \phi)} d\alpha. \quad (2.8)\]

The first term involving \( \left[ R^o_{e,h} - R^n_{e,h} \right] \) on the RHS of (2.8) is \( 2\pi j \) times the residue arising from crossing the GO pole of \( \sec \left( \frac{\alpha + \phi'}{2} \right) \) at \( \alpha \equiv \alpha_{go} = \pi - \phi' \) in deforming \( C_\alpha \) to SDP. Likewise, the second term \( u^sw_{pw} \) is the surface wave (SW) field launched on the \( o \)-face via diffraction by the discontinuity at “0”; it is given by \( 2\pi j \) times the residue arising from a capture of the SW pole of \( G_{e,h}^-(k \cos \alpha) \) at \( \alpha = \alpha_{sw} \) in this contour deformation. The \( U(\cdot) \) is the Heaviside step function whose value is unity for positive arguments and zero for negative arguments. Also \( U_{sw} \) is a step function which is unity if a surface wave pole is captured; otherwise it is zero. It is noted that the right side of (2.8) is continuous even when the SDP (which moves with the observation point at \((\rho, \phi)\)) crosses the poles at \( \alpha = \alpha_{go} \) and \( \alpha = \alpha_{sw} \), respectively.
A heuristic approximation, based on a set of physical arguments enumerated below, can be introduced in (2.8) to remove the cumbersome W-H split functions $G_{e,h}$. In the vicinity of the RSB, where the saddle point $\alpha_s = \phi$ approaches $\pi - \phi'$, the dominant contribution to the SDP integral in (2.8) comes from a region where $\sec \left( \frac{\alpha + \phi'}{2} \right)$ approaches a singularity, and within this region the bracketed term that involves the ratio of the W-H split functions $\frac{G_{e,h}(k \cos \alpha_s)}{G_{e,h}(-k \cos \phi)}$ can be approximated by unity. Thus, one obtains a simplified form for $u_{pw}^p$ as follows:

$$u_{pw}^p \sim -\frac{1}{2\pi j} \int_{C_{\alpha}} d\alpha \left[ R_{e,h}^{o}(\alpha) - R_{e,h}^{n}(\alpha) \right] \left( \pm \frac{1}{2} \right) \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) \pm \sec \left( \frac{\alpha + \phi'}{2} \right) \right] e^{-jk\rho \cos(\alpha - \phi)}. \quad (2.9)$$

It is important to note that the $R_{e,h}^{o}(\phi')$ and $R_{e,h}^{n}(\phi')$ in (2.5) are now replaced by $R_{e,h}^{o}(\alpha)$ and $R_{e,h}^{n}(\alpha)$, respectively, in (2.9). The latter is necessary because $\alpha_{sw}$ is a pole of $G_{e,h}^-(\alpha)$ in the integrand of (2.5), and to preserve this important property in the approximate integrand of (2.9) (which is now devoid of $G_{e,h}^-(\alpha)$) it is necessary to have it manifest as a pole at $\alpha = \alpha_{sw}$ of the spectral reflection coefficient $R_{e,h}^{o}(\alpha)$ in (2.9) for the $o$-face. Of course, $R_{e,h}^{o}(\alpha) = \mp 1$, as before for the PEC $n$-face. Deforming $C_{\alpha}$ into the SDP contour allows one to express (2.9) as

$$u_{pw}^p \approx -\frac{1}{2\pi j} \int_{SDP} d\alpha \left[ R_{e,h}^{o}(\alpha) - R_{e,h}^{n}(\alpha) \right] \left( \pm \frac{1}{2} \right) \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) \pm \sec \left( \frac{\alpha + \phi'}{2} \right) \right] e^{-jk\rho \cos(\alpha - \phi)}. \quad (2.10)$$

One notes that the $\tilde{u}_{pw}^{sw}$ in (2.10) is now an approximation to $u_{pw}^{sw}$ of (2.8); likewise, the SDP integral in (2.10) is an approximation to the SDP integral of (2.8). Nevertheless, the approximation result in (2.10) contains the same GO pole contribution and the
surface wave propagation constant as does the exact W-H result in (2.8). Also, a closed form evaluation of the SDP integral in (2.10) via the non-uniform steepest descent method, yields the diffracted field \( \tilde{u}_{pw} \) given by

\[
\tilde{u}_{pw} \sim -\frac{e^{-j\pi/4}}{\sqrt{2\pi k}} \left[ R_{e,h}^o(\phi) - R_{e,h}^n(\phi) \right] \left( \pm \frac{1}{2} \right) \left[ \sec \left( \frac{\phi - \phi'}{2} \right) \pm \sec \left( \frac{\phi + \phi'}{2} \right) \right] e^{-jk\rho} \sqrt{\rho}, \quad (2.11)
\]

which still continues to satisfy the PEC boundary condition on the \( n \)-face and the Karp-Karal lemma on the \( o \)-face, respectively despite the approximations used to arrive at (2.9). Thus, the solution in (2.10) (and (2.11)), which is based on the approximate expression of (2.9), clearly retains many of the important physical properties which are present in the corresponding exact W-H result of (2.8), thereby lending more confidence to the heuristic approximation of (2.9). In contrast, a solution based on a Kirchhoff type approximation generally will not retain most of the above properties.

While \( u_{pw} \), based on the W-H method, satisfies reciprocity, the approximate diffracted \( \tilde{u}_{pw} \) of (2.11) does not; it will be shown later in Section 2.1.2 how reciprocity can be restored into \( \tilde{u}_{pw} \) in very simple fashion. As expected, the non-uniform results for \( u_{pw} \) and \( \tilde{u}_{pw} \), respectively, become unbounded at the RSB. Bounded results for these diffracted fields can be easily obtained in terms of the UTD Fresnel integral type transition functions via a uniform asymptotic evaluation of the SDP integrals. The latter uniform approach has not been incorporated above as it is not essential for arriving at the desired ansatz; it will be employed later in Section 2.1.2 when developing the UTD solution for the original problem in Fig. 2.1(a). The desired ansatz is now established by the set of equations (2.1)-(2.4) and (2.9), respectively.
2.1.2 Extension to treat the uniform line source excitation case

The problem treated below is that of a uniform line source excitation of a junction between two semi-infinite, thin, planar DPS/DNG material slabs of different electrical properties and thickness on a PEC ground plane as shown in Fig. 2.1. The incident, \( \hat{z} \)-directed, electric field, \( E_z^i \) (or the magnetic field, \( H_z^i \)) at an observer location \( \bar{\rho}(\rho, \phi) \), which is produced by a uniform electric (or magnetic) line source of strength \( I_o \) (or \( M_o \)) at \( \bar{\rho}'(\rho', \phi') \), respectively, can be expressed as \[41\]

\[
\begin{align*}
  u^i &\equiv \left\{ E_z^i \middle| H_z^i \right\} = -jk \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} G_o(k|\bar{\rho} - \bar{\rho}'|) \quad (2.12) \\
  G_o(k|\bar{\rho} - \bar{\rho}'|) &= \frac{-j}{4} H_o^{(2)}(k|\bar{\rho} - \bar{\rho}'|) \quad (2.13)
\end{align*}
\]

where \( H_o^{(2)} \) is a Hankel function of the second kind and order zero. The \( \bar{\rho} \) and \( \bar{\rho}' \) are shown in Fig. 2.1(a). For sufficiently large \( k\rho' \) (i.e. for source not close to the discontinuity at “0” which is assumed true), \( G_o \) may be replaced by its large argument form, namely

\[
G_o(ks^i) \sim \frac{-j}{4} \sqrt{\frac{2j}{\pi k}} e^{-jk s^i} \quad (2.14)
\]

where \( |\bar{\rho} - \bar{\rho}'| = s^i \), and

\[
s^i = \sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos(\phi - \phi')} \quad (2.15)
\]

The solution for the total field \( u \), which corresponds to \( \hat{z}E_z \) (or \( \hat{z}H_z \)) for the TE (or TM) case, for the problem Fig. 2.1(a), may be based on the ansatz established in (2.1)-(2.4) and (2.9) as described above. Following (2.1), one may express

\[
u = u^i + u^s \quad (2.16)
\]
where the scattered field $u^s$ for the geometry in Fig. 2.1(a) can be decomposed as in (2.3), if one assumes that the line source is sufficiently far from the $o$ and $n$ faces, respectively. Thus, under the latter assumption,

$$
u^s \sim -\frac{k}{4} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} R_{e,h}^{co}(\phi') \frac{e^{-jks'}}{s'} + u^p(\rho, \phi) \quad (2.17)$$

where, as in (2.3), the first term on the RHS of (2.17) represents the field scattered from the “unperturbed” structure, which is assumed to be an infinite planar structure consists of a thin PEC backed material that is identical (in its geometrical and electrical properties) to the original PEC backed material pertaining to the $o$-face in Fig. 2.1(a). Under the present assumption of source far from the surface at $y = 0$, one can show that the unperturbed scattered field is asymptotically given by the first term on the RHS of (2.17) which is the GO reflected field, where $R_{e,h}^{co}$ is the Fresnel reflection coefficient (FRC) for this unperturbed material surface with PEC ground plane, and $s'$ is the GO ray path corresponding to the GO field reflected from that unperturbed surface, where

$$R_{e,h}^{co}(\phi') = \frac{P_{e,h}^{co}(\phi')}{Q_{e,h}^{co}(\phi')}, \quad (2.18)$$

with

$$P_{e,h}^{co}(\phi') = \{[\sin \phi' - \eta_{e,h} N(\phi')] \mp [\sin \phi' + \eta_{e,h} N(\phi')]) e^{-j2k\tau_o N(\phi')}\} e^{j2k\tau_o \sin \phi'} \quad (2.19)$$

and

$$Q_{e,h}^{co}(\phi') = [\sin \phi' + \eta_{e,h} N(\phi') \mp [\sin \phi' - \eta_{e,h} N(\phi')]) e^{-j2k\tau_o N(\phi')} \quad (2.20)$$

Also,

$$N(\phi') = \sqrt{\mu_r \epsilon_r - \cos^2 \phi'} \quad (2.21)$$

$27$
In the above, \( \tau_o \) is the material thickness for the \( o \)-face, and \( \eta_e = 1/\mu_r \) for the TE (or \( e \)) case, while \( \eta_h = 1/\epsilon_r \) for the TM (or \( h \)) case, respectively. Also,

\[
s' = \sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos(\phi + \phi')}.
\] (2.22)

As in (2.3), the \( u^p(\rho, \phi) \) represents the “perturbation” to the first term on the RHS of (2.17); it arises from the fact that the actual geometry in Fig. 2.1(a) is composed of two different PEC backed materials on the \( o \) and \( n \) faces, instead of a single “unperturbed” surface. The \( u^p(\rho, \phi) \) can be expressed as a CWS integral [36] by

\[
u^p = -\frac{1}{2\pi j} \int_{C_\alpha} A_{e,h}(\alpha, \phi') G_0[ks(\alpha)] d\alpha.
\] (2.23)

In (2.23), the \( A_{e,h}(\alpha) \) is the appropriate spectral amplitude or weight function, and \( G_0[ks(\alpha)] \) denotes the CWS kernel based on the free space line source Green’s function, namely,

\[
G_0[ks(\alpha)] = -\frac{j}{4} H^{(2)}_o[ks(\alpha)]
\] (2.24)

with

\[
s(\alpha) = \sqrt{\rho^2 + \rho'^2 + 2\rho \rho' \cos(\alpha - \phi)}.
\] (2.25)

It is important to note that if the line source is not assumed to be sufficiently far from the \( o \) and \( n \) faces, then additional contributions (not present in (2.17)) must be included. Such additional contributions arise because the line source can excite SWs directly in the material; these SWs become incident on the discontinuity at “0” to produce a reflected SW and a transmitted SW, as well as a diffracted space wave. The reflected and transmitted SWs can be deduced from the W-H solution to appropriate, simpler, canonical two-part diffraction problems in which the excitation is an incident SW. In the radiation problem, these reflected and transmitted SW
effects are not significant. The latter will be reported in a separate paper. Only the
diffraction of the incident SW by the discontinuity contributes to the radiation field;
its effect is discussed separately in Section 2.4. The $G_o[ks(\alpha)]$ in (2.23) may now be
replaced by its large argument form valid for large $kp'$ (or $k\rho$) as

$$
G_o[ks(\alpha)] \sim -\frac{j}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jk\rho}}{\sqrt{s(\alpha)}}.
$$

(2.26)

The spectral function $A_{e,h}$ is proportional to the strength of the line source, and may
be expressed as

$$
A_{e,h}(\alpha, \phi') \equiv -jk \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} D_{e,h}^c(\alpha, \phi')
$$

(2.27)

where the unknown spectral weight $D_{e,h}^c$ is to be determined using the ansatz of
Section 2.1.1 based on the special canonical problem which retains all the features of
the original problem in Fig. 2.1(a). In order to identify $D_{e,h}^c$, the exponential in (2.26)
may be approximated by the first two terms of its binomial expansion for large $k\rho'$,
which is assumed here to be the large parameter (for the asymptotic development).

Then, (2.23) becomes

$$
u_p(\rho, \phi) = -\frac{1}{2\pi j} \int_{C_o} A_{e,h}(\alpha, \phi') \left( -\frac{j}{4} \sqrt{\frac{2j}{\pi k}} \right) e^{-jk(\rho+\rho')} e^{jk\rho' \rho' [1-\cos(\alpha-\phi)]} d\alpha.
$$

(2.28)

If the line source is allowed to receded to infinity, i.e., if $\rho' \to \infty$, while $\rho$ is kept finite,
then one obtains the scattered field $u_{pw}^p$ due to plane wave illumination, namely

$$
u_p^p(\rho, \phi) \sim c_o(k\rho') u_{pw}^p
$$

(2.29)

where $c_o$ is the line source factor given by

$$
c_o(k\rho') = -jk \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} -\frac{j}{4} \sqrt{\frac{2j}{\pi k}} \frac{e^{-jk\rho'}}{\sqrt{\rho'}}
$$

(2.30)
and
\[ u_{pw}^p = -\frac{1}{2\pi j} \int_{C_\alpha} \mathcal{D}_{e,h}^c(\alpha, \phi') e^{-jk\rho \cos(\alpha-\phi)} \, d\alpha. \] (2.31)

By directly comparing (2.31) with the desired ansatz in (2.9), one can easily identify \( \mathcal{D}_{e,h}^c \) by inspection to be
\[ \mathcal{D}_{e,h}^c(\alpha, \phi') = \pm \frac{1}{2} \left[ R_{e,h}^{co} - R_{e,h}^{cn} \right] \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) \pm \sec \left( \frac{\alpha + \phi'}{2} \right) \right], \] (2.32)
except that new material FRCs \( R_{e,h}^{co,cn} \) must now be used in (2.32) to replace \( R_{e,h}^{o,n} \) of (2.9) pertaining to the two part impedance boundary approximation of the W-H solution. The \( R_{e,h}^{cn}(\alpha) \) is defined in (2.18) with \( \phi' \) replaced by \( \alpha \) in (2.32), and \( R_{e,h}^{co}(\alpha) \) is likewise the spectral FRC for the \( n \)-face at \((x < 0, y = 0)\). Here \( R_{e,h}^{co,cn}(\alpha) \) is given by
\[ R_{e,h}^{co,cn}(\alpha) = \frac{P_{e,h}^{co,cn}(\alpha)}{Q_{e,h}^{co,cn}(\alpha)} \] (2.33)
where
\[ P_{e,h}^{co,cn}(\alpha) = \{ \sin \alpha - \eta_{e,h} N(\alpha) \} \mp \{ \sin \alpha + \eta_{e,h} N(\alpha) \} e^{-j2k\tau N(\alpha)} e^{j2k\tau \sin \alpha} \]
and
\[ Q_{e,h}^{co,cn}(\alpha) = \{ \sin \alpha + \eta_{e,h} N(\alpha) \} \mp \{ \sin \alpha - \eta_{e,h} N(\alpha) \} e^{-j2k\tau N(\alpha)} \]
with the material slab thickness \( \tau = \tau_o \) for the \( o \)-face and \( \tau = \tau_n \) for the \( n \)-face. It is noted that if one removes the material slab for \( x > 0 \) (or \( x < 0 \)), then the \( R_{e,h}^{co} \) (or \( R_{e,h}^{cn} \)) automatically reduces to \((\mp 1)\) for the PEC case pertaining to the \((e,h)\) polarization. The \( \eta_e = 1/\mu_r \) for TE \((e)\) case, \( \eta_h = 1/\epsilon_r \) for TM \((h)\) case, and \( N(\alpha) = \sqrt{\epsilon_r \mu_r - \cos^2 \alpha} \) as before. The \( R_{e,h}^{co,cn}(\alpha) \) term yields the proper material FRC for describing the GO reflected field from the residue of the GO pole at \( \alpha = \alpha_{go} = \pi - \phi' \) in (2.28) (together with (2.32)) where the function \( \sec(\frac{\alpha + \phi'}{2}) \) in \( \mathcal{D}_{e,h}^c(\alpha, \phi') \) becomes singular.
After deforming the integral contour of (2.28) to the steepest descent path (SDP) through the saddle point at $\alpha \equiv \alpha_s = \phi$ as shown in Fig. 2.2, one defines the SDP integral, which yields the diffracted field $u^d$, to be

$$u^d = \int_{SDP} \mathcal{F}_{e,h}(\alpha, \phi') e^{\kappa f(\alpha)} d\alpha,$$  

(2.34)

where

$$\mathcal{F}_{e,h}(\alpha, \phi') = \frac{k}{8\pi j} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} D_{e,h}^c(\alpha, \phi') \frac{e^{-jk(\rho + \rho')}}{s(\alpha)},$$

the $\kappa$ denotes $k \frac{\rho'}{\rho + \rho'}$, and $f(\alpha) = j[1 - \cos(\alpha - \phi)]$. It is noted that in (2.28) (together with (2.32)), the $\alpha = \alpha_{sw}$ marks the location of the SW pole where the denominator of $\mathcal{R}_{e,h}^{co,cn}$ in $D_{e,h}^c(\alpha, \phi')$ vanishes. This leads to an exact form of the transcendental or characteristic equation for the SWs which may be of the FSW or BSW type, respectively. Since the saddle point at $\alpha \equiv \alpha_s = \phi$ moves with the observation point, the pole at $\alpha_{go}$ is captured to provide a non zero GO reflected field $u^r$ where $u^r = u^r_{ro}$.
for the $o$-face when $\phi + \phi' < \pi$ and $u' = u'^o$ for the $n$-face when $\phi + \phi' > \pi$. The residue from the pole at $\alpha_{sw}$ yields either a FSW or a BSW field contribution, $u^{sw}$. The integral along the SDP in (2.34) may be evaluated asymptotically in a uniform fashion for large $\kappa$ to yield the UTD closed form expression for the diffracted field contribution $u^d$. It is desirable to decompose the spectral function in the integrand of (2.34) into a term containing only GO pole singularities $\mathcal{D}^{cgo}_{e,h}$ and a term containing only SW pole singularities $\mathcal{D}^{cw}_{e,h}$. In particular, (2.32) can be decomposed as $\mathcal{D}^c_{e,h} = \mathcal{D}^{cgo}_{e,h} + \mathcal{D}^{cw}_{e,h}$. Such a decomposition allows one to conveniently obtain the GO dominant UTD diffraction coefficient from the spectral part containing the GO type pole in a simple form using the Pauli-Clemmow (PC) approach [36], while the remainder spectral part can be treated by the Van der Waerden (VDW) approach [41]. The total field $u$ for corresponding DPS/DNG material configuration at an observation point $(P)$ or at $(\rho, \phi)$ may be expressed via (2.16), (2.17) and (2.34) as the sum of the classical line source incident field (with target absent) and scattered field, i.e., $u(\rho, \phi) = u^i + u^s$, in which $u^s = u^r + u^{sw} + u^d$. The $u^d$ denotes the first order diffracted field emanating from the material discontinuity at “0”, and $u^{sw}$ denotes the FSW/BSW field along the $o$ or $n$-face after being launched at “0”. Note that the classical incident field is given asymptotically (for $k\rho' \gg 1$) by (2.12) (together with (2.14)). Since $0 < \phi, \phi' < \pi$, the $u^i$ is also the GO incident field for $y > 0$. The reflected field $u^r$ is given by the sum of the “unperturbed” GO reflected field contained in the first term on the RHS of (2.17) and the pole contribution from $\alpha = \alpha_{go} = \pi - \phi'$ in (2.28) (together with (2.27) and (2.32)) given by $k \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} \left[ \Re^{co}_{e,h}(\phi') - \Re^{cn}_{e,h}(\phi') \right] U[\phi - (\pi - \phi')]$ as

$$u^r(\rho, \phi) = -\frac{k}{4} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} \Re^{c}_{e,h}(\phi') e^{-jkS_r} \sqrt{S_r}$$

(2.35)
where \( S_r = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi + \phi')} \), and

\[
R_{e,h}^c(\phi') = \begin{cases} 
\frac{P_{e,h}^o(\phi')}{Q_{e,h}^o(\phi')} & \text{if } \phi + \phi' < \pi \\
\frac{P_{e,h}^n(\phi')}{Q_{e,h}^n(\phi')} & \text{if } \phi + \phi' > \pi 
\end{cases}
\]  
\tag{2.36}

in which \( R_{e,h}^{co,cn} \) denotes the FRC. Also, \( u_{sw} \) is given by the pole contribution from \( \alpha = \alpha_{sw} \) to (2.28) as

\[
u_{sw}(\rho, \phi) = -\frac{k}{4} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} \left[ \frac{P_{e,h}^{swu}(\alpha_{sw}, \phi')}{\sqrt{s(\alpha_{sw}, \phi)}} U(\phi_{sw}^c - \phi) - \frac{R_{e,h}^{cswn}(\alpha_{sw}^c, \phi')}{\sqrt{s(\alpha_{sw}^c, \phi)}} U(\phi_{sw}^n - |\pi - \phi|) \right]
\]  
\tag{2.37}

where \( \phi_{sw}^{co,cn} \) (and \( \alpha_{sw}^{co,cn} \)) denote \( \phi_{sw}^c \) (and \( \alpha_{sw}^c \)) for the \((o, n)\) face. Here,

\[
\phi_{sw}^c = \begin{cases} 
\cos^{-1}\left(\frac{1}{\cosh \xi_{sw}}\right) & \text{for } \alpha_{sw}^c = -j\xi_{sw}^c \\
\pi - \cos^{-1}\left(\frac{1}{\cosh \xi_{sw}}\right) & \text{for } \alpha_{sw}^c = \pi + j\xi_{sw}^c 
\end{cases}
\]  
\tag{2.38}

It is noted that \( \alpha_{sw}^c = -j\xi_{sw}^c \) case is for FSW, and \( \alpha_{sw}^c = \pi + j\xi_{sw}^c \) is for BSW, respectively. Also,

\[
R_{e,h}^{swu}(\alpha_{sw}^c, \phi') = \pm \frac{P_{e,h}^o(\alpha_{sw}^c)}{2Q_{e,h}^o(\alpha_{sw}^c)} \left[ \sec \left( \frac{\alpha_{sw}^c - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{sw}^c + \phi'}{2} \right) \right].
\]  
\tag{2.39}

The \( Q_{e,h}^{o}(\alpha_{sw}) \) is the derivative of \( Q_{e,h}^{o}(\alpha) \) with respect to \( \alpha \) and evaluated at \( \alpha = \alpha_{sw}^c \). The \( U(\cdot) \) denotes the Heaviside unit step function as before. Also, \( R_{e,h}^{cswn}(\alpha_{sw}^n, \phi') \) is given by (2.39) with \( o \) replaced by \( n \), likewise, \( s(\alpha_{sw}^o, \phi) = \sqrt{\rho^2 + \rho'^2 + 2\rho\rho' \cos(\alpha_{sw}^o - \phi)} \) and \( s(\alpha_{sw}^n, \phi) \) can be found similarly. It is noted that the lowest SW mode, which is denoted here as the TM mode, has no low-frequency cutoff. There are several surface wave modes which can exist for sufficiently large value of the material thickness. However, throughout this work, one assumes that the material is thin (typically only a small fraction (e.g., one tenth) of the free space wavelength) in which \( \tau/\lambda < 0.25 \sqrt{\epsilon r \mu r - 1} \), so that only the dominant SW exists if the material is lossless.
The expressions for the UTD first order diffracted field is given by

\[ u^d(\rho, \phi) = u^i(0)D_{e,h}^c(\phi, \phi') e^{-jk\rho/\sqrt{\rho}} \]  

(2.40)

where \( D_{e,h}^c = D_{e,h}^{cgo} + D_{e,h}^{csw} \). Here \( u^i(0) \) denotes the GO incident field at the diffraction point corresponding to the discontinuity at “0” (see Fig. 2.1(a)). The \( D_{e,h}^{cgo} \) is based on the PC method and \( D_{e,h}^{csw} \) is based on the VDW method as explained previously; they are given by

\[ D_{e,h}^{cgo}(\phi, \phi') = \pm e^{-j\pi/4} \frac{2\sqrt{2\pi k}}{\sqrt{2\pi k}} \left[ \Gamma_{e,h}^{co}(\phi, \phi') - \Gamma_{e,h}^{cn}(\phi, \phi') \right] \left[ \sec \left( \frac{\phi - \phi'}{2} \right) F_{KP} \left( kLa^-_{go} \right) \right. 
\[ \left. \pm \sec \left( \frac{\phi + \phi'}{2} \right) F_{KP} \left( kLa^+_{go} \right) \right] \]  

(2.41)

where \( a^\pm_{go} = 2 \cos^2 \left( \frac{\phi + \phi'}{2} \right) \) and \( L = \frac{\rho'}{\rho + \rho} \). The function \( F_{KP}(x) \) is the well-known UTD edge transition function defined in [1]. The proper branch of \( \sqrt{kLa} \) is chosen such that \( -\frac{3\pi}{4} < \arg(\sqrt{kLa}) < \frac{\pi}{4} \), where \( a = a^\pm_{go} \), to satisfy the radiation condition.

The \( \Gamma_{e,h}^{co,\alpha}(\phi, \phi') \) is an ad hoc modification to \( \mathcal{R}_{e,h}^{co,\alpha}(\alpha) \) such that \( \sin \alpha \) in the latter is split into \( 2 \sin(\alpha/2) \sin(\phi'\alpha/2) \) so as to preserve reciprocity (symmetry) in \( D_{e,h}^{cgo} \) with respect to \( \phi \) and \( \phi' \) when \( \alpha = \phi \) at the saddle point, and to also let \( \Gamma_{e,h}^{co,\alpha}(\phi, \phi') \) reduces exactly to \( \mathcal{R}_{e,h}^{co,\alpha}(\alpha) \) at the GO reflection shadow boundary \( (\alpha = \phi = \pi - \phi') \) as it should. Thus,

\[ \Gamma_{e,h}^{co,\alpha}(\phi, \phi') = \frac{\zeta - \eta_{e,h}N}{\zeta + \eta_{e,h}N} \left[ \sec \left( \frac{\phi - \phi'}{2} \right) F_{KP} \left( kLa^-_{go} \right) \right. 
\[ \left. \pm \sec \left( \frac{\phi + \phi'}{2} \right) F_{KP} \left( kLa^+_{go} \right) \right] \]  

(2.42)

where \( \zeta = 2 \sin(\phi/2) \sin(\phi'/2) \), and \( N = \sqrt{\epsilon_r \mu_r} - 1 + \zeta^2 \).

\[ D_{e,h}^{csw}(\phi, \phi'; \alpha^c_{sw}) = \pm \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[ \frac{R_{e,h}^{cswo}(\alpha^c_{sw}, \phi')}{\sin \left( \frac{\alpha^c_{sw} - \phi}{2} \right)} [1 - F_{KP} \left( kLa^c_{sw} \right))] + d_{e,h}^{cswo}(\phi, \phi'; \alpha^c_{sw}) \right. 
\[ \left. + \frac{R_{e,h}^{cswn}(\alpha^c_{sw}, \phi')}{\sin \left( \frac{\alpha^c_{sw} - \phi}{2} \right)} [1 - F_{KP} \left( kLa^c_{sw} \right))] + d_{e,h}^{cswn}(\phi, \phi'; \alpha^c_{sw}) \right] \]  

(2.43)
\[ a_{sw}^{co} = 2 \sin^2 \left( \frac{\alpha_{sw} - \phi}{2} \right), \quad a_{sw}^{cn} = 2 \sin^2 \left( \frac{\alpha_{sw} + \phi}{2} \right). \]

Also
\[ c_{sw}^{co} = 2 \sin^2 \left( \frac{\alpha_{sw} - \phi}{2} \right), \quad c_{sw}^{cn} = 2 \sin^2 \left( \frac{\alpha_{sw} + \phi}{2} \right). \]

\[ \frac{d_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^{co})}{Q_{e,h}^{co}(\phi)} = \pm \frac{ \sec \left( \frac{\alpha_{sw} - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{sw} + \phi'}{2} \right) }{2} \] \quad (2.44)

likewise, \( d_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^{cn}) \) can be found by replacing \( o \) by \( n \) in (2.44). The solution for the plane wave excitation case can be obtained by letting \( \rho' \to \infty \) in the solution for the uniform line source illumination presented above.

### 2.2 Diffraction by a junction between two different planar material slabs on a PMC ground plane

The geometry of interest in this Section is a junction between two different planar material slabs, shown as in Fig. 2.1(a) except the ground plane is now PMC instead of PEC. Since the electric (or magnetic) field surrounding this geometry satisfies the same type of boundary conditions as the magnetic (or electric) field in Section 2.1.2, it follows that the UTD solution for a junction between two different planar material slabs on a PMC ground plane can be simply obtained from the results for the PEC ground plane case discussed in Section 2.1.2 by duality [42]. Thus, one can follow the same ansatz in Section 2.1.2 and replace \( D_{e,h}^{m} \) in (2.31) with \( D_{e,h}^{m} \), where

\[ D_{e,h}^{m}(\alpha, \phi') = \mp \frac{1}{2} \left[ R_{e,h}^{mo}(\alpha) - R_{e,h}^{mn}(\alpha) \right] \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) \pm \sec \left( \frac{\alpha + \phi'}{2} \right) \right]. \quad (2.45) \]

Here \( R_{e,h}^{mo,mm}(\alpha) \) is the appropriate FRC for material coating with PMC ground plane and given by

\[ R_{e,h}^{mo,mm}(\alpha) = \frac{P_{e,h}^{mo,mm}(\alpha)}{Q_{e,h}^{mo,mm}(\alpha)} \quad (2.46) \]

in which,

\[
\begin{align*}
P_{e,h}^{mo,mm}(\alpha) &= \{ [\sin \alpha - \eta_{e,h} N(\alpha)] \pm [\sin \alpha + \eta_{e,h} N(\alpha)] e^{-j2k\tau N(\alpha)} \} e^{j2k\tau \sin \alpha}
\end{align*}
\]
and
\[ Q_{e,h}^{mo,mn}(\alpha) = [\sin \alpha + \eta_{e,h} N(\alpha)] \pm [\sin \alpha - \eta_{e,h} N(\alpha)] e^{-j2kN(\alpha)} \]

It is noted that if one removes the material slab for \( x > 0 \) (or \( x < 0 \)), then the \( R_{e,h}^{mo} \) (or \( R_{e,h}^{mn} \)) automatically reduces to \((\pm 1)\) for the PMC case pertaining to the \((e,h)\) polarization. The \( R_{e,h}^{mo,mn}(\alpha) \) term yields the proper material FRC for describing the GO reflected field from the residue of the GO pole at \( \alpha = \alpha_{go} = \pi - \phi' \) in (2.28) (together with (2.45)) where the \( \sec(\frac{\alpha + \phi'}{2}) \) function in \( D_{e,h}^m(\alpha, \phi') \) becomes singular.

The scattered field for corresponding DPS/DNG material with PMC ground plane can be given as before by \( u^s = u^r + u^{sw} + u^d \). The \( u^r \) is given by (2.35) with \( R_{c,e,h} \) replaced by \( R_{e,h}^{m} \) as

\[
\begin{align*}
R_{e,h}^{m}(\phi') = \begin{cases} 
R_{e,h}^{mo}(\phi') = \frac{P_{e,h}^{mo}(\phi')}{Q_{e,h}^{mo}(\phi')} & \text{if } \phi + \phi' < \pi \\
R_{e,h}^{mn}(\phi') = \frac{P_{e,h}^{mn}(\phi')}{Q_{e,h}^{mn}(\phi')} & \text{if } \phi + \phi' > \pi 
\end{cases} 
\end{align*}
\] (2.47)

in which \( R_{e,h}^{mo,mn} \) denotes the appropriate FRC for the material with PMC ground plane. The \( u^{sw} \) is given by

\[
u^{sw}(\rho, \phi) = -\frac{k}{4} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} \left[ P_{e,h}^{mswo}(\alpha_{sw}^{mo}, \phi') \frac{e^{-jks(\alpha_{sw}^{mo}, \phi)}}{s(\alpha_{sw}^{mo}, \phi)} U(\phi_{sw}^{mo} - \phi) \\
- R_{e,h}^{mswn}(\alpha_{sw}^{mn}, \phi') \frac{e^{-jks(\alpha_{sw}^{mn}, \phi)}}{s(\alpha_{sw}^{mn}, \phi)} U(\phi_{sw}^{mn} - [\pi - \phi]) \right] \] (2.48)

where \( \phi_{sw}^{mo,mn} \) (and \( \alpha_{sw}^{mo,mn} \)) denote \( \phi_{sw}^{m} \) (and \( \alpha_{sw}^{m} \)) for the \((o, n)\) face. Here,

\[
\phi_{sw}^{m} = \begin{cases} 
\cos^{-1}\left(\frac{1}{\cosh \xi_{sw}^{m}}\right) & \text{for } \alpha_{sw}^{m} = -j\xi_{sw}^{m} \\
\pi - \cos^{-1}\left(\frac{1}{\cosh \xi_{sw}^{m}}\right) & \text{for } \alpha_{sw}^{m} = \pi + j\xi_{sw}^{m}. 
\end{cases} 
\] (2.49)

It is noted that \( \alpha_{sw}^{m} = -j\xi_{sw}^{m} \) case is for the FSW, and \( \alpha_{sw}^{m} = \pi + j\xi_{sw}^{m} \) is for the BSW, respectively. Also,

\[
R_{e,h}^{mswo}(\alpha_{sw}^{mo}, \phi') = \pm \frac{P_{e,h}^{mo}(\alpha_{sw}^{mo})}{2Q_{e,h}^{mo}(\alpha_{sw}^{mo})} \left[ \sec \left(\frac{\alpha_{sw}^{mo} - \phi'}{2}\right) \pm \sec \left(\frac{\alpha_{sw}^{mo} + \phi'}{2}\right) \right]. \] (2.50)

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The \( Q_{\alpha_{sw}}^m \) is the derivative of \( Q_{\alpha_{sw}}(\alpha) \) with respect to \( \alpha \) and evaluated at \( \alpha = \alpha_{sw} \). The \( R_{e,h}^{m_{sw}}(\alpha_{sw}, \phi') \) can be found from (2.50) by replacing \( o \) by \( n \). It is important to note that within the same thin material coating assumption of Section 2.1.2, one can expect that only the dominant TE surface wave can propagate in a material slab on a PMC ground plane.

The first order diffracted field \( u^d \) can be expressed as

\[
u^d(\rho, \phi) = u^i(0) D_{e,h}^m(\phi, \phi') e^{-j\rho} \]

(2.51)

where

\[
D_{e,h}^m = D_{e,h}^{m_{go}} + D_{e,h}^{m_{sw}}
\]

in which

\[
D_{e,h}^{m_{go}}(\phi, \phi') = \pm e^{-j\pi/4} \frac{2}{2\sqrt{2\pi k}} \left[ \Gamma_{e,h}(\phi, \phi') - \Gamma_{e,h}(\phi, \phi') \right] \left( F_{KP} \left( kL_{go}^{-} \right) \right) + \sec \left( \frac{\phi + \phi'}{2} \right) F_{KP} \left( kL_{go}^{+} \right) \]

(2.52)

with

\[
\Gamma_{e,h}^{mo, mn}(\phi, \phi') = \left[ \zeta - \eta_{e,h} \right] \pm \left[ \zeta + \eta_{e,h} \right] e^{-j2k_{o,n}N} e^{j2k_{o,n}N} \]

(2.53)

and

\[
D_{e,h}^{m_{sw}}(\phi, \phi'; \alpha_{sw}) = \pm e^{-j\pi/4} \frac{2}{2\sqrt{2\pi k}} \left[ \frac{R_{e,h}^{m_{sw}}(\alpha_{sw}, \phi')}{\sin \left( \frac{\alpha_{sw} - \phi}{2} \right)} \left[ 1 - F_{KP} \left( kL_{sw}^{mo} \right) \right] + d_{e,h}^{m_{sw}}(\phi, \phi'; \alpha_{sw}^{mo}) \right]
\]

\[
+ \frac{R_{e,h}^{m_{sw}}(\alpha_{sw}, \phi')}{\sin \left( \frac{\alpha_{sw} - \phi}{2} \right)} \left[ 1 - F_{KP} \left( kL_{sw}^{mn} \right) \right] + d_{e,h}^{m_{sw}}(\phi, \phi'; \alpha_{sw}^{mn}) \]

(2.54)

, where

\[
\alpha_{sw}^{mo} = 2 \sin^2 \left( \frac{\alpha_{sw} - \phi}{2} \right), \quad \alpha_{sw}^{mn} = 2 \sin^2 \left( \frac{\alpha_{sw} - \phi}{2} \right).
\]

Also

\[
d_{e,h}^{m_{sw}}(\phi, \phi'; \alpha_{sw}) = \frac{P_{e,h}^{m_{sw}}(\alpha_{sw})}{Q_{e,h}(\phi)} \left[ \sec \left( \frac{\alpha_{sw}^{mo} - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{sw}^{mo} + \phi'}{2} \right) \right] \]

(2.55)

likewise,

\[
d_{e,h}^{m_{sw}}(\phi, \phi'; \alpha_{sw}) \]

can be found from (2.55) by replacing \( o \) by \( n \).
2.3 Slope diffraction contribution

When the source is a line dipole as shown in Fig. 2.3, the dipole axis could be oriented to produce an incident field with a pattern null either in the direction of the discontinuity or close to it. In the latter case, the slope diffraction field dominates over the first order UTD diffracted field. It is thus important to extend the UTD solution (given in Section 2.1.2 for PEC ground plane and 2.2 for PMC ground plane) to include a slope diffraction term. To do so, let the line dipole source be of the magnetic type at \( P' \) whose density is given by \( \bar{M}_d = \hat{d} m_o \delta(x'' - x') \delta(y'' - y') \) with \( \hat{d} \times \hat{z} = 0 \), and \( m_o \) is a known constant. Note also that \( \hat{d} \cdot \hat{\rho}' = -\cos(\phi' + \phi_s) \) and \( \hat{d} \cdot \hat{\phi}' = -\sin(\phi' + \phi_s) \). One may invoke the reciprocity theorem to find the electric field \( \bar{E} \), which is produced by \( \bar{M}_d \), and thus directly use the solution developed in Section 2.1.2 for the problem of a junction between two different planar material slabs on a PEC ground plane or 2.2 for the problem of a junction between two different planar material slabs on a PMC ground plane to accomplish this task. Specifically, since the electric field \( \bar{E} \) from a magnetic line dipole source \( \bar{M}_d \) is entirely \( \hat{z} \)-directed, it can “react” with a \( \hat{z} \)-directed uniform electric line test source \( \bar{J}' = \hat{z} I'_o \delta(x'' - x) \delta(y'' - y) \) at \( P \), with \( I'_o \) being a known constant. The test source \( \bar{J}' \) produces the fields \( (\bar{E}', \bar{H}') \) in which \( \bar{E}' = \hat{z} \bar{E}' \) can be obtained from Section 2.1.2 or 2.2, i.e. \( \bar{E}'(\rho, \phi) = \bar{E}_i' + \bar{E}_s' \), where \( \bar{E}_i' \) and \( \bar{E}_s' \) represent the \( \hat{z} \)-directed incident and scattered fields, respectively. Now, one can find \( \bar{E}(\rho, \phi) \) from a knowledge of \( \bar{E}'(\rho', \phi') \) via the reciprocity (or “reaction”) theorem as:

\[
\iint_{\Omega} \bar{E} \cdot \bar{J}' \, ds'' = - \iint_{\Omega} \bar{H}' \cdot \bar{M}_d \, ds''
\]  

(2.56)
Figure 2.3: Line dipole source illumination of a junction between two different thin, planar DPS/DNG material slabs of different thickness on a PEC ground plane.

with $ds'' = dx'' dy''$, and the closed region $\Omega$ is defined for $(|x| < \infty, 0 < y < \infty)$ as shown in Fig. 2.3. Substituting for $\bar{J}'$ and $\bar{M}_d$ it then follows that

$$\bar{E}(\rho, \phi) \cdot \hat{z} = -m_o \frac{\hat{d} \cdot \bar{H}'}{P_o}.$$ 

A UTD field description for $\bar{E}$ can be written symbolically as

$$\bar{E}(\rho, \phi) \sim \bar{E}^i + \bar{E}^r + \bar{E}^{sw} + \bar{E}^{sd}$$

(2.57)

where the superscripts have the same meaning as in Section 2.1.2. The “tilde” on the diffracted field $\bar{E}^{sd}$ on the RHS of (2.57) is employed here to denote that it includes both ordinary plus slope diffraction effects. One obtains the $\bar{E}^i$, $\bar{E}^r$, $\bar{E}^{sw}$, and $\bar{E}^{sd}$ after evaluating the CWS integral for $\bar{H}'$ on the RHS of (2.56) asymptotically. As indicated above, $\bar{E}^{sd}$ contains a superposition of the first order diffracted UTD space wave field originating from “0”, and a slope diffracted UTD space wave field from
\[ E_z^d(\rho, \phi) = \left[ E_z^i(0) \left\{ D_e^{go}(\phi', \phi) + D_e^{sw}(\phi', \phi; \alpha_{sw}) \right\} + \frac{1}{jk\rho'} \frac{\partial}{\partial \phi'} E_z^i(0) \left\{ D_e^{sd}(\phi', \phi) + D_e^{swd}(\phi', \phi; \alpha_{sw}) \right\} \right] e^{-jk\rho/\sqrt{\rho}} \] (2.58)

where \( E_z^i(0) \) denotes the incident field at “0”, which is given by

\[ E_z^i(0) = -\frac{k}{4\pi} \sqrt{2} m_o \sin(\phi' + \phi_s) e^{-jk\rho'/\sqrt{\rho'}}. \]

In (2.58), the \( D_e^{go} \) and \( D_e^{sw} \) terms are the ordinary UTD contribution for the ray diffracted into space from “0” as discussed previously in (2.41) and (2.43) for PEC ground plane case (or (2.52) and (2.54) for PMC ground plane case), while the \( D_e^{sd} \) and \( D_e^{swd} \) refer to the slope effects (in the UTD ray context). In particular,

\[ D_e^{sd}(\phi', \phi) = -\frac{e^{-j\pi/4}}{4\sqrt{2\pi}k} \left\{ \Re_e^{o}(\phi) - \Re_e^{n}(\phi) \right\} \cdot \left\{ \frac{\sin\left(\frac{\phi' - \phi}{2}\right) - \sin\left(\frac{\phi' + \phi}{2}\right)}{\cos^2\left(\frac{\phi' - \phi}{2}\right)} F_s\left(kLa_{go}^-\right) + \frac{\sin\left(\frac{\phi' + \phi}{2}\right)}{\cos^2\left(\frac{\phi' + \phi}{2}\right)} F_s\left(kLa_{go}^+\right) \right\}, \] (2.59)

with \( F_s(\chi) = 2j\chi[1 - F_{KP}(\chi)] \). Also, the \( D_e^{swd} = D_e^{swdo} + D_e^{swdn} \) with

\[ D_e^{swdo}(\phi', \phi; \alpha_{sw}^o) = \frac{e^{-j\pi/4}}{2\sqrt{2\pi}k} \left[ R_e^{swdo}(\alpha_{sw}^o, \phi) \frac{1 - F_{KP}\left(kLa_{sw}^o\right)}{\sin\left(\frac{\alpha_{sw}^o - \phi}{2}\right)} + d_e^{swdo}(\phi', \phi; \alpha_{sw}^o) \right] \] (2.60)

where

\[ R_e^{swdo}(\alpha_{sw}^o, \phi) = -\frac{P_e(\alpha_{sw}^o)}{2\left[Q_e(\alpha_{sw}^o)^c\right]^2} \frac{Q_e(\alpha_{sw}^o)}{\sin\left(\frac{\alpha_{sw}^o - \phi}{2}\right)} \left\{ \sec\left(\frac{\alpha_{sw}^o - \phi}{2}\right) + \sec\left(\frac{\alpha_{sw}^o + \phi}{2}\right) \right\} \] (2.61)

\[ d_e^{swdo}(\phi', \phi; \alpha_{sw}^o) = -\frac{P_e(\alpha_{sw}^o)}{\left[Q_e(\phi')^c\right]^2} Q_e(\phi') \left\{ \sec\left(\frac{\alpha_{sw}^o - \phi}{2}\right) + \sec\left(\frac{\alpha_{sw}^o + \phi}{2}\right) \right\} \] (2.62)
Note that all of the terms corresponding to $n$-face are the same as for the $o$-face case with $o$ replaced by $n$, and the $(\epsilon_{ro}, \mu_{ro})$ for the $o$-face replaced by its material values $(\epsilon_{rn}, \mu_{rn})$ for the $n$-face.

If the magnetic line dipole source $\bar{M}_d$ in the above analysis is replaced by an electric line dipole source $\bar{J}_d$, then instead of $\bar{M}_d$ which radiates $\bar{E} = \hat{z} E_z$, the $\bar{J}_d$ will now produce a magnetic field $\bar{H} = \hat{z} H_z$, which is entirely $\hat{z}$ polarized. Hence, the test source in this case would have to be a $\hat{z}$-directed uniform magnetic line source of strength $M'_o$ at $P$ and the diffraction coefficients in (2.58) are now replaced by $D_{ho}^{go}$, $D_{ho}^{ew}$, $D_{ho}^{sd}$, and $D_{ho}^{swd}$, respectively. It is also important to note that the result obtained in (2.58) for the $\bar{M}_d$ case is equally applicable to the case of a uniform electric line source of strength $I_o$ when it is located directly on the material. In the latter case the $\tilde{E}_z^d$ in (2.58) is now produced by the slope diffraction of $E_z^i$ incident from $I_o$. This $E_z^i$ vanishes on the surface; hence, $\frac{\partial}{\partial \phi} E_z^i$ represents its slope which is non zero. Likewise, the result for the $\bar{J}_d$ case will be directly applicable to the case of a uniform magnetic line source of strength $M_o$ when it moves on to the material surface.

It is important to note that the slope effects in (2.59) and (2.60) can be obtained for both PEC and PMC ground plane cases since they are related by duality with appropriate substitution of the corresponding parameters for each ground plane case.

### 2.4 Surface wave diffraction

Expressions for the launching of conventional FSWs, and BSWs (on the material slabs of Fig. 2.1) due to diffraction at “0” of the wave incident from an external line source or line dipole source are presented above in Sections 2.1.2, 2.2 and 2.3, respectively. However, if the line source (or line dipole source) is placed very close to or
on the thin material slab as shown in Fig. 2.4(a), but far from “0”, then the source can noticeably and directly excite a FSW/BSW on the material slab as indicated earlier in Section 2.1.2 or 2.2. Such a FSW/BSW carries power directly from the source to the discontinuity at “0” from where it can be diffracted into space. This is particularly true for the DPS (or DNG) junctions. The latter interaction is simply reciprocal to the launching of an FSW/BSW on the slab by diffraction of the wave incident at “0” from the source which is off the slab surfaces at \( y = 0 \) as in Fig. 2.4(b). Hence, the diffraction of an FSW/BSW at “0” which launches a diffracted ray into space in Fig. 2.4(a) is found directly via reciprocity from the result given in Sections 2.1.2, 2.2 and 2.3, for the launching of an FSW/BSW along the material slab in Fig. 2.4(b) due to the diffraction at “0” of a wave incident from the source. This problem is useful in the design of surface wave antennas with DPS/DNG media.
2.5 Diffraction by a thin DPS/DNG material half plane

As indicated in the introduction, a bisection method as described in [34] can be employed to directly synthesize a CWS integral for the problem in Fig. 2.5 in terms of the CWS integral pertaining to the problem in Fig. 2.1. This bisection method requires the solution to the problem in Fig. 2.1 with the PEC ground plane as shown in Section 2.1.2, and also requires a second solution for the case when the PEC ground plane is replaced by a PMC as shown in Section 2.2. The solutions to the two PEC and PMC problems are then superposed as illustrated in [34] to obtain the diffraction by the material half plane as shown in Fig. 2.6. The total field can be expressed as

\[ u = u^i + u^r + u^{sw} + u^d, \]

where \( u^t \) denotes the transmitted electric field \( \hat{E}_z^t \) for the TE case (or the transmitted magnetic field \( \hat{H}_z^t \) for the TM case). The observation angle \( \phi \) and incident angle \( \phi' \) can now lie within the interval \( 0 \leq \phi \leq 2\pi \) and \( 0 < \phi' < 2\pi \), respectively. The reflected field \( u^r \) is given by

\[
u_r(\rho, \phi) = u_o \{ R_{e,h}(\phi')U(\phi - \phi' + \pi) + R_{e,h}(2\pi - \phi')U(\phi - 3\pi + \phi')\} e^{-jkS_r}/\sqrt{S_r} \tag{2.63}
\]

where \( u_o = -\frac{k}{4} \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \sqrt{\frac{2j}{\pi k}} \) and

\[
R_{e,h}(\phi') = \frac{P_{e,h}(\phi')}{Q_{e,h}(\phi')}, \tag{2.64}
\]

in which \( R_{e,h} \) denotes the FRC of material surface without the ground plane backing, where

\[
P_{e,h}(\phi') = [\sin^2 \phi' + \eta_{e,h}^2 N^2(\phi')][1 - e^{-j2krN}]e^{jkr \sin \phi'}
\]

and

\[
Q_{e,h}(\phi') = [\sin \phi' + \eta_{e,h} N]^2 - [\sin \phi' - \eta_{e,h} N]^2 e^{-j2krN}.
\]
The transmitted field is also given by

\[ u_t(\rho, \phi) = u_o \left\{ \mathcal{T}_{e,h}(\phi') U(\phi - \phi' - \pi) + \mathcal{T}_{e,h}(2\pi - \phi') U(\phi' - \phi - \pi) \right\} e^{-jkS} \sqrt{S} \]  

where

\[ \mathcal{T}_{e,h}(\phi') = \frac{P_{e,h}(\phi')}{Q_{e,h}(\phi')} \]  

in which \( \mathcal{T}_{e,h} \) denotes the Fresnel transmission coefficient (FTC) of material surface without the ground plane backing, and

\[ P_{e,h}(\phi') = 4 \sin \phi' \eta_{e,h} N e^{-jk\tau N} e^{jk\tau \sin \phi'} \]

The \( N \) and \( \eta_{e,h} \) are defined in Section 2.1.2. As mentioned in Sections 2.1.2 and 2.2, one can obtain the surface wave field for the TM case \( \hat{z}H_{sw}^z \) from (2.37) and the surface wave field for the TE case \( \hat{z}E_{sw}^z \) from (2.48). Thus the surface wave field
Figure 2.6: Relation between material two-part problem and the material half plane problem.
\( u^{sw} \) propagating on the material half plane without any ground plane backing is given by

\[
\begin{align*}
    u^{sw}(\rho, \phi) &= \frac{u_o}{2} R^{sw}_{e,h}(\alpha^{m,c}_{sw}, \phi') \left[ \frac{e^{-jk s(\alpha^{m,c}_{sw}, \phi)}}{\sqrt{s(s^{m,c}_{sw}, \phi)}} U(\phi^{m,c}_{sw} - \phi) 
    
    + \frac{e^{-jk s(\alpha^{m,c}_{sw}, -\phi)}}{\sqrt{s(s^{m,c}_{sw}, -\phi)}} U(\phi^{m,c}_{sw} + \phi - 2\pi) \right] 
\end{align*}
\] (2.67)

It is noted that \( R^{sw}_e = R^{mswo}_e \) from (2.50) and \( R^{sw}_h = R^{csw}_h \) from (2.39). The expressions for the UTD first order diffracted field is given by

\[
    u^d(\rho, \phi) = u^i(0) D_{e,h}(\phi, \phi') e^{-jk \rho} / \sqrt{\rho} 
\] (2.68)

where

\[
    D_{e,h}(\phi, \phi') = \frac{D^e_{e,h}(\phi, \phi')}{2} + \frac{D^m_{e,h}(\phi, \phi')}{2} 
\] (2.69)

in which \( D^e_{e,h} = D^{ego}_{e,h} + D^{csw}_{e,h} \) and \( D^m_{e,h} = D^{mgo}_{e,h} + D^{msw}_{e,h} \) as defined previously in (2.40), (2.41), (2.43), (2.51), (2.52) and (2.54). One can then express the diffraction coefficient for a thin material half plane as \( D_{e,h} = D^{ego}_{e,h} + D^{sw}_{e,h} \), where

\[
    D^{ego}_{e,h}(\phi, \phi') = \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left\{ 1 - T_{e,h}(\phi, \phi') \right\} \sec \left( \frac{\phi - \phi'}{2} \right) F_{KP} \left( k \alpha_{go}^- \right) + \Gamma_{e,h}(\phi, \phi') \sec \left( \frac{\phi + \phi'}{2} \right) F_{KP} \left( k \alpha_{go}^+ \right), 
\] (2.70)

in which

\[
    T_{e,h}(\phi, \phi') = \frac{4\zeta_\eta e, h N e^{-jk\pi N e^{jk\pi \zeta}}}{\zeta + \eta e, h N} \left[ 1 - \left( \frac{\zeta}{\eta e, h N} \right)^2 \right] e^{-2jk\pi N} \] (2.71)

and

\[
    \Gamma_{e,h}(\phi, \phi') = \frac{\left[ \zeta^2 - \eta^2 e, h N^2 \right]}{\zeta + \eta e, h N} \left[ 1 - e^{-2jk\pi N} \right] e^{jk\pi \zeta} \frac{e^{-2jk\pi N}}{\zeta - \eta e, h N} \] (2.72)

It is noted that the \( T_{e,h}(\phi, \phi') \) and \( \Gamma_{e,h}(\phi, \phi') \) reduce exactly to \( T_{e,h}(\phi') \) and \( \mathcal{R}_{e,h}(\phi') \) at the GO reflection shadow boundary (\( \alpha_i = \pi + \phi' \) and \( \alpha_r = \pi - \phi' \)), respectively.
Also,

\[
D_{e,h}(\phi, \phi'; \alpha_{sw}) = \mp \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[ \frac{R_{e}^{sw}(\alpha_{sw}^m, \phi')}{\sin(\alpha_{sw}^m - \phi)} [1 - F_{KP}(kL\alpha_{sw}^m)] + d_{e}^{sw}(\phi, \phi'; \alpha_{sw}^m) \right] \\
+ \frac{R_{h}^{sw}(\alpha_{sw}^c, \phi')}{\sin(\alpha_{sw}^c - \phi)} [1 - F_{KP}(kL\alpha_{sw}^c)] + d_{h}^{sw}(\phi, \phi'; \alpha_{sw}^c) \right] 
\]

(2.73)

where \(\alpha_{sw}^c = 2\sin^2\left(\frac{\alpha_{sw}^c - \phi}{2}\right)\), \(\alpha_{sw}^m = 2\sin^2\left(\frac{\alpha_{sw}^m - \phi}{2}\right)\). Also

\[
d_{e,h}(\phi, \phi'; \alpha_{sw}^{m,c}) = \left[ -\frac{P_{e,h}^{r}(\alpha_{sw}^{m,c})}{Q_{e,h}(\phi)} \sec\left(\frac{\alpha_{sw}^{m,c} - \phi}{2}\right) + \frac{P_{e,h}^{r}(\alpha_{sw}^{m,c})}{Q_{e,h}(\phi)} \sec\left(\frac{\alpha_{sw}^{m,c} + \phi}{2}\right) \right].
\]

(2.74)

Finally the UTD slope diffraction contribution may be obtained for the problem in Fig. 2.5 via reciprocity in a manner similar to that obtained earlier in Section 2.3 for the problem in Fig. 2.1. Furthermore, by employing reciprocity property of the solution as shown in Section 2.4, one can simply obtain the surface wave diffraction for the material half plane without the ground plane backing. However, the reflected field \(u^r\) which is a function of the observation angle \(\phi\) has a fast spatial variation as shown in Fig. 2.7(a). This causes a “kink” in the scattered field plot at the reflection shadow boundaries (RSBs) as shown in Fig. 2.7(b). This is due to the fact that the slope diffraction contribution obtained in Section 2.3 can deal only with the fast spatial variation of the incident field and this slope contribution vanishes in the present case where the incident field is produced by a uniform line source and thus has no spatial variation in its radiation pattern. Obviously therefore the slope diffraction solution from Section 2.3 can not treat such a fast spatial variation of the reflected field which is present at the RSB. Thus it is necessary to treat the slope diffraction of the incident and reflected fields separately; however, this is easily done since the reflected field can be considered to be like an incident field on the edge which arrives from an image.
source. Such a separation is not necessary for the case of the problem in Section 2.3 because the reflected field has a slow spatial variation there, thus one needs to treat the spatial variation of only the incident field in that particular problem. It is noted that a similar idea is applied for the slope diffraction on a curved surface as explained in [43].

Thus the slope diffracted field \( u^{sd} \) for the present problem can be expressed as

\[
    u^{sd}(\rho, \phi) = \frac{1}{jk\rho^2} \left[ \left\{ \frac{\partial}{\partial \phi'} u_z^i(0) \right\} d_{e,h}^{sd} \right. \\
    \left. - \left\{ \frac{\partial}{\partial \phi'} u_z^r(0) \right\} d_{e,h}^{sd} \right] e^{-jk\rho} \tag{2.75}
\]

in which \( d_{e,h}^{sd} \) and \( d_{e,h}^{sd} \) are the slope diffraction coefficients for the incident and reflected GO fields, respectively, to account for a rapid spatial of both these GO fields as discussed above. In particular,

\[
    d_{e,h}^{sd} = -\frac{e^{-j\pi/4}}{4\sqrt{2\pi k}} \{1 - \mathcal{T}_{e,h}(\phi)\} \frac{\sin \left( \frac{\phi'-\phi}{2} \right)}{\cos^2 \left( \frac{\phi'-\phi}{2} \right)} \mathcal{F}_{s} \left( kLa_{go}^\pm \right) \tag{2.76}
\]

\[
    d_{e,h}^{sd} = -\frac{e^{-j\pi/4}}{4\sqrt{2\pi k}} \mathcal{R}_{e,h}(\phi) \frac{\sin \left( \frac{\phi'+\phi}{2} \right)}{\cos^2 \left( \frac{\phi'+\phi}{2} \right)} \mathcal{F}_{s} \left( kLa_{go}^\pm \right) \tag{2.77}
\]

By applying (2.75) together with (2.68), one can now obtain, in contrast to the non-smooth result in Fig. 2.7, a smooth and continuous scattered field as shown in Fig. 2.8.
Figure 2.7: Thin, DPS Material strip with $\lambda/10$ and $(\epsilon_r = 3.4, \mu_r = 2)$ illuminated by a line source at $\rho' = 0.5\lambda$. The observation point is in the far zone.
Figure 2.8: Thin, DPS Material strip with $\lambda/10$ and $(\varepsilon_r = 3.4, \mu_r = 2)$ illuminated by a line source at $\rho' = 0.5\lambda$. The observation point is in far zone.
CHAPTER 3

ANALYSIS OF 3-D EM WAVE DIFFRACTION AT A JUNCTION BETWEEN TWO DIFFERENT THIN PLANAR MATERIAL SLABS ON A PEC GROUND PLANE

In this chapter, it is of interest to extend the normal incidence solution as discussed in Chapter 2 in order to treat the more general case of skew (or oblique) incidence (three-dimensional 3D). Plane wave (for oblique or skew incidence) and spherical wave illumination are considered here. The geometry of the problem is shown in Fig. 3.1. It is well known that the normal field components $E_y$ and $H_y$ satisfy the Helmholtz scalar equation and impedance boundary conditions independently. Hence, it is convenient to first begin the ansatz, based on the simplification of the W-H solution [17] for the normal field components in the case of a unit amplitude, plane wave at skew incidence as done in Section 3.1 below for the related problem in Fig. 3.1 where the $n$-face ($x < 0, y = 0, z$) is assumed to be a PEC for now. One can then use the ansatz developed for this plane wave excitation case to subsequently treat the spherical wave excitation case as described next in Section 3.2.
3.1 Ansatz for the obliquely incident plane wave illumination case with one face being PEC

Following the approach in Chapter 2, one can develop a UTD solution for the diffraction of a plane wave at skew (or oblique) incidence on a thin, planar material half plane on an entire PEC ground plane shown in Fig. 3.1, by employing a similar ansatz for the normal field components $E_y$ and $H_y$. Each of these normal field components independently satisfy the Helmholtz scalar equation and impedance boundary conditions as shown in [17] and [30]. This leads to a decoupled solution separately for $E_y$ and $H_y$. Thus it is convenient to start an ansatz, based on the simplification of a related effective 2-D W-H solution [17] for the normal field components when it is applied to the special case in Fig. 3.1. The normal components of total field for $y > 0$ (free space) for the problem of interest may be expressed as

$$\bar{U}_y = \bar{U}_y^i + \bar{U}_y^s \quad (3.1)$$

where $\bar{U}_y$ denotes $\hat{y} \begin{bmatrix} E_y \\ \eta_o H_y \end{bmatrix}$. Here $E_y$ represents the total electric field for the TE case and $H_y$ represents the total magnetic field for the TM case. Also $\eta_o$ is the intrinsic impedance of free space as before. The $\bar{U}_y^s$ is $\hat{y} \begin{bmatrix} E_y^s \\ \eta_o H_y^s \end{bmatrix}$. Note that $E_y^s$ (or $H_y^s$) is the $\hat{y}$-directed electric (or magnetic) scattered field. The incident uniform plane wave $\bar{U}_y^i$ is $\hat{y} \begin{bmatrix} E_y^i \\ \eta_o H_y^i \end{bmatrix}$, where $E_y^i$ (or $H_y^i$) is the $\hat{y}$-directed electric (or magnetic) incident field, which is given by

$$\bar{U}_y^i = \bar{U}_{oy} e^{jk'_x x + jk'_y y + jk'_z z} \quad (3.2)$$

where $\bar{U}_{oy}$ denotes $\hat{y} \begin{bmatrix} E_{oy} \\ \eta_o H_{oy} \end{bmatrix}$. The $E_{oy}$ and $H_{oy}$ are assumed to be unity for convenience. The $k'_x$, $k'_y$, and $k'_z$ are given by

$$k'_x = k \sin \beta'_o \cos \phi' ; \quad k'_y = k \sin \beta'_o \sin \phi' ; \quad k'_z = k \cos \beta'_o \quad (3.3)$$
Figure 3.1: Thin, planar DPS/DNG material half plane on an entire PEC ground plane illuminated by a skew incident plane wave excitation.

with \( 0 < \beta_o' < \pi \) and \( 0 \leq \phi' \leq \pi \). Following the form of the W-H solution for the canonical two part problem in [17], the scattered field \( \vec{U}^s_y \) can also be expressed as

\[
\vec{U}^s_y = \tilde{\mathbf{R}}^o(\phi') \vec{U}_{oy} e^{j k'_x x - j k'_y y + j k'_z z} + \vec{U}^p_y
\]  

(3.4)

where \( \tilde{\mathbf{R}}^o(\phi') \) is the \( o \)-face FRC, namely

\[
\tilde{\mathbf{R}}^o(\phi') = \begin{bmatrix} \mathcal{R}^o_e(\phi') & 0 \\ 0 & \mathcal{R}^o_h(\phi') \end{bmatrix}
\]  

(3.5)

where

\[
\mathcal{R}^o_{e,h}(\phi') = \frac{P^o_{e,h}(\phi')}{Q^o_{e,h}(\phi')}
\]  

(3.6)

with

\[
P^o_{e,h}(\phi') = \sin \phi' - \delta^o_{e,h}/ \sin \beta_o'
\]  

(3.7)
and

\[ Q^o_{e,h}(\phi') = \sin \phi' + \delta^o_{e,h} / \sin \beta'_o \] (3.8)

where

\[ \delta^o_e = -jY_d\eta \cot(N\tau d) \quad , \quad \delta^o_h = jZ_d\eta \tan(N\tau d) \] (3.9)

with \( k_d = k\sqrt{\epsilon_r\mu_r}, \ Z_d = \sqrt{\mu_r/\epsilon_r}, \ Y_d = 1/Z_d, \ N = \sqrt{1 - \eta^2 \sin^2 \beta'_o \sin^2 \phi'} \) and \( \eta = 1/\mu_r\epsilon_r \). The first term on the RHS of (3.4) is chosen here to correspond to the field reflected from an “unperturbed” surface of a thin material slab of infinite extent on a PEC plane with the same material and thickness as that on the \( o \)-face \((x > 0, y = 0, z)\). The second term on the RHS of (3.4) constitutes “perturbation” to the first term which results from the fact that the actual problem in Fig. 3.1 contains a PEC for the \( n \)-face \((x < 0, y = 0, z)\). The “perturbed” field \( \tilde{U}^p_y \) can be expressed in a manner similar to that done earlier in Chapter 2 for the 2-D case. One can start to rewrite (2.9) as

\[ u^p_z(\rho, \phi) \approx -\frac{1}{2\pi j} \int_{C_o} d\alpha (\alpha) \left[ \frac{u_{oz}}{\cos \alpha + \cos \phi'} \right] e^{-jk_D \rho \cos(\alpha - \phi')} \] (3.10)

where the following identity has been employed, namely:

\[ \sec \left( \frac{\alpha + \phi'}{2} \right) \mp \sec \left( \frac{\alpha - \phi'}{2} \right) = \frac{4}{\cos \alpha + \cos \phi'} \left\{ \sin \alpha / 2 \sin \phi' / 2 \right\} \] (3.11)

with

\[ \Re_e(\alpha) = [\Re^o_e(\alpha) - \Re^n_e(\alpha)] 2 \cos \alpha / 2 \cos \phi' / 2 \]

\[ \Re_h(\alpha) = [\Re^o_h(\alpha) - \Re^n_h(\alpha)] 2 \sin \alpha / 2 \sin \phi' / 2 \]

and \( u_{oz} \) is \( E_{oz} \) (or \( H_{oz} \)) for the TE (or TM) case. Then the integral in (3.10) can be expressed in the \( \tilde{k}_x \) plane (rectangular coordinate system) as

\[ u^p_z(x, y) \approx -\frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{d\tilde{k}_x}{k_y} (\alpha) \left[ \frac{u_{oz}}{k_x + k'_x} \right] e^{-jk_x x - jk_y y}. \] (3.12)
in which \( \tilde{k}_x = k \cos \alpha, \tilde{k}_y = k \sin \alpha \) and \( \tilde{k}'_x = k \cos \phi' \). The \( u_p \) denotes the perturbed field corresponding to \( E_p \) (or \( H_p \)) for TE (or TM) and the \( u_{oz} \) denotes \( E_{oz} \) (or \( H_{oz} \)) for TE (or TM) which is assumed to be unity here. Next one can obtain the 2-D normal component of the perturbed field \( u_p^y \) from \( u_p^z \) in (3.12) by using Maxwell’s equations, namely:

\[
E_{p_y}^p(x, y) = -\frac{1}{jk_{o}} \frac{\partial}{\partial x} H_{p_z}^p(x, y) \quad ; \quad H_{p_y}^p(x, y) = \frac{1}{jk_{o}} \frac{\partial}{\partial x} E_{p_z}^p(x, y).
\]

One can then employ the inverse Fourier transform and with \( k \) replaced by \( k_t = k \sin \beta_o \). The 3-D PWS integral can be conjectured from 2-D PWS integral as

\[
\tilde{U}_y^p(x, y, z) \sim -\frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{dk_x}{k_y} \tilde{R}(\alpha) \cdot \left[ \frac{\tilde{U}_{oy}^p}{k_x + k'_x} + \tilde{W} \right] e^{(-jk_x x - jk_y y + jk'_z z)} \quad (3.13)
\]

where \( \tilde{U}_y^p \) denotes \( \hat{y} \left[ \begin{array}{c} E_{p_y}^p \\ \eta_0 H_{p_y}^p \end{array} \right] \). The \( k_x \) and \( k_y \) are given by

\[
k_x = k \sin \beta_o \cos \alpha \quad ; \quad k_y = k \sin \beta_o \sin \alpha.
\]

The \( \tilde{W} \) is an unknown constant column vector which was absent in the 2-D situation and \( \tilde{W} \) is \( \hat{y} \left[ \begin{array}{c} A \\ B \end{array} \right] \). It is necessary to introduce this unknown constant in this 3-D situation to suppress the nonphysical poles, which may occur in the tangential field components \( E_z \) and \( H_z \) for the skew incidence case. In addition, this unknown constant \( \tilde{W} \) will make the 3-D PWS integral in (3.13) to recover the 2-D PWS integral when \( \beta_o' \rightarrow \pi/2 \) or at normal incidence. This unknown constant \( \tilde{W} \) will be determined later. It is noted that one can obtain the same PWS integral as shown in (3.13) if the same ansatz as explained in Section 2.1.1 is used to heuristically synthesize the PWS integral from the available 3-D W-H in [17]. The \( \tilde{R} \) in (3.13) is given by

\[
\tilde{R}(\alpha) = \begin{bmatrix} \mathcal{R}_h(\alpha) & 0 \\ 0 & \mathcal{R}_e(\alpha) \end{bmatrix}
\quad (3.14)
\]
In the above, the reflection coefficient \( R_n(\alpha) = -1 \) and \( R_h(\alpha) = 1 \) for the \( n \)-face because it is PEC in this special canonical problem. It is important to note that the \( \bar{U}_p(x, y, z) \) in (3.13) can be recovered from the \( u_p(x, y) \) when the plane wave is normal incident (\( \beta'_o = \pi/2 \)) where \( \cos \beta'_o = \hat{s} \cdot \hat{z} \). From (3.13), one can easily obtain the vector potentials \( \bar{A} = \hat{y}A_y \) and \( \bar{F} = \hat{y}F_y \) directly in terms of \( E_p \) and \( H_p \) from the usual relations between fields and potentials [42], namely:

\[
A_y = \frac{j\omega \mu_o \epsilon_o}{k^2 - k_y^2} E^p_y ; \quad F_y = \frac{j\omega \mu_o \epsilon_o}{k^2 - k_y^2} H^p_y
\]

for the problem in Fig. 3.1. Thus, from (3.13), one can obtain

\[
A_y = -\frac{\omega \mu_o \epsilon_o}{2\pi} \int_{-\infty}^{\infty} \frac{dk_x}{k_y} R_h(\alpha) \left[ \frac{E_{oy}}{k_x + k'_x} + A \right] \frac{1}{k^2 - k_y^2} e^{(-jk_x x - jk_y y + jk'_z z)} \quad (3.15)
\]

and

\[
F_y = -\frac{\omega \mu_o \epsilon_o}{2\pi \eta_o} \int_{-\infty}^{\infty} \frac{dk_x}{k_y} R_e(\alpha) \left[ \frac{\eta_o H_{oy}}{k_x + k'_x} + B \right] \frac{1}{k^2 - k_y^2} e^{(-jk_x x - jk_y y + jk'_z z)} \quad (3.16)
\]

where \( \epsilon_o \) and \( \mu_o \) are the permittivity and permeability of free space as usual. Next all the remaining components of the external electric and magnetic fields can be found from these vector potentials by using

\[
\vec{E} = \frac{1}{j\omega \epsilon_o \mu_o} \nabla \times \nabla \times \bar{A} - \frac{1}{\epsilon_o} \nabla \times \bar{F}
\]

\[
\vec{H} = \frac{1}{\mu_o} \nabla \times \bar{A} + \frac{1}{j\omega \epsilon_o \mu_o} \nabla \times \nabla \times \bar{F}.
\]

It follows that the “perturbed” tangential field components \( E^p_z \) and \( H^p_z \) can thus be expressed as

\[
E^p_z(x, y, z) \sim \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{dk_x}{k_y(k^2 - k_y^2)} \left\{ k k_x R_e(\alpha) \left[ \frac{\eta_o H_{oy}}{k_x + k'_x} + B \right] \right. \\
+ k_x k_y R_h(\alpha) \left[ \frac{E_{oy}}{k_x + k'_x} + A \right] \left\} e^{(-jk_x x - jk_y y + jk'_z z)} \quad (3.17)
\]

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\[ \eta_0 H_z^p(x, y, z) \sim \frac{1}{2\pi j} \int_{-\infty}^{\infty} \frac{dk_x}{k_y(k^2 - k_y^2)} \left\{ k_y k_z R_e(\alpha) \left[ \frac{\eta_0 H_{oy}}{k_x + k_y'} + B \right] - k_x R_h(\alpha) \left[ \frac{E_{oy}}{k_x + k_y'} + A \right] \right\} e^{-jk_x x - jk_y y + jk_z z} \]  

(3.18)

where \( k_y^2 = k_i^2 - k_x^2 \) and \( k_z^2 = k^2 - k_i^2 \). One notes that from the two preceding equations there are two poles at \( k_x = \pm jk_y' \) in (3.17) and (3.18), whose residues introduce spurious field contributions which do not have a physical meaning. Therefore, one needs to remove those spurious residues. The latter can be suppressed by using the unknown constant \( W \), which was introduced earlier in (3.13), only for this purpose, namely to suppress those spurious two poles. It follows from (3.17) that

\[ \pm j R_{\pm}^e(\alpha) \left[ \frac{\eta_0 H_{oy}}{k_x^\pm + k_y'} + B \right] = -R_{\pm}^h(\alpha) \left[ \frac{E_{oy}}{k_x^\pm + k_y'} + A \right] \]  

(3.19)

where the superscript \( \pm \) corresponds to the residue at the pole location \( k_x = \pm jk_y' \). One can solve for the unknown constants \( A \) and \( B \) from (3.19).

It is more convenient to evaluate the integration in the angular spectral domain; hence, one introduces a transformation, \( k_x = k \sin \beta_o \cos \alpha, k_y = k \sin \beta_o \sin \alpha, \) and \( k_z = k \cos \beta_o \), in which the \( \alpha \) is a complex angular spectral variable. Also one replaces the \( x \) and \( y \) by the cylindrical polar coordinate quantities \( \rho \cos \phi \) and \( \rho \sin \phi \), respectively with \( \rho = \sqrt{x^2 + y^2} \). The expressions in (3.17) and (3.18) can be expressed in the new \( \alpha \)-domain as follows:

\[ E_z^p(\vec{r}) \sim -\frac{1}{2\pi j} \int_{C_\alpha} \frac{d\alpha}{\Delta(\alpha)} \left\{ \cos \alpha R_e(\alpha) \left[ \frac{\eta_0 H_{oy}}{\cos \alpha + \cos \phi'} + B \right] + \cos \beta_o \sin \alpha R_h(\alpha) \left[ \frac{E_{oy}}{\cos \alpha + \cos \phi'} + A \right] \right\} e^{-j\rho \sin \beta_o \cos(\alpha - \phi)} e^{jk \cos \beta_o \phi} \]  

(3.20)
\[ \eta_0 H^p_z(\vec{r}) \sim -\frac{1}{2\pi j} \int d\alpha \frac{\sin \beta_0}{\Delta(\alpha)} \left\{ -\cos \alpha \Re(\alpha) \left[ \frac{E_{\text{oy}}}{\cos \alpha + \cos \phi'} + A \right] \right. \\
+ \cos \beta_0 \sin \alpha \Re(\alpha) \left[ \frac{\eta_0 H_{\text{oy}}}{\cos \alpha + \cos \phi'} + B \right] \left. \right\} e^{-jkp \sin \beta_0 \cos(\alpha - \phi)} e^{jkz \cos \beta'_0} \] (3.21)

where \( \Delta(\alpha) = 1 - \sin^2 \beta_0 \sin^2 \phi' \) and

\[
A = \frac{\sin \beta_0 \cos \beta_0}{\Delta(\phi')} \left[ \xi \Re^+ \Re^\alpha - \{ \cos \phi' \tan \beta_0 + j \zeta \} \eta_0 H_{\text{oy}} \right]
\]

\[
B = -\frac{\sin \beta_0 \cos \beta_0}{\Delta(\phi')} \left[ \{ \cos \phi' \tan \beta_0 + j \zeta \} E_{\text{oy}} + \xi \eta_0 H_{\text{oy}} \right]
\]

with \( \xi = \frac{2}{\Re^+ + \Re^-} \), \( \zeta = \frac{2}{\Re^+ + \Re^-} \), and \( \Re^\pm = \frac{\Re}{\Re^+} \), in which

\[
\Re^\pm = \frac{[\Re_i(\alpha^\pm_0) - 1] \sin \alpha^\pm_0 / 2 \sin \phi' / 2}{[\Re_e(\alpha^\pm_0) + 1] \cos \alpha^\pm_0 / 2 \cos \phi' / 2}
\]

with \( \alpha^\pm_0 = \pi/2 \mp j \ln(\cot \beta_0/2) \). On using the identity,

\[
\frac{4}{\cos \alpha + \cos \phi'} \left\{ \frac{\sin \alpha}{2} \sin \phi' / 2 \right\} = \sec \left( \frac{\alpha + \phi'}{2} \right) \mp \sec \left( \frac{\alpha - \phi'}{2} \right) \] (3.22)

and after some manipulations, one can have the “perturbed” field in a compact form, namely

\[
\tilde{U}^p_{\text{pw}}(\vec{r}) \sim -\frac{1}{2\pi j} \int d\alpha \frac{\sin \beta_0}{\Delta(\alpha) \Delta(\phi')} \left\{ C(\alpha) \tilde{T}(\alpha) \cdot \tilde{D}^c(\alpha, \phi') \right. \\
+ \tilde{U}_u(\alpha) \cdot \tilde{T}(\alpha, \phi') + \tilde{T}_v(\alpha) \cdot \tilde{V}(\alpha, \phi') \left. \right\} \cdot \tilde{T}(\phi') \cdot \tilde{U}_{av} e^{-jkp \sin \beta_0 \cos(\alpha - \phi)} e^{jkz \cos \beta'_0} \] (3.23)

where \( C(\alpha) = \cos^2 \beta_0 - \sin^2 \beta_0 \cos \alpha \cos \phi' \), \( \tilde{U}^p_{\text{pw}} = \tilde{z} \left[ \frac{E^p_z}{\eta_0 H^p_z} \right] \), and

\[
\tilde{T}(\alpha) = \begin{bmatrix} \cos \alpha & \cos \beta_0 \sin \alpha \\ \cos \beta_0 \sin \alpha & -\cos \alpha \end{bmatrix} ; \quad \tilde{T}_u(\alpha) = \begin{bmatrix} -\cos \alpha & \cos \beta_0 \sin \alpha \\ \cos \beta_0 \sin \alpha & \cos \alpha \end{bmatrix} \]

\[
\tilde{T}_v(\alpha) = \begin{bmatrix} -\cos \beta_0 \sin \alpha & \cos \alpha \\ \cos \alpha & \cos \beta_0 \sin \alpha \end{bmatrix} \]

\[
\tilde{U}(\alpha, \phi') = \begin{bmatrix} U_c & 0 \\ 0 & U_h \end{bmatrix} ; \quad \tilde{V}(\alpha, \phi') = \begin{bmatrix} V_h & 0 \\ 0 & V_c \end{bmatrix}
\]

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\[ \tilde{D}^c(\alpha, \phi') = \begin{bmatrix} D^c & 0 \\ 0 & D^h \end{bmatrix} ; \quad \tilde{U}_{oz} = \hat{z} \begin{bmatrix} E_{oz} \\ \eta_o H_{oz} \end{bmatrix}, \]

with

\[ U_e = \sin \beta_o \cos \beta_o 2 \cos \alpha / 2 \cos \phi' / 2(\mathcal{R}_e + 1)j\zeta \]
\[ U_h = \sin \beta_o \cos \beta_o 2 \sin \alpha / 2 \sin \phi' / 2(\mathcal{R}_h - 1)j\zeta \]
\[ V_e = \sin \beta_o \cos \beta_o 2 \cos \alpha / 2 \cos \phi' / 2(\mathcal{R}_e + 1)\xi \mathcal{R}^+ \mathcal{R}^- \]
\[ V_h = \sin \beta_o \cos \beta_o 2 \sin \alpha / 2 \sin \phi' / 2(\mathcal{R}_h - 1)j\xi. \]

The dyads \( \overline{T} \), \( \overline{T}_u \), \( \overline{T}_v \), \( \overline{U} \) and \( \overline{V} \) in the above are expressed in matrix form for convenience. The \( D^c,e,h \) are defined in (2.32) with \( \mathcal{R}^c,e,h(\alpha) \) from (3.6) and \( \mathcal{R}^c,e,h(\alpha) = \mp 1 \). It is noted that one can write \( \tilde{U}_{oy} \) in terms of \( \tilde{U}_{oz} \) by employing incident vector potentials, \( A^i_z \) and \( F^i_z \), which can be obtained via inspection, namely:

\[ A^i_z = \frac{j \omega \mu_0 \epsilon_0}{k^2 - k'^2} E^i_z \quad ; \quad F^i_z = \frac{j \omega \mu_0 \epsilon_0}{k^2 - k'^2} H^i_z \]

where \( E^i_z = E_{oz}e^{(jk'_x x + jk'_y y + jk'_z z)} \) and \( H^i_z = H_{oz}e^{(jk'_x x + jk'_y y + jk'_z z)} \). The normal field components of incident field denoted by \( \tilde{U}_{oy} \) can then be obtained from the incident vector potentials, \( A^i_z \) and \( F^i_z \). The result provides the transformation matrix \( \overline{T}(\phi') \) given above.

Next, (3.23) can be evaluated by using the SDP asymptotic integration technique when \( \kappa \) is large, where \( \kappa = k \rho \sin \beta_o \). One can rewrite (3.23) symbolically as

\[ \tilde{U}_{pvw} \sim \int_{C_\alpha} d\alpha \overline{F}(\alpha)e^{\kappa f(\alpha)} , \quad 0 \leq \phi \leq \pi \] (3.24)

where

\[ \overline{F} = -\frac{1}{2\pi j} \frac{\sin \beta_o}{\Delta(\alpha) \Delta(\phi')} \left\{ C(\alpha) \overline{T}(\alpha) \cdot \tilde{D}^c(\alpha, \phi') + \overline{T}_u(\alpha) \cdot \overline{U}(\alpha, \phi') \right\} \cdot \overline{T}(\phi') \cdot \tilde{U}_{oz}e^{jk \cos \beta_o z} \]

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and \( f(\alpha) = -j \cos(\alpha - \phi) \). It is noted that the \( \mathcal{F} \) has poles at \( \alpha_r = \pi - \phi' \), and \( \alpha_{swo} \).

Deforming the contour \( C_\alpha \) to the SDP contour allows one to express (3.23) as

\[
\bar{U}_{pw}^d \sim -2\pi j \left[ \text{Res} \left\{ \mathcal{F}(\alpha_r)e^{\kappa f(\alpha_r)} \right\} U(\alpha_r - \phi) \right. \\
+ \left. \text{Res} \left\{ \{\mathcal{F}(\alpha_{swo})e^{\kappa f(\alpha_{swo})}\} U(\alpha_{swo} - \phi) \right\} \right] + \int_{\text{SDP}} d\alpha \mathcal{F}(\alpha)e^{\kappa f(\alpha)}
\]

(3.25)

where \( \alpha_r \) is the GO reflected wave pole which provides the GO reflected field contributions, \( \bar{U}_{pw}^r \) and \( \bar{U}_{pnw}^r \). The \( \alpha_{swo} \) is the SW pole, which yields either a FSW or a BSW field contribution, \( \bar{U}_{pw}^s \). The \( U(\cdot) \) is the usual Heaviside unit step function. Applying Cauchy’s residue theorem, one can obtain \( \bar{U}_{pw}^r \), \( \bar{U}_{pw}^n \), and \( \bar{U}_{pw}^s \) as follows:

\[
\bar{U}_{pw}^r = -\frac{1}{\Delta(\phi')} \bar{\mathcal{T}}(\pi - \phi') \cdot \bar{\mathcal{T}}(\phi') \cdot \bar{U}_{ow} e^{jk\rho \sin \beta_o \cos(\phi + \phi')} e^{jkz \cos \beta} U(\pi - \phi' - \phi)
\]

(3.26)

\[
\bar{U}_{pw}^n = \bar{R}^{\pi} \cdot \bar{U}_{ow} e^{j\kappa \rho \sin \beta_o \cos(\phi + \phi')} e^{jkz \cos \beta} U(\pi - \phi' - \phi)
\]

(3.27)

\[
\bar{U}_{pw}^s = \frac{C(\alpha_{swo}^o)}{\Delta(\alpha_{swo}^o)\Delta(\phi')} \bar{\mathcal{T}}(\alpha_{swo}^o) \cdot \bar{R}_{swo} \cdot \bar{\mathcal{T}}(\phi') \cdot \bar{U}_{ow} e^{j\kappa \rho \sin \beta_o (\alpha_{swo}^o - \phi)} e^{jkz \cos \beta} U(\alpha_{swo}^o - \phi)
\]

(3.28)

where \( \bar{\mathcal{T}} \) is defined in (3.5), \( \bar{\mathcal{T}} = [-1 \ 0] \), and \( \bar{R}_{swo} = \begin{bmatrix} R_{e,swo} & 0 \\ 0 & R_{h,swo} \end{bmatrix} \). The \( R_{swo} \) is defined as

\[
R_{e,h}(\alpha_{swo}^o, \phi') = \pm \frac{P_{e,h}(\alpha_{swo}^o)}{2Q_{e,h}(\alpha_{swo}^o)} \left[ \sec \left( \frac{\alpha_{swo}^o - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{swo}^o + \phi'}{2} \right) \right].
\]

(3.29)

The \( Q_{e,h}(\alpha_{swo}^o) \) is the derivative of \( Q_{e,h}(\alpha) \) in (3.8) with respect to \( \alpha \) and evaluated at \( \alpha = \alpha_{swo}^o \). The \( \bar{U}_{pw}^r \) and \( \bar{U}_{pw}^n \) represents the GO fields reflected from \( o \) and \( n \)-face, respectively. A closed form evaluation of the SDP integral in (3.25) via the non-uniform steepest descent method, yields the non uniform diffracted field \( \bar{U}_{pw}^d \) as

\[
\bar{U}_{pw}^d \sim \frac{1}{\Delta(\phi)\Delta(\phi')} \left[ C(\phi, \phi') \bar{\mathcal{T}}(\phi) \cdot \bar{\mathcal{D}}(\phi, \phi') \cdot \bar{\mathcal{T}}(\phi') + \bar{\mathcal{D}} \right] \cdot \bar{U}_{ow} e^{-jks} \sqrt{8}
\]

(3.30)
where $\mathcal{D}^c = \begin{bmatrix} \mathcal{D}_c^e & 0 \\ 0 & \mathcal{D}_h \end{bmatrix}$ with $\mathcal{D}^c_{e,h}$ is defined as

$$
\mathcal{D}^c_{e,h}(\alpha, \phi') = \pm \frac{1}{2} \left[ \mathcal{R}^o_{e,h}(\alpha) - \mathcal{R}^o_{e,h}(\alpha) \right] \left[ \sec \left( \frac{\alpha - \phi'}{2} \right) \pm \sec \left( \frac{\alpha + \phi'}{2} \right) \right],
$$

(3.31)

and where $\mathcal{R}^o_{e,h}$ are defined above. The $\mathbf{W}$ is given by

$$
\mathbf{W} = e^{-j\pi/4} \sqrt{2\pi k} \left[ \mathbf{T}_u(\phi) \cdot \mathbf{U} + \mathbf{T}_v(\phi) \cdot \mathbf{V} \right] \cdot \mathbf{T}(\phi').
$$

(3.32)

The total field $\mathbf{U}_{pw}$ can be obtained by

$$
\mathbf{U}_{pw} \sim \mathbf{U}_{pw}^i + \mathbf{U}_{pw}^s.
$$

(3.33)

The $\hat{z}$-directed tangential component of a uniform incident plane wave $\mathbf{U}_{pw}^i$ is

$$
\mathbf{U}_{pw}^i = \hat{z} \begin{bmatrix} E_{pw}^i \\ \eta_o H_{pw}^i \end{bmatrix},
$$

where $E_{pw}^i$ (or $H_{pw}^i$) is the electric (or magnetic) incident field, which is given by

$$
\mathbf{U}_{pw}^i = \hat{z} e^{j(k_{x'} x + k_{y'} y + k_{z'} z)}
$$

(3.34)

where as before $\mathbf{U}_{oz}$ denotes

$$
\hat{z} \begin{bmatrix} E_{oz} \\ \eta_o H_{oz} \end{bmatrix}.
$$

The $\mathbf{U}_{pw}^s$ can be found by using the same approach for finding the $\mathbf{U}_{pw}^p$ from $\mathbf{U}_{y}$.

One can then obtain the $\mathbf{U}_{pw}^s$ as

$$
\mathbf{U}_{pw}^s \sim \frac{1}{\Delta(\phi')} \mathbf{T}(\pi - \phi') \cdot \mathbf{R}^o(\phi') \cdot \mathbf{T}(\phi') \cdot \mathbf{U}_{oz} \hat{z} e^{jk_z \sin \beta_o \cos(\phi + \phi')} e^{jk_z \cos \beta_o} + \mathbf{U}_{pw}^p.
$$

(3.35)

By substituting (3.35) and (3.25) (together with (3.26)-(3.30)) into (3.33), one can obtain the total $\hat{z}$-directed tangential field components $\mathbf{U}_{pw}$.

It is important to note that the solution in (3.30) still satisfies all the crucial physical properties, such as the PEC boundary condition on the $n$-face, the Karp-Karal lemma on the $o$-face despite the approximations used to arrive at (3.23). Furthermore, the approximated solution in (3.30) still recovers the PEC solution when the material slab is removed. Thus, the solution in (3.25) (and (3.30)), which is based on
the approximate expression of (3.23), clearly retains many of the important physical properties, thereby lending more confidence to the heuristic approximation of (3.23). However, the analytical expression for the approximate diffracted $\bar{U}_d^{\text{pw}}$ in (3.30) does not satisfy reciprocity, but is expected to provide numerical results which nearly satisfy the reciprocity principle. On the other hand, the technique shown in Section 2.1.2 can be easily applied to restore the reciprocity condition into the $\bar{U}_d^{\text{pw}}$ in (3.30). The desired ansatz is now established by the set of equations (3.33)-(3.35) and (3.23), respectively which allows one to obtain a corresponding solution for the case of spherical wave incidence.

### 3.2 Extension to treat the case of spherical wave excitation with one face being PEC

A UTD solution for a thin planar material half plane on an entire PEC ground plane illuminated by spherical wave or an elemental current moment is developed in this section. A spherical wave with an arbitrary field polarization transverse to the incident ray direction, $\hat{s}^i$, can be created by superimposing the fields of $\hat{z}$-directed electric and magnetic current moments at the origin of the spherical wave. The incident, $\hat{z}$-directed, electric field, $E_z^i$, (or the magnetic field, $H_z^i$) at an observer location $\bar{r}(\rho, \phi, z)$, which is produced by an electric (or magnetic) current moment of strength $d\bar{p}_e = \hat{z}dp_{ez}$ (or $d\bar{p}_m = \hat{z}dp_{mz}$) at $\bar{r}'(\rho', \phi', z)$, respectively, can be expressed as

$$\bar{U}_z^i \equiv \hat{z} \left\{ \begin{array}{c} E_z^i \\ H_z^i \end{array} \right\} = -jk\hat{z} \left\{ \begin{array}{c} Z_0 dp_{ez} \\ Y_0 dp_{mz} \end{array} \right\} \sin^2 \beta_0 \tilde{G}_o(k|\bar{r} - \bar{r}'|)$$ \hspace{1cm} (3.36)

and

$$\tilde{G}_o(k|\bar{r} - \bar{r}'|) = e^{-jks'^i} \frac{4\pi S^i}{4\pi S^i}$$ \hspace{1cm} (3.37)
If one assumes that the current moment is sufficiently far from the $o$ face so that the incident spherical wave generates “locally” plane at the line of discontinuity (or edge), then one may employ (3.33) and (3.35) obtained for the skew incident plane wave case to provide the $\hat{z}$ components of the scattered field for the spherical wave incidence case as:

$$
\bar{U}_{s}z \sim -jk \left\{ \frac{Z_o dp_{ez}}{Y_o dp_{mz}} \sin^2 \beta_o \frac{1}{\Delta (\phi')} \cdot \overline{T}(\pi - \phi') \cdot \overline{R}^o (\phi') \cdot \overline{T}(\phi') \cdot \bar{U}_{oz} e^{-j S r} \right\} + \bar{U}_p^z
$$

(3.39)

where the first term on the RHS of (3.39) represents as before the field scattered from the “unperturbed” structure, which is assumed to be a thin planar material slabs of infinite extent with PEC backing that has the same thickness and electrical parameters as the PEC backed material pertaining to the $o$-face in the actual or original problem geometry of Fig. 3.1. Also the $\bar{U}_{oz}$ in (3.39) is defined in Section 3.1.

Under the present assumption of source far from the surface at $y = 0$, one can show that the unperturbed scattered field is asymptotically given by the first term on the RHS of (3.39) which is the GO reflected field, where $\overline{R}^o$ is the FRC for this unperturbed material surface with PEC ground plane, and $S^r$ is the GO ray path corresponding to the GO field reflected from that unperturbed surface. The $\overline{R}^o (\phi')$ is defined in (3.5). Also,

$$
S^r = \sqrt{\rho^2 + \rho'^2 - 2\rho \rho' \cos (\phi + \phi') + (z - z')^2}.
$$

(3.40)

The $\bar{U}_p^z$ in (3.39) represents the “perturbation” to the first term on the RHS of (3.39). By employing an inverse Fourier integral transformation, one can find the $\bar{U}_p^z$ for the 3-D spherical wave incidence case from the $w_p^z$ in (2.23) for the 2-D cylindrical wave
incidence case as explained in [36], namely
\[
\widetilde{U}_z^p = \frac{1}{2\pi} \int_{\infty}^{\infty} \left[ \int_{\mathcal{C}_\alpha} \left( -\frac{1}{2\pi j} \right) \tilde{A}(\alpha, \phi') G_o[k_t s(\alpha)] e^{-jk_z(z' - z)} d\alpha \right] dk_z. \tag{3.41}
\]

The \(\tilde{A}(\alpha, \phi')\) is an appropriate spectral amplitude or weight function, and \(G_o[k_t s(\alpha)]\) is defined in (2.24) with \(k\) replaced by \(k_t\), in which \(k_t^2 = k^2 - k_z^2\). By following the steps in [36], one can further rewrite (3.41) in terms of the modified cylindrical Bessel function of the second kind and of order zero, \(K_o[jk_t s(\alpha)]\), to yield
\[
\widetilde{U}_z^p = -\frac{1}{8\pi^3 j} \int_{\mathcal{C}_\alpha} d\alpha \tilde{A}(\alpha, \phi') \int_{-\infty}^{\infty} \frac{dk_z}{k_z} K_o[jk_t s(\alpha)] e^{-jk_z(z' - z)} \tag{3.42}
\]
where for large \(k_t s(\alpha)\)
\[
K_o[j\vartheta(\varsigma)] = \sqrt{\frac{\pi}{2j\vartheta(\varsigma)}} e^{-j\vartheta(\varsigma)}. \tag{3.43}
\]

It is noted that \(K_o(j\vartheta) = -j\frac{\pi}{2} H_o^{(2)}(\vartheta)\). One can next apply the following identity
\[
\int_{-\infty}^{\infty} dk_z H_o^{(2)}[k_t s(\alpha)] e^{-jk_z(z' - z)} \equiv 2j \frac{e^{-jk_S(\alpha)}}{S(\alpha)} \tag{3.44}
\]
in (3.42). This leads to
\[
\tilde{U}_z^p = -\frac{1}{2\pi j} \int_{\mathcal{C}_\alpha} d\alpha \tilde{A}(\alpha, \phi') \frac{e^{-jk_S(\alpha)}}{4\pi S(\alpha)} \tag{3.45}
\]
with
\[
S(\alpha) = \sqrt{\rho^2 + \rho'^2 + 2\rho\rho' \cos(\alpha - \phi) + (z - z')^2}. \tag{3.46}
\]

It is important to note that if the current moment is not assumed to be sufficiently far from the \(o\) and \(n\) faces, then additional contributions (not present in (3.39)) must be included. Such additional contributions arise because the current moment can in some situations strongly excite SWs directly in the material; these SWs become incident on the discontinuity at “0” to produce a reflected SW and a transmitted SW, as well
as a diffracted space wave. The reflected and transmitted SWs can be deduced from
the W-H solution to appropriate, simpler, canonical two-part diffraction problems in
which the excitation is an incident SW. In the radiation problem, these SW effects
are not significant. Only the diffraction of the incident SW by the discontinuity
contributes to the radiation field; its effect is discussed in Section 2.4 for a line source
excitation. The spectral function $\bar{A}$ is proportional to the strength of the current
moment, and it may be expressed as

$$\bar{A}(\alpha, \phi') \equiv -jk \left\{ \frac{Z_o dp_{e_2}}{Y_o dp_{m_2}} \right\} \sin^2 \beta_o \bar{D}(\alpha, \phi') \cdot U_{oz} \quad (3.46)$$

where the unknown spectral weight $\bar{D}$ is to be determined using the ansatz of Sec-
tion 3.1. Let $\rho' = s' \sin \beta_o$, $\rho = s \sin \beta_o$, and $z = -s \cos \beta_o$. Thus,

$$S^2(\alpha) = (s + s')^2 \left\{ 1 - \frac{2ss'}{(s + s')^2} \sin^2 \beta_o [1 - \cos(\alpha - \phi)] \right\} \quad (3.47)$$

In order to identify $\bar{D}$, the exponential in (3.44) with (3.47) may be approximated
by the first two terms of its binomial expansion for large $k \frac{ss'}{s + s'} \sin^2 \beta_o$, which is as-
sumed here to be the large parameter (for the asymptotic development). Then, (3.44)
becomes

$$\bar{U}_{p z} \sim -\frac{1}{2\pi j} \int_{C_o} A(\alpha, \phi') e^{-jk(s + s')} e^{jk \frac{ss'}{s + s'} \sin^2 \beta_o [1 - \cos(\alpha - \phi)]} d\alpha. \quad (3.48)$$

If the current moment is allowed to receded to infinity, i.e., if $s' \to \infty$, while $s$ is
kept finite, then one obtains the scattered field $\bar{U}_{p w}$ due to plane wave illumination,

$$\bar{U}_{p z} \sim C_o(k s') \bar{U}_{p w} \quad (3.49)$$

where $C_o$ is the current moment factor given by

$$C_o(k s') = -jk \left\{ \frac{Z_o dp_{e_2}}{Y_o dp_{m_2}} \right\} \sin^2 \beta'_o \frac{e^{-jk s'}}{4\pi s'} \quad (3.50)$$
and
\[
\bar{U}_{pw}^p = -\frac{1}{2\pi j} \int_{C_\alpha} \bar{D}(\alpha, \phi') \cdot \bar{U}_{oz} e^{-jk_p \sin \beta_o \cos(\alpha - \phi)} e^{jk_z \cos \beta'_o} \, d\alpha. \tag{3.51}
\]

By directly comparing (3.51) with the desired ansatz in (3.23), one can easily identify \(\bar{D}\) by inspection to be
\[
\bar{D}(\alpha, \phi') = \frac{\sin \beta_o}{\Delta(\alpha) \Delta(\phi')} \left\{ C(\alpha) \bar{T}(\alpha) \cdot \bar{D}^c(\alpha, \phi') + \bar{T}_u(\alpha) \cdot \bar{U}(\alpha, \phi') + \bar{T}_v(\alpha) \cdot \bar{V}(\alpha, \phi') \right\} \cdot \bar{U}(\phi'). \tag{3.52}
\]

### 3.3 Asymptotic Analysis

Next one can asymptotically evaluate, in closed form, the integral in (3.48) by using the steepest descent method. One can start this evaluation by rewriting (3.48) symbolically as
\[
\bar{U}_{pz}^p \sim \int_{C_\alpha} d\alpha \mathcal{F}(\alpha) e^{\kappa f(\alpha)}, \quad 0 \leq \phi \leq \pi \tag{3.53}
\]
where the \(\kappa\) denotes \(k \frac{\beta'_o}{s + s'} \sin \beta_o^2\), \(f(\alpha) = j[1 - \cos(\alpha - \phi)]\) and
\[
\mathcal{F} = -\frac{1}{8\pi^2 j} \left\{ Z_o dp_{oz} \right\} \sin^2 \beta_o^2 \frac{\sin \beta_o}{\Delta(\alpha) \Delta(\phi')} \left\{ C(\alpha) \bar{T}(\alpha) \cdot \bar{D}^c(\alpha, \phi') + \bar{T}_u(\alpha) \cdot \bar{U}(\alpha, \phi') + \bar{T}_v(\alpha) \cdot \bar{V}(\alpha, \phi') \right\} \cdot \bar{U}(\phi') \cdot \frac{e^{-jk(s+s')}}{S(\alpha)}.
\]

After deforming the integral contour of (3.53) to the steepest descent path (SDP) through the saddle point at \(\alpha \equiv \alpha_s = \phi\) as shown in Fig. 2.2, it allows one to express (3.53) for large \(\kappa\) as
\[
\bar{U}_{pz}^p \sim -2\pi j \text{Res}\{\mathcal{F}(\alpha_{s}) e^{\kappa f(\alpha_{s})}\} U(\alpha_{s} - \phi) - 2\pi j \text{Res}\{\mathcal{F}(\alpha_{sw}) e^{\kappa f(\alpha_{sw})}\} U(\alpha_{sw} - \phi) + \int_{SDP} d\alpha \mathcal{F}(\alpha) e^{\kappa f(\alpha)}. \tag{3.54}
\]

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The reflected field $\bar{U}_r^z$ is given by the sum of the “unperturbed” GO reflected field contained in the first term on the RHS of (3.39) together with the residue contribution from the pole at $\alpha = \alpha_r = \pi - \phi'$ in (3.54), as

$$\bar{U}_r^z = -j k \left\{ \frac{Z_o d p_{e z}}{Y_o d p_{m z}} \right\} \sin^2 \beta'_o \frac{1}{\Delta(\phi')} \bar{T}(\pi - \phi') \cdot \bar{R}(\phi') \cdot \bar{T}(\phi') \cdot \bar{U}_{o z} e^{-j k S_r} \frac{e^{-j k S_r}}{4\pi S_r} \tag{3.55}$$

where $S_r$ is defined in (3.40), and

$$\bar{R}(\phi') = \begin{cases} \bar{R}^o(\phi') & \text{if } \phi + \phi' < \pi \\ \bar{R}^n(\phi') & \text{if } \phi + \phi' > \pi \end{cases} \tag{3.56}$$

in which $\bar{R}^{o,n}$ denotes the FRC as defined earlier. Also, $\bar{U}_{zsw}^z$ is given by the residue arising from the SW pole $\alpha = \alpha_{sw}^o$ in (3.54) as

$$\bar{U}_{zsw}^z = \frac{C(\alpha_{sw}^o)}{\Delta(\alpha_{sw}^o) \Delta(\phi')} \bar{T}(\alpha_{sw}^o) \cdot \bar{R}_{sw}^{so}(\alpha_{sw}^o) \cdot \bar{T}(\phi') \cdot \bar{U}_{o z} e^{-j k S_{sw}} \frac{e^{-j k S_{sw}(\alpha_{sw}^o, \phi)}}{4\pi S(\alpha_{sw}^o, \phi)} U(\alpha_{sw}^o - \phi) \tag{3.57}$$

As mentioned in Section 2.1.2, one assumes that the material slab is sufficiently thin so only the lowest TM surface wave can propagate for the material slab with PEC ground plane. Thus, the $\bar{R}_{sw}^{so} = \begin{bmatrix} 0 & 0 \\ 0 & R_{sw}^{ho} \end{bmatrix}$. The SDP integral, which yields the diffracted field $\bar{U}_d^z$ symbolically is given by

$$\bar{U}_d^z = \int_{SDP} \bar{T}(\alpha, \phi') e^{\kappa f(\alpha)} d\alpha. \tag{3.58}$$

As for the normal incidence case discussed in Chapter 2, one can decompose the spectral function $\bar{F}(\alpha)$ in the integrand of (3.58) in to one contains the GO type pole and remaining part containing the SW type pole. The former can be conveniently evaluated by using the PC approach while the latter can be performed by the VDW approach. The expression for the UTD first order diffracted field is then found to have the general form as

$$\bar{U}_d^z = \bar{U}_d^z(Q_e) \cdot \bar{D}(\phi, \phi') A(s, s') e^{-j k s} \tag{3.59}$$

67
where $\bar{D} = \bar{D}^{go} + \bar{D}^{sw}$. Here $\bar{U}_2^i(Q_e)$ represents the incident field at the point of diffraction $Q_e$, and $A(s, s')$ is a spread factor given by $A(s, s') = \sqrt{\frac{s'}{s(s + s')}}$. Here $s$ is the distance from $Q_e$ to an observation point, and the $s'$ is the distance from $Q_e$ to the source point. The $\bar{D}^{go}$ is based on the PC method and $\bar{D}^{sw}$ is based on the VDW method; they are given by

$$\bar{D}^{go} = \frac{1}{\Delta(\phi)\Delta(\phi') \sin \beta_o} \left[ C(\phi, \phi') \bar{T}(\phi) \cdot \bar{D}^c(\phi, \phi') \cdot \bar{T}(\phi') + W \right],$$  \hspace{1cm} (3.60)$$

and

$$\bar{D}^{sw} = \frac{1}{\Delta(\alpha^o_{sw})\Delta(\phi') \sin \beta_o} \left[ C(\alpha^o_{sw}, \phi') \bar{T}(\alpha^o_{sw}) \cdot \bar{D}^{sw}(\alpha^o_{sw}, \phi') \cdot \bar{T}(\phi') \right] \hspace{1cm} (3.61)$$

where the $\bar{D}^c = \begin{bmatrix} D_e^c & 0 \\ 0 & D_h^c \end{bmatrix}$ and $\bar{D}^{sw} = \begin{bmatrix} D_e^{sw} & 0 \\ 0 & D_h^{sw} \end{bmatrix}$ with $D_{e,h}^c$ and $D_{e,h}^{sw}$ are given by

$$D_{e,h}^c(\phi, \phi') = \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[ \Gamma_{e,h}^c(\phi, \phi') - \Gamma_{e,h}^n(\phi, \phi') \right] \left[ \sec \left( \frac{\phi - \phi'}{2} \right) F_{KP} (k L^a_{go}) \pm \sec \left( \frac{\phi + \phi'}{2} \right) F_{KP} (k L^b_{go}) \right],$$  \hspace{1cm} (3.62)$$

and

$$D_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^o) = \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[ \frac{R_{e,h}^{sw}(\alpha_{sw}^o, \phi')}{\sin \left( \frac{\alpha_{sw}^o - \phi'}{2} \right)} \left[ 1 - F_{KP} (k L^a_{sw}) + d_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^o) \right] \right] \hspace{1cm} (3.63)$$

$$d_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^o) = \frac{P_{e,h}^{sw}(\alpha_{sw}^o)}{Q_{e,h}^o(\phi)} \left[ \sec \left( \frac{\alpha_{sw}^o - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{sw}^o + \phi'}{2} \right) \right].$$  \hspace{1cm} (3.64)$$

The $\Gamma_{e,h}^o$ is an ad hoc modification to $\mathcal{R}_{e,h}^o$ so as to preserve reciprocity. It is given by

$$\Gamma_{e,h}^o(\phi') = \frac{2 \sin \frac{\phi}{2} \sin \frac{\phi'}{2} - \delta_{e,h}^o / \sin \beta_o}{2 \sin \frac{\phi}{2} \sin \frac{\phi'}{2} + \delta_{e,h}^o / \sin \beta_o} \hspace{1cm} (3.65)$$

where

$$\delta_e^o = -j \gamma_d N \cot (N \tau k_d), \hspace{1cm} \delta_h^o = j \gamma_d N \tan (N \tau k_d) \hspace{1cm} (3.66)$$
with $N = \sqrt{1 - \eta^4 \sin^2 \beta_o \sin^2 \frac{\phi}{2} \sin^2 \frac{\phi'}{2}}$. The $\Gamma_{e,h}^n = \mp 1$ because the $n$-face is PEC. It is noted that as in the UTD solution for normal incidence case given in Chapter 2, a UTD solution for a thin planar material half plane on an entire PMC ground plane at skew incidence can be again directly obtained from the PEC case by simply using the duality theorem. Furthermore, the material half plane without ground plane at skew incidence can be obtained by using the bisection theorem. Also one can apply the same idea as shown in Section 2.3 and 2.5 to find the slope diffraction term for material junction with ground plane and material half plane, respectively for the skew incidence case as for the 2-D case. It is important to note that the UTD solutions for a junction between two different planar material slabs on a PEC (and PMC) ground plane at skew incidence as shown in Fig. 3.2 can be easily found by replacing $\Re_{e,h}^n(\phi')$ in $\mathcal{D}$ of $\mathcal{F}$ in (3.53) with

$$\Re_{e,h}^n(\phi') = \frac{P_{e,h}^n(\phi')}{Q_{e,h}^n(\phi')}$$ (3.67)

with

$$P_{e,h}^n(\phi') = \sin \phi' - \delta_{e,h}^n / \sin \beta_o$$ (3.68)

and

$$Q_{e,h}^n(\phi') = \sin \phi' + \delta_{e,h}^n / \sin \beta_o$$ (3.69)

where

$$\delta_{c}^n = -j \frac{Y_{d}}{k_{d}} N \cot(N \tau_{n} k_{d}^{n}) , \quad \delta_{h}^n = j \frac{Z_{d}}{k_{d}} N \tan(N \tau_{n} k_{d}^{n})$$ (3.70)

with $k_{d} = k \sqrt{\epsilon_{rn} \mu_{rn}}$, $Z_{d} = \sqrt{\mu_{rn}/\epsilon_{rn}}$, $Y_{d} = 1/Z_{d}$, $N = \sqrt{1 - \eta_{n} \sin^2 \beta_o \sin^2 \phi'}$ and $\eta_{n} = 1/\mu_{rn} \epsilon_{rn}$. Then one can asymptotically evaluate the integral in (3.53) with the
replacement of $\mathcal{R}_{e,h}$ defined in (3.67). The result can be summarized as follows:

$$
\bar{U}_p^p \sim -2\pi j \text{Res}\{\mathcal{F}(\alpha) e^{\kappa f(\alpha)}\} U(\alpha_0 - \phi) - 2\pi j \text{Res}\{\mathcal{F}(\alpha_{sw}^o) e^{\kappa f(\alpha_{sw}^o)}\} U(\alpha_{sw}^o - \phi)
- 2\pi j \text{Res}\{\mathcal{F}(\alpha_{sw}^n) e^{\kappa f(\alpha_{sw}^n)}\} U(\alpha_{sw}^n - \phi) + \int_{\mathcal{SDP}} d\alpha \mathcal{F}(\alpha) e^{\kappa f(\alpha)}.
$$

(3.71)

The GO reflected field is the same as (3.55) except $\bar{\mathcal{R}}^n(\phi')$ in (3.56) is now

$$
\bar{\mathcal{R}}^n(\phi') = \begin{bmatrix} \mathcal{R}_{e,h}^n(\phi') & 0 \\ 0 & \mathcal{R}_h^n(\phi') \end{bmatrix}
$$

(3.72)

where $\mathcal{R}_{e,h}^n(\phi')$ are defined above in (3.67). The SW fields are given by the residues arising from the SW poles $\alpha = \alpha_{sw}^o$ and $\alpha = \alpha_{sw}^n$ in (3.71). The SW field contribution on the $o$-face remains the same as (3.57). The SW field on the $n$-face $\bar{U}_{sw}^n$ is given by

$$
\bar{U}_{sw}^n = \frac{C(\alpha_{sw}^n)}{\Delta(\alpha_{sw}^n)\Delta(\phi')} \bar{\mathcal{F}}(\alpha_{sw}^n) \cdot \bar{R}_{sw}^n(\alpha_{sw}^n) \cdot \mathcal{F}(\phi') \cdot \bar{U}_{oz} e^{-jkS(\alpha_{sw}^n, \phi')} 4\pi S(\alpha_{sw}^n, \phi) U(\alpha_{sw}^n - \phi)
$$

(3.73)

where $\bar{R}_{sw}^n = \begin{bmatrix} 0 & 0 \\ 0 & \mathcal{R}_h^{sw} \end{bmatrix}$. The first order UTD diffracted field is given in the same form as (3.59)-(3.62) except the $\Gamma_{e,h}^n(\phi')$ is now

$$
\Gamma_{e,h}^n(\phi') = \frac{2 \sin \frac{\phi}{2} \sin \frac{\phi'}{2} - \delta_{e,h}^n / \sin \beta_o}{2 \sin \frac{\phi}{2} \sin \frac{\phi'}{2} + \delta_{e,h}^n / \sin \beta_o}
$$

(3.74)

where

$$
\delta_e^n = -jY_d N^n \cot(N^n \tau_n k_d^n) \quad , \quad \delta_h^n = jZ_d^n N^n \tan(N^n \tau_n k_d^n)
$$

(3.75)

with $N^n = \sqrt{1 - \eta_4 \sin^2 \beta_o \sin^2 \frac{\phi}{2} \sin^2 \frac{\phi'}{2}}$. The $D_{e,h}^{sw}$ is the same as (3.63) and $D_{e,h}^{sw}$ is given by

$$
D_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^n) = \frac{e^{-j\pi/4}}{2\sqrt{2\pi k}} \left[ \frac{R_{e,h}^{sw}(\alpha_{sw}^n, \phi')}{\sin \left( \frac{\alpha_{sw}^n - \phi'}{2} \right)} [1 - F_{KP}(kL_{sw}^n)] + d_{e,h}^{sw}(\phi, \phi'; \alpha_{sw}^n) \right]
$$

(3.76)
Figure 3.2: 3-D junction between two different, thin, planar DPS/DNG material slabs on a PEC ground plane illuminated by a $\hat{z}$-directed current moment.

and

$$d_{e,h}^{\text{swn}}(\phi, \phi'; \alpha_{\text{sw}}^{n}) = \frac{P_{e,h}^{n}(\alpha_{\text{SW}}^{n})}{Q_{e,h}^{n}(\phi)} \left[ \sec \left( \frac{\alpha_{\text{sw}}^{n} - \phi'}{2} \right) \pm \sec \left( \frac{\alpha_{\text{sw}}^{n} + \phi'}{2} \right) \right].$$  \hspace{1cm} (3.77)
CHAPTER 4

ANALYSIS OF 2-D AND 3-D EM WAVE DIFFRACTION
BY A MATERIAL COATED PEC WEDGE

The work described in the previous chapters is restricted to treating problems of EM wave diffraction by planar configurations involving thin, uniform material discontinuities as shown in Figs. 1.3(a), 1.3(b), and 1.3(c), respectively. However, for analysis of EM radiation/scattering by antennas on or near complex material coated electrically large metallic platforms such as modern aircraft, naval ships, etc., a UTD solution for the EM wave diffraction by a material coated PEC wedge of arbitrary angle is generally required. Thus, the spectral synthesis approach of the previous chapters for solving the planar two part diffraction problems is extended in this chapter for developing an approximate solution for wave diffraction by a PEC wedge with a thin material coating on the two faces in which the wedge angle (WA) is arbitrary, and in which the materials may be different on the two faces and may also have different thicknesses on the two faces, respectively. Plane wave, cylindrical wave (or line source) and spherical wave (point source) illumination of the wedge are considered here. Furthermore, the solution developed here is also valid for antennas placed directly on the coated PEC wedge. The excitation by sources on the coated wedge surface requires one to include slope diffraction effects because the first order
UTD vanishes for this case; hence, the relevant UTD slope diffraction term is also developed in this work. It is noted that slope diffraction is also necessary to properly describe the fields around a wedge excited by a source off the wedge surface when that source exhibits a rapidly varying radiation pattern near the edge.

The development of the UTD solution for the EM wave diffraction by a material coated metallic wedge begins in Section 4.1 of this Chapter with an exact spectral integral formulation presented in [14] which is originally based on the Maliuzhinets (MZ) method for describing the fields surrounding an impedance wedge illuminated by a uniform plane wave. This work [14] is then extended to treat a uniform electric and magnetic line source excitation in [15]. This 2-D formulation in [15] reduces automatically to the case of plane wave illumination of the wedge at normal incidence if the line source is allowed to recede to infinity. This particular formulation in [14] is expressed in a form that is highly reminiscent of the physically appealing spectral integral formulation of [36] which earlier led to a well established UTD solution for the corresponding special case of a PEC wedge illuminated by a uniform line source. It is important to note that this formulation for the impedance wedge is next heuristically transformed in a simple fashion to allow for a thin material coating on a PEC wedge instead of the surface impedance boundary condition. The first order UTD space wave diffraction by, and edge excited surface waves on the PEC wedge with a thin material coating are then explicitly extracted along with the GO ray contributions by asymptotically evaluating the spectral integral formulation for the total field. Next, the slope diffraction effects are found, in Section 4.2, from the solution for the uniform line source excitation case, by superposition of the fields of two oppositely directed parallel line sources spaced an infinitesimal distance apart to simulate an electric line.
source doublet, which is oriented so as to produce a pattern null at the edge so that
the first order UTD diffraction vanishes leaving only the higher order slope diffracted
field. Finally in Section 4.3, a fully 3-D UTD solution of the problem diffraction by a
PEC wedge with a thin material coating on both faces when it is illuminated by an
external spherical EM wave, or excited by an antenna on the wedge face, is developed.
This approximate 3-D UTD solution for the wedge with arbitrary wedge angle includes
slope diffraction contributions as well. To date no other exact or asymptotic closed
form solutions have been reported for this general external wedge diffraction problem
where both faces are coated and the WA is restricted to only exterior wedge problems
\((0 < W A = (2 - n)\pi < \pi)\) but is otherwise arbitrary, and which also allow antennas
to be placed directly on the coated wedge. Thus, the present 3-D UTD solution is
expected to fill an important need for practical applications.

### 4.1 Development of a 2-D solution for a material coated PEC
wedge illuminated by a uniform line source

First consider a wedge with surface impedance faces illuminated by a \(\hat{z}\)-directed
uniform line source as shown in Fig. 4.1(a). Two different impedance boundary
conditions are imposed on the two faces of the wedge. The total field, \(u_z = \begin{bmatrix} E_z \\ H_z \end{bmatrix}\),
surrounding the wedge can be expressed in a spectral integral form [14] together
with [36] over the complex contour \(\mathcal{L} - \mathcal{L}'\) of Fig. 4.2, as follows:

\[
\begin{align*}
  u_z &= -jk \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \frac{1}{8\pi^2 jn} \int_{\mathcal{L} - \mathcal{L}'} d\xi \frac{\tilde{\Psi}(\frac{n\pi}{2} + \xi - \phi)}{\Psi(\frac{n\pi}{2} - \phi')} \\
  &\quad \times \left[ \cot \left( \frac{\xi - \beta^-}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] K_0[jks(\xi)] 
\end{align*}
\] (4.1)
where $\beta^\pm = \phi \pm \phi'$, and $\tilde{\Psi}(\alpha)$ is given by

$$\tilde{\Psi}(\alpha) = \frac{2\Psi(\alpha)}{\cos \frac{\alpha}{n} + \sin \frac{\theta_{e,h}}{n}}.$$  

In (4.1), $K_o[j\vartheta]$ is the modified cylindrical Bessel function of the second kind, of order zero. Also, $\sin \theta_{e,n}^o = \eta_o/Z_{o,n}$ and $\sin \theta_{h,n}^o = Z_{o,n}/\eta_o$ in which $Z_o$ and $Z_n$ denote the value of the surface impedance for the $o$-face (at $\phi = 0$) and the $n$-face (at $\phi = n\pi$) of the wedge with an internal angle of $(2 - n)\pi$, respectively. As usual, $\eta_o$ is the free

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space intrinsic impedance. Also,

\[ s(\xi) = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \xi} \quad (4.3) \]

The \( \Psi(\alpha) \) is an auxiliary function which is given in terms of the MZ functions \( \psi_n(\alpha) \) as follows:

\[ \Psi(\alpha) = \psi_n(\alpha + \frac{n\pi}{2} + \frac{\pi}{2} - \theta_{e,h}^o) \psi_n(\alpha + \frac{n\pi}{2} - \frac{\pi}{2} + \theta_{e,h}^o) \]
\[ \psi_n(\alpha - \frac{n\pi}{2} - \frac{\pi}{2} + \theta_{e,h}^o) \psi_n(\alpha - \frac{n\pi}{2} + \frac{\pi}{2} - \theta_{e,h}^o). \quad (4.4) \]

This MZ function, \( \psi_n(\alpha) \), is a special meromorphic function which is in the form of an integral. More details on this MZ function \( \psi_n(\alpha) \) including some useful identities can be found in [30] and they are also discussed in Appendix A. As shown in [36], if the argument \( |ks(\xi)| \) of \( K_o \) in (4.1) is large with respect to the order of \( K_o[jks(\xi)] \) then one may use the large argument approximation of \( K_o[jks(\xi)] \) which contains an exponential function. One can then approximate this exponential function by the first two terms of its binomial expansion for large \( k\rho\rho' \). The \( u_z \) in (4.1) can now be expressed for large \( k\rho\rho' \) as

\[ u_z = u_{oz} \frac{1}{8\pi^2 jn} \int_{L-L'} d\xi \sqrt{\frac{j}{2} \frac{\Psi(\frac{n\pi}{2} + \xi - \phi)}{\Psi(\frac{n\pi}{2} - \phi')}} \left[ \cot \left( \frac{\xi - \beta^-}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] \frac{e^{-jk(\rho+\rho')}}{\sqrt{s(\xi)}} e^{ik\rho\rho'(1+\cos \xi)} \quad (4.5) \]

where \( u_{oz} = -jk \left\{ \frac{Z_o I_o}{Y_o M_o} \right\} \) and \( I_o \) (or \( M_o \)) is the strength of the electric (or magnetic) line source which excites the wedge. The contour of the integration in (4.5) can be closed using the steepest descent paths, or \( SDP(\pm \pi) \), which pass through the saddle points \( \xi_s = \pm \pi \) as shown in Fig. 4.2, and the result after applying the identity \( \psi_n(\alpha + n\pi) = \cot \left( \frac{\alpha + \pi/2}{2} \right) \psi_n(\alpha - n\pi) \) (more details in Appendix A) yields
Figure 4.2: Steepest descent paths SDP(±π) and the complex ξ plane topology
\[ u_z = 2\pi j \sum_{q=1}^{q=7} \text{Res}(\xi = \xi_q) U(\phi_q) - \int_{SDP(\pi)} d\xi F_o(\xi, \beta) e^{\phi(\xi)} - \int_{SDP(-\pi)} d\xi F_n(\xi, \beta) e^{\phi(\xi)} \] (4.6)

where

\[ F_o(\xi, \beta) = u_{oz} \sqrt{\frac{\pi}{2 j k s(\xi)}} \left[ \frac{\tilde{\Psi}(\frac{n\pi}{2} + \xi - \phi)}{\tilde{\Psi}(\frac{n\pi}{2} - \phi') \cot \left( \frac{\xi - \beta^-}{2n} \right)} \right] e^{-j(k_r + \nu')}, \] (4.7)

and

\[ F_n(\xi, \beta) = u_{oz} \sqrt{\frac{\pi}{2 j k s(\xi)}} \left[ \frac{\tilde{\Psi}(\frac{n\pi}{2} + \xi - \phi)}{\tilde{\Psi}(\frac{n\pi}{2} + \phi') \cot \left( \frac{\xi - \beta^+}{2n} \right)} \right] e^{-j(k_r + \nu')}, \] (4.8)

with \( \kappa \) denotes \( k \rho \rho' \) and \( f(\xi) = j(1 + \cos \xi) \). The poles \( \xi_q \) of the integrand are listed below:

\[ \xi_1|_{SDP(\pi)} = \xi_{io} = \phi - \phi' \quad \text{or} \quad \xi_1|_{SDP(-\pi)} = \xi_{in} = \phi - \phi', \]

\[ \xi_2 = \xi_{ro} = \phi + \phi' \quad \xi_3 = \xi_{rn} = \phi + \phi' - 2n\pi, \] (4.9)

\[ \xi_{4,5} = \xi_{e,h}^{e,w} = \phi + \theta_{e,h}^o + \pi \quad \xi_{6,7} = \xi_{e,h}^{e,n} = \phi - n\pi - \pi - \theta_{e,h}^n, \]

\[ \phi_1 = \pi + \phi - \phi' \quad \phi_1 = -\pi + \phi - \phi', \]

\[ \phi_2 = \pi - \phi + \phi' \quad \phi_3 = \pi + \phi + \phi' - 2n\pi, \] (4.10)

\[ \phi_{4,5} = \phi_{e,h}^{e,w} - \phi \quad \phi_{6,7} = \phi - \phi_{e,h}^{e,n} \]

with \( \phi_{e,h}^{e,w} = -\xi_{e,h}^o + \cos^{-1}(1/\cosh \nu_{e,h}^o) \) and \( \phi_{e,h}^{e,n} = n\pi + \xi_{e,h}^n - \cos^{-1}(1/\cosh \nu_{e,h}^n) \).

Here \( \theta_{e,h}^{o,n} = \theta_{e,h}^o + j\nu_{e,h}^{o,n} \). The derivation leading to the pole locations given above can
be found in Appendix A. The $\mathcal{R}^{o,n}$ denote the FRCs, namely

$$
\mathcal{R}^o_{e,h}(\phi') = \frac{\sin \phi' - \sin \theta^o_{e,h}}{\sin \phi' + \sin \theta^o_{e,h}} \quad (4.11a)
$$

$$
\mathcal{R}^n_{e,h}(n\pi - \phi') = \frac{\sin(n\pi - \phi') - \sin \theta^n_{e,h}}{\sin(n\pi - \phi') + \sin \theta^n_{e,h}} \quad (4.11b)
$$

As mentioned earlier, one can now heuristically restore the integrity of the actual material coating as shown in Fig. 4.1(b). If one compares (4.11) with (B.13a) and (B.16a) in Appendix B where the latter are obtained from the actual material but the ones from (4.11) are from approximate impedance boundary conditions, one can conjecture the $\sin \theta^{o,n}_{e,h}$ for the impedance boundary condition in (4.11) by $\delta^{o,n}_{e,h}$ for the actual material parameters obtained from (B.13a) and (B.16a) in Appendix B with corresponding $o$ or $n$-face material parameters. This leads one to have

$$
\theta^{o,n}_{e,h} = \sin^{-1} \delta^{o,n}_{e,h} \quad (4.12)
$$

It is important to note that preserving the actual material properties allows one to recover not only the exact FRCs, but also it allows one to obtain exact SW propagation and attenuation constants. In (4.6), the residues of $\mathcal{F}$ evaluated at the real poles $\xi_1, \xi_2,$ and $\xi_3$ must be included in (4.6) if any of these poles are captured when the original integration contour $\mathcal{L} - \mathcal{L}'$ of (4.5) is deformed into the $SDP(\pm \pi)$ paths as indicated in (4.6). These poles at $\xi_1, \xi_2,$ and $\xi_3$ give rise to residues which provide the GO field contribution, i.e., $u^i_z$, $u^{ro}_z$, and $u^{rn}_z$, respectively, where $u^i_z$ is incident field, $u^{ro}_z$ and $u^{rn}_z$ are the fields reflected from the $o$ and $n$-faces of the wedge, respectively. Similarly, for the complex poles $\xi_4, \xi_5, \xi_6$ and $\xi_7$, the residues of $\mathcal{F}$ pertaining to $\xi_{4,5}$ (and $\xi_{6,7}$) are the edge excited surface wave fields $u^{swo}_z$ (and $u^{swn}_z$) traveling away from the edge on the $o$-face (and $n$-face), respectively. All of those surface wave fields are also defined in Appendix A.
In the RHS of (4.6), the SDP integral terms can be seen to provide the diffracted field, \( u^d_z \). As in the preceding chapter, it is of interest to decompose the spectral function in the integrand of (4.6) into two terms, namely

\[
    u^d_z = u^{dgo}_z + u^{dsw}_z
\]

in which \( u^{dgo}_z \) contains only the GO type poles and \( u^{dsw}_z \) contains only the SW type poles. Such a decomposition allows one to conveniently evaluate (4.6) asymptotically for large \( \kappa \) and thereby obtaining the GO dominant UTD diffraction coefficient in a simple form by using the PC method, while the remainder part which contains only the SW type poles, which are complex poles, can be treated by the VDW method in a manner similar to that employed in the development of the solution of the two part diffraction problems of the preceding chapter. However, the quantities \( \frac{\Psi(n\pi + \xi_{ro} - \phi)}{\Psi(n\pi + \phi')} \), and \( \frac{\Psi(n\pi + \xi_{rn} - \phi)}{\Psi(-3n\pi + \phi')} \) are unity at the GO poles, \( \xi_i \), \( \xi_{ro} \), and \( \xi_{rn} \). This leads to a violation of the Karp-Karal lemma for the first order diffracted field. Thus, to avoid the above problem one needs to ad hoc decompose the spectral function by evaluating the MZ functions at the corresponding saddle points \( \pm \xi \) only for the \( u^{dgo}_z \) term. The evaluation of \( u^{dsw}_z \) remains the same as that developed in Chapter 2. Thus the \( u^{dgo}_z \) and \( u^{dsw}_z \) are given by

\[
    u^{dgo}_z = - \frac{1}{8 \pi^2 j n} \int_{SDP(\pi)} d\xi e^{\pi f(\xi)} \sqrt{\frac{\pi}{2 j k s(\xi)}} \left\{ \frac{\tilde{\Psi}(\frac{n\pi}{2} + \pi - \phi)}{\Psi(\frac{n\pi}{2} - \phi')} \cot \left( \frac{\xi - \beta^-}{2n} \right) \right. \\
    + \mathcal{R}(\phi') \frac{\tilde{\Psi}(\frac{n\pi}{2} + \pi - \phi)}{\Psi(\frac{n\pi}{2} + \phi')} \cot \left( \frac{\xi - \beta^+}{2n} \right) \right\} e^{-jk(\rho + \rho') u_{oz}} \\
    - \frac{1}{8 \pi^2 j n} \int_{SDP(-\pi)} d\xi e^{\pi f(\xi)} \sqrt{\frac{\pi}{2 j k s(\xi)}} \left\{ \frac{\tilde{\Psi}(\frac{n\pi}{2} - \pi - \phi)}{\Psi(\frac{n\pi}{2} - \phi')} \cot \left( \frac{\xi - \beta^-}{2n} \right) \right. \\
    + \mathcal{R}(n\pi - \phi') \frac{\tilde{\Psi}(\frac{n\pi}{2} - \pi - \phi)}{\Psi(-3n\pi + \phi')} \cot \left( \frac{\xi - \beta^+}{2n} \right) \right\} e^{-jk(\rho + \rho') u_{oz}}.
\]

(4.14)
\[ u^d_{sw} = -\frac{1}{8\pi^2jn} \int_{SDP(\pi)} d\xi e^{\nu f(\xi)} \sqrt{\frac{\pi}{2jks(\xi)}} \left\{ \tilde{\Psi}\left(\frac{n\pi}{2} + \xi - \phi\right) \cot\left(\frac{\xi_{sw} - \beta^-}{2n}\right) + \mathcal{R}^o(\phi') \frac{\tilde{\Psi}\left(\frac{n\pi}{2} + \xi + \phi\right)}{\tilde{\Psi}\left(\frac{n\pi}{2} + \phi'\right)} \cot\left(\frac{\xi_{sw} + \beta^+}{2n}\right) \right\} e^{-jk(\rho + \rho')u_{oz}} \]
\[ + \frac{1}{8\pi^2jn} \int_{SDP(-\pi)} d\xi e^{\nu f(\xi)} \sqrt{\frac{\pi}{2jks(\xi)}} \left\{ \tilde{\Psi}\left(\frac{n\pi}{2} + \xi + \phi\right) \cot\left(\frac{\xi_{sw} + \beta^-}{2n}\right) + \mathcal{R}^o(n\pi - \phi') \frac{\tilde{\Psi}\left(\frac{n\pi}{2} + \xi - \phi\right)}{\tilde{\Psi}\left(-\frac{3n\pi}{2} + \phi'\right)} \cot\left(\frac{\xi_{sw} - \beta^+}{2n}\right) \right\} e^{-jk(\rho + \rho')u_{oz}}. \]

Next a uniform asymptotic evaluation of the integrals in (4.14) by the PC method and of those in (4.15) by the VDW method, respectively, for large \( \kappa \), yields:

\[ u^d_z = u^i_z(0) D_{e,h}(\phi, \phi') e^{-jk\rho} \]

where \( D_{e,h} = D^{go}_{e,h} + D^{sw}_{e,h} \). The \( u^i_z(0) \) denotes the GO incident field at the diffraction point “0” and it is given by

\[ u^i_z(0) = u_{oz} - j \frac{e^{-j\pi/4}}{2j\pi k s^i} \sqrt{\frac{2j}{\pi k s^i}} e^{-jks^i} \]

where \( s^i = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')} \). The \( D^{go}_{e,h} \) is given by

\[ D^{go}_{e,h} = -\frac{e^{-j\pi/4}}{2\pi \sqrt{2j\pi k}} \left\{ \tilde{\Psi}\left(\frac{n\pi}{2} + \pi - \phi\right) \cot\left(\frac{\pi - \beta^-}{2n}\right) F_{KP} \left[ kLa^-\left(\beta^-\right) \right] \right. \]
\[ + \frac{\tilde{\Psi}\left(\frac{n\pi}{2} + \pi - \phi\right)}{\tilde{\Psi}\left(\frac{n\pi}{2} - \phi'\right)} \cot\left(\frac{\pi + \beta^-}{2n}\right) F_{KP} \left[ kLa^+\left(\beta^-\right) \right] \]
\[ + \mathcal{R}^{o,h}(\phi') \frac{\tilde{\Psi}\left(\frac{n\pi}{2} + \pi + \phi\right)}{\tilde{\Psi}\left(\frac{n\pi}{2} + \phi'\right)} \cot\left(\frac{\pi - \beta^+}{2n}\right) F_{KP} \left[ kLa^+\left(\beta^+\right) \right] \]
\[ \left. + \mathcal{R}^{o,h}(n\pi - \phi') \frac{\tilde{\Psi}\left(\frac{n\pi}{2} + \pi - \phi\right)}{\tilde{\Psi}\left(-\frac{3n\pi}{2} + \phi'\right)} \cot\left(\frac{\pi + \beta^+}{2n}\right) F_{KP} \left[ kLa^+\left(\beta^+\right) \right] \right\} \]
and $D_{e,h}^{sw}$ can be expressed as

$$D_{e,h}^{sw} = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k}} \left[ \Psi\left(\frac{\pi}{2} - \pi - \phi\right) \begin{pmatrix} \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \\ \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \end{pmatrix} - \frac{\Psi\left(\frac{\pi}{2} - \pi - \phi\right)}{\Psi\left(\frac{\pi}{2} - \phi'\right)} \begin{pmatrix} \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \\ \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \end{pmatrix} \right]$$

where $\Psi(\chi)$ is again the well-known UTD edge transition function defined in [1] and

$$F_{KP}(\chi) = 2 \cos^2 \left(\frac{2n\pi N^\pm - \beta}{2}\right)$$

in which $N^\pm$ are the integers which most nearly satisfy the equations

$$2n\pi N^+ - \beta = \pi$$

$$2n\pi N^- - \beta = -\pi.$$  

The $s_{sw}^2 = 2 \cos^2 \frac{\xi_{sw}}{2}$. Here $L = \frac{\rho' \rho}{\rho + \rho'}$. Also

$$R_{e,h}^{sw}(\xi_{sw}) = \frac{\sin \frac{\pi}{2n}}{\Psi\left(\frac{\pi}{2} - \phi'\right)} \psi_n(n\pi - \frac{\pi}{2}) \psi_n(n\pi + \frac{\pi}{2} + 2\theta e,h)$$

$$\begin{pmatrix} \psi_n(\theta e,h + \theta e,h + \frac{\pi}{2}) \psi_n(\theta e,h - \theta e,h + \frac{3\pi}{2}) \\ \psi_n(\theta e,h + \theta e,h + \frac{\pi}{2}) \psi_n(\theta e,h - \theta e,h + \frac{3\pi}{2}) \end{pmatrix} \begin{pmatrix} \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \\ \cot \left(\frac{\xi_{sw} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sw} - \beta^+}{2n}\right) \end{pmatrix}$$

$$R_{e,h}^{sun}(\xi_{sun}) = \frac{\sin \frac{\pi}{2n}}{\Psi\left(\frac{\pi}{2} - \phi'\right)} \psi_n(n\pi - \frac{\pi}{2}) \psi_n(n\pi + \frac{\pi}{2} + 2\theta e,h)$$

$$\begin{pmatrix} \psi_n(\theta e,h + \theta e,h + \frac{\pi}{2}) \psi_n(\theta e,h - \theta e,h + \frac{3\pi}{2}) \\ \psi_n(\theta e,h + \theta e,h + \frac{\pi}{2}) \psi_n(\theta e,h - \theta e,h + \frac{3\pi}{2}) \end{pmatrix} \begin{pmatrix} \cot \left(\frac{\xi_{sun} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sun} - \beta^+}{2n}\right) \\ \cot \left(\frac{\xi_{sun} - \beta^-}{2n}\right) - \cot \left(\frac{\xi_{sun} - \beta^+}{2n}\right) \end{pmatrix}$$

It is noted that the above solution for the line source excitation case automatically recovers the solution for the plane wave excitation by allowing $\rho'$ to recede to infinity (i.e. by letting $\rho' \to \infty$) and by suppressing the line source factor. Also the solution for the line source excitation can be extended to treat the spherical wave excitation case by using a Fourier transformation as discussed later in section 4.3.
4.2 Slope diffraction for a material coated wedge

It is well known that the first order UTD diffraction solution is proportional to the incident field at the point of edge diffraction “0” as can be seen from (4.16). Thus the first order diffraction becomes less dominant or even vanishes when a source produces an incident field with a pattern null either in the direction toward the edge (or the discontinuity) or close to it. The next higher order UTD diffraction solution, which in this case is the slope diffraction solution, becomes dominant in the latter case. Furthermore, when the amplitude of the incident field has a rapid spatial variation at and near the point of edge diffraction, then the total field which includes only a first order diffraction will now exhibit a slope discontinuity at the shadow boundaries. The UTD slope diffraction contribution then restores continuity to the slope of the total field, while the first order diffraction only provides continuity for the total high frequency UTD field. On the other hand, if the incident real ray field vanishes at the edge, as may happen if the source is assumed to exhibit a pattern null exactly in the direction of the edge when the source is off the wedge surface, then the first order diffraction vanishes everywhere. The latter situation also arises when a source or antenna is placed directly on the material coated wedge surface because the space wave or ray optical field launched by such a source vanishes to first order along a direction which is coated surface on which the source is placed; however, the slope of this incident space wave is non zero along the normal to that surface. Of course, it is possible that the source may also excite a surface wave strongly when it is placed on the coated wedge in which case this incident surface wave (SW) diffracts from the edge, while the vanishing incident space wave field produces slope diffraction at the
edge. Thus slope diffraction effects are studied in this Section for the case of the coated wedge geometry.

As discussed previously in Section 2.3, the slope diffraction solution for the coated wedge case can also be developed by employing the reciprocity theorem. However, in this section, an alternative way to develop the slope diffraction is shown following that done in [44] for the special PEC wedge. One starts the latter development by considering an electric line source doublet which produces a non-uniform incident electric field as shown in Fig. 4.3. This electric line source doublet imitates the pattern of a magnetic line dipole source as discussed in Section 2.3 which contains a pattern null that can be aimed either directly on the edge of the wedge or close to it by properly orienting the line source doublet. One can use the UTD solution developed in Section 4.1 for the uniform electric line source case to develop the slope
diffraction solution as follows. First consider the SDP integral in (4.6), which can be expressed as

\[
E_z^d = -\frac{1}{4\pi j n} \int_{SDP(\pi)} d\xi e^{i\ell(\xi)} \left[ \tilde{\Psi}(\frac{\pi}{2} + \xi - \phi) \frac{\Psi(\frac{\pi}{2} + \phi')}{\Psi(\frac{\pi}{2} - \phi')} \cot \left( \frac{\xi - \beta^-}{2n} \right) 
+ R^o(\phi') \tilde{\Psi}(\frac{\pi}{2} + \xi - \phi') \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] e^{-jk\rho} \sqrt{s(\xi)} \rho' E_i^d(0) 

- \frac{1}{4\pi j n} \int_{SDP(-\pi)} d\xi e^{i\ell(\xi)} \left[ \tilde{\Psi}(\frac{\pi}{2} + \xi - \phi') \frac{\Psi(\frac{\pi}{2} - \phi + \phi')}{\Psi(\frac{\pi}{2} + \phi')} \cot \left( \frac{\xi - \beta^+}{2n} \right) 
+ R^o(n\pi - \phi') \tilde{\Psi}(\frac{\pi}{2} + \xi - \phi') \frac{\Psi(-\frac{3\pi}{2} + \phi')}{\Psi(-\frac{\pi}{2} + \phi')} \cot \left( \frac{\xi - \beta^-}{2n} \right) \right] e^{-jk\rho} \sqrt{s(\xi)} \rho' E_i^d(0)
\]

(4.24)

where the \(E_i(0)\) denotes the GO incident electric field at the diffraction point “0” defined in (4.17) as \(E_i(0) = -jkZo I_o e^{-jk\rho} \). By employing (4.24) to obtain the diffracted field from each of the two electric line sources of strength \(I_o\) and \(-I_o\), respectively, comprising the electric line source doublet of Fig. 4.3, one can by superposition of these two fields directly obtain the field diffracted by the coated wedge when it is illuminated by an electric line source doublet. It is noted that the \(I_o\) and \(-I_o\) sources forming the doublet are separated by a very small (almost infinitesimal) distance, \(h\).

If at first, for convenience, the doublet is oriented such that it produces a null directly on the edge of the coated wedge, then only slope diffracted field exists in which case the above superposition yields the slope diffracted field \(E_{sd}^s\) as

\[
E_{sd}^s = E_i^d(0)[\mathcal{D}_e(\phi, \phi' + \epsilon, \xi) - \mathcal{D}_e(\phi, \phi' - \epsilon, \xi)]
\]

(4.25)
where

\[
\mathcal{D}_e(\phi, \phi', \xi) = -\frac{1}{4\pi jn} \int_{SDP(\pi)} d\xi e^{\gamma f(\xi)} \left[ \tilde{M}_1(\phi, \phi', \xi) \cot \left( \frac{\xi - \beta^-}{2n} \right) + \tilde{M}_2(\phi, \phi', \xi) \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] e^{-j\rho \sqrt{\frac{s(\xi)}{\rho'}}},
\]

\[
-\frac{1}{4\pi jn} \int_{SDP(-\pi)} d\xi e^{\gamma f(\xi)} \left[ \tilde{M}_3(\phi, \phi', \xi) \cot \left( \frac{\xi - \beta^-}{2n} \right) + \tilde{M}_4(\phi, \phi', \xi) \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] e^{-j\rho \sqrt{\frac{s(\xi)}{\rho'}}}.
\] (4.26)

with

\[
\tilde{M}_1(\phi, \phi', \xi) = \frac{\tilde{\Psi}(\frac{\pi}{2} + \xi - \phi)}{\tilde{\Psi}(\frac{\pi}{2} - \phi')},
\]

\[
\tilde{M}_2(\phi, \phi', \xi) = \Re^{o}(\phi') \frac{\tilde{\Psi}(\frac{\pi}{2} + \xi - \phi)}{\tilde{\Psi}(\frac{\pi}{2} + \phi')},
\]

\[
\tilde{M}_3(\phi, \phi', \xi) = \frac{\tilde{\Psi}(\frac{\pi}{2} + \xi - \phi)}{\tilde{\Psi}(\frac{3\pi}{2} - \phi')},
\]

\[
\tilde{M}_4(\phi, \phi', \xi) = \Re^{n}(n\pi - \phi') \frac{\tilde{\Psi}(\frac{\pi}{2} + \xi - \phi)}{\tilde{\Psi}(n\pi + \phi')}.
\] (4.27)

Next, one can expand the \(\mathcal{D}_e(\phi, \phi', \xi)\) about \(\phi'\) by using Taylor series as

\[
\mathcal{D}_e(\phi, \phi' \pm \varepsilon, \xi) \sim \mathcal{D}_e(\phi, \phi', \xi) \pm \varepsilon \frac{\partial}{\partial \phi'} \mathcal{D}_e(\phi, \phi', \xi) \text{ (4.28)}
\]

where only first two terms are retained in (4.28). Then the \(E^{sd}_z\) in (4.25) can be expressed as

\[
E^{sd}_z \sim 2\varepsilon E^i_z(0) \frac{\partial}{\partial \phi'} \mathcal{D}_e(\phi, \phi', \xi) \text{ (4.29)}
\]

Next consider the incident field \(E^i_z(Q_n)\) produced by an electric line source doublet at \(Q_n\) as shown in Fig. 4.3. One obtains

\[
E^i_z(Q_n) = -jkZ_0I_o e^{-jk\rho''} e^{jk\frac{\xi}{2}\sin \Delta} - e^{-jk\frac{\xi}{2}\sin \Delta} \] (4.30)
which can be further reduced for $\Delta \to 0$ as

$$E_z^i(Q_n) = -jk I_o \frac{e^{-jk\rho'}}{\sqrt{\rho'}} [jk h].$$

(4.31)

The partial derivative of $E_z^i(Q_n)$ in (4.31) with respect to the distance $\rho'$ in the direction of $\hat{\Delta}$ is defined as

$$\frac{1}{\rho'} \frac{\partial}{\partial \Delta} E_z^i(Q_n) = 2\varepsilon j k Z_o I_o \frac{e^{-jk\rho'}}{\sqrt{\rho'}}$$

$$= 2\varepsilon j k E_z^i(0)$$

(4.32)

where $2\varepsilon = \frac{h}{\rho'}$. Also,

$$\left. \frac{1}{\rho'} \frac{\partial}{\partial \Delta} E_z^i(Q_n) \right|_{\Delta \to 0} = \frac{\partial}{\partial n} E_z^i(0).$$

(4.33)

Thus one can express the $E_{sd}^z$ in (4.29) as

$$E_{sd}^z \sim \frac{1}{jk} \frac{\partial}{\partial n} E_z^i(0) \frac{\partial}{\partial \phi'} D_e(\phi, \phi', \xi)$$

(4.34)

Now consider the terms which will occur in the partial derivative of $D_e(\phi, \phi', \xi)$ with respect to $\phi'$ in the (4.34) above. It is noted from (4.26) that the $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{M}_3$, $\tilde{M}_4$, and $\cot(\cdot)$ functions in $D_e(\phi, \phi', \xi)$ are functions of $\phi'$. The $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{M}_3$, and $\tilde{M}_4$ functions are slowly varying with $\phi'$ and the $\cot(\cdot)$ functions become strongest at the shadow boundaries. For example, as shown in Fig. 4.4, the $\tilde{M}_1$ function is slowly varying in $\phi'$. Also the $\cot\left(\frac{\pi - \beta - 2n}{2n}\right)$ function becomes strongest at the shadow boundary, namely, at $\phi = 210^\circ$ for this example in Fig. 4.4. One concludes that the derivatives of $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{M}_3$, and $\tilde{M}_4$ with respective to $\phi'$ are less important than the $\phi'$ derivatives of the $\cot(\cdot)$ functions which become very strong especially around the shadow boundaries (SBs). Also the spatial rate of change of the incident field becomes important at the SBs where the $\tilde{M}_1$ and $\tilde{M}_3$ become unity. Thus one can approximate the $\tilde{M}_1$ and $\tilde{M}_3$ by unity for the slope diffraction case. In addition,
as mentioned in Section 2.5, if the reflected field has a rapid spatial variation at the edge, this may cause a problem at the RBs unless one accounts for the slope diffraction effects associated with the rapid variation of not only the incident but also the reflected field. One can apply the same technique as done in Section 2.5 to treat the spatial variation of the reflected fields, which becomes important at the RBs where the $\tilde{M}_2$ and $\tilde{M}_4$ become $R^o$ and $R^n$. One can therefore also approximate the $\tilde{M}_2$ and $\tilde{M}_4$ by $R^o(\phi)$ and $R^n(\phi)$, respectively. Note that the $R^o(\phi)$ and $R^n(\phi)$ are now functions of $\phi$. Thus the partial derivative of $D_ε(\phi, \phi', \xi)$ with respect to $\phi'$ is
finally given, after incorporating the above approximations, by the following:

\[ \frac{\partial}{\partial \phi'} D_e(\phi, \phi', \xi) \sim - \frac{1}{8\pi jn} \int_{SDP(\pi)} d\xi e^{\kappa f(\xi)} \left[ - \csc^2 \left( \frac{\xi - \beta^-}{2n} \right) \right. \]

\[ + R_e^o(\phi) \csc^2 \left( \frac{\xi - \beta^+}{2n} \right) \left. \right] \frac{e^{-jk\rho}}{\sqrt{s(\xi)/\rho}} \]

\[ - \frac{1}{8\pi jn} \int_{SDP(-\pi)} d\xi e^{\kappa f(\xi)} \left[ - \csc^2 \left( \frac{\xi - \beta^-}{2n} \right) \right. \]

\[ + R_e^o(n\pi - \phi) \csc^2 \left( \frac{\xi - \beta^+}{2n} \right) \left. \right] \frac{e^{-jk\rho}}{\sqrt{s(\xi)/\rho}} \]

(4.35)

It is noted that the integrand in SDP integral of (4.35) has a double pole due to the \( \csc^2(\cdot) \). One needs to treat the double pole properly when performing the uniform asymptotic evaluation. The details of the uniform asymptotic evaluation when the integrand has a double pole are shown in Appendix C. If one assumes that \( \sin \theta_{o,n} \) is large enough such that the SW pole location is far from saddle points \( \xi_s = \pm \pi \) even when the source is located on the faces of wedge, and the observation angle \( \phi \) approaches the wedge faces, then one may use the PC method to evaluate the SDP integral. After uniform asymptotic evaluation of the SDP integral in (4.35) for this latter situation, one obtains

\[ \frac{\partial}{\partial \phi'} D_e(\phi, \phi') \sim - \frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \left[ - \csc^2 \left( \frac{\pi - \beta^-}{2n} \right) F_s[kLa^-(\beta^-)] \right. \]

\[ + R_e^o(\phi) \csc^2 \left( \frac{\pi - \beta^+}{2n} \right) F_s[kLa^-(\beta^+)] \]

\[ + \csc^2 \left( \frac{\pi + \beta^-}{2n} \right) F_s[kLa^+(\beta^-)] \]

\[ - R_e^o(n\pi - \phi) \csc^2 \left( \frac{\pi + \beta^+}{2n} \right) F_s[kLa^+(\beta^+)] \left. \right] \frac{e^{-jk\rho}}{\sqrt{\rho}} \]

(4.36)

where \( F_s[\chi] = 2j\chi \{ 1 - F_{KP}[\chi] \} \). Then (4.34) now becomes

\[ E_{sd}^z \sim \frac{1}{jk} \left\{ \frac{\partial}{\partial n} E_z^i(0) \frac{\partial}{\partial \phi'} D_i^e + \frac{\partial}{\partial n} E_z^{ro}(0) \frac{\partial}{\partial \phi'} D^{ro}_e + \frac{\partial}{\partial n} E_z^{rn}(0) \frac{\partial}{\partial \phi'} D^{rn}_e \right\} \frac{e^{-jk\rho}}{\sqrt{\rho}} \]

(4.37)
in which
\[
\frac{\partial}{\partial \phi'} D^i_e \equiv D^sdi \sim -\frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \left\{ -\csc^2\left(\frac{\pi - \beta^-}{2n}\right) F_s \left[ kLa^-(\beta^-) \right] + \csc^2\left(\frac{\pi + \beta^-}{2n}\right) F_s \left[ kLa^+(\beta^-) \right] \right\}
\] (4.38)

\[
\frac{\partial}{\partial \phi'} D^{ro}_e \equiv D^sdro \sim -\frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \frac{\mathcal{R}_h(\phi)}{\csc^2\left(\frac{\pi - \beta^-}{2n}\right)} F_s \left[ kLa^-(\beta^-) \right]
\]

\[
\frac{\partial}{\partial \phi'} D^{rn}_e \equiv D^sdrn \sim \frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \frac{\mathcal{R}_h(n\pi - \phi)}{\csc^2\left(\frac{\pi + \beta^-}{2n}\right)} F_s \left[ kLa^+(\beta^-) \right]
\] (4.39)

Likewise, for the magnetic line source doublet, the slope diffracted field \( H^sd_z \) can be obtained in the same manner. Indeed, one can show that
\[
H^sd_z \sim \frac{1}{jk} \left\{ \frac{\partial}{\partial n} H^i_z(0) \frac{\partial}{\partial \phi'} D^i_h + \frac{\partial}{\partial n} H^{ro}_z(0) \frac{\partial}{\partial \phi'} D^{ro}_h + \frac{\partial}{\partial n} H^{rn}_z(0) \frac{\partial}{\partial \phi'} D^{rn}_h \right\} e^{-jk\rho} \sqrt{\rho}
\] (4.40)

where
\[
\frac{\partial}{\partial \phi'} D^i_h \equiv D^sdi \sim -\frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \left\{ -\csc^2\left(\frac{\pi - \beta^-}{2n}\right) F_s \left[ kLa^-(\beta^-) \right] + \csc^2\left(\frac{\pi + \beta^-}{2n}\right) F_s \left[ kLa^+(\beta^-) \right] \right\}
\] (4.41)

\[
\frac{\partial}{\partial \phi'} D^{ro}_h \equiv D^sdro \sim -\frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \frac{\mathcal{R}_h(\phi)}{\csc^2\left(\frac{\pi - \beta^-}{2n}\right)} F_s \left[ kLa^-(\beta^-) \right]
\]

\[
\frac{\partial}{\partial \phi'} D^{rn}_h \equiv D^sdrn \sim \frac{e^{-jk\pi/4}}{4n^2\sqrt{2\pi k}} \frac{\mathcal{R}_h(n\pi - \phi)}{\csc^2\left(\frac{\pi + \beta^-}{2n}\right)} F_s \left[ kLa^+(\beta^-) \right]
\] (4.42)

In general, the diffracted field \( u^d_z \) can be expressed as
\[
u^d \sim \left[ u^i(0)D_{e,h}(\phi, \phi') + \frac{\partial}{\partial n} u^i_z(0) \right] \frac{1}{jk} D^sdi \left( \phi, \phi' \right) + \left[ \frac{\partial}{\partial n} u^{ro}_z(0) \right] \frac{1}{jk} D^{sro} \left( \phi, \phi' \right) + \left[ \frac{\partial}{\partial n} u^{rn}_z(0) \right] \frac{1}{jk} D^{sdrn} \left( \phi, \phi' \right) e^{-jk\rho} \sqrt{\rho}
\] (4.43)

It is noted that the general result above in (4.43) includes both, the first order UTD as well as the UTD slope diffraction contributions. Thus, (4.43) is now also applicable
to line source doublet excitation for which the pattern null is not necessarily aimed
directly at the edge even though the original development leading to (4.37) and (4.40)
was based on the null of the doublet as being aimed at the edge for convenience of
the analysis. The first order UTD in (4.43) is expressed in terms of $D_{e,h}$ where

$$D_{e,h}(\phi, \phi') = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k}} \left\{ \tilde{M}_1(\phi, \phi', \pi) \cot \left( \frac{\pi - \beta^-}{2n} \right) F_{KP} \left[ kLa^- (\beta^-) \right] 
+ \tilde{M}_2(\phi, \phi', \pi) \cot \left( \frac{\pi - \beta^+}{2n} \right) F_{KP} \left[ kLa^- (\beta^+) \right] 
+ \tilde{M}_3(\phi, \phi', -\pi) \cot \left( \frac{\pi + \beta^-}{2n} \right) F_{KP} \left[ kLa^+ (\beta^-) \right] 
+ \tilde{M}_4(\phi, \phi', -\pi) \cot \left( \frac{\pi + \beta^+}{2n} \right) F_{KP} \left[ kLa^+ (\beta^+) \right] \right\}$$  \hspace{1cm} (4.44)

and $D_{sdi,e,h}$ are obtained from (4.38) and (4.41), respectively. The $D_{sdr,e,h}$ and $D_{sdn,e,h}$ are
obtained from (4.39) and (4.42), respectively. Here $a^\pm(\beta)$ and $L$ are the same as in
the previous section.

4.3 Development of a 3-D solution for a material coated PEC
wedge illuminated by a current moment

Consider the 3-D diffraction problem of a material coated PEC wedge which is
illuminated by a $\hat{z}$-directed infinitesimal electric or magnetic current moment $\hat{z}dp =
\hat{z}dp$ as shown in Fig. 4.5. It is of interest to develop a UTD solution for this 3-
D problem in the present Section. For the sake of simplicity, the slope diffraction
effects are initially neglected in this development; they will be added separately to
the solution later on. By using the same technique as presented in [36], one can
first extend the 2D solution for the $\hat{z}$ component $u_z$ corresponding to \( \left\{ E_z \right\} \) to the
corresponding 3D solution for $U_z$ by using the inverse Fourier transformation, in
which \( \hat{z} \) is directed along the edge, namely,

\[
U_z(s, s') = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{u}_z(\phi, \phi'; k_z) e^{jk_z z'} dk_z
\]  

(4.45)

where \( \hat{u}_z(\phi, \phi'; k_z) e^{jk_z z'} = u_z(\phi, \phi'; k_t) \) with \( k_t = \sqrt{k^2 - k_z^2} \). Thus from (4.1), one can rewrite the total field \( u_z \) with \( k \) replaced by the effective transverse wave number \( k_t \) as

\[
u_z(\phi, \phi'; k_t) = -\frac{1}{16\pi n} \int_{\xi - \xi'} d\xi \tilde{M}(\phi, \phi', \xi) \left[ \cot \left( \frac{\xi - \beta}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] H_o^{(2)}[k_t s(\xi)] u_{oz} \]

(4.46)

in which \( \tilde{M}(\phi, \phi', \xi) = \frac{\Psi(n\pi + \xi - \phi)}{\Psi(n\pi - \phi')} \). One can then obtain the total field \( \bar{U}_z = \hat{z} U_z \) by

![Figure 4.5: A material coated PEC wedge illuminated by a \( \hat{z} \)-directed infinitesimal current moment of strength \( dp \)](image)
applying the inverse Fourier transformation in (4.45) together with (4.46) as
\[
\mathcal{U}_z (\bar{s}, \bar{s}') = -\frac{1}{32\pi^2 n} \int_{\mathcal{L}-\mathcal{L}'} d\xi \bar{\mathcal{L}}(\phi, \phi', \xi) \left[ \cot \left( \frac{\xi - \beta^-}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] \cdot \mathcal{U}_{\omega z} \int_{-\infty}^{\infty} dk_z H_0^{(2)}(k_z s(\xi)) e^{-jk_z(z'-z)}, \tag{4.47}
\]
where \( \bar{L} \) is an unknown dyad which will be determined shortly. Using (3.43) in (4.47) yields:
\[
\mathcal{U}_z (\bar{s}, \bar{s}') = \frac{1}{16\pi^2 j n} \int_{\mathcal{L}-\mathcal{L}'} d\xi \bar{\mathcal{L}}(\phi, \phi', \xi) \left[ \cot \left( \frac{\xi - \beta^-}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] e^{-jk_s(z'-z)} \cdot \mathcal{U}_{\omega z} \tag{4.48}
\]
where \( S(\xi) \) is defined as
\[
S(\xi) = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos \xi + (z - z')^2}. \tag{4.49}
\]
With \( \rho = s \sin \beta_o, \rho' = s' \sin \beta_o, \) and \( |z - z'| = (s + s') \cos \beta_o, \) it can be written as
\[
S^2(\xi) = (s + s')^2 \left\{ 1 - \frac{2ss'}{(s + s')^2} \sin^2 \beta_o (1 + \cos \xi) \right\}. \tag{4.50}
\]
Then the exponential term in (4.48) can be further approximated by the first two terms of its binomial expansion for large \( k \frac{ss'}{s+s'} \sin^2 \beta_o, \) which yields
\[
S(\xi) \sim s + s' \left\{ 1 - \frac{ss'}{(s + s')^2} \sin^2 \beta_o (1 + \cos \xi) \right\}. \tag{4.51}
\]
Now one can write (4.48) symbolically as
\[
\mathcal{U}_z (\bar{s}, \bar{s}') = \int_{\mathcal{L}-\mathcal{L}'} d\xi \bar{\mathcal{F}}(\xi) e^{\kappa f(\xi)} \tag{4.52}
\]
where \( \kappa = k \frac{ss'}{s+s'} \sin^2 \beta_o, \ f(\xi) = j(1 + \cos \xi) \) and
\[
\bar{\mathcal{F}}(\xi) = -\frac{1}{16\pi^2 j n} \bar{\mathcal{L}}(\phi, \phi', \xi) \left[ \cot \left( \frac{\xi - \beta^-}{2n} \right) - \cot \left( \frac{\xi - \beta^+}{2n} \right) \right] \frac{e^{-jk_s(s+s')}}{S(\xi)} \cdot \mathcal{U}_{\omega z}. \tag{4.53}
\]
From the GO fields, the GO incident field $\bar{U}_z^i$ is given by

$$\bar{U}_z^i = -jk \left[ \frac{Z_o d p_{e z}}{Y_o d p_{m z}} \right] \sin^2 \beta_o e^{-jk S^i} \left( \frac{\sin 2 \beta_o e^{-jk S^i}}{4 \pi S^i} \right)$$

(4.54)

and from Appendix B, the GO reflected field $\bar{U}_z^{ro}$ from $o$-face is given by

$$\bar{U}_z^{ro} = -\bar{T}(\phi') \cdot \bar{R}^o(\phi') \cdot \bar{T}(\phi') \cdot \bar{U}_z^i(Q_{ro}) e^{-jk S_{ro}} S_{ro}.$$  

(4.55)

It is noted that all of the angles need to be measured from its own face for example in case of the GO reflected field from $o$-face $\bar{U}_z^{ro}$, the angles ($\phi, \phi'$) are measured from the $o$-face as shown in Fig. 4.1. Likewise, for the GO reflected field from $n$-face $\bar{U}_z^{rn}$ the angles ($\phi, \phi'$) need to be measured from the $n$-face, thus those angles ($\phi, \phi'$) are given by ($n\pi - \phi, n\pi - \phi'$). Thus, the $\bar{U}_z^{rn}$ can be expressed as

$$\bar{U}_z^{rn} = -\bar{T}(n\pi - \phi') \cdot \bar{R}^n(n\pi - \phi') \cdot \bar{T}(n\pi - \phi') \cdot \bar{U}_z^i(Q_{rn}) e^{-jk S_{rn}} S_{rn}.$$  

(4.56)

$$\bar{T}(\phi') = \frac{1}{\Delta(\phi')} \begin{bmatrix} \cos \beta_o \sin \phi' & -\cos \phi' \\ \cos \phi' & \cos \beta_o \sin \phi' \end{bmatrix}$$

(4.57)

where the $\bar{U}_z^i(Q_{ro})$ (and $\bar{U}_z^i(Q_{rn})$) represent the incident field radiated by the $z$-directed current moment evaluated at the reflection point on the $o$-face at $Q_{ro}$ if the $o$-face is illuminated (and on the $n$-face at $Q_{rn}$ if the $n$-face is illuminated), respectively. The $S_{ro}$ (and $S_{rn}$) is the distance between the point of reflection $Q_{ro}$ (and $Q_{rn}$) on the $o$-face (and $n$-face) to an observation point. The $\Delta(\phi') = \sqrt{1 - \sin^2 \beta_o \sin^2 \phi'}$ and $\bar{R}^o$ and $\bar{R}^n$ are given by

$$\bar{R}^o(\phi') = \begin{bmatrix} R^o_h & 0 \\ 0 & R^o_e \end{bmatrix} \quad \text{and} \quad \bar{R}^n(n\pi - \phi') = \begin{bmatrix} R^n_h & 0 \\ 0 & R^n_e \end{bmatrix}$$

(4.58)

The $\mathcal{R}_{e,h}^{o,n}$ are the FRCs of $o$ and $n$-face which can be found in (B.16a) (B.13a), respectively with corresponding $o$ and $n$-face material electrical parameters $(\epsilon_r, \mu_r)$.  

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The residues corresponding to the real poles $\xi_1$, $\xi_2$, and $\xi_3$ of $\mathcal{F}(\xi)$ in (4.53) whose locations are listed in (4.9) yield the GO fields, which must be identical to the GO fields shown in (4.54)-(4.56). In particular, it is well known [23] that the $TE_z$ and $TM_z$ solutions are coupled at the edge of a coated PEC wedge (while they are uncoupled for the PEC wedge). This coupling between the $TE_z$ and $TM_z$ reflected GO solutions is contained within the transformation $T$ of (4.57). Thus one can identify $\tilde{L}(\phi, \phi', \xi)$ in $\mathcal{F}(\xi)$ by appropriately introducing a transformation matrix related to $\mathcal{T}(\phi')$ into the original $\mathcal{F}(\xi)$ of (4.53) such that it can at least recover the incident and reflected reflected GO field contribution from the residues. This leads one to a modified version of (4.53) with $\tilde{L}$ to be $\bar{M}$ for the cot $\left(\frac{\xi-\beta}{2n}\right)$ term, and $\tilde{L}$ to be $\mathcal{T}_d(\nu)\cdot \bar{M}_r(\phi, \phi', \xi)\cdot \mathcal{T}_d(\nu)$ for the cot $\left(\frac{\xi-\beta^+}{2n}\right)$ term, respectively:

$$
\mathcal{F}(\xi) = \frac{1}{16\pi^2 jn} \left[ \bar{M}(\phi, \phi', \xi) \cot \left(\frac{\xi-\beta^-}{2n}\right) - \mathcal{T}_d(\nu) \cdot \bar{M}_r(\phi, \phi', \xi) \cdot \mathcal{T}_d(\nu) \cot \left(\frac{\xi-\beta^+}{2n}\right) \right] e^{-jk(s+s')} S(\xi) \cdot \bar{U}_{oz}. 
$$

(4.59)

in which

$$
\bar{M}_r(\phi, \phi', \xi) = \begin{bmatrix} \tilde{M}_h & 0 \\ 0 & \tilde{M}_e \end{bmatrix} 
$$

(4.60)

It is important to note that the $\mathcal{T}(\phi')$ of (4.57) is now replaced by $\mathcal{T}_d(\nu)$ in (4.59) where $\nu$ is a function of $\phi$, of course, at the reflection shadow boundaries $\mathcal{T}_d(\nu)$ reduces to $\mathcal{T}(\phi')$ and it also provides the correct GO reflected fields from the residues at the GO reflection poles in the integrand of (4.59). The precise form of $\mathcal{T}_d(\nu)$ is defined later in (4.66) in which $\nu$ is obtained by using polynomial interpolation which allows sufficient degrees of freedom so that the 3-D solution can properly satisfy certain important physical conditions. The latter important physical requirements are, for example, that it satisfies the Karp-Karal lemma as well as the continuity of the total
field at the reflection shadow boundaries after asymptotic evaluation of (4.52). It is useful to categorize the physical requirements separately for three distinct situations, namely for the case when only the \(o\)-face is illuminated designated as \(m = o\) case, or when only the \(n\)-face is illuminated designated as the \(m = n\) case, or when both faces are illuminated which is designated as the \(m = 2\) case. Thus \(\nu\) is given by

\[\nu = \phi - p_m(\phi) \quad , \quad m = o, n, 2 \] (4.61)

where the polynomial, \(p_m(\phi)\) is defined as following. For the \(m = 0\) case, let

\[p_o(\phi) = a_0 + a_1 \phi + a_2 \phi^2\]

where \(a_0\), \(a_1\), and \(a_2\) are sufficient to allow the following three conditions to be true when only the \(o\)-face is illuminated, namely that \(\nu = \phi - p_o(\phi) = -\phi'\) at the reflection shadow boundary (\(RSB_o\) \(\phi_{ro}\) for the \(o\)-face, and that the Karp-Karal lemma be satisfied independently on the \(o\) and \(n\) wedge faces, respectively. The latter lemma requires \(\overline{T}_d(\nu) \rightarrow 0\) on the \(o\) and \(n\) faces in the \(m = 0\) case. It can be shown that \(a_o = 0\), and

\[p_o(\phi) = \frac{(\pi - \phi_{ro})\phi^2 + (\phi_{ro}^2 - n\pi^2)\phi}{\phi_{ro}(\phi_{ro} - n\pi)} \] (4.62)

where \(\phi_{ro} = \pi - \phi'\). For the \(m = n\) case, let

\[p_n(\phi) = b_o + b_1 \phi + b_2 \phi^2.\]

Again, \(b_o\), \(b_1\), and \(b_2\) are sufficient to allow the following three conditions to be true when only the \(n\)-face is illuminated, namely that \(\nu = \phi - p_n(\phi) = (n - 1)\pi - \phi'\) at the reflection shadow boundary (\(RSB_n\) \(\phi_{rn}\) for the \(n\)-face, and that the Karp-Karal lemma be satisfied independently on the two faces when \(\overline{T}_d \rightarrow 0\), respectively. It can
be also shown that $b_o = 0$, and

$$p_n(\phi) = \frac{(n\pi - \phi_{rn})\phi^2 + (\phi_{rn}^2 - \{n\pi\}^2)\phi}{\phi_{rn}(\phi_{rn} - n\pi)}$$

(4.63)

where $\phi_{rn} = (2n - 1)\pi - \phi'$. For the $m = 2$ case, one requires

$$p_2(\phi) = c_o + c_1\phi + c_2\phi^2 + c_3\phi^3.$$  

Since four conditions need to be met when both faces are illuminated namely that the Karp-Karal lemma must be satisfied on both wedge faces as before, and that the $\nu = -\phi'$ on $RSB_o$ while $\nu = (n - 1)\pi - \phi'$ on $RSB_n$. This leads to $c_o = 0$ and

$$p_2(\phi) = c_3\phi^3 + c_2\phi^2 + c_1\phi$$  

(4.64)

where the unknown coefficients $a_1, a_2$ and $a_3$ in (4.64) can be found from

$$\begin{bmatrix} \phi_{ro}^3 & \phi_{ro}^2 & \phi_{ro} \\ \phi_{rn}^3 & \phi_{rn}^2 & \phi_{rn} \\ \{n\pi\}^3 & \{n\pi\}^2 & n\pi \end{bmatrix} \begin{bmatrix} c_3 \\ c_2 \\ c_1 \end{bmatrix} = \begin{bmatrix} \pi \\ n\pi \\ n\pi \end{bmatrix}.$$  

(4.65)

The $\bar{T}_d(\nu)$ with $\nu$ as in (4.61) is thus given by

$$\bar{T}_d(\nu) = \frac{1}{\Delta(\nu)} \begin{bmatrix} -\cos \beta_o \sin \nu & \cos \nu \\ -\cos \nu & -\cos \beta_o \sin \nu \end{bmatrix}$$  

(4.66)

It is also important to note that the conditions used to synthesize the spectrum in (4.59) are obtained only from space wave considerations. As such the $\bar{T}_d(\nu)$ in (4.59) does not contain any information on the SW poles; this fact may lead to some inaccuracies in calculating the residues associated with the SW poles which in turn provide the SW contributions that are excited on the two faces via diffraction of the incident wave at the edge. Thus, an improved SW residue calculation can be heuristically obtained following the work in [23] by using $\bar{T}(\alpha)$ instead of $\bar{T}_d(\nu)$ in (4.59) only for calculating these SW contributions; the $\alpha$ in $\bar{T}(\alpha)$ is evaluated at
specific values given by [23]. In particular, the edge-excited surface wave field $\bar{U}_{z\text{owo}}$ on the $\alpha$-face is given by

$$\bar{U}_{z\text{owo}} = -\mathbf{T}(\xi_{\text{owo}} - \phi + \frac{n\pi}{2}) \cdot \bar{R}_{e,h} \cdot \mathbf{T}(\frac{n\pi}{2} - \phi') \cdot \bar{U}_{oz} e^{-jks(\xi_{\text{owo}})}.$$  (4.67)

where $\mathbf{T}(\cdot)$ is defined in (4.57) and

$$\bar{R}_{h} = \left[ \begin{array}{cc} R_{h} & 0 \\ 0 & 0 \end{array} \right]$$  (4.68)

and

$$\bar{R}_{e} = \left[ \begin{array}{cc} 0 & 0 \\ 0 & R_{e} \end{array} \right]$$  (4.69)

The $\bar{U}_{z\text{owo}}$, which is the SW field launched on the $n$-face by edge diffraction of the incident wave, can be found by only replacing each $o$ by $n$ in (4.67). After closing the contour $\mathcal{L} - \mathcal{L}'$ by using the SDP$(\pm \pi)$ which pass through the saddle points $\xi_s = \pm \pi$ as shown in Fig. 4.2, one can obtain the following expression for (4.52) in which $\mathbf{F}$ is now given by (4.59):

$$\bar{U}_z = 2\pi j \sum_{q=1}^{q=7} \text{Res} (\xi = \xi_q) U(\phi_q) - \int_{\text{SDP}(\pi)} d\xi \mathbf{F}(\xi, \beta) e^{\kappa_f(\xi)} - \int_{\text{SDP}(-\pi)} d\xi \mathbf{F}(\xi, \beta) e^{\kappa_f(\xi)}$$  (4.70)

where $q = 1, 2, ..., 7$ because there are GO poles corresponding to the incident GO field, the reflected GO fields from the two faces, and the $TE_z$ as well as $TM_z$ SW fields both of which are launched on each of the two faces respectively. The last two terms in (4.70) represents the diffracted field $\bar{U}_z^d$. After asymptotic evaluation of the SDP integral, one obtains the first order diffracted wave to be

$$\bar{U}_z^d = \bar{U}_z (Q_e) \cdot \mathbf{D}(\phi, \phi', \beta) A(s, s') e^{-jks}.$$  (4.71)
where

\[
\overline{D}(\phi, \phi', \beta_o) = -\frac{e^{-j\pi/4}}{2n\sqrt{2\pi k \sin \beta_o}} \left\{ \overline{M}(\phi, \phi', \pi) \cot \left( \frac{\pi - \beta^-}{2n} \right) F_{KP} \left[ kLa^- (\beta^-) \right] + \overline{M}(\phi, \phi', -\pi) \cot \left( \frac{\pi + \beta^-}{2n} \right) F_{KP} \left[ kLa^+ (\beta^-) \right] - \overline{T}_d(\nu) \cdot \overline{M}_r(\phi, \phi', \pi) \cdot \overline{T}_d(\nu) \cot \left( \frac{\pi - \beta^+}{2n} \right) F_{KP} \left[ kLa^- (\beta^+) \right] - \overline{T}_d(\nu) \cdot \overline{M}_r(\phi, \phi', -\pi) \cdot \overline{T}_d(\nu) \cot \left( \frac{\pi + \beta^+}{2n} \right) F_{KP} \left[ kLa^+ (\beta^+) \right] \right\}
\]

(4.72)

where \( \overline{U}_z^i(Q_e) \) is the incident field in (4.54) evaluated at the point of diffraction \( Q_e \) on the edge as shown in Fig. 4.5 and \( A(s, s') = \sqrt{\frac{s'}{s(s+s')}} \). Without going through details, one may express the UTD diffracted field from point \( Q_e \) on the edge which now also includes slope diffraction can be expressed in the following general form, namely:

\[
\begin{align*}
\overline{U}_z^d & \sim \left[ \overline{U}_z^i(Q_e) \cdot \overline{D}(\phi, \phi', \beta_o) + \frac{\partial}{\partial n} \overline{U}_z^i(Q_e) \cdot \frac{1}{jk} \overline{D}^{sdi}(\phi, \phi', \beta_o) + \frac{\partial}{\partial n} \overline{U}_z^r(Q_e) \cdot \frac{1}{jk} \overline{D}^{sdro}(\phi, \phi', \beta_o) \right] A(s, s')e^{-jks}
\end{align*}
\]

(4.73)

\[
\overline{D}^{sdi} = -\frac{e^{-j\pi/4}}{4n^2\sqrt{2\pi k \sin \beta_o}} \overline{D}_d(\nu) \left\{ -\csc^2 \left( \frac{\pi - \beta^-}{2n} \right) F_s \left[ kLa^- (\beta^-) \right] + \csc^2 \left( \frac{\pi + \beta^-}{2n} \right) F_s \left[ kLa^+ (\beta^-) \right] \right\}
\]

(4.74)

\[
\overline{D}^{sdro} = -\frac{e^{-j\pi/4}}{4n^2\sqrt{2\pi k \sin \beta_o}} \overline{T}_d(\nu) \cdot \overline{D}_d(\nu) \csc^2 \left( \frac{\pi - \beta^+}{2n} \right) F_s \left[ kLa^- (\beta^+) \right]
\]

\[
\overline{D}^{sdrn} = \frac{e^{-j\pi/4}}{4n^2\sqrt{2\pi k \sin \beta_o}} \overline{T}_d(\nu) \cdot \overline{n}(\pi - \phi) \cdot \overline{T}_d(\nu) \csc^2 \left( \frac{\pi + \beta^+}{2n} \right) F_s \left[ kLa^+ (\beta^+) \right]
\]

(4.75)

where \( a^\pm(\beta) \) is defined in (4.20) and here \( L = \frac{s'}{s+s'} \sin^2 \beta_o \). The \( \overline{I} \) represents an identity matrix.

In case of an arbitrary-polarized infinitesimal current moment \( d\overline{p} \), the diffracted field \( \overline{U}_z^d \) can be decomposed into a component in the edge-fixed plane coordinate
system [1], namely, \( E^d_z = \sin \beta_o E^d_\beta \) and \( \eta_o H^d_z = \sin \beta_o E^d_\phi \). Finally, knowing \( \bar{U}^d \), one can write the complete vector field polarized transverse to the diffracted ray as [23, 36]:

\[
\bar{U}^d \sim \left[ \bar{U}^i(Q_e) \cdot \bar{D}(\phi, \phi', \beta_o) + \frac{\partial}{\partial n} \bar{U}^i(Q_e) \cdot \frac{1}{jk} \bar{D}_{sdi}(\phi, \phi', \beta_o) \right. \\
\left. + \frac{\partial}{\partial n} \bar{U}^{ro}(Q_e) \cdot \frac{1}{jk} \bar{D}_{sdro}(\phi, \phi', \beta_o) + \frac{\partial}{\partial n} \bar{U}^{rn}(Q_e) \cdot \frac{1}{jk} \bar{D}_{sdrn}(\phi, \phi', \beta_o) \right] A(s, s') e^{-jks} 
\]  \hspace{1cm} (4.76)

where

\[
\bar{U}^d = \begin{bmatrix} E^d_\beta \\ E^d_\phi \end{bmatrix} ; \quad \bar{U}^i = \begin{bmatrix} E^i_\beta \\ E^i_\phi \end{bmatrix} ; \quad \bar{U}^{ro} = \begin{bmatrix} E^{ro}_\beta \\ E^{ro}_\phi \end{bmatrix} ; \quad \bar{U}^{rn} = \begin{bmatrix} E^{rn}_\beta \\ E^{rn}_\phi \end{bmatrix}. 
\]  \hspace{1cm} (4.77)

The \( \hat{\beta}_o, \hat{\phi}, \hat{\beta}'_o, \) and \( \hat{\phi}' \) are defined in [1, 36].
Numerical results are presented in this chapter to demonstrate both the utility and the accuracy of the present UTD solutions developed in Chapters 2 through 4. In particular, the radiation and scattering associated with the source excited canonical problems of interest in Fig. 1.3 are calculated by using the present UTD solutions, and are shown to compare very well some other available reference solutions such as ray solutions based on the W-H \cite{17, 33, 34} and MZ \cite{23} methods which may employ either rigorous or approximate formulations satisfying impedance boundary conditions on the canonical surfaces of interest. Also, an available numerical solution based on the Finite Element Boundary Integral method (FE-BI) is employed as a reference solution for providing comparison with the present UTD solutions when the latter are employed to study the scattering by a thin material slab of finite extent excited by a line source.

The results in Figs. 5.1-5.4 show the validity of the relatively simpler, but approximate, UTD solutions for some 2-D problems of diffraction by thin planar material discontinuities as developed in Chapter 2. First consider the canonical two part problem illuminated by a nearby uniform line source as shown in Fig. 2.1. For this case, it is seen from Fig. 5.1 that a very good agreement is obtained between the diffracted...
field calculated by the present approximate UTD and a reference W-H solution. The
W-H solution which was based on the use of approximate impedance boundary con-
ditions is obtained from [34]. It is important to note that the present approximate
UTD based first order diffracted field for both polarizations shown in Fig. 5.1 satisfies the boundary condition on the PEC and the Karp-Karal lemma. Fig. 5.2 shows
the total field for TM plane wave scattering by the material half plane on a PEC
entire plane of Fig. 2.1. Fig. 5.2(a) pertains to a DPS material, while Fig. 5.2(b)
is for a DNG material. In both cases, the simpler and approximate UTD solution
agrees very well with the W-H solution of [34] which had to be modified so that the
impedance boundary condition on which it is based allows a thin DNG material to
also be approximated by a surface impedance in that W-H solution. Fig. 5.3(a) shows
the result for the total fields surrounding the geometry of Fig. 2.5 which corresponds
to just only a material half plane without a PEC entire plane backing. It is noted
that the magnetic line source case is removed to infinity ($\rho' \to \infty$) to simulate a
TM plane wave illumination in the vicinity of the discontinuity at “0” for the UTD
solution in Fig. 5.3. The material thickness and electrical parameters are shown in all
the figures; it is noted that the coating consists of a thin negative (or DNG) material
for the cases in Figs. 5.2(b) and 5.3(a). It is clear that the approximate impedance
boundary conditions in the W-H solution become valid for the extremely thin ma-
terial half planes chosen here for all the numerical comparisons. It is also possible
to compare the present simpler but approximate UTD solutions with corresponding
completely independent numerical solution based on the FE-BI method for the case
of 2-D diffraction by a thin material strip which is excited by an external line source.
The scattered field calculated by the present UTD solutions with and without slope
diffraction are compared with the corresponding FE-BI of [45] in Fig. 5.4 for this 2-D problem. It is noted that the “kink” in Fig. 5.4(a) is due to the non-vanishing derivative of the reflected field on the shadow boundary. This is eliminated by adding the slope diffracted field contribution arising from a rapidly varying reflected field at the edges that may be viewed as a rapidly varying field incident from image space as explained in Chapter 2. The inclusion of this slope diffraction results in a smooth and continuous scattered field as shown in Fig. 5.4(b). The results obtained by these two independent methods are shown to agree very well, thereby lending confidence to the new UTD results which also include the higher order slope diffraction effects.
Figure 5.1: Comparison of UTD and W-H solutions for diffracted fields at $\rho = 10\lambda$ from two-part DPS material plane excited by a uniform line source at $\rho' = 4\lambda$. The material is $\lambda/20$ thick with $\varepsilon_r = 2$ and $\mu_r = 3$. 

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(a) A two-part DPS material plane with $\epsilon_r = 3$ and $\mu_r = 1$ at $\phi' = 45^\circ$

(b) A two-part DNG material plane with $\epsilon_r = -2$ and $\mu_r = -3$ at $\phi' = 135^\circ$

(c) A plane wave excitation

Figure 5.2: Comparison of UTD and W-H solutions for total magnetic fields at $\rho = 10\lambda$ from a two-part DNG and DPS material plane. The material is $\lambda/20$ thick. The illumination is a TM plane wave incident.
Figure 5.3: Comparison of UTD and W-H solutions for total magnetic fields at $\rho = 10\lambda$ from a DNG material half plane. The material is $\lambda/20$ thick with $\epsilon_r = -2$ and $\mu_r = -3$. The illumination is a TM plane wave incident at $\phi' = 135^\circ$. 

(a) A DNG material half plane

(b) A plane wave excitation
Figure 5.4: Comparison of UTD and FE-BI solutions for scattered fields from a DPS material strip excited by an electric line source at $y' = 1\lambda$. The material is $\lambda/10$ thick with $\varepsilon_r = 3.4$ and $\mu_r = 2$. 

(a) Without slope diffraction in UTD

(b) With slope diffraction in UTD

(c) An electric line source $\mathbf{J}$ excitation
One recall that relative simpler but approximate UTD solutions for the 3-D problems of diffraction by a thin, planar DPS material junction excited by a uniform skew incident plane wave, or by a spherical wave, were developed in Chapter 3 and 4. The UTD solutions for the planar two part problems developed in Chapter 3 which do not contain the W-H or MZ functions referred to as simply UTD, while the new UTD solutions for the more general wedge problems developed in Chapter 4 are now referred to as UTD-MZ, respectively in all the plots throughout the remainder of this Chapter for the sake of clarity and convenience in describing the numerical results. The scattered and total fields are calculated by the present UTD as well as UTD-MZ solutions and compared in Figs. 5.5-5.14 where possible, with the results of the MZ solution [23]. There is a very good agreement between all three methods, with less than $\pm 1$ dB differences between them, for the case of $o$-face illumination in Fig. 5.5 and for the case of $n$-face illumination in Fig. 5.6. It is noted that the illumination is a plane wave at skew incidence in Figs. 5.5 and 5.6.

In Figs. 5.7 and 5.8, only the UTD solutions developed in Chapter 3 and 4 are compared to each other; since here the wedge solutions of Chapter 4 can be easily specialized to also directly provide solutions for two part problems by simply opening up the wedge so that the internal (or external) wedge angle is $\pi$ as is also done for the cases in Figs. 5.5 and 5.6. The results agree very well for both the scattered and total fields in Figs. 5.7 and 5.8. One notes that the excitation for the problem in Fig. 5.8 is a current moment ($d\hat{p}_u = \hat{z}dp_{ez}$ or $d\hat{p}_m = \hat{z}dp_{mz}$), which produces a spherical wave, whereas the excitation is a skew incident plane wave for Figs. 5.5-5.7. It is important to note that surface wave effects are purposely neglected in Figs. 5.5-5.8 in order to clearly test if the boundary conditions are properly satisfied by the
first order diffracted fields in the new UTD and UTD-MZ solutions as compared to
the reference MZ solutions, or the new UTD solutions compared to UTD-MZ. If the
surface wave effects were included in Figs. 5.5-5.8, they would mask the behavior
of the diffracted fields near the boundaries. The next set of results provide a nice
comparison between the present UTD and the MZ solutions in Figs. 5.9 and 5.10 for
the special case of a PEC half plane with different materials on each face. It is also
observed that the edge excited surface wave field is included in these plots; the effect
of this surface wave is stronger near the material surface as expected.

It is of interest to see the comparison between the UTD-MZ and the MZ solutions
for all wave components to demonstrate the cross polarization effects that result from
the coupling between the $TE_z$ and $TM_z$ solutions; these comparisons are shown in
Figs. 5.11 and 5.12.

The UTD solution developed in Chapter 4 is compared with the available MZ
solution of [23], in which the latter is based on the approximate impedance boundary
condition that is used here to simulate the thin coating on one face, for a right
angle wedge excited by a uniform skew incident plane wave in Figs. 5.13 and 5.14.
The results from both solutions agree well. Any small discrepancy present in those
comparisons in Fig. 5.14 is due to the fact that the polynomial interpolation which is
used in the present UTD solution of Chapter 4 may be slightly impaired in calculating
the scattered fields (see Figs.5.14(a) and 5.14(c)) when both faces of the wedge are
illuminated. However, the above mentioned discrepancy occurs for very low levels of
the scattered field (typically below -25 dB) as seen from the plots; certainly, the total
field completely swamp out these small errors in the scattered field as is evident from
Figs. 5.14(b) and 5.14(d). As mentioned above, the exact MZ based solutions with

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an approximate impedance boundary condition for a material coated metallic wedge at skew incidence have been found only the following special cases: an entire plane impedance junction \((n = 1)\) [17, 25, 26], an impedance half plane \((n = 2)\) with different face impedances [24] and a right angle interior \((n = 0.5)\) and exterior \((n = 1.5)\) wedges with one face PEC [23, 26–28]; also an approximate solution for an impedance wedge at skew incidence is found in [29, 30], based on MZ half plane solution, but it loses accuracy for large wedge angle. In contrast, the present UTD-MZ solution can deal with an arbitrary PEC wedge angle, with arbitrary but thin uniform material coating which may be different on the two faces, and with plane, cylindrical or spherical wave excitation. More importantly, the present UTD-MZ solution allows for the source to be placed directly on the coated wedge faces because it also contains slope diffraction effects which are absent in [17, 23, 25–30]. A numerical example for a material coated metallic PEC wedge \((WA = 54^\circ)\) excited by a \(\hat{z}\)-directed current moment (of electric and magnetic types) is shown in Fig. 5.15. In Fig. 5.15, the \(n\)-face is PEC which is so chosen in order to demonstrate that the diffracted UTD-MZ field satisfies the proper PEC boundary condition there, and it satisfies the Karp-Karal lemma on the coated \(o\)-face.
Figure 5.5: Comparison of UTD-MZ, UTD and MZ solutions for a DPS material junction excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 45^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 4, \mu_{ro} = 2)$ and $(\epsilon_{rn} = 5, \mu_{rn} = 1)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.6: Comparison of UTD-MZ, UTD and MZ solutions for a DPS material junction excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 135^\circ$ and $\beta_0' = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 4, \mu_{ro} = 2)$ and $(\epsilon_{rn} = 5, \mu_{rn} = 1)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.7: Comparison of UTD-MZ and UTD solutions for a grounded DPS material half plane with PEC ground plane excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 60^\circ$ and $\beta'_o = 120^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/10$ thick with $(\epsilon_{ro} = 4, \mu_{ro} = 2)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.8: Comparison of UTD-MZ and UTD solutions for a DPS material junction excited by (a)-(b) a $\hat{z}$-directed electric current moment $d_{pe}^{z}$ and (c)-(d) a $\hat{z}$-directed magnetic current moment $d_{pm}^{z}$ at $r' = 7\lambda$, $\phi' = 45^\circ$ and $\theta' = 55^\circ$. The fields are observed at $r = 15\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 12, \mu_{ro} = 8)$ and $(\epsilon_{rn} = 1, \mu_{rn} = 4)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.9: Comparison of UTD-MZ and MZ solutions for a material coated PEC half plane (WA=0°) excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 45^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 2, \mu_{ro} = 3)$ and $(\epsilon_{rn} = 4, \mu_{rn} = 2)$. 
Figure 5.10: Comparison of UTD-MZ and MZ solutions for a material coated PEC half plane (WA=0°) excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 135^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 2, \mu_{ro} = 3)$ and $(\epsilon_{rn} = 4, \mu_{rn} = 2)$. 
Figure 5.11: Comparison of UTD-MZ and MZ solutions for a DPS material junction excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 45^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 4, \mu_{ro} = 2)$ and $(\epsilon_{rn} = 2, \mu_{rn} = 5)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.12: Comparison of UTD-MZ and MZ solutions for a material coated PEC half plane (WA=0°) excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 45^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with $(\epsilon_{ro} = 4, \mu_{ro} = 2)$ and $(\epsilon_{rn} = 2, \mu_{rn} = 5)$. Note that surface wave effects are neglected in these plots in order to test the boundary conditions on the first order UTD diffracted fields; otherwise the surface waves would have masked the behavior of the diffracted fields near the boundaries.
Figure 5.13: Comparison of UTD-MZ and MZ solutions for a material coated PEC wedge (WA=90°) excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 45^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{rn} = 4, \mu_{rn} = 2$).
Figure 5.14: Comparison of UTD-MZ and MZ solutions for a material coated PEC wedge (WA=90°) excited by a uniform skew incident plane wave (a)-(b) TE and (c)-(d) TM at $\phi' = 135^\circ$ and $\beta'_o = 65^\circ$. The fields are observed at $r = 5\lambda$ on the Keller cone of diffraction. The material is $\lambda/20$ thick with ($\epsilon_{rn} = 4, \mu_{rn} = 2$).
Figure 5.15: Effect of slope diffraction on the magnitude of electric field for a material coated PEC wedge (WA=54°). The excitation is a \( \hat{z} \)-directed current moment (\( d\bar{p} = dp_{ez} \) for TE and \( d\bar{p} = dp_{mz} \) for TM located at \( (r' = 5\lambda, \phi' = 117^\circ, \beta_o' = 66^\circ) \)). The fields are observed at \( r = 15\lambda \) on the Keller cone of diffraction. The material coated \( o \)-face of PEC wedge is \( \lambda/20 \) thick with \( (\epsilon_{ro} = 2.4, \mu_{ro} = 8) \).
Next set of numerical results shown in Figs. 5.16-5.20 demonstrate the effect of slope diffraction when a 2-D line or line dipole source is located on a finite material strip with or without an infinite ground plane backing. These sources on the surface provide a pattern null of the GO field (incident+reflected) at the edges due to the fact that this field must satisfy the Karp-Karal lemma or that it must vanish to first order on the material surface. In particular, Figs. 5.16-5.19 show the pattern of an electric line source placed on a DPS material strip of finite size. In the case of Figs. 5.16 and 5.17 the DPS material strip is on a PEC ground plane while in the case of Figs. 5.18 and 5.19 the PEC ground plane is absent. Fig. 5.20 shows the scattered and total field patterns of a magnetic line dipole source placed tangentially on a DPS material strip of finite size on a PEC entire plane. These cases in Figs. 5.16-5.20 are interesting because there is no SW excited by this type of a source for the chosen thickness of the strip. Hence, the UTD slope diffraction effects only remain in the absence of SWs because the first order UTD field diffracted from both edges also vanishes in these cases. If the slope diffraction effects were not included, then only the pattern directly radiated by the source remains as would be true only if the strip edges were absent! In Figs. 5.16 and 5.17 or Figs. 5.18 and 5.19, the strip parameters such as the length $L$, the thickness $\tau$ and material electrical properties ($\epsilon_r, \mu_r$) are changed to study the effect of those parameters on the slope diffraction contribution. It is seen that the slope diffraction effect depends strongly on both material thickness and material electrical properties ($\epsilon_r, \mu_r$). The length of the material strip controls the number of lobes in the pattern as expected; the larger the strip the more the number of lobes. The total fields in Figs. 5.20(b) and 5.20(d) are quite different from Fig. 5.17(b) and 5.17(d) due to the difference in the radiation patterns of a line dipole
source and a uniform line source. Figs. 5.21-5.24 show the pattern of an electric line
dipole located tangentially on the $o$-face of a DPS material coated metallic wedge
with various wedge angles. Since UTD type slope diffraction has generally not been
treated in other ray solutions available in the literature, no comparisons with other
ray solutions is therefore possible for the cases in Figs. 5.16-5.24
Figure 5.16: Effect of slope diffraction on the magnitude of total TE fields at far field distance with (a)-(b) $L = 5\lambda$ and (c)-(d) $L = 10\lambda$ of material strip on a PEC entire plane. The material strip has $\epsilon_r = 3.4$ and $\mu_r = 10$ with thickness $\lambda/20$. The excitation is a $\hat{z}$-directed electric line source located on the center of the strip.
Figure 5.17: Effect of slope diffraction on the magnitude of total TE fields at far field distance with (a)-(b) $L = 5\lambda$ and (c)-(d) $L = 10\lambda$ of material strip on a PEC entire plane. The material strip has $\epsilon_r = 1.6$ and $\mu_r = 5.2$ with thickness $\lambda/10$. The excitation is a $\hat{z}$-directed electric line source located on the center of the strip.
Figure 5.18: Effect of slope diffraction on the magnitude of total TE fields at far field distance with (a)-(b) $L = 5\lambda$ and (c)-(d) $L = 10\lambda$ of a DPS material strip. The material strip has $\epsilon_r = 2.4$ and $\mu_r = 26$ with thickness $\lambda/20$. The excitation is a $\hat{z}$-directed electric line source located on the center of the strip.
Figure 5.19: Effect of slope diffraction on the magnitude of total TE fields at far field distance with (a)-(b) $L = 5\lambda$ and (c)-(d) $L = 10\lambda$ of material strip. The material strip has $\epsilon_r = 2.4$ and $\mu_r = 12.5$ with thickness $\lambda/10$. The excitation is a $\hat{z}$-directed electric line source located on the center of the strip.
Figure 5.20: Effect of slope diffraction on the magnitude of total TE fields at far field distance with (a)-(b) $L = 5\lambda$ and (c)-(d) $L = 10\lambda$ of material strip on a PEC entire plane. The material strip has $\epsilon_r = 1.6$ and $\mu_r = 5.2$ with thickness $\lambda/10$. The excitation is a $\hat{x}$-directed magnetic line dipole source located on the center of the strip.
Figure 5.21: Effect of slope diffraction on the magnitude of electric fields for a material coated PEC wedge (WA=126°). The excitation is a $\hat{z}$-directed electric current moment located on the surface of $o$-face material at $s' = 3\lambda$ and $\beta_o' = 45^\circ$. The fields are observed at far field distance on the Keller cone of diffraction. The material coated both faces of PEC wedge is $\lambda/20$ thick with $(\epsilon_{ro} = 2.4, \mu_{ro} = 10)$ and $(\epsilon_{rn} = 1, \mu_{rn} = 4)$. 
Figure 5.22: Effect of slope diffraction on the magnitude of electric fields for a material coated PEC wedge (WA=54°). The excitation is a $\hat{z}$-directed electric current moment located on the surface of $o$-face material at $s' = 3\lambda$ and $\beta'_o = 45^\circ$. The fields are observed at far field distance on the Keller cone of diffraction. The material coated both faces of PEC wedge is $\lambda/20$ thick with $(\epsilon_{ro} = 2.4, \mu_{ro} = 10)$ and $(\epsilon_{rn} = 1, \mu_{rn} = 4)$. 
Figure 5.23: Effect of slope diffraction on the magnitude of electric field for a material coated PEC wedge (WA=180°). The excitation is a $\hat{z}$-directed electric current moment located on the surface of $o$-face material at $s'=3\lambda$ and $\beta'_o=45^\circ$. The fields are observed at far field distance on the Keller cone of diffraction. The material coated $o$-face of PEC wedge is $\lambda/20$ thick with ($\epsilon_{ro}=2.4, \mu_{ro}=10$).
Figure 5.24: Effect of slope diffraction on the magnitude of electric field for a material coated PEC wedge (WA=0°). The excitation is a $\hat{z}$-directed electric current moment located on the surface of $o$-face material at $s' = 3\lambda$ and $\beta'_o = 45^\circ$. The fields are observed at far field distance on the Keller cone of diffraction. The material coated $o$-face of PEC wedge is $\lambda/20$ thick with $(\epsilon_{ro} = 2.4, \mu_{ro} = 10)$ and $(\epsilon_{rn} = 1, \mu_{rn} = 4)$. 
It is of interest to study surface wave diffraction effects in problems of radiation/scattering by edges in material coated metallic surfaces, and by a material half plane. The effects of surface wave diffraction are demonstrated here in Figs. 5.25-5.30 for 2-D configurations only. Fig. 5.25 illustrates the patterns of a surface wave antenna (or magnetic line source here) on a material half plane placed directly over a PEC entire plane as a special case of the geometry in Fig. 2.1. In Figs. 5.25(a) and 5.25(b), the material is positive (DPS) and its thickness and electric properties are indicated in the figure; in contrast, the material is negative (DNG) for the case in Figs. 5.25(c) and 5.25(d). It is noted that the magnetic line source excites a FSW in the case of Figs. 5.25(a) and 5.25(b), while in the case of Figs. 5.25(c) and 5.25(d) it excites a BSW. These SWs excited by the source undergo diffraction at an edge. The BSW diffraction in the DNG case produces an antenna pattern which is markedly different from the FSW diffraction for the DPS case. Fig. 5.26 shows the patterns of a surface wave antenna (magnetic line source) on a material half plane without a PEC ground plane backing. The antenna patterns in Fig. 5.26 are shown in both the lit region and the shadow region because the material half plane is penetrable. Figs. 5.27-5.30 demonstrate the effects of surface wave diffraction present in the case of a finite and thin material strip with or without an infinite PEC ground plane backing. Figs. 5.29 and 5.30 show the surface wave diffraction effect in case of source off the surface while those in Figs. 5.27 and 5.26 pertain to a line source on the material strip. It is noted from these figures that the surface wave diffraction becomes important even when source is off the surface. Also the number of lobed in the surface diffraction pattern is also dependent on the strip length, again as expected.
Figure 5.25: Forward (Backward) surface antenna patterns (far field) with the antenna (unit magnetic line source) at $\rho' = 5\lambda$, and $\phi' = 0^\circ$ on a positive (negative) material half plane of thickness $\lambda/20$ over PEC entire plane. (a)-(b) is for a DPS half plane ($\epsilon_r = 3$ and $\mu_r = 2$) on a PEC entire plane and (c)-(d) is for a DNG half plane ($\epsilon_r = -18$ and $\mu_r = -19$) on a PEC entire plane.
Figure 5.26: Forward (Backward) surface antenna patterns (far field) with the antenna (unit magnetic line source) at \( \rho' = 5\lambda \), and \( \phi' = 0^\circ \) on a positive (negative) material half plane of thickness \( \lambda/20 \). (a)-(b) is for a DPS half plane (\( \epsilon_r = 3 \) and \( \mu_r = 2 \)) and (c)-(d) is for a DNG half plane (\( \epsilon_r = -18 \) and \( \mu_r = -19 \)).
(a) FSW diffraction from a DPS strip ($\epsilon_r = 3.4$ and $\mu_r = 2$) on a PEC entire plane

(b) BSW diffraction from a DNG strip ($\epsilon_r = -18$ and $\mu_r = -19$) on a PEC entire plane

Figure 5.27: Forward (Backward) surface antenna patterns (total field) with the antenna (unit magnetic line source) on the center of a positive (negative) material strip of thickness $\lambda/20$ and length $5\lambda$ over PEC entire plane.
Figure 5.28: Forward (Backward) surface antenna patterns (total field) with the antenna (unit magnetic line source) on the center of a positive (negative) material strip of thickness $\lambda/20$ and length $5\lambda$. 

(a) FSW diffraction from a DPS strip ($\epsilon_r = 3.4$ and $\mu_r = 2$) 

(b) BSW diffraction from a DNG strip ($\epsilon_r = -18$ and $\mu_r = -19$)
(a) FSW diffraction from a DPS strip ($\epsilon_r = 3.4$ and $\mu_r = 2$) on a PEC entire plane

(b) BSW diffraction from a DNG half plane ($\epsilon_r = -18$ and $\mu_r = -19$) on a PEC entire plane

Figure 5.29: Forward (Backward) surface antenna patterns (total field) with the antenna (unit magnetic line source) at $y' = 0.5\lambda$ above a positive (negative) material strip of thickness $\lambda/20$ and length $5\lambda$ over PEC entire plane.
(a) FSW diffraction from a DPS strip ($\epsilon_r = 3.4$ and $\mu_r = 2$)

(b) BSW diffraction from a DNG half plane ($\epsilon_r = -18$ and $\mu_r = -19$)

Figure 5.30: Forward (Backward) surface antenna patterns (total field) with the antenna (unit magnetic line source) at $y' = 0.5\lambda$ above a positive (negative) material strip of thickness $\lambda/20$ and length $5\lambda$. 

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CHAPTER 6

CONCLUSIONS AND FUTURE WORK

Accurate but approximate and relatively simple closed form asymptotic high frequency solutions are presented in the UTD format for the canonical problems of diffraction by thin planar positive (or negative) material coated metallic wedge structures. These solutions can deal with different kinds of incident fields such as a plane wave, a cylindrical wave corresponding to a line/line dipole source excitation, a spherical wave (a current moment), and a forward (or backward) surface wave. The material coated metallic wedge structures can be a material junction with PEC or PMC ground plane, material half plane without ground plane, and material coated metallic wedge of internal wedge angle $\leq \pi$. In addition, one can apply this UTD solutions to analyze the finite material structures such as material strip with and without PEC ground plane.

The present UTD solutions can be used to describe the radiation by (and coupling between) antennas near or on the material. The latter requires one to include higher UTD slope diffraction terms which are developed here in addition to the UTD fields diffracted to first order. Almost all previous related solutions do not contain slope diffraction effects because they deal with only uniform plane wave illumination. The slope diffraction contribution is developed to also include the case when the reflected
field, which is considered to be incident on the edge from image space, has a rapid spatial variation near the edge. It is thus necessary to treat the diffraction of the incident and reflected fields separately as described in Chapter 2. Additional weaker higher order terms could be obtained if desired, using the approach in [46] which was developed for the special PEC wedge problem; the latter includes the diffraction of waves by sources extremely close to the edge and hence, in particular, it contains the diffraction of a wave with a significant field component polarized along the incident ray.

The previous solutions on material coated metallic wedge structures are typically based on W-H or MZ methods, respectively, and use approximate impedance type boundary conditions. In contrast, the present asymptotic solutions of the wave equation, which are developed here via a heuristic spectral synthesis approach, provide UTD diffracted fields which satisfy reciprocity to first order and the PEC boundary conditions as well as the Karp-Karal lemma. These UTD solutions also recover the proper FRTCs and the exact surface wave propagation constants, whereas the W-H or MZ based solutions do not; consequently, the present solutions may be applicable to slightly thicker materials than is possible by the latter methods. Interesting radiation properties are observed for BSW antennas associated with negative (DNG) materials.

Additional work is also required to obtain better cross polar fields diffracted by a coated metallic wedge for the case of 3-D illumination; currently, these cross-polar components are predicted reasonably well only for the special half plane and two part planar configurations. This proposed improvement to be incorporated in the near future would involve a more careful interpolation of the \( \nu(\phi) \) function in (4.61).
It is noted that due to time constraints, the UTD-MZ solution developed here for the metallic coated wedge diffraction problem does not as yet contain the reflection of the SW excited by a source on one of the wedge faces; it also does not contain the transmission of the SW launched on the other face in this case.

It is worth extending the present UTD solutions to material coated metallic curved surfaces including curved edges and corners as shown in Fig. 6.1 which become very important in the practical antenna applications, as for example, for the analysis of flush mounted antennas on a complex platform such as an aircraft or a modern naval ship. One can also combine the present UTD solutions with numerical methods such as MoM, FE-BI, or FEM to treat complex antennas in the presence of complex platforms [8]. For example in Fig. 6.2 the hybrid MoM-UTD is applied in [47] to show the mutual coupling effects between a transmitted array (XMTR) and a receiving array (RCVR) in the presence of a naval ship tower which is assumed to be made of PEC. If the metallic surface of the naval ship contains a material coating such as an absorber or some non-metallic material, the present UTD solutions may be useful for analyzing such structures. Also it is interesting to investigate the effect of finite size thin uniform material patches on a PEC planar and convex surface as shown in Figs. 6.3 where the present UTD diffraction coefficient is extended to treat the edges and corners in planar and curved material patches.
Figure 6.1: Antenna with the presence of a material coated metallic curve wedge including a corner.

Figure 6.2: 31 × 31 Slot arrays coupling with the presence of PEC naval ship tower.
Figure 6.3: Diffraction from edges and corners in finite material coating on planar and convexly curved PEC surfaces.
APPENDIX A

MALIUZHINETS FUNCTIONS AND ITS SURFACE
WAVE POLES

This appendix shows brief details of Maliuzhinets (MZ) function, which is used in the derivation of (4.7) and (4.8) leading one to (4.6). In addition, the details of the surface wave poles occurred in the MZ functions is described, which is useful in arriving (4.9).

A.1 Maliuzhinets function

In [13], Maliuzhinets introduced a solution to the problem of a plane wave diffraction by a 2-D impedance wedge. In [13], an auxiliary function $\Psi(\alpha)$ which contains no poles and zeros within the strip $|\Re\{\alpha\}| < n\pi/2$ is first introduced in a system of difference equations:

\[
\begin{align*}
\sin \alpha + \sin \theta^o \Psi(\alpha + n\pi/2) &= -\sin \alpha + \sin \theta^o \Psi(-\alpha + n\pi/2) \quad (A.1) \\
\sin \alpha - \sin \theta^o \Psi(\alpha - n\pi/2) &= -\sin \alpha - \sin \theta^o \Psi(-\alpha - n\pi/2) \quad (A.2)
\end{align*}
\]

By replacing $\alpha$ in (A.1) with $\alpha + n\pi/2$ and $\alpha$ in (A.2) with $\alpha - n\pi/2$, one obtains

\[
\begin{align*}
\{\sin(\alpha + n\pi/2) + \sin \theta^o\} \Psi(\alpha + n\pi) &= -\{\sin(\alpha + n\pi/2) + \sin \theta^o\} \Psi(-\alpha) \quad (A.3) \\
\{\sin(\alpha - n\pi/2) - \sin \theta^o\} \Psi(\alpha - n\pi) &= -\{\sin(\alpha - n\pi/2) - \sin \theta^o\} \Psi(-\alpha) \quad (A.4)
\end{align*}
\]
Dividing (A.3) by (A.4) yields the first order functional difference equation, namely:

\[
\{\sin(\alpha + n\pi/2) + \sin \theta^o\}\{\sin(\alpha - n\pi/2) + \sin \theta^n\}\Psi(\alpha + n\pi) = \{\sin(\alpha - n\pi/2) + \sin \theta^o\}\{\sin(\alpha - n\pi/2) - \sin \theta^n\}\Psi(\alpha - n\pi). \tag{A.5}
\]

The \(\Psi(\alpha)\) is given in terms of a product of four meromorphic MZ functions \(\psi_n(\alpha)\) as follows:

\[
\Psi(\alpha) = \psi_n(\alpha + \frac{n\pi}{2} + \frac{\pi}{2} - \theta^o_{e,h})\psi_n(\alpha + \frac{n\pi}{2} - \frac{\pi}{2} + \theta^o_{e,h}) \tag{A.6}
\]

\[
\psi_n(\alpha - \frac{n\pi}{2} - \frac{\pi}{2} + \theta^o_{e,h})\psi_n(\alpha - \frac{n\pi}{2} + \frac{\pi}{2} - \theta^o_{e,h})
\]

in which the \(\psi_n\) is a special meromorphic function so called the MZ function, which can be expressed as

\[
\psi_n(\alpha) = \exp \left\{ -\frac{1}{2} \int_0^\infty \frac{\cosh(\alpha t) - 1}{t \cosh \frac{nt}{2} \sinh(n\pi t)} \, dt \right\} \tag{A.7}
\]

The following is the list of \(\psi_n(\alpha)\) in special cases:

\[
\psi_1(\alpha) = \exp \left\{ \frac{1}{4\pi} \int_0^\alpha \frac{2u - \pi \sin u}{\cos u} \, du \right\} \tag{A.8a}
\]

\[
\psi_{1.5}(\alpha) = \frac{4 \cos \frac{\alpha - \pi}{6} \cos \frac{\alpha + \pi}{6}}{3 \cos \frac{\alpha}{6}} \tag{A.8b}
\]

\[
\psi_2(\alpha) = \exp \left\{ -\frac{1}{8\pi} \int_0^\alpha \frac{\pi \sin u - 2\sqrt{2} \sin \frac{u}{2} + 2u}{\cos u} \, du \right\} \tag{A.8c}
\]

where \(n = 1\) in (A.8a) denotes the full plane (or wedge angle (WA) is \(\pi\)), \(n = 1.5\) in (A.8b) is the right angle wedge (or WA is \(\frac{\pi}{2}\)), and \(n = 2\) in (A.8c) is a half plane.
structure (or WA is zero). Some of the useful identities are listed as follows:

\[ \psi_n(-\alpha) = \psi_n(\alpha) \quad \text{(A.9a)} \]

\[ \psi_n(\alpha^*) = \psi_n(\alpha) \quad \text{(A.9b)} \]

\[ \psi_n(\alpha + \frac{\pi}{2})\psi_n(\alpha - \frac{\pi}{2}) = \{\psi_n(\frac{\pi}{2})\}^2 \cos \frac{\alpha}{2n} \quad \text{(A.9c)} \]

\[ \psi_n(\alpha + n\pi/2)\psi_n(\alpha - n\pi/2) = \{\psi_n(n\pi/2)\}^2 \psi_n(\alpha) \quad \text{(A.9d)} \]

\[ \psi_n(\alpha + n\pi) = \cot \left( \frac{\alpha + \pi}{2} \right) \psi_n(\alpha - n\pi) \quad \text{(A.9e)} \]

\[ \psi_n(\alpha \pm n\pi \pm \frac{3\pi}{2}) = \pm \frac{\sin \left\{ \frac{1}{2n}(\pi \pm \alpha) \right\}}{\sin \frac{\alpha}{2n}} \psi_n(\alpha \pm n\pi \mp \frac{\pi}{2}) \quad \text{(A.9f)} \]

where the asterisk (*) in (A.9b) denotes the complex conjugate. This means that the \( \psi_n(\alpha) \) is a real even function of \( \alpha \).

Now one considers the \( \tilde{\Psi}(\frac{n\pi}{2} - \phi') \) in (4.5), it can be expressed in terms of MZ functions as

\[ \tilde{\Psi}(\frac{n\pi}{2} - \phi') = \psi_n(n\pi - \phi' + \theta_{c,h}^o - \frac{\pi}{2})\psi_n(n\pi - \phi' + \theta_{c,h}^o + \frac{\pi}{2}) \quad \text{(A.10)} \]

By applying the identity in (A.9c) with the first two MZ functions in (A.10), one obtains

\[ \tilde{\Psi}(\frac{n\pi}{2} - \phi') = -\frac{1}{R_n(\phi')} \tilde{\Psi}(\frac{n\pi}{2} + \phi') \quad \text{(A.11)} \]

Also if one applies the identity in (A.9e) and (A.9a) with the last two Maliuzhinets functions in (A.10), one have

\[ \tilde{\Psi}(\frac{n\pi}{2} - \phi') = -\frac{1}{R_n(n\pi - \phi')} \tilde{\Psi}(-\frac{3n\pi}{2} + \phi') \quad \text{(A.12)} \]

By using (A.11) and (A.12) in (4.5), this allows one to obtain (4.7) and (4.8), respectively.
A.2 Surface wave poles occurred in Maliuzhinets function

In addition to the real poles \( \xi_i = \phi - \phi' \), \( \xi_{ro} = \phi + \phi' \), \( \xi_{rn} = \phi + \phi' - 2n\pi \) from the cotangent functions in (4.5), there are complex poles \( \xi_{sw} \) from \( \tilde{\Psi}(\frac{n\pi}{2} + \xi - \phi) \) function, which can be expressed in terms of Maliuzhinets functions as

\[
\tilde{\Psi}(\frac{n\pi}{2} + \xi - \phi) = \psi_n(n\pi + \xi - \phi + \theta_{e,h}^o - \frac{\pi}{2})\psi_n(n\pi + \xi - \phi - \theta_{e,h}^o + \frac{\pi}{2})
\]

(A.13)

From the identity in (A.9f), the poles which are closest to the saddle point \( \xi = \pm \pi \) are from the second and third factors on RHS of (A.13), namely

\[
\xi_{sw}^{e,h} = \phi + \theta_{e,h}^o + \pi \quad ; \quad \xi_{sw}^{e,h} = \phi - n\pi - \theta_{e,h}^n - \pi
\]

It is noted that the pole \( \xi_{sw}^{e,h} \) is closest to the saddle point \( \xi = \pi \) and the pole \( \xi_{sw}^{e,h} \) is closest to the saddle point \( \xi = -\pi \). If the pole \( \xi_{sw}^{e,h} \) is captured, its residue contribution represents an edge excited surface wave \( u_{zsw}^{e,h} \) traveling along the \( o \)-face, away from the edge.

\[
u_{zsw}^{e,h} = 2\pi j \text{Res}(\xi = \xi_{sw}^{e,h})
\]

\[
= \frac{\sin \frac{\pi}{2n}}{\Psi(\frac{n\pi}{2} - \phi') \psi_n(n\pi - \frac{\pi}{2})\psi_n(n\pi + 2\theta_{e,h}^o + \frac{\pi}{2})\psi_n(\theta_{e,h}^o + \theta_{e,h}^n + \frac{\pi}{2})\psi_n(\theta_{e,h}^o - \theta_{e,h}^n + 3\pi/2)}
\]

\[
\left[ \cot \left( \frac{\pi + \theta_{e,h}^o + \phi'}{2n} \right) - \cot \left( \frac{\pi + \theta_{e,h}^o - \phi'}{2n} \right) \right] e^{-jk(\rho + \rho')} e^{jk\frac{\rho'}{\rho + \rho'}}(1 + \cos \xi_{sw}^{e,h}) U(\phi_{sw}^{e,h} - \phi)
\]

(A.14)

Likewise, if the pole \( \xi_{sw}^{n} \) is captured, its residue contribution represents an edge excited surface wave \( u_{zsw}^{n} \) traveling along the \( n \)-face away from the edge.

\[
u_{zsw}^{n} = \frac{\sin \frac{\pi}{2n}}{\Psi(\frac{n\pi}{2} - \phi') \psi_n(n\pi - \frac{\pi}{2})\psi_n(n\pi + 2\theta_{e,h}^n + \frac{\pi}{2})\psi_n(\theta_{e,h}^o + \theta_{e,h}^n + \frac{\pi}{2})\psi_n(\theta_{e,h}^o - \theta_{e,h}^n - 3\pi/2)}
\]

\[
\left[ \cot \left( \frac{\pi + \theta_{e,h}^n + \pi + \phi'}{2n} \right) - \cot \left( \frac{\pi + \theta_{e,h}^n + \pi - \phi'}{2n} \right) \right] e^{-jk(\rho + \rho')} e^{jk\frac{\rho'}{\rho + \rho'}}(1 + \cos \xi_{sw}^{n}) U(\phi - \phi_{sw}^{n})
\]

(A.15)
where $U(\cdot)$ denotes the unit step function. In general, if one let $\delta_{e,h}^{o,n} = \sin \theta_{e,h}^{o,n}$ with $\delta_{e}^{o,n} = -jY_{d}^{o,n}N^{o,n}\cot(k_{d}^{o,n}N^{o,n}\tau)$ and $\delta_{h}^{o,n} = jZ_{d}^{o,n}N^{o,n}\tan(k_{d}^{o,n}N^{o,n}\tau)$. If one first considers the $o$-face case, one have

$$
\theta_{e}^{o} = \sin^{-1}(-jY_{d}^{o,n}N^{o}\cot(k_{d}^{o,n}N^{o}\tau)) \tag{A.16a}
$$

$$
\theta_{h}^{o} = \sin^{-1}(jZ_{d}^{o,n}N^{o}\tan(k_{d}^{o,n}N^{o}\tau)) \tag{A.16b}
$$

Then, one can evaluate the $\sin^{-1}(\omega)$, where the $\omega = a + jb$ is a complex number, from [48]

$$
\sin^{-1}(a + jb) = \sin^{-1} \zeta + j \ln(\chi + \sqrt{\chi^2 + 1}), \tag{A.17}
$$

where $\chi = \frac{1}{2} \sqrt{(a + 1)^2 + b^2} + \frac{1}{2} \sqrt{(a - 1)^2 + b^2}$ and $\zeta = \frac{1}{2} \sqrt{(a + 1)^2 + b^2} - \frac{1}{2} \sqrt{(a - 1)^2 + b^2}$. Likewise, for the $n$-face case, one can obtain $\theta_{e,n}^{o} = \eta_{su}^{e} + j\nu_{su}^{e}$ and $\theta_{e,n}^{n} = \eta_{su}^{e} + j\nu_{su}^{e}$. Next one can find the $\phi_{su}^{e}$ and $\phi_{su}^{n}$ in (4.10). The SDP equations are found from the condition that

$$
\Im\{f(\xi)\} = \Im\{f(\xi_{s})\}
$$

where $f(\xi) = j \cos \xi$ and $\xi_{s}$ is the saddle point which is $\pm \pi$. Thus, $\Im\{f(\xi)\} = -1$. At $\xi = \xi_{su}^{e} = \phi + \pi + \theta_{e,h}^{o}$, one can have

$$
\phi_{su}^{e} = -\eta_{su}^{e} + \cos^{-1}(1/ \cosh \nu_{su}^{e}) \tag{A.18}
$$

Likewise, at $\xi = \xi_{su}^{n} = \phi - n\pi - \pi - \theta_{e,h}^{n}$, one obtains

$$
\phi_{su}^{e} = n\pi + \eta_{su}^{e} - \cos^{-1}(1/ \cosh \nu_{su}^{e}) \tag{A.19}
$$

The surface wave poles $\xi_{su}^{e,h}$ for the $o$-face case can be expressed as

$$
\xi_{e}^{o} = \phi + \pi + \eta_{su}^{e} + j\nu_{su}^{e} \tag{A.20a}
$$

$$
\xi_{h}^{o} = \phi + \pi + \eta_{su}^{h} + j\nu_{su}^{h} \tag{A.20b}
$$
and for the \(n\)-face case, one can have

\[
\xi^e_n = \phi - n\pi - \pi - \eta^e_{swn} - j\nu^e_{swn} \quad \text{(A.21a)}
\]
\[
\xi^h_n = \phi - n\pi - \pi - \eta^h_{swn} - j\nu^h_{swn} \quad \text{(A.21b)}
\]

For a given value of \(\mu_r\) and \(\epsilon_r\), only one surface wave pole can be captured if \(\Im\{\xi_{swoo}\} > 0\) for the \(o\)-face case or if \(\Im\{\xi_{sun}\} < 0\) for the \(n\)-face case as shown in Fig. 4.2.
APPENDIX B

RADIATION OF A CURRENT MOMENT IN THE PRESENCE OF AN INFINITE, PLANAR MATERIAL WITH PEC GROUND PLANE

Radiation of a current moment in the presence of an infinite, planar material with PEC ground plane is summarized in this appendix. This solution provides an important transformation matrix for the solution in Chapter 4. The current moment with an arbitrary polarization is allowed to be on or even in the infinite, planar material. A few examples are provided at the end of this appendix. Electric field $\vec{E}$ radiated by a current moment $d\vec{p}_{e,m}$ in the presence of the infinite, material configuration shown in Fig. B.1(a) can be found by using reciprocity theorem by relating the electric field $\vec{E}$ to the fields $(\vec{E}_t, \vec{H}_t)$ generated in the test problem as follows:

$$\int_{\Omega} \vec{E} \cdot \vec{J}_t ds'' = \int_{\Omega} \left( \vec{E}_t \cdot d\vec{p}_e - \vec{H}_t \cdot d\vec{p}_m \right)$$

where $d\vec{p}_e$ denotes an electric current moment and $d\vec{p}_m$ is a magnetic current moment. It is noted that magnetic field $\vec{H}$ radiated by the current moment $d\vec{p}_{e,m}$ can be found directly from the electric field $\vec{E}$ by using Maxwell’s equations. A test (impressed) current $\vec{J}_t = \hat{u} \delta(\vec{r}'' - \vec{r})$ at $\vec{r}$ as shown in Fig. B.1(b) radiates in the presence of the infinite, planar material with PEC ground plane when the dipole moment $d\vec{p}_{e,m}$ from
Figure B.1: An infinite, planar material with PEC ground plane illuminated by a current moment $d\hat{p}$ in the original problem and a test current $\bar{J}_t$ in the test problem.

The original problem is turned off. Let the test source $\bar{J}_t$ at $\bar{r}$ generate the fields $(\bar{E}_t, \bar{H}_t)$ as

$$\bar{E}_t = \bar{E}^i_t + \bar{E}^s_t \quad ; \quad \bar{H}_t = \bar{H}^i_t + \bar{H}^s_t$$ (B.2a)

where $(\bar{E}^i_t, \bar{H}^i_t)$ are produced by $\bar{J}_t$ in the absence of the material structure. The $(\bar{E}^s_t, \bar{H}^s_t)$ are the fields scattered by the material structure. If it is of interest to evaluate the fields in far zone with $k|\bar{\xi}| \gg 1$, where $\bar{\xi} = \bar{r}'' - \bar{r}$, then the incident fields
Figure B.2: A spherical coordinate system used in this problem

$\vec{r} = \hat{x}\cos\phi\sin\theta + \hat{y}\sin\phi\sin\theta + \hat{z}\cos\theta$

$\hat{\phi} = [-\hat{x}\cos\theta + \hat{z}\sin\theta\cos\phi]\frac{1}{\Delta}$

$\hat{\theta} = [\hat{x}\sin^2\theta\cos\phi - \hat{y}\Delta^2$

$+ \hat{z}\sin\theta\cos\theta\sin\phi]\frac{1}{\Delta}$

$\hat{\rho} = \frac{\hat{z}}{\rho}$

$\cos\vartheta = \sin\theta\cos\phi$

$\sin\vartheta = \sqrt{1 - \sin^2\theta\sin^2\phi}$

$\Delta = \frac{\cos\theta}{\sin\theta\cos\phi}$

$\cos\varphi = \frac{\sin\theta\cos\phi}{\Delta}$

$(\vec{E}_i^i, \vec{H}_i^i)$ are locally plane over the material structure and it can be expressed as

$$E_i^i(r'') \sim \frac{jk\eta_0}{4\pi} \hat{\xi} \times \hat{\xi} \times \hat{\xi} \times \hat{\xi} e^{-jk\xi} \xi$$ (B.3a)

$$H_i^i(r'') \sim -\frac{jk}{4\pi} \hat{\xi} \times \hat{\xi} \hat{\xi} e^{-jk\xi} \xi.$$. (B.3b)

Also here $r \gg r''$ thus one can have $\frac{e^{-jk\xi}}{\xi} \sim \frac{e^{-jk\xi}}{r} e^{jk\varphi''}$ and $\hat{\xi} \sim -\hat{\rho}$. The incident fields $(\vec{E}_i^i, \vec{H}_i^i)$ can be written as

$$\vec{E}_i^i(\vec{r}'') \sim \frac{jk\eta_0}{4\pi} [\hat{\rho} \times -\hat{\rho} \times \hat{\vartheta}] e^{-jk\varphi''} \hat{\vartheta}$$ (B.4a)

$$\vec{H}_i^i(\vec{r}'') \sim -\frac{jk}{4\pi} [\hat{\rho} \times \hat{\vartheta}] e^{-jk\varphi''} \hat{\vartheta}$$ (B.4b)

where $\hat{\rho} = \hat{x}\cos\varphi\sin\vartheta + \hat{y}\cos\vartheta + \hat{z}\sin\varphi\sin\vartheta$ or $\hat{\rho} = \hat{\rho}_{zz}\sin\vartheta + \hat{\rho}_{y}\cos\vartheta$ according to the coordinate system shown in Fig. B.2. The $\vec{r}'' = \hat{\rho}_{zz}\rho_{zz} + \hat{y}\varphi''$ and $\hat{\rho} \cdot \vec{r}'' = \rho_{zz}\sin\vartheta + \varphi''$ cos $\vartheta$ with $\rho_{zz}'' = x''$ cos $\varphi + z''$ sin $\varphi$.

If $\hat{\vartheta} = \hat{\vartheta}$, then

$$\hat{\rho} \times \hat{\vartheta} = \hat{\vartheta} ; \hat{\rho} \times \hat{\rho} \times \hat{\vartheta} = \hat{\rho} \times \hat{\vartheta} = -\hat{\vartheta}.$$ If $\hat{\vartheta} = \hat{\varphi}$, then

$$\hat{\rho} \times \hat{\varphi} = -\hat{\vartheta} ; \hat{\rho} \times \hat{\rho} \times \hat{\varphi} = \hat{\rho} \times -\hat{\vartheta} = -\hat{\varphi}.$$
It is now of interest to find the scattered fields \((\bar{E}_s, \bar{H}_s)\) and hence \((\bar{E}_t, \bar{H}_t)\) by using the local plane wave behavior of \((\bar{E}_i, \bar{H}_i)\) near the material structure.

### B.1 Case 1: \(\hat{u} = \hat{\vartheta}\)

The incident fields \((\bar{E}_i, \bar{H}_i)\) in this case can be expressed as

\[
\bar{E}_i \sim -\hat{\vartheta} E_o e^{jk\rho''_{xz} \sin \vartheta} e^{jk'y' \cos \vartheta}
\]  
\[\eta_o \bar{H}_i \sim -\hat{\varphi} E_o e^{jk\rho''_{xz} \sin \vartheta} e^{jk'y' \cos \vartheta}
\]

where \(E_o = \frac{jk\eta_o}{4\pi} e^{-jk_r} \). As shown in Fig. B.3, the reflected fields \((\bar{E}_r, \bar{H}_r)\) can be expressed as

\[
\bar{E}_r \sim -\hat{\vartheta} R e^{jk\rho''_{xz} \sin \vartheta} e^{-jk'y' \cos \vartheta}
\]
\[\eta_o \bar{H}_r \sim -\hat{\varphi} R e^{jk\rho''_{xz} \sin \vartheta} e^{-jk'y' \cos \vartheta}
\]
where \( \hat{\vartheta}^r = -\hat{\rho}_{xx} \cos \vartheta^r - \hat{y} \sin \vartheta^r \). Furthermore, the fields \((\hat{E}^d, \hat{H}^d)\) inside the material can be expressed as

\[
\begin{align*}
\hat{E}^d_t &\sim \hat{\vartheta}^d_t E_o T_a e^{ijk_dz'z''} \sin \vartheta^d e^{jk_ay''} \cos \vartheta^d + \hat{\vartheta}^d_u E_o T_b e^{ijk_dz'z''} \sin \vartheta^d e^{-jk_ay''} \cos \vartheta^d, \\
\eta_o \hat{H}^d_t &\sim -\hat{\varphi} E_o T_a e^{jk_dz'z''} \sin \vartheta^d e^{jk_ay''} \cos \vartheta^d - \hat{\varphi} E_o T_b e^{jk_dz'z''} \sin \vartheta^d e^{-jk_ay''} \cos \vartheta^d.
\end{align*}
\]  

(B.7a) \[ \text{(B.7b)} \]

where \( z_d = \sqrt{\mu \epsilon} \), \( k_d = k \sqrt{\mu \epsilon} \), and \( \eta_o \) is the intrinsic impedance of free space. The \( \hat{\vartheta}^d_t = -\hat{\rho}_{xx} \cos \vartheta^d + \hat{y} \sin \vartheta^d \) and \( \hat{\vartheta}^d_u = \hat{\rho}_{xx} \cos \vartheta^d + \hat{y} \sin \vartheta^d \). By enforcing phase matching condition of all waves along the boundary; i.e., matching the tangential component \( \hat{\rho}_{xx} \) phase velocities of the waves in the air region with that of waves in the material slab region at \( y'' = 0 \), yields: \( \vartheta = \vartheta^r \) and \( k \sin \vartheta = k_d \sin \vartheta^d \). One can then write all field expressions from (B.5)-(B.7b) as

\[
\begin{align*}
\hat{E}^d_t &\sim -\hat{\vartheta} E_o e^{jk_xx''} e^{jk_yy''} e^{jk_zz''}, \\
\eta_o \hat{H}^d_t &\sim -\hat{\varphi} E_o e^{jk_xx''} e^{jk_yy''} e^{jk_zz''}, \\
\hat{E}^u_t &\sim -\hat{\vartheta} E_o R_h e^{jk_xx''} e^{-jk_yy''} e^{jk_zz''}, \\
\eta_o \hat{H}^u_t &\sim -\hat{\varphi} E_o R_h e^{jk_xx''} e^{-jk_yy''} e^{jk_zz''}, \\
\hat{E}^d_T &\sim \hat{\vartheta}^d T_a e^{jk_xx''} e^{jk_dNy''} e^{jk_zz''} + \hat{\vartheta}^d_u T_b e^{jk_xx''} e^{-jk_dNy''} e^{jk_zz''}, \\
\eta_o \hat{H}^d_T &\sim -\hat{\varphi} E_o T_a e^{jk_xx''} e^{jk_dNy''} e^{jk_zz''} - \hat{\varphi} E_o T_b e^{jk_xx''} e^{-jk_dNy''} e^{jk_zz''}.
\end{align*}
\]  

(B.8a) \[ \text{(B.8b)} \]

(B.8c) \[ \text{(B.8d)} \]

(B.8e) \[ \text{(B.8f)} \]

where \( N = \sqrt{1 - \eta \sin^2 \vartheta} \) with \( \eta = 1 / \sqrt{\mu \epsilon} \). Also \( k_x = k \cos \vartheta \), \( k_y = k \sin \vartheta \), and \( k_z = k \cos \vartheta \). The unknown coefficients \( R_h, T_a, \) and \( T_b \) in (B.8) can be found by enforcing the boundary conditions at \( y'' = 0 \) and \( y'' = -\tau \), respectively. First, the boundary condition which requires the tangential component of the total electric field be continuous at \( y'' = 0 \) yields

\[
E_o \cos \vartheta (1 - R_h) = E_o \cos \vartheta^d (T_a - T_b).
\]

(B.9)
The continuity of the total tangential magnetic field at \( y'' = 0 \) requires

\[
E_o (1 + \mathcal{R}_h) = \frac{E_o}{Z_d} (T_a + T_b). \tag{B.10}
\]

Finally, the vanishing of the total tangential electric field at \( y'' = -\tau \) requires

\[
E_o T_a \cos \vartheta' e^{-jk_d N \tau} - E_o T_b \cos \vartheta' e^{-jk_d N \tau} = 0. \tag{B.11}
\]

Summarizing the results in (B.9)-(B.11) of the above three boundary conditions gives:

\[
1 - \mathcal{R}_h = (T_a - T_b) \frac{\cos \vartheta'}{\cos \vartheta}, \tag{B.12a}
\]

\[
1 + \mathcal{R}_h = (T_a + T_b) \frac{1}{Z_d}, \tag{B.12b}
\]

\[
T_a = T_b e^{jk_d N \tau}. \tag{B.12c}
\]

After solving for \( \mathcal{R}_h, T_a, \) and \( T_b \) from (B.12), one can obtain

\[
\mathcal{R}_h (\vartheta) = \frac{\cos \vartheta - jZ_d N \tan(k_d N \tau)}{\cos \vartheta + jZ_d N \tan(k_d N \tau)} \cos \vartheta', \tag{B.13a}
\]

\[
T_a (\vartheta') = \frac{Z_d N \tan(k_d N \tau) \cos \vartheta e^{jk_d N \tau}}{\cos \vartheta' \sin(k_d N \tau) (\cos \vartheta + jZ_d N \tan(k_d N \tau))}, \tag{B.13b}
\]

\[
T_b (\vartheta') = \frac{Z_d N \tan(k_d N \tau) \cos \vartheta e^{-jk_d N \tau}}{\cos \vartheta' \sin(k_d N \tau) (\cos \vartheta + jZ_d N \tan(k_d N \tau))}, \tag{B.13c}
\]

or if one applies \( \cos \vartheta = \sin \theta' \sin \phi' \) and \( \sin \vartheta = \sqrt{1 - \sin^2 \theta' \sin^2 \phi'} \), one obtains

\[
\mathcal{R}_h (\phi') = \frac{\sin \phi' - \delta_h / \sin \theta'}{\sin \phi' + \delta_h / \sin \theta'}, \tag{B.13a}
\]

\[
T_a (\phi') = \frac{-j \sin \phi' \delta_h e^{jk_d N \tau}}{N \sin(k_d N \tau) (\sin \phi' + \delta_h / \sin \theta')}, \tag{B.13b}
\]

\[
T_b (\phi') = \frac{-j \sin \phi' \delta_h e^{-jk_d N \tau}}{N \sin(k_d N \tau) (\sin \phi' + \delta_h / \sin \theta')}, \tag{B.13c}
\]

where \( \delta_h = jZ_d N \tan(k_d N \tau) \) and \( N = \sqrt{1 - \eta (1 - \sin^2 \theta' \sin^2 \phi')} \). The total fields \( (\vec{E}_t, \vec{H}_t) \) produced by the test source \( \vec{J}_t \) when \( \hat{u} = \hat{\vartheta} \) can be obtained by using (B.13) in (B.8) together with (B.2) and (B.5).

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B.2 Case 2: $\hat{u} = \hat{\varphi}$

Following the procedure employed for Case 1 ($\hat{u} = \hat{\vartheta}$), one can write all field expressions, without any detail, as follows:

\begin{align}
\vec{E}_i^t &\sim -\hat{\varphi} E_o e^{jk_s x''} e^{jk_y y''} e^{jk_z z''} \tag{B.14a} \\
\eta_o \vec{H}_t^i &\sim \hat{\vartheta} E_o e^{jk_s x''} e^{jk_y y''} e^{jk_z z''} \tag{B.14b} \\
\vec{E}_i^r &\sim -\hat{\varphi} E_o R e^{jk_s x''} e^{-jk_y y''} e^{jk_z z''} \tag{B.14c} \\
\eta_o \vec{H}_t^i &\sim \hat{\vartheta} E_o R e^{jk_s x''} e^{-jk_y y''} e^{jk_z z''} \tag{B.14d} \\
\vec{E}_i^d &\sim -\hat{\varphi} E_o T e^{jk_s x''} e^{jk_d N y''} e^{jk_z z''} - \hat{\varphi} E_o T d e^{jk_s x''} e^{-jk_d N y''} e^{jk_z z''} \tag{B.14e} \\
\eta_o \vec{H}_t^i &\sim -\hat{\vartheta} E_o T e^{jk_s x''} e^{jk_d N y''} e^{jk_z z''} - \hat{\vartheta} E_o T d e^{jk_s x''} e^{-jk_d N y''} e^{jk_z z''}. \tag{B.14f} \\
\end{align}

The unknown coefficients $R_e$, $T_c$, and $T_d$ are found by enforcing the same boundary conditions at $y'' = 0$ and $y'' = -\tau$ as before in Section B.1. Thus,

\begin{align}
1 + R_e &= (T_c + T_d) \tag{B.15a} \\
1 - R_e &= (T_c - T_d) \frac{Z_d \cos \vartheta_d}{\cos \vartheta} \tag{B.15b} \\
T_c &= -T_d e^{jk_d N \tau} \tag{B.15c} \\
\end{align}

where $Y_d = 1/Z_d$. Then after solving for $R_e$, $T_c$, and $T_d$ from (B.15), one can have

\begin{align}
R_e (\phi') &= \frac{\sin \phi' - \delta_e/ \sin \theta'}{\sin \phi' + \delta_e/ \sin \theta'} \tag{B.16a} \\
T_c (\phi') &= \frac{-j \sin \phi' e^{jk_d N \tau}}{\sin(k_d N \tau)(\sin \phi' + \delta_e/ \sin \theta')} \tag{B.16b} \\
T_d (\phi') &= \frac{j \sin \phi' e^{-jk_d N \tau}}{\sin(k_d N \tau)(\sin \phi' + \delta_e/ \sin \theta')} \tag{B.16c} \\
\end{align}

where $\delta_e = -j Y_d N \cot(k_d N \tau)$. The total fields ($\vec{E}_t$, $\vec{H}_t$) produced by the test source $\vec{J}_t$ when $\hat{u} = \hat{\varphi}$ can be obtained by using (B.16) in (B.14) together with (B.2).
B.3 Transformation matrix

It is well known [36] that the radiation from an arbitrary current moment may be created by superimposing the fields of \( \hat{z} \)-directed electric and magnetic current moment. This allows one to find a general field expression via a superposition of the TE and TM wave solutions. Thus it is important to find the fields radiated by a \( \hat{z} \)-directed dipole moments \( \hat{p}_{e,m} \), in which,

\[
d\hat{p}_e = \hat{z}\delta(\vec{r}' - \vec{r}')d\bar{p}_{ez} \\
d\hat{p}_m = \hat{z}\delta(\vec{r}' - \vec{r}')d\bar{p}_{mz}
\]

(B.17a)

(B.17b)

where \( d\bar{p}_{ez} \) and \( d\bar{p}_{mz} \) are a known constant. Also it is of interest to find a transformation matrix for the reflected fields \((\vec{E}^r_{\beta_o}, \vec{E}^r_{\phi})\) in the edge fixed plane coordinate system. By applying (B.1) to the reflected field produced from \( \hat{p}_{e,m} \), one can have

\[
\vec{E}^r_e = \frac{\sin\theta}{\Delta} (\hat{\vartheta} \cos \theta \sin \phi \mathcal{R}_h - \hat{\phi} \cos \phi \mathcal{R}_e) E_{o} d\bar{p}_{ez} e^{jk_x'x'} e^{-jk_y'y'}e^{jk_z'z'}
\]

\[
\vec{E}^r_m = \frac{\sin\theta}{\Delta} (\hat{\vartheta} \cos \phi \mathcal{R}_h + \hat{\phi} \cos \theta \sin \phi \mathcal{R}_e) \frac{E_{o}}{\eta_o} d\bar{p}_{mz} e^{jk_x'x'} e^{-jk_y'y'}e^{jk_z'z'}
\]

(B.18a)

(B.18b)

with \( \theta' = \beta_o \),

\[
\hat{\vartheta} = [\hat{x} \sin^2 \beta_o \cos \phi \sin \phi - \hat{y} \Delta^2 + \hat{z} \sin \beta_o \cos \beta_o \sin \phi] \frac{1}{\Delta} \\
\hat{\phi} = [-\hat{x} \cos \beta_o - \hat{z} \sin \beta_o \cos \phi] \frac{1}{\Delta}.
\]

Also, from the edge fixed plane coordinate system [1], one have

\[
\hat{\beta}_o = -\hat{x} \cos \beta_o \cos \phi - \hat{y} \cos \beta_o \sin \phi - \hat{z} \sin \beta_o \\
\hat{\phi} = \hat{x} \sin \phi - \hat{y} \cos \phi.
\]
Then,

\[
\begin{align*}
\vec{E}_e \cdot \hat{\beta}_o &= \frac{\sin \beta_o}{\Delta^2} (\cos^2 \beta_o \sin^2 \phi R_h - \cos^2 \phi R_e) E_o d_p e z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'} \quad (B.19a) \\
\vec{E}_e \cdot \hat{\phi} &= -\frac{\sin \beta_o}{\Delta^2} \cos \beta_o \cos \phi \sin \phi (R_e + R_h) E_o x p e z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'} \quad (B.19b) \\
\vec{E}_m \cdot \hat{\beta}_o &= \frac{\sin \beta_o}{\Delta^2} \cos \beta_o \cos \phi \sin \phi (R_e + R_h) \frac{E_o}{\eta_o} d m z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'} \quad (B.19c) \\
\vec{E}_m \cdot \hat{\phi} &= \frac{\sin \beta_o}{\Delta^2} (\cos^2 \beta_o \sin^2 \phi R_e - \cos^2 \phi R_h) \frac{E_o}{\eta_o} d m z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'}. \quad (B.19d)
\end{align*}
\]

One can write (B.19) in a compact form as

\[
\begin{bmatrix}
\vec{E}_{\beta_o} \\
\vec{E}_{\phi}
\end{bmatrix} = \frac{\sin \beta_o}{\Delta^2} \begin{bmatrix}
R_h \cos^2 \beta_o \sin^2 \phi - R_e \cos^2 \phi & -(R_e + R_h) \cos \beta_o \cos \phi \sin \phi \\
(R_e + R_h) \cos \beta_o \cos \phi \sin \phi & R_e \cos^2 \beta_o \sin^2 \phi - R_h \cos^2 \phi
\end{bmatrix} \begin{bmatrix}
E_o d p e z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'} \\
E_o \eta_o d m z e^{j k_z x'} e^{-j k_y y'} e^{j k_z z'}
\end{bmatrix}.
\]

At the reflection point \( \vartheta^r = \vartheta \) and \( \varphi^r = \pi + \phi \) as shown in Fig. B.3, one can also have \( \phi = \pi + \phi' \). This leads to

\[
\vec{E}^r = -\frac{1}{\Delta^2} \overline{T}(\phi') \cdot \overline{R}(\phi') \cdot \overline{T}(\phi') \cdot \vec{u}^i(Q_r) e^{j k_r p}
\]

where

\[
\overline{T}(\phi') = \begin{bmatrix}
\cos \beta_o \sin \phi' & -\cos \phi' \\
\cos \phi' & \cos \beta_o \sin \phi'
\end{bmatrix}
\]

and

\[
\overline{R}(\phi') = \begin{bmatrix}
R_h(\phi') & 0 \\
0 & R_e(\phi')
\end{bmatrix}
\]

\( \vec{u}^i(Q_r) = \begin{bmatrix}
E_{\beta_o}^i(Q_r) \\
E_{\phi}^i(Q_r)
\end{bmatrix} \) in which \( Q_r \) is point of reflection on the material structure.

### B.4 Examples of antenna mounted on or in an infinite planar material with PEC ground plane

In practical, most antennas are mounted on or sometimes embedded in the surface of the platform, which are usually an absorber, non-metallic materials. Thus, it is
of interest to analyze these problems. This section shows some examples of antenna (current moment) mounted on or in an infinite planar material with PEC backing.

**B.4.1 \( \hat{z} \)-directed current moment on or in an infinite planar material with PEC ground plane**

Let first consider a \( \hat{z} \)-directed current moment of strength \( d\vec{p}_e = \hat{z} d\vec{p}_{ez} \) (or \( d\vec{p}_m = \hat{z} d\vec{p}_{mz} \)) located on a material surface as shown in Fig. B.4. By applying (B.1), the

![Diagram of \( \hat{z} \)-directed current moment](image)

Figure B.4: A \( \hat{z} \)-directed current moment mounted on an infinite, planar material with PEC ground plane.
Figure B.5: A $\hat{z}$-directed current moment embedded in an infinite, planar material with PEC ground plane

Fields radiated by the current moment $d\vec{p}_{e,m}$ can be expressed as

\[
\vec{E}_e = \left[ \hat{\vartheta} (\vec{E}_i + \vec{E}_r) + \hat{\varphi} (\vec{E}_i + \vec{E}_r^T) \right] \cdot \hat{z} dp_{ez}
\]

\[
\vec{E}_m = \left[ \hat{\vartheta} (\vec{H}_i + \vec{H}_r^T) + \hat{\varphi} (\vec{H}_i + \vec{H}_r^T) \right] \cdot \hat{z} dp_{mz}
\]

where $\vec{E}_e$ denotes the electric field radiated by an electric current moment $d\vec{p}_e$ and $\vec{E}_m$ represents the electric field radiated by a magnetic current moment $d\vec{p}_m$. Next if the $\hat{z}$-directed current moment is embedded in material as shown in Fig. B.5, the fields radiated by the dipole moment can be expressed as

\[
\vec{E}_e = \left[ \hat{\vartheta} \tilde{E}_i^T + \hat{\varphi} \tilde{E}_i^T \right] \cdot \hat{z} dp_{ez} = 0
\]  

\[
\eta_o \vec{E}_m = - \left[ \hat{\vartheta} \tilde{H}_i^T + \hat{\varphi} \tilde{H}_i^T \right] \cdot \hat{z} dp_{mz}
\]

\[
= 2 \frac{E_o}{Z_{zd}} dp_{mz} \frac{1}{\Delta} \left[ \hat{\vartheta} \sin \theta \cos \phi T_b + \hat{\varphi} \cos \theta NT_d \right] e^{jk \xi N \tau}
\]

As expected the field $\vec{E}_e$ radiated by an electric current moment $d\vec{p}_e$ vanishes but the field $\vec{E}_m$ radiated by a magnetic current moment $d\vec{p}_m$ exists and is doubled due to the image theorem.
B.4.2 $\hat{x}$-directed current moment on or in an infinite planar material with PEC ground plane

The case of $\hat{x}$-directed current moment source $\vec{d}\vec{\rho}_e = \hat{x}d\rho_{ex}$ (or $\vec{d}\vec{\rho}_m = \hat{x}d\rho_{mx}$) mounted on or in the material surface is discussed in this section. Let first consider the case of the source is on the material as shown in Fig. B.6 Similarly, by applying

Figure B.6: A $\hat{x}$-directed current moment mounted on an infinite, planar material with PEC ground plane.
Figure B.7: A \( \hat{x} \)-directed current moment embedded in an infinite, planar material with PEC ground plane

(B.1), the fields radiated by the current moment \( d\vec{p}_{e,m} \) can be expressed as

\[
\vec{E}_e = [\hat{\vartheta}(\vec{E}^i + \vec{E}^r) + \hat{\varphi}(\vec{E}^i + \vec{E}^r)] \cdot \hat{x} dp_{ex} \\
= -E_o dp_{ex} \frac{1}{\Delta} [\hat{\vartheta} \sin^2 \theta \cos \phi \sin \phi (1 - R_h) - \hat{\varphi} \cos \theta (1 + R_e)] 
\]

\[
\eta_o \vec{E}_m = -[\hat{\vartheta}(\vec{H}^i + \vec{H}^r) + \hat{\varphi}(\vec{H}^i + \vec{H}^r)] \cdot \hat{x} dp_{mx} \\
= \frac{E_o}{\eta_o} dp_{mx} \frac{1}{\Delta} [\hat{\vartheta} \cos \theta (1 + R_h) - \hat{\varphi} \sin^2 \theta \cos \phi \sin \phi (1 - R_e)] 
\]

(B.25a)

(B.25b)

Next if the \( \hat{x} \)-directed current moment is embedded in material as shown in Fig. B.7, the fields radiated by the current moment can be expressed as

\[
\vec{E}_e = [\hat{\vartheta} \vec{E}^d + \hat{\varphi} \vec{E}^d] \cdot \hat{x} dp_{ex} = 0 
\]

\[
\eta_o \vec{E}_m = -[\hat{\vartheta} \vec{H}^d + \hat{\varphi} \vec{H}^d] \cdot \hat{x} dp_{mx} \\
= -2 \frac{E_o}{\varepsilon_o} dp_{mx} \frac{1}{\Delta} [\hat{\vartheta} \cos \theta T_b - \hat{\varphi} \sin \theta \cos \phi N T_d] e^{jk d N \tau} 
\]

(B.26a)

(B.26b)

Note that the field \( \vec{E}_e \) radiated by an electric current moment \( d\vec{p}_e \) vanishes but the field \( \vec{E}_m \) radiated by an magnetic current moment \( d\vec{p}_m \) exists and is doubled due to the image theorem.
It is also noted that by using (B.1) together with the fields in (B.8) and (B.14) one can obtain the solution with arbitrary polarization of current moment $d\bar{\rho}_{e,m}$, which can be located on or even in the material structure.
APPENDIX C

ASYMPTOTIC EVALUATION OF AN INTEGRAL WITH A DOUBLE POLE

A uniform asymptotic evaluation of an integral, containing an isolated first order saddle point and the integrand has a double pole, is summarized in this appendix for the sake of completeness. This is also explained in the unpublished class note [49].

The method of steepest descents is used to evaluate the integral. Let consider an integral of the form,

\[ I(\kappa) = \int_{C} \mathcal{F}(\xi) e^{\kappa f(\xi)} d\xi \]  

where \( C \) denotes the integration path in the complex \( \xi \) plane. Here \( \kappa \) is real, positive, and large. The \( \mathcal{F}(\xi) \) and \( f(\xi) \) are the complex functions of \( \xi \). It is of interest to evaluate the integral containing an isolated first order saddle point at \( \xi_s \) with \( f''(\xi_s) \neq 0 \). It is also assumed that \( \mathcal{F}(\xi) \) and \( f(\xi) \) are analytic over the path \( C \) in the complex \( \xi \) plane. Thus, one can deform the integration path \( C \) into the SDP which passes through the saddle point \( \xi_s \), yielding the result to be

\[ I_s(\kappa) = \int_{SDP} \mathcal{F}(\xi) e^{\kappa f(\xi)} d\xi. \]  

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To simplify the integral above, one can introduce the following mapping \( f(\xi) = f(\xi_s) - s^2 \), then one can write the (C.2) as

\[
I_s(\kappa) = \int_{-\infty}^{\infty} F(\xi) \frac{d \xi}{ds} e^{\kappa f(\xi_s) - s^2} ds
\]

\[
= e^{\kappa f(\xi_s)} \int_{-\infty}^{\infty} G(s) e^{-\kappa s^2} ds
\]

(C.3)

where \( G(s) = f(\xi) \frac{d \xi}{ds} \) and

\[
\frac{d \xi}{ds} = \frac{-2s}{f'(\xi)}.
\]

(C.4)

It is note that the saddle point \( \xi_s \) and the pole \( \xi_p \) in \( \xi \) plane are now mapped to \( s = 0 \) and \( s = s_p \) in the \( s \) plane, respectively. The pole \( s_p \) is defined by

\[
s_p^2 = f(\xi_s) - f(\xi_p) \equiv -ja
\]

(C.5)

where \( a = js_p^2 \) is a distance between \( \xi_s \) and \( \xi_p \). Also, if one takes the derivative with respective to \( s \) both sides of (C.4), one can have

\[
\frac{d^2 \xi}{ds^2} f'(\xi) + \left( \frac{d \xi}{ds} \right)^2 f''(\xi) = -2.
\]

(C.6)

Then, since \( f'(\xi_s) = 0 \), one have

\[
\left. \frac{d \xi}{ds} \right|_{\xi = \xi_s} = \sqrt{-\frac{2}{f''(\xi_s)}}
\]

(C.7)

If one let \( G_2(s) = G(s)(s - s_p)^2 \), where \( G_2(s) \) is an analytic function, then one can write (C.3) as

\[
I_s(\kappa) = e^{\kappa f(\xi_s)} \int_{-\infty}^{\infty} \frac{G_2(s)}{(s - s_p)^2} e^{-\kappa s^2} ds.
\]

(C.8)

By using the modified Pauli-Clemmow method [36], one expresses the \( G_2(s) \) in terms of a Taylor series

\[
G_2(s) = \sum_{n=0}^{\infty} \frac{d^n G_2(s)}{ds^n} \bigg|_{s=0} \frac{s^n}{n!}.
\]

(C.9)
One can then apply (C.9) in (C.8) and retain only the leading term \( n = 0 \) for the first order approximation. The (C.8) can be approximated by

\[
I_s(\kappa) \sim G_2(0)e^{\kappa f(\xi_s)} \int_{-\infty}^{\infty} \frac{e^{-\kappa s^2}}{(s-s_p)^2} ds
\]

where

\[
G_2(0) = \mathcal{F}(\xi_s) \left. \frac{d\xi}{ds} \right|_{s=0} = \mathcal{F}(\xi_s) \sqrt{-\frac{2}{f''(\xi_s)}} s_p^2
\]

which follows via (C.7). One notes that

\[
\int_{-\infty}^{\infty} e^{-\kappa s^2} (s-s_p)^2 ds = -\frac{\partial}{\partial s_p} \left[ s_p \int_{-\infty}^{\infty} \frac{e^{-\kappa s^2}}{s^2-s_p^2} ds \right].
\]

It is also shown in [49] that

\[
\int_{-\infty}^{\infty} \frac{e^{-\kappa s^2}}{s^2-s_p^2} ds = 2\sqrt{\frac{\pi}{js_p^2}} \int_{-\infty}^{\infty} e^{-j\tau^2} d\tau
\]

and

\[
F_{KP}(\chi) = 2j\sqrt{\chi} e^{j\chi} \int_{\sqrt{\chi}}^{\infty} e^{-j\tau^2} d\tau
\]

where \( \chi = j\kappa s_p^2 \). The \( F_{KP}(\chi) \) is well known transition function defined in [1]. By using (C.14) in (C.13), one can write (C.12) as

\[
\int_{-\infty}^{\infty} \frac{e^{-\kappa s^2}}{(s-s_p)^2} ds = -\frac{\pi}{\kappa} \partial \left[ \frac{F_{KP}(\chi)}{s_p} \right]^{b(z)}_{a(z)} = -\frac{\pi}{\kappa} \left[ \frac{1}{s_p} \partial F_{KP}(\chi) - \frac{F_{KP}(\chi)}{s_p^2} \right].
\]

After performing the derivative of the first term in RHS of (C.15) by using the Leibniz integral rule:

\[
\frac{\partial}{\partial z} \int_{a(z)}^{b(z)} g\{x, z\} dx = \int_{a(z)}^{b(z)} \frac{\partial}{\partial z} g\{x, z\} dx + g\{b(z), z\} \frac{\partial}{\partial z} b(z) - g\{a(z), z\} \frac{\partial}{\partial z} a(z),
\]

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one obtains
\[ \int_{-\infty}^{\infty} \frac{e^{-\kappa s^2}}{(s - s_p)^2} ds = 2\sqrt{\pi j} \frac{\kappa s_p}{\sqrt{\chi}} [F_{KP}(\chi) - 1]. \] (C.17)
Substituting (C.17) into (C.10) yields
\[ I_s(\kappa) \sim F(\xi_s) \sqrt{-\frac{2}{f''(\xi_s)}} e^{\kappa f(\xi_s)} \left\{ 2\sqrt{\pi j} \frac{\kappa s_p^3}{\sqrt{\chi}} [F_{KP}(\chi) - 1] \right\}. \] (C.18)
Then one can write (C.18) as
\[ I_s(\kappa) \sim F(\xi_s) \sqrt{-\frac{2}{f''(\xi_s)}} e^{\kappa f(\xi_s)} F_s(\chi) \] (C.19)
where \( F_s(\chi) = 2j\chi[1 - F_{KP}(\chi)] \). This transition function is very useful for the UTD slope diffraction problem. The behavior of the \( F_s(\chi) \) can be shown in Fig. C.1

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Figure C.1: The transition function \( F_s(\chi) \), where \( \chi \) is real argument.


